JEE Main 2022 Paper 1 (B.E./B.Tech) Jun 27 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 1 (June), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 27 June 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The area of the polygon, whose vertices are the non-real roots of the equation $\overline{z} = iz^2$ is : [4 marks]

(A) $\frac{3\sqrt{3}}{4}$

(B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{3}{2}$

(D) $\frac{3}{4}$

Correct Answer: (A) $\frac{3\sqrt{3}}{4}$

Solution:

Concept:

- Write a nonzero complex number as $z = re^{i\theta}$, where $r > 0$. Its conjugate has the same modulus and argument $-\theta$.
- Three points equally spaced on a circle form an equilateral triangle. Its area is $\sqrt{3}s^2/4$, where s is its side length.

Step 1: Find the nonzero roots Taking moduli of $\overline{z} = iz^2$ gives $r = r^2$. The root $z = 0$ is real and is excluded. Thus every required root has $r = 1$. Comparing arguments modulo 2π,

$$-\theta \equiv \frac{\pi}{2} + 2\theta \pmod{2\pi}.$$

Hence the three distinct arguments can be taken as $-\pi/6, \pi/2, 7\pi/6$. The roots are $\sqrt{3}/2 - i/2$, i, and $-\sqrt{3}/2 - i/2$; all are non-real.

Step 2: Calculate the triangle area The last and first roots have the same imaginary part, so the horizontal base has length $\sqrt{3}$. The top vertex has imaginary part 1, whereas the base has imaginary part $-1/2$, giving height $3/2$. Thus

$$A = \frac{1}{2} \times \sqrt{3} \times \frac{3}{2} = \frac{3\sqrt{3}}{4}.$$

This also equals the equilateral-triangle formula with side $\sqrt{3}$. Option (B) doubles this area, while (C) and (D) omit the required irrational factor from the base length.

Step 3: Conclusion The polygon is an equilateral triangle of area $\boxed{3\sqrt{3}/4}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: Use moduli first to fix the circle, then compare arguments to locate the vertices; do not include the real root zero.

2. Let the system of linear equations

$x + 2y + z = 2$,

$\alpha x + 3y - z = \alpha$,

$$-\alpha x + y + 2z = -\alpha$$

be inconsistent. Then α is equal to : [4 marks]

(A) $\frac{5}{2}$

(B) $-\frac{5}{2}$

(C) $\frac{7}{2}$

(D) $-\frac{7}{2}$

Correct Answer: (D) $-\frac{7}{2}$

Solution:

Concept:

- A linear system is inconsistent when elimination produces a false equation such as $0 = k$ with $k \neq 0$. A zero coefficient determinant alone is only a necessary condition.
- Adding equations with opposite coefficients of x can eliminate the parameter-dependent terms directly.

Step 1: Eliminate two variables Add the second and third equations. The αx terms and the constants cancel, giving

$$4y + z = 0, \qquad z = -4y.$$

Substitute this in the first equation:

$$x + 2y - 4y = 2, \qquad x = 2 + 2y.$$

These two relations hold for every possible value of α, with no division by the parameter, so they cannot discard an exceptional case.

Step 2: Identify the contradictory parameter Substitute $x = 2 + 2y$ and $z = -4y$ in the second equation:

$$\alpha(2 + 2y) + 3y + 4y = \alpha,$$

$$(2\alpha + 7)y = -\alpha.$$

If $2\alpha + 7 \neq 0$, this determines y, and then x, z; hence the system is consistent. If $\alpha = -7/2$, it becomes $0 = 7/2$, a contradiction. The third equation is already enforced by the sum relation, so there is no further exceptional case. The values $5/2, -5/2, 7/2$ give nonzero coefficients of y, ruling out (A), (B) and (C).

Step 3: Conclusion The inconsistent system requires $\boxed{\alpha = -7/2}$. Hence, the correct option is (D).

Quick Tip to Solve: After finding a parameter that makes a coefficient vanish, check the right-hand side: a nonzero remainder proves inconsistency.

3. $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$, **where a, b, c are in A.P. and |a| < 1, |b| < 1, |c| < 1, abc ≠ 0, then :** [4 marks]

(A) x, y, z are in A.P.

(B) x, y, z are in G.P.

(C) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

(D) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

Correct Answer: (C) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

Solution:

Concept:

- For $|r| < 1$, the infinite geometric sum beginning with 1 is $\sum_{n=0}^{\infty} r^n = 1/(1-r)$.
- Three numbers u, v, w are in arithmetic progression exactly when $u + w = 2v$. Here $a + c = 2b$.

Step 1: Evaluate and invert the three sums The convergence conditions allow the geometric-sum formula for each ratio separately:

$$x = \frac{1}{1-a}, \qquad y = \frac{1}{1-b}, \qquad z = \frac{1}{1-c}.$$

None of these values is zero. Their reciprocals are therefore

$$\frac{1}{x} = 1 - a, \qquad \frac{1}{y} = 1 - b, \qquad \frac{1}{z} = 1 - c.$$

Taking a reciprocal has converted the sums into linear expressions in the given A.P. terms.

Step 2: Test the progression and the alternatives Since $a + c = 2b$,

$$\frac{1}{x} + \frac{1}{z} = 2 - (a + c) = 2 - 2b = \frac{2}{y}.$$

Thus the reciprocals are in A.P. The original sums need not be in A.P. or G.P.: for $a = 1/4, b = 1/2, c = 3/4$, they are $4/3, 2, 4$. Here $4/3 + 4 \neq 4$ and $(4/3)4 \neq 2^2$, disproving the general claims in (A) and (B). Also, the actual reciprocal sum is $3 - (a + b + c)$, not $1 - (a + b + c)$, so (D) is false.

Step 3: Conclusion The numbers $\boxed{1/x, 1/y, 1/z}$ are in arithmetic progression. Hence, the correct option is (C).

Quick Tip to Solve: When geometric sums are formed from A.P. ratios, take their reciprocals to recover affine expressions in those ratios.

4. Let a be an integer such that $\lim_{x\to 7} \frac{18-[1-x]}{[x-3a]}$ exists, where [t] is greatest integer $\leq$ t. Then a is equal to : [4 marks]

(A) -6

(B) -2

(C) 2

(D) 6

Correct Answer: (A) -6

Solution:

Concept:

- The greatest integer function $[t]$ is constant between consecutive integers and changes at integers. Its left and right values must be checked separately.
- A finite two-sided limit exists only when its two one-sided limits agree and the expressions are defined in a punctured neighbourhood.

Step 1: Evaluate the two one-sided expressions Put $x = 7 - h$ with $0 < h < 1$. Then $1 - x = -6 + h$, so $[1 - x] = -6$; because a is an integer, $[x - 3a] = 6 - 3a$. From the right, put $x = 7 + h$. Now $[1 - x] = -7$ and $[x - 3a] = 7 - 3a$. Therefore

$$L_- = \frac{24}{6 - 3a}, \qquad L_+ = \frac{25}{7 - 3a}.$$

In particular $a = 2$ gives a zero denominator throughout a left-hand interval and cannot provide the required limit.

Step 2: Equate the limits and check the result For an admissible a, equality of the one-sided limits gives

$$24(7 - 3a) = 25(6 - 3a),$$

$$168 - 72a = 150 - 75a, \qquad 3a = -18.$$

Thus $a = -6$. The denominators then become 24 and 25, respectively, and both one-sided limits equal 1. The value at $x = 7$ is irrelevant to existence of the limit. The equation has a unique solution, so the remaining candidate values -2 and 6 cannot satisfy it.

Step 3: Conclusion The required integer is $\boxed{a = -6}$, and the limit is 1. Hence, the correct option is (A).

Quick Tip to Solve: For a floor-function limit at an integer, write $x = 7 - h$ and $x = 7 + h$; substituting the endpoint alone misses the jumps.

5. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is : [4 marks]

(A) 4

(B) 2

(C) 1

(D) 0

Correct Answer: (B) 2

Solution:

Concept:

- For a polynomial, derivative signs determine intervals of strict monotonicity. A strictly monotone interval contains at most one root.
- A continuous function that changes sign on an interval has a root there by the intermediate value theorem.

Step 1: Locate the unique turning point Let $f(x) = x^4 - 4x + 1$. Its derivative factors as

$$f\prime(x) = 4x^3 - 4 = 4(x-1)(x^2 + x + 1).$$

The quadratic factor is always positive, since $x^2 + x + 1 = (x + 1/2)^2 + 3/4 > 0$. Hence f is strictly decreasing for $x < 1$, and strictly increasing for $x > 1$. Its only minimum is at $x = 1$, where $f(1) = -2$. This point is not itself a root.

Step 2: Count the crossings on each side The values $f(0) = 1$, $f(1) = -2$, and $f(2) = 9$ show a sign change in both $(0, 1)$ and $(1, 2)$. Continuity gives at least one root in each interval. Strict decrease on $(-\infty, 1)$ and strict increase on $(1, \infty)$ show that each of these intervals contains at most one root. Thus these are exactly two distinct real roots. A quartic need not have four real roots, so its degree does not establish option (A). The two sign changes exclude the counts 1 and 0 in (C) and (D).

Step 3: Conclusion The equation has $\boxed{2}$ distinct real roots. Hence, the correct option is (B).

Quick Tip to Solve: Combine derivative signs with a few function values: monotonicity gives the upper bound on roots and sign changes give existence.

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6. If $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$, $|y| < 2$, then : [4 marks]

(A) $x^2y'' + xy' - 25y = 0$

(B) $x^2y'' - xy' - 25y = 0$

(C) $x^2y'' - xy' + 25y = 0$

(D) $x^2y'' + xy' + 25y = 0$

Correct Answer: (D) $x^2y'' + xy' + 25y = 0$

Solution:

Concept:

- If $u = \log x$, then $d/dx = (1/x)d/du$. Trigonometric functions of $\log x$ lead to Euler-type equations.

- The logarithm in the question requires $x > 0$. Taking cosine of $\cos^{-1}(y/2) = t$ gives $y = 2\cos t$ on the stated inverse-cosine branch.

Step 1: Express the dependent variable explicitly Write

$$t = \log_e \left(\frac{x}{5}\right)^5 = 5\log\left(\frac{x}{5}\right).$$

Since the original left side is an inverse cosine, its principal value restricts $0 < t < \pi$ when $|y| < 2$. On this interval,

$$y = 2\cos t, \qquad \frac{dt}{dx} = \frac{5}{x}, \qquad y\prime = -\frac{10\sin t}{x}.$$

The chain-rule factor 5 is essential; it produces the coefficient 25 after the second differentiation.

Step 2: Differentiate and eliminate the angle Using the product rule once more,

$$y\prime\prime = \frac{10\sin t}{x^2} - \frac{50\cos t}{x^2}.$$

Consequently

$$x^2y\prime\prime + xy\prime = (10\sin t - 50\cos t) - 10\sin t = -50\cos t.$$

Since $y = 2\cos t$, the last expression is $-25y$. Thus the required equation has both a positive $xy\prime$ term and a positive $25y$ term after everything is placed on the left. Options (B) and (C) have the wrong first-derivative sign; (A) has the wrong sign on the final term.

Step 3: Conclusion The identity is $\boxed{x^2y\prime\prime + xy\prime + 25y = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: For $\cos(k\log x)$, use the operator $x\,d/dx$: applying it twice gives $-k^2$ times the original function.

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7. If $\int \frac{(x^2+1)e^x}{(x+1)^2}dx = f(x)e^x + C$, where C is a constant, then $\frac{d^3f}{dx^3}$ at x = 1 is equal to : [4 marks]

(A) $-\frac{3}{4}$

(B) $\frac{3}{4}$

(C) $-\frac{3}{2}$

(D) $\frac{3}{2}$

Correct Answer: (B) $\frac{3}{4}$

Solution:

Concept:

- The derivative $(e^x f(x))\prime = e^x(f + f\prime)$ lets us recognize integrands containing an exponential times a rational function.
- For $(x+1)^{-1}$, each differentiation decreases the exponent by one and multiplies by the old exponent. Work away from $x = -1$.

Step 1: Recognize the rational factor Differentiate the asserted antiderivative to obtain

$$f(x) + f\prime(x) = \frac{x^2+1}{(x+1)^2}.$$

Try $f(x) = (x-1)/(x+1) = 1 - 2/(x+1)$. Its derivative is $2/(x+1)^2$, and direct combination verifies

$$f + f\prime = \frac{(x-1)(x+1)+2}{(x+1)^2} = \frac{x^2+1}{(x+1)^2}.$$

Therefore the antiderivative is $e^x(x-1)/(x+1) + C$. The arbitrary additive integration constant is already represented by C.

Step 2: Differentiate the factor three times Use the simplified form $f(x) = 1 - 2(x+1)^{-1}$:

$$f\prime(x) = \frac{2}{(x+1)^2}, \qquad f\prime\prime(x) = -\frac{4}{(x+1)^3},$$

$$f\prime\prime\prime(x) = \frac{12}{(x+1)^4}.$$

At $x = 1$, this gives $12/2^4 = 12/16 = 3/4$. The third derivative is positive here, excluding (A) and (C). Option (D) doubles the value because it effectively misses one factor of 2 in the fourth power of the denominator.

Step 3: Conclusion The required value is $\boxed{f\prime\prime\prime(1) = 3/4}$. Hence, the correct option is (B).

Quick Tip to Solve: Before integrating an exponential times a rational function, test whether the rational factor is $f + f\prime$; then differentiate the simplified f.

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8. The value of the integral $\int_{-2}^{2} \frac{|x^3+x|}{(e^{x|x|}+1)}dx$ is equal to : [4 marks]

(A) $5e^2$

(B) $3e^{-2}$

(C) 4

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- For an integral over a symmetric interval, pair the contributions at $x = t$ and $x = -t$. Here $x^3 + x = x(x^2+1)$ has the sign of x, so its absolute value becomes $t^3 + t$ in both halves.
- The identity $\frac{1}{1+e^u} + \frac{1}{1+e^{-u}} = 1$ removes complementary exponential denominators. It follows by multiplying the second fraction by e^u/e^u.

Step 1: Split the integral at zero Write the required integral as I. On the positive half put $x = t$; on the negative half put $x = -t$, reversing the limits to absorb $dx = -dt$. For $t \geq 0$, the two numerators are both $t^3 + t$, while $x|x|$ equals t^2 and $-t^2$, respectively. Therefore

$$I = \int_0^2 (t^3 + t)\left(\frac{1}{1 + e^{t^2}} + \frac{1}{1 + e^{-t^2}}\right) dt.$$

The absolute value is essential: replacing the numerator by an odd function would incorrectly suggest cancellation.

Step 2: Combine the paired fractions and integrate Apply the complementary-denominator identity with $u = t^2$. The bracket is exactly one at every point of the interval, including $t = 0$. Thus

$$I = \int_0^2 (t^3 + t)\, dt = \left[\frac{t^4}{4} + \frac{t^2}{2}\right]_0^2 = 4 + 2 = 6.$$

There is no surviving exponential endpoint term, so options (A) and (B) do not follow from the paired integrand. Option (C), namely 4, counts only the cubic contribution and misses the integral of t.

Step 3: Conclusion The given definite integral equals 6. Hence, the correct option is (D).

Quick Tip to Solve: With $e^{x|x|}$ on a symmetric interval, pair positive and negative inputs before attempting any exponential substitution.

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9. If $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0$, x, y > 0, y(1) = 1, then y(2) is equal to : [4 marks]

(A) $2 + \log_2 3$

(B) $2 + \log_3 2$

(C) $2 - \log_3 2$

(D) $2 - \log_2 3$

Correct Answer: (D) $2 - \log_2 3$

Solution:

Concept:

- An equation involving 2^y can be simplified by setting $v = 2^y$, for which $dv/dx = (\ln 2)2^y\, dy/dx$. The given positivity makes the logarithms used below well defined.
- After separation, use $\int 2^x/(2^x - 1)\, dx = \ln(2^x - 1)/\ln 2$. The initial condition determines the integration constant; it must be used before finding the requested value.

Step 1: Separate the transformed equation First rewrite $2^{x-y}(2^y - 1) = 2^x(1 - 2^{-y})$. Multiplication by $(\ln 2)2^y$ gives

$$\frac{dv}{dx} = -\frac{(\ln 2)2^x}{2^x - 1}(v - 1).$$

Since $y > 0$, we have $v - 1 > 0$, so division by this factor is permitted. Separating and integrating yields

$$\ln(v - 1) = -\ln(2^x - 1) + C.$$

Equivalently, $(2^y - 1)(2^x - 1) = K$, with a positive constant K.

Step 2: Apply the condition and evaluate At $x = 1$, the prescribed value $y = 1$ gives $K = (2 - 1)(2 - 1) = 1$. At $x = 2$, therefore,

$$3(2^{y(2)} - 1) = 1, \qquad 2^{y(2)} = \frac{4}{3}.$$

Taking logarithms to base 2 gives $y(2) = \log_2(4/3) = 2 - \log_2 3$. The differential equation has negative derivative throughout $x, y > 0$, so the answer must be below 1. Options (A) and (B) exceed 2; option (C) is also above 1 since $\log_3 2 < 1$. Only (D) meets both the exact equation and this monotonicity check.

Step 3: Conclusion $y(2) = 2 - \log_2 3$. Hence, the correct option is (D).

Quick Tip to Solve: Use $v = 2^y$ to expose separation, and check that the resulting value decreases from the initial value when the derivative is negative.

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10. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is 2x + y = 4. Let the point B lie on the line x + 3y = 7. If (α, β) is the centroid of ΔABC, then 15(α + β) is equal to : [4 marks]

(A) 39

(B) 41

(C) 51

(D) 63

Correct Answer: (C) 51

Solution:

Concept:

- In an isosceles triangle, the perpendicular from the vertex to the base also bisects the base. Thus the foot M from A to BC is the midpoint of B and C.
- The foot of (x_0, y_0) on $ax + by + d = 0$ is $(x_0 - aT, y_0 - bT)$, where $T = (ax_0 + by_0 + d)/(a^2 + b^2)$. The centroid is $G = (A + B + C)/3 = (A + 2M)/3$.

Step 1: Find the midpoint of the base The base is $2x + y - 4 = 0$, and the vertex is $A = (6, 1)$. Its line-expression value is $2(6) + 1 - 4 = 9$, while the squared normal length is $2^2 + 1^2 = 5$. Hence the perpendicular foot is

$$M = \left(6 - \frac{18}{5}, 1 - \frac{9}{5}\right) = \left(\frac{12}{5}, -\frac{4}{5}\right).$$

Substitution gives $2(12/5) - 4/5 = 4$, checking that the point lies on the base. The isosceles condition is what makes this foot its midpoint.

Step 2: Compute the centroid and requested expression Using $B + C = 2M$, we obtain

$$(\alpha, \beta) = \frac{1}{3}\left((6, 1) + \left(\frac{24}{5}, -\frac{8}{5}\right)\right) = \left(\frac{18}{5}, -\frac{1}{5}\right).$$

Therefore $15(\alpha+\beta) = 15(17/5) = 51$. The additional line determines B uniquely as $(1, 2)$; reflecting it in M gives $C = (19/5, -18/5)$, whose direct coordinate average gives the same centroid. Thus no endpoint assumption has been introduced. Options 39, 41 and 63 disagree with this checked coordinate sum.

Step 3: Conclusion $15(\alpha+\beta) = 51$. Hence, the correct option is (C).

Quick Tip to Solve: For the centroid of an isosceles triangle, project the vertex onto the base and use $G = (A + 2M)/3$; finding both endpoints is optional.

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11. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$, be $\frac{1}{4}$. If this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then $a^2 + b^2$ is equal to : [4 marks]

(A) 29

(B) 31

(C) 32

(D) 34

Correct Answer: (B) 31

Solution:

Concept:

- For an ellipse with major semiaxis $a > b$, eccentricity satisfies $e^2 = 1 - b^2/a^2$. Squaring the given eccentricity is essential when finding the semiaxis ratio.
- A point on the ellipse satisfies $x^2/a^2 + y^2/b^2 = 1$. Combining this point condition with the semiaxis ratio determines both squared semiaxes without taking unnecessary square roots.

Step 1: Express one squared semiaxis in terms of the other Use $e = 1/4$ in the eccentricity relation:

$$\frac{b^2}{a^2} = 1 - \left(\frac{1}{4}\right)^2 = \frac{15}{16}, \qquad b^2 = \frac{15}{16}a^2.$$

This ratio is less than one, consistent with the stated major axis along the x-direction. Replacing e^2 by e would give a different and incorrect ratio. The point has $x^2 = 16(2/5) = 32/5$ and $y^2 = 9$; the negative sign of x disappears on squaring.

Step 2: Substitute the point and check the values The point condition becomes

$$\frac{32}{5a^2} + \frac{9 \cdot 16}{15a^2} = 1.$$

The two numerator contributions are $32/5$ and $48/5$, so $16/a^2 = 1$. Thus $a^2 = 16$ and $b^2 = 15$. As a check, the original point gives $(32/5)/16 + 9/15 = 2/5 + 3/5 = 1$, and $\sqrt{1 - 15/16} = 1/4$, verifying both supplied conditions. Consequently the sum is 31. The other options 29, 32 and 34 cannot satisfy these uniquely determined squared semiaxes.

Step 3: Conclusion $a^2 + b^2 = 16 + 15 = 31$. Hence, the correct option is (B).

Quick Tip to Solve: Find $b^2/a^2 = 1 - e^2$ first, then substitute the given point; work entirely with squared semiaxes until the requested sum is found.

12. If two straight lines whose direction cosines are given by the relations $l+m-n=0$, $3l^2+m^2+cnl=0$ are parallel, then the positive value of c is : [4 marks]

(A) 6

(B) 4

(C) 3

(D) 2

Correct Answer: (A) 6

Solution:

Concept:

- The direction cosines (l, m, n) of a line specify its direction up to simultaneous reversal. Parallel lines have proportional direction triples, so the two direction ratios from the given homogeneous equations must coincide.
- Eliminate n with $n = l + m$. The resulting quadratic in m/l must have a repeated real root for the two directions to be parallel; its discriminant must therefore vanish.

Step 1: Reduce the equations to a direction-ratio quadratic From the linear relation, $n = l + m$. Substituting into the quadratic gives

$$m^2 + clm + (3 + c)l^2 = 0.$$

We may divide by l^2: if $l = 0$, this equation forces $m = 0$, and then the linear relation forces $n = 0$, contradicting $l^2 + m^2 + n^2 = 1$. Thus with $t = m/l$, the possible directions are governed by

$$t^2 + ct + (3 + c) = 0.$$

Each root gives one proportional triple $(1, t, 1 + t)$, which can then be normalized.

Step 2: Enforce coincident directions and choose the positive value Parallelism requires the two roots to coincide. Therefore

$$c^2 - 4(3 + c) = 0, \qquad c^2 - 4c - 12 = (c - 6)(c + 2) = 0.$$

The admissible values are $c = 6$ and $c = -2$; the positive one is 6. At $c = 6$, the quadratic is $(t + 3)^2 = 0$, giving the single direction ratio $(1, -3, -2)$, so the parallel condition is explicitly verified. For the other listed positive values 4, 3 and 2 the discriminant is not zero, so they cannot produce the required repeated direction.

Step 3: Conclusion The positive value of c is 6. Hence, the correct option is (A).

Quick Tip to Solve: For two directions defined by a linear and a quadratic homogeneous relation, eliminate one cosine and impose a zero discriminant for parallelism.

13. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in$ {1, 2, ..., 10} is : [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (A) 0

Solution:

Concept:

- A cross product $\vec{b} \times \vec{c}$ is perpendicular to $\vec{c}$, regardless of $\vec{b}$. Consequently $(\vec{b} \times \vec{c}) \cdot \vec{c} = 0$ is a necessary condition for any prescribed cross product.
- Before applying an additional magnitude restriction, test whether the vector equation itself has a solution. If the prescribed vector is not perpendicular to $\vec{c}$, the solution set is empty for all magnitudes.

Step 1: Test the necessary perpendicularity condition If $\vec{b} \times \vec{c} = \vec{a}$, taking the scalar product with $\vec{c}$ on both sides gives $\vec{a} \cdot \vec{c} = 0$. Here the coordinates are $\vec{a} = (1, 1, -1)$ and $\vec{c} = (2, -3, 2)$, so

$$\vec{a} \cdot \vec{c} = (1)(2) + (1)(-3) + (-1)(2) = 2 - 3 - 2 = -3.$$

This nonzero value contradicts the compulsory orthogonality of a cross product. In particular, the negative k-component of $\vec{a}$ must be retained in this calculation.

Step 2: Apply the magnitude restriction to the empty solution set Because the vector equation already has no solution, there is no vector $\vec{b}$ of any length that satisfies it. Restricting $|\vec{b}|$ to the integers $1, 2, \ldots, 10$ cannot introduce a solution, and hence the number remains zero. As an alternative check, the component equations for $\vec{b} = (p, q, r)$ would be $2q + 3r = 1$, $2r - 2p = 1$, and $-3p - 2q = -1$. Combining them would force the same impossible scalar-product condition. Options 1, 2 and 3 each assert the existence of a vector, which the orthogonality test excludes.

Step 3: Conclusion There are 0 such vectors $\vec{b}$. Hence, the correct option is (A).

Quick Tip to Solve: Before solving a cross-product equation componentwise, dot the proposed result with each factor; a nonzero dot product immediately proves inconsistency.

14. Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3, ..., 18 and are arranged in the increasing order $(x_1 < x_2 < x_3 < x_4 < x_5)$. The probability that $x_2 = 7$ and $x_4 = 11$ is : [4 marks]

(A) $\frac{1}{136}$
(B) $\frac{1}{72}$
(C) $\frac{1}{68}$
(D) $\frac{1}{34}$

Correct Answer: (C) $\frac{1}{68}$

Solution:

Concept:

- Choosing five distinct numbers uniformly from 18 gives $\binom{18}{5}$ equally likely subsets. Arranging each chosen subset increasingly is unique, so no factor of $5!$ is needed.
- To prescribe the second and fourth order statistics, fix their values and count the choices in the three disjoint intervals before, between and after them. Strict inequalities determine exactly how many numbers each interval supplies.

Step 1: Count the favourable subsets by intervals The event fixes $x_2 = 7$ and $x_4 = 11$. Since the list is strictly increasing, x_1 must be chosen from $1, \ldots, 6$, giving 6 choices. The middle number x_3 must be one of $8, 9, 10$, giving 3 choices. Finally x_5 is chosen from $12, \ldots, 18$, giving 7 choices. These choices are independent in the counting sense because the intervals are disjoint. Thus

$$N_{\text{fav}} = \binom{6}{1}\binom{3}{1}\binom{7}{1} = 6 \cdot 3 \cdot 7 = 126.$$

The fixed numbers 7 and 11 add no further choice factors.

Step 2: Divide by the total number of selections The total number of five-element subsets is

$$N_{\text{all}} = \binom{18}{5} = \frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14}{120} = 8568.$$

Therefore $P = 126/8568 = 1/68$, since $126 \cdot 68 = 8568$. Using ordered selections in the denominator alone would incorrectly undercount the probability: each subset already produces just one increasing list. Options $1/136$, $1/72$ and $1/34$ disagree with the exact interval count and binomial sample space; there is no symmetry factor to halve or double the favourable count.

Step 3: Conclusion The required probability is $\frac{1}{68}$. Hence, the correct option is (C).

Quick Tip to Solve: When order statistics are fixed, partition the available integers into open intervals around them and count the required selections in each interval.

15. The value of $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$ is equal to : [4 marks]

(A) -1

(B) $-\frac{1}{2}$

(C) $-\frac{1}{3}$

(D) $-\frac{1}{4}$

Correct Answer: (B) $-\frac{1}{2}$

Solution:

Concept:

- The seventh roots of unity satisfy $1+z+z^2+\cdots+z^6=0$ when $z=e^{2\pi i/7}\neq 1$. Taking real parts converts this algebraic sum into a cosine sum.
- Conjugate roots have equal real parts: $\cos(2\pi-\theta)=\cos\theta$. Thus the six non-real seventh roots contribute twice the three cosines requested.

Step 1: Write the complete root sum Put $z=e^{2\pi i/7}$. Since $z^7=1$ but $z\neq 1$, the finite geometric-series formula gives

$$1+z+z^2+z^3+z^4+z^5+z^6=\frac{z^7-1}{z-1}=0.$$

Taking real parts preserves equality and produces

$$1+\sum_{r=1}^{6}\cos\frac{2r\pi}{7}=0.$$

The initial 1 must be retained: it comes from the real root and is what determines the sign and size of the answer.

Step 2: Pair the equal cosine terms The terms with indices $r=1,6$, $r=2,5$, and $r=3,4$ are equal in pairs, because their angles add to 2π. Therefore, if the required sum is S,

$$1+2\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)=0, \qquad 1+2S=0.$$

Hence $S=-1/2$. Each of the other offered values fails the exact identity: -1 gives $1+2S=-1$, $-1/3$ gives $1/3$, and $-1/4$ gives $1/2$, rather than zero.

Step 3: Conclusion The required sum is $\boxed{-\frac{1}{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equally spaced cosine angles, complete the full root-of-unity sum and pair conjugate angles before calculating individual terms.

16. $\sin^1\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ **is equal to :** [4 marks]

(A) $\frac{11\pi}{12}$

(B) $\frac{17\pi}{12}$

(C) $\frac{31\pi}{12}$

(D) $-\frac{3\pi}{4}$

Correct Answer: The displayed $\sin^1$ gives no listed option; replacing it by inverse sine would give option (A).

Solution:

Concept:

- The principal range of $\cos^{-1}$ is $[0,\pi]$, and the principal range of $\tan^{-1}$ is $(-\pi/2,\pi/2)$. An inverse function returns the angle in its principal range, rather than the original angle automatically.
- The displayed first factor is $\sin^1$, which denotes sine to the first power, not $\sin^{-1}$. Its literal value must be distinguished from the inverse-sine interpretation suggested by the options.

Step 1: Evaluate the expression as displayed The two unambiguous inverse-function terms are

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)=\frac{5\pi}{6},\qquad \tan^{-1}\left(\tan\frac{3\pi}{4}\right)=-\frac{\pi}{4}.$$

The first follows because $\cos(7\pi/6)=\cos(5\pi/6)$; the second follows because $\tan(3\pi/4)=-1$. With the exponent 1 as displayed, the first term is $\sin(\sin(2\pi/3))=\sin(\sqrt{3}/2)$. Therefore the literal total is

$$L=\sin\left(\frac{\sqrt{3}}{2}\right)+\frac{7\pi}{12}.$$

It is not any offered value.

Step 2: Check the options and the inverse-sine interpretation Since $0<\sin(\sqrt{3}/2)<\sqrt{3}/2<\pi/3$,

$$\frac{7\pi}{12}<L<\frac{11\pi}{12}.$$

Thus option (A) is already too large; (B) and (C) are larger still, while (D) is negative. There is no correct listed option under the literal notation. If the first symbol were instead $\sin^{-1}$, its principal value would be $\pi/3$, giving

$$\frac{\pi}{3}+\frac{5\pi}{6}-\frac{\pi}{4}=\frac{11\pi}{12}.$$

This explains option (A) conditionally, but it does not make $\sin^1$ an inverse sine. The distinction is essential because a positive exponent and an inverse-function superscript have different mathematical meanings.

Step 3: Conclusion As displayed, the value is $\boxed{\sin(\sqrt{3}/2)+7\pi/12}$, and none of (A)–(D) is correct. If $\sin^{-1}$ was intended in the first term, the value would be $11\pi/12$, corresponding to (A).

Quick Tip to Solve: Check the superscript before applying a principal-value rule: sin to the first power and inverse sine are different functions.

17. Let $\frac{dy}{dx} = \frac{ax-by+a}{bx+cy+a}$, where a, b, c are constants, represent a circle passing through the point (2, 5). Then the shortest distance of the point (11, 6) from this circle is : [4 marks]

(A) 10

(B) 8

(C) 7

(D) 5

Correct Answer: (B) 8

Solution:

Concept:

- A circle with centre (h, k) satisfies $(x-h)^2 + (y-k)^2 = R^2$, so implicit differentiation gives $dy/dx = -(x-h)/(y-k)$.
- For a point outside a circle, the shortest distance to its circumference is the distance from its centre minus its radius. The radius and the external point must be compared using the same centre.

Step 1: Identify the coefficients required by a circle For a circle, the tangent direction is perpendicular to its radius. Substitution of the given derivative into this condition yields

$$(ax - by + a)(y - k) + (bx + cy + a)(x - h) = 0.$$

This quadratic vanishes on the circle, and so its quadratic part must be a multiple of $x^2 + y^2$. Its coefficients of x^2, y^2, xy are respectively $b, -b, a + c$. Equality of the first two and absence of a mixed term give $b = 0$ and $c = -a$. The nondegenerate circle has $a \neq 0$, and therefore

$$\frac{dy}{dx} = -\frac{x+1}{y-1}.$$

Thus its centre is $(-1, 1)$.

Step 2: Find the radius and the nearest distance Separating the differential equation gives $(y-1)\,dy = -(x+1)\,dx$, and integration gives

$$(x+1)^2 + (y-1)^2 = R^2.$$

The point $(2, 5)$ lies on the circle, so $R^2 = 3^2 + 4^2 = 25$ and $R = 5$. For the point $(11, 6)$, the centre distance is

$$d = \sqrt{(11+1)^2 + (6-1)^2} = \sqrt{144 + 25} = 13.$$

Since $13 > 5$, the point is outside and the shortest distance is $13 - 5 = 8$. The values 10, 7 and 5 cannot equal this radial difference; 5 is only the radius, not the required distance.

Step 3: Conclusion The shortest distance is $\boxed{8}$ units. Hence, the correct option is (B).

Quick Tip to Solve: Compare a circle differential equation with the slope perpendicular to the radius, then subtract the radius from the external centre distance.

18. The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. If for x = k, the area of the triangle is maximum, then $3k^2$ is equal to : [4 marks]

(A) 5

(B) 8

(C) 10

(D) 12

Correct Answer: (C) 10

Solution:

Concept:

- For an isosceles triangle with equal sides a and base b, the altitude bisects the base. Its height is $h = \sqrt{a^2 - (b/2)^2}$, and its area is $bh/2$.
- A nonnegative area and its square attain their maxima at the same point. Setting $t = x^2$ makes the geometric constraint and the derivative simple.

Step 1: Express the area in terms of a nonnegative variable Let $t = x^2$. Then each equal side is $10 + t$, and the base is $20 - 2t$. A nondegenerate triangle requires $0 < t < 10$: the base must be positive, and it must be smaller than twice an equal side. The altitude therefore satisfies

$$h^2 = (10 + t)^2 - (10 - t)^2 = 40t.$$

Consequently the area is

$$A(t) = (10 - t)\sqrt{40t}, \qquad A(t)^2 = 40t(10 - t)^2.$$

This formula incorporates the dependence of both the base and height on x; maximizing only the base would miss their competing effects.

Step 2: Maximize the squared area Differentiate on $0 < t < 10$:

$$\frac{d}{dt}A^2 = 40\big((10 - t)^2 - 2t(10 - t)\big) = 40(10 - t)(10 - 3t).$$

The interior stationary value is $t = 10/3$. The derivative is positive below this value and negative above it up to 10, so this is the maximum. At the degenerate boundary values 0 and 10 the area tends to zero, which confirms the global maximum. Thus $k^2 = 10/3$, and $3k^2 = 10$. Either sign of k gives the same area and the same requested value. Options 5, 8 and 12 do not satisfy the maximizing equation $10 - 3k^2 = 0$.

Step 3: Conclusion At maximum area, $\boxed{3k^2 = 10}$. Hence, the correct option is (C).

Quick Tip to Solve: When side lengths depend on a square, set that square equal to a new variable and maximize the area squared to avoid radicals.

19. Let X be a random variable having binomial distribution B(7, p). If P(X = 3) = 5P(x = 4), then the sum of the mean and the variance of X is : [4 marks]

(A) $\frac{105}{16}$
(B) $\frac{7}{16}$
(C) $\frac{77}{36}$
(D) $\frac{49}{16}$

Correct Answer: (C) $\frac{77}{36}$

Solution:

Concept:

- For $X \sim B(n,p)$, $P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$. Adjacent probabilities are often best compared by taking their ratio.
- The mean and variance of a binomial variable are np and $np(1-p)$, respectively. They are separate quantities and must both be included in the requested sum.

Step 1: Use the adjacent-probability condition Here $n = 7$. For an interior parameter $0 < p < 1$,

$$\frac{P(X=3)}{P(X=4)} = \frac{\binom{7}{3}p^3(1-p)^4}{\binom{7}{4}p^4(1-p)^3} = \frac{1-p}{p},$$

because $\binom{7}{3} = \binom{7}{4} = 35$. The given ratio is 5, so

$$1-p = 5p, \qquad p = \frac{1}{6}, \qquad 1-p = \frac{5}{6}.$$

In particular, do not reverse the ratio: the probability of three successes is the larger one, so a small success probability is consistent with the condition.

Step 2: Calculate and add the two moments Substitute into the standard formulas:

$$\mu = np = \frac{7}{6}, \qquad \sigma^2 = np(1-p) = 7 \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{35}{36}.$$

Using a common denominator gives

$$\mu + \sigma^2 = \frac{42}{36} + \frac{35}{36} = \frac{77}{36}.$$

The other options $105/16$, $7/16$, and $49/16$ have denominator 16 and cannot follow from the parameter $p = 1/6$; none equals the computed sum. The nondegenerate binomial interpretation is used here, as required by the question asking for a determined numerical value; at the excluded endpoint parameters the ratio of these two probabilities is undefined.

Step 3: Conclusion The sum of the mean and variance is $\boxed{\frac{77}{36}}$. Hence, the correct option is (C).

Quick Tip to Solve: For adjacent binomial probabilities, cancel the common powers and binomial coefficients first; calculate the mean and variance only after finding p.

20. The boolean expression $(\sim (p \wedge q)) \vee q$ is equivalent to : [4 marks]

(A) $q \rightarrow (p \wedge q)$

(B) $p \rightarrow q$

(C) $p \rightarrow (p \rightarrow q)$

(D) $p \rightarrow (p \vee q)$

Correct Answer: (D) $p \rightarrow (p \vee q)$

Solution:

Concept:

- De Morgan's law gives $\neg(p \wedge q) \equiv \neg p \vee \neg q$. The complement law gives $q \vee \neg q \equiv \mathrm{T}$.
- An implication is equivalent to $u \rightarrow v \equiv \neg u \vee v$. Two expressions are logically equivalent only if they have the same truth value for every assignment of their variables.

Step 1: Simplify the given expression Apply De Morgan's law before combining the disjunctions:

$$\begin{aligned} \neg(p \wedge q) \vee q &\equiv (\neg p \vee \neg q) \vee q \\ &\equiv \neg p \vee (\neg q \vee q) \\ &\equiv \neg p \vee \mathrm{T} \equiv \mathrm{T}. \end{aligned}$$

Thus the expression is a tautology: if q is true the last disjunct makes it true, and if q is false then $p \wedge q$ is false, making its negation true. We need the option that is true in all four cases, not merely one that agrees for a particular assignment.

Step 2: Test each proposed equivalent Option (D) simplifies to

$$p \rightarrow (p \vee q) \equiv \neg p \vee p \vee q \equiv \mathrm{T},$$

so it is equivalent. Option (A) is $\neg q \vee (p \wedge q) \equiv \neg q \vee p$, which is false for $p = \mathrm{F}, q = \mathrm{T}$. Option (B), $\neg p \vee q$, is false for $p = \mathrm{T}, q = \mathrm{F}$. Option (C) reduces to $\neg p \vee (\neg p \vee q) \equiv \neg p \vee q$, and has the same false case as (B). These counterexamples rule out all three competitors.

Step 3: Conclusion Both the given expression and $p \rightarrow (p \vee q)$ are tautologies. Hence, the correct option is (D).

Quick Tip to Solve: Simplify the question first; once it is a tautology, reject each competing option using just one assignment that makes it false.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let f : R → R be a function defined by $f(x) = \frac{2e^{2x}}{e^{2x}+e}$. Then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+ \ldots\ldots + f\left(\frac{99}{100}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- A sum over the equally spaced arguments $1/100, \ldots, 99/100$ is unchanged when each argument x is replaced by $1 - x$. Pairing these arguments can avoid adding 99 terms individually.
- For the given function, dividing numerator and denominator by e^{2x} yields $f(x) = 2/(1 + e^{1-2x})$. This form makes the symmetry about $x = 1/2$ visible.

Step 1: Establish the complementary-argument identity Write $t = e^{2x} > 0$. Since $e^{2(1-x)} = e^2/t$,

$$f(1-x) = \frac{2e^2/t}{e^2/t + e} = \frac{2e}{e+t}.$$

The original function is $f(x) = 2t/(t + e)$, so

$$f(x) + f(1-x) = \frac{2t+2e}{t+e} = 2.$$

The denominator never vanishes, and this identity is valid for every real x, including all 99 arguments in the sum. Notice that the pairing uses $1 - x$, not $-x$, because the fixed term in the denominator is e.

Step 2: Pair all terms and evaluate the middle term The first and last arguments add to 1, as do the second and next-to-last. The 98 noncentral terms form 49 disjoint pairs:

$$\left(\frac{1}{100}, \frac{99}{100}\right), \ldots, \left(\frac{49}{100}, \frac{51}{100}\right).$$

Each pair contributes 2. The only unpaired argument is $50/100 = 1/2$, and

$$f\left(\frac{1}{2}\right) = \frac{2e}{e+e} = 1.$$

Thus the sum is $49 \cdot 2 + 1 = 99$. Equivalently, writing the sum once in forward order and once in reverse order gives twice the sum as $99 \cdot 2$, providing a direct independent check on the term count.

Step 3: Conclusion The required sum is $\boxed{99}$, a dimensionless number.

Quick Tip to Solve: Before adding exponential fractions at evenly spaced arguments, test whether values at x and 1 minus x have a constant sum.

22. If the sum of all the roots of the equation

$e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ **is** $\log_e p$**, then p is equal to _____.** [4 marks]

Solution:

Concept:

- The substitution $t = e^x$ converts this exponential equation into a polynomial equation. For a real value of x, the substituted variable necessarily satisfies $t > 0$.
- After factorisation, only positive real polynomial roots may be converted back using $x = \ln t$. Nonreal polynomial roots cannot represent e^x for real x.

Step 1: Convert to a cubic Multiply the equation by e^x, which is strictly positive and therefore introduces or removes no real roots. With $t = e^x$, we obtain

$$t^3 - 11t^2 + \frac{81}{2}t - 45 = 0.$$

Equivalently, $2t^3 - 22t^2 + 81t - 90 = 0$. Substitution of $t = 2$ gives zero, so division by $t - 2$ is possible.

Step 2: Select the admissible root The exact factorisation is

$$(t - 2)(2t^2 - 18t + 45) = 0.$$

The discriminant of the quadratic factor is

$$(-18)^2 - 4(2)(45) = 324 - 360 = -36 < 0.$$

Thus that factor has no real roots. Its two nonreal roots cannot equal the positive real number e^x. Consequently the original real equation has exactly one solution, $x = \ln 2$.

Step 3: Compare the sum with the stated logarithm The sum over all real roots is therefore $\ln 2$. Since the real logarithm is one-to-one, $\ln p = \ln 2$ implies $p = 2$. Multiplying all three cubic roots would give 45 by Vieta, but two of those roots are nonreal and are inadmissible under $t = e^x > 0$.

Step 4: Conclusion For the equation as printed, the real-root answer is $\boxed{p = 2}$, a dimensionless number. This differs from the supplied key value 45: that value includes the two nonreal cubic roots and does not give the sum of the real solutions of the exponential equation.

Quick Tip to Solve: After substituting $e^x = t$, enforce $t > 0$ before using a product-of-roots formula; Vieta alone does not impose the domain.

23. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _____.

[4 marks]

Solution:

Concept:

- Cubes of the same colour are indistinguishable, so an arrangement is specified by the counts of blue cubes in the gaps before, between and after the red cubes.
- The number of nonnegative integer solutions of $z_1 + \cdots + z_r = s$ is $\binom{s+r-1}{r-1}$, by the stars-and-bars formula.

Step 1: Reserve the compulsory separating cubes There are $16 - 11 = 5$ red cubes. Their left-to-right order has four internal gaps. Each internal gap must contain at least two blue cubes, so first reserve

$$4 \times 2 = 8$$

blue cubes. This accounts for the entire separation condition: if adjacent red cubes are separated by two blue cubes, any more widely separated pair is automatically separated by at least two blue cubes.

Step 2: Distribute the remaining blue cubes After reserving eight cubes, $11 - 8 = 3$ blue cubes remain. There are six available gaps: one before the first red cube, four internal gaps, and one after the last red cube. If their additional blue counts are $z_0, z_1, \ldots, z_5$, then

$$z_0 + z_1 + z_2 + z_3 + z_4 + z_5 = 3, \qquad z_i \geq 0.$$

Every solution gives exactly one valid colour arrangement, and every valid arrangement yields exactly one such solution by subtracting two from each internal gap.

Step 3: Evaluate the combination Therefore stars and bars gives

$$N = \binom{3+6-1}{6-1} = \binom{8}{5} = \binom{8}{3} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56.$$

No factorial for permuting the red or blue cubes is included, because cubes of a fixed colour are identical. The two end gaps have no minimum requirement.

Step 4: Conclusion The required number of arrangements is $\boxed{56}$, a dimensionless count.

Quick Tip to Solve: Place the red cubes first, reserve two blues in each internal gap, then apply stars and bars to the six gaps including both ends.

24. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}}+\frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k \cdot l$, where l, k $\in$ N and l is co-prime to 5, then k is equal to _____. [4 marks]

Solution:

Concept:

- The general term of $(a+b)^{60}$, with r copies of b, is $\binom{60}{r}a^{60-r}b^r$. Match the exponent of x before calculating the coefficient.
- The exponent of a prime 5 in $n!$ is $v_5(n!) = \lfloor n/5 \rfloor + \lfloor n/25 \rfloor + \cdots$. If the remaining factor is coprime to five, every factor of five must be included in 5^k.

Step 1: Find the term containing the required power Take $a = x^{1/2}5^{-1/4}$ and $b = 5^{1/2}x^{-1/3}$. The term indexed by r is

$$T_{r+1} = \binom{60}{r} x^{(60-r)/2-r/3}5^{-(60-r)/4+r/2}.$$

Its exponent of x must equal ten. Hence

$$30 - \frac{5r}{6} = 10 \quad \Longrightarrow \quad r = 24.$$

This index lies between zero and sixty, so the required term exists uniquely.

Step 2: Separate the explicit power of five At $r = 24$, the exponent of five already present in the powers of a and b is

$$-\frac{36}{4} + \frac{24}{2} = -9 + 12 = 3.$$

Thus the coefficient of x^{10} is $5^3\binom{60}{24}$. The binomial coefficient may itself contain additional factors of five, which must not be overlooked.

Step 3: Count the remaining factors of five Legendre's formula gives

$$\begin{aligned} v_5(60!) &= 12 + 2 = 14, \\ v_5(24!) &= 4, \\ v_5(36!) &= 7 + 1 = 8. \end{aligned}$$

Therefore $v_5\binom{60}{24} = 14 - 4 - 8 = 2$. The total exponent in the coefficient is $k = 3 + 2 = 5$; after removing these five factors of five, the remaining integer is coprime to five.

Step 4: Conclusion The required exponent is $\boxed{k = 5}$, a dimensionless integer.

Quick Tip to Solve: Match the power of x first, then add the explicit power of five to the five-adic valuation of the binomial coefficient.

25. Let

$A_1 = \{(x, y) : |x| \le y^2, |x| + 2y \le 8\}$ **and**

$A_2 = \{(x, y) : |x| + |y| \le k\}$**. If 27 (Area A_1) = 5 (Area A_2), then k is equal to :** [4 marks]

Solution:

Concept:

- For a fixed ordinate y, the inequalities require $0 \le |x| \le \min(y^2, 8 - 2y)$. Thus the horizontal width is twice this minimum whenever it is nonnegative.
- An area integral must cover the entire stated set. If a region extends indefinitely with a non-integrable positive width, its area is infinite, even if a bounded portion has a finite area.

Step 1: Check the full range of ordinates The second inequality allows $y \le 4$, with no lower bound. For $y \le -4$,

$$y^2 - (8 - 2y) = (y + 4)(y - 2) \ge 0.$$

Consequently the upper bound on $|x|$ in this entire lower tail is $8 - 2y$. Every horizontal segment from $x = -(8 - 2y)$ to $x = 8 - 2y$ belongs to A_1, so its width is $16 - 4y$.

Step 2: Evaluate the lower-tail area For $M > 4$, the area in the strip $-M \le y \le -4$ alone is

$$\int_{-M}^{-4} (16 - 4y)\, dy = 2M^2 + 16M - 96.$$

As $M \to \infty$, this tends to infinity. Hence $\text{Area}(A_1) = \infty$. A finite positive k gives the diamond A_2 area $2k^2$, so the printed equality cannot hold for a finite real k.

Step 3: Identify why the supplied finite key does not follow The supplied value six would follow if an additional restriction $y \ge 0$ were present: in that restricted region the width changes at $y = 2$, and

$$2\int_0^2 y^2\, dy + 2\int_2^4 (8 - 2y)\, dy = \frac{40}{3}.$$

Then $27(40/3) = 5(2k^2)$ gives $k = 6$. However the stated set contains no restriction $y \ge 0$, so this restricted integral is not its area.

Step 4: Conclusion As printed, A_1 has infinite area and there is $\boxed{\text{no finite real value of } k}$. The supplied key 6 requires the extra condition $y \ge 0$, which is absent from the question.

Quick Tip to Solve: Before integrating a region defined by inequalities, test very negative as well as positive coordinates; a visible bounded portion need not be the whole set.

26. A rectangle R with end points of one of its sides as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is 2x − y + 4 = 0, then the area of R is _____. [4 marks]

Solution:

Concept:

- The centre of a circle lies on the perpendicular bisector of every chord. For a rectangle inscribed in a circle, the centre is also the intersection of the rectangle's diagonals.
- If one side of an inscribed rectangle is a chord of length L, and its line is at distance d from the circle centre, the opposite side is its reflection in the centre. The other side length is therefore $2d$.

Step 1: Locate the centre from the chord Let the given adjacent vertices be $P = (1, 2)$, $Q = (3, 6)$. Their midpoint is $M = (2, 4)$, and the chord direction is $(2, 4)$. Writing the centre as $C = (h, j)$, perpendicularity gives

$$2(h - 2) + 4(j - 4) = 0 \quad \Longrightarrow \quad h + 2j = 10.$$

The centre also lies on the stated diameter line, so $2h - j + 4 = 0$.

Step 2: Solve for the centre and its distance to the side From the diameter equation, $j = 2h + 4$. Substitute into the perpendicular-bisector equation to obtain

$$h + 2(2h + 4) = 10 \quad \Longrightarrow \quad h = \frac{2}{5}, \quad j = \frac{24}{5}.$$

The line through P and Q is $2x - y = 0$. Its perpendicular distance from C is

$$d = \frac{|2(2/5) - 24/5|}{\sqrt{2^2 + (-1)^2}} = \frac{4}{\sqrt{5}}.$$

Step 3: Calculate both side lengths and the area The given side has length

$$PQ = \sqrt{(3 - 1)^2 + (6 - 2)^2} = \sqrt{20} = 2\sqrt{5}.$$

The opposite side lies equally far on the other side of the centre, so the rectangle's width is $2d = 8/\sqrt{5}$. Thus

$$\text{Area}(R) = (2\sqrt{5})\frac{8}{\sqrt{5}} = 16.$$

Using just d would measure only half the width and would incorrectly halve the area.

Step 4: Conclusion The area of the rectangle is $\boxed{16 \text{ square units}}$.

Quick Tip to Solve: For an inscribed rectangle, find the centre from the side's perpendicular bisector, then double its distance from that side to get the width.

27. The positive value of the determinant of the matrix A, whose

Adj(Adj(A)) = $\begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}$**, is _____.** [4 marks]

Solution:

Concept:

- For an $n \times n$ matrix, $\det(\text{adj}\, A) = (\det A)^{n-1}$. Applying the identity twice gives $\det(\text{adj}(\text{adj}\, A)) = (\det A)^{(n-1)^2}$.
- Factoring a common scalar c from each row of a three-by-three determinant extracts a factor c^3. The positive-root condition fixes the sign after taking an even power.

Step 1: Express the supplied determinant in terms of the unknown Let $d = \det A$, and denote the given matrix by $B = \text{adj}(\text{adj}\, A)$. Since the matrix is three-by-three,

$$\det B = (\det(\text{adj}\, A))^2 = (d^2)^2 = d^4.$$

Thus it is enough to evaluate $\det B$; there is no need to determine all nine entries of A.

Step 2: Evaluate the determinant of the numerical matrix Extract fourteen from each row:

$$\det B = 14^3 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}.$$

Expanding the smaller determinant along its first row gives

$$\begin{aligned} D &= 1(1+2) - 2(-1-4) + (-1)(1-2) \\ &= 3 + 10 + 1 = 14. \end{aligned}$$

The negative cofactor sign of the middle entry and the negative final entry both matter in this expansion. Consequently $\det B = 14^3 \cdot 14 = 14^4$.

Step 3: Use the requested positive sign The determinant identity now reads $d^4 = 14^4$. Over the real numbers this permits $d = 14$ or $d = -14$. The problem specifically asks for the positive value, so select $d = 14$. Notice that taking a square root only once would produce d^2, not d; two applications of the adjugate identity produce the fourth power.

Step 4: Conclusion The positive value of $\det A$ is $\boxed{14}$, a dimensionless number.

Quick Tip to Solve: For a three-by-three matrix, the determinant of its double adjugate is the fourth power of its determinant; factor common row scalars before expanding.

28. If the sum of the first ten terms of the series

$$\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+\ ....$$

is $\frac{m}{n}$, where m and n are co-prime numbers, then m + n is equal to _____. [4 marks]

Solution:

Concept:

- The displayed denominators follow $4r^4+1$, so the general term is $r/(4r^4+1)$. Factoring this quartic exposes neighbouring quadratic expressions.
- In a telescoping sum, write the term as $c(u_r - u_{r+1})$; all intermediate terms then cancel, leaving only the first and the final boundary values.

Step 1: Factor the denominator of the general term The first five denominators are respectively $4(1)^4+1$, $4(2)^4+1$, $4(3)^4+1$, $4(4)^4+1$, and $4(5)^4+1$. Use the difference-of-squares identity

$$4r^4+1=(2r^2+1)^2-(2r)^2=(2r^2-2r+1)(2r^2+2r+1).$$

Both factors are positive for positive integer r, so the following decomposition is valid for every term in the sum.

Step 2: Write a difference of consecutive expressions The difference of the reciprocals of these factors has numerator $4r$. Therefore

$$\frac{r}{4r^4+1}=\frac{1}{4}\left(\frac{1}{2r^2-2r+1}-\frac{1}{2r^2+2r+1}\right).$$

The second denominator at r is exactly the first denominator at $r+1$, since

$$2(r+1)^2-2(r+1)+1=2r^2+2r+1.$$

Hence cancellation occurs between each pair of adjacent summands.

Step 3: Evaluate the ten-term sum and reduce the fraction On adding from $r=1$ through $r=10$, all the intermediate reciprocal terms cancel:

$$S_{10}=\frac{1}{4}\left(1-\frac{1}{2(10)^2+2(10)+1}\right)=\frac{1}{4}\left(1-\frac{1}{221}\right)=\frac{55}{221}.$$

Now $55=5\cdot 11$, whereas $221=13\cdot 17$, so this fraction is already in lowest terms. Thus $m=55$ and $n=221$.

Step 4: Conclusion The required dimensionless integer is $\boxed{m+n=55+221=276}$.

Quick Tip to Solve: For denominators $4r^4+1$, factor into $2r^2\pm 2r+1$; their reciprocal difference has exactly the numerator needed for telescoping.

29. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$. Then $(4\alpha - 8)^2$ is equal to _____.

[4 marks]

Solution:

Concept:

- The vertex and focus of $y^2 = 4ax$ are $(0, 0)$ and $(a, 0)$. A circle through two points has its centre on their perpendicular bisector.
- Tangency requires the two curves to have the same tangent at their common point. Translating $u = x - \frac{1}{4}$ makes the symmetry axis common to the circle and the second parabola.

Step 1: Locate the centre of the circle Here $4a = 2$, so the vertex and focus are $(0, 0)$ and $(\frac{1}{2}, 0)$. Their midpoint is $(\frac{1}{4}, 0)$; hence write the centre as $(\frac{1}{4}, k)$. The radius is 2, so the distance from the centre to the vertex gives

$$\frac{1}{16} + k^2 = 4, \qquad k^2 = \frac{63}{16}.$$

Thus the circle is $u^2 + (y - k)^2 = 4$, while the parabola is $y = u^2 + \alpha$.

Step 2: Apply the tangency condition For a nonvertical tangent, differentiating the circle gives $y\prime = -u/(y - k)$, whereas the parabola gives $y\prime = 2u$. Therefore tangency requires $u = 0$ or $y - k = -\frac{1}{2}$. In the latter case, the circle yields $u^2 = \frac{15}{4}$, and then the parabola gives $\alpha = k - \frac{17}{4} < 0$, since $|k| < 2$. This is excluded. At $u = 0$, the common point is the parabola vertex, so $\alpha = k \pm 2$. Positivity excludes $k - 2$; therefore $\alpha = k + 2$. Vertical circle tangents cannot match this parabola, whose derivative is finite everywhere.

Step 3: Evaluate the required expression Using $\alpha - 2 = k$ removes the need to choose the sign of the centre coordinate:

$$(4\alpha - 8)^2 = 16(\alpha - 2)^2 = 16k^2 = 16\left(\frac{63}{16}\right) = 63.$$

Both allowed signs of k give a positive $\alpha = k + 2$ and the same requested squared quantity.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{63}$.

Quick Tip to Solve: Use the perpendicular bisector first; for tangency, test off-axis contacts before assuming that the vertices touch.

30. Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$. If $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to _____.

[4 marks]

Solution:

Concept:

- The displacement from a point to its reflection in a plane is parallel to the plane normal. The midpoint of the point and its image lies in the plane.
- For $Ax + By + Cz + D = 0$, the reflected point is obtained by subtracting $2(Aa + Bb + Cc + D)(A, B, C)/(A^2 + B^2 + C^2)$ from (a, b, c).

Step 1: Use the known change in the first coordinate The normal vector is $(3, -4, 12)$, with squared length $9 + 16 + 144 = 169$. Set $S = 3a - 4b + 12c + 19$. The reflected first coordinate satisfies

$$a - \frac{6S}{169} = a - 6.$$

Hence $S = 169$, and therefore $3a - 4b + 12c = 150$. The minus sign in the reflection formula is essential: the image lies on the opposite side of the plane at equal perpendicular distance.

Step 2: Find the other reflected coordinates With $S/169 = 1$, the same reflection formula yields

$$\beta = b + 8, \qquad \gamma = c - 24.$$

These changes form the vector $(-6, 8, -24) = -2(3, -4, 12)$, confirming that the segment joining the two points is normal to the mirror plane. Its midpoint is $(a - 3, b + 4, c - 12)$, whose substitution in the plane gives $S - 169 = 0$, providing a second check.

Step 3: Eliminate the original coordinates Use the supplied condition $a + b + c = 5$, so $a = 5 - b - c$. Substitution gives

$$3(5 - b - c) - 4b + 12c = 150, \qquad -7b + 9c = 135.$$

Thus $7b - 9c = -135$. Finally,

$$7\beta - 9\gamma = 7(b + 8) - 9(c - 24) = -135 + 56 + 216 = 137.$$

There is no need to determine the three original coordinates individually; only this linear combination is fixed.

Step 4: Conclusion The required numerical value is $\boxed{137}$.

Quick Tip to Solve: One known coordinate change determines the normal displacement factor; then eliminate only the coordinate combination actually requested.

31. A projectile is launched at an angle 'α' with the horizontal with a velocity 20 ms^{-1}. After 10 s, its inclination with horizontal is 'β'. The value of tanβ will be : (g = 10 ms^{-2}). [4 marks]

(A) tanα + 5secα

(B) tanα − 5secα

(C) 2tanα − 5secα

(D) 2tanα + 5secα

Correct Answer: (B) tanα − 5secα

Solution:

Concept:

- Without air resistance, horizontal velocity remains constant and vertical velocity changes by $-gt$, taking upward as positive.
- The signed tangent of the velocity inclination is $\tan\beta = v_y/v_x$. The velocity components, rather than the projectile position, determine the instantaneous direction.

Step 1: Resolve the launch velocity The launch speed is $20\,\mathrm{m\,s^{-1}}$ at angle α. Therefore

$$u_x = 20\cos\alpha, \qquad u_y = 20\sin\alpha.$$

Gravity acts vertically downward, so there is no horizontal acceleration. The horizontal component stays $v_x = 20\cos\alpha$. This is why the denominator in the slope of the velocity is unchanged with time.

Step 2: Calculate the vertical component after ten seconds Using $v_y = u_y - gt$, with the given $g = 10\,\mathrm{m\,s^{-2}}$, gives

$$v_y = 20\sin\alpha - (10)(10) = 20\sin\alpha - 100.$$

The subtraction represents the downward change in velocity. The velocity can point downward at this instant; the signed inclination is measured consistently with the horizontal velocity direction.

Step 3: Form the tangent and compare the options Dividing the components gives

$$\tan\beta = \frac{20\sin\alpha - 100}{20\cos\alpha} = \frac{\sin\alpha}{\cos\alpha} - \frac{5}{\cos\alpha} = \tan\alpha - 5\sec\alpha.$$

Option (A) has the wrong sign for the gravitational contribution. Options (C) and (D) incorrectly double the initial vertical-to-horizontal ratio; (D) also has the wrong gravitational sign. Only option (B) reproduces both components and the correct change with time.

Step 4: Conclusion $\boxed{\tan\beta = \tan\alpha - 5\sec\alpha}$. Hence, the correct option is (B).

Quick Tip to Solve: For a projectile inclination, divide the instantaneous vertical velocity by the unchanged horizontal velocity; do not divide the displacements.

32. A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2}$ kmh^{-1}, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is : [4 marks]

(A) 30 kmh^{-1}

(B) $\frac{25}{\sqrt{2}}$ kmh^{-1}

(C) $\frac{30}{\sqrt{2}}$ kmh^{-1}

(D) 25 kmh^{-1}

Correct Answer: (C) $\frac{30}{\sqrt{2}}$ kmh^{-1}

Solution:

Concept:

- The velocity of rain relative to a person is $\vec{v}_{rg} = \vec{v}_r - \vec{v}_g$. Subtract horizontal and vertical components separately.
- A rain direction at 45° to the vertical means that its horizontal and downward vertical speed components have equal magnitudes.

Step 1: Resolve the rain velocity for the stationary girl Initially the girl is at rest, so she sees the actual rain velocity. Let its horizontal component, along her eventual running direction, be u, and its downward vertical component be v. The umbrella angle gives

$$\tan 45^\circ = \frac{u}{v} = 1, \qquad u = v.$$

The angle is with the vertical, so this ratio compares horizontal motion with downward motion; it does not directly give the total rain speed.

Step 2: Use the vertical appearance while she runs When the girl runs horizontally at $15\sqrt{2}\,\mathrm{km\,h^{-1}}$, the rain strikes her head vertically. Thus its relative horizontal component is zero:

$$u - 15\sqrt{2} = 0, \qquad u = 15\sqrt{2}\,\mathrm{km\,h^{-1}}.$$

Since she has no vertical velocity, the relative vertical component remains $v = u = 15\sqrt{2}\,\mathrm{km\,h^{-1}}$. Her motion cancels the horizontal component only.

Step 3: Find the relative speed and identify the option The relative velocity is entirely vertical, so its magnitude is

$$v_{rg} = 15\sqrt{2} = \frac{30}{\sqrt{2}}\,\mathrm{km\,h^{-1}}.$$

The actual ground-frame rain speed would be $\sqrt{u^2 + v^2} = 30\,\mathrm{km\,h^{-1}}$, which explains the tempting option (A), but that is not the requested relative speed. Options (B) and (D) do not equal the unchanged vertical component and fail the given cancellation condition.

Step 4: Conclusion The rain speed relative to the moving girl is $\boxed{\frac{30}{\sqrt{2}}\,\mathrm{km\,h^{-1}}}$, directed vertically downward. Hence, the correct option is (C).

Quick Tip to Solve: If running makes rain appear vertical, the running speed equals the rain horizontal component; the remaining relative speed is its vertical component.

33. A silver wire has a mass (0.6 ± 0.006) g, radius (0.5 ± 0.005) mm and length (4 ± 0.04) cm. The maximum percentage error in the measurement of its density will be : [4 marks]

(A) 4%

(B) 3%

(C) 6%

(D) 7%

Correct Answer: (A) 4%

Solution:

Concept:

- The density of a cylindrical wire is $\rho = m/(\pi r^2 L)$, where m, r, and L denote mass, radius, and length.
- For maximum percentage error in a product or quotient, add the absolute fractional errors weighted by the magnitudes of the powers. The radius contribution is doubled because area contains r^2.

Step 1: Write the density dependence The wire volume is its circular cross-sectional area times its length:

$$V = \pi r^2 L, \qquad \rho = \frac{m}{\pi r^2 L}.$$

The constant π contributes no measurement uncertainty. Although the given mass, radius, and length use different units, each uncertainty is in the same unit as its own measurement. Their fractional errors can therefore be evaluated directly without converting all measurements to SI units.

Step 2: Calculate the individual percentage errors The mass percentage error is

$$\frac{0.006}{0.6} \times 100 = 1\%.$$

The radius percentage error is

$$\frac{0.005}{0.5} \times 100 = 1\%,$$

and the length percentage error is

$$\frac{0.04}{4} \times 100 = 1\%.$$

These are uncertainties in the measured quantities themselves, before accounting for the powers appearing in density.

Step 3: Add the maximum contributions Applying the first-order maximum-error rule,

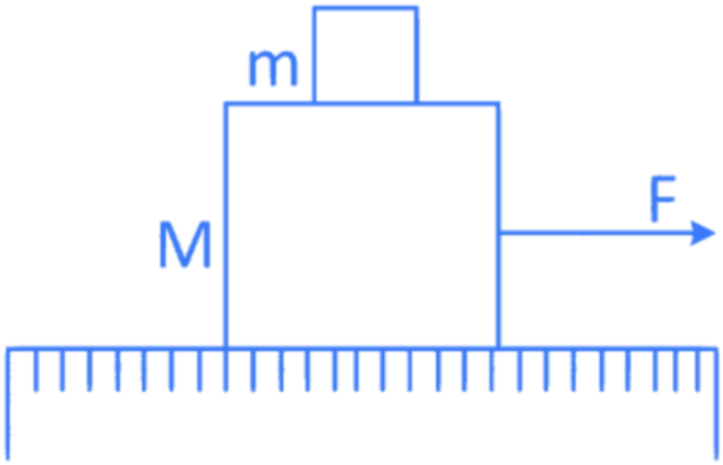
$$\frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta m}{m} \times 100 + 2\frac{\Delta r}{r} \times 100 + \frac{\Delta L}{L} \times 100 = 1 + 2(1) + 1 = 4\%.$$

Option (B), 3%, misses the second radius contribution. Options (C) and (D) overcount the uncertainties. A maximum-error estimate uses the sum of magnitudes, so errors are not subtracted merely because radius and length occur in the denominator.

Step 4: Conclusion The maximum percentage error in density is $\boxed{4\%}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the measured quantity as powers of the inputs first; each power magnitude multiplies that input fractional error.

34. A system of two blocks of masses m = 2 kg and M = 8 kg is placed on a smooth table as shown in figure. The coefficient of static friction between two blocks is 0.5. The maximum horizontal force F that can be applied to the block of mass M so that the blocks move together will be : [4 marks]



(A) 9.8 N

(B) 39.2 N

(C) 49 N

(D) 78.4 N

Correct Answer: (C) 49 N

Solution:

Concept:

- For the two blocks to move together, they must have the same horizontal acceleration. The only horizontal force accelerating the upper block is static friction.
- The maximum available static friction is $f_{\max} = \mu_s N = \mu_s mg$. The table is smooth, so the total two-block system has acceleration $a = F/(M + m)$.

Step 1: Find the force required by the upper block The upper block has mass $m = 2\,\text{kg}$. Its weight and the normal force balance vertically, giving $N = mg$. Horizontally, static friction must supply the acceleration of this block:

$$f = ma.$$

This friction acts to the right on the upper block, and an equal opposite friction acts on the lower block. These two forces cancel when both blocks are treated as one system.

Step 2: Impose the no-slipping condition Static friction adjusts to the required value only up to $\mu_s mg$. Therefore

$$ma \le \mu_s mg, \qquad a \le \mu_s g.$$

Using $\mu_s = 0.5$ and $g = 9.8\,\text{m s}^{-2}$, the largest common acceleration is

$$a_{\max} = 0.5(9.8) = 4.9\,\text{m s}^{-2}.$$

The upper-block mass cancels in this limiting acceleration, but it must still be included in the total

mass accelerated by the applied force.

Step 3: Calculate the maximum applied force The smooth table contributes no horizontal resistance. Newton's second law for both blocks together gives

$$F_{\max} = (M + m)a_{\max} = (8 + 2)(4.9) = 49\,\text{N}.$$

Option (A), $9.8\,\text{N}$, is the limiting friction on the upper block, not the applied force. Option (B) uses only the lower mass with the limiting acceleration. Option (D) would require friction greater than its static limit and cause slipping.

Step 4: Conclusion The maximum force is $\boxed{49\,\text{N}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the upper block to find the friction-limited acceleration, then multiply that acceleration by the combined mass.

---

35. Two blocks of masses 10 kg and 30 kg are placed on the same straight line with coordinates (0, 0) cm and (x, 0) cm respectively. The block of 10 kg is moved on the same line through a distance of 6 cm towards the other block. The distance through which the block of 30 kg must be moved to keep the position of centre of mass of the system unchanged is : [4 marks]

(A) 4 cm towards the 10 kg block

(B) 2 cm away from the 10 kg block

(C) 2 cm towards the 10 kg block

(D) 4 cm away from the 10 kg block

Correct Answer: (C) 2 cm towards the 10 kg block

Solution:

Concept:

- The centre of mass coordinate of two particles is $X = (m_1x_1 + m_2x_2)/(m_1 + m_2)$. Constant centre of mass position requires zero change in the mass-weighted position sum.
- For fixed masses, $m_1\Delta x_1 + m_2\Delta x_2 = 0$. The two displacements must therefore have opposite signs and magnitudes inversely proportional to the masses.

Step 1: Choose a consistent direction Take the positive direction from the initial position of the $10\,\text{kg}$ block toward the $30\,\text{kg}$ block. The lighter block moves $6\,\text{cm}$ in this positive direction. Let the signed displacement of the heavier block be d cm. We seek the displacement of each block, so the unknown initial separation x need not be known.

Step 2: Keep the centre of mass unchanged Initially the mass-weighted position sum is $10(0) + 30x$. After the shifts it becomes $10(6) + 30(x + d)$. Equating the initial and final centre of mass coordinates gives

$$\frac{30x}{40} = \frac{60 + 30x + 30d}{40}.$$

Cancel the unchanged initial-position term to obtain

$$60 + 30d = 0, \qquad d = -2\,\text{cm}.$$

The negative sign means that the heavier block moves opposite to the lighter block displacement, toward the lighter block.

Step 3: Check the alternatives A $2\,\text{cm}$ displacement toward the lighter block changes the heavier block moment by $-60\,\text{kg cm}$, exactly balancing the lighter block change of $+60\,\text{kg cm}$. A $4\,\text{cm}$ shift would overcompensate, eliminating options (A) and (D). Motion away from the lighter block has the wrong sign and reinforces the centre of mass change, eliminating option (B). Thus only option (C) preserves the centre of mass position.

Step 4: Conclusion The $30\,\text{kg}$ block must move $\boxed{2\,\text{cm}}$ toward the $10\,\text{kg}$ block. Hence, the correct option is (C).

Quick Tip to Solve: For unchanged centre of mass, balance mass times signed displacement; the heavier mass needs the smaller displacement in the opposite direction.

36. A 72 Ω galvanometer is shunted by a resistance of 8 Ω. The percentage of the total current which passes through the galvanometer is : [4 marks]

(A) 0.1%

(B) 10%

(C) 25%

(D) 0.25%

Correct Answer: (B) 10%

Solution:

Concept:

- A shunt and a galvanometer are connected in parallel, so the potential difference across them is the same.
- For galvanometer resistance G, shunt resistance S, and branch currents I_g, I_s, Ohm's law gives $I_g G = I_s S$. The total current is $I = I_g + I_s$.

Step 1: Set up the branch-current ratio Take $G = 72\,\Omega$ and $S = 8\,\Omega$. Equal branch voltages give

$$\frac{I_s}{I_g} = \frac{G}{S} = \frac{72}{8} = 9.$$

Thus the shunt carries nine times the galvanometer current. This is physically reasonable: the smaller resistance provides the easier parallel path, so most of the current bypasses the delicate galvanometer coil.

Step 2: Express the galvanometer current as a fraction of the total Using current conservation at the

junction,

$$I = I_g + 9I_g = 10I_g, \qquad \frac{I_g}{I} = \frac{1}{10}.$$

Equivalently, the current-divider relation is $I_g/I = S/(G+S) = 8/80$. Notice that the denominator contains both resistances; using $8/72$ would compare the two branch currents rather than the galvanometer current with the total.

Step 3: Convert the fraction to a percentage The required percentage is

$$\frac{I_g}{I} \times 100 = \frac{1}{10} \times 100 = 10\%.$$

The decimal fraction 0.1 is not 0.1 percent; that is why option (A) fails. Options (C) and (D) also fail the measured current ratio: a 25 percent fraction implies a shunt-to-coil current ratio of 3, while a 0.25 percent fraction implies a ratio of 399, instead of 9.

Step 4: Conclusion The galvanometer carries 10% of the total current. Hence, the correct option is (B).

Quick Tip to Solve: In a parallel shunt, the fraction through the galvanometer is $S/(G+S)$, with the shunt resistance in the numerator.

37. Given below are two statements :

Statement I : The law of gravitation holds good for any pair of bodies in the universe.

Statement II : The weight of any person becomes zero when the person is at the centre of the earth.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Gravitation is a universal attractive interaction between masses. For point masses, its Newtonian form is $F = Gm_1m_2/r^2$; extended bodies can be treated by summing the forces from their elements.
- Weight is the gravitational force $W = mg$. At the centre of a spherically symmetric Earth the net gravitational field vanishes by symmetry.

Step 1: Evaluate Statement I Every pair of material bodies attracts gravitationally. The interaction does not require either body to be charged, magnetic, or in contact with the other. For extended bodies the force is obtained by adding the interactions of all their mass elements. The simple point-mass formula can be applied directly when the bodies are pointlike or suitably spherically symmetric; that detail does not restrict the universality of gravity itself. Therefore Statement I is true in the Newtonian

framework of this question.

Step 2: Evaluate Statement II At the Earth's centre, for each mass element producing a field in one direction there is a corresponding element in the opposite direction. Their vector contributions cancel. In the standard spherical-Earth model,

$$g_{\text{centre}} = 0, \qquad W_{\text{centre}} = mg_{\text{centre}} = 0.$$

The person still has mass and inertia; zero weight does not mean zero mass. Therefore Statement II is also true.

Step 3: Compare the truth values with the choices We have found that both statements are true. Option (B) incorrectly rejects both universal attraction and the cancellation of the field at the centre. Option (C) incorrectly assigns a nonzero weight at the centre, while option (D) incorrectly denies universal gravitation. This is a two-statement truth-value question, so no additional assertion that one statement explains the other is needed.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish mass from weight: mass remains finite at the centre, but the vector gravitational field and hence mg vanish.

---

38. What percentage of kinetic energy of a moving particle is transferred to a stationary particle when it strikes the stationary particle of 5 times its mass?

(Assume the collision to be head-on elastic collision) [4 marks]

(A) 50.0%

(B) 66.6%

(C) 55.6%

(D) 33.3%

Correct Answer: (C) 55.6%

Solution:

Concept:

- In an isolated elastic collision, momentum and kinetic energy are conserved. The relative speed of separation equals the relative speed of approach.
- Let the incident mass be m, its initial speed be u, and the initially stationary target have mass $M = 5m$. Denote their final velocities by v_1, v_2.

Step 1: Find the final target speed Momentum conservation and the elastic relative-speed relation give

$$mu = mv_1 + Mv_2, \qquad v_2 - v_1 = u.$$

Substitute $v_1 = v_2 - u$ into the momentum equation:

$$2mu = (m + M)v_2.$$

With $M = 5m$, this becomes

$$v_2 = \frac{2mu}{6m} = \frac{u}{3}.$$

The projectile velocity is $v_1 = -2u/3$; its negative sign shows that the lighter projectile rebounds.

Step 2: Calculate the fraction transferred to the target The incident kinetic energy is $K_i = mu^2/2$, and the initially stationary target starts with zero energy. Its final kinetic energy is

$$K_2 = \frac{1}{2}(5m)\left(\frac{u}{3}\right)^2 = \frac{5mu^2}{18}.$$

Consequently,

$$\frac{K_2}{K_i} = \frac{5mu^2/18}{mu^2/2} = \frac{5}{9}.$$

This is the transferred fraction, rather than the fraction remaining with the projectile.

Step 3: Express and check the percentage The percentage transferred is $100(5/9) = 55.555\ldots\%$, or 55.6% to the precision of the options. The projectile retains the fraction $v_1^2/u^2 = 4/9$, so the fractions add to one as energy conservation requires. None of 50.0 percent, 66.6 percent, or 33.3 percent equals the target-energy fraction; in particular, the target speed fraction $1/3$ must not be mistaken for an energy fraction.

Step 4: Conclusion The stationary particle receives 55.6% of the incident kinetic energy. Hence, the correct option is (C).

Quick Tip to Solve: For a stationary target in a head-on elastic collision, the transferred fraction is $4mM/(m+M)^2$; it is largest when the masses are equal.

39. The velocity of a small ball of mass 'm' and density d_1, when dropped in a container filled with glycerin, becomes constant after some time. If the density of glycerin is d_2, then the viscous force acting on the ball, will be : [4 marks]

(A) $mg\left(1-\frac{d_1}{d_2}\right)$

(B) $mg\left(1-\frac{d_2}{d_1}\right)$

(C) $mg\left(\frac{d_1}{d_2}-1\right)$

(D) $mg\left(\frac{d_2}{d_1}-1\right)$

Correct Answer: (B) $mg\left(1-\frac{d_2}{d_1}\right)$

Solution:

Concept:

- At terminal speed the ball has zero acceleration, even though it is still moving, so the net force on it is zero.
- The downward weight is mg, the upward buoyancy is d_2Vg, and viscous drag opposes the downward motion. The ball volume is $V = m/d_1$.

Step 1: Identify the forces at constant speed The ball falls downward through the glycerin. Its weight acts downward, while the displaced liquid exerts an upward buoyant force. Viscous resistance also acts upward because it opposes the relative motion of the ball through the liquid. Since the speed has become constant, Newton's second law gives

$$mg - F_b - F_v = 0.$$

The viscous force is therefore the effective weight after subtracting buoyancy; it is not the entire gravitational weight.

Step 2: Substitute the ball volume into the buoyant force The ball's density is $d_1 = m/V$, hence $V = m/d_1$. Archimedes' principle then gives

$$F_b = d_2 V g = \frac{m d_2 g}{d_1}.$$

Substitution into the balance equation yields

$$F_v = mg - \frac{m d_2 g}{d_1} = mg\left(1 - \frac{d_2}{d_1}\right).$$

This expression has force units because the density ratio is dimensionless.

Step 3: Check the sign and reject the other expressions For the stated downward settling, $d_1 > d_2$. The bracket in the derived expression is positive and less than one, so the drag magnitude is positive and smaller than mg. Option (A) reverses the density ratio and gives a negative value for this case. Option (D) is the negative of the required result. Option (C) uses the inverted ratio and can exceed the weight, contradicting force balance. The formula needs neither the radius nor the viscosity because terminal equilibrium already determines the drag.

Step 4: Conclusion The upward viscous force has magnitude $mg(1 - d_2/d_1)$. Hence, the correct option is (B).

Quick Tip to Solve: At terminal fall, write weight = buoyancy + drag before substituting densities; use the ball density to convert its mass into volume.

40. The susceptibility of a paramagnetic material is 99. The permeability of the material in Wb/A-m, is :

[Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m] [4 marks]

(A) $4\pi \times 10^{-7}$

(B) $4\pi \times 10^{-4}$

(C) $4\pi \times 10^{-5}$

(D) $4\pi \times 10^{-6}$

Correct Answer: (C) $4\pi \times 10^{-5}$

Solution:

Concept:

- Magnetic susceptibility χ_m is defined by $M = \chi_m H$, where M is magnetisation and H is magnetic field intensity.
- For a linear magnetic material, $B = \mu_0(H + M) = \mu H$, so the relative permeability is $\mu_r = 1 + \chi_m$, and the absolute permeability is $\mu = \mu_0 \mu_r$.

Step 1: Obtain the relative permeability Use the given susceptibility exactly as stated, $\chi_m = 99$. Then

$$\mu_r = 1 + 99 = 100.$$

The quantity 99 is dimensionless and is not itself the absolute permeability. Although ordinary paramagnets generally have small positive susceptibilities, the requested calculation follows directly from the specified value and the defining relation; no alteration of the given data is needed.

Step 2: Multiply by the free-space permeability The supplied free-space permeability is $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{Wb\,A^{-1}\,m^{-1}}$. Hence

$$\begin{aligned}\mu &= 100\left(4\pi \times 10^{-7}\right)\\ &= 4\pi \times 10^{2-7}\\ &= 4\pi \times 10^{-5}\,\mathrm{Wb\,A^{-1}\,m^{-1}}.\end{aligned}$$

Multiplication by the dimensionless relative permeability leaves the units unchanged.

Step 3: Compare the exponents in the options Option (A) is simply the permeability of free space and would require zero susceptibility. Option (B) is 1000 times the free-space value and would correspond to $\chi_m = 999$, not 99. Option (D) is only 10 times the free-space value and would correspond to $\chi_m = 9$. Only option (C) represents the factor 100 required here. This exponent check also guards against mistaking susceptibility for a percentage or adding 99 directly to a quantity with units.

Step 4: Conclusion The permeability is $4\pi \times 10^{-5}\,\mathrm{Wb\,A^{-1}\,m^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: First calculate the dimensionless factor $1 + \chi_m$, then multiply μ_0; adding susceptibility directly to absolute permeability is dimensionally wrong.

41. The current flowing through an ac circuit is given by

$I = 5 \sin(120\pi t)$A

How long will the current take to reach the peak value starting from zero? [4 marks]

(A) $\frac{1}{60}$ s

(B) 60 s

(C) $\frac{1}{120}$ s

(D) $\frac{1}{240}$ s

Correct Answer: (D) $\frac{1}{240}$ s

Solution:

Concept:

- A sinusoidal current can be written as $I = I_0 \sin(\omega t)$. Here the amplitude is $I_0 = 5\,\text{A}$, and the angular frequency is $\omega = 120\pi\,\text{rad}\,\text{s}^{-1}$.
- Starting from zero with positive slope, a sine wave first reaches its positive peak at phase $\pi/2$, one quarter of a full cycle.

Step 1: Set the sine factor equal to one The peak value of the current is its amplitude, $5\,\text{A}$. It first occurs when

$$5\sin(120\pi t) = 5, \qquad \sin(120\pi t) = 1.$$

Starting from $t = 0$, the earliest positive phase satisfying this condition is $120\pi t = \pi/2$. Later solutions differ by full periods and do not answer how long the current first takes to reach the peak.

Step 2: Solve for the first peak time Divide the phase equation by 120π:

$$t = \frac{\pi/2}{120\pi} = \frac{1}{240}\,\text{s}.$$

The amplitude cancels because changing the current scale does not change the phase or timing. As a separate check, $f = \omega/(2\pi) = 60\,\text{Hz}$, so the period is $T = 1/60\,\text{s}$. The required time is $T/4 = 1/240\,\text{s}$, consistent with the direct calculation.

Step 3: Reject the other time intervals At $t = 1/60\,\text{s}$, the phase is 2π, so the current has completed a full cycle and is zero again; option (A) is the period. At $t = 1/120\,\text{s}$, the phase is π, another zero, so option (C) is a half-period. The interval 60 seconds in option (B) confuses frequency with time and spans many cycles. Only the quarter-period corresponds to the first positive maximum.

Step 4: Conclusion The current first reaches its peak after $1/240\,\text{s}$. Hence, the correct option is (D).

Quick Tip to Solve: For $I = I_0 \sin \omega t$ starting at zero, the first peak is at $t = \pi/(2\omega) = T/4$, rather than at a full period.

42. Match List-I with List-II :

	List - I		List - II
(a)	Ultraviolet rays	(i)	Study crystal structure
(b)	Microwaves	(ii)	Greenhouse effect
(c)	Infrared rays	(iii)	Sterilizing surgical instrument
(d)	X-rays	(iv)	Radar system

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(B) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(C) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(D) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Correct Answer: (A) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Solution:

Concept:

- Electromagnetic waves have different practical applications because their wavelengths, absorption properties and interactions with matter differ.
- For this matching problem, assign each listed radiation to its characteristic application before comparing the four complete answer codes.

Step 1: Match ultraviolet rays and microwaves Ultraviolet radiation can damage the genetic material of microorganisms, so it is used for sterilizing suitable surgical instruments and exposed surfaces. Therefore (a) matches (iii). Microwaves are used in radar: they can be transmitted in directed beams and their reflected pulses are detected to locate objects and determine distances. Therefore (b) matches (iv). These two assignments already eliminate codes that put microwaves with crystal-structure studies or ultraviolet with radar.

Step 2: Match infrared rays and X-rays The greenhouse effect involves absorption and re-emission of the Earth's outgoing thermal infrared radiation by greenhouse gases. Thus infrared rays match the greenhouse effect, giving (c)-(ii). X-rays have wavelengths comparable to atomic separations in crystals. Their diffraction pattern reveals the arrangement of the crystal lattice, so X-rays match study of crystal structure, giving (d)-(i). Crystal diffraction is not the characteristic infrared application listed here.

Step 3: Assemble and test the full code The complete matching is

$$(a)\text{-(iii)}, \quad (b)\text{-(iv)}, \quad (c)\text{-(ii)}, \quad (d)\text{-(i)}.$$

Option (B) exchanges the microwave and X-ray applications. Option (C) exchanges the ultraviolet and microwave applications. Option (D) assigns crystal-structure study to infrared and the greenhouse effect to X-rays. Each of these contradicts the physical applications just identified; only option (A) preserves all four pairs simultaneously.

Step 4: Conclusion All four radiation-application pairs agree with the first answer code. Hence, the correct option is (A).

Quick Tip to Solve: Remember the application chain: ultraviolet for sterilization, microwaves for radar, infrared for thermal trapping, and X-rays for crystal diffraction.

43. An α particle and a carbon 12 atom has same kinetic energy K. The ratio of their de-Broglie wavelengths $(\lambda_\alpha : \lambda_{C12})$ is : [4 marks]

(A) $1 : \sqrt{3}$

(B) $\sqrt{3} : 1$

(C) $3 : 1$

(D) $2 : \sqrt{3}$

Correct Answer: (B) $\sqrt{3} : 1$

Solution:

Concept:

- The de Broglie wavelength of a particle is $\lambda = h/p$, where h is Planck's constant and p is its momentum.
- For nonrelativistic kinetic energy $K = p^2/(2m)$, equal kinetic energies give $\lambda = h/\sqrt{2mK}$, so wavelength varies inversely as the square root of mass.

Step 1: Express both wavelengths at the common energy The alpha particle is a helium nucleus of mass approximately $4u$, while the carbon-12 atom has mass $12u$, where u is the atomic mass unit. The same kinetic energy does not mean the same momentum: the heavier carbon atom has the larger momentum. Thus

$$\lambda_\alpha = \frac{h}{\sqrt{2(4u)K}}, \qquad \lambda_C = \frac{h}{\sqrt{2(12u)K}}.$$

The common factors will cancel in the ratio, so their numerical values are unnecessary.

Step 2: Take the inverse square-root mass ratio Dividing in the order requested gives

$$\frac{\lambda_\alpha}{\lambda_C} = \sqrt{\frac{12u}{4u}} = \sqrt{3}.$$

The lighter alpha particle must have the longer wavelength, which also checks the direction of the ratio. Option (A) reverses it. Option (C) uses the mass ratio without its square root. Option (D) neither equals the inverse square-root mass ratio nor follows the common-energy momentum relation.

Step 3: Conclusion The wavelength ratio is $\lambda_\alpha : \lambda_{C12} = \sqrt{3} : 1$. It is dimensionless, and the alpha particle has the longer wavelength. Hence, the correct option is (B).

Quick Tip to Solve: At equal kinetic energy use $\lambda \propto m^{-1/2}$; at equal momentum the wavelengths would instead be equal.

44. A force of 10 N acts on a charged particle placed between two plates of a charged capacitor. If one plate of capacitor is removed, then the force acting on that particle will be. [4 marks]

(A) 5 N

(B) 10 N

(C) 20 N

(D) Zero

Correct Answer: (A) 5 N

Solution:

Concept:

- A large uniformly charged sheet produces an electric field of magnitude $\sigma/(2\varepsilon_0)$ on either side, where σ is its surface charge density.
- Between the oppositely charged plates of an ideal capacitor, their fields point in the same direction and add. The force magnitude on a charge q is $F = |q|E$.

Step 1: Separate the two equal field contributions Let the magnitude of the field due to either original plate be $E_0 = \sigma/(2\varepsilon_0)$. Inside the capacitor, the positive plate field points toward the negative plate, and the negative plate field points in that same direction. Therefore

$$E_{\text{initial}} = E_0 + E_0 = 2E_0.$$

The stated force is consequently

$$F_{\text{initial}} = |q|\,2E_0 = 10\,\text{N}.$$

Each plate supplies half of that total field at the particle.

Step 2: Retain the field of the remaining charged plate After one plate is removed, the remaining charged plate still produces its own field. In the ideal charged-sheet model used here, its charge density is retained, and edge effects are neglected. Thus

$$F_{\text{final}} = |q|E_0 = \frac{F_{\text{initial}}}{2} = 5\,\text{N}.$$

Option (B) incorrectly retains both field contributions. Option (C) would require the field to double when a source is removed. Option (D) incorrectly treats the remaining charged sheet as producing no field.

Step 3: Conclusion The remaining plate exerts a force of magnitude $5\,\text{N}$ on the particle. Hence, the correct option is (A).

Quick Tip to Solve: An isolated large charged plate contributes half the field between two oppositely charged capacitor plates; superpose the fields before applying $F = qE$.

45. The displacement of simple harmonic oscillator after 3 seconds starting from its mean position is equal to half of its amplitude. The time period of harmonic motion is : [4 marks]

(A) 6 s

(B) 8 s

(C) 12 s

(D) 36 s

Correct Answer: (D) 36 s

Solution:

Concept:

- For an oscillator starting at its mean position, choose $x = A\sin(\omega t)$, where A is the amplitude and $\omega = 2\pi/T$ is the angular frequency.
- The earliest displacement $A/2$ on departure from the mean corresponds to a phase $\pi/6$. The phase, rather than the fraction of displacement, determines the elapsed fraction of a period.

Step 1: Relate the stated displacement to the phase Take the initial velocity in the positive direction, which simply fixes the sign convention for displacement. Substituting the stated time gives

$$\frac{x(3)}{A} = \sin(3\omega) = \frac{1}{2}.$$

For the first arrival at this displacement after starting from the mean,

$$3\omega = \frac{\pi}{6}, \qquad \omega = \frac{\pi}{18}\,\mathrm{rad\,s^{-1}}.$$

This first-arrival reading is the one represented by the offered choices; later arrivals would involve additional phase branches.

Step 2: Calculate the period and check the choices Use the angular-frequency relation to obtain

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/18} = 36\,\mathrm{s}.$$

At $t = 3\,\mathrm{s}$, this gives $x/A = \sin(\pi/6) = 1/2$, as required. For periods 6, 8, and $12\,\mathrm{s}$, the same displacement formula gives respectively 0, $\sqrt{2}/2$, and 1, so options (A), (B), and (C) fail the stated condition. Half the amplitude must not be confused with half a period.

Step 3: Conclusion The time period corresponding to the first arrival at half the amplitude is $T = 36\,\mathrm{s}$. It is also the only offered period satisfying the displacement condition. Hence, the correct option is (D).

Quick Tip to Solve: Starting at the mean, reaching $A/2$ for the first time takes $T/12$, since its phase is $\pi/6$.

46. Consider a light ray travelling in air is incident into a medium of refractive index $\sqrt{2n}$. The incident angle is twice that of refracting angle. Then, the angle of incidence will be : [4 marks]

(A) $\sin^{-1}\left(\sqrt{n}\right)$

(B) $\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

(C) $\sin^{-1}\left(\sqrt{2n}\right)$

(D) $2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

Correct Answer: (D) $2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

Solution:

Concept:

- Snell's law for air into a medium is $\sin i = \mu \sin r$, with i and r measured from the normal and $\mu = \sqrt{2n}$.
- The condition $i = 2r$, together with $\sin 2r = 2\sin r \cos r$, converts the refraction equation to an equation for the refracted angle.

Step 1: Insert the relation between the two angles For the nonzero incident ray described, substitute $i = 2r$ into Snell's law:

$$\sin(2r) = \sqrt{2n}\sin r.$$

Expanding the double angle gives

$$2\sin r \cos r = \sqrt{2n}\sin r.$$

Cancelling $\sin r$ for this oblique ray leads to

$$\cos r = \frac{\sqrt{2n}}{2} = \sqrt{\frac{n}{2}}.$$

Cancelling would not determine an angle at normal incidence; the intended solution concerns the nonzero branch.

Step 2: Recover the incidence angle and compare options The refracted angle is

$$r = \cos^{-1}\left(\sqrt{\frac{n}{2}}\right).$$

The requested angle is twice this value:

$$i = 2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right).$$

Option (B) gives r, not i. Options (A) and (C) do not follow the derived cosine relation. For a physical oblique ray entering this medium with $i \leq 90°$, the corresponding parameter range is $1 \leq n < 2$; the limiting normal ray occurs as $n \to 2$.

Step 3: Conclusion The angle of incidence is $i = 2\cos^{-1}\left(\sqrt{n/2}\right)$, with the inverse cosine chosen for the physical refracted angle. Hence, the correct option is (D).

Quick Tip to Solve: When incidence is twice refraction, first find $\cos r = \mu/2$, then double r; stopping at r selects the tempting wrong option.

47. A hydrogen atom in its ground state absorbs 10.2 eV of energy. The angular momentum of electron of the hydrogen atom will increase by the value of :

(Given, Planck's constant = 6.6 $\times$ 10^{-34} Js). [4 marks]

(A) 2.10 $\times$ 10^{-34} Js

(B) 1.05 $\times$ 10^{-34} Js

(C) 3.15 $\times$ 10^{-34} Js

(D) 4.2 $\times$ 10^{-34} Js

Correct Answer: (B) 1.05 $\times$ 10^{-34} Js

Solution:

Concept:

- Hydrogen has Bohr energies $E_n = -13.6/n^2\,\text{eV}$, with the ground state at $n = 1$. Absorbed energy raises the energy to a less negative value.
- Bohr angular momentum is $L_n = nh/(2\pi) = n\hbar$. The increase is $\Delta L = L_f - L_i$, which differs from the final angular momentum.

Step 1: Find the excited state from the absorbed energy The initial energy is $-13.6\,\text{eV}$. After absorption,

$$E_f = -13.6 + 10.2 = -3.4\,\text{eV}.$$

Equating this to the Bohr energy yields

$$-\frac{13.6}{n_f^2} = -3.4, \qquad n_f^2 = 4, \qquad n_f = 2.$$

Thus the electron makes the first excitation, from $n_i = 1$ to $n_f = 2$. It is not ionised, since the final energy is still negative.

Step 2: Subtract the initial angular momentum The angular momenta before and after absorption are $h/(2\pi)$ and $2h/(2\pi)$. Therefore

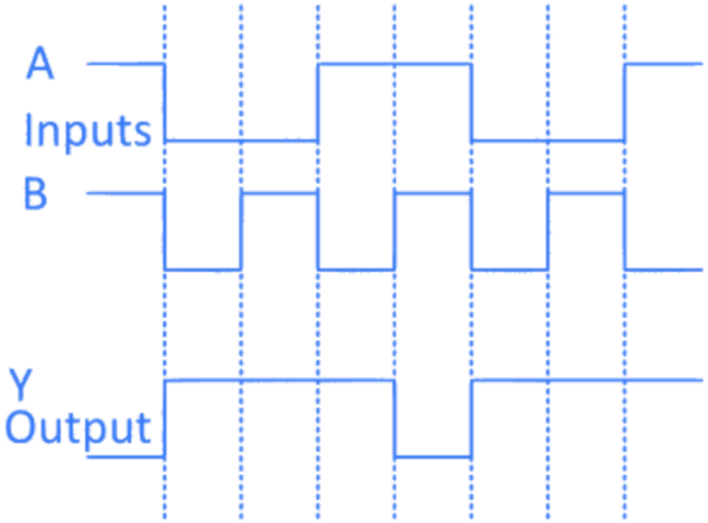
$$\Delta L = (2-1)\frac{h}{2\pi} = \frac{6.6 \times 10^{-34}}{2\pi} = 1.05 \times 10^{-34}\,\text{J s}.$$

Option (A) is the final angular momentum, not its increase. Options (C) and (D) correspond to three or four units of $\hbar$, inconsistent with the single-step increase in the principal quantum number. The unit J s is the same as $\text{kg m}^2\,\text{s}^{-1}$.

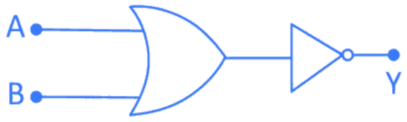
Step 3: Conclusion The angular momentum increases by $1.05 \times 10^{-34}\,\text{J s}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the absorbed energy to determine n_f, then calculate $(n_f - n_i)\hbar$; do not report $n_f\hbar$ when the question asks for an increase.

48. Identify the correct Logic Gate for the following output (Y) of two inputs A and B. [4 marks]



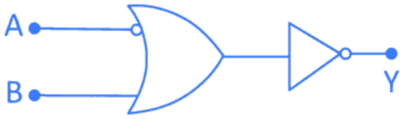
(A) $Y = \overline{A + B}$ (OR followed by NOT)



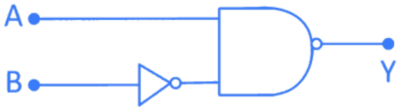
(B) $Y = \overline{AB}$ (AND followed by NOT)



(C) $Y = \overline{\overline{A} + B}$ (inverted A, then NOR)



(D) $Y = \overline{A\overline{B}}$ (inverted input B to NAND)



Correct Answer: (B)

$Y = \overline{AB}$ (AND followed by NOT)



Solution:

Concept:

- For a two-input NAND gate, $Y = \overline{AB}$: the output is low only when both inputs are high.
- A timing diagram is read interval by interval between its vertical transition lines. High and low plateaus represent binary 1 and 0; transition instants themselves are not additional truth-table cases.

Step 1: Read the stable input and output combinations In the interval after the first simultaneous falling edges, both A and B are low, while Y is high. In the next interval $(A, B) = (0, 1)$, and Y remains high. The following interval has $(A, B) = (1, 0)$, again with high output. Only the interval

where both inputs are high has a low output. The resulting truth table is

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

These four cases establish the complete two-input Boolean function.

Step 2: Identify the matching circuit and reject the others The truth table is the complement of AND, hence $Y = \overline{AB}$. Option (B) shows an AND gate followed by a NOT gate, exactly implementing this function. Option (A) is NOR, $\overline{A+B}$, which would give low output for either mixed input. Option (C) implements $\overline{\overline{A}+B} = A\overline{B}$, failing at input 00. Option (D) implements $\overline{A\overline{B}}$, which is low at 10 rather than 11. Therefore only the second circuit matches every stable interval.

Step 3: Conclusion The output is $Y = \overline{AB}$, the NAND function, realised by an AND gate followed by an inverter. Hence, the correct option is (B).

Quick Tip to Solve: Locate the single low-output interval first: if it occurs only for input 11, the required function is NAND.

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49. A mixture of hydrogen and oxygen has volume 2000 cm^3, temperature 300 K, pressure 100 kPa and mass 0.76 g. The ratio of number of moles of hydrogen to number of moles of oxygen in the mixture will be:

[Take gas constant R = 8.3 JK^{-1}mol^{-1}] [4 marks]

(A) $\frac{1}{3}$

(B) $\frac{3}{1}$

(C) $\frac{1}{16}$

(D) $\frac{16}{1}$

Correct Answer: (B) $\frac{3}{1}$

Solution:

Concept:

- For an ideal mixture the total amount is $n_{\text{tot}} = PV/(RT)$; both gases share the stated pressure, volume and temperature.
- If the amounts of hydrogen and oxygen are n_H and n_O, the mass equation is $2n_H + 32n_O = 0.76$, with molar masses in g mol^{-1}.

Step 1: Calculate the total amount in SI units Convert the volume and pressure before applying the gas law:

$$V = 2000\,\text{cm}^3 = 2.000 \times 10^{-3}\,\text{m}^3, \qquad P = 1.00 \times 10^5\,\text{Pa}.$$

Then

$$n_H + n_O = \frac{(1.00 \times 10^5)(2.000 \times 10^{-3})}{8.3(300)} = 0.08032\,\text{mol} \simeq 0.080\,\text{mol}.$$

The mass is quoted to two significant figures, so retaining approximately $0.080\,\text{mol}$ is appropriate for the offered simple ratios.

Step 2: Combine the mass and mole equations Using $n_H + n_O \simeq 0.080$, subtract twice this equation from the mass equation:

$$30n_O \simeq 0.76 - 0.160 = 0.600, \qquad n_O \simeq 0.020\,\text{mol}.$$

Thus $n_H \simeq 0.060\,\text{mol}$, and $n_H/n_O \simeq 3$. Keeping every input digit gives about 3.02, reflecting the rounding of the printed data; the compatible offered simple ratio is $3:1$. Ratios $1:3$ and $1:16$ would make the mixture much heavier, while $16:1$ would make it much lighter. The measured average molar mass, about $9.46\,\text{g mol}^{-1}$, is close to 9.5 for $3:1$.

Step 3: Conclusion The mole ratio is approximately $n_H : n_O = 3:1$, the simple ratio consistent with the rounded gas data. Hence, the correct option is (B).

Quick Tip to Solve: Find total moles from PV/RT first, then use the mass balance; the hydrogen-to-oxygen mass ratio is not their mole ratio.

50. An observer moves towards a stationary source of sound with a velocity equal to one-fifth of the velocity of sound. The percentage change in the frequency will be : [4 marks]

(A) 20%

(B) 10%

(C) 5%

(D) 0%

Correct Answer: (A) 20%

Solution:

Concept:

- For a stationary source and an observer approaching it, the observed frequency is $f' = f(v + v_o)/v$, where v is the speed of sound in the medium and v_o is the observer speed.
- The moving observer meets wavefronts more frequently, while their wavelength in the medium remains $\lambda = v/f$. Percentage change is measured relative to the emitted frequency f.

Step 1: Identify the correct moving-observer formula The source stays at rest, so it emits successive crests with separation $\lambda = v/f$. The observer moves towards those crests at speed $v_o = v/5$, making the speed of encounter $v + v_o$. Therefore the change enters the numerator; the denominator remains the speed of sound because the source has not changed the wavelength.

Step 2: Calculate the fractional increase Substitute the given observer speed into the Doppler formula:

$$f' = f\frac{v + v/5}{v} = \frac{6}{5}f = 1.2f.$$

Hence the increase is $\Delta f = f' - f = 0.2f$, and

$$\frac{\Delta f}{f} \times 100 = \frac{0.2f}{f} \times 100 = 20\%.$$

The frequency increases rather than decreases because the observer approaches the source. The 10% and 5% options would require observer speeds $v/10$ and $v/20$, respectively; 0% would require no motion along the line joining source and observer.

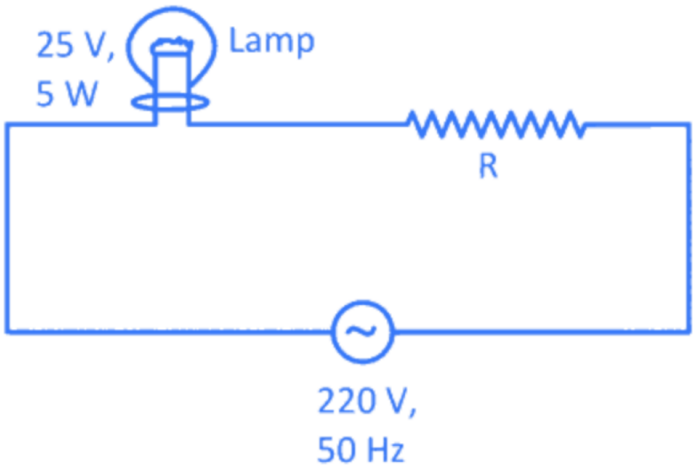
Step 3: Conclusion The observed frequency increases by 20%. Hence, the correct option is (A).

Quick Tip to Solve: For an approaching observer with a fixed source, the fractional frequency increase is directly v_o/v; multiply this fraction by 100.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A 220 V, 50 Hz AC source is connected to a 25 V, 5 W lamp and an additional resistance R in series (as shown in figure) to run the lamp at its peak brightness, then the value of R (in ohm) will be _____. [4 marks]



Solution:

Concept:

- A lamp operating at its rated brightness must have its rated voltage across it and dissipate its rated power. For a resistive lamp, $P_L = V_L I$ using rms voltage and current.
- In a series circuit the same current flows through the lamp and resistor. Since both components are resistive, their rms voltage drops add arithmetically: $V_s = V_L + IR$.

Step 1: Find the rated lamp current and resistance The lamp rating is $25\,\text{V}, 5\,\text{W}$. To run it at the specified brightness, use these ratings rather than the full source voltage:

$$I = \frac{P_L}{V_L} = \frac{5}{25} = 0.20\,\text{A}.$$

Its operating resistance is consequently

$$R_L = \frac{V_L^2}{P_L} = \frac{25^2}{5} = 125\,\Omega.$$

The AC voltage ratings are rms values. The word brightness refers to the lamp operating at its rated power; it does not require replacing rms values by peak amplitudes.

Step 2: Find the voltage drop across the series resistor The series resistor must remove the excess voltage while allowing the rated current through the lamp. Because there is no reactive component, the current and each voltage drop are in phase:

$$V_R = V_s - V_L = 220 - 25 = 195\,\text{V}.$$

Ohm's law gives

$$R = \frac{V_R}{I} = \frac{195}{0.20} = 975\,\Omega.$$

As a check, the total resistance is $975 + 125 = 1100\,\Omega$, so the source supplies $220/1100 = 0.20\,\text{A}$, giving precisely $25\,\text{V}$ across the lamp. The frequency does not alter this purely resistive division.

Step 3: Conclusion The required additional resistance is $R = 975\,\Omega$; the numerical entry is 975.

Quick Tip to Solve: First compute the lamp's rated current from P/V, then divide only the excess supply voltage by that current.

52. In Young's double slit experiment the two slits are 0.6 mm distance apart. Interference pattern is observed on a screen at a distance 80 cm from the slits. The first dark fringe is observed on the screen directly opposite to one of the slits. The wavelength of light will be _____ nm. [4 marks]

Solution:

Concept:

- In Young's double-slit experiment, the path difference at transverse displacement y is approximately $\delta = dy/D$, where d is slit separation and D is screen distance.
- The first dark fringe adjacent to the central bright fringe satisfies $\delta = \lambda/2$, so its transverse position is $y = \lambda D/(2d)$.

Step 1: Locate the point opposite one slit The central bright fringe lies opposite the midpoint of the two slits. A point on the screen directly opposite either slit is displaced from that midpoint by half the slit separation:

$$|y| = \frac{d}{2} = \frac{0.6\,\text{mm}}{2} = 0.3\,\text{mm}.$$

This is the transverse distance from the centre, rather than the screen distance D. The given distances in SI units are $d = 6.0 \times 10^{-4}\,\text{m}$ and $D = 0.80\,\text{m}$. The small ratio d/D supports the usual paraxial approximation.

Step 2: Apply the first-minimum condition At the stated point the first dark fringe occurs. Thus

$$\frac{dy}{D} = \frac{\lambda}{2}.$$

Inserting $y = d/2$ simplifies this relation to

$$\lambda = \frac{d^2}{D} = \frac{(6.0 \times 10^{-4})^2}{0.80}\,\mathrm{m} = 4.5 \times 10^{-7}\,\mathrm{m}.$$

Convert metres to nanometres using $1\,\mathrm{nm} = 10^{-9}\,\mathrm{m}$:

$$\lambda = 450\,\mathrm{nm}.$$

The implied fringe width is $\lambda D/d = 0.6\,\mathrm{mm}$; its half-width is $0.3\,\mathrm{mm}$, agreeing with the specified location.

Step 3: Conclusion The wavelength is $450\,\mathrm{nm}$, so the required numerical entry is 450.

Quick Tip to Solve: When the first dark fringe lies opposite a slit, put its displacement equal to $d/2$, giving the shortcut $\lambda = d^2/D$.

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53. A beam of monochromatic light is used to excite the electron in $Li^{+\,+}$ from the first orbit to the third orbit. The wavelength of monochromatic light is found to be x $\times$ 10^{-10} m. The value of x is _____.

[Given hc = 1242 eV nm] [4 marks]

Solution:

Concept:

- Lithium with two electrons removed, Li^{2+}, is a one-electron ion with nuclear charge $Z = 3$. Its Bohr energies are $E_n = -13.6Z^2/n^2\,\mathrm{eV}$.
- An absorbed photon supplies the energy difference between the final and initial levels: $hc/\lambda = E_f - E_i$. The supplied value $hc = 1242\,\mathrm{eV\,nm}$ gives wavelength directly in nanometres.

Step 1: Calculate the excitation energy The electron rises from $n_i = 1$ to $n_f = 3$, so the final state is less strongly bound. The initial and final energies are

$$E_1 = -13.6(3)^2 = -122.4\,\mathrm{eV}, \qquad E_3 = -\frac{13.6(3)^2}{9} = -13.6\,\mathrm{eV}.$$

The positive energy which must be absorbed is

$$\Delta E = E_3 - E_1 = (-13.6) - (-122.4) = 108.8\,\mathrm{eV}.$$

Retaining the nuclear-charge factor is essential: applying the hydrogen energy spacing without the factor $Z^2 = 9$ would produce the wrong wavelength.

Step 2: Convert photon energy into the requested coefficient Use the given photon-energy product:

$$\lambda = \frac{hc}{\Delta E} = \frac{1242}{108.8}\,\text{nm} \approx 11.4154\,\text{nm}.$$

Since $1\,\text{nm} = 10^{-9}\,\text{m} = 10 \times 10^{-10}\,\text{m}$,

$$\lambda \approx 114.154 \times 10^{-10}\,\text{m}.$$

The dimensionless coefficient is therefore $x \approx 114.154$, which gives 114 to the nearest integer for the numerical entry. This wavelength is in the ultraviolet region, consistent with the large excitation energy of a hydrogen-like ion.

Step 3: Conclusion $\lambda \approx 1.14154 \times 10^{-8}\,\text{m}$, hence $x \approx 114.154$ and the integer numerical answer is 114.

Quick Tip to Solve: Keep the Z^2 factor for hydrogen-like ions, and convert nm to the requested $10^{-10}\,\text{m}$ scale before rounding.

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54. The current density in a cylindrical wire of radius 4 mm is 4×10^6 Am^{-2}. The current through the outer portion of the wire between radial distances $\frac{R}{2}$ and R is _____ π A. [4 marks]

Solution:

Concept:

- For uniform current density J, the current crossing a specified part of the wire is $I = JA$, where A is its cross-sectional area normal to the flow.
- The outer portion between radii $R/2$ and R is an annulus of area $\pi[R^2 - (R/2)^2] = 3\pi R^2/4$; it is not half the total area.

Step 1: Find the area of the outer annulus The wire radius is $R = 4\,\text{mm} = 4 \times 10^{-3}\,\text{m}$, while the inner boundary of the required portion is $R/2 = 2 \times 10^{-3}\,\text{m}$. Subtract the inner disk from the whole disk:

$$A = \pi\left[(4 \times 10^{-3})^2 - (2 \times 10^{-3})^2\right].$$

Squaring the radii gives

$$A = \pi(16 - 4) \times 10^{-6} = 12\pi \times 10^{-6}\,\text{m}^2.$$

Halving a radius quarters a circular area, so the requested region contains three quarters of the full cross-section.

Step 2: Multiply the area by the current density The specified current density is $J = 4 \times 10^6\,\text{A m}^{-2}$. Therefore

$$I = JA = (4 \times 10^6)(12\pi \times 10^{-6}) = 48\pi\,\text{A}.$$

The area unit cancels the inverse-area factor in J, leaving amperes. A useful check is that the total wire current would be

$$I_{\text{total}} = J\pi R^2 = 64\pi\,\text{A},$$

and three quarters of it is exactly 48π A. The blank precedes an already printed π A, so enter only the coefficient, not the full decimal value of the current.

Step 3: Conclusion The current in the outer portion is 48π A, giving the numerical entry 48.

Quick Tip to Solve: For a uniform current density, use the fraction of circular area: the annulus from $R/2$ to R carries $3/4$ of the current.

55. A capacitor of capacitance 50 pF is charged by 100 V source. It is then connected to another uncharged identical capacitor. Electrostatic energy loss in the process is _____ nJ. [4 marks]

Solution:

Concept:

- The charge and electrostatic energy of a capacitor are $Q = CV$ and $U = CV^2/2$. The source charges the first capacitor before the charge-sharing stage.
- On connecting two identical capacitors with one initially uncharged, total charge is conserved and both reach the same voltage. Their combined capacitance is $2C$, so their final common voltage is $V/2$.

Step 1: Calculate the initial stored energy Initially only the first capacitor carries charge. Its capacitance is $C = 50\,\text{pF} = 50 \times 10^{-12}\,\text{F}$, and its voltage is $V = 100\,\text{V}$. Thus

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}(50 \times 10^{-12})(100)^2 = 2.5 \times 10^{-7}\,\text{J} = 250\,\text{nJ}.$$

The second capacitor contributes no initial electrostatic energy because it is uncharged. The initial total charge is $Q = CV = 5.0 \times 10^{-9}\,\text{C}$.

Step 2: Conserve charge and compare the final energy Charge redistributes after the source is removed and the two capacitors are connected together. The common voltage becomes

$$V_f = \frac{Q}{2C} = \frac{V}{2} = 50\,\text{V}.$$

Each capacitor stores $CV_f^2/2$, so the energy in the pair is

$$U_f = 2\left(\frac{1}{2}CV_f^2\right) = C(50)^2 = 1.25 \times 10^{-7}\,\text{J} = 125\,\text{nJ}.$$

Consequently,

$$\Delta U = U_i - U_f = 250 - 125 = 125\,\text{nJ}.$$

Total charge remains conserved; electrostatic field energy does not. The missing field energy is dissipated during the transient through resistance and electromagnetic radiation.

Step 3: Conclusion The electrostatic energy lost is $125\,\text{nJ}$, so the numerical answer is 125.

Quick Tip to Solve: For charge sharing between identical capacitors, one initially uncharged, the final voltage halves and exactly half the initial field energy is lost.

56. The area of cross-section of a large tank is 0.5 m^2. It has a narrow opening near the bottom having area of cross-section 1 cm^2. A load of 25 kg is applied on the water at the top in the tank. Neglecting the speed of water in the tank, the velocity of the water, coming out of the opening at the time when the height of water level in the tank is 40 cm above the bottom, will be _____ cms^{-1}. [Take g = 10 ms^{-2}] [4 marks]

Solution:

Concept:

- A load M distributed over surface area A adds pressure Mg/A above atmospheric pressure at the upper water surface.
- Bernoulli's equation between the upper surface and the outlet gives $v^2/2 = gh + \Delta p/\rho$ when the upper-surface speed is negligible. Water has density $\rho = 1000\,\mathrm{kg\,m^{-3}}$.

Step 1: Calculate the pressure head from the load The tank area is $A = 0.5\,\mathrm{m^2}$ and the applied load is $M = 25\,\mathrm{kg}$. Its additional surface pressure is

$$\Delta p = \frac{Mg}{A} = \frac{25 \times 10}{0.5} = 500\,\mathrm{Pa}.$$

This is equivalent to an extra water head

$$h_L = \frac{\Delta p}{\rho g} = \frac{500}{1000 \times 10} = 0.05\,\mathrm{m}.$$

The outlet is open to atmosphere, so atmospheric pressure appears at both ends and cancels. The applied load adds to the gravitational head rather than subtracting from it.

Step 2: Apply Bernoulli and convert the exit speed The water surface is $h = 40\,\mathrm{cm} = 0.40\,\mathrm{m}$ above the outlet. The total driving head is therefore $h + h_L = 0.45\,\mathrm{m}$. Hence

$$v = \sqrt{2g(h + h_L)} = \sqrt{2(10)(0.45)} = \sqrt{9} = 3\,\mathrm{m\,s^{-1}}.$$

The opening area is $1\,\mathrm{cm^2} = 10^{-4}\,\mathrm{m^2}$, much smaller than the tank area; their ratio is 2×10^{-4}, consistent with neglecting the surface speed. It does not otherwise enter the exit-speed calculation. Finally,

$$3\,\mathrm{m\,s^{-1}} = 300\,\mathrm{cm\,s^{-1}}.$$

Neglecting the load would instead give $\sqrt{8}\,\mathrm{m\,s^{-1}}$, so retaining its pressure contribution is necessary.

Step 3: Conclusion The speed of the emerging water is $300\,\mathrm{cm\,s^{-1}}$; the numerical entry is 300.

Quick Tip to Solve: Convert an applied surface load into the pressure head $M/(\rho A)$, add it to the water depth, then use Torricelli's formula.

57. A pendulum of length 2 m consists of a wooden bob of mass 50 g. A bullet of mass 75 g is fired towards the stationary bob with a speed v. The bullet emerges out of the bob with a speed $\frac{v}{3}$ and the bob just completes the vertical circle. The value of v is _____ ms^{-1}. (if g = 10 m/s^2). [4 marks]

Solution:

Concept:

- During the very short bullet-bob interaction, external impulse along the bullet direction is negligible. Linear momentum in that direction is conserved, although mechanical energy is not conserved in penetration.
- For a bob on a flexible string to just complete a vertical circle of radius L, tension is zero at the highest point. Hence $u_{\text{top}}^2 = gL$. Mechanical energy is conserved during the subsequent swing.

Step 1: Find the minimum speed after impact Write the energy equation from the bottom to the top, where the bob rises through $2L$. If u is its speed immediately after the bullet exits, then

$$\frac{1}{2}Mu^2 = \frac{1}{2}MgL + Mg(2L).$$

Thus $u^2 = 5gL$. Substituting the given length and acceleration,

$$u = \sqrt{5 \times 10 \times 2} = 10\,\text{m}\,\text{s}^{-1}.$$

This is the minimum bottom speed compatible with a taut string throughout the circle; a smaller speed would make the string slack before completing it.

Step 2: Apply momentum conservation during penetration The bullet has mass $m = 0.075\,\text{kg}$, and the wooden bob has mass $M = 0.050\,\text{kg}$. The bullet exits forward at $v/3$, while the bob is initially at rest. Therefore,

$$mv = m\frac{v}{3} + Mu \quad \Longrightarrow \quad \frac{2m}{3}v = Mu.$$

Insert the masses and the minimum bob speed:

$$\frac{2(0.075)}{3}v = 0.050(10), \qquad 0.050v = 0.500.$$

Do not equate the bullet's lost kinetic energy to the bob's gained energy: penetration dissipates energy into deformation and heat.

Step 3: Conclusion The required incident bullet speed is $v = 10\,\text{m}\,\text{s}^{-1}$. The number to enter is $\boxed{10}$.

Quick Tip to Solve: First obtain the bob speed from the vertical-circle condition; then use momentum, rather than energy, across the bullet-bob collision.

58. In a carnot engine, the temperature of reservoir is 527°C and that of sink is 200 K. If the work done by the engine when it transfers heat from reservoir to sink is 12000 kJ, the quantity of heat absorbed by the engine from reservoir is _____ $\times 10^6$ J. [4 marks]

Solution:

Concept:

- A reversible Carnot engine operating between absolute temperatures T_h and T_c has efficiency $\eta = 1 - T_c/T_h$. Temperatures must be measured in kelvin in this ratio.
- The thermal efficiency also equals useful work divided by heat absorbed: $\eta = W/Q_h$. Over a complete cycle the change in internal energy is zero, so $Q_h = W + Q_c$.

Step 1: Convert the temperatures and find efficiency The hot reservoir is at $527^\circ\mathrm{C}$. Convert it to the absolute scale before using the Carnot relation:

$$T_h = 527 + 273 = 800\,\mathrm{K}, \qquad T_c = 200\,\mathrm{K}.$$

Hence

$$\eta = 1 - \frac{200}{800} = \frac{3}{4} = 0.75.$$

The cold-reservoir temperature is already in kelvin; adding 273 to it would be incorrect. The engine converts three quarters of its absorbed heat into work and rejects the remaining quarter to the sink.

Step 2: Calculate the absorbed heat and its coefficient The stated work is $12000\,\mathrm{kJ}$, not $12000\,\mathrm{J}$. In joules this is

$$W = 12000 \times 10^3 = 12 \times 10^6\,\mathrm{J}.$$

Use $Q_h = W/\eta$, since the work is only a fraction of the heat absorbed:

$$Q_h = \frac{12 \times 10^6}{3/4} = 16 \times 10^6\,\mathrm{J}.$$

As an independent energy check, the rejected heat is $Q_c = Q_h - W = 4 \times 10^6\,\mathrm{J}$. Its ratio to the input heat is $4/16 = 1/4 = T_c/T_h$, consistent with reversible operation. The required blank is the coefficient multiplying $10^6\,\mathrm{J}$, not the full number of joules.

Step 3: Conclusion The heat absorbed is $\boxed{16 \times 10^6\,\mathrm{J}}$, so the numerical entry is $\boxed{16}$.

Quick Tip to Solve: Convert Celsius to kelvin first, then divide the work by the efficiency. Finally extract the coefficient of the power of ten specified in the question.

59. A cell, shunted by a 8 Ω resistance, is balanced across a potentiometer wire of length 3 m. The balancing length is 2 m when the cell is shunted by 4 Ω resistance. The value of internal resistance of the cell will be _____ Ω. [4 marks]

Solution:

Concept:

- A potentiometer at null balance measures the cell's terminal potential difference without drawing additional current from it. For an unchanged potential gradient k, the balancing length l gives $V = kl$.
- For a cell of emf E and internal resistance r, supplying an external shunt resistance R, the terminal voltage is $V = ER/(R + r)$. The shunt causes current and therefore an internal voltage drop.

Step 1: Write the two loaded terminal voltages With the $8\,\Omega$ resistor connected across the cell, the terminal voltage and balance relation are

$$V_1 = \frac{8E}{8 + r} = 3k.$$

For the $4\,\Omega$ resistor they become

$$V_2 = \frac{4E}{4 + r} = 2k.$$

Both measurements use the same cell and unchanged potentiometer arrangement, so the emf and potential gradient cancel on taking their ratio. Neither balance is an open-circuit measurement of the emf; using the usual single-loaded-length formula with 3 m as the emf length would therefore be inappropriate.

Step 2: Eliminate the emf and solve for internal resistance Divide the first balance equation by the second:

$$\frac{V_1}{V_2} = \frac{3}{2} = \frac{8}{8 + r}\frac{4 + r}{4} = \frac{2(4 + r)}{8 + r}.$$

Cross multiplication gives

$$3(8 + r) = 4(4 + r), \qquad 24 + 3r = 16 + 4r.$$

Consequently $r = 8\,\Omega$. Check the result directly: the terminal voltages are then $E/2$ and $E/3$, whose ratio is indeed $3/2$. Decreasing the shunt resistance increases current and internal voltage loss, explaining why the second balancing length is shorter.

Step 3: Conclusion The cell's internal resistance is $\boxed{8\,\Omega}$. The numerical entry is $\boxed{8}$.

Quick Tip to Solve: When both balance lengths are measured with shunts connected, use the ratio of the two loaded voltages; neither length alone measures the emf.

60. The height of a transmitting antenna at the top of a tower is 25 m and that of receiving antenna is, 49 m. The maximum distance between them, for satisfactory communication in LOS (Line-Of-Sight) is K$\sqrt{5} \times 10^2$ m. The value of K is _____.

(Assume radius of Earth is 64 $\times$ 10^{+5} m) [Calculate upto nearest integer value] [4 marks]

Solution:

Concept:

- For an antenna at height $h \ll R$ above the Earth, its horizon distance is approximately $d = \sqrt{2Rh}$, where R is the Earth's radius. Both lengths must use consistent units.
- Two elevated antennas can communicate along a line of sight up to the sum of their horizon distances: $d_{\max} = \sqrt{2Rh_t} + \sqrt{2Rh_r}$. Each antenna extends the reachable distance beyond the other's individual horizon.

Step 1: Substitute the two antenna heights Use the given radius $R = 64 \times 10^5\,\text{m} = 6.4 \times 10^6\,\text{m}$, transmitter height $h_t = 25\,\text{m}$, and receiver height $h_r = 49\,\text{m}$. The small-height approximation is justified because both heights are negligible compared with the Earth's radius. Thus

$$d_{\max} = \sqrt{2R}(\sqrt{25} + \sqrt{49}) = 12\sqrt{128 \times 10^5}\,\text{m}.$$

It is the sum of the square roots of the heights that appears. Replacing it by $\sqrt{25 + 49}$ would omit the two separate horizon contributions.

Step 2: Express the distance in the required form Simplify the square root without introducing premature rounding:

$$128 \times 10^5 = 12{,}800{,}000 = 256 \times 5 \times 10^4.$$

Therefore

$$\sqrt{128 \times 10^5} = 16\sqrt{5} \times 10^2\,\text{m},$$

and the total limiting separation is

$$d_{\max} = 12(16\sqrt{5} \times 10^2) = 192\sqrt{5} \times 10^2\,\text{m}.$$

This is approximately $4.293 \times 10^4\,\text{m}$, or $42.93\,\text{km}$, a reasonable terrestrial radio horizon for antennas of these heights. Compare the exact coefficient with the question's expression before rounding.

Step 3: Conclusion The comparison gives $K = \boxed{192}$, already an integer. The corresponding maximum distance is $192\sqrt{5} \times 10^2\,\text{m}$.

Quick Tip to Solve: Factor out $\sqrt{2R}$ and add $\sqrt{h_t}$ and $\sqrt{h_r}$. Keep radicals exact until matching the coefficient requested.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : At 10°C, the density of a 5 M solution of KCl [atomic masses of K & Cl are 39 & 35.5 g mol^{-1} respectively], is 'x' g ml^{-1}. The solution is cooled to -21°C. The molality of the solution will remain unchanged.

Reason (R) : The molality of a solution does not change with temperature as mass remains unaffected with temperature.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Correct Answer: (A) Both (A) and (R) are true and (R) is the correct explanation of (A).

Solution:

Concept:

- Molality m is the number of moles of solute per kilogram of solvent: $m = n_{\text{solute}}/m_{\text{solvent}}$. Both quantities refer to amounts of matter, rather than the volume occupied by the solution.
- Molarity is moles of solute per litre of solution. Its numerical value may change when temperature changes because solution volume and density can change, even though the masses remain fixed.

Step 1: Evaluate the assertion using a fixed quantity of solution Consider $1\,\text{L}$ of the given solution at 10°C. It contains $5\,\text{mol}$ of KCl. Its total mass is $1000x\,\text{g}$, and the solute mass is

$$5(39 + 35.5) = 372.5\,\text{g}.$$

The solvent mass is therefore $(1000x - 372.5)\,\text{g}$, giving

$$m = \frac{5 \times 1000}{1000x - 372.5}\,\text{mol kg}^{-1}.$$

Cooling a fixed-composition solution changes its volume and density, but not the solute amount or solvent mass used in this expression. Its molality remains unchanged. Assertion (A) is true for the solution described.

Step 2: Evaluate the reason and its explanatory link Reason (R) states precisely why molality is temperature independent: mass is unaffected by thermal expansion or contraction. Since no volume occurs in the definition of molality, a change in volume does not change it. Thus (R) is true and directly explains (A). Option (B) incorrectly denies this explanatory link. Option (C) incorrectly makes the mass-based reason false, and option (D) incorrectly makes the assertion false. The supplied density is useful for calculating an initial molality, but no numerical density is needed to decide whether it changes with temperature.

Step 3: Conclusion Both statements are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

Quick Tip to Solve: Check the denominator: concentrations based on mass remain independent of thermal expansion, while concentrations based on solution volume generally vary with temperature.

62. Based upon VSEPR theory, match the shape (geometry) of the molecules in List-I with the molecules in List-II and select the most appropriate option.

	List-I (Shape)		List-II (Molecules)
(A)	T-shaped	(I)	XeF_4
(B)	Trigonal planar	(II)	SF_4
(C)	Square planar	(III)	ClF_3
(D)	See-saw	(IV)	BF_3

[4 marks]

(A) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

(B) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

(C) (A) - (III), (B) - (IV), (C) - (I), (D) - (I)

(D) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

Correct Answer: (B) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Solution:

Concept:

- VSEPR theory counts bonding domains and lone pairs around the central atom. Multiple bonds count as single domains. Electron-pair geometry includes lone pairs, whereas molecular shape describes only the positions of bonded atoms.
- Lone pairs repel more strongly than bonding pairs. In a trigonal bipyramidal arrangement they preferentially occupy equatorial sites, where there are fewer close 90° interactions.

Step 1: Assign the shapes of the three fluorides with lone pairs In ClF_3, chlorine has seven valence electrons and forms three bonds, leaving two lone pairs: AX_3E_2. Five electron domains give trigonal bipyramidal electron geometry; two equatorial lone pairs leave a T-shaped molecule. Thus (A) matches (III). In XeF_4, xenon forms four bonds and retains two lone pairs: AX_4E_2. Six domains form an octahedral arrangement, with the lone pairs opposite each other. Four fluorine atoms then lie in one square plane, so (C) matches (I). In SF_4, sulfur has four bonds and one lone pair: AX_4E. The five-domain arrangement with an equatorial lone pair gives a see-saw shape, so (D) matches (II).

Step 2: Complete the matching and eliminate the alternatives Boron in BF_3 has three bonding domains and no lone pair, so it is AX_3, with a trigonal planar shape and bond angles of 120°. Hence (B) matches (IV). Combining all four assignments gives

(A)-(III), (B)-(IV), (C)-(I), (D)-(II).

This is option (B). Option (A) would make XeF_4 T-shaped and SF_4 trigonal planar, contradicting their

domain counts. Option (C) incorrectly gives XeF_4 the see-saw assignment as well as the square-planar one. Option (D) exchanges the T-shaped and trigonal-planar species. Counting lone pairs resolves these tempting confusions without relying on memorized names alone.

Step 3: Conclusion The complete matching is the combination given in the second alternative. Hence, the correct option is (B).

Quick Tip to Solve: Count both bonds and lone pairs before naming the shape: five domains can give either T-shaped or see-saw geometry, depending on the number of lone pairs.

63. Match List-I with List-II.

	List-I		List-II
(A)	**Spontaneous process**	**(I)**	$\Delta H < 0$
(B)	**Process with $\Delta P = 0$, $\Delta T = 0$**	**(II)**	$\Delta G_{T,P} < 0$
(C)	$\Delta H_{\text{reaction}}$	**(III)**	**Isothermal and isobaric process**
(D)	**Exothermic process**	**(IV)**	**[Bond energies of molecules in reactants] − [Bond energies of product molecules**

Choose the correct answer from the options given below: [4 marks]

(A) (A) - (III), (B) - (II), (C) - (IV), (D) - (I)

(B) (A) - (II), (B) - (III), (C) - (IV), (D) - (I)

(C) (A) - (II), (B) - (III), (C) - (I), (D) - (IV)

(D) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

Correct Answer: (B) (A) - (II), (B) - (III), (C) - (IV), (D) - (I)

Solution:

Concept:

- At constant temperature and pressure, the criterion for a spontaneous process is $\Delta G < 0$, where G is the Gibbs energy. A negative enthalpy change alone does not establish spontaneity.
- An isothermal process has constant temperature, and an isobaric process has constant pressure. Exothermic processes release heat and have $\Delta H < 0$.
- Reaction enthalpy can be estimated from bond energies as energy required to break reactant bonds minus energy released on forming product bonds.

Step 1: Match spontaneity and the stated constraints The Gibbs-energy criterion at fixed T and P gives

$$\text{Spontaneous process} \longleftrightarrow \Delta G_{T,P} < 0.$$

Thus (A) matches (II). For the second entry, $\Delta P = 0$ specifies constant pressure and $\Delta T = 0$ specifies constant temperature. Together these mean an isothermal and isobaric process, so (B) matches (III). Do not interchange the process conditions with the spontaneity criterion: a process can be carried out at constant temperature and pressure without being spontaneous in its chosen direction.

Step 2: Match reaction enthalpy and exothermicity Breaking bonds consumes energy, whereas

forming bonds releases energy. The enthalpy estimate is consequently

$$\Delta H_{\text{reaction}} = \sum E_{\text{bonds broken}} - \sum E_{\text{bonds formed}}.$$

This is the reactant-bond-energy minus product-bond-energy expression in (IV), giving (C)-(IV). Heat release in an exothermic process lowers the enthalpy of the system, so (D)-(I). The complete set is therefore (A)-(II), (B)-(III), (C)-(IV), (D)-(I), which occurs in option (B). Option (A) exchanges the first two assignments; option (C) confuses a general reaction-enthalpy expression with the sign for exothermicity; option (D) misassigns the process constraints and the enthalpy entries.

Step 3: Conclusion The complete matching is the combination given in the second alternative. Hence, the correct option is (B).

Quick Tip to Solve: Keep three ideas separate: constant temperature and pressure name the process conditions, negative Gibbs energy gives spontaneity, and negative enthalpy identifies heat release.

64. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The ionic radii of O^{2-} and Mg^{2+} are same.

Reason (R) : Both O^{2-} and Mg^{2+} are isoelectronic species.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Correct Answer: (D) (A) is false but (R) is true.

Solution:

Concept:

- Isoelectronic species have the same total number of electrons, but need not have the same nuclear charge or size.
- Within an isoelectronic series, increasing nuclear charge pulls the same electron cloud closer and decreases the ionic radius.

Step 1: Count the electrons Oxygen has atomic number 8, so O^{2-} has $8+2 = 10$ electrons. Magnesium has atomic number 12, so Mg^{2+} has $12 - 2 = 10$ electrons. Both therefore have the closed-shell configuration $1s^2 2s^2 2p^6$, the same as neon. Reason (R), which describes equality of electron counts, is true.

Step 2: Compare their nuclear attractions The oxide ion has 8 protons attracting these 10 electrons, whereas the magnesium ion has 12 protons attracting the same number of electrons. The larger

nuclear charge of magnesium contracts its electron cloud more strongly. Thus

$$r(O^{2-}) > r(Mg^{2+}).$$

Equality of electronic configuration does not imply equality of radius. Assertion (A), which claims equal ionic radii, is false.

Step 3: Select the truth combination Options (A) and (B) both require a true assertion and so fail. Option (C) also reverses the actual truth values, requiring a true assertion and a false reason. Only option (D) states the established combination: false assertion and true reason. The reason is a valid fact, but cannot establish the incorrect equality in the assertion.

Step 4: Conclusion The assertion is false and the reason is true: Mg^{2+} is smaller than O^{2-} despite their equal electron counts. Hence, the correct option is (D).

Quick Tip to Solve: For equal electron counts, compare atomic numbers: the ion with more protons has the smaller radius.

65. Which of the following will have maximum stabilization due to crystal field? [4 marks]

(A) $[Ti(H_2O)_6]^{3+}$

(B) $[Co(H_2O)_6]^{2+}$

(C) $[Co(CN)_6]^{3-}$

(D) $[Cu(NH_3)_4]^{2+}$

Correct Answer: (C) $[Co(CN)_6]^{3-}$

Solution:

Concept:

- In an octahedral field, each t_{2g} electron contributes $-0.4\Delta_o$ and each e_g electron contributes $+0.6\Delta_o$.
- A strong-field ligand such as CN^- favours low-spin filling; higher metal oxidation state also generally increases splitting.

Step 1: Find oxidation states and electron counts Water and ammonia are neutral ligands. The titanium ion in $[Ti(H_2O)_6]^{3+}$ is Ti^{3+}, with d^1. The aqua cobalt complex contains Co^{2+}, with d^7. In $[Co(CN)_6]^{3-}$, six cyanide ions contribute charge -6, so cobalt is +3 and has d^6. The copper ion in the ammonia complex is +2 and has d^9.

Step 2: Compare the orbital stabilizations The titanium aqua complex is t_{2g}^1, giving $-0.4\Delta_o$. The weak-field cobalt(II) aqua complex is high spin, $t_{2g}^5 e_g^2$, giving $-0.8\Delta_o$. Cyanide produces low-spin cobalt(III), $t_{2g}^6 e_g^0$, giving

$$CFSE = 6(-0.4\Delta_o) = -2.4\Delta_o.$$

Its six d electrons all occupy the lower-energy set; none cancels that stabilization by occupying the upper set.

Step 3: Identify the largest stabilization The cobalt(III) cyanide complex combines the largest

favourable occupancy with the strong ligand field of cyanide and the higher cobalt charge. The copper(II) d9 complex has substantial occupation of antibonding upper orbitals and cannot obtain the filled-lower-set stabilization of low-spin d6. Thus titanium aqua, cobalt aqua and copper ammonia do not compete with the low-spin cobalt(III) cyanide complex for maximum crystal-field stabilization. Pairing energy affects spin-state selection; the stated comparison concerns crystal-field stabilization.

Step 4: Conclusion $[Co(CN)_6]^{3-}$ has the maximum crystal-field stabilization, with low-spin t_{2g}^6 occupation. Hence, the correct option is (C).

Quick Tip to Solve: Look first for a strong-field low-spin d6 complex: its filled lower octahedral set gives a stabilization of magnitude $2.4\Delta_o$.

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66. 'A' and 'B' respectively are:

$$\text{A} \xrightarrow[\textbf{(2) Zn–}H_2O]{\textbf{(1) }O_3} \textbf{Ethane-1,2-dicarbaldehyde + Glyoxal/Oxaldehyde},$$

$$\text{B} \xrightarrow[\textbf{(2) Zn–}H_2O]{\textbf{(1) }O_3} \textbf{5-oxohexanal}.$$

[4 marks]

(A) 1-methylcyclohex-1,3-diene & cyclopentene.

(B) Cyclohex-1,3-diene & cyclopentene

(C) 1-methylcyclohex-1,4-diene & 1-methylcyclopent-1-ene

(D) Cyclohex-1,3-diene & 1-methylcyclopent-1-ene

Correct Answer: (D) Cyclohex-1,3-diene & 1-methylcyclopent-1-ene

Solution:

Concept:

- Reductive ozonolysis cleaves each carbon-carbon double bond and converts each alkene carbon into a carbonyl carbon.
- An alkene carbon carrying hydrogen produces an aldehyde; an alkene carbon carrying two carbon groups produces a ketone. Carbon skeletons can be reconstructed by joining carbonyl carbons pairwise.

Step 1: Reconstruct compound A A gives ethane-1,2-dicarbaldehyde and glyoxal. Their structures are

$$OHC-CH_2-CH_2-CHO \quad \text{and} \quad OHC-CHO.$$

Together they contain six carbons and four aldehyde groups. Joining their carbonyl ends pairwise restores two double bonds in a six-membered ring. One ring segment contains the two adjacent double-bond carbons which form glyoxal, while the other contains two intervening methylene groups. This gives cyclohexa-1,3-diene. The absence of any ketone group also rules out a methyl substituent on its double-bond carbon.

Step 2: Reconstruct compound B The other product is 5-oxohexanal, whose condensed structure is

$$OHC-CH_2-CH_2-CH_2-CO-CH_3.$$

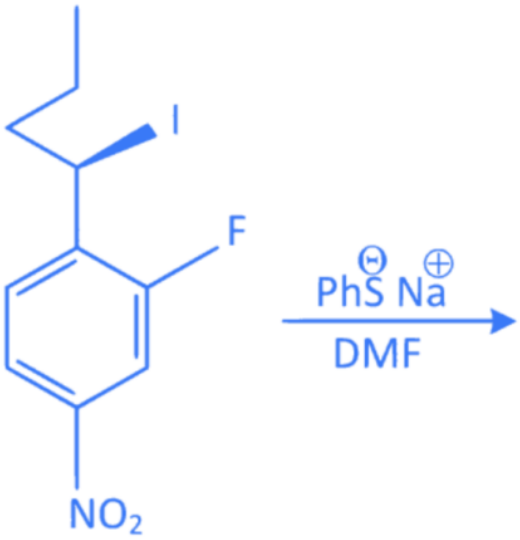
The aldehyde carbon and the ketone carbon were the two carbons of the original double bond. Joining them closes a five-membered ring: these two carbons plus the three intervening methylene carbons. The methyl group remains on the ketone-derived alkene carbon, so B is 1-methylcyclopent-1-ene.

Step 3: Check the alternatives Cyclopentene without a methyl substituent would yield pentane-1,5-dial, containing five carbons and two aldehyde ends; it cannot yield the stated six-carbon ketoaldehyde. Therefore options (A) and (B) fail for B. Option (C) proposes a methyl-substituted 1,4-diene for A, inconsistent both with the all-aldehyde fragments and with the isolated glyoxal fragment. Option (D) has both reconstructed starting compounds.

Step 4: Conclusion A is cyclohexa-1,3-diene and B is 1-methylcyclopent-1-ene. Hence, the correct option is (D).

Quick Tip to Solve: To reverse ozonolysis, join the carbonyl carbons; an aldehyde end carries H, whereas a ketone end retains its carbon substituent.

67. The major product of the following reaction is : [4 marks]



(A) Benzylic iodide replaced by SPh on a dashed bond; aryl fluorine retained.

(B) Aryl fluorine replaced by SPh; benzylic iodide retained on its solid wedge.

(C) Benzylic iodide replaced by SPh on a solid wedge; aryl fluorine retained.

(D) Both benzylic iodide and aryl fluorine replaced by SPh; benzylic SPh on a solid wedge.

Correct Answer: (A) Benzylic iodide replaced by SPh on a dashed bond; aryl fluorine retained.

Solution:

Concept:

- Phenylthiolate, PhS^-, is a strong sulfur nucleophile and attacks an accessible benzylic carbon readily.
- A concerted S_N2 displacement attacks from the side opposite the leaving group and causes inversion at a stereogenic carbon. Polar aprotic DMF favours this pathway.

Step 1: Identify the reactive bond The starting compound has a secondary benzylic carbon attached to iodide, a propyl chain and the substituted aromatic ring. Its iodide is an excellent leaving group, and the adjacent aromatic system makes the benzylic position favourable for nucleophilic substitution. Sodium phenylthiolate supplies PhS^-; sodium is a spectator ion. The incoming nucleophilic atom is sulfur, so the new group at that carbon is $-\mathrm{SPh}$.

Step 2: Apply inversion at the benzylic centre In DMF the nucleophile attacks the benzylic carbon from the back of the carbon-iodine bond while iodide departs. The printed starting iodide is on a solid wedge toward the viewer. Inversion places the newly attached phenylthio group on the opposite side, represented by the dashed bond in option (A). The carbon skeleton remains intact; no rearrangement or change in chain length is needed.

Step 3: Exclude aromatic replacement and retention The aromatic fluorine is meta to the nitro group, so the nitro group does not provide the usual ortho/para resonance activation for addition-elimination displacement at that fluorinated carbon. The readily substituted benzylic iodide reacts preferentially. Option (B) incorrectly replaces aryl fluorine while retaining iodide. Option (C) replaces iodide but depicts retention with a solid wedge instead of inversion. Option (D) additionally substitutes aryl fluorine and has the wrong benzylic stereochemistry. Only option (A) has both the correct site and inverted configuration.

Step 4: Conclusion The major product retains aryl fluorine and replaces benzylic iodide by SPh with inversion, shown by a dashed bond. Hence, the correct option is (A).

Quick Tip to Solve: For thiolate in DMF, locate the good leaving group on an sp3 carbon, then reverse its wedge/dash orientation when applying SN2.

68. Which of the following reactions will yield benzaldehyde as a product?

(A) $C_6H_5COOH \xrightarrow[\text{(ii) } H_2/Pd-BaSO_4]{\text{(i) } SOCl_2,\ \textbf{Quinoline}} ?$

(B) $C_6H_5CH_2OH \xrightarrow{CrO_3/H_2SO_4} ?$

(C) $C_6H_5COOCH_3 \xrightarrow[\text{(ii) PCC}]{\text{(i) } NaBH_4} ?$

(D) $C_6H_5CH_3 \xrightarrow[\text{(ii) } H_3O^+,\ \Delta]{\text{(i) } CrO_3,\ (CH_3CO)_2O} ?$

[4 marks]

(A) (B) and (C)

(B) (C) and (D)

(C) (A) and (D)

(D) (A) and (C)

Correct Answer: (C) (A) and (D)

Solution:

Concept:

- Rosenmund reduction converts an acid chloride to an aldehyde using hydrogen and poisoned palladium catalyst.
- The oxidation outcome depends on the reagent: strong aqueous chromium(VI) oxidizes primary alcohols to acids, whereas protected side-chain oxidation followed by hydrolysis can stop at an aldehyde.

Step 1: Assess reactions A and B In reaction (A), thionyl chloride converts benzoic acid to benzoyl chloride. Hydrogen with palladium supported on barium sulfate gives the Rosenmund reduction; poisoning suppresses further reduction of the aldehyde. Thus

$$C_6H_5COOH \xrightarrow{SOCl_2} C_6H_5COCl \xrightarrow{H_2/Pd-BaSO_4} C_6H_5CHO.$$

Reaction (A) yields benzaldehyde. In (B), benzyl alcohol is treated with CrO_3/H_2SO_4, a strong oxidizing system. The primary alcohol is oxidized onward to benzoic acid, so it is not a selective preparation of benzaldehyde.

Step 2: Assess reactions C and D Reaction (C) starts with methyl benzoate. Ordinary $NaBH_4$ does not reduce an ester under the conditions implied by this reagent; it reduces aldehydes and ketones readily. Hence the first step does not produce benzyl alcohol for PCC to oxidize, and this sequence does not yield benzaldehyde. In (D), toluene is oxidized with CrO_3 in acetic anhydride. The aldehyde oxidation level is trapped as benzylidene diacetate, $C_6H_5CH(OCOCH_3)_2$. Acidic hydrolysis on heating releases C_6H_5CHO. Thus (D) is successful.

Step 3: Match the correct pair Only reactions (A) and (D) provide benzaldehyde. The pair (B),(C) contains two unsuccessful routes; (C),(D) contains the unsuccessful ester sequence; and (A),(C) also includes that unsuccessful sequence. The third printed option alone contains the two successful reactions.

Step 4: Conclusion The acid-chloride reduction route and the protected toluene oxidation route yield benzaldehyde. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish NaBH4 from stronger ester-reducing agents, and remember that aqueous chromium(VI) carries primary alcohol oxidation past the aldehyde.

69. Given below are two statements :

Statement I : In Hofmann degradation reaction, the migration of only an alkyl group takes place from carbonyl carbon of the amide to the nitrogen atom.

Statement II : The group is migrated in Hofmann degradation reaction to electron deficient atom.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (D) Statement I is incorrect but Statement II is correct.

Solution:

Concept:

- Hofmann degradation changes a primary amide $\mathrm{RCONH_2}$ into $\mathrm{RNH_2}$, removing the carbonyl carbon from the final amine skeleton.
- The migrating group moves with its bonding electron pair from the carbonyl carbon to electron-deficient nitrogen during rearrangement to an isocyanate.

Step 1: Test the restriction in Statement I The group attached to the amide carbonyl carbon can be alkyl or aryl. Both types can migrate in Hofmann rearrangement. For example, benzamide gives aniline:

$$\mathrm{C_6H_5CONH_2} \xrightarrow{\mathrm{Br_2/NaOH}} \mathrm{C_6H_5NH_2}.$$

Here the migrating group is phenyl, an aryl group, rather than an alkyl group. Consequently, the word "only" makes Statement I incorrect. The direction from carbonyl carbon to nitrogen is correct, but the imposed restriction on the identity of the migrating group is not.

Step 2: Identify the atom receiving the group After N-bromination and deprotonation, rearrangement transfers the R group from the carbonyl carbon to nitrogen as bromide leaves. Nitrogen is the electron-deficient migration terminus in this process. The migration produces an isocyanate, $\mathrm{R{-}N{=}C{=}O}$, which undergoes hydrolysis and decarboxylation to give the primary amine. Thus Statement II correctly characterizes migration to an electron-deficient atom; the migrating bond supplies the electron pair needed to form the new carbon-nitrogen bond.

Step 3: Choose the statement combination Statement I is false and Statement II is true. Option (A) incorrectly accepts the restriction to alkyl groups. Option (B) incorrectly rejects the electron-deficient nature of the migration terminus. Option (C) reverses both truth values. Option (D) alone retains the

valid mechanistic statement while rejecting the overly restrictive first statement.

Step 4: Conclusion Statement I is incorrect because aryl groups also migrate, while Statement II is correct because migration is to electron-deficient nitrogen. Hence, the correct option is (D).

Quick Tip to Solve: In rearrangement statements, check words such as "only": benzamide-to-aniline immediately tests whether aryl migration has been wrongly excluded.

70. L-isomer of a compound 'A' ($C_4H_8O_4$) gives a positive test with $[Ag(NH_3)_2]^+$. Treatment of 'A' with acetic anhydride yields triacetate derivative. Compound 'A' produces an optically active compound (B) and an optically inactive compound (C) on treatment with bromine water and HNO_3 respectively. Compound (A) is : [4 marks]

(A) Fischer projection: CHO at top, CH_2OH at bottom; OH left and H right at both C2 and C3.

(B) Fischer projection: CHO at top, CH_2OH at bottom; OH right at C2 and left at C3.

(C) Fischer projection: CHO at top, CH_2OH at bottom; OH right and H left at both C2 and C3.

(D) Fischer projection: CHO at top, CH_2OH at bottom; OH left at C2 and right at C3.

Correct Answer: (A) Fischer projection: CHO at top, CH_2OH at bottom; OH left and H right at both C2 and C3.

Solution:

Concept:

- A positive Tollens test together with three acetylatable hydroxyl groups identifies an aldotetrose, $CHO-CH(OH)-CH(OH)-CH_2OH$.
- Bromine water oxidizes the aldehyde end to an aldonic acid. Nitric acid oxidizes both terminal groups to the same carboxylic-acid group, allowing a meso product when the internal stereochemistry has symmetry.

Step 1: Use the functional-group tests The formula $C_4H_8O_4$ and formation of a triacetate show three hydroxyl groups. With the positive silver-ammonia test, the consistent open-chain structure is an aldehyde with two secondary alcohol groups and one terminal primary alcohol group. Thus the choices are the four stereoisomers of an aldotetrose. All must be distinguished by the relative positions of their hydroxyl groups and by the L designation, rather than by their gross formula alone.

Step 2: Apply the oxidation and symmetry condition Bromine water changes only CHO to $COOH$, giving $HOOC-CH(OH)-CH(OH)-CH_2OH$. Its unequal ends do not create an internal symmetry plane, and the product remains optically active. Nitric acid oxidizes both ends to give tartaric acid, $HOOC-CH(OH)-CH(OH)-COOH$. For this product to be optically inactive as a single stereoisomer, it must be meso. In its Fischer projection, the two hydroxyl groups must lie on the same side, producing the internal symmetry of meso-tartaric acid.

Step 3: Select the L member and reject the alternatives With the aldehyde at the top and CH_2OH at the bottom, the L designation requires the hydroxyl group on the lowest stereogenic carbon, C3, to lie on the left. Combining this with the same-side requirement puts both hydroxyl groups on the left: option (A), L-erythrose. Option (B) has opposite-side hydroxyl groups and gives active tartaric acid. Option (C) has the same-side arrangement but is the D isomer. Option (D) is also an opposite-side

arrangement and is D at C3. None meets all the stated tests.

Step 4: Conclusion Compound A is L-erythrose, whose Fischer projection has both hydroxyl groups on the left. Its nitric-acid oxidation gives meso-tartaric acid. Hence, the correct option is (A).

Quick Tip to Solve: For an aldotetrose, first use meso-tartaric-acid formation to require same-side OH groups; then use the lowest stereocentre to choose L or D.

71. Match List-I with List-II

	List - I		List - II
(A)	Lyophilic colloid	(I)	Liquid-liquid colloid
(B)	Emulsion	(II)	Protective colloid
(C)	Positively charged colloid	(III)	$FeCl_3$ + NaOH
(D)	Negatively charged colloid	(IV)	$FeCl_3$ + hot water

Choose the correct answer from the options given below : [4 marks]

(A) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

(B) (A) - (III), (B) - (I), (C) - (IV), (D) - (II)

(C) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

(D) (A) - (III), (B) - (II), (C) - (I), (D) - (IV)

Correct Answer: (A) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

Solution:

Concept:

- Lyophilic sols have a strong affinity for the dispersion medium. Their solvated particles can protect a lyophobic sol against coagulation.
- An emulsion has a liquid dispersed in another immiscible liquid. The charge on a sol is determined by preferential adsorption of ions from its preparation medium.

Step 1: Match the colloid types A lyophilic colloid forms a solvated protective layer around lyophobic particles, so (A) matches protective colloid (II). An emulsion is specifically a liquid-liquid colloid, so (B) matches (I). These assignments concern the type of dispersion and do not depend on the particular iron sol used in the remaining entries.

Step 2: Identify the charges of the iron hydroxide sols Hydrolysis of $FeCl_3$ in hot water produces a $Fe(OH)_3$ sol in a medium containing excess ferric ions; adsorption of Fe^{3+} gives a positively charged sol. Thus (C) matches (IV). Preparation with NaOH gives the negatively charged sol through adsorption of hydroxide ions, so (D) matches (III). The complete sequence is therefore II, I, IV, III. Option B wrongly assigns III to the lyophilic colloid, C reverses the two iron-sol charges, and D even misidentifies the emulsion.

Step 3: Conclusion The matching sequence for List-I is II, I, IV, III. Hence, the correct option is (A).

Quick Tip to Solve: For a sol-charge matching question, identify which ions are available for preferential adsorption rather than assigning the charge of the neutral hydroxide formula.

72. Match List-I with List-II

	List - I		List - II
(A)	Concentration of gold ore	(I)	Aniline
(B)	Leaching of alumina	(II)	NaOH
(C)	Froth stabiliser	(III)	SO_2
(D)	Blister copper	(IV)	NaCN

Choose the correct answer from the options given below : [4 marks]

(A) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

(B) (A) - (IV), (B) - (II), (C) - (I), (D) - (III)

(C) (A) - (III), (B) - (II), (C) - (I), (D) - (IV)

(D) (A) - (II), (B) - (IV), (C) - (III), (D) - (I)

Correct Answer: (B) (A) - (IV), (B) - (II), (C) - (I), (D) - (III)

Solution:

Concept:

- Leaching selectively dissolves a desired metal or compound by forming a soluble complex or salt, leaving suitable gangue behind.
- Froth stabilisers preserve the foam in flotation. Blister copper gets its appearance from escaping gas during solidification.

Step 1: Match the leaching reagents Gold is concentrated by cyanide leaching: cyanide forms the soluble dicyanoaurate complex. The relevant reaction is

$$4\,Au + 8\,NaCN + O_2 + 2\,H_2O \longrightarrow 4\,Na[Au(CN)_2] + 4\,NaOH.$$

Hence (A) matches NaCN, entry IV. Alumina is amphoteric and dissolves in concentrated sodium hydroxide as aluminate, so (B) matches entry II. These are different selective dissolution processes; sulfur dioxide is not the reagent for leaching alumina.

Step 2: Match froth stabilisation and blistering Aniline is used as a froth stabiliser, so (C) matches I. During copper extraction,

$$Cu_2S + 2\,Cu_2O \longrightarrow 6\,Cu + SO_2.$$

Escaping SO_2 produces the blistered appearance of the copper, so (D) matches III. The complete sequence is IV, II, I, III. Option A incorrectly puts SO_2 with alumina, option C puts SO_2 with gold, and option D puts NaOH with gold. Only the second option preserves all four chemical roles.

Step 3: Conclusion The matching sequence for List-I is IV, II, I, III. Hence, the correct option is (B).

Quick Tip to Solve: Associate cyanide with gold, caustic soda with alumina, aniline with stable froth, and escaping sulfur dioxide with blister copper.

73. Addition of H_2SO_4 to BaO_2 produces : [4 marks]

(A) BaO, SO_2 and H_2O

(B) $BaHSO_4$ and O_2

(C) $BaSO_4$, H_2 and O_2

(D) $BaSO_4$ and H_2O_2

Correct Answer: (D) $BaSO_4$ and H_2O_2

Solution:

Concept:

- A metal peroxide contains the peroxide ion ${O_2}^{2-}$, whose oxygen-oxygen bond is retained when it is protonated by a dilute acid.
- Sulfuric acid precipitates barium ions as insoluble $BaSO_4$; the peroxide ion gives H_2O_2 rather than hydrogen gas.

Step 1: Write the acid-peroxide reaction Treat barium peroxide as $Ba^{2+}{O_2}^{2-}$. Two protons convert its peroxide ion to hydrogen peroxide:

$$ {O_2}^{2-} + 2\,H^+ \longrightarrow H_2O_2. $$

The sulfate ion combines with barium to give an insoluble sulfate. Combining these changes gives the balanced preparation reaction

$$ BaO_2 + H_2SO_4 \longrightarrow BaSO_4\downarrow + H_2O_2. $$

Each side contains one barium atom, one sulfur atom, two hydrogen atoms and six oxygen atoms.

Step 2: Compare the listed products The precipitate removes barium from the solution and leaves hydrogen peroxide as the other product. Option A incorrectly proposes reduction of sulfuric acid to SO_2; this is acidification of a peroxide, not that reduction. Option B gives neither the correct sulfate composition nor the peroxide product. Option C incorrectly splits hydrogen peroxide into hydrogen and oxygen; its formation does not involve production of H_2. Although hydrogen peroxide can subsequently decompose to water and oxygen, that is a separate change and does not alter the products asked here.

Step 3: Conclusion The products are $BaSO_4$ and H_2O_2. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish a peroxide from an oxide: acid plus a peroxide yields hydrogen peroxide, whereas acid plus an ordinary oxide gives water.

74. $BeCl_2$ reacts with $LiAlH_4$ to give : [4 marks]

(A) Be + $Li[AlCl_4]$ + H_2

(B) Be + AlH_3 + LiCl + HCl

(C) BeH_2 + LiCl + $AlCl_3$

(D) BeH_2 + $Li[AlCl_4]$

Correct Answer: (C) BeH_2 + LiCl + $AlCl_3$

Solution:

Concept:

- Lithium aluminium hydride supplies hydride for conversion of suitable covalent metal chlorides into their hydrides.
- Beryllium remains in oxidation state +2 in $BeCl_2$ and BeH_2; the transformation replaces chloride ligands by hydride.

Step 1: Balance the hydride substitution One $LiAlH_4$ unit provides four hydride hydrogens, enough to form two units of BeH_2. The two $BeCl_2$ units supply four chlorides, distributed between lithium chloride and aluminium chloride. Thus the standard preparation equation is

$$2\,BeCl_2 + LiAlH_4 \longrightarrow 2\,BeH_2 + LiCl + AlCl_3.$$

This equation conserves two beryllium atoms, one lithium atom, one aluminium atom, four chlorine atoms and four hydrogen atoms. It is the usual preparation of covalent beryllium hydride from its chloride.

Step 2: Select the product combination The hydride reagent changes the groups attached to beryllium; it does not produce elemental beryllium in this preparation. Options A and B therefore fail because they contain Be rather than BeH_2. Option B also proposes hydrogen chloride, incompatible with the intended hydride substitution. The standard reaction lists LiCl and $AlCl_3$ as the accompanying products, exactly as option C states. Option D groups the chlorides into lithium tetrachloroaluminate instead of giving the standard product set asked here. Although chloroaluminate complex formation is possible in suitable media, it does not replace the conventional preparation equation or justify choosing D for this question.

Step 3: Conclusion The standard reaction gives BeH_2, LiCl and $AlCl_3$. Hence, the correct option is (C).

Quick Tip to Solve: Balance four hydride hydrogens against two beryllium dihydride units, then write the conventional chloride products as lithium chloride and aluminium chloride.

75. Match List-I with List-II.

	List-I (Si-compounds)		List-II (products)
(A)	$(CH_3)_4Si$	**(I)**	**Chain silicone**
(B)	$CH_3Si(OH)_3$	**(II)**	**Dimeric silicone**
(C)	$(CH_3)_2Si(OH)_2$	**(III)**	**Silane**
(D)	$(CH_3)_3SiOH$	**(IV)**	**2D-silicone**

Choose the correct answer from the options given below: [4 marks]

(A) (A) - (III), (B) - (II), (C) - (I), (D) - (IV)

(B) (A) - (IV), (B) - (I), (C) - (II), (D) - (III)

(C) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

(D) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (D) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Solution:

Concept:

- Condensation of silanol groups creates siloxane links: $\mathrm{Si{-}OH + HO{-}Si \longrightarrow Si{-}O{-}Si + H_2O}$. The number of hydroxyl groups controls connectivity.
- A silanol with one hydroxyl group can terminate by dimerisation; two hydroxyl groups allow chains, while three allow a cross-linked network.

Step 1: Classify the first two silicon compounds Tetramethylsilane, $(CH_3)_4Si$, has no hydroxyl groups to undergo silanol condensation. It is an organosilane rather than a chain-forming silanol, giving (A)-(III). Methylsilanetriol, $CH_3Si(OH)_3$, has three condensable hydroxyl groups per silicon atom. Condensation can therefore connect a silicon centre to several neighbours, giving the network product listed as 2D-silicone, so (B)-(IV).

Step 2: Use functionality to match chain and dimer Dimethylsilanediol, $(CH_3)_2Si(OH)_2$, has two reactive ends. Repeated condensation links these units into a chain silicone, so (C)-(I). Trimethylsilanol, $(CH_3)_3SiOH$, has only one hydroxyl group. Two molecules condense once to give

$$2\,(CH_3)_3SiOH \longrightarrow (CH_3)_3Si{-}O{-}Si(CH_3)_3 + H_2O.$$

No hydroxyl groups remain for extension, hence (D)-(II). Option A swaps the triol and monool products. Options B and C misclassify the compound with no hydroxyl groups and also fail the functionality test. The last option gives all four matches correctly.

Step 3: Conclusion The matching sequence for List-I is III, IV, I, II. Hence, the correct option is (D).

Quick Tip to Solve: Count Si-OH groups first: zero identifies the silane, one a dimer, two a chain, and three an extended network.

76. Heating white phosphorus with conc. NaOH solution gives mainly : [4 marks]

(A) Na_3P and H_2O

(B) H_3PO and NaH

(C) $P(OH)_3$ and NaH_2PO_4

(D) PH_3 and NaH_2PO_2

Correct Answer: (D) PH_3 and NaH_2PO_2

Solution:

Concept:

- White phosphorus reacts with hot concentrated alkali by disproportionation: phosphorus in oxidation state zero is both reduced and oxidised.
- Phosphorus has oxidation state -3 in PH_3 and +1 in the hypophosphite ion $H_2PO_2^-$; these are the characteristic products.

Step 1: Identify the reduced and oxidised products White phosphorus is molecular P_4. On heating with concentrated aqueous NaOH, part of it is reduced to phosphine, PH_3, while the rest is oxidised to sodium hypophosphite, NaH_2PO_2. The balanced equation is

$$P_4 + 3\,NaOH + 3\,H_2O \longrightarrow PH_3 + 3\,NaH_2PO_2.$$

The count is four phosphorus atoms, three sodium atoms, nine hydrogen atoms and six oxygen atoms on each side.

Step 2: Check the electron balance and options One phosphorus atom changes from 0 to -3, gaining three electrons. Three phosphorus atoms each change from 0 to +1, together losing three electrons. This confirms the disproportionation ratio of one phosphine to three hypophosphites. Option A gives a phosphide instead of the hydrolysis-resistant salt/gas pair formed in aqueous alkali. Options B and C include NaH, which cannot persist in water because it reacts to give hydrogen and hydroxide. Option C also substitutes phosphate for the required hypophosphite. Only D has both characteristic products.

Step 3: Conclusion Heating white phosphorus with concentrated NaOH mainly gives PH_3 and NaH_2PO_2. Hence, the correct option is (D).

Quick Tip to Solve: Check the phosphorus oxidation states: one atom at -3 balances three at +1, fixing the phosphine-to-hypophosphite ratio.

77. **Given below are two Statements:**

Statement I : Classical smog occurs in cool humid climate. It is a reducing mixture of smoke, fog and sulphur dioxide.

Statement II : Photochemical smog has components, ozone, nitric oxide, acrolein, formaldehyde, PAN etc.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (A) Both Statement I and Statement II are correct.

Solution:

Concept:

- Classical smog is associated with cool, humid conditions and consists of smoke, fog and sulfur dioxide; its overall character is reducing.
- Photochemical smog develops through sunlight-driven reactions involving nitrogen oxides and hydrocarbons. Ozone, aldehydes and peroxyacetyl nitrate are characteristic constituents.

Step 1: Evaluate Statement I Cool humid air promotes fog, and suspended smoke particles combine with this fog in the presence of SO_2. This mixture is the classical, or sulfurous, type of smog. It is conventionally described as reducing smog, in contrast to the oxidising photochemical type. Thus every feature asserted in Statement I agrees with the recognised definition: the climate, the smoke-fog mixture and sulfur dioxide all belong to classical smog.

Step 2: Evaluate Statement II and combine the verdicts Photochemical smog contains a mixture of nitrogen oxides and secondary pollutants produced through sunlight-driven atmospheric reactions. The listed components include ozone, nitric oxide, acrolein, formaldehyde and PAN, meaning peroxyacetyl nitrate. Nitric oxide participates in the nitrogen oxide cycle, while ozone, aldehydes and PAN are important products in the photochemical mixture. Thus Statement II is also correct. Option B rejects two valid statements, C incorrectly rejects the photochemical components, and D incorrectly rejects the classical-smog description. The appropriate combination is that both statements are correct.

Step 3: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: Use climate and chemical character as anchors: cool humid sulfurous smog is reducing, whereas sunny photochemical smog contains ozone and PAN.

78. Which of the following is structure of a separating funnel? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- A separating funnel separates two immiscible liquids into layers according to their densities. The liquids must be allowed to settle before either layer is withdrawn.
- The identifying features are a closed, stoppered reservoir, a narrow lower outlet and a stopcock that controls drainage. The stopcock, rather than the general funnel shape, is decisive.

Step 1: Identify the controlled outlet Option (A) has a pear-shaped reservoir with a stopper at its upper neck and a tap near its lower end. Two immiscible liquid layers can remain inside the reservoir while the tap is closed. Opening the tap drains the denser lower layer first; closing it as the interface reaches the outlet prevents the upper layer from mixing with the collected liquid. These are precisely the structural features needed for differential extraction.

Step 2: Eliminate the other apparatus Option (B) is an ordinary open conical funnel. It guides liquid into a vessel or supports filter paper, but has neither a closed reservoir nor a stopcock to control a liquid interface. Option (C) has a porous filtering plate and is a filtration funnel, suited to retaining a solid rather than separating two liquid layers. Option (D) is a bent adapter with a side arm, not a vessel in which two liquid layers can settle and be selectively drained. Thus neither an open stem, a porous plate nor a side arm serves the function of the tap shown in (A).

Step 3: Conclusion The stoppered, tap-controlled vessel in (A) is the separating funnel. Hence, the correct option is (A).

Quick Tip to Solve: Look first for a stopcock beneath a closed reservoir: a simple conical stem identifies a filter funnel instead.

79. Match List - I with List - II.

	List - I (Polymer)		List - II (Used in)
(A)	Bakelite	(I)	Radio and television cabinets
(B)	Glyptal	(II)	Electrical switches
(C)	PVC	(III)	Paints and Lacquers
(D)	Polystyrene	(IV)	Water pipes

Choose the correct answer from the options given below : [4 marks]

(A) (A) - (II), (B) - (III), (C) - (IV), (D) - (I)

(B) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

(C) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

(D) (A) - (II), (B) - (III), (C) - (I), (D) - (IV)

Correct Answer: (A) (A) - (II), (B) - (III), (C) - (IV), (D) - (I)

Solution:

Concept:

- A polymer is selected for an application by its mechanical, thermal, insulating and film-forming properties. Match each named polymer independently before comparing the complete answer codes.
- Bakelite is a thermosetting phenol-formaldehyde resin; glyptal is a polyester resin. PVC and polystyrene are addition polymers with different characteristic applications.

Step 1: Match Bakelite and glyptal Bakelite is rigid, heat resistant and a good electrical insulator. It is used to make electrical switches, giving (A)-(II). Glyptal is obtained by condensation involving ethylene glycol and phthalic acid; its resin forms protective films and is used in paints and lacquers, giving (B)-(III). These two matches already reject option (B), which pairs Bakelite with cabinets and glyptal with switches, and option (C), which pairs Bakelite with water pipes.

Step 2: Complete the remaining matches Polyvinyl chloride, or PVC, is durable, chemically resistant and readily fabricated into water pipes, so (C)-(IV). Polystyrene is moulded into radio and television cabinets, so (D)-(I). The complete sequence is therefore II, III, IV, I in List-I order. Option (D) has the first two matches correct but interchanges the characteristic uses of PVC and polystyrene, so it fails the last two checks. Only option (A) satisfies all four pairings simultaneously.

Step 3: Conclusion The matching sequence in List-I order is II, III, IV, I. Hence, the correct option is (A).

Quick Tip to Solve: Recall the anchor pairs Bakelite-switches, glyptal-paints and PVC-pipes; the remaining cabinet use belongs to polystyrene.

80. Match List - I with List - II.

	List - I		List - II
(A)	$[CH_3(CH_2)_{15}N(CH_3)_3]^+ Br^-$	**(I)**	**Dishwashing powder**
(B)	$CH_3(CH_2)_{11}-C_6H_4-SO_3^- Na^+$ **(para-substituted benzene)**	**(II)**	**Toothpaste**
(C)	$C_{17}H_{35}COO^- Na^+ + Na_2CO_3$ **+ Rosinate**	**(III)**	**Laundry soap**
(D)	$CH_3(CH_2)_{16}COO(CH_2CH_2O)_nCH_2CH_2OH$	**(IV)**	**Hair conditioner**

Choose the correct answer from the options given below : [4 marks]

(A) (A) - (III), (B) - (II), (C) - (IV), (D) - (I)

(B) (A) - (IV), (B) - (II), (C) - (III), (D) - (I)

(C) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

(D) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (B) (A) - (IV), (B) - (II), (C) - (III), (D) - (I)

Solution:

Concept:

- The charge on the surface-active part classifies a detergent: a quaternary ammonium cation is cationic, an alkylbenzene sulphonate anion is anionic, and a polyoxyethylene chain without an ionic head is non-ionic.
- Cationic detergents are used in hair conditioners, anionic detergents in toothpaste, and non-ionic detergents in dishwashing preparations. Soap mixed with washing soda and rosin is a laundry formulation.

Step 1: Identify the first two detergents In (A), nitrogen is bonded to a long alkyl chain and three methyl groups. The positively charged quaternary ammonium ion is the surface-active species, with bromide as counterion. This is a cationic detergent used in hair conditioner, hence (A)-(IV). In (B), the alkylbenzene sulphonate group carries a negative charge and sodium is the counterion. This is an anionic detergent used in toothpaste, hence (B)-(II). Option (A) and option (D) immediately fail because they assign (A) to laundry soap rather than hair conditioner.

Step 2: Identify the soap and non-ionic material Entry (C) explicitly contains sodium stearate, sodium carbonate and rosinate. Sodium stearate is a soap; the carbonate and rosin-containing components are characteristic of laundry soap, so (C)-(III). Entry (D) contains a long hydrocarbon chain and a polyoxyethylene chain ending in an alcohol group, without an ionic head. It is a non-ionic detergent used in dishwashing powder, hence (D)-(I). Option (C) incorrectly interchanges the toothpaste and laundry-soap assignments. The complete sequence IV, II, III, I occurs only in option (B).

Step 3: Conclusion The matching sequence in List-I order is IV, II, III, I. Hence, the correct option is (B).

Quick Tip to Solve: Classify the charged head first: ammonium means cationic, sulphonate means anionic, and the uncharged ether chain means non-ionic.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. If the uncertainty in velocity and position of a minute particle in space are, 2.4 $\times$ 10^{-26} (m s^{-1}) and 10^{-7} (m) respectively. The mass of the particle in g is _____. (Nearest integer)
(Given : h = 6.626 $\times$ 10^{-34} Js) [4 marks]

Solution:

Concept:

- Heisenberg's relation is $\Delta x\,\Delta p \geq h/(4\pi)$. Here Δx is position uncertainty and Δp is momentum uncertainty along the same direction.
- For a particle of fixed mass m, $\Delta p = m\Delta v$. The limiting uncertainty estimate used to determine mass sets the product equal to $h/(4\pi)$; SI inputs initially give mass in kilograms.

Step 1: Express the limiting mass Using the limiting form of the uncertainty relation, substitute the momentum uncertainty in terms of velocity uncertainty:

$$m\Delta x\Delta v = \frac{h}{4\pi}, \qquad m = \frac{h}{4\pi\Delta x\Delta v}.$$

The position uncertainty is $10^{-7}\,\mathrm{m}$ and the velocity uncertainty is $2.4 \times 10^{-26}\,\mathrm{m\,s^{-1}}$. Their product is $2.4 \times 10^{-33}\,\mathrm{m^2\,s^{-1}}$. The very small velocity uncertainty must be used with its negative exponent unchanged.

Step 2: Substitute and convert to grams Insert the supplied Planck constant and simplify the powers of ten:

$$m = \frac{6.626 \times 10^{-34}}{4\pi(2.4 \times 10^{-33})}\,\mathrm{kg} = 2.197 \times 10^{-2}\,\mathrm{kg}.$$

Since $1\,\mathrm{kg} = 10^3\,\mathrm{g}$, this is $21.97\,\mathrm{g}$, whose nearest integer is 22. The units check as $\mathrm{J\,s/(m^2\,s^{-1})} = \mathrm{kg}$. Strictly, the general inequality gives a minimum mass of this size; the numerical determination uses its limiting equality, as in the standard uncertainty estimate.

Step 3: Conclusion The limiting uncertainty mass is $21.97\,\mathrm{g}$, so the requested nearest-integer answer is $\boxed{22\,\mathrm{g}}$.

Quick Tip to Solve: Use momentum uncertainty as mass times velocity uncertainty, retain 4π in the denominator, and convert kilograms to grams only at the end.

82. 2 g of a non-volatile non-electrolyte solute is dissolved in 200 g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 8. The elevation in boiling points of A and B are in the ratio $\frac{x}{y}$ (x : y). The value of y is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- For a dilute solution, the elevation of boiling point is $\Delta T_b = iK_b m$, where K_b is the solvent's ebullioscopic constant and m is the solute molality.
- A non-electrolyte has van't Hoff factor $i = 1$. Equal masses of the same solute in equal masses of different solvents have equal molalities, even though the solvent constants differ.

Step 1: Compare the solute molalities Let the molar mass of the common solute be $M\,\mathrm{g\,mol^{-1}}$. Each solution contains $2/M$ moles of solute and $200\,\mathrm{g} = 0.200\,\mathrm{kg}$ of solvent. Consequently,

$$m_A = m_B = \frac{2/M}{0.200} = \frac{10}{M}\,\mathrm{mol\,kg^{-1}}.$$

No value of the solute molar mass is needed because the same unknown appears in both solutions. The mass of solvent, rather than the total solution mass, is used in the definition of molality.

Step 2: Take the ratio of boiling-point elevations Apply the colligative-property relation separately to the two solutions and divide:

$$\frac{\Delta T_{b,A}}{\Delta T_{b,B}} = \frac{K_{b,A} m_A}{K_{b,B} m_B} = \frac{K_{b,A}}{K_{b,B}} = \frac{1}{8}.$$

Thus the simplest integer ratio is $x : y = 1 : 8$. Solvent B has the larger constant, so its elevation must be larger; this gives an immediate check that the ratio has not been inverted. The question asks for the second entry y, rather than the first entry x or the numerical quotient x/y.

Step 3: Conclusion The ratio is $1 : 8$, giving $\boxed{y = 8}$. This is a dimensionless integer.

Quick Tip to Solve: When solute amount and solvent mass are identical, cancel molality before doing any arithmetic: the elevation ratio equals the constant ratio.

83. $2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$

In an experiment, 2.0 moles of NOCl was placed in a one-litre flask and the concentration of NO after equilibrium established, was found to be 0.4 mol/L. The equilibrium constant at 30°C is _____ $\times 10^{-4}$. [4 marks]

Solution:

Concept:

- For $2\,\mathrm{NOCl(g)} \rightleftharpoons 2\,\mathrm{NO(g)} + \mathrm{Cl_2(g)}$, the concentration equilibrium constant is $K_c = [\mathrm{NO}]^2[\mathrm{Cl_2}]/[\mathrm{NOCl}]^2$. Exponents follow the balanced equation.
- Reaction concentration changes follow stoichiometric ratios: forming two units of NO consumes two units of NOCl and forms one unit of chlorine. In a one-litre flask, mole amounts and molar concentrations have the same numerical values.

Step 1: Find all equilibrium concentrations Initially, only 2.0 moles of NOCl are present in $1\,\mathrm{L}$, so $[\mathrm{NOCl}]_0 = 2.0\,\mathrm{mol\,L^{-1}}$. The measured equilibrium NO concentration is $0.4\,\mathrm{mol\,L^{-1}}$. The equal coefficients of NOCl and NO mean that NOCl consumption is also $0.4\,\mathrm{mol\,L^{-1}}$, while chlorine formation is half as large. Hence

$$[\mathrm{NOCl}] = 1.6, \qquad [\mathrm{NO}] = 0.4, \qquad [\mathrm{Cl_2}] = 0.2$$

with all three concentrations in $\mathrm{mol\,L^{-1}}$.

Step 2: Evaluate the concentration quotient Substitute equilibrium concentrations, not the initial NOCl concentration:

$$K_c = \frac{(0.4)^2(0.2)}{(1.6)^2} = \frac{0.032}{2.56} = 0.0125 = 125 \times 10^{-4}.$$

The increase in gaseous stoichiometric coefficients is one, so the conventional concentration-unit form of K_c is $\mathrm{mol\,L^{-1}}$. No conversion to K_p is required: concentrations are given directly and the requested expression is obtained using K_c. The stated temperature specifies the equilibrium under consideration; it does not introduce a further factor into this calculation.

Step 3: Conclusion $K_c = 0.0125\,\mathrm{mol\,L^{-1}} = 125 \times 10^{-4}\,\mathrm{mol\,L^{-1}}$. The required numerical coefficient is $\boxed{125}$.

Quick Tip to Solve: Use the NO concentration to obtain the extent first; chlorine is half of NO, while the NOCl decrease equals NO formation.

84. The limiting molar conductivities of NaI, $NaNO_3$ and $AgNO_3$ are 12.7, 12.0 and 13.3 mS m^2 mol^{-1}, respectively (all at 25°C). The limiting molar conductivity of AgI at this temperature is _____ mS m^2 mol^{-1}. [4 marks]

Solution:

Concept:

- Kohlrausch's law of independent ionic migration states that limiting molar conductivity is the sum of the limiting ionic contributions, multiplied by their stoichiometric coefficients.
- For these 1:1 electrolytes each ion appears once. Combine known electrolyte conductivities so that the unwanted sodium and nitrate contributions cancel, leaving silver and iodide.

Step 1: Write the three ionic sums Denote the limiting ionic conductivity of ion X by λ°_X. At the common temperature of $25^\circ\mathrm{C}$,

$$\Lambda^\circ_{\mathrm{NaI}} = \lambda^\circ_{\mathrm{Na^+}} + \lambda^\circ_{\mathrm{I^-}},$$
$$\Lambda^\circ_{\mathrm{NaNO_3}} = \lambda^\circ_{\mathrm{Na^+}} + \lambda^\circ_{\mathrm{NO_3^-}},$$
$$\Lambda^\circ_{\mathrm{AgNO_3}} = \lambda^\circ_{\mathrm{Ag^+}} + \lambda^\circ_{\mathrm{NO_3^-}}.$$

The desired $\Lambda^\circ_{\mathrm{AgI}}$ is the sum of the silver-ion and iodide-ion terms. These are limiting quantities, so independent ionic contributions can be combined even for a sparingly soluble salt.

Step 2: Cancel spectator contributions and substitute Add the first and third sums, then subtract the

second sum. Sodium cancels once and nitrate cancels once, leaving precisely the ions of AgI:

$$\Lambda^\circ_{\mathrm{AgI}} = \Lambda^\circ_{\mathrm{NaI}} + \Lambda^\circ_{\mathrm{AgNO_3}} - \Lambda^\circ_{\mathrm{NaNO_3}}.$$

All values already use the same units, so substitute without a unit conversion:

$$\Lambda^\circ_{\mathrm{AgI}} = 12.7 + 13.3 - 12.0 = 14.0\,\mathrm{mS\,m^2\,mol^{-1}}.$$

Adding all three would count sodium and nitrate twice and therefore would not represent AgI. The subtraction is fixed by the ionic cancellation, rather than by guessing an average of the given values.

Step 3: Conclusion The limiting molar conductivity is $\boxed{14.0\,\mathrm{mS\,m^2\,mol^{-1}}}$; the numerical entry is 14.

Quick Tip to Solve: Write each electrolyte as an ionic sum and choose plus or minus signs to cancel the ions absent from the required salt.

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85. The rate constant for a first order reaction is given by the following equation:

$$\ln k = 33.24 - \frac{2.0 \times 10^4\,K}{T}$$

The activation energy for the reaction is given by _____ kJ mol^{-1}. (In nearest integer) (Given : R = 8.3 J K^{-1} mol^{-1}) [4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where A is the pre-exponential factor, E_a the activation energy, R the gas constant and T the absolute temperature.
- Taking natural logarithms gives $\ln k = \ln A - (E_a/R)(1/T)$. Thus the coefficient of $1/T$, including its sign, identifies $-E_a/R$.

Step 1: Compare the two natural-logarithm equations The given equation is

$$\ln k = 33.24 - \frac{2.0 \times 10^4\,\mathrm{K}}{T}.$$

Comparing it directly with the Arrhenius form gives

$$\ln A = 33.24, \qquad \frac{E_a}{R} = 2.0 \times 10^4\,\mathrm{K}.$$

The constant 33.24 fixes the pre-exponential factor; it is not used to calculate the activation energy. No factor 2.303 is required because both equations use natural logarithms.

Step 2: Calculate the energy and convert its unit Multiply the coefficient by the supplied gas constant.

Keeping the units visible,

$$\begin{aligned} E_a &= (2.0 \times 10^4\,\mathrm{K})(8.3\,\mathrm{J\,K^{-1}\,mol^{-1}}) \\ &= 1.66 \times 10^5\,\mathrm{J\,mol^{-1}} \\ &= 166\,\mathrm{kJ\,mol^{-1}}. \end{aligned}$$

Kelvin cancels between the coefficient and the gas constant. The negative coefficient of reciprocal temperature corresponds to a positive activation energy: increasing temperature makes the negative term smaller in magnitude, so the rate constant increases.

Step 3: Conclusion The activation energy, expressed in the requested unit and rounded to the nearest integer, is $\boxed{166\,\mathrm{kJ\,mol^{-1}}}$. The numerical entry is $\boxed{166}$.

Quick Tip to Solve: For a graph or formula of $\ln k$ versus $1/T$, multiply the magnitude of the slope by R; introduce 2.303 only when the logarithm is base ten.

86. The number of statements correct from the following for Copper (at. no. 29) is/are _____.

(A) Cu(II) complexes are always paramagnetic.

(B) Cu(I) complexes are generally colourless

(C) Cu(I) is easily oxidized

(D) In Fehling solution, the active reagent has Cu(I) [4 marks]

Solution:

Concept:

- Copper has ground-state configuration $[\mathrm{Ar}]3d^{10}4s^1$. Removing the outer electron gives $\mathrm{Cu^+}$, with $3d^{10}$, while $\mathrm{Cu^{2+}}$ has $3d^9$.
- Unpaired electrons cause paramagnetism. A filled d subshell has no ordinary d-d transitions, so copper(I) complexes are generally colourless; charge-transfer transitions can be exceptions.

Step 1: Judge statements A and B from the electron counts For copper(II), the nine electrons occupy five d orbitals. At least one electron is unpaired, regardless of whether the ligand field has a strong or weak splitting; hence ordinary mononuclear copper(II) complexes are paramagnetic. Statement (A) is correct in the coordination-chemistry sense tested here. Copper(I) is d^{10}, so all its d orbitals are filled and a d-d excitation is unavailable. Its complexes are therefore generally colourless, making statement (B) correct. The word generally allows colour arising through other kinds of electronic transition.

Step 2: Judge statements C and D chemically Copper(I) is readily oxidised to copper(II), particularly in aqueous conditions; the increased hydration stabilisation of $\mathrm{Cu^{2+}}$ is important. Thus statement (C) is correct. In Fehling solution, the active oxidising species is an alkaline copper(II)-tartrate complex. On reacting with a reducing sugar or an aldehyde, copper(II) is reduced to copper(I), which precipitates as brick-red $\mathrm{Cu_2O}$. Copper(I) is therefore found in the reduction product, not in the initial active reagent. Statement (D) is false.

Step 3: Conclusion Statements (A), (B) and (C) are correct, whereas (D) is incorrect. The required number of correct statements is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: For copper questions, write $\mathrm{Cu^+}$: d^{10} and $\mathrm{Cu^{2+}}$: d^9 first, and distinguish the blue Fehling reagent from its red $\mathrm{Cu_2O}$ product.

87. Acidified potassium permanganate solution oxidises oxalic acid. The spin-only magnetic moment of the manganese product formed from the above reaction is _____ B.M. (Nearest integer)

[4 marks]

Solution:

Concept:

- In acidic solution, permanganate is reduced from manganese(VII) to manganese(II), while oxalic acid is oxidised to carbon dioxide.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons and μ_B is the Bohr magneton (B.M.).

Step 1: Identify the manganese product in acidic medium The reduction half-reaction is

$$\mathrm{MnO_4^-} + 8\,\mathrm{H^+} + 5\,\mathrm{e^-} \longrightarrow \mathrm{Mn^{2+}} + 4\,\mathrm{H_2O}.$$

Combining this with oxalic-acid oxidation gives

$$2\,\mathrm{MnO_4^-} + 5\,\mathrm{H_2C_2O_4} + 6\,\mathrm{H^+} \longrightarrow 2\,\mathrm{Mn^{2+}} + 10\,\mathrm{CO_2} + 8\,\mathrm{H_2O}.$$

Both the atoms and the total charge balance. Thus the manganese product is $\mathrm{Mn^{2+}}$, not $\mathrm{MnO_2}$, which is associated with neutral or weakly alkaline reduction conditions. The acidic medium is decisive for selecting the oxidation state.

Step 2: Count the unpaired electrons and evaluate the moment Neutral manganese has atomic number 25 and configuration $[\mathrm{Ar}]3d^5 4s^2$. The two $4s$ electrons are removed first, giving

$$\mathrm{Mn^{2+}} : [\mathrm{Ar}]3d^5.$$

In the aqueous product this is a high-spin configuration, with one electron in each of the five d orbitals. Hence $n = 5$, and

$$\mu = \sqrt{5(5+2)}\,\mu_B = \sqrt{35}\,\mu_B \simeq 5.916\,\mu_B.$$

Rounding is performed only after evaluating the square root, since the question asks for the nearest integer.

Step 3: Conclusion The spin-only magnetic moment of the manganese product is approximately 5.92 B.M., which rounds to $\boxed{6\ \mathrm{B.M.}}$. The numerical entry is $\boxed{6}$.

Quick Tip to Solve: Choose the permanganate reduction product from the reaction medium before counting electrons: acidic permanganate gives high-spin $\mathrm{Mn^{2+}}$, with five unpaired electrons.

88. Two elements A and B which form 0.15 moles of A_2B and AB_3 type compounds. If both A_2B and AB_3 weigh equally, then the atomic weight of A is _____ times of atomic weight of B. [4 marks]

Solution:

Concept:

- The mass of a compound is $m = nM$, where n is its amount in moles and M is its molar mass. Equal mole amounts with equal masses must have equal molar masses.
- Let the atomic weights of A and B be a and b. The formulae give molar masses proportional to $2a + b$ for A_2B and $a + 3b$ for AB_3.

Step 1: Express the two sample masses using their formulae Each compound is present in an amount of 0.15 mol. Therefore its mass is obtained by multiplying the formula molar mass by the same mole amount:

$$m(A_2B) = 0.15(2a + b),$$
$$m(AB_3) = 0.15(a + 3b).$$

Here the atomic weights are used numerically as the corresponding atomic molar masses in $\mathrm{g\,mol^{-1}}$, so both expressions give masses in grams. The stoichiometric subscripts multiply the appropriate atomic contribution; they do not change the stated mole amount of compound.

Step 2: Use the equality of masses and solve the ratio The problem states that the two samples weigh equally. Consequently,

$$0.15(2a + b) = 0.15(a + 3b),$$
$$2a + b = a + 3b,$$
$$a = 2b.$$

The common factor 0.15 cancels because the numbers of formula units are equal. Dividing by the positive atomic weight b gives $a/b = 2$. As a check, substituting $a = 2b$ makes each formula mass equal to $5b$, so equal 0.15 mol samples indeed have equal masses.

Step 3: Conclusion The atomic weight of A is $\boxed{2}$ times the atomic weight of B. The requested ratio is dimensionless, so the numerical entry is $\boxed{2}$.

Quick Tip to Solve: When two samples have both equal mole amounts and equal masses, equate their formula molar masses directly; the common number of moles cancels.

89. Total number of possible stereoisomers of dimethyl cyclopentane is _____. [4 marks]

Solution:

Concept:

- Two identical methyl substituents on different carbons of cyclopentane can have cis or trans relative configurations. A cis pair lies on the same face of the ring; a trans pair lies on opposite faces.
- A chiral configuration and its mirror image count as two stereoisomers. A configuration with an internal symmetry element is achiral and counts once. Rapidly interconverting ring conformations are not counted separately.

Step 1: List the distinct arrangements that exhibit stereoisomerism With one methyl group at carbon 1, the second may be at carbon 2 or carbon 3. Arrangements called 1,5 and 1,4 are identical to 1,2 and 1,3, respectively, by reversing the ring numbering. The geminal 1,1 arrangement has both identical methyl groups on one carbon and has no cis-trans pair or asymmetric substitution pattern. Thus the configurational stereoisomer count concerns the 1,2 and 1,3 arrangements, rather than counting 1,1-dimethylcyclopentane as an additional stereoisomer of a different constitution.

Step 2: Count the cis and trans configurations for each arrangement For 1,2-dimethylcyclopentane, the cis configuration has identical substituents on the same face and an internal symmetry relation between its substituted carbons. It is achiral and counts once. The trans configuration is chiral and exists as an enantiomeric pair, counting twice. Its subtotal is therefore $1 + 2 = 3$. For 1,3-dimethylcyclopentane the same count applies: cis is an achiral form and trans has two mirror-image configurations. Its subtotal is also three. Ring puckering changes conformations, but does not supply additional stable configurations for this count.

Step 3: Conclusion The total number of configurational stereoisomers for the dimethylcyclopentane arrangements that exhibit stereoisomerism is

$$N = 3\ (1,2) + 3\ (1,3) = \boxed{6}.$$

This is a dimensionless count.

Quick Tip to Solve: For identical disubstituted cycloalkanes, first remove equivalent position numbers, then check whether each cis/trans form is achiral or an enantiomeric pair; do not multiply every form by two.

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90. Metal deficiency defect is shown by $Fe_{0.93}O$. In the crystal, some Fe^{2+} cations are missing and loss of positive charge is compensated by the presence of Fe^{3+} ions. The percentage of Fe^{2+} ions in the $Fe_{0.93}O$ crystals is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- A macroscopic ionic crystal is electrically neutral. In $Fe_{0.93}O$, each oxide ion contributes charge -2, which must be balanced by the iron ions present.
- If iron occurs as Fe^{2+} and Fe^{3+}, the stoichiometric iron amount gives one equation and charge balance gives another. The percentage of a species is calculated relative to the total iron ions actually present.

Step 1: Set up the composition and charge equations Take one mole of oxide ions as the basis. Let x mol of the iron be Fe^{2+} and y mol be Fe^{3+}. The formula fixes the total iron amount:

$$x + y = 0.93.$$

Since the oxide charge is -2 per formula basis, electrical neutrality requires

$$2x + 3y = 2.$$

The vacancies carry no positive ionic charge; their effect is already expressed by the reduced iron content 0.93. Counting them as additional iron ions would give an incorrect denominator.

Step 2: Find the iron(II) amount and calculate its percentage Subtract twice the first equation from the charge equation:

$$\begin{aligned}(2x + 3y) - 2(x + y) &= 2 - 2(0.93),\\ y &= 0.14,\\ x &= 0.93 - 0.14 = 0.79.\end{aligned}$$

Check the positive charge: $2(0.79) + 3(0.14) = 1.58 + 0.42 = 2.00$, exactly balancing oxide. The fraction of iron ions that are divalent is therefore

$$\frac{x}{x + y} = \frac{0.79}{0.93}.$$

Multiplying by 100 gives

$$\%\mathrm{Fe}^{2+} = \frac{0.79}{0.93} \times 100 = 84.946\ldots\%.$$

The value 79 would refer to divalent iron per 100 ideal iron sites, not its percentage among the iron ions present.

Step 3: Conclusion The percentage of Fe^{2+} ions among the iron ions in $\mathrm{Fe}_{0.93}\mathrm{O}$, rounded to the nearest integer, is $\boxed{85\%}$. The required numerical entry is $\boxed{85}$.

Quick Tip to Solve: For $\mathrm{Fe}_{1-\delta}\mathrm{O}$, charge balance gives 2δ ferric ions and $1 - 3\delta$ ferrous ions per oxide; divide by $1 - \delta$, the actual total iron content, for percentages.