JEE Main 2022 Paper 1 (B.E./B.Tech) Jun 29 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 1 (June), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 29 June 2022, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then, the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to : [4 marks]

(A) 1

(B) α

(C) $1 + \alpha$

(D) $1 + 2\alpha$

Correct Answer: (A) 1

Solution:

Concept:

- The identity $(x^2 - 1)(x^4 + x^2 + 1) = x^6 - 1$ relates the given equation to sixth roots of unity.
- If $\alpha^6 = 1$, powers of α can be reduced modulo 6. Since the constant term is nonzero, $\alpha \neq 0$.

Step 1: Establish the power relation Using the given equation, multiply by $\alpha^2 - 1$:

$$(\alpha^2 - 1)(1 + \alpha^2 + \alpha^4) = \alpha^6 - 1 = 0.$$

Hence $\alpha^6 = 1$. The values $\alpha = 1$ and $\alpha = -1$ do not satisfy the original equation, because each gives the sum 3. It is unnecessary to find the four remaining complex roots individually.

Step 2: Reduce the three exponents The exponents have remainders 3, 0 and 3 on division by 6:

$$1011 = 6(168) + 3, \qquad 2022 = 6(337), \qquad 3033 = 6(505) + 3.$$

Therefore

$$\alpha^{1011} = \alpha^3, \qquad \alpha^{2022} = 1, \qquad \alpha^{3033} = \alpha^3.$$

The first and third terms cancel with their printed opposite signs. Thus the expression is $\alpha^3 + 1 - \alpha^3 = 1$, independently of which root is chosen. The expressions in the other options retain a nonzero multiple of α, so they cannot equal this constant result for these roots.

Step 3: Conclusion The value is $\boxed{1}$. Hence, the correct option is (A).

Quick Tip to Solve: For a polynomial containing consecutive even powers, multiply by $x^2 - 1$ before reducing large exponents.

2. Let arg(z) represent the principal argument of the complex number z. Then, |z| = 3 and arg(z − 1) − arg(z + 1) = $\frac{\pi}{4}$ intersect : [4 marks]

(A) exactly at one point.

(B) exactly at two points.

(C) nowhere.

(D) at infinitely many points.

Correct Answer: (C) nowhere.

Solution:

Concept:

- For nonzero complex numbers, the argument of a quotient agrees modulo 2π with the difference of their arguments.
- Write $z = x + iy$. A complex number of argument $\pi/4$ has equal positive real and imaginary parts.

Step 1: Express the argument condition algebraically If the printed difference of principal arguments is $\pi/4$, then $(z-1)/(z+1)$ must have argument $\pi/4$. The points $z = \pm 1$ are not on $|z| = 3$, so this quotient is defined and nonzero. Rationalising gives

$$\frac{z-1}{z+1} = \frac{x^2 + y^2 - 1 + 2iy}{(x+1)^2 + y^2}.$$

The denominator is a positive real number, so it does not change the direction of the numerator.

Step 2: Combine with the modulus condition On the circle $|z| = 3$, we have $x^2 + y^2 = 9$. Consequently the numerator above is $8 + 2iy$. To have argument $\pi/4$, its imaginary part must equal its positive real part:

$$2y = 8 \quad \Longrightarrow \quad y = 4.$$

But a point on $x^2 + y^2 = 9$ must satisfy $-3 \le y \le 3$. Substitution would give $x^2 = 9 - 16 = -7$, impossible for real x. This necessary condition already rules out every point; issues of adding multiples of 2π cannot create a solution. Thus the alternatives claiming one, two or infinitely many intersections are all excluded.

Step 3: Conclusion The two loci have no common point: they intersect nowhere. Hence, the correct option is (C).

Quick Tip to Solve: Rationalise $(z-1)/(z+1)$ and impose the modulus first; the argument condition then becomes a simple condition on the imaginary coordinate.

3. The value of $\lim_{x\to 1} \frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to: [4 marks]

(A) $\frac{\pi^2}{6}$

(B) $\frac{\pi^2}{3}$

(C) $\frac{\pi^2}{2}$

(D) π^2

Correct Answer: (D) π^2

Solution:

Concept:

- The standard limit is $\lim_{u\to 0} \sin u/u = 1$, with angles measured in radians.
- Factor polynomial zeros before cancelling in a limit. Cancellation is valid on a punctured neighbourhood of the limiting point.

Step 1: Factor the denominator exactly The denominator vanishes at $x = 1$, and its factorisation is

$$x^4 - 2x^3 + 2x - 1 = (x-1)^3(x+1).$$

Also, $x^2 - 1 = (x-1)(x+1)$. For $x \neq 1$ sufficiently near 1, the ratio therefore simplifies to

$$\frac{(x^2-1)\sin^2(\pi x)}{x^4 - 2x^3 + 2x - 1} = \frac{\sin^2(\pi x)}{(x-1)^2}.$$

This cancellation removes the common polynomial factors, but it does not mean substituting $x = 1$ into the original zero-over-zero expression.

Step 2: Apply the sine limit after shifting the variable Put $h = x - 1$, so that $h \to 0$. Since $\sin(\pi + \pi h) = -\sin(\pi h)$, its square is unchanged. Hence

$$\lim_{x\to 1} \frac{\sin^2(\pi x)}{(x-1)^2} = \lim_{h\to 0} \frac{\sin^2(\pi h)}{h^2}$$
$$= \pi^2 \lim_{h\to 0} \left(\frac{\sin(\pi h)}{\pi h}\right)^2 = \pi^2.$$

Both one-sided limits give this positive value. The other proposed fractions would require an extra numerical factor in the surviving polynomial coefficient; the exact factorisation shows that coefficient is one.

Step 3: Conclusion The required limit is $\boxed{\pi^2}$. Hence, the correct option is (D).

Quick Tip to Solve: When a denominator has a repeated zero, factor it fully; this exposes the precise power needed for the standard sine limit.

4. Let f be a real valued continuous function on [0, 1] and $f(x) = x + \int_0^1 (x-t)f(t)dt$.

Then, which of the following points (x, y) lies on the curve y = f(x) ? [4 marks]

(A) (2, 4)

(B) (1, 2)

(C) (4, 17)

(D) (6, 8)

Correct Answer: (D) (6, 8)

Solution:

Concept:

- In a definite integral with variable t, the quantity x is constant and may be taken outside the integral.
- The two integrals $\int_0^1 f(t)\,dt$ and $\int_0^1 tf(t)\,dt$ are constants. Thus the displayed equation forces an affine expression for $f(x)$.

Step 1: Introduce the two constant integrals Let

$$A = \int_0^1 f(t)\,dt, \qquad B = \int_0^1 tf(t)\,dt.$$

The given equation becomes $f(x) = (1 + A)x - B$. Write this as $f(x) = ax + b$, where $a = 1 + A$ and $b = -B$. The continuity assumption ensures that both defining integrals exist. We now determine these coefficients from the equation itself, rather than fitting a line to one of the options.

Step 2: Solve for the coefficients Integrating the affine expression on $[0, 1]$ gives

$$A = \frac{a}{2} + b, \qquad B = \frac{a}{3} + \frac{b}{2}.$$

Therefore

$$\frac{a}{2} - b = 1, \qquad \frac{a}{3} + \frac{3b}{2} = 0.$$

The second equation gives $a = -9b/2$. In the first, this yields $-13b/4 = 1$, so $b = -4/13$ and $a = 18/13$. Thus the curve prescribed by the displayed expression is

$$y = \frac{18x - 4}{13}.$$

Testing the proposed coordinates gives values $32/13$, $14/13$, $68/13$, and $104/13 = 8$ at $x = 2, 1, 4, 6$, respectively. Only the last equals its proposed ordinate. The interval specifies where the integrals are evaluated; the resulting affine curve supplies the point test requested.

Step 3: Conclusion The listed point on the curve is $\boxed{(6, 8)}$. Hence, the correct option is (D).

Quick Tip to Solve: For a kernel $x - t$, replace the two definite integrals by constants; the unknown function becomes a straight line immediately.

5. If $\int_0^2 \left(\sqrt{2x} - \sqrt{2x - x^2}\right) dx = \int_0^1 \left(1 - \sqrt{1 - y^2} - \frac{y^2}{2}\right) dy + \int_1^2 \left(2 - \frac{y^2}{2}\right) dy + I$**, then I equals** [4 marks]

(A) $\int_0^1 \left(1 + \sqrt{1 - y^2}\right) dy$

(B) $\int_0^1 \left(\frac{y^2}{2} - \sqrt{1 - y^2} + 1\right) dy$

(C) $\int_0^1 \left(1 - \sqrt{1 - y^2}\right) dy$

(D) $\int_0^1 \left(\frac{y^2}{2} + \sqrt{1 - y^2} + 1\right) dy$

Correct Answer: (C) $\int_0^1 \left(1 - \sqrt{1 - y^2}\right) dy$

Solution:

Concept:

- The curve $\sqrt{2x - x^2} = \sqrt{1 - (x - 1)^2}$ is the upper half of a unit circle centred at $(1, 0)$.
- The integral $\int_0^1 \sqrt{1 - y^2}\, dy = \pi/4$ is the area of a quarter unit circle; elementary powers are integrated directly.

Step 1: Evaluate the left-hand side Complete the square in the second radical: $2x - x^2 = 1 - (x - 1)^2$. Its integral from 0 to 2 is the area of a semicircle of radius one, namely $\pi/2$. The other integral is

$$\int_0^2 \sqrt{2x}\, dx = \sqrt{2}\left[\frac{2}{3}x^{3/2}\right]_0^2 = \frac{8}{3}.$$

Consequently the left-hand side is $8/3 - \pi/2$. No sign change is needed because both radicals are nonnegative over the full interval.

Step 2: Subtract the two explicit right-hand integrals The first specified integral equals

$$\int_0^1 \left(1 - \sqrt{1 - y^2} - \frac{y^2}{2}\right) dy = 1 - \frac{\pi}{4} - \frac{1}{6} = \frac{5}{6} - \frac{\pi}{4}.$$

The second is

$$\int_1^2 \left(2 - \frac{y^2}{2}\right) dy = \left[2y - \frac{y^3}{6}\right]_1^2 = \frac{5}{6}.$$

Isolating the unknown therefore gives

$$I = \left(\frac{8}{3} - \frac{\pi}{2}\right) - \left(\frac{5}{3} - \frac{\pi}{4}\right) = 1 - \frac{\pi}{4}.$$

Option (C) has exactly this value. Options (A), (B) and (D) give $1 + \pi/4$, $7/6 - \pi/4$, and $7/6 + \pi/4$, respectively, so none equals the required difference.

Step 3: Conclusion $I = \int_0^1 \left(1 - \sqrt{1 - y^2}\right) dy = 1 - \frac{\pi}{4}$. Hence, the correct option is (C).

Quick Tip to Solve: Before changing the order of an area integral, check whether semicircle and quarter-circle areas evaluate every term directly.

6. If y = y(x) is the solution of the differential equation $\left(1+e^{2x}\right)\frac{dy}{dx}+2\left(1+y^2\right)e^x=0$ and y (0) = 0, then $6\left(y'(0)+\left(y\left(\log_e\sqrt{3}\right)\right)^2\right)$ is equal to [4 marks]

(A) 2

(B) -2

(C) -4

(D) -1

Correct Answer: (C) -4

Solution:

Concept:

- Separate the variables using $dy/(1+y^2)$, whose antiderivative is $\tan^{-1} y$.
- The substitution $u=e^x$ converts $2e^x\,dx/(1+e^{2x})$ into $2\,du/(1+u^2)$.

Step 1: Integrate and use the initial value The equation gives

$$\frac{dy}{1+y^2}=-\frac{2e^x}{1+e^{2x}}\,dx.$$

Integration yields $\tan^{-1} y=-2\tan^{-1}(e^x)+C$. At $x=0$, the initial value $y=0$ gives $C=\pi/2$. Therefore

$$\tan^{-1} y=\frac{\pi}{2}-2\tan^{-1}(e^x).$$

For positive e^x, the right side lies strictly between $-\pi/2$ and $\pi/2$, so taking the tangent is consistent with the principal inverse tangent.

Step 2: Find the derivative and the requested function value Using the double-angle identity,

$$y=\cot\left(2\tan^{-1}(e^x)\right)=\frac{1-e^{2x}}{2e^x}=\frac{e^{-x}-e^x}{2}.$$

This gives $y'(0)=(-1-1)/2=-1$, also obtained directly from the original differential equation. At $x=\log_e\sqrt{3}$, we have $e^x=\sqrt{3}$ and

$$y=\frac{1-3}{2\sqrt{3}}=-\frac{1}{\sqrt{3}},\qquad y^2=\frac{1}{3}.$$

The required expression is consequently

$$6\left(-1+\frac{1}{3}\right)=-4.$$

In particular the square term is positive but smaller than one, so the total must be negative; keeping its exact value distinguishes this answer from the other negative choices.

Step 3: Conclusion The required value is $\boxed{-4}$. Hence, the correct option is (C).

Quick Tip to Solve: Find $y'(0)$ directly from the differential equation, then use the integrated solution only for the nonzero argument.

7. Let a triangle ABC be inscribed in the circle $x^2 - \sqrt{2}(x+y) + y^2 = 0$ such that $\angle BAC = \frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the ΔABC is equal to : [4 marks]

(A) 1

(B) $\left(\sqrt{6}+\sqrt{3}\right)/2$

(C) $\left(3+\sqrt{3}\right)/4$

(D) $\left(\sqrt{6}+2\sqrt{3}\right)/4$

Correct Answer: (A) 1

Solution:

Concept:

- Completing squares puts a circle into the form $(x-h)^2 + (y-k)^2 = R^2$, where R is its radius.
- A right angle subtended by a chord at the circumference implies that chord is a diameter. The area of a right triangle is half the product of its perpendicular legs.

Step 1: Find the circumradius and the hypotenuse The given equation can be written as

$$\left(x - \frac{1}{\sqrt{2}}\right)^2 + \left(y - \frac{1}{\sqrt{2}}\right)^2 = 1.$$

Thus the circumcircle has centre $(1/\sqrt{2}, 1/\sqrt{2})$ and radius $R = 1$. Since $\angle BAC = \pi/2$, its opposite side BC is the diameter of this circle. Hence $BC = 2$, rather than one. The two perpendicular legs meet at A, so they are AB and AC.

Step 2: Calculate the second leg and the area Apply Pythagoras in the right triangle:

$$AB^2 + AC^2 = BC^2.$$

The specified value $AB = \sqrt{2}$ gives

$$2 + AC^2 = 4, \qquad AC^2 = 2, \qquad AC = \sqrt{2}.$$

The positive square root is taken because this is a side length. Therefore

$$\text{Area}(\triangle ABC) = \frac{1}{2}(AB)(AC) = \frac{1}{2}\sqrt{2}\sqrt{2} = 1.$$

This also shows the triangle is right isosceles. As a check, for fixed hypotenuse 2, the maximum right-triangle area is one and occurs at equal legs, exactly as here. Each of the other listed radical expressions is greater than one, so those alternatives cannot be the area of this triangle.

Step 3: Conclusion The area is $\boxed{1}$ square unit. Hence, the correct option is (A).

Quick Tip to Solve: When an inscribed angle is a right angle, identify the diameter as the hypotenuse before using the given chord length.

8. The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3}, \frac{7}{3}\right)$ is : [4 marks]

(A) $\sqrt{2}$

(B) 2

(C) $2\sqrt{2}$

(D) 4

Correct Answer: (C) $2\sqrt{2}$

Solution:

Concept:

- The intersection of two sides gives one vertex. An altitude through another vertex is perpendicular to the opposite side and passes through the orthocentre.
- For vertices $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$, the centroid is $G = ((x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3)$. Its distance from the origin is $\sqrt{x_G^2 + y_G^2}$.

Step 1: Find the common vertex Solve the given side equations simultaneously. From $x - 2y + 1 = 0$, $x = 2y - 1$. Substitution in $2x - y - 1 = 0$ gives $3y = 3$. Thus the common vertex is $A = (1, 1)$. Call the side of slope $1/2$ the line AB and that of slope 2 the line AC.

Step 2: Recover the other two vertices Write $H = (7/3, 7/3)$. The altitude from C is perpendicular to AB, so its slope is -2. Through H its equation is $y = -2x + 7$. Intersecting this with AC, $y = 2x - 1$, gives $C = (2, 3)$. Similarly the altitude from B has slope $-1/2$ and equation $y = -x/2 + 7/2$. Its intersection with AB, $y = (x + 1)/2$, is $B = (3, 2)$. These points lie on the required sides and their altitudes pass through H.

Step 3: Average the coordinates and measure the distance Therefore

$$G = \left(\frac{1 + 3 + 2}{3}, \frac{1 + 2 + 3}{3}\right) = (2, 2).$$

Hence

$$OG = \sqrt{2^2 + 2^2} = 2\sqrt{2}.$$

The alternatives $\sqrt{2}$, 2 and 4 do not equal this distance; in particular, the coordinate value 2 is not itself the radial distance.

Step 4: Conclusion The distance is $\boxed{2\sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the two known sides to locate their vertex, then intersect each side with the altitude perpendicular to the other side.

9. Let A, B, C be three points whose position vectors respectively are

$$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$$

$$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k},\ \alpha \in R$$

$$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$$

If α is the smallest positive integer for which $\vec{a}$, $\vec{b}$, $\vec{c}$ are noncollinear, then the length of the median, in ΔABC, through A is : [4 marks]

(A) $\frac{\sqrt{82}}{2}$

(B) $\frac{\sqrt{62}}{2}$

(C) $\frac{\sqrt{69}}{2}$

(D) $\frac{\sqrt{66}}{2}$

Correct Answer: (A) $\frac{\sqrt{82}}{2}$

Solution:

Concept:

- Three points are collinear exactly when the displacement vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are proportional.
- The midpoint of BC has position vector $(\vec{b}+\vec{c})/2$. The median through A joins A to that midpoint; its length is $\left|(\vec{b}+\vec{c})/2 - \vec{a}\right|$.

Step 1: Exclude the collinear value The coordinates are $A = (1, 4, 3)$, $B = (2, \alpha, 4)$ and $C = (3, -2, 5)$. Thus

$$\overrightarrow{AB} = (1, \alpha - 4, 1), \qquad \overrightarrow{AC} = (2, -6, 2).$$

The first and third coordinates force the proportionality factor to be $1/2$. The second coordinate then requires $\alpha - 4 = -3$, or $\alpha = 1$. Conversely this value does make the two displacement vectors proportional, so it must be excluded.

Step 2: Select the least admissible positive integer The positive integers start with 1, but 1 gives a degenerate triangle. The next value is $\alpha = 2$, which is noncollinear because $-2 \neq -3$. Therefore $B = (2, 2, 4)$. Noncollinearity concerns the three points, so using proportionality of their position vectors alone would test the wrong condition.

Step 3: Find the midpoint and median length The midpoint of BC is

$$M = \left(\frac{5}{2}, 0, \frac{9}{2}\right).$$

Thus

$$\overrightarrow{AM} = \left(\frac{3}{2}, -4, \frac{3}{2}\right), \qquad AM^2 = \frac{9}{4} + 16 + \frac{9}{4} = \frac{82}{4}.$$

Taking the positive square root gives $AM = \sqrt{82}/2$. Squaring the four proposed lengths distinguishes them immediately: only the option containing 82 has squared length $82/4$.

Step 4: Conclusion The median has length $\boxed{\sqrt{82}/2}$. Hence, the correct option is (A).

Quick Tip to Solve: Test collinearity with vectors measured from the same vertex before substituting the smallest allowed parameter into the midpoint formula.

10. The probability that a relation R from {x, y} to {x, y} is both symmetric and transitive, is equal to : [4 marks]

(A) $\frac{5}{16}$
(B) $\frac{9}{16}$
(C) $\frac{11}{16}$
(D) $\frac{13}{16}$

Correct Answer: (A) $\frac{5}{16}$

Solution:

Concept:

- A relation on $S = \{x, y\}$ is any subset of $S \times S$. There are four ordered pairs, giving $2^4 = 16$ equally likely relations.
- Symmetry requires (x, y) and (y, x) to occur together. Transitivity requires (a, c) whenever both (a, b) and (b, c) occur. Neither condition alone requires reflexivity on all of S.

Step 1: Separate the off-diagonal possibilities The ordered pairs are $(x, x), (x, y), (y, x), (y, y)$. A symmetric relation either contains both off-diagonal pairs or contains neither. A relation containing exactly one of $(x, y), (y, x)$ cannot be favourable, regardless of the two diagonal choices. We count the two symmetric cases separately so that the empty relation is not accidentally omitted.

Step 2: Count relations without off-diagonal pairs When neither off-diagonal pair occurs, either diagonal pair can be included independently. The four possibilities are the empty relation, $\{(x, x)\}$, $\{(y, y)\}$, and $\{(x, x), (y, y)\}$. Each is symmetric. Each is also transitive: any available chain only repeats a diagonal pair, whose required composite is already present. Thus this case contributes four relations.

Step 3: Count relations containing both off-diagonal pairs Now $(x, y), (y, x) \in R$. Their compositions require both (x, x) and (y, y). Consequently the only transitive relation in this case is the full relation $S \times S$. There are $4 + 1 = 5$ favourable relations, so the probability is $5/16$. The numerators 9, 11 and 13 overcount relations that violate symmetry or transitivity.

Step 4: Conclusion The required probability is $\boxed{5/16}$. Hence, the correct option is (A).

Quick Tip to Solve: For two elements, first decide whether the pair of off-diagonal arrows is present; if it is, transitivity forces both loops.

11. The number of values of a $\in$ N such that the variance of 3, 7, 12, a, 43 $-$ a is a natural number is : [4 marks]

(A) 0

(B) 2

(C) 5

(D) infinite

Correct Answer: (A) 0

Solution:

Concept:

- For five observations, the variance is $\sigma^2 = \frac{1}{5}\sum x_i^2 - \bar{x}^2$, where $\bar{x}$ is their arithmetic mean.
- An integer polynomial divided by 5 is an integer only if its numerator is zero modulo 5. Checking the five residue classes exhausts every natural-number input.

Step 1: Compute the fixed mean The five observations have sum

$$3 + 7 + 12 + a + (43 - a) = 65.$$

Hence the mean is $\bar{x} = 13$, independent of a. The two variable observations compensate in the sum, but not in the sum of squares. No restriction that $43 - a$ be positive is printed; we therefore consider every natural value of a, as requested.

Step 2: Express the variance as a polynomial The sum of the fixed squares is $9 + 49 + 144 = 202$. Therefore

$$\sigma^2 = \frac{202 + a^2 + (43 - a)^2}{5} - 169$$
$$= \frac{2a^2 - 86a + 1206}{5}.$$

This is the population variance of the five listed numbers, rather than the sample-variance formula with denominator four. To be a natural number it must first be an integer.

Step 3: Check divisibility by five Modulo 5 the numerator is $2a^2 - a + 1$. For $a \equiv 0, 1, 2, 3, 4 \pmod 5$, its residues are respectively $1, 2, 2, 1, 4$. None is zero. Thus the variance is never even an integer and cannot be a natural number. This rules out both finite positive counts 2 and 5 as well as the claim of infinitely many values. The conclusion is unchanged by whether zero is included in the convention for natural numbers.

Step 4: Conclusion There are $\boxed{0}$ admissible natural values of a. Hence, the correct option is (A).

Quick Tip to Solve: When the mean is fixed and variance must be integral, reduce its numerator modulo the denominator before solving inequalities.

12. Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1(adj\ A) + {}^5C_2(adj\ A)^2 - \ \ldots\ \ - {}^5C_5(adj\ A)^5$, then the sum of all elements of the matrix B is [4 marks]

(A) -5

(B) -6

(C) -7

(D) -8

Correct Answer: (C) -7

Solution:

Concept:

- The identity commutes with every square matrix, so the ordinary binomial expansion gives $(I - M)^5 = \sum_{k=0}^{5} (-1)^k \binom{5}{k} M^k$.
- For a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, its adjugate is $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. A matrix N with $N^2 = 0$ makes all higher binomial terms vanish.

Step 1: Recognise the binomial expression The displayed alternating expression includes the identity and all powers of $\text{adj}\ A$ through the fifth. Its coefficients and signs are exactly those of $(I - \text{adj}\ A)^5$. It is not the entrywise fifth power, and the adjugate should not be confused with the inverse; no division by the determinant is made here.

Step 2: Calculate the adjugate and isolate the nilpotent part For the given triangular matrix,

$$\text{adj}\ A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}.$$

Set $N = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. Then $N^2 = 0$ and

$$I - \text{adj}\ A = -I - N = -(I + N).$$

The positive upper-right entry of the adjugate is essential: it is the negative of the original upper-right entry, which is -1.

Step 3: Raise to the fifth power and sum the entries Because five is odd,

$$B = -(I + N)^5 = -(I + 5N) = \begin{pmatrix} -1 & -5 \\ 0 & -1 \end{pmatrix}.$$

Every term involving N^2 or a higher power is zero. The requested sum is $-1 - 5 + 0 - 1 = -7$. In particular, summing just the diagonal gives -2, not the requested sum of all entries; the other listed totals do not match the explicit matrix.

Step 4: Conclusion The sum of all elements of B is $\boxed{-7}$. Hence, the correct option is (C).

Quick Tip to Solve: For an upper triangular matrix with equal diagonal entries, separate a scalar identity and a nilpotent matrix before taking high powers.

13. The sum of the infinite series $1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+\frac{35}{6^4}+\frac{51}{6^5}+\frac{70}{6^6}+$ ..... is equal to : [4 marks]

(A) $\frac{425}{216}$

(B) $\frac{429}{216}$

(C) $\frac{288}{125}$

(D) $\frac{280}{125}$

Correct Answer: (C) $\frac{288}{125}$

Solution:

Concept:

- The numerator sequence $1, 5, 12, 22, 35, \ldots$ has first differences $4, 7, 10, 13, \ldots$, so its general term is quadratic.
- For $|r| < 1$, $\sum_{n\geq 0} r^n = 1/(1-r)$, $\sum_{n\geq 0} nr^n = r/(1-r)^2$, and $\sum_{n\geq 0} n^2 r^n = r(1+r)/(1-r)^3$. These identities follow by differentiating the geometric series.

Step 1: Identify the numerator at each power Index the first term by $n = 0$, so its denominator is 6^0. Let its numerator be u_n. The constant second difference is 3, giving

$$u_n = 1 + 4n + \frac{3}{2}n(n-1) = \frac{3n^2+5n+2}{2}.$$

This gives 1, 5, 12, 22 and 35 for $n = 0, 1, 2, 3, 4$, and also gives the printed later terms 51 and 70. This check prevents an index shift.

Step 2: Use the geometric-series identities Put $r = 1/6$. Since $|r| < 1$, the series with a quadratic numerator converges absolutely, and the standard differentiated geometric sums apply. Thus

$$S = \frac{1}{2}\left\{\frac{3r(1+r)}{(1-r)^3} + \frac{5r}{(1-r)^2} + \frac{2}{1-r}\right\}.$$

The summations include $n = 0$; that term contributes only through the constant part.

Step 3: Simplify and compare the options At $r = 1/6$,

$$\frac{r(1+r)}{(1-r)^3} = \frac{42}{125}, \quad \frac{r}{(1-r)^2} = \frac{6}{25}, \quad \frac{1}{1-r} = \frac{6}{5}.$$

Therefore

$$S = \frac{1}{2}\left(\frac{126}{125} + \frac{150}{125} + \frac{300}{125}\right) = \frac{288}{125}.$$

The first four terms alone exceed 2, ruling out the two options with denominator 216; direct evaluation distinguishes this result from $280/125$.

Step 4: Conclusion The sum is $\boxed{288/125}$. Hence, the correct option is (C).

Quick Tip to Solve: Constant second differences signal a quadratic numerator; write it in terms of the exponent index before applying geometric sums.

14. Let f : R → R be a function defined by f(x) = (x − 3)^{n_1} (x − 5)^{n_2}, n_1, n_2 ∈ N. Then, which of the following is NOT true? [4 marks]

(A) For $n_1 = 3$, $n_2 = 4$, there exists $\alpha \in (3, 5)$ where f attains local maxima.

(B) For $n_1 = 4$, $n_2 = 3$, there exists $\alpha \in (3, 5)$ where f attains local minima.

(C) For $n_1 = 3$, $n_2 = 5$, there exists $\alpha \in (3, 5)$ where f attains local maxima.

(D) For $n_1 = 4$, $n_2 = 6$, there exists $\alpha \in (3, 5)$ where f attains local maxima.

Correct Answer: (C) For $n_1 = 3$, $n_2 = 5$, there exists $\alpha \in (3, 5)$ where f attains local maxima.

Solution:

Concept:

- Inside $(3, 5)$, the factors $x-3$ and $x-5$ are nonzero. Logarithmic differentiation gives $f'(x)/f(x) = n_1/(x-3) + n_2/(x-5)$.
- A stationary point is a local maximum when the derivative changes from positive to negative, and a local minimum when it changes from negative to positive. The question asks for the statement that is NOT true.

Step 1: Locate the unique interior stationary point Combining the two fractions gives

$$\frac{f'(x)}{f(x)} = \frac{(n_1+n_2)x - (5n_1+3n_2)}{(x-3)(x-5)}.$$

Hence the only interior stationary point is

$$\alpha = \frac{5n_1+3n_2}{n_1+n_2} = 3 + \frac{2n_1}{n_1+n_2}.$$

Since both exponents are positive, $3 < \alpha < 5$. The numerator changes from negative to positive at this point, while the denominator remains negative.

Step 2: Determine the extremum from the parity Consequently f'/f is positive before α and negative after it. On the interval, $x - 3 > 0$ and $x - 5 < 0$, so the sign of f is $(-1)^{n_2}$. If n_2 is even, f is positive and its derivative changes from positive to negative, giving a local maximum. If n_2 is odd, f is negative and the derivative changes from negative to positive, giving a local minimum.

Step 3: Test all four assertions In (A), $n_2 = 4$ is even, so the stated maximum exists. In (B), $n_2 = 3$ is odd, so the stated minimum exists. In (C), $n_2 = 5$ is odd: the stationary point is a minimum, not a maximum. In (D), $n_2 = 6$ is even, so the maximum exists. Since the interior stationary point is unique, there is no additional local maximum that could make (C) true.

Step 4: Conclusion The statement in (C) is NOT true. Hence, the correct option is (C).

Quick Tip to Solve: Between the two roots, the parity of the exponent on the negative factor decides whether the single interior turning point is a maximum or minimum.

15. Let P : $y^2 = 4ax$, $a > 0$ be a parabola with focus S. Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line $y = 3x + 5$ touch the parabola P at A and B. Then the value of a for which A, B and S are collinear is : [4 marks]

(A) 8 only

(B) 2 only

(C) $\frac{1}{4}$ only

(D) any $a > 0$

Correct Answer: (D) any $a > 0$

Solution:

Concept:

- For $y^2 = 4ax$, the point with parameter t is $(at^2, 2at)$, its tangent has slope $1/t$, and its focus is $S = (a, 0)$.
- Two distinct parametric points are endpoints of a focal chord precisely when $t_1 t_2 = -1$. This condition is independent of the positive scale parameter a.

Step 1: Find the two tangent slopes Let a tangent have slope m. The line $y = 3x + 5$ has slope 3; its intercept does not affect the angle. Hence

$$\left|\frac{m-3}{1+3m}\right| = \tan\frac{\pi}{4} = 1.$$

Solving $m - 3 = 1 + 3m$ gives $m = -2$, while $m - 3 = -(1 + 3m)$ gives $m = 1/2$. Both are finite, distinct slopes, and each yields a tangent to the parabola for every $a > 0$.

Step 2: Check that their chord contains the focus The contact parameters are therefore $t_1 = -1/2$ and $t_2 = 2$. Their product is -1, already the focal-chord condition. Explicitly, the chord joining two parametric points has equation

$$(t_1 + t_2)y = 2x + 2at_1t_2.$$

Here this becomes $\frac{3}{2}y = 2x - 2a$. Substituting the focus $(a, 0)$ gives zero on both sides, so A, B, S are collinear. The verification has imposed no additional equation on a. The three isolated values in the other options all work, but none is the only possible value.

Step 3: Conclusion Collinearity holds for every $a > 0$. Hence, the correct option is (D).

Quick Tip to Solve: For a parabola, first multiply the tangent slopes: product -1 gives contact parameters with product -1, hence a focal chord.

16. Let $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z+3}{-1}$ lie on the plane $px - qy + z = 5$, for some p, q $\in$ R. The shortest distance of the plane from the origin is : [4 marks]

(A) $\sqrt{\frac{3}{109}}$

(B) $\sqrt{\frac{5}{142}}$

(C) $\frac{5}{\sqrt{71}}$

(D) $\frac{1}{\sqrt{142}}$

Correct Answer: (B) $\sqrt{\frac{5}{142}}$

Solution:

Concept:

- A line contained in a plane supplies two conditions: one point lies on the plane, and the line direction is perpendicular to the plane normal.
- The perpendicular distance of the origin from $Ax + By + Cz = D$ is $|D|/\sqrt{A^2 + B^2 + C^2}$. All coefficients must be used with their signs.

Step 1: Impose the point and direction conditions The given line passes through $(2, -1, -3)$ with direction vector $(3, -2, -1)$. The plane normal is $(p, -q, 1)$, because the coefficient of y is $-q$. Substitution of the point gives

$$2p + q - 3 = 5, \qquad 2p + q = 8.$$

The direction condition is

$$(p, -q, 1) \cdot (3, -2, -1) = 0, \qquad 3p + 2q = 1.$$

These two independent equations encode the requirement for the entire line, rather than just one of its points, to belong to the plane.

Step 2: Calculate the normal and distance From $q = 8-2p$, the second equation becomes $3p+16-4p = 1$, so $p = 15$ and $q = -22$. Thus the plane is $15x + 22y + z = 5$. Its normal has squared length

$$15^2 + 22^2 + 1^2 = 225 + 484 + 1 = 710.$$

Consequently,

$$d = \frac{5}{\sqrt{710}}, \qquad d^2 = \frac{25}{710} = \frac{5}{142}.$$

Distance is nonnegative, so its simplified form is $\sqrt{5/142}$. The other expressions have different squares, and therefore cannot represent this same perpendicular distance.

Step 3: Conclusion The shortest distance is $\sqrt{5/142}$. Hence, the correct option is (B).

Quick Tip to Solve: For a line lying in a plane, use both its point and its direction before applying the plane-distance formula.

17. Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane x + 2y + 2z = 16. Let T be a plane passing through the point Q and contains the line $\vec{r} = -\hat{k} + \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right)$, $\lambda \in R$. Then, which of the following points lies on T? [4 marks]

(A) (2, 1, 0)

(B) (1, 2, 1)

(C) (1, 2, 2)

(D) (1, 3, 2)

Correct Answer: (B) (1, 2, 1)

Solution:

Concept:

- Reflection of a point P in $\mathbf{n} \cdot \mathbf{r} = d$ gives $Q = P - 2(\mathbf{n} \cdot P - d)\mathbf{n}/|\mathbf{n}|^2$.
- A plane through a line and an additional point has a normal perpendicular to the line direction and the vector from a point on the line to that additional point.

Step 1: Find the reflected point For the reflecting plane, $\mathbf{n} = (1, 2, 2)$ and $d = 16$. At $P = (1, 2, 1)$, the residual is $1 + 4 + 2 - 16 = -9$, while $|\mathbf{n}|^2 = 9$. Therefore

$$Q = (1, 2, 1) - \frac{2(-9)}{9}(1, 2, 2) = (3, 6, 5).$$

As a check, the midpoint $(2, 4, 3)$ satisfies $2 + 8 + 6 = 16$, and $Q - P = (2, 4, 4)$ is parallel to the reflecting-plane normal, as required.

Step 2: Construct the required plane and test the points The contained line passes through $A = (0, 0, -1)$ with direction $\mathbf{u} = (1, 1, 2)$. The second in-plane vector is $Q - A = (3, 6, 6)$. Their cross product is

$$\mathbf{u} \times (Q - A) = (-6, 0, 3).$$

A proportional normal is $(-2, 0, 1)$; hence T has equation $-2x + z + 1 = 0$. It contains Q, since $-6 + 5 + 1 = 0$, and every line point $(\lambda, \lambda, -1 + 2\lambda)$ also satisfies it. For the listed points, the residuals $-2x + z + 1$ are respectively $-3, 0, 1, 1$. Exactly the second point lies in T.

Step 3: Conclusion The point $(1, 2, 1)$ lies on T. Hence, the correct option is (B).

Quick Tip to Solve: Check a reflected point by its midpoint and normal displacement; then obtain the new plane from two in-plane vectors.

18. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is : [4 marks]

(A) $15\sqrt{3}$

(B) $20\sqrt{3}$

(C) $20 + 10\sqrt{3}$

(D) 30

Correct Answer: (D) 30

Solution:

Concept:

- For vertical structures on level ground, the tangent of an elevation or depression angle is vertical separation divided by horizontal separation.
- The angle subtended by the pole at the tower top is the angle between the rays to the pole base and pole top; it is not the depression angle to the pole top itself.

Step 1: Express the tower height using the base angle Let the tower height be H metres and let the horizontal distance between the bases be $d > 0$ metres. From the pole base, the elevation angle to the tower top is $60°$. Therefore

$$\frac{H}{d} = \tan 60° = \sqrt{3}, \qquad H = \sqrt{3}\, d.$$

Equivalently, the ray from the tower top to the pole base makes a depression angle of $60°$. The given subtended angle is measured between this ray and the ray to the pole top.

Step 2: Use the angle subtended by the pole Because the pole top is 20 metres above its base, its ray from the tower top lies above the base ray. Reducing the depression by the subtended $30°$ gives a depression angle of $30°$ to the pole top. The vertical difference is $H - 20$, so

$$\frac{H - 20}{d} = \tan 30° = \frac{1}{\sqrt{3}}.$$

Substitute $d = H/\sqrt{3}$:

$$H - 20 = \frac{H}{3}, \qquad \frac{2H}{3} = 20, \qquad H = 30.$$

This also gives $d = 10\sqrt{3}$ and a 10-metre height difference, whose ratio is $1/\sqrt{3}$, confirming the second angle. None of the other proposed heights satisfies both tangent equations.

Step 3: Conclusion The tower is $30\,\mathrm{m}$ high. Hence, the correct option is (D).

Quick Tip to Solve: Convert the subtended angle into a difference of depression angles before writing the second tangent equation.

19. Negation of the Boolean statement $(p \vee q) \Rightarrow ((\sim r) \vee p)$ is equivalent to : [4 marks]

(A) $p \wedge (\sim q) \wedge r$

(B) $(\sim p) \wedge (\sim q) \wedge r$

(C) $(\sim p) \wedge q \wedge r$

(D) $p \wedge q \wedge (\sim r)$

Correct Answer: (C) $(\sim p) \wedge q \wedge r$

Solution:

Concept:

- An implication $A \Rightarrow B$ is false exactly when A is true and B is false. Therefore its negation is $A \wedge \neg B$.
- De Morgan's law gives $\neg(X \vee Y) = (\neg X) \wedge (\neg Y)$. Distributivity and contradiction then simplify the expression.

Step 1: Negate the complete implication Take $A = p \vee q$ and $B = (\neg r) \vee p$. Negate the implication as a whole rather than negating each symbol separately:

$$\neg(A \Rightarrow B) = A \wedge \neg B.$$

Now apply De Morgan's law to the consequent:

$$\neg((\neg r) \vee p) = r \wedge (\neg p).$$

Thus the required statement is $(p \vee q) \wedge r \wedge (\neg p)$. In particular, falsifying the consequent forces r true and p false.

Step 2: Simplify and compare the truth conditions Use distributivity on the two factors involving p:

$$(p \vee q) \wedge (\neg p) = (p \wedge \neg p) \vee (q \wedge \neg p) = q \wedge \neg p.$$

The first disjunct is always false. Hence the complete negation is $(\neg p) \wedge q \wedge r$. A direct truth check confirms this: with p false, q true and r true, the antecedent is true and the consequent is false. Option (A) instead has p true, making the consequent true. Option (B) has both p, q false, so its antecedent is false. Option (D) again has p true. None of these can make the implication false.

Step 3: Conclusion The negation is $(\neg p) \wedge q \wedge r$. Hence, the correct option is (C).

Quick Tip to Solve: To negate an implication, preserve its antecedent and negate its consequent; only then apply Boolean simplification.

20. Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\alpha$ and $6^n - 5n - 1 = 25\beta$, then $\alpha - \beta$ is equal to [4 marks]

(A) $1 + {}^nC_2\,(8-5) + {}^nC_3\,(8^2-5^2) + \ldots\ldots + {}^nC_n\,(8^{n-1}-5^{n-1})$

(B) $1 + {}^nC_3\,(8-5) + {}^nC_4\,(8^2-5^2) + \ldots\ldots + {}^nC_n\,(8^{n-2}-5^{n-2})$

(C) ${}^nC_3\,(8-5) + {}^nC_4\,(8^2-5^2) + \ldots\ldots + {}^nC_n\,(8^{n-2}-5^{n-2})$

(D) ${}^nC_4\,(8-5) + {}^nC_5\,(8^2-5^2) + \ldots\ldots + {}^nC_n\,(8^{n-3}-5^{n-3})$

Correct Answer: (C) ${}^nC_3\,(8-5) + {}^nC_4\,(8^2-5^2) + \ldots\ldots + {}^nC_n\,(8^{n-2}-5^{n-2})$

Solution:

Concept:

- The binomial theorem gives $(1+x)^n = 1 + nx + \sum_{k=2}^{n} \binom{n}{k} x^k$ for a positive integer n.
- Subtracting the constant and linear terms and dividing by x^2 starts the remaining expansion with $\binom{n}{2}$. This common term cancels when the two results are subtracted.

Step 1: Expand both powers and identify the quotients Write $9^n = (1+8)^n$. Removing $1 + 8n$ and dividing by $64 = 8^2$ gives

$$\alpha = \sum_{k=2}^{n} \binom{n}{k} 8^{k-2}.$$

Similarly, write $6^n = (1+5)^n$; removing $1 + 5n$ and dividing by $25 = 5^2$ gives

$$\beta = \sum_{k=2}^{n} \binom{n}{k} 5^{k-2}.$$

The powers shift down by exactly two, because both denominators are squares. This shift determines both the first surviving term and the final exponent.

Step 2: Subtract and cancel the constant terms Term-by-term subtraction yields

$$\alpha - \beta = \sum_{k=2}^{n} \binom{n}{k} (8^{k-2} - 5^{k-2}).$$

At $k = 2$, the bracket is $1 - 1 = 0$, so the sum starts at $k = 3$:

$$\alpha - \beta = \binom{n}{3}(8-5) + \binom{n}{4}(8^2-5^2) + \cdots + \binom{n}{n}(8^{n-2}-5^{n-2}).$$

There is no standalone 1; therefore option (B) is too large by 1. Option (A) starts with the wrong coefficient and exponent, while option (D) omits the nonzero $k = 3$ contribution and shifts subsequent powers incorrectly. The third option matches every term.

Step 3: Conclusion The expansion is $\binom{n}{3}(8-5) + \binom{n}{4}(8^2-5^2) + \cdots + \binom{n}{n}(8^{n-2}-5^{n-2})$. Hence, the correct option is (C).

Quick Tip to Solve: After dividing a binomial remainder by a square, shift every exponent down by two and check whether the first constants cancel.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} + \left(\vec{b} \times \vec{c}\right) = \vec{0}$ and $\vec{b} \cdot \vec{c} = 5$. Then the value of $3\left(\vec{c} \cdot \vec{a}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- A cross product $\mathbf{b} \times \mathbf{c}$ is perpendicular to both $\mathbf{b}$ and $\mathbf{c}$. Thus $\mathbf{a} = -(\mathbf{b} \times \mathbf{c})$ requires $\mathbf{a} \cdot \mathbf{b} = 0$.
- Before calculating a requested value from simultaneous vector equations, check that the prescribed vectors permit a solution. A consequence of inconsistent equations does not define a valid vector.

Step 1: Test the necessary orthogonality condition The first equation is $\mathbf{b} \times \mathbf{c} = -\mathbf{a}$. Dot both sides with $\mathbf{b}$. Since a cross product is perpendicular to its first factor,

$$\mathbf{b} \cdot (\mathbf{b} \times \mathbf{c}) = 0 = -\mathbf{b} \cdot \mathbf{a}.$$

However, the supplied vectors give

$$\mathbf{a} \cdot \mathbf{b} = 1(1) + (-2)(1) + 3(1) = 2.$$

The necessary condition would therefore say $0 = -2$, which is impossible. This contradiction already rules out every possible $\mathbf{c}$, regardless of the additional condition $\mathbf{b} \cdot \mathbf{c} = 5$.

Step 2: Explain why a formal zero is not a valid answer Dotting the first equation with $\mathbf{c}$ would formally imply $\mathbf{c} \cdot \mathbf{a} = 0$, since $\mathbf{c} \cdot (\mathbf{b} \times \mathbf{c}) = 0$. But that argument presupposes a vector satisfying the first equation, and the preceding orthogonality check proves that no such vector exists. It cannot supply a numerical answer to this inconsistent problem. In components, if $\mathbf{c} = (x, y, z)$, then $\mathbf{b} \times \mathbf{c} = (z - y, x - z, y - x)$, whose component sum is identically zero. The vector $-\mathbf{a} = (-1, 2, -3)$ has component sum -2, providing an independent check of the same contradiction.

Step 3: Conclusion No vector $\mathbf{c}$ satisfies the printed conditions, so $3(\mathbf{c} \cdot \mathbf{a})$ has no defined value. The official final key dropped this question and awarded bonus marks to all.

Quick Tip to Solve: Check the dot product of the known vectors before solving a cross-product equation; a nonzero value can expose inconsistency immediately.

22. Let y = y(x), x > 1, be the solution of the differential equation $(x-1)\frac{dy}{dx} + 2xy = \frac{1}{x-1}$, with $y(2) = \frac{1+e^4}{2e^4}$. If $y(3) = \frac{e^\alpha + 1}{\beta e^\alpha}$, then the value of $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- A linear equation $y' + P(x)y = Q(x)$ is solved using the integrating factor $\mu(x) = e^{\int P(x)\,dx}$. Multiplication then turns the left side into the derivative of μy.

- The condition $x > 1$ permits division by $x - 1$, and gives $\log|x-1| = \log(x-1)$. The prescribed value at $x = 2$ determines the integration constant.

Step 1: Put the equation in linear form Divide each term by $x - 1$. Since $x = (x-1)+1$, the resulting equation is

$$y' + \left(2 + \frac{2}{x-1}\right) y = \frac{1}{(x-1)^2}.$$

Thus the integrating factor is

$$\mu(x) = \exp\left(2x + 2\log(x-1)\right) = e^{2x}(x-1)^2.$$

The square on $x - 1$ is essential: it cancels the denominator of the right side after multiplication.

Step 2: Integrate and apply the initial value Multiplying by this factor and applying the product rule in reverse gives

$$\frac{d}{dx}\left[e^{2x}(x-1)^2 y\right] = e^{2x}.$$

Consequently,

$$e^{2x}(x-1)^2 y = \frac{e^{2x}}{2} + C.$$

At $x = 2$, the left side equals $(1 + e^4)/2$, whereas the first term on the right is $e^4/2$. Subtraction gives $C = 1/2$. Therefore the particular solution is

$$y(x) = \frac{e^{2x} + 1}{2e^{2x}(x-1)^2}.$$

Step 3: Evaluate the required parameters Substituting $x = 3$ gives

$$y(3) = \frac{e^6 + 1}{2e^6 \cdot 4} = \frac{e^6 + 1}{8e^6}.$$

Comparison with the specified expression yields $\alpha = 6$ and $\beta = 8$. Both the exponential factor and the squared distance from $x = 1$ contribute to the denominator; dropping the latter would give an incorrect parameter.

Step 4: Conclusion The required numerical value is $\boxed{\alpha + \beta = 6 + 8 = 14}$.

Quick Tip to Solve: After finding the integrating factor, check that its product with the right side becomes simply e^{2x}; this catches a missing power of $x - 1$.

23. Let 3, 6, 9, 12, ... upto 78 terms and 5, 9, 13, 17, ... upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to _____. [4 marks]

Solution:

Concept:

- An arithmetic progression with first term a and difference d has term $a + (n-1)d$. Its sum is $n(a+l)/2$, where l is its last term.
- Common terms of two integer arithmetic progressions satisfy both congruences. Their separation is the least common multiple of the two differences, but their range must respect both finite lists.

Step 1: Find the finite ranges The first progression consists of the first 78 positive multiples of 3. Its last term is

$$3 + 77 \cdot 3 = 234.$$

The second progression has first term 5, difference 4 and 59 terms. Its last term is

$$5 + 58 \cdot 4 = 237.$$

A common term must therefore be at most 234, the smaller endpoint. It must also occur after the start of each list; using an infinite progression without these endpoint checks would overcount.

Step 2: Identify the common progression Write a common term as $3k$. Belonging to the second list requires $3k \equiv 1 \pmod 4$. Since 3 is its own multiplicative inverse modulo 4, this implies $k \equiv 3 \pmod 4$. Hence the common terms have the form

$$9 + 12j, \qquad j = 0, 1, 2, \ldots.$$

The first common term is 9, not 3, because 3 does not belong to the second list. The common difference is $\operatorname{lcm}(3, 4) = 12$.

Step 3: Count and sum the common terms The endpoint condition is $9 + 12j \le 234$, so $j \le 18.75$. The largest allowable integer is 18, giving last common term $9 + 12 \cdot 18 = 225$. The next candidate, 237, belongs to the second list but exceeds the first list. Thus there are 19 common terms, including both endpoints, and their sum is

$$S = \frac{19}{2}(9 + 225) = 19 \cdot 117 = 2223.$$

This uses a count of 19 rather than 18 because the parameter begins at zero.

Step 4: Conclusion The sum of all terms common to the two finite series is $\boxed{2223}$.

Quick Tip to Solve: Solve the two congruences first, then cap the common progression at the smaller last term. The larger endpoint alone can add a spurious term.

24. For real numbers a, b (a > b > 0), let

Area $\left\{(x, y) : x^2 + y^2 \leq a^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \geq 1\right\} = 30\pi$

and

Area $\left\{(x, y) : x^2 + y^2 \leq b^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\right\} = 18\pi$

Then, the value of $(a - b)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The ellipse $x^2/a^2 + y^2/b^2 \leq 1$ with $a > b > 0$ lies inside the circle of radius a and contains the circle of radius b, all centred at the origin.
- The areas of an ellipse and a circle are πab and πr^2, respectively. Once containment is proved, areas of the specified sets become differences or the full area of an inner region.

Step 1: Verify the geometric containments For a point in the ellipse,

$$x^2 + y^2 \leq x^2 + \frac{a^2}{b^2}y^2 \leq a^2.$$

Thus the entire ellipse lies inside the larger circle. Conversely, if $x^2 + y^2 \leq b^2$, then

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq \frac{x^2 + y^2}{b^2} \leq 1.$$

So the smaller disk lies entirely inside the ellipse. Boundaries have zero area, hence their inclusion in either inequality does not affect the numerical areas.

Step 2: Translate the two given areas The first set is the part of the larger disk outside the ellipse. Therefore its area gives

$$\pi a^2 - \pi ab = 30\pi, \qquad a(a - b) = 30.$$

The second set is the intersection of the smaller disk with the ellipse. Since that disk is contained in the ellipse, the intersection is just the whole smaller disk. Hence

$$\pi b^2 = 18\pi, \qquad b^2 = 18.$$

No subtraction of ellipse area is needed for the second condition.

Step 3: Determine the difference of semiaxes Let $d = a - b > 0$, so that $a = b + d$. The first relation becomes $d^2 + bd = 30$. Since $b = 3\sqrt{2}$, it follows that

$$d^2 + 3\sqrt{2}\,d - 30 = 0.$$

The discriminant is $18 + 120 = 138$, so the positive solution is

$$d = \frac{\sqrt{138} - 3\sqrt{2}}{2}.$$

Squaring gives $d^2 = 39 - 3\sqrt{69}$. This follows directly from both displayed area conditions; the value 12 would instead require $a(a - b) = 12 + 6\sqrt{6}$, which is not 30.

Step 4: Conclusion For the inequalities and areas as printed, $\boxed{(a-b)^2 = 39 - 3\sqrt{69}}$. This differs from the supplied answer-key value 12; substituting 12 into the stated conditions does not satisfy them.

Quick Tip to Solve: Prove containment before subtracting areas. The disk of radius b is wholly inside the ellipse, so the second condition fixes b^2 directly.

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25. Let f and g be twice differentiable even functions on $(-2, 2)$ such that $f\left(\frac{1}{4}\right) = 0$, $f\left(\frac{1}{2}\right) = 0$, $f(1) = 1$ and $g\left(\frac{3}{4}\right) = 0$, $g(1) = 2$. Then, the minimum number of solutions of $f(x)g''(x) + f'(x)g'(x) = 0$ in $(-2, 2)$ is equal to _____. [4 marks]

Solution:

Concept:

- For twice differentiable f, g, the expression in the equation is $H\prime(x)$, where $H(x) = f(x)g\prime(x)$. Rolle's theorem supplies a zero of the derivative between each pair of distinct zeros of H.
- The derivative of a differentiable even function is odd and vanishes at zero. Evenness also reflects each nonzero zero across the origin. To establish a minimum, a lower bound should be accompanied by an example attaining it.

Step 1: Produce five zeros of the product Because f is even and vanishes at $1/4$ and $1/2$, it also vanishes at $-1/4$ and $-1/2$. As g is differentiable and even, $g\prime(0) = 0$. Therefore the differentiable function $H = fg\prime$ has zeros at

$$-\frac{1}{2}, \quad -\frac{1}{4}, \quad 0, \quad \frac{1}{4}, \quad \frac{1}{2}.$$

All these points are strictly inside the prescribed domain. Their values are zero regardless of any other zeros the functions might have.

Step 2: Apply Rolle theorem on consecutive intervals Apply Rolle's theorem to each of the four closed intervals between consecutive displayed points. Differentiability on $(-2, 2)$ ensures continuity on each of these intervals. In each open interval there is a point at which

$$H\prime(x) = f\prime(x)g\prime(x) + f(x)g\prime\prime(x) = 0.$$

The four intervals are disjoint, so these points are distinct. Thus every allowable pair of functions gives at least four solutions of the required equation.

Step 3: Show that four solutions can be attained Choose the even polynomials

$$f(x) = \frac{64}{45}\left(x^2 - \frac{1}{16}\right)\left(x^2 - \frac{1}{4}\right), \qquad g(x) = \frac{32}{7}\left(x^2 - \frac{9}{16}\right).$$

They satisfy all five prescribed values, including $f(1) = 1$ and $g(1) = 2$. For this choice, $H(x)$ is a nonzero constant times $x(x^2 - 1/16)(x^2 - 1/4)$, a polynomial of degree five. Its derivative has degree four, and therefore has at most four distinct real zeros. The preceding Rolle argument already provides four in $(-1/2, 1/2)$, so this example has exactly four in the full domain.

Step 4: Conclusion The lower bound is attainable. Hence the minimum number of solutions in $(-2, 2)$

is $\boxed{4}$.

Quick Tip to Solve: When the equation looks like a product derivative, name that product first. Use evenness to reflect zeros, and check sharpness with simple even polynomials.

26. Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)^{15}, x > 0$, be m and n respectively. If r is a positive integer such that $mn^2 = {}^{15}C_r \cdot 2^r$, then the value of r is equal to _____. [4 marks]

Solution:

Concept:

- In $(A+B)^{15}$, the term indexed by k is $\binom{15}{k}A^{15-k}B^k$, with $0 \le k \le 15$. The exponent identifies the needed term before its coefficient is evaluated.
- With $A = 2x^{1/5}$ and $B = -x^{-1/5}$, each choice of the second factor lowers the exponent of x by $2/5$. The sign of the coefficient is $(-1)^k$.

Step 1: Write the general term The binomial term containing k copies of the negative second term is

$$T_{k+1} = \binom{15}{k} 2^{15-k}(-1)^k x^{(15-2k)/5}.$$

It is necessary to track the coefficient and exponent separately, since the requested powers of x are negative even though the expansion has a positive integer number of factors. The exponent decreases from 3 to -3 as k runs from 0 to 15.

Step 2: Extract both requested coefficients For x^{-1}, solve $(15-2k)/5 = -1$, giving $k = 10$. This is even, and hence

$$m = \binom{15}{10} 2^5 = \binom{15}{5} 2^5.$$

For x^{-3}, solve $(15-2k)/5 = -3$, giving $k = 15$. This is the final term, whose coefficient is

$$n = \binom{15}{15} 2^0 (-1)^{15} = -1.$$

In particular, $n^2 = 1$; the negative sign does not survive the square.

Step 3: Compare with the prescribed expression Combining the coefficients gives

$$mn^2 = \binom{15}{5} 2^5.$$

This has exactly the requested form with $r = 5$. To check no other positive index gives the same value, put $A_r = \binom{15}{r} 2^r$. Its successive ratio is

$$\frac{A_{r+1}}{A_r} = \frac{2(15-r)}{r+1}.$$

The sequence increases up to indices 10 and then decreases. For indices 1 through 4 it is less than A_5.

For indices 6 through 14 it is at least the smaller endpoint value $A_{14} = 245760 > A_5 = 96096$; finally $A_{15} = 32768 < A_5$. Thus 5 is the only admissible index.

Step 4: Conclusion The positive integer satisfying the coefficient relation is $\boxed{r = 5}$.

Quick Tip to Solve: Solve the exponent equation first. The coefficient of the lowest power comes from choosing the second term in every factor, which makes it especially easy to check.

27. The total number of four digit numbers such that each of first three digits is divisible by the last digit, is equal to _____. [4 marks]

Solution:

Concept:

- Fixing the last digit makes the divisibility condition independent for each of the first three positions. The multiplication principle counts choices for these positions, and the addition principle combines distinct final digits.
- The thousands digit cannot be zero, whereas the hundreds and tens digits can be zero. Zero is divisible by any nonzero integer, but divisibility by zero is undefined, so the last digit cannot be zero.

Step 1: Count choices for a fixed last digit Let the units digit be $d \in \{1, 2, \ldots, 9\}$, and set $k = \lfloor 9/d \rfloor$. The nonzero digits divisible by d are $d, 2d, \ldots, kd$. Thus there are k choices for the first digit and $k + 1$ for each of the next two digits, because those positions also admit zero. Repetition is not forbidden by the question, so the choices are independent. For this final digit the number of four-digit numbers is therefore

$$k(k + 1)^2.$$

Step 2: Group final digits having the same count For $d = 1$, $k = 9$, giving $9 \cdot 10^2 = 900$. For $d = 2$, $k = 4$, giving $4 \cdot 5^2 = 100$. For $d = 3$, $k = 3$, giving $3 \cdot 4^2 = 48$. For $d = 4$, $k = 2$, giving $2 \cdot 3^2 = 18$. Finally, for each of $d = 5, 6, 7, 8, 9$, only one nonzero digit is divisible by d, namely d itself. Each such case contributes $1 \cdot 2^2 = 4$ numbers.

Step 3: Add the disjoint cases Numbers with different units digits are different, so there is no overlap between the nine cases. Their total is

$$900 + 100 + 48 + 18 + 5 \cdot 4 = 1086.$$

The initial position was kept nonzero throughout, so no counted string has fewer than four digits. Allowing zero in the middle positions is essential: for example, a number ending in 5 may have either 0 or 5 in each of those positions, but it must begin with 5. The condition concerns individual digits, not divisibility of the complete number by the units digit.

Step 4: Conclusion The total number of four-digit numbers satisfying the digitwise divisibility condition is $\boxed{1086}$.

Quick Tip to Solve: Fix the units digit and use $\lfloor 9/d \rfloor (\lfloor 9/d \rfloor + 1)^2$. Include zero in the middle positions, but exclude it from the first and final positions.

28. Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number an $N = \sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is _____. [4 marks]

Solution:

Concept:

- The given skew-symmetric matrix satisfies $M^2 = -\alpha^2 I$. Every even power is therefore a scalar multiple of the identity, reducing the matrix equation to one real-variable equation.
- A finite matrix geometric sum can be telescoped by multiplication with $I - M^2$. Its intermediate powers cancel exactly; no matrix inverse or infinite-series convergence is required.

Step 1: Reduce the even powers to scalar powers Multiplying the two matrices explicitly gives

$$M^2 = \begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{bmatrix} = -\alpha^2 I.$$

Writing $t = \alpha^2 > 0$, we have $M^{2k} = (-t)^k I$. Thus N is a scalar multiple of I. The nonzero-real condition ensures $t > 0$, which will let us identify the unique solution by monotonicity.

Step 2: Telescope the finite sum Use the definition $N = M^2 + M^4 + \cdots + M^{98}$. Multiplication by $I - M^2$ yields

$$(I - M^2)N = M^2 - M^{100}.$$

All powers between M^4 and M^{98} cancel in pairs. Since $M^{100} = (M^2)^{50} = t^{50} I$, the stated equation becomes

$$(-t - t^{50})I = -2I, \qquad t + t^{50} = 2.$$

The final exponent is 50 rather than 49 because multiplying the last term of the sum introduces one additional factor of M^2.

Step 3: Solve the positive scalar equation The number $t = 1$ satisfies $t + t^{50} = 2$. It is the only positive solution: if $0 < t < 1$, both t and t^{50} are less than 1, so their sum is less than 2; if $t > 1$, both exceed 1 and their sum exceeds 2. Consequently $\alpha^2 = 1$, giving the two real possibilities $\alpha = \pm 1$. The question asks for the positive integral value, which selects $\alpha = 1$. As a direct check, when $M^2 = -I$, the 49-term alternating sum $-I + I - \cdots - I$ equals $-I$, and $(I - M^2)N = 2I(-I) = -2I$.

Step 4: Conclusion The required positive integral value is $\boxed{\alpha = 1}$.

Quick Tip to Solve: Compute M^2 before working on a long power sum. Multiplication by $I - M^2$ leaves only the first term and one extra final power.

29. Let f(x) and g(x) be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$ and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is _____. [4 marks]

Solution:

Concept:

- Write the linear polynomial as $g(x) = ax + b$, with $a \neq 0$, and the quadratic as $f(x) = cx^2 + dx + e$, with $c \neq 0$. Equality of polynomials means equality of corresponding coefficients.
- The compositions have different constant and linear terms: $g(f(x)) = acx^2 + adx + ae + b$, while $f(g(x)) = ca^2x^2 + (2cab + da)x + cb^2 + db + e$.

Step 1: Compare the quadratic coefficients From $g(f(x)) = 4x^2 + 6x + 1$, we have $ac = 4$, $ad = 6$, and $ae + b = 1$. From the other composition, $ca^2 = 8$. Dividing this equation by $ac = 4$, which is nonzero, gives $a = 2$. Consequently, $c = 2$ and $d = 3$. This determines both leading coefficients before we use any constant terms.

Step 2: Determine the remaining coefficients The coefficient of x in $f(g(x))$ is -2. Therefore

$$2(2)(2)b + 3(2) = -2, \qquad 8b + 6 = -2,$$

so $b = -1$. Now $ae + b = 1$ becomes $2e - 1 = 1$, hence $e = 1$. Thus $f(x) = 2x^2 + 3x + 1$ and $g(x) = 2x - 1$. As a check, the constant term of $f(g(x))$ is $2 - 3 + 1 = 0$, as required, and direct substitution gives $8x^2 - 2x$.

Step 3: Evaluate both functions at the requested argument The compositions were only used to find the functions; the question now asks for their separate values. We obtain

$$f(2) = 2(2)^2 + 3(2) + 1 = 15, \qquad g(2) = 2(2) - 1 = 3.$$

Adding these values gives $15 + 3 = 18$. No choice of an alternative sign for a remains, because its value followed from the ratio of two nonzero coefficient equations.

Step 4: Conclusion The required numerical value is $\boxed{f(2) + g(2) = 18}$.

Quick Tip to Solve: Compare the leading coefficients of the two compositions first: their ratio immediately gives the slope of the linear function.

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30. The number of solutions of the equation $\sin x = \cos^2 x$ in the interval (0, 10) is _____. [4 marks]

Solution:

Concept:

- Use $\cos^2 x = 1 - \sin^2 x$ to turn the equation into a quadratic in $\sin x$. Any root must lie in the range $[-1, 1]$.
- For $\sin x = s$ with $0 < s < 1$, the two solution families are $x = \alpha + 2k\pi$ and $x = \pi - \alpha + 2k\pi$, where $\alpha = \sin^{-1} s$ and k is an integer. Angles here are in radians.

Step 1: Solve the quadratic and reject the inadmissible root Put $s = \sin x$. The given equation

becomes

$$s = 1 - s^2, \qquad s^2 + s - 1 = 0.$$

Thus $s = (-1 \pm \sqrt{5})/2$. The negative root is less than -1, so it cannot be a sine value. Only $s = (\sqrt{5} - 1)/2$, approximately 0.618, is allowed. Keeping both algebraic roots would incorrectly count extra trigonometric solutions.

Step 2: Count the solutions in the specified open interval Let $\alpha = \sin^{-1}((\sqrt{5} - 1)/2)$, approximately 0.666. For $k = 0$, the two roots are α and $\pi - \alpha$, approximately 0.666 and 2.475. For $k = 1$, they are $2\pi + \alpha$ and $3\pi - \alpha$, approximately 6.950 and 8.759. All four lie strictly between 0 and 10.

Step 3: Exclude all remaining periods For $k \leq -1$, even $\pi - \alpha + 2k\pi$ is negative. For $k \geq 2$, the smaller root $\alpha + 2k\pi$ exceeds $4\pi > 10$. Hence no other family member belongs to the interval. The endpoints themselves do not create a counting issue because the interval is open and the four roots found are interior points.

Step 4: Conclusion There are $\boxed{4}$ solutions in $(0, 10)$.

Quick Tip to Solve: After solving for sine, list both branches in each period; counting only the principal inverse-sine value misses half the roots.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A small toy starts moving from the position of rest under a constant acceleration. If it travels a distance of 10m in t s, the distance travelled by the toy in the next t s will be : [4 marks]

(A) 10 m

(B) 20 m

(C) 30 m

(D) 40 m

Correct Answer: (C) 30 m

Solution:

Concept:

- For motion from rest with constant acceleration a, the displacement at elapsed time T is $s(T) = \frac{1}{2}aT^2$. The distance in a later interval is the difference of total displacements.
- The toy continues to accelerate during the next interval. Its velocity at time t is at, so the second interval does not begin from rest.

Step 1: Use the first interval to express the acceleration The initial speed is zero and the first distance is $10\,\mathrm{m}$. Applying the constant-acceleration equation gives

$$10 = \frac{1}{2}at^2, \qquad at^2 = 20\,\mathrm{m}.$$

We do not need the separate values of a and t: the product at^2 is sufficient. Since the acceleration is along the direction in which the toy starts moving, there is no reversal of motion.

Step 2: Subtract the displacement at the interval boundaries The end of the next interval is at total elapsed time $2t$, measured from the original start. Thus

$$s(2t) = \frac{1}{2}a(2t)^2 = 4\left(\frac{1}{2}at^2\right) = 40\,\text{m}.$$

This is the total distance in both intervals. The required distance in the next t seconds alone is

$$s(2t) - s(t) = 40 - 10 = 30\,\text{m}.$$

Equivalently, start the second interval with speed at: its distance is $(at)t + \frac{1}{2}at^2 = \frac{3}{2}at^2$.

Step 3: Check the alternatives against the time dependence Option (A) would apply to equal intervals at unchanged speed, which contradicts constant nonzero acceleration from rest. Option (B) treats the distance as proportional to time rather than time squared. Option (D) is the cumulative distance after $2t$, not the distance during the second interval. The difference calculation identifies option (C).

Step 4: Conclusion The toy travels $\boxed{30\,\text{m}}$ during the next t seconds. Hence, the correct option is (C).

Quick Tip to Solve: For a start from rest, distances in successive equal time intervals follow the ratio 1:3:5; distinguish an interval distance from the total distance.

32. At what temperature a gold ring of diameter 6.230 cm be heated so that it can be fitted on a wooden bangle of diameter 6.241 cm ? Both the diameters have been measured at room temperature (27°C).

(Given : coefficient of linear thermal expansion of gold $\alpha_L = 1.4 \times 10^{-5}\ \text{K}^{-1}$) [4 marks]

(A) 125.7°C

(B) 91.7°C

(C) 425.7°C

(D) 152.7°C

Correct Answer: (D) 152.7°C

Solution:

Concept:

- The diameter of a freely heated ring expands by the same fractional amount as any linear dimension: $D = D_0(1 + \alpha_L \Delta T)$. Its opening grows on heating.
- The required increase in diameter is the difference between the bangle diameter and the initial ring diameter. A temperature difference has the same numerical value in kelvin and degrees Celsius.

Step 1: Find the required change in diameter Only the gold ring is heated; the wooden bangle provides the target diameter. The ring must reach at least $6.241\,\text{cm}$, starting from $6.230\,\text{cm}$ at $27°\text{C}$. Therefore

$$\Delta D = 6.241 - 6.230 = 0.011\,\text{cm}.$$

The coefficient supplied is a linear expansion coefficient, so it applies directly to the diameter; an

area-expansion factor of two is not appropriate.

Step 2: Calculate the temperature rise Using $\Delta D = \alpha_L D_0 \Delta T$,

$$\Delta T = \frac{0.011}{(1.4 \times 10^{-5})(6.230)} \approx 126.1\,\text{K}.$$

The centimetre units cancel in this ratio, so no conversion to metres is required. This is a rise above room temperature, rather than the final Celsius temperature. To the precision of the answer choices, the intended rise is about 125.7 to 126 degrees.

Step 3: Convert the rise into the final temperature Adding the starting temperature gives

$$T \approx 27 + 126.1 = 153.1^\circ\text{C}.$$

The nearest listed temperature is 152.7°C, option (D), with the small difference attributable to numerical rounding in the listed value. Option (A) is close to the rise alone and omits the initial 27°C. Option (B) supplies too little expansion, and option (C) heats the ring far beyond the required size.

Step 4: Conclusion The calculated temperature is approximately 153°C; the matching listed value is $\boxed{152.7^\circ\text{C}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the diameter with the linear coefficient, then add the initial Celsius temperature; the expansion formula first gives a temperature rise.

33. Two point charges Q each are placed at a distance d apart. A third point charge q is placed at a distance x from mid-point on the perpendicular bisector. The value of x at which charge q will experience the maximum Coulomb's force is : [4 marks]

(A) x = d

(B) $x = \frac{d}{2}$

(C) $x = \frac{d}{\sqrt{2}}$

(D) $x = \frac{d}{2\sqrt{2}}$

Correct Answer: (D) $x = \frac{d}{2\sqrt{2}}$

Solution:

Concept:

- The two equal charges have equal distances $r = \sqrt{x^2 + (d/2)^2}$ from the test charge. Their field components parallel to the line joining the source charges cancel by symmetry.
- The perpendicular components add. Coulomb force magnitude is $|q|E$, so its maximum occurs at the same position as the maximum of E, for fixed nonzero charges.

Step 1: Express the force as a function of position Write $a = d/2$ and $k = 1/(4\pi\varepsilon_0)$. Each charge produces field magnitude $k|Q|/(x^2 + a^2)$. The component along the perpendicular bisector multiplies

this by $x/\sqrt{x^2+a^2}$. Thus, for distance $x \geq 0$,

$$F(x) = \frac{2k|Qq|x}{(x^2+a^2)^{3/2}}.$$

The signs of the charges determine direction, but not the position maximizing this magnitude.

Step 2: Differentiate the position-dependent factor The constant $2k|Qq|$ can be left outside the derivative. Applying the product rule gives

$$\frac{d}{dx}\left[x(x^2+a^2)^{-3/2}\right] = \frac{a^2-2x^2}{(x^2+a^2)^{5/2}}.$$

Its denominator is positive. The derivative vanishes when $2x^2 = a^2$, so $x = a/\sqrt{2} = d/(2\sqrt{2})$. The derivative changes from positive to negative there, establishing a maximum.

Step 3: Check the boundary behaviour and options At the midpoint $x = 0$, the two fields cancel and the force is zero. As x becomes very large, the force tends to zero as $1/x^2$. Hence the stationary point between these limits is the global maximum. Options (A), (B), and (C) all give larger values of x than the maximum position and do not satisfy the derivative equation $a^2 = 2x^2$.

Step 4: Conclusion The maximum Coulomb force occurs at $\boxed{x = d/(2\sqrt{2})}$. Hence, the correct option is (D).

Quick Tip to Solve: On the perpendicular bisector, add field components first; maximizing the single-charge field alone misses the projection factor.

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34. The speed of light in media 'A' and 'B' are 2.0×10^{10} cm/s and 1.5×10^{10} cm/s respectively. A ray of light enters from the medium B to A at an incident angle 'θ'. If the ray suffers total internal reflection, then [4 marks]

(A) $\theta = \sin^{-1}\left(\frac{3}{4}\right)$
(B) $\theta > \sin^{-1}\left(\frac{2}{3}\right)$
(C) $\theta < \sin^{-1}\left(\frac{3}{4}\right)$
(D) $\theta > \sin^{-1}\left(\frac{3}{4}\right)$

Correct Answer: (D) $\theta > \sin^{-1}\left(\frac{3}{4}\right)$

Solution:

Concept:

- Refractive index and light speed satisfy $n = c/v$. Therefore the medium with the lower light speed has the higher refractive index and is optically denser.
- For light going from a denser medium to a rarer medium, the critical angle satisfies $\sin\theta_c = n_{\text{rarer}}/n_{\text{denser}}$. Total internal reflection requires incidence strictly above this angle.

Step 1: Identify the direction of the refractive-index change The given speeds are $v_A = 2.0 \times 10^{10}\,\text{cm}\,\text{s}^{-1}$ and $v_B = 1.5 \times 10^{10}\,\text{cm}\,\text{s}^{-1}$. Since $v_B < v_A$, we have $n_B > n_A$. The ray travels from B to A, so it travels from the optically denser medium to the rarer one. This satisfies the first condition for

total internal reflection; incidence from A to B would not do so.

Step 2: Find the critical angle from Snell's law At the critical angle, the refracted ray travels along the interface, at 90° to the normal. Snell's law gives

$$n_B \sin\theta_c = n_A, \qquad \sin\theta_c = \frac{n_A}{n_B} = \frac{v_B}{v_A} = \frac{1.5}{2.0} = \frac{3}{4}.$$

Consequently $\theta_c = \sin^{-1}(3/4)$, approximately 48.6°. The common speed units cancel in the ratio.

Step 3: Apply the strict inequality and test the alternatives Total internal reflection occurs for $\theta > \theta_c$. Equality in option (A) describes the limiting refracted ray along the boundary, not total internal reflection. Option (C) gives incidence below the critical angle, where an ordinary refracted ray exists. Option (B) uses $2/3$, an incorrect ratio; that lower threshold includes angles at which refraction still occurs. Only option (D) states the required critical-angle condition.

Step 4: Conclusion The necessary condition is $\boxed{\theta > \sin^{-1}(3/4)}$. Hence, the correct option is (D).

Quick Tip to Solve: For a critical-angle calculation using speeds, take the slower-medium speed divided by the faster-medium speed; the ratio must be below one.

35. The electric field at a point associated with a light wave is given by

$E = 200\ [\sin(6 \times 10^{15})t + \sin(9 \times 10^{15})t]\ Vm^{-1}$

Given : $h = 4.14 \times 10^{-15}$ eVs

If this light falls on a metal surface having a work function of 2.50 eV, the maximum kinetic energy of the photoelectrons will be [4 marks]

(A) 1.90 eV

(B) 3.27 eV

(C) 3.60 eV

(D) 3.42 eV

Correct Answer: (D) 3.42 eV

Solution:

Concept:

- In a term $\sin(\omega t)$, the coefficient of time is angular frequency, so the ordinary frequency is $\nu = \omega/(2\pi)$. Photon energy is $h\nu$, not $h\omega$.
- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the work function. With two incident frequencies, the higher frequency gives the fastest emitted electrons.

Step 1: Choose the frequency giving maximum photon energy The field contains components with angular frequencies $6 \times 10^{15}\,\mathrm{s}^{-1}$ and $9 \times 10^{15}\,\mathrm{s}^{-1}$. A photoelectron absorbs the energy of a photon from one component; the two photon energies are not added merely because the electric fields superpose. The higher angular frequency therefore determines the maximum kinetic energy. Its

ordinary frequency is

$$\nu_{\max} = \frac{9 \times 10^{15}}{2\pi}\,\text{Hz}.$$

Step 2: Subtract the work function from the photon energy The supplied Planck constant is already in electron-volt seconds. Hence

$$h\nu_{\max} = \frac{(4.14 \times 10^{-15})(9 \times 10^{15})}{2\pi}\,\text{eV} \approx 5.93\,\text{eV}.$$

Subtracting the work function $2.50\,\text{eV}$ gives

$$K_{\max} \approx 5.93 - 2.50 = 3.43\,\text{eV},$$

which matches the listed value $3.42\,\text{eV}$ to the precision of the approximations used.

Step 3: Distinguish the lower-frequency electrons and other options The lower component has photon energy about $3.95\,\text{eV}$, so its emitted electrons have maximum energy only about $1.45\,\text{eV}$. They do not determine the maximum over all emitted electrons. The field amplitude controls the rate of photon arrival and the photocurrent, not this energy cutoff. Values 1.90, 3.27, and $3.60\,\text{eV}$ do not follow from the higher-frequency photon energy minus the given work function.

Step 4: Conclusion The maximum photoelectron kinetic energy is approximately $\boxed{3.42\,\text{eV}}$. Hence, the correct option is (D).

Quick Tip to Solve: Always divide the time coefficient in a sine wave by 2π before using $E = h\nu$; use the higher frequency for the maximum cutoff.

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36. A capacitor is discharging through a resistor R. Consider in time t_1, the energy stored in the capacitor reduces to half of its initial value and in time t_2, the charge stored reduces to one eighth of its initial value. The ratio t_1/t_2 will be [4 marks]

(A) 1/2

(B) 1/3

(C) 1/4

(D) 1/6

Correct Answer: (D) 1/6

Solution:

Concept:

- The charge on a discharging capacitor obeys $Q(t) = Q_0 e^{-t/(RC)}$, where RC is the circuit time constant.
- The stored energy is $U = Q^2/(2C)$. Consequently its decay factor is the square of the charge decay factor, not the same factor.

Step 1: Find the energy half-time Write the energy in terms of its initial value:

$$\frac{U(t)}{U_0} = \left(\frac{Q(t)}{Q_0}\right)^2 = e^{-2t/(RC)}.$$

At the specified instant, this ratio is one half. Taking natural logarithms gives

$$e^{-2t_1/(RC)} = \frac{1}{2}, \qquad t_1 = \frac{RC}{2}\ln 2.$$

The factor of two in the exponent is essential because energy is quadratic in charge.

Step 2: Find the charge decay time For charge to become one eighth of its initial value, use the charge equation directly:

$$e^{-t_2/(RC)} = \frac{1}{8} = 2^{-3}.$$

Taking logarithms yields

$$t_2 = RC\ln 8 = 3RC\ln 2.$$

This is three charge half-lives, whereas the energy half-life is only half of one charge half-life.

Step 3: Compare the two intervals The resistance, capacitance and logarithmic factor cancel in the ratio:

$$\frac{t_1}{t_2} = \frac{(RC/2)\ln 2}{3RC\ln 2} = \frac{1}{6}.$$

Thus the ratio is independent of the initial charge and circuit parameters. The alternatives one half, one third and one fourth omit either the squared dependence of energy or the three charge half-lives.

Step 4: Conclusion The required ratio is $t_1/t_2 = 1/6$. Hence, the correct option is (D).

Quick Tip to Solve: For capacitor discharge, energy decays twice as fast in the exponent as charge: first write the fraction for the quantity actually specified.

37. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2, if the process is purely adiabatic and W_3 if the process is purely isobaric. Then, choose the correct option [4 marks]

(A) $W_1 < W_2 < W_3$

(B) $W_2 < W_3 < W_1$

(C) $W_3 < W_1 < W_2$

(D) $W_2 < W_1 < W_3$

Correct Answer: (D) $W_2 < W_1 < W_3$

Solution:

Concept:

- The work done by an expanding gas is $W = \int_{V_1}^{V_2} P\,dV$; it is determined by the pressure throughout the expansion.
- For the same initial state (P_1, V_1, T_1), isothermal expansion obeys $PV = P_1V_1$, while reversible

adiabatic expansion obeys $PV^\gamma = P_1V_1^\gamma$, with $\gamma > 1$.

Step 1: Compare the pressures at an intermediate volume Put $x = V/V_1$. During expansion, $x > 1$. The pressures for the three processes are

$$P_{\text{iso}} = P_1x^{-1}, \quad P_{\text{ad}} = P_1x^{-\gamma}, \quad P_{\text{bar}} = P_1.$$

Since $\gamma > 1$, the adiabatic pressure falls more rapidly than the isothermal pressure. Hence

$$P_{\text{ad}} < P_{\text{iso}} < P_{\text{bar}} \qquad (V > V_1).$$

All three begin at the same pressure, so the comparison concerns the subsequent paths.

Step 2: Integrate over the common volume interval The final volume V_2 is the same for all three expansions, and dV is positive. Integrating the ordered positive pressures therefore preserves their order:

$$\int_{V_1}^{V_2} P_{\text{ad}}\,dV < \int_{V_1}^{V_2} P_{\text{iso}}\,dV < \int_{V_1}^{V_2} P_{\text{bar}}\,dV.$$

These are respectively W_2, W_1, W_3. Physically, an adiabatically expanding gas cools, reducing its pressure and the work below the isothermal value. Constant pressure gives the greatest area under the pressure-volume path. Every other option reverses at least one of these pressure-area comparisons.

Step 3: Conclusion The correct ordering is $W_2 < W_1 < W_3$. Hence, the correct option is (D).

Quick Tip to Solve: For expansion from one common initial state, compare the pressure curves before calculating work: isobaric lies highest and adiabatic lowest.

38. Two long current carrying conductors are placed to each other at a distance of 8 cm between them. The magnitude of magnetic field produced at mid-point between the two conductors due to current flowing in them is 300 μT. The equal current flowing in the two conductors is : [4 marks]

(A) 30A in the same direction.

(B) 30A in the opposite direction.

(C) 60A in the opposite direction.

(D) 300A in the opposite direction.

Correct Answer: (B) 30A in the opposite direction.

Solution:

Concept:

- The magnetic field at perpendicular distance r from a long straight wire carrying current I has magnitude $B = \mu_0 I/(2\pi r)$.
- Magnetic fields add as vectors. At the midpoint of two identical parallel wires, equal currents in the same direction produce cancelling fields; opposite currents produce adding fields.

Step 1: Determine the direction relation The wire separation is $d = 8\,\text{cm} = 0.08\,\text{m}$, so the midpoint lies $r = d/2 = 0.04\,\text{m}$ from each wire. The two field magnitudes there are equal. By the right-hand

rule, same-direction currents give oppositely directed midpoint fields and a zero resultant. Since the stated resultant is nonzero, the two currents must run in opposite directions. In that case their fields point in the same direction at the midpoint.

Step 2: Use the resultant field to calculate the current Add the two equal contributions:

$$B_{\text{net}} = 2\frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{\pi r}.$$

Substitute $B_{\text{net}} = 300 \times 10^{-6}\,\text{T}$ and $\mu_0 = 4\pi \times 10^{-7}\,\text{T m A}^{-1}$:

$$I = \frac{B_{\text{net}}\pi r}{\mu_0} = \frac{300 \times 10^{-6} \times 0.04}{4 \times 10^{-7}} = 30\,\text{A}.$$

Each wire thus contributes $150\,\mu\text{T}$. A current of 60 A would double the required resultant and 300 A would make it ten times too large; 30 A in the same direction would cancel instead.

Step 3: Conclusion Each wire carries $30\,\text{A}$, and the currents are in opposite directions. Hence, the correct option is (B).

Quick Tip to Solve: At the midpoint, first decide whether the two fields add or cancel; use half the wire separation as the distance from either wire.

39. The time period of a satellite revolving around earth in a given orbit is 7 hours. If the radius of orbit is increased to three times its previous value, then approximate new time period of the satellite will be [4 marks]

(A) 40 hours

(B) 36 hours

(C) 30 hours

(D) 25 hours

Correct Answer: (B) 36 hours

Solution:

Concept:

- For a satellite orbiting the same central body, Kepler's third law gives $T^2 \propto r^3$, where r is the orbital radius measured from the centre of the Earth.
- Equivalently, $T = 2\pi\sqrt{r^3/(GM)}$. Taking ratios removes the gravitational constant and Earth's mass.

Step 1: Form the ratio of orbital periods Let the original period and radius be T_1 and r_1, and the new quantities be T_2 and r_2. Here $T_1 = 7\,\text{h}$ and $r_2 = 3r_1$. Apply the period-radius law:

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3 = 3^3 = 27.$$

Take the positive square root because a time period is positive:

$$\frac{T_2}{T_1} = \sqrt{27} = 3\sqrt{3}.$$

Radius itself, rather than height above the surface, is tripled here.

Step 2: Evaluate and select the nearest listed value The new period is therefore

$$T_2 = 7(3\sqrt{3}) = 21\sqrt{3}\,\text{h} \approx 36.37\,\text{h}.$$

To the approximation used in the choices, this is 36 hours. The value 40 hours is farther from the calculated result, while 30 and 25 hours underestimate the increase. A linear scaling of period with radius would give 21 hours, which is also inconsistent: orbital speed decreases as the radius increases, so the period grows faster than the orbit circumference alone.

Step 3: Conclusion The new orbital period is approximately $36\,\text{h}$. Hence, the correct option is (B).

Quick Tip to Solve: When an orbital radius is multiplied by a factor k, multiply the period by $k^{3/2}$, not by k or k^2.

40. The motion of a simple pendulum executing S.H.M. is represented by the following equation. $y = A\sin(\pi t + \phi)$, where time is measured in second. The length of pendulum is [4 marks]

(A) 97.23 cm

(B) 25.3 cm

(C) 99.4 cm

(D) 406.1 cm

Correct Answer: (C) 99.4 cm

Solution:

Concept:

- Simple harmonic motion has the form $y = A\sin(\omega t + \phi)$. The coefficient of time inside the phase is the angular frequency ω, in radians per second.
- For a simple pendulum in the small-angle approximation, $\omega = \sqrt{g/L}$, where L is its length and g the gravitational acceleration.

Step 1: Read the angular frequency correctly Compare the given equation

$$y = A\sin(\pi t + \phi)$$

with the standard SHM equation. Since time is in seconds,

$$\omega = \pi\,\text{rad}\,\text{s}^{-1}.$$

The amplitude and initial phase do not change the period of ideal small-amplitude pendulum oscilla-

tions. The corresponding period is

$$T = \frac{2\pi}{\omega} = 2\,\text{s}.$$

In particular, the angular frequency is not the ordinary frequency; the latter is $f = 1/T = 0.5\,\text{Hz}$.

Step 2: Calculate the length and compare choices Rearrange the pendulum frequency formula:

$$L = \frac{g}{\omega^2} = \frac{9.81}{\pi^2}\,\text{m} \approx 0.994\,\text{m}.$$

Converting metres to centimetres gives

$$L \approx 99.4\,\text{cm}.$$

Here the standard value $g = 9.81\,\text{m}\,\text{s}^{-2}$ has been used. The 25.3 cm alternative is approximately one quarter of this length and results from an incorrect frequency factor; 406.1 cm is about four times too large. Of the two values near a metre, 99.4 cm matches the computed length, whereas 97.23 cm does not.

Step 3: Conclusion The pendulum length is approximately $99.4\,\text{cm}$. Hence, the correct option is (C).

Quick Tip to Solve: In a sine equation, identify the coefficient of t as angular frequency; then use $L = g/\omega^2$ directly to avoid extra factors of 2π.

41. A vessel contains 16g of hydrogen and 128g of oxygen at standard temperature and pressure. The volume of the vessel in cm^3 is : [4 marks]

(A) 72×10^5

(B) 32×10^5

(C) 27×10^4

(D) 54×10^4

Correct Answer: (C) 27×10^4

Solution:

Concept:

- For an ideal mixture at a common temperature and pressure, the gas law uses the total mole number: $PV = (n_{H_2} + n_{O_2})RT$.
- At standard temperature and pressure, one mole of ideal gas occupies approximately $22.4\,\text{L}$. Hydrogen and oxygen here are diatomic molecular gases, with molar masses 2 and 32 grams per mole.

Step 1: Convert the masses into molecular mole numbers The amounts of the two gases are calculated separately:

$$n_{H_2} = \frac{16\,\text{g}}{2\,\text{g}\,\text{mol}^{-1}} = 8\,\text{mol}, \qquad n_{O_2} = \frac{128\,\text{g}}{32\,\text{g}\,\text{mol}^{-1}} = 4\,\text{mol}.$$

Hence the vessel contains a total of

$$n = 8 + 4 = 12\,\text{mol}.$$

The molecules are counted using their molecular masses, not the atomic masses of hydrogen and oxygen. No chemical reaction or ignition is specified, so the sample is treated as the stated gas mixture.

Step 2: Find the common volume at STP Use the total mole number in the ideal gas law, or multiply it by the molar volume:

$$V = 12 \times 22.4 = 268.8\,\mathrm{L}.$$

Since one litre is $10^3\,\mathrm{cm}^3$,

$$V = 268.8 \times 10^3\,\mathrm{cm}^3 = 2.688 \times 10^5\,\mathrm{cm}^3 \approx 27 \times 10^4\,\mathrm{cm}^3.$$

Thus the listed value is an approximation to the calculated result. The first two choices are over an order of magnitude too large; the last choice is about twice the required volume, as can occur if atomic rather than molecular masses are used.

Step 3: Conclusion The volume is approximately $27 \times 10^4\,\mathrm{cm}^3$. Hence, the correct option is (C).

Quick Tip to Solve: For a gas mixture, add moles rather than masses before using molar volume, and remember that hydrogen and oxygen gases are diatomic.

42. Given below are two statements :

Statement I : The electric force changes the speed of the charged particle and hence changes its kinetic energy; whereas the magnetic force does not change the kinetic energy of the charged particle.

Statement II : The electric force accelerates the positively charged particle perpendicular to the direction of electric field. The magnetic force accelerates the moving charged particle along the direction of magnetic field.

In the light of the above statements, choose the most appropriate answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (C) Statement I is correct but Statement II is incorrect.

Solution:

Concept:

- The Lorentz force is $\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$. The electric force on a positive charge points along the electric field.
- The rate of change of kinetic energy is $dK/dt = \mathbf{F} \cdot \mathbf{v}$. A force perpendicular to instantaneous velocity does no work.

Step 1: Evaluate Statement I using the work-energy relation The magnetic contribution to the power

is

$$P_B = q(\mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} = 0,$$

because the cross product is perpendicular to $\mathbf{v}$. A magnetic field therefore cannot change the speed or kinetic energy, although it can change the velocity direction. The electric contribution is

$$P_E = q\mathbf{E} \cdot \mathbf{v},$$

which can be nonzero: an electric field can do work and change the speed and kinetic energy. Statement I expresses this standard distinction and is correct; it does not require the electric power to be nonzero at every instant of every possible trajectory.

Step 2: Check each directional claim in Statement II For a positive particle, $\mathbf{a}_E = q\mathbf{E}/m$, so the electric acceleration is parallel to the electric field, not perpendicular to it. The magnetic acceleration is

$$\mathbf{a}_B = \frac{q}{m}(\mathbf{v} \times \mathbf{B}),$$

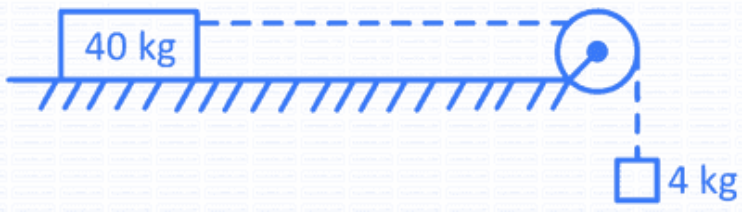
so, whenever nonzero, it is perpendicular to both the velocity and the magnetic field, not along the magnetic field. Both direction claims in Statement II are consequently wrong. Options saying both statements are correct, both are incorrect, or only Statement II is correct fail these separate checks.

Step 3: Conclusion Statement I is correct, whereas Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: Check magnetic forces with both dot and cross products: the cross product gives direction, and its zero dot product with velocity proves zero work.

43. A block of mass 40 kg slides over a surface, when a mass of 4 kg is suspended through an inextensible massless string passing over frictionless pulley as shown below.

The coefficient of kinetic friction between the surface and block is 0.02. The acceleration of block is. (Given g = 10 ms^{-2}.) [4 marks]



(A) 1 ms^{-2}

(B) 1/5 ms^{-2}

(C) 4/5 ms^{-2}

(D) 8/11 ms^{-2}

Correct Answer: (D) 8/11 ms^{-2}

Solution:

Concept:

- For a sliding block, kinetic friction has magnitude $f_k = \mu_k N$ and acts opposite the motion. The vertical forces on the horizontal block balance, so $N = Mg$.
- A massless inextensible string gives both masses the same acceleration magnitude. Adding their Newton-law equations cancels the internal tension.

Step 1: Calculate the opposing friction The horizontal block has mass $M = 40\,\text{kg}$. Its normal reaction and friction are

$$N = 40(10) = 400\,\text{N}, \qquad f_k = 0.02(400) = 8\,\text{N}.$$

The hanging mass supplies a downward weight of $4(10) = 40\,\text{N}$, which is greater than the opposing friction, so it accelerates downward while the block moves towards the pulley.

Step 2: Apply Newton's law to both masses Let T be the string tension and a the common acceleration. For the horizontal block and hanging mass respectively,

$$T - 8 = 40a, \qquad 40 - T = 4a.$$

Adding eliminates tension and gives

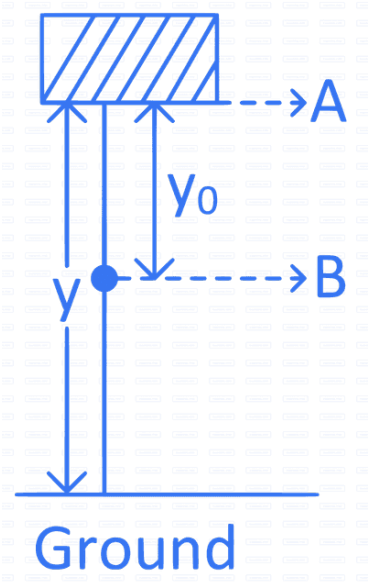
$$32 = 44a \quad \Rightarrow \quad a = \frac{8}{11}\,\text{m}\,\text{s}^{-2}.$$

The denominator includes both moving masses. Using only the block mass would give $4/5$, option (C); omitting friction would give a different value. Options (A) and (B) do not satisfy the combined force equation.

Step 3: Conclusion The acceleration is $\boxed{\frac{8}{11}\,\text{m}\,\text{s}^{-2}}$, with the block moving towards the pulley. Hence, the correct option is (D).

Quick Tip to Solve: Add the two force equations before finding tension: the net external driving force is hanging weight minus friction, and the accelerated mass is the sum of both masses.

44. In the given figure, the block of mass m is dropped from the point 'A'. The expression for kinetic energy of block when it reaches point 'B' is [4 marks]



(A) $\frac{1}{2}mg\,{y_0}^2$

(B) $\frac{1}{2}mg\,y^2$

(C) $mg(y - y_0)$

(D) mgy_0

Correct Answer: (D) mgy_0

Solution:

Concept:

- The work-energy theorem gives $K_B - K_A = W_g$. For a freely falling body the work done by gravity equals its weight times its downward displacement.
- Gravitational potential energy relative to the ground is $U = mgh$. Only the difference in heights matters for the kinetic energy gained.

Step 1: Identify the distance fallen from the figure The figure marks the total height of A above the ground as y. The segment from A down to B is labelled y_0, so B is at height $y - y_0$ above the ground. Thus y_0, rather than y, is the downward displacement during the motion being asked about. The word dropped means the initial speed at A is zero, giving $K_A = 0$.

Step 2: Convert the loss of potential energy into kinetic energy Neglecting air resistance, gravity is the only force doing work during the fall. Conservation of mechanical energy therefore gives

$$mgy = K_B + mg(y - y_0).$$

Subtracting the potential energy at B,

$$K_B = mg\{y - (y - y_0)\} = mgy_0.$$

This is positive for a downward fall and becomes zero when B coincides with A. Options (A) and (B)

contain the square of a length, so their units are not energy. Option (C) is the remaining potential energy at B relative to the ground, not the kinetic energy gained since release.

Step 3: Conclusion The kinetic energy at B is $\boxed{mgy_0}$. Hence, the correct option is (D).

Quick Tip to Solve: Read the arrow endpoints carefully: the energy gained is weight times the distance fallen from the release point, not weight times the height still above the ground.

45. A block of mass M placed inside a box descends vertically with acceleration 'a'. The block exerts a force equal to one-fourth of its weight on the floor of the box. The value of 'a' will be [4 marks]

(A) $\frac{g}{4}$

(B) $\frac{g}{2}$

(C) $\frac{3g}{4}$

(D) g

Correct Answer: (C) $\frac{3g}{4}$

Solution:

Concept:

- The normal reaction N is the contact force exerted by the box floor on the block. Newton's third law makes its magnitude equal to the force the block exerts on the floor.
- For downward acceleration a, Newton's second law for the block gives $Mg - N = Ma$. Its true weight Mg remains unchanged.

Step 1: Express the normal reaction from the given apparent weight The block pushes downward on the floor with one-fourth of its weight. The floor therefore pushes upward on the block with

$$N = \frac{Mg}{4}.$$

This contact force is the apparent weight. It is smaller than the true weight because the block and the box accelerate downward together; it is not the net force acting on the block.

Step 2: Solve the vertical force equation Take the downward direction as positive. Gravity acts downward and the floor reaction upward, hence

$$Mg - N = Ma.$$

Substituting the specified contact force gives

$$Mg - \frac{Mg}{4} = Ma \quad \Rightarrow \quad a = \frac{3g}{4}.$$

The mass cancels, so the result does not depend on the choice of block. Acceleration $g/4$ would leave a reaction $3Mg/4$, and $g/2$ would leave $Mg/2$, ruling out (A) and (B). At $a = g$, the reaction is zero, the free-fall case, which rules out (D).

Step 3: Conclusion The box descends with acceleration $\boxed{a = 3g/4}$. Hence, the correct option is (C).

Quick Tip to Solve: In a descending accelerating lift, use apparent weight $N = M(g - a)$. A specified fraction of weight is the reaction, not the acceleration fraction.

46. If the electric potential at any point (x, y, z) m in space is given by V = $3x^2$ volt. The electric field at the point (1, 0, 3) m will be : [4 marks]

(A) 3 Vm^{-1}, directed along positive x-axis.

(B) 3 Vm^{-1}, directed along negative x-axis.

(C) 6 Vm^{-1}, directed along positive x-axis.

(D) 6 Vm^{-1}, directed along negative x-axis.

Correct Answer: (D) 6 Vm^{-1}, directed along negative x-axis.

Solution:

Concept:

- The electric field is the negative gradient of electric potential: $\mathbf{E} = -\nabla V$. Its Cartesian components are the negatives of the corresponding partial derivatives.
- A potential depending only on x produces no y- or z-component of electric field. Field units are $\mathrm{V\,m^{-1}}$, equivalent to $\mathrm{N\,C^{-1}}$.

Step 1: Differentiate the potential before substituting coordinates The potential is $V = 3x^2$. Since there is no dependence on y or z,

$$E_x = -\frac{\partial V}{\partial x} = -6x, \qquad E_y = 0, \qquad E_z = 0.$$

Thus the field varies with the x-coordinate alone. Substituting the point into V first would give a scalar potential value and lose the spatial variation needed to calculate the field.

Step 2: Evaluate the field vector at the specified point At $(x, y, z) = (1, 0, 3)\,\mathrm{m}$, the x-coordinate is 1 m, giving

$$\mathbf{E} = -6\,\hat{\mathbf{i}}\,\mathrm{V\,m^{-1}}.$$

Its magnitude is $6\,\mathrm{V\,m^{-1}}$, and the minus sign means the negative x-direction. At positive x the potential increases with x, so the field must point towards decreasing x. The values $y = 0$ and $z = 3$ introduce no additional components. Options (A) and (B) have the wrong magnitude; option (C) has the correct magnitude but reverses the negative-gradient direction.

Step 3: Conclusion The field is $\boxed{6\,\mathrm{V\,m^{-1}}}$, directed along the negative x-axis. Hence, the correct option is (D).

Quick Tip to Solve: Always differentiate V first and insert the point afterwards; then use the negative sign to establish the field direction.

47. The combination of two identical cells, whether connected in series or parallel combination provides the same current through an external resistance of 2Ω. The value of internal resistance of each cell is [4 marks]

(A) 2Ω

(B) 4Ω

(C) 6Ω

(D) 8Ω

Correct Answer: (A) 2Ω

Solution:

Concept:

- For cells supplying an external resistance R, current is the equivalent emf divided by the sum of external and equivalent internal resistance.
- Two identical cells of emf $\mathcal{E}$ and internal resistance r have series values $(2\mathcal{E}, 2r)$ and parallel values $(\mathcal{E}, r/2)$, when connected with matching polarity.

Step 1: Write the external current in each connection For the series combination the two emfs add, as do the two internal resistances. For the parallel combination the emf stays equal to that of one cell while the two internal resistances combine in parallel. Therefore

$$I_s = \frac{2\mathcal{E}}{R + 2r}, \qquad I_p = \frac{\mathcal{E}}{R + r/2}.$$

The same external resistor $R = 2\,\Omega$ is used in both cases.

Step 2: Use equality of the two currents Equating the currents and cancelling the nonzero cell emf gives

$$\frac{2}{R + 2r} = \frac{1}{R + r/2}.$$

Cross-multiplication yields

$$2R + r = R + 2r \quad \Rightarrow \quad r = R = 2\,\Omega.$$

As a check, for $r = R = 2\,\Omega$ the series current is $2\mathcal{E}/6 = \mathcal{E}/3$, and the parallel current is $\mathcal{E}/(2+1) = \mathcal{E}/3$. Each of the other proposed resistances, 4, 6 and 8 ohms, violates the necessary condition $r = R$, so none produces equal currents.

Step 3: Conclusion The internal resistance of each cell is $\boxed{2\,\Omega}$. Hence, the correct option is (A).

Quick Tip to Solve: For two identical cells, equal current from series and parallel connections occurs when the external resistance equals the internal resistance of one cell.

48. A person can throw a ball upto a maximum range of 100 m. How high above the ground he can throw the same ball? [4 marks]

(A) 25 m

(B) 50 m

(C) 100 m

(D) 200 m

Correct Answer: (B) 50 m

Solution:

Concept:

- For projectile motion between points at the same height, the horizontal range is $R = u^2 \sin 2\theta/g$, where u is launch speed.
- The greatest vertical height for a given launch speed comes from throwing vertically upward. With final speed zero at the top, $H = u^2/(2g)$.

Step 1: Infer the available launch speed from the maximum range Let u be the greatest speed the person can impart to the ball. The range is largest when $\sin 2\theta = 1$, corresponding to a launch angle of $45°$. Thus

$$R_{\max} = \frac{u^2}{g} = 100\,\text{m}.$$

This relation supplies u^2/g without needing to choose a numerical value of g. The same available launch speed is used when the person aims upward.

Step 2: Maximise the vertical rise For a launch at an arbitrary angle, the initial vertical speed is $u \sin\theta$; it cannot exceed u. The greatest height is therefore obtained by a vertical throw. Applying $v^2 = u^2 - 2gH$ with $v = 0$ at the top,

$$H_{\max} = \frac{u^2}{2g} = \frac{R_{\max}}{2} = 50\,\text{m}.$$

The height 25 m is only the peak height of the particular $45°$ throw that maximises range, not the greatest possible height. Values 100 m and 200 m would require more launch energy than the range information allows.

Step 3: Conclusion The greatest height is $\boxed{50\,\text{m}}$, taking the launch level as the ground. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish maximum height of a vertical throw from the height of a maximum-range trajectory: they are $R_{\max}/2$ and $R_{\max}/4$, respectively.

49. In the following nuclear reaction,

$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^-} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\gamma} D_4$$

Mass number of D is 182 and atomic number is 74. Mass number and atomic number of D_4 respectively will be _____. [4 marks]

(A) 174 and 71

(B) 174 and 69

(C) 172 and 69

(D) 172 and 71

Correct Answer: (A) 174 and 71

Solution:

Concept:

- An alpha particle is a helium nucleus with mass number 4 and atomic number 2. Each alpha emission reduces the parent mass number A by 4 and atomic number Z by 2.
- In beta-minus decay a neutron becomes a proton, so A stays fixed and Z increases by 1. Gamma emission removes excitation energy without changing A or Z.

Step 1: Track the first alpha and the beta-minus decays The parent D has $(A, Z) = (182, 74)$. After the first alpha decay,

$$D_1: \quad A_1 = 182 - 4 = 178, \qquad Z_1 = 74 - 2 = 72.$$

The following beta-minus emission changes a neutron to a proton inside the nucleus. The emitted electron does not reduce the nuclear mass number, so

$$D_2: \quad (A_2, Z_2) = (178, 73).$$

The increase of Z is essential: beta-minus decay is not a loss of a proton.

Step 2: Apply the remaining alpha and gamma transitions The second alpha decay subtracts another four from A and two from Z:

$$D_3: \quad (A_3, Z_3) = (178 - 4, 73 - 2) = (174, 71).$$

The gamma photon carries no nucleon number or charge. Consequently D_4 has the same A and Z as D_3. Equivalently, the net changes are $\Delta A = -8$ and $\Delta Z = -4 + 1 = -3$. Options with A = 172 wrongly change the mass number beyond the two alpha losses. The value Z = 69 uses a decrease instead of an increase for the beta-minus step.

Step 3: Conclusion The mass number and atomic number of D_4 are respectively $\boxed{174 \text{ and } 71}$. Hence, the correct option is (A).

Quick Tip to Solve: Keep separate A and Z columns through a decay chain: alpha changes both, beta-minus only raises Z, and gamma changes neither.

50. The TV transmission tower at a particular station has a height of 125 m. For doubling the coverage of its range, the height of the tower should be increased by [4 marks]

(A) 125 m

(B) 250 m

(C) 375 m

(D) 500 m

Correct Answer: (C) 375 m

Solution:

Concept:

- Terrestrial TV signals propagate approximately along straight lines, so the horizon limits the line-of-sight transmission range.
- For a tower of height h above a spherical Earth of radius R, the horizon distance is $d = \sqrt{2Rh}$, since $h \ll R$. Thus range is proportional to the square root of tower height.

Step 1: Relate the new range to the old range The original tower height is $h_1 = 125\,\text{m}$. Doubling its range means that the new limiting distance satisfies $d_2 = 2d_1$. The Earth radius is unchanged, and it cancels when we divide the two horizon formulae. Consequently, the comparison does not require a numerical value of the Earth radius.

$$\frac{d_2}{d_1} = \sqrt{\frac{h_2}{h_1}} = 2.$$

Step 2: Calculate the required increase in height Squaring the range ratio gives the height ratio, rather than merely doubling the height:

$$h_2 = 4h_1 = 4(125) = 500\,\text{m}.$$

The question asks for the increase above the existing height, so subtract the original value:

$$\Delta h = h_2 - h_1 = 500 - 125 = 375\,\text{m}.$$

Option (D) is the final height, not the increase. An increase of 125 m gives only a factor $\sqrt{2}$ in range; an increase of 250 m gives a factor $\sqrt{3}$. Neither doubles the range.

Step 3: Conclusion The tower must be increased in height by $\boxed{375\,\text{m}}$. Hence, the correct option is (C).

Quick Tip to Solve: For horizon-limited range, square the required distance factor to get the height factor, then subtract the old height.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The Vernier constant of Vernier callipers is 0.1 mm and it has zero error of (−0.05) cm. While measuring diameter of a sphere, the main scale reading is 1.7 cm and coinciding vernier division is 5. The corrected diameter will be _____ $\times 10^{-2}$ cm. [4 marks]

Solution:

Concept:

- The observed vernier reading equals the main-scale reading plus the coinciding vernier division multiplied by the vernier constant.
- The corrected measurement is the observed reading minus the signed zero error. A negative zero error therefore requires a positive correction.

Step 1: Convert the vernier constant and obtain the observed diameter The main-scale reading is already in centimetres, so express the vernier constant in the same unit before adding the two contributions. Since $10\,\text{mm} = 1\,\text{cm}$,

$$\text{VC} = 0.1\,\text{mm} = 0.01\,\text{cm}.$$

The fifth vernier division coincides with the main scale, giving a vernier contribution of $5(0.01) = 0.05\,\text{cm}$. Hence the uncorrected diameter is

$$D_{\text{obs}} = 1.7 + 0.05 = 1.75\,\text{cm}.$$

Step 2: Apply the zero correction and express the requested coefficient The zero error is $-0.05\,\text{cm}$. This means the instrument underreads by that amount when compared with the true value. Subtracting the negative error increases the observed diameter:

$$D = D_{\text{obs}} - e_0 = 1.75 - (-0.05) = 1.80\,\text{cm}.$$

Do not confuse the vernier contribution, which is added to the main-scale reading, with the separate correction for the instrument zero. Finally, express the result in the form printed in the question:

$$1.80\,\text{cm} = 180 \times 10^{-2}\,\text{cm}.$$

Step 3: Conclusion The corrected diameter is $1.80\,\text{cm}$, so the required numerical entry is $\boxed{180}$.

Quick Tip to Solve: Write the signed zero error explicitly in observed reading minus zero error; this prevents reversing the correction.

52. A small spherical ball of radius 0.1 mm and density 10^4 kg m^{-3} falls freely under gravity through a distance h before entering a tank of water. If, after entering the water the velocity of ball does not change and it continue to fall with same constant velocity inside water, then the value of h will be _____ m.

(Given g = 10 ms^{-2}, viscosity of water = 1.0×10^{-5} N-sm^{-2}). [4 marks]

Solution:

Concept:

- A sphere moving at constant speed in a viscous liquid has zero resultant force: its weight is balanced by buoyancy and viscous drag.
- Stokes drag is $6\pi\eta r v$; hence the terminal speed is $v_t = 2r^2(\rho_s - \rho_w)g/(9\eta)$. Free fall from rest through height h gives $v^2 = 2gh$.

Step 1: Find the constant speed inside water The ball enters water without a subsequent change of speed, so its entry speed equals the terminal speed. Use $r = 0.1\,\text{mm} = 10^{-4}\,\text{m}$, ball density $\rho_s = 10^4\,\text{kg}\,\text{m}^{-3}$, and water density $\rho_w = 10^3\,\text{kg}\,\text{m}^{-3}$. Balancing the forces gives

$$\frac{4}{3}\pi r^3(\rho_s - \rho_w)g = 6\pi\eta r v_t.$$

With the given viscosity $\eta = 10^{-5}\,\text{N}\,\text{s}\,\text{m}^{-2}$,

$$v_t = \frac{2(10^{-4})^2(10^4 - 10^3)(10)}{9(10^{-5})} = 20\,\text{m}\,\text{s}^{-1}.$$

Step 2: Match the free-fall speed to the terminal speed Before reaching the water the sphere falls freely from rest. Its loss of gravitational potential energy becomes kinetic energy, giving the same relation as the constant-acceleration equation:

$$mgh = \frac{1}{2}mv_t^2, \qquad h = \frac{v_t^2}{2g}.$$

The mass cancels because the free-fall acceleration does not depend on the sphere mass. Therefore

$$h = \frac{20^2}{2(10)} = 20\,\text{m}.$$

Buoyancy must be retained when finding the terminal speed: using the ball density alone would give a different height.

Step 3: Conclusion The required free-fall distance is $\boxed{h = 20\,\text{m}}$, so the numerical entry is $\boxed{20}$.

Quick Tip to Solve: No speed change on entering a liquid means the entry speed is already terminal; equate it to the free-fall speed.

53. In an experiment to determine the velocity of sound in air at room temperature using a resonance tube, the first resonance is observed when the air column has a length of 20.0 cm for a tuning fork of frequency 400 Hz is used. The velocity of the sound at room temperature is 336 ms^{-1}. The third resonance is observed when the air column has a length of _____ cm. [4 marks]

Solution:

Concept:

- A resonance tube with one closed end supports odd quarter wavelengths: $L_n + e = (2n-1)\lambda/4$, where e is the open-end correction.
- The wavelength is $\lambda = v/f$. Successive resonances differ by $\lambda/2$, so the first and third resonance lengths differ by one full wavelength.

Step 1: Calculate the wavelength and end correction The sound speed and tuning-fork frequency refer to the same room-temperature experiment. Thus

$$\lambda = \frac{v}{f} = \frac{336}{400} = 0.84\,\text{m} = 84\,\text{cm}.$$

The effective length for the first resonance is $\lambda/4 = 21\,\text{cm}$. The observed physical air-column length is only 20.0 cm, so the end correction is

$$e = \frac{\lambda}{4} - L_1 = 21 - 20 = 1\,\text{cm}.$$

The effective vibrating column is slightly longer than the measured tube column because the antinode extends beyond the open end.

Step 2: Use the third resonance condition The first, second and third resonances correspond respectively to effective lengths $\lambda/4$, $3\lambda/4$, and $5\lambda/4$. Therefore

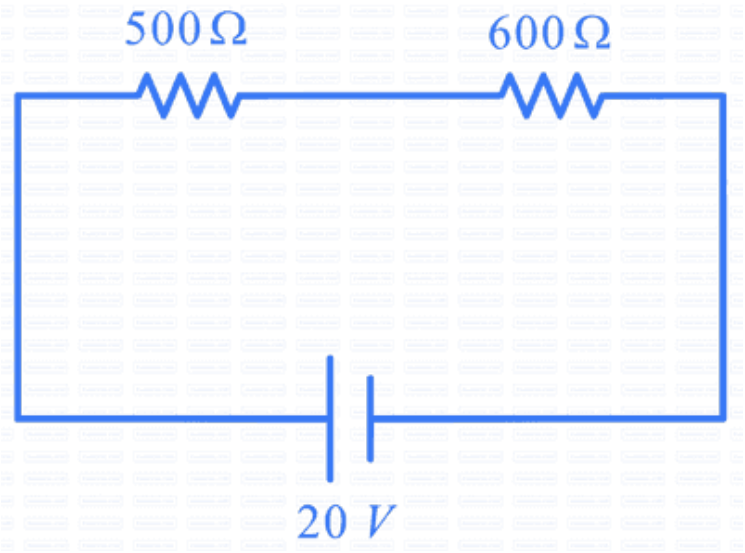
$$L_3 + e = \frac{5\lambda}{4} = 105\,\text{cm}, \qquad L_3 = 105 - 1 = 104\,\text{cm}.$$

Equivalently, subtract the first resonance equation from the third: the same end correction cancels, yielding $L_3 - L_1 = \lambda$. This checks the calculation directly: $20 + 84 = 104\,\text{cm}$. Using $3\lambda/4$ would locate the second resonance, not the third.

Step 3: Conclusion The third resonance occurs at an air-column length of $\boxed{104\,\text{cm}}$. The required entry is $\boxed{104}$.

Quick Tip to Solve: Count resonances as odd multiples of a quarter wavelength; differences cancel the open-end correction.

54. Two resistors are connected in series across a battery as shown in figure. If a voltmeter of resistance 2000 Ω is used to measure the potential difference across 500 Ω resistor, the reading of the voltmeter will be _____ V. [4 marks]



Solution:

Concept:

- A voltmeter is connected in parallel with the component whose potential difference is measured. A finite-resistance voltmeter changes the circuit resistance.
- For resistors in parallel, $R_p = R_1R_2/(R_1 + R_2)$; for series elements, the source voltage divides in proportion to resistance.

Step 1: Combine the voltmeter with the measured resistor The figure shows a 20 V battery with 500 Ω and 600 Ω resistors in series. Connecting the 2000 Ω voltmeter across the 500 Ω resistor makes these two resistances parallel. Their equivalent resistance is

$$R_p = \frac{500(2000)}{500 + 2000} = 400\,\Omega.$$

The 600 Ω resistor remains in series with this parallel combination. The voltage across the meter is the voltage across the entire 400 Ω equivalent branch; it is not calculated from the original unloaded series circuit.

Step 2: Find the current and the meter voltage The battery now sees

$$R_{\text{total}} = 400 + 600 = 1000\,\Omega.$$

Taking the battery shown as an ideal 20 V source, the total current is

$$I = \frac{20}{1000} = 0.020\,\text{A}.$$

The voltage across the parallel branch, and thus across the voltmeter, is

$$V_{\text{meter}} = IR_p = 0.020(400) = 8\,\text{V}.$$

As a check, the other resistor has a drop $0.020(600) = 12\,\text{V}$, and the two series drops add to the source voltage of 20 V. The meter current is $8/2000 = 0.004\,\text{A}$, while the 500 Ω resistor carries 0.016 A; these sum to the series current.

Step 3: Conclusion The voltmeter reads $\boxed{8\,\text{V}}$, so the required numerical entry is $\boxed{8}$.

Quick Tip to Solve: Replace the measured resistor and finite-resistance voltmeter by their parallel equivalent before applying voltage division.

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55. A potential barrier of 0.4 V exists across a p-n junction. An electron enters the junction from the n-side with a speed of 6.0 × 10^5 ms^{-1}. The speed with which electron enters the p side will be $\frac{x}{3} \times 10^5$ ms^{-1} the value of x is _____.

(Given mass of electron = 9 × 10^{-31} kg, charge on electron = 1.6 × 10^{-19} C.) [4 marks]

Solution:

Concept:

- The junction electric field opposes an electron crossing from the n region to the p region. Its kinetic energy decreases by eV_b, where V_b is the barrier potential.
- Energy conservation gives $\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - eV_b$. Use the positive magnitude e of the electron charge for the barrier energy.

Step 1: Calculate the initial energy and barrier energy The incident speed is $v_i = 6.0 \times 10^5\,\text{m s}^{-1}$ and the given electron mass is $9 \times 10^{-31}\,\text{kg}$. Therefore

$$K_i = \frac{1}{2}(9 \times 10^{-31})(6.0 \times 10^5)^2 = 1.62 \times 10^{-19}\,\text{J}.$$

The work needed to cross the 0.4 V potential barrier is

$$eV_b = (1.6 \times 10^{-19})(0.4) = 6.4 \times 10^{-20}\,\text{J}.$$

Since the initial energy exceeds the barrier energy, the electron can cross the junction; it emerges with a smaller speed.

Step 2: Find the exit speed and the requested coefficient The remaining kinetic energy is

$$K_f = 1.62 \times 10^{-19} - 0.64 \times 10^{-19} = 9.8 \times 10^{-20}\,\text{J}.$$

Consequently,

$$v_f^2 = \frac{2K_f}{m} = \frac{2(9.8 \times 10^{-20})}{9 \times 10^{-31}} = \frac{196}{9} \times 10^{10}\,\text{m}^2\,\text{s}^{-2}.$$

Taking the positive square root for speed gives

$$v_f = \frac{14}{3} \times 10^5\,\text{m s}^{-1}.$$

Compare this expression directly with the stated form $(x/3) \times 10^5\,\text{m s}^{-1}$. The quantity requested is

the coefficient x, not the numerical speed in SI units.

Step 3: Conclusion The exit speed is $\frac{14}{3} \times 10^5\,\mathrm{m\,s^{-1}}$. Therefore $\boxed{x = 14}$.

Quick Tip to Solve: Treat a junction barrier as an energy cost e times barrier voltage, and compare the final speed with the exact form requested.

---

56. The displacement current of 4.425 μA is developed in the space between the plates of parallel plate capacitor when voltage is changing at a rate of 10^6 Vs^{-1}. The area of each plate of the capacitor is 40 cm^2. The distance between each plate of the capacitor is x $\times$ 10^{-3} m. The value of x is _____. (Permittivity of free space, E_0 = 8.85 $\times$ 10^{-12} C^2 N^{-1} m^{-2}). [4 marks]

Solution:

Concept:

- For a parallel-plate capacitor with air or vacuum between its plates, $C = \varepsilon_0 A/d$, where A is plate area and d is separation.
- The displacement current is $I_d = dQ/dt = C\,dV/dt$ for fixed capacitor geometry. Thus the changing electric flux in the gap gives the same current as the charging circuit.

Step 1: Obtain the capacitance from the displacement current The given current is $I_d = 4.425\,\mu\mathrm{A} = 4.425 \times 10^{-6}\,\mathrm{A}$, and the voltage changes at $10^6\,\mathrm{V\,s^{-1}}$. Divide current by the voltage-change rate:

$$C = \frac{I_d}{dV/dt} = \frac{4.425 \times 10^{-6}}{10^6} = 4.425 \times 10^{-12}\,\mathrm{F}.$$

The factor of 10^6 is a rate, not a voltage to substitute directly into $Q = CV$; using the derivative relation keeps the dimensions correct.

Step 2: Use the plate area to determine the separation Convert the area carefully: because a centimetre is 10^{-2} metre, a square centimetre is 10^{-4} square metre. Hence

$$A = 40\,\mathrm{cm^2} = 40 \times 10^{-4} = 4.0 \times 10^{-3}\,\mathrm{m^2}.$$

Rearranging the parallel-plate capacitance expression gives

$$d = \frac{\varepsilon_0 A}{C} = \frac{(8.85 \times 10^{-12})(4.0 \times 10^{-3})}{4.425 \times 10^{-12}} = 8.0 \times 10^{-3}\,\mathrm{m}.$$

The result is a length, since $\varepsilon_0 A/C$ has units of metres. Comparing with $d = x \times 10^{-3}\,\mathrm{m}$ identifies the requested coefficient.

Step 3: Conclusion The separation is $8.0 \times 10^{-3}\,\mathrm{m}$, so $\boxed{x = 8}$.

Quick Tip to Solve: First calculate capacitance from current divided by voltage-change rate, then use C equals epsilon times area divided by separation.

---

57. The moment of inertia of a uniform thin rod about a perpendicular axis passing through one end is I_1. The same rod is bent into a ring and its moment of inertia about a diameter is I_2. If $\frac{I_1}{I_2}$ is $\frac{x\pi^2}{3}$, then the value of x will be _____. [4 marks]

Solution:

Concept:

- For a uniform thin rod of mass M and length L, the moment of inertia about an axis perpendicular to the rod through an end is $ML^2/3$.
- For a thin circular ring of radius R, the moment of inertia about its symmetry axis is MR^2. The perpendicular-axis theorem and symmetry give $MR^2/2$ about a diameter.

Step 1: Relate the ring radius to the rod length Bending changes the shape but does not change the mass or length of the thin rod. The original length becomes the circumference of the ring, so

$$L = 2\pi R, \qquad R = \frac{L}{2\pi}.$$

The axis specified for the ring is a diameter in its plane; it is not the axis through its centre perpendicular to the plane.

Step 2: Express both inertias using the same length For the straight rod, $I_1 = ML^2/3$. For the bent rod,

$$I_2 = \frac{1}{2}MR^2 = \frac{ML^2}{8\pi^2}.$$

The factor one-half is essential: using the ring's perpendicular central axis would give twice this inertia. Both expressions have dimensions mass times length squared.

Step 3: Compare the required coefficients Taking the ratio cancels the unchanged mass and the square of the original length:

$$\frac{I_1}{I_2} = \frac{ML^2/3}{ML^2/(8\pi^2)} = \frac{8\pi^2}{3}.$$

Comparison with $x\pi^2/3$ therefore gives a dimensionless coefficient $x = 8$.

Step 4: Conclusion The required value is $\boxed{x = 8}$.

Quick Tip to Solve: First replace the ring radius by $L/(2\pi)$, and check whether its axis is a diameter or the perpendicular central axis.

58. In a double slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the plane of slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-3} cm. If the distance between the slits is 1 mm, then the wavelength of the light will be _____ nm. [4 marks]

Solution:

Concept:

- For slit separation d, wavelength λ and screen distance D, the fringe width is $\beta = \lambda D/d$ in the usual small-angle approximation.
- With wavelength and slit separation fixed, the magnitude of the change in fringe width satisfies $|\Delta\beta| = \lambda|\Delta D|/d$. Moving the screen nearer reduces the fringe width.

Step 1: Convert all quantities to SI units The displacement of the screen is $|\Delta D| = 5 \times 10^{-2}\,\text{m}$. The slit separation is $d = 1\,\text{mm} = 10^{-3}\,\text{m}$. The given fringe-width change is in centimetres:

$$|\Delta\beta| = 3 \times 10^{-3}\,\text{cm} = 3 \times 10^{-5}\,\text{m}.$$

Converting centimetres before substitution avoids an error by a factor of one hundred.

Step 2: Subtract the two fringe-width equations Initially $\beta_1 = \lambda D/d$, while after the movement $\beta_2 = \lambda(D - 0.05)/d$. Subtraction eliminates the unspecified initial screen distance:

$$\beta_1 - \beta_2 = \frac{\lambda(0.05)}{d}, \qquad \lambda = \frac{d|\Delta\beta|}{|\Delta D|}.$$

The positive difference is the amount by which the width decreases.

Step 3: Evaluate and convert to nanometres Inserting the three SI quantities gives

$$\lambda = \frac{(10^{-3})(3 \times 10^{-5})}{5 \times 10^{-2}} = 6 \times 10^{-7}\,\text{m}.$$

Since $1\,\text{nm} = 10^{-9}\,\text{m}$, this is $600\,\text{nm}$, a visible-light wavelength.

Step 4: Conclusion The wavelength is $\boxed{600\,\text{nm}}$; enter 600.

Quick Tip to Solve: Use the change form of the fringe-width formula: the unknown original screen distance cancels when the two equations are subtracted.

59. An inductor of 0.5 mH, a capacitor of 200 μF and a resistor of 2 Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the frequency of ac source will be _____ $\times 10^2$ Hz. [4 marks]

Solution:

Concept:

- The impedance of a series LCR circuit is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$. The phase angle obeys $\tan\phi = (\omega L - 1/(\omega C))/R$.

- Current is in phase with the applied emf at series resonance, where inductive and capacitive reactances cancel: $\omega_0 L = 1/(\omega_0 C)$. Thus $f_0 = 1/(2\pi\sqrt{LC})$.

Step 1: Apply the in-phase condition The stated phase relation means $\phi = 0$, so the net reactance must be zero. Consequently,

$$\omega_0^2 = \frac{1}{LC}, \qquad f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

The resistance determines the resonant current but does not enter the ideal series-resonance frequency. The source voltage likewise does not set this frequency.

Step 2: Convert the inductance and capacitance The SI values are

$$L = 0.5 \times 10^{-3} = 5 \times 10^{-4}\,\text{H}, \qquad C = 200 \times 10^{-6} = 2 \times 10^{-4}\,\text{F}.$$

Their product is $LC = 10^{-7}\,\text{s}^2$, giving $\sqrt{LC} = 3.1623 \times 10^{-4}\,\text{s}$. The conversion of milli and micro prefixes must be done separately.

Step 3: Find the frequency and requested coefficient Therefore,

$$f_0 = \frac{1}{2\pi(3.1623 \times 10^{-4})} \approx 503.3\,\text{Hz} = 5.033 \times 10^2\,\text{Hz}.$$

The numerical blank asks for the coefficient of 10^2 Hz, not the frequency in hertz itself. To the nearest integer that coefficient is 5.

Step 4: Conclusion The frequency is approximately 5.03×10^2 Hz; the required integer entry is $\boxed{5}$.

Quick Tip to Solve: For an in-phase series LCR current, equate the two reactances first; divide the resulting frequency by the power of ten printed after the blank.

60. The half life of a radioactive substance is 5 years. After x years a given sample of the radioactive substance gets reduced to 6.25% of its initial value. The value of x is _____. [4 marks]

Solution:

Concept:

- Radioactive decay obeys $N = N_0 e^{-\lambda t}$, where N_0 is the initial number of undecayed nuclei and λ is the decay constant.
- In terms of the half-life $T_{1/2}$, the fraction remaining is $N/N_0 = (1/2)^{t/T_{1/2}}$. Each completed half-life halves the amount remaining, rather than removing a fixed amount.

Step 1: Convert the stated percentage to a fraction The sample is reduced to 6.25% of its initial value, so this percentage describes what remains, not what has decayed. Therefore,

$$\frac{N}{N_0} = \frac{6.25}{100} = 0.0625 = \frac{1}{16}.$$

The fraction that has decayed would instead be $15/16$, which is not the quantity to put on the left side of the decay law.

Step 2: Count the number of half-lives Because $16 = 2^4$, the required fraction can be written as

$$\frac{1}{16} = \left(\frac{1}{2}\right)^4.$$

The successive remaining percentages are 50%, 25%, 12.5% and 6.25%. Thus exactly four half-lives have elapsed; the process is multiplicative rather than linear.

Step 3: Convert the half-life count into elapsed time With $T_{1/2} = 5$ years and the requested time denoted by x,

$$\left(\frac{1}{2}\right)^{x/5} = \left(\frac{1}{2}\right)^4, \qquad \frac{x}{5} = 4, \qquad x = 20 \text{ years}.$$

As a direct check, after twenty years the exponential law gives $2^{-20/5} = 2^{-4} = 0.0625$, exactly the specified fraction.

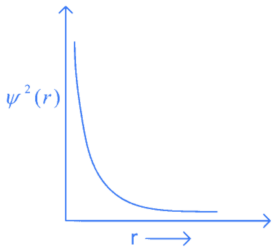
Step 4: Conclusion The elapsed time is $\boxed{20 \text{ years}}$; enter 20.

Quick Tip to Solve: Recognise common remaining fractions as powers of one-half: $6.25\% = 1/16$ immediately means four half-lives.

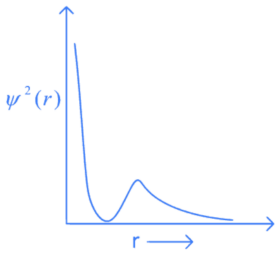
Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Which of the following is the correct plot for the probability density ψ^2 (r) as a function of distance 'r' of the electron from the nucleus for 2s orbital? [4 marks]

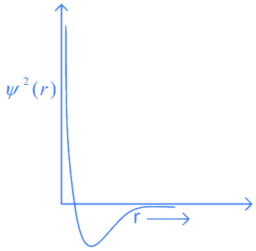
(A)



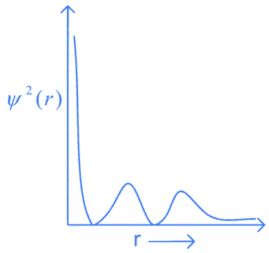
(B)



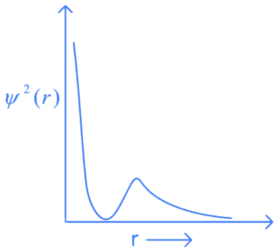
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- For an s orbital the probability density at a point is $|\psi(r)|^2$, which is non-negative and need not vanish at the nucleus. It differs from the radial probability distribution $4\pi r^2|\psi(r)|^2$.
- The number of radial nodes is $n - l - 1$. A 2s orbital has $n = 2$, $l = 0$, hence exactly one radial node at a positive distance from the nucleus.

Step 1: Determine the features required of the plot For a hydrogen-like 2s orbital, a positive constant multiplies

$$|\psi_{2s}(r)|^2 \propto \left(2 - \frac{Zr}{a_0}\right)^2 e^{-Zr/a_0}.$$

Here Z is nuclear charge and a_0 the Bohr radius. At $r = 0$ this expression is finite and nonzero. At $r = 2a_0/Z$ it vanishes, but being a square it cannot become negative.

Step 2: Follow the density beyond the node Beyond the radial node, the density rises into a smaller outer maximum and then approaches zero at large distance because the exponential decay dominates. Thus the graph starts above zero, falls to one zero minimum, rises once more, and finally decays. The node is a zero of the density, not a region where probability becomes negative.

Step 3: Compare the four graphical options Option (B) shows precisely one node and the smaller outer lobe. Option (A) simply decreases and has no node, so it resembles the 1s density. Option (C) crosses below zero, which is impossible for a squared wavefunction. Option (D) shows two radial nodes and too many lobes for a 2s orbital. These checks distinguish density from a signed wavefunction as well as from a higher s state.

Step 4: Conclusion The plot with one radial node and non-negative density throughout is (B). Hence, the correct option is (B).

Quick Tip to Solve: Check three features before choosing an orbital-density graph: its value at the nucleus, non-negativity, and the radial-node count $n - l - 1$.

62. Consider the species CH_4, NH_4^+ and BH_4^-. Choose the correct option with respect to the these species. [4 marks]

(A) They are isoelectronic and only two have tetrahedral structures.

(B) They are isoelectronic and all have tetrahedral structures.

(C) Only two are isoelectronic and all have tetrahedral structures.

(D) Only two are isoelectronic and only two have tetrahedral structures.

Correct Answer: (B) They are isoelectronic and all have tetrahedral structures.

Solution:

Concept:

- Isoelectronic species have the same total number of electrons. Count atomic numbers and add one electron per negative charge or subtract one per positive charge.
- According to VSEPR theory, four bonding pairs and no lone pair on a central atom give an AX_4 tetrahedral structure, with ideal bond angles near $109.5°$.

Step 1: Count the electrons in each species For the three species, the total electron counts are

$$\begin{aligned} CH_4: &\quad 6+4(1)=10, \\ NH_4{}^+: &\quad 7+4(1)-1=10, \\ BH_4{}^-: &\quad 5+4(1)+1=10. \end{aligned}$$

The positive charge of ammonium removes one electron from the neutral-atom sum; the negative charge of borohydride adds one. All three therefore have ten electrons and are mutually isoelectronic.

Step 2: Identify the electron-pair arrangement Each central atom forms four sigma bonds to hydrogen and has no lone pair in these species. Methane has four shared carbon-hydrogen pairs. In ammonium, nitrogen has four nitrogen-hydrogen bonds after protonation of ammonia; its former lone pair has become bonding. The extra electron in borohydride permits four boron-hydrogen bonds. Each central environment is consequently AX_4.

Step 3: Select the statement satisfying both tests Four bonding domains repel into a tetrahedral arrangement. Thus all three species, not just two, have tetrahedral structures. Option (A) fails the shape test; option (C) fails the electron-count test; option (D) fails both. Option (B) alone states that they are all isoelectronic and all tetrahedral.

Step 4: Conclusion All three species are isoelectronic and tetrahedral. Hence, the correct option is (B).

Quick Tip to Solve: Include ionic charge in the electron count, then count central-atom lone pairs separately; equal electron counts alone do not generally establish identical shapes.

63. 4.0 moles of argon and 5.0 moles of PCl_5 are introduced into an evacuated flask of 100 litre capacity at 610 K. The system is allowed to equilibrate. At equilibrium, the total pressure of mixture was found to be 6.0 atm. The K_p for the reaction is :

[Given : R = 0.082 L atm K^{-1} mol^{-1}] [4 marks]

(A) 2.25

(B) 6.24

(C) 12.13

(D) 15.24

Correct Answer: (A) 2.25

Solution:

Concept:

- The dissociation is $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$. If ξ moles dissociate, each mole of reaction increases the total gas amount by one mole.
- The ideal-gas equation gives $n_{tot} = PV/(RT)$. For the reacting gases, $K_p = p_{PCl_3}p_{Cl_2}/p_{PCl_5}$; inert argon contributes to total pressure but not directly to this quotient.

Step 1: Use total pressure to determine dissociation The initial total amount is $4 + 5 = 9$ mol. At equilibrium the amounts are 4 mol argon, $5 - \xi$ mol PCl_5, and ξ mol each of PCl_3 and Cl_2. Thus

$$9 + \xi = \frac{6.0(100)}{0.082(610)} = 11.9952, \qquad \xi \approx 2.9952 \text{ mol}.$$

At the precision of the given data this is approximately 3.0 mol dissociated.

Step 2: Calculate the reacting partial pressures Each gas has partial pressure $p_i = n_i RT/V$, with $RT/V = 0.5002\,\text{atm}\,\text{mol}^{-1}$. Hence

$$p_{PCl_3} = p_{Cl_2} \approx 1.4984\,\text{atm}, \qquad p_{PCl_5} \approx 1.0028\,\text{atm}.$$

Argon has partial pressure $2.0008\,\text{atm}$; adding all four partial pressures recovers $6.0\,\text{atm}$.

Step 3: Evaluate the equilibrium quotient Using these partial pressures,

$$K_p = \frac{(1.4984)^2}{1.0028} \approx 2.24.$$

To the intended precision, $RT \approx 50\,\text{L}\,\text{atm}\,\text{mol}^{-1}$, so the partial pressures are $1.5, 1.5, 1.0$ atm and $K_p \approx 2.25$. This matches option (A). The much larger values in (B), (C) and (D) are incompatible with the product and reactant partial pressures.

Step 4: Conclusion The equilibrium constant is approximately 2.25 in the pressure convention used in the options. Hence, the correct option is (A).

Quick Tip to Solve: Subtract inert-gas moles only after using the measured total pressure to find total moles; include argon in the mole balance but omit it from K_p.

64. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The first ionization enthalpy for oxygen is lower than that of nitrogen.

Reason R : The four electrons in 2p orbitals of oxygen experience more electron-electron repulsion.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both A and R are are correct and R is the correct explanation of A.

(B) Both A and R are are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Correct Answer: (A) Both A and R are are correct and R is the correct explanation of A.

Solution:

Concept:

- First ionization enthalpy is the energy required to remove an electron from an isolated gaseous atom. Subshell occupancy can cause exceptions to the general increase across a period.
- Hund's rule gives nitrogen a half-filled $2p^3$ subshell; oxygen has $2p^4$, containing one doubly occupied orbital.

Step 1: Evaluate the assertion Nitrogen has electronic configuration $1s^2 2s^2 2p^3$, whereas oxygen has $1s^2 2s^2 2p^4$. Although oxygen has a larger nuclear charge, its first electron is removed from the doubly occupied $2p$ orbital. The first ionization enthalpy of oxygen is therefore lower than that of nitrogen. Assertion A is correct.

Step 2: Evaluate the reason In nitrogen, the three $2p$ electrons occupy separate orbitals with parallel spins. In oxygen, the fourth electron must pair in one orbital. Two electrons sharing the same orbital experience additional electrostatic repulsion, so removing one of that pair is easier. Reason R is correct.

Step 3: Connect the reason to the trend Ionization of oxygen relieves the pairing repulsion and leaves the stable half-filled $2p^3$ arrangement in O^+. Ionizing nitrogen instead disrupts its half-filled arrangement. Thus the repulsion described in R directly explains the lower value in A. Option B incorrectly denies the explanatory link, while C and D each reject a true statement.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (A). The supplied answer key lists the second option, but the printed reason directly explains the assertion through the extra pairing repulsion in oxygen.

Quick Tip to Solve: For the N/O ionization exception, draw the three p-orbital boxes: the paired electron in oxygen is the easier one to remove.

65. Given below are two statements.

• Statement I : In $CuSO_4 . 5H_2O$, Cu-O bonds are present.

• Statement II : In $CuSO_4 . 5H_2O$, ligands coordinating with Cu(II) ion are O-and S-based ligands.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (C) Statement I is correct but Statement II is incorrect.

Solution:

Concept:

- A ligand is classified by the atom directly donating an electron pair to the metal; the mere presence of sulfur elsewhere in a ligand does not make it S-donating.
- In hydrated copper sulphate, water binds through oxygen and sulfate interactions with copper also involve oxygen, not sulfur.

Step 1: Identify the copper coordination bonds In $CuSO_4 \cdot 5\,H_2O$, copper(II) is surrounded by oxygen donors. Four water molecules provide the prominent equatorial water-oxygen coordination bonds; the remaining water participates in the hydrogen-bonded crystal network. Longer axial contacts to sulfate oxygen atoms complete the distorted environment. Therefore Cu–O bonds are present and Statement I is correct.

Step 2: Check the donor atoms in Statement II Water donates an oxygen lone pair to Cu^{2+}. Sulfate also contacts copper through its oxygen atoms; its central sulfur atom is bonded to oxygen and is not the donor directly attached to copper. Consequently, the crystal does not contain the claimed combination of O-based and S-based donor ligands. Statement II is incorrect.

Step 3: Select the matching combination It is essential to distinguish the composition of sulfate from its coordination mode. Calling sulfate S-based simply because its formula contains S would predict a Cu–S bond, which is not the bonding described here. Option A makes that error. Options B and D wrongly deny the real Cu–O bonds. Only option C retains Statement I while rejecting Statement II.

Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: Name a ligand donor by the atom bonded directly to the metal: sulfate is oxygen-donating even though it contains sulfur.

66. Number of lone pair(s) of electrons on central atom and the shape BrF_3 molecule respectively, are [4 marks]

(A) 0, triangular planar.

(B) 1, pyramidal.

(C) 2, bent T-shape.

(D) 1, bent T-shape.

Correct Answer: (C) 2, bent T-shape.

Solution:

Concept:

- Bromine has seven valence electrons. Each single bond to a monovalent fluorine uses one bromine electron in a bonding pair.
- VSEPR distinguishes the arrangement of all electron pairs from molecular shape, which records only atomic positions.

Step 1: Count the bromine lone pairs Three bromine electrons participate in the three $\mathrm{Br}-\mathrm{F}$ bonds. The remaining $7-3=4$ electrons form two lone pairs on bromine. Thus the central atom has three bonding pairs and two lone pairs: AX_3E_2. The steric number is $3+2=5$, so the electron-pair arrangement is trigonal bipyramidal.

Step 2: Place the lone pairs In a trigonal bipyramid an axial site has three close 90° interactions, whereas an equatorial site has only two. Lone pairs exert stronger repulsion than bonding pairs, so both occupy equatorial sites to reduce unfavorable interactions. The three fluorine atoms consequently occupy two axial positions and the remaining equatorial position.

Step 3: Read the molecular shape The atomic arrangement resembles a T, with small deviations from ideal bond angles due to lone-pair repulsion; it is described as a bent T-shape. The electron-pair geometry must not be reported as the molecular shape. Option A has no lone pairs and the wrong planar arrangement. B corresponds to neither the correct count nor shape. D gets the T description but misses one lone pair.

Step 4: Conclusion $\mathrm{BrF_3}$ has two lone pairs on bromine and a bent T-shaped molecular geometry. Hence, the correct option is (C).

Quick Tip to Solve: For a halogen trifluoride, use AX3E2: five electron domains, two equatorial lone pairs, and three atoms forming a T.

67. Sulphur dioxide is one of the components of polluted air. SO_2 is also a major contributor to acid rain. The correct and complete reaction to represent acid rain caused by SO_2 is : [4 marks]

(A) $2SO_2 + O_2 \rightarrow 2SO_3$

(B) $SO_2 + O_3 \rightarrow SO_3 + O_2$

(C) $SO_2 + H_2O_2 \rightarrow H_2SO_4$

(D) $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$

Correct Answer: (D) $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$

Solution:

Concept:

- Acid rain formation involves oxidation of dissolved sulfur dioxide and formation of sulfuric acid in atmospheric moisture.
- A complete overall reaction includes the pollutant, atmospheric oxidant and water, rather than stopping at the acid anhydride.

Step 1: Write the oxidation and hydration stages Sulfur dioxide can first be oxidized to sulfur trioxide, which then reacts with water. The balanced stages are

$$2\,SO_2 + O_2 \longrightarrow 2\,SO_3$$

$$2\,SO_3 + 2\,H_2O \longrightarrow 2\,H_2SO_4.$$

These equations distinguish production of the oxide from actual production of the acid in rainwater.

Step 2: Obtain the overall reaction Add the two equations and cancel the intermediate SO_3 from both sides. This gives

$$2\,SO_2 + O_2 + 2\,H_2O \longrightarrow 2\,H_2SO_4.$$

There are two sulfur atoms, four hydrogen atoms and eight oxygen atoms on each side. Sulfur is oxidized from +4 to +6; oxygen supplies the oxidizing capacity and water supplies the hydrogen of the acid.

Step 3: Compare the alternatives Options A and B stop at sulfur trioxide and do not display acid formation. Option C is a chemically possible aqueous oxidation by hydrogen peroxide and can contribute to atmospheric sulfur oxidation, but it specifies a particular peroxide pathway rather than the complete overall oxygen-and-water reaction requested here. Option D explicitly combines oxidation with hydration to give sulfuric acid, matching the intended overall acid-rain representation.

Step 4: Conclusion The complete overall representation is $2\,SO_2 + O_2 + 2\,H_2O \longrightarrow 2\,H_2SO_4$. Hence, the correct option is (D).

Quick Tip to Solve: For an overall acid-rain equation, continue beyond SO3: include water and end with sulfuric acid, then balance atoms.

68. Which of the following carbocations is most stable? [4 marks]

(A) $CH_2{=}C(OCH_3){-}CH_2{}^+$

(B) $CH_2{=}C(OCH_3){-}CH{=}CH{-}CH_2{}^+$

(C) $CH_3O{-}CH{=}CH{-}CH_2{}^+$

(D) $CH_3O{-}CH{=}CH{-}CH{=}CH{-}CH_2{}^+$

Correct Answer: (D) $CH_3O{-}CH{=}CH{-}CH{=}CH{-}CH_2{}^+$

Solution:

Concept:

- An allylic carbocation is stabilized by overlap between its empty p orbital and an adjacent conjugated pi system.
- A methoxy oxygen can donate a lone pair by resonance when conjugation connects it to the cation; this can yield an oxonium contributor with complete carbon octets.

Step 1: Compare the conjugated pathways In A, $CH_2{=}C(OCH_3){-}CH_2{}^+$, the allylic charge can move between the terminal carbon atoms, but the oxygen is attached to the middle carbon. Its lone pair cannot terminate the charge-delocalization sequence as an octet-satisfying oxonium contributor in the same way as an end methoxy substituent. B extends the carbon chain but retains this less favorable placement of methoxy.

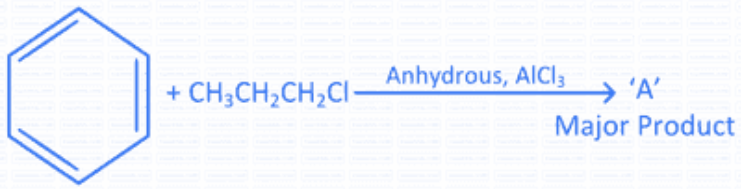
Step 2: Locate direct oxygen donation In C, the structure $CH_3O{-}CH{=}CH{-}CH_2{}^+$ allows allylic charge delocalization to the carbon bonded to oxygen. Oxygen can then donate a lone pair and carry the positive charge. D has the same useful terminal oxygen arrangement but a longer, uninterrupted conjugated chain: $CH_3O{-}CH{=}CH{-}CH{=}CH{-}CH_2{}^+$.

Step 3: Choose the most delocalized ion For D, moving the pi electrons successively places positive charge at alternating carbon positions along the pentadienyl chain, including the oxygen-bearing carbon. Lone-pair donation then gives an additional oxonium contributor. Thus D combines extended conjugation with effective methoxy resonance donation. C has a shorter delocalization path, while A and B do not place the oxygen at the charge-bearing end of the corresponding resonance sequence.

Step 4: Conclusion The carbocation in D has the greatest effective resonance delocalization, including donation from methoxy oxygen. Hence, the correct option is (D).

Quick Tip to Solve: Trace alternating pi bonds from the positive carbon; an oxygen lone pair stabilizes most effectively when the trace reaches its attached carbon.

69. The stable carbocation formed in the above reaction is [4 marks]



(A) $CH_3CH_2CH_2^+$

(B) $CH_3CH_2^+$

(C) $(CH_3)_2CH^+$

(D) $C_6H_5-CH^+-CH_2CH_3$

Correct Answer: (C) $(CH_3)_2CH^+$

Solution:

Concept:

- Anhydrous $AlCl_3$ is a Lewis acid that activates an alkyl chloride for electrophilic aromatic substitution.
- A primary alkyl carbocation can undergo a 1,2-hydride shift if a more stable secondary carbocation results, with the carbon skeleton retained.

Step 1: Identify the initially generated electrophile The reagent is 1-chloropropane, $CH_3CH_2CH_2Cl$, reacting with benzene in anhydrous $AlCl_3$. Coordination of chlorine to the Lewis acid polarizes the C–Cl bond. The initially developed n-propyl electrophilic species is represented as $CH_3CH_2CH_2^+$, a primary carbocation and therefore relatively unstable.

Step 2: Allow the hydride rearrangement A hydride shifts from the neighboring middle carbon to the terminal electron-deficient carbon. The positive charge moves onto the middle carbon:

$$CH_3CH_2CH_2^+ \longrightarrow (CH_3)_2CH^+.$$

The resulting isopropyl carbocation is secondary. Its two methyl groups stabilize the positive center through electron donation and hyperconjugation, making it more stable than the unrearranged primary ion.

Step 3: Check the printed choices Option A is the initial primary propyl ion, not the stable rearranged one. B has only two carbon atoms and is not obtained by this hydride shift. C is the secondary isopropyl ion and subsequently gives isopropylbenzene after aromatic substitution and deprotonation. D already contains a phenyl group bonded to the charged carbon, so it is not the alkyl electrophile generated from the three-carbon alkyl chloride before benzene attacks.

Step 4: Conclusion The stable carbocation formed is the isopropyl ion, $(CH_3)_2CH^+$. Hence, the correct option is (C).

Quick Tip to Solve: Before predicting a Friedel–Crafts alkylation product, check whether its initially primary electrophile can become secondary by a hydride shift.

70. Two isomers (A) and (B) with Molar mass 184 g/mol and elemental composition C, 52.2%; H, 4.9% and Br 42.9% gave benzoic acid and p-bromobenzoic acid, respectively on oxidation with $KMnO_4$. Isomer 'A' is optically active and gives a pale yellow precipitate when warmed with alcoholic $AgNO_3$. Isomer 'A' and 'B' are, respectively [4 marks]

(A) $C_6H_5CH(Br)CH_3$ and ortho-$CH_3C_6H_4CH_2Br$

(B) ortho-$CH_3C_6H_4CH_2Br$ and para-$BrC_6H_4CH_2CH_3$

(C) $C_6H_5CH(Br)CH_3$ and para-$BrC_6H_4CH_2CH_3$

(D) para-$BrC_6H_4CH_2CH_3$ and $C_6H_5CH(Br)CH_3$

Correct Answer: (C) $C_6H_5CH(Br)CH_3$ and para-$BrC_6H_4CH_2CH_3$

Solution:

Concept:

- Elemental mass percentages and molar mass determine a molecular formula. Oxidation of an alkylbenzene with benzylic hydrogen converts its side chain to a carboxyl group.
- Optical activity requires a stereogenic center here; alcoholic silver nitrate tests the availability of the halide for ionization, forming pale-yellow $AgBr$.

Step 1: Determine the formula For one mole of mass 184 g, the element masses are approximately 96 g C, 9 g H and 79 g Br. Dividing by atomic masses gives eight carbon atoms, nine hydrogen atoms and one bromine atom:

$$\text{Molecular formula} = C_8H_9Br.$$

This is consistent with a brominated ethylbenzene or a brominated side-chain isomer.

Step 2: Identify the optically active isomer A Oxidation of A gives benzoic acid, so bromine is not retained as a ring substituent. The structure $C_6H_5CH(Br)CH_3$ has a benzylic carbon bonded to four different groups: phenyl, methyl, hydrogen and bromine. It is stereogenic and can be optically active. Its benzylic C–Br bond ionizes readily in alcoholic $AgNO_3$, giving pale-yellow $AgBr$.

Step 3: Identify B and compare the pairs To give para-bromobenzoic acid, B must retain bromine at the para position on the benzene ring while an ethyl side chain oxidizes to $COOH$. Thus B is para-bromoethylbenzene, p-$BrC_6H_4CH_2CH_3$. Option A has the wrong second isomer and ring substitution. B places an achiral ortho-substituted compound first. D reverses the required order, placing the aryl bromide before the optically active benzylic bromide.

Step 4: Conclusion A is $C_6H_5CH(Br)CH_3$ and B is para-$BrC_6H_4CH_2CH_3$. Hence, the correct option is (C).

Quick Tip to Solve: Use oxidation to locate ring bromine first, then use chirality and alcoholic AgNO3 to identify the benzylic bromide.

71. In Friedel-Crafts alkylation of aniline, one gets [4 marks]

(A) alkylated product with ortho and para substitution.

(B) secondary amine after acidic treatment.

(C) an amide product.

(D) positively charged nitrogen at benzene ring.

Correct Answer: (D) positively charged nitrogen at benzene ring.

Solution:

Concept:

- Aniline, $C_6H_5NH_2$, is a Lewis base because nitrogen has a lone pair. The Friedel-Crafts catalyst $AlCl_3$ is a Lewis acid and accepts this electron pair.
- Coordination changes the electronic effect of the amino group: a nitrogen bearing positive formal charge withdraws electron density and strongly deactivates the benzene ring.

Step 1: Identify the competing Lewis acid-base reaction In an ordinary Friedel-Crafts alkylation, an alkyl halide and $AlCl_3$ generate the electrophile for aromatic substitution. With aniline, however, the amino nitrogen itself strongly binds the Lewis acid. Its lone pair is donated to aluminium, forming an aniline–aluminium chloride complex. In a coordinate-bond representation, nitrogen has four bonds and bears positive formal charge.

Step 2: Apply the consequence to the ring The coordinated amino group can no longer donate its lone pair into the ring in the usual way. The positively charged nitrogen makes the aromatic ring strongly electron deficient, so aniline does not undergo the normal Friedel-Crafts ring alkylation. Thus the usual ortho/para directing argument for free aniline cannot be used in these conditions.

Step 3: Eliminate the other proposed products Option (A) incorrectly assumes the free amino group retains its activating effect in the presence of the Lewis acid. Option (B) describes an N-alkylation outcome, not this catalyst complex; acidic treatment does not establish a secondary amine here. Option (C) requires formation of an acyl–nitrogen bond, whereas alkylation supplies no acyl group.

Step 4: Conclusion The Lewis-acid complex has positively charged nitrogen attached to the benzene ring, and normal Friedel-Crafts alkylation is inhibited. Hence, the correct option is (D).

Quick Tip to Solve: Before applying aromatic directing effects, check whether the substituent binds the Lewis-acid catalyst; aniline consumes $AlCl_3$ through its nitrogen lone pair.

72. The structure of protein that is unaffected by heating is [4 marks]

(A) secondary structure

(B) tertiary structure

(C) primary structure

(D) quaternary structure

Correct Answer: (C) primary structure

Solution:

Concept:

- The primary structure is the covalent sequence of amino-acid residues joined by peptide bonds. It specifies the order of residues along the polypeptide chain.
- Heating can denature a protein by disturbing the interactions responsible for its folded shape. Ordinary thermal denaturation does not hydrolyse the peptide bonds.

Step 1: Separate sequence from spatial organisation The primary structure records which amino acid follows which; changing it would require cleavage or rearrangement of peptide bonds. Secondary structure describes regular local folding, such as an alpha helix or beta sheet, supported largely by hydrogen bonding. Tertiary structure is the three-dimensional folding of a chain, and quaternary structure is the association of multiple chains.

Step 2: Determine the effect of heating On heating, the weak interactions stabilising the native conformation are disturbed. Consequently the protein may unfold, lose biological activity and coagulate. This is denaturation. The local and overall folding can therefore change even though the amino-acid sequence remains the same. The familiar coagulation of egg protein illustrates a loss of native conformation rather than a rewriting of its sequence.

Step 3: Choose the unaffected level Secondary, tertiary and quaternary structures all concern spatial arrangement or subunit association and may be disrupted by heating. They cannot be chosen as the level preserved during ordinary denaturation. Primary structure survives because its peptide-bond backbone is retained; peptide hydrolysis under suitable chemical conditions is a different process.

Step 4: Conclusion The amino-acid sequence, or primary structure, remains unaffected during ordinary thermal denaturation. Hence, the correct option is (C).

Quick Tip to Solve: For denaturation questions, distinguish unfolding from hydrolysis: loss of folding changes higher structures, while cleavage of peptide bonds changes the primary structure.

73. A white precipitate was formed when $BaCl_2$ was added to water extract of an inorganic salt. Further, a gas 'X' with characteristic odour was released when the formed white precipitate was dissolved in dilute HCl. The anion present in the inorganic salt is [4 marks]

(A) I^-

(B) SO_3^{2-}

(C) S^{2-}

(D) NO_2^-

Correct Answer: (B) SO_3^{2-}

Solution:

Concept:

- An anion is identified using the complete sequence of observations, not only the colour of a precipitate. Several barium salts are white, but their behaviour with dilute acid differs.
- Sulphite ions form white $BaSO_3$, which reacts with dilute acid to release pungent sulphur dioxide, SO_2.

Step 1: Identify the white barium salt Adding $BaCl_2$ supplies Ba^{2+}. With sulphite, the insoluble white precipitate is barium sulphite:

$$Ba^{2+} + SO_3^{2-} \longrightarrow BaSO_3(s).$$

This accounts for the first observation. The gas must then be generated by acid acting on this precipitate, rather than by an unrelated reaction of the original solution.

Step 2: Use the acid and odour test Barium sulphite dissolves in dilute hydrochloric acid as sulphur dioxide escapes:

$$BaSO_3(s) + 2\,HCl \longrightarrow BaCl_2 + SO_2(g) + H_2O.$$

Sulphur dioxide has a characteristic pungent, choking odour. At the ionic level, sulphite is protonated to unstable sulphurous acid, which decomposes into water and sulphur dioxide. This explains both disappearance of the solid and evolution of gas X.

Step 3: Reject the remaining anions Barium iodide and barium nitrite are soluble, so iodide and nitrite do not give the required white precipitate in this test. Sulphide can evolve rotten-egg-smelling hydrogen sulphide with acid, but it does not produce the specified insoluble barium sulphite precipitate. The precipitation step and gas test together select sulphite uniquely.

Step 4: Conclusion The anion is SO_3^{2-}, and gas X is SO_2. Hence, the correct option is (B).

Quick Tip to Solve: Pair the barium precipitation test with dilute acid: sulphite precipitates dissolve with pungent SO_2, whereas a white precipitate alone is not a complete identification.

74. Using the rules for significant figures, the correct answer for the expression $\frac{0.02858\times0.112}{0.5702}$ will be [4 marks]

(A) 0.005613

(B) 0.00561

(C) 0.0056

(D) 0.006

Correct Answer: (B) 0.00561

Solution:

Concept:

- For multiplication and division, the final result has as many significant figures as the measured factor with the fewest significant figures. This is different from the decimal-place rule for addition.
- Leading zeros locate the decimal point and are not significant. Nonzero digits, and zeros between significant digits, do count. Keep guard digits until the final rounding.

Step 1: Count the significant figures in the input In 0.02858, the leading zeros do not count, leaving the four significant digits 2, 8, 5 and 8. The factor 0.112 contains three significant digits. The denominator 0.5702 contains four significant digits: its zero lies between significant digits and therefore counts. The least precise factor is consequently 0.112, so the answer must have three significant figures.

Step 2: Evaluate before rounding Carry the full intermediate product into the division:

$$0.02858 \times 0.112 = 0.00320096,$$

$$\frac{0.00320096}{0.5702} = 0.005613750\ldots.$$

The first three significant digits of this result are 5, 6 and 1. The next digit is 3, which is less than 5, so the third significant digit stays unchanged.

Step 3: Compare the offered precisions The correctly rounded result is 0.00561. Option (A), 0.005613, retains too many significant figures. Option (C), 0.0056, gives only two, and option (D), 0.006, gives only one. Neither leading zeros nor the number of displayed decimal places changes the required count of three significant digits.

Step 4: Conclusion The expression equals 0.00561 to three significant figures. Hence, the correct option is (B).

Quick Tip to Solve: Write a small significant-figure count above each factor, use the smallest count for the final answer, and round only after completing both multiplication and division.

75. A 42.12% (w/v) solution of NaCl causes precipitation of a certain sol in 10 hours. The coagulating value of NaCl for the sol in 2 hours is :

[Given : Molar mass : Na = 23.0 g mol^{-1} ; Cl = 35.5 g mol^{-1}] [4 marks]

(A) 36 m mol L^{-1}

(B) 36 mol L^{-1}

(C) 1440 mol L^{-1}

(D) 1440 m mol L^{-1}

Correct Answer: (D) 1440 m mol L^{-1}

Solution:

Concept:

- Coagulating value is the minimum electrolyte concentration, usually in millimoles per litre, needed to coagulate a sol in a specified time. Its definition alone does not supply a concentration–time proportionality.
- A concentration of 42.12% (w/v) means $42.12\,\mathrm{g}$ of NaCl per $100\,\mathrm{mL}$ solution. Its molar mass is the sum of the stated atomic molar masses.

Step 1: Convert the mass concentration into molar concentration The molar mass of sodium chloride is

$$M = 23.0 + 35.5 = 58.5\,\mathrm{g\,mol^{-1}}.$$

A litre of the given solution contains $42.12 \times 10 = 421.2\,\mathrm{g}$ of sodium chloride. Thus the electrolyte amount on a litre basis for the stated ten-hour interval is

$$\frac{421.2}{58.5} = 7.2\,\mathrm{mol\,L^{-1}} = 7200\,\mathrm{mmol\,L^{-1}}.$$

The factor of ten comes from converting 100 mL to one litre; it must be applied before comparing the units in the options.

Step 2: Express the result on the standard two-hour basis The accepted answer is obtained by directly normalising the stated ten-hour amount to two hours:

$$7200 \times \frac{2}{10} = 1440\,\mathrm{mmol\,L^{-1}}.$$

This is the time-scaling assumption implicit in the accepted key, not an established kinetic law or a consequence of the definition of coagulating value. With this intended convention, option (D) is selected.

Step 3: Distinguish the keyed convention from the literal data The initial concentration, $7.2\,\mathrm{mol\,L^{-1}}$, and a single coagulation time do not by themselves determine the concentration required for a different time. A concentration–time relation or a dosage-rate assumption is needed. For example, imposing an inverse concentration–time law instead would give $7.2(10/2) = 36\,\mathrm{mol\,L^{-1}}$, option (B), demonstrating that different extra assumptions give different options. That inverse law is also not supplied. Thus the literal problem is underdetermined even though its accepted key clearly indicates the direct-normalisation convention. In the keyed result, the unit is millimol per litre; option (C) differs by a factor of one thousand.

Step 4: Conclusion Under the direct time-normalisation implicit in the accepted key, the result is $1440\,\text{mmol}\,\text{L}^{-1}$. Hence, the correct option is (D) on that convention. Strictly, the printed concentration and one coagulation time do not establish a unique result without an additional concentration–time relation.

Quick Tip to Solve: For this keyed exercise, $7200(2/10) = 1440$ gives (D); distinguish that intended normalisation from a general coagulation rate law, which the statement does not provide.

76. Match List I with List II.

	List-IOre		List-IIComposition
(A)	Siderite	(I)	$FeCO_3$
(B)	Malachite	(II)	$CuCO_3$. $Cu(OH)_2$
(C)	Sphalerite	(III)	ZnS
(D)	Calamine	(IV)	$ZnCO_3$

Choose the correct answer from the options given below : [4 marks]

(A) A-I, B-II, C-III, D-IV

(B) A-III, B-IV, C-II, D-I

(C) A-IV, B-III, C-I, D-II

(D) A-I, B-II, C-IV, D-III

Correct Answer: (A) A-I, B-II, C-III, D-IV

Solution:

Concept:

- An ore name refers to a characteristic mineral composition, not merely to the metal present. Carbonate and sulphide ores of the same metal must therefore be distinguished.
- Siderite is iron(II) carbonate, malachite is basic copper(II) carbonate, sphalerite is zinc sulphide and calamine is zinc carbonate in the classification used in this question.

Step 1: Match the iron and copper minerals Siderite has formula $FeCO_3$, so entry A matches entry I. Its iron is in the $+2$ oxidation state, balancing carbonate. Malachite is the green basic carbonate of copper, written $CuCO_3 \cdot Cu(OH)_2$, so entry B matches entry II. The hydroxide component is essential: malachite is not simply an anhydrous copper carbonate.

Step 2: Distinguish the two zinc ores Sphalerite, also called zinc blende, has composition ZnS. Hence C matches III. Calamine is listed here as $ZnCO_3$, so D matches IV. Both contain zinc in the $+2$ oxidation state, but the anions differ. Merely identifying zinc would not distinguish these two rows; the sulphide versus carbonate distinction settles the last two matches.

Step 3: Check the complete option Combining the four independently identified pairs gives A–I, B–II, C–III, D–IV. Option (B) incorrectly associates siderite with zinc sulphide, while option (C) also fails the iron carbonate match. Option (D) gets the first two rows right but interchanges the zinc sulphide and zinc carbonate entries. Only the first option preserves all four mineral formulae.

Step 4: Conclusion The correct matching is A–I, B–II, C–III, D–IV. Hence, the correct option is (A).

Quick Tip to Solve: Use the anion to separate similar ore names: zinc blende/sphalerite is the sulphide ZnS, while calamine in this table is the carbonate $ZnCO_3$.

77. Amongst baking soda, caustic soda and washing soda, carbonate anion is present in : [4 marks]

(A) washing soda only.

(B) washing soda and caustic soda only.

(C) washing soda and baking soda only.

(D) baking soda, caustic soda and washing soda.

Correct Answer: (A) washing soda only.

Solution:

Concept:

- Common names must first be converted into chemical formulae: baking soda is sodium hydrogencarbonate, caustic soda is sodium hydroxide, and washing soda is hydrated sodium carbonate.
- The carbonate ion CO_3^{2-} and hydrogencarbonate ion HCO_3^- are distinct anions. Water of crystallisation does not change which anion is present.

Step 1: Write the formula of each substance Baking soda is $NaHCO_3$, containing Na^+ and HCO_3^-. Caustic soda is $NaOH$, containing sodium and hydroxide ions. Washing soda is $Na_2CO_3 \cdot 10\,H_2O$, containing two sodium ions for each carbonate ion, together with ten waters of crystallisation. Listing the formulae removes the ambiguity caused by the shared word soda.

Step 2: Identify the anion without changing the compounds The question asks which substances contain carbonate anion as given. Washing soda contains CO_3^{2-}. Baking soda contains hydrogencarbonate, whose hydrogen and single negative charge distinguish it from carbonate. Although heating baking soda can produce sodium carbonate, that conversion does not mean carbonate is the anion of the original baking soda. Caustic soda contains OH^-, not any carbon-containing anion.

Step 3: Reject the combined choices Option (B) wrongly includes caustic soda. Option (C) wrongly treats hydrogencarbonate as carbonate. Option (D) includes both of those errors. The ten water molecules in washing soda are neutral crystallisation water and do not remove or replace its carbonate ion, so washing soda alone meets the condition.

Step 4: Conclusion Carbonate anion is present in washing soda only. Hence, the correct option is (A).

Quick Tip to Solve: Write the formula before choosing: $NaHCO_3$, $NaOH$, and $Na_2CO_3 \cdot 10\,H_2O$ contain hydrogencarbonate, hydroxide, and carbonate respectively.

78. Aqueous solution of which of the following boron compounds will be strongly basic in nature?

[4 marks]

(A) $NaBH_4$

(B) $LiBH_4$

(C) B_2H_6

(D) $Na_2B_4O_7$

Correct Answer: (D) $Na_2B_4O_7$

Solution:

Concept:

- Borax, $Na_2B_4O_7$, is a salt of a strong base and a weak acid. Hydrolysis of its anion generates hydroxide ions in water.
- Boric acid behaves as a weak Lewis acid by accepting hydroxide from water; it does not behave like a strong proton-donating mineral acid.

Step 1: Identify the acid and base corresponding to borax Sodium tetraborate gives sodium ions and tetraborate ions in water. The sodium ion does not appreciably hydrolyse, whereas the borate species interact with water. The acid corresponding to the borate system is weak boric acid, while sodium hydroxide is a strong base. Consequently, the solution is alkaline rather than neutral. This is the familiar alkaline hydrolysis of borax.

Step 2: Write the hydrolysis and compare the alternatives The net hydrolysis may be represented by

$$B_4O_7{}^{2-} + 7\,H_2O \rightleftharpoons 4\,H_3BO_3 + 2\,OH^-.$$

The formation of hydroxide ions explains the basic character. Equivalently, the reaction is often written as formation of boric acid and sodium hydroxide. The borohydrides in options (A) and (B) are principally reducing hydrides; their reactions with water involve hydrogen evolution and are not the characteristic borax hydrolysis tested here. Diborane hydrolyses to boric acid and hydrogen, so it does not supply the required alkaline salt solution. Thus the borate salt in (D) is the intended choice.

Step 3: Conclusion The aqueous solution of $Na_2B_4O_7$ is basic because of anionic hydrolysis. Hence, the correct option is (D).

Quick Tip to Solve: For borax in water, write the tetraborate hydrolysis equation: the appearance of OH^- immediately identifies the alkaline solution.

79. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Dacron is an example of polyester polymer.

Reason R : Dacron is made up of ethylene glycol and terephthalic acid monomers.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Correct Answer: (A) Both A and R are correct and R is the correct explanation of A.

Solution:

Concept:

- A polyester has repeated ester linkages, $-COO-$, along its main polymer chain. Classification depends on the linkage, not merely on its use as a fibre.
- A diol reacts with a dicarboxylic acid by condensation: hydroxyl and carboxyl groups form ester bonds with elimination of water.

Step 1: Evaluate Assertion A Dacron is polyethylene terephthalate, commonly abbreviated PET. Its repeating chain contains ester groups connecting the ethylene glycol residue and the terephthalate residue. Since these ester bonds occur repeatedly in the backbone, Dacron belongs to the polyester class. Assertion A is therefore correct. It is a condensation polymer rather than a polymer formed simply by addition across an alkene double bond.

Step 2: Evaluate Reason R The monomers are ethylene glycol, $HOCH_2CH_2OH$, and terephthalic acid, $HOOC-C_6H_4-COOH$, with the two acid groups in the para positions. Both monomers are bifunctional, allowing chain growth at both ends. Their condensation produces a chain segment of the form $-OCH_2CH_2OCO-C_6H_4-CO-$. The alcohol and acid groups form the ester links, releasing water. Reason R is therefore correct.

Step 3: Test whether the reason explains the assertion The stated monomers specifically explain why ester linkages occur in the chain: a diol and a dicarboxylic acid undergo repeated esterification. Thus the reason does more than state an unrelated fact about Dacron. Option (B) incorrectly denies this explanatory connection, while (C) and (D) incorrectly reject one of the two true statements.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: In polymer assertion-reason questions, identify the functional groups of both monomers and the new bond they form; that bond determines the polymer class.

80. The mixture of chloroxylenol and terpineol is an example of [4 marks]

(A) antiseptic

(B) pesticide

(C) disinfectant

(D) narcotic analgesic

Correct Answer: (A) antiseptic

Solution:

Concept:

- Antiseptics are antimicrobial agents used on living tissues, such as skin, to prevent infection. Their intended use distinguishes them from disinfectants used on non-living surfaces.
- Chloroxylenol and terpineol form the characteristic antiseptic mixture associated with Dettol in the standard chemistry classification.

Step 1: Identify the named mixture The question specifies two ingredients together: chloroxylenol and terpineol. Chloroxylenol is a substituted phenolic antimicrobial compound, and terpineol is a terpene alcohol. This combination is the familiar antiseptic formulation discussed in chemistry in everyday life. Its purpose is to inhibit or destroy microorganisms in a preparation suitable for antiseptic use, rather than to relieve pain or control agricultural pests.

Step 2: Distinguish the close alternative Antiseptics and disinfectants can share antimicrobial chemistry, and the action of a substance can depend on its concentration and application. However, the named chloroxylenol-terpineol mixture is classified in this question as an antiseptic. A disinfectant is ordinarily applied to non-living articles, floors or surfaces. Therefore, simply knowing that chloroxylenol kills microorganisms does not justify selecting option (C); the standard identity of the mixture decides the choice.

Step 3: Eliminate the remaining options A pesticide is used against pests such as insects or unwanted organisms in an agricultural or related setting, which is not the use identifying this mixture. A narcotic analgesic relieves severe pain through effects on the nervous system; morphine is a typical example. Neither description fits the antiseptic role of these ingredients. Consequently (B), (C) and (D) do not give the requested classification.

Step 4: Conclusion The mixture of chloroxylenol and terpineol is an antiseptic. Hence, the correct option is (A).

Quick Tip to Solve: Remember the named pair chloroxylenol plus terpineol as an antiseptic formulation, and distinguish living-tissue use from surface disinfection.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. A box contains 0.90 g of liquid water in equilibrium with water vapour at 27°C. The equilibrium vapour pressure of water at 27°C is 32.0 Torr. When the volume of the box is increased, some of the liquid water evaporates to maintain the equilibrium pressure. If all the liquid water evaporates, then the volume of the box must be _____ litre. [nearest integer]

(Given : R = 0.082 L atm K^{-1} mol^{-1})

(Ignore the volume of the liquid water and assume water vapours behave as an ideal gas.) [4 marks]

Solution:

Concept:

- At a fixed temperature, liquid water in equilibrium with its vapour maintains the saturation vapour pressure while some liquid remains.
- At the limiting volume where the last liquid evaporates, use $PV = nRT$. Pressure must be in atmospheres when $R = 0.082\,\mathrm{L\,atm\,K^{-1}\,mol^{-1}}$.

Step 1: Determine the available amount of water The molar mass of water is $18\,\mathrm{g\,mol^{-1}}$. The amount corresponding to the stated water mass is therefore

$$n = \frac{0.90}{18} = 0.050\,\mathrm{mol}.$$

As the box expands, evaporation supplies additional vapour. The last drop disappears at the limiting saturated state; increasing the volume beyond that point would lower the vapour pressure because no liquid remains to supply more vapour.

Step 2: Convert the temperature and pressure The absolute temperature is $T = 27 + 273 = 300\,\mathrm{K}$. Since $760\,\mathrm{Torr} = 1\,\mathrm{atm}$, the given saturation pressure becomes

$$P = \frac{32.0}{760}\,\mathrm{atm}.$$

Using 32 directly with the supplied gas constant would mix Torr and atmosphere units and give the wrong volume.

Step 3: Calculate the limiting volume Substitute the amount, temperature and equilibrium pressure into the ideal gas equation:

$$V = \frac{nRT}{P} = \frac{0.050 \times 0.082 \times 300}{32.0/760} = 29.2125\,\mathrm{L}.$$

The liquid volume is neglected as instructed. The requested response is the nearest integer, so rounding is performed only after this calculation. The result is consistent with a relatively large vapour volume at a pressure much smaller than one atmosphere.

Step 4: Conclusion The required limiting box volume is approximately $29.21\,\mathrm{L}$; to the nearest integer, the answer is $\boxed{29}$.

Quick Tip to Solve: At the instant the last drop evaporates, set the gas pressure equal to the saturation vapour pressure; convert Torr to atm before using the supplied gas constant.

82. 2.2 g of nitrous oxide (N_2O) gas is cooled at a constant pressure of 1 atm from 310 K to 270 K causing the compression of the gas from 217.1 mL to 167.75 mL. The change in internal energy of the process, ΔU is '$-$x' J. The value of 'x' is _____. [nearest integer]

(Given : atomic mass of N = 14 g mol^{-1} and of O = 16 g mol^{-1}. Molar heat capacity of N_2O is 100 J K^{-1} mol^{-1}) [4 marks]

Solution:

Concept:

- Using the chemistry sign convention, the first law is $\Delta U = q + w$, where compression gives positive work done on the gas.
- At constant pressure, heat is $q_p = nC_{p,m}\Delta T$, and pressure-volume work is $w = -P_{\text{ext}}\Delta V$. Use the given volumes for the work.

Step 1: Calculate the heat released on cooling The molar mass of nitrous oxide is $2(14) + 16 = 44\,\text{g mol}^{-1}$, so

$$n = \frac{2.2}{44} = 0.050\,\text{mol}.$$

The temperature change is $270 - 310 = -40\,\text{K}$. At the stated constant pressure, the given molar heat capacity yields

$$q_p = 0.050 \times 100 \times (-40) = -200\,\text{J}.$$

The negative sign denotes heat transferred out of the gas.

Step 2: Calculate the compression work The final volume is smaller than the initial volume:

$$\Delta V = 167.75 - 217.1 = -49.35\,\text{mL} = -0.04935\,\text{L}.$$

Thus

$$w = -(1\,\text{atm})(-0.04935\,\text{L}) = 0.04935\,\text{L atm}.$$

With $1\,\text{L atm} = 101.325\,\text{J}$, this is $w \approx +5.000\,\text{J}$. Compression transfers energy into the system, so a positive sign is essential.

Step 3: Apply the first law and identify x Combine heat and work with their signs intact:

$$\Delta U = -200 + 5.000 \approx -195\,\text{J}.$$

The question writes the result as $-x$ J, so x is the positive magnitude, not the signed internal-energy change. Use the supplied volume change rather than imposing an additional ideal-gas relation on these numerical data. Rounding the magnitude gives the requested integer.

Step 4: Conclusion $\Delta U \approx -195\,\text{J}$, so the requested value is $\boxed{x = 195}$.

Quick Tip to Solve: For cooling with compression, heat is negative but work is positive. Compute both separately before adding them to obtain the internal-energy change.

83. Elevation in boiling point for 1.5 molal solution of glucose in water is 4 K. The depression in freezing point for 4.5 molal solution of glucose in water is 4 K. The ratio of molal elevation constant to molal depression constant (K_b/K_f) is _____. [4 marks]

Solution:

Concept:

- For a dilute solution, boiling-point elevation is $\Delta T_b = iK_b m$, where m is molality and i is the van't Hoff factor.
- Freezing-point depression obeys $\Delta T_f = iK_f m$. Glucose is a non-electrolyte, so its factor is $i = 1$ in both solutions.

Step 1: Use the boiling-point elevation data The first solution is 1.5 molal and its boiling point is elevated by $4\,\mathrm{K}$. Substitution into the elevation relation gives

$$4 = K_b(1.5), \qquad K_b = \frac{4}{1.5}\,\mathrm{K\,kg\,mol^{-1}}.$$

There is no dissociation factor to introduce because glucose remains as molecular solute. Use the numerical data supplied by the question rather than replacing them with tabulated constants for water.

Step 2: Use the freezing-point depression data The second solution is 4.5 molal and has a freezing-point depression of $4\,\mathrm{K}$. The depression is its positive magnitude, so

$$4 = K_f(4.5), \qquad K_f = \frac{4}{4.5}\,\mathrm{K\,kg\,mol^{-1}}.$$

Although freezing occurs at a lower temperature, the symbol ΔT_f in this formula denotes the magnitude of the lowering; a negative constant is neither required nor appropriate.

Step 3: Divide the constants Dividing the expressions cancels both the common temperature change and the common units:

$$\frac{K_b}{K_f} = \frac{4/1.5}{4/4.5} = \frac{4.5}{1.5} = 3.$$

For equal temperature shifts, the required molality is inversely proportional to the corresponding constant. The smaller molality for boiling-point elevation therefore correctly corresponds to the larger constant, providing a direct check that the ratio was not inverted.

Step 4: Conclusion The required dimensionless ratio is $\boxed{K_b/K_f = 3}$.

Quick Tip to Solve: When the two temperature shifts and solute factors are equal, cancel them first: the ratio of constants is the inverse ratio of the molalities.

84. The cell potential for the given cell at 298 K

Pt| H_2 (g, 1 bar) | H^+ (aq) || Cu^{2+} (aq) | Cu(s)

is 0.31 V. The pH of the acidic solution is found to be 3, whereas the concentration of Cu^{2+} is 10^{-x} M. The value of x is _____.

(Given : $E^{\Theta}_{Cu^{2+}/Cu}$ = 0.34 V and $\frac{2.303\,RT}{F}$ = 0.06 V) [4 marks]

Solution:

Concept:

- The left-hand hydrogen electrode is the anode and the right-hand copper electrode is the cathode. The overall reaction transfers two electrons.
- At 298 K, $E = E^\circ - (0.06/n)\log_{10} Q$. Pure solid copper and hydrogen gas at its standard pressure have unit activity in this expression.

Step 1: Write the overall cell reaction At the anode, hydrogen is oxidised: $\mathrm{H_2} \longrightarrow 2\,\mathrm{H^+} + 2\,\mathrm{e^-}$. At the cathode, copper ions are reduced: $\mathrm{Cu^{2+}} + 2\,\mathrm{e^-} \longrightarrow \mathrm{Cu}$. Adding the half-reactions gives

$$\mathrm{H_2(g)} + \mathrm{Cu^{2+}(aq)} \longrightarrow 2\,\mathrm{H^+(aq)} + \mathrm{Cu\,(s)}.$$

Thus $n = 2$, and the standard cell potential is $0.34 - 0 = 0.34\,\mathrm{V}$. The reaction quotient must follow this reaction direction.

Step 2: Express the reaction quotient using pH The hydrogen pressure is 1 bar and the solid has unit activity. With the concentration approximation used in the problem,

$$Q = \frac{[\mathrm{H^+}]^2}{[\mathrm{Cu^{2+}}]}.$$

The pH is 3, so $[\mathrm{H^+}] = 10^{-3}\,\mathrm{M}$. If $[\mathrm{Cu^{2+}}] = 10^{-x}\,\mathrm{M}$, then

$$Q = \frac{10^{-6}}{10^{-x}} = 10^{x-6}.$$

Squaring the hydrogen-ion concentration is required by its stoichiometric coefficient.

Step 3: Substitute in the Nernst equation Use the given logarithmic coefficient and the observed cell voltage:

$$0.31 = 0.34 - \frac{0.06}{2}(x - 6).$$

Therefore $0.03 = 0.03(x - 6)$, giving $x - 6 = 1$ and $x = 7$. As a check, this concentration gives $Q = 10$, whose positive logarithm lowers the cell potential by $0.03\,\mathrm{V}$, exactly as observed. Treating the hydrogen electrode as having zero potential at pH 3 would miss this concentration effect.

Step 4: Conclusion The copper-ion concentration is $10^{-7}\,\mathrm{M}$, so the required value is $\boxed{x = 7}$.

Quick Tip to Solve: Write the full cell reaction before forming Q. Here the squared hydrogen-ion term contributes -6 to $\log Q$, while copper contributes $+x$.

85. The equation

$k = (6.5 \times 10^{12} s^{-1}) e^{-26000K/T}$

is followed for the decomposition of compound A. The activation energy for the reaction is _____ kJ mol^{-1}. [nearest integer]

(Given : R = 8.314 J K^{-1} mol^{-1}) [4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where A is the pre-exponential factor, E_a is the molar activation energy, R is the gas constant and T is the absolute temperature.
- The exponent of an exponential is dimensionless. Thus the coefficient of $1/T$ equals E_a/R and has the unit kelvin.

Step 1: Compare the exponential factors The given expression is $k = (6.5\times10^{12}\,\mathrm{s^{-1}})\exp[-(26000\,\mathrm{K})/T]$. Compare its exponent directly with $-E_a/(RT)$, retaining the negative sign on both sides. Hence

$$\frac{E_a}{R} = 26000\,\mathrm{K}.$$

The prefactor determines the frequency factor, not the activation energy; it need not be used in this calculation. The positive value of the coefficient gives a positive activation barrier.

Step 2: Calculate and convert the energy Multiply the coefficient by the supplied gas constant:

$$E_a = (26000\,\mathrm{K})(8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}) = 216164\,\mathrm{J\,mol^{-1}}.$$

The kelvin units cancel. Since one kilojoule is one thousand joules,

$$E_a = 216.164\,\mathrm{kJ\,mol^{-1}}.$$

Round only this final value, rather than rounding the gas constant or the product prematurely.

Step 3: Conclusion The activation energy is $216.164\,\mathrm{kJ\,mol^{-1}}$, so the required nearest-integer entry is $\boxed{216}$.

Quick Tip to Solve: For a rate law written as $Ae^{-B/T}$, first use $E_a = BR$; then convert joules to kilojoules before rounding.

86. Spin only magnetic moment of $[MnBr_6]^{4-}$ is _____ B.M. (round off to the closest integer)

[4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mathrm{B.M.}$, where n is the number of unpaired electrons on the central metal ion.
- Bromide is a weak-field ligand. An octahedral complex with a d^5 metal ion is therefore high spin,

with one electron in each of the five metal d orbitals.

Step 1: Find the oxidation state and d-electron count Let the oxidation state of manganese be z. Each of the six bromide ligands has charge -1, while the complex has charge -4. Therefore

$$z + 6(-1) = -4 \quad \Longrightarrow \quad z = +2.$$

Neutral manganese has the configuration $[\mathrm{Ar}]3d^5 4s^2$. Removing the two outer 4s electrons gives Mn^{2+} with configuration $[\mathrm{Ar}]3d^5$. The complex charge must not be mistaken for the oxidation state of manganese.

Step 2: Count unpaired electrons and evaluate the moment The six ligands produce an octahedral field. Because Br^- causes a small splitting, pairing is not favoured before all five d orbitals are singly occupied. The high-spin arrangement is $t_{2g}^3 e_g^2$, giving $n = 5$. Thus

$$\mu = \sqrt{5(5+2)} = \sqrt{35} = 5.916\,\mathrm{B.M.}$$

This is the spin-only result; no orbital contribution is added. Rounding the calculated moment to the closest integer gives 6, rather than rounding the unpaired-electron count itself.

Step 3: Conclusion The spin-only magnetic moment is approximately $5.92\,\mathrm{B.M.}$. The required closest-integer entry is $\boxed{6}$.

Quick Tip to Solve: Calculate the metal oxidation state before counting d electrons; weak-field bromide keeps a d^5 ion high spin.

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87. For the reaction given below :

$CoCl_3$. xNH_3 + $AgNO_3$ (aq) $\rightarrow$

If two equivalents of AgCl precipitate out, then the value of x will be _____. [4 marks]

Solution:

Concept:

- In Werner complexes, chloride ions outside the coordination sphere are ionisable and precipitate as AgCl when aqueous silver nitrate is added.
- Cobalt(III) ammine complexes have coordination number six. Ammonia is a neutral monodentate ligand, and each coordinated chloride occupies one coordination site.

Step 1: Identify the ionisable chloride ions Two equivalents of AgCl precipitating per equivalent of the cobalt compound mean that two chloride ions are outside the square brackets. The formula contains three chlorides in total, so one chloride remains coordinated to cobalt. The complex can therefore be represented as

$$[\mathrm{Co(NH_3)_x Cl}]\mathrm{Cl_2}.$$

The two outer chlorides are released in water and react immediately with Ag^+; the inner chloride belongs to the coordination sphere and is not counted as a free chloride in this test.

Step 2: Complete the six-coordinate sphere One coordinated chloride occupies one of the six sites

around cobalt(III). Each ammonia molecule occupies one further site, so

$$x + 1 = 6 \quad \Longrightarrow \quad x = 5.$$

The compound is thus $[Co(NH_3)_5Cl]Cl_2$. Its complex ion carries charge $+2$, consistent with cobalt in oxidation state $+3$, one inner chloride of charge -1, and neutral ammonia ligands. Its precipitation reaction is

$$[Co(NH_3)_5Cl]Cl_2 + 2\,AgNO_3 \longrightarrow [Co(NH_3)_5Cl](NO_3)_2 + 2\,AgCl\downarrow.$$

This checks both the chloride count and the stated amount of precipitate.

Step 3: Conclusion There are five coordinated ammonia molecules, so $\boxed{x = 5}$.

Quick Tip to Solve: Subtract the precipitated chloride count from the total chloride count to find inner-sphere chlorides, then fill the remaining six coordination sites with ammonia.

88. The number of chiral alcohol(s) with molecular formula $C_4H_{10}O$ is _____. [4 marks]

Solution:

Concept:

- A tetrahedral carbon is a stereogenic centre when it is bonded to four different substituents. A molecule with one such centre has a pair of enantiomers.
- For $C_4H_{10}O$, count alcohols only: the oxygen must occur in a hydroxyl group. Ether structures do not satisfy the stated functional-group requirement.

Step 1: List the possible alcohol connectivities The four constitutional alcohol isomers are butan-1-ol, butan-2-ol, 2-methylpropan-1-ol and 2-methylpropan-2-ol. Their condensed formulae are, respectively,

$$CH_3CH_2CH_2CH_2OH, \quad CH_3CH(OH)CH_2CH_3,$$

$$(CH_3)_2CHCH_2OH, \quad (CH_3)_3COH.$$

These exhaust the straight-chain and branched four-carbon skeletons with distinct hydroxyl positions. Merely having four carbon atoms or an alcohol group does not establish chirality.

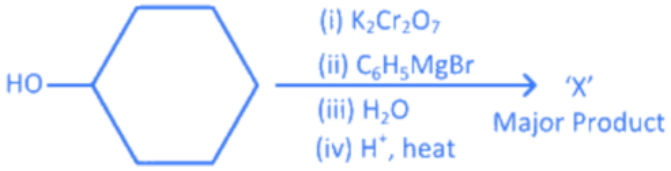
Step 2: Test each structure and count enantiomers In butan-1-ol, every potentially relevant carbon is either a methyl or methylene carbon, so none has four different groups. In 2-methylpropan-1-ol the branched carbon bears two identical methyl groups. In 2-methylpropan-2-ol the hydroxyl-bearing carbon bears three identical methyl groups. Both are achiral. In butan-2-ol, carbon 2 bears H, OH, CH_3 and CH_2CH_3, which are four distinct substituents. It therefore exists as two non-superimposable mirror images, the (R) and (S) forms. Count these individual chiral alcohols rather than counting only their shared constitutional formula.

Step 3: Conclusion Butan-2-ol gives two enantiomeric chiral alcohols. Hence the required number is $\boxed{2}$.

Quick Tip to Solve: After finding a chiral connectivity, count its stereoisomers: one isolated stereogenic centre gives two enantiomers, not just one chiral alcohol.

89. In the given reaction,

the number of sp^2 hybridised carbon(s) in compound 'X' is _____. [4 marks]



Solution:

Concept:

- A secondary alcohol is oxidised to a ketone by dichromate. A Grignard reagent adds its carbon group to the carbonyl carbon, and hydrolysis produces an alcohol.
- Acid-catalysed dehydration removes water from an alcohol to form an alkene. Each carbon of a carbon-carbon double bond is sp^2-hybridised; all six carbons of a benzene ring are also sp^2-hybridised.

Step 1: Follow oxidation and Grignard addition The starting compound is cyclohexanol, a secondary alcohol. Oxidation with $K_2Cr_2O_7$ gives cyclohexanone without changing the six-membered carbon skeleton. Phenylmagnesium bromide, C_6H_5MgBr, then adds a phenyl group to the ketone carbonyl carbon. Hydrolysis with water protonates the alkoxide, giving 1-phenylcyclohexan-1-ol. The phenyl group retains its aromatic benzene ring throughout these operations. At this intermediate stage, the hydroxyl-bearing ring carbon is tetrahedral.

Step 2: Dehydrate and count the planar carbons Heating the tertiary alcohol in acid eliminates water. Removal of a beta hydrogen from either adjacent ring carbon leads to the same alkene by ring symmetry: 1-phenylcyclohex-1-ene. The new carbon-carbon double bond lies in the cyclohexene ring and is conjugated with the attached phenyl ring. Count the carbons by structural unit:

$$N(sp^2) = 6 \text{ (phenyl ring)} + 2 \text{ (cyclohexene double bond)} = 8.$$

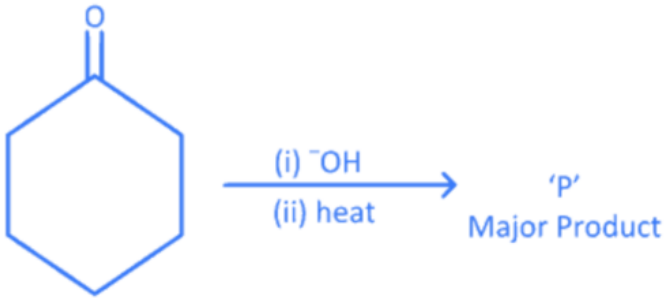
The other four carbons of the six-membered nonaromatic ring remain saturated and sp^3-hybridised. No aromatic carbon is counted twice, since the phenyl ring and the cyclohexene ring have distinct carbon atoms.

Step 3: Conclusion Compound X contains $\boxed{8}$ sp^2-hybridised carbon atoms.

Quick Tip to Solve: Follow every reagent before counting hybridisation: dehydration creates two extra sp^2 carbons in addition to the six already present in the phenyl ring.

90. In the given reaction,

The number of π electrons present in the product 'P' is _____. [4 marks]



Solution:

Concept:

- A ketone with alpha hydrogens undergoes base-catalysed aldol addition: an enolate attacks the carbonyl group of another molecule to form a beta-hydroxy ketone.
- On heating, the beta-hydroxy carbonyl compound loses water to form an alpha,beta-unsaturated carbonyl compound. A double bond contains one pi bond, holding two pi electrons.

Step 1: Identify the self-aldol reaction The reactant is cyclohexanone. It has alpha hydrogens on the carbons adjacent to its carbonyl group, so hydroxide produces a small equilibrium concentration of the enolate. The alpha carbon of this enolate attacks the carbonyl carbon of another cyclohexanone molecule. Protonation gives a beta-hydroxy ketone containing the two six-membered rings joined by a new carbon-carbon bond. The donor molecule retains its carbonyl group; the attacked carbonyl group becomes an alcohol in the aldol addition intermediate.

Step 2: Apply heating and count pi electrons The heating step favours dehydration of the beta-hydroxy ketone. Loss of the hydroxyl group and an alpha hydrogen creates a carbon-carbon double bond between the two ring systems. The product is 2-cyclohexylidenecyclohexan-1-one, with a conjugated $\mathrm{C{=}C{-}C{=}O}$ unit. It therefore has one $\mathrm{C{=}C}$ pi bond and one $\mathrm{C{=}O}$ pi bond:

$$N(\pi \text{ bonds}) = 1 + 1 = 2, \qquad N(\pi \text{ electrons}) = 2 \times 2 = 4.$$

The remaining ring bonds are single sigma bonds and add no pi electrons. Count the retained carbonyl as well as the alkene formed on dehydration; counting only the new bond would omit half the required electrons.

Step 3: Conclusion The major condensation product P contains two pi bonds, hence $\boxed{4}$ pi electrons.

Quick Tip to Solve: In a heated self-aldol condensation, the product retains one carbonyl and gains one alkene; each contributes two pi electrons.