JEE Main 2022 Paper 1 (B.E./B.Tech) Jun 25 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 1 (June), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 25 June 2022, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $A = \{x \in R : |x + 1| < 2\}$ and $B = \{x \in R : |x - 1| \geq 2\}$. Then which one of the following statements is NOT true? [4 marks]

(A) $A - B = (-1, 1)$

(B) $B - A = R - (-3, 1)$

(C) $A \cap B = (-3, -1]$

(D) $A \cup B = R - [1, 3)$

Correct Answer: (B) $B - A = R - (-3, 1)$

Solution:

Concept:

- An inequality involving an absolute value gives an interval or the exterior of an interval. Strict inequalities exclude endpoints.
- The difference $A - B$ contains elements of A which do not belong to B; intersection requires membership of both sets.

Step 1: Resolve the absolute values From $-2 < x + 1 < 2$, obtain $A = (-3, 1)$. The inequality $|x - 1| \geq 2$ gives $x \leq -1$ or $x \geq 3$, so $B = (-\infty, -1] \cup [3, \infty)$. Notice that -1 is included in B, while neither endpoint of A is included.

Step 2: Check the intersection and difference The common part is $A \cap B = (-3, -1]$, confirming (C). Removing this part from A gives $A - B = (-1, 1)$, confirming (A). In the other direction, removing A from B leaves

$$B - A = (-\infty, -3] \cup [3, \infty).$$

Option (B) instead includes every number at least 1; for instance, 2 is in its proposed set but is not in B.

Step 3: Check the union The union covers every real number below 1 and every real number at least 3. Thus $A \cup B = \mathbb{R} - [1, 3)$, so (D) is true. The question asks for the statement which is NOT true, making the incorrect set difference the required selection.

Step 4: Conclusion The statement $B - A = \mathbb{R} - (-3, 1)$ is false. Hence, the correct option is (B).

Quick Tip to Solve: Check a proposed set difference with a point in its disputed interval: here 2 immediately disproves option (B).

2. Let a, b $\in$ R be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to : [4 marks]

(A) 37

(B) 58

(C) 68

(D) 92

Correct Answer: (B) 58

Solution:

Concept:

- A quadratic with a repeated root has zero discriminant, and its root is the negative linear coefficient divided by twice the leading coefficient.
- For roots α, β of a monic quadratic, use their sum and product to compute $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$.

Step 1: Use the repeated-root condition The first equation is a genuine quadratic, so $a \neq 0$. Its discriminant is $4b^2 - 60a = 0$, giving $b^2 = 15a$. Its repeated root is $\alpha = b/a$. Consequently

$$b\alpha = \frac{b^2}{a} = 15.$$

This relation is useful because the second equation also contains the product $b\alpha$.

Step 2: Substitute the shared root Since α is also a root of the second equation,

$$\alpha^2 - 2b\alpha + 21 = 0.$$

Substituting $b\alpha = 15$ gives $\alpha^2 = 30 - 21 = 9$. Also $b = 15/\alpha$, hence $b^2 = 225/9 = 25$. Both possible signs of α lead to this same value; there is no need to choose a sign.

Step 3: Evaluate the required symmetric expression For the second equation, $\alpha+\beta = 2b$ and $\alpha\beta = 21$. Therefore

$$\alpha^2 + \beta^2 = 4b^2 - 42 = 100 - 42 = 58.$$

The other proposed values 37, 68 and 92 do not satisfy this identity with the forced value $b^2 = 25$.

Step 4: Conclusion $\alpha^2 + \beta^2 = 58$. Hence, the correct option is (B).

Quick Tip to Solve: When a root is shared by two quadratics, substitute it into the second equation after simplifying products with the first equation.

3. Let z_1 and z_2 be two complex numbers such that $z_1 = iz_2$ and $\arg\left(\frac{z_1}{z_2}\right) = \pi$. Then: [4 marks]

(A) $\arg z_2 = \frac{\pi}{4}$

(B) $\arg z_2 = -\frac{3\pi}{4}$

(C) $\arg z_1 = \frac{\pi}{4}$

(D) $\arg z_1 = -\frac{3\pi}{4}$

Correct Answer: The two printed hypotheses contradict each other; no pair satisfies them and no listed option is determined.

Solution:

Concept:

- The principal argument of the nonzero complex number i is $\pi/2$, since it lies on the positive imaginary axis.
- The argument of a quotient is defined only when its denominator and numerator are nonzero. Both printed conditions must hold for the same pair.

Step 1: Use the first printed relation The argument condition requires $z_2 \neq 0$; otherwise the quotient is undefined. Dividing the first relation $z_1 = iz_2$ by z_2 therefore gives exactly

$$\frac{z_1}{z_2} = i.$$

This identity does not depend on the modulus or argument chosen for z_2, so changing that number cannot change the quotient.

Step 2: Compare the resulting argument The number $i = 0 + 1i$ has principal argument $\pi/2$. Its general arguments are $\pi/2 + 2n\pi$, where n is an integer. None equals π, because that would require $n = 1/4$. Thus

$$\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2} \neq \pi.$$

The second printed condition instead places the quotient on the negative real axis, contradicting the first relation.

Step 3: Assess the answer choices Options (A) and (B) specify particular arguments of z_2, and (C) and (D) specify particular arguments of z_1. For every nonzero candidate satisfying the first relation, the quotient remains i, so none satisfies the second relation. Choosing zero cannot resolve the contradiction because its argument is undefined. Consequently there is no admissible pair from which any listed assertion could be selected. This conclusion follows from the printed relations without introducing conjugates or changing the angle.

Step 4: Conclusion The printed conditions are inconsistent: they force the quotient to be i, whose argument is $\pi/2$, while requiring argument π. No listed option is determined by an admissible pair.

Quick Tip to Solve: Before manipulating argument differences, simplify a quotient using any direct relation between the two complex numbers; this can reveal inconsistent premises immediately.

4. The system of equations

$$-kx + 3y - 14z = 25$$

$$-15x + 4y - kz = 3$$

$$-4x + y + 3z = 4$$

is consistent for all k in the set [4 marks]

(A) R

(B) R $-$ {-11, 13}

(C) R $-$ {13}

(D) R $-$ {-11, 11}

Correct Answer: (D) R $-$ {-11, 11}

Solution:

Concept:

- A linear system is uniquely solvable whenever the determinant of its coefficient matrix is nonzero.
- At parameter values making the determinant zero, inspect the reduced equations: a statement $0 = c$ with $c \neq 0$ proves inconsistency.

Step 1: Eliminate the variable y The third equation gives $y = 4 + 4x - 3z$. Substituting into the first two equations gives

$$(12 - k)x - 23z = 13, \qquad x - (k + 12)z = -13.$$

These operations are reversible and introduce no division by a parameter expression, so they remain valid even at exceptional values of k.

Step 2: Find the exceptional parameter values From the second reduced equation, $x = -13+(k+12)z$. Substitute into the first to obtain

$$\big((12 - k)(k + 12) - 23\big)z = 13 + 13(12 - k),$$

and hence

$$(121 - k^2)z = 13(13 - k).$$

For $k \neq -11, 11$, this determines exactly one z; then the preceding equations determine x and y. The original system is therefore consistent for all such values.

Step 3: Test the singular cases At $k = 11$, the last equation reads $0 = 26$, which is impossible. At $k = -11$, it reads $0 = 312$, also impossible. Both exceptional values must be excluded. In particular, $k = 13$ is allowed because the coefficient on the left is nonzero. Thus (A) wrongly keeps the singular values, while (B) and (C) wrongly exclude 13 and fail to exclude 11.

Step 4: Conclusion The consistency set is $\mathbb{R} - \{-11, 11\}$. Hence, the correct option is (D).

Quick Tip to Solve: Never exclude all determinant-zero values without checking them: substitute into a parameter-free elimination equation to decide consistency.

5. $\lim\limits_{x\to\frac{\pi}{2}}\left(\tan^2 x\left((2\sin^2 x+3\sin x+4)^{\frac{1}{2}}-(\sin^2 x+6\sin x+2)^{\frac{1}{2}}\right)\right)$ **is equal to** [4 marks]

(A) $\frac{1}{12}$

(B) $-\frac{1}{18}$

(C) $-\frac{1}{12}$

(D) $\frac{1}{6}$

Correct Answer: (A) $\frac{1}{12}$

Solution:

Concept:

- Rationalising a difference of square roots exposes a polynomial factor that can cancel a vanishing trigonometric denominator.
- With $s=\sin x$, the identity $\tan^2 x = s^2/(1-s^2)$ converts the limit into an algebraic limit as $s\to 1$.

Step 1: Rationalise the radical difference Put $s=\sin x$. Both radicands tend to 9 as $x\to\pi/2$, so the sum of the square roots tends to 6 and is positive nearby. Rationalisation gives

$$\sqrt{2s^2+3s+4}-\sqrt{s^2+6s+2}=\frac{s^2-3s+2}{\sqrt{2s^2+3s+4}+\sqrt{s^2+6s+2}}.$$

The numerator factors as $(s-1)(s-2)$.

Step 2: Cancel the vanishing factor For nearby points where the original expression is defined,

$$\frac{s^2(s-1)(s-2)}{(1-s^2)(\sqrt{2s^2+3s+4}+\sqrt{s^2+6s+2})}=\frac{-s^2(s-2)}{(1+s)(\sqrt{2s^2+3s+4}+\sqrt{s^2+6s+2})}.$$

Here $1-s^2=-(s-1)(1+s)$; keeping this minus sign avoids reversing the answer.

Step 3: Evaluate the continuous expression The simplified denominator tends to $2\times 6=12$, while the numerator tends to $-1(1-2)=1$. The limit is therefore $1/12$. It is positive, ruling out (B) and (C). Option (D) would miss the factor $1+s\to 2$ from the trigonometric denominator.

Step 4: Conclusion The limit equals $1/12$. Hence, the correct option is (A).

Quick Tip to Solve: In radical limits near $\sin x=1$, rationalise first and factor $1-\sin^2 x$; this often cancels the apparent singularity.

6. The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is [4 marks]

(A) $\frac{1}{3}$

(B) $\frac{1}{6}$

(C) $\frac{2}{3}$

(D) $\frac{3}{4}$

Correct Answer: (A) $\frac{1}{3}$

Solution:

Concept:

- For curves expressed as x in terms of y, area is the integral of right boundary minus left boundary with respect to y.
- The intersections determine the integration limits. Symmetry about the x-axis allows twice the integral over the positive half of the region.

Step 1: Locate the intersection points Equating the two expressions for y^2 gives $2x - 1 = 4x - 3$, so $x = 1$. Then $y^2 = 1$, giving intersections $(1, 1)$ and $(1, -1)$. Thus the bounded region occupies $-1 \le y \le 1$. Expressing the boundaries as horizontal coordinates avoids splitting the area into upper and lower branches.

Step 2: Identify the horizontal width The first parabola gives $x_1 = (y^2 + 1)/2$, and the second gives $x_2 = (y^2 + 3)/4$. Their difference is

$$x_2 - x_1 = \frac{1 - y^2}{4}.$$

This is nonnegative throughout $[-1, 1]$, so x_2 is the right boundary and x_1 the left boundary. At $y = 0$, for example, their coordinates are $3/4$ and $1/2$, confirming the orientation.

Step 3: Integrate the width The required area is

$$\mathcal{A} = \int_{-1}^{1} \frac{1 - y^2}{4}\, dy = \frac{1}{2}\int_{0}^{1} (1 - y^2)\, dy = \frac{1}{2}\left[y - \frac{y^3}{3}\right]_0^1 = \frac{1}{3}.$$

Option (B) counts only one half of the region. Option (C) doubles the full area, and (D) does not equal the definite integral. The units are square units because the quantity is an area.

Step 4: Conclusion The enclosed area is $1/3$ square units. Hence, the correct option is (A).

Quick Tip to Solve: When both parabolas open sideways, integrate a horizontal strip with respect to y rather than handling separate square-root branches.

7. The coefficient of x^{101} in the expression $(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+\ .....\ +x^{500}$, $x>0$, is [4 marks]

(A) $^{501}C_{101}\ (5)^{399}$

(B) $^{501}C_{101}\ (5)^{400}$

(C) $^{501}C_{100}\ (5)^{400}$

(D) $^{500}C_{101}\ (5)^{399}$

Correct Answer: (A) $^{501}C_{101}\ (5)^{399}$

Solution:

Concept:

- A finite sum $a^n+a^{n-1}b+\cdots+b^n$ equals $(a^{n+1}-b^{n+1})/(a-b)$ when $a\neq b$.
- The coefficient of x^r in $(5+x)^n$ is $\binom{n}{r}5^{n-r}$, by the binomial theorem.

Step 1: Recognise the finite geometric identity Let $S=(5+x)^{500}+x(5+x)^{499}+\cdots+x^{500}$. Multiplication by $5+x$ and by x gives two sums whose interior terms cancel on subtraction. Thus

$$((5+x)-x)S=(5+x)^{501}-x^{501}.$$

Since the difference of the two bases is exactly 5,

$$S=\frac{(5+x)^{501}-x^{501}}{5}.$$

This is a polynomial identity, and the stated restriction $x>0$ does not affect coefficient extraction.

Step 2: Extract the required coefficient The subtracted term x^{501} has no contribution to the coefficient of x^{101}. The binomial theorem gives

$$[x^{101}](5+x)^{501}=\binom{501}{101}5^{501-101}=\binom{501}{101}5^{400}.$$

Dividing by 5 from the geometric identity therefore leaves $\binom{501}{101}5^{399}$.

Step 3: Compare the proposed expressions Option (B) forgets the division by 5. Option (C) selects the wrong binomial coefficient and also has the wrong power of 5. Option (D) uses exponent 500 rather than 501, as if only the first summand contributed. The other summands do contribute, and the finite geometric identity includes all of them exactly.

Step 4: Conclusion The coefficient is $\binom{501}{101}5^{399}$. Hence, the correct option is (A).

Quick Tip to Solve: For a sum with descending powers of one expression and ascending powers of another, apply the finite geometric identity before expanding any term.

8. Water is being filled at the rate of 1 cm^3 / sec in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm^2 / sec) at which the wet conical surface area of the vessel increases is [4 marks]

(A) 5

(B) $\frac{\sqrt{21}}{5}$

(C) $\frac{\sqrt{26}}{5}$

(D) $\frac{\sqrt{26}}{10}$

Correct Answer: (C) $\frac{\sqrt{26}}{5}$

Solution:

Concept:

- Similarity fixes the ratio of the radius to the water depth: $r/h = 7/35 = 1/5$. Both radius and slant height vary as water enters.
- The wet surface is the curved surface in contact with water, $S = \pi r\ell$, whereas the volume is $V = \pi r^2 h/3$. The circular top is a free water surface and is not part of the wet vessel.

Step 1: Express the geometry using water depth Let h be the water depth and r its surface radius, in centimetres. Similar right triangles give $r = h/5$. By Pythagoras the slant length from the vertex to the water surface is

$$\ell = \sqrt{h^2 + r^2} = \frac{\sqrt{26}}{5}h.$$

Therefore the two varying quantities can be written using only h:

$$S = \frac{\pi\sqrt{26}}{25}h^2, \qquad V = \frac{\pi}{75}h^3.$$

Step 2: Differentiate and use the inflow rate Differentiating with respect to time gives

$$\frac{dS}{dt} = \frac{2\pi\sqrt{26}}{25}h\frac{dh}{dt}, \qquad \frac{dV}{dt} = \frac{\pi}{25}h^2\frac{dh}{dt}.$$

Divide these expressions to eliminate the changing depth rate:

$$\frac{dS}{dt} = \frac{2\sqrt{26}}{h}\frac{dV}{dt}.$$

At $h = 10$ and $dV/dt = 1$, this is $\sqrt{26}/5$. The rate is positive because the wetted surface grows. Option D loses a factor of two; B has the wrong slant-height factor, while A does not follow the cone geometry.

Step 3: Conclusion The wet curved surface increases at $\frac{\sqrt{26}}{5}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Express both area and volume in the same depth variable, then divide their derivatives to eliminate the unknown rise rate.

9. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx$, $n \in N$, then [4 marks]

(A) $b_3 - b_2$, $b_4 - b_3$, $b_5 - b_4$ are in A.P. with common difference -2

(B) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference 2

(C) $b_3 - b_2$, $b_4 - b_3$, $b_5 - b_4$ are in a G.P.

(D) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference -2

Correct Answer: The integrals diverge at zero; the listed differences are undefined. A common-cutoff interpretation yields (D).

Solution:

Concept:

- A definite integral whose denominator vanishes at an endpoint must first be tested for convergence. Here $\cos^2(nx) \to 1$ and $\sin x \sim x$ as $x \to 0^+$.
- Subtracting two divergent improper integrals is undefined. Subtracting their integrands before integration defines a different, convergent expression and must be identified explicitly.

Step 1: Test the printed definition For every positive integer n, continuity gives $\cos^2(nx) > 1/2$ for sufficiently small positive x. Also $\sin x \le x$. Hence

$$\frac{\cos^2(nx)}{\sin x} \ge \frac{1}{2x}.$$

The integral of $1/(2x)$ from zero diverges, so every printed b_n is $+\infty$, rather than a real number. Thus none of the displayed differences $b_{n+1} - b_n$ is defined under the stated definition.

Step 2: Explain the formal finite-difference result If a common lower cutoff $\varepsilon > 0$ is imposed before taking a difference, the cancellation instead gives

$$\cos^2((n+1)x) - \cos^2(nx) = -\sin((2n+1)x)\sin x.$$

Its integral tends to

$$-\int_0^{\pi/2} \sin((2n+1)x)\, dx = -\frac{1}{2n+1}.$$

The three such regularized differences are $-1/5, -1/7, -1/9$, whose reciprocals are $-5, -7, -9$. They form an A.P. with difference -2, suggesting D under that additional convention. Their original values form neither the A.P. in A nor the G.P. in C, and B reverses the reciprocal difference. However, the printed definition does not specify this regularization.

Step 3: Conclusion As printed, every b_n diverges, so no option is mathematically defined. Option (D) is the formal result only if differences are interpreted using a common endpoint cutoff before the limit; it is not a difference of the stated real-valued integrals.

Quick Tip to Solve: Check endpoint convergence before subtracting integrals; cancellation is legitimate only before the improper limit, with that convention explicitly stated.

10. If $y = y(x)$ is the solution of the differential equation $2x^2\frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then y(1) is equal to : [4 marks]

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 3

Correct Answer: (B) $\frac{2}{3}$

Solution:

Concept:

- The terms x^2y', xy and y^2 suggest substituting $y = vx$, where v is a function of x. Then $y' = v + xv'$.
- The initial value is given at $x = e > 0$, so use the positive interval and $\log x$. The nonzero initial value permits division by v^2 on the solution interval.

Step 1: Reduce to a separable equation Substitute $y = vx$ into the differential equation without dropping the derivative of v:

$$2x^2(v + xv') - 2x(vx) + 3v^2x^2 = 0.$$

The two linear terms cancel. Dividing by x^2 gives

$$2x\frac{dv}{dx} + 3v^2 = 0, \qquad \frac{dv}{v^2} = -\frac{3}{2}\frac{dx}{x}.$$

Integration therefore yields

$$-\frac{1}{v} = -\frac{3}{2}\log x + C, \qquad \frac{1}{v} = \frac{3}{2}\log x + C_1.$$

Step 2: Apply the initial condition and evaluate At $x = e$, $v = y/x = 1/3$. Therefore

$$3 = \frac{3}{2}\log e + C_1, \qquad C_1 = \frac{3}{2}.$$

Thus

$$v = \frac{2}{3(1 + \log x)}, \qquad y = \frac{2x}{3(1 + \log x)}.$$

The interval $x > e^{-1}$ contains both the initial point and $x = 1$, so this expression is valid at the requested point. Substituting gives $y(1) = 2/3$. Option A confuses the initial ratio with the final value, C is its reciprocal, and D is incompatible with the integrated initial condition.

Step 3: Conclusion $y(1) = \frac{2}{3}$. Hence, the correct option is (B).

Quick Tip to Solve: For homogeneous equations substitute y=vx and keep both terms in its derivative; apply the initial condition to v=y/x.

11. The value of 2sin (12°) − sin (72°) is : [4 marks]

(A) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$

(B) $\frac{1-\sqrt{5}}{8}$

(C) $\frac{\sqrt{3}(1-\sqrt{5})}{2}$

(D) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

Correct Answer: (D) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

Solution:

Concept:

- The angle $12^\circ = 72^\circ - 60^\circ$ is useful because doubling the sine-difference formula cancels the given $\sin 72^\circ$.
- The exact pentagon angle value is $\cos 72^\circ = (\sqrt{5}-1)/4$. Using an exact identity avoids choosing between nearby negative radicals numerically.

Step 1: Arrange the cancellation Apply $\sin(A-B) = \sin A\cos B - \cos A\sin B$ with $A = 72^\circ$ and $B = 60^\circ$:

$$\sin 12^\circ = \frac{1}{2}\sin 72^\circ - \frac{\sqrt{3}}{2}\cos 72^\circ.$$

Multiply the entire identity by two, then subtract the sine appearing in the question:

$$2\sin 12^\circ - \sin 72^\circ = -\sqrt{3}\cos 72^\circ.$$

This already determines that the answer is negative, since the remaining cosine is positive.

Step 2: Evaluate the exact cosine and compare For completeness let $c = \cos 36^\circ$. The identities $\cos 108^\circ = -\cos 72^\circ$ and $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ give

$$(c+1)(4c^2 - 2c - 1) = 0.$$

Since $c > 0$, $c = (1+\sqrt{5})/4$, and the double-angle formula yields $\cos 72^\circ = 2c^2 - 1 = (\sqrt{5}-1)/4$. Therefore

$$-\sqrt{3}\cos 72^\circ = \frac{\sqrt{3}(1-\sqrt{5})}{4}.$$

Option C doubles this exact result. Neither A nor B has the coefficient obtained from the sine-difference cancellation, so both fail the exact identity.

Step 3: Conclusion The value is $\frac{\sqrt{3}(1-\sqrt{5})}{4}$. Hence, the correct option is (D).

Quick Tip to Solve: When two trigonometric terms differ by 60 degrees, expand the smaller angle as a difference and look for cancellation.

12. A biased die is marked with numbers 2, 4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is : [4 marks]

(A) $\frac{7}{2^{11}}$

(B) $\frac{7}{2^{12}}$

(C) $\frac{3}{2^{10}}$

(D) $\frac{13}{2^{12}}$

Correct Answer: (D) $\frac{13}{2^{12}}$

Solution:

Concept:

- The two faces labelled 32 are distinct faces. Each has probability $1/32$, so the probability of the numerical outcome 32 is $2/32 = 1/16$.
- Independent throws allow probabilities to be multiplied. Disjoint arrangements giving the required sum can then be added; repeated outcomes require the appropriate number of distinct permutations.

Step 1: List every possible unordered triple The available numbers are $2, 4, 8, 16, 32$. If one throw is 32, the other two must sum to 16; the only permitted pair is $8 + 8$. Two 32s already exceed 48. If no 32 occurs, each throw is at most 16, so equality with 48 forces all three to be 16. Thus the only numerical triples are

$$(32, 8, 8) \quad \text{and} \quad (16, 16, 16).$$

The first has three distinct orders because the position of 32 can be chosen in three ways. The second has just one order.

Step 2: Calculate and add the probabilities The face probabilities sum correctly:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{32} = 1.$$

Using the combined probability for the repeated label gives

$$P(32, 8, 8 \text{ in any order}) = 3 \cdot \frac{1}{16}\left(\frac{1}{8}\right)^2 = \frac{3}{1024},$$

$$P(16, 16, 16) = \left(\frac{1}{16}\right)^3 = \frac{1}{4096}.$$

These are disjoint, hence the total is $12/4096 + 1/4096 = 13/4096$. Option C omits the all-16 outcome. Option B counts only one of the two 32 faces, while A does not equal the sum of the exhaustive cases.

Step 3: Conclusion The required probability is $\frac{13}{4096} = \frac{13}{2^{12}}$. Hence, the correct option is (D).

Quick Tip to Solve: Combine probabilities of identically labelled faces before listing numerical triples, then count only distinct orders of each triple.

13. The value of $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to : [4 marks]

(A) $-\frac{\pi}{4}$

(B) $-\frac{\pi}{8}$

(C) $-\frac{5\pi}{12}$

(D) $-\frac{4\pi}{9}$

Correct Answer: (B) $-\frac{\pi}{8}$

Solution:

Concept:

- Cosine is periodic with period 2π, so the large angle can first be reduced modulo 2π without changing the quotient.
- The principal range of $\tan^{-1}$ is $(-\pi/2, \pi/2)$. Recognizing a tangent value is sufficient only after verifying that the resulting angle belongs to this interval.

Step 1: Simplify the argument Reduce $15\pi/4$ by 2π:

$$\cos\frac{15\pi}{4} = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}, \qquad \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}.$$

Consequently the exact argument of the inverse tangent is

$$\frac{\cos(15\pi/4)-1}{\sin(\pi/4)} = \frac{\sqrt{2}/2-1}{\sqrt{2}/2} = 1-\sqrt{2}.$$

It is negative and has magnitude less than one. Thus its principal angle must lie between $-\pi/4$ and zero, which already excludes A, C and D.

Step 2: Recognize the half-angle tangent The half-angle identity gives a direct exact comparison:

$$\tan\frac{\pi}{8} = \frac{\sin(\pi/4)}{1+\cos(\pi/4)} = \frac{\sqrt{2}}{2+\sqrt{2}} = \sqrt{2}-1.$$

Since tangent is odd, $\tan(-\pi/8) = 1-\sqrt{2}$. The angle $-\pi/8$ is inside the principal range, so no multiple of π is added. In particular, option A has tangent -1, rather than the computed value; C and D have angles below $-\pi/4$, where the magnitude of the negative tangent exceeds one. The exact half-angle calculation singles out B.

Step 3: Conclusion The principal value is $-\frac{\pi}{8}$. Hence, the correct option is (B).

Quick Tip to Solve: Reduce the ordinary trigonometric angle first; only then use the inverse function and enforce its principal range.

14. The line y = x + 1 meets the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to : [4 marks]

(A) 20

(B) 12

(C) 11

(D) 8

Correct Answer: (A) 20

Solution:

Concept:

- Substituting the equation of the line into the ellipse gives a quadratic whose roots are the x-coordinates of the intersection points.
- If both points lie on $y = x + 1$, their y-coordinate difference equals their x-coordinate difference. Thus their squared distance is twice the squared difference of the quadratic roots.

Step 1: Find the quadratic for the intersections Insert $y = x + 1$ into the ellipse and multiply by four:

$$x^2 + 2(x+1)^2 = 4, \qquad 3x^2 + 4x - 2 = 0.$$

Let the two distinct roots be x_1, x_2, corresponding to P and Q. Vieta formulas give

$$x_1 + x_2 = -\frac{4}{3}, \qquad x_1 x_2 = -\frac{2}{3}.$$

It follows without calculating either root individually that

$$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2 = \frac{16}{9} + \frac{8}{3} = \frac{40}{9}.$$

Step 2: Convert the chord to the radius The two intersection points are $(x_1, x_1 + 1)$ and $(x_2, x_2 + 1)$. The distance formula therefore gives

$$PQ^2 = (x_1 - x_2)^2 + ((x_1 + 1) - (x_2 + 1))^2 = \frac{80}{9}.$$

The question makes PQ the diameter, so the radius is $r = PQ/2$, not PQ. Hence

$$r^2 = \frac{20}{9}, \qquad (3r)^2 = 9r^2 = 20.$$

All lengths are positive and the positive discriminant confirms two intersections. The other numerical options 12, 11 and 8 do not satisfy the distance relation obtained from the line slope and the ellipse quadratic; no rounding or approximation is involved.

Step 3: Conclusion $(3r)^2 = 20$. Hence, the correct option is (A).

Quick Tip to Solve: For a chord on a known line, use the squared root difference from Vieta formulas and multiply by one plus the slope squared.

15. The sum 1 + 2 . 3 + 3 . 3^2 + ... + 10 . 3^9 is equal to : [4 marks]

(A) $\frac{2 \cdot 3^{12}+10}{4}$

(B) $\frac{19 \cdot 3^{10}+1}{4}$

(C) $5 \cdot 3^{10} - 2$

(D) $\frac{9 \cdot 3^{10}+1}{2}$

Correct Answer: (B) $\frac{19 \cdot 3^{10}+1}{4}$

Solution:

Concept:

- An arithmetic-geometric series has terms formed by multiplying an arithmetic progression by a geometric progression. Here the required sum is $S = \sum_{j=1}^{10} j3^{j-1}$.
- Multiplication by the common ratio aligns consecutive powers. Subtracting the two expressions leaves a geometric series, which can be evaluated using $1 + r + \cdots + r^{n-1} = (r^n - 1)/(r - 1)$.

Step 1: Align the powers and subtract Write the complete pattern before subtracting, so that the last term is not lost:

$$S = 1 + 2 \cdot 3 + 3 \cdot 3^2 + \cdots + 10 \cdot 3^9,$$
$$3S = 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \cdots + 10 \cdot 3^{10}.$$

For each power from 3 to 3^9, the difference of its coefficients is one. Thus

$$S - 3S = 1 + 3 + 3^2 + \cdots + 3^9 - 10 \cdot 3^{10}.$$

The negative term is the final term of $3S$, which has no counterpart in the original sum.

Step 2: Evaluate and compare the expressions The geometric part has ten terms, beginning with 1 and ending with 3^9. Therefore

$$-2S = \frac{3^{10} - 1}{2} - 10 \cdot 3^{10},$$
$$4S = 20 \cdot 3^{10} - 3^{10} + 1,$$
$$S = \frac{19 \cdot 3^{10} + 1}{4}.$$

This is exactly expression (B). For a direct numerical check, $3^{10} = 59049$, giving $S = 280483$. The values of (A), (C) and (D) are respectively 265723, 295243 and 265721; none equals this sum.

Step 3: Conclusion The required sum is $\boxed{\frac{19 \cdot 3^{10} + 1}{4}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a finite arithmetic-geometric sum, write the final term of both S and 3S explicitly before subtracting; it determines the boundary correction.

16. Let p be the plane passing through the intersection of the planes $\vec{r} \cdot \left(\hat{i} + 3\hat{j} - \hat{k}\right) = 5$ and $\vec{r} \cdot \left(2\hat{i} - \hat{j} + \hat{k}\right) = 3$, and the point (2, 1, −2). Let the position vectors of the points X and Y be $\hat{i} - 2\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 2\hat{k}$ respectively. Then the points : [4 marks]

(A) X and X + Y are on the same side of P

(B) Y and Y − X are on the opposite sides of P

(C) X and Y are on the opposite sides of P

(D) X + Y and X − Y are on the same side of P

Correct Answer: (C) X and Y are on the opposite sides of P

Solution:

Concept:

- A plane through the line of intersection of $F_1 = 0$ and $F_2 = 0$ can be written $F_1 + \lambda F_2 = 0$, with its parameter fixed by the additional point.
- For a fixed plane $F(x, y, z) = 0$, two points lie on the same side when their nonzero F-values have the same sign, and on opposite sides when those signs differ.

Step 1: Find the plane through the given point The two plane expressions are $F_1 = x + 3y - z - 5$ and $F_2 = 2x - y + z - 3$. At $(2, 1, -2)$, their values are

$$F_1 = 2 + 3 + 2 - 5 = 2, \qquad F_2 = 4 - 1 - 2 - 3 = -2.$$

Consequently $2 - 2\lambda = 0$, so $\lambda = 1$. Adding the original equations gives the required plane

$$P: \quad 3x + 2y - 8 = 0.$$

The z-coefficient cancels; this does not prevent the plane from containing the given intersection line.

Step 2: Test every proposed pair with one signed expression Use $F(x, y, z) = 3x + 2y - 8$ for every point, including the vector sums and differences. Since $X = (1, -2, 4)$ and $Y = (5, -1, 2)$,

$$\begin{aligned} F(X) &= 3 - 4 - 8 = -9, \\ F(Y) &= 15 - 2 - 8 = 5, \\ F(X + Y) &= F(6, -3, 6) = 18 - 6 - 8 = 4, \\ F(Y - X) &= F(4, 1, -2) = 12 + 2 - 8 = 6, \\ F(X - Y) &= F(-4, -1, 2) = -12 - 2 - 8 = -22. \end{aligned}$$

Thus X and Y have opposite signs, proving (C). In (A), the signs are negative and positive, so they are opposite, not the same. In (B), both signs are positive. In (D), the signs are positive and negative, again opposite. Only (C) survives all these tests.

Step 3: Conclusion The points X and Y are on opposite sides of $P : 3x + 2y - 8 = 0$. Hence, the correct option is (C).

Quick Tip to Solve: Do not add signed plane values to evaluate a vector sum: the constant term would be counted twice. Substitute the summed coordinates into the plane equation.

17. A circle touches both the y-axis and the line x + y = 0. Then the locus of its center is : [4 marks]

(A) $y = \sqrt{2}x$

(B) $x = \sqrt{2}y$

(C) $y^2 - x^2 = 2xy$

(D) $x^2 - y^2 = 2xy$

Correct Answer: (D) $x^2 - y^2 = 2xy$

Solution:

Concept:

- The perpendicular distance from the centre of a circle to any tangent equals its radius. For a centre (h, k), the distance to the y-axis is $|h|$.
- The distance from (h, k) to $ax + by + c = 0$ is $|ah + bk + c|/\sqrt{a^2 + b^2}$. Equating distances to two lines gives their angle bisectors.

Step 1: Equate the two distances to the tangents Let the radius be $r > 0$. Tangency to the y-axis gives $r = |h|$. Tangency to $x + y = 0$ gives

$$r = \frac{|h + k|}{\sqrt{1^2 + 1^2}} = \frac{|h + k|}{\sqrt{2}}.$$

Hence

$$|h| = \frac{|h + k|}{\sqrt{2}}.$$

The absolute values are essential because the circle may lie in either of the regions formed by the two tangent lines; a centre need not have positive coordinates.

Step 2: Eliminate the radius and identify the locus Both sides of the distance equality are nonnegative, so squaring preserves equivalence:

$$\begin{aligned} 2h^2 &= (h + k)^2, \\ 2h^2 &= h^2 + 2hk + k^2, \\ h^2 - k^2 &= 2hk. \end{aligned}$$

Replacing the centre coordinates by the running coordinates gives $x^2 - y^2 = 2xy$. It represents the pair of angle bisectors, $y = (-1 + \sqrt{2})x$ and $y = (-1 - \sqrt{2})x$. The origin is excluded as a centre of a nondegenerate circle because it would give radius zero. Options (A) and (B) give different single slopes and omit one bisector. Option (C) reverses the sign of the quadratic relation; for instance the valid centre $(1, \sqrt{2} - 1)$ does not satisfy it.

Step 3: Conclusion The locus is $\boxed{x^2 - y^2 = 2xy}$, with the origin excluded for positive radius. Hence, the correct option is (D).

Quick Tip to Solve: When a circle touches two lines, eliminate its radius by equating the perpendicular distances from its centre to those lines; retain absolute values until squaring.

18. If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t \cos t)$, $y = 12(1 + \sin t)^2$, $0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to: [4 marks]

(A) $6\left(3 + 2\sqrt{2}\right)$

(B) $3\left(7 + 4\sqrt{3}\right)$

(C) 27

(D) 48

Correct Answer: (C) 27

Solution:

Concept:

- For parametric equations $x = x(t)$, $y = y(t)$, the tangent slope is $dy/dx = (dy/dt)/(dx/dt)$, provided $dx/dt \neq 0$.
- A tangent making an angle θ with the positive x-axis has slope $\tan\theta$. Here $0 < t < \pi/2$, so both sine and cosine are positive.

Step 1: Differentiate both parametric equations For $x = 12(t + \sin t \cos t)$, the product rule gives

$$\frac{dx}{dt} = 12(1 + \cos^2 t - \sin^2 t) = 24\cos^2 t.$$

For $y = 12(1 + \sin t)^2$, the chain rule gives

$$\frac{dy}{dt} = 24(1 + \sin t)\cos t.$$

Since cosine is positive in the given interval, division is valid and

$$\frac{dy}{dx} = \frac{1 + \sin t}{\cos t}.$$

In particular the tangent slope is positive, in agreement with the stated angle.

Step 2: Use the tangent angle to determine the parameter The required slope is $\tan(\pi/3) = \sqrt{3}$. Put $s = \sin t$. Squaring $1 + s = \sqrt{3}\cos t$ gives

$$\begin{aligned} (1 + s)^2 &= 3(1 - s^2), \\ 4s^2 + 2s - 2 &= 0, \\ (2s - 1)(s + 1) &= 0. \end{aligned}$$

The root $s = -1$ is outside $0 < s < 1$, leaving $s = 1/2$, or $t = \pi/6$. This also satisfies the unsquared equation, so no extraneous root is retained. Thus

$$y_0 = 12\left(1 + \frac{1}{2}\right)^2 = 12 \cdot \frac{9}{4} = 27.$$

The other values, $6(3 + 2\sqrt{2})$, $3(7 + 4\sqrt{3})$ and 48, are all different from 27 and therefore fail the parameter condition.

Step 3: Conclusion The required ordinate is $\boxed{y_0 = 27}$. Hence, the correct option is (C).

Quick Tip to Solve: Simplify the parametric slope before inserting the angle. After squaring a trigonometric equation, check both the parameter interval and the original equation.

19. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to :

[4 marks]

(A) $p \Rightarrow q$

(B) $q \Rightarrow p$

(C) $\sim (p \Rightarrow q)$

(D) $\sim (q \Rightarrow p)$

Correct Answer: (C) $\sim (p \Rightarrow q)$

Solution:

Concept:

- An implication $A \Rightarrow B$ is false only when A is true and B is false. Thus $\neg(A \Rightarrow B) \equiv A \wedge \neg B$.
- De Morgan's law gives $\neg(U \vee V) \equiv (\neg U) \wedge (\neg V)$. Also $\neg\neg p \equiv p$, and repeating a proposition in a conjunction does not change it.

Step 1: Negate the complete implication Here the antecedent and consequent are

$$A = (\neg q) \wedge p, \qquad B = (\neg p) \vee q.$$

We must negate the whole implication, rather than negate its two sides separately. Using the implication rule,

$$\neg(A \Rightarrow B) \equiv ((\neg q) \wedge p) \wedge \neg((\neg p) \vee q).$$

The antecedent requires p to be true and q to be false; in that same case the consequent is false, suggesting that this is precisely the condition sought.

Step 2: Apply De Morgan's law and compare options Apply De Morgan's law to the final factor:

$$\neg((\neg p) \vee q) \equiv p \wedge (\neg q).$$

Therefore

$$\begin{aligned} \neg(A \Rightarrow B) &\equiv ((\neg q) \wedge p) \wedge (p \wedge (\neg q)) \\ &\equiv p \wedge (\neg q) \\ &\equiv \neg(p \Rightarrow q). \end{aligned}$$

For a direct distinction among the choices, take p true and q false. The required expression is true, while (A), $p \Rightarrow q$, is false and (D), $\neg(q \Rightarrow p)$, is false. Option (B) is also not equivalent: for p false and q false, $q \Rightarrow p$ is true, but the required conjunction is false. Hence the only matching expression is (C).

Step 3: Conclusion The negation is logically equivalent to $\boxed{\neg(p \Rightarrow q)}$. Hence, the correct option is (C).

Quick Tip to Solve: To negate an implication, keep its antecedent true and negate its consequent. Only then simplify the internal conjunctions and disjunctions.

20. If the line $y = 4 + kx,\ k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is : [4 marks]

(A) $\frac{3}{2}$

(B) $\frac{26}{9}$

(C) $\frac{5}{2}$

(D) $\frac{23}{6}$

Correct Answer: (C) $\frac{5}{2}$

Solution:

Concept:

- At a point of tangency the line and parabola intersect in a repeated root. Thus substituting the line into the parabola gives a quadratic with zero discriminant.
- The vertex of $y = x - x^2 = 1/4 - (x - 1/2)^2$ is $V = (1/2, 1/4)$. The slope through distinct points is their difference in ordinates divided by their difference in abscissae.

Step 1: Find the tangent and its point of contact Equating $4 + kx$ and $x - x^2$ yields

$$x^2 + (k - 1)x + 4 = 0.$$

For tangency,

$$(k - 1)^2 - 16 = 0, \qquad k - 1 = \pm 4.$$

Thus $k = 5$ or $k = -3$. The given condition $k > 0$ selects $k = 5$. The repeated root is

$$x_P = -\frac{k - 1}{2} = -2, \qquad y_P = 4 + 5(-2) = -6.$$

Hence $P = (-2, -6)$. As a check, the derivative of the parabola is $1 - 2x$, which at $x = -2$ is indeed 5.

Step 2: Calculate the slope from the contact point to the vertex Completing the square gives

$$y = \frac{1}{4} - \left(x - \frac{1}{2}\right)^2,$$

so $V = (1/2, 1/4)$. The desired line is the chord joining P to the vertex, not the tangent itself. Its slope is

$$m_{PV} = \frac{\frac{1}{4} - (-6)}{\frac{1}{2} - (-2)} = \frac{25/4}{5/2} = \frac{5}{2}.$$

Both coordinate differences are positive, so the slope must be positive. Option (C) is exactly this ratio. The other positive candidates $3/2$, $26/9$ and $23/6$ do not equal $(25/4)/(5/2)$; none gives the line through both computed points. The discarded negative value of k would give a different point and would violate the stated condition.

Step 3: Conclusion The required slope is $\boxed{\frac{5}{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the positive-slope condition immediately after the discriminant equation; then compute the chord slope separately from the tangent slope.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $A = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$. Then the number of elements in the set {(n, m) : n, m $\in$ {1, 2, ..., 10} and $nA^n + mB^m = I$} is _____. [4 marks]

Solution:

Concept:

- A matrix M is idempotent when $M^2 = M$. Induction then gives $M^r = M$ for every positive integer r, which reduces expressions involving large positive powers.
- Matrix equality means equality of corresponding entries. After simplifying the powers, the off-diagonal entries are often enough to impose a relation between unknown coefficients.

Step 1: Compute the squares and simplify all positive powers Direct multiplication gives

$$A^2 = \begin{pmatrix} 4-2 & -4+2 \\ 2-1 & -2+1 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = A$$

and

$$B^2 = \begin{pmatrix} 1-2 & -2+4 \\ 1-2 & -2+4 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix} = B.$$

If $A^r = A$, then $A^{r+1} = A^r A = A^2 = A$, and the same reasoning applies to B. Since n and m are drawn from 1 through 10, no zero-power exception arises. Thus the stated equation reduces exactly to $nA + mB = I$.

Step 2: Compare entries and count the allowed ordered pairs The reduced matrix equation is

$$\begin{pmatrix} 2n-m & -2n+2m \\ n-m & -n+2m \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The lower-left entry gives $n - m = 0$, so $n = m$. The top-left entry then gives $2n - n = 1$, or $n = 1$. Therefore $m = 1$ as well. The remaining two entries are consistent: $-2 + 2 = 0$ and $-1 + 2 = 1$. Conversely, the pair $(1, 1)$ works because

$$A + B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$$

Both coordinates belong to the allowed set, and the entry equations prove there are no other solutions. Because the elements being counted are ordered pairs, this unique solution contributes one element, not two.

Step 3: Conclusion The set is $\{(1, 1)\}$, so its number of elements is $\boxed{1}$ (dimensionless).

Quick Tip to Solve: Before expanding powers of a small matrix repeatedly, compute its square. Idempotence can reduce every positive power to the original matrix.

22. **Let $f(x) = \left[2x^2 + 1\right]$ and $g(x) = \begin{cases} 2x - 3, & x < 0 \\ 2x + 3, & x \geq 0 \end{cases}$, where [t] is the greatest integer $\leq$ t. Then, in the open interval $(-1, 1)$, the number of points where fog is discontinuous is equal to _____.**

[4 marks]

Solution:

Concept:

- The composition is $f(g(x)) = \lfloor 2g(x)^2 + 1 \rfloor$. A greatest integer function jumps when its continuous argument crosses an integer.
- One must count discontinuities separately on each side of zero and then test zero itself. The endpoints of the given open interval are excluded.

Step 1: Find the argument on each side For negative x, $g(x) = 2x - 3$, so its square increases as x moves from zero towards -1. For nonnegative x, $g(x) = 2x + 3$. Thus the common expression in terms of distance from zero is

$$H(x) = 2(2|x| + 3)^2 + 1.$$

It has value 19 at zero, and tends to 51 at either excluded endpoint. On each of $(-1, 0)$ and $(0, 1)$, it is strictly monotonic, so every integer strictly between 19 and 51 is crossed exactly once.

Step 2: Count the jumps and check zero The crossed integers are $20, 21, \ldots, 50$, giving $50 - 20 + 1 = 31$ points on each side. At a crossing, the floor changes between two consecutive integers, so the composition is discontinuous. Near zero, $H(x) \geq 19$ and is less than 20 on both sides; consequently the composition equals 19 throughout a neighbourhood of zero. Although g itself jumps there, its square removes that jump. Hence zero adds no discontinuity:

$$N = 31 + 31 = 62.$$

Step 3: Conclusion The number of points of discontinuity is $\boxed{62}$. This is a dimensionless count.

Quick Tip to Solve: For a floor of a composition, count integer crossings of the inner expression; test a jump of the intermediate function separately, because squaring can remove it.

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23. **The value of b $>$ 3 for which $12 \int_3^b \frac{1}{(x^2-1)(x^2-4)} dx = \log_e \left(\frac{49}{40}\right)$, is equal to _____.** [4 marks]

Solution:

Concept:

- For $x > 3$, all factors are positive and the integrand is continuous. Partial fractions reduce the integral to logarithms.
- The function $12 \int_3^b [(x^2 - 1)(x^2 - 4)]^{-1} \, dx$ is strictly increasing in $b > 3$, so any solution found is unique.

Step 1: Decompose and integrate the rational function The two quadratic denominators differ by 3,

which gives

$$\frac{12}{(x^2-1)(x^2-4)} = \frac{4}{x^2-4} - \frac{4}{x^2-1}.$$

Using $\int dx/(x^2-a^2) = \frac{1}{2a}\log((x-a)/(x+a))$ on this positive domain, an antiderivative is

$$F(x) = \log\frac{x-2}{x+2} - 2\log\frac{x-1}{x+1} = \log\left[\frac{(x-2)(x+1)^2}{(x+2)(x-1)^2}\right].$$

No absolute-value ambiguity occurs because $x > 3$.

Step 2: Apply the lower limit and determine the upper limit At the lower limit, the argument of the logarithm is $(1/5)(4/2)^2 = 4/5$. Therefore the required condition is

$$\frac{(b-2)(b+1)^2}{(b+2)(b-1)^2} = \frac{4}{5}\frac{49}{40} = \frac{49}{50}.$$

At $b = 6$, the left side equals $(4/8)(7/5)^2 = 49/50$. Thus 6 satisfies the original equation exactly. Since the derivative with respect to the upper limit is positive for every $b > 3$, no second value in the permitted domain can satisfy it. Substitution gives $F(6) - F(3) = \log(49/40)$, as required.

Step 3: Conclusion The required value is $\boxed{b = 6}$, a dimensionless real number.

Quick Tip to Solve: After integrating, combine the logarithms into one ratio; verify a candidate exactly and use positivity of the integrand to establish uniqueness.

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24. If the sum of the co-efficient of all the positive even powers of x in the binomial expansion of $\left(2x^3 + \frac{3}{x}\right)^{10}$ is $5^{10} - \beta \cdot 3^9$, then β is equal to _____. [4 marks]

Solution:

Concept:

- The general term obtained by choosing r factors of $3/x$ is $\binom{10}{r}2^{10-r}3^r x^{30-4r}$.
- All powers here are even. Only strictly positive powers are included, so negative powers and any zero power must be excluded from the coefficient sum.

Step 1: Identify the coefficients to retain The exponent condition is $30 - 4r > 0$, hence the allowed integer indices are $r = 0, 1, \ldots, 7$. There is no constant term because $30 - 4r = 0$ would require the noninteger index $r = 7.5$. The sum of every coefficient in this Laurent expansion is found by substituting $x = 1$:

$$\sum_{r=0}^{10}\binom{10}{r}2^{10-r}3^r = (2+3)^{10} = 5^{10}.$$

Substitution at 1 is legitimate even for the negative powers, whose coefficient contribution is then also counted.

Step 2: Subtract the three negative-power coefficients The omitted terms have $r = 8, 9, 10$, with

exponents $-2, -6, -10$. Their total coefficient is

$$\begin{aligned} T &= \binom{10}{8}2^2 3^8 + \binom{10}{9}2\,3^9 + 3^{10} \\ &= 180\,3^8 + 20\,3^9 + 3\,3^9 \\ &= (60 + 20 + 3)3^9 = 83\,3^9. \end{aligned}$$

Thus the positive-even-power coefficient sum is $5^{10} - 83\,3^9$. Comparing this expression directly with the form in the question determines the requested coefficient without expanding the large powers numerically.

Step 3: Conclusion The required coefficient is $\boxed{\beta = 83}$, which is dimensionless.

Quick Tip to Solve: Use the exponent of the general term to determine the included indices, then subtract a short excluded tail from the total coefficient sum.

25. If the mean deviation about the mean of the numbers 1, 2, 3, ..., n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to _____. [4 marks]

Solution:

Concept:

- For consecutive integers $1, 2, \ldots, n$, the arithmetic mean is $(n+1)/2$. Mean deviation is the sum of absolute deviations divided by n.
- When n is odd, there is one central observation and equal numbers of observations on either side, allowing deviations to be paired.

Step 1: Write the deviations using an odd integer parameter Put $n = 2k+1$, where k is a nonnegative integer. The mean is then $k+1$, an actual member of the list. The distances below the mean are $k, k-1, \ldots, 1$; those above it are $1, 2, \ldots, k$, while the central observation contributes zero. Thus

$$\sum_{j=1}^{n} \left| j - \frac{n+1}{2} \right| = 2(1 + 2 + \cdots + k) = k(k+1).$$

Dividing by the total number of observations gives the mean deviation, rather than a sum of deviations.

Step 2: Equate the mean deviation to the stated value In terms of n, $k(k+1) = (n^2-1)/4$, and hence

$$\frac{n^2 - 1}{4n} = \frac{5(n+1)}{n}.$$

Since n is a positive number of observations, multiplying by $4n$ is valid. Also $n + 1 \neq 0$, so

$$(n-1)(n+1) = 20(n+1) \quad \Longrightarrow \quad n - 1 = 20.$$

This gives $n = 21$, which is indeed odd. As a check, the absolute-deviation sum is $2(1+\cdots+10) = 110$; dividing by 21 agrees with $5(21+1)/21 = 110/21$.

Step 3: Conclusion The number of observations is $\boxed{n = 21}$, a dimensionless positive integer.

Quick Tip to Solve: For an odd consecutive list, pair equal distances on opposite sides of the middle observation before applying the mean-deviation formula.

26. Let $\overrightarrow{b} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\lambda \in$ R. If $\overrightarrow{a}$ is a vector such that $\overrightarrow{a} \times \overrightarrow{b} = 13\hat{i} - \hat{j} - 4\hat{k}$ and $\overrightarrow{a} \cdot \overrightarrow{b} + 21 = 0$, then $\left(\overrightarrow{b} - \overrightarrow{a}\right) \cdot \left(\hat{k} - \hat{j}\right) + \left(\overrightarrow{b} + \overrightarrow{a}\right) \cdot \left(\hat{i} - \hat{k}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- A cross product is perpendicular to each of its factors. Therefore $(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$ determines the unknown component of $\vec{b}$.
- Writing vectors in Cartesian components turns the cross-product and dot-product data into linear equations. The final scalar is computed from the requested projections.

Step 1: Determine the vector b Use perpendicularity before solving for the three components of $\vec{a}$:

$$(13, -1, -4) \cdot (1, 1, \lambda) = 13 - 1 - 4\lambda = 0.$$

Thus $\lambda = 3$ and $\vec{b} = (1, 1, 3)$. The dot-product condition is $\vec{a} \cdot \vec{b} = -21$; its sign must be retained when solving the equations.

Step 2: Solve for the components of a Let $\vec{a} = (u, v, w)$. Expanding the cross product gives

$$(3v - w,\ w - 3u,\ u - v) = (13, -1, -4).$$

The last two components yield $v = u + 4$ and $w = 3u - 1$. Substituting these into the dot product produces

$$u + v + 3w = -21 \quad \Longrightarrow \quad 11u + 1 = -21.$$

Hence $u = -2$, $v = 2$, $w = -7$. The unused cross-product component checks correctly: $3v - w = 6 + 7 = 13$.

Step 3: Evaluate the requested scalar Now $\vec{b} - \vec{a} = (3, -1, 10)$ and $\vec{b} + \vec{a} = (-1, 3, -4)$. Therefore

$$(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) = 10 - (-1) = 11,$$
$$(\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k}) = -1 - (-4) = 3.$$

Adding the two projections gives $11 + 3 = 14$. This calculation also avoids confusing the order $\vec{a} \times \vec{b}$ with its negative.

Step 4: Conclusion The requested scalar is $\boxed{14}$; no physical unit is specified.

Quick Tip to Solve: First dot the given cross product with the known vector to find its unknown parameter; this usually simplifies all later component equations.

27. The total number of three-digit numbers, with one digit repeated exactly two times, is _____. [4 marks]

Solution:

Concept:

- A three-digit number cannot begin with zero. Exactly one repeated pair means its digit multiset is $\{a, a, b\}$, with $a \neq b$.
- Count separately according to whether zero is the repeated digit, the singleton, or absent; these cases are disjoint and exhaustive.

Step 1: Count numbers using only nonzero digits Choose the repeated digit in 9 ways and the different singleton digit in 8 ways. The singleton can occupy any of the three positions; once it is placed, the remaining two positions contain the repeated digit. Since no digit is zero, every arrangement is a valid three-digit number. This gives

$$N_1 = 9 \cdot 8 \cdot 3 = 216.$$

There is no double counting by interchanging the two chosen digits, because their roles are different: one occurs twice and the other occurs once.

Step 2: Count the cases involving zero If zero is repeated, the digits are $0, 0, b$ with $b \in \{1, \ldots, 9\}$. Only $b00$ has a nonzero hundreds digit, giving $N_2 = 9$. If zero is the singleton, the digits are $a, a, 0$, where a has 9 choices. Zero may be in the tens or units position, giving $N_3 = 9 \cdot 2 = 18$. The arrangement beginning with zero is excluded. Numbers with all three digits equal do not enter any case, since the singleton was required to differ from the repeated digit.

Step 3: Combine the disjoint cases Every permitted number either has no zero or has a zero with one of the two multiplicities just considered. Thus

$$N = N_1 + N_2 + N_3 = 216 + 9 + 18 = 243.$$

As an independent check, count all length-three strings with exactly a pair: $10 \cdot 9 \cdot 3 = 270$. Of these, 18 start with zero when zero is repeated (the forms $00b$ and $0b0$) and 9 start with zero when it is the singleton (the form $0aa$), so $270 - 27 = 243$.

Step 4: Conclusion The total number of such three-digit numbers is $\boxed{243}$, a dimensionless count.

Quick Tip to Solve: Whenever repeated digits include zero, separate zero as the repeated digit from zero as the singleton; the valid leading-digit restrictions differ.

28. Let $f(x) = |(x-1)(x^2-2x-3)| + x - 3$, $x \in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to _____. [4 marks]

Solution:

Concept:

- Write the polynomial inside the absolute value as $P(x) = (x-1)(x-3)(x+1)$. Its sign determines the differentiable formula on each interval.
- A local minimum occurs where the derivative changes from negative to positive, and a local maximum where it changes from positive to negative. Corners must also be checked.

Step 1: Remove the absolute value on the relevant intervals On $(0,1)$, $P > 0$; on $(1,3)$, $P < 0$; and on $(3,4)$, $P > 0$. Since $P = x^3 - 3x^2 - x + 3$, the function becomes

$$f(x) = \begin{cases} x^3 - 3x^2, & 0 < x < 1, \\ -x^3 + 3x^2 + 2x - 6, & 1 < x < 3, \\ x^3 - 3x^2, & 3 < x < 4. \end{cases}$$

The original absolute-value expression is continuous at 1 and 3, so derivative signs on their adjacent intervals determine whether these corners are extrema.

Step 2: Find and classify the smooth stationary point In the outside intervals the derivative is $3x(x-2)$: it is negative on $(0,1)$ and positive on $(3,4)$, with no zero inside either interval. In the middle interval,

$$f'(x) = -3x^2 + 6x + 2 = 0 \quad \Longrightarrow \quad x = 1 \pm \frac{\sqrt{15}}{3}.$$

Only $c = 1 + \sqrt{15}/3$ lies in $(1,3)$. The derivative is positive before c and negative after it, making c a local maximum.

Step 3: Classify the corners and count At $x = 1$, the derivative changes from negative on the left to positive on the right; hence 1 is a local minimum. At $x = 3$, it changes from negative on the left to positive on the right; hence 3 is another local minimum. There are no other sign changes. The endpoints 0 and 4 are excluded by the open interval. Consequently $m = 2$, $M = 1$, and $m + M = 3$.

Step 4: Conclusion The required sum is $\boxed{m + M = 3}$, a dimensionless count.

Quick Tip to Solve: For an absolute value, split at its internal zeros and inspect derivative signs at the corners as well as at smooth stationary points.

29. **Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and l_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to _____.** [4 marks]

Solution:

Concept:

- The intercept form identifies two points on each line. A line through position vector $\mathbf{a}$, parallel to a nonzero vector $\mathbf{u}$, has equation $\mathbf{r} = \mathbf{a} + t\mathbf{u}$.
- For nonparallel lines, their shortest separation is the projection of a connecting vector onto the common perpendicular: $d = |(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{u} \times \mathbf{v})|/|\mathbf{u} \times \mathbf{v}|$.

Step 1: Choose points and directions Take the x-intercepts as the base points: $\mathbf{a} = (1/8, 0, 0)$ and $\mathbf{b} = (-1/8, 0, 0)$. The other intercept of the first line is $(0, 1/(4\sqrt{2}), 0)$; subtracting its base point and multiplying by $8\sqrt{2}$ gives $\mathbf{u} = (-\sqrt{2}, 2, 0)$. For the second line the other intercept is $(0, 0, -1/(6\sqrt{3}))$. Its displacement, multiplied by $24\sqrt{3}$, gives $\mathbf{v} = (3\sqrt{3}, 0, -4)$. Scaling direction vectors does not change either line.

Step 2: Find the common perpendicular Take the cross product, retaining all three components:

$$\mathbf{u} \times \mathbf{v} = (-8, -4\sqrt{2}, -6\sqrt{3}).$$

Its squared magnitude is $64 + 32 + 108 = 204$, so its magnitude is $2\sqrt{51}$. The nonzero cross product confirms that the lines are not parallel.

Step 3: Project the connecting vector The base-point displacement is $\mathbf{b} - \mathbf{a} = (-1/4, 0, 0)$. Its scalar product with the cross product is $(-1/4)(-8) = 2$. Therefore

$$d = \frac{2}{2\sqrt{51}} = \frac{1}{\sqrt{51}}, \qquad d^{-2} = 51.$$

The absolute value in the distance formula makes the answer independent of the direction chosen for either line.

Step 4: Conclusion The shortest distance is $d = 1/\sqrt{51}$; hence the requested dimensionless value is $\boxed{51}$.

Quick Tip to Solve: Use the two x-intercepts as base points: their connecting vector has only an x-component, so the scalar triple product becomes a single multiplication.

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30. **Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x + \beta y = \lambda$, then $\lambda - \beta$ is equal to _____.** [4 marks]

Solution:

Concept:

- For the hyperbola $x^2/a^2 - y^2/b^2 = 1$, its eccentricity satisfies $e^2 = 1 + b^2/a^2$. Thus the eccentricity fixes the ratio of the squared semi-axes.

- Implicit differentiation gives the tangent slope $dy/dx = b^2x/(a^2y)$. At a regular point with nonzero tangent slope, the normal slope is its negative reciprocal.

Step 1: Determine the semi-axis ratio Using $e = 5/4$, obtain

$$\frac{b^2}{a^2} = \frac{25}{16} - 1 = \frac{9}{16}.$$

The supplied point belongs to the hyperbola. Substitution also checks the parameters:

$$a^2 = x_1^2 - \frac{16}{9}y_1^2 = \frac{64}{5} - \frac{16}{9}\frac{144}{25} = \frac{64}{25}, \qquad b^2 = \frac{36}{25}.$$

Both squared semi-axes are positive, as required.

Step 2: Calculate the normal slope At $(x_1, y_1) = (8/\sqrt{5}, 12/5)$, the tangent slope is

$$m_t = \frac{9}{16}\frac{8/\sqrt{5}}{12/5} = \frac{15}{8\sqrt{5}}.$$

Consequently the perpendicular normal has slope $m_n = -8\sqrt{5}/15$. The negative sign is essential: the tangent rises at this point, whereas the normal falls.

Step 3: Match the required equation Write the point-slope equation and multiply by 15:

$$15\left(y - \frac{12}{5}\right) = -8\sqrt{5}\left(x - \frac{8}{\sqrt{5}}\right).$$

Rearranging gives $8\sqrt{5}x + 15y = 100$. Its x-coefficient already equals the prescribed coefficient, so there is no further rescaling. Thus $\beta = 15$ and $\lambda = 100$, and subtraction yields $100 - 15 = 85$.

Step 4: Conclusion The normal is $8\sqrt{5}x + 15y = 100$. Hence $\lambda - \beta = \boxed{85}$, a dimensionless value.

Quick Tip to Solve: For a normal, first find the tangent slope and then take the negative reciprocal. Match the specified x-coefficient before reading off the remaining constants.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Two identical balls A and B thrown with same velocity 'u' at two different angles with horizontal attained the same range R. IF A and B reached the maximum height h_1 and h_2 respectively, then $R = 4\sqrt{h_1 h_2}$

Reason R : Product of said heights.

$$h_1 h_2 = \left(\frac{u^2 \sin^2\theta}{2g}\right) \cdot \left(\frac{u^2 \cos^2\theta}{2g}\right)$$

Choose the correct answer : [4 marks]

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Correct Answer: (A) Both A and R are true and R is the correct explanation of A.

Solution:

Concept:

- For a projectile launched and landing at the same level, the horizontal range is $R = u^2 \sin 2\theta/g$, while the maximum height is $h = u^2 \sin^2\theta/(2g)$. Air resistance is neglected.
- Two different upward projection angles giving the same range at the same speed are complementary: θ and $\pi/2 - \theta$. Thus the corresponding squared sine factors are $\sin^2\theta$ and $\cos^2\theta$.

Step 1: Evaluate the assertion Use the two complementary angles. Their heights are

$$h_1 = \frac{u^2 \sin^2\theta}{2g}, \qquad h_2 = \frac{u^2 \cos^2\theta}{2g}.$$

Taking the positive square root of the product is appropriate because both heights are nonnegative. For ordinary upward projection, $\sin\theta$ and $\cos\theta$ are nonnegative, so

$$4\sqrt{h_1 h_2} = \frac{2u^2 \sin\theta \cos\theta}{g} = \frac{u^2 \sin 2\theta}{g} = R.$$

The assertion is therefore true.

Step 2: Evaluate the reason The reason writes exactly the product of the two maximum-height expressions for complementary angles. It correctly substitutes $\cos^2\theta$ for the squared sine of the second angle. Thus the stated product is true; no extra factor of two is missing in either height.

Step 3: Check the explanatory connection The product in the reason leads directly to the asserted range relation after taking its square root and using $2\sin\theta\cos\theta = \sin 2\theta$. Therefore it explains the assertion, rather than merely providing an unrelated true statement. Option B wrongly denies this link. Options C and D wrongly declare one of the two correct statements false.

Step 4: Conclusion Both Assertion A and Reason R are true, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: For equal-speed projectiles with equal ranges, replace the second launch angle by the complement of the first before comparing their heights.

32. Two buses P and Q start from a point at the same time and move in a straight line and their positions are represented by $X_P(t) = \alpha t + \beta t^2$ and $X_Q(t) = ft - t^2$. At what time, both the buses have same velocity? [4 marks]

(A) $\frac{\alpha - f}{1+\beta}$

(B) $\frac{\alpha + f}{2(\beta - 1)}$

(C) $\frac{\alpha + f}{2(1+\beta)}$

(D) $\frac{f - \alpha}{2(1+\beta)}$

Correct Answer: (D) $\frac{f - \alpha}{2(1+\beta)}$

Solution:

Concept:

- Instantaneous velocity is the time derivative of displacement: $v(t) = dX(t)/dt$. Equality of velocities concerns the slopes of the position-time curves, not necessarily equal positions.
- Differentiate each power of time separately: the derivative of at is a, and that of bt^2 is $2bt$, when the coefficients are constant.

Step 1: Differentiate both positions For bus P the position is $X_P = \alpha t + \beta t^2$, and hence

$$v_P = \alpha + 2\beta t.$$

For bus Q the position is $X_Q = ft - t^2$, giving

$$v_Q = f - 2t.$$

The minus sign in the quadratic term of Q must be retained: it makes its velocity decrease linearly with time. The coefficient f is a constant in the given position function.

Step 2: Set the instantaneous velocities equal At the required instant,

$$\alpha + 2\beta t = f - 2t.$$

Collect the time-dependent terms on the left and the constants on the right:

$$2(1 + \beta)t = f - \alpha.$$

Thus, for $1 + \beta \neq 0$,

$$t = \frac{f - \alpha}{2(1 + \beta)}.$$

This gives a physically later instant when the resulting time is nonnegative. The question presumes parameters for which such an instant exists.

Step 3: Compare the options Option D has both the correct initial-velocity difference and the factor of two from differentiation. Option A reverses the numerator and loses that factor. Options B and C use the sum $\alpha + f$, whereas equating velocities requires their difference; B also uses the wrong denominator. As a limiting check, $f = \alpha$ gives equal initial velocities and the result $t = 0$, as expected.

Step 4: Conclusion The buses have the same velocity at $t = \dfrac{f - \alpha}{2(1 + \beta)}$. Hence, the correct option is (D).

Quick Tip to Solve: Differentiate first, then equate velocities. Equating the positions instead answers when the buses meet and gives a different time.

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33. A disc with a flat small bottom beaker placed on it at a distance R from its center is revolving about an axis passing through the center and perpendicular to its plane with an angular velocity ω. The coefficient of static friction between the bottom of the beaker and the surface of the disc is μ. The beaker will revolve with the disc if : [4 marks]

(A) $R \leq \frac{\mu g}{2\omega^2}$
(B) $R \leq \frac{\mu g}{\omega^2}$
(C) $R \geq \frac{\mu g}{2\omega^2}$
(D) $R \geq \frac{\mu g}{\omega^2}$

Correct Answer: (B) $R \leq \frac{\mu g}{\omega^2}$

Solution:

Concept:

- A body co-rotating at radius R with angular velocity ω needs an inward centripetal acceleration $a_c = \omega^2 R$, and hence an inward force $m\omega^2 R$.
- Static friction adjusts to the required value up to its maximum $f_{s,\max} = \mu N$. On a horizontal disc with no vertical acceleration, the normal reaction is $N = mg$.

Step 1: Identify the force keeping the beaker in its circle The weight of the beaker acts vertically downward, and the normal reaction acts vertically upward. These balance, so $N = mg$. In the horizontal plane, friction between the small flat base and the disc provides the inward force. For the beaker to remain at the same point relative to the rotating disc, it must execute circular motion at the disc angular speed.

Step 2: Apply the no-slipping condition The required friction is $f_s = m\omega^2 R$. It cannot exceed the available maximum static friction, so

$$m\omega^2 R \leq \mu mg.$$

Cancel the positive mass and divide by ω^2:

$$R \leq \frac{\mu g}{\omega^2}.$$

At equality the beaker is on the verge of sliding; at smaller radii the required friction is smaller. It is incorrect to set friction equal to its limiting value at every radius.

Step 3: Reject the other inequalities Option A imposes an unnecessary extra factor of two and excludes allowed radii. Options C and D require a lower bound on radius, although the required centripetal force grows as the radius grows. A beaker sufficiently near the centre needs very little friction, which immediately contradicts those lower-bound options. The units also check: g/ω^2 has units of length.

Step 4: Conclusion The beaker revolves with the disc without slipping when $R \leq \frac{\mu g}{\omega^2}$. Hence, the correct option is (B).

Quick Tip to Solve: Use static friction as an upper bound, not automatically as μmg. The limiting radius comes from equating the required centripetal force to that upper bound.

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34. A solid metallic cube having total surface area 24 m^2 is uniformly heated. If its temperature is increased by 10°C, calculate the increase in volume of the cube. (Given α = 5.0 × 10^{-4} °C^{-1}).

[4 marks]

(A) 2.4 × 10^6 cm^3

(B) 1.2 × 10^5 cm^3

(C) 6.0 × 10^4 cm^3

(D) 4.8 × 10^5 cm^3

Correct Answer: (B) 1.2 × 10^5 cm^3

Solution:

Concept:

- For an isotropic solid undergoing a small temperature change, the coefficient of volume expansion is approximately $\gamma = 3\alpha$, where α is its coefficient of linear expansion.
- The first-order increase in volume is $\Delta V = 3\alpha V_0 \Delta T$. A cube of side a has total surface area $6a^2$ and volume a^3.

Step 1: Calculate the initial volume The quoted area is the total area of all six faces, so

$$6a^2 = 24\,\mathrm{m}^2, \qquad a^2 = 4\,\mathrm{m}^2, \qquad a = 2\,\mathrm{m}.$$

Therefore the original cube volume is $V_0 = a^3 = 8\,\mathrm{m}^3$. Using the surface area itself as a volume would be dimensionally incorrect.

Step 2: Apply the volume-expansion coefficient The temperature rise is $\Delta T = 10\,^\circ\mathrm{C}$. The given coefficient is linear, so

$$\gamma = 3(5.0 \times 10^{-4}) = 1.5 \times 10^{-3}\,^\circ\mathrm{C}^{-1}.$$

Using the usual small-expansion approximation gives

$$\Delta V = (1.5 \times 10^{-3})(8)(10) = 0.12\,\mathrm{m}^3.$$

Here $\alpha\Delta T = 0.005 \ll 1$; higher powers of this small quantity are neglected in the standard coefficient formula used by the options.

Step 3: Convert to the listed units Since $1\,\text{m} = 100\,\text{cm}$, one cubic metre equals $10^6\,\text{cm}^3$. Therefore

$$0.12\,\text{m}^3 = 1.2 \times 10^5\,\text{cm}^3.$$

Only option B matches. Options A, C and D give respectively twenty times, half, and four times this increase; none follows from the correct cube geometry and coefficient conversion.

Step 4: Conclusion The increase in volume is $\boxed{1.2 \times 10^5\,\text{cm}^3}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the side from all six faces before finding the volume. Then replace the linear expansion coefficient by three times its value for volume expansion.

35. A copper block of mass 5.0 kg is heated to a temperature of 500°C and is placed on a large ice block. What is the maximum amount of ice that can melt? [Specific heat of copper : 0.39 J g^{-1} °C^{-1} and latent heat of fusion of water : 335 J g^{-1}] [4 marks]

(A) 1.5 kg

(B) 5.8 kg

(C) 2.9 kg

(D) 3.8 kg

Correct Answer: (C) 2.9 kg

Solution:

Concept:

- The heat released when a solid cools without a change of state is $Q = mc\Delta T$, where m is its mass and c is its specific heat capacity.
- Melting ice at $0\,^\circ\text{C}$ requires latent heat $Q = m_i L_f$. For the maximum mass melted, all heat lost by the hot copper is used for fusion, with no heat lost to the surroundings.

Step 1: Identify the final copper temperature The ice block is large, so ice remains while melting takes place. The copper can therefore cool from $500\,^\circ\text{C}$ to the ice-water equilibrium temperature $0\,^\circ\text{C}$. Its temperature fall is $500\,^\circ\text{C}$. No temperature rise of the melted water is included when calculating the maximum mass of ice melted.

Step 2: Calculate the heat available The specific heat is supplied per gram, so convert the copper mass: $5.0\,\text{kg} = 5000\,\text{g}$. Then

$$Q = (5000)(0.39)(500) = 975000\,\text{J}.$$

The units cancel as $\text{g} \times \text{J}\,\text{g}^{-1}\,^\circ\text{C}^{-1} \times\,^\circ\text{C} = \text{J}$. Keeping kilograms with the stated gram-based heat capacity would give a result too small by a factor of one thousand.

Step 3: Equate the released heat to latent heat With $L_f = 335\,\mathrm{J\,g^{-1}}$,

$$m_i = \frac{Q}{L_f} = \frac{975000}{335}\,\mathrm{g} \simeq 2910.45\,\mathrm{g} = 2.91045\,\mathrm{kg}.$$

To the precision of the choices this is $2.9\,\mathrm{kg}$. Option A would use only about half the available heat; option B would require approximately twice it, and option D also requires more heat than the copper releases. Thus none of those alternatives satisfies the heat balance.

Step 4: Conclusion The maximum ice mass melted is approximately $\boxed{2.9\,\mathrm{kg}}$. Hence, the correct option is (C).

Quick Tip to Solve: When specific heat and latent heat are quoted per gram, perform the heat balance in grams and convert the final melted mass to kilograms.

36. The ratio of specific heats $\left(\frac{C_P}{C_V}\right)$ in terms of degree of freedom (f) is given by : [4 marks]

(A) $\left(1+\frac{f}{3}\right)$

(B) $\left(1+\frac{2}{f}\right)$

(C) $\left(1+\frac{f}{2}\right)$

(D) $\left(1+\frac{1}{f}\right)$

Correct Answer: (B) $\left(1+\frac{2}{f}\right)$

Solution:

Concept:

- Equipartition assigns an average energy $k_BT/2$ to each independent quadratic degree of freedom. For one mole with f active degrees, $U = fRT/2$.
- Molar heat capacities obey $C_V = (\partial U/\partial T)_V$ and Mayer's relation $C_P - C_V = R$ for an ideal gas. The heat-capacity ratio is dimensionless.

Step 1: Obtain the constant-volume heat capacity At fixed volume no expansion work is done, so the supplied heat increases internal energy. In the temperature interval where the active degrees of freedom do not change, differentiating $U = fRT/2$ gives

$$C_V = \frac{fR}{2}.$$

Here f is a pure number, and C_V has units of energy per mole per kelvin. It is important not to identify $f/2$ itself with the ratio of heat capacities.

Step 2: Include the expansion work at constant pressure Heating one mole at constant pressure requires the internal-energy increment plus the work of expansion. Since $p\,dV = R\,dT$, the additional heat capacity is R. Thus

$$C_P = C_V + R = \frac{(f+2)R}{2}.$$

Dividing the two capacities cancels the common factor $R/2$:

$$\gamma = \frac{C_P}{C_V} = \frac{f+2}{f} = 1 + \frac{2}{f}.$$

The numerator contribution is 2, not 1 or $f^2/2$. For example, $f = 3$ gives $5/3$, as required for a monatomic ideal gas; options A, C and D fail this check.

Step 3: Conclusion $C_P/C_V = 1 + 2/f$. Hence, the correct option is (B).

Quick Tip to Solve: Remember $C_V = fR/2$, then add R before taking the ratio; do not add to f without the factor of two.

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37. For a particle in uniform circular motion, the acceleration $\vec{a}$ at any point P(R, θ) on the circular path of radius R is (when θ is measured from the positive x-axis and v is uniform speed) : [4 marks]

(A) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$
(B) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$
(C) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$
(D) $-\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

Correct Answer: (C) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

Solution:

Concept:

- Uniform speed does not imply constant velocity: its direction changes continuously around the circle, producing centripetal acceleration of magnitude v^2/R.
- With the centre as origin, the outward radial unit vector at polar angle θ is $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$. Acceleration points opposite to it.

Step 1: Resolve the inward radial acceleration The position vector is $\vec{r} = R\cos\theta\hat{i} + R\sin\theta\hat{j}$. Since the particle moves at constant speed, the tangential component of acceleration vanishes. The entire acceleration is inward, so

$$\vec{a} = -\frac{v^2}{R}\hat{r}.$$

Substitution of the radial unit vector therefore gives

$$\vec{a} = -\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}.$$

Both components reverse the corresponding components of position.

Step 2: Check the direction and reject the alternatives At $\theta = 0$, the particle lies on the positive x-axis, so its acceleration must point along negative x. At $\theta = \pi/2$, it must point along negative y. Option C satisfies both tests and has magnitude $\sqrt{(v^2\cos\theta/R)^2 + (v^2\sin\theta/R)^2} = v^2/R$. Option A is tangential rather than inward. Option B gives a positive y component in the first quadrant, so it is not directed towards the centre. Option D is independent of position and has magnitude $\sqrt{2}v^2/R$, both inconsistent with circular motion.

Step 3: Conclusion $\vec{a} = -(v^2/R)\cos\theta\,\hat{i} - (v^2/R)\sin\theta\,\hat{j}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the outward radial unit vector first, then multiply it by $-v^2/R$; a check at the top of the circle catches the wrong y sign.

38. Two metallic plates form a parallel plate capacitor. The distance between the plates is 'd'. A metal sheet of thickness $\frac{d}{2}$ and of area equal to area of each plate is introduced between the plates. What will be the ratio of the new capacitance to the original capacitance of the capacitor? [4 marks]

(A) 2 : 1

(B) 1 : 2

(C) 1 : 4

(D) 4 : 1

Correct Answer: (A) 2 : 1

Solution:

Concept:

- A parallel-plate capacitor with plate area A and vacuum or air gap d has capacitance $C_0 = \varepsilon_0 A/d$, neglecting edge effects.
- The electric field inside an isolated metal slab is zero in electrostatic equilibrium. Only the remaining air gaps contribute to the potential difference between the capacitor plates.

Step 1: Find the effective field-filled separation Let the metal thickness be $t = d/2$. If the two air gaps on its sides have widths d_1 and d_2, their sum is

$$d_1 + d_2 = d - t = \frac{d}{2}.$$

For plate charge magnitude Q, the field in either air gap is $E = Q/(\varepsilon_0 A)$. The induced charges on the slab surfaces make its interior field zero; no voltage drop accumulates across the metal itself. Consequently the slab need not be centred for this result to hold, provided it does not short the plates.

Step 2: Calculate the new capacitance and compare The total voltage drop across the two gaps is

$$V = E(d_1 + d_2) = \frac{Q(d-t)}{\varepsilon_0 A}.$$

Hence

$$C_{\text{new}} = \frac{Q}{V} = \frac{\varepsilon_0 A}{d-t} = \frac{2\varepsilon_0 A}{d} = 2C_0.$$

The ratio requested is new to original, so it is $2 : 1$, rather than its reciprocal. Option B would incorrectly suggest that inserting metal reduces capacitance. Options C and D introduce a factor of four, although the field-filled distance is only halved. The result also agrees with the series combination of capacitors formed by the two remaining air gaps.

Step 3: Conclusion The capacitance doubles: $C_{\text{new}} : C_0 = 2 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: For a metal slab spanning the plate area, replace the gap d by $d - t$; the conductor contributes zero potential drop.

39. Two cells of same emf but different internal resistances r_1 and r_2 are connected in series with a resistance R. The value of resistance R, for which the potential difference across second cell is zero, is : [4 marks]

(A) $r_2 - r_1$

(B) $r_1 - r_2$

(C) r_1

(D) r_2

Correct Answer: (A) $r_2 - r_1$

Solution:

Concept:

- For two identical emfs $\mathcal{E}$ connected in series aiding, the total emf is $2\mathcal{E}$ and the total circuit resistance is $R + r_1 + r_2$.
- The terminal potential difference of a cell supplying current I is $V = \mathcal{E} - Ir$. Its internal resistance causes an internal voltage drop distinct from the external load voltage.

Step 1: Write the current and the zero-voltage condition Using the sum of the emfs and resistances for the complete circuit gives

$$I = \frac{2\mathcal{E}}{R + r_1 + r_2}.$$

The second cell has internal resistance r_2, so the potential difference between its terminals is

$$V_2 = \mathcal{E} - Ir_2.$$

For $V_2 = 0$, the internal drop must equal its emf: $Ir_2 = \mathcal{E}$, or $I = \mathcal{E}/r_2$. Zero terminal voltage does not imply zero current; the generated emf is then completely balanced by the internal resistance drop.

Step 2: Solve for the load resistance Equate the two expressions for current and cancel the nonzero emf:

$$\frac{2}{R + r_1 + r_2} = \frac{1}{r_2}.$$

Therefore

$$R + r_1 + r_2 = 2r_2, \qquad R = r_2 - r_1.$$

This is a passive external resistance only when $r_2 \geq r_1$; for a strictly positive resistor, $r_2 > r_1$. Substitution gives total resistance $2r_2$, current $\mathcal{E}/r_2$, and indeed zero second-cell terminal voltage. Option B reverses the required difference, while C and D do not in general satisfy the terminal-voltage equation. The difference of resistances also has the correct unit, ohm.

Step 3: Conclusion $R = r_2 - r_1$, with a positive load possible for $r_2 > r_1$. Hence, the correct option is (A).

Quick Tip to Solve: Apply $V_2 = \mathcal{E} - Ir_2$ to the named cell before using the total circuit; zero terminal voltage still allows a current.

40. Given below are two statements :

Statement I : Susceptibilities of paramagnetic and ferromagnetic substances increase with decrease in temperature.

Statement II : Diamagnetism is a result of orbital motions of electrons developing magnetic moments opposite to the applied magnetic field.

Choose the correct answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (A) Both Statement I and Statement II are true.

Solution:

Concept:

- Magnetic susceptibility χ relates magnetisation to the magnetising field: $M = \chi H$. Thermal agitation competes with alignment of permanent magnetic moments.
- Paramagnets have a small positive susceptibility; ferromagnets exhibit strong cooperative alignment. Diamagnetic response is induced opposite to the applied field and has negative susceptibility.

Step 1: Evaluate the temperature statement For an ordinary paramagnet, Curie's law gives $\chi = C/T$, where C is the Curie constant and T is absolute temperature. Lowering temperature reduces thermal randomisation, so the susceptibility increases. Ferromagnetic materials likewise show stronger magnetic ordering on cooling in the usual textbook description. Above their Curie temperature the Curie-Weiss expression $\chi = C/(T - T_C)$ also exhibits an increasing response as temperature falls towards T_C. Thus the temperature trend stated in Statement I is the accepted one for these substances; it is not the nearly temperature-independent behaviour characteristic of diamagnetism.

Step 2: Evaluate the induced orbital response An applied magnetic field changes the orbital motion of bound electrons. The resulting induced magnetic moment opposes the applied field, consistent with Lenz's law. This orbital response is the origin of diamagnetism, so Statement II is also true. It does not require permanent unpaired-electron moments, unlike paramagnetism, and it does not arise from spontaneous alignment of ferromagnetic domains. Since neither statement is false, the alternatives claiming that both are false or that exactly one is false are rejected. The question asks for the truth of the two statements, rather than whether one explains the other.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Associate cooling with reduced thermal disorder for para- and ferromagnets; associate diamagnetism with an induced moment opposite to the field.

41. A long solenoid carrying a current produces a magnetic field B along its axis. If the current is doubled and the number of turns per cm is halved, the new value of magnetic field will be equal to [4 marks]

(A) B

(B) 2B

(C) 4B

(D) $\frac{B}{2}$

Correct Answer: (A) B

Solution:

Concept:

- Inside an ideal long solenoid the axial field is $B = \mu n I$, where $n = N/\ell$ is the number of turns per unit length, I the current and μ the permeability.
- With unchanged core material and geometry appropriate to a long solenoid, the field depends on the product of turn density and current, rather than either factor alone.

Step 1: Express both changes as multiplicative factors Let the original turn density be n, whether expressed per metre or per centimetre. The new density is $n\prime = n/2$, and the new current is $I\prime = 2I$. The same length unit must be used before and after the change. If converting turns per centimetre to turns per metre, both densities acquire the same factor of 100, which cancels from the ratio. The core and hence its permeability are unchanged, so there is no extra multiplicative factor from the material.

Step 2: Compare the fields directly Apply the solenoid formula to the modified winding:

$$B\prime = \mu n\prime I\prime = \mu \left(\frac{n}{2}\right)(2I) = \mu n I = B.$$

Equivalently,

$$\frac{B\prime}{B} = \frac{n\prime}{n}\frac{I\prime}{I} = \frac{1}{2} \times 2 = 1.$$

Doubling the current alone would give option B, but halving the turn density cancels that increase. Option D includes only the density change and ignores the increased current. Option C would require both factors to double, whereas one is actually halved. The new field therefore has the same magnitude as the original; its direction is also unchanged if the current direction is unchanged.

Step 3: Conclusion The new magnetic field equals B. Hence, the correct option is (A).

Quick Tip to Solve: For a long solenoid compare nI: doubling current and halving turns per unit length cancel exactly.

42. A sinusoidal voltage V(t) = 210 sin 3000 t volt is applied to a series LCR circuit in which L = 10 mH, C = 25 μF and R = 100 Ω. The phase difference (Φ) between the applied voltage and resultant current will be : [4 marks]

(A) $\tan^{-1}(0.17)$

(B) $\tan^{-1}(9.46)$

(C) $\tan^{-1}(0.30)$

(D) $\tan^{-1}(13.33)$

Correct Answer: (A) $\tan^{-1}(0.17)$

Solution:

Concept:

- For a sinusoidal series LCR circuit, $X_L = \omega L$, $X_C = 1/(\omega C)$, and $\tan\phi = (X_L - X_C)/R$. The sign determines whether voltage leads or lags current.
- The angular frequency is the coefficient of time inside the sine, not the peak voltage: $V(t) = V_0 \sin \omega t$. Reactances and resistance must all be expressed in ohms.

Step 1: Calculate the two reactances in SI units From the voltage expression, $\omega = 3000\,\text{rad}\,\text{s}^{-1}$. Convert $L = 10\,\text{mH} = 0.010\,\text{H}$ and $C = 25\,\mu\text{F} = 25 \times 10^{-6}\,\text{F}$. The inductive reactance is

$$X_L = 3000(0.010) = 30\,\Omega.$$

The capacitive reactance is

$$X_C = \frac{1}{3000(25 \times 10^{-6})} = \frac{40}{3}\,\Omega \simeq 13.33\,\Omega.$$

The two contributions oppose in the impedance, so they must be subtracted rather than added.

Step 2: Divide the net reactance by the resistance The net reactance is positive:

$$X_L - X_C = 30 - \frac{40}{3} = \frac{50}{3}\,\Omega.$$

Using $R = 100\,\Omega$,

$$\tan\phi = \frac{50/3}{100} = \frac{1}{6} \simeq 0.167.$$

Thus $\phi = \tan^{-1}(0.17)$, about $9.46°$; the current lags the voltage because the circuit is net inductive. Option B incorrectly puts an angle close to the degree value inside the inverse tangent. Option C uses X_L/R and omits capacitance; option D uses a reactance value without the required resistance division. The peak voltage 210 V does not enter this phase ratio.

Step 3: Conclusion $\phi = \tan^{-1}(1/6) \simeq \tan^{-1}(0.17)$, with current lagging voltage. Hence, the correct option is (A).

Quick Tip to Solve: Read ω from the sine argument, convert mH and microfarads, then evaluate $(X_L - X_C)/R$ before taking the inverse tangent.

43. The electromagnetic waves travel in a medium at a speed of 2.0×10^8 m/s. The relative permeability of the medium is 1.0. The relative permittivity of the medium will be : [4 marks]

(A) 2.25

(B) 4.25

(C) 6.25

(D) 8.25

Correct Answer: (A) 2.25

Solution:

Concept:

- The wave speed in a linear medium is $v = 1/\sqrt{\mu\varepsilon}$, while in vacuum $c = 1/\sqrt{\mu_0\varepsilon_0}$. Here $\mu = \mu_r\mu_0$ and $\varepsilon = \varepsilon_r\varepsilon_0$.
- Therefore $v = c/\sqrt{\mu_r\varepsilon_r}$. Relative permeability and relative permittivity are dimensionless; the refractive index is c/v.

Step 1: Express the speed ratio The given speed is $v = 2.0 \times 10^8\,\mathrm{m\,s^{-1}}$. Using $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$, obtain

$$\frac{c}{v} = \frac{3.0 \times 10^8}{2.0 \times 10^8} = 1.5.$$

The ratio is greater than one, consistently with propagation slower than in vacuum.

Step 2: Solve for the relative permittivity Square the speed relation and insert the stated $\mu_r = 1.0$:

$$\mu_r\varepsilon_r = \left(\frac{c}{v}\right)^2 = 1.5^2 = 2.25, \qquad \varepsilon_r = \frac{2.25}{1.0} = 2.25.$$

The speed reduction depends on the square root of permittivity, so taking only c/v would miss the required square.

Step 3: Check the alternatives For $\mu_r = 1$, the options 4.25, 6.25 and 8.25 would instead give speeds $3.0 \times 10^8/\sqrt{\varepsilon_r}$, approximately 1.46×10^8, 1.20×10^8 and $1.04 \times 10^8\,\mathrm{m\,s^{-1}}$. None equals the stated speed.

Step 4: Conclusion The relative permittivity is $\boxed{\varepsilon_r = 2.25}$, with no unit. Hence, the correct option is (A).

Quick Tip to Solve: First calculate the refractive index c/v, then square it and divide by μ_r; relative permittivity is not simply the refractive index.

44. The interference pattern is obtained with two coherent light sources of intensity ratio 4 : 1. And the ratio $\frac{I_{max}+I_{min}}{I_{max}-I_{min}}$ is $\frac{5}{x}$. Then, the value of x will be equal to : [4 marks]

(A) 3

(B) 4

(C) 2

(D) 1

Correct Answer: (B) 4

Solution:

Concept:

- Intensity is proportional to the square of the wave amplitude. Thus an intensity ratio $4 : 1$ corresponds to amplitude ratio $2 : 1$, not $4 : 1$.
- For coherent waves of intensities I_1 and I_2, the extrema are $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$.

Step 1: Calculate the extreme intensities Write $I_1 = 4I_0$ and $I_2 = I_0$, where $I_0 > 0$. Constructive interference adds the amplitudes and destructive interference subtracts them:

$$I_{max} = (2\sqrt{I_0} + \sqrt{I_0})^2 = 9I_0, \qquad I_{min} = (2\sqrt{I_0} - \sqrt{I_0})^2 = I_0.$$

Unequal amplitudes leave a nonzero minimum.

Step 2: Form the specified ratio The question asks for the sum divided by the difference, rather than the visibility (its reciprocal). Hence

$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{9I_0 + I_0}{9I_0 - I_0} = \frac{10}{8} = \frac{5}{4}.$$

The common intensity scale cancels.

Step 3: Compare with the given expression Equating this result to the printed expression gives

$$\frac{5}{x} = \frac{5}{4} \quad \Rightarrow \quad x = 4.$$

The other proposed values $3, 2, 1$ would produce ratios $5/3, 5/2, 5$, respectively; each differs from the directly calculated $5/4$.

Step 4: Conclusion The requested value is $\boxed{x = 4}$. Hence, the correct option is (B).

Quick Tip to Solve: Take square roots of the source intensities before finding the maximum and minimum; also check whether the requested ratio is visibility or its reciprocal.

45. A light whose electric field vectors are completely removed by using a good polaroid, allowed to incident on the surface of the prism at Brewster's angle. Choose the most suitable option for the phenomenon related to the prism. [4 marks]

(A) Reflected and refracted rays will be perpendicular to each other.

(B) Wave will propagate along the surface of prism.

(C) No refraction, and there will be total reflection of light.

(D) No reflection, and there will be total transmission of light.

Correct Answer: (D) No reflection, and there will be total transmission of light.

Solution:

Concept:

- At Brewster incidence on a transparent dielectric, the reflection coefficient for the electric-field component parallel to the plane of incidence (p polarization) vanishes.
- The perpendicular (s-polarized) component generally remains reflected. A polaroid removes a component of electric field, not every component of a nonzero transmitted light wave.

Step 1: Identify the required polarization condition The statement is imprecise: if every electric-field vector were literally removed, no light would remain to be incident on the prism. The Brewster transmission interpretation requires that the polaroid removes the component perpendicular to the plane of incidence, leaving p-polarized incident light. Without that orientation, complete absence of reflection cannot be inferred.

Step 2: Apply Brewster law to the transmitted component For p-polarized light at the first interface, Brewster law gives $\tan i_B = n_2/n_1$. Combining it with Snell law gives $i_B + r = 90^\circ$ and a zero p-polarized Fresnel reflection coefficient. Thus, neglecting absorption, all incident energy enters the refracted beam at that surface. This is a statement about the incident interface, not every subsequent face of a prism.

Step 3: Assess the listed phenomena Option (D) describes the intended p-polarized Brewster effect. Propagation along the surface would require a grazing transmitted ray, which Brewster incidence does not give, so (B) fails. Total reflection in (C) is also not the Brewster phenomenon. The perpendicular reflected/refracted geometry in (A) is the usual Brewster geometry when a reflected component exists; it does not express the zero reflected intensity of p-polarized incidence.

Step 4: Conclusion The intended Brewster transmission answer is (D): no reflected p-polarized light at the incident interface. Hence, the correct option is (D) under that polarization interpretation. Literally, removal of all electric-field vectors leaves no incident light, and the stated wording does not specify the polaroid orientation, so a unique physical conclusion needs that clarification.

Quick Tip to Solve: At Brewster angle, zero reflection applies to p polarization only. Check the polarizer orientation before concluding that all incident energy is transmitted.

46. A proton, a neutron, an electron and an α particle have same energy. If λ_p, λ_n, λ_e and λ_a are the de Broglie's wavelengths of proton, neutron, electron and α particle respectively, then choose the correct relation from the following : [4 marks]

(A) $\lambda_p = \lambda_n > \lambda_e > \lambda_a$

(B) $\lambda_a < \lambda_n < \lambda_p < \lambda_e$

(C) $\lambda_e < \lambda_p = \lambda_n > \lambda_a$

(D) $\lambda_e = \lambda_p = \lambda_n = \lambda_a$

Correct Answer: (B) $\lambda_a < \lambda_n < \lambda_p < \lambda_e$

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$. For a nonrelativistic particle with kinetic energy K, $p = \sqrt{2mK}$, so $\lambda = h/\sqrt{2mK}$.
- At fixed kinetic energy, wavelength decreases as the inverse square root of mass. The relevant rest masses obey $m_e < m_p < m_n < m_\alpha$; the neutron is slightly heavier than the proton.

Step 1: Compare the lightest and heaviest particles The electron has by far the smallest mass and hence the largest wavelength. The alpha particle consists of two protons and two neutrons and has roughly four nucleon masses, so its wavelength is the smallest:

$$\lambda_\alpha < \lambda_p, \lambda_n < \lambda_e.$$

Equal energy must not be mistaken for equal momentum; the heavier particle needs larger momentum for the same kinetic energy.

Step 2: Distinguish the proton from the neutron The small mass difference matters because the options distinguish equality from inequality. Since $m_n > m_p$,

$$\frac{\lambda_n}{\lambda_p} = \sqrt{\frac{m_p}{m_n}} < 1.$$

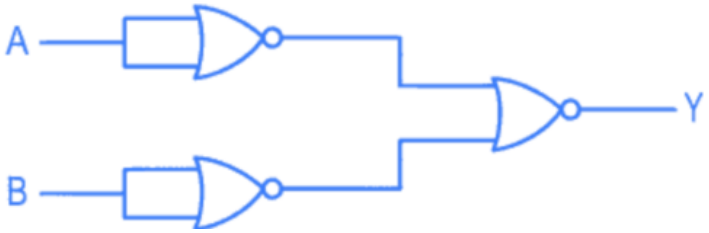
Thus $\lambda_n < \lambda_p$, although the wavelengths are close. Using equal nucleon masses would erase a physically relevant distinction.

Step 3: Check all four relations Combining the comparisons gives $\lambda_\alpha < \lambda_n < \lambda_p < \lambda_e$. Option (A) incorrectly makes the electron wavelength smaller than the nucleon wavelengths. Option (C) also places the electron below them and equates proton and neutron. Option (D) would require equal momenta, which equal kinetic energy for different masses does not supply.

Step 4: Conclusion The ordering is $\boxed{\lambda_\alpha < \lambda_n < \lambda_p < \lambda_e}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal kinetic energies, order wavelengths in the reverse order of masses; retain the small neutron-proton mass difference if an option tests it.

47. Identify the logic operation performed by the given circuit: [4 marks]



(A) AND gate

(B) OR gate

(C) NOR gate

(D) NAND gate

Correct Answer: (A) AND gate

Solution:

Concept:

- A NOR gate gives the complement of the OR of its inputs: $Y = \overline{P + Q}$. The symbol $+$ denotes Boolean OR, while multiplication denotes Boolean AND.
- Tying both inputs of a NOR gate to the same signal produces an inverter: $\overline{A + A} = \overline{A}$. De Morgan's law converts a NOR of complemented inputs to an AND.

Step 1: Reduce the two input gates The upper NOR gate has both inputs connected to A, and the lower NOR gate has both inputs connected to B. Their outputs are therefore

$$P = \overline{A + A} = \overline{A}, \qquad Q = \overline{B + B} = \overline{B}.$$

The repeated connection is important: these are single-signal inversions, rather than two independent OR operations.

Step 2: Simplify the final NOR operation The final gate is another NOR, so

$$Y = \overline{P + Q} = \overline{\overline{A} + \overline{B}} = A\,B.$$

The last equality is De Morgan's law. The inversion bubbles on the first two gates and the final gate must all be included; neglecting one changes the result.

Step 3: Verify with the input combinations For inputs $(A, B) = (0, 0), (0, 1), (1, 0), (1, 1)$, the expression AB gives outputs $0, 0, 0, 1$. An OR gate instead gives 1 for either mixed pair; a NOR gives 1 at $(0, 0)$; a NAND gives 0 only at $(1, 1)$. Thus none of those three alternatives reproduces the circuit truth table.

Step 4: Conclusion The complete circuit performs $\boxed{Y = A\,B}$, the AND operation. Hence, the correct option is (A).

Quick Tip to Solve: First replace each tied-input NOR gate by NOT, then apply De Morgan's law to the last gate; do not identify the complete circuit by its last symbol alone.

48. If n represents the actual number of deflections in a converted galvanometer of resistance G and shunt resistance S. Then the total current I when its figure of merit is K will be: [4 marks]

(A) $\frac{KS}{(S+G)}$

(B) $\frac{(G+S)}{nKS}$

(C) $\frac{nKS}{(G+S)}$

(D) $\frac{nK(G+S)}{S}$

Correct Answer: (D) $\frac{nK(G+S)}{S}$

Solution:

Concept:

- The figure of merit K of a galvanometer is the current needed per scale division. A deflection of n divisions therefore means galvanometer current $I_g = nK$.
- A shunt of resistance S is connected in parallel with galvanometer resistance G. Parallel branches have the same potential difference, and the total current is the sum of their branch currents.

Step 1: Determine the galvanometer branch current Using the definition of figure of merit directly gives

$$I_g = Kn.$$

Here n is a scale reading in divisions and K has units of amperes per division, so their product is a current. The figure of merit is the sensitivity parameter converting the scale reading into branch current.

Step 2: Find the current through the shunt Let I_s be the shunt current. Equal voltages across the parallel branches require

$$I_g G = I_s S \quad \Rightarrow \quad I_s = \frac{I_g G}{S} = \frac{nKG}{S}.$$

The lower-resistance shunt takes the larger share when $S < G$, which is the usual ammeter conversion.

Step 3: Add the currents and check the options Kirchhoff's current law at the branch junction gives

$$I = I_g + I_s = nK + \frac{nKG}{S} = \frac{nK(G+S)}{S}.$$

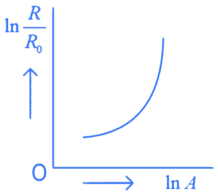
Option (A) lacks the required proportionality to deflection. Option (B) is inverse in nK, while (C) is smaller than nK although the total must exceed the galvanometer branch current. Only (D) has the correct dependence and limiting behaviour.

Step 4: Conclusion The total current is $\boxed{I = \frac{nK(G+S)}{S}}$, in amperes when K is in amperes per division. Hence, the correct option is (D).

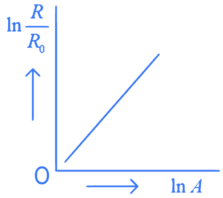
Quick Tip to Solve: Write $I_g = nK$ first, then use equal parallel-branch voltages; the total ammeter current must be larger than I_g.

49. Which of the following figure represents the variation of $l_n\left(\frac{R}{R_0}\right)$ with l_nA (if R = radius of a nucleus and A = its mass number) [4 marks]

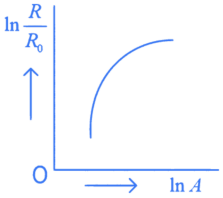
(A) An increasing curve with increasing slope.



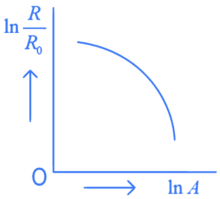
(B) A straight line through the origin with positive slope.



(C) An increasing curve with decreasing slope.

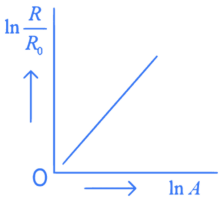


(D) A decreasing curve.



Correct Answer: (B)

A straight line through the origin with positive slope.



Solution:

Concept:

- The empirical nuclear-radius law is $R = R_0A^{1/3}$, where R_0 is the nuclear radius constant and A is the dimensionless mass number.
- Taking logarithms converts a power law into a straight-line relation: $\ln(u^a) = a\ln u$. The plotted ordinate is $\ln(R/R_0)$, not $\ln R$ alone.

Step 1: Write the dimensionless radius ratio Divide the nuclear-radius law by R_0:

$$\frac{R}{R_0} = A^{1/3}.$$

Both sides are dimensionless, so their natural logarithms are well defined for positive mass number. The radius increases with mass number, but it does so as a cube root rather than linearly.

Step 2: Convert to the plotted variables Apply the natural logarithm to obtain

$$\ln\left(\frac{R}{R_0}\right) = \ln(A^{1/3}) = \frac{1}{3}\ln A.$$

If $y = \ln(R/R_0)$ and $x = \ln A$, this is $y = x/3$. The slope is the constant $1/3$, and the intercept is zero. At $A = 1$, both plotted quantities vanish.

Step 3: Match the graphical options Option (B) is the straight increasing line through the origin and therefore matches the derived relation. Option (A) curves upward with changing slope; (C) increases with decreasing slope. Neither can represent a constant logarithmic slope. Option (D) decreases, contradicting the positive coefficient $1/3$. The cube-root curvature in ordinary R-versus-A axes disappears on the stated logarithmic axes.

Step 4: Conclusion The plot is a straight line through the origin with slope $\boxed{1/3}$. Hence, the correct option is (B).

Quick Tip to Solve: Before choosing a graph, rewrite the physical law in the exact axis variables; a power law becomes a straight line when both axes use logarithms.

50. Match List I with List II

	List - I		List -II
A.	Facsimile	I.	Static Document Image
B.	Guided media Channel	II.	Local Broadcast Radio
C.	Frequency Modulation	III.	Rectangular wave
D.	Digital Signal	IV.	Optical Fiber

Choose the correct answer from the following options: [4 marks]

(A) A-IV, B-III, C-II, D-I

(B) A-I, B-IV, C-II, D-III

(C) A-IV, B-II, C-III, D-I

(D) A-I, B-II, C-III, D-IV

Correct Answer: (B) A-I, B-IV, C-II, D-III

Solution:

Concept:

- Facsimile transmits a reproduction of a stationary document or picture. It is associated with a static document image, rather than an audio broadcast.
- A guided communication channel confines the signal to a physical path. An optical fibre guides light, while radio waves used for broadcasting travel through free space.
- Frequency modulation is used for local radio broadcasting. A digital signal takes discrete levels, commonly represented by rectangular pulses.

Step 1: Match the physical media and applications Facsimile sends a scanned document, so A matches I. Optical fibre is a guided medium, so B matches IV. The word guided refers to the path followed by

the wave, not to the shape of the signal. A radio broadcast is an unguided transmission and cannot be the match for guided media.

Step 2: Match the modulation and signal form Frequency modulation varies carrier frequency with the message and is used for local broadcast radio, giving C-II. A digital waveform switches between discrete levels and is represented by a rectangular wave, giving D-III. A static document is a message, not a digital pulse shape.

Step 3: Check the offered combinations The complete matching is A-I, B-IV, C-II, D-III. Option (A) incorrectly associates facsimile with optical fibre as its defining application. Option (C) also starts with that incorrect association. Option (D) puts guided media with radio broadcasting and digital signals with optical fibre, confusing transmission medium with signal shape.

Step 4: Conclusion The matching is A-I, B-IV, C-II, D-III. Hence, the correct option is (B).

Quick Tip to Solve: Separate message type, transmission medium, modulation method and signal shape before matching the lists.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. For $z = a^2x^3y^{\frac{1}{2}}$, where 'a' is a constant. If percentage error in measurement of 'x' and 'y' are 4% and 12% respectively, then the percentage error for 'z' will be _____%. [4 marks]

Solution:

Concept:

- For a measured product $z = Cx^py^q$, maximum fractional errors add after multiplication by the absolute values of the powers: $\Delta z/z = |p|\Delta x/x + |q|\Delta y/y$.
- The constant a is not a measured variable here. Its factor a^2 therefore contributes no uncertainty to the error requested.

Step 1: Identify the powers of the measured variables The expression is $z = a^2x^3y^{1/2}$. Thus the sensitivity to a small fractional change in x is three times that change, whereas the sensitivity to a fractional change in y is one half of that change. The square on the constant does not create an extra error term.

Step 2: Add the maximum percentage contributions Using the given percentage errors directly,

$$\frac{\Delta z}{z} \times 100 = 3(4) + \frac{1}{2}(12) = 12 + 6 = 18\%.$$

The contribution from x is 12%, and that from y is 6%. Both contributions are positive in the maximum-error rule, irrespective of the sign of an individual experimental deviation.

Step 3: Check the interpretation of the result The question supplies percentage errors, so there is no need to know numerical values of a, x, y. We are asked for the standard maximum percentage error, not a statistical standard deviation; hence the errors are added linearly rather than combined by a root-sum-square formula. The final blank already carries the percent sign.

Step 4: Conclusion The percentage error in z is 18%; the required numerical entry is 18.

Quick Tip to Solve: For products of powers, multiply each percentage error by its exponent and add; exact constants contribute zero.

52. A curved in a level road has a radius 75 m. The maximum speed of a car turning this curved road can be 30 m/s without skidding. If radius of curved road is changed to 48 m and the coefficient of friction between the tyres and the road remains same, then maximum allowed speed would be _____ m/s. [4 marks]

Solution:

Concept:

- On a level road, the normal reaction is $N = mg$, and limiting static friction $\mu_s mg$ supplies the centripetal force.
- At the maximum safe speed, $mv_{\max}^2/r = \mu_s mg$, so $v_{\max} = \sqrt{\mu_s g r}$. The mass cancels.

Step 1: Relate the two limiting speeds For the first bend, $v_1 = 30\,\mathrm{m\,s^{-1}}$ and $r_1 = 75\,\mathrm{m}$. For the second bend, $r_2 = 48\,\mathrm{m}$. Since the tyres, road coefficient and gravitational acceleration are unchanged, both bends have the same value of $v_{\max}^2/r$.

Step 2: Evaluate the radius scaling Dividing the limiting-force equations gives

$$\frac{v_2^2}{v_1^2} = \frac{r_2}{r_1} = \frac{48}{75} = \frac{16}{25}.$$

Taking the positive root because speed is nonnegative,

$$v_2 = 30\sqrt{\frac{16}{25}} = 30\frac{4}{5} = 24\,\mathrm{m\,s^{-1}}.$$

The square root is essential: speed is not directly proportional to the radius.

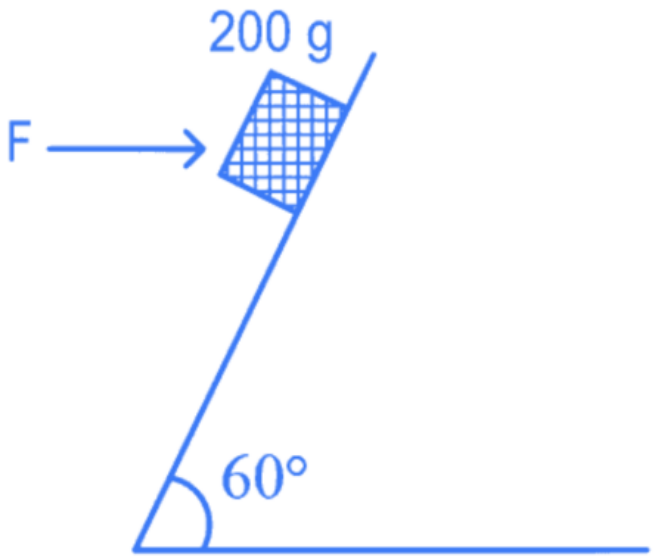
Step 3: Check against the friction limit The original road requires $\mu_s g = 30^2/75 = 12\,\mathrm{m\,s^{-2}}$. The proposed speed on the new road requires $24^2/48 = 12\,\mathrm{m\,s^{-2}}$, exactly the same limiting acceleration. A smaller turning radius therefore permits a lower maximum speed, consistent with the result.

Step 4: Conclusion The maximum allowed speed is $24\,\mathrm{m\,s^{-1}}$.

Quick Tip to Solve: For unchanged friction on level bends, compare speeds using the square root of the radius ratio.

53. A block of mass 200 g is kept stationary on a smooth inclined plane by applying a minimum horizontal force F = $\sqrt{x}$N as shown in figure.

The value of x = _____. [4 marks]



Solution:

Concept:

- On a smooth incline there is no friction. Equilibrium requires zero resultant force both parallel and perpendicular to the plane.
- For a horizontal applied force and an incline of angle θ, the uphill component is $F\cos\theta$, while gravity contributes $mg\sin\theta$ downhill.

Step 1: Resolve the forces along the incline The figure gives $\theta = 60^\circ$ and $m = 200\,\text{g} = 0.200\,\text{kg}$. The horizontal force points towards the plane. Its component along the upward incline is $F\cos 60^\circ$. The normal reaction has no component along the plane, so it cannot balance the downhill component of gravity.

Step 2: Apply equilibrium and calculate the force The along-plane condition is

$$F\cos 60^\circ = mg\sin 60^\circ, \qquad F = mg\tan 60^\circ.$$

Using the standard value $g = 10\,\text{m s}^{-2}$,

$$F = 0.200(10)\sqrt{3} = 2\sqrt{3}\,\text{N} = \sqrt{12}\,\text{N}.$$

The horizontal force must have this value to prevent sliding on the frictionless surface.

Step 3: Compare with the form requested The question writes the force as $F = \sqrt{x}\,\text{N}$. Squaring the numerical coefficient gives $x = (2\sqrt{3})^2 = 12$. As a perpendicular check, $N = mg\cos 60^\circ + F\sin 60^\circ = 1 + 3 = 4\,\text{N}$, which is positive, so contact with the plane is maintained. The parameter x is the numerical value under the radical.

Step 4: Conclusion $F = \sqrt{12}\,\text{N}$, so the required value is $x = 12$.

Quick Tip to Solve: For a horizontal push holding a block on a smooth incline, resolve along the plane to obtain F = mg tan(theta).

54. Moment of Inertia (M.I.) of four bodies having same mass 'M' and radius '2R' are as follows:

I_1 = M.I. of solid sphere about its diameter

I_2 = M.I. of solid cylinder about its axis

I_3 = M.I. of solid circular disc about its diameter

I_4 = M.I. of thin circular ring about its diameter

If $2(I_2 + I_3) + I_4 = x \cdot I_1$, then the value of x will be _____. [4 marks]

Solution:

Concept:

- For radius r, the solid sphere about a diameter has $I = 2Mr^2/5$, and the solid cylinder about its symmetry axis has $I = Mr^2/2$.
- A circular disc about an in-plane diameter has $I = Mr^2/4$; a thin ring about an in-plane diameter has $I = Mr^2/2$. The perpendicular-axis theorem relates these to their perpendicular central axes.

Step 1: Use the common radius consistently All four bodies have radius $2R$, so $r^2 = 4R^2$. Substitution into the standard formulas gives

$$I_1 = \frac{8}{5}MR^2, \quad I_2 = 2MR^2, \quad I_3 = MR^2, \quad I_4 = 2MR^2.$$

The diameter of the disc and ring is an axis lying in their plane, not the axis perpendicular to their plane.

Step 2: Form the prescribed combination Insert the four expressions without changing their assigned axes:

$$2(I_2 + I_3) + I_4 = 2(2MR^2 + MR^2) + 2MR^2 = 8MR^2.$$

The multiplier 2 acts on both I_2 and I_3, whereas I_4 is added once.

Step 3: Extract the dimensionless ratio By the stated equation this sum equals xI_1. Hence

$$x = \frac{8MR^2}{(8/5)MR^2} = 5.$$

Both mass and squared length cancel, as they must because x multiplies one moment of inertia to give another. The result is independent of the numerical mass and radius shared by the bodies.

Step 4: Conclusion The required dimensionless value is $x = 5$.

Quick Tip to Solve: Write the actual rotation axis beside each inertia formula; a disc diameter uses one half of its perpendicular-axis inertia.

55. Two satellites S_1 and S_2 are revolving in circular orbits around a planet with radius R_1 = 3200 km and R_2 = 800 km respectively. The ratio of speed of satellite S_1 to be speed of satellite S_2 in their respective orbits would be $\frac{1}{x}$ where x = _____. [4 marks]

Solution:

Concept:

- For a satellite in a circular orbit of radius r, gravitational attraction supplies centripetal force: $GMm/r^2 = mv^2/r$.
- Therefore $v = \sqrt{GM/r}$, where M is the planet mass. For satellites around the same planet, orbital speed varies as $r^{-1/2}$, independently of satellite mass.

Step 1: Form the ratio for the common planet Let v_1 and v_2 be the speeds of the two satellites. The same gravitational parameter GM occurs in both speed formulas. Thus

$$\frac{v_1}{v_2} = \sqrt{\frac{GM/R_1}{GM/R_2}} = \sqrt{\frac{R_2}{R_1}}.$$

The question supplies orbit radii, so no planet radius is added to them.

Step 2: Substitute the given orbital radii With $R_1 = 3200\,\text{km}$ and $R_2 = 800\,\text{km}$,

$$\frac{v_1}{v_2} = \sqrt{\frac{800}{3200}} = \sqrt{\frac{1}{4}} = \frac{1}{2}.$$

The distances may remain in kilometres here because only their dimensionless ratio is used. Converting both to metres would give the identical ratio.

Step 3: Compare with the specified reciprocal form The required form is $v_1/v_2 = 1/x$. Equating $1/x = 1/2$ gives $x = 2$. This result also passes the physical check: the satellite in the larger orbit moves more slowly, so the first speed must be less than the second. The ratio is one half rather than one quarter because the speed depends on the inverse square root of radius.

Step 4: Conclusion $v_1/v_2 = 1/2$, and the required dimensionless value is $x = 2$.

Quick Tip to Solve: For circular orbits around one planet, use the inverse square root of orbital radius for the speed ratio.

56. When a gas filled in a closed vessel is heated by raising the temperature by 1°C, its pressure increases by 0.4%. The initial temperature of the gas is _____ K. [4 marks]

Solution:

Concept:

- For a fixed amount of ideal gas in a rigid closed vessel, $PV = nRT$. With n and V constant, pressure is proportional to absolute temperature.
- An increase of 1°C is an increase of $1\,\text{K}$. Fractional pressure changes must be related to the initial temperature expressed in kelvin.

Step 1: Write the initial and final gas equations Let the initial pressure and absolute temperature be P and T. After heating, the temperature is $T + 1\,\mathrm{K}$ and the pressure is $P + \Delta P$. Dividing the two constant-volume gas equations gives

$$(P + \Delta P)/P = (T + \Delta T)/T.$$

Subtracting unity yields the exact relation $\Delta P/P = \Delta T/T$.

Step 2: Convert the percentage increase to a fraction The stated pressure increase is 0.4% of the original pressure, so

$$\frac{\Delta P}{P} = \frac{0.4}{100} = 0.004.$$

With $\Delta T = 1\,\mathrm{K}$,

$$0.004 = \frac{1\,\mathrm{K}}{T}, \qquad T = \frac{1}{0.004}\,\mathrm{K} = 250\,\mathrm{K}.$$

Using 0.4 directly would confuse a percentage with a fractional change.

Step 3: Check the initial-temperature interpretation At the calculated temperature, heating from $250\,\mathrm{K}$ to $251\,\mathrm{K}$ makes the pressure ratio $251/250 = 1.004$, exactly a 0.4% rise. The answer requested is the initial temperature; the final temperature would instead be $251\,\mathrm{K}$. An absolute temperature, rather than a Celsius reading, belongs in the gas-law ratio.

Step 4: Conclusion The initial temperature of the gas is $250\,\mathrm{K}$.

Quick Tip to Solve: Use kelvin for the initial temperature and a fractional pressure increase of 0.004 for 0.4 percent.

57. 27 identical drops are charged at 22V each. They combine to form a bigger drop. The potential of the bigger drop will be _____ V. [4 marks]

Solution:

Concept:

- The surface potential of a spherical conducting drop is $V = kq/r$, with zero potential at infinity. Here q and r denote the charge and radius of one small drop, and $k = 1/(4\pi\varepsilon_0)$.
- Coalescence conserves both the total liquid volume and the total electric charge. The larger sphere therefore has radius proportional to the cube root of the number of drops, while its charge is proportional to that number.

Step 1: Find the radius after combining the drops Let the larger radius be R. Since the 27 identical spherical drops combine without loss of liquid, their volumes add:

$$\frac{4\pi}{3}R^3 = 27\frac{4\pi}{3}r^3.$$

Cancelling the common factor gives $R^3 = 27r^3$, so $R = 3r$. The radius does not become 27 times the original radius; it is volume that is additive.

Step 2: Use charge conservation to calculate the new potential Each original drop has potential

$kq/r = 22\,\text{V}$. All drops have the same charge, so the combined charge is $Q = 27q$. Hence

$$V_{\text{big}} = \frac{kQ}{R} = \frac{k(27q)}{3r} = 9\frac{kq}{r} = 9(22) = 198\,\text{V}.$$

The increase of radius reduces the factor of 27 in charge to a factor of nine in potential. This calculation uses the final spherical drop and its own radius, rather than adding the original potentials.

Step 3: Conclusion The potential of the bigger drop is $\boxed{198\,\text{V}}$. The required numerical entry is 198.

Quick Tip to Solve: For n identical equally charged drops, combine charge and volume conservation to get $V_{\text{big}} = n^{2/3}V_{\text{small}}$.

58. The length of a given cylindrical wire is increased to double of its original length. The percentage increase in the resistance of the wire will be _____ %. [4 marks]

Solution:

Concept:

- For a uniform cylindrical wire, $R = \rho l/A$, where ρ is resistivity, l is length and A is the cross-sectional area.
- In stretching the same wire at unchanged temperature, its material volume Al and its resistivity are taken to remain constant. Both the change in length and the resulting change in area must be included.

Step 1: Apply conservation of the wire volume Write the original dimensions as l, A, and the final dimensions as l', A'. The length doubles, so $l' = 2l$. Since the amount of metal is unchanged,

$$Al = A'l' = A'(2l), \qquad A' = \frac{A}{2}.$$

Thus the wire becomes thinner while it becomes longer. Holding the original area fixed would describe a different wire with twice the volume, not a stretched version of this wire.

Step 2: Calculate the resistance ratio and percentage change Using the same resistivity before and after stretching gives

$$R' = \frac{\rho(2l)}{A/2} = 4\frac{\rho l}{A} = 4R.$$

The increase is $R' - R = 3R$, so the percentage increase, measured relative to the original resistance, is

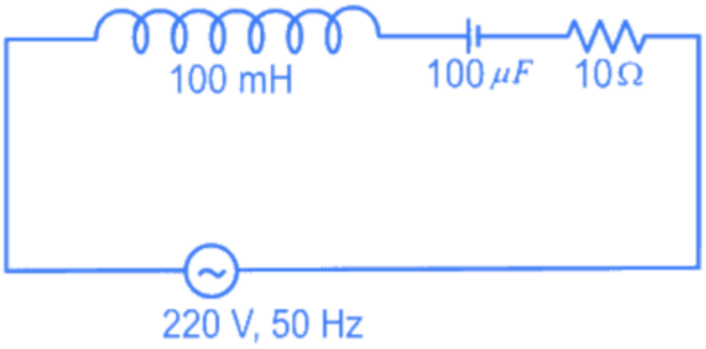
$$\frac{R' - R}{R}\,100\% = \frac{3R}{R}\,100\% = 300\%.$$

The final resistance is 400 percent of the original value, whereas the increase is 300 percent; these two percentages should not be confused.

Step 3: Conclusion The resistance increases by $\boxed{300\%}$. The required numerical entry is 300.

Quick Tip to Solve: For stretching at constant volume and temperature, $R \propto l^2$; square the length ratio, then subtract one before converting to a percentage.

59. In a series LCR circuit, the inductance, capacitance and resistance are L = 100 mH, C = 100 μF and R = 10 Ω respectively. They are connected to an AC source of voltage 220 V and frequency of 50 Hz. The approximate value of current in the circuit will be _____ A. [4 marks]



Solution:

Concept:

- The inductive and capacitive reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$, respectively, where $\omega = 2\pi f$. Their effects oppose each other in a series circuit.
- The impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. For an AC voltage given as its rms value, the rms current is $I = V/Z$. A small net reactance makes the circuit nearly resonant.

Step 1: Convert the components and evaluate both reactances The component values in SI units are $L = 100\,\text{mH} = 0.100\,\text{H}$, $C = 100\,\mu\text{F} = 1.00 \times 10^{-4}\,\text{F}$, and $R = 10\,\Omega$. At $f = 50\,\text{Hz}$,

$$\omega = 100\pi\,\text{rad}\,\text{s}^{-1}, \qquad X_L = 10\pi\,\Omega \simeq 31.416\,\Omega.$$

$$X_C = \frac{1}{100\pi \times 10^{-4}}\,\Omega = \frac{100}{\pi}\,\Omega \simeq 31.831\,\Omega.$$

Their difference is about $-0.415\,\Omega$, much smaller in magnitude than the resistance.

Step 2: Use the impedance to find the approximate current The slightly capacitive net reactance must be squared when calculating the impedance magnitude:

$$Z = \sqrt{10^2 + (-0.415)^2}\,\Omega \simeq 10.0086\,\Omega.$$

Therefore

$$I = \frac{220}{10.0086}\,\text{A} \simeq 21.98\,\text{A} \simeq 22\,\text{A}.$$

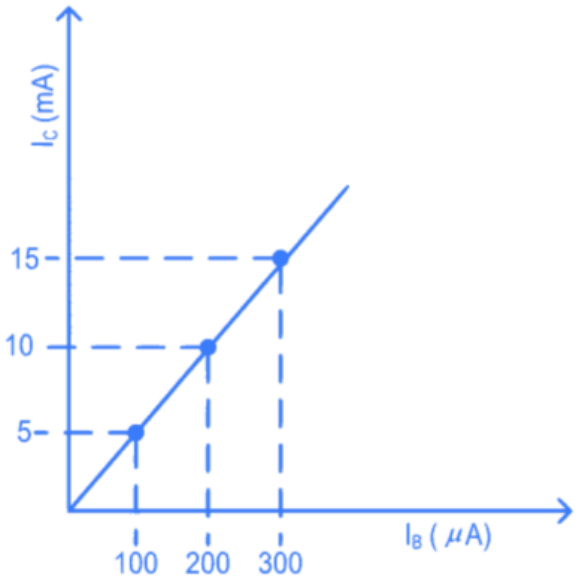
The near cancellation of reactances explains why the current is almost V/R. Adding X_L and X_C as positive resistances would miss this cancellation and predict an incorrect current.

Step 3: Conclusion The approximate current is $\boxed{22\,\text{A}}$, so the numerical entry is 22.

Quick Tip to Solve: Compare X_L and X_C before doing the square root: near resonance their difference is small and I is close to V/R.

60. In an experiment of CE configuration of n-p-n transistor, the transfer characteristics are observed as given in figure.

If the input resistance is 200 Ω and output resistance is 60 Ω, the voltage gain in this experiment will be _____. [4 marks]



Solution:

Concept:

- The small-signal current gain is $\beta_{\text{ac}} = \Delta I_C/\Delta I_B$, obtained from the slope of the transfer characteristic, with both currents expressed in the same units.
- The voltage-gain magnitude is $|A_v| = \beta_{\text{ac}} R_{\text{out}}/R_{\text{in}}$. A common-emitter amplifier inverts its signal, so its signed voltage gain is negative even though numerical gain questions conventionally ask for the magnitude.

Step 1: Read the slope of the transfer characteristic The graph gives $I_C = 5\,\text{mA}$ at $I_B = 100\,\mu\text{A}$, and $I_C = 15\,\text{mA}$ at $I_B = 300\,\mu\text{A}$. Taking the change between these two points gives

$$\Delta I_C = 10\,\text{mA}, \qquad \Delta I_B = 200\,\mu\text{A} = 0.200\,\text{mA}.$$

Hence

$$\beta_{\text{ac}} = \frac{10}{0.200} = 50.$$

The milliampere and microampere scales differ by a factor of one thousand, so they cannot be divided as bare numbers without conversion.

Step 2: Convert current gain into voltage gain The input voltage change has magnitude $\Delta I_B R_{\text{in}}$, while the output voltage change has magnitude $\Delta I_C R_{\text{out}}$. Their ratio is therefore

$$|A_v| = \frac{\Delta I_C R_{\text{out}}}{\Delta I_B R_{\text{in}}} = 50\frac{60}{200} = 15.$$

The resistance ratio is dimensionless, as is voltage gain. The common-emitter phase inversion would give $A_v = -15$ under a signed output-to-input convention; the gain magnitude requested for the

numerical entry is positive.

Step 3: Conclusion The voltage-gain magnitude is $\boxed{15}$ (dimensionless). The signed common-emitter voltage gain is -15.

Quick Tip to Solve: Convert the transfer-graph currents to common units first; then multiply its slope by $R_{\text{out}}/R_{\text{in}}$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The minimum energy that must be possessed by photons in order to produce the photoelectric effect with platinum metal is :

[Given : The threshold frequency of platinum is 1.3×10^{15} s^{-1} and h = 6.6×10^{-34} J s.] [4 marks]

(A) 3.21×10^{-14} J

(B) 6.24×10^{-16} J

(C) 8.58×10^{-19} J

(D) 9.76×10^{-20} J

Correct Answer: (C) 8.58×10^{-19} J

Solution:

Concept:

- A photon of frequency ν has energy $E = h\nu$. The threshold frequency ν_0 is the lowest frequency capable of releasing an electron from the metal surface.
- The corresponding work function is $\phi = h\nu_0$. In Einstein's photoelectric equation $K_{\max} = h\nu - \phi$, the maximum kinetic energy is zero at threshold and becomes positive above threshold.

Step 1: Identify the energy required at threshold For platinum the supplied threshold frequency is $\nu_0 = 1.3 \times 10^{15}\,\text{s}^{-1}$. The photon must supply at least the work function to overcome the surface binding. Thus the required minimum is $E_{\min} = h\nu_0$, with no extra kinetic-energy term. Using a frequency smaller than ν_0 would not cause photoemission even if the light intensity were increased.

Step 2: Substitute the given constants and compare the options Insert $h = 6.6 \times 10^{-34}\,\text{J s}$:

$$E_{\min} = (6.6 \times 10^{-34})(1.3 \times 10^{15})\,\text{J} = 8.58 \times 10^{-19}\,\text{J}.$$

The exponent is $-34 + 15 = -19$, and the seconds cancel. Option (C) matches both the coefficient and power of ten. Options (A) and (B) have energies much larger than the threshold value, so they are not the minimum. Option (D) is smaller than the work function and cannot eject electrons from platinum at all.

Step 3: Conclusion The minimum photon energy is $\boxed{8.58 \times 10^{-19}\,\text{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: At the threshold frequency set $K_{\max} = 0$; multiply $h\nu_0$ and add the powers of ten separately to avoid exponent errors.

62. At 25°C and 1 atm pressure, the enthalpy of combustion of benzene (I) and acetylene (g) are -3268 kJ mol^{-1} and -1300 kJ mol^{-1}, respectively. The change in enthalpy for the reaction 3 C_2H_2(g) $\rightarrow$ C_6H_6 (I), is : [4 marks]

(A) $+324$ kJ mol^{-1}

(B) $+632$ kJ mol^{-1}

(C) $-$ 632 kJ mol^{-1}

(D) $-$ 732 kJ mol^{-1}

Correct Answer: (C) $-$ 632 kJ mol^{-1}

Solution:

Concept:

- Enthalpy is a state function: Hess's law permits reaction enthalpies to be added when the corresponding chemical equations are added.
- Multiply an equation's enthalpy by its stoichiometric multiplier. Reverse its sign when reversing the reaction. Combusting three moles of acetylene and one mole of benzene produces the same amounts of carbon dioxide and liquid water.

Step 1: Write the two combustion routes to identical products Three moles of acetylene undergo

$$3\,C_2H_2(g) + (15/2)\,O_2(g) \longrightarrow 6\,CO_2(g) + 3\,H_2O\,(l),$$

$$\Delta H_1 = 3(-1300) = -3900\,\text{kJ}.$$

For one mole of liquid benzene,

$$C_6H_6(l) + (15/2)\,O_2(g) \longrightarrow 6\,CO_2(g) + 3\,H_2O\,(l),$$

$$\Delta H_2 = -3268\,\text{kJ}.$$

Both routes have exactly the same combustion products, including their physical states, at the stated temperature and pressure.

Step 2: Subtract the benzene combustion equation Reverse the benzene combustion equation so its enthalpy becomes $+3268$ kJ. Adding it to the first equation cancels oxygen, carbon dioxide and water, leaving

$$3\,C_2H_2(g) \longrightarrow C_6H_6(l).$$

Therefore

$$\Delta H = -3900 + 3268 = -632\,\text{kJ mol}^{-1}.$$

Here the molar enthalpy refers to one mole of reaction as written, producing one mole of benzene. Option (B) has the correct magnitude but the wrong sign; (A) and (D) do not equal the stoichiometrically weighted difference of the given combustion enthalpies.

Step 3: Conclusion The reaction enthalpy is $\boxed{-632\,\text{kJ mol}^{-1}}$; trimerisation is exothermic. Hence, the correct option is (C).

Quick Tip to Solve: When reactants and product combust to the same final substances, subtract product combustion enthalpy from the stoichiometric reactant total.

63. Solute A associates in water. When 0.7 g of solute A is dissolved in 42.0 g of water, it depresses the freezing point by 0.2°C. The percentage association of solute A in water, is :
[Given : Molar mass of A = 93 g mol^{-1}. Molal depression constant of water is 1.86 K kg mol^{-1}.]

[4 marks]

(A) 50%

(B) 60%

(C) 70%

(D) 80%

Correct Answer: 80% for dimerisation (D); the question does not specify the aggregate size, so other percentages, including 60% for trimerisation (B), also fit.

Solution:

Concept:

- Freezing-point depression measures the number of dissolved particles: $\Delta T_f = iK_f m$, where m is calculated from the unassociated solute formula mass and i is the van't Hoff factor.
- If a fraction α of monomer units associates into aggregates containing n units each, particle counting gives $i = 1 - \alpha + \alpha/n$. The aggregation number is needed to convert i into a percentage association.

Step 1: Calculate the observed particle factor The initial amount of A is $0.7/93\,\text{mol}$, and the solvent mass is $42.0\,\text{g} = 0.0420\,\text{kg}$. Thus

$$m = \frac{0.7/93}{0.0420} = \frac{50}{279}\,\text{mol kg}^{-1}.$$

Without association,

$$K_f m = 1.86\frac{50}{279}\,\text{K} = \frac{1}{3}\,\text{K}.$$

The measured depression is $0.2\,\text{K}$, so

$$i = \frac{0.2}{1/3} = 0.60.$$

This value establishes a reduction in the particle count but does not by itself specify the aggregate size.

Step 2: Examine the association model needed for a percentage Under the usual dimerisation model $2\,\text{A} \rightleftharpoons \text{A}_2$, $n = 2$, so $0.60 = 1 - \alpha/2$ and $\alpha = 0.80$. This gives 80 percent, option (D). However, the printed question says only that A associates; it does not specify dimers. For trimerisation, $n = 3$ gives

$$0.60 = 1 - \frac{2\alpha}{3}, \qquad \alpha = 0.60,$$

which gives 60 percent, option (B), with the same observed depression. Option (A) would require $n = 5$, also not excluded by the stated data. Option (C) would require a nonintegral single aggregate size $n = 7/3$. Therefore selecting one percentage requires an additional chemical assumption.

Step 3: Conclusion The measured van't Hoff factor is $\boxed{i = 0.60}$. Assuming dimerisation, the expected answer is $\boxed{80\%}$, option (D). As printed, the aggregation number is unspecified, so the percentage is

not uniquely determined; trimerisation instead gives 60%, option (B).

Quick Tip to Solve: Determine i from the colligative data first. Use $\alpha = (1-i)/(1-1/n)$ only after identifying the aggregate size n.

64. The K_{sp} for bismuth sulphide (Bi_2S_3) is 1.08×10^{-73}. The solubility of Bi_2S_3 in mol L^{-1} at 298 K is : [4 marks]

(A) 1.0×10^{-15}

(B) 2.7×10^{-12}

(C) 3.2×10^{-10}

(D) 4.2×10^{-8}

Correct Answer: (A) 1.0×10^{-15}

Solution:

Concept:

- The molar solubility s is the number of moles of the salt dissolved per litre of saturated solution. Ion concentrations follow the stoichiometric coefficients of its dissolution equation.
- The solubility product is the product of equilibrium ion concentrations, each raised to its coefficient. For Bi_2S_3, this gives $K_{sp} = (2s)^2(3s)^3 = 108s^5$.

Step 1: Write the dissolution equilibrium One formula unit produces two bismuth ions and three sulphide ions:

$$Bi_2S_3(s) \rightleftharpoons 2\,Bi^{3+}(aq) + 3\,S^{2-}(aq).$$

If the solubility is $s\,\text{mol}\,L^{-1}$, then $[Bi^{3+}] = 2s$ and $[S^{2-}] = 3s$. The solid does not occur in the expression because its activity is constant.

Step 2: Substitute the solubility product Using the stated value at 298 K,

$$1.08 \times 10^{-73} = 108s^5, \qquad s^5 = 1.0 \times 10^{-75}.$$

Taking the fifth root gives

$$s = (10^{-75})^{1/5} = 1.0 \times 10^{-15}\,\text{mol}\,L^{-1}.$$

The factor 108 must be retained; treating the salt as a 1:1 electrolyte would give a wrong power and a wrong value. Substitution of this result returns exactly $108 \times 10^{-75} = 1.08 \times 10^{-73}$, whereas the other proposed solubilities are much too large.

Step 3: Conclusion The solubility is $1.0 \times 10^{-15}\,\text{mol}\,L^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: For a salt giving two cations and three anions, write $108s^5$ before taking any root; do not use the 1:1 formula $K_{sp} = s^2$.

65. The correct order of electron gain enthalpies of Cl, F, Te and Po is [4 marks]

(A) $F < Cl < Te < Po$

(B) $Po < Te < F < Cl$

(C) $Te < Po < Cl < F$

(D) $Cl < F < Te < Po$

Correct Answer: (D) $Cl < F < Te < Po$

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change for $X(g) + e^- \longrightarrow X^-(g)$. An exothermic electron addition has a negative value, so a more negative number is smaller in an algebraic ordering.
- Halogens generally have more negative electron gain enthalpies than group 16 elements. Chlorine is more negative than fluorine because the compact fluorine 2p shell causes greater electron-electron repulsion.

Step 1: Compare chlorine and fluorine Although fluorine is smaller and more electronegative, the added electron enters its very compact 2p subshell. Repulsion from the electrons already present reduces the energy released. Chlorine accepts the electron into the more spacious 3p subshell. Thus chlorine has the more negative electron gain enthalpy:

$$\Delta_{eg}H(Cl) < \Delta_{eg}H(F).$$

Electronegativity and electron gain enthalpy must not be treated as identical properties.

Step 2: Place tellurium and polonium Tellurium and polonium are group 16 elements and their electron gain enthalpies are less negative than those of the halogens here. Down this part of the group, increasing size reduces the attraction for the added electron, so polonium is less negative than tellurium. Combining these comparisons yields

$$Cl < F < Te < Po.$$

Option A incorrectly reverses the exceptional chlorine-fluorine comparison. Option B reverses the signed ordering, and option C incorrectly places the group 16 elements before the halogens.

Step 3: Conclusion The signed electron gain enthalpy order is $Cl < F < Te < Po$. Hence, the correct option is (D). This differs from the supplied reversed-order key: that reversed order compares magnitudes of energy released, whereas the printed electron gain enthalpies are signed negative quantities.

Quick Tip to Solve: Write the sign convention first: more energy released means a more negative, hence numerically smaller, electron gain enthalpy.

66. Given below are two statements one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : The amphoteric nature of water is explained by using Lewis acid/base concept.

Reason R : Water acts as an acid with NH_3 and as a base with H_2S.

In the light of the above statements choose the correct answer from the options given below :

[4 marks]

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Correct Answer: (D) A is false but R is true.

Solution:

Concept:

- In the Brønsted–Lowry description, an acid donates a proton and a base accepts a proton. A substance capable of both actions is amphoteric in proton-transfer reactions.
- A Lewis base donates an electron pair and a Lewis acid accepts an electron pair. Water readily donates a lone pair, but its amphoteric proton-transfer behaviour is conventionally explained with the Brønsted–Lowry concept.

Step 1: Evaluate the assertion Water can donate H^+ to a suitable base or accept H^+ from a suitable acid. The acid-base framework that directly explains these two roles is the Brønsted–Lowry proton-transfer concept. Calling this explanation the Lewis acid/base concept confuses proton donation with acceptance of an electron pair by the whole water molecule. Therefore Assertion A, as stated in this classification question, is false.

Step 2: Evaluate the reason With ammonia, water supplies a proton:

$$NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-.$$

Here water is the acid and ammonia is the base. With hydrogen sulphide, water accepts a proton:

$$H_2S + H_2O \rightleftharpoons H_3O^+ + HS^-.$$

Here water is the base. Both examples in Reason R therefore correctly describe water's behaviour, making R true.

Step 3: Select the truth-value combination Since A is false and R is true, neither option claiming both statements true can apply. The option claiming A true and R false reverses both evaluations. The required combination is precisely the fourth option; there is no need to assess an explanatory relationship when the assertion itself is false.

Step 4: Conclusion Assertion A is false, but Reason R is true. Hence, the correct option is (D).

Quick Tip to Solve: For water acting with ammonia and hydrogen sulphide, track the proton: donation makes water the acid, and acceptance makes it the base.

67. The correct order of reduction potentials of the following pairs is

A. Cl_2/Cl^-

B. I^2/I^-

C. Ag^+/Ag

D. Na^+/Na

E. Li^+/Li

Choose the correct answer from the options given below. [4 marks]

(A) A > C > B > D > E

(B) A > B > C > D > E

(C) A > C > B > E > D

(D) A > B > C > E > D

Correct Answer: (A) A > C > B > D > E

Solution:

Concept:

- A reduction potential refers to the tendency of a species to gain electrons. A more positive standard reduction potential means that the reduction is more favourable relative to the standard hydrogen electrode.
- The relevant standard values in volts are approximately Cl_2/Cl^- : $+1.36$, Ag^+/Ag : $+0.80$, I_2/I^- : $+0.54$, Na^+/Na : -2.71, and Li^+/Li : -3.04.

Step 1: Rank the positive potentials Chlorine has the largest positive value among the listed couples and therefore comes first. Silver ion is reduced more readily than iodine under standard conditions, so the sequence of positive reduction potentials is

$$E^\circ(Cl_2/Cl^-) > E^\circ(Ag^+/Ag) > E^\circ(I_2/I^-).$$

In terms of the question's labels, this is $A > C > B$. The unusual superscript in the printed iodine label denotes the iodine/iodide couple in this electrochemical comparison.

Step 2: Rank the alkali-metal couples Both alkali-metal reduction potentials are negative. The value -2.71 V for sodium is larger than -3.04 V for lithium, so $D > E$. Lithium has a very negative aqueous standard reduction potential because of the energetics of forming its strongly hydrated ion; its place cannot be decided merely from atomic ionisation energy. Combining all five gives

$$A > C > B > D > E.$$

Options B and D incorrectly place iodine above silver. Option C gets those three right but reverses sodium and lithium.

Step 3: Conclusion The decreasing order of reduction potentials is $A > C > B > D > E$. Hence, the correct option is (A).

Quick Tip to Solve: Compare the signed standard potentials: among negative values, -2.71 is greater than -3.04. Do not rank their absolute magnitudes.

68. The metal ion (in gaseous state) with lowest spin-only magnetic moment value is : [4 marks]

(A) V^{2+}

(B) Ni^{2+}

(C) Cr^{2+}

(D) Fe^{2+}

Correct Answer: (B) Ni^{2+}

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons and μ_B is the Bohr magneton. It increases as n increases.
- For transition-metal cations, remove 4s electrons before 3d electrons. In an isolated gaseous ion, fill the five degenerate d orbitals according to Hund's rule; no ligand-field pairing is imposed.

Step 1: Find the d-electron configurations Vanadium has the neutral configuration $[\mathrm{Ar}]3d^3 4s^2$, giving $V^{2+} : 3d^3$. Nickel gives $Ni^{2+} : 3d^8$. Chromium starts as $[\mathrm{Ar}]3d^5 4s^1$; removing two electrons gives $Cr^{2+} : 3d^4$. Iron gives $Fe^{2+} : 3d^6$. The chromium exception must be handled before counting the unpaired electrons.

Step 2: Count spins and compare moments For free gaseous ions, d^3 has three unpaired electrons, d^8 has two, d^4 has four, and d^6 also has four. Hence

$$\mu(V^{2+}) = \sqrt{15}\,\mu_B,$$
$$\mu(Ni^{2+}) = \sqrt{8}\,\mu_B,$$
$$\mu(Cr^{2+}) = \mu(Fe^{2+}) = \sqrt{24}\,\mu_B.$$

The smallest is $\sqrt{8} \approx 2.83\,\mu_B$, belonging to nickel(II). Vanadium has one more unpaired electron, while chromium(II) and iron(II) have two more; none can have a smaller spin-only moment.

Step 3: Conclusion Ni^{2+} has the lowest spin-only magnetic moment, $\sqrt{8}\,\mu_B \approx 2.83\,\mu_B$. Hence, the correct option is (B).

Quick Tip to Solve: The words gaseous ion mean use free-ion Hund filling; first remove 4s electrons and then count the unpaired 3d electrons.

69. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : A mixture contains benzoic acid and napthalene. The pure benzoic acid can be separated out by the use of benzene.

Reason R : Benzoic acid is soluble in hot water.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Correct Answer: (D) A is false but R is true.

Solution:

Concept:

- A solvent separates a mixture by selective dissolution only when its components have sufficiently different solubilities. Dissolving both substances together does not isolate a pure component.
- Benzoic acid dissolves well in hot water and much less in cold water, whereas naphthalene is practically insoluble in water. This difference permits hot filtration followed by crystallisation on cooling.

Step 1: Evaluate the assertion Benzoic acid and naphthalene are both organic substances that dissolve in benzene. Thus using benzene does not selectively take benzoic acid into solution while leaving naphthalene behind. Merely evaporating such a solution would recover the mixture rather than establish pure benzoic acid. The proposed use of benzene as the separating solvent is therefore incorrect, so Assertion A is false.

Step 2: Evaluate the reason and describe the useful separation Reason R is true: benzoic acid is soluble in hot water. Treat the mixture with a suitable small quantity of hot water so the benzoic acid dissolves. Naphthalene remains as the insoluble material and can be removed by hot filtration. On cooling the filtrate, benzoic acid crystallises because its solubility falls. Filter and dry these crystals. This account also shows why the choice of solvent, rather than solubility in an arbitrary organic solvent, is decisive.

Step 3: Choose the matching option The two truth values are A false and R true. Options A and B both require a true assertion and therefore fail immediately. Option C requires the opposite truth-value combination. Only option D correctly states that the suggested benzene separation is false while the hot-water solubility statement is true.

Step 4: Conclusion Assertion A is false, but Reason R is true. Hence, the correct option is (D).

Quick Tip to Solve: To separate benzoic acid from naphthalene, exploit hot-water solubility and cooling crystallisation; benzene dissolves both components.

70. During halogen test, sodium fusion extract is boiled with concentrated HNO_3 to [4 marks]

(A) remove unreacted sodium

(B) decompose cyanide or sulphide of sodium

(C) extract halogen from organic compound

(D) maintain the pH of extract.

Correct Answer: (B) decompose cyanide or sulphide of sodium

Solution:

Concept:

- Sodium fusion converts covalently bound halogens into water-soluble sodium halides, which give silver halide precipitates with silver nitrate after suitable acidification.
- If nitrogen or sulphur is present, the extract may also contain $NaCN$ or Na_2S. Cyanide and sulphide react with silver ions and would interfere with identifying the halide precipitate.

Step 1: Identify the interfering precipitates Adding silver nitrate directly to an untreated extract could precipitate not only a silver halide but also silver cyanide or silver sulphide:

$$Ag^+ + CN^- \longrightarrow AgCN\,(s), \qquad 2\,Ag^+ + S^{2-} \longrightarrow Ag_2S\,(s).$$

These unwanted precipitates can be mistaken for, or conceal, the result of the halogen test. The extract therefore needs pretreatment before the diagnostic silver nitrate reagent is added.

Step 2: Explain the nitric-acid treatment Boiling the extract with concentrated nitric acid decomposes the sodium cyanide and sodium sulphide present and removes their interference. Nitric acid is used rather than hydrochloric acid, since hydrochloric acid would itself introduce chloride ions and cause a misleading silver chloride precipitate. The necessary conversion of organic halogen into sodium halide has already occurred during sodium fusion.

Step 3: Eliminate the other purposes The aim is not to remove metallic sodium: the aqueous fusion extract is prepared only after the fused material has been worked up with water. Nor does nitric acid perform the original extraction of halogen from the organic molecule. Although acidification changes pH, merely maintaining a pH is not the specific analytical reason for boiling with concentrated acid. Decomposition of cyanide and sulphide is the required purpose.

Step 4: Conclusion The extract is boiled with concentrated HNO_3 to decompose sodium cyanide or sodium sulphide. Hence, the correct option is (B).

Quick Tip to Solve: Before adding silver nitrate, destroy cyanide and sulphide interference with nitric acid; avoid hydrochloric acid because it supplies chloride.

71. Amongst the following, the major product of the given chemical reaction is [4 marks]



(A) Trans-3-bromo-2-methoxytetrahydropyran: Br dashed at C3 and OCH3 wedged at C2.

(B) Cis-3-bromo-2-methoxytetrahydropyran: Br and OCH3 both wedged.

(C) Trans-2-bromo-3-methoxytetrahydropyran: OCH3 dashed at C3 and Br wedged at C2.

(D) Trans-2-bromo-3-methoxytetrahydropyran: OCH3 wedged at C3 and Br dashed at C2.

Correct Answer: (A) Trans-3-bromo-2-methoxytetrahydropyran: Br dashed at C3 and OCH3 wedged at C2.

Solution:

Concept:

- An enol ether has an electron-rich carbon–carbon double bond adjacent to oxygen. The oxygen lone pair stabilises positive charge on the neighbouring carbon through resonance.
- Bromine in methanol adds as Br and methoxy across the double bond. Opening of a bromonium-like intermediate by solvent gives anti addition.

Step 1: Locate the two alkene carbons Number the ring oxygen as position 1. The double bond is between C2, adjacent to oxygen, and C3. Addition must remove this double bond while retaining the six-membered ring. Methanol is present as solvent and nucleophile; therefore the expected product is a bromo ether rather than a simple vicinal dibromide.

Step 2: Determine the regiochemistry Electrophilic bromine attacks the double bond. The developing positive charge is better accommodated at C2 because it is adjacent to the ring oxygen. Lone-pair donation produces an oxocarbenium resonance contribution. Methanol consequently attacks C2, and bromine remains attached at C3. Deprotonation of the attacking methanol oxygen leaves an OCH3 substituent.

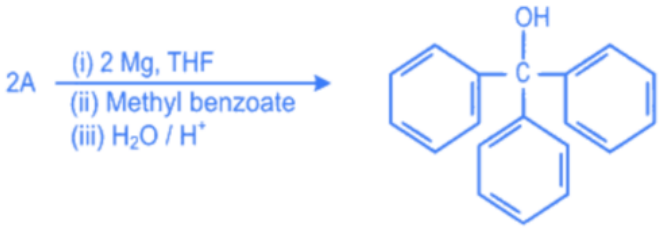
Step 3: Check the stereochemistry and alternatives Attack opposite the bridging bromine gives the anti product: Br at C3 and OCH3 at C2 lie on opposite faces of the ring. Option (A) shows this arrangement. Option (B) has the same connectivity but cis substituents, contrary to anti addition. Options (C) and (D) put methoxy at C3 and bromine at C2, reversing the oxygen-directed regiochemistry. Attack on either enantiotopic face may give the corresponding enantiomer; the displayed anti regioisomer is the required choice.

Step 4: Conclusion The major product is trans-3-bromo-2-methoxytetrahydropyran, with methoxy next to the ring oxygen. Hence, the correct option is (A).

Quick Tip to Solve: For halogen addition to an enol ether in an alcohol, place OR on the carbon adjacent to oxygen and check that Br and OR are anti.

72. In the given reaction

'A' can be [4 marks]



(A) benzyl bromide

(B) bromobenzene

(C) cyclohexyl bromide

(D) methyl bromide

Correct Answer: (B) bromobenzene

Solution:

Concept:

- An ester reacts with two equivalents of a Grignard reagent to give a tertiary alcohol after acidic work-up. The ester alkoxy group is lost during the first addition.
- For $C_6H_5COOCH_3$, the original phenyl group remains on the carbinol carbon. The two other carbon groups are supplied by the Grignard reagent.

Step 1: Read the carbon skeleton of the product The drawn product is triphenylmethanol, $(C_6H_5)_3COH$. The OH-bearing carbon is bonded directly to three phenyl groups. Methyl benzoate supplies one of these groups through its benzoyl part. Therefore each of the two equivalents of reagent must supply a phenyl group, not benzyl, methyl or cyclohexyl.

Step 2: Form the required reagent Bromobenzene reacts with magnesium in dry THF to form phenylmagnesium bromide:

$$C_6H_5Br + Mg \longrightarrow C_6H_5MgBr.$$

Its phenyl carbon attacks the ester carbonyl. Elimination of methoxide gives benzophenone, which undergoes a second addition of phenylmagnesium bromide. Protonation of the resulting magnesium alkoxide then gives triphenylmethanol.

Step 3: Exclude the remaining halides Benzyl bromide would produce benzylmagnesium bromide and introduce CH2C6H5 groups, giving extra methylene links absent from the product. Cyclohexyl bromide would introduce saturated cyclohexyl groups instead of aromatic phenyl groups. Methyl bromide would give methylmagnesium bromide and hence two methyl substituents on the alcohol carbon. Only bromobenzene supplies phenyl groups directly, so only its carbon skeleton fits the drawing.

Step 4: Conclusion A is bromobenzene, which forms phenylmagnesium bromide in magnesium/THF. Hence, the correct option is (B).

Quick Tip to Solve: For ester plus two equivalents of RMgX, retain the ester acyl substituent and attach R twice to the final OH-bearing carbon.

73. Which of the following conditions or reaction sequence will NOT give acetophenone as the major product? [4 marks]

(A) (a) $C_6H_5CHO + CH_3MgBr$; (b) $Na_2Cr_2O_7,\ H^+$.

(B) (a) $CH_3CHO + C_6H_5MgBr$; (b) PCC, DCM.

(C) $C_6H_5COOC_2H_5 + 2\,CH_3MgBr$.

(D) $C_6H_5COCl + CH_3MgBr + CdCl_2$.

Correct Answer: (C) $C_6H_5COOC_2H_5 + 2\,CH_3MgBr$.

Solution:

Concept:

- Acetophenone is $C_6H_5COCH_3$, an aryl methyl ketone. Oxidation of the secondary alcohol 1-phenylethanol gives this ketone.
- Esters ordinarily consume two equivalents of a Grignard reagent: the ketone intermediate is more reactive and undergoes further addition. Organocadmium reagents can convert acid chlorides into ketones.

Step 1: Test the two aldehyde routes In (A), methylmagnesium bromide adds a methyl group to benzaldehyde. After protonation the product is $C_6H_5CH(OH)CH_3$. Acidified dichromate oxidises this secondary alcohol to acetophenone. In (B), phenylmagnesium bromide adds phenyl to ethanal and produces the same secondary alcohol; PCC in dichloromethane also oxidises it to acetophenone. Thus both (A) and (B) give the desired ketone.

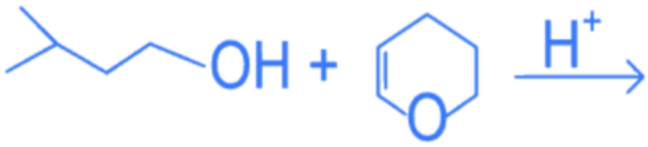
Step 2: Follow both additions to the ester In (C), ethyl benzoate first reacts with methylmagnesium bromide to form acetophenone after ethoxide is eliminated. This ketone does not remain unchanged in the presence of the second equivalent: another methyl group adds to its carbonyl. Acidic work-up gives $C_6H_5C(OH)(CH_3)_2$, a tertiary alcohol. It cannot be oxidised to acetophenone by the simple secondary-alcohol pathway because the carbinol carbon has no hydrogen.

Step 3: Check the acid chloride route In (D), cadmium chloride converts methylmagnesium bromide to an organocadmium reagent. This less reactive carbon nucleophile acylates benzoyl chloride and stops at the ketone stage. The methyl group replaces chlorine, furnishing $C_6H_5COCH_3$. Therefore (D) is also a ketone preparation, while the ester with two Grignard equivalents is the exceptional route.

Step 4: Conclusion The ester reaction in (C) gives a tertiary alcohol rather than acetophenone as the major final product. Hence, the correct option is (C).

Quick Tip to Solve: When an ester meets two equivalents of a Grignard reagent, carry the mechanism past the transient ketone to the tertiary alcohol.

74. The major product formed in the following reaction, is [4 marks]



(A) 3-(3-Methylbutoxy)tetrahydropyran.

(B) $(CH_3)_2CHCH_2CH_2OCH_2CH_2CH_2CH{=}CHOH$.

(C) $(CH_3)_2CHCH_2CH_2OCH_2CH_2CH_2CH_2CHO$.

(D) 2-(3-Methylbutoxy)tetrahydropyran.

Correct Answer: (D) 2-(3-Methylbutoxy)tetrahydropyran.

Solution:

Concept:

- An alcohol reacts with 3,4-dihydro-2H-pyran under acid catalysis to form its tetrahydropyranyl (THP) ether. This is a common reversible protection of an alcohol.
- Protonation of an enol ether generates an oxocarbenium ion stabilised by the adjacent ring oxygen. Alcohol attack occurs at the carbon next to that oxygen.

Step 1: Identify the alcohol and enol ether The alcohol is 3-methylbutan-1-ol, $(CH_3)_2CHCH_2CH_2OH$. The second reactant is dihydropyran, a cyclic enol ether. The reaction conditions show acid catalysis, so the alcohol oxygen acts as the nucleophile. Its carbon chain remains intact throughout the protection reaction; cleavage of the alcohol carbon skeleton is not involved.

Step 2: Generate the stabilised electrophile Proton addition to the double bond places the positive charge on the carbon adjacent to the pyran oxygen. The ring oxygen donates a lone pair and stabilises this oxocarbenium centre. The alcohol oxygen attacks that centre, creating a new bond between RO and C2 of the tetrahydropyran ring. Loss of a proton regenerates the acid catalyst and yields the neutral acetal.

Step 3: Compare the displayed structures The product retains the six-membered oxygen-containing ring and bears the 3-methylbutoxy group at C2, directly adjacent to the ring oxygen. This is the structure in (D). Option (A) puts the alkoxy substituent at a different ring carbon and does not represent attack at the resonance-stabilised centre. Options (B) and (C) open the cyclic system into a chain containing an enol or aldehyde, respectively. Neither is the THP ether formed under these alcohol-protection conditions.

Step 4: Conclusion The major product is the 2-tetrahydropyranyl ether of 3-methylbutan-1-ol, shown in (D). Hence, the correct option is (D).

Quick Tip to Solve: In THP protection, keep the pyran ring closed and attach the alcohol oxygen to the ring carbon directly next to the ring oxygen.

75. Which of the following ketone will NOT give enamine on treatment with secondary amines? [where t-Bu is $-C(CH_3)_3$] [4 marks]

(A) $C_2H_5COC_2H_5$.

(B) $C_2H_5COCH_3$.

(C) $(CH_3)_3CCOC(CH_3)_3$.

(D) Cyclohexanone.

Correct Answer: (C) $(CH_3)_3CCOC(CH_3)_3$.

Solution:

Concept:

- A secondary amine reacts with a carbonyl compound through a carbinolamine and then an iminium ion. Removal of an alpha hydrogen converts the iminium intermediate into an enamine.
- An alpha carbon is a carbon directly bonded to the carbonyl carbon. Enamine formation requires at least one hydrogen on such a carbon, so that a C=C bond can form next to nitrogen.

Step 1: Apply the structural requirement The nitrogen in a secondary amine already carries two carbon substituents. After condensation, the iminium ion cannot simply lose an N–H proton to form a neutral imine. Instead, a base removes an alpha C–H proton. The electrons form a double bond between the former carbonyl carbon and the alpha carbon, while the C=N electrons return to nitrogen. A ketone lacking alpha hydrogens therefore cannot complete this enamine-forming step.

Step 2: Inspect the acyclic ketones Option (A), diethyl ketone, has a CH2 group on each side of the carbonyl, providing alpha hydrogens. Option (B), ethyl methyl ketone, has alpha hydrogens on both the methyl and methylene groups. In (C), each carbon directly attached to the carbonyl is the central carbon of a tert-butyl group. Each of these carbons is already bonded to three methyl groups and the carbonyl carbon; it has no hydrogen.

Step 3: Check the cyclic ketone and conclude Cyclohexanone in (D) has methylene groups directly adjacent to the carbonyl, so it too can form an enamine. The methyl hydrogens of the tert-butyl groups in (C) are not alpha hydrogens: their carbon atoms are one bond farther away. They cannot be removed to make the required adjacent double bond. Thus (A), (B) and (D) meet the necessary structural condition, while (C) fails it irrespective of steric effects.

Step 4: Conclusion Di-tert-butyl ketone has no alpha hydrogen and will not give an enamine with a secondary amine. Hence, the correct option is (C).

Quick Tip to Solve: Circle the carbons immediately touching C=O, then count their hydrogens; hydrogens on more distant methyl groups do not qualify.

76. Match List I with List II

	List - I		List -II
A.	Zymase	I.	Stomach
B.	Diastase	II.	Yeast
C.	Urease	III.	Malt
D.	Pepsin	IV.	Soyabean

Choose the correct answer from the options given below : [4 marks]

(A) A-II, B-III, C-I, D-IV

(B) A-II, B-III, C-IV, D-I

(C) A-III, B-II, C-IV, D-I

(D) A-III, B-II, C-I, D-IV

Correct Answer: (B) A-II, B-III, C-IV, D-I

Solution:

Concept:

- Enzymes are biological catalysts associated with characteristic organisms or tissues. A matching question is solved by identifying each enzyme source independently.
- Zymase is associated with alcoholic fermentation by yeast; diastase with malt; urease with soyabean; and pepsin with the stomach.

Step 1: Match the fermentation and starch enzymes Zymase is the enzyme complex present in yeast that catalyses alcoholic fermentation of sugars. It therefore matches II, yeast. Diastase refers to starch-hydrolysing enzyme activity found in malt, where it converts starch into smaller carbohydrates during germination and malting. Thus diastase matches III. These two observations fix A–II and B–III before the remaining enzymes are considered.

Step 2: Match the remaining enzymes Urease catalyses the hydrolysis of urea to ammonia and carbon dioxide and is obtained from sources such as soyabean. It matches IV. Pepsin is a proteolytic enzyme of gastric juice; it acts on proteins in the acidic environment of the stomach. It matches I. The completed correspondence is therefore A–II, B–III, C–IV, D–I.

Step 3: Check every proposed combination Option (B) reproduces all four correspondences exactly. Option (A) retains the first two but exchanges the sources of urease and pepsin, incorrectly assigning pepsin to soyabean and urease to the stomach. Options (C) and (D) exchange the first two sources, incorrectly assigning zymase to malt and diastase to yeast; (D) also reverses the last two. Recognising just one pair may narrow the choices, but checking the complete mapping prevents accepting a partially correct option.

Step 4: Conclusion The matching is A–II, B–III, C–IV, D–I. Hence, the correct option is (B).

Quick Tip to Solve: Use pepsin–stomach as a quick anchor, then verify zymase–yeast and diastase–malt before choosing the complete sequence.

77. **Given below are two statements.**

Statement I : During electrolytic refining, blister copper deposits precious metals.

Statement II : In the process of obtaining pure copper by electrolysis method, copper blister is used to make the anode.

In the light of the above statements, choose the correct from the options given below. [4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (A) Both Statement I and Statement II are true.

Solution:

Concept:

- Electrolytic refining uses impure blister copper as the anode, a thin sheet of pure copper as the cathode, and an acidified copper sulphate solution as electrolyte.
- Copper dissolves anodically and deposits cathodically. Noble metal impurities do not dissolve with copper under these conditions and accumulate beneath the anode as anode mud.

Step 1: Assess Statement I Blister copper contains valuable impurities such as silver and gold. During electrorefining, copper is selectively oxidised from the impure anode, whereas these less reactive precious metals remain undissolved and settle as anode mud. Thus the statement that blister copper deposits precious metals refers to their collection as a solid deposit during refining. They are recovered from the anode mud, rather than co-deposited as the intended pure copper at the cathode. In this sense Statement I is true.

Step 2: Assess Statement II The impure copper must be the anode because oxidation occurs there:

$$\mathrm{Cu} \longrightarrow \mathrm{Cu}^{2+} + 2\,\mathrm{e}^{-}.$$

Copper ions migrate through the electrolyte and are reduced at the pure copper cathode:

$$\mathrm{Cu}^{2+} + 2\,\mathrm{e}^{-} \longrightarrow \mathrm{Cu}.$$

Consequently the anode gradually loses copper and the cathode gains refined copper. Blister copper is therefore correctly used to make the anode, so Statement II is true.

Step 3: Select the statement combination Both the precious-metal residue and the anode construction agree with the electrorefining process. Option (A) accepts both statements and is correct. Option (B) rejects both verified facts. Option (C) wrongly rejects the anode role of blister copper, while (D) wrongly denies the precious-metal deposit obtained as anode mud. Distinguishing this residue from cathodic copper deposition resolves the potentially confusing wording of Statement I.

Step 4: Conclusion Both Statement I and Statement II are true; precious metals collect as anode mud and blister copper forms the anode. Hence, the correct option is (A).

Quick Tip to Solve: In copper electrorefining, remember: impure anode dissolves, pure cathode grows, and noble-metal impurities settle as anode mud.

78. The number of bridged oxygen atoms present in compound B formed from the following reactions is

$\mathbf{Pb(NO_3)_2} \xrightarrow{673\,K} \mathbf{A + PbO + O_2}$

$\mathbf{A} \xrightarrow{Dimerise} \mathbf{B}$ [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (A) 0

Solution:

Concept:

- Thermal decomposition of a heavy-metal nitrate produces the metal oxide, nitrogen dioxide and oxygen. Atom conservation identifies the gaseous oxide without confusing it with nitrogen monoxide.
- A bridging oxygen is bonded to two central atoms. An oxygen bonded to only one nitrogen is terminal, irrespective of the bond order shown in a resonance structure.

Step 1: Identify the decomposition product Lead nitrate decomposes on heating according to the balanced equation

$$2\,\mathrm{Pb(NO_3)_2} \xrightarrow{673\,\mathrm{K}} 2\,\mathrm{PbO} + 4\,\mathrm{NO_2} + \mathrm{O_2}.$$

Thus A is nitrogen dioxide, $\mathrm{NO_2}$. The coefficient 4 specifies the amount formed in this balanced equation; it does not mean that the molecule contains four nitrogen atoms. The nitrogen-containing gas, rather than the solid lead oxide or released oxygen, undergoes the stated dimerisation.

Step 2: Identify the bond that joins the dimer Two nitrogen dioxide molecules combine to give dinitrogen tetroxide:

$$2\,\mathrm{NO_2} \rightleftharpoons \mathrm{N_2O_4}.$$

In B, the two nitrogen centres are directly joined by an N–N bond. Its connectivity is $\mathrm{O_2N{-}NO_2}$: each oxygen belongs to one of the two nitrogen centres, and no oxygen lies between them. Resonance within each nitro group distributes the N–O bonding but does not create an N–O–N bridge. Therefore all four oxygens are terminal. The alternatives 1, 2 and 3 incorrectly replace a direct nitrogen–nitrogen connection with oxygen bridges.

Step 3: Conclusion The dimer B is $\mathrm{N_2O_4}$, which has zero bridging oxygen atoms. Hence, the correct option is (A).

Quick Tip to Solve: For a dimer, identify the new inter-monomer bond first. An N–N bond in $\mathrm{N_2O_4}$ creates no oxygen bridge.

79. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Polluted water may have a value of BOD of the order of 17 ppm.

Reason R : BOD is a measure of oxygen required to oxidise both the bio-degradable an non-biodegradable organic material in water.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Correct Answer: (C) A is correct but R is not correct.

Solution:

Concept:

- Biochemical oxygen demand (BOD) measures dissolved oxygen consumed by microorganisms while decomposing biodegradable organic matter under specified test conditions.
- A larger BOD indicates a larger load of biologically oxidisable waste. It must be distinguished from chemical oxygen demand, which uses a strong chemical oxidant.

Step 1: Evaluate the assertion Clean water has a low BOD because it contains little biodegradable waste for microorganisms to oxidise. Polluted water may have a BOD of about $17\,\mathrm{ppm}$; this relatively high oxygen demand is consistent with appreciable organic pollution. In dilute water, ppm is approximately $\mathrm{mg\,L^{-1}}$, so this value represents a significant demand on the available dissolved oxygen. The assertion says that polluted water may have this value, not that every polluted sample has exactly this BOD. Assertion A is therefore correct.

Step 2: Evaluate the reason The reason extends BOD to both biodegradable and non-biodegradable organic matter. This is incorrect: the biological test measures oxygen consumption associated with material that microorganisms can degrade within the test conditions. Material resistant to biodegradation is not automatically included in this measure. A chemical oxidation test and a biological oxidation test must not be treated as interchangeable definitions.

Step 3: Select the logical combination Since A is true and R is false, R cannot explain A. Options (A) and (B) both wrongly require the reason to be correct. Option (D) reverses the truth values by rejecting the valid assertion and accepting the incorrect reason. Only option (C) gives the required combination.

Step 4: Conclusion Assertion A is correct, but Reason R is not correct. Hence, the correct option is (C).

Quick Tip to Solve: The word biological in BOD is the key: restrict its organic load to biodegradable matter before evaluating an assertion–reason pair.

80. An antiseptic dettol is a mixture of two compounds 'A' and 'B' where A has 6π electrons and B has 2π electrons. What is 'B'? [4 marks]

(A) Bithionol

(B) Terpineol

(C) Chloroxylenol

(D) Chloramphenicol

Correct Answer: (B) Terpineol

Solution:

Concept:

- Dettol contains chloroxylenol and terpineol. The former has an aromatic benzene ring, while the latter contains a single carbon–carbon double bond.
- A carbon–carbon double bond contains one sigma bond and one pi bond. The pi bond contributes two pi electrons; an aromatic benzene ring contains six pi electrons.

Step 1: Use the six-electron clue to identify A Chloroxylenol is a substituted phenol: its benzene ring supplies three delocalised pi bonds, or six pi electrons. It therefore corresponds to compound A in the question. The chlorine, methyl and hydroxyl substituents do not turn this aromatic six-electron system into the two-electron system requested for B. This assignment also agrees with the known antiseptic composition of Dettol.

Step 2: Use the two-electron clue to identify B Terpineol is a monoterpene alcohol with one carbon–carbon double bond. That one pi bond contains two electrons, so it fits compound B. Its hydroxyl group is attached through single bonds and does not add another pi bond. The electron count refers to electrons in pi bonds, not all valence electrons or all oxygen lone-pair electrons in the molecule.

Step 3: Eliminate the remaining names Bithionol is an antiseptic containing aromatic rings and is not the two-pi-electron component of Dettol. Chloroxylenol is present in Dettol but is A, not B, because its aromatic ring has six pi electrons. Chloramphenicol is an antibiotic with a different molecular structure and is not this Dettol ingredient. Thus the chemical composition and the electron-count clue both single out terpineol.

Step 4: Conclusion Compound B is terpineol, whose single C=C bond contains two pi electrons. Hence, the correct option is (B).

Quick Tip to Solve: First recall the two Dettol ingredients; then use aromatic ring = six pi electrons and isolated C=C = two pi electrons to assign A and B.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. A protein 'A' contains 0.30% of glycine (molecular weight 75). The minimum molar mass of the protein 'A' is _____ $\times 10^3$ g mol^{-1} [nearest integer] [4 marks]

Solution:

Concept:

- Mass percentage relates the mass of an identified component to the total mass: percentage = component mass divided by total mass, multiplied by 100.
- The minimum molecular mass arises when one protein molecule contains the smallest positive integral number of the stated glycine units. Use the given molecular weight 75 for this mass-percentage calculation.

Step 1: Set up the minimum composition Let the protein molar mass be $M\,\mathrm{g\,mol^{-1}}$. A mole of protein molecules must contain at least one mole of the glycine component specified in the question. The minimum corresponds to exactly one such unit per protein molecule; two or more would multiply the required total mass while leaving the percentage fixed. With the supplied glycine molecular weight, this minimum amount contributes $75\,\mathrm{g}$ per mole of protein.

Step 2: Apply the given percentage The stated glycine content is 0.30%, which as a mass fraction is 0.0030. Hence

$$\frac{75}{M} \times 100 = 0.30.$$

Solving with the percentage kept in its correct form gives

$$M = \frac{75 \times 100}{0.30} = 25000\,\mathrm{g\,mol^{-1}}.$$

Equivalently, dividing 75 by the decimal fraction 0.0030 gives the same result. No additional water-loss correction is introduced: the calculation uses the component molecular weight explicitly supplied in the question.

Step 3: Express the requested coefficient The answer blank multiplies $10^3\,\mathrm{g\,mol^{-1}}$. Since $25000 = 25 \times 10^3$, the number entered is 25. It is already an integer, so rounding does not change it. A check gives $75/25000 = 0.0030$, reproducing the required mass fraction.

Step 4: Conclusion The minimum molar mass is $25 \times 10^3\,\mathrm{g\,mol^{-1}}$. The required nearest-integer coefficient is $\boxed{25}$.

Quick Tip to Solve: Use one constituent unit for the minimum mass, divide its molecular weight by the decimal mass fraction, and then convert to the units beside the blank.

82. Amongst BeF_2, BF_3, H_2O, NH_3, CCl_4 and HCl, the number of molecules with non-zero net dipole moment is _____. [4 marks]

Solution:

Concept:

- The molecular dipole moment is the vector sum of bond dipole moments. Polar bonds alone do not guarantee that the whole molecule has a non-zero dipole.
- Symmetrically arranged identical bonds cancel in linear, trigonal planar and tetrahedral geometries. Bent and pyramidal geometries generally prevent complete cancellation.

Step 1: Identify the molecules with cancelling dipoles BeF_2 is linear in its molecular form. Its two equal Be–F dipoles point in opposite directions and cancel. BF_3 is trigonal planar; its three equal B–F dipoles at 120° have zero vector sum. CCl_4 is tetrahedral with four equivalent C–Cl bonds. Their symmetric spatial arrangement likewise gives zero net dipole. Thus these three molecules do not contribute to the requested count, despite containing polar bonds.

Step 2: Identify the molecules with surviving dipoles H_2O is bent because oxygen has two lone pairs in addition to its two bonding pairs. The O–H bond dipoles do not oppose each other directly, so their resultant is non-zero. NH_3 has a trigonal pyramidal structure with a lone pair on nitrogen; the three N–H dipoles have a non-zero resultant along the molecular symmetry axis. HCl is a heteronuclear diatomic molecule with one polar bond, so there is no second bond vector to cancel its dipole.

Step 3: Count the non-polar and polar subsets The zero-dipole subset is BeF_2, BF_3, CCl_4, and the non-zero subset is H_2O, NH_3, HCl. Each listed species is counted once. There are three members in the second subset; no numerical dipole values are required to decide the count.

Step 4: Conclusion The molecules with non-zero net dipole moment are H_2O, NH_3 and HCl. The required number is $\boxed{3}$.

Quick Tip to Solve: Draw the geometry mentally before adding bond dipoles: symmetry cancels identical bond vectors, while bent and pyramidal shapes retain a resultant.

83. At 345 K, the half life for the decomposition of a sample of a gaseous compound initially at 55.5 kPa was 340 s. When the pressure was 27.8 kPa, the half life was found to be 170 s. The order of the reaction is _____. [integer answer] [4 marks]

Solution:

Concept:

- At a fixed temperature and volume, the initial pressure of a gaseous reactant is proportional to its initial concentration. Pressure ratios can therefore be used in half-life comparisons.
- For a zero-order reaction, $t_{1/2} = [A]_0/(2k)$. For a first-order reaction it is independent of initial concentration, while for a second-order reaction it varies inversely with initial concentration.

Step 1: Compare the two experimental ratios The first experiment starts at $55.5\,\text{kPa}$ and has half-life $340\,\text{s}$; the second starts at $27.8\,\text{kPa}$ and has half-life $170\,\text{s}$. The initial pressure has approximately

halved, and the half-life has also halved:

$$\frac{55.5}{27.8} \simeq 2, \qquad \frac{340}{170} = 2.$$

The small departure of the pressure ratio from exactly 2 is consistent with the reported decimal precision. Both experiments are at 345 K, so the rate constant is compared at the same temperature.

Step 2: Match the half-life dependence For a power-law reaction of order n, the half-life scales as $t_{1/2} \propto [A]_0^{1-n}$, with the first-order case giving concentration-independent half-life. The observed direct proportionality gives $1 - n = 1$, hence $n = 0$. The zero-order integrated law provides the same conclusion directly: $[A] = [A]_0 - kt$, so putting $[A] = [A]_0/2$ gives $t_{1/2} = [A]_0/(2k)$.

Step 3: Check against alternative orders A first-order reaction would retain the same half-life after the starting pressure is halved. A second-order reaction would instead double its half-life. Neither trend matches the data. As a consistency check, the pressure decrease per half-life is $27.75/340 \simeq 0.0816\,\mathrm{kPa\,s^{-1}}$ and $13.9/170 \simeq 0.0818\,\mathrm{kPa\,s^{-1}}$, agreeing within the given precision.

Step 4: Conclusion The reaction is zero order. The required dimensionless integer is $\boxed{0}$.

Quick Tip to Solve: When halving the initial concentration also halves the half-life, recognise zero order; unchanged half-life signals first order instead.

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84. A solution of $Fe_2(SO_4)_3$ is electrolyzed for 'x' min with a current of 1.5 A to deposit 0.3482 g of Fe. The value of x is _____. [nearest integer]

Given : 1 F = 96500 C mol^{-1}

Atomic mass of Fe = 56 g mol^{-1} [4 marks]

Solution:

Concept:

- Faraday's law relates deposited mass to charge: $m = (M/zF)It$, where M is molar mass and z is the number of electrons required per deposited atom.
- Ferric sulfate contains iron in oxidation state +3. Deposition as metallic iron therefore requires three moles of electrons per mole of iron, not two.

Step 1: Write the cathode electron requirement In $Fe_2(SO_4)_3$, each sulfate ion carries charge -2, so the two iron ions together carry $+6$. Thus each iron ion is Fe^{3+}, and the overall cathode reduction for deposition is

$$Fe^{3+} + 3\,e^- \longrightarrow Fe.$$

Even if reduction passes through Fe^{2+}, the total electron requirement from ferric ion to metal remains three per iron atom. Consequently the electrochemical equivalent must use $z = 3$.

Step 2: Calculate the charge needed The deposited amount of iron is

$$n(Fe) = \frac{0.3482}{56}\,\mathrm{mol}.$$

The charge required is therefore

$$Q = 3F\, n(\mathrm{Fe}) = \frac{3 \times 96500 \times 0.3482}{56}\,\mathrm{C} \simeq 1800.07\,\mathrm{C}.$$

The charge is positive here because it represents the magnitude supplied by the current. The sulfate ions do not alter the number of electrons needed to make one atom of metallic iron.

Step 3: Convert charge into minutes Using $Q = It$ with $I = 1.5\,\mathrm{C\,s^{-1}}$,

$$t = \frac{1800.07}{1.5}\,\mathrm{s} \simeq 1200.05\,\mathrm{s}.$$

Since the question asks for minutes, divide by 60:

$$x = \frac{1200.05}{60} = 20.0009\,\mathrm{min}.$$

Rounding to the nearest integer gives 20. Keeping seconds and minutes separate avoids a factor-of-60 error in the requested entry.

Step 4: Conclusion The electrolysis time is approximately $20.0009\,\mathrm{min}$, so the requested nearest-integer value of x is $\boxed{20}$ (time in minutes).

Quick Tip to Solve: Find the metal oxidation state first, then use $t = mzF/(MI)$ in seconds and divide by 60 only at the final conversion.

85. Amongst $FeCl_3.3H_2O$, $K_3[Fe(CN)_6]$ and $[Co(NH_3)_6]Cl_3$, the spin-only magnetic moment value of the inner-orbital complex that absorbs light at shortest wavelength is _____ B.M. [nearest integer] [4 marks]

Solution:

Concept:

- For octahedral complexes, the energy of absorbed light is the crystal-field splitting: $\Delta_o = hc/\lambda$. A larger splitting therefore corresponds to a shorter absorption wavelength.
- The spin-only magnetic moment depends on the number n of unpaired electrons: $\mu = \sqrt{n(n+2)}\,\mathrm{B.M.}$. Strong-field ligands can produce electron pairing and inner-orbital complexes.

Step 1: Compare the ligand fields Chloride and water produce weaker ligand fields than cyanide. Ammonia is stronger than water but weaker than cyanide in the spectrochemical series. The cyanide complex $[\mathrm{Fe(CN)_6}]^{3-}$ therefore has the largest relevant splitting and absorbs the shortest wavelength. Both ferricyanide and the cobalt(III) ammine are inner-orbital complexes, but the cyanide ligand distinguishes the required one.

Step 2: Count the unpaired electrons In $\mathrm{K_3[Fe(CN)_6]}$, six cyanide ligands contribute charge -6, so iron has oxidation state $+3$. Thus $\mathrm{Fe^{3+}}$ is $3d^5$. Strong-field cyanide pairs the electrons into the lower octahedral set, giving $t_{2g}^5 e_g^0$, with one unpaired electron. In contrast, low-spin cobalt(III) is d^6 and diamagnetic; its moment is not the one asked for.

Step 3: Calculate and round the moment Insert $n = 1$, rather than the five unpaired electrons of a high-spin iron(III) ion:

$$\mu = \sqrt{1(1+2)} = \sqrt{3} \simeq 1.73\,\text{B.M.}$$

Rounding this value to the nearest integer gives 2.

Step 4: Conclusion The required inner-orbital complex is $[\text{Fe(CN)}_6]^{3-}$, and its spin-only moment is $\boxed{2\,\text{B.M.}}$, to the nearest integer.

Quick Tip to Solve: First select the complex using ligand-field strength; only then count its unpaired electrons. Do not confuse the diamagnetic cobalt complex with the shortest-wavelength absorber.

86. How many of the given compounds will give a positive Biuret test _____ ?

Glycine, Glycylalanine, Tripeptide, Biuret [4 marks]

Solution:

Concept:

- The Biuret test is the violet colour produced when suitable peptide linkages coordinate copper(II) ions in alkaline solution. A normal peptide requires at least two peptide bonds for a positive test.
- A peptide bond is the amide linkage $-\text{CO}-\text{NH}-$ between amino-acid residues. The test detects the arrangement of coordinating groups, not merely the presence of nitrogen or an amino group.

Step 1: Check the amino acid and dipeptide Glycine is a single amino acid, $\text{NH}_2\text{CH}_2\text{COOH}$, and contains no peptide bond. It therefore gives no positive Biuret test. Glycylalanine is formed from two amino acids with loss of one water molecule, and has only one peptide linkage. One peptide bond is insufficient for the characteristic violet complex, so this compound is also excluded.

Step 2: Check the tripeptide and biuret A tripeptide joins three amino-acid residues in one chain. The number of peptide bonds is one less than the number of residues, so it has $3 - 1 = 2$ peptide linkages and gives a positive test. Biuret itself, $\text{NH}_2\text{CONHCONH}_2$, also gives the characteristic positive result. It is the compound after which the test is named; it need not be a protein or a chain of amino acids to respond.

Step 3: Count the positive compounds The four entries therefore have responses: glycine, negative; glycylalanine, negative; tripeptide, positive; and biuret, positive. Counting the two positive entries rather than counting amino acids gives

$$N_{\text{positive}} = 1 + 1 = 2.$$

No molar-mass or concentration calculation is required for this qualitative test.

Step 4: Conclusion Both the tripeptide and biuret give a positive Biuret test. The required number is $\boxed{2}$ (a dimensionless count).

Quick Tip to Solve: For ordinary peptides count peptide bonds, not residues: a dipeptide has one bond and is negative, whereas a tripeptide has two and is positive.

87. The neutralization occurs when 10 mL of 0.1M acid 'A' is allowed to react with 30 mL of 0.05 M base $M(OH)_2$. The basicity of the acid 'A' is _____.

[M is a metal] [4 marks]

Solution:

Concept:

- At complete neutralization the number of moles of replaceable acidic protons equals the number of moles of hydroxide ions supplied by the base.
- If an acid has basicity b, its acid equivalents are bMV. Each formula unit of $\mathrm{M(OH)_2}$ supplies two hydroxide equivalents; volume must be in litres when calculating moles.

Step 1: Calculate the hydroxide amount The base concentration refers to moles of $\mathrm{M(OH)_2}$, not directly to moles of hydroxide. Its amount is

$$n_{\text{base}} = 0.05 \times \frac{30}{1000} = 1.5 \times 10^{-3}\,\text{mol}.$$

Because there are two hydroxide groups in each formula unit, the amount available to neutralize acidic protons is

$$n_{\mathrm{OH^-}} = 2(1.5 \times 10^{-3}) = 3.0 \times 10^{-3}\,\text{mol}.$$

Step 2: Relate this amount to the acid The acid amount is

$$n_A = 0.1 \times \frac{10}{1000} = 1.0 \times 10^{-3}\,\text{mol}.$$

Let each mole of the acid supply b moles of ionisable protons. Complete neutralization requires

$$b(1.0 \times 10^{-3}) = 3.0 \times 10^{-3}.$$

The factor of two belongs to the hydroxide formula and must be retained; simply equating the molarities times volumes would miss it.

Step 3: Obtain the basicity Dividing the hydroxide-equivalent amount by the acid amount gives

$$b = \frac{3.0 \times 10^{-3}}{1.0 \times 10^{-3}} = 3.$$

Thus each acid molecule can furnish three protons under complete neutralization. This determines its basicity without requiring the identity or atomic mass of the metal.

Step 4: Conclusion The acid is tribasic: its basicity is $\boxed{3}$, a dimensionless number of ionisable protons per molecule.

Quick Tip to Solve: Use acid equivalents equal to base equivalents. For a hydroxide with two OH groups, multiply its molarity by two before comparing neutralizing capacities.

88. A rigid nitrogen tank stored inside a laboratory has a pressure of 30 atm at 06:00 am when the temperature is 27°C. At 03:00 pm, when the temperature is 45°, the pressure in the tank will be _____ atm. [nearest integer] [4 marks]

Solution:

Concept:

- For a fixed quantity of gas in a rigid tank, both the amount n and volume V are constant. The ideal gas equation then implies $P/T = nR/V =$ constant.
- Temperature in a gas-law ratio must be absolute temperature in kelvin: $T(\mathrm{K}) = t(^\circ\mathrm{C}) + 273$. The pressure increases on heating at constant volume.

Step 1: Convert both temperatures The morning temperature is

$$T_1 = 27 + 273 = 300\,\mathrm{K}.$$

The afternoon temperature is

$$T_2 = 45 + 273 = 318\,\mathrm{K}.$$

The times identify the two states; their difference does not enter the equilibrium pressure relation. Using the ratio $45/27$ would incorrectly treat the arbitrary Celsius zero as an absolute zero.

Step 2: Apply the constant-volume relation The tank is rigid, so it does not expand as the gas warms, and the stored gas amount is unchanged. Write

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}, \qquad P_2 = P_1\frac{T_2}{T_1}.$$

Substituting the stated initial pressure gives

$$P_2 = 30 \times \frac{318}{300} = 31.8\,\mathrm{atm}.$$

The temperature increases by six per cent on the kelvin scale, so the pressure also increases by six per cent.

Step 3: Round as requested The question asks for the nearest integer in atmospheres. Since the decimal part of 31.8 exceeds 0.5, the numerical entry is 32. Using the more precise conversion with 273.15 also gives approximately 31.80 atm and the same rounded answer; there is no change in the reported integer.

Step 4: Conclusion The afternoon pressure is $31.8\,\mathrm{atm}$, or $\boxed{32\,\mathrm{atm}}$ to the nearest integer.

Quick Tip to Solve: For a sealed rigid tank use P proportional to absolute temperature. A small Celsius rise near room temperature produces only a small percentage pressure rise.

89. Consider the following reactions:

$PCl_3 + H_2O \rightarrow A + HCl$

$A + H_2O \rightarrow B + HCl$

The number of ionisable protons present in the product B is _____. [4 marks]

Solution:

Concept:

- Hydrolysis can be represented formally by replacing P-Cl groups with P-OH groups and releasing hydrogen chloride. Each replacement consumes one water molecule.
- For phosphorus compounds, O-H protons and P-H protons are chemically different. An oxo/P-H tautomer can therefore change the count inferred from a formal hydroxyl structure. The structure of the product must be specified before claiming its basicity.

Step 1: Follow the formal substitution sequence Replacing one chlorine in the first reaction gives the formal hydroxyl product

$$\mathrm{PCl_3 + H_2O \longrightarrow PCl_2(OH) + HCl.}$$

A second replacement then gives

$$\mathrm{PCl_2(OH) + H_2O \longrightarrow PCl(OH)_2 + HCl.}$$

Counting the two OH groups in this formal representation of B leads to 2, the intended numerical entry. HCl is a separate product and is not included in the count.

Step 2: Check the oxo tautomer interpretation The formal trivalent structure $\mathrm{PCl(OH)_2}$ is not enough to establish two acidic hydrogens unconditionally. The oxo/P-H tautomer is $\mathrm{HP(O)(OH)Cl}$. It has the same elemental composition but only one O-H group and one P-H bond. On that structural interpretation, only one proton is ionisable in the usual oxyacid sense; the P-H hydrogen is not an acidic hydroxyl hydrogen.

Step 3: Distinguish the intended entry from the chemical limitation The displayed reactions give neither a structure for A or B nor a statement that the formal hydroxyl form is to be used. Thus the purely sequential replacement convention supports the supplied entry 2, while accounting for the oxo/P-H form gives an O-H count of 1. Fully hydrolysed phosphorous acid, $\mathrm{HP(O)(OH)_2}$, has two acidic OH groups, but that fact cannot be transferred to the partially hydrolysed chloride without identifying B.

Step 4: Conclusion The intended formal-substitution answer is $\boxed{2}$ (dimensionless). The literal product specification is ambiguous: the oxo/P-H structure $\mathrm{HP(O)(OH)Cl}$ instead contains one ionisable O-H proton.

Quick Tip to Solve: Count O-H protons only after checking the phosphorus structure. A formal hydroxyl substitution formula can hide an oxo/P-H tautomer and lead to a different acidic-proton count.

90. The Novolac polymer has mass of 963 g. The number of monomer units present in it are [4 marks]

Solution:

Concept:

- Novolac is a phenol-formaldehyde condensation polymer. Its aromatic phenol units are joined by methylene bridges, and water is eliminated during bridge formation.
- A molecular mass is needed to infer the number of units in one polymer molecule. A bulk sample mass alone does not specify a degree of polymerization, because the amount of polymer molecules must also be known.

Step 1: Establish the mass contribution of a bridge The molar masses of phenol and formaldehyde are approximately 94 and $30\,\mathrm{g\,mol^{-1}}$, respectively. Forming one methylene bridge from formaldehyde eliminates water, so its net contribution to the phenol framework is

$$30 - 18 = 12\,\mathrm{g\,mol^{-1}}.$$

The commonly used interior repeat composition is $\mathrm{C_7H_6O}$, whose formula mass is 106, not 107.

Step 2: Check a finite linear chain A linear molecule containing n phenol rings has $n-1$ bridges. For that specified architecture its molar mass would be

$$M = 94n + 12(n-1) = 106n - 12\,\mathrm{g\,mol^{-1}}.$$

Even interpreting the printed 963 g as a molecular molar mass gives

$$n = \frac{963 + 12}{106} = \frac{975}{106} \simeq 9.20,$$

which is not an integer. A nine-ring chain in this model has molar mass $942\,\mathrm{g\,mol^{-1}}$, so the printed value does not give an exact nine-unit molecule.

Step 3: State the numerical convention and its limit The supplied numerical answer is 9, which is also the nearest integer to the finite-chain estimate above. Thus 9 can be reported as the intended numerical entry. However, no exact number of monomer units follows from a bulk mass of 963 g as written, and the question supplies neither a chain structure nor an instruction to round. Dividing 963 by an invented repeat-unit mass of 107 would give 9 but would not be chemically justified.

Step 4: Conclusion The intended numerical entry is $\boxed{9}$ monomer units (dimensionless). The literal mass statement is insufficient for an exact degree of polymerization; the condensation mass balance above explains this limitation.

Quick Tip to Solve: For condensation polymers include both the lost water and the end groups. A bulk mass must not be treated automatically as the mass of one mole of polymer molecules.