JEE Main 2022 Paper 1 (B.E./B.Tech) Jun 24 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 1 (June), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 24 June 2022, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $x * y = x^2 + y^3$ and $(x * 1) * 1 = x * (1 * 1)$.

Then a value of $2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)$ is : [4 marks]

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{6}$

Correct Answer: (B) $\frac{\pi}{3}$

Solution:

Concept:

- The operation is defined by $u * v = u^2 + v^3$. It must be applied to each pair of operands in its written order; ordinary multiplication is not intended.
- The principal value of $\sin^{-1} t$ lies in $[-\pi/2, \pi/2]$. A polynomial relation may determine the complete expression without requiring the individual values of x.

Step 1: Expand the nested operations First use the definition on the inner pair, then square that result when it becomes the first operand of the outer operation. Thus

$$x * 1 = x^2 + 1, \qquad (x * 1) * 1 = (x^2 + 1)^2 + 1.$$

On the other side, the inner operation is $1 * 1 = 2$; since this is the second operand, its cube is required. Therefore

$$x * (1 * 1) = x * 2 = x^2 + 8.$$

The imposed equality becomes

$$x^4 + 2x^2 + 2 = x^2 + 8.$$

Step 2: Find the combination appearing in the argument Rearrange the equality rather than solving separately for x:

$$x^4 + x^2 = 6.$$

This value is valid for every real solution of the original condition. Indeed, writing $u = x^2$ gives $u^2 + u - 6 = (u - 2)(u + 3) = 0$, and the admissible value is $u = 2$. In particular the denominator in the inverse-sine argument is nonzero.

Step 3: Evaluate the principal inverse sine Substitute the known combination directly:

$$\frac{x^4 + x^2 - 2}{x^4 + x^2 + 2} = \frac{6 - 2}{6 + 2} = \frac{1}{2}.$$

Because $\pi/6$ belongs to the principal inverse-sine interval and has sine $1/2$,

$$2\sin^{-1}\left(\frac{1}{2}\right) = 2 \cdot \frac{\pi}{6} = \frac{\pi}{3}.$$

The other listed angles would require different inverse-sine arguments, so they do not satisfy this evaluation.

Step 4: Conclusion The required value is $\boxed{\pi/3}$. Hence, the correct option is (B).

Quick Tip to Solve: Expand a defined operation from the innermost brackets first, then look for the same polynomial combination in the requested expression.

2. The sum of all the real roots of the equation

$(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$ **is** [4 marks]

(A) $\log_e 3$

(B) $-\log_e 3$

(C) $\log_e 6$

(D) $-\log_e 6$

Correct Answer: (B) $-\log_e 3$

Solution:

Concept:

- The substitution $t = e^x$ turns the exponential equation into a polynomial equation, with the essential restriction $t > 0$.
- Each positive value of t gives exactly one real value $x = \ln t$. The sum of logarithms equals the logarithm of the product of their positive arguments.

Step 1: Factor in the positive variable Put $t = e^x > 0$, so that $e^{2x} = t^2$. The given equation becomes

$$(t^2 - 4)(6t^2 - 5t + 1) = 0.$$

Factor both polynomials:

$$(t - 2)(t + 2)(3t - 1)(2t - 1) = 0.$$

This is a product equal to zero, so at least one of its four factors must vanish. The candidates for t are therefore $2, -2, 1/3, 1/2$.

Step 2: Retain exactly the real exponential solutions The value $t = -2$ is impossible because an exponential with real exponent is strictly positive. Each of the other three values is positive and hence gives one real root, with no duplication:

$$x_1 = \ln 2, \qquad x_2 = -\ln 3, \qquad x_3 = -\ln 2.$$

Substituting these values makes, respectively, the first factor or one of the factors of the quadratic vanish. Thus none has been introduced by the substitution and no real root has been lost.

Step 3: Add all three roots Using the logarithm product law gives

$$x_1 + x_2 + x_3 = \ln\left(2 \cdot \frac{1}{3} \cdot \frac{1}{2}\right) = \ln\left(\frac{1}{3}\right) = -\ln 3.$$

Equivalently the $\ln 2$ and $-\ln 2$ terms cancel. The negative sign rules out the two positive logarithms among the options. The remaining negative alternative $-\ln 6$ would result from omitting the positive root $\ln 2$, and therefore is not the sum of all real roots.

Step 4: Conclusion The sum is $\boxed{-\log_e 3}$. Hence, the correct option is (B).

Quick Tip to Solve: After setting $t = e^x$, discard nonpositive polynomial roots before taking logarithms; then add the logs by multiplying their arguments.

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3. Let the system of linear equations

$x + y + \alpha z = 2$

$3x + y + z = 4$

$x + 2z = 1$

have a unique solution (x^*, y^*, z^*). If (α, x^*), (y^*, α) and $(x^*, -y^*)$ are collinear points, then the sum of absolute values of all possible values of α is [4 marks]

(A) 4

(B) 3

(C) 2

(D) 1

Correct Answer: (C) 2

Solution:

Concept:

- A square linear system has a unique solution only when the determinant of its coefficient matrix is nonzero.
- Three points are collinear when the determinant formed by their two coordinates and a column of ones vanishes. This criterion also handles coincident points without dividing by coordinate differences.

Step 1: Use the unique-solution condition The coefficient determinant is

$$\begin{vmatrix} 1 & 1 & \alpha \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -\alpha - 3.$$

Thus uniqueness requires $\alpha \neq -3$. Subtract the first equation from the second and use $x + 2z = 1$:

$$2x + (1 - \alpha)z = 2, \qquad x = 1 - 2z.$$

Substituting the latter in the former yields

$$-(\alpha + 3)z = 0.$$

Since the multiplier is nonzero, $z^* = 0$, and the remaining equations give $x^* = 1$ and $y^* = 1$.

Step 2: Apply the collinearity criterion The three points therefore reduce to $(\alpha, 1), (1, \alpha), (1, -1)$. Their signed double-area must be zero:

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0.$$

Expanding gives

$$\alpha(\alpha + 1) - (1 - 1) + (-1 - \alpha) = \alpha^2 - 1 = 0.$$

Hence the possible values are $\alpha = 1$ and $\alpha = -1$. The determinant approach is preferable to an ordinary slope equation because the latter could divide by zero when two x-coordinates agree.

Step 3: Check admissibility and add magnitudes Both values satisfy $\alpha \neq -3$, so both retain the required unique solution. At $\alpha = 1$, the first two listed points coincide and all three lie on $x = 1$; at $\alpha = -1$, the last two coincide and lie on the line with the first. They satisfy the stated collinearity determinant. Therefore

$$|1| + |-1| = 2.$$

The other choices do not equal the sum of the magnitudes of these two admissible values.

Step 4: Conclusion The sum of absolute values is $\boxed{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the linear equations to simplify the coordinates before imposing collinearity, and keep the determinant restriction on the parameter throughout.

4. Let x, y > 0. If $x^3y^2 = 2^{15}$, then the least value of 3x + 2y is [4 marks]

(A) 30

(B) 32

(C) 36

(D) 40

Correct Answer: (D) 40

Solution:

Concept:

- For positive numbers, the arithmetic mean is at least their geometric mean. Equality occurs exactly when all numbers in the list are equal.
- The coefficients of $3x + 2y$ suggest a list of five positive terms: three copies of x and two copies of y. Their product is the given constraint.

Step 1: Apply AM-GM to five terms Write the target as $x + x + x + y + y$. All five terms are positive, so the arithmetic-geometric mean inequality gives

$$\frac{x + x + x + y + y}{5} \geq (x^3y^2)^{1/5}.$$

The constraint fixes the product, hence

$$\frac{3x+2y}{5} \geq (2^{15})^{1/5} = 2^3 = 8.$$

Multiplying by five establishes the universal lower bound

$$3x + 2y \geq 40.$$

This bound applies to every positive pair satisfying the constraint; it does not depend on an approximation to the variables.

Step 2: Verify that the lower bound is attainable An inequality gives a candidate minimum only if equality can actually occur under the original constraint. Here equality in the five-term AM-GM relation requires $x = x = x = y = y$, which is simply $x = y$. With this condition, the constraint becomes

$$x^5 = 2^{15}.$$

Since $x > 0$, its solution is $x = 8$, and consequently $y = 8$. These numbers satisfy

$$8^3 \cdot 8^2 = 8^5 = 2^{15}.$$

The corresponding objective value is

$$3(8) + 2(8) = 24 + 16 = 40.$$

Step 3: Compare with the listed values The lower bound is attained, so it is the actual least value rather than merely an infimum. The values 30, 32 and 36 all lie below the proven bound and are therefore impossible for any admissible positive pair. In particular, applying AM-GM directly to only $3x$ and $2y$ would not use the fixed product x^3y^2 correctly; the five-term choice is what makes the constraint useful.

Step 4: Conclusion The least value is $\boxed{40}$, attained at $x = y = 8$. Hence, the correct option is (D).

Quick Tip to Solve: When an objective has integer weights matching exponents in a product constraint, repeat each variable that many times before applying AM-GM.

5. **Let** $f(x) = \begin{cases} \frac{\sin(x-[x])}{x-[x]} & , x \in (-2,-1) \\ \max\{2x, 3[|x|]\} & , |x| < 1 \\ 1 & , otherwise \end{cases}$

where [t] denotes greatest integer $\leq$ t. If m is the number of points where f is not continuous and n is the number of points where f is not differentiable, then the ordered pair (m, n) is : [4 marks]

(A) (3, 3)

(B) (2, 4)

(C) (2, 3)

(D) (3, 4)

Correct Answer: (C) (2, 3)

Solution:

Concept:

- Check continuity at each boundary of the defining intervals, then check differentiability at every continuous junction and at any internal corner.
- For $|x| < 1$, $[|x|] = 0$. Thus the middle branch simplifies to $\max\{2x, 0\}$, equal to zero for negative x and $2x$ for positive x.

Step 1: Simplify on the open intervals For $-2 < x < -1$, the greatest integer is $[x] = -2$, giving

$$f(x) = \frac{\sin(x+2)}{x+2}.$$

For $-1 < x < 0$, the function is zero; for $0 < x < 1$, it is $2x$. Outside the special intervals the value is one. Each of these expressions is differentiable in its open interval. Therefore only $-2, -1, 0, 1$ need junction checks; there are no other possible failures.

Step 2: Count the discontinuities At -2, the right limit of $\sin(x+2)/(x+2)$ is one, matching both the left limit and $f(-2) = 1$. At -1, the left limit is $\sin 1$, the right limit is zero and the assigned value is one, so continuity fails. At zero, both one-sided limits and the function value are zero. At one, the left limit is two whereas the right limit and $f(1)$ are one. Thus the discontinuities are precisely -1 and 1, giving $m = 2$.

Step 3: Check derivatives at the continuous junctions At zero, the left derivative is zero and the right derivative is two, so there is a corner. At -2, the left derivative is zero because the function is locally constant on the left. The right difference quotient is

$$\lim_{h \to 0^+} \frac{\sin h/h - 1}{h} = \lim_{h \to 0^+} \frac{\sin h - h}{h^2} = 0,$$

using $\sin h = h - h^3/6 + O(h^5)$. Thus the derivatives agree at -2. The two discontinuities are automatically nondifferentiability points, and zero contributes one more. Hence $n = 3$; this excludes both choices with four failures and the choice with three discontinuities.

Step 4: Conclusion The ordered pair is $\boxed{(m,n) = (2,3)}$. Hence, the correct option is (C).

Quick Tip to Solve: First remove the greatest-integer notation on each open interval. Count discontinuities before testing the extra corners where the function stays continuous.

6. The value of the integral

$\int_{-\pi/2}^{\pi/2} \frac{dx}{(1+e^x)(\sin^6 x+\cos^6 x)}$ **is equal to** [4 marks]

(A) 2π

(B) 0

(C) π

(D) $\frac{\pi}{2}$

Correct Answer: (C) π

Solution:

Concept:

- On a symmetric interval, replace x by $-x$ and add the two integrals. Here $\frac{1}{1+e^x} + \frac{1}{1+e^{-x}} = 1$.
- The identity $\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^2 x = 1 - \frac{3}{4}\sin^2 2x$ reduces the remaining trigonometric denominator.

Step 1: Eliminate the exponential using symmetry Let the required integral be I. The trigonometric denominator $g(x) = \sin^6 x + \cos^6 x$ is even. Replacing x by $-x$ and adding gives

$$2I = \int_{-\pi/2}^{\pi/2} \frac{dx}{g(x)}.$$

Since the remaining integrand is even,

$$I = \int_0^{\pi/2} \frac{dx}{1 - \frac{3}{4}\sin^2 2x}.$$

The exponential factor has disappeared exactly; simply calling the original integrand odd would incorrectly suggest zero.

Step 2: Reduce to a standard tangent integral Put $u = 2x$, so $dx = du/2$. Reflection about $u = \pi/2$ then gives

$$I = \frac{1}{2}\int_0^{\pi} \frac{du}{1 - \frac{3}{4}\sin^2 u} = \int_0^{\pi/2} \frac{du}{1 - \frac{3}{4}\sin^2 u}.$$

Now take $t = \tan u$. Since $\sin^2 u = t^2/(1+t^2)$ and $du = dt/(1+t^2)$,

$$I = \int_0^{\infty} \frac{dt}{1 + t^2/4}.$$

The transformed bounds are zero and infinity, and the denominator is strictly positive throughout.

Step 3: Evaluate and check the options An antiderivative is $2\tan^{-1}(t/2)$. Consequently

$$I = \left[2\tan^{-1}\left(\frac{t}{2}\right)\right]_0^{\infty} = 2 \cdot \frac{\pi}{2} = \pi.$$

The integral is positive, excluding zero. The factor one-half from pairing the original integrals is already included; omitting it produces the incorrect 2π, while applying it twice produces $\pi/2$. Thus

only the listed value π agrees with the full calculation.

Step 4: Conclusion The value of the integral is $\boxed{\pi}$. Hence, the correct option is (C).

Quick Tip to Solve: Pair x and $-x$ before making a trigonometric substitution when an even factor multiplies $1 + e^x$ in a symmetric definite integral.

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7. A particle is moving in the xy-plane along a curve C passing through the point (3, 3). The tangent to the curve C at the point P meets the x-axis at Q. If the y-axis bisects the segment PQ, then C is a parabola with : [4 marks]

(A) length of latus rectum 3

(B) length of latus rectum 6

(C) focus $\left(\frac{4}{3}, 0\right)$

(D) focus $\left(0, \frac{3}{4}\right)$

Correct Answer: (A) length of latus rectum 3

Solution:

Concept:

- The tangent slope at a point $P = (x, y)$ is dy/dx. A midpoint condition on the tangent intercept translates directly into a differential equation.
- The standard parabola $y^2 = 4ax$ has focus $(a, 0)$ and length of latus rectum $4a$ when $a > 0$.

Step 1: Translate the bisection condition into a slope Let the tangent meet the x-axis at $Q = (q, 0)$. Its midpoint with $P = (x, y)$ is $((x + q)/2, y/2)$. Since this midpoint lies on the y-axis, its x-coordinate is zero, which gives $q = -x$. Thus the slope of the tangent through P and Q is

$$\frac{dy}{dx} = \frac{y - 0}{x - (-x)} = \frac{y}{2x}.$$

This relation holds on the portion through $(3, 3)$, where both coordinates are nonzero.

Step 2: Solve and use the given point Separate variables to obtain

$$\frac{dy}{y} = \frac{dx}{2x}.$$

Integration gives

$$\ln |y| = \frac{1}{2} \ln |x| + C.$$

On the branch containing $(3, 3)$, this has the squared form $y^2 = kx$. Substitution of the given point fixes the constant:

$$3^2 = k(3) \quad \Longrightarrow \quad k = 3.$$

Hence the containing parabola is

$$y^2 = 3x.$$

The positive branch passes through the specified point; the equation identifies the complete parabola.

Step 3: Read the parabola parameters and compare Comparison with $y^2 = 4ax$ gives $4a = 3$, hence $a = 3/4$. Its axis is the x-axis, its focus is $(3/4, 0)$, and the length of its latus rectum is three. Thus a latus rectum of six is incorrect. The point $(4/3, 0)$ uses the reciprocal of the correct focal distance, and $(0, 3/4)$ places the focus on the wrong axis. The tangent check $yy_1 = \frac{3}{2}(x + x_1)$ also gives the x-intercept $-x_1$, as required.

Step 4: Conclusion The curve is $y^2 = 3x$, with latus rectum of length $\boxed{3}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the tangent intercept as a coordinate and use the midpoint formula first; it often supplies the differential equation with very little algebra.

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8. Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$, $a > 2$, having one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the y-axis, be $6\sqrt{3}$. Then the eccentricity of the ellipse is : [4 marks]

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\sqrt{3}}{4}$

Correct Answer: (A) $\frac{\sqrt{3}}{2}$

Solution:

Concept:

- For an ellipse with semiaxes $a > b$, the eccentricity is $e = \sqrt{1 - b^2/a^2}$. Here the minor semiaxis is $b = 2$.
- The triangle area is half the product of a base and its perpendicular altitude. Maximise this expression over the possible vertical chords.

Step 1: Express the area using the vertical chord By symmetry choose the fixed vertex as $(a, 0)$. The vertical side has endpoints $(x, \pm 2\sqrt{1 - x^2/a^2})$, where $-a \le x \le a$. Its length is $4\sqrt{1 - x^2/a^2}$, and its perpendicular distance from the fixed vertex is $a - x$. Thus, writing $u = x/a$,

$$A = 2(a - x)\sqrt{1 - x^2/a^2} = 2a(1 - u)\sqrt{1 - u^2}.$$

The nondegenerate triangles correspond to interior values of this interval.

Step 2: Find the interior maximum Since area is nonnegative, maximise its square instead. Apart from the positive constant $4a^2$, this is $(1 - u)^3(1 + u)$. Its derivative is

$$\frac{d}{du}\left[(1 - u)^3(1 + u)\right] = -2(1 - u)^2(1 + 2u).$$

The only interior critical value is $u = -1/2$. The derivative is positive before this point and negative

afterwards; both endpoint areas are zero. Therefore it gives the global maximum,

$$A_{\max} = 2a\left(\frac{3}{2}\right)\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}a}{2}.$$

Step 3: Determine the semiaxis and eccentricity Equating the specified maximum with this expression gives $3\sqrt{3}a/2 = 6\sqrt{3}$, hence $a = 4$, which satisfies $a > 2$. Consequently

$$e = \sqrt{1 - \frac{4}{16}} = \frac{\sqrt{3}}{2}.$$

The alternatives $1/2$, $1/\sqrt{2}$ and $\sqrt{3}/4$ would correspond to different major semiaxes and do not satisfy the specified maximum area.

Step 4: Conclusion The eccentricity is $\boxed{\sqrt{3}/2}$. Hence, the correct option is (A).

Quick Tip to Solve: For a triangle based on a chord perpendicular to the major axis, use the dimensionless coordinate $u = x/a$; its maximum occurs at $u = -1/2$.

9. Let the area of the triangle with vertices A(1, α), B(α, 0) and C(0, α) be 4 sq. units. If the points (α, $-\alpha$), ($-\alpha$, α) and (α^2, β) are collinear, then β is equal to : [4 marks]

(A) 64

(B) -8

(C) -64

(D) 512

Correct Answer: (C) -64

Solution:

Concept:

- The area of a triangle is half the absolute value of the determinant of two side vectors. Absolute values are essential because either orientation is possible.
- Two distinct points determine a line; every third collinear point satisfies its equation.

Step 1: Use a horizontal side to obtain the area The vertices $A = (1, \alpha)$ and $C = (0, \alpha)$ lie on the horizontal line $y = \alpha$, and their separation is exactly one unit. The perpendicular distance of $B = (\alpha, 0)$ from this line is $|\alpha|$. Therefore

$$\Delta = \frac{1}{2}(1)|\alpha| = 4, \qquad |\alpha| = 8.$$

In particular $\alpha \neq 0$, and $\alpha^2 = 64$. There is no need to choose one sign for α, because the final result depends only on its square.

Step 2: Find the line through the first two collinear points Both $(\alpha, -\alpha)$ and $(-\alpha, \alpha)$ satisfy $x + y = 0$. They are distinct since $\alpha \neq 0$, so this is their unique line. Substitution of the third point (α^2, β)

therefore gives

$$\alpha^2 + \beta = 0, \qquad \beta = -\alpha^2 = -64.$$

This also directly checks collinearity: the coordinate sum of each of the three points is zero.

Step 3: Compare the four values The positive value 64 has the wrong sign, while -8 incorrectly uses $-|\alpha|$ instead of $-\alpha^2$. The value 512 would involve a cube that does not occur in the equation of the line. Only -64 obeys both the area condition and the collinearity condition.

Step 4: Conclusion $\boxed{\beta = -64}$. Hence, the correct option is (C).

Quick Tip to Solve: Check for a horizontal or vertical side before expanding an area determinant; here the unit horizontal side immediately gives $|\alpha| = 8$.

10. The number of distinct real roots of the equation
$x^7 - 7x - 2 = 0$ is [4 marks]

(A) 5

(B) 7

(C) 1

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- A continuous function changing sign on an interval has at least one zero there. Strict monotonicity ensures that it has at most one zero on that interval.
- Critical points of $f(x) = x^7 - 7x - 2$ occur where $f\prime(x) = 0$; use them to partition the whole real line.

Step 1: Locate every critical point Differentiate the polynomial:

$$f\prime(x) = 7x^6 - 7 = 7(x^6 - 1).$$

For real x, it vanishes only at $x = -1$ and $x = 1$. It is positive when $|x| > 1$ and negative when $|x| < 1$. Hence f increases strictly on $(-\infty, -1)$, decreases strictly on $(-1, 1)$, and increases strictly on $(1, \infty)$. There are no further turning points on the real axis.

Step 2: Check signs on each monotonic interval The extremum values are

$$f(-1) = -1 + 7 - 2 = 4, \qquad f(1) = 1 - 7 - 2 = -8.$$

Also $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$. Thus the left increasing interval goes from a negative value to a positive one, the middle decreasing interval goes from positive to negative, and the right increasing interval goes from negative to positive. Continuity supplies one root in each interval, while strict monotonicity makes each root unique.

Step 3: Count and exclude the other choices Neither critical point itself is a root, because its function value is nonzero. The three open intervals exhaust every other real number, so there are exactly three distinct real roots. The degree seven permits complex roots and does not force seven real roots. Five and seven would require additional crossings on the strictly monotonic intervals, while one ignores the other two sign changes.

Step 4: Conclusion There are $\boxed{3}$ distinct real roots. Hence, the correct option is (D).

Quick Tip to Solve: For $x^{2n+1} - (2n+1)x - c$, examine the values at $x = \pm 1$; the derivative already limits the number of real crossings to three.

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11. A random variable X has the following probability distribution :

X	0	1	2	3	4
P(X)	k	2k	4k	6k	8k

The value of P(1 < X < 4 | X ≤ 2) is equal to : [4 marks]

(A) $\frac{4}{7}$

(B) $\frac{2}{3}$

(C) $\frac{3}{7}$

(D) $\frac{4}{5}$

Correct Answer: (A) $\frac{4}{7}$

Solution:

Concept:

- For events E, F with $P(F) > 0$, conditional probability is $P(E \mid F) = P(E \cap F)/P(F)$.
- Probabilities over the complete support add to one. In a ratio with a common probability scale k, the scale cancels.

Step 1: Normalise and identify the conditioning event The table gives the five probabilities $k, 2k, 4k, 6k, 8k$ for the respective values $0, 1, 2, 3, 4$. Therefore

$$k + 2k + 4k + 6k + 8k = 21k = 1, \qquad k = \frac{1}{21}.$$

Let $F = \{X \leq 2\}$. Its allowed outcomes are $0, 1, 2$, so

$$P(F) = k + 2k + 4k = 7k > 0.$$

Conditioning restricts the sample space to these three outcomes; outcomes 3 and 4 cannot contribute to the conditional numerator.

Step 2: Intersect the requested event with the condition The strict inequalities $1 < X < 4$ give $E = \{2, 3\}$. Intersecting with F leaves only the outcome $X = 2$. Consequently

$$P(E \cap F) = P(X = 2) = 4k, \qquad P(E \mid F) = \frac{4k}{7k} = \frac{4}{7}.$$

This is a conditional probability, so using $P(E)$ alone or normalising over all five outcomes would answer a different question.

Step 3: Check the alternatives The denominator must be the total weight $1+2+4=7$ of the conditioning outcomes, and the numerator must be the weight 4 of outcome 2. The value $3/7$ is the conditional probability of its complement, namely outcomes 0 and 1. Neither $2/3$ nor $4/5$ uses the required pair of event weights. All entries in the table are nonnegative, and the result lies between zero and one as required.

Step 4: Conclusion $\boxed{P(1<X<4 \mid X \le 2)=4/7}$. Hence, the correct option is (A).

Quick Tip to Solve: For a conditional probability table, first cross out outcomes excluded by the condition, then divide the desired surviving weight by the total surviving weight.

12. If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible value of λ is : [4 marks]

(A) 16

(B) 6

(C) 12

(D) 15

Correct Answer: (A) 16

Solution:

Concept:

- For lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$ with nonparallel directions $\mathbf{b}_1, \mathbf{b}_2$, their distance is $d = |(\mathbf{a}_2-\mathbf{a}_1)\cdot(\mathbf{b}_1\times\mathbf{b}_2)|/|\mathbf{b}_1\times\mathbf{b}_2|$.
- After squaring a distance equation, verify the denominator is nonzero and substitute the resulting parameter values back into the original formula.

Step 1: Form the direction and separation vectors From the symmetric forms choose

$$\mathbf{a}_1=(1,2,3), \quad \mathbf{a}_2=(2,4,5), \quad \mathbf{b}_1=(2,3,\lambda), \quad \mathbf{b}_2=(1,4,5).$$

Then $\mathbf{a}_2-\mathbf{a}_1=(1,2,2)$, and

$$\mathbf{b}_1\times\mathbf{b}_2=(15-4\lambda,\lambda-10,5).$$

Its third component is always 5, so the directions are never parallel and the distance formula is valid for every real λ.

Step 2: Apply the given distance The scalar triple product is

$$(15-4\lambda)+2(\lambda-10)+10=5-2\lambda.$$

The squared magnitude of the cross product is

$$(15-4\lambda)^2+(\lambda-10)^2+25=17\lambda^2-140\lambda+350.$$

Thus $d = 1/\sqrt{3}$ gives

$$3(5-2\lambda)^2 = 17\lambda^2 - 140\lambda + 350.$$

Rearranging and dividing by 5 yields

$$\lambda^2 - 16\lambda + 55 = 0 = (\lambda - 5)(\lambda - 11).$$

Step 3: Check the roots and sum them For $\lambda = 5$, the numerator is 5 and the denominator is $5\sqrt{3}$; for $\lambda = 11$, they are 17 and $17\sqrt{3}$. Both therefore give precisely the required positive distance. Their sum is $5 + 11 = 16$. The other listed sums 6, 12 and 15 differ from both this direct sum and the coefficient sum 16 obtained by Vieta's formula.

Step 4: Conclusion The sum of all admissible values is $\boxed{16}$. Hence, the correct option is (A).

Quick Tip to Solve: Once the squared distance equation is a quadratic, use Vieta for the sum, but still check that its roots do not make the cross product vanish.

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13. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $|(\hat{a}+\hat{b}) + 2(\hat{a}\times\hat{b})| = 2$. If $\theta \in (0, \pi)$ is the angle between $\hat{a}$ and $\hat{b}$, then among the statements :

(S1) : $2|\hat{a}\times\hat{b}| = |\hat{a} - \hat{b}|$

(S2) : The projection of $\hat{a}$ on $(\hat{a} + \hat{b})$ is $\frac{1}{2}$ [4 marks]

(A) Only (S1) is true.

(B) Only (S2) is true.

(C) Both (S1) and (S2) are true.

(D) Both (S1) and (S2) are false.

Correct Answer: (C) Both (S1) and (S2) are true.

Solution:

Concept:

- For unit vectors with angle θ, write $c = \cos\theta$; then $|\hat{a}+\hat{b}|^2 = 2 + 2c$, $|\hat{a}-\hat{b}|^2 = 2 - 2c$, and $|\hat{a}\times\hat{b}|^2 = 1 - c^2$.
- The cross product is perpendicular to each original vector and therefore to their sum. The scalar projection of $\mathbf{u}$ on $\mathbf{v}$ is $\mathbf{u}\cdot\mathbf{v}/|\mathbf{v}|$.

Step 1: Obtain the permitted angle The two vectors inside the given magnitude are perpendicular, so squaring that magnitude introduces no cross term:

$$(2 + 2c) + 4(1 - c^2) = 4.$$

Therefore

$$2c^2 - c - 1 = 0, \qquad (2c+1)(c-1) = 0.$$

The value $c = 1$ would give $\theta = 0$, excluded by the stated open interval. Hence $c = -1/2$ and $\theta = 2\pi/3$. This domain check is necessary before testing the statements.

Step 2: Test the cross-product identity in S1 For the permitted angle,

$$2|\widehat{a}\times\widehat{b}| = 2\sqrt{1-\frac{1}{4}} = \sqrt{3}, \qquad |\widehat{a}-\widehat{b}| = \sqrt{2-2(-1/2)} = \sqrt{3}.$$

Both sides coincide, so S1 is true. All magnitudes are nonnegative, so comparing these values introduces no sign ambiguity.

Step 3: Evaluate the scalar projection in S2 The sum vector has magnitude $\sqrt{2+2c} = 1$, while

$$\widehat{a}\cdot(\widehat{a}+\widehat{b}) = 1+c = \frac{1}{2}.$$

Its scalar projection is therefore $(1/2)/1 = 1/2$, proving S2 true as well. Options A and B each discard one verified statement, and option D discards both. Only the option asserting both statements is consistent with the permitted angle.

Step 4: Conclusion Both S1 and S2 are true. Hence, the correct option is (C).

Quick Tip to Solve: Whenever a sum contains a vector sum and its cross product, use perpendicularity before expanding the norm; then enforce the given angle interval.

14. If $y = \tan^{-1}\left(\sec x^3 - \tan x^3\right)$, $\frac{\pi}{2} < x^3 < \frac{3\pi}{2}$, then [4 marks]

(A) $xy'' + 2y' = 0$

(B) $x^2y'' - 6y + \frac{3\pi}{2} = 0$

(C) $x^2y'' - 6y + 3\pi = 0$

(D) $xy'' - 4y' = 0$

Correct Answer: (B) $x^2y'' - 6y + \frac{3\pi}{2} = 0$

Solution:

Concept:

- The identity $\sec t - \tan t = (1-\sin t)/\cos t = \tan(\pi/4 - t/2)$ holds wherever the original expression is defined.
- The principal range of $\tan^{-1}$ is $(-\pi/2, \pi/2)$. Reduce an inverse tangent using its principal branch before differentiating.

Step 1: Check the inverse-tangent branch Put $t = x^3$. The given interval $\pi/2 < t < 3\pi/2$ contains no zero of $\cos t$ in its interior, so the original expression is defined throughout it. Furthermore

$$-\frac{\pi}{2} < \frac{\pi}{4} - \frac{t}{2} < 0.$$

This entire interval is within the principal range of the inverse tangent. Hence no multiple of π needs to be added and

$$y = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x^3}{2}\right)\right] = \frac{\pi}{4} - \frac{x^3}{2}.$$

Step 2: Compute the derivatives Differentiating with respect to x gives

$$y' = -\frac{3}{2}x^2, \qquad y'' = -3x.$$

In particular $x^2y'' = -3x^3$ and $-6y = -3\pi/2 + 3x^3$. The cubic terms cancel when these quantities are added, leaving the constant $-3\pi/2$. Therefore

$$x^2y'' - 6y + \frac{3\pi}{2} = 0.$$

Step 3: Rule out the other equations For option A, substitution gives $xy'' + 2y' = -6x^2 \neq 0$; for option D it gives $xy'' - 4y' = 3x^2 \neq 0$, since the interval implies $x > 0$. Replacing the constant $3\pi/2$ by 3π, as in option C, leaves $3\pi/2 \neq 0$. Thus precisely the equation with constant $3\pi/2$ is identically valid on the stated domain.

Step 4: Conclusion $\boxed{x^2y'' - 6y + 3\pi/2 = 0}$. The printed option B contains this equation; the key label C refers to a different option order and does not satisfy the printed equation. Hence, the correct option is (B).

Quick Tip to Solve: Before cancelling $\tan^{-1}(\tan u)$, check that u lies in the principal interval; an overlooked branch changes the constant in the differential equation.

15. Let λ^* be the largest value of λ for which the function $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$ is increasing for all x $\in$ R. Then $f_{\lambda^*}(1) + f_{\lambda^*}(-1)$ is equal to : [4 marks]

(A) 36

(B) 48

(C) 64

(D) 72

Correct Answer: (D) 72

Solution:

Concept:

- A differentiable function increasing on the entire real line must have a nonnegative derivative everywhere. For this polynomial, nonnegativity is also sufficient, because its derivative can vanish only at isolated points.
- For a quadratic with a positive leading coefficient, completing the square gives its minimum. The parameter must satisfy the minimum condition as well as the sign condition at large positive and negative arguments.

Step 1: Differentiate and isolate the parameter Differentiate each term with respect to x:

$$f'_\lambda(x) = 12\lambda x^2 - 72\lambda x + 36 = 12\lambda(x-3)^2 + 36 - 108\lambda.$$

If $\lambda < 0$, the derivative tends to a negative value without bound as $|x|$ increases, so this cannot give

an increasing function on all of $\mathbb{R}$. If $\lambda = 0$, the derivative is the positive constant 36.

Step 2: Find the largest admissible value For $\lambda > 0$, the smallest derivative occurs at $x = 3$. Thus

$$36 - 108\lambda \geq 0 \quad \Longrightarrow \quad \lambda \leq \frac{1}{3}.$$

Consequently $\lambda^* = 1/3$. At this value the derivative is $4(x-3)^2$, positive except at the single point 3; hence the function really is increasing and the boundary value is admissible.

Step 3: Add the two function values Adding values at opposite arguments cancels the cubic and linear terms:

$$f_\lambda(1) + f_\lambda(-1) = 96 - 72\lambda.$$

Substituting the largest admissible parameter gives

$$96 - 72\left(\frac{1}{3}\right) = 72.$$

The remaining choices 36, 48 and 64 do not equal this sum at the required boundary parameter; substituting an arbitrary allowed parameter would not answer the largest-value condition.

Step 4: Conclusion The required sum is 72. Hence, the correct option is (D).

Quick Tip to Solve: Complete the square in the derivative first; then add values at opposite arguments to cancel all odd powers.

16. $\lim\limits_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)} + \frac{n^2}{(n^2+4)(n+2)} + \frac{n^2}{(n^2+9)(n+3)} + \ldots + \frac{n^2}{(n^2+n^2)(n+n)}\right)$ **is equal to :** [4 marks]

(A) $\frac{\pi}{8} + \frac{1}{4}\log_e 2$

(B) $\frac{\pi}{4} + \frac{1}{8}\log_e 2$

(C) $\frac{\pi}{4} - \frac{1}{8}\log_e 2$

(D) $\frac{\pi}{8} + \log_e\sqrt{2}$

Correct Answer: (A) $\frac{\pi}{8} + \frac{1}{4}\log_e 2$

Solution:

Concept:

- A sum of the form $\frac{1}{n}\sum_{k=1}^{n} g(k/n)$ converges to $\int_0^1 g(t)\,dt$ when g is continuous on the interval.
- Partial fractions split the rational integrand into a logarithmic term, a derivative-of-denominator term, and an inverse-tangent term. All logarithms here are natural logarithms.

Step 1: Rescale the general summand The term indexed by k is

$$\frac{n^2}{(n^2+k^2)(n+k)} = \frac{1}{n}\frac{1}{(1+(k/n)^2)(1+k/n)}.$$

Thus the given limit is a right-endpoint Riemann sum. The integrand has no zero denominator on

$[0,1]$, so continuity justifies the passage to the integral:

$$I = \int_0^1 \frac{dt}{(1+t^2)(1+t)}.$$

Step 2: Decompose and integrate Comparing coefficients, or multiplying both sides by $(1+t)(1+t^2)$, gives

$$\frac{1}{(1+t)(1+t^2)} = \frac{1}{2(1+t)} + \frac{1-t}{2(1+t^2)}.$$

Integrating each part yields

$$I = \left[\frac{1}{2}\ln(1+t) - \frac{1}{4}\ln(1+t^2) + \frac{1}{2}\tan^{-1}t\right]_0^1.$$

The minus sign on the second logarithm comes from the numerator $-t$, whose integral uses the derivative $2t$ of $1+t^2$.

Step 3: Evaluate the endpoints and compare At the lower endpoint every displayed term is zero. At the upper endpoint,

$$I = \frac{1}{2}\ln 2 - \frac{1}{4}\ln 2 + \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8} + \frac{1}{4}\ln 2.$$

Options B and C have the wrong coefficient of π; option D uses $\ln\sqrt{2} = \frac{1}{2}\ln 2$, twice the required logarithmic contribution. Only option A retains both coefficients correctly.

Step 4: Conclusion The limit is $\frac{\pi}{8} + \frac{1}{4}\log_e 2$. Hence, the correct option is (A).

Quick Tip to Solve: Factor powers of n from every denominator before identifying the Riemann sum; do not lose its outside factor 1/n.

17. The number of solutions of the equation

$\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}\cos^2 2x$, $x \in [-3\pi, 3\pi]$ **is :** [4 marks]

(A) 8

(B) 5

(C) 6

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- The product identity $\cos(a+x)\cos(a-x) = \cos^2 a - \sin^2 x$ converts the shifted-angle product to a single squared trigonometric function.
- After solving an algebraic equation in $\cos 2x$, return to the original interval and count distinct real angles, including endpoints when the interval is closed.

Step 1: Reduce the shifted cosines With $a = \pi/3$, the left side becomes

$$\cos\left(x + \frac{\pi}{3}\right)\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4} - \sin^2 x = \frac{2\cos 2x - 1}{4}.$$

This identity is valid for every real x; no division by a trigonometric expression has been used, so no zero-case solutions have been lost.

Step 2: Solve the resulting quadratic Substitute into the given equation and multiply by 4:

$$2\cos 2x - 1 = \cos^2 2x.$$

Rearranging and factoring,

$$\cos^2 2x - 2\cos 2x + 1 = 0 \quad \Longrightarrow \quad (\cos 2x - 1)^2 = 0.$$

Thus $\cos 2x = 1$. The repeated algebraic root is a single trigonometric condition, not two separate families of solutions.

Step 3: Count the permitted angles The complete real solution is

$$2x = 2k\pi \quad \Longrightarrow \quad x = k\pi, \qquad k \in \mathbb{Z}.$$

The closed interval $[-3\pi, 3\pi]$ permits

$$k = -3, -2, -1, 0, 1, 2, 3.$$

There are seven distinct angles. Both endpoints satisfy the original equation and must be included. Choices 5 and 6 omit valid boundary or central solutions, while 8 would count a nonexistent extra solution or double-count a repeated root.

Step 4: Conclusion The number of solutions is 7. Hence, the correct option is (D).

Quick Tip to Solve: Factor the equation in cos(2x), then count integer multiples of pi inside the closed interval, checking both endpoints.

18. Let the points on the plane P be equidistant from the points (−4, 2, 1) and (2, −2, 3). Then the acute angle between the plane P and the plane 2x + y + 3z = 1 is : [4 marks]

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{5\pi}{12}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Concept:

- The locus of points equidistant from two fixed distinct points is their perpendicular-bisector plane. Its normal vector is parallel to the vector joining those points.
- The acute angle θ between planes satisfies $\cos\theta = |\mathbf{n}_1 \cdot \mathbf{n}_2|/(|\mathbf{n}_1||\mathbf{n}_2|)$, where the vectors are normals to the planes.

Step 1: Find a normal to the equidistant plane Let the fixed points be $A = (-4, 2, 1)$ and $B = (2, -2, 3)$. Their joining vector is

$$\overrightarrow{AB} = (6, -4, 2) = 2(3, -2, 1).$$

Therefore a convenient normal to the required plane P is $\mathbf{n}_1 = (3, -2, 1)$. This follows directly by expanding $|X - A|^2 = |X - B|^2$: the quadratic terms in the variable coordinates cancel, leaving a linear equation whose coefficients are proportional to $B - A$.

Step 2: Compute the angle using dot products The given plane $2x + y + 3z = 1$ has normal $\mathbf{n}_2 = (2, 1, 3)$. Hence

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = 3(2) + (-2)(1) + 1(3) = 7,$$

$$|\mathbf{n}_1| = \sqrt{9 + 4 + 1} = \sqrt{14}, \qquad |\mathbf{n}_2| = \sqrt{4 + 1 + 9} = \sqrt{14}.$$

It follows that

$$\cos\theta = \frac{7}{\sqrt{14}\sqrt{14}} = \frac{1}{2}.$$

The acute angle is therefore $\theta = \pi/3$.

Step 3: Check the angle convention Planes have no preferred orientation, so reversing either normal must leave their acute angle unchanged; this is why the dot-product formula uses an absolute value. The values $\pi/6$, $\pi/4$ and $5\pi/12$ have cosines different from $1/2$, so they do not satisfy the calculated relation. We require the angle between the planes themselves, which is the acute angle between their normals, rather than the angle between a line and a plane.

Step 4: Conclusion The acute angle is $\pi/3$. Hence, the correct option is (C).

Quick Tip to Solve: For an equidistant-plane problem, use the joining vector as its normal; finding the full plane equation is unnecessary.

19. Consider the following statements:

A : Rishi is a judge.

B : Rishi is honest.

C : Rishi is not arrogant.

The negation of the statement "if Rishi is a judge and he is not arrogant, then he is honest" is [4 marks]

(A) B $\rightarrow$ (A $\vee$ C)

(B) ($\sim$B) $\wedge$ (A $\wedge$ C)

(C) B $\rightarrow$ (($\sim$A) $\vee$ ($\sim$C))

(D) B $\rightarrow$ (A $\wedge$ C)

Correct Answer: (B) ($\sim$B) $\wedge$ (A $\wedge$ C)

Solution:

Concept:

- An implication $P \rightarrow Q$ is false exactly when its antecedent P is true and its consequent Q is false. Thus $\neg(P \rightarrow Q) \equiv P \wedge \neg Q$.
- The statement C already means that Rishi is not arrogant. It must be used as written in the antecedent, without adding another negation.

Step 1: Translate the complete antecedent The phrase "Rishi is a judge and he is not arrogant" is the conjunction $A \wedge C$. The phrase "he is honest" is B. Consequently the original sentence is

$$(A \wedge C) \rightarrow B.$$

The word "and" joins two requirements in the antecedent; it must not be replaced by a disjunction. Also, the existing negative word in the meaning of C is already part of that proposition.

Step 2: Negate the implication Using the false-case rule for an implication,

$$\neg\big((A \wedge C) \rightarrow B\big) = (A \wedge C) \wedge \neg B = (\neg B) \wedge (A \wedge C).$$

In ordinary language, the negation says that Rishi is a judge, is not arrogant, and is not honest. All three facts must hold simultaneously to contradict the proposed implication.

Step 3: Rule out the other expressions Option B states exactly this conjunction. Options A, C and D all retain an implication with antecedent B. For example, if B is false and A is false, each of those implications is true because its antecedent is false, whereas the required negation is false because $A \wedge C$ is not true. This counterexample rules out all three without relying only on their surface wording. Conjunction is commutative, so placing $\neg B$ first does not change the correct expression.

Step 4: Conclusion The negation is $(\sim B) \wedge (A \wedge C)$. Hence, the correct option is (B).

Quick Tip to Solve: To negate an implication, keep its entire antecedent true and negate only the consequent; do not reverse the arrow.

20. The slope of normal at any point (x, y), x > 0, y > 0 on the curve y = y(x) is given by $\frac{x^2}{xy-x^2y^2-1}$. If the curve passes through the point (1, 1), then e . y(e) is equal to [4 marks]

(A) $\frac{1-\tan(1)}{1+\tan(1)}$

(B) tan(1)

(C) 1

(D) $\frac{1+\tan(1)}{1-\tan(1)}$

Correct Answer: The formal expression is option (D), but the stated positive solution through (1,1) has a pole before x=e, so the requested value does not exist.

Solution:

Concept:

- Tangent and normal slopes are negative reciprocals: $dy/dx = -1/m_n$. The product substitution $u = xy$ absorbs the linear term in the resulting differential equation.
- The separable equation $du/(1+u^2) = dx/x$ has an inverse-tangent solution. Its maximal interval must exclude poles of the tangent expression.

Step 1: Convert the normal slope to a differential equation Taking the negative reciprocal of the stated normal slope gives

$$y' = \frac{x^2y^2 - xy + 1}{x^2} = y^2 - \frac{y}{x} + \frac{1}{x^2}.$$

Set $u = xy$. Since $u' = y + xy'$, substitution cancels the terms linear in y:

$$u' = y + x\left(y^2 - \frac{y}{x} + \frac{1}{x^2}\right) = \frac{u^2+1}{x}.$$

Step 2: Integrate and use the initial point For $x > 0$, separation gives

$$\tan^{-1} u = \ln x + C.$$

At $(x, y) = (1, 1)$, we have $u = 1$, hence $C = \pi/4$ on the branch through the initial point. Therefore

$$xy = \tan\left(\ln x + \frac{\pi}{4}\right).$$

Formal substitution of $x = e$, followed by the tangent addition formula, gives

$$e\,y(e) = \tan\left(1 + \frac{\pi}{4}\right) = \frac{1+\tan 1}{1-\tan 1}.$$

This is the algebraic expression listed in option D; the other options have the wrong addition formula or omit the initial phase.

Step 3: Check the stated positive-domain restriction There is a domain inconsistency in the printed question. The branch through $(1, 1)$ with $y > 0$ exists on

$$e^{-\pi/4} < x < e^{\pi/4}.$$

Its tangent expression has a pole at $x = e^{\pi/4} < e$. Thus no positive differentiable solution through

$(1,1)$ reaches $x = e$. Moreover, the formal expression in option D is negative, since $1 > \pi/4$ implies $\tan 1 > 1$. Consequently the formal multiple-choice answer must be distinguished from the literal positive-domain initial-value problem.

Step 4: Conclusion Formally, $e\,y(e) = (1+\tan 1)/(1-\tan 1)$, which corresponds to option (D). Under the printed condition $y > 0$, however, $y(e)$ is undefined on the solution branch through $(1,1)$; no listed option is a valid value for that literal initial-value problem.

Quick Tip to Solve: After separating an equation that produces a tangent, locate its nearest pole before evaluating the solution at another point.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let S = {z ∈ C : |z − 3| ≤ 1 and z(4 + 3i) + $\bar{z}$(4 − 3i) ≤ 24}. If α + iβ is the point in S which is closest to 4i, then 25(α + β) is equal to _____. [4 marks]

Solution:

Concept:

- Writing $z = x + iy$ converts $|z-3| \le 1$ into the closed disc $(x-3)^2 + y^2 \le 1$. The additional real inequality cuts this disc by a half-plane.
- The nearest point of a closed disc to an exterior point lies on the radius directed from the disc centre toward that point. If it also satisfies the half-plane restriction, it remains the nearest point of their intersection.

Step 1: Translate both conditions into Cartesian form Put $z = x + iy$, where x, y are real. The modulus condition gives the unit disc with centre $C = (3,0)$. Also,

$$z(4+3i) + \bar{z}(4-3i) = 2\,\mathrm{Re}((x+iy)(4+3i)) = 8x - 6y.$$

Thus the second restriction is

$$8x - 6y \le 24 \quad \Longleftrightarrow \quad 4x - 3y \le 12.$$

The target point $4i$ has Cartesian coordinates $T = (0,4)$, a distance 5 from the centre.

Step 2: Find the nearest point of the disc The centre-to-target vector is $T - C = (-3,4)$, whose length is 5. Therefore the unit-radius point on this ray is

$$Q = C + \frac{T-C}{5} = (3,0) + \left(-\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{12}{5}, \frac{4}{5}\right).$$

The reverse triangle inequality shows every disc point X obeys $|T - X| \ge |T - C| - |X - C| \ge 4$. Equality occurs at Q, so this construction proves global minimality rather than merely giving a stationary boundary point.

Step 3: Check feasibility and evaluate the answer For Q, the half-plane expression is

$$4\left(\frac{12}{5}\right) - 3\left(\frac{4}{5}\right) = \frac{36}{5} \leq 12.$$

Hence Q belongs to S, and imposing the half-plane does not remove the nearest point of the disc. Therefore $\alpha = 12/5$ and $\beta = 4/5$, giving

$$25(\alpha + \beta) = 25\left(\frac{16}{5}\right) = 80.$$

This is a dimensionless numerical value because the complex-plane coordinates carry no specified physical units.

Step 4: Conclusion The nearest point is $\alpha + i\beta = 12/5 + (4/5)i$, and the required dimensionless value is $\boxed{80}$.

Quick Tip to Solve: Find the nearest point on the full disc first, then test the extra linear inequality before doing constrained boundary calculations.

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22. Let $S = \left\{ \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix} ; a, b \in \{1, 2, 3, ...100\} \right\}$ **and let** $T_n = \{A \in S : A^{n(n+1)} = I\}$**. Then the number of elements in** $\bigcap_{n=1}^{100} T_n$ **is _____.** [4 marks]

Solution:

Concept:

- A matrix belongs to an intersection only when it satisfies every defining condition. In particular, membership in T_1 requires $A^2 = I$.
- Once $A^2 = I$, every even power of A is the identity. The product of consecutive integers $n(n+1)$ is always even.

Step 1: Use the condition for the first set Write the matrix with its given positive integer parameters and multiply it by itself:

$$A = \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}, \qquad A^2 = \begin{pmatrix} 1 & a(b-1) \\ 0 & b^2 \end{pmatrix}.$$

For this to equal the identity, we require $b^2 = 1$ and $a(b-1) = 0$. Since b belongs to $\{1, 2, \ldots, 100\}$, it is positive, so the first equation forces $b = 1$. The second equation then holds for every permitted value of a; there is no further restriction on it.

Step 2: Check all the remaining powers and count For $b = 1$, the calculation above gives $A^2 = I$ irrespective of a. For each integer n from 1 to 100, let $m = n(n+1)/2$, which is an integer. Then

$$A^{n(n+1)} = (A^2)^m = I^m = I.$$

Thus every one of these matrices lies in every T_n, not merely in T_1. Conversely, a matrix with $b \neq 1$

already fails the necessary T_1 condition and cannot lie in the intersection. There are 100 distinct choices for the upper-right entry a, with the other entries fixed.

Step 3: Conclusion The intersection contains exactly $\boxed{100}$ matrices. The required numerical answer is 100.

Quick Tip to Solve: For an intersection of matrix-power conditions, test the smallest exponent first; if it gives an involution, all even exponents follow immediately.

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23. The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is _____. [4 marks]

Solution:

Concept:

- A number is divisible by 11 exactly when the sum of its digits in odd positions minus the sum in even positions is a multiple of 11.
- All seven digits are distinct and nonzero. After choosing the four odd-position digits, they can be arranged in $4!$ ways and the remaining three in $3!$ ways.

Step 1: Find the allowed sums in odd positions The total of the seven given digits is $1+2+3+4+5+7+9=31$. If s is the sum in positions 1, 3, 5 and 7, the sum in positions 2, 4 and 6 is $31-s$. Therefore the alternating difference is $2s-31$. The least possible four-digit sum is 10 and the greatest is 25, so this difference lies between -11 and 19. The only multiples of 11 in that interval compatible with its being odd are -11 and 11. Hence

$$2s-31=-11 \text{ or } 11, \qquad s=10 \text{ or } 21.$$

Step 2: List the groups and arrange their digits The four-digit subsets with these sums are

$$s=10:\{1,2,3,4\},$$
$$s=21:\{1,4,7,9\},\ \{2,3,7,9\},\ \{3,4,5,9\}.$$

For completeness, a sum of 21 without 9 would require all four of the largest remaining digits, whose sum is only 19. With 9 included, the other three must sum to 12, giving exactly the three triples displayed above. Each of the four admissible subsets produces $4!\,3!=24\times 6=144$ arrangements. Their odd-position digit sets differ, so no number is counted twice. No leading-zero exclusion is needed.

Step 3: Conclusion The required count is $4\times 144=\boxed{576}$ seven-digit multiples of 11.

Quick Tip to Solve: Choose the digit sets in alternate positions before arranging them; the divisibility-by-11 test becomes a small subset-sum problem.

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24. The sum of all the elements of the set $\{\alpha \in \{1, 2, ..., 100\} : HCF(\alpha, 24) = 1\}$ is _____.

[4 marks]

Solution:

Concept:

- The prime factors of 24 are only 2 and 3. Thus $\gcd(\alpha, 24) = 1$ means that α is divisible by neither 2 nor 3.
- An integer coprime to 6 is congruent to 1 or 5 modulo 6. These two residue classes allow the required sum to be evaluated by arithmetic progressions.

Step 1: Identify the integers to include Since $24 = 2^3 \cdot 3$, no power of 2 or 3 may divide an allowed integer. This excludes all even integers and every multiple of 3. Among the six possible residues modulo 6, residues 0, 2 and 4 are even and residue 3 is divisible by 3; only residues 1 and 5 remain. Thus the eligible integers form the two progressions

$$1, 7, 13, \ldots, 97 \quad \text{and} \quad 5, 11, 17, \ldots, 95.$$

The endpoints respect the inclusive upper bound 100: the next terms would be 103 and 101, respectively. Also 1 is included because its greatest common divisor with 24 is 1.

Step 2: Sum the two progressions The first progression has

$$\frac{97 - 1}{6} + 1 = 17$$

terms, so its sum is $17(1 + 97)/2 = 833$. The second progression has $(95 - 5)/6 + 1 = 16$ terms, so its sum is $16(5 + 95)/2 = 800$. Adding the two disjoint residue classes gives

$$833 + 800 = 1633.$$

As a check, in each of the 16 complete blocks $6j + 1, \ldots, 6j + 6$, for $0 \le j \le 15$, the allowed pair contributes $12j + 6$. These contributions total 1536, and the remaining eligible integer 97 brings the same total to 1633.

Step 3: Conclusion The sum of all elements of the set is $\boxed{1633}$.

Quick Tip to Solve: Replace coprimality with exclusion of the prime factors; when those factors are 2 and 3, group the integers by their residues modulo 6.

25. The remainder on dividing 1 + 3 + 3^2 + 3^3 + ... + 3^{2021} by 50 is _____. [4 marks]

Solution:

Concept:

- The sum of the geometric progression is $S = (3^{2022} - 1)/(3 - 1) = (3^{2022} - 1)/2$.
- To find the remainder of a quotient by 2 modulo 50, first work with the numerator modulo 100. Division by 2 is then justified in an ordinary integer equation.

Step 1: Reduce the large power modulo 100 The powers of 3 have a convenient period modulo 100. In particular,

$$3^{10} = 59049 \equiv 49 \pmod{100}, \qquad 3^{20} \equiv 49^2 = 2401 \equiv 1 \pmod{100}.$$

The exponent in the geometric-sum numerator is 2022, since the original series starts at 3^0 and ends at 3^{2021}. Decompose it as $2022 = 20 \cdot 101 + 2$. Therefore

$$3^{2022} = (3^{20})^{101} 3^2 \equiv 1^{101} \cdot 9 = 9 \pmod{100}.$$

This calculation uses modulus 100 rather than 50 so that the later division cannot lose parity information.

Step 2: Divide the numerator and identify the remainder The congruence means that $3^{2022} = 100k + 9$ for some integer k. Substituting into the exact geometric-sum expression gives

$$S = \frac{100k + 9 - 1}{2} = 50k + 4.$$

The second term is already between 0 and 49, so it is the remainder on division by 50. This also explains why one should not simply cancel the factor 2 inside a congruence modulo 50: 2 has no multiplicative inverse modulo 50. Writing the numerator as $100k + 8$ before division makes the remainder determination rigorous.

Step 3: Conclusion The remainder on division by 50 is $\boxed{4}$.

Quick Tip to Solve: For a geometric-sum formula with denominator 2, compute the numerator modulo twice the desired modulus before dividing.

26. The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is _____. [4 marks]

Solution:

Concept:

- For horizontal slices, the area between curves is $\int (x_{\text{right}} - x_{\text{left}})\, dy$, with limits given by their intersections.
- The parabola is $x = y^2/2$, while the line is $x = 4 - y$. Using y as the integration variable avoids splitting the parabola into its upper and lower branches.

Step 1: Find the intersection levels and the slice width At an intersection, both expressions for x agree:

$$\frac{y^2}{2} = 4 - y \quad \Longrightarrow \quad y^2 + 2y - 8 = 0 \quad \Longrightarrow \quad (y + 4)(y - 2) = 0.$$

Thus the limits are $y = -4$ and $y = 2$, corresponding to intersection points $(8, -4)$ and $(2, 2)$. Between these levels, the line is the right boundary: for example, at $y = 0$, its x-coordinate is 4 while the parabola's is 0. More generally the width is $(2 - y)(y + 4)/2$, which is nonnegative throughout the

interval. This confirms the order of subtraction.

Step 2: Integrate the horizontal widths The enclosed area is consequently

$$\mathcal{A} = \int_{-4}^{2} \left(4 - y - \frac{y^2}{2}\right) dy$$
$$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6}\right]_{-4}^{2}.$$

The upper endpoint contributes $8-2-8/6 = 14/3$, and the lower endpoint contributes $-16-8+64/6 = -40/3$. Their difference is

$$\mathcal{A} = \frac{14}{3} - \left(-\frac{40}{3}\right) = 18.$$

Both branches of the parabola are accounted for because the entire vertical range from -4 to 2 was integrated.

Step 3: Conclusion The required enclosed area is $\boxed{18 \text{ square units}}$.

Quick Tip to Solve: When a parabola is given as a quadratic in y, horizontal strips often give a single integral where vertical strips would require two.

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27. Let a circle C : $(x - h)^2 + (y - k)^2 = r^2$, $k > 0$, touch the x-axis at (1, 0). If the line $x + y = 0$ intersects the circle C at P and Q such that the length of the chord PQ is 2, then the value of $h + k + r$ is equal to _____. [4 marks]

Solution:

Concept:

- A circle tangent to the x-axis at $(1, 0)$ has its centre vertically above or below that point. Since $k > 0$, its centre is $(1, k)$ and its radius is $r = k$.
- For a chord at perpendicular distance d from the centre, half its length satisfies $(L/2)^2 = r^2 - d^2$. This follows from the right triangle through the chord midpoint.

Step 1: Use the tangency and calculate the distance to the line The radius to the point of contact is perpendicular to the x-axis. Therefore the centre's horizontal coordinate is exactly $h = 1$. The distance from $(1, k)$ to the x-axis is k, which equals the positive radius r. For the chord line $x + y = 0$, the perpendicular distance from the centre is

$$d = \frac{|1 + k|}{\sqrt{1^2 + 1^2}} = \frac{1 + k}{\sqrt{2}},$$

since $k > 0$. The perpendicular from the centre bisects a chord, so the given chord length 2 produces a right triangle with one leg 1, the other leg d, and hypotenuse k.

Step 2: Solve for the radius and evaluate the requested sum Applying Pythagoras yields

$$\begin{aligned} k^2 - \frac{(1+k)^2}{2} &= 1, \\ 2k^2 - (1 + 2k + k^2) &= 2, \\ k^2 - 2k - 3 &= 0, \\ (k-3)(k+1) &= 0. \end{aligned}$$

The algebraic roots are 3 and -1, but $k > 0$ excludes the latter. Thus $k = 3$, $r = 3$ and $h = 1$. Checking the chord condition, $d = 4/\sqrt{2} = 2\sqrt{2}$, so $2\sqrt{r^2 - d^2} = 2\sqrt{9-8} = 2$, as required. Finally the requested sum is $1 + 3 + 3$.

Step 3: Conclusion The value of $h + k + r$ is $\boxed{7}$.

Quick Tip to Solve: Tangency to an axis fixes one centre coordinate and makes the other centre coordinate equal to the radius in magnitude; use this before the chord formula.

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28. Let the hyperbola $H : \frac{x^2}{a^2} - y^2 = 1$ and the ellipse $E : 3x^2 + 4y^2 = 12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12\left(e_H^2 + e_E^2\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- For an ellipse with major semiaxis A and minor semiaxis B, the latus rectum has length $2B^2/A$ and $e^2 = 1 - B^2/A^2$.
- For a hyperbola $x^2/a^2 - y^2/b^2 = 1$, the latus rectum length is $2b^2/a$ and $e^2 = 1 + b^2/a^2$. All semiaxis lengths are taken positive.

Step 1: Read the ellipse parameters and its latus rectum Divide the ellipse equation by 12 to put it in standard form:

$$\frac{x^2}{4} + \frac{y^2}{3} = 1.$$

The larger denominator is 4, so its major semiaxis is $A = 2$, while $B^2 = 3$. Its latus rectum length and eccentricity squared are therefore

$$\ell_E = \frac{2 \cdot 3}{2} = 3, \qquad e_E^2 = 1 - \frac{3}{4} = \frac{1}{4}.$$

The distinction between a semiaxis and its square matters here: the numerator uses $B^2 = 3$, whereas the denominator in the latus rectum formula uses $A = 2$, not 4.

Step 2: Use equality of latus recta and combine eccentricities In the hyperbola $x^2/a^2 - y^2 = 1$, the coefficient of y^2 shows that $b^2 = 1$. Equating its latus rectum with the ellipse's gives

$$\frac{2 \cdot 1}{a} = 3 \implies a = \frac{2}{3}, \qquad a^2 = \frac{4}{9}.$$

Thus

$$e_H^2 = 1 + \frac{1}{4/9} = 1 + \frac{9}{4} = \frac{13}{4}.$$

This exceeds 1, as a hyperbola's eccentricity should, while the ellipse's eccentricity squared is less than 1. The requested expression becomes

$$12(e_H^2 + e_E^2) = 12\left(\frac{13}{4} + \frac{1}{4}\right) = 12 \cdot \frac{14}{4} = 42.$$

Step 3: Conclusion The required value is $\boxed{42}$.

Quick Tip to Solve: Write both conics in standard form first, and keep semiaxis lengths separate from their squares when equating latus rectum lengths.

29. Let P_1 be a parabola with vertex (3, 2) and focus (4, 4) and P_2 be its mirror image with respect to the line x + 2y = 6. Then the directrix of P_2 is x + 2y = _____. [4 marks]

Solution:

Concept:

- A parabola consists of points equidistant from its focus and directrix. Its vertex lies halfway between its focus and the directrix along the axis.
- Reflection preserves distances, so reflecting both a focus and its directrix produces the focus and directrix of the reflected parabola. Parallel lines can be tracked using their common normal.

Step 1: Find the original axis and directrix The vertex is $V = (3, 2)$ and focus is $F = (4, 4)$. Thus the axis direction is $F - V = (1, 2)$, and a directrix perpendicular to this direction has equation $x + 2y = c$. The point on the directrix nearest the vertex is the reflection of the focus about the vertex:

$$D = 2V - F = (2, 0).$$

Substituting this point gives $c = 2$. Therefore the original directrix is $x + 2y = 2$, and its distance from the vertex is $5/\sqrt{5} = \sqrt{5}$, equal to the focus distance.

Step 2: Reflect the line about the given mirror Write $s = x + 2y$. Reflection about the parallel line $s = 6$ sends a line $s = c$ to $s = 12 - c$, since the two signed normal distances from the mirror must be equal and opposite. Hence

$$c_{\text{reflected}} = 2(6) - 2 = 10.$$

As a check, the vertex has $s = 7$ and reflects to $s = 5$, while the focus has $s = 12$ and reflects to $s = 0$. The reflected vertex is again midway between the reflected focus and the line $s = 10$.

Step 3: Conclusion The directrix of P_2 is $\boxed{x + 2y = 10}$. The required numerical entry is $\boxed{10}$.

Quick Tip to Solve: When the mirror is parallel to the directrix, reflect its line constant using $c \mapsto 2c_{\text{mirror}} - c$; find the original directrix first.

30. In an examination, there are 10 true-false type questions. Out of 10, a student can guess the answer of 4 questions correctly with probability $\frac{3}{4}$ and the remaining 6 questions correctly with probability $\frac{1}{4}$. If the probability that the student guesses the answers of exactly 8 questions correctly out of 10 is $\frac{27k}{4^{10}}$, then k is equal to _____. [4 marks]

Solution:

Concept:

- For independent guesses, multiply the probabilities of the individual outcomes and use binomial coefficients to count which questions have those outcomes.
- Exactly eight correct answers out of ten is equivalent to exactly two wrong answers. The two groups have different success probabilities, so they must be counted separately.

Step 1: List the possible placements of two failures Let j be the number of failures among the four questions answered correctly with probability $3/4$. Then $2-j$ failures occur among the other six questions, whose success probability is $1/4$. The possible values are $j=0,1,2$. For a fixed value, the number of arrangements is $\binom{4}{j}\binom{6}{2-j}$. The first group supplies $4-j$ successes and j failures, whereas the second supplies $4+j$ successes and $2-j$ failures.

Step 2: Add the three mutually exclusive probabilities Every term has denominator 4^{10}. The numerator receives a factor of 3 from each success in the first group and each failure in the second group. Therefore

$$P=\frac{1}{4^{10}}\sum_{j=0}^{2}\binom{4}{j}\binom{6}{2-j}3^{6-2j}.$$

Evaluating each term gives

$$P=\frac{15(729)+24(81)+6(9)}{4^{10}}=\frac{10935+1944+54}{4^{10}}.$$

Thus $P=12933/4^{10}$. These cases exhaust the possibilities without overlap: both failures are in the second group, one in each group, or both in the first group.

Step 3: Conclusion Comparing $12933/4^{10}$ with $27k/4^{10}$ gives $k=12933/27=\boxed{479}$. This is a dimensionless numerical entry.

Quick Tip to Solve: For a large number of correct answers, count the few failures instead. Keep groups with different success probabilities separate until the final sum.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Identify the pair of physical quantities that have same dimensions: [4 marks]

(A) velocity gradient and decay constant

(B) Wien's constant and Stefan constant

(C) angular frequency and angular momentum

(D) wave number and Avogadro number

Correct Answer: (A) velocity gradient and decay constant

Solution:

Concept:

- Dimensions express a quantity in terms of mass M, length L, time T, temperature Θ, and, where needed, amount of substance.
- Two quantities can have equal dimensions even when their physical meanings differ. Derive each dimension from the defining equation rather than comparing names.

Step 1: Check the velocity gradient and decay constant A velocity gradient is a change of velocity per unit distance, such as dv/dx. Thus

$$\left[\frac{dv}{dx}\right] = \frac{LT^{-1}}{L} = T^{-1}.$$

In radioactive decay, $N = N_0 e^{-\lambda t}$. The exponent must be dimensionless, which requires $[\lambda t] = 1$, hence $[\lambda] = T^{-1}$. Both members of the first pair therefore have the same dimensions, although one describes spatial variation of velocity and the other describes a fractional decay rate.

Step 2: Rule out the remaining pairs Wien's displacement law gives $\lambda_{\max}\Theta = b$, so $[b] = L\Theta$. Stefan's law for power radiated per area is $P/A = \sigma\Theta^4$, giving $[\sigma] = MT^{-3}\Theta^{-4}$; these differ. Angular frequency has dimensions T^{-1}, while angular momentum has dimensions ML^2T^{-1}, so the third pair also fails. Wave number has dimensions L^{-1}, whereas Avogadro number is a pure count (and the Avogadro constant is measured per mole). Neither interpretation supplies an inverse length, so the fourth pair fails as well.

Step 3: Conclusion Velocity gradient and decay constant both have dimensions $\boxed{T^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: An exponential argument must be dimensionless: this immediately gives the dimensions of any decay or growth constant.

32. The distance between Sun and Earth is R. The duration of year if the distance between Sun and Earth becomes 3R will be : [4 marks]

(A) $\sqrt{3}$ years

(B) 3 years

(C) 9 years

(D) $3\sqrt{3}$ years

Correct Answer: (D) $3\sqrt{3}$ years

Solution:

Concept:

- For orbits about the same central mass, Kepler's third law states that the square of the orbital period is proportional to the cube of the semi-major axis: $T^2 \propto R^3$.
- Comparing two orbits around the Sun cancels the gravitational constant and solar mass. A year here is the original Earth orbital period.

Step 1: Form a ratio of the periods Let the original period be $T_1 = 1\,\text{year}$ at distance $R_1 = R$, and the new period be T_2 at $R_2 = 3R$. Treating the given distances as the orbital radii, or equivalently as semi-major axes for the comparison, Kepler's law gives

$$\frac{T_2^2}{T_1^2} = \left(\frac{R_2}{R_1}\right)^3 = 3^3 = 27.$$

The central mass remains the Sun, so the same proportionality constant applies in both cases.

Step 2: Take the positive square root and compare options An orbital duration is positive, so

$$\frac{T_2}{T_1} = \sqrt{27} = 3\sqrt{3}.$$

Therefore $T_2 = 3\sqrt{3}\,\text{years}$, approximately 5.20 years. The alternatives $\sqrt{3}$, 3, and 9 years would give squared period ratios of 3, 9, and 81 respectively; none equals the required radius-cube ratio 27. Increasing the orbit size increases both the circumference and the time taken per unit distance, because orbital speed decreases with increasing radius.

Step 3: Conclusion The new duration of the year is $\boxed{3\sqrt{3}\,\text{years}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the exponent $3/2$ directly: multiplying orbital radius by a multiplies its period by $a^{3/2}$.

33. A stone of mass m, tied to a string is being whirled in a vertical circle with a uniform speed. The tension in the string is [4 marks]

(A) the same throughout the motion.

(B) minimum at the highest position of the circular path.

(C) minimum at the lowest position of the circular path.

(D) minimum when the rope is in the horizontal position.

Correct Answer: (B) minimum at the highest position of the circular path.

Solution:

Concept:

- For circular motion, the net inward radial force must be mv^2/r. Uniform speed makes this required radial force constant; it does not make each individual force constant.
- Tension always acts toward the centre, while the radial component of weight depends on the stone's position. At the highest point gravity helps supply the inward force most strongly.

Step 1: Write the equations at the highest and lowest points Let the circular radius be r and the given uniform speed be v. At the highest point both tension and weight are directed toward the centre, hence

$$T_{\text{top}} + mg = \frac{mv^2}{r}, \qquad T_{\text{top}} = \frac{mv^2}{r} - mg.$$

At the lowest point tension acts inward and weight outward, so

$$T_{\text{bottom}} - mg = \frac{mv^2}{r}, \qquad T_{\text{bottom}} = \frac{mv^2}{r} + mg.$$

Thus the bottom tension exceeds the top tension by $2mg$.

Step 2: Compare all positions on the circle When the string is horizontal, gravity is perpendicular to the inward radius, so $T_{\text{side}} = mv^2/r$. More generally, if θ is measured from the upward vertical radius, the inward component of weight is $mg\cos\theta$. Consequently

$$T = \frac{mv^2}{r} - mg\cos\theta.$$

This is smallest when $\cos\theta = 1$, at the top. It is therefore neither constant nor smallest at the bottom or side. This radial conclusion uses the stated uniform speed and presumes the string remains taut throughout the specified motion, requiring $v^2/r \geq g$ at the top.

Step 3: Conclusion The tension is minimum at the highest position of the circular path. Hence, the correct option is (B).

Quick Tip to Solve: At the top, gravity helps tension provide centripetal force; at the bottom it opposes tension, so the bottom tension is larger.

34. Two identical charged particles each having a mass 10 g and charge 2.0×10^{-7}C are placed on a horizontal table with a separation of L between them such that they stay in limited equilibrium. If the coefficient of friction between each particle and the table is 0.25, find the value of L. [Use g = 10 ms^{-2}] [4 marks]

(A) 12 cm

(B) 10 cm

(C) 8 cm

(D) 5 cm

Correct Answer: (A) 12 cm

Solution:

Concept:

- The repulsive force between equal charges q separated by L is $F_e = k_e q^2/L^2$, where $k_e = 9.0 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$.
- In limiting equilibrium, static friction has its maximum value $f_{\max} = \mu N$. The electric force here is horizontal, so vertical equilibrium gives $N = mg$ for each particle.

Step 1: Calculate the maximum friction on one particle Convert the mass before substituting: $m = 10\,\mathrm{g} = 0.010\,\mathrm{kg}$. With $g = 10\,\mathrm{m\,s^{-2}}$, the normal reaction is

$$N = mg = 0.010(10) = 0.10\,\mathrm{N}.$$

Hence the limiting friction is

$$f_{\max} = 0.25(0.10) = 0.025\,\mathrm{N}.$$

Analyse either particle separately: each experiences one outward Coulomb force and an equal inward friction force. There is no factor of two in this single-particle balance.

Step 2: Balance the repulsion and solve for separation At the threshold of slipping,

$$\frac{9.0 \times 10^9 (2.0 \times 10^{-7})^2}{L^2} = 0.025.$$

Squaring the charge first gives $q^2 = 4.0 \times 10^{-14}\,\mathrm{C^2}$, so

$$L^2 = \frac{3.6 \times 10^{-4}}{0.025} = 0.0144\,\mathrm{m^2}, \qquad L = 0.12\,\mathrm{m}.$$

In centimetres this is 12 cm. At 10, 8, or 5 cm the inverse-square repulsive force would be larger than the limiting friction, and equilibrium would be impossible; larger separations could allow non-limiting equilibrium.

Step 3: Conclusion The limiting separation is $\boxed{L = 12\,\mathrm{cm}}$. Hence, the correct option is (A).

Quick Tip to Solve: Balance forces on just one charge, convert grams to kilograms, and square the charge before evaluating the Coulomb force.

35. Two massless springs with spring constants 2 k and 9 k, carry 50 g and 100 g masses at their free ends. These two masses oscillate vertically such that their maximum velocities are equal. Then, the ratio of their respective amplitudes will be : [4 marks]

(A) 1 : 2

(B) 3 : 2

(C) 3 : 1

(D) 2 : 3

Correct Answer: (B) 3 : 2

Solution:

Concept:

- A mass on an ideal spring oscillates about its equilibrium position with angular frequency $\omega = \sqrt{K/m}$, where K is the spring constant.
- The maximum speed in simple harmonic motion is $v_{\max} = A\omega$. Gravity shifts the equilibrium of a vertical spring but does not alter this frequency.

Step 1: Compare the angular frequencies For the first oscillator, $K_1 = 2k$ and $m_1 = 50\,\text{g}$; for the second, $K_2 = 9k$ and $m_2 = 100\,\text{g}$. Both masses may be left in grams in this ratio because the conversion factor cancels:

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{K_2/m_2}{K_1/m_1}} = \sqrt{\frac{9k}{100}\frac{50}{2k}} = \sqrt{\frac{9}{4}} = \frac{3}{2}.$$

Although the second mass is larger, the greater spring stiffness more than compensates, giving the second oscillator the larger angular frequency.

Step 2: Use the equality of maximum speeds The stated equality is $A_1\omega_1 = A_2\omega_2$. Rearranging it yields

$$\frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \frac{3}{2}.$$

Thus the lower-frequency oscillator needs a larger amplitude to reach the same maximum speed. The inverse ratio $2 : 3$ would apply to the frequencies in the order first to second, not to the amplitudes. Ratios $1 : 2$ and $3 : 1$ neglect the combined square-root dependence on both stiffness and mass and do not satisfy the speed equality.

Step 3: Conclusion The respective amplitudes are in the ratio $\boxed{A_1 : A_2 = 3 : 2}$. Hence, the correct option is (B).

Quick Tip to Solve: Equal maximum speeds mean amplitudes are inversely proportional to angular frequencies: use $A \propto \sqrt{m/K}$.

36. What will be the most suitable combination of three resistors A = 2Ω, B = 4Ω, C = 6Ω so that $\left(\frac{22}{3}\right)\Omega$ is equivalent resistance of combination? [4 marks]

(A) Parallel combination of A and C connected in series with B.

(B) Parallel combination of A and B connected in series with C.

(C) Series combination of A and C connected in parallel with B.

(D) Series combination of B and C connected in parallel with A.

Correct Answer: (B) Parallel combination of A and B connected in series with C.

Solution:

Concept:

- For resistors in series, the same current passes through each and their equivalent resistance is the sum: $R_s = R_1 + R_2$.
- For two resistors in parallel, their potential differences are equal and $R_p = R_1R_2/(R_1 + R_2)$. Reduce the parallel pair first, then add any series resistor.

Step 1: Calculate the proposed parallel pair The target resistance is $22/3\,\Omega$, which exceeds $6\,\Omega$. Try the combination with the largest resistor, C, in series with the parallel pair A and B. Their equivalent resistance is

$$R_{AB} = \frac{2 \times 4}{2+4} = \frac{8}{6} = \frac{4}{3}\,\Omega.$$

This is smaller than either member of the parallel pair, as required physically.

Step 2: Add C and compare every alternative Putting C in series gives

$$R = R_{AB} + R_C = \frac{4}{3} + 6 = \frac{22}{3}\,\Omega.$$

For option (A), the result instead is $(2 \times 6)/(2 + 6) + 4 = 11/2\,\Omega$. For option (C), the series pair has resistance $8\,\Omega$, so its parallel combination with B is $8 \times 4/(8 + 4) = 8/3\,\Omega$. For option (D), the corresponding value is $10 \times 2/(10 + 2) = 5/3\,\Omega$. Thus none of these equals the target; the placement of the series resistor is decisive.

Step 3: Conclusion The parallel combination of A and B must be connected in series with C, giving $R_{\text{eq}} = 22/3\,\Omega$. Hence, the correct option is (B).

Quick Tip to Solve: When the target exceeds every individual resistance, first try the largest resistor in series with a reduced parallel pair; a wholly parallel final stage cannot give that target.

37. The soft-iron is a suitable material for making an electromagnet. This is because soft-iron has

[4 marks]

(A) low coercivity and high retentivity.

(B) low coercivity and low permeability.

(C) high permeability and low retentivity.

(D) high permeability and high retentivity.

Correct Answer: (C) high permeability and low retentivity.

Solution:

Concept:

- Permeability measures how readily a magnetic field is established inside a material. A core with high permeability gives a large magnetic induction for a modest magnetising field.
- Retentivity describes the magnetisation left after the magnetising field is removed. An electromagnet should lose most of its magnetisation when its current is switched off.

Step 1: Identify the property needed while current flows A current in the coil produces a magnetising field H. The soft-iron core concentrates the magnetic flux because its permeability is high; in the approximately linear description, $B = \mu H$. Thus a large μ makes a strong electromagnet without requiring an excessively large coil current. Low permeability, as stated in option (B), would work against this purpose.

Step 2: Identify the property needed when current stops An electromagnet must be controllable: it should attract strongly when energised and release its load when de-energised. Soft iron has low retentivity in the standard comparison with permanent-magnet materials, so little residual magnetisation remains after the applied field is removed. Its low coercivity also makes its magnetisation easy to reverse or remove, but option (A) incorrectly couples that feature with high retentivity. Option (D) likewise specifies high retentivity, which is useful for a permanent magnet rather than a readily switched electromagnet. Only option (C) combines easy magnetisation with low residual magnetism.

Step 3: Conclusion Soft iron is suitable because it has high permeability and low retentivity. Hence, the correct option is (C).

Quick Tip to Solve: For an electromagnet, ask two separate questions: does the core magnetise strongly with current, and does it release its magnetism when the current stops?

38. A proton, a deutron and an α-particle with same kinetic energy enter into a uniform magnetic field at right angle to magnetic field. The ratio of the radii of their respective circular paths is : [4 marks]

(A) $1 : \sqrt{2} : \sqrt{2}$

(B) $1 : 1 : \sqrt{2}$

(C) $\sqrt{2} : 1 : 1$

(D) $1 : \sqrt{2} : 1$

Correct Answer: (D) $1 : \sqrt{2} : 1$

Solution:

Concept:

- A particle entering perpendicular to a uniform magnetic field follows a circle because the magnetic force provides centripetal force: $|q|vB = mv^2/r$. Hence $r = mv/(|q|B)$.
- For nonrelativistic motion, $K = mv^2/2$, so the momentum is $mv = \sqrt{2mK}$. At equal kinetic energy and field, the radius is proportional to $\sqrt{m}/|q|$.

Step 1: Express each radius relative to the proton radius Let the proton mass and charge magnitude be m and e. The deuteron has approximately mass $2m$ and charge e, while the alpha particle has mass $4m$ and charge $2e$. With the same K and B,

$$r_p = \frac{\sqrt{2mK}}{eB}, \qquad \frac{r_d}{r_p} = \sqrt{2}.$$

The greater mass increases the deuteron momentum at the fixed energy.

Step 2: Include the alpha particle charge For the alpha particle the mass factor raises the momentum by $\sqrt{4} = 2$, but its charge is also doubled. These effects cancel:

$$\frac{r_\alpha}{r_p} = \frac{\sqrt{4}}{2} = 1.$$

Therefore the three radii, in the stated order proton, deuteron and alpha particle, are $1 : \sqrt{2} : 1$. Options (A) and (B) give the alpha particle a larger radius than the proton and ignore this cancellation. Option (C) makes the proton radius larger than the deuteron radius, contrary to the $\sqrt{m}$ dependence at equal energy.

Step 3: Conclusion The required ratio is $r_p : r_d : r_\alpha = 1 : \sqrt{2} : 1$. Hence, the correct option is (D).

Quick Tip to Solve: Equal kinetic energy means use $\sqrt{m}/q$, whereas equal speed would mean use m/q; do not interchange these two conditions.

39. Given below are two statements :

Statement I : The reactance of an ac circuit is zero. It is possible that the circuit contains a capacitor and an inductor.

Statement II : In ac circuit, the average power delivered by the source never becomes zero.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (C) Statement I is true but Statement II is false.

Solution:

Concept:

- For a series circuit containing inductance L and capacitance C, the net reactance is $X = \omega L - 1/(\omega C)$. The individual reactances have opposite signs.
- The average power delivered by a sinusoidal source is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$, where ϕ is the phase difference between voltage and current.

Step 1: Test Statement I at resonance At the frequency satisfying $\omega L = 1/(\omega C)$, the inductive and capacitive reactances cancel. Equivalently,

$$\omega = \frac{1}{\sqrt{LC}}, \qquad X = 0.$$

Both components are still physically present; zero net reactance does not mean that neither component exists. A series RLC circuit at resonance therefore supplies an explicit example proving Statement I true.

Step 2: Test the universal claim in Statement II A purely inductive or a purely capacitive ideal AC circuit has $|\phi| = 90^\circ$. Consequently,

$$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos 90^\circ = 0.$$

Energy is stored in the field during part of the cycle and returned to the source during another part; it is not dissipated over a complete cycle. This counterexample is sufficient to disprove the word "never" in Statement II. Thus option (A) is excluded because II is false, option (B) because I is true, and option (D) because it reverses both truth values.

Step 3: Conclusion Statement I is true, but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: A claim that average AC power can never vanish is immediately tested using an ideal pure inductor or capacitor, for which the power factor is zero.

40. Potential energy as a function of r is given by $U = \frac{A}{r^{10}} - \frac{B}{r^5}$, where r is the interatomic distance, A and B are positive constants. The equilibrium distance between the two atoms will be : [4 marks]

(A) $\left(\frac{A}{B}\right)^{\frac{1}{5}}$

(B) $\left(\frac{B}{A}\right)^{\frac{1}{5}}$

(C) $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$

(D) $\left(\frac{B}{2A}\right)^{\frac{1}{5}}$

Correct Answer: (C) $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$

Solution:

Concept:

- For a central conservative interaction, the radial force is $F_r = -dU/dr$. A finite equilibrium separation therefore satisfies $dU/dr = 0$.
- A stable equilibrium is a local minimum of the potential energy, verified by $d^2U/dr^2 > 0$. The physical separation must be a positive real number.

Step 1: Differentiate and set the force to zero Treat A and B as fixed positive constants and differentiate each inverse power:

$$\frac{dU}{dr} = -\frac{10A}{r^{11}} + \frac{5B}{r^6}.$$

At equilibrium the derivative vanishes. Multiplying by the positive quantity r^{11} gives

$$-10A + 5Br^5 = 0, \qquad r^5 = \frac{2A}{B}.$$

Taking the positive fifth root selects the physically meaningful interatomic distance.

Step 2: Confirm stability and eliminate the other expressions The second derivative is

$$\frac{d^2U}{dr^2} = \frac{110A}{r^{12}} - \frac{30B}{r^7}.$$

At $r = r_0$, the equilibrium relation gives $Br_0^5 = 2A$, so

$$U''(r_0) = \frac{110A - 60A}{r_0^{12}} = \frac{50A}{r_0^{12}} > 0.$$

Thus this stationary point is a stable minimum. Option (A) omits the factor 2 generated by differentiating the powers 10 and 5. Options (B) and (D) invert the ratio of the constants and do not satisfy the zero-force equation.

Step 3: Conclusion The equilibrium distance is $r_0 = (2A/B)^{1/5}$, and it corresponds to stable equilibrium. Hence, the correct option is (C).

Quick Tip to Solve: Differentiate before balancing the two inverse-power terms: equilibrium balances their forces, so the exponents contribute numerical coefficients.

41. An object of mass 5 kg is thrown vertically upwards from the ground. The air resistance produces a constant retarding force of 10 N throughout the motion. The ratio of time of ascent to the time of descent will be equal to : [Use g = 10 ms^{-2}]. [4 marks]

(A) 1 : 1

(B) $\sqrt{2} : \sqrt{3}$

(C) $\sqrt{3} : \sqrt{2}$

(D) 2 : 3

Correct Answer: (B) $\sqrt{2} : \sqrt{3}$

Solution:

Concept:

- Air resistance opposes the instantaneous velocity. It acts downwards during ascent and upwards during descent, while weight acts downwards throughout.
- For constant acceleration starting or ending at rest over a height h, the travel time is $t = \sqrt{2h/a}$, with a the appropriate positive acceleration magnitude.

Step 1: Find the acceleration on each leg The weight is $mg = 5 \times 10 = 50\,\text{N}$. During upward motion, both weight and the $10\,\text{N}$ resistance point downwards. The upward speed therefore decreases with magnitude

$$a_{\text{up}} = \frac{50 + 10}{5} = 12\,\text{m s}^{-2}.$$

During downward motion, resistance is upward, so the net downward acceleration is

$$a_{\text{down}} = \frac{50 - 10}{5} = 8\,\text{m s}^{-2}.$$

The reversal of drag is essential; it cannot be assigned the same downward direction on both legs.

Step 2: Compare times over the common maximum height Let h be the height reached above the ground. At the top the vertical velocity is zero. Applying constant-acceleration kinematics backwards from that point for ascent and forwards for descent gives

$$h = \tfrac{1}{2} a_{\text{up}} t_{\text{up}}^2 = \tfrac{1}{2} a_{\text{down}} t_{\text{down}}^2.$$

Hence

$$\frac{t_{\text{up}}}{t_{\text{down}}} = \sqrt{\frac{8}{12}} = \sqrt{\frac{2}{3}}.$$

Equal times in (A) ignore drag; (C) reverses the faster and slower legs. Option (D) uses the acceleration ratio directly instead of its square root.

Step 3: Conclusion Ascent is faster because its deceleration is larger. The ratio of ascent time to descent time is $\sqrt{2} : \sqrt{3}$. Hence, the correct option is (B).

Quick Tip to Solve: For unequal constant accelerations over the same height, times scale as the inverse square root of acceleration, not its inverse.

42. A fly wheel is accelerated uniformly from rest and rotates through 5 rad in the first second. The angle rotated by the fly wheel in the next second, will be : [4 marks]

(A) 7.5 rad

(B) 15 rad

(C) 20 rad

(D) 30 rad

Correct Answer: (B) 15 rad

Solution:

Concept:

- For constant angular acceleration α, angular displacement from an initial angular velocity ω_0 is $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$.
- An angle traversed during a particular interval is the difference between cumulative angular displacements at its ending and starting times. Here "the next second" means from $t = 1$ to $t = 2\,\text{s}$.

Step 1: Determine the angular acceleration from the first second The flywheel starts from rest, so $\omega_0 = 0$. Using the displacement $5\,\text{rad}$ in the first $1\,\text{s}$,

$$5 = \tfrac{1}{2}\alpha(1)^2, \qquad \alpha = 10\,\text{rad}\,\text{s}^{-2}.$$

This value remains constant in the next second. The angular velocity at one second is $\omega(1) = \alpha(1) = 10\,\text{rad}\,\text{s}^{-1}$, rather than zero.

Step 2: Subtract the cumulative angles At two seconds the angle measured from the initial position is

$$\theta(2) = \tfrac{1}{2}(10)(2)^2 = 20\,\text{rad}.$$

Therefore the angle during the next second alone is

$$\Delta\theta = \theta(2) - \theta(1) = 20 - 5 = 15\,\text{rad}.$$

As a check, the angular velocity rises from 10 to $20\,\text{rad}\,\text{s}^{-1}$ over that interval, giving average angular velocity $15\,\text{rad}\,\text{s}^{-1}$. Option (C) is the cumulative angle after two seconds, not the requested interval angle. Neither 7.5 nor $30\,\text{rad}$ satisfies the constant-acceleration interval calculation.

Step 3: Conclusion The flywheel rotates through $15\,\text{rad}$ in the next second. Hence, the correct option is (B).

Quick Tip to Solve: From rest under uniform acceleration, angles in successive equal time intervals are in the ratio $1 : 3 : 5 : \cdots$; the second-second angle is three times the first.

43. A 100 g of iron nail is hit by a 1.5 kg hammer striking at a velocity of 60 ms^{-1}. What will be the rise in the temperature of the nail if one fourth of energy of the hammer goes into heating the nail? [Specific heat capacity of iron = 0.42 $Jg^{-1}\,°C^{-1}$] [4 marks]

(A) 675°C

(B) 1600°C

(C) 16.07°C

(D) 6.75°C

Correct Answer: (C) 16.07°C

Solution:

Concept:

- The hammer initially carries translational kinetic energy $K = \frac{1}{2}Mv^2$, where M is its mass and v its speed immediately before striking.
- Heat raises the temperature of the nail according to $Q = mc\Delta T$. Only the stated fraction of the hammer energy is transferred as heat to the nail; the remaining energy does not enter this heat balance.

Step 1: Calculate the energy transferred The hammer mass is $M = 1.5\,\text{kg}$ and its speed is $60\,\text{m}\,\text{s}^{-1}$. Thus its kinetic energy is

$$K = \frac{1}{2}(1.5)(60)^2 = 2700\,\text{J}.$$

One fourth heats the nail, so the heat received is

$$Q = \frac{2700}{4} = 675\,\text{J}.$$

This is an energy, not a temperature rise; selecting the numerical value 675 as a temperature would ignore the nail heat capacity.

Step 2: Apply the nail heat capacity The specific heat is given per gram, so use the nail mass as $m = 100\,\text{g}$ together with $c = 0.42\,\text{J}\,\text{g}^{-1}\,°\text{C}^{-1}$. Its heat capacity is

$$mc = (100)(0.42) = 42\,\text{J}\,°\text{C}^{-1}.$$

Hence

$$\Delta T = \frac{Q}{mc} = \frac{675}{42} = 16.0714\ldots\,°\text{C}.$$

Using grams with this specific heat avoids an erroneous factor of 1000. Options (A), (B), and (D) do not satisfy $mc\Delta T = 675\,\text{J}$.

Step 3: Conclusion The temperature rise of the nail is $\boxed{16.07\,°\text{C}}$. Hence, the correct option is (C).

Quick Tip to Solve: First reduce the hammer energy by the heating fraction, then divide by the nail heat capacity. Keep grams with a specific heat quoted per gram.

44. If the charge on a capacitor is increased by 2 C, the energy stored in it increases by 44%. The original charge on the capacitor is (in C) [4 marks]

(A) 10

(B) 20

(C) 30

(D) 40

Correct Answer: (A) 10

Solution:

Concept:

- For a capacitor of fixed capacitance C, the energy is $U = Q^2/(2C)$, where Q denotes the magnitude of the charge on either plate.
- A percentage increase of 44 percent makes the final energy $1.44U$. Because energy depends on the square of charge, the charge ratio is the positive square root of the energy ratio.

Step 1: Write the ratio of energies Let the original charge be Q coulombs. After adding $2\,\mathrm{C}$, the final charge magnitude is $Q + 2$. The capacitance is unchanged, so it cancels from the ratio:

$$\frac{U_f}{U_i} = \frac{(Q+2)^2/(2C)}{Q^2/(2C)} = \left(\frac{Q+2}{Q}\right)^2.$$

The stated increase means $U_f - U_i = 0.44U_i$, or $U_f/U_i = 1.44$. It does not mean that the final energy is only 44 percent of the initial energy.

Step 2: Solve for the positive charge magnitude Taking the positive square root, since both charge magnitudes are positive, gives

$$\frac{Q+2}{Q} = \sqrt{1.44} = 1.2.$$

Therefore

$$2 = 0.2Q, \qquad Q = 10\,\mathrm{C}.$$

As a direct check, the charge changes from 10 to 12 C, and $(12/10)^2 - 1 = 0.44$. The charges 20, 30 and 40 C would give fractional energy increases 0.21, approximately 0.1378 and 0.1025 respectively. Thus none of the other offered values produces the required increase.

Step 3: Conclusion The original charge is $\boxed{10\,\mathrm{C}}$. Hence, the correct option is (A).

Quick Tip to Solve: For fixed capacitance, take the square root of the energy multiplier before finding the charge increment; a 44 percent energy rise means a 20 percent charge rise.

45. A long cylindrical volume contains a uniformly distributed charge of density ρ. The radius of cylindrical volume is R. A charge particle (q) revolves around the cylinder in a circular path. The kinetic energy of the particle is : [4 marks]

(A) $\frac{\rho q R^2}{4\varepsilon_0}$

(B) $\frac{\rho q R^2}{2\varepsilon_0}$

(C) $\frac{q\rho}{4\varepsilon_0 R^2}$

(D) $\frac{4\varepsilon_0 R^2}{q\rho}$

Correct Answer: (A) $\frac{\rho q R^2}{4\varepsilon_0}$

Solution:

Concept:

- Outside a long uniformly charged cylinder, a coaxial Gaussian surface gives $E(r) = \rho R^2/(2\varepsilon_0 r)$ in magnitude, where R is the cylinder radius and $r \geq R$ the orbital radius.
- For circular motion, the inward electric force supplies mv^2/r. The kinetic energy is $K = mv^2/2$. A freely revolving particle must be attracted to the cylinder; the charge product in the options represents the magnitude of that attraction.

Step 1: Find the electric field outside the cylinder Take a Gaussian cylinder of radius r and length L, coaxial with the charged volume. Its enclosed charge magnitude is $\rho\pi R^2 L$. The curved surface contributes the flux, while the end faces contribute none. Thus

$$E(2\pi rL) = \frac{\rho\pi R^2 L}{\varepsilon_0}, \qquad E = \frac{\rho R^2}{2\varepsilon_0 r}.$$

The radius of the charged material remains R; it must not be replaced by the orbital radius in the enclosed charge.

Step 2: Use the centripetal force condition Equate the magnitude of the attractive electric force to the required centripetal force:

$$\frac{mv^2}{r} = qE = \frac{q\rho R^2}{2\varepsilon_0 r}.$$

The orbital radius cancels, leaving

$$mv^2 = \frac{q\rho R^2}{2\varepsilon_0}, \qquad K = \frac{1}{2}mv^2 = \frac{q\rho R^2}{4\varepsilon_0}.$$

Option (B) is twice the kinetic energy because it omits the final factor one half. Options (C) and (D) have the wrong dependence on R, and do not have the dimensions of energy. If signed charges are used, the positive energy contains $|q\rho|$.

Step 3: Conclusion The kinetic energy, with $q\rho$ understood as the magnitude of the attractive charge product, is $\boxed{\rho q R^2/(4\varepsilon_0)}$. Hence, the correct option is (A).

Quick Tip to Solve: Outside a long charged cylinder the electric force varies as inverse orbital radius, so multiplying the centripetal equation by that radius removes it from the kinetic energy.

46. An electric bulb is rated as 200 W. What will be the peak magnetic field at 4 m distance produced by the radiations coming from this bulb? Consider this bulb as a point source with 3.5% efficiency.

[4 marks]

(A) 1.19×10^{-8}T

(B) 1.71×10^{-8}T

(C) 0.84×10^{-8}T

(D) 3.36×10^{-8}T

Correct Answer: (B) 1.71×10^{-8}T

Solution:

Concept:

- An isotropic point source of radiated power P gives average intensity $I = P/(4\pi r^2)$ at distance r. The useful radiated power is the efficiency times the electrical input power.
- For an electromagnetic wave, average intensity and peak magnetic field obey $I = cB_0^2/(2\mu_0)$. The factor two distinguishes peak amplitude B_0 from its root mean square value.

Step 1: Calculate the intensity of the radiation The electrical rating is $200\,\mathrm{W}$, but only 3.5 percent is emitted as the radiation under consideration. Therefore

$$P = 0.035(200) = 7\,\mathrm{W}.$$

At $r = 4\,\mathrm{m}$, this power is spread over a spherical surface of area $4\pi(4)^2 = 64\pi\,\mathrm{m}^2$. Hence

$$I = \frac{7}{64\pi}\,\mathrm{W\,m^{-2}} \approx 0.0348\,\mathrm{W\,m^{-2}}.$$

Using all 200 W would disregard the efficiency and overestimate the field.

Step 2: Convert intensity to the peak field Rearrange the wave intensity relation and substitute $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$ and $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$:

$$B_0^2 = \frac{2\mu_0 I}{c} = \frac{2(4\pi \times 10^{-7})}{3 \times 10^8}\frac{7}{64\pi}.$$

$$B_0^2 = 2.9167 \times 10^{-16}\,\mathrm{T}^2, \qquad B_0 = 1.7078 \times 10^{-8}\,\mathrm{T}.$$

The value in option (A) is close to the root mean square field, rather than the requested peak field. Options (C) and (D) also fail the intensity relation. Taking a square root only after substituting all factors avoids confusing field and intensity ratios.

Step 3: Conclusion The peak magnetic field is $\boxed{1.71 \times 10^{-8}\,\mathrm{T}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the radiated power after applying efficiency, and remember that the peak magnetic field satisfies an intensity formula containing a factor one half.

47. The light of two different frequencies whose photons have energies 3.8 eV and 1.4 eV respectively, illuminate a metallic surface whose work function is 0.6 eV successively. The ratio of maximum speeds of emitted electrons for the two frequencies respectively will be : [4 marks]

(A) 1 : 1

(B) 2 : 1

(C) 4 : 1

(D) 1 : 4

Correct Answer: (B) 2 : 1

Solution:

Concept:

- Einstein photoelectric equation gives the maximum emitted kinetic energy as $K_{\max} = h\nu - \phi$, where $h\nu$ is the photon energy and ϕ the work function.
- For the same electron mass, $K_{\max} = m_e v_{\max}^2/2$, so the ratio of maximum speeds is the square root of the ratio of excess photon energies above the work function.

Step 1: Subtract the work function from each photon energy Both incident photon energies exceed $\phi = 0.6\,\text{eV}$, so both illuminations emit electrons. For the first frequency,

$$K_1 = 3.8 - 0.6 = 3.2\,\text{eV}.$$

For the second frequency,

$$K_2 = 1.4 - 0.6 = 0.8\,\text{eV}.$$

The work function must be subtracted separately in each case: the photon energy ratio itself is not the kinetic energy ratio. The energies can stay in electronvolts because the common joule conversion cancels in the ratio.

Step 2: Take the square root of the kinetic energy ratio The electron mass is the same in both experiments. Dividing their kinetic energy equations gives

$$\frac{K_1}{K_2} = \frac{v_1^2}{v_2^2} = \frac{3.2}{0.8} = 4.$$

Consequently

$$\frac{v_1}{v_2} = \sqrt{4} = 2.$$

The faster electrons are emitted by the higher-energy photons, as expected. Option (A) would require equal excess energies. Option (C) confuses the kinetic energy ratio with the speed ratio, while option (D) reverses the trend and also omits the square root. These checks select a unique speed ratio.

Step 3: Conclusion The ratio of maximum emitted electron speeds, in the order given, is $\boxed{2:1}$. Hence, the correct option is (B).

Quick Tip to Solve: Subtract the work function first, then take the square root of the excess-energy ratio. Photon-energy ratios do not directly give electron-speed ratios.

48. Two light beams of intensities in the ratio of 9 : 4 are allowed to interfere. The ratio of the intensity of maxima and minima will be : [4 marks]

(A) 2 : 3

(B) 16 : 81

(C) 25 : 169

(D) 25 : 1

Correct Answer: (D) 25 : 1

Solution:

Concept:

- For coherent beams, intensity is proportional to the square of the wave amplitude. Thus $I_1 : I_2 = 9 : 4$ corresponds to an amplitude ratio $a_1 : a_2 = 3 : 2$.
- Constructive interference adds amplitudes and destructive interference subtracts them. Therefore $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$.

Step 1: Determine the amplitude ratio Write the separate intensities as $I_1 = 9I_0$ and $I_2 = 4I_0$, where I_0 is a common positive intensity scale. The corresponding amplitude factors are proportional to

$$\sqrt{I_1} = 3\sqrt{I_0}, \qquad \sqrt{I_2} = 2\sqrt{I_0}.$$

It is these square roots, rather than the numbers 9 and 4 themselves, that must be combined. Equal-amplitude cancellation is impossible here because the incident intensities differ, so the minimum intensity is nonzero.

Step 2: Form the maximum and minimum intensities At a maximum the waves have zero relative phase modulo 2π, giving

$$I_{\max} = (3\sqrt{I_0} + 2\sqrt{I_0})^2 = 25I_0.$$

At a minimum they have relative phase π, giving

$$I_{\min} = (3\sqrt{I_0} - 2\sqrt{I_0})^2 = I_0.$$

Dividing removes the arbitrary intensity scale:

$$I_{\max} : I_{\min} = 25 : 1.$$

A maximum-to-minimum ratio must exceed one, which immediately rejects (A), (B) and (C); each of those is less than one. Direct calculation confirms that (D) also has the correct magnitude, rather than merely the correct ordering.

Step 3: Conclusion The ratio of maximum to minimum intensity is $\boxed{25 : 1}$. Hence, the correct option is (D).

Quick Tip to Solve: Take square roots of the separate intensities, add or subtract those amplitudes, and only then square. The minimum vanishes only for equal incident intensities.

49. In Bohr's atomic model of hydrogen, let K, P and E are the kinetic energy, potential energy and total energy of the electron respectively. Choose the correct option when the electron undergoes transitions to a higher level : [4 marks]

(A) All K, P and E increase.

(B) K decreases, P and E increase.

(C) P decreases, K and E increase.

(D) K increases, P and E decrease.

Correct Answer: (B) K decreases, P and E increase.

Solution:

Concept:

- For hydrogen in Bohr level n, $K_n = 13.6/n^2\,\text{eV}$, $P_n = -27.2/n^2\,\text{eV}$, and $E_n = -13.6/n^2\,\text{eV}$. The potential energy reference is zero at infinite separation.
- The Coulomb bound orbit obeys $P = -2K$ and $E = -K$. A higher level means a larger principal quantum number, a larger orbital radius, and a less tightly bound electron.

Step 1: Compare the kinetic energies Let the electron move from n_1 to n_2, with $n_2 > n_1$. Since $1/n_2^2 < 1/n_1^2$,

$$K_{n_2} = \frac{13.6}{n_2^2}\,\text{eV} < \frac{13.6}{n_1^2}\,\text{eV} = K_{n_1}.$$

The kinetic energy is positive and decreases as the orbit expands. Thus statements that kinetic energy increases, namely options (A), (C), and (D), cannot describe the transition.

Step 2: Track the signed potential and total energies Both potential and total energies have negative coefficients:

$$P_n = -\frac{27.2}{n^2}\,\text{eV}, \qquad E_n = -\frac{13.6}{n^2}\,\text{eV}.$$

Increasing n decreases their absolute magnitudes, making each signed value less negative and therefore larger. For example, a transition from $n = 1$ to $n = 2$ changes P from -27.2 to -6.8 eV and E from -13.6 to -3.4 eV. This illustrates the distinction between a decrease in binding-energy magnitude and an increase in actual total energy. Energy absorption is required for the upward transition, consistently giving $\Delta E > 0$.

Step 3: Conclusion On transition to a higher level, K decreases, while P and E increase. Hence, the correct option is (B).

Quick Tip to Solve: Keep the signs of bound-state energies: moving a negative value towards zero is an increase, even though its magnitude decreases.

50. A Carnot engine takes 5000 kcal of heat from a reservoir at 727°C and gives heat to a sink at 127°C. The work done by the engine is [4 marks]

(A) 3×10^6 J

(B) Zero

(C) 12.6×10^6 J

(D) 8.4×10^6 J

Correct Answer: (C) 12.6×10^6 J

Solution:

Concept:

- The efficiency of a reversible Carnot engine depends on the absolute reservoir temperatures: $\eta = 1 - T_C/T_H$. Temperatures must be in kelvin, because heat ratios cannot be formed from Celsius readings.
- The work delivered per cycle is $W = \eta Q_H$, where Q_H is the heat absorbed. Energy conservation also gives $Q_H = W + Q_C$, with Q_C the rejected heat.

Step 1: Convert the reservoir temperatures The hot and cold reservoir temperatures are respectively

$$T_H = 727 + 273 = 1000\,\text{K}, \qquad T_C = 127 + 273 = 400\,\text{K}.$$

Therefore the fraction of the input heat that can become useful work is

$$\eta = 1 - \frac{400}{1000} = 0.60.$$

The remaining fraction, 0.40, must be rejected to the sink even for this reversible engine.

Step 2: Convert heat to joules and calculate work Using $1\,\text{kcal} = 4.2 \times 10^3$ J, the heat absorbed is

$$Q_H = 5000(4.2 \times 10^3) = 21.0 \times 10^6\,\text{J}.$$

Hence

$$W = 0.60(21.0 \times 10^6) = 12.6 \times 10^6\,\text{J}.$$

Zero work would require equal reservoir temperatures, so option (B) is excluded. Option (D), 8.4×10^6 J, is the heat rejected rather than the work; option (A) agrees with neither the efficiency nor the heat balance.

Step 3: Conclusion The engine delivers 12.6×10^6 J of work. Hence, the correct option is (C).

Quick Tip to Solve: Convert Celsius to kelvin before finding Carnot efficiency; multiply input heat by efficiency, not by the rejected-heat fraction.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A body is projected from the ground at an angle of 45° with the horizontal. Its velocity after 2s is 20 ms^{-1}. The maximum height reached by the body during its motion is _____ m. (use g = 10 ms^{-2}) [4 marks]

Solution:

Concept:

- In projectile motion without air resistance the horizontal velocity is constant, while the vertical velocity decreases by gt. At a launch angle of 45°, the two initial components are equal.
- The maximum height above the launch point follows from vertical motion: $H = u_y^2/(2g)$. A stated speed at a later instant is the magnitude of both velocity components together.

Step 1: Use the speed after two seconds Let each initial velocity component be $a = u/\sqrt{2}$, so $u_x = u_y = a$. After $2\,\mathrm{s}$, gravity has reduced only the vertical component:

$$v_x = a, \qquad v_y = a - 10(2) = a - 20.$$

The given speed is $20\,\mathrm{m\,s^{-1}}$, and hence

$$a^2 + (a-20)^2 = 20^2.$$

Expanding gives

$$2a^2 - 40a = 0, \qquad a(a-20) = 0.$$

A body projected upward at the stated nonzero angle has $a > 0$, so $a = 20\,\mathrm{m\,s^{-1}}$; the zero root does not describe the stated launch.

Step 2: Find the height from the initial vertical component At the highest point the vertical velocity is zero. Apply $v_y^2 = u_y^2 - 2gH$, using the vertical component rather than the total launch speed:

$$0 = 20^2 - 2(10)H.$$

Thus

$$H = \frac{400}{20} = 20\,\mathrm{m}.$$

As a check, the time to the highest point is $u_y/g = 20/10 = 2\,\mathrm{s}$, exactly the given observation time. There the entire speed is horizontal, of magnitude $20\,\mathrm{m\,s^{-1}}$, so the result satisfies the supplied speed as well.

Step 3: Conclusion The maximum height is $20\,\mathrm{m}$; the numerical entry is $\boxed{20}$.

Quick Tip to Solve: For a 45-degree projectile set the initial components equal, then square and add the later horizontal and vertical components before using the height formula.

52. Two travelling waves of equal amplitudes and equal frequencies move in opposite directions along a string. They interfere to produce a stationary wave whose equation is given by $y = (10\cos\pi x \sin\frac{2\pi t}{T})$ cm

The amplitude of the particle at $x = \frac{4}{3}$ cm will be _____ cm. [4 marks]

Solution:

Concept:

- A stationary wave can be written as $y = A(x)\sin(\omega t)$, where its amplitude depends on position. The amplitude is the nonnegative magnitude of the coefficient of the time-dependent sine.
- A negative spatial coefficient reverses the phase of oscillation; it does not give a negative amplitude. Nodes have zero coefficient and antinodes have its largest magnitude.

Step 1: Identify the spatial amplitude factor The equation supplied is

$$y = 10\cos(\pi x)\sin\left(\frac{2\pi t}{T}\right)\ \text{cm}.$$

At any fixed position x, the cosine factor is a constant while the sine varies between -1 and $+1$. Consequently the greatest distance from equilibrium is

$$A(x) = 10|\cos(\pi x)|\ \text{cm}.$$

The value 10 is the antinode amplitude of the resulting stationary wave, so there is no additional factor of two to insert. The interference has already been incorporated into the given equation.

Step 2: Substitute the stated position Use $x = 4/3$ in the coordinate convention of the supplied equation, with x measured in centimetres:

$$\cos\left(\pi\frac{4}{3}\right) = \cos\left(\pi + \frac{\pi}{3}\right) = -\frac{1}{2}.$$

Thus the displacement of this particular particle is

$$y = -5\sin\left(\frac{2\pi t}{T}\right)\ \text{cm}.$$

Its displacement ranges from $-5\,\text{cm}$ to $+5\,\text{cm}$. The coefficient sign signifies a phase difference of π relative to a particle with positive spatial factor; the positive amplitude is therefore

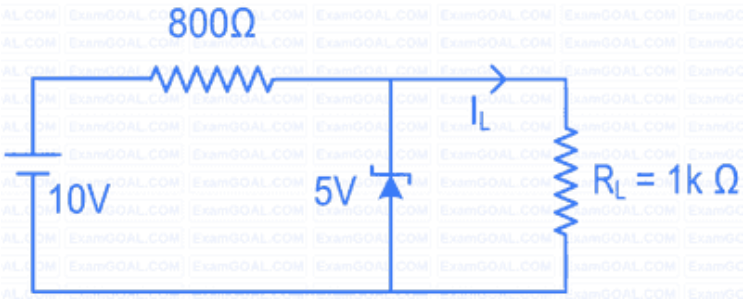
$$A = |-5| = 5\,\text{cm}.$$

Neither the wavelength nor the period is needed to evaluate this spatial amplitude.

Step 3: Conclusion The particle amplitude is $5\,\text{cm}$; the numerical entry is $\boxed{5}$.

Quick Tip to Solve: In a stationary-wave equation take the absolute value of the spatial coefficient; its sign changes phase, not amplitude.

53. In the given circuit, the value of current I_L will be _____ mA. (When R_L = 1kΩ) [4 marks]



Solution:

Concept:

- A reverse-biased Zener diode in breakdown maintains approximately its rated voltage across a parallel load. Here the diode is rated at $5\,\text{V}$, and the load is $1\,\text{k}\Omega$.
- The breakdown assumption must be checked using current conservation: the current through the series resistor must supply the load, with a nonnegative remainder flowing through the Zener diode.

Step 1: Find the load voltage and load current The circuit places the $5\,\text{V}$ Zener across R_L, with its cathode toward the positive supply node. If it operates in breakdown, the load voltage is therefore

$$V_L = V_Z = 5\,\text{V}.$$

Ohm's law gives

$$I_L = \frac{V_L}{R_L} = \frac{5}{1000}\,\text{A} = 5.0 \times 10^{-3}\,\text{A}.$$

The $800\,\Omega$ resistor is a series current-limiting resistor and does not form a simple series pair with the load while the diode conducts; the branch current must be checked separately.

Step 2: Verify that the Zener remains in breakdown The supply is $10\,\text{V}$, so the voltage across the $800\,\Omega$ resistor is $10 - 5 = 5\,\text{V}$. Its current is

$$I_s = \frac{10 - 5}{800} = 6.25 \times 10^{-3}\,\text{A} = 6.25\,\text{mA}.$$

At the upper junction, $I_s = I_L + I_Z$, giving

$$I_Z = 6.25 - 5.00 = 1.25\,\text{mA} > 0.$$

Thus the supply can provide the required load current and still pass current through the Zener. The ideal breakdown model is self-consistent, and no diode-off voltage-divider calculation is needed. Finally, converting amperes to milliamperes multiplies the value by 10^3.

Step 3: Conclusion The load current is $I_L = 5\,\text{mA}$; the numerical entry is $\boxed{5}$.

Quick Tip to Solve: After using the Zener voltage across the load, check that the series current exceeds the load current so the breakdown assumption is valid.

54. A ray of light is incident at an angle of incidence 60° on the glass slab of refractive index $\sqrt{3}$. After refraction, the light ray emerges out from other parallel faces and lateral shift between incident ray and emergent ray is $4\sqrt{3}$ cm. The thickness of the glass slab is _____ cm. [4 marks]

Solution:

Concept:

- Snell's law at the air-glass surface is $\sin i = \mu \sin r$, where i and r are measured from the normal and μ is the glass refractive index.
- For a slab of normal thickness t with parallel faces, the emergent ray is parallel to the incident ray. Their perpendicular separation is $d = t\sin(i-r)/\cos r$.

Step 1: Calculate the refraction angle The incident angle is $i = 60°$ and $\mu = \sqrt{3}$. Taking the refractive index of air as unity, Snell's law gives

$$\sin r = \frac{\sin 60°}{\sqrt{3}} = \frac{\sqrt{3}/2}{\sqrt{3}} = \frac{1}{2}.$$

The physical refracted angle lies between zero and $90°$, so

$$r = 30°.$$

The ray bends toward the normal when entering glass. The incidence on the second face is consequently also $30°$, and refraction back into air restores the original direction.

Step 2: Relate the shift to the slab thickness The distance travelled inside the slab along the ray is $t/\cos r$. The perpendicular component of this length relative to the original ray direction gives the lateral shift:

$$d = \frac{t\sin(i-r)}{\cos r}.$$

Insert $d = 4\sqrt{3}\,\text{cm}$, $i - r = 30°$, and $r = 30°$:

$$4\sqrt{3} = \frac{t(1/2)}{\sqrt{3}/2} = \frac{t}{\sqrt{3}}.$$

Multiplication by $\sqrt{3}$ gives

$$t = 4(\sqrt{3})^2 = 12\,\text{cm}.$$

The answer exceeds the shift, as expected for these angles; the normal thickness must not be confused with the longer oblique path inside the slab.

Step 3: Conclusion The glass slab thickness is $12\,\text{cm}$; the numerical entry is $\boxed{12}$.

Quick Tip to Solve: Find the internal angle first, then use lateral shift as a perpendicular separation: it is not the distance travelled inside the slab.

55. A circular coil of 1000 turns each with area 1m^2 is rotated about its vertical diameter at the rate of one revolution per second in a uniform horizontal magnetic field of 0.07T. The maximum voltage generation will be _____ V. [4 marks]

Solution:

Concept:

- Faraday's law states that the induced emf equals the negative time derivative of the magnetic flux linkage: $\mathcal{E} = -d(N\Phi)/dt$. A coil rotating uniformly has a sinusoidal flux.
- For a coil with N turns and area A rotating at angular speed ω in field B, the peak emf is $\mathcal{E}_{\max} = NAB\omega$. Rotation frequency f must first be converted using $\omega = 2\pi f$.

Step 1: Write the time-dependent flux linkage The rotation axis is the vertical diameter, while the field is horizontal. Thus the coil's area vector rotates relative to the magnetic field, attaining both parallel and perpendicular orientations. Choose the time origin so the area vector is initially parallel to the field. Then

$$N\Phi = NAB\cos(\omega t).$$

Differentiation yields

$$\mathcal{E} = NAB\omega\sin(\omega t).$$

The maximum magnitude occurs when the sine has magnitude one, so it is the peak value rather than the root-mean-square value that the question requests.

Step 2: Substitute the rotation rate and coil data One revolution per second means $f = 1\,\text{Hz}$, hence

$$\omega = 2\pi\,\text{rad}\,\text{s}^{-1}.$$

Each turn has area $1\,\text{m}^2$, and the field is $0.07\,\text{T}$. Therefore

$$\mathcal{E}_{\max} = 1000(1)(0.07)(2\pi) = 140\pi\,\text{V}.$$

Numerically this is approximately $439.82\,\text{V}$, whose nearest whole-volt value is $440\,\text{V}$. Omitting 2π would incorrectly treat revolutions per second as radians per second, while dividing by $\sqrt{2}$ would give the rms voltage rather than the requested maximum.

Step 3: Conclusion The maximum induced voltage is $140\pi\,\text{V} \approx 440\,\text{V}$; the numerical entry is $\boxed{440}$.

Quick Tip to Solve: For generator peak emf use angular speed in radians per second: one revolution per second contributes a factor of 2π.

56. A monoatomic gas performs a work of $\frac{Q}{4}$ where Q is the heat supplied to it. The molar heat capacity of the gas will be _____ R during this transformation. Where R is the gas constant.

[4 marks]

Solution:

Concept:

- For an ideal monatomic gas, internal energy depends only on temperature and $\Delta U = nC_V\Delta T$, with $C_V = 3R/2$. This remains true even when the actual process is not at constant volume.
- With work done by the gas positive, the first law reads $Q = \Delta U + W$. The molar heat capacity of the stated transformation is $C = Q/(n\Delta T)$, rather than necessarily C_P or C_V.

Step 1: Apply the supplied work-to-heat relation The gas performs work, so the stated $W = Q/4$ is positive under the work-by-gas convention. Insert it into the first law:

$$\Delta U = Q - W = Q - \frac{Q}{4} = \frac{3Q}{4}.$$

Thus three quarters of the supplied heat increases the internal energy, while one quarter leaves as mechanical work. This sign choice matters: adding the work to the heat would represent work done on the gas and would not match the wording.

Step 2: Express the temperature change and heat capacity For n moles of the monatomic ideal gas,

$$\Delta U = \frac{3}{2}nR\Delta T.$$

Equating this to the first-law result gives

$$\frac{3}{2}nR\Delta T = \frac{3}{4}Q.$$

Multiply by $4/3$ to obtain

$$Q = 2nR\Delta T.$$

Consequently the heat supplied per mole per unit temperature increase is

$$C = \frac{Q}{n\Delta T} = 2R.$$

The mole number cancels, as it must for a molar quantity. The result lies between $C_V = 1.5R$ and $C_P = 2.5R$; it describes the specified work fraction rather than an isochoric or isobaric process. Its units are $\mathrm{J\,mol^{-1}\,K^{-1}}$.

Step 3: Conclusion The molar heat capacity is $2R$, so the dimensionless coefficient to enter in the blank is $\boxed{2}$.

Quick Tip to Solve: For a specified work fraction first find the fraction of heat stored as internal energy, then compare it with $nC_V\Delta T$.

57. An antenna is placed in a dielectric medium of dielectric constant 6.25. If the maximum size of that antenna is 5.0 mm, it can radiate a signal of minimum frequency of _____ GHz. (Given $\mu_r = 1$ for dielectric medium) [4 marks]

Solution:

Concept:

- An antenna radiates efficiently when its size is at least one quarter of the wavelength: $L \geq \lambda/4$. At the limiting minimum frequency its available size is $L = \lambda_{\max}/4$.
- In a medium of relative permittivity ε_r and relative permeability μ_r, electromagnetic waves travel at $v = c/\sqrt{\varepsilon_r \mu_r}$, and their frequency is $f = v/\lambda$.

Step 1: Calculate the wave speed in the dielectric The dielectric reduces the wave speed relative to vacuum. Using the given relative permeability of unity and relative permittivity 6.25 gives

$$v = \frac{3.0 \times 10^8}{\sqrt{6.25 \times 1}} = \frac{3.0 \times 10^8}{2.5} = 1.2 \times 10^8 \ \mathrm{m\,s^{-1}}.$$

Frequency is related to this speed, rather than directly to the vacuum speed.

Step 2: Find the longest wavelength and minimum frequency Convert the available antenna size to SI units: $L = 5.0 \times 10^{-3}$ m. The longest wavelength compatible with the quarter-wave criterion is

$$\lambda_{\max} = 4L = 2.0 \times 10^{-2} \ \mathrm{m}.$$

Since frequency varies inversely with wavelength at fixed speed, this longest wavelength gives the lowest radiated frequency:

$$f_{\min} = \frac{1.2 \times 10^8}{2.0 \times 10^{-2}} = 6.0 \times 10^9 \ \mathrm{Hz}.$$

Step 3: Conclusion The minimum frequency is 6 GHz. Therefore the required numerical entry is $\boxed{6}$.

Quick Tip to Solve: Use the quarter-wave antenna length and the wave speed in the medium together; here the dielectric divides the vacuum wave speed by 2.5.

58. A potentiometer wire of length 10 m and resistance 20 Ω is connected in series with a 25 V battery and an external resistance 30 Ω. A cell of emf E in secondary circuit is balanced by 250 cm long potentiometer wire. The value of E (in volt) is $\frac{x}{10}$. The value of x is _____. [4 marks]

Solution:

Concept:

- In the primary potentiometer circuit, the wire and the external resistance carry the same current: $I = V/(R_{\text{wire}} + R_{\text{ext}})$. The battery voltage is shared between the two resistances.
- For a uniform potentiometer wire the potential gradient is $k = IR_{\text{wire}}/L$. At balance, no current is drawn from the test cell and its emf is $E = k\ell$, where ℓ is the balancing length.

Step 1: Determine the primary current and wire voltage The total series resistance, including the full

potentiometer wire, is

$$R_{\text{total}} = 20 + 30 = 50\ \Omega.$$

Thus the battery drives a current

$$I = \frac{25}{50} = 0.50\ \text{A}.$$

The voltage across the potentiometer wire alone is therefore

$$V_{\text{wire}} = IR_{\text{wire}} = 0.50 \times 20 = 10\ \text{V}.$$

Using the whole 25 V across the wire would incorrectly ignore the external resistor.

Step 2: Use the balance length and extract x The 10 m wire has a constant potential gradient

$$k = \frac{10\ \text{V}}{10\ \text{m}} = 1.0\ \text{V m}^{-1}.$$

Convert the balance length: $250\ \text{cm} = 2.50\ \text{m}$. At the null point the cell emf equals the drop across this length:

$$E = k\ell = 1.0 \times 2.50 = 2.50\ \text{V}.$$

Comparing with $E = (x/10)\ \text{V}$, multiply the numerical emf in volts by ten to obtain $x = 25$.

Step 3: Conclusion The cell emf is $2.5\ \text{V}$, and the requested value is $\boxed{x = 25}$.

Quick Tip to Solve: First find the voltage across the potentiometer wire, then multiply it by the fraction 2.5/10 of the wire used at balance.

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59. A sample contains 10^{-2} kg each of two substances A and B with half lives 4 s and 8 s respectively. The ratio of their atomic weights is 1 : 2. The ratio of the amounts of A and B after 16 s is $\frac{x}{100}$. The value of x is _____. [4 marks]

Solution:

Concept:

- The mass of undecayed radioactive material follows the same fractional decay as its number of nuclei: $m(t) = m_0(1/2)^{t/T_{1/2}}$. Here $T_{1/2}$ denotes the half-life.
- The given equal initial amounts are masses in kilograms. Their remaining mass ratio is determined by the decay fractions; atomic weights would be needed for a ratio of numbers of atoms or moles instead.

Step 1: Find the surviving mass of each substance Substance A has half-life 4 s, so 16 s contains four of its half-lives. Its surviving mass is

$$m_A = 10^{-2}\left(\frac{1}{2}\right)^4 = \frac{10^{-2}}{16}\ \text{kg}.$$

Substance B has half-life 8 s and therefore undergoes two half-lives in the same interval:

$$m_B = 10^{-2}\left(\frac{1}{2}\right)^2 = \frac{10^{-2}}{4}\ \text{kg}.$$

A decays more rapidly and must consequently leave the smaller remaining mass.

Step 2: Form the ratio of remaining amounts Dividing the remaining masses cancels their equal initial mass:

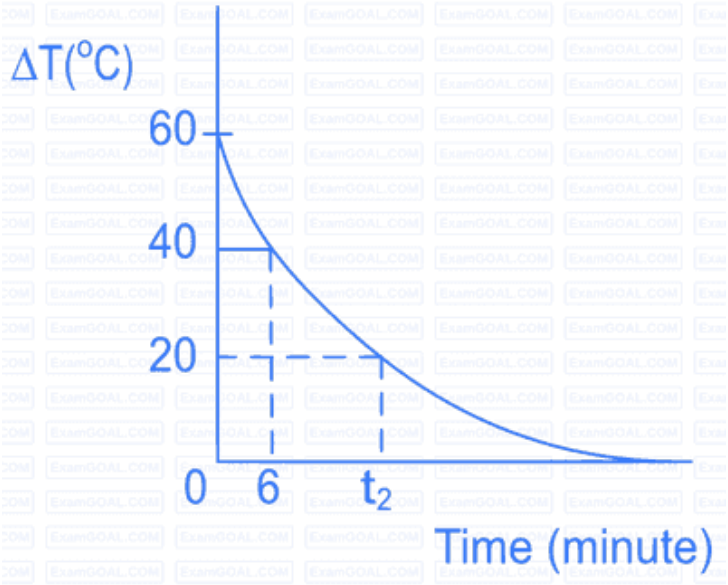
$$\frac{m_A}{m_B} = \frac{10^{-2}/16}{10^{-2}/4} = \frac{4}{16} = \frac{1}{4} = \frac{25}{100}.$$

The atomic-weight ratio does not enter this mass calculation. In particular, multiplying by the inverse atomic-weight ratio would change the question to a number-of-nuclei ratio, which is a different quantity from the remaining kilogram amounts specified here. Comparing with $x/100$ directly identifies the required number.

Step 3: Conclusion The remaining mass ratio is $m_A : m_B = 1 : 4$, so $\boxed{x = 25}$, a dimensionless number.

Quick Tip to Solve: Count half-lives separately before taking the ratio. Equal starting masses cancel; do not convert a mass ratio into an atom-count ratio.

60. In an experiment to verify Newton's law of cooling, a graph is plotted between the temperature difference (ΔT) of the water and surroundings and time as shown in figure. The initial temperature of water is taken as 80°C. The value of t_2 as mentioned in the graph will be _____. [4 marks]



Solution:

Concept:

- Newton's law of cooling states $d(\Delta T)/dt = -k\Delta T$, where $\Delta T = T - T_s$ is the excess over the constant surroundings temperature and $k > 0$ is the cooling constant.

- The resulting excess-temperature curve is $\Delta T(t) = \Delta T(0)e^{-kt}$. Ratios of excess temperatures determine time intervals without needing the heat capacity or mass of the water.

Step 1: Read the temperature differences from the graph The vertical axis shows excess temperature rather than the water temperature itself. Initially $\Delta T = 60\,^{\circ}\mathrm{C}$, while the water is at $80\,^{\circ}\mathrm{C}$; hence the surroundings are at $20\,^{\circ}\mathrm{C}$. The graph gives $\Delta T = 40\,^{\circ}\mathrm{C}$ at 6 minutes and $\Delta T = 20\,^{\circ}\mathrm{C}$ at t_2. These are the three excess-temperature values to use in the exponential law.

Step 2: Calculate the cooling constant and the requested time Using the first two readings eliminates the initial excess:

$$\frac{40}{60} = e^{-6k}, \qquad k = \frac{\ln(3/2)}{6}\ \mathrm{min}^{-1}.$$

For the reading at t_2,

$$\frac{20}{60} = e^{-kt_2}, \qquad t_2 = \frac{\ln 3}{k} = \frac{6\ln 3}{\ln(3/2)} \approx 16.26\ \mathrm{min}.$$

Thus the integer time represented by the graph is 16 minutes. The interval from 40 to 20 degrees of excess is longer than the initial 6-minute interval because cooling slows as the temperature approaches that of the surroundings.

Step 3: Conclusion The graph gives $t_2 \approx 16$ min to the nearest minute. The required numerical entry is $\boxed{16}$.

Quick Tip to Solve: Use excess temperatures 60, 40 and 20, rather than absolute water temperatures; the exponential ratios then give the time directly.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. 120 g of an organic compound that contains only carbon and hydrogen gives 330 g of CO_2 and 270 g of water on complete combustion. The percentage of carbon and hydrogen, respectively are [4 marks]

(A) 25 and 75

(B) 40 and 60

(C) 60 and 40

(D) 75 and 25

Correct Answer: (D) 75 and 25

Solution:

Concept:

- On complete combustion, all carbon in a hydrocarbon appears in CO_2 and all hydrogen appears in H_2O. Product masses therefore reveal the masses of these elements in the original sample.
- The carbon mass fraction in CO_2 is $12/44$, while the hydrogen mass fraction in H_2O is $2/18$. The percentage of an element is its mass divided by the sample mass, multiplied by 100.

Step 1: Extract the carbon and hydrogen masses Each 44 g of carbon dioxide contains 12 g of carbon.

Consequently the 330 g of carbon dioxide contains

$$m_C = 330 \times \frac{12}{44} = 90 \text{ g}.$$

Similarly, each 18 g of water contains 2 g of hydrogen, so the 270 g of water contains

$$m_H = 270 \times \frac{2}{18} = 30 \text{ g}.$$

The sum $90 + 30 = 120$ g equals the original sample mass, as expected for a compound containing only these two elements. Oxygen in the combustion products was supplied during combustion.

Step 2: Convert the elemental masses to percentages The denominator is the original 120 g compound, not the combined mass of the combustion products:

$$\%C = \frac{90}{120} \times 100 = 75, \qquad \%H = \frac{30}{120} \times 100 = 25.$$

Option (A) reverses the required element percentages. Options (B) and (C) also fail because their carbon fractions would not yield the measured 330 g of carbon dioxide from this sample. Only the ordered pair 75 and 25 reproduces both product masses.

Step 3: Conclusion The percentages of carbon and hydrogen are 75% and 25%, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Obtain carbon from carbon dioxide using 12/44 and hydrogen from water using 2/18; their masses should add to the hydrocarbon sample mass.

62. The energy of one mole of photons of radiation of wavelength 300 nm is
(Given : h = 6.63 $\times$ 10^{-34} J s, N_A = 6.02 $\times$ 10^{23} mol^{-1}, c = 3 $\times$ 10^8 m s^{-1}) [4 marks]

(A) 235 kJ mol^{-1}

(B) 325 kJ mol^{-1}

(C) 399 kJ mol^{-1}

(D) 435 kJ mol^{-1}

Correct Answer: (C) 399 kJ mol^{-1}

Solution:

Concept:

- A photon of wavelength λ has energy $\varepsilon = hc/\lambda$. Planck's constant h connects the photon frequency to its energy, and c/λ is that frequency.
- One mole contains N_A photons, so its energy is $E_{\text{mol}} = N_A hc/\lambda$. Convert nanometres to metres before substitution, then joules to kilojoules for comparison with the options.

Step 1: Calculate the energy of one photon The stated wavelength in SI units is

$$\lambda = 300 \times 10^{-9} = 3.00 \times 10^{-7} \text{ m}.$$

Substitute the given constants without changing their significant figures:

$$\varepsilon = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{3.00 \times 10^{-7}} = 6.63 \times 10^{-19}\ \text{J}.$$

This is the energy of a single photon. It must be multiplied by the number of photons in a mole; it cannot be compared directly with a molar-energy option.

Step 2: Scale to one mole and compare the answers Multiply by Avogadro's constant:

$$E_{\text{mol}} = (6.63 \times 10^{-19})(6.02 \times 10^{23}) = 3.99126 \times 10^{5}\ \text{J mol}^{-1}.$$

Since $1000\ \text{J} = 1\ \text{kJ}$, this is

$$E_{\text{mol}} = 399.126\ \text{kJ mol}^{-1} \approx 399\ \text{kJ mol}^{-1}.$$

The 235, 325 and 435 alternatives do not match the direct substitution with the given wavelength and constants; no additional factor of two or conversion factor is required.

Step 3: Conclusion The molar photon energy is approximately $399\ \text{kJ mol}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: For a molar photon energy, include Avogadro's constant and check both conversions: nanometres to metres and joules to kilojoules.

63. The correct order of bond orders of C_2^{2-}, N_2^{2-} and O_2^{2-} is, respectively [4 marks]

(A) $C_2^{2-} < N_2^{2-} < O_2^{2-}$
(B) $O_2^{2-} < N_2^{2-} < C_2^{2-}$
(C) $C_2^{2-} < O_2^{2-} < N_2^{2-}$
(D) $N_2^{2-} < C_2^{2-} < O_2^{2-}$

Correct Answer: (B) $O_2^{2-} < N_2^{2-} < C_2^{2-}$

Solution:

Concept:

- Molecular-orbital bond order is $\text{BO} = (N_b - N_a)/2$, where N_b and N_a are the numbers of electrons in bonding and antibonding molecular orbitals.
- For these second-period diatomic ions, the filled core orbitals and the filled $2s$ bonding/antibonding pair cancel in the bond-order difference. The occupation of the bonding and antibonding $2p$ orbitals decides the comparison.

Step 1: Count electrons and determine each bond order The ion C_2^{2-} has 10 valence electrons. Its $2p$ bonding orbitals contain six electrons and its $2p$ antibonding orbitals are empty, giving

$$\text{BO}(\text{C}_2^{2-}) = \frac{6-0}{2} = 3.$$

The ion N_2^{2-} has 12 valence electrons: six in bonding $2p$ orbitals and two in antibonding $2p$ orbitals. Thus

$$\text{BO}(N_2^{2-}) = \frac{6-2}{2} = 2.$$

Finally O_2^{2-} has 14 valence electrons, with six bonding and four antibonding $2p$ electrons, giving bond order $(6-4)/2 = 1$.

Step 2: Arrange the values in increasing order The numerical values are 3 for the carbon ion, 2 for the nitrogen ion and 1 for the oxygen ion. Therefore

$$\text{BO}(O_2^{2-}) < \text{BO}(N_2^{2-}) < \text{BO}(C_2^{2-}).$$

Option (A) reverses the trend. Option (C) wrongly places the carbon ion below the oxygen ion, while option (D) wrongly makes the oxygen ion the largest. The added electrons in these comparisons increasingly occupy antibonding orbitals, reducing the bond order rather than increasing it.

Step 3: Conclusion The correct increasing order is $O_2^{2-} < N_2^{2-} < C_2^{2-}$, with bond orders $1 < 2 < 3$. Hence, the correct option is (B).

Quick Tip to Solve: Count valence electrons for each charged species before filling orbitals; every pair added to antibonding orbitals lowers bond order by one.

64. At 25°C and 1 atm pressure, the enthalpies of combustion are as given below :

Substance	H_2	**C (graphite)**	$C_2H_6(g)$
$\frac{\Delta_c H^\ominus}{kJ\,mol^{-1}}$	-286.0	-394.0	-1560.0

The enthalpy of formation of ethane is [4 marks]

(A) $+54.0$ kJ mol^{-1}

(B) -68.0 kJ mol^{-1}

(C) -86.0 kJ mol^{-1}

(D) $+97.0$ kJ mol^{-1}

Correct Answer: (C) -86.0 kJ mol^{-1}

Solution:

Concept:

- Hess's law states that enthalpy change depends only on the initial and final states. Reversing a reaction changes the sign of its enthalpy; multiplying it multiplies the enthalpy by the same factor.
- The standard formation reaction makes exactly one mole of a substance from its elements in their reference states. For ethane this is $2\,C(\text{graphite}) + 3\,H_2(g) \longrightarrow C_2H_6(g)$.

Step 1: Write a common combustion endpoint Two moles of graphite and three moles of hydrogen burn to the same products as one mole of ethane: $2\,CO_2(g) + 3\,H_2O\,(l)$. Their combined combustion

enthalpy is

$$2(-394.0) + 3(-286.0) = -1646.0\ \text{kJ mol}^{-1}.$$

The coefficients are essential: ethane contains two carbon atoms and six hydrogen atoms.

Step 2: Apply the enthalpy cycle Forming ethane first and then burning it must give the same net enthalpy as burning the elements directly. Thus

$$\Delta_f H^\circ(\text{C}_2\text{H}_6) + (-1560.0) = -1646.0.$$

$$\Delta_f H^\circ = -1646.0 + 1560.0 = -86.0\ \text{kJ mol}^{-1}.$$

The negative sign means formation is exothermic. Values +54.0 and +97.0 have the wrong sign, and -68.0 does not satisfy this balanced Hess cycle.

Step 3: Conclusion The enthalpy of formation of gaseous ethane is $\boxed{-86.0\ \text{kJ mol}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Balance formation of one mole first; then use combustion of the elements minus combustion of the compound.

65. For a first order reaction, the time required for completion of 90% reaction is 'x' times the half life of the reaction. The value of 'x' is

(Given : ln 10 = 2.303 and log 2 = 0.3010) [4 marks]

(A) 1.12

(B) 2.43

(C) 3.32

(D) 33.31

Correct Answer: (C) 3.32

Solution:

Concept:

- For a first-order reaction, $kt = \ln([A]_0/[A]_t)$, where k is the rate constant and the concentrations refer to the remaining reactant.
- The half-life is $t_{1/2} = \ln 2/k$, independent of initial concentration. A ratio of two reaction times therefore eliminates both the concentration scale and the rate constant.

Step 1: Use the fraction remaining Completion of 90% means that only 10% remains: $[A]_t = 0.10[A]_0$. Consequently

$$kt_{90} = \ln \frac{[A]_0}{0.10[A]_0} = \ln 10, \qquad t_{90} = \frac{\ln 10}{k}.$$

Do not insert 0.90 as the fraction remaining; that would describe only 10% completion. The stated initial amount has no effect on the desired ratio.

Step 2: Divide by the half-life Writing $t_{90} = xt_{1/2}$, obtain

$$x = \frac{\ln 10/k}{\ln 2/k} = \frac{\ln 10}{\ln 2} = \frac{\log 10}{\log 2} = \frac{1}{0.3010} = 3.322\ldots.$$

The ratio is dimensionless. After three half-lives 12.5% remains, and after four 6.25% remains, so 90% completion must occur between three and four half-lives. This rejects 1.12, 2.43 and 33.31 immediately.

Step 3: Conclusion The required multiplier is $\boxed{x \simeq 3.32}$. Hence, the correct option is (C).

Quick Tip to Solve: For a first-order reaction, divide log of the reciprocal remaining fraction by log 2 to count half-lives.

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66. Metals generally melt at very high temperature. Amongst the following, the metal with the highest melting point will be : [4 marks]

(A) Hg

(B) Ag

(C) Ga

(D) Cs

Correct Answer: (B) Ag

Solution:

Concept:

- A metal melts when thermal motion overcomes the cohesive interactions maintaining its crystal lattice. The melting point depends on metallic bonding and crystal structure, rather than merely on whether an element is a metal.
- The options include three familiar low-melting metals, mercury, gallium and caesium, and the much higher-melting transition metal silver.

Step 1: Identify the low-melting candidates Mercury, Hg, melts near $-39^{\circ}\mathrm{C}$, so it is already liquid at ordinary room temperature. Gallium, Ga, melts near $30^{\circ}\mathrm{C}$, and caesium, Cs, near $28^{\circ}\mathrm{C}$. These approximate temperatures identify all three as low-melting metals; none requires the high temperature needed to melt silver. The introductory word "generally" allows such exceptions.

Step 2: Compare with silver Silver, Ag, has a melting point of about $962^{\circ}\mathrm{C}$, far above the other three. Its cohesive solid lattice persists until a much higher temperature. The relevant comparison is therefore

$$T_m(\mathrm{Ag}) \gg T_m(\mathrm{Ga}),\ T_m(\mathrm{Cs}),\ T_m(\mathrm{Hg}).$$

The precise ordering of gallium and caesium is unnecessary for selecting the maximum. Options Hg, Ga and Cs are excluded by their low melting points, leaving Ag as the unique choice.

Step 3: Conclusion Among the listed metals, $\boxed{\mathrm{Ag}}$ has the highest melting point. Hence, the correct option is (B).

Quick Tip to Solve: Remember Hg, Ga and Cs as low-melting exceptions; silver melts at a much higher temperature.

67. PCl_5 is well known, but NCl_5 is not. because, [4 marks]

(A) nitrogen is less reactive than phosphorus.

(B) nitrogen doesn't have d-orbitals in its valence shell.

(C) catenation tendency is weaker in nitrogen than phosphorus.

(D) size of phosphorus is larger than nitrogen.

Correct Answer: (B) nitrogen doesn't have d-orbitals in its valence shell.

Solution:

Concept:

- Nitrogen is a second-period element with valence-shell configuration $2s^2 2p^3$. Its valence shell has only s and p orbitals and cannot accommodate an expanded octet in the conventional valence-bond description.
- Phosphorus lies in the third period. The syllabus explanation for its pentahalides contrasts the available third-shell orbitals with the absence of valence d orbitals in nitrogen.

Step 1: Count the electrons around the central atom Five ordinary covalent N–Cl bonds would place ten bonding electrons around nitrogen. This exceeds the octet available to a second-period central atom. Nitrogen has no $2d$ subshell: for principal quantum number $n = 2$, only $l = 0$ and $l = 1$ are possible. Thus the proposed pentachloride cannot be described by the expanded-valence picture used for phosphorus pentachloride.

Step 2: Select the syllabus explanation In the conventional account, phosphorus can exhibit covalency five, whereas nitrogen cannot expand its octet because it lacks d orbitals in its valence shell. This is the distinction directly stated in option B. A general claim of lower reactivity in A does not establish a coordination limit. Catenation in C concerns bonds between atoms of the same element, not formation of five N–Cl bonds. Phosphorus is larger, as D says, but size alone does not give the valence-shell restriction tested here.

Step 3: Conclusion In the conventional syllabus explanation, nitrogen lacks d orbitals in its valence shell and cannot expand its octet to form NCl_5. Hence, the correct option is (B).

Quick Tip to Solve: For second-period central atoms, check the octet limit before considering expanded-valence compounds.

68. Transition metal complex with highest value of crystal field splitting (Δ_0) will be : [4 marks]

(A) $[Cr(H_2O)_6]^{3+}$

(B) $[Mo(H_2O)_6]^{3+}$

(C) $[Fe(H_2O)_6]^{3+}$

(D) $[Os(H_2O)_6]^{3+}$

Correct Answer: (D) $[Os(H_2O)_6]^{3+}$

Solution:

Concept:

- In an octahedral complex, the metal d orbitals split into lower-energy t_{2g} and higher-energy e_g sets. Their energy separation is the octahedral crystal-field splitting Δ_o.
- The splitting depends on the ligand, metal charge and transition series. For comparable ligands and charges, the greater radial extent of 4d and especially 5d orbitals generally gives larger splitting than 3d orbitals.

Step 1: Hold ligand and oxidation state fixed Every option has six neutral water ligands and overall charge $+3$. The metal oxidation state is therefore $+3$ in each complex, and the ligand identity and octahedral coordination are common. There is no strong-field versus weak-field ligand comparison to make here: the discriminating factor is the identity and orbital shell of the metal ion.

Step 2: Compare the transition series Chromium and iron are 3d metals, molybdenum is a 4d metal, and osmium is a 5d metal. The more extended 5d orbitals interact more strongly with the surrounding ligands and produce the greatest splitting in this comparison. Thus $[Os(H_2O)_6]^{3+}$ is preferred over the 4d molybdenum and 3d chromium or iron complexes. The number of d electrons determines occupation of the split levels, but is not itself the criterion for the largest separation. Options A, B and C omit the 5d ion that gives the strongest splitting here.

Step 3: Conclusion The largest Δ_o is for $\boxed{[Os(H_2O)_6]^{3+}}$. Hence, the correct option is (D).

Quick Tip to Solve: When ligand and oxidation state are fixed, identify the transition row: 5d complexes generally split more strongly than 4d and 3d analogues.

69. Arrange the following carbocations in decreasing order of stability. [4 marks]



(A) B > A > C

(B) A > B > C

(C) C > B > A

(D) C > A > B

Correct Answer: (A) B > A > C

Solution:

Concept:

- A carbocation is stabilized when an adjacent lone pair can overlap with its vacant p orbital and delocalize the positive charge by resonance.
- Oxygen also withdraws electron density through sigma bonds by its negative inductive effect. Whether resonance donation is possible depends on the position of the carbocation relative to oxygen.

Step 1: Identify the resonance-stabilized cation In B the positively charged carbon is directly bonded to the ring oxygen. An oxygen lone pair can form a pi bond to that carbon, giving an oxonium resonance contributor. This delocalizes the positive charge and supplies an octet at the initially electron-deficient carbon. The resonance contribution is more stabilizing than the accompanying inductive withdrawal, so B is the most stable of the three structures.

Step 2: Compare the remaining two cations A is a cyclopentyl carbocation without an oxygen atom in the ring. C has a ring oxygen, but its positive carbon is not directly adjacent to oxygen: a saturated carbon interrupts the path, so direct lone-pair resonance donation is absent. Oxygen instead withdraws electron density inductively and destabilizes the positive charge relative to A. Therefore the order is

$$B > A > C.$$

Option B incorrectly places A above the resonance-stabilized B. Options C and D incorrectly make the inductively destabilized C the most stable.

Step 3: Conclusion The decreasing stability order is $\boxed{B > A > C}$. Hence, the correct option is (A).

Quick Tip to Solve: Check direct adjacency to a lone-pair donor first; a saturated carbon between oxygen and a carbocation blocks resonance donation.

70. Given below are two statements :

Statement I : The presence of weaker π-bonds make alkenes less stable than alkanes.

Statement II : The strength of the double bond is greater than that of carbon-carbon single bond.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (A) Both Statement I and Statement II are correct.

Solution:

Concept:

- A carbon–carbon double bond consists of one sigma bond and one pi bond. The pi component is weaker and more easily attacked than a sigma bond because its sideways overlap is less effective.
- The total energy needed to break a complete double bond includes both components. A weaker pi component does not imply that the whole double bond is weaker than a single bond.

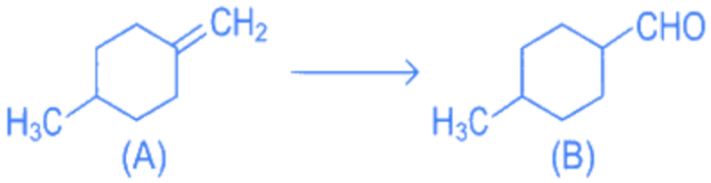
Step 1: Evaluate Statement I Alkenes contain an exposed, relatively weak pi component that readily participates in addition reactions. Hydrogenation replaces the unsaturation with strong sigma bonds and is normally exothermic. In the intended comparison of unsaturated compounds with their corresponding saturated products, this makes alkenes less stable and more reactive than alkanes. Thus Statement I is correct in the standard syllabus sense; it refers to the reactive pi component and the stabilization on saturation.

Step 2: Evaluate Statement II separately The carbon–carbon double bond is stronger overall than a carbon–carbon single bond. Typical bond dissociation energies are roughly $610\ \text{kJ}\,\text{mol}^{-1}$ for $\text{C}{=}\text{C}$ and $350\ \text{kJ}\,\text{mol}^{-1}$ for $\text{C}{-}\text{C}$. The additional pi contribution increases the total strength, even though that contribution alone is weaker than a sigma bond. Statement II is therefore also correct. Options B, C and D each label at least one true statement as false; there is no contradiction between the two statements.

Step 3: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish the weakness of the pi component from the total strength of the complete carbon–carbon double bond.

71. Which of the following reagents / reactions will convert 'A' to 'B' ? [4 marks]



(A) PCC oxidation

(B) Ozonolysis

(C) BH_3, $H_2O_2/^-OH$ followed by PCC oxidation

(D) HBr, hydrolysis followed by oxidation by $K_2Cr_2O_7$.

Correct Answer: (C) BH_3, $H_2O_2/^-OH$ followed by PCC oxidation

Solution:

Concept:

- Hydroboration followed by alkaline peroxide gives anti-Markovnikov hydration of an alkene: the OH group goes to the less substituted carbon.
- PCC oxidises a primary alcohol to an aldehyde under anhydrous conditions, without the further oxidation to a carboxylic acid typical of stronger aqueous oxidants.

Step 1: Identify the required carbon skeleton The starting structure has an exocyclic methylene group, whereas the product has a formyl group attached to the same ring carbon. Thus the terminal carbon of the double bond must be retained and converted into the aldehyde carbon. The methyl substituent on the opposite side of the cyclohexane ring remains unchanged throughout the required conversion.

Step 2: Hydrate the double bond in the correct direction With BH_3, boron attaches preferentially to the terminal CH_2 carbon. Oxidation using H_2O_2/OH^- replaces the carbon-boron bond by a carbon-oxygen bond. The terminal carbon therefore becomes CH_2OH, giving the primary alcohol attached to the cyclohexane ring. Hydrogen is added to the ring carbon of the original double bond.

Step 3: Oxidise the alcohol and test the alternatives PCC converts the side chain $-CH_2OH$ into $-CHO$, exactly as required. PCC alone does not hydrate the starting alkene. Ozonolysis cleaves its carbon-carbon double bond, producing a ring ketone and methanal instead of retaining the side chain. Ordinary HBr addition puts bromine at the more substituted ring carbon; hydrolysis gives a tertiary alcohol there, which cannot yield the required aldehyde by dichromate oxidation.

Step 4: Conclusion Hydroboration-oxidation followed by PCC converts the exocyclic methylene group into the formyl group while retaining the carbon skeleton. Hence, the correct option is (C).

Quick Tip to Solve: For $R_2C{=}CH_2 \longrightarrow R_2CH{-}CHO$, first make the terminal primary alcohol by hydroboration-oxidation, then use PCC.

72. Hex-4-ene-2-ol on treatment with PCC gives 'A'. 'A' on reaction with sodium hypoiodite gives 'B', which on further heating with soda lime gives 'C'. The compound 'C' is : [4 marks]

(A) 2-pentene

(B) proponaldehyde

(C) 2-butene

(D) 4-methylpent-2-ene

Correct Answer: (C) 2-butene

Solution:

Concept:

- PCC converts a secondary alcohol into a ketone without normally attacking an isolated carbon-carbon double bond.
- A methyl ketone undergoes the haloform reaction with sodium hypoiodite, giving iodoform and a carboxylate. Soda lime then removes the carboxylate carbon as carbonate.

Step 1: Oxidise the secondary alcohol Write hex-4-en-2-ol as $CH_3CH(OH)CH_2CH{=}CHCH_3$. The alcohol-bearing carbon is secondary, so PCC produces the methyl ketone A without changing the alkene position:

$$CH_3CH(OH)CH_2CH{=}CHCH_3 \xrightarrow{PCC} CH_3COCH_2CH{=}CHCH_3.$$

A is hex-4-en-2-one.

Step 2: Apply the haloform reaction The CH_3CO^- group is the diagnostic methyl-ketone unit. Successive iodination of its methyl group and cleavage by hydroxide give CHI_3 and the sodium salt B, $NaOOCCH_2CH{=}CHCH_3$. The methyl carbon originally next to the carbonyl leaves as iodoform; it is not retained in B. The remaining chain contains five carbons, counting the carboxylate carbon.

Step 3: Decarboxylate and identify the alkene Heating the sodium carboxylate with soda lime replaces $-COONa$ by hydrogen:

$$NaOOCCH_2CH{=}CHCH_3 + NaOH \xrightarrow{CaO,\ \Delta} CH_3CH{=}CHCH_3 + Na_2CO_3.$$

The four-carbon alkene is but-2-ene. Pent-2-ene would retain one carbon too many; propanal introduces a carbonyl absent after soda-lime decarboxylation; and 4-methylpent-2-ene requires a branched six-carbon skeleton that none of the reagents creates.

Step 4: Conclusion The sequence gives hex-4-en-2-one, sodium pent-3-enoate, and finally but-2-ene (2-butene). Hence, the correct option is (C).

Quick Tip to Solve: Track both carbon losses: a methyl ketone loses its methyl carbon as iodoform, then its carboxylate carbon during soda-lime heating.

73. The conversion of propan-1-ol to n-butylamine involves the sequential addition of reagents. The correct sequential order of reagents is [4 marks]

(A) (i) $SOCl_2$ (ii) KCN (iii) H_2/Ni, Na(Hg)/C_2H_5OH

(B) (i) HCl (ii) H_2/Ni, Na(Hg)/C_2H_5OH

(C) (i) $SOCl_2$ (ii) KCN (iii) CH_3NH_2

(D) (i) HCl (ii) CH_3NH_2

Correct Answer: (A) (i) $SOCl_2$ (ii) KCN (iii) H_2/Ni, Na(Hg)/C_2H_5OH

Solution:

Concept:

- A primary alcohol can be converted into a primary chloroalkane with thionyl chloride, $SOCl_2$.
- Alcoholic KCN replaces halide by a carbon-bonded cyanide group, increasing the chain length by one. Reduction of $R-CN$ gives $R-CH_2NH_2$.

Step 1: Convert the alcohol into a suitable leaving-group derivative Propan-1-ol has three carbon atoms, whereas n-butylamine has four. Direct replacement of OH by an amino group cannot accomplish the required chain extension. First use thionyl chloride to form 1-chloropropane:

$$CH_3CH_2CH_2OH + SOCl_2 \longrightarrow CH_3CH_2CH_2Cl + SO_2 + HCl.$$

The carbon chain is unchanged in this step.

Step 2: Introduce the extra carbon with cyanide Treat the primary chloroalkane with KCN. Nucleophilic substitution introduces the carbon end of cyanide:

$$CH_3CH_2CH_2Cl + KCN \longrightarrow CH_3CH_2CH_2CN + KCl.$$

This product is butanenitrile. Its nitrile carbon is the fourth carbon required in the target amine; cyanide is therefore essential to the stated conversion.

Step 3: Reduce the nitrile and eliminate wrong sequences Catalytic hydrogenation with H_2/Ni, or the indicated sodium amalgam/ethanol reduction, converts the nitrile into a primary amino group:

$$CH_3CH_2CH_2CN + 2\,H_2 \xrightarrow{Ni} CH_3CH_2CH_2CH_2NH_2.$$

The reducing systems are alternatives for the last transformation. Option B has no carbon-extending cyanide step. Methylamine in C does not reduce a nitrile into n-butylamine, and in D its alkylation with a propyl halide would give a secondary N-methylpropylamine rather than a straight-chain primary amine.

Step 4: Conclusion The required sequence is $SOCl_2$, then KCN, followed by nitrile reduction using H_2/Ni or $Na(Hg)/C_2H_5OH$. Hence, the correct option is (A).

Quick Tip to Solve: When a primary amine must contain one carbon more than the starting alcohol, use halide formation, cyanide substitution, then nitrile reduction.

74. In the flame test of a mixture of salts, a green flame with blue centre was observed. Which one of the following cations may be present? [4 marks]

(A) Cu^{2+}

(B) Sr^{2+}

(C) Ba^{2+}

(D) Ca^{2+}

Correct Answer: (A) Cu^{2+}

Solution:

Concept:

- Heating a volatile metal salt in a flame excites electrons. Their return to lower energy levels produces characteristic visible emission.
- The observed colour is a qualitative identification test; both the main flame colour and a distinctive centre help distinguish ions that give similar green shades.

Step 1: Interpret the specified flame appearance The question describes a green flame with a blue centre, the characteristic blue-green flame associated with copper salts. On heating, copper-containing species emit visible radiation that gives this appearance. Thus the observation is consistent with the presence of Cu^{2+}. The wording "may be present" is appropriate because a mixture can contain other ions whose colours are masked.

Step 2: Distinguish the other listed cations Strontium salts give a crimson-red flame, so Sr^{2+} does not match the green observation. Calcium salts give a brick-red flame and are likewise inconsistent. Barium salts do give an apple-green flame, making Ba^{2+} the tempting alternative, but the explicitly stated blue centre identifies the copper-type flame rather than the usual apple-green barium test. The full colour description must be used, not just the word green.

Step 3: State the scope of the identification A flame test is based on characteristic emission rather than on precipitation or the bulk colour of the solid salt. Consequently the presence of a blue or green solid is not the criterion here; it is the colour generated in the flame. For the four given choices, the reported green flame with a blue centre singles out copper(II), while the red-flame ions and the apple-green barium alternative fail the stated observation.

Step 4: Conclusion The cation that may be present is Cu^{2+}, which gives the characteristic green flame with a blue centre. Hence, the correct option is (A).

Quick Tip to Solve: Remember copper as blue-green and barium as apple-green; strontium is crimson and calcium is brick red.

75. Which of the following chemical reactions represents Hall-Heroult Process? [4 marks]

(A) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

(B) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

(C) $FeO + CO \rightarrow Fe + CO_2$

(D) $2[Au(CN)_2]^-_{(aq)} + Zn(s) \rightarrow 2Au(s) + [Zn(CN_4)]^{2-}$

Correct Answer: (B) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

Solution:

Concept:

- The Hall-Heroult process extracts aluminium by electrolytic reduction of alumina dissolved in molten cryolite.
- Aluminium forms at the cathode. Oxide ions discharge at carbon anodes, and the oxygen consumes the carbon, giving the overall carbon-dioxide equation.

Step 1: Identify the metal and the role of electrolysis The named process belongs to aluminium metallurgy. Alumina is dissolved in a molten cryolite-based bath, which allows electrolysis at a lower operating temperature than pure alumina would require and provides an electrically conducting melt. Aluminium ions are reduced at the cathode:

$$\mathrm{Al^{3+} + 3\,e^- \longrightarrow Al.}$$

Carbon is the consumable anode, not a metal-displacement reagent.

Step 2: Combine the electrode changes For four aluminium atoms, twelve electrons are consumed at the cathode. Oxide ions supply the corresponding anodic oxidation, and the oxygen formed reacts with the carbon anode. The overall process is represented by

$$\mathrm{2\,Al_2O_3 + 3\,C \longrightarrow 4\,Al + 3\,CO_2.}$$

This conserves four aluminium atoms, six oxygen atoms and three carbon atoms on the two sides. The anodes must be replaced because they are consumed.

Step 3: Classify the competing reactions Option A uses aluminium to reduce chromium(III) oxide; that is an aluminothermic reaction, not electrolytic extraction of aluminium. Option C is reduction of iron(II) oxide by carbon monoxide, associated with iron metallurgy. Option D uses zinc to displace gold from its cyanide complex during cyanide extraction. Neither reaction produces aluminium. Only option B expresses alumina consumption, aluminium production and carbon-anode oxidation together.

Step 4: Conclusion The Hall-Heroult overall reaction is $\mathrm{2\,Al_2O_3 + 3\,C \longrightarrow 4\,Al + 3\,CO_2}$. Hence, the correct option is (B).

Quick Tip to Solve: In Hall-Heroult electrolysis, look for alumina becoming aluminium while the carbon anode becomes carbon dioxide.

76. In the industrial production of which of the following, molecular hydrogen is obtained as a byproduct? [4 marks]

(A) NaOH

(B) NaCl

(C) Na metal

(D) Na_2CO_3

Correct Answer: (A) NaOH

Solution:

Concept:

- The chlor-alkali process electrolyses concentrated aqueous sodium chloride solution (brine) to obtain sodium hydroxide, chlorine and hydrogen.
- In aqueous brine, water is reduced at the cathode in preference to sodium ions; the cathodic products are hydrogen gas and hydroxide ions.

Step 1: Write the cathode and anode reactions At the cathode, reduction of water liberates molecular hydrogen:

$$2\,H_2O + 2\,e^- \longrightarrow H_2 + 2\,OH^-.$$

At the anode, chloride ions are oxidised:

$$2\,Cl^- \longrightarrow Cl_2 + 2\,e^-.$$

Sodium ions remain in the solution and accompany the hydroxide ions produced at the cathode, giving the sodium hydroxide solution collected as the principal alkali product.

Step 2: Combine the reactions for industrial manufacture Adding the electrode equations and including spectator sodium ions gives

$$2\,NaCl + 2\,H_2O \longrightarrow 2\,NaOH + H_2 + Cl_2.$$

Thus molecular hydrogen is obtained alongside the industrial production of $NaOH$. Separation of the anode and cathode products prevents chlorine from reacting with the hydroxide and keeps hydrogen separate from the chlorine stream.

Step 3: Reject the remaining choices Sodium chloride is the feedstock in this process, rather than the manufactured alkali. Sodium metal is obtained by electrolysis of a molten salt; in the absence of water the cathode reduces Na^+ to metal instead of evolving hydrogen. Sodium carbonate is made by processes such as the Solvay process, whose usual overall chemistry does not generate molecular hydrogen. Hence none of B, C or D describes the hydrogen-producing manufacture asked for.

Step 4: Conclusion Hydrogen is a byproduct of the chlor-alkali manufacture of sodium hydroxide, $NaOH$. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish aqueous and molten sodium chloride: aqueous brine gives hydrogen and NaOH, whereas molten salt gives sodium metal.

77. Which one of the following compounds is used as a chemical in certain type of fire extinguishers?

[4 marks]

(A) Baking soda

(B) Soda ash

(C) Washing soda

(D) Caustic soda

Correct Answer: (A) Baking soda

Solution:

Concept:

- Baking soda is sodium hydrogen carbonate, $NaHCO_3$. It releases carbon dioxide when reacted with an acid.
- Carbon dioxide does not support ordinary combustion and forms a blanket over a fire, reducing contact between the fuel and atmospheric oxygen.

Step 1: Identify the active chemical Certain fire extinguishers use sodium hydrogen carbonate as the carbon-dioxide-generating chemical. Its common name is baking soda. In the traditional soda-acid extinguisher, an acid and a hydrogen-carbonate solution are brought into contact when the extinguisher is operated. Their reaction generates gas pressure and carbon dioxide for discharge onto the fire.

Step 2: Show how carbon dioxide is generated The essential ionic change is

$$HCO_3^- + H^+ \longrightarrow CO_2 + H_2O.$$

For example, using sulphuric acid the overall reaction is

$$2\,NaHCO_3 + H_2SO_4 \longrightarrow Na_2SO_4 + 2\,CO_2 + 2\,H_2O.$$

The carbon dioxide evolved helps exclude oxygen around the burning material. Sodium hydrogen carbonate is also useful in appropriate dry chemical formulations, but the simple acid reaction explains its familiar textbook application.

Step 3: Compare the other sodium compounds Soda ash is anhydrous Na_2CO_3, washing soda is $Na_2CO_3 \cdot 10\,H_2O$, and caustic soda is $NaOH$. These names must not be interchanged with sodium hydrogen carbonate. Although carbonates can also release carbon dioxide with acids, the standard chemical identified in this fire-extinguisher application is baking soda. Caustic soda contains no carbonate or hydrogen carbonate group and cannot produce carbon dioxide by acid neutralisation.

Step 4: Conclusion The compound used in the stated fire-extinguisher application is baking soda, $NaHCO_3$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the common names separate: baking soda is NaHCO3, washing soda is hydrated sodium carbonate, and caustic soda is NaOH.

78. Some gases are responsible for heating of atmosphere (green house effect). Identify from the following the gaseous species which does not cause it. [4 marks]

(A) CH_4

(B) O_3

(C) H_2O

(D) N_2

Correct Answer: (D) N_2

Solution:

Concept:

- The greenhouse effect arises when atmospheric gases absorb outgoing terrestrial infrared radiation and re-emit it, reducing the rate at which the surface loses heat.
- A molecular vibration absorbs infrared radiation only if it produces a change in the electric dipole moment. The abundance of a gas alone does not decide its greenhouse activity.

Step 1: Identify the infrared-active gases Methane, CH_4, has infrared-active vibrational modes and is a greenhouse gas. Ozone, O_3, is a bent molecule with infrared absorption bands and also contributes to atmospheric heating. Water vapour, H_2O, is another important greenhouse gas, with rotational and vibrational absorption in the infrared. Thus options (A), (B) and (C) all name species which can cause the effect in the question.

Step 2: Examine molecular nitrogen Nitrogen, N_2, is a homonuclear diatomic molecule. Its ordinary bond vibration does not create a changing electric dipole, because the two identical atoms remain electrically equivalent. It consequently does not absorb terrestrial infrared radiation by the ordinary dipole-active mechanism responsible for the greenhouse effect. In the classification asked here, nitrogen is therefore excluded even though it forms the largest fraction of dry air.

Step 3: Conclusion The gaseous species that does not cause the greenhouse effect is N_2. Hence, the correct option is (D).

Quick Tip to Solve: For greenhouse-gas questions, check infrared absorption rather than atmospheric abundance; a homonuclear diatomic gas such as nitrogen is the usual exception.

79. Which of the following is not an example of a condensation polymer? [4 marks]

(A) Nylon 6,6

(B) Dacron

(C) Buna-N

(D) Silicone

Correct Answer: (C) Buna-N

Solution:

Concept:

- Condensation polymerisation links bifunctional or polyfunctional monomers through successive reactions, commonly eliminating a small molecule such as water or hydrogen chloride.
- Addition polymerisation joins unsaturated monomers by opening their carbon-carbon multiple bonds. A copolymer can be an addition polymer: the presence of two monomers does not imply condensation.

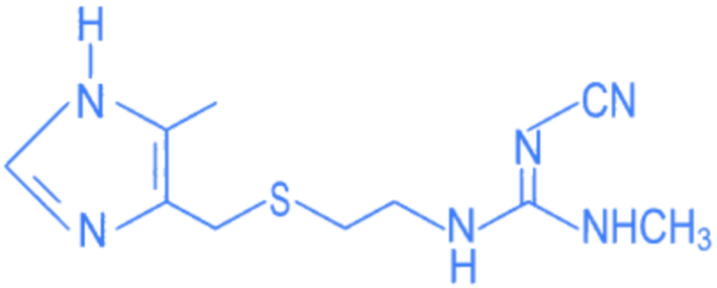
Step 1: Classify the three condensation products Nylon 6,6 forms from hexamethylenediamine and adipic acid; amide links form with loss of water. Dacron, a polyester, forms from ethylene glycol and terephthalic acid, again eliminating water while ester links are produced. Silicones form by condensation of hydrolysed organosilicon compounds, giving the characteristic $\mathrm{Si-O-Si}$ backbone. These are the condensation polymers represented by options (A), (B) and (D).

Step 2: Identify the addition copolymer Buna-N is the copolymer of 1,3-butadiene and acrylonitrile. Their unsaturated carbon-carbon bonds participate in chain growth, incorporating the monomers into the polymer chain without the repeated elimination of small molecules that characterises condensation. It is a synthetic rubber and an addition copolymer. The nitrile side group identifies the acrylonitrile contribution; it does not create a condensation linkage.

Step 3: Conclusion Buna-N is an addition copolymer, rather than a condensation polymer. Hence, the correct option is (C).

Quick Tip to Solve: Identify the linkage and monomers: amide, ester and siloxane backbones suggest condensation, whereas Buna-N comes from addition of two unsaturated monomers.

80. The structure shown below is of which well-known drug molecule? [4 marks]



(A) Ranitidine

(B) Seldane

(C) Cimetidine

(D) Codeine

Correct Answer: (C) Cimetidine

Solution:

Concept:

- A drug can be identified by its characteristic ring system and functional groups. Similar therapeutic use alone is insufficient when a molecular structure is supplied.
- Cimetidine is a histamine H_2-receptor antagonist. Its recognisable structure contains a substituted imidazole ring linked through a sulfur-containing chain to a cyanoguanidine group.

Step 1: Read the structural features The five-membered ring at the left contains two nitrogen atoms, one shown bearing hydrogen. This is the imidazole ring, and it carries a methyl substituent. A side chain connects the ring to a sulfur atom and then to a two-carbon chain terminating at nitrogen. On the right is the substituted guanidine unit with an $N-CN$ group and an $NHCH_3$ substituent. Taken together, these features are characteristic of cimetidine.

Step 2: Eliminate the other drug names Ranitidine is also an acid-suppressing drug, but its recognisable heterocycle is a furan ring, rather than this two-nitrogen imidazole ring, and its terminal functionality differs. Seldane is an antihistamine with bulky phenyl-containing structural units; that framework is absent here. Codeine has the fused morphinan ring system of an opioid, not an open sulfur-containing chain joined to imidazole. Thus neither common use nor a single nitrogen atom can justify the other alternatives.

Step 3: Conclusion The displayed molecule is cimetidine. Hence, the correct option is (C).

Quick Tip to Solve: Look for the combination of an imidazole ring, a thioether chain and a cyanoguanidine end group to recognise cimetidine quickly.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. A company dissolves 'x' amount of CO_2 at 298 K in 1 litre of water to prepare soda water. X = _____ $\times 10^{-3}$ g. (nearest integer)
(Given : partial pressure of CO_2 at 298 K = 0.835 bar.
Henry's law constant for CO_2 at 298 K = 1.67 kbar.
Atomic mass of H, C and O is 1, 12, and 6 g mol^{-1}, respectively) [4 marks]

Solution:

Concept:

- Henry's law in mole-fraction form is $p = K_H\chi$, where p is the partial pressure of carbon dioxide and χ its mole fraction in solution. Pressure and K_H must use the same unit.
- The printed atomic mass of oxygen is 6, which conflicts with the standard value 16. This affects both the molar mass of water and that of carbon dioxide. The two interpretations must be distinguished rather than silently replacing the given number.

Step 1: Find the dissolved-gas mole fraction Convert the constant to bar and substitute the stated pressure:

$$K_H = 1.67 \times 10^3\,\text{bar}, \qquad \chi = \frac{0.835}{1670} = 5.00 \times 10^{-4}.$$

For n_w moles of water and n_g moles of gas, $\chi = n_g/(n_w + n_g)$, so $n_g = \chi n_w/(1-\chi)$. As usual for this dilute-water calculation, take one litre of water as approximately 1000 g.

Step 2: Calculate from the atomic masses actually given With the printed oxygen mass 6, the molar masses are $M(H_2O) = 8$ and $M(CO_2) = 24$, in g mol^{-1}. Therefore

$$n_w = \frac{1000}{8} = 125\,\text{mol},$$

$$m(CO_2) = \frac{0.0005 \times 125}{0.9995} \times 24 = 1.50075\,\text{g}.$$

The requested coefficient, rounded to the nearest integer, is consequently 1501. Even the dilute approximation would give 1500, not the key value.

Step 3: Explain the intended conventional calculation Using the standard oxygen mass 16, the conventional dilute calculation takes $n_w \simeq 55.5\,\text{mol}$ and $M(CO_2) = 44\,\text{g mol}^{-1}$. It then gives

$$m \simeq (5.00 \times 10^{-4})(55.5)(44) = 1.221\,\text{g}.$$

This produces the keyed coefficient 1221, but requires the standard atomic mass and the stated dilute approximation rather than the literal printed mass.

Step 4: Conclusion The intended keyed answer is 1221, corresponding to 1221×10^{-3} g of CO_2 with standard atomic masses and the conventional dilute approximation. The question as printed with oxygen mass 6 instead gives approximately 1501×10^{-3} g by the exact mole-fraction equation; thus that printed datum is inconsistent with the key.

Quick Tip to Solve: Keep Henry-law pressures in the same unit and distinguish the gas mole fraction from its mole count; check supplied atomic masses before converting moles to grams.

82. PCl_5 dissociates as

$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

5 moles of PCl_5 are placed in a 200 litre vessel which contains 2 moles of N_2 and is maintained at 600 K. The equilibrium pressure is 2.46 atm. The equilibrium constant K_p for the dissociation of PCl_5 is _____ $\times 10^{-3}$. (nearest integer)

(Given : R = 0.082 L atm K^{-1} mol^{-1}; Assume ideal gas behaviour) [4 marks]

Solution:

Concept:

- The ideal-gas equation gives the total equilibrium amount: $n_{\text{tot}} = PV/(RT)$. An inert gas contributes to total pressure but does not enter the equilibrium-constant expression.
- For $\text{PCl}_5(\text{g}) \rightleftharpoons \text{PCl}_3(\text{g}) + \text{Cl}_2(\text{g})$, the pressure equilibrium constant is $K_p = p_{\text{PCl}_3} p_{\text{Cl}_2}/p_{\text{PCl}_5}$. At fixed temperature and volume, each partial pressure is $n_i RT/V$.

Step 1: Calculate the extent of dissociation The equilibrium total number of moles is

$$n_{\text{tot}} = \frac{2.46 \times 200}{0.082 \times 600} = 10.$$

Initially the vessel contains 5 mol of PCl_5 and 2 mol of inert N_2, a total of 7 mol. If ξ mol of PCl_5 dissociates, one mole becomes two, so the net increase is ξ. Hence $7 + \xi = 10$, giving $\xi = 3\,\text{mol}$. The equilibrium amounts are therefore $2, 3, 3, 2$ mol for PCl_5, PCl_3, Cl_2 and N_2, respectively.

Step 2: Use the reactive-gas partial pressures The pressure per mole in this vessel is

$$\frac{RT}{V} = \frac{0.082 \times 600}{200} = 0.246\,\text{atm}\,\text{mol}^{-1}.$$

Thus $p_{\text{PCl}_5} = 0.492\,\text{atm}$, while each product has partial pressure $0.738\,\text{atm}$. Their sum with the nitrogen pressure $0.492\,\text{atm}$ is $2.46\,\text{atm}$, checking the mole balance. Now

$$K_p = \frac{0.738 \times 0.738}{0.492} = 1.107\,\text{atm} = 1107 \times 10^{-3}\,\text{atm}.$$

Nitrogen must not be included in this quotient.

Step 3: Conclusion The required nearest-integer coefficient is $\boxed{1107}$; the pressure-form constant is $K_p = 1.107\,\text{atm}$.

Quick Tip to Solve: Count the inert gas when extracting the dissociation extent from total pressure, then omit it from the reaction equilibrium quotient.

83. The resistance of a conductivity cell containing 0.01 M KCl solution at 298 K is 1750 Ω. If the conductivity of 0.01 M KCl solution at 298 K is 0.152 $\times$ 10^{-3} S cm^{-1}, then the cell constant of the conductivity cell is _____ $\times$ 10^{-3} cm^{-1}. [4 marks]

Solution:

Concept:

- The conductance of a cell is the reciprocal of its resistance: $G = 1/R$. Conductance depends on the dimensions of the cell as well as on the solution.
- Conductivity and conductance are related by $\kappa = G(l/A)$, where l is electrode separation and A is electrode area. The geometrical factor l/A is called the cell constant and has unit cm^{-1} when conductivity is in $\mathrm{S\,cm}^{-1}$.

Step 1: Rearrange the conductivity relation Substituting $G = 1/R$ into $\kappa = G(l/A)$ gives

$$\kappa = \frac{1}{R}\frac{l}{A}, \qquad \frac{l}{A} = \kappa R.$$

Thus the cell constant is obtained by multiplying the known conductivity by the measured resistance. It is not obtained by dividing conductivity by resistance. The concentration and temperature specify the reference solution for which the conductivity value is supplied; no molar-conductivity calculation is needed.

Step 2: Substitute and express the requested coefficient Use $R = 1750\,\Omega$ and $\kappa = 0.152 \times 10^{-3}\,\mathrm{S\,cm}^{-1}$:

$$\frac{l}{A} = (0.152 \times 10^{-3})(1750) = 0.266\,\mathrm{cm}^{-1}.$$

The units check because $\mathrm{S}\,\Omega = 1$. Rewriting the result in the form printed in the question gives

$$0.266\,\mathrm{cm}^{-1} = 266 \times 10^{-3}\,\mathrm{cm}^{-1}.$$

The cell constant describes the geometry of this cell, rather than a concentration-dependent molar property of potassium chloride.

Step 3: Conclusion The cell constant is $0.266\,\mathrm{cm}^{-1}$, so the required coefficient is $\boxed{266}$.

Quick Tip to Solve: Use cell constant $= \kappa R$; checking that siemens times ohms cancels prevents a reciprocal mistake.

84. Manganese (VI) has ability to disproportionate in acidic solution. The difference in oxidation states of two ions it forms in acidic solution is _____. [4 marks]

Solution:

Concept:

- Disproportionation is a redox process in which the same initial oxidation state is converted into both a higher and a lower oxidation state. One fraction is oxidised while another is reduced.
- Manganese(VI) is present in manganate, $\mathrm{MnO_4}^{2-}$. In acidic solution it disproportionates to perman-

ganate, MnO_4^-, and manganese dioxide, MnO_2, containing manganese(VII) and manganese(IV), respectively.

Step 1: Identify the oxidation and reduction products For manganate, let the oxidation state of manganese be s. Since oxygen has oxidation state -2, $s + 4(-2) = -2$, so $s = +6$. In permanganate, $s + 4(-2) = -1$, giving $+7$. In manganese dioxide, $s + 2(-2) = 0$, giving $+4$. The balanced acidic disproportionation is

$$3\,MnO_4^{2-} + 4\,H^+ \longrightarrow 2\,MnO_4^- + MnO_2 + 2\,H_2O.$$

Two manganese atoms each lose one electron, while the third gains two, checking the electron balance.

Step 2: Take the difference between the final states The two final manganese oxidation states are $+7$ and $+4$, so their difference is

$$\Delta s = 7 - 4 = 3.$$

Do not subtract each product state separately from the initial $+6$: those changes are $+1$ and -2, whereas the question asks for the separation of the two final states. Although the wording calls both products ions, the lower oxidation-state product is the oxide precipitate MnO_2; its manganese oxidation state is still unambiguously $+4$.

Step 3: Conclusion The difference between the two product oxidation states is $\boxed{3}$ (dimensionless).

Quick Tip to Solve: For acidic manganate disproportionation remember the pair: purple permanganate has Mn(VII), while manganese dioxide has Mn(IV).

85. 0.2 g of an organic compound was subjected to estimation of nitrogen by Dumas method in which volume of N_2 evolved (at STP) was found to be 22.400 mL. The percentage of nitrogen in the compound is _____. [nearest integer]

(Given : Molar mass of N_2 is 28 g mol^{-1}. Molar volume of N_2 at STP : 22.4 L) [4 marks]

Solution:

Concept:

- In the Dumas method, nitrogen in an organic compound is converted into molecular nitrogen, N_2. The mass of the collected nitrogen gas equals the mass of nitrogen originally present in the sample.
- At the stated STP, one mole of N_2 occupies $22.4\,L$ and has mass $28\,g$. Percentage by mass is the mass of nitrogen divided by the sample mass, multiplied by 100.

Step 1: Convert the gas volume to litres The volume is given in millilitres, whereas the molar volume is given in litres. Use the same volume unit before calculating the amount of gas.

$$V = 22.400\,mL = 0.022400\,L.$$

The stated volume is already at STP, so a temperature or pressure correction is unnecessary.

Step 2: Calculate the nitrogen mass The number of moles and the corresponding mass of nitrogen are

$$n(\mathrm{N_2}) = \frac{0.022400}{22.4} = 1.000 \times 10^{-3}\,\mathrm{mol},$$

$$m(\mathrm{N_2}) = 1.000 \times 10^{-3} \times 28 = 0.0280\,\mathrm{g}.$$

The molar mass 28 already includes both nitrogen atoms. Multiplying this mass by two again would count the same nitrogen twice.

Step 3: Find the mass percentage The original organic sample has mass $0.200\,\mathrm{g}$. Hence

$$\%\mathrm{N} = \frac{0.0280}{0.200} \times 100 = 14.0\%.$$

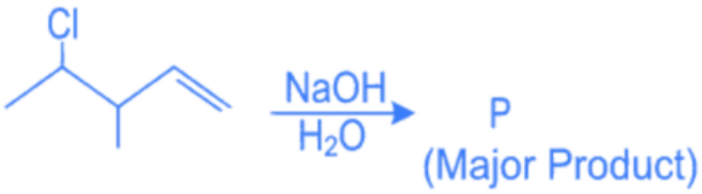
This is a mass percentage, rather than a mole percentage or the fraction of the collected gas volume. Its nearest integer is unchanged.

Step 4: Conclusion The nitrogen content is 14% by mass. The required numerical entry is $\boxed{14}$.

Quick Tip to Solve: Convert mL to L first, then use 28 g per mole of nitrogen gas; do not introduce another factor of two.

86. Consider the above reaction. The number of π electrons present in the product 'P' is _____.

[4 marks]



Solution:

Concept:

- Aqueous sodium hydroxide supplies hydroxide ions, which replace the chlorine atom in an alkyl chloride to give an alcohol. Aqueous conditions favour nucleophilic substitution over the elimination promoted by alcoholic hydroxide.
- A carbon–carbon double bond consists of one sigma bond and one pi bond. Each occupied pi bond contains two electrons; saturated carbon–carbon and carbon–oxygen single bonds contribute no pi electrons.

Step 1: Identify the functional groups in the reactant The displayed structure contains a chlorine substituent on a saturated carbon and a terminal alkene elsewhere in the chain. Reading from the left, its condensed structure is $\mathrm{CH_3CH(Cl)CH(CH_3)CH{=}CH_2}$. There is one carbon–carbon double bond before reaction, and the chlorine is not bonded directly to an alkene carbon.

Step 2: Apply the aqueous hydroxide reaction The reagent is NaOH in water, so replace the chlorine

on the saturated carbon by a hydroxyl group. The substitution product can be written as

$$\mathrm{CH_3CH(Cl)CH(CH_3)CH{=}CH_2 + NaOH}$$

$$\longrightarrow \mathrm{CH_3CH(OH)CH(CH_3)CH{=}CH_2 + NaCl.}$$

The remote terminal double bond is retained. A second double bond must not be introduced by treating the specified aqueous reagent as alcoholic hydroxide.

Step 3: Count only the pi electrons The major product has one alkene double bond and no carbonyl or aromatic pi system. Thus

$$N_{\pi \text{ bonds}} = 1, \qquad N_{\pi \text{ electrons}} = 1 \times 2 = 2.$$

The oxygen lone pairs are nonbonding electrons, not pi electrons in this alcohol; the numerous sigma-bond electrons also do not enter the requested count.

Step 4: Conclusion Product P contains one pi bond and therefore $\boxed{2}$ pi electrons. The numerical count has no unit.

Quick Tip to Solve: First distinguish aqueous hydroxide from alcoholic hydroxide, then count two electrons for each pi bond that remains.

87. In alanylglycyl leucyl alanyl valine, the number of peptide linkages is _____. [4 marks]

Solution:

Concept:

- A peptide linkage is the amide group $-\mathrm{CO}-\mathrm{NH}-$ formed when the carboxyl group of one amino acid condenses with the amino group of the next, eliminating water.
- A single open chain containing n amino-acid residues has $n-1$ peptide linkages. Each linkage joins a neighbouring pair of residues; terminal groups are not additional linkages.

Step 1: Read the sequence residue by residue The name alanylglycyl leucyl alanyl valine represents the sequence Ala–Gly–Leu–Ala–Val. It contains alanine, glycine, leucine, a second alanine, and valine, giving five residues in total. The repeated alanine must be counted as a separate residue because it occupies a separate place in the chain.

Step 2: Count the joins between neighbouring residues Write one join between each adjacent pair: Ala–Gly, Gly–Leu, Leu–Ala, and Ala–Val. These are four distinct condensation connections. Equivalently, for this pentapeptide,

$$N_{\text{peptide}} = n - 1 = 5 - 1 = 4.$$

The suffixes alanyl, glycyl and leucyl describe residues incorporated into the peptide; they do not introduce extra amino acids.

Step 3: Check the terminal groups The first residue supplies the free amino terminus and the final valine supplies the free carboxyl terminus in an ordinary linear peptide. Neither terminus connects

back to the other, so there is no fifth peptide bond. Forming the chain from the five free amino acids would eliminate four water molecules, one for each of the four amide linkages counted above.

Step 4: Conclusion The pentapeptide contains $\boxed{4}$ peptide linkages. This is a dimensionless count.

Quick Tip to Solve: Count residues, including repeated amino acids, then subtract one for a linear peptide; count a closing bond only for an explicitly cyclic peptide.

88. At 300 K, a sample of 3.0 g of gas A occupies the same volume as 0.2 g of hydrogen at 200 K at the same pressure. The molar mass of gas A is _____ g mol^{-1}. (nearest integer) Assume that the behaviour of gases as ideal.

(Given : The molar mass of hydrogen (H_2) gas is 2.0 g mol^{-1}.) [4 marks]

Solution:

Concept:

- The ideal-gas equation is $PV = nRT$, where P is pressure, V volume, n amount in moles and T absolute temperature.
- For gases occupying the same volume at the same pressure, nT is the same. Using $n = m/M$, the useful comparison is $m_A T_A/M_A = m_H T_H/M_H$; equal volumes imply equal moles only at equal temperatures.

Step 1: Find the amount of hydrogen The hydrogen sample is molecular hydrogen, H_2, with the stated molar mass $2.0\,\text{g mol}^{-1}$. Therefore

$$n_H = \frac{0.2}{2.0} = 0.100\,\text{mol}.$$

Its temperature is $200\,\text{K}$, while gas A is at $300\,\text{K}$. These different temperatures must be retained in the volume comparison.

Step 2: Compare the two ideal-gas equations Both samples have identical P and V, and the gas constant is universal. Thus

$$n_A RT_A = n_H RT_H \quad \Rightarrow \quad n_A = \frac{n_H T_H}{T_A}.$$

$$n_A = \frac{0.100 \times 200}{300} = \frac{1}{15}\,\text{mol}.$$

The hotter gas needs fewer moles to produce the same pressure in the same volume, consistent with this result.

Step 3: Calculate the molar mass of gas A Divide the stated sample mass by the calculated amount:

$$M_A = \frac{m_A}{n_A} = \frac{3.0}{1/15} = 45\,\text{g mol}^{-1}.$$

As a check, $n_A T_A = (1/15) \times 300 = 20\,\text{mol K}$, matching 0.100×200 for hydrogen. Using equal mole numbers instead would incorrectly discard the temperature information.

Step 4: Conclusion The molar mass is $\boxed{45\,\text{g mol}^{-1}}$. Rounded to the nearest integer, the requested entry is 45.

Quick Tip to Solve: At equal pressure and volume compare nT, not n alone; insert kelvin temperatures before solving for molar mass.

89. When 200 mL of 0.2 M acetic acid is shaken with 0.6 g of wood charcoal, the final concentration of acetic acid after adsorption is 0.1 M. The mass of acetic acid adsorbed per gram of carbon is _____ g. [4 marks]

Solution:

Concept:

- Adsorption removes solute from the bulk solution onto the surface of the solid. The amount adsorbed is obtained from the loss of solute in solution, rather than from the final concentration alone.
- For a fixed solution volume V, the moles adsorbed are $V(C_i - C_f)$. If x is the adsorbed mass and m the charcoal mass, the required adsorption per gram is x/m.

Step 1: Calculate the loss of dissolved acetic acid Convert the solution volume to litres to match the molarity units:

$$V = 200\,\text{mL} = 0.200\,\text{L}.$$

Initially there are $0.200 \times 0.2 = 0.040\,\text{mol}$, and after adsorption there are $0.200 \times 0.1 = 0.020\,\text{mol}$. Hence

$$n_{\text{ads}} = 0.040 - 0.020 = 0.020\,\text{mol}.$$

The concentration drop gives the amount transferred to the charcoal.

Step 2: Convert adsorbed moles into mass Acetic acid is CH_3COOH, with molar mass

$$M = 2(12) + 4(1) + 2(16) = 60\,\text{g mol}^{-1}.$$

Therefore the mass adsorbed by the entire charcoal sample is

$$x = 0.020 \times 60 = 1.20\,\text{g}.$$

This is the total uptake by $0.6\,\text{g}$ of charcoal; it is not yet the requested mass per gram.

Step 3: Divide by the charcoal mass The adsorption expressed per gram of carbon is

$$\frac{x}{m} = \frac{1.20\,\text{g}}{0.6\,\text{g}} = 2.0\,\text{g g}^{-1}.$$

Thus one gram of carbon corresponds to two grams of adsorbed acetic acid under these conditions. No gas-law volume conversion or adsorption isotherm constant is needed because the before-and-after concentrations directly determine the uptake.

Step 4: Conclusion The mass adsorbed is $\boxed{2\,\text{g}}$ per gram of carbon, equivalent to $2\,\text{g g}^{-1}$. The

numerical entry is 2.

Quick Tip to Solve: Use the decrease in molarity times volume to get moles adsorbed, convert to grams, and only then divide by charcoal mass.

90. (a) Baryte, (b) Galena, (c) Zinc blende and (d) Copper pyrites. How many of these minerals are sulphide based? [4 marks]

Solution:

Concept:

- A sulphide mineral contains sulphur in sulphide form combined with a metal. A sulphate contains the polyatomic ion $SO_4{}^{2-}$ and must not be counted as a sulphide merely because it contains sulphur.
- Identify each mineral by its chemical formula before counting. Baryte is barium sulphate; galena and zinc blende are the common sulphides of lead and zinc, while copper pyrites is a copper–iron sulphide.

Step 1: Separate baryte from the sulphides Baryte has the formula $BaSO_4$. Its sulphur belongs to the sulphate ion, where sulphur has oxidation state $+6$, rather than to sulphide. Thus baryte is excluded. The presence of the letter S in a formula is not by itself a sufficient test for a sulphide-based mineral.

Step 2: Identify the other three minerals Galena is lead sulphide, PbS, so it is sulphide based. Zinc blende is zinc sulphide, ZnS, and also qualifies. Copper pyrites, also called chalcopyrite, has formula $CuFeS_2$; it is classified as a sulphide mineral despite containing two different metals. These classifications follow the chemical form of sulphur, not the number of metal atoms present.

Step 3: Count the qualifying entries The list therefore contains one sulphate and three sulphide-based minerals. In the question's order, the qualifying entries are (b) galena, (c) zinc blende and (d) copper pyrites.

$$N_{\text{sulphide minerals}} = 1 + 1 + 1 = 3.$$

Count minerals rather than sulphur atoms: the two sulphur atoms in $CuFeS_2$ still correspond to just one listed mineral.

Step 4: Conclusion Galena, zinc blende and copper pyrites are sulphide based. The required dimensionless count is $\boxed{3}$.

Quick Tip to Solve: Write the ore formula first: sulfate groups such as SO4 are excluded, while PbS, ZnS and CuFeS2 each count as one sulphide mineral.