JEE Main 2022 Paper 1 (B.E./B.Tech) Jun 29 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 1 (June), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 29 June 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The probability that a randomly chosen 2×2 matrix with all the entries from the set of first 10 primes, is singular, is equal to : [4 marks]

(A) $\frac{133}{10^4}$
(B) $\frac{18}{10^3}$
(C) $\frac{19}{10^3}$
(D) $\frac{271}{10^4}$

Correct Answer: (C) $\frac{19}{10^3}$

Solution:

Concept:

- A matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is singular exactly when its determinant $ad - bc$ is zero.
- Unique prime factorisation implies that equal products of two primes have the same unordered pair of factors. All four positions are chosen independently, so there are 10^4 equally likely matrices.

Step 1: Translate singularity into a counting condition Write the entries as a, b, c, d, each one of the first ten primes. We require

$$ad = bc.$$

If $a = d = p$, the product on the left is p^2, and unique factorisation forces $b = c = p$. Thus the repeated-prime case contributes ten matrices, one for each prime. No other pair of prime entries can produce p^2.

Step 2: Count products of two different primes There are $\binom{10}{2} = 45$ unordered pairs $\{p, q\}$ with $p \neq q$. For each such pair, (a, d) can be (p, q) or (q, p); independently, (b, c) has the same two choices. Therefore this case contributes

$$45 \cdot 2 \cdot 2 = 180$$

matrices. The two cases do not overlap, so the favourable count is $10 + 180 = 190$.

Step 3: Divide by the full sample space The required probability is

$$P = \frac{190}{10000} = \frac{19}{1000}.$$

Equivalently, the two patterns $a = b, d = c$ and $a = c, d = b$ each give 100 matrices, with the ten all-equal matrices counted twice. This confirms the count 190. Options (A), (B), and (D) correspond to different favourable counts and therefore fail.

Step 4: Conclusion The probability is $19/10^3$. Hence, the correct option is (C).

Quick Tip to Solve: For equal products of prime entries, match the unordered prime pairs; count the repeated-prime case separately to avoid double counting.

2. Let the solution curve of the differential equation

$x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ **be** $y = y(x)$**. Then y(2) is equal to:** [4 marks]

(A) 15

(B) 11

(C) 13

(D) 17

Correct Answer: (A) 15

Solution:

Concept:

- The substitution $v = y/x$ gives $y = xv$ and $xy\prime - y = x^2 v\prime$. It removes the homogeneous combination on the left.
- On the interval from 1 to 2, $x > 0$, so $\sqrt{x^2(v^2 + 16)} = x\sqrt{v^2 + 16}$. The sign of x must be retained when simplifying a square root.

Step 1: Separate the variables Substitute $y = xv$ into the equation. Since $y\prime = v + xv\prime$, we obtain

$$x^2\frac{dv}{dx} = x\sqrt{v^2 + 16}, \qquad \frac{dv}{\sqrt{v^2 + 16}} = \frac{dx}{x}.$$

The initial value becomes $v(1) = 3$. There is no division by zero on the interval under consideration.

Step 2: Integrate and apply the initial condition Using the standard logarithmic integral,

$$\log\left(v + \sqrt{v^2 + 16}\right) = \log x + C.$$

The logarithm argument is positive because $\sqrt{v^2 + 16} > |v|$. At $x = 1, v = 3$, the argument is $3 + 5 = 8$. Hence

$$v + \sqrt{v^2 + 16} = 8x.$$

Isolating the square root and squaring yields $16 = 64x^2 - 16xv$, so $v = 4x - 1/x$.

Step 3: Recover the curve and evaluate Multiplying by x gives $y = 4x^2 - 1$. Direct substitution checks the original sign condition: $xy\prime - y = 4x^2 + 1 > 0$, and

$$y^2 + 16x^2 = (4x^2 + 1)^2.$$

The curve also gives $y(1) = 3$. Therefore $y(2) = 16 - 1 = 15$. The other proposed values 11, 13 and 17 do not lie on this initial-value solution at $x = 2$.

Step 4: Conclusion $y(2) = 15$. Hence, the correct option is (A).

Quick Tip to Solve: For an equation containing $xy\prime - y$, try $y = xv$; simplify $\sqrt{x^2}$ as $|x|$ before using the initial interval.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \max\{t^3 - 3t : t \le x\}, & x \le 2, \\ x^2 + 2x - 6, & 2 < x < 3, \\ \lfloor x - 3 \rfloor + 9, & 3 \le x \le 5, \\ 2x + 1, & x > 5. \end{cases}$$

Here $\lfloor t \rfloor$ is the greatest integer less than or equal to t. Let m be the number of points where f is not differentiable and $I = \int_{-2}^{2} f(x)\,dx$. Then the ordered pair (m, I) is equal to: [4 marks]

(A) $\left(3, \frac{27}{4}\right)$
(B) $\left(3, \frac{23}{4}\right)$
(C) $\left(4, \frac{27}{4}\right)$
(D) $\left(4, \frac{23}{4}\right)$

Correct Answer: (C) $\left(4, \frac{27}{4}\right)$

Solution:

Concept:

- A running maximum $\max_{t \le x} g(t)$ follows an increasing record of g and remains constant while g stays below its previous maximum.
- Differentiability requires continuity and equality of one-sided derivatives. Integer-part expressions have jumps at their integer thresholds.

Step 1: Resolve the maximum branch Let $g(t) = t^3 - 3t$. Its derivative $3(t^2 - 1)$ is positive for $t < -1$, negative for $-1 < t < 1$, and positive for $t > 1$. The local maximum is $g(-1) = 2$. On $[-1, 2]$, the cubic never exceeds 2 and reaches 2 again at $t = 2$. Thus

$$f(x) = x^3 - 3x \ (x < -1), \qquad f(x) = 2 \ (-1 \le x \le 2).$$

At -1, both value and derivative match: the derivatives are zero on both sides.

Step 2: Check every possible exceptional point At $x = 2$, the function is continuous, but its left derivative is 0 and its right derivative from the quadratic is 6. At $x = 3$, both limiting values are 9, while the derivatives are 8 and 0. The integer-part branch equals 9 on $[3, 4)$, 10 on $[4, 5)$, and 11 at 5. Therefore there are jumps at 4 and 5. The right-hand linear branch approaches 11 at 5, but the left limit there is 10. All other branch interiors are differentiable. The complete exceptional set is $\{2, 3, 4, 5\}$, giving $m = 4$.

Step 3: Integrate the resolved first branch Only the running-maximum branch contributes on the integration interval:

$$I = \int_{-2}^{-1} (x^3 - 3x)\,dx + \int_{-1}^{2} 2\,dx.$$

The first integral is

$$\left[\frac{x^4}{4} - \frac{3x^2}{2}\right]_{-2}^{-1} = \frac{3}{4},$$

and the second is 6. Hence $I = 27/4$. Options (A) and (B) miss an exceptional point, while (D) has the wrong integral.

Step 4: Conclusion $(m, I) = (4, 27/4)$. Hence, the correct option is (C).

Quick Tip to Solve: First replace a running maximum by explicit branches. A smooth extremum can join its constant plateau differentiably, whereas a later branch junction may not.

4. Let $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in R$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ is equal to : [4 marks]

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (A) 3

Solution:

Concept:

- The scalar projection of $\vec{a}$ on $\vec{c}$ is $(\vec{a} \cdot \vec{c})/|\vec{c}|$, whereas the vector projection would include a direction vector.
- The cross product is evaluated by its signed determinant expansion; comparing individual components determines unknown parameters.

Step 1: Use the scalar projection to find alpha For $\vec{c} = (1, 2, -2)$, its length is

$$|\vec{c}| = \sqrt{1 + 4 + 4} = 3.$$

The dot product is $\vec{a} \cdot \vec{c} = \alpha + 6 + 2 = \alpha + 8$. The given projection therefore implies

$$\frac{\alpha + 8}{3} = \frac{10}{3}, \qquad \alpha = 2.$$

The contribution from the two negative third components is positive; losing this sign would change the answer.

Step 2: Expand the cross product to find beta With $\vec{b} = (3, -\beta, 4)$, the cross product is

$$\vec{b} \times \vec{c} = (2\beta - 8)\hat{i} + 10\hat{j} + (6 + \beta)\hat{k}.$$

For example, the middle component is $4 \cdot 1 - 3(-2) = 10$, including the determinant sign. Comparing the first component with the given -6 gives $2\beta - 8 = -6$, hence $\beta = 1$. Independently, the last component gives $6 + \beta = 7$, confirming the same value.

Step 3: Check and combine the parameters The recovered vectors give $\vec{a} \cdot \vec{c} = 10$, so the projection is indeed $10/3$. They also give every specified cross-product component, $(-6, 10, 7)$, rather than merely

one of them. Consequently

$$\alpha + \beta = 2 + 1 = 3.$$

The alternatives 4, 5 and 6 cannot satisfy both recovered parameter values. In particular, using a squared norm for a scalar projection would apply the vector-projection coefficient instead.

Step 4: Conclusion $\alpha + \beta = 3$. Hence, the correct option is (A).

Quick Tip to Solve: Find the norm before using a projection, and check the unknown from two cross-product components to catch a determinant sign error.

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5. The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}\,x$ that lies outside the triangle formed by $y = \sqrt{2}\,x$, $x = 1$, $y = 2\sqrt{2}$, is equal to: [4 marks]

(A) $\frac{16\sqrt{2}}{6}$

(B) $\frac{11\sqrt{2}}{6}$

(C) $\frac{13\sqrt{2}}{6}$

(D) $\frac{5\sqrt{2}}{6}$

Correct Answer: (C) $\frac{13\sqrt{2}}{6}$

Solution:

Concept:

- The bounded area between two curves is the integral of right boundary minus left boundary with respect to y, or upper minus lower with respect to x.
- The requested area excludes the part occupied by the specified triangle. Verify that the whole triangle lies inside the curved region before subtracting its full area.

Step 1: Find the curved region and its intersections The line is $y = \sqrt{2}x$, so $x = y/\sqrt{2}$; the parabola gives $x = y^2/8$. Their intersections satisfy $y^2 = 8y/\sqrt{2}$, giving $y = 0, 4\sqrt{2}$, with $x = 0, 4$. Between these values the parabola is the left boundary and the line the right boundary. Therefore

$$A = \int_0^{4\sqrt{2}} \left(\frac{y}{\sqrt{2}} - \frac{y^2}{8} \right) dy = \frac{8\sqrt{2}}{3}.$$

Step 2: Locate the triangle and calculate its area The triangle has vertices $(1, \sqrt{2})$, $(1, 2\sqrt{2})$, and $(2, 2\sqrt{2})$. Its horizontal base has length 1 and its vertical height is $\sqrt{2}$, hence

$$T = \frac{1}{2} \cdot 1 \cdot \sqrt{2} = \frac{\sqrt{2}}{2}.$$

For $\sqrt{2} \le y \le 2\sqrt{2}$, this triangle occupies $1 \le x \le y/\sqrt{2}$. In the same range $y^2/8 \le 1$; thus every point of the triangle belongs to the curved region. Its entire area can be removed.

Step 3: Subtract only the excluded area The area lying outside the triangle is consequently

$$A - T = \frac{8\sqrt{2}}{3} - \frac{\sqrt{2}}{2} = \frac{13\sqrt{2}}{6}.$$

All quantities are areas in square units. Option (A) retains the full curved area, while (B) and (D) do not equal the verified subtraction. Using a horizontal strip avoids taking square roots of the parabola and makes containment easy to check.

Step 4: Conclusion The required area is $13\sqrt{2}/6$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a right-opening parabola, integrate horizontally. Check triangle containment before subtracting its base-times-height area.

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6. If the system of linear equations

$$\begin{aligned} 2x + y - z &= 7, \\ x - 3y + 2z &= 1, \\ x + 4y + \delta z &= k, \end{aligned}$$

where $\delta, k \in \mathbb{R}$, has infinitely many solutions, then $\delta + k$ is equal to: [4 marks]

(A) -3

(B) 3

(C) 6

(D) 9

Correct Answer: (B) 3

Solution:

Concept:

- Infinitely many solutions require equal coefficient and augmented ranks, both smaller than the number of unknowns.
- If two coefficient rows are independent, a dependent third equation must be the same linear combination of their right-hand sides as of their coefficients.

Step 1: Express the third row as a combination The first two rows $(2, 1, -1)$ and $(1, -3, 2)$ are not proportional, so they are independent. For infinitely many solutions, write the third coefficient row as a times the first plus b times the second. Comparing the coefficients of x and y gives

$$2a + b = 1, \qquad a - 3b = 4.$$

Substitute $b = 1 - 2a$ into the second equation: $7a = 7$. Therefore $a = 1, b = -1$.

Step 2: Impose dependence on the remaining entries The coefficient of z and the right-hand side must follow exactly the same combination. Thus

$$\delta = 1(-1) - 1(2) = -3, \qquad k = 1(7) - 1(1) = 6.$$

Indeed, subtracting the second original equation from the first yields $x + 4y - 3z = 6$, precisely the third equation with these values. Making only the coefficient determinant zero without imposing the right-hand side could instead produce an inconsistent system.

Step 3: Verify that a free parameter remains Let $z = t$. The first two equations then yield

$$y = \frac{5 + 5t}{7}, \qquad x = \frac{22 + t}{7}.$$

These expressions satisfy the third equation for every real t, so there are genuinely infinitely many solutions. The requested sum is $\delta + k = -3 + 6 = 3$. Option (A) gives only δ, option (C) gives only k, and option (D) incorrectly changes the sign of δ.

Step 4: Conclusion $\delta + k = 3$. Hence, the correct option is (B).

Quick Tip to Solve: When one equation depends on the others, carry the same row combination through to the constant column; determinant zero alone is insufficient.

7. Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to : [4 marks]

(A) 50

(B) 250

(C) 1250

(D) 1500

Correct Answer: (A) 50

Solution:

Concept:

- Both roots of $x^2 + (2i - 1) = 0$ satisfy $x^2 = 1 - 2i$. An even power can therefore be evaluated without finding either square root.
- The modulus is multiplicative: $|zw| = |z||w|$ and $|z^n| = |z|^n$. For real a, b, $|a + ib| = \sqrt{a^2 + b^2}$.

Step 1: Reduce both eighth powers to the same base For each of the roots, the quadratic gives

$$\alpha^2 = \beta^2 = 1 - 2i.$$

Raising these equalities to the fourth power yields $\alpha^8 = \beta^8 = (1 - 2i)^4$. Therefore

$$\alpha^8 + \beta^8 = 2(1 - 2i)^4.$$

The roots are negatives of one another, but an even power removes that sign. Thus their eighth powers add; they do not cancel. No choice of a complex square-root branch affects this calculation.

Step 2: Apply the modulus law The modulus of the common base is

$$|1 - 2i| = \sqrt{1^2 + (-2)^2} = \sqrt{5}.$$

Taking the modulus of the sum already factored above, we obtain

$$|\alpha^8 + \beta^8| = |2|\,|1-2i|^4 = 2(\sqrt{5})^4 = 2 \cdot 25 = 50.$$

We may factor before taking the modulus because the two summands are identical. One should not replace the modulus of an arbitrary sum by the sum of moduli.

Step 3: Verify by direct squaring and compare options For an independent arithmetic check,

$$(1-2i)^2 = -3-4i, \qquad (1-2i)^4 = -7+24i.$$

Hence the sum is $-14+48i$, whose modulus is

$$\sqrt{(-14)^2+48^2} = \sqrt{2500} = 50.$$

The values 250, 1250 and 1500 would require additional powers or factors not present in the expression. Both the factorisation method and the explicit complex calculation give the same result.

Step 4: Conclusion $|\alpha^8 + \beta^8| = 50$. Hence, the correct option is (A).

Quick Tip to Solve: For high even powers of quadratic roots, use the equation to replace the square first; then take the modulus after combining equal terms.

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8. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all i, j = 1, 2, 3. Then, the matrix $A^2 + A^3 + \ldots\ldots + A^{10}$ is equal to : [4 marks]

(A) $\left(\frac{3^{10}-3}{2}\right) A$

(B) $\left(\frac{3^{10}-1}{2}\right) A$

(C) $\left(\frac{3^{10}+1}{2}\right) A$

(D) $\left(\frac{3^{10}+3}{2}\right) A$

Correct Answer: (A) $\left(\frac{3^{10}-3}{2}\right) A$

Solution:

Concept:

- The (i,j) entry of a product is the sum of row-column products: $(A^2)_{ij} = \sum_{r=1}^{3} a_{ir}a_{rj}$.
- If $A^2 = cA$, repeated multiplication gives $A^m = c^{m-1}A$ for every integer $m \geq 1$. A geometric sum then replaces a long matrix calculation.

Step 1: Compute the square entry by entry For any fixed i, j, each summand has the same value because the intermediate index cancels in its exponent. Thus

$$(A^2)_{ij} = \sum_{r=1}^{3} 2^{r-i}2^{j-r} = \sum_{r=1}^{3} 2^{j-i} = 3\,2^{j-i} = 3a_{ij}.$$

This holds for all nine entries, so $A^2 = 3A$. No assumption that the matrix is diagonal or invertible is needed.

Step 2: Reduce the powers and sum the progression Multiplying the identity by A gives $A^3 = 3A^2 = 3^2A$. Continuing inductively, $A^{10} = 3^9A$. Hence

$$A^2 + A^3 + \cdots + A^{10} = (3 + 3^2 + \cdots + 3^9)A.$$

The scalar progression has nine terms, first term 3 and common ratio 3. Therefore

$$3 + 3^2 + \cdots + 3^9 = \frac{3(3^9 - 1)}{3 - 1} = \frac{3^{10} - 3}{2}.$$

Option B includes the extra coefficient 1, corresponding to adding A; C and D are still larger and cannot equal this sum.

Step 3: Conclusion The required matrix is $\frac{3^{10} - 3}{2}A$. Hence, the correct option is (A).

Quick Tip to Solve: Look for cancellation of the intermediate index in matrix multiplication before computing all nine entries separately.

9. Let a set $A = A_1 \cup A_2 \cup ..... \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j$, $1 \leq j, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i, 1 \leq i \leq k\}$. Then, R is : [4 marks]

(A) reflexive, symmetric but not transitive.

(B) reflexive, transitive but not symmetric.

(C) reflexive but not symmetric and transitive.

(D) an equivalence relation.

Correct Answer: (D) an equivalence relation.

Solution:

Concept:

- A relation on a set is an equivalence relation exactly when it is reflexive, symmetric and transitive.
- The disjoint sets A_i partition A: two related elements have the same membership in every block, or equivalently belong to the same block.

Step 1: Check reflexivity and symmetry Take any $x \in A$. For every block A_i, the statement $x \in A_i$ is equivalent to itself, so $(x, x) \in R$. Therefore R is reflexive. Next suppose $(x, y) \in R$. The defining condition says that $y \in A_i$ if and only if $x \in A_i$. Reversing an if-and-only-if statement preserves its truth, so $x \in A_i$ if and only if $y \in A_i$. Hence $(y, x) \in R$, proving symmetry.

Step 2: Check transitivity and compare the options Suppose $(x, y) \in R$ and $(y, z) \in R$. For every i, membership of x in A_i is equivalent to membership of y, which in turn is equivalent to membership of z. Consequently membership of x and z is equivalent in every block. Thus $(x, z) \in R$, establishing transitivity. The same argument can be expressed by noting that x, y lie in one block and y, z lie in one block; disjointness forces those blocks to coincide. Option A incorrectly denies transitivity, B

incorrectly denies symmetry, and C denies properties that have both been proved.

Step 3: Conclusion The relation is reflexive, symmetric and transitive, and therefore is an equivalence relation. Hence, the correct option is (D).

Quick Tip to Solve: For relations defined by belonging to the same class or block, test the three equivalence properties through common membership.

10. The distance between the two points A and A' which lie on y = 2 such that both the line segments AB and A' B (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to : [4 marks]

(A) 10

(B) $\frac{48}{5}$

(C) $\frac{52}{5}$

(D) 3

Correct Answer: (C) $\frac{52}{5}$

Solution:

Concept:

- For two planar vectors u, v with positive dot product, $\tan\theta = |u_x v_y - u_y v_x|/(u \cdot v)$, where θ is the angle between the rays.
- Points on the horizontal line $y = 2$ have coordinates $(x, 2)$; the distance between two such points is the absolute difference of their abscissae.

Step 1: Convert the angle requirement into an equation Write a candidate point as $P = (x, 2)$. Its position vector is $(x, 2)$, while $\overrightarrow{OB} = (2, 3)$. Since $\angle POB = \pi/4$ is acute, the dot product must be positive. Thus

$$2x + 6 > 0, \qquad \frac{|3x - 4|}{2x + 6} = \tan\frac{\pi}{4} = 1.$$

The absolute value accounts for the two rays making the specified angle with OB. Keeping the positivity condition avoids accidentally accepting the supplementary angle.

Step 2: Find both points and their separation The equation is $|3x - 4| = 2x + 6$, with $x > -3$. Its two linear cases give

$$3x - 4 = 2x + 6 \implies x = 10,$$

$$-(3x - 4) = 2x + 6 \implies x = -\frac{2}{5}.$$

Both roots satisfy $x > -3$, so both correspond to the required acute angle. Therefore the points are $(10, 2)$ and $(-2/5, 2)$. Their vertical coordinates are equal, and

$$AA' = \sqrt{\left(10 + \frac{2}{5}\right)^2 + (2 - 2)^2} = \frac{52}{5}.$$

The other options 10, $48/5$, and 3 do not equal the separation; 10 is just one abscissa rather than the distance between the two points.

Step 3: Conclusion The distance is $52/5$ coordinate units. Hence, the correct option is (C).

Quick Tip to Solve: When an angle condition gives two rays, retain the dot-product sign and solve both cases of the determinant absolute value.

11. A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is : [4 marks]

(A) $\frac{22}{9+4\sqrt{3}}$
(B) $\frac{66}{9+4\sqrt{3}}$
(C) $\frac{22}{4+9\sqrt{3}}$
(D) $\frac{66}{4+9\sqrt{3}}$

Correct Answer: (B) $\frac{66}{9+4\sqrt{3}}$

Solution:

Concept:

- For a square of side s, area is s^2 and perimeter is $4s$. An equilateral triangle of side t has area $\sqrt{3}\,t^2/4$ and perimeter $3t$.
- Express the constraint in one variable, differentiate the total area, and verify that its second derivative is positive before declaring a minimum.

Step 1: Express both areas using the triangle side Let t metres be the side of the equilateral triangle. It uses $3t$ metres of wire, leaving $22-3t$ metres for the square. The square therefore has side $(22-3t)/4$, with $0 < t < 22/3$. The combined area is

$$F(t) = \frac{(22-3t)^2}{16} + \frac{\sqrt{3}}{4}t^2.$$

The constraint concerns perimeters, so the remaining wire length must be divided by four before its square is used as an area.

Step 2: Locate and verify the minimum Differentiating the quadratic gives

$$F\prime(t) = -\frac{3(22-3t)}{8} + \frac{\sqrt{3}}{2}t.$$

Set this equal to zero and multiply by 8:

$$-66 + 9t + 4\sqrt{3}\,t = 0, \qquad t = \frac{66}{9+4\sqrt{3}}.$$

Moreover,

$$F\prime\prime(t) = \frac{9}{8} + \frac{\sqrt{3}}{2} > 0.$$

Thus the quadratic is strictly convex and its stationary point is the unique global minimum. The computed side is positive and less than $22/3$, so both pieces have positive length. Options A and C

have an incorrect numerator, and option D has an incorrect denominator; none satisfies the stationary equation $(9+4\sqrt{3})t = 66$.

Step 3: Conclusion The equilateral triangle must have side $\dfrac{66}{9+4\sqrt{3}}$ m. Hence, the correct option is (B).

Quick Tip to Solve: Use side lengths consistently: a wire segment supplies a perimeter, not a side, and the final second-derivative check confirms the minimum.

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12. The domain of the function $\cos^{-1}\left(\dfrac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is : [4 marks]

(A) $R - \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

(B) $(-\infty, -1] \cup [1, \infty) \cup \{0\}$

(C) $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$

(D) $\left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

Correct Answer: (D) $\left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

Solution:

Concept:

- The real inverse sine $\sin^{-1} u$ requires $-1 \le u \le 1$ and has range $[-\pi/2, \pi/2]$.
- The real inverse cosine requires its argument in $[-1, 1]$. Check nested functions from the innermost expression outward, retaining equality at valid endpoints.

Step 1: Apply the inner inverse-sine restriction The denominator must be nonzero, and the argument of the inverse sine must have absolute value at most 1. Thus

$$\left|\frac{1}{4x^2-1}\right| \le 1 \quad \Longleftrightarrow \quad |4x^2-1| \ge 1.$$

This condition already excludes a zero denominator. Resolve the absolute value into its two alternatives:

$$4x^2 - 1 \ge 1 \quad \text{or} \quad 4x^2 - 1 \le -1.$$

The first gives $x^2 \ge 1/2$, hence $x \le -1/\sqrt{2}$ or $x \ge 1/\sqrt{2}$. The second gives $x^2 \le 0$, so it contributes only $x = 0$.

Step 2: Check the outer inverse cosine and endpoints Whenever the inner inverse sine is real, its range implies

$$-1 \le \frac{2}{\pi}\sin^{-1}\left(\frac{1}{4x^2-1}\right) \le 1.$$

Therefore the outer inverse cosine imposes no additional restriction. At $x = 0$, the inner argument is -1; at $x = \pm 1/\sqrt{2}$, it is 1. All three endpoint cases are allowed. Option A admits invalid inner arguments near $1/2$, B omits valid values between $1/\sqrt{2}$ and 1, and C includes values just larger than $1/2$, where the inner argument exceeds 1.

Step 3: Conclusion The domain is $\left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$. Hence, the correct option is (D).

Quick Tip to Solve: The factor $2/\pi$ maps the entire principal range of inverse sine into the admissible interval for inverse cosine.

13. If the constant term in the expansion of $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ is $2^k.l$, where l is an odd integer, then the value of k is equal to: [4 marks]

(A) 6

(B) 7

(C) 8

(D) 9

Correct Answer: (D) 9

Solution:

Concept:

- Choosing the three terms a, b, c times in a tenth power requires $a + b + c = 10$; the corresponding multinomial coefficient is $10!/(a!b!c!)$.
- A constant term has total exponent zero. To write its coefficient as $2^k\ell$ with odd ℓ, count all factors of 2, including those in the multinomial coefficient.

Step 1: Find the only combination giving exponent zero Choose $3x^3$ exactly a times, $-2x^2$ exactly b times, and $5x^{-5}$ exactly c times. The conditions are

$$a + b + c = 10, \qquad 3a + 2b - 5c = 0.$$

Eliminate $b = 10 - a - c$ to obtain $a = 7c - 20$, and hence $b = 30 - 8c$. Nonnegative integer counts require $c \geq 3$ and $c \leq 3$. Thus

$$c = 3, \qquad a = 1, \qquad b = 6.$$

There is only one constant contribution, so no cancellation or addition of different terms needs to be considered.

Step 2: Calculate the coefficient and its power of two The constant coefficient is

$$\frac{10!}{1!\,6!\,3!}\,3\,(-2)^6\,5^3 = 840 \cdot 3 \cdot 64 \cdot 125.$$

Factor the even parts as $840 = 2^3 \cdot 105$ and $64 = 2^6$. The remaining factors 105, 3 and 125 are all odd. Consequently

$$840 \cdot 3 \cdot 64 \cdot 125 = 2^9(105 \cdot 3 \cdot 125) = 2^9 \cdot 39375.$$

Thus the odd factor is indeed odd and no additional factor of two remains. Option A counts only the six factors from $(-2)^6$; options B and C also miss factors contributed by 840.

Step 3: Conclusion The coefficient is $2^9 \cdot 39375$, so $k = 9$. Hence, the correct option is (D).

Quick Tip to Solve: After imposing exponent zero, include the multinomial coefficient when counting factors of two; its contribution can change the answer.

14. $\int_0^5 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right) dx$,

where [t] denotes greatest integer less than or equal to t, is equal to: [4 marks]

(A) -3

(B) -2

(C) 2

(D) 0

Correct Answer: (D) 0

Solution:

Concept:

- The greatest-integer function $\lfloor x/2 \rfloor$ is constant on intervals bounded by consecutive even integers.
- Changing an integrand at isolated endpoints does not change its definite integral. Integrate on each open interval using the appropriate constant floor value.

Step 1: Split the interval at the floor-function jumps On $[0, 2)$, $\lfloor x/2 \rfloor = 0$; on $[2, 4)$, it equals 1; and on $[4, 5]$, it equals 2. Therefore the required integral is

$$I = \int_0^2 \cos(\pi x)\, dx + \int_2^4 \cos(\pi(x-1))\, dx + \int_4^5 \cos(\pi(x-2))\, dx.$$

At the jump points 2 and 4, the one-sided formulas can give different values, but the values at these finitely many points do not affect the integral. The floor is not $x/2$, so it cannot simply be replaced by that expression inside the cosine.

Step 2: Evaluate the three elementary integrals Each antiderivative is a sine divided by π. Hence

$$I_1 = \frac{\sin(2\pi) - \sin 0}{\pi} = 0,$$
$$I_2 = \frac{\sin(3\pi) - \sin \pi}{\pi} = 0,$$
$$I_3 = \frac{\sin(3\pi) - \sin(2\pi)}{\pi} = 0.$$

Adding them gives $I = 0$. The first two intervals span complete cosine cycles with integer shifts, while the final interval has integer-multiple-of-π sine values at both ends, so its signed area also vanishes. In particular a positive or negative integrand on part of an interval does not mean its integral has that sign; positive and negative contributions cancel. Thus the proposed nonzero results $-3, -2, 2$ are all excluded.

Step 3: Conclusion The definite integral equals 0. Hence, the correct option is (D).

Quick Tip to Solve: Split where the floor changes, then evaluate the sine at the interval endpoints; isolated discontinuities do not contribute area.

15. Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point (3, 0). Let the line segment PQ be also a focal chord of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a^2 > b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is equal to : [4 marks]

(A) $1 + \sqrt{2}$

(B) $3 + 2\sqrt{2}$

(C) $1 + 2\sqrt{3}$

(D) $4 + 5\sqrt{3}$

Correct Answer: (B) $3 + 2\sqrt{2}$

Solution:

Concept:

- A focal chord of $y^2 = 4x$ joins the parametric points $(t^2, 2t)$ and $(s^2, 2s)$, where $ts = -1$. Its focus is $(1, 0)$.
- Perpendicular vectors have zero scalar product. For an ellipse with horizontal major axis, $c^2 = a^2 - b^2$ and $e = c/a$, where c is its focal distance.

Step 1: Use the right angle to determine the chord Put $s = -1/t$, with $t \neq 0$, so that the focal-chord endpoints are

$$P = (t^2, 2t), \qquad Q = (t^{-2}, -2/t).$$

The angle is measured at $R = (3, 0)$, so it is $\overrightarrow{RP} \cdot \overrightarrow{RQ}$, not the product of vectors from the focus, that must vanish:

$$(t^2 - 3)(t^{-2} - 3) + (2t)(-2/t) = 0.$$

Expanding gives $6 - 3(t^2 + t^{-2}) = 0$. Hence

$$t^2 + t^{-2} = 2, \qquad (t - t^{-1})^2 = 0.$$

Thus $t = \pm 1$; the endpoints are $(1, 2)$ and $(1, -2)$. In particular, the chord is the vertical segment $x = 1$.

Step 2: Apply the ellipse conditions and select the admissible root The same segment passes through an ellipse focus. Since its foci are $(\pm c, 0)$, the focus on $x = 1$ gives $c = 1$, so $a^2 - b^2 = 1$. Write $A = a^2$, so $b^2 = A - 1 > 0$. Substitution of either endpoint into the ellipse gives

$$\frac{1}{A} + \frac{4}{A - 1} = 1.$$

Multiplying by $A(A - 1)$ yields $A^2 - 6A + 1 = 0$, whence

$$A = 3 \pm 2\sqrt{2}.$$

The smaller root is less than 1 and would make $b^2 < 0$, so only $A = 3 + 2\sqrt{2}$ describes the stated ellipse. Finally $e^2 = c^2/a^2 = 1/A$. The other listed expressions therefore do not satisfy the required value of a^2.

Step 3: Conclusion $1/e^2 = 3 + 2\sqrt{2}$. Hence, the correct option is (B).

Quick Tip to Solve: First determine a focal chord from the parabola; then use its line to identify the ellipse focus before substituting an endpoint.

16. Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively. If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a + x_5)$ is equal to: [4 marks]

(A) 13

(B) 15

(C) 17

(D) 18

Correct Answer: (B) 15

Solution:

Concept:

- For n observations, $\bar{x} = \sum x_i/n$ and $\sigma^2 = \sum x_i^2/n - \bar{x}^2$. The variance here is the population variance, with denominator n.
- Removing one observation subtracts its value from the sum and its square from the sum of squares. Recover these two sums before calculating the new variance.

Step 1: Recover the missing observation and total square sum The sum of all five observations is $5(24/5) = 24$, while the sum of the first four is $4(7/2) = 14$. Therefore

$$x_5 = 24 - 14 = 10.$$

Do not average the two stated means directly: they refer to groups of different sizes. From the five-observation variance,

$$\frac{1}{5}\sum_{i=1}^{5} x_i^2 = \frac{194}{25} + \left(\frac{24}{5}\right)^2 = \frac{194 + 576}{25} = \frac{154}{5}.$$

Consequently $\sum_{i=1}^{5} x_i^2 = 154$. This square sum is independent of the ordering of the first four observations, which need not be found separately.

Step 2: Calculate the first-four variance and the required expression Subtract the square of the observation removed:

$$\sum_{i=1}^{4} x_i^2 = 154 - 10^2 = 54.$$

The first-four mean is already given as $7/2$. Hence their variance is

$$a = \frac{54}{4} - \left(\frac{7}{2}\right)^2 = \frac{54 - 49}{4} = \frac{5}{4}.$$

It is nonnegative, as required for a variance. The requested combination is therefore

$$4a + x_5 = 4\left(\frac{5}{4}\right) + 10 = 5 + 10 = 15.$$

As a consistency check, the first four have total squared deviation $4a = 5$, not variance 5. Confusing these quantities would change the requested result. The values 13, 17 and 18 disagree with the exact

expression obtained from both supplied moments.

Step 3: Conclusion $4a + x_5 = 15$. Hence, the correct option is (B).

Quick Tip to Solve: Convert means and variances into sums and sums of squares; remove an observation only after squaring its value for the second sum.

17. If the mirror image of the point (2, 4, 7) in the plane 3x − y + 4z = 2 is (a, b, c), then 2a + b + 2c is equal to : [4 marks]

(A) 54

(B) 50

(C) -6

(D) -42

Correct Answer: (C) -6

Solution:

Concept:

- A plane $\mathbf{n} \cdot \mathbf{r} = d$ has normal vector $\mathbf{n}$. The displacement from a point to its mirror image is parallel to this normal.
- If P is reflected in the plane, then $P' = P - 2(\mathbf{n} \cdot P - d)\mathbf{n}/\|\mathbf{n}\|^2$. The factor 2 carries the point across the perpendicular foot.

Step 1: Evaluate the signed plane residual For $3x - y + 4z = 2$, take $\mathbf{n} = (3, -1, 4)$, $d = 2$, and $P = (2, 4, 7)$. Then

$$\|\mathbf{n}\|^2 = 3^2 + (-1)^2 + 4^2 = 26,$$

$$\mathbf{n} \cdot P - d = 3(2) - 4 + 4(7) - 2 = 28.$$

The positive residual means that the reflection is obtained by moving opposite to the chosen normal. Write the reflected point as $P' = P - \lambda\mathbf{n}$. Its midpoint with P must lie on the plane, which gives $\lambda = 2(28)/26 = 28/13$. This midpoint condition also explains why using only $14/13$ would give the foot, not the mirror image.

Step 2: Find the image and evaluate the linear combination Substituting the normal displacement gives

$$a = 2 - 3(28/13) = -58/13,$$

$$b = 4 + 28/13 = 80/13,$$

$$c = 7 - 4(28/13) = -21/13.$$

Thus

$$2a + b + 2c = \frac{-116 + 80 - 42}{13} = \frac{-78}{13} = -6.$$

One may check the reflection without recomputing distances: the residual of the image is $3a - b + 4c - 2 = -28$, the negative of the original residual, and the joining segment is parallel to $(3, -1, 4)$. These establish equal perpendicular distances on opposite sides. The other choices 54, 50 and -42 fail this calculated combination.

Step 3: Conclusion $2a + b + 2c = -6$. Hence, the correct option is (C).

Quick Tip to Solve: For a requested linear combination, apply it directly to the reflection formula to avoid unnecessary coordinate arithmetic.

18. Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q) \Delta ((p \vee q) \Rightarrow q)$ is a tautology. Then Δ is equal to : [4 marks]

(A) $\wedge$

(B) $\vee$

(C) $\Rightarrow$

(D) $\Leftrightarrow$

Correct Answer: (C) $\Rightarrow$

Solution:

Concept:

- A tautology is true for every truth assignment of its component statements. An implication $A \Rightarrow B$ is false only when A is true and B is false.
- The implication law gives $U \Rightarrow V \equiv \neg U \vee V$. Conjunction requires both statements, disjunction requires at least one, and equivalence requires equal truth values.

Step 1: Simplify the right-hand statement Let $A = p \wedge q$ and $B = (p \vee q) \Rightarrow q$. Applying implication and De Morgan laws,

$$\begin{aligned} B &\equiv \neg(p \vee q) \vee q \\ &\equiv (\neg p \wedge \neg q) \vee q \\ &\equiv \neg p \vee q. \end{aligned}$$

If A is true, both p and q are true, so B is certainly true. Therefore there can be no truth assignment in which A is true and B is false. This already proves that $A \Rightarrow B$ is a tautology; it remains to exclude the other proposed connectives.

Step 2: Check all assignments and rule out the alternatives For completeness, the truth values are

p	q	A	B	$A \Rightarrow B$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

The conjunction $A \wedge B$ is false at $(p, q) = (T, F)$, since both components are false. The disjunction $A \vee B$ is also false at that assignment. The equivalence $A \Leftrightarrow B$ is false at (F, T), since then $A = F$ while $B = T$. Thus each competing connective has an explicit counterexample. The implication alone remains true in all four rows, including the rows with false antecedent, where an implication is true by definition.

Step 3: Conclusion $\Delta = \Rightarrow$. Hence, the correct option is (C).

Quick Tip to Solve: To test an implication for tautology, look only for a true antecedent with a false consequent; one counterexample rejects another connective.

19. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all n $\geq$ 0. Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to: [4 marks]

(A) $\frac{6}{343}$

(B) $\frac{7}{216}$

(C) $\frac{8}{343}$

(D) $\frac{49}{216}$

Correct Answer: (B) $\frac{7}{216}$

Solution:

Concept:

- A constant second difference produces a quadratic sequence. Defining $d_n = a_{n+1} - a_n$ converts this recurrence into a first-difference relation.
- For $|x| < 1$, the geometric series can be differentiated twice to obtain $\sum_{n=2}^{\infty} n(n-1)x^n = 2x^2/(1-x)^3$.

Step 1: Solve the recurrence using first differences Rearrange the recurrence as

$$a_{n+2} - a_{n+1} = (a_{n+1} - a_n) + 1.$$

Thus $d_{n+1} = d_n + 1$, and the starting values give $d_0 = a_1 - a_0 = 0$. Consequently $d_n = n$. Summing these differences from 0 to $n-1$ gives

$$a_n - a_0 = \sum_{k=0}^{n-1} k = \frac{n(n-1)}{2}.$$

Since $a_0 = 0$, the sequence is $a_n = n(n-1)/2$. In particular $a_2 = 1$, $a_3 = 3$, and $a_4 = 6$, which agree with the original recurrence and confirm the indexing.

Step 2: Sum the convergent series Start from $\sum_{n=0}^{\infty} x^n = (1-x)^{-1}$. Differentiating twice and multiplying by $x^2/2$ yields

$$\sum_{n=2}^{\infty} \frac{n(n-1)}{2} x^n = \frac{x^2}{(1-x)^3}.$$

These operations are valid for $|x| < 1$; the required $x = 1/7$ lies in this interval. Therefore

$$\sum_{n=2}^{\infty} \frac{a_n}{7^n} = \frac{1/49}{(6/7)^3} = \frac{1}{49}\frac{343}{216} = \frac{7}{216}.$$

The terms behave like a quadratic divided by 7^n, so convergence is also clear by the ratio test. The first nonzero term is $1/49$ and the rest are positive; this immediately excludes $6/343 < 1/49$. The exact sum excludes $8/343$ and $49/216$, which do not equal the evaluated generating function.

Step 3: Conclusion The sum is $7/216$. Hence, the correct option is (B).

Quick Tip to Solve: For a recurrence with constant second difference, solve its first differences before summing; then use differentiated geometric series.

20. Let the tangent to the circle $C_1 : x^2 + y^2 = 2$ at the point $M(-1, 1)$ intersect the circle $C_2 : (x - 3)^2 + (y - 2)^2 = 5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to : [4 marks]

(A) $\frac{1}{2}$
(B) $\frac{2}{3}$
(C) $\frac{1}{6}$
(D) $\frac{5}{3}$

Correct Answer: (C) $\frac{1}{6}$

Solution:

Concept:

- The tangent to $x^2 + y^2 = r^2$ at (x_1, y_1) is $xx_1 + yy_1 = r^2$. A circle chord at distance d from the centre has half-length $\sqrt{R^2 - d^2}$.
- If tangents at the endpoints of a chord meet at N, its chord of contact is perpendicular to ON, and $OH \cdot ON = R^2$, where H is the chord midpoint.

Step 1: Find the common chord line and its length The tangent to the first circle at $M = (-1, 1)$ is

$$-x + y = 2, \qquad y = x + 2.$$

For the second circle the centre is $O = (3, 2)$ and radius is $R = \sqrt{5}$. The perpendicular distance from O to this line is

$$d = \frac{|-3 + 2 - 2|}{\sqrt{(-1)^2 + 1^2}} = \frac{3}{\sqrt{2}}.$$

Since $d^2 = 9/2 < 5$, there are indeed two distinct intersection points. If H is their midpoint, then

$$AH = BH = \sqrt{5 - \frac{9}{2}} = \frac{1}{\sqrt{2}}, \qquad AB = \sqrt{2}.$$

This avoids solving separately for the two endpoint coordinates.

Step 2: Locate the tangent intersection and calculate the area Translate the origin to O by writing $u = x - 3$, $v = y - 2$. The circle becomes $u^2 + v^2 = 5$, while the chord line becomes $-u + v = 3$. If N has translated coordinates (u_N, v_N), its chord of contact is $uu_N + vv_N = 5$. Comparing this with the chord line gives

$$u_N = -\frac{5}{3}, \qquad v_N = \frac{5}{3}, \qquad ON = \frac{5\sqrt{2}}{3}.$$

The point N lies beyond H, so the altitude of triangle ANB to base AB is

$$NH = ON - OH = \frac{5\sqrt{2}}{3} - \frac{3}{\sqrt{2}} = \frac{1}{3\sqrt{2}}.$$

Hence

$$[ANB] = \frac{1}{2}(\sqrt{2})\frac{1}{3\sqrt{2}} = \frac{1}{6}.$$

Using ON itself as the altitude would give a wrong area, because the base does not pass through the

centre. The exact altitude and base rule out the other three listed areas.

Step 3: Conclusion The area is $1/6$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Use the distance of the chord from the centre and its polar relation; the tangent triangle altitude is $R^2/d - d$.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $S = \{z \in C : |z-2| \le 1,\ z(1+i) + \bar{z}(1-i) \le 2\}$. Let $|z-4i|$ attains minimum and maximum values, respectively, at $z_1 \in S$ and $z_2 \in S$. If $5(|z_1|^2 + |z_2|^2) = \alpha + \beta\sqrt{5}$, where α and β are integers, then the value of $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- For $z = x + iy$, $|z-2| \le 1$ is the closed unit disc centred at $C = (2, 0)$, and $|z-4i|$ is distance from $A = (0, 4)$.
- A linear inequality in x, y cuts a disc into a circular segment. Distance extrema on its boundary are checked on the circular arc and on its straight chord.

Step 1: Describe the feasible circular segment Expanding the second inequality gives

$$z(1+i) + \bar{z}(1-i) = 2(x-y) \le 2,$$

so the feasible set is the part of $(x-2)^2 + y^2 \le 1$ satisfying $y \ge x - 1$. The cutting line $y = x - 1$ meets the circle where

$$(x-2)^2 + (x-1)^2 = 1,$$

that is, at $B = (1, 0)$ and $D = (2, 1)$. Thus the boundary consists of the upper-left circular arc from B to D and its joining chord. The set is closed and bounded, so its distance minimum and maximum are attained.

Step 2: Find the nearest point on the retained arc The centre-to-target vector is $A - C = (-2, 4)$, of length $2\sqrt{5}$. The nearest point of the full disc to A is

$$z_1 = 2 - \frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}}i.$$

It satisfies $y - x + 1 = 3/\sqrt{5} - 1 > 0$, so it belongs to the retained segment and is also the constrained minimiser. Its squared modulus is

$$|z_1|^2 = \left(2 - \frac{1}{\sqrt{5}}\right)^2 + \frac{4}{5} = 5 - \frac{4}{\sqrt{5}}.$$

Step 3: Find the farthest point and extract the integers On the circle, $x^2 + y^2 = 4x - 3$, so the squared distance from A is $4x - 8y + 13$. On the retained arc its only interior stationary distance point is

the nearest point just found; the full-circle farthest point lies outside the segment. Hence the arc maximum occurs at an endpoint. On the chord write $y = x - 1$, $1 \le x \le 2$; the squared distance $x^2 + (x-5)^2$ decreases on this interval. Finally

$$|B - A|^2 = 17 > 13 = |D - A|^2.$$

Thus $z_2 = 1$ and $|z_2|^2 = 1$. Consequently

$$5(|z_1|^2 + |z_2|^2) = 5\left(6 - \frac{4}{\sqrt{5}}\right) = 30 - 4\sqrt{5}.$$

Therefore $\alpha = 30$ and $\beta = -4$; the sign of the radical coefficient must be retained.

Step 4: Conclusion $\alpha + \beta = 30 - 4 = \boxed{26}$, a dimensionless integer.

Quick Tip to Solve: Find the nearest point of the full disc first, test its half-plane condition, and check both chord endpoints for the farthest point.

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22. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} + \frac{\sqrt{2}y}{2\cos^4 x - \cos^2 x} = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}$, $0 < x < \frac{\pi}{2}$ with $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}$. If $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18}e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to _____. [4 marks]

Solution:

Concept:

- A classical solution of a first-order differential equation must be differentiable on its interval and must satisfy the equation at every point where it is asserted. In particular, its coefficients must be defined at the initial point.
- Before applying an integrating factor, factor the printed denominator. Here $2\cos^4 x - \cos^2 x = \cos^2 x \cos 2x$, so its zero inside the stated interval must be checked against the initial condition.

Step 1: Check the coefficient at the initial point At $x_0 = \pi/4$, the trigonometric values are $\cos^2 x_0 = 1/2$ and $\cos 2x_0 = 0$. Consequently,

$$2\cos^4 x_0 - \cos^2 x_0 = 2(1/4) - 1/2 = 0.$$

The numerator of the coefficient term is nonzero because the imposed initial value gives $\sqrt{2}\,y(x_0) = \sqrt{2}\pi^2/32 \neq 0$. Thus the differential equation as printed is undefined at the very point at which the initial condition is imposed.

Step 2: Test whether a continuous extension can rescue the condition Even on a punctured neighbourhood, continuity with this nonzero initial value is incompatible with the equation. Put $h = x - \pi/4$. Then $\cos^2 x \cos 2x = -h + O(h^2)$, whereas the right-hand side $x\exp(\tan^{-1}(\sqrt{2}\cot 2x))$ remains finite. The equation would therefore require

$$y'(x) = \frac{\sqrt{2}\,y(x_0)}{h} + o(1/|h|).$$

This nonzero simple-pole behaviour integrates to a logarithmic divergence, contradicting a finite

continuous value $y(x_0)$. Merely deleting the initial point from the differential equation does not produce the stated initial-value solution.

Step 3: Conclusion The literal printed initial-value problem has no classical solution, so α, and hence $3\alpha^2$, is not defined. The supplied answer key lists 2, but that value cannot follow from the printed equation with this nonzero initial condition.

Quick Tip to Solve: Factor the coefficient denominator and test the initial point before choosing an integrating factor; a nonzero prescribed value at a pole can invalidate the problem.

23. $50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right)+2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}(2\sqrt{2})\right)$ **is equal to _____.** [4 marks]

Solution:

Concept:

- The tangent addition identity is $\tan(u+v)=(\tan u+\tan v)/(1-\tan u\tan v)$, provided the denominator is nonzero. Integer multiples can be reduced with the double- and triple-angle identities.
- Principal inverse angles determine signs: $\cos^{-1}(1/\sqrt{5})$ and $\tan^{-1}(2\sqrt{2})$ are acute. For an acute angle C, $\tan(C/2)=\sin C/(1+\cos C)$, with the positive square-root values.

Step 1: Reduce the first tangent Let $A=\tan^{-1}(1/2)$ and $B=\cos^{-1}(1/\sqrt{5})$. Both are acute, and $\tan A=1/2$, $\sin B=2/\sqrt{5}$, so $\tan B=2$. Therefore

$$\tan 3A=\frac{3(1/2)-(1/2)^3}{1-3(1/2)^2}=\frac{11}{2},\qquad \tan 2B=\frac{2(2)}{1-2^2}=-\frac{4}{3}.$$

The addition formula is applicable since its denominator is not zero. It gives

$$\tan(3A+2B)=\frac{11/2-4/3}{1+(11/2)(4/3)}=\frac{25/6}{25/3}=\frac{1}{2}.$$

There is no need to determine the quadrant of the combined angle separately because the tangent identity already gives its signed value.

Step 2: Reduce the half-angle tangent and combine Put $C=\tan^{-1}(2\sqrt{2})$, so $0<C<\pi/2$. A right triangle with adjacent side 1 and opposite side $2\sqrt{2}$ has hypotenuse 3. Thus $\cos C=1/3$ and $\sin C=2\sqrt{2}/3$. It follows that

$$\tan\frac{C}{2}=\frac{2\sqrt{2}/3}{1+1/3}=\frac{\sqrt{2}}{2}.$$

Substitution into the original expression, retaining its factors 50 and $4\sqrt{2}$, now yields

$$50\left(\frac{1}{2}\right)+4\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)=25+4=29.$$

All inverse-angle branches used here are the principal branches specified by the standard inverse functions.

Step 3: Conclusion The required numerical value is $\boxed{29}$, a dimensionless number.

Quick Tip to Solve: Find tangents of the inverse angles first, and use the principal range only to choose the positive half-angle value.

24. Let c, k $\in$ R. If $f(x) = (c+1)x^2 + (1-c^2)x + 2k$ and $f(x+y) = f(x) + f(y) - xy$, for all x, y $\in$ R, then the value of $|2(f(1) + f(2) + f(3) + \ldots\ldots + f(20))|$ is equal to _____. [4 marks]

Solution:

Concept:

- An identity valid for every real x, y permits coefficient comparison after expanding both sides. For a quadratic $Ax^2 + Bx + C$, its additive defect is $f(x+y) - f(x) - f(y) = 2Axy - C$.
- The finite sums are $\sum_{r=1}^{n} r = n(n+1)/2$ and $\sum_{r=1}^{n} r^2 = n(n+1)(2n+1)/6$. Apply the absolute value after evaluating the whole signed expression.

Step 1: Find the polynomial coefficients Here $A = c+1$, $B = 1-c^2$, and $C = 2k$. Expanding the identity and cancelling the pure x, y, x^2, and y^2 terms gives

$$2(c+1)xy - 2k = -xy \qquad \text{for every real } x, y.$$

Taking $x = y = 0$ first gives $k = 0$. Equating the coefficients of xy next gives

$$2(c+1) = -1 \quad \Longrightarrow \quad c = -\frac{3}{2}.$$

The linear coefficient is consequently $1 - c^2 = 1 - 9/4 = -5/4$, so the uniquely determined function is

$$f(x) = -\frac{1}{2}x^2 - \frac{5}{4}x.$$

Its cross term in $f(x+y)$ is indeed $-xy$, which checks the functional equation directly.

Step 2: Sum the values and take the absolute value For the first 20 positive integers,

$$\sum_{r=1}^{20} r = \frac{20 \cdot 21}{2} = 210, \qquad \sum_{r=1}^{20} r^2 = \frac{20 \cdot 21 \cdot 41}{6} = 2870.$$

Linearity of summation therefore gives

$$\sum_{r=1}^{20} f(r) = -\frac{1}{2}(2870) - \frac{5}{4}(210) = -\frac{6790}{4}.$$

The requested quantity includes an outside factor of 2 and an absolute value, so

$$\left|2\sum_{r=1}^{20} f(r)\right| = \left|-2870 - \frac{5}{2}(210)\right| = |-3395| = 3395.$$

Keeping that outer factor until the last step avoids confusing the sum itself with the quantity asked for.

Step 3: Conclusion The required value is $\boxed{3395}$, dimensionless.

Quick Tip to Solve: For a quadratic functional equation, compute the additive defect first; its mixed term finds the quadratic coefficient immediately.

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25. Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, a > 0, b > 0, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4(2\sqrt{2} + \sqrt{14})$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then the value of $a^2 + b^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, the full transverse and conjugate axis lengths are $2a$ and $2b$. The parameters a, b are positive semiaxis lengths.
- If the focal distance from the centre is c, then $c^2 = a^2 + b^2$ and the eccentricity is $e = c/a$. Consequently $e^2 = 1 + b^2/a^2$; this fixes the ratio of the semiaxes.

Step 1: Use the eccentricity to obtain the axis ratio The given eccentricity is $e = \sqrt{11}/2$. Squaring and substituting into the hyperbola relation gives

$$\frac{11}{4} = 1 + \frac{b^2}{a^2} \quad \Longrightarrow \quad \frac{b^2}{a^2} = \frac{7}{4}.$$

As both parameters are strictly positive, the positive square root must be taken:

$$\frac{b}{a} = \frac{\sqrt{7}}{2}, \qquad b = \frac{\sqrt{7}}{2}a.$$

The negative ratio is inadmissible, even though the preceding equation only involves squares.

Step 2: Apply the sum of the full axis lengths The printed sum refers to full axes, not semiaxes. Hence

$$2a + 2b = 4(2\sqrt{2} + \sqrt{14}), \qquad a + b = 2(2\sqrt{2} + \sqrt{14}).$$

Notice that $2\sqrt{2} + \sqrt{14} = \sqrt{2}(2 + \sqrt{7})$. Substituting the ratio just obtained gives

$$a\left(1 + \frac{\sqrt{7}}{2}\right) = 2\sqrt{2}(2 + \sqrt{7}).$$

The common positive factor $2 + \sqrt{7}$ cancels, leaving $a = 4\sqrt{2}$. Thus $a^2 = 32$, $b^2 = (7/4)32 = 56$, and $b = 2\sqrt{14}$. These semiaxes also reproduce the given axis sum, providing a direct check of the factor of 2.

Step 3: Conclusion Therefore $a^2 + b^2 = 32 + 56 = \boxed{88}$, in square coordinate units.

Quick Tip to Solve: Convert the stated full-axis sum into a semiaxis sum before substituting the eccentricity ratio.

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26. Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i \in \{1, 2, 3, ..., 100\}$ for $1 \le i \le 4$ and $b_i \ne b_j$ for $i \ne j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1b_2b_3b_4$ is equal to _____. [4 marks]

Solution:

Concept:

- For two conditions A, B, inclusion-exclusion gives $|A \cup B| = |A| + |B| - |A \cap B|$. This counts permutations satisfying either condition, including those satisfying both only once.
- The consecutive integers in the stated order form an increasing run $r, r+1, r+2$. All four entries must be distinct, so a free fourth entry cannot be any of the three entries in that run.

Step 1: Count each placement of a consecutive triple Let A consist of the permutations whose first three entries are consecutive, so $(b_1, b_2, b_3) = (r, r+1, r+2)$. The starting integer r ranges from 1 to 98 so that the last member does not exceed 100. Once these three positions are fixed, b_4 can be any of the other $100 - 3 = 97$ integers. Thus

$$|A| = 98 \cdot 97 = 9506.$$

Similarly, for B, put $(b_2, b_3, b_4) = (s, s+1, s+2)$, where $1 \le s \le 98$; the first entry has 97 choices. Therefore $|B| = 9506$ as well. The entries are ordered positions, so there is no further division by a factorial.

Step 2: Find the overlap and subtract it once If both conditions hold, the middle entries force the starts to agree in the appropriate shifted manner. From the first condition $b_2 = b_1 + 1$ and $b_3 = b_1 + 2$; the second then gives $b_4 = b_3 + 1 = b_1 + 3$. Hence the common permutations are precisely

$$(r, r+1, r+2, r+3), \qquad 1 \le r \le 97.$$

There are 97 such permutations. They are counted twice by $|A| + |B|$, so exactly one copy must be removed. Inclusion-exclusion now gives

$$|A \cup B| = 2(98 \cdot 97) - 97 = 19012 - 97 = 18915.$$

The overlap is not 98, because the run of four cannot start at 98 without using the excluded integer 101.

Step 3: Conclusion The number of permutations is $\boxed{18915}$, agreeing with the supplied key for consecutive entries in the given order.

Quick Tip to Solve: For overlapping consecutive-position conditions, write out both runs explicitly; their intersection usually forces one longer run.

27. Let d be the distance between the foot of perpendiculars of the points P(1, 2, −1) and Q(2, −1, 3) on the plane −x + y + z = 1. Then d^2 is equal to _____. [4 marks]

Solution:

Concept:

- A plane $\mathbf{n} \cdot \mathbf{r} = c$ has normal vector $\mathbf{n}$. The foot of a point $\mathbf{r}$ is $\mathbf{r} - [(\mathbf{n} \cdot \mathbf{r} - c)/\|\mathbf{n}\|^2]\mathbf{n}$.
- For two projected points, the squared distance equals $\|\mathbf{v}\|^2 - (\mathbf{n} \cdot \mathbf{v})^2/\|\mathbf{n}\|^2$, where $\mathbf{v}$ is the original displacement. Projection removes only the normal component.

Step 1: Compute the normal components of the two points The plane is $-x + y + z = 1$, so choose $\mathbf{n} = (-1, 1, 1)$, with $\|\mathbf{n}\|^2 = 3$. For $P = (1, 2, -1)$,

$$\mathbf{n} \cdot P = -1 + 2 - 1 = 0.$$

For $Q = (2, -1, 3)$,

$$\mathbf{n} \cdot Q = -2 - 1 + 3 = 0.$$

Thus both original points have the same signed plane residual, -1. Their feet are $P\prime = P + \mathbf{n}/3$ and $Q\prime = Q + \mathbf{n}/3$. The identical correction vector means their mutual displacement is unchanged by the projection.

Step 2: Evaluate the squared distance and check tangency Subtracting the foot vectors gives

$$Q\prime - P\prime = Q - P = (1, -3, 4).$$

Consequently,

$$d^2 = 1^2 + (-3)^2 + 4^2 = 1 + 9 + 16 = 26.$$

As a separate check, the displacement is parallel to the plane since

$$(-1, 1, 1) \cdot (1, -3, 4) = -1 - 3 + 4 = 0.$$

Therefore its normal component is zero, and the general projection-distance formula also reduces to $d^2 = \|Q - P\|^2 = 26$. The constant 1 in the plane equation influences each foot separately but cancels when their displacement is formed, which is why computing the full foot coordinates is unnecessary here.

Step 3: Conclusion The requested squared distance is $\boxed{26}$ square coordinate units.

Quick Tip to Solve: First compare the two signed plane residuals; if they agree, the projection shifts both points by the same vector and preserves their distance.

28. The number of elements in the set $S = \{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$ is _____. [4 marks]

Solution:

Concept:

- The double-angle identity $\cos 2\theta = 2\cos^2\theta - 1$ gives $\cos^2\theta = (1+\cos 2\theta)/2$. Substituting it converts this equation to a polynomial in one trigonometric quantity.
- For $-1 < a < 1$, $\cos u = a$ has two distinct roots in each full period of length 2π. When counting on a closed interval, check its endpoints to avoid including a shared boundary twice.

Step 1: Reduce and factor the equation Write $t = \cos 2\theta$. Then the printed equation becomes

$$3t^2 + 6t - 10\left(\frac{1+t}{2}\right) + 5 = 0.$$

Simplifying the constant and linear terms gives

$$3t^2 + t = t(3t+1) = 0.$$

It follows that either $\cos 2\theta = 0$ or $\cos 2\theta = -1/3$. Both right-hand sides lie strictly between -1 and 1, so both cases produce real angles. They cannot overlap, because the same cosine cannot simultaneously be 0 and $-1/3$.

Step 2: Count both families within the closed interval Put $u = 2\theta$. The allowed interval $-4\pi \le \theta \le 4\pi$ becomes $-8\pi \le u \le 8\pi$, whose length is 16π, or eight complete cosine periods. Each equation $\cos u = 0$ and $\cos u = -1/3$ has two solutions per period. Neither endpoint is a solution because $\cos(\pm 8\pi) = 1$. Thus each family contributes $8 \cdot 2 = 16$ distinct angles. The map $u = 2\theta$ is one-to-one on the interval, so this count is unchanged on returning to θ. Adding the disjoint families gives $16 + 16 = 32$. Equivalently, the original equations have period π in θ, and the interval contains eight such periods.

Step 3: Conclusion The set has $\boxed{32}$ elements.

Quick Tip to Solve: Reduce everything to $\cos 2\theta$, then count full periods in the doubled-angle variable and check the two endpoints explicitly.

29. The number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$ in R is equal to _____. [4 marks]

Solution:

Concept:

- An equation has exactly one real solution if its left-hand side is continuous, strictly monotonic, and changes sign.
- The derivative of $\cos^2\theta$ is $-\sin 2\theta$. The bound $-1 \le \sin 2\theta \le 1$ holds for every real angle.

Step 1: Define the function and differentiate Put $f(\theta) = 2\theta - \cos^2\theta + \sqrt{2}$. Both the linear and

trigonometric terms are continuous and differentiable throughout the real line. Differentiation gives

$$f\prime(\theta) = 2 + \sin 2\theta.$$

Since the sine function never falls below -1, we have $f\prime(\theta) \geq 1 > 0$ for every real θ. Thus the function is strictly increasing on its entire domain, so it can cross the horizontal axis at most once.

Step 2: Establish the existence of a zero Check two simple arguments on opposite sides of the origin. At zero,

$$f(0) = \sqrt{2} - 1 > 0.$$

At $\theta = -1$,

$$f(-1) = -2 - \cos^2 1 + \sqrt{2} \leq -2 + \sqrt{2} < 0.$$

The intermediate value theorem therefore gives at least one zero in $(-1, 0)$. Strict increase proves that this zero is the only one. No periodic repetition creates further zeros: the nonperiodic linear term dominates the bounded cosine term.

Step 3: Conclusion The equation has exactly $\boxed{1}$ real solution. The requested count is dimensionless.

Quick Tip to Solve: For a linear term plus a bounded trigonometric term, bound the derivative first; then use one sign change to prove existence.

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30. Let $P_1 : \vec{r} \cdot \left(2\hat{i} + \hat{j} - 3\hat{k}\right) = 4$ be a plane. Let P_2 be another plane which passes through the points (2, −3, 2), (2, −2, −3) and (1, −4, 2). If the direction ratios of the line of intersection of P_1 and P_2 be 16, α, β, then the value of $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- A normal to a plane through three noncollinear points is the cross product of two independent vectors lying in that plane.
- The direction of the intersection of two planes is perpendicular to both plane normals and is therefore proportional to their cross product.

Step 1: Find a normal to the second plane Let $A = (2, -3, 2)$, $B = (2, -2, -3)$, and $C = (1, -4, 2)$. The two vectors in P_2 are

$$\overrightarrow{AB} = (0, 1, -5), \qquad \overrightarrow{AC} = (-1, -1, 0).$$

Their cross product is

$$\boldsymbol{n}_2 = \overrightarrow{AB} \times \overrightarrow{AC} = (-5, 5, 1).$$

It is nonzero, so the three given points determine a plane. The first plane has normal $\boldsymbol{n}_1 = (2, 1, -3)$, read directly from its vector equation. The constant 4 locates that plane but does not affect its normal direction.

Step 2: Cross the normals and normalize the ratios The required line direction is

$$\begin{aligned}\boldsymbol{d} &= \boldsymbol{n}_1 \times \boldsymbol{n}_2 \\ &= (1\cdot 1-(-3)5,\ (-3)(-5)-2\cdot 1,\ 2\cdot 5-1(-5)) \\ &= (16, 13, 15).\end{aligned}$$

Its first component already equals the specified first direction ratio 16. Therefore no further scale factor is required: $\alpha = 13$, $\beta = 15$. As a check, $2(16)+13-3(15) = 0$ and $-5(16)+5(13)+15 = 0$, so this direction is perpendicular to both normals.

Step 3: Conclusion $\alpha+\beta = 13+15 = \boxed{28}$. Direction ratios and their requested sum are dimensionless.

Quick Tip to Solve: Cross the two normals, then scale the entire resulting vector to the specified component before reading the other direction ratios.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Two balls A and B are placed at the top of 180 m tall tower. Ball A is released from the top at t = 0 s. Ball B is thrown vertically down with an initial velocity 'u' at t = 2 s. After a certain time, both balls meet 100 m above the ground. Find the value of 'u' in ms^{-1}. [use g = 10 ms^{-2}] : [4 marks]

(A) 10

(B) 15

(C) 20

(D) 30

Correct Answer: (D) 30

Solution:

Concept:

- For motion with constant acceleration, displacement is $s = ut + \frac{1}{2}gt^2$, with downward chosen positive.
- Each ball has its own elapsed flight time. A delay in release must be subtracted from the common meeting time.

Step 1: Find the meeting time using ball A The meeting point is 100 m above ground, whereas the top is 180 m above ground. Each ball therefore descends $180-100 = 80\,\text{m}$. Ball A starts from rest at $t = 0$. Hence

$$80 = \frac{1}{2}(10)t^2 = 5t^2.$$

The physically relevant positive root is $t = 4\,\text{s}$; the negative algebraic root does not describe a time after release. This is the time measured from the release of A, not from the launch of B.

Step 2: Use the shorter flight time of ball B Ball B is launched at $t = 2\,\text{s}$, so it travels for $4-2 = 2\,\text{s}$

before meeting A. Substituting this duration gives

$$80 = 2u + \frac{1}{2}(10)(2)^2 = 2u + 20, \qquad u = 30\,\mathrm{m\,s^{-1}}.$$

The other choices do not give the required fall in 2 s: initial speeds 10, 15 and 20 produce downward displacements 40, 50 and 60 m respectively. Thus they would leave B above the stated meeting point. A useful consistency check is that the computed launch is fast enough to make up A's earlier head start.

Step 3: Conclusion The required initial downward speed is $\boxed{30\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the meeting time from the ball released from rest, then subtract the launch delay before writing the equation for the second ball.

32. A body of mass M at rest explodes into three pieces, in the ratio of masses 1 : 1 : 2. Two smaller pieces fly off perpendicular to each other with velocities of 30 ms^{-1} and 40 ms^{-1} respectively. The velocity of the third piece will be : [4 marks]

(A) 15 ms^{-1}

(B) 25 ms^{-1}

(C) 35 ms^{-1}

(D) 50 ms^{-1}

Correct Answer: (B) 25 ms^{-1}

Solution:

Concept:

- During a short explosion with negligible external impulse, total vector momentum is conserved; the initial momentum here is zero.
- Perpendicular momentum components combine by the Pythagorean theorem. Speeds themselves cannot be added as scalars when their directions differ.

Step 1: Assign masses and resolve momentum Write the piece masses as $m, m, 2m$, where $m = M/4$. Choose the x direction along the first smaller piece and the y direction along the second. Their momenta are

$$\boldsymbol{p}_1 = 30m\,\hat{\boldsymbol{i}}, \qquad \boldsymbol{p}_2 = 40m\,\hat{\boldsymbol{j}}.$$

Conservation of momentum requires the third piece to have

$$\boldsymbol{p}_3 = -(\boldsymbol{p}_1 + \boldsymbol{p}_2) = -30m\,\hat{\boldsymbol{i}} - 40m\,\hat{\boldsymbol{j}}.$$

The minus signs specify that its components oppose those of the two smaller fragments.

Step 2: Divide the resultant momentum by the larger mass The magnitude of the required momentum is

$$|\boldsymbol{p}_3| = m\sqrt{30^2 + 40^2} = 50m.$$

The third piece has mass $2m$, not m, so

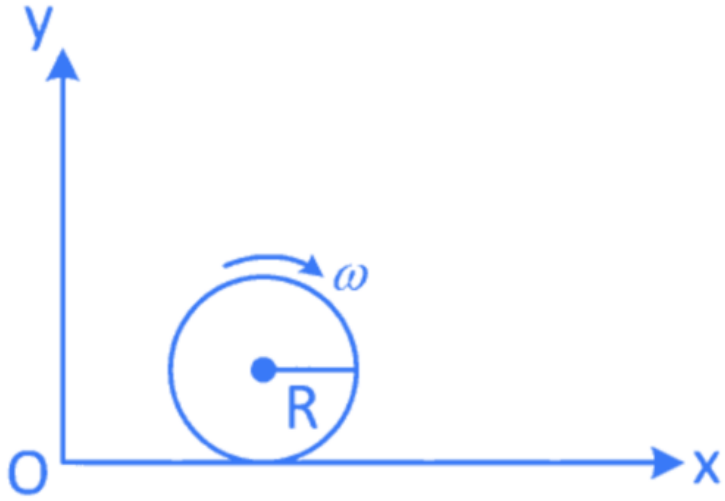
$$v_3 = \frac{50m}{2m} = 25\,\mathrm{m\,s^{-1}}.$$

Equivalently, its velocity components are -15 and $-20\,\mathrm{m\,s^{-1}}$, whose resultant magnitude is 25. Option 50 incorrectly uses a small-fragment mass for the third piece. The values 15 and 35 do not give the momentum magnitude needed to cancel the two perpendicular momenta; 15 is only one velocity component.

Step 3: Conclusion The third piece has speed $\boxed{25\,\mathrm{m\,s^{-1}}}$, directed opposite to the resultant momentum of the two smaller pieces. Hence, the correct option is (B).

Quick Tip to Solve: Use momentum vectors first. After taking their resultant, divide by the mass of the actual fragment whose speed is required.

33. A spherical shell of 1 kg mass and radius R is rolling with angular speed ω on horizontal plane (as shown in figure). The magnitude of angular momentum of the shell about the origin O is $\frac{a}{3}$ $R^2\omega$. The value of a will be : [4 marks]



(A) 2

(B) 3

(C) 5

(D) 4

Correct Answer: (C) 5

Solution:

Concept:

- Angular momentum about a fixed origin is the sum of centre-of-mass orbital angular momentum and spin angular momentum about the centre.

- A thin spherical shell has $I_{\text{CM}} = \frac{2}{3}mR^2$. Pure rolling gives $v_{\text{CM}} = R\omega$.

Step 1: Evaluate the orbital contribution about O The centre is at height R above the horizontal plane. For clockwise rolling it moves right, so write $\boldsymbol{r}_{\text{CM}} = x\hat{\boldsymbol{i}} + R\hat{\boldsymbol{j}}$ and $\boldsymbol{p} = mR\omega\hat{\boldsymbol{i}}$. The cross product is

$$\boldsymbol{L}_{\text{orb}} = \boldsymbol{r}_{\text{CM}} \times \boldsymbol{p} = -mR^2\omega\hat{\boldsymbol{k}}.$$

The horizontal coordinate x does not contribute because that component of the position is parallel to the momentum. The perpendicular distance from O to the centre's line of motion is R.

Step 2: Add the spin contribution with its direction The clockwise angular velocity is $-\omega\hat{\boldsymbol{k}}$. Therefore

$$\boldsymbol{L}_{\text{spin}} = -\frac{2}{3}mR^2\omega\hat{\boldsymbol{k}}.$$

Both contributions point into the plane, so they add rather than subtract:

$$|\boldsymbol{L}_O| = \left(1 + \frac{2}{3}\right) mR^2\omega = \frac{5}{3}R^2\omega$$

because $m = 1\,\text{kg}$. Comparing with $(a/3)R^2\omega$ gives $a = 5$. The value 2 counts spin alone; 3 counts orbital angular momentum alone; 4 is inconsistent with the shell's moment of inertia. The total angular momentum has SI units $\text{kg}\,\text{m}^2\,\text{s}^{-1}$.

Step 3: Conclusion The dimensionless coefficient is $\boxed{a = 5}$. Hence, the correct option is (C).

Quick Tip to Solve: For rolling about a ground-level origin, check the directions of orbital and spin angular momentum before adding their magnitudes.

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34. A cylinder of fixed capacity of 44.8 litres contains helium gas at standard temperature and pressure. The amount of heat needed to raise the temperature of gas in the cylinder by 20.0°C will be :

(Given gas constant R = 8.3 JK^{-1}-mol^{-1}) [4 marks]

(A) 249 J

(B) 415 J

(C) 498 J

(D) 830 J

Correct Answer: (C) 498 J

Solution:

Concept:

- At standard temperature and pressure, one mole of an ideal gas occupies 22.4 litres.
- Helium is monatomic, with molar constant-volume heat capacity $C_V = \frac{3}{2}R$. At fixed volume, heat increases internal energy: $Q = nC_V\Delta T$.

Step 1: Find the amount of gas and identify the constraint The cylinder contains 44.8 litres at STP, so

$$n = \frac{44.8}{22.4} = 2\,\text{mol}.$$

The cylinder has fixed capacity, which means its volume remains constant during heating. Thus the gas performs no expansion work: $W = \int P\,dV = 0$. We must use C_V, not C_P. A temperature rise of 20.0°C is a rise of 20.0 K; no 273 is added to a temperature difference.

Step 2: Calculate the heat using the given gas constant Helium has three translational degrees of freedom and no active molecular rotational modes. Hence

$$C_V = \frac{3}{2}(8.3) = 12.45\,\text{J mol}^{-1}\,\text{K}^{-1}.$$

Substitution gives

$$Q = (2)(12.45)(20.0) = 498\,\text{J}.$$

The energy is positive because the gas is heated. Option 249 would correspond to only one mole at constant volume. Option 830 uses $C_P = 5R/2$, appropriate to a constant-pressure process rather than this rigid cylinder. Option 415 combines the one-mole amount with constant-pressure heat capacity; it also violates the stated volume and amount of gas.

Step 3: Conclusion The required heat is $\boxed{498\,\text{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: A rigid gas container means use constant-volume heat capacity; convert the STP volume to moles before substituting the temperature rise.

35. A wire of length L is hanging from a fixed support. The length changes to L_1 and L_2 when masses 1 kg and 2 kg are suspended respectively from its free end. Then the value of L is equal to : [4 marks]

(A) $\sqrt{L_1 L_2}$

(B) $\frac{L_1+L_2}{2}$

(C) $2L_1 - L_2$

(D) $3L_1 - 2L_2$

Correct Answer: (C) $2L_1 - L_2$

Solution:

Concept:

- Within the elastic range, a wire of unchanged material and cross-section has extension proportional to applied force.
- The measured stretched length is the original length plus extension. Doubling the suspended mass doubles the added extension, not the entire length.

Step 1: Write the extensions for the two loads Let x denote the additional extension caused by the 1 kg suspended mass. By Hooke's law, $\Delta L = FL/(AY)$, where A is cross-sectional area and Y is Young's modulus. Since the same wire is used, the factor multiplying the applied force is unchanged. The 2 kg

mass exerts twice the force, so its added extension is $2x$. Thus the two observations are

$$L_1 = L + x, \qquad L_2 = L + 2x.$$

If the wire's own weight is included in its unloaded hanging length, its fixed contribution is common to both measurements and does not alter this elimination.

Step 2: Eliminate the unknown extension Double the first equation and subtract the second:

$$\begin{aligned} 2L_1 - L_2 &= 2(L + x) - (L + 2x) \\ &= L. \end{aligned}$$

Therefore the original length is obtained directly without needing the area, Young's modulus or g. Check the other expressions by substituting the same two lengths: the arithmetic mean is $L + 3x/2$, while $3L_1 - 2L_2 = L - x$. The geometric mean $\sqrt{(L + x)(L + 2x)}$ also exceeds L for a positive extension. None reproduces the original length for a general nonzero load. Finally, the result behaves correctly at vanishing load, when $L_1 = L_2 = L$.

Step 3: Conclusion The unloaded length is $\boxed{L = 2L_1 - L_2}$, in the same length unit as L_1 and L_2. Hence, the correct option is (C).

Quick Tip to Solve: Represent the added extension for one unit of load by x, write both measured lengths, and eliminate x using a linear combination.

36. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : The photoelectric effect does not takes place, if the energy of the incident radiation is less than the work function of a metal.

Reason R : Kinetic energy of the photoelectrons is zero, if the energy of the incident radiation is equal to the work function of a metal.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is not the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Correct Answer: (B) Both A and R are correct but R is not the correct explanation of A.

Solution:

Concept:

- Photoelectric emission requires a photon energy at least equal to the work function ϕ, the minimum energy needed to liberate an electron from the metal surface.
- Einstein's energy balance is $K_{\max} = h\nu - \phi$, provided emission is energetically possible. The

threshold frequency is $\nu_0 = \phi/h$.

Step 1: Evaluate Assertion A An electron absorbs energy from a photon. If $h\nu < \phi$, that energy cannot overcome the binding energy at the surface. Therefore no photoelectron can escape, regardless of the intensity of monochromatic light. Assertion A correctly states this necessary threshold condition. A formal negative value from Einstein's expression represents forbidden emission, not a physically negative kinetic energy.

Step 2: Evaluate Reason R At the threshold, $h\nu = \phi$. Substitution into the energy balance gives

$$K_{\max} = h\nu - \phi = 0.$$

Thus an electron just escapes with no excess kinetic energy. Reason R correctly describes the limiting case at the threshold. It is a statement about the equality case, rather than a statement of the energy requirement for escape below the threshold.

Step 3: Test the explanatory link Both statements are true, so options (C) and (D), each declaring one of them false, are ruled out. However, R only states the kinetic energy when the incident energy equals the work function. By itself it does not explain why insufficient photon energy prevents liberation. That explanation is the minimum escape-energy requirement stated in the concept. Hence option (A), which asserts a direct explanatory relationship, is not the appropriate choice.

Step 4: Conclusion Both A and R are correct, but R is not the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: In assertion-reason items, distinguish a true limiting-case result from the physical principle that explains the assertion.

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37. A particle of mass 500 gm is moving in a straight line with velocity v = b $x^{5/2}$. The work done by the net force during its displacement from x = 0 to x = 4 m is : (Take b = 0.25 $m^{-3/2}$ s^{-1}). [4 marks]

(A) 2 J

(B) 4 J

(C) 8 J

(D) 16 J

Correct Answer: (D) 16 J

Solution:

Concept:

- The work done by the net force equals the change in kinetic energy: $W_{\text{net}} = \frac{1}{2}m(v_f^2 - v_i^2)$. This holds even when acceleration varies with position.
- The given velocity law $v = bx^{5/2}$ determines the endpoint speeds directly. Use SI units before evaluating kinetic energy.

Step 1: Find the endpoint speeds The mass is $500\,\text{g} = 0.500\,\text{kg}$. At the starting point $x_i = 0$, the law

gives $v_i = 0$. At $x_f = 4\,\text{m}$,

$$v_f = 0.25(4)^{5/2} = \frac{1}{4}\left(4^2\sqrt{4}\right) = \frac{1}{4}(32) = 8\,\text{m s}^{-1}.$$

The exponent acts on the displacement before multiplication by the coefficient. Thus $4^{5/2}$ is 32, not 8 or 16.

Step 2: Calculate the net work Only the endpoint kinetic energies are needed; no assumption of constant force is required. Applying the work-energy theorem,

$$W_{\text{net}} = \frac{1}{2}(0.500)(8^2 - 0^2) = 0.250(64) = 16\,\text{J}.$$

The positive result agrees with the increase in speed. The particle starts from rest and gains kinetic energy, so the net work cannot be negative.

Step 3: Check the alternatives As a separate consistency check, $v^2 = b^2x^5$, so the work from the origin is $\frac{1}{2}mb^2x^5$. Substituting $b = 0.25$ and $x = 4$ again gives $\frac{1}{2}(0.5)(0.0625)(1024) = 16\,\text{J}$. The values 2 J, 4 J and 8 J in options (A), (B) and (C) do not equal this kinetic-energy change; they result from omitting a factor in the power or energy calculation.

Step 4: Conclusion The net force does 16 J of work. Hence, the correct option is (D).

Quick Tip to Solve: When velocity is given as a function of position, find the two speeds and use the work-energy theorem instead of deriving the force first.

38. A charge particle moves along circular path in a uniform magnetic field in a cyclotron. The kinetic energy of the charge particle increases to 4 times its initial value. What will be the ratio of new radius to the original radius of circular path of the charge particle : [4 marks]

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 1 : 4

Correct Answer: (C) 2 : 1

Solution:

Concept:

- For motion perpendicular to a uniform magnetic field, magnetic force supplies centripetal force: $|q|vB = mv^2/r$, hence $r = mv/(|q|B)$.
- In the nonrelativistic cyclotron model, $K = \frac{1}{2}mv^2$. At fixed mass, charge and field, radius is proportional to momentum and therefore to $\sqrt{K}$.

Step 1: Express radius using kinetic energy Eliminating speed avoids confusing a change in energy

with the same change in velocity. From $v = \sqrt{2K/m}$,

$$r = \frac{m}{|q|B}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2mK}}{|q|B}.$$

The particle and the uniform field are unchanged, so the factors m, $|q|$ and B are identical for the two circular orbits.

Step 2: Take the radius ratio The new kinetic energy is $K_f = 4K_i$. Therefore

$$\frac{r_f}{r_i} = \sqrt{\frac{K_f}{K_i}} = \sqrt{4} = 2.$$

Equivalently, quadrupling kinetic energy doubles speed; the magnetic force then doubles, but the required centripetal acceleration changes consistently with an orbit of twice the radius. The ratio requested is new radius to original radius, in that order.

Step 3: Reject the other ratios The ratio $1 : 1$ in (A) would mean unchanged speed and unchanged kinetic energy. The ratios $1 : 2$ and $1 : 4$ in (B) and (D) imply a smaller new radius, whereas increasing kinetic energy increases momentum and hence radius. Neither decreasing ratio can describe this orbit. A fourfold radius increase would require sixteen times the kinetic energy, not four times.

Step 4: Conclusion The ratio of the new radius to the original radius is $2 : 1$. Hence, the correct option is (C).

Quick Tip to Solve: For a fixed magnetic field, remember the chain $r \propto v \propto \sqrt{K}$; an energy multiplier must be square-rooted for the radius.

39. For a series LCR circuit, I vs ω curve is shown :

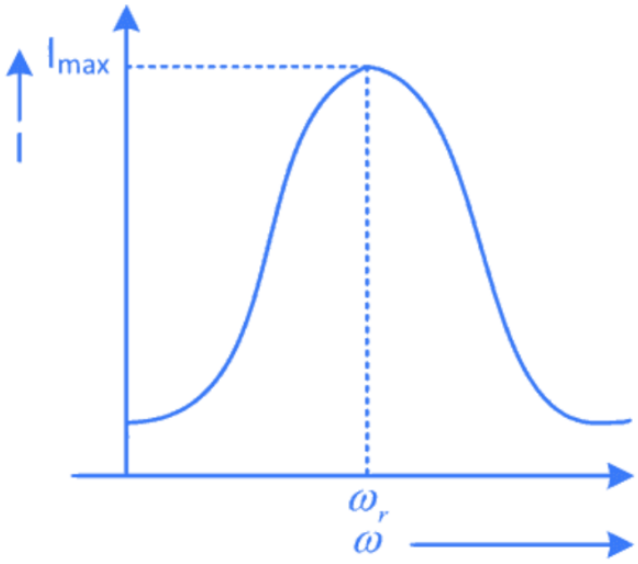
(a) To the left of ω_r, the circuit is mainly capacitive.

(b) To the left of ω_r, the circuit is mainly inductive.

(c) At ω_r, impedance of the circuit is equal to the resistance of the circuit.

(d) At ω_r, impedance of the circuit is 0.

Choose the most appropriate answer from the options given below : [4 marks]



(A) (a) and (d) only.

(B) (b) and (d) only.

(C) (a) and (c) only.

(D) (b) and (c) only.

Correct Answer: (C) (a) and (c) only.

Solution:

Concept:

- In a series LCR circuit, $X_L = \omega L$, $X_C = 1/(\omega C)$, and $Z = \sqrt{R^2 + (X_L - X_C)^2}$. The current amplitude is $I = V/Z$.
- Resonance occurs at $\omega_r = 1/\sqrt{LC}$, where the inductive and capacitive reactances cancel. Resistance remains present even at resonance.

Step 1: Examine frequencies below resonance To the left of the current maximum, $\omega < \omega_r$. Consequently $\omega^2 LC < 1$, which implies

$$\omega L < \frac{1}{\omega C}, \qquad X_L < X_C.$$

The net reactance $X_L - X_C$ is negative and the circuit behaves capacitively. Thus statement (a) is true and statement (b) is false. The current maximum shown in the graph marks resonance, not the whole low-frequency region.

Step 2: Evaluate the impedance at resonance At $\omega = \omega_r$, the two reactances have equal magnitudes,

so

$$Z_r = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2} = R.$$

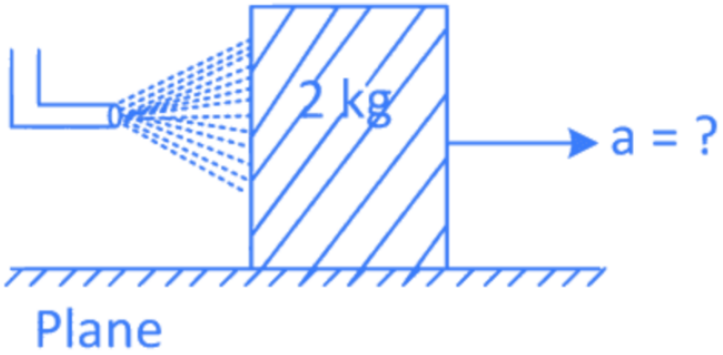
The cancellation removes only the reactive contribution. It does not remove the resistor. The finite maximum current is $I_{\max} = V/R$. Therefore statement (c) is true, while statement (d), claiming zero impedance, is false for the resistive circuit shown.

Step 3: Select the correct combination The complete truth pattern is (a) true, (b) false, (c) true, (d) false. Option (A) contains the false statement (d); option (B) contains both false statements (b) and (d); option (D) contains false statement (b). Only option (C) includes exactly the two valid statements. Notice that zero net reactance and zero impedance are different conditions because impedance also contains resistance.

Step 4: Conclusion The circuit is capacitive below resonance, and its impedance equals its resistance at resonance. Hence, the correct option is (C).

Quick Tip to Solve: Below resonance a series LCR circuit is capacitive; at resonance write $Z = R$, never $Z = 0$ unless resistance is explicitly zero.

40. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1 kgs^{-1} and at a speed of 10 ms^{-1}. Then, the initial acceleration of the block, in ms^{-2}, will be : [4 marks]



(A) 3

(B) 6

(C) 5

(D) 4

Correct Answer: (C) 5

Solution:

Concept:

- The force exerted by a steady jet is the rate at which it transfers momentum. If the incoming normal velocity is u and the outgoing normal velocity is zero, $F = \dot{m}u$.
- Initially the block is stationary. On the frictionless horizontal plane its initial horizontal acceleration follows $a_0 = F_0/M$, where M is the block mass.

Step 1: Find the initial momentum flux The water hits the vertical face normally. At the instant considered the block has not acquired any velocity, so the incident water speed relative to the block is $10\,\mathrm{m\,s^{-1}}$. Water spreads along the face after impact and loses its incoming horizontal velocity. In a small time Δt, mass $\dot{m}\Delta t$ loses horizontal momentum $(\dot{m}\Delta t)u$. Dividing by Δt gives the force magnitude.

Step 2: Apply Newton's second law The force on the block is opposite to the force that stops the incoming horizontal momentum of the water. It acts to the right, along the jet:

$$F_0 = (1\,\mathrm{kg\,s^{-1}})(10\,\mathrm{m\,s^{-1}}) = 10\,\mathrm{N}.$$

With block mass $M = 2\,\mathrm{kg}$,

$$a_0 = \frac{F_0}{M} = \frac{10}{2} = 5\,\mathrm{m\,s^{-2}}.$$

Weight and the normal reaction balance vertically and contribute no horizontal acceleration.

Step 3: Check the physical interpretation There is no stated elastic reversal of the water jet, so the momentum change is not $2\dot{m}u$. Nor should the water mass rate be added directly to the block mass: it has units of mass per time. The requested acceleration is initial; a later moving block would meet a different relative water speed. The alternatives 3, 6 and 4 do not follow from the given momentum flux divided by the stated block mass.

Step 4: Conclusion The initial acceleration is $5\,\mathrm{m\,s^{-2}}$, to the right. Hence, the correct option is (C).

Quick Tip to Solve: For a jet striking a stationary flat face, first calculate force as mass flow rate times the lost normal velocity, then divide by block mass.

---

41. In van der Waal equation $\left[P + \frac{a}{V^2}\right]$ [V − b] = RT; P is pressure, V is volume, R is universal gas constant and T is temperature. The ratio of constants $\frac{a}{b}$ is dimensionally equal to : [4 marks]

(A) $\frac{P}{V}$

(B) $\frac{V}{P}$

(C) PV

(D) PV^3

Correct Answer: (C) PV

Solution:

Concept:

- Terms added or subtracted in a physically meaningful equation must have the same dimensions. Thus P and a/V^2 share dimensions, as do V and b.
- Pressure has dimension $ML^{-1}T^{-2}$ and volume has dimension L^3. Ratios of the constants can be obtained without knowing their numerical values.

Step 1: Determine the dimensions of each constant From the pressure correction in the first bracket,

$$\left[\frac{a}{V^2}\right] = [P] \quad \Rightarrow \quad [a] = [P][V]^2.$$

From the volume correction in the second bracket, $[b] = [V]$. These statements follow from addition and subtraction inside the brackets, so no rearrangement of the gas constant or temperature is needed. The dimensions are not the same for a and b.

Step 2: Form the requested ratio Dividing gives

$$\left[\frac{a}{b}\right] = \frac{[P][V]^2}{[V]} = [P][V].$$

In fundamental dimensions this is

$$ML^{-1}T^{-2}\,L^3 = ML^2T^{-2}.$$

It is the dimension of energy, which is also the dimension of pressure multiplied by volume. The question asks for dimensional equality, not a numerical equation $a/b = PV$ for all gas states.

Step 3: Compare all four options Option (A), P/V, has dimension $ML^{-4}T^{-2}$. Option (B), V/P, has dimension $M^{-1}L^4T^2$. Option (D), PV^3, has dimension ML^8T^{-2}. None equals ML^2T^{-2}. Option (C), PV, matches the derived ratio exactly. Comparing the powers of volume is sufficient once $[a] = [P][V]^2$ and $[b] = [V]$ are established.

Step 4: Conclusion The ratio a/b is dimensionally equal to PV. Hence, the correct option is (C).

Quick Tip to Solve: In an equation with corrected pressure and volume, infer constant dimensions from the terms inside each bracket before multiplying the brackets.

42. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A}$ + $\vec{B}$ is equal to two times the magnitude of $\vec{A} - \vec{B}$, then the angle between $\vec{A}$ and $\vec{B}$ will be : [4 marks]

(A) $\sin^{-1}\left(\frac{3}{5}\right)$
(B) $\sin^{-1}\left(\frac{1}{3}\right)$
(C) $\cos^{-1}\left(\frac{3}{5}\right)$
(D) $\cos^{-1}\left(\frac{1}{3}\right)$

Correct Answer: (C) $\cos^{-1}\left(\frac{3}{5}\right)$

Solution:

Concept:

- For two vectors of equal nonzero magnitude a and included angle θ, $|\vec{A} + \vec{B}|^2 = 2a^2(1 + \cos\theta)$.
- The corresponding difference satisfies $|\vec{A} - \vec{B}|^2 = 2a^2(1 - \cos\theta)$. Squaring a relation between nonnegative magnitudes preserves the required equality.

Step 1: Square the stated magnitude relation The condition is $|\vec{A} + \vec{B}| = 2|\vec{A} - \vec{B}|$. The factor of 2 must become 4 upon squaring. Therefore

$$2a^2(1 + \cos\theta) = 4\,[2a^2(1 - \cos\theta)].$$

The common magnitude is nonzero because an angle between zero vectors is undefined. We can

consequently cancel $2a^2$ without losing a valid angular solution.

Step 2: Solve for the cosine After cancellation,

$$1 + \cos\theta = 4 - 4\cos\theta.$$

Collecting the cosine terms gives $5\cos\theta = 3$, so

$$\cos\theta = \frac{3}{5}, \qquad \theta = \cos^{-1}\left(\frac{3}{5}\right).$$

This is an acute angle, consistent with the sum being larger than the difference. Indeed, substituting the cosine gives squared magnitudes in the ratio $(1 + 3/5)/(1 - 3/5) = 4$, hence magnitudes in the ratio 2.

Step 3: Check the alternatives Option (A) gives a sine of $3/5$, corresponding to cosine $4/5$, and fails the derived equation. Option (B) gives sine $1/3$, also inconsistent with cosine $3/5$. Option (D) has cosine $1/3$, which yields a squared magnitude ratio 2 rather than 4. Only option (C) satisfies the original factor-of-two condition. An optional check is $\tan(\theta/2) = 1/2$, giving the same cosine.

Step 4: Conclusion The included angle is $\cos^{-1}(3/5)$, approximately $53.13°$. Hence, the correct option is (C).

Quick Tip to Solve: Square vector-magnitude ratios before using dot-product formulae; a factor 2 between magnitudes becomes a factor 4 between their squares.

43. The escape velocity of a body on a planet 'A' is 12 kms^{-1}. The escape velocity of the body on another planet 'B', whose density is four times and radius is half of the planet 'A', is : [4 marks]

(A) 12 kms^{-1}

(B) 24 kms^{-1}

(C) 36 kms^{-1}

(D) 6 kms^{-1}

Correct Answer: (A) 12 kms^{-1}

Solution:

Concept:

- The escape speed from the surface of a spherical planet is $v_e = \sqrt{2GM/R}$, where G is the gravitational constant, M the planet mass and R its radius.
- For a planet of mean density ρ, its mass is $M = (4\pi/3)\rho R^3$. Thus the speed depends on both density and radius, not on density alone.

Step 1: Express the speed in terms of density Substitute the spherical volume into the escape-speed formula. The mass of the escaping body cancels from the energy equation, so the same comparison

applies to the given body on either planet.

$$v_e = \sqrt{\frac{2G}{R}\frac{4\pi}{3}\rho R^3} = R\sqrt{\frac{8\pi G\rho}{3}}.$$

Therefore the required proportionality is $v_e \propto R\sqrt{\rho}$.

Step 2: Compare the two planets The density ratio is $\rho_B/\rho_A = 4$, while the radius ratio is $R_B/R_A = 1/2$. Hence

$$\frac{v_B}{v_A} = \frac{R_B}{R_A}\sqrt{\frac{\rho_B}{\rho_A}} = \frac{1}{2}\sqrt{4} = 1.$$

$$v_B = 1 \times 12 = 12\,\mathrm{km\,s^{-1}}.$$

The density increase contributes a factor two, exactly cancelled by the radius decrease. Thus 24 and 36 overestimate the speed, while 6 incorrectly neglects the density change.

Step 3: Conclusion The escape velocity on planet B is $12\,\mathrm{km\,s^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: When density and radius are compared, first use $v_e \propto R\sqrt{\rho}$; using a mass ratio directly without the volume factor gives the wrong scaling.

44. A longitudinal wave is represented by $x = 10\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$ cm. The maximum particle velocity will be four times the wave velocity if the determined value of wavelength is equal to : [4 marks]

(A) 2π

(B) 5π

(C) π

(D) $\frac{5\pi}{2}$

Correct Answer: (B) 5π

Solution:

Concept:

- For a sinusoidal progressive wave with displacement amplitude a and frequency n, the maximum particle speed is $u_{\max} = 2\pi na$. Particle speed is a time derivative at a fixed equilibrium position.
- The propagation speed of the wave is $v = n\lambda$. The wave speed and the oscillating particle speed are different quantities even for a longitudinal wave.

Step 1: Identify the amplitude and differentiate The coefficient of the sine gives the displacement amplitude $a = 10\,\mathrm{cm}$, and the time-dependent phase gives angular frequency $\omega = 2\pi n$. Keeping the equilibrium position fixed during differentiation, the particle velocity has cosine dependence:

$$u = 2\pi na\cos\left[2\pi\left(nt - \frac{x}{\lambda}\right)\right].$$

Since the largest magnitude of cosine is one, $u_{\max} = 20\pi n\,\mathrm{cm\,s^{-1}}$.

Step 2: Apply the given speed ratio The maximum particle speed is four times the propagation speed,

not four times the frequency. With wavelength measured in centimetres,

$$20\pi n = 4(n\lambda).$$

For a propagating wave $n \neq 0$, so cancellation gives

$$\lambda = \frac{20\pi}{4} = 5\pi \text{ cm}.$$

As a check, this wavelength makes $u_{\max}/v = 20\pi/(5\pi) = 4$. The other proposed values give ratios 10, 20 and 8 respectively, so none meets the stated condition.

Step 3: Conclusion The required wavelength is 5π cm. Hence, the correct option is (B).

Quick Tip to Solve: Use $u_{\max}/v = 2\pi a/\lambda$: the frequency cancels, but amplitude and wavelength must use the same length unit.

45. A parallel plate capacitor filled with a medium of dielectric constant 10, is connected across a battery and is charged. The dielectric slab is replaced by another slab of dielectric constant 15. Then the energy of capacitor will : [4 marks]

(A) increase by 50%

(B) decrease by 15%

(C) increase by 25%

(D) increase by 33%

Correct Answer: (A) increase by 50%

Solution:

Concept:

- A completely filled parallel-plate capacitor has capacitance $C = K\varepsilon_0 A/d$, where K is the dielectric constant and the plate geometry stays fixed.
- For a capacitor connected across a battery, the potential difference V stays fixed. Its stored electrostatic energy is $U = \frac{1}{2}CV^2$, so at fixed voltage $U \propto C \propto K$.

Step 1: Identify the fixed electrical quantity The capacitor is connected across the battery, and no disconnection is specified. The battery therefore maintains the same potential difference when the dielectric is replaced. Charge need not remain fixed: the battery supplies additional charge as the capacitance rises. The expression $Q^2/(2C)$ must not be used with the old charge held constant in this situation. With unchanged area and separation,

$$\frac{C_f}{C_i} = \frac{K_f}{K_i} = \frac{15}{10} = \frac{3}{2}.$$

Step 2: Calculate the fractional energy increase Because the voltage is common to both states, its

squared factor cancels in the ratio:

$$\frac{U_f}{U_i} = \frac{\frac{1}{2}C_f V^2}{\frac{1}{2}C_i V^2} = \frac{3}{2}.$$

Thus the final stored energy is 150 percent of the initial stored energy. The increase relative to the initial value is

$$\frac{U_f - U_i}{U_i} \times 100\% = \left(\frac{3}{2} - 1\right) 100\% = 50\%.$$

A decrease would correspond to an inappropriate fixed-charge assumption; neither a 25 percent nor a 33 percent increase equals the calculated ratio.

Step 3: Conclusion The stored energy increases by 50%. Hence, the correct option is (A).

Quick Tip to Solve: Before comparing capacitor energies, check whether the battery remains connected: fixed voltage gives $U \propto K$, whereas an isolated fixed charge gives $U \propto 1/K$.

---

46. A positive charge particle of 100 mg is thrown in opposite direction to a uniform electric field of strength 1×10^5 NC^{-1}. If the charge on the particle is 40 μC and the initial velocity is 200 ms^{-1}, how much distance it will travel before coming to the rest momentarily : [4 marks]

(A) 1 m

(B) 5 m

(C) 10 m

(D) 0.5 m

Correct Answer: (D) 0.5 m

Solution:

Concept:

- A positive charge in a uniform electric field experiences a constant force $F = qE$ along the field. Motion initially opposite the field is uniformly decelerated until the particle stops.
- The stopping distance can be found from $v^2 = u^2 + 2as$, with a negative acceleration along the initial-motion axis, or from loss of kinetic energy $qEs = \frac{1}{2}mu^2$.

Step 1: Convert the data and find the retardation Take the initial velocity direction as positive. The electric force then points in the negative direction because the charge is positive and the field opposes the motion. Convert the given quantities to SI units:

$$m = 100\,\text{mg} = 1.0 \times 10^{-4}\,\text{kg}, \qquad q = 40 \times 10^{-6}\,\text{C}.$$

$$F = qE = (40 \times 10^{-6})(10^5) = 4\,\text{N}.$$

$$a = -\frac{F}{m} = -\frac{4}{10^{-4}} = -4.0 \times 10^4\,\text{m s}^{-2}.$$

Step 2: Find the distance to the turning point At the momentary stop, $v = 0$, while $u = 200\,\text{m s}^{-1}$.

Substitute into the constant-acceleration relation:

$$0 = (200)^2 - 2(4.0 \times 10^4)s.$$

$$s = \frac{40000}{80000} = 0.5\,\text{m}.$$

The initial kinetic energy is $\frac{1}{2}(10^{-4})(200)^2 = 2\,\text{J}$, and the field does negative work $-4(0.5) = -2\,\text{J}$, independently checking the result. Distances 1, 5 and 10 m would require more work than the initial kinetic energy and therefore cannot be the first stopping distance.

Step 3: Conclusion The particle travels $0.5\,\text{m}$ before momentarily coming to rest. Hence, the correct option is (D).

Quick Tip to Solve: Convert milligrams to kilograms before calculating acceleration: $1\,\text{mg} = 10^{-6}\,\text{kg}$. Check a stopping distance with $qEs = \frac{1}{2}mu^2$.

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47. Using Young's double slit experiment, a monochromatic light of wavelength 5000 $\overset{o}{A}$ produces fringes of fringe width 0.5 mm. If another monochromatic light of wavelength 6000 $\overset{o}{A}$ is used and the separation between the slits is doubled, then the new fringe width will be : [4 marks]

(A) 0.5 mm

(B) 1.0 mm

(C) 0.6 mm

(D) 0.3 mm

Correct Answer: (D) 0.3 mm

Solution:

Concept:

- In Young's double-slit experiment, fringe width is $\beta = \lambda D/d$, where λ is wavelength, D is slit-to-screen distance and d is slit separation.
- For the same apparatus with unchanged screen position, $\beta \propto \lambda/d$. A larger wavelength widens the fringes, whereas a larger slit separation narrows them.

Step 1: Form the ratio for the two arrangements Let the original quantities carry subscript 1 and the new quantities carry subscript 2. The wavelength changes from 5000 to 6000 angstrom, while the slit separation becomes twice its original value. The screen distance is unchanged, so

$$\frac{\beta_2}{\beta_1} = \frac{\lambda_2 D/d_2}{\lambda_1 D/d_1} = \frac{\lambda_2}{\lambda_1}\frac{d_1}{d_2}.$$

Using a ratio avoids unnecessary conversion of both wavelengths into metres: their common unit cancels exactly.

Step 2: Apply both changes and compare the options Substitute the wavelength ratio and the doubled

separation together:

$$\frac{\beta_2}{\beta_1} = \frac{6000}{5000}\frac{1}{2} = \frac{6}{5}\frac{1}{2} = \frac{3}{5}.$$

$$\beta_2 = \frac{3}{5}(0.5\,\text{mm}) = 0.3\,\text{mm}.$$

A value of 0.6 mm would include only the wavelength increase and omit the doubled slit separation. A value of 1.0 mm reverses the effect of increasing the separation. A value of 0.5 mm would require the two scale factors to cancel completely, whereas their product is 0.6 here. The net effect must be a reduction from the original fringe width.

Step 3: Conclusion The new fringe width is $0.3\,\text{mm}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the fringe-width ratio before inserting numbers; multiply by the wavelength ratio and by the inverse slit-separation ratio.

48. The activity of a radioactive material is 2.56×10^{-3} Ci. If the half life of the material is 5 days, after how many days the activity will become 2×10^{-5} Ci ? [4 marks]

(A) 30 days

(B) 35 days

(C) 40 days

(D) 25 days

Correct Answer: (B) 35 days

Solution:

Concept:

- Radioactive activity is proportional to the number of undecayed nuclei: $A = \lambda_d N$, where λ_d is the decay constant. Hence activity follows the same exponential decay law as the nuclei.
- After time t, activity is $A = A_0(1/2)^{t/T_{1/2}}$, where $T_{1/2}$ is the half-life. Every elapsed half-life halves the current activity, not the original activity.

Step 1: Find the required reduction factor The initial and final activities are both stated in curies, so conversion to becquerels is unnecessary in their ratio. Carefully combine the numerical coefficients and powers of ten:

$$\frac{A_0}{A} = \frac{2.56 \times 10^{-3}}{2 \times 10^{-5}} = 1.28 \times 10^2 = 128.$$

Thus the activity must fall to $1/128$ of its initial value. The difference of the activities does not determine the time because radioactive decay is exponential rather than linear.

Step 2: Count the half-lives and calculate the time Since $128 = 2^7$, the decay law requires

$$2^{t/T_{1/2}} = 2^7 \quad \Longrightarrow \quad t/T_{1/2} = 7.$$

Using the given half-life of 5 days gives

$$t = 7 \times 5 = 35\,\text{days}.$$

Direct substitution checks the final activity: $(2.56 \times 10^{-3})/128 = 2.00 \times 10^{-5}$ Ci. The other options correspond to six, eight and five half-lives, giving reduction factors 64, 256 and 32 respectively; none yields the specified activity.

Step 3: Conclusion The activity reaches the stated value after $35\,\text{days}$. Hence, the correct option is (B).

Quick Tip to Solve: When the activity ratio is a power of two, count half-lives directly. Keep both activities in the same unit so the unit conversion cancels.

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49. At a certain place the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.5 G. The earth's total magnetic field (in G), at that certain place, is : [4 marks]

(A) $\frac{1}{\sqrt{3}}$

(B) $\frac{1}{2}$

(C) $\sqrt{3}$

(D) 1

Correct Answer: (A) $\frac{1}{\sqrt{3}}$

Solution:

Concept:

- The angle of dip δ is measured between the total terrestrial magnetic field B and the horizontal plane. Thus its horizontal component is $B_H = B\cos\delta$.
- The vertical component is $B_V = B\sin\delta$, and $B^2 = B_H^2 + B_V^2$. These component relations also provide a check that the total field cannot be smaller than a known component.

Step 1: Relate the known component to the total field The question supplies the horizontal component $B_H = 0.5\,\text{G}$ and dip $\delta = 30°$. Because dip is measured from the horizontal, the cosine multiplies the total field to give the horizontal projection. Rearranging,

$$B = \frac{B_H}{\cos\delta}.$$

Using sine instead would incorrectly treat the dip as an angle measured from the vertical. No gauss-to-tesla conversion is needed because the answer is requested in gauss.

Step 2: Substitute the angle and check the options With $\cos 30° = \sqrt{3}/2$,

$$B = \frac{0.5}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}\,\text{G} \simeq 0.577\,\text{G}.$$

The answer is slightly larger than its horizontal component, as required by vector resolution. The option $1/2\,\text{G}$ would make total and horizontal fields equal, implying zero dip rather than 30 degrees. For $\sqrt{3}\,\text{G}$, the horizontal projection would be $1.5\,\text{G}$; for $1\,\text{G}$, it would be $\sqrt{3}/2\,\text{G}$. Both contradict the given 0.5 G. As an additional check, the vertical component is $B_H\tan 30° = 0.5/\sqrt{3}\,\text{G}$.

Step 3: Conclusion The earth's total magnetic field is $1/\sqrt{3}\,\text{G}$. Hence, the correct option is (A).

Quick Tip to Solve: For dip measured from the horizontal, use $B_H = B\cos\delta$. Check that the computed total field exceeds the given horizontal component.

50. Only 2% of the optical source frequency is the available channel bandwidth for an optical communicating system operating at 1000 nm. If an audio signal requires a bandwidth of 8 kHz, how many channels can be accommodated for transmission : [4 marks]

(A) 375×10^7

(B) 75×10^7

(C) 375×10^8

(D) 75×10^9

Correct Answer: (B) 75×10^7

Solution:

Concept:

- The carrier frequency and wavelength are related by $\nu = c/\lambda$, where c is the speed of light. Convert the wavelength into metres before using this equation.
- The usable frequency interval is the stated fraction of the carrier frequency. The number of independent channels is the total available bandwidth divided by the bandwidth required for one channel.

Step 1: Calculate the optical carrier frequency The operating wavelength is $1000\,\text{nm} = 1000 \times 10^{-9}\,\text{m} = 10^{-6}\,\text{m}$. Hence the optical frequency is

$$\nu = \frac{3 \times 10^8}{10^{-6}} = 3 \times 10^{14}\,\text{Hz}.$$

This frequency is the carrier frequency, not the available bandwidth; only the specified small fraction can be used for the channels.

Step 2: Find the available bandwidth Two percent means a factor $2/100 = 0.02$. Thus

$$B = 0.02\nu = 0.02(3 \times 10^{14}) = 6 \times 10^{12}\,\text{Hz}.$$

The bandwidth of each audio channel is $8\,\text{kHz} = 8 \times 10^3\,\text{Hz}$, so the two bandwidths are now expressed in the same unit.

Step 3: Count channels and test the options The number accommodated is

$$N = \frac{B}{8 \times 10^3} = \frac{6 \times 10^{12}}{8 \times 10^3} = 7.5 \times 10^8 = 75 \times 10^7.$$

Option A is five times this result, option C is fifty times it, and option D is one hundred times it. Those values require a bandwidth larger than the available interval and therefore fail the bandwidth check.

Step 4: Conclusion The system accommodates $7.5 \times 10^8 = 75 \times 10^7$ channels, a dimensionless count. Hence, the correct option is (B).

Quick Tip to Solve: First take 2 percent of the optical frequency, then divide by 8000 Hz; dividing the full carrier frequency overcounts channels by a factor of 50.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Two coils require 20 minutes and 60 minutes respectively to produce same amount of heat energy when connected separately to the same source. If they are connected in parallel arrangement to the same source; the time required to produce same amount of heat by the combination of coils, will be _____ min. [4 marks]

Solution:

Concept:

- For a steady source voltage V, a resistive coil delivers heat at rate $P = V^2/R$. For a fixed required heat H, the individual time satisfies $P = H/t$.
- Parallel coils each retain the full source voltage. Their powers add, so the time for the combination follows from adding rates of production of the same heat, rather than adding times.

Step 1: Express the separate powers Let H be the heat amount specified in the question. Using minutes consistently, the powers in heat units per minute are

$$P_1 = \frac{H}{20}, \qquad P_2 = \frac{H}{60}.$$

The first coil heats three times as fast as the second. We need not calculate either resistance, because the measured times already give the relative and absolute heating rates in terms of H.

Step 2: Add the powers in parallel In a parallel arrangement each coil has the same voltage across it as when it was connected separately. Thus neither individual power changes, and the total rate is

$$P_{\parallel} = \frac{H}{20} + \frac{H}{60} = \frac{3H + H}{60} = \frac{H}{15}.$$

This sum is valid for the total heat released by both coils together; it is not the time for each coil to release H separately.

Step 3: Determine the common heating time If the required time is t, then $H = P_{\parallel} t$. Substitution gives

$$t = \frac{H}{H/15} = 15\,\text{min}.$$

Equivalently, $1/t = 1/20 + 1/60$, so $t = (20 \times 60)/(20 + 60)$. The answer is shorter than 20 minutes, as it must be, because adding the second coil increases total power.

Step 4: Conclusion The parallel combination produces the required heat in $\boxed{15\,\text{min}}$. The numerical entry is 15.

Quick Tip to Solve: For a fixed heat target and unchanged voltage, parallel heating times combine through reciprocal times: $1/t = 1/t_1 + 1/t_2$.

52. The intensity of the light from a bulb incident on a surface is 0.22 W/m^2. The amplitude of the magnetic field in this light-wave is _____ $\times 10^{-9}$ T.

(Given : Permittivity of vacuum $\in_0$ = 8.85 $\times 10^{-12}$ C^2 N^{-1}-m^{-2}, speed of light in vacuum c = 3 $\times 10^8$ ms^{-1}) [4 marks]

Solution:

Concept:

- The intensity I of a sinusoidal electromagnetic wave is its time-averaged energy flux: $I = \frac{1}{2}\varepsilon_0 c E_0^2$. The factor one-half distinguishes amplitude from root-mean-square field.
- The electric and magnetic field amplitudes obey $E_0 = cB_0$. Thus $I = \frac{1}{2}\varepsilon_0 c^3 B_0^2$, where B_0 is the requested magnetic amplitude.

Step 1: Isolate the magnetic amplitude Substitute $E_0 = cB_0$ in the intensity relation:

$$I = \frac{1}{2}\varepsilon_0 c(cB_0)^2 = \frac{1}{2}\varepsilon_0 c^3 B_0^2.$$

It follows that

$$B_0 = \sqrt{\frac{2I}{\varepsilon_0 c^3}}.$$

We take the positive square root because an amplitude is a nonnegative magnitude. No assumption about the bulb distance or surface area is needed, since the incident intensity is already supplied.

Step 2: Substitute the stated constants Using $I = 0.22\,\mathrm{W\,m^{-2}}$, $\varepsilon_0 = 8.85 \times 10^{-12}$ in SI units and $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$,

$$B_0^2 = \frac{0.44}{(8.85 \times 10^{-12})(3 \times 10^8)^3} = \frac{0.44}{2.3895 \times 10^{14}}\,\mathrm{T^2}.$$

Therefore $B_0^2 \simeq 1.8414 \times 10^{-15}\,\mathrm{T^2}$.

Step 3: Express the answer in the requested scale Taking the square root gives

$$B_0 \simeq 4.291 \times 10^{-8}\,\mathrm{T} = 42.91 \times 10^{-9}\,\mathrm{T}.$$

The requested numerical coefficient, expressed as an integer, is therefore 43. Using $I = \varepsilon_0 c^3 B_0^2$ instead would omit time averaging and give the rms magnetic field instead of the amplitude.

Step 4: Conclusion The magnetic field amplitude is approximately $\boxed{43 \times 10^{-9}\,\mathrm{T}}$; enter 43.

Quick Tip to Solve: Keep the factor of 2 under the square root when intensity is given and peak field amplitude is requested; the rms field lacks that factor.

53. As per the given figure, two plates A and B of thermal conductivity K and 2 K are joined together to form a compound plate. The thickness of plates are 4.0 cm and 2.5 cm respectively and the area of cross-section is 120 cm^2 for each plate. The equivalent thermal conductivity of the compound plate is $\left(1+\frac{5}{\alpha}\right)$ K, then the value of α will be _____. [4 marks]



Solution:

Concept:

- The thermal resistance of a plate of thickness L, area A, and conductivity k is $R_{\text{th}} = L/(kA)$. In steady conduction the same heat current passes through plates placed in series.
- For equal-area layers placed one after another along the heat flow, resistances add: $(L_1 + L_2)/(k_{\text{eq}}A) = L_1/(k_1A) + L_2/(k_2A)$.

Step 1: Identify the series arrangement The figure shows heat flowing from the $100°\text{C}$ end of plate A to the $0°\text{C}$ end of plate B through their common interface. Therefore the plates are in series, not parallel. Their respective thicknesses are $L_1 = 4.0\,\text{cm}$ and $L_2 = 2.5\,\text{cm}$, with conductivities K and $2K$. Both have the same cross-sectional area $120\,\text{cm}^2$.

Step 2: Compute the equivalent conductivity For a single compound plate of thickness $6.5\,\text{cm}$, equality of thermal resistances gives

$$\frac{6.5}{k_{\text{eq}}A} = \frac{4}{KA} + \frac{2.5}{2KA} = \frac{5.25}{KA}.$$

The common area cancels; there is no need to calculate the heat current. Thus

$$k_{\text{eq}} = \frac{6.5}{5.25}K = \frac{26}{21}K.$$

This lies between K and $2K$, which is consistent with the conductivities of the two layers.

Step 3: Compare the specified coefficient The question writes this conductivity as $(1+5/\alpha)K$. Divide both expressions by K and subtract 1:

$$1 + \frac{5}{\alpha} = \frac{26}{21}, \qquad \frac{5}{\alpha} = \frac{5}{21}.$$

Consequently $\alpha = 21$. It is a dimensionless parameter because it appears in a dimensionless coefficient multiplying K.

Step 4: Conclusion The equivalent conductivity is $(26/21)K$, so $\boxed{\alpha = 21}$. The numerical entry is 21.

Quick Tip to Solve: For equal-area layers in series use total thickness divided by the sum of thickness/conductivity terms; an arithmetic average is generally incorrect.

54. A body is performing simple harmonic with an amplitude of 10 cm. The velocity of the body was tripled by air jet when it is at 5 cm from its mean position. The new amplitude of vibration is $\sqrt{x}$ cm. The value of x is _____. [4 marks]

Solution:

Concept:

- In simple harmonic motion with angular frequency ω, amplitude A, and displacement y, the speed obeys $v^2 = \omega^2(A^2 - y^2)$.
- An instantaneous impulse changes the velocity at the current position. After the jet ends, the restoring system and hence ω remain the same, while the energy and amplitude change.

Step 1: Find the original speed at the stated position Use centimetres consistently for amplitude and displacement: $A = 10\,\text{cm}$ and $y = 5\,\text{cm}$. Before the air jet acts,

$$v^2 = \omega^2(10^2 - 5^2) = 75\omega^2.$$

Here the length factors are in cm^2. The body is not at the mean position, so its speed is not the maximum speed ωA.

Step 2: Apply the tripling of velocity Immediately after the jet, the speed is $v_{\text{new}} = 3v$, so its square becomes $v_{\text{new}}^2 = 9v^2 = 675\omega^2$. The displacement is still $5\,\text{cm}$, because the velocity change occurs at that location. If A_{new} is the new amplitude, the SHM speed relation gives

$$675\omega^2 = \omega^2(A_{\text{new}}^2 - 25).$$

The angular frequency cancels because the restoring force law has not changed.

Step 3: Determine the new amplitude parameter Solving the preceding relation,

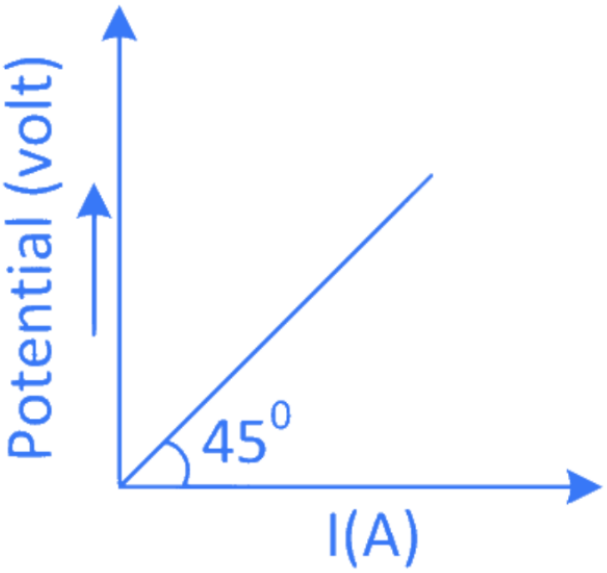
$$A_{\text{new}}^2 = 675 + 25 = 700\,\text{cm}^2, \qquad A_{\text{new}} = \sqrt{700}\,\text{cm}.$$

The question specifies $A_{\text{new}} = \sqrt{x}\,\text{cm}$, so $x = 700$. This differs from simply tripling the original amplitude: the potential energy at the nonzero displacement stays unchanged while only the kinetic energy is multiplied by nine.

Step 4: Conclusion The new amplitude is $\sqrt{700}\,\text{cm}$; hence $\boxed{x = 700}$.

Quick Tip to Solve: At a fixed displacement, a speed multiplication by n gives $A_{\text{new}}^2 = y^2 + n^2(A^2 - y^2)$; only at the mean position does amplitude scale by n.

55. The variation of applied potential and current flowing through a given wire is shown in figure. The length of wire is 31.4 cm. The diameter of wire is measured as 2.4 cm. The resistivity of the given wire is measured as x $\times$ 10^{-3} Ω cm. The value of x is _____. [Take π = 3.14] [4 marks]



Solution:

Concept:

- For an ohmic wire, the slope of a potential-versus-current graph gives resistance: $R = \Delta V/\Delta I$. The order of the plotted axes matters.
- A uniform wire obeys $R = \rho L/A$, with circular cross-section $A = \pi d^2/4$. Consequently $\rho = RA/L$; using lengths in centimetres gives resistivity in $\Omega\,\mathrm{cm}$.

Step 1: Read the resistance from the graph The vertical axis is potential in volts and the horizontal axis is current in amperes. The line passes through the origin and makes 45° with the current axis, so its numerical slope in the indicated axes is

$$R = \frac{V}{I} = \tan 45^\circ = 1\,\Omega.$$

Taking the reciprocal slope would correspond to conductance rather than resistance, even though in this particular graph the numerical value is also one.

Step 2: Calculate the cross-sectional area The stated diameter is $d = 2.4\,\mathrm{cm}$, so the radius is $1.2\,\mathrm{cm}$. With the specified value $\pi = 3.14$,

$$A = \pi r^2 = 3.14(1.2)^2 = 4.5216\,\mathrm{cm}^2.$$

Use the radius in this expression, not the diameter. Keep the length as $L = 31.4\,\mathrm{cm}$ because the requested resistivity is in ohm-centimetres.

Step 3: Find and scale the resistivity Rearranging the wire-resistance relation gives

$$\rho = \frac{RA}{L} = \frac{(1)(4.5216)}{31.4} = 0.144\,\Omega\,\mathrm{cm}.$$

Since $0.144 = 144 \times 10^{-3}$, comparison with $x \times 10^{-3}\,\Omega\,\text{cm}$ yields $x = 144$. The dimensions are $\Omega \times \text{cm}^2/\text{cm} = \Omega\,\text{cm}$, confirming the requested unit.

Step 4: Conclusion The resistivity is $\boxed{144 \times 10^{-3}\,\Omega\,\text{cm}}$, so the numerical entry is $\boxed{144}$.

Quick Tip to Solve: For a V-I graph use its slope as resistance, then use $\pi d^2/4$ for area; retaining centimetres directly produces the requested ohm-centimetre unit.

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56. $\sqrt{d_1}$ and $\sqrt{d_2}$ are the impact parameters corresponding to scattering angles 60° and 90° respectively, when an α particle is approaching a gold nucleus. For $d_1 = x\, d_2$, the value of x will be _____. [4 marks]

Solution:

Concept:

- For Rutherford scattering at fixed incident kinetic energy E, the impact parameter is $b = C\cot(\theta/2)$, where $C = kZze^2/(2E)$ is the same for both scattering events.
- The given impact parameters are $\sqrt{d_1}$ and $\sqrt{d_2}$, not d_1 and d_2. Thus the ratio of the quantities called d is the square of the impact-parameter ratio.

Step 1: Relate impact parameter to scattering angle For repulsive Coulomb scattering by the same gold nucleus, with the same incident alpha-particle energy, all factors in C are identical. Therefore

$$\frac{b_1}{b_2} = \frac{\cot(\theta_1/2)}{\cot(\theta_2/2)}.$$

A smaller scattering angle corresponds to a larger impact parameter: a particle passing farther from the nucleus experiences a smaller deflection. This provides a qualitative check on the ratio.

Step 2: Substitute the two half-angles The scattering angles are $\theta_1 = 60°$ and $\theta_2 = 90°$, so their half-angles are $30°$ and $45°$. Consequently

$$\frac{b_1}{b_2} = \frac{\cot 30°}{\cot 45°} = \frac{\sqrt{3}}{1} = \sqrt{3}.$$

Using the full angles inside the cotangents would not be the Rutherford relation and would give an incorrect comparison.

Step 3: Convert the parameter ratio into the requested ratio Because $b_1 = \sqrt{d_1}$ and $b_2 = \sqrt{d_2}$, we have

$$\sqrt{\frac{d_1}{d_2}} = \sqrt{3}.$$

Squaring both sides gives $d_1/d_2 = 3$, or $d_1 = 3d_2$. The requested coefficient x is dimensionless, since it is a ratio of two quantities having the same dimensions of length squared.

Step 4: Conclusion The relation is $d_1 = 3d_2$; hence $\boxed{x = 3}$.

Quick Tip to Solve: Use the half-scattering angle in the cotangent, and then check whether the stated impact parameter is d or $\sqrt{d}$ before forming the final ratio.

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57. A parallel beam of light is allowed to fall on a transparent spherical globe of diameter 30 cm and refractive index 1.5. The distance from the centre of the globe at which the beam of light can converge is _____ mm. [4 marks]

Solution:

Concept:

- For refraction at one spherical surface, $n_2/v - n_1/u = (n_2 - n_1)/R$, with distances measured from that surface using the Cartesian sign convention.
- The two surfaces must be treated successively: the first image acts as the object for the second. The requested distance is from the centre, not from the rear surface.

Step 1: Locate the image formed by the front surface Take incident light from left to right as positive. The radius is $r = 15\,\text{cm}$, and the first surface has $R_1 = +15\,\text{cm}$. Parallel incident rays have $u_1 = -\infty$. Therefore

$$\frac{1.5}{v_1} - \frac{1}{-\infty} = \frac{1.5 - 1}{15}, \qquad v_1 = 45\,\text{cm}.$$

This image position is measured from the front vertex inside the continued glass medium.

Step 2: Use that image as the object for the rear surface The rear vertex is 30 cm from the front vertex, so the converging beam would meet 15 cm to its right. It is thus a virtual object for the rear surface, with $u_2 = +15\,\text{cm}$. Here $n_1 = 1.5$, $n_2 = 1$, and $R_2 = -15\,\text{cm}$. Consequently

$$\frac{1}{v_2} - \frac{1.5}{15} = \frac{1 - 1.5}{-15} = \frac{1}{30}.$$

Thus $1/v_2 = 1/10 + 1/30 = 2/15$, giving $v_2 = 7.5\,\text{cm}$ beyond the rear vertex.

Step 3: Measure the distance from the centre The centre is 15 cm before the rear vertex. Hence the centre-to-focus distance is

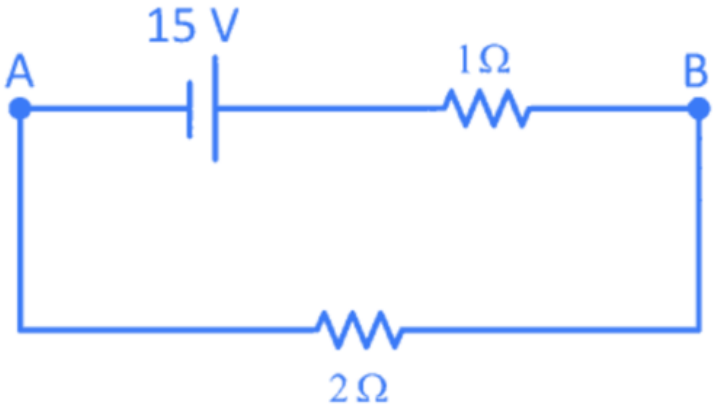
$$d = 15 + 7.5 = 22.5\,\text{cm} = 225\,\text{mm}.$$

Using 7.5 cm alone would give the back focal distance, a different quantity.

Step 4: Conclusion The beam converges $225\,\text{mm}$ from the centre of the globe. The required numerical entry is $\boxed{225}$.

Quick Tip to Solve: For a thick spherical globe, transfer the first image position to the second vertex before applying refraction again; finally convert to the reference point asked for.

58. For the network shown below, the value of $V_B - V_A$ is _____ V. [4 marks]



Solution:

Concept:

- Kirchhoff's loop law states that the algebraic sum of potential changes around a closed circuit is zero. A resistor produces a drop IR along the current.
- The longer plate of a cell denotes its positive terminal. Potential difference between two nodes can be computed along either path connecting them.

Step 1: Identify the current direction and series resistance The cell in the upper branch has its long plate on the right, so it raises the potential by 15 V when traversed from A toward B. Current therefore travels clockwise: from A through the cell and the $1\,\Omega$ resistor to B, then through the lower $2\,\Omega$ resistor back to A. There is only one closed loop, so both resistors carry the same current; they are in series, despite being drawn on different branches.

Step 2: Apply the loop equation Following the current around the loop gives

$$15 - I(1) - I(2) = 0.$$

The total resistance is $1 + 2 = 3\,\Omega$, hence

$$I = \frac{15}{3} = 5\,\text{A}.$$

No separate current division is needed because there is no junction with three conducting branches.

Step 3: Calculate the node voltage and check it Along the upper path A to B, the cell gives a 15 V rise and the resistor gives a $5 \times 1 = 5\,\text{V}$ drop. Thus

$$V_B - V_A = 15 - 5 = 10\,\text{V}.$$

Equivalently, the lower resistor carries current from B to A, so B is higher than A by $I(2) = 10\,\text{V}$. Both paths give the same sign and magnitude. Choosing A as the zero reference would therefore assign B a potential of 10 V.

Step 4: Conclusion The required potential difference is $\boxed{V_B - V_A = 10\,\text{V}}$, so the numerical entry is 10.

Quick Tip to Solve: Check the polarity using the lower resistor: current flows from B to A, so B must have the greater potential.

59. 300 cal. of heat is given to a heat engine and it rejects 225 cal. If source temperature is 227°C, then the temperature of sink will be _____ °C. [4 marks]

Solution:

Concept:

- For a cyclic heat engine, the work output is $W = Q_H - Q_C$, where Q_H is heat absorbed and Q_C is heat rejected.
- For the ideal reversible heat-engine relation used to determine reservoir temperatures, $Q_C/Q_H = T_C/T_H$. Temperatures in this ratio must be absolute, in kelvin.

Step 1: Calculate the heat rejected as a fraction of heat supplied The heat supplied is $Q_H = 300\,\text{cal}$, while $Q_C = 225\,\text{cal}$ is rejected. The work delivered per cycle is

$$W = 300 - 225 = 75\,\text{cal}.$$

Therefore the efficiency is

$$\eta = \frac{W}{Q_H} = \frac{75}{300} = 0.25,$$

and the rejected fraction is $Q_C/Q_H = 0.75$. Calories cancel from either ratio, so conversion to joules is unnecessary.

Step 2: Convert the source temperature and use the ideal-engine relation The source temperature is

$$T_H = 227 + 273 = 500\,\text{K}.$$

Using the reversible engine relation gives

$$T_C = T_H \frac{Q_C}{Q_H} = 500\left(\frac{225}{300}\right) = 375\,\text{K}.$$

The kelvin scale is essential: multiplying the Celsius value 227 by 0.75 would incorrectly treat the Celsius zero as absolute zero.

Step 3: Express the sink temperature on the requested scale Converting back gives

$$t_C = 375 - 273 = 102\,^\circ\text{C}.$$

The answer is below the source temperature, consistent with heat flow and positive work. Strictly, a general irreversible engine need not satisfy equality between heat and temperature ratios; without the ideal reversible interpretation, these data only bound the sink temperature rather than determine it uniquely.

Step 4: Conclusion Using the ideal reversible heat-engine relation intended in this numerical problem, the sink temperature is $\boxed{102\,^\circ\text{C}}$. The required entry is 102.

Quick Tip to Solve: First form the dimensionless heat ratio, then multiply the source temperature in kelvin; convert to Celsius only at the end.

60. A transistor is used in an amplifier circuit in common emitter mode. If the base current changes by 100 μA, it brings a change of 10 mA in collector current. If the load resistance is 2 kΩ and input resistance is 1 kΩ, the value of power gain is x $\times$ 10^4. The value of x is _____. [4 marks]

Solution:

Concept:

- The small-signal current gain in common-emitter mode is $\beta = \Delta I_C/\Delta I_B$. It is a ratio of currents expressed in identical units.
- The magnitude of voltage gain is $A_V = \beta R_L/R_{\text{in}}$. Hence the power gain is $A_P = \beta A_V = \beta^2 R_L/R_{\text{in}}$.

Step 1: Calculate the current gain Convert the collector-current change to microamperes to match the base-current change:

$$\Delta I_C = 10\,\text{mA} = 10000\,\mu\text{A}, \qquad \Delta I_B = 100\,\mu\text{A}.$$

Consequently

$$\beta = \frac{10000}{100} = 100.$$

Using 10/100 without converting units would underestimate this gain by a factor of one thousand.

Step 2: Calculate the voltage and power gains The load and input resistances are $R_L = 2\,\text{k}\Omega$ and $R_{\text{in}} = 1\,\text{k}\Omega$, so their ratio is 2. The magnitude of voltage gain is

$$|A_V| = 100 \times 2 = 200.$$

The output voltage is phase inverted in common-emitter operation, but this does not give negative power gain. The positive power ratio is

$$A_P = \beta|A_V| = 100 \times 200 = 20000.$$

Equivalently, the ratio of signal powers $(\Delta I_C)^2 R_L/[(\Delta I_B)^2 R_{\text{in}}]$ yields the same expression.

Step 3: Match the requested coefficient Express the dimensionless result in the stated form:

$$20000 = 2 \times 10^4.$$

Comparing with $x \times 10^4$ directly gives $x = 2$. The resistance units cancel because only their ratio occurs, just as the common current units cancel in the gain.

Step 4: Conclusion The power gain is 2×10^4, so the requested dimensionless value is $\boxed{x = 2}$.

Quick Tip to Solve: For a CE amplifier, power gain contains the square of current gain: use $\beta^2 R_L/R_{\text{in}}$, not just $\beta R_L/R_{\text{in}}$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Production of iron in blast furnace follows the following equation

$Fe_3O_4(s) + 4CO(g) \rightarrow 3Fe(l) + 4CO_2(g)$

when 4.640 kg of Fe_3O_4 and 2.520 kg of CO are allowed to react then the amount of iron (in g) produced is :

[Given : Molar Atomic mass (g mol^{-1}) : Fe = 56, Molar Atomic mass (g mol^{-1}) : O = 16, Molar Atomic mass (g mol^{-1}) : C = 12] [4 marks]

(A) 1400

(B) 2200

(C) 3360

(D) 4200

Correct Answer: (C) 3360

Solution:

Concept:

- The limiting reagent is identified by comparing available moles divided by the corresponding balanced-equation coefficient.
- For $Fe_3O_4 + 4\,CO \longrightarrow 3\,Fe + 4\,CO_2$, one mole of magnetite consumes four moles of carbon monoxide and produces three moles of iron.

Step 1: Convert the given masses to moles Using the supplied atomic masses,

$$M(Fe_3O_4) = 3(56) + 4(16) = 232\,g\,mol^{-1}.$$

$$M(CO) = 12 + 16 = 28\,g\,mol^{-1}.$$

The masses are $4640\,g$ of magnetite and $2520\,g$ of carbon monoxide. Thus

$$n(Fe_3O_4) = \frac{4640}{232} = 20\,mol,$$

$$n(CO) = \frac{2520}{28} = 90\,mol.$$

Masses cannot be compared directly because the reagents have different molar masses and stoichiometric coefficients.

Step 2: Find the limiting reagent and product mass Twenty moles of magnetite require $4 \times 20 = 80\,mol$ of CO. Since 90 mol is available, CO is in excess and magnetite is limiting. Equivalently, the reaction-extent limits are 20 and $90/4 = 22.5$ mol, of which 20 is smaller. Therefore

$$n(Fe) = 3 \times 20 = 60\,mol, \qquad m(Fe) = 60 \times 56 = 3360\,g.$$

There are 10 mol of unreacted CO, which confirms that all the magnetite can react.

Step 3: Compare with the options The calculated mass equals 3360 g, option (C). The values 1400 g and 2200 g are below the iron yield from complete consumption of the limiting magnetite, while

4200 g exceeds it. Using all 90 mol of CO to calculate iron would also overestimate the yield because there is insufficient magnetite for that extent.

Step 4: Conclusion The amount of iron produced is $\boxed{3360\,\text{g}}$. Hence, the correct option is (C).

Quick Tip to Solve: Divide moles by reaction coefficients before calculating product: the smaller quotient sets the reaction extent.

62. Which of the following statements are correct?

(A) The electronic configuration of Cr is [Ar] $3d^5$ $4s^1$.

(B) The magnetic quantum number may have a negative value.

(C) In the ground state of an atom, the orbitals are filled in order of their increasing energies.

(D) The total number of nodes are given by n − 2.

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A), (C) and (D) only

(B) (A) and (B) only

(C) (A) and (C) only

(D) (A), (B) and (C) only

Correct Answer: (D) (A), (B) and (C) only

Solution:

Concept:

- The chromium ground-state configuration is $[\text{Ar}]\,3d^5 4s^1$; the half-filled d subshell makes this more stable than the simple predicted $3d^4 4s^2$ arrangement.
- The magnetic quantum number takes integer values from $-l$ to $+l$, including zero. An orbital has $n-1$ total nodes: radial nodes plus angular nodes.

Step 1: Check statements A and B Statement (A) gives the experimentally established chromium configuration $[\text{Ar}]3d^5 4s^1$, so it is correct. The promotion of one 4s electron yields the particularly stable half-filled 3d subshell. Statement (B) is also correct: for instance, for a p orbital $l = 1$, so the allowed magnetic quantum numbers are $-1, 0, +1$. A negative magnetic quantum number is fully permitted; it describes orbital orientation rather than a negative principal energy level.

Step 2: Check statements C and D Statement (C) is the Aufbau filling principle for the atomic ground state: electrons occupy orbitals in increasing order of energy, subject to the Pauli principle and Hund rule. It is correct. For statement (D), the number of radial nodes is $n - l - 1$, and the number of angular nodes is l. Their sum is

$$(n - l - 1) + l = n - 1,$$

not $n - 2$. Thus (D) is false; a 1s orbital with zero nodes is a simple counterexample to the proposed formula.

Step 3: Select the complete combination The true statements are exactly (A), (B), and (C), which appear together in option (D). Option (A) wrongly includes statement (D), while options (B) and (C)

omit one of the other true statements. The question asks for the complete correct combination rather than just any subset of true statements.

Step 4: Conclusion The first three statements only are correct, while the proposed node count is false. Hence, the correct option is (D).

Quick Tip to Solve: For node-count questions, add radial nodes $n - l - 1$ and angular nodes l; the total always simplifies to $n - 1$.

63. The solubility of AgCl will be maximum in which of the following? [4 marks]

(A) 0.01 M KCl

(B) 0.01 M HCl

(C) 0.01 M $AgNO_3$

(D) Deionised water

Correct Answer: (D) Deionised water

Solution:

Concept:

- The dissolution equilibrium is $\mathrm{AgCl(s)} \rightleftharpoons \mathrm{Ag^+\,(aq) + Cl^-\,(aq)}$, with $K_{\mathrm{sp}} = [\mathrm{Ag^+}][\mathrm{Cl^-}]$ at fixed temperature.
- Adding a soluble salt that supplies either constituent ion shifts dissolution equilibrium toward the solid and lowers the solubility: this is the common-ion effect.

Step 1: Compare the chloride-containing solutions Both $0.01\,\mathrm{M}$ KCl and $0.01\,\mathrm{M}$ HCl supply approximately $0.01\,\mathrm{M}$ chloride ions before any AgCl dissolves. If the additional dissolved AgCl concentration is s, then

$$K_{\mathrm{sp}} = s(0.01 + s).$$

The large pre-existing chloride concentration suppresses dissolution compared with pure water. HCl is not expected to remove chloride by protonation because chloride is the conjugate base of a strong acid.

Step 2: Compare silver nitrate with deionised water Silver nitrate supplies the other common ion, $\mathrm{Ag^+}$, at $0.01\,\mathrm{M}$. Its solubility equation is again $K_{\mathrm{sp}} = s(0.01 + s)$. In deionised water neither constituent ion is initially present, so dissolution gives equal concentrations:

$$[\mathrm{Ag^+}] = [\mathrm{Cl^-}] = s_0, \qquad K_{\mathrm{sp}} = s_0^2.$$

For any positive s, the common-ion product $s(0.01 + s)$ is greater than s^2, hence it reaches the fixed solubility product at a smaller dissolved concentration than s_0.

Step 3: Choose the medium with the greatest solubility Options (A) and (B) suppress solubility through chloride; option (C) suppresses it through silver ions. Deionised water, option (D), has no such common-ion suppression and therefore has the greatest AgCl solubility among the listed media. This comparison uses the ordinary dilute-solution solubility-product treatment intended here.

Step 4: Conclusion The solubility of AgCl is maximum in deionised water. Hence, the correct option is (D).

Quick Tip to Solve: Look for either common ion, not just the anion: both chloride salts and silver salts suppress the dissolution of AgCl.

64. The electronic configuration of Pt (atomic number 78) is : [4 marks]

(A) [Xe] $4f^{14}$ $5d^{9}$ $6s^{1}$

(B) [Kr] $4f^{14}$ $5d^{10}$

(C) [Xe] $4f^{14}$ $5d^{10}$

(D) [Xe] $4f^{14}$ $5d^{8}$ $6s^{2}$

Correct Answer: (A) [Xe] $4f^{14}$ $5d^{9}$ $6s^{1}$

Solution:

Concept:

- The ground-state configuration is the electron distribution with the lowest total energy. The simple Aufbau order can have exceptions when the outer s and d levels are close in energy.
- The xenon core contains 54 electrons. Platinum has atomic number 78, so its neutral atom must contain 24 electrons beyond this core.

Step 1: Count the electrons outside the core The filled inner $4f$ subshell contributes 14 electrons, leaving ten to distribute between $5d$ and $6s$. Thus both $[\mathrm{Xe}]4f^{14}5d^{8}6s^{2}$ and $[\mathrm{Xe}]4f^{14}5d^{9}6s^{1}$ have the required electron count. Counting alone therefore cannot distinguish these two candidates.

Step 2: Use the observed ground-state arrangement For platinum the balance of the closely spaced $5d$ and $6s$ orbital energies and electron interactions favours $5d^{9}6s^{1}$. The actual configuration is consequently $[\mathrm{Xe}]4f^{14}5d^{9}6s^{1}$, rather than the naive $5d^{8}6s^{2}$ filling. This is an exceptional configuration; a rule that every d block atom must have two outer s electrons is unreliable.

Step 3: Reject the other configurations Option (B) has only $36 + 14 + 10 = 60$ electrons because its core is krypton, so it cannot describe neutral Pt. Option (C) has $54 + 14 + 10 = 78$ electrons, but places both outer electrons in the d subshell and is not the platinum ground state. Option (D) also totals 78, yet gives the uncorrected Aufbau prediction rather than the observed configuration.

Step 4: Conclusion The electronic configuration of platinum is $\boxed{[\mathrm{Xe}]4f^{14}5d^{9}6s^{1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the total electron count first, then recall the exceptional ground-state distribution of Pt; a correct total alone does not prove a configuration.

65. Two isomers 'A' and 'B' with molecular formula C_4H_8 give different products on oxidation with $KMnO_4$ in acidic medium. Isomer 'A' on reaction with $KMnO_4/H^+$ results in effervescence of a gas and gives ketone. The compound 'A' is [4 marks]

(A) But-1-ene.

(B) cis-But-2-ene.

(C) trans-But-2-ene.

(D) 2-methyl propene.

Correct Answer: (D) 2-methyl propene.

Solution:

Concept:

- Hot acidic $KMnO_4$ cleaves the carbon-carbon double bond. An alkene carbon bearing no hydrogen gives a ketone; a carbon bearing one hydrogen gives a carboxylic acid.
- A terminal $=CH_2$ carbon is oxidised completely to CO_2, whose evolution accounts for effervescence.

Step 1: Translate the product clues into a double bond The simultaneous formation of a ketone and carbon dioxide requires one double-bond carbon with two carbon substituents and the other as a terminal methylene group. The needed pattern is $R_2C{=}CH_2$. With four carbon atoms in total, the two groups on the substituted carbon must both be methyl groups.

Step 2: Identify the alkene and write its oxidation The structure is $(CH_3)_2C{=}CH_2$, named 2-methylpropene. Cleavage retains the two methyl groups on the substituted carbon, producing propanone, while the terminal carbon becomes carbon dioxide. A balanced overall oxygen-equivalent equation is

$$(CH_3)_2C{=}CH_2 + 4\,[O] \longrightarrow (CH_3)_2CO + CO_2 + H_2O.$$

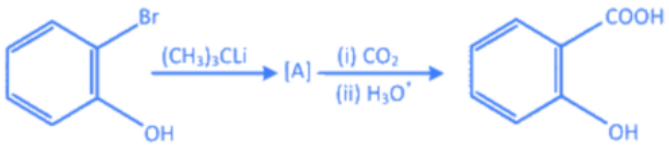
Thus both the ketone and gas conditions are fulfilled.

Step 3: Compare all remaining isomers But-1-ene also contains a terminal methylene group and can evolve carbon dioxide, but its other double-bond carbon has a hydrogen, so it yields propanoic acid rather than a ketone. Both cis-but-2-ene and trans-but-2-ene have one hydrogen on each alkene carbon. Their cleavage gives ethanoic acid, with neither the required ketone nor terminal-carbon carbon dioxide. Stereochemistry does not change this product classification.

Step 4: Conclusion Compound A is $\boxed{\text{2-methylpropene}}$, which yields propanone and CO_2. Hence, the correct option is (D).

Quick Tip to Solve: For oxidative cleavage, count hydrogens on each alkene carbon: zero gives a ketone, one gives an acid, and terminal CH2 gives carbon dioxide.

66. In the given conversion the compound A is : [4 marks]



(A) o-LiC_6H_4OH

(B) o-LiC_6H_4OLi

(C) o-$BrC_6H_4OC(CH_3)_3$

(D) o-$(CH_3)_3CC_6H_4OH$

Correct Answer: (B) o-LiC_6H_4OLi

Solution:

Concept:

- An organolithium reagent is both a strong base and a source of a nucleophilic carbon. A phenolic proton is removed rapidly to form a lithium phenoxide.
- Lithium-halogen exchange replaces an aryl bromine by lithium. The aryllithium carbon attacks CO_2, and acidic work-up gives a carboxylic acid.

Step 1: Account for the phenolic hydrogen The starting compound is ortho-bromophenol. tert-Butyllithium first removes its acidic $O-H$ proton, giving the O^-Li^+ group. An intermediate having a strongly basic aryl-lithium bond alongside an unchanged phenolic OH group would undergo proton transfer, so the structure with both aryl Li and OH is not the appropriate intermediate.

Step 2: Locate the carbon-lithium bond The bromine on the adjacent ring carbon undergoes lithium-halogen exchange with the organolithium reagent. Intermediate A therefore has lithium attached to that carbon and lithium also on the phenoxide oxygen: o-LiC_6H_4OLi. The two Li positions have different roles: carbon-bound Li marks the nucleophilic site, while oxygen-bound Li results from deprotonation.

Step 3: Check the stated final product and options The aryl carbon attacks carbon dioxide to give an ortho-carboxylate. Acidic work-up protonates both carboxylate and phenoxide, furnishing salicylic acid, o-HOC_6H_4COOH. Option (B) has precisely the two lithium positions needed. Option (A) incorrectly retains the acidic OH; (C) is an aryl tert-butyl ether retaining Br and has no carbon-lithium site for carboxylation; (D) replaces Br by tert-butyl and likewise cannot give the depicted carboxylic acid by this sequence.

Step 4: Conclusion A is the dilithiated intermediate $\boxed{\text{o-}LiC_6H_4OLi}$. Hence, the correct option is (B).

Quick Tip to Solve: Before using an organolithium reagent for carbon-carbon bond formation, check whether an OH or NH proton first consumes it by acid-base reaction.

67. Given below are two statements :

Statement I : The esterification of carboxylic acid with an alcohol is a nucleophilic acyl substitution.

Statement II : Electron withdrawing groups in the carboxylic acid will increase the rate of esterification reaction.

Choose the most appropriate option : [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (A) Both Statement I and Statement II are correct.

Solution:

Concept:

- In Fischer esterification a carboxylic acid reacts reversibly with an alcohol under acid catalysis to give an ester and water.
- Nucleophilic acyl substitution replaces a substituent at an acyl carbon through addition to the carbonyl followed by elimination. Electron withdrawal increases the electrophilic character of that carbon.

Step 1: Assess Statement I from the reaction mechanism Protonation activates the carboxylic-acid carbonyl oxygen. The alcohol oxygen attacks the carbonyl carbon, forming a tetrahedral intermediate. Proton transfers make water a suitable leaving group; elimination restores the carbonyl, and deprotonation gives the ester. The overall change replaces the acid hydroxyl group by the alcohol-derived alkoxy group, so it is a nucleophilic acyl substitution.

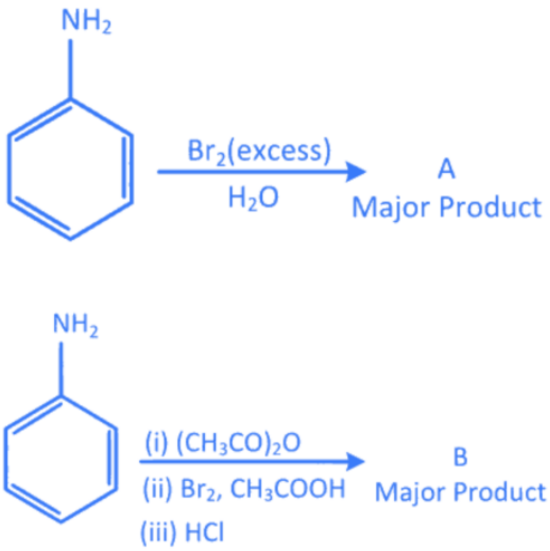
Step 2: Assess the electron-withdrawing effect An electron-withdrawing group draws electron density away from the carboxyl group through its inductive effect. Under comparable conditions this makes the carbonyl carbon more susceptible to nucleophilic attack by the alcohol and increases the esterification rate in the substituent comparison intended here. This kinetic effect concerns attack at the acyl carbon; it should not be confused with the equilibrium conversion, which also depends on water removal and reagent concentrations.

Step 3: Combine the truth values Statement I correctly classifies the addition-elimination mechanism. Statement II correctly states the activating effect of electron-withdrawing substituents on the acyl carbon in this comparison. Therefore the alternatives that reject either statement do not fit: (B) rejects both correct statements, (C) incorrectly rejects II, and (D) incorrectly rejects I.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: For acyl substitution, track the group replaced at the carbonyl carbon; for substituent effects, ask whether that carbon becomes more electrophilic.

68. Consider the above reactions, the product A and product B respectively are [4 marks]



(A) 4-bromoaniline and 4-bromoaniline

(B) 2,4,6-tribromoaniline and 2,4,6-tribromoaniline

(C) 2,4,6-tribromoaniline and 4-bromoaniline

(D) 4-bromoaniline and 2,4,6-tribromoaniline

Correct Answer: (C) 2,4,6-tribromoaniline and 4-bromoaniline

Solution:

Concept:

- The $-NH_2$ group strongly activates a benzene ring by resonance and directs electrophilic substitution to the ortho and para positions.
- Acetylation changes $-NH_2$ into $-NHCOCH_3$. The amide group remains ortho/para directing but activates the ring less strongly, allowing controlled bromination.

Step 1: Determine product A from direct bromination Aniline reacts readily with excess bromine water. Both ortho sites and the para site relative to the amino group are substituted, giving 2,4,6-tribromoaniline as the major product. The net substitution is

$$C_6H_5NH_2 + 3\,Br_2 \longrightarrow C_6H_2Br_3NH_2 + 3\,HBr.$$

Consequently A must be the tribrominated structure, which immediately excludes the options showing A as only para-bromoaniline.

Step 2: Follow the protected route to product B Acetic anhydride first converts aniline into acetanilide. In acetanilide the nitrogen lone pair is partly involved with the amide carbonyl, reducing its donation to the aromatic ring. Bromination in acetic acid then predominantly introduces one bromine at the para position, where steric crowding is less than at an ortho site. The final acidic hydrolysis removes the acetyl protecting group and restores $-NH_2$, yielding para-bromoaniline.

Step 3: Match the product pair in order The requested order is A followed by B: 2,4,6-tribromoaniline

followed by 4-bromoaniline. Option (C) shows this pair. Option (A) incorrectly assigns monobromination to the unprotected aniline. Option (B) incorrectly makes the protected route tribrominate as well. Option (D) reverses the direct and protected outcomes. The order matters even though both named products appear among the choices.

Step 4: Conclusion $\boxed{A = \text{2,4,6-tribromoaniline}, \quad B = \text{4-bromoaniline}}$. Hence, the correct option is (C).

Quick Tip to Solve: Protect aniline as acetanilide to control bromination; direct bromine water gives the 2,4,6-tribromo product, while protection favours para monobromination.

69. Sugar moiety in DNA and RNA molecules respectively are [4 marks]

(A) β-D-2-deoxyribose, β-D-deoxyribose.

(B) β-D-2-deoxyribose, β-D-ribose

(C) β-D-ribose, β-D-2-deoxyribose.

(D) β-D-deoxyribose, β-D-2-deoxyribose.

Correct Answer: (B) β-D-2-deoxyribose, β-D-ribose

Solution:

Concept:

- DNA is deoxyribonucleic acid and contains 2-deoxyribose; RNA is ribonucleic acid and contains ribose. Both sugars have five carbon atoms.
- In their nucleosides these sugars have the beta-D configuration and are present in the furanose form. The distinguishing oxygen is on the sugar carbon numbered $2'$.

Step 1: Identify the DNA sugar The sugar in DNA is beta-D-2-deoxyribose. The prefix 2-deoxy means that the hydroxyl group of ribose at the second carbon has been replaced by hydrogen. Thus the DNA sugar has H rather than OH at $2'$. The term deoxy refers to this oxygen difference, not to the loss of a carbon from the pentose skeleton.

Step 2: Identify the RNA sugar RNA contains beta-D-ribose and retains the $2'$-hydroxyl group. The nitrogenous base is linked to the anomeric $1'$ carbon by a beta glycosidic linkage. The sugars therefore differ at a specific position while maintaining the same five-carbon framework; RNA does not contain deoxyribose as its standard sugar.

Step 3: Respect the order and eliminate distractors The question asks for DNA first and RNA second, so the pair must be beta-D-2-deoxyribose followed by beta-D-ribose. Option (B) has this order. Option (A) incorrectly gives a deoxy sugar for RNA. Option (C) reverses the correct assignments, placing ribose in DNA. Option (D) again assigns a deoxy sugar to RNA and does not provide the required ribose second member. Recognising only one member of a pair is therefore insufficient.

Step 4: Conclusion DNA contains β-D-2-deoxyribose and RNA contains β-D-ribose. Hence, the correct option is (B).

Quick Tip to Solve: Use the name DNA to remember deoxyribose, then check the pair order: DNA lacks the 2-prime OH that is present in RNA.

70. Given below are two statements :

Statement I : Phenols are weakly acidic.

Statement II : Therefore they are freely soluble in NaOH solution and are weaker acids than alcohols and water.

Choose the most appropriate option : [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (C) Statement I is correct but Statement II is incorrect.

Solution:

Concept:

- Phenol is a weak acid: it ionises only partially in water, but its phenoxide conjugate base is stabilised by resonance.
- Acid strength increases when the conjugate base is more stable. Phenol is more acidic than water and ordinary aliphatic alcohols, whose alkoxide ions lack comparable resonance stabilisation.

Step 1: Evaluate Statement I Phenol can donate the hydrogen of its hydroxyl group to form phenoxide. The negative charge in phenoxide is delocalised over the oxygen and the aromatic system, making proton loss easier than from an aliphatic alcohol. Its approximate pK_a is 10, so it remains a weak acid rather than a completely ionised strong acid. Statement I is therefore correct.

Step 2: Separate the two claims in Statement II Phenol does dissolve in aqueous sodium hydroxide through acid-base reaction, forming the ionic salt sodium phenoxide:

$$C_6H_5OH + NaOH \longrightarrow C_6H_5ONa + H_2O.$$

However, the statement then says phenols are weaker acids than alcohols and water. This reverses the correct comparison. Water has pK_a about 15.7 and common aliphatic alcohols have values around 16 or higher; their larger values indicate weaker acids than phenol.

Step 3: Judge the complete statement A statement joined by "and" must have both claims correct to be accepted as a whole. Although sodium hydroxide solubility is correctly described, the acid-strength comparison is false, so Statement II is incorrect. Hence (A) fails because it accepts II, (B) fails because it rejects I, and (D) reverses the actual truth values. The correct combination accepts I and rejects II.

Step 4: Conclusion Statement I is correct but Statement II is incorrect: phenols are stronger acids than water and ordinary alcohols. Hence, the correct option is (C).

Quick Tip to Solve: A weak acid can still be stronger than another weak acid. Compare conjugate-base stability or pKa rather than treating the word weak as an absolute ranking.

71. Arrange the following in the decreasing order of their covalent character :

(A) LiCl

(B) NaCl

(C) KCl

(D) CsCl

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A) > (C) > (B) > (D)

(B) (B) > (A) > (C) > (D)

(C) (A) > (B) > (C) > (D)

(D) (A) > (B) > (D) > (C)

Correct Answer: (C) (A) > (B) > (C) > (D)

Solution:

Concept:

- Fajans' rules relate covalent character to the polarising power of a cation and the polarisability of an anion. Distortion of the anion electron cloud favours electron sharing.
- For cations having the same charge, a smaller ionic radius gives stronger polarising power. Comparing salts with the same anion therefore isolates the effect of cation size.

Step 1: Compare the ions being varied All four compounds are chlorides of alkali metals. Each has a singly charged metal cation and the same chloride anion, Cl^-. Thus neither anion size nor cation charge accounts for the differences. Down Group 1, added electron shells increase the ionic radius:

$$r(Li^+) < r(Na^+) < r(K^+) < r(Cs^+).$$

Lithium ion has the greatest charge density, while caesium ion has the least.

Step 2: Apply polarisation and compare the choices The smallest cation distorts the chloride electron cloud most strongly. Consequently, the covalent contribution decreases in the order

$$LiCl > NaCl > KCl > CsCl.$$

Translating the compound labels gives (A) greater than (B), then (C), then (D). Option (A) incorrectly puts potassium chloride before sodium chloride. Option (B) incorrectly places sodium chloride above lithium chloride. Option (D) incorrectly puts caesium chloride above potassium chloride. Only option (C) preserves every adjacent comparison.

Step 3: Conclusion The decreasing covalent-character order is $LiCl > NaCl > KCl > CsCl$. Hence, the correct option is (C).

Quick Tip to Solve: When the anion and cation charge are fixed, order the cations from smallest to largest; covalent character usually follows that same sequence.

72. Which of the following is a correct statement? [4 marks]

(A) Brownian motion destabilises sols.

(B) Any amount of dispersed phase can added to emulsion without destablising it.

(C) Mixing two oppositely charged sols in equal amount neutralises charges and stabilises colloids.

(D) Presence of equal and similar charges on colloidal particles provides stability to the colloidal solution.

Correct Answer: (D) Presence of equal and similar charges on colloidal particles provides stability to the colloidal solution.

Solution:

Concept:

- Particles in a stable sol generally carry charges of the same sign. Their electrostatic repulsion opposes aggregation and subsequent coagulation.
- Brownian motion is the irregular motion of colloidal particles due to collisions with molecules of the dispersion medium. It helps oppose settling rather than causing coagulation.

Step 1: Test the statements about motion and emulsions Option (A) reverses the usual role of Brownian motion. Random movement keeps the particles distributed and helps counter settling under gravity; it does not itself destabilise the sol. Option (B) makes an unlimited claim. An emulsion can remain stable only over a suitable range of composition and with suitable stabilisation. Adding arbitrarily large amounts of the dispersed phase can cause droplets to merge or the emulsion to break, so that statement cannot be accepted.

Step 2: Examine the role of surface charge When oppositely charged sols are mixed in suitable proportions, the charges can neutralise. This removes the electrostatic repulsion that previously kept particles apart. The particles may then aggregate, giving mutual coagulation rather than improved stability; hence option (C) is false. In contrast, equal and similar charges on the particles generate mutual repulsion. This suppresses close approach and aggregation, explaining why a sol with similarly charged particles remains stable. That is precisely the mechanism described in option (D).

Step 3: Conclusion Similar charges on colloidal particles produce repulsion and thereby help stabilise the colloidal solution. Hence, the correct option is (D).

Quick Tip to Solve: For sol stability, track repulsion: like surface charges preserve dispersion, while charge neutralisation encourages coagulation.

73. In isolation of which one of the following metals from their ores, the use of cyanide salt is not commonly involved? [4 marks]

(A) Zinc

(B) Gold

(C) Silver

(D) Copper

Correct Answer: (D) Copper

Solution:

Concept:

- Cyanide can form soluble coordination complexes with certain metals. In metallurgy it is used in cyanide leaching and also as a selective depressant during froth flotation.
- An extraction question can refer to ore concentration as well as metal production. A reagent need not reduce the metal ion directly in order to be involved in its isolation.

Step 1: Identify the familiar cyanide processes Gold and silver are commonly extracted by cyanide leaching. In the presence of air, the metal dissolves by forming a stable soluble dicyanido complex, such as $[Au(CN)_2]^-$ or $[Ag(CN)_2]^-$. The dissolved precious metal is subsequently recovered, for example by displacement with zinc. Thus both gold and silver are clearly associated with cyanide salts, eliminating options (B) and (C).

Step 2: Include concentration of zinc ore and select the exception Sodium cyanide also serves as a depressant in the selective froth flotation of sulphide ores. In a mixture containing zinc sulphide and lead sulphide, it prevents zinc sulphide from entering the froth by forming a cyanide complex, allowing selective separation. Therefore cyanide is involved in the isolation route of zinc, even though zinc is not normally obtained by precious-metal-style cyanide leaching. Common copper extraction proceeds through concentration, roasting/smelting and conversion, followed by refining; cyanide salt is not a common reagent in that standard route. Hence copper is the exception among the four listed metals.

Step 3: Conclusion The metal whose usual isolation from its ores does not commonly involve cyanide salt is copper. Hence, the correct option is (D).

Quick Tip to Solve: Do not eliminate zinc just because it is not cyanide-leached: sodium cyanide is a depressant used in separating zinc sulphide during froth flotation.

74. Which one of the following reactions indicates the reducing ability of hydrogen peroxide in basic medium? [4 marks]

(A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$

(C) $2MnO_4^- + 3H_2O_2 \rightarrow 2MnO_2 + 3O_2 + 2H_2O + 2OH^-$

(D) $Mn^{2+} + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$

Correct Answer: (C) $2MnO_4^- + 3H_2O_2 \rightarrow 2MnO_2 + 3O_2 + 2H_2O + 2OH^-$

Solution:

Concept:

- Oxygen has oxidation number -1 in H_2O_2. When peroxide acts as a reducing agent, its oxygen is oxidised to elemental O_2, where the oxidation number is zero.
- To identify the medium as well as the redox role, inspect the complete reaction. A basic-medium reduction of permanganate commonly yields MnO_2, with hydroxide appearing in the balanced equation.

Step 1: Identify oxidation of peroxide in the correct medium In option (C), peroxide produces oxygen gas and therefore loses electrons:

$$H_2O_2 + 2\,OH^- \longrightarrow O_2 + 2\,H_2O + 2\,e^-.$$

Simultaneously manganese in MnO_4^- is reduced from $+7$ to $+4$ in MnO_2. Combining the relevant half-reactions gives

$$2\,MnO_4^- + 3\,H_2O_2 \longrightarrow 2\,MnO_2 + 3\,O_2 + 2\,H_2O + 2\,OH^-.$$

The production of hydroxide and manganese dioxide is consistent with the basic-medium form specified.

Step 2: Rule out the remaining reactions Option (A) also shows peroxide yielding oxygen, but it produces H_3O^+ and is the acidic-medium representation, so it fails the medium requirement. In option (B), sulphur in PbS is oxidised from -2 to $+6$ in sulphate; peroxide is the oxidising agent. In option (D), manganese is oxidised from $+2$ to $+4$, so peroxide again acts as an oxidising agent. Merely finding hydroxide is insufficient: both the oxidation-state change and the medium must match.

Step 3: Conclusion Option (C) shows H_2O_2 reducing permanganate to MnO_2 while itself being oxidised to O_2 in basic medium. Hence, the correct option is (C).

Quick Tip to Solve: First check whether peroxide becomes oxygen gas; then check the medium. Peroxide forming water or hydroxide instead usually indicates its oxidising role.

75. Match List-I with List-II.

	List-I (Metal)		List-II (Emitted light wavelength (nm))
(A)	Li	(I)	670.8
(B)	Na	(II)	589.2
(C)	Rb	(III)	780.0
(D)	Cs	(IV)	455.5

Choose the most appropriate answer from the options given below: [4 marks]

(A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(B) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(C) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(D) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Correct Answer: (A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Solution:

Concept:

- Heating an alkali-metal compound excites its atoms. When the excited electrons return to lower energy levels, light of characteristic wavelengths is emitted.
- The photon energy and wavelength obey $E = hc/\lambda$, where h is Planck's constant and c is the speed of light. Flame-test line positions are characteristic atomic data rather than a simple monotonic group trend.

Step 1: Match lithium and sodium first Lithium gives its characteristic crimson-red emission near $670.8\,\text{nm}$, so the lithium entry (A) matches (I). Sodium gives an intense yellow emission, represented in this list by $589.2\,\text{nm}$, so (B) matches (II). The yellow sodium line is a particularly useful anchor because it is familiar and sharply distinguishes the sodium match from the red lithium line.

Step 2: Complete and verify the mapping For the remaining listed characteristic lines, rubidium corresponds to $780.0\,\text{nm}$, giving (C)-(III), while caesium corresponds to $455.5\,\text{nm}$, giving (D)-(IV). Thus all four pairs are (A)-(I), (B)-(II), (C)-(III), (D)-(IV). Option (B) interchanges the lithium and rubidium entries. Option (C) assigns the sodium wavelength to rubidium and the lithium wavelength to sodium. Option (D) assigns the caesium wavelength to lithium and the rubidium wavelength to caesium. The wavelength data should be matched directly; ordering wavelengths by atomic size would not reproduce the listed spectra.

Step 3: Conclusion The matches are Li–$670.8\,\text{nm}$, Na–$589.2\,\text{nm}$, Rb–$780.0\,\text{nm}$, and Cs–$455.5\,\text{nm}$. Hence, the correct option is (A).

Quick Tip to Solve: Anchor sodium at the yellow line near 589 nm and lithium at the crimson line near 671 nm before checking the remaining spectral-line matches.

76. Match List-I with List-II.

	List-I (Metal)		List-II (Application)
(A)	Cs	(I)	High temperature thermometer
(B)	Ga	(II)	Water repellent sprays
(C)	B	(III)	Photoelectric cells
(D)	Si	(IV)	Bullet proof vest

Choose the most appropriate answer from the options given below: [4 marks]

(A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(B) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(C) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(D) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

Correct Answer: (A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- An element's applications follow characteristic physical or chemical properties. Low ionisation energy favours photoelectric emission, while a wide liquid range favours thermometer use.
- An application listed against an element can involve one of its compounds. Boron-containing ceramics provide hard protective materials, while silicone compounds of silicon provide water-repellent coatings.

Step 1: Use the distinctive physical properties Caesium has a low ionisation energy and readily releases electrons when illuminated, making it suitable for photoelectric cells. Therefore (A) matches (III). Gallium melts close to room temperature but has a very high boiling point, so it remains liquid over a wide temperature range. This makes it useful in high-temperature thermometry, giving (B)-(I). These two property-based matches already separate the correct mapping from the alternatives.

Step 2: Identify the materials applications and reject alternatives Boron carbide is an extremely hard, lightweight ceramic used in protective armour, including bullet-resistant vests; the boron entry therefore matches (IV). Silicone materials, based on silicon-oxygen frameworks with organic groups, are used in water-repellent sprays, giving (D)-(II). The completed mapping is (A)-(III), (B)-(I), (C)-(IV), (D)-(II). Option (B) incorrectly assigns caesium to protective armour and gallium to photoelectric cells. Option (C) incorrectly assigns caesium to water repellents and silicon to thermometry. Option (D) assigns all four entries to inappropriate applications. Only option (A) is consistent with all four materials and their properties.

Step 3: Conclusion Cs matches photoelectric cells, Ga matches high-temperature thermometers, B matches bullet-proof vests, and Si matches water-repellent sprays. Hence, the correct option is (A).

Quick Tip to Solve: Recall two anchors: easy electron emission links Cs to photocells, and a broad liquid range links Ga to high-temperature thermometers.

77. The oxoacid of phosphorus that is easily obtained from a reaction of alkali and white phosphorus and has two P-H bonds, is : [4 marks]

(A) Phosphonic acid

(B) Phosphinic acid

(C) Pyrophosphorus acid

(D) Hypophosphoric acid

Correct Answer: (B) Phosphinic acid

Solution:

Concept:

- In phosphorus oxoacids, hydrogen directly bonded to oxygen is ionisable, while hydrogen directly bonded to phosphorus is not normally released as an acidic proton.
- Hypophosphorous acid, systematically called phosphinic acid, has formula H_3PO_2 and connectivity $H_2P(=O)OH$. It contains two phosphorus-hydrogen bonds and one hydroxyl group.

Step 1: Follow the reaction of white phosphorus with alkali White phosphorus reacts with hot aqueous alkali by disproportionation, producing phosphine and a hypophosphite salt. With sodium hydroxide the balanced reaction is

$$P_4 + 3\,NaOH + 3\,H_2O \longrightarrow PH_3 + 3\,NaH_2PO_2.$$

The salt NaH_2PO_2 corresponds to the acid H_3PO_2, obtained on acidification. Phosphorus is reduced in phosphine and oxidised in hypophosphite, explaining the two different products.

Step 2: Check the bonds and distinguish the names The acid H_3PO_2 has two P–H bonds, one P=O bond and one P–OH bond. Therefore it satisfies the stated bond-count condition and is named phosphinic acid. Phosphonic acid, H_3PO_3, has only one P–H bond and two hydroxyl groups, so option (A) fails. Pyrophosphorous acid is a condensed oxoacid rather than the hypophosphite product of this reaction. Hypophosphoric acid, $H_4P_2O_6$, contains a phosphorus-phosphorus bond and does not have the required two P–H bonds. Both the preparation and connectivity thus identify the same acid.

Step 3: Conclusion The required oxoacid is phosphinic acid (hypophosphorous acid), H_3PO_2, with two P–H bonds. Hence, the correct option is (B).

Quick Tip to Solve: Count P–H and P–OH bonds separately: hypophosphorous acid has two P–H bonds but only one ionisable hydroxyl hydrogen.

78. The acid that is believed to be mainly responsible for the damage of Taj Mahal is : [4 marks]

(A) sulfuric acid.

(B) hydrofluoric acid.

(C) phosphoric acid.

(D) hydrochloric acid.

Correct Answer: (A) sulfuric acid.

Solution:

Concept:

- Acid rain forms when atmospheric oxides of sulphur and nitrogen are oxidised and dissolve in water. The acids attack carbonate stone rather than merely depositing dust on its surface.
- The marble of the Taj Mahal contains calcium carbonate, $CaCO_3$. An acid reacts with carbonate to form a salt, carbon dioxide and water, progressively damaging the marble.

Step 1: Identify the relevant atmospheric acid Sulphur dioxide released by industrial activity is oxidised in air and ultimately produces sulphuric acid in atmospheric moisture. The sequence can be represented by the balanced reactions

$$2\,SO_2 + O_2 \longrightarrow 2\,SO_3, \qquad SO_3 + H_2O \longrightarrow H_2SO_4.$$

Rain containing this acid comes into contact with the marble surface. This connects sulphur pollution with the deterioration called marble cancer.

Step 2: Connect the acid with marble damage The carbonate reacts according to

$$CaCO_3 + H_2SO_4 \longrightarrow CaSO_4 + CO_2 + H_2O.$$

The conversion of the original marble into calcium sulphate damages its surface; hydrated sulphate deposits may also form. Hydrofluoric acid is well known for attacking glass, but it is not the main acid implicated here. Phosphoric acid is not the characteristic sulphur-pollution product, and hydrochloric acid is likewise not the principal cause identified for this monument.

Step 3: Conclusion The acid mainly responsible is sulphuric acid, H_2SO_4. Hence, the correct option is (A).

Quick Tip to Solve: For marble damage, first identify the carbonate in the stone and then connect sulphur oxide pollution with sulphuric acid.

79. The polymer, which can be stretched and remains its original status on releasing the force is [4 marks]

(A) Bakelite.

(B) Nylon 6, 6.

(C) Buna-N.

(D) Terylene.

Correct Answer: (C) Buna-N.

Solution:

Concept:

- An elastomer is a polymer that can undergo substantial reversible stretching. Its chains extend under an applied force and tend to return to their coiled arrangement when the force is removed.
- Weak intermolecular attractions allow chain movement, while a limited number of cross-links prevent permanent sliding and help the stretched material recover its original dimensions.

Step 1: Identify the polymer class described The stated property is elastic recovery: the material can be stretched and returns to its original state after release. This describes an elastomer rather than a rigid thermosetting resin or a highly oriented fibre. Reversible extension requires flexible chains and an arrangement that does not permanently lock them into the stretched configuration.

Step 2: Classify the four listed materials Buna-N is a synthetic rubber produced by copolymerising buta-1,3-diene and acrylonitrile. It belongs to the elastomer class and therefore has the required rubber-like response. Bakelite is a heavily cross-linked phenol-formaldehyde thermosetting polymer; its rigid network is unsuitable for large reversible stretching. Nylon 6,6 is a polyamide fibre with strong interchain hydrogen bonding. Terylene is a polyester fibre with comparatively strong intermolecular attractions and oriented chains. These fibre-forming materials are valued for tensile strength rather than the large reversible deformation characteristic of rubber.

Step 3: Conclusion Buna-N is the elastomer that can be stretched and recover its original state on releasing the force. Hence, the correct option is (C).

Quick Tip to Solve: Use the class distinction: Buna rubbers are elastomers, Bakelite is a thermoset, and nylon and Terylene are fibres.

80. Which of the following compound does not contain sulphur atom? [4 marks]

(A) Cimetidine

(B) Ranitidine

(C) Histamine

(D) Saccharin

Correct Answer: (C) Histamine

Solution:

Concept:

- The presence of sulphur is determined by the molecular structure, not by whether a substance is a drug or by its therapeutic use. A sulphur atom may occur in a thioether linkage or in a sulphonyl group.
- Histamine consists of an imidazole ring with an ethylamine side chain. Its molecular formula is $C_5H_9N_3$, so its constituent elements are carbon, hydrogen and nitrogen.

Step 1: Identify the sulphur-free structure Histamine has two nitrogen atoms in its five-membered imidazole ring and a third nitrogen in its amino side chain. There is no sulphur-containing substituent or linkage. Its formula $C_5H_9N_3$ provides a direct check: the element symbol S does not occur. Thus histamine meets the demand that the compound contain no sulphur atom.

Step 2: Eliminate the sulphur-containing alternatives Cimetidine contains a sulphur atom in a thioether linkage connecting parts of its side chain. Ranitidine also contains a thioether sulphur atom, even though its heterocyclic ring itself is a furan ring. Saccharin contains a sulphonyl group, $-SO_2-$, within its cyclic sulphonamide structure. Each of these three compounds therefore contains sulphur. The relevant distinction is the actual atom present, so the fact that cimetidine and ranitidine are antihistaminic drugs does not make their elemental compositions identical to histamine.

Step 3: Conclusion Histamine does not contain a sulphur atom. Hence, the correct option is (C).

Quick Tip to Solve: Remember histamine as imidazole plus an ethylamine side chain; neither fragment contains sulphur.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. 17.0 g of NH_3 completely vapourises at -33.42°C and 1 bar pressure and the enthalpy change in the process is 23.4 kJ mol^{-1}. The enthalpy change for the vapourisation of 85 g of NH_3 under the same conditions is _____ kJ. [4 marks]

Solution:

Concept:

- The molar enthalpy of vaporisation, $\Delta_{vap}H_m$, is the enthalpy required to convert one mole of liquid into vapour at the stated temperature and pressure.
- Enthalpy is extensive. At identical conditions, vaporising n moles requires $\Delta H = n\Delta_{vap}H_m$. The

sign is positive because vaporisation absorbs energy.

Step 1: Convert the ammonia mass to moles The molar mass of ammonia is

$$M(\mathrm{NH_3}) = 14 + 3(1) = 17\,\mathrm{g\,mol^{-1}}.$$

Thus the original 17.0 g is one mole, consistent with the stated molar enthalpy. For the required sample,

$$n = \frac{85\,\mathrm{g}}{17\,\mathrm{g\,mol^{-1}}} = 5\,\mathrm{mol}.$$

The larger sample contains five times as many molecules undergoing the same phase change.

Step 2: Scale the molar enthalpy to the sample Both samples vaporise completely at the same temperature and pressure, so the enthalpy per mole remains $23.4\,\mathrm{kJ\,mol^{-1}}$. Therefore

$$\Delta H = (5\,\mathrm{mol})(23.4\,\mathrm{kJ\,mol^{-1}}) = 117\,\mathrm{kJ}.$$

The mole unit cancels, leaving the total enthalpy in kilojoules. No temperature correction or heat-capacity term is needed: the problem changes the amount of ammonia, not the conditions of the phase transition. The negative Celsius temperature does not make the vaporisation enthalpy negative.

Step 3: Conclusion The enthalpy change for vaporising 85 g of $\mathrm{NH_3}$ is $\boxed{+117\,\mathrm{kJ}}$; the required numerical entry is 117.

Quick Tip to Solve: At unchanged conditions, scale the phase-change enthalpy by the mass ratio: here $85/17 = 5$.

82. 1.2 mL of acetic acid is dissolved in water to make 2.0 L of solution. The depression in freezing point observed for this strength of acid is 0.0198°C. The percentage of dissociation of the acid is _____. (Nearest integer)

[Given : Density of acetic acid is 1.02 g mL^{-1}, Molar mass of acetic acid is 60 g mol^{-1}, $K_f(H_2O)$ = 1.85 K kg mol^{-1}] [4 marks]

Solution:

Concept:

- For a dilute solution, freezing-point depression is $\Delta T_f = iK_f m$, where m is molality and i is the van't Hoff factor counting the effective dissolved particles.
- Acetic acid dissociates as $\mathrm{CH_3COOH} \rightleftharpoons \mathrm{H^+} + \mathrm{CH_3COO^-}$. One initial particle gives two on dissociation, so for degree of dissociation α, $i = 1 + \alpha$.

Step 1: Calculate the analytical molality The mass of acetic acid is

$$w = (1.2\,\mathrm{mL})(1.02\,\mathrm{g\,mL^{-1}}) = 1.224\,\mathrm{g}.$$

Its amount is

$$n = \frac{1.224}{60} = 0.0204\,\mathrm{mol}.$$

For this very dilute aqueous solution, 2.0 L corresponds to approximately 2.0 kg of solvent. The tiny solute amount does not appreciably change this conversion. Hence

$$m \simeq \frac{0.0204}{2.0} = 0.0102\,\text{mol kg}^{-1}.$$

Step 2: Obtain the dissociated fraction from the particle count Without dissociation, the predicted depression would be

$$K_f m = (1.85)(0.0102) = 0.01887\,\text{K}.$$

Using the measured depression,

$$i = \frac{0.0198}{0.01887} = 1.04928\ldots.$$

Therefore

$$\alpha = i - 1 = 0.04928\ldots, \qquad 100\alpha = 4.928\ldots\%.$$

The observed depression is slightly larger than the undissociated value, as expected when a weak acid forms additional ions. Rounding is performed only after calculating the percentage.

Step 3: Conclusion The percentage dissociation is approximately 4.93%, which rounds to $\boxed{5\%}$. The required numerical entry is 5.

Quick Tip to Solve: For a weak acid yielding two ions, use $\alpha = \Delta T_f/(K_f m) - 1$, then multiply by 100 before rounding.

83. A dilute solution of sulphuric acid is electrolysed using a current of 0.10 A for 2 hours to produce hydrogen and oxygen gas. The total volume of gases produced a STP is _____ cm^3. (Nearest integer)

[Given : Faraday constant F = 96500 C mol^{-1} at STP, molar volume of an ideal gas is 22.7 L mol^{-1}]

[4 marks]

Solution:

Concept:

- Faraday's law relates the amount of electrons transferred to charge: $n_e = Q/F$, with $Q = It$. Here $F = 96500\,\text{C mol}^{-1}$.
- In acidic water electrolysis, two electrons produce one molecule of hydrogen and four electrons correspond to one molecule of oxygen. Thus $n(\text{H}_2) = Q/(2F)$ and $n(\text{O}_2) = Q/(4F)$.

Step 1: Calculate the charge passed through the electrolyte Convert the duration to seconds before using the current in amperes:

$$t = 2(60)(60) = 7200\,\text{s}.$$

The charge is

$$Q = (0.10\,\text{C s}^{-1})(7200\,\text{s}) = 720\,\text{C}.$$

The sulphuric acid provides ions for conduction; the net gaseous products arise from water decomposition. The cathode reaction is $2\,\text{H}^+ + 2\,\text{e}^- \longrightarrow \text{H}_2$, and the anode reaction is $2\,\text{H}_2\text{O} \longrightarrow$

$O_2 + 4\,H^+ + 4\,e^-$.

Step 2: Add the gas amounts and convert to volume The same charge passes through both electrodes, so

$$n_{\text{total}} = \frac{Q}{2F} + \frac{Q}{4F} = \frac{3Q}{4F} = \frac{3(720)}{4(96500)}\ \text{mol}.$$

Using the specified STP molar volume rather than replacing it by 22.4 L,

$$V = \frac{2160}{386000}(22.7)(1000) = 127.026\ldots\ \text{cm}^3.$$

The factor 1000 converts litres to cubic centimetres. Hydrogen contributes twice the volume of oxygen, in agreement with the overall reaction $2\,H_2O \longrightarrow 2\,H_2 + O_2$.

Step 3: Conclusion The total gas volume at STP, rounded to the nearest integer, is $\boxed{127\,\text{cm}^3}$. The required numerical entry is 127.

Quick Tip to Solve: For water electrolysis, add both gases as $3Q/(4F)$ moles and use the molar volume explicitly supplied.

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84. The activation energy of one of the reactions in a biochemical process is 532611 J mol^{-1}. When the temperature falls from 310 K to 300 K, the change in rate constant observed is $k_{300} = x \times 10^{-3}\,k_{310}$. The value of x is _____.

[Given : $\ln 10 = 2.3$**, R = 8.3 J K^{-1} mol^{-1}]** [4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$. For the same reaction with constant activation energy and pre-exponential factor over the temperature interval, it gives a ratio independent of A.
- For cooling from T_h to T_c, $\ln(k_c/k_h) = -(E_a/R)(1/T_c - 1/T_h)$. A positive activation energy makes the rate constant smaller at the lower temperature.

Step 1: Insert the two temperatures with the correct sign Here the colder temperature is 300 K and the warmer temperature is 310 K. Therefore

$$\ln\frac{k_{300}}{k_{310}} = -\frac{532611}{8.3}\left(\frac{1}{300} - \frac{1}{310}\right).$$

The reciprocal-temperature difference is positive:

$$\frac{1}{300} - \frac{1}{310} = \frac{10}{93000} = \frac{1}{9300}\ \text{K}^{-1}.$$

Consequently the logarithm must be negative, which is a useful sign check for a cooling process.

Step 2: Convert the logarithm into the requested multiplier Simplifying with the supplied constants

gives

$$\ln \frac{k_{300}}{k_{310}} = -\frac{532611}{8.3(9300)} = -6.9.$$

Since the problem supplies $\ln 10 = 2.3$, this is $-3 \ln 10$. Hence

$$\frac{k_{300}}{k_{310}} = e^{-3 \ln 10} = 10^{-3}.$$

The energy and gas-constant units cancel consistently because both are expressed in joules per mole. Comparing this result with $k_{300} = x \times 10^{-3} k_{310}$ now isolates the dimensionless factor x.

Step 3: Conclusion The rate constant decreases by a factor of 10^3, so $\boxed{x = 1}$. The requested value is dimensionless.

Quick Tip to Solve: On cooling, check that $k_c/k_h < 1$; convert a logarithm of -6.9 to $-3 \ln 10$ using the given approximation.

85. The number of terminal oxygen atoms present in the product B obtained from the following reaction is _____.

$FeCr_2O_4 + Na_2CO_3 + O_2 \rightarrow A + Fe_2O_3 + CO_2$

$A + H^+ \rightarrow B + H_2O + Na^+$ [4 marks]

Solution:

Concept:

- Roasting chromite ore, $FeCr_2O_4$, with sodium carbonate in oxygen oxidises chromium from +3 to +6 and produces sodium chromate.
- Acidification converts chromate ions to dichromate ions: $2\,{CrO_4}^{2-} + 2\,H^+ \longrightarrow {Cr_2O_7}^{2-} + H_2O$. A terminal oxygen is bonded to only one chromium atom; a bridging oxygen joins both chromium centres.

Step 1: Identify the chromate formed during roasting The alkaline oxidative fusion of chromite produces the yellow chromate salt A. The balanced reaction is

$$4\,FeCr_2O_4 + 8\,Na_2CO_3 + 7\,O_2 \longrightarrow 8\,Na_2CrO_4 + 2\,Fe_2O_3 + 8\,CO_2.$$

Thus A is Na_2CrO_4, not a lower oxide of chromium. Iron appears separately as ferric oxide, as the given scheme requires.

Step 2: Identify B after acidification Adding acid to chromate shifts the chromate–dichromate equilibrium towards the orange dichromate ion:

$$2\,{CrO_4}^{2-} + 2\,H^+ \longrightarrow {Cr_2O_7}^{2-} + H_2O.$$

The sodium ions are spectators in this change. Therefore the chromium-containing product B is dichromate, ${Cr_2O_7}^{2-}$. The chromium oxidation state remains +6 in this second reaction; acidification changes the oxoanion's aggregation, not its oxidation state.

Step 3: Count the terminal oxygen atoms Dichromate consists of two chromium-centred tetrahedra sharing one oxygen vertex. The shared oxygen has Cr–O–Cr connectivity and is the only bridging oxygen. Each chromium also has three oxygen atoms bonded only to that chromium. Consequently,

$$N_{\text{terminal O}} = 3 + 3 = 7 - 1 = 6.$$

Count atoms, rather than the number of Cr–O bonds: the bridging atom is shared by the tetrahedra and must not be counted twice.

Step 4: Conclusion Product B is $Cr_2O_7^{2-}$, which contains $\boxed{6}$ terminal oxygen atoms. The required numerical entry is 6.

Quick Tip to Solve: For dichromate, picture two tetrahedra sharing one corner: one bridging oxygen leaves six terminal oxygens.

86. An acidified manganate solution undergoes disproportionation reaction. The spin-only magnetic moment value of the product having manganese in higher oxidation state is _____ B.M. (Nearest integer) [4 marks]

Solution:

Concept:

- Disproportionation is simultaneous oxidation and reduction of the same element. Acidified manganate(VI), MnO_4^{2-}, yields permanganate(VII) and manganese(IV) oxide.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons. An ion with no unpaired electrons is diamagnetic.

Step 1: Identify the two disproportionation products Manganese is initially in the +6 state in manganate because four oxygen atoms contribute a total charge of -8 while the ion has charge -2. On acidification the balanced disproportionation is

$$3\,MnO_4^{2-} + 4\,H^+ \longrightarrow 2\,MnO_4^- + MnO_2 + 2\,H_2O.$$

The equation conserves three manganese atoms and twelve oxygen atoms. Its net charge is -2 on each side, which also verifies the acid and water coefficients.

Step 2: Select the product with the higher oxidation state For permanganate, $x - 8 = -1$, giving manganese oxidation state +7. For MnO_2, $x - 4 = 0$, giving +4. Therefore the requested product is MnO_4^-, not manganese dioxide. Two manganese atoms are oxidised from +6 to +7 while one is reduced from +6 to +4; the electron loss and gain are both two.

Step 3: Determine the electronic configuration and moment Neutral manganese has configuration $[Ar]3d^5 4s^2$. In the +7 oxidation state it has a formal d^0 configuration: the two 4s electrons and five 3d electrons have been removed in ionic electron counting. There are consequently no unpaired d electrons, so $n = 0$. Substitution into the specified spin-only expression gives

$$\mu = \sqrt{0(0+2)} = 0 \text{ B.M.}$$

No high-spin versus low-spin distinction is needed for a d^0 species.

Step 4: Conclusion The higher-oxidation-state product is permanganate(VII), with spin-only magnetic moment $\boxed{0\ \text{B.M.}}$. The nearest-integer entry is 0.

Quick Tip to Solve: First choose Mn(VII), rather than Mn(IV); a d-zero ion has zero spin-only magnetic moment without any ligand-field calculation.

87. Kjeldahl's method was used for the estimation of nitrogen in an organic compound. The ammonia evolved from 0.55 g of the compound neutralised 12.5 mL of 1 M H_2SO_4 solution. The percentage of nitrogen in the compound is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- In Kjeldahl analysis, nitrogen in the sample is converted to ammonia. Each mole of collected NH_3 corresponds to one mole of nitrogen atoms in the original compound.
- Sulfuric acid is dibasic: $2\,NH_3 + H_2SO_4 \longrightarrow (NH_4)_2SO_4$. Hence one mole of H_2SO_4 neutralises two moles of ammonia, and the nitrogen percentage is $100 m_N / m_{\text{sample}}$.

Step 1: Calculate the moles of sulfuric acid consumed The stated volume is 12.5 mL, so it must first be converted to litres when using molarity. The number of moles of acid actually neutralised by the evolved ammonia is

$$n(H_2SO_4) = CV = 1 \times \frac{12.5}{1000} = 0.0125 \text{ mol}.$$

The question gives the acid volume neutralised directly; there is no excess-acid back-titration to subtract.

Step 2: Use the acid–ammonia stoichiometric ratio The balanced neutralisation equation is

$$2\,NH_3 + H_2SO_4 \longrightarrow (NH_4)_2SO_4.$$

Therefore

$$n(NH_3) = 2(0.0125) = 0.0250 \text{ mol}.$$

Because every ammonia molecule contains one nitrogen atom, this is also the amount of nitrogen atoms in the 0.55 g organic sample. Using a 1:1 acid-to-ammonia ratio would omit one acidic proton of sulfuric acid and halve the answer.

Step 3: Convert nitrogen mass to percentage and round Taking the atomic molar mass of nitrogen as $14\ \text{g mol}^{-1}$,

$$m_N = 0.0250 \times 14 = 0.350 \text{ g}.$$

The nitrogen mass fraction is measured relative to the original organic compound, not to the acid solution:

$$\%N = \frac{0.350}{0.55} \times 100 = 63.636\ldots\%.$$

Retain the unrounded fraction until this final step, then round to the nearest integer as requested.

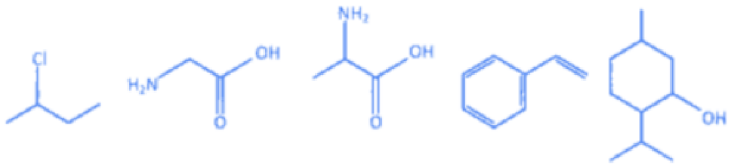
Step 4: Conclusion The nitrogen content is approximately 63.64%, so the required nearest-integer percentage is $\boxed{64\%}$, with numerical entry 64.

Quick Tip to Solve: For Kjeldahl calculations with sulfuric acid, count two ammonia molecules for every acid molecule before finding the nitrogen mass.

88. Observe structures of the following compounds

The total number of structures/compounds which possess asymmetric carbon atoms is _____.

[4 marks]



Solution:

Concept:

- An asymmetric carbon is a tetrahedral, sp^3-hybridised carbon attached to four different substituents. A carbon with two identical groups cannot be asymmetric.
- For a carbon in a substituted ring, compare the two directions around the ring as complete paths. Count compounds possessing at least one asymmetric carbon, rather than the total number of asymmetric carbon centres.

Step 1: Examine the first three structures from left to right The first compound is 2-chlorobutane. Its chlorine-bearing carbon is attached to Cl, H, methyl and ethyl groups, so all four substituents differ: this compound qualifies. The second compound is glycine, H_2NCH_2COOH. Its methylene carbon has two hydrogen atoms and its carboxyl carbon is trigonal planar, so it has no asymmetric carbon. The third compound is alanine, $CH_3CH(NH_2)COOH$. Its central carbon bears H, amino, methyl and carboxyl groups, so this compound qualifies.

Step 2: Check the aromatic and cyclic structures The fourth compound is styrene, $C_6H_5CH{=}CH_2$. Both the benzene-ring carbons and the alkene carbons are sp^2 hybridised. Thus none can be a tetrahedral asymmetric carbon. The last structure is the substituted cyclohexanol shown. At the carbon bearing OH, the four attachments are OH, H and the two ring paths. Those paths differ because one direction encounters the isopropyl-bearing carbon immediately, whereas the other direction encounters a differently substituted part of the ring. Hence this structure possesses at least one asymmetric carbon.

Step 3: Count qualifying compounds once each The qualifying structures are the first (2-chlorobutane), third (alanine) and fifth (the substituted cyclohexanol). Glycine and styrene do not qualify. Thus

$$N_{\text{compounds with asymmetric carbon}} = 1 + 1 + 1 = 3.$$

The cyclic compound may contain more than one such centre, but that does not increase the requested

count because the question asks for the number of structures or compounds.

Step 4: Conclusion Three of the displayed compounds possess asymmetric carbon atoms. The required numerical entry is $\boxed{3}$.

Quick Tip to Solve: Test each compound for one tetrahedral carbon with four different groups; in rings compare both paths, then count the compound only once.

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89. $C_6H_{12}O_6 \xrightarrow{\text{Zymase}} A \xrightarrow[\Delta]{NaOI} B + CHI_3$

The number of carbon atoms present in the product B is _____. [4 marks]

Solution:

Concept:

- Zymase catalyses the fermentation of glucose to ethanol and carbon dioxide: $C_6H_{12}O_6 \longrightarrow 2\,C_2H_5OH + 2\,CO_2$.
- Ethanol undergoes the iodoform reaction with alkaline iodine or hypoiodite. It is oxidised to ethanal, whose methyl group becomes CHI_3, leaving the remaining carbon as formate.

Step 1: Identify the product of glucose fermentation The first reagent is zymase, the enzyme mixture responsible for alcoholic fermentation. Its organic product A is ethanol:

$$C_6H_{12}O_6 \xrightarrow{\text{Zymase}} 2\,CH_3CH_2OH + 2\,CO_2.$$

Although glucose has six carbon atoms, they are distributed between two ethanol molecules and two carbon dioxide molecules. Therefore A is a two-carbon compound; carrying all six glucose carbons into the next reaction would give the wrong count.

Step 2: Follow ethanol through the iodoform reaction Sodium hypoiodite provides the alkaline oxidising and iodinating conditions of the iodoform test. Ethanol is first oxidised to ethanal, CH_3CHO. Its methyl hydrogens are replaced by iodine, followed by cleavage of the triiodomethyl group as iodoform, CHI_3. The other product is sodium formate, $HCOONa$. A balanced overall hypoiodite equation is

$$CH_3CH_2OH + 4\,NaOI \longrightarrow HCOONa + CHI_3 + NaI + 2\,NaOH + H_2O.$$

This accounts for all four iodine atoms: three appear in iodoform and one remains as iodide when the usual ionic formulation is used.

Step 3: Count carbon atoms in B The ethanol carbon bonded to oxygen becomes the carbon of the formate group. The other ethanol carbon is removed into CHI_3. Thus B is $HCOONa$, which has a single carbon atom. Its sodium ion and the two oxygen atoms do not contribute to the carbon count. The two carbons of each ethanol molecule are conserved as one in formate and one in iodoform.

Step 4: Conclusion B is sodium formate, $HCOONa$, so it contains $\boxed{1}$ carbon atom. The required numerical entry is 1.

Quick Tip to Solve: Fermentation gives ethanol; the iodoform reaction removes its methyl carbon as CHI3 and leaves the other carbon in formate.

90. Geraniol, a volatile organic compound, is a component of rose oil. The density of the vapour is 0.46 gL^{-1} at 257°C and 100 mm Hg. The molar mass of geraniol is _____ g mol^{-1}. (Nearest Integer)

[Given : R = 0.082 L atm K^{-1} mol^{-1}] [4 marks]

Solution:

Concept:

- The ideal gas equation is $PV = nRT = (m/M)RT$. With mass density $d = m/V$, rearrangement gives $M = dRT/P$.
- When $R = 0.082\ \mathrm{L\,atm\,K^{-1}\,mol^{-1}}$, use density in $\mathrm{g\,L^{-1}}$, pressure in atmospheres and absolute temperature in kelvin. The resulting molar mass is in $\mathrm{g\,mol^{-1}}$.

Step 1: Convert the temperature and pressure consistently The temperature appearing in the gas equation must be absolute:

$$T = 257 + 273 = 530\ \mathrm{K}.$$

Convert the given mercury pressure using $760\ \mathrm{mm\,Hg} = 1\ \mathrm{atm}$:

$$P = \frac{100}{760}\ \mathrm{atm}.$$

The density is already given as $0.46\ \mathrm{g\,L^{-1}}$, matching the litre unit in the supplied gas constant. Using 100 directly with this R would mix mm Hg and atmospheres.

Step 2: Derive and evaluate the molar mass From $PV = (m/M)RT$, divide by volume and solve for M:

$$M = \frac{mRT}{PV} = \frac{dRT}{P}.$$

Substitute the converted quantities without rounding the pressure prematurely:

$$\begin{aligned} M &= \frac{0.46 \times 0.082 \times 530}{100/760} \\ &= 0.46 \times 0.082 \times 530 \times 7.6 \\ &= 151.93776\ \mathrm{g\,mol^{-1}}. \end{aligned}$$

The factor $760/100$ multiplies the numerator because pressure is in the denominator. This also makes physical sense: at a fixed temperature and measured density, a lower pressure corresponds to a greater molecular molar mass.

Step 3: Apply the requested final rounding The question asks for the nearest integer, so round only the final molar mass. The value 151.93776 is nearer to 152 than to 151. The result is a molar mass, not the dimensionless relative vapour density sometimes defined with respect to hydrogen or air; no extra factor of two should be applied to the stated mass-per-volume density.

Step 4: Conclusion The molar mass of geraniol is $\boxed{152\ \mathrm{g\,mol^{-1}}}$ to the nearest integer. The required numerical entry is 152.

Quick Tip to Solve: Use M=dRT/P with kelvin and atmospheres; the given density is mass per litre, so do not multiply it by two as a relative vapour density.