JEE Main 2022 Paper 1 (B.E./B.Tech) Jun 25 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 1 (June), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 25 June 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The value of $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ is equal to: [4 marks]

(A) $\frac{\pi^2}{4}$

(B) $\frac{\pi^2}{2}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

Correct Answer: (C) $\frac{\pi}{4}$

Solution:

Concept:

- The reflection identity is $\int_0^a F(x)\,dx = \int_0^a F(a-x)\,dx$. It allows two complementary exponential factors to be added before integration.
- Under $x \mapsto \pi - x$, sine is unchanged and cosine changes sign. The substitution $u = \cos x$ then gives $du = -\sin x\,dx$, while $\int du/(1+u^2) = \tan^{-1} u + C$.

Step 1: Reflect the integral Denote the given integral by I. The denominator is unchanged on replacing x by $\pi - x$, because cosine is squared in its first factor and the exponential terms exchange places in the second. Therefore,

$$I = \int_0^{\pi} \frac{e^{-\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}\,dx.$$

The limits remain 0 and π after the direction of substitution is reversed. This reflection removes the difficulty caused by the exponential numerator.

Step 2: Add and evaluate Add this expression to the original integral. The two exponential terms in the numerator cancel the corresponding sum in the denominator, giving

$$2I = \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx = \int_{-1}^{1} \frac{du}{1+u^2}.$$

Thus $2I = [\tan^{-1} u]_{-1}^{1} = \pi/4 - (-\pi/4) = \pi/2$, so $I = \pi/4$. The factor 2 on the left is essential: $\pi/2$ is the sum of two equal integrals, not the required integral itself. Options (A) and (B) incorrectly introduce an additional factor of π; option (D) omits this division by 2.

Step 3: Conclusion The integral equals $\boxed{\pi/4}$. Hence, the correct option is (C).

Quick Tip to Solve: When the denominator contains $e^{\cos x} + e^{-\cos x}$, pair the integral with its reflection about $\pi/2$ before making a substitution.

2. Let f : N → R be a function such that $f(x+y) = 2f(x)f(y)$ for natural numbers x and y. If f(1) = 2, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha+k) = \frac{512}{3}(2^{20}-1)$$

holds, is : [4 marks]

(A) 2

(B) 3

(C) 4

(D) 6

Correct Answer: (C) 4

Solution:

Concept:

- For a function defined on natural numbers, inserting $y = 1$ into the given functional equation yields a recurrence for successive values.
- A geometric progression with first term a, common ratio $r \neq 1$ and n terms has sum $a(r^n-1)/(r-1)$. Here the recurrence fixes the common ratio uniquely.

Step 1: Determine the function on natural numbers Set $y = 1$ and use $f(1) = 2$. Then

$$f(x+1) = 2f(x)f(1) = 4f(x).$$

Starting with $f(1) = 2$, successive values are $2, 8, 32, \ldots$. Induction therefore gives

$$f(n) = 2 \cdot 4^{n-1} = \frac{4^n}{2} \qquad (n \in \mathbb{N}).$$

This expression also satisfies the original equation, since $2(4^x/2)(4^y/2) = 4^{x+y}/2$. All four proposed values of α make the arguments $\alpha + k$ natural numbers.

Step 2: Sum the ten terms and compare coefficients The first term is $f(\alpha+1) = 2 \cdot 4^{\alpha}$, and every following term is four times the preceding one. Consequently,

$$\sum_{k=1}^{10} f(\alpha+k) = \frac{2 \cdot 4^{\alpha}}{3}(4^{10}-1).$$

Since $4^{10} = 2^{20}$, the required equality reduces, after cancelling the positive common factor $(2^{20}-1)/3$, to

$$2 \cdot 4^{\alpha} = 512 = 2^9.$$

Thus $2\alpha + 1 = 9$, giving $\alpha = 4$. For the other offered values $2, 3, 6$, the coefficient $2 \cdot 4^{\alpha}$ is respectively $32, 128, 8192$, rather than 512. Hence none of those options satisfies the sum.

Step 3: Conclusion $\boxed{\alpha = 4}$. Hence, the correct option is (C).

Quick Tip to Solve: First set one argument equal to 1 in a natural-number functional equation; the resulting recurrence often turns the requested sum directly into a geometric progression.

3. Let A be a 3 × 3 real matrix such that

$A\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}; A\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$ **and** $A\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$.

If $X=(x_1,x_2,x_3)^T$ **and I is an identity matrix of order 3, then the system** $(A-2I)X=\begin{pmatrix}4\\1\\1\end{pmatrix}$ **has :**

[4 marks]

(A) no solution

(B) infinitely many solutions

(C) unique solution

(D) exactly two solutions

Correct Answer: (B) infinitely many solutions

Solution:

Concept:

- The columns of a matrix A are Ae_1, Ae_2, Ae_3, where e_i are the standard basis vectors. Linearity lets us recover them from the images of sums of basis vectors.
- A linear system has infinitely many solutions when its equations are consistent and leave at least one free variable. A singular coefficient matrix alone does not establish consistency.

Step 1: Recover the columns and form the system The last given image gives $Ae_3=(1,1,2)^T$. Subtracting it from the image of e_1+e_3 gives $Ae_1=(-2,-1,-1)^T$. Subtracting that vector from the image of e_1+e_2 then gives $Ae_2=(3,2,1)^T$. Hence

$$A=\begin{pmatrix}-2&3&1\\-1&2&1\\-1&1&2\end{pmatrix},\qquad A-2I=\begin{pmatrix}-4&3&1\\-1&0&1\\-1&1&0\end{pmatrix}.$$

The order of these recovered vectors matters: each is a column, not a row.

Step 2: Solve and check consistency The second and third equations of the required system are

$$-x_1+x_3=1,\qquad -x_1+x_2=1.$$

Thus $x_2=x_1+1$ and $x_3=x_1+1$. On substitution into the first equation,

$$-4x_1+3(x_1+1)+(x_1+1)=4,$$

which is the identity $4=4$. There is no extra restriction on x_1. Writing $x_1=t$, every vector $(t,t+1,t+1)^T$, with real t, is a solution. This explicitly verifies consistency and the free parameter. Option (A) fails because solutions exist, while (C) and (D) fail because the real parameter produces infinitely many distinct solutions rather than one or two.

Step 3: Conclusion The solution set is $\boxed{X=(t,t+1,t+1)^T,\ t\in\mathbb{R}}$, so the system has infinitely many solutions. Hence, the correct option is (B).

Quick Tip to Solve: After recovering a matrix from vector images, solve the simplest rows first; an identity in the remaining row reveals a free parameter and proves consistency.

4. Let f : R → R be defined as $f(x) = x^3 + x - 5$. If g(x) is a function such that $f(g(x)) = x, \forall' x' \in R$, then g'(63) is equal to _____. [4 marks]

(A) $\frac{1}{49}$

(B) $\frac{3}{49}$

(C) $\frac{43}{49}$

(D) $\frac{91}{49}$

Correct Answer: (A) $\frac{1}{49}$

Solution:

Concept:

- If $f(g(x)) = x$, the chain rule gives $f\prime(g(x))g\prime(x) = 1$. Thus the derivative of the inverse is the reciprocal of the original derivative at the corresponding input.
- Here $f\prime(t) = 3t^2 + 1 > 0$ for every real t. The cubic is strictly increasing and has range $\mathbb{R}$, so its inverse is well-defined and the reciprocal formula applies.

Step 1: Find the inverse value at 63 Put $x = 63$ in the identity $f(g(x)) = x$. If $u = g(63)$, then

$$u^3 + u - 5 = 63 \quad \Longrightarrow \quad u^3 + u = 68.$$

The value $u = 4$ satisfies this equation because $4^3 + 4 = 64 + 4 = 68$. Since the function $u^3 + u$ has positive derivative $3u^2 + 1$, it is strictly increasing. There cannot be another real root. Hence $g(63) = 4$. This inverse value, rather than 63 itself, must be substituted into the derivative of f.

Step 2: Differentiate the composition Differentiate the given relation with respect to x:

$$\big(3g(x)^2 + 1\big)g\prime(x) = 1.$$

Using the value already obtained,

$$g\prime(63) = \frac{1}{3 \cdot 4^2 + 1} = \frac{1}{48 + 1} = \frac{1}{49}.$$

As a direct check, $f\prime(4)g\prime(63) = 49 \cdot (1/49) = 1$, exactly as the differentiated identity demands. The numerators 3, 43 and 91 in the other options would make this product 3, 43 and 91 respectively, so each contradicts the chain rule. The inverse derivative is positive, consistently with both functions being increasing.

Step 3: Conclusion $\boxed{g\prime(63) = 1/49}$. Hence, the correct option is (A).

Quick Tip to Solve: For an inverse derivative at a large input, solve $f(u) = x$ for the smaller preimage u first, then evaluate $1/f\prime(u)$.

5. If $\frac{1}{2.3^{10}} + \frac{1}{2^2.3^9} + \ldots\ldots + \frac{1}{2^{10}.3} = \frac{K}{2^{10}.3^{10}}$, then the remainder when K is divided by 6 is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- Multiplying a finite sum by its common denominator converts the problem into an integer sum. The numerator of the term $1/(2^j 3^{11-j})$ becomes $2^{10-j}3^{j-1}$.
- The finite identity $a^{n-1} + a^{n-2}b + \cdots + b^{n-1} = (a^n - b^n)/(a-b)$ follows by multiplying the sum by $a - b$ and cancelling successive terms.

Step 1: Find the integer numerator There are ten terms, indexed by $j = 1, \ldots, 10$. Multiplication by $2^{10}3^{10}$ yields

$$K = 2^9 + 2^8 \cdot 3 + 2^7 \cdot 3^2 + \cdots + 3^9.$$

This is a finite geometric sum with successive ratio $3/2$, although the displayed expression consists entirely of integers. Using the difference-of-powers identity gives

$$K = \frac{3^{10} - 2^{10}}{3 - 2} = 3^{10} - 2^{10}.$$

The first numerator is 2^9, not 2^{10}, because the original first denominator contains one factor of 2.

Step 2: Reduce the powers modulo 6 For every positive exponent, a power of 3 is congruent to 3 modulo 6: it is odd and divisible by 3. The positive even powers of 2 are congruent to 4 modulo 6, because powers of 2 alternate between residues 2 and 4. Therefore,

$$K \equiv 3 - 4 \equiv -1 \equiv 5 \pmod 6.$$

The required remainder must be the nonnegative residue less than 6, so it is 5. As an independent check, K is odd and is congruent to $-1 \equiv 2 \pmod 3$; among the six possible remainders only 5 has both properties. Options 1 and 3 fail the modulo-3 test, and option 2 fails the parity test.

Step 3: Conclusion The remainder when K is divided by 6 is $\boxed{5}$. Hence, the correct option is (D).

Quick Tip to Solve: When a common denominator is already displayed, multiply through first and recognize a difference of powers; then use parity and modulo 3 to obtain the residue modulo 6.

6. Let f(x) be a polynomial function such that $f(x)+f'(x)+f''(x)=x^5+64$. Then, the value of $\lim_{x\to1}\frac{f(x)}{x-1}$ is equal to: [4 marks]

(A) -15

(B) -60

(C) 60

(D) 15

Correct Answer: (A) -15

Solution:

Concept:

- Equality of two polynomials for all x means their coefficients of each power are equal. Differentiation lowers the degree, so the leading term of f must match the leading term x^5 on the right.
- If a polynomial satisfies $f(1)=0$, then $\lim_{x\to1} f(x)/(x-1)=f\prime(1)$ by the definition of its derivative. This condition must be checked before using the formula.

Step 1: Determine the polynomial Write $f(x)=a_5x^5+a_4x^4+a_3x^3+a_2x^2+a_1x+a_0$. No higher degree is possible, since its leading term could not cancel with the lower-degree derivatives. Matching coefficients in the given identity successively gives

$$\begin{aligned} a_5=1, \qquad & a_4+5=0, \quad a_3+4a_4+20=0,\\ a_2+3a_3+12a_4=0, \quad & a_1+2a_2+6a_3=0,\\ a_0+a_1+2a_2=64. & \end{aligned}$$

Thus $a_4=-5, a_3=0, a_2=60, a_1=-120, a_0=64$, and

$$f(x)=x^5-5x^4+60x^2-120x+64.$$

Step 2: Check the zero and evaluate the derivative At the point of the limit,

$$f(1)=1-5+60-120+64=0.$$

We may therefore replace the numerator by $f(x)-f(1)$ and use the derivative definition. Differentiation gives

$$f\prime(x)=5x^4-20x^3+120x-120.$$

Consequently,

$$\lim_{x\to1}\frac{f(x)}{x-1}=f\prime(1)=5-20+120-120=-15.$$

This is a two-sided finite limit because polynomials are differentiable. The positive option 15 loses the sign of $5-20$; the alternatives -60 and 60 do not equal the computed derivative and are ruled out by direct substitution into this exact expression.

Step 3: Conclusion The limit is $\boxed{-15}$. Hence, the correct option is (A).

Quick Tip to Solve: For a polynomial relation involving derivatives, compare coefficients from the highest power downward; verify the numerator vanishes at the limit point before using the derivative definition.

7. Let E_1 and E_2 be two events such that the conditional probabilities $P(E_1|E_2) = \frac{1}{2}$, $P(E_2|E_1) = \frac{3}{4}$ and $P(E_1 \cap E_2) = \frac{1}{8}$. Then : [4 marks]

(A) $P(E_1 \cap E_2) = P(E_1) \, . \, P(E_2)$

(B) $P(E_1' \cap E_2') = P(E_1') \, . \, P(E_2)$

(C) $P(E_1 \cap E_2') = P(E_1) \, . \, P(E_2)$

(D) $P(E_1' \cap E_2) = P(E_1) \, . \, P(E_2)$

Correct Answer: (C) $P(E_1 \cap E_2') = P(E_1) \, . \, P(E_2)$

Solution:

Concept:

- Conditional probability obeys $P(E_1 \mid E_2) = P(E_1 \cap E_2)/P(E_2)$, and the analogous formula with the events interchanged determines the other marginal probability.
- The events $E_1 \cap E_2$ and $E_1 \cap E_2'$ partition E_1. Thus their probabilities add to $P(E_1)$. Complement and inclusion-exclusion identities similarly determine the other intersections.

Step 1: Recover the marginal probabilities Use the given intersection probability $1/8$ in the two conditional probability formulas:

$$P(E_2) = \frac{1/8}{1/2} = \frac{1}{4}, \qquad P(E_1) = \frac{1/8}{3/4} = \frac{1}{6}.$$

Their product is therefore

$$P(E_1)P(E_2) = \frac{1}{6} \cdot \frac{1}{4} = \frac{1}{24}.$$

In particular, this differs from $P(E_1 \cap E_2) = 1/8$, so the events are not independent and option (A) is false. The product must be computed rather than inferred from the conditional probabilities.

Step 2: Check the complementary intersections Subtract the intersection from each marginal probability:

$$P(E_1 \cap E_2') = \frac{1}{6} - \frac{1}{8} = \frac{1}{24},$$

$$P(E_1' \cap E_2) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}.$$

Also, inclusion-exclusion and complementation give

$$P(E_1' \cap E_2') = 1 - \left(\frac{1}{6} + \frac{1}{4} - \frac{1}{8}\right) = \frac{17}{24}.$$

Thus the intersection with only E_2 complemented equals the product $1/24$. The intersection with both events complemented is $17/24$, rejecting (B), and the intersection with only E_1 complemented is $1/8$, rejecting (D). These four intersection probabilities also sum to 1, providing a check on the partition of the sample space.

Step 3: Conclusion $\boxed{P(E_1 \cap E_2') = P(E_1)P(E_2) = 1/24}$. Hence, the correct option is (C).

Quick Tip to Solve: Compute the marginals from conditional probabilities first, then fill the four cells formed by the events and their complements; this makes every proposed equality directly testable.

8. Let $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is : [4 marks]

(A) a non-identity symmetric matrix

(B) a skew-symmetric matrix

(C) neither symmetric nor skew-symmetric matrix

(D) an identity matrix

Correct Answer: (A) a non-identity symmetric matrix

Solution:

Concept:

- A matrix is symmetric when its transpose equals itself. Every real scalar multiple of the identity matrix is symmetric; a nonzero such matrix is not skew-symmetric.
- First reduce the powers using $A^2 = -4I$. Even powers are scalar multiples of I, whereas odd powers are scalar multiples of A.

Step 1: Compute the basic square Multiply the given matrix by itself, keeping the negative sign in the upper-right entry:

$$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I.$$

Consequently, $A^{2k} = (-4)^k I$ and $A^{2k-1} = (-4)^{k-1} A$. This avoids multiplying twenty separate matrices.

Step 2: Express both sums with one scalar Put $s = \sum_{j=0}^{9} (-4)^j$. The finite geometric progression gives

$$s = \frac{1 - (-4)^{10}}{1 - (-4)} = \frac{1 - 4^{10}}{5} = -209715.$$

Therefore

$$M = -4sI, \qquad N = sA, \qquad N^2 = s^2 A^2 = -4s^2 I.$$

The scalar s is negative and nonzero; its sign will also exclude the identity matrix.

Step 3: Classify the required product Since scalar matrices commute with every matrix,

$$MN^2 = (-4sI)(-4s^2 I) = 16s^3 I.$$

Its transpose is itself, so it is symmetric. Its scalar coefficient is negative, whereas the identity has coefficient 1; hence it is not the identity. A nonzero scalar identity matrix cannot satisfy $X^T = -X$, so option (B) fails. Option (C) fails because symmetry has already been established, and option (D) fails because the coefficient is not one.

Step 4: Conclusion $MN^2 = 16(-209715)^3 I$ is a non-identity symmetric matrix. Hence, the correct option is (A).

Quick Tip to Solve: For a matrix with $A^2 = cI$, reduce every even and odd power before summing; test the final scalar coefficient separately to distinguish a scalar matrix from the identity.

9. Let $g:(0,\infty)\to R$ be a differentiable function such that $\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{x\,g(x)}{e^x+1}+c$, for all x > 0, where c is an arbitrary constant. Then :

[4 marks]

(A) g is decreasing in $\left(0,\frac{\pi}{4}\right)$

(B) g' is increasing in $\left(0,\frac{\pi}{4}\right)$

(C) g + g' is increasing in $\left(0,\frac{\pi}{2}\right)$

(D) g − g' is increasing in $\left(0,\frac{\pi}{2}\right)$

Correct Answer: (D) g − g' is increasing in $\left(0,\frac{\pi}{2}\right)$

Solution:

Concept:

- Differentiate an indefinite-integral identity to equate its integrand with the derivative of the stated antiderivative. The arbitrary constant then disappears.
- A differentiable function increases where its derivative is positive. For combinations such as $g-g\prime$, test $g\prime-g\prime\prime$, not the signs of the two functions separately.

Step 1: Differentiate the identity Using the product rule on $xg(x)/(e^x+1)$,

$$\frac{d}{dx}\left(\frac{xg(x)}{e^x+1}\right)=\frac{xg\prime(x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}.$$

Compare this with the given integrand. The terms multiplying $g(x)$ are identical and cancel. Because $x>0$, the remaining factor $x/(e^x+1)$ is nonzero.

Step 2: Find the derivative information Cancellation therefore yields

$$g\prime(x)=\cos x-\sin x,\qquad g\prime\prime(x)=-\sin x-\cos x.$$

On $(0,\pi/4)$, cosine exceeds sine, so $g\prime>0$: g increases and option (A) is false. On that interval $g\prime\prime<0$, so $g\prime$ decreases and option (B) is false. These signs are independent of the additive constant in g.

Step 3: Test the two combinations On $(0,\pi/2)$, both sine and cosine are positive. Hence

$$(g+g\prime)\prime=g\prime+g\prime\prime=-2\sin x<0,$$
$$(g-g\prime)\prime=g\prime-g\prime\prime=2\cos x>0.$$

Thus the sum is strictly decreasing, excluding (C), while the difference is strictly increasing throughout the entire interval required in (D). No endpoint test is needed because the intervals are open.

Step 4: Conclusion $g-g\prime$ is increasing on $(0,\pi/2)$. Hence, the correct option is (D).

Quick Tip to Solve: In an integral identity with g in both sides, differentiate first and cancel matching product-rule terms before attempting to integrate for g.

10. Let $f : R \to R$ and $g : R \to R$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x) = \frac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$ holds ? [4 marks]

(A) (2, 3)

(B) $(-2, -1)$

(C) (1, 2)

(D) $(-1, 1)$

Correct Answer: (A) (2, 3)

Solution:

Concept:

- For a strictly increasing outer function, comparing function values is equivalent to comparing its inputs. A strictly decreasing inner function reverses the inequality.
- To establish monotonicity on all real numbers, check the derivative on both negative and nonnegative inputs; positivity on only one half-line is insufficient.

Step 1: Prove that the outer function increases Differentiate the given expression:

$$f\prime(x) = \frac{2x}{1+x^2} + e^{-x}.$$

If $x \geq 0$, the first term is nonnegative and the exponential is positive. If $x < 0$, the inequality $(x+1)^2 \geq 0$ gives $2x/(1+x^2) \geq -1$, while $e^{-x} > 1$. Thus $f\prime(x) > 0$ in this case as well. Therefore f is strictly increasing on $\mathbb{R}$.

Step 2: Determine the direction for the composite Rewrite $g(x) = e^{-x} - 2e^x$. Then

$$g\prime(x) = -e^{-x} - 2e^x < 0$$

for every real x. Accordingly g is strictly decreasing. Since f is increasing, the composite $f \circ g$ is strictly decreasing. Its larger value must correspond to the smaller input, with a strict comparison because both functions are strictly monotonic.

Step 3: Solve the resulting quadratic inequality The required inequality is therefore equivalent to

$$\frac{(\alpha-1)^2}{3} < \alpha - \frac{5}{3}.$$

Multiplying by the positive number 3 preserves its direction. Expanding and factoring gives

$$\alpha^2 - 5\alpha + 6 < 0 \quad \Longleftrightarrow \quad (\alpha-2)(\alpha-3) < 0.$$

The product is negative exactly between its roots. Both endpoints give equality and are excluded. Options (B), (C) and (D) lie outside this interval and therefore cannot satisfy the strict inequality.

Step 4: Conclusion The required range is $2 < \alpha < 3$. Hence, the correct option is (A).

Quick Tip to Solve: Before substituting complicated arguments into logarithms or exponentials, establish whether the composite increases or decreases; then compare the arguments directly.

11. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $a_i > 0$, $i = 1, 2, 3$ be a vector which makes equal angles with the coordinate axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i} + 4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a}$, $\vec{b}$ and x-axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is equal to: [4 marks]

(A) $\sqrt{7}$

(B) $\sqrt{2}$

(C) 2

(D) 7

Correct Answer: (B) $\sqrt{2}$

Solution:

Concept:

- Equal angles with the three positive coordinate axes give equal direction cosines, so the positive components of $\vec{a}$ are equal.
- A rotation through $90°$ preserves magnitude and produces a perpendicular vector. Coplanarity with the x-axis restricts the rotated vector to the plane spanned by $\vec{a}$ and $\hat{i}$.

Step 1: Find the original vector Write $\vec{a} = c(\hat{i} + \hat{j} + \hat{k})$, with $c > 0$. The vector $\vec{v} = 3\hat{i} + 4\hat{j}$ has magnitude 5. The given projection is

$$\frac{\vec{a} \cdot \vec{v}}{|\vec{v}|} = \frac{7c}{5} = 7,$$

so $c = 5$. Thus $\vec{a} = (5, 5, 5)$ and $|\vec{a}|^2 = 75$. The positivity condition ensures that the original vector points toward the positive octant.

Step 2: Use coplanarity and the right angle The plane through the x-axis and $(5, 5, 5)$ is $y = z$. Hence write $\vec{b} = (u, v, v)$. Perpendicularity and preservation of length imply

$$5(u + 2v) = 0, \qquad u^2 + 2v^2 = 75.$$

Substituting $u = -2v$ gives $6v^2 = 75$, so

$$v = \pm\frac{5}{\sqrt{2}}, \qquad u = \mp 5\sqrt{2}.$$

The two signs correspond to the two possible senses of the quarter-turn in this plane.

Step 3: Compute the projection length The signed scalar component along $\vec{v}$ is

$$\frac{\vec{b} \cdot \vec{v}}{5} = \frac{3u + 4v}{5} = \frac{-2v}{5} = \pm\sqrt{2}.$$

Therefore the length of the projection, which is the nonnegative value represented by the options, is $\sqrt{2}$. The sign depends on rotation sense, which was not specified, but its magnitude is fixed. None of $\sqrt{7}$, 2, or 7 equals this magnitude, excluding (A), (C), and (D).

Step 4: Conclusion The projection length of $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is $\sqrt{2}$. Hence, the correct option is (B).

Quick Tip to Solve: When rotation is confined to a plane, use the plane equation, the perpendicular dot product and the preserved norm; no rotation matrix is necessary.

12. Let $y = y(x)$ be the solution of the differential equation $(x+1)y' - y = e^{3x}(x+1)^2$, with $y(0) = \frac{1}{3}$. Then, the point $x = -\frac{4}{3}$ for the curve $y = y(x)$ is : [4 marks]

(A) not a critical point

(B) a point of local minima

(C) a point of local maxima

(D) a point of inflection

Correct Answer: (B) a point of local minima

Solution:

Concept:

- The quotient rule gives $\frac{d}{dx}[y/(x+1)] = ((x+1)y\prime - y)/(x+1)^2$, turning this differential equation into a direct integration.
- A stationary point has $y\prime = 0$. The sign change of the first derivative, or a positive second derivative there, distinguishes a local minimum from a maximum.

Step 1: Integrate the quotient derivative For $x \neq -1$, divide by $(x+1)^2$ to obtain

$$\frac{d}{dx}\left(\frac{y}{x+1}\right) = e^{3x}.$$

Integration yields

$$\frac{y}{x+1} = \frac{e^{3x}}{3} + C, \qquad y = (x+1)\left(\frac{e^{3x}}{3} + C\right).$$

This expression can be differentiated directly to check that it satisfies the original equation.

Step 2: Apply the initial value and differentiate At $x = 0$, the condition $y(0) = 1/3$ gives $1/3 = 1/3 + C$, hence $C = 0$. The solution curve is

$$y = \frac{(x+1)e^{3x}}{3}.$$

This expression also extends smoothly through $x = -1$ and satisfies the undivided equation there. Differentiating by the product rule,

$$y\prime = \frac{e^{3x}}{3}(3x+4).$$

The exponential factor is always positive, so the only zero of the derivative is $x = -4/3$.

Step 3: Classify the stationary point Immediately to the left of $-4/3$, the factor $3x+4$ is negative, and immediately to the right it is positive. Thus the curve changes from decreasing to increasing. Equivalently,

$$y\prime\prime = e^{3x}(3x+5), \qquad y\prime\prime(-4/3) = e^{-4} > 0.$$

This proves a strict local minimum. Option (A) is excluded by the vanishing first derivative; (C) has the opposite derivative change; and (D) fails because the second derivative is positive, with no change of concavity in a sufficiently small neighbourhood.

Step 4: Conclusion The point with abscissa $x = -4/3$ is a point of local minimum. Hence, the correct option is (B).

Quick Tip to Solve: Look for a quotient derivative when a linear differential equation contains $(x+1)y\prime - y$; apply the initial condition before classifying extrema.

13. If the solution curve $y = y(x)$ of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$, which passes through the point (1, 1) and intersects the line $y = \sqrt{3}x$ at the point $(\alpha, \sqrt{3}\alpha)$, then value of $\log_e(\sqrt{3}\alpha)$ is equal to : [4 marks]

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{12}$

(D) $\frac{\pi}{6}$

Correct Answer: (C) $\frac{\pi}{12}$

Solution:

Concept:

- A homogeneous equation in x and y can be simplified by using their ratio. Here treating x as a function of y makes the substitution $v = x/y$ especially short.
- The identities $\int dv/(1 + v^2) = \tan^{-1} v$ and $\int dy/y = \ln|y|$ separate the variables. The initial point fixes the integration constant and the relevant branch.

Step 1: Substitute the ratio of the coordinates Near the initial point $(1, 1)$, $y > 0$. Write $x = vy$, so $dx = v\,dy + y\,dv$. Substituting into the equation gives

$$y^2(v\,dy + y\,dv) + (v^2y^2 - vy^2 + y^2)\,dy = 0.$$

The terms $vy^2\,dy$ cancel, leaving

$$y^3\,dv + y^2(1 + v^2)\,dy = 0.$$

Divide by $y^3(1 + v^2)$, which is nonzero on this positive-y branch.

Step 2: Integrate and use the initial point The separated equation is

$$\frac{dv}{1 + v^2} + \frac{dy}{y} = 0.$$

Therefore

$$\tan^{-1} v + \ln y = C.$$

At $(x, y) = (1, 1)$, the ratio $v = 1$, so $C = \pi/4$. The required solution branch thus obeys

$$\tan^{-1}\left(\frac{x}{y}\right) + \ln y = \frac{\pi}{4}.$$

No absolute value is needed on this branch because y is positive.

Step 3: Evaluate at the specified line At the intersection $(\alpha, \sqrt{3}\alpha)$, the ratio $x/y = 1/\sqrt{3}$. The logarithm in the question also requires $\alpha > 0$, consistently with the positive-y branch. Since $\tan^{-1}(1/\sqrt{3}) = \pi/6$,

$$\ln(\sqrt{3}\alpha) = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}.$$

The values $\pi/3$, $\pi/2$, and $\pi/6$ in the other options omit or miscombine one of the two angles and do not satisfy the integrated relation with the initial condition.

Step 4: Conclusion $\log_e(\sqrt{3}\alpha) = \pi/12$. Hence, the correct option is (C).

Quick Tip to Solve: For homogeneous equations written with dx and dy, try both ratios mentally; choosing x/y here cancels the mixed terms immediately.

---

14. Let $x = 2t$, $y = \frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that $SA \perp BA$, where A is any point on the conic. If k is the ordinate of the centroid of the ΔSAB, then $\lim_{t \to 1} k$ is equal to : [4 marks]

(A) $\frac{17}{18}$
(B) $\frac{19}{18}$
(C) $\frac{11}{18}$
(D) $\frac{13}{18}$

Correct Answer: (D) $\frac{13}{18}$

Solution:

Concept:

- Eliminating the parameter identifies the parabola and its focus: $x^2 = 4ay$ has axis the y-axis and focus $(0, a)$.
- Perpendicular line segments have zero dot product. The ordinate of a triangle centroid is the arithmetic mean of its three vertex ordinates.

Step 1: Identify the focus and variable points From $x = 2t$ and $y = t^2/3$,

$$x^2 = 4t^2 = 12y.$$

Comparing with $x^2 = 4ay$ gives $a = 3$, so $S = (0, 3)$. The point on the parabola is $A = (2t, t^2/3)$. Since B lies on the axis, write $B = (0, b)$, where the ordinate b is determined by the perpendicularity condition.

Step 2: Impose the right-angle condition Use vectors from A to the other vertices:

$$\overrightarrow{AS} = (-2t, 3 - t^2/3), \quad \overrightarrow{AB} = (-2t, b - t^2/3).$$

Their dot product must vanish, giving

$$4t^2 + \left(3 - \frac{t^2}{3}\right)\left(b - \frac{t^2}{3}\right) = 0.$$

For t near 1, the factor $9 - t^2$ is nonzero, so

$$b = \frac{t^2}{3} - \frac{12t^2}{9 - t^2}.$$

The negative sign is essential: B lies below the vertex when $t = 1$.

Step 3: Average the ordinates and take the limit The centroid ordinate is

$$k = \frac{3 + t^2/3 + b}{3}.$$

All terms are continuous near $t = 1$, permitting direct substitution. At that limit,

$$b \longrightarrow \frac{1}{3} - \frac{12}{8} = -\frac{7}{6},$$

and therefore

$$k \longrightarrow \frac{3 + 1/3 - 7/6}{3} = \frac{13}{18}.$$

The focus ordinate, the point ordinate and the axis-intercept ordinate must all be included in this average. The other three listed fractions do not equal this value and hence fail the perpendicularity-and-centroid calculation.

Step 4: Conclusion $\lim_{t \to 1} k = 13/18$. Hence, the correct option is (D).

Quick Tip to Solve: For a parameterized parabola, eliminate the parameter first; then a dot product avoids slope exceptions when one point lies on the axis.

---

15. Let a circle C in complex plane pass through the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then $arg(z)$ is equal to : [4 marks]

(A) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \pi$

(B) $\tan^{-1}\left(\frac{24}{7}\right) - \pi$

(C) $\tan^{-1}(3) - \pi$

(D) $\tan^{-1}\left(\frac{3}{4}\right) - \pi$

Correct Answer: (B) $\tan^{-1}\left(\frac{24}{7}\right) - \pi$

Solution:

Concept:

- A complex number $z = x + iy$ represents the point (x, y). A circle with centre at the origin has equation $x^2 + y^2 = \rho^2$, where ρ is its radius.
- Perpendicular nonvertical lines have slopes whose product is -1. The principal argument must also be placed in the quadrant determined by the signs of both coordinates.

Step 1: Identify the circle through the three points The points are $(3, 4), (4, 3), (0, 5)$. Each satisfies $x^2 + y^2 = 25$, and they are noncollinear, so their unique circle is

$$x^2 + y^2 = 25.$$

This gives the circle directly, without solving three simultaneous equations for its centre.

Step 2: Find the perpendicular chord line The line joining the second and third points has slope

$(5-3)/(0-4) = -1/2$. Thus the line through $(3,4)$ perpendicular to it has slope 2. Its equation is

$$y - 4 = 2(x-3), \qquad y = 2x - 2.$$

The desired point is the second intersection of this line with the circle; the known intersection must be excluded.

Step 3: Calculate the second intersection and select its argument Substituting the line equation into the circle gives

$$x^2 + (2x-2)^2 = 25 \quad \Longrightarrow \quad 5x^2 - 8x - 21 = 0.$$

Hence

$$(x-3)(5x+7) = 0.$$

Excluding $x = 3$ leaves $x = -7/5$, and then $y = -24/5$. Both coordinates are negative, so the point lies in the third quadrant. Its positive reference angle has tangent $24/7$. The other offered reference-angle ratios, $2/\sqrt{5}, 3, 3/4$, do not equal $24/7$, so they cannot give this direction.

Step 4: Conclusion The point is $z = -\frac{7}{5} - \frac{24}{5}i$, and its principal argument is $\tan^{-1}(24/7) - \pi$. Hence, the correct option is (B).

Quick Tip to Solve: When several complex points have the same modulus, test a circle centred at the origin first; determine the argument quadrant after finding the second intersection.

16. Let a circle C touch the lines $L_1 : 4x - 3y + K_1 = 0$ and $L_2 = 4x - 3y + K_2 = 0$, $K_1, K_2 \in R$. If a line passing through the centre of the circle C intersects L_1 at $(-1, 2)$ and L_2 at $(3, -6)$, then the equation of the circle C is : [4 marks]

(A) $(x-1)^2 + (y-2)^2 = 4$

(B) $(x+1)^2 + (y-2)^2 = 4$

(C) $(x-1)^2 + (y+2)^2 = 16$

(D) $(x-1)^2 + (y-2)^2 = 16$

Correct Answer: (C) $(x-1)^2 + (y+2)^2 = 16$

Solution:

Concept:

- A circle tangent to two distinct parallel lines has its centre on the parallel line halfway between them, and its radius is half their perpendicular separation.
- The perpendicular distance from (h,k) to $Ax + By + C = 0$ is $|Ah + Bk + C|/\sqrt{A^2 + B^2}$. The circle equation is $(x-h)^2 + (y-k)^2 = r^2$.

Step 1: Recover the equations of the two tangent lines Substitute the supplied points into their respective lines. For $(-1, 2)$,

$$-4 - 6 + K_1 = 0 \quad \Longrightarrow \quad K_1 = 10.$$

For $(3, -6)$,

$$12 + 18 + K_2 = 0 \quad \Longrightarrow \quad K_2 = -30.$$

Therefore the tangents are $4x - 3y + 10 = 0$ and $4x - 3y - 30 = 0$. They have the same normal vector and are distinct, as required.

Step 2: Locate the centre on the given transversal The locus equidistant from these parallel lines is their midline, $4x - 3y - 10 = 0$. A line through the centre meets the two parallel tangents at the given endpoints. Its intersection with the midline is the midpoint of those endpoints, because the signed linear expression $4x - 3y$ changes linearly along any transversal. Thus

$$(h, k) = \left(\frac{-1+3}{2}, \frac{2-6}{2}\right) = (1, -2).$$

The transversal need not be perpendicular to the tangents; its full length is therefore not the diameter.

Step 3: Calculate the radius and compare the choices The distance between the parallel tangents is

$$\frac{|10 - (-30)|}{\sqrt{4^2 + (-3)^2}} = \frac{40}{5} = 8.$$

Consequently $r = 4$ and $r^2 = 16$. As a check, the distance from $(1, -2)$ to the first line is $|4+6+10|/5 = 4$. Options (A), (B) and (D) have incorrect centres; (A) and (B) also use the incorrect squared radius 4.

Step 4: Conclusion The required equation is $\boxed{(x-1)^2 + (y+2)^2 = 16}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the midpoint of the two intersections to locate the centre, but use the perpendicular separation of the parallel lines to calculate the diameter.

---

17. Let a, b and c be the length of sides of a triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of $\frac{R}{r}$ is equal to : [4 marks]

(A) $\frac{5}{2}$

(B) 2

(C) $\frac{3}{2}$

(D) 1

Correct Answer: (A) $\frac{5}{2}$

Solution:

Concept:

- The common value in a chain of ratios may be denoted by a positive parameter. Adding the pairwise side sums then determines the semiperimeter and each side.
- For a triangle of area Δ and semiperimeter s, $r = \Delta/s$ and $R = abc/(4\Delta)$. A right triangle also has circumradius equal to half its hypotenuse.

Step 1: Convert the given ratios into side lengths Let the common value be $k > 0$. Then

$$a + b = 7k, \qquad b + c = 8k, \qquad c + a = 9k.$$

Adding these equations gives $2(a + b + c) = 24k$, so $a + b + c = 12k$. Subtract the equation for the other two sides to obtain

$$a = 4k, \qquad b = 3k, \qquad c = 5k.$$

These lengths satisfy the triangle inequalities, and the unknown scale will cancel in the radius ratio.

Step 2: Recognise the right triangle and find its area The lengths obey

$$(3k)^2 + (4k)^2 = (5k)^2.$$

Thus the triangle is right angled, with perpendicular sides $3k, 4k$ and hypotenuse $5k$. Its area and semiperimeter are

$$\Delta = \frac{1}{2}(3k)(4k) = 6k^2, \qquad s = \frac{3k + 4k + 5k}{2} = 6k.$$

The largest side must be used as the hypotenuse; interchanging its role with a smaller side would produce an incorrect circumradius.

Step 3: Evaluate both radii and their ratio The incircle radius follows from the area relation:

$$r = \frac{\Delta}{s} = \frac{6k^2}{6k} = k.$$

Since the circumcentre of a right triangle is the midpoint of its hypotenuse,

$$R = \frac{5k}{2}, \qquad \frac{R}{r} = \frac{5k/2}{k} = \frac{5}{2}.$$

The right-triangle identity $r = (3k + 4k - 5k)/2 = k$ independently confirms the inradius. One can check the circumradius with $abc/(4\Delta) = 60k^3/(24k^2) = 5k/2$. The value 2 applies to an equilateral triangle, which these unequal sides do not form. The values $3/2$ and 1 also disagree with the explicitly calculated radii.

Step 4: Conclusion The scale cancels and $\boxed{R/r = 5/2}$. Hence, the correct option is (A).

Quick Tip to Solve: From pairwise sums, first find the total side sum; the resulting 3:4:5 ratio makes both the area and circumradius immediate.

18. Consider the following two propositions:

$$P1 :\sim (p \rightarrow \sim q)$$

$$P2 : (p \wedge \sim q) \wedge ((\sim p) \vee q)$$

If the proposition $p \rightarrow ((\sim p) \vee q)$ is evaluated as FALSE, then : [4 marks]

(A) P1 is TRUE and P2 is FALSE

(B) P1 is FALSE and P2 is TRUE

(C) Both P1 and P2 are FALSE

(D) Both P1 and P2 are TRUE

Correct Answer: (C) Both P1 and P2 are FALSE

Solution:

Concept:

- An implication $A \rightarrow B$ is false only when its antecedent A is true and its consequent B is false. Negation reverses a truth value.
- A disjunction is false only if both components are false; a conjunction is true only if both components are true. Apply these rules to each bracket before combining them.

Step 1: Deduce the values of the original statements The given proposition is

$$p \rightarrow ((\sim p) \vee q).$$

Since it is false, p must be TRUE and $(\sim p) \vee q$ must be FALSE. A false disjunction requires $\sim p$ to be FALSE and q to be FALSE. Hence the only compatible assignment is

$$p = \text{TRUE}, \qquad q = \text{FALSE}.$$

There is no need to construct a full four-row truth table: the false implication already singles out this row.

Step 2: Evaluate the first proposition For $P_1 =\sim (p \rightarrow \sim q)$, the value $q = \text{FALSE}$ gives $\sim q = \text{TRUE}$. Thus $p \rightarrow \sim q$ is TRUE, because its antecedent and consequent are both TRUE. Negating this result gives

$$P_1 =\sim (\text{TRUE}) = \text{FALSE}.$$

Equivalently, $\sim (p \rightarrow \sim q)$ simplifies to $p \wedge q$, which is also FALSE under the derived assignment.

Step 3: Evaluate the second proposition and compare the options The first factor of P_2 is $p \wedge \sim q = \text{TRUE} \wedge \text{TRUE} = \text{TRUE}$. Its second factor is $(\sim p) \vee q = \text{FALSE} \vee \text{FALSE} = \text{FALSE}$. Therefore

$$P_2 = \text{TRUE} \wedge \text{FALSE} = \text{FALSE}.$$

Options (A) and (D) incorrectly make P_1 TRUE. Options (B) and (D) incorrectly make P_2 TRUE. Only the choice declaring both propositions FALSE matches the evaluation.

Step 4: Conclusion Both P_1 and P_2 are FALSE. Hence, the correct option is (C).

Quick Tip to Solve: For a false implication, write antecedent TRUE and consequent FALSE immediately; then evaluate the smaller logical brackets one at a time.

---

19. If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to : [4 marks]

(A) $3 + 4\sqrt{2}$

(B) $-5 + 6\sqrt{2}$

(C) $-4 + 3\sqrt{2}$

(D) $7 + 6\sqrt{2}$

Correct Answer: (C) $-4 + 3\sqrt{2}$

Solution:

Concept:

- A nonvertical line $y = mx + c$ is tangent to a parabola when substitution gives a quadratic with zero discriminant. This criterion fixes the intercept in terms of the slope.
- A line touches a circle when the perpendicular distance from its centre to the line equals its radius. Here the circle has centre $(0, 0)$ and radius $\sqrt{2}$.

Step 1: Impose tangency to the parabola Substitute $y = mx + c$ into $y^2 = x$:

$$m^2x^2 + (2mc - 1)x + c^2 = 0.$$

For tangency the discriminant vanishes:

$$(2mc - 1)^2 - 4m^2c^2 = 1 - 4mc = 0.$$

Hence $c = 1/(4m)$. In particular $m \neq 0$; a horizontal line intersects this parabola once but is not its tangent at a finite point, and does not satisfy the discriminant condition above.

Step 2: Impose tangency to the circle The line is $mx - y + c = 0$. Its distance from the origin must equal $\sqrt{2}$, so

$$\frac{|c|}{\sqrt{1 + m^2}} = \sqrt{2} \quad \Longrightarrow \quad c^2 = 2(1 + m^2).$$

Substituting the parabola condition gives

$$\frac{1}{16m^2} = 2(1 + m^2).$$

Put $t = m^2 > 0$. After multiplying through,

$$32t^2 + 32t - 1 = 0.$$

Both conditions are required; using only one would describe tangents to just one of the curves.

Step 3: Find the product of the two slopes The quadratic gives

$$t = \frac{-32 \pm \sqrt{1152}}{64} = \frac{-4 \pm 3\sqrt{2}}{8}.$$

Only the plus sign gives a positive squared slope. The two distinct slopes are therefore $m_1 = \sqrt{t}$ and $m_2 = -\sqrt{t}$, whence

$$8|m_1 m_2| = 8t = -4 + 3\sqrt{2}.$$

The negative root for t cannot represent a real line. Numerically the required value is about 0.243; the values in (A), (B) and (D) are all different and do not satisfy the derived slope equation.

Step 4: Conclusion The required value is $\boxed{-4 + 3\sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a common tangent, obtain the parabola intercept from a zero discriminant, then use the circle distance condition; solve for the squared slope to avoid unnecessary signs.

20. Let Q be the mirror image of the point P(1, 0, 1) with respect to the plane S : x + y + z = 5. If a line L passing through (1, −1, −1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to : [4 marks]

(A) 2

(B) 5

(C) 7

(D) 11

Correct Answer: (B) 5

Solution:

Concept:

- The segment joining a point to its mirror image in a plane is normal to that plane, and its midpoint lies on the plane. For $x + y + z = 5$, a normal vector is $(1, 1, 1)$.
- A line parallel to a normal vector can be parameterised by adding equal parameters to all three coordinates. The squared distance between two points is the sum of the squared coordinate differences.

Step 1: Find the reflected point Write the reflected point as

$$Q = P + \lambda(1, 1, 1) = (1 + \lambda, \lambda, 1 + \lambda).$$

The midpoint of P and Q is $(1 + \lambda/2, \lambda/2, 1 + \lambda/2)$. It lies on the reflecting plane, giving

$$2 + \frac{3\lambda}{2} = 5 \quad \Longrightarrow \quad \lambda = 2.$$

Therefore $Q = (3, 2, 3)$. The midpoint $(2, 1, 2)$ indeed has coordinate sum 5, verifying the reflection.

Step 2: Intersect the parallel line with the plane The direction of PQ is $(2, 2, 2)$, equivalent to $(1, 1, 1)$.

The line through $(1, -1, -1)$ parallel to it is

$$(x, y, z) = (1 + t, -1 + t, -1 + t).$$

Substitute this into $x + y + z = 5$:

$$-1 + 3t = 5 \quad \Longrightarrow \quad t = 2.$$

Thus $R = (3, 1, 1)$. Its coordinate sum is 5, so the intersection satisfies the required plane equation.

Step 3: Calculate the requested squared length The vector from R to Q is

$$Q - R = (0, 1, 2).$$

Consequently

$$QR^2 = 0^2 + 1^2 + 2^2 = 5.$$

The question asks for the squared length, not the length $\sqrt{5}$. The value 2 would count the nonzero differences without squaring them, while 7 and 11 likewise do not equal the sum of the squared coordinate differences. Thus none of those other choices matches the computed geometry.

Step 4: Conclusion The required squared distance is $\boxed{QR^2 = 5}$ square coordinate units. Hence, the correct option is (B).

Quick Tip to Solve: For reflection in a plane, move along its normal and impose the midpoint condition; use a separate parameter for the parallel line meeting the plane.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is _____.

[4 marks]

Solution:

Concept:

- Write a three-digit odd number as $100a + 10b + c$, where $1 \le a \le 9$, $0 \le b \le 9$, and $c \in \{1, 3, 5, 7, 9\}$. The hundreds digit cannot be zero.
- The digit sum must be a multiple of seven. Its minimum is 2 and maximum is 27, so the only possible sums are $7, 14, 21$. Count disjoint cases according to the units digit and sum.

Step 1: Derive a count for the first two digits For a chosen digit sum S and odd units digit c, put $t = S - c$. We need $a + b = t$, with $1 \le a \le 9$ and $0 \le b \le 9$. Since $b = t - a$, the admissible integers a lie in

$$\max(1, t - 9) \le a \le \min(9, t).$$

Thus the number of pairs is

$$N(t) = \max\{0, \min(9, t) - \max(1, t - 9) + 1\}.$$

This incorporates both the no-leading-zero restriction and the upper bound on the tens digit.

Step 2: Count the possibilities for each digit sum Using the odd units digits in the order $1, 3, 5, 7, 9$, the counts are

$S\backslash c$	1	3	5	7	9	Total
7	6	4	2	0	0	12
14	6	8	9	7	5	35
21	0	1	3	5	7	16

For example, when $S = 14, c = 1$, the first two digits sum to 13: the hundreds digit ranges from 4 to 9, giving six choices. When $S = 21, c = 3$, their sum is 18, so only $(a, b) = (9, 9)$ is possible.

Step 3: Add the disjoint cases Every admissible number has exactly one units digit and exactly one digit sum, so the table has neither overlap nor omission. Adding the row totals gives

$$12 + 35 + 16 = 63.$$

A digit sum of zero is impossible for a three-digit positive number, and the next multiple 28 exceeds the maximum possible digit sum 27. No further cases are needed. Repeated digits are permitted because the question imposes no distinct-digit condition.

Step 4: Conclusion The number of such three-digit odd numbers is $\boxed{63}$, a dimensionless count.

Quick Tip to Solve: Fix the odd units digit first. Then count hundreds digits in an interval, rather than listing numbers or accidentally allowing a leading zero.

---

22. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4$, $|\vec{b}| = 3$ and $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$. Then $\left|\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right)\right|^2 + 4\left(\vec{a} \cdot \vec{b}\right)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The cross product is distributive and antisymmetric: $\mathbf{b} \times \mathbf{a} = -\mathbf{a} \times \mathbf{b}$, while $\mathbf{a} \times \mathbf{a} = \mathbf{0}$.
- For vectors with angle θ, $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$ and $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$.

Step 1: Expand the vector product Apply distributivity to all four terms before taking its magnitude. The self-products vanish, and the two mixed products have the same sign after antisymmetry is used:

$$\begin{aligned}(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} + \mathbf{b}) &= \mathbf{a} \times \mathbf{a} + \mathbf{a} \times \mathbf{b} - \mathbf{b} \times \mathbf{a} - \mathbf{b} \times \mathbf{b} \\ &= 2\mathbf{a} \times \mathbf{b}.\end{aligned}$$

Thus its squared magnitude is four times the squared magnitude of the original cross product; it is not merely twice that quantity.

Step 2: Substitute the two vector magnitudes Here $|\mathbf{a}||\mathbf{b}| = 4 \times 3 = 12$. Hence the two contributions to the requested expression are

$$|2\mathbf{a} \times \mathbf{b}|^2 = 4(12 \sin\theta)^2 = 576 \sin^2\theta$$

$$4(\mathbf{a} \cdot \mathbf{b})^2 = 4(12 \cos\theta)^2 = 576 \cos^2\theta.$$

The given open interval for the angle is consistent with these formulas, but no particular angle in that interval needs to be selected.

Step 3: Use the Pythagorean identity Adding the two contributions gives $576(\sin^2\theta + \cos^2\theta) = 576$. The dependence on the angle cancels exactly. In particular, substituting an endpoint of the interval is unnecessary and would not be justified by the strict inequalities.

Step 4: Conclusion The required numerical value is $\boxed{576}$. This is dimensionless for the abstract vectors specified.

Quick Tip to Solve: Expand the cross product first: the expression becomes four times the sum of squared cross and dot products, so the angle cancels.

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23. Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to _____. [4 marks]

Solution:

Concept:

- If $P = (x_1, y_1)$ and $Q = (x_2, y_2)$, the circle with diameter PQ has equation $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.
- Vieta's formulas give the sum and product of roots directly, without finding the individual roots or deciding how the coordinates are paired.

Step 1: Find the coordinate sums and products The abscissae are roots of $2t^2 - rt + p = 0$, where t is a dummy variable. Therefore

$$x_1 + x_2 = \frac{r}{2}, \qquad x_1 x_2 = \frac{p}{2}.$$

Similarly, the ordinates are roots of $t^2 - st - q = 0$, so

$$y_1 + y_2 = s, \qquad y_1 y_2 = -q.$$

The minus sign in the ordinate product is essential: the constant term of the second polynomial is $-q$, not q.

Step 2: Form and compare the circle equation Expanding the diameter equation and inserting these sums and products yields

$$x^2 + y^2 - \frac{r}{2}x - sy + \frac{p}{2} - q = 0.$$

Multiplying the entire equation by 2 puts it in the same normalization as the supplied equation:

$$2(x^2+y^2)-rx-2sy+p-2q=0.$$

Comparing coefficients with $2(x^2+y^2)-11x-14y-22=0$ gives $r=11$, $s=7$, and $p-2q=-22$.

Step 3: Evaluate only the combination required The individual values of p and q are not needed. The target already contains their determined combination:

$$2r+s-2q+p=2(11)+7+(p-2q)=22+7-22=7.$$

This also shows why any ambiguity in pairing the two abscissae with the two ordinates cannot affect the requested result.

Step 4: Conclusion The required value of $2r+s-2q+p$ is $\boxed{7}$, a dimensionless number.

Quick Tip to Solve: For a circle on a diameter, compare coordinate sums and the sum of coordinate products; avoid solving for the endpoints.

24. Let $f: R \to R$ be a function defined by $f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)(2+x^{25})\right)^{\frac{1}{50}}$. If the function $g(x)=f(f(f(x)))+f(f(x))$, then the greatest integer less than or equal to g(1) is _____. [4 marks]

Solution:

Concept:

- Simplifying the radicand gives $f(x)=(4-x^{50})^{1/50}$ wherever this expression is real. The real root is the nonnegative fiftieth root.
- To evaluate a nested composition at 1, follow its successive values. The stated formula is real for $|x| \le 4^{1/50}$, and all the arguments used here lie in that interval.

Step 1: Simplify before composing Put $t=x^{25}$. The product inside the root satisfies

$$2\left(1-\frac{t}{2}\right)(2+t)=(2-t)(2+t)=4-t^2.$$

Therefore $f(x)^{50}=4-x^{50}$, provided the real value exists. Although the printed declaration uses all real numbers, the formula is real only on $[-4^{1/50}, 4^{1/50}]$. This domain issue does not obstruct the requested composition, since its arguments remain within this interval.

Step 2: Calculate the successive values at 1 Let $a=3^{1/50}$, the positive real fiftieth root of 3. Then

$$f(1)=(4-1)^{1/50}=a.$$

Since $a^{50}=3$, the next application gives

$$f(f(1))=f(a)=(4-3)^{1/50}=1.$$

Applying f once more gives $f(f(f(1))) = f(1) = a$. Thus the successive inputs alternate between 1 and a; neither introduces a negative radicand.

Step 3: Locate the value between consecutive integers We have $g(1) = a + 1$. Because $1^{50} < 3 < 2^{50}$ and taking the positive fiftieth root preserves order, $1 < a < 2$. Consequently $2 < g(1) < 3$, so its greatest integer is 2. A decimal approximation is unnecessary and could obscure how close the positive root is to 1.

Step 4: Conclusion The greatest integer less than or equal to $g(1)$ is $\boxed{2}$. The requested value is dimensionless.

Quick Tip to Solve: Raise each intermediate value to the fiftieth power: this exposes the two-cycle $1 \mapsto 3^{1/50} \mapsto 1$.

25. Let A be a 3×3 matrix having entries from the set $\{-1, 0, 1\}$. The number of all such matrices A having sum of all the entries equal to 5, is _____. [4 marks]

Solution:

Concept:

- A 3×3 matrix has nine distinct positions. Assigning entries to these labelled positions is counted by multinomial coefficients.
- If there are a entries equal to 1, b equal to -1, and c equal to 0, then $a + b + c = 9$ and the entry sum is $a - b$.

Step 1: Find all possible entry counts The required sum is 5, so $a - b = 5$, or $a = b + 5$. Substituting into the total-position equation gives

$$c = 9 - (b + 5) - b = 4 - 2b.$$

Both b and c must be nonnegative integers. Thus b can only be 0, 1, or 2. The resulting triples (a, b, c) are $(5, 0, 4)$, $(6, 1, 2)$, and $(7, 2, 0)$. Values $b \geq 3$ would require a negative number of zero entries and are impossible.

Step 2: Count matrices for each triple For each fixed triple, select the positions occupied by the three entry values. Since every matrix position is distinguishable, the counts are

$$N_0 = \frac{9!}{5!\,0!\,4!} = \binom{9}{5} = 126,$$

$$N_1 = \frac{9!}{6!\,1!\,2!} = \binom{9}{1}\binom{8}{2} = 252,$$

$$N_2 = \frac{9!}{7!\,2!\,0!} = \binom{9}{2} = 36.$$

In the middle case one first chooses the negative entry, then the two zeros; all six remaining entries are 1.

Step 3: Add the disjoint cases These cases cannot overlap because they contain different numbers of

negative entries, and the nonnegativity argument shows they exhaust all possibilities. Hence

$$N = N_0 + N_1 + N_2 = 126 + 252 + 36 = 414.$$

There is no division by row or column permutations: interchanging positions generally produces a different matrix.

Step 4: Conclusion There are $\boxed{414}$ such matrices. This is a dimensionless count.

Quick Tip to Solve: Use the numbers of positive, negative and zero entries as variables; the sum condition reduces the count to only three multinomial cases.

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26. Let C_r denote the binomial coefficient of x^r in the expansion of $(1+x)^{10}$. If for $\alpha, \beta \in R$, $C_1 + 3.2C_2 + 5.3C_3 +$ ..... upto 10 terms $= \frac{\alpha \times 2^{11}}{2^\beta - 1}\left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots.. upto\, 10\, terms\right)$ then the value of $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- For $C_r = \binom{10}{r}$, differentiation of $(1+x)^{10}$ gives weighted sums involving rC_r and $r(r-1)C_r$.
- The printed equation has real parameters α, β; one equation for two real parameters does not generally determine their sum. The number of terms on the right also matters.

Step 1: Evaluate the left-hand weighted sum The displayed coefficients are $1, 3 \cdot 2, 5 \cdot 3, \ldots$, so the general term is $r(2r-1)C_r$, for $r = 1, \ldots, 10$. Using derivatives of the binomial expansion at $x = 1$,

$$\sum rC_r = 10\,2^9, \qquad \sum r(r-1)C_r = 90\,2^8.$$

Since $r(2r-1) = 2r(r-1) + r$, the left side is $180\,2^8 + 10\,2^9 = 51200 = 25\,2^{11}$.

Step 2: Check the printed ten-term sum on the right Ten terms starting with C_0 end at $C_9/10$. The identity $C_r/(r+1) = \binom{11}{r+1}/11$ therefore gives

$$S = \sum_{r=0}^{9} \frac{C_r}{r+1} = \frac{2^{11} - 2}{11} = 186.$$

The literal equation reduces to $\alpha/(2^\beta - 1) = 25/186$. For example $\beta = 1$ gives $\alpha = 25/186$, whereas $\beta = 2$ gives $\alpha = 75/186$. Both are allowed real pairs and their sums differ; consequently the literal demand has no unique answer.

Step 3: Explain the conventional keyed evaluation If the parenthesis is instead read as the complete eleven-term sum, including $C_{10}/11$, its value is $(2^{11} - 1)/11$. The usual coefficient comparison then chooses $\alpha = 275$, $\beta = 11$, producing $\alpha + \beta = 286$. This requires an additional term and a specific parameter convention; it is not a consequence of the printed ten-term equation with arbitrary real parameters.

Step 4: Conclusion The recorded answer key is $\boxed{286}$, obtained by the conventional complete-sum interpretation above. As literally printed, the ten-term sum and real-parameter condition do not

determine a unique value of $\alpha + \beta$. All quantities here are dimensionless.

Quick Tip to Solve: Write the summation limits explicitly before using a binomial identity: starting at C_0, ten terms stop at $C_9/10$, not $C_{10}/11$.

27. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\,\mathrm{cos}\,ec^2x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is _____. [4 marks]

Solution:

Concept:

- Use $\csc^2 x = 1/\sin^2 x$ and $\cos^2 x = 1 - \sin^2 x$. The original equation requires $\sin x \neq 0$.
- After substituting $u = \sin^2 x$, retain only roots in $(0, 1]$, then count the corresponding angles in the specified open interval.

Step 1: Reduce the equation to a quadratic Let $u = \sin^2 x$, with $u > 0$ because the cosecant is defined. Then

$$\frac{14}{u} - 2u = 21 - 4(1 - u) = 17 + 4u.$$

Multiplication by the nonzero u gives $14 = 17u + 6u^2$, or

$$6u^2 + 17u - 14 = (3u - 2)(2u + 7) = 0.$$

Thus $u = 2/3$ or $u = -7/2$. The negative root cannot be a squared sine and is discarded.

Step 2: Find the reference angle Define $a = \sin^{-1}\sqrt{2/3}$, with the inverse sine in its principal interval. Since $\sqrt{2/3} > 1/\sqrt{2}$ but is less than 1,

$$\frac{\pi}{4} < a < \frac{\pi}{2}.$$

The equation $\sin^2 x = 2/3$ has four solutions in $[0, 2\pi)$: a, $\pi - a$, $\pi + a$, and $2\pi - a$. These come from the two sine signs; keeping only positive sine would miss half the solutions.

Step 3: Check the open interval The first two solutions lie respectively in $(\pi/4, \pi/2)$ and $(\pi/2, 3\pi/4)$. The latter two lie in $(5\pi/4, 3\pi/2)$ and $(3\pi/2, 7\pi/4)$. Hence every one lies strictly between the requested endpoints. None has zero sine, so multiplying by u introduced no extraneous solution among them.

Step 4: Conclusion The number of values of x in the given interval is $\boxed{4}$, a dimensionless count.

Quick Tip to Solve: After solving for $\sin^2 x$, include both signs of sine and test the interval endpoints using the reference angle.

28. For a natural number n, let $\alpha_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8}$ is _____. [4 marks]

Solution:

Concept:

- For powers of two bases a, b, the difference sequence $a^n - b^n$ satisfies a second-order recurrence with coefficients $a + b$ and ab.
- Here the bases are 19 and 12, so their sum is 31 and their product is 228. Recognizing these values avoids calculating large powers.

Step 1: Derive the recurrence directly Start with the definition $\alpha_n = 19^n - 12^n$. For any positive integer n, expand

$$\begin{aligned} 31\alpha_n - \alpha_{n+1} &= 31(19^n - 12^n) - (19^{n+1} - 12^{n+1}) \\ &= (31 - 19)19^n - (31 - 12)12^n \\ &= 12\,19^n - 19\,12^n. \end{aligned}$$

Factoring one copy of each base gives $228(19^{n-1} - 12^{n-1}) = 228\alpha_{n-1}$. This derivation fixes the signs without relying on a memorized recurrence.

Step 2: Apply the identity at the requested index Take $n = 9$ in the derived identity. The numerator in the question is exactly

$$31\alpha_9 - \alpha_{10} = 228\alpha_8.$$

Therefore the requested ratio becomes

$$\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8} = \frac{228\alpha_8}{57\alpha_8}.$$

Since $19 > 12 > 0$, $19^8 - 12^8 > 0$; hence α_8 is nonzero and cancellation is valid.

Step 3: Simplify the remaining constants Cancel the common sequence term, then divide the remaining integers: $228/57 = 4$, since $57 \times 4 = 228$. The recurrence also explains why the particular exponent 9 does not require a separate numerical calculation: the same constant ratio results whenever the denominator uses the preceding sequence term.

Step 4: Conclusion The value of the given expression is $\boxed{4}$, a dimensionless number.

Quick Tip to Solve: Look for the sum and product of the two exponential bases: they generate a recurrence that cancels the large powers.

29. Let the lines

$$L_1 : \overrightarrow{r} = \lambda\left(\widehat{i} + 2\widehat{j} + 3\widehat{k}\right),\ \lambda \in R$$

$L_2 : \overrightarrow{r} = \left(\widehat{i} + 3\widehat{j} + \widehat{k}\right) + \mu\left(\widehat{i} + \widehat{j} + 5\widehat{k}\right);\ \mu \in R$**,**
intersect at the point S. If a plane ax + by − z + d = 0 passes through S and is parallel to both the lines L_1 and L_2, then the value of a + b + d is equal to _____. [4 marks]

Solution:

Concept:

- A plane with equation $ax + by - z + d = 0$ has normal vector $\vec{n} = (a, b, -1)$. Each direction vector parallel to this plane is perpendicular to its normal.
- The intersection point must satisfy both vector equations. Substituting that point into the plane determines its constant term d.

Step 1: Find the common point of the lines On the first line a general point is $(\lambda, 2\lambda, 3\lambda)$, while on the second it is $(1 + \mu, 3 + \mu, 1 + 5\mu)$. Equate the first two coordinates:

$$\lambda = 1 + \mu, \qquad 2\lambda = 3 + \mu.$$

Subtracting gives $\lambda = 2$, and hence $\mu = 1$. The third coordinates then agree because $3\lambda = 6 = 1 + 5\mu$. Therefore the common point is $S = (2, 4, 6)$; checking the third coordinate is essential because lines in space need not intersect merely because two coordinate equations agree.

Step 2: Use the two parallel directions The direction vectors are $\vec{u} = (1, 2, 3)$ and $\vec{v} = (1, 1, 5)$. Their scalar products with the normal vanish:

$$a + 2b - 3 = 0, \qquad a + b - 5 = 0.$$

Subtract the second equation from the first to obtain $b = -2$. Then $a + b = 5$ gives $a = 7$. These directions are not proportional, so they determine a unique plane direction. The fixed coefficient of z sets the scale of the normal.

Step 3: Determine the constant and required sum Insert $S = (2, 4, 6)$ into $7x - 2y - z + d = 0$:

$$14 - 8 - 6 + d = 0 \quad \Rightarrow \quad d = 0.$$

Thus the plane is $7x - 2y - z = 0$. Both entire lines lie in it, consistently with their intersection and the two perpendicularity conditions. Finally,

$$a + b + d = 7 - 2 + 0 = 5.$$

Step 4: Conclusion The required numerical value is $\boxed{5}$.

Quick Tip to Solve: For a plane parallel to two lines, first dot its normal with both line directions; use their intersection only to find the constant term.

30. The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}$, ... is equal to _____. [4 marks]

Solution:

Concept:

- The displayed denominators are 3^n. Subtracting each numerator from its denominator gives $2, 4, 8, 16$, so the general term is $t_n = 1 - (2/3)^n$.
- For a geometric progression of ratio $r \neq 1$, $\sum_{n=1}^{N} r^n = r(1 - r^N)/(1 - r)$. The greatest integer function is determined by exact bounds, not by rounding.

Step 1: Identify the general term Write the four given fractions as deficits from unity:

$$\frac{1}{3} = 1 - \frac{2}{3}, \quad \frac{5}{9} = 1 - \frac{4}{9}, \quad \frac{19}{27} = 1 - \frac{8}{27}, \quad \frac{65}{81} = 1 - \frac{16}{81}.$$

Their deficits form a geometric progression with first term $2/3$ and common ratio $2/3$. Thus the indicated sequence has $t_n = 1 - (2/3)^n$, for $n = 1, 2, \ldots$. Checking all four displayed terms guards against treating the fractions themselves as a geometric progression; their ratios are not constant.

Step 2: Sum the first hundred terms exactly Let S_{100} denote the required sum before taking its greatest integer. Sum the constant parts and the geometric deficits separately:

$$\begin{aligned} S_{100} &= 100 - \sum_{n=1}^{100} \left(\frac{2}{3}\right)^n \\ &= 100 - \frac{\frac{2}{3}\left[1 - (\frac{2}{3})^{100}\right]}{1 - \frac{2}{3}} \\ &= 98 + 2\left(\frac{2}{3}\right)^{100}. \end{aligned}$$

The correction term is positive, so the finite sum is slightly greater than 98, rather than equal to the limiting expression 98.

Step 3: Bound the correction before taking the integer part Since $0 < 2/3 < 1$, increasing the exponent decreases its power. In particular,

$$0 < 2\left(\frac{2}{3}\right)^{100} < 2\left(\frac{2}{3}\right)^{2} = \frac{8}{9} < 1.$$

Consequently $98 < S_{100} < 99$. Exactly one integer can be its greatest integer less than or equal to it, namely 98; no decimal approximation of the tiny correction is needed.

Step 4: Conclusion The greatest integer less than or equal to the sum is $\boxed{98}$.

Quick Tip to Solve: When summing terms close to one, sum their deficits as a geometric series and bound the remainder before applying the greatest integer function.

Physics – Section A: Multiple Choice Questions (80 marks)

31. If $Z = \frac{A^2B^3}{C^4}$, then the relative error in Z will be : [4 marks]

(A) $\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

(B) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} - \frac{4\Delta C}{C}$

(C) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}$

(D) $\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{\Delta C}{C}$

Correct Answer: (C) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}$

Solution:

Concept:

- For a product of measured powers, maximum relative errors add, with coefficients equal to the absolute values of the powers. Here the powers of A, B, C are $2, 3, -4$, respectively.
- Measurement uncertainties are nonnegative bounds. The negative exponent of a denominator contributes its positive magnitude to the maximum possible error; it does not cancel another uncertainty.

Step 1: Take the logarithmic differential Express the measured quantity as $Z = A^2B^3C^{-4}$. For small signed changes, logarithmic differentiation gives

$$\frac{dZ}{Z} = 2\frac{dA}{A} + 3\frac{dB}{B} - 4\frac{dC}{C}.$$

The coefficient 2 arises because A is squared, the coefficient 3 because B is cubed, and the coefficient 4 because C occurs to the fourth power in the denominator. This equation describes signed changes, not yet a maximum error bound.

Step 2: Convert signed changes into the relative error bound Errors can occur with either sign independently. To estimate the largest possible magnitude, add the magnitudes of all three contributions:

$$\frac{\Delta Z}{Z} = 2\frac{\Delta A}{A} + 3\frac{\Delta B}{B} + 4\frac{\Delta C}{C}.$$

As usual, the symbols in an error expression denote the positive measurement magnitudes and their positive uncertainties. For example, an underestimate of C increases Z, so its uncertainty can reinforce increases caused by errors in A and B. It cannot safely be subtracted.

Step 3: Compare the proposed expressions Option (C) has the required coefficients 2, 3 and 4 and adds every contribution. Option (A) incorrectly assigns unit coefficients despite the powers. Option (B) retains the minus sign from the signed differential, which is inappropriate for maximum uncertainty. Option (D) makes both mistakes: it omits the powers and subtracts the denominator contribution. Relative error is dimensionless because each uncertainty is divided by its measured quantity.

Step 4: Conclusion $\boxed{\Delta Z/Z = 2\Delta A/A + 3\Delta B/B + 4\Delta C/C}$. Hence, the correct option is (C).

Quick Tip to Solve: Differentiate logarithmically to read off the powers, then take their absolute values when adding maximum relative measurement errors.

32. $\overrightarrow{A}$ **is a vector quantity such that** $|\overrightarrow{A}|$ **= non-zero constant. Which of the following expression is true for** $\overrightarrow{A}$ **?** [4 marks]

(A) $\overrightarrow{A} \cdot \overrightarrow{A} = 0$

(B) $\overrightarrow{A} \times \overrightarrow{A} < 0$

(C) $\overrightarrow{A} \times \overrightarrow{A} = 0$

(D) $\overrightarrow{A} \times \overrightarrow{A} > 0$

Correct Answer: (C) $\overrightarrow{A} \times \overrightarrow{A} = 0$

Solution:

Concept:

- For vectors $\vec{u}, \vec{v}$, the cross-product magnitude is $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin\phi$, where ϕ is the angle between them.
- A nonzero vector has $\vec{A} \cdot \vec{A} = |\vec{A}|^2 > 0$. In contrast, its cross product with itself is the zero vector because the angle between identical vectors is zero.

Step 1: Evaluate the self dot product Let the constant magnitude be $A_0 > 0$. The scalar product of $\vec{A}$ with itself is

$$\vec{A} \cdot \vec{A} = A_0^2 \cos 0 = A_0^2 > 0.$$

Therefore option (A), which sets this scalar product to zero, cannot be true. The qualification that the magnitude is nonzero rules out the only case where a self dot product could vanish. Its being constant does not imply its square is zero.

Step 2: Evaluate the self cross product The two factors in $\vec{A} \times \vec{A}$ point in the same direction, so their included angle is 0. Therefore

$$|\vec{A} \times \vec{A}| = A_0^2 \sin 0 = 0, \qquad \vec{A} \times \vec{A} = \vec{0}.$$

Equivalently, antisymmetry gives $\vec{A} \times \vec{A} = -\vec{A} \times \vec{A}$, which forces the result to vanish. This identity is algebraic and holds even if the vector direction changes with time; no derivative occurs in the printed expression.

Step 3: Select the identity and reject the alternatives Option (C) states precisely this zero cross-product identity. Options (B) and (D) propose negative or positive values for a self cross product, while the actual result is zero. In addition, an ordinary vector cross product is not ordered by the scalar inequalities $<$ or $>$. Thus neither inequality can replace the vector equality. The constancy of magnitude is consistent with the answer but is not needed to establish this self-product property.

Step 4: Conclusion For the printed expression, $\boxed{\vec{A} \times \vec{A} = \vec{0}}$. Hence, the correct option is (C).

Quick Tip to Solve: A vector crossed with itself always vanishes; a nonzero vector dotted with itself gives its positive squared magnitude. Check which product is printed.

33. Which of the following relations is true for two unit vector $\widehat{A}$ and $\widehat{B}$ making an angle θ to each other? [4 marks]

(A) $|\widehat{A}+\widehat{B}| = |\widehat{A}-\widehat{B}| \tan\frac{\theta}{2}$
(B) $|\widehat{A}-\widehat{B}| = |\widehat{A}+\widehat{B}| \tan\frac{\theta}{2}$
(C) $|\widehat{A}+\widehat{B}| = |\widehat{A}-\widehat{B}| cos\frac{\theta}{2}$
(D) $|\widehat{A}-\widehat{B}| = |\widehat{A}+\widehat{B}| \cos\frac{\theta}{2}$

Correct Answer: (B) $|\widehat{A}-\widehat{B}| = |\widehat{A}+\widehat{B}| \tan\frac{\theta}{2}$

Solution:

Concept:

- For unit vectors $\widehat{A}, \widehat{B}$ with included angle θ, their scalar product is $\widehat{A}\cdot\widehat{B} = \cos\theta$.
- The half-angle identities $1+\cos\theta = 2\cos^2(\theta/2)$ and $1-\cos\theta = 2\sin^2(\theta/2)$ simplify the sum and difference magnitudes.

Step 1: Calculate the two squared magnitudes Expand each squared norm using scalar products and the fact that each vector has magnitude one:

$$|\widehat{A}+\widehat{B}|^2 = 1+1+2\cos\theta = 2(1+\cos\theta),$$
$$|\widehat{A}-\widehat{B}|^2 = 1+1-2\cos\theta = 2(1-\cos\theta).$$

The plus combination is largest for parallel vectors, whereas the minus combination vanishes in that case. These limiting behaviours are useful checks on the signs.

Step 2: Apply the half-angle formulae For an included angle $0 \le \theta \le \pi$, the sine and cosine of $\theta/2$ are nonnegative. Hence taking positive square roots yields

$$|\widehat{A}+\widehat{B}| = 2\cos(\theta/2), \qquad |\widehat{A}-\widehat{B}| = 2\sin(\theta/2).$$

For $\theta < \pi$, divide the second magnitude by the first to obtain

$$\frac{|\widehat{A}-\widehat{B}|}{|\widehat{A}+\widehat{B}|} = \tan(\theta/2).$$

The equivalent multiplied relation is the one displayed in option (B).

Step 3: Check the option and its endpoint domain Option (A) reverses the required ratio. For example at $\theta = 60^\circ$, the sum magnitude is $\sqrt{3}$ and the difference magnitude is 1, so only (B) gives the correct tangent factor. The cosine factors in (C) and (D) do not give this ratio either. At exactly $\theta = \pi$, the tangent form is undefined because the sum is zero; the two separate half-angle magnitude identities remain valid, and the relation in (B) is understood on its defined domain.

Step 4: Conclusion $\boxed{|\widehat{A}-\widehat{B}| = |\widehat{A}+\widehat{B}| \tan(\theta/2)}$, wherever the tangent is defined. Hence, the correct option is (B).

Quick Tip to Solve: For two unit vectors, remember sum magnitude $2\cos(\theta/2)$ and difference magnitude $2\sin(\theta/2)$; their ratio is the tangent.

34. If force $\overrightarrow{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be : [4 marks]

(A) $3\hat{i} + 4\hat{j} - 2\hat{k}$

(B) $-10\hat{i} + 10\hat{j} + 5\hat{k}$

(C) $10\hat{i} + 5\hat{j} - 10\hat{k}$

(D) $10\hat{i} + \hat{j} - 5\hat{k}$

Correct Answer: (B) $-10\hat{i} + 10\hat{j} + 5\hat{k}$

Solution:

Concept:

- The torque of a force about the origin is $\vec{\tau} = \vec{r} \times \vec{F}$, with position vector first and force second.
- For components, $\vec{r} \times \vec{F} = (r_y F_z - r_z F_y)\hat{i} + (r_z F_x - r_x F_z)\hat{j} + (r_x F_y - r_y F_x)\hat{k}$.

Step 1: Insert the position and force components Here $\vec{r} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$. The moment arm is measured from the origin to the particle, exactly the given position vector. Thus

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 3 & 4 & -2 \end{vmatrix}.$$

Keeping the order $\vec{r} \times \vec{F}$ is essential: reversing the factors reverses the entire torque vector.

Step 2: Expand the determinant carefully The three components are

$$\begin{aligned} \tau_x &= 1(-2) - 2(4) = -10, \\ \tau_y &= 2(3) - 2(-2) = 10, \\ \tau_z &= 2(4) - 1(3) = 5. \end{aligned}$$

Therefore $\vec{\tau} = -10\hat{i} + 10\hat{j} + 5\hat{k}$. The middle component is positive; one may compute it directly as $r_z F_x - r_x F_z$ to avoid forgetting the minus sign in a cofactor expansion. If the position and force use SI units, torque is expressed in newton metres.

Step 3: Verify the direction and compare the options A cross product must be perpendicular to both input vectors. Indeed,

$$(-10, 10, 5) \cdot (2, 1, 2) = -20 + 10 + 10 = 0,$$

and

$$(-10, 10, 5) \cdot (3, 4, -2) = -30 + 40 - 10 = 0.$$

Option (B) matches all three calculated components. Option (A) merely repeats the force vector, and options (C) and (D) have component values and signs inconsistent with the determinant. The perpendicularity checks independently support the computed direction.

Step 4: Conclusion The torque is $\boxed{-10\hat{i} + 10\hat{j} + 5\hat{k}}$, in $\mathrm{N\,m}$ for SI data. Hence, the correct option is (B).

Quick Tip to Solve: Use position crossed with force, then verify your torque has zero dot product with each input vector; this catches most sign errors.

35. The height of any point P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at point P will be : (Given g = acceleration due to gravity at the surface of earth). [4 marks]

(A) g/2

(B) g/4

(C) g/3

(D) g/9

Correct Answer: (D) g/9

Solution:

Concept:

- Outside a spherical Earth, gravitational acceleration at distance r from its centre is $g(r) = GM/r^2$, where M is Earth's mass.
- At the surface $g = GM/R^2$. A point at height h above the surface is a distance $R + h$ from the centre, so $g(h)/g = [R/(R + h)]^2$.

Step 1: Convert the height into the distance from the centre Let Earth's radius be R. Its diameter is $2R$, and the stated height above the surface is therefore $h = 2R$. The distance used in the inverse-square law is measured from the centre, not from the surface:

$$r = R + h = R + 2R = 3R.$$

Using $2R$ directly as the radial distance would confuse altitude with distance from the gravitating centre and would produce the incorrect factor $1/4$.

Step 2: Take the ratio to surface gravity At the surface and at the point P, respectively,

$$g = \frac{GM}{R^2}, \qquad g_P = \frac{GM}{(3R)^2}.$$

Dividing eliminates the same Earth mass and gravitational constant:

$$\frac{g_P}{g} = \frac{R^2}{9R^2} = \frac{1}{9}.$$

Hence $g_P = g/9$. The acceleration points toward Earth's centre; the options ask for its magnitude. No near-surface approximation is appropriate because the given height is comparable to, and indeed exceeds, the radius.

Step 3: Distinguish the distractors Option (D) supplies the inverse square of the correct distance factor, 3^2. Option (C), $g/3$, would result from incorrectly treating gravity as inversely proportional to distance rather than distance squared. Option (B), $g/4$, treats the height as the entire radial distance. Option (A), $g/2$, follows neither the correct radius conversion nor the inverse-square law. The computed reduction also has the expected sign: gravity decreases as the point moves outward.

Step 4: Conclusion The acceleration due to gravity at P is $\boxed{g/9}$, directed toward Earth's centre. Hence, the correct option is (D).

Quick Tip to Solve: Always add Earth's radius to the altitude before applying the inverse-square law; here height $2R$ means radial distance $3R$.

36. The terminal velocity (v_t) of the spherical rain drop depends on the radius (r) of the spherical rain drop as : [4 marks]

(A) $r^{1/2}$

(B) r

(C) r^2

(D) r^3

Correct Answer: (C) r^2

Solution:

Concept:

- Terminal speed is reached when the weight of a falling drop is balanced by buoyancy and viscous drag. Its acceleration then becomes zero although it continues to fall.
- For a small sphere of radius r moving slowly through a fluid of viscosity η, Stokes drag is $F_d = 6\pi\eta r v$. The effective weight depends on the density difference $\rho_d - \rho_a$.

Step 1: Write the force balance Take downward as positive. The drop has volume $V = 4\pi r^3/3$, so its weight is $\rho_d V g$, while the upward buoyant force is $\rho_a V g$. At terminal speed the downward resultant must vanish. Therefore,

$$\frac{4}{3}\pi r^3(\rho_d - \rho_a)g = 6\pi\eta r v_t.$$

The buoyant correction changes the coefficient, but does not change the dependence on radius.

Step 2: Isolate the radius dependence Divide by $6\pi\eta r$ to obtain

$$v_t = \frac{2(\rho_d - \rho_a)g}{9\eta}\, r^2.$$

For the same liquid, surrounding air and temperature, density, gravity and viscosity are fixed. Thus the only variable factor is r^2. Doubling the radius makes the effective gravitational force eight times larger, but the drag coefficient only twice as large; terminal speed must consequently become four times larger. Option (D) confuses the volume dependence of weight with speed. Options (A) and (B) also fail this Stokes-force balance; the square-root law corresponds to a different drag model rather than the viscous terminal-speed relation used here.

Step 3: Conclusion The terminal speed varies as $v_t \propto r^2$. Hence, the correct option is (C).

Quick Tip to Solve: In the Stokes regime, divide the effective weight proportional to radius cubed by the drag coefficient proportional to radius; the remaining power is two.

37. The relation between root mean square speed (v_{rms}) and most probable sped (v_p) for the molar mass M of oxygen gas molecule at the temperature of 300 K will be : [4 marks]

(A) $v_{rms} = \sqrt{\frac{2}{3}} v_p$

(B) $v_{rms} = \sqrt{\frac{3}{2}} v_p$

(C) $v_{rms} = v_p$

(D) $v_{rms} = \sqrt{\frac{1}{3}} v_p$

Correct Answer: (B) $v_{rms} = \sqrt{\frac{3}{2}} v_p$

Solution:

Concept:

- For an ideal gas at temperature T, the root mean square speed is $v_{\text{rms}} = \sqrt{3RT/M}$, where R is the molar gas constant and M is the molar mass in kilograms per mole.
- The most probable speed is $v_p = \sqrt{2RT/M}$. It is the speed at the maximum of the Maxwell speed distribution, whereas the rms speed is obtained by averaging the square of speed and then taking a square root.

Step 1: Write the two speeds for oxygen At the specified temperature $T = 300\,\text{K}$, both quantities refer to the same oxygen gas. Their formulas are

$$v_{\text{rms}} = \sqrt{\frac{3R(300)}{M}}, \qquad v_p = \sqrt{\frac{2R(300)}{M}}.$$

There is no need to substitute a numerical value of the oxygen molar mass because the question asks for a relation between the speeds, rather than either individual speed.

Step 2: Divide and compare with the options Take the ratio before evaluating either square root:

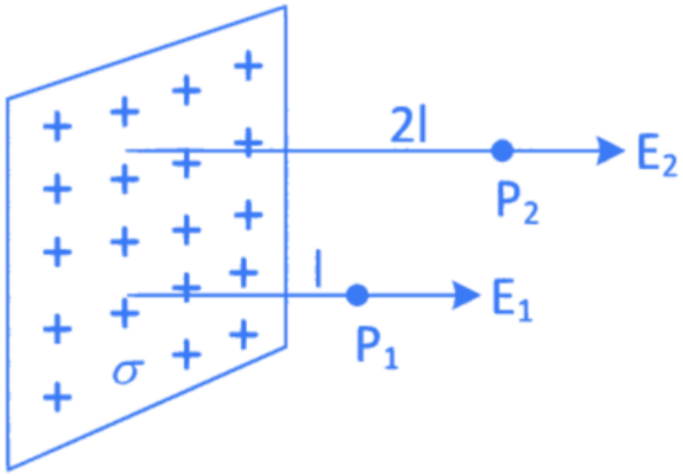
$$\frac{v_{\text{rms}}}{v_p} = \sqrt{\frac{3RT/M}{2RT/M}} = \sqrt{\frac{3}{2}}.$$

The common temperature and molar mass cancel. Consequently the rms speed is larger than the most probable speed, as expected because the higher-speed tail contributes strongly to a mean of squared speeds. Option (A) reverses this ratio and would put the rms speed below the most probable speed. Option (C) incorrectly treats these different statistical measures as equal. Option (D) also predicts an rms speed smaller than the most probable speed and has the wrong numerical factor. The cancellation shows that the relation holds for any ideal gas at a common temperature, not just oxygen at this temperature.

Step 3: Conclusion The required relation is $v_{\text{rms}} = \sqrt{3/2}\, v_p$. Hence, the correct option is (B).

Quick Tip to Solve: Remember the coefficients under the square roots: most probable, mean and rms speeds use 2, 8/pi and 3 respectively; form ratios before substituting mass or temperature.

38. In the figure, a very large plane sheet of positive charge is shown. P_1 and P_2 are two points at distance l and 2l from the charge distribution. If σ is the surface charge density, then the magnitude of electric fields E_1 and E_2 at P_1 and P_2 respectively are : [4 marks]



(A) $E_1 = \sigma/\varepsilon_0,\ E_2 = \sigma/2\varepsilon_0$

(B) $E_1 = 2\sigma/\varepsilon_0,\ E_2 = \sigma/\varepsilon_0$

(C) $E_1 = E_2 = \sigma/2\varepsilon_0$

(D) $E_1 = E_2 = \sigma/\varepsilon_0$

Correct Answer: (C) $E_1 = E_2 = \sigma/2\varepsilon_0$

Solution:

Concept:

- For a very large uniformly charged plane sheet, edge effects are neglected. Symmetry makes the electric field perpendicular to the sheet and equal in magnitude on its two sides.
- Gauss's law is $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\varepsilon_0$. A thin cylindrical pillbox through the sheet has two faces contributing to electric flux.

Step 1: Apply Gauss law to a pillbox Choose the flat faces of a cylindrical Gaussian surface parallel to the sheet, each of area A. The field is normal to these faces. The curved surface has no flux because the field is parallel to it. For a positive sheet the field points outward on both sides, so the total outward flux is $EA + EA = 2EA$. The enclosed charge is σA. Therefore,

$$2EA = \frac{\sigma A}{\varepsilon_0}, \qquad E = \frac{\sigma}{2\varepsilon_0}.$$

Step 2: Compare the two observation points The derived expression has no distance from the sheet. Hence moving from l to $2l$, while remaining in the region where the large-sheet approximation applies, does not change the field magnitude. Both arrows in the figure point away from the positively charged sheet, consistently with this result. Thus

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}.$$

Options (A) and (B) introduce a distance dependence which the planar symmetry and Gauss-law result do not permit. Option (D) makes the fields equal but misses the factor of two arising from flux through both pillbox faces. The expression σ/ε_0 is associated with a different boundary situation, such as the field just outside a conductor when its local surface charge is σ.

Step 3: Conclusion The two magnitudes are $E_1 = E_2 = \sigma/(2\varepsilon_0)$. Hence, the correct option is (C).

Quick Tip to Solve: For a single nonconducting infinite sheet, count two flux faces; the field has a factor of one-half and does not decrease when the distance doubles.

39. Match List-I with List-II.

	List - I		List -II
(A)	AC generator	(I)	Detects the presence of current in the circuit
(B)	Galvanometer	(II)	Converts mechanical energy into electrical energy
(C)	Transformer	(III)	Works on the principle of resonance in AC circuit
(D)	Metal detector	(IV)	Changes an alternating voltage for smaller or greater value

Choose the correct answer from the options given below : [4 marks]

(A) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

(B) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

(C) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

(D) (A) - (III), (B) - (I), (C) - (II), (D) - (IV)

Correct Answer: (A) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

Solution:

Concept:

- Electromagnetic induction produces an emf when the magnetic flux linked with a circuit changes. An AC generator uses mechanical rotation to change flux, while a transformer uses the varying magnetic field of an alternating current.
- A galvanometer detects small electric currents through the magnetic torque on its coil. A resonant AC circuit responds strongly near its natural frequency, a principle used in the metal detector described here.

Step 1: Match the generator and galvanometer with their functions In an AC generator, an external mechanical agent rotates a coil in a magnetic field. The induced emf drives an alternating current, so mechanical energy is converted into electrical energy. Hence (A) matches (II). A galvanometer gives a deflection when current flows through it; the deflection signals the presence of current and can also indicate its direction. Thus (B) matches (I). These two matches already reject options (C) and (D), which incorrectly associate the generator with resonance.

Step 2: Match the transformer and metal detector and select the option A transformer transfers energy between two coils through a varying magnetic flux. Its turns ratio determines whether an alternating voltage is stepped up or stepped down, so (C) matches (IV). The metal detector in the list

uses the resonance behaviour of an AC circuit: nearby metal changes the effective inductance and therefore the resonant response, enabling detection. Thus (D) matches (III). Option (B) interchanges these last two functions, incorrectly making the transformer the resonance device and the detector the voltage-changing device. Only option (A) contains all four correct pairings simultaneously.

Step 3: Conclusion The generator converts mechanical energy, the galvanometer detects current, the transformer changes AC voltage, and the metal detector uses resonance. Hence, the correct option is (A).

Quick Tip to Solve: First match the two distinctive functions: a generator converts mechanical energy, and a transformer changes AC voltage. Then use the remaining functions to complete the matching.

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40. A long straight wire with a circular cross-section having radius R, is carrying a steady current I. The current I is uniformly distributed across this cross-section. Then the variation of magnetic field due to current I with distance r (r < R) from its centre will be : [4 marks]

(A) B $\propto$ r^2

(B) B $\propto$ r

(C) B $\propto \frac{1}{r^2}$

(D) B $\propto \frac{1}{r}$

Correct Answer: (B) B $\propto$ r

Solution:

Concept:

- Ampere's circuital law gives $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$ for a steady current. Cylindrical symmetry makes the magnetic field tangential and constant along a circular path concentric with the wire.
- Uniform current density means $J = I/(\pi R^2)$. A path of radius $r < R$ encloses only the current through the inner area πr^2, rather than the full wire current.

Step 1: Find the enclosed current Because every equal cross-sectional area carries the same current, the fraction of current enclosed equals the fraction of area enclosed:

$$I_{\text{enc}} = J\pi r^2 = I\frac{r^2}{R^2}.$$

The enclosed current therefore increases quadratically with the distance from the centre. This is the point that distinguishes the inside-wire field from the outside-wire result.

Step 2: Apply Ampere law inside the wire For a circular Amperian loop, the line integral is $B(2\pi r)$. Consequently,

$$B(2\pi r) = \mu_0 I\frac{r^2}{R^2}, \qquad B = \frac{\mu_0 I}{2\pi R^2}\, r.$$

With I and R fixed, the field is directly proportional to r. It vanishes at the axis and rises linearly to $\mu_0 I/(2\pi R)$ at the surface. Option (A) retains the quadratic dependence of enclosed current but forgets the loop circumference in Ampere law. Options (C) and (D) diverge at the centre, contrary to symmetry for a uniform finite current density. In particular, the inverse-distance law applies outside

the wire, where the enclosed current has already reached the full constant current I, not inside it.

Step 3: Conclusion For $r < R$, $B = \mu_0 Ir/(2\pi R^2)$, so $B \propto r$. Hence, the correct option is (B).

Quick Tip to Solve: Always determine enclosed current first: inside a uniform wire it grows as radius squared, and dividing by the circular path length leaves a linear field.

41. If wattless current flows in the AC circuit, then the circuit is : [4 marks]

(A) Purely Resistive circuit

(B) Purely Inductive circuit

(C) LCR series circuit

(D) RC series circuit only

Correct Answer: (B) Purely Inductive circuit

Solution:

Concept:

- Average power in a sinusoidal AC circuit is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$, where ϕ is the phase difference between voltage and current.
- Wattless current transfers no net energy over a full cycle. For nonzero rms voltage and current, this requires zero power factor, $\cos\phi = 0$, or a phase difference of 90°.

Step 1: Identify the power factor of an inductor An ideal inductor stores energy in its magnetic field during one part of the cycle and returns it during another. Its current lags the applied voltage by $\pi/2$. Thus

$$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos(\pi/2) = 0.$$

There can still be a nonzero alternating current; wattless describes zero average power consumption, not zero current or zero instantaneous power.

Step 2: Eliminate the other circuits In a purely resistive circuit, voltage and current are in phase. Hence $\cos\phi = 1$ and the resistor dissipates positive average power, eliminating (A). A series LCR circuit containing a nonzero resistance has $P_{\text{av}} = I_{\text{rms}}^2 R > 0$; at resonance it is purely resistive rather than wattless. This rules out (C) as the stated general circuit. A series RC circuit also dissipates $I_{\text{rms}}^2 R$ in its resistor, eliminating (D). A purely capacitive ideal circuit would also be wattless, but it is not one of the listed options. Among the offered choices, only a purely inductive circuit has no resistive loss and a phase difference of exactly a quarter-cycle. The word purely is essential here: a real coil with resistance consumes some average power.

Step 3: Conclusion A purely inductive circuit supports wattless current because its power factor is zero. Hence, the correct option is (B).

Quick Tip to Solve: Use average power rather than instantaneous power: a reactive element can carry current and exchange energy while its cycle-average power remains zero.

42. The electric field in an electromagnetic wave is given by E = 56.5 sin ω(t − x/c) NC^{-1}. Find the intensity of the wave if it is propagating along x-axis in the free space.

(Given : ε_0 = 8.85 × $10^{-12}C^2N^{-1}m^{-2}$) [4 marks]

(A) 5.65 Wm^{-2}

(B) 4.24 Wm^{-2}

(C) 1.9 × 10^{-7} Wm^{-2}

(D) 56.5 Wm^{-2}

Correct Answer: (B) 4.24 Wm^{-2}

Solution:

Concept:

- The intensity of a sinusoidal plane electromagnetic wave in free space is its time-averaged energy flux, $I = \frac{1}{2}\varepsilon_0 c E_0^2$, where E_0 is the peak electric-field amplitude.
- The electric and magnetic fields carry equal average energy. The factor one-half in the intensity formula results from averaging $\sin^2$ over a full cycle; the given sinusoidal coefficient is a peak value, not an rms value.

Step 1: Read the amplitude and select the formula Comparing the given field with $E = E_0 \sin[\omega(t - x/c)]$, the amplitude is $E_0 = 56.5\,\mathrm{N\,C^{-1}}$. The wave propagates along the positive x-direction, and its mean energy flux magnitude is

$$I = \frac{1}{2}\varepsilon_0 c E_0^2.$$

Use $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$ and $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$.

Step 2: Substitute and compare with the options The square of the amplitude is $56.5^2 = 3192.25$. Therefore,

$$\begin{aligned} I &= \frac{1}{2}(8.85 \times 10^{-12})(3.0 \times 10^8)(3192.25) \\ &= 4.237711875\,\mathrm{W\,m^{-2}} \\ &\simeq 4.24\,\mathrm{W\,m^{-2}}. \end{aligned}$$

This is the average intensity; it is not the instantaneous maximum. The units are power per unit area, as required for energy transported through a surface each second. Options (A) and (D) are not obtained by squaring the amplitude and including the free-space factor. Option (D) simply repeats the field amplitude as though field and intensity were the same physical quantity. Option (C) has an incorrect order of magnitude: multiplying $\varepsilon_0 c$, of order 10^{-3}, by E_0^2, of order 10^3, must produce an intensity of order unity.

Step 3: Conclusion The wave intensity is $I \simeq 4.24\,\mathrm{W\,m^{-2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a sinusoidal field, square the peak amplitude and retain the factor one-half; using the rms amplitude instead removes that factor.

43. The two light beams having intensities I and 9I interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point P and π at point Q. Then the difference between the resultant intensities at P and Q will be : [4 marks]

(A) 2 I

(B) 6 I

(C) 5 I

(D) 7 I

Correct Answer: (B) 6 I

Solution:

Concept:

- For two coherent beams of intensities I_1 and I_2, the resultant intensity at phase difference ϕ is $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$. Intensities add directly only when the interference term vanishes.
- Intensity is proportional to the square of wave amplitude. Thus intensities I and $9I$ correspond to amplitude ratio $1:3$, and their interference coefficient is $2\sqrt{I(9I)} = 6I$.

Step 1: Evaluate the intensity at P Insert the two given intensities into the interference formula before substituting either phase. This gives

$$I_R = 10I + 6I\cos\phi.$$

At P the phase difference is $\pi/2$, whose cosine is zero. Consequently

$$I_P = 10I + 6I\cos(\pi/2) = 10I.$$

The waves are in quadrature here, so the interference contribution vanishes even though the beams remain coherent.

Step 2: Evaluate the intensity at Q and subtract At Q the phase difference is π, so the interference term is negative and maximum in magnitude:

$$I_Q = 10I + 6I\cos\pi = 10I - 6I = 4I.$$

Unequal amplitudes do not cancel completely; destructive interference therefore leaves a nonzero minimum. The requested difference is

$$I_P - I_Q = 10I - 4I = 6I.$$

The value $2I$ does not follow from the amplitude or intensity relation, while $5I$ and $7I$ are also inconsistent with the calculated cross term. The result has the same intensity units as the specified I.

Step 3: Conclusion The resultant intensities at P and Q are $10I$ and $4I$, respectively, so their difference is $\boxed{6I}$. Hence, the correct option is (B).

Quick Tip to Solve: Compute the interference coefficient $2\sqrt{I_1 I_2}$ first. Between phases $\pi/2$ and π, the intensity difference equals this coefficient.

44. A light wave travelling linearly in a medium of dielectric constant 4, incidents on the horizontal interface separating medium with air. The angle of incidence for which the total intensity of incident wave will be reflected back into the same medium will be :

(Given : relative permeability of medium $\mu_r = 1$) [4 marks]

(A) 10°

(B) 20°

(C) 30°

(D) 60°

Correct Answer: (D) 60°

Solution:

Concept:

- The refractive index of a transparent medium is $n = \sqrt{\varepsilon_r \mu_r}$, where ε_r is its dielectric constant and μ_r its relative permeability. Air has refractive index approximately one.
- For light travelling from a denser to a rarer medium, the critical angle obeys $\sin i_c = n_{\text{rarer}}/n_{\text{denser}}$. Total internal reflection occurs when the incidence angle, measured from the normal, exceeds i_c.

Step 1: Calculate the refractive index and critical angle The given dielectric constant is 4 and relative permeability is 1, so

$$n = \sqrt{4 \times 1} = 2.$$

The light travels from this medium to air, satisfying the necessary denser-to-rarer condition. At the critical angle the refracted ray just grazes the interface. Apply Snell's law with refraction angle $90°$:

$$2 \sin i_c = 1 \sin 90°, \qquad \sin i_c = \frac{1}{2}.$$

Hence $i_c = 30°$.

Step 2: Choose the angle greater than the critical angle Compare every offered incidence angle with $30°$. The angles $10°$ and $20°$ are smaller, so a transmitted ray enters air and the total incident intensity is not reflected. The angle $30°$ is the critical-angle boundary, where the refracted direction is along the surface; the usual total-internal-reflection condition is the strict inequality $i > i_c$. Of the listed choices, only $60°$ exceeds the critical angle. It therefore gives total reflection back into the original medium under the ideal lossless-interface model used here.

Step 3: Conclusion The critical angle is $30°$, and $60°$ satisfies $i > i_c$. The required offered angle is $\boxed{60°}$. Hence, the correct option is (D).

Quick Tip to Solve: First convert dielectric constant into refractive index by taking its square root when $\mu_r = 1$; then select an incidence angle greater than the critical angle.

45. The ratio for the speed of the electron in the 3rd orbit of He^+ to the speed of the electron in the 3rd orbit of hydrogen atom will be : [4 marks]

(A) 1 : 1

(B) 1 : 2

(C) 4 : 1

(D) 2 : 1

Correct Answer: (D) 2 : 1

Solution:

Concept:

- A hydrogen-like atom or ion contains one electron. In the Bohr model its allowed electron speed is $v_n = Z\alpha c/n$, where Z is atomic number, n is orbit number, α is the fine-structure constant and c is the speed of light.
- At the same principal quantum number the electron speed is directly proportional to nuclear charge Z. The Z^2 dependence belongs to the binding energy, not to the orbital speed.

Step 1: Identify the nuclear charges and orbit numbers A helium nucleus has charge $+2e$, and He^+ has just one bound electron. It is therefore a hydrogen-like ion with $Z_{He} = 2$. A hydrogen atom has one proton and one electron, so $Z_H = 1$. Both specified electrons occupy the third orbit: $n_{He} = n_H = 3$. The ionic charge $+1$ of He^+ is its net charge; it must not be confused with the charge $+2e$ of its nucleus.

Step 2: Form the speed ratio and test the choices Substitute the two nuclear charges into the speed law:

$$v_{He} = \frac{2\alpha c}{3}, \qquad v_H = \frac{\alpha c}{3}.$$

Dividing cancels the common constants and the equal orbit number:

$$\frac{v_{He}}{v_H} = \frac{2\alpha c/3}{\alpha c/3} = 2.$$

Thus the helium-ion electron moves twice as fast. The ratio $1 : 1$ incorrectly ignores the nuclear charge, $1 : 2$ reverses the requested order, and $4 : 1$ incorrectly uses the energy dependence Z^2 for a speed. No absolute numerical value of α or c is needed.

Step 3: Conclusion The requested ratio, helium-ion speed to hydrogen speed, is $\boxed{2 : 1}$. Hence, the correct option is (D).

Quick Tip to Solve: For hydrogen-like systems remember $v \propto Z/n$, whereas $|E| \propto Z^2/n^2$. Keeping these two scalings distinct avoids the $4 : 1$ distractor.

46. The photodiode is used to detect the optical signals. These diodes are preferably operated in reverse biased mode because : [4 marks]

(A) fractional change in majority carriers produce higher forward bias current

(B) fractional change in majority carriers produce higher reverse bias current

(C) fractional change in minority carriers produce higher forward bias current

(D) fractional change in minority carriers produce higher reverse bias current

Correct Answer: (D) fractional change in minority carriers produce higher reverse bias current

Solution:

Concept:

- A photodiode is a p-n junction used to convert incident optical power into an electrical current. Photons of sufficient energy generate electron-hole pairs in or near the depletion region.
- Reverse current in a p-n junction is associated with minority carriers. Their equilibrium population is small, so illumination can produce a large fractional change in the reverse current and a sensitive optical response.

Step 1: Explain how reverse bias collects photocarriers Reverse bias widens the depletion layer and establishes a strong electric field across it. When absorbed light generates an electron-hole pair, this field drives the electron towards the n side and the hole towards the p side before they recombine. The collected carriers produce a photocurrent in the reverse direction. Increasing the incident optical intensity generates more pairs per unit time, so the magnitude of this photocurrent increases and provides a measurable optical signal.

Step 2: Compare the current response with forward bias In the dark, the reverse current is small because the minority-carrier population is small. The additional carriers generated by light can therefore cause a substantial fractional increase relative to this dark current. In forward bias, the diode already carries a large injection current dominated by majority-carrier transport; the light-induced contribution is comparatively difficult to distinguish. Thus it is the minority-carrier contribution to reverse current that makes reverse operation preferable. Options (A) and (B) incorrectly attribute the sensitive response to majority carriers, while option (C) identifies the wrong bias direction. The wording of option (D) expresses the intended increase in reverse-current response due to the light-generated minority carriers.

Step 3: Conclusion A small minority-carrier population makes the fractional light-induced change in reverse current large and readily detectable. Hence, the correct option is (D).

Quick Tip to Solve: Connect photodiode sensitivity with a small dark current: light-generated carriers are easy to detect against the low reverse-current background, unlike the large forward current.

47. The difference of speed of light in the two media A and B ($v_A - v_B$) is 2.6×10^7 m/s. If the refractive index of medium B is 1.47, then the ratio of refractive index of medium B to medium A is : (Given : speed of light in vacuum c = 3×10^8 ms^{-1}) [4 marks]

(A) 1.303

(B) 1.318

(C) 1.13

(D) 0.12

Correct Answer: (C) 1.13

Solution:

Concept:

- The refractive index of a medium is $n = c/v$, with v the speed of light in the medium. A larger refractive index therefore means a smaller speed.
- The required ratio is $n_B/n_A = (c/v_B)/(c/v_A) = v_A/v_B$. Use the given signed speed difference $v_A - v_B > 0$ so that the final ratio must exceed one.

Step 1: Express the speed in A using the known speed in B Let $\Delta v = v_A - v_B = 2.6 \times 10^7\,\mathrm{m\,s^{-1}}$. The given refractive index of B gives

$$v_B = \frac{c}{n_B} = \frac{3 \times 10^8}{1.47}\,\mathrm{m\,s^{-1}}.$$

Because the difference is positive, light moves faster in A: $v_A = v_B + \Delta v$. Consequently A must have a smaller refractive index than B, which is a useful check on the order of the requested ratio.

Step 2: Evaluate the ratio without rounding the intermediate speed Substitute $v_A = v_B + \Delta v$ into the ratio:

$$\begin{aligned}\frac{n_B}{n_A} = \frac{v_A}{v_B} &= 1 + \frac{\Delta v}{v_B} \\ &= 1 + \frac{n_B \Delta v}{c} \\ &= 1 + \frac{1.47(2.6 \times 10^7)}{3 \times 10^8} \\ &= 1 + 0.1274 = 1.1274.\end{aligned}$$

To the precision represented by the options this is 1.13. The alternatives 1.303 and 1.318 would require substantially larger speed differences. The value 0.12 resembles only the increment above unity and cannot be the ratio itself; it also contradicts $n_B > n_A$. The ratio is dimensionless because the two speeds have identical units.

Step 3: Conclusion The refractive-index ratio is $n_B/n_A = 1.1274 \approx \boxed{1.13}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $n_B/n_A = 1 + n_B(v_A - v_B)/c$ to avoid rounding v_B. Check that the ratio is above one when $v_A > v_B$.

48. A teacher in his physics laboratory allotted an experiment to determine the resistance (G) of a galvanometer. Students took the observations for $\frac{1}{3}$ deflection in the galvanometer. Which of the below is true for measuring value of G? [4 marks]

(A) $\frac{1}{3}$ deflection method cannot be used for determining the resistance of the galvanometer.

(B) $\frac{1}{3}$ deflection method can be used and in this case the G equals to twice the value of shunt resistances.

(C) $\frac{1}{3}$ deflection method can be used and in this case, the G equals to three times the value of shunt resistances.

(D) $\frac{1}{3}$ deflection method can be used and in this case the G value equals to the shunt resistances.

Correct Answer: (B) $\frac{1}{3}$ deflection method can be used and in this case the G equals to twice the value of shunt resistances.

Solution:

Concept:

- For a moving-coil galvanometer, deflection is proportional to the current through its coil. Reducing the deflection to one third reduces the galvanometer current to one third of its original value.
- In the fractional-deflection experiment, a shunt resistance S is placed parallel to a galvanometer of resistance G. With a sufficiently large series resistance, the total current is nearly unchanged when the shunt is connected.

Step 1: Relate the shunt and galvanometer currents Let the initial galvanometer current be I, when no shunt is connected. On connecting the shunt, one-third deflection means $I_g = I/3$. Under the usual large-series-resistance condition, the current supplied to the parallel combination remains approximately I. Therefore the shunt carries the remainder:

$$I_s = I - I_g = \frac{2I}{3}.$$

The shunt carries twice the current through the galvanometer, so its resistance must be half the galvanometer resistance.

Step 2: Use equal parallel voltages and compare the alternatives The galvanometer and shunt have the same potential difference because they are parallel. Thus

$$I_g G = I_s S, \qquad \frac{I}{3}G = \frac{2I}{3}S,$$

giving

$$G = 2S.$$

More exactly, if the series resistance is R, the current ratio is $I_g/I = S(R+G)/[R(G+S)+GS]$. Setting this to $1/3$ gives $G = 2RS/(R-2S)$, which approaches $2S$ when $R \gg S$, as required by the standard experimental method. Hence one-third deflection is usable, rejecting (A). The values $3S$ and S in (C) and (D) do not produce the required current division in this approximation.

Step 3: Conclusion In the standard fractional-deflection method, $\boxed{G = 2S}$, twice the shunt resistance. Hence, the correct option is (B).

Quick Tip to Solve: If the deflection is reduced to a fraction f, use $I_g = fI$, $I_s = (1-f)I$, then $G/S = (1-f)/f$ under the large-series-resistance approximation.

49. Given below are two statements :

Statement I : Davisson-Germer experiment establishes the wave nature of electrons.

Statement II : If electrons have wave nature, they can interfere and show diffraction.

In the light of the above statements choose the correct answer from the option given below : [4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (A) Both Statement I and Statement II are true.

Solution:

Concept:

- De Broglie associated a wavelength $\lambda = h/p$ with a moving particle of momentum p. Electron diffraction tests this matter-wave prediction directly.
- Interference and diffraction arise from wave superposition. A regular crystal lattice provides a periodic arrangement of scattering centres that can produce constructive interference of electron waves.

Step 1: Evaluate Statement I using the experiment In the Davisson-Germer experiment an electron beam was scattered by a nickel crystal. The scattered intensity showed pronounced maxima at particular directions rather than the smooth distribution expected from a purely classical particle description. The wavelength inferred from the diffraction condition agreed with the de Broglie wavelength h/p calculated from the electrons' momentum. The agreement established experimentally that electrons exhibit wave behaviour. Statement I is therefore true; it does not claim that the particle aspects of electrons disappear.

Step 2: Evaluate Statement II and select the truth combination If electrons have wave nature, their probability amplitudes can superpose. Where coherent amplitudes reinforce one another the detected intensity rises; where they cancel it decreases. The resulting interference pattern, including diffraction by an aperture or a crystal lattice, is characteristic of waves. Thus electrons can show interference and diffraction, making Statement II true as well. The question asks for the truth values of the two statements, not for a separate assertion-reason explanation code. Option (A) gives both as true. Option (B) rejects both experimentally supported statements; option (C) wrongly denies electron diffraction, and option (D) wrongly denies what the Davisson-Germer experiment established.

Step 3: Conclusion Both Statement I and Statement II are true: the Davisson-Germer experiment confirms electron wave nature, and electron waves can interfere and diffract. Hence, the correct option is (A).

Quick Tip to Solve: For matter-wave statement questions, connect Davisson-Germer with crystal diffraction and $\lambda = h/p$; then evaluate each statement before selecting their truth combination.

50. A signal of 100 THz frequency can be transmitted with maximum efficiency by : [4 marks]

(A) Coaxial cable

(B) Optical fibre

(C) Twisted pair of copper wires

(D) Water

Correct Answer: (B) Optical fibre

Solution:

Concept:

- The medium used to carry a signal must support its frequency with sufficiently small attenuation and dispersion. Electrical wire transmission is appropriate at much lower carrier frequencies than optical transmission.
- The wavelength of an electromagnetic wave in vacuum is related to its frequency by $\lambda = c/f$, where $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$. Optical fibres guide light through a transparent dielectric core.

Step 1: Identify the spectral range The frequency given is $100\,\mathrm{THz} = 100 \times 10^{12}\,\mathrm{Hz} = 10^{14}\,\mathrm{Hz}$. Therefore its free-space wavelength is

$$\lambda = \frac{3 \times 10^8}{10^{14}} = 3 \times 10^{-6}\,\mathrm{m} = 3\,\mu\mathrm{m}.$$

This is an infrared electromagnetic signal. It belongs to the optical-frequency range rather than the radio-frequency range ordinarily carried as electrical signals along copper conductors.

Step 2: Compare the four transmission media An optical fibre guides an optical signal along its dielectric core by total internal reflection, with a suitable core and cladding selected for the wavelength. This makes optical fibre the appropriate transmission medium among those listed. A coaxial cable and a twisted pair carry electrical signals, but conductor and dielectric losses become severe at such extremely high frequencies; neither is an efficient optical waveguide. Water does not provide the controlled, low-loss guiding path of an optical fibre and absorbs infrared radiation strongly. Thus the infrared frequency selects the fibre option rather than either copper-wire arrangement or water.

Step 3: Conclusion The suitable medium for maximum transmission efficiency among the choices is optical fibre. Hence, the correct option is (B).

Quick Tip to Solve: Convert a very large carrier frequency to wavelength first: frequencies of order $10^{14}\,\mathrm{Hz}$ point to optical transmission rather than ordinary copper-wire communication.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A uniform chain of 6 m length is placed on a table such that a part of its length is hanging over the edge of the table. The system is at rest. The co-efficient of static friction between the chain and the surface of the table is 0.5, the maximum length of the chain hanging from the table is _____ m. [4 marks]

Solution:

Concept:

- For a uniform chain, mass is proportional to length. If its linear mass density is λ, a length l has weight $\lambda l g$.
- The maximum available static friction is $f_{\max} = \mu_s N$. At the limiting hanging length, the hanging weight equals this limiting friction on the portion resting on the horizontal table.

Step 1: Express the forces using the hanging length Let x metres hang vertically over the edge. The length supported by the table is $6 - x$ metres. Its weight, balanced by the normal force, gives

$$N = \lambda(6 - x)g.$$

The hanging part has weight $\lambda x g$. For the chain to remain at rest, the tension at the edge balances this hanging weight. The friction on the tabletop portion must supply the same tension, so the required friction is $f = \lambda x g$.

Step 2: Apply the condition for impending sliding As the hanging length increases, the driving weight increases while the tabletop normal reaction, and hence the maximum friction, decrease. The greatest possible static hanging length is therefore obtained at equality:

$$\lambda x g = \mu_s \lambda (6 - x) g.$$

Cancel the common positive factors λ and g, then substitute $\mu_s = 0.5$:

$$x = 0.5(6 - x), \qquad 1.5x = 3, \qquad x = 2\,\text{m}.$$

As a check, the supported length is $4\,\text{m}$, whose limiting friction is $0.5\lambda(4)g = 2\lambda g$, exactly the weight of the hanging portion. Any larger length would require more friction than is available.

Step 3: Conclusion The maximum length hanging from the table is $\boxed{2\,\text{m}}$. The numerical entry is 2.

Quick Tip to Solve: For a uniform chain of total length L, write $x = \mu_s(L - x)$; the density and gravitational acceleration cancel, giving $x = \mu_s L/(1 + \mu_s)$.

52. A 0.5 kg block moving at a speed of 12 ms^{-1} compresses a spring through a distance 30 cm when its speed is halved. The spring constant of the spring will be _____ Nm^{-1}. [4 marks]

Solution:

Concept:

- The kinetic energy of a block of mass m and speed v is $K = \frac{1}{2}mv^2$. Halving speed reduces kinetic energy to one quarter of its original value.
- A spring compressed by x from its natural length stores elastic potential energy $U = \frac{1}{2}kx^2$. The decrease of block kinetic energy supplies this stored energy.

Step 1: Calculate the kinetic energy transferred The initial speed is $u = 12\,\mathrm{m\,s^{-1}}$, and the speed after the stated compression is $v = u/2 = 6\,\mathrm{m\,s^{-1}}$. The mass is $m = 0.5\,\mathrm{kg}$. Thus

$$K_i = \tfrac{1}{2}(0.5)(12)^2 = 36\,\mathrm{J},$$

$$K_f = \tfrac{1}{2}(0.5)(6)^2 = 9\,\mathrm{J}.$$

The block has not stopped at this compression: it retains $9\,\mathrm{J}$ of kinetic energy. The energy transferred to the spring is therefore $36 - 9 = 27\,\mathrm{J}$, rather than the entire initial energy.

Step 2: Use the spring compression to find its stiffness Convert the compression before substitution: $x = 30\,\mathrm{cm} = 0.30\,\mathrm{m}$. Mechanical-energy conservation gives

$$\tfrac{1}{2}kx^2 = K_i - K_f = 27\,\mathrm{J}.$$

Hence

$$k = \frac{2(27)}{(0.30)^2} = \frac{54}{0.09} = 600\,\mathrm{N\,m^{-1}}.$$

Equivalently, $k = m(u^2 - v^2)/x^2$. Substitution into the spring energy gives $\frac{1}{2}(600)(0.09) = 27\,\mathrm{J}$, so the final kinetic and spring energies sum to the original $36\,\mathrm{J}$.

Step 3: Conclusion The spring constant is $\boxed{600\,\mathrm{N\,m^{-1}}}$. The numerical entry is 600.

Quick Tip to Solve: Use the loss of kinetic energy, not the initial kinetic energy, whenever a block still has a nonzero speed after compressing a spring.

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53. The velocity of upper layer of water in a river is 36 kmh^{-1}. Shearing stress between horizontal layers of water is 10^{-3} Nm^{-2}. Depth of the river is _____ m. (Co-efficient of viscosity of water is 10^{-2} Pa.s) [4 marks]

Solution:

Concept:

- Newton's law of viscosity relates the magnitude of shearing stress to the velocity gradient: $\tau = \eta\,|dv/dy|$, where η is the dynamic viscosity.
- For the river-layer model, the water at the bed is at rest and the speed rises uniformly to the surface value. For depth h, the velocity gradient is v_{surface}/h.

Step 1: Convert the surface speed and identify the gradient The upper layer moves at $36\,\mathrm{km\,h^{-1}}$. In SI units,

$$v = 36 \times \frac{1000}{3600} = 10\,\mathrm{m\,s^{-1}}.$$

The bed is the stationary boundary, so the speed difference between the bed and the top layer is $\Delta v = 10\,\mathrm{m\,s^{-1}}$. Taking the velocity change across the full depth, the relevant gradient is

$$\frac{\Delta v}{\Delta y} = \frac{10}{h}\,\mathrm{s^{-1}}.$$

The shear stress is already in SI units: $\tau = 10^{-3}\,\mathrm{N\,m^{-2}}$.

Step 2: Apply the viscous stress relation The given viscosity is $\eta = 10^{-2}\,\mathrm{Pa\,s}$. Substitute the velocity gradient into $\tau = \eta \Delta v/h$:

$$10^{-3} = 10^{-2}\frac{10}{h}.$$

Rearranging yields

$$h = \frac{\eta \Delta v}{\tau} = \frac{10^{-2} \times 10}{10^{-3}} = 100\,\mathrm{m}.$$

The dimensions also check: $(\mathrm{Pa\,s})(\mathrm{m\,s^{-1}})/\mathrm{Pa} = \mathrm{m}$. A verification of the gradient gives $10/100 = 0.1\,\mathrm{s^{-1}}$, and multiplying by the viscosity returns the stated stress of $10^{-3}\,\mathrm{Pa}$.

Step 3: Conclusion The depth of the river in the stated layer-flow model is $\boxed{100\,\mathrm{m}}$. The numerical entry is 100.

Quick Tip to Solve: In $\tau = \eta v/h$, convert the surface speed to SI units before solving for depth; the viscosity is dynamic viscosity, not kinematic viscosity.

54. The first overtone frequency of an open organ pipe is equal to the fundamental frequency of a closed organ pipe. If the length of the closed organ pipe is 20 cm. The length of the open organ pipe is _____ cm. [4 marks]

Solution:

Concept:

- An organ pipe open at both ends supports harmonics $f_n = nv/(2L_o)$. Its first overtone is its second harmonic, with frequency v/L_o.
- An organ pipe closed at one end has fundamental frequency $f_c = v/(4L_c)$. Here v is the common speed of sound and L_c is the closed-pipe length.

Step 1: Identify the two frequencies being compared Write L_o for the unknown open-pipe length and $L_c = 20\,\mathrm{cm}$ for the closed-pipe length. The open pipe has displacement antinodes at both ends. Its fundamental wavelength is $2L_o$, while its first overtone wavelength is L_o, so

$$f_{o,\text{first overtone}} = \frac{v}{L_o}.$$

The closed pipe has a displacement node at the closed end and an antinode at the open end. Its

fundamental wavelength is $4L_c$, giving

$$f_{c,\text{fundamental}} = \frac{v}{4L_c}.$$

Step 2: Equate the frequencies and solve for the length The question states that these frequencies are equal. Therefore

$$\frac{v}{L_o} = \frac{v}{4L_c}.$$

Both pipes contain the same medium under the same conditions, so the common nonzero sound speed cancels. Consequently

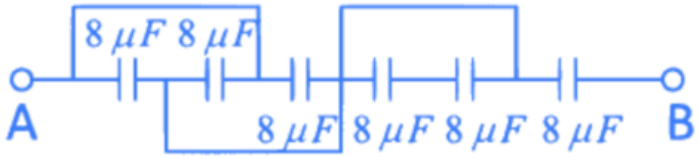
$$L_o = 4L_c = 4(20\,\text{cm}) = 80\,\text{cm}.$$

One can check the result directly: the open pipe of length $80\,\text{cm}$ has first-overtone wavelength $80\,\text{cm}$, while the $20\,\text{cm}$ closed pipe has fundamental wavelength $4(20) = 80\,\text{cm}$. Equal wavelengths at the same sound speed mean equal frequencies, as required.

Step 3: Conclusion The length of the open organ pipe is $\boxed{80\,\text{cm}}$. The numerical entry is 80.

Quick Tip to Solve: For an open pipe, the first overtone is the second harmonic. Use v/L_o, rather than its fundamental value $v/(2L_o)$, in the frequency equality.

55. The equivalent capacitance between points A and B in below shown figure will be _____ μF. [4 marks]



Solution:

Concept:

- Points joined by an ideal conducting wire are the same electrical node, even when the wire bypasses other circuit elements. Redraw the node connections mentally before deciding whether capacitors are in series or parallel.
- Capacitors connected across the same two nodes add in parallel: $C_p = \sum C_i$. For two capacitors in series, $C_{\text{eq}} = C_1C_2/(C_1 + C_2)$.

Step 1: Identify the nodes joined by the bypass wires Number the six $8\,\mu$F capacitors along the horizontal line from left to right as C_1 through C_6. The upper wire on the left connects terminal A to the junction after C_2. The lower wire connects the junction after C_1 to the junction after C_3. The upper wire on the right connects the junction after C_3 to the junction after C_5. Thus C_1, C_2, C_3 are all connected between A and the same intermediate node X. The junction between C_4 and C_5 is connected to X through these two capacitors only; both outer ends of that pair are at X, so it has no potential difference imposed across it and does not affect the terminal capacitance.

Step 2: Combine the active parallel group with the final capacitor The three capacitors between A and X are in parallel, giving

$$C_{AX} = 8 + 8 + 8 = 24\,\mu\text{F}.$$

The final capacitor $C_6 = 8\,\mu\text{F}$ connects X to B, so it is in series with this parallel group. Therefore

$$C_{AB} = \frac{C_{AX}C_6}{C_{AX} + C_6} = \frac{24 \times 8}{24 + 8}\,\mu\text{F} = 6\,\mu\text{F}.$$

This is smaller than the smaller active series capacitance of $8\,\mu\text{F}$, as a series combination must be.

Step 3: Conclusion The equivalent capacitance between A and B is $\boxed{6\,\mu\text{F}}$. The numerical entry is 6.

Quick Tip to Solve: Label every junction joined by a bypass wire with the same node letter first. Physical positions along a straight line do not by themselves make capacitors a series chain.

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56. A resistor develops 300 J of thermal energy in 15 s, when a current of 2 A is passed through it. If the current increases to 3 A, the energy developed in 10 s is _____ J. [4 marks]

Solution:

Concept:

- The electrical energy dissipated as heat in a resistor is $H = I^2Rt$, where I is current, R is resistance and t is the duration.
- For the same resistor with unchanged resistance, heat is proportional to I^2t. A current increase therefore changes the heating rate according to the square of the current ratio.

Step 1: Determine the resistance from the initial heating In the first situation, $H_1 = 300\,\text{J}$, $I_1 = 2\,\text{A}$ and $t_1 = 15\,\text{s}$. Apply Joule's law:

$$300 = (2)^2R(15).$$

Hence

$$R = \frac{300}{4 \times 15} = 5\,\Omega.$$

The initial electrical power is $300/15 = 20\,\text{W}$, consistent with $I_1^2R = 4(5) = 20\,\text{W}$. This determines the property of the resistor that must be retained when the current changes.

Step 2: Calculate the heat for the new current and duration The new current is $I_2 = 3\,\text{A}$, acting for $t_2 = 10\,\text{s}$. With the same $R = 5\,\Omega$, the new heating power is

$$P_2 = I_2^2R = 3^2(5) = 45\,\text{W}.$$

Therefore the energy developed is

$$H_2 = P_2t_2 = 45(10) = 450\,\text{J}.$$

The direct ratio gives an independent check:

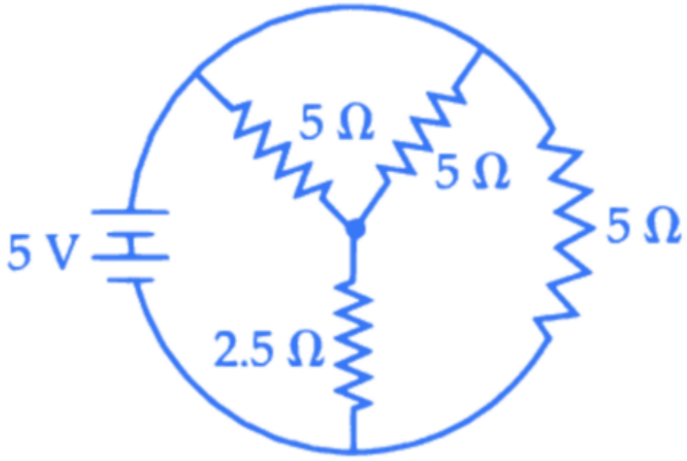
$$\frac{H_2}{H_1} = \frac{I_2^2 t_2}{I_1^2 t_1} = \frac{9(10)}{4(15)} = \frac{3}{2}.$$

Multiplying the original $300\,\text{J}$ by $3/2$ again yields $450\,\text{J}$. Although the new duration is shorter, the larger squared current increases the total energy.

Step 3: Conclusion The thermal energy developed in the second situation is $\boxed{450\,\text{J}}$. The numerical entry is 450.

Quick Tip to Solve: For the same resistor, compare heats using $H_2/H_1 = (I_2/I_1)^2(t_2/t_1)$; do not scale heat linearly with current.

57. The total current supplied to the circuit as shown in figure by the 5 V battery is _____ A. [4 marks]



Solution:

Concept:

- Ideal conducting wires form equipotential nodes. Resistors are parallel when both their ends connect to the same two nodes, regardless of their appearance in the drawing.
- For parallel resistances, $1/R_p = 1/R_1 + 1/R_2$; series resistances add. The total battery current is $I = V/R_{\text{eq}}$.

Step 1: Identify the two inner parallel resistors The upper circular arc is one continuous wire attached to the upper battery terminal. Both inner $5\,\Omega$ resistors connect this same upper node to the central junction. They are therefore in parallel, not in series. Their equivalent resistance is

$$R_1 = \frac{5 \times 5}{5 + 5} = 2.5\,\Omega.$$

No current is lost at the central junction; the currents from these two resistors combine there.

Step 2: Reduce the two complete battery branches The inner equivalent $2.5\,\Omega$ connects in series with the lower $2.5\,\Omega$ resistor, giving

$$R_{\text{inner}} = 2.5 + 2.5 = 5\,\Omega.$$

The outer right-hand resistor is also $5\,\Omega$, directly between the battery terminals. Thus the inner branch and outer branch are in parallel:

$$R_{\text{eq}} = \frac{5 \times 5}{5 + 5} = 2.5\,\Omega.$$

Each complete branch has the full $5\,\text{V}$ across it.

Step 3: Apply Ohm's law to the complete circuit The battery supplies the sum of the two branch currents. Each branch carries $5/5 = 1\,\text{A}$, so

$$I = \frac{5}{2.5} = 2\,\text{A}.$$

This also checks Kirchhoff's junction rule: the total current is larger than the current in either single branch.

Step 4: Conclusion The total current supplied by the battery is $\boxed{2\,\text{A}}$. The numerical entry is 2.

Quick Tip to Solve: Mark the upper arc as one node and the lower arc as another before combining resistors; the two slanting resistors share both endpoints electrically.

58. The current in a coil of self inductance 2.0 H is increasing according to I = 2 sin(t^2) A. The amount of energy spent during the period when current changes from 0 to 2 A is _____ J. [4 marks]

Solution:

Concept:

- An inductor of constant self-inductance L stores magnetic energy $U = \frac{1}{2}LI^2$. The energy depends on the current endpoints, not on the detailed time dependence.
- The applied voltage overcoming self-induced emf is $V = L\,dI/dt$. Hence the power delivered to the magnetic field is $P = VI = LI\,dI/dt$.

Step 1: Express the work in terms of current The current increases from $I_i = 0$ to $I_f = 2\,\text{A}$, and the self-inductance is $L = 2.0\,\text{H}$. During an infinitesimal time, the supplied work is

$$dW = P\,dt = LI\,dI.$$

Thus we can integrate directly with respect to current. The expression $I = 2\sin(t^2)$ specifies how quickly the current changes, but cannot change the energy needed to reach a given current in an ideal inductor.

Step 2: Integrate between the stated limits For constant inductance,

$$W = \int_0^2 LI\,dI = \frac{1}{2}L\left(I_f^2 - I_i^2\right).$$

Substituting the numerical values gives

$$W = \frac{1}{2}(2.0)(2^2 - 0^2) = 4.0\,\text{J}.$$

The initial magnetic energy is zero and the final magnetic energy is $4.0\,\text{J}$, so the positive difference is the energy supplied against the back emf.

Step 3: Check the sign and interpretation An increasing positive current produces a back emf opposing its increase according to Lenz's law. External work is therefore positive and is stored in the magnetic field. No resistance or heat-loss term is specified, so an extra Joule-heating contribution must not be added. In SI units, $\text{H}\,\text{A}^2 = \text{J}$, consistent with the result.

Step 4: Conclusion The energy spent in increasing the current is $\boxed{4\,\text{J}}$. The numerical entry is 4.

Quick Tip to Solve: For a fixed inductance, use the change in $\frac{1}{2}LI^2$; do not integrate a complicated current-time law when only the initial and final currents are needed.

59. A force on an object of mass 100 g is $\left(10\hat{i} + 5\hat{j}\right)$ N. The position of that object at t = 2 s is $\left(a\hat{i} + b\hat{j}\right)$ m after starting from rest. The value of $\frac{a}{b}$ will be _____. [4 marks]

Solution:

Concept:

- Newton's second law gives constant acceleration $\vec{a}_{\text{acc}} = \vec{F}/m$ when both force and mass are constant.
- For motion starting from rest at the coordinate origin, the position vector is $\vec{r} = \frac{1}{2}\vec{a}_{\text{acc}}t^2$. The two Cartesian components can be treated independently.

Step 1: Convert the mass and calculate acceleration The mass must be expressed in kilograms before using SI forces:

$$m = 100\,\text{g} = 0.100\,\text{kg}.$$

The applied force is $\vec{F} = (10\hat{i} + 5\hat{j})\,\text{N}$. Therefore

$$\vec{a}_{\text{acc}} = \frac{10\hat{i} + 5\hat{j}}{0.100} = (100\hat{i} + 50\hat{j})\,\text{m}\,\text{s}^{-2}.$$

Both acceleration components are positive and constant.

Step 2: Evaluate the position after two seconds Taking the initial position as the origin implicit in the stated position-from-rest problem, the initial velocity is zero. At $t = 2\,\text{s}$,

$$\vec{r} = \frac{1}{2}(100\hat{i} + 50\hat{j})(2)^2 = (200\hat{i} + 100\hat{j})\,\text{m}.$$

Comparison with $(a\hat{i} + b\hat{j})\,\text{m}$ gives $a = 200$ and $b = 100$. The symbols a, b here denote position coefficients, not acceleration.

Step 3: Form the requested dimensionless ratio The common mass and time factor cancels between

the two displacement components:

$$\frac{a}{b} = \frac{\frac{1}{2}(F_x/m)t^2}{\frac{1}{2}(F_y/m)t^2} = \frac{F_x}{F_y} = \frac{10}{5} = 2.$$

The explicit component calculation gives the same result, $200/100 = 2$. Since the numerator and denominator are both position coefficients, their ratio is dimensionless.

Step 4: Conclusion The requested value is $\boxed{a/b = 2}$, a dimensionless ratio.

Quick Tip to Solve: From rest under a constant force, displacement is parallel to the force. Its component ratio equals the force-component ratio without separately calculating the time factor.

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60. A steam engine intakes 50 g of steam at 100°C per minute and cools it down to 20°C. If latent heat of vaporization of steam is 540 cal g^{-1}, then the heat rejected by the steam engine per minute is _____ $\times$ 10^3 cal.

(Given : specific heat capacity of water : 1 cal g^{-1} $°C^{-1}$) [4 marks]

Solution:

Concept:

- Steam at its condensation temperature first releases latent heat $Q_1 = mL$ on becoming liquid water at the same temperature.
- The resulting water then releases sensible heat $Q_2 = mc\Delta T$ as its temperature falls. Both contributions must be added for the total heat rejected.

Step 1: Calculate the heat released on condensation The engine receives $50\,\mathrm{g}$ of steam in one minute. Since it begins at $100°\mathrm{C}$, condensation to water at $100°\mathrm{C}$ releases

$$Q_1 = mL = (50)(540) = 27000\,\mathrm{cal}.$$

The temperature does not change during this phase transition, so a specific-heat expression alone would omit this main contribution.

Step 2: Calculate the heat released on cooling the water After condensation, the same $50\,\mathrm{g}$ of water cools from $100°\mathrm{C}$ to $20°\mathrm{C}$. The magnitude of the temperature fall is

$$\Delta T = 100 - 20 = 80°\mathrm{C}.$$

Using $c = 1\,\mathrm{cal\,g^{-1}\,°C^{-1}}$, the additional heat rejected is

$$Q_2 = mc\Delta T = (50)(1)(80) = 4000\,\mathrm{cal}.$$

There is no further phase change because the final temperature is above the freezing point.

Step 3: Add the contributions for one minute Both stages release heat from the working steam/water,

so their positive rejected-heat magnitudes add:

$$Q = Q_1 + Q_2 = 27000 + 4000 = 31000\,\text{cal} = 31 \times 10^3\,\text{cal}.$$

The stated mass is already the amount received per minute; it should not be multiplied by an additional factor of 60. The result refers to heat rejected in one minute, rather than power in calories per second.

Step 4: Conclusion The heat rejected per minute is $\boxed{31 \times 10^3\,\text{cal}}$. Hence the numerical entry is 31.

Quick Tip to Solve: For steam cooling below its boiling point, add condensation heat and subsequent liquid cooling heat: $m[L + c(100 - T_f)]$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Bonding in which of the following diatomic molecule(s) become(s) stronger, on the basis of MO Theory, by removal of an electron?

(A) NO

(B) N_2

(C) O_2

(D) C_2

(E) B_2

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A), (B), (C) only

(B) (B), (C), (E) only

(C) (A), (C) only

(D) (D) only

Correct Answer: (C) (A), (C) only

Solution:

Concept:

- Bond order is $\text{BO} = (N_b - N_a)/2$, where N_b and N_a count bonding and antibonding electrons, respectively.
- Removing one electron from an antibonding orbital increases bond order by $1/2$; removing one from a bonding orbital decreases bond order by $1/2$. Higher bond order corresponds to stronger bonding.

Step 1: Identify the molecules with antibonding outer electrons In NO, the highest occupied molecular orbital is antibonding π^*. Removing its electron changes the bond order from 2.5 to 3, strengthening the bond. In O_2, the highest electrons also occupy antibonding π^* orbitals; removing one changes the bond order from 2 to 2.5. Therefore species (A) and (C) satisfy the condition.

Step 2: Check each remaining molecule In N_2, the electron is removed from a bonding σ_{2p} orbital, reducing bond order from 3 to 2.5. In C_2, removal is from a bonding π_{2p} orbital, reducing bond order

from 2 to 1.5. In B_2, removal is likewise from a bonding π_{2p} orbital, reducing bond order from 1 to 0.5. Thus (B), (D) and (E) do not become more strongly bonded.

Step 3: Select the matching combination The required set consists of NO and O_2, namely entries (A) and (C) in the molecular list. Option (A) incorrectly includes N_2; option (B) incorrectly includes both N_2 and B_2 and excludes NO; option (D) selects C_2, whose bond weakens. Only option (C) gives precisely the two qualifying molecules.

Step 4: Conclusion Only NO and O_2 become more strongly bonded on electron removal. Hence, the correct option is (C).

Quick Tip to Solve: Locate the highest occupied MO before changing bond order: ionisation strengthens a bond only when the removed electron was antibonding.

62. The pair, in which ions are isoelectronic with Al^{3+} is : [4 marks]

(A) Br^- and Be^{2+}

(B) Cl^- and Li^+

(C) S^{2-} and K^+

(D) O^{2-} and Mg^{2+}

Correct Answer: (D) O^{2-} and Mg^{2+}

Solution:

Concept:

- Isoelectronic species have the same total number of electrons. Different nuclear charges do not prevent ions from being isoelectronic.
- A cation has $Z - q$ electrons for positive charge q; an anion of charge magnitude q has $Z + q$ electrons. Count each ion separately before choosing a pair.

Step 1: Count the electrons in the reference ion Aluminium has atomic number $Z = 13$, so a neutral aluminium atom contains 13 electrons. Formation of Al^{3+} removes three electrons:

$$N(Al^{3+}) = 13 - 3 = 10.$$

Its electronic configuration is $1s^2\,2s^2\,2p^6$, the neon configuration. Both ions in a correct option must therefore have ten electrons, not merely the same number as each other.

Step 2: Test every proposed pair For option (A), Br^- has $35 + 1 = 36$ electrons, whereas Be^{2+} has $4 - 2 = 2$. For option (B), Cl^- has $17 + 1 = 18$ and Li^+ has $3 - 1 = 2$. Neither pair matches the ten-electron reference. In option (C), S^{2-} has $16 + 2 = 18$ and K^+ has $19 - 1 = 18$; these ions are isoelectronic with each other, but with argon rather than with the reference ion.

Step 3: Verify the ten-electron pair For option (D), oxygen has atomic number 8 and gains two electrons, while magnesium has atomic number 12 and loses two:

$$N(O^{2-}) = 8 + 2 = 10, \qquad N(Mg^{2+}) = 12 - 2 = 10.$$

Both attain the same electron count and closed-shell configuration as Al^{3+}. Their different ionic charges and ionic sizes do not alter this electron-count criterion.

Step 4: Conclusion The pair is O^{2-} and Mg^{2+}, each with ten electrons. Hence, the correct option is (D).

Quick Tip to Solve: Always compare a proposed isoelectronic pair with the reference ion: two ions may match each other while both have the wrong electron count.

63. Number of electron deficient molecules among the following

PH_3, B_2H_6, CCl_4, NH_3, LiH and BCl_3 is [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (C) 2

Solution:

Concept:

- An electron deficient covalent molecule lacks enough valence electrons for all its bonds to be represented by ordinary two-centre, two-electron bonds with complete octets.
- Boron compounds commonly show electron deficiency. Distinguish an incomplete covalent octet from ionic bonding: a simple ionic hydride is not counted as an electron deficient covalent molecule.

Step 1: Identify the electron deficient boron compounds In BCl_3, boron forms three ordinary covalent bonds and has only six electrons in its valence shell. It therefore has an incomplete octet and is electron deficient. In B_2H_6, the available valence electrons cannot form all the apparent connections as conventional two-centre bonds. Two bridging hydrogen atoms form three-centre, two-electron B–H–B bonds. This multicentre bonding is characteristic of its electron deficiency.

Step 2: Exclude the remaining listed compounds In PH_3, phosphorus has three bonding pairs and one lone pair, giving an octet. The central carbon in CCl_4 has four bonding pairs and a complete octet. The central nitrogen in NH_3 has three bonds and a lone pair, also completing its octet. LiH is an ionic hydride, represented as $Li^+ H^-$; the hydride ion has a filled $1s^2$ shell. It is not an additional electron deficient covalent molecule.

Step 3: Count and compare the options The qualifying compounds are exactly B_2H_6 and BCl_3. Thus the number is two. Option (A) overlooks both boron compounds, while option (B) misses one of them. Option (D) would require incorrectly including a third species, such as treating ionic LiH as electron deficient merely because lithium has few valence electrons. The actual bonding description excludes that interpretation.

Step 4: Conclusion There are $\boxed{2}$ electron deficient molecules: B_2H_6 and BCl_3. Hence, the correct option is (C).

Quick Tip to Solve: Check the central atom's octet and the bonding type. Boron hydrides may use multicentre bonds, whereas an ionic hydride such as LiH is not counted here.

64. White precipitate of AgCl dissolves in aqueous ammonia solution due to formation of : [4 marks]

(A) $[Ag(NH_3)_4]Cl_2$

(B) $[Ag(Cl)_2(NH_3)_2]$

(C) $[Ag(NH_3)_2]Cl$

(D) $[Ag(NH_3)Cl]Cl$

Correct Answer: (C) $[Ag(NH_3)_2]Cl$

Solution:

Concept:

- Silver chloride is sparingly soluble: $AgCl(s) \rightleftharpoons Ag^+(aq) + Cl^-(aq)$. Its dissolution equilibrium depends on the concentration of free silver ions.
- Ammonia is a neutral ligand that binds silver(I) to give the soluble, linear diamminesilver(I) ion $[Ag(NH_3)_2]^+$. Complex formation can drive dissolution of a precipitate.

Step 1: Identify the oxidation state and ligand In $AgCl$, silver is in the $+1$ oxidation state. Each ammonia molecule donates its nitrogen lone pair without changing the oxidation state of silver. Under these conditions silver(I) forms a complex with two ammonia ligands, rather than a four-ammonia silver(II) species. The complex ion therefore has charge $+1$.

Step 2: Combine the equilibria Complexation removes free Ag^+ from the dissolution equilibrium. More solid consequently dissolves to restore equilibrium. The net process is

$$AgCl(s) + 2\,NH_3(aq) \rightleftharpoons [Ag(NH_3)_2]^+(aq) + Cl^-(aq).$$

Writing the counter-ion alongside the complex gives $[Ag(NH_3)_2]Cl$. The chloride is outside the coordination sphere.

Step 3: Eliminate the other formulae Option (A) has four ammonia ligands and two chloride counter-ions, inconsistent with the silver(I) diamine complex. Option (B) puts two chlorides within the coordination sphere and is not the soluble diamminesilver(I) species. Option (D) includes coordinated chloride and an extra chloride counter-ion, again giving the wrong composition.

Step 4: Conclusion The white precipitate dissolves through formation of soluble $[Ag(NH_3)_2]Cl$, containing $[Ag(NH_3)_2]^+$. Hence, the correct option is (C).

Quick Tip to Solve: For $AgCl$ in aqueous ammonia, remember the linear diamminesilver(I) ion; chloride remains its counter-ion.

65. Cerium (IV) has a noble gas configuration. Which of the following is correct statement about it? [4 marks]

(A) It will not prefer to undergo redox reactions.

(B) It will prefer to gain electron and act as an oxidizing agent.

(C) It will prefer to give away an electron and behave as reducing agent.

(D) It acts as both, oxidizing and reducing agent.

Correct Answer: (B) It will prefer to gain electron and act as an oxidizing agent.

Solution:

Concept:

- An oxidizing agent accepts electrons and is reduced. A reducing agent donates electrons and is oxidized.
- Although Ce^{4+} has the xenon configuration, the +3 oxidation state is generally characteristic of lanthanoids; aqueous Ce^{4+} is a strong oxidizing agent.

Step 1: Write the electron-transfer process Cerium has atomic number 58. The relevant configurations are Ce^{4+} : [Xe] and Ce^{3+} : $[Xe]4f^1$. Cerium(IV) readily accepts an electron according to

$$Ce^{4+} + e^- \longrightarrow Ce^{3+}.$$

Its standard reduction potential in acidic aqueous solution is strongly positive (about $1.6\,V$), so this reduction is favourable when coupled to oxidation of a suitable species.

Step 2: Interpret the role of cerium(IV) The electron is gained by cerium, whose oxidation state decreases from +4 to +3. The other reactant loses that electron and becomes oxidized. Thus cerium(IV) acts as the oxidizing agent. A noble-gas electronic configuration by itself does not establish inertness: the overall free-energy change, including hydration and the relative stability of oxidation states, determines redox behaviour.

Step 3: Reject the incompatible claims Option (A) incorrectly infers that the xenon configuration prevents redox reactions. Option (C) reverses the direction of electron transfer: donating an electron would produce the unusually high +5 state. Option (D) overlooks the strongly preferred reduction of cerium(IV) to cerium(III) in its characteristic chemistry.

Step 4: Conclusion Cerium(IV) prefers to gain an electron and is reduced to cerium(III); it acts as an oxidizing agent. Hence, the correct option is (B).

Quick Tip to Solve: Assign the redox role from electron transfer: $Ce^{4+} \longrightarrow Ce^{3+}$ is electron gain, so cerium(IV) is the oxidant.

66. Among the following, which is the strongest oxidizing agent? [4 marks]

(A) Mn^{3+}

(B) Fe^{3+}

(C) Ti^{3+}

(D) Cr^{3+}

Correct Answer: (A) Mn^{3+}

Solution:

Concept:

- For ions in the same oxidation state, compare their tendency to gain an electron: $M^{3+} + e^- \longrightarrow M^{2+}$. A more positive reduction potential indicates a stronger oxidizing agent.
- The stability of the product configuration helps explain the trend. Mn^{2+} has a particularly stable half-filled $3d^5$ configuration.

Step 1: Compare the relevant reductions For the aqueous couples, representative standard potentials are

$$E^\circ(Mn^{3+}/Mn^{2+}) \approx +1.51\,V,$$
$$E^\circ(Fe^{3+}/Fe^{2+}) \approx +0.77\,V,$$
$$E^\circ(Cr^{3+}/Cr^{2+}) \approx -0.41\,V,$$
$$E^\circ(Ti^{3+}/Ti^{2+}) \approx -0.37\,V.$$

The manganese couple has the greatest electron-accepting tendency of the listed ions.

Step 2: Relate the result to electronic structure Mn^{3+} is $3d^4$; gaining one electron gives Mn^{2+}, which is $3d^5$. The extra stability of the half-filled product supports the strong oxidizing behaviour of manganese(III). In contrast, Fe^{3+} itself is $3d^5$; reduction destroys this half-filled configuration, so it is a weaker oxidant than manganese(III).

Step 3: Eliminate the remaining ions Cr^{3+} has a stable $3d^3$ configuration, and its reduction to chromium(II) is much less favourable. Ti^{3+} also has a negative potential for reduction to titanium(II), and often instead behaves as a reducing agent by becoming titanium(IV). Neither rivals the positive manganese(III)/manganese(II) couple.

Step 4: Conclusion Among the given ions, Mn^{3+} is the strongest oxidizing agent. Hence, the correct option is (A).

Quick Tip to Solve: When comparing $3+$ transition-metal oxidants, examine the $2+$ product; forming half-filled Mn^{2+} strongly favours reduction.

67. Phenol on reaction with dilute nitric acid, gives two products. Which method will be most efficient for large scale separation? [4 marks]

(A) Chromatographic separation

(B) Fractional Crystallisation

(C) Steam distillation

(D) Sublimation

Correct Answer: (C) Steam distillation

Solution:

Concept:

- Dilute nitric acid nitrates phenol to give ortho-nitrophenol and para-nitrophenol. The products differ greatly in their hydrogen bonding and steam volatility.
- Intramolecular hydrogen bonding reduces intermolecular association; intermolecular hydrogen bonding increases association and suppresses volatility. Steam distillation exploits this difference.

Step 1: Identify the two products The hydroxyl group of phenol activates the ring and directs electrophilic substitution mainly to the ortho and para positions. Therefore dilute HNO_3 produces a mixture of 2-nitrophenol and 4-nitrophenol. The separation should exploit a physical property that differs markedly between these positional isomers.

Step 2: Compare their hydrogen bonding In ortho-nitrophenol, the neighbouring hydroxyl and nitro groups form an intramolecular hydrogen bond. This reduces association between separate molecules, making the ortho isomer relatively volatile in steam. In para-nitrophenol the groups are too far apart for such an internal bond, so intermolecular hydrogen-bonded association predominates. The para isomer is consequently much less steam volatile.

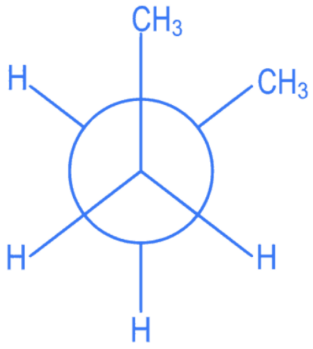
Step 3: Choose an efficient large-scale method On passing steam through the mixture, ortho-nitrophenol distils with the steam while para-nitrophenol largely remains in the vessel. The distillate is condensed and the ortho product recovered. This directly exploits the volatility difference without heating each compound to its normal boiling point. Chromatography is less economical for this large-scale separation; fractional crystallisation relies on solubility differences and repeated operations. Sublimation is not the preferred selective process for this pair.

Step 4: Conclusion Steam distillation efficiently separates steam-volatile ortho-nitrophenol from the much less volatile para isomer. Hence, the correct option is (C).

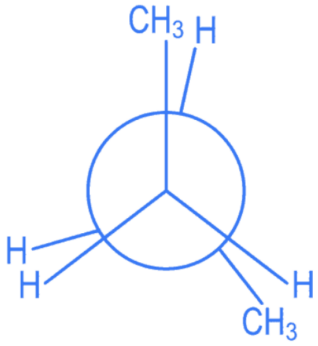
Quick Tip to Solve: For ortho- versus para-nitrophenol, connect intramolecular hydrogen bonding with steam volatility: the ortho isomer distils first.

68. In the following structures, which on is having staggered conformation with maximum dihedral angle? [4 marks]

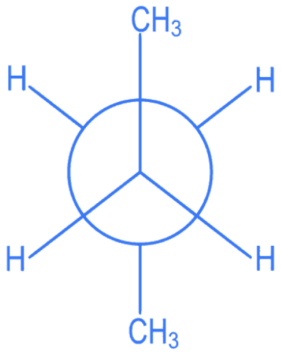
(A)



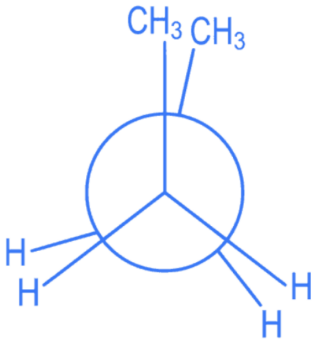
(B)



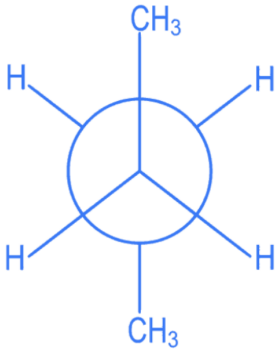
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- A Newman projection views a carbon-carbon single bond along its axis. The dihedral angle is measured between a bond on the front carbon and a bond on the rear carbon.
- In staggered butane, the largest methyl-methyl dihedral angle is 180°. This anti conformation keeps the two methyl groups farthest apart and minimizes their steric interaction.

Step 1: Locate the two methyl groups Each projection represents rotation about the central bond of butane. The three bonds emerging from the centre belong to the front carbon, while the three bonds from the circumference belong to the rear carbon. To compare conformations, follow the two CH_3 groups, rather than an arbitrary pair of carbon-hydrogen bonds.

Step 2: Read the staggered projections In option (A), the two methyl bonds are separated by 60°, giving a gauche staggered conformation. In option (C), the front methyl points upwards and the rear methyl points downwards. Their directions are opposite, so

$$\phi(CH_3{-}C{-}C{-}CH_3) = 180^\circ.$$

The other bonds are staggered as well, making this the anti conformation.

Step 3: Compare the eclipsed alternatives Options (B) and (D) depict eclipsed arrangements: front and rear bonds are drawn nearly along the same directions to keep both visible. In (B), a methyl group eclipses a hydrogen; in (D), the methyl groups eclipse one another. They have torsional strain and do not satisfy the stated staggered condition. Among the staggered choices (A) and (C), only (C) has the maximum methyl-methyl dihedral angle and the greatest separation of bulky groups.

Step 4: Conclusion Option (C) is anti-staggered butane, with a methyl-methyl dihedral angle of 180°. Hence, the correct option is (C).

Quick Tip to Solve: In a Newman projection of butane, first identify the two methyl groups: opposite methyl bonds mean anti, with 180° dihedral angle.

69. The product formed in the following reaction.

$$(CH_3)_2C{=}CH_2 + (CH_3)_3CH \xrightarrow{H^+} ?$$

[4 marks]

(A) $(CH_3)_2CH{-}CH_2{-}CH_2{-}CH(CH_3)_2$

(B) $(CH_3)_2CH{-}CH_2{-}C(CH_3)_3$

(C) $CH_3{-}CH(CH_3){-}CH(CH_3){-}CH(CH_3){-}CH_3$

(D) $(CH_3)_3C{-}C(CH_3)_3$

Correct Answer: (B) $(CH_3)_2CH{-}CH_2{-}C(CH_3)_3$

Solution:

Concept:

- Acid-catalysed addition to an alkene begins by protonation of its double bond. The preferred orientation forms the more stable carbocation.
- Isobutane can transfer a hydride ion from its tertiary carbon. Alkylation of isobutane with isobutene gives the branched eight-carbon alkane 2,2,4-trimethylpentane, commonly called isooctane.

Step 1: Generate the reactive carbocation The alkene shown is 2-methylpropene, $(CH_3)_2C{=}CH_2$, and the alkane is isobutane, $(CH_3)_3CH$. Protonation at the terminal carbon of the alkene produces a tertiary tert-butyl carbocation, $(CH_3)_3C^+$. Forming a primary carbocation by the reverse orientation would be less favourable.

Step 2: Form the carbon-carbon bond and transfer hydride The tert-butyl cation adds to the terminal carbon of another 2-methylpropene molecule, giving the tertiary eight-carbon cation $(CH_3)_3C{-}CH_2{-}C^+(CH_3)_2$. Hydride transfer from isobutane then produces

$$(CH_3)_3C{-}CH_2{-}CH(CH_3)_2.$$

That transfer also regenerates the tert-butyl cation, allowing the acid-catalysed alkylation chain to continue. The net reaction combines one molecule each of the alkene and isobutane.

Step 3: Identify the printed product Read the longest chain of the product as pentane and number it from the quaternary-carbon end. There are methyl substituents at positions 2, 2 and 4, so the product is 2,2,4-trimethylpentane. This is exactly structure (B), shown in the opposite direction as $(CH_3)_2CH{-}CH_2{-}C(CH_3)_3$. Option (A) is 2,5-dimethylhexane, (C) is 2,3,4-trimethylpentane and (D) is 2,2,3,3-tetramethylbutane; these have different carbon connectivity.

Step 4: Conclusion The product is isooctane, $(CH_3)_3C{-}CH_2{-}CH(CH_3)_2$, or 2,2,4-trimethylpentane. Hence, the correct option is (B).

Quick Tip to Solve: Acid-catalysed alkylation of isobutane with isobutene gives isooctane; check for a quaternary carbon separated by CH_2 from $CH(CH_3)_2$.

70. The IUPAC name of ethylidene chloride is : [4 marks]

(A) 1-Chloroethene

(B) 1-Chloroethyne

(C) 1,2-Dichloroethane

(D) 1,1-Dichloroethane

Correct Answer: (D) 1,1-Dichloroethane

Solution:

Concept:

- A geminal dihalide has both halogen atoms attached to the same carbon atom. A vicinal dihalide has halogens on two neighbouring carbon atoms.
- Ethylidene chloride is the common name of CH_3CHCl_2. IUPAC naming uses the saturated two-carbon parent ethane and assigns the lowest possible locants to the two chloro substituents.

Step 1: Translate the common name into structure The name ethylidene chloride denotes replacement of two hydrogens on the same carbon of ethane by chlorine. Its condensed formula is CH_3-CHCl_2, with molecular formula $C_2H_4Cl_2$. The word ethylidene should not be confused with ethylene: ethylene chloride is the common name for the different vicinal compound $ClCH_2-CH_2Cl$.

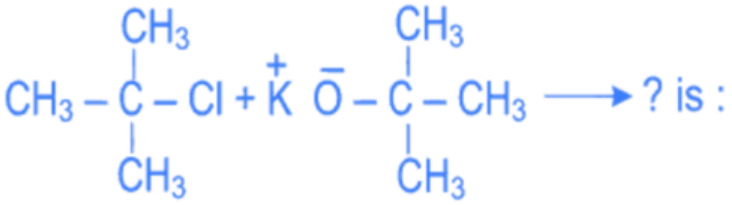
Step 2: Select the parent and number the chain The carbon skeleton contains two carbon atoms linked by a single bond, so the parent hydrocarbon is ethane. Numbering from the $CHCl_2$ end places both chlorines at carbon 1. Numbering from the other end would give locants 2,2, which are larger. Thus the substituent prefix is 1,1-dichloro and the complete name is 1,1-dichloroethane.

Step 3: Reject the alternative names Options (A) and (B), 1-chloroethene and 1-chloroethyne, describe unsaturated compounds with only one chlorine atom and cannot represent CH_3CHCl_2. Option (C), 1,2-dichloroethane, has one chlorine on each carbon and is the vicinal isomer. Option (D) alone preserves the saturated ethane skeleton and places both chlorines on the same carbon.

Step 4: Conclusion The IUPAC name of ethylidene chloride, CH_3CHCl_2, is 1,1-dichloroethane. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish ethylidene chloride (geminal, 1,1-) from ethylene chloride (vicinal, 1,2-) before assigning the IUPAC name.

71. The major product in the reaction [4 marks]



(A) t-Butyl ethyl ether

(B) 2,2-Dimethyl butane

(C) 2-Methyl pent-1-ene

(D) 2-Methyl prop-1-ene

Correct Answer: (D) 2-Methyl prop-1-ene

Solution:

Concept:

- A tertiary alkyl halide reacts with a strong bulky base predominantly by beta-elimination. The base removes a hydrogen from a carbon adjacent to the carbon bearing the leaving group.
- Potassium tert-butoxide, $KOC(CH_3)_3$, is sterically hindered: proton abstraction is easier than attack at a crowded tertiary carbon. An E2 elimination forms a carbon-carbon double bond.

Step 1: Identify the substrate and reagent The substrate shown is tert-butyl chloride, $(CH_3)_3CCl$, and the reagent is potassium tert-butoxide. The carbon bonded to chlorine is attached to three methyl groups. Consequently a backside substitution attack at this carbon is strongly obstructed, so a Williamson ether synthesis is not the favoured pathway. The alkoxide is a strong base as well as a potential nucleophile, and here its basic behaviour dominates.

Step 2: Remove a beta-hydrogen All three methyl groups of tert-butyl chloride are equivalent. Removing a hydrogen from any one of them while chloride leaves gives the same alkene. The net reaction is

$$(CH_3)_3CCl + KOC(CH_3)_3 \longrightarrow (CH_3)_2C{=}CH_2 + HOC(CH_3)_3 + KCl.$$

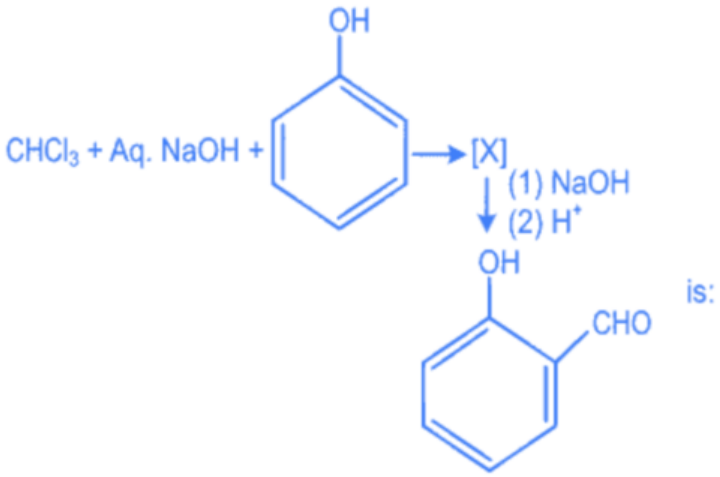
The alkene retains the four-carbon skeleton of the alkyl halide; the base does not donate its own carbon skeleton to the product.

Step 3: Compare the products offered The product $CH_2{=}C(CH_3)_2$ is 2-methylprop-1-ene. Option (A) requires ether formation and even contains an ethyl group absent from the reagents. Options (B) and (C) contain six carbons and would require carbon-carbon bond formation, whereas this reaction removes hydrogen chloride from the four-carbon halide. Neither a coupling reagent nor an ethyl-carbon source is present.

Step 4: Conclusion The major product is $CH_2{=}C(CH_3)_2$, 2-methylprop-1-ene. Hence, the correct option is (D).

Quick Tip to Solve: For a tertiary halide with tert-butoxide, look for loss of HX and an alkene before considering ether formation; count the substrate carbons as a quick check.

72. The intermediate X, in the reaction : [4 marks]



(A) o-$HOC_6H_4CCl_3$

(B) o-$HOC_6H_4CHCl_2$

(C) o-$NaOC_6H_4CHCl_2$

(D) o-$NaOC_6H_4CCl_3$

Correct Answer: (C) o-$NaOC_6H_4CHCl_2$

Solution:

Concept:

- Phenol reacts with chloroform and aqueous alkali in the Reimer-Tiemann reaction. The reactive electrophile is dichlorocarbene, $:CCl_2$, generated by base-induced elimination from chloroform.
- In alkaline solution phenol exists mainly as phenoxide. Ortho introduction of a dichloromethyl group is followed by hydrolysis to an aldehyde; acidic work-up restores the phenolic OH group.

Step 1: Generate the attacking species Hydroxide removes the proton from chloroform. The resulting trichloromethyl anion eliminates chloride, producing dichlorocarbene:

$$CHCl_3 + OH^- \longrightarrow {CCl_3}^- + H_2O, \qquad {CCl_3}^- \longrightarrow :CCl_2 + Cl^-.$$

At the same time, phenol is converted into sodium phenoxide. Its negatively charged oxygen activates the ring, especially the ortho and para positions, towards the electrophilic carbene.

Step 2: Identify X before hydrolysis The major ortho pathway installs a $-CHCl_2$ group next to the phenoxide oxygen. Because the reaction medium is aqueous sodium hydroxide and acidification is shown only later, X retains the sodium phenoxide form:

$$X = o\text{-}NaOC_6H_4CHCl_2.$$

Two chlorine atoms remain on the side-chain carbon at this stage, not three. The carbene that attacked had only two chlorine atoms.

Step 3: Check the final product and alternatives Hydrolysis of the gem-dichloride converts $-CHCl_2$ into $-CHO$, and the final H^+ work-up protonates phenoxide to give salicylaldehyde, the product

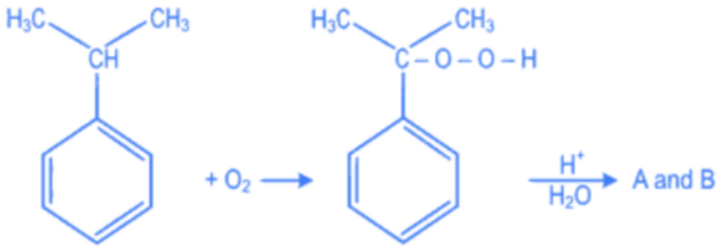
shown. Option (B) has the correct side chain but incorrectly shows the protonated phenol before the acid work-up. Options (A) and (D) contain $-CCl_3$, which is inconsistent with dichlorocarbene addition and formation of the displayed aldehyde.

Step 4: Conclusion X is sodium ortho-(dichloromethyl)phenoxide, $o\text{-}NaOC_6H_4CHCl_2$. Hence, the correct option is (C).

Quick Tip to Solve: In Reimer-Tiemann questions, track both changes separately: the side chain is CHCl2 before hydrolysis, and the oxygen stays phenoxide until acidification.

73. In the following reaction :

The compounds A and B respectively are : [4 marks]



(A) Toluene and CH_3COOH

(B) Benzene-1,3-diol and CH_3COCH_3

(C) Phenol and CH_3COCH_3

(D) Benzene-1,2-diol and CH_3COCH_3

Correct Answer: (C) Phenol and CH_3COCH_3

Solution:

Concept:

- Cumene is isopropylbenzene, $C_6H_5CH(CH_3)_2$. Oxidation with oxygen gives cumene hydroperoxide, $C_6H_5C(CH_3)_2OOH$.
- Acid-catalysed cleavage of cumene hydroperoxide produces phenol and acetone. In the rearrangement the phenyl group migrates to oxygen; the two methyl groups remain on the carbonyl carbon of acetone.

Step 1: Recognise the hydroperoxide The first structure has a benzene ring attached to $CH(CH_3)_2$, so it is cumene. The intermediate shown contains $-OOH$ on the benzylic carbon. This is not an ordinary alcohol oxidation: the oxygen-oxygen linkage is crucial because acid promotes its cleavage and rearrangement. The substrate therefore identifies the industrial cumene process for making phenol.

Step 2: Follow the acid-catalysed cleavage Protonation of the hydroperoxide enables oxygen-oxygen bond cleavage with migration of the phenyl group onto oxygen. Hydrolysis of the rearranged

intermediate yields one molecule each of phenol and acetone. The overall product equation is

$$C_6H_5C(CH_3)_2OOH \xrightarrow[H_2O]{H^+} C_6H_5OH + CH_3COCH_3.$$

The aromatic fragment acquires an OH group while the original benzylic carbon becomes the carbonyl carbon of acetone.

Step 3: Eliminate the other pairs Phenol and acetone together preserve the nine carbons of cumene: six are in phenol and three in acetone. Option (A), toluene and acetic acid, does not represent hydroperoxide rearrangement and puts no hydroxyl group on the ring. Options (B) and (D) show dihydroxybenzenes; the cumene process introduces only one oxygen substituent onto the aromatic ring, not a second ring hydroxyl group. Thus the monohydric phenol pair is selected.

Step 4: Conclusion The products A and B are phenol, C_6H_5OH, and acetone, CH_3COCH_3, respectively. Hence, the correct option is (C).

Quick Tip to Solve: An isopropylbenzene hydroperoxide followed by acid is the signature of the cumene process: write phenol plus acetone without oxidising the aromatic ring further.

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74. The reaction of $RCONH_2$ with bromine and KOH gives RNH_2 as the end product. Which one of the following is the intermediate product formed in this reaction? [4 marks]

(A) $RCONHBr$

(B) $RNHBr$

(C) $R-N=C=O$

(D) $RCONBr_2$

Correct Answer: (C) $R-N=C=O$

Solution:

Concept:

- Hofmann bromamide degradation converts a primary amide, $RCONH_2$, into a primary amine, RNH_2, using bromine and alkali. The carbonyl carbon is lost from the organic product.
- The characteristic rearrangement intermediate is an isocyanate, $R-N=C=O$. Hydrolysis gives a carbamic acid, which loses carbon dioxide to form the amine.

Step 1: Recognise the reaction The combination of a primary amide with Br_2 and KOH, ending in RNH_2, identifies the Hofmann bromamide reaction. Bromination initially forms an N-bromoamide, and removal of its N-H proton allows rearrangement. The group R migrates from the carbonyl carbon to nitrogen; it is this migration that establishes the new R-N bond of the final amine.

Step 2: Write the rearrangement intermediate After migration and departure of bromide, the product is the isocyanate

$$R-N=C=O.$$

It contains the original carbonyl carbon, but now R is attached directly to nitrogen. Water then

converts the isocyanate into a carbamic acid:

$$R{-}N{=}C{=}O + H_2O \longrightarrow RNHCOOH \longrightarrow RNH_2 + CO_2.$$

In alkaline medium the carbon dioxide is captured as carbonate. This sequence explains why the amine contains one carbon fewer than the starting amide.

Step 3: Interpret the choices carefully Option (C) is the isocyanate formed in the rearrangement and subsequently hydrolysed to the amine. Option (A), $RCONHBr$, is an earlier N-bromination species, and is therefore also an intermediate in this same reaction. The wording does not specify the intermediate formed after rearrangement, so it cannot exclude (A) on chemical grounds. The single-answer key selects (C), the characteristic rearrangement intermediate. Option (B), $RNHBr$, lacks the carbonyl carbon before hydrolysis and is not the isocyanate. Option (D), the N,N-dibromoamide, is not the characteristic intermediate on the standard Hofmann rearrangement pathway.

Step 4: Conclusion The characteristic intermediate formed after rearrangement is the isocyanate $R{-}N{=}C{=}O$, which hydrolyses to the primary amine. The official key selects option (C). However, as printed, the word intermediate also includes the N-bromoamide in (A), so the literal question is ambiguous between (A) and (C).

Quick Tip to Solve: Trace the migrating group in Hofmann degradation: R moves from the carbonyl carbon onto nitrogen, giving R-N=C=O before hydrolysis removes the carbonyl carbon.

75. Incorrect statement for Tyndall effect is : [4 marks]

(A) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.

(B) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(C) During projection of movies in the cinemas hall, Tyndall effect is noticed.

(D) It is used to distinguish a true solution from a colloidal solution.

Correct Answer: (B) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

Solution:

Concept:

- The Tyndall effect is the scattering of light by colloidal particles, making the path of a beam visible when viewed from the side.
- For a conspicuous Tyndall cone, the dispersed particles must be sufficiently large relative to the wavelength of light and the refractive indices of the dispersed phase and dispersion medium must differ appreciably.

Step 1: State the size requirement A colloid contains dispersed particles larger than the molecules or ions in a true solution. These particles scatter an incident beam strongly enough for its path to be visible. The usual condition stated for an observable Tyndall effect is that the particle diameter is not much smaller than the wavelength of the light employed. If particles are far smaller, visible scattering is weak and the beam path does not become conspicuous in the same way.

Step 2: Check the refractive-index and example statements Option (A) correctly requires a substantial refractive-index contrast: light is redirected at the boundary between the dispersed particles and the medium. Option (C) is also correct. In a cinema hall the projector beam can be seen because suspended dust and other fine particles scatter the light towards the observer. The observation is a standard practical illustration of the Tyndall effect.

Step 3: Identify the incorrect statement Option (D) correctly states an application. A colloidal solution shows a visible beam path, whereas a true solution does not show a visible Tyndall cone under comparable conditions because its molecular-sized solute particles are too small. Option (B) asserts that the dispersed-particle diameter is much smaller than the wavelength, reversing the customary size condition for a conspicuous Tyndall cone. Since the question asks for the incorrect statement, that is the statement to select.

Step 4: Conclusion The incorrect statement is that the diameter of the dispersed particles is much smaller than the wavelength of light used. Hence, the correct option is (B).

Quick Tip to Solve: For a question asking the incorrect Tyndall statement, test particle size and refractive-index contrast first; the visible cinema beam and colloid test are standard correct examples.

76. Leaching of gold with dilute aqueous solution of NaCN in presence of oxygen gives complex [A], which on reaction with zinc forms the elemental gold and another complex [B]. [A] and [B], respectively are : [4 marks]

(A) $[Au(CN)_4]^-$ and $[Zn(CN)_2(OH)_2]^{2-}$

(B) $[Au(CN)_2]^-$ and $[Zn(OH)_4]^{2-}$

(C) $[Au(CN)_2]^-$ and $[Zn(CN)_4]^{2-}$

(D) $[Au(CN)_4]^{2-}$ and $[Zn(CN)_6]^{4-}$

Correct Answer: (C) $[Au(CN)_2]^-$ and $[Zn(CN)_4]^{2-}$

Solution:

Concept:

- During cyanide leaching, oxygen oxidises gold to the $+1$ state and cyanide stabilises it as the soluble dicyanidoaurate(I) ion, $[\mathrm{Au(CN)_2}]^-$.
- Zinc displaces gold from the cyanide complex: zinc is oxidised to $+2$ and forms tetracyanidozincate(II), $[\mathrm{Zn(CN)_4}]^{2-}$, while gold(I) is reduced to elemental gold.

Step 1: Write the leaching reaction In dilute aqueous sodium cyanide, oxygen allows metallic gold to dissolve as a cyanide complex. The ionic equation is

$$4\,\mathrm{Au} + 8\,\mathrm{CN^-} + \mathrm{O_2} + 2\,\mathrm{H_2O} \longrightarrow 4\,[\mathrm{Au(CN)_2}]^- + 4\,\mathrm{OH^-}.$$

There are two cyanide ligands for each gold atom. Taking each cyanide ligand as -1, a gold oxidation state of $+1$ gives the overall complex charge $+1 - 2 = -1$. Therefore complex A is $[\mathrm{Au(CN)_2}]^-$.

Step 2: Recover the gold with zinc Zinc supplies the electrons needed to reduce the dissolved gold(I)

to the metal. The balanced displacement is

$$2\,[\mathrm{Au(CN)_2}]^- + \mathrm{Zn} \longrightarrow [\mathrm{Zn(CN)_4}]^{2-} + 2\,\mathrm{Au}.$$

Two gold(I) ions each gain one electron while one zinc atom loses two electrons. The four cyanide ligands are retained in the soluble zinc complex. Its charge is $+2 - 4 = -2$, confirming the formula of B.

Step 3: Compare both complexes with the options Only option (C) gives dicyanidoaurate(I) and tetracyanidozincate(II) together. Option (B) gives the correct gold complex but replaces the cyanide zinc complex by a hydroxido complex, despite cyanide being the complexing ligand in this extraction. Options (A) and (D) give the wrong gold cyanide species for the stated leaching process; option (D) also gives the wrong zinc coordination formula. Both stages, not just the first complex, must match.

Step 4: Conclusion $[A] = [\mathrm{Au(CN)_2}]^-$ and $[B] = [\mathrm{Zn(CN)_4}]^{2-}$. Hence, the correct option is (C).

Quick Tip to Solve: For gold cyanidation remember the ligand counts 2 then 4: Au(I) binds two cyanides, and the zinc used for displacement ends as Zn(II) with four cyanides.

77. Which one of the following alkaline earth metal ions has the highest ionic mobility in its aqueous solution? [4 marks]

(A) Be^{2+}

(B) Mg^{2+}

(C) Ca^{2+}

(D) Sr^{2+}

Correct Answer: (D) Sr^{2+}

Solution:

Concept:

- Ionic mobility in aqueous solution depends on the effective size of the moving hydrated ion, rather than just the radius of the bare ion.
- For ions of the same charge, a smaller bare radius means a greater charge density and stronger hydration. The strongly bound hydration shell increases resistance to motion through water.

Step 1: Distinguish bare and hydrated size The bare ionic radii increase down Group 2:

$$\mathrm{Be^{2+} < Mg^{2+} < Ca^{2+} < Sr^{2+}}.$$

However, all four ions carry charge +2, so the smaller ones have a stronger electric field at their surfaces. They attract surrounding water molecules more strongly and hold a larger, more tightly associated hydration shell. A bare-radius comparison alone would therefore give the wrong mobility trend in aqueous solution.

Step 2: Apply the hydration trend Hydration is strongest for $\mathrm{Be^{2+}}$ and decreases down the group. The effective hydrated size and solvent drag consequently decrease as we move towards strontium

among the listed ions. For ions under the same electric field, greater viscous resistance means a smaller drift speed. Conversely, the ion with less restrictive hydration moves faster and has the higher ionic mobility.

Step 3: Select among the four ions Beryllium in option (A) is the smallest bare ion but is the most strongly hydrated, so it cannot be chosen merely for its small unhydrated radius. Magnesium in option (B) is less strongly hydrated than beryllium, and calcium in option (C) less strongly than magnesium, but strontium is the least strongly hydrated of the choices. Thus the aqueous mobility order is

$$u(\mathrm{Be}^{2+}) < u(\mathrm{Mg}^{2+}) < u(\mathrm{Ca}^{2+}) < u(\mathrm{Sr}^{2+}).$$

The question asks only among the given ions, so the final member is selected.

Step 4: Conclusion Sr^{2+} has the highest ionic mobility in aqueous solution among the ions listed. Hence, the correct option is (D).

Quick Tip to Solve: For aqueous mobility of same-charge ions, compare hydration first: the smaller bare ion binds water more strongly and can move more slowly despite its small crystal radius.

78. The eutrophication of water body results in : [4 marks]

(A) loss of Biodiversity.

(B) breakdown of organic matter.

(C) increase in biodiversity.

(D) decrease in BOD.

Correct Answer: (A) loss of Biodiversity.

Solution:

Concept:

- Eutrophication is excessive nutrient enrichment of a water body, particularly by nitrates and phosphates. It promotes rapid growth of algae and other aquatic plants.
- Biochemical oxygen demand (BOD) measures the oxygen required by microorganisms to decompose biodegradable organic matter. A larger decomposing organic load tends to increase BOD and deplete dissolved oxygen.

Step 1: Trace the effect of nutrient enrichment Fertilisers and domestic waste can supply excess nutrients to a lake. Algae then multiply rapidly, producing an algal bloom. The bloom reduces light penetration, so submerged plants may die. The increase in algal biomass is not the same thing as an increase in the variety of species: a few tolerant organisms may become dominant while many others disappear.

Step 2: Connect decomposition to oxygen depletion When the algae and plants die, microorganisms decompose their remains and consume dissolved oxygen. Fish and other oxygen-dependent organisms cannot survive prolonged oxygen depletion. Consequently, species diversity falls. The organic breakdown in option (B) is a process in this chain, but option (A) identifies the characteristic detrimental ecological result of eutrophication.

Step 3: Eliminate the remaining options Option (C) reverses the ecological outcome: a bloom can increase biomass while reducing biodiversity. Option (D) is also opposite to the usual trend, since abundant decaying material raises oxygen demand rather than reducing BOD. Thus nutrient enrichment, decomposition and oxygen depletion together explain the loss of biodiversity.

Step 4: Conclusion Eutrophication causes oxygen depletion and loss of biodiversity in the affected water body. Hence, the correct option is (A).

Quick Tip to Solve: Follow the chain: excess nutrients, algal bloom, decay, oxygen depletion, then loss of aquatic species. Do not confuse more algal biomass with greater biodiversity.

79. Using very little soap while washing clothes, does not serve the purpose of cleaning of clothes, because : [4 marks]

(A) soap particles remain floating in water as ions.

(B) the hydrophobic part of soap is not able to take away grease.

(C) the micelles are not formed due to concentration of soap, below its CMC value.

(D) colloidal structure of soap in water is completely disturbed.

Correct Answer: (C) the micelles are not formed due to concentration of soap, below its CMC value.

Solution:

Concept:

- Soap molecules are amphiphilic: each has a hydrophobic hydrocarbon tail and a hydrophilic ionic head. Their tails associate with grease while their heads interact with water.
- Micelles form in appreciable numbers only above the critical micelle concentration (CMC), at a suitable temperature. Their hydrophobic interiors can incorporate oily dirt and disperse it in water.

Step 1: Identify what effective cleaning requires Grease does not dissolve readily in water. Soap assists its removal by forming aggregates in which the hydrocarbon tails point towards the oily material and the ionic heads remain exposed to water. These micelles help emulsify and suspend the grease so that it can be washed away. Thus the cleaning action depends on collective aggregation, not merely on the presence of a few dissolved soap ions.

Step 2: Apply the concentration condition If very little soap is added to a large amount of water, its concentration can remain below the CMC. In this region soap exists predominantly as individual ions or molecules, and there are too few micelles for effective removal of grease. Adding an adequate quantity of soap raises the concentration above the micelle-formation threshold, making cleaning efficient.

Step 3: Compare all four explanations Option (A) mentions dispersed ions but does not identify the concentration threshold responsible for ineffective cleaning. Option (B) suggests an intrinsic failure of the hydrophobic tail, whereas that tail can associate with grease when suitable aggregates form. Option (D) implies destruction of an existing colloidal structure; below the CMC the central problem is that sufficient micelles have not yet formed. Option (C) states this cause directly.

Step 4: Conclusion The soap concentration is below its CMC, so sufficient micelles are not formed to

remove grease effectively. Hence, the correct option is (C).

Quick Tip to Solve: For a question about using too little surfactant, check the CMC first: individual soap ions below the threshold cannot provide effective micellar cleaning.

80. Which one of the following is an example of artificial sweetner? [4 marks]

(A) Bithional

(B) Alitame

(C) Salvarsan

(D) Lactose

Correct Answer: (B) Alitame

Solution:

Concept:

- Artificial sweeteners are substances used to impart sweetness, commonly in much smaller amounts than ordinary sugars. Their chemical identity and use distinguish them from antiseptics and other medicinal compounds.
- Alitame is a high-intensity artificial sweetener. Lactose is instead a naturally occurring disaccharide, while bithional and salvarsan are associated with antimicrobial applications.

Step 1: Identify the food additive in the list Alitame belongs to the group of artificial sweetening agents. It is a peptide-based, high-intensity sweetener and is far sweeter than sucrose, so a small quantity can produce the required sweet taste. Its classification is based on its use as a sweetener; the fact that other listed substances are organic compounds does not place them in the same functional category.

Step 2: Distinguish the medicinal compounds Bithional is an antimicrobial compound associated with antiseptic use, including its addition to soaps in the textbook discussion. Salvarsan is the arsenic-containing drug historically used in the treatment of syphilis. Neither is classified as an artificial sweetener. Their presence as distractors tests recognition of the purpose of a chemical, rather than recognition of an unfamiliar name alone.

Step 3: Separate a natural sugar from an artificial sweetener Lactose is the natural milk sugar and is a disaccharide made from glucose and galactose units. Although it can contribute sweetness, it is not an artificial sweetener. Therefore options (A), (C) and (D) belong to different categories, leaving alitame as the only correct choice among the four substances.

Step 4: Conclusion Alitame is the artificial sweetener in the list. Hence, the correct option is (B).

Quick Tip to Solve: Classify each name by use: alitame is a sweetener, lactose is milk sugar, and bithional and salvarsan are antimicrobial compounds.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The number of N atoms in 681 g of $C_7H_5N_3O_6$ is x $\times$ 10^{21}. The value of x is (N_A = 6.02 $\times$ 10^{23} mol^{-1}) (Nearest Integer) [4 marks]

Solution:

Concept:

- The molar mass of a molecular compound is the sum of the atomic masses of all atoms in its molecular formula. Using C = 12, H = 1, N = 14 and O = 16 gives the mass of one mole.
- The number of molecules is the amount in moles multiplied by Avogadro's constant. Multiply this molecular count by the number of nitrogen atoms per molecule to obtain the nitrogen-atom count.

Step 1: Calculate the molar mass and amount For $C_7H_5N_3O_6$, the contributions of carbon, hydrogen, nitrogen and oxygen must all be included:

$$M = 7(12) + 5(1) + 3(14) + 6(16) = 227\,\text{g mol}^{-1}.$$

The given mass therefore contains

$$n = \frac{681\,\text{g}}{227\,\text{g mol}^{-1}} = 3\,\text{mol}$$

of the compound. This is the amount of whole molecules, not yet the amount of nitrogen atoms.

Step 2: Count nitrogen atoms and express the requested coefficient Each molecule contains three nitrogen atoms, so three moles of molecules contain nine moles of nitrogen atoms. Hence

$$N_{\text{N}} = 3 \times 3 \times 6.02 \times 10^{23} = 5.418 \times 10^{24}.$$

Convert the power of ten to the one specified in the question:

$$5.418 \times 10^{24} = 5418 \times 10^{21}.$$

The coefficient is already an integer. The factor of three from the molecular formula must not be omitted: doing so would count molecules rather than nitrogen atoms. Also, the requested coefficient is not the scientific-notation coefficient 5.418, because the question fixes the exponent at 21.

Step 3: Conclusion The number of nitrogen atoms is $5.418 \times 10^{24} = 5418 \times 10^{21}$. Therefore the nearest-integer value is $\boxed{x = 5418}$, a dimensionless coefficient of the atom count.

Quick Tip to Solve: First count moles of molecules, then multiply by the subscript of the required atom. Finally match the exact power of ten requested.

82. The longest wavelength of light that can be used for the ionisation of lithium atom (Li) in its ground state is x $\times$ 10^{-8} m. The value of x is _____. (Nearest Integer).

(Given : Energy of the electron in the first shell of the hydrogen atom is -2.2×10^{-18} J ; h = 6.63 $\times$ 10^{-34} Js and c = 3 $\times$ 10^{8} ms^{-1}) [4 marks]

Solution:

Concept:

- The photon threshold satisfies $hc/\lambda_{\max} = E_{\text{ion}}$. Larger wavelengths have smaller photon energies and cannot ionise an electron with the specified binding energy.
- The unscreened Bohr expression is $E_n = -2.2 \times 10^{-18} Z^2/n^2$ J. This expression is exact only for a one-electron species within the model; applying it to neutral lithium neglects screening and electron interactions.

Step 1: State the model implicit in the supplied numerical data Lithium has nuclear charge $Z = 3$ and ground-state electron configuration $1s^2 2s^1$. The electron removed on first ionisation occupies the second shell, so the simple unscreened Bohr estimate uses $n = 2$. With the hydrogen energy supplied in the question, this gives

$$|E_2| = 2.2 \times 10^{-18} \frac{3^2}{2^2} = 4.95 \times 10^{-18}\,\text{J}.$$

Neutral lithium is a multi-electron atom, so this calculation is an approximation, not an accurate determination of its actual first ionisation energy. The supplied hydrogen energy alone does not specify the screening correction.

Step 2: Calculate the threshold wavelength in that model Set the photon energy equal to the estimated binding energy:

$$\lambda_{\max} = \frac{hc}{|E_2|} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{4.95 \times 10^{-18}} = 4.018 \times 10^{-8}\,\text{m}.$$

Thus the coefficient rounds from 4.018 to 4. Using $n = 1$ would remove an inner-shell electron rather than the outer ground-state electron. Treating the atom as a hydrogen-like ion would also change the physical species. The value obtained here is specifically the unscreened second-shell calculation implied by the numerical answer, with its limitation stated explicitly.

Step 3: Conclusion The supplied Bohr-model estimate gives $\lambda_{\max} \simeq 4.018 \times 10^{-8}\,\text{m}$, hence $\boxed{x = 4}$, matching the numerical key. For literal neutral Li, screening is essential and this is not its accurate physical first-ionisation threshold.

Quick Tip to Solve: Use the outer occupied shell for first ionisation, and state the approximation when applying a hydrogenic energy formula to a multi-electron atom.

83. The standard entropy change for the reaction

$4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$ is -550 J K^{-1} at 298 K.

[Given : The standard enthalpy change for the reaction is -165 kJ mol^{-1}]. The temperature in K at which the reaction attains equilibrium is _____. (Nearest Integer) [4 marks]

Solution:

Concept:

- The standard Gibbs energy is $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$. The standard thermodynamic crossover occurs when $\Delta G^\circ = 0$, so $T = \Delta H^\circ / \Delta S^\circ$.
- Enthalpy and entropy must use compatible energy units. Treat the supplied standard changes as approximately constant over the small temperature interval of interest.

Step 1: Put the reaction changes in consistent units The data refer to the reaction as written,

$$4\,\mathrm{Fe(s)} + 3\,\mathrm{O_2(g)} \longrightarrow 2\,\mathrm{Fe_2O_3(s)}.$$

Use both changes for that same stoichiometric reaction; do not divide only one of them by a coefficient. Convert the enthalpy into joules:

$$\Delta H^\circ = -165 \times 10^3\,\mathrm{J\,mol^{-1}}, \qquad \Delta S^\circ = -550\,\mathrm{J\,K^{-1}\,mol^{-1}}.$$

The negative entropy change is consistent with consuming a gaseous reactant and forming solid product.

Step 2: Set the standard Gibbs energy to zero At the equilibrium crossover intended by the question,

$$0 = \Delta H^\circ - T\Delta S^\circ, \qquad T = \frac{-165000}{-550} = 300\,\mathrm{K}.$$

Both numerator and denominator are negative, so the temperature is positive. As a check, at 298 K the standard Gibbs change is $-165000 + 298(550) = -1100\,\mathrm{J\,mol^{-1}}$, slightly negative, and it reaches zero when the temperature rises to 300 K. Strictly, actual equilibrium means $\Delta G = 0$; the calculated standard-state crossover has $\Delta G^\circ = 0$ and thus equilibrium constant unity. This is the temperature determined from the supplied standard quantities.

Step 3: Conclusion The standard equilibrium crossover temperature is $\boxed{300\,\mathrm{K}}$, to the nearest integer.

Quick Tip to Solve: For a Gibbs-energy crossover, divide enthalpy by entropy only after converting kJ to J; matching negative signs give a positive temperature.

84. 1 L aqueous solution of H_2SO_4 contains 0.02 m mol H_2SO_4. 50% of this solution is diluted with deionized water to give 1 L solution (A). In solution (A), 0.01 m mol of H_2SO_4 are added. Total m mols of H_2SO_4 in the final solution is _____ $\times 10^3$ m mols. [4 marks]

Solution:

Concept:

- Dilution adds solvent and changes concentration, but it does not change the amount of solute already present. Taking half of a homogeneous solution takes half of its solute.
- The total amount after adding more solute is the amount in the aliquot plus the added amount. Millimoles are amounts, whereas millimoles per litre would be concentrations.

Step 1: Track the sulfuric acid through the aliquot and dilution The initial one-litre solution contains $0.02\,\text{mmol}$ of H_2SO_4. Taking 50 percent of the homogeneous solution takes 50 percent of that amount:

$$n_{\text{aliquot}} = \tfrac{1}{2}(0.02) = 0.01\,\text{mmol}.$$

Deionized water is then added until the volume becomes one litre. Water contributes no sulfuric acid, so solution (A) still contains $0.01\,\text{mmol}$. Its concentration has decreased, but dilution does not create or remove acid formula units.

Step 2: Add the second portion and match the specified scale Adding another $0.01\,\text{mmol}$ gives

$$n_{\text{final}} = 0.01 + 0.01 = 0.02\,\text{mmol}.$$

If the blank is x in the expression $x \times 10^3\,\text{mmol}$, then

$$x = \frac{0.02}{10^3} = 2 \times 10^{-5} = 0.00002.$$

The exact coefficient is therefore not zero. The numerical key gives 0, which is its nearest-integer rounding; that must not be interpreted as absence of acid. The printed question does not explicitly request nearest-integer rounding here, so keeping the exact coefficient also makes the actual amount clear. No dissociation factor is used because the question asks for sulfuric-acid amount, not the total count of dissolved ionic species.

Step 3: Conclusion The final solution contains $\boxed{0.02\,\text{mmol}}$ of H_2SO_4, corresponding to the exact coefficient $x = 2 \times 10^{-5}$. Rounded to the integer used by the numerical key, $\boxed{x = 0}$; this rounded entry does not mean zero acid is present.

Quick Tip to Solve: Keep a running solute balance through each aliquot, dilution and addition; divide by the printed power of ten only at the end.

85. The standard free energy change (ΔG°) for 50% dissociation of N_2O_4 into NO_2 at 27°C and 1 atm pressure is $-$ x J mol^{-1}. The value of x is _____. (Nearest Integer)

[Given : R = 8.31 J K^{-1} mol^{-1}, log 1.33 = 0.1239 ln 10 = 2.3] [4 marks]

Solution:

Concept:

- For the gas equilibrium $\mathrm{N_2O_4(g)} \rightleftharpoons 2\,\mathrm{NO_2(g)}$, the degree of dissociation α determines the equilibrium mole fractions. Partial pressure is mole fraction multiplied by total pressure.
- The standard Gibbs energy and equilibrium constant obey $\Delta G^\circ = -RT\ln K_p$. The pressure equilibrium constant is written with each pressure divided by its standard pressure; the stated atmosphere convention is used here.

Step 1: Set up the equilibrium amounts Start with one mole of $\mathrm{N_2O_4}$. If half dissociates, its remaining amount is $1-\alpha = 0.5$ mol and the amount of $\mathrm{NO_2}$ formed is $2\alpha = 1.0$ mol. The total amount is $1+\alpha = 1.5$ mol. Consequently, at total pressure 1 atm, the partial pressures are

$$p_{\mathrm{N_2O_4}} = \frac{0.5}{1.5} = \frac{1}{3}\,\mathrm{atm}, \qquad p_{\mathrm{NO_2}} = \frac{1}{1.5} = \frac{2}{3}\,\mathrm{atm}.$$

Step 2: Evaluate the equilibrium constant Use the stoichiometric power two on the nitrogen dioxide pressure, rather than merely taking a ratio of the pressures. With the pressures expressed relative to 1 atm,

$$K_p = \frac{(2/3)^2}{1/3} = \frac{4}{3} \simeq 1.33.$$

Since this exceeds unity, its logarithm is positive and the standard Gibbs energy change is negative.

Step 3: Calculate the magnitude The absolute temperature is $27+273 = 300\,\mathrm{K}$. Using exactly the logarithm and conversion factor supplied,

$$\begin{aligned}\Delta G^\circ &= -8.31(300)(2.3)(0.1239)\\ &= -710.427\,\mathrm{J\,mol^{-1}}.\end{aligned}$$

The question defines this quantity as $-x$, so the requested number is its positive magnitude, rounded to an integer.

Step 4: Conclusion The standard free energy change is approximately $-710\,\mathrm{J\,mol^{-1}}$; hence $\boxed{x = 710}$.

Quick Tip to Solve: For dissociation into two gas molecules, use $K_p = 4\alpha^2 P/(1-\alpha^2)$, then check the sign of $-RT\ln K_p$.

86. In a cell, the following reactions take place

$$Fe^{2+} \rightarrow Fe^{3+} + e^- \quad E^o_{Fe^{3+}/Fe^{2+}} = 0.77\,V$$
$$2I^- \rightarrow I_2 + 2e^- \quad E^o_{I_2/I^-} = 0.54\,V$$

The standard electrode potential for the spontaneous reaction in the cell is x $\times$ 10^{-2} V 298 K. The value of x is _____. (Nearest Integer) [4 marks]

Solution:

Concept:

- Tabulated $E^\circ_{\text{oxidised/reduced}}$ values are standard reduction potentials, even when an accompanying half-equation is printed in the oxidation direction.
- A spontaneous galvanic reaction has $E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}} > 0$. Electrode potentials are intensive quantities and are not multiplied when a half-reaction is scaled.

Step 1: Choose the reduction and oxidation The reduction potential for $\text{Fe}^{3+}/\text{Fe}^{2+}$ is $0.77\,\text{V}$, greater than the $0.54\,\text{V}$ for $\text{I}_{2/}\text{I}^-$. Thus ferric ions are reduced at the cathode, and iodide ions are oxidised at the anode. The first printed oxidation half-equation must be reversed to obtain the spontaneous combination. The relevant processes are

$$\text{Fe}^{3+} + \text{e}^- \longrightarrow \text{Fe}^{2+}, \qquad 2\,\text{I}^- \longrightarrow \text{I}_2 + 2\,\text{e}^-.$$

Step 2: Balance the transferred electrons Multiply the iron half-reaction by two so that it consumes the two electrons released by iodide. Addition gives

$$2\,\text{Fe}^{3+} + 2\,\text{I}^- \longrightarrow 2\,\text{Fe}^{2+} + \text{I}_2.$$

Both charge and atoms balance: the total charge on each side is $+4$. The coefficient two changes the amount of material reacting, but does not double the iron electrode potential.

Step 3: Find the positive cell potential Subtract the anode reduction potential from the cathode reduction potential:

$$E^\circ_{\text{cell}} = 0.77 - 0.54 = 0.23\,\text{V}.$$

This positive result verifies the selected spontaneous direction. Reversing the complete reaction would give $-0.23\,\text{V}$, which would require an external drive under standard conditions. Finally write $0.23 = 23 \times 10^{-2}$ to match the requested form.

Step 4: Conclusion The spontaneous cell has standard potential $0.23\,\text{V} = 23 \times 10^{-2}\,\text{V}$. Therefore $\boxed{x = 23}$.

Quick Tip to Solve: First identify whether the listed values are reduction potentials. Balance electrons separately; never multiply an electrode potential by a stoichiometric coefficient.

87. For a given chemical reaction

$\gamma_1 A + \gamma_2 B \rightarrow \gamma_3 C + \gamma_4 D$

Concentration of C changes from 10 mmol dm^{-3} to 20 mmol dm^{-3} in 10 seconds. Rate of appearance of D is 1.5 times the rate of disappearance of B which is twice the rate of disappearance A. The rate of appearance of D has been experimentally determined to be 9 mmol dm^{-3} s^{-1}. Therefore, the rate of reaction is _____ mmol dm^{-3} s^{-1}. (Nearest Integer) [4 marks]

Solution:

Concept:

- For $\gamma_1 A + \gamma_2 B \rightarrow \gamma_3 C + \gamma_4 D$, the reaction rate is each signed species rate divided by its stoichiometric coefficient.
- Species disappear or appear in the ratios of their coefficients. Use the conventional smallest whole-number stoichiometric equation before assigning a numerical reaction rate.

Step 1: Calculate the measured species rates The average appearance rate of C over the stated interval is

$$v_C = \frac{20 - 10}{10} = 1\,\text{mmol}\,\text{dm}^{-3}\,\text{s}^{-1}.$$

All appearance and disappearance rates in the following ratios denote positive magnitudes. Given $v_D = 9$ in the same units and $v_D = 1.5 v_B$, obtain

$$v_B = \frac{9}{1.5} = 6\,\text{mmol}\,\text{dm}^{-3}\,\text{s}^{-1}.$$

Step 2: Obtain the stoichiometric ratio The disappearance rate of B is twice that of A, so

$$v_A = \frac{v_B}{2} = 3\,\text{mmol}\,\text{dm}^{-3}\,\text{s}^{-1}.$$

Combining all four rates gives

$$v_A : v_B : v_C : v_D = 3 : 6 : 1 : 9.$$

Since the C entry is already unity, these are the smallest positive whole-number coefficients. Thus the normalised reaction is $3\,\text{A} + 6\,\text{B} \longrightarrow \text{C} + 9\,\text{D}$. The coefficient of D is nine, not its given rate regarded as a coefficient without comparison.

Step 3: Divide by the appropriate coefficient The stoichiometrically normalised rate is

$$r = \frac{v_A}{3} = \frac{v_B}{6} = \frac{v_C}{1} = \frac{v_D}{9}$$
$$= 1\,\text{mmol}\,\text{dm}^{-3}\,\text{s}^{-1}.$$

Checking every species yields the same rate, as required by a single reaction. In particular, 9 is the appearance rate of D, whereas the reaction rate includes division by its coefficient.

Step 4: Conclusion The reaction rate is $\boxed{1\,\text{mmol}\,\text{dm}^{-3}\,\text{s}^{-1}}$. The nearest-integer entry is $\boxed{1}$.

Quick Tip to Solve: Write the four positive species rates in a single ratio, reduce it to whole-number coefficients, and only then divide to obtain the reaction rate.

88. If $[Cu(H_2O)_4]^{2+}$ absorbs a light of wavelength 600 nm for d-d transition, then the value of octahedral crystal field splitting energy for $[Cu(H_2O)_6]^{2+}$ will be _____ $\times 10^{-21}$ J. [Nearest Integer]

(Given : h = 6.63 $\times 10^{-34}$ Js and c = 3.08 $\times 10^{8}$ ms^{-1}) [4 marks]

Solution:

Concept:

- The energy of a photon is $E = hc/\lambda$, with wavelength expressed in metres. A d–d transition matching the crystal-field gap absorbs this energy per complex.
- For the same metal, oxidation state and ligand, tetrahedral and octahedral splittings are related by $\Delta_t = \frac{4}{9}\Delta_o$. Therefore $\Delta_o = \frac{9}{4}\Delta_t$, not $\frac{4}{9}\Delta_t$.

Step 1: Identify the two splitting energies The four-coordinate aqua complex provides the tetrahedral splitting in the crystal-field relation used here. Its absorption wavelength determines Δ_t. The requested six-coordinate aqua complex is octahedral, so its gap is larger by the factor $9/4$. Both contain copper in oxidation state $+2$ and the same water ligand; the comparison changes the coordination geometry rather than the metal oxidation state.

Step 2: Compute the photon energy Convert the given wavelength:

$$\lambda = 600\,\text{nm} = 600 \times 10^{-9}\,\text{m}.$$

Substitute the supplied values of both constants, including $c = 3.08 \times 10^{8}\,\text{m}\,\text{s}^{-1}$:

$$\begin{aligned}\Delta_t &= \frac{(6.63 \times 10^{-34})(3.08 \times 10^{8})}{600 \times 10^{-9}} \\ &= 3.4034 \times 10^{-19}\,\text{J}.\end{aligned}$$

This is an energy for one complex, not a molar energy, so no Avogadro factor is required.

Step 3: Convert to the octahedral gap Apply the geometry factor before rounding:

$$\begin{aligned}\Delta_o &= \frac{9}{4}(3.4034 \times 10^{-19}) \\ &= 7.65765 \times 10^{-19}\,\text{J} \\ &= 765.765 \times 10^{-21}\,\text{J}.\end{aligned}$$

The coefficient of 10^{-21} J is the number to round. Rounding the initial photon energy too early, or using a different speed of light, would change the stated result.

Step 4: Conclusion The octahedral splitting is approximately $\boxed{766 \times 10^{-21}\,\text{J}}$, so the nearest-integer entry is $\boxed{766}$.

Quick Tip to Solve: Convert wavelength to metres, calculate the photon energy, then multiply by $9/4$ when passing from tetrahedral to octahedral splitting.

89. Number of grams of bromine that will completely react with 5.0 g of pent-1-ene is _____ $\times 10^{-2}$ g. (Atomic mass of Br = 80 g/mol) [Nearest Integer] [4 marks]

Solution:

Concept:

- A carbon–carbon double bond consumes one molecule of bromine in an addition reaction. Pent-1-ene has one double bond, so its mole ratio with Br_2 is 1:1.
- Mass and amount are related by $n = m/M$. Use the molecular molar mass of bromine, $M(Br_2) = 2(80) = 160\,g\,mol^{-1}$, rather than the atomic mass alone.

Step 1: Write the balanced addition reaction Pent-1-ene is $CH_2{=}CHCH_2CH_2CH_3$, molecular formula C_5H_{10}. Bromine adds across its single carbon–carbon double bond:

$$CH_2{=}CHCH_2CH_2CH_3 + Br_2 \longrightarrow CH_2BrCHBrCH_2CH_2CH_3.$$

The product is 1,2-dibromopentane. Both bromine atoms come from one molecule of Br_2; they do not represent two separately consumed molecules.

Step 2: Find the required bromine amount Using the usual atomic masses of carbon and hydrogen,

$$M(C_5H_{10}) = 5(12) + 10(1) = 70\,g\,mol^{-1}.$$

Hence the amount of alkene in the sample is

$$n(C_5H_{10}) = \frac{5.0}{70} = \frac{1}{14}\,mol.$$

The balanced reaction requires the same amount, $1/14$ mol, of bromine. No additional bromine is consumed once the single double bond has been saturated.

Step 3: Convert the amount to the requested mass form The mass of bromine is

$$m(Br_2) = \frac{1}{14}(160) = \frac{80}{7} = 11.428571\ldots\ g.$$

If this is written as $x \times 10^{-2}$ g, then

$$x = \frac{11.428571\ldots}{10^{-2}} = 1142.8571\ldots.$$

Round this coefficient to the nearest integer at the end. The number 11 is not the requested entry because the question has already supplied the factor 10^{-2}.

Step 4: Conclusion The required mass is approximately $11.43\,g = 1143 \times 10^{-2}$ g. Thus the entry is $\boxed{1143}$.

Quick Tip to Solve: Count double bonds to fix the bromine mole ratio, then use 160 g per mole of Br_2; rescale the final mass before rounding.

90. The distance between Na^+ and Cl^- ions in solid NaCl of density 43.1 g cm^{-3} is _____ $\times$ 10^{-10} m. (Nearest Integer)

(Given : N_A = 6.02 $\times$ 10^{23} mol^{-1}) [4 marks]

Solution:

Concept:

- Sodium chloride has the rock-salt structure with $Z = 4$ formula units in a cubic unit cell. Its density is $\rho = ZM/(N_A a^3)$, where a is the unit-cell edge.
- The nearest sodium and chloride ions are separated by half a cube edge: $d = a/2$. If density is in $\mathrm{g\,cm^{-3}}$ and molar mass in $\mathrm{g\,mol^{-1}}$, the calculated edge is in centimetres.

Step 1: Calculate the mass and volume of one cell Take $M(\mathrm{NaCl}) = 23 + 35.5 = 58.5\,\mathrm{g\,mol^{-1}}$. Four formula units per cell give

$$m_{\text{cell}} = \frac{4(58.5)}{6.02 \times 10^{23}}\,\mathrm{g}.$$

Use the density as printed, $43.1\,\mathrm{g\,cm^{-3}}$. The volume is mass divided by density, so

$$a^3 = \frac{4(58.5)}{(43.1)(6.02 \times 10^{23})}\,\mathrm{cm^3}.$$

Step 2: Extract the cube edge Numerically,

$$a^3 \simeq 9.0185 \times 10^{-24}\,\mathrm{cm^3}, \qquad a \simeq 2.0815 \times 10^{-8}\,\mathrm{cm}.$$

The cube root applies to the numerical factor and to the power of ten. This edge is determined by the density of a complete cell, so it must not yet be interpreted as the nearest unlike-ion separation. The count four already includes the sharing of corner and face-centre positions.

Step 3: Find the unlike-ion distance and rescale Along a cube edge a corner ion and the adjacent edge-centre ion are separated by $a/2$. Thus

$$\begin{aligned} d &= 1.0408 \times 10^{-8}\,\mathrm{cm} \\ &= 1.0408 \times 10^{-10}\,\mathrm{m}. \end{aligned}$$

We used $1\,\mathrm{cm} = 10^{-2}\,\mathrm{m}$. The requested coefficient is therefore approximately 1.0408, whose nearest integer is 1. Using a face diagonal distance instead would connect the wrong neighbours in this lattice.

Step 4: Conclusion The nearest $\mathrm{Na^+}$–$\mathrm{Cl^-}$ distance is approximately $1.04 \times 10^{-10}\,\mathrm{m}$. The requested nearest-integer entry is $\boxed{1}$.

Quick Tip to Solve: For rock salt, first obtain the edge using four formula units per cell, then halve it and convert centimetres to metres.