JEE Main 2022 Paper 1 (B.E./B.Tech) Jul 29 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 2 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 29 July 2022, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If $z \neq 0$ be a complex number such that $\left|z - \frac{1}{z}\right| = 2$, then the maximum value of $|z|$ is : [4 marks]

(A) $\sqrt{2}$

(B) 1

(C) $\sqrt{2} - 1$

(D) $\sqrt{2} + 1$

Correct Answer: (D) $\sqrt{2} + 1$

Solution:

Concept:

- For complex numbers, the reverse triangle inequality states $\big||u| - |v|\big| \leq |u - v|$. Apply it to $u = z$ and $v = 1/z$.
- If $r = |z| > 0$, then $|1/z| = 1/r$. A bound for r must also be attainable by a complex number satisfying the original condition.

Step 1: Bound the modulus Put $r = |z|$. The given equality and the reverse triangle inequality give

$$\left|r - \frac{1}{r}\right| \leq 2.$$

In particular, $r - 1/r \leq 2$. Multiplication by the positive number r preserves the inequality, giving

$$r^2 - 2r - 1 \leq 0.$$

The roots are $1 - \sqrt{2}$ and $1 + \sqrt{2}$; since $r > 0$, we obtain $r \leq 1 + \sqrt{2}$.

Step 2: Check equality and the choices Take the positive real number $z = 1 + \sqrt{2}$. Its reciprocal is $\sqrt{2} - 1$, so

$$\left|z - \frac{1}{z}\right| = |(1 + \sqrt{2}) - (\sqrt{2} - 1)| = 2.$$

Thus the upper bound is achieved, rather than merely approached. The values 1, $\sqrt{2}$, and $\sqrt{2} - 1$ are all smaller than this attainable value and therefore cannot be the maximum. No phase of a complex number can give a larger modulus because the bound applied to every nonzero complex z.

Step 3: Conclusion The maximum modulus is $\boxed{1 + \sqrt{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the reverse triangle inequality to bound the modulus, then test a positive real number to prove that the bound is attained.

2. Which of the following matrices can NOT be obtained from the matrix $\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ by a single elementary row operation ? [4 marks]

(A) $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

Correct Answer: (C) $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$

Solution:

Concept:

- A single elementary row operation is a row interchange, multiplication of one row by a nonzero scalar, or addition of a scalar multiple of another row to one row.
- A row replacement $R_2 \mapsto R_2 + cR_1$ leaves R_1 unchanged and uses the same scalar c in both entries.

Step 1: Identify three permitted operations Write the original rows as $R_1 = (-1, 2)$ and $R_2 = (1, -1)$. For (A), replace R_1 by $R_1 + R_2$; this gives $(0, 1)$, while the second row remains $(1, -1)$. For (B), interchange R_1 and R_2, obtaining exactly the displayed matrix. For (D), use

$$R_2 \mapsto R_2 + 2R_1 = (1, -1) + 2(-1, 2) = (-1, 3).$$

Each construction uses precisely one elementary operation, so these three choices are possible.

Step 2: Exclude every single operation for option C In (C), the first row is unchanged and the proposed second row is $(-2, 7)$. A row interchange would change the first row, so it cannot work. Scaling $(1, -1)$ to make its first entry -2 requires a factor -2, but then its second entry becomes 2, not 7. For a row replacement, equate entries:

$$(1, -1) + c(-1, 2) = (-2, 7).$$

The first entry requires $1 - c = -2$, hence $c = 3$. The second would then be $-1 + 2(3) = 5$, not 7. An operation on the first row cannot produce the required change in the second row. These possibilities exhaust single elementary row operations.

Step 3: Conclusion The matrix in (C) cannot be obtained by one elementary row operation. Hence, the correct option is (C).

Quick Tip to Solve: For a row replacement, determine the multiplier from one entry and check the other entry with that same multiplier.

3. If the system of equations

$$x + y + z = 6$$
$$2x + 5y + \alpha z = \beta$$
$$x + 2y + 3z = 14$$

has infinitely many solutions, then $\alpha + \beta$ is equal to [4 marks]

(A) 8

(B) 36

(C) 44

(D) 48

Correct Answer: (C) 44

Solution:

Concept:

- Infinitely many solutions require the equations to be consistent and linearly dependent, with coefficient rank smaller than the number of unknowns.
- If one equation is a linear combination of two independent equations, its constant term must be the identical linear combination of their constants.

Step 1: Find the dependence of the left sides The first and third coefficient rows $(1, 1, 1)$ and $(1, 2, 3)$ are independent, because they are not scalar multiples. For the second equation to be dependent on these, let its left side be a times the first plus b times the third. Matching the coefficients of x and y gives

$$a + b = 2, \qquad a + 2b = 5.$$

Subtracting gives $b = 3$, and hence $a = -1$. The coefficient of z must consequently satisfy

$$\alpha = a + 3b = -1 + 9 = 8.$$

Step 2: Impose consistency on the constants Apply precisely the same combination to the right sides:

$$\beta = -6 + 3(14) = 36.$$

Thus the second equation is three times the third minus the first, on both sides. The two remaining independent equations in three unknowns leave one free parameter; for example, putting $z = t$ gives $y = 8 - 2t$ and $x = t - 2$. Hence there are indeed infinitely many solutions. The values 8 and 36 in the choices are the individual parameters, not their requested sum, and 48 fails the computed relation.

Step 3: Conclusion We have $\alpha + \beta = 8 + 36 = \boxed{44}$. Hence, the correct option is (C).

Quick Tip to Solve: When two coefficient columns fix a row dependence, use that same dependence for the remaining coefficient and the constant term.

4.

Let the function $f(x) = \begin{cases} \frac{\log_e(1+5x)-\log_e(1+\alpha x)}{x} & ; \text{ if } x \neq 0 \\ 10 & ; \text{ if } x = 0 \end{cases}$ **be continuous at** $x = 0$.

Then α **is equal to** [4 marks]

(A) 10

(B) -10

(C) 5

(D) -5

Correct Answer: (D) -5

Solution:

Concept:

- Continuity at zero means that the two-sided limit $\lim_{x\to 0} f(x)$ exists and equals the assigned value $f(0) = 10$.
- The standard logarithmic limit is $\lim_{u\to 0} \ln(1+u)/u = 1$, so $\lim_{x\to 0} \ln(1+cx)/x = c$ for a fixed real constant c.

Step 1: Evaluate the limit of the nonzero branch Keep the two logarithms separate so that the standard limit applies directly:

$$\lim_{x\to 0} \frac{\ln(1+5x) - \ln(1+\alpha x)}{x} = \lim_{x\to 0} \frac{\ln(1+5x)}{x} - \lim_{x\to 0} \frac{\ln(1+\alpha x)}{x}$$
$$= 5 - \alpha.$$

These logarithms are defined in an open neighbourhood of zero for every fixed finite α, because their arguments tend to the positive number 1. Thus the two-sided limit calculation is legitimate.

Step 2: Use the assigned value and test the options Continuity requires the preceding limit to equal the value at zero, rather than merely to be finite. Therefore

$$5 - \alpha = 10 \quad \Longrightarrow \quad \alpha = -5.$$

For this value the quotient becomes $[\ln(1+5x) - \ln(1-5x)]/x$, whose limit is $5-(-5) = 10$, confirming continuity. The proposed values 10, -10, and 5 instead produce limits -5, 15, and 0, respectively. None equals $f(0)$. Notice that the minus sign between the logarithms is essential: it makes the second limit subtract α.

Step 3: Conclusion The required parameter is $\boxed{\alpha = -5}$. Hence, the correct option is (D).

Quick Tip to Solve: For a logarithmic difference divided by x, subtract the linear coefficients inside the logarithms before imposing continuity.

5. If $[t]$ denotes the greatest integer $\leq t$, then the value of $\int_0^1 \left[2x - \left|3x^2 - 5x + 2\right| + 1\right] dx$ is : [4 marks]

(A) $\frac{\sqrt{37}+\sqrt{13}-4}{6}$

(B) $\frac{\sqrt{37}-\sqrt{13}-4}{6}$

(C) $\frac{-\sqrt{37}-\sqrt{13}+4}{6}$

(D) $\frac{-\sqrt{37}+\sqrt{13}+4}{6}$

Correct Answer: (A) $\frac{\sqrt{37}+\sqrt{13}-4}{6}$

Solution:

Concept:

- The greatest-integer function $[t]$ is constant whenever its argument stays between consecutive integers. Its integral is the sum of rectangle areas on these intervals.
- Factor $3x^2 - 5x + 2 = (3x - 2)(x - 1)$ to remove the absolute value separately on $[0, 2/3]$ and $[2/3, 1]$.

Step 1: Find where the argument crosses integers Let $g(x) = 2x - |3x^2 - 5x + 2| + 1$. On the integration interval,

$$g(x) = \begin{cases} -3x^2 + 7x - 1, & 0 \leq x \leq 2/3, \\ 3x^2 - 3x + 3, & 2/3 \leq x \leq 1. \end{cases}$$

The first expression increases from -1 to $7/3$. Its crossings of $0, 1, 2$ occur respectively at

$$a = \frac{7 - \sqrt{37}}{6}, \qquad \frac{1}{3}, \qquad b = \frac{7 - \sqrt{13}}{6}.$$

The second expression increases from $7/3$ to 3, so its floor is 2 except at the endpoint $x = 1$.

Step 2: Integrate the constant pieces A finite collection of endpoints does not affect the integral. Thus the floor of g is -1 on $(0, a)$, 0 on $(a, 1/3)$, 1 on $(1/3, b)$, and 2 on $(b, 1)$. Consequently,

$$\begin{aligned} \int_0^1 [g(x)]\, dx &= -a + \left(b - \frac{1}{3}\right) + 2(1 - b) \\ &= \frac{5}{3} - a - b \\ &= \frac{\sqrt{37} + \sqrt{13} - 4}{6}. \end{aligned}$$

The positive contributions dominate the short negative interval. Options (B) and (C) are negative, and (D) is a different positive value resulting from incorrect interval signs; the direct sum fixes the signs of both radicals.

Step 3: Conclusion The integral equals $\boxed{(\sqrt{37} + \sqrt{13} - 4)/6}$. Hence, the correct option is (A).

Quick Tip to Solve: First remove the absolute value; then solve for every integer crossing of its argument and integrate the resulting constant floor values.

6. For $I(x)=\int \frac{\sec^2 x-2022}{\sin^{2022} x}dx$, if $I\left(\frac{\pi}{4}\right)=2^{1011}$, then [4 marks]

(A) $3^{1010}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(B) $3^{1010}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

(C) $3^{1011}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(D) $3^{1011}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

Correct Answer: (A) $3^{1010}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

Solution:

Concept:

- The integrand combines a secant-squared term with a power of sine, suggesting the product $\tan x\ \sin^{-2022} x$.
- The product rule and $\tan x \cot x = 1$ make the derivative of this product match the two terms of the integrand exactly.

Step 1: Recognise and verify the antiderivative On an interval where sine and cosine are nonzero, differentiate:

$$\frac{d}{dx}\left(\frac{\tan x}{\sin^{2022} x}\right)=\frac{\sec^2 x}{\sin^{2022} x}-2022\tan x\frac{\cos x}{\sin^{2023} x}$$
$$=\frac{\sec^2 x-2022}{\sin^{2022} x}.$$

Therefore $I(x)=\tan x/\sin^{2022} x+C$. At $x=\pi/4$, the product already equals 2^{1011}, since $\tan(\pi/4)=1$ and $\sin^{2022}(\pi/4)=2^{-1011}$. The given value fixes $C=0$ on the interval containing all three angles.

Step 2: Compare the two required values Using the exact trigonometric values gives

$$I(\pi/3)=\sqrt{3}\left(\frac{4}{3}\right)^{1011},\qquad I(\pi/6)=\frac{4^{1011}}{\sqrt{3}}.$$

Divide the second value by the first:

$$\frac{I(\pi/6)}{I(\pi/3)}=\frac{3^{1011}}{3}=3^{1010}.$$

Thus the multiple in (A) is correct. Option (C) has an extra factor of 3, while (B) and (D) reverse the larger and smaller integrals. Both values are positive, so none of those alternative relations can vanish.

Step 3: Conclusion The identity is $3^{1010}I(\pi/3)-I(\pi/6)=0$. Hence, the correct option is (A).

Quick Tip to Solve: When a high power accompanies a derivative term, test a product derivative before attempting repeated integration by parts.

7. If the solution curve of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passes through the points $(2,1)$ and $(k+1,2), k>0$, then [4 marks]

(A) $2\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(k^2+1\right)$

(B) $\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(k^2+1\right)$

(C) $2\tan^{-1}\left(\frac{1}{k+1}\right) = \log_e\left(k^2+2k+2\right)$

(D) $2\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(\frac{k^2+1}{k^2}\right)$

Correct Answer: (A) $2\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(k^2+1\right)$

Solution:

Concept:

- Translating the intersection of the linear expressions to the origin converts this equation into a homogeneous differential equation.
- For $X = x-1$, $Y = y-1$, use $v = Y/X$ and $dY/dX = v + X\,dv/dX$, then separate variables.

Step 1: Translate and integrate the equation The substitution $X = x-1$, $Y = y-1$ gives

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}.$$

Writing $Y = vX$, we obtain

$$X\frac{dv}{dX} = \frac{1+v}{1-v} - v = \frac{1+v^2}{1-v}.$$

Separate and integrate:

$$\int \frac{1-v}{1+v^2}\,dv = \int \frac{dX}{X},$$

so $\tan^{-1} v - \frac{1}{2}\ln(1+v^2) = \ln|X| + C$. On the branch with $X>0$, this is equivalently

$$2\tan^{-1}(Y/X) = \ln(X^2+Y^2) + C_1.$$

Step 2: Apply both points on the curve The first point $(x,y) = (2,1)$ becomes $(X,Y) = (1,0)$, so $C_1 = 0$. The second point becomes $(X,Y) = (k,1)$. Since $k>0$, it lies on the positive-X branch, and substitution yields

$$2\tan^{-1}(1/k) = \ln(k^2+1).$$

The factor 2 comes from multiplying the integrated equation by 2; omitting it gives (B). Failing to translate x gives the incorrect $k+1$ in (C). Retaining an extra $\ln k^2$ after combining logarithms gives (D), whereas that term has already been absorbed into $\ln(X^2+Y^2)$.

Step 3: Conclusion The required relation is $\boxed{2\tan^{-1}(1/k) = \ln(k^2+1)}$. Hence, the correct option is (A).

Quick Tip to Solve: Translate both x and y by the common zero of the two linear forms, and combine the logarithms before inserting the second point.

8. Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right) y = \frac{(x+3)}{x+1}, x > -1$, which passes through the point $(0, 1)$. Then $y(1)$ is equal to : [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $\frac{7}{2}$

Correct Answer: (B) $\frac{3}{2}$

Solution:

Concept:

- A first-order linear equation $y\prime + P(x)y = Q(x)$ has integrating factor $I(x) = e^{\int P(x)\,dx}$, so that $(Iy)\prime = IQ$.
- Factor the cubic coefficient denominator before integrating. The condition $x > -1$ makes all three linear factors positive and avoids singularities; the initial condition fixes the integration constant.

Step 1: Factor the denominator and find the integrating factor The denominator factors as $(x+1)(x+2)(x+3)$. Partial fractions give

$$P(x) = \frac{2}{x+1} + \frac{1}{x+2} - \frac{1}{x+3}.$$

Indeed, putting these fractions over the common denominator recovers $2x^2 + 11x + 13$. Hence an integrating factor is

$$I(x) = \exp\{2\log(x+1) + \log(x+2) - \log(x+3)\} = \frac{(x+1)^2(x+2)}{x+3}.$$

Step 2: Integrate the transformed equation and impose the initial value Multiplication by the integrating factor turns the left side into an exact product derivative. The right side simplifies substantially:

$$\frac{d}{dx}(Iy) = \frac{(x+1)^2(x+2)}{x+3}\frac{x+3}{x+1} = (x+1)(x+2).$$

Thus

$$Iy = \frac{x^3}{3} + \frac{3x^2}{2} + 2x + C.$$

At $x = 0$, $I(0) = 2/3$ and $y(0) = 1$; consequently $C = 2/3$. This condition must be applied to Iy, rather than directly to the polynomial alone.

Step 3: Evaluate at the requested point At $x = 1$, the integrating factor equals $4 \cdot 3/4 = 3$, while

$$I(1)y(1) = \frac{1}{3} + \frac{3}{2} + 2 + \frac{2}{3} = \frac{9}{2}.$$

Dividing by $I(1)$ gives $y(1) = 3/2$. The alternatives $1/2$, $5/2$ and $7/2$ would give Iy values inconsistent with this initial-value solution.

Step 4: Conclusion $y(1) = \frac{3}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: When the right side cancels factors of the integrating factor, integrate the simplified product first; remember to divide by the integrating factor at the end.

9. Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2+11a+3\left(m_1^2+m_2^2\right)=220$. If one vertex of the square is $(10(\cos\alpha-\sin\alpha), 10(\sin\alpha+\cos\alpha))$, where $\alpha \in \left(0, \frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos\alpha-\sin\alpha)x+(\sin\alpha+\cos\alpha)y=10$, then $72\left(\sin^4\alpha+\cos^4\alpha\right)+a^2-3a+13$ is equal to : [4 marks]

(A) 119

(B) 128

(C) 145

(D) 155

Correct Answer: (B) 128

Solution:

Concept:

- The perpendicular distance from a vertex to the diagonal through the other two vertices of a square equals half its diagonal, namely $a/\sqrt{2}$.
- Adjacent sides are perpendicular, so their slopes are m and $-1/m$. If their inclination is α, the squared slope sum is $\tan^2\alpha+\cot^2\alpha$.

Step 1: Find the side length from the vertex and diagonal Write $c=\cos\alpha$ and $s=\sin\alpha$. The given vertex is $V=(10(c-s), 10(s+c))$. Substitution in the diagonal's left side gives

$$(c-s)10(c-s)+(s+c)10(s+c)=20.$$

It is therefore not on the diagonal whose equation has right side 10. Since $(c-s)^2+(s+c)^2=2$, its distance from that diagonal is

$$d=\frac{|20-10|}{\sqrt{2}}=5\sqrt{2}.$$

The vertex lies on the other diagonal, so $d=a/\sqrt{2}$, giving $a=10$.

Step 2: Use the slope condition to determine the trigonometric expression The normal to the specified diagonal has inclination $\alpha+\pi/4$, and the diagonal itself has inclination $\alpha-\pi/4$, modulo π. Square sides make angles of $\pi/4$ with its diagonals, so their inclinations are α and $\alpha+\pi/2$. Hence

$$100+110+3(\tan^2\alpha+\cot^2\alpha)=220,$$

which yields $\tan^2\alpha+\cot^2\alpha=10/3$. Using $s^2+c^2=1$,

$$\tan^2\alpha+\cot^2\alpha=\frac{s^4+c^4}{s^2c^2}=\frac{1}{s^2c^2}-2.$$

Therefore $s^2c^2=3/16$ and $s^4+c^4=1-2(3/16)=5/8$.

Step 3: Compute the requested value Insert both results without solving for the angle itself:

$$72\left(\frac{5}{8}\right)+10^2-3(10)+13=45+100-30+13=128.$$

The other numerical choices cannot satisfy this evaluation; the side length and the fourth-power sum

have already been fixed by independent conditions.

Step 4: Conclusion The required value is 128. Hence, the correct option is (B).

Quick Tip to Solve: Use the distance from the supplied vertex to the opposite diagonal to find the side; the angle can then be eliminated through $\sin^2\alpha\cos^2\alpha$.

10. Let $\mathrm{A}(\alpha,-2),\mathrm{B}(\alpha,6)$ and $\mathrm{C}\left(\frac{\alpha}{4},-2\right)$ be vertices of a $\triangle\mathrm{ABC}$. If $\left(5,\frac{\alpha}{4}\right)$ is the circumcentre of $\triangle\mathrm{ABC}$, then which of the following is NOT correct about $\triangle\mathrm{ABC}$? [4 marks]

(A) area is 24

(B) perimeter is 25

(C) circumradius is 5

(D) inradius is 2

Correct Answer: (B) perimeter is 25

Solution:

Concept:

- If two sides of a triangle are perpendicular, the circumcentre is the midpoint of its hypotenuse and the circumradius is half the hypotenuse.
- For a right triangle with legs p,q and hypotenuse h, the area is $pq/2$ and the inradius is $(p+q-h)/2$. These identities allow all four statements to be tested.

Step 1: Locate the right angle and determine the parameter Points A and B have the same first coordinate, so AB is vertical. Points A and C have the same second coordinate, so AC is horizontal. Thus the right angle is at A, and BC is the hypotenuse. Its midpoint is

$$\left(\frac{\alpha+\alpha/4}{2},\frac{6-2}{2}\right)=\left(\frac{5\alpha}{8},2\right).$$

Equating this to the given circumcentre $(5,\alpha/4)$ gives $5\alpha/8=5$ and $\alpha/4=2$. Both coordinates consistently give $\alpha=8$.

Step 2: Calculate the three side lengths The vertices are consequently $A=(8,-2)$, $B=(8,6)$ and $C=(2,-2)$. The perpendicular side lengths are

$$AB=8,\qquad AC=6.$$

By the distance formula, or Pythagoras,

$$BC=\sqrt{(8-2)^2+(6+2)^2}=\sqrt{36+64}=10.$$

All lengths are positive, so the triangle is nondegenerate and the right-triangle centre formulas apply.

Step 3: Test every printed statement and select the false one The area is $\frac{1}{2}(6)(8)=24$, so (A) is true. The perimeter is $6+8+10=24$, which contradicts the value 25 in (B). The circumradius is $10/2=5$,

so (C) is true. Finally,

$$r = \frac{6 + 8 - 10}{2} = 2,$$

so (D) is also true. As a cross-check, the semiperimeter is 12 and $\Delta/s = 24/12 = 2$, giving the same inradius. Since the question asks what is NOT correct, the perimeter statement is the required selection.

Step 4: Conclusion The perimeter is 24, not 25. Hence, the correct option is (B).

Quick Tip to Solve: First check for horizontal and vertical sides. A right angle makes the circumcentre and both radii much faster to calculate than general coordinate formulas.

11. Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is : [4 marks]

(A) $\frac{4}{9}$
(B) $\frac{5}{18}$
(C) $\frac{1}{6}$
(D) $\frac{3}{10}$

Correct Answer: (B) $\frac{5}{18}$

Solution:

Concept:

- Bayes theorem reverses conditioning: $P(R \mid B) = P(R)P(B \mid R)/P(B)$, where R means a red transfer and B means a black final draw.
- The probability of a black final draw is found by partitioning according to the transferred colour. The second bag contains ten balls after every possible transfer.

Step 1: List the transfer cases and their conditional probabilities The first bag contains ten balls. Let R, K, W denote transfer of a red, black or white ball, respectively. Their probabilities are

$$P(R) = \frac{3}{10}, \quad P(K) = \frac{4}{10}, \quad P(W) = \frac{3}{10}.$$

The second bag initially contains nine balls, five of which are black. A red or white transfer leaves the black count at five, whereas a black transfer raises it to six. Its total count becomes ten in all cases. Therefore

$$P(B \mid R) = \frac{5}{10}, \quad P(B \mid K) = \frac{6}{10}, \quad P(B \mid W) = \frac{5}{10}.$$

Step 2: Calculate the probability of the observed black draw The transfer cases are mutually exclusive and exhaustive, so the law of total probability gives

$$\begin{aligned} P(B) &= \frac{3}{10}\frac{5}{10} + \frac{4}{10}\frac{6}{10} + \frac{3}{10}\frac{5}{10} \\ &= \frac{15 + 24 + 15}{100} = \frac{54}{100}. \end{aligned}$$

The joint event of a red transfer followed by a black draw has probability $15/100$. This distinction between a joint event and the observed event is essential: the question does not merely ask the initial chance of a red transfer.

Step 3: Condition on the observation Bayes theorem now yields

$$P(R \mid B) = \frac{P(R \cap B)}{P(B)} = \frac{15/100}{54/100} = \frac{5}{18}.$$

Option (D), $3/10$, is the prior probability and ignores the information provided by the black draw. The posterior is slightly smaller because transferring a black ball makes that observed draw more likely. Options (A) and (C) likewise do not equal the calculated joint-to-total ratio.

Step 4: Conclusion The required conditional probability is $\frac{5}{18}$. Hence, the correct option is (B).

Quick Tip to Solve: After a transfer, update both the total number of balls and the black count in each case before applying Bayes theorem.

12. Let $S = \{z = x + iy : |z - 1 + i| \geq |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x, for which $w = 2x + iy \in S$ for some $y \in \mathbb{R}$, is : [4 marks]

(A) $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

(C) $\left(-\sqrt{2}, \frac{1}{2}\right]$

(D) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

Correct Answer: (B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

Solution:

Concept:

- For a point $U + iV$, squared complex moduli are Cartesian squared distances. Squaring an equality or inequality between moduli is valid because both sides are nonnegative.
- Membership of $w = 2x + iy$ in the specified set must be tested using $U = 2x$ and $V = y$. The real coordinate of w is not x, and this factor changes both interval endpoints.

Step 1: Express the defining loci in Cartesian coordinates Use a generic member $U + iV$ of S. The first condition becomes

$$(U - 1)^2 + (V + 1)^2 \geq U^2 + V^2,$$

which simplifies to $V - U + 1 \geq 0$. The modulus bound is $U^2 + V^2 < 4$. Finally, the equal-distance condition gives

$$U^2 + (V + 1)^2 = (U - 1)^2 + V^2,$$

so $2V = -2U$, or $V = -U$. Thus the locus lies on a straight line through the origin, inside an open disk, and on one side of a line.

Step 2: Intersect the three conditions On substituting $V = -U$, the half-plane condition becomes

$$-U - U + 1 \geq 0 \quad \Longrightarrow \quad U \leq \frac{1}{2}.$$

The disk condition becomes $2U^2 < 4$, or $-\sqrt{2} < U < \sqrt{2}$. Their intersection is therefore

$$-\sqrt{2} < U \leq \frac{1}{2}.$$

The left endpoint is excluded by the strict modulus inequality. The right endpoint is allowed because its squared modulus is only $1/2 < 4$, while equality is permitted in the half-plane condition.

Step 3: Convert the real coordinate to the requested variable For $w = 2x + iy$, put $U = 2x$. Dividing the last interval by the positive number 2 gives

$$-\frac{1}{\sqrt{2}} < x \leq \frac{1}{4}.$$

Conversely, any x in this interval works by choosing $y = -2x$, so no additional existence condition is missing. Choices (A) and (C) retain an incorrect left endpoint, while (D) has an incorrect upper endpoint. The interval in (B) has both the correct bounds and the correct open/closed endpoints.

Step 4: Conclusion The set is $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$. Hence, the correct option is (B).

Quick Tip to Solve: Solve the locus for a generic real coordinate first, then substitute the actual real part $2x$; this avoids losing the factor of two.

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13. Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) + (\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = 168$, then $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is equal to : [4 marks]

(A) 10

(B) 14

(C) 16

(D) 18

Correct Answer: (C) 16

Solution:

Concept:

- Three nonzero coplanar vectors with equal pairwise angles have common angle $120°$, unless they all point in the same direction. The latter case gives zero cross products and is excluded by the stated nonzero sum.
- The identity $(\mathbf{u} \times \mathbf{v}) \cdot (\mathbf{w} \times \mathbf{t}) = (\mathbf{u} \cdot \mathbf{w})(\mathbf{v} \cdot \mathbf{t}) - (\mathbf{u} \cdot \mathbf{t})(\mathbf{v} \cdot \mathbf{w})$ reduces the expression to magnitudes.

Step 1: Determine the common angle Write the positive magnitudes as A, B, C, with $ABC = 14$, and let the common angle be θ. The Gram matrix of the three unit direction vectors has diagonal entries 1

and off-diagonal entries $t = \cos\theta$. Coplanarity implies its determinant vanishes:

$$(1-t)^2(1+2t) = 0.$$

If $t = 1$, all cross products vanish, contradicting the given sum 168. Hence $t = -1/2$, so $\theta = 120°$. In particular, the pairwise scalar products are $-AB/2, -BC/2, -CA/2$.

Step 2: Evaluate one term and then sum cyclically Apply the scalar product identity carefully in the printed order:

$$\begin{aligned}(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c}) &= (\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c}) - (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{b}) \\ &= \frac{AB^2C}{4} + \frac{AB^2C}{2} = \frac{3}{4}AB^2C.\end{aligned}$$

The other two cyclic terms are $3ABC^2/4$ and $3A^2BC/4$. All are positive; reversing just one cross product would incorrectly reverse its sign. Therefore the full sum is

$$\frac{3}{4}ABC(A+B+C) = 168.$$

Step 3: Use the given product of magnitudes Substituting $ABC = 14$ gives

$$\frac{3}{4}(14)(A+B+C) = 168, \qquad A+B+C = \frac{168\cdot 4}{42} = 16.$$

No individual magnitude is needed. The values 10, 14 and 18 would instead produce sums 105, 147 and 189 in the stated cross-product equation, respectively, so they do not satisfy it.

Step 4: Conclusion $|\vec{a}| + |\vec{b}| + |\vec{c}| = 16$. Hence, the correct option is (C).

Quick Tip to Solve: For equal-angle coplanar vectors, determine the angle first, then factor the cyclic expression as the magnitude product times the magnitude sum.

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14. The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ is : [4 marks]

(A) $[1, \infty)$

(B) $[-1, 2]$

(C) $[-1, \infty)$

(D) $(-\infty, 2]$

Correct Answer: (C) $[-1, \infty)$

Solution:

Concept:

- The real inverse sine function is defined precisely for inputs in $[-1, 1]$. Both inequalities must hold, including equality at the endpoints.
- Before multiplying inequalities by a polynomial denominator, establish its sign. Completing the square here gives $x^2 + 2x + 7 = (x+1)^2 + 6 > 0$ for every real x.

Step 1: Check the denominator and impose the upper bound Let $N = x^2 - 3x + 2$ and $D = x^2 + 2x + 7$. Since $D = (x+1)^2 + 6$ is strictly positive, the rational expression is defined for every real number and multiplying by D cannot reverse an inequality. The upper inverse-sine condition is

$$\frac{N}{D} \leq 1 \quad \Longleftrightarrow \quad N \leq D.$$

Subtracting the common quadratic terms gives

$$-3x + 2 \leq 2x + 7 \quad \Longleftrightarrow \quad -5x \leq 5 \quad \Longleftrightarrow \quad x \geq -1.$$

The direction reverses in the last step because division is by a negative number.

Step 2: Check the lower bound independently The lower condition is

$$\frac{N}{D} \geq -1 \quad \Longleftrightarrow \quad N + D \geq 0.$$

Here

$$N + D = 2x^2 - x + 9 = 2\left(x - \frac{1}{4}\right)^2 + \frac{71}{8} > 0$$

for every real x. Thus the lower bound adds no restriction; it must still be checked, since imposing only the upper bound is not generally enough for an inverse-sine expression.

Step 3: Combine the restrictions and check the boundary The intersection is $x \geq -1$. At $x = -1$, the numerator and denominator both equal 6, so the input is 1 and the function is defined. For $x < -1$, $N - D = -5(x+1) > 0$, so the input exceeds 1 and is inadmissible. Option (A) unnecessarily removes valid inputs between -1 and 1; option (B) unnecessarily removes inputs above 2; option (D) includes forbidden values below -1 and also cuts off valid values above 2.

Step 4: Conclusion The domain is $[-1, \infty)$. Hence, the correct option is (C).

Quick Tip to Solve: Complete the denominator square first; then solve $N \leq D$ and $N \geq -D$ separately, checking whether either condition is automatic.

15.

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and

$$a_{n+2} = 3a_{n+1} - 2a_n + 1, \forall n \geq 0.$$

Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to [4 marks]

(A) 483

(B) 528

(C) 575

(D) 624

Correct Answer: (B) 528

Solution:

Concept:

- The expression groups as a product of two differences of the form $a_k - 2a_{k-1}$. This combination also occurs naturally in the given recurrence.
- If $b_k = a_k - 2a_{k-1}$, subtracting the recurrence terms gives an arithmetic progression for b_k. Its initial term follows from $a_0 = a_1 = 0$.

Step 1: Choose a difference adapted to the recurrence For integers $k \geq 1$, define $b_k = a_k - 2a_{k-1}$. Rewrite the given relation without trying to compute the large individual terms:

$$a_{n+2} - 2a_{n+1} = a_{n+1} - 2a_n + 1.$$

Consequently $b_{n+2} = b_{n+1} + 1$ for every $n \geq 0$. The sequence of these differences, starting with $b_1 = a_1 - 2a_0 = 0$, therefore has common difference one. It follows that $b_k = k - 1$.

Step 2: Factor the requested expression Group the first two terms and the last two terms by their common factors. This avoids expanding any powers that might appear in a closed formula for a_n:

$$\begin{aligned} E &= a_{25}(a_{23} - 2a_{22}) - 2a_{24}(a_{23} - 2a_{22}) \\ &= (a_{25} - 2a_{24})(a_{23} - 2a_{22}) \\ &= b_{25}b_{23}. \end{aligned}$$

The indices are important: the first difference is b_{25}, while the second is b_{23}, not two consecutive differences.

Step 3: Substitute the two arithmetic-progression terms From $b_k = k - 1$, the required factors are $b_{25} = 24$ and $b_{23} = 22$. Thus $E = 24 \times 22 = 528$. This exact factorisation fixes the value uniquely; 483, 575 and 624 would require different differences and so do not satisfy the recurrence-based evaluation.

Step 4: Conclusion The expression equals 528. Hence, the correct option is (B).

Quick Tip to Solve: Before solving a recurrence explicitly, factor the target expression and look for a difference such as $a_k - 2a_{k-1}$.

16. $\sum_{r=1}^{20} \left(r^2+1\right)(r!)$ **is equal to** [4 marks]

(A) $22! - 21!$

(B) $22! - 2(21!)$

(C) $21! - 2(20!)$

(D) $21! - 20$!

Correct Answer: (B) $22! - 2(21!)$

Solution:

Concept:

- Factorial identities such as $(r+1)! = (r+1)r!$ let a polynomial times $r!$ be written as a difference of successive terms.
- In a telescoping sum, each interior positive term cancels the following negative term. Only the endpoints remain; the lower endpoint must also be checked.

Step 1: Find a successive-term difference Seek a telescoping expression whose factorial part is $r!$. The useful identity is

$$\begin{aligned} r(r+1)! - (r-1)r! &= \{r(r+1) - (r-1)\}r! \\ &= (r^2+1)r!. \end{aligned}$$

Thus, on defining $F(r) = (r-1)r!$, the summand is exactly $F(r+1) - F(r)$. The linear factor in F is essential: the simpler difference $(r+1)! - r! = r\,r!$ would not give the printed quadratic polynomial.

Step 2: Cancel all interior terms Apply the identity separately for every integer from 1 through 20:

$$\begin{aligned} \sum_{r=1}^{20}(r^2+1)r! &= \sum_{r=1}^{20}\{F(r+1) - F(r)\} \\ &= F(21) - F(1) \\ &= 20 \cdot 21! - 0 \cdot 1! \\ &= 20 \cdot 21!. \end{aligned}$$

In particular, the lower boundary contributes zero, so no extra constant is subtracted. As a small check, for upper limit one the same rule gives $1 \cdot 2! = 2$, agreeing with the first summand.

Step 3: Express the endpoint in the listed form Using $22! = 22 \cdot 21!$, we obtain $20 \cdot 21! = 22! - 2(21!)$. Option (A) instead equals $21 \cdot 21!$, while options (C) and (D) are respectively $19 \cdot 20!$ and $20 \cdot 20!$, both much smaller than the required endpoint. Only the second listed expression matches the telescoping result.

Step 4: Conclusion The sum is $22! - 2(21!)$. Hence, the correct option is (B).

Quick Tip to Solve: For $(r^2+1)r!$, try the difference $r(r+1)! - (r-1)r!$; do not expand the factorials numerically.

17. The number of elements in the set $S = \left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}\right\}$ is : [4 marks]

(A) 1

(B) 3

(C) 0

(D) infinite

Correct Answer: (A) 1

Solution:

Concept:

- For every real angle t, $-1 \le \cos t \le 1$, so the left side of the equation cannot exceed 2.
- For any positive real u, $u + u^{-1} \ge 2$, with equality only at $u = 1$. A shared boundary forces equality on both sides.

Step 1: Bound both sides independently The trigonometric side satisfies

$$2\cos\left(\frac{x^2+x}{6}\right) \le 2$$

for all real x. On the other hand, set $u = 4^x > 0$. Then

$$4^x + 4^{-x} - 2 = u + \frac{1}{u} - 2 = \frac{(u-1)^2}{u} \ge 0.$$

Therefore the exponential side is at least 2. Since one side is bounded above by 2 and the other below by 2, equality of the two sides is possible only when both are exactly 2. This conclusion uses the positivity of u, so multiplying or dividing by u does not change an inequality sign.

Step 2: Impose the equality condition Equality in the nonnegative-square identity requires $u - 1 = 0$. Hence $4^x = 1$, which has the unique real solution $x = 0$, because an exponential function with base 4 is strictly increasing. Thus no other real value can belong to the set, regardless of the periodicity of the cosine. In particular, merely solving $\cos((x^2+x)/6) = 1$ would generate candidates that fail the exponential condition.

Step 3: Verify the candidate in the original equation For $x = 0$, the angle $(x^2+x)/6$ is zero. The left side is $2\cos 0 = 2$, and the right side is $4^0 + 4^0 = 1 + 1 = 2$. Thus the candidate really is a solution and $S = \{0\}$. The set is neither empty, nor a three-element set, nor infinite; its cardinality is exactly one.

Step 4: Conclusion There is exactly 1 element in S. Hence, the correct option is (A).

Quick Tip to Solve: When a cosine expression equals a positive exponential sum, compare their maximum and minimum before solving any angle equation.

18. Let Q be the foot of perpendicular drawn from the point $P(1,2,3)$ to the plane $x+2y+z=14$. If R is a point on the plane such that $\angle PRQ = 60^\circ$, then the area of $\triangle PQR$ is equal to : [4 marks]

(A) $\frac{\sqrt{3}}{2}$

(B) $\sqrt{3}$

(C) $2\sqrt{3}$

(D) 3

Correct Answer: (B) $\sqrt{3}$

Solution:

Concept:

- The perpendicular distance from (x_0, y_0, z_0) to $ax + by + cz + d = 0$ is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2+b^2+c^2}$.
- A line perpendicular to a plane is perpendicular to every line in that plane through its foot. Thus $PQ \perp QR$, and the triangle is right angled at Q.

Step 1: Find the perpendicular height Write the plane as $x+2y+z-14=0$. Substitution of $P(1,2,3)$ into the distance formula gives

$$PQ = \frac{|1+2(2)+3-14|}{\sqrt{1^2+2^2+1^2}} = \frac{6}{\sqrt{6}} = \sqrt{6}.$$

The absolute value ensures that this geometric distance is positive. The normal vector is $(1,2,1)$, whose squared length is 6; using the unnormalised numerator alone would incorrectly give a distance of 6.

Step 2: Use the specified angle at R Both Q and R lie in the given plane, while PQ is normal to it. Hence $\angle PQR = 90^\circ$. In this right triangle, the angle $\angle PRQ = 60^\circ$ is at R, so its opposite side is PQ, and its adjacent leg is QR. Therefore

$$\tan 60^\circ = \frac{PQ}{QR}, \qquad QR = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}.$$

The hypotenuse is PR; it must not be substituted for the base QR in the area formula.

Step 3: Compute the area from the perpendicular legs The area is half the product of the two perpendicular sides:

$$[\triangle PQR] = \frac{1}{2} PQ \cdot QR = \frac{1}{2}\sqrt{6}\sqrt{2} = \frac{\sqrt{12}}{2} = \sqrt{3}.$$

Although the plane contains many possible points R, the given angle fixes the length QR, so the area is the same for all such choices. The other proposed areas differ by a factor or do not match this product; in particular, forgetting the factor one-half gives $2\sqrt{3}$.

Step 4: Conclusion The area is $\sqrt{3}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Locate the given angle carefully: here PQ is opposite the angle at R, so $QR = PQ/\tan 60^\circ$.

19. If $(2,3,9),(5,2,1),(1,\lambda,8)$ and $(\lambda,2,3)$ are coplanar, then the product of all possible values of λ is: [4 marks]

(A) $\frac{21}{2}$
(B) $\frac{59}{8}$
(C) $\frac{57}{8}$
(D) $\frac{95}{8}$

Correct Answer: (D) $\frac{95}{8}$

Solution:

Concept:

- Four points are coplanar exactly when the three displacement vectors from any one of them have zero scalar triple product.
- For a quadratic $a\lambda^2+b\lambda+c=0$, the product of its two roots, counted with multiplicity, is c/a. Check that the parameter roots are real when real coordinates are required.

Step 1: Form three vectors from the same point Name the points $A=(2,3,9)$, $B=(5,2,1)$, $C=(1,\lambda,8)$ and $D=(\lambda,2,3)$. Their displacement vectors from A are

$$\overrightarrow{AB}=(3,-1,-8),\quad \overrightarrow{AC}=(-1,\lambda-3,-1),\quad \overrightarrow{AD}=(\lambda-2,-1,-6).$$

All three vectors must use the same starting point. Coplanarity means that the parallelepiped determined by them has zero volume, so the determinant with these vectors as its rows vanishes.

Step 2: Expand the coplanarity determinant The required equation is

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda-3 & -1 \\ \lambda-2 & -1 & -6 \end{vmatrix}=0.$$

Expanding along the first row carefully, including the alternating cofactor signs, gives

$$\begin{aligned} 0 &= 3[-6(\lambda-3)-1]+(\lambda+4) \\ &\quad -8[1-(\lambda-3)(\lambda-2)] \\ &= 8\lambda^2-57\lambda+95. \end{aligned}$$

The middle cofactor contributes $\lambda+4$, because both the matrix entry and its cofactor sign are negative.

Step 3: Read the product and check admissibility The discriminant is $57^2-4(8)(95)=3249-3040=209>0$, so there are two distinct real parameter values. Their product is $95/8$ by the coefficient relation; computing either root separately is unnecessary. None of $21/2$, $59/8$ or $57/8$ equals this coefficient ratio. The numerator 57 belongs to the sum of roots, not their product.

Step 4: Conclusion The product of all possible values of λ is $95/8$. Hence, the correct option is (D).

Quick Tip to Solve: For a product of parameter values, derive the determinant equation and use the constant-to-leading-coefficient ratio instead of solving the quadratic.

20. The statement $(p \Rightarrow q) \vee (p \Rightarrow r)$ is NOT equivalent to [4 marks]

(A) $(p \wedge (\sim r)) \Rightarrow q$

(B) $(\sim q) \Rightarrow ((\sim r) \vee p)$

(C) $p \Rightarrow (q \vee r)$

(D) $(p \wedge (\sim q)) \Rightarrow r$

Correct Answer: (B) $(\sim q) \Rightarrow ((\sim r) \vee p)$

Solution:

Concept:

- The implication $u \Rightarrow v$ is equivalent to $(\sim u) \vee v$. De Morgan's law changes the negation of a conjunction into a disjunction.
- Equivalent statements have the same truth value for every assignment. A single truth assignment on which two statements differ proves they are not equivalent.

Step 1: Reduce the given statement Replace the two implications separately and combine the repeated disjunction:

$$(p \Rightarrow q) \vee (p \Rightarrow r) = (\sim p \vee q) \vee (\sim p \vee r)$$
$$\equiv \sim p \vee q \vee r.$$

Call this reduced expression T. The question asks for the option that is NOT equivalent to T, so identifying an equivalent expression is not sufficient to choose the answer.

Step 2: Simplify the three equivalent options For option (A), implication elimination followed by De Morgan's law gives

$$(p \wedge \sim r) \Rightarrow q \equiv \sim p \vee r \vee q = T.$$

Option (C) is immediately $p \Rightarrow (q \vee r) \equiv \sim p \vee q \vee r = T$. Similarly, option (D) becomes

$$(p \wedge \sim q) \Rightarrow r \equiv \sim p \vee q \vee r = T.$$

Thus (A), (C) and (D) all give the same condition as the printed statement for every possible truth assignment.

Step 3: Disprove equivalence of the remaining option Option (B) reduces differently:

$$(\sim q) \Rightarrow ((\sim r) \vee p) \equiv q \vee \sim r \vee p.$$

Choose p true and q, r both false. Then $T = \sim p \vee q \vee r$ is false, since all three of its terms are false. But option (B) is true, since its term p is true (and $\sim r$ is also true). This explicit difference in truth values establishes the required failure of equivalence.

Step 4: Conclusion Option (B) is not equivalent to the given statement. Hence, the correct option is (B).

Quick Tip to Solve: First replace every implication by a disjunction; for a NOT-equivalent question, confirm the exceptional option with one counterexample.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $\alpha, \beta(\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in \mathrm{N}$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to _____. [4 marks]

Solution:

Concept:

- Each root t of $t^2 - t - 4 = 0$ obeys $t^2 = t + 4$. Multiplying by t^n and subtracting the identities for the two roots generates a recurrence for their power difference.
- Factoring a bilinear expression in neighbouring sequence terms can expose exactly the differences present in that recurrence. This avoids computing very large powers.

Step 1: Derive the recurrence for P For either root $t = \alpha$ or $t = \beta$, multiply $t^2 = t + 4$ by t^n to obtain $t^{n+2} = t^{n+1} + 4t^n$. Subtract the identity for β from that for α:

$$P_{n+2} = P_{n+1} + 4P_n.$$

In particular,

$$P_{16} - P_{15} = 4P_{14}, \qquad P_{15} - P_{14} = 4P_{13}.$$

The signs of these differences follow directly from the plus sign in the recurrence; changing that sign would give a different result.

Step 2: Factor the numerator before substituting Let N denote the printed numerator. Group together terms carrying the same factor:

$$\begin{aligned} N &= P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14}) \\ &= (P_{15} - P_{14})(P_{16} - P_{15}) \\ &= (4P_{13})(4P_{14}) \\ &= 16P_{13}P_{14}. \end{aligned}$$

This uses precisely the two recurrence differences obtained above, so neither explicit radical roots nor direct expansion of fifteenth and sixteenth powers is needed.

Step 3: Check the denominator before cancellation The quadratic roots are $\alpha = (1 + \sqrt{17})/2$ and $\beta = (1 - \sqrt{17})/2$. They have opposite signs and $\alpha > |\beta| > 0$. For odd exponent 13, $P_{13} = \alpha^{13} - \beta^{13} > 0$; for even exponent 14, $\alpha^{14} > \beta^{14}$, so $P_{14} > 0$ as well. Therefore the denominator does not vanish and cancellation is valid:

$$\frac{N}{P_{13}P_{14}} = 16.$$

Step 4: Conclusion The required numerical value is 16.

Quick Tip to Solve: For power differences of quadratic roots, derive their second-order recurrence and factor the target numerator into consecutive differences.

22. Let $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ **and** $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$**. For** $k \in N$**, if** $X'A^kX = 33$**, then** k **is equal to _____.**

[4 marks]

Solution:

Concept:

- Use the parity of the exponent after computing A^2. A matrix of the form $I + cE$, where $E^2 = 0$, satisfies $(I + cE)^m = I + mcE$.
- For the all-ones column vector X, the scalar $X'BX$ is the sum of every entry of B. Here $X'X = 3$.

Step 1: Compute the square and identify the nilpotent part Direct multiplication gives

$$A^2 = \begin{pmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I + 6E, \qquad E = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The only nonzero entry of E is in position $(1, 3)$, so $E^2 = 0$. The binomial expansion therefore terminates after its linear term; no higher powers contribute.

Step 2: Solve for an even exponent Write $k = 2m$, where m is a positive integer. Then

$$A^{2m} = (I + 6E)^m = I + 6mE.$$

Since $X'EX = 1$, the given scalar condition becomes

$$X'A^{2m}X = 3 + 6m = 33.$$

Thus $m = 5$, giving the candidate $k = 10$.

Step 3: Exclude all odd exponents If $k = 2m + 1$, where $m \geq 0$, use $EA = -E$ to obtain

$$A^{2m+1} = (I + 6mE)A = A - 6mE.$$

The sum of the entries of A is 10, and hence

$$X'A^{2m+1}X = 10 - 6m \leq 10.$$

It cannot equal 33. The even candidate is therefore the only natural-number solution, and substitution gives $3 + 6(5) = 33$.

Step 4: Conclusion The required exponent is $\boxed{k = 10}$.

Quick Tip to Solve: For a triangular matrix with diagonal entries ± 1, compute its square first: it may become identity plus a square-zero matrix.

23. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits $2, 3, 4, 5, 6$ (repetition of digits is not allowed) and divisible by 55 is _____. [4 marks]

Solution:

Concept:

- A number divisible by $55 = 5 \times 11$ must satisfy both divisibility tests. With the available digits, divisibility by 5 forces its final digit to be 5.
- The difference between the sums of alternate digits must be a multiple of 11. Distinct digits are required, so each possible arrangement must use a digit at most once.

Step 1: Determine the possible lengths The interval $(1012, 23421)$ contains relevant four-digit and five-digit numbers only. A three-digit number is too small and a six-digit number is too large. Every five-digit candidate must use all five digits. The smallest such arrangement is 23456, already larger than 23421. Thus no five-digit arrangement qualifies, and only four-digit numbers need to be counted.

Step 2: Apply the divisibility test to four-digit numbers Write a four-digit candidate as $\overline{abc5}$, with a, b, c distinct members of $\{2, 3, 4, 6\}$. The 11-test gives

$$a - b + c - 5 \equiv 0 \pmod{11}.$$

Its possible values lie between -6 and 3, so the only multiple of 11 possible is zero. Consequently

$$a + c = b + 5.$$

We can count by the middle digit b; the two outer positions are ordered.

Step 3: List the valid ordered arrangements For $b = 2$, the pair $\{a, c\} = \{3, 4\}$ gives $3245, 4235$. For $b = 3$, the pair $\{2, 6\}$ gives $2365, 6325$. For $b = 4$, the pair $\{3, 6\}$ gives $3465, 6435$. For $b = 6$, the required sum is 11, impossible using two distinct digits from $\{2, 3, 4\}$. Thus the count is

$$2 + 2 + 2 + 0 = 6.$$

All listed numbers exceed 1012, lie below 23421, end in 5 and pass the 11-test. No other lengths or middle digits remain.

Step 4: Conclusion The number of qualifying natural numbers is $\boxed{6}$.

Quick Tip to Solve: First impose the final-digit restriction for divisibility by 5, then count alternate-digit sums for divisibility by 11.

24. If $[t]$ denotes the greatest integer $\leq t$, then the number of points, at which the function $f(x) = 4|2x+3| + 9\left[x+\frac{1}{2}\right] - 12[x+20]$ is not differentiable in the open interval $(-20, 20)$, is _____.

[4 marks]

Solution:

Concept:

- A differentiable function is continuous. A nonzero jump in either floor term therefore gives a point of non-differentiability unless another term cancels the jump.
- The modulus term $4|2x+3|$ is continuous everywhere and has its sole corner at $x = -3/2$. Away from its corner it is linear.

Step 1: Locate the integer jumps The floor $[x+20]$ jumps by 1 exactly when x is an integer, since 20 is an integer. Within the open interval $(-20, 20)$, these points are

$$-19, -18, \ldots, 18, 19.$$

There are 39 of them. At each, $-12[x+20]$ jumps by -12. The other floor term has no jump there and the modulus term is continuous, so none of these jumps can cancel.

Step 2: Locate the half-integer jumps The floor $[x+1/2]$ jumps at $x = m - 1/2$, with m an integer. Inside the specified interval these are

$$-19.5, -18.5, \ldots, 18.5, 19.5.$$

There are 40 such points, disjoint from the integer set. At each the term $9[x+1/2]$ has a nonzero jump of $+9$; the other floor term is locally constant and the modulus is continuous. Thus all 40 are also points of non-differentiability.

Step 3: Account for the corner without double counting The modulus changes slope only when $2x + 3 = 0$, that is, at $x = -3/2$. This is already one of the half-integers counted above, where a nonzero floor jump persists. It must not be added again. At every other point outside the two jump sets, both floors are locally constant and the modulus is locally linear, so the function is differentiable there. The endpoints are excluded by the open interval. Therefore the complete count is

$$39 + 40 = 79.$$

Step 4: Conclusion The number of points of non-differentiability is $\boxed{79}$.

Quick Tip to Solve: Count integer and shifted-integer jump sets separately, then check whether a modulus corner is already included before adding it.

25. Let AB be a chord of length 12 of the circle $(x-2)^2+(y+1)^2=\frac{169}{4}$. If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to _____. [4 marks]

Solution:

Concept:

- The perpendicular from the centre of a circle to a chord bisects that chord. The right triangle formed by a radius, half-chord and perpendicular distance determines the distance from the centre.
- If tangents at the ends of a chord meet at P, the chord of contact is perpendicular to the line joining P to the centre O. If its midpoint is M, then $OM \cdot OP = r^2$.

Step 1: Find the radius and the centre-to-chord distance The given circle has centre $O=(2,-1)$ and radius $r=13/2$. Let M be the midpoint of AB; then $AM=6$ and $OM \perp AB$. Applying Pythagoras in OMA gives

$$OM^2 = r^2 - AM^2 = \frac{169}{4} - 36 = \frac{25}{4}, \qquad OM = \frac{5}{2}.$$

The positive root is used because this is a distance.

Step 2: Locate the intersection of tangents Tangents from P have equal lengths, and $OA=OB$, so both P and O lie on the perpendicular bisector of AB. Hence O, M, P are collinear. In the right triangle OAP, the radius OA is perpendicular to tangent AP, and AM is the altitude to hypotenuse OP. The right-triangle projection relation gives

$$OA^2 = OM \cdot OP.$$

It follows that

$$OP = \frac{169/4}{5/2} = \frac{169}{10}.$$

Step 3: Subtract along the common perpendicular The foot M lies between O and P. The shortest distance from P to the chord line is therefore

$$PM = OP - OM = \frac{169}{10} - \frac{5}{2} = \frac{144}{10} = \frac{72}{5}.$$

The requested quantity is five times this distance, rather than OP or the tangent length. Thus

$$5PM = 5\left(\frac{72}{5}\right) = 72.$$

Step 4: Conclusion Five times the distance of P from chord AB is $\boxed{72}$.

Quick Tip to Solve: For a chord of length $2a$, set $d=\sqrt{r^2-a^2}$; the tangent-intersection distance from the chord is $r^2/d - d = a^2/d$.

26. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2, \vec{a}\cdot\vec{b} = 3$ and $|\vec{a}\times\vec{b}|^2 = 75$. Then $|\vec{a}|^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The squared magnitude of a sum satisfies $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}$. The dot-product term records the angle between the vectors.
- Lagrange's identity relates their cross and dot products: $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$. It eliminates the angle without finding the vectors themselves.

Step 1: Use the magnitude-of-sum condition Expand the left side of the stated equality, then equate it to the given right side:

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + 2|\vec{b}|^2.$$

After cancelling $|\vec{a}|^2$ and subtracting $|\vec{b}|^2$, this yields

$$|\vec{b}|^2 = 2\vec{a}\cdot\vec{b} = 2(3) = 6.$$

This determines the squared magnitude of $\vec{b}$, not its magnitude; keeping that distinction avoids an unnecessary square root.

Step 2: Substitute into the cross-product identity Use the specified squared cross-product magnitude 75 and dot product 3 directly:

$$75 = |\vec{a}|^2(6) - 3^2.$$

Consequently

$$6|\vec{a}|^2 = 75 + 9 = 84, \qquad |\vec{a}|^2 = \frac{84}{6} = 14.$$

The square on the dot product is essential: the term being added is 9 rather than 3.

Step 3: Check the recovered squared magnitudes The product of the squared magnitudes is $14\times6 = 84$. Subtracting the squared dot product gives $84 - 9 = 75$, exactly the prescribed squared cross-product magnitude. Also the expanded sum has value $14 + 6 + 2(3) = 26$, equal to the stated expression $14 + 2(6) = 26$. Both original vector conditions are therefore satisfied, and no direction or angle is needed.

Step 4: Conclusion The required value is $\boxed{|\vec{a}|^2 = 14}$.

Quick Tip to Solve: Use the sum identity to find one squared norm, then apply dot-cross Lagrange identity to find the other without resolving components.

27. Let

$$S = \left\{(x, y) \in \mathbb{N} \times \mathbb{N} : 9(x-3)^2 + 16(y-4)^2 \leq 144\right\}$$

and

$$T = \left\{(x, y) \in \mathbb{R} \times \mathbb{R} : (x-7)^2 + (y-4)^2 \leq 36\right\}.$$

Then $n(S \cap T)$ is equal to _____. [4 marks]

Solution:

Concept:

- The first inequality defines a closed ellipse centred at $(3, 4)$, with horizontal semiaxis 4 and vertical semiaxis 3. Only positive integer coordinates are permitted in S.
- The second inequality defines a closed disk centred at $(7, 4)$ with radius 6. Count integer points column by column and retain only those satisfying both inequalities.

Step 1: Bound the integer columns using the ellipse Rewrite the ellipse as

$$\frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} \leq 1.$$

Thus $-1 \leq x \leq 7$ and $1 \leq y \leq 7$. Since x, y are natural numbers, the possible columns are $x = 1, 2, 3, 4, 5, 6, 7$. For each column the ellipse gives

$$(y-4)^2 \leq \frac{144 - 9(x-3)^2}{16}.$$

Because the inequalities are non-strict, points on either boundary are included.

Step 2: Enumerate the allowed rows in every column Applying the ellipse bound and then the disk bound $(y-4)^2 \leq 36 - (x-7)^2$ gives

x	ellipse rows y	rows in both	count
1	$2, 3, 4, 5, 6$	4	1
2	$2, 3, 4, 5, 6$	$2, 3, 4, 5, 6$	5
3	$1, 2, 3, 4, 5, 6, 7$	$1, 2, 3, 4, 5, 6, 7$	7
4	$2, 3, 4, 5, 6$	$2, 3, 4, 5, 6$	5
5	$2, 3, 4, 5, 6$	$2, 3, 4, 5, 6$	5
6	$3, 4, 5$	$3, 4, 5$	3
7	4	4	1

For example, at $x = 1$ the disk requires $(y-4)^2 \leq 0$, so only $y = 4$ survives. For all later columns every listed ellipse row lies in the disk.

Step 3: Add the disjoint column counts Each point has exactly one integer x-coordinate, so the column counts do not overlap. All possible positive-integer columns have been exhausted. Therefore

$$n(S \cap T) = 1 + 5 + 7 + 5 + 5 + 3 + 1 = 27.$$

The extreme ellipse column $x = -1$ and the column $x = 0$ are not admissible because the definition

requires natural-number coordinates.

Step 4: Conclusion The number of points is $\boxed{n(S \cap T) = 27}$.

Quick Tip to Solve: For lattice points in an ellipse intersected with a disk, tabulate integer vertical columns; this makes boundary inclusion and positivity explicit.

28. The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. Then the number of trials in the binomial distribution is _____. [4 marks]

Solution:

Concept:

- For a binomial random variable with n trials and success probability p, the mean is $\mu = np$ and variance is $v = np(1-p)$. Thus $v \leq \mu$.
- When the sum and product of two quantities are known, they are the roots of a quadratic. Here the inequality between mean and variance identifies which root represents each quantity.

Step 1: Recover the mean and variance from their sum and product The data give $\mu + v = 82.5$ and $\mu v = 1350$. Hence each of μ, v is a root of

$$t^2 - 82.5t + 1350 = 0.$$

The factorisation is

$$(t - 60)(t - 22.5) = 0,$$

since $60 + 22.5 = 82.5$ and $60 \times 22.5 = 1350$. These two roots are positive, as required for the non-degenerate binomial parameters.

Step 2: Assign the roots and obtain the success probability A binomial variance equals its mean multiplied by $1 - p$, a number between 0 and 1. It cannot be larger than the mean. Therefore

$$\mu = 60, \qquad v = 22.5.$$

Taking the ratio gives

$$1 - p = \frac{v}{\mu} = \frac{22.5}{60} = \frac{3}{8}, \qquad p = \frac{5}{8}.$$

Interchanging the two roots would make $1 - p > 1$ and yield an invalid probability, so that assignment is excluded.

Step 3: Calculate and verify the number of trials From $np = 60$,

$$n = \frac{60}{5/8} = 60 \cdot \frac{8}{5} = 96.$$

This is a positive integer as a number of trials must be. As a final check,

$$np = 96 \cdot \frac{5}{8} = 60, \qquad np(1-p) = 60 \cdot \frac{3}{8} = 22.5.$$

Their sum is 82.5 and their product is 1350, recovering both given conditions without rounding. The question asks for n, rather than either quadratic root.

Step 4: Conclusion The number of trials is $\boxed{96}$.

Quick Tip to Solve: After finding the two roots, assign the larger one to the mean; then use $n = \mu^2/(\mu - v)$ to obtain the trial count directly.

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29. If $\sum_{k=1}^{10} K^2 \left(10_{C_K}\right)^2 = 22000L$, then L is equal to _____. [4 marks]

Solution:

Concept:

- The identity $k\binom{n}{k} = n\binom{n-1}{k-1}$ turns a weighted binomial coefficient into an ordinary one. It follows directly by cancelling factorials.
- Vandermonde's identity gives $\sum_{r=0}^{m} \binom{m}{r}^2 = \binom{2m}{m}$. It follows by comparing the coefficient of x^m in $(1+x)^m(1+x)^m$.

Step 1: Remove the squared weight Let the sum in the question be S. Apply the factorial identity separately to each of its two identical factors. Since k runs from 1 to 10, the new index $r = k - 1$ runs from 0 to 9:

$$S = \sum_{k=1}^{10} \left[k\binom{10}{k}\right]^2 = 100 \sum_{k=1}^{10} \binom{9}{k-1}^2 = 100 \sum_{r=0}^{9} \binom{9}{r}^2.$$

The factor is 100, rather than 10, because the entire weighted coefficient is squared.

Step 2: Evaluate the unweighted sum Use symmetry $\binom{9}{r} = \binom{9}{9-r}$. The sum is therefore the coefficient of x^9 in $(1+x)^9(1+x)^9 = (1+x)^{18}$. Thus

$$S = 100\binom{18}{9} = 100(48620) = 4862000.$$

This coefficient counts all possible complementary selections, so no endpoint term is omitted.

Step 3: Compare with the given multiple The printed equation is $S = 22000L$, so divide the evaluated sum by that coefficient:

$$L = \frac{4862000}{22000} = 221.$$

As a direct check, $22000 \times 221 = 4862000$, exactly the sum above.

Step 4: Conclusion The required numerical value is $\boxed{L = 221}$.

Quick Tip to Solve: Absorb a factor k into each binomial coefficient before using Vandermonde; squaring also squares the factor n.

---

30. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a, b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point (2, −1), then $|2a + 9b|$ is equal to _____. [4 marks]

Solution:

Concept:

- A tangent at $x = a$ has slope equal to the derivative there and passes through $(a, f(a))$. A common tangent must satisfy both the slope and the point condition.
- For $f(x) = x^3 - x^2 + x$, the derivative is $f\prime(x) = 3x^2 - 2x + 1$; for $g(x) = 5x^2 + 2x - 25$, it is $g\prime(x) = 10x + 2$.

Step 1: Find the fixed tangent line At the specified point of the parabola, the slope is $g\prime(2) = 22$. The point lies on the curve since $g(2) = 20 + 4 - 25 = -1$. Hence its tangent is

$$y + 1 = 22(x - 2), \qquad y = 22x - 45.$$

Both the slope and the intercept of this particular line must be retained when checking the cubic.

Step 2: Find the candidate points of tangency For the cubic tangent to have slope 22, its abscissa must satisfy

$$3a^2 - 2a + 1 = 22, \qquad 3a^2 - 2a - 21 = (a - 3)(3a + 7) = 0.$$

Thus $a = 3$ or $a = -7/3$. Equal slopes alone establish only parallel tangents, so these two candidates still require the intercept check.

Step 3: Enforce the same intercept Since $b = a^3 - a^2 + a$, the cubic tangent's intercept is $b - 22a$. Using its derivative condition, it can equivalently be written as $-2a^3 + a^2$. At $a = 3$, this is $-54 + 9 = -45$, which agrees with the fixed line. At $a = -7/3$, it is $833/27$, which does not agree. Therefore only $a = 3$ gives the common tangent, and

$$b = 27 - 9 + 3 = 21, \qquad |2a + 9b| = |6 + 189| = 195.$$

Step 4: Conclusion The common tangency point is $(a, b) = (3, 21)$, giving $\boxed{195}$.

Quick Tip to Solve: After matching derivatives, check the tangent intercept: equal slopes can produce different parallel lines.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Two identical metallic spheres A and B when placed at certain distance in air repel each other with a force of F. Another identical uncharged sphere C is first placed in contact with A and then in contact with B and finally placed at midpoint between spheres A and B. The force experienced by sphere C will be: [4 marks]

(A) 3F/2

(B) 3F/4

(C) F

(D) 2F

Correct Answer: (B) 3F/4

Solution:

Concept:

- Identical isolated conducting spheres share their total charge equally on contact. Coulomb force between charges separated by r has magnitude $k_e|q_1q_2|/r^2$.
- The intended equal-charge arrangement takes the initial charges on A and B as q each. At the midpoint, their repulsive forces on C point in opposite directions and must be subtracted.

Step 1: Track both contacts Write the original separation as r, so the original force is $F = k_e q^2/r^2$. When the initially uncharged C contacts A, conservation of charge gives $q/2$ on each. C then touches B, whose charge is still q. Their total $3q/2$ is divided equally. Thus the final charges are

$$q_A = \frac{q}{2}, \qquad q_B = q_C = \frac{3q}{4}.$$

The charges remain on the spheres when C is subsequently placed midway between A and B.

Step 2: Calculate the opposite midpoint forces The distance from C to either endpoint sphere is $r/2$, so each Coulomb force has a factor 4 relative to using separation r. Their magnitudes are

$$F_{A\to C} = \frac{k_e(q/2)(3q/4)}{(r/2)^2} = \frac{3F}{2}, \qquad F_{B\to C} = \frac{k_e(3q/4)^2}{(r/2)^2} = \frac{9F}{4}.$$

The repulsion from B is larger, so the net force on C points towards A.

Step 3: Subtract and compare the choices The resultant magnitude is

$$F_C = \frac{9F}{4} - \frac{3F}{2} = \frac{3F}{4}.$$

Choice (A) is only the contribution from A; choice (C) and choice (D) do not equal the difference of the two actual forces. Adding the magnitudes would also be incorrect because the forces at a midpoint oppose each other.

Step 4: Check the qualification in the wording Identical spheres have equal radii, but this does not by itself fix their initial charges. If the initial charges are $u, v > 0$, the contacts leave $u/2$ on A and

$(u + 2v)/4$ on both B and C. Therefore

$$\frac{F_C}{F} = \frac{|4v^2 - u^2|}{4uv}.$$

This is $3/4$ when $u = v$, but is zero when $u = 2v$. Thus the listed answer uses equal initial charges, an assumption omitted from the literal statement.

Step 5: Conclusion Under the intended equal-initial-charge interpretation, the correct option is (B), $\boxed{3F/4}$, directed towards A. As literally worded without equal initial charges, the force is not uniquely determined.

Quick Tip to Solve: Write the charge on every sphere after each contact, then use the halved midpoint distance before subtracting the forces.

32. Match List I with List II.

	List I		List II
A.	**Torque**	**I.**	$\mathrm{N\,m\,s^{-1}}$
B.	**Stress**	**II.**	$\mathrm{J\,kg^{-1}}$
C.	**Latent Heat**	**III.**	$\mathrm{N\,m}$
D.	**Power**	**IV.**	$\mathrm{N\,m^{-2}}$

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-II, C-I, D-IV

(B) A-III, B-IV, C-II, D-I

(C) A-IV, B-I, C-III, D-II

(D) A-II, B-III, C-I, D-IV

Correct Answer: (B) A-III, B-IV, C-II, D-I

Solution:

Concept:

- Derive a compound SI unit from the defining equation of its physical quantity. A newton (N) measures force, a joule (J) measures energy, and a second (s) measures time.
- Torque is force times perpendicular distance, stress is force per area, specific latent heat is energy per mass, and power is work per time.

Step 1: Match torque and stress The magnitude of torque is $\tau = Fr_\perp$, so its SI unit is $\mathrm{N\,m}$. This is entry III in List II, and hence A matches III. Although the dimensions coincide with energy, torque is conventionally expressed in newton metre. For stress,

$$\sigma = \frac{F}{A}, \qquad [\sigma] = \mathrm{N\,m^{-2}}.$$

Thus B matches IV. Stress is force divided by area, not force multiplied by a distance.

Step 2: Match latent heat and power The latent-heat entry uses the specific latent heat L, defined by

$Q = mL$, so

$$L = \frac{Q}{m}, \qquad [L] = \mathrm{J\,kg^{-1}}.$$

This gives C-II. Power is the rate of doing work:

$$P = \frac{W}{t}, \qquad [P] = \mathrm{J\,s^{-1}} = \mathrm{N\,m\,s^{-1}}.$$

It therefore gives D-I. Do not confuse the total heat Q, measured in joules, with the specific quantity listed here.

Step 3: Assemble the complete matching The four independently derived matches combine to give

$$\mathrm{A-III}, \quad \mathrm{B-IV}, \quad \mathrm{C-II}, \quad \mathrm{D-I}.$$

This is exactly choice (B). Choice (A) incorrectly assigns stress an energy-per-mass unit, choice (C) gives torque a pressure unit, and choice (D) gives torque an energy-per-mass unit; each fails a direct defining-equation check.

Step 4: Conclusion The matching is A-III, B-IV, C-II, D-I. Hence, the correct option is (B).

Quick Tip to Solve: Use the defining equation to derive each unit; recognise that the latent-heat unit here is per kilogram.

33. Two identical thin metal plates has charge q_1 and q_2 respectively such that $q_1 > q_2$. The plates were brought close to each other to form a parallel plate capacitor of capacitance C. The potential difference between them is : [4 marks]

(A) $\frac{(q_1+q_2)}{C}$
(B) $\frac{(q_1-q_2)}{C}$
(C) $\frac{(q_1-q_2)}{2C}$
(D) $\frac{2(q_1-q_2)}{C}$

Correct Answer: (C) $\frac{(q_1-q_2)}{2C}$

Solution:

Concept:

- For two large parallel plates of area A separated by d, neglecting edge effects, the capacitance is $C = \varepsilon_0 A/d$. The voltage is the electric field in the gap times d.
- A uniformly charged sheet produces field $\sigma/(2\varepsilon_0)$ on either side. In the gap, the fields associated with the two net plate charges oppose, so their difference determines the voltage.

Step 1: Resolve the field between the plates Take the positive direction from the plate carrying q_1 towards the plate carrying q_2. The net-charge sheets contribute $q_1/(2\varepsilon_0 A)$ and $-q_2/(2\varepsilon_0 A)$ in this direction. Thus

$$E_{\mathrm{gap}} = \frac{q_1 - q_2}{2\varepsilon_0 A}.$$

This can also be understood by splitting the charges into a common part $(q_1 + q_2)/2$ on each plate

and opposite differential parts $\pm(q_1 - q_2)/2$. Only the differential parts produce the capacitor's gap voltage.

Step 2: Convert the field into voltage The plates are equipotentials and their separation is d, so

$$V_1 - V_2 = E_{\text{gap}} d = \frac{(q_1 - q_2)d}{2\varepsilon_0 A} = \frac{q_1 - q_2}{2C}.$$

The stated inequality $q_1 > q_2$ makes this voltage positive in the chosen order. The charge entering $Q = CV$ is the differential charge $(q_1 - q_2)/2$, not the full difference.

Step 3: Check the factors and alternatives For equal plate charges the voltage must vanish, immediately excluding a sum $q_1 + q_2$. As another check, put $q_1 = Q$ and $q_2 = -Q$: the result becomes Q/C, the familiar capacitor voltage. Omitting the factor 2 would instead predict $2Q/C$, and the factor in option (D) is larger still. Only option (C) passes both checks.

Step 4: Conclusion The potential difference is $\boxed{(q_1 - q_2)/(2C)}$. Hence, the correct option is (C).

Quick Tip to Solve: Decompose the net charges into a common charge and opposite differential charges; use only the differential charge in Q=CV.

34. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Alloys such as constantan and manganin are used in making standard resistance coils.

Reason R: Constantan and manganin have very small value of temperature coefficient of resistance.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Correct Answer: (A) Both A and R are true and R is the correct explanation of A.

Solution:

Concept:

- For a small temperature change, resistance obeys $R(T) = R(T_0)[1 + \alpha(T - T_0)]$, where α is the temperature coefficient of resistance.
- A standard resistance should remain as constant as possible when the ambient temperature changes. Constantan and manganin are alloys with a very small temperature coefficient near ordinary operating temperatures.

Step 1: Evaluate the assertion Standard resistance coils provide a stable reference resistance in electrical measurements. If their resistance drifted strongly with room temperature or small heating currents, a calibration made at one temperature would no longer be reliable at another. Constantan and manganin are used for these coils because their resistance remains comparatively stable over a

useful temperature range. The stated practical use is therefore correct, so Assertion A is true.

Step 2: Evaluate the reason The fractional resistance change in the linear approximation is

$$\frac{\Delta R}{R(T_0)} = \alpha \Delta T.$$

For the same temperature change ΔT, a much smaller magnitude of α gives a much smaller fractional change. Constantan and manganin have precisely this small coefficient compared with ordinary pure conducting metals in the relevant range. Reason R is therefore also true. A small coefficient does not mean zero resistance; it means weak temperature dependence.

Step 3: Test the explanatory connection The property named in R directly supplies the stability required for the application named in A. Therefore R explains why these alloys are chosen for standard coils, rather than merely stating an unrelated material property. Choice (B) misses this causal link, choice (C) wrongly rejects the small-coefficient property, and choice (D) wrongly rejects the actual use of these alloys. The two truth values and their connection select choice (A).

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: For resistance standards, look for a small temperature coefficient; high resistance by itself does not guarantee temperature stability.

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35. A $1\ \mathrm{m}$ long wire is broken into two unequal parts X and Y. The X part of the wire is streched into another wire W. Length of W is twice the length of X and the resistance of W is twice that of Y. Find the ratio of length of X and Y. [4 marks]

(A) 1 : 4

(B) 1 : 2

(C) 4 : 1

(D) 2 : 1

Correct Answer: (B) 1 : 2

Solution:

Concept:

- For a uniform wire, $R = \rho\ell/A$, with resistivity ρ, length ℓ and cross-sectional area A. Initially both cut pieces have the same material and area.
- Stretching conserves the wire volume when no material is lost. Hence doubling a piece's length halves its area, making its resistance four times the original value at unchanged resistivity.

Step 1: Express the initial resistances Let the initial lengths be x and y, so $x + y = 1\ \mathrm{m}$. With original cross-sectional area A,

$$R_X = \frac{\rho x}{A}, \qquad R_Y = \frac{\rho y}{A}.$$

Their resistance ratio initially equals their length ratio, because cutting a uniform wire changes neither the cross-section nor the material resistivity.

Step 2: Account for stretching both dimensions The stretched wire W has length $2x$. Volume conservation gives

$$Ax = A_W(2x), \qquad A_W = \frac{A}{2}.$$

Therefore its resistance is

$$R_W = \frac{\rho(2x)}{A/2} = \frac{4\rho x}{A} = 4R_X.$$

Using only the doubled length while leaving the area unchanged would give an incorrect factor of two. The statement concerns a stretched piece, not a new wire with the original cross-section.

Step 3: Apply the stated resistance relation Given $R_W = 2R_Y$, substitute the two expressions:

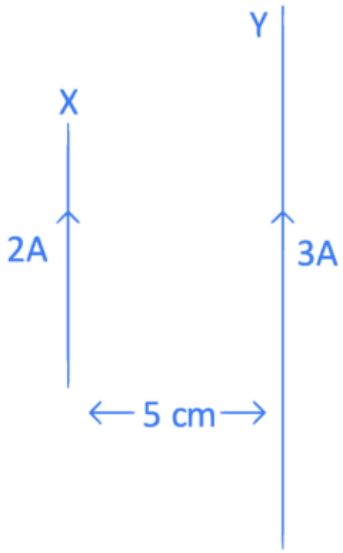
$$\frac{4\rho x}{A} = \frac{2\rho y}{A}, \qquad 4x = 2y, \qquad x : y = 1 : 2.$$

The total length gives $x = 1/3\,\text{m}$ and $y = 2/3\,\text{m}$, confirming both are positive unequal pieces. Choice (A) would make $R_W = R_Y$, while choices (C) and (D) make the stretched resistance far too large. Choice (B) alone gives the required factor of two.

Step 4: Conclusion The ratio of the original lengths is $\boxed{X : Y = 1 : 2}$. Hence, the correct option is (B).

Quick Tip to Solve: For volume-conserving stretching, resistance scales as the square of the length multiplier, so doubling length multiplies resistance by four.

36. A wire X of length $50\ \text{cm}$ carrying a current of $2\ \text{A}$ is placed parallel to a long wire Y of length $5\ \text{m}$. The wire Y carries a current of $3\ \text{A}$. The distance between two wires is $5\ \text{cm}$ and currents flow in the same direction. The force acting on the wire Y is [4 marks]



(A) 1.2×10^{-5} N directed towards wire X.

(B) 1.2×10^{-4} N directed away from wire X.

(C) 1.2×10^{-4} N directed towards wire X.

(D) 2.4×10^{-5} N directed towards wire X.

Correct Answer: (A) 1.2×10^{-5} N directed towards wire X.

Solution:

Concept:

- A long straight wire carrying current I_Y produces the field $B_Y = \mu_0 I_Y/(2\pi d)$ at perpendicular distance d, where $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$.
- A straight wire of length l, perpendicular to a magnetic field, experiences force $F = IlB$. Parallel currents in the same direction attract; the two wires exert equal and opposite forces on one another.

Step 1: Use the length of the shorter wire The 5 m wire Y is long compared with the 0.50 m wire X and their 0.05 m separation. Calculate its approximately uniform field along X. The interacting length in $I_X l_X B_Y$ is $l_X = 0.50\,\mathrm{m}$, not the full 5 m length of Y. This gives the force on X first; action and reaction then give the required force on Y.

Step 2: Calculate the mutual force Insert the currents and separation in SI units:

$$B_Y = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times 0.05} = 1.2 \times 10^{-5}\,\mathrm{T}.$$

$$F_X = 2 \times 0.50 \times 1.2 \times 10^{-5} = 1.2 \times 10^{-5}\,\mathrm{N}.$$

Consequently $F_Y = F_X$ in magnitude. Using 5 m instead of 0.50 m would overestimate the force tenfold.

Step 3: Determine the direction and compare the options Same-direction currents attract, so Y is pulled towards X. Option B has both an incorrect magnitude and repulsive direction. Option C incorrectly uses the long-wire length, while D doubles the correct force without justification. Only A has the correct magnitude and direction.

Step 4: Conclusion The force on Y is $1.2 \times 10^{-5}\,\mathrm{N}$, directed towards X. Hence, the correct option is (A).

Quick Tip to Solve: For unequal parallel wires, find the force on the shorter wire in the long wire's field, then use equal and opposite mutual forces.

37. A juggler throws balls vertically upwards with same initial velocity in air. When the first ball reaches its highest position, he throws the next ball. Assuming the juggler throws n balls per second, the maximum height the balls can reach is [4 marks]

(A) g/2n

(B) g/n

(C) 2gn

(D) $g/2n^2$

Correct Answer: (D) $g/2n^2$

Solution:

Concept:

- For vertical projection at speed u, the upward velocity is $v = u - gt$; at the highest point $v = 0$, giving ascent time $t_{\text{up}} = u/g$.
- The maximum height above the launch point is $H = u^2/(2g)$. A rate of n launches per second means the interval between successive launches is $1/n$, not n.

Step 1: Relate the launch interval to the ascent time The next ball is thrown just when the preceding ball reaches its highest point. Thus the time between two successive throws equals the time taken for one ball to rise to its apex. Because the throwing rate is n per second,

$$t_{\text{up}} = \Delta t = \frac{1}{n}.$$

Use the ascent time alone: the time for a complete up-and-down flight would be twice as large and is not the specified interval.

Step 2: Find the common launch speed Set the velocity equal to zero at the apex:

$$0 = u - g\left(\frac{1}{n}\right) \quad \Longrightarrow \quad u = \frac{g}{n}.$$

Every ball is thrown with this same initial speed, so every ball has the same maximum rise. The acceleration is downward throughout the flight, including at the instant when its velocity is zero.

Step 3: Substitute in the height formula Apply $v^2 = u^2 - 2gH$ with $v = 0$:

$$H = \frac{u^2}{2g} = \frac{(g/n)^2}{2g} = \frac{g}{2n^2}.$$

Since n has units s^{-1}, this expression has units of length. Options A and B have the wrong power of the launch rate and option C increases with that rate, contrary to the shorter ascent time.

Step 4: Conclusion The maximum height is $\boxed{g/(2n^2)}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert a throwing frequency to its reciprocal time interval before using the equations of vertical motion; here the interval is the ascent time.

38. A circuit element X when connected to an a.c. supply of peak voltage 100 V gives a peak current of 5 A which is in phase with the voltage. A second element Y when connected to the same a.c. supply also gives the same value of peak current which lags behind the voltage by $\frac{\pi}{2}$. If X and Y are connected in series to the same supply, what will be the rms value of the current in ampere? [4 marks]

(A) $\frac{10}{\sqrt{2}}$

(B) $\frac{5}{\sqrt{2}}$

(C) $5\sqrt{2}$

(D) $\frac{5}{2}$

Correct Answer: (D) $\frac{5}{2}$

Solution:

Concept:

- Current in phase with voltage identifies a pure resistance: $R = V_0/I_0$, with both voltage and current taken as peak values.
- Current lagging voltage by $\pi/2$ identifies an ideal inductor. In a series RL circuit, $Z = \sqrt{R^2 + X_L^2}$ and $I_{\text{rms}} = V_0/(\sqrt{2}Z)$.

Step 1: Identify the two elements For X, current and voltage are in phase; its opposition is resistive. For Y, current lags voltage by a quarter cycle; its opposition is inductive, rather than capacitive (which would make current lead). Both are tested with the same peak voltage and each gives the same peak current, so

$$R = \frac{100}{5} = 20\,\Omega, \qquad X_L = \frac{100}{5} = 20\,\Omega.$$

These are properties at the given supply frequency.

Step 2: Combine their impedances in series The resistor and inductor carry the same instantaneous current in series. Their voltage phasors are perpendicular, so their opposition cannot be added as two ordinary resistances. Instead,

$$Z = \sqrt{20^2 + 20^2} = 20\sqrt{2}\,\Omega.$$

The resulting phase angle is $45°$; this also checks that both contributions are equally large.

Step 3: Convert the series peak current to rms The supply still has peak voltage 100 V. Therefore

$$I_0 = \frac{100}{20\sqrt{2}} = \frac{5}{\sqrt{2}}\,\text{A}, \qquad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{5}{2}\,\text{A}.$$

Option B is the series peak current, not its rms value. Options A and C exceed even the original 5 A peak current, although adding an element in series increases the impedance.

Step 4: Conclusion The rms current is $\boxed{2.5\,\text{A}}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep peak and rms values separate: first combine the resistance and reactance in quadrature, then divide the peak current by $\sqrt{2}$.

39. An unpolarised light beam of intensity $2I_0$ is passed through a polaroid P and then through another polaroid Q which is oriented in such a way that its passing axis makes an angle of 30° relative to that of P. The intensity of the emergent light is [4 marks]

(A) $\frac{I_0}{4}$

(B) $\frac{I_0}{2}$

(C) $\frac{3I_0}{4}$

(D) $\frac{3I_0}{2}$

Correct Answer: (C) $\frac{3I_0}{4}$

Solution:

Concept:

- An ideal polaroid transmits half the incident intensity of unpolarised light, because the squared component along its passing axis averages to $1/2$.
- For linearly polarised light incident on an analyser at angle θ, Malus law gives $I_{\text{out}} = I_{\text{in}} \cos^2\theta$, where the angle is between the two passing axes.

Step 1: Account for the first polaroid The incident intensity is $2I_0$, and the beam is initially unpolarised. After passing through P it becomes linearly polarised along the passing axis of P, with intensity

$$I_P = \frac{2I_0}{2} = I_0.$$

This first factor of one-half is necessary before applying the angle-dependent transmission at Q. It does not mean that every later polaroid must halve the beam again.

Step 2: Apply Malus law at the second polaroid The incident beam at Q is already polarised. The axis of Q makes 30° with the polarisation direction, so

$$I_Q = I_P \cos^2 30^\circ = I_0 \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3I_0}{4}.$$

The squared cosine describes the intensity reduction because intensity is proportional to the square of the transmitted electric-field amplitude.

Step 3: Check the result against the alternatives The result must be less than $I_P = I_0$, since a passive analyser cannot amplify the beam. Thus option D is impossible and corresponds to forgetting the first polaroid's reduction. Option A would correspond to 60°, while option B would correspond to 45°, rather than the stated angle. The value $3I_0/4$ is consistent with nearly aligned axes.

Step 4: Conclusion The emergent intensity is $\boxed{3I_0/4}$. Hence, the correct option is (C).

Quick Tip to Solve: Halve the intensity once for initially unpolarised light; after that, apply the squared cosine of each relative axis angle.

40. An α particle and a proton are accelerated from rest through the same potential difference. The ratio of linear momenta acquired by above two particles will be: [4 marks]

(A) $\sqrt{2}:1$

(B) $2\sqrt{2}:1$

(C) $4\sqrt{2}:1$

(D) $8:1$

Correct Answer: (B) $2\sqrt{2}:1$

Solution:

Concept:

- A particle of charge q, accelerated from rest through a potential difference V, acquires kinetic energy $K = qV$.
- In the nonrelativistic regime, $K = p^2/(2m)$, so $p = \sqrt{2mqV}$. An alpha particle has charge $2e$ and mass approximately $4m_p$, where m_p is the proton mass.

Step 1: Express momentum in terms of mass and charge The work done by the electric field becomes kinetic energy because the particles start from rest. Combining the energy and momentum relations gives

$$\frac{p^2}{2m} = qV \quad \Longrightarrow \quad p = \sqrt{2mqV}.$$

Equal accelerating potential differences do not imply equal kinetic energies for particles with different charges. In particular, the alpha particle gains twice as much energy as the proton.

Step 2: Insert the particle properties in the ratio The common potential difference and the factor 2 cancel between the two momenta:

$$\frac{p_\alpha}{p_p} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{(4m_p)(2e)}{m_p e}} = \sqrt{8} = 2\sqrt{2}.$$

Both the greater mass and the greater charge contribute inside the square root; neither ratio should be multiplied outside it without taking that root.

Step 3: Eliminate the other ratios Option A accounts only for the charge ratio and ignores the alpha particle's fourfold mass. Option D gives the product of mass and charge ratios without the required square root. Option C introduces an extra factor of two. The ratio is dimensionless, as it must be when comparing momenta, and is independent of the common accelerating voltage.

Step 4: Conclusion The momentum ratio is $\boxed{p_\alpha : p_p = 2\sqrt{2}:1}$. Hence, the correct option is (B).

Quick Tip to Solve: At equal accelerating voltage compare $\sqrt{mq}$; at equal kinetic energy compare only $\sqrt{m}$.

41. An object of mass $1\ \text{kg}$ is taken to a height from the surface of earth which is equal to three times the radius of earth. The gain in potential energy of the object will be [If, $g = 10\ \text{ms}^{-2}$ and radius of earth $= 6400\ \text{km}$] [4 marks]

(A) 48 MJ

(B) 24 MJ

(C) 36 MJ

(D) 12 MJ

Correct Answer: (A) 48 MJ

Solution:

Concept:

- Outside a spherical Earth, gravitational potential energy with zero at infinity is $U(r) = -GMm/r$, where r is measured from Earth's centre.
- The surface acceleration satisfies $g = GM/R^2$. For heights comparable to the Earth's radius, use the inverse-distance potential rather than the near-surface approximation mgh.

Step 1: Locate the initial and final positions The object starts at the surface, so $r_i = R$. Its height above the surface is $h = 3R$, hence its final distance from the centre is

$$r_f = R + h = 4R.$$

The stated height is not the final centre-to-object distance. Confusing these two distances changes the potential-energy difference.

Step 2: Calculate the change in potential energy Subtract the initial negative potential energy from the final negative potential energy:

$$\Delta U = -\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right) = \frac{3GMm}{4R}.$$

With $GM = gR^2$, this becomes

$$\Delta U = \frac{3}{4}mgR.$$

The positive sign is expected: lifting the object works against gravity and moves its energy closer to zero.

Step 3: Evaluate and compare with the options Convert the radius to metres before substitution:

$$R = 6400\,\text{km} = 6.4 \times 10^6\,\text{m}.$$

$$\Delta U = \frac{3}{4}(1)(10)(6.4 \times 10^6) = 4.8 \times 10^7\,\text{J} = 48\,\text{MJ}.$$

The energy needed to take this mass all the way to infinity is $mgR = 64\,\text{MJ}$, so the calculated gain is smaller, as required. The other listed gains do not equal the inverse-distance potential difference for $r_f = 4R$.

Step 4: Conclusion The gain in potential energy is $\boxed{48\,\text{MJ}}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the final radius as $R + h$ before calculating gravitational potential energy; use mgh only when $h \ll R$.

42. A ball is released from a height h. If t_1 and t_2 be the time required to complete first half and second half of the distance respectively. Then, choose the correct relation between t_1 and t_2. [4 marks]

(A) $t_1 = (\sqrt{2})t_2$

(B) $t_1 = (\sqrt{2}-1)t_2$

(C) $t_2 = (\sqrt{2}+1)t_1$

(D) $t_2 = (\sqrt{2}-1)t_1$

Correct Answer: (D) $t_2 = (\sqrt{2}-1)t_1$

Solution:

Concept:

- A body released from rest falls through distance $s = \frac{1}{2}gt^2$ after elapsed time t, assuming uniform gravitational acceleration and negligible air resistance.
- The time for the second segment of a fall is the total elapsed time minus the time for the first segment. Equal distances need not take equal times when the speed changes.

Step 1: Find the time for the first half The ball begins at rest and covers $h/2$ in time t_1. Therefore

$$\frac{h}{2} = \frac{1}{2}gt_1^2 \quad \Longrightarrow \quad t_1 = \sqrt{\frac{h}{g}}.$$

At this point the ball is already moving downward. The next half of the fall therefore starts with a nonzero speed, unlike the first half.

Step 2: Find the total time of the fall Let T denote the elapsed time from release until the ball has covered the full height h. Applying the same equation from the original release point gives

$$h = \frac{1}{2}gT^2 \quad \Longrightarrow \quad T = \sqrt{\frac{2h}{g}} = \sqrt{2}\,t_1.$$

This total time includes both halves: $T = t_1 + t_2$. It must not be identified directly with t_2.

Step 3: Subtract to obtain the second-half time Use the interval relation:

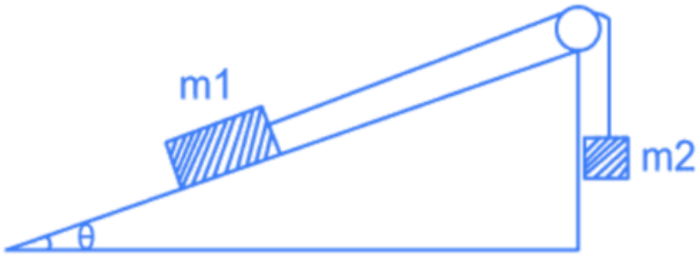
$$t_2 = T - t_1 = (\sqrt{2}-1)t_1.$$

Since $\sqrt{2}-1 \simeq 0.414$, the second half takes less time than the first, consistent with increasing speed. Options B and C instead make the second interval longer; option A predicts a different ratio. No height or numerical value of g remains because both intervals scale with $\sqrt{h/g}$.

Step 4: Conclusion The required relation is $\boxed{t_2 = (\sqrt{2}-1)t_1}$. Hence, the correct option is (D).

Quick Tip to Solve: For successive equal distances in free fall, calculate cumulative times from release and subtract; never restart with zero speed at the midpoint.

43. Two bodies of masses $m_1 = 5$ kg and $m_2 = 3$ kg are connected by a light string going over a smooth light pulley on a smooth inclined plane as shown in the figure. The system is at rest. The force exerted by the inclined plane on the body of mass m_1 will be : [Take $g = 10\ ms^{-2}$] [4 marks]



(A) 30 N

(B) 40 N

(C) 50 N

(D) 60 N

Correct Answer: (B) 40 N

Solution:

Concept:

- For a body in equilibrium, the resultant force is zero along each independent direction. A smooth plane exerts only a normal reaction, perpendicular to its surface.
- A light string over a smooth light pulley has the same tension throughout. Resolve the weight of the block on an incline into components parallel and perpendicular to the plane.

Step 1: Find the tension from the hanging body The hanging mass is at rest, so its upward tension balances its weight. Hence

$$T = m_2 g = 3 \times 10 = 30\,\text{N}.$$

There is no acceleration to include in this equation; the pulley merely redirects the string tension.

Step 2: Use equilibrium along the incline Let the inclination be θ, as indicated in the figure. For the mass on the plane, tension acts up the slope and the component of weight acts down it. Thus

$$m_1 g \sin\theta = T, \qquad 50 \sin\theta = 30.$$

Therefore $\sin\theta = 3/5$ and, since the incline is acute, $\cos\theta = 4/5$.

Step 3: Calculate the force exerted by the plane The string is parallel to the plane, so it contributes no perpendicular component. Normal equilibrium gives

$$N = m_1 g \cos\theta = 50\left(\frac{4}{5}\right) = 40\,\text{N}.$$

The value 30 N is the tension, not the plane reaction. The value 50 N is the full weight before resolving it. A reaction of 60 N would exceed the weight even though there is no extra force pressing the block into the plane.

Step 4: Conclusion The plane exerts a normal force of $\boxed{40\,\text{N}}$ on m_1. Hence, the correct option is (B).

Quick Tip to Solve: When both bodies are at rest, first use the hanging mass to find tension; then resolve the inclined block weight to obtain the normal reaction.

44. If momentum of a body is increased by 20%, then its kinetic energy increases by [4 marks]

(A) 36%

(B) 40%

(C) 44%

(D) 48%

Correct Answer: (C) 44%

Solution:

Concept:

- For a nonrelativistic body of fixed mass m, momentum is $p = mv$ and kinetic energy is $K = \frac{1}{2}mv^2 = p^2/(2m)$.
- For percentage changes, compare the final value with the original value. The percentage increase is $(K_f/K_i - 1) \times 100$, rather than the final percentage itself.

Step 1: Express the new momentum as a multiple Let the initial momentum magnitude be p. Increasing it by 20% gives

$$p_f = p + 0.20p = 1.20p.$$

The mass remains the same because the question changes the momentum of the same body. Consequently its speed also becomes 1.20 times its original speed.

Step 2: Find the kinetic-energy ratio Use the squared dependence on momentum while keeping the mass fixed:

$$\frac{K_f}{K_i} = \frac{p_f^2/(2m)}{p^2/(2m)} = \left(\frac{p_f}{p}\right)^2 = (1.20)^2 = 1.44.$$

Thus the final energy is 144% of the original energy. Only the excess over 100% is the increase requested.

Step 3: Convert the ratio into the percentage increase The fractional change is

$$\frac{K_f - K_i}{K_i} = 1.44 - 1 = 0.44,$$

so the percentage increase is $0.44 \times 100 = 44\%$. The approximation $2 \times 20\% = 40\%$ omits the square term and is not exact here. Indeed, for a fractional momentum increase x, the energy increase is $2x + x^2$. With $x = 0.20$, the additional $x^2 = 0.04$ contributes four percentage points. The proposed increases 36% and 48% also disagree with the exact ratio 1.44.

Step 4: Conclusion The kinetic energy increases by $\boxed{44\%}$. Hence, the correct option is (C).

Quick Tip to Solve: For a fixed mass, square the momentum multiplier: a 20% rise means multiplying kinetic energy by 1.2^2, then subtracting the original 100%.

45. The torque of a force $5\hat{i}+3\hat{j}-7\hat{k}$ about the origin is τ. If the force acts on a particle whose position vector is $2i+2j+k$, then the value of τ will be [4 marks]

(A) $11\hat{i}+19\hat{j}-4\hat{k}$

(B) $-11\hat{i}+9\hat{j}-16\hat{k}$

(C) $-17\hat{i}+19\hat{j}-4\hat{k}$

(D) $17\hat{i}+9\hat{j}+16\hat{k}$

Correct Answer: (C) $-17\hat{i}+19\hat{j}-4\hat{k}$

Solution:

Concept:

- The torque of a force about the origin is the vector cross product $\boldsymbol{\tau}=\mathbf{r}\times\mathbf{F}$, where $\mathbf{r}$ points from the origin to the point of application.
- The cross product follows the right-hand rule and is perpendicular to both vectors. Its order matters: $\mathbf{F}\times\mathbf{r}=-\mathbf{r}\times\mathbf{F}$.

Step 1: Insert the vector components The position and force vectors have components

$$\mathbf{r}=(2,2,1), \qquad \mathbf{F}=(5,3,-7).$$

The minus sign in the third force component must be retained. Using the determinant form,

$$\boldsymbol{\tau}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & 1\\ 5 & 3 & -7\end{vmatrix}.$$

The middle cofactor carries a minus sign.

Step 2: Evaluate each component The component form avoids ambiguity in the determinant signs:

$$\begin{aligned}\tau_x &= r_yF_z-r_zF_y=2(-7)-1(3)=-17,\\ \tau_y &= r_zF_x-r_xF_z=1(5)-2(-7)=19,\\ \tau_z &= r_xF_y-r_yF_x=2(3)-2(5)=-4.\end{aligned}$$

Combining the components gives the torque vector in the same Cartesian basis as the data.

Step 3: Check the result and compare the options As a sign and arithmetic check, the dot products are

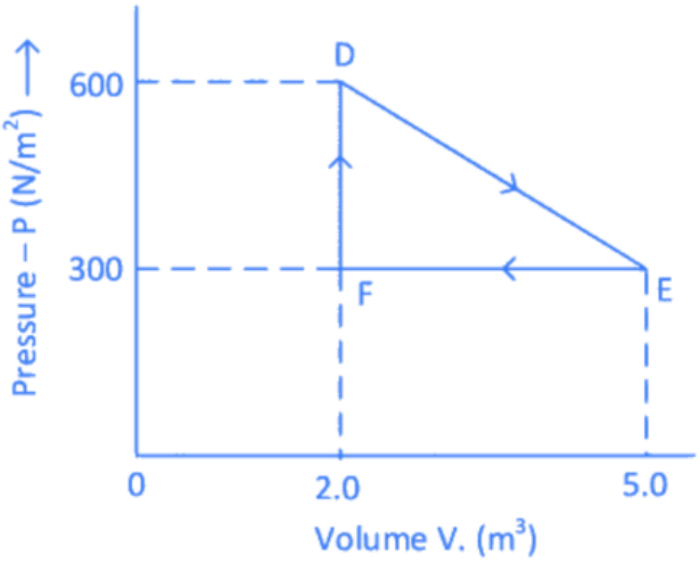
$$\boldsymbol{\tau}\cdot\mathbf{r}=-34+38-4=0, \qquad \boldsymbol{\tau}\cdot\mathbf{F}=-85+57+28=0.$$

Both vanish, as required for a cross product. Only option (C) has all three computed components. Option (A) has the wrong first component, while (B) and (D) have incorrect second and third components; they cannot represent the required torque about the origin.

Step 4: Conclusion $\boxed{\boldsymbol{\tau}=-17\hat{i}+19\hat{j}-4\hat{k}}$, in torque units corresponding to the supplied force and position units. Hence, the correct option is (C).

Quick Tip to Solve: Calculate torque as position cross force, then check that the result has zero dot product with both input vectors to catch component-sign errors.

46. A thermodynamic system is taken from an original state D to an intermediate state E by the linear process shown in the figure. Its volume is then reduced to the original volume from E to F by an isobaric process. The total work done by the gas from D to E to F will be [4 marks]



(A) -450 J

(B) 450 J

(C) 900 J

(D) 1350 J

Correct Answer: (B) 450 J

Solution:

Concept:

- The work done by a gas in a quasistatic process is $W = \int P\,dV$. Expansion contributes positive work; compression contributes negative work.
- On a pressure-volume diagram, work is the signed area under the process curve. For a straight line, the area is the mean of the endpoint pressures multiplied by the volume change.

Step 1: Read the endpoint coordinates The graph gives $D : (V = 2.0\,\mathrm{m}^3, P = 600\,\mathrm{N\,m^{-2}})$, $E : (5.0, 300)$, and $F : (2.0, 300)$ in the same SI units. The gas expands by $3.0\,\mathrm{m}^3$ from D to E, then compresses by the same volume from E to F. No work is associated with a vertical return F to D, were one added, because volume would then be constant.

Step 2: Calculate the work along the straight expansion Pressure falls linearly, so the expansion area is a trapezium:

$$W_{DE} = \frac{P_D + P_E}{2}(V_E - V_D) = \frac{600 + 300}{2}(5 - 2) = 1350\,\mathrm{J}.$$

This is positive because volume increases. The SI product $\mathrm{N\,m^{-2}} \times \mathrm{m}^3$ equals $\mathrm{N\,m} = \mathrm{J}$.

Step 3: Add the compression work with its sign For the isobaric return,

$$W_{EF} = P_E(V_F - V_E) = 300(2 - 5) = -900\,\mathrm{J}.$$

Therefore

$$W_{DEF} = 1350 - 900 = 450\,\text{J}.$$

The enclosed triangular area independently gives $\frac{1}{2}(600-300)(5-2) = 450\,\text{J}$. The clockwise direction makes it positive. Option (A) reverses the net sign; 900 J is only the magnitude of compression work, and 1350 J is only the expansion work.

Step 4: Conclusion The total work done by the gas is $\boxed{450\,\text{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a straight expansion followed by isobaric compression, the net work is the signed triangular area between the paths; clockwise motion gives positive work.

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47. The root mean square speed of smoke particles of mass 5×10^{-17} kg in their Brownian motion in air at NTP is approximately. [Given $\text{k} = 1.38 \times 10^{-23}\text{JK}^{-1}$] [4 marks]

(A) 60 mm s^{-1}

(B) 12 mm s^{-1}

(C) 15 mm s^{-1}

(D) 36 mm s^{-1}

Correct Answer: (C) 15 mm s^{-1}

Solution:

Concept:

- A suspended particle in thermal equilibrium has mean translational kinetic energy $\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT$, where k is Boltzmann constant.
- The root mean square speed is $v_{\text{rms}} = \sqrt{\langle v^2\rangle} = \sqrt{3kT/m}$. Use the mass of a single smoke particle, not a molar mass, in this expression.

Step 1: Choose the absolute temperature At the normal temperature used in the question, take $T \simeq 273\,\text{K}$. The specified mass is $m = 5 \times 10^{-17}\,\text{kg}$, and $k = 1.38 \times 10^{-23}\,\text{J K}^{-1}$. Brownian motion is random, so its mean velocity can vanish while its mean square speed and kinetic energy remain nonzero.

Step 2: Substitute into the equipartition result The mean square speed is

$$v_{\text{rms}}^2 = \frac{3(1.38 \times 10^{-23})(273)}{5 \times 10^{-17}} = 2.26044 \times 10^{-4}\,\text{m}^2\,\text{s}^{-2}.$$

The temperature factor cancels the kelvin in Boltzmann constant. Dividing the energy by the particle mass leaves the required squared-speed units.

Step 3: Take the square root and convert units Taking the positive square root gives

$$v_{\text{rms}} \simeq 1.503 \times 10^{-2}\,\text{m}\,\text{s}^{-1} = 15.03\,\text{mm}\,\text{s}^{-1}.$$

The factor for converting metres to millimetres is 10^3, applied after taking the square root. This is approximately 15 mm/s. Values 12, 36 and 60 mm/s do not follow from the three-dimensional mean

translational energy. In particular, the factor 3 under the root accounts for all three independent velocity components; using only one component would answer a different question.

Step 4: Conclusion The root mean square speed is approximately $\boxed{15\,\mathrm{mm\,s^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: For Brownian particles use $3kT/m$ with the single-particle mass, take the square root in SI units, and only then convert metres per second to millimetres per second.

48. Light enters from air into a given medium at an angle of 45° with interface of the air-medium surface. After refraction, the light ray is deviated through an angle of 15° from its original direction. The refractive index of the medium is: [4 marks]

(A) 1.732

(B) 1.333

(C) 1.414

(D) 2.732

Correct Answer: (C) 1.414

Solution:

Concept:

- Snell law for refraction from air to a medium is $n = \sin i / \sin r$, taking the refractive index of air as unity. Both angles are measured from the normal.
- For a ray bending towards the normal at a plane interface, the deviation from its incident direction is $\delta = i - r$. The angle with the surface is complementary to the incidence angle.

Step 1: Convert the angle with the interface The incoming ray makes 45° with the interface, so its angle with the perpendicular normal is

$$i = 90^\circ - 45^\circ = 45^\circ.$$

Although these two angles happen to be equal here, Snell law requires the angle to the normal. The specified deviation is 15°, not the refracted angle itself.

Step 2: Determine the refracted angle On entering a medium with index greater than that of air, the ray bends towards the normal. Thus

$$\delta = i - r, \qquad r = 45^\circ - 15^\circ = 30^\circ.$$

All proposed refractive indices are greater than one and therefore correspond to this bending direction. Taking $r = 60^\circ$ would instead imply a medium less optically dense than air, contrary to these choices.

Step 3: Apply Snell law and compare the choices Substituting the two angles gives

$$n = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2} \simeq 1.414.$$

This ratio is dimensionless. The value 1.732 is $\sqrt{3}$, but it would require a different pair of angles. The values 1.333 and 2.732 likewise fail the specific sine ratio. A quick consistency check is that $r < i$, so the computed index must exceed one, as the result does.

Step 4: Conclusion The refractive index is $\boxed{\sqrt{2} \simeq 1.414}$. Hence, the correct option is (C).

Quick Tip to Solve: First convert a surface angle to a normal angle; then subtract the deviation to get the refraction angle before applying Snell law.

49. Read the following statements :

(A) Volume of the nucleus is directly proportional to the mass number.

(B) Volume of the nucleus is independent of mass number.

(C) Density of the nucleus is directly proportional to the mass number.

(D) Density of the nucleus is directly proportional to the cube root of the mass number.

(E) Density of the nucleus is independent of the mass number.

Choose the correct option from the following options. [4 marks]

(A) (A) and (D) only.

(B) (A) and (E) only.

(C) (B) and (E) only.

(D) (A) and (C) only.

Correct Answer: (B) (A) and (E) only.

Solution:

Concept:

- The empirical nuclear radius relation is $R = R_0 A^{1/3}$, where A is mass number and R_0 is approximately constant for nuclei.
- The spherical nuclear volume is $V = 4\pi R^3/3$, while its mass is approximately $M = Am_N$, with m_N the average mass per nucleon. Nuclear density is the ratio M/V.

Step 1: Find how the volume depends on mass number Cube the radius law before comparing the volume:

$$V = \frac{4\pi}{3}(R_0 A^{1/3})^3 = \frac{4\pi R_0^3}{3} A.$$

The factor in front of A is independent of the nucleus. Therefore volume is directly proportional to mass number. Statement (A) is true, while statement (B), which says that volume is independent of mass number, is false.

Step 2: Find how the density depends on mass number Divide the approximate mass by the calculated volume:

$$\rho = \frac{Am_N}{(4\pi R_0^3/3)A} = \frac{3m_N}{4\pi R_0^3}.$$

The mass number cancels exactly within this standard nuclear model. Increasing the number of nucleons increases nuclear mass and volume in the same proportion, so the density stays approximately

constant. Statement (E) is therefore true.

Step 3: Eliminate the incorrect statement pairs Statement (C) incorrectly makes density proportional to A; statement (D) incorrectly uses the cube-root dependence belonging to the radius. Both fail because the volume already grows in proportion to nuclear mass. The only correct pair is (A) and (E). Option (A) includes the false statement (D), option (C) includes the false statement (B), and option (D) includes the false statement (C). Small real nuclear variations do not alter the standard scaling relation tested here.

Step 4: Conclusion Statements A and E only are correct. Hence, the correct option is (B).

Quick Tip to Solve: Remember the sequence $R \propto A^{1/3}$, $V \propto A$, and $M \propto A$; dividing mass by volume removes the mass-number dependence.

50. The vertical component of the earth's magnetic field is 6×10^{-5} T at any place where the angle of dip is 37°. The earth's resultant magnetic field at that place will be (Given $\tan 37^\circ = \frac{3}{4}$) [4 marks]

(A) 8×10^{-5} T

(B) 6×10^{-5} T

(C) 5×10^{-4} T

(D) 1×10^{-4} T

Correct Answer: (D) 1×10^{-4} T

Solution:

Concept:

- The angle of dip δ is the angle the resultant terrestrial magnetic field B makes with the horizontal. Its components are $B_H = B\cos\delta$ and $B_V = B\sin\delta$.
- The ratio of vertical to horizontal components is $\tan\delta = B_V/B_H$. The components are perpendicular, so $B^2 = B_H^2 + B_V^2$.

Step 1: Resolve the field using the given dip The specified quantity is the vertical component, not the total field. With $\tan 37^\circ = 3/4$, a right triangle has vertical, horizontal and resultant sides in the ratio $3:4:5$. Thus $\sin 37^\circ = 3/5$ and $\cos 37^\circ = 4/5$. The tangent alone is not the factor by which the vertical component gives the resultant.

Step 2: Calculate the horizontal component and resultant First use the component ratio to obtain

$$B_H = \frac{B_V}{\tan 37^\circ} = \frac{6 \times 10^{-5}}{3/4} = 8 \times 10^{-5}\,\text{T}.$$

Now add perpendicular components vectorially:

$$B = \sqrt{(8 \times 10^{-5})^2 + (6 \times 10^{-5})^2} = 10 \times 10^{-5}\,\text{T}.$$

Equivalently, $B = B_V/\sin 37^\circ$ gives the same value. Option (A) is only the horizontal component; option (B) repeats the vertical component. Option (C) is five times the resultant and does not satisfy the component relation.

Step 3: Conclusion The earth's resultant magnetic field is $B = 1 \times 10^{-4}\,\mathrm{T}$. Hence, the correct option is (D).

Quick Tip to Solve: For a dip with tangent $3/4$, use the $3 : 4 : 5$ component triangle: vertical, horizontal, resultant.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A tube of length $50\ \mathrm{cm}$ is filled completely with an incompressible liquid of mass $250\ \mathrm{g}$ and closed at both ends. The tube is then rotated in horizontal plane about one of its ends with a uniform angular velocity $x\sqrt{F}\,\mathrm{rad\,s^{-1}}$. If F be the force exerted by the liquid at the other end then the value of x will be _____. [4 marks]

Solution:

Concept:

- A liquid co-rotating with its tube needs centripetal acceleration $\omega^2 r$ at radius r. The radial pressure gradient supplies this acceleration: $dp/dr = \rho\omega^2 r$.
- For uniform cross-sectional area A, the liquid mass is $m = \rho AL$. The rotational force contribution at the outer end is the pressure rise multiplied by A.

Step 1: Set up radial force balance Convert the dimensions to SI units: $L = 0.50\,\mathrm{m}$ and $m = 0.250\,\mathrm{kg}$. Consider a thin slice of liquid of length dr at radius r. Its mass is $\rho A\,dr$. The difference of pressure forces across its ends must provide its centripetal force, so $A\,dp = \rho A\,dr\,\omega^2 r$. The pressure therefore increases towards the closed outer end; the acceleration is not the same for every slice.

Step 2: Integrate the pressure rise along the tube Integrate from the rotation axis to the outer end:

$$\Delta p = \int_0^L \rho\omega^2 r\,dr = \frac{\rho\omega^2 L^2}{2}.$$

The corresponding force is

$$F = A\Delta p = \frac{\rho AL\,\omega^2 L}{2} = \frac{m\omega^2 L}{2}.$$

The factor one-half reflects the variation of radius along the liquid column. Treating all the liquid as concentrated at the outer end would miss this factor.

Step 3: Compare with the specified angular velocity Substitute the mass and length:

$$F = \frac{0.250 \times 0.50}{2}\omega^2 = \frac{\omega^2}{16}.$$

Consequently $\omega = 4\sqrt{F}$ when force and angular velocity are expressed in the stated SI units. Comparing this expression with $\omega = x\sqrt{F}$ identifies the requested numerical coefficient directly.

Step 4: Conclusion The required coefficient is $\boxed{x = 4}$.

Quick Tip to Solve: For a uniform radial liquid column, integrate $r\,dr$; its effective radius in the total force is $L/2$, not L.

52. Nearly 10% of the power of a $110\ \mathrm{W}$ light bulb is converted to visible radiation. The change in average intensities of visible radiation, at a distance of $1\ \mathrm{m}$ from the bulb to a distance of $5\ \mathrm{m}$ is $a \times 10^{-2}\ \mathrm{W/m^2}$. The value of 'a' will be _____. [4 marks]

Solution:

Concept:

- For a source radiating uniformly in all directions, power spreads over a sphere of area $4\pi r^2$. The average intensity is $I = P/(4\pi r^2)$.
- Only the visible fraction of the bulb's power contributes to visible intensity. Intensity is power per unit area and has SI unit $\mathrm{W\,m^{-2}}$.

Step 1: Find the power carried by visible light The electrical rating of the bulb is $110\,\mathrm{W}$, but the question states that only about ten percent emerges as visible radiation. Hence

$$P_{\text{vis}} = 0.10 \times 110 = 11\,\mathrm{W}.$$

Using the full rating would include non-visible radiation and would make the requested change ten times too large. The same visible power passes through both concentric spherical surfaces in the ideal spreading model.

Step 2: Calculate the intensity decrease between the two radii At the nearer and farther points the intensities are respectively

$$I_1 = \frac{11}{4\pi}\,\mathrm{W\,m^{-2}}, \qquad I_5 = \frac{11}{100\pi}\,\mathrm{W\,m^{-2}}.$$

The farther intensity is one twenty-fifth of the nearer intensity because intensity follows the inverse square of distance. Thus the positive magnitude of the decrease is

$$I_1 - I_5 = \frac{11}{4\pi}\left(1 - \frac{1}{25}\right) = \frac{2.64}{\pi}\,\mathrm{W\,m^{-2}}.$$

This gives approximately $0.840\,\mathrm{W\,m^{-2}}$, consistent with a decrease rather than an increase.

Step 3: Express the result in the requested form Writing the decrease as $a \times 10^{-2}\,\mathrm{W\,m^{-2}}$ gives $a = 100(2.64/\pi) \simeq 84.0$. The inverse-square dependence has already accounted for both observation distances, so no additional factor of their separation is needed.

Step 4: Conclusion The visible intensity decreases by approximately $84 \times 10^{-2}\,\mathrm{W\,m^{-2}}$, so $\boxed{a = 84}$.

Quick Tip to Solve: Apply the visible fraction first, then subtract $P_{\text{vis}}/(4\pi r^2)$ at the two radii; do not use their distance difference.

53. A metal wire of length $0.5\ \text{m}$ and cross-sectional area $10^{-4}\ \text{m}^2$ has breaking stress $5 \times 10^8\ \text{Nm}^{-2}$. A block of $10\ \text{kg}$ is attached at one end of the string and is rotating in a horizontal circle. The maximum linear velocity of block will be _____ ms^{-1}. [4 marks]

Solution:

Concept:

- Tensile stress is force per cross-sectional area, $\sigma = T/A$. The largest allowed tension before breaking is $T_{\max} = \sigma_{\text{break}} A$.
- For uniform circular motion of a mass m at speed v on a circle of radius r, the inward force is mv^2/r. Here the wire provides that force and its length specifies the radius.

Step 1: Convert breaking stress to maximum force The wire can sustain a maximum stress of $5 \times 10^8\ \text{N m}^{-2}$, and its cross-sectional area is $10^{-4}\ \text{m}^2$. Therefore

$$T_{\max} = (5 \times 10^8)(10^{-4}) = 5 \times 10^4\ \text{N}.$$

Stress itself is not the centripetal force: multiplying by area is necessary. The powers of ten combine to give 10^4, while the area units cancel those in stress.

Step 2: Use tension as the centripetal force At the limiting speed, the required tension reaches the breaking value. With $m = 10\,\text{kg}$ and $r = 0.50\,\text{m}$,

$$\frac{mv_{\max}^2}{r} = T_{\max}, \qquad v_{\max}^2 = \frac{(5 \times 10^4)(0.50)}{10} = 2500\,\text{m}^2\,\text{s}^{-2}.$$

The motion is specified as a horizontal circle, so there is no changing vertical circular-motion tension to consider in this model. The supplied wire length is used for the circle radius; no extension information is provided.

Step 3: Take the positive root and check the limit The speed is non-negative, so $v_{\max} = \sqrt{2500} = 50\,\text{m s}^{-1}$. Substitution gives $10(50)^2/0.50 = 5 \times 10^4\,\text{N}$, exactly the limiting tension. A larger speed would require a larger stress because the required centripetal force increases as the square of speed.

Step 4: Conclusion The maximum linear speed of the block is $\boxed{50\,\text{m s}^{-1}}$.

Quick Tip to Solve: First multiply breaking stress by wire area to get force; only then set that force equal to mv^2/r.

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54. The velocity of a small ball of mass $0.3\ \text{g}$ and density $8\ \text{g/cc}$ when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is $1.3\ \text{g/cc}$, then the value of viscous force acting on the ball will be $x \times 10^{-4}\ \text{N}$, The value of x is _____. [use $\text{g} = 10\ \text{m/s}^2$] [4 marks]

Solution:

Concept:

- At terminal speed the acceleration is zero. The upward viscous force and buoyant force together balance the downward weight: $F_v + F_b = mg$.

- For a fully submerged ball of volume V, Archimedes' principle gives $F_b = \rho_f V g$, while $m = \rho_b V$. Thus $F_v = mg(1 - \rho_f/\rho_b)$.

Step 1: Identify the forces after the speed becomes constant The ball is moving downwards, so viscosity opposes its motion and acts upwards. Buoyancy also acts upwards. Since the speed has become constant, the resultant force vanishes; the viscous force is the apparent weight, not the whole weight. There is no need to know the ball radius or glycerine viscosity, because terminal force balance already determines the drag.

Step 2: Evaluate the weight and buoyant correction Convert the mass to kilograms:

$$m = 0.3\,\mathrm{g} = 3.0 \times 10^{-4}\,\mathrm{kg}, \qquad mg = 3.0 \times 10^{-3}\,\mathrm{N}.$$

The density ratio can be taken directly in the common units given:

$$\frac{\rho_f}{\rho_b} = \frac{1.3}{8} = 0.1625.$$

Consequently $F_b = 0.1625\,mg = 4.875 \times 10^{-4}\,\mathrm{N}$. The ball is denser than glycerine, so the remaining apparent weight is positive and it can settle downwards.

Step 3: Find the viscous force and numerical coefficient Subtract buoyancy from weight:

$$F_v = 3.0 \times 10^{-3} - 4.875 \times 10^{-4} = 2.5125 \times 10^{-3}\,\mathrm{N}.$$

In the requested form this is $25.125 \times 10^{-4}\,\mathrm{N}$. Therefore the coefficient before numerical rounding is 25.125, whose nearest integer is 25. The upward direction of the drag is consistent with opposition to the downward motion.

Step 4: Conclusion The viscous force is $2.5125 \times 10^{-3}\,\mathrm{N}$ upward, giving $x = 25.125$, or $\boxed{25}$ to the nearest integer.

Quick Tip to Solve: At terminal speed use drag = weight minus buoyancy. A density ratio lets you compute buoyancy without first finding the ball radius.

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55. The speed of a transverse wave passing through a string of length $50\ \mathrm{cm}$ and mass $10\ \mathrm{g}$ is $60\ \mathrm{ms}^{-1}$. The area of cross-section of the wire is $2.0\ \mathrm{mm}^2$ and its Young's modulus is $1.2 \times 10^{11}\mathrm{Nm}^{-2}$. The extension of the wire over its natural length due to its tension will be $x \times 10^{-5}\ \mathrm{m}$. The value of x is _____.

[4 marks]

Solution:

Concept:

- Transverse wave speed on a stretched wire is $v = \sqrt{T/\mu}$, where T is tension and $\mu = m/L$ is mass per unit length.
- Young's modulus is longitudinal stress divided by strain: $Y = (T/A)/(\Delta L/L)$. Hence the elastic extension is $\Delta L = TL/(AY)$.

Step 1: Determine the linear density and tension The wire length is $L = 0.50\,\mathrm{m}$, and its mass is

$m = 0.010\,\text{kg}$. Thus

$$\mu = \frac{0.010}{0.50} = 0.020\,\text{kg m}^{-1}.$$

Squaring the given wave speed and multiplying by linear density gives

$$T = \mu v^2 = 0.020(60)^2 = 72\,\text{N}.$$

The wave speed measures the tension indirectly; it is not a speed of longitudinal stretching of the wire.

Step 2: Apply the stress-strain relation The cross-sectional area must be expressed in square metres: $A = 2.0\,\text{mm}^2 = 2.0 \times 10^{-6}\,\text{m}^2$. Therefore

$$\Delta L = \frac{72 \times 0.50}{(2.0 \times 10^{-6})(1.2 \times 10^{11})} = \frac{36}{2.4 \times 10^5}\,\text{m} = 1.5 \times 10^{-4}\,\text{m}.$$

This relation assumes the wire is within its elastic range. Its strain is only $\Delta L/L = 3.0 \times 10^{-4}$, so using the given length for the linear density is consistent with the small-extension model.

Step 3: Read off the requested coefficient Express the extension using the power of ten in the question:

$$1.5 \times 10^{-4}\,\text{m} = 15 \times 10^{-5}\,\text{m}.$$

Thus the coefficient is fifteen. As a unit check, AY has units of force, so $TL/(AY)$ indeed has units of length. Confusing square millimetres with millimetres would introduce an erroneous factor of one thousand.

Step 4: Conclusion The extension is $15 \times 10^{-5}\,\text{m}$, so $\boxed{x = 15}$.

Quick Tip to Solve: Use wave speed to find $T = \mu v^2$, then insert that tension into $\Delta L = TL/(AY)$; convert mm^2 to $10^{-6}\,\text{m}^2$.

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56. The metallic bob of simple pendulum has the relative density 5. The time period of this pendulum is 10 s. If the metallic bob is immersed in water, then the new time period becomes $5\sqrt{x}$ s. The value of x will be _____. [4 marks]

Solution:

Concept:

- For small oscillations in air, a simple pendulum of length L has period $T = 2\pi\sqrt{L/g}$. The bob's inertial mass is unchanged on immersion.
- Buoyancy reduces the restoring weight of a fully immersed bob. Ignoring viscous damping, the effective gravitational acceleration is $g_{\text{eff}} = g(1 - \rho_w/\rho_b)$.

Step 1: Calculate the effective weight in water Relative density five means $\rho_b/\rho_w = 5$. For bob volume V, its weight is $\rho_b V g$ while the upward buoyant force is $\rho_w V g$. The net downward force is therefore

$$mg - F_b = mg\left(1 - \frac{1}{5}\right) = \frac{4}{5}mg.$$

Both gravity and buoyancy are vertical, so their resultant supplies the pendulum's restoring force with the same angular dependence as in air.

Step 2: Compare the immersed and original periods For a small angular displacement θ, the tangential equation becomes $mL\ddot{\theta} = -(4/5)mg\theta$. Thus the restoring acceleration is obtained by replacing g with $4g/5$, while leaving the inertial mass unchanged. At the same pendulum length,

$$\frac{T_w}{T} = \sqrt{\frac{g}{4g/5}} = \sqrt{\frac{5}{4}}.$$

The ratio exceeds one, as expected when buoyancy weakens the restoring force. Replacing both inertial and gravitational masses by apparent mass would incorrectly cancel this effect.

Step 3: Match the given expression for the new period Using the original period $T = 10\,\text{s}$,

$$T_w = 10\sqrt{\frac{5}{4}} = 5\sqrt{5}\,\text{s}.$$

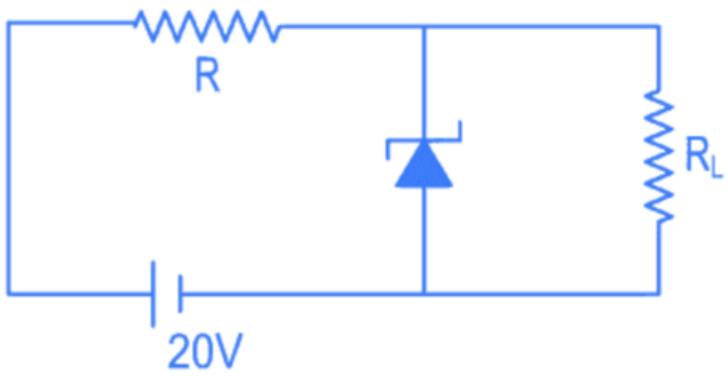
This is approximately $11.18\,\text{s}$. Comparing it with the form $5\sqrt{x}\,\text{s}$ gives $x = 5$. The result uses the usual ideal pendulum approximation in which fluid damping and added fluid inertia are neglected.

Step 4: Conclusion The new period is $5\sqrt{5}\,\text{s}$, hence $\boxed{x = 5}$.

Quick Tip to Solve: Immersion changes restoring weight but not bob inertia: use $g_{\text{eff}} = g(1-1/\text{relative density})$ in the period formula.

57. A $8\ \text{V}$ Zener diode along with a series resistance R is connected across a $20\ \text{V}$ supply (as shown in the figure). If the maximum Zener current is $25\ \text{mA}$, then the minimum value of R will be _____ Ω.

[4 marks]



Solution:

Concept:

- A reverse-biased Zener diode in breakdown maintains approximately its Zener voltage, here $V_Z = 8\,\text{V}$, across itself and the parallel load.
- The series resistor limits current: $I_R = (V_S - V_Z)/R$. At the junction, $I_R = I_Z + I_L$, where I_L is

the load current.

Step 1: Find the potential difference across the resistor The supply and the Zener voltages must not both be placed across the series resistor. The conducting Zener holds the output at its breakdown voltage, so Kirchhoff's voltage law gives

$$V_R = V_S - V_Z = 20 - 8 = 12\,\text{V}.$$

The same output voltage appears across the load because it is connected in parallel with the Zener.

Step 2: Apply the maximum-current safety condition The Zener current is greatest when the load draws the least current. To protect the diode even at zero load current, take the limiting case $I_L = 0$, so the resistor current equals the Zener current. A nonzero load diverts some current and therefore reduces the current in the diode. The stated maximum safe current is

$$I_{Z,\text{max}} = 25\,\text{mA} = 25 \times 10^{-3}\,\text{A}.$$

Step 3: Calculate the smallest safe resistance For the worst-case Zener current not to exceed its rating,

$$\frac{12}{R} \le 25 \times 10^{-3}.$$

Consequently,

$$R \ge \frac{12}{25 \times 10^{-3}} = 480\,\Omega.$$

At this limiting resistance the no-load current is exactly the rated maximum; choosing a smaller resistor would allow an excessive no-load current.

Step 4: Conclusion The minimum safe series resistance is $\boxed{480\,\Omega}$. The numerical entry is 480.

Quick Tip to Solve: For a Zener current-limiting resistor, use the supply voltage minus the Zener voltage, and test diode safety at zero load current.

58. A capacitor of capacitance 500 μF is charged completely using a dc supply of 100 V. It is now connected to an inductor of inductance 50 mH to form an LC circuit. The maximum current in LC circuit will be _____ A. [4 marks]

Solution:

Concept:

- In an ideal LC circuit, energy alternates between the electric field of the capacitor and the magnetic field of the inductor, while total energy remains constant.
- The capacitor stores $U_C = \frac{1}{2}CV^2$, and the inductor stores $U_L = \frac{1}{2}LI^2$. The current is maximum when the capacitor is momentarily discharged.

Step 1: Convert the circuit values to SI units The capacitor initially has the full charging voltage of

100 V. Its capacitance and the inductance must be in farads and henries before substituting:

$$C = 500 \times 10^{-6} = 5.00 \times 10^{-4}\,\text{F}, \qquad L = 50 \times 10^{-3} = 5.00 \times 10^{-2}\,\text{H}.$$

The dc supply charges the capacitor first; the subsequent LC oscillations exchange this initially stored energy.

Step 2: Equate the initial and maximum magnetic energies Initially the current is zero, so the energy resides in the charged capacitor. At maximum current, the capacitor charge and voltage pass through zero, and all the stored energy is magnetic. Therefore,

$$\frac{1}{2}CV_0^2 = \frac{1}{2}LI_{\max}^2, \qquad I_{\max} = V_0\sqrt{\frac{C}{L}}.$$

The capacitance-to-inductance ratio has units that make the final result an electric current.

Step 3: Evaluate the maximum current Substitute the converted values without dropping the square root:

$$I_{\max} = 100\sqrt{\frac{5.00 \times 10^{-4}}{5.00 \times 10^{-2}}} = 100\sqrt{10^{-2}} = 10\,\text{A}.$$

As a check, the initial electric energy is $\frac{1}{2}(5 \times 10^{-4})(100)^2 = 2.5\,\text{J}$. The magnetic energy at 10 A is also $\frac{1}{2}(0.05)(10)^2 = 2.5\,\text{J}$, confirming the calculation.

Step 4: Conclusion The maximum current is $\boxed{10\,\text{A}}$. The numerical entry is 10.

Quick Tip to Solve: At maximum LC current the capacitor is discharged: equate its initial electric energy to the maximum magnetic energy, rather than equating voltage and current directly.

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59. A modulating signal $2\sin\left(6.28 \times 10^6\right)t$ is added to the carrier signal $4\sin\left(12.56 \times 10^9\right)t$ for amplitude modulation. The combined signal is passed through a non-linear square law device. The output is then passed through a band pass filter. The bandwidth of the output signal of band pass filter will be _____ MHz. [4 marks]

Solution:

Concept:

- For a sinusoid written as $A\sin(\omega t)$, the coefficient of time is angular frequency, so ordinary frequency is $f = \omega/(2\pi)$.
- A square-law modulator generates product terms. The signal-carrier product produces sidebands at $f_c - f_m$ and $f_c + f_m$, which are passed by a filter centred on the carrier.

Step 1: Extract the modulating frequency The modulating signal is $2\sin[(6.28 \times 10^6)t]$. Thus its angular frequency, rather than its ordinary frequency, is $\omega_m = 6.28 \times 10^6\,\text{rad}\,\text{s}^{-1}$. Using $2\pi \simeq 6.28$,

$$f_m = \frac{6.28 \times 10^6}{2\pi} \simeq 10^6\,\text{Hz} = 1\,\text{MHz}.$$

The amplitude 2 affects the modulation strength, not the separation of the sideband frequencies.

Step 2: Identify the frequencies selected by the filter When the sum of the modulating wave and the carrier enters the nonlinear device, its squared term contains their cross product. The identity

$$2\sin\omega_m t\sin\omega_c t=\cos[(\omega_c-\omega_m)t]-\cos[(\omega_c+\omega_m)t]$$

shows the two sidebands. The carrier frequency is about $2\times10^9\,\text{Hz}$; components at zero, the modulating frequency and unwanted harmonics are rejected by the appropriate carrier-band filter.

Step 3: Subtract the band edges The useful transmitted frequency band extends from the lower sideband to the upper sideband, so

$$B=(f_c+f_m)-(f_c-f_m)=2f_m=2(1\,\text{MHz})=2\,\text{MHz}.$$

Although the carrier frequency is much larger, it cancels in this subtraction. The bandwidth is twice the modulating frequency, not twice the carrier frequency.

Step 4: Conclusion The output bandwidth is $\boxed{2\,\text{MHz}}$. The numerical entry is 2.

Quick Tip to Solve: Convert the coefficient of time to hertz first; an AM band containing both sidebands has width twice the highest modulating frequency.

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60. Two radioactive materials A and B have decay constants 25λ and 16λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of B to that of A will be "e" after a time $\frac{1}{a\lambda}$. The value of a is _____. [4 marks]

Solution:

Concept:

- Radioactive decay obeys $N(t)=N_0e^{-kt}$, where k is the decay constant and N_0 is the initial population.
- For equal initial populations, their ratio removes N_0. Its time dependence is controlled by the difference of the two decay constants, not by their sum.

Step 1: Write each surviving population Let the common initial number of nuclei be N_0. Material A decays with constant 25λ, while material B decays with constant 16λ. At a time t,

$$N_A=N_0e^{-25\lambda t},\qquad N_B=N_0e^{-16\lambda t}.$$

Both numbers decrease, but A decreases faster because its decay constant is larger. Hence the specified ratio B to A should increase from its initial value of one.

Step 2: Impose the required ratio Divide in the order asked in the question:

$$\frac{N_B}{N_A}=\frac{N_0e^{-16\lambda t}}{N_0e^{-25\lambda t}}=e^{(25-16)\lambda t}=e^{9\lambda t}.$$

The common starting population cancels exactly. Setting the ratio equal to $e=e^1$ and equating real

exponents gives

$$9\lambda t = 1, \qquad t = \frac{1}{9\lambda}.$$

Step 3: Compare the coefficient with the stated time The problem expresses this same time as $t = 1/(a\lambda)$. Comparison with the derived result gives

$$\frac{1}{a\lambda} = \frac{1}{9\lambda} \quad \Rightarrow \quad a = 9.$$

The result has a positive time and therefore agrees with the physical check that B to A grows. Taking A to B instead would produce a decreasing exponential and would not reach e at a positive time.

Step 4: Conclusion The required coefficient is $\boxed{a = 9}$. The numerical entry is 9.

Quick Tip to Solve: For equal initial radioactive populations, form the ratio in the requested order before subtracting decay constants; the faster-decaying sample belongs in the denominator here.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Consider the reaction

$$4\,HNO_3(l) + 3\,KCl(s) \longrightarrow Cl_2(g) + NOCl(g) + 2\,H_2O(g) + 3\,KNO_3(s)$$

The amount of HNO_3 required to produce 110.0 g of KNO_3 is
(Given: Atomic masses of H, O, N and K are $1, 16, 14$ and 39, respectively.) [4 marks]

(A) 32.2 g
(B) 69.4 g
(C) 91.5 g
(D) 162.5 g

Correct Answer: (C) 91.5 g

Solution:

Concept:

- A balanced equation gives mole ratios between substances. For this reaction, four moles of HNO_3 produce three moles of KNO_3.
- Convert a substance's mass to moles using $n = m/M$, where M is its molar mass, and convert the required reactant moles back to mass.

Step 1: Calculate both molar masses Use the atomic masses provided, so the mass calculation is consistent with the question:

$$M(KNO_3) = 39 + 14 + 3(16) = 101\,g\,mol^{-1},$$

$$M(HNO_3) = 1 + 14 + 3(16) = 63\,g\,mol^{-1}.$$

The coefficients 4 and 3 in the balanced reaction multiply mole amounts; they do not mean that the two masses are in the ratio 4 to 3.

Step 2: Find the required nitric acid from the mole ratio The stated product amount corresponds to

$$n(\mathrm{KNO_3}) = \frac{110.0}{101}\,\mathrm{mol}.$$

For every three moles of this product, four moles of nitric acid are consumed. Hence

$$n(\mathrm{HNO_3}) = \frac{4}{3}\left(\frac{110.0}{101}\right)\,\mathrm{mol}.$$

This uses the balanced equation directly and assumes the required stoichiometric amount for the given product yield.

Step 3: Convert to mass and compare the choices Multiplying by the nitric acid molar mass gives

$$m(\mathrm{HNO_3}) = \frac{4 \times 110.0 \times 63}{3 \times 101} = \frac{9240}{101} \simeq 91.5\,\mathrm{g}.$$

Equivalently, $303\,\mathrm{g}$ of potassium nitrate requires $252\,\mathrm{g}$ of nitric acid. Scaling that pair to $110.0\,\mathrm{g}$ gives the same result. The masses 32.2, 69.4 and $162.5\,\mathrm{g}$ do not satisfy this required mass ratio.

Step 4: Conclusion The nitric acid required is $\boxed{91.5\,\mathrm{g}}$. Hence, the correct option is (C).

Quick Tip to Solve: Attach molar masses to the balanced coefficients first: here the reactant-to-product mass ratio is $(4 \times 63)/(3 \times 101)$.

---

62. Given below are the quantum numbers for 4 electrons.

A. $\mathrm{n} = 3, l = 2,\ \mathrm{m_l} = 1,\ \mathrm{m_s} = +1/2$

B. $\mathrm{n} = 4, l = 1,\ \mathrm{m_l} = 0,\ \mathrm{m_s} = +1/2$

C. $\mathrm{n} = 4, l = 2,\ \mathrm{m_l} = -2,\ \mathrm{m_s} = -1/2$

D. $\mathrm{n} = 3, l = 1,\ \mathrm{m_l} = -1,\ \mathrm{m_s} = +1/2$

The correct order of increasing energy is : [4 marks]

(A) D < B < A < C

(B) D < A < B < C

(C) B < D < A < C

(D) B < D < C < A

Correct Answer: (B) D < A < B < C

Solution:

Concept:

- For the usual multi-electron atomic energy ordering, the subshell with smaller $n + l$ has lower energy. If $n + l$ is equal, the subshell with smaller n is lower.
- The orbital angular momentum quantum numbers $l = 0, 1, 2, 3$ denote s, p, d and f subshells.

Without an applied field, magnetic and spin projections do not change the subshell order used here.

Step 1: Identify the four subshells Read each electron's principal and orbital quantum numbers, keeping the letter labels from the question:

$$A : 3d\ (n = 3, l = 2), \qquad B : 4p\ (n = 4, l = 1),$$

$$C : 4d\ (n = 4, l = 2), \qquad D : 3p\ (n = 3, l = 1).$$

The supplied magnetic quantum numbers choose orientations within each subshell, and the spin quantum numbers specify spin projections. They do not replace the $n + l$ comparison.

Step 2: Calculate the energy-ordering sums The relevant sums are

$$(n + l)_D = 3 + 1 = 4, \qquad (n + l)_A = 3 + 2 = 5,$$

$$(n + l)_B = 4 + 1 = 5, \qquad (n + l)_C = 4 + 2 = 6.$$

Thus D is the lowest and C is the highest of these subshells. This already rules out choices placing B below D or C below A; those arrangements contradict the respective sums.

Step 3: Resolve the tie between A and B A and B share the same $n+l = 5$, so use the smaller principal quantum number as the tie-breaker. A has $n = 3$, whereas B has $n = 4$; consequently $3d$ lies below $4p$ in this ordering. Combining both comparisons gives

$$3p < 3d < 4p < 4d, \qquad D < A < B < C.$$

Choice (A) incorrectly reverses the tied pair A and B, while choices (C) and (D) incorrectly place B before D.

Step 4: Conclusion The increasing energy order is $\boxed{D < A < B < C}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the subshell names and their n plus l sums in a short list; use the smaller n only when two sums are equal.

63.

$$\mathrm{C(s) + O_2(\ g) \rightarrow CO_2(\ g) + 400\ kJ}$$

$$\mathrm{C(s) + \frac{1}{2}O_2(\ g) \rightarrow CO(g) + 100\ kJ}$$

When coal of purity 60% is allowed to burn in presence of insufficient oxygen, 60% of carbon is converted into 'CO' and the remaining is converted into 'CO_2'. The heat generated when 0.6 kg of coal is burnt is _____. [4 marks]

(A) 1600 kJ

(B) 3200 kJ

(C) 4400 kJ

(D) 6600 kJ

Correct Answer: (D) 6600 kJ

Solution:

Concept:

- Purity determines the mass of actual carbon in coal. Only that carbon enters the given combustion equations; the stated impurity contributes no specified combustion heat.
- The two equations release $100\,\mathrm{kJ}$ per mole of carbon forming CO, and $400\,\mathrm{kJ}$ per mole forming CO_2. Add the heat released by the two portions.

Step 1: Find the amount of pure carbon The coal sample has mass $0.6\,\mathrm{kg} = 600\,\mathrm{g}$, but its purity is only 60%. Therefore the carbon mass is

$$m_C = 0.60 \times 600 = 360\,\mathrm{g}.$$

Using the molar mass of carbon, $12\,\mathrm{g\,mol^{-1}}$, the available carbon is

$$n_C = \frac{360}{12} = 30\,\mathrm{mol}.$$

The purity percentage applies to the original coal mass before the carbon is divided between the combustion products.

Step 2: Divide the carbon between the two products Of these 30 moles of carbon, 60% forms carbon monoxide and the remaining 40% forms carbon dioxide. Since each molecule of either product contains one carbon atom,

$$n_{\mathrm{CO}} = 0.60 \times 30 = 18\,\mathrm{mol}, \qquad n_{\mathrm{CO_2}} = 0.40 \times 30 = 12\,\mathrm{mol}.$$

The two portions add to the original 30 moles, so no carbon is counted twice or omitted.

Step 3: Add the heat contributions The heat values in the printed equations refer to one mole of carbon in each pathway. Thus

$$Q = 18(100) + 12(400) = 1800 + 4800 = 6600\,\mathrm{kJ}.$$

The heat generated is quoted as a positive released amount; the corresponding enthalpy change would

be negative. The alternatives 1600, 3200 and 4400 kJ do not reproduce this sum. Treating all the carbon as forming either one product would ignore the specified 60:40 division.

Step 4: Conclusion The heat generated is $\boxed{6600\,\text{kJ}}$. Hence, the correct option is (D).

Quick Tip to Solve: Apply the coal purity first, then split only the pure carbon between CO and CO2; the two percentages describe different stages of the calculation.

64. $200\ \text{mL}$ **of** $0.01\ \text{M}\ \text{HCl}$ **is mixed with** $400\ \text{mL}$ **of** $0.01\ \text{M}\ \text{H}_2\text{SO}_4$**. The** pH **of the mixture is _____.**

Given: $\log 2 = 0.30, \log 3 = 0.48, \log 5 = 0.70, \log 7 = 0.84, \log 11 = 1.04$ [4 marks]

(A) 1.14

(B) 1.78

(C) 2.34

(D) 3.02

Correct Answer: (B) 1.78

Solution:

Concept:

- For strong acids, calculate the amount of hydrogen ions supplied before dividing by the total solution volume. Hydrochloric acid supplies one proton per formula unit; in the strong-acid treatment used here, sulfuric acid supplies two.
- The pH is $\text{pH} = -\log_{10}[\text{H}^+]$, where the numerical concentration is in $\text{mol}\,\text{L}^{-1}$. Acid mixtures do not undergo acid-base neutralisation with one another.

Step 1: Count the hydrogen ions from each acid Convert the volumes to litres. The amount supplied by hydrochloric acid is

$$n_1 = 0.200(0.01) = 0.002\ \text{mol}.$$

Sulfuric acid has amount $0.400(0.01) = 0.004\ \text{mol}$, and its two acidic hydrogens give

$$n_2 = 2(0.004) = 0.008\ \text{mol}.$$

Thus the combined hydrogen-ion amount is $0.010\ \text{mol}$; adding the acid molarities directly would miss both the different volumes and the diprotic contribution.

Step 2: Find the diluted concentration and take its logarithm Assuming the volumes are additive, the final volume is $0.600\ \text{L}$. Hence

$$[\text{H}^+] = \frac{0.010}{0.600} = \frac{1}{60}\ \text{mol}\,\text{L}^{-1}.$$

Therefore

$$\text{pH} = \log 60 = 1 + \log 2 + \log 3 = 1 + 0.30 + 0.48 = 1.78.$$

The other numerical choices do not equal the logarithm of the concentration obtained from the total proton amount. In particular, this mixture contains no base, so no subtraction of hydrogen-ion amounts is justified.

Step 3: Conclusion The pH of the mixture is $\boxed{1.78}$. Hence, the correct option is (B).

Quick Tip to Solve: For mixed strong acids, add MV times the number of acidic protons, divide by the total volume, and only then calculate pH.

65. In liquation process used for tin (Sn), the metal : [4 marks]

(A) is reacted with acid.

(B) is dissolved in water.

(C) is brought to molten form which is made to flow on a slope.

(D) is fused with NaOH

Correct Answer: (C) is brought to molten form which is made to flow on a slope.

Solution:

Concept:

- Liquation is a physical refining method based on a difference in melting points between a low-melting metal and its comparatively infusible impurities.
- Tin has a sufficiently low melting point to be selectively melted. Its liquid can be separated by gravity while solid impurities remain behind; no chemical conversion of the metal is required.

Step 1: Identify what heating separates The impure tin is placed on a sloping hearth and heated to a temperature high enough to melt the tin but not the higher-melting impurities. Selective melting is the essential operation. It changes the physical state of tin rather than converting it into an acid-soluble salt or a water-soluble compound. The impurities therefore remain mainly as solid material on the hearth.

Step 2: Follow the molten metal down the slope Once tin becomes liquid, gravity makes the molten metal flow down the inclined surface into a receiver. The solid residue is retained higher on the hearth. The slope makes mechanical collection of the melt possible and is specifically characteristic of liquation. Repeating the separation can improve purity, although impurities with similar melting behaviour cannot be removed efficiently by this method.

Step 3: Compare the four descriptions Option (C) describes exactly this melting and flow on an incline. Reaction with acid, option (A), is a chemical treatment rather than selective melting. Dissolution in water, option (B), is neither the basis of liquation nor the behaviour used to refine tin. Fusion with NaOH, option (D), would be a chemical fusion process; adding an alkali is not a required step of liquation.

Step 4: Conclusion In liquation, tin is brought to molten form and made to flow down a slope. Hence, the correct option is (C).

Quick Tip to Solve: Associate liquation with a low-melting metal, a sloping hearth, and the flow of liquid metal away from solid impurities.

66. Given below are two statements.

Statement I : Stannane is an example of a molecular hydride.

Statement II : Stannane is a planar molecule.

In the light of the above statement, choose the most appropriate answer from the options given below. [4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (C) Statement I is true but Statement II is false.

Solution:

Concept:

- Stannane is SnH_4, the covalent hydride of tin. A molecular hydride consists of discrete covalently bonded molecules, unlike saline hydrides with an extended ionic lattice.
- In the VSEPR description of SnH_4, the central tin atom has four bonding electron pairs and no lone pair: an AX_4 arrangement. Four such electron domains favour a tetrahedral geometry.

Step 1: Evaluate Statement I using the bonding Tin is a group 14 element and forms four covalent $Sn-H$ bonds in stannane. The formula represents a discrete molecular species, comparable in its basic bonding pattern to methane, silane and germane. It is therefore classified as a molecular or covalent hydride. Statement I is true; it should not be confused with a saline hydride such as NaH, or with an interstitial hydride of a transition metal.

Step 2: Evaluate Statement II using the shape The four $Sn-H$ bond directions arrange themselves tetrahedrally around tin to reduce electron-pair repulsion. A tetrahedral molecule is three-dimensional: the central atom and all four attached hydrogen atoms cannot lie in one plane. Its ideal bond angle is about $109.5°$, rather than the angles of a square-planar arrangement. Statement II is consequently false.

Step 3: Match the truth values to the options The required combination is true for Statement I and false for Statement II. Option (A) incorrectly accepts planarity. Option (B) incorrectly rejects the molecular-hydride classification. Option (D) reverses both established truth values. Option (C) alone reproduces the correct evaluation; no causal relationship between the statements is being asked for here.

Step 4: Conclusion Statement I is true but Statement II is false: SnH_4 is a tetrahedral molecular hydride. Hence, the correct option is (C).

Quick Tip to Solve: For a group 14 tetrahydride, separate the classification from the shape: discrete covalent molecules can still have a nonplanar tetrahedral geometry.

67. Which of the following $3d$-metal ion will give the lowest enthalpy of hydration $(\Delta_{hyd}\,H)$ when dissolved in water ? [4 marks]

(A) Cr^{2+}

(B) Mn^{2+}

(C) Fe^{2+}

(D) Co^{2+}

Correct Answer: (B) Mn^{2+}

Solution:

Concept:

- Hydration of divalent transition-metal ions involves both electrostatic attraction to water and the crystal-field stabilisation of their aqua complexes. Comparisons near a half-filled shell must include the crystal-field contribution.
- Water is a weak-field ligand, so these octahedral aqua ions are high spin. For an octahedral complex, each t_{2g} electron contributes $-0.4\Delta_o$ and each e_g electron contributes $+0.6\Delta_o$ to the crystal-field energy.

Step 1: Write the divalent-ion configurations Using the configurations of the divalent ions gives $Cr^{2+}: 3d^4$, $Mn^{2+}: 3d^5$, $Fe^{2+}: 3d^6$, and $Co^{2+}: 3d^7$. In weak-field octahedral water complexes their occupancies are respectively $t_{2g}^3e_g^1$, $t_{2g}^3e_g^2$, $t_{2g}^4e_g^2$, and $t_{2g}^5e_g^2$. The half-filled manganese ion is the special case.

Step 2: Compare the extra stabilisation on hydration The corresponding crystal-field energies are $-0.6\Delta_o$, zero, $-0.4\Delta_o$, and $-0.8\Delta_o$. For manganese,

$$3(-0.4\Delta_o) + 2(0.6\Delta_o) = 0.$$

Thus Mn^{2+} receives no extra crystal-field stabilisation, whereas the chromium, iron and cobalt ions do. This accounts for the anomalously small heat evolved on hydrating Mn^{2+}.

Step 3: Interpret the hydration comparison The question uses lowest hydration enthalpy in the customary comparative sense of the smallest magnitude of the exothermic hydration effect. That is Mn^{2+}, rather than one of the ions with a nonzero crystal-field stabilisation. Hydration enthalpies themselves are negative, so it is useful to state this magnitude convention explicitly; the selected ion has the least exothermic hydration among these choices.

Step 4: Conclusion Mn^{2+} has the smallest magnitude of hydration enthalpy in the intended comparison, owing to its high-spin d^5 configuration. Hence, the correct option is (B).

Quick Tip to Solve: In hydration-enthalpy comparisons, check d^5: a high-spin octahedral half-filled shell has zero CFSE and an unusually small hydration-energy magnitude.

68. Octahedral complexes of copper(II) undergo structural distortion (Jahn-Teller). Which one of the given copper (II) complexes will show the maximum structural distortion? (en - ethylenediamine; $H_2\ N_CH_2 - CH_2 - NH_2$) [4 marks]

(A) $[Cu\,(H_2O)_6]\,SO_4$

(B) $[Cu(en)\,(H_2O)_4]\,SO_4$

(C) cis-$[Cu\,(en)_2Cl_2]$

(D) trans-$[Cu\,(en)_2Cl_2]$

Correct Answer: (A) $[Cu\,(H_2O)_6]\,SO_4$

Solution:

Concept:

- Copper(II) is $3d^9$. In an octahedral ligand field it has the configuration $t_{2g}^6 e_g^3$; the unequal occupancy of the two e_g orbitals gives a Jahn-Teller instability.
- Jahn-Teller distortion removes orbital degeneracy by changing metal-ligand bond lengths. The freedom of ligands to move is important: monodentate aqua ligands are less geometrically constrained than the donor atoms tied together in a chelate ring.

Step 1: Identify why all the copper complexes can distort Removing two electrons from copper gives the d^9 ion. In an octahedral arrangement the e_g orbitals contain three electrons, so one is doubly occupied and the other singly occupied. A tetragonal change of the coordination octahedron splits these orbitals and lowers the energy. Thus the presence of copper(II) establishes the tendency in all four options, but does not by itself select one of them.

Step 2: Compare ligand freedom with chelate restrictions The hexaaqua ion $[Cu(H_2O)_6]^{2+}$ contains six individual water ligands. Its axial and equatorial bonds can adjust without the restrictions of chelate rings, producing the characteristic pronounced tetragonal distortion of hydrated copper(II). In the en-containing complexes each ethylenediamine ligand binds through two nitrogen donors connected by a short carbon chain. This imposes a bite-angle and ring constraint that limits the available structural deformation.

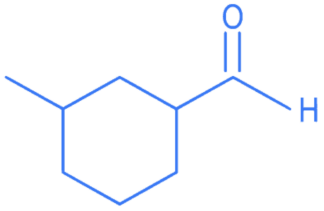
Step 3: Distinguish Jahn-Teller deformation from ligand differences Option (B) already introduces one chelate ring, and both (C) and (D) contain two en ligands. Changing from cis to trans placement of the chloride ligands changes the coordination geometry, but a difference between copper-chloride and copper-nitrogen bond lengths is not by itself a measure of a larger Jahn-Teller deformation. The fully aqua complex, with no chelate constraints, is the choice with the maximum structural distortion in this comparison. The sulfate is a counter-ion and does not occupy a site of the copper coordination octahedron.

Step 4: Conclusion The maximum Jahn-Teller structural distortion among these choices is shown by $[Cu(H_2O)_6]SO_4$. Hence, the correct option is (A).

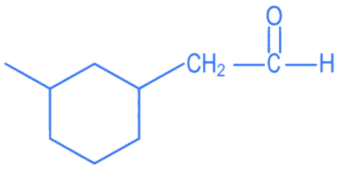
Quick Tip to Solve: For these copper(II) choices, identify the common d^9 instability first, then compare unconstrained water ligands with the geometric restrictions of en chelation.

69. Correct structure of γ-methylcyclohexane carbaldehyde is [4 marks]

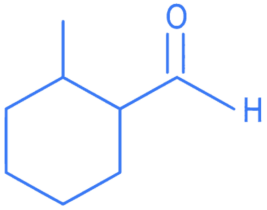
(A) 3-Methylcyclohexane-1-carbaldehyde.



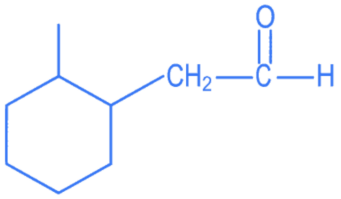
(B) 2-(3-Methylcyclohexyl)ethanal.



(C) 2-Methylcyclohexane-1-carbaldehyde.

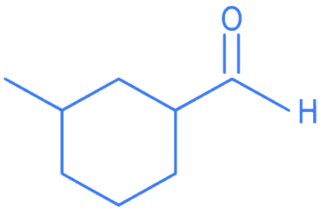


(D) 2-(2-Methylcyclohexyl)ethanal.



Correct Answer: (A)

3-Methylcyclohexane-1-carbaldehyde.



Solution:

Concept:

- The suffix carbaldehyde means that the $-\mathrm{CHO}$ carbon is outside the ring and directly attached to a ring carbon. It does not mean a $-\mathrm{CH_2CHO}$ side chain.
- For Greek locants measured from an aldehyde carbonyl, the carbon directly attached to the $-\mathrm{CHO}$ carbon is the α carbon; the next is β, and the next is γ.

Step 1: Locate the aldehyde and identify the alpha carbon The target has a cyclohexane ring bearing a directly attached $-\mathrm{CHO}$ group. In the displayed options this requirement is satisfied by (A) and (C). In (B) and (D), an extra $\mathrm{CH_2}$ group separates the ring from the aldehyde carbonyl, so those structures have a different carbon framework and do not represent the specified cyclohexane carbaldehyde.

Step 2: Count around the ring to the methyl group Call the ring carbon directly bonded to $-\mathrm{CHO}$ carbon 1. It is the α carbon. Its adjacent ring carbon is carbon 2, or β; the next is carbon 3, or γ. A methyl group on that third ring carbon therefore fulfils the required γ-methyl description. In modern

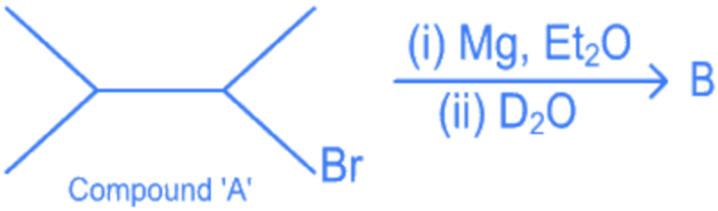
numerical locants, the required constitution is 3-methylcyclohexane-1-carbaldehyde.

Step 3: Compare the positions in the drawings Option (A) has one intervening ring carbon between the aldehyde-bearing carbon and the methyl-bearing carbon: the substituents are at ring positions 1 and 3. Option (C) places them on adjacent ring carbons, positions 1 and 2, and would be the β-methyl compound. Option (B) has both the wrong side-chain length and a different reference sequence; option (D) also contains the unwanted methylene spacer. Only (A) satisfies both the functional-group attachment and the Greek locant.

Step 4: Conclusion The required structure is the 1,3-substituted cyclohexane bearing directly attached $-\mathrm{CHO}$ and methyl groups, as shown in (A). Hence, the correct option is (A).

Quick Tip to Solve: For a carbaldehyde, keep the carbonyl carbon outside the ring and start Greek counting at the ring carbon bonded directly to it.

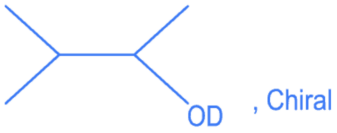
70. Compound 'A' undergoes following sequence of reactions to give compound 'B'.
The correct structure and chirality of compound 'B' is
[where Et is $-\mathrm{C_2H_5}$] [4 marks]



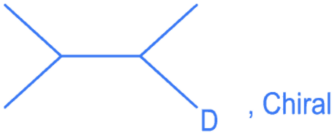
(A) $(\mathrm{CH_3})_2\mathrm{CH}-\mathrm{CH(D)}-\mathrm{CH_3}$, achiral.



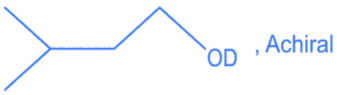
(B) $(\mathrm{CH_3})_2\mathrm{CH}-\mathrm{CH(OD)}-\mathrm{CH_3}$, chiral.



(C) $(\mathrm{CH_3})_2\mathrm{CH}-\mathrm{CH(D)}-\mathrm{CH_3}$, chiral.

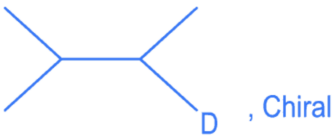


(D) $(\mathrm{CH_3})_2\mathrm{CH}-\mathrm{CH_2}-\mathrm{CH_2OD}$, achiral.



Correct Answer: (C)

$(\mathrm{CH_3})_2\mathrm{CH}-\mathrm{CH(D)}-\mathrm{CH_3}$, chiral.



Solution:

Concept:

- A haloalkane reacts with magnesium in dry ether to form a Grignard reagent, $R-MgBr$. Reaction of this reagent with D_2O replaces the carbon-magnesium bond by a carbon-deuterium bond.
- A tetrahedral carbon is a stereogenic centre when its four attached groups are different. Hydrogen and deuterium are different isotopes and count as distinct substituents for this purpose.

Step 1: Follow the carbon bearing bromine The starting skeleton is $(CH_3)_2CH-CH(Br)-CH_3$. In dry diethyl ether, magnesium inserts into the carbon-bromine bond without adding oxygen to the carbon skeleton:

$$(CH_3)_2CH-CH(Br)-CH_3 \xrightarrow[Et_2O]{Mg} (CH_3)_2CH-CH(MgBr)-CH_3.$$

The location of the reactive bond stays at the original bromine-bearing carbon; no rearrangement of the chain is part of this reaction.

Step 2: Quench the Grignard reagent with heavy water The carbon-magnesium bond abstracts deuterium from D_2O, giving

$$(CH_3)_2CH-CH(MgBr)-CH_3 + D_2O \longrightarrow (CH_3)_2CH-CH(D)-CH_3 + MgBrOD.$$

Thus the product contains $C-D$, not $C-OD$. The ether is the solvent and its ethyl groups are not incorporated into the organic product. Options showing an OD substituent describe a different transformation.

Step 3: Test the substituted carbon for chirality The carbon now bearing D is bonded to D, H, CH_3, and $CH(CH_3)_2$. These four groups are distinct, so it is a stereogenic centre and the product structure is chiral. Option (A) has the correct connectivity but the wrong achiral description. Option (B) incorrectly introduces OD, while option (D) changes the connectivity and also introduces OD. Option (C) alone combines the correct deuterated structure with the correct chirality.

Step 4: Conclusion Product B is 2-Deuterio-3-methylbutane, a chiral molecule. Hence, the correct option is (C).

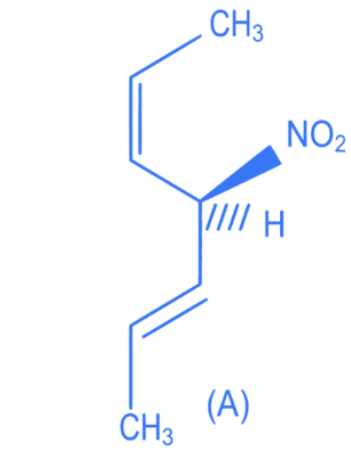
Quick Tip to Solve: With D_2O, a Grignard reagent gains D directly on carbon; remember that replacing one of two hydrogens by D can create a stereogenic centre.

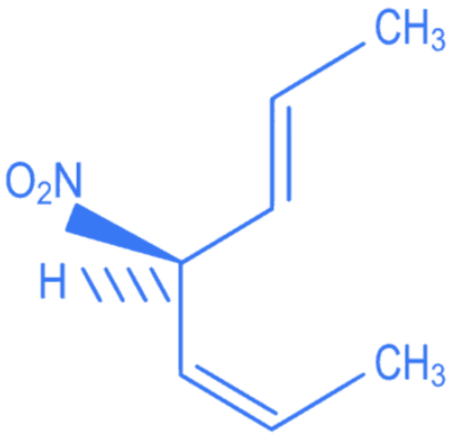
71. Given below are two statements.

Statement I : The compound is optically active.

Statement II : is mirror image of above compound A.

In the light of the above statement, choose the most appropriate answer from the options given below. [4 marks]





(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (C) Statement I is correct but Statement II is incorrect.

Solution:

Concept:

- Optical activity requires chirality: a tetrahedral carbon attached to four different groups can be a stereogenic centre.
- Groups with identical connectivity but different alkene geometry, E and Z, are distinguishable substituents. A mirror image reverses configuration at a stereogenic centre but preserves each double bond's geometry.

Step 1: Inspect the four attachments The central tetrahedral carbon bears NO_2, hydrogen, and two

propenyl groups. In the upper arm of A, the methyl group and the central-carbon substituent lie on the same side of the double bond, so this arm is Z. In the lower arm they lie on opposite sides, so that arm is E. Consequently the two carbon chains are stereochemically different even though their constitutions agree. All four attachments are therefore different and compound A is chiral. Statement I is correct.

Step 2: Compare configurations rather than page orientation The second drawing has the E arm above and the Z arm below. It also puts the solid-wedge nitro group to the left rather than the right, with hydrogen still behind. Interchanging the two distinct carbon arms reverses the orientation once, and reversing the positions of the nitro group relative to them reverses it again. These two reversals preserve the absolute configuration. Thus the second drawing represents the same stereoisomer after reorientation; it is not A's enantiomer. A simple reflected drawing would retain the identity of each upper and lower alkene arm while reversing handedness.

Step 3: Select the statement combination Statement I is true and Statement II is false. The choices claiming that both statements are true, that both are false, or that only Statement II is true each contradict at least one of the stereochemical checks. The distinction between the two arms must be made before judging the wedge and dash representation.

Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: When two substituents look constitutionally identical, check their E/Z geometry before testing chirality; then compare configurations using the same labelled groups.

72. When ethanol is heated with conc. H_2SO_4, a gas is produced. The compound formed, when this gas is treated with cold dilute aqueous solution of Baeyer's reagent, is [4 marks]

(A) formaldehyde

(B) formic acid

(C) glycol

(D) ethanoic acid

Correct Answer: (C) glycol

Solution:

Concept:

- Concentrated sulphuric acid dehydrates ethanol on sufficient heating to form ethene; elimination removes one molecule of water.
- Cold dilute alkaline potassium permanganate, Baeyer's reagent, hydroxylates an alkene to a vicinal diol without cleaving the carbon skeleton.

Step 1: Identify the gas from ethanol Heating ethanol with concentrated H_2SO_4 under the conditions giving a gas produces ethene. The acid acts as a dehydrating agent. The transformation is

$$CH_3CH_2OH \xrightarrow[\text{heat}]{\text{conc.}\ H_2SO_4} CH_2{=}CH_2 + H_2O.$$

Although ethanol can form ether under other conditions, the stated gaseous product identifies the alkene in this problem. Ethene has two carbon atoms and one carbon–carbon double bond available for the next reaction.

Step 2: Apply the mild oxidation conditions Baeyer's reagent is cold, dilute alkaline $KMnO_4$. Under these mild conditions one hydroxyl group is introduced on each of the two double-bond carbons. Thus the double bond becomes a single bond and the product is ethane-1,2-diol:

$$CH_2{=}CH_2 + H_2O + [O] \longrightarrow HOCH_2CH_2OH.$$

This compound is commonly called ethylene glycol or simply glycol. The purple permanganate colour is discharged during the test for unsaturation.

Step 3: Reject the carbonyl and acid alternatives Formaldehyde would require a carbonyl product with only one carbon atom, while formic acid also contains one carbon. Neither is the vicinal diol formed by adding hydroxyl groups across ethene. Ethanoic acid can result from oxidation of ethanol under suitable conditions, but the substance treated here is the gas ethene, and the reagent conditions specify hydroxylation rather than oxidation of the original alcohol.

Step 4: Conclusion The final compound is ethylene glycol, $HOCH_2CH_2OH$. Hence, the correct option is (C).

Quick Tip to Solve: Follow the intermediate first: ethanol gives ethene, and cold dilute alkaline permanganate converts its double bond into adjacent hydroxyl groups.

73. The Hinsberg reagent is [4 marks]

(A) Benzenesulphonyl chloride, $C_6H_5SO_2Cl$

(B) Benzenediazonium chloride, $C_6H_5N_2{}^+Cl^-$

(C) Potassium phthalimide

(D) p-Hydroxyazobenzene, $HO{-}C_6H_4{-}N{=}N{-}C_6H_5$

Correct Answer: (A) Benzenesulphonyl chloride, $C_6H_5SO_2Cl$

Solution:

Concept:

- The Hinsberg reagent is benzenesulphonyl chloride, $C_6H_5SO_2Cl$, used in alkaline medium to distinguish primary, secondary and tertiary amines.
- The diagnostic functional group is the sulphonyl chloride group $-SO_2Cl$, in which sulphur carries two oxygens and a chlorine atom.

Step 1: Read the functional group in each option Option (A) shows a benzene ring bonded to sulphur, two sulphur–oxygen double bonds and a sulphur–chlorine bond. Its condensed formula is $C_6H_5SO_2Cl$, so it is benzenesulphonyl chloride. Option (B) instead contains the diazonium group and chloride counterion; option (C) is potassium phthalimide; option (D) contains an azo linkage between aromatic rings and a phenolic hydroxyl group. Only the first structure has a sulphonyl chloride group.

Step 2: Connect the reagent to its amine test The nitrogen of a primary or secondary amine attacks

the electrophilic sulphonyl group and replaces chloride. A primary amine gives an N-substituted benzenesulphonamide containing an acidic nitrogen–hydrogen bond:

$$C_6H_5SO_2Cl + RNH_2 \longrightarrow C_6H_5SO_2NHR + HCl.$$

Its product dissolves in alkali through salt formation. A secondary amine forms a sulphonamide without an $N-H$ bond, so that product does not dissolve in alkali. A tertiary amine lacks the required nitrogen–hydrogen bond for this sulphonamide formation. These contrasting behaviours are the basis of the Hinsberg test.

Step 3: Eliminate other familiar amine reagents Benzenediazonium chloride belongs to diazotization and coupling chemistry. Potassium phthalimide is the nitrogen reagent of Gabriel synthesis, rather than a reagent for this classification test. The azo compound is a coupling product and does not supply the reactive sulphonyl chloride function. Familiarity with nitrogen-containing structures must therefore not override the specific identity of the named reagent.

Step 4: Conclusion Hinsberg reagent is benzenesulphonyl chloride, $C_6H_5SO_2Cl$. Hence, the correct option is (A).

Quick Tip to Solve: Identify Hinsberg reagent by the benzene–SO_2Cl unit; distinguish it from potassium phthalimide, which is associated with Gabriel synthesis.

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74. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Amylose is insoluble in water.

Reason R: Amylose is a long linear molecule with more than 200 glucose units.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.

(C) **A** is correct but **R** is not correct

(D) **A** is not correct but **R** is correct.

Correct Answer: (D) **A** is not correct but **R** is correct.

Solution:

Concept:

- Amylose is the water-soluble component of starch and consists of long, essentially unbranched chains of α-D-glucose units.
- In assertion–reason questions, first judge the truth of each statement separately; an explanation link is considered only when both are true.

Step 1: Evaluate the assertion Assertion A says that amylose is insoluble in water. In the standard classification of starch components, amylose is the water-soluble component, while amylopectin is the branched component described as insoluble in water. Thus the assertion has attributed the insolubility

property to the wrong starch fraction. Assertion A is false. The fact that amylose is a large polymer does not by itself justify changing the stated textbook classification.

Step 2: Evaluate the reason independently Amylose contains a long chain of glucose residues joined mainly through $\alpha(1 \rightarrow 4)$ glycosidic bonds. It is described as an unbranched or linear polymer with more than 200 glucose units; the chain can adopt a helical conformation, but this does not make its connectivity branched. Reason R therefore gives a correct description of the molecular chain. A helical shape and a linear connectivity are compatible statements, since the latter refers to how the glucose residues are linked.

Step 3: Choose the truth-value combination Since A is false and R is true, the question does not reach the stage of testing whether R explains a true assertion. Options (A) and (B) both require a true assertion and are excluded. Option (C) requires a true assertion and a false reason, reversing both conclusions. The only matching combination is the one stating that A is not correct but R is correct. Do not infer insolubility merely from a large number of glucose units.

Step 4: Conclusion Assertion A is incorrect, whereas Reason R is correct. Hence, the correct option is (D).

Quick Tip to Solve: Remember the starch pair: amylose is unbranched and water-soluble; amylopectin is branched and water-insoluble in the standard textbook classification.

75. A compound 'X' is a weak acid and it exhibits colour change at pH close to the equivalence point during neutralization of NaOH with CH_3COOH. Compound 'X' exists in ionized form in basic medium. The compound 'X' is [4 marks]

(A) methyl orange

(B) methyl red

(C) phenolphthalein

(D) erichrome Black T

Correct Answer: (C) phenolphthalein

Solution:

Concept:

- An acid–base indicator is a weak acid or base whose ionized and unionized forms have different colours.
- The equivalence solution of a weak acid with a strong base is basic because the conjugate base hydrolyses; an indicator must change colour near that basic endpoint.

Step 1: Determine the endpoint region Sodium hydroxide is a strong base, whereas ethanoic acid is a weak acid. At equivalence the solution contains sodium ethanoate. Ethanoate ions react with water:

$$CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + OH^-.$$

Production of hydroxide ions makes the equivalence-point pH greater than 7. The direction of adding acid to base changes which colour disappears, but does not change the basic character of the salt

solution at equivalence.

Step 2: Match the weak acidic indicator Phenolphthalein acts as a weak acidic indicator and changes from colourless in acidic or nearly neutral solution to pink in basic solution. Its simplified equilibrium may be represented by

$$HIn \rightleftharpoons H^+ + In^-.$$

In basic medium removal of H^+ shifts this equilibrium towards the ionized coloured form. Its useful transition interval is approximately pH 8.2–10, which matches the basic region of this weak-acid–strong-base titration. This agrees with both the colour-change clue and the stated ionized form in base.

Step 3: Compare the remaining choices Methyl orange changes colour in the acidic region, approximately pH 3.1–4.4, and methyl red also changes in an acidic interval, approximately pH 4.4–6.2. They do not match the stated basic endpoint as well as phenolphthalein. Eriochrome Black T is chiefly a metal-ion indicator in complexometric titrations, notably with EDTA; it is not the conventional endpoint indicator for this neutralization. The endpoint and indicator mechanism together select one choice.

Step 4: Conclusion Compound X is phenolphthalein. Hence, the correct option is (C).

Quick Tip to Solve: For a weak acid titrated against a strong base, identify the basic equivalence solution first, then choose an indicator with a basic transition interval.

76. Given below are the critical temperatures of some of the gases:

Gas	Critical temperature (K)
He	5.2
CH_4	190.0
CO_2	304.2
NH_3	405.5

The gas showing least adsorption on a definite amount of charcoal is [4 marks]

(A) He

(B) CH_4

(C) CO_2

(D) NH_3

Correct Answer: (A) He

Solution:

Concept:

- Physical adsorption on charcoal depends on the ease with which a gas is liquefied, since stronger intermolecular attractions favour adsorption.
- A higher critical temperature indicates greater ease of liquefaction; under comparable conditions such gases are generally adsorbed more strongly.

Step 1: Interpret the critical temperatures The critical temperature is the highest temperature at which a gas can be liquefied by applying pressure. A gas with a very low critical temperature has weak attractive interactions and is difficult to liquefy at an ordinary temperature. Its interaction with a charcoal surface is also comparatively weak. Thus, for the given comparison on a definite amount of charcoal, the lowest critical temperature identifies the least readily adsorbed gas. The question gives all four temperatures, so no molecular-mass estimate is needed.

Step 2: Arrange the gases using the supplied data The printed values give the increasing order

$$T_c(\mathrm{He}) = 5.2\,\mathrm{K} < T_c(\mathrm{CH_4}) = 190.0\,\mathrm{K}$$

$$< T_c(\mathrm{CO_2}) = 304.2\,\mathrm{K} < T_c(\mathrm{NH_3}) = 405.5\,\mathrm{K}.$$

Helium lies far below the other three gases. It is the most difficult to liquefy and therefore has the weakest tendency towards physical adsorption in this comparison. The corresponding qualitative adsorption tendency increases from helium towards ammonia.

Step 3: Exclude the gases with stronger adsorption Methane has a much greater critical temperature than helium, so it is more readily adsorbed under comparable conditions. Carbon dioxide and ammonia have still higher critical temperatures and cannot be selected as the least adsorbed gases. In particular, choosing ammonia would reverse the relation between ease of liquefaction and adsorption. The word least is decisive: select the smallest critical temperature, rather than the largest one.

Step 4: Conclusion Helium, He, shows the least adsorption on charcoal. Hence, the correct option is (A).

Quick Tip to Solve: For comparable physisorption of gases, use critical temperature as the shortcut: lower critical temperature means harder liquefaction and weaker adsorption.

77. Portland cement contains 'X' to enhance the setting time. What is 'X'? [4 marks]

(A) $\mathrm{CaSO_4 \cdot \frac{1}{2}H_2O}$

(B) $\mathrm{CaSO_4 \cdot 2H_2O}$

(C) $\mathrm{CaSO_4}$

(D) $\mathrm{CaCO_3}$

Correct Answer: (B) $\mathrm{CaSO_4 \cdot 2H_2O}$

Solution:

Concept:

- Gypsum, $\mathrm{CaSO_4 \cdot 2\,H_2O}$, is added to Portland cement as a setting retarder.
- Increasing setting time means slowing the initial setting process, allowing the cement paste to be mixed, transported and placed before it becomes rigid.

Step 1: Identify the purpose of the additive Fresh cement can react rapidly with water, particularly through the tricalcium aluminate component. If this initial hydration is uncontrolled, the paste can undergo an undesirably rapid or flash set. A small quantity of gypsum is therefore mixed with cement

clinker during grinding. It regulates the rapid aluminate hydration and extends the workable period. The phrase enhance the setting time refers to increasing this time interval, rather than accelerating the setting reaction.

Step 2: Write the chemical identity of gypsum Gypsum is calcium sulphate dihydrate, with two water molecules per formula unit:

$$\text{Gypsum} = \text{CaSO}_4 \cdot 2\,\text{H}_2\text{O}.$$

The water of crystallization is part of the identity being tested. The first option, $\text{CaSO}_4 \cdot \frac{1}{2}\text{H}_2\text{O}$, is the hemihydrate associated with plaster of Paris; it is not gypsum. The third option is anhydrous calcium sulphate, which also lacks the specified dihydrate composition. The second option alone gives gypsum's formula.

Step 3: Distinguish raw materials from the retarder Calcium carbonate is an important limestone constituent used as a raw material in cement manufacture, but it is not the gypsum added to control the final cement's initial setting. Thus the presence of calcium in all four choices is insufficient to identify X. One must match both its role as a retarder and its hydration state. Gypsum provides sulphate that controls aluminate hydration, explaining why its addition increases the time available for working with cement.

Step 4: Conclusion X is gypsum, calcium sulphate dihydrate, $\text{CaSO}_4 \cdot 2\,\text{H}_2\text{O}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the hydrates distinct: gypsum is $\text{CaSO}_4 \cdot 2\,\text{H}_2\text{O}$, whereas plaster of Paris is the hemihydrate; cement uses gypsum to retard setting.

78. When borax is heated with CoO on a platinum loop, blue coloured bead formed is largely due to : [4 marks]

(A) B_2O_3

(B) $\text{Co}\,(\text{BO}_2)_2$

(C) CoB_4O_7

(D) $\text{Co}\,[\text{B}_4\text{O}_5(\text{OH})_4]$

Correct Answer: (B) $\text{Co}\,(\text{BO}_2)_2$

Solution:

Concept:

- The borax bead test identifies certain transition-metal ions through the characteristic colour of their metaborates dissolved in a glassy bead.
- On strong heating, hydrated borax loses water and gives sodium metaborate and boric anhydride. The latter reacts with the metal oxide to produce a coloured metaborate.

Step 1: Describe the formation of the glassy bead Borax first loses its water of crystallisation. The anhydrous salt then decomposes on strong heating according to

$$\text{Na}_2\text{B}_4\text{O}_7 \longrightarrow 2\,\text{NaBO}_2 + \text{B}_2\text{O}_3.$$

The resulting transparent fused mass provides the medium for the metal oxide. Thus B_2O_3 is a reagent and glass former in this test; it does not itself account for the characteristic blue colour.

Step 2: Identify the cobalt compound Cobalt(II) oxide combines with the boric anhydride in the hot bead:

$$CoO + B_2O_3 \longrightarrow Co(BO_2)_2.$$

The cobalt(II) metaborate formed in this reaction imparts a deep blue colour to the bead. The platinum loop supports the molten bead and does not supply the blue colouring species.

Step 3: Distinguish the other options Option (A) is boric anhydride, which participates in the reaction but is not the blue cobalt product. Option (C) is a cobalt tetraborate formula rather than the metaborate produced in the standard borax bead reaction. Option (D) contains hydroxyl groups and represents a hydrated borate formulation; strong heating instead gives the fused metaborate bead.

Step 4: Conclusion The blue colour is largely due to cobalt(II) metaborate, $Co(BO_2)_2$. Hence, the correct option is (B).

Quick Tip to Solve: For a borax bead question, first form boric anhydride, then combine it with the metal oxide to identify the coloured metaborate.

79. Dinitrogen is a robust compound, but reacts at high altitudes to form oxides. The oxide of nitrogen that can damage plant leaves and retard photosynthesis is : [4 marks]

(A) NO

(B) NO_3^-

(C) NO_2

(D) NO_2^-

Correct Answer: (C) NO_2

Solution:

Concept:

- The strong triple bond in N_2 makes dinitrogen relatively unreactive. Very high temperatures or electrical discharges can nevertheless enable it to react with oxygen.
- NO_2, nitrogen dioxide, is an atmospheric pollutant that injures vegetation, damages plant leaves and suppresses photosynthetic activity.

Step 1: Follow the formation of nitrogen oxides At the high temperatures produced by lightning, nitrogen and oxygen can combine:

$$N_2 + O_2 \longrightarrow 2\,NO.$$

Nitric oxide can then be oxidised further in air:

$$2\,NO + O_2 \longrightarrow 2\,NO_2.$$

This explains how stable dinitrogen can ultimately contribute to the formation of nitrogen dioxide in the atmosphere.

Step 2: Identify the pollutant affecting leaves Nitrogen dioxide is a reactive gaseous oxide. Exposure can damage leaf tissue and interfere with the functioning of the photosynthetic apparatus, thereby retarding photosynthesis. These are the characteristic effects identified in the question. The required species is the neutral molecule NO_2, rather than an ion containing nitrogen and oxygen.

Step 3: Eliminate the remaining choices NO is nitric oxide, an important precursor of nitrogen dioxide, but it is not the oxide singled out for the stated leaf damage. NO_3^- and NO_2^- are nitrate and nitrite ions respectively; they are not the neutral gaseous oxide being described. Their superscript negative charges distinguish them chemically from nitrogen dioxide.

Step 4: Conclusion The oxide is nitrogen dioxide, NO_2. Hence, the correct option is (C).

Quick Tip to Solve: Read charges carefully: nitrogen dioxide is neutral NO2, whereas nitrite and nitrate are negatively charged ions.

80. Which of the following is NOT a natural polymer? [4 marks]

(A) Protein

(B) Starch

(C) Rubber

(D) Rayon

Correct Answer: (D) Rayon

Solution:

Concept:

- Natural polymers are macromolecules formed by biological processes and obtained from natural sources, such as proteins, starch and natural rubber.
- Rayon is a regenerated cellulose fibre prepared by chemical processing of natural cellulose. It is classified as a semi-synthetic polymer rather than a natural polymer.

Step 1: Classify the three natural materials Proteins are natural polypeptides formed from amino-acid units in living organisms. Starch is a natural polysaccharide built from glucose units and stored by plants. Natural rubber is predominantly cis-1,4-polyisoprene obtained from latex. Therefore protein, starch and rubber all belong to the natural-polymer category used in this classification question.

Step 2: Examine how rayon is produced Rayon starts from cellulose obtained from a natural source, but the cellulose is subjected to chemical processing and subsequently regenerated as fibres. This manufacturing process gives a man-made fibre. Its natural starting material does not make rayon itself a naturally occurring polymer material in the classification being tested.

Step 3: Apply the word NOT in the question The question asks for the exception to the natural-polymer group. Options (A), (B) and (C) are directly recognised natural polymers. Rayon is the only listed material classified as semi-synthetic. It should not be confused with fully synthetic polymers made by polymerisation of petrochemical monomers; being semi-synthetic is already enough to exclude it from the natural group.

Step 4: Conclusion Rayon is the material that is not classified as a natural polymer. Hence, the correct

option is (D).

Quick Tip to Solve: A natural raw material can yield a semi-synthetic fibre: chemically processed, regenerated cellulose is rayon.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Consider, $PF_5, BrF_5, PCl_3, SF_6, [ICl_4]^-, ClF_3$ and IF_5.

Amongst the above molecule(s)/ion(s), the number of molecule(s)/ion(s) having $sp^3\ d^2$ hybridisation is _____. [4 marks]

Solution:

Concept:

- In the conventional hybridisation model, sp^3d^2 corresponds to six electron domains around the central atom: an octahedral electron-pair arrangement.
- The steric number is the number of sigma bonds plus lone pairs on the central atom. A lone pair changes the molecular shape but still counts as an electron domain.

Step 1: Count domains in phosphorus and sulfur species In PF_5, phosphorus forms five sigma bonds and has no lone pair, so the steric number is 5 and the conventional hybridisation is sp^3d. In PCl_3, phosphorus has three bonds and one lone pair, giving steric number 4 and sp^3. Sulfur in SF_6 has six bonds and no lone pair, giving steric number 6 and sp^3d^2. Thus only SF_6 qualifies in this group.

Step 2: Count domains in the halogen species Bromine in BrF_5 and iodine in IF_5 each have five sigma bonds and one lone pair, so each has six domains and sp^3d^2 hybridisation. In ICl_4^-, iodine has four bonds and two lone pairs, again six domains. The negative charge provides the additional electron needed in this count. Chlorine in ClF_3 has three bonds and two lone pairs, only five domains, so it is sp^3d.

Step 3: Collect the qualifying species The qualifying list is

$$BrF_5, \quad SF_6, \quad ICl_4^-, \quad IF_5.$$

Their molecular shapes need not all be octahedral: the two pentafluorides are square pyramidal and the tetrachloroiodate ion is square planar, while their electron-domain count remains six.

Step 4: Conclusion The number of molecules or ions having sp^3d^2 hybridisation is $\boxed{4}$.

Quick Tip to Solve: Count sigma bonds plus lone pairs; do not infer the electron-domain count from the number of ligands alone.

82. $1.80\ \text{g}$ **of solute A was dissolved in** $62.5\ \text{cm}^3$ **of ethanol and freezing point of the solution was found to be** $155.1\ \text{K}$**. The molar mass of solute A is _____ g** mol^{-1}**.**

[Given : Freezing point of ethanol is 156.0 K.

Density of ethanol is 0.80 g cm^{-3}**.**

Freezing point depression constant of ethanol is 2.00 K kg mol^{-1}**]** [4 marks]

Solution:

Concept:

- For a dilute solution of a non-electrolyte, freezing-point depression obeys $\Delta T_f = K_f m$, where m is the number of moles of solute per kilogram of solvent.
- Convert the solvent volume into mass with $m_{\text{solvent}} = \rho V$. Molality uses the solvent mass in kilograms, not the solution volume.

Step 1: Calculate the depression and solvent mass The solution freezes below pure ethanol by

$$\Delta T_f = 156.0 - 155.1 = 0.9\,\text{K}.$$

Using the given density and volume, the mass of ethanol is

$$m_{\text{solvent}} = 0.80 \times 62.5 = 50.0\,\text{g} = 0.0500\,\text{kg}.$$

The $1.80\,\text{g}$ of solute is not added to this solvent mass because the colligative-property relation here uses molality.

Step 2: Find the moles of dissolved solute The molality required to produce this depression is

$$m = \frac{\Delta T_f}{K_f} = \frac{0.9}{2.00} = 0.45\,\text{mol kg}^{-1}.$$

Multiplying by the mass of ethanol in kilograms gives

$$n_A = 0.45 \times 0.0500 = 0.0225\,\text{mol}.$$

No dissociation or association factor is specified, so the stated dilute-solution relation is used directly.

Step 3: Determine and check the molar mass Divide the given solute mass by the calculated number of moles:

$$M_A = \frac{1.80}{0.0225} = 80\,\text{g mol}^{-1}.$$

As a reverse check, $1.80/80 = 0.0225$ mol in 0.0500 kg gives molality 0.45, whose freezing-point depression is $2.00 \times 0.45 = 0.90$ K, exactly the observed difference.

Step 4: Conclusion The molar mass of solute A is $\boxed{80\,\text{g mol}^{-1}}$; the required numerical entry is 80.

Quick Tip to Solve: Convert solvent volume to kilograms before using molality; using grams would introduce a factor of one thousand.

83. For a cell, $\mathrm{Cu(s)}\,|\mathrm{Cu^{2+}(0.001\,M)}\|\mathrm{Ag^{+}(0.01\,M)}|\,\mathrm{Ag(s)}$

the cell potential is found to be 0.43 V **at** 298 K**. The magnitude of standard electrode potential for** $\mathrm{Cu^{2+}/Cu}$ **is** _____ $\times 10^{-2}$ V**.**

[Given : $E^{\ominus}_{Ag^{+}/Ag}$ **= 0.80 V and** $\frac{2.303RT}{F}$ **= 0.06 V]** [4 marks]

Solution:

Concept:

- The cell reaction follows the written cell notation: oxidation occurs at the left copper electrode and reduction at the right silver electrode.
- At the given temperature, $E_{\text{cell}} = E^{\circ}_{\text{cell}} - (0.06/n)\log Q$, and $E^{\circ}_{\text{cell}} = E^{\circ}_{\text{cathode}} - E^{\circ}_{\text{anode}}$, using reduction potentials.

Step 1: Balance the reaction and obtain the quotient The electrode reactions are

$$\mathrm{Cu} \longrightarrow \mathrm{Cu^{2+}} + 2\,\mathrm{e^{-}}, \qquad 2\,\mathrm{Ag^{+}} + 2\,\mathrm{e^{-}} \longrightarrow 2\,\mathrm{Ag}.$$

Thus the net reaction transfers $n = 2$ electrons. Pure solids have unit activity, so the concentration quotient is

$$Q = \frac{[\mathrm{Cu^{2+}}]}{[\mathrm{Ag^{+}}]^2} = \frac{0.001}{(0.01)^2} = 10.$$

The silver concentration must be squared because its stoichiometric coefficient is two.

Step 2: Recover the standard cell potential Apply the Nernst equation with the supplied value $2.303RT/F = 0.06$ V:

$$0.43 = E^{\circ}_{\text{cell}} - \frac{0.06}{2}\log 10.$$

Since $\log 10 = 1$,

$$E^{\circ}_{\text{cell}} = 0.43 + 0.03 = 0.46\,\mathrm{V}.$$

The quotient exceeds unity, so the nonstandard cell potential is lower than the standard value, consistent with this result.

Step 3: Calculate the copper reduction potential For the stated electrodes,

$$0.46 = 0.80 - E^{\circ}_{\mathrm{Cu^{2+}/Cu}}.$$

It follows that

$$E^{\circ}_{\mathrm{Cu^{2+}/Cu}} = 0.34\,\mathrm{V} = 34 \times 10^{-2}\,\mathrm{V}.$$

This is already positive, so taking its magnitude does not change the value. The question asks for the coefficient of 10^{-2} V, rather than the potential expressed directly in volts.

Step 4: Conclusion The magnitude is $0.34\,\mathrm{V} = \boxed{34} \times 10^{-2}\,\mathrm{V}$. The required entry is 34.

Quick Tip to Solve: Balance the electron transfer before writing Q: two silver ions mean a squared silver concentration and n equal to two.

84. Assuming $1\ \mu g$ of trace radioactive element X with a half life of 30 years is absorbed by a growing tree. The amount of X remaining in the tree after 100 years is _____ $\times 10^{-1} \mu g$.

[Given : ln 10 = 2.303; log 2 = 0.30] [4 marks]

Solution:

Concept:

- Radioactive decay follows first-order kinetics: $m_t = m_0 e^{-kt}$. The decay constant is related to half-life by $k = \ln 2/t_{1/2}$.
- An equivalent base-ten expression is $\log(m_0/m_t) = (t/t_{1/2}) \log 2$. The mass and the number of radioactive nuclei decrease by the same fraction.

Step 1: Find the number of half-lives elapsed The element remains in the tree for 100 years and has a half-life of 30 years. The number of half-lives is therefore

$$\frac{t}{t_{1/2}} = \frac{100}{30} = \frac{10}{3}.$$

The fractional part matters: simply counting three complete half-lives would overestimate the mass left after the full interval.

Step 2: Use the supplied logarithm With the given approximation $\log 2 = 0.30$,

$$\log\left(\frac{m_0}{m_t}\right) = \frac{100}{30} \times 0.30 = 1.00.$$

Taking the base-ten antilogarithm gives

$$\frac{m_0}{m_t} = 10, \qquad m_t = \frac{m_0}{10}.$$

The initial radioactive mass is $m_0 = 1\ \mu g$, so the remaining radioactive element has mass $0.10\ \mu g$.

Step 3: Express the answer in the requested scale The printed blank is the coefficient multiplying $10^{-1}\ \mu g$. Hence

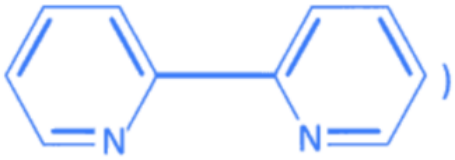
$$m_t = 0.10\ \mu g = 1 \times 10^{-1}\ \mu g.$$

Using a more precise value of $\log 2$ gives approximately $0.0992\ \mu g$, which is consistent with the value obtained using the constants provided. The growth of the tree does not change the radioactive decay law for the absorbed nuclei.

Step 4: Conclusion The amount remaining is $\boxed{1 \times 10^{-1}\ \mu g}$. Thus the required numerical coefficient is 1.

Quick Tip to Solve: Use fractional half-lives rather than rounding their number, then convert the mass into the power-of-ten scale printed beside the blank.

85. Sum of oxidation state (magnitude) and coordination number of cobalt in $\mathrm{Na}\,[\mathrm{Co(bpy)Cl_4}]$ is _____. [4 marks]

(Given : bpy = )

Solution:

Concept:

- The oxidation state is found by balancing the charge of the entire coordination ion. Sodium is present as Na^+, so the complex ion has charge -1.
- Coordination number counts donor atoms directly bonded to the metal, rather than the number of ligand molecules. The ligand bpy (2,2′-bipyridine) is neutral and bidentate.

Step 1: Determine the charge and oxidation state Let the oxidation state of cobalt be z. Each coordinated chloride has charge -1, while bpy contributes no net charge. The sodium ion balances one negative charge on the coordination entity. Therefore,

$$z + 0 + 4(-1) = -1, \qquad z = +3.$$

The required magnitude of the oxidation state is thus 3; the minus sign on the complex is not the oxidation state of cobalt.

Step 2: Count the directly coordinated donor atoms The displayed bpy structure contains two pyridyl nitrogen atoms. Both donate their lone pairs to the same cobalt centre, so the one bpy molecule occupies two coordination sites. Each of the four chloride ligands is monodentate and supplies one donor atom. Hence,

$$\text{Coordination number} = 2 + 4 = 6.$$

Counting bpy as one would incorrectly give five; denticity, rather than the number of ligand names in the formula, controls this count.

Step 3: Add the two requested quantities The question asks for the sum of the magnitude of the oxidation state and the coordination number. These are different quantities: the former follows from charge balance and the latter from the metal–donor bonds. Thus,

$$|z| + \text{coordination number} = 3 + 6 = 9.$$

Step 4: Conclusion The required numerical answer is $\boxed{9}$.

Quick Tip to Solve: Find charge balance first, then count donor atoms separately: a neutral chelating ligand changes coordination number without changing the charge balance.

86. A 1.84 mg sample of polyhydric alcoholic compound 'X' of molar mass 92.0 g/mol gave 1.344 mL of H_2 gas at STP. The number of alcoholic hydrogens present in compound 'X' is _____. [4 marks]

Solution:

Concept:

- An alcoholic hydrogen is the hydrogen bonded to oxygen in an alcohol group. In the usual active-hydrogen reaction with sodium, two such hydrogens furnish one molecule of H_2.
- At STP, one mole of an ideal gas occupies $22.4\,\mathrm{L}$. Divide the moles of active hydrogen by the moles of the compound to obtain the number per molecule.

Step 1: Calculate the amount of compound First express the sample mass in grams, since the molar mass is given in grams per mole:

$$m_X = 1.84\,\mathrm{mg} = 1.84 \times 10^{-3}\,\mathrm{g}.$$

The amount of compound is therefore

$$n_X = \frac{1.84 \times 10^{-3}}{92.0} = 2.00 \times 10^{-5}\,\mathrm{mol}.$$

This is the number of moles of molecules, not the number of moles of hydroxyl groups.

Step 2: Convert the gas volume to moles The evolved gas volume is $1.344\,\mathrm{mL} = 1.344 \times 10^{-3}\,\mathrm{L}$. Consequently,

$$n_{H_2} = \frac{1.344 \times 10^{-3}}{22.4} = 6.00 \times 10^{-5}\,\mathrm{mol}.$$

The volume must be converted from millilitres to litres before using the molar gas volume.

Step 3: Use the hydrogen stoichiometry The active-hydrogen stoichiometry is

$$2\,\mathrm{ROH} + 2\,\mathrm{Na} \longrightarrow 2\,\mathrm{RONa} + \mathrm{H_2}.$$

If each molecule contains k alcoholic hydrogens, one mole of the compound produces $k/2$ moles of hydrogen. Thus,

$$\frac{k}{2} = \frac{n_{H_2}}{n_X} = \frac{6.00 \times 10^{-5}}{2.00 \times 10^{-5}} = 3, \qquad k = 6.$$

The factor of two is essential because each gas molecule contains two hydrogen atoms.

Step 4: Conclusion The data give $\boxed{6}$ alcoholic hydrogens per molecule of X.

Quick Tip to Solve: Use $k = 2n(H_2)/n(X)$: the gas-to-compound mole ratio alone gives half the number of alcoholic hydrogens.

87. The number of stereoisomers formed in a reaction of $(\pm)\mathrm{Ph(C=O)C(OH)(CN)Ph}$ with HCN is _____.

[where Ph is $-\mathrm{C_6H_5}$] [4 marks]

Solution:

Concept:

- Addition of HCN to a carbonyl group gives a cyanohydrin: the carbonyl carbon gains CN and its oxygen becomes an OH group.
- Two stereogenic centres usually permit $2^2 = 4$ configurations, but identical substituent sets and internal symmetry can make the RS and SR descriptions represent one meso stereoisomer.

Step 1: Identify the structure after addition The starting material is $\mathrm{Ph{-}CO{-}C(OH)(CN){-}Ph}$. Its existing tetrahedral carbon has four different groups, and the prefix $(\pm)$ indicates a racemic starting sample. Addition of hydrogen cyanide at the remaining carbonyl produces

$$\mathrm{Ph{-}C(OH)(CN){-}C(OH)(CN){-}Ph.}$$

The original carbonyl carbon now becomes a second stereogenic centre. Each of the two adjacent centres bears phenyl, hydroxyl, cyano and the other substituted carbon.

Step 2: Enumerate the configurations Label the two centres temporarily as C1 and C2. The formal configuration assignments are (R,R), (S,S), (R,S) and (S,R). Because the ends of the product are identical, interchanging C1 and C2 does not give a new constitution. In the opposite-configuration arrangement, the molecule has internal symmetry; (R,S) and (S,R) are the same meso form, rather than distinct enantiomers.

Step 3: Count the distinct stereoisomers The (R,R) and (S,S) products are distinct mirror-image stereoisomers. The opposite-configuration products together contribute only one achiral meso stereoisomer. Starting with both enantiomers and allowing addition at the planar carbonyl gives these possibilities. Therefore,

$$N = 2 \text{ enantiomers} + 1 \text{ meso form} = 3.$$

The simple count of four must be reduced because it double-counts the meso structure.

Step 4: Conclusion The number of distinct stereoisomers formed is $\boxed{3}$.

Quick Tip to Solve: After cyanohydrin formation, compare both ends of the product. Identical ends with two equivalent stereogenic centres suggest an enantiomeric pair plus a meso form.

88. 'x' g of molecular oxygen (O_2) is mixed with 200 g of neon (Ne). The total pressure of the non-reactive mixture of O_2 and Ne in the cylinder is 25 bar. The partial pressure of Ne is 20 bar at the same temperature and volume. The value of 'x' is _____.

[Given: Molar mass of $O_2 = 32$ g mol^{-1}.

Molar mass of Ne $= 20$ g mol^{-1}] [4 marks]

Solution:

Concept:

- For non-reacting ideal gases at the same temperature and volume, $p_i = n_i RT/V$. Therefore, the ratio of partial pressures equals the ratio of mole amounts.
- Dalton's law gives the total pressure as the sum of the component partial pressures; converting a mole amount to mass requires its own molar mass.

Step 1: Find the partial pressure of oxygen There are only two gases in the cylinder, so their partial pressures add to the stated total:

$$P_{\text{total}} = p_{O_2} + p_{\text{Ne}}.$$

Using the supplied pressures,

$$p_{O_2} = 25 - 20 = 5\,\text{bar}.$$

The oxygen pressure is not the total pressure; it is the remainder after subtracting the contribution from neon.

Step 2: Calculate the mole amounts The given neon mass and molar mass give

$$n_{\text{Ne}} = \frac{200\,\text{g}}{20\,\text{g mol}^{-1}} = 10\,\text{mol}.$$

Since both gases share the same temperature and cylinder volume, the common factor RT/V cancels in the ratio:

$$\frac{n_{O_2}}{n_{\text{Ne}}} = \frac{p_{O_2}}{p_{\text{Ne}}} = \frac{5}{20} = \frac{1}{4}.$$

It follows that $n_{O_2} = 10/4 = 2.5\,\text{mol}$.

Step 3: Convert the oxygen amount to mass Use the molecular molar mass of oxygen, $32\,\text{g mol}^{-1}$, rather than the atomic mass of oxygen:

$$x = n_{O_2} M_{O_2} = 2.5 \times 32 = 80\,\text{g}.$$

As a check, the total amount is $12.5\,\text{mol}$, and the neon mole fraction is $10/12.5 = 0.8$. Its partial pressure is accordingly $0.8 \times 25 = 20\,\text{bar}$, as required.

Step 4: Conclusion The oxygen mass is $80\,\text{g}$, so the requested value of x is $\boxed{80}$.

Quick Tip to Solve: At a common temperature and volume, use the pressure ratio as a mole ratio; do not equate it to a mass ratio unless the molar masses are equal.

89. Consider the following sulphur based oxoacids.

$H_2SO_3, H_2SO_4, H_2S_2O_8$ **and** $H_2S_2O_7$**.**

Amongst these oxoacids, the number of those with peroxo $(O-O)$ **bond is _____.** [4 marks]

Solution:

Concept:

- A peroxo linkage is a direct oxygen–oxygen single bond, $-O-O-$. An oxygen bridge between sulphur atoms, $S-O-S$, is a different linkage and is not peroxo.
- The conventional structures of the sulphur oxoacids distinguish ordinary terminal oxo or hydroxyl oxygen atoms from peroxide oxygen atoms. Formula oxygen counts alone do not identify an oxygen–oxygen bond.

Step 1: Examine sulphurous and sulphuric acids Sulphurous acid, H_2SO_3, has the conventional connectivity $HO-S(=O)-OH$. Sulphuric acid, H_2SO_4, has connectivity $HO-S(=O)_2-OH$. In each, the oxygen atoms attach directly to sulphur or form hydroxyl groups; none is bonded directly to another oxygen. Thus neither of these two acids contributes to the required count.

Step 2: Distinguish the two disulphur acids Peroxodisulphuric acid, $H_2S_2O_8$, contains the connectivity

$$HO-S(=O)_2-O-O-S(=O)_2-OH.$$

Its central $O-O$ linkage is a peroxide bond. In contrast, pyrosulphuric acid, $H_2S_2O_7$, is represented by

$$HO-S(=O)_2-O-S(=O)_2-OH.$$

Here a single oxygen bridges the two sulphur centres, so there is no direct oxygen–oxygen bond.

Step 3: Count acids rather than oxygen atoms Among the four listed compounds, only $H_2S_2O_8$ contains a peroxo bond. It contains one such linkage, but the question specifically asks how many of the acids possess that feature. H_2SO_3, H_2SO_4 and $H_2S_2O_7$ are all excluded. The resulting number of qualifying oxoacids is therefore one.

Step 4: Conclusion Only $H_2S_2O_8$ has a peroxo bond; the numerical answer is $\boxed{1}$.

Quick Tip to Solve: Keep $S-O-O-S$ and $S-O-S$ distinct: only the first contains the direct oxygen–oxygen bond of a peroxoacid.

90. The number of chlorine atoms in bithionol is _____. [4 marks]

Solution:

Concept:

- Bithionol is a sulphur-linked chlorinated phenolic compound used as an antiseptic. Its structural name is 2,2′-thiobis(4,6-dichlorophenol).
- In a name containing bis, count the substituents on both repeated units. Each 4,6-dichlorophenol unit has two chlorine substituents on its benzene ring.

Step 1: Identify the two phenolic units The structure of bithionol consists of two substituted phenol rings joined through a sulphur atom. The sulphur links the carbon at position 2 of one phenol ring with the corresponding position of the other ring. Each ring retains its hydroxyl group. The linking sulphur is separate from the halogen substituents and adds no chlorine to the molecule.

Step 2: Count chlorines on each ring The first phenolic ring has chlorine at positions 4 and 6, giving two chlorine atoms. The second phenolic ring has the same substitution pattern, so it contributes another two. The word dichloro describes one repeated ring unit; it must not be taken as the chlorine count of the entire molecule when that unit occurs twice.

Step 3: Add both contributions and check the formula The total chlorine count is

$$N_{\mathrm{Cl}} = 2 + 2 = 2 \times 2 = 4.$$

This agrees with the molecular formula $\mathrm{C_{12}H_6Cl_4O_2S}$: the subscript on chlorine is four. The oxygen subscript two comes from the two phenolic groups, and the single sulphur atom is the bridge. These other heteroatoms are not included because the question asks specifically for chlorine atoms.

Step 4: Conclusion Bithionol contains $\boxed{4}$ chlorine atoms.

Quick Tip to Solve: Remember bithionol as two identical dichlorophenol units joined by sulphur; two chlorines on each ring give four in total.