JEE Main 2022 Paper 1 (B.E./B.Tech) Jun 24 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 1 (June), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 24 June 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $A = \{z \in C : 1 \le |z - (1+i)| \le 2\}$
and $B = \{z \in A : |z - (1-i)| = 1\}$**. Then, B :** [4 marks]

(A) is an empty set

(B) contains exactly two elements

(C) contains exactly three elements

(D) is an infinite set

Correct Answer: (D) is an infinite set

Solution:

Concept:

- A modulus $|z - z_0|$ is the distance from the point representing z to the fixed point z_0 in the Argand plane.
- The condition defining A is a closed annulus about $1 + i$, while $|z - (1 - i)| = 1$ describes a circle of radius one.

Step 1: Compare the two centres The centres are $C = 1 + i$ and $D = 1 - i$, whose separation is $|C - D| = 2$. For a point on the circle of centre D and radius one, its distance from C ranges continuously from $2 - 1 = 1$ to $2 + 1 = 3$. Thus no point on this circle lies inside the inner boundary of the annulus. The remaining restriction is that its distance from C must not exceed two.

Step 2: Exhibit an entire arc of admissible points Write a point on the circle as $z = 1 - i + e^{it}$, with $0 \le t < 2\pi$. Its other squared distance is

$$|z - (1+i)|^2 = |e^{it} - 2i|^2 = 5 - 4\sin t.$$

The annulus condition becomes

$$1 \le 5 - 4\sin t \le 4, \qquad \sin t \ge \frac{1}{4}.$$

In particular, every parameter in $[\sin^{-1}(1/4), \pi - \sin^{-1}(1/4)]$ supplies a point of B. This interval has positive length, and distinct parameters in it give distinct points. Hence the intersection contains a whole circular arc, rather than only boundary intersection points.

Step 3: Conclusion The set B has infinitely many elements. It is neither empty nor a set of two or three points; those finite counts would confuse the annulus with just its boundary circles. Hence, the correct option is (D).

Quick Tip to Solve: For an intersection with an annulus, retain the entire portion of the circle between the two distance bounds; do not count only boundary intersections.

2. The remainder when 3^{2022} is divided by 5 is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- Two integers are congruent modulo five if they leave the same remainder on division by five.
- If a power of an integer is congruent to one, exponents can be reduced by that power to identify the repeating remainder cycle.

Step 1: Find the repeating power cycle The successive powers of three have remainders

$$3^1 \equiv 3, \quad 3^2 \equiv 4, \quad 3^3 \equiv 2, \quad 3^4 \equiv 1 \pmod 5.$$

In particular, $3^4 = 81 = 5(16) + 1$. Multiplying by 3^4 therefore repeats the remainder. This reduces the calculation to a small exponent and avoids constructing the very large integer 3^{2022}.

Step 2: Reduce the exponent and identify the remainder Divide the exponent by the cycle length four:

$$2022 = 4(505) + 2.$$

Consequently,

$$3^{2022} = (3^4)^{505}3^2 \equiv 1^{505} \cdot 9 \equiv 4 \pmod 5.$$

Equivalently, expanding $(80 + 1)^{505}$ shows that every term except its constant term contains a factor five; multiplication by nine leaves remainder four. A valid remainder on division by five is one of $0, 1, 2, 3, 4$, so the congruence already gives the required remainder without any further reduction. The other listed remainders belong to exponents congruent to zero, three or one modulo four, respectively; none corresponds to $2022 \equiv 2 \pmod 4$.

Step 3: Conclusion The remainder when 3^{2022} is divided by five is 4. Hence, the correct option is (D).

Quick Tip to Solve: Write the exponent as a multiple of the remainder-cycle length plus its residue; here the residue is two, so only the remainder of 3^2 matters.

3. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds, it becomes 7 units, then its radius after 9 seconds is : [4 marks]

(A) 9

(B) 10

(C) 11

(D) 12

Correct Answer: (A) 9

Solution:

Concept:

- The surface area of a sphere of radius r is $S = 4\pi r^2$. A constant rate of increase of S makes S, and therefore r^2, linear in time.
- The radius is positive, so recovering it from r^2 requires the positive square root. Constant surface-area growth does not imply constant radial growth.

Step 1: Determine the constant rate from the two observations Let t be time in seconds and let c be the constant rate of increase of r^2. Initially $r = 3$, so

$$r^2 = 9 + ct.$$

At $t = 5$, the radius is seven, giving

$$49 = 9 + 5c, \qquad c = \frac{49 - 9}{5} = 8.$$

The corresponding surface-area rate is $4\pi c = 32\pi$ square units per second. The squared radii, rather than the radii themselves, must be used to find the rate.

Step 2: Evaluate the radius at nine seconds Substitute $t = 9$ into the squared-radius relation:

$$r^2 = 9 + 8(9) = 81.$$

Since the balloon radius cannot be negative,

$$r = \sqrt{81} = 9 \text{ units.}$$

As a check, the area rises from 36π initially to 196π after five seconds and to 324π after nine seconds; both increments correspond to the same rate 32π. The alternative radii ten, eleven and twelve would give squared radii one hundred, one hundred twenty-one and one hundred forty-four, none equal to eighty-one.

Step 3: Conclusion After nine seconds, the radius is 9 units. Hence, the correct option is (A).

Quick Tip to Solve: When a spherical area changes uniformly, interpolate r^2 against time. Interpolating r instead gives an incorrect growth law.

4. Bag A contains 2 white, 1 black and 3 red balls and bag B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to _____. [4 marks]

(A) 13

(B) 6

(C) 4

(D) 3

Correct Answer: (C) 4

Solution:

Concept:

- For unordered drawing without replacement, the probability of one red and one black ball is the number of red-black pairs divided by the total number of pairs.
- Bayes theorem combines the likelihood of an observed event with the prior probability of each possible source bag.

Step 1: Calculate the likelihood in each bag Let E denote the event of drawing one red and one black ball. Bag A has six balls and 3×1 favourable pairs, so

$$P(E \mid A) = \frac{3 \cdot 1}{\binom{6}{2}} = \frac{1}{5}.$$

Bag B contains $n + 5$ balls, with $2 \times 3 = 6$ favourable pairs, giving

$$P(E \mid B) = \frac{6}{\binom{n+5}{2}} = \frac{12}{(n+5)(n+4)}.$$

Both expressions treat the draw as an unordered pair; no extra factor two belongs in their numerators.

Step 2: Use the stated posterior probability The bag is selected at random, so $P(A) = P(B) = 1/2$. These equal factors cancel in Bayes theorem. Put $L = (n + 5)(n + 4)$. Then

$$\frac{P(E \mid A)}{P(E \mid A) + P(E \mid B)} = \frac{1/5}{1/5 + 12/L} = \frac{6}{11}.$$

Cross multiplication gives

$$11L = 6(L + 60), \quad L = 72.$$

Thus

$$n^2 + 9n + 20 = 72, \quad (n - 4)(n + 13) = 0.$$

A number of balls is nonnegative, excluding $n = -13$, and leaving $n = 4$. This gives nine balls in bag B and likelihood $6/36 = 1/6$. Substitution checks the posterior: $(1/5)/(1/5 + 1/6) = 6/11$, as required.

Step 3: Conclusion The number of white balls in bag B is $n = 4$. The other proposed counts thirteen, six and three do not satisfy $(n + 5)(n + 4) = 72$. Hence, the correct option is (C).

Quick Tip to Solve: Equal bag probabilities cancel in Bayes theorem; calculate each likelihood using combinations consistently.

5. The number of values of α for which the system of equations :

$x + y + z = \alpha$

$\alpha x + 2\alpha y + 3z = -1$

$x + 3\alpha y + 5z = 4$

is inconsistent, is [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- A square linear system has a unique solution whenever its coefficient determinant is nonzero. Inconsistency can occur only where that determinant vanishes.
- A zero coefficient determinant alone does not prove inconsistency: the reduced equations must be checked for contradictory right-hand sides.

Step 1: Find all possible singular parameter values Let M be the coefficient matrix. Subtract α times its first row from its second, and its first row from its third. These operations preserve the determinant and give

$$\det M = \begin{vmatrix} 1 & 1 & 1 \\ 0 & \alpha & 3-\alpha \\ 0 & 3\alpha - 1 & 4 \end{vmatrix}.$$

Expanding along the first column,

$$\det M = 4\alpha - (3-\alpha)(3\alpha - 1) = 3(\alpha - 1)^2.$$

Therefore every $\alpha \neq 1$ gives a nonsingular matrix and a consistent system with a unique solution. Only the value $\alpha = 1$ needs further investigation.

Step 2: Check the remaining parameter directly At $\alpha = 1$, the equations become

$$x + y + z = 1, \quad x + 2y + 3z = -1, \quad x + 3y + 5z = 4.$$

Subtracting the first equation from the second and third gives

$$y + 2z = -2, \qquad 2y + 4z = 3.$$

Twice the first reduced equation requires $2y + 4z = -4$, contradicting the second reduced equation. Hence no triple (x, y, z) can solve the system for this parameter. The repeated factor in the determinant is a double root but represents just one distinct parameter value; it must not be counted twice.

Step 3: Conclusion The system is inconsistent exactly at $\alpha = 1$, so the number of values is 1. Zero misses this contradiction, while two counts the double root twice and three has no determinant basis.

Hence, the correct option is (B).

Quick Tip to Solve: Count distinct zeros of the coefficient determinant, then test each singular system: a repeated determinant root is still only one parameter value.

6. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to : [4 marks]

(A) 18

(B) 24

(C) 36

(D) 96

Correct Answer: (B) 24

Solution:

Concept:

- For the quadratic $3x^2+\lambda x-1=0$, Vieta relations give the root sum $s=\alpha+\beta=-\lambda/3$ and product $p=\alpha\beta=-1/3$.
- Use symmetric identities $\alpha^2+\beta^2=s^2-2p$ and $\alpha^3+\beta^3=s^3-3ps$, avoiding the individual roots.

Step 1: Translate the reciprocal condition into the root sum Because the product $p=-1/3$ is nonzero, both reciprocals exist. Combining them over a common denominator gives

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}=\frac{s^2-2p}{p^2}.$$

Substitute the given value and the known product:

$$15=\frac{s^2+2/3}{1/9}=9s^2+6.$$

Hence $9s^2=9$, so $s^2=1$. There is no need to decide between $s=1$ and $s=-1$, because the final expression is squared.

Step 2: Calculate the cubic sum and its square Apply the cubic identity with $p=-1/3$:

$$\alpha^3+\beta^3=s^3-3ps=s^3+s=s(s^2+1)=2s.$$

Squaring removes the unresolved sign of s:

$$(\alpha^3+\beta^3)^2=4s^2=4.$$

Multiplication by the printed factor six now yields

$$6(\alpha^3+\beta^3)^2=6\cdot 4=24.$$

As a consistency check, $\lambda^2=9s^2=9$; either possible coefficient gives the same squared cubic sum.

The other proposed totals eighteen, thirty-six and ninety-six cannot result from this identity with $s^2 = 1$.

Step 3: Conclusion The required value is 24. Hence, the correct option is (B).

Quick Tip to Solve: When a target is symmetric in two quadratic roots, compute their sum and product first; a squared target often removes any sign ambiguity immediately.

7. The set of all values of k for which

$(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = k\pi^3,\ x \in R$**, is the interval :** [4 marks]

(A) $\left[\frac{1}{32}, \frac{7}{8}\right)$

(B) $\left(\frac{1}{24}, \frac{13}{16}\right)$

(C) $\left[\frac{1}{48}, \frac{13}{16}\right]$

(D) $\left[\frac{1}{32}, \frac{9}{8}\right)$

Correct Answer: (A) $\left[\frac{1}{32}, \frac{7}{8}\right)$

Solution:

Concept:

- Use the principal branches $\tan^{-1} x \in (-\pi/2, \pi/2)$ and $\cot^{-1} x \in (0, \pi)$, so their sum is $\pi/2$ for every real x.
- A sum of two cubes with fixed sum reduces to a quadratic. Its vertex and the open domain endpoints determine the exact range, including endpoint inclusion.

Step 1: Express the sum of cubes using one angle Put $t = \tan^{-1} x$, so $\cot^{-1} x = \pi/2 - t$. Expanding and completing the square gives

$$\begin{aligned} t^3 + (\pi/2 - t)^3 &= \frac{\pi^3}{8} - \frac{3\pi^2 t}{4} + \frac{3\pi t^2}{2} \\ &= \frac{\pi^3}{32} + \frac{3\pi}{2}\left(t - \frac{\pi}{4}\right)^2. \end{aligned}$$

Divide by π^3, which is positive, to obtain

$$k = \frac{1}{32} + \frac{3}{2}\left(\frac{t}{\pi} - \frac{1}{4}\right)^2.$$

The allowed domain remains the open interval $-\pi/2 < t < \pi/2$.

Step 2: Determine the minimum and upper bound The squared term vanishes at $t = \pi/4$, an allowed angle corresponding to $x = 1$. Thus the minimum $1/32$ is attained. Across the domain, $t/\pi - 1/4$ ranges over $(-3/4, 1/4)$, whose squared values fill $[0, 9/16)$. The upper value is approached as $t \to -\pi/2$, equivalently $x \to -\infty$, but never attained by finite x. Therefore

$$\frac{1}{32} \le k < \frac{1}{32} + \frac{3}{2} \cdot \frac{9}{16} = \frac{7}{8}.$$

Continuity of the quadratic ensures every intermediate value is obtained. The alternative intervals either have a wrong lower bound or a wrong upper bound.

Step 3: Conclusion The required interval is $\left[\frac{1}{32}, \frac{7}{8}\right)$. Hence, the correct option is (A).

Quick Tip to Solve: Replace inverse cotangent by $\pi/2 - \tan^{-1} x$, complete the square, and retain the open endpoints of the inverse-tangent range.

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8. For the function

$f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5,\ x > 1$, **which one of the following is NOT correct?** [4 marks]

(A) f is increasing in (1, 2) and decreasing in (2, ∞)

(B) f(x) = -1 has exactly two solutions

(C) $f'(e) - f''(2) < 0$

(D) f(x) = 0 has a root in the interval (e, e + 1)

Correct Answer: (C) $f'(e) - f''(2) < 0$

Solution:

Concept:

- Differentiate to find monotonicity. For a continuous function, opposite signs at two endpoints guarantee a zero between them.
- A strictly increasing or decreasing branch takes any value in its range exactly once; its endpoint limits determine that range.

Step 1: Differentiate and identify the branches For $x > 1$, differentiation gives

$$f'(x) = \frac{4}{x-1} - 4x + 4 = \frac{4x(2-x)}{x-1}, \qquad f''(x) = -\frac{4}{(x-1)^2} - 4.$$

The denominator of f' is positive. Thus $f' > 0$ on $(1, 2)$ and $f' < 0$ on $(2, \infty)$. The function has maximum $f(2) = 5$. Option (A) is correct.

Step 2: Check the horizontal level and the root interval As $x \to 1^+$, the logarithm tends to $-\infty$, and as $x \to \infty$, the negative quadratic dominates, again giving $-\infty$. Each strictly monotone branch therefore crosses the level -1 once, making (B) correct. Since $2 < e < 3$ and the function decreases after 2,

$$f(e) > f(3) = 4\ln 2 - 1 > 0, \qquad f(e+1) = 7 - 2e^2 < 0.$$

Continuity gives a zero in $(e, e+1)$, so (D) is also correct.

Step 3: Test the derivative comparison At the required points,

$$f'(e) - f''(2) = \frac{4}{e-1} - 4e + 12.$$

Both $4/(e-1)$ and $12 - 4e$ are positive because $1 < e < 3$. Consequently this difference is strictly positive, whereas (C) claims it is negative. The question asks for the statement that is NOT correct.

Step 4: Conclusion The proposed negative derivative difference is incorrect: the difference is strictly positive. Hence, the correct option is (C).

Quick Tip to Solve: For a NOT-correct question, check all statements: one derivative sign chart often settles several options at once.

9. The sum of absolute maximum and absolute minimum values of the function $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval [0, 1] is : [4 marks]

(A) $3 + \frac{\sin(1)\cos^2\left(\frac{1}{2}\right)}{2}$

(B) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$

(C) $5 + \frac{1}{2}(\sin(1) + \sin(2))$

(D) $2 + \sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

Correct Answer: (B) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$

Solution:

Concept:

- At a zero of the expression inside a modulus, split the domain before differentiating. Absolute extrema on a closed interval include endpoints and nondifferentiable points.
- Use $\sin x \cos x = \frac{1}{2}\sin 2x$ and $\sin 2x = 2\sin x \cos x$ to compare the result with the options.

Step 1: Split at the zero of the quadratic Factor the expression inside the modulus:

$$2x^2 + 3x - 2 = (2x - 1)(x + 2).$$

It is negative for $0 \le x < 1/2$ and positive for $1/2 < x \le 1$. Hence the function is

$$f(x) = \begin{cases} -2x^2 - 3x + 2 + \frac{1}{2}\sin 2x, & 0 \le x \le 1/2, \\ 2x^2 + 3x - 2 + \frac{1}{2}\sin 2x, & 1/2 \le x \le 1. \end{cases}$$

The join $x = 1/2$ must be checked even though the derivative may not exist there.

Step 2: Find the minimum and maximum On the left branch, $f'(x) = -4x - 3 + \cos 2x \le -2 < 0$. On the right branch, $f'(x) = 4x + 3 + \cos 2x \ge 4x + 2 > 0$. Thus the minimum is at the join:

$$f_{\min} = f(1/2) = \tfrac{1}{2}\sin 1.$$

The maximum is one of the endpoints. Since $f(0) = 2$ and $f(1) = 3 + \frac{1}{2}\sin 2 > 3$, the absolute maximum is $3 + \frac{1}{2}\sin 2$. All trigonometric arguments here are in radians.

Step 3: Add and match the option The required sum is therefore

$$f_{\max} + f_{\min} = 3 + \tfrac{1}{2}(\sin 1 + \sin 2) = 3 + \tfrac{1}{2}(1 + 2\cos 1)\sin 1.$$

This is exactly (B). Option (A) has an extra factor involving $\cos^2(1/2)$ and is smaller; (C) has constant 5 instead of 3. Option (D) uses the left endpoint value 2 rather than the actual maximum, so it also

fails.

Step 4: Conclusion The sum is $3 + \frac{1}{2}(1 + 2\cos 1)\sin 1$. Hence, the correct option is (B).

Quick Tip to Solve: For an absolute-value function, test the kink along with endpoints; differentiating only one polynomial branch misses the minimum.

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10. If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to : [4 marks]

(A) 48

(B) 96

(C) 92

(D) 104

Correct Answer: (B) 96

Solution:

Concept:

- For an arithmetic progression of common difference 1, each even-indexed term exceeds the immediately preceding odd-indexed term by 1.
- The total sum is the sum of the even-indexed terms and the odd-indexed terms. Pairwise subtraction eliminates the unknown first term.

Step 1: Separate the even and odd sums Let E denote the sum of the even-indexed terms and O the sum of the odd-indexed terms. Because n is even, these two groups contain the same number $n/2$ of terms and together contain every term exactly once. The data give

$$E = 120, \qquad E + O = 192,$$

so

$$O = 192 - 120 = 72.$$

This separation requires no assumption that the first term is positive.

Step 2: Subtract paired terms Write the difference as a sum of adjacent pairs:

$$E - O = (a_2 - a_1) + (a_4 - a_3) + \cdots + (a_n - a_{n-1}).$$

Every bracket equals the common difference 1, and there are $n/2$ brackets. Therefore

$$120 - 72 = \frac{n}{2}, \qquad n = 96.$$

The value 48 is the number of pairs, not the number of terms. Options 92 and 104 would give differences 46 and 52 respectively, contradicting the observed difference 48.

Step 3: Verify consistency with an arithmetic progression For $n = 96$, the total-sum formula gives

$$192 = \frac{96}{2}(2a_1 + 95), \qquad a_1 = -\frac{91}{2}.$$

The even terms run from $-89/2$ to $99/2$. Their sum is $48\big((-89/2 + 99/2)/2\big) = 120$, confirming that the given sums are consistent. Negative initial terms are allowed in an arithmetic progression.

Step 4: Conclusion The number of terms is $n = 96$. Hence, the correct option is (B).

Quick Tip to Solve: Subtract the odd-position sum from the even-position sum: for common difference d, the difference is nd/2.

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11. If x = x(y) is the solution of the differential equation
$y\frac{dx}{dy} = 2x + y^3(y+1)e^y$, $x(1) = 0$**; then x(e) is equal to :** [4 marks]

(A) $e^3(e^e - 1)$

(B) $e^e(e^3 - 1)$

(C) $e^2(e^e + 1)$

(D) $e^e(e^2 - 1)$

Correct Answer: (A) $e^3(e^e - 1)$

Solution:

Concept:

- A first-order linear equation $x\prime + P(y)x = Q(y)$ uses integrating factor $e^{\int P(y)\,dy}$, with y as the independent variable.
- The initial value at $y = 1$ fixes the integration constant. Work on $y > 0$, which contains both 1 and e.

Step 1: Put the equation in linear form The dependent variable is x, not y. Divide the equation by y on the positive interval containing the initial point:

$$\frac{dx}{dy} - \frac{2}{y}x = y^2(y+1)e^y.$$

The coefficient of x is $-2/y$, so the integrating factor is

$$\text{IF} = e^{\int -2/y\,dy} = y^{-2}.$$

The negative sign in this exponent is essential; using y^2 would not produce the product derivative required below.

Step 2: Integrate the product derivative Multiplication by y^{-2} yields

$$\frac{d}{dy}\left(\frac{x}{y^2}\right) = (y+1)e^y.$$

Since $\frac{d}{dy}(ye^y) = (y+1)e^y$, integration gives

$$\frac{x}{y^2} = ye^y + C, \qquad x = y^3e^y + Cy^2.$$

Apply $x(1) = 0$: $0 = e + C$, hence $C = -e$. The resulting solution $x = y^3e^y - ey^2$ satisfies both the equation and the initial condition.

Step 3: Evaluate at the specified argument Substitute $y = e$, distinguishing the argument e from the exponential e^y:

$$x(e) = e^3e^e - e^3 = e^3(e^e - 1).$$

Only (A) has the required factor e^3 and subtraction term $-e^3$. Option (B) interchanges the factors in the subtraction, (C) gives the wrong sign and power, and (D) has a different subtraction term.

Step 4: Conclusion The required value is $e^3(e^e - 1)$. Hence, the correct option is (A).

Quick Tip to Solve: When x is a function of y, form the integrating factor using y; then recognize $(y+1)e^y$ as the derivative of ye^y.

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12. Let $\hat{a}$, $\hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|6\vec{c}|^2$ is equal to : [4 marks]

(A) $6\left(3 - \sqrt{3}\right)$

(B) $3 + \sqrt{3}$

(C) $6\left(3 + \sqrt{3}\right)$

(D) $6\left(\sqrt{3} + 1\right)$

Correct Answer: (C) $6\left(3 + \sqrt{3}\right)$

Solution:

Concept:

- The vector $\vec{c} \times \hat{a}$ is perpendicular to $\vec{c}$, so their scalar product is zero.
- For unit $\hat{a}$, $|\vec{c} \times \hat{a}| = |\vec{c}| \sin\theta$, where θ is the angle between them.

Step 1: Square the given vector identity Let $r = |\vec{c}|$ and $\theta = \pi/12$. Using $|\hat{b}| = 1$, square the magnitude of the given sum:

$$1 = |\vec{c} + 2(\vec{c} \times \hat{a})|^2 = r^2 + 4|\vec{c} \times \hat{a}|^2.$$

The mixed term vanishes because a cross product is perpendicular to its first factor. This is why one must add the squared magnitudes rather than add the magnitudes and then square.

Step 2: Evaluate the trigonometric factor Since $\hat{a}$ is a unit vector,

$$1 = r^2(1 + 4\sin^2(\pi/12)).$$

Use the half-angle identity and $\cos(\pi/6) = \sqrt{3}/2$:

$$\sin^2(\pi/12) = \frac{2-\sqrt{3}}{4}, \qquad 1 + 4\sin^2(\pi/12) = 3 - \sqrt{3}.$$

Thus $r^2 = 1/(3-\sqrt{3})$. The denominator is positive, as required for a squared magnitude.

Step 3: Scale and rationalize Multiplication of a vector by 6 multiplies its squared magnitude by 36. Consequently

$$|6\vec{c}|^2 = 36r^2 = \frac{36}{3-\sqrt{3}} = \frac{36(3+\sqrt{3})}{9-3} = 6(3+\sqrt{3}).$$

Option (A) has the conjugate with the wrong sign, (B) omits the factor 6, and (D) changes the constant term. None equals the squared magnitude computed from the unit-vector condition.

Step 4: Conclusion $|6\vec{c}|^2 = 6(3+\sqrt{3})$. Hence, the correct option is (C).

Quick Tip to Solve: Before expanding a vector magnitude, check perpendicularity; a cross product often removes the mixed term immediately.

13. The domain of the function

$f(x) = \frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\log_e(x^2-3x+2)}$ **is :** [4 marks]

(A) $(-\infty, 1) \cup (2, \infty)$

(B) $(2, \infty)$

(C) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty)$

(D) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{3, \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

Correct Answer: (D) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{3, \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

Solution:

Concept:

- For real $\cos^{-1} u$, its argument must lie in $[-1, 1]$. An original rational denominator must remain nonzero even after cancellation.
- A logarithm requires a positive argument. When a logarithm appears in a denominator, its argument must also differ from 1.

Step 1: Impose the inverse-cosine restrictions Factor the rational argument:

$$u = \frac{(x-2)(x-3)}{(x-3)(x+3)} = \frac{x-2}{x+3}, \qquad x \neq -3, 3.$$

The upper bound $u \leq 1$ is equivalent to $-5/(x+3) \leq 0$, hence $x > -3$. On this range the lower bound $u \geq -1$ gives

$$\frac{2x+1}{x+3} \geq 0 \quad \Longrightarrow \quad x \geq -\frac{1}{2}.$$

Thus inverse cosine requires $x \in [-1/2, \infty)$, retaining the original exclusion $x = 3$. The excluded

value -3 already lies outside this interval.

Step 2: Apply both logarithmic conditions The logarithm argument must be positive:

$$x^2 - 3x + 2 = (x-1)(x-2) > 0 \quad \Longrightarrow \quad x < 1 \text{ or } x > 2.$$

Intersecting with the inverse-cosine interval gives $[-1/2, 1) \cup (2, \infty)$, with 3 still removed. In addition, the logarithm in the denominator cannot be zero:

$$x^2 - 3x + 2 \neq 1 \quad \Longrightarrow \quad x \neq \frac{3 \pm \sqrt{5}}{2}.$$

Both roots lie in the otherwise permitted intervals, so both exclusions matter.

Step 3: Match the complete domain The domain is

$$\left(\left[-\frac{1}{2}, 1\right) \cup (2, \infty)\right) \setminus \left\{3, \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}.$$

Option (A) does not enforce the inverse-cosine restriction, (B) loses valid points below 1, and (C) forgets the three excluded values. Only (D) imposes every condition simultaneously.

Step 4: Conclusion The complete domain is the set specified in option (D). Hence, the correct option is (D).

Quick Tip to Solve: For a quotient involving log and inverse cosine, list three conditions separately: cosine argument range, positive log argument, and nonzero log value.

14. Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then A + C is equal to _____. [4 marks]

(A) 16

(B) 88/5

(C) 72

(D) -8

Correct Answer: (A) 16

Solution:

Concept:

- Two curves tangent at a common point have the same tangent line there. Implicit differentiation gives the circle slope $-(2x + A)/(2y + B)$.
- A point on a circle satisfies its equation. Use both common-point equations before imposing the tangent-slope condition.

Step 1: Use the two points on the circle Because the circle passes through $(0, 6)$,

$$36 + 6B + C = 0.$$

Tangency to the parabola at $(2, 4)$ means this point also lies on the circle:

$$20 + 2A + 4B + C = 0.$$

Subtract the first relation from the second to eliminate C:

$$2A - 2B - 16 = 0, \qquad A - B = 8.$$

The condition that the two curves share a point alone is insufficient; their slopes must agree as well.

Step 2: Equate the tangent slopes For the parabola $y = x^2$, $dy/dx = 2x$, so its slope at $(2, 4)$ is 4. Differentiating the circle gives

$$2x + A + (2y + B)\frac{dy}{dx} = 0.$$

At the tangency point, substitution of the slope 4 gives

$$4 + A + 4(8 + B) = 0, \qquad A + 4B = -36.$$

Combine this with $A = B + 8$: $5B = -44$, hence $B = -44/5$ and $A = -4/5$. The circle's slope denominator is $8 + B = -4/5 \neq 0$, consistent with the finite parabola slope.

Step 3: Find the required coefficient sum Return to the equation at $(0, 6)$:

$$C = -36 - 6B = -36 + \frac{264}{5} = \frac{84}{5}.$$

Therefore

$$A + C = -\frac{4}{5} + \frac{84}{5} = 16.$$

Only option (A) equals 16. The values $88/5$, 72 and -8 in the other options fail the coefficient equations obtained from the two points and the common tangent. No additional geometric drawing is needed to determine these coefficients.

Step 4: Conclusion $A + C = 16$. Hence, the correct option is (A).

Quick Tip to Solve: For circle-parabola tangency, use the common point and equal slopes; subtracting the two point equations first eliminates C.

15. Let S = $\{\sqrt{n} : 1 \le n \le 50$ and n is odd$\}$.

Let a $\in$ S and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$.

If $\sum_{a \in S} \det(adj\ A) = 100\lambda$, then λ is equal to : [4 marks]

(A) 218

(B) 221

(C) 663

(D) 1717

Correct Answer: (B) 221

Solution:

Concept:

- For a square matrix of order m, $\det(\text{adj}\ A) = (\det A)^{m-1}$. Here the matrix has order three, so the exponent is two.
- The elements of S are square roots of the 25 odd positive integers from 1 to 49. Writing $n = 2k - 1$, with $1 \le k \le 25$, turns the required sum into a sum of squares.

Step 1: Evaluate the determinant using the sparse column The second column is $(0, 1, 0)^T$, so expansion along it leaves the first and third rows and columns. The cofactor sign for its nonzero entry is positive. Thus

$$\det A = \begin{vmatrix} 1 & a \\ -a & 1 \end{vmatrix} = 1 + a^2.$$

Since $a^2 = n$, this determinant is $n + 1$, always positive for the given set. Consequently, the adjugate determinant is $(n + 1)^2$, rather than $n + 1$ itself.

Step 2: Convert the sum into a standard finite sum Put $n = 2k - 1$. Each admissible value of n occurs once, and $n + 1 = 2k$. Therefore

$$\sum_{a \in S} \det(\text{adj}\ A) = 4 \sum_{k=1}^{25} k^2.$$

Using $\sum_{k=1}^{m} k^2 = m(m + 1)(2m + 1)/6$,

$$4\frac{25 \cdot 26 \cdot 51}{6} = 4(5525) = 22100.$$

The prescribed expression equals 100λ, so division by 100 gives $\lambda = 221$. The other proposed values, 218, 663 and 1717, do not reproduce this determinant sum.

Step 3: Conclusion $\lambda = 221$. Hence, the correct option is (B).

Quick Tip to Solve: Expand along a column with two zeros before using the adjugate identity; for a three-by-three matrix, square the determinant, not the matrix entries.

16. If the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve : [4 marks]

(A) $x^2 + \frac{y^2}{81} = 2$

(B) $\frac{y^2}{9} - x^2 = 8$

(C) $y = 4x^2 + 5$

(D) $\frac{x}{3} - y^2 = 2$

Correct Answer: (D) $\frac{x}{3} - y^2 = 2$

Solution:

Concept:

- The tangent at $(t, f(t))$ is $y - f(t) = f\prime(t)(x - t)$. Passing through the origin imposes $f(t) = tf\prime(t)$.
- After finding every real point of tangency, substitute its coordinates into each proposed curve. The question asks for the curve on which that point does not lie.

Step 1: Impose the condition that the tangent contains the origin For $f(x) = x^3 + 3x^2 + 5$, differentiation gives $f\prime(x) = 3x^2 + 6x$. Let the abscissa of tangency be t. Substitution of the origin into the tangent equation gives

$$t^3 + 3t^2 + 5 = t(3t^2 + 6t).$$

Moving terms and factoring,

$$2t^3 + 3t^2 - 5 = (t - 1)(2t^2 + 5t + 5) = 0.$$

The quadratic factor has discriminant $25 - 40 = -15$, so it has no real root. The only real point of tangency is therefore $(x_1, y_1) = (1, 9)$. As a check, its slope is 9 and its tangent is $y = 9x$, which indeed passes through the origin.

Step 2: Check all four proposed equations For option (A), substitution gives $1 + 81/81 = 2$, so the point lies on this ellipse. For option (B), it gives $81/9 - 1 = 8$, so the point also lies on that hyperbola. For option (C), the right side is $4(1)^2 + 5 = 9$, agreeing with its ordinate. For option (D), however,

$$\frac{x_1}{3} - y_1^2 = \frac{1}{3} - 81 = -\frac{242}{3} \neq 2.$$

Thus the first three equations contain the tangent point, whereas the fourth fails its defining equation.

Step 3: Conclusion The point $(1, 9)$ does not lie on $x/3 - y^2 = 2$. Hence, the correct option is (D).

Quick Tip to Solve: Use $f(t) = tf\prime(t)$ for a tangent through the origin, and check that any extra algebraic roots actually correspond to real tangency points.

17. Let $\lambda x-2y=\mu$ be a tangent to the hyperbola $a^2x^2-y^2=b^2$. Then $\left(\frac{\lambda}{a}\right)^2-\left(\frac{\mu}{b}\right)^2$ is equal to : [4 marks]

(A) -2

(B) -4

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- A tangent meets a nondegenerate conic at a repeated intersection point. Substituting the line into the conic therefore produces a quadratic with discriminant zero.
- The line $\lambda x - 2y = \mu$ has $y = (\lambda x - \mu)/2$. Retaining the coefficient 2 is essential because its square determines the final constant.

Step 1: Form the intersection equation Substitute the expression for y directly into the hyperbola, without converting the parameters into semiaxes:

$$a^2x^2 - \left(\frac{\lambda x - \mu}{2}\right)^2 = b^2.$$

Multiplication by four and collection of powers of x gives

$$(4a^2 - \lambda^2)x^2 + 2\lambda\mu x - (\mu^2 + 4b^2) = 0.$$

For an ordinary tangent this quadratic must have one repeated real root. The nondegenerate hyperbola has nonzero a and b; the zero-leading-coefficient limiting lines are asymptotes or single crossings, rather than finite tangents, so they do not supply additional tangent cases.

Step 2: Set the discriminant equal to zero and simplify The repeated-root condition is

$$(2\lambda\mu)^2 + 4(4a^2 - \lambda^2)(\mu^2 + 4b^2) = 0.$$

Divide by four, expand, and cancel the opposite $\lambda^2\mu^2$ terms:

$$4a^2\mu^2 + 16a^2b^2 - 4\lambda^2b^2 = 0.$$

Hence $\lambda^2b^2 - a^2\mu^2 = 4a^2b^2$. Dividing by a^2b^2 yields

$$\frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 4.$$

This exact identity excludes both negative alternatives and the value 2; no choice of the tangent point changes the constant.

Step 3: Conclusion $(\lambda/a)^2 - (\mu/b)^2 = 4$. Hence, the correct option is (D).

Quick Tip to Solve: Substitution and a zero discriminant avoid confusion between the coefficients of a hyperbola and its semiaxes; square the line coefficients carefully.

18. If a random variable X follows the Binomial distribution B(33, p) such that $3P(X=0)=P(X=1)$, then the value of $\frac{P(X=15)}{P(X=18)}-\frac{P(X=16)}{P(X=17)}$ is equal to : [4 marks]

(A) 1320

(B) 1088

(C) $\frac{120}{1331}$

(D) $\frac{1088}{1089}$

Correct Answer: (A) 1320

Solution:

Concept:

- If $X \sim B(n,p)$, then $P(X=r)=\binom{n}{r}p^r q^{n-r}$, where $q=1-p$. The supplied nontrivial probability condition determines the ratio q/p.
- Binomial coefficients satisfy $\binom{n}{r}=\binom{n}{n-r}$. Thus probabilities at complementary indices have a ratio involving only powers of p/q.

Step 1: Determine the success probability Using $n=33$, the stated equality is

$$3q^{33}=33pq^{32}.$$

For a nondegenerate binomial distribution, $0<p<1$, divide by $3q^{32}$ to obtain $q=11p$. Combining this with $p+q=1$ gives $p=1/12$ and $q=11/12$. The endpoint $p=1$ would make the requested probability quotients undefined, so it cannot give a value of the expression. The endpoint $p=0$ does not satisfy the initial equality.

Step 2: Use complementary indices to cancel coefficients Since $15+18=33$, the binomial coefficients in the first quotient are equal. Therefore

$$\frac{P(X=15)}{P(X=18)}=\frac{p^{15}q^{18}}{p^{18}q^{15}}=\left(\frac{q}{p}\right)^3=11^3=1331.$$

Similarly, $16+17=33$, and hence

$$\frac{P(X=16)}{P(X=17)}=\frac{p^{16}q^{17}}{p^{17}q^{16}}=\frac{q}{p}=11.$$

The required difference is $1331-11=1320$. Option (B) gives neither this difference nor either quotient. Options (C) and (D) correspond to small reciprocal-type values and are inconsistent with the first ratio being 1331.

Step 3: Conclusion The required value is 1320. Hence, the correct option is (A).

Quick Tip to Solve: Before expanding any factorials, check whether the two indices add to n; complementary binomial coefficients cancel immediately.

19. Let $S = \left\{\theta \in [-\pi, \pi] - \left\{\pm \frac{\pi}{2}\right\} : \sin\theta \tan\theta + \tan\theta = \sin 2\theta\right\}$**.**

If $T = \sum_{\theta \in S} \cos 2\theta$**, then T + n(S) is equal to :** [4 marks]

(A) $7 + \sqrt{3}$

(B) 9

(C) $8 + \sqrt{3}$

(D) 10

Correct Answer: (B) 9

Solution:

Concept:

- Use $\tan\theta = \sin\theta/\cos\theta$ only where $\cos\theta \neq 0$. The two excluded angles ensure this condition on the stated interval.
- The identity $\sin 2\theta = 2\sin\theta\cos\theta$ reduces the equation to a polynomial in $\sin\theta$. Keep the zero-sine solutions instead of dividing by $\sin\theta$.

Step 1: Factor the equation in terms of sine Write $s = \sin\theta$. Multiply the given equation by $\cos\theta$, which is nonzero throughout its domain. Then

$$s^2 + s = 2s\cos^2\theta = 2s(1 - s^2).$$

Thus

$$s(2s^2 + s - 1) = s(2s - 1)(s + 1) = 0.$$

The possibilities are $s = 0$, $s = 1/2$, and $s = -1$. The last gives $\theta = -\pi/2$, an excluded angle at which the original tangent is undefined, so it is rejected.

Step 2: List the admissible angles and add their contributions On the closed interval $[-\pi, \pi]$, the equation $\sin\theta = 0$ gives $-\pi, 0, \pi$. Both endpoints are distinct elements of this set and must be counted. The equation $\sin\theta = 1/2$ gives $\pi/6$ and $5\pi/6$. Consequently

$$S = \{-\pi, 0, \pi, \pi/6, 5\pi/6\}, \qquad n(S) = 5.$$

For the first three angles $\cos 2\theta = 1$. For the last two it equals $1/2$. Hence

$$T = 1 + 1 + 1 + \frac{1}{2} + \frac{1}{2} = 4, \qquad T + n(S) = 9.$$

The irrational choices cannot match this rational sum, while 10 would overcount either a solution or its cosine contribution.

Step 3: Conclusion $T + n(S) = 9$. Hence, the correct option is (B).

Quick Tip to Solve: Never cancel a sine factor before listing its zero solutions, and keep both endpoints when the interval includes $-\pi$ and π.

20. The number of choices for $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, such that $(p\Delta q) \Rightarrow ((p\Delta \sim q) \vee ((\sim p)\Delta q))$ is a tautology, is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Concept:

- An implication is false exactly when its antecedent is true and its consequent is false. A tautology must remain true for all four assignments of p and q.
- Test each possible connective separately, using $p \Rightarrow q \equiv \neg p \vee q$. A single counterexample is enough to eliminate a connective.

Step 1: Reject conjunction and biconditional with counterexamples First choose $\Delta = \wedge$ and let $p = q = \mathrm{T}$. The antecedent $p \wedge q$ is true, but each of $p \wedge \neg q$ and $\neg p \wedge q$ is false. Their disjunction is false, so the whole implication is false. Next choose $\Delta = \Leftrightarrow$ with the same assignment. The antecedent $p \Leftrightarrow q$ is true, but $p \Leftrightarrow \neg q$ and $\neg p \Leftrightarrow q$ are both false. Thus biconditional also fails. Neither of these two connectives qualifies.

Step 2: Prove that disjunction and implication always work For $\Delta = \vee$, the consequent becomes

$$(p \vee \neg q) \vee (\neg p \vee q).$$

It contains both $p \vee \neg p$ and $q \vee \neg q$, and is therefore always true. An implication with this consequent is a tautology. For $\Delta = \Rightarrow$, rewrite its consequent as

$$(\neg p \vee \neg q) \vee (p \vee q),$$

which is again always true. This second connective therefore also qualifies, irrespective of the antecedent. Exactly two of the four choices work: disjunction and implication. Counts 1, 3 and 4 are excluded by these explicit positive proofs and counterexamples.

Step 3: Conclusion The number of admissible connective choices is 2. Hence, the correct option is (B).

Quick Tip to Solve: To test a compound implication, first try making its antecedent true and consequent false; if the consequent simplifies to a tautology, the whole expression is settled.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The number of one-one functions f : {a, b, c, d} $\to$ {0, 1, 2, ..., 10} such that 2f(a) $-$ f(b) + 3f(c) + f(d) = 0 is _____. [4 marks]

Solution:

Concept:

- Write $A = f(a), B = f(b), C = f(c), D = f(d)$. A one-one function requires these four integers to be pairwise distinct and to belong to $\{0, 1, \ldots, 10\}$.
- The equation determines $B = 2A + 3C + D$. Count ordered triples (A, C, D) satisfying this upper bound and distinctness, without multiplying by any permutation factor: the domain labels are already fixed.

Step 1: Bound the possible value of C All values are nonnegative and $B \leq 10$, so $3C \leq 10$, giving $C = 0, 1, 2, 3$. For fixed C, require $0 \leq D \leq 10 - 3C - 2A$, excluding $D = A$ and $D = C$; also exclude $A = C$. In all admissible cases below, B is strictly larger than A, C, D, which verifies the remaining injectivity conditions. In particular, when $C = 0$, both A and D are positive, so $B = 2A + D$ exceeds each of them.

Step 2: Count the admissible pairs for each C The full enumeration is shown by listing the allowable D values for each A:

C	A	allowable D	count
0	1	$2, 3, 4, 5, 6, 7, 8$	7
0	2	$1, 3, 4, 5, 6$	5
0	3	$1, 2, 4$	3
0	4	$1, 2$	2
1	0	$2, 3, 4, 5, 6, 7$	6
1	2	$0, 3$	2
1	3	0	1
2	0	$1, 3, 4$	3
2	1	0	1
3	0	1	1

No higher A is possible in any row group because the upper bound on D, together with distinctness, leaves no admissible integer. Thus the totals for $C = 0, 1, 2, 3$ are respectively $17, 9, 4, 1$. Each listed pair determines exactly one B in range and hence exactly one function. The total is

$$17 + 9 + 4 + 1 = 31.$$

Step 3: Conclusion There are 31 one-one functions. The requested numerical entry is 31, a dimensionless count.

Quick Tip to Solve: Solve the linear constraint for the value with a negative coefficient, then enforce injectivity while enumerating the remaining small nonnegative integers.

22. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, -2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is _____. [4 marks]

Solution:

Concept:

- Let c, w, u be the numbers of correct, wrong and unattempted questions. They are nonnegative integers satisfying $c + w + u = 5$.
- The score condition is $3c - 2w = 5$. After choosing the questions in each category, each wrong answer has two possible choices; a correct answer and an omission each have only one.

Step 1: Find all feasible score combinations From the score equation,

$$w = \frac{3c - 5}{2}.$$

Thus c must be odd, and $w \geq 0$ requires $c \geq 2$. Since there are only five questions, the possible odd values are $c = 3$ and $c = 5$. For $c = 3$, we get $w = 2$, so $u = 0$. For $c = 5$, we get $w = 5$, which would require ten attempted questions and is impossible. Hence the only feasible distribution is three correct responses and two wrong responses.

Step 2: Count the actual answer selections Choose the three questions to answer correctly in $\binom{5}{3} = 10$ ways. Their correct choices are fixed. On each of the remaining two questions, exactly two of the three choices are incorrect, giving $2^2 = 4$ ways to choose the wrong answers. Therefore,

$$N = \binom{5}{3} 2^2 = 10 \times 4 = 40.$$

No additional factor is needed for ordering the questions, since their positions are already distinct. Also, no omission is possible in the only feasible distribution. The score checks as $3(3) - 2(2) = 9 - 4 = 5$.

Step 3: Conclusion The number of ways is $\boxed{40}$. This is a count of response patterns and has no physical unit.

Quick Tip to Solve: First solve the integer score equation with the total-question constraint; then multiply by two for each wrong response.

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23. Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$, $a > 0$, be a fixed point in the xy-plane. The image of A in y-axis be B and the image of B in x-axis be C. If $D(3\cos\theta, a\sin\theta)$ is a point in the fourth quadrant such that the maximum area of ΔACD is 12 square units, then a is equal to _____. [4 marks]

Solution:

Concept:

- Reflection in the y-axis reverses the x-coordinate; reflection in the x-axis reverses the y-coordinate.
- When two vertices are opposite position vectors, $C = -A$, the triangle area is $\frac{1}{2}|\det(C - A, D -$

$A)| = |\det(A, D)|$.

- For fourth-quadrant angles, $\cos\theta > 0$ and $\sin\theta < 0$; the largest value of $\cos\theta - \sin\theta$ is $\sqrt{2}$.

Step 1: Locate the reflected vertices and express the area The two reflections give

$$B = \left(-\frac{3}{\sqrt{a}}, \sqrt{a}\right), \qquad C = \left(-\frac{3}{\sqrt{a}}, -\sqrt{a}\right) = -A.$$

Using the determinant formula with $D = (3\cos\theta, a\sin\theta)$,

$$\begin{aligned}[ACD] &= |\det(A, D)| \\ &= \left|\frac{3}{\sqrt{a}}\, a\sin\theta - \sqrt{a}\, 3\cos\theta\right| \\ &= 3\sqrt{a}\,|\sin\theta - \cos\theta|.\end{aligned}$$

Because D lies in the fourth quadrant and $a > 0$, this is $3\sqrt{a}(\cos\theta - \sin\theta)$.

Step 2: Maximise the trigonometric factor and solve By the identity

$$\cos\theta - \sin\theta = \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right),$$

the maximum possible factor is $\sqrt{2}$. Equality occurs at $\theta = -\pi/4$, which does put D strictly in the fourth quadrant. Thus this maximum is attained within the allowed set, rather than being merely a bound. The maximum area is therefore

$$3\sqrt{2a} = 12 \implies \sqrt{2a} = 4 \implies a = 8.$$

The positive solution respects $a > 0$; substituting it back gives the required area of 12 square units.

Step 3: Conclusion The parameter is $\boxed{a = 8}$.

Quick Tip to Solve: When successive reflections place two vertices opposite about the origin, reduce the area to one determinant before maximising.

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24. Let a line having direction ratios, 1, −4, 2 intersect the lines $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the points A and B. Then $(AB)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Parameterise each given line separately. A point on the first line is $(7 + 3t, 1 - t, -2 + t)$, and a point on the second is $(2s, 7 + 3s, s)$.
- If the joining line has direction ratios $(1, -4, 2)$, its displacement satisfies $\overrightarrow{AB} = k(1, -4, 2)$. Therefore $AB^2 = k^2(1 + 16 + 4)$.

Step 1: Set up and solve the intersection equations Writing $B - A = k(1, -4, 2)$ component by

component gives

$$2s - 7 - 3t = k,$$
$$6 + 3s + t = -4k,$$
$$s + 2 - t = 2k.$$

The last equation yields $t = s + 2 - 2k$. Substitute it into the first equation:

$$-s - 13 + 6k = k \quad \Longrightarrow \quad s = 5k - 13.$$

Substitute the same expression for t into the second equation:

$$8 + 4s - 2k = -4k \quad \Longrightarrow \quad s = -2 - \frac{k}{2}.$$

Equating these expressions gives $5k - 13 = -2 - k/2$, hence $k = 2$, $s = -3$ and $t = -5$.

Step 2: Find the points and calculate the squared distance At $t = -5$, the first line gives $A = (-8, 6, -7)$. At $s = -3$, the second gives $B = (-6, -2, -3)$. Thus

$$\overrightarrow{AB} = (2, -8, 4) = 2(1, -4, 2),$$

which verifies the required direction ratios directly. The squared distance is

$$AB^2 = 2^2 + (-8)^2 + 4^2 = 4 + 64 + 16 = 84.$$

This also agrees with the direction-vector calculation $k^2(1^2 + (-4)^2 + 2^2) = 4 \times 21$. We do not take a square root, because the question explicitly asks for the square of the length.

Step 3: Conclusion $\boxed{(AB)^2 = 84}$, in square coordinate units.

Quick Tip to Solve: Use one scalar multiple of the specified direction vector for the displacement; it turns the intersection problem into three linear equations.

25. The number of points where the function

$$f(x) = \begin{cases} |2x^2 - 3x - 7| & if \quad x \leq -1 \\ [4x^2 - 1] & if \quad -1 < x < 1 \\ |x + 1| + |x - 2| & if \quad x \geq 1 \end{cases}$$

[t] denotes the greatest integer $\leq$ t, is discontinuous is _____. [4 marks]

Solution:

Concept:

- Absolute values of polynomial or linear functions are continuous everywhere. The possible discontinuities outside the floor-function interval are therefore only the junctions $x = -1, 1$.
- Inside $(-1, 1)$, the expression $4x^2 - 1$ takes values in $[-1, 3)$. A floor-function jump can occur where this expression crosses an integer; a tangency must be checked separately.

Step 1: Count the jumps strictly between the junctions For integer levels $0, 1, 2$, the equations $4x^2 - 1 = n$ give

$$x = \pm\frac{1}{2}, \qquad x = \pm\frac{1}{\sqrt{2}}, \qquad x = \pm\frac{\sqrt{3}}{2}.$$

All six points lie in $(-1, 1)$, and the inner quadratic crosses the corresponding integer at each one. The floor therefore jumps at every one of them. The remaining attained integer level is -1, reached at $x = 0$. Near zero, $-1 \le 4x^2 - 1 < 0$, so the floor is identically -1, including at zero. Hence $x = 0$ is continuous. The level 3 is not attained in the open interval.

Step 2: Check both junctions independently At $x = -1$, the first branch has value

$$f(-1) = |2 + 3 - 7| = 2.$$

Its left limit is also 2. Approaching from the right, $4x^2 - 1$ approaches 3 from below, so its floor is 2. Both one-sided limits and the function value agree: there is no discontinuity at -1. At $x = 1$, the left limit is again 2, but the right branch has

$$f(1) = |2| + |-1| = 3.$$

Its right limit is 3. Thus $x = 1$ adds one discontinuity. The absolute-value corners elsewhere cause no jump because their component functions remain continuous.

Step 3: Conclusion There are $6 + 1 = \boxed{7}$ points of discontinuity.

Quick Tip to Solve: For a floor of a quadratic, count integer crossings, exclude a minimum-level tangency, and then test each piecewise endpoint separately.

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26. Let $f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t)dt$**. Then the value of** $\left|\int_0^{\pi/2} f(\theta)d\theta\right|$ **is _____.**

[4 marks]

Solution:

Concept:

- The integration variable t is independent of θ. Hence each definite integral involving only t is a constant coefficient in the expression for $f(\theta)$.
- On a symmetric interval, an odd integrand integrates to zero. This eliminates $\sin t$ and $t\cos t$, while $\cos t$ and $t\sin t$ are even.

Step 1: Reduce the integral equation to two coefficients Define

$$I = \int_{-\pi/2}^{\pi/2} f(t)\,dt, \qquad J = \int_{-\pi/2}^{\pi/2} tf(t)\,dt.$$

The given equation becomes

$$f(\theta) = (1 + I)\sin\theta + J\cos\theta.$$

Write this as $f(\theta) = A\sin\theta + B\cos\theta$, where $A = 1 + I$ and $B = J$. Integrating over the symmetric interval gives $I = 2B$, since the sine term is odd and the cosine integral equals 2. Consequently,

$$A = 1 + 2B.$$

Step 2: Determine the coefficients and evaluate the requested integral Multiplying by t and integrating gives

$$J = A\int_{-\pi/2}^{\pi/2} t\sin t\,dt.$$

The omitted $Bt\cos t$ term is odd. By integration by parts,

$$\int_{-\pi/2}^{\pi/2} t\sin t\,dt = [-t\cos t + \sin t]_{-\pi/2}^{\pi/2} = 2.$$

Thus $B = J = 2A$. Combining this with $A = 1 + 2B$ yields $A = 1 + 4A$, so $A = -1/3$, $B = -2/3$. On $[0, \pi/2]$, the integrals of both sine and cosine equal 1. Therefore,

$$\int_0^{\pi/2} f(\theta)\,d\theta = A + B = -1.$$

The question asks for its absolute value, which removes this negative sign.

Step 3: Conclusion The requested value is $\boxed{1}$, a dimensionless number.

Quick Tip to Solve: For a kernel linear in $\sin\theta$ and $\cos\theta$, solve for their constant coefficients using symmetry before integrating again.

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27. Let $\underset{0\le x\le 2}{Max}\left\{\frac{9-x^2}{5-x}\right\} = \alpha$ **and** $\underset{0\le x\le 2}{Min}\left\{\frac{9-x^2}{5-x}\right\} = \beta$**.**

If $\int_{\beta-\frac{8}{3}}^{2\alpha-1} Max\left\{\frac{9-x^2}{5-x}, x\right\}dx = \alpha_1 + \alpha_2\log_e\left(\frac{8}{15}\right)$ **then** $\alpha_1 + \alpha_2$ **is equal to _____.** [4 marks]

Solution:

Concept:

- For $g(x) = (9 - x^2)/(5 - x)$, use $g(x) = x + 5 - 16/(5 - x)$. This form makes both differentiation and integration direct.
- A maximum of two functions is integrated by first identifying their crossing points and determining which function is larger on each side.

Step 1: Find the extrema and the integration limits For $0 \le x \le 2$,

$$g'(x) = 1 - \frac{16}{(5-x)^2}.$$

It is positive for $x < 1$, zero at 1, and negative for $x > 1$. Thus the maximum is $g(1) = 2$. The endpoint

values are $g(0) = 9/5$ and $g(2) = 5/3$, so the minimum is $5/3$. Hence

$$\alpha = 2, \quad \beta = \frac{5}{3}, \quad \beta - \frac{8}{3} = -1, \quad 2\alpha - 1 = 3.$$

Step 2: Split the maximum at its crossing point Throughout $[-1, 3]$, the denominator $5 - x$ is positive. Since

$$g(x) - x = \frac{9 - 5x}{5 - x},$$

we have $g(x) \geq x$ up to $x = 9/5$, and $x \geq g(x)$ afterwards. The integral is therefore

$$\int_{-1}^{9/5} \left(x + 5 - \frac{16}{5 - x} \right) dx + \int_{9/5}^{3} x \, dx.$$

An antiderivative for the first integrand is $x^2/2 + 5x + 16 \log_e(5 - x)$. All logarithm arguments here are positive.

Step 3: Evaluate and identify the two coefficients The quadratic terms at the internal boundary cancel between the two integrals. Their combined quadratic contribution is $(3^2 - (-1)^2)/2 = 4$, and the constant-term contribution is $5(9/5 + 1) = 14$. The logarithmic contribution is

$$16 \log_e \left(\frac{5 - 9/5}{5 - (-1)} \right) = 16 \log_e \left(\frac{8}{15} \right).$$

Thus the integral equals $18 + 16 \log_e(8/15)$, giving $\alpha_1 = 18$ and $\alpha_2 = 16$.

Step 4: Conclusion The required coefficient sum is $\boxed{\alpha_1 + \alpha_2 = 34}$.

Quick Tip to Solve: Divide the rational function first; then compare it with the other function by subtracting, so the maximum changes at an exact algebraic crossing.

28. Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of areas R_1, R_2. If $\max\{R_1, R_2\} = R_2$, then $\frac{R_2}{R_1}$ is equal to _____. [4 marks]

Solution:

Concept:

- The curves $y = x^3$ and $y^2 = x$ intersect where $x^6 = x$. Their real intersection points are $(0, 0)$ and $(1, 1)$.
- The bounded region has $0 \leq x \leq 1$ and $x^3 \leq y \leq \sqrt{x}$. The negative branch $y = -\sqrt{x}$ does not enclose another bounded region with the cubic.

Step 1: Locate the dividing line inside the bounded region Since $x \geq 0$ in this region, $y = 2|x|$ reduces to $y = 2x$. Its nonzero intersection with the upper boundary satisfies

$$2x = \sqrt{x} \quad \Longrightarrow \quad 4x^2 = x \quad \Longrightarrow \quad x = \frac{1}{4}.$$

On $0 < x < 1/4$, we have $x^3 < 2x < \sqrt{x}$. On $1/4 < x \leq 1$, the line lies above the entire region. Also,

$x^3 = 2x$ has no positive root in $(0, 1]$, because its nonzero positive root is $\sqrt{2}$. Thus the line cuts off just the small cap between $2x$ and $\sqrt{x}$ for $0 \le x \le 1/4$.

Step 2: Calculate the small cap and the remaining area The cap area is

$$\begin{aligned} R_{\text{cap}} &= \int_0^{1/4} (\sqrt{x} - 2x)\, dx \\ &= \left[\frac{2}{3}x^{3/2} - x^2\right]_0^{1/4} \\ &= \frac{1}{12} - \frac{1}{16} = \frac{1}{48}. \end{aligned}$$

The total area between the original curves is

$$S = \int_0^1 (\sqrt{x} - x^3)\, dx = \frac{2}{3} - \frac{1}{4} = \frac{5}{12} = \frac{20}{48}.$$

The remaining area is $20/48 - 1/48 = 19/48$, which is larger than the cap. Therefore the condition $\max\{R_1, R_2\} = R_2$ identifies $R_2 = 19/48$ and $R_1 = 1/48$.

Step 3: Conclusion The ratio is $\boxed{\frac{R_2}{R_1} = 19}$. The area units cancel, so the ratio is dimensionless.

Quick Tip to Solve: Compute the small region cut off by the line, subtract it from the total area, and use the stated size ordering only at the end.

29. If the shortest distance between the lines

$$\vec{r} = \left(-\hat{i} + 3\hat{k}\right) + \lambda\left(\hat{i} - a\hat{j}\right)$$

and $\vec{r} = \left(-\hat{j} + 2\hat{k}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$ **is** $\sqrt{\frac{2}{3}}$**, then the integral value of a is equal to _____.**

[4 marks]

Solution:

Concept:

- For lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$, with nonparallel direction vectors $\mathbf{b}_1, \mathbf{b}_2$, the shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.
- The cross product points along the common perpendicular. Its scalar product with the joining vector gives the perpendicular separation; the absolute value makes distance nonnegative.

Step 1: Identify the vectors The fixed points are $\mathbf{a}_1 = (-1, 0, 3)$, $\mathbf{a}_2 = (0, -1, 2)$. Thus $\mathbf{a}_2 - \mathbf{a}_1 = (1, -1, -1)$. The directions are $\mathbf{b}_1 = (1, -a, 0)$, $\mathbf{b}_2 = (1, -1, 1)$. Keep the parameter a distinct from these position-vector labels.

Step 2: Calculate the perpendicular separation Expanding the cross product and then taking the

scalar product gives

$$\mathbf{b}_1 \times \mathbf{b}_2 = (-a, -1, a-1), \qquad (1,-1,-1)\cdot(-a,-1,a-1) = 2-2a.$$

Its squared length is $a^2 + 1 + (a-1)^2 = 2a^2 - 2a + 2$, which is always positive, so the lines are never parallel.

Step 3: Apply the distance and retain the integer root Square the distance equation; its nonnegative sides make this operation valid:

$$\frac{4(a-1)^2}{2a^2-2a+2} = \frac{2}{3} \quad \Longrightarrow \quad 6(a-1)^2 = 2a^2 - 2a + 2.$$

Therefore

$$4a^2 - 10a + 4 = 0, \qquad 2(2a-1)(a-2) = 0.$$

The possibilities are $a = 1/2$ and $a = 2$. Only the latter is integral. Substitution gives $d = 2/\sqrt{6} = \sqrt{2/3}$, confirming it in the original formula.

Step 4: Conclusion The required integral value is $\boxed{a = 2}$. The other algebraic root, $1/2$, is excluded because it is not an integer.

Quick Tip to Solve: Use the scalar triple product for skew-line distance, then impose the integer condition only after solving the complete quadratic.

30. If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$ equals _____. [4 marks]

Solution:

Concept:

- A tangent of finite nonzero slope m to $y^2 = 4x$ has equation $y = mx + 1/m$. A point (α, β) on it consequently satisfies $\alpha m^2 - \beta m + 1 = 0$.
- Vieta's formulae connect the sum and product of the two tangent slopes to the external point. The ellipse equation supplies the second relation, allowing the requested expression to be evaluated without radical coordinates.

Step 1: Relate the two tangent slopes Let the slopes be m and $4m$. Their product is nonzero, since the constant term of the slope quadratic is 1. By sum and product of roots,

$$5m = \frac{\beta}{\alpha}, \qquad 4m^2 = \frac{1}{\alpha}.$$

There are two finite slopes as stated, so $\alpha \neq 0$. Squaring the sum relation and using the product relation eliminates m.

Step 2: Combine the slope relation with the ellipse The preceding equations give

$$\frac{\beta^2}{\alpha^2} = \frac{25}{4\alpha}, \qquad 4\beta^2 = 25\alpha.$$

In particular $\alpha > 0$. Substitute this into $25\alpha^2 + 4\beta^2 = 1$:

$$25\alpha^2 + 25\alpha = 1, \qquad \alpha^2 + \alpha = \frac{1}{25}.$$

We need this combined relation, rather than separate values of the coordinates.

Step 3: Evaluate the requested sum of squares First,

$$(10\alpha + 5)^2 = 100(\alpha^2 + \alpha) + 25 = 29.$$

Also $16\beta^2 = 100\alpha$, hence

$$(16\beta^2 + 50)^2 = (100\alpha + 50)^2 = 100(10\alpha + 5)^2 = 2900.$$

Adding the two terms gives $29 + 2900 = 2929$. The expression is independent of the sign of β, as expected because the ellipse and the squared slopes relation allow symmetric points above and below the axis.

Step 4: Conclusion The required numerical value is $\boxed{2929}$.

Quick Tip to Solve: For tangent-slope ratios, use Vieta first. Evaluate combinations such as $\alpha^2 + \alpha$ directly to avoid unnecessary radical calculations.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The bulk modulus of a liquid is 3×10^{10} Nm^{-2}. The pressure required to reduce the volume of liquid by 2% is : [4 marks]

(A) 3×10^8 Nm^{-2}

(B) 9×10^8 Nm^{-2}

(C) 6×10^8 Nm^{-2}

(D) 12×10^8 Nm^{-2}

Correct Answer: (C) 6×10^8 Nm^{-2}

Solution:

Concept:

- Bulk modulus is pressure increase divided by fractional decrease in volume: $B = -\Delta P/(\Delta V/V)$. The minus sign accounts for decreasing volume under increasing pressure.
- For the magnitude of a compression, write $\Delta P = B\,|\Delta V|/V$. A percentage must be divided by 100 before substitution; pressure and bulk modulus have the same SI units.

Step 1: Convert the compression into volume strain The volume is reduced by 2%, so the final volume

is $0.98V$ and $\Delta V = -0.02V$. Consequently the magnitude of the fractional volume change is

$$\frac{|\Delta V|}{V} = \frac{2}{100} = 0.02.$$

This is a dimensionless strain, not a change of two cubic metres. The initial volume need not be known because it cancels in the ratio.

Step 2: Calculate the applied pressure increase Treat the stated bulk modulus as constant over this small compression. The required pressure increase is

$$\Delta P = (3 \times 10^{10})(0.02) = 6 \times 10^{8}\,\mathrm{N\,m^{-2}}.$$

The sign of the increase is positive: external pressure must rise to decrease the volume. Equivalently, using signed volume strain in the definition gives $-B(-0.02)$, the same result.

Step 3: Check the choices and scale Option (C) contains exactly $6 \times 10^{8}\,\mathrm{N\,m^{-2}}$. Option (A) would produce only 1% compression; option (B) would give 3%; option (D) would give 4%, since each pressure divided by 3×10^{10} gives its volume strain. The pressure is much smaller than the bulk modulus because the specified fractional compression is small.

Step 4: Conclusion The required pressure increase is $\boxed{6 \times 10^{8}\,\mathrm{N\,m^{-2}}}$. Hence, the correct option is (C).

Quick Tip to Solve: Write percentage compression as a fraction before multiplying by bulk modulus; a 2 percent decrease means a strain magnitude of 0.02.

32. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In an uniform magnetic field, speed and energy remains the same for a moving charged particle.

Reason (R) : Moving charged particle experiences magnetic force perpendicular to its direction of motion. [4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is NOT the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Correct Answer: (A) Both (A) and (R) are true and (R) is the correct explanation of (A).

Solution:

Concept:

- The magnetic part of the Lorentz force is $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. It is perpendicular to velocity whenever it is nonzero.
- Instantaneous power is $P = \mathbf{F} \cdot \mathbf{v}$. A force perpendicular to velocity does no work and cannot change kinetic energy or speed, although it can change the velocity direction.

Step 1: Evaluate the assertion For a particle moving under the magnetic force alone, take the scalar product of its equation of motion with velocity:

$$\frac{dK}{dt} = \mathbf{F} \cdot \mathbf{v} = q(\mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} = 0.$$

Therefore kinetic energy stays constant. Since the mass is fixed and $K = mv^2/2$, the magnitude of velocity also stays constant. The assertion is true; constant speed does not mean constant vector velocity.

Step 2: Evaluate the reason The cross-product rule gives a magnetic force normal to both the magnetic field and the particle velocity. Thus the reason correctly states the geometrical property of the magnetic force. When the particle moves parallel to the field the force vanishes; this limiting case still gives zero power and unchanged speed. For transverse motion the force changes direction continuously, producing circular motion rather than a speed increase.

Step 3: Connect the reason to the assertion Perpendicularity makes the power vanish at every instant. It therefore directly explains why no kinetic energy is transferred and why speed remains unchanged. Option (B) is excluded because the reason does explain the assertion. Options (C) and (D) each declare a true statement false. The conserved energy here is the particle's kinetic energy in a magnetic-only field; no electric force or other work-producing interaction is specified.

Step 4: Conclusion Both Assertion (A) and Reason (R) are true, and (R) correctly explains (A). Hence, the correct option is (A).

Quick Tip to Solve: For magnetic-force energy questions, calculate $\mathbf{F} \cdot \mathbf{v}$: zero power fixes speed even when the trajectory curves.

33. Two identical cells each of emf 1.5 V are connected in parallel across a parallel combination of two resistors each of resistance 20 Ω. A voltmeter connected in the circuit measures 1.2 V. The internal resistance of each cell is : [4 marks]

(A) 2.5 Ω

(B) 4 Ω

(C) 5 Ω

(D) 10 Ω

Correct Answer: (C) 5 Ω

Solution:

Concept:

- Identical cells connected in parallel retain the emf of one cell, while their internal resistances combine in parallel. Thus two cells of emf E and resistance r are equivalent to emf E and resistance $r/2$.
- Terminal voltage under load obeys $V = E - Ir_{\text{eq}}$, where I is the total load current. An ideal voltmeter measures the same voltage across both parallel external resistors.

Step 1: Reduce the external and internal parallel combinations The two $20\,\Omega$ resistors in parallel give

$$R = \frac{20 \times 20}{20 + 20} = 10\,\Omega.$$

The source equivalent has $E = 1.5\,\text{V}$ and internal resistance $r_{\text{eq}} = r/2$. The emf is not doubled by parallel connection. Because the cells are identical they share the load current equally.

Step 2: Find the current and internal voltage drop The voltmeter reads the terminal voltage $V = 1.2\,\text{V}$, so the external current is

$$I = \frac{V}{R} = \frac{1.2}{10} = 0.12\,\text{A}.$$

The internal voltage drop of the equivalent source is $E - V = 1.5 - 1.2 = 0.3\,\text{V}$. Thus

$$r_{\text{eq}} = \frac{E - V}{I} = \frac{0.3}{0.12} = 2.5\,\Omega.$$

Step 3: Recover each cell resistance and check Since $r/2 = 2.5\,\Omega$, each cell has $r = 5\,\Omega$. Each cell carries $0.06\,\text{A}$, giving its own internal drop $0.06 \times 5 = 0.3\,\text{V}$, consistent with the measured voltage. Option (A) is the equivalent resistance, not that of each cell. Options (B) and (D) would give terminal voltages $1.25\,\text{V}$ and $1.00\,\text{V}$, respectively, using $V = ER/(R + r/2)$, so neither fits.

Step 4: Conclusion The internal resistance of each cell is $\boxed{5\,\Omega}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep the parallel-cell emf unchanged and halve the internal resistance; distinguish the equivalent source resistance from that of each cell.

34. Identify the pair of physical quantities which have different dimensions: [4 marks]

(A) Wave number and Rydberg's constant

(B) Stress and Coefficient of elasticity

(C) Coercivity and Magnetisation

(D) Specific heat capacity and Latent heat

Correct Answer: (D) Specific heat capacity and Latent heat

Solution:

Concept:

- Dimensions express a quantity in base units, independent of its numerical value. Two quantities may have different physical meanings but identical dimensions.
- Specific heat capacity is energy per unit mass per unit temperature rise. Specific latent heat is energy per unit mass for a phase change, with no temperature-change factor.

Step 1: Check the inverse-length and elastic pairs Wave number has dimension L^{-1}, whether defined as $1/\lambda$ or $2\pi/\lambda$. Rydberg's constant also has dimension L^{-1}, as seen from the spectral formula for $1/\lambda$. Thus option (A) is not the required unequal pair. Stress is force per unit area, of dimension $ML^{-1}T^{-2}$. An elastic modulus is stress divided by dimensionless strain, so it has that same dimension. Option (B) is therefore excluded.

Step 2: Check the magnetic pair Coercivity is the magnitude of reverse magnetic field strength H required to remove magnetisation. Its SI unit is ampere per metre. Magnetisation is magnetic dipole moment per volume; since dipole moment has unit $\mathrm{A\,m^2}$, magnetisation also has unit $\mathrm{A\,m^{-1}}$. Both dimensions are IL^{-1}, where I denotes electric current. Coercivity must not be confused with magnetic flux density measured in tesla.

Step 3: Compare the thermal pair Write the defining energy relations $Q = mc\Delta T$ and $Q = mL_s$, with L_s the specific latent heat. They give

$$[c] = L^2T^{-2}\Theta^{-1}, \qquad [L_s] = L^2T^{-2}.$$

Here Θ is the base dimension of temperature. The extra inverse-temperature factor in specific heat capacity makes these dimensions different. Even if latent heat denotes total energy rather than its specific value, it still differs dimensionally from specific heat capacity.

Step 4: Conclusion Specific heat capacity and latent heat have different dimensions. Hence, the correct option is (D).

Quick Tip to Solve: Derive thermal dimensions from $Q = mc\Delta T$ and $Q = mL_s$; the inverse-temperature factor distinguishes heat capacity from latent heat.

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35. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be :

[use g = 10 m/s^2] [4 marks]

(A) $\frac{1}{2}\sin^{-1}\left(\frac{5t^2}{4R}\right)$

(B) $\frac{1}{2}\sin^{-1}\left(\frac{4R}{5t^2}\right)$

(C) $\tan^{-1}\left(\frac{4t^2}{5R}\right)$

(D) $\cot^{-1}\left(\frac{R}{20t^2}\right)$

Correct Answer: (D) $\cot^{-1}\left(\frac{R}{20t^2}\right)$

Solution:

Concept:

- For a projectile launched at speed u and angle θ, vertical velocity is $v_y = u\sin\theta - gt$, while horizontal velocity remains $u\cos\theta$.
- Horizontal inclination becomes zero at the highest point, where vertical velocity vanishes. For the standard range on the launch level, the full flight time is twice the ascent time.

Step 1: Use the horizontal direction at time t At the stated instant the velocity is horizontal, so $v_y = 0$. Hence

$$u\sin\theta = gt, \qquad 25\sin\theta = 10t.$$

Thus t is the time of ascent, not the full time of flight. Gravity continues to act at the highest point even though the instantaneous vertical velocity is zero.

Step 2: Express the range using the ascent time The total flight time is $T = 2t$, and the constant horizontal velocity is $u\cos\theta$. Therefore

$$R = 2ut\cos\theta.$$

Divide this by $2gt^2$ and use $gt = u\sin\theta$:

$$\frac{R}{2gt^2} = \frac{u\cos\theta}{gt} = \frac{\cos\theta}{\sin\theta} = \cot\theta.$$

With $g = 10\,\mathrm{m\,s^{-2}}$, the ratio is $R/(20t^2)$, using SI numerical values.

Step 3: Identify the inverse trigonometric form For a projectile with positive horizontal range and an acute launch angle, inversion gives

$$\theta = \cot^{-1}\left(\frac{R}{20t^2}\right).$$

The launch speed cancels because both the range and ascent time already encode the horizontal and vertical velocity components. Option (C) has the wrong coefficient for the equivalent tangent form, which is $\tan^{-1}(20t^2/R)$. The sine expressions in (A) and (B) do not follow from $R/(2gt^2) = \cot\theta$ and cannot represent the required relation generally.

Step 4: Conclusion The launch angle is $\boxed{\theta = \cot^{-1}(R/(20t^2))}$, with the quantities expressed in SI units. Hence, the correct option is (D).

Quick Tip to Solve: Zero inclination identifies the highest point. Use total flight time $2t$, then divide range by $2gt^2$ to obtain $\cot\theta$.

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36. A block of mass 10 kg starts sliding on a surface with an initial velocity of 9.8 ms^{-1}. The coefficient of friction between the surface and block is 0.5. The distance covered by the block before coming to rest is :

[use g = 9.8 ms^{-2}] [4 marks]

(A) 4.9 m

(B) 9.8 m

(C) 12.5 m

(D) 19.6 m

Correct Answer: (B) 9.8 m

Solution:

Concept:

- Kinetic friction opposes sliding: $f_k = \mu_k N$. On the horizontal surface, vertical balance gives $N = mg$, where m is the block mass and g is gravitational acceleration.
- The frictional force is constant, so the horizontal acceleration is constant. Use $v^2 = u^2 + 2as$, with s the distance travelled before the final speed v becomes zero.

Step 1: Find the frictional deceleration The block is already sliding, so use the given coefficient of kinetic friction. There is no vertical acceleration and therefore the normal force is $N = 10(9.8) = 98\,\mathrm{N}$.

Taking the initial direction of motion as positive,

$$f_k = 0.5(98) = 49\,\text{N}, \qquad a = -\frac{49}{10} = -4.9\,\text{m s}^{-2}.$$

The minus sign shows that the force reduces the speed; the mass cancels from the general relation $a = -\mu_k g$.

Step 2: Apply the stopping condition At rest, $v = 0$, while the initial speed is $u = 9.8\,\text{m s}^{-1}$. Substituting into the constant-acceleration relation gives

$$0 = (9.8)^2 - 2(4.9)s, \qquad s = \frac{96.04}{9.8} = 9.8\,\text{m}.$$

This is the distance along the surface, rather than the stopping time. The time would be $u/(\mu_k g) = 2\,\text{s}$.

Step 3: Check energy and the options The initial kinetic energy is $\frac{1}{2}(10)(9.8)^2 = 480.2\,\text{J}$. Friction removes $49(9.8) = 480.2\,\text{J}$, confirming the stopping distance. The alternatives 4.9, 12.5 and $19.6\,\text{m}$ do not remove exactly the initial kinetic energy.

Step 4: Conclusion The block travels $9.8\,\text{m}$ before coming to rest. Hence, the correct option is (B).

Quick Tip to Solve: For horizontal sliding with constant friction, write $s = u^2/(2\mu_k g)$; the block mass cancels.

37. A boy ties a stone of mass 100 g to the end of a 2 m long string and whirls it around in a horizontal plane. The string can withstand the maximum tension of 80 N. If the maximum speed with which the stone can revolve is $\frac{K}{\pi}$ rev./min. The value of K is :

(Assume the string is massless and unstretchable) [4 marks]

(A) 400

(B) 300

(C) 600

(D) 800

Correct Answer: (C) 600

Solution:

Concept:

- The string tension supplies the radial force for horizontal circular motion: $T = m\omega^2 r$, where r is the radius and ω is angular speed in radians per second.
- One revolution equals 2π radians. Thus a rotation frequency n in revolutions per minute corresponds to $\omega = 2\pi n/60$.

Step 1: Use the limiting string tension The mass in SI units is $100\,\text{g} = 0.100\,\text{kg}$, and the circular-path radius is the string length, $r = 2\,\text{m}$. The greatest allowed angular speed occurs at the greatest allowed tension, because the required radial force grows as the square of speed. Therefore

$$80 = 0.100\,\omega_{\text{max}}^2(2), \qquad \omega_{\text{max}}^2 = 400\,\text{s}^{-2}.$$

The positive square root is used because a speed is nonnegative.

Step 2: Convert to revolutions per minute The limiting angular speed is $\omega_{\max} = 20\,\text{rad}\,\text{s}^{-1}$. Dividing by 2π gives the number of revolutions each second; multiplying by 60 converts seconds to minutes:

$$n_{\max} = \frac{20}{2\pi}(60) = \frac{600}{\pi}\,\text{rev}\,\text{min}^{-1}.$$

Comparing with the form K/π specified in the question identifies $K = 600$. The angular speed and the revolutions-per-minute value are different quantities.

Step 3: Verify the limiting force The tangential speed is $v = \omega r = 40\,\text{m}\,\text{s}^{-1}$, so $mv^2/r = 0.100(1600)/2 = 80\,\text{N}$, as required. Since tension is proportional to K^2, the values 400 and 300 give speeds below the limit, whereas 800 would exceed the permitted tension. Only 600 describes the maximum.

Step 4: Conclusion The coefficient is $K = 600$. Hence, the correct option is (C).

Quick Tip to Solve: Find angular speed from $T = mr\omega^2$, then multiply by $60/(2\pi)$ to obtain revolutions per minute.

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38. A vertical electric field of magnitude 4.9×10^5 N/C just prevents a water droplet of a mass 0.1 g from falling. The value of charge on the droplet will be :
(Given : g = 9.8 m/s^2) [4 marks]

(A) 1.6×10^{-9} C
(B) 2.0×10^{-9} C
(C) 3.2×10^{-9} C
(D) 0.5×10^{-9} C

Correct Answer: (B) 2.0×10^{-9} C

Solution:

Concept:

- A charge of magnitude $|q|$ in an electric field of magnitude E experiences electric force of magnitude $|q|E$. Its direction depends on the sign of the charge.
- For a droplet just suspended against gravity, the upward electric force balances the downward weight: $|q|E = mg$. Use kilograms for the mass in this SI equation.

Step 1: Convert the mass and calculate weight The given mass is $0.1\,\text{g}$, not $0.1\,\text{kg}$. Since one gram equals $10^{-3}\,\text{kg}$,

$$m = 0.1 \times 10^{-3} = 1.0 \times 10^{-4}\,\text{kg}.$$

Its weight is consequently

$$mg = (1.0 \times 10^{-4})(9.8) = 9.8 \times 10^{-4}\,\text{N}.$$

The phrase just prevents falling means that the droplet can remain in vertical equilibrium rather than accelerate downward.

Step 2: Balance the electric force against gravity The electric force must point upward and have the same magnitude as the weight. Inserting the specified field into $|q| = mg/E$ gives

$$|q| = \frac{9.8 \times 10^{-4}}{4.9 \times 10^{5}} = \frac{9.8}{4.9} \times 10^{-9} = 2.0 \times 10^{-9}\,\text{C}.$$

The exponent is $-4 - 5 = -9$. The question specifies only that the field is vertical, so its upward or downward orientation would determine the charge sign; the options give its magnitude.

Step 3: Check equilibrium and the alternatives The selected magnitude produces force $(2.0 \times 10^{-9})(4.9 \times 10^{5}) = 9.8 \times 10^{-4}\,\text{N}$, exactly the weight. The values 1.6×10^{-9} and $0.5 \times 10^{-9}\,\text{C}$ produce too little force, while $3.2 \times 10^{-9}\,\text{C}$ produces too much for balance.

Step 4: Conclusion The required charge magnitude is $2.0 \times 10^{-9}\,\text{C}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert grams to kilograms before using $|q| = mg/E$; determine the charge sign separately from the field direction.

39. A particle experiences a variable force $\overrightarrow{F} = \left(4x\hat{i} + 3y^2\hat{j}\right)$ in a horizontal x-y plane. Assume distance in meters and force is newton. If the particle moves from point (1, 2) to point (2, 3) in the x-y plane, then Kinetic Energy changes by : [4 marks]

(A) 50.0 J

(B) 12.5 J

(C) 25.0 J

(D) 0 J

Correct Answer: (C) 25.0 J

Solution:

Concept:

- The work-energy theorem states that the change in kinetic energy equals net work: $\Delta K = \int \vec{F} \cdot d\vec{r}$. Positive work increases kinetic energy.
- Here $\vec{F} \cdot d\vec{r} = 4x\,dx + 3y^2\,dy$. Each force component depends only on its corresponding coordinate, so the integral depends on the endpoints, with primitive $2x^2 + y^3$.

Step 1: Set up the work between the endpoints The initial point is $(1, 2)$ and the final point is $(2, 3)$, with all coordinates measured in metres. Resolve the infinitesimal displacement as $d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$. The dot product retains matching components only:

$$\Delta K = \int_1^2 4x\,dx + \int_2^3 3y^2\,dy.$$

No path needs to be specified: there are no mixed-coordinate terms, and both integrals are exact endpoint differences.

Step 2: Evaluate the two contributions The work associated with the horizontal component is

$$W_x = [2x^2]_1^2 = 2(4-1) = 6\,\text{J}.$$

The work associated with the vertical coordinate in the horizontal plane is

$$W_y = [y^3]_2^3 = 27 - 8 = 19\,\text{J}.$$

Both contributions are positive as the coordinates increase. The y-direction here lies in the stated horizontal plane and is not a vertical displacement against gravity.

Step 3: Add the work and identify the energy change The total work is $W = 6 + 19 = 25\,\text{J}$, so the kinetic energy increases by this amount. This is a change in energy, and does not require the initial speed or mass. The choices 50.0 and $12.5\,\text{J}$ are respectively twice and half the computed work; zero would require cancellation, whereas both endpoint contributions are positive.

Step 4: Conclusion The kinetic energy increases by $25.0\,\text{J}$. Hence, the correct option is (C).

Quick Tip to Solve: For $F_x(x)\hat{i} + F_y(y)\hat{j}$, integrate the two components separately between their corresponding coordinate limits.

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40. The approximate height from the surface of earth at which the weight of the body becomes $\frac{1}{3}$ of its weight on the surface of earth is :

[Radius of earth R = 6400 km and $\sqrt{3}$ = 1.732] [4 marks]

(A) 3840 km

(B) 4685 km

(C) 2133 km

(D) 4267 km

Correct Answer: (B) 4685 km

Solution:

Concept:

- Outside a spherical Earth, gravitational acceleration at distance r from its centre is $g(r) = GM/r^2$. A body at height h above the surface has $r = R + h$.
- The ratio of weights for the same body is $W_h/W_0 = [R/(R+h)]^2$, since the body mass and Earth mass cancel. This inverse-square relation remains valid when the height is comparable to the Earth radius.

Step 1: Form the specified weight ratio On the surface, $W_0 = GMm/R^2$, while at height h, $W_h = GMm/(R+h)^2$. Setting their ratio equal to the fraction printed in the question gives

$$\frac{1}{3} = \frac{R^2}{(R+h)^2}.$$

Both radii are positive, so taking the positive square root gives $R + h = \sqrt{3}\,R$. The distance in this

equation is measured from Earth's centre, not from its surface.

Step 2: Subtract the Earth radius The altitude is the excess over the surface radius:

$$h = (\sqrt{3} - 1)R = (1.732 - 1)(6400)\,\text{km}.$$

Using the supplied approximation,

$$h = 0.732(6400) = 4684.8\,\text{km} \approx 4685\,\text{km}.$$

Keeping kilometres throughout is valid because the formula multiplies the given radius by a dimensionless factor.

Step 3: Check the result and reject the other choices At this altitude the centre-to-body distance is approximately $11084.8\,\text{km} = 1.732R$, whose square is about $3R^2$. Thus the weight is one-third of its surface value. The proposed altitudes 3840, 2133 and 4267 km do not give this ratio. The small-height linear approximation is unsuitable because the computed altitude is about $0.732R$, which is not small compared with R.

Step 4: Conclusion The required height is approximately $4685\,\text{km}$ above the surface. Hence, the correct option is (B).

Quick Tip to Solve: Use $R + h$, not h, in the inverse-square law; subtract R only after finding the distance from Earth's centre.

41. A resistance of 40 Ω is connected to a source of alternating current rated 220 V, 50 Hz. Find the time taken by the current to change from its maximum value to the rms value : [4 marks]

(A) 2.5 ms

(B) 1.25 ms

(C) 2.5 s

(D) 0.25 s

Correct Answer: (A) 2.5 ms

Solution:

Concept:

- For sinusoidal current, the rms value is $I_{\text{rms}} = I_0/\sqrt{2}$, where I_0 is the maximum instantaneous current.
- The angular frequency is $\omega = 2\pi f$. Choosing time zero at a positive current maximum gives $i(t) = I_0 \cos(\omega t)$, so the first descent to the rms value corresponds to phase $\pi/4$.

Step 1: Choose the time origin at the maximum The resistance is purely resistive, so its current is sinusoidal at the applied frequency $f = 50\,\text{Hz}$. Measure elapsed time from the positive maximum, giving $i(0) = I_0$. The amplitude is not needed to find the requested interval because the target is a fixed fraction of that same amplitude. In particular, $I_{\text{rms}} = 220/40 = 5.5\,\text{A}$ and $I_0 = 5.5\sqrt{2}\,\text{A}$.

Step 2: Find the phase change to the rms level The first time after the maximum when the instanta-

neous current equals the positive rms current satisfies

$$I_0\cos(\omega t) = \frac{I_0}{\sqrt{2}}, \qquad \cos(\omega t) = \frac{1}{\sqrt{2}}.$$

On the descending part of the cycle, the smallest positive phase is $\omega t = \pi/4$. This is an eighth of a complete 2π cycle, rather than a quarter-cycle; the latter would take the current to zero.

Step 3: Convert the phase interval into time Using the given frequency,

$$t = \frac{\pi/4}{2\pi(50)} = \frac{1}{400}\,\text{s} = 0.0025\,\text{s} = 2.5\,\text{ms}.$$

The period is $1/50 = 20\,\text{ms}$, confirming that the interval is $T/8$. The option $1.25\,\text{ms}$ is only $T/16$; the two intervals quoted in seconds are much longer than one cycle and do not describe the first change from maximum to rms.

Step 4: Conclusion The elapsed time is $2.5\,\text{ms}$. Hence, the correct option is (A).

Quick Tip to Solve: For the first drop of a sine-wave current from its positive maximum to rms, the phase changes by $\pi/4$, so use $T/8$.

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42. The equations of two waves are given by :

y_1 = 5 sin 2π(x - vt) cm

y_2 = 3 sin 2π(x − vt + 1.5) cm

These waves are simultaneously passing through a string. The amplitude of the resulting wave is : [4 marks]

(A) 2 cm

(B) 4 cm

(C) 5.8 cm

(D) 8 cm

Correct Answer: (A) 2 cm

Solution:

Concept:

- For two waves of the same frequency and propagation direction, the resultant amplitude obeys $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$, where ϕ is their constant phase difference.
- The phase difference must include the coefficient multiplying the bracket in a sine function. A phase change of any odd multiple of π reverses the sign of the sine and produces destructive interference.

Step 1: Extract the complete phase difference Write the common phase as $\theta = 2\pi(x - vt)$. The second equation becomes

$$y_2 = 3\sin(\theta + 2\pi \times 1.5)\,\text{cm} = 3\sin(\theta + 3\pi)\,\text{cm}.$$

Thus the constant phase difference is 3π radians, equivalent to π modulo 2π. The number 1.5 lies

inside the bracket and is not itself the phase in radians.

Step 2: Apply superposition directly Since $\sin(\theta+3\pi) = -\sin\theta$, the second displacement is $-3\sin\theta\,\text{cm}$. Adding displacements at the same position and time gives

$$y = y_1 + y_2 = (5-3)\sin\theta\,\text{cm} = 2\sin\theta\,\text{cm}.$$

The amplitude is the nonnegative coefficient of the resulting sine, hence $A = 2\,\text{cm}$. The waves partly cancel but do not cancel completely because their amplitudes differ.

Step 3: Check the amplitude formula and alternatives As an independent check, $\cos 3\pi = -1$, giving

$$A^2 = 5^2 + 3^2 + 2(5)(3)(-1) = 25 + 9 - 30 = 4\,\text{cm}^2.$$

Therefore $A = 2\,\text{cm}$. The value 8 cm would require the waves to be in phase; approximately 5.8 cm would arise from a phase difference with zero cosine. The value 4 cm also contradicts the fixed antiphase relation in the printed equations.

Step 4: Conclusion The resultant wave amplitude is $2\,\text{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: Expand the entire sine argument before comparing phases; an extra 3π makes the two waves opposite in phase, so subtract their amplitudes.

---

43. A plane electromagnetic wave travels in a medium of relative permeability 1.61 and relative permittivity 6.44. If magnitude of magnetic intensity is 4.5 $\times$ 10^{-2} Am^{-1} at a point, what will be the approximate magnitude of electric field intensity at that point?

(Given : Permeability of free space μ_0 = 4π $\times$ 10^{-7} NA^{-2}, speed of light in vacuum c = 3 $\times$ 10^8 ms^{-1}) [4 marks]

(A) 16.96 Vm^{-1}

(B) 2.25 $\times$ 10^{-2} Vm^{-1}

(C) 8.48 Vm^{-1}

(D) 6.75 $\times$ 10^6 Vm^{-1}

Correct Answer: (C) 8.48 Vm^{-1}

Solution:

Concept:

- For a plane electromagnetic wave, the electric field and magnetic induction obey $E = vB$, where v is the wave speed in the medium.
- Magnetic intensity H differs from magnetic induction: $B = \mu H$. The wave impedance is therefore $E/H = \sqrt{\mu/\varepsilon} = \mu_0 c\sqrt{\mu_r/\varepsilon_r}$.

Step 1: Find the wave impedance The relative permeability and permittivity describe the same medium, so use their ratio rather than their product in the impedance. Since $c = 1/\sqrt{\mu_0\varepsilon_0}$, the

impedance of free space is $\mu_0 c$. Thus

$$Z = \frac{E}{H} = (4\pi \times 10^{-7})(3 \times 10^{8})\sqrt{\frac{1.61}{6.44}}.$$

Here $6.44 = 4(1.61)$, making the square-root factor exactly $1/2$.

Step 2: Calculate the electric field First simplify the impedance, and then multiply by the given magnetic intensity:

$$Z = 120\pi \times \frac{1}{2} = 60\pi\ \Omega, \qquad E = ZH = 60\pi(4.5 \times 10^{-2}).$$

$$E = 2.7\pi\ \mathrm{V\,m^{-1}} \simeq 8.48\ \mathrm{V\,m^{-1}}.$$

The units are consistent because $\Omega\,\mathrm{A\,m^{-1}} = \mathrm{V\,m^{-1}}$.

Step 3: Check the alternatives Option (A), $16.96\ \mathrm{V\,m^{-1}}$, is twice the answer and results from using the vacuum impedance without the medium correction. Option (B) does not include the required impedance factor. Option (D) is far too large and is inconsistent with the given magnetic intensity and the calculated impedance. Only (C) agrees with the wave relations.

Step 4: Conclusion The approximate electric field intensity is $8.48\ \mathrm{V\,m^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: When the data give magnetic intensity in $\mathrm{A\,m^{-1}}$, use $E = ZH$; use $E = vB$ only for magnetic induction in tesla.

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44. Choose the correct option from the following options given below : [4 marks]

(A) In the ground state of Rutherford's model electrons are in stable equilibrium. While in Thomson's model electrons always experience a net-force.

(B) An atom has a nearly continuous mass distribution in a Rutherford's model but has a highly non-uniform mass distribution in Thomson's model.

(C) A classical atom based on Rutherford's model is doomed to collapse.

(D) The positively charged part of the atom possesses most of the mass in Rutherford's model but not in Thomson's model.

Correct Answer: (C) A classical atom based on Rutherford's model is doomed to collapse.

Solution:

Concept:

- Rutherford places the positive charge and almost all atomic mass in a tiny nucleus, with electrons moving around it.
- A charge moving in a circular orbit has centripetal acceleration. Classical electrodynamics predicts that an accelerating charge radiates energy.

Step 1: Apply the classical radiation argument An orbiting electron is accelerated even when its speed is constant, because its velocity continuously changes direction. It therefore emits electromagnetic

radiation according to classical theory. The radiation carries away energy, so the electron cannot retain a fixed orbital energy indefinitely. This is a failure of the classical Rutherford atom; the model itself does not contain the stationary-state postulate introduced later by Bohr.

Step 2: Follow the loss of orbital energy For an electron bound to a nucleus of charge $+Ze$, a circular orbit has total mechanical energy

$$E = -\frac{kZe^2}{2r},$$

where r is the orbital radius and $k = 1/(4\pi\varepsilon_0)$. As energy is radiated, E becomes more negative and r decreases. The electron consequently spirals toward the nucleus instead of remaining in a stable classical orbit. This establishes the statement in (C).

Step 3: Reject the other model comparisons Option (A) reverses the stability issue: a classical Rutherford orbit is not a stable equilibrium configuration, whereas embedded electrons can be in equilibrium in the Thomson picture. Option (B) also reverses the mass distributions: Rutherford concentrates mass in the nucleus, while Thomson spreads positive matter through the atom. Option (D) is false because the positive part accounts for nearly all atomic mass in both pictures; their distinction is how that positive charge and mass are distributed.

Step 4: Conclusion A classical atom based on Rutherford's model loses energy by radiation and is doomed to collapse. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish a model's charge distribution from its stability: a concentrated nucleus explains scattering, but classical orbiting electrons still radiate.

45. Nucleus A is having mass number 220 and its binding energy per nucleon is 5.6 MeV. It splits in two fragments 'B' and 'C' of mass numbers 105 and 115. The binding energy of nucleons in 'B' and 'C' is 6.4 MeV per nucleon. The energy Q released per fission will be : [4 marks]

(A) 0.8 MeV

(B) 275 MeV

(C) 220 MeV

(D) 176 MeV

Correct Answer: (D) 176 MeV

Solution:

Concept:

- Total binding energy equals mass number times binding energy per nucleon: $B = A\bar{b}$. The per-nucleon value is not the energy of the entire nucleus.
- For fission with the same total nucleon content before and after, $Q = B_{\text{products}} - B_{\text{parent}}$. Greater binding of the products means energy is released.

Step 1: Calculate the parent binding energy The initial nucleus contains 220 nucleons and has binding energy $5.6\ \text{MeV}$ per nucleon. Its total binding energy is therefore

$$B_A = 220 \times 5.6 = 1232\ \text{MeV}.$$

The mass numbers of the fragments satisfy $105 + 115 = 220$, so the given reaction conserves the total number of nucleons. No missing emitted nucleon needs to be included in this calculation.

Step 2: Add the binding energies of both fragments Each fragment has binding energy 6.4 MeV per nucleon. Their individual total binding energies are

$$B_B = 105 \times 6.4 = 672 \text{ MeV},$$

$$B_C = 115 \times 6.4 = 736 \text{ MeV}.$$

Thus the products have total binding energy $672 + 736 = 1408$ MeV. Their larger binding energy means that their combined rest energy is smaller than the initial rest energy.

Step 3: Find and compare the released energy The difference in binding energy becomes the energy released per fission:

$$Q = 1408 - 1232 = 176 \text{ MeV}.$$

Equivalently, $Q = 220(6.4 - 5.6) = 220(0.8)$ MeV. The value 0.8 MeV in (A) is only the gain per nucleon. Option (C) uses the nucleon count as an energy, and (B) is not the difference of the total binding energies.

Step 4: Conclusion The energy released per fission is $Q = 176$ MeV. Hence, the correct option is (D).

Quick Tip to Solve: When all fission fragments have the same binding energy per nucleon, multiply its increase by the conserved total mass number.

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46. A parallel plate capacitor is formed by two plates each of area 30π cm^2 separated by 1 mm. A material of dielectric strength 3.6×10^7 Vm^{-1} is filled between the plates. If the maximum charge that can be stored on the capacitor without causing any dielectric breakdown is 7×10^{-6}C, the value of dielectric constant of the material is :

[Use $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2 C^{-2}] [4 marks]

(A) 1.66

(B) 1.75

(C) 2.25

(D) 2.33

Correct Answer: (D) 2.33

Solution:

Concept:

- A parallel-plate capacitor filled completely with a dielectric of relative permittivity K has capacitance $C = K\varepsilon_0 A/d$.
- Dielectric strength is the largest sustainable electric field, so $V_{\max} = E_{\max} d$ and $Q_{\max} = K\varepsilon_0 A E_{\max}$.

Step 1: Express the maximum charge in terms of the field At the threshold of breakdown, the field

between the plates reaches 3.6×10^7 V m^{-1}. Use $Q = CV$ with the limiting voltage:

$$Q_{\max} = \frac{K\varepsilon_0 A}{d}(E_{\max} d) = K\varepsilon_0 A E_{\max}.$$

The separation cancels. As a check, the given separation is 10^{-3} m, so the limiting voltage itself is 3.6×10^4 V.

Step 2: Convert the area and substitute the constant The area must be expressed in square metres, using $1\ \text{cm}^2 = 10^{-4}\ \text{m}^2$:

$$A = 30\pi \times 10^{-4} = 3\pi \times 10^{-3}\ \text{m}^2.$$

The supplied Coulomb constant gives $\varepsilon_0 = 1/(36\pi \times 10^9)$ F m^{-1}. Hence

$$\varepsilon_0 A E_{\max} = \frac{(3\pi \times 10^{-3})(3.6 \times 10^7)}{36\pi \times 10^9} = 3.0 \times 10^{-6}\ \text{C}.$$

Step 3: Determine the dielectric constant Rearranging the limiting-charge relation gives

$$K = \frac{7 \times 10^{-6}}{3.0 \times 10^{-6}} = \frac{7}{3} \simeq 2.33.$$

The dielectric constant is dimensionless. For comparison, values 1.66, 1.75 and 2.25 would permit charges only 4.98, 5.25 and 6.75 microcoulombs at the specified field, respectively, rather than the stated 7 microcoulombs.

Step 4: Conclusion The dielectric constant is approximately $K = 2.33$. Hence, the correct option is (D).

Quick Tip to Solve: For breakdown-limited charge, combine $C \propto 1/d$ with $V_{\max} \propto d$ first; the plate separation cancels.

47. The magnetic field at the centre of a circular coil of radius r, due to current I flowing through it, is B. The magnetic field at a point along the axis at a distance $\frac{r}{2}$ from the centre is : [4 marks]

(A) B/2

(B) 2B

(C) $\left(\frac{2}{\sqrt{5}}\right)^3 B$

(D) $\left(\frac{2}{\sqrt{3}}\right)^3 B$

Correct Answer: (C) $\left(\frac{2}{\sqrt{5}}\right)^3 B$

Solution:

Concept:

- For a circular coil of radius r, carrying current I in N turns, the axial magnetic field at distance x is $B_x = \mu_0 N I r^2/[2(r^2 + x^2)^{3/2}]$.
- At the centre, $B_0 = \mu_0 N I/(2r)$. Taking a ratio eliminates current, number of turns and permeability.

Step 1: Form the ratio to the central field The stated field B refers to the centre, so identify $B_0 = B$. Dividing the axial-field expression by this central value gives

$$\frac{B_x}{B} = \frac{\mu_0 N I r^2}{2(r^2+x^2)^{3/2}} \frac{2r}{\mu_0 N I} = \frac{r^3}{(r^2+x^2)^{3/2}}.$$

This may also be written $[1+(x/r)^2]^{-3/2}$. The coil may have one or several turns; the result is unchanged because N cancels.

Step 2: Substitute the axial distance At the required point, $x = r/2$. Therefore

$$r^2 + x^2 = r^2 + \frac{r^2}{4} = \frac{5}{4}r^2,$$

$$\frac{B_x}{B} = \left(\frac{4}{5}\right)^{3/2} = \frac{8}{5\sqrt{5}} = \left(\frac{2}{\sqrt{5}}\right)^3.$$

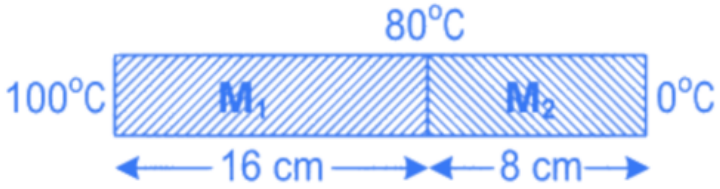
The radius is positive, so taking the fractional power introduces no sign ambiguity. The field points along the same axial direction as the central field.

Step 3: Check the magnitude against all choices The ratio is approximately 0.716, so the off-centre field is smaller than B, as expected on moving away from the centre. It is not $B/2$, eliminating (A). Option (B) would increase the field to $2B$, and option (D) gives a ratio greater than one because $2/\sqrt{3} > 1$; both contradict the decreasing axial-field expression.

Step 4: Conclusion The required field is $B_x = \left(2/\sqrt{5}\right)^3 B$. Hence, the correct option is (C).

Quick Tip to Solve: For comparisons along a coil axis, use $B_x/B_0 = [1+(x/r)^2]^{-3/2}$ directly and check that the ratio is below one.

48. Two metallic blocks M_1 and M_2 of same area of cross-section are connected to each other (as shown in figure). If the thermal conductivity of M_2 is K then the thermal conductivity of M_1 will be : [Assume steady state heat conduction] [4 marks]



(A) 10 K

(B) 8 K

(C) 12.5 K

(D) 2 K

Correct Answer: (B) 8 K

Solution:

Concept:

- In steady conduction through blocks in series, no heat accumulates at their junction, so the same heat current passes through each block.
- Fourier's law gives the heat-current magnitude $\dot{Q} = kA\Delta T/L$, where k is thermal conductivity, A cross-sectional area and L length.

Step 1: Read the temperature drops and lengths The left face of M_1 is at 100°C, the common junction at 80°C, and the right face of M_2 at 0°C. Thus the temperature drops along the direction of heat flow are

$$\Delta T_1 = 100 - 80 = 20 \text{ K}, \qquad \Delta T_2 = 80 - 0 = 80 \text{ K}.$$

The lengths are $L_1 = 16$ cm and $L_2 = 8$ cm.

Step 2: Equate the steady heat currents Let the unknown thermal conductivity of M_1 be k_1, while the conductivity of M_2 is the given K. Both blocks have the same cross-sectional area A, hence

$$\frac{k_1 A(20)}{16} = \frac{KA(80)}{8}.$$

Cancel the common area and solve:

$$\frac{k_1}{K} = \frac{80}{20}\frac{16}{8} = 4 \times 2 = 8.$$

Step 3: Check the physical interpretation and options The longer first block carries the same heat current with a much smaller temperature drop, so its conductivity must be considerably larger. Thus $8K$ fits the data; $10K$, $12.5K$ and $2K$ would not produce equal heat currents at these junction temperatures.

Step 4: Conclusion The thermal conductivity of block M_1 is $k_1 = 8K$, where K denotes the conductivity of M_2. Hence, the correct option is (B).

Quick Tip to Solve: For equal-area blocks in series, compare $\Delta T/L$: the block with the smaller gradient has the greater conductivity.

49. A baseband signal of 3.5 MHz frequency is modulated with a carrier signal of 3.5 GHz frequency using amplitude modulation method. What should be the minimum size of antenna required to transmit the modulated signal? [4 marks]

(A) 42.8 m

(B) 42.8 mm

(C) 21.4 mm

(D) 21.4 m

Correct Answer: (C) 21.4 mm

Solution:

Concept:

- An efficient transmitting antenna has a minimum characteristic length of order one quarter of the wavelength: $L_{\min} = \lambda/4$.
- Amplitude modulation translates the information to radio frequencies near the carrier. The relevant wavelength is therefore approximately $\lambda = c/f_c$, rather than the wavelength of the baseband signal.

Step 1: Choose the frequency that is transmitted The carrier frequency is $f_c = 3.5\ \text{GHz} = 3.5 \times 10^9\ \text{Hz}$, while the baseband frequency is only 3.5×10^6 Hz. An amplitude-modulated wave contains the carrier and sidebands at $f_c \pm f_m$. Here $f_m/f_c = 10^{-3}$, so all transmitted frequencies are close to the carrier and the usual quarter-wave antenna estimate is based on f_c.

Step 2: Find the carrier wavelength and quarter wavelength Using the speed of electromagnetic waves in air as $c = 3.0 \times 10^8\ \text{m s}^{-1}$,

$$\lambda_c = \frac{3.0 \times 10^8}{3.5 \times 10^9} = 0.0857\ \text{m} = 85.7\ \text{mm}.$$

Thus the approximate minimum antenna size is

$$L_{\min} = \frac{\lambda_c}{4} = 0.0214\ \text{m} = 21.4\ \text{mm}.$$

The tiny relative sideband shift does not change the listed approximate answer.

Step 3: Distinguish the incorrect length scales Option (B), 42.8 mm, is approximately half the carrier wavelength instead of one quarter. Option (D), 21.4 m, comes from applying the quarter-wavelength formula to the unmodulated baseband frequency; modulation is precisely what avoids needing such a long antenna. Option (A), 42.8 m, is half the baseband wavelength. None of these is the requested minimum carrier-frequency estimate.

Step 4: Conclusion The minimum antenna size is approximately 21.4 mm. Hence, the correct option is (C).

Quick Tip to Solve: In an AM antenna question, convert the carrier frequency to hertz first, then evaluate $c/(4f_c)$; do not use the audio or baseband frequency.

50. A Carnot engine whose heat sinks at 27°C, has an efficiency of 25%. By how many degrees should the temperature of the source be changed to increase the efficiency by 100% of the original efficiency? [4 marks]

(A) Increases by 18°C

(B) Increases by 200°C

(C) Increases by 120°C

(D) Increases by 73°C

Correct Answer: (B) Increases by 200°C

Solution:

Concept:

- For a reversible Carnot engine, $\eta = 1 - T_c/T_h$, where T_c and T_h are the absolute temperatures of the sink and source.
- A temperature interval has the same numerical value in kelvin and degrees Celsius. An increase by 100% of the original efficiency means doubling the efficiency, not making it 100%.

Step 1: Find the initial source temperature The sink remains at $T_c = 27 + 273 = 300\,\text{K}$. Initially $\eta_1 = 0.25$, so

$$0.25 = 1 - \frac{300}{T_{h1}}, \qquad T_{h1} = \frac{300}{0.75} = 400\,\text{K}.$$

Absolute temperatures must be used in the Carnot formula; directly inserting 27 would give an incorrect ratio.

Step 2: Calculate the source temperature for doubled efficiency The new efficiency is $\eta_2 = 2(0.25) = 0.50$. With the same sink,

$$T_{h2} = \frac{300}{1 - 0.50} = 600\,\text{K}.$$

Thus $\Delta T_h = 600 - 400 = 200\,\text{K} = 200\,^\circ\text{C}$. Increases of 18, 120 and 73 degrees give source temperatures 418, 520 and 473 K respectively; none yields efficiency 0.50.

Step 3: Conclusion The source temperature must increase by $200\,^\circ\text{C}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert both reservoir temperatures to kelvin first, and interpret a percentage increase relative to the original efficiency.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. 0.056 kg of Nitrogen is enclosed in a vessel at a temperature of 127°C. Th amount of heat required to double the speed of its molecules is _____ k cal.
Take R = 2 cal mole^{-1} K^{-1}) [4 marks]

Solution:

Concept:

- For a fixed gas, molecular speeds vary as $\sqrt{T}$, where T is absolute temperature. Doubling the speed therefore requires four times the temperature.
- For diatomic nitrogen in the ideal-gas treatment with translational and rotational modes, $C_V = 5R/2$. In a rigid vessel there is no volume work, so $Q = \Delta U = nC_V\Delta T$.

Step 1: Find the amount and the temperature change The gas mass is $0.056\,\text{kg} = 56\,\text{g}$. Nitrogen has molar mass $28\,\text{g}\,\text{mol}^{-1}$, hence $n = 56/28 = 2\,\text{mol}$. Initially $T_1 = 127 + 273 = 400\,\text{K}$. Since $v_2/v_1 = 2 = \sqrt{T_2/T_1}$,

$$T_2 = 4T_1 = 1600\,\text{K}, \qquad \Delta T = 1200\,\text{K}.$$

Step 2: Calculate the heat at constant volume Use the specified $R = 2\,\text{cal}\,\text{mol}^{-1}\text{K}^{-1}$, so $C_V = 5\,\text{cal}\,\text{mol}^{-1}\text{K}^{-1}$. Therefore

$$Q = 2 \times 5 \times 1200 = 12000\,\text{cal} = 12\,\text{kcal}.$$

Using C_P would incorrectly include expansion work; the containing vessel keeps the volume fixed. The required temperature change is 1200 K, rather than the initial temperature or twice that temperature.

Step 3: Conclusion The heat required is $12\,\text{kcal}$, so the numerical entry is $\boxed{12}$.

Quick Tip to Solve: When a molecular speed is multiplied by a factor, square that factor to find the temperature ratio before calculating the heat.

52. Two identical thin biconvex lens of focal length 15 cm and refractive index 1.5 are in contact with each other. The space between the lenses is filled with a liquid of refractive index 1.25. The focal length of the combination is _____ cm. [4 marks]

Solution:

Concept:

- The power of a thin refracting surface is $(\mu_2 - \mu_1)/R$, using the signed radius. Powers add when the refracting surfaces are close together.
- For a glass lens in air, $P_g = (\mu_g - 1)(1/R_1 - 1/R_2)$. The liquid filling the gap between convex faces acts as a biconcave lens and reduces the combined converging power.

Step 1: Express the curvature factor from the glass-lens focal length Let $S = 1/R_1 - 1/R_2$ for either

identical biconvex lens, with signed radii $R_1 > 0$ and $R_2 < 0$. The lens maker formula in air gives

$$\frac{1}{15} = (1.5 - 1)S, \qquad S = \frac{2}{15}\,\text{cm}^{-1}.$$

There is no need to calculate individual radii. Each glass lens has positive power $1/15\,\text{cm}^{-1}$, so their powers without the liquid would add to $2/15\,\text{cm}^{-1}$.

Step 2: Add the negative power of the intervening liquid The liquid has the complementary biconcave shape: its curvature factor is $-S$. Its equivalent thin-lens power in air is therefore

$$P_l = -(1.25 - 1)S = -0.25\left(\frac{2}{15}\right) = -\frac{1}{30}\,\text{cm}^{-1}.$$

Adding the powers of the two glass lenses and the liquid lens gives

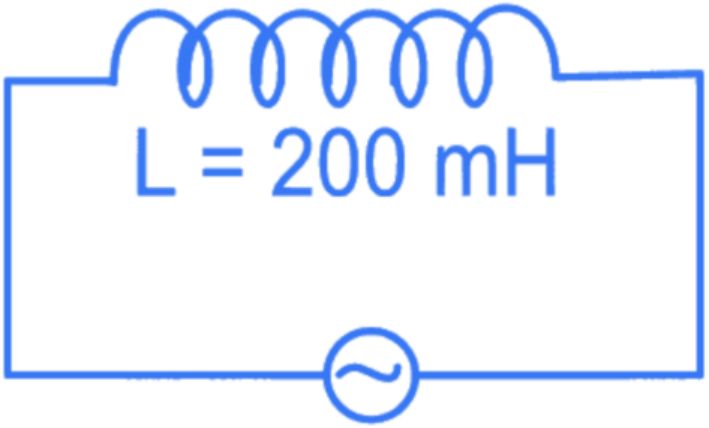
$$\frac{1}{F} = \frac{1}{15} + \frac{1}{15} - \frac{1}{30} = \frac{1}{10}\,\text{cm}^{-1}.$$

This equivalent-power method accounts for the glass-liquid boundaries. Treating the whole system as immersed in liquid would incorrectly change the outer air-glass surfaces as well.

Step 3: Conclusion The focal length of the combination is $F = +10\,\text{cm}$, so the numerical entry is $\boxed{10}$.

Quick Tip to Solve: Treat the liquid trapped between two convex lens faces as an extra biconcave lens; its negative power lengthens the focal length of the pair.

53. As shown in the figure an inductor of inductance 200 mH is connected to an AC source of emf 220 V and frequency 50 Hz. The instantaneous voltage of the source is 0 V when the peak value of current is $\frac{\sqrt{a}}{\pi}$ A. The value of a is _____. [4 marks]



Solution:

Concept:

- The inductive reactance is $X_L = \omega L = 2\pi f L$. For a pure inductor, $I_{\text{rms}} = V_{\text{rms}}/X_L$.
- A sinusoidal current has peak value $I_0 = \sqrt{2} I_{\text{rms}}$. Current lags voltage by $\pi/2$, so its magnitude is a

maximum when the instantaneous voltage is zero.

Step 1: Find the reactance and rms current The quoted AC voltage $220\,\text{V}$ is its rms value. Convert the inductance to henry: $L = 200\,\text{mH} = 0.200\,\text{H}$. With $f = 50\,\text{Hz}$,

$$X_L = 2\pi(50)(0.200) = 20\pi\,\Omega.$$

Hence

$$I_{\text{rms}} = \frac{220}{20\pi} = \frac{11}{\pi}\,\text{A}.$$

No resistance is specified, so the impedance is purely inductive.

Step 2: Convert to peak current and identify the coefficient The peak current is

$$I_0 = \sqrt{2}\frac{11}{\pi} = \frac{11\sqrt{2}}{\pi} = \frac{\sqrt{242}}{\pi}\,\text{A}.$$

In the form stated in the question, $I_0 = \sqrt{a}/\pi\,\text{A}$, comparison gives $a = 11^2 \times 2 = 242$. The condition of zero instantaneous voltage is consistent with this peak current because of the quarter-cycle phase difference; it does not imply that the current is zero.

Step 3: Conclusion The peak current is $\sqrt{242}/\pi\,\text{A}$, so $\boxed{a = 242}$ (dimensionless).

Quick Tip to Solve: In an ideal inductor, voltage zero corresponds to maximum current magnitude; use the rms-to-peak factor only after calculating the reactance.

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54. Sodium light of wavelengths 650 nm and 655 nm is used to study diffraction at a single slit of aperture 0.5 mm. The distance between the slit and the screen is 2.0 m. The separation between the positions of the first maxima of diffraction pattern obtained in the two cases is _____ $\times 10^{-5}$ m. [4 marks]

Solution:

Concept:

- For single-slit diffraction, $I = I_0(\sin\beta/\beta)^2$, where $\beta = \pi b \sin\theta/\lambda$ and b is the slit width.
- The first secondary maximum lies approximately at $b\sin\theta = 3\lambda/2$. For a distant screen and small angle, its position is $y \simeq 3D\lambda/(2b)$.

Step 1: Express the separation for the two wavelengths The central maximum is at the same position for both wavelengths. The first secondary maxima on the same side occur at positions proportional to wavelength. Their separation is therefore

$$\Delta y \simeq \frac{3D}{2b}(\lambda_2 - \lambda_1).$$

Convert the data consistently: $b = 0.5\,\text{mm} = 5 \times 10^{-4}\,\text{m}$, $D = 2.0\,\text{m}$, and $\Delta\lambda = 5\,\text{nm} = 5 \times 10^{-9}\,\text{m}$.

Step 2: Evaluate the coefficient in the requested unit Substitution gives

$$\Delta y \simeq \frac{3(2.0)(5\times10^{-9})}{2(5\times10^{-4})} = 3.0\times10^{-5}\,\mathrm{m}.$$

This refers to the first secondary bright fringe, not the first dark fringe at $D\lambda/b$. More exactly, $\tan\beta = \beta$ has its first nonzero maximum at $\beta \simeq 4.493$, giving $\Delta y \simeq 2.86\times10^{-5}\,\mathrm{m}$; its requested integer coefficient is also 3.

Step 3: Conclusion The separation is approximately $3\times10^{-5}\,\mathrm{m}$, so the numerical entry is $\boxed{3}$.

Quick Tip to Solve: Distinguish a secondary maximum from a minimum: the first secondary maximum is near 1.5 times the first-minimum distance.

55. When light of frequency twice the threshold frequency is incident on the metal plate, the maximum velocity of emitted electron is v_1. When the frequency of incident radiation is increased to five times the threshold value, the maximum velocity of emitted electron becomes v_2. If $v_2 = x\,v_1$, the value of x will be _____. [4 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = h(\nu - \nu_0)$, where ν_0 is the threshold frequency and $\phi = h\nu_0$.
- For a nonrelativistic emitted electron, $K_{\max} = m_e v_{\max}^2/2$. Speed ratios are the square roots of the corresponding excess-frequency ratios.

Step 1: Write the two maximum kinetic energies For the first light frequency $\nu_1 = 2\nu_0$, the photon must first supply the work function $h\nu_0$. Thus

$$\frac{1}{2}m_e v_1^2 = h(2\nu_0 - \nu_0) = h\nu_0.$$

At the second frequency $\nu_2 = 5\nu_0$,

$$\frac{1}{2}m_e v_2^2 = h(5\nu_0 - \nu_0) = 4h\nu_0.$$

The excess energy above threshold, rather than the entire photon energy, becomes kinetic energy.

Step 2: Take the ratio and extract the speed factor Dividing the second energy equation by the first cancels both Planck's constant and the electron mass:

$$\frac{v_2^2}{v_1^2} = \frac{4h\nu_0}{h\nu_0} = 4.$$

Speeds are nonnegative, so $v_2/v_1 = \sqrt{4} = 2$. It would be incorrect to use $\sqrt{5/2}$, because that ratio ignores the energy needed to overcome the work function.

Step 3: Conclusion The relation is $v_2 = 2v_1$, hence $\boxed{x = 2}$ (dimensionless).

Quick Tip to Solve: Subtract the threshold frequency before comparing photoelectron energies, then take a square root to obtain the speed ratio.

56. From the top of a tower, a ball is thrown vertically upward which reaches the ground in 6 s. A second ball thrown vertically downward from the same position with the same speed reaches the ground in 1.5 s. A third ball released, from the rest from the same location, will reach the ground in _____ s. [4 marks]

Solution:

Concept:

- For constant gravitational acceleration and a downward-positive axis, displacement is $H = u_d t + gt^2/2$, where u_d is the signed initial velocity.
- The two thrown balls have initial velocities $-u$ and $+u$ in this axis. The released ball has zero initial velocity, so its fall time satisfies $H = gT^2/2$.

Step 1: Write the two equations for the common height Let the tower height be H and the common launch speed be u. The upward-thrown ball takes $t_1 = 6\,\text{s}$, giving

$$H = -ut_1 + \frac{1}{2}gt_1^2.$$

The downward-thrown ball takes $t_2 = 1.5\,\text{s}$, giving

$$H = ut_2 + \frac{1}{2}gt_2^2.$$

Both displacements are the same positive height even though the initial velocities have opposite signs.

Step 2: Eliminate the launch speed and calculate the free-fall time Divide the first equation by t_1 and the second by t_2, then add:

$$H\left(\frac{1}{t_1} + \frac{1}{t_2}\right) = \frac{g}{2}(t_1 + t_2).$$

Cancelling $t_1 + t_2$ gives $H = gt_1t_2/2$. For release from rest,

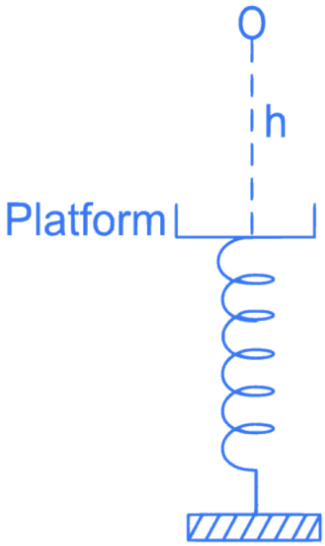
$$\frac{1}{2}gT^2 = \frac{1}{2}gt_1t_2, \qquad T = \sqrt{6 \times 1.5} = 3\,\text{s}.$$

Gravity cancels, so no numerical value of g is required.

Step 3: Conclusion The third ball reaches the ground in $\boxed{3\,\text{s}}$; the numerical entry is 3.

Quick Tip to Solve: For equal and opposite launch speeds from the same height, the release-from-rest time is the geometric mean of the two measured times.

57. A ball of mass 100 g is dropped from a height h = 10 cm on a platform fixed at the top of a vertical spring (as shown in figure). The ball stays on the platform and the platform is depressed by a distance $\frac{h}{2}$. The spring constant is _____ Nm^{-1}.
(Use g = 10 ms^{-2}) [4 marks]



Solution:

Concept:

- The gravitational potential energy lost by the ball is converted into elastic potential energy of the spring at maximum compression: $mg\Delta y = \frac{1}{2}kx^2$.
- The total downward displacement includes both the free fall and the compression: $\Delta y = h + x$. At maximum compression the instantaneous speed is zero.

Step 1: Convert the data into SI units The ball has mass $m = 100\,\text{g} = 0.100\,\text{kg}$. Its initial height above the platform is $h = 10\,\text{cm} = 0.100\,\text{m}$. The spring compression is half this height, so

$$x = \frac{h}{2} = 0.050\,\text{m}.$$

The ball continues moving downward after it first touches the platform; the initial height alone is therefore not its total fall.

Step 2: Apply energy conservation up to maximum compression Take the initial platform position as the reference level. The ball loses gravitational energy during the entire fall $h + x$. Initially the spring is uncompressed, while at the lowest position the ball and platform are instantaneously at rest. Thus

$$mg(h + x) = \frac{1}{2}kx^2.$$

With the stated gravitational acceleration,

$$0.100 \times 10 \times (0.100 + 0.050) = \frac{1}{2}k(0.050)^2.$$

The energy stored is $0.150\,\text{J}$, and hence

$$k = \frac{2(0.150)}{0.0025} = 120\,\text{N m}^{-1}.$$

The dimensions are energy divided by length squared, which correctly give force per unit length.

Step 3: Conclusion The required spring constant is $\boxed{120\,\text{N m}^{-1}}$. The numerical entry is $\boxed{120}$.

Quick Tip to Solve: At maximum spring compression, include gravity acting during compression too: use the total fall $h + x$, rather than just h.

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58. A metre scale is balanced on a knife edge at its centre. When two coins, each of mass 10 g are put one on the top of the other at the 10.0 cm mark the scale is found to be balanced at 40.0 cm mark. The mass of the metre scale is found to be x $\times$ 10^{-2} kg. The value of x is _____. [4 marks]

Solution:

Concept:

- A metre scale balanced at its centre has its centre of mass at the $50.0\,\text{cm}$ mark. Its weight acts there even when the support is moved.
- In rotational equilibrium, clockwise and anticlockwise moments about the knife edge are equal. A weight contributes moment mgd, where d is its perpendicular distance from the pivot.

Step 1: Locate the weights relative to the new pivot The two coins are stacked at the same $10.0\,\text{cm}$ mark, so their combined mass is $m_c = 2 \times 10 = 20\,\text{g}$. The new balance point is at $40.0\,\text{cm}$. Therefore the coins act $40.0 - 10.0 = 30.0\,\text{cm}$ to the left of the pivot, while the scale acts $50.0 - 40.0 = 10.0\,\text{cm}$ to its right. The weights provide opposite torques. The upward support reaction has zero moment about the knife edge.

Step 2: Equate the moments and express the mass in the requested form Let M be the scale mass. Taking moments about the new pivot gives

$$Mg(10.0) = 20g(30.0),$$

when masses are in grams and distances in centimetres. The common gravitational acceleration cancels. Thus

$$M = 20\frac{30.0}{10.0} = 60\,\text{g} = 0.060\,\text{kg}.$$

Both coins must be included; stacking them does not change their horizontal lever arm. Comparing with the stated form

$$M = x \times 10^{-2}\,\text{kg} = 6 \times 10^{-2}\,\text{kg}$$

gives the dimensionless coefficient directly.

Step 3: Conclusion The mass of the metre scale is $0.060\,\text{kg}$, so $\boxed{x = 6}$.

Quick Tip to Solve: Use distances measured from the new knife edge. The scale weight stays at its original centre of mass, while stacked coins share one lever arm.

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59. A transistor is used in common-emitter mode in an amplifier circuit. When a signal of 10 mV is added to the base-emitter voltage, the base current changes by 10 μA and the collector current changes by 1.5 mA. The load resistance is 5 kΩ. The voltage gain of the transistor will be _____.

[4 marks]

Solution:

Concept:

- The small-signal current gain is $\beta = \Delta I_C/\Delta I_B$, and the input resistance is $r_{\text{in}} = \Delta V_{BE}/\Delta I_B$.
- The voltage gain magnitude for a collector load R_L is $|A_v| = \beta R_L/r_{\text{in}} = \Delta I_C R_L/\Delta V_{BE}$. A common-emitter output is inverted.

Step 1: Find the input resistance and current gain The voltage change applied at the input is $\Delta V_{BE} = 10\,\text{mV} = 0.010\,\text{V}$, and the corresponding base-current change is $10\,\mu\text{A} = 10^{-5}\,\text{A}$. Hence

$$r_{\text{in}} = \frac{0.010}{10^{-5}} = 1000\,\Omega.$$

The collector-current change is $1.5\,\text{mA} = 1.5 \times 10^{-3}\,\text{A}$, giving

$$\beta = \frac{1.5 \times 10^{-3}}{10^{-5}} = 150.$$

This is a current ratio, not yet the requested voltage ratio.

Step 2: Calculate the output voltage change and gain magnitude The load resistance is $R_L = 5\,\text{k}\Omega = 5000\,\Omega$. The magnitude of the output signal is therefore

$$|\Delta V_o| = \Delta I_C R_L = (1.5 \times 10^{-3})(5000) = 7.5\,\text{V}.$$

Dividing by the input voltage change gives

$$|A_v| = \frac{7.5}{0.010} = 750.$$

Equivalently, $150 \times 5000/1000 = 750$. An increasing collector current increases the voltage drop across the collector load, so the collector voltage falls. Thus the signed common-emitter gain is negative, while the numerical gain requested is its positive magnitude.

Step 3: Conclusion The voltage gain magnitude is $\boxed{750}$, a dimensionless ratio. The common-emitter amplifier has signed gain $A_v = -750$, indicating phase inversion.

Quick Tip to Solve: Compute $\Delta I_C R_L$ first, then divide by the input signal voltage. Do not confuse the current gain β with the voltage gain.

60. In a potentiometer arrangement, a cell gives a balancing point at 75 cm length of wire. This cell is now replaced by another cell of unknown emf. If the ratio of the emf's of two cells respectively is 3 : 2, the difference in the balancing length of the potentiometer wire in above two cases will be _____ cm. [4 marks]

Solution:

Concept:

- At the null point of a potentiometer, the cell EMF equals the potential gradient times the balancing length: $E = kl$. No current is drawn from the balanced cell.
- With the same potentiometer arrangement, the potential gradient k is unchanged, so $E_1/E_2 = l_1/l_2$. The ratio must follow the same order for cells and lengths.

Step 1: Associate each EMF with its balancing length Let E_1 be the EMF of the first cell and E_2 that of its replacement. The first balance length is $l_1 = 75\,\text{cm}$. The question gives the EMFs in this order as $E_1 : E_2 = 3 : 2$. The replacement therefore has smaller EMF and must balance at a shorter length. Since the driving arrangement has not changed, the same potential gradient applies to both measurements.

Step 2: Use the EMF ratio to find the second balance length The two balance conditions are $E_1 = kl_1$ and $E_2 = kl_2$. Dividing them eliminates the potential gradient:

$$\frac{3}{2} = \frac{75}{l_2}.$$

Thus

$$l_2 = 75\frac{2}{3} = 50\,\text{cm}.$$

Using centimetres throughout is valid because the ratio is dimensionless. The unknown EMF itself need not be calculated to determine the wire length.

Step 3: Calculate the difference requested The difference between the balancing lengths is the positive separation between the two null points:

$$\Delta l = |l_1 - l_2| = |75 - 50| = 25\,\text{cm}.$$

The check $75 : 50 = 3 : 2$ reproduces the EMF ratio and confirms the association of the two cells. Inverting the ratio would instead give an incorrectly longer length for the weaker cell.

Step 4: Conclusion The difference in the balancing lengths is $\boxed{25\,\text{cm}}$. The numerical entry is $\boxed{25}$.

Quick Tip to Solve: Under unchanged potentiometer current, EMF and balance length are proportional. Keep the first and second cell in the same order in both ratios.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. If a rocket runs on a fuel ($C_{15}H_{30}$) and liquid oxygen, the weight of oxygen required and CO_2 released for every litre of fuel respectively are :
(Given : density of the fuel is 0.756 g/mL) [4 marks]

(A) 1188 g and 1296 g
(B) 2376 g and 2592 g
(C) 2592 g and 2376 g
(D) 3429 g and 3142 g

Correct Answer: (C) 2592 g and 2376 g

Solution:

Concept:

- Complete combustion converts all carbon in a hydrocarbon into CO_2 and all hydrogen into H_2O. Atom balance fixes the oxygen demand.
- Use $m = \rho V$, then $n = m/M$, where m is mass, ρ is density, V is volume and M is molar mass. Reaction coefficients give mole ratios, not mass ratios.

Step 1: Balance combustion and find the amount of fuel For one mole of the stated fuel, carbon requires 15 moles of carbon dioxide and hydrogen requires 15 moles of water. The balanced equation is

$$C_{15}H_{30} + (45/2)\,O_2 \longrightarrow 15\,CO_2 + 15\,H_2O.$$

The fuel molar mass is $15(12) + 30(1) = 210\,\text{g mol}^{-1}$. A litre is 1000 mL, so its mass is

$$m_f = 0.756 \times 1000 = 756\,\text{g}, \qquad n_f = \frac{756}{210} = 3.6\,\text{mol}.$$

Step 2: Calculate oxygen consumed and carbon dioxide produced Each mole of fuel needs 22.5 moles of oxygen and produces 15 moles of carbon dioxide. Thus

$$n(O_2) = 3.6(22.5) = 81\,\text{mol}, \qquad m(O_2) = 81(32) = 2592\,\text{g}.$$

Likewise,

$$n(CO_2) = 3.6(15) = 54\,\text{mol}, \qquad m(CO_2) = 54(44) = 2376\,\text{g}.$$

The question asks for oxygen first and carbon dioxide second, which is exactly option (C). Option (B) reverses these masses. Option (A) is half that reversed pair, while option (D) does not satisfy the balanced-reaction calculation. The water produced has mass $54(18) = 972\,\text{g}$; total product mass $2376 + 972 = 3348\,\text{g}$ equals $756 + 2592\,\text{g}$, providing a mass-conservation check.

Step 3: Conclusion The oxygen required is $2592\,\text{g}$ and the carbon dioxide released is $2376\,\text{g}$ per litre of fuel. Hence, the correct option is (C).

Quick Tip to Solve: Balance combustion before using density. For C_xH_y, the oxygen coefficient is $x + y/4$; keep the requested order of product and reactant masses.

62. Consider the following pairs of electrons

(A) (a) n = 3, l = 1, m_1 = 1, m_s = + $\frac{1}{2}$

(b) n = 3, l = 2, m_1 = 1, m_s = + $\frac{1}{2}$

(B) (a) n = 3, l = 2, m_1 = −2, m_s = $-\frac{1}{2}$

(b) n = 3, l = 2, m_1 = −1, m_s = $-\frac{1}{2}$

(C) (a) n = 4, l = 2, m_1 = 2, m_s = + $\frac{1}{2}$

(b) n = 3, l = 2, m_1 = 2, m_s = + $\frac{1}{2}$

The pairs of electrons present in degenerate orbitals is/are : [4 marks]

(A) Only (A)

(B) Only (B)

(C) Only (C)

(D) (B) and (C)

Correct Answer: (B) Only (B)

Solution:

Concept:

- Degenerate orbitals have equal energy. In the ordinary many-electron-atom treatment, orbitals of a given subshell have the same n and l, with different allowed m_l values.
- The principal quantum number n identifies the shell and l identifies the subshell. The spin quantum number m_s describes electron spin; matching spins alone does not make two orbitals degenerate.

Step 1: Compare the first two pairs In pair (A), both electrons have $n = 3$, but the first has $l = 1$ and the second has $l = 2$. They occupy $3p$ and $3d$ orbitals, respectively. These are different subshells, so they do not constitute the required degenerate pair. In pair (B), both have $n = 3$ and $l = 2$, while their magnetic quantum numbers are -2 and -1. They therefore occupy two different orbitals of the same $3d$ subshell. Those orbitals have equal energy in the absence of an external field. Both values of m_l are allowed because $-l \leq m_l \leq l$.

Step 2: Check the third pair and select the complete answer In pair (C), the common $l = 2$ identifies d orbitals, but the principal quantum numbers are 4 and 3. These are $4d$ and $3d$, which belong to different shells and do not have equal energy. The equal values of m_l and m_s do not remove this difference. Thus only pair (B) is degenerate. Option (A) incorrectly accepts different subshells; option (C) incorrectly accepts different principal shells. An option combining (B) with (C) would also be wrong because pair (C) fails. No Pauli exclusion violation occurs in pair (B): its electrons have different magnetic quantum numbers even though their spins agree.

Step 3: Conclusion Only pair (B) consists of electrons in degenerate orbitals of the same $3d$ subshell. Hence, the correct option is (B).

Quick Tip to Solve: For orbital degeneracy in this setting, compare n and l first. Different m_l values within one subshell label distinct orbitals of equal energy.

63. Match List - I with List - II :

	List - I		List -II
(A)	Sphalerite	(I)	$FeCO_3$
(B)	Calamine	(II)	PbS
(C)	Galena	(III)	$ZnCO_3$
(D)	Siderite	(IV)	ZnS

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(B) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(C) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer: (A) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Solution:

Concept:

- An ore name must be matched with its characteristic chemical composition: sphalerite and galena are sulphide ores, whereas calamine and siderite are carbonate ores.
- The listed divalent metal ions combine with S^{2-} or ${CO_3}^{2-}$ in a one-to-one formula ratio. Identify the metal as well as the anion before selecting a matching code.

Step 1: Identify the two zinc ores Sphalerite, also called zinc blende, is zinc sulphide, ZnS. In List II this formula is entry (IV), giving (A)-(IV). Calamine is zinc carbonate, $ZnCO_3$. This appears as entry (III), giving (B)-(III). Both minerals contain zinc, but their anions differ: sulphide in sphalerite and carbonate in calamine. Consequently, simply identifying both as zinc ores is insufficient; the compound formula must also be matched.

Step 2: Identify the lead and iron ores and check each code Galena is lead sulphide, PbS, corresponding to entry (II). Hence (C)-(II). Siderite is iron(II) carbonate, $FeCO_3$, corresponding to entry (I), so (D)-(I). Combining all four identifications gives

$$(A)-(IV), \quad (B)-(III), \quad (C)-(II), \quad (D)-(I).$$

This is the first option. The second option interchanges the calamine and siderite assignments; the third incorrectly identifies sphalerite as lead sulphide and galena as iron carbonate. The fourth interchanges the two zinc ores. Checking all four matches avoids accepting a code merely because one familiar mineral is paired correctly.

Step 3: Conclusion The correct matching is sphalerite–ZnS, calamine–$ZnCO_3$, galena–PbS, and siderite–$FeCO_3$. Hence, the correct option is (A).

Quick Tip to Solve: Group these ores by anion: sphalerite and galena are sulphides; calamine and siderite are carbonates. Then use the metal to distinguish each pair.

64. For a reaction at equilibrium

A(g) $\rightleftharpoons$ B(g) + $\frac{1}{2}$ C(g)

the relation between dissociation constant (K), degree of dissociation (α) and equilibrium pressure (p) is given by : [4 marks]

(A) $K = \frac{\alpha^{\frac{1}{2}} p^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(B) $K = \frac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$

(C) $K = \frac{(\alpha p)^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(D) $K = \frac{(\alpha p)^{\frac{3}{2}}}{(1+\alpha)(1-\alpha)^{\frac{1}{2}}}$

Correct Answer: (B) $K = \frac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$

Solution:

Concept:

- For an ideal gaseous equilibrium, each partial pressure equals the total equilibrium pressure multiplied by its mole fraction. The total number of moles must include every species formed.
- For $\mathrm{A(g)} \rightleftharpoons \mathrm{B(g)} + \frac{1}{2}\,\mathrm{C(g)}$, the pressure equilibrium constant is $K = p_B p_C^{1/2}/p_A$. Begin with one mole of pure A and let its degree of dissociation be α.

Step 1: Write the equilibrium composition Initially take one mole of A and no products. When a fraction α dissociates, the equilibrium amounts are $n_A = 1-\alpha$, $n_B = \alpha$, and $n_C = \alpha/2$. Thus the total amount is

$$n_{\text{total}} = 1 - \alpha + \alpha + \frac{\alpha}{2} = 1 + \frac{\alpha}{2}.$$

The half coefficient of C is essential: one mole of dissociated A does not produce one mole of C.

Step 2: Convert mole fractions into partial pressures At the stated total equilibrium pressure p, divide each amount by $1 + \alpha/2$. This gives

$$p_A = \frac{(1-\alpha)p}{1+\alpha/2}, \quad p_B = \frac{\alpha p}{1+\alpha/2}, \quad p_C = \frac{\alpha p}{2+\alpha}.$$

These pressures sum to p, providing a useful check on the mole balance.

Step 3: Substitute and simplify the constant Using the stoichiometric powers in the equilibrium expression,

$$\begin{aligned} K &= \frac{p_B}{p_A}\sqrt{p_C} \\ &= \frac{\alpha}{1-\alpha}\sqrt{\frac{\alpha p}{2+\alpha}} \\ &= \frac{\alpha^{3/2} p^{1/2}}{\sqrt{2+\alpha}(1-\alpha)}. \end{aligned}$$

Options A and C incorrectly use $1 + 3\alpha/2$, and A also has the wrong power of α. Options A, C and D have $p^{3/2}$, whereas the net change in gaseous moles is only $1/2$, so the required pressure factor is $p^{1/2}$.

Step 4: Conclusion

$$K = \frac{\alpha^{3/2} p^{1/2}}{\sqrt{2+\alpha}(1-\alpha)}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Use an initial amount of one mole, then calculate total equilibrium moles before writing partial pressures; fractional stoichiometric coefficients affect both quantities.

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65. Given below are the oxides:

Na_2O, As_2O_3, N_2O, NO and Cl_2O_7

Number of amphoteric oxides is : [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- An amphoteric oxide reacts with both acids and bases. Classification concerns acid-base behaviour, rather than simply whether the element has more than one oxidation state.
- Metal oxides such as Na_2O are strongly basic; several nonmetal oxides are acidic, while NO and N_2O are neutral. Arsenic(III) oxide is amphoteric.

Step 1: Identify the amphoteric member Among the five listed oxides, As_2O_3 shows both kinds of acid-base behaviour. It reacts with hydrochloric acid to form arsenic(III) chloride and with alkali to form arsenite. Representative equations are

$$As_2O_3 + 6\,HCl \longrightarrow 2\,AsCl_3 + 3\,H_2O$$

and

$$As_2O_3 + 2\,NaOH \longrightarrow 2\,NaAsO_2 + H_2O.$$

These reactions establish amphoterism directly, rather than inferring it from the oxidation state of arsenic alone.

Step 2: Classify the remaining four oxides Na_2O is a basic oxide: it forms sodium hydroxide with water and neutralises acids. It is not amphoteric. Both N_2O and NO are neutral oxides; they are not classified as acid anhydrides or amphoteric oxides in this comparison. Cl_2O_7 is acidic, being the anhydride of perchloric acid:

$$Cl_2O_7 + H_2O \longrightarrow 2\,HClO_4.$$

It therefore belongs in the acidic category, even though chlorine has a high oxidation state.

Step 3: Count only the amphoteric category The classification is: one basic oxide, two neutral oxides,

one acidic oxide and one amphoteric oxide. Only As_2O_3 contributes to the required count. Thus zero would omit arsenic(III) oxide, while two or three would incorrectly include one or more neutral, acidic or basic oxides. The number of oxygen atoms in a formula does not determine this count.

Step 4: Conclusion There is exactly one amphoteric oxide, As_2O_3. Hence, the correct option is (B).

Quick Tip to Solve: For oxide-count questions, assign each formula to acidic, basic, neutral or amphoteric before counting; remember that NO and nitrous oxide are neutral.

66. The most stable trihalide of nitrogen is : [4 marks]

(A) NF_3

(B) NCl_3

(C) NBr_3

(D) NI_3

Correct Answer: (A) NF_3

Solution:

Concept:

- Nitrogen trihalides have the formula NX_3, with three nitrogen-halogen bonds and a lone pair on nitrogen. Their relative stability depends strongly on the strength of those bonds.
- The compact second-period orbitals of nitrogen and fluorine overlap efficiently. The resulting strong $N-F$ bonds make NF_3 substantially more stable than the heavier nitrogen trihalides.

Step 1: Compare the sizes and orbital overlap Nitrogen is a small second-period atom, and fluorine also uses compact second-period valence orbitals. The $2p$-$2p$ overlap in an $N-F$ bond is effective. Chlorine, bromine and iodine have successively larger, more diffuse valence orbitals, so their overlap with the small nitrogen orbitals is poorer. The nitrogen-halogen bonds in their trihalides are consequently much weaker than the nitrogen-fluorine bonds.

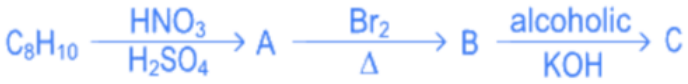
Step 2: Relate bond strength to decomposition Decomposition of a nitrogen trihalide can produce very stable molecular nitrogen. Whether that occurs readily depends on how easily the original nitrogen-halogen bonds can be broken. Strong $N-F$ bonds resist this process, and NF_3 is a stable compound under ordinary conditions. In contrast, NCl_3, NBr_3 and NI_3 are much less stable and can decompose explosively. The increasing size of the attached halogen does not stabilise these compounds.

Step 3: Eliminate the heavier-halogen alternatives Option B selects nitrogen trichloride, which is sensitive and less stable than nitrogen trifluoride. Option C selects nitrogen tribromide, whose weaker bonds do not provide the required stability. Option D selects nitrogen triiodide, well known for its extreme sensitivity; it is not the most stable trihalide. The appropriate comparison is the actual nitrogen-halogen bonding, rather than the stability trend of unrelated hydrogen halides or other group-15 compounds.

Step 4: Conclusion The most stable nitrogen trihalide is NF_3. Hence, the correct option is (A).

Quick Tip to Solve: When comparing nitrogen trihalides, focus on the strong N–F bond and effective overlap between two small second-period atoms.

67. In the given reaction sequence, the major product 'C' is : [4 marks]



(A) $C_6H_5C(NO_2)=CH_2$

(B) p-$NO_2C_6H_4CH=CH_2$

(C) p-$NO_2C_6H_4C(NO_2)=CH_2$

(D) o-$NO_2C_6H_4CH=CH_2$

Correct Answer: (B) p-$NO_2C_6H_4CH=CH_2$

Solution:

Concept:

- An ethyl group activates an aromatic ring and directs electrophilic nitration to the ortho and para positions; the less crowded para product is the major product in the indicated sequence.
- Bromination with heat favours free-radical substitution at a benzylic carbon. Alcoholic KOH then removes hydrogen bromide by elimination to produce an alkene.

Step 1: Identify the nitration product A The sequence and the offered aromatic products identify the starting C_8H_{10} hydrocarbon as ethylbenzene, $C_6H_5CH_2CH_3$. Mixed concentrated nitric and sulfuric acids nitrate the aromatic ring. The ethyl substituent is ortho/para directing, and para substitution gives the major product because the entering nitro group is farther from the ethyl group. Thus A is mainly p-nitroethylbenzene, p-$NO_2C_6H_4CH_2CH_3$.

Step 2: Locate bromination in B With Br_2 and heat, substitution occurs through a radical pathway. The benzylic radical is stabilised by delocalisation into the aromatic ring, so bromination replaces a benzylic hydrogen:

$$p\text{-}NO_2C_6H_4CH_2CH_3 \longrightarrow p\text{-}NO_2C_6H_4CHBrCH_3.$$

The nitro group stays attached to the ring; this step neither adds a second nitro group nor transfers the existing nitro group onto the side chain.

Step 3: Eliminate hydrogen bromide to obtain C Alcoholic potassium hydroxide promotes removal of HBr from the adjacent carbons of the side chain. Loss of bromine from the benzylic carbon and hydrogen from the terminal methyl group gives

$$p\text{-}NO_2C_6H_4CHBrCH_3 \longrightarrow p\text{-}NO_2C_6H_4CH=CH_2.$$

This is para-nitrostyrene. Option A has a side-chain nitro group absent from the sequence. Option C introduces an extra nitro group. Option D is the ortho isomer rather than the major para product selected by nitration.

Step 4: Conclusion The major final product is p-nitrostyrene, p-$NO_2C_6H_4CH=CH_2$. Hence, the correct option is (B).

Quick Tip to Solve: Track ring substitution separately from side-chain reactions: mixed-acid nitration acts on the ring, heated bromine on the benzylic position, and alcoholic base forms the vinyl group.

68. Two statements are given below :

Statement I : The melting point of monocarboxylic acid with even number of carbon atoms is higher than that of with odd number of carbon atoms acid immediately below and above it in the series.

Statement II : The solubility of monocarboxylic acids in water decreases with increase in molar mass.

Choose the most appropriate option : [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (A) Both Statement I and Statement II are correct.

Solution:

Concept:

- Carboxylic acids form strong intermolecular hydrogen bonds. Their melting points also depend on how efficiently molecules pack in the crystal, producing an odd-even alternation in a homologous series.
- Water solubility reflects competition between the hydrophilic carboxyl group and the hydrophobic hydrocarbon portion; extending the carbon chain makes the nonpolar contribution more important.

Step 1: Evaluate the melting-point statement Even-carbon monocarboxylic acids pack more efficiently in the solid state than their neighbouring odd-carbon homologues. The orientation of terminal groups in an extended carbon chain produces this alternating packing pattern. More effective crystal packing stabilises the solid and requires a higher temperature for melting. Consequently, an even-carbon acid generally has a higher melting point than the immediately preceding and succeeding odd-carbon acids in the homologous series. Statement I expresses this standard odd-even effect and is correct.

Step 2: Evaluate the solubility statement The carboxyl group can both donate and accept hydrogen bonds with water, making the lower members appreciably soluble. As the homologous series is ascended, extra methylene units increase the molar mass and enlarge the nonpolar hydrocarbon region, while there remains just one carboxyl group per molecule. Water cannot compensate as effectively for this increasing hydrophobic part, so solubility decreases. Statement II is therefore also correct for the monocarboxylic-acid homologous series under comparison.

Step 3: Match the two truth values to an option Both independent tests give a true statement. Option B would require both statements to be false, contrary to crystal-packing and solvation behaviour. Option C incorrectly denies the falling water-solubility trend, and option D incorrectly denies the odd-even melting-point effect. No claim that Statement II explains Statement I is needed here: the question asks only for the truth values of the two statements.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: Keep the two effects separate: odd-even melting-point variation is a crystal-packing effect, while falling water solubility follows the growing hydrocarbon chain.

69. Which of the following is an example of conjugated diketone? [4 marks]

(A) $CH_3CO(CH_2)_3COCH_3$

(B) 4-(2-Oxopropyl)cyclohexan-1-one

(C) Cyclohexa-2,5-diene-1,4-dione (1,4-benzoquinone)

(D) $C_6H_5COCH_2CH_2COCH_3$

Correct Answer: (C) Cyclohexa-2,5-diene-1,4-dione (1,4-benzoquinone)

Solution:

Concept:

- A conjugated system has a continuous chain of adjacent p orbitals, usually represented by alternating multiple and single bonds. An intervening saturated carbon interrupts this overlap.
- For the two ketone groups to be conjugated with each other, trace a continuous unsaturated path from one carbonyl to the other. A carbonyl conjugated only with a phenyl ring does not by itself make both carbonyls mutually conjugated.

Step 1: Inspect the first two structures Option A is $CH_3CO(CH_2)_3COCH_3$. Its carbonyl groups are separated by three saturated methylene groups, which interrupt p-orbital overlap. It is an isolated diketone. In option B, the side-chain carbonyl is attached through a methylene group and saturated cyclohexane carbons to the ring carbonyl. That route likewise contains saturated centres and cannot carry continuous conjugation between the two ketone groups.

Step 2: Trace the unsaturated path in option C Option C is 1,4-benzoquinone, or cyclohexa-2,5-diene-1,4-dione. Each side of its ring provides the sequence

$$O{=}C{-}C{=}C{-}C{=}O.$$

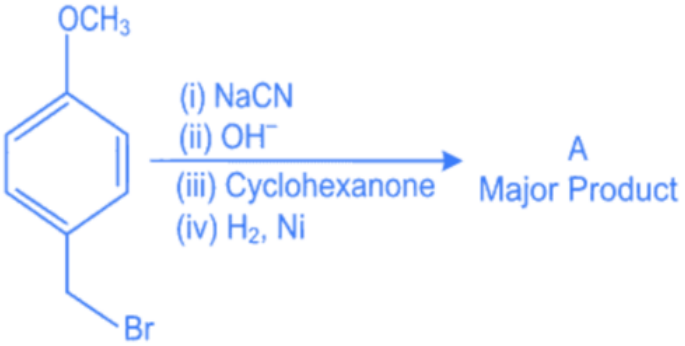
Along this sequence, adjacent carbon atoms have p orbitals, and the single bonds separate multiple bonds without a saturated carbon interrupting the path. Both carbonyl groups are therefore part of the conjugated unsaturated system. The ring is a quinone; it need not be described as an aromatic benzene ring to establish this conjugation.

Step 3: Check why the phenyl-containing alternative fails Option D is $C_6H_5COCH_2CH_2COCH_3$. The first carbonyl is conjugated with the phenyl ring, but the two ketone groups themselves are separated by CH_2CH_2. Those saturated carbons break conjugation along the connection to the second carbonyl. Thus A, B and D fail the required continuous p-orbital test, whereas C passes. Merely seeing an aromatic ring or two carbonyl groups is not sufficient.

Step 4: Conclusion The conjugated diketone is 1,4-benzoquinone, shown in option (C). Hence, the correct option is (C).

Quick Tip to Solve: Trace the path connecting the two carbonyl groups; any intervening tetrahedral saturated carbon breaks their conjugation, even if one carbonyl is attached to an aromatic ring.

70. The major product of the above reactions is : [4 marks]



(A) 2-Cyclohexyl-2-(4-methoxyphenyl)ethan-1-amine

(B) 1-[Amino(4-methoxyphenyl)methyl]cyclohexan-1-ol

(C) 2-Cyclohexyl-2-(4-methoxyphenyl)acetamide

(D) 1-[2-Amino-1-(4-methoxyphenyl)ethyl]cyclohexan-1-ol

Correct Answer: (D) 1-[2-Amino-1-(4-methoxyphenyl)ethyl]cyclohexan-1-ol

Solution:

Concept:

- Cyanide substitutes a benzylic bromide through its carbon end to form a nitrile, extending the chain by one carbon. The alpha hydrogen of a benzyl cyanide can be removed to form a resonance-stabilised nucleophile.
- The alpha-nitrile carbanion adds to a ketone carbonyl, forming a carbon-carbon bond and an alcohol on protonation. Catalytic hydrogenation converts $-\mathrm{CN}$ into $-\mathrm{CH_2NH_2}$.

Step 1: Form the substituted benzyl cyanide Let $\mathrm{Ar} = p\text{-}\mathrm{CH_3OC_6H_4}$, preserving the para methoxy group throughout. The starting bromide is $\mathrm{ArCH_2Br}$. Sodium cyanide replaces bromide to give $\mathrm{ArCH_2CN}$. The carbon of the cyanide group becomes an extra carbon in the side chain; it is not lost in the later reactions. The methoxy substituent is unchanged by the listed reagents.

Step 2: Add the alpha-nitrile nucleophile to cyclohexanone Base removes an alpha hydrogen from $\mathrm{ArCH_2CN}$, giving the stabilised species represented as $\mathrm{ArCH^-CN}$. Its nucleophilic carbon attacks the carbonyl carbon of cyclohexanone. The carbonyl pi bond opens to oxygen, and protonation produces an alcohol. The former cyclohexanone carbonyl carbon is now bonded to OH and to $\mathrm{CH(Ar)CN}$, while remaining part of the six-membered ring. This fixes the location of the hydroxyl group on the ring carbon.

Step 3: Reduce the nitrile and compare all products With $\mathrm{H_2/Ni}$, the nitrile is hydrogenated according to

$$\mathrm{R{-}CN + 2\,H_2 \longrightarrow R{-}CH_2NH_2.}$$

Therefore the side chain attached to the hydroxyl-bearing ring carbon becomes $\mathrm{CH(Ar)CH_2NH_2}$. Option D has exactly this connectivity. Option A incorrectly omits the hydroxyl group. Option B attaches amino directly to the aryl-bearing carbon and loses the extra carbon introduced by cyanide. Option C retains an amide carbonyl, whereas catalytic reduction gives an amine, not an amide.

Step 4: Conclusion The product is 1-[2-amino-1-(4-methoxyphenyl)ethyl]cyclohexan-1-ol, with the OH group on the ring carbon and the terminal CH_2NH_2 group in its side chain. Hence, the correct option is (D).

Quick Tip to Solve: Label the cyanide carbon when it enters: it becomes the terminal methylene of the primary amine after hydrogenation, while the cyclohexanone oxygen becomes OH.

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71. A polysaccharide 'X' on boiling with dil H_2SO_4 at 393 K under 2-3 atm pressure yields 'Y'. 'Y' on treatment with bromine water gives gluconic acid. 'X' contains β-glycosidic linkages only. Compound 'X' is : [4 marks]

(A) starch

(B) cellulose

(C) amylose

(D) amylopectin

Correct Answer: (B) cellulose

Solution:

Concept:

- Polysaccharides undergo acid-catalysed hydrolysis: water cleaves glycosidic bonds and releases the constituent monosaccharides.
- Bromine water oxidises the aldehyde group of glucose to gluconic acid. The configuration of the glycosidic linkage distinguishes cellulose from starch.

Step 1: Identify the hydrolysis product The product Y gives gluconic acid on treatment with bromine water. Thus Y is glucose: its open-chain aldehyde group is oxidised to a carboxyl group without changing the carbon skeleton. The given acid, temperature and pressure promote cleavage of the polymer into glucose units. This test identifies the monomer, but does not by itself distinguish the four glucose polymers in the options.

Step 2: Use the linkage configuration Cellulose is a straight-chain polymer of β-D-glucose units joined by $\beta(1 \rightarrow 4)$ glycosidic bonds. It therefore satisfies the decisive condition that X contains only beta linkages. Starch consists of amylose and amylopectin: amylose has $\alpha(1 \rightarrow 4)$ bonds, while amylopectin has $\alpha(1 \rightarrow 4)$ chain bonds and $\alpha(1 \rightarrow 6)$ branch bonds. These alpha-linked alternatives do not satisfy the stated condition, even though hydrolysis also yields glucose.

Step 3: Conclusion X is cellulose and Y is glucose. The combination of glucose as the hydrolysis product and exclusively β-glycosidic linkages identifies cellulose. Hence, the correct option is (B).

Quick Tip to Solve: First identify the monomer from its chemical test; then use alpha versus beta linkage to distinguish starch from cellulose.

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72. During the qualitative analysis of salt with cation y^{2+}, addition of a reagent (X) to alkaline solution of the salt gives a bright red precipitate. The reagent (X) and the cation (y^{2+}) present respectively are : [4 marks]

(A) Dimethylglyoxime and Ni^{2+}

(B) Dimethylglyoxime and Co^{2+}

(C) Nessler's reagent and Hg^{2+}

(D) Nessler's reagent and Ni^{2+}

Correct Answer: (A) Dimethylglyoxime and Ni^{2+}

Solution:

Concept:

- Dimethylglyoxime (DMG) is a selective reagent for nickel(II) in ammoniacal or alkaline solution.
- Deprotonated dimethylglyoxime chelates Ni^{2+}, producing a sparingly soluble bright red nickel dimethylglyoximate complex.

Step 1: Identify the characteristic precipitate The bright red precipitate in an alkaline solution is the characteristic confirmatory test for nickel(II) with dimethylglyoxime. Each DMG molecule loses one proton and binds through its donor nitrogen atoms; two ligands surround one nickel ion. Writing the neutral reagent as H_2DMG, the precipitation may be summarised as

$$Ni^{2+} + 2\,H_2DMG + 2\,OH^- \longrightarrow Ni(HDMG)_2\downarrow + 2\,H_2O.$$

The alkaline medium helps remove the ligand protons and favours formation of the insoluble chelate.

Step 2: Compare the proposed reagent-ion pairs The dimethylglyoxime–nickel(II) pair therefore matches both the reagent and the red precipitate. Cobalt(II) does not give this characteristic nickel precipitate under the stated test conditions, so replacing nickel with cobalt is incorrect. Nessler's reagent is alkaline potassium tetraiodomercurate(II), used to detect ammonia or ammonium ions through a yellow-brown colour or brown precipitate. It is not the reagent for the bright red nickel chelate, and pairing it with mercury(II) or nickel(II) does not describe the test.

Step 3: Conclusion X is dimethylglyoxime and the cation is Ni^{2+}. Hence, the correct option is (A).

Quick Tip to Solve: Associate DMG in ammoniacal solution with a bright red nickel precipitate; associate Nessler reagent with ammonia.

73. Given below are two statements :

Statement I : Emulsions of oil in water are unstable and sometimes they separate into two layers on standing.

Statement II : For stabilisation of an emulsion, excess of electrolyte is added.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (C) Statement I is correct but Statement II is incorrect.

Solution:

Concept:

- An emulsion disperses droplets of one liquid in another immiscible liquid; oil-in-water emulsions have water as the continuous phase.
- Emulsifiers such as soaps, detergents and proteins protect dispersed droplets against coalescence. Excess electrolyte can destroy this protection.

Step 1: Evaluate Statement I Oil and water are immiscible. Mechanical agitation can disperse oil as tiny droplets in water, but the large interfacial area is energetically unfavourable. Without sufficient stabilisation the droplets collide, coalesce into larger drops, and eventually form a separate oil layer. Thus an oil-in-water emulsion can separate into two layers on standing. Statement I describes this instability correctly; the word "sometimes" accommodates differences in how effectively emulsifiers protect the droplets.

Step 2: Evaluate Statement II and select the pair An emulsion is stabilised by an emulsifying agent, which forms a protective interfacial layer around droplets and helps prevent coalescence. Adding an excess of electrolyte is not the general method for stabilisation: it can screen or neutralise droplet charges and promote coagulation or breaking of the emulsion. Statement II is therefore incorrect. Option (A) incorrectly accepts II, option (B) incorrectly rejects I, and option (D) reverses the two truth values. Only the choice stating I true and II false survives.

Step 3: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish an emulsifier from an electrolyte: protective films stabilise droplets, whereas excess electrolyte may break the dispersion.

74. Match List - I with List - II :

	List - I		List -II
(A)	$[PtCl_4]^{2-}$	**(I)**	sp^3d
(B)	BrF_5	**(II)**	d^2sp^3
(C)	PCl_5	**(III)**	dsp^2
(D)	$[Co(NH_3)_6]^{3+}$	**(IV)**	sp^3d^2

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(D) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Correct Answer: (B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Solution:

Concept:

- The hybridisation follows the geometry and coordination environment. Square planar complexes use dsp^2, while octahedral species use d^2sp^3 or sp^3d^2.
- For main-group compounds, count bonding pairs and lone pairs together to obtain the steric number; a lone pair changes molecular shape without changing that count.

Step 1: Assign the platinum and cobalt complexes In $[\mathrm{PtCl_4}]^{2-}$, platinum has oxidation state +2 and a $5d^8$ configuration. This platinum(II) complex is square planar, corresponding to dsp^2, so A matches III. In $[\mathrm{Co(NH_3)_6}]^{3+}$, cobalt is +3 and hence d^6. The ammonia ligands with cobalt(III) produce a low-spin inner-orbital octahedral complex. Pairing leaves two inner d orbitals for d^2sp^3 hybridisation, so D matches II.

Step 2: Count electron pairs for the main-group compounds In $\mathrm{BrF_5}$, bromine has five bonding pairs and one lone pair. Its six electron-pair directions give an octahedral electron-pair arrangement with sp^3d^2 hybridisation; the molecular shape is square pyramidal. Thus B matches IV. In $\mathrm{PCl_5}$, phosphorus has five bonding pairs and no lone pair, giving a trigonal bipyramidal arrangement and sp^3d hybridisation. Thus C matches I. The full match is A–III, B–IV, C–I, D–II. The other choices either assign octahedral hybridisation to square planar platinum or interchange the steric numbers of bromine and phosphorus.

Step 3: Conclusion The matching follows square planar platinum, six electron-pair directions around bromine, trigonal bipyramidal phosphorus and inner-orbital octahedral cobalt. Hence, the correct option is (B).

Quick Tip to Solve: For main-group species count lone pairs as well as bonds; for metal complexes check oxidation state, d configuration and actual geometry.

75. The highest industrial consumption of molecular hydrogen is to produce compounds of element :

[4 marks]

(A) Carbon

(B) Nitrogen

(C) Oxygen

(D) Chlorine

Correct Answer: (B) Nitrogen

Solution:

Concept:

- Hydrogen is a major industrial feedstock. Its largest individual chemical use in the standard industrial comparison is synthesis of ammonia.
- The Haber process combines nitrogen and hydrogen: $N_2 + 3\,H_2 \rightleftharpoons 2\,NH_3$. Ammonia supplies nitrogen for fertilisers and other chemicals.

Step 1: Connect hydrogen consumption with ammonia manufacture In the Haber process, one mole of nitrogen reacts with three moles of hydrogen to form two moles of ammonia. The enormous demand for nitrogen fertilisers makes ammonia manufacture a principal large-scale sink for molecular hydrogen. The element whose compound is being produced in this process is nitrogen: ammonia is its hydrogen compound. The question asks for the element, rather than for the name of the product or the composition of the feed mixture.

Step 2: Distinguish the other listed elements Carbon compounds also use hydrogen in methanol manufacture and hydrogenation, and hydrogen is used in petroleum refining. These uses do not replace ammonia as the expected highest-consumption chemical-manufacture application in this comparison. Oxygen would suggest water formation, which is not the major industrial purpose for consuming manufactured hydrogen. Chlorine reacts with hydrogen to form hydrogen chloride, but the scale of this application is not the largest one identified here. Thus nitrogen is the only listed element corresponding to the standard industrial use being tested.

Step 3: Conclusion The highest industrial consumption of molecular hydrogen in this comparison is for manufacturing ammonia, a nitrogen compound. Hence, the correct option is (B).

Quick Tip to Solve: When asked about the major industrial chemical use of hydrogen, recall ammonia synthesis and then identify nitrogen as the other element.

76. Which of the following statements are correct?

(A) Both LiCl and $MgCl_2$ are soluble in ethanol.

(B) The oxides Li_2O and MgO combine with excess of oxygen to give superoxide.

(C) LiF is less soluble in water than other alkali metal fluorides.

(D) Li_2O is more soluble in water than other alkali metal oxides.

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A) and (C) only

(B) (A), (C) and (D) only

(C) (B) and (C) only

(D) (A) and (D) only

Correct Answer: (A) (A) and (C) only

Solution:

Concept:

- Lithium and magnesium show a diagonal relationship because their small cations have comparable polarising power. Several salts consequently show increased covalent character.
- The stability of oxides, peroxides and superoxides depends strongly on cation size. Lithium preferentially forms the normal oxide, rather than a superoxide.

Step 1: Test statements A and B Statement A is correct: both $LiCl$ and $MgCl_2$ dissolve in ethanol, reflecting their appreciable covalent character and the diagonal relationship. Statement B is incorrect. Lithium forms the normal oxide Li_2O, while magnesium forms MgO; neither oxide simply combines with excess oxygen to give the superoxide claimed. Superoxide formation is favoured by the larger alkali-metal cations such as potassium, rubidium and caesium, which stabilise the large superoxide anion.

Step 2: Test statements C and D and compare combinations Statement C is correct: LiF is much less soluble in water than the other alkali-metal fluorides. Its small oppositely charged ions give a particularly strong crystal lattice; hydration does not fully overcome this lattice-energy cost. Statement D is incorrect: lithium oxide is not more soluble in water than the other alkali-metal oxides. Their interaction with water and the generally increasing solubility trend down the alkali group do not support the asserted lithium maximum. Only A and C are correct. The other combinations include the false B or D, or omit the true A.

Step 3: Conclusion The correct statements are A and C only. Hence, the correct option is (A).

Quick Tip to Solve: Use lithium anomalies together: covalent chloride, sparingly soluble fluoride and normal oxide; do not assign the heavier alkali-metal superoxide behaviour to lithium.

77. Identify the correct statement for B_2H_6 from those given below :

(A) In B_2H_6, all B-H bonds are equivalent.

(B) In B_2H_6, there are four 3-centre-2-electron bonds.

(C) B_2H_6 is a Lewis acid.

(D) B_2H_6 can be synthesized from both BF_3 and $NaBH_4$.

(E) B_2H_6 is a planar molecule.

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A) and (E) only

(B) (B), (C) and (E) only

(C) (C) and (D) only

(D) (C) and (E) only

Correct Answer: (C) (C) and (D) only

Solution:

Concept:

- Diborane, B_2H_6, is electron deficient. It contains four terminal two-centre two-electron B–H bonds and two bridging three-centre two-electron bonds.
- Electron-deficient boranes accept electron pairs and act as Lewis acids. The bridging hydrogens also make diborane nonplanar.

Step 1: Assess the bonding and geometry statements Statement A is false because terminal B–H bonds and bridging B–H interactions are not equivalent; their lengths and bonding descriptions differ. Statement B is false because there are two, not four, three-centre two-electron B–H–B bridge bonds. The other four B–H bonds are ordinary terminal two-centre two-electron bonds. Statement E is false: the two bridging hydrogen atoms lie on opposite sides of the plane containing the boron atoms and four terminal hydrogens. The whole molecule is therefore not planar.

Step 2: Check acidity and preparation routes Statement C is correct: diborane is electron deficient and accepts electron pairs from Lewis bases. Statement D is also correct. Reduction of boron trifluoride with lithium aluminium hydride yields diborane, for example

$$4\,BF_3 + 3\,LiAlH_4 \longrightarrow 2\,B_2H_6 + 3\,LiF + 3\,AlF_3.$$

Oxidation of sodium borohydride with iodine also gives it:

$$2\,NaBH_4 + I_2 \longrightarrow B_2H_6 + 2\,NaI + H_2.$$

Hence C and D are true. Choices containing A, B or E are rejected by the bonding and geometry checks; only the C-and-D combination remains.

Step 3: Conclusion Only statements C and D are correct. Hence, the correct option is (C).

Quick Tip to Solve: For diborane remember four terminal bonds plus two bridge bonds; two hydrogens above and below the terminal plane rule out planarity.

78. Which one of the following elemental forms is not present in the enamel of the teeth? [4 marks]

(A) Ca^{2+}

(B) P^{3+}

(C) F^{-}

(D) P^{5+}

Correct Answer: (B) P^{3+}

Solution:

Concept:

- Tooth enamel contains calcium phosphate in an apatite lattice. Hydroxyapatite has the formula $Ca_{10}(PO_4)_6(OH)_2$. Calcium is present in oxidation state $+2$.
- Phosphorus in phosphate is in oxidation state $+5$, whereas fluoride can replace hydroxide to give fluorapatite, $Ca_{10}(PO_4)_6F_2$. Oxidation-state labels describe the elements within these compounds, not isolated phosphorus cations in enamel.

Step 1: Identify the calcium and fluorine forms Calcium is a group 2 element and occurs as Ca^{2+} in enamel minerals, so option (A) is present. Fluorine occurs as F^{-} in fluoride-containing apatite. Replacement of hydroxide by fluoride increases resistance to acid attack; therefore option (C) is also a form associated with enamel.

Step 2: Find the oxidation state of phosphorus For a phosphate ion, let the oxidation state of phosphorus be x. Oxygen contributes -2 per atom, and the total charge is -3:

$$x + 4(-2) = -3 \quad \Rightarrow \quad x = +5.$$

Thus the phosphorus form represented in option (D) is present in phosphate. The $+3$ state in option (B) would correspond to a different phosphorus chemistry and is not the phosphate constituent of tooth enamel.

Step 3: Conclusion The form not present in tooth enamel is phosphorus in the $+3$ state, represented by P^{3+}. Hence, the correct option is (B).

Quick Tip to Solve: Recognise enamel as an apatite calcium phosphate: calcium is +2, phosphorus is +5, and fluoride is -1. Determine the phosphorus state from phosphate rather than from the element alone.

79. Which of the following is an example of polyester? [4 marks]

(A) Butadiene-styrene copolymer

(B) Melamine polymer

(C) Neoprene

(D) Poly-β-hydroxybutyrate-co-β-hydroxyvalerate

Correct Answer: (D) Poly-β-hydroxybutyrate-co-β-hydroxyvalerate

Solution:

Concept:

- A polyester contains recurring ester linkages, $-C(=O)-O-$, within its polymer backbone. The functional group joining repeat units determines this classification.
- Poly-β-hydroxybutyrate-co-β-hydroxyvalerate (PHBV) is a biodegradable copolyester made from two hydroxy acids. Each monomer contains both a hydroxyl and a carboxylic acid group.

Step 1: Locate the ester-forming monomers In PHBV, the hydroxyl group of one hydroxy-acid unit links with the carboxylic acid group of another, producing an ester bond. Repetition gives a chain containing $-O-CH(R)-CH_2-C(=O)-$ units, with different alkyl substituents in the butyrate and valerate units. Because the ester bonds lie in the main chain, the copolymer is classified as a polyester.

Step 2: Eliminate the other polymer classes Butadiene-styrene copolymer is Buna-S, an addition copolymer with a carbon-chain backbone and no recurring ester linkage. Neoprene is the addition polymer of chloroprene, so its backbone also lacks ester links. Melamine polymer is a cross-linked melamine-formaldehyde resin; its network forms through reactions of amino groups with formaldehyde, not through hydroxy-acid esterification. Thus options (A), (B) and (C) fail the defining structural test for a polyester.

Step 3: Conclusion Poly-β-hydroxybutyrate-co-β-hydroxyvalerate is a polyester. Hence, the correct option is (D).

Quick Tip to Solve: Classify a polymer by its backbone linkage. The word copolymer says that two monomers are used; PHBV is still a polyester because the joining groups are esters.

80. Which of the following is not a broad spectrum antibiotic? [4 marks]

(A) Vancomycin

(B) Ampicillin

(C) Ofloxacin

(D) Penicillin G

Correct Answer: (D) Penicillin G

Solution:

Concept:

- Antibiotic spectrum describes the range of bacteria against which a drug is effective. In the prescribed examination classification, broad-spectrum antibiotics act against a wider range of Gram-positive and Gram-negative bacteria.
- Penicillin G is classified as a narrow-spectrum antibiotic. The NCERT listing used for this question groups ampicillin, ofloxacin and vancomycin among broad-spectrum examples; the question tests that textbook classification.

Step 1: Apply the textbook antibiotic classification Ampicillin is a broad-spectrum penicillin derivative and therefore does not satisfy the negative wording of the question. Ofloxacin is also listed as a broad-spectrum antibacterial agent. Vancomycin is included in the broad-spectrum list used in the prescribed textbook classification, so it is not the expected selection here. These observations eliminate options (A), (B) and (C) within the classification being tested.

Step 2: Identify the narrow-spectrum example Penicillin G is the standard narrow-spectrum example in this comparison. Its activity is restricted to a more limited range of susceptible bacteria than the broad-spectrum examples listed with it. Hence it is the one that answers the phrase "not a broad spectrum antibiotic. The name penicillin alone should not be used to group all its derivatives together: ampicillin has a broader spectrum than penicillin G. The requested property is spectrum of activity, rather than whether the compound is an antibiotic at all.

Step 3: Conclusion Penicillin G is the narrow-spectrum antibiotic in the prescribed textbook comparison. Hence, the correct option is (D).

Quick Tip to Solve: For this textbook classification, remember penicillin G as the narrow-spectrum example and distinguish it from the broader-spectrum derivative ampicillin.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. $2O_3(g) \rightleftharpoons 3O_2(g)$

At 300 K, ozone is fifty percent dissociated. The standard free energy change at this temperature and 1 atm pressure is (−) _____ J mol^{-1}. (Nearest integer)

[Given : ln 1.35 = 0.3 and R = 8.3 J K^{-1} mol^{-1}] [4 marks]

Solution:

Concept:

- For an ideal gas equilibrium, K_p is formed from equilibrium partial pressures divided by the standard pressure. For $2\,O_3(g) \rightleftharpoons 3\,O_2(g)$, $K_p = (p_{O_2}/p^\circ)^3/(p_{O_3}/p^\circ)^2$.
- The standard reaction Gibbs energy satisfies $\Delta_r G^\circ = -RT \ln K_p$. A fractional dissociation changes both the gas composition and the total number of moles, so mole fractions must use the equilibrium total.

Step 1: Calculate the equilibrium partial pressures Start with one mole of ozone and take its degree of dissociation as $\alpha = 0.50$. The remaining ozone is $1 - \alpha = 0.50$ mol. The stoichiometry produces $3\alpha/2 = 0.75$ mol of oxygen, giving a total of 1.25 mol. At total pressure $1\,\text{atm}$,

$$p_{O_3} = \frac{0.50}{1.25} = 0.40\,\text{atm}, \qquad p_{O_2} = \frac{0.75}{1.25} = 0.60\,\text{atm}.$$

Step 2: Evaluate the equilibrium constant and free energy Using the stated 1 atm pressure reference, the pressure ratios give

$$K_p = \frac{(0.60)^3}{(0.40)^2} = \frac{0.216}{0.16} = 1.35.$$

Now use the supplied logarithm without replacing it by a separately calculated approximation:

$$\Delta_r G^\circ = -(8.3)(300)(0.3) = -747\,\text{J mol}^{-1}.$$

Since $K_p > 1$, the negative sign is consistent with products being favoured at the standard-state comparison.

Step 3: Conclusion The standard free energy change is $-747\,\text{J mol}^{-1}$. Since the blank already carries a minus sign, the nearest-integer entry is $\boxed{747}$.

Quick Tip to Solve: Use equilibrium mole fractions after dissociation, not the initial mole number. For this 2-to-3 gas reaction, half dissociation makes the equilibrium total 1.25 mol per initial mole.

82. The osmotic pressure of blood is 7.47 bar at 300 K. To inject glucose to a patient intravenously, it has to be isotonic with blood. The concentration of glucose solution in gL^{-1} is _____.
(Molar mass of glucose = 180 g mol^{-1}, R = 0.083 L bar K^{-1} mol^{-1}) (Nearest integer) [4 marks]

Solution:

Concept:

- Isotonic solutions at the same temperature have equal osmotic pressures. For dilute glucose solution, $\pi = iCRT$, where C is molarity and $i = 1$ because glucose does not ionise.
- Mass concentration in $\mathrm{g\,L^{-1}}$ equals molarity in $\mathrm{mol\,L^{-1}}$ multiplied by molar mass in $\mathrm{g\,mol^{-1}}$. The gas constant must have pressure units matching the given osmotic pressure.

Step 1: Find the required molar concentration The glucose solution must have osmotic pressure $7.47\,\mathrm{bar}$ at $300\,\mathrm{K}$. With $R = 0.083\,\mathrm{L\,bar\,K^{-1}\,mol^{-1}}$, the pressure units already match, so no conversion to atmospheres or pascals is needed. Therefore

$$C = \frac{\pi}{RT} = \frac{7.47}{0.083 \times 300} = \frac{7.47}{24.9} = 0.300\,\mathrm{mol\,L^{-1}}.$$

Step 2: Convert molarity into grams per litre Each mole of glucose has mass $180\,\mathrm{g}$, so the required mass per litre is

$$c_m = CM = (0.300)(180) = 54.0\,\mathrm{g\,L^{-1}}.$$

As a check, dividing this mass concentration by the molar mass gives $54/180 = 0.300\,\mathrm{mol\,L^{-1}}$; substituting back into CRT returns $7.47\,\mathrm{bar}$. The answer is already an integer, so the requested rounding does not change it.

Step 3: Conclusion The required glucose concentration is $\boxed{54\,\mathrm{g\,L^{-1}}}$; the nearest-integer entry is 54.

Quick Tip to Solve: First find molarity from osmotic pressure, then multiply by molar mass. Glucose is a nonelectrolyte, so do not insert an ionic dissociation factor.

83. The cell potential for the following cell
Pt $|H_2(g)|H^+$ (aq)$||$ Cu^{2+} (0.01 M)$|$Cu(s)
is 0.576 V at 298 K. The pH of the solution is _____. (Nearest integer)
(Given : $E^o_{Cu^{2+}/Cu} = 0.34$ V and $\frac{2.303\,RT}{F} = 0.06$ V) [4 marks]

Solution:

Concept:

- Cell emf is the right-hand electrode reduction potential minus the left-hand electrode reduction potential. Here copper is reduced at the cathode and hydrogen is oxidised at the anode.
- The Nernst equation is $E = E^\circ - (0.06/n)\log Q$ at the stated temperature. The hydrogen electrode uses the conventional unit hydrogen pressure in this cell; $\mathrm{pH} = -\log[\mathrm{H^+}]$.

Step 1: Write the reaction and reaction quotient The electrode processes are $\mathrm{H_2} \longrightarrow 2\,\mathrm{H^+} + 2\,\mathrm{e^-}$

and $Cu^{2+} + 2\,e^- \longrightarrow Cu$. Hence the overall reaction transfers two electrons:

$$H_2(g) + Cu^{2+}(aq) \longrightarrow 2\,H^+(aq) + Cu(s).$$

Taking the solid activity and hydrogen pressure as unity gives $Q = [H^+]^2/[Cu^{2+}]$. Also $E^\circ_{cell} = 0.34 - 0 = 0.34\,V$.

Step 2: Substitute the concentration and solve for pH The Nernst equation becomes

$$0.576 = 0.34 - \frac{0.06}{2}\log\left(\frac{[H^+]^2}{0.01}\right).$$

Since $\log[H^+] = -pH$, the logarithm is $-2pH + 2$. Thus

$$0.576 = 0.34 - 0.03(-2pH + 2) = 0.28 + 0.06pH.$$

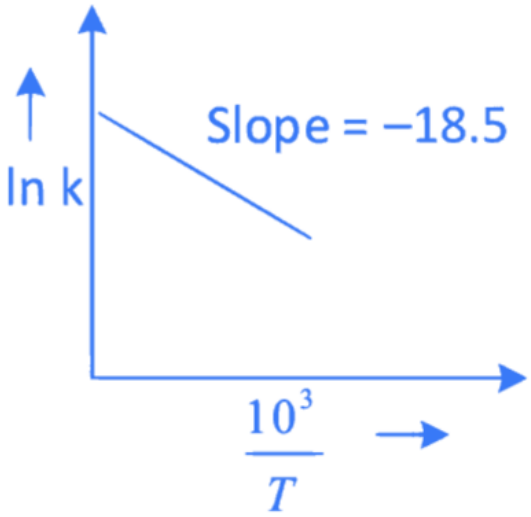
Therefore $pH = 0.296/0.06 = 4.933\ldots$. The copper electrode itself has reduction potential $0.28\,V$, which provides the same result using the hydrogen electrode potential directly.

Step 3: Conclusion The solution has $pH \approx 4.93$, so the required nearest-integer entry is $\boxed{5}$. The pH is dimensionless.

Quick Tip to Solve: Calculate the copper electrode potential at its actual concentration first. At unit hydrogen pressure, subtracting the hydrogen electrode adds 0.06 times pH to that copper potential.

84. The rate constants for decomposition of acetaldehyde have been measured over the temperature range 700 - 1000 K. The data has been analysed by plotting ln k vs $\frac{10^3}{T}$ graph. The value of activation energy for the reaction is _____ kJ mol^{-1}. (Nearest integer)

(Given : R = 8.31 J K^{-1} mol^{-1}) [4 marks]



Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where E_a is the activation energy and A is the pre-exponential factor. Taking natural logarithms gives $\ln k = \ln A - E_a/(RT)$.
- The horizontal variable in the given graph is $x = 10^3/T$, rather than $1/T$. The slope must therefore be related to E_a after accounting for this scale factor. Natural logarithms require no extra factor of 2.303.

Step 1: Express the Arrhenius equation in the plotted variable From $x = 10^3/T$, it follows that $1/T = x/10^3$. Substitute this into the logarithmic Arrhenius equation:

$$\ln k = \ln A - \frac{E_a}{10^3 R}\, x.$$

Comparing with the straight-line form $y = c + mx$, the plotted slope is $m = -E_a/(10^3 R)$. The graph explicitly labels this slope as -18.5. Its negative sign is consistent with a positive activation energy.

Step 2: Calculate and convert the activation energy Equating the two slope expressions gives

$$-18.5 = -\frac{E_a}{10^3(8.31)}.$$

Therefore

$$E_a = (18.5)(10^3)(8.31) = 153735\,\text{J mol}^{-1} = 153.735\,\text{kJ mol}^{-1}.$$

Dividing by 1000 changes joules to kilojoules. Finally, rounding 153.735 to the nearest integer gives 154. Omitting the horizontal scale factor would make the energy a thousand times too small before the unit conversion.

Step 3: Conclusion The activation energy is $\boxed{154\,\text{kJ mol}^{-1}}$ to the nearest integer, so the numerical entry is 154.

Quick Tip to Solve: Read both axis labels before using an Arrhenius slope. For ln k against 1000/T, multiply the slope magnitude by 1000R to obtain activation energy in joules per mole.

85. The difference in oxidation state of chromium in chromate and dichromate salts is _____.

[4 marks]

Solution:

Concept:

- The algebraic sum of oxidation numbers equals the charge on an ion. Oxygen has oxidation number -2 in both of the oxoanions considered here.
- Chromate is $CrO_4{}^{2-}$, whereas dichromate is $Cr_2O_7{}^{2-}$. An oxidation number refers to each chromium atom, not the combined contribution of all chromium atoms in an ion.

Step 1: Calculate chromium in chromate Let the oxidation number of chromium in $CrO_4{}^{2-}$ be x.

Four oxygen atoms contribute -8, and the ion has total charge -2. Thus

$$x + 4(-2) = -2 \quad \Rightarrow \quad x = +6.$$

The negative charge is the charge of the entire ion; it is not the oxidation number of its chromium atom.

Step 2: Calculate chromium in dichromate Both chromium atoms in $Cr_2O_7^{2-}$ have the same oxidation number, say y. Include the factor of two explicitly:

$$2y + 7(-2) = -2 \quad \Rightarrow \quad 2y = 12 \quad \Rightarrow \quad y = +6.$$

The value $+12$ is the combined contribution of two chromium atoms, so it must be divided by two before comparing with chromate.

Step 3: Compare the individual oxidation states The chromium oxidation state is $+6$ in each ion. Therefore the difference is

$$|(+6) - (+6)| = 0.$$

Interconversion of chromate and dichromate upon changing acidity is consequently not a redox change of chromium; changing the number of atoms in the ion does not itself imply a change in oxidation state.

Step 4: Conclusion Chromium is in oxidation state $+6$ in both salts. The required numerical difference is $\boxed{0}$, a dimensionless quantity.

Quick Tip to Solve: For a polyatomic ion, first equate the oxidation-number sum to its charge; divide the metal contribution by the number of metal atoms before comparing oxidation states.

86. In the cobalt-carbonyl complex : $[Co_2(CO)_8]$, number of Co-Co bonds is "X" and terminal CO ligands is "Y". X + Y = _____. [4 marks]

Solution:

Concept:

- Dicobalt octacarbonyl, $Co_2(CO)_8$, has the familiar bridged structure with one cobalt–cobalt bond, two bridging carbonyl ligands and six terminal carbonyl ligands.
- A terminal CO is attached to one metal atom; a bridging CO connects the two metals. Each carbonyl molecule counts as one ligand, even when it has two metal contacts.

Step 1: Count the metal–metal bond The two cobalt centres are connected directly by a single $Co-Co$ bond. This is distinct from their indirect connection through the bridging carbonyls, so

$$X = 1.$$

One must count the direct bond between the metal atoms, not the number of bridging paths or the total number of metal–carbon bonds.

Step 2: Separate terminal from bridging carbonyls Of the eight carbonyl ligands in the standard

bridged form, two are bridging ligands. The remaining carbonyls are terminal, distributed as three on each cobalt centre:

$$Y = 8 - 2 = 6 = 3 + 3.$$

Counting all eight as terminal would disregard the two bridging carbonyl groups and would describe a different structural form rather than the familiar bridged structure tested here.

Step 3: Add the two requested numbers The question asks for the sum of the number of direct cobalt–cobalt bonds and the number of terminal ligands. Therefore

$$X + Y = 1 + 6 = 7.$$

As a structural check, the two metals and eight carbonyl groups account for the complete formula. The two bridging carbonyl ligands have not disappeared from the complex; they are excluded only from the terminal-ligand count. This distinction also prevents counting a bridging ligand twice just because it attaches to both metals.

Step 4: Conclusion For the standard bridged structure of $Co_2(CO)_8$, $X = 1$, $Y = 6$, and hence $\boxed{X + Y = 7}$. This is a dimensionless count.

Quick Tip to Solve: For a bridged metal carbonyl, list metal–metal bonds, bridging CO groups and terminal CO groups separately before adding the quantities asked for.

87. A 0.166 g sample of an organic compound was digested with conc. H_2SO_4 and then distilled with NaOH. The ammonia gas evolved was passed through 50.0 mL of 0.5 N H_2SO_4. The used acid required 30.0 mL of 0.25 N NaOH for complete neutralization. The mass percentage of nitrogen in the organic compound is _____. [4 marks]

Solution:

Concept:

- In Kjeldahl analysis, each mole of nitrogen produces one mole of NH_3. Each mole of ammonia consumes one acid equivalent: $NH_3 + H^+ \longrightarrow NH_4{}^+$.
- Normality already measures equivalents per litre. Initial acid equivalents minus residual acid equivalents give the equivalents consumed by ammonia; no additional factor of two is applied for H_2SO_4.

Step 1: Calculate initial and residual acid equivalents The initial acid has volume $50.0\,\text{mL}$ and normality $0.5\,\text{eq}\,\text{L}^{-1}$. Its equivalents are

$$E_i = 0.0500(0.5) = 0.0250\,\text{eq}.$$

The acid left after ammonia absorption is neutralized by $30.0\,\text{mL}$ of $0.25\,\text{N}$ sodium hydroxide:

$$E_r = 0.0300(0.25) = 0.00750\,\text{eq}.$$

In a back titration, this latter value represents residual acid, not acid consumed by ammonia.

Step 2: Determine nitrogen from the acid consumed The acid consumed is

$$E_{NH_3} = E_i - E_r = 0.0250 - 0.00750 = 0.01750\,\text{eq}.$$

Thus $n(NH_3) = n(N) = 0.01750\,\text{mol}$, and

$$m(N) = 0.01750 \times 14 = 0.2450\,\text{g}.$$

This already exceeds the printed total sample mass, $0.166\,\text{g}$, which exposes a numerical inconsistency.

Step 3: Evaluate the printed data and check the reported value Literal substitution gives

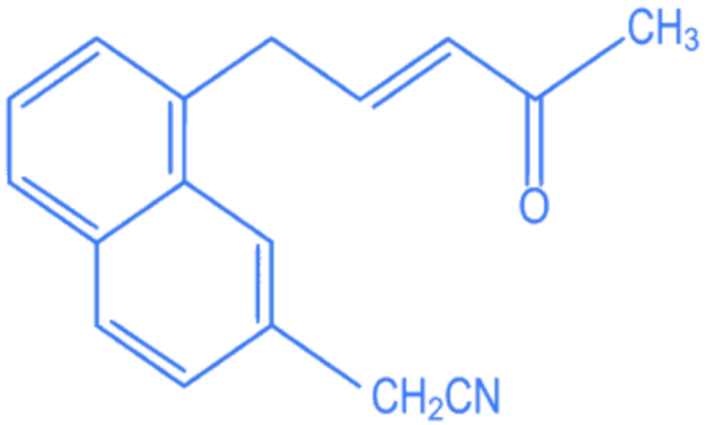
$$\%N = \frac{0.2450}{0.166} \times 100 = 147.590\ldots\%.$$

A mass percentage above 100% is physically impossible. The reported answer 63 instead comes from $0.00750 \times 14 \times 100/0.166 = 63.253\ldots$, which incorrectly identifies residual-acid equivalents with ammonia equivalents under the usual back-titration interpretation. The printed values therefore do not yield a physically valid nitrogen percentage.

Step 4: Conclusion The printed back-titration data formally give $\boxed{147.59\%}$ nitrogen, an impossible mass percentage. Thus no physically consistent percentage follows from these data as written; the reported value 63% is incompatible with the acid-equivalent balance.

Quick Tip to Solve: Always subtract the residual acid found by back titration from the initial acid. Check that the calculated nitrogen mass does not exceed the mass of the whole sample.

88. Number of electrophilic centres in the given compound is _____. [4 marks]



Solution:

Concept:

- An electrophilic centre is electron deficient and can accept an electron pair from a nucleophile. Polarization and resonance determine such centres; simply counting all unsaturated carbon atoms is incorrect.
- The printed compound contains an α,β-unsaturated ketone and a nitrile group. Conjugation

extends the electron-deficient character of the carbonyl system to its β-carbon.

Step 1: Identify the carbonyl carbon In the ketone unit $C{=}O$, oxygen withdraws electron density from carbon. The resonance contribution $C^{+}{-}O^{-}$ displays the carbonyl carbon as an electron-pair acceptor. Hence the carbonyl carbon of the side-chain ketone is the first electrophilic centre. Oxygen, by contrast, is the electron-rich end of this polarized bond.

Step 2: Identify the conjugate-addition site The side chain contains $-CH{=}CH{-}C(=O){-}CH_3$. Electron withdrawal through this conjugated system gives a resonance contributor with positive charge at the carbon two positions away from the carbonyl carbon, namely the β-carbon. Nucleophiles may therefore add at the carbonyl carbon by direct addition or at this β-carbon by conjugate addition. The β-carbon is the second electrophilic centre; the other alkene carbon is not an additional independent electron-deficient site.

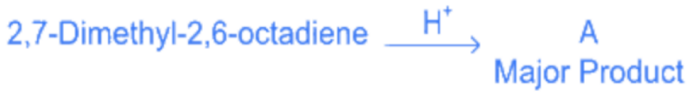
Step 3: Identify the nitrile carbon and total the sites The substituent $-CH_2{-}C{\equiv}N$ has its triple bond polarized towards nitrogen. The nitrile carbon consequently bears partial positive charge and accepts nucleophilic attack; it is the third centre. The intervening methylene carbon and ordinary aromatic carbons are not counted as electrophilic centres in this neutral structure. Thus the count is

$$1 \text{ (carbonyl)} + 1 \text{ (enone } \beta\text{-carbon)} + 1 \text{ (nitrile)} = 3.$$

Step 4: Conclusion The electrophilic centres are the ketone carbonyl carbon, the β-carbon of the conjugated enone, and the nitrile carbon. Their number is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: For an enone, count both the carbonyl carbon and its conjugate-addition beta carbon. Then inspect other polarized multiple bonds, such as the carbon of a nitrile, separately.

89. The major product 'A' of the following given reaction has _____ sp^2 hybridized carbon atoms. [4 marks]



Solution:

Concept:

- Protonation of an alkene generates a carbocation. In a suitably connected diene, the other double bond can attack intramolecularly, producing a cyclic carbocation which loses a proton to form an alkene.
- Each carbon of an ordinary carbon–carbon double bond is sp^2 hybridized. A saturated carbon with four sigma bonds is sp^3, including a carbocation centre after it has formed its final fourth bond.

Step 1: Recognize the two reacting double bonds The substrate is $2,7$-dimethylocta-$2,6$-diene, which can be represented as

$$(CH_3)_2C{=}CH{-}CH_2{-}CH_2{-}CH{=}C(CH_3)_2.$$

It initially contains two carbon–carbon double bonds and hence four sp^2 carbon atoms. The acid shown over the reaction arrow initiates electrophilic addition at an alkene; the question asks about the final major product rather than this starting count.

Step 2: Follow cyclization and proton loss Protonation creates a stabilized carbocation at one end of the diene. The remaining alkene supplies its pi electrons for intramolecular carbon–carbon bond formation, closing a ring and leaving a new carbocation. Loss of a proton from this cation restores a double bond and regenerates the acid catalyst. Overall, one of the two initial units of unsaturation becomes a ring and the other remains a carbon–carbon double bond in the major cyclic product.

Step 3: Count only the alkene carbons in the product The neutral product contains one C=C bond. Both of its carbon atoms are trigonal planar and sp^2 hybridized, whereas the saturated ring and substituent carbons are sp^3. Therefore

$$n(sp^2 \text{ carbon atoms}) = 1 \times 2 = 2.$$

A ring does not itself require sp^2 hybridization: a carbon belongs to this count only when its bonding in the final product contains the alkene pi bond. Retaining the initial count of four would ignore the cyclization reaction.

Step 4: Conclusion The major cyclic product retains one carbon–carbon double bond, so it has $\boxed{2}$ sp^2-hybridized carbon atoms.

Quick Tip to Solve: After an acid-catalysed diene cyclization, count double bonds in the final neutral product. Count two sp2 carbon atoms per C=C; do not count a saturated ring as an extra double bond.

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90. Atoms of element X form hcp lattice and those of element Y occupy $\frac{2}{3}$ of its tetrahedral voids. The percentage of element X in the lattice is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- If a close-packed lattice contains N host atoms, it has $2N$ tetrahedral voids and N octahedral voids. These counts hold for hcp as well as cubic close packing.
- The requested percentage of atoms is the number of atoms of element X divided by the total number of atoms of both elements, multiplied by 100. No atomic masses are supplied, so this is an atomic percentage.

Step 1: Calculate the number of guest atoms Let the hcp host lattice contain N atoms of X. It then has $2N$ tetrahedral voids. Since Y occupies two-thirds of these voids,

$$n_Y = \frac{2}{3}(2N) = \frac{4N}{3}.$$

The occupancy fraction is applied to the number of tetrahedral voids, not directly to the number of host atoms. Each occupied void contributes one atom of Y.

Step 2: Find the atomic ratio and total atom count The host and guest populations are

$$n_X : n_Y = N : \frac{4N}{3} = 3 : 4.$$

Thus the simplest composition is X_3Y_4, and the total population corresponding to N host atoms is

$$n_{\text{total}} = N + \frac{4N}{3} = \frac{7N}{3}.$$

Empty tetrahedral voids are spaces and do not add atoms to this denominator.

Step 3: Calculate and round the fraction of X The atomic fraction of the host is

$$x_X = \frac{N}{7N/3} = \frac{3}{7}.$$

Converting to a percentage gives

$$\%X = \frac{3}{7} \times 100 = 42.857\ldots\%.$$

The problem explicitly asks for the nearest integer, so this rounds upward to 43. As a check, the guest fraction is $4/7$; together the two atomic fractions sum to one.

Step 4: Conclusion The percentage of X atoms is $42.857\ldots\%$, giving the requested nearest-integer answer $\boxed{43}$ (i.e. 43%).

Quick Tip to Solve: For N close-packed atoms, start from 2N tetrahedral voids. Apply the occupied fraction to 2N, then divide the host count by the combined host and guest atom count.