JEE Main 2022 Paper 1 (B.E./B.Tech) Jun 27 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 1 (June), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 27 June 2022, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The number of points of intersection of

$|z-(4+3i)|=2$ **and** $|z|+|z-4|=6$**, z** $\in$ **C, is :** [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (C) 2

Solution:

Concept:

- Write $z=x+iy$. The modulus $|z-(a+ib)|$ is the distance of (x,y) from (a,b), so the first locus is a circle.
- A constant sum of distances from two fixed points defines an ellipse. Here the foci are $(0,0),(4,0)$, the centre is $(2,0)$, and $a=3, c=2, b^2=a^2-c^2=5$.

Step 1: Express the two curves in Cartesian form The circle and ellipse respectively have equations

$$(x-4)^2+(y-3)^2=4, \qquad \frac{(x-2)^2}{9}+\frac{y^2}{5}=1.$$

The circle has $1\le y\le 5$, whereas the ellipse has $|y|\le\sqrt{5}<3$. Thus any intersection must lie on the upper half of the ellipse and the lower semicircle. Their common possible horizontal interval is $2\le x\le 5$.

Step 2: Count the crossings without solving a quartic On that interval define

$$g(x)=3-\sqrt{4-(x-4)^2}-\sqrt{5\left(1-\frac{(x-2)^2}{9}\right)}.$$

The first curve is strictly convex and the ellipse upper arc is strictly concave, so their difference is strictly convex in the interior. It can have at most two zeros: three zeros would contradict the strict convexity inequality at the middle one. Now

$$g(2)=3-\sqrt{5}>0, \quad g(4)=1-\frac{5}{3}<0, \quad g(5)=3-\sqrt{3}>0.$$

Continuity gives one zero in $(2,4)$ and one in $(4,5)$. There are no intersections outside these arcs or this interval. Options giving zero, one or three therefore fail.

Step 3: Conclusion There are exactly 2 distinct points of intersection. Hence, the correct option is (C).

Quick Tip to Solve: For a circle meeting an ellipse, restrict to the possible arcs first; opposite signs at three ordered points, together with convexity, can count intersections exactly.

2. If a_1, a_2, a_3 ..... and b_1, b_2, b_3 ..... are A.P., and $a_1 = 2$, $a_{10} = 3$, $a_1b_1 = 1 = a_{10}b_{10}$, then $a_4\, b_4$ is equal to - [4 marks]

(A) $\frac{35}{27}$

(B) 1

(C) $\frac{27}{28}$

(D) $\frac{28}{27}$

Correct Answer: (D) $\frac{28}{27}$

Solution:

Concept:

- For an arithmetic progression, $a_n = a_1 + (n-1)d_a$, where d_a is its constant common difference.
- The products at two positions determine the corresponding terms of the second progression; the products themselves need not form an arithmetic progression.

Step 1: Determine both common differences From $a_1 = 2$ and $a_{10} = 3$, there are nine equal increments, hence

$$d_a = \frac{3-2}{9} = \frac{1}{9}.$$

The product conditions give

$$b_1 = \frac{1}{a_1} = \frac{1}{2}, \qquad b_{10} = \frac{1}{a_{10}} = \frac{1}{3}.$$

Consequently the second common difference is negative:

$$d_b = \frac{b_{10} - b_1}{9} = \frac{1/3 - 1/2}{9} = -\frac{1}{54}.$$

This sign is consistent with the decrease of the second progression.

Step 2: Evaluate the fourth terms and their product The fourth term lies three increments after the first, not four. Thus

$$a_4 = 2 + 3\left(\frac{1}{9}\right) = \frac{7}{3}, \qquad b_4 = \frac{1}{2} + 3\left(-\frac{1}{54}\right) = \frac{4}{9}.$$

Multiplying the actual terms yields

$$a_4 b_4 = \frac{7}{3} \cdot \frac{4}{9} = \frac{28}{27}.$$

As a check, a_4 is one third of the way from 2 to 3, and b_4 is one third of the way from $1/2$ to $1/3$. The product is slightly greater than 1, so neither 1 nor its reciprocal $27/28$ works; $35/27$ is not the product of these fourth terms.

Step 3: Conclusion $a_4 b_4 = \frac{28}{27}$. Hence, the correct option is (D).

Quick Tip to Solve: Interpolate each progression separately. Equal endpoint products do not imply equal products at intermediate positions.

3. If m and n respectively are the number of local maximum and local minimum points of the function $f(x) = \int_0^{x^2} \frac{t^2-5t+4}{2+e^t}dt$, then the ordered pair (m, n) is equal to [4 marks]

(A) (3, 2)

(B) (2, 3)

(C) (2, 2)

(D) (3, 4)

Correct Answer: (B) (2, 3)

Solution:

Concept:

- By the fundamental theorem of calculus and the chain rule, $\frac{d}{dx}\int_0^{x^2} h(t)\,dt = 2x\,h(x^2)$.
- A local maximum occurs when the derivative changes from positive to negative; a local minimum occurs for a negative-to-positive change. A zero derivative alone does not decide the type.

Step 1: Differentiate and locate all stationary points The denominator $2 + e^t$ is always positive, so the integrand is continuous for every real t. Differentiating the given function gives

$$f\prime(x) = \frac{2x(x^4 - 5x^2 + 4)}{2 + e^{x^2}} = \frac{2x(x^2 - 1)(x^2 - 4)}{2 + e^{x^2}}.$$

Therefore its only stationary points are $x = -2, -1, 0, 1, 2$. The positive denominator cannot change the sign, and each of these numerator roots is simple.

Step 2: Classify the stationary points by sign changes In the consecutive intervals separated by these five points, the derivative signs are

x	$(-\infty,-2)$	$(-2,-1)$	$(-1,0)$	$(0,1)$	$(1,2)$	$(2,\infty)$
$f\prime(x)$	$-$	$+$	$-$	$+$	$-$	$+$

Thus $-2, 0, 2$ are local minima and $-1, 1$ are local maxima. The function is even, so its extrema away from the origin should occur in symmetric pairs, which checks the classification. Counting only positive roots would omit genuine extrema, while interchanging maxima and minima reverses the requested ordering. The other offered pairs do not match this complete sign analysis.

Step 3: Conclusion The counts are $m = 2$ and $n = 3$, so $(m, n) = (2, 3)$. Hence, the correct option is (B).

Quick Tip to Solve: For an upper limit x^2, remember the chain-rule factor $2x$; it creates the stationary point at the origin.

4. Let f be a differentiable function in $\left(0, \frac{\pi}{2}\right)$. If $\int_{\cos x}^{1} t^2 f(t)dt = \sin^3 x + \cos x$, then $\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to [4 marks]

(A) $6 - 9\sqrt{2}$

(B) $6 - \frac{9}{\sqrt{2}}$

(C) $\frac{9}{2} - 6\sqrt{2}$

(D) $\frac{9}{\sqrt{2}} - 6$

Correct Answer: (B) $6 - \frac{9}{\sqrt{2}}$

Solution:

Concept:

- For a variable lower limit, $\frac{d}{dx}\int_{u(x)}^{1} h(t)\,dt = -h(u(x))u\prime(x)$. The two negative signs cancel when $u(x) = \cos x$.
- After differentiation, put $u = \cos x$, with $\sin x = \sqrt{1-u^2}$ on $0 < x < \pi/2$. This expresses the required derivative in terms of its actual argument.

Step 1: Differentiate the stated relation Applying the fundamental theorem of calculus on the open interval gives

$$\cos^2 x\, f(\cos x) \sin x = 3\sin^2 x \cos x - \sin x.$$

Since $\sin x > 0$, divide by it to obtain

$$\cos^2 x\, f(\cos x) = 3\sin x \cos x - 1.$$

With $u = \cos x$, the resulting necessary expression is

$$f(u) = \frac{3\sqrt{1-u^2}}{u} - \frac{1}{u^2}.$$

In particular the positive square root is required; changing its sign would change the answer.

Step 2: Differentiate with respect to the argument and substitute The derivative of the first quotient simplifies because $u^2 + (1-u^2) = 1$:

$$f\prime(u) = -\frac{3}{u^2\sqrt{1-u^2}} + \frac{2}{u^3}.$$

Multiplying by u, which at the required point equals $1/\sqrt{3}$, gives

$$uf\prime(u) = -\frac{3}{u\sqrt{1-u^2}} + \frac{2}{u^2}.$$

At $u = 1/\sqrt{3}$, the square root is $\sqrt{2/3}$, so the expression becomes

$$\frac{1}{\sqrt{3}} f\prime\left(\frac{1}{\sqrt{3}}\right) = 6 - \frac{9}{\sqrt{2}}.$$

The other expressions have either the wrong radical factor or the opposite sign and do not result from

this differentiation.

Step 3: Conclusion The derivative calculation requested by the relation gives $6 - \frac{9}{\sqrt{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: First express $f(u)$ in its own variable; differentiating $f(\cos x)$ again without tracking the chain factor is a common source of a wrong radical.

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5. The integral $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}} dx$, where [.] denotes the greatest integer function, is equal to [4 marks]

(A) $1 + 6\log_e\left(\frac{6}{7}\right)$

(B) $1 - 6\log_e\left(\frac{6}{7}\right)$

(C) $\log_e\left(\frac{7}{6}\right)$

(D) $1 - 7\log_e\left(\frac{6}{7}\right)$

Correct Answer: (A) $1 + 6\log_e\left(\frac{6}{7}\right)$

Solution:

Concept:

- For an integer $n \geq 1$, $\lfloor 1/x \rfloor = n$ on $1/(n+1) < x \leq 1/n$, apart from endpoint conventions that do not affect an integral.
- The convergent power series $\sum_{n=1}^{\infty} r^n/n = -\log(1-r)$ for $|r| < 1$ converts the resulting series into a logarithm.

Step 1: Split the interval at the reciprocal integers On the interval $(1/(n+1), 1/n]$, the integrand is the constant 7^{-n}. Its contribution is its height times its width, so

$$I = \sum_{n=1}^{\infty} \frac{1}{7^n}\left(\frac{1}{n} - \frac{1}{n+1}\right).$$

The integrand tends to zero as $x \to 0^+$, and each summand is positive and bounded by 7^{-n}, ensuring convergence. The isolated endpoints where the floor value jumps have no effect on the area.

Step 2: Sum the two logarithmic series Set $r = 1/7$ and $L = -\log(1-r)$. Then

$$\sum_{n=1}^{\infty} \frac{r^n}{n} = L, \qquad \sum_{n=1}^{\infty} \frac{r^n}{n+1} = \frac{L-r}{r}.$$

Subtracting gives

$$I = L - \frac{L-r}{r} = 1 - 6L = 1 + 6\log\left(\frac{6}{7}\right).$$

As a numerical check this is about 0.0751, consistent with a positive integrand at most $1/7$. The second and fourth options exceed 1; the third is $\log(7/6) > 1/7$, and hence is also impossible. Thus only the first expression has the required value and bound.

Step 3: Conclusion $I = 1 + 6\log_e(6/7)$. Hence, the correct option is (A).

Quick Tip to Solve: When a floor contains $1/x$, partition at $x = 1/n$; the interval widths are $1/n - 1/(n+1)$.

6. If the solution curve of the differential equation $((\tan^{-1}y) - x)dy = (1 + y^2)dx$ passes through the point (1, 0), then the abscissa of the point on the curve whose ordinate is tan(1), is [4 marks]

(A) 2e

(B) $\frac{2}{e}$

(C) 2

(D) $\frac{1}{e}$

Correct Answer: (B) $\frac{2}{e}$

Solution:

Concept:

- Treat x as a function of y. The equation becomes linear in x, unlike its appearance when one tries to solve directly for $y(x)$.
- The substitution $t = \tan^{-1} y$ has $dt = dy/(1 + y^2)$, reducing the equation to $dx/dt + x = t$. Its integrating factor is e^t.

Step 1: Transform and integrate the linear equation Dividing the stated differential equation by $1 + y^2$ gives

$$dx = (\tan^{-1} y - x)\frac{dy}{1 + y^2}.$$

Put $t = \tan^{-1} y$. Then

$$\frac{dx}{dt} + x = t, \qquad \frac{d}{dt}(xe^t) = te^t.$$

Integration by parts gives

$$xe^t = e^t(t - 1) + C, \qquad x = t - 1 + Ce^{-t}.$$

This form also makes it straightforward to check the transformed equation: differentiating it yields $dx/dt = 1 - Ce^{-t} = t - x$.

Step 2: Apply the given point and evaluate the required ordinate At the point $(x, y) = (1, 0)$, the substituted variable is $t = \tan^{-1} 0 = 0$. Hence

$$1 = -1 + C, \qquad C = 2.$$

The particular curve is therefore

$$x = \tan^{-1} y - 1 + 2e^{-\tan^{-1} y}.$$

For $y = \tan 1$, the angle 1 radian lies in the principal arctangent interval $(-\pi/2, \pi/2)$, so $t = 1$. It follows that

$$x = 1 - 1 + 2e^{-1} = \frac{2}{e}.$$

The choices $2e$, 2 and $1/e$ respectively have the wrong exponential sign, omit the exponential factor, or miss the integration constant.

Step 3: Conclusion The required abscissa is $\frac{2}{e}$. Hence, the correct option is (B).

Quick Tip to Solve: If the equation contains both $\tan^{-1} y$ and $dy/(1+y^2)$, substitute the angle first and solve linearly for x.

7. If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x + y - 29 = 0$, is $x^2 + ay^2 + bxy + cx + dy + k = 0$, then $a + b + c + d + k$ is equal to : [4 marks]

(A) 575

(B) -575

(C) 576

(D) -576

Correct Answer: (D) -576

Solution:

Concept:

- The vertex lies halfway between the focus and its perpendicular projection onto the directrix. The axis is normal to the directrix.
- A parabola consists of points equidistant from the focus and the directrix. For $Ax + By + C = 0$, the perpendicular distance is $|Ax + By + C|/\sqrt{A^2 + B^2}$.

Step 1: Find the foot of the perpendicular and the focus At the vertex $V = (5, 4)$, the directrix expression is $3(5) + 4 - 29 = -10$, and the squared length of its normal $(3, 1)$ is 10. Thus the perpendicular foot is

$$H = (5, 4) - \frac{-10}{10}(3, 1) = (8, 5).$$

Reflecting this foot in the vertex gives the focus

$$F = 2V - H = (10, 8) - (8, 5) = (2, 3).$$

Both VF and VH have length $\sqrt{10}$, confirming that the vertex is correctly positioned on the axis.

Step 2: Use equal distances and compare the coefficients For an arbitrary point (x, y), the squared-distance equality is

$$(x - 2)^2 + (y - 3)^2 = \frac{(3x + y - 29)^2}{10}.$$

Expanding after multiplication by 10 gives

$$x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0.$$

The coefficient of x^2 is already 1, exactly as in the required normalisation. Hence

$$a = 9, \quad b = -6, \quad c = 134, \quad d = -2, \quad k = -711.$$

Their sum is

$$9 - 6 + 134 - 2 - 711 = -576.$$

Substituting the vertex in the expanded equation gives zero, an additional check on the signs. The positive choices have the wrong sign and -575 differs by one from the requested sum, which excludes the coefficient of x^2.

Step 3: Conclusion $a + b + c + d + k = -576$. Hence, the correct option is (D).

Quick Tip to Solve: Find the focus by reflecting the directrix foot in the vertex; normalise the x^2 coefficient before adding the requested coefficients.

8. The set of values of k, for which the circle $C : 4x^2 + 4y^2 - 12x + 8y + k = 0$ lies inside the fourth quadrant and the point $\left(1, -\frac{1}{3}\right)$ lies on or inside the circle C, is : [4 marks]

(A) an empty set

(B) $\left(6, \frac{65}{9}\right]$

(C) $\left[\frac{80}{9}, 10\right)$

(D) $\left(9, \frac{92}{9}\right]$

Correct Answer: (D) $\left(9, \frac{92}{9}\right]$

Solution:

Concept:

- Complete squares to read the centre and radius of a circle. A circle wholly inside the fourth quadrant has positive horizontal coordinates and negative vertical coordinates at every point.
- For centre (h, k_0), its radius must be smaller than both positive distances from the centre to the coordinate axes. A point lies on or inside a circle when its squared distance from the centre is at most the squared radius.

Step 1: Find the centre and radius Divide the equation by four and complete the squares, preserving the constant parameter k:

$$x^2 + y^2 - 3x + 2y + \frac{k}{4} = 0,$$

$$\left(x - \frac{3}{2}\right)^2 + (y + 1)^2 = \frac{13 - k}{4}.$$

Thus the centre is $(3/2, -1)$ and $r^2 = (13 - k)/4$. The centre itself is in the required quadrant.

Step 2: Impose the quadrant condition The distances of the centre from the vertical and horizontal axes are $3/2$ and 1, respectively. The horizontal axis is the nearer boundary, so the entire circle remains strictly inside the quadrant precisely when $r < 1$. Therefore

$$\frac{13 - k}{4} < 1 \quad \Longrightarrow \quad k > 9.$$

The point condition below also ensures a real, positive radius.

Step 3: Include the specified point The squared distance of $(1, -1/3)$ from the centre is

$$\left(1-\frac{3}{2}\right)^2+\left(-\frac{1}{3}+1\right)^2=\frac{1}{4}+\frac{4}{9}=\frac{25}{36}.$$

Consequently,

$$\frac{25}{36}\le\frac{13-k}{4}\quad\Longrightarrow\quad k\le\frac{92}{9}.$$

Intersecting this closed upper constraint with $k>9$ leaves a nonempty interval. The endpoint $k=9$ is excluded because the circle touches the horizontal axis, whereas $k=92/9$ is allowed because the specified point may lie on the circumference. None of the other listed intervals satisfies these two bounds together.

Step 4: Conclusion The required set is $\boxed{(9, 92/9]}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the smaller centre-to-axis distance to bound the radius, then intersect with the point-in-circle inequality.

9. The shortest distance between the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$, is : [4 marks]

(A) $\frac{18}{\sqrt{5}}$

(B) $\frac{22}{3\sqrt{5}}$

(C) $\frac{46}{3\sqrt{5}}$

(D) $6\sqrt{3}$

Correct Answer: (A) $\frac{18}{\sqrt{5}}$

Solution:

Concept:

- For two nonparallel lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$ with direction vectors $\mathbf{b}_1, \mathbf{b}_2$, the common perpendicular is parallel to $\mathbf{b}_1 \times \mathbf{b}_2$.
- The shortest distance is the magnitude of the projection of $\mathbf{a}_2 - \mathbf{a}_1$ on that perpendicular: $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.

Step 1: Read points and directions From the numerators, take points $P = (3, 2, 1)$ and $Q = (-3, 6, 5)$. The second numerator is $x + 3$, so its point has horizontal coordinate -3, not 3. The denominators give

$$\mathbf{b}_1 = (2, 3, -1), \qquad \mathbf{b}_2 = (2, 1, 3), \qquad \overrightarrow{PQ} = (-6, 4, 4).$$

Changing the point on either line does not change its perpendicular projection.

Step 2: Find the perpendicular direction Expand the cross product component by component:

$$\mathbf{b}_1 \times \mathbf{b}_2 = (3 \cdot 3 - (-1) \cdot 1,\ (-1) \cdot 2 - 2 \cdot 3,\ 2 \cdot 1 - 3 \cdot 2) = (10, -8, -4).$$

Its magnitude is

$$\sqrt{10^2 + (-8)^2 + (-4)^2} = \sqrt{180} = 6\sqrt{5}.$$

The nonzero cross product shows the lines are not parallel.

Step 3: Project the joining vector The scalar triple product is

$$(-6, 4, 4) \cdot (10, -8, -4) = -60 - 32 - 16 = -108.$$

Distance is nonnegative, so use the absolute value:

$$d = \frac{|-108|}{6\sqrt{5}} = \frac{18}{\sqrt{5}}.$$

As a check, $d^2 = 324/5$. The squares of the remaining choices are $484/45$, $2116/45$, and 108, respectively; none equals $324/5$. The nonzero triple product also confirms the lines are skew rather than intersecting.

Step 4: Conclusion The shortest distance is $\boxed{18/\sqrt{5}}$ coordinate units. Hence, the correct option is (A).

Quick Tip to Solve: Take the direction ratios from the denominators, but take each point coordinate by setting its numerator equal to zero.

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10. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute, $|\vec{a}| = 1$, and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$. If $\vec{c} = 2\sqrt{2}\left(\vec{a} \times \vec{b}\right) - 2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is : [4 marks]

(A) $\frac{\pi}{4}$

(B) $-\frac{\pi}{4}$

(C) $\frac{5\pi}{6}$

(D) $\frac{3\pi}{4}$

Correct Answer: (D) $\frac{3\pi}{4}$

Solution:

Concept:

- The area of a parallelogram is half the magnitude of the cross product of its diagonal vectors: $A = \frac{1}{2}|\mathbf{a} \times \mathbf{b}|$.
- For an acute angle θ, equality of the magnitudes of dot and cross products gives $\cos\theta = \sin\theta$. Also, $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{b}$, allowing orthogonal components to be combined by Pythagoras.

Step 1: Find the angle and the second diagonal The area condition gives

$$|\mathbf{a} \times \mathbf{b}| = 2(2\sqrt{2}) = 4\sqrt{2}.$$

Since θ is acute, the dot product is positive. Cancel the nonzero factor $|\mathbf{a}||\mathbf{b}|$ from the given equality

to obtain $\cos\theta = \sin\theta$, so $\theta = \pi/4$. Using $|\mathbf{a}| = 1$,

$$4\sqrt{2} = |\mathbf{b}|\sin(\pi/4) = \frac{|\mathbf{b}|}{\sqrt{2}}, \qquad |\mathbf{b}| = 8.$$

Step 2: Resolve perpendicular components In $\mathbf{c} = 2\sqrt{2}(\mathbf{a}\times\mathbf{b}) - 2\mathbf{b}$, the first component is perpendicular to $\mathbf{b}$, while the second points opposite to it. Their magnitudes are

$$2\sqrt{2}(4\sqrt{2}) = 16, \qquad 2|\mathbf{b}| = 16.$$

Thus

$$|\mathbf{c}| = \sqrt{16^2 + 16^2} = 16\sqrt{2}.$$

The equal magnitudes are useful, but the negative parallel component must be retained when choosing the angle.

Step 3: Evaluate the required angle Let $\phi \in [0, \pi]$ be the angle between $\mathbf{b}$ and $\mathbf{c}$. Orthogonality gives

$$\mathbf{b}\cdot\mathbf{c} = 0 - 2|\mathbf{b}|^2 = -128,$$

$$\cos\phi = \frac{-128}{8(16\sqrt{2})} = -\frac{1}{\sqrt{2}}.$$

Therefore $\phi = 3\pi/4$. The acute option $\pi/4$ has positive cosine; a negative angle is outside the standard vector-angle range, and $5\pi/6$ has cosine $-\sqrt{3}/2$, which does not match.

Step 4: Conclusion The required angle is $\boxed{3\pi/4}$. Hence, the correct option is (D).

Quick Tip to Solve: When a vector combines a cross product with a multiple of one factor, those two components are perpendicular.

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11. The mean and variance of the data 4, 5, 6, 6, 7, 8, x, y, where x $<$ y, are 6 and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to : [4 marks]

(A) 162

(B) 320

(C) 674

(D) 420

Correct Answer: (B) 320

Solution:

Concept:

- For N observations with mean $\bar{x}$, the variance is $\sigma^2 = \frac{1}{N}\sum x_i^2 - \bar{x}^2$. The stated data variance uses the divisor N, rather than the sample divisor $N-1$.
- The identities $(x+y)^2 = x^2+y^2+2xy$ and $t^2-(x+y)t+xy = 0$ recover two unknown observations from their sum and sum of squares.

Step 1: Use the mean to obtain the sum There are eight observations. The six known values sum to

$$4 + 5 + 6 + 6 + 7 + 8 = 36.$$

Their total including x, y must be $8 \cdot 6 = 48$, and hence

$$x + y = 48 - 36 = 12.$$

Keep both occurrences of 6, since repeated observations each contribute to both the sum and the sum of squares.

Step 2: Use the variance to obtain the sum of squares The total sum of squares is

$$\sum x_i^2 = 8\left(\frac{9}{4} + 6^2\right) = 8 \cdot \frac{153}{4} = 306.$$

The known contributions are

$$4^2 + 5^2 + 6^2 + 6^2 + 7^2 + 8^2 = 16 + 25 + 36 + 36 + 49 + 64 = 226.$$

Thus $x^2 + y^2 = 306 - 226 = 80$. Squaring their sum now gives

$$144 = 80 + 2xy, \qquad xy = 32.$$

Step 3: Identify the ordered pair and evaluate The two observations are roots of

$$t^2 - 12t + 32 = (t - 4)(t - 8) = 0.$$

The restriction $x < y$ fixes $x = 4$ and $y = 8$; interchanging them would change the requested expression because the powers differ. Therefore

$$x^4 + y^2 = 4^4 + 8^2 = 256 + 64 = 320.$$

As an independent variance check, the squared deviations of the six known values from 6 sum to $4 + 1 + 0 + 0 + 1 + 4 = 10$, and the two new deviations contribute $4 + 4 = 8$. Their total 18, divided by 8, is $9/4$, as required. Only the second listed value agrees; 162, 674 and 420 do not.

Step 4: Conclusion The required value is $\boxed{320}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the mean for the sum and the variance for the sum of squares before solving for individual observations.

12. If a point A(x, y) lies in the region bounded by the y-axis, straight lines 2y + x = 6 and 5x − 6y = 30, then the probability that y < 1 is : [4 marks]

(A) $\frac{1}{6}$
(B) $\frac{5}{6}$
(C) $\frac{2}{3}$
(D) $\frac{6}{7}$

Correct Answer: (B) $\frac{5}{6}$

Solution:

Concept:

- For a point chosen uniformly from a planar region, the probability of a subregion is its area divided by the total area. Boundary lines themselves have zero area.
- The area of a triangle is half the product of a chosen base and its perpendicular height. The complement $y \geq 1$ is a simpler triangular cap in this problem.

Step 1: Find the complete bounded region The vertical axis is $x = 0$. Its intersections with $x + 2y = 6$ and $5x - 6y = 30$ are $P = (0, 3)$ and $Q = (0, -5)$. To intersect the oblique lines, put $x = 6 - 2y$ in the second equation:

$$5(6 - 2y) - 6y = 30 \quad \Longrightarrow \quad -16y = 0.$$

Thus their intersection is $R = (6, 0)$, and the bounded region is triangle PQR. Its vertical base PQ has length 8, and its perpendicular height is the horizontal distance 6.

Step 2: Find the total area and cap The total area is

$$[PQR] = \frac{1}{2} \cdot 8 \cdot 6 = 24.$$

The horizontal line $y = 1$ meets the vertical axis at $S = (0, 1)$, and the side PR, whose equation is $x + 2y = 6$, at $T = (4, 1)$. The part above this line is triangle PST. Its vertical height is $3 - 1 = 2$ and its horizontal base has length 4, giving

$$[PST] = \frac{1}{2} \cdot 4 \cdot 2 = 4.$$

The other oblique side runs between heights -5 and 0, so it contributes no additional piece above height 1.

Step 3: Take the area ratio for the desired event The desired subregion lies below the horizontal line, so its area is $24 - 4 = 20$. Consequently,

$$\mathbb{P}(y < 1) = \frac{20}{24} = \frac{5}{6}.$$

Including or excluding the boundary $y = 1$ does not change this ratio. The fraction $1/6$ is the probability of the complementary cap, while $2/3$ and $6/7$ give neither the cap-to-total nor the desired area-to-total ratio.

Step 4: Conclusion The required probability is $\boxed{5/6}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the triangle vertices first; a horizontal cutoff near a vertex is often easiest to handle by subtracting its triangular cap.

13. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ **is :** [4 marks]

(A) $\frac{26}{25}$

(B) $\frac{25}{26}$

(C) $\frac{50}{51}$

(D) $\frac{52}{51}$

Correct Answer: (A) $\frac{26}{25}$

Solution:

Concept:

- The tangent subtraction formula gives $\tan(A-B)=(\tan A-\tan B)/(1+\tan A\tan B)$. Checking the ranges of the angles is necessary before identifying principal inverse tangents.
- A telescoping sum cancels adjacent intermediate terms. Once the angle is reduced to a difference of two inverse tangents, compute its tangent and then take the reciprocal for the cotangent.

Step 1: Express each summand as a difference For every integer $n\geq 1$, set $A=\tan^{-1}(n+1)$ and $B=\tan^{-1}n$. Both angles are positive and smaller than $\pi/2$, with $A>B$, so their difference lies in the principal interval $(0,\pi/2)$. Therefore

$$\tan(A-B)=\frac{(n+1)-n}{1+n(n+1)}=\frac{1}{n^2+n+1},$$

$$\tan^{-1}\left(\frac{1}{n^2+n+1}\right)=\tan^{-1}(n+1)-\tan^{-1}n.$$

No extra multiple of π is needed because of the range check.

Step 2: Telescope the fifty terms Denote the angle inside the cotangent by S. Substituting the difference identity produces

$$\begin{aligned}S&=(\tan^{-1}2-\tan^{-1}1)+(\tan^{-1}3-\tan^{-1}2)+\cdots\\&\quad+(\tan^{-1}51-\tan^{-1}50)\\&=\tan^{-1}51-\tan^{-1}1.\end{aligned}$$

There are fifty differences but the final uncancelled upper argument is 51, not 50. In particular, S is positive and smaller than $\pi/4$.

Step 3: Compute the cotangent Applying the subtraction formula once more gives

$$\tan S=\frac{51-1}{1+51\cdot 1}=\frac{50}{52}=\frac{25}{26}.$$

The question asks for cotangent, so take the reciprocal:

$$\cot S=\frac{26}{25}.$$

The second option is the tangent rather than the cotangent, the third loses the denominator correction, and the fourth does not follow from the telescoping endpoints. Also, $S<\pi/4$ requires $\cot S>1$,

which immediately excludes the two options below one.

Step 4: Conclusion The value is $\boxed{26/25}$. Hence, the correct option is (A).

Quick Tip to Solve: Look for a denominator of the form $1 + n(n+1)$: it signals the difference of consecutive inverse tangents.

14. $\alpha = \sin 36^\circ$ **is a root of which of the following equation?** [4 marks]

(A) $16x^4 - 10x^2 - 5 = 0$

(B) $16x^4 + 20x^2 - 5 = 0$

(C) $16x^4 - 20x^2 + 5 = 0$

(D) $4x^4 - 10x^2 + 5 = 0$

Correct Answer: (C) $16x^4 - 20x^2 + 5 = 0$

Solution:

Concept:

- Multiple-angle identities convert special-angle conditions into algebraic equations. For $\theta = 36^\circ$, the relation $3\theta = 180^\circ - 2\theta$ implies $\sin 3\theta = \sin 2\theta$.
- Use $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, $\sin 2\theta = 2\sin\theta\cos\theta$, and $\sin^2\theta + \cos^2\theta = 1$. The sign of cosine is fixed because this angle is acute.

Step 1: Derive the exact cosine Write $s = \sin 36^\circ$ and $c = \cos 36^\circ$. Since $\sin 108^\circ = \sin 72^\circ$,

$$3s - 4s^3 = 2sc.$$

Here $s \neq 0$, so dividing by s and replacing s^2 by $1 - c^2$ yields

$$3 - 4(1 - c^2) = 2c, \qquad 4c^2 - 2c - 1 = 0.$$

The quadratic formula gives $c = (1 \pm \sqrt{5})/4$. Only the positive root is admissible for an acute angle, hence $c = (1 + \sqrt{5})/4$.

Step 2: Obtain a polynomial for the sine Now compute the square of s:

$$s^2 = 1 - c^2 = 1 - \frac{(1+\sqrt{5})^2}{16} = \frac{5 - \sqrt{5}}{8}.$$

Thus $8s^2 - 5 = -\sqrt{5}$. Squaring this verified identity gives

$$(8s^2 - 5)^2 = 5,$$

$$64s^4 - 80s^2 + 20 = 0.$$

Divide by 4 to obtain $16s^4 - 20s^2 + 5 = 0$. Squaring may produce other roots of the polynomial, but that is harmless: the task only asks for a polynomial having this specified sine as a root.

Step 3: Check the other listed equations Let $u = s^2 = (5 - \sqrt{5})/8 > 0$. From the derived equation, $16u^2 = 20u - 5$. Substitution into the first proposed polynomial gives $10u - 10 \neq 0$, since $u < 1$. The

second gives $40u - 10 = 15 - 5\sqrt{5} \neq 0$. For the fourth,

$$4u^2 - 10u + 5 = \frac{15}{4} - 5u = \frac{5 + 5\sqrt{5}}{8} > 0.$$

Therefore the other three equations do not vanish at the given value. The third is exactly the polynomial derived from the angle identities.

Step 4: Conclusion $\alpha = \sin 36^\circ$ satisfies $\boxed{16x^4 - 20x^2 + 5 = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: To derive the exact value at 36°, equate $\sin 3\theta$ and $\sin 2\theta$ before eliminating cosine.

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15. Let $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a \in R$. **Then the sum of the squares of all the values of a, for which** $2f'(10) - f'(5) + 100 = 0$**, is** [4 marks]

(A) 117

(B) 106

(C) 125

(D) 136

Correct Answer: (C) 125

Solution:

Concept:

- Expand a determinant along the row containing a zero to reduce the calculation to two minors. The parameter a is constant when differentiating with respect to x.
- For two roots a_1, a_2, their squared sum is $a_1^2 + a_2^2 = (a_1 + a_2)^2 - 2a_1a_2$.

Step 1: Expand the determinant The third entry of the first row is zero. The cofactor sign for the second entry is negative, which cancels its entry -1. Therefore

$$f(x) = a(a^2 + ax) + (a^2x + ax^2) = a^3 + 2a^2x + ax^2.$$

Keeping this sign is essential: subtracting the second minor would incorrectly cancel the terms linear in x.

Step 2: Apply the derivative condition Differentiate with respect to x, holding a fixed:

$$f\prime(x) = 2a^2 + 2ax.$$

Consequently,

$$\begin{aligned} 2f\prime(10) - f\prime(5) + 100 &= 2(2a^2 + 20a) - (2a^2 + 10a) + 100 \\ &= 2a^2 + 30a + 100. \end{aligned}$$

Setting this equal to zero and dividing by 2 gives $a^2 + 15a + 50 = (a + 5)(a + 10) = 0$. Both roots are

real, as required.

Step 3: Add the squared values The admissible values are -5 and -10. Hence

$$(-5)^2 + (-10)^2 = 25 + 100 = 125.$$

As a check, Vieta gives $(-15)^2 - 2(50) = 125$. The other offered values 117, 106 and 136 do not satisfy this squared-sum identity; the sum of the roots itself is not what is requested.

Step 4: Conclusion The required sum of squares is $\boxed{125}$. Hence, the correct option is (C).

Quick Tip to Solve: Expand before differentiating, and remember that the parameter is constant. Vieta checks the squared sum without separately squaring the roots.

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16. Let for some real numbers α and β, $a = \alpha - i\beta$. If the system of equations $4ix + (1+i)y = 0$ and $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + ay = 0$ has more than one solution, then $\frac{\alpha}{\beta}$ is equal to [4 marks]

(A) $-2\sqrt{3}$

(B) $2 - \sqrt{3}$

(C) $2 + \sqrt{3}$

(D) $-2 - \sqrt{3}$

Correct Answer: No printed option is correct: $\alpha/\beta = \sqrt{3} - 2$. Option (B) requires $\bar{a}y$ instead of ay.

Solution:

Concept:

- A homogeneous system always has the zero solution. It has more than one solution precisely when its coefficient determinant vanishes.
- Here $a = \alpha - i\beta$, so its real part is α and its imaginary part is $-\beta$. The signs must be retained when comparing components.

Step 1: Set the determinant to zero The printed second equation contains ay. Thus the coefficient determinant condition is

$$4ia - 8(1+i)\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 0.$$

The first coefficient $4i$ is nonzero, so this condition makes the two equations dependent and gives infinitely many solutions over the complex numbers.

Step 2: Determine the real parameters Solve for a, using $(1+i)/i = 1 - i$:

$$\begin{aligned} a &= 2(1-i)\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \\ &= (\sqrt{3} - 1) + i(\sqrt{3} + 1). \end{aligned}$$

Comparison with $a = \alpha - i\beta$ gives $\alpha = \sqrt{3} - 1$ and $\beta = -(\sqrt{3} + 1)$. Therefore

$$\frac{\alpha}{\beta} = -\frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \sqrt{3} - 2.$$

The ratio is negative and has magnitude less than one.

Step 3: Check the listed choices and the key The value $\sqrt{3} - 2$ is about -0.268, whereas A and D are less than -3, and B and C are positive. Thus none of the four options equals the result of the printed equations. The listed answer B, $2 - \sqrt{3}$, would follow if the second equation contained $\overline{a}y$ instead of ay: conjugation reverses the imaginary part and hence the sign of β. That is a different equation, so it cannot be silently substituted for the printed one.

Step 4: Conclusion As printed, $\boxed{\alpha/\beta = \sqrt{3} - 2}$, which is absent from the options. The keyed option (B) applies to the conjugated-coefficient version, not to the literal printed coefficient.

Quick Tip to Solve: After solving for a complex coefficient, compare its imaginary part with minus beta, not beta. Check conjugation marks before choosing an option.

17. Let A and B be two 3 × 3 matrices such that $AB = I$ and $|A| = \frac{1}{8}$. Then $|adj\,(B\,adj(2A))|$ is equal to [4 marks]

(A) 16

(B) 32

(C) 64

(D) 128

Correct Answer: (C) 64

Solution:

Concept:

- For square matrices, $\det(XY) = \det X \det Y$. From $AB = I$, both matrices are invertible and their determinants are reciprocal.
- For an $n \times n$ matrix, $\det(\text{adj}\, X) = (\det X)^{n-1}$, whereas $\det(cX) = c^n \det X$. Here $n = 3$.

Step 1: Find the determinant of B Take determinants of $AB = I$:

$$|A||B| = 1.$$

Since $|A| = 1/8$, this gives $|B| = 8$. The nonzero determinant ensures that all inverse and adjugate identities used below are applicable.

Step 2: Evaluate the inner adjugate determinant Multiplying a three-row matrix by 2 multiplies each row by 2, so

$$|2A| = 2^3|A| = 8\left(\frac{1}{8}\right) = 1.$$

Thus

$$|\operatorname{adj}(2A)| = |2A|^2 = 1.$$

The exponent is two for the adjugate of a matrix of order three, not three. Confusing this rule with scalar multiplication is a common source of an incorrect power.

Step 3: Apply the outer adjugate identity Put $M = B\operatorname{adj}(2A)$. The determinant of this product is

$$|M| = |B||\operatorname{adj}(2A)| = 8 \cdot 1 = 8.$$

The requested expression is the determinant of its adjugate, not merely $|M|$. Therefore

$$|\operatorname{adj} M| = |M|^2 = 8^2 = 64.$$

The alternatives 16, 32 and 128 correspond to different powers and do not follow from the order-three adjugate formula. No entries of either matrix need to be calculated.

Step 4: Conclusion The determinant is $\boxed{64}$. Hence, the correct option is (C).

Quick Tip to Solve: Work from the innermost brackets outward. At every adjugate of a 3 by 3 matrix, square its determinant; at scalar multiplication, cube the scalar.

18. Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots\ldots$ Then 4S is equal to [4 marks]

(A) $\left(\frac{7}{3}\right)^2$
(B) $\frac{7^3}{3^2}$
(C) $\left(\frac{7}{3}\right)^3$
(D) $\frac{7^2}{3^3}$

Correct Answer: (C) $\left(\frac{7}{3}\right)^3$

Solution:

Concept:

- The numerators are successive products $n(n+1)$, starting at $n = 1$. The denominator of the nth term is 7^{n-1}.
- The geometric series $\sum_{n=0}^{\infty} r^n = (1-r)^{-1}$ can be differentiated for $|r| < 1$. This yields sums with polynomial factors in n.

Step 1: Express the series using one variable Set $r = 1/7$. The series is

$$S = \sum_{n=1}^{\infty} n(n+1)r^{n-1}.$$

This reproduces $2, 6r, 12r^2, 20r^3, 30r^4$, so the exponent offset and the starting index are both correct. Absolute convergence follows from $|r| < 1$.

Step 2: Obtain the generating identity Multiply the geometric series by r^2:

$$\sum_{n=0}^{\infty} r^{n+2} = \frac{r^2}{1-r}.$$

Differentiating twice gives

$$\sum_{n=0}^{\infty}(n+2)(n+1)r^n = \frac{2}{(1-r)^3}.$$

Replacing $n+1$ by the index of the required series shows that $S = 2/(1-r)^3$. Termwise differentiation is valid inside the interval of convergence.

Step 3: Substitute and compare the options At $r = 1/7$,

$$S = \frac{2}{(6/7)^3} = \frac{343}{108}.$$

The question asks for four times this sum:

$$4S = \frac{343}{27} = \left(\frac{7}{3}\right)^3.$$

Option A has the wrong exponent, B has denominator 9 instead of 27, and D has numerator 49 instead of 343. As a numerical check, the first few terms sum to a little above 3, so $4S$ must be near 12.7.

Step 4: Conclusion $\boxed{4S = (7/3)^3}$. Hence, the correct option is (C).

Quick Tip to Solve: For numerators 2, 6, 12, 20, identify n(n+1). The matching generating sum is 2/(1-r) cubed, with the denominator starting at r to the power zero.

19. Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane $3x + 4y + 12z + 23 = 0$ is : [4 marks]

(A) 5

(B) $\frac{50}{13}$

(C) 4

(D) $\frac{63}{13}$

Correct Answer: (A) 5

Solution:

Concept:

- A point on the line is $P = (-2 + 4t, 1 + 2t, -1 + 3t)$, whose direction vector is $\mathbf{d} = (4, 2, 3)$.
- At a perpendicular foot, $(P - Q) \cdot \mathbf{d} = 0$. The distance of (x_0, y_0, z_0) from $Ax + By + Cz + D = 0$ is $|Ax_0 + By_0 + Cz_0 + D|/\sqrt{A^2 + B^2 + C^2}$.

Step 1: Locate the perpendicular foot Use $Q = (1, 2, 4)$. The vector from Q to the general point on

the line is

$$P - Q = (-3 + 4t, -1 + 2t, -5 + 3t).$$

Its scalar product with $(4, 2, 3)$ must vanish. Thus

$$4(-3 + 4t) + 2(-1 + 2t) + 3(-5 + 3t) = 0,$$

which simplifies to $-29 + 29t = 0$. Therefore $t = 1$, and substituting in the line gives $P = (2, 3, 2)$.

Step 2: Check the projection condition The displacement $P - Q = (1, 1, -2)$ has scalar product $4+2-6 = 0$ with the line direction. Also, the three line ratios evaluated at P are all one. These checks confirm both that the point lies on the specified line and that the joining segment is perpendicular to it.

Step 3: Compute the point-plane distance For the plane normal $(3, 4, 12)$, the normal length is

$$\sqrt{3^2 + 4^2 + 12^2} = \sqrt{169} = 13.$$

The numerator at the foot is

$$|3(2) + 4(3) + 12(2) + 23| = |6 + 12 + 24 + 23| = 65.$$

Hence the distance is $65/13 = 5$ coordinate units. The alternatives $50/13$, 4 and $63/13$ use incorrect substituted coordinates or omit part of the plane constant.

Step 4: Conclusion The required distance is $\boxed{5 \text{ units}}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the foot using a dot product before applying the plane-distance formula. The original external point is not the point whose distance is requested.

20. Which of the following statement is a tautology? [4 marks]

(A) $((\sim q) \wedge p) \wedge q$

(B) $((\sim q) \wedge p) \wedge (p \wedge (\sim p))$

(C) $((\sim q) \wedge p) \vee (p \vee (\sim p))$

(D) $(p \wedge q) \wedge (\sim p \wedge q))$

Correct Answer: (C) $((\sim q) \wedge p) \vee (p \vee (\sim p))$

Solution:

Concept:

- A tautology is true for every possible assignment of truth values to its component propositions. A contradiction is false for every assignment.
- The complement laws give $p \vee (\sim p) = T$ and $p \wedge (\sim p) = F$. Also $X \vee T = T$ and $X \wedge F = F$.

Step 1: Test the expression containing excluded middle In option C, the second part is $p \vee (\sim p)$,

which is always true. Whatever truth value the first part $(\sim q) \wedge p$ takes,

$$\big((\sim q) \wedge p\big) \vee \big(p \vee (\sim p)\big) = X \vee T = T.$$

Therefore this option is true independently of both p and q, and is a tautology.

Step 2: Reject the conjunctions in the other options Option A contains both q and $\sim q$ joined by conjunction. By associativity and commutativity it becomes $p \wedge ((\sim q) \wedge q) = F$. Option B explicitly contains $p \wedge (\sim p)$; conjoining any proposition with that contradiction is again false. Option D also contains p and $\sim p$ in a conjunction, so it simplifies to $(p \wedge \sim p) \wedge q \wedge q = F$. None can be true for all assignments.

Step 3: Confirm that no truth assignment is exceptional If $p = T$, the disjunction $p \vee \sim p$ is true because its first term is true. If $p = F$, it is true because its second term is true. These are the only two possibilities, and q never changes this subexpression. Thus the conclusion for C covers all four combinations of p and q, rather than merely one successful example.

Step 4: Conclusion Option C is always true. Hence, the correct option is (C).

Quick Tip to Solve: Look first for p or not-p inside a disjunction: it forces truth. The same pair inside a conjunction forces falsity.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Define f : S → S as

$$f(n) = \begin{cases} 2n & , \quad if\ n = 1, 2, 3, 4, 5 \\ 2n - 11 & , \quad if\ n = 6, 7, 8, 9, 10 \end{cases}.$$

Let g : S → S be a function such that $fog(n) = \begin{cases} n+1 & , \quad if\ n\ is\ odd \\ n-1 & , \quad if\ n\ is\ even \end{cases}$.

Then $g(10)g(1) + g(2) + g(3) + g(4) + g(5))$ **is equal to _____.** [4 marks]

Solution:

Concept:

- The first branch of f maps inputs 1 through 5 to the even elements of S; the second maps inputs 6 through 10 to the odd elements. Thus f is a bijection.
- To obtain $g(n)$, first calculate the prescribed value $(f \circ g)(n)$, then apply the inverse of f. Composition is not pointwise multiplication.

Step 1: Write the inverse mapping If the output m is even, $f^{-1}(m) = m/2$. If it is odd, solve $2k - 11 = m$, giving $f^{-1}(m) = (m + 11)/2$. These inverse values lie in the correct input branches, so every required $g(n)$ is uniquely determined.

Step 2: Calculate the needed values of g For odd n, the composition output is $n + 1$, which is even. Hence $g(n) = (n + 1)/2$. For even n, the output is $n - 1$, which is odd, so $g(n) = (n + 10)/2$. In

particular,

$$g(1) = 1, \quad g(2) = 6, \quad g(3) = 2,$$
$$g(4) = 7, \quad g(5) = 3, \quad g(10) = 10.$$

For example $f(g(10)) = f(10) = 9$, exactly as the even branch requires.

Step 3: Evaluate the literal expression and identify the bracket issue The printed expression has an unmatched final closing parenthesis after $g(5)$. Taking the multiplication and additions exactly as visible gives

$$g(10)g(1) + g(2) + g(3) + g(4) + g(5) = 10(1) + 6 + 2 + 7 + 3 = 28.$$

The official keyed value 190 instead corresponds to placing an opening parenthesis immediately after $g(10)$:

$$g(10)\big(g(1) + g(2) + g(3) + g(4) + g(5)\big) = 10(1 + 6 + 2 + 7 + 3) = 190.$$

Thus the key identifies the intended grouping, but the visible ungrouped product does not itself have that value. Both evaluations are dimensionless.

Step 4: Conclusion The intended grouped expression gives $\boxed{190}$ (dimensionless), agreeing with the key. Read literally without the missing opening bracket, the printed expression gives $\boxed{28}$; the parenthesis defect makes the notation ambiguous.

Quick Tip to Solve: Invert f separately on even and odd outputs. Once g is known, keep track of whether g(10) multiplies only the first term or the complete sum.

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22. Let α, β be the roots of the equation $x^2 - 4\lambda x + 5 = 0$ and α, γ be the roots of the equation $x^2 - \left(3\sqrt{2} + 2\sqrt{3}\right)x + 7 + 3\lambda\sqrt{3} = 0$, $\lambda > 0$. If $\beta + \gamma = 3\sqrt{2}$, then $(\alpha + 2\beta + \gamma)^2$ is equal to _____.

[4 marks]

Solution:

Concept:

- For a monic quadratic $x^2 - sx + p = 0$, the sum of its roots is s and their product is p. Apply these relations separately to the two equations, retaining the root common to both.
- The identity $(\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$ determines the sum without solving a quadratic formula. The condition $\lambda > 0$ selects its positive sign.

Step 1: Compare the two root sums For the first equation, $\alpha + \beta = 4\lambda$ and $\alpha\beta = 5$. For the second, $\alpha + \gamma = 3\sqrt{2} + 2\sqrt{3}$. Subtract the given relation $\beta + \gamma = 3\sqrt{2}$ from the latter relation to eliminate γ:

$$\alpha - \beta = 2\sqrt{3}.$$

Step 2: Find the individual roots The product relation now gives

$$(\alpha + \beta)^2 = (2\sqrt{3})^2 + 4(5) = 32.$$

Because $\alpha+\beta=4\lambda>0$, take $\alpha+\beta=4\sqrt{2}$. Adding and subtracting the sum and difference yields

$$\alpha=2\sqrt{2}+\sqrt{3}, \qquad \beta=2\sqrt{2}-\sqrt{3}.$$

Then $\gamma=3\sqrt{2}-\beta=\sqrt{2}+\sqrt{3}$. As a consistency check, $\lambda=\sqrt{2}$ and $\alpha\gamma=7+3\sqrt{6}=7+3\lambda\sqrt{3}$, as required by the second equation.

Step 3: Evaluate the requested expression Group the expression as $(\alpha+\beta)+(\beta+\gamma)$. Both sums have already been determined, so

$$\alpha+2\beta+\gamma=4\sqrt{2}+3\sqrt{2}=7\sqrt{2}.$$

Squaring this real number gives $(7\sqrt{2})^2=49\cdot 2=98$.

Step 4: Conclusion The requested numerical value is $\boxed{98}$. This is dimensionless.

Quick Tip to Solve: When two quadratics share a root, subtract their root-sum relations first. Here the requested expression is itself the sum of two known root sums.

23. Let A be a matrix of order 2 × 2, whose entries are from the set {0, 1, 2, 3, 4, 5}. If the sum of all the entries of A is a prime number p, 2 < p < 8, then the number of such matrices A is _____.

[4 marks]

Solution:

Concept:

- A 2×2 matrix has four distinct positions, so its entries form an ordered quadruple. Equal entries are allowed; different placements generally give different matrices.
- The number of nonnegative integer solutions of $a+b+c+d=s$ without an upper bound is $\binom{s+3}{3}$. Subtract solutions violating the entry bound $0\le a,b,c,d\le 5$.

Step 1: List the admissible prime sums The strict inequalities $2<p<8$ restrict the prime number to $p=3,5,7$. These three sums are disjoint cases. Write the four matrix entries in fixed order as a,b,c,d; each lies in $\{0,1,2,3,4,5\}$. We count each possible sum separately rather than choosing four entries without regard to their positions.

Step 2: Count the sums three and five For a total of 3, no nonnegative entry can exceed 5. Stars and bars therefore gives

$$N_3=\binom{3+3}{3}=\binom{6}{3}=20.$$

Likewise, a total of 5 automatically obeys the upper bound, including the possibility that one entry is 5 and all others are zero:

$$N_5=\binom{5+3}{3}=\binom{8}{3}=56.$$

No correction is needed in either case.

Step 3: Correct the count for a sum of seven Ignoring the upper bound initially gives $\binom{10}{3}=120$. If a chosen position has an entry at least 6, subtract 6 from that entry. The four resulting nonnegative

variables have sum 1, giving $\binom{4}{3} = 4$ violations for each of the four positions. Two positions cannot both be at least 6 because their sum would be at least 12. Thus no intersections have been subtracted twice:

$$N_7 = 120 - 4 \cdot 4 = 104.$$

Finally, add the disjoint cases: $N = 20 + 56 + 104 = 180$.

Step 4: Conclusion The number of such matrices is $\boxed{180}$, a dimensionless count.

Quick Tip to Solve: Treat matrix positions as ordered variables. For the sum 7, the only forbidden entries are 6 or 7; shifting one offending entry by 6 counts both possibilities at once.

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24. Let [t] denote the greatest integer $\leq$ t and {t} denote the fractional part of t. The integral value of α for which the left hand limit of the function $f(x) = [1+x] + \frac{\alpha^{2[x]+\{x\}+[x]-1}}{2[x]+\{x\}}$ at x = 0 is equal to $\alpha - \frac{4}{3}$, is _____. [4 marks]

Solution:

Concept:

- For $-1 < x < 0$, $[x] = -1$, $\{x\} = x + 1$, and $[1 + x] = 0$. Use the interval on the specified side of the limit before simplifying.
- A real exponential α^u for all real exponents near the limiting point is defined in the usual real sense for $\alpha > 0$. Its continuity then allows direct substitution once the denominator has a nonzero limit.

Step 1: Write the expression just to the left of zero Approach zero through $-1 < x < 0$. The integer and fractional parts give

$$2[x] + \{x\} = -2 + (x + 1) = x - 1.$$

Also $[x] - 1 = -2$, while $[1 + x] = 0$. Hence the function on this interval is exactly

$$f(x) = \frac{\alpha^{x-1} - 2}{x - 1}.$$

The denominator tends to -1, so this is not an indeterminate quotient and does not require differentiation.

Step 2: Equate the limiting value to the condition For a positive base α, the exponential term tends to α^{-1}. Therefore

$$\lim_{x \to 0^-} f(x) = \frac{\alpha^{-1} - 2}{-1} = 2 - \frac{1}{\alpha}.$$

The given condition becomes

$$2 - \frac{1}{\alpha} = \alpha - \frac{4}{3}.$$

Multiply by 3α, which is nonzero on the real exponential domain, and rearrange to obtain $3\alpha^2 - 10\alpha + 3 = 0$.

Step 3: Select the integral root Factor the quadratic:

$$3\alpha^2 - 10\alpha + 3 = (3\alpha - 1)(\alpha - 3).$$

Its positive roots are $\alpha = 1/3$ and $\alpha = 3$. Only 3 is an integer. Substitution checks the selected root: the left-hand limit is $2 - 1/3 = 5/3$, and the required value is $3 - 4/3 = 5/3$. The zero base is excluded by the negative exponents near the limit; a negative base does not define this function for all real nearby x.

Step 4: Conclusion The integral value is $\boxed{\alpha = 3}$.

Quick Tip to Solve: For a left-hand limit at zero, floor x equals -1 and the fractional part equals 1+x. Substituting floor x = 0 would incorrectly use the other side.

25. If the area of the region $\left\{(x, y) : x^{\frac{2}{3}} + y^{\frac{2}{3}} \leq 1, x + y \geq 0, y \geq 0\right\}$ is A, then $\frac{256A}{\pi}$ is equal to _____. [4 marks]

Solution:

Concept:

- The real curve $x^{2/3} + y^{2/3} = 1$ is an astroid, symmetric about both coordinate axes and the lines $y = x$ and $y = -x$. Each coordinate power means the square of the real cube root.
- In the first quadrant use $x = \cos^3 t$, $y = \sin^3 t$, $0 \leq t \leq \pi/2$. Area is computed as $\int y\, dx$ with limits ordered to give a positive value.

Step 1: Identify the included portions of the astroid The restriction $y \geq 0$ keeps the upper half. In the first quadrant, $x + y \geq 0$ holds everywhere, so the entire first-quadrant astroid region contributes. In the second quadrant the same inequality is $y \geq -x$. Reflection in $y = -x$ preserves the second-quadrant astroid region and divides it into two equal areas. Thus the desired region consists of one whole quadrant and half of an adjacent quadrant. If a quadrant has area Q, then $A = 3Q/2$.

Step 2: Evaluate one quadrant area On the first-quadrant boundary, $x = \cos^3 t$, $y = \sin^3 t$, and $dx = -3\cos^2 t \sin t\, dt$. Reversing the limits gives

$$Q = 3\int_0^{\pi/2} \sin^4 t \cos^2 t\, dt.$$

The standard even-power integral, or the power-reduction identities, yields

$$\int_0^{\pi/2} \sin^4 t \cos^2 t\, dt = \frac{3 \cdot 1 \cdot 1}{6 \cdot 4 \cdot 2}\frac{\pi}{2} = \frac{\pi}{32}.$$

Consequently $Q = 3\pi/32$.

Step 3: Scale the selected area The first quadrant contributes $3\pi/32$, and the selected half of the

second contributes $3\pi/64$. Their sum and the requested scaled value are

$$A = \frac{9\pi}{64}, \qquad \frac{256A}{\pi} = \frac{256}{\pi}\frac{9\pi}{64} = 36.$$

The line boundaries themselves have zero area and do not affect the calculation.

Step 4: Conclusion The requested dimensionless value is $\boxed{36}$.

Quick Tip to Solve: Use the diagonal symmetry before integrating: the two half-plane conditions retain three eighths of the full astroid, so only a single quadrant integral is needed.

---

26. If $y(x) = (x^x)^x$, $x > 0$, then $\frac{d^2x}{dy^2} + 20$ at x = 1 is equal to _____. [4 marks]

Solution:

Concept:

- For $x > 0$, the expression $(x^x)^x = x^{x^2}$ can be differentiated using $\log y = x^2 \log x$. Logarithmic differentiation handles the variable base and exponent together.
- Where $y\prime(x) \neq 0$, a local inverse exists and $\frac{d^2x}{dy^2} = -\frac{y\prime\prime(x)}{(y\prime(x))^3}$. A second inverse derivative is not simply the reciprocal of the original second derivative.

Step 1: Differentiate the logarithm Put $g(x) = \log y = x^2 \log x$, with the logarithm natural. Product differentiation gives

$$g\prime(x) = 2x \log x + x, \qquad g\prime\prime(x) = 2\log x + 3.$$

At $x = 1$, $g(1) = 0$, $g\prime(1) = 1$, and $g\prime\prime(1) = 3$. Hence $y(1) = e^0 = 1$. Keeping both the outer exponential and the inner logarithmic derivative is necessary when differentiating y.

Step 2: Find the first two derivatives of y From $y = e^{g(x)}$, the chain and product rules imply

$$y\prime = yg\prime, \qquad y\prime\prime = y\big((g\prime)^2 + g\prime\prime\big).$$

Thus at the specified point

$$y\prime(1) = 1 \cdot 1 = 1, \qquad y\prime\prime(1) = 1(1^2 + 3) = 4.$$

The nonzero first derivative confirms that the inverse derivative is well defined locally.

Step 3: Differentiate with respect to y Because $dx/dy = 1/y\prime$, differentiation with respect to y means applying $(1/y\prime)d/dx$. Consequently

$$\frac{d^2x}{dy^2} = \frac{1}{y\prime}\frac{d}{dx}\left(\frac{1}{y\prime}\right) = -\frac{y\prime\prime}{(y\prime)^3}.$$

At $x = 1$, this equals $-4/1^3 = -4$. Adding the given constant gives $-4 + 20 = 16$.

Step 4: Conclusion The required value is $\boxed{16}$, a dimensionless number.

Quick Tip to Solve: Compute y prime and y double prime at the point first, then apply the inverse-second-derivative formula. The minus sign and the cube of y prime are both essential.

27. Let $y = y(x)$ be the solution of the differential equation $(1-x^2)dy = \left(xy + (x^3+2)\sqrt{1-x^2}\right)dx, -1 < x < 1$, and $y(0) = 0$. If $\int_{\frac{-1}{2}}^{\frac{1}{2}} \sqrt{1-x^2}y(x)dx = k$, then k^{-1} is equal to _____. [4 marks]

Solution:

Concept:

- Write a first-order linear equation in the form $y' + P(x)y = Q(x)$. Multiplication by the integrating factor $e^{\int P(x)dx}$ turns the left side into a product derivative.
- On $-1 < x < 1$, $\sqrt{1-x^2}$ is positive. Over symmetric limits the integral of an odd function is zero, whereas an even function contributes twice its integral over the positive half.

Step 1: Find the integrating factor Divide the differential equation by $1 - x^2 > 0$ and rearrange:

$$y' - \frac{x}{1-x^2}y = \frac{x^3+2}{\sqrt{1-x^2}}.$$

The integrating factor is

$$I(x) = \exp\left(\int \frac{-x}{1-x^2}\,dx\right) = \sqrt{1-x^2}.$$

Multiplication therefore gives

$$\frac{d}{dx}\left(y\sqrt{1-x^2}\right) = x^3 + 2.$$

This product is exactly the combination appearing in the required definite integral.

Step 2: Use the initial value Integrating the product derivative gives

$$y(x)\sqrt{1-x^2} = \frac{x^4}{4} + 2x + C.$$

At $x = 0$, the left side is zero by $y(0) = 0$, so $C = 0$. In particular, the solution itself is $y(x) = (x^4/4 + 2x)/\sqrt{1-x^2}$, valid on the stated open interval. Substituting the product form into the requested integral avoids unnecessary manipulation of the square root.

Step 3: Evaluate k and take its reciprocal The odd term $2x$ cancels over $[-1/2, 1/2]$. The remaining even term gives

$$k = \int_{-1/2}^{1/2}\left(\frac{x^4}{4} + 2x\right)dx = \frac{1}{2}\int_0^{1/2} x^4\,dx.$$

$$k = \frac{1}{2}\left[\frac{x^5}{5}\right]_0^{1/2} = \frac{1}{10}\frac{1}{32} = \frac{1}{320}.$$

This value is positive, so its reciprocal is well defined and equals 320.

Step 4: Conclusion The required dimensionless value is $\boxed{k^{-1} = 320}$.

Quick Tip to Solve: When the integrating factor times y already appears under the requested integral, retain that product instead of dividing it out. Then exploit the symmetric limits.

28. Let S = {E_1, E_2, ..., E_8} be a sample space of a random experiment such that $P(E_n) = \frac{n}{36}$ for every n = 1, 2, ..., 8. Then the number of elements in the set $\left\{A \subseteq S : P(A) \geq \frac{4}{5}\right\}$ is _____.

[4 marks]

Solution:

Concept:

- An event is a subset of the eight elementary outcomes. Because these outcomes are disjoint, its probability is the sum of their assigned weights $n/36$. The total weight $1 + 2 + \cdots + 8 = 36$ normalizes the sample space.
- Complementation is a bijection on subsets and satisfies $P(A^c) = 1 - P(A)$. It converts a large-probability requirement into counting subsets with a small integer weight sum.

Step 1: Translate the probability bound Let $B = S \setminus A$. Then $P(A) \geq 4/5$ is equivalent to $P(B) \leq 1/5$. Multiplying by 36, the sum of the integer labels of the outcomes in B must be at most $36/5 = 7.2$. Since that sum is an integer, it is at most 7. Outcome E_8 cannot belong to B. Thus count subsets of the distinct labels $1, 2, \ldots, 7$ whose sum is at most 7.

Step 2: Count by the number of selected outcomes The empty subset contributes 1, and any singleton from $1, \ldots, 7$ contributes, giving 7 more. For pairs $a < b$, those starting with 1 have $b = 2, 3, 4, 5, 6$, giving 5. Those starting with 2 have $b = 3, 4, 5$, giving 3; the only pair starting with 3 is $(3, 4)$, giving 1. Thus there are $5 + 3 + 1 = 9$ pairs. Each is counted once because the smaller member fixes its group.

Step 3: Finish with triples and exclude larger subsets The smallest triple has sum $1 + 2 + 3 = 6$. The triples of sum at most 7 are exactly $\{1, 2, 3\}$ and $\{1, 2, 4\}$, giving 2. Any other increasing triple has sum at least 8. A subset of four or more labels has sum at least $1 + 2 + 3 + 4 = 10$, so none qualifies. Hence the number of complements, and therefore of events A, is

$$1 + 7 + 9 + 2 = 19.$$

The empty complement corresponds to the full event $A = S$ and must be retained.

Step 4: Conclusion The required number of events is $\boxed{19}$, a dimensionless count.

Quick Tip to Solve: Count the excluded outcomes rather than the included ones when the event probability is close to one. Convert the probability bound into an integer weight threshold before enumerating.

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29. If the sum of the coefficients of all the positive powers of x, in the Binomial expansion of $\left(x^n + \frac{2}{x^5}\right)^7$ is 939, then the sum of all the possible integral values of n is _____.

[4 marks]

Solution:

Concept:

- The term obtained by choosing the second term in r of the seven factors is $T_{r+1} = \binom{7}{r} 2^r x^{7n-(n+5)r}$, where $0 \leq r \leq 7$. Its coefficient is always positive.
- Only terms with strictly positive exponent contribute to the stated sum. A term with exponent zero is constant and must be excluded.

Step 1: List the coefficients in order The coefficients for $r = 0, 1, \ldots, 7$ are respectively

$$1,\ 14,\ 84,\ 280,\ 560,\ 672,\ 448,\ 128.$$

For $n \leq 0$, every exponent $(7-r)n - 5r$ is nonpositive, so the sum could not be 939. Thus $n > 0$, and the exponents decrease strictly as r increases. The required coefficients therefore form an initial segment of this list.

Step 2: Identify the last positive-power term The successive coefficient sums through $r = 0, 1, 2, 3, 4$ are $1, 15, 99, 379, 939$. Since all coefficients are positive, the required sum is obtained exactly when the term for $r = 4$ has positive exponent but the term for $r = 5$ does not. This also excludes every later term because the exponents are decreasing.

Step 3: Apply the strict and non-strict inequalities The two boundary exponents give

$$7n - 4(n+5) = 3n - 20 > 0,$$

$$7n - 5(n+5) = 2n - 25 \leq 0.$$

Hence $20/3 < n \leq 25/2$. The allowed integers are $7, 8, 9, 10, 11, 12$. Their sum is

$$\frac{6(7+12)}{2} = 57.$$

A constant term at the boundary would remain excluded; this is why the second inequality is non-strict.

Step 4: Conclusion The possible integral values of n are $7, 8, 9, 10, 11, 12$, whose sum is $\boxed{57}$. This is a dimensionless integer.

Quick Tip to Solve: First match the cumulative coefficient sum, then bound the exponent of the last included term and the first excluded term.

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30. Let a circle C of radius 5 lie below the x-axis. The line L_1 : 4x + 3y + 2 = 0 passes through the centre P of the circle C and intersects the line L_2 = 3x − 4y − 11 = 0 at Q. The line L_2 touches C at the point Q. Then the distance of P from the line 5x − 12y + 51 = 0 is _____. [4 marks]

Solution:

Concept:

- The radius to a point of tangency is perpendicular to the tangent. Thus PQ is normal to L_2, and its length equals the radius 5.
- The distance from (h, k) to $ax + by + c = 0$ is $|ah + bk + c|/\sqrt{a^2 + b^2}$. The location below the x-axis selects the appropriate centre.

Step 1: Locate the point of tangency The point Q lies on both given lines, so solve

$$4x + 3y = -2, \qquad 3x - 4y = 11.$$

Multiplying the first equation by 4 and the second by 3 and adding gives $25x = 25$, hence $x = 1$.

Substitution gives $y = -2$, so $Q = (1, -2)$. The normal to L_2 has direction $(3, -4)$, whose length is 5.

Step 2: Select the centre below the x-axis A displacement of length 5 along either normal direction is therefore $\pm(3, -4)$. The possible centres are

$$P = Q \pm (3, -4) = (4, -6) \text{ or } (-2, 2).$$

Both lie on L_1, since the two given lines are perpendicular. The circle lies below the x-axis, so its centre is $(4, -6)$: its highest point has ordinate $-6 + 5 = -1 < 0$. The other centre has positive ordinate and cannot describe the stated circle.

Step 3: Evaluate the requested perpendicular distance For the line $5x - 12y + 51 = 0$, substitute $P = (4, -6)$:

$$d = \frac{|5(4) - 12(-6) + 51|}{\sqrt{5^2 + (-12)^2}}$$

$$= \frac{|20 + 72 + 51|}{13} = \frac{143}{13} = 11.$$

The absolute value ensures a nonnegative distance; the denominator normalizes the line coefficients.

Step 4: Conclusion The distance of the centre P from the given line is $\boxed{11}$ coordinate units.

Quick Tip to Solve: Intersect the two lines to get the tangency point, move one radius along the tangent normal, and use the stated side of the axis to choose the centre.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The SI unit of a physical quantity is pascal-second. The dimensional formula of this quantity will be : [4 marks]

(A) $[ML^{-1}T^{-1}]$

(B) $[ML^{-1}T^{-2}]$

(C) $[ML^{2}T^{-1}]$

(D) $[M^{-1}L^{3}T^{0}]$

Correct Answer: (A) $[ML^{-1}T^{-1}]$

Solution:

Concept:

- Pressure is force per unit area. Newton$= \mathrm{kg\,m\,s^{-2}}$, so pascal$= \mathrm{kg\,m^{-1}\,s^{-2}}$.
- Dimensions multiply when units multiply. A second contributes one additional power of T; it does not change the mass or length powers.

Step 1: Reduce the pascal to base units By Newton's second law, the dimension of force is

$$[F] = [M][LT^{-2}] = [MLT^{-2}].$$

Dividing by the dimension $[L^2]$ of area gives

$$[\text{pressure}] = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}].$$

The negative length exponent follows from the two powers of length in the denominator and the one in force. This avoids confusing pressure with force or energy.

Step 2: Include the time factor The question specifies pascal-second, which is a product of pressure and time. Therefore

$$[\text{quantity}] = [ML^{-1}T^{-2}][T]$$
$$= [ML^{-1}T^{-1}].$$

In SI base units this is $\mathrm{kg\,m^{-1}\,s^{-1}}$. Such a unit is also the unit of dynamic viscosity; identifying that quantity is a useful consistency check, although the dimensional reduction alone decides the answer.

Step 3: Check the proposed alternatives Option (A) has the required powers $M^1L^{-1}T^{-1}$. Option (B) retains T^{-2}, so it represents pressure alone and omits the multiplying second. Option (C) has L^2, whereas division by area requires the length power to be -1. Option (D) has inverse mass and no time dependence, both inconsistent with the base-unit product. Thus only one option survives all three exponent checks.

Step 4: Conclusion The dimensional formula is $\boxed{[ML^{-1}T^{-1}]}$. Hence, the correct option is (A).

Quick Tip to Solve: Write Pa as N per square metre first; then multiply by s and add the time exponents.

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32. The distance of the Sun from earth is 1.5×10^{11} m and its angular diameter is (2000) s when observed from the earth. The diameter of the Sun will be : [4 marks]

(A) 2.45×10^{10} m

(B) 1.45×10^{10} m

(C) 1.45×10^{9} m

(D) 0.14×10^{9} m

Correct Answer: (C) 1.45×10^{9} m

Solution:

Concept:

- For a small angular diameter θ in radians, the physical diameter is $D \simeq R\theta$, where R is the distance to the object.
- One degree contains 3600 seconds of arc, so 1 arcsecond $= \pi/(180 \times 3600)$ radian. The symbol s here denotes angular seconds, not elapsed time.

Step 1: Convert the angular diameter to radians The observed angular diameter is 2000 seconds of arc. Convert through degrees:

$$\theta = \frac{2000}{3600}\frac{\pi}{180}$$

$$= \frac{\pi}{324} \text{ rad} \simeq 9.696 \times 10^{-3} \text{ rad}.$$

The small value in radians justifies using the small-angle relation. Using 2000 directly would mix angular seconds with radians and give an incorrect scale.

Step 2: Multiply by the distance Using $R = 1.5 \times 10^{11}$ m, obtain

$$D \simeq R\theta = (1.5 \times 10^{11})(9.696 \times 10^{-3})\,\text{m}$$

$$= 1.4544 \times 10^{9}\,\text{m} \simeq 1.45 \times 10^{9}\,\text{m}.$$

The angle given is the diameter, so no extra factor of two is needed. Equivalently, the exact geometry gives $D = 2R\tan(\theta/2)$, which reduces to this expression at the stated small angle.

Step 3: Compare the magnitude with the choices The calculated diameter is roughly one hundredth of the distance because the angular diameter is about 0.01 radian. This check supports the power 10^9, rather than 10^{10}. Options (A) and (B) are much too large. Option (D), 0.14×10^9 m, is about ten times smaller than the result. Option (C) agrees with both the coefficient and the power of ten.

Step 4: Conclusion The diameter of the Sun is approximately $\boxed{1.45 \times 10^{9}\,\text{m}}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert angular seconds to radians before applying diameter = distance times angular diameter; do not introduce an extra factor of two.

33. When a ball is dropped into a lake from a height 4.9 m above the water level, it hits the water with a velocity v and then sinks to the bottom with the constant velocity v. It reaches the bottom of the lake 4.0 s after it is dropped. The approximate depth of the lake is : [4 marks]

(A) 19.6 m

(B) 29.4 m

(C) 39.2 m

(D) 73.5 m

Correct Answer: (B) 29.4 m

Solution:

Concept:

- For a body dropped from rest, $h = \frac{1}{2}gt^2$ and $v = gt$, taking downward as positive.
- The underwater motion is explicitly at constant velocity. Its distance is therefore speed times the remaining time, rather than a free-fall displacement.

Step 1: Find the time spent above the water The ball begins from rest and falls through $h = 4.9\,\text{m}$. Taking $g = 9.8\,\text{m s}^{-2}$,

$$4.9 = \frac{1}{2}(9.8)t_1^2,$$

$$t_1^2 = 1\,\text{s}^2, \qquad t_1 = 1.0\,\text{s}.$$

Choose the positive root because time elapsed is positive. This first second is part of the stated total time of four seconds and must not be counted again underwater.

Step 2: Determine the speed and underwater duration The speed at the water surface is

$$v = gt_1 = 9.8(1.0) = 9.8\,\mathrm{m\,s^{-1}}.$$

The same speed is maintained while sinking, as the problem specifies. The time spent in the lake is

$$t_2 = 4.0 - 1.0 = 3.0\,\mathrm{s}.$$

Acceleration under gravity was used only during the fall through air; introducing another gt_2 contribution underwater would contradict the constant-speed condition.

Step 3: Calculate the lake depth and check the options The lake depth is the underwater displacement:

$$d = vt_2 = (9.8)(3.0) = 29.4\,\mathrm{m}.$$

Option (A) would correspond to only two seconds of sinking. Option (C) uses the entire four seconds underwater and ignores the initial fall time. Option (D) is also inconsistent with a speed of $9.8\,\mathrm{m\,s^{-1}}$ maintained for three seconds. Thus the specified two-stage motion selects option (B).

Step 4: Conclusion The approximate depth of the lake is $\boxed{29.4\,\mathrm{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the entry speed and subtract the air-fall time from the total time before multiplying speed by the sinking time.

34. One end of a massless spring of spring constant k and natural length l_0 is fixed while the other end is connected to a small object of mass m lying on a frictionless table. The spring remains horizontal on the table. If the object is made to rotate at an angular velocity ω about an axis passing through fixed end, then the elongation of the spring will be : [4 marks]

(A) $\frac{k-m\omega^2 l_0}{m\omega^2}$

(B) $\frac{m\omega^2 l_0}{k+m\omega^2}$

(C) $\frac{m\omega^2 l_0}{k-m\omega^2}$

(D) $\frac{k+m\omega^2 l_0}{m\omega^2}$

Correct Answer: (C) $\frac{m\omega^2 l_0}{k-m\omega^2}$

Solution:

Concept:

- If the extension is x, Hooke's law gives spring tension kx. The circular orbit radius is the stretched length $l_0 + x$, not the natural length alone.
- The radial force required at angular speed ω is $m\omega^2 r$. On the horizontal frictionless table, weight and normal reaction cancel vertically.

Step 1: Set up the radial force balance The only horizontal force on the mass is the inward spring

force. Calling the extension x, the distance from the fixed end to the mass is

$$r = l_0 + x.$$

For steady rotation at angular speed ω, Newton's second law along the radius gives

$$kx = m\omega^2(l_0 + x).$$

The extension affects both sides: it determines the tension and increases the radius of the circular path. Replacing the radius with l_0 would neglect the latter effect.

Step 2: Collect the extension terms Expand and move the term containing x to the left:

$$kx = m\omega^2 l_0 + m\omega^2 x,$$

$$(k - m\omega^2)x = m\omega^2 l_0.$$

Thus

$$x = \frac{m\omega^2 l_0}{k - m\omega^2}.$$

A finite positive extension requires $k > m\omega^2$. As the angular speed approaches $\sqrt{k/m}$ from below, the ideal model requires an increasingly large extension; at or above this limit no finite positive steady extension satisfies the equation.

Step 3: Check the expression against the alternatives Both k and $m\omega^2$ have units of force per length, so the result has units of length. It tends to zero as $\omega \to 0$, as a stationary mass on a frictionless table should. Option (B) has a plus sign because it would move the radius correction to the wrong side. Options (A) and (D) even combine unlike dimensions in their numerators. Option (C) alone satisfies the radial equation and these checks.

Step 4: Conclusion The elongation is $\boxed{\frac{m\omega^2 l_0}{k - m\omega^2}}$, for $k > m\omega^2$. Hence, the correct option is (C).

Quick Tip to Solve: Use the stretched radius $l_0 + x$ in the centripetal force; then collect all terms containing the extension on the same side.

35. A stone tide to a spring of length L is whirled in a vertical circle with the other end of the spring at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of change in its velocity, as it reaches a position where the string is horizontal, is $\sqrt{x(u^2 - gL)}$. The value of x is - [4 marks]

(A) 3

(B) 2

(C) 1

(D) 5

Correct Answer: (B) 2

Solution:

Concept:

- In vertical circular motion with a string of fixed length L, tension is radial and does no work. Mechanical energy relates speeds at different heights.
- Velocity is a vector. The tangents at the lowest point and at the horizontal-radius position are perpendicular, so the squared change in velocity is the sum of the squared speeds.

Step 1: Find the speed at the horizontal position Let the speed at the lowest point be u, and the speed when the string becomes horizontal be v. The stone rises through height L. Taking the lowest point as zero gravitational potential energy, conservation of energy gives

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgL.$$

Cancel the mass and rearrange:

$$v^2 = u^2 - 2gL.$$

The speed decreases because kinetic energy supplies the increase in gravitational potential energy. The question describes the stone reaching this position, so the motion is understood to permit it.

Step 2: Use the directions of the two velocities At the bottom, the velocity is horizontal because it is tangent to the circle. When the radius is horizontal, the tangent and therefore the velocity are vertical. Thus $\mathbf{u} \cdot \mathbf{v} = 0$. The magnitude of their vector difference is

$$|\Delta\mathbf{v}|^2 = |\mathbf{v} - \mathbf{u}|^2$$

$$= v^2 + u^2 - 2\mathbf{u} \cdot \mathbf{v} = v^2 + u^2.$$

Subtracting speed magnitudes would ignore the right-angle change of direction and would not answer the question.

Step 3: Match the required form Insert the energy result:

$$|\Delta\mathbf{v}|^2 = (u^2 - 2gL) + u^2$$

$$= 2(u^2 - gL).$$

Consequently,

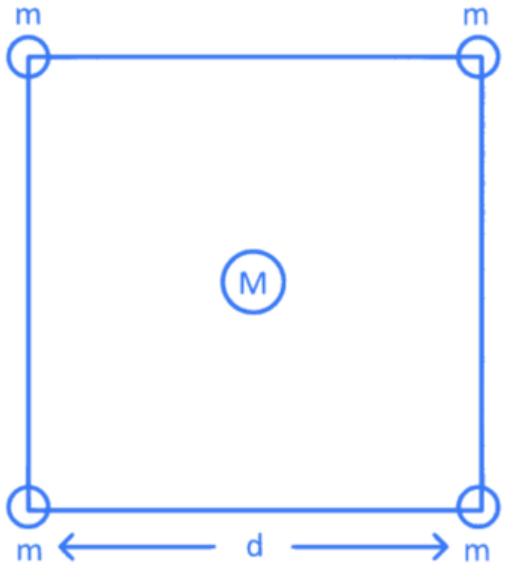
$$|\Delta \mathbf{v}| = \sqrt{2(u^2 - gL)}.$$

Comparison with $\sqrt{x(u^2 - gL)}$ gives $x = 2$. The alternatives 3, 1 and 5 would give the wrong coefficient of u^2, as can also be checked by setting gravity to zero: two perpendicular velocities of equal speed have difference $\sqrt{2}u$.

Step 4: Conclusion The dimensionless coefficient is $\boxed{x = 2}$. Hence, the correct option is (B).

Quick Tip to Solve: Combine energy conservation with the angle between the initial and final velocity vectors; a change in velocity is not just a change in speed.

36. Four spheres each of mass m from a square of side d (as shown in figure). A fifth sphere of mass M is situated at the centre of square. The total gravitational potential energy of the system is : [4 marks]



(A) $-\frac{Gm}{d}\left[(4+\sqrt{2})m + 4\sqrt{2}M\right]$
(B) $-\frac{Gm}{d}\left[(4+\sqrt{2})M + 4\sqrt{2}m\right]$
(C) $-\frac{Gm}{d}\left[3m^2 + 4\sqrt{2}M\right]$
(D) $-\frac{Gm}{d}\left[6m^2 + 4\sqrt{2}M\right]$

Correct Answer: (A) $-\frac{Gm}{d}\left[(4+\sqrt{2})m + 4\sqrt{2}M\right]$

Solution:

Concept:

- The gravitational potential energy of two point masses separated by r is $-Gm_1m_2/r$, taking zero energy at infinite separation.
- For a collection of masses, add the energies of every distinct pair once. Five spheres give $\binom{5}{2} = 10$ pairs: six corner pairs and four centre–corner pairs.

Step 1: Count the side and diagonal pairs The four sides each join two masses m at separation d.

The two diagonals have length $\sqrt{2}d$. Thus the energy of the corner spheres is

$$U_{\text{corner}} = -\frac{4Gm^2}{d} - \frac{2Gm^2}{\sqrt{2}d} = -\frac{Gm^2}{d}(4+\sqrt{2}).$$

The diagonal contribution must be included even though no line joining those spheres is explicitly drawn.

Step 2: Add the interactions with the central mass The distance from the square centre to each vertex is half a diagonal, $r = d/\sqrt{2}$. There are four such pairs, each involving M and m, so

$$U_{\text{centre}} = 4\left(-\frac{GMm}{d/\sqrt{2}}\right) = -\frac{4\sqrt{2}GMm}{d}.$$

No self-energy term is included: the question asks for the mutual potential energy of the sphere system.

Step 3: Factor the total and compare options Adding these contributions and extracting $-Gm/d$ gives

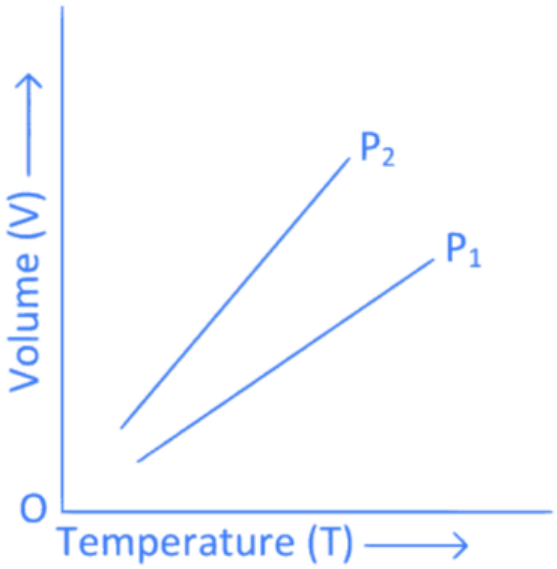
$$U = -\frac{Gm}{d}\left[(4+\sqrt{2})m + 4\sqrt{2}M\right].$$

Option B interchanges the roles of the central and corner masses. Options C and D have an incorrect corner contribution and even mix different mass powers inside their brackets. Every pair contributes negatively, as expected for attractive gravity.

Step 4: Conclusion The total gravitational potential energy is $-\frac{Gm}{d}[(4+\sqrt{2})m + 4\sqrt{2}M]$. Hence, the correct option is (A).

Quick Tip to Solve: Group pairs by separation: four sides, two diagonals and four centre–vertex distances. Check that the pair counts add to ten.

37. For a perfect gas, two pressures P_1 and P_2 are shown in figure. The graph shows : [4 marks]



(A) $P_1 > P_2$

(B) $P_1 < P_2$

(C) $P_1 = P_2$

(D) Insufficient data to draw any conclusion

Correct Answer: (A) $P_1 > P_2$

Solution:

Concept:

- For a fixed amount n of ideal gas, $PV = nRT$. On a volume–absolute-temperature plot an isobar obeys $V = (nR/P)T$.
- The slope of an isobar is nR/P: for the same gas sample the slope is inversely proportional to its pressure.

Step 1: Read the axes and relative slopes The vertical axis is volume V, and the horizontal axis is temperature T. The straight line labelled P_2 is steeper than the line labelled P_1. Therefore its change in volume for the same change in temperature is larger:

$$s_2 = \left(\frac{\Delta V}{\Delta T}\right)_{P_2} > s_1 = \left(\frac{\Delta V}{\Delta T}\right)_{P_1}.$$

It is the inclination on this particular pair of axes, rather than the pressure label position, that determines the comparison.

Step 2: Translate the slope comparison into pressures Using the ideal-gas relation for the same fixed amount of gas gives

$$\frac{s_2}{s_1} = \frac{nR/P_2}{nR/P_1} = \frac{P_1}{P_2} > 1.$$

Consequently $P_1 > P_2$. Equivalently, at any common temperature the upper line has the larger volume, and a larger volume means lower pressure when n and T are fixed.

Step 3: Check the remaining choices The inequality $P_1 < P_2$ would require the P_1 line to be steeper, opposite to the graph. Equal pressures would give equal slopes and coincident isobars for a fixed gas sample. Numerical scales are unnecessary because only a pressure comparison is requested; the relative slopes already supply the needed information. Thus the insufficient-data option is also excluded.

Step 4: Conclusion The shallower isobar represents the higher pressure, so $P_1 > P_2$. Hence, the correct option is (A).

Quick Tip to Solve: On a V-versus-T graph, write the slope as nR/P before comparing lines; higher slope means lower pressure.

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38. According to kinetic theory of gases,

A. The motion of the gas molecules freezes at 0°C.

B. The mean free path of gas molecules decreases if the density of molecules is increased.

C. The mean free path of gas molecules increases if temperature is increased keeping pressure constant.

D. Average kinetic energy per molecule per degree of freedom is $\frac{3}{2}k_BT$ (for monoatomic gases).

Choose the most appropriate answer from the options given below : [4 marks]

(A) A and C only

(B) B and C only

(C) A and B only

(D) C and D only

Correct Answer: (B) B and C only

Solution:

Concept:

- For molecules of collision diameter d and number density $n = N/V$, the mean free path is $\lambda = 1/(\sqrt{2}\pi d^2 n)$.
- At pressure P, the ideal-gas relation gives $n = P/(k_BT)$, hence $\lambda = k_BT/(\sqrt{2}\pi d^2 P)$. Each independent quadratic degree of freedom has average energy $k_BT/2$.

Step 1: Test statements A and B Statement A is false. A temperature of $0^\circ\mathrm{C}$ is approximately $273\,\mathrm{K}$, so a gas still has thermal molecular motion. The classical kinetic energy would vanish only at absolute zero, not at the Celsius zero. Statement B is true: increasing molecular number density reduces the distance travelled between successive collisions. From the mean-free-path expression, $\lambda \propto 1/n$ when molecular size is unchanged.

Step 2: Test statements C and D Statement C is true. At constant pressure, heating the gas reduces its number density because the volume increases. Substituting the ideal-gas relation shows directly that

$$\lambda = \frac{k_BT}{\sqrt{2}\pi d^2 P} \propto T \qquad (P \text{ constant}).$$

Statement D is false: $3k_BT/2$ is the total mean translational kinetic energy of one monatomic molecule, which has three translational degrees of freedom. Per degree of freedom the mean energy is only $k_BT/2$.

Step 3: Select the correct pair Only B and C survive all the checks. The combinations A and C or A and B include the false Celsius-zero statement. The combination C and D includes the false energy-per-degree-of-freedom statement. Distinguishing total translational energy from energy in one coordinate is essential here.

Step 4: Conclusion Statements B and C alone are correct. Hence, the correct option is (B).

Quick Tip to Solve: Keep $k_BT/2$ per quadratic degree of freedom distinct from $3k_BT/2$ for all three translational coordinates.

---

39. A lead bullet penetrates into a solid object and melts. Assuming that 40% of its kinetic energy is used to heat it, the initial speed of bullet is :

(Given : initial temperature of the bullet = 127°C, Melting point of the bullet = 327°C, Latent heat of fusion of lead = 2.5 × 10^4 J kg^{-1}, Specific heat capacity of lead = 125 J/kg K) [4 marks]

(A) 125 ms^{-1}

(B) 500 ms^{-1}

(C) 250 ms^{-1}

(D) 600 ms^{-1}

Correct Answer: (B) 500 ms^{-1}

Solution:

Concept:

- To melt an initially solid bullet, first supply sensible heat $mc\Delta T$, then latent heat mL_f at the melting point.
- Only the specified fraction of the initial kinetic energy heats the bullet: $0.40(\frac{1}{2}mv^2) = mc\Delta T + mL_f$.

Step 1: Calculate the temperature rise The bullet starts at 127°C and melts at 327°C. Its required temperature increase is

$$\Delta T = 327 - 127 = 200\,\text{K}.$$

A Celsius temperature difference has the same numerical value as a kelvin difference. The sensible heat required per kilogram is therefore

$$c\Delta T = 125 \times 200 = 25000\,\text{J}\,\text{kg}^{-1}.$$

The lead must reach its melting point before its phase can change.

Step 2: Include fusion and apply the energy fraction Adding the latent heat gives the total thermal energy per kilogram:

$$\frac{Q}{m} = c\Delta T + L_f = 25000 + 25000 = 50000\,\text{J}\,\text{kg}^{-1}.$$

The mass cancels when the heat gained is equated to forty percent of the kinetic energy:

$$0.40\left(\frac{1}{2}v^2\right) = 50000.$$

The remaining kinetic energy is transferred elsewhere during penetration and cannot be counted as heat of the bullet.

Step 3: Solve for the initial speed and check Thus

$$v^2 = \frac{50000}{0.20} = 250000\,\mathrm{m^2\,s^{-2}}, \qquad v = 500\,\mathrm{m\,s^{-1}}.$$

At this speed the initial kinetic energy per kilogram is $125000\,\mathrm{J\,kg^{-1}}$; forty percent is exactly $50000\,\mathrm{J\,kg^{-1}}$. The speeds 125 and 250 supply too little heat, while 600 supplies more than the required melting energy.

Step 4: Conclusion The initial speed of the lead bullet is $500\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Add $c\Delta T$ and latent heat before dividing by the heating fraction. The unknown bullet mass cancels out.

40. The equation of a particle executing simple harmonic motion is given by $x = \sin\pi\left(t + \frac{1}{3}\right) m$. At t = 1s, the speed of particle will be

(Given : π = 3.14) [4 marks]

(A) 0 cm s^{-1}

(B) 157 cm s^{-1}

(C) 272 cm s^{-1}

(D) 314 cm s^{-1}

Correct Answer: (B) 157 cm s^{-1}

Solution:

Concept:

- Velocity is the time derivative of displacement. For $x = A\sin(\omega t + \phi)$, $v = A\omega\cos(\omega t + \phi)$.
- Speed is the magnitude $|v|$, so a negative cosine changes the direction of motion but does not make speed negative.

Step 1: Differentiate the given displacement The displacement is

$$x = \sin\left[\pi\left(t + \frac{1}{3}\right)\right]\,\mathrm{m}.$$

Its amplitude is $A = 1\,\mathrm{m}$, and its angular frequency is $\omega = \pi\,\mathrm{s^{-1}}$. Applying the chain rule yields

$$v = \pi\cos\left[\pi\left(t + \frac{1}{3}\right)\right]\,\mathrm{m\,s^{-1}}.$$

The factor π comes from differentiating the phase; omitting it would give both the wrong magnitude and the wrong dimensions.

Step 2: Evaluate the phase at the stated instant At $t = 1\,\text{s}$, the phase is $\pi(1 + 1/3) = 4\pi/3$, whose cosine is $-1/2$. Therefore

$$v(1) = -\frac{\pi}{2}\,\text{m s}^{-1}.$$

The negative sign indicates motion toward decreasing displacement. Since the question asks for speed, take the positive magnitude:

$$|v(1)| = \frac{3.14}{2} = 1.57\,\text{m s}^{-1}.$$

Step 3: Convert units and compare choices The options use centimetres per second. Multiplying by 100 gives

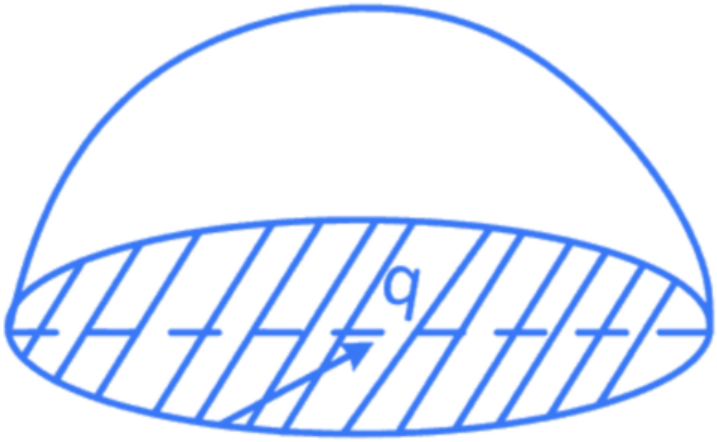
$$1.57\,\text{m s}^{-1} = 157\,\text{cm s}^{-1}.$$

Zero speed occurs at an extreme displacement, not at this phase. The value 272 results from using the sine rather than the cosine in the velocity. The value 314 is the maximum speed $A\omega$, achieved when the magnitude of the cosine is one.

Step 4: Conclusion The speed at $t = 1\,\text{s}$ is $157\,\text{cm s}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Differentiate first, substitute the phase second, and take the magnitude last. Convert metres to centimetres only after finding the speed.

41. If a charge q is placed at the centre of a closed hemispherical non-conducting surface, the total flux passing through the flat surface would be : [4 marks]



(A) $\frac{q}{\varepsilon_0}$
(B) $\frac{q}{2\varepsilon_0}$
(C) $\frac{q}{4\varepsilon_0}$
(D) $\frac{q}{2\pi\varepsilon_0}$

Correct Answer: (B) $\frac{q}{2\varepsilon_0}$

Solution:

Concept:

- An isolated point charge produces total electric flux q/ε_0 over all directions. The flux assigned to a surface subtending solid angle Ω is $q\Omega/(4\pi\varepsilon_0)$.
- A charge exactly on a Gaussian boundary requires a limiting prescription because its field is singular there. The usual symmetric boundary convention assigns half the full flux to a smooth boundary point.

Step 1: Identify the special position of the charge The diagram places q at the centre of the flat circular face, which is also the centre of the sphere from which the hemisphere is cut. The field is tangent to that plane at every ordinary point of the disk. Consequently one must not apply the ordinary enclosed-charge formula at the singular point without specifying how the surface is approached. This boundary position is different from a charge located strictly inside a closed hemisphere.

Step 2: Apply the boundary flux convention used by the options Take the disk as the limit of a parallel face approached from one side of the charge, or equivalently assign the half-space solid angle $\Omega = 2\pi$ at the charge. The magnitude of the flux through this limiting flat face is

$$|\Phi| = \frac{q}{4\pi\varepsilon_0}(2\pi) = \frac{q}{2\varepsilon_0}.$$

A small displacement of the charge into the hemisphere makes this limiting value explicit; the disk then captures half the flux as that displacement tends to zero.

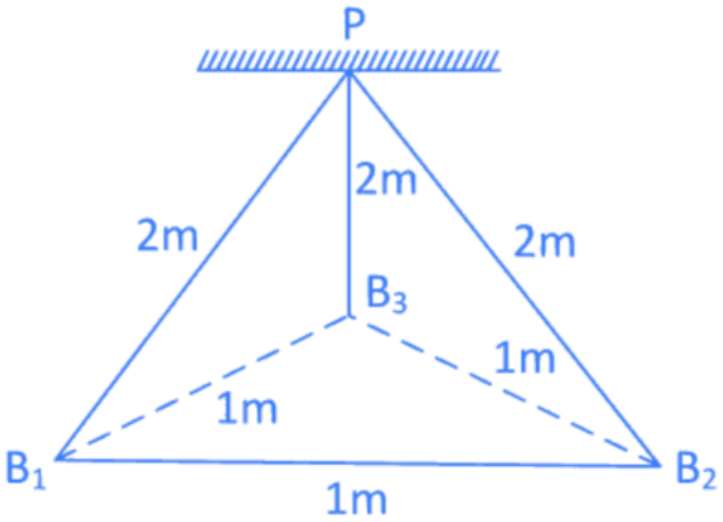
Step 3: State the interpretation and eliminate alternatives With the standard one-sided limiting interpretation, the flux magnitude is one half of the full spherical flux. It is not q/ε_0, which corresponds to all 4π steradians, or $q/(4\varepsilon_0)$, which corresponds to a quarter of that angular coverage. The factor 2π cancels against 4π and does not remain in the denominator. Strictly, the unregularised field integral on the exact plane excludes a singular point; the selected answer uses the conventional limiting flux magnitude.

Step 4: Conclusion Using the usual boundary-charge limiting convention, the flux magnitude through the flat face is $q/(2\varepsilon_0)$. Hence, the correct option is (B).

Quick Tip to Solve: For a charge lying on a Gaussian surface, state the limiting convention before applying Gauss's law; direct substitution of enclosed charge is unsafe.

42. Three identical charged balls each of charge 2 C are suspended from a common point P by silk threads of 2 m each (as shown in figure). They form an equilateral triangle of side 1m.

The ratio of net force on a charged ball to the force between any two charged balls will be : [4 marks]



(A) 1 : 1

(B) 1 : 4

(C) $\sqrt{3}$: 2

(D) $\sqrt{3}$: 1

Correct Answer: (D) $\sqrt{3}$: 1

Solution:

Concept:

- Coulomb's law gives the magnitude of the force between two charges: $F = kq^2/a^2$ for equal charges q separated by a.
- Two vectors of equal magnitude F with angle θ between them have a resultant along their angle bisector, with magnitude $R = 2F\cos(\theta/2)$.

Step 1: Find the directions and magnitudes of the two forces Choose the ball at one vertex of the equilateral triangle. The other two balls repel it along the lines joining their centres. Since all three charges are identical and every side is $a = 1\,\text{m}$, both repulsive forces have the same magnitude

$$F = \frac{k(2\,\text{C})^2}{(1\,\text{m})^2}.$$

Their numerical magnitude is unnecessary because the question asks for a ratio. Reversing both side directions to represent repulsion preserves the included angle of 60°.

Step 2: Add the forces as vectors Using the cosine rule for the vector sum gives

$$R^2 = F^2 + F^2 + 2F^2\cos 60^\circ = 3F^2, \qquad R = \sqrt{3}F.$$

The resultant lies along the outward angle bisector at the chosen vertex. One may also resolve each

force along this bisector: the transverse components cancel and the longitudinal components add to $2F\cos 30^\circ = \sqrt{3}F$.

Step 3: Form the ratio and clarify the force being compared The requested net force is the resultant electrostatic force from the other two charged balls, so

$$R : F = \sqrt{3} : 1.$$

The thread length affects the suspension geometry but cancels from this comparison, since the side lengths already fix both electrostatic forces. The total mechanical force including gravity and tension is zero in equilibrium; that is not the electrostatic resultant compared by the listed options. The other ratios do not equal the vector sum of two equal forces at 60°.

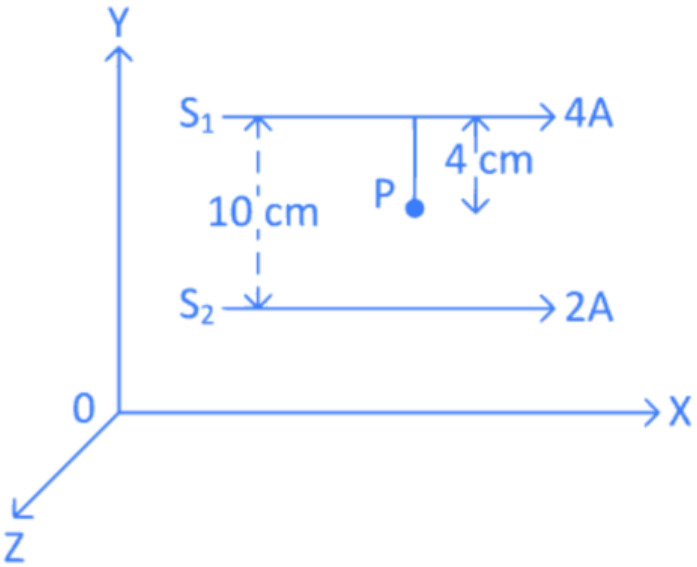
Step 4: Conclusion The net electrostatic force to pairwise force ratio is $\sqrt{3} : 1$. Hence, the correct option is (D).

Quick Tip to Solve: For two equal repulsions at a triangle vertex, use the angle between the force vectors. In an equilateral triangle the resultant is $\sqrt{3}F$, not $2F$.

43. Two long parallel conductors S_1 and S_2 are separated by a distance 10 cm and carrying currents of 4A and 2A respectively. The conductors are placed along x-axis in X-Y plane. There is a point P located between the conductors (as shown in figure).

A charge particle of 3π coulomb is passing through the point P with velocity $\vec{v} = (2\hat{i} + 3\hat{j})$ m/s; where $\hat{i}$ and $\hat{j}$ represents unit vector along x & y axis respectively.

The force acting on the charge particle is $4\pi \times 10^{-5}(-x\hat{i} + 2\hat{j})$ N. The value of x is : [4 marks]



(A) 2

(B) 1

(C) 3

(D) -3

Correct Answer: (C) 3

Solution:

Concept:

- The magnetic field of a long straight wire is $B = \mu_0 I/(2\pi r)$, where r is the perpendicular distance. The right-hand rule gives its direction.
- The magnetic Lorentz force is $\vec{F} = q(\vec{v} \times \vec{B})$. Fields from the two wires add vectorially, not as unsigned magnitudes.

Step 1: Find the field at the point The upper wire carries $4\,\text{A}$ along positive x and lies $0.04\,\text{m}$ above P. Its field is along negative z. P is $0.10 - 0.04 = 0.06\,\text{m}$ above the lower wire, whose $2\,\text{A}$ current gives a positive-z field. Thus

$$\vec{B} = 2 \times 10^{-7}\left(-\frac{4}{0.04} + \frac{2}{0.06}\right)\hat{k} = -\frac{4}{3} \times 10^{-5}\hat{k}\,\text{T}.$$

Step 2: Evaluate the vector product Insert the positive charge $q = 3\pi\,\text{C}$ and the given velocity. Since $\hat{i} \times \hat{k} = -\hat{j}$ and $\hat{j} \times \hat{k} = \hat{i}$,

$$\vec{F} = 3\pi(2\hat{i} + 3\hat{j}) \times \left(-\frac{4}{3} \times 10^{-5}\hat{k}\right)$$
$$= 4\pi \times 10^{-5}(-3\hat{i} + 2\hat{j})\,\text{N}.$$

Step 3: Compare the coefficients The positive y component agrees with the supplied force, and the x coefficient is -3, so $x = 3$. Values 1 and 2 give the wrong ratio of force components; $x = -3$ reverses the x force. As a further check, $\vec{F} \cdot \vec{v}$ is proportional to $(-3)(2) + (2)(3) = 0$, as a magnetic force must be perpendicular to velocity.

Step 4: Conclusion The required dimensionless value is $\boxed{x = 3}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $\vec{F} \cdot \vec{v} = 0$ first: here $-2x + 6 = 0$ gives x immediately; the wire fields verify the sign and magnitude.

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44. If L, C and R are the self inductance, capacitance and resistance respectively, which of the following does not have the dimension of time? [4 marks]

(A) RC

(B) $\frac{L}{R}$

(C) $\sqrt{LC}$

(D) $\frac{L}{C}$

Correct Answer: (D) $\frac{L}{C}$

Solution:

Concept:

- A dimension is the physical nature of a quantity, independent of the choice of units. In an equation, all terms added or equated must have the same dimensions.
- Use $V = IR$, $Q = CV$, $I = Q/t$ and the inductor relation $V = L\,dI/dt$. These determine dimensions without memorising the full base-unit expressions.

Step 1: Express inductance and capacitance in useful units From the inductor equation, $[L] = [V][T]/[I] = [R][T]$, so a henry is an ohm-second. From charge storage, $[C] = [Q]/[V] = [I][T]/[V] = [T]/[R]$, so a farad is a second per ohm. Resistance itself is not dimensionless: $[R] = [M\mathcal{L}^2T^{-3}I^{-2}]$, where $\mathcal{L}$ denotes length and I current.

Step 2: Test the three time expressions Using those relations gives

$$[RC] = [R]\frac{[T]}{[R]} = [T],$$
$$[L/R] = \frac{[R][T]}{[R]} = [T],$$
$$[\sqrt{LC}] = \sqrt{[R][T]\frac{[T]}{[R]}} = [T].$$

Thus options A, B and C all genuinely have the dimension of time. These also agree with the RC and RL relaxation times and the LC oscillation time scale; the square root in the third expression is essential.

Step 3: Check the remaining quotient For the fourth expression the time factors cancel instead:

$$\left[\frac{L}{C}\right] = \frac{[R][T]}{[T]/[R]} = [R]^2.$$

Its SI unit is Ω^2, or equivalently $\mathrm{kg^2\,m^4\,s^{-6}\,A^{-4}}$. This cannot equal the unit second, so the fourth expression is the exception.

Step 4: Conclusion $\boxed{L/C}$ has the dimensions of resistance squared, not time. Hence, the correct option is (D).

Quick Tip to Solve: Remember $\mathrm{H} = \Omega\,\mathrm{s}$ and $\mathrm{F} = \mathrm{s}/\Omega$; all four checks then reduce to simple cancellation.

45. Given below are two statements :

Statement I : A time varying electric field is a source of changing magnetic field and vice-versa. Thus a disturbance in electric or magnetic field creates EM waves.

Statement II : In a material medium, the EM wave travels with speed $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$. In the light of the above statements, choose the correct answer from the options given below. [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is correct but Statement II is false

(D) Statement I is incorrect but Statement II is true

Correct Answer: (C) Statement I is correct but Statement II is false

Solution:

Concept:

- Maxwell's equations couple electric and magnetic fields: a time-varying magnetic flux induces an electric field, and a time-varying electric field contributes to the magnetic field through displacement current.
- In a homogeneous nonconducting medium, electromagnetic wave speed is $v = 1/\sqrt{\mu\varepsilon}$, with the medium's permeability μ and permittivity ε.

Step 1: Evaluate Statement I Faraday's law is $\nabla \times \vec{E} = -\partial \vec{B}/\partial t$. In a source-free region the Ampere-Maxwell law is $\nabla \times \vec{B} = \mu\varepsilon\, \partial \vec{E}/\partial t$. A propagating electromagnetic disturbance therefore has coupled changing electric and magnetic fields. The statement describes this mutual generation and is true in the standard electromagnetic-wave description.

Step 2: Evaluate Statement II The subscript zero denotes vacuum values. Their combination gives the vacuum speed,

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.$$

In a material medium, substitute $\mu = \mu_r \mu_0$ and $\varepsilon = \varepsilon_r \varepsilon_0$:

$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}.$$

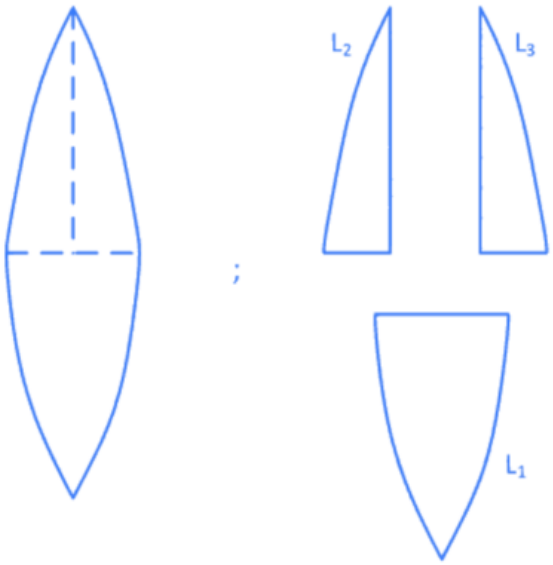
Material response therefore affects the speed. In a usual nonmagnetic transparent medium, μ_r is approximately 1 and the refractive index accounts primarily for the electric response. Vacuum constants alone do not give the general speed in matter, so Statement II is false.

Step 3: Select the matching truth combination The first statement is true and the second false. Option A wrongly accepts the vacuum expression for a material; B wrongly rejects the field-coupling statement; D reverses both assessments. Only C keeps the two conclusions in their correct order. The error in the second statement is its use of vacuum parameters, not the inverse-square-root form itself.

Step 4: Conclusion Statement I is correct but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Whenever an electromagnetic-wave speed contains μ_0, ε_0, identify it as the vacuum value; a material requires μ, ε.

46. A convex lens has power P. It is cut into two halves along its principal axis. Further one piece (out of the two halves) is cut into two halves perpendicular to the principal axis (as shown in figures). Choose the incorrect option for the reported pieces. [4 marks]



(A) Power of $L_1 = \frac{P}{2}$

(B) Power of $L_2 = \frac{P}{2}$

(C) Power of $L_3 = \frac{P}{2}$

(D) Power of L_1 = P

Correct Answer: (A) Power of $L_1 = \frac{P}{2}$

Solution:

Concept:

- For a thin lens in air, lens-maker's formula is $P = (\mu - 1)(1/R_1 - 1/R_2)$. Power depends on refractive index and surface curvature, not on aperture area.
- A cut along the principal axis reduces the transmitting aperture while preserving both curved surfaces. A perpendicular cut through the centre of the symmetric lens shown replaces one curved surface of each resulting piece by a plane.

Step 1: Analyse the piece cut along the axis The lower piece labelled L_1 retains the original front and rear radii of curvature. Each narrow ray bundle passing through it experiences the same paraxial refraction as before. Thus its focal length is the original focal length, and

$$P_1 = P.$$

The smaller aperture admits less light and can produce an off-axis image, but neither change halves the focal power. This establishes that the assertion in option D is true.

Step 2: Analyse the two pieces cut across the axis For the symmetric biconvex lens in the drawing,

take the original radii as $+R$ and $-R$. Then

$$P = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(\mu - 1)}{R}.$$

Each upper piece, L_2 or L_3, is plano-convex: the plane has infinite radius, so its curvature term vanishes. Hence

$$P_2 = P_3 = \frac{\mu - 1}{R} = \frac{P}{2}.$$

Reversing a plano-convex piece in air does not alter this thin-lens power.

Step 3: Identify the incorrect statement Options B and C correctly assign half the power to the plano-convex pieces, while option D correctly retains P for L_1. Option A instead assigns $P/2$ to the piece whose two curved faces remain unchanged. That is the incorrect claim requested. The distinction is between removing aperture and removing a curved refracting surface, not simply between pieces of different sizes.

Step 4: Conclusion The incorrect statement is $P_1 = P/2$; actually $P_1 = P$. Hence, the correct option is (A).

Quick Tip to Solve: After cutting a lens, count the curved surfaces along the light path. Cutting aperture alone does not change thin-lens power.

47. If a wave gets refracted into a denser medium, then which of the following is true? [4 marks]

(A) wavelength, speed and frequency decreases.

(B) wavelength increases, sped decreases and frequency remains constant.

(C) wavelength and speed decreases but frequency remains constant.

(D) wavelength, speed and frequency increases.

Correct Answer: (C) wavelength and speed decreases but frequency remains constant.

Solution:

Concept:

- At a stationary interface, the incident and transmitted waves have the same frequency. The source fixes the oscillation rate; the interface cannot continuously create or remove wave crests.
- Wave speed and wavelength obey $v = f\lambda$. For light, optical density is described by refractive index $n = c/v$, so a larger index means a smaller propagation speed.

Step 1: Determine the frequency after refraction Let the incident wave have frequency f_1 and the transmitted wave frequency f_2. The boundary particles or fields must oscillate at one common frequency to join the two waves continuously in time. Thus

$$f_2 = f_1 = f.$$

Refraction changes propagation direction and speed in general; it does not change the source frequency. In the usual refraction context of this question, denser means optically denser.

Step 2: Compare speed and wavelength If the refractive indices are $n_2 > n_1$, then

$$\frac{v_2}{v_1} = \frac{n_1}{n_2} < 1.$$

Since frequency remains constant,

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2/f}{v_1/f} = \frac{n_1}{n_2} < 1.$$

Both speed and wavelength therefore decrease together in the denser medium. A shorter distance is travelled during each unchanged oscillation period, which explains the reduced spacing between successive crests.

Step 3: Eliminate the other options Option A incorrectly decreases frequency as well as wavelength and speed. Option B retains frequency and reduces speed but increases wavelength; that contradicts $v = f\lambda$ for a fixed f. Option D increases all three quantities, contradicting both the boundary-frequency condition and the lower speed of the optically denser medium. Option C alone preserves frequency and decreases the other two quantities consistently.

Step 4: Conclusion Wavelength and speed decrease, while frequency remains constant. Hence, the correct option is (C).

Quick Tip to Solve: Keep frequency fixed at a stationary boundary, then use $\lambda = v/f$: wavelength must change in the same ratio as speed.

48. Given below are two statements :

Statement I : In hydrogen atom, the frequency of radiation emitted when an electron jumps from lower energy orbit (E_1) to higher energy orbit (E_2), is given as $hf = E_1 - E_2$

Statement II : The jumping of electron from higher energy orbit (E_2) to lower energy orbit (E_1) is associated with frequency of radiation given as $f = (E_2 - E_1)/h$

This condition is Bohr's frequency condition.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is correct but Statement II is false.

(D) Statement I is incorrect but Statement II is true.

Correct Answer: (D) Statement I is incorrect but Statement II is true.

Solution:

Concept:

- A photon has positive energy hf, where h is Planck's constant and $f > 0$ is its frequency. Energy conservation relates photon energy to the difference between atomic levels.
- An upward transition requires absorption of energy; a downward transition releases energy as radiation. For levels ordered $E_2 > E_1$, their positive energy separation is $E_2 - E_1$.

Step 1: Test the claimed emission in Statement I The transition described goes from the lower level E_1 to the higher level E_2. The atom must gain energy, so it absorbs a photon rather than emitting one. The required absorption energy is

$$hf = E_2 - E_1 > 0.$$

The printed expression $E_1 - E_2$ is negative for this ordering and cannot be the energy of an emitted photon of positive frequency. Both the emission description and its sign are wrong. Statement I is false.

Step 2: Apply conservation of energy to Statement II For a transition from E_2 down to E_1, the loss of atomic energy appears in the emitted photon:

$$E_2 = E_1 + hf.$$

Rearranging gives

$$f = \frac{E_2 - E_1}{h}.$$

The numerator is positive, and division by Planck's constant gives units of inverse seconds, as a frequency requires. This is Bohr's frequency condition for emission, so Statement II is true. Hydrogen levels may have negative energies, but the ordering of higher and lower levels and the positive difference remain the same.

Step 3: Match the pair of verdicts The truth values are false for I and true for II, which is option D. Option A accepts the incorrect upward-emission claim, B rejects the correct downward frequency relation, and C reverses the actual results. The sign must be decided from energy flow rather than from whether an individual bound-state energy is negative.

Step 4: Conclusion Statement I is incorrect and Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Upward atomic jumps absorb energy; downward jumps emit. Always take the photon energy as the positive level separation.

49. For a transistor to act as a switch, it must be operated in [4 marks]

(A) Active region.

(B) Saturation state only.

(C) Cut-off state only.

(D) Saturation and cut-off state.

Correct Answer: (D) Saturation and cut-off state.

Solution:

Concept:

- A transistor used as a switch represents two distinct circuit states: an OFF state with negligible collector current and an ON state with a small collector-emitter voltage.
- For an ordinary bipolar junction transistor, these states are obtained in cutoff and saturation respectively. The active region is used mainly for approximately linear current amplification.

Step 1: Identify the OFF state In cutoff, the base-emitter junction is not sufficiently forward biased to inject an appreciable base current. Apart from small leakage, the collector current is negligible:

$$I_B \simeq 0, \qquad I_C \simeq 0.$$

The collector-emitter path then behaves like an open switch. In a simple common-emitter switching circuit with a collector load, almost the full supply voltage can appear across that path. Cutoff supplies one required state, but cannot by itself provide switching between OFF and ON.

Step 2: Identify the ON state With sufficient base drive, the transistor enters saturation. Both junctions are forward biased and the collector-emitter voltage falls to a small saturation value:

$$V_{CE} \simeq V_{CE(\text{sat})}.$$

The collector current is then primarily limited by the external load and supply, approximately $I_C = (V_{CC} - V_{CE(\text{sat})})/R_C$ for a resistive collector load. This low-voltage conducting path behaves like a closed switch. The ideal closed-switch voltage is zero; the practical transistor retains a small nonzero voltage.

Step 3: Distinguish switching from amplification In the active region the base-emitter junction is forward biased and the collector-base junction reverse biased; collector current varies approximately as βI_B. This controlled intermediate state suits amplification rather than either limiting switch state. Thus A is unsuitable. B specifies only ON, while C specifies only OFF. A complete switching operation must alternate between saturation and cutoff, as stated in D.

Step 4: Conclusion The transistor must operate between saturation (ON) and cutoff (OFF). Hence, the correct option is (D).

Quick Tip to Solve: Associate cutoff with an open switch and saturation with a closed switch; active operation belongs to amplification.

50. We do not transmit low frequency signal to long distances because-

(a) The size of the antenna should be comparable to signal wavelength which is unreal solution for a signal of longer wavelength.

(b) Effective power radiated by a long wavelength baseband signal would be high.

(c) We want to avoid mixing up signals transmitted by different transmitter simultaneously.

(d) Low frequency signal can be sent to long distances by superimposing with a high frequency wave as well.

Therefore, the most suitable option will be : [4 marks]

(A) All statements are true

(B) (a), (b) and (c) are true only

(C) (a), (c) and (d) are true only

(D) (b), (c) and (d) are true only

Correct Answer: (C) (a), (c) and (d) are true only

Solution:

Concept:

- An efficient transmitting antenna must have a length comparable to the wavelength, commonly about $\lambda/4$, where $\lambda = c/f$. A low frequency therefore demands an impractically large antenna.
- Modulation superimposes the information signal on a high-frequency carrier. It improves radiation efficiency and allows different transmitters to occupy separate carrier-frequency bands.

Step 1: Assess antenna size and radiated power Statement (a) is true: a low-frequency baseband signal has a very long wavelength, so an efficient antenna would be impractically large. For example, at $f = 1\,\text{kHz}$, $\lambda = 3 \times 10^5\,\text{m}$, and a quarter-wave antenna would be $75\,\text{km}$ long. For a fixed short antenna, radiated power varies as $(l/\lambda)^2$. Thus increasing the wavelength reduces the radiated power; it does not make it high. Statement (b) is false.

Step 2: Assess separation and modulation Statement (c) is true: direct transmission of several overlapping low-frequency information signals makes it difficult to separate them at the receiver. Assigning different high-frequency carriers provides frequency separation. Statement (d) is also true: modulation places the information on a high-frequency wave suitable for transmission over a long distance. Therefore the accepted set must include (a), (c) and (d), and exclude (b). Options (A), (B) and (D) all contain the false statement (b), so each is rejected.

Step 3: Conclusion The antenna-size, channel-separation and carrier-modulation statements are true. Hence, the correct option is (C).

Quick Tip to Solve: For communication questions, check all three motivations for modulation: practical antenna length, efficient radiation and separation of simultaneous signals.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A mass of 10 kg is suspended vertically by a rope of length 5 m from the roof. A force of 30 N is applied at the middle point of rope in horizontal direction. The angle made by upper half of the rope with vertical is $\theta = \tan^{-1}(x \times 10^{-1})$. The value of x is _____.
(Given, g = 10 m/s^2) [4 marks]

Solution:

Concept:

- A suspended body in static equilibrium has zero resultant force. For the $10\,\text{kg}$ load, the tension in the lower vertical half of the light rope equals mg.
- At the midpoint, the upper tension balances both the horizontal applied force and the downward lower-rope tension. Resolving it gives $\tan\theta = F/(mg)$.

Step 1: Find the tension below the point of application The load is acted on by its weight and the tension of the lower half of the rope. Since there is no horizontal force on the load, that lower half remains vertical. Its tension is therefore

$$T_{\text{lower}} = mg = 10 \times 10 = 100\,\text{N}.$$

The horizontal force acts at the midpoint, so it changes the direction and magnitude of the tension above that point; it does not add to the weight of the suspended load.

Step 2: Resolve the upper tension Let T be the upper-half tension, making angle θ with the vertical. Equilibrium of the midpoint requires

$$T\sin\theta = 30\,\text{N}, \qquad T\cos\theta = 100\,\text{N}.$$

Dividing eliminates the unknown tension:

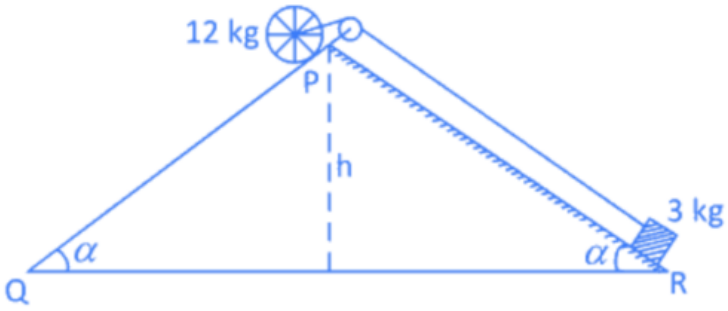
$$\tan\theta = \frac{30}{100} = 0.3 = 3 \times 10^{-1}.$$

Consequently $\theta = \tan^{-1}(3 \times 10^{-1})$. The rope length affects the geometry of the displacement but is unnecessary for this force ratio.

Step 3: Conclusion Comparing with $\theta = \tan^{-1}(x \times 10^{-1})$, we obtain $\boxed{x = 3}$, a dimensionless number.

Quick Tip to Solve: When an angle is measured from the vertical, divide the horizontal force by the vertical force to obtain its tangent.

52. A rolling wheel of 12 kg is on an inclined plane at position P and connected to a mass of 3 kg through a string of fixed length and pulley as shown in figure. Consider PR as friction free surface. The velocity of centre of mass of the wheel when it reaches at the bottom Q of the inclined plane PQ will be $\frac{1}{2}\sqrt{xgh}$ m/s. The value of x is _____. [4 marks]



Solution:

Concept:

- For the wheel represented by its rim and spokes, use the thin-wheel moment of inertia $I = MR^2$. Rolling without slipping gives $\omega = v/R$, so its total kinetic energy is Mv^2.
- The fixed-length string joins the wheel axle to the block over a fixed pulley. Their speeds are equal. Since the two inclines have the same angle, when the wheel descends through height h, the block rises through height h.

Step 1: Calculate the decrease in gravitational potential energy The wheel has mass $M = 12\,\text{kg}$, while the block has mass $m = 3\,\text{kg}$. The wheel loses potential energy $12gh$, and the block gains $3gh$. Therefore the energy available for motion is

$$\Delta E = (12 - 3)gh = 9gh.$$

The surface PR is frictionless, so the block has translational kinetic energy only. The wheel on PQ both translates and rotates; static friction in ideal rolling supplies the torque without dissipating energy.

Step 2: Include both translation and wheel rotation If both bodies have speed v, conservation of mechanical energy gives

$$\begin{aligned} 9gh &= \frac{1}{2}(12)v^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}(3)v^2 \\ &= 6v^2 + 6v^2 + \frac{3}{2}v^2 \\ &= \frac{27}{2}v^2. \end{aligned}$$

Hence

$$v^2 = \frac{2}{3}gh, \qquad v = \sqrt{\frac{2gh}{3}}.$$

Comparing its square with the stated parametrisation $v = \frac{1}{2}\sqrt{xgh}$ yields $x/4 = 2/3$, or $x = 8/3$. The integer numerical response is the nearest integer, rather than an assertion that the unrounded coefficient is exactly three.

Step 3: Conclusion The wheel speed is $\sqrt{2gh/3}\,\text{m}\,\text{s}^{-1}$ when g, h are in SI units. The coefficient is

$x = 8/3$, giving the integer numerical entry $\boxed{3}$ on rounding.

Quick Tip to Solve: Include the rotating wheel and the translating block in the same energy equation. Round the requested numerical coefficient only after comparing with the given speed form.

53. A diatomic gas (γ = 1.4) does 400J of work when it is expanded isobarically. The heat given to the gas in the process is _____ J. [4 marks]

Solution:

Concept:

- For an ideal gas undergoing an isobaric process, the work done is $W = P\Delta V = nR\Delta T$, while the heat supplied is $Q = nC_P\Delta T$.
- The heat capacities obey $C_P - C_V = R$ and $\gamma = C_P/C_V$, giving $C_P/R = \gamma/(\gamma - 1)$. Work done by the gas and heat supplied to it are positive here.

Step 1: Express the heat in terms of the work The expansion occurs at constant pressure, so the ideal-gas equation gives

$$P\Delta V = nR\Delta T = 400\,\text{J}.$$

For the same temperature increase, the heat received is $Q = nC_P\Delta T$. Dividing these expressions avoids the need to know the amount of gas or its initial temperature:

$$\frac{Q}{W} = \frac{C_P}{R}.$$

Because the gas also gains internal energy, the heat supplied must exceed the work done.

Step 2: Use the heat-capacity ratio and check the energy balance Using $C_P = \gamma C_V$ in Mayer's relation gives

$$(\gamma - 1)C_V = R, \qquad \frac{C_P}{R} = \frac{\gamma}{\gamma - 1} = \frac{1.4}{0.4} = 3.5.$$

Hence

$$Q = 3.5 \times 400 = 1400\,\text{J}.$$

As a check, $\Delta U = nC_V\Delta T = W/(\gamma - 1) = 1000\,\text{J}$, and the first law gives

$$Q = \Delta U + W = 1000 + 400 = 1400\,\text{J}.$$

The positive sign agrees with heating during expansion.

Step 3: Conclusion The heat supplied to the gas is $\boxed{1400\,\text{J}}$; the numerical entry is 1400.

Quick Tip to Solve: For an isobaric ideal-gas process, use $Q/W = \gamma/(\gamma - 1)$; for internal energy alone, use $\Delta U/W = 1/(\gamma - 1)$.

54. A particle executes simple harmonic motion. Its amplitude is 8 cm and time period is 6 s. The time it will take to travel from its position of maximum displacement to the point corresponding to half of its amplitude, is _____ s. [4 marks]

Solution:

Concept:

- When timing starts at the positive extreme of simple harmonic motion, the displacement is $y = A\cos(\omega t)$, where A is the amplitude and $\omega = 2\pi/T$.
- The first arrival at half the positive amplitude obeys $\cos(\omega t) = 1/2$, with phase between 0 and $\pi/2$. SHM does not cover equal displacements in equal times.

Step 1: Choose the initial phase at maximum displacement Take $t = 0$ when the particle is at $y = +A$. Its velocity is zero there, so the cosine form is appropriate:

$$y = 8\cos\left(\frac{2\pi t}{6}\right)\ \text{cm}.$$

The required point on the first inward journey is $y = A/2 = 4\,\text{cm}$. Dividing by the amplitude gives

$$\frac{y}{A} = \frac{1}{2} = \cos\left(\frac{\pi t}{3}\right).$$

The amplitude cancels because the requested destination is specified as a fraction of it.

Step 2: Select the first inward crossing On the first journey from the extreme toward the mean position, the phase increases from 0 to $\pi/2$. In that interval,

$$\cos\phi = \frac{1}{2} \quad \Rightarrow \quad \phi = \frac{\pi}{3}.$$

Therefore

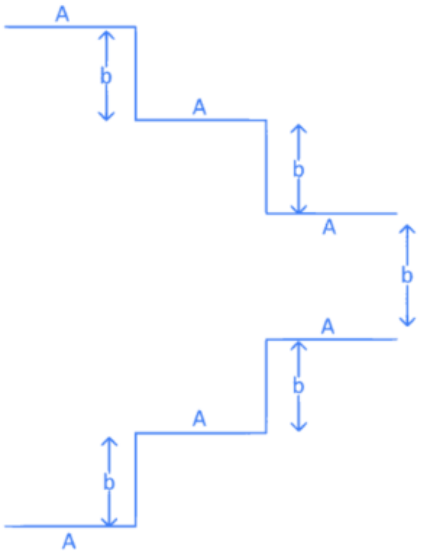
$$\frac{\pi t}{3} = \frac{\pi}{3} \quad \Rightarrow \quad t = 1\,\text{s}.$$

Equivalently, this time is $T/6$. It is less than the extreme-to-mean time $T/4 = 1.5\,\text{s}$, as expected; the particle speeds up as it approaches the mean position.

Step 3: Conclusion The required first travel time is $\boxed{1\,\text{s}}$; the numerical entry is 1.

Quick Tip to Solve: Starting at an extreme, use a cosine and solve for the phase. Reaching half the amplitude takes $T/6$, not half of $T/4$.

55. A parallel plate capacitor is made up of stair like structure with a plate area A of each stair and that is connected with a wire of length b, as shown in the figure. The capacitance of the arrangement is $\frac{x}{15}\frac{\varepsilon_0 A}{b}$. The value of x is _____. [4 marks]



Solution:

Concept:

- The capacitance of a parallel-plate region of overlap area A and separation d is $C = \varepsilon_0 A/d$, neglecting edge effects.
- All terraces on one continuous metal staircase are at the same potential. Corresponding terraces on the opposite staircase therefore form capacitors in parallel, even though their separations differ.

Step 1: Identify the three parallel regions The upper staircase is one conducting plate, and the lower staircase is the other. Opposing horizontal terraces have the same overlap area A. From right to left, their separations are b, $3b$ and $5b$: moving one terrace left raises the top by b and lowers the bottom by another b. Thus each step increases the gap by $2b$. The vertical connecting parts join terraces at a common potential; they are not separate floating plates forming a series combination.

Step 2: Add the capacitances at the common voltage The three facing regions experience the same potential difference, so their charges add at fixed voltage:

$$\begin{aligned} C_{\text{eq}} &= \frac{\varepsilon_0 A}{b} + \frac{\varepsilon_0 A}{3b} + \frac{\varepsilon_0 A}{5b} \\ &= \frac{\varepsilon_0 A}{b}\left(1 + \frac{1}{3} + \frac{1}{5}\right) \\ &= \frac{23}{15}\frac{\varepsilon_0 A}{b}. \end{aligned}$$

The equivalent capacitance exceeds that of the closest terrace alone, as a parallel combination should. Treating the terraces as series capacitors would incorrectly assign different potential differences to pieces of the same conductor.

Step 3: Conclusion Comparing $C_{\text{eq}} = (23/15)(\varepsilon_0 A/b)$ with the given form gives $\boxed{x = 23}$, a dimen-

sionless number.

Quick Tip to Solve: First trace which surfaces are electrically connected. For opposing staircases, measure the full gap at each overlap before adding their parallel capacitances.

56. The current density in a cylindrical wire of radius r = 4.0 mm is 1.0×10^6 A/m^2. The current through the outer portion of the wire between radial distances $\frac{r}{2}$ and r is xπ A; where x is _____.

[4 marks]

Solution:

Concept:

- For a uniform current density J normal to a cross-section, the current through area S is $I = JS$. An annulus has area equal to its outer disk area minus its inner disk area.
- The region from radius $r/2$ to r has area $\pi[r^2 - (r/2)^2] = (3/4)\pi r^2$. The radius must be converted from millimetres to metres before squaring.

Step 1: Calculate the conducting annular area The outer radius is $r = 4.0\,\text{mm} = 4.0 \times 10^{-3}\,\text{m}$, and the inner radius of the requested region is $r/2 = 2.0 \times 10^{-3}\,\text{m}$. Its area is

$$\begin{aligned} S &= \pi\left[(4.0 \times 10^{-3})^2 - (2.0 \times 10^{-3})^2\right] \\ &= \pi(16 - 4) \times 10^{-6} \\ &= 12\pi \times 10^{-6}\,\text{m}^2. \end{aligned}$$

Using the entire disk would include the central portion that the question excludes.

Step 2: Multiply by the uniform current density With $J = 1.0 \times 10^6\,\text{A m}^{-2}$, the current in that annulus is

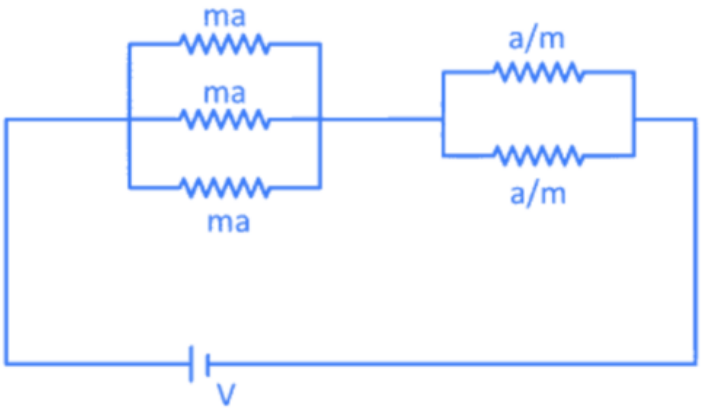
$$I = JS = (1.0 \times 10^6)(12\pi \times 10^{-6}) = 12\pi\,\text{A}.$$

The powers of ten cancel, leaving amperes. As a check, the current in the whole wire is 16π A, while the current in its central disk is 4π A. Their difference is 12π A, consistent with the annular calculation. The outer half of the radius carries three quarters of the total current, not one half.

Step 3: Conclusion The annular current is $\boxed{12\pi\,\text{A}}$. Thus the dimensionless coefficient is $\boxed{x = 12}$.

Quick Tip to Solve: For uniform current density, current fractions equal area fractions. Removing the inner half-radius disk removes only one quarter of the current.

57. In the given circuit 'a' is an arbitrary constant. The value of m for which the equivalent circuit resistance is minimum, will be $\sqrt{\frac{x}{2}}$. The value of x is _____. [4 marks]



Solution:

Concept:

- For equal resistors in parallel, the equivalent resistance is the resistance of one divided by their number. The two parallel groups in this circuit are connected in series.
- Physical resistances are positive. With $a > 0$ and $m > 0$, minimise the resulting function of m by differentiation; a positive second derivative confirms a minimum.

Step 1: Reduce the two parallel groups The left group has three resistors, each of resistance ma, sharing the same two nodes. The right group has two resistors, each of resistance a/m, sharing another pair of nodes. Therefore,

$$R_L = \frac{ma}{3}, \qquad R_R = \frac{a}{2m}.$$

The current passes through these groups successively, so their equivalent resistances add, rather than combine in parallel.

Step 2: Minimise the series equivalent The total resistance is

$$R(m) = a\left(\frac{m}{3} + \frac{1}{2m}\right).$$

Differentiating while holding the arbitrary constant a fixed gives

$$\frac{dR}{dm} = a\left(\frac{1}{3} - \frac{1}{2m^2}\right) = 0.$$

Hence $m^2 = 3/2$. The physical root is $m = \sqrt{3/2}$, since a negative value would not describe the positive resistors shown. Also,

$$\frac{d^2R}{dm^2} = \frac{a}{m^3} > 0.$$

Thus the stationary point is a minimum. The resistance increases without bound at either end of the domain, as $m \to 0^+$ or $m \to \infty$, confirming this is the global minimum.

Step 3: Conclusion Comparing $m = \sqrt{3/2}$ with $\sqrt{x/2}$, the required dimensionless value is $\boxed{x = 3}$.

Quick Tip to Solve: Reduce each parallel group before minimising. At the optimum here, the two series contributions $am/3$ and $a/(2m)$ are equal.

---

58. A deuteron and a proton moving with equal kinetic energy enter into a uniform magnetic field at right angle to the field. If r_d and r_p are the radii of their circular paths respectively, then the ratio $\frac{r_d}{r_p}$ will be $\sqrt{x}$: 1 where x is _____. [4 marks]

Solution:

Concept:

- A charge entering a uniform magnetic field perpendicular to the field undergoes circular motion because the magnetic force is perpendicular to its velocity.
- Equating magnetic and centripetal forces gives $r = p/(qB)$. For nonrelativistic kinetic energy K, momentum is $p = \sqrt{2mK}$.

Step 1: Express the radius in terms of kinetic energy Let m, v and q denote the particle mass, speed and charge magnitude. The force balance is

$$qvB = \frac{mv^2}{r}, \qquad r = \frac{mv}{qB}.$$

Since $K = mv^2/2$, this becomes

$$r = \frac{\sqrt{2mK}}{qB}.$$

Equal kinetic energies do not mean equal speeds: the more massive particle is slower, but has the larger momentum. It is momentum, rather than speed alone, that sets the magnetic radius.

Step 2: Compare the deuteron and proton A proton and a deuteron both have charge magnitude e. The deuteron contains a proton and a neutron, so its mass is approximately twice the proton mass, $m_d = 2m_p$, as used in this comparison. The field and kinetic energy are common to the particles. Therefore,

$$\frac{r_d}{r_p} = \frac{\sqrt{2m_dK}/(eB)}{\sqrt{2m_pK}/(eB)} = \sqrt{\frac{m_d}{m_p}} = \sqrt{2}.$$

The common charge, energy and magnetic field cancel. A factor of two in mass produces a factor of $\sqrt{2}$, rather than two, in radius for equal kinetic energies.

Step 3: Conclusion The radius ratio is $r_d : r_p = \sqrt{2} : 1$. Thus $\boxed{x = 2}$, a dimensionless number.

Quick Tip to Solve: For equal kinetic energies in the same field, compare $\sqrt{m}/q$; for equal speeds, compare m/q. Keep these two conditions distinct.

---

59. A metallic rod of length 20 cm is placed in North-South direction and is moved at a constant speed of 20 m/s towards East. The horizontal component of the Earth's magnetic field at that place is 4×10^{-3} T and the angle of dip is 45°. The emf induced in the rod is _____ mV. [4 marks]

Solution:

Concept:

- The motional emf across a straight moving rod is $\varepsilon = \int (\mathbf{v} \times \mathbf{B}) \cdot d\boldsymbol{\ell}$. Only the force component along the rod separates charge between its ends.
- At dip angle δ, the vertical and horizontal components obey $B_V = B_H \tan\delta$. The rod and its velocity are perpendicular here.

Step 1: Choose the magnetic component that acts along the rod The rod lies north-south and moves east. The horizontal component of the Earth field lies north-south, parallel to the rod. Its cross product with the eastward velocity is vertical, so it does not contribute to the emf measured between the rod ends. The vertical component instead gives a north-south magnetic force on moving charges and contributes along the full rod length. Thus the relevant field magnitude is B_V, not the total Earth field.

Step 2: Calculate the emf and convert the unit The dip angle is $45°$, so

$$B_V = B_H \tan 45° = 4 \times 10^{-3} \text{ T}.$$

Convert the rod length before substituting: $\ell = 20 \text{ cm} = 0.20 \text{ m}$. The relevant field, velocity and rod directions are mutually perpendicular. Therefore,

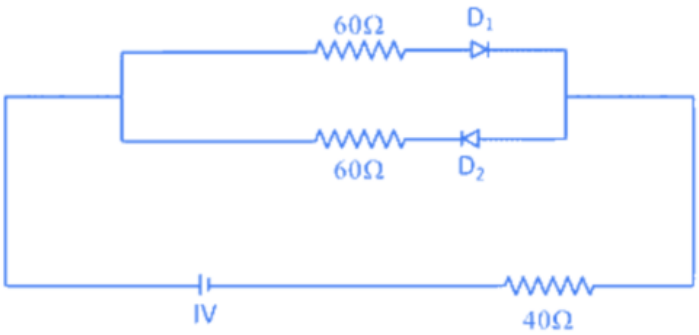
$$\varepsilon = B_V v \ell = (4 \times 10^{-3})(20)(0.20) \text{ V} = 0.016 \text{ V}.$$

Using $1 \text{ V} = 1000 \text{ mV}$, this is 16 mV. Using the total field would introduce an incorrect factor of $\sqrt{2}$, because its horizontal part has no component of magnetic force along the rod.

Step 3: Conclusion The induced emf magnitude is $\boxed{16 \text{ mV}}$. The number to enter is $\boxed{16}$.

Quick Tip to Solve: In three-dimensional motional-emf problems, project $\mathbf{v} \times \mathbf{B}$ along the rod before using $Bv\ell$.

60. The cut-off voltage of the diodes (shown in figure) in forward bias is 0.6 V. The current through the resister of 40 Ω is _____ mA. [4 marks]



Solution:

Concept:

- In the constant-voltage diode model, a forward-biased conducting diode has a fixed drop of 0.6 V; a reverse-biased diode is treated as an open circuit.
- After selecting the conducting branch, apply Kirchhoff's voltage law. Resistors in the single conducting loop carry the same current.

Step 1: Identify the conducting diode branch The longer battery plate is on the left, so conventional current leaves the positive terminal, goes up the left side and crosses the upper branch from left to right. Diode D_1 is forward biased in this direction. The oppositely oriented diode D_2 is reverse biased, so no steady current passes through its lower $60\ \Omega$ branch. Consequently the two $60\ \Omega$ resistors must not be combined as a parallel pair: one branch is blocked by its diode.

Step 2: Apply the loop equation The conducting loop contains the 1 V battery, diode D_1, its $60\ \Omega$ resistor, and the common $40\ \Omega$ resistor. The net voltage available across the resistors is

$$V_R = 1.0 - 0.6 = 0.4 \text{ V}.$$

The resistors are in series along this path, so

$$R = 60 + 40 = 100\ \Omega, \qquad I = \frac{0.4}{100} = 0.004 \text{ A}.$$

This same current passes through the $40\ \Omega$ resistor. The voltage drops across the resistors are 0.24 V and 0.16 V, which add to 0.40 V; including the diode drop gives the full battery voltage.

Step 3: Conclusion The current through the $40\ \Omega$ resistor is $\boxed{4 \text{ mA}}$. The required numerical entry is $\boxed{4}$.

Quick Tip to Solve: Decide diode bias before reducing resistor networks. A resistor in a reverse-biased diode branch does not join the conducting parallel combination.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Identify the incorrect statement for PCl_5 from the following. [4 marks]

(A) In this molecule, orbitals of phosphorous are assumed to undergo sp^3d hybridization.

(B) The geometry of PCl_5 is trigonal bipyramidal.

(C) PCl_5 has two axial bonds stronger than three equatorial bonds.

(D) The three equatorial bonds of PCl_5 lie in a plane.

Correct Answer: (C) PCl_5 has two axial bonds stronger than three equatorial bonds.

Solution:

Concept:

- In the conventional valence-bond description of gaseous $\mathrm{PCl_5}$, five bond pairs occupy a trigonal-bipyramidal arrangement, commonly represented by sp^3d hybridisation.
- Electron-pair repulsion is greater at 90° than at 120°. Axial bonds experience more close-angle interactions than equatorial bonds and are longer and weaker.

Step 1: Compare the two types of bonds Three equatorial $\mathrm{P-Cl}$ bonds lie in a single plane with angles of 120° between them. Two axial bonds point above and below that plane, at 180° to one another and 90° to each equatorial bond. An axial bond therefore has three 90° bond-pair interactions. An equatorial bond has only two such interactions, with the axial bonds; its other interactions are at 120°. The greater repulsion makes the axial bonds less strongly held.

Step 2: Test all four statements Option (A) states the standard assumed sp^3d hybridisation description and is correct in the model used by the question. Option (B), trigonal-bipyramidal geometry, is also correct. Option (D) correctly places the three equatorial bonds in one plane. Option (C) reverses the actual bond-strength comparison: the two axial bonds are longer and weaker than the three equatorial bonds, rather than stronger. The word "incorrect" means we must select this reversed statement, not one of the three true descriptions. The distinction concerns the isolated molecule described here, rather than an ionic structural description of solid phosphorus pentachloride.

Step 3: Conclusion The incorrect statement is that the axial bonds are stronger than the equatorial bonds. Hence, the correct option is (C).

Quick Tip to Solve: In a trigonal bipyramid, count the 90° interactions: more close-angle repulsions mean the axial bonds are longer and weaker.

62. The correct order of increasing intermolecular hydrogen bond strength is : [4 marks]

(A) $HCN < H_2O < NH_3$

(B) $HCN < CH_4 < NH_3$

(C) $CH_4 < HCN < NH_3$

(D) $CH_4 < NH_3 < HCN$

Correct Answer: (C) $CH_4 < HCN < NH_3$

Solution:

Concept:

- Hydrogen bonding depends on the availability and polarity of a donor bond and a suitable lone-pair acceptor. Molecular polarity by itself does not guarantee a strong hydrogen bond.
- For the comparison asked here, methane has negligible hydrogen bonding; hydrogen cyanide has weak association through its polar $C-H$ end and nitrogen acceptor, while ammonia forms conventional $N-H\cdots N$ hydrogen bonds.

Step 1: Place methane at the weak end In CH_4, carbon and hydrogen have only a small electronegativity difference, and the tetrahedral molecule is nonpolar. It also lacks a nitrogen, oxygen or fluorine lone-pair acceptor. Its intermolecular attraction is mainly dispersion, rather than a conventional hydrogen-bond network. Thus methane belongs at the beginning of the increasing sequence; any option putting it above hydrogen cyanide is inconsistent with this comparison.

Step 2: Compare hydrogen cyanide and ammonia, then test the options Although HCN is strongly polar and its terminal nitrogen can accept an interaction, its hydrogen is bonded to carbon. The interaction assigned to it in this question is weaker than conventional hydrogen bonding involving a hydrogen directly attached to nitrogen. Ammonia has polar $N-H$ donor bonds and a lone pair on nitrogen, allowing $N-H\cdots N$ association. The resulting increasing order is

$$CH_4 < HCN < NH_3.$$

Option (C) is exactly this sequence. Option (B) reverses methane and hydrogen cyanide, and option (D) reverses hydrogen cyanide and ammonia. Option (A) places water below ammonia, whereas the more polar $O-H$ bond normally gives stronger hydrogen bonding than $N-H$.

Step 3: Conclusion The required order is $CH_4 < HCN < NH_3$. Hence, the correct option is (C).

Quick Tip to Solve: Do not rank hydrogen bonds from dipole moment alone. Identify the atom carrying hydrogen and the available lone-pair acceptor first.

63. The correct order of increasing ionic radii is [4 marks]

(A) $Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

(B) $N^{3-} < O^{2-} < F^{-} < Na^{+} < Mg^{2+}$

(C) $F^{-} < Na^{+} < O^{2-} < Mg^{2+} < N^{3-}$

(D) $Na^{+} < F^{-} < Mg^{2+} < O^{2-} < N^{3-}$

Correct Answer: (A) $Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

Solution:

Concept:

- An isoelectronic series contains species with the same number of electrons. Here every listed ion has ten electrons and the same closed-shell configuration.
- For isoelectronic ions, increasing nuclear charge attracts the common electron cloud more strongly, decreasing its radius. Compare proton counts after checking electron counts.

Step 1: Check that the ions are isoelectronic The neutral atomic numbers are $Z = 12, 11, 9, 8, 7$ for magnesium, sodium, fluorine, oxygen and nitrogen respectively. Forming the listed ions gives

$$N_e(Mg^{2+}) = 12 - 2 = 10, \quad N_e(Na^{+}) = 11 - 1 = 10,$$
$$N_e(F^{-}) = 9 + 1 = 10, \quad N_e(O^{2-}) = 8 + 2 = 10,$$
$$N_e(N^{3-}) = 7 + 3 = 10.$$

All five therefore have the configuration $1s^2\,2s^2\,2p^6$. The comparison is not between different occupied principal shells.

Step 2: Order the radii by nuclear attraction The electron count and broad shielding pattern are the same, but the nuclear charge decreases in the sequence $12, 11, 9, 8, 7$. Magnesium therefore holds the ten-electron cloud most tightly, while nitrogen holds it least tightly. Reversing the nuclear-charge order gives the increasing-radius sequence

$$Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}.$$

This is option (A). Option (B) gives the reverse order. Options (C) and (D) incorrectly place magnesium or sodium above anions with smaller nuclear charge, despite their identical electron count. Ionic charge alone is not the full rule; the decisive comparison is nuclear charge within this established isoelectronic series.

Step 3: Conclusion The correct increasing-radius order is $Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$. Hence, the correct option is (A).

Quick Tip to Solve: Count electrons first. If all species are isoelectronic, the ion with the most protons has the smallest radius.

64. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Fluorine forms one oxoacid.

Reason R : Fluorine has smallest size amongst all halogens and is highly electronegative.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Correct Answer: (A) Both A and R are correct and R is the correct explanation of A.

Solution:

Concept:

- Fluorine differs from the other halogens because of its very small atomic size and its exceptionally high electronegativity. It does not display the usual positive oxidation states of the heavier halogens.
- The only oxoacid of fluorine is hypofluorous acid, HOF, which contains an $H-O-F$ linkage. Assertion–reason questions require the truth of both statements and their causal connection to be checked separately.

Step 1: Evaluate the assertion Hypofluorous acid, HOF, is the single oxoacid formed by fluorine. The assertion says one oxoacid, rather than no oxoacid, and is therefore correct. The heavier halogens can form several oxoacids with different oxygen contents, whereas fluorine does not form such a series.

Step 2: Evaluate the reason Fluorine is the smallest halogen atom and has the highest electronegativity. Both properties stated in the reason are correct. In particular, its electronegativity exceeds that of oxygen; the oxidation state of fluorine remains -1, even in HOF. With hydrogen at $+1$, oxygen consequently has oxidation state zero in this compound.

Step 3: Connect the statements and eliminate the alternatives The exceptional size and electronegativity of fluorine account for its restricted oxoacid chemistry, so the reason explains the assertion. Option (B) incorrectly denies this explanation. Options (C) and (D) reject a true statement and are therefore excluded. Do not apply the positive halogen oxidation-state pattern of chlorine oxoacids to fluorine.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Fluorine forms only HOF. Hence, the correct option is (A).

Quick Tip to Solve: For fluorine oxoacids, remember the exception HOF: fluorine remains -1, even though oxygen is present.

65. In 3d series, the metal having the highest M^{2+}/M standard electrode potential is : [4 marks]

(A) Cr

(B) Fe

(C) Cu

(D) Zn

Correct Answer: (C) Cu

Solution:

Concept:

- The standard potential written $E^\circ(M^{2+}/M)$ is the reduction potential for $M^{2+} + 2\,e^- \longrightarrow M$, measured relative to the standard hydrogen electrode.
- A more positive reduction potential means a greater thermodynamic tendency for the aqueous ion to be reduced to the metal under standard conditions. Atomisation, ionisation and hydration energies jointly determine this value.

Step 1: Compare the given metals The approximate standard reduction potentials of the listed couples are

$$E^\circ(Cr^{2+}/Cr) = -0.91\,V,$$
$$E^\circ(Fe^{2+}/Fe) = -0.44\,V,$$
$$E^\circ(Cu^{2+}/Cu) = +0.34\,V,$$
$$E^\circ(Zn^{2+}/Zn) = -0.76\,V.$$

Copper is the only one of these four with a positive value.

Step 2: Identify the largest value The comparison is of signed potentials, not their absolute magnitudes. Thus $+0.34$ is larger than every negative number in the list. The relevant order is

$$Cr^{2+}/Cr < Zn^{2+}/Zn < Fe^{2+}/Fe < Cu^{2+}/Cu.$$

Chromium does not win merely because its negative potential has a large magnitude.

Step 3: Explain and reject the other choices The energy needed to atomise copper and produce its divalent gaseous ion is not sufficiently compensated by hydration, giving its unusual positive reduction potential. Chromium, iron and zinc all have negative values for the stated couples and therefore cannot be the highest. Zinc having a filled d subshell does not reverse this experimental comparison.

Step 4: Conclusion Copper has the highest M^{2+}/M standard electrode potential among the listed metals: about $+0.34\,V$. Hence, the correct option is (C).

Quick Tip to Solve: Read an electrode couple as a reduction and compare signed values; a positive copper potential exceeds the negative values of the other listed metals.

66. The 'f' orbitals are half and completely filled, respectively in lanthanide ions :
[Given : Atomic no. Eu, 63; Sm, 62; Tm, 69; Tb, 65; Yb, 70; Dy, 66] [4 marks]

(A) Eu^{2+} and Tm^{2+}

(B) Sm^{2+} and Tm^{3+}

(C) Tb^{4+} and Yb^{2+}

(D) Dy^{3+} and Yb^{3+}

Correct Answer: (C) Tb^{4+} and Yb^{2+}

Solution:

Concept:

- The f subshell contains seven orbitals and can accommodate fourteen electrons. Half-filled and completely filled configurations are $4f^7$ and $4f^{14}$, respectively.
- For the lanthanide ions here, use the xenon core of 54 electrons and subtract the ionic charge from the atomic number. These ions have no remaining outer $6s$ electrons; their remaining electrons occupy the 4f subshell.

Step 1: Find the required pair For terbium with atomic number 65, Tb^{4+} has $65 - 4 = 61$ electrons. Relative to xenon, $61 - 54 = 7$ electrons remain:

$$Tb^{4+} : [Xe]4f^7.$$

For ytterbium with atomic number 70, Yb^{2+} has 68 electrons, giving

$$Yb^{2+} : [Xe]4f^{14}.$$

These are half-filled and filled, in the required order.

Step 2: Check options A and B Europium(II) does have $63 - 2 - 54 = 7$ f electrons, but thulium(II) has $69 - 2 - 54 = 13$, not fourteen. Thus option (A) has only its first member correct. In option (B), samarium(II) has $62 - 2 - 54 = 6$ f electrons and thulium(III) has $69 - 3 - 54 = 12$; neither matches the required count.

Step 3: Check option D Dysprosium(III) has $66 - 3 - 54 = 9$ f electrons, and ytterbium(III) has $70 - 3 - 54 = 13$. Hence option (D) contains neither a half-filled first ion nor a completely filled second ion. The word respectively is essential: the first ion must be $4f^7$ and the second must be $4f^{14}$, not merely a stable ion of either type.

Step 4: Conclusion The required ions are Tb^{4+} with $4f^7$ and Yb^{2+} with $4f^{14}$. Hence, the correct option is (C).

Quick Tip to Solve: For these lanthanide ions, calculate $Z - \text{charge} - 54$ and look for 7 followed by 14; neutral-atom configurations alone can mislead.

67. Arrange the following coordination compounds in the increasing order of magnetic moments. (Atomic numbers : Mn = 25; Fe = 26)

A. $[FeF_6]^{3-}$

B. $[Fe(CN)_6]^{3-}$

C. $[MnCl_6]^{3-}$ (high spin)

D. $[Mn(CN)_6]^{3-}$

Choose the correct answer from the options given below : [4 marks]

(A) A < B < D < C

(B) B < D < C < A

(C) A < C < D < B

(D) B < D < A < C

Correct Answer: (B) B < D < C < A

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons. It increases monotonically with n.
- Fluoride and chloride are weak-field ligands, while cyanide is a strong-field ligand. In octahedral complexes, strong-field cyanide favours low-spin configurations for the metal ions in this question.

Step 1: Determine oxidation states and d counts Each complex has six singly negative ligands and overall charge -3, so every metal is in oxidation state $+3$. Iron(III) is $3d^5$, whereas manganese(III) is $3d^4$. The neutral-atom d counts must not be used for these ions.

Step 2: Count unpaired electrons Complex A, $[FeF_6]^{3-}$, is high-spin d^5, with $t_{2g}^3 e_g^2$ and five unpaired electrons. Complex B, $[Fe(CN)_6]^{3-}$, is low-spin d^5, with t_{2g}^5 and one unpaired electron. Complex C, high-spin $[MnCl_6]^{3-}$, has $t_{2g}^3 e_g^1$ and four unpaired electrons. Complex D, $[Mn(CN)_6]^{3-}$, is low-spin d^4, with t_{2g}^4 and two unpaired electrons.

Step 3: Compare the moments and the options The corresponding moments are

$$\mu_B = \sqrt{3}\,\mu_B, \quad \mu_D = \sqrt{8}\,\mu_B, \quad \mu_C = \sqrt{24}\,\mu_B, \quad \mu_A = \sqrt{35}\,\mu_B.$$

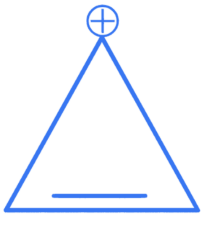
Therefore B precedes D, and C precedes A. Options (A) and (C) incorrectly start with A, the largest moment; option (D) incorrectly places A below C.

Step 4: Conclusion The increasing order of magnetic moments is $\boxed{B < D < C < A}$. Hence, the correct option is (B).

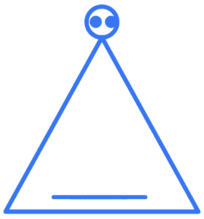
Quick Tip to Solve: Find oxidation state, d count and ligand strength before counting unpaired electrons; for ordering, compare n without calculating square roots.

68. Which of the following is most stable? [4 marks]

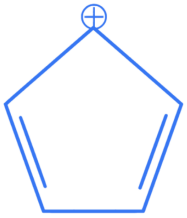
(A) Cyclopropenyl cation



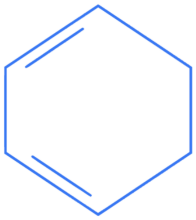
(B) Cyclopropenyl anion



(C) Cyclopentadienyl cation

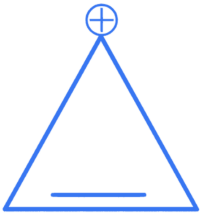


(D) Cyclohexa-1,3-diene



Correct Answer: (A)

Cyclopropenyl cation



Solution:

Concept:

- Aromatic stabilisation requires a cyclic, planar, continuously conjugated system with $4n + 2$ pi electrons, where n is a non-negative integer.
- A planar conjugated ring with $4n$ pi electrons is antiaromatic. A ring with an interruption in its p-orbital array is non-aromatic, even if some of its bonds are conjugated.

Step 1: Analyse the cyclopropenyl cation In option (A), the two alkene carbons and the positively charged carbon each supply a p orbital around the three-membered ring. The cationic carbon has an empty p orbital, so it contributes no electron. The one double bond supplies two pi electrons. The ring is planar and continuously conjugated, and

$$2 = 4(0) + 2.$$

It therefore has aromatic stabilisation with the positive charge delocalised around the ring.

Step 2: Compare the other charged rings In option (B), the cyclopropenyl anion has two electrons from the double bond plus two from its lone pair, giving four pi electrons if that pair participates in the cyclic conjugation. This is the antiaromatic $4n$ count. In option (C), the cyclopentadienyl cation

has four pi electrons from two double bonds and an empty p orbital at the cationic centre, again giving an antiaromatic count rather than aromatic stabilisation.

Step 3: Check the neutral diene Option (D), cyclohexa-1,3-diene, has two double bonds, but its saturated carbons interrupt conjugation around the full ring. It is non-aromatic: the presence of a six-membered ring does not by itself make it benzene-like. Among the structures presented, only the cyclopropenyl cation has the closed, delocalised aromatic pi system that supplies the decisive extra stability.

Step 4: Conclusion The cyclopropenyl cation has an aromatic two-pi-electron system and is the most stable of the given choices. Hence, the correct option is (A).

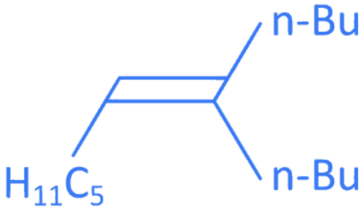
Quick Tip to Solve: Count only electrons in the cyclic p-orbital system: an empty cationic p orbital contributes zero, while a participating anionic lone pair contributes two.

69. What will be the major product of following sequence of reactions?

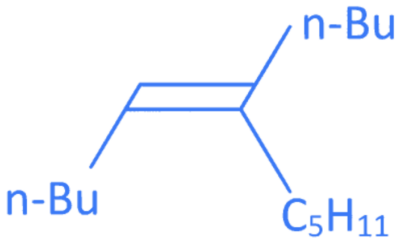
$$\text{n-}C_4H_9-C\equiv CH \xrightarrow[\text{(ii) Lindlar catalyst, H2}]{\text{(i) n-BuLi, n-C5H11Cl}} \text{Product}$$

[4 marks]

(A) $C_5H_{11}CH=C(\text{n-}C_4H_9)_2$



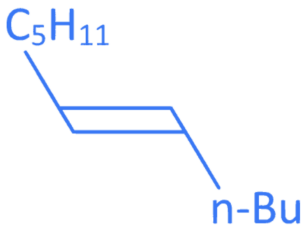
(B) $\text{n-}C_4H_9CH=C(\text{n-}C_4H_9)C_5H_{11}$



(C) $\text{cis-n-}C_4H_9CH=CHC_5H_{11}$



(D) $\text{trans-n-}C_4H_9CH=CHC_5H_{11}$



Correct Answer: (C)

$\text{cis-n-}C_4H_9CH=CHC_5H_{11}$



Solution:

Concept:

- A terminal alkyne has an acidic hydrogen on its sp-hybridised carbon. A strong base such as n-butyllithium generates the acetylide ion, which alkylates a primary alkyl chloride by nucleophilic substitution.
- Hydrogen with Lindlar catalyst partially reduces an alkyne by syn addition. The reaction stops at the cis alkene, placing the two pre-existing carbon substituents on the same side.

Step 1: Generate and alkylate the acetylide The starting material is $\text{n-C}_4\text{H}_9-\text{C}\equiv\text{CH}$. n-Butyllithium acts as a base, removing the terminal proton; it does not add a new butyl substituent to the carbon skeleton in this step. The resulting acetylide attacks n-pentyl chloride:

$$\text{n-C}_4\text{H}_9-\text{C}\equiv\text{C}^- + \text{Cl}-\text{C}_5\text{H}_{11} \longrightarrow \text{n-C}_4\text{H}_9-\text{C}\equiv\text{C}-\text{C}_5\text{H}_{11} + \text{Cl}^-.$$

This gives an internal alkyne with one n-butyl group and one n-pentyl group.

Step 2: Apply Lindlar reduction The internal alkyne adsorbs on the poisoned metal catalyst and accepts two hydrogen atoms from the same face. Thus the product is

$$\text{cis-n-C}_4\text{H}_9-\text{CH}=\text{CH}-\text{C}_5\text{H}_{11}.$$

Both large alkyl groups lie on the same side of the double bond, as drawn in option (C). The product is the Z isomer of undec-5-ene; the carbon count is $4 + 2 + 5 = 11$.

Step 3: Eliminate the other structures Options (A) and (B) each show an additional n-butyl group attached to an alkene carbon. Such products would incorrectly treat n-butyllithium as a carbon-addition reagent in this terminal-alkyne deprotonation. Option (D) has the correct two alkyl groups but opposite-side geometry. It is the trans product associated with a dissolving-metal reduction, rather than the syn reduction by Lindlar catalyst.

Step 4: Conclusion The major product is $\text{cis-n-C}_4\text{H}_9\text{CH}=\text{CHC}_5\text{H}_{11}$, or (Z)-undec-5-ene. Hence, the correct option is (C).

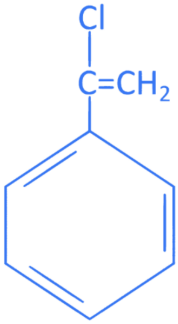
Quick Tip to Solve: Separate the jobs of the reagents: n-BuLi removes the terminal proton, the primary halide adds its alkyl group, and Lindlar catalyst fixes cis geometry.

70. Product 'A' of following sequence of reactions is

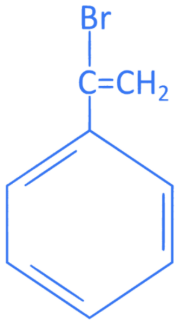
$$\textbf{Ethylbenzene} \xrightarrow{\textbf{(a)}\ \text{Br}_2,\text{Fe}} \xrightarrow{\textbf{(b)}\ \text{Cl}_2,\Delta} \xrightarrow{\textbf{(c) alcoholic}\ \text{KOH}} A\ (\textbf{major product}).$$

[4 marks]

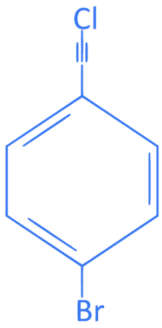
(A)



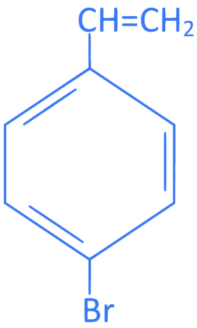
(B)



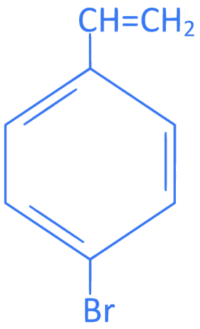
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- An alkyl group activates a benzene ring towards electrophilic substitution and directs an incoming halogen to the ortho and para positions. The para product is favoured when steric crowding matters.
- Chlorination with heat gives free-radical side-chain substitution, especially at the benzylic carbon.

Alcoholic potassium hydroxide then promotes elimination of hydrogen halide to form an alkene.

Step 1: Carry out ring bromination In the presence of iron, bromine undergoes electrophilic substitution on the aromatic ring of ethylbenzene. The ethyl group is ortho/para directing, and the major para isomer is $\text{p-Br}-\text{C}_6\text{H}_4-\text{CH}_2\text{CH}_3$. Bromine is therefore attached to the ring, not to the side-chain carbon. This distinguishes this reagent combination from radical side-chain bromination.

Step 2: Chlorinate the benzylic position Heating with chlorine replaces a benzylic hydrogen in the ethyl side chain. The benzylic radical intermediate is resonance-stabilised, favouring substitution at the carbon directly bonded to the ring:

$$\text{p-Br}-\text{C}_6\text{H}_4-\text{CH}_2\text{CH}_3 \xrightarrow{\text{Cl}_2,\,\Delta} \text{p-Br}-\text{C}_6\text{H}_4-\text{CH(Cl)CH}_3.$$

The aryl bromine introduced in the first step is retained.

Step 3: Eliminate and match the structure Alcoholic KOH removes a beta hydrogen from the terminal methyl group while chloride leaves the neighbouring benzylic carbon. Elimination gives

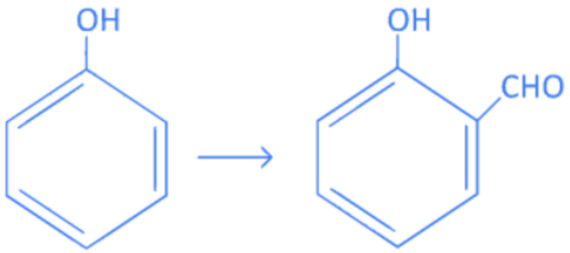
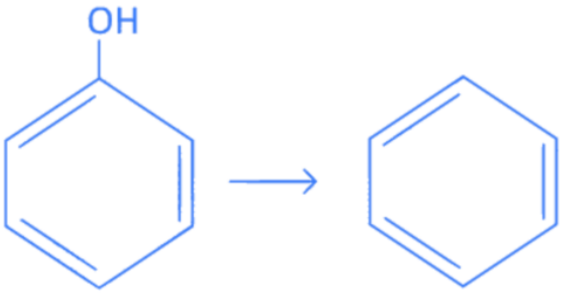
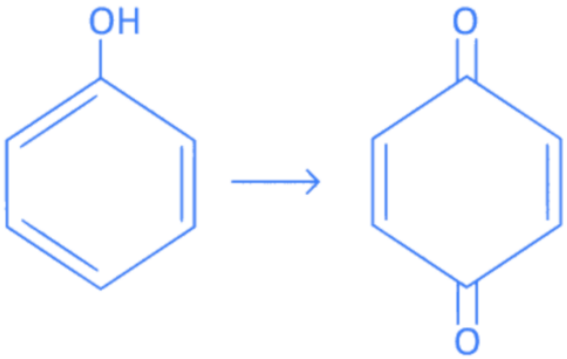
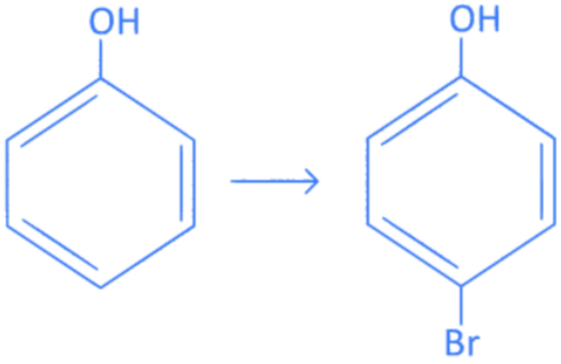
$$\text{p-Br}-\text{C}_6\text{H}_4-\text{CH(Cl)CH}_3 \xrightarrow{\text{alc}\cdot\text{KOH}} \text{p-Br}-\text{C}_6\text{H}_4-\text{CH}=\text{CH}_2.$$

The side-chain chlorine is lost, and the aromatic bromine remains. Options (A) and (B) put a halogen on the vinyl side chain and omit para ring bromine. Option (C) retains a chlorine-bearing unsaturated side chain, unlike the alkene formed by this dehydrohalogenation.

Step 4: Conclusion Product A is para-bromostyrene, $\text{p-Br}-\text{C}_6\text{H}_4-\text{CH}=\text{CH}_2$. Hence, the correct option is (D).

Quick Tip to Solve: Track the reaction site at each stage: Br_2/Fe acts on the ring, hot Cl_2 on the benzylic side chain, and alcoholic base forms the alkene.

71. Match List-I with List-II. Choose the correct answer from the options given below: [4 marks]

	List-I		List-II
(A)		I.	Br_2 in CS_2
(B)		II.	$Na_2Cr_2O_7/H_2SO_4$
(C)		III.	Zn
(D)		IV.	$CHCl_3/NaOH$

(A) A-IV, B-III, C-II, D-I

(B) A-IV, B-III, C-I, D-II

(C) A-II, B-III, C-I, D-IV

(D) A-IV, B-II, C-III, D-I

Correct Answer: (A) A-IV, B-III, C-II, D-I

Solution:

Concept:

- Phenol undergoes electrophilic substitution mainly at the ortho and para positions because its hydroxyl group activates these positions by resonance.
- The reagent must supply the change actually shown: formylation, removal of oxygen, oxidation to a quinone, or bromination. Distinguishing these transformations is more reliable than memorising a matching code.

Step 1: Match the formylation and reduction In (A), phenol acquires an ortho aldehyde group while retaining its hydroxyl group. This is the Reimer–Tiemann reaction: $CHCl_3/NaOH$, followed by acidification, produces salicylaldehyde (2-hydroxybenzaldehyde). Hence A–IV. In (B), the phenolic oxygen is removed and benzene is obtained. Heating phenol with zinc dust causes this reduction:

$$C_6H_5OH + Zn \longrightarrow C_6H_6 + ZnO.$$

Thus B–III. Zinc is consumed by accepting the oxygen atom; it is not the oxidant needed to form the quinone.

Step 2: Match the oxidation and substitution In (C), phenol changes to 1,4-benzoquinone, containing two carbonyl groups. Acidified sodium dichromate is the oxidising reagent, so C–II. In (D), one ring hydrogen is replaced by bromine without removing the hydroxyl group. Bromine in the non-polar solvent carbon disulphide gives monobromination, predominantly at the para position, so D–I. This differs from bromine water, which readily gives 2,4,6-tribromophenol. Collecting the four independent matches gives A–IV, B–III, C–II, D–I. Option B wrongly exchanges the oxidation and bromination reagents; option C assigns the oxidant to formylation; option D assigns dichromate to deoxygenation. None reproduces all four transformations.

Step 3: Conclusion The correct matching is A–IV, B–III, C–II, D–I. Hence, the correct option is (A).

Quick Tip to Solve: First label the changes as formylation, deoxygenation, oxidation and bromination; then match each with its characteristic reagent.

72. Decarboxylation of all six possible forms of diaminobenzoic acids $C_6H_3(NH_2)_2COOH$ yields three products A, B and C. Three acids give a product 'A', two acids gives a product 'B' and one acid give a product 'C'. The melting point of product 'C' is [4 marks]

(A) 63°C

(B) 90°C

(C) 104°C

(D) 142°C

Correct Answer: (D) 142°C

Solution:

Concept:

- Decarboxylation replaces the carboxyl group on the benzene ring by hydrogen. It preserves the relative positions of the two amino groups, so only ortho-, meta- and para-phenylenediamine can result.
- Symmetry must be applied while counting diaminobenzoic acids: fix the carboxyl-bearing carbon as carbon 1, and identify positions related by reflection through carbons 1 and 4.

Step 1: Enumerate the six distinct acids With COOH at carbon 1, the two amino groups can occupy the distinct pairs $(2,3),(2,4),(2,5),(2,6),(3,4),(3,5)$. Reflection makes the other possible pairs equivalent to one of these. After removing COOH, the pairs $(2,3)$ and $(3,4)$ are adjacent and yield ortho-phenylenediamine. The pairs $(2,4),(2,6),(3,5)$ have one intervening ring carbon, taking the shorter path, and yield meta-phenylenediamine. The pair $(2,5)$ lies opposite on the ring and yields para-phenylenediamine.

Step 2: Identify C and its melting point The multiplicities are therefore three acids giving the meta product, two giving the ortho product, and one giving the para product. Thus A is meta-phenylenediamine, B is ortho-phenylenediamine, and C is para-phenylenediamine. The para isomer has the most symmetric arrangement and packs efficiently in the crystal, consistent with its high melting point, approximately $142°C$. The $63°C$ and $104°C$ choices correspond approximately to the meta and ortho isomers respectively, rather than C. The $90°C$ choice does not identify para-phenylenediamine. The product count identifies the isomer before any physical-property value is selected.

Step 3: Conclusion Product C is para-phenylenediamine, with melting point $142°C$. Hence, the correct option is (D).

Quick Tip to Solve: For decarboxylation isomer counts, keep the relative amino-group separation and erase only the carboxyl group; count symmetry-equivalent starting pairs once.

73. Given below are two statements.

Statement I : Maltose has two α-D-glucose units linked at C_1 and C_4 and is a reducing sugar.

Statement II : Maltose has two monosaccharides : α-D-glucose and β-D-glucose linked at C_1 and C_6 and it is a non-reducing sugar.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (C) Statement I is true but Statement II is false.

Solution:

Concept:

- Maltose consists of two glucose residues joined by an $\alpha(1 \rightarrow 4)$ glycosidic linkage: the anomeric carbon of the first residue is linked through oxygen to carbon 4 of the second.
- A sugar is reducing when it retains a free hemiacetal or anomeric centre that can open to a carbonyl form. A glycosidic bond involving only one of the two anomeric carbons leaves a reducing end.

Step 1: Evaluate Statement I The linkage in maltose is between carbon 1 of one glucose unit and carbon 4 of the other, with the glycosidic configuration at the first carbon being alpha. This is the conventional description of maltose as two alpha-D-glucose units. The second glucose residue retains its free anomeric carbon, so it can undergo ring opening and mutarotation. Its open-chain form contains an aldehyde group and can reduce appropriate mild oxidising reagents. Therefore the linkage description and reducing character in Statement I are correct. The reducing-end anomer can interconvert in solution; this does not change the alpha configuration of the glycosidic bond.

Step 2: Evaluate Statement II and compare the choices Statement II changes the linkage to carbon 1–carbon 6 and calls maltose non-reducing. Maltose does not have that linkage: its bond is carbon 1–carbon 4. Moreover, its free anomeric centre makes it reducing. A description using one alpha and one beta unit does not remove that free centre or convert the actual glycosidic linkage into a 1–6 bond. Hence Statement II is false. Option A fails because it accepts this incorrect statement. Option B fails because Statement I is true. Option D reverses the correct truth values. Only option C gives a true first statement and a false second statement.

Step 3: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: To test whether a disaccharide is reducing, locate both anomeric carbons and check whether either one remains a free hemiacetal.

74. Match List I with List II.

	List-I (Anion)		List-II (gas evolved on reaction with dil.H_2SO_2)
(A)	**CO_3^{2-}**	**I.**	**Colourless gas which turns lead acetate paper black.**
(B)	**S^{2-}**	**II.**	**Colourless gas which turns acidified potassium dichromate solution green.**
(C)	**SO_3^{2-}**	**III.**	**Brown fumes which turns acidified KI solution containing starch blue.**
(D)	**NO_2^-**	**IV.**	**Colourless gas evolved with brisk effervescence, which turns lime water milky.**

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-I, C-II, D-IV

(B) A-II, B-I, C-IV, D-III

(C) A-IV, B-I, C-III, D-II

(D) A-IV, B-I, C-II, D-III

Correct Answer: (D) A-IV, B-I, C-II, D-III

Solution:

Concept:

- Acid treatment of carbonate, sulphide, sulphite and nitrite ions evolves characteristic gases. Identify the gas first, then match its confirmatory observation.
- Carbon dioxide turns lime water milky; hydrogen sulphide blackens lead acetate; sulphur dioxide reduces acidified dichromate; nitrogen dioxide gives brown fumes and oxidises iodide.

Step 1: Match carbonate and sulphide Carbonate reacts with hydrogen ions to evolve carbon dioxide:

$$CO_3{}^{2-} + 2\,H^+ \longrightarrow CO_2 + H_2O.$$

The colourless gas produces brisk effervescence and forms a white precipitate of $CaCO_3$ in lime water. Thus A–IV. Sulphide is protonated to hydrogen sulphide:

$$S^{2-} + 2\,H^+ \longrightarrow H_2S.$$

This gas gives black lead sulphide with lead acetate paper, so B–I. These observations distinguish the two colourless gases without relying on their odour.

Step 2: Match sulphite and nitrite Sulphite evolves sulphur dioxide in acid:

$$SO_3{}^{2-} + 2\,H^+ \longrightarrow SO_2 + H_2O.$$

Sulphur dioxide reduces orange dichromate(VI) to green chromium(III); hence C–II. Nitrite gives nitrous acid on acidification. Its decomposition gives nitrogen oxides, including brown NO_2; these oxidising fumes liberate iodine from acidified iodide, and iodine gives the blue starch complex.

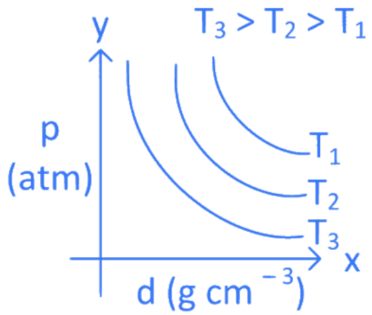
Thus D–III. Combining the observations gives A–IV, B–I, C–II, D–III. Option A incorrectly assigns brown fumes to carbonate. Option B assigns dichromate reduction to carbonate and lime-water milkiness to sulphite. Option C interchanges the sulphite and nitrite tests. Only option D keeps all four characteristic reactions consistent.

Step 3: Conclusion The correct matching is A–IV, B–I, C–II, D–III. Hence, the correct option is (D).

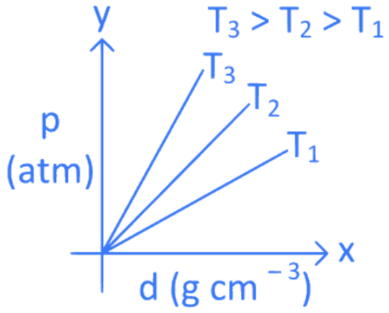
Quick Tip to Solve: Remember the diagnostic sequence: carbonate–lime water, sulphide–black lead sulphide, sulphite–green dichromate test, nitrite–brown nitrogen dioxide.

75. Which amongst the given plots is the correct plot for pressure (p) vs density (d) for an ideal gas? [4 marks]

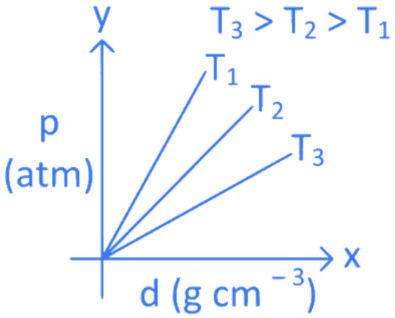
(A)



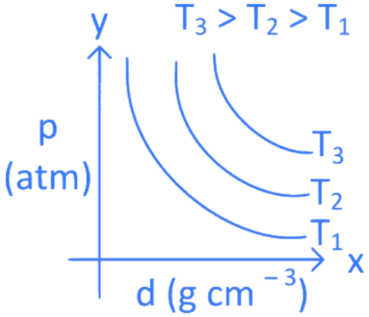
(B)



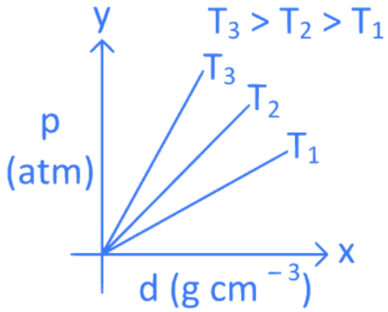
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- For an ideal gas, $pV = nRT$, with pressure p, volume V, amount n, absolute temperature T, and gas constant R.
- If the gas mass is m and its molar mass is M, then $n = m/M$ and density is $d = m/V$. A pressure–density graph at fixed temperature therefore has slope RT/M.

Step 1: Express pressure in terms of density Substitute $n = m/M$ into the ideal-gas equation and divide through by volume:

$$pV = \frac{m}{M}RT, \qquad p = \frac{m}{V}\frac{RT}{M} = d\frac{RT}{M}.$$

At a fixed temperature and for the same gas, R, T, M are constants. Pressure is directly proportional to density, rather than inversely proportional to it. Consequently each isotherm in a pressure–density plot must be a straight line passing through the origin. The origin is understood as the limiting state of vanishing density and pressure.

Step 2: Order the slopes by temperature The slope is $p/d = RT/M$. For $T_3 > T_2 > T_1$, the slope at T_3 is the largest and that at T_1 is the smallest. Thus the steepest straight line must be labelled T_3, the intermediate line T_2, and the shallowest line T_1. Option B shows exactly this arrangement. Options A and D are curved decreasing plots, which would represent an inverse dependence and contradict $p \propto d$. Option C uses straight lines but reverses the temperature ordering: it places the coolest isotherm at the largest slope. Checking both linearity and temperature ordering is necessary; checking just one does not distinguish B from every other plot.

Step 3: Conclusion The straight lines through the origin with slopes increasing from T_1 to T_3 give the required plot. Hence, the correct option is (B).

Quick Tip to Solve: Write $p = dRT/M$: the graph is straight, and hotter gas has a steeper pressure–density line at fixed molar mass.

76. Statement I : Leaching of gold with cyanide ion in absence of air / O_2 leads to cyano complex of Au(III).

Statement II : Zinc is oxidized during the displacement reaction carried out for gold extraction.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (D) Statement I is incorrect but Statement II is correct.

Solution:

Concept:

- Cyanide leaching dissolves gold in the presence of air or oxygen as the soluble dicyanoaurate(I) complex, $[Au(CN)_2]^-$. Oxygen supplies the oxidising action required for dissolution.
- A more reactive metal can recover gold by displacement from the cyanide complex. The displacing metal is oxidised while gold(I) is reduced to metallic gold.

Step 1: Evaluate the proposed leaching reaction The cyanide process for gold uses air or oxygen, with an overall reaction

$$4\,Au + 8\,CN^- + O_2 + 2\,H_2O \longrightarrow 4\,[Au(CN)_2]^- + 4\,OH^-.$$

In $[Au(CN)_2]^-$, each cyanide ligand has charge -1, so the gold oxidation state is $+1$, not $+3$. Statement I is therefore incorrect for two independent reasons: it excludes the oxygen needed for normal oxidative leaching and identifies the product as a gold(III) complex. Formation of a coordination complex does not by itself establish an oxidation state of three.

Step 2: Check zinc in the recovery step Zinc displaces dissolved gold according to

$$2\,[Au(CN)_2]^- + Zn \longrightarrow [Zn(CN)_4]^{2-} + 2\,Au.$$

Zinc changes from oxidation state zero in the metal to $+2$ in the tetracyanozincate complex. It loses two electrons and is oxidised. Each gold atom changes from $+1$ to zero, so two gold ions accept the two electrons supplied by one zinc atom. Statement II is correct. Option A fails because Statement I is false; option B fails because Statement II is true; option C reverses both truth values. Option D alone gives an incorrect first statement and a correct second statement.

Step 3: Conclusion Statement I is incorrect, whereas Statement II is correct. Hence, the correct option is (D).

Quick Tip to Solve: Calculate gold oxidation state from the complex charge before judging the statement; then track zinc from zero to plus two during displacement.

77. The gas produced by treating an aqueous solution of ammonium chloride with sodium nitrite is : [4 marks]

(A) NH_3

(B) N_2

(C) N_2O

(D) Cl_2

Correct Answer: (B) N_2

Solution:

Concept:

- Ammonium nitrite is unstable and decomposes into nitrogen gas and water. It is conveniently generated in situ by mixing an ammonium salt with a soluble nitrite.
- Ammonium nitrogen has oxidation state -3, whereas nitrite nitrogen has oxidation state $+3$. Their internal redox reaction forms elemental nitrogen, whose oxidation state is zero.

Step 1: Form the unstable ammonium salt In aqueous solution, ammonium chloride and sodium nitrite provide $NH_4{}^+$ and $NO_2{}^-$ ions. Their reaction is conventionally represented by the double-displacement equation

$$NH_4Cl + NaNO_2 \longrightarrow NH_4NO_2 + NaCl.$$

The sodium and chloride ions are spectators in the gas-forming process. The important intermediate is ammonium nitrite, not ammonium nitrate. The difference between nitrite and nitrate is decisive because the two salts produce different nitrogen-containing gases on decomposition.

Step 2: Decompose ammonium nitrite and eliminate the alternatives Ammonium nitrite decomposes as

$$NH_4NO_2 \longrightarrow N_2 + 2\,H_2O.$$

Combining the two equations gives

$$NH_4Cl + NaNO_2 \longrightarrow N_2 + NaCl + 2\,H_2O.$$

Both nitrogen atoms and all four hydrogen atoms balance, and the two oxygen atoms become water. Thus the evolved gas is nitrogen. Ammonia, option A, would characteristically be released from an ammonium salt by a strong base, which is not the reaction here. Nitrous oxide, option C, is associated with controlled thermal decomposition of ammonium nitrate rather than ammonium nitrite. Chlorine, option D, would require oxidation of chloride and is not a product of this internal nitrogen redox reaction. Only option B matches the balanced process.

Step 3: Conclusion The gas evolved is N_2, nitrogen. Hence, the correct option is (B).

Quick Tip to Solve: Keep the two ammonium salts distinct: ammonium nitrite gives nitrogen, while ammonium nitrate on suitable heating gives nitrous oxide.

78. On the surface of polar stratospheric clouds, hydrolysis of chlorine nitrite gives A and B while its reaction with its reaction with HCl produces B and C. A, B and C are, respectively : [4 marks]

(A) HOCl, HNO_3, Cl_2

(B) Cl_2, HNO_3, HOCl

(C) $HClO_2$, HNO_2, HOCl

(D) HOCl, HNO_2, Cl_2O

Correct Answer: The printed word nitrite gives no correct option; the standard chlorine-nitrate reaction gives option (A).

Solution:

Concept:

- Polar stratospheric cloud surfaces activate chlorine reservoirs through hydrolysis and reaction with hydrogen chloride. The nitrogen oxyacid formed depends on whether the reservoir is chlorine nitrate or chlorine nitrite.
- Chlorine nitrate is $ClONO_2$, whereas chlorine nitrite has composition ClONO. Conservation of oxygen distinguishes their reactions; nitrate and nitrite cannot be interchanged in a balanced equation.

Step 1: Check the compound named in the question The printed name is chlorine nitrite. Hydrolysis of this species would give hypochlorous acid and nitrous acid, not nitric acid:

$$ClONO + H_2O \longrightarrow HOCl + HNO_2.$$

Its corresponding chlorine-activation reaction with hydrogen chloride is

$$ClONO + HCl \longrightarrow Cl_2 + HNO_2.$$

Thus the literal wording requires A = HOCl, B = HNO_2, and C = Cl_2.

Step 2: Compare the literal and conventional interpretations No option lists that complete triple. Option (D) has the first two products but gives Cl_2O instead of chlorine. Option (C) has the wrong chlorine oxyacid and third product. Options (A) and (B) contain nitric acid and therefore cannot describe hydrolysis of the named nitrite. For the usual polar-cloud reservoir, chlorine nitrate, the balanced equations are

$$ClONO_2 + H_2O \longrightarrow HOCl + HNO_3,$$

$$ClONO_2 + HCl \longrightarrow Cl_2 + HNO_3.$$

These conventional nitrate reactions give the triple in (A); (B) reverses the identities of A and C.

Step 3: Conclusion As written with chlorine nitrite, no listed option is fully correct: the products are HOCl, HNO_2, and Cl_2. If the intended reservoir is chlorine nitrate, the answer is option (A), HOCl, HNO_3, Cl_2.

Quick Tip to Solve: Check the nitrogen oxyacid by balancing oxygen: nitrate produces nitric acid, whereas nitrite produces nitrous acid. Do not decide from the chlorine products alone.

79. Which is true about Buna-N? [4 marks]

(A) It is a linear polymer of 1, 3-butadiene.

(B) It is obtained by copolymerization of 1, 3-butadiene and styrene.

(C) It is obtained by copolymerization of 1, 3-butadiene and acrylonitrile.

(D) The suffix N in Buna-N stands for its natural occurrence.

Correct Answer: (C) It is obtained by copolymerization of 1, 3-butadiene and acrylonitrile.

Solution:

Concept:

- Buna-N is a synthetic elastomer produced by addition copolymerisation of two different unsaturated monomers: 1,3-butadiene and acrylonitrile.
- A copolymer contains units derived from more than one monomer. The nitrile group $-\mathrm{CN}$ supplied by acrylonitrile distinguishes nitrile rubber from the styrene-containing rubber Buna-S.

Step 1: Identify the two monomers Write the monomers as

$$\mathrm{CH_2{=}CH{-}CH{=}CH_2} \quad \text{and} \quad \mathrm{CH_2{=}CH{-}CN}.$$

The first is 1,3-butadiene; the second is acrylonitrile. Their double bonds participate in addition polymerisation and create a chain containing units from both. The butadiene component provides the flexible hydrocarbon backbone, while the acrylonitrile component introduces polar nitrile groups. Hence the product is the butadiene-acrylonitrile copolymer described by option (C).

Step 2: Eliminate the other descriptions Option (A) describes a polymer made from butadiene alone; it omits the second monomer required for Buna-N. Option (B) uses styrene as the comonomer, which identifies Buna-S rather than Buna-N. In the naming distinction, S refers to styrene and N identifies the nitrile-containing comonomer. Option (D) therefore gives an incorrect explanation: Buna-N is a synthetic rubber and its suffix does not indicate natural occurrence. These checks distinguish the polymers by chemical composition rather than merely by their shared rubber-like properties.

Step 3: Conclusion Buna-N is obtained by copolymerisation of 1,3-butadiene and acrylonitrile. Hence, the correct option is (C).

Quick Tip to Solve: Associate Buna-N with the nitrile group of acrylonitrile and Buna-S with styrene; the common butadiene monomer does not distinguish them.

80. Match List-I with List-II

	List-I		List-II
(A)	Antipyretic	I.	Reduces pain
(B)	Analgesic	II.	Reduces stress
(C)	Tranquilizer	III.	Reduces fever
(D)	Antacid	IV.	Reduces acidity (stomach)

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-I, C-II, D-IV

(B) A-III, B-I, C-IV, D-II

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-III, C-II, D-IV

Correct Answer: (A) A-III, B-I, C-II, D-IV

Solution:

Concept:

- Drug classes are identified by their principal therapeutic action: an antipyretic lowers fever, whereas an analgesic relieves pain.
- Tranquilizers reduce stress and anxiety through their action on the nervous system. Antacids reduce excess stomach acidity by neutralising gastric acid.

Step 1: Match fever and pain with their drug classes An antipyretic reduces an abnormally elevated body temperature, so entry A matches III, reduces fever. An analgesic is a pain-relieving drug, so entry B matches I, reduces pain. These actions are distinct even though a particular drug can sometimes possess both properties. For this classification question, match each class to the defining action stated in the lists:

$$\mathrm{A}-\mathrm{III}, \qquad \mathrm{B}-\mathrm{I}.$$

In particular, the first entry is not assigned to pain simply because some fever medicines also relieve pain.

Step 2: Match stress and acidity, then compare all options Tranquilizers reduce stress and anxiety, giving C-II. Antacids neutralise excess hydrochloric acid in the stomach, giving D-IV. Combining all four matches gives

$$\mathrm{A}-\mathrm{III},\ \mathrm{B}-\mathrm{I},\ \mathrm{C}-\mathrm{II},\ \mathrm{D}-\mathrm{IV}.$$

This is exactly option (A). Option (B) exchanges the actions of tranquilizers and antacids. Option (C) wrongly assigns pain reduction to antipyretics and acidity reduction to analgesics. Option (D) exchanges the fever and pain actions for A and B. Thus each alternative fails at least one defining therapeutic function.

Step 3: Conclusion The matching is A-III, B-I, C-II, D-IV. Hence, the correct option is (A).

Quick Tip to Solve: Use the word roots: pyretic refers to fever, analgesic to pain, and antacid to acid. The remaining stress-reducing class is the tranquilizer.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. 116 g of a substance upon dissociation reaction, yields 7.5 g of hydrogen, 60 g of oxygen and 48.5 g of carbon. Given that the atomic masses of H, O and C are 1, 16 and 12, respectively. The data agrees with how many formulae of the following?

A. CH_3COOH, B. HCHO, C. CH_3OOCH_3, D. CH_3CHO [4 marks]

Solution:

Concept:

- For a pure compound, the mass fractions of its constituent elements are fixed by its molecular formula. Divide each measured elemental mass by its atomic mass to compare mole ratios.
- Two molecular formulae agree with the same elemental analysis only when their simplest atomic ratios coincide. Rounding must not erase a substantial discrepancy in measured composition.

Step 1: Calculate the atomic ratio from the stated masses The given element masses add to the sample mass: $7.5 + 60 + 48.5 = 116$ g. However, conservation of total mass alone does not identify a formula. The amounts of atoms are

$$n_{\mathrm{H}} = 7.5, \quad n_{\mathrm{O}} = \frac{60}{16} = 3.75, \quad n_{\mathrm{C}} = \frac{48.5}{12} = \frac{97}{24} \quad \text{mol}.$$

Dividing by the oxygen amount gives

$$\mathrm{C} : \mathrm{H} : \mathrm{O} = \frac{97}{90} : 2 : 1.$$

The carbon-to-oxygen atomic ratio is therefore about 1.0778 rather than exactly 1.

Step 2: Test each proposed formula against that ratio For A, CH_3COOH, the simplest ratio is C:H:O = 1:2:1. B, HCHO, has the same ratio. C, CH_3OOCH_3, has ratio 1:3:1, and D, CH_3CHO, has ratio 2:4:1. None equals $97/90 : 2 : 1$. In particular, A and B would require equal moles of carbon and oxygen: 60 g oxygen would correspond to $3.75 \times 12 = 45$ g carbon and 7.5 g hydrogen, totalling 112.5 g, not the printed 48.5 g carbon and 116 g sample. The discrepancy cannot be removed by ordinary final-digit rounding.

Step 3: Conclusion The literal stated analysis agrees exactly with $\boxed{0}$ of the formulae; this count is dimensionless. The answer key gives 2, corresponding to A and B and the intended ratio 1:2:1, but that requires 45 g carbon and 112.5 g total rather than the printed masses.

Quick Tip to Solve: Compare atomic ratios before counting formulae. Agreement of the sum of elemental masses with the sample mass checks conservation, not the empirical formula.

82. Consider the following set of quantum numbers.

	n	l	m_l
A.	**3**	**3**	**−3**
B.	**3**	**2**	**−2**
C.	**2**	**1**	**+1**
D.	**2**	**2**	**+2**

The number of correct sets of quantum numbers is _____. [4 marks]

Solution:

Concept:

- Allowed quantum numbers satisfy $n = 1, 2, 3, \ldots$, $l = 0, 1, \ldots, n-1$, and $m_l = -l, -l+1, \ldots, l$.
- The principal quantum number first bounds the orbital angular momentum quantum number. Only after that condition is satisfied should the magnetic quantum number be checked.

Step 1: Test the angular momentum quantum number For row A, $n = 3$ allows only $l = 0, 1, 2$, so the listed $l = 3$ is forbidden. The value $m_l = -3$ being within the range for $l = 3$ cannot rescue the row because that value of l is itself impossible in the third shell. For row B, $n = 3, l = 2$ is allowed. For row C, $n = 2, l = 1$ is allowed. For row D, $n = 2$ allows only $l = 0, 1$, so $l = 2$ makes this row invalid before its magnetic value is considered.

Step 2: Check magnetic values for the surviving rows For B, $l = 2$ gives the possible magnetic quantum numbers

$$m_l = -2, -1, 0, 1, 2.$$

The listed -2 is permitted. For C, $l = 1$ gives

$$m_l = -1, 0, 1.$$

The listed $+1$ is permitted. Thus both B and C satisfy all supplied restrictions, while A and D violate $l \leq n - 1$. A spin quantum number is not listed in this question; no extra spin restriction needs to be invented to assess these three-number sets. Count each permitted row once.

Step 3: Conclusion Only sets B and C are correct, so the number of correct sets is $\boxed{2}$, a dimensionless count.

Quick Tip to Solve: Check $l < n$ before $|m_l| \leq l$. A plausible magnetic quantum number cannot make an impossible subshell valid.

83. When 5 moles of He gas expand isothermally and reversibly at 300 K from 10 litre to 20 litre, the magnitude of the maximum work obtained is _____ J. [nearest integer] (Given : R = 8.3 J K^{-1} mol^{-1} and log 2 = 0.3010) [4 marks]

Solution:

Concept:

- For reversible isothermal expansion of an ideal gas, the work done by the gas is $W_{\text{by}} = nRT\ln(V_f/V_i)$. Reversibility gives the maximum work for the specified initial and final states.
- In the chemistry sign convention, $w = -nRT\ln(V_f/V_i)$ for expansion. The question requests the magnitude, so the final reported value is positive.

Step 1: Substitute the amount, temperature and volume ratio Helium is treated as an ideal gas at the stated temperature. Since both volumes are given in litres, their units cancel directly:

$$\frac{V_f}{V_i} = \frac{20}{10} = 2.$$

With $n = 5$ mol, $T = 300$ K and the specified gas constant,

$$nRT = 5 \times 8.3 \times 300 = 12450 \text{ J}.$$

No pressure is needed because the reversible pressure changes continuously and has already been integrated in the logarithmic work formula. Using a constant external pressure here would not describe this reversible path.

Step 2: Evaluate the logarithm and round the magnitude The work magnitude is

$$|w_{\max}| = 12450 \ln 2.$$

Using $\ln 2 \simeq 0.693147$, consistent with the supplied rounded common logarithm, gives

$$|w_{\max}| \simeq 8629.68 \text{ J}.$$

It rounds to 8630 J. Equivalently, the usual textbook conversion $\ln 2 \simeq 2.303 \times 0.3010$ gives 8630.38 J, which yields the same nearest integer. Keep adequate precision until the final rounding. The system work itself is negative because the expanding gas transfers energy to the surroundings; only its positive magnitude is requested.

Step 3: Conclusion The magnitude of the maximum work obtained is $\boxed{8630 \text{ J}}$, to the nearest integer.

Quick Tip to Solve: For reversible isothermal ideal-gas work, use the volume ratio inside a natural logarithm and round only at the end; expansion work on the gas has a negative sign.

84. A solution containing 2.5 $\times$ 10^{-3} kg of a solute dissolved in 75 $\times$ 10^{-3} kg of water boils at 373.535 K. The molar mass of the solute is _____ g mol^{-1}. [nearest integer] (Given : $K_b(H_2O)$ = 0.52 K kg mol^{-1} and boiling point of water = 373.15 K) [4 marks]

Solution:

Concept:

- For a dilute solution of a nonvolatile nonelectrolyte, the elevation of boiling point is $\Delta T_b = K_b m$, where m is moles of solute per kilogram of solvent.
- The molar mass follows from $M = w_2/n_2$. Use the mass of water as the solvent mass, not the total mass of solution, in calculating molality.

Step 1: Find the boiling-point elevation and molality The boiling-point elevation is the difference between the solution and pure solvent boiling points:

$$\Delta T_b = 373.535 - 373.15 = 0.385 \text{ K}.$$

With the supplied water ebullioscopic constant,

$$m = \frac{\Delta T_b}{K_b} = \frac{0.385}{0.52} = 0.7403846 \text{ mol kg}^{-1}.$$

The temperature difference may be calculated directly in kelvin. It is not necessary to convert the two temperatures to degrees Celsius, and using the absolute boiling temperature itself in the numerator would be incorrect.

Step 2: Calculate moles and the solute molar mass The water mass is $75 \times 10^{-3} = 0.075$ kg, so

$$n_2 = m \times 0.075 = 0.05552885 \text{ mol}.$$

The solute mass is 2.5×10^{-3} kg $= 2.5$ g. Therefore

$$M = \frac{2.5}{0.05552885} = 45.0216 \text{ g mol}^{-1}.$$

As a direct check, the combined relation gives

$$M = \frac{0.52 \times 2.5}{0.385 \times 0.075} \text{ g mol}^{-1},$$

with the same result. Keep the solvent mass in kilograms when using the stated units of K_b, and round the final molar mass only after the division.

Step 3: Conclusion The molar mass of the solute is $\boxed{45 \text{ g mol}^{-1}}$, to the nearest integer.

Quick Tip to Solve: Subtract the pure-solvent boiling point first. In $M = K_b w_2/(\Delta T_b w_1)$, use solute mass in grams and solvent mass in kilograms.

85. pH value of 0.001 M NaOH solution is _____. [4 marks]

Solution:

Concept:

- NaOH is a strong base and dissociates completely in dilute aqueous solution, furnishing one hydroxide ion per formula unit.
- At ordinary laboratory temperature, $K_w = 10^{-14}$, so $\mathrm{pH} + \mathrm{pOH} = 14$. The logarithm in these definitions is to base ten.

Step 1: Find the hydroxide concentration The solution contains 0.001 mole of NaOH per litre. Complete dissociation gives

$$\mathrm{NaOH} \longrightarrow \mathrm{Na^+} + \mathrm{OH^-}$$

Hence $[\mathrm{OH^-}] = 0.001 = 10^{-3}\,\mathrm{mol\,L^{-1}}$. The hydroxide supplied by water is negligible compared with this concentration; it must not be added as 10^{-7} independently because the common-ion effect suppresses water ionisation.

Step 2: Calculate pOH and convert to pH Using the concentration obtained above,

$$\mathrm{pOH} = -\log_{10}(10^{-3}) = 3.$$

Therefore, under the standard $25^\circ\mathrm{C}$ convention,

$$\mathrm{pH} = 14 - 3 = 11.$$

This is greater than seven, as required for an alkaline solution. The value three is the pOH and should not be entered as the pH.

Step 3: Conclusion The pH of the solution is $\boxed{11}$ (a dimensionless quantity).

Quick Tip to Solve: For a dilute strong monobasic base, first take the negative logarithm of its concentration to get pOH; subtract that result from 14 to obtain pH.

86. For the reaction taking place in the cell :

Pt (s)| H_2 (g)|H^+(aq) || Ag^+(aq) |Ag (s)

E^o_{cell} = + 0.5332 V.

The value of $\Delta_f G^\circ$ is _____ kJ mol^{-1}. (in nearest integer) [4 marks]

Solution:

Concept:

- The cell is written with oxidation on the left and reduction on the right. A positive standard cell emf corresponds to a negative standard Gibbs energy change: $\Delta_r G^\circ = -nFE^\circ_{\mathrm{cell}}$.
- The electron count n must refer to the explicitly balanced reaction. Multiplying a reaction by two doubles its Gibbs energy change but leaves its emf unchanged.

Step 1: Balance the electrode reactions At the hydrogen electrode, $\mathrm{H_2} \longrightarrow 2\,\mathrm{H^+} + 2\,\mathrm{e^-}$; at the silver

electrode, $Ag^+ + e^- \longrightarrow Ag$. Per mole of silver formed, the overall reaction is

$$(1/2)\,H_2 + Ag^+ \longrightarrow H^+ + Ag.$$

It transfers one mole of electrons, so $n = 1$. For the integer-coefficient reaction $H_2 + 2\,Ag^+ \longrightarrow 2\,H^+ + 2\,Ag$, the corresponding count is $n = 2$.

Step 2: Calculate the signed Gibbs energy With $F = 96485\,C\,mol^{-1}$ and the given emf,

$$\Delta_r G^\circ = -(1)(96485)(0.5332)\,J\,mol^{-1} = -51.446\,kJ\,mol^{-1}.$$

Thus the nearest integer per mole of silver is -51. For the doubled reaction it is -102.892, which rounds to $-103\,kJ\,mol^{-1}$. The negative sign is required by the positive emf; changing the reaction normalisation cannot change that sign.

Step 3: Interpret the requested entry The printed symbol $\Delta_f G^\circ$ is a formation-energy symbol, whereas the supplied cell emf directly determines the Gibbs energy of the cell reaction. It does not provide a general standard formation energy for an unspecified substance. The numerical key accepts 51 or 103, the magnitudes for the two reaction normalisations; these positive entries are not the signed Gibbs energy changes of the spontaneous forward reaction.

Step 4: Conclusion For the forward cell reaction, $\boxed{\Delta_r G^\circ = -51\,kJ\,mol^{-1}}$ per mole of silver, or $\boxed{-103\,kJ\,mol^{-1}}$ for the doubled reaction. The accepted numerical entries 51 or 103 represent the corresponding magnitudes.

Quick Tip to Solve: Always write the balanced cell reaction before choosing n. A positive cell potential means a negative forward-reaction Gibbs energy, even when a numerical key records only its magnitude.

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87. It has been found that for a chemical reaction with rise in temperature by 9 K the rate constant gets doubled. Assuming a reaction to be occurring at 300 K, the value of activation energy is found to be _____ kJ mol^{-1}. [nearest integer]

(Given ln10 = 2.3, R = 8.3 J K^{-1} mol^{-1}, log 2 = 0.30) [4 marks]

Solution:

Concept:

- The Arrhenius relation is $k = A\exp[-E_a/(RT)]$. Over the small stated temperature interval, the pre-exponential factor and activation energy are treated as constant.
- For two temperatures, $\ln(k_2/k_1) = (E_a/R)(1/T_1 - 1/T_2)$. Temperatures must be in kelvin, and the logarithm must be natural when using this form.

Step 1: Identify the two temperatures and rate ratio The initial temperature is $T_1 = 300\,K$. A rise of $9\,K$ makes $T_2 = 309\,K$, and doubling gives $k_2/k_1 = 2$. Convert the supplied common logarithm using

$$\ln 2 = (\ln 10)\log_{10} 2 = (2.3)(0.30) = 0.69.$$

The final temperature is 309 K, not 9 K; only the difference is nine.

Step 2: Rearrange and substitute in the Arrhenius equation The reciprocal-temperature difference is

$$\frac{1}{300} - \frac{1}{309} = \frac{9}{300 \times 309}\,\mathrm{K}^{-1}.$$

Therefore

$$E_a = \frac{(8.3)(0.69)(300)(309)}{9}\,\mathrm{J\,mol^{-1}} = 58988.1\,\mathrm{J\,mol^{-1}}.$$

Converting joules to kilojoules gives $58.9881\,\mathrm{kJ\,mol^{-1}}$. The positive result is consistent with the rate increasing as temperature rises.

Step 3: Conclusion The activation energy to the nearest integer is $\boxed{59\,\mathrm{kJ\,mol^{-1}}}$; the numerical entry is 59.

Quick Tip to Solve: Use the product T1 T2 divided by their difference when rearranging the two-temperature Arrhenius equation; convert common logarithms and joules before rounding.

88. 0.25 g of an organic compound containing chlorine gave 0.40 g of silver chloride in Carius estimation. The percentage of chlorine present in the compound is _____. [in nearest integer] (Given : Molar mass of Ag is 108 g mol^{-1} and that of Cl is 35.5 g mol^{-1}) [4 marks]

Solution:

Concept:

- In Carius estimation, chlorine in the organic sample is converted quantitatively into precipitated silver chloride. Each mole of AgCl contains one mole of chlorine atoms.
- The chlorine mass is the precipitate mass multiplied by its chlorine mass fraction, $M(\mathrm{Cl})/M(\mathrm{AgCl})$. Percentage is then calculated relative to the original sample mass.

Step 1: Calculate the chlorine mass in the precipitate The given atomic molar masses give

$$M(\mathrm{AgCl}) = 108 + 35.5 = 143.5\,\mathrm{g\,mol^{-1}}.$$

Thus $0.40\,\mathrm{g}$ of AgCl contains

$$m(\mathrm{Cl}) = 0.40\left(\frac{35.5}{143.5}\right) = 0.09895\,\mathrm{g}.$$

The silver mass is included in the precipitate mass and must be removed through this mass-fraction conversion.

Step 2: Express chlorine as a percentage of the sample The organic sample weighed $0.25\,\mathrm{g}$, so

$$\%\mathrm{Cl} = \frac{0.09895}{0.25} \times 100 = 39.58\%.$$

Equivalently, substitute directly as

$$\%\mathrm{Cl} = \frac{0.40 \times 35.5 \times 100}{143.5 \times 0.25}.$$

The denominator uses the original compound mass, not the mass of precipitate. Round only the final percentage to the nearest integer.

Step 3: Conclusion The chlorine content is $\boxed{40\%}$ to the nearest integer; the required entry is 40.

Quick Tip to Solve: For a halogen precipitated as AgX, multiply the precipitate mass by M(X)/M(AgX) before dividing by the organic sample mass.

89. BeO reacts with HF in presence of ammonia to give [A] which on thermal decomposition produces [B] and ammonium fluoride. Oxidation state of Be in [A] is _____. [4 marks]

Solution:

Concept:

- Beryllium oxide forms a soluble fluoroberyllate complex in the presence of fluoride and ammonium ions. Formation of a coordination complex does not itself imply a change in oxidation state.
- Fluorine has oxidation state -1 in these compounds. The sum of oxidation states within an ion equals the ionic charge, which determines the central-metal oxidation state.

Step 1: Identify the fluoroberyllate intermediate The product $[A]$ is ammonium tetrafluoroberyllate, $(NH_4)_2[BeF_4]$. Its formation can be represented by the balanced equation

$$BeO + 4\,HF + 2\,NH_3 \longrightarrow (NH_4)_2[BeF_4] + H_2O.$$

The two ammonium cations together supply charge $+2$, so the complex anion must be $[BeF_4]^{2-}$. Ammonia supplies ammonium ions in the acidic medium rather than reducing beryllium.

Step 2: Calculate the oxidation state and check decomposition Let the oxidation state of beryllium in the anion be x. With four fluoride ligands,

$$x + 4(-1) = -2, \qquad x = +2.$$

On heating,

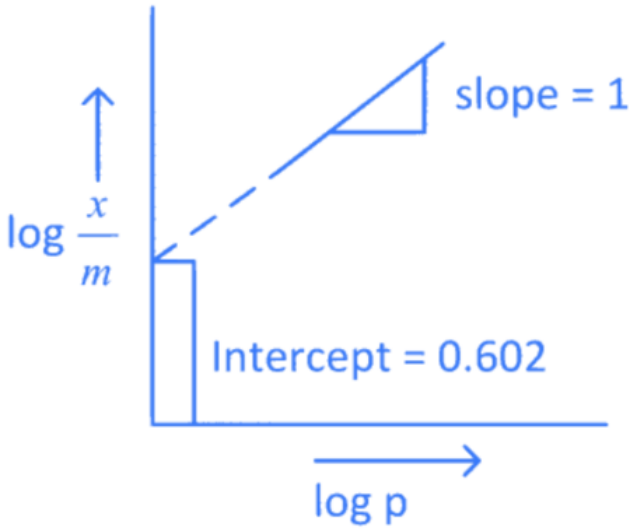
$$(NH_4)_2[BeF_4] \longrightarrow BeF_2 + 2\,NH_4F.$$

Thus $[B]$ is BeF_2, in which beryllium also has oxidation state $+2$. This is complex decomposition, not a redox reaction, so the oxidation-state result is consistent at both stages.

Step 3: Conclusion The oxidation state of Be in $[A]$ is $\boxed{+2}$; the numerical entry is 2.

Quick Tip to Solve: Find the charge on the complex ion from its counterions first, then add the fluoride oxidation states; coordination number four does not mean oxidation state four.

90. If the initial pressure of a gas is 0.03 atm, the mass of the gas adsorbed per gram of the adsorbent is _____ $\times 10^{-2}$ g. [4 marks]



Solution:

Concept:

- The Freundlich adsorption isotherm is $x/m = kp^{1/n}$, where x is the mass adsorbed, m is the mass of adsorbent, and p is the equilibrium pressure.
- Taking common logarithms gives $\log_{10}(x/m) = \log_{10} k + (1/n)\log_{10} p$. Thus a plot of these two logarithms has slope $1/n$ and vertical intercept $\log_{10} k$.

Step 1: Read the constants from the graph The supplied straight line has slope 1 and vertical intercept 0.602. Therefore

$$\frac{1}{n} = 1, \qquad \log_{10} k = 0.602.$$

Taking the antilogarithm yields $k = 10^{0.602} \simeq 4$, since $\log_{10} 4 \simeq 0.602$. The intercept is the logarithm of k, not k itself. In the numerical units used by the graph, the isotherm reduces to $x/m = 4p$.

Step 2: Substitute the pressure and match the requested scale Using the stated pressure $p = 0.03\,\text{atm}$,

$$\frac{x}{m} = 4(0.03) = 0.12\,\text{g}\,\text{g}^{-1}.$$

Hence one gram of the adsorbent holds $0.12\,\text{g}$ of gas. Write this in the form requested in the question:

$$0.12\,\text{g} = 12 \times 10^{-2}\,\text{g}.$$

The blank therefore contains 12, rather than 0.12; the power of ten is already printed after the blank.

Step 3: Conclusion The mass adsorbed per gram of adsorbent is $\boxed{12 \times 10^{-2}\,\text{g}}$; the numerical entry is $\boxed{12}$.

Quick Tip to Solve: In a logarithmic Freundlich plot, take the antilogarithm of the intercept to get k, and convert the calculated mass to the exact power-of-ten form printed beside the blank.