JEE Main 2022 Paper 1 (B.E./B.Tech) Jun 26 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 1 (June), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 26 June 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $f(x) = \frac{x-1}{x+1}$, $x \in R - \{0, -1, 1\}$. If $f^{n+1}(x) = f(f^n(x))$ for all n $\in$ N, then $f^6(6) + f^7(7)$ is equal to : [4 marks]

(A) $\frac{7}{6}$

(B) $-\frac{3}{2}$

(C) $\frac{7}{12}$

(D) $-\frac{11}{12}$

Correct Answer: (B) $-\frac{3}{2}$

Solution:

Concept:

- The superscript in $f^n(x)$ denotes repeated composition, not an ordinary numerical power. Compute successive iterates to identify a repeating cycle.
- If $f^4(x) = x$ throughout the stated domain, the iteration number can be reduced modulo four. The exclusions keep the denominators in these compositions nonzero.

Step 1: Find the first four iterates Substitute the entire expression for the inner function into the outer function and simplify numerator and denominator separately:

$$f^2(x) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = -\frac{1}{x}.$$

Then

$$f^3(x) = \frac{-1/x - 1}{-1/x + 1} = \frac{x+1}{1-x}, \qquad f^4(x) = \frac{\frac{x+1}{1-x} - 1}{\frac{x+1}{1-x} + 1} = x.$$

Thus every block of four applications returns the original input; the function has a four-step composition cycle on its domain.

Step 2: Reduce the iteration counts and add Since $6 = 4 + 2$ and $7 = 4 + 3$, use the second and third expressions in the cycle at their respective inputs:

$$f^6(6) = f^2(6) = -\frac{1}{6}, \qquad f^7(7) = f^3(7) = \frac{8}{-6} = -\frac{4}{3}.$$

Consequently,

$$f^6(6) + f^7(7) = -\frac{1}{6} - \frac{4}{3} = -\frac{9}{6} = -\frac{3}{2}.$$

The inputs 6 and 7 and their iterates avoid the excluded values, so no substitution crosses an undefined expression. The positive alternatives (A) and (C) have the wrong sign, while (D) does not equal this exact sum.

Step 3: Conclusion The required value is $-\frac{3}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: For a fractional linear function, compute its first few compositions before trying to evaluate a high iterate; a short cycle can eliminate nearly all the work.

2. Let $A = \left\{z \in C : \left|\frac{z+1}{z-1}\right| < 1\right\}$ and $B = \left\{z \in C : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}\right\}$. Then A ∩ B is : [4 marks]

(A) a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

(B) a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

(C) an empty

(D) a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

Correct Answer: (B) a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

Solution:

Concept:

- For $z = x + iy$, an inequality between $|z+1|$ and $|z-1|$ compares distances to the real-axis points -1 and 1. Squaring is valid because both distances are nonnegative.
- An argument of $2\pi/3$ requires a negative real part and a positive imaginary part. The tangent equation alone also admits the opposite ray, so the signs must be retained.

Step 1: Convert the modulus inequality to a half-plane The denominator in the definition of A is nonzero. Its condition is

$$|z+1|^2 < |z-1|^2 \iff (x+1)^2 + y^2 < (x-1)^2 + y^2 \iff x < 0.$$

Hence only the open left half-plane can belong to the intersection; points on the imaginary axis are excluded.

Step 2: Find the argument locus with its sign restrictions Multiplication by the conjugate gives

$$\frac{z-1}{z+1} = \frac{x^2 + y^2 - 1 + 2iy}{(x+1)^2 + y^2}.$$

The denominator is positive wherever the ratio is defined. Therefore the prescribed argument implies $y > 0$, $x^2 + y^2 - 1 < 0$, and

$$\frac{2y}{x^2 + y^2 - 1} = -\sqrt{3}.$$

Completing the square yields

$$x^2 + \left(y + \frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3}.$$

Conversely, on this circle with $y > 0$, its equation gives a negative real numerator and the required tangent, so the angle is exactly $2\pi/3$.

Step 3: Intersect the circle and the half-plane Combining $x < 0$ with $y > 0$ retains only the second-quadrant portion of the circle centred at $(0, -1/\sqrt{3})$. This arc is nonempty; for example $(-1/\sqrt{3}, 1 - 1/\sqrt{3})$ lies on it. Option (A) incorrectly includes third-quadrant points; (D) keeps the wrong quadrant, and (C) incorrectly declares the intersection empty.

Step 4: Conclusion The intersection is the second-quadrant portion of the circle centred at $(0, -1/\sqrt{3})$. Hence, the correct option is (B).

Quick Tip to Solve: After using the tangent of an argument to obtain a circle, restore the signs of its real and imaginary parts to select the correct arc.

3. The ordered pair (a, b), for which the system of linear equations

$3x - 2y + z = b$

$5x - 8y + 9z = 3$

$2x + y + az = -1$

has no solution, is : [4 marks]

(A) $\left(3, \frac{1}{3}\right)$

(B) $\left(-3, \frac{1}{3}\right)$

(C) $\left(-3, -\frac{1}{3}\right)$

(D) $\left(3, -\frac{1}{3}\right)$

Correct Answer: (C) $\left(-3, -\frac{1}{3}\right)$

Solution:

Concept:

- A nonzero determinant of the coefficient matrix guarantees a unique solution for every right-hand side. A system can be inconsistent only when its coefficient rows are linearly dependent.
- When one coefficient row is a combination of the others, the right-hand sides must obey the identical combination. Failure of that equality gives a contradiction and hence no solution.

Step 1: Find the singular value of the parameter Write the coefficient matrix and expand its determinant:

$$M = \begin{pmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{pmatrix}, \qquad \det M = 3(-8a - 9) + 2(5a - 18) + 21 = -14(a + 3).$$

Thus $a = 3$, appearing in options (A) and (D), gives a nonzero determinant and therefore cannot produce no solution. Only $a = -3$ needs a compatibility test.

Step 2: Test the dependent equations For $a = -3$, denote the coefficient rows by R_1, R_2, R_3. They satisfy

$$R_2 = 3R_1 - 2R_3,$$

since $3(3, -2, 1) - 2(2, 1, -3) = (5, -8, 9)$. The same operation on the constants requires

$$3 = 3b - 2(-1) = 3b + 2,$$

so consistency requires $b = 1/3$. For $b = -1/3$, the row operation $E_2 - 3E_1 + 2E_3$ instead gives

$$0 = 3 - 3(-1/3) + 2(-1) = 2,$$

which is impossible. For $b = 1/3$, that row reduces to $0 = 0$; the first and third coefficient rows remain independent, so the remaining two equations are compatible. Hence option (B) has infinitely many solutions rather than no solution.

Step 3: Conclusion The inconsistent pair is $(a, b) = (-3, -1/3)$. Hence, the correct option is (C).

Quick Tip to Solve: After setting the determinant to zero, use a row dependence to test the constants; singularity alone does not distinguish no solution from infinitely many solutions.

4. The remainder when $(2021)^{2023}$ is divided by 7 is : [4 marks]

(A) 1

(B) 2

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Concept:

- If two integers differ by a multiple of 7, their positive integer powers have the same remainder upon division by 7. Reduce the base before dealing with a large exponent.
- Powers of a nonzero residue repeat modulo 7. Here it is enough to find a small power of the reduced base that is congruent to 1 and then divide the exponent by that period.

Step 1: Reduce the large base Use ordinary division to obtain

$$2021 = 7 \cdot 288 + 5, \qquad 2021 \equiv 5 \pmod 7.$$

Therefore $2021^{2023} \equiv 5^{2023} \pmod 7$. The reduction applies to the base, not to the exponent; replacing the entire expression by the remainder of 2023 would not preserve the power. We can also regard the base as -2 modulo 7, but using 5 keeps every intermediate remainder nonnegative.

Step 2: Establish the power cycle and reduce the exponent Successive reductions give

$$5^2 \equiv 4, \quad 5^3 \equiv 6, \quad 5^6 \equiv 6^2 \equiv 1 \pmod 7.$$

Now divide the exponent by 6:

$$2023 = 6 \cdot 337 + 1.$$

It follows that

$$5^{2023} = (5^6)^{337} \cdot 5 \equiv 1^{337} \cdot 5 \equiv 5 \pmod 7.$$

The cycle returns to 1 at the sixth power, whereas the given exponent is one beyond a multiple of six. Thus the final residue is 5 rather than 1. The residue 6 occurs at the third power, not at this exponent, and 2 is also not the computed residue. Since 5 lies between 0 and 6, it is already the ordinary division remainder and no further adjustment is needed.

Step 3: Conclusion The remainder on division by 7 is 5. Hence, the correct option is (C).

Quick Tip to Solve: Reduce the base modulo the divisor, identify a power equal to 1 modulo that divisor, and reduce the exponent by the resulting cycle length.

5. $\lim\limits_{x\to\frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1}x)-x}{1-\tan(\cos^{-1}x)}$ **is equal to :** [4 marks]

(A) $\sqrt{2}$

(B) $-\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $-\frac{1}{\sqrt{2}}$

Correct Answer: (D) $-\frac{1}{\sqrt{2}}$

Solution:

Concept:

- For $-1 \le x \le 1$, the principal value of $\cos^{-1} x$ belongs to $[0, \pi]$. Hence $\sin(\cos^{-1} x) = \sqrt{1-x^2}$, with the nonnegative square root.
- A limit may be evaluated after cancelling a common factor on a punctured neighbourhood of the limiting point. The simplified expression need not be defined by the original formula at that point.

Step 1: Express both trigonometric functions in terms of x Put $s = \sqrt{1-x^2}$. Around $x = 1/\sqrt{2}$, we have $x > 0$, so

$$\sin(\cos^{-1} x) = s, \qquad \tan(\cos^{-1} x) = \frac{s}{x}.$$

The expression becomes

$$\frac{s-x}{1-s/x}.$$

At the limiting point $s = x = 1/\sqrt{2}$, the numerator and denominator both vanish. Direct substitution therefore gives an indeterminate form, rather than a value of zero or an infinite limit.

Step 2: Cancel the identical difference with its sign For nearby $x \neq 1/\sqrt{2}$, the denominator can be written as

$$1 - \frac{s}{x} = \frac{x-s}{x} = -\frac{s-x}{x}.$$

Thus the quotient simplifies exactly to

$$\frac{s-x}{1-s/x} = \frac{x(s-x)}{x-s} = -x.$$

There is no other zero of $s - x$ in a sufficiently small neighbourhood: $s = x$ with $x > 0$ implies $1 - x^2 = x^2$, so $x = 1/\sqrt{2}$. Consequently the cancellation is valid for the limit from either side within the domain. Taking the limit now gives $-1/\sqrt{2}$. The positive alternatives lose the minus sign, while $-\sqrt{2}$ would arise from incorrectly reversing the factor of x.

Step 3: Conclusion The limit equals $-\frac{1}{\sqrt{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: When the numerator is $s - x$ and the denominator contains $1 - s/x$, multiply the denominator by x first; the same difference appears with the opposite sign.

6. Let f, g : R → R be two real valued functions defined as $f(x) = \begin{cases} -|x+3| & , \quad x < 0 \\ e^x & , \quad x \geq 0 \end{cases}$ **and** $g(x) = \begin{cases} x^2 + k_1 x & , \quad x < 0 \\ 4x + k_2 & , \quad x \geq 0 \end{cases}$**, where k_1 and k_2 are real constants. If (gof) is differentiable at x = 0, then (gof) (− 4) + (gof) (4) is equal to :** [4 marks]

(A) $4(e^4 + 1)$

(B) $2(2e^4 + 1)$

(C) $4e^4$

(D) $2(2e^4 - 1)$

Correct Answer: (D) $2(2e^4 - 1)$

Solution:

Concept:

- Differentiability at a point requires both continuity and equality of the two one-sided derivatives. Apply these requirements to $h = g \circ f$, rather than to the inner function alone.
- The branch of the outer function is selected using the sign of $f(x)$, not the sign of x. Near zero on the left $f(x) = -(x+3) < 0$, while on the right $f(x) = e^x > 0$.

Step 1: Write the composite near zero and impose continuity For negative x sufficiently close to zero, $x + 3 > 0$. Hence

$$h(x) = (x+3)^2 - k_1(x+3) \quad (x < 0), \qquad h(x) = 4e^x + k_2 \quad (x \geq 0).$$

The left limit is $9 - 3k_1$, and the value at zero is $4 + k_2$. Continuity therefore requires

$$9 - 3k_1 = 4 + k_2.$$

Step 2: Match the one-sided derivatives The left polynomial has derivative $2(x+3) - k_1$, whose limit at zero is $6 - k_1$. The right expression has derivative $4e^x$, which is 4 at zero. Equality gives

$$6 - k_1 = 4 \quad \Longrightarrow \quad k_1 = 2.$$

Substitution in the continuity equation then gives $k_2 = -1$. These two conditions also make the two difference quotients agree, so the required differentiability is satisfied.

Step 3: Evaluate the two requested inputs At $x = -4$, the absolute value must be recomputed: $f(-4) = -|-1| = -1$. Therefore

$$h(-4) = g(-1) = 1 - 2 = -1.$$

At $x = 4$, we have $f(4) = e^4 > 0$, giving

$$h(4) = 4e^4 - 1, \qquad h(-4) + h(4) = 4e^4 - 2 = 2(2e^4 - 1).$$

The other alternatives have the wrong constant term: they equal $4e^4 + 4$, $4e^4 + 2$, and $4e^4$, respectively.

Step 4: Conclusion The required sum is $2(2e^4 - 1)$. Hence, the correct option is (D).

Quick Tip to Solve: Find the composite formula only near the differentiability point, then return to the original absolute-value definition when evaluating a distant input such as -4.

7. The sum of the absolute minimum and the absolute maximum values of the function f(x) = |3x − x² + 2| − x in the interval [−1, 2] is : [4 marks]

(A) $\frac{\sqrt{17}+3}{2}$

(B) $\frac{\sqrt{17}+5}{2}$

(C) 5

(D) $\frac{9-\sqrt{17}}{2}$

Correct Answer: (A) $\frac{\sqrt{17}+3}{2}$

Solution:

Concept:

- On a closed interval a continuous function attains its absolute extrema. For a modulus expression, check endpoints, stationary points of each smooth branch, and zeros of the expression inside the modulus.
- If $p(x) = 3x - x^2 + 2$, then $|p(x)|$ equals $p(x)$ where $p(x) \geq 0$, and $-p(x)$ where $p(x) < 0$. The resulting branches must be restricted to their own intervals.

Step 1: Locate the change of sign Solve $p(x) = 0$:

$$x = \frac{3 \pm \sqrt{17}}{2}.$$

Only $\alpha = (3 - \sqrt{17})/2$ lies in $[-1, 2]$. Since the quadratic opens downwards, it is negative on $[-1, \alpha)$ and nonnegative on $[\alpha, 2]$. Thus

$$f(x) = \begin{cases} x^2 - 4x - 2, & -1 \leq x \leq \alpha, \\ -x^2 + 2x + 2, & \alpha \leq x \leq 2. \end{cases}$$

Step 2: Find the extrema on each branch On the first interval, $f'(x) = 2x - 4 < 0$, so the function decreases from

$$f(-1) = 3 \quad \text{to} \quad f(\alpha) = -\alpha = \frac{\sqrt{17} - 3}{2}.$$

On the second interval,

$$f(x) = 3 - (x - 1)^2.$$

Its stationary point $x = 1$ lies inside the interval and gives a maximum of 3. Its endpoint values are $f(\alpha) = (\sqrt{17} - 3)/2$ and $f(2) = 2$, so its minimum is at α.

Step 3: Compare the candidates and add The absolute maximum over the entire interval is therefore 3, attained at both -1 and 1. The absolute minimum is $(\sqrt{17} - 3)/2$, attained at the modulus corner

α. Their sum is

$$3 + \frac{\sqrt{17} - 3}{2} = \frac{\sqrt{17} + 3}{2}.$$

This distinguishes the requested sum from a difference of extrema: option (D) would subtract the minimum from the maximum, option (B) adds an extra 1, and option (C) uses 2 as the minimum while missing the corner point.

Step 4: Conclusion The sum is $\frac{\sqrt{17} + 3}{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Include every zero of the expression inside a modulus among your extremum candidates; an absolute minimum can occur at a corner where the derivative does not exist.

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8. The area bounded by the curve $y = |x^2 - 9|$ and the line $y = 3$ is : [4 marks]

(A) $4(2\sqrt{3} + \sqrt{6} - 4)$

(B) $4(4\sqrt{3} + \sqrt{6} - 4)$

(C) $8(4\sqrt{3} + 3\sqrt{6} - 9)$

(D) $8(4\sqrt{3} + 2\sqrt{6} - 9)$

Correct Answer: (D) $8(4\sqrt{3} + 2\sqrt{6} - 9)$

Solution:

Concept:

- The area between two graphs is the integral of their vertical separation, with the upper graph determined on each interval.
- The graph $y = |x^2 - 9|$ is even and changes formula at $x = \pm 3$: it is $9 - x^2$ for $|x| \leq 3$ and $x^2 - 9$ outside. All bounded components must be included.

Step 1: Locate intersections and divide the regions Solve $|x^2 - 9| = 3$. The four intersection abscissae are $\pm\sqrt{6}$ and $\pm 2\sqrt{3}$. Between $-\sqrt{6}$ and $\sqrt{6}$, the curve lies above the line. Between $\sqrt{6}$ and $2\sqrt{3}$, the line lies above the curve, with a formula change at 3. An identical side region lies on the left. These three regions are all bounded by the given graphs.

Step 2: Integrate the central component Using symmetry about the vertical axis, the central area is

$$A_c = 2\int_0^{\sqrt{6}} (9 - x^2 - 3)\, dx = 2\left[6x - \frac{x^3}{3}\right]_0^{\sqrt{6}} = 8\sqrt{6}.$$

The absolute value must be resolved before integrating; integrating $x^2 - 9$ throughout would give an incorrect region.

Step 3: Add the two side components The combined side area is

$$\begin{aligned} A_s &= 2\left[\int_{\sqrt{6}}^{3}(x^2-6)\,dx + \int_{3}^{2\sqrt{3}}(12-x^2)\,dx\right] \\ &= 2\left[(-9+4\sqrt{6}) + (16\sqrt{3}-27)\right] \\ &= 8\sqrt{6}+32\sqrt{3}-72. \end{aligned}$$

Therefore $A = A_c + A_s = 16\sqrt{6}+32\sqrt{3}-72$. The other three expressions do not equal this sum; omitting the central component or one side component does not give the total bounded area.

Step 4: Conclusion The total area is $\boxed{8(4\sqrt{3}+2\sqrt{6}-9)}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: For an absolute-value parabola, mark both its zeros and all intersections with the horizontal line before selecting integration limits.

9. Let R be the point (3, 7) and let P and Q be two points on the line x + y = 5 such that PQR is an equilateral triangle. Then the area of ΔPQR is : [4 marks]

(A) $\frac{25}{4\sqrt{3}}$

(B) $\frac{25\sqrt{3}}{2}$

(C) $\frac{25}{\sqrt{3}}$

(D) $\frac{25}{2\sqrt{3}}$

Correct Answer: (D) $\frac{25}{2\sqrt{3}}$

Solution:

Concept:

- The perpendicular distance from (x_0, y_0) to $Ax+By+C=0$ is $|Ax_0+By_0+C|/\sqrt{A^2+B^2}$.
- For an equilateral triangle of side s, its altitude is $h=\sqrt{3}s/2$, and its area is $sh/2 = h^2/\sqrt{3}$.

Step 1: Find the altitude from the third vertex Both P and Q lie on $x+y=5$, so this line contains the base PQ. In an equilateral triangle, the perpendicular from the opposite vertex R to the base is its altitude and also meets the midpoint of the base. Thus the altitude is fixed even without locating P and Q separately. With $R=(3,7)$,

$$h = \frac{|3+7-5|}{\sqrt{1^2+1^2}} = \frac{5}{\sqrt{2}}.$$

Step 2: Recover the side and calculate the area From $h=\sqrt{3}s/2$, we obtain $s = 2h/\sqrt{3} = 10/\sqrt{6}$. Consequently,

$$A = \frac{1}{2}sh = \frac{1}{2}\cdot\frac{10}{\sqrt{6}}\cdot\frac{5}{\sqrt{2}} = \frac{25}{2\sqrt{3}}.$$

Equivalently, $A = h^2/\sqrt{3} = (25/2)/\sqrt{3}$. The altitude has been squared before dividing by $\sqrt{3}$, so no sign or orientation of the base enters the result. Option (A) is half this area, option (C) is twice it, and

option (B) is three times it; each has an incorrect geometric factor.

Step 3: Conclusion The area of $\triangle PQR$ is $\boxed{25/(2\sqrt{3})}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: When two vertices of an equilateral triangle lie on a given line, first find the third vertex's distance from that line; the area is immediately $h^2/\sqrt{3}$.

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10. If the two lines $l_1 : \frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $l_2 : \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$ are perpendicular, then an angle between the lines l_2 and $l_3 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is : [4 marks]

(A) $\cos^{-1}\left(\frac{29}{4}\right)$

(B) $\sec^{-1}\left(\frac{29}{4}\right)$

(C) $\cos^{-1}\left(\frac{2}{29}\right)$

(D) $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Correct Answer: (B) $\sec^{-1}\left(\frac{29}{4}\right)$

Solution:

Concept:

- Direction ratios come from a parametric form of the line. A coefficient multiplying a coordinate must be accounted for before reading these ratios.
- Perpendicular directions satisfy $\mathbf{u} \cdot \mathbf{v} = 0$. For the acute angle between lines, $\cos\theta = |\mathbf{u} \cdot \mathbf{v}|/(|\mathbf{u}||\mathbf{v}|)$.

Step 1: Determine the unknown direction ratio For l_1, the coordinate z is constant, so a direction vector is $(3, -2, 0)$. Setting each ratio in l_2 equal to t gives $x = 1+t$, $y = -3/2+\alpha t/2$, and $z = -5+2t$. Thus its direction is $(1, \alpha/2, 2)$. Perpendicularity gives

$$3 - 2\left(\frac{\alpha}{2}\right) = 0, \qquad \alpha = 3.$$

An equivalent integer direction for l_2 is now $\mathbf{u} = (2, 3, 4)$.

Step 2: Read the third direction and take the scalar product For l_3, write $(1-x)/3 = (2y-1)/(-4) = z/4 = t$. This gives $x = 1-3t$, $y = 1/2-2t$, and $z = 4t$, so $\mathbf{v} = (-3, -2, 4)$. Therefore

$$\mathbf{u} \cdot \mathbf{v} = -6 - 6 + 16 = 4, \qquad |\mathbf{u}| = |\mathbf{v}| = \sqrt{29}.$$

$$\cos\theta = \frac{4}{29}, \qquad \sec\theta = \frac{29}{4}.$$

Option (A) has an impossible real inverse-cosine argument. Options (C) and (D) use incorrect normalising factors; the product of the two vector lengths is 29, not $\sqrt{29}$.

Step 3: Conclusion An angle between the lines is $\boxed{\sec^{-1}(29/4)}$, equivalently $\cos^{-1}(4/29)$. Hence, the correct option is (B).

Quick Tip to Solve: Convert every symmetric equation to a parameter first: $2y+3 = \alpha t$ contributes $\alpha/2$, while $1-x = 3t$ contributes -3.

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11. The mean of the numbers a, b, 8, 5, 10 is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then 25 M is equal to : [4 marks]

(A) 60

(B) 55

(C) 50

(D) 45

Correct Answer: (A) 60

Solution:

Concept:

- For N observations with mean $\bar{x}$, population variance is $\sigma^2 = N^{-1}\sum x_i^2 - \bar{x}^2$.
- Mean deviation about the mean uses absolute deviations: $M = N^{-1}\sum |x_i - \bar{x}|$. Squared deviations and absolute deviations measure different quantities.

Step 1: Use the given mean and variance There are five observations. The mean equation is

$$\frac{a+b+8+5+10}{5} = 6, \qquad a+b = 7.$$

Using the population variance stated in the question,

$$\frac{a^2+b^2+64+25+100}{5} - 36 = 6.8.$$

Hence the total sum of squares is $5(42.8) = 214$, and therefore $a^2+b^2 = 214 - 189 = 25$. No sample correction is used because the variance describes the five numbers themselves.

Step 2: Find the missing observations The identity $(a+b)^2 = a^2+b^2+2ab$ gives $49 = 25 + 2ab$, so $ab = 12$. Thus a and b are roots of

$$t^2 - 7t + 12 = (t-3)(t-4) = 0.$$

Their order is immaterial: the five observations are $3, 4, 8, 5, 10$.

Step 3: Compute the absolute deviations About the mean 6, the absolute deviations are $3, 2, 2, 1, 4$. Their sum is 12, so

$$M = \frac{12}{5}, \qquad 25M = 25 \cdot \frac{12}{5} = 60.$$

The other values $55, 50, 45$ would require total absolute deviations of $11, 10, 9$, respectively, none of which equals the computed sum. In particular, signed deviations would sum to zero and cannot be used here.

Step 4: Conclusion We obtain $\boxed{25M = 60}$. Hence, the correct option is (A).

Quick Tip to Solve: Use mean and variance to determine the missing numbers first, then recompute absolute deviations; variance itself is not the mean deviation.

12. Let $f(x) = 2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10$, $x \in [-1, 1]$. If [a, b] is the range of the function f, then 4a − b is equal to : [4 marks]

(A) 11

(B) $11 - \pi$

(C) $11 + \pi$

(D) $15 - \pi$

Correct Answer: (B) $11 - \pi$

Solution:

Concept:

- For $-1 < x < 1$, $d(\cos^{-1} x)/dx = -1/\sqrt{1-x^2}$, and $d(\cot^{-1} x)/dx = -1/(1+x^2)$.
- A continuous strictly decreasing function on a closed interval takes its minimum at the right endpoint and its maximum at the left endpoint. Here $\cot^{-1}$ has principal range $(0, \pi)$.

Step 1: Establish the sign of the derivative Differentiating in the open interval yields

$$f\prime(x) = -\frac{2}{\sqrt{1-x^2}} - \frac{4}{1+x^2} - 6x - 2.$$

For $0 \le x < 1$, every term is nonpositive and the first terms are strictly negative. For $-1 < x < 0$, put $t = -x$, so $0 < t < 1$. Since $2/\sqrt{1-t^2} \ge 2$ and $4/(1+t^2) \ge 2$,

$$f\prime(x) \le -2 - 2 + 6t - 2 = 6t - 6 < 0.$$

Thus the derivative is strictly negative throughout $(-1, 1)$; no interior extremum is missed.

Step 2: Evaluate the two endpoint values Continuity holds on $[-1, 1]$. At the right endpoint, $\cos^{-1}(1) = 0$ and $\cot^{-1}(1) = \pi/4$, hence

$$a = f(1) = \pi - 3 - 2 + 10 = \pi + 5.$$

At the left endpoint, $\cos^{-1}(-1) = \pi$ and $\cot^{-1}(-1) = 3\pi/4$, giving

$$b = f(-1) = 2\pi + 3\pi - 3 + 2 + 10 = 5\pi + 9.$$

The endpoints of the range are therefore in increasing order as $[\pi + 5, 5\pi + 9]$.

Step 3: Form the requested expression Substitute these endpoint values:

$$4a - b = 4(\pi + 5) - (5\pi + 9) = 11 - \pi.$$

Option (A) incorrectly cancels all trigonometric contributions, option (C) reverses the remaining sign of π, and option (D) has the wrong constant term.

Step 4: Conclusion The required value is $\boxed{11 - \pi}$. Hence, the correct option is (B).

Quick Tip to Solve: For sums of decreasing inverse-trigonometric terms and a polynomial, bound the derivative first; this can avoid solving a complicated critical-point equation.

13. Let A be a 3 × 3 invertible matrix. If |adj (24A)| = |adj (3 adj (2A))|, then $|A|^2$ is equal to : [4 marks]

(A) 6^6

(B) 2^{12}

(C) 2^6

(D) 1

Correct Answer: (C) 2^6

Solution:

Concept:

- For a matrix B of order three, $\det(\text{adj}\, B) = (\det B)^2$, and $\det(kB) = k^3 \det B$.
- The matrix A is invertible, so $d = \det A \neq 0$. This allows division by powers of d, but one must still track each nested scalar factor.

Step 1: Evaluate the determinant on the left Let $d = \det A$. Scaling a third-order matrix by 24 multiplies its determinant by 24^3. Taking the adjugate then squares that determinant:

$$\det(\text{adj}(24A)) = \left(24^3 d\right)^2 = 24^6 d^2.$$

The exponent six arises from applying both rules, rather than treating an adjugate as a scalar multiple of its original matrix.

Step 2: Evaluate the nested expression on the right First calculate

$$\det(\text{adj}(2A)) = \left(2^3 d\right)^2 = 2^6 d^2.$$

Next, multiplying this third-order adjugate matrix by 3 gives

$$\det(3\,\text{adj}(2A)) = 3^3\, 2^6 d^2.$$

Finally, the outer adjugate squares this complete determinant, so

$$\det(\text{adj}(3\,\text{adj}(2A))) = 3^6\, 2^{12} d^4.$$

All three operations must be retained; omitting the outer square changes the answer.

Step 3: Equate and simplify The hypothesis therefore reads $24^6 d^2 = 3^6\, 2^{12} d^4$. Divide by the nonzero factor $3^6\, 2^{12} d^2$:

$$d^2 = \frac{24^6}{3^6\, 2^{12}} = \frac{(3 \cdot 2^3)^6}{3^6\, 2^{12}} = 2^6.$$

Thus neither 6^6, 2^{12}, nor 1 satisfies the required equation; they retain an extra scalar factor or lose the power arising from the nesting.

Step 4: Conclusion We find $\boxed{|A|^2 = 2^6}$. Hence, the correct option is (C).

Quick Tip to Solve: Work from the innermost matrix outwards, writing its determinant at each stage; every adjugate squares the determinant for a matrix of order three.

14. Let S be the set of all the natural numbers, for which the line $\frac{x}{a}+\frac{y}{b}=2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b), ab $\neq$ 0. Then : [4 marks]

(A) S = ϕ

(B) n(S) = 1

(C) S = {2k : k $\in$ N}

(D) S = N

Correct Answer: (D) S = N

Solution:

Concept:

- For an implicit curve $F(x,y)=0$, the tangent at a regular point (x_0,y_0) is $F_x(x_0,y_0)(x-x_0)+F_y(x_0,y_0)(y-y_0)=0$.
- Every positive integer power is differentiable. The conditions $ab\neq 0$ and $n\in\mathbb{N}$ make the gradient of this curve at (a,b) nonzero.

Step 1: Check the point and differentiate the curve Write $F(x,y)=(x/a)^n+(y/b)^n-2$. At (a,b), its value is $1+1-2=0$, so the stated point is on the curve for every natural number n. The partial derivatives there are

$$F_x(a,b)=\frac{n}{a},\qquad F_y(a,b)=\frac{n}{b}.$$

Both are defined and nonzero because $n\geq 1$ and $a,b\neq 0$. Thus the ordinary tangent formula is applicable for every such n.

Step 2: Write and simplify the tangent equation The tangent equation becomes

$$\frac{n}{a}(x-a)+\frac{n}{b}(y-b)=0.$$

Divide by the common nonzero factor n and expand:

$$\frac{x}{a}-1+\frac{y}{b}-1=0,\qquad \frac{x}{a}+\frac{y}{b}=2.$$

This is exactly the specified line. Its equation does not depend on whether n is even or odd, and there is no restriction to a single power.

Step 3: Determine the set of permissible powers Every positive integer satisfies the tangent condition, including $n=1$. In that case the curve is itself a straight line and its tangent coincides with the same line, so it is not excluded. Consequently S is neither empty nor a singleton, and restricting S to even natural numbers would discard valid odd powers such as $n=3$. There are no additional natural-number cases to check.

Step 4: Conclusion The tangent condition holds for all natural numbers, so $\boxed{S=\mathbb{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: At the equal-ratio point (a,b), both powers reduce to one, so the same factor n appears in both gradient components and cancels from the tangent equation.

15. Let C be a circle passing through the points A(2, −1) and B(3, 4). The line segment AB s not a diameter of C. If r is the radius of C and its centre lies on the circle $(x-5)^2+(y-1)^2=\frac{13}{2}$, then r^2 is equal to : [4 marks]

(A) 32

(B) $\frac{65}{2}$

(C) $\frac{61}{2}$

(D) 30

Correct Answer: (B) $\frac{65}{2}$

Solution:

Concept:

- The centre of a circle passing through two distinct points lies on the perpendicular bisector of their chord. Thus its distances from A and B are equal.
- For a centre (h,k), the squared radius is $r^2=(h-2)^2+(k+1)^2$. The midpoint is the centre only when the chord is a diameter.

Step 1: Find the locus of the centre Equating the squared distances to the two given points gives

$$(h-2)^2+(k+1)^2=(h-3)^2+(k-4)^2,$$

so $h+5k=10$. The midpoint of the chord is $M=(5/2,3/2)$, and the direction $(-5,1)$ is perpendicular to $AB=(1,5)$. Hence every possible centre has the form

$$(h,k)=\left(\frac{5}{2}-5t,\frac{3}{2}+t\right), \qquad t\in\mathbb{R}.$$

Step 2: Apply the second circle condition The centre must also lie on the stated circle. Substitution yields

$$\left(-\frac{5}{2}-5t\right)^2+\left(\frac{1}{2}+t\right)^2=\frac{13}{2}.$$

The left side is $26t^2+26t+13/2$; therefore $26t(t+1)=0$. The candidate centres correspond to $t=0$ and $t=-1$. The first is exactly M, which would make AB a diameter, contrary to the given restriction. Thus the permitted centre is $(15/2,1/2)$.

Step 3: Compute and compare the radius Using the distance to $A=(2,-1)$,

$$r^2=\left(\frac{15}{2}-2\right)^2+\left(\frac{1}{2}+1\right)^2=\frac{121+9}{4}=\frac{65}{2}.$$

The distance to B gives the same value. Consequently 32, $61/2$, and 30 cannot be the squared radius of the permitted circle.

Step 4: Conclusion The squared radius is $\boxed{r^2=65/2}$. Hence, the correct option is (B).

Quick Tip to Solve: Parameterise the perpendicular bisector from the chord midpoint; the parameter zero is immediately excluded when the chord is not a diameter.

16. Let the normal at the point on the parabola $y^2 = 6x$ pass through the point $(5, -8)$. If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is : [4 marks]

(A) -3

(B) $-\frac{9}{4}$

(C) $-\frac{5}{2}$

(D) -2

Correct Answer: (B) $-\frac{9}{4}$

Solution:

Concept:

- For $y^2 = 4ax$, a point is $P = (at^2, 2at)$, and its normal is $y = -tx + 2at + at^3$. Here $a = 3/2$.
- The tangent at parameter t is $ty = x + at^2$, and the directrix is $x = -a$. The required ordinate is found by intersecting these two lines.

Step 1: Determine the parameter from the normal Put $(5, -8)$ into the normal equation with $a = 3/2$:

$$-8 = -5t + 3t + \frac{3}{2}t^3, \qquad 3t^3 - 4t + 16 = 0.$$

The value $t = -2$ is a root. Factoring the polynomial gives

$$3t^3 - 4t + 16 = (t + 2)(3t^2 - 6t + 8).$$

The quadratic factor has discriminant $36 - 96 = -60 < 0$, so there is only one real parameter and no second real point to consider.

Step 2: Write the tangent and directrix With $t = -2$, the point on the parabola is $P = (6, -6)$. Its tangent is

$$-2y = x + 6.$$

This equation also follows by differentiating $y^2 = 6x$: the tangent slope at P is $3/y = -1/2$. The normal slope is therefore 2, and the line through P with that slope passes through $(5, -8)$, independently checking the selected parameter.

Step 3: Intersect the tangent with the directrix The directrix is $x = -3/2$. At its intersection Q with the tangent,

$$-2y_Q = -\frac{3}{2} + 6 = \frac{9}{2}, \qquad y_Q = -\frac{9}{4}.$$

The other proposed ordinates $-3, -5/2, -2$ do not satisfy the tangent equation at $x = -3/2$, so none represents the requested intersection.

Step 4: Conclusion The ordinate of Q is $\boxed{-9/4}$. Hence, the correct option is (B).

Quick Tip to Solve: After finding the normal parameter, use $y_Q = a(t^2 - 1)/t$ to intersect the tangent directly with $x = -a$.

17. Let the plane 2x + 3y + z + 20 = 0 be rotated through a right angle about its line of intersection with the plane x − 3y + 5z = 8. If the mirror image of the point $\left(2, -\frac{1}{2}, 2\right)$ in the rotated plane is B(a, b, c), then : [4 marks]

(A) $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$
(B) $\frac{a}{4} = \frac{b}{5} = \frac{c}{-2}$
(C) $\frac{a}{8} = \frac{b}{-5} = \frac{c}{4}$
(D) $\frac{a}{4} = \frac{b}{5} = \frac{c}{2}$

Correct Answer: (A) $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$

Solution:

Concept:

- A plane through the intersection of two planes belongs to their linear family. A right-angle rotation makes its normal perpendicular to the original normal.
- The reflection of a point $\mathbf{p}$ in $\mathbf{n} \cdot \mathbf{x} + d = 0$ is $\mathbf{p} - 2(\mathbf{n} \cdot \mathbf{p} + d)\mathbf{n}/\|\mathbf{n}\|^2$.

Step 1: Find the perpendicular plane in the family Let $P_1 = 2x+3y+z+20$ and $P_2 = x-3y+5z-8$. A plane through their common line is $P_2+\lambda P_1 = 0$. Its normal is $(1+2\lambda, -3+3\lambda, 5+\lambda)$. Perpendicularity to the original normal $(2, 3, 1)$ requires

$$2(1 + 2\lambda) + 3(-3 + 3\lambda) + (5 + \lambda) = 0,$$

which reduces to $-2 + 14\lambda = 0$. Hence $\lambda = 1/7$.

Step 2: Simplify the rotated plane Multiplying the family equation by seven gives $P_1 + 7P_2 = 0$, or

$$9x - 18y + 36z - 36 = 0.$$

Thus the rotated plane is $x - 2y + 4z - 4 = 0$, with normal $\mathbf{n} = (1, -2, 4)$ and $\|\mathbf{n}\|^2 = 21$. Its normal has dot product $2 - 6 + 4 = 0$ with the original normal, confirming the right angle; membership in the family preserves the rotation axis.

Step 3: Reflect the given point and test the ratios For $\mathbf{p} = (2, -1/2, 2)$, the plane expression is $2 + 1 + 8 - 4 = 7$. Therefore

$$(a, b, c) = \left(2, -\frac{1}{2}, 2\right) - \frac{14}{21}(1, -2, 4) = \left(\frac{4}{3}, \frac{5}{6}, -\frac{2}{3}\right).$$

Consequently $a/8 = b/5 = c/(-4) = 1/6$. In option B the first two ratios differ, in option C the second ratio has the wrong sign, and in option D the third ratio has the wrong sign.

Step 4: Conclusion $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4} = \frac{1}{6}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the family of planes to preserve the rotation axis, impose perpendicular normals, and only then apply the point-reflection formula.

18. If $\vec{a} \cdot \vec{b} = 1$, $\vec{b} \cdot \vec{c} = 2$ and $\vec{c} \cdot \vec{a} = 3$, then the value of

$$\left[\vec{a} \times \left(\vec{b} \times \vec{c}\right), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times \left(\vec{b} \times \vec{a}\right)\right]$$

is : [4 marks]

(A) 0

(B) $-6\vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$

(C) $-12\vec{c} \cdot \left(\vec{a} \times \vec{b}\right)$

(D) $-12\vec{b} \cdot (\vec{c} \times \vec{a})$

Correct Answer: (A) 0

Solution:

Concept:

- The vector triple-product identity is $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \mathbf{v}(\mathbf{u} \cdot \mathbf{w}) - \mathbf{w}(\mathbf{u} \cdot \mathbf{v})$.
- The scalar triple product $[\mathbf{u}, \mathbf{v}, \mathbf{w}]$ is a determinant. It vanishes when its three vector arguments are linearly dependent.

Step 1: Expand the first two vector arguments Write the three arguments as $\mathbf{U}, \mathbf{V}, \mathbf{W}$, in their printed order. Using $\mathbf{a} \cdot \mathbf{b} = 1$, $\mathbf{b} \cdot \mathbf{c} = 2$, and $\mathbf{c} \cdot \mathbf{a} = 3$, we obtain

$$\mathbf{U} = \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = 3\mathbf{b} - \mathbf{c},$$
$$\mathbf{V} = \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) = \mathbf{c} - 2\mathbf{a}.$$

The order of the vectors inside each cross product matters, so the identity is applied without interchanging its arguments.

Step 2: Expand the third argument and compare The third printed argument contains $\mathbf{b} \times \mathbf{a}$, not $\mathbf{a} \times \mathbf{b}$. Hence

$$\mathbf{W} = \mathbf{c} \times (\mathbf{b} \times \mathbf{a}) = \mathbf{b}(\mathbf{c} \cdot \mathbf{a}) - \mathbf{a}(\mathbf{c} \cdot \mathbf{b}) = 3\mathbf{b} - 2\mathbf{a}.$$

Adding the two preceding expressions shows $\mathbf{U} + \mathbf{V} = \mathbf{W}$. This relation establishes dependence without needing the lengths or individual coordinates of the original vectors.

Step 3: Evaluate the determinant By linearity in its third argument,

$$[\mathbf{U}, \mathbf{V}, \mathbf{W}] = [\mathbf{U}, \mathbf{V}, \mathbf{U}] + [\mathbf{U}, \mathbf{V}, \mathbf{V}] = 0.$$

Each determinant on the right has two equal vector arguments. The nonzero multiples of $[\mathbf{a}, \mathbf{b}, \mathbf{c}]$ appearing in the other options are not the general identity here; they would coincide with zero only in special dependent cases.

Step 4: Conclusion The required scalar triple product is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: Expand the nested cross products first and look for a linear relation among the three resulting arguments before computing any determinant.

19. Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting atmost two heads is : [4 marks]

(A) $\frac{275}{6^5}$

(B) $\frac{36}{5^4}$

(C) $\frac{181}{5^5}$

(D) $\frac{46}{6^4}$

Correct Answer: (D) $\frac{46}{6^4}$

Solution:

Concept:

- For five independent tosses with head probability p, the number X of heads is binomial: $P(X = k) = \binom{5}{k}p^k(1-p)^{5-k}$.
- At most two heads means the disjoint cases $X = 0, 1, 2$; their probabilities must be added. A biased coin has unequal head and tail probabilities.

Step 1: Determine the head probability For a nondegenerate coin, $0 < p < 1$. The stated equality gives

$$\binom{5}{4}p^4(1-p) = \binom{5}{5}p^5.$$

Dividing by the positive factor p^4, we obtain $5(1-p) = p$, so

$$p = \frac{5}{6}, \qquad 1 - p = \frac{1}{6}.$$

Thus the coin favours heads strongly; treating it as a fair coin would contradict the equality of the two specified probabilities. Indeed, both $P(X = 4)$ and $P(X = 5)$ now equal $5^5/6^5$.

Step 2: Add the three required cases Using the common denominator 6^5,

$$P(X \le 2) = \left(\frac{1}{6}\right)^5 + 5\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^4 + 10\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^3$$
$$= \frac{1 + 25 + 250}{6^5} = \frac{276}{6^5}.$$

The binomial coefficients count the possible positions of the heads. Omitting them would count only one ordering in each case rather than every sequence with the required head count.

Step 3: Simplify and compare the options Since $276 = 6 \cdot 46$, cancellation yields

$$P(X \le 2) = \frac{46}{6^4}.$$

Option A has numerator 275 over 6^5, missing the all-tail case of probability $1/6^5$. Options B and C have different values and arise from neither the correct head probability nor the complete binomial sum.

Step 4: Conclusion The probability of at most two heads is $\boxed{46/6^4}$. Hence, the correct option is (D).

Quick Tip to Solve: Compare adjacent binomial probabilities to find p, then place all terms over the common denominator before adding their numerators.

20. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $p \nabla q \Rightarrow ((p \Delta q) \nabla r)$ is a tautology. Then $(p \nabla q) \Delta r$ is logically equivalent to : [4 marks]

(A) $(p \Delta r) \vee q$

(B) $(p \Delta r) \wedge q$

(C) $(p \wedge r) \Delta q$

(D) $(p \nabla r) \wedge q$

Correct Answer: (A) $(p \Delta r) \vee q$

Solution:

Concept:

- An implication $A \Rightarrow B$ is false exactly when A is true and B is false. A tautology must avoid that case for every assignment.
- Conjunction $\wedge$ requires both propositions to be true, while disjunction $\vee$ requires at least one. Disjunction is associative and commutative.

Step 1: Determine the outer connective Suppose first that $\nabla = \wedge$. Choose $p = q = \mathrm{T}$ and $r = \mathrm{F}$. Then the antecedent $p \nabla q$ is true, whereas the consequent $(p \Delta q) \wedge r$ is false, regardless of Δ. This contradicts the tautology condition. Therefore ∇ cannot be conjunction, and we must have $\nabla = \vee$.

Step 2: Determine the remaining connective Now the implication becomes

$$(p \vee q) \Rightarrow ((p \Delta q) \vee r).$$

If $\Delta = \wedge$, the assignment $p = \mathrm{T}, q = \mathrm{F}, r = \mathrm{F}$ makes its antecedent true and its consequent false. Hence $\Delta = \vee$. With both connectives equal to disjunction, the implication is indeed a tautology: whenever $p \vee q$ is true, $p \vee q \vee r$ is also true, independently of r.

Step 3: Rewrite the requested expression The expression in question is therefore

$$(p \nabla q) \Delta r = (p \vee q) \vee r = p \vee q \vee r.$$

Option A becomes $(p \vee r) \vee q$, which is identical by associativity and commutativity. Options B and D reduce to $(p \vee r) \wedge q$; with p true and q, r false they fail. Option C becomes $(p \wedge r) \vee q$, which also fails for that assignment.

Step 4: Conclusion The equivalent proposition is $(p \Delta r) \vee q$, with $\Delta = \nabla = \vee$. Hence, the correct option is (A).

Quick Tip to Solve: Test an implication with a true antecedent and set r false; this quickly eliminates connectives that could make the consequent false.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The sum of the cubes of all the roots of the equation

$x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ **is _____.** [4 marks]

Solution:

Concept:

- For roots $\alpha_1, \ldots, \alpha_4$, let $s_k = \sum_{i=1}^{4} \alpha_i^k$. Newton identities obtain these power sums directly from the coefficients.
- For $x^4 - e_1x^3 + e_2x^2 - e_3x + e_4$, Vieta gives the elementary symmetric sums e_1, e_2, e_3, e_4. In particular $s_3 = e_1s_2 - e_2s_1 + 3e_3$.

Step 1: Read the symmetric sums from the polynomial Compare the given equation with the monic quartic in the concept:

$$x^4 - 3x^3 - 2x^2 + 3x + 1 = 0.$$

Its coefficients give

$$e_1 = 3, \qquad e_2 = -2, \qquad e_3 = -3, \qquad e_4 = 1.$$

The sign in e_3 is especially important: the coefficient of x is $-e_3$, so the printed $+3x$ means $e_3 = -3$. We seek the sum over all four roots, counting their multiplicities, and do not need their individual values.

Step 2: Compute the first two power sums The first Newton identity gives $s_1 = e_1 = 3$. Squaring the sum of roots, or using the second identity, gives

$$s_2 = e_1^2 - 2e_2 = 3^2 - 2(-2) = 13.$$

This identity follows because $(\sum \alpha_i)^2$ contains each squared root once and each pairwise product twice. It applies to complex as well as real roots, so no assumptions about the nature of the quartic roots are required.

Step 3: Apply the third Newton identity For the cubes, the identity is

$$s_3 - e_1s_2 + e_2s_1 - 3e_3 = 0.$$

Substituting the values already determined,

$$s_3 = 3(13) - (-2)(3) + 3(-3) = 39 + 6 - 9 = 36.$$

As a direct check, the equivalent symmetric formula gives $s_3 = e_1^3 - 3e_1e_2 + 3e_3 = 27 + 18 - 9 = 36$. Both calculations preserve the alternating coefficient signs.

Step 4: Conclusion The sum of the cubes of all the roots is $\boxed{36}$. The required numerical entry is 36.

Quick Tip to Solve: For a cubic power sum, stop at e_3: use $s_3 = e_1^3 - 3e_1e_2 + 3e_3$, carefully reversing the sign of the linear coefficient in a quartic.

22. There are ten boys B_1, B_2, ..., B_{10} and five girls G_1, G_2, ..., G_5 in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is _____. [4 marks]

Solution:

Concept:

- A group is an unordered selection, so choose its members with binomial coefficients, not permutations. The boys and girls are selected independently.
- Use complementary counting: subtract groups containing both specified boys from all groups of three boys and three girls. The restriction forbids their joint presence, while either boy alone is permitted.

Step 1: Count all unrestricted groups There are ten boys, of whom exactly three must be chosen, and five girls, of whom exactly three must be chosen. Since every valid choice of boys can be combined with every choice of girls, the multiplication principle gives

$$N_{\text{all}} = \binom{10}{3}\binom{5}{3} = 120 \times 10 = 1200.$$

No ordering within either selection is counted: exchanging the positions of two members does not create another group.

Step 2: Subtract the forbidden groups Fix both B_1 and B_2 in a forbidden group. Its third boy can be any one of the remaining eight boys, and its three girls can still be any three of the five girls. Therefore

$$N_{\text{bad}} = \binom{8}{1}\binom{5}{3} = 8 \times 10 = 80.$$

Each forbidden group is counted exactly once because its third boy and its set of girls determine it uniquely. Subtraction removes precisely the groups prohibited by the statement:

$$N_{\text{valid}} = 1200 - 80 = 1120.$$

As a check, choosing neither specified boy or exactly one gives $\binom{8}{3} + 2\binom{8}{2} = 56 + 56 = 112$ boy selections; multiplying by ten again gives the same result.

Step 3: Conclusion The number of permitted groups is $\boxed{1120}$.

Quick Tip to Solve: When two named members must not occur together, subtract the selections containing both; do not exclude groups containing just one of them.

23. **Let $f(x) = \max\{|x+1|, |x+2|, \ldots, |x+5|\}$. Then $\int_{-6}^{0} f(x)\,dx$ is equal to _____.** [4 marks]

Solution:

Concept:

- The quantity $|x+k|$ is the distance between $-x$ and k on the real line. Of the five points $1, 2, 3, 4, 5$, an extreme point gives the greatest distance.
- A definite integral of a continuous piecewise linear function can be evaluated by integrating its expressions on the intervals where the maximum changes.

Step 1: Reduce the maximum to two distances The points $2, 3, 4$ lie between 1 and 5. Their distances from any given point cannot exceed both endpoint distances. Therefore

$$f(x) = \max\{|x+1|, |x+5|\}.$$

To find where the larger expression changes, compare their squares. Squaring preserves the order since absolute values are nonnegative:

$$(x+1)^2 - (x+5)^2 = -8(x+3).$$

The first distance is larger when $x < -3$, the second when $x > -3$, and they tie at $x = -3$. Thus the transition lies inside the integration interval.

Step 2: Identify the two linear expressions If $-6 \le x \le -3$, then $x+1 < 0$, giving $|x+1| = -x-1$. If $-3 \le x \le 0$, then $x+5 > 0$, giving $|x+5| = x+5$. Hence

$$f(x) = \begin{cases} -x-1, & -6 \le x \le -3, \\ x+5, & -3 \le x \le 0. \end{cases}$$

At the joining point both values equal two, so there is no discontinuity. The interior absolute values need no further breakpoints because none is the maximum.

Step 3: Integrate and check the areas Split only at the transition point and evaluate each elementary antiderivative:

$$\int_{-6}^{0} f(x)\,dx = \left[-\frac{x^2}{2} - x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 5x\right]_{-3}^{0}$$
$$= \frac{21}{2} + \frac{21}{2} = 21.$$

Each subinterval has width three, and its linear graph has endpoint heights five and two. The trapezium area $3(5+2)/2 = 21/2$ independently checks each contribution.

Step 4: Conclusion The integral in the printed question is $\boxed{21}$. This follows directly from its two linear pieces, even though the supplied numerical key for this label gives a different value.

Quick Tip to Solve: For the maximum of distances to collinear points, retain only the two extremes and split at their midpoint before integrating.

24. Let the solution curve y = y(x) of the differential equation

$(4+x^2)dy-2x(x^2+3y+4)dx=0$ **pass through the origin. Then y(2) is equal to _____.** [4 marks]

Solution:

Concept:

- A first-order linear equation $y' + P(x)y = Q(x)$ is solved using the integrating factor $e^{\int P(x)\,dx}$. Multiplication by it converts the left side to a product derivative.
- Passing through the origin means $y(0) = 0$. Apply this initial condition to determine the integration constant before evaluating the solution at $x = 2$.

Step 1: Put the equation in linear form Since $x^2 + 4$ is positive for every real x, divide the given equation by $(x^2 + 4)\,dx$ without losing any real solution. Rearranging gives

$$y' - \frac{6x}{x^2+4}y = 2x.$$

The coefficient of y is $-6x/(x^2 + 4)$, so the integrating factor is

$$\text{I.F.} = \exp\left(\int \frac{-6x}{x^2+4}\,dx\right) = (x^2+4)^{-3}.$$

Here the substitution $u = x^2 + 4$, with $du = 2x\,dx$, produces the logarithm and the exponent -3.

Step 2: Integrate and impose the initial condition Multiplying by the integrating factor yields

$$\frac{d}{dx}\left(\frac{y}{(x^2+4)^3}\right) = \frac{2x}{(x^2+4)^3}.$$

Integrating with the same substitution,

$$\frac{y}{(x^2+4)^3} = -\frac{1}{2(x^2+4)^2} + C.$$

At the origin this becomes $0 = -1/32 + C$, so $C = 1/32$. Therefore the required solution curve is

$$y = \frac{(x^2+4)^3}{32} - \frac{x^2+4}{2}.$$

It satisfies $y(0) = 64/32 - 4/2 = 0$, as required.

Step 3: Evaluate at the requested point At $x = 2$, the quantity $x^2 + 4$ equals eight. Substitute in the fully determined solution:

$$y(2) = \frac{8^3}{32} - \frac{8}{2} = 16 - 4 = 12.$$

The equation and initial condition have both been used, so no arbitrary constant remains.

Step 4: Conclusion The requested numerical value is $\boxed{y(2) = 12}$.

Quick Tip to Solve: After dividing by $x^2 + 4$, simplify the right side to $2x$ before finding the integrating factor; keep the negative sign in its exponent.

25. If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ) = \alpha - \frac{1}{16}\sin(10^\circ)$, then $16 + \alpha^{-1}$ is equal to _____.

[4 marks]

Solution:

Concept:

- Use the triple-angle product identity $4\sin\theta\sin(60^\circ+\theta)\sin(60^\circ-\theta) = \sin 3\theta$. All angles in this question are in degrees.
- Product-to-sum identities reduce the remaining factor: $2\sin u\sin v = \cos(u-v) - \cos(u+v)$ and $2\sin u\cos v = \sin(u+v) + \sin(u-v)$.

Step 1: Reduce three factors of the product Write $s = \sin 10^\circ$. Taking $\theta = 10^\circ$ in the triple-angle product identity gives

$$\sin 10^\circ \sin 70^\circ \sin 50^\circ = \frac{\sin 30^\circ}{4} = \frac{1}{8}.$$

Thus the entire left-hand side, denoted by P, becomes

$$P = \frac{1}{8}\sin 10^\circ \sin 20^\circ \sin 40^\circ = \frac{s}{8}\sin 20^\circ \sin 40^\circ.$$

One factor $\sin 10^\circ$ remains because the original expression contains its square.

Step 2: Express the remaining product in terms of sine ten degrees The first product-to-sum identity gives

$$\sin 20^\circ \sin 40^\circ = \frac{\cos 20^\circ - \cos 60^\circ}{2} = \frac{\cos 20^\circ}{2} - \frac{1}{4}.$$

Also

$$2s\cos 20^\circ = \sin 30^\circ + \sin(-10^\circ) = \frac{1}{2} - s.$$

It follows that

$$s\sin 20^\circ \sin 40^\circ = \frac{1}{8} - \frac{s}{2}, \qquad P = \frac{1}{64} - \frac{s}{16}.$$

All reductions are exact, so no numerical approximation to the angles is needed.

Step 3: Identify alpha and evaluate the expression Compare the obtained identity with the stated equation $P = \alpha - \sin 10^\circ/16$. The sine terms are identical, leaving $\alpha = 1/64$. Therefore

$$16 + \alpha^{-1} = 16 + 64 = 80.$$

The reciprocal applies to α itself; it is not a reciprocal trigonometric function.

Step 4: Conclusion The required value is $\boxed{80}$.

Quick Tip to Solve: Group $\sin 10^\circ \sin 50^\circ \sin 70^\circ$ first; its exact value $1/8$ removes three factors at once.

26. Let A = {n ∈ N : H.C.F. (n, 45) = 1} and
Let B = {2k : k ∈ {1, 2, ..., 100}}. Then the sum of all the elements of A ∩ B is _____.

[4 marks]

Solution:

Concept:

- The condition $\gcd(n, 45) = 1$ is equivalent to divisibility by neither three nor five, because $45 = 3^2 \cdot 5$. Only the distinct prime factors matter for coprimality.
- Every element of B has the form $2k$, with $1 \le k \le 100$. Since two is coprime to 45, the admissible indices are precisely those divisible by neither three nor five.

Step 1: Translate the intersection into an index condition For $n = 2k$, multiplication by two introduces no factor three or five. Hence $\gcd(2k, 45) = 1$ if and only if $3 \nmid k$ and $5 \nmid k$. The required sum is therefore twice the sum of the integers from one to one hundred after removing multiples of three and five. The map $k \mapsto 2k$ is one-to-one, so each eligible element of the intersection appears exactly once.

Step 2: Apply inclusion-exclusion to the sums The sum of all indices is $100 \cdot 101/2 = 5050$. There are 33 multiples of three, 20 multiples of five, and six multiples of fifteen in this range. Their respective sums are

$$S_3 = 3\frac{33 \cdot 34}{2} = 1683,$$
$$S_5 = 5\frac{20 \cdot 21}{2} = 1050,$$
$$S_{15} = 15\frac{6 \cdot 7}{2} = 315.$$

A multiple of fifteen was removed in both previous subtractions, although it must be excluded just once. Add its sum back once to obtain

$$S_{\text{eligible}} = 5050 - 1683 - 1050 + 315 = 2632.$$

The upper limits use integer parts of $100/3$, $100/5$, and $100/15$; numbers beyond one hundred are not included.

Step 3: Convert back to elements of B The eligible indices are k, whereas the elements being summed are $2k$. Multiply the eligible-index sum by two:

$$\sum_{n \in A \cap B} n = 2S_{\text{eligible}} = 2(2632) = 5264.$$

This last factor is necessary because the largest possible element of B is two hundred, not one hundred.

Step 4: Conclusion The sum of all elements of $A \cap B$ is $\boxed{5264}$.

Quick Tip to Solve: For a coprimality condition, exclude multiples of the distinct prime factors; apply inclusion-exclusion to their sums, then restore any scale factor such as $n = 2k$.

27. The value of the integral

$\frac{48}{\pi^4}\int_0^{\pi}\left(\frac{3\pi x^2}{2}-x^3\right)\frac{\sin x}{1+\cos^2 x}dx$ **is equal to _____.** [4 marks]

Solution:

Concept:

- On $[0,\pi]$, the kernel $w(x)=\sin x/(1+\cos^2 x)$ is unchanged by reflection $x\mapsto\pi-x$. A symmetric kernel allows the polynomial factor to be replaced by the average of its reflected values.
- The substitution $u=\cos x$, $du=-\sin x\,dx$, evaluates the remaining integral using $\int du/(1+u^2)=\arctan u$.

Step 1: Reflect the integral about its midpoint Denote the unscaled integral by

$$I=\int_0^{\pi}p(x)w(x)\,dx,\qquad p(x)=\frac{3\pi x^2}{2}-x^3.$$

Under $x\mapsto\pi-x$, sine remains the same and cosine changes sign; its square therefore remains the same. Thus

$$I=\int_0^{\pi}p(\pi-x)w(x)\,dx.$$

Adding the original and reflected expressions is useful because the cubic and quadratic terms in their polynomial sum cancel.

Step 2: Simplify the reflected polynomial Expand carefully:

$$\begin{aligned}p(\pi-x)&=\frac{3\pi}{2}(\pi^2-2\pi x+x^2)\\&\quad-(\pi^3-3\pi^2x+3\pi x^2-x^3)\\&=\frac{\pi^3}{2}-\frac{3\pi x^2}{2}+x^3.\end{aligned}$$

Consequently $p(x)+p(\pi-x)=\pi^3/2$, and averaging the two integrals gives

$$I=\frac{\pi^3}{4}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\,dx.$$

The factor $1/2$ comes from averaging the two integrals.

Step 3: Evaluate and restore the prefactor With $u=\cos x$, the limits become one and negative one. Hence

$$\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\,dx=\int_{-1}^{1}\frac{du}{1+u^2}=\frac{\pi}{2}.$$

Therefore $I=\pi^4/8$, and the given scaled expression is

$$\frac{48}{\pi^4}I=\frac{48}{\pi^4}\frac{\pi^4}{8}=6.$$

The denominator of the kernel is everywhere positive, so the substitutions and integrals have no singularity on this interval.

Step 4: Conclusion The value of the given expression is $\boxed{6}$.

Quick Tip to Solve: Before integrating a polynomial times a symmetric trigonometric kernel, compute $p(x)+p(\pi-x)$; it may be constant even when p is cubic.

28. Let $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$ and $B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$. Then $A+B$ is equal to _____. [4 marks]

Solution:

Concept:

- For any real numbers u, v, their minimum and maximum together are the same two numbers, so $\min\{u,v\}+\max\{u,v\}=u+v$.
- Finite sums are linear. The sum of the first n positive integers is $n(n+1)/2$, and a term independent of an index is repeated for each value of that index.

Step 1: Combine the sums for the same ordered pair Both A and B use precisely the same bounds: i and j each range independently from one to ten. If $i \le j$, the minimum is i and the maximum is j. If $i > j$, those roles reverse. In either case their sum is $i+j$; this also holds on the diagonal $i=j$. By adding term by term,

$$A+B=\sum_{i=1}^{10}\sum_{j=1}^{10}(i+j).$$

This identity avoids evaluating triangular portions of the two arrays separately and prevents double counting their diagonal.

Step 2: Sum first over the inner index Hold i fixed. The term i occurs ten times, while the values of j add to fifty-five. Therefore

$$\sum_{j=1}^{10}(i+j)=10i+\frac{10\cdot 11}{2}=10i+55.$$

There are ten possible choices of the outer index. Consequently, both the variable contribution and the constant contribution must be summed over all ten values. The original sums include one hundred ordered pairs, rather than only ten pairs with equal indices.

Step 3: Evaluate the outer sum Apply the arithmetic-series formula again:

$$\begin{aligned}A+B&=\sum_{i=1}^{10}(10i+55)\\&=10\frac{10\cdot 11}{2}+10(55)\\&=550+550=1100.\end{aligned}$$

For a second check, the average of each index over its ten values is 5.5. Hence the average of $i+j$ over the one hundred pairs is eleven, which gives the same total 100×11.

Step 4: Conclusion The printed double sums give $\boxed{A+B=1100}$. The termwise identity proves this result even though the supplied numerical key for this label gives a different value.

Quick Tip to Solve: Add matching minimum and maximum expressions before summing; they collapse to the sum of their two arguments.

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29. Let $S=(0,2\pi)-\left\{\frac{\pi}{2},\frac{3\pi}{4},\frac{3\pi}{2},\frac{7\pi}{4}\right\}$. Let $y=y(x)$, x ∈ S, be the solution curve of the differential equation $\frac{dy}{dx}=\frac{1}{1+\sin 2x}$, $y\left(\frac{\pi}{4}\right)=\frac{1}{2}$. If the sum of abscissas of all the points of intersection of the curve y = y(x) with the curve $y=\sqrt{2}\sin x$ is $\frac{k\pi}{12}$, then k is equal to _____. [4 marks]

Solution:

Concept:

- Use $1+\sin 2x=(\sin x+\cos x)^2$ to integrate the given derivative.
- Intersections satisfy both equations. When factoring an intersection equation, keep the zero-factor solutions before dividing.

Step 1: Integrate and apply the initial condition The derivative of $\sin x/(\sin x+\cos x)$ is $1/(\sin x+\cos x)^2$. Thus the required expression is

$$y=\frac{\sin x}{\sin x+\cos x}+C.$$

At $x=\pi/4$, the fraction equals $1/2$; the initial value therefore gives $C=0$. This form also stays finite at points where a tangent-based expression would need special handling. The denominator is nonzero on the stated set.

Step 2: Factor the intersection equation Substitute the solution into the second curve and multiply by the nonzero denominator:

$$\sin x\{1-\sqrt{2}(\sin x+\cos x)\}=0.$$

The first factor gives $x=\pi$ in $(0,2\pi)$. For the other factor,

$$\sin x+\cos x=\frac{1}{\sqrt{2}},\qquad \sin(x+\pi/4)=\frac{1}{2}.$$

The solutions in the required interval are $x=7\pi/12$ and $x=23\pi/12$. None of these three abscissas is one of the four excluded values.

Step 3: Add all the abscissas Keeping the root from the first factor as well as both roots from the second factor, the sum is

$$\pi+\frac{7\pi}{12}+\frac{23\pi}{12}=\frac{42\pi}{12}.$$

Comparison with the stipulated expression $k\pi/12$ now determines the integer directly.

Step 4: Conclusion The sum of the intersection abscissas is $42\pi/12$, hence $\boxed{k=42}$.

Quick Tip to Solve: Factor before dividing by $\sin x$; otherwise the valid intersection at $x=\pi$ is lost.

---

30. Let the common tangents to the curves $4(x^2+y^2)=9$ and $y^2=4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latus rectum of this ellipse, then $\frac{l}{e^2}$ is equal to _____. [4 marks]

Solution:

Concept:

- A tangent of slope m to $y^2=4ax$ is $y=mx+a/m$. Here $a=1$ and $m\neq 0$.
- For an ellipse with semi-axes $a>b$, $e^2=1-b^2/a^2$ and the full latus rectum has length $l=2b^2/a$.

Step 1: Determine the common tangents The circle has centre $O=(0,0)$ and radius $3/2$. A tangent to the parabola is $mx-y+1/m=0$. It touches this circle when its distance from the origin is $3/2$:

$$\frac{|1/m|}{\sqrt{m^2+1}}=\frac{3}{2}.$$

Squaring and writing $u=m^2$ gives $u(u+1)=4/9$, or $(3u-1)(3u+4)=0$. Only $u=1/3$ is admissible because a real slope has nonnegative square. The vertical parabola tangent $x=0$ passes through the circle centre and is not a circle tangent.

Step 2: Find their intersection and the ellipse axes The two lines are

$$y=\frac{x}{\sqrt{3}}+\sqrt{3}, \qquad y=-\frac{x}{\sqrt{3}}-\sqrt{3}.$$

Adding their equations gives $y=0$, and then $x=-3$. Consequently $Q=(-3,0)$, so $OQ=3$. The ellipse has $b=3$ and $a=6$; its orientation does not affect its eccentricity or latus rectum length.

Step 3: Evaluate the requested ratio Use the semi-axis lengths, rather than their full axis lengths:

$$e^2=1-\frac{9}{36}=\frac{3}{4}, \qquad l=\frac{2(9)}{6}=3.$$

It follows that $l/e^2=3/(3/4)=4$.

Step 4: Conclusion The required value is $\boxed{l/e^2=4}$.

Quick Tip to Solve: Use the distance from the circle centre to a parabola tangent; this determines both common slopes without solving simultaneous contact-point equations.

Physics – Section A: Multiple Choice Questions (80 marks)

31. An expression for a dimensionless quantity P is given by $P = \frac{\alpha}{\beta}\log_e\left(\frac{kt}{\beta x}\right)$; where α and β are constants, x is distance; k is Boltzmann constant and t is the temperature. Then the dimensions of α will be : [4 marks]

(A) [M^0 L^{-1} T^0]

(B) [M L^0 T^{-2}]

(C) [M L T^{-2}]

(D) [M L^2 T^{-2}]

Correct Answer: (C) [M L T^{-2}]

Solution:

Concept:

- The argument of a logarithm must be dimensionless. The logarithm itself then carries no physical dimensions.
- Boltzmann constant times absolute temperature is an energy: $[kt] = ML^2T^{-2}$. The symbol t here denotes temperature, not time.

Step 1: Apply the logarithm condition The quantity $kt/(\beta x)$ must have dimensions unity. Therefore $[\beta x] = [kt]$. Since x is a distance, dividing energy dimensions by length gives

$$[\beta] = \frac{ML^2T^{-2}}{L} = MLT^{-2}.$$

Thus β has the dimensions of force. This inference does not require the numerical value of Boltzmann constant or the temperature.

Step 2: Apply the dimensionless prefactor condition The problem states that P is dimensionless. Since its logarithmic factor is dimensionless, the multiplicative factor α/β must also be dimensionless. Hence

$$[\alpha] = [\beta] = MLT^{-2}.$$

Both restrictions are necessary: considering only the logarithm determines β, while considering the prefactor transfers that result to α.

Step 3: Compare all the options Option (C) is exactly the force dimension obtained above. Option (A) is inverse length and lacks both the mass and time factors. Option (B) lacks the required length factor. Option (D) is energy, which is the dimension of kt, before division by the distance x. These alternatives cannot satisfy both dimensional conditions simultaneously.

Step 4: Conclusion The dimensions of α are $\boxed{MLT^{-2}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a logarithmic expression, first make the argument dimensionless, then separately check the prefactor.

32. A person is standing in an elevator. In which situation, he experiences weight loss? [4 marks]

(A) When the elevator moves upward with constant acceleration

(B) When the elevator moves downward with constant acceleration

(C) When the elevator moves upward with uniform velocity

(D) When the elevator moves downward with uniform velocity

Correct Answer: (B) When the elevator moves downward with constant acceleration

Solution:

Concept:

- Apparent weight is the normal reaction N exerted by the elevator floor, rather than a change in gravitational force mg.
- With upward positive, Newton's second law gives $N - mg = ma_y$, where a_y is the vertical acceleration.

Step 1: Write the force equation The person has gravity mg downward and the floor reaction N upward. If the elevator accelerates downward with magnitude a, then $a_y = -a$. Consequently

$$N - mg = -ma, \qquad N = m(g - a).$$

For an ordinary downward acceleration smaller than g, the contact force remains positive but is less than mg. This smaller supporting force is the weight loss experienced by the person. In the limiting case $a = g$, the reaction becomes zero.

Step 2: Compare acceleration and uniform-velocity cases For upward acceleration, $a_y = +a$, so $N = m(g + a) > mg$; option (A) describes a weight gain. For either upward or downward uniform velocity the acceleration is zero. The force equation then gives $N = mg$, so neither option (C) nor option (D) produces weight loss. Motion direction alone is insufficient: it is the direction of acceleration that changes the floor reaction.

Step 3: Identify the specified situation Option (B) states downward motion with constant acceleration, which in the intended accelerated descent has acceleration directed downward. Its reaction is therefore reduced according to the expression just derived. The gravitational mass and the local gravitational field remain unchanged during this comparison.

Step 4: Conclusion The person experiences apparent weight loss during downward acceleration. Hence, the correct option is (B).

Quick Tip to Solve: Use the acceleration direction in $N - mg = ma_y$; upward or downward velocity by itself does not determine apparent weight.

33. An object is thrown vertically upwards. At its maximum height, which of the following quantity becomes zero? [4 marks]

(A) Momentum

(B) Potential Energy

(C) Acceleration

(D) Force

Correct Answer: (A) Momentum

Solution:

Concept:

- Linear momentum is $\boldsymbol{p} = m\boldsymbol{v}$. It vanishes whenever the instantaneous velocity is zero.
- For vertical motion near Earth with air resistance neglected, gravitational acceleration remains downward with magnitude g, including at the highest point.

Step 1: Examine the turning point As the object rises, its upward velocity decreases under gravity. At the maximum height the velocity is momentarily zero before it reverses direction. Hence

$$v_{\text{top}} = 0, \qquad p_{\text{top}} = mv_{\text{top}} = 0.$$

This instant of rest does not imply that all dynamical quantities vanish. The force must still act to change the velocity from upward to downward.

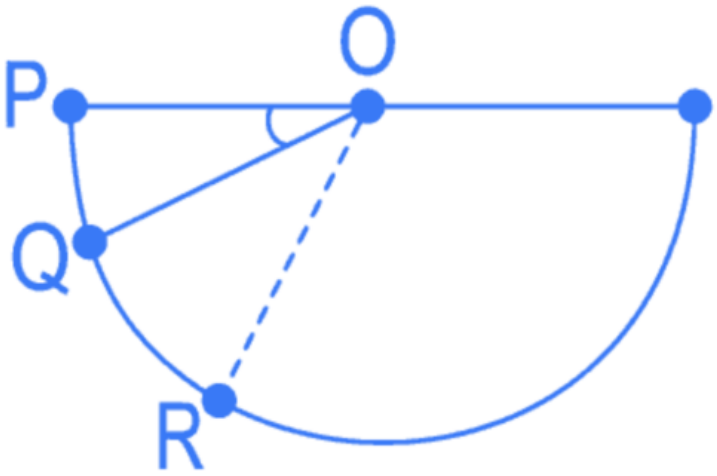
Step 2: Check acceleration and force The gravitational acceleration at the top is still $\boldsymbol{a} = -g\hat{\boldsymbol{y}}$ if upward is positive. The net force is $\boldsymbol{F} = m\boldsymbol{a} = -mg\hat{\boldsymbol{y}}$. Neither is zero; therefore options (C) and (D) are incorrect. A turning point is a zero of velocity, rather than a zero of acceleration.

Step 3: Check potential energy With the launch level chosen as the zero of potential energy, the object has $U = mgh_{\max}$ at its maximum height, a positive value for a nonzero upward launch. It is the largest gravitational potential energy along this flight. More generally potential energy depends on a chosen reference level and is not forced to vanish at a turning point. Thus option (B) is not the universally vanishing quantity requested here.

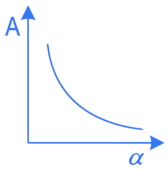
Step 4: Conclusion The object has zero momentum at its maximum height. Hence, the correct option is (A).

Quick Tip to Solve: At the top of a vertical throw set velocity to zero, but retain the downward acceleration g and force mg.

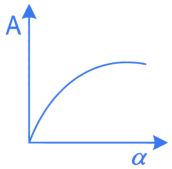
34. A ball is released from rest from point P of a smooth semi-spherical vessel as shown in figure. The ratio of the centripetal force and normal reaction on the ball at point Q is A while angular position of point Q is α with respect to point P. Which of the following graphs represent the correct relation between A and α when ball goes from Q to R? [4 marks]



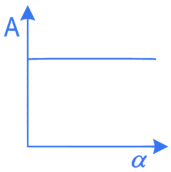
(A) Decreasing convex curve of A against α.



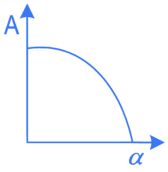
(B) Increasing concave-down curve of A against α.



(C) Horizontal line: A is constant as α changes.

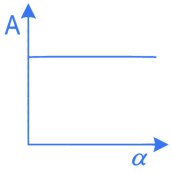


(D) Decreasing concave-down curve of A against α.



Correct Answer: (C)

Horizontal line: A is constant as α changes.



Solution:

Concept:

- On a smooth vessel, mechanical energy is conserved because the normal force is perpendicular to the instantaneous motion and does no work.
- The centripetal force mv^2/r is the net inward radial force. It must not be identified with the normal reaction alone.

Step 1: Find the speed at angular position alpha Let the vessel radius be r and the ball mass be m. The angle α is measured downward from the horizontal radius OP. The vertical drop from the release point is therefore $r\sin\alpha$. Starting from rest and using energy conservation,

$$\frac{1}{2}mv^2 = mgr\sin\alpha, \qquad \frac{mv^2}{r} = 2mg\sin\alpha.$$

The inward radial direction points from the ball to the centre of the vessel.

Step 2: Resolve the radial forces The normal reaction N points inward. Gravity has an outward radial component of magnitude $mg\sin\alpha$. The radial equation is consequently

$$N - mg\sin\alpha = \frac{mv^2}{r} = 2mg\sin\alpha.$$

It follows that $N = 3mg\sin\alpha$. The requested ratio of the centripetal force to the normal reaction is

$$A = \frac{mv^2/r}{N} = \frac{2mg\sin\alpha}{3mg\sin\alpha} = \frac{2}{3}.$$

This holds at positions after release where the denominator is nonzero, including the segment from Q to R.

Step 3: Select the graph The ratio is independent of α, even though both the speed and the normal force change as the ball descends. Thus the graph must be a horizontal line at a positive value. Option (C) shows this relation. Options (A) and (D) decrease with angle, while option (B) increases; none of those variable ratios agrees with the constant value obtained from the force equation.

Step 4: Conclusion The ratio is $\boxed{A = 2/3}$, independent of α. Hence, the correct option is (C).

Quick Tip to Solve: First find speed from the vertical drop, then resolve gravity radially; the normal reaction is larger than the net centripetal force here.

35. A thin circular ring of mass M and radius R is rotating with a constant angular velocity 2 rads^{-1} in a horizontal plane about an axis vertical to its plane and passing through the center of the ring. If two objects each of mass m be attached gently to the opposite ends of a diameter of ring, the ring will then rotate with an angular velocity (in rads^{-1}). [4 marks]

(A) $\frac{M}{(M+m)}$

(B) $\frac{(M+2m)}{2M}$

(C) $\frac{2M}{(M+2m)}$

(D) $\frac{2(M+2m)}{M}$

Correct Answer: (C) $\frac{2M}{(M+2m)}$

Solution:

Concept:

- In the absence of an external torque about the rotation axis, total angular momentum is conserved during the attachment.
- A thin ring has moment of inertia MR^2 about its central normal axis; each point mass at radius R contributes mR^2.

Step 1: Calculate the initial angular momentum The original ring rotates at $\omega_i = 2\,\mathrm{rad\,s^{-1}}$. Its moment of inertia is $I_i = MR^2$, so

$$L_i = I_i\omega_i = 2MR^2.$$

The two masses are attached gently and initially supply no angular momentum about the axis. Contact forces that bring them into rotation are internal to the combined ring-and-masses system.

Step 2: Calculate the final moment of inertia Both attached masses lie at opposite ends of a diameter, a distance R from the axis. Their moments of inertia add even though they are on opposite sides, since the distance is squared. Thus

$$I_f = MR^2 + mR^2 + mR^2 = (M+2m)R^2.$$

Conserving angular momentum gives

$$(M+2m)R^2\omega_f = 2MR^2, \qquad \omega_f = \frac{2M}{M+2m}\,\mathrm{rad\,s^{-1}}.$$

Mechanical kinetic energy need not be conserved while the masses are attached.

Step 3: Check the options and limiting behaviour The expression in option (C) has the required result. When $m = 0$, it returns the initial angular speed 2; options (A) and (B) instead give 1 and $1/2$, respectively. For positive m, the angular speed must decrease as the inertia grows. Option (D) increases above 2, contrary to angular-momentum conservation. The cancellation of R^2 explains why the final result is independent of the ring radius.

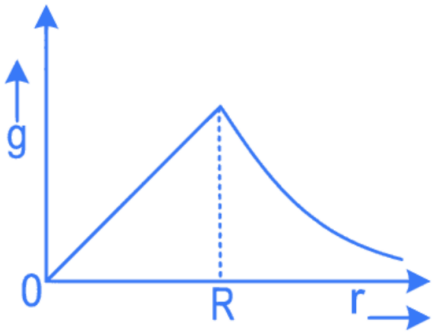
Step 4: Conclusion The final angular speed is $\boxed{2M/(M+2m)\,\mathrm{rad\,s^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: Add both point-mass contributions mR^2; opposite positions do not cancel moments of inertia.

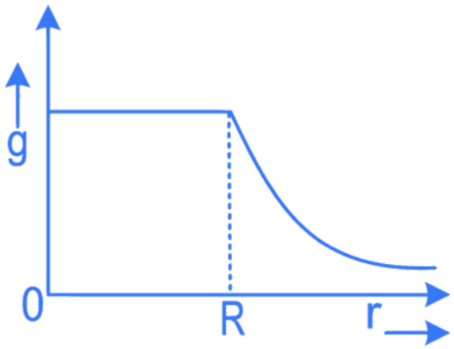
36. The variation of acceleration due to gravity (g) with distance (r) from the center of the earth is correctly represented by :

(Given R = radius of earth) [4 marks]

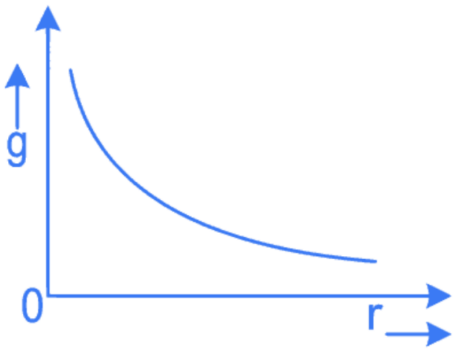
(A)



(B)



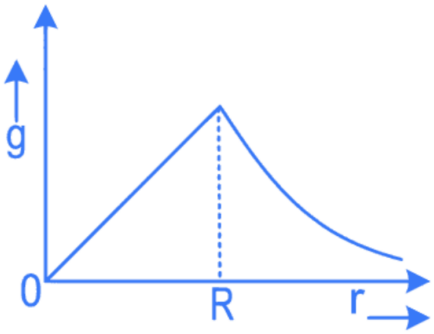
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- For the usual uniform-density model of the Earth, the gravitational field at radius $r < R$ depends only on the mass enclosed within that radius. The spherical shells outside it have zero net field inside.
- Outside a spherical Earth of mass M, its field is the same as that of a point mass at its centre: $g(r) = GM/r^2$. Here G is the gravitational constant and R is the Earth's radius.

Step 1: Find the field inside the Earth Let the uniform density be ρ. The enclosed mass is $M_r = 4\pi\rho r^3/3 = Mr^3/R^3$. Substituting this in the gravitational field gives

$$g(r) = \frac{GM_r}{r^2} = \frac{GM}{R^3}r, \qquad 0 \le r \le R.$$

Thus the field is zero at the centre by symmetry and rises along a straight line to $g(R) = GM/R^2$ at the surface. The increase is linear, rather than quadratic, because enclosed mass grows as r^3 while the inverse-square factor contributes r^{-2}.

Step 2: Join the exterior field and test the options For $r \ge R$, the whole Earth is enclosed, so

$$g(r) = \frac{GM}{r^2} = g(R)\left(\frac{R}{r}\right)^2.$$

The curve is continuous at the surface, falls nonlinearly outside, and approaches zero at large distance. Graph (A) has precisely a straight rise followed by an inverse-square fall. Graph (B) incorrectly makes the interior field constant. Graph (C) extends the exterior trend into the interior, failing the central zero. Graph (D) incorrectly makes the interior rise curved rather than linear.

Step 3: Conclusion The field rises linearly from zero to its maximum at $r = R$, then decreases as $1/r^2$. Hence, the correct option is (A).

Quick Tip to Solve: Check the centre, the surface and infinity: $g(0) = 0$, the peak is at R, and the exterior curve tends to zero. The interior segment must be straight in the uniform-density model.

37. Time period of a simple pendulum in a stationary lift is 'T'. If the lift accelerates with $\frac{g}{6}$ vertically upwards then the time period will be :

(Where g = acceleration due to gravity) [4 marks]

(A) $\sqrt{\frac{6}{5}}T$

(B) $\sqrt{\frac{5}{6}}T$

(C) $\sqrt{\frac{6}{7}}T$

(D) $\sqrt{\frac{7}{6}}T$

Correct Answer: (C) $\sqrt{\frac{6}{7}}T$

Solution:

Concept:

- A simple pendulum of length l, oscillating through a small angle, has period $T = 2\pi\sqrt{l/g}$ in a stationary lift. Its period depends on the effective downward acceleration.
- In the frame of a lift accelerating upward with acceleration a, the bob experiences a downward pseudo-force ma. Its effective weight is $m(g + a)$, so replace g by $g + a$.

Step 1: Determine the effective gravity The lift accelerates upward with $a = g/6$. Since the pseudo-force in its frame points downward, it reinforces the actual gravitational force rather than opposing it. Therefore

$$g_{\text{eff}} = g + \frac{g}{6} = \frac{7g}{6}.$$

The mass of the bob cancels from its angular equation of motion, and the pendulum length stays unchanged. For a small displacement, the restoring angular acceleration is larger than it was in the stationary lift, so the oscillation should become faster and its period shorter.

Step 2: Compare the two periods Using the same length before and after acceleration,

$$T' = 2\pi\sqrt{\frac{l}{7g/6}}, \qquad \frac{T'}{T} = \sqrt{\frac{g}{7g/6}} = \sqrt{\frac{6}{7}}.$$

Hence $T' = \sqrt{6/7}\,T$. Option (D) reverses this ratio and would predict a longer period even though effective gravity has increased. Option (A) uses the reduction in gravity appropriate to downward acceleration. Although (B) also gives a shorter period, its factor does not follow from the stated upward acceleration $g/6$.

Step 3: Conclusion The new time period is $\boxed{T' = \sqrt{6/7}\,T}$. Hence, the correct option is (C).

Quick Tip to Solve: First decide whether the lift adds to or subtracts from effective gravity. Upward acceleration increases g_{eff}, and the period decreases in the inverse square-root ratio.

38. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats 1.4. Vessel is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surrounding and vessel temperature of the gas increases by :

(R = universal gas constant) [4 marks]

(A) $\frac{Mv^2}{7R}$

(B) $\frac{Mv^2}{5R}$

(C) $2\frac{Mv^2}{7R}$

(D) $7\frac{Mv^2}{5R}$

Correct Answer: (B) $\frac{Mv^2}{5R}$

Solution:

Concept:

- The bulk translational kinetic energy of a gas moving with its vessel is distinct from its random molecular internal energy. When the vessel is suddenly stopped and the gas settles, that bulk motion is dissipated into internal energy under the stated idealisation.
- For n moles of an ideal gas, $\Delta U = nC_V\Delta T$, with $C_V = R/(\gamma - 1)$. The gas mass is nM, where M is its molar mass in units consistent with R.

Step 1: Express the available kinetic energy Consider n moles of the gas. Initially the entire gas has bulk speed v, so its translational kinetic energy relative to the ground is

$$K_{\text{bulk}} = \frac{1}{2}nMv^2.$$

After stopping and equilibration, the mean bulk velocity is zero. With thermal insulation and no energy absorbed by the vessel in the problem's idealisation, this energy becomes thermal energy of the gas. This is not a reversible adiabatic compression, so a pressure-volume adiabatic relation is unnecessary; the relevant calculation is an energy balance.

Step 2: Use the heat capacity to obtain the rise The stated ratio of specific heats is $\gamma = 1.4 = 7/5$. Hence

$$C_V = \frac{R}{1.4 - 1} = \frac{5R}{2}.$$

Equating the loss of bulk kinetic energy to the gain of internal energy,

$$n\frac{5R}{2}\Delta T = \frac{1}{2}nMv^2 \quad \Longrightarrow \quad \Delta T = \frac{Mv^2}{5R}.$$

Both the amount of gas and the factor $1/2$ cancel. The denominator contains 5 because the internal-energy heat capacity is C_V, not $C_P = 7R/2$. The alternatives involving 7 use an inappropriate heat-capacity factor, and the remaining numerical factors violate the energy balance.

Step 3: Conclusion The gas temperature increases by $\boxed{\Delta T = Mv^2/(5R)}$. Hence, the correct option is (B).

Quick Tip to Solve: For a suddenly stopped insulated ideal gas, equate its lost bulk kinetic energy to $nC_V\Delta T$. Use C_V, since the gain being calculated is internal energy.

39. Two capacitors having capacitance C_1 and C_2 respectively are connected as shown in figure. Initially, capacitor C_1 is charged to a potential difference V volt by a battery. The battery is then removed and the charged capacitor C_1 is now connected to uncharged capacitor C_2 by closing the switch S. The amount of charge on the capacitor C_2, after equilibrium, is : [4 marks]



(A) $\frac{C_1C_2}{(C_1+C_2)}V$

(B) $\frac{(C_1+C_2)}{C_1C_2}V$

(C) $(C_1+C_2)V$

(D) $(C_1-C_2)V$

Correct Answer: (A) $\frac{C_1C_2}{(C_1+C_2)}V$

Solution:

Concept:

- The charge on a capacitor is $Q = CV$. Initially only C_1 is charged, so the total charge available on the connected positive-plate node is $Q_0 = C_1V$.
- After the battery is removed, charge is conserved. Closing the switch joins the capacitors in parallel, so they acquire the same final potential difference V_f; their individual charges need not be equal.

Step 1: Apply charge conservation at equilibrium Initially the uncharged capacitor C_2 contributes no charge, and the battery has been disconnected. After closing the switch, the top plates form one isolated conducting node and the bottom plates form the other. Charge redistributes until both capacitors have the same voltage:

$$Q_{1f} = C_1V_f, \qquad Q_{2f} = C_2V_f.$$

Conservation of charge on either node then gives

$$Q_{1f} + Q_{2f} = C_1V \quad \Longrightarrow \quad (C_1 + C_2)V_f = C_1V.$$

Thus the common voltage is smaller than the initial voltage for any positive C_2.

Step 2: Find the charge acquired by the second capacitor Substituting the common voltage into $Q_{2f} = C_2V_f$,

$$V_f = \frac{C_1}{C_1 + C_2}V, \qquad Q_{2f} = \frac{C_1C_2}{C_1 + C_2}V.$$

The result tends to zero as $C_2 \to 0$, and it is always less than the initially available charge C_1V.

Option (B) has units of voltage divided by capacitance, not charge. Option (C) exceeds the available charge. Option (D) would give zero for equal capacitors, whereas equal capacitors must share the initial charge equally. Energy need not be conserved in the capacitors alone during the transient; charge conservation is the essential condition.

Step 3: Conclusion The equilibrium charge on C_2 is $\boxed{Q_{2f} = C_1C_2V/(C_1 + C_2)}$. Hence, the correct option is (A).

Quick Tip to Solve: For isolated capacitor sharing, conserve charge and impose a common final voltage. Do not keep the initial battery voltage fixed after the battery has been removed.

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40. Given below two statements : One is labelled as Assertion (A) and other is labelled as Reason (R).

Assertion (A) : Non-polar materials do not have any permanent dipole moment.

Reason (R) : When a non-polar material is placed in an electric field, the centre of the positive charge distribution of it's individual atom or molecule coincides with the centre of the negative charge distribution.

In the light of above statements, choose the most appropriate answer from the options given below.

[4 marks]

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(B) Both (A) and (R) are correct and (R) is not the correct explanation of (A).

(C) (A) is correct but (R) is not correct.

(D) (A) is not correct but (R) is correct.

Correct Answer: (C) (A) is correct but (R) is not correct.

Solution:

Concept:

- A non-polar atom or molecule has zero permanent electric dipole moment: in its undisturbed state, the centres of positive and negative charge coincide. A polar molecule has a permanent dipole even in the absence of an applied field.
- An applied electric field polarises a non-polar dielectric by displacing the positive and negative charge distributions in opposite directions. The resulting induced dipole is commonly written $\mathbf{p} = \alpha\mathbf{E}$, with polarisability α.

Step 1: Evaluate the assertion Assertion (A) states that non-polar materials have no permanent dipole moment. This is correct: absence of a permanent molecular dipole is the defining feature of a non-polar material. The word permanent is essential. It does not mean that the material can never acquire a dipole moment; an externally induced dipole can appear when a field is applied.

Step 2: Evaluate the reason Reason (R) claims that the centres of the positive and negative charge distributions coincide when the non-polar material is placed in an electric field. This is incorrect. An external field exerts opposite forces on positive and negative charges, causing a small relative displacement of their charge centres. If q is the effective separated charge and $\mathbf{d}$ is the displacement from the negative centre to the positive centre, the induced dipole is $\mathbf{p} = q\mathbf{d}$. Coincidence of charge

centres describes the unpolarised state, rather than the field-induced state asserted in the reason.

Step 3: Select the assertion-reason combination The assertion is true and the reason is false. Consequently neither option (A) nor option (B), both of which require a true reason, can hold. Option (D) incorrectly rejects the true assertion and accepts the false reason. Option (C) alone has the correct truth values; there is no explanatory relation to test once the reason itself is false.

Step 4: Conclusion The assertion is correct, but the reason is not correct. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish permanent from induced dipoles. Non-polar molecules have coincident charge centres before applying a field; the field separates the centres and induces polarisation.

41. The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi = (5t^3 + 4t^2 + 2t - 5)$ Weber. If the resistance of the coil is 5 ohm, then the induced current through the coil at t = 2 s will be, [4 marks]

(A) 15.6 A

(B) 16.6 A

(C) 17.6 A

(D) 18.6 A

Correct Answer: (A) 15.6 A

Solution:

Concept:

- Faraday's law gives induced emf $\mathcal{E} = -d\phi/dt$ when ϕ is the linked magnetic flux specified for the coil. The negative sign expresses Lenz's law; the current magnitude is $I = |\mathcal{E}|/\mathcal{R}$.
- The induced current at a particular instant depends on the instantaneous rate of change of flux, not on the flux divided by time. Here the coil resistance is $\mathcal{R} = 5\,\Omega$.

Step 1: Differentiate the given flux The linked flux is

$$\phi(t) = 5t^3 + 4t^2 + 2t - 5 \quad \text{Wb}.$$

Differentiate term by term before inserting the required time:

$$\frac{d\phi}{dt} = 15t^2 + 8t + 2.$$

The constant term contributes zero because a constant flux produces no induced emf. The quadratic term in the flux contributes $8t$, while the cubic term contributes $15t^2$. There is no additional factor for turns because the stated linked flux is the quantity used in the question.

Step 2: Evaluate the emf and use Ohm's law At $t = 2\,\text{s}$,

$$\left.\frac{d\phi}{dt}\right|_{t=2} = 15(2)^2 + 8(2) + 2 = 60 + 16 + 2 = 78\,\text{Wb}\,\text{s}^{-1}.$$

Thus the emf magnitude is $78\,\mathrm{V}$, and

$$I = \frac{78}{5} = 15.6\,\mathrm{A}.$$

The derivative is positive at this instant, so the induced field opposes the increase of the linked flux. No reference direction for current has been prescribed, so the requested positive option is its magnitude. The values 16.6 A, 17.6 A and 18.6 A would require emfs of 83 V, 88 V and 93 V, respectively, none of which equals the correct derivative.

Step 3: Conclusion The induced current has magnitude $\boxed{15.6\,\mathrm{A}}$. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate the complete flux polynomial first, substitute the time second, and divide the emf magnitude by the coil resistance. A constant flux term never contributes.

42. An aluminium wire is stretched to make its length, 0.4% larger. The percentage change in resistance is : [4 marks]

(A) 0.4%

(B) 0.2%

(C) 0.8%

(D) 0.6%

Correct Answer: (C) 0.8%

Solution:

Concept:

- For a uniform wire, $\mathcal{R} = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area. In ordinary small stretching problems, the material volume and temperature are taken as unchanged.
- With volume $\mathcal{V} = AL$ fixed and resistivity unchanged, $\mathcal{R} = \rho L^2/\mathcal{V}$. Thus a small fractional increase of length produces approximately twice that fractional increase of resistance.

Step 1: Account for the decrease of cross-section The new length is $L' = 1.004L$. Conservation of volume requires

$$A'L' = AL \quad \Longrightarrow \quad A' = \frac{A}{1.004}.$$

Therefore the length increase and area decrease both increase the resistance. Treating the area as fixed would miss one of these effects and incorrectly give only a 0.4 percent rise. The stated small deformation is treated geometrically; changes of resistivity from temperature or microscopic strain are excluded in this standard model.

Step 2: Calculate the percentage increase Using the same resistivity before and after stretching,

$$\frac{\mathcal{R}'}{\mathcal{R}} = \frac{L'}{L}\frac{A}{A'} = (1.004)^2 = 1.008016.$$

Hence the exact geometrical increase is

$$\frac{\mathcal{R}' - \mathcal{R}}{\mathcal{R}} \times 100\% = 0.8016\% \simeq 0.8\%.$$

Equivalently, to first order, $\Delta\mathcal{R}/\mathcal{R} = 2\Delta L/L$. Option (A) neglects the area change; option (B) incorrectly halves the length percentage. Option (D) is inconsistent with the square-law dependence. The approximation is appropriate because the omitted squared fractional strain is only 0.000016.

Step 3: Conclusion The resistance increases by approximately $\boxed{0.8\%}$. Hence, the correct option is (C).

Quick Tip to Solve: For stretching at constant volume, remember $\mathcal{R} \propto L^2$. Double the small percentage increase in length; using only $\mathcal{R} \propto L$ neglects the shrinking area.

43. A proton and an alpha particle of the same velocity enter in a uniform magnetic field which is acting perpendicular to their direction of motion. The ratio of the radii of the circular paths described by the alpha particle and proton is : [4 marks]

(A) 1 : 4

(B) 4 : 1

(C) 2 : 1

(D) 1 : 2

Correct Answer: (C) 2 : 1

Solution:

Concept:

- A magnetic force perpendicular to velocity provides the centripetal force: $|q|vB = mv^2/r$, so $r = mv/(|q|B)$. Here m is particle mass, q its charge and B the magnetic-field magnitude.
- A proton has mass approximately m_p and charge $+e$. An alpha particle is a helium nucleus, with mass approximately $4m_p$ and charge $+2e$.

Step 1: Compare the quantities held fixed Both particles enter the same uniform field with the same speed, and both velocities are perpendicular to the field. Thus the full magnetic force acts in each case. The field does no work, so each speed remains constant during circular motion. The radius is proportional to mass divided by charge under these conditions; equal speed does not imply equal momentum or equal kinetic energy.

Step 2: Substitute the particle masses and charges For the proton and alpha particle respectively,

$$r_p = \frac{m_p v}{eB}, \qquad r_\alpha = \frac{4m_p v}{2eB} = \frac{2m_p v}{eB}.$$

Dividing cancels the common speed and field:

$$\frac{r_\alpha}{r_p} = \frac{4}{2} = 2.$$

The factor of four from mass is reduced by the factor of two from charge. Taking only the mass ratio would incorrectly give option (B); reversing the requested order would incorrectly give option (D). Option (A) also contradicts the mass-to-charge comparison.

Step 3: Conclusion The requested order is alpha particle to proton: $r_\alpha : r_p = 2 : 1$. Hence, the correct

option is (C).

Quick Tip to Solve: For equal speeds in the same transverse magnetic field, compare $m/|q|$; first write the order of the requested ratio to avoid its reciprocal.

44. If Electric field intensity of a uniform plane electromagnetic wave is given as $E = -301.6\sin(kz - \omega t)\hat{a}_x + 452.4\sin(kz - \omega t)\hat{a}_y \frac{V}{m}$. Then magnetic intensity 'H' of this wave in Am^{-1} will be :
[Given : Speed of light in vacuum $c = 3 \times 10^8$ ms^{-1}, Permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}]

[4 marks]

(A) $+0.8\sin(kz - \omega t)\hat{a}_y + 0.8\sin(kz - \omega t)\hat{a}_x$
(B) $+1.0 \times 10^{-6}\sin(kz - \omega t)\hat{a}_y + 1.5 \times 10^{-6}(kz - \omega t)\hat{a}_x$
(C) $-0.8\sin(kz - \omega t)\hat{a}_y - 1.2\sin(kz - \omega t)\hat{a}_x$
(D) $-1.0 \times 10^{-6}\sin(kz - \omega t)\hat{a}_y - 1.5 \times 10^{-6}\sin(kz - \omega t)\hat{a}_x$
Correct Answer: (C) $-0.8\sin(kz - \omega t)\hat{a}_y - 1.2\sin(kz - \omega t)\hat{a}_x$

Solution:

Concept:

- In vacuum the wave impedance is $Z_0 = \mu_0 c$, and the magnetic intensity satisfies $\mathbf{H} = \hat{\mathbf{k}} \times \mathbf{E}/Z_0$. Magnetic intensity H has units $\mathrm{A\,m^{-1}}$.
- The phase $kz - \omega t$ represents propagation along $+z$. The ordered triad $\mathbf{E}, \mathbf{H}, \hat{\mathbf{k}}$ is right handed, with $\mathbf{E} \times \mathbf{H}$ along propagation.

Step 1: Find the impedance and propagation direction At fixed phase, $z = (\omega/k)t +$ constant; the propagation direction is therefore $+\hat{a}_z$. Using the supplied constants gives

$$Z_0 = (4\pi \times 10^{-7})(3 \times 10^8) = 120\pi\ \Omega \simeq 377\ \Omega.$$

Write $s = \sin(kz - \omega t)$, so $\mathbf{E} = (-301.6\hat{a}_x + 452.4\hat{a}_y)s$. The whole field shares this same phase factor.

Step 2: Take the cross product and divide by impedance Use $\hat{a}_z \times \hat{a}_x = \hat{a}_y$ and $\hat{a}_z \times \hat{a}_y = -\hat{a}_x$. Hence

$$\mathbf{H} = \frac{-301.6\hat{a}_y - 452.4\hat{a}_x}{377}s.$$

To the precision used in the options, the coefficients are -0.8 and -1.2. Both components are negative. Option (A) has wrong signs and unequal-to-required amplitudes, while (B) and (D) have the scale of magnetic flux density rather than magnetic intensity. Dividing by c alone would find B, not H.

Step 3: Conclusion

$$\mathbf{H} = -0.8\sin(kz - \omega t)\hat{a}_y - 1.2\sin(kz - \omega t)\hat{a}_x\ \mathrm{A\,m^{-1}}.$$

Hence, the correct option is (C).

Quick Tip to Solve: Distinguish $B = E/c$ from $H = E/(\mu_0 c)$, and use the cross product with the propagation direction to fix both component signs.

45. In free space, an electromagnetic wave of 3 GHz frequency strikes over the edge of an object of size $\frac{\lambda}{100}$, where λ is the wavelength of the wave in free space. The phenomenon, which happens there will be : [4 marks]

(A) Reflection

(B) Refraction

(C) Diffraction

(D) Scattering

Correct Answer: (D) Scattering

Solution:

Concept:

- The vacuum wavelength is $\lambda = c/f$, where c is the speed of light and f the frequency. The ratio of object size d to wavelength decides the relevant interaction regime.
- An object much smaller than the incident wavelength produces small-particle, or Rayleigh-type, scattering. Pronounced edge or aperture diffraction is normally compared with an obstacle or opening of size comparable to the wavelength.

Step 1: Compare the wavelength with the object size The frequency is $3\ \text{GHz} = 3 \times 10^9\ \text{Hz}$. With $c = 3 \times 10^8\ \text{m s}^{-1}$,

$$\lambda = \frac{3 \times 10^8}{3 \times 10^9} = 0.10\ \text{m}.$$

The stated size is

$$d = \frac{\lambda}{100} = 1.0 \times 10^{-3}\ \text{m}, \qquad \frac{d}{\lambda} = 0.01.$$

Thus the object is only one hundredth of a wavelength across, rather than a macroscopic reflecting surface or a wavelength-sized aperture.

Step 2: Identify the small-object interaction The electromagnetic field drives charges in the small object, which reradiate the wave into surrounding directions. This is scattering. The unusually small size in the question is the decisive information. Reflection would describe a returned wave at an extended interface, and refraction describes propagation into another medium with a changed phase speed. Although diffraction broadly concerns wave spreading and edge effects, the specified subwavelength object is classified here in the scattering regime; choosing diffraction solely from the word edge misses the given scale.

Step 3: Conclusion The relevant phenomenon for an object of size $\lambda/100$ is scattering. Hence, the correct option is (D).

Quick Tip to Solve: Calculate d/λ before naming the phenomenon: a very small isolated object indicates scattering, whereas a wavelength-sized slit highlights diffraction.

46. An electron with speed v and a photon with speed c have the same de-Broglie wavelength. If the kinetic energy and momentum of electron are E_e and p_e and that of photon are E_{ph} and p_{ph} respectively. Which of the following is correct? [4 marks]

(A) $\frac{E_e}{E_{ph}} = \frac{2c}{v}$

(B) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$

(C) $\frac{p_e}{p_{ph}} = \frac{2c}{v}$

(D) $\frac{p_e}{p_{ph}} = \frac{v}{2c}$

Correct Answer: (B) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$. Equal wavelengths therefore mean equal momentum magnitudes, even for particles with different mass or speed.
- For the nonrelativistic electron in the standard de Broglie treatment, $E_e = p_e^2/(2m_e) = p_e v/2$. For a photon, $E_{\text{ph}} = p_{\text{ph}}c$; photon energy is not $mv^2/2$.

Step 1: Use the common wavelength to compare momenta Let the common wavelength be λ. Applying the momentum-wavelength relation separately gives

$$p_e = \frac{h}{\lambda}, \qquad p_{\text{ph}} = \frac{h}{\lambda}.$$

Consequently $p_e/p_{\text{ph}} = 1$. The speeds do not enter this equality, so neither option (C) nor option (D) gives the required momentum ratio. Equal wavelength does not by itself imply equal energy.

Step 2: Form the ratio of electron and photon energies Write the common momentum as p. Using the nonrelativistic electron kinetic energy and the photon energy gives

$$E_e = \frac{pv}{2}, \qquad E_{\text{ph}} = pc.$$

Division now yields

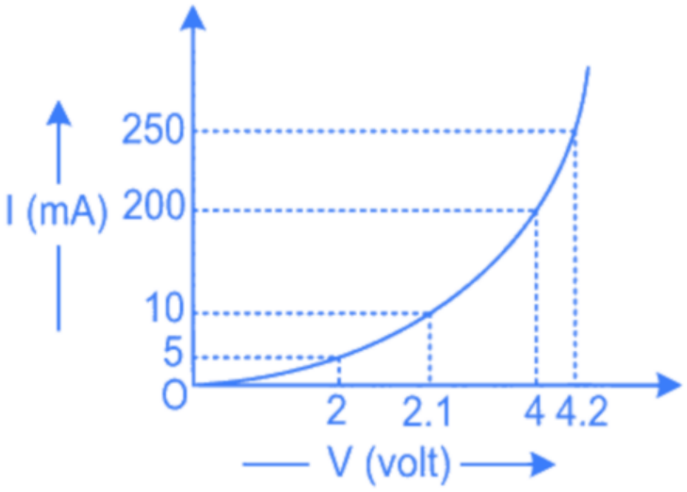
$$\frac{E_e}{E_{\text{ph}}} = \frac{pv/2}{pc} = \frac{v}{2c}.$$

The common momentum cancels. This is less than unity, as expected for a slow electron compared with a photon of the same momentum. Option (A) is the reciprocal of the required ratio. The factor of two comes from the electron kinetic-energy expression, not from the wavelength relation.

Step 3: Conclusion In the nonrelativistic de Broglie treatment, $E_e/E_{\text{ph}} = v/(2c)$, while $p_e/p_{\text{ph}} = 1$. Hence, the correct option is (B).

Quick Tip to Solve: For an electron and photon of equal wavelength, cancel their equal momenta first, then use the separate energy laws; do not assign the photon a massive-particle kinetic energy.

47. The I-V characteristics of a p-n junction diode in forward bias is shown in the figure. The ratio of dynamic resistance, corresponding to forward bias voltage of 2 V and 4 V respectively, is : [4 marks]



(A) 1 : 2

(B) 5 : 1

(C) 1 : 40

(D) 20 : 1

Correct Answer: (B) 5 : 1

Solution:

Concept:

- Dynamic resistance near an operating point is the local inverse slope of the current-voltage curve: $r_d = \Delta V/\Delta I$ for the nearby points supplied on the graph.
- Static resistance V/I is a different quantity. In a forward-biased diode the current rises steeply as voltage increases, so dynamic resistance decreases at the higher operating voltage.

Step 1: Read the nearby points at the lower voltage Near 2 V, the graph labels the current as 5 mA, increasing to 10 mA at 2.1 V. Therefore

$$r_d(2) = \frac{2.1 - 2.0}{(10 - 5) \times 10^{-3}} = \frac{0.1}{0.005} = 20\ \Omega.$$

The current difference must be converted from milliamperes to amperes. These nearby differences estimate the local slope; using the coordinates relative to the origin instead would calculate static resistance.

Step 2: Read the higher-voltage increment and form the ratio Near 4 V, the current changes from 200 mA to 250 mA as the voltage changes from 4.0 to 4.2 V. Thus

$$r_d(4) = \frac{4.2 - 4.0}{(250 - 200) \times 10^{-3}} = \frac{0.2}{0.050} = 4\ \Omega.$$

The requested lower-voltage to higher-voltage ratio is

$$r_d(2) : r_d(4) = 20 : 4 = 5 : 1.$$

The resistance does decrease at the steeper part of the curve. None of the ratios in (A), (C), or (D) follows from these two local increments.

Step 3: Conclusion The dynamic resistances are $20\ \Omega$ and $4\ \Omega$, giving the ratio $5 : 1$. Hence, the correct option is (B).

Quick Tip to Solve: For dynamic resistance, take the voltage difference divided by the current difference between neighbouring marked points; never use V/I from the origin.

48. The efficiency of a Carnot's engine, working between steam point and ice point, will be : [4 marks]

(A) 26.81%

(B) 37.81%

(C) 47.81%

(D) 57.81%

Correct Answer: (A) 26.81%

Solution:

Concept:

- The efficiency of a reversible Carnot engine is $\eta = 1 - T_c/T_h$, where T_h and T_c are the absolute temperatures of the hot and cold reservoirs.
- The steam and ice points correspond approximately to $100°\text{C}$ and $0°\text{C}$, or $373\ \text{K}$ and $273\ \text{K}$. Temperature ratios in thermodynamics require kelvin.

Step 1: Identify the hot and cold reservoir temperatures The engine receives heat at the steam point and rejects heat at the ice point. Converting each Celsius temperature gives

$$T_h = 100 + 273 = 373\ \text{K}, \qquad T_c = 0 + 273 = 273\ \text{K}.$$

The difference is $100\ \text{K}$, but the absolute temperatures are not 100 and zero. Taking a ratio of Celsius readings would incorrectly suggest complete conversion of heat into work.

Step 2: Substitute in the Carnot efficiency formula The reversible heat-transfer relation gives $Q_c/Q_h = T_c/T_h$, and $\eta = (Q_h - Q_c)/Q_h$. Thus

$$\eta = 1 - \frac{273}{373} = \frac{100}{373} \simeq 0.2681.$$

Express this fraction as a percentage by multiplying by 100:

$$\eta \times 100 \simeq 26.81\%.$$

The remaining energy is rejected to the cold reservoir, so efficiency is well below unity. Keeping the

conventional rounded kelvin values reproduces option (A); options (B), (C), and (D) are all larger than the value fixed by these two reservoir temperatures.

Step 3: Conclusion The Carnot efficiency is approximately 26.81%. Hence, the correct option is (A).

Quick Tip to Solve: Use $\eta = (T_h - T_c)/T_h$ with the hot temperature in kelvin; Celsius temperature differences are valid, but Celsius temperature ratios are not.

49. How many alpha and beta particles are emitted when Uranium $_{92}U^{238}$ decays to lead $_{82}Pb^{206}$?

[4 marks]

(A) 3 alpha particles and 5 beta particles

(B) 6 alpha particles and 4 beta particles

(C) 4 alpha particles and 5 beta particles

(D) 8 alpha particles and 6 beta particles

Correct Answer: (D) 8 alpha particles and 6 beta particles

Solution:

Concept:

- An alpha emission reduces the nuclear mass number A by four and the atomic number Z by two. A beta-minus emission leaves A unchanged and increases Z by one.
- Use conservation of mass number first to count alpha particles; then use the atomic-number balance to count beta-minus particles. The emitted beta particle is an electron.

Step 1: Determine the alpha emissions from mass numbers Let the numbers of alpha and beta-minus emissions be a and b. The starting mass number is 238, while the final mass number is 206. Beta emissions do not change this number, so

$$238 - 4a = 206, \qquad 4a = 32, \qquad a = 8.$$

This immediately rules out options (A), (B), and (C): their alpha counts would not reduce uranium to a nucleus with mass number 206.

Step 2: Balance atomic numbers after the alpha decays The eight alpha emissions reduce the nuclear atomic number from 92 to

$$92 - 2(8) = 76.$$

The final lead nucleus has atomic number 82, so six increases of one are needed. Hence

$$92 - 2a + b = 82, \qquad 76 + b = 82, \qquad b = 6.$$

These must be beta-minus, rather than beta-plus, decays because the nuclear atomic number must rise. As a check using emitted charges, the final nuclear charge plus the emitted particle charges is $82 + 8(2) - 6 = 92$, which agrees with the initial charge. Both the mass and charge balances therefore give the same result.

Step 3: Conclusion The decay chain emits eight alpha particles and six beta-minus particles. Hence,

the correct option is (D).

Quick Tip to Solve: Count alpha emissions from the change in mass number first; then correct the resulting atomic number with beta-minus emissions, each of which raises it by one.

50. Choose the correct statement for amplitude modulation : [4 marks]

(A) Amplitude of modulating signal is varied in accordance with the information signal.

(B) Amplitude of modulated signal is varied in accordance with the information signal.

(C) Amplitude of carrier signal is varied in accordance with the information signal.

(D) Amplitude of modulated signal is varied in accordance with the modulating signal.

Correct Answer: (C) Amplitude of carrier signal is varied in accordance with the information signal.

Solution:

Concept:

- Amplitude modulation encodes a low-frequency information signal by changing the amplitude of a high-frequency carrier wave.
- If the information signal is $m(t)$, the modulated wave can be written $s(t) = [A_c + k_a m(t)] \cos(\omega_c t)$. Here A_c is the unmodulated carrier amplitude, k_a is the amplitude sensitivity and ω_c is the carrier angular frequency.

Step 1: Identify the signal that is controlled The modulating signal is the information supplied to the transmitter. It controls a parameter of the carrier; it is not itself changed to encode that same information. In amplitude modulation, the controlled parameter is the carrier amplitude. The rapid oscillations remain at the carrier frequency while their envelope follows the information signal.

Step 2: Compare the four statements Option (C) states the defining operation precisely: the amplitude of the carrier varies in accordance with the information signal. Option (A) incorrectly makes the modulating signal the quantity being varied. Options (B) and (D) refer to the already modulated output rather than identifying the carrier parameter on which modulation is performed; they do not give the standard definition. Frequency and phase are not the parameters intentionally varied in this process.

Step 3: Conclusion The amplitude of the carrier signal is varied in accordance with the information signal. Hence, the correct option is (C).

Quick Tip to Solve: For modulation definitions, identify the carrier parameter that changes: amplitude for AM, frequency for FM and phase for PM.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A fighter jet is flying horizontally at a certain altitude with a speed of 200 ms^{-1}. When it passes directly overhead an anti-aircraft gun, a bullet is fired from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is 400 m/s, the value of θ will be _____°. [4 marks]

Solution:

Concept:

- Horizontal motion is uniform when air resistance is neglected. For launch speed u and elevation θ, the horizontal component is $u\cos\theta$.
- Two objects starting with the same horizontal coordinate can meet at a later time only if their horizontal displacements are equal at that time.

Step 1: Write the horizontal positions Choose the gun as the origin and let the positive horizontal direction be the direction of flight. At firing, the jet is directly above the gun, so both horizontal positions are zero. If they meet after time $t > 0$, the jet has travelled $x_j = 200t$, while the bullet has travelled $x_b = 400\cos\theta\, t$. Gravity changes the vertical component of the bullet velocity, but not its horizontal component.

Step 2: Apply the interception condition Equate the two horizontal positions at impact and cancel the nonzero time:

$$400\cos\theta\, t = 200t, \qquad \cos\theta = \frac{1}{2}.$$

The bullet must be fired upward and forward, so the relevant angle above the horizontal is $\theta = 60°$. The unknown altitude affects the possible impact time through vertical motion; it does not change this necessary horizontal-velocity matching condition. A vertical shot would leave the bullet behind the moving jet, while a horizontal shot would have too large a forward velocity.

Step 3: Conclusion The required launch angle is $\boxed{\theta = 60°}$. The numerical entry is $\boxed{60}$.

Quick Tip to Solve: When a projectile is fired as a target passes directly overhead, first equate their horizontal velocities; the unknown interception time cancels.

52. A ball of mass 0.5 kg is dropped from the height of 10 m. The height, at which the magnitude of velocity becomes equal to the magnitude of acceleration due to gravity, is _____ m. [Use g = 10 m/s^2] [4 marks]

Solution:

Concept:

- For motion with constant acceleration, $v^2 = u^2 + 2gs$, where s is the downward distance fallen.
- The speed and acceleration have different dimensions. The wording asks for equality of their numerical magnitudes in the stated SI units, so use a speed of $10\,\mathrm{m\,s^{-1}}$.

Step 1: Determine the distance fallen The ball is dropped, so its initial speed is $u = 0$. Use

$g = 10\,\mathrm{m\,s^{-2}}$ and the required speed $v = 10\,\mathrm{m\,s^{-1}}$. Then

$$10^2 = 0 + 2(10)s, \qquad s = \frac{100}{20} = 5\,\mathrm{m}.$$

This is the distance travelled downward from the release point, not the height above the ground. The mass $0.5\,\mathrm{kg}$ is irrelevant because free-fall acceleration is independent of mass.

Step 2: Convert the fall distance to height The initial height above the ground is $H = 10\,\mathrm{m}$. After falling $s = 5\,\mathrm{m}$, the remaining height is

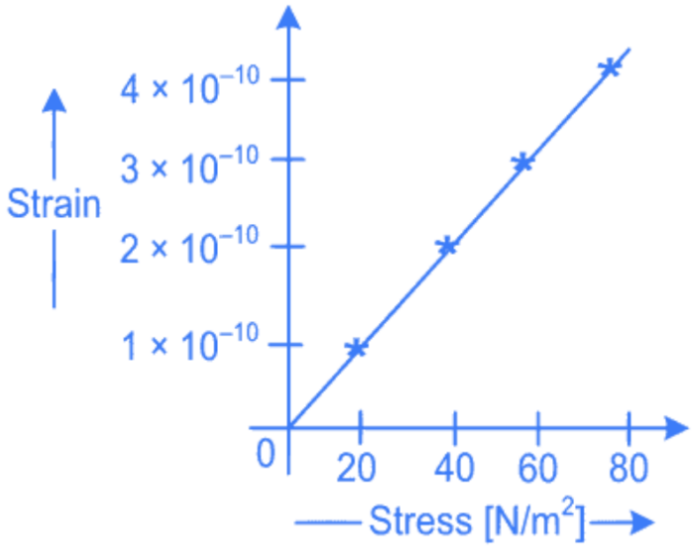
$$h = H - s = 10 - 5 = 5\,\mathrm{m}.$$

As a check, the time to reach this speed is $t = v/g = 1\,\mathrm{s}$. The distance $\frac{1}{2}gt^2 = 5\,\mathrm{m}$ agrees with the result. The event therefore occurs before the ball reaches the ground. Literal equality of the physical quantities velocity and acceleration would be dimensionally meaningless; the numerical interpretation is the one implied by the question.

Step 3: Conclusion The ball is at a height of $\boxed{5\,\mathrm{m}}$ above the ground. The numerical entry is $\boxed{5}$.

Quick Tip to Solve: Use the kinematic equation to find distance fallen, then subtract it from the starting height to obtain height above the ground.

53. The elastic behaviour of material for linear stress and linear strain, is shown in the figure. The energy density for a linear strain of 5×10^{-4} is _____ kJ/m^3. Assume that material is elastic upto the linear strain of 5×10^{-4}. [4 marks]



Solution:

Concept:

- For a linearly elastic material, Young's modulus is $Y = \sigma/\varepsilon$, where σ is stress and ε is longitudinal strain.
- The elastic energy stored per unit volume is $u = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}Y\varepsilon^2$. It is the area under a stress-versus-strain curve.

Step 1: Read the modulus from the graph The graph places strain on the vertical axis and stress on the horizontal axis, so its slope is $1/Y$, not Y. A clearly marked point gives stress $20\,\mathrm{N\,m^{-2}}$ at strain 1×10^{-10}. Hence

$$Y = \frac{20}{1 \times 10^{-10}} = 2 \times 10^{11}\,\mathrm{N\,m^{-2}}.$$

The other marked points, such as stress 40 with strain 2×10^{-10}, give the same ratio. The straight line through the origin therefore represents Hooke's law.

Step 2: Calculate the energy at the specified strain The stated elastic range permits using the same modulus at $\varepsilon = 5 \times 10^{-4}$, even though that strain lies outside the small portion of the graph shown. Its corresponding stress is $Y\varepsilon = 10^8\,\mathrm{Pa}$. Thus

$$u = \frac{1}{2}(2 \times 10^{11})(5 \times 10^{-4})^2 = 2.5 \times 10^4\,\mathrm{J\,m^{-3}}.$$

Since $1\,\mathrm{kJ} = 10^3\,\mathrm{J}$, divide by 1000 to express the result in the units requested. The factor one half is necessary because the force increases gradually from zero during elastic extension.

Step 3: Conclusion The elastic energy density is $\boxed{25\,\mathrm{kJ\,m^{-3}}}$. The numerical entry is $\boxed{25}$.

Quick Tip to Solve: Check the graph axes before taking a slope: a strain-versus-stress graph has slope $1/Y$; energy density still equals $Y\varepsilon^2/2$.

54. The elongation of a wire on the surface of the earth is 10^{-4} m. The same wire of same dimensions is elongated by 6×10^{-5} m on another planet. The acceleration due to gravity on the planet will be _____ $\mathrm{ms^{-2}}$. (Take acceleration due to gravity on the surface of earth = 10 $\mathrm{ms^{-2}}$) [4 marks]

Solution:

Concept:

- For a wire under a tensile load, $\Delta L = FL/(AY)$, where L, A and Y are its length, cross-sectional area and Young's modulus.
- For the same gravitational loading arrangement, $F = mg$, so extension is proportional to local gravitational acceleration when all other quantities remain unchanged.

Step 1: Form an extension ratio Compare the same wire and loading on Earth and on the other planet. Its material and dimensions remain unchanged, so L, A and Y cancel. The mass producing

the gravitational tension is also the same. Therefore

$$\frac{\Delta L_p}{\Delta L_E} = \frac{mg_pL/(AY)}{mg_EL/(AY)} = \frac{g_p}{g_E}.$$

Even for extension under the wire's own weight, the fixed geometry factor would cancel in this ratio. The important point is that gravitational tension, and hence elastic elongation, scales directly with g.

Step 2: Substitute the measured elongations The Earth extension is $\Delta L_E = 10^{-4}\,\mathrm{m}$, and the planet extension is $\Delta L_p = 6 \times 10^{-5}\,\mathrm{m}$. Consequently

$$g_p = 10\frac{6 \times 10^{-5}}{10^{-4}} = 10(0.6) = 6\,\mathrm{m\,s^{-2}}.$$

The shorter extension on the planet is consistent with a smaller gravitational acceleration. Using the inverse ratio would incorrectly predict stronger gravity for the smaller extension. No numerical values of the wire length, cross-sectional area, modulus or suspended mass are required because the comparison removes them.

Step 3: Conclusion The acceleration due to gravity on the planet is $\boxed{6\,\mathrm{m\,s^{-2}}}$. The numerical entry is $\boxed{6}$.

Quick Tip to Solve: For the same wire and mass, take ratios of extensions before inserting numbers; every elastic and geometrical parameter cancels.

55. A 10 Ω, 20 mH coil carrying constant current is connected to a battery of 20 V through a switch. Now after switch is opened current becomes zero in 100 μs. The average e.m.f. induced in the coil is _____ V. [4 marks]

Solution:

Concept:

- After a sufficiently long time on a DC supply, an inductor has no inductive voltage drop because $dI/dt = 0$; the steady current is limited by the coil resistance.
- The average self-induced emf has magnitude $|\mathcal{E}_{\text{av}}| = L|\Delta I|/\Delta t$. Its direction opposes the change in current, as required by Lenz's law.

Step 1: Find the current before opening the switch The coil carries constant current before the switch is opened. Its resistance is $R = 10\,\Omega$ and the battery voltage is $V = 20\,\mathrm{V}$. Hence the initial current is

$$I_i = \frac{V}{R} = \frac{20}{10} = 2\,\mathrm{A}.$$

The inductance does not prevent a steady current; it opposes a change of current. Thus it would be incorrect to omit the resistance when determining the initial current or to assume the current is zero merely because an inductor is present.

Step 2: Calculate the average emf during interruption The final current is zero, so $\Delta I = I_f - I_i = -2\,\mathrm{A}$.

Convert the inductance and time to SI units: $L = 20\,\text{mH} = 0.020\,\text{H}$ and $\Delta t = 100\,\mu\text{s} = 10^{-4}\,\text{s}$. Then

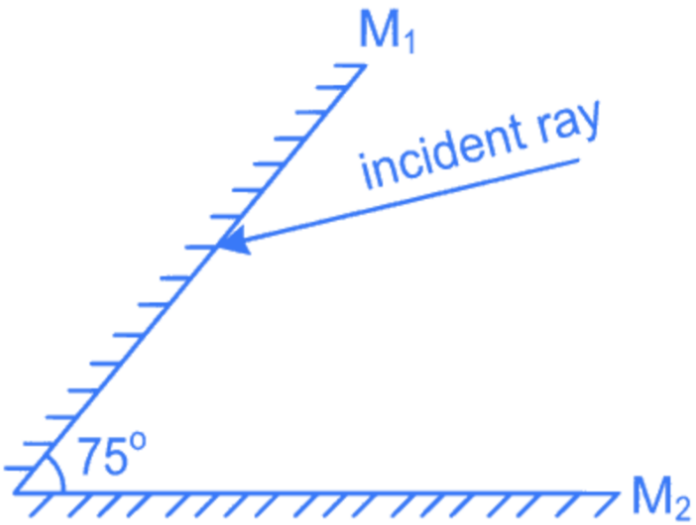
$$|\mathcal{E}_{\text{av}}| = 0.020\frac{2}{10^{-4}} = 400\,\text{V}.$$

This average depends only on the net current change and its duration; the current need not fall linearly at every instant. The induced emf acts to sustain the original current and can substantially exceed the battery voltage because the interruption occurs very rapidly.

Step 3: Conclusion The average induced emf has magnitude $\boxed{400\,\text{V}}$. The numerical entry is $\boxed{400}$.

Quick Tip to Solve: For a switched coil, use resistance to find the initial steady current, then use inductance and the current-change time to find the induced emf.

56. A light ray is incident, at an incident angle θ_1, on the system of tow plane mirrors M_1 and M_2 having an inclination angle 75° between them (as shown in figure). After reflecting from mirror M_1 it gets reflected back by the mirror M_2 with an angle of reflection 30°. The total deviation of the ray will be _____ degree. [4 marks]



Solution:

Concept:

- At a plane mirror the angle of reflection equals the angle of incidence, both measured from the normal. A single reflection turns a ray through $\delta = 180° - 2i$.
- The ray segment between two mirrors and the two mirror surfaces form a triangle. Its angles determine the incidence angles; directed turns must be added consistently.

Step 1: Determine the first incidence angle At the horizontal mirror M_2, the reflection angle is $r_2 = 30°$. Therefore $i_2 = 30°$, and the approaching ray makes $90° - 30° = 60°$ with that mirror. The triangle enclosed by the two mirror segments and the inter-mirror ray has angles $75°$ at the vertex

and 60° at M_2. Its third angle is

$$180^\circ - 75^\circ - 60^\circ = 45^\circ.$$

This is the angle of the reflected ray with M_1. Its angle with the normal, and hence the incidence angle at M_1, is also 45°.

Step 2: Add the deviations in the actual ray path The turn at the first mirror is $\delta_1 = 180^\circ - 2(45^\circ) = 90^\circ$. The turn at the second mirror is $\delta_2 = 180^\circ - 2(30^\circ) = 120^\circ$. In the pictured arrangement both changes of ray direction have the same sense, so the total deviation along the two-reflection path is

$$\delta = \delta_1 + \delta_2 = 90^\circ + 120^\circ = 210^\circ.$$

Equivalently it is $360^\circ - 2(75^\circ)$. The smaller angle between the initial and final directions would be 150°; that is not the accumulated deviation asked for here.

Step 3: Conclusion The total deviation of the ray is $\boxed{210^\circ}$. The numerical entry is $\boxed{210}$.

Quick Tip to Solve: For two reflections, calculate each $180^\circ - 2i$ turn and track its sense; do not automatically replace a total above 180° by its smaller angle.

57. In a vernier callipers, each cm on the main scale is divided into 20 equal parts. If tenth vernier scale division coincides with nineth main scale division. Then the value of vernier constant will be _____ $\times$ 10^{-2} mm. [4 marks]

Solution:

Concept:

- The main-scale division is the distance between two consecutive main-scale graduations. Here one centimetre contains 20 such equal intervals, so the value of one division must first be found in millimetres.
- For a direct vernier, the vernier constant or least count is the difference between one main-scale division (MSD) and one vernier-scale division (VSD): $\mathrm{VC} = 1\,\mathrm{MSD} - 1\,\mathrm{VSD}$.

Step 1: Find the main-scale division Convert the stated centimetre to millimetres before dividing by the number of intervals. Thus

$$1\,\mathrm{MSD} = \frac{1\,\mathrm{cm}}{20} = \frac{10\,\mathrm{mm}}{20} = 0.50\,\mathrm{mm}.$$

The number 20 refers to divisions within a centimetre; it does not mean that the main scale has a least division of one millimetre.

Step 2: Use the vernier coincidence The tenth vernier division coincides with the ninth main-scale division when measured from their common starting point. Therefore ten VSD occupy the same length as nine MSD:

$$10\,\mathrm{VSD} = 9\,\mathrm{MSD}, \qquad 1\,\mathrm{VSD} = \frac{9}{10}(0.50) = 0.45\,\mathrm{mm}.$$

Each vernier division is slightly shorter than a main-scale division, as required for a direct vernier.

Step 3: Calculate the vernier constant Take the difference of the two division lengths, rather than

their ratio:

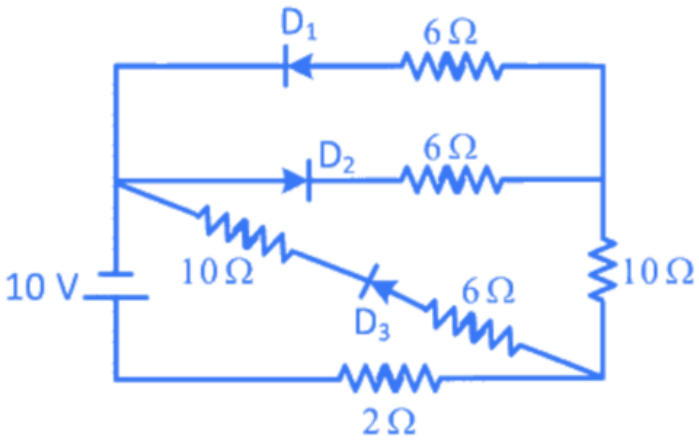
$$\text{VC} = 0.50 - 0.45 = 0.05\,\text{mm} = 5 \times 10^{-2}\,\text{mm}.$$

This is the smallest displacement represented by the change of one coinciding graduation.

Step 4: Conclusion The vernier constant is $5 \times 10^{-2}\,\text{mm}$. Hence the numerical value to enter in the blank is $\boxed{5}$.

Quick Tip to Solve: For n vernier divisions equal to $(n-1)$ main divisions, use least count $= 1\,\text{MSD}/n$, after converting the main-scale unit.

58. As per the given circuit, the value of current through the battery will be _____ A. [4 marks]



Solution:

Concept:

- An ideal diode conducts when forward biased and acts as a short circuit; a reverse-biased ideal diode acts as an open circuit. The bar side of a diode symbol is its cathode.
- After replacing each diode by its appropriate circuit model, combine resistors in series and parallel. The current through the battery is set by the total resistance seen between its terminals.

Step 1: Determine the conducting diodes The longer battery plate is at the bottom, so its lower terminal is positive. Conventional current leaves this terminal, traverses the bottom $2\,\Omega$ resistor, and reaches the lower-right junction. From there current can return to the negative, left-hand junction through either the diagonal branch or the upper branch. In these directions D_3 and D_1 are forward biased. The middle diode D_2 is reverse biased and its $6\,\Omega$ branch carries no current.

Step 2: Reduce the two return paths The upper conducting route contains the right-hand $10\,\Omega$ resistor and the top $6\,\Omega$ resistor in series. Its resistance is $16\,\Omega$. The diagonal route also contains $10\,\Omega$ and $6\,\Omega$ in series, giving the same resistance. Both routes connect the same pair of junctions:

$$R_{\parallel} = \frac{16 \times 16}{16 + 16} = 8\,\Omega.$$

There is no third conducting parallel route through the middle resistor.

Step 3: Calculate the battery current The bottom resistor is common to the whole circuit and is in series with this parallel combination:

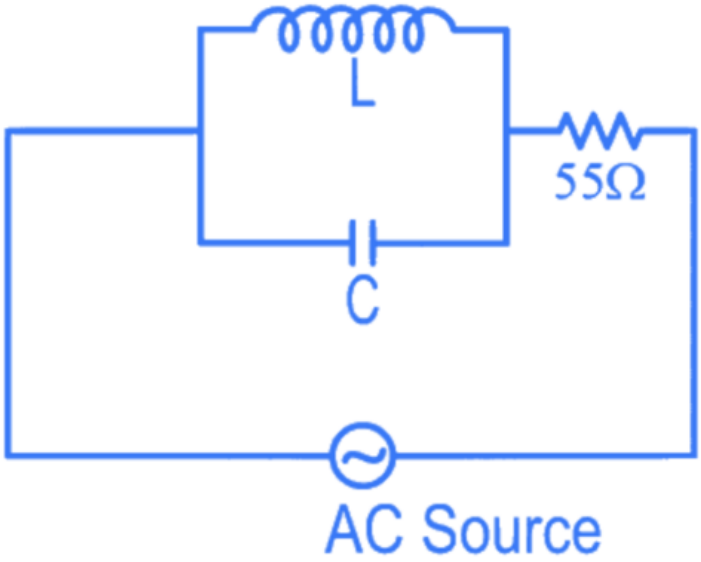
$$R_{\text{eq}} = 2 + 8 = 10\,\Omega, \qquad I = \frac{10\,\text{V}}{10\,\Omega} = 1\,\text{A}.$$

As a consistency check, the return current splits equally between the two identical $16\,\Omega$ paths, giving $0.5\,\text{A}$ in each.

Step 4: Conclusion The current through the battery is $\boxed{1\,\text{A}}$. The required numerical entry is 1.

Quick Tip to Solve: Mark the positive battery terminal first, then trace allowed current through each diode before reducing the resistor network.

59. A 110 V, 50 Hz, AC source is connected in the circuit (as shown in figure). The current through the resistance 55 Ω, at resonance in the circuit, will be _____ A. [4 marks]



Solution:

Concept:

- The figure contains an inductor and capacitor in parallel, with the parallel combination in series with the $55\,\Omega$ resistor. It is therefore essential to distinguish parallel resonance from series resonance.
- For ideal parallel LC branches, the admittance is $Y_{LC} = 1/(j\omega L) + j\omega C = j(\omega C - 1/(\omega L))$, where $\omega = 2\pi f$. At resonance the two reactive admittances cancel.

Step 1: Identify the resonant combination The inductor and capacitor share both end junctions in the diagram, so they have the same voltage and their currents add at the terminals. They are not in series. The resonance condition is

$$\omega_0 C = \frac{1}{\omega_0 L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}.$$

This condition is applied to the parallel tank connected ahead of the resistor.

Step 2: Find the terminal current of the tank At resonance, the capacitor current leads the tank voltage by a quarter cycle, whereas the inductor current lags it by a quarter cycle. Their magnitudes are equal, so their phasor sum is zero:

$$I_{LC} = V_{LC}Y_{LC} = 0.$$

Equivalently, an ideal parallel resonant tank presents infinite impedance to the source. Individual branch currents may exist, but they cancel at its external terminals.

Step 3: Apply the series-current condition The $55\,\Omega$ resistor is in series with the parallel tank, so the resistor current equals the current entering that tank from the AC supply. Consequently

$$I_R = I_{\text{source}} = 0, \qquad V_R = I_R R = 0.$$

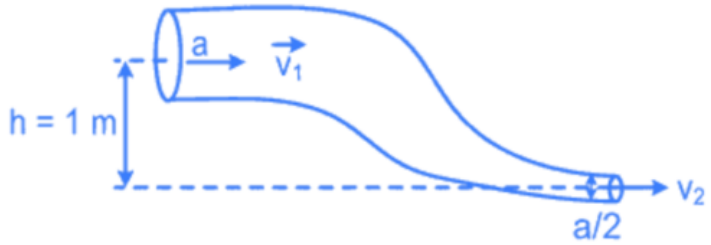
The quotient $110/55 = 2\,\text{A}$ would assume the entire source voltage appears across the resistor, which is false here: at ideal resonance it appears across the tank instead.

Step 4: Conclusion At resonance the current through the $55\,\Omega$ resistance is $\boxed{0\,\text{A}}$. Thus the numerical entry is 0.

Quick Tip to Solve: Before using resonance shortcuts, check the connections: ideal parallel LC blocks source current at resonance, whereas series LC cancels its net reactance.

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60. An ideal fluid of density 800 kgm^{-3}, flows smoothly through a bent pipe (as shown in figure) that tapers in cross-sectional area from a to $\frac{a}{2}$. The pressure difference between the wide and narrow sections of pipe is 4100 Pa. At wider section, the velocity of fluid is $\frac{\sqrt{x}}{6}$ ms^{-1} for x = _____. (Given g = 10 ms^{-2}) [4 marks]



Solution:

Concept:

- For steady incompressible flow, conservation of volume gives $A_1v_1 = A_2v_2$. The narrow area is half the wider area, so the narrow-section speed is twice the wider-section speed.
- Along a streamline in an ideal fluid, Bernoulli's equation is $p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$. Both the speed change and the one-metre fall shown in the figure must be included.

Step 1: Relate the two speeds Let the wide-section speed be v. Since its area is a, while the

narrow-section area is $a/2$, continuity gives

$$av = \frac{a}{2}v_2, \qquad v_2 = 2v.$$

Set the height of the narrow section to zero. The centre of the wide section is one metre above it, so $h_1 - h_2 = 1\,\text{m}$.

Step 2: Apply the pressure and height balance Writing Bernoulli's equation between these two sections and moving the kinetic and gravitational terms to the appropriate sides gives

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v^2) - \rho g(h_1 - h_2).$$

Substitute the stated pressure difference, density and height:

$$4100 = \frac{1}{2}(800)(4v^2 - v^2) - 800(10)(1).$$

Hence

$$1200v^2 = 12100, \qquad v^2 = \frac{121}{12}\,\text{m}^2\text{s}^{-2}.$$

The fall contributes to increasing the downstream kinetic energy; omitting it would give the wrong speed.

Step 3: Extract the requested parameter The question writes $v = \sqrt{x}/6$, with speed measured in $\text{m}\,\text{s}^{-1}$. Square this form and compare it with the computed value:

$$\frac{x}{36} = \frac{121}{12}, \qquad x = 36\left(\frac{121}{12}\right) = 363.$$

The positive square root is used because the requested quantity is a speed.

Step 4: Conclusion The wide-section speed is $\sqrt{363}/6\,\text{m}\,\text{s}^{-1}$, so the required parameter is $\boxed{x = 363}$.

Quick Tip to Solve: Use continuity first to eliminate one speed, then include the signed height difference in Bernoulli's equation before inserting the pressure difference.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. A commercially sold conc. HCl is 35% HCl by mass. If the density of this commercial acid is 1.46 g/mL, the molarity of this solution is:
(Atomic mass : Cl = 35.5 amu, H = 1 amu) [4 marks]

(A) 10.2 M
(B) 12.5 M
(C) 14.0 M
(D) 18.2 M

Correct Answer: (C) 14.0 M

Solution:

Concept:

- A mass percentage of 35 means that every $100\,\mathrm{g}$ of solution contains $35\,\mathrm{g}$ of HCl; the percentage refers to the total solution mass, not to the solvent mass.
- Molarity is moles of solute per litre of solution: $M = n/V$. Density connects the chosen solution volume with its total mass, and the molar mass converts solute mass to moles.

Step 1: Choose one litre of the solution Take $1.000\,\mathrm{L} = 1000\,\mathrm{mL}$ of the commercial acid. Its density is $1.46\,\mathrm{g\,mL^{-1}}$, so the total solution mass is

$$m_{\text{solution}} = \rho V = 1.46 \times 1000 = 1460\,\mathrm{g}.$$

Choosing a litre makes the final number of moles directly equal to the numerical molarity; the concentration is independent of this convenient sample size.

Step 2: Calculate the amount of hydrogen chloride The solution is 35 percent solute by mass. Thus

$$m_{\mathrm{HCl}} = \frac{35}{100}(1460) = 511\,\mathrm{g}.$$

With the given atomic masses,

$$M_{\text{molar}}(\mathrm{HCl}) = 1 + 35.5 = 36.5\,\mathrm{g\,mol^{-1}}, \qquad n = \frac{511}{36.5} = 14.0\,\mathrm{mol}.$$

The water mass is not needed because the denominator in molarity is solution volume.

Step 3: Find and compare the molarity These 14.0 moles occupy the one-litre sample selected at the start, giving

$$M = \frac{14.0\,\mathrm{mol}}{1.000\,\mathrm{L}} = 14.0\,\mathrm{mol\,L^{-1}}.$$

Therefore only 14.0 M matches both the density and the mass fraction. The proposed 10.2 M, 12.5 M and 18.2 M values would imply different solute masses per litre and cannot describe this composition.

Step 4: Conclusion The molarity of the commercial acid is $\boxed{14.0\,\mathrm{M}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a mass percentage w and density ρ in $\mathrm{g\,mL^{-1}}$, use $M = 10\rho w/M_{\text{molar}}$, with $w = 35$ here.

62. If the radius of the 3rd Bohr's orbit of hydrogen atom is r_3 and the radius of 4th Bohr's orbit is r_4. Then : [4 marks]

(A) $r_4 = \frac{9}{16} r_3$

(B) $r_4 = \frac{16}{9} r_3$

(C) $r_4 = \frac{3}{4} r_3$

(D) $r_4 = \frac{4}{3} r_3$

Correct Answer: (B) $r_4 = \frac{16}{9} r_3$

Solution:

Concept:

- In the Bohr model, the radius of the nth permitted orbit of a hydrogen-like species is $r_n = a_0 n^2/Z$. Here a_0 is the Bohr radius and Z is the nuclear atomic number.
- Both orbits belong to the same hydrogen atom, so $Z = 1$ and the same constant a_0 occurs in both radii. Their ratio therefore depends only on the squares of the principal quantum numbers.

Step 1: Write the two orbital radii Insert the principal quantum numbers into the radius law rather than into the energy or velocity laws:

$$r_3 = a_0(3)^2 = 9a_0, \qquad r_4 = a_0(4)^2 = 16a_0.$$

The fourth orbit lies farther from the nucleus than the third orbit. This observation already rules out any proposed ratio smaller than unity for r_4/r_3.

Step 2: Take the ratio and rearrange Dividing the second radius by the first eliminates the common atomic length scale:

$$\frac{r_4}{r_3} = \frac{16a_0}{9a_0} = \frac{16}{9}.$$

Multiplication by r_3 gives

$$r_4 = \frac{16}{9} r_3.$$

The factor is dimensionless, as it must be when relating two lengths, and is approximately 1.78, consistent with the larger orbit having the larger radius.

Step 3: Reject the alternative relations The factors $9/16$ and $3/4$ in (A) and (C) would make the fourth orbit smaller than the third and have the direction reversed. Option (D), with factor $4/3$, preserves the direction but assumes a linear dependence on n. Radius instead scales as n^2, so that factor must be squared. Only option (B) satisfies the Bohr radius relation exactly.

Step 4: Conclusion The required relation is $\boxed{r_4 = \frac{16}{9} r_3}$. Hence, the correct option is (B).

Quick Tip to Solve: For two orbits of the same hydrogen-like ion, form $r_m/r_n = (m/n)^2$; no numerical value of the Bohr radius is required.

63. Consider the ions/molecule

O_2^+, O_2, O_2^-, O_2^{2-}

For increasing bond order the correct option is : [4 marks]

(A) $O_2^{2-} < O_2^- < O_2 < O_2^+$

(B) $O_2^- < O_2^{2-} < O_2 < O_2^+$

(C) $O_2^- < O_2^{2-} < O_2^+ < O_2$

(D) $O_2^- < O_2^+ < O_2^{2-} < O_2$

Correct Answer: (A) $O_2^{2-} < O_2^- < O_2 < O_2^+$

Solution:

Concept:

- Molecular-orbital bond order is $\mathrm{BO} = (N_b - N_a)/2$, where N_b and N_a are the numbers of electrons in bonding and antibonding molecular orbitals.
- For the oxygen species in the question, electrons are removed from or added to the highest occupied antibonding π^* orbitals. Adding one such electron decreases bond order by one half; removing one increases it by one half.

Step 1: Start from neutral oxygen Neutral O_2 has eight valence electrons in bonding orbitals and four in antibonding orbitals. Consequently

$$\mathrm{BO}(O_2) = \frac{8-4}{2} = 2.$$

The core orbitals do not change this result because their bonding and antibonding contributions cancel. The two singly occupied π^* orbitals are the ones relevant to forming the stated ions.

Step 2: Calculate the bond orders of the ions Formation of ${O_2}^+$ removes one antibonding electron, whereas formation of ${O_2}^-$ or ${O_2}^{2-}$ adds one or two antibonding electrons. Therefore

$$\mathrm{BO}({O_2}^+) = \frac{8-3}{2} = 2.5,$$
$$\mathrm{BO}({O_2}^-) = \frac{8-5}{2} = 1.5,$$
$$\mathrm{BO}({O_2}^{2-}) = \frac{8-6}{2} = 1.$$

None of these operations changes the bonding-electron count.

Step 3: Arrange and check the choices The increasing numerical order is $1 < 1.5 < 2 < 2.5$, which translates into

$${O_2}^{2-} < {O_2}^- < O_2 < {O_2}^+.$$

Option (B) reverses the peroxide and superoxide positions. Option (C) also places the cation below neutral oxygen, and (D) incorrectly raises the peroxide above species with fewer antibonding electrons. These contradictions eliminate all three alternatives.

Step 4: Conclusion The increasing bond-order sequence is $\boxed{{O_2}^{2-} < {O_2}^- < O_2 < {O_2}^+}$. Hence, the correct option is (A).

Quick Tip to Solve: For oxygen species, track electrons in π^*: every extra antibonding electron lowers bond order by $1/2$, so the more negative ions have weaker bonds.

64. The $\left(\frac{\partial E}{\partial T}\right)_P$ of different types of half cells are as follows:

A	B	C	D
1×10^{-4}	2×10^{-4}	0.1×10^{-4}	0.2×10^{-4}

(Where E is the electromotive force)

Which of the above half cells would be preferred to be used as reference electrode? [4 marks]

(A) A

(B) B

(C) C

(D) D

Correct Answer: (C) C

Solution:

Concept:

- A reference electrode supplies a reproducible potential against which another electrode potential can be measured. Its potential should change as little as possible when laboratory temperature changes.
- At constant pressure the temperature coefficient is $\alpha = (\partial E/\partial T)_P$. For a small temperature change, $\Delta E \simeq \alpha \Delta T$, so the smallest magnitude of α gives the most temperature-stable reference.

Step 1: Compare all coefficients on one scale The four given numbers are positive. Express them with the same power of ten:

$$\alpha_A = 10 \times 10^{-5}, \quad \alpha_B = 20 \times 10^{-5}, \quad \alpha_C = 1 \times 10^{-5}, \quad \alpha_D = 2 \times 10^{-5}.$$

Thus the order of temperature sensitivity is $C < D < A < B$. A small decimal coefficient must be interpreted together with its exponent; the entry for C is ten times smaller than that for A.

Step 2: Apply the reference-electrode criterion For the same temperature fluctuation, the reference half cell C has the smallest potential drift. For example, multiplying every coefficient by the same positive temperature increment preserves the order, so C still gives the smallest absolute error in the measured cell emf. A and B have respectively ten and twenty times the coefficient of C, while D has twice its coefficient. Hence none of those three gives the best temperature stability among the listed choices. This comparison concerns reproducibility, rather than selecting a half cell with the largest potential.

Step 3: Conclusion Half cell C is preferred because its temperature coefficient is the smallest, 0.1×10^{-4}. Hence, the correct option is (C).

Quick Tip to Solve: Convert every coefficient to the same power of ten, then choose the smallest absolute temperature coefficient for a reference electrode.

65. Choose the correct stability order of group 13 elements in their + 1 oxidation state : [4 marks]

(A) Al $<$ Ga $<$ In $<$ Tl

(B) Tl $<$ In $<$ Ga $<$ Al

(C) Al $<$ Ga $<$ Tl $<$ In

(D) Al $<$ Tl $<$ Ga $<$ In

Correct Answer: (A) Al $<$ Ga $<$ In $<$ Tl

Solution:

Concept:

- Group 13 atoms have the outer electronic configuration ns^2np^1. Loss of the single np electron gives the $+1$ state, whereas participation of all three valence electrons gives the $+3$ state.
- The inert-pair effect is the increasing reluctance of the outer ns^2 pair to participate in bonding down the heavier p-block elements. It increases the relative stability of the lower oxidation state.

Step 1: Relate the oxidation states to valence electrons In aluminium the valence s pair participates readily in bonding, and the $+3$ state is strongly favoured. The $+1$ state requires the s pair to remain unused. On descending through gallium, indium and thallium, the s electrons become progressively more resistant to participation. Poor shielding by intervening d and f electrons and the stronger binding of the heavy-element s pair contribute to this trend. Consequently retaining the pair, and therefore forming the $+1$ oxidation state, becomes more favourable.

Step 2: Order the four elements and reject the alternatives The elements occur down group 13 in the order Al, Ga, In and Tl. Their increasing inert-pair effect gives

$$\text{stability of } +1 \text{ state: } \mathrm{Al} < \mathrm{Ga} < \mathrm{In} < \mathrm{Tl}.$$

Thallium therefore belongs at the most-stable end, not before aluminium or gallium. Option B reverses the trend and makes aluminium the most stable $+1$ species. Option C incorrectly places indium above thallium in $+1$-state stability. Option D also places thallium too low. Only A preserves the increasing stability of the lower oxidation state down the group. The trend asked for is not the stability of the $+3$ state, which shows the opposite tendency relative to $+1$.

Step 3: Conclusion The correct order is $\mathrm{Al} < \mathrm{Ga} < \mathrm{In} < \mathrm{Tl}$. Hence, the correct option is (A).

Quick Tip to Solve: For the lower oxidation state of a heavy p-block group, first check the inert-pair effect: retaining the s pair becomes more favourable down the group.

66. Given below are two statements :

Statement I : In 'Lassaigne's Test', when both nitrogen and sulphur are present in an organic compound, sodium thiocyanate is formed.

Statement II : If both nitrogen and sulphur are present in an organic compound, then the excess of sodium used in sodium fusion will decompose the sodium thiocyanate formed to give NaCN and Na_2S.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (A) Both Statement I and Statement II are correct.

Solution:

Concept:

- Sodium fusion converts covalently bound heteroatoms into water-soluble ionic salts. If nitrogen and sulphur occur together, sodium thiocyanate, $NaSCN$, can form in the fusion mixture.
- The fusion product depends on the amount of sodium. Excess sodium converts the thiocyanate into sodium cyanide, $NaCN$, and sodium sulphide, Na_2S.

Step 1: Evaluate Statement I When an organic compound containing both nitrogen and sulphur is fused with sodium, the carbon, nitrogen and sulphur can be retained together in the thiocyanate ion. The product is sodium thiocyanate, $NaSCN$, so Statement I correctly identifies the characteristic combined product. This explains the familiar blood-red ferric thiocyanate response when the extract containing thiocyanate is treated appropriately with ferric ions. Statement I does not assert that the thiocyanate must survive any possible excess of sodium.

Step 2: Evaluate Statement II using the fusion reaction In the presence of excess sodium, the thiocyanate undergoes further conversion:

$$NaSCN + 2\,Na \longrightarrow NaCN + Na_2S.$$

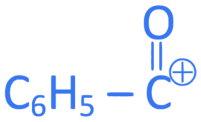
The equation conserves three sodium atoms, one carbon atom, one nitrogen atom and one sulphur atom on each side. It shows exactly the products named in Statement II, so that statement is also correct. The two statements describe successive possibilities under different sodium amounts and are consistent rather than contradictory. Option B rejects both valid facts; C wrongly rejects the excess-sodium reaction; D wrongly rejects thiocyanate formation. Therefore the option accepting both statements is required.

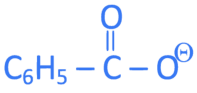
Step 3: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

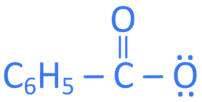
Quick Tip to Solve: In a combined nitrogen–sulphur test, distinguish initial $NaSCN$ formation from its further reaction with excess sodium to give $NaCN$ and Na_2S.

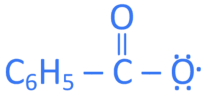
67. $(C_7H_5O_2)_2 \xrightarrow{h\nu}$ [X] $\rightarrow 2C_6H_5 + 2CO_2$

Consider the above reaction and identify the intermediate 'X' [4 marks]

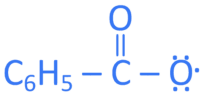
(A) $C_6H_5CO^+$ 

(B) $C_6H_5COO^-$ 

(C) C_6H_5COO, terminal oxygen with two lone pairs 

(D) $C_6H_5COO^\bullet$ 

Correct Answer: (D)

$C_6H_5COO^\bullet$ 

Solution:

Concept:

- Benzoyl peroxide contains a weak peroxide O−O bond. Irradiation causes homolytic cleavage, in which each oxygen retains one electron from the broken bond.
- The initial fragments are benzoyloxy radicals, $C_6H_5COO^\bullet$. Each loses carbon dioxide to give a phenyl radical; this is a radical decarboxylation.

Step 1: Identify the bond broken by light The formula $(C_7H_5O_2)_2$ represents benzoyl peroxide, with the connectivity $C_6H_5CO{-}O{-}O{-}COC_6H_5$. Its central peroxide bond is especially susceptible to photolytic homolysis. The initial stage can be expressed as

$$(C_6H_5COO)_2 \xrightarrow{h\nu} 2\,C_6H_5COO^\bullet.$$

The radical centre is on the terminal oxygen of each benzoyloxy fragment. Homolysis produces neutral radicals, rather than an oppositely charged pair of ions. Thus the intermediate type must contain an unpaired electron.

Step 2: Check the next step and compare the structures A benzoyloxy radical then gives a phenyl radical and carbon dioxide:

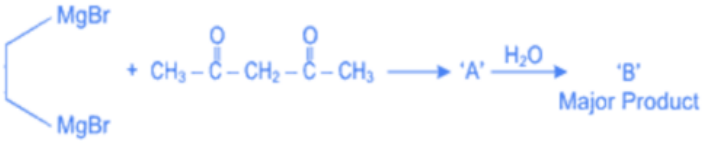
$$C_6H_5COO^\bullet \longrightarrow C_6H_5^\bullet + CO_2.$$

Two such fragments account for the two phenyl units and two carbon dioxide molecules shown. Option D depicts the benzoyloxy radical, with the unpaired electron on oxygen. A is an acylium cation, whose oxygen count and charge do not match the homolysis fragment. B is a benzoate anion, an ionic species rather than the required radical. C does not show the unpaired electron required for this radical intermediate. Conservation of the radical electron as well as the carbon dioxide loss distinguishes D.

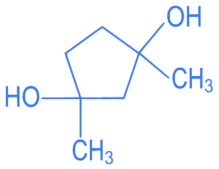
Step 3: Conclusion The intermediate is the benzoyloxy radical, $C_6H_5COO^{\bullet}$, shown in option D. Hence, the correct option is (D).

Quick Tip to Solve: For photolysis of a peroxide, break its $O-O$ bond homolytically first; for an acyloxy radical, then check whether loss of CO_2 explains the next product.

68. Consider the above reaction sequence and identify the product B. [4 marks]



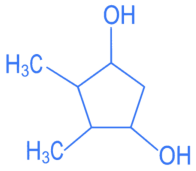
(A) 1,3-Dimethylcyclopentane-1,3-diol



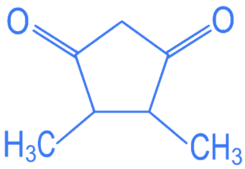
(B) Cyclohexane-1,4-diol



(C) 4,5-Dimethylcyclopentane-1,3-diol

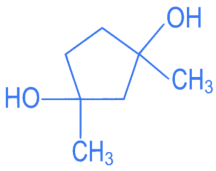


(D) 4,5-Dimethylcyclopentane-1,3-dione



Correct Answer: (A)

1,3-Dimethylcyclopentane-1,3-diol



Solution:

Concept:

- A Grignard carbon is nucleophilic and adds to a ketone carbonyl carbon, converting $C=O$ into an alkoxide. Hydrolysis protonates the alkoxide to an alcohol.
- A di-Grignard reagent has two reactive carbon ends. Addition of both ends to a diketone can close

a ring; the original carbonyl carbons retain their substituents.

Step 1: Trace the two carbon–carbon bond formations The drawn reagent is $BrMgCH_2CH_2MgBr$, and the diketone is $CH_3COCH_2COCH_3$. One nucleophilic end adds to one carbonyl carbon. The second end then adds intramolecularly to the remaining carbonyl carbon. Trace the newly closed ring: first former carbonyl carbon, the intervening diketone methylene, second former carbonyl carbon, and the two methylene carbons from the di-Grignard reagent. These are five ring atoms. The two original carbonyl carbons occupy the 1 and 3 positions of the resulting cyclopentane ring.

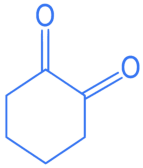
Step 2: Hydrolyse the alkoxides and retain substituents Each carbonyl carbon originally carried a methyl group. After addition it bears both that methyl group and an alkoxide oxygen. Treatment with water converts both alkoxides to hydroxyl groups, without removing methyl groups or changing the carbon skeleton. The product is therefore 1,3-dimethylcyclopentane-1,3-diol, represented by A. Option B incorrectly gives a six-membered ring and omits the methyl groups. In C the hydroxyl and methyl groups are on different ring carbons, inconsistent with carbonyl addition. D retains ketone carbonyls, although the two Grignard additions have converted them into alcohol groups. These connectivity checks select A independently of how the pentagon is drawn.

Step 3: Conclusion Product B is 1,3-dimethylcyclopentane-1,3-diol, shown in option A. Hence, the correct option is (A).

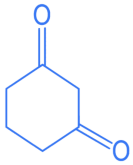
Quick Tip to Solve: Trace the closed ring through both original carbonyl carbons and both Grignard carbons; after hydrolysis, put OH and the retained methyl group on each former carbonyl carbon.

69. Which will have the highest enol content? [4 marks]

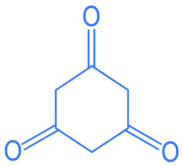
(A) Cyclohexane-1,2-dione



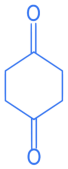
(B) Cyclohexane-1,3-dione



(C) Cyclohexane-1,3,5-trione

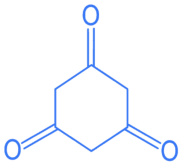


(D) Cyclohexane-1,4-dione



Correct Answer: (C)

Cyclohexane-1,3,5-trione



Solution:

Concept:

- Keto–enol tautomerism transfers an alpha hydrogen to oxygen while creating a carbon–carbon double bond. The proportion of enol increases when the enol gains special stabilization.
- Conjugation, intramolecular hydrogen bonding and especially aromaticity can stabilize an enol. Aromatic stabilization requires a cyclic, continuously conjugated system with $4n + 2$ pi electrons.

Step 1: Identify the especially stabilized enol Option C is cyclohexane-1,3,5-trione. Enolization of all three carbonyl groups creates three hydroxyl groups and three alternating double bonds around the six-membered ring. The resulting enol is benzene-1,3,5-triol, commonly called phloroglucinol. Its ring has six delocalized pi electrons, meeting the $4n + 2$ condition with $n = 1$. Thus the enol form gains aromatic stabilization. This is much stronger than simply forming one isolated alkene next to an alcohol group.

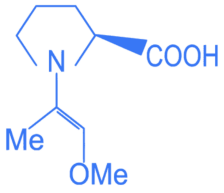
Step 2: Compare the other carbonyl arrangements A is cyclohexane-1,2-dione, B is cyclohexane-1,3-dione, and D is cyclohexane-1,4-dione. B can form an enol stabilized by conjugation and intramolecular hydrogen bonding, so it is more favourable for enolization than an ordinary isolated ketone. However, none of these diones gives the fully aromatic tri-enol available to C by ordinary keto–enol tautomerism. The ring size alone cannot decide enol content, because all four structures have six-membered rings. It is the placement of the three carbonyl groups in C that permits the alternating double-bond pattern and aromatic stabilization. Therefore selecting B solely because it is a beta-diketone would miss the stronger aromatic effect in C.

Step 3: Conclusion Cyclohexane-1,3,5-trione has the highest enol content because its tri-enol form is aromatic. Hence, the correct option is (C).

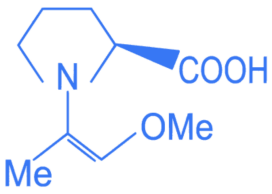
Quick Tip to Solve: Before comparing beta-dicarbonyl hydrogen bonding, ask whether complete enolization produces an aromatic ring; the 1,3,5-trione here gives phloroglucinol.

70. Among the following structures, which will show the most stable enamine formation? (Where Me is $-CH_3$) [4 marks]

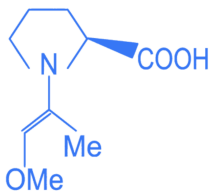
(A)

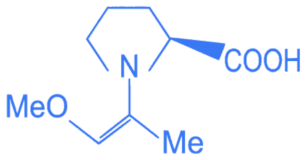


(B)

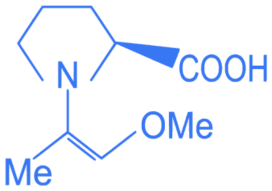


(C)



(D) 

Correct Answer: (B)



Solution:

Concept:

- An enamine contains the conjugated $N-C=C$ arrangement. Different geometries and conformations can have different stability because their substituents approach one another differently.
- The carboxylic acid group can donate a hydrogen bond to the methoxy oxygen. A geometry that brings these groups close enough for intramolecular hydrogen bonding gains stabilization.

Step 1: Compare the substituent orientations All four drawings contain the same basic enamine functionality, a methyl group and a methoxy substituent, together with the chiral carboxylic acid-bearing ring. They must therefore be distinguished by spatial arrangement rather than simply by counting atoms or double bonds. In B the methoxy group lies on the side of the enamine that faces the carboxylic acid group. This allows the acidic hydroxyl hydrogen to approach the methoxy oxygen and form an intramolecular $O-H\cdots O$ interaction. The nitrogen-containing enamine remains conjugated while this extra interaction stabilizes the structure.

Step 2: Reject orientations that lose the favourable contact In A the methoxy group points away on the opposite alkene geometry, so the donor and acceptor are less favourably positioned. C and D place the enamine substituent on the other side of the nitrogen substituent arrangement, moving the methoxy group away from the carboxylic acid-bearing side of the ring. They do not give the same favourable close donor–acceptor arrangement as B. Although all are enamines and can possess nitrogen-to-alkene conjugation, that common feature does not distinguish their relative stability. The additional intramolecular hydrogen bond available in B is the deciding feature. The solid wedge on the carboxylic acid-bearing carbon is relevant when comparing these drawn orientations.

Step 3: Conclusion The arrangement in B is most stabilized by intramolecular hydrogen bonding between the carboxylic acid OH and methoxy oxygen. Hence, the correct option is (B).

Quick Tip to Solve: For enamine geometrical alternatives with COOH and OMe groups, locate the acid hydrogen and methoxy oxygen first; choose the arrangement that lets them form an internal hydrogen bond.

71. Which statement is not true with respect to nitrate ion test? [4 marks]

(A) A dark brown ring is formed at the junction of two solutions.

(B) Ring is formed due to nitroferrous sulphate complex.

(C) The brown complex is $[Fe(H_2O)_5\ (NO)]SO_4$.

(D) Heating the nitrate salt with conc. H_2SO_4, light brown fumes are evolved.

Correct Answer: (B) Ring is formed due to nitroferrous sulphate complex.

Solution:

Concept:

- The nitrate brown-ring test first reduces nitrate ions to nitric oxide using freshly prepared iron(II) sulphate in acidic medium. Nitric oxide then acts as a ligand towards an iron aqua complex.
- The coloured species is the nitrosyl complex $[Fe(H_2O)_5(NO)]^{2+}$, commonly written as its sulphate salt $[Fe(H_2O)_5(NO)]SO_4$. A nitrosyl ligand must be distinguished from a nitro ligand.

Step 1: Identify how the ring develops Concentrated sulphuric acid is carefully added down the side of the test tube, so that it forms a separate lower layer rather than mixing throughout immediately. At the interface, the acidic conditions generate nitric oxide from nitrate in the presence of iron(II):

$$NO_3^- + 3\,Fe^{2+} + 4\,H^+ \longrightarrow NO + 3\,Fe^{3+} + 2\,H_2O.$$

The freshly produced nitric oxide is trapped by iron(II), giving the characteristic dark brown ring. Thus statement (A) correctly identifies where the ring is observed.

Step 2: Check the complex name and the other statements The coordination product contains molecular NO, not the nitro ligand NO_2^-. Therefore calling it nitroferrous sulphate is incorrect; the conventional name is nitrosoferrous sulphate, or pentaaquanitrosyliron sulphate. Statement (C) gives the corresponding brown-complex formula correctly. Heating a nitrate with concentrated sulphuric acid liberates nitric acid, whose decomposition can give brown NO_2 fumes, so statement (D) is consistent with the nitrate dry test. The question asks for the false statement rather than the correct complex formula.

Step 3: Conclusion The ring is due to a nitrosyl, conventionally nitrosoferrous, complex, not nitroferrous sulphate. Hence, the correct option is (B).

Quick Tip to Solve: In the nitrate ring test, remember that the bound ligand is NO: nitrosyl and nitro refer to different ligands.

72. An evacuated glass vessel weighs 40.0 g when empty, 135.0 g when filled with a liquid of density 0.95 g mL^{-1} and 40.5 g when filled with an ideal gas at 0.82 atm at 250 K. The molar mass of the gas in g mol^{-1} is :

(Given : R = 0.082 L atm K^{-1} mol^{-1}) [4 marks]

(A) 35

(B) 50

(C) 75

(D) 125

Correct Answer: (D) 125

Solution:

Concept:

- The vessel volume equals the volume of liquid that fills it. Use $V = m/\rho$, subtracting the mass of the empty glass vessel before calculating either fluid mass.
- For the ideal gas, $PV = nRT$ and $n = m/M$. Hence the molar mass is $M = mRT/(PV)$, with volume in litres for the supplied value of R.

Step 1: Find the vessel volume and gas mass The mass of liquid alone is $135.0 - 40.0 = 95.0\,\text{g}$. Divide this by the given liquid density:

$$V = \frac{95.0\,\text{g}}{0.95\,\text{g mL}^{-1}} = 100\,\text{mL} = 0.100\,\text{L}.$$

The mass of gas alone is

$$m = 40.5 - 40.0 = 0.5\,\text{g}.$$

Both fillings occupy the same internal volume; the mass of glass must not be included in the gas mass.

Step 2: Apply the ideal-gas law and compare options At the stated pressure and temperature, the amount of gas is

$$n = \frac{PV}{RT} = \frac{0.82 \times 0.100}{0.082 \times 250} = 0.00400\,\text{mol}.$$

Thus

$$M = \frac{m}{n} = \frac{0.5}{0.00400} = 125\,\text{g mol}^{-1}.$$

Equivalently, substitution into $mRT/(PV)$ gives the same result directly. As a check, 0.00400 mol of a gas of molar mass 125 has mass 0.500 g, exactly the measured increase. The alternatives 35, 50, and 75 would give masses 0.140, 0.200, and 0.300 g at this pressure, volume and temperature; none matches the data.

Step 3: Conclusion The molar mass of the ideal gas is $\boxed{125\,\text{g mol}^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Subtract the empty-vessel mass twice, once for each filling, and convert the liquid-derived volume from mL to L before using R.

73. Given below are two statements :

Statement I : According to Ellingham diagram, any metal oxide with higher ΔG° is more stable than the one with lower ΔG°.

Statement II : The metal involved in the formation of oxide placed lower in the Ellingham diagram can reduce the oxide of a metal placed higher in the diagram.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (D) Statement I is incorrect but Statement II is correct.

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy change of oxide formation against temperature. Oxidation reactions must be compared on a consistent oxygen basis at the same temperature.
- A lower, more negative ΔG° of formation indicates a more thermodynamically stable oxide. A metal forming that oxide has a greater tendency to combine with oxygen.

Step 1: Evaluate Statement I Formation of a stable oxide is favoured by a large decrease in Gibbs energy. Consequently, an oxide whose formation line is lower in the Ellingham diagram is more stable with respect to its constituent elements under the stated standard conditions. A higher value of ΔG° means formation is less favourable, rather than more favourable. Statement I reverses this relation and is therefore incorrect. The important comparison is the numerical value, not simply the magnitude without its sign.

Step 2: Evaluate Statement II using a reaction difference Let the formation Gibbs energies, on the same oxygen basis, be g_A for the lower oxide line and g_B for the higher line, so $g_A < g_B$. To transfer oxygen from the oxide of B to metal A, add oxidation of A to the reverse of oxidation of B. The overall Gibbs energy is

$$\Delta G^\circ_{\text{reduction}} = g_A - g_B < 0.$$

Thus the metal associated with the lower line can thermodynamically reduce the oxide associated with the higher line at that temperature. Statement II is correct. Options (A) and (C) incorrectly accept Statement I, and option (B) incorrectly rejects Statement II. This is a thermodynamic criterion; actual experimental rates may additionally depend on kinetic factors.

Step 3: Conclusion Statement I is incorrect, whereas Statement II is correct. Hence, the correct option is (D).

Quick Tip to Solve: At a fixed temperature, subtract the higher oxide-formation line from the lower one: a negative difference favours oxygen transfer to the lower-line metal.

74. Consider the following reaction:

$$2HSO_4^-(aq) \xrightarrow[(2)\ Hydrolysis]{(1)\ Electrolysis} 2HSO_4^- + 2H^+ + A$$

The dihedral angle in product A in its solid phase at 110 K is : [4 marks]

(A) 104°

(B) 111.5°

(C) 90.2°

(D) 111.0°

Correct Answer: (C) 90.2°

Solution:

Concept:

- Electrolytic oxidation of hydrogen sulphate ions produces peroxodisulphate, which contains a peroxide linkage. Hydrolysis of the peroxo compound yields hydrogen peroxide.
- H_2O_2 has a non-planar, open-book structure. Its H–O–O–H dihedral angle depends on the phase; distinguish this torsional angle from an H–O–O bond angle.

Step 1: Identify product A from the preparation In the electrolytic step, hydrogen sulphate ions undergo oxidation to peroxodisulphate:

$$2\,HSO_4^- \longrightarrow S_2O_8^{2-} + 2\,H^+ + 2\,e^-.$$

Subsequent hydrolysis produces hydrogen peroxide and regenerates hydrogen sulphate:

$$S_2O_8^{2-} + 2\,H_2O \longrightarrow 2\,HSO_4^- + H_2O_2.$$

Therefore the product designated A is H_2O_2. Identifying this peroxide is necessary before using the structural information; neither ordinary water nor a sulphur oxide is the required product.

Step 2: Use the solid-state dihedral angle The dihedral angle is the angle between the two planes that contain the O–O bond and one of the O–H bonds. In solid hydrogen peroxide at $110\,K$, the H–O–O–H dihedral angle is $90.2°$. Hydrogen bonding and crystal packing make the solid-state conformation different from the isolated gas-phase conformation. The value near $111.5°$ is associated with the gas-phase dihedral angle and must not be used for this solid sample. A value near $104°$ concerns a bond-angle comparison rather than the solid-state torsion asked here. The remaining $111.0°$ likewise fails to specify the measured solid-phase value. The stated temperature and phase thus select a unique option.

Step 3: Conclusion Product A is hydrogen peroxide, with solid-state dihedral angle $\boxed{90.2°}$ at $110\,K$. Hence, the correct option is (C).

Quick Tip to Solve: For hydrogen peroxide, keep the two dihedral values separate: about 111.5 degrees in the gas and 90.2 degrees in the solid.

75. The correct order of melting point is : [4 marks]

(A) Be > Mg > Ca > Sr

(B) Sr > Ca > Mg > Be

(C) Be > Ca > Mg > Sr

(D) Be > Ca > Sr > Mg

Correct Answer: (D) Be > Ca > Sr > Mg

Solution:

Concept:

- The melting point of a metal depends on metallic bonding and the crystal structure. It cannot always be predicted by a single monotonic trend in atomic size down a group.
- Among these alkaline-earth metals, beryllium has an exceptionally high melting point. Magnesium melts below calcium and strontium, so the actual order must retain this irregularity.

Step 1: Compare the relevant melting points The approximate melting temperatures of the listed metals are

$$\mathrm{Be} : 1287^{\circ}\mathrm{C}, \quad \mathrm{Ca} : 842^{\circ}\mathrm{C},$$
$$\mathrm{Sr} : 777^{\circ}\mathrm{C}, \quad \mathrm{Mg} : 650^{\circ}\mathrm{C}.$$

The numerical values are used only to establish the order; small differences in tabulated rounding do not change any comparison. Beryllium is clearly highest, calcium exceeds strontium, and both exceed magnesium.

Step 2: Arrange the elements and eliminate alternatives Arranging the temperatures from largest to smallest gives

$$\mathrm{Be} > \mathrm{Ca} > \mathrm{Sr} > \mathrm{Mg}.$$

Option (A) wrongly places magnesium above both calcium and strontium. Option (B) makes beryllium the lowest, contradicting its particularly strong metallic cohesion and its high melting point. Option (C) gets the first two elements correct but places magnesium above strontium, reversing their measured order. Only option (D) preserves all three successive comparisons. A simple rule that melting point must steadily decrease down group 2 would therefore give the wrong answer here; structural and bonding changes must be allowed for.

Step 3: Conclusion The correct melting-point order is $\boxed{\mathrm{Be} > \mathrm{Ca} > \mathrm{Sr} > \mathrm{Mg}}$. Hence, the correct option is (D).

Quick Tip to Solve: Remember the group-2 exception: magnesium melts below both calcium and strontium, while beryllium is highest among these four.

76. The correct order of melting points of hydrides of group 16 elements is : [4 marks]

(A) $H_2S < H_2Se < H_2Te < H_2O$

(B) $H_2O < H_2S < H_2Se < H_2Te$

(C) $H_2S < H_2Te < H_2Se < H_2O$

(D) $H_2Se < H_2S < H_2Te < H_2O$

Correct Answer: (A) $H_2S < H_2Se < H_2Te < H_2O$

Solution:

Concept:

- Melting requires disruption of intermolecular attractions in a solid. The relevant comparison is between molecules, rather than the strength of the covalent E–H bonds within each molecule.
- For H_2S, H_2Se, and H_2Te, increasing size and polarizability strengthen dispersion attractions. Water additionally forms an extensive intermolecular hydrogen-bond network.

Step 1: Order the heavier hydrides The melting points rise through the heavier hydrides in the sequence

$$H_2S < H_2Se < H_2Te.$$

Approximate temperatures are 188, 208, and $224\,K$, respectively. The electron clouds become larger and more easily distorted from sulphur to tellurium, so dispersion interactions become stronger. These compounds do not form a hydrogen-bonded lattice comparable to that of water; a trend based solely on decreasing E–H bond strength would confuse an intramolecular property with the melting process.

Step 2: Place water and check the proposed orders Water melts at approximately $273\,K$, above all three heavier hydrides. Its unusually high melting point is explained by the interconnected hydrogen bonds between its molecules in ice. Thus the complete increasing order is

$$H_2S < H_2Se < H_2Te < H_2O.$$

Option (B) incorrectly places water lowest, ignoring hydrogen bonding. Option (C) incorrectly reverses the order of hydrogen selenide and hydrogen telluride. Option (D) incorrectly puts hydrogen selenide below hydrogen sulphide. Only option (A) combines the heavier-hydride order with the anomalously high value for water. No covalent O–H bond needs to be broken to melt ice.

Step 3: Conclusion The correct order is $\boxed{H_2S < H_2Se < H_2Te < H_2O}$. Hence, the correct option is (A).

Quick Tip to Solve: First order sulphur, selenium and tellurium hydrides by increasing polarizability, then place water highest because of its hydrogen-bonded solid.

77. Consider the following reaction :

A + alkali $\rightarrow$ B (Major Product)

If B is an oxoacid of phosphorus with no P-H bond, then A is : [4 marks]

(A) White P_4

(B) Red P_4

(C) P_2O_3

(D) H_3PO_3

Correct Answer: (B) Red P_4

Solution:

Concept:

- The oxoacids of phosphorus differ in their P–H bonds: hypophosphorous acid has two, phosphorous acid has one, and orthophosphoric acid has none.
- Treatment with alkali produces the corresponding oxoacid salts. The phosphorus starting material determines whether the oxygen-containing product is hypophosphite, phosphite or phosphate.

Step 1: Identify the required phosphorus oxoacid family An oxoacid of phosphorus with no P–H bond is orthophosphoric acid, H_3PO_4, whose hydrogens are attached to oxygen. In alkaline solution its corresponding salt is a phosphate. In contrast, H_3PO_2 contains two P–H bonds and H_3PO_3 contains one. The wording about oxoacid B therefore identifies the phosphate-forming reaction, even though the alkaline reaction mixture contains the salt rather than free acid.

Step 2: Compare the four starting substances Red phosphorus reacts with hot concentrated alkali to give phosphine and phosphate; the oxygen-containing major product corresponds to orthophosphoric acid. A balanced representation is

$$8\,P + 9\,NaOH + 3\,H_2O \longrightarrow 3\,Na_3PO_4 + 5\,PH_3.$$

White phosphorus instead gives hypophosphite under its usual alkaline disproportionation:

$$P_4 + 3\,NaOH + 3\,H_2O \longrightarrow PH_3 + 3\,NaH_2PO_2.$$

Hence white phosphorus gives an oxoacid family retaining P–H bonds. Phosphorus(III) oxide, conventionally represented as P_2O_3 in the option, gives phosphite on reaction with alkali; phosphorous acid itself also gives phosphite by neutralisation. Both belong to the H_3PO_3 family and retain a P–H bond. Thus options (A), (C), and (D) fail the specified absence of P–H bonding, whereas red phosphorus gives the required phosphate family.

Step 3: Conclusion A is red phosphorus, represented as red P_4 in the options. Its phosphate product corresponds to an oxoacid with no P–H bond. Hence, the correct option is (B).

Quick Tip to Solve: Use the oxygen-containing product to distinguish phosphorus allotropes: white phosphorus gives hypophosphite; red phosphorus with hot alkali gives phosphate.

78. Polar stratospheric clouds facilitate the formation of : [4 marks]

(A) $ClONO_2$

(B) HOCl

(C) ClO

(D) CH_4

Correct Answer: (B) HOCl

Solution:

Concept:

- Polar stratospheric clouds provide surfaces for heterogeneous reactions of chlorine reservoir compounds in the cold stratosphere. A reservoir compound temporarily stores chlorine in a relatively inactive form.
- One important surface reaction is hydrolysis of chlorine nitrate: $ClONO_2 + H_2O \longrightarrow HOCl + HNO_3$. Thus hypochlorous acid, HOCl, is formed directly in the cloud-mediated reaction.

Step 1: Identify the cloud-mediated conversion In the polar stratosphere, ice and other particles supply surfaces on which reactions can occur more readily than in the surrounding gas. Chlorine nitrate, $ClONO_2$, reacts with water on these surfaces. Its chlorine-containing product is hypochlorous acid, HOCl, while the nitrogen-containing product is nitric acid, HNO_3. Therefore the species asked for is the product of this surface reaction, not its chlorine nitrate reactant.

Step 2: Distinguish the other chlorine species Chlorine nitrate is consumed in this conversion, so option (A) reverses the role of the reservoir and product. Chlorine monoxide, ClO, is important in ozone destruction, but it is produced subsequently when reactive chlorine attacks ozone: $Cl + O_3 \longrightarrow ClO + O_2$. Methane, CH_4, is not the chlorine-containing activation product and does not arise from hydrolysis of chlorine nitrate. Among the listed substances, only hypochlorous acid identifies the cloud-mediated product directly.

Step 3: Conclusion Polar stratospheric clouds facilitate formation of HOCl, hypochlorous acid. Hence, the correct option is (B).

Quick Tip to Solve: Connect polar cloud surfaces with $ClONO_2 + H_2O \longrightarrow HOCl + HNO_3$; distinguish this activation step from later ozone-destruction reactions.

79. Which of the following sets are correct regarding polymer?

(A) Copolymer : Buna-S

(B) Condensation polymer : Nylon-6,6

(C) Fibres : Nylon-6,6

(D) Thermosetting polymer : Terylene

(E) Homopolymer : Buna-N

Choose the correct answer from given options below : [4 marks]

(A) (A), (B) and (C) are correct

(B) (B), (C) and (D) are correct

(C) (A), (C) and (E) are correct

(D) (A), (B) and (D) are correct

Correct Answer: (A) (A), (B) and (C) are correct

Solution:

Concept:

- A copolymer contains units derived from at least two different monomers; a homopolymer is formed from a single kind of monomer. Buna-S and Buna-N must be classified by their two starting monomers.
- Condensation polymers form through reactions of functional groups with elimination of small molecules. Fibre-forming polymers have strong intermolecular attractions and suitable chain alignment; thermosetting polymers form permanent cross-linked networks.

Step 1: Check statements A, B and C Buna-S is made from 1,3-butadiene and styrene, so it contains two types of monomer units and statement (A) is correct. Nylon-6,6 is made by condensation of hexamethylenediamine with adipic acid, with elimination of water as amide links form; statement (B) is correct. The amide groups in nylon-6,6 permit hydrogen bonding between chains. Its strong, ordered chains make it suitable for fibres, so statement (C) is also correct.

Step 2: Reject statements D and E Terylene is a polyester formed from ethylene glycol and terephthalic acid. It is a thermoplastic, rather than the permanently cross-linked thermosetting material claimed in (D), so that statement is false. Buna-N is formed from 1,3-butadiene and acrylonitrile, so it is a copolymer and cannot be the homopolymer asserted in (E). Consequently any answer combination containing (D) or (E) must be rejected. The combination of statements (A), (B) and (C) is precisely the correct set.

Step 3: Conclusion The first three statements are correct, while the final two statements are incorrect. Hence, the correct option is (A).

Quick Tip to Solve: For Buna-S and Buna-N, remember the second monomers: styrene and acrylonitrile. Both are copolymers, while nylon-6,6 is both a condensation polymer and a fibre.

80. A chemical which stimulates the secretion of pepsin is : [4 marks]

(A) Anti histamine

(B) Cimetidine

(C) Histamine

(D) Zantac

Correct Answer: (C) Histamine

Solution:

Concept:

- Histamine is a chemical messenger involved in gastric secretion. In the chemistry of drugs, it is associated with stimulation of gastric acid and pepsin secretion.
- An antagonist blocks the action of a messenger at its receptor. The acid-reducing drugs cimetidine and ranitidine act by blocking histamine action at gastric H_2 receptors, so they oppose rather than supply the stimulatory signal.

Step 1: Identify the stimulatory messenger The question asks for the chemical that stimulates secretion, not a drug prescribed to suppress excessive gastric secretion. Histamine is the relevant physiological messenger. Its action on the gastric system promotes the secretory response associated with acid and pepsin. Therefore the substance to select is histamine itself; the word "anti" in another option signals that the effect of histamine is being opposed.

Step 2: Eliminate the inhibitory drugs An antihistamine blocks or reduces histamine-mediated action and does not identify the messenger responsible for stimulation, so option (A) is excluded. Cimetidine is an H_2-receptor antagonist used to reduce gastric acid secretion, which rules out option (B). Zantac is the trade name associated with ranitidine, another histamine H_2-receptor antagonist, and is therefore also on the inhibitory side rather than the stimulatory side. These distinctions leave histamine as the only listed chemical with the requested role.

Step 3: Conclusion The chemical is histamine. Hence, the correct option is (C).

Quick Tip to Solve: Separate the natural messenger histamine from drugs that block it: cimetidine and ranitidine reduce its gastric action rather than reproduce its stimulation.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. For complete combustion of methanol

$CH_3OH(I) + \frac{3}{2}O_2(g) \rightarrow CO_2(g) + 2H_2O(I)$

the amount of heat produced as measured by bomb calorimeter is 726 kJ mol^{-1} at 27°C. The enthalpy of combustion for the reaction is $-x$ kJ mol^{-1}, where x is _____. (Nearest integer)

(Given : R = 8.3 JK^{-1} mol^{-1}) [4 marks]

Solution:

Concept:

- A bomb calorimeter operates at constant volume. The heat of reaction measured under these conditions equals the internal-energy change, $q_V = \Delta U$, when only pressure-volume work is relevant.
- For a reaction involving ideal gases at temperature T, $\Delta H = \Delta U + \Delta n_g RT$. Here Δn_g is moles of gaseous products minus moles of gaseous reactants; pure liquids are omitted from this count.

Step 1: Assign the heat sign and count gas moles The combustion produces heat, so the reacting system loses energy. Thus $\Delta U = -726\,\text{kJ mol}^{-1}$, not a positive value. For $\text{CH}_3\text{OH}\,(\text{l}) + \frac{3}{2}\,\text{O}_2(\text{g}) \longrightarrow \text{CO}_2(\text{g}) + 2\,\text{H}_2\text{O}\,(\text{l})$, only oxygen and carbon dioxide contribute to the gas-mole correction. Methanol and the product water are liquids. Therefore

$$\Delta n_g = 1 - \frac{3}{2} = -\frac{1}{2}, \qquad T = 27 + 273 = 300\,\text{K}.$$

Step 2: Apply the enthalpy correction and round Convert the gas constant to kilojoule units before adding the correction: $R = 0.0083\,\text{kJ K}^{-1}\,\text{mol}^{-1}$. Hence

$$\Delta n_g RT = -\frac{1}{2}(0.0083)(300) = -1.245\,\text{kJ mol}^{-1}.$$

It follows that

$$\Delta H = -726 - 1.245 = -727.245\,\text{kJ mol}^{-1}.$$

The enthalpy is slightly more negative than the measured internal-energy change because the reaction reduces the number of gas moles. As the problem writes the result as $-x$, the positive magnitude is rounded only at the end.

Step 3: Conclusion $\Delta H = -727.245\,\text{kJ mol}^{-1}$, so $x = 727$ to the nearest integer.

Quick Tip to Solve: For bomb calorimetry start with ΔU, count only gases in Δn_g, and convert R to the same energy unit as the measured heat.

82. A 0.5 percent solution of potassium chloride was found to freeze at -0.24°C. The percentage dissociation of potassium chloride is _____. (Nearest integer)

(Molal depression constant for water is 1.80 K kg mol^{-1} and molar mass of KCl is 74.6 g mol^{-1})

[4 marks]

Solution:

Concept:

- For a mass-percent solution, 0.5 percent means $0.5\,\mathrm{g}$ of solute in $100\,\mathrm{g}$ of solution. The solvent mass is therefore $99.5\,\mathrm{g}$, not the total solution mass.
- Freezing-point depression obeys $\Delta T_f = iK_f m$, with molality m in moles per kilogram of solvent. For $\mathrm{KCl} \longrightarrow \mathrm{K^+} + \mathrm{Cl^-}$, each dissociated formula unit produces two ions, so $i = 1 + \alpha$, where α is the fraction dissociated.

Step 1: Calculate the molality on a mass basis Choose $100\,\mathrm{g}$ of solution, containing $0.5\,\mathrm{g}$ of potassium chloride and $99.5\,\mathrm{g} = 0.0995\,\mathrm{kg}$ of water. The amount of salt is

$$n = \frac{0.5}{74.6}\,\mathrm{mol}.$$

Hence the molality is

$$m = \frac{0.5/74.6}{0.0995} = 0.06736\,\mathrm{mol\,kg^{-1}}.$$

Keeping the solvent mass distinct from solution mass is necessary before the final nearest-integer rounding.

Step 2: Find the particle factor and dissociation The depression from pure water is $\Delta T_f = 0 - (-0.24) = 0.24\,\mathrm{K}$. Therefore

$$i = \frac{0.24}{1.80m} = \frac{0.24(74.6)(0.0995)}{1.80(0.5)} = 1.979386\ldots.$$

Since potassium chloride yields two ions,

$$\alpha = i - 1 = 0.979386\ldots, \qquad 100\alpha = 97.9386\ldots\%.$$

This value is less than complete dissociation and gives a van't Hoff factor below the limiting value 2, which provides a physical check.

Step 3: Conclusion The percentage dissociation of KCl is 97.94%, giving the required nearest-integer answer 98.

Quick Tip to Solve: For a mass-percent solution, subtract solute mass from solution mass before calculating molality; even a small difference can change the rounded dissociation percentage.

83. 50 mL of 0.1 M CH_3COOH is being titrated against 0.1 M NaOH. When 25 mL of NaOH has been added, the pH of the solution will be _____ $\times 10^{-2}$. (Nearest integer)

(Given : pK_a (CH_3COOH) = 4.76)

log 2 = 0.30

log 3 = 0.48

log 5 = 0.69

log 7 = 0.84

log 11 = 1.04

[4 marks]

Solution:

Concept:

- Acetic acid is a weak monoprotic acid, and hydroxide neutralises it in a 1:1 mole ratio: $CH_3COOH + OH^- \longrightarrow CH_3COO^- + H_2O$.
- After partial neutralisation, the remaining acid and its conjugate base form a buffer. The Henderson–Hasselbalch relation is $\mathrm{pH} = \mathrm{p}K_a + \log([CH_3COO^-]/[CH_3COOH])$; at half-neutralisation their concentrations are equal.

Step 1: Determine the acid and hydroxide amounts The initial acid amount and the added base amount are

$$n_{\text{acid}} = 0.050(0.1) = 0.0050\,\text{mol}, \qquad n_{\text{OH}^-} = 0.025(0.1) = 0.0025\,\text{mol}.$$

Hydroxide is consumed completely and neutralises exactly half the initial acid. Thus $0.0025\,\text{mol}$ of acetic acid remains and $0.0025\,\text{mol}$ of acetate is produced. This is the half-equivalence point, not the equivalence point, so a strong-base excess calculation is inappropriate.

Step 2: Apply the buffer ratio and requested scale Both species are present in the same total solution volume, $75\,\text{mL}$, so the concentration ratio equals their mole ratio:

$$[CH_3COO^-]/[CH_3COOH] = 0.0025/0.0025 = 1.$$

Therefore

$$\mathrm{pH} = 4.76 + \log 1 = 4.76.$$

No value from the supplied logarithm list is needed because the ratio is unity. The blank in the question multiplies 10^{-2}, so it is not asking for 4.76 directly. Writing the pH on the requested scale gives

$$4.76 = N \times 10^{-2}, \qquad N = 476.$$

Step 3: Conclusion The solution has $\mathrm{pH} = 4.76 = 476 \times 10^{-2}$. The required integer is 476.

Quick Tip to Solve: At half the equivalence volume for a weak acid and strong base, acid and conjugate base are equal, so $\mathrm{pH} = \mathrm{p}K_a$; then apply the scale printed beside the blank.

84. A flask is filled with equal moles of A and B. The half lives of A and B are 100 s and 50 s respectively and are independent of the initial concentration. The time required for the concentration of A to be four times that of B is _____ s.

(Given : ln 2 = 0.693) [4 marks]

Solution:

Concept:

- A half-life independent of initial concentration identifies first-order kinetics. For either species, $[X]_t = [X]_0 e^{-k_X t}$ and $k_X = \ln 2 / t_{1/2,X}$.
- The two species begin with equal mole amounts in the same flask, so their initial concentrations are equal. Their concentration ratio at time t is consequently determined by the difference in their first-order rate constants.

Step 1: Find the two rate constants Using the given half-lives,

$$k_A = \frac{0.693}{100} = 0.00693\,\mathrm{s}^{-1}, \qquad k_B = \frac{0.693}{50} = 0.01386\,\mathrm{s}^{-1}.$$

Species B decays faster because its half-life is shorter. Thus $[A]/[B]$ starts at 1 and increases with time, consistent with the requested ratio of 4. The relevant rate is the difference $k_B - k_A$, not either rate constant alone.

Step 2: Solve for the concentration ratio Dividing the two first-order laws and using equal initial concentrations gives

$$\frac{[A]_t}{[B]_t} = \frac{[A]_0 e^{-k_A t}}{[B]_0 e^{-k_B t}} = e^{(k_B - k_A)t}.$$

Set this ratio equal to four and take natural logarithms:

$$\ln 4 = (k_B - k_A)t, \qquad t = \frac{2(0.693)}{0.01386 - 0.00693} = 200\,\mathrm{s}.$$

As a direct check, after 200 s A has undergone two half-lives and retains one-quarter of its initial concentration, whereas B has undergone four half-lives and retains one-sixteenth. Their ratio is therefore $(1/4)/(1/16) = 4$, exactly as required.

Step 3: Conclusion The required time is $200\,\mathrm{s}$.

Quick Tip to Solve: For equal initial amounts undergoing separate first-order decays, take their concentration ratio first; its exponential contains $k_B - k_A$, which simplifies the calculation.

85. The spin-only magnetic moment value of the most basic oxide of vanadium among V_2O_3, V_2O_4 and V_2O_5 is _____ B.M. (Nearest integer) [4 marks]

Solution:

Concept:

- For oxides of the same transition metal, basic character generally decreases as the metal oxidation state increases: the low oxidation state favours ionic, basic oxides, whereas high oxidation states favour covalent, acidic oxides.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$, where n is the number of unpaired electrons of the metal ion.

Step 1: Identify the most basic oxide Taking oxygen as oxidation state -2, the vanadium oxidation states in V_2O_3, V_2O_4 and V_2O_5 are respectively $+3$, $+4$ and $+5$. The oxide with the lowest oxidation state, V_2O_3, is therefore the most basic of the three. This comparison must precede the magnetic calculation; using the highest oxide would select the wrong ion.

Step 2: Count the unpaired electrons of the selected ion Neutral vanadium has atomic number 23 and configuration $[\text{Ar}]3d^3 4s^2$. Remove the two $4s$ electrons before removing a $3d$ electron. Thus V^{3+} has configuration $[\text{Ar}]3d^2$. The two electrons occupy separate available d orbitals with parallel spins, so $n = 2$. Neither the neutral atom configuration nor the d^0 configuration of V^{5+} describes the ion in the chosen oxide.

Step 3: Calculate and round the moment Insert the unpaired-electron count into the spin-only expression:

$$\mu = \sqrt{2(2+2)} = \sqrt{8} = 2.828\ldots\ \text{B.M.}$$

This is closer to 3 than to 2. Round the magnetic moment only at the final stage, as the question asks for the nearest integer.

Step 4: Conclusion The most basic oxide is V_2O_3, and its vanadium ion has spin-only moment $\sqrt{8}\,\text{B.M.}$. The required nearest-integer value is $\boxed{3}\,\text{B.M.}$.

Quick Tip to Solve: First use the oxide oxidation state to select the ion, then count its unpaired electrons; the most basic vanadium oxide here contains the lowest oxidation state.

86. The spin-only magnetic moment value of an octahedral complex among $CoCl_3.4NH_3$, $NiCl_2.6H_2O$ and $PtCl_4.2HCl$, which upon reaction with excess of $AgNO_3$ gives 2 moles of AgCl is _____ B.M. (Nearest integer) [4 marks]

Solution:

Concept:

- Excess aqueous $AgNO_3$ precipitates chloride ions outside the coordination sphere as $AgCl$. Chloride ligands bound inside a stable coordination sphere are not counted as immediately ionisable chloride.
- For an octahedral d^8 ion the occupation is $t_{2g}^6 e_g^2$, with two unpaired electrons. The spin-only moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$.

Step 1: Choose the compound with two ionisable chlorides The octahedral coordination formulations

are $[Co(NH_3)_4Cl_2]Cl$, $[Ni(H_2O)_6]Cl_2$, and $H_2[PtCl_6]$. Their numbers of chloride counterions are respectively 1, 2 and 0. Consequently, the nickel compound gives two moles of silver chloride per mole of compound:

$$[Ni(H_2O)_6]Cl_2 + 2\,AgNO_3 \longrightarrow [Ni(H_2O)_6](NO_3)_2 + 2\,AgCl\downarrow$$

The cobalt compound gives only one mole in this test, while the platinum compound has all six chlorides coordinated.

Step 2: Find the nickel electron configuration Water is neutral and the two chloride counterions contribute charge -2, so the complex cation contains Ni^{2+}. Neutral nickel is $[Ar]3d^8 4s^2$; loss of the two $4s$ electrons gives $[Ar]3d^8$. In an octahedral field six electrons fill t_{2g}, and the remaining two occupy the two e_g orbitals singly. Thus there are two unpaired electrons, not eight.

Step 3: Evaluate the spin-only moment With $n = 2$,

$$\mu = \sqrt{2(2+2)} = \sqrt{8} = 2.828\ldots \text{ B.M.}$$

The requested answer is the nearest integer, so this is rounded upward to 3. The precipitation criterion identifies the complex before its magnetic moment is calculated.

Step 4: Conclusion The selected octahedral complex is $[Ni(H_2O)_6]Cl_2$. Its nearest-integer spin-only magnetic moment is $\boxed{3}$ B.M..

Quick Tip to Solve: Count chloride counterions, not total chlorine atoms, when applying the silver-nitrate test; then calculate the moment of the selected metal ion.

87. On complete combustion 0.30 g of an organic compound gave 0.20 g of carbon dioxide and 0.10 g of water. The percentage of carbon in the given organic compound is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- In complete combustion, all carbon atoms in the organic sample are converted into CO_2. Each mole of carbon dioxide contains one mole of carbon atoms.
- The carbon mass fraction of CO_2 is $12/44$. Elemental percentage is the mass of that element divided by the original sample mass, multiplied by 100.

Step 1: Extract the carbon mass from carbon dioxide The measured carbon dioxide mass is $0.20\,g$. Carbon contributes $12\,g$ in every $44\,g$ of carbon dioxide, so

$$m_C = 0.20 \times \frac{12}{44} = 0.054545\ldots \text{ g.}$$

The entire $0.20\,g$ is not carbon: most of its mass belongs to oxygen added during combustion. Therefore directly dividing 0.20 by the sample mass would overestimate the carbon percentage.

Step 2: Compare with the original sample mass The original organic compound weighed $0.30\,g$, so

this is the denominator in the percentage calculation:

$$\%\mathrm{C} = \frac{0.054545\ldots}{0.30} \times 100 = 18.1818\ldots\%.$$

Equivalently, combine the conversion and percentage operations as $(0.20 \times 12 \times 100)/(44 \times 0.30)$. The water mass is relevant to hydrogen analysis, but it does not change the carbon recovered as carbon dioxide.

Step 3: Round the requested percentage The value $18.1818\ldots$ lies nearer to 18 than to 19. Hence report the nearest integer after evaluating the full expression. As a consistency check, the recovered carbon mass is much smaller than the sample mass, and the resulting percentage is between zero and one hundred, as required for a mass percentage.

Step 4: Conclusion The percentage of carbon is $18.1818\ldots\%$, which to the nearest integer is $\boxed{18}\%$.

Quick Tip to Solve: For combustion carbon analysis, multiply the carbon dioxide mass by $12/44$ before dividing by the sample mass; water is unnecessary for the carbon calculation.

88. Compound 'P' on nitration with dil. HNO_3 yields two isomers (A) and (B). These isomers can be separated by steam distillation. Isomers (A) and (B) show the intramolecular and intermolecular hydrogen bonding respectively. Compound (P) on reaction with conc. HNO_3 yields a yellow compound 'C', a strong acid. The number of oxygen atoms is present in compound 'C' _____. [4 marks]

Solution:

Concept:

- The hydroxyl group of phenol activates its aromatic ring and directs nitration to the ortho and para positions. Dilute nitric acid therefore gives ortho-nitrophenol and para-nitrophenol.
- Ortho-nitrophenol can form intramolecular hydrogen bonding, whereas para-nitrophenol forms intermolecular hydrogen bonding. Strong nitration of phenol produces 2,4,6-trinitrophenol, also called picric acid.

Step 1: Identify the starting phenol from the isomers The two products described are o-nitrophenol (A) and p-nitrophenol (B), so compound P is phenol, $\mathrm{C_6H_5OH}$. In the ortho isomer the adjacent nitro and hydroxyl groups allow hydrogen bonding within one molecule. In the para isomer their separation prevents this internal bonding, so molecules associate by intermolecular hydrogen bonding. These distinct association patterns explain why steam distillation separates the two nitration products.

Step 2: Identify the yellow strongly acidic product Concentrated nitric acid introduces nitro groups at all three activated positions, namely 2, 4 and 6 relative to the phenolic hydroxyl group. Product C is therefore picric acid:

$$\mathrm{C_6H_5OH} + 3\,\mathrm{HNO_3} \longrightarrow \mathrm{C_6H_2(NO_2)_3OH} + 3\,\mathrm{H_2O}.$$

The three strongly electron-withdrawing nitro groups stabilise its conjugate base, explaining the strong acidity mentioned in the question and confirming the identification.

Step 3: Count oxygen atoms in the product There are three NO_2 groups, each contributing two oxygen atoms, and one retained phenolic OH group contributing one oxygen atom. Thus

$$N_O = 3 \times 2 + 1 = 7.$$

Counting only the nitro oxygens would miss the oxygen already present in phenol. The formula $C_6H_3N_3O_7$ independently displays the same oxygen total.

Step 4: Conclusion Compound C is 2,4,6-trinitrophenol (picric acid), containing $\boxed{7}$ oxygen atoms per molecule.

Quick Tip to Solve: After identifying picric acid, count one oxygen from the original hydroxyl group as well as six from its three nitro groups.

89. The number of oxygens present in a nucleotide formed from a base, that is present only in RNA is _____. [4 marks]

Solution:

Concept:

- The nitrogenous base present in RNA but absent from DNA is uracil. Its nucleotide consists of uracil, ribose and one phosphate group; the corresponding nucleoside is uridine.
- Formation of the glycosidic bond and the phosphate ester involves condensation. Count oxygens in the final nucleotide, rather than simply adding those in the unreacted components.

Step 1: Identify the nucleotide components Uracil is the RNA-specific base; adenine, guanine and cytosine occur in both RNA and DNA, while thymine is characteristic of DNA. An RNA nucleotide formed from uracil is uridine monophosphate. Uracil contributes two carbonyl oxygen atoms. Ribose has molecular formula $C_5H_{10}O_5$, and phosphoric acid has formula H_3PO_4. The nucleotide contains one sugar and one phosphate group, not a nucleotide triphosphate.

Step 2: Account for the condensation steps Joining uracil to ribose through the N-glycosidic bond eliminates one molecule of water. Uridine therefore contains $2 + 5 - 1 = 6$ oxygen atoms. Esterification of a ribose hydroxyl group with phosphoric acid removes one further water molecule. Adding phosphate to the nucleoside consequently adds $4 - 1 = 3$ oxygen atoms. The total is

$$N_O = 2 + 5 + 4 - 2 = 9.$$

Both waters must be accounted for when starting from the three separate components.

Step 3: Check against the molecular formula Adding $C_4H_4N_2O_2$, $C_5H_{10}O_5$ and H_3PO_4, then subtracting $2H_2O$, gives $C_9H_{13}N_2O_9P$. Its oxygen subscript is 9. This also distinguishes the nucleotide from uridine alone, which has six oxygens and lacks phosphorus; confusing nucleoside and nucleotide would therefore give the wrong count.

Step 4: Conclusion The uracil-containing RNA nucleotide, uridine monophosphate, has $\boxed{9}$ oxygen atoms.

Quick Tip to Solve: Uracil contributes two oxygens; making uridine removes one sugar oxygen, and adding a phosphate ester contributes three more oxygens overall.

90. 2.0 g of H_2 gas is adsorbed on 2.5 g of platinum powder at 300 K and 1 bar pressure. The volume of the gas adsorbed per gram of the adsorbent is _____ mL.

(Given : R = 0.083 L bar K^{-1} mol^{-1}) [4 marks]

Solution:

Concept:

- Use the ideal-gas equation $PV = nRT$, with pressure in bar and volume in litres when $R = 0.083\,\mathrm{L\,bar\,K^{-1}\,mol^{-1}}$. The amount of gas is $n = m/M$.
- The requested specific adsorption volume is the volume of adsorbed gas divided by the mass of platinum adsorbent. Convert litres to millilitres using $1\,\mathrm{L} = 1000\,\mathrm{mL}$.

Step 1: Convert the hydrogen mass into moles The adsorbed gas is molecular hydrogen, H_2, whose molar mass is $2.0\,\mathrm{g\,mol^{-1}}$ for the given data. Therefore

$$n = \frac{2.0\,\mathrm{g}}{2.0\,\mathrm{g\,mol^{-1}}} = 1.0\,\mathrm{mol}.$$

Use the molecular molar mass, not the atomic mass of hydrogen, because the supplied mass refers to hydrogen gas. This determines the total amount adsorbed on the entire platinum sample.

Step 2: Calculate the gas volume at the stated conditions At $T = 300\,\mathrm{K}$ and $P = 1\,\mathrm{bar}$, the equivalent gas volume is

$$V = \frac{nRT}{P} = \frac{1.0 \times 0.083 \times 300}{1} = 24.9\,\mathrm{L}.$$

The pressure is already compatible with the supplied gas constant, so no atmosphere-to-bar conversion is needed. Also, 300 K is already absolute temperature and must not have 273 added to it.

Step 3: Divide by the adsorbent mass and convert units This volume is adsorbed on $2.5\,\mathrm{g}$ of platinum. Thus the volume per gram is

$$\frac{24.9}{2.5} = 9.96\,\mathrm{L\,g^{-1}} = 9960\,\mathrm{mL\,g^{-1}}.$$

Dividing by the hydrogen mass instead would answer a different question: the denominator must be the mass of the solid adsorbent. Multiplication by 1000 changes litres into the requested millilitres without changing the amount of adsorbed gas.

Step 4: Conclusion The volume of gas adsorbed per gram of platinum is $\boxed{9960}\,\mathrm{mL\,g^{-1}}$; the value to enter is 9960.

Quick Tip to Solve: Compute the total gas volume first, then divide by platinum mass and multiply by 1000; keep bar with the supplied litre-bar gas constant.