JEE Main 2022 Paper 1 (B.E./B.Tech) Jul 29 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 2 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 29 July 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let R be a relation from the set $\{1, 2, 3, \ldots, 60\}$ to itself such that $R = \{(a, b) : b = pq$, where $p, q \geqslant 3$ are prime numbers$\}$. Then, the number of elements in R is : [4 marks]

(A) 600

(B) 660

(C) 540

(D) 720

Correct Answer: (B) 660

Solution:

Concept:

- A relation is a set of ordered pairs: distinct choices of the first coordinate give distinct pairs, even when their second coordinates agree.
- The condition restricts only b. Both coordinates belong to $\{1, \ldots, 60\}$, and $b = pq$ with odd primes $p, q \geq 3$. Repeated primes are allowed.

Step 1: List the products without duplication Since the order of the prime factors does not change the integer b, take $p \leq q$. Then $p^2 \leq 60$, so the smaller prime can only be $3, 5, 7$. The possibilities are

$$\begin{aligned} p = 3: &\quad b = 9, 15, 21, 33, 39, 51, 57, \\ p = 5: &\quad b = 25, 35, 55, \\ p = 7: &\quad b = 49. \end{aligned}$$

For the first row the larger prime is at most 20, for the second it is at most 12, and for the third it is at most $60/7$. Thus the lists are exhaustive, with $7 + 3 + 1 = 11$ distinct second coordinates. Products such as $15 = 3 \cdot 5 = 5 \cdot 3$ are counted once, and $9 = 3^2$ is valid.

Step 2: Count the ordered pairs For each one of these eleven values of b, the first coordinate a is completely unrestricted within the sixty-element set. The multiplication principle therefore gives

$$|R| = 60 \times 11 = 660.$$

The options $600, 540, 720$ would require respectively ten, nine or twelve admissible values of b, whereas the explicit exhaustive list contains eleven. There is no requirement that a itself be prime or a product of primes.

Step 3: Conclusion The relation contains 660 elements. Hence, the correct option is (B).

Quick Tip to Solve: Count distinct products pq with $p \leq q$, then multiply by the unrestricted number of first coordinates; do not count factor order twice.

2. If $z = 2 + 3i$, then $z^5 + (\bar{z})^5$ is equal to : [4 marks]

(A) 244

(B) 224

(C) 245

(D) 265

Correct Answer: (A) 244

Solution:

Concept:

- Conjugation commutes with integer powers: $(\bar{z})^5 = \overline{z^5}$. Thus their sum is twice the real part of z^5.
- Use $i^2 = -1$ when multiplying complex numbers. The imaginary parts cancel in the sum of a number and its conjugate.

Step 1: Compute the powers by multiplication Starting from $z = 2 + 3i$, square and then square again:

$$\begin{aligned} z^2 &= (2 + 3i)^2 = -5 + 12i, \\ z^4 &= (-5 + 12i)^2 = 25 - 120i - 144 \\ &= -119 - 120i. \end{aligned}$$

Multiplying by z once more gives

$$\begin{aligned} z^5 &= (-119 - 120i)(2 + 3i) \\ &= -238 - 357i - 240i - 360i^2 \\ &= 122 - 597i. \end{aligned}$$

In particular, the real contribution from the product of the imaginary terms is positive because $-360i^2 = 360$.

Step 2: Add the conjugate and check the choices The fifth power of $\bar{z} = 2 - 3i$ is $122 + 597i$. Hence

$$z^5 + (\bar{z})^5 = (122 - 597i) + (122 + 597i) = 244.$$

This is real, as a conjugate sum must be. As an independent arithmetic check, the real part from the binomial expansion is $2^5 - \binom{5}{2}2^3 3^2 + \binom{5}{4}2 \cdot 3^4 = 32 - 720 + 810 = 122$, giving the same doubled value. The remaining proposed values $224, 245, 265$ do not equal twice this real part, so none satisfies the given expression.

Step 3: Conclusion $z^5 + (\bar{z})^5 = 244$. Hence, the correct option is (A).

Quick Tip to Solve: For a sum of conjugate powers, calculate only the real part of one power and double it; the imaginary terms cancel automatically.

3. Let A and B be two 3×3 non-zero real matrices such that AB is a zero matrix. Then [4 marks]

(A) the system of linear equations $AX = 0$ has a unique solution

(B) the system of linear equations $AX = 0$ has infinitely many solutions

(C) B is an invertible matrix

(D) $\mathrm{adj(A)}$ is an invertible matrix

Correct Answer: (B) the system of linear equations $AX = 0$ has infinitely many solutions

Solution:

Concept:

- The columns of AB are A multiplied by the corresponding columns of B. A nonzero column of B therefore gives a nonzero solution of $AX = 0$.
- A homogeneous real linear system with a nonzero solution has infinitely many solutions, because every real scalar multiple is also a solution.

Step 1: Find a nontrivial null vector Write $B = [b_1\ b_2\ b_3]$, where each b_j is a column vector. The equation $AB = 0$ says

$$Ab_1 = 0, \qquad Ab_2 = 0, \qquad Ab_3 = 0.$$

Since B is a nonzero matrix, at least one column, say b_j, is nonzero. Thus the homogeneous system $AX = 0$ has both the zero solution and the nonzero solution $X = b_j$. In fact $X = tb_j$ is a solution for every $t \in \mathbb{R}$, and these vectors are distinct for distinct t. The system therefore has infinitely many solutions, ruling out uniqueness.

Step 2: Rule out the invertibility claims If A were invertible, left multiplication of $AB = 0$ by A^{-1} would imply $B = 0$, a contradiction. Hence $\det A = 0$. Likewise, if B were invertible, right multiplication by B^{-1} would imply $A = 0$, also a contradiction. Finally, for a 3×3 matrix,

$$\det(\mathrm{adj}\, A) = (\det A)^2 = 0.$$

Thus the adjugate is singular as well. Options (A), (C) and (D) each contradict consequences of the given nonzero matrices and their zero product.

Step 3: Conclusion The system $AX = 0$ has infinitely many real solutions. Hence, the correct option is (B).

Quick Tip to Solve: When a nonzero matrix multiplies to zero, use a nonzero column of the other factor as an explicit null vector instead of relying only on determinants.

4. If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \ldots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$, then the maximum value of a is : [4 marks]

(A) 198

(B) 202

(C) 212

(D) 218

Correct Answer: (C) 212

Solution:

Concept:

- Partial fractions turn consecutive reciprocal products into a telescoping difference: $1/[u(u+20)] = (1/20)(1/u - 1/(u+20))$.
- The original denominators require $a \neq 20, 40, \ldots, 200$. Any roots obtained after clearing denominators must be checked against these exclusions.

Step 1: Telescope the nine terms Apply the identity with $u = 20 - a, 40 - a, \ldots, 180 - a$. Every intermediate reciprocal cancels, leaving

$$\begin{aligned} S &= \frac{1}{20}\left(\frac{1}{20-a} - \frac{1}{200-a}\right) \\ &= \frac{1}{20}\frac{180}{(20-a)(200-a)} \\ &= \frac{9}{(20-a)(200-a)}. \end{aligned}$$

There are nine products, rather than ten: their first factors run from $20 - a$ to $180 - a$ in steps of twenty. This cancellation is valid for every a allowed in the original expression.

Step 2: Solve the resulting quadratic and select the maximum Set the telescoped value equal to the printed right-hand side $1/256$:

$$\begin{aligned} \frac{9}{(20-a)(200-a)} &= \frac{1}{256}, \\ (20-a)(200-a) &= 2304, \\ a^2 - 220a + 1696 &= 0, \\ (a-8)(a-212) &= 0. \end{aligned}$$

Both 8 and 212 avoid every forbidden denominator value, so both solve the original sum. The larger is 212. The other choices $198, 202, 218$ are not roots of this quadratic and therefore cannot satisfy the original equation. Taking the maximum is essential because telescoping produces two valid values.

Step 3: Conclusion The maximum admissible value is $a = 212$. Hence, the correct option is (C).

Quick Tip to Solve: For equally spaced consecutive factors, use their difference in partial fractions; the long sum reduces to just its two endpoint reciprocals.

5. If $\lim\limits_{x\to 0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\frac{2}{3}$**, where** $\alpha,\beta,\gamma\in$ **R, then which of the following is NOT correct?** [4 marks]

(A) $\alpha^2+\beta^2+\gamma^2=6$

(B) $\alpha\beta+\beta\gamma+\gamma\alpha+1=0$

(C) $\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=0$

(D) $\alpha^2-\beta^2+\gamma^2=4$

Correct Answer: (C) $\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=0$

Solution:

Concept:

- Near zero, $e^x=1+x+x^2/2+x^3/6+O(x^4)$ and $\sin x=x-x^3/6+O(x^5)$.
- The denominator $x\sin^2 x$ has leading term x^3. A finite limit forces every numerator coefficient of degree below three to vanish.

Step 1: Determine the three constants Expanding the numerator and collecting the powers that control the limit gives

$$N(x)=(\alpha+\beta)+(\alpha-\beta+\gamma)x + \frac{\alpha+\beta}{2}x^2+\frac{\alpha-\beta-\gamma}{6}x^3+O(x^4).$$

Since the denominator is $x^3+O(x^5)$, finiteness requires $\alpha+\beta=0$ and $\alpha-\beta+\gamma=0$. Consequently $\beta=-\alpha$ and $\gamma=-2\alpha$. The cubic coefficient is then $4\alpha/6=2\alpha/3$. The given limit $2/3$ therefore yields

$$\alpha=1,\qquad \beta=-1,\qquad \gamma=-2.$$

Step 2: Test each statement, remembering NOT Substitute these constants directly into all four statements:

$$\alpha^2+\beta^2+\gamma^2=1+1+4=6,$$
$$\alpha\beta+\beta\gamma+\gamma\alpha+1=-1+2-2+1=0,$$
$$\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=1-4-2+3=-2,$$
$$\alpha^2-\beta^2+\gamma^2=1-1+4=4.$$

Thus (A), (B) and (D) hold exactly, while (C) claims a zero value for an expression that actually equals -2. The question asks for the statement that is not correct, so the false equality is the required selection.

Step 3: Conclusion The statement in (C) is not correct. Hence, the correct option is (C).

Quick Tip to Solve: First match the order of the vanishing denominator, cancel all lower-order numerator terms, and only then compare the first surviving coefficient.

6. The integral $\int_0^{\frac{\pi}{2}} \frac{1}{3+2\sin x+\cos x}\,dx$ is equal to : [4 marks]

(A) $\tan^{-1}(2)$

(B) $\tan^{-1}(2) - \frac{\pi}{4}$

(C) $\frac{1}{2}\tan^{-1}(2) - \frac{\pi}{8}$

(D) $\frac{1}{2}$

Correct Answer: (B) $\tan^{-1}(2) - \frac{\pi}{4}$

Solution:

Concept:

- The substitution $t = \tan(x/2)$ gives $\sin x = 2t/(1+t^2)$, $\cos x = (1-t^2)/(1+t^2)$, and $dx = 2\,dt/(1+t^2)$.
- Completing the square reduces a positive quadratic denominator to the standard antiderivative $\int du/(1+u^2) = \tan^{-1} u + C$.

Step 1: Transform the integrand and limits At $x = 0$, $t = 0$; at $x = \pi/2$, $t = 1$. On this interval the substitution is continuous and increasing. The denominator becomes

$$3 + 2\sin x + \cos x = \frac{3(1+t^2) + 4t + 1 - t^2}{1+t^2} = \frac{2t^2 + 4t + 4}{1+t^2}.$$

The factor $1 + t^2$ cancels with the one in the differential, yielding

$$I = \int_0^1 \frac{dt}{t^2 + 2t + 2} = \int_0^1 \frac{dt}{(t+1)^2 + 1}.$$

Keeping the differential factor two is important; it cancels the common factor two in the transformed denominator.

Step 2: Integrate and distinguish the options The standard inverse-tangent integral gives

$$I = \left[\tan^{-1}(t+1)\right]_0^1 = \tan^{-1}(2) - \tan^{-1}(1) = \tan^{-1}(2) - \frac{\pi}{4}.$$

This is option (B). Option (A) omits the lower-limit contribution; option (C) incorrectly halves the whole answer. Also $(t+1)^2 + 1 > 2$ for $t > 0$, so the transformed integral is strictly less than $1/2$, excluding (D). The integrand is positive throughout the interval, consistent with the positive difference of the two inverse tangents.

Step 3: Conclusion $I = \tan^{-1}(2) - \pi/4$. Hence, the correct option is (B).

Quick Tip to Solve: For a constant plus a linear combination of sine and cosine, the tangent half-angle substitution converts the entire denominator into a quadratic.

7. Let the solution curve $y = y(x)$ of the differential equation $\left(1 + \mathrm{e}^{2x}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x} + y\right) = 1$ pass through the point $\left(0, \frac{\pi}{2}\right)$. Then, $\lim\limits_{x\to\infty} \mathrm{e}^{x} y(x)$ is equal to : [4 marks]

(A) $\frac{\pi}{4}$

(B) $\frac{3\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{3\pi}{2}$

Correct Answer: (B) $\frac{3\pi}{4}$

Solution:

Concept:

- A first-order linear equation $y' + P(x)y = Q(x)$ has integrating factor $e^{\int P(x)dx}$, which converts its left side into a product derivative.
- Here $P(x) = 1$, so the required quantity $e^{x}y(x)$ is exactly the product obtained using the integrating factor e^{x}.

Step 1: Integrate the product derivative The factor $1 + e^{2x}$ is positive for every real x, so divide by it to obtain

$$y' + y = \frac{1}{1 + e^{2x}}.$$

Multiplication by e^{x} gives

$$\frac{d}{dx}(e^{x}y) = \frac{e^{x}}{1 + e^{2x}}.$$

Use $u = e^{x}$, with $du = e^{x}dx$. Hence

$$e^{x}y = \int \frac{du}{1 + u^{2}} = \tan^{-1}(e^{x}) + C.$$

The derivative of the right side is $e^{x}/(1 + e^{2x})$, checking the transformed differential equation directly.

Step 2: Apply the initial condition and take the limit At $x = 0$, the given value is $y(0) = \pi/2$. Thus

$$\frac{\pi}{2} = \tan^{-1}(1) + C = \frac{\pi}{4} + C, \qquad C = \frac{\pi}{4}.$$

It follows that

$$\lim_{x\to\infty} e^{x}y(x) = \lim_{x\to\infty}\left(\tan^{-1}(e^{x}) + \frac{\pi}{4}\right) = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}.$$

Option (A) is only the integration constant, and (C) omits this constant when taking the limit. Option (D) doubles the correct value without justification. The requested limit is the product $e^{x}y$; although y itself tends to zero, this product has the nonzero finite limit just calculated.

Step 3: Conclusion $\lim\limits_{x\to\infty} e^{x}y(x) = 3\pi/4$. Hence, the correct option is (B).

Quick Tip to Solve: If a limit asks for the integrating-factor product, solve directly for that product and apply the initial condition before taking its limit.

8. Let a line L pass through the point of intersection of the lines $bx + 10y - 8 = 0$ and $2x - 3y = 0$, $b \in \mathbf{R} - \left\{\frac{4}{3}\right\}$. If the line L also passes through the point $(1, 1)$ and touches the circle $17\left(x^2 + y^2\right) = 16$, then the eccentricity of the ellipse $\frac{x^2}{5} + \frac{y^2}{b^2} = 1$ is : [4 marks]

(A) $\frac{2}{\sqrt{5}}$

(B) $\sqrt{\frac{3}{5}}$

(C) $\frac{1}{\sqrt{5}}$

(D) $\sqrt{\frac{2}{5}}$

Correct Answer: (B) $\sqrt{\frac{3}{5}}$

Solution:

Concept:

- The pencil of lines through the intersection of two given lines is formed by a linear combination of their equations. Requiring the line to pass through another point determines the combination.
- A line is tangent to a circle when its perpendicular distance from the centre equals the radius. For an ellipse with squared semiaxes $a^2 > B^2$, its eccentricity is $e = \sqrt{1 - B^2/a^2}$.

Step 1: Form the required line Write the line through the given intersection as $bx + 10y - 8 + \lambda(2x - 3y) = 0$. Substituting $(1, 1)$ gives $b + 2 - \lambda = 0$, hence $\lambda = b + 2$. Therefore the line is

$$(3b + 4)x + (4 - 3b)y - 8 = 0.$$

This equation also covers a vertical line if one occurs, so no slope case has been lost.

Step 2: Impose the tangency condition The circle has centre $(0, 0)$ and radius $4/\sqrt{17}$. Thus

$$\frac{8}{\sqrt{(3b + 4)^2 + (4 - 3b)^2}} = \frac{4}{\sqrt{17}}.$$

Squaring and simplifying, with a positive denominator, gives

$$(3b + 4)^2 + (4 - 3b)^2 = 68, \qquad 18b^2 + 32 = 68.$$

Consequently $b^2 = 2$; both signs satisfy the stated exclusion on b.

Step 3: Identify the major axis and compare the choices The ellipse is now $x^2/5 + y^2/2 = 1$. Its major axis lies along the x-axis because $5 > 2$. Therefore

$$e^2 = 1 - \frac{2}{5} = \frac{3}{5}.$$

The other listed eccentricities have squares $4/5, 1/5, 2/5$, respectively; none equals the required $3/5$. In particular, $\sqrt{2/5}$ is the minor-to-major semiaxis ratio, not the eccentricity.

Step 4: Conclusion The eccentricity is $\sqrt{3/5}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the line pencil before applying the distance formula; the cross terms cancel and leave an equation directly in b^2.

9. Let the circumcentre of a triangle with vertices A(a, 3), B(b, 5) and C(a, b), ab $>$ 0 be P(1,1). If the line AP intersects the line BC at the point Q(k_1, k_2), then $k_1 + k_2$ is equal to : [4 marks]

(A) 2

(B) $\frac{4}{7}$

(C) $\frac{2}{7}$

(D) 4

Correct Answer: (B) $\frac{4}{7}$

Solution:

Concept:

- The circumcentre is equidistant from all three vertices, so compare squared distances to avoid square roots. A triangle must have three distinct non-collinear vertices.
- After determining the vertices, solve the two linear equations for AP and BC. The sign restriction $ab > 0$ must be applied before selecting coordinates.

Step 1: Determine b from equal radii Since $PA = PC$, the common x-coordinate a cancels in the squared-distance equation:

$$(a-1)^2 + (3-1)^2 = (a-1)^2 + (b-1)^2.$$

Thus $(b-1)^2 = 4$, giving $b = 3$ or $b = -1$. If $b = 3$, then $C = (a, 3) = A$, which cannot form the stated triangle. Hence $b = -1$. This geometric exclusion is essential even though the distance equation itself allows both values.

Step 2: Determine a using the sign condition Now $B = (-1, 5)$, so $PB^2 = (-2)^2 + 4^2 = 20$. Equating PA^2 to this gives

$$(a-1)^2 + 4 = 20, \qquad (a-1)^2 = 16.$$

Therefore $a = 5$ or $a = -3$. Since $ab > 0$ and $b = -1$, a must be negative. The vertices are consequently $A = (-3, 3)$, $B = (-1, 5)$ and $C = (-3, -1)$.

Step 3: Intersect AP and BC The slope of AP is $(1-3)/(1+3) = -1/2$, giving $x + 2y = 3$. The slope of BC is $(5+1)/(-1+3) = 3$, giving $y = 3x + 8$. Substituting yields

$$7x + 16 = 3, \qquad x = -\frac{13}{7}, \qquad y = \frac{17}{7}.$$

Both line equations are satisfied by these coordinates. Thus $k_1 + k_2 = 4/7$, whereas the other proposed sums $2, 2/7, 4$ do not equal this intersection sum.

Step 4: Conclusion We obtain $Q = (-13/7, 17/7)$, so $k_1 + k_2 = 4/7$. Hence, the correct option is (B).

Quick Tip to Solve: When equal-radius equations produce two roots, check for coincident vertices before using the sign restriction to choose the remaining coordinate.

10. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$. If θ is the angle between the vectors $(\hat{a}+\hat{b})$ and $(\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b}))$, then the value of $164\cos^2\theta$ is equal to : [4 marks]

(A) $90+27\sqrt{2}$

(B) $45+18\sqrt{2}$

(C) $90+3\sqrt{2}$

(D) $54+90\sqrt{2}$

Correct Answer: (A) $90+27\sqrt{2}$

Solution:

Concept:

- For unit vectors with included angle $\pi/4$, write $c=\hat{a}\cdot\hat{b}=1/\sqrt{2}$. Their cross product is perpendicular to both vectors and has squared magnitude $1-c^2=1/2$.
- For two nonzero vectors u, v, $\cos^2\theta=(u\cdot v)^2/(|u|^2|v|^2)$. Orthogonality makes all mixed terms involving the cross product vanish.

Step 1: Calculate the scalar product Let $u=\hat{a}+\hat{b}$ and $v=\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b})$. The cross-product contribution to $u\cdot v$ is zero. Expanding the remaining terms gives

$$u\cdot v=1+2c+c+2=3(1+c).$$

This retains the two different coefficients of $\hat{b}$; replacing v by $\hat{a}+\hat{b}$ would give an incorrect angle.

Step 2: Calculate the squared lengths First, $|u|^2=2+2c$. For the second vector, its in-plane part and perpendicular part are orthogonal, so

$$\begin{aligned}|v|^2&=|\hat{a}+2\hat{b}|^2+4|\hat{a}\times\hat{b}|^2\\&=5+4c+4(1-c^2)=7+4c.\end{aligned}$$

The last equality uses $c^2=1/2$. In particular, the cross-product term adds 2 to the squared length; it cannot simply be omitted.

Step 3: Simplify the requested expression Therefore

$$164\cos^2\theta=\frac{164\cdot 9(1+c)^2}{(2+2c)(7+4c)}=\frac{738(1+c)}{7+4c}.$$

Put $c=\sqrt{2}/2$ and rationalise the denominator:

$$\frac{738(1+\sqrt{2}/2)}{7+2\sqrt{2}}=\frac{738(5+3\sqrt{2}/2)}{41}=90+27\sqrt{2}.$$

The other options have different constant or radical coefficients and do not match this exact value.

Step 4: Conclusion The required value is $90+27\sqrt{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Separate the in-plane and perpendicular parts before finding a norm; orthogonality removes several mixed terms immediately.

11. If $f(\alpha)=\int_{1}^{\alpha}\frac{\log_{10}t}{1+t}dt, \alpha>0$, **then** $f\left(e^{3}\right)+f\left(e^{-3}\right)$ **is equal to :** [4 marks]

(A) 9

(B) $\frac{9}{2}$

(C) $\frac{9}{\log_e(10)}$

(D) $\frac{9}{2\log_e(10)}$

Correct Answer: (D) $\frac{9}{2\log_e(10)}$

Solution:

Concept:

- Use $\log_{10} t = \ln t/\ln 10$; changing the base is necessary because the endpoints involve powers of e. The integrand is continuous on every positive interval between the stated endpoints.
- The substitution $t = 1/u$ maps the interval from 1 to $1/a$ onto the interval from 1 to a, with the orientation handled explicitly.

Step 1: Transform the reciprocal-endpoint integral For $a > 1$, write

$$f(1/a) = \frac{1}{\ln 10}\int_1^{1/a}\frac{\ln t}{1+t}\,dt.$$

Set $t = 1/u$, so $dt = -du/u^2$, $\ln t = -\ln u$, and the new upper endpoint is $u = a$. The two minus signs in the transformed integrand cancel, giving

$$f(1/a) = \frac{1}{\ln 10}\int_1^{a}\frac{\ln u}{u(1+u)}\,du.$$

The original integral is positive because its limits are reversed and its logarithm is negative.

Step 2: Add the integrands on common limits Rename the integration variable in $f(a)$ as u. Then

$$\begin{aligned} f(a)+f(1/a) &= \frac{1}{\ln 10}\int_1^{a}\ln u\left(\frac{1}{1+u}+\frac{1}{u(1+u)}\right)du \\ &= \frac{1}{\ln 10}\int_1^{a}\frac{\ln u}{u}\,du \\ &= \frac{(\ln a)^2}{2\ln 10}. \end{aligned}$$

The simplification follows from adding the two fractions before integrating, rather than trying to find separate antiderivatives.

Step 3: Evaluate at the given endpoint Take $a = e^3$, for which $\ln a = 3$. The sum is $9/(2\ln 10)$. Options 9 and $9/2$ omit the base-conversion factor; $9/\ln 10$ additionally misses the factor $1/2$ from integrating $(\ln u)/u$. The logarithm in the denominator is exactly $\log_e(10)$, as used in option (D).

Step 4: Conclusion $f(e^3)+f(e^{-3}) = 9/(2\log_e 10)$. Hence, the correct option is (D).

Quick Tip to Solve: For paired positive endpoints a and $1/a$, use a reciprocal substitution in the second integral and combine the integrands first.

12. The area of the region

$\left\{(x, y) : |x-1| \le y \le \sqrt{5-x^2}\right\}$ **is equal to :** [4 marks]

(A) $\frac{5}{2}\sin^{-1}\left(\frac{3}{5}\right) - \frac{1}{2}$

(B) $\frac{5\pi}{4} - \frac{3}{2}$

(C) $\frac{3\pi}{4} + \frac{3}{2}$

(D) $\frac{5\pi}{4} - \frac{1}{2}$

Correct Answer: (D) $\frac{5\pi}{4} - \frac{1}{2}$

Solution:

Concept:

- The upper boundary is the upper semicircle $y = \sqrt{5-x^2}$, and the lower boundary is $y = |x-1|$. Area is the integral of upper minus lower over their common interval.
- Split the absolute value at $x = 1$. For the circle integral use $\int \sqrt{R^2-x^2}\,dx = \frac{1}{2}[x\sqrt{R^2-x^2} + R^2\sin^{-1}(x/R)]$.

Step 1: Find the intersection limits Both boundary functions are nonnegative, so their equality can be squared without introducing a sign ambiguity:

$$(x-1)^2 = 5-x^2, \qquad 2x^2-2x-4=0.$$

The solutions are $x = -1, 2$. The inequality $(x-1)^2 \le 5-x^2$ holds between these roots, so the desired area is

$$A = \int_{-1}^{2}\sqrt{5-x^2}\,dx - \int_{-1}^{2}|x-1|\,dx.$$

The entire interval also lies inside the semicircle domain.

Step 2: Integrate the upper boundary Applying the stated antiderivative gives

$$I = \left[\frac{x\sqrt{5-x^2}}{2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2}$$
$$= 2 + \frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right).$$

Both inverse sines are acute. Their arguments are respectively the sine and cosine of the same acute angle, so their sum is $\pi/2$. Hence $I = 2 + 5\pi/4$.

Step 3: Subtract the lower area and check the choices The absolute-value integral must be split at its vertex:

$$\int_{-1}^{1}(1-x)\,dx + \int_{1}^{2}(x-1)\,dx = 2 + \frac{1}{2} = \frac{5}{2}.$$

Thus $A = 2 + 5\pi/4 - 5/2 = 5\pi/4 - 1/2$. Option (B) subtracts one extra unit, while (C) uses a different circular contribution. Option (A) has a different angle and does not equal the evaluated integral.

Step 4: Conclusion The area is $5\pi/4 - 1/2$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Find the boundary intersections first, then split at the absolute-value vertex; the two inverse-sine angles here are complementary.

13. Let the focal chord of the parabola $\mathrm{P}: y^2 = 4x$ along the line $\mathrm{L}: y = \mathrm{m}x + \mathrm{c}, \mathrm{m} > 0$ meet the parabola at the points M and N. Let the line L be a tangent to the hyperbola $\mathrm{H}: x^2 - y^2 = 4$. If O is the vertex of P and F is the focus of H on the positive x-axis, then the area of the quadrilateral OMFN is : [4 marks]

(A) $2\sqrt{6}$

(B) $2\sqrt{14}$

(C) $4\sqrt{6}$

(D) $4\sqrt{14}$

Correct Answer: (B) $2\sqrt{14}$

Solution:

Concept:

- The focus of $y^2 = 4x$ is $(1, 0)$, so a focal chord on $y = mx + c$ satisfies $c = -m$. A line $y = mx + c$ tangent to $x^2 - y^2 = 4$ satisfies $c^2 = 4(m^2 - 1)$.
- The positive focus of the hyperbola is $(2\sqrt{2}, 0)$. A quadrilateral whose opposite vertices lie on the x-axis can be divided into two triangles with that common diagonal as base.

Step 1: Determine the focal-chord line Passing through $(1, 0)$ gives $L: y = m(x - 1)$. Combining $c = -m$ with the tangent condition yields

$$m^2 = 4(m^2 - 1), \qquad m^2 = \frac{4}{3}.$$

Since $m > 0$, $m = 2/\sqrt{3}$. Thus the required line is $y = 2(x - 1)/\sqrt{3}$. The tangent condition can also be obtained by setting the discriminant to zero after substituting the line into the hyperbola.

Step 2: Find the vertical separation of the chord endpoints Substitute the line into $y^2 = 4x$:

$$\frac{4}{3}(x - 1)^2 = 4x, \qquad x^2 - 5x + 1 = 0.$$

The endpoint abscissae are $x_+ = (5 + \sqrt{21})/2$ and $x_- = (5 - \sqrt{21})/2$. They lie on opposite sides of $x = 1$, so the endpoint ordinates have opposite signs. Their difference is

$$y_+ - y_- = \frac{2}{\sqrt{3}}(x_+ - x_-) = 2\sqrt{7}.$$

There is no need to calculate each ordinate separately.

Step 3: Calculate the two-triangle area The diagonal OF has length $2\sqrt{2}$, because $O = (0, 0)$ and the positive hyperbola focus is $F = (2\sqrt{2}, 0)$. The two triangles on this diagonal have heights y_+ and $-y_-$. Their total area is

$$\frac{1}{2}(2\sqrt{2})(y_+ - y_-) = \sqrt{2} \cdot 2\sqrt{7} = 2\sqrt{14}.$$

Options with $\sqrt{6}$ use an incorrect focus or endpoint separation; $4\sqrt{14}$ doubles the triangle-area factor.

Step 4: Conclusion The area of $OMFN$ is $2\sqrt{14}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Use the common diagonal OF as the base; the difference of the chord endpoint ordinates gives the sum of the two triangle heights.

14. The number of points, where the function $f : \mathbf{R} \to \mathbf{R}$,
$f(x) = |x-1|\cos|x-2|\sin|x-1| + (x-3)\left|x^2-5x+4\right|$**, is NOT differentiable, is :** [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Concept:

- An absolute value can create a corner at a simple zero of its argument, but multiplication by a vanishing factor can remove the corner. Therefore the apparent breakpoints must be checked.
- Evenness gives $\cos|u| = \cos u$, and $|u|\sin|u| = u\sin u$. These identities can turn a seemingly nonsmooth term into a smooth function everywhere.

Step 1: Simplify the trigonometric term Put $u = x-1$. If $u \geq 0$, then $|u|\sin|u| = u\sin u$. If $u < 0$, oddness of sine gives $(-u)\sin(-u) = u\sin u$ as well. Also $\cos|x-2| = \cos(x-2)$. Thus the first term becomes

$$(x-1)\sin(x-1)\cos(x-2),$$

a product of smooth functions on all of $\mathbf{R}$. In particular, neither $x = 1$ nor $x = 2$ introduces a corner through this term.

Step 2: Locate the remaining possible corners Factor the polynomial in the second term:

$$g(x) = (x-3)|(x-1)(x-4)|.$$

Away from $x = 1, 4$, the polynomial inside the absolute value has a fixed sign locally, so g is locally a polynomial and is differentiable. The zero of the outside factor at $x = 3$ causes no difficulty: the inside polynomial is nonzero there. Hence only 1 and 4 need one-sided derivative tests.

Step 3: Verify that both corners survive For $p(x) = x^2 - 5x + 4$, the roots are simple: $p\prime(1) = -3$ and $p\prime(4) = 3$. Near a root r, $|p(x)| = |p\prime(r)||x-r| + O((x-r)^2)$. The coefficient of the corner after multiplication is $(r-3)|p\prime(r)|$, which equals -6 at 1 and 3 at 4; both are nonzero. The smooth first term has equal one-sided derivatives and cannot cancel either jump. Thus exactly two points remain, ruling out the counts one, three and four.

Step 4: Conclusion The function fails to be differentiable exactly at $x = 1$ and $x = 4$, giving two points. Hence, the correct option is (B).

Quick Tip to Solve: Simplify even and odd trigonometric functions before counting absolute-value breakpoints; apparent corners may disappear identically.

15. Let $S = \{1, 2, 3, \ldots, 2022\}$. Then the probability, that a randomly chosen number n from the set S such that $\mathrm{HCF}\,(n, 2022) = 1$, is : [4 marks]

(A) $\frac{128}{1011}$
(B) $\frac{166}{1011}$
(C) $\frac{127}{337}$
(D) $\frac{112}{337}$

Correct Answer: (D) $\frac{112}{337}$

Solution:

Concept:

- For equally likely choices from a finite set, probability equals the number of favourable integers divided by the total number of integers.
- Euler's totient $\phi(N)$ counts the integers from 1 to N that are relatively prime to N. For distinct prime factors p, $\phi(N) = N\prod_{p|N}(1 - 1/p)$.

Step 1: Factor the given integer We require integers having no common prime divisor with 2022. Its prime factorisation is

$$2022 = 2 \times 1011 = 2 \times 3 \times 337.$$

The integer 337 is prime: none of the primes 2, 3, 5, 7, 11, 13 or 17, which are all the primes up to its square root, divides it. Thus there are exactly three distinct prime factors to exclude.

Step 2: Count and divide by the sample size The totient formula gives

$$\phi(2022) = 2022\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{337}\right) = 1 \times 2 \times 336 = 672.$$

The integer 1 is included among these favourable outcomes, whereas 2022 is not. Since all 2022 elements of the set are equally likely,

$$P = \frac{672}{2022} = \frac{112}{337}.$$

For comparison using a common denominator, the four proposed values are $128/1011$, $166/1011$, $381/1011$ and $336/1011$. Our result is $336/1011$, so the first three do not equal the required favourable fraction.

Step 3: Conclusion The required probability is $\boxed{112/337}$, a dimensionless fraction. Hence, the correct option is (D).

Quick Tip to Solve: To count integers coprime to a large endpoint, first factor the endpoint into distinct primes; apply each exclusion factor only once, even if a prime is repeated.

16. Let $f(x) = 3^{(x^2-2)^3+4}, x \in \mathrm{R}$. Then which of the following statements are true?

P : $x = 0$ **is a point of local minima of** f

Q : $x = \sqrt{2}$ **is a point of inflection of** f

R : f' **is increasing for** $x > \sqrt{2}$ [4 marks]

(A) Only P and Q

(B) Only P and R

(C) Only Q and R

(D) All P, Q and R

Correct Answer: (D) All P, Q and R

Solution:

Concept:

- An exponential with base greater than 1 increases with its exponent. The sign of the first derivative determines local increase or decrease.
- A point of inflection requires a change of concavity. Check the sign of the second derivative on both sides, rather than merely checking that it is zero.

Step 1: Test the minimum at zero Put $g(x) = (x^2 - 2)^3 + 4$, so that $f(x) = 3^{g(x)}$. By the chain rule,

$$g'(x) = 6x(x^2 - 2)^2, \qquad f'(x) = 6x(x^2 - 2)^2 \ln 3\, f(x).$$

The factor $f(x) \ln 3$ is strictly positive. Near zero, $(x^2 - 2)^2 > 0$, so f' is negative to the left and positive to the right. Thus zero is a strict local minimum, and P is true.

Step 2: Test concavity at and beyond the stated point Differentiating once more gives

$$g''(x) = 6(x^2 - 2)(5x^2 - 2),$$

$$f''(x) = f(x) \ln 3 \left[g''(x) + \ln 3\, (g'(x))^2\right].$$

Set $u = x^2 - 2$. The bracket factors as

$$6u \left[(5x^2 - 2) + 6x^2u^3 \ln 3\right].$$

At $x = \sqrt{2}$, the expression in square brackets is 8 and remains positive in a sufficiently small neighbourhood. The sign therefore changes from negative to positive as u crosses zero: Q is true. For every $x > \sqrt{2}$, both u and $5x^2 - 2$ are positive, giving $f''(x) > 0$. Hence f' is increasing there, so R is true. Each of the first three options omits one statement that has just been proved.

Step 3: Conclusion All three statements P, Q and R are true. Hence, the correct option is (D).

Quick Tip to Solve: For $f = a^{g(x)}$, keep $f'' = f \ln a\, [g'' + (\ln a)(g')^2]$ factored; near a repeated stationary point, the lowest-order sign-changing factor decides concavity.

17. If the foot of the perpendicular from the point $\mathrm{A}(-1,4,3)$ on the plane $\mathrm{P}: 2x+\mathrm{m}y+\mathrm{n}z=4$, is $\left(-2,\frac{7}{2},\frac{3}{2}\right)$, then the distance of the point A from the plane P, measured parallel to a line with direction ratios $3,-1,-4$, is equal to : [4 marks]

(A) 1

(B) $\sqrt{26}$

(C) $2\sqrt{2}$

(D) $\sqrt{14}$

Correct Answer: (B) $\sqrt{26}$

Solution:

Concept:

- The segment joining a point to its perpendicular foot on a plane is parallel to the normal vector of that plane.
- To measure distance along a prescribed direction $\mathbf{d}$, intersect the line $\mathbf{A}+t\mathbf{d}$ with the plane. The length is $|t|\|\mathbf{d}\|$, rather than the perpendicular-distance formula.

Step 1: Recover the plane normal Let the given foot be $H=(-2,7/2,3/2)$. The displacement from A to H is

$$\overrightarrow{AH}=\left(-1,-\frac{1}{2},-\frac{3}{2}\right)=-\frac{1}{2}(2,1,3).$$

It is parallel to the normal $(2,m,n)$. Matching the first coordinate fixes the proportionality factor, and then comparison of the other coordinates gives $m=1,n=3$. The plane is therefore

$$2x+y+3z=4.$$

As a consistency check, H gives $-4+7/2+9/2=4$, so the stated foot lies on this plane.

Step 2: Intersect the line through A A line through A with the required direction ratios is

$$(x,y,z)=(-1,4,3)+t(3,-1,-4).$$

Substitute $x=-1+3t$, $y=4-t$, $z=3-4t$ in the recovered plane:

$$2(-1+3t)+(4-t)+3(3-4t)=4.$$

$$11-7t=4 \quad\Longrightarrow\quad t=1.$$

Thus its intersection is $B=(2,3,-1)$, which indeed satisfies $4+3-3=4$. The requested length is

$$AB=\sqrt{3^2+(-1)^2+(-4)^2}=\sqrt{26}.$$

The values 1, $2\sqrt{2}$ and $\sqrt{14}$ have squares 1, 8 and 14, respectively, whereas the actual squared displacement is 26. In particular, the parameter value 1 is not itself the distance because the direction vector is not a unit vector.

Step 3: Conclusion The distance measured in the specified direction is $\boxed{\sqrt{26}}$ units. Hence, the correct option is (B).

Quick Tip to Solve: After finding a line parameter, multiply its absolute value by the magnitude of the direction vector; a parameter equals a distance only for a unit direction vector.

18. Let $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. Let $\vec{c}$ be a vector satisfying $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda\vec{c}$. If $\vec{b}$ and $\vec{c}$ are non-parallel, then the value of λ is : [4 marks]

(A) -5

(B) 5

(C) 1

(D) -1

Correct Answer: (A) -5

Solution:

Concept:

- The vector triple-product identity is $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$. The order of the two terms determines the sign.
- Two non-parallel vectors are linearly independent. A linear combination of them is zero only if both scalar coefficients vanish.

Step 1: Expand the nested product The vectors are $\mathbf{a} = (3, 1, 0)$ and $\mathbf{b} = (1, 2, 1)$. First compute their dot product:

$$\mathbf{a} \cdot \mathbf{b} = 3(1) + 1(2) + 0(1) = 5.$$

Using the triple-product identity, the given equation becomes

$$(\mathbf{a} \cdot \mathbf{c})\mathbf{b} - 5\mathbf{c} = \mathbf{b} + \lambda\mathbf{c}.$$

Move every term to the left without changing either vector:

$$(\mathbf{a} \cdot \mathbf{c} - 1)\mathbf{b} + (-5 - \lambda)\mathbf{c} = \mathbf{0}.$$

This converts a nested cross product into an equation in just the two independent vectors named in the question.

Step 2: Compare the independent coefficients Because $\mathbf{b}$ and $\mathbf{c}$ are non-parallel, neither coefficient can be cancelled by a multiple of the other vector. Consequently,

$$\mathbf{a} \cdot \mathbf{c} - 1 = 0, \qquad -5 - \lambda = 0.$$

It follows that $\mathbf{a} \cdot \mathbf{c} = 1$ and $\lambda = -5$. These conditions are consistent: for instance, $\mathbf{c} = (0, 1, 0)$ has $\mathbf{a} \cdot \mathbf{c} = 1$ and is not parallel to $\mathbf{b}$. Substituting this scalar condition into the expanded identity gives exactly $\mathbf{b} - 5\mathbf{c}$, verifying the result. Option (B) reverses the essential minus sign of the identity. Options (C) and (D) confuse the coefficient coming from $\mathbf{a} \cdot \mathbf{c} = 1$ with the coefficient of $\mathbf{c}$, which is fixed by $\mathbf{a} \cdot \mathbf{b} = 5$.

Step 3: Conclusion The scalar is $\boxed{\lambda = -5}$. Hence, the correct option is (A).

Quick Tip to Solve: For a nested cross product, write the triple-product identity before substituting components. Non-parallel vectors then let you equate their coefficients directly.

19. The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then $\cot\alpha$ is equal to : [4 marks]

(A) $\frac{6}{5}$
(B) $\frac{9}{5}$
(C) $\frac{4}{3}$
(D) $\frac{7}{3}$

Correct Answer: (A) $\frac{6}{5}$

Solution:

Concept:

- For the angle of elevation θ to a vertical tower of height h, $\tan\theta = h/d$, where d is the horizontal distance from the observation point to the tower base.
- North and west are perpendicular horizontal directions. Thus the ground distances here form a right triangle, independently of the vertical right triangles used for elevation.

Step 1: Find the horizontal distance from A Let O be the base of the tower and T its top. Since A is due north of O and B is due west of A, $OA \perp AB$. The given distance of B from the tower is the ground distance $OB = 15$, while $AB = 9$. By Pythagoras in the horizontal triangle OAB,

$$OA^2 = OB^2 - AB^2 = 15^2 - 9^2 = 225 - 81 = 144.$$

$$OA = 12.$$

We use the positive root because this quantity is a distance. The right angle is at A, so the 15-unit side is the horizontal hypotenuse of this triangle.

Step 2: Recover the tower height from B Write $\beta = \cos^{-1}(3/\sqrt{13})$. This is an acute elevation angle, so

$$\cos\beta = \frac{3}{\sqrt{13}}, \qquad \sin\beta = \frac{2}{\sqrt{13}}, \qquad \tan\beta = \frac{2}{3}.$$

For the vertical triangle BOT, the height is therefore

$$OT = OB\tan\beta = 15\left(\frac{2}{3}\right) = 10.$$

Finally, in triangle AOT,

$$\cot\alpha = \frac{OA}{OT} = \frac{12}{10} = \frac{6}{5}.$$

The alternatives $9/5, 4/3, 7/3$ cannot equal this ratio of the derived horizontal distance 12 and tower height 10. The 9-unit distance AB joins the observers; it must not be substituted as the distance of A from the tower.

Step 3: Conclusion The requested dimensionless ratio is $\boxed{\cot\alpha = 6/5}$. Hence, the correct option is (A).

Quick Tip to Solve: Separate the ground triangle from the elevation triangles: first use the compass directions to get horizontal distances, then use the elevation angle to find the height.

20. The statement $(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to : [4 marks]

(A) $q \Rightarrow (p \wedge r)$

(B) $p \Rightarrow (\mathrm{p} \wedge \mathrm{r})$

(C) $(\mathrm{p} \wedge \mathrm{r}) \Rightarrow (\mathrm{p} \wedge \mathrm{q})$

(D) $(p \wedge q) \Rightarrow r$

Correct Answer: (D) $(p \wedge q) \Rightarrow r$

Solution:

Concept:

- An implication $A \Rightarrow B$ is equivalent to $\neg A \vee B$; it is false only when its antecedent is true and its consequent is false.
- Use De Morgan's law and distributivity to simplify compound statements. Logical equivalence requires agreement for every truth assignment, so one counterexample rules out an option.

Step 1: Simplify the original implication Let $E = (p \wedge q) \Rightarrow (p \wedge r)$. Replacing the implication and applying De Morgan's law gives

$$E = \neg(p \wedge q) \vee (p \wedge r) = (\neg p \vee \neg q) \vee (p \wedge r).$$

Set $U = \neg p \vee \neg q$ and use $U \vee (p \wedge r) = (U \vee p) \wedge (U \vee r)$:

$$E = (\neg p \vee \neg q \vee p) \wedge (\neg p \vee \neg q \vee r).$$

The first bracket is always true, leaving

$$E = \neg p \vee \neg q \vee r = \neg(p \wedge q) \vee r.$$

Converting this last expression back into implication form produces $(p \wedge q) \Rightarrow r$.

Step 2: Rule out the remaining statements The logical reason is that whenever $p \wedge q$ is true, p is already guaranteed; asking for $p \wedge r$ then adds only the requirement r. This simplification does not allow us to discard p from the antecedent. For option (A), take $p = F, q = T, r = F$: the original is true, while $q \Rightarrow (p \wedge r)$ is false. For option (B), take $p = T, q = F, r = F$: the original is true, but $p \Rightarrow (p \wedge r)$ is false. For option (C), use $p = T, q = F, r = T$: the original is true, whereas $(p \wedge r) \Rightarrow (p \wedge q)$ is false. Each counterexample disproves equivalence with that option.

Step 3: Conclusion The equivalent statement is $\boxed{(p \wedge q) \Rightarrow r}$. Hence, the correct option is (D).

Quick Tip to Solve: When an implication repeats a conjunct from its antecedent in its consequent, that repeated conjunct is already guaranteed whenever the antecedent is true.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let the mean and the variance of 20 observations $x_1, x_2, \ldots, x_{20}$ be 15 and 9 , respectively. For $\alpha \in$ R, if the mean of $(x_1+\alpha)^2, (x_2+\alpha)^2, \ldots, (x_{20}+\alpha)^2$ is 178 , then the square of the maximum value of α is equal to _____. [4 marks]

Solution:

Concept:

- For observations with mean $\bar{x}$, the population variance is $\sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$. This gives the second moment directly.
- Expand the shifted square before averaging: $\frac{1}{n}\sum(x_i+\alpha)^2 = \frac{1}{n}\sum x_i^2 + 2\alpha\bar{x} + \alpha^2$. A constant shift preserves variance.

Step 1: Recover the original second moment The data have $n = 20$, mean $\bar{x} = 15$ and variance $\sigma^2 = 9$. Thus

$$\frac{1}{20}\sum_{i=1}^{20} x_i^2 = \sigma^2 + \bar{x}^2 = 9 + 225 = 234.$$

We need neither the individual observations nor their order: the given mean and variance already determine their mean square. The variance here is the variance of the complete listed observations, so its defining average uses 20, not the sample-estimation divisor 19.

Step 2: Apply the condition on the shifted squares Averaging the expansion of each square gives

$$\frac{1}{20}\sum_{i=1}^{20}(x_i+\alpha)^2 = 234 + 2\alpha(15) + \alpha^2.$$

Equating this to the supplied mean 178,

$$\alpha^2 + 30\alpha + 234 = 178,$$

$$\alpha^2 + 30\alpha + 56 = 0 = (\alpha+2)(\alpha+28).$$

Hence the two real possibilities are $\alpha = -2$ and $\alpha = -28$. The larger of these signed numbers is -2. The question asks us to select that larger value first and only then square it; comparing the squares before selecting the maximum would give the wrong operation. As a check, the shifted mean for $\alpha = -2$ is 13 and the unchanged variance is 9, so its mean square is $13^2 + 9 = 178$, as required.

Step 3: Conclusion The maximum permissible value is $\alpha = -2$, so its square is $\boxed{4}$, dimensionless. This agrees with the supplied numerical answer key.

Quick Tip to Solve: Use $\text{mean}[(x+\alpha)^2] = \sigma^2 + (\bar{x}+\alpha)^2$ to obtain the two shifts at once, then select the larger signed shift before squaring it.

22. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to _____. [4 marks]

Solution:

Concept:

- An arithmetic progression has general term $a_r = a + (r-1)d$, where $a = a_1$ and d is its common difference.
- For $|x| < 1$, the geometric series gives $\sum_{r=1}^{\infty} x^r = x/(1-x)$ and its differentiated form gives $\sum_{r=1}^{\infty} rx^r = x/(1-x)^2$.

Step 1: Express the weighted sum using the AP parameters Substitute the general term into the given series. Because the weights decrease geometrically and the AP grows at most linearly, both resulting series converge absolutely, so they may be separated:

$$\sum_{r=1}^{\infty} \frac{a_r}{2^r} = a \sum_{r=1}^{\infty} \frac{1}{2^r} + d \sum_{r=1}^{\infty} \frac{r-1}{2^r}.$$

The second sum uses $r-1$, rather than r, because the first term of the progression corresponds to $r = 1$.

Step 2: Evaluate the two numerical series Put $x = 1/2$ into the geometric-series identities:

$$\sum_{r=1}^{\infty} 2^{-r} = 1, \qquad \sum_{r=1}^{\infty} r2^{-r} = 2.$$

Consequently,

$$\sum_{r=1}^{\infty} (r-1)2^{-r} = 2 - 1 = 1.$$

The given equation therefore becomes $a + d = 4$. This determines the second term directly; finding a and d separately is unnecessary and is not possible from this one condition alone.

Step 3: Identify the requested term The second term of an AP is $a_2 = a + d$, so the condition already says $a_2 = 4$. Multiply this equality by the factor requested:

$$4a_2 = 4 \times 4 = 16.$$

This argument also covers a constant progression, since it never divides by d.

Step 4: Conclusion Thus $\boxed{4a_2 = 16}$. The requested numerical value is 16.

Quick Tip to Solve: For an AP weighted by 2^{-r}, the infinite sum is exactly its second term: the weights sum to 1 and their weighted index sums to 2.

23. Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$, in the increasing powers of $\frac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6}:1$. If the sixth term from the beginning is $\frac{\alpha}{\sqrt[4]{3}}$, then α is equal to _____. [4 marks]

Solution:

Concept:

- In $(a+b)^n$, the $(r+1)$th term is $T_{r+1}=\binom{n}{r}a^{n-r}b^r$, when terms are arranged in increasing powers of b.
- The fifth term from the end has index $n-3$ from the beginning. Symmetry gives $\binom{n}{4}=\binom{n}{n-4}$.

Step 1: Write the two terms in the given ratio Set $a=2^{1/4}$ and $b=3^{-1/4}$. The fifth term from the beginning corresponds to $r=4$, while the fifth term from the end corresponds to $r=n-4$. Thus

$$T_5=\binom{n}{4}a^{n-4}b^4, \qquad T_{n-3}=\binom{n}{n-4}a^4b^{n-4}.$$

Both coefficients are equal and nonzero. Their cancellation is the useful feature of comparing terms equally far from the two ends.

Step 2: Determine the exponent of the expansion Taking the ratio gives

$$\frac{T_5}{T_{n-3}}=\left(\frac{a}{b}\right)^{n-8}=6^{(n-8)/4}.$$

The stated ratio is $6^{1/4}:1$, so

$$6^{(n-8)/4}=6^{1/4} \quad\Longrightarrow\quad n-8=1.$$

Therefore $n=9$. Since the base is positive and different from 1, equality of these real powers implies equality of exponents.

Step 3: Evaluate the sixth term The sixth term has $r=5$, not $r=6$. Substituting $n=9$ gives

$$\begin{aligned}T_6&=\binom{9}{5}(2^{1/4})^4(3^{-1/4})^5\\&=126\cdot 2\cdot 3^{-5/4}\\&=\frac{252}{3\sqrt[4]{3}}=\frac{84}{\sqrt[4]{3}}.\end{aligned}$$

Comparing with the prescribed form $\alpha/\sqrt[4]{3}$ determines its numerator uniquely.

Step 4: Conclusion The sixth term is $84/\sqrt[4]{3}$, so $\boxed{\alpha=84}$.

Quick Tip to Solve: For the kth terms from opposite ends, cancel the equal binomial coefficients first. Then use the term index minus one as the exponent of the second summand.

24. The number of matrices of order 3×3, whose entries are either 0 or 1 and the sum of all the entries is a prime number, is _____. [4 marks]

Solution:

Concept:

- A 3×3 matrix has nine distinct positions. A matrix with entries 0 or 1 is determined completely by the subset of positions occupied by 1.
- If exactly k entries are 1, the sum of all entries is k, and the number of such matrices is $\binom{9}{k}$.

Step 1: List the permissible sums Since each entry is either 0 or 1, the total sum is an integer between 0 and 9 inclusive. The prime integers in this interval are

$$2,\ 3,\ 5,\ 7.$$

The value 1 is not prime. Neither 0 nor any even integer greater than 2 contributes. The problem imposes no row-sum, column-sum, symmetry or invertibility condition, so these four possible totals are the complete restrictions.

Step 2: Count matrices for each permissible sum To obtain total k, choose exactly k of the nine positions for the 1s; all remaining positions must contain 0. This gives a one-to-one correspondence between position subsets and matrices. In particular, different placements count separately even if their entries can be obtained by permuting rows or columns. Therefore the four counts are

$$\binom{9}{2} = 36, \quad \binom{9}{3} = 84, \quad \binom{9}{5} = 126, \quad \binom{9}{7} = 36.$$

For example, $\binom{9}{5} = \binom{9}{4} = 9 \cdot 8 \cdot 7 \cdot 6/24 = 126$.

Step 3: Add the disjoint cases A matrix cannot have two different total sums, so the four classes do not overlap. The addition principle applies without any subtraction:

$$N = \binom{9}{2} + \binom{9}{3} + \binom{9}{5} + \binom{9}{7} = 36 + 84 + 126 + 36 = 282.$$

The count is smaller than $2^9 = 512$, the number of all binary matrices, as expected because only certain totals qualify.

Step 4: Conclusion The required number of matrices is $\boxed{282}$.

Quick Tip to Solve: For binary-entry arrays, convert a condition on the sum into a condition on the number of 1s. Choose their positions; the zeros then require no further choices.

25. Let p and p + 2 be prime numbers and let

$$\Delta = \begin{vmatrix} \mathrm{p}! & (\mathrm{p}+1)! & (\mathrm{p}+2)! \\ (\mathrm{p}+1)! & (\mathrm{p}+2)! & (\mathrm{p}+3)! \\ (\mathrm{p}+2)! & (\mathrm{p}+3)! & (\mathrm{p}+4)! \end{vmatrix}$$

Then the sum of the maximum values of α and β, such that p^{α} and $(\mathrm{p}+2)^{\beta}$ divide Δ, is _____.

[4 marks]

Solution:

Concept:

- Factorials satisfy $(m+1)! = (m+1)m!$. Extracting common factors from the rows simplifies a determinant without changing its prime factors unpredictably.
- The largest exponent of a prime dividing an integer is the total number of copies of that prime in its factorisation. The primes p and $p+2$ here are distinct.

Step 1: Extract the row factors Factor $p!$, $(p+1)!$ and $(p+2)!$ from the first, second and third rows respectively. This gives

$$\Delta = p!(p+1)!(p+2)! \begin{vmatrix} 1 & p+1 & (p+1)(p+2) \\ 1 & p+2 & (p+2)(p+3) \\ 1 & p+3 & (p+3)(p+4) \end{vmatrix}.$$

Subtract the first row from the second and third rows. Expanding along the first column reduces the remaining determinant to

$$\begin{vmatrix} 1 & 2(p+2) \\ 2 & 4p+10 \end{vmatrix} = (4p+10) - 4(p+2) = 2.$$

Hence $\Delta = 2p!(p+1)!(p+2)!$.

Step 2: Count the factors of p The case $p = 2$ is impossible because then $p+2 = 4$ is not prime. Thus $p \geq 3$ is odd. In each of the factorials $p!$, $(p+1)!$, $(p+2)!$, the only multiple of p is p, since $p+2 < 2p$. Also $p^2 > p+2$, so this factor contributes just one copy. The prefactor 2 contributes none. Therefore

$$\alpha = 1+1+1 = 3.$$

Step 3: Count the factors of the other prime Let $q = p+2$. The factorials $p!$ and $(p+1)!$ contain no multiple of q; the factorial $(p+2)! = q!$ contains exactly one, namely q itself. Since $q \geq 5$, the prefactor 2 is coprime to q. It follows that

$$\beta = 1, \qquad \alpha + \beta = 3 + 1 = 4.$$

These are exact valuations, so they are the maximum exponents requested, rather than merely lower bounds.

Step 4: Conclusion The maximum exponents are $\alpha = 3$ and $\beta = 1$. Thus $\boxed{\alpha + \beta = 4}$.

Quick Tip to Solve: Factor factorial determinants by rows before counting prime powers. Check the small possible prime separately so that inequalities such as $p+2 < 2p$ are justified.

26. Let $S = \{4, 6, 9\}$ and $T = \{9, 10, 11, \ldots, 1000\}$. If $A = \{a_1 + a_2 + \ldots + a_k : k \in \mathbf{N}, a_1, a_2, a_3, \ldots, a_k \epsilon S\}$, then the sum of all the elements in the set $T - A$ is equal to _____. [4 marks]

Solution:

Concept:

- The set A consists of all finite nonempty sums of 4, 6 and 9, with repetitions allowed. Thus it contains numbers $4u + 6v + 9w$ for nonnegative integers u, v, w not all zero.
- The difference $T - A$ contains elements of T that cannot be written in this form. Once four consecutive integers are representable, adding 4 proves all later integers are representable.

Step 1: Check the smallest elements of T The set T begins at 9. The first few values have representations

$$9 = 9, \quad 10 = 4 + 6, \quad 12 = 6 + 6, \quad 13 = 9 + 4, \quad 14 = 4 + 4 + 6, \quad 15 = 9 + 6.$$

The only candidate missing between 9 and 15 is 11. To test it, note that a sum containing no 9 is even, since both 4 and 6 are even. It therefore cannot equal 11. A sum containing a 9 leaves 2, which is not a sum of 4s and 6s; two 9s would already exceed 11. Hence $11 \notin A$.

Step 2: Prove that no larger exceptions exist The four consecutive integers 12, 13, 14 and 15 are all in A, as shown explicitly above. Every integer $n \geq 12$ can be written uniquely as

$$n = 12 + r + 4m, \qquad r \in \{0, 1, 2, 3\}, \quad m \geq 0.$$

Choose the displayed representation of $12 + r$ and append m summands equal to 4. This constructs a valid representation of n using only members of S. It proves coverage all the way to 1000 and beyond, without relying on a long numerical search.

Step 3: Form the set difference and its sum Both 9 and 10 belong to A, and every integer from 12 through 1000 belongs to A. The proof that 11 is impossible completes the classification:

$$T - A = \{11\}.$$

A singleton set has just its one element in the requested sum, so the sum equals 11.

Step 4: Conclusion The only element of $T - A$ is 11. Therefore the required sum is $\boxed{11}$.

Quick Tip to Solve: For sums generated by numbers including 4, find four consecutive attainable totals. Adding 4 then covers every later total, so only the earlier cases need inspection.

27. Let the mirror image of a circle $c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$ in line $y = x + 1$ be $c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$. If r is the radius of circle c_2, then $\alpha + 6r^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The centre of $x^2 + y^2 - 2hx - 2ky + c = 0$ is (h, k), and its squared radius is $h^2 + k^2 - c$. First normalise any equation whose quadratic coefficient is not 1.

- Reflection preserves distances and therefore the radius of a circle. The reflection in $y = x + 1$ maps a point (x, y) to $(y - 1, x + 1)$.

Step 1: Reflect the first centre Completing squares in the equation of c_1 gives

$$(x - 1)^2 + (y - 3)^2 = 10 - \alpha.$$

Its centre is $C_1 = (1, 3)$. Translate the coordinates downward by 1 so that the mirror line becomes $y = x$, interchange coordinates, and translate back. Equivalently, use the map $(x, y) \mapsto (y - 1, x + 1)$. It sends the centre to

$$C_2 = (3 - 1, 1 + 1) = (2, 2).$$

The midpoint $(3/2, 5/2)$ lies on the mirror line, and the joining segment has slope -1, providing a direct reflection check.

Step 2: Find the second radius from its equation Divide the equation of c_2 by 5:

$$x^2 + y^2 + 2gx + 2fy + \frac{38}{5} = 0.$$

Its centre is $(-g, -f)$, so $g = f = -2$. Hence its squared radius is

$$r^2 = g^2 + f^2 - \frac{38}{5} = 4 + 4 - \frac{38}{5} = \frac{2}{5}.$$

The positive value confirms that the reflected curve is a real circle, as required.

Step 3: Use preservation of radius Reflection does not scale the circle. Consequently the squared radius found from c_1 must equal that found from c_2:

$$10 - \alpha = \frac{2}{5} \quad \Longrightarrow \quad \alpha = \frac{48}{5}.$$

Now substitute into the requested expression, keeping the exact fractions:

$$\alpha + 6r^2 = \frac{48}{5} + 6\left(\frac{2}{5}\right) = \frac{60}{5} = 12.$$

Step 4: Conclusion The required value is $\boxed{\alpha + 6r^2 = 12}$.

Quick Tip to Solve: Reflect the centre and keep the radius unchanged. Normalise the second circle equation before reading its centre or radius; its leading coefficient here is 5.

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28. Let $S = \{\theta \in (0, 2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\}$. Then, the sum of roots of all the equations $x^2 - 2\left(\tan^2\theta + \cot^2\theta\right)x + 6\sin^2\theta = 0, \theta \in S$, is _____. [4 marks]

Solution:

Concept:

- Writing $u = \cos^2\theta$ gives $\sin^2\theta = 1 - u$ and $\cos 2\theta = 2u - 1$, with the restriction $0 \le u \le 1$.

- By Vieta's relation, the sum of the two roots of $x^2 - Bx + C = 0$ is B. Apply this to every admissible value of θ.

Step 1: Reduce the equation defining S Substitute the identities into the defining condition:

$$7u - 3(1-u) - 2(2u-1)^2 = 2.$$

Expanding and collecting terms gives

$$8u^2 - 18u + 7 = 0 \quad \Longrightarrow \quad (4u-7)(2u-1) = 0.$$

The root $u = 7/4$ is impossible for a squared cosine. Thus the only permissible value is $\cos^2\theta = 1/2$, which also implies $\sin^2\theta = 1/2$.

Step 2: Find every angle in the specified interval In the open interval $(0, 2\pi)$, the squared-cosine condition has exactly four solutions:

$$S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}.$$

At all four angles both sine and cosine are nonzero, so tangent and cotangent are defined. Moreover,

$$\tan^2\theta = \cot^2\theta = 1.$$

No endpoints are lost or gained because neither 0 nor 2π satisfies the condition.

Step 3: Sum the roots for all four angles For each of these four choices of θ, the equation becomes

$$x^2 - 2(1+1)x + 6\left(\frac{1}{2}\right) = 0, \qquad x^2 - 4x + 3 = 0.$$

Its roots are 1 and 3, and their sum is 4, also obtained directly from Vieta's relation. The question asks for the sum over all equations indexed by $\theta \in S$. Thus each of the four angle choices contributes its own root sum, even though the resulting quadratic is the same:

$$\text{Total} = 4 \times (1+3) = 16.$$

Step 4: Conclusion The sum of the roots of all four equations, counting each equation for its corresponding angle, is $\boxed{16}$.

Quick Tip to Solve: Reject squared-trigonometric values outside $[0, 1]$, count the admissible angles, and then use Vieta. Repeated quadratics still contribute once for each angle in the stated family.

29. Let a line with direction ratios $a, -4a, -7$ be perpendicular to the lines with direction ratios $3, -1, 2b$ and $b, a, -2$. If the point of intersection of the line $\frac{x+1}{a^2+b^2} = \frac{y-2}{a^2-b^2} = \frac{z}{1}$ and the plane $x - y + z = 0$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to _____. [4 marks]

Solution:

Concept:

- Two lines are perpendicular when the scalar product of their direction vectors vanishes. This supplies two equations for the parameters a and b.
- A symmetric line equation can be written in parametric form using a common parameter. Substitution in the plane equation gives the parameter of the intersection; no distance formula is needed.

Step 1: Use the two perpendicularity conditions The direction vector $(a, -4a, -7)$ is orthogonal to each of the other direction vectors. Hence

$$3a + 4a - 14b = 0, \qquad ab - 4a^2 + 14 = 0.$$

The first equation gives $a = 2b$. Substituting into the second gives

$$2b^2 - 16b^2 + 14 = 0 \quad \Longrightarrow \quad b^2 = 1.$$

Consequently $a^2 = 4$. Either simultaneous sign choice gives the same squared parameters, so both possibilities lead to the same line in the question.

Step 2: Parameterise the line and impose the plane Using $a^2 + b^2 = 5$ and $a^2 - b^2 = 3$, put the three symmetric ratios equal to t. Then

$$x = -1 + 5t, \qquad y = 2 + 3t, \qquad z = t.$$

The plane condition becomes

$$(-1 + 5t) - (2 + 3t) + t = 0 \quad \Longrightarrow \quad 3t - 3 = 0.$$

Thus $t = 1$, and the coordinates of intersection are $(4, 5, 1)$. As a direct check, these coordinates satisfy $4 - 5 + 1 = 0$, and all three line ratios equal one. The coefficient of t in the plane condition is nonzero, confirming a unique intersection.

Step 3: Conclusion Therefore $\alpha + \beta + \gamma = 4 + 5 + 1 = \boxed{10}$. This is a dimensionless numerical answer.

Quick Tip to Solve: When a line contains only squared parameters, determine their squares first; carrying unnecessary sign choices into the intersection calculation doubles the work.

30. If $\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\frac{1}{4\times5\times6}+\ldots+\frac{1}{100\times101\times102}=\frac{k}{101}$, then 34 k is equal to _____. [4 marks]

Solution:

Concept:

- The identity $\frac{1}{n(n+1)(n+2)}=\frac{1}{2}\left[\frac{1}{n(n+1)}-\frac{1}{(n+1)(n+2)}\right]$ turns consecutive triple products into a telescoping sum.
- In a telescoping sum, every interior term cancels with its neighbour. Keep the first and last terms with the original summation limits before comparing with the given fraction.

Step 1: Express each summand as a difference Write the required sum as $S=\sum_{n=2}^{100}1/[n(n+1)(n+2)]$. Subtracting the two adjacent double products gives

$$\frac{1}{n(n+1)}-\frac{1}{(n+1)(n+2)}=\frac{(n+2)-n}{n(n+1)(n+2)}.$$

The numerator is 2, which explains the essential factor $1/2$. Therefore

$$S=\frac{1}{2}\left[\left(\frac{1}{2\cdot3}-\frac{1}{3\cdot4}\right)+\cdots+\left(\frac{1}{100\cdot101}-\frac{1}{101\cdot102}\right)\right].$$

The negative double-product term in each bracket is cancelled by the positive term in the next bracket.

Step 2: Evaluate the surviving endpoints and find the requested value Only the initial positive term and the final negative term remain:

$$S=\frac{1}{2}\left(\frac{1}{6}-\frac{1}{101\cdot102}\right)=\frac{1}{12}-\frac{1}{20604}.$$

Since $S=k/101$, multiply by 101 before simplifying:

$$k=\frac{101}{12}-\frac{1}{204}=\frac{1717-1}{204}=\frac{1716}{204}=\frac{143}{17}.$$

The question asks for $34k$, rather than for k itself, so the denominator 17 cancels exactly when the final multiplication is performed.

Step 3: Conclusion The required dimensionless value is $34k=34\cdot143/17=\boxed{286}$.

Quick Tip to Solve: For three consecutive factors in the denominator, try half the difference of adjacent two-factor reciprocals; write the final uncancelled endpoint carefully.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Given below are two statements : One is labelled as Assertion (A) and other is labelled as Reason (R).

Assertion (A) : Time period of oscillation of a liquid drop depends on surface tension (S), if density of the liquid is ρ and radius of the drop is r, then $\mathrm{T} = \mathrm{K}\sqrt{\rho \mathrm{r}^3/\mathrm{S}^{3/2}}$ is dimensionally correct, where K is dimensionless.

Reason (R) : Using dimensional analysis we get R.H.S. having different dimension than that of time period.

In the light of above statements, choose the correct answer from the options given below. [4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

Correct Answer: (D) (A) is false but (R) is true

Solution:

Concept:

- Surface tension is force per unit length, so $[S] = MT^{-2}$. Liquid density has dimensions $[\rho] = ML^{-3}$, and radius has dimensions $[r] = L$.
- A dimensionless constant cannot correct a dimensional mismatch. In an Assertion-Reason problem, first evaluate each statement independently, then select the option for their truth values.

Step 1: Check the dimensions in the assertion The proposed formula contains $S^{3/2}$ inside the square root; that exponent must be retained. First,

$$[\rho r^3] = (ML^{-3})L^3 = M, \qquad [S^{3/2}] = (MT^{-2})^{3/2} = M^{3/2}T^{-3}.$$

Consequently the dimensional formula of the proposed right-hand side is

$$\left[K\sqrt{\frac{\rho r^3}{S^{3/2}}}\right] = \left(M^{-1/2}T^3\right)^{1/2} = M^{-1/4}T^{3/2}.$$

A time period must have dimensions T, with zero mass exponent. Both the mass and time exponents here disagree. The assertion that this expression is dimensionally correct is therefore false.

Step 2: Evaluate the reason and choose the truth-value pair The reason states that dimensional analysis gives dimensions on the right-hand side different from a time period. The calculation above establishes exactly that statement, so the reason is true. Options (A) and (B) both require a true assertion and are excluded. Option (C) requires a true assertion and a false reason, also the opposite of what the dimensional calculation shows. Only option (D) has a false assertion and a true reason. For comparison, the familiar dimensionally admissible scaling $T \propto \sqrt{\rho r^3/S}$ would have dimensions T; this does not rescue the distinct expression actually printed.

Step 3: Conclusion The assertion is false and the reason is true. Hence, the correct option is (D).

Quick Tip to Solve: Take fractional powers of the entire dimensional formula. A square root halves every exponent, including any unwanted mass exponent.

32. A ball is thrown up vertically with a certain velocity so that, it reaches a maximum height h. Find the ratio of the times in which it is at height $\frac{h}{3}$ while going up and coming down respectively. [4 marks]

(A) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

(B) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(C) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

(D) $\frac{1}{3}$

Correct Answer: (B) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Solution:

Concept:

- For upward projection with initial speed u, the height at time t is $y = ut - \frac{1}{2}gt^2$, with constant downward acceleration g.
- The maximum height is $h = u^2/(2g)$. Solving the height equation gives two times: the smaller root for ascent and the larger root for descent, both measured from launch.

Step 1: Write the quadratic equation at the specified height The target height is $h/3 = u^2/(6g)$. Substituting this into the displacement equation gives

$$\frac{u^2}{6g} = ut - \frac{1}{2}gt^2, \qquad gt^2 - 2ut + \frac{u^2}{3g} = 0.$$

Using the quadratic formula,

$$t = \frac{2u \pm \sqrt{4u^2 - 4u^2/3}}{2g} = \frac{u}{g}\left(1 \pm \sqrt{\frac{2}{3}}\right).$$

Both roots are positive, as expected for reaching a height between the launch point and the maximum. Their midpoint is u/g, the time of reaching the highest point.

Step 2: Take the ascent-to-descent time ratio The earlier time is $t_{\text{up}} = (u/g)(1 - \sqrt{2/3})$, whereas the later time is $t_{\text{down}} = (u/g)(1 + \sqrt{2/3})$. Therefore

$$\frac{t_{\text{up}}}{t_{\text{down}}} = \frac{1 - \sqrt{2/3}}{1 + \sqrt{2/3}} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}.$$

The factors containing the launch speed and gravity cancel, so the ratio is independent of both. Option (A) uses a different height fraction, and option (C) also has the wrong root factor. Option (D) is not the ratio of the two roots of the height equation; height and elapsed time are not proportional under acceleration.

Step 3: Conclusion The required ratio is $\boxed{(\sqrt{3} - \sqrt{2})/(\sqrt{3} + \sqrt{2})}$. Hence, the correct option is (B).

Quick Tip to Solve: For a fraction f of the maximum height, the two times are $(u/g)(1 \pm \sqrt{1 - f})$. Always use the smaller root in the numerator for ascent.

33. If $t = \sqrt{x} + 4$, **then** $\left(\frac{dx}{dt}\right)_{t=4}$ **is :** [4 marks]

(A) 4

(B) zero

(C) 8

(D) 16

Correct Answer: (B) zero

Solution:

Concept:

- Instantaneous velocity is the derivative of position with respect to time: $v = dx/dt$. A position-time equation can often be inverted before differentiating.
- The real principal square root is nonnegative. Thus the stated relation $t = \sqrt{x} + 4$ has $t \geq 4$, and $t = 4$ corresponds to $x = 0$.

Step 1: Express position explicitly as a function of time Subtract 4 from the given relation to obtain $\sqrt{x} = t - 4$. On the allowed branch $t \geq 4$, squaring gives the equivalent position law

$$x = (t - 4)^2.$$

Writing position explicitly avoids dividing by $\sqrt{x}$ at $x = 0$. It also keeps track of the domain: the point $t = 4$ is the initial endpoint of this branch, so the physical instantaneous velocity there is obtained as the derivative from later times. There is no arbitrary nonzero initial velocity hidden in the time offset 4.

Step 2: Differentiate and evaluate at the requested instant Differentiating the explicit position law gives

$$\frac{dx}{dt} = 2(t - 4).$$

At $t = 4$, this is zero. The endpoint can also be checked directly through the difference quotient. For $\Delta t > 0$,

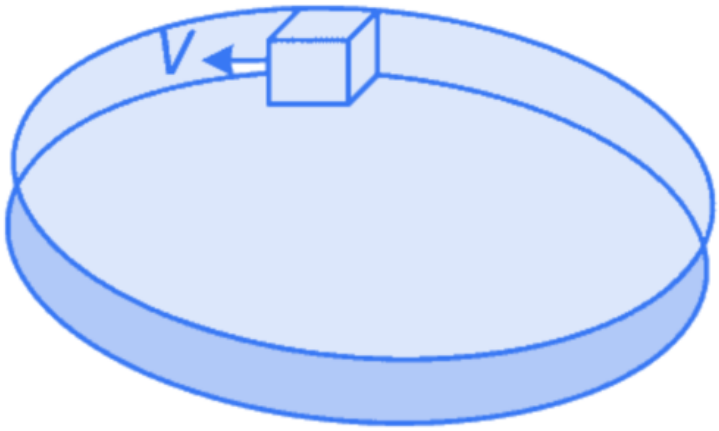
$$\frac{x(4 + \Delta t) - x(4)}{\Delta t} = \frac{(\Delta t)^2}{\Delta t} = \Delta t \longrightarrow 0.$$

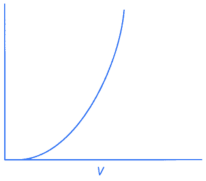
Thus the velocity starts from zero on this branch. The nonzero choices 4, 8 and 16 cannot follow from the derivative; they confuse the time offset, an unsubtracted time value, or a squared quantity with velocity. In particular, differentiating the constant 4 contributes nothing.

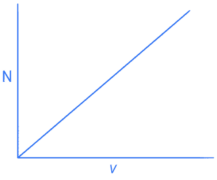
Step 3: Conclusion At $t = 4$, $\boxed{dx/dt = 0}$, i.e. zero velocity. Hence, the correct option is (B).

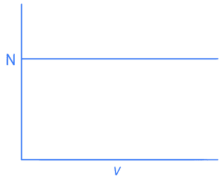
Quick Tip to Solve: Invert a square-root time relation before differentiating, especially at the point where the square root vanishes; the explicit polynomial avoids a singular intermediate expression.

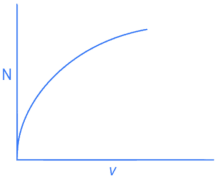
34. A smooth circular groove has a smooth vertical wall as shown in figure. A block of mass m moves against the wall with a speed v. Which of the following curve represents the correct relation between the normal reaction on the block by the wall (N) and speed of the block (v) ? [4 marks]

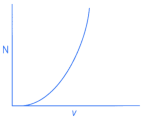


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- Uniform circular motion of speed v along a fixed circle of radius R needs an inward radial acceleration v^2/R. Newton's second law therefore requires a net radial force mv^2/R.
- The smooth vertical wall exerts a horizontal normal reaction on the block. Weight is vertical and is balanced by the groove's support; it does not provide the horizontal centripetal force.

Step 1: Identify the force that turns the block Choose the inward horizontal radial direction toward the circle's centre. Because the contact surfaces are smooth, there is no frictional force to add to the radial equation. The wall's normal reaction acts along this radial direction and provides the acceleration needed to change the direction of the velocity. Thus, with the groove radius R fixed,

$$N = m\frac{v^2}{R}.$$

The force is positive for any nonzero speed and vanishes when the speed is zero. A larger speed requires a stronger force even if the magnitude of that speed remains constant during any individual revolution.

Step 2: Compare the predicted graph with the four options For a fixed mass and radius, m/R is a positive constant, so $N \propto v^2$. The graph of N against nonnegative v is an upward-curving parabola through the origin. Its slope is $dN/dv = 2mv/R$: zero at the origin and increasing with speed. This is graph (A). Graph (B) is a straight line and would represent $N \propto v$, not v^2. Graph (C) would imply a constant reaction independent of speed. Graph (D) bends downward and has a decreasing slope, opposite to the quadratic dependence. Doubling the speed must quadruple the reaction, which provides a quick check of the curvature.

Step 3: Conclusion The wall reaction is $\boxed{N = mv^2/R}$, so graph (A) is correct. Hence, the correct option is (A).

Quick Tip to Solve: Check both the origin and curvature: a centripetal force proportional to speed squared gives a convex upward curve, whereas plotting the same force against v^2 would give a straight line.

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35. A ball is projected with kinetic energy E, at an angle of 60° to the horizontal. The kinetic energy of this ball at the highest point of its flight will become : [4 marks]

(A) Zero

(B) $\frac{E}{2}$

(C) $\frac{E}{4}$

(D) E

Correct Answer: (C) $\frac{E}{4}$

Solution:

Concept:

- Ignoring air resistance, projectile motion has constant horizontal velocity and uniformly changing vertical velocity under gravity. At the highest point only the vertical component vanishes.
- Kinetic energy is $K = \frac{1}{2}m(v_x^2 + v_y^2)$. For initial speed u, the initial kinetic energy is $E = \frac{1}{2}mu^2$.

Step 1: Resolve the initial velocity and identify what remains The initial velocity components are $u_x = u\cos 60^\circ = u/2$ horizontally and $u_y = u\sin 60^\circ = \sqrt{3}u/2$ vertically. Gravity acts vertically, so there is no horizontal acceleration and $v_x = u/2$ throughout the flight. At the highest point, $v_y = 0$, because the vertical motion changes there from ascent to descent. The speed at that point is consequently $u/2$; it is not zero. The ball still travels horizontally even while its height is momentarily stationary.

Step 2: Calculate the remaining kinetic energy and eliminate other choices Substitute the highest-point speed into the kinetic-energy expression:

$$K_{\text{top}} = \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}mu^2\right) = \frac{E}{4}.$$

Equivalently, $K_{\text{top}}/E = \cos^2 60^\circ = 1/4$. Option (A), zero, would be appropriate for a purely vertical launch, but here the horizontal motion remains. Option (B) would require $\cos^2\theta = 1/2$, as for a 45° launch. Option (D) would mean that no kinetic energy had been converted into gravitational potential

energy, which contradicts the rise in height. In fact the lost kinetic energy is $3E/4$, exactly the initial vertical kinetic-energy contribution.

Step 3: Conclusion The kinetic energy at the highest point is $\boxed{E/4}$. Hence, the correct option is (C).

Quick Tip to Solve: At the apex, retain the horizontal velocity before squaring it. The kinetic-energy fraction is $\cos^2\theta$, rather than $\cos\theta$.

---

36. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i}+2\hat{j}+\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$ respectively. The magnitude of position vector of centre of mass of this system will be similar to the magnitude of vector : [4 marks]

(A) $\hat{i}+2\hat{j}+\hat{k}$

(B) $-3\hat{i}-2\hat{j}+\hat{k}$

(C) $-2\hat{i}+2\hat{k}$

(D) $2\hat{i}-\hat{j}+2\hat{k}$

Correct Answer: (A) $\hat{i}+2\hat{j}+\hat{k}$

Solution:

Concept:

- The centre-of-mass position is the mass-weighted average, $\vec{r}_{\text{CM}} = (m_1\vec{r}_1 + m_2\vec{r}_2)/(m_1+m_2)$. The weighting must be applied to each Cartesian component.
- The magnitude of a Cartesian vector $x\hat{i}+y\hat{j}+z\hat{k}$ is $\sqrt{x^2+y^2+z^2}$. Equal magnitudes do not require equal vectors or equal directions.

Step 1: Calculate the weighted position Here $m_1 = 1\,\text{kg}$ and $m_2 = 3\,\text{kg}$, so the total mass is $4\,\text{kg}$. Substitute the given vectors before taking a magnitude:

$$\vec{r}_{\text{CM}} = \frac{(\hat{i}+2\hat{j}+\hat{k}) + 3(-3\hat{i}-2\hat{j}+\hat{k})}{4} = -2\hat{i}-\hat{j}+\hat{k}.$$

Thus the centre of mass is closer to the heavier particle. Averaging the two position vectors without the masses would give a different and incorrect location.

Step 2: Compare magnitudes with all options The squared magnitude of the centre-of-mass position is

$$|\vec{r}_{\text{CM}}|^2 = (-2)^2 + (-1)^2 + 1^2 = 6.$$

For options (A), (B), (C) and (D), the squared magnitudes are respectively

$$1+4+1=6, \qquad 9+4+1=14, \qquad 4+4=8, \qquad 4+1+4=9.$$

Only option (A) has squared magnitude 6. Although its vector differs from the centre-of-mass vector, the question asks for the magnitude alone; rejecting it on directional grounds would misread the demand.

Step 3: Conclusion The centre-of-mass position has magnitude $\sqrt{6}$, equal to the magnitude of

$\hat{i} + 2\hat{j} + \hat{k}$. Hence, the correct option is (A).

Quick Tip to Solve: For a magnitude comparison, compare sums of squared components; there is no need to evaluate any square roots.

37. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Clothes containing oil or grease stains cannot be cleaned by water wash.

Reason (R): Because the angle of contact between the oil/grease and water is obtuse.

In the light of the above statements, choose the correct answer from the option given below. [4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

Correct Answer: (A) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- The contact angle measures wetting at the contact of a liquid with a surface. An obtuse contact angle, greater than 90°, indicates poor wetting.
- Oil and grease repel water, so water alone cannot spread effectively over an oily fabric surface and remove its adhering greasy stain. A detergent changes this wetting behaviour.

Step 1: Judge the assertion An oil or grease coating on fibres is not readily wetted by plain water. The water tends to collect in drops rather than spread into close contact over the greasy layer. Furthermore, oil does not dissolve in water, so ordinary water washing does not readily detach and carry away the stain. In the usual surface-tension context of the question, the assertion that such clothes cannot be cleaned by a water wash is therefore true. It refers to washing with water alone, without a soap or detergent.

Step 2: Judge the reason Water on the greasy surface has an obtuse angle of contact. This is the non-wetting situation: the cohesive tendency of water prevents it from spreading over the oil-coated fibres. Thus the reason is true in the wetting interpretation intended here. An acute angle would instead indicate that water wets and spreads over the surface more easily, which is contrary to the situation described.

Step 3: Connect the two statements The obtuse contact angle directly explains the poor wetting responsible for the failure of a plain-water wash. Soap or detergent promotes wetting and helps disperse the greasy matter, allowing it to be carried away. Thus the reason is an explanation of the assertion, rather than an unrelated true fact. Option (B) misses this causal link; options (C) and (D) incorrectly reject one of the true statements.

Step 4: Conclusion Both the assertion and the reason are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

Quick Tip to Solve: In wetting questions, associate an obtuse contact angle with water beading up on a greasy surface; detergents help it spread.

38. If the length of a wire is made double and radius is halved of its respective values. Then, the Young's modulus of the material of the wire will : [4 marks]

(A) remain same

(B) become 8 times its initial value

(C) become $\frac{1}{4}$ of its initial value

(D) become 4 times its initial value

Correct Answer: (A) remain same

Solution:

Concept:

- Young's modulus is the ratio of longitudinal stress to longitudinal strain in the elastic regime: $Y = (F/A)/(\Delta L/L) = FL/(A\Delta L)$.
- The modulus is a property of the material at fixed physical conditions. Length and area determine the stiffness of a particular wire, but do not by themselves change its material modulus.

Step 1: Distinguish material modulus from wire stiffness For a wire of length L, radius r, area $A = \pi r^2$ and small extension ΔL, Hooke's law gives

$$F = \frac{YA}{L}\Delta L.$$

Consequently its force constant is $k = YA/L$. This equation separates the material factor Y from the geometrical factors A and L. A change of geometry changes how far the wire stretches under a specified load, not the intrinsic stress-to-strain ratio of its material.

Step 2: Apply the changed dimensions consistently The new length is $L' = 2L$, and halving the radius gives $A' = \pi(r/2)^2 = A/4$. Hence, for the same material,

$$k' = \frac{Y(A/4)}{2L} = \frac{k}{8}.$$

Under the same small tensile force the extension becomes $\Delta L' = 8\Delta L$. Substitution into the defining expression confirms

$$Y' = \frac{F(2L)}{(A/4)(8\Delta L)} = \frac{FL}{A\Delta L} = Y.$$

Treating the extension as unchanged would incorrectly give a factor of eight. The other proposed factors similarly confuse geometrical changes with a material property. No change of material or temperature is specified.

Step 3: Conclusion The Young's modulus remains unchanged: $Y' = Y$. Hence, the correct option is (A).

Quick Tip to Solve: Keep Y, a material property, separate from YA/L, the stiffness of a particular wire; dimensional changes alter the latter.

39. The time period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle, which moves without friction down an inclined plane of inclination α, is given by : [4 marks]

(A) $2\pi\sqrt{\mathrm{L}/(\mathrm{g}\cos\alpha)}$

(B) $2\pi\sqrt{\mathrm{L}/(\mathrm{g}\sin\alpha)}$

(C) $2\pi\sqrt{\mathrm{L}/\mathrm{g}}$

(D) $2\pi\sqrt{\mathrm{L}/(\mathrm{g}\tan\alpha)}$

Correct Answer: (A) $2\pi\sqrt{\mathrm{L}/(\mathrm{g}\cos\alpha)}$

Solution:

Concept:

- A frictionless vehicle on an incline of angle α accelerates down the plane with magnitude $a = g\sin\alpha$.
- In the vehicle frame the pendulum experiences effective gravity $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}$. For small oscillations about its new equilibrium, $T = 2\pi\sqrt{L/g_{\text{eff}}}$.

Step 1: Resolve gravity relative to the incline Take one axis down the plane and another perpendicular to it. Gravity has a downslope component $g\sin\alpha$ and an inward normal component $g\cos\alpha$. The vehicle accelerates downslope at $g\sin\alpha$, because there is no friction. In its frame the pseudo-acceleration acts upslope with exactly the same magnitude. Therefore the downslope components cancel:

$$(g_{\text{eff}})_{\parallel} = g\sin\alpha - a = 0, \qquad |(g_{\text{eff}})_{\perp}| = g\cos\alpha.$$

The string's equilibrium direction is along this effective gravity, perpendicular to the incline rather than vertically downward.

Step 2: Use the small-oscillation equation If θ is a small angular displacement from the new equilibrium, the restoring torque is $-mg_{\text{eff}}L\sin\theta$. With moment of inertia mL^2 about the suspension point, the equation becomes

$$\ddot{\theta} + \frac{g\cos\alpha}{L}\theta = 0.$$

It follows that $\omega^2 = g\cos\alpha/L$. The factor $g\sin\alpha$ is the cancelled component, so option (B) cannot be used. Option (C) ignores the vehicle's acceleration, while option (D) has no basis in the component resolution. At $\alpha = 0$, the expression correctly reduces to the usual pendulum period.

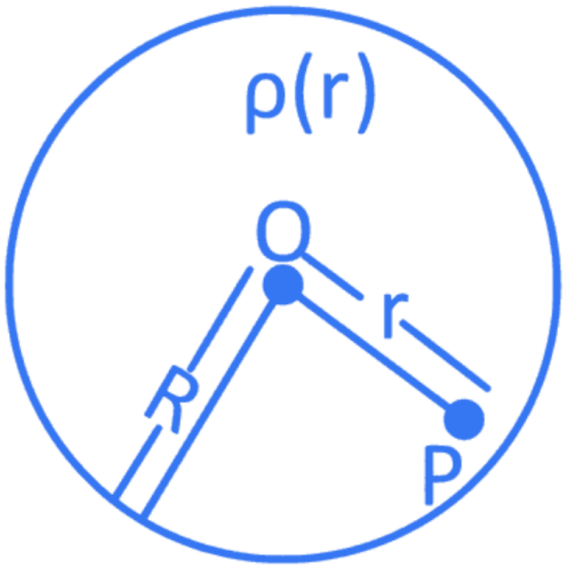
Step 3: Conclusion The period is $T = 2\pi\sqrt{L/(g\cos\alpha)}$. Hence, the correct option is (A).

Quick Tip to Solve: For a pendulum in a freely sliding vehicle, cancel the gravity component along the vehicle acceleration before using the pendulum formula.

40. A spherically symmetric charge distribution is considered with charge density varying as

$$\rho(r) = \begin{cases} \rho_0\left(\frac{3}{4} - \frac{r}{R}\right) & \text{for } r \le R \\ \text{zero} & \text{for } r > R \end{cases}$$

Where, $r(r < R)$ is the distance from the centre O (as shown in figure). The electric field at point P will be: [4 marks]



(A) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{3}{4} - \frac{r}{R}\right)$
(B) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{3}{4} - \frac{r}{R}\right)$
(C) $\frac{\rho_0 r}{4\varepsilon_0}\left(1 - \frac{r}{R}\right)$
(D) $\frac{\rho_0 r}{5\varepsilon_0}\left(1 - \frac{r}{R}\right)$

Correct Answer: (C) $\frac{\rho_0 r}{4\varepsilon_0}\left(1 - \frac{r}{R}\right)$

Solution:

Concept:

- Spherical symmetry makes the electric field radial and of constant magnitude on a concentric Gaussian sphere: $4\pi r^2 E = Q_{\text{enc}}/\varepsilon_0$.
- For a radially varying volume density, each thin shell contributes $dq = \rho(s)4\pi s^2 ds$. Use a dummy radius s when integrating up to the observation radius r.

Step 1: Integrate the enclosed charge Since P lies at $r < R$, only charge inside radius r enters Gauss's law. The density is not constant, so it cannot be taken outside the integral. Instead,

$$\begin{aligned} Q_{\text{enc}} &= 4\pi\rho_0 \int_0^r \left(\frac{3}{4} - \frac{s}{R}\right) s^2\, ds \\ &= 4\pi\rho_0 \left(\frac{r^3}{4} - \frac{r^4}{4R}\right) = \pi\rho_0 r^3 \left(1 - \frac{r}{R}\right). \end{aligned}$$

The charge density becomes negative in the outer region $s > 3R/4$, but this does not invalidate the

integral; its signed contribution must be retained.

Step 2: Apply the Gaussian surface relation Dividing the enclosed charge by $4\pi\varepsilon_0 r^2$ gives

$$E(r) = \frac{\rho_0 r}{4\varepsilon_0}\left(1 - \frac{r}{R}\right).$$

For positive ρ_0 the field is outward for $0 < r < R$, and it vanishes both at the centre and at $r = R$. The latter check follows because the total charge of the distribution is zero. Options (A) and (B) retain the local-density factor rather than the correctly integrated factor and do not vanish at R. Option (D) has the right radial form but the wrong coefficient; the shell integral gives $1/4$, not $1/5$.

Step 3: Conclusion At P, $\vec{E} = \frac{\rho_0 r}{4\varepsilon_0}\left(1 - \frac{r}{R}\right)\hat{r}$. Hence, the correct option is (C).

Quick Tip to Solve: Never substitute the local density into the uniform-sphere formula. First integrate $4\pi s^2\rho(s)$, then divide by the Gaussian area.

41. Given below are two statements.

Statement I : Electric potential is constant within and at the surface of each conductor.

Statement II : Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- In electrostatic equilibrium the electric field inside the conducting material is zero; otherwise its free charges would continue to move.
- The electric field and potential obey $\vec{E} = -\nabla V$. A vanishing interior field and vanishing tangential surface field make a connected conductor an equipotential.

Step 1: Evaluate Statement I Consider two points within a conductor in electrostatic equilibrium. Their potential difference is

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l} = 0,$$

because the field inside the conductor is zero. The potential is continuous on approaching its surface, so the same value applies there as well. Thus every connected conductor has a constant potential throughout its material and over its surface. This constant need not be zero and different conductors need not have equal potentials. Statement I is therefore correct in the electrostatic setting of the

question.

Step 2: Evaluate Statement II and select the pair If the field just outside the conductor had a nonzero component tangent to its surface, the free surface charges would be driven along the surface. This would contradict electrostatic equilibrium. Charges redistribute until the tangential component vanishes:

$$E_{\text{tangent}} = 0.$$

Hence any nonzero electric field immediately outside a smooth charged surface is normal to it, with its direction outward for positive local surface charge and inward for negative local surface charge. This establishes Statement II. The statement concerns direction, not uniform field magnitude: the field can be stronger at sharply curved regions. Since both statements hold, options (B), (C) and (D), which reject at least one of them, are excluded.

Step 3: Conclusion Both Statement I and Statement II are correct for a conductor in electrostatic equilibrium. Hence, the correct option is (A).

Quick Tip to Solve: Use zero interior field for equipotentiality and zero tangential surface field for normality; these are two consequences of charge equilibrium.

42. Two metallic wires of identical dimensions are connected in series. If σ_1 and σ_2 are the conductivities of the these wires respectively, the effective conductivity of the combination is :

[4 marks]

(A) $\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}$
(B) $\frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$
(C) $\frac{\sigma_1+\sigma_2}{2\sigma_1\sigma_2}$
(D) $\frac{\sigma_1+\sigma_2}{\sigma_1\sigma_2}$

Correct Answer: (B) $\frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

Solution:

Concept:

- A uniform wire of length L, cross-sectional area A, and conductivity σ has resistance $R = L/(\sigma A)$.
- Resistances add in series. The equivalent uniform wire has the combined length $2L$ and the same area A; its conductivity must be defined using that full geometry.

Step 1: Add the resistances of the two wires Let each identical-sized wire have length L and cross-sectional area A. Their individual resistances are

$$R_1 = \frac{L}{\sigma_1 A}, \qquad R_2 = \frac{L}{\sigma_2 A}.$$

The same current flows through both wires, while their voltage drops add. Thus

$$R_{\text{series}} = \frac{L}{A}\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right).$$

Conductivities themselves do not add in series. It is the resistance, or equivalently the resistivity weighted by length, that must first be combined.

Step 2: Identify the effective conductivity The series combination occupies a total length $2L$ with unchanged cross-section A. If σ_{eff} denotes its effective conductivity, equate its resistance to the sum:

$$\frac{2L}{\sigma_{\text{eff}} A} = \frac{L}{A}\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right).$$

Cancelling L/A and taking the reciprocal gives

$$\sigma_{\text{eff}} = \frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}.$$

Option (A) misses the factor of two from the full length. Options (C) and (D) have resistivity dimensions, so they cannot represent conductivity. As a final check, setting $\sigma_1 = \sigma_2 = \sigma$ gives $\sigma_{\text{eff}} = \sigma$, as required for a longer piece of the same material.

Step 3: Conclusion The effective conductivity is $\sigma_{\text{eff}} = \dfrac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$, the harmonic mean. Hence, the correct option is (B).

Quick Tip to Solve: After adding series resistances, use the total length in the equivalent wire; missing that step loses the factor of two.

43. An alternating emf $\mathrm{E} = 440\sin 100\pi\mathrm{t}$ is applied to a circuit containing an inductance of $\frac{\sqrt{2}}{\pi}\mathrm{H}$. If an a.c. ammeter is connected in the circuit, its reading will be : [4 marks]

(A) 4.4 A

(B) 1.55 A

(C) 2.2 A

(D) 3.11 A

Correct Answer: (C) 2.2 A

Solution:

Concept:

- For a sinusoidal voltage $E = E_0 \sin\omega t$, the coefficient E_0 is the peak voltage, whereas an ordinary a.c. ammeter reads the root-mean-square current.
- An ideal inductor has reactance $X_L = \omega L$. The peak and rms currents are $I_0 = E_0/X_L$ and $I_{\text{rms}} = I_0/\sqrt{2}$, respectively.

Step 1: Read the voltage parameters Compare the given expression with the standard sinusoidal form. Its amplitude is $E_0 = 440\,\text{V}$, and its angular frequency is $\omega = 100\pi\,\text{rad}\,\text{s}^{-1}$. The factor multiplying time is already angular frequency; it must not be multiplied by 2π again. The given inductance is $L = \sqrt{2}/\pi\,\text{H}$.

Step 2: Calculate the reactance and peak current The opposition offered by the inductance is

$$X_L = (100\pi)\left(\frac{\sqrt{2}}{\pi}\right) = 100\sqrt{2}\,\Omega.$$

Hence the current amplitude is

$$I_0 = \frac{440}{100\sqrt{2}} = \frac{4.4}{\sqrt{2}}\,\text{A} \simeq 3.11\,\text{A}.$$

This is the maximum instantaneous current, rather than the meter reading.

Step 3: Convert to the ammeter reading For sinusoidal current, the mean-square value is half the square of the amplitude. Therefore

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{440}{100\sqrt{2}\sqrt{2}} = 2.2\,\text{A}.$$

Option (D) gives the peak current and must be rejected. Option (A) misses the two factors of $\sqrt{2}$; option (B) introduces an extra conversion factor. Only option (C) equals the effective current.

Step 4: Conclusion The a.c. ammeter reads $2.2\,\text{A}$. Hence, the correct option is (C).

Quick Tip to Solve: Calculate X_L using angular frequency, then use rms voltage divided by reactance; this avoids confusing the peak current with the meter reading.

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44. A coil of inductance 1 H and resistance $100\,\Omega$ is connected to a battery of 6 V. Determine approximately :

(a) The time elapsed before the current acquires half of its steady - state value.

(b) The energy stored in the magnetic field associated with the coil at an instant 15 ms after the circuit is switched on. (Given $\ln 2 = 0.693, \text{e}^{-3/2} = 0.25$) [4 marks]

(A) t = 10 ms; U = 2 mJ

(B) t = 10 ms; U = 1 mJ

(C) t = 7 ms; U = 1 mJ

(D) t = 7 ms; U = 2 mJ

Correct Answer: (C) t = 7 ms; U = 1 mJ

Solution:

Concept:

- After a d.c. source is connected to an initially unenergised series RL circuit, $i(t) = I_\infty(1 - e^{-t/\tau})$, where $I_\infty = V/R$ and $\tau = L/R$.
- The magnetic energy at any instant is $U = \frac{1}{2}Li^2$, using the current at that instant, not necessarily its final steady value.

Step 1: Find the steady current and time constant The resistance limits the eventual current because an ideal inductance has no induced emf once the current becomes steady. With $V = 6\,\text{V}$, $R = 100\,\Omega$

and $L = 1\,\text{H}$,

$$I_\infty = \frac{6}{100} = 0.06\,\text{A}, \qquad \tau = \frac{1}{100} = 0.01\,\text{s} = 10\,\text{ms}.$$

The time constant itself is not the half-current time.

Step 2: Calculate the half-current time Set $i = I_\infty/2$ in the growth equation:

$$\frac{1}{2} = 1 - e^{-t/\tau} \quad \Rightarrow \quad e^{-t/\tau} = \frac{1}{2}.$$

Taking logarithms gives

$$t = \tau \ln 2 = (10\,\text{ms})(0.693) = 6.93\,\text{ms} \simeq 7\,\text{ms}.$$

Thus the choices with $10\,\text{ms}$ fail the first requirement.

Step 3: Evaluate the energy at the specified instant At $t = 15\,\text{ms}$, $t/\tau = 1.5$. Use the supplied approximate exponential value as requested:

$$i = 0.06(1 - 0.25) = 0.045\,\text{A}.$$

The stored energy is therefore

$$U = \frac{1}{2}(1)(0.045)^2 = 1.0125 \times 10^{-3}\,\text{J} \simeq 1\,\text{mJ}.$$

Using the final current here would instead give the final energy, about $1.8\,\text{mJ}$, and would incorrectly suggest $2\,\text{mJ}$. Option (D) has that energy error, while (A) and (B) already have the wrong time.

Step 4: Conclusion The approximate values are $t = 7\,\text{ms}$ and $U = 1\,\text{mJ}$. Hence, the correct option is (C).

Quick Tip to Solve: In RL growth problems distinguish τ, $\tau \ln 2$, and the actual observation time before substituting the current into $Li^2/2$.

45. Match List - I with List - II :

	List - I		List - II
(a)	UV rays	(i)	Diagnostic tool in medicine
(b)	X-rays	(ii)	Water purification
(c)	Microwave	(iii)	Communication, Radar
(d)	Infrared wave	(iv)	Improving visibility in foggy days

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

(B) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(C) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

(D) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

Correct Answer: (B) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Solution:

Concept:

- Electromagnetic radiation has different applications because its wavelength controls penetration, interaction with matter and the degree of scattering. The matching must use the stated applications of each band.
- Ultraviolet radiation has germicidal action; X-rays penetrate soft tissue; microwaves are useful for directed communication and radar; infrared radiation suffers less scattering than visible light in fog.

Step 1: Match ultraviolet rays and X-rays Ultraviolet rays can destroy microorganisms by damaging their biological molecules. This germicidal action is used in water purification, so (a) matches (ii). X-rays are sufficiently penetrating to pass through soft tissue, while denser structures absorb them more strongly. This gives contrast in medical radiography, so (b) matches (i). The diagnostic application in this list therefore belongs to X-rays rather than to the ultraviolet or infrared entry.

Step 2: Match microwave and infrared applications Microwaves can be transmitted as narrow beams and used to carry signals. Their reflection from objects is used to determine position or motion in radar. Consequently (c) matches (iii), communication and radar. Infrared has a longer wavelength than visible radiation and is scattered less strongly by the small particles responsible for poor visibility. In the application stated in the table, this makes it suitable for improving visibility in foggy conditions, giving (d) matched with (iv).

Step 3: Compare the complete sequence with the choices Collecting the four independent matches gives

$$(a) \to (ii), \qquad (b) \to (i), \qquad (c) \to (iii), \qquad (d) \to (iv).$$

Option (A) misassigns UV, X-rays and microwaves. Option (C) puts the diagnostic application against infrared and fog visibility against X-rays. Option (D) interchanges the UV and microwave applications. Only option (B) preserves all four assignments simultaneously.

Step 4: Conclusion UV rays match water purification, X-rays match medical diagnosis, microwaves match communication and radar, and infrared matches fog visibility. Hence, the correct option is (B).

Quick Tip to Solve: Anchor this matching with UV sterilisation and X-ray radiography; microwave radar and infrared fog imaging then fix the remaining two pairs.

46. The kinetic energy of emitted electron is E when the light incident on the metal has wavelength λ. To double the kinetic energy, the incident light must have wavelength: [4 marks]

(A) $\frac{hc}{E\lambda - hc}$
(B) $\frac{hc\lambda}{E\lambda + hc}$
(C) $\frac{h\lambda}{E\lambda + hc}$
(D) $\frac{hc\,\lambda}{E\lambda - hc}$

Correct Answer: (B) $\frac{hc\lambda}{E\lambda + hc}$

Solution:

Concept:

- For the same metal, the work function ϕ is fixed. Einstein's equation relates the emitted electron energy to photon energy by $K = hc/\lambda - \phi$.
- Doubling the electron kinetic energy increases photon energy by the original kinetic energy E, rather than simply doubling the photon energy; the work function is paid in both cases.

Step 1: Write the equation for the initial illumination For the original wavelength and kinetic energy,

$$E = \frac{hc}{\lambda} - \phi.$$

Thus $\phi = hc/\lambda - E$. Here h is Planck's constant and c is the speed of light. The work function is a property of the metal, so it remains the same when the wavelength is changed. It must not be set to zero or scaled with the desired electron energy.

Step 2: Eliminate the work function Let the required wavelength be λ'. The demanded kinetic energy is $2E$, so

$$2E = \frac{hc}{\lambda'} - \phi.$$

Subtract the initial equation from this equation to remove the unknown material constant:

$$E = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right).$$

Rearranging,

$$\frac{1}{\lambda'} = \frac{E}{hc} + \frac{1}{\lambda} = \frac{E\lambda + hc}{hc\lambda}.$$

Step 3: Invert and check the expression Taking the reciprocal yields

$$\lambda' = \frac{hc\lambda}{E\lambda + hc}.$$

The positive extra term in the denominator makes $\lambda' < \lambda$ for $E > 0$, as required for greater photon energy. Options (A) and (C) do not even have dimensions of length. Option (D) has a minus sign; it would reduce the photon energy, or even produce a negative wavelength, instead of increasing the kinetic energy. Only option (B) has the correct dimensions and physical trend.

Step 4: Conclusion The required wavelength is $\frac{hc\lambda}{E\lambda + hc}$. Hence, the correct option is (B).

Quick Tip to Solve: Write Einstein's equation twice and subtract: the unknown work function cancels immediately, preventing the incorrect shortcut of halving the wavelength.

47. Find the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its (i) second permitted energy level to the first level, and (ii) the highest permitted energy level to the first permitted level. [4 marks]

(A) 3 : 4

(B) 4 : 3

(C) 1 : 4

(D) 4 : 1

Correct Answer: (A) 3 : 4

Solution:

Concept:

- The permitted hydrogen energies in the Bohr model are $E_n = -13.6/n^2\,\text{eV}$. An emitted photon carries the positive difference between the initial and final atomic energies.
- The highest permitted level means the limiting level $n \to \infty$, whose energy approaches zero. Both transitions in this question terminate at the ground state $n = 1$.

Step 1: Evaluate the photon energy for the second level The second permitted level is $n = 2$; it is not the second excited state. The initial and final energies are

$$E_2 = -\frac{13.6}{4} = -3.4\,\text{eV}, \qquad E_1 = -13.6\,\text{eV}.$$

Consequently the emitted photon energy is

$$\varepsilon_{2\to1} = E_2 - E_1 = (-3.4) - (-13.6) = 10.2\,\text{eV}.$$

Equivalently it is $13.6(1 - 1/4)\,\text{eV}$.

Step 2: Evaluate the limiting transition energy At the top of the allowed bound-state sequence, n tends to infinity and $1/n^2$ tends to zero. Therefore

$$E_\infty = 0, \qquad \varepsilon_{\infty\to1} = E_\infty - E_1 = 13.6\,\text{eV}.$$

This is the largest photon energy for a transition ending in the first level. The atom loses energy in each emission, while the photon energy itself remains positive.

Step 3: Take the ratio in the stated order The question lists the finite transition first and the limiting transition second. Hence

$$\frac{\varepsilon_{2\to1}}{\varepsilon_{\infty\to1}} = \frac{10.2}{13.6} = 1 - \frac{1}{4} = \frac{3}{4}.$$

Option (B) reverses this ratio. Option (C) uses the magnitude of the second-level energy rather than the energy gap, and option (D) reverses that incorrect result. Since the first transition has the smaller gap, ratios greater than one also fail the physical check.

Step 4: Conclusion The energies of the photons are in the ratio $3 : 4$. Hence, the correct option is (A).

Quick Tip to Solve: For transitions to $n = 1$, divide by the series-limit energy first: the ratio for a starting level n is simply $1 - 1/n^2$.

48. A travelling microscope has 20 divisions per cm on the main scale while its vernier scale has total 50 divisions and 25 vernier scale divisions are equal to 24 main scale divisions, what is the least count of the travelling microscope? [4 marks]

(A) 0.001 cm

(B) 0.002 mm

(C) 0.002 cm

(D) 0.005 cm

Correct Answer: (C) 0.002 cm

Solution:

Concept:

- For a direct vernier, the least count is one main scale division minus one vernier scale division: $\mathrm{LC} = 1\,\mathrm{MSD} - 1\,\mathrm{VSD}$.
- The relation between the lengths of individual divisions determines least count. The total number of marks drawn on the vernier does not replace the stated calibration relation.

Step 1: Find the length of one main scale division There are 20 equal divisions along one centimetre of the main scale. Hence each interval represents

$$1\,\mathrm{MSD} = \frac{1}{20}\,\mathrm{cm} = 0.05\,\mathrm{cm}.$$

This is also $0.5\,\mathrm{mm}$. It is the spacing between adjacent main-scale marks, rather than the total centimetre, that must be compared with one vernier interval.

Step 2: Use the calibration of the vernier divisions The given correspondence is $25\,\mathrm{VSD} = 24\,\mathrm{MSD}$, so

$$1\,\mathrm{VSD} = \frac{24}{25}\,\mathrm{MSD}.$$

Therefore the difference between the two individual interval lengths is

$$\mathrm{LC} = \left(1 - \frac{24}{25}\right)\mathrm{MSD} = \frac{1}{25}\,\mathrm{MSD}.$$

The fact that the full vernier carries 50 divisions merely means that it contains two groups of these 25 divisions; it does not halve this difference.

Step 3: Substitute and compare the units Using the main-scale spacing,

$$\text{LC} = \frac{0.05}{25}\,\text{cm} = 0.002\,\text{cm} = 0.02\,\text{mm}.$$

Option (A) is obtained by dividing by 50 without using the actual vernier relation. Option (B), $0.002\,\text{mm}$, is ten times smaller than the result after the proper conversion. Option (D) is larger than the actual division difference. Option (C) is exactly the calibrated least count.

Step 4: Conclusion The least count is $0.002\,\text{cm}$, or $0.02\,\text{mm}$. Hence, the correct option is (C).

Quick Tip to Solve: Always derive one VSD from the printed relation before finding the difference; a vernier with 50 divisions need not have least count equal to MSD divided by 50.

49. In an experiment to find out the diameter of wire using screw gauge, the following observations were noted :

(A) Screw moves 0.5 mm on main scale in one complete rotation

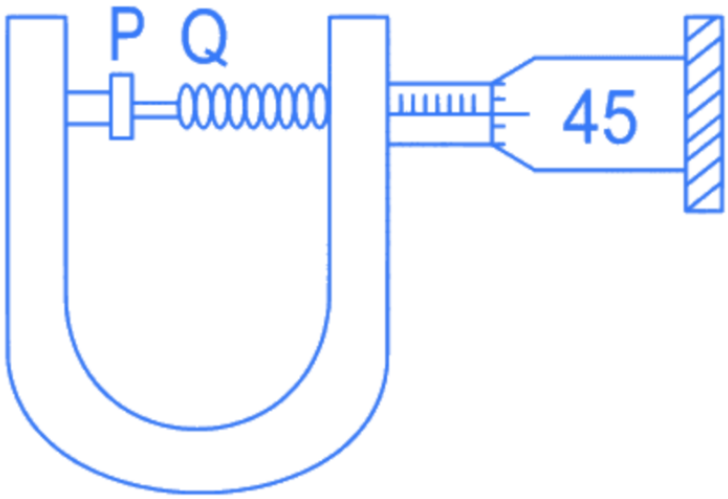
(B) Total divisions on circular scale $= 50$

(C) Main scale reading is 2.5 mm

(D) 45^{th} division of circular scale is in the pitch line

(E) Instrument has 0.03 mm negative error

Then the diameter of wire is : [4 marks]



(A) 2.92 mm

(B) 2.54 mm

(C) 2.98 mm

(D) 3.45 mm

Correct Answer: (C) 2.98 mm

Solution:

Concept:

- The pitch is the axial distance moved by the screw in one full rotation. Least count is pitch divided by the number of circular-scale divisions.
- The observed reading is main-scale reading plus circular-scale reading times least count. The corrected reading equals observed reading minus the signed zero error.

Step 1: Calculate the least count The screw advances $0.5\,\text{mm}$ in one rotation, and that rotation is divided into 50 circular divisions. Thus

$$\text{LC} = \frac{0.5\,\text{mm}}{50} = 0.01\,\text{mm}.$$

The pitch must not be added directly to the main-scale reading: only the fraction of a rotation indicated by the circular scale contributes to the measurement.

Step 2: Combine the main and circular readings The main-scale reading is $2.5\,\text{mm}$. Since the 45th circular division coincides with the pitch line, its contribution is

$$45 \times 0.01 = 0.45\,\text{mm}.$$

Hence the uncorrected diameter is

$$d_{\text{obs}} = 2.5 + 0.45 = 2.95\,\text{mm}.$$

This value is still subject to the instrument's zero error and is not yet the final result.

Step 3: Apply the signed zero correction The stated negative error has signed value $e = -0.03\,\text{mm}$. The correction is the opposite of the error, so

$$d = d_{\text{obs}} - e = 2.95 - (-0.03) = 2.98\,\text{mm}.$$

A negative zero error means the instrument under-reads, which independently confirms that the correction must increase the reading. Option (A), $2.92\,\text{mm}$, subtracts the magnitude with the wrong sign. Option (B) fails to use the 45 circular divisions correctly, while option (D) is incompatible with the pitch and circular contribution. Option (C) alone incorporates both scales and the signed correction.

Step 4: Conclusion The corrected diameter of the wire is $2.98\,\text{mm}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep zero error as a signed number and use corrected reading = observed reading minus error; a negative zero error therefore adds to the observation.

50. Find the modulation index of an AM wave having $8\ \mathrm{V}$ variation where maximum amplitude of the AM wave is $9\ \mathrm{V}$. [4 marks]

(A) 0.8

(B) 0.5

(C) 0.2

(D) 0.1

Correct Answer: (A) 0.8

Solution:

Concept:

- The envelope amplitude of an amplitude-modulated carrier varies between $A_{\max} = A_c + A_m$ and $A_{\min} = A_c - A_m$, where A_c is the carrier amplitude and A_m is the modulating amplitude.
- The modulation index is the dimensionless ratio $\mu = A_m/A_c = (A_{\max} - A_{\min})/(A_{\max} + A_{\min})$. The stated variation is the difference between the maximum and minimum envelope amplitudes.

Step 1: Find the minimum envelope amplitude The largest amplitude is $9\,\mathrm{V}$, and the total envelope variation is $8\,\mathrm{V}$. Therefore, the difference of the two extreme amplitudes, rather than the carrier amplitude itself, is given.

$$A_{\max} - A_{\min} = 8\,\mathrm{V}, \qquad A_{\min} = 9 - 8 = 1\,\mathrm{V}.$$

The positive minimum is consistent with an envelope that never reaches zero, so the wave is not overmodulated.

Step 2: Calculate the carrier and modulating amplitudes Add the two envelope limits to isolate the carrier, and subtract them to isolate the modulating signal:

$$A_c = \frac{9+1}{2} = 5\,\mathrm{V}, \qquad A_m = \frac{9-1}{2} = 4\,\mathrm{V}.$$

Hence $\mu = 4/5 = 0.8$. Equivalently, direct substitution gives $(9-1)/(9+1) = 8/10$. The values 0.5, 0.2 and 0.1 would give envelope variations $2A_c\mu$ of 5 V, 2 V and 1 V for this carrier, respectively, and so do not reproduce the stated 8 V variation.

Step 3: Conclusion The modulation index is $\boxed{0.8}$, corresponding to 80 percent modulation. Hence, the correct option is (A).

Quick Tip to Solve: When envelope variation and maximum amplitude are given, first obtain the minimum by subtraction; use the sum of the two limits in the denominator.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. An object is projected in the air with initial velocity u at an angle θ. The projectile motion is such that the horizontal range R, is maximum. Another object is projected in the air with a horizontal range half of the range of first object. The initial velocity remains same in both the case. The value of the angle of projection, at which the second object is projected, will be _____ degree. [4 marks]

Solution:

Concept:

- For projection and landing at the same height in uniform gravity, the horizontal range is $R = u^2 \sin(2\theta)/g$, with u the launch speed and g the gravitational acceleration.
- At a fixed speed the largest range is $R_{\max} = u^2/g$, because $\sin(2\theta) \leq 1$. Complementary acute launch angles have the same range.

Step 1: Express the required range relative to the maximum The first projectile is launched at the angle giving maximum range, so $2\theta = 90^\circ$ and $\theta = 45^\circ$. Its range is therefore u^2/g. The second projectile must cover half of this distance while retaining the same initial speed. If its angle is ϕ, then

$$\frac{u^2 \sin(2\phi)}{g} = \frac{1}{2}\frac{u^2}{g}.$$

Cancel the common positive factor u^2/g; neither the mass nor the numerical speed is required.

Step 2: Solve for both admissible launch angles The reduced condition is $\sin(2\phi) = 1/2$. For a forward projectile launched above the horizontal, $0 < \phi < 90^\circ$, so $0 < 2\phi < 180^\circ$. Sine is positive in both quadrants of this interval, giving

$$2\phi = 30^\circ \quad \text{or} \quad 150^\circ, \qquad \phi = 15^\circ \quad \text{or} \quad 75^\circ.$$

Substituting either angle yields $\sin 30^\circ = \sin 150^\circ = 1/2$. The lower trajectory has a shorter flight time and the higher trajectory has a smaller horizontal speed; both effects produce the same required horizontal displacement.

Step 3: Conclusion The possible projection angles are $\boxed{15^\circ \text{ or } 75^\circ}$. Both accepted numerical answers satisfy the required half-maximum range.

Quick Tip to Solve: For a given speed, divide the required range by the maximum range to get $\sin 2\phi$; remember the supplementary solution for 2ϕ.

52. If the acceleration due to gravity experienced by a point mass at a height h above the surface of earth is same as that of the acceleration due to gravity at a depth $\alpha h\,(h << R_e)$ from the earth surface. The value of α will be _____.

(use $R_e = 6400$ km) [4 marks]

Solution:

Concept:

- At height h above Earth, $g_h = g_0[R_e/(R_e + h)]^2$. Here g_0 is surface gravity and R_e is the radius of Earth.
- For a uniform-density Earth, the gravitational acceleration at depth d is $g_d = g_0(1 - d/R_e)$. For $h \ll R_e$, a first-order binomial expansion gives $g_h \simeq g_0(1 - 2h/R_e)$.

Step 1: Expand the altitude expression to first order Write the height dependence in a form suitable for the small-height condition printed in the question:

$$g_h = g_0\left(1 + \frac{h}{R_e}\right)^{-2} \simeq g_0\left(1 - \frac{2h}{R_e}\right).$$

Terms containing $(h/R_e)^2$ and higher powers are negligible. Thus moving up by h lowers the acceleration from its surface value by approximately $2g_0h/R_e$. The factor two arises from the inverse-square dependence on distance from the centre.

Step 2: Equate the acceleration at the specified depth The depth is $d = \alpha h$, so $g_d = g_0(1 - \alpha h/R_e)$. Equating the two accelerations and cancelling g_0 gives

$$1 - \frac{2h}{R_e} = 1 - \frac{\alpha h}{R_e}, \qquad \frac{2h}{R_e} = \frac{\alpha h}{R_e}.$$

For nonzero small h, cancellation of h/R_e yields $\alpha = 2$. The given radius 6400 km cancels, so no numerical unit conversion is necessary. This result is the leading-order value under the stated small-height approximation; gravity decreases twice as rapidly with a small altitude as with an equal small depth.

Step 3: Conclusion The required ratio is $\boxed{\alpha = 2}$, a dimensionless quantity. The matching depth is approximately $2h$.

Quick Tip to Solve: Near Earth's surface, use $\Delta g/g_0 \simeq 2h/R_e$ for altitude and d/R_e for depth, then equate the fractional decreases.

53. The pressure P_1 and density d_1 of diatomic gas $\left(\gamma = \frac{7}{5}\right)$ changes suddenly to $\mathrm{P}_2\,(>\mathrm{P}_1)$ and d_2 respectively during an adiabatic process. The temperature of the gas increases and becomes _____ times of its initial temperature. (given $\frac{\mathrm{d}_2}{\mathrm{d}_1} = 32$) [4 marks]

Solution:

Concept:

- For an ideal gas undergoing the adiabatic change described by $PV^{\gamma} =$ constant, a fixed mass has $V \propto 1/d$, where d denotes its mass density.
- Combining the adiabatic relation with $PV = nRT$ gives $TV^{\gamma-1} =$ constant, or $T \propto d^{\gamma-1}$. Temperature ratios must use the absolute scale.

Step 1: Eliminate volume using the density ratio Let the unchanged mass of gas be M. Since $d_1 = M/V_1$ and $d_2 = M/V_2$, the given compression implies

$$\frac{V_1}{V_2} = \frac{d_2}{d_1} = 32.$$

Using $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$, divide by $T_1 V_2^{\gamma-1}$ to obtain

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{d_2}{d_1}\right)^{\gamma-1}.$$

The final density exceeds the initial density, so the temperature ratio should exceed unity.

Step 2: Evaluate the exponent for the diatomic gas The stated heat-capacity ratio gives $\gamma - 1 = 7/5 - 1 = 2/5$. Therefore,

$$\frac{T_2}{T_1} = 32^{2/5} = (2^5)^{2/5} = 2^2 = 4.$$

As a check, the corresponding pressure ratio is $P_2/P_1 = 32^{7/5} = 128$. The ideal-gas identity $T \propto P/d$ then gives $T_2/T_1 = 128/32 = 4$, consistent with the temperature-density calculation and with the specified increase in pressure. It is the exponent $\gamma - 1$, rather than γ, that controls the temperature ratio.

Step 3: Conclusion The final absolute temperature is $\boxed{4}$ times the initial temperature: $T_2 = 4T_1$. The requested numerical factor has no unit.

Quick Tip to Solve: For adiabatic compression with density data, use $T_2/T_1 = (d_2/d_1)^{\gamma-1}$; the pressure exponent γ would give a different ratio.

54. One mole of a monoatomic gas is mixed with three moles of a diatomic gas. The molecular specific heat of mixture at constant volume is $\frac{\alpha^2}{4}\mathrm{R\,J/mol\,K}$; then the value of α will be _____. (Assume that the given diatomic gas has no vibrational mode). [4 marks]

Solution:

Concept:

- A monatomic ideal gas has three translational degrees of freedom, giving molar heat capacity $C_{V,m} = 3R/2$. A diatomic gas without active vibration has five degrees of freedom, giving $C_{V,d} = 5R/2$.
- The total constant-volume heat capacity is additive: $C_{V,\text{total}} = \sum n_i C_{V,i}$. Divide by the total number of moles to obtain the molar heat capacity of the mixture.

Step 1: Add the heat capacities of the two components For a common temperature increase ΔT, each gas gains internal energy $\Delta U_i = n_i C_{V,i}\Delta T$. Thus one mole of monatomic gas and three moles of diatomic gas give

$$C_{V,\text{total}} = 1\left(\frac{3}{2}R\right) + 3\left(\frac{5}{2}R\right) = \frac{3+15}{2}R = 9R.$$

This is the heat capacity of all four moles, rather than the molar heat capacity sought. The diatomic vibrational contribution is excluded exactly as specified, so no additional degrees of freedom enter.

Step 2: Form the molar average and determine alpha The mixture contains $n = 1 + 3 = 4$ moles. Its constant-volume molar heat capacity is therefore

$$C_{V,\text{mix}} = \frac{9R}{4}.$$

Compare this with the given form $\alpha^2 R/4$:

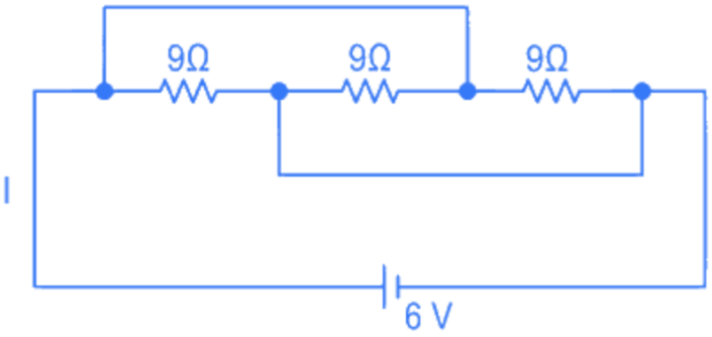
$$\frac{\alpha^2 R}{4} = \frac{9R}{4} \quad \Rightarrow \quad \alpha^2 = 9.$$

The required positive numerical parameter is $\alpha = 3$. The average $9R/4 = 2.25R$ lies between $1.5R$ and $2.5R$, and is closer to the diatomic value because that component supplies three quarters of the moles.

Step 3: Conclusion The molar heat capacity is $9R/4$, so the requested dimensionless value is $\boxed{\alpha = 3}$.

Quick Tip to Solve: Weight each component's molar heat capacity by its mole number, add, and divide by total moles; do not simply add the two molar values.

55. The current I flowing through the given circuit will be _____A. [4 marks]



Solution:

Concept:

- Points joined by ideal connecting wires have the same potential and represent a single electrical node. The appearance of resistors in a straight row does not establish a series connection.
- Resistors connected between the same two nodes are in parallel: $1/R_{\text{eq}} = \sum_i 1/R_i$. The battery current is then determined by Ohm's law, $I = V/R_{\text{eq}}$.

Step 1: Identify the two nodes in the drawing Label the four junctions along the upper resistor row, from left to right, as A, B, C, D. The upper wire connects A directly to C, so $V_A = V_C$. The lower connecting wire joins B directly to D, so $V_B = V_D$. The three $9\,\Omega$ resistors lie between A and B, between B and C, and between C and D. On replacing wire-connected points by their common node, all three resistors connect the same pair of nodes. None has both ends on a single wire-connected node.

Step 2: Reduce the parallel combination and find the current The three equal resistors are therefore in parallel across the $6\,\text{V}$ battery:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3}\,\Omega^{-1}, \qquad R_{\text{eq}} = 3\,\Omega.$$

The current marked I is in the external supply lead, before the current splits among the resistor branches. Hence

$$I = \frac{6}{3} = 2\,\text{A}.$$

Each resistor carries a current of magnitude $6/9 = 2/3\,\text{A}$; their sum is $2\,\text{A}$, checking the supply current by Kirchhoff's junction rule.

Step 3: Conclusion The current in the indicated supply lead is $\boxed{I = 2\,\text{A}}$.

Quick Tip to Solve: Mark equal-potential nodes before reducing a circuit with crossing or bypass wires; here the three drawn consecutive resistors actually share both end nodes.

56. A closely wounded circular coil of radius 5 cm produces a magnetic field of 37.68×10^{-4} T at its center. The current through the coil is _____A.

[Given, number of turns in the coil is 100 and $\pi = 3.14$] [4 marks]

Solution:

Concept:

- The field at the centre of one circular current loop of radius r is $\mu_0 I/(2r)$. For N closely wound identical turns, their fields add to give $B = \mu_0 NI/(2r)$.
- Use SI units: r in metres, B in tesla, current in amperes, and $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$. Close winding permits all turns to be treated as having the same radius.

Step 1: Convert the data and isolate the current The radius is $5\,\mathrm{cm} = 0.05\,\mathrm{m}$, the number of turns is $N = 100$, and the centre field is $37.68 \times 10^{-4}\,\mathrm{T}$. Rearranging the circular-coil field formula gives

$$I = \frac{2rB}{\mu_0 N}.$$

The factor two belongs in the numerator after rearrangement. This expression applies at the centre of a circular coil; the long-solenoid formula would involve turns per unit length and is not appropriate to this geometry.

Step 2: Substitute and simplify the powers of ten Using the supplied value $\pi = 3.14$,

$$I = \frac{2(0.05)(37.68 \times 10^{-4})}{(4 \times 3.14 \times 10^{-7})(100)} = \frac{3.768 \times 10^{-4}}{1.256 \times 10^{-4}} = 3\,\mathrm{A}.$$

Both numerator and denominator contain the same power of ten, which cancels. The units also reduce correctly: $\mathrm{m\,T/(T\,m\,A^{-1})} = \mathrm{A}$. Substituting 3 A back gives $B = (4 \times 3.14 \times 10^{-7})(100)(3)/(0.10) = 3.768 \times 10^{-3}\,\mathrm{T}$, equal to the printed field.

Step 3: Conclusion The current through the circular coil is $\boxed{3\,\mathrm{A}}$.

Quick Tip to Solve: For a closely wound circular coil, multiply the single-turn centre field by the number of turns, and convert the radius from centimetres before substitution.

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57. Two light beams of intensities 4I and 9I interfere on a screen. The phase difference between these beams on the screen at point A is zero and at point B is π. The difference of resultant intensities, at the point A and B, will be _____ I. [4 marks]

Solution:

Concept:

- For two coherent beams, resultant intensity is $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, where ϕ is their phase difference. Intensities add with an interference term because electric-field amplitudes, rather than intensities, superpose.
- The electric-field amplitude is proportional to the square root of intensity. Hence the incident intensities $4I$ and $9I$ correspond to amplitudes in the ratio $2 : 3$. The common factor I is an

intensity, not an amplitude.

Step 1: Calculate the intensity at A At A the phase difference is zero, so the interference term is positive and the amplitudes add. Substituting the two incident intensities gives

$$I_A = 4I + 9I + 2\sqrt{(4I)(9I)}\cos 0 = 13I + 12I = 25I.$$

This is the maximum intensity, equivalently $(\sqrt{4I} + \sqrt{9I})^2 = 25I$.

Step 2: Calculate the intensity at B and subtract At B the phase difference is π, so $\cos\pi = -1$. The amplitudes oppose each other, giving

$$I_B = 4I + 9I - 2\sqrt{(4I)(9I)} = 13I - 12I = I.$$

The minimum is not zero because the two amplitudes are unequal. Therefore

$$I_A - I_B = 25I - I = 24I.$$

Both values are positive; reversing the sign of the interference term exchanges maximum and minimum, rather than making intensity negative.

Step 3: Conclusion The difference of resultant intensities is $\boxed{24I}$. Thus the number to enter in the blank is $\boxed{24}$.

Quick Tip to Solve: Use $I_{\max} - I_{\min} = 4\sqrt{I_1 I_2}$. Unequal incident intensities give a nonzero minimum, so do not set the dark-fringe intensity to zero.

58. A wire of length 314 cm carrying current of 14 A is bent to form a circle. The magnetic moment of the coil is _____ A $-\mathrm{m}^2$. [Given $\pi = 3.14$] [4 marks]

Solution:

Concept:

- The magnetic dipole moment magnitude of a planar current loop is $m = NIA$, where N is the number of turns, I the current and A the area enclosed by each turn. Here the wire is bent into one circular turn, so $N = 1$.
- The wire length is the circumference of the circle: $L = 2\pi r$. The enclosed area is $A = \pi r^2 = L^2/(4\pi)$. Length must be expressed in metres to obtain magnetic moment in A m^2.

Step 1: Find the radius and area Convert the complete wire length before using it in the circular geometry:

$$L = 314\,\mathrm{cm} = 3.14\,\mathrm{m}.$$

Using the given value $\pi = 3.14$,

$$r = \frac{3.14}{2(3.14)} = 0.50\,\mathrm{m}, \qquad A = 3.14(0.50)^2 = 0.785\,\mathrm{m}^2.$$

The quoted length surrounds the circle; treating it as a diameter would give an incorrect area.

Step 2: Calculate the magnetic moment The current is $14\,\mathrm{A}$, and there is one turn. Thus

$$m = IA = 14(0.785) = 10.99\,\mathrm{A\,m^2}.$$

This is the magnitude requested. Its direction would be perpendicular to the circular plane according to the right-hand rule, but no orientation is specified or needed for this numerical blank. To the nearest integer the magnitude is $11\,\mathrm{A\,m^2}$. The unit follows directly from current multiplied by area.

Step 3: Conclusion The magnetic moment is $10.99\,\mathrm{A\,m^2}$, or $\boxed{11\,\mathrm{A\,m^2}}$ to the nearest integer. The numerical entry is $\boxed{11}$.

Quick Tip to Solve: For a single circular loop made from a wire of length L, use $m = IL^2/(4\pi)$ after converting the length to metres.

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59. The X-Y plane be taken as the boundary between two transparent media $\mathrm{M_1}$ and $\mathrm{M_2}$. $\mathrm{M_1}$ in $Z \geqslant 0$ has a refractive index of $\sqrt{2}$ and M_2 with $Z < 0$ has a refractive index of $\sqrt{3}$. A ray of light travelling in $\mathrm{M_1}$ along the direction given by the vector $\overrightarrow{\mathrm{P}} = 4\sqrt{3}\hat{i} - 3\sqrt{3}\hat{j} - 5\hat{k}$, is incident on the plane of separation. The value of difference between the angle of incident in $\mathrm{M_1}$ and the angle of refraction in $\mathrm{M_2}$ will be _____ degree. [4 marks]

Solution:

Concept:

- The normal to the xy-plane is along the z-axis. The angle of incidence is measured from the normal, not from the interface. The negative z-component of the incident vector correctly points towards the boundary from $z \geq 0$.
- Snell's law is $n_1 \sin i = n_2 \sin r$, where i and r are the acute incident and refracted angles to the normal. Passing into a medium of larger refractive index bends the ray towards the normal, so $r < i$.

Step 1: Extract the angle of incidence from the vector For the given direction vector, the magnitude is

$$|\vec{P}| = \sqrt{(4\sqrt{3})^2 + (-3\sqrt{3})^2 + (-5)^2} = \sqrt{48 + 27 + 25} = 10.$$

The magnitude of its normal component is 5. Hence

$$\cos i = \frac{|P_z|}{|\vec{P}|} = \frac{5}{10} = \frac{1}{2}, \qquad i = 60^\circ.$$

The signs of the in-plane components determine the azimuth of the ray; they do not change this angle to the normal.

Step 2: Apply Snell's law and compare the angles Here $n_1 = \sqrt{2}$, $n_2 = \sqrt{3}$, and $\sin 60^\circ = \sqrt{3}/2$. Therefore

$$\sin r = \frac{\sqrt{2}}{\sqrt{3}}\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}}, \qquad r = 45^\circ.$$

The physical transmitted ray uses the acute angle 45°. Its smaller angle is consistent with entry into

the optically denser medium. The difference is

$$i - r = 60^\circ - 45^\circ = 15^\circ.$$

There is no total internal reflection when light passes from the lower to the higher refractive index.

Step 3: Conclusion The difference between incidence and refraction angles is $\boxed{15^\circ}$. Thus the numerical entry in degrees is $\boxed{15}$.

Quick Tip to Solve: For an interface specified by a coordinate plane, first select its perpendicular component: at an xy-boundary, use $|P_z|/|\vec{P}|$ to find $\cos i$.

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60. If the potential barrier across a p-n junction is 0.6 V. Then the electric field intensity, in the depletion region having the width of 6×10^{-6} m,
will be _____ $\times 10^5$ N/C. [4 marks]

Solution:

Concept:

- Electric field is the negative spatial rate of change of potential: $E_x = -dV/dx$. For the field intensity inferred from a given barrier height ΔV and depletion width d, the magnitude is $E = \Delta V/d$, the average field across the region.
- Potential difference is measured in volts and width in metres, so $\mathrm{V\,m^{-1}}$ is equivalent to $\mathrm{N\,C^{-1}}$. The requested blank is the coefficient of $10^5\,\mathrm{N\,C^{-1}}$, rather than the full field magnitude.

Step 1: Insert the barrier height and depletion width The potential barrier is $\Delta V = 0.6\,\mathrm{V}$, and the width of the depletion region is $d = 6 \times 10^{-6}\,\mathrm{m}$. Consequently,

$$E = \frac{0.6}{6 \times 10^{-6}}\,\mathrm{V\,m^{-1}}.$$

The potential barrier is an energy per unit charge, so dividing by distance gives force per unit charge. There is no need to multiply by the electronic charge when finding field intensity.

Step 2: Simplify and express the requested coefficient Separate the numerical ratio from the power of ten:

$$E = \left(\frac{0.6}{6}\right) 10^6 = 0.1 \times 10^6 = 1.0 \times 10^5\,\mathrm{N\,C^{-1}}.$$

Comparison with the specified form, blank $\times 10^5$ N/C, gives a coefficient of 1. Only the magnitude is asked; a signed component would additionally require a chosen coordinate direction. A detailed junction profile could have a varying local field, but the given barrier and width determine this average field directly.

Step 3: Conclusion The field intensity inferred across the depletion region is $\boxed{1 \times 10^5\,\mathrm{N\,C^{-1}}}$. Hence the blank is $\boxed{1}$.

Quick Tip to Solve: Compute $\Delta V/d$, then rewrite it using the power of ten already printed after the blank; enter only the remaining coefficient.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Which of the following pair of molecules contain odd electron molecule and an expanded octet molecule? [4 marks]

(A) BCl_3 and SF_6

(B) NO and H_2SO_4

(C) SF_6 and H_2SO_4

(D) BCl_3 and NO

Correct Answer: (B) NO and H_2SO_4

Solution:

Concept:

- An odd-electron molecule has an odd total number of valence electrons, preventing all electrons from being paired in a conventional Lewis structure. Nitric oxide, NO, has $5 + 6 = 11$ valence electrons and is a standard example.
- An expanded-octet molecule is represented in the usual Lewis treatment with more than eight electrons around a central atom from the third or a later period. Sulfur in SF_6 and in the conventional Lewis structure of H_2SO_4 is an expanded-octet example.

Step 1: Identify the odd-electron species Count the total valence electrons in nitric oxide:

$$N_{\text{valence}}(\text{NO}) = 5 + 6 = 11.$$

Thus NO supplies the odd-electron member required by the question. In contrast, BCl_3 is an electron-deficient molecule: three B–Cl bonds leave only six electrons around boron. Electron deficiency is a different exception from an odd total electron count. Indeed, its total valence electron count is $3 + 3(7) = 24$, which is even.

Step 2: Find the pair with the required second member In the conventional Lewis representation $HO{-}S(=O)_2{-}OH$, sulfur has two single and two double bonds, corresponding to twelve shared electrons around it. Thus H_2SO_4 supplies the expanded-octet member, and the pair is NO, H_2SO_4. Option A combines an incomplete-octet species with an expanded-octet species and lacks an odd-electron molecule. Option C contains two expanded-octet examples and lacks an odd-electron molecule. Option D combines incomplete-octet and odd-electron examples but lacks the required expanded-octet molecule.

Step 3: Conclusion The required pair is $\boxed{\text{NO and } H_2SO_4}$. Hence, the correct option is (B).

Quick Tip to Solve: Separate the three octet exceptions: BCl_3 is incomplete-octet, NO is odd-electron, and sulfur compounds such as SF_6 can have expanded octets.

62.

$$\underset{20\,\mathrm{g}}{\mathrm{N_2(g)}} + \underset{5\,\mathrm{g}}{3\,\mathrm{H_2(g)}} \rightleftharpoons 2\,\mathrm{NH_3(g)}$$

Consider the above reaction, the limiting reagent of the reaction and number of moles of $\mathrm{NH_3}$ formed respectively are : [4 marks]

(A) $\mathrm{H_2}, 1.42$ moles

(B) $\mathrm{H_2}, 0.71$ moles

(C) $\mathrm{N_2}, 1.42$ moles

(D) $\mathrm{N_2}, 0.71$ moles

Correct Answer: (C) $\mathrm{N_2}, 1.42$ moles

Solution:

Concept:

- For $\mathrm{N_2} + 3\,\mathrm{H_2} \longrightarrow 2\,\mathrm{NH_3}$, one mole of nitrogen reacts with three moles of hydrogen to form two moles of ammonia. Stoichiometric comparisons use moles divided by their coefficients, rather than the reactant masses alone.
- The limiting reagent has the smaller value of n/ν, where n is its initial amount and ν its coefficient. The ammonia amount calculated here is the stoichiometric amount obtained upon consumption of the limiting reagent, as intended by the options.

Step 1: Convert the supplied masses into amounts Using molar masses $28\,\mathrm{g\,mol^{-1}}$ for nitrogen and $2\,\mathrm{g\,mol^{-1}}$ for hydrogen,

$$n(\mathrm{N_2}) = \frac{20}{28} = \frac{5}{7}\,\mathrm{mol}, \qquad n(\mathrm{H_2}) = \frac{5}{2} = 2.5\,\mathrm{mol}.$$

Compare the amounts per reaction coefficient:

$$\frac{n(\mathrm{N_2})}{1} = \frac{5}{7} \simeq 0.714, \qquad \frac{n(\mathrm{H_2})}{3} = \frac{5}{6} \simeq 0.833.$$

The smaller value belongs to nitrogen, so hydrogen is present in excess.

Step 2: Calculate the ammonia yield and test the options Two moles of ammonia form for each mole of nitrogen consumed. Thus

$$n(\mathrm{NH_3}) = 2\left(\frac{5}{7}\right) = \frac{10}{7}\,\mathrm{mol} \simeq 1.43\,\mathrm{mol}.$$

The printed option uses 1.42 moles, obtained by taking $n(\mathrm{N_2}) \simeq 0.71$ before multiplying by two. This identifies option C. Options A and B name hydrogen as the limiting reagent, contradicting the mole-ratio test. Option D uses approximately the nitrogen amount itself as the ammonia amount and misses the product coefficient of two.

Step 3: Conclusion $\mathrm{N_2}$ is the limiting reagent and the stoichiometric ammonia yield is $10/7\,\mathrm{mol}$, approximately $1.43\,\mathrm{mol}$. The listed value 1.42 moles reflects early rounding. Hence, the correct option is (C).

Quick Tip to Solve: Divide each reactant amount by its balanced-equation coefficient first. Then multiply the smallest result by the coefficient of the desired product.

63. The first ionization enthalpy of Na, Mg and Si, respectively, are : 496, 737 and $786\ \text{kJ mol}^{-1}$. The first ionization enthalpy (kJ mol^{-1}) of Al is : [4 marks]

(A) 487

(B) 768

(C) 577

(D) 856

Correct Answer: (C) 577

Solution:

Concept:

- First ionization enthalpy is the energy needed to remove one electron from an isolated gaseous atom. Across a period it generally increases as effective nuclear charge increases, but subshell energies and filled-subshell stability cause specific exceptions.
- The outer configurations are $\text{Mg} : 3s^2$, $\text{Al} : 3s^2 3p^1$, and $\text{Si} : 3s^2 3p^2$. Removing the first electron from aluminium means removing its higher-energy, less penetrating $3p$ electron, whereas magnesium loses a $3s$ electron from a filled subshell.

Step 1: Place aluminium relative to its neighbours The transition from Mg to Al introduces the first $3p$ electron. This electron is easier to remove than a $3s$ electron from magnesium, so the usual across-period rise has a dip at aluminium. Thus its first ionization enthalpy must be below the given magnesium value $737\,\text{kJ mol}^{-1}$, rather than being obtained by interpolating monotonically between magnesium and silicon. Aluminium still has a greater first ionization enthalpy than sodium, whose single outer $3s$ electron is readily removed.

Step 2: Select the value consistent with the subshell dip The required value should lie above the given sodium value $496\,\text{kJ mol}^{-1}$ and below the magnesium value $737\,\text{kJ mol}^{-1}$. Of the options, $577\,\text{kJ mol}^{-1}$ satisfies this relationship and is the standard first ionization enthalpy of aluminium. The value 487 would put aluminium below sodium. The values 768 and 856 would both put it above magnesium and miss the $3s$-to-$3p$ exception; 856 would even exceed the supplied silicon value of 786. These alternatives are inconsistent with the relevant electronic configurations.

Step 3: Conclusion The first ionization enthalpy of aluminium is $\boxed{577\,\text{kJ mol}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Remember the period-3 Mg-to-Al dip: a $3p$ electron in Al is removed more readily than an electron from Mg's filled $3s^2$ subshell.

64. The reaction of zinc with excess of aqueous alkali, evolves hydrogen gas and gives : [4 marks]

(A) $Zn(OH)_2$

(B) ZnO

(C) $[Zn(OH)_4]^{2-}$

(D) $[ZnO_2]^{2-}$

Correct Answer: (C) $[Zn(OH)_4]^{2-}$

Solution:

Concept:

- Aqueous hydroxide reacts with amphoteric zinc to produce a soluble hydroxo complex. The word aqueous specifies the chemical environment of the product.
- Zinc is oxidized from oxidation state zero to $+2$, while water supplies the hydrogen that is reduced to hydrogen gas. Hydroxide stabilizes the resulting zinc(II) ion by complex formation.

Step 1: Identify the role of excess alkali Zinc dissolves in excess aqueous alkali because zinc(II) can bind hydroxide ligands. A precipitate of $Zn(OH)_2$, if formed initially, dissolves in excess hydroxide rather than remaining as the final product. The soluble species contains four hydroxide ligands around one zinc ion.

Step 2: Write the balanced aqueous reaction The net ionic equation is

$$Zn + 2\,OH^- + 2\,H_2O \longrightarrow [Zn(OH)_4]^{2-} + H_2.$$

Atoms and charge are balanced: both sides contain one zinc atom, four oxygen atoms, six hydrogen atoms and a total charge of -2. Zinc supplies two electrons, giving one molecule of hydrogen.

Step 3: Compare the proposed products Option (A) is the hydroxide before dissolution in excess base, and option (B) is zinc oxide, not the soluble complex formed here. Option (D), the oxide representation of zincate, is associated with a different formulation; in aqueous excess hydroxide the appropriate species is tetrahydroxozincate(II), as in option (C).

Step 4: Conclusion The product in excess aqueous alkali is $[Zn(OH)_4]^{2-}$, with evolution of H_2. Hence, the correct option is (C).

Quick Tip to Solve: For an amphoteric metal in excess aqueous base, look for a soluble hydroxo complex rather than the initially formed hydroxide precipitate.

65. Number of lone pairs of electrons in the central atom of SCl_2, O_3, ClF_3 and SF_6, respectively, are : [4 marks]

(A) 0, 1, 2 and 2

(B) 2, 1, 2 and 0

(C) 1, 2, 2 and 0

(D) 2, 1, 0 and 2

Correct Answer: (B) 2, 1, 2 and 0

Solution:

Concept:

- A lone pair is a nonbonding pair on the specified central atom; lone pairs on terminal atoms do not enter this count.
- Count bonding and nonbonding electrons from a valid Lewis structure. A multiple bond changes bond order but remains one electron-domain direction in VSEPR reasoning.

Step 1: Count the pairs on sulphur and central oxygen Sulphur in SCl_2 has six valence electrons. Two single bonds use one electron from sulphur apiece, leaving four nonbonding electrons, or two lone pairs. In a resonance structure of ozone, the central oxygen has one single bond and one double bond. It carries one lone pair and a positive formal charge; the terminal singly bonded oxygen carries the compensating negative charge. Resonance interchanges the terminal bonds without changing the central lone-pair count.

Step 2: Count the pairs in the fluorides Chlorine in ClF_3 has seven valence electrons. Three single bonds to fluorine leave four nonbonding electrons on chlorine, giving two lone pairs. These occupy equatorial positions in the five-domain arrangement. In SF_6, sulphur makes six sulphur–fluorine bonds and has no central lone pair; the six bond directions form an octahedron.

Step 3: Match the ordered sequence Keeping the compounds in the question's order gives

$$SCl_2 : 2, \qquad O_3 : 1, \qquad ClF_3 : 2, \qquad SF_6 : 0.$$

Option (A) incorrectly removes both sulphur lone pairs in the first compound; option (C) interchanges the first two counts. Option (D) incorrectly assigns no lone pairs to chlorine trifluoride and two to sulphur hexafluoride.

Step 4: Conclusion The required sequence is $2, 1, 2, 0$. Hence, the correct option is (B).

Quick Tip to Solve: Draw or count the central atom's electrons first; do not count the many lone pairs on terminal halogen atoms.

66. In following pairs, the one in which both transition metal ions are colourless is : [4 marks]

(A) Sc^{3+}, Zn^{2+}

(B) Ti^{4+}, Cu^{2+}

(C) V^{2+}, Ti^{3+}

(D) Zn^{2+}, Mn^{2+}

Correct Answer: (A) Sc^{3+}, Zn^{2+}

Solution:

Concept:

- The usual colour of an aqueous first-row metal ion arises from absorption associated with transitions between split d orbitals.
- An empty d^0 shell or a completely filled d^{10} shell cannot undergo ordinary d–d transitions. For these simple ions this gives colourless solutions; a partially filled d shell can give colour.

Step 1: Find the configurations of the candidate colourless ions On ion formation the outer 4s electrons are removed before 3d electrons. Scandium has atomic number 21, so Sc^{3+} is $[Ar]3d^0$. Zinc has atomic number 30, so Zn^{2+} is $[Ar]3d^{10}$. Both therefore lack an available ordinary d–d excitation and are colourless in their usual aqueous forms.

Step 2: Check every remaining pair Titanium(IV) is d^0 and colourless, but copper(II) is d^9 and coloured, so (B) does not qualify. Vanadium(II) is d^3, and titanium(III) is d^1; both have partially filled d shells and coloured ions, rejecting (C). In (D), zinc(II) is colourless but manganese(II) is d^5 and pale pink. Its weak, spin-forbidden absorption does not make it truly colourless.

Step 3: Apply the condition to both members The question requires that both members of one pair be colourless. Finding just one colourless ion is insufficient. Only the scandium(III)–zinc(II) pair has the empty-or-full d-shell condition for each member; the other three pairs each contain at least one coloured ion.

Step 4: Conclusion Both Sc^{3+} and Zn^{2+} are colourless. Hence, the correct option is (A).

Quick Tip to Solve: Write the ionic d count, removing 4s electrons first. In a pair question, check both ions; pale pink manganese(II) is still coloured.

67. In neutral or faintly alkaline medium, $KMnO_4$ being a powerful oxidant can oxidize, thiosulphate almost quantitatively, to sulphate. In this reaction overall change in oxidation state of manganese will be : [4 marks]

(A) 5

(B) 1

(C) 0

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- Permanganate contains manganese in oxidation state $+7$. Its reduction product depends on the medium, so identify the medium before calculating the change.
- In neutral or faintly alkaline solution, permanganate is normally reduced to manganese dioxide, in which manganese has oxidation state $+4$. The magnitude of oxidation-state change equals electrons gained per manganese atom.

Step 1: Calculate the initial oxidation state Let the oxidation state of manganese in MnO_4^- be x. Oxygen is -2, so

$$x + 4(-2) = -1 \quad \Rightarrow \quad x = +7.$$

Potassium is a spectator ion and does not affect the redox change of manganese.

Step 2: Use the product appropriate to this medium In neutral or faintly alkaline medium, the manganese-containing reduction product is MnO_2. Oxygen contributes -4, so manganese is $+4$. The reduction half-reaction can be written as

$$MnO_4^- + 2\,H_2O + 3\,e^- \longrightarrow MnO_2 + 4\,OH^-.$$

It balances atoms and charge and explicitly shows the gain of three electrons. Thiosulphate provides the reducing equivalents as it is oxidized to sulphate.

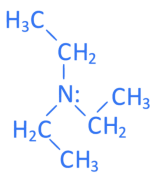
Step 3: Calculate the change and eliminate other choices The signed change is $+4 - (+7) = -3$, so the magnitude asked for in the options is 3. A change of 5 would describe reduction to Mn^{2+} in acid, not this medium. A change of 1 would describe manganate formation under strongly alkaline conditions; zero would imply no manganese reduction and is incompatible with its oxidizing action.

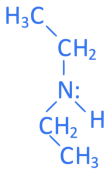
Step 4: Conclusion Manganese changes from $+7$ to $+4$; its oxidation state decreases by 3. Hence, the correct option is (D).

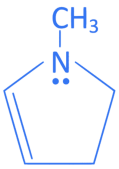
Quick Tip to Solve: Associate permanganate products with the medium: acid gives $+2$, neutral or faintly alkaline gives $+4$, and strongly alkaline reduction can give $+6$.

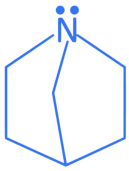
68. Which among the following is the strongest Bronsted base? [4 marks]

(A) $(C_2H_5)_3N$ (triethylamine)

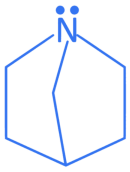


(B) $(C_2H_5)_2NH$ (diethylamine) 

(C) The depicted N-methyl five-membered unsaturated cyclic amine 

(D) The depicted bridgehead bicyclic tertiary amine 

Correct Answer: (D)

The depicted bridgehead bicyclic tertiary amine 

Solution:

Concept:

- A Bronsted base accepts a proton by donating its nitrogen lone pair. Basicity depends on how available that lone pair is and how readily the conjugate acid forms and is stabilized.
- Electron donation by alkyl groups increases basicity, whereas conjugation of the lone pair with a neighbouring double bond reduces its availability. Steric accessibility and the geometry of a rigid amine also matter.

Step 1: Identify the four nitrogen environments Option (A) is triethylamine, with three ethyl groups around nitrogen; option (B) is diethylamine, with two ethyl groups and one hydrogen. Option (C) contains a nitrogen adjacent to a carbon–carbon double bond in a five-membered ring, so its lone pair can interact with the unsaturated system. Option (D) is a rigid saturated bicyclic amine with nitrogen at a bridgehead and its lone pair directed outside the cage.

Step 2: Compare lone-pair availability and protonation In (D), the sigma-bonded alkyl framework supplies electron density to nitrogen, while the outward-directed lone pair is accessible to an incoming proton. The rigid cage keeps the carbon framework away from that approach direction. In (A), the three freely oriented ethyl groups crowd the nitrogen region more. Although (B) is a strong aliphatic base, the exposed bridgehead lone pair in (D) gives the strongest proton-accepting centre among the depicted structures.

Step 3: Exclude the unsaturated competitor The nitrogen lone pair of (C) is partly delocalized toward the adjacent double bond. Protonation at nitrogen removes that lone pair from conjugation, making proton capture less favourable than for the saturated bridgehead amine. Thus the ranking cannot be decided merely by counting carbon substituents on nitrogen: availability, conjugation and geometry

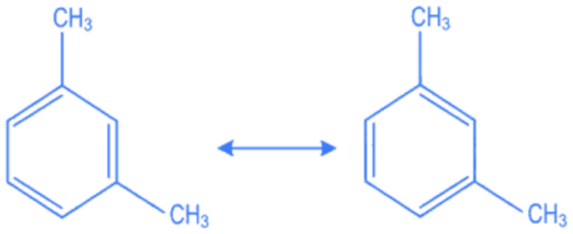
must be compared together. Neither the crowded triethylamine nor diethylamine exceeds (D) here.

Step 4: Conclusion The bridgehead bicyclic amine shown in (D) is the strongest Bronsted base. Hence, the correct option is (D).

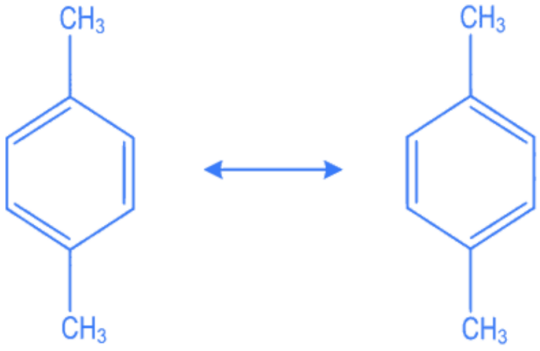
Quick Tip to Solve: For cyclic amines, inspect where the lone pair points and whether it is conjugated; alkyl-group counting alone can miss a strongly basic bridgehead nitrogen.

69. Which among the following pairs of the structures will give different products on ozonolysis? (Consider the double bonds in the structures are rigid and not delocalized.) [4 marks]

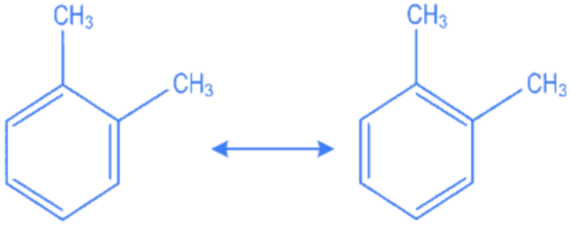
(A) The two depicted fixed double-bond arrangements of meta-xylene



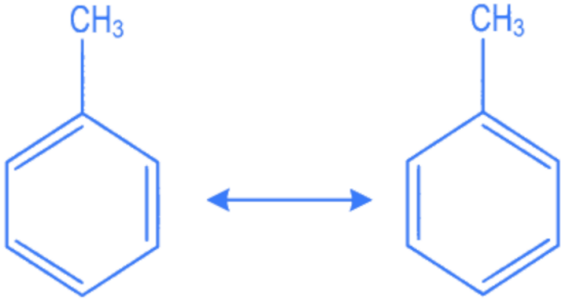
(B) The two depicted fixed double-bond arrangements of para-xylene



(C) The two depicted fixed double-bond arrangements of ortho-xylene

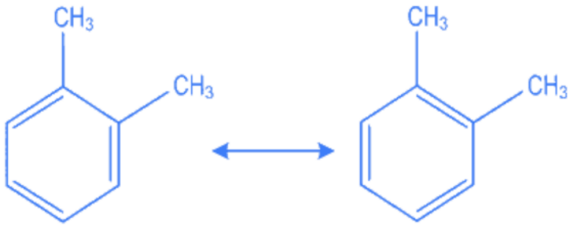


(D) The two depicted fixed double-bond arrangements of toluene



Correct Answer: (C)

The two depicted fixed double-bond arrangements of ortho-xylene



Solution:

Concept:

- Ozonolysis cleaves each carbon–carbon double bond, converting its two carbons into carbonyl centres. The question explicitly treats the depicted double bonds as fixed, so compare the remaining single-bonded carbon pairs.
- In a six-membered ring with three alternating double bonds, complete cleavage yields three two-carbon dicarbonyl fragments. A fragment with zero, one or two methyl-substituted ends gives glyoxal, methylglyoxal or diacetyl, respectively.

Step 1: Compare the meta and para pairs For meta-xylene (A), the two methyl-bearing ring carbons are not adjacent and therefore cannot form one surviving carbon pair. Either fixed arrangement yields two molecules of methylglyoxal, CH_3COCHO, and one of glyoxal, $OHCCHO$. The same argument applies to the opposite methyl groups of para-xylene (B): either arrangement gives two methylglyoxal molecules and one glyoxal molecule. Their two drawn forms therefore do not give different product sets.

Step 2: Trace the adjacent methyl groups in option C In ortho-xylene the methyl-bearing carbons are adjacent. In one fixed arrangement the bond joining them is single and survives cleavage of the three double bonds. Those two carbons stay together as diacetyl, $CH_3COCOCH_3$, while the other two fragments are glyoxal. In the other arrangement, the bond joining the methyl-bearing carbons is double and is cleaved. Each methyl-bearing carbon then belongs to a separate fragment, giving two methylglyoxal molecules and one glyoxal molecule. These product sets are different.

Step 3: Check the singly substituted pair For toluene (D), only one ring carbon bears a methyl group. Whichever alternating double-bond arrangement is cleaved, exactly one resulting two-carbon fragment contains that substituent. Both forms therefore yield one methylglyoxal and two glyoxal molecules. This rules out the final alternative and leaves only the ortho-disubstituted pair.

Step 4: Conclusion Only the two fixed double-bond arrangements of ortho-xylene give different ozonolysis product sets. Hence, the correct option is (C).

Quick Tip to Solve: Erase every drawn double bond and examine the carbon pairs left connected by single bonds. Two adjacent substituted carbons stay together only when their joining bond survives.

70.

$$CH_3CH_2Cl \xrightarrow[C_2H_5OH-H_2O]{AgCN} A \text{ (major product)}$$

$$CH_3CH_2Cl \xrightarrow[C_2H_5OH-H_2O]{NaCN} B \text{ (major product)}$$

Considering the above reactions, the compound 'A' and compound 'B' respectively are: [4 marks]

(A) $C_2H_5-N^+\equiv C^-$, $C_2H_5-N^+\equiv C^-$

(B) $C_2H_5-C\equiv N$, $C_2H_5-C\equiv N$

(C) $C_2H_5-N^+\equiv C^-$, $C_2H_5-C\equiv N$

(D) $C_2H_5-C\equiv N$, $C_2H_5-N^+\equiv C^-$

Correct Answer: (C) $C_2H_5-N^+\equiv C^-$, $C_2H_5-C\equiv N$

Solution:

Concept:

- Cyanide is an ambident nucleophile: it can form a new bond through carbon or through nitrogen. Carbon attachment gives a nitrile, $R-CN$, whereas nitrogen attachment gives an isocyanide, $R-NC$.
- Ionic sodium cyanide provides free cyanide with the carbon end available for substitution. In substantially covalent silver cyanide, carbon is bound to silver, and the nitrogen end gives the major alkylation product.

Step 1: Identify the alkyl group undergoing substitution The starting halide in the reaction scheme is chloroethane, CH_3CH_2Cl. Substitution replaces chlorine without changing the ethyl skeleton. The two routes differ in which atom of the cyanide reagent becomes bonded to the ethyl group, not in the number of carbon atoms already present in the substrate.

Step 2: Find compound A from silver cyanide With $AgCN$, substitution mainly produces an isocyanide because nitrogen is the available attacking end of the covalently bonded cyanide reagent. Thus

$$C_2H_5Cl + AgCN \longrightarrow C_2H_5NC + AgCl.$$

Compound A is ethyl isocyanide. Its bond connectivity is ethyl–nitrogen–carbon; the charged Lewis representation $C_2H_5-N^+\equiv C^-$ in the option shows that same connectivity.

Step 3: Find compound B and compare the pairs With ionic $NaCN$, the carbon end of free cyanide attacks to give the nitrile:

$$C_2H_5Cl + NaCN \longrightarrow C_2H_5CN + NaCl.$$

Compound B is propanenitrile (ethyl cyanide), with ethyl–carbon–nitrogen connectivity. Options (A) and (B) incorrectly make the two products identical. Option (D) reverses the reagents' characteristic connectivities. The ordered pair required is therefore isocyanide first and nitrile second.

Step 4: Conclusion Compound A is C_2H_5NC and compound B is C_2H_5CN. Hence, the correct option is (C).

Quick Tip to Solve: Track the atom joining the alkyl group: NaCN normally adds through carbon to give RCN, whereas AgCN mainly adds through nitrogen to give RNC.

71.

$$p\text{-HO}-\text{C}_6\text{H}_4-\text{CHO} \xrightarrow{\text{Br}_2} \text{A} \xrightarrow{\text{NH}_2\text{OH}} \text{B} \xrightarrow{\text{P}_2\text{O}_5,\ \Delta} \text{C}$$

Consider the above reaction sequence, the Product 'C' is: [4 marks]

(A) 3,5-Dibromobenzonitrile

(B) 3,5-Dibromo-4-hydroxybenzaldehyde

(C) 3,5-Dibromobenzaldehyde

(D) 3,5-Dibromo-4-hydroxybenzonitrile

Correct Answer: (D) 3,5-Dibromo-4-hydroxybenzonitrile

Solution:

Concept:

- Phenolic hydroxyl activates the aromatic ring at its ortho and para positions. The para position is already occupied by the aldehyde group, so bromination occurs at the two ortho positions.
- An aldehyde reacts with hydroxylamine to form an aldoxime. A dehydrating agent such as P_2O_5 converts $\text{Ar}-\text{CH}=\text{NOH}$ into $\text{Ar}-\text{CN}$, retaining the original aldehyde carbon.

Step 1: Identify the bromination product The starting compound is 4-hydroxybenzaldehyde. Taking the carbon bearing CHO as carbon 1 places OH at carbon 4. Its two free ortho positions are carbons 3 and 5. Hence bromination gives 3,5-dibromo-4-hydroxybenzaldehyde, labelled A in the reaction sequence. These positions are also meta to the deactivating aldehyde group, so both substituents direct bromination consistently.

Step 2: Convert the aldehyde into the nitrile Hydroxylamine condenses with the aldehyde carbonyl, eliminating water and forming the corresponding aldoxime B. Write Ar for the 3,5-dibromo-4-hydroxyphenyl group:

$$\text{Ar}-\text{CHO} + \text{NH}_2\text{OH} \longrightarrow \text{Ar}-\text{CH}=\text{NOH} + \text{H}_2\text{O}.$$

Heating the oxime with P_2O_5 removes another molecule of water:

$$\text{Ar}-\text{CH}=\text{NOH} \longrightarrow \text{Ar}-\text{CN} + \text{H}_2\text{O}.$$

The phenolic hydroxyl and the two ring bromines are retained; this reagent sequence does not remove the hydroxyl group.

Step 3: Compare the four products Option A has the nitrile but incorrectly omits the phenolic hydroxyl. Option B is the intermediate brominated aldehyde and has not undergone oxime formation and dehydration. Option C both retains the aldehyde and omits the hydroxyl. Only option D contains all three required features: two bromines, the retained hydroxyl, and the newly formed nitrile.

Step 4: Conclusion Product C is 3,5-dibromo-4-hydroxybenzonitrile. Hence, the correct option is (D).

Quick Tip to Solve: For an aldehyde followed by hydroxylamine and a dehydrating agent, replace CHO by CN while retaining the other ring substituents.

72.

$$\text{A } (C_8H_6Cl_2O) \xrightarrow{NH_3} C_8H_8ClNO \xrightarrow{Br_2/NaOH}$$

4-chloro-3-methylaniline.

Consider the above reaction, the compound 'A' is: [4 marks]

(A) 4-Chloro-2-methylbenzoyl chloride

(B) 3-Chloro-4-methylbenzoyl chloride

(C) 4-Chloro-3-methylbenzoyl chloride

(D) 4-Chloro-3-(chloromethyl)benzaldehyde

Correct Answer: (C) 4-Chloro-3-methylbenzoyl chloride

Solution:

Concept:

- An acid chloride reacts with ammonia by nucleophilic acyl substitution to give a primary amide: $RCOCl \longrightarrow RCONH_2$. The chlorine on an aromatic ring is unaffected under these conditions.
- Hofmann bromamide degradation with $Br_2/NaOH$ converts $RCONH_2$ to RNH_2. The carbonyl carbon is lost, while the substituent arrangement within R is retained.

Step 1: Work backwards from the aromatic amine Take the ring carbon bearing NH_2 in the final drawing as carbon 1. The methyl group is at carbon 3 and chlorine is at carbon 4. The product is therefore 4-chloro-3-methylaniline. In a Hofmann degradation the ring carbon that carried the amide substituent ends up bonded directly to nitrogen. Thus the required amide is 4-chloro-3-methylbenzamide.

Step 2: Recover compound A and check its formula To obtain this amide from ammonia, replace the amide $-CONH_2$ group by the acid-chloride group $-COCl$. This gives 4-chloro-3-methylbenzoyl chloride, with ring substituents unchanged. Its formula is $C_8H_6Cl_2O$: six ring carbons, one methyl carbon and one carbonyl carbon; one aryl chlorine and one acyl chlorine. Ammonolysis gives C_8H_8ClNO, exactly the intermediate formula printed in the scheme.

Step 3: Reject the positional and functional alternatives Option A would put methyl ortho to the eventual amino group rather than meta. Option B has chlorine meta and methyl para, also inconsistent with the final ring. Option D is an aldehyde with a chloromethyl substituent; it is not the acid chloride needed to generate the stated primary amide with ammonia. Option C alone preserves the printed positions and gives the correct loss of one carbon in the Hofmann step.

Step 4: Conclusion Compound A is 4-chloro-3-methylbenzoyl chloride. Hence, the correct option is (C).

Quick Tip to Solve: Trace Hofmann degradation backwards by replacing the final NH2 group with CONH2, then use the preceding reagent to identify its acyl precursor.

73.

$$C_6H_5{-}N_2{}^+\,Cl^- + A \xrightarrow{NaOH} \text{Product}$$

The product is an orange-red dye. Which among the following represent reagent 'A'? [4 marks]

(A) β-Naphthol (naphthalen-2-ol)

(B) α-Naphthol (naphthalen-1-ol)

(C) Aniline

(D) Naphthalen-1-amine

Correct Answer: (A) β-Naphthol (naphthalen-2-ol)

Solution:

Concept:

- Benzenediazonium chloride contains the electrophilic diazonium group $-N_2{}^+$. It couples with sufficiently activated aromatic rings to form coloured azo compounds containing $-N{=}N-$.
- In alkaline medium a naphthol forms a naphthoxide ion, whose oxygen donates electron density into the aromatic system. β-Naphthol is the standard coupling component giving an orange-red azo dye with benzenediazonium chloride.

Step 1: Use the alkaline medium and colour The reaction explicitly shows $NaOH$, so a phenolic coupling component is converted to its more strongly activating phenoxide form. In particular, naphthalen-2-ol, also called β-naphthol, gives the familiar orange-red azo dye on coupling with benzenediazonium chloride. The diazonium reagent supplies one phenyl group and the azo linkage, while the naphthol supplies the second aromatic system.

Step 2: Locate the coupling site For β-naphthoxide, carbon 1 is a strongly activated site adjacent to the hydroxyl-bearing carbon 2. Electrophilic coupling there, followed by loss of a proton and work-up, gives 1-(phenylazo)naphthalen-2-ol. The extended conjugated system containing the two aromatic portions and $-N{=}N-$ absorbs visible light and accounts for the orange-red product. This is coupling, not replacement of the diazonium group by hydroxyl.

Step 3: Distinguish the proposed reagents Option A shows the hydroxyl on the beta position of naphthalene and therefore is β-naphthol. Option B is the alpha isomer, naphthalen-1-ol, which is not the coupling component for the standard orange-red dye identified here. Options C and D are aromatic amines, aniline and naphthalen-1-amine. They do not produce the beta-naphthol-derived orange-red dye shown by this alkaline coupling test; aromatic amine coupling is normally carried out under suitably controlled, weakly acidic conditions.

Step 4: Conclusion Reagent A is β-naphthol, or naphthalen-2-ol. Hence, the correct option is (A).

Quick Tip to Solve: An orange-red azo dye with benzenediazonium salt in alkali is the characteristic beta-naphthol coupling reaction.

74. Consider the following reaction sequence:

$$p\text{-}H_2N-C_6H_4-CN \xrightarrow[(ii)\ H_2O]{(i)\ AlH(i\text{-}Bu)_2} A$$

$$A \xrightarrow[dil.\ NaOH,\ \Delta]{CH_3CHO} B \text{ (Major Product).}$$

The product 'B' is: [4 marks]

(A) p-$OHC-C_6H_4-N=CH-CH_3$

(B) p-$H_2N-C_6H_4-CH=CH-CHO$

(C) p-$H_2N-C_6H_4-CH_2-N=CH-CH_3$

(D) p-$H_2N-C_6H_4-C(=O)-N=CH-CHO$

Correct Answer: (B) p-$H_2N-C_6H_4-CH=CH-CHO$

Solution:

Concept:

- Diisobutylaluminium hydride, $(i\text{-}Bu)_2AlH$, reduces a nitrile to an imine derivative that yields an aldehyde on hydrolysis. It does not require reduction all the way to a primary amine.
- An aromatic aldehyde without an alpha hydrogen can undergo crossed aldol condensation with acetaldehyde in dilute alkali. Acetaldehyde supplies the enolate; heating favours dehydration to the conjugated alpha,beta-unsaturated aldehyde.

Step 1: Identify intermediate A The starting compound is 4-aminobenzonitrile. Reduction of its nitrile group with diisobutylaluminium hydride followed by water converts $-CN$ into $-CHO$, while retaining the para amino group. Thus A is 4-aminobenzaldehyde, p-$H_2N-C_6H_4-CHO$. The carbon of the nitrile remains the aldehyde carbon; it is not removed and is not reduced to a methylene group in this controlled conversion.

Step 2: Carry out the crossed aldol condensation In dilute $NaOH$, acetaldehyde forms its enolate, which attacks the aromatic aldehyde carbonyl. Protonation first gives the beta-hydroxy aldehyde

$$p\text{-}H_2N-C_6H_4-CH(OH)-CH_2-CHO.$$

Heating removes water to form the conjugated product:

$$p\text{-}H_2N-C_6H_4-CHO + CH_3CHO \longrightarrow$$

$$p\text{-}H_2N-C_6H_4-CH=CH-CHO + H_2O.$$

The aldehyde group in the final side chain belongs to acetaldehyde; the original aromatic aldehyde carbon becomes the carbon attached directly to the ring.

Step 3: Check the alternatives Option B is exactly this para-amino substituted cinnamaldehyde. Option A instead consumes the amino group in an imine while retaining the original ring aldehyde, ignoring the base-promoted carbon-carbon condensation. Option C requires nitrile reduction to an aminomethyl group, inconsistent with the given selective reagent and hydrolysis. Option D introduces an amide-type nitrogen-containing linkage, which neither step of this sequence generates.

Step 4: Conclusion The major product B is p-$H_2N-C_6H_4-CH=CH-CHO$. Hence, the correct option is (B).

Quick Tip to Solve: After DIBAL-H and water, write the nitrile as an aldehyde first; then let acetaldehyde provide the enolate in the base-promoted condensation.

75. A compound 'X' is acidic and it is soluble in NaOH solution, but insoluble in $NaHCO_3$ solution. Compound 'X' also gives violet colour with neutral $FeCl_3$ solution. The compound 'X' is : [4 marks]

(A) Cyclohexanol

(B) Phenol

(C) Benzyl alcohol

(D) Cyclohexylmethanol

Correct Answer: (B) Phenol

Solution:

Concept:

- Phenol is a weak acid because its conjugate base, phenoxide, is stabilised by resonance. A strong base such as $NaOH$ converts it to a soluble sodium phenoxide salt.
- Phenol is weaker than carbonic acid and therefore does not liberate carbon dioxide from $NaHCO_3$. A violet colour with neutral $FeCl_3$ is characteristic of the ferric complex of phenolic compounds.

Step 1: Apply the two acid-base observations The compound dissolves in sodium hydroxide, so it must be acidic enough to form an ionic sodium salt under these conditions. However, it does not dissolve in sodium hydrogen carbonate. This distinguishes a phenol from an ordinary carboxylic acid: a carboxylic acid generally reacts with bicarbonate to release CO_2, whereas phenol does not. Thus the two observations place X in the phenolic acidity range.

Step 2: Use the ferric chloride confirmation Phenol reacts with neutral ferric chloride to form a coloured coordination complex, commonly violet for phenol itself. The combined evidence is stronger than the colour observation alone: X shows the acid-base behaviour of a weak phenolic acid and also contains a phenolic oxygen able to coordinate ferric ions. The relevant neutralisation with strong base is

$$C_6H_5OH + NaOH \longrightarrow C_6H_5ONa + H_2O.$$

The soluble salt explains the reported solubility in sodium hydroxide.

Step 3: Compare where the hydroxyl group is attached Option B has hydroxyl directly bonded to an aromatic ring and is phenol. Option A is cyclohexanol, a normal aliphatic alcohol with no resonance-stabilised phenoxide. In option C, benzyl alcohol, an intervening methylene separates oxygen from the aromatic ring; it is not a phenol. Option D, cyclohexylmethanol, is also an aliphatic alcohol. These three do not combine the stated sodium hydroxide solubility and characteristic violet ferric chloride test.

Step 4: Conclusion Compound X is phenol, C_6H_5OH. Hence, the correct option is (B).

Quick Tip to Solve: Check the bond to oxygen: an OH directly on an aromatic ring gives phenolic behaviour; a CH2 group between the ring and OH makes it an alcohol.

76. $100\ \mathrm{mL}$ **of** $5\%\ (\mathrm{w/v})$ **solution of** NaCl **in water was prepared in** $250\ \mathrm{mL}$ **beaker. Albumin from the egg was poured into** NaCl **solution and stirred well. This resulted in a/an :** [4 marks]

(A) Lyophilic sol

(B) Lyophobic sol

(C) Emulsion

(D) Precipitate

Correct Answer: (A) Lyophilic sol

Solution:

Concept:

- Lyophilic colloids have a strong affinity between the dispersed substance and the dispersion medium. Proteins such as albumin form hydrated colloidal particles in water.
- A sol contains colloid-sized particles dispersed in a liquid. An emulsion instead consists of droplets of one liquid dispersed in a second, immiscible liquid.

Step 1: Identify the dispersed substance and medium The dispersion medium is water containing sodium chloride, and the added material is egg albumin, a protein. Albumin is a macromolecular substance with numerous polar and ionisable groups that interact with water. On stirring, the hydrated protein molecules disperse through the aqueous medium. This is the standard example of a protein forming a lyophilic sol; the word hydrophilic is used when the lyophilic medium is water.

Step 2: Interpret the preparation conditions A $5\%\ (\mathrm{w/v})$ solution contains $5\ \mathrm{g}$ of NaCl in each $100\ \mathrm{mL}$ of solution. The $250\ \mathrm{mL}$ value describes the beaker capacity and does not increase the solution volume or change that concentration. Albumin is stirred into this solution without a stated heating step, denaturing treatment, or sufficient salting-out treatment to establish precipitation. The intended classification is governed by the hydrated protein dispersed in water.

Step 3: Eliminate the other classes A lyophobic sol would require weak affinity between dispersed material and water, unlike the strong hydration of albumin. An emulsion would require two liquid phases, such as oil and water; protein particles dispersed in water do not meet that definition. A precipitate is a separate insoluble phase that settles or can be filtered as bulk material, rather than the hydrated colloidal dispersion produced by the stated preparation. Therefore the correct classification among the options is a lyophilic sol.

Step 4: Conclusion Albumin forms a lyophilic sol in the aqueous medium. Hence, the correct option is (A).

Quick Tip to Solve: Proteins, starch and gums are standard lyophilic colloids in water; classify by affinity for the medium rather than the container size.

77. In metallurgy the term "gangue" is used for : [4 marks]

(A) Contamination of undesired earthy materials.

(B) Contamination of metals, other than desired metal.

(C) Minerals which are naturally occurring in pure form

(D) Magnetic impurities in an ore.

Correct Answer: (A) Contamination of undesired earthy materials.

Solution:

Concept:

- An ore is a naturally occurring material from which a metal can be extracted economically. Along with its valuable mineral, an ore generally contains unwanted earthy or rocky material.
- The unwanted earthy impurities accompanying an ore, such as sand, clay and siliceous matter, are collectively called gangue. Their removal is a main purpose of concentration or dressing of the ore.

Step 1: State the meaning of the term In metallurgy, gangue is the commercially unwanted earthy material associated with the valuable ore mineral. It includes substances such as sand, clay and rocky matter that are mined together with the metal-bearing portion. The term describes these impurities as a class; it does not refer to the desired metal or to a pure mineral. Concentration enriches the useful mineral by separating as much gangue as possible before chemical extraction.

Step 2: Relate the definition to separation methods The suitable concentration method depends on a physical or chemical difference between the valuable mineral and its gangue. For example, differences in density may permit hydraulic washing, differences in magnetic behaviour may permit magnetic separation, and differences in surface wettability are used in froth flotation. These methods illustrate why gangue is not defined by a single physical property such as being magnetic: different ores contain different unwanted earthy substances.

Step 3: Compare the options precisely Option A explicitly identifies undesired earthy contamination, which is the definition of gangue. Option B refers generally to other metals; although an ore can contain such impurities, that wording is not the meaning of gangue. Option C describes naturally occurring pure minerals and omits the unwanted earthy component. Option D is too restrictive: gangue need not be magnetic, and magnetic separation is only one possible way of separating it. The broad earthy-impurity description is therefore the appropriate one.

Step 4: Conclusion Gangue means the undesired earthy materials associated with an ore. Hence, the correct option is (A).

Quick Tip to Solve: Remember ore as valuable mineral plus gangue; gangue names the unwanted earthy portion, while its physical properties determine the separation method.

78. Lithium nitrate and sodium nitrate, when heated separately, respectively, give : [4 marks]

(A) $LiNO_2$ and $NaNO_2$

(B) Li_2O and Na_2O

(C) Li_2O and $NaNO_2$

(D) $LiNO_2$ and Na_2O

Correct Answer: (C) Li_2O and $NaNO_2$

Solution:

Concept:

- On heating, most alkali-metal nitrates form the corresponding nitrites and release oxygen. Lithium nitrate is the important exception to this general pattern.
- The small Li^+ ion has a comparatively high polarising power. Its nitrate decomposes to the oxide, nitrogen dioxide and oxygen, resembling the behaviour of alkaline-earth metal nitrates.

Step 1: Identify the special behaviour of lithium nitrate Lithium does not follow sodium in this thermal decomposition. Its small cation strongly polarises the nitrate ion, making further decomposition to the oxide favourable. The balanced equation is

$$4\,LiNO_3 \xrightarrow{\Delta} 2\,Li_2O + 4\,NO_2 + O_2.$$

Thus the solid lithium-containing product is Li_2O, not $LiNO_2$. Nitrogen dioxide and oxygen are the gaseous products; the question asks for the corresponding lithium and sodium compounds.

Step 2: Apply the usual rule to sodium nitrate Sodium nitrate undergoes the usual alkali-metal nitrate decomposition:

$$2\,NaNO_3 \xrightarrow{\Delta} 2\,NaNO_2 + O_2.$$

The sodium-containing solid is therefore $NaNO_2$, not sodium oxide. Preserve the order in the question: lithium product first and sodium product second. Option (A) incorrectly treats lithium like sodium; option (B) incorrectly makes sodium oxide; option (D) makes both of these mistakes. Only option (C) combines the two correct products.

Step 3: Conclusion The products are Li_2O and $NaNO_2$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: For alkali-metal nitrate decomposition, check lithium separately: lithium gives the oxide, whereas sodium and the heavier members give nitrites.

79. Which among the following pairs has only herbicides? [4 marks]

(A) Aldrin and Dieldrin

(B) Sodium chlorate and Aldrin

(C) Sodium arsinate and Dieldrin

(D) Sodium chlorate and sodium arsinite

Correct Answer: (D) Sodium chlorate and sodium arsinite

Solution:

Concept:

- Herbicides are chemicals used to destroy unwanted plants or weeds. Insecticides target insects; both may be pesticides, but they are different functional classes.
- Sodium chlorate and sodium arsenite are herbicides. Aldrin and dieldrin are chlorinated organic insecticides, so a pair containing either one cannot be a pair of only herbicides.

Step 1: Classify the substances by their use The decisive word in the question is *only*: both members of the selected pair must control unwanted vegetation. Sodium chlorate is an inorganic herbicide. The arsenic-containing sodium herbicide is sodium arsenite, named as sodium arsinite in option (D). By contrast, aldrin and dieldrin belong to the organochlorine insecticides. Their membership in the broad pesticide category does not make them herbicides; the target organism determines the classification required here.

Step 2: Eliminate each mixed or insecticide pair Option (A), aldrin and dieldrin, has two insecticides and therefore fails. Option (B) contains the herbicide sodium chlorate but also aldrin, an insecticide, so it is a mixed pair. Option (C) contains dieldrin and therefore fails regardless of the first substance in that pair. Option (D) contains sodium chlorate and the sodium arsenic herbicide, so both substances belong to the herbicide class. No reaction calculation is needed: this is a classification question, and each complete pair must be checked rather than selecting a familiar pesticide name.

Step 3: Conclusion Sodium chlorate and sodium arsinite, the pair printed in option (D), are the herbicides. Hence, the correct option is (D).

Quick Tip to Solve: For a question asking for only herbicides, reject a pair immediately if it contains aldrin or dieldrin: these are insecticides.

80. Which of the following compounds is an example of hypnotic drug? [4 marks]

(A) Seldane

(B) Amytal

(C) Aspartame

(D) Prontosil

Correct Answer: (B) Amytal

Solution:

Concept:

- Hypnotic drugs induce sleep. Barbiturates are a familiar class of sedative-hypnotic drugs; their action on the central nervous system distinguishes them from antihistamines and antimicrobial drugs.
- Amytal is a barbiturate hypnotic. Seldane is an antihistamine, aspartame is an artificial sweetener, and Prontosil is an antibacterial drug.

Step 1: Identify the sleep-inducing drug The question asks for a hypnotic, so look for the compound used to induce sleep rather than merely any substance used in medicine. Amytal is a barbiturate and is included among the sedative-hypnotic drugs. A sedative reduces excitement, while a hypnotic produces sleep; barbiturates are described in this therapeutic grouping because of their central nervous system depressant action. Consequently Amytal directly matches the function asked for.

Step 2: Check the therapeutic class of the other options Seldane belongs to the antihistamine class: its relevant use is relief from allergic responses, so it is not the hypnotic in this list. Aspartame is an artificial sweetening agent and does not belong to a sleep-inducing drug class. Prontosil is associated with antibacterial treatment and the development of sulfonamide drugs, rather than hypnotic action. These distinct functions rule out options (A), (C) and (D), respectively. The selection is based on pharmacological classification; the presence of a chemical name alone does not imply that the compound is a hypnotic.

Step 3: Conclusion Amytal is the example of a hypnotic drug. Hence, the correct option is (B).

Quick Tip to Solve: Associate Amytal with barbiturates and sleep; distinguish the other names by their classes: antihistamine, sweetener and antibacterial drug.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Resistance of a conductivity cell (cell constant $129\ \mathrm{m^{-1}}$) filled with $74.5\ \mathrm{ppm}$ solution of KCl is $100\ \Omega$ (labelled as solution 1). When the same cell is filled with KCl solution of $149\ \mathrm{ppm}$, the resistance is $50\ \Omega$ (labelled as solution 2). The ratio of molar conductivity of solution 1 and solution 2 is i.e. $\frac{\Lambda_1}{\Lambda_2} = x \times 10^{-3}$. The value of x is _____. (Nearest integer)

Given, molar mass of KCl is $74.5\ \mathrm{g\ mol^{-1}}$. [4 marks]

Solution:

Concept:

- The conductivity of a solution is $\kappa = G_{\text{cell}}/R$, where $G_{\text{cell}} = l/A$ is the cell constant and R is the measured resistance.
- For concentration c in $\mathrm{mol\,m^{-3}}$, molar conductivity is $\Lambda_m = \kappa/c$. In a ratio using the same cell, its cell constant cancels.

Step 1: Convert the two concentrations consistently For these dilute aqueous solutions, ppm gives milligrams of solute per litre. Using the molar mass $74.5\ \mathrm{g\,mol^{-1}}$, the first concentration is

$$c_1 = \frac{0.0745\ \mathrm{g\,L^{-1}}}{74.5\ \mathrm{g\,mol^{-1}}} = 0.001\ \mathrm{mol\,L^{-1}} = 1\ \mathrm{mol\,m^{-3}}.$$

Similarly, $149\ \mathrm{ppm}$ gives $c_2 = 0.002\ \mathrm{mol\,L^{-1}} = 2\ \mathrm{mol\,m^{-3}}$. Thus the second solution has twice the molar concentration.

Step 2: Calculate the conductivities and their molar ratio Using the stated common cell constant,

$$\kappa_1 = \frac{129}{100} = 1.29\ \mathrm{S\,m^{-1}}, \qquad \kappa_2 = \frac{129}{50} = 2.58\ \mathrm{S\,m^{-1}}.$$

Therefore $\Lambda_1 = 1.29/1 = 1.29\ \mathrm{S\,m^2\,mol^{-1}}$ and $\Lambda_2 = 2.58/2 = 1.29\ \mathrm{S\,m^2\,mol^{-1}}$. Equivalently, the ratio directly is

$$\frac{\Lambda_1}{\Lambda_2} = \frac{R_2 c_2}{R_1 c_1} = \frac{50 \times 2}{100 \times 1} = 1.$$

Equate this with $x \times 10^{-3}$, giving $x = 10^3$.

Step 3: Conclusion The ratio is $1 = 1000 \times 10^{-3}$. The required nearest integer is $\boxed{x = 1000}$, which is dimensionless.

Quick Tip to Solve: In the same conductivity cell, use $\Lambda_1/\Lambda_2 = R_2 c_2/(R_1 c_1)$; resistance alone does not give the molar-conductivity ratio.

82. The minimum uncertainty in the speed of an electron in an one dimensional region of length $2a_o$ (Where a_o = Bohr radius $52.9\,\text{pm}$) is _____ km s^{-1}.

(Given : Mass of electron = 9.1×10^{-31} kg, Planck's constant h = 6.63×10^{-34} Js) [4 marks]

Solution:

Concept:

- The position-momentum uncertainty relation is $\Delta x\,\Delta p \geq h/(4\pi)$. For the minimum uncertainty permitted by this relation, use equality.
- For an electron of fixed mass m_e, $\Delta p = m_e \Delta v$, so $\Delta v_{\text{min}} = h/(4\pi m_e \Delta x)$. Use the full stated one-dimensional region length as Δx.

Step 1: Express the confinement length in SI units The electron occupies a one-dimensional region of length $2a_0$, not a_0. With the supplied Bohr radius,

$$\Delta x = 2(52.9\,\text{pm}) = 105.8 \times 10^{-12}\,\text{m} = 1.058 \times 10^{-10}\,\text{m}.$$

The uncertainty relation pairs this position uncertainty with the momentum uncertainty along the same direction. The mass remains $9.1 \times 10^{-31}\,\text{kg}$; no factor associated with three spatial directions is required for this one-dimensional statement.

Step 2: Apply the equality limit and convert speed units Substitution into the uncertainty relation gives

$$\Delta v_{\text{min}} = \frac{6.63 \times 10^{-34}}{4\pi(9.1 \times 10^{-31})(1.058 \times 10^{-10})}\,\text{m s}^{-1}.$$

The denominator is approximately 1.210×10^{-39} in SI units, so

$$\Delta v_{\text{min}} \approx 5.48 \times 10^{5}\,\text{m s}^{-1} = 548\,\text{km s}^{-1}.$$

The conversion to kilometres per second divides the SI speed by 10^3. Using a_0 instead of $2a_0$, or using $h/(2\pi)$ instead of $h/(4\pi)$, would incorrectly double the result.

Step 3: Conclusion The minimum uncertainty in speed is approximately $\boxed{548\,\text{km s}^{-1}}$. The required numerical entry is 548.

Quick Tip to Solve: Write the uncertainty relation before substituting: the printed length is $2a_0$, and the lower bound contains 4π, not 2π.

83. When 600 mL of 0.2 M HNO_3 is mixed with $400\ \text{mL}$ of 0.1 M NaOH solution in a flask, the rise in temperature of the flask is _____ $\times 10^{-2}\,^{\circ}\text{C}$.

(Enthalpy of neutralisation $= 57\ \text{kJ mol}^{-1}$ and Specific heat of water $= 4.2\,\text{JK}^{-1}\,\text{g}^{-1}$) (Neglect heat capacity of flask) [4 marks]

Solution:

Concept:

- A strong acid and strong base neutralise in a one-to-one mole ratio: $\text{HNO}_3 + \text{NaOH} \longrightarrow \text{NaNO}_3 + \text{H}_2\text{O}$. The limiting reagent determines the moles of water formed.
- The released heat is $q = n|\Delta H_{\text{neut}}|$, and the temperature rise obeys $q = mc\Delta T$. Treat the dilute solution mass as its total volume in grams, using water density $1\,\text{g mL}^{-1}$.

Step 1: Find the amount actually neutralised Convert the volumes to litres before using molarity:

$$n(\text{HNO}_3) = 0.600 \times 0.2 = 0.120\,\text{mol},$$

$$n(\text{NaOH}) = 0.400 \times 0.1 = 0.040\,\text{mol}.$$

The base is the limiting reactant. Only $0.040\,\text{mol}$ of acid reacts and $0.040\,\text{mol}$ of water forms; the excess acid does not release additional neutralisation heat. Thus the magnitude of heat delivered to the mixture is

$$q = 0.040 \times 57 \times 10^3 = 2280\,\text{J}.$$

Step 2: Use the heat capacity of the whole mixture The combined volume is $600 + 400 = 1000\,\text{mL}$, corresponding to approximately $1000\,\text{g}$ of solution. With the flask heat capacity neglected, all released heat warms this total mass. Therefore

$$\Delta T = \frac{2280}{1000 \times 4.2} = 0.542857\,\text{K}.$$

A temperature difference has the same numerical value in kelvin and degrees Celsius. In the requested form,

$$\Delta T = 54.2857 \times 10^{-2}\,^{\circ}\text{C}.$$

The coefficient, rounded to the integer used for the numerical answer, is 54.

Step 3: Conclusion The rise in temperature is approximately $0.543\,^{\circ}\text{C} = 54.3 \times 10^{-2}\,^{\circ}\text{C}$. The required integer coefficient is $\boxed{54}$.

Quick Tip to Solve: Use the smaller acid/base mole amount for released heat, but use the combined solution mass in $mc\Delta T$.

84. If O_2 gas is bubbled through water at 303 K, the number of millimoles of O_2 gas that dissolve in 1 litre of water is _____. (Nearest Integer)

(Given : Henry's Law constant for O_2 at 303 K is 46.82 k bar and partial pressure of $O_2 = 0.920$ bar)

(Assume solubility of O_2 in water is too small, nearly negligible) [4 marks]

Solution:

Concept:

- Henry law in mole-fraction form is $p = K_H x_{O_2}$, where p is the oxygen partial pressure and K_H must be expressed in the same pressure unit.
- For very low gas solubility, $x_{O_2} = n_{O_2}/(n_{H_2O} + n_{O_2}) \approx n_{O_2}/n_{H_2O}$. One litre of water has approximately $1000/18$ moles.

Step 1: Calculate the oxygen mole fraction The Henry constant is given in kilobar. Express it in bar to match the oxygen partial pressure:

$$K_H = 46.82 \times 10^3 \text{ bar} = 46820 \text{ bar}.$$

Henry law then gives

$$x_{O_2} = \frac{0.920}{46820} = 1.965 \times 10^{-5}.$$

This small mole fraction is consistent with the stated negligible solubility approximation. Using 46.82 bar instead would overestimate the solubility by a factor of 10^3.

Step 2: Convert mole fraction into millimoles dissolved For one litre of water, taking its density as 1 g mL^{-1},

$$n_{H_2O} = \frac{1000}{18} = 55.56 \text{ mol}.$$

Since dissolved oxygen contributes negligibly to the total mole count,

$$n_{O_2} \approx x_{O_2} n_{H_2O} = (1.965 \times 10^{-5})(55.56) = 1.092 \times 10^{-3} \text{ mol}.$$

Multiplying by 1000 converts moles to millimoles, giving approximately 1.092 mmol. The exact expression $n_{O_2} = x n_{H_2O}/(1 - x)$ changes this negligibly and yields the same nearest integer.

Step 3: Conclusion Approximately 1.09 mmol of O_2 dissolves in one litre of water. To the nearest integer, the answer is $\boxed{1 \text{ mmol}}$, so the numerical entry is 1.

Quick Tip to Solve: Convert kilobar to bar first, and remember that Henry law gives mole fraction; multiply by the water mole amount before converting to millimoles.

85. If the solubility product of PbS is 8×10^{-28}, then the solubility of PbS in pure water at 298 K is x $\times 10^{-16}$ mol L^{-1}. The value of x is _____. (Nearest Integer)

[Given : $\sqrt{2}$ = 1.41] [4 marks]

Solution:

Concept:

- The solubility product is the product of equilibrium ion concentrations, each raised to its stoichiometric coefficient. For the 1:1 salt PbS, $K_{sp} = [Pb^{2+}][S^{2-}]$.
- If the molar solubility in pure water is s, one mole of dissolved salt gives one mole of each ion, so $K_{sp} = s^2$. Use the supplied approximation $\sqrt{2} = 1.41$.

Step 1: Write the dissolution equilibrium Lead sulfide dissociates according to

$$PbS\,(s) \rightleftharpoons Pb^{2+}(aq) + S^{2-}(aq).$$

The solid has unit activity and does not enter the solubility-product expression. In pure water there is no initially added lead or sulfide common ion. Consequently, dissolving $s\,\text{mol}\,L^{-1}$ produces equal concentrations $[Pb^{2+}] = [S^{2-}] = s$.

Step 2: Take the square root and express the requested coefficient Substitute the given value and take the positive root, since a solubility cannot be negative:

$$s = \sqrt{8 \times 10^{-28}} = 2\sqrt{2} \times 10^{-14}\,\text{mol}\,L^{-1}.$$

With the approximation specified in the question,

$$s = 2(1.41) \times 10^{-14} = 2.82 \times 10^{-14}\,\text{mol}\,L^{-1}.$$

To write this with 10^{-16}, increase the coefficient by a factor of 100:

$$s = 282 \times 10^{-16}\,\text{mol}\,L^{-1}.$$

It is the coefficient x, rather than the concentration in ordinary decimal notation, that must be entered.

Step 3: Conclusion The molar solubility is $2.82 \times 10^{-14}\,\text{mol}\,L^{-1}$. Thus the requested dimensionless coefficient is $\boxed{x = 282}$ (nearest integer).

Quick Tip to Solve: For a 1:1 salt use $s = \sqrt{K_{sp}}$, then convert to the power of ten printed beside the answer blank before rounding.

86. The reaction between X and Y is first order with respect to X and zero order with respect to Y.

Experiment	$\frac{[X]}{mol\,L^{-1}}$	$\frac{[Y]}{mol\,L^{-1}}$	$\frac{Initial\,rate}{mol\,L^{-1}\,min^{-1}}$
I	**0.1**	**0.1**	2×10^{-3}
I	**L**	**0.2**	4×10^{-3}
III	**0.4**	**0.4**	$M \times 10^{-3}$
IV	**0.1**	**0.2**	2×10^{-3}

Examine the data of table and calculate ratio of numerical values of M and L. (Nearest Integer)

[4 marks]

Solution:

Concept:

- For a reaction first order in X and zero order in Y, the initial rate is $r = k[X]^1[Y]^0 = k[X]$. Changing $[Y]$ alone does not affect the rate.
- The same rate constant applies to every experiment at the same temperature. Here L is the numerical concentration in the second row, whereas M is the numerical coefficient multiplying 10^{-3} in the third-row rate.

Step 1: Determine the rate constant and the unknown concentration Using experiment I,

$$k = \frac{r}{[X]} = \frac{2 \times 10^{-3}}{0.1} = 2 \times 10^{-2}\,\text{min}^{-1}.$$

The second row has twice the first-row rate. Because the reaction is first order in X, its X concentration must also be twice as large; the change in Y contributes no rate factor. Equivalently,

$$4 \times 10^{-3} = (2 \times 10^{-2})L, \qquad L = 0.2.$$

Thus the concentration represented by L is $0.2\,\text{mol}\,\text{L}^{-1}$.

Step 2: Find the rate coefficient and form the numerical ratio For experiment III, $[X] = 0.4\,\text{mol}\,\text{L}^{-1}$, four times the first-row concentration. Therefore

$$r_{\text{III}} = k[X] = (2 \times 10^{-2})(0.4) = 8 \times 10^{-3}\,\text{mol}\,\text{L}^{-1}\,\text{min}^{-1}.$$

Comparison with $M \times 10^{-3}$ gives $M = 8$. Experiment IV provides a check: it has the original X concentration and the original rate despite doubled Y. Finally, the requested ratio of numerical values is

$$\frac{M}{L} = \frac{8}{0.2} = 40.$$

Do not use 8×10^{-3} in the numerator: M already denotes its coefficient.

Step 3: Conclusion $L = 0.2$ and $M = 8$; hence the ratio of their numerical values is $\boxed{40}$ (nearest integer). The requested numerical ratio has no concentration or rate unit.

Quick Tip to Solve: Cancel zero-order reactants from rate comparisons first. Distinguish a rate coefficient such as M from the full rate $M \times 10^{-3}$.

87. In a linear tetrapeptide (Constituted with different amino acids), (number of amino acids) − (number of peptide bonds) is _____. [4 marks]

Solution:

Concept:

- A peptide bond is an amide linkage, $-\mathrm{CO}-\mathrm{NH}-$, formed by condensation of the carboxyl group of one amino acid with the amino group of another.
- A linear peptide containing n amino acid residues has $n-1$ peptide bonds. Its two terminal residues are not joined to each other, unlike a cyclic peptide.

Step 1: Count the amino acid residues The prefix tetra indicates four amino acids. Label their residues A_1, A_2, A_3, A_4 in the order in which they occur along the chain. The fact that they are different affects the possible amino acid sequences, but not how many residues the tetrapeptide contains. Thus

$$N_{\text{amino acids}} = 4.$$

In this question the count refers to the amino acid units constituting the peptide, not to free amino acid molecules remaining after condensation.

Step 2: Count the connecting linkages A linear chain joins A_1 to A_2, A_2 to A_3, and A_3 to A_4. These are three separate peptide linkages, so

$$N_{\text{peptide bonds}} = 4 - 1 = 3.$$

Each new amino acid added to an existing linear chain supplies one new linkage and eliminates one water molecule. There is no fourth linkage between the last and first residues because the molecule is expressly linear. Subtract the two counts in the order asked:

$$N_{\text{amino acids}} - N_{\text{peptide bonds}} = 4 - 3 = 1.$$

Counting all covalent bonds in the molecule would be irrelevant; only the amide linkages connecting adjacent amino acid residues are peptide bonds.

Step 3: Conclusion The linear tetrapeptide has four amino acid residues and three peptide bonds. The required dimensionless difference is $\boxed{1}$.

Quick Tip to Solve: In any single linear peptide chain, count the connections between neighbouring residues: n residues give $n-1$ peptide bonds.

88. In bromination of Propyne, with Bromine, 1, 1, 2, 2-tetrabromopropane is obtained in 27% yield. The amount of 1, 1, 2, 2-tetrabromopropane obtained from 1 g of Bromine in this reaction is _____ $\times 10^{-1}$ g. (Nearest integer)

(Molar Mass : Bromine = 80 g/mol) [4 marks]

Solution:

Concept:

- Complete bromination adds two molecules of Br_2 across the triple bond of one propyne molecule, producing one molecule of $C_3H_4Br_4$.
- The actual product mass equals the theoretical product mass multiplied by the fractional yield. The given bromine atomic molar mass is $80\,\text{g mol}^{-1}$; molecular Br_2 has molar mass $160\,\text{g mol}^{-1}$.

Step 1: Balance the addition and determine the theoretical amount The reaction is

$$CH_3C{\equiv}CH + 2\,Br_2 \longrightarrow CH_3CBr_2CHBr_2.$$

One mole of 1,1,2,2-tetrabromopropane therefore requires two moles of bromine. From 1 g of bromine,

$$n(Br_2) = \frac{1}{160} = 0.00625\,\text{mol}, \qquad n_{\text{product,theoretical}} = \frac{0.00625}{2} = 0.003125\,\text{mol}.$$

Propyne supplies the carbon and hydrogen skeleton; its mass is not part of the given bromine mass.

Step 2: Apply the yield and convert to the answer scale The product formula is $C_3H_4Br_4$, so its molar mass is

$$M = 3(12) + 4(1) + 4(80) = 360\,\text{g mol}^{-1}.$$

The theoretical mass is

$$m_{\text{theoretical}} = 0.003125(360) = 1.125\,\text{g}.$$

At a 27 percent yield,

$$m_{\text{actual}} = 1.125\left(\frac{27}{100}\right) = 0.30375\,\text{g} = 3.0375 \times 10^{-1}\,\text{g}.$$

The answer blank is the coefficient multiplying 10^{-1} g. Rounding that coefficient, rather than rounding the gram amount directly, gives 3.

Step 3: Conclusion The actual product mass is $0.30375\,\text{g}$. Written on the required scale it is $3.0375 \times 10^{-1}\,\text{g}$, so the nearest-integer entry is $\boxed{3}$.

Quick Tip to Solve: Use two moles of Br_2 per mole of tetrabromo product, apply the percentage yield last, and round only the requested coefficient.

89. $[Fe(CN)_6]^{3-}$ **should be an inner orbital complex. Ignoring the pairing energy, the value of crystal field stabilization energy for this complex is** $(-)$ **_____** Δ_0**. (Nearest integer)** [4 marks]

Solution:

Concept:

- In an octahedral crystal field the three t_{2g} orbitals each contribute $-0.4\Delta_o$ per electron and the two e_g orbitals contribute $+0.6\Delta_o$ per electron.
- Cyanide is a strong-field ligand. The stated inner-orbital complex is low spin, so electrons pair in the lower t_{2g} set before occupying the higher e_g set. Pairing-energy contributions are explicitly excluded here.

Step 1: Find the metal oxidation state and d-electron count Let the iron oxidation state be z. Each of the six cyanide ligands has charge -1, while the complex has charge -3. Thus

$$z + 6(-1) = -3, \qquad z = +3.$$

Neutral iron has configuration $[Ar]3d^6 4s^2$. Removing the two $4s$ electrons and one $3d$ electron gives Fe^{3+} a $3d^5$ configuration. The d-electron count must follow the metal oxidation state, not the overall charge of the complex.

Step 2: Fill the low-spin orbitals and calculate the stabilization For low-spin octahedral d^5, the occupancy is

$$t_{2g}^5 e_g^0.$$

Two lower orbitals contain electron pairs and the third contains one electron; there are no electrons in the upper set. Relative to the unsplit d-orbital barycentre, the crystal field contribution is

$$\text{CFSE} = 5(-0.4\Delta_o) + 0(+0.6\Delta_o) = -2.0\Delta_o.$$

Although forming pairs would introduce a pairing-energy term in a full energy comparison, the question instructs us to ignore it. The blank follows an already printed minus sign, so it asks for the positive magnitude of the coefficient.

Step 3: Conclusion The crystal field stabilization energy, ignoring pairing energy, is $-2\Delta_o$. Therefore the nearest-integer coefficient in the printed expression is $\boxed{2}$, which is dimensionless.

Quick Tip to Solve: Determine oxidation state before counting d electrons. For a low-spin octahedral d^5 complex use t_{2g}^5, then retain the negative stabilization sign.

90. Ionic radii of cation A^+ and anion B^- are 102 and $181\,pm$ respectively. These ions are allowed to crystallize into an ionic solid. This crystal has cubic close packing for B^-. A^+ is present in all octahedral voids. The edge length of the unit cell of the crystal AB is _____ pm. (Nearest Integer) [4 marks]

Solution:

Concept:

- A cubic close-packed array of anions with all octahedral holes occupied by cations has the rock-salt structure. Anions occupy face centres and corners; cations occupy the body centre and edge centres.
- Along a cube edge, an anion at a corner touches a cation at the edge centre. Their centre-to-centre distance is $a/2$, where a is the unit-cell edge length.

Step 1: Identify the nearest unlike-ion contact Consider an edge from one corner of the cubic unit cell to the next. There is a B^- anion at each corner and a A^+ cation halfway along the edge, occupying an octahedral site. The corner-to-edge-centre distance is half the edge length. In the ionic hard-sphere description this equals the sum of the two ionic radii:

$$\frac{a}{2} = r_{A^+} + r_{B^-}.$$

This is the nearest cation-anion contact relevant to the given radii. The face-diagonal relation for touching equal spheres should not be substituted for this unlike-ion contact.

Step 2: Substitute the radii and find the edge length The supplied radii are $r_{A^+} = 102\,pm$ and $r_{B^-} = 181\,pm$. Hence

$$r_{A^+} + r_{B^-} = 102 + 181 = 283\,pm.$$

There are two of these nearest-neighbour separations along the complete cube edge, so

$$a = 2(283) = 566\,pm.$$

No conversion of units is needed because both radii and the requested edge length are in picometres. Occupation of all octahedral sites also gives equal numbers of cations and anions, consistent with the stated formula AB.

Step 3: Conclusion The unit-cell edge length is $\boxed{566\,pm}$. Thus the nearest-integer entry in the blank is 566.

Quick Tip to Solve: For the rock-salt arrangement, use the corner-anion to edge-centre-cation contact: $a = 2(r_+ + r_-)$, not the pure-fcc equal-sphere radius formula.