JEE Main 2022 Paper 1 (B.E./B.Tech) Jun 30 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 1 (June), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 30 June 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $S_1 = \left\{x \in R - \{1,2\} : \frac{(x+2)(x^2+3x+5)}{-2+3x-x^2} \geq 0\right\}$ **and** $S_2 = \{x \in R : 3^{2x} - 3^{x+1} - 3^{x+2} + 27 \leq 0\}$**. Then,** $S_1 \cup S_2$ **is equal to :** [4 marks]

(A) $(-\infty, -2] \cup (1, 2)$

(B) $(-\infty, -2] \cup [1, 2]$

(C) $(-2, 1] \cup [2, \infty)$

(D) $(-\infty, 2]$

Correct Answer: (B) $(-\infty, -2] \cup [1, 2]$

Solution:

Concept:

- The quadratic $x^2 + 3x + 5$ is positive for every real x, since its discriminant is $9 - 20 = -11 < 0$ and its leading coefficient is positive.
- For an exponential inequality, put $t = 3^x > 0$. The exponential function is strictly increasing, so bounds for t translate directly into bounds for x.

Step 1: Solve the rational inequality The denominator factors as $-2 + 3x - x^2 = -(x-1)(x-2)$. Its zeros are excluded from S_1. The sign changes can therefore occur only at $-2, 1, 2$. On $(-\infty, -2)$, the numerator and denominator are both negative, so the quotient is positive. On $(-2, 1)$ it is negative, on $(1, 2)$ positive, and on $(2, \infty)$ negative. The zero at -2 is allowed. Thus

$$S_1 = (-\infty, -2] \cup (1, 2).$$

Step 2: Solve the exponential inequality and take the union Using $3^{x+1} + 3^{x+2} = 12 \cdot 3^x$, the second inequality becomes

$$t^2 - 12t + 27 = (t-3)(t-9) \leq 0.$$

Hence $3 \leq t \leq 9$, or $1 \leq x \leq 2$. Therefore $S_2 = [1, 2]$. Taking a union fills the excluded endpoints of the second interval of S_1; it does not fill the gap $(-2, 1)$. Option (A) omits these endpoints, option (C) reverses the relevant intervals, and option (D) incorrectly includes this gap.

Step 3: Conclusion

$$S_1 \cup S_2 = (-\infty, -2] \cup [1, 2].$$

Hence, the correct option is (B).

Quick Tip to Solve: Keep denominator exclusions for the first set, then check whether the second set supplies those endpoints before forming the union.

2. The real part of the complex number $\frac{(1+2i)^8 \cdot (1-2i)^2}{(3+2i) \cdot \overline{(4-6i)}}$ is equal to : [4 marks]

(A) $\frac{500}{13}$

(B) $\frac{110}{13}$

(C) $\frac{55}{6}$

(D) $\frac{550}{13}$

Correct Answer: (D) $\frac{550}{13}$

Solution:

Concept:

- Conjugation reverses the sign of the imaginary part: $\overline{4-6i} = 4+6i$, where $i^2 = -1$.
- Pair conjugate factors before expanding high powers: $(1+2i)(1-2i) = 1+4 = 5$. A purely imaginary denominator can be divided out using $1/i = -i$.

Step 1: Simplify the numerator and denominator Use two factors of $(1+2i)$ to pair with $(1-2i)^2$. The numerator is then

$$[(1+2i)(1-2i)]^2(1+2i)^6 = 25(1+2i)^6.$$

Independently, the denominator is

$$(3+2i)(4+6i) = 12+18i+8i+12i^2 = 26i.$$

The real terms cancel. In particular, dropping the conjugate bar would give an entirely different denominator, so it must be applied before multiplication.

Step 2: Evaluate the remaining power and extract the real part Compute the cube first: $(1+2i)^2 = -3+4i$, and $(1+2i)^3 = (-3+4i)(1+2i) = -11-2i$. Squaring this yields

$$(1+2i)^6 = (-11-2i)^2 = 117+44i.$$

Thus the required complex number is

$$\frac{25(117+44i)}{26i} = \frac{1100-2925i}{26}.$$

Its real part is $1100/26 = 550/13$. The other listed quantities, $500/13$, $110/13$ and $55/6$, are unequal to this exact real coefficient; they cannot result from the correctly conjugated denominator.

Step 3: Conclusion The real part is $\boxed{550/13}$. Hence, the correct option is (D).

Quick Tip to Solve: When the denominator becomes purely imaginary, the numerator's imaginary coefficient supplies the real part after division.

3. Let S be the set of all integral values of α for which the sum of squares of two real roots of the quadratic equation $3x^2 + (\alpha - 6)x + (\alpha + 3) = 0$ is minimum. Then S : [4 marks]

(A) is an empty set

(B) is a singleton

(C) contains exactly two elements

(D) contains more than two elements

Correct Answer: (B) is a singleton

Solution:

Concept:

- For roots r, s, Vieta's formulae give $r + s = -b/a$, $rs = c/a$, and $r^2 + s^2 = (r + s)^2 - 2rs$.
- The roots are real only when the discriminant is nonnegative. Minimise over this admissible parameter set, rather than over every real parameter.

Step 1: Determine the permissible parameter values For $3x^2 + (\alpha - 6)x + (\alpha + 3) = 0$, the discriminant is

$$D = (\alpha - 6)^2 - 12(\alpha + 3) = \alpha(\alpha - 24).$$

Therefore the two roots are real, including a repeated real root, exactly when $\alpha \leq 0$ or $\alpha \geq 24$. Values between 0 and 24 are inadmissible even if they make a formal expression smaller.

Step 2: Minimise the sum on the admissible set Vieta's formulae give

$$r^2 + s^2 = \frac{(6 - \alpha)^2}{9} - \frac{2(\alpha + 3)}{3} = \frac{\alpha^2 - 18\alpha + 18}{9} = \frac{(\alpha - 9)^2 - 63}{9}.$$

The unrestricted vertex $\alpha = 9$ is outside the real-root domain. On $(-\infty, 0]$, the expression decreases toward its endpoint and has minimum 2 at $\alpha = 0$. On $[24, \infty)$, it increases from its endpoint value 18. Thus the global admissible minimum is attained only at $\alpha = 0$, which is an integer. At this value the equation is $3(x - 1)^2 = 0$, confirming real roots and sum of squares 2. The set is neither empty nor a set of two or more elements.

Step 3: Conclusion $S = \{0\}$, so it is a singleton. Hence, the correct option is (B). The supplied empty-set key disagrees with the real-root constrained minimum: $\alpha = 0$ explicitly gives two coincident real roots and achieves the unique admissible minimum.

Quick Tip to Solve: For optimisation involving real quadratic roots, impose the discriminant condition before choosing the vertex of the parameter expression.

4. Let $A = \begin{bmatrix} 1 & -2 & \alpha \\ \alpha & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & \alpha \\ -1 & 2 \\ 4 & -5 \end{bmatrix}$, $\alpha \in C$. Then the absolute value of the sum of all values of α for which det(AB) = 0 is : [4 marks]

(A) 3

(B) 4

(C) 2

(D) 5

Correct Answer: (A) 3

Solution:

Concept:

- The product of a 2×3 matrix and a 3×2 matrix is 2×2; compute this square product before taking its determinant.
- For a quadratic $az^2 + bz + c = 0$, the sum of its two complex roots is $-b/a$. The absolute value of a real sum is its nonnegative magnitude.

Step 1: Multiply the matrices in the stated order Taking row-by-column scalar products gives

$$AB = \begin{bmatrix} 4 + 4\alpha & -4 - 4\alpha \\ 2\alpha - 6 & \alpha^2 + 9 \end{bmatrix}.$$

For example the top right entry is $\alpha - 4 - 5\alpha = -4 - 4\alpha$, and the bottom right entry is $\alpha^2 + 4 + 5 = \alpha^2 + 9$. The rectangular factors themselves have no ordinary determinant, so a product rule for their determinants would not apply.

Step 2: Factor the determinant and add every allowed root The determinant condition is

$$\begin{aligned} \det(AB) &= 4(1 + \alpha)(\alpha^2 + 9) + 4(1 + \alpha)(2\alpha - 6) \\ &= 4(1 + \alpha)(\alpha^2 + 2\alpha + 3) = 0. \end{aligned}$$

One root is -1, and the remaining quadratic has roots $-1 + i\sqrt{2}$ and $-1 - i\sqrt{2}$. All three must be included because the question explicitly allows $\alpha \in \mathbb{C}$. The quadratic roots sum to -2, so the total is $-1 - 2 = -3$. Taking the magnitude gives 3. Options 4, 2 and 5 are different from this value; 2 in particular counts only the magnitude of the quadratic-root sum and omits the linear root.

Step 3: Conclusion The absolute value of the sum is $\boxed{3}$. Hence, the correct option is (A).

Quick Tip to Solve: Factor out the common linear factor in the determinant, then use Vieta's formula for the quadratic factor instead of expanding a cubic.

5. For two positive real numbers a and b such that $\frac{1}{a^2}+\frac{1}{b^3}=4$, then minimum value of the constant term in the expansion of $\left(ax^{\frac{1}{8}}+bx^{-\frac{1}{12}}\right)^{10}$ is : [4 marks]

(A) $\frac{105}{2}$
(B) $\frac{105}{4}$
(C) $\frac{105}{8}$
(D) $\frac{105}{16}$

Correct Answer: (C) $\frac{105}{8}$

Solution:

Concept:

- The general term of $(u+v)^{10}$ is $\binom{10}{r}u^{10-r}v^r$, with $r=0,1,\ldots,10$. The constant term has net exponent zero.
- For positive u,v of fixed sum, $uv\le((u+v)/2)^2$, with equality exactly when $u=v$.

Step 1: Find the term independent of the variable The term indexed by r in the given expansion is

$$\binom{10}{r}a^{10-r}b^r x^{(10-r)/8-r/12}.$$

Its exponent vanishes when $3(10-r)-2r=0$, giving $r=6$. Consequently the constant term is

$$T=\binom{10}{6}a^4b^6=210a^4b^6.$$

Both positivity and the exponents matter: the coefficient is a positive quantity to minimise, and there is exactly one zero-exponent term.

Step 2: Use the constraint to minimise the coefficient Put $u=1/a^2>0$ and $v=1/b^3>0$. The constraint gives $u+v=4$, while $a^4b^6=1/(u^2v^2)$. Therefore

$$uv\le 4,\qquad T=\frac{210}{(uv)^2}\ge\frac{210}{16}=\frac{105}{8}.$$

Equality is attained at $u=v=2$, corresponding to $a=1/\sqrt{2}$ and $b=2^{-1/3}$, which are permitted positive numbers. Hence this is an attained minimum, not merely a lower bound. Options (A) and (B) are larger than the attained minimum, while (D) is below the proved lower bound and cannot occur.

Step 3: Conclusion The minimum constant term is $\boxed{105/8}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert reciprocal-power constraints into new positive variables; minimising the reciprocal product then becomes maximising a fixed-sum product.

6. If xy^4 attains maximum value at the point (x, y) on the line passing through the points (50 + α, 0) and (0, 50 + α), $\alpha > 0$, then (x, y) also lies on the line : [4 marks]

(A) y = 4x

(B) x = 4y

(C) y = 4x + α

(D) x = 4y − α

Correct Answer: (A) y = 4x

Solution:

Concept:

- The intercept form of the line is $x + y = L$, where $L = 50 + \alpha > 0$. Eliminate one variable before differentiating the objective.
- At an interior local maximum of a differentiable function, the first derivative vanishes; a sign change from positive to negative confirms that maximum.

Step 1: Locate the stationary point on the line Use $x = L - y$. Then the objective becomes

$$g(y) = (L - y)y^4 = Ly^4 - y^5, \qquad g\prime(y) = y^3(4L - 5y).$$

The nonzero stationary point is $y = 4L/5$, and its corresponding coordinate is $x = L/5$. Thus $y = 4x$. Near this positive stationary point, $y^3 > 0$; the factor $4L - 5y$ changes from positive to negative, establishing a local maximum.

Step 2: Check the meaning of maximum and the choices On the segment between the two positive intercepts, $0 \le y \le L$, the endpoint products are zero and the interior value is positive, so this stationary point is also the segment's absolute maximum. It satisfies (A). Option (B) reverses the ratio, and (C) would require $\alpha = 0$, contrary to the question. Substitution in (D) would give $\alpha = 3L$, impossible because $L = 50 + \alpha$ and $\alpha > 0$. Strictly, on the entire unbounded line there is no absolute maximum: for $y = -t$, $x = L + t$, and $xy^4 = (L + t)t^4 \to \infty$. Thus the stated maximum is the local maximum, or equivalently the maximum on the intercept segment; the unrestricted-line reading has no such point.

Step 3: Conclusion The maximum point in the local or intercept-segment sense lies on $\boxed{y = 4x}$. Hence, the correct option is (A). On the entire unbounded line an absolute maximum does not exist.

Quick Tip to Solve: After eliminating a variable, classify stationary points and check whether the domain is a bounded segment or the full line.

7. Let $f(x) = 4x^3 - 11x^2 + 8x - 5$, $x \in R$. Then f : [4 marks]

(A) has a local minina at $x = \frac{1}{2}$

(B) has a local minima at $x = \frac{3}{4}$

(C) is increasing in $\left(\frac{1}{2}, \frac{3}{4}\right)$

(D) is decreasing in $\left(\frac{1}{2}, \frac{4}{3}\right)$

Correct Answer: (D) is decreasing in $\left(\frac{1}{2}, \frac{4}{3}\right)$

Solution:

Concept:

- A differentiable function is increasing where its derivative is positive and decreasing where its derivative is negative.
- A local minimum requires a change of derivative sign from negative to positive; a local maximum requires the reverse sign change.

Step 1: Factor the first derivative Differentiating term by term yields

$$f\prime(x) = 12x^2 - 22x + 8 = 2(2x - 1)(3x - 4).$$

The stationary points are $x = 1/2$ and $x = 4/3$, not $3/4$. The leading coefficient of the derivative is positive, so the derivative is positive outside these two roots and negative between them.

Step 2: Classify the points and test each statement For $x < 1/2$, both linear factors are negative and their product is positive. For $1/2 < x < 4/3$, the first factor is positive and the second is negative, giving $f\prime(x) < 0$. For $x > 4/3$, both are positive. Hence $x = 1/2$ is a local maximum, making (A) false. The local minimum occurs at $4/3$, so (B), which states $3/4$, is false. In fact $f\prime(3/4) = -7/4 \neq 0$. The interval $(1/2, 3/4)$ is contained within the decreasing interval, so (C) is false as well. Throughout $(1/2, 4/3)$, the derivative is strictly negative, which establishes (D). The additive constant -5 shifts the graph vertically but has no effect on these monotonicity intervals.

Step 3: Conclusion The function is decreasing on $\boxed{(1/2, 4/3)}$. Hence, the correct option is (D).

Quick Tip to Solve: Factor the derivative and write its sign on the three intervals; this simultaneously checks monotonicity and the type of each stationary point.

8. Let m and M respectively be the minimum and the maximum values of $f(x) = \sin^{-1}2x + \sin 2x + \cos^{-1}2x + \cos 2x$, $x \in \left[0, \frac{\pi}{8}\right]$. Then m + M is equal to : [4 marks]

(A) $1 + \sqrt{2} + \pi$

(B) $\left(1 + \sqrt{2}\right)\pi$

(C) $\pi + \sqrt{2}$

(D) $1 + \pi$

Correct Answer: (A) $1 + \sqrt{2} + \pi$

Solution:

Concept:

- For a real number $u \in [-1, 1]$, the principal inverse functions satisfy $\sin^{-1} u + \cos^{-1} u = \pi/2$. This identity applies to the same argument, regardless of its value in that domain.
- On an interval, a continuous function attains its extreme values at an endpoint or a stationary point. Here $\sin t + \cos t$ increases for $0 \le t \le \pi/4$.

Step 1: Check the domain and simplify The given interval gives $0 \le 2x \le \pi/4 < 1$, so both inverse functions are real and their sum is constant. Therefore the function reduces to

$$f(x) = \frac{\pi}{2} + \sin 2x + \cos 2x.$$

The inverse sine and inverse cosine must not be differentiated separately; their complementary-value identity removes both terms exactly.

Step 2: Locate and evaluate the extreme values Differentiate the remaining elementary expression:

$$f'(x) = 2(\cos 2x - \sin 2x).$$

For $0 \le 2x \le \pi/4$, cosine is at least sine. Thus the derivative is nonnegative, and it vanishes only at the right endpoint. The minimum is at $x = 0$, and the maximum is at $x = \pi/8$:

$$m = \frac{\pi}{2} + 1, \qquad M = \frac{\pi}{2} + \sqrt{2}.$$

Adding gives $m + M = \pi + 1 + \sqrt{2}$. Option B incorrectly multiplies the trigonometric contribution by π; option C omits the minimum endpoint contribution 1, and option D omits the maximum contribution $\sqrt{2}$.

Step 3: Conclusion $m + M = 1 + \sqrt{2} + \pi$. Hence, the correct option is (A).

Quick Tip to Solve: Combine inverse sine and inverse cosine with the same argument before finding extrema; then check the original domain and both endpoints.

9. Let α_1, α_2 ($\alpha_1 < \alpha_2$) be the values of α fo the points (α, -3), (2, 0) and (1, α) to be collinear. Then the equation of the line, passing through (α_1, α_2) and making an angle of $\frac{\pi}{3}$ with the positive direction of the x-axis, is : [4 marks]

(A) $x - \sqrt{3}y - 3\sqrt{3} + 1 = 0$

(B) $\sqrt{3}x - y + \sqrt{3} + 3 = 0$

(C) $x - \sqrt{3}y + 3\sqrt{3} + 1 = 0$

(D) $\sqrt{3}x - y + \sqrt{3} - 3 = 0$

Correct Answer: (B) $\sqrt{3}x - y + \sqrt{3} + 3 = 0$

Solution:

Concept:

- Three points are collinear when the determinant formed from their coordinates and a final column of ones is zero. This avoids division by a possibly zero coordinate difference.
- A line inclined at angle θ to the positive x-axis has slope $\tan\theta$. Through (x_0, y_0), its equation is $y - y_0 = \tan\theta(x - x_0)$.

Step 1: Determine the two parameter values Apply the collinearity condition to the three stated points:

$$\begin{vmatrix} \alpha & -3 & 1 \\ 2 & 0 & 1 \\ 1 & \alpha & 1 \end{vmatrix} = 0.$$

Expansion along the first row gives

$$-\alpha^2 + 3 + 2\alpha = 0, \qquad \alpha^2 - 2\alpha - 3 = 0.$$

Factorising, $(\alpha - 3)(\alpha + 1) = 0$. Since the values are ordered, $\alpha_1 = -1$ and $\alpha_2 = 3$. These are parameter values, and together they form the single point through which the required line passes.

Step 2: Use the required inclination The required point is $(-1, 3)$, while the inclination is $\pi/3$. Hence its slope is $\sqrt{3}$, not its reciprocal. Substituting into the point-slope formula gives

$$y - 3 = \sqrt{3}(x + 1).$$

Moving all terms to the left produces

$$\sqrt{3}x - y + \sqrt{3} + 3 = 0.$$

This is option B, whereas option D has the wrong constant sign; substituting $(-1, 3)$ in D gives -6. Options A and C both have slope $1/\sqrt{3}$, so their inclination is $\pi/6$, not the angle demanded. Thus both the point and the inclination are necessary checks.

Step 3: Conclusion The required line is $\sqrt{3}x - y + \sqrt{3} + 3 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Order the two parameter roots before using them as coordinates, and test a proposed line for both its slope and its passage through the required point.

10. Let the eccentricity of the ellipse $x^2 + a^2y^2 = 25a^2$ be b times the eccentricity of the hyperbola $x^2 - a^2y^2 = 5$, where a is the minimum distance between the curves y = e^x and y = $\log_e x$. Then $a^2 + \frac{1}{b^2}$ is equal to : [4 marks]

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) 3

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- The curves $y = e^x$ and $y = \ln x$ are reflections in $y = x$. The inequality $e^t \geq 1 + t$, with equality at $t = 0$, separates them by parallel supporting lines.
- For an ellipse with major and minor semiaxes A, B, $e_E^2 = 1 - B^2/A^2$. For a hyperbola with transverse and conjugate semiaxes A, B, $e_H^2 = 1 + B^2/A^2$.

Step 1: Find the minimum distance between the curves Every point $P = (t, e^t)$ on the exponential satisfies $y - x = e^t - t \geq 1$. For $Q = (s, \ln s)$, $s > 0$, the inequality $\ln s \leq s - 1$ gives $y - x \leq -1$. Projection onto the unit normal $(-1, 1)/\sqrt{2}$ therefore shows that any joining segment has length at least $2/\sqrt{2} = \sqrt{2}$. Equality is achieved by $P = (0, 1)$ and $Q = (1, 0)$, whose joining segment is normal to these supporting lines. Consequently $a = \sqrt{2}$ and $a^2 = 2$. This establishes the global minimum, rather than just a stationary distance.

Step 2: Compute the eccentricity ratio Put both conics in standard form using $a^2 = 2$:

$$\frac{x^2}{50} + \frac{y^2}{25} = 1, \qquad \frac{x^2}{5} - \frac{y^2}{5/2} = 1.$$

Thus $e_E^2 = 1 - 25/50 = 1/2$, whereas $e_H^2 = 1 + (5/2)/5 = 3/2$. Since $e_E = be_H$,

$$b^2 = \frac{e_E^2}{e_H^2} = \frac{1}{3}, \qquad a^2 + \frac{1}{b^2} = 2 + 3 = 5.$$

The ellipse uses subtraction and the hyperbola uses addition; interchanging those rules would not describe these conics. Options A, B and C do not equal the computed sum 5.

Step 3: Conclusion $a^2 + 1/b^2 = 5$. Hence, the correct option is (D).

Quick Tip to Solve: Bound the distance by projecting onto the normal to parallel supporting lines, then exhibit points attaining the bound before computing the conic eccentricities.

11. Let $\alpha = \tan\left(\frac{5\pi}{16}\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)$ and $\beta = \cos\left(\sin^{-1}\left(\frac{4}{5}\right) + \sec^{-1}\left(\frac{5}{3}\right)\right)$ where the inverse trigonometric functions take principal values. Then, the equation whose roots are α and β is :

[4 marks]

(A) $15x^2 - 8x - 7 = 0$

(B) $5x^2 - 12x + 7 = 0$

(C) $25x^2 - 18x - 7 = 0$

(D) $25x^2 - 32x + 7 = 0$

Correct Answer: (C) $25x^2 - 18x - 7 = 0$

Solution:

Concept:

- Principal inverse sine and inverse cosine determine the signs of the associated sine and cosine. Use $\sin 2t = 2\sin t\cos t$ and $\cos(u+v) = \cos u\cos v - \sin u\sin v$.
- If the roots of a quadratic are α, β, its monic form is $x^2 - (\alpha+\beta)x + \alpha\beta = 0$. Multiplying by a nonzero constant preserves its roots.

Step 1: Evaluate alpha with the double-angle identity Let $t = \cos^{-1}(1/\sqrt{5})$. Its principal value is in the first quadrant, so $\cos t = 1/\sqrt{5}$ and $\sin t = 2/\sqrt{5}$. Thus

$$\sin 2t = 2\frac{1}{\sqrt{5}}\frac{2}{\sqrt{5}} = \frac{4}{5}.$$

The angle inside the tangent is therefore $(5\pi/16)(4/5) = \pi/4$. Hence $\alpha = \tan(\pi/4) = 1$. The nested inverse function is removed before evaluating the outer tangent.

Step 2: Evaluate beta and form the quadratic Write $u = \sin^{-1}(4/5)$ and $v = \sec^{-1}(5/3)$. Both principal angles are acute, with $\cos u = 3/5$, $\cos v = 3/5$, and $\sin v = 4/5$. Consequently

$$\beta = \frac{3}{5}\frac{3}{5} - \frac{4}{5}\frac{4}{5} = -\frac{7}{25}.$$

The sum is $18/25$ and the product is $-7/25$, giving

$$x^2 - \frac{18}{25}x - \frac{7}{25} = 0, \qquad 25x^2 - 18x - 7 = 0.$$

Option A has the different second root $-7/15$; options B and D have positive products and hence cannot have the negative root found here. The equation in C also factors as $(x-1)(25x+7) = 0$, directly checking both roots.

Step 3: Conclusion The required equation is $25x^2 - 18x - 7 = 0$. Hence, the correct option is (C).

Quick Tip to Solve: Use the principal-value quadrants to fix square-root signs, then check a quadratic using both the sum and product of its roots.

12. Let A and B be two square matrices of order 2. If $det\,(A) = 2$, $det\,(B) = 3$ and

$$\det\left((\det\ 5(det\ A)B)A^2\right) = 2^a 3^b 5^c$$

for some a, b, c, $\in$ N, then a + b + c is equal to : [4 marks]

(A) 10

(B) 12

(C) 13

(D) 14

Correct Answer: (B) 12

Solution:

Concept:

- For a square matrix of order two, $\det(kC) = k^2 \det C$. Multiplying a matrix by a scalar multiplies each of its two rows by that scalar.
- Determinants are multiplicative: $\det(CD) = \det C \det D$. In particular, $\det(A^2) = (\det A)^2$. A determinant already evaluated is a scalar.

Step 1: Evaluate the inner determinant first The inner matrix is $5(\det A)B = 10B$, since $\det A = 2$. As B is of order two,

$$k = \det\big(5(\det A)B\big) = \det(10B) = 10^2 \det B = 300.$$

In prime factors, $k = 2^2 \cdot 3 \cdot 5^2$. It is important that the factor 10 is squared: it acts on both rows of B. The result k is now a number multiplying the matrix A^2, rather than a matrix or a remaining determinant operation.

Step 2: Collect the prime powers The requested determinant is $\det(kA^2)$. Apply the same scalar rule once again, and apply multiplicativity to A^2:

$$\det(kA^2) = k^2 \det(A^2) = 300^2 (\det A)^2.$$

Consequently

$$\det(kA^2) = (2^2 \cdot 3 \cdot 5^2)^2 \cdot 2^2 = 2^6 \cdot 3^2 \cdot 5^4.$$

By uniqueness of prime factorisation, $a = 6$, $b = 2$, and $c = 4$, so their sum is 12. Option A would miss the extra factor $(\det A)^2 = 4$; options C and D likewise have incorrect exponent totals. No commutativity assumption about A and B is required, since the inner determinant is a scalar.

Step 3: Conclusion $a + b + c = 6 + 2 + 4 = 12$. Hence, the correct option is (B).

Quick Tip to Solve: Evaluate nested determinants from inside out, and write the matrix order beside each scalar rule so that every scalar receives the correct exponent.

13. The value of $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ \ldots.\ +\ \frac{1}{1+2+3+\ \ldots..\ +11}$ **is equal to:** [4 marks]

(A) $\frac{20}{11}$

(B) $\frac{11}{6}$

(C) $\frac{241}{132}$

(D) $\frac{21}{11}$

Correct Answer: (B) $\frac{11}{6}$

Solution:

Concept:

- The sum of the first k positive integers is $k(k+1)/2$. Therefore the reciprocal of this triangular number is $2/[k(k+1)]$.
- The partial fraction identity $1/[k(k+1)] = 1/k - 1/(k+1)$ makes the finite series telescope. Its first and last surviving terms determine the total.

Step 1: Express all terms using one index The initial 1 is itself $1/(1)$, so it is the term with $k = 1$; it must not be added again after summation. The final denominator ends at 11, so the index runs through 11, inclusive:

$$S = \sum_{k=1}^{11} \frac{1}{1+2+\cdots+k} = \sum_{k=1}^{11} \frac{2}{k(k+1)}.$$

Each denominator is a triangular number, not simply its final integer k. For example, the last denominator is $11 \cdot 12/2 = 66$, agreeing with the printed final term.

Step 2: Cancel consecutive reciprocal terms Use partial fractions to write the finite sum as

$$S = 2\sum_{k=1}^{11} \left(\frac{1}{k} - \frac{1}{k+1}\right).$$

Expanding the first and last terms displays the cancellation:

$$S = 2\left(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots + \frac{1}{11} - \frac{1}{12}\right).$$

All interior reciprocal terms cancel pairwise, leaving

$$S = 2\left(1 - \frac{1}{12}\right) = \frac{11}{6}.$$

Option A corresponds to stopping at $k = 10$; C is smaller than the exact result, and D is larger. In particular, a quick bound $1 < S < 2$ is consistent with the answer, but the exact endpoint cancellation is what selects B uniquely.

Step 3: Conclusion The value of the series is $11/6$. Hence, the correct option is (B).

Quick Tip to Solve: Treat the leading 1 as the first triangular-number reciprocal, and remember that a last index of 11 leaves the uncancelled reciprocal $1/12$.

14. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{r}{2r^2-7rn+6n^2}$ **is equal to :** [4 marks]

(A) $\log_e\left(\frac{\sqrt{3}}{2}\right)$

(B) $\log_e\left(\frac{3\sqrt{3}}{4}\right)$

(C) $\log_e\left(\frac{27}{4}\right)$

(D) $\log_e\left(\frac{4}{3}\right)$

Correct Answer: (B) $\log_e\left(\frac{3\sqrt{3}}{4}\right)$

Solution:

Concept:

- A sum of the form $(1/n)\sum_{r=1}^{n} f(r/n)$ tends to $\int_0^1 f(x)\,dx$ when f is continuous on the interval. Here the mesh size is $1/n$.
- Factoring a quadratic denominator allows partial fractions and logarithmic integration. Absolute values in logarithms are retained while integrating factors that are negative on the interval.

Step 1: Convert the limit into an integral Divide numerator and denominator of each summand by n^2:

$$\frac{r}{2r^2-7rn+6n^2}=\frac{1}{n}\frac{r/n}{2(r/n)^2-7(r/n)+6}.$$

Thus the limit is

$$I=\int_0^1\frac{x}{2x^2-7x+6}\,dx.$$

The denominator factors as $(2x-3)(x-2)$. Its zeros are $3/2$ and 2, both outside $[0,1]$, so the integrand is continuous and the Riemann-sum conversion is justified. Both factors are negative within this interval, making the integrand nonnegative.

Step 2: Integrate by partial fractions and evaluate Equating coefficients yields the decomposition

$$\frac{x}{(2x-3)(x-2)}=-\frac{3}{2x-3}+\frac{2}{x-2}.$$

Indeed, its combined numerator is $-3(x-2)+2(2x-3)=x$. Hence

$$I=\left[-\frac{3}{2}\ln|2x-3|+2\ln|x-2|\right]_0^1.$$

The upper endpoint contributes zero, and the lower endpoint contributes $-\frac{3}{2}\ln 3+2\ln 2$. Therefore

$$I=\frac{3}{2}\ln 3-2\ln 2=\ln\left(\frac{3\sqrt{3}}{4}\right).$$

Option A is negative and contradicts positivity. Options C and D have different logarithmic factors; neither matches the exact endpoint evaluation. Retaining the factor 1/2 when integrating $1/(2x-3)$ is essential.

Step 3: Conclusion The limit equals $\ln(3\sqrt{3}/4)$. Hence, the correct option is (B).

Quick Tip to Solve: Extract exactly one factor 1/n to recognise the Riemann sum, then check that the denominator has no zero on the integration interval.

15. Let $\frac{dy}{dx} = \frac{ax-by+a}{bx+cy+a}$, $a, b, c \in R$, represents a circle with center (α, β). Then, $\alpha + 2\beta$ is equal to :

[4 marks]

(A) -1

(B) 0

(C) 1

(D) 2

Correct Answer: (C) 1

Solution:

Concept:

- A circle with centre (α, β) satisfies $(x-\alpha)^2+(y-\beta)^2 = R^2$, so its tangent slope is $-(x-\alpha)/(y-\beta)$.
- The radius is perpendicular to the tangent. Equating the two slope expressions produces a quadratic identity on the circle; its squared-coordinate coefficients must agree.

Step 1: Compare the differential equation with a circular tangent Implicit differentiation of the circle gives $(x-\alpha)+(y-\beta)y' = 0$. Substitute the given differential equation and clear its denominator on arcs where the slope is finite:

$$(x-\alpha)(bx+cy+a)+(y-\beta)(ax-by+a) = 0.$$

This relation extends to the whole circle by continuity. Its homogeneous quadratic terms are $bx^2 + (a+c)xy - by^2$. A nonzero quadratic vanishing on a circle must be a constant multiple of its circle equation, whose x^2 and y^2 coefficients are equal and whose xy coefficient is zero. Hence $b = -b$, giving $b = 0$, and $c = -a$.

Step 2: Integrate and identify the centre The differential equation must be defined, so $a \neq 0$. It now reduces to

$$\frac{dy}{dx} = \frac{x+1}{1-y}.$$

Separate the variables and integrate:

$$(y-1)\,dy = -(x+1)\,dx, \qquad \frac{(y-1)^2}{2} + \frac{(x+1)^2}{2} = C.$$

Thus the circle has centre $(-1, 1)$. Therefore $\alpha + 2\beta = -1 + 2 = 1$. The numbers $-1, 0, 2$ in the other options do not equal this combination; in particular, -1 is only the first coordinate, not the requested expression.

Step 3: Conclusion The centre is $(-1, 1)$, and $\alpha + 2\beta = \boxed{1}$. Hence, the correct option is (C).

Quick Tip to Solve: For a circle, differentiate its centre-radius equation before comparing a given slope; the negative sign between the radial coordinates is essential.

16. Consider three circles:

$$C_1 : x^2 + y^2 = r^2$$

$$C_2 : (x-1)^2 + (y-1)^2 = r^2$$

$$C_3 : (x-2)^2 + (y-1)^2 = r^2$$

If a line L : y = mx + c be a common tangent to C_1, C_2 and C_3 such that C_1 and C_3 lie on one side of line L while C_2 lies on other side, then the value of $20(r^2 + c)$ is equal to : [4 marks]

(A) 23

(B) 15

(C) 12

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- The perpendicular distance from (x_0, y_0) to $mx - y + c = 0$ is $|mx_0 - y_0 + c|/\sqrt{m^2+1}$. Tangency makes this distance the radius.
- Equal circles on the same side of a common tangent have equal signed distances; circles on opposite sides have opposite signed distances.

Step 1: Use the specified sides of the tangent Let $F(x, y) = mx - y + c$. The centres of the three circles are $O = (0, 0)$, $A = (1, 1)$, and $B = (2, 1)$. Since C_1 and C_3 lie on the same side and have equal radii, their signed distances agree:

$$F(O) = F(B), \qquad c = 2m - 1 + c.$$

Hence $m = 1/2$. Since C_2 lies on the opposite side, $F(A) = -F(O)$, which gives

$$m - 1 + c = -c, \qquad 2c = 1 - m = \frac{1}{2}.$$

Therefore $c = 1/4$. The signs are fixed by the side information; merely squaring all three distances would hide that information.

Step 2: Find the radius and evaluate the expression The tangent touches the circle centred at the origin, so

$$r = \frac{|c|}{\sqrt{1+m^2}} = \frac{1/4}{\sqrt{5/4}} = \frac{1}{2\sqrt{5}}, \qquad r^2 = \frac{1}{20}.$$

Check the signed values directly: $F(O) = 1/4$, $F(A) = -1/4$, and $F(B) = 1/4$, exactly as required. Now

$$20(r^2 + c) = 20\left(\frac{1}{20} + \frac{1}{4}\right) = 1 + 5 = 6.$$

Thus 23, 15 and 12 fail the common-radius and side conditions together; none is the evaluated value for the admissible tangent.

Step 3: Conclusion We obtain $r^2 = 1/20$, $c = 1/4$, and $20(r^2 + c) = \boxed{6}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep signed distances until the same-side and opposite-side conditions have determined the tangent; take an absolute value only when computing the radius.

17. The distance of the point (3, 2, −1) from the plane $3x-y+4z+1=0$ along the line $\frac{2-x}{2} = \frac{y-3}{2} = \frac{z+1}{1}$ is equal to : [4 marks]

(A) 9

(B) 6

(C) 3

(D) 2

Correct Answer: (C) 3

Solution:

Concept:

- A distance measured along a prescribed line direction uses a line through the given point parallel to that direction; it is generally different from perpendicular distance.
- For $P + t\mathbf{d}$, intersection with $\mathbf{n} \cdot \mathbf{x} + k = 0$ gives $t = -(\mathbf{n} \cdot P + k)/(\mathbf{n} \cdot \mathbf{d})$. The travelled distance is $|t|\,|\mathbf{d}|$.

Step 1: Extract the direction and construct the required line The displayed symmetric line can be parametrised as $x = 2 - 2s$, $y = 3 + 2s$, $z = -1 + s$. Its direction vector is therefore $\mathbf{d} = (-2, 2, 1)$, with length $\sqrt{4+4+1} = 3$. The phrase along the line specifies this direction. Through the given point $P = (3, 2, -1)$, use

$$(x, y, z) = (3, 2, -1) + t(-2, 2, 1).$$

The supplied reference line need not pass through P; translating it parallel to itself is the required directional construction.

Step 2: Intersect with the plane and compute the length Substitute $x = 3 - 2t$, $y = 2 + 2t$, $z = -1 + t$ into the plane:

$$3(3 - 2t) - (2 + 2t) + 4(-1 + t) + 1 = 0.$$

Collecting constants and coefficients yields $4 - 4t = 0$, so $t = 1$. The intersection is $Q = (1, 4, 0)$, and $3(1) - 4 + 4(0) + 1 = 0$ confirms it lies on the plane. Thus

$$PQ = \sqrt{(1-3)^2 + (4-2)^2 + (0+1)^2} = \sqrt{9} = 3.$$

The options 9, 6 and 2 do not equal the length: 9 is the squared length, while using the wrong sign for the first direction ratio also changes the intersection incorrectly.

Step 3: Conclusion The distance along the specified direction is $\boxed{3}$ coordinate units. Hence, the

correct option is (C).

Quick Tip to Solve: Rewrite $(2-x)/2$ as $(x-2)/(-2)$ before reading direction ratios; then multiply the line parameter by the direction vector length.

18. Let a vector $\vec{c}$ be coplanar with the vectors $\vec{a} = -\hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{j}-\hat{k}$. If the vector $\vec{c}$ also satisfies the conditions $\vec{c}\,.\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)\right] = -42$ and $\left(\vec{c}\times\left(\vec{a}-\vec{b}\right)\right).\hat{k} = 3$, then the value of $|\vec{c}|^2$ is equal to : [4 marks]

(A) 24

(B) 29

(C) 35

(D) 42

Correct Answer: (C) 35

Solution:

Concept:

- Two nonparallel vectors span their plane. Coplanarity allows $\mathbf{c} = u\mathbf{a} + v\mathbf{b}$, reducing the unknown vector to two scalar coefficients.
- For vectors $\mathbf{p}, \mathbf{q}$, the $\hat{k}$ component of $\mathbf{p}\times\mathbf{q}$ is $p_x q_y - p_y q_x$. Dot products and cross products are linear in each argument.

Step 1: Translate the cross-product condition into coefficients Here $\mathbf{a} = (-1,1,1)$, $\mathbf{b} = (2,1,-1)$, and $\mathbf{a}\times\mathbf{b} = (-2,1,-3) \neq \mathbf{0}$. Write

$$\mathbf{c} = u\mathbf{a} + v\mathbf{b} = (-u+2v, u+v, u-v).$$

Also $\mathbf{a}-\mathbf{b} = (-3,0,2)$. The last component of $\mathbf{c}\times(\mathbf{a}-\mathbf{b})$ is $c_x(0) - c_y(-3) = 3c_y$. The second given condition therefore gives $3(u+v) = 3$, or $u+v = 1$.

Step 2: Use the other scalar condition and find the norm Compute the remaining vector in the stated order:

$$(\mathbf{a}+\mathbf{b})\times(\mathbf{a}\times\mathbf{b}) = (1,2,0)\times(-2,1,-3) = (-6,3,5).$$

Its dot products with $\mathbf{a}$ and $\mathbf{b}$ are 14 and -14, respectively. Hence $14u - 14v = -42$, so $u - v = -3$. Solving together with $u+v=1$ gives $u=-1$, $v=2$. Consequently

$$\mathbf{c} = -\mathbf{a}+2\mathbf{b} = (5,1,-3), \qquad |\mathbf{c}|^2 = 25+1+9 = 35.$$

A direct check gives $(5,1,-3)\cdot(-6,3,5) = -42$ and the required cross-product component 3. The other options 24, 29 and 42 fail this uniquely determined squared norm.

Step 3: Conclusion The vector is $5\hat{i}+\hat{j}-3\hat{k}$, with squared magnitude $\boxed{35}$. Hence, the correct option is (C).

Quick Tip to Solve: Use coplanarity first to write a two-coefficient linear combination; checking the order of each cross product prevents a sign reversal.

19. If a random variable X follows the Binomial distribution B(5, p) such that P(X = 0) = P(X = 1), then $\frac{P(X=2)}{P(X=3)}$ is equal to : [4 marks]

(A) 1

(B) 10

(C) 25

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- For $X \sim B(5,p)$, $P(X=k) = \binom{5}{k}p^k q^{5-k}$, where $q = 1-p$ is the failure probability.
- A ratio of binomial probabilities is simplified by cancelling common factors before substituting the parameter. The denominator must be nonzero for the stated ratio to exist.

Step 1: Determine the success probability The given equality reads

$$q^5 = 5pq^4.$$

Since the question asks for $P(X=2)/P(X=3)$, its denominator is nonzero. In particular, neither $p=0$ nor $q=0$ can be used. We may therefore cancel q^4, obtaining $q = 5p$. Combining this with $p+q=1$ yields

$$6p = 1, \qquad p = \frac{1}{6}, \qquad q = \frac{5}{6}.$$

The formal endpoint $p=1$ makes both sides of the original equality zero, but also makes the requested ratio $0/0$, so it is inadmissible here.

Step 2: Cancel the binomial probabilities carefully Use the distribution formula at the two requested values:

$$\frac{P(X=2)}{P(X=3)} = \frac{\binom{5}{2}p^2q^3}{\binom{5}{3}p^3q^2} = \frac{10}{10}\frac{q}{p}.$$

Since the binomial coefficients are equal, only one power each of q and p survives. Thus the ratio is $(5/6)/(1/6) = 5$. This also agrees with the adjacent-probability formula $P(X=k)/P(X=k+1) = (k+1)q/((5-k)p)$ at $k=2$. Option 1 would incorrectly assume equal success and failure probabilities; 10 retains a coefficient that cancels, and 25 incorrectly squares q/p.

Step 3: Conclusion The probability ratio is $\boxed{5}$, a dimensionless quantity. Hence, the correct option is (D).

Quick Tip to Solve: In a binomial probability ratio, cancel factorial coefficients and common powers first; this often reduces the computation to a single factor q/p.

20. The conditional statement

$((p \wedge q) \rightarrow ((\sim p) \vee r)) \vee (((\sim p) \vee r) \rightarrow (p \wedge q))$ **is :** [4 marks]

(A) a tautology

(B) a contadiction

(C) equivalent to $p \wedge q$

(D) equivalent to $(\sim p) \vee r$

Correct Answer: (A) a tautology

Solution:

Concept:

- An implication $A \rightarrow B$ is equivalent to $(\sim A) \vee B$; it is false only when its antecedent is true and its consequent false.
- A tautology is true for every assignment of the underlying propositions. Complementary statements satisfy $A \vee (\sim A) = \mathrm{T}$.

Step 1: Replace the compound propositions by simpler symbols Put $A = p \wedge q$ and $B = (\sim p) \vee r$. The complete statement is then

$$(A \rightarrow B) \vee (B \rightarrow A).$$

These are two implications joined by OR, rather than by AND. They therefore do not express a biconditional. Applying the implication identity separately gives

$$((\sim A) \vee B) \vee ((\sim B) \vee A).$$

No truth values of p, q, r have been assumed, so any identity obtained at this stage applies to all of them.

Step 2: Simplify the disjunction and exclude the alternatives Associativity and commutativity of OR permit regrouping as

$$((\sim A) \vee A) \vee (B \vee (\sim B)) = \mathrm{T} \vee \mathrm{T} = \mathrm{T}.$$

Equivalently, if the first implication were false then A would be true and B false, making the reverse implication true. Hence the two implications cannot both be false. This rules out a contradiction. It is not equivalent to $p \wedge q$: choose p false and q true, so that conjunction is false while our expression stays true. It is also not equivalent to $(\sim p) \vee r$: choose p true and r false, making that expression false while the full statement remains true. Thus each remaining option is excluded by a direct truth assignment.

Step 3: Conclusion The statement is true for every truth assignment, so it is $\boxed{\text{a tautology}}$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever two reverse implications are joined by OR, replace each implication by a disjunction; complementary terms immediately show a tautology.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The number of 6-digit numbers made by using the digits 1, 2, 3, 4, 5, 6, 7, without repetition and which are multiple of 15 is _____. [4 marks]

Solution:

Concept:

- A number is divisible by 15 precisely when it is divisible by both 3 and 5. Divisibility by 3 depends on the digit sum, and divisibility by 5 depends on the final digit.
- Choosing six distinct digits from seven is equivalent to choosing the single omitted digit. Once the final digit is fixed, five distinct remaining digits can be arranged in $5!$ ways.

Step 1: Fix the units digit and identify possible omissions The available digits are $1, 2, 3, 4, 5, 6, 7$, with no zero. A multiple of 5 must therefore end in 5. Their total sum is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28.$$

If the omitted digit is d, the six selected digits have sum $28 - d$. To make this divisible by 3 we need

$$28 - d \equiv 0 \pmod 3, \qquad d \equiv 1 \pmod 3.$$

The available possibilities are $d = 1, 4, 7$. Each is allowed because none is the compulsory final digit 5. The resulting sums are respectively 27, 24 and 21, all divisible by 3.

Step 2: Arrange each admissible digit set For each omitted digit, the number ends in 5, and its other five positions contain the five remaining distinct digits in any order. All such orders give six-digit numbers because none starts with zero. Therefore each admissible omission contributes

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

numbers. The three cases are disjoint: different omitted digits give different sets of digits, so no number is counted twice. Conversely, every number satisfying the conditions must end in 5 and must omit one of 1, 4 or 7, so the counting includes every valid number. Thus

$$N = 3 \times 120 = 360.$$

We do not multiply by six choices for the last digit, since divisibility by 5 has already fixed that digit uniquely.

Step 3: Conclusion The required number of six-digit multiples of 15 is $\boxed{360}$. This is a count of numbers and has no physical unit.

Quick Tip to Solve: When almost all available digits are used, count the omitted digit instead of choosing the whole subset; then fix the last digit before arranging the rest.

22. Suppose $\lim_{x\to 0}\frac{F(x)}{x^3}$ exists and is equal to L, where

$$F(x)=\begin{vmatrix} a+\sin\frac{x}{2} & -b\cos x & 0 \\ -b\cos x & 0 & a+\sin\frac{x}{2} \\ 0 & a+\sin\frac{x}{2} & -b\cos x \end{vmatrix}.$$

Then, -112 L is equal to _____. [4 marks]

Solution:

Concept:

- For a finite limit of $F(x)/x^3$, the constant, linear and quadratic terms in the expansion of the numerator must vanish.
- Expand the determinant first; the elementary limit $\lim_{u\to 0}\sin u/u = 1$ then evaluates the surviving cubic term.

Step 1: Simplify the determinant Write $A = a + \sin(x/2)$ and $B = b\cos x$. Expansion along the first row gives

$$F(x) = A(0 - A^2) + B(B^2) = B^3 - A^3.$$

Thus the apparently complicated determinant reduces to $b^3\cos^3 x - (a + \sin(x/2))^3$. This exact simplification also fixes the sign of the eventual answer.

Step 2: Use the existence of the limit Continuity and the finite limit require $F(0) = b^3 - a^3 = 0$. Since the parameters are real, $a = b$. The coefficient of x in $b^3\cos^3 x$ is zero, whereas that in $-(a + \sin(x/2))^3$ is $-3a^2/2$. A nonzero linear coefficient would make division by x^3 divergent, so $a = 0$, and consequently $b = 0$. The quadratic term then vanishes automatically.

Step 3: Evaluate the remaining limit With these forced parameter values, $F(x) = -\sin^3(x/2)$. Therefore

$$L = -\left(\lim_{x\to 0}\frac{\sin(x/2)}{x}\right)^3 = -\left(\frac{1}{2}\right)^3 = -\frac{1}{8}.$$

Multiplication by the requested negative factor gives $-112L = 112/8$.

Step 4: Conclusion The required dimensionless value is $\boxed{14}$.

Quick Tip to Solve: For a determinant divided by a small power of x, simplify the determinant before expanding; eliminate the lower-order terms using existence of the limit.

23. If for some $\alpha > 0$, the area of the region $\{(x, y) : |x + \alpha| \le y \le 2 - |x|\}$ is equal to $\frac{3}{2}$, then the area of the region $\{(x, y) : 0 \le y \le x + 2\alpha,\ |x| \le 1\}$ is equal to _____. [4 marks]

Solution:

Concept:

- The area between an upper curve $y = u(x)$ and a lower curve $y = l(x)$ is $\int(u - l)\,dx$ over the interval where $u \ge l$.
- Absolute-value expressions change form at their zeros. Here those points are $-\alpha$ and 0, since $\alpha > 0$.

Step 1: Locate the first region The first region exists where $|x+\alpha|+|x| \le 2$. Its minimum left side is α, so positive area requires $0 < \alpha < 2$. The extreme intersections are $x = -1 - \alpha/2$ and $x = 1 - \alpha/2$. Between $-\alpha$ and 0, the vertical height is constant, equal to $2 - \alpha$.

Step 2: Calculate its area and find the parameter The two outside pieces are triangles, each having horizontal base $1 - \alpha/2$ and height $2 - \alpha$; the middle piece is a rectangle of width α. Hence

$$A_1 = 2 \cdot \frac{1}{2}(1 - \alpha/2)(2 - \alpha) + \alpha(2 - \alpha) = 2 - \frac{\alpha^2}{2}.$$

Setting this equal to $3/2$ gives $\alpha^2 = 1$. The condition $\alpha > 0$ selects $\alpha = 1$.

Step 3: Integrate over the second region Now the upper line is $y = x + 2$, and $|x| \le 1$ means $-1 \le x \le 1$. On this whole interval $x + 2 \ge 1$, so the lower bound $y = 0$ is compatible everywhere. Thus

$$A_2 = \int_{-1}^{1} (x+2)\,dx = \left[\frac{x^2}{2} + 2x\right]_{-1}^{1} = 4.$$

The odd term x contributes zero over the symmetric interval.

Step 4: Conclusion The required area is $\boxed{4}$ square units.

Quick Tip to Solve: Split absolute values at their zeros; in the second area use symmetry to cancel the odd part of the integrand.

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24. Let $f(t) = \int_0^t e^{x^3}\left(\frac{x^8}{(x^6+2x^3+2)^2}\right)dx$. If $f(1) + f'(1) = \alpha e - \frac{1}{6}$, then the value of 150α is equal to _____. [4 marks]

Solution:

Concept:

- The substitution $u = x^3$ gives $x^8 dx = u^2 du/3$, converting the integrand into an exponential times a rational function.
- The fundamental theorem of calculus gives $f\prime(t)$ by replacing x with t in the continuous integrand.

Step 1: Identify an exact derivative Put $u = x^3$. The denominator becomes $(u^2 + 2u + 2)^2$. For $Q(u) = u^2 + 2u + 2$, the quotient rule gives

$$\frac{d}{du}\left(\frac{e^u}{Q(u)}\right) = \frac{e^u(Q - Q\prime)}{Q^2} = \frac{e^u u^2}{(u^2+2u+2)^2}.$$

The cancellation in $Q - Q\prime = u^2$ is the reason this substitution is useful.

Step 2: Evaluate the integral at one The bounds $x = 0, 1$ become $u = 0, 1$, respectively. Consequently

$$f(1) = \frac{1}{3}\int_0^1 \frac{e^u u^2}{(u^2+2u+2)^2}\,du = \frac{1}{3}\left[\frac{e^u}{u^2+2u+2}\right]_0^1 = \frac{e}{15} - \frac{1}{6}.$$

The constant term comes from the lower limit and must not be omitted.

Step 3: Add the derivative value and compare coefficients By the fundamental theorem,

$$f\prime(1) = \frac{e \cdot 1^8}{(1^6 + 2 \cdot 1^3 + 2)^2} = \frac{e}{25}.$$

Thus

$$f(1) + f\prime(1) = \left(\frac{1}{15} + \frac{1}{25}\right) e - \frac{1}{6} = \frac{8}{75}e - \frac{1}{6}.$$

Comparing this with $\alpha e - 1/6$ gives $\alpha = 8/75$, and hence $150\alpha = 16$.

Step 4: Conclusion The required dimensionless value is $\boxed{16}$.

Quick Tip to Solve: When an exponential is divided by a squared polynomial, differentiate the exponential divided by the unsquared polynomial and look for cancellation.

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25. A hostel has 100 students. On a certain day (consider it day zero) it was found that two students are infected with some virus. Assume that the rate at which the virus spreads is directly proportional to the product of the number of infected students and the number of non-infected students. If the number of infected students on 4th day is 30, then number of infected students on 8th day will be _____. [4 marks]

Solution:

Concept:

- If $I(t)$ is the number infected, the number not infected is $100 - I(t)$. The stated spread law is $dI/dt = kI(100 - I)$, with $k > 0$.
- Separating variables makes the odds ratio $R = I/(100 - I)$ exponential in time: $R(t) = R(0)e^{100kt}$.

Step 1: Solve the spread equation Separate variables and use partial fractions:

$$\frac{dI}{I(100 - I)} = k\,dt, \qquad \frac{1}{I(100 - I)} = \frac{1}{100}\left(\frac{1}{I} + \frac{1}{100 - I}\right).$$

Integration yields $\log(I/(100 - I)) = 100kt + C$. Therefore the infected-to-uninfected ratio grows by the same multiplicative factor over equal time intervals.

Step 2: Find the factor for four days Initially $I(0) = 2$, so $R(0) = 2/98 = 1/49$. On the fourth day $I(4) = 30$, giving $R(4) = 30/70 = 3/7$. Hence

$$e^{400k} = \frac{R(4)}{R(0)} = \frac{3/7}{1/49} = 21.$$

It is the odds ratio, rather than the number of students itself, that multiplies by 21.

Step 3: Advance by another four days The time from day four to day eight is the same as from day zero to day four. Thus

$$R(8) = 21R(4) = 21 \cdot \frac{3}{7} = 9.$$

Solving $I(8)/(100 - I(8)) = 9$ gives $I(8) = 900 - 9I(8)$, and hence $10I(8) = 900$. This value remains

below the total population, as required by the model.

Step 4: Conclusion The number infected on the eighth day is $\boxed{90}$ students.

Quick Tip to Solve: In a logistic-growth problem, propagate the ratio infected/uninfected over equal time intervals instead of calculating the rate constant explicitly.

26. Let PQ be a focal chord of length 6.25 units of the parabola $y^2 = 4x$. If O is the vertex of the parabola, then 10 times the area (in sq. units) of $\triangle POQ$ is equal to _____. [4 marks]

Solution:

Concept:

- On $y^2 = 4ax$, the parametric point is $(at^2, 2at)$. Here $a = 1$, so it is $(t^2, 2t)$.
- The endpoints of a focal chord have parameters t, s satisfying $ts = -1$. The area with the origin follows from half the absolute coordinate determinant.

Step 1: Obtain the focal-chord condition Take $P = (t^2, 2t)$ and $Q = (s^2, 2s)$. Their chord has equation $2x - (t + s)y + 2ts = 0$. Since the focus is $(1, 0)$, substituting it gives $2 + 2ts = 0$, or $ts = -1$. In particular the two parameters have opposite signs and neither is zero.

Step 2: Express the chord length using the parameters The distance formula gives

$$PQ^2 = (t^2 - s^2)^2 + 4(t - s)^2 = (t - s)^2\big((t + s)^2 + 4\big).$$

Because $ts = -1$, the second factor equals $(t - s)^2$. Hence $PQ = (t - s)^2$. The supplied length $6.25 = 25/4$ therefore gives $|t - s| = 5/2$. This relation uses the chord length itself, not its square.

Step 3: Calculate the triangle area As the vertex is $O = (0, 0)$, the determinant formula yields

$$\text{Area}(\triangle POQ) = \frac{1}{2}\left|t^2(2s) - s^2(2t)\right| = |ts(t - s)| = |t - s| = \frac{5}{2}.$$

Multiplying the area by the requested factor of ten gives $10 \cdot 5/2 = 25$. The absolute value ensures the area does not depend on which endpoint is named first.

Step 4: Conclusion Ten times the area is $\boxed{25}$ square units.

Quick Tip to Solve: For a focal chord of $y^2 = 4x$, use $ts = -1$; the chord length is $(t - s)^2$ and the triangle area with the vertex is $|t - s|$.

27. Consider a triangle ABC whose vertices are A(0, α, α), B(α, 0, α) and C(α, α, 0), $\alpha > 0$. Let D be a point moving on the line x + z − 3 = 0 = y and G be the centroid of $\triangle$ABC. If the minimum length of GD is $\sqrt{\frac{57}{2}}$, then α is equal to _____. [4 marks]

Solution:

Concept:

- The centroid of a triangle is obtained by averaging the three vertices coordinate by coordinate.
- The shortest distance to a line is found by minimizing the squared distance to its general point; a sum of squares makes the minimizing parameter transparent.

Step 1: Write the centroid and a general point on the line The centroid is

$$G = \left(\frac{2\alpha}{3}, \frac{2\alpha}{3}, \frac{2\alpha}{3}\right).$$

Write $c = 2\alpha/3$. The line is the intersection of $y = 0$ with $x + z = 3$, so every point on it can be represented as $D = (t, 0, 3 - t)$, with real parameter t.

Step 2: Minimize the squared distance The squared distance is

$$GD^2 = (t - c)^2 + c^2 + (3 - t - c)^2.$$

Completing the square in t gives

$$GD^2 = 2\left(t - \frac{3}{2}\right)^2 + 2\left(\frac{3}{2} - c\right)^2 + c^2.$$

The first term is nonnegative and becomes zero at $t = 3/2$. Thus the minimum is attained at $D = (3/2, 0, 3/2)$.

Step 3: Use the given minimum to find the parameter Substitute the specified minimum squared distance:

$$2\left(\frac{3}{2} - c\right)^2 + c^2 = \frac{57}{2}.$$

Expansion gives $3c^2 - 6c + 9/2 = 57/2$, or $c^2 - 2c - 8 = 0$. Therefore $(c - 4)(c + 2) = 0$. Since $\alpha > 0$, also $c > 0$, so $c = 4$. Finally $2\alpha/3 = 4$ implies $\alpha = 6$; the root $c = -2$ is inadmissible.

Step 4: Conclusion The required value is $\boxed{\alpha = 6}$, a dimensionless coordinate parameter.

Quick Tip to Solve: For a line such as $x + z = 3, y = 0$ and a point with equal x- and z-coordinates, symmetry places the closest point at $x = z = 3/2$.

28. The probability distribution of X is :

X	0	1	2	3
P(X)	$\frac{1-d}{4}$	$\frac{1+2d}{4}$	$\frac{1-4d}{4}$	$\frac{1+3d}{4}$

For the minimum possible value of d, sixty times the mean of X is equal to _____. [4 marks]

Solution:

Concept:

- Every probability must be nonnegative and the probabilities must sum to one. These constraints determine the allowed interval for d.
- The mean of a discrete random variable is $E[X] = \sum_x xP(X = x)$; include each possible value with its own probability.

Step 1: Find all constraints on the parameter The four numerators must be nonnegative:

$$1 - d \geq 0, \quad 1 + 2d \geq 0, \quad 1 - 4d \geq 0, \quad 1 + 3d \geq 0.$$

These give $d \leq 1$, $d \geq -1/2$, $d \leq 1/4$, and $d \geq -1/3$, respectively. Taking their intersection gives $-1/3 \leq d \leq 1/4$. The minimum possible value is therefore $d = -1/3$, rather than $-1/2$.

Step 2: Check normalization and compute the mean The sum of the probabilities is

$$(1 - d + 1 + 2d + 1 - 4d + 1 + 3d)/4 = 1$$

for every d. Thus there is no additional constraint from normalization. The expectation is

$$E[X] = \frac{0(1-d) + 1(1+2d) + 2(1-4d) + 3(1+3d)}{4} = \frac{6+3d}{4}.$$

The outcome zero contributes zero to this sum but its probability still had to satisfy the nonnegativity constraint.

Step 3: Substitute the minimum and apply the requested factor At $d = -1/3$, the probabilities in order are $1/3, 1/12, 7/12, 0$. They are all nonnegative and sum to one, confirming that this endpoint is allowed. The mean is

$$E[X] = \frac{6-1}{4} = \frac{5}{4}.$$

It follows that $60E[X] = 60 \cdot 5/4 = 75$. The mean lies between the smallest and largest possible outcomes, as expected.

Step 4: Conclusion Sixty times the mean is the dimensionless value $\boxed{75}$.

Quick Tip to Solve: Intersect every nonnegativity constraint before choosing the smallest parameter; checking only one probability can give an invalid endpoint.

29. Let for $f(x) = a_0x^2 + a_1x + a_2$, $f'(0) = 1$ and $f'(1) = 0$. If a_0, a_1, a_2 are in an arithmatico-geometric progression, whose corresponding A.P. has common difference 1 and corresponding G.P. has common ratio 2, then f(4) is equal to _____. [4 marks]

Solution:

Concept:

- An arithmetico-geometric progression is formed by multiplying corresponding terms of an arithmetic progression and a geometric progression. With first factors A and B, its first three terms are AB, 2B(A+1), and 4B(A+2).
- The derivative of a quadratic polynomial is linear: $f\prime(x) = 2a_0x + a_1$. The two derivative conditions therefore determine the quadratic and linear coefficients before the progression is used.

Step 1: Use the derivative conditions Differentiating and substituting zero gives $a_1 = f\prime(0) = 1$. The second condition gives $2a_0 + a_1 = 0$, hence $a_0 = -1/2$. Thus the first two progression terms, in their stated order, are $-1/2$ and 1; their order must be retained when determining the third term.

Step 2: Find the third progression term Write the corresponding A.P. as $A, A + 1, A + 2$ and the G.P. as $B, 2B, 4B$. Then

$$AB = -\frac{1}{2}, \qquad 2B(A + 1) = 1.$$

Subtracting twice the first equation from the second yields $2B = 2$, so $B = 1$ and $A = -1/2$. The third product is consequently

$$a_2 = 4B(A + 2) = 4\left(-\frac{1}{2} + 2\right) = 6.$$

This directly checks the required common difference and common ratio rather than treating the products themselves as an A.P.

Step 3: Evaluate the requested polynomial The polynomial is now $f(x) = -x^2/2 + x + 6$. Substitute the requested argument only after finding all coefficients:

$$f(4) = -\frac{16}{2} + 4 + 6 = -8 + 10 = 2.$$

As a check, its derivative is $1 - x$, which equals one at zero and zero at one, exactly as required.

Step 4: Conclusion The required numerical value is $\boxed{f(4) = 2}$. This is a dimensionless polynomial value.

Quick Tip to Solve: For an arithmetico-geometric progression, divide the successive terms by the known geometric factors first; the resulting terms must be in an A.P.

30. Let $S_1 = \{x \in [0, 12\pi] : \sin^5 x + \cos^5 x = 1\}$
and $S_2 = \{x \in [0, 8\pi] : \sin^7 x + \cos^7 x = 1\}$
Then $n(S_1) - n(S_2)$ **is equal to _____.** [4 marks]

Solution:

Concept:

- For any real $t \in [-1, 1]$ and odd integer $p > 2$, $t^p \leq t^2$, with equality only at $t = 0$ or $t = 1$. For negative t the left side is negative, while the square is positive.
- The identity $\sin^2 x + \cos^2 x = 1$ bounds the sum of the higher odd powers. Equality in the sum requires equality for both components individually.

Step 1: Determine all equality cases Apply the preceding inequality to $t = \sin x$ and $t = \cos x$. For either exponent five or seven,

$$\sin^p x + \cos^p x \leq \sin^2 x + \cos^2 x = 1.$$

Each difference on the right is nonnegative. Therefore a sum equal to one is possible only if both differences vanish. Thus each of sine and cosine is zero or one; because their squares add to one, the only pairs are $(0, 1)$ and $(1, 0)$. Negative unit values do not qualify for an odd power.

Step 2: Count the first set The complete solution families for either equation are

$$x = 2k\pi \quad \text{or} \quad x = \frac{\pi}{2} + 2k\pi, \qquad k \in \mathbb{Z}.$$

On the closed interval $[0, 12\pi]$, the first family has $k = 0, 1, \ldots, 6$, giving seven points. The second has $k = 0, 1, \ldots, 5$, giving six points. The families are disjoint, so $n(S_1) = 7 + 6 = 13$.

Step 3: Count the second set and subtract On $[0, 8\pi]$, the first family has $k = 0, 1, \ldots, 4$, giving five points. The second has $k = 0, 1, 2, 3$, giving four points. Thus

$$n(S_2) = 5 + 4 = 9, \qquad n(S_1) - n(S_2) = 13 - 9 = 4.$$

The terminal multiples of 2π are included because both intervals are closed; omitting them would miscount each individual set.

Step 4: Conclusion The required difference is $\boxed{4}$, a dimensionless count.

Quick Tip to Solve: Bound each odd power by the corresponding square; equality conditions reduce the trigonometric equation to two simple periodic families.

31. At t = 0, truck, starting from rest, moves in the positive x-direction at uniform acceleration of 5 ms^{-2}. At t = 20 s, a ball is released from the top of the truck. The ball strikes the ground in 1 s after the release. The velocity of the ball, when it strikes the ground, will be :

(Given g = 10 ms^{-2}) [4 marks]

(A) $100\hat{i} - 10\hat{j}$

(B) $10\hat{i} - 100\hat{j}$

(C) $100\hat{i}$

(D) $-10\hat{j}$

Correct Answer: (A) $100\hat{i} - 10\hat{j}$

Solution:

Concept:

- At release, a freely dropped object retains the horizontal velocity of its carrier. With no horizontal force after release, that component remains constant throughout the flight.
- The vertical and horizontal motions are independent. Taking upward as positive gives $v_y = u_y - g\tau$, where τ is time measured from release.

Step 1: Find the velocity at release The truck begins from rest and accelerates for twenty seconds before the ball is released. Its instantaneous velocity then is

$$u_x = 0 + at = 5 \times 20 = 100\,\mathrm{m\,s^{-1}}.$$

The ball has this same horizontal component at the instant it leaves the truck. Releasing it does not remove the forward motion it already possesses. Since it is released rather than projected vertically, its initial vertical component is zero.

Step 2: Evolve the two components separately After release the truck may continue accelerating, but it no longer supplies a force to the ball. Neglecting air resistance, horizontal acceleration of the ball is zero. During the one second before impact,

$$v_x = 100\,\mathrm{m\,s^{-1}}, \qquad v_y = 0 - 10(1) = -10\,\mathrm{m\,s^{-1}}.$$

The negative sign indicates downward motion, consistently with the choice of positive y upward.

Step 3: Compare the listed vectors The components combine as $\vec{v} = (100\hat{i} - 10\hat{j})\,\mathrm{m\,s^{-1}}$. Option B interchanges the component magnitudes and cannot follow from either time interval. Option C omits the downward velocity acquired under gravity, while option D omits the inherited horizontal velocity. The relevant fall time is one second, not the twenty seconds for which the ball was carried by the truck.

Step 4: Conclusion The impact velocity is $\boxed{\vec{v} = (100\hat{i} - 10\hat{j})\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the truck velocity at release for the horizontal component, and start a fresh clock at release for the vertical component.

32. If n main scale divisions coincide with (n + 1) vernier scale divisions. The least count of vernier callipers, when each centimetre on the main scale is divided into five equal parts, will be : [4 marks]

(A) $\frac{2}{n+1}$ mm

(B) $\frac{5}{n+1}$ mm

(C) $\frac{1}{2n}$ mm

(D) $\frac{1}{5n}$ mm

Correct Answer: (A) $\frac{2}{n+1}$ mm

Solution:

Concept:

- For a direct vernier, the least count is the difference between one main-scale division and one vernier-scale division: $\text{LC} = 1\,\text{MSD} - 1\,\text{VSD}$.
- When n main-scale divisions span the same length as n+1 vernier-scale divisions, a vernier division is smaller than a main-scale division by the factor implied by that coincidence.

Step 1: Convert the main-scale spacing One centimetre contains ten millimetres. Since each centimetre is divided into five equal parts, one main-scale division is

$$1\,\text{MSD} = \frac{10}{5}\,\text{mm} = 2\,\text{mm}.$$

It is essential to use the stated graduation rather than assume the common one-millimetre spacing of another instrument.

Step 2: Use the coincidence condition The same physical length is represented on the two scales, so

$$n(1\,\text{MSD}) = (n+1)(1\,\text{VSD}).$$

Therefore

$$1\,\text{VSD} = \frac{n}{n+1}(1\,\text{MSD}) = \frac{2n}{n+1}\,\text{mm}.$$

This is indeed smaller than two millimetres for positive n, as expected for the direct vernier described.

Step 3: Subtract the spacings and check the options The smallest measurable increment is consequently

$$\text{LC} = 2 - \frac{2n}{n+1} = \frac{2}{n+1}\,\text{mm}.$$

Option B uses an incorrect main-scale spacing of five millimetres. Options C and D have n alone in the denominator and do not satisfy the given n versus n+1 coincidence condition. The result also tends to zero for an increasingly fine vernier, providing a useful consistency check.

Step 4: Conclusion The least count is $\boxed{\frac{2}{n+1}\,\text{mm}}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert one main-scale division into the requested unit first, then multiply it by the fractional mismatch between the two scales.

33. The radii of two planets A and B are in the ratio 2 : 3. Their densities are 3ρ and 5ρ respectively. The ratio of their acceleration due to gravity is : [4 marks]

(A) 9 : 4

(B) 9 : 8

(C) 9 : 10

(D) 2 : 5

Correct Answer: (D) 2 : 5

Solution:

Concept:

- For a spherical planet of mass M and radius R, surface gravity is $g = GM/R^2$. Its mass is $M = (4\pi/3)\rho_p R^3$, where ρ_p denotes its mean density.
- Combining these relations gives $g = (4\pi G/3)\rho_p R$. Thus surface gravity is proportional to the product of density and radius, rather than inversely proportional to radius squared when density is specified.

Step 1: Express gravity using density Substitute the volume-times-density expression for the planetary mass into the gravitational formula:

$$g = \frac{G}{R^2}\left(\frac{4\pi R^3}{3}\rho_p\right) = \frac{4\pi G}{3}\rho_p R.$$

The cancellation of two powers of radius is crucial. The larger planet contains more material; its mass cannot be treated as fixed while comparing its radius with that of the other planet.

Step 2: Form the ratio for the given planets For A the density is 3ρ, while for B it is 5ρ. Also $R_A/R_B = 2/3$. The universal constants and common density factor cancel, giving

$$\frac{g_A}{g_B} = \frac{(3\rho)R_A}{(5\rho)R_B} = \frac{3}{5}\times\frac{2}{3} = \frac{2}{5}.$$

Both the smaller radius and the smaller density make the surface gravity of A lower in this comparison.

Step 3: Check the alternatives The ratio must therefore be less than one. Options A and B are greater than one, immediately contradicting the derived density-radius dependence. Option C is less than one but equals $9/10$, which would result from an incorrect combination of the geometric and density factors. Only option D reproduces the exact product $(3/5)(2/3)$, so the cancellation and ordering of the ratio are both verified.

Step 4: Conclusion The surface-gravity ratio is $\boxed{g_A : g_B = 2 : 5}$. Hence, the correct option is (D).

Quick Tip to Solve: When densities are given, replace planetary mass by density times volume before taking the gravity ratio; the dependence becomes density times radius.

34. Two projectiles P_1 and P_2 thrown with speed in the ratio $\sqrt{3} : \sqrt{2}$, attain the same height during their motion. If P_2 is thrown at an angle of 60° with the horizontal, the angle of projection of P_1 with horizontal will be : [4 marks]

(A) 15°

(B) 30°

(C) 45°

(D) 60°

Correct Answer: (C) 45°

Solution:

Concept:

- The maximum height of a projectile is $H = u^2 \sin^2\theta/(2g)$. Only the initial vertical component of velocity determines this height.
- Equal maximum heights under the same gravitational acceleration imply equal squared initial vertical components: $u_1^2 \sin^2\theta_1 = u_2^2 \sin^2\theta_2$.

Step 1: Translate equal heights into an equation Let the two speeds be $u_1 = \sqrt{3}\,k$ and $u_2 = \sqrt{2}\,k$, with k a common positive speed factor. The second projectile is launched at 60°. Equating their maximum heights cancels both the common factor $2g$ and k^2:

$$3\sin^2\theta_1 = 2\sin^2 60^\circ.$$

The speeds themselves are not equal, so the launch angles cannot simply be equated.

Step 2: Calculate the first launch angle Using $\sin 60^\circ = \sqrt{3}/2$, we obtain

$$3\sin^2\theta_1 = 2\left(\frac{3}{4}\right) = \frac{3}{2}, \qquad \sin^2\theta_1 = \frac{1}{2}.$$

For the acute angles of projection in the options, $\sin\theta_1 = 1/\sqrt{2}$, hence $\theta_1 = 45^\circ$. The same conclusion follows by equating the positive vertical components directly.

Step 3: Verify against the choices At 45°, the squared vertical speed of the first projectile is $3k^2/2$, exactly that of the second projectile at sixty degrees. A first launch angle of thirty degrees would instead give $3k^2/4$, and sixty degrees would give $9k^2/4$. Fifteen degrees gives an even smaller height than thirty degrees. Thus the other three choices do not satisfy the specified equal-height condition.

Step 4: Conclusion The first projectile must be launched at $\boxed{45^\circ}$ to the horizontal. Hence, the correct option is (C).

Quick Tip to Solve: Equal projectile heights mean equal initial vertical speeds; compare u sin(theta), rather than the total speeds or horizontal components.

35. An air bubble of negligible weight having radius r rises steadily through a solution of density σ at speed v. The coefficient of viscosity of the solution is given by : [4 marks]

(A) $\eta = \frac{4r\sigma g}{9v}$

(B) $\eta = \frac{2r^2\sigma g}{9v}$

(C) $\eta = \frac{2\pi r^2\sigma g}{9v}$

(D) $\eta = \frac{2r^2\sigma g}{3\pi v}$

Correct Answer: (B) $\eta = \frac{2r^2\sigma g}{9v}$

Solution:

Concept:

- At steady terminal speed, the upward buoyant force balances the downward viscous drag when the bubble weight is negligible. Buoyancy equals the weight of the displaced liquid.
- Using Stokes drag for a small spherical object, $F_d = 6\pi\eta rv$, while buoyancy is $F_b = (4\pi r^3/3)\sigma g$. Here η is dynamic viscosity and σ is liquid density.

Step 1: Identify and balance the forces The bubble rises at constant speed, so its acceleration is zero and the resultant force must vanish. The solution exerts an upward buoyant force equal to the weight of a sphere of liquid of radius r:

$$F_b = \frac{4}{3}\pi r^3\sigma g.$$

The viscous force opposes the upward motion and is therefore downward. With bubble weight neglected, there is no additional downward weight term to retain.

Step 2: Solve the terminal-force equation Applying the Stokes-law model gives

$$6\pi\eta rv = \frac{4}{3}\pi r^3\sigma g.$$

Divide both sides by $6\pi rv$ to isolate the viscosity:

$$\eta = \frac{(4/3)\pi r^3\sigma g}{6\pi rv} = \frac{2r^2\sigma g}{9v}.$$

One factor of r cancels against the drag radius, leaving the square of the radius in the final expression.

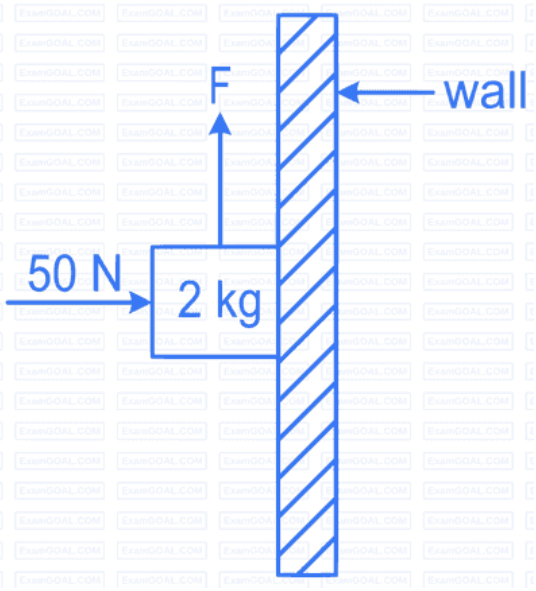
Step 3: Check dimensions and reject other factors The dimensions of $r^2\sigma g/v$ are $\mathrm{kg\,m^{-1}\,s^{-1}}$, the dimensions of dynamic viscosity. Option A has only one power of r and is dimensionally inconsistent. In the force balance the factors of pi cancel completely, so the extra pi in option C and the residual inverse pi in option D cannot occur. The numerical factor $2/9$ comes directly from the volume and drag coefficients.

Step 4: Conclusion The coefficient of viscosity is $\boxed{\eta = \frac{2r^2\sigma g}{9v}}$, in $\mathrm{Pa\,s}$. Hence, the correct option is (B).

Quick Tip to Solve: At terminal speed, equate buoyancy with drag before cancelling factors; r cubed from volume divided by r from Stokes drag leaves r squared.

36. A 2 kg block is pushed against a vertical wall by applying a horizontal force of 50 N. The coefficient of static friction between the block and the wall is 0.5. A force F is also applied on the block vertically upward (as shown in figure). The maximum value of F applied, so that the block does not move upward, will be :

(Given : g = 10 ms^{-2}) [4 marks]



(A) 10 N

(B) 20 N

(C) 25 N

(D) 45 N

Correct Answer: (D) 45 N

Solution:

Concept:

- The normal reaction is set by horizontal equilibrium. Static friction adjusts to oppose impending relative motion and satisfies $|f_s| \leq \mu_s N$.
- At the greatest upward applied force compatible with rest, the block is about to rise. Friction then acts downward, in the same direction as the weight mg.

Step 1: Find the contact force and friction limit The applied horizontal push is $50\,\text{N}$. There is no other horizontal force apart from the wall reaction, and the block has no horizontal acceleration. Hence $N = 50\,\text{N}$. The largest available static friction is therefore

$$f_{s,\max} = \mu_s N = 0.5 \times 50 = 25\,\text{N}.$$

This is a limiting value, not a downward friction force that must act for every possible value of the vertical applied force.

Step 2: Balance forces at impending upward motion The weight is $mg = 2 \times 10 = 20\,\text{N}$, downward. As the upward force is increased past the weight, the tendency is to slide upward, so the wall friction

opposes that tendency by acting downward. At the upper threshold for equilibrium,

$$F_{\max} - mg - f_{s,\max} = 0.$$

Thus

$$F_{\max} = 20 + 25 = 45\,\text{N}.$$

For an even larger applied force, the downward friction required would exceed its permitted maximum, and the block would accelerate upward. The alternatives 10 N and 20 N do not exhaust the downward friction available; 25 N is only the friction limit, not the total upward force that can be supported.

Step 3: Conclusion The maximum upward applied force is $\boxed{45\,\text{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: For impending upward sliding on a vertical wall, add the weight and the limiting friction; first obtain the normal force from the horizontal push.

37. Two bodies A and B of masses 5 kg and 8 kg are moving such that the momentum of body B is twice that of the body A. The ratio of their kinetic energies will be : [4 marks]

(A) 4 : 5

(B) 2 : 5

(C) 5 : 4

(D) 5 : 2

Correct Answer: (B) 2 : 5

Solution:

Concept:

- For nonrelativistic motion, $K = \frac{1}{2}mv^2$ and momentum magnitude is $p = mv$. Eliminating velocity gives $K = p^2/(2m)$.
- Kinetic energy depends on the square of momentum, so doubling momentum multiplies its contribution by four. A comparison must also include the unequal masses.

Step 1: Express both energies using one momentum Let the momentum magnitude of body A be p. The specified relation gives $p_B = 2p$, while $m_A = 5\,\text{kg}$ and $m_B = 8\,\text{kg}$. Therefore

$$K_A = \frac{p^2}{2(5)}, \qquad K_B = \frac{(2p)^2}{2(8)}.$$

No value of either speed is needed: using momentum already incorporates their different speeds and prevents the mistaken assumption that they move equally fast.

Step 2: Form the requested ratio in A-to-B order Divide the first expression by the second, cancelling the common momentum factor and the factor of two:

$$\frac{K_A}{K_B} = \frac{p_A^2}{p_B^2}\frac{m_B}{m_A} = \frac{1}{4}\frac{8}{5} = \frac{2}{5}.$$

Thus body B has the larger kinetic energy, by a factor $5/2$. The option 5:2 reverses the requested order, while 4:5 and 5:4 do not follow the squared-momentum relation. As a separate check, the velocities obey $v_B/v_A = (p_B/p_A)(m_A/m_B) = 5/4$; substituting this into $m_A v_A^2 : m_B v_B^2$ gives the same ratio 2:5. The direction of motion does not change this energy comparison because kinetic energy depends on speed squared.

Step 3: Conclusion The ratio is $\boxed{K_A : K_B = 2 : 5}$. Hence, the correct option is (B).

Quick Tip to Solve: When a question gives momentum ratios and different masses, use $K = p^2/(2m)$; square the momentum ratio before including the inverse mass ratio.

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38. The pressure of the gas in a constant volume gas thermometer is 100 cm of mercury when placed in melting ice at 1 atm. When the bulb is placed in a liquid, the pressure becomes 180 cm of mercury. Temperature of the liquid is :

(Given 0°C = 273 K) [4 marks]

(A) 300 K

(B) 400 K

(C) 600 K

(D) 491 K

Correct Answer: (D) 491 K

Solution:

Concept:

- For a fixed amount of ideal gas at constant volume, $PV = nRT$ gives $P/T =$ constant. Temperature in this law must be measured on the kelvin scale.
- Pressure readings in the same mercury-column units can be divided directly; converting each one to pascals would introduce a common factor that cancels.

Step 1: Identify the calibration state The bulb in melting ice has temperature $T_1 = 273\,\text{K}$, as supplied. Its gas pressure is $P_1 = 100\,\text{cm of Hg}$. In the liquid, the same bulb contains the same quantity of gas at the same volume, with pressure $P_2 = 180\,\text{cm of Hg}$. The statement about melting ice at 1 atm specifies the ice-point condition; it is not an instruction to add atmospheric pressure to either quoted gas pressure.

Step 2: Use absolute-temperature proportionality Apply the gas law between the two fixed-volume states:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}, \qquad T_2 = T_1 \frac{P_2}{P_1}.$$

Substitution gives

$$T_2 = 273 \times \frac{180}{100} = 491.4\,\text{K}.$$

The available answer is 491 K, the nearest kelvin. For context, this is $218.4\,^\circ\text{C}$, but the options ask for the absolute temperature. The answers 300 K, 400 K and 600 K would require pressure ratios of approximately 1.10, 1.47 and 2.20 respectively; none matches the given ratio 1.80. Applying

proportionality to degrees Celsius would incorrectly make the ice-point calibration zero and cannot describe an ideal-gas thermometer.

Step 3: Conclusion The liquid temperature is $491.4\,\mathrm{K}$, approximately $\boxed{491\,\mathrm{K}}$. Hence, the correct option is (D).

Quick Tip to Solve: In a constant-volume gas thermometer, multiply the known kelvin temperature by the pressure ratio; do not use Celsius in the proportionality.

39. A coil of n number of turns wound tightly in the form of a spiral with inner and outer radii r_1 and r_2 respectively. When a current of strength I is passed through the coil, the magnetic field at its centre will be : [4 marks]

(A) $\frac{\mu_0 nI}{2(r_2-r_1)}$

(B) $\frac{\mu_0 nI}{r_2}$

(C) $\frac{\mu_0 nI}{r_2-r_1}\log_e\frac{r_1}{r_2}$

(D) $\frac{\mu_0 nI}{2(r_2-r_1)}\log_e\frac{r_2}{r_1}$

Correct Answer: (D) $\frac{\mu_0 nI}{2(r_2-r_1)}\log_e\frac{r_2}{r_1}$

Solution:

Concept:

- A circular turn of radius r carrying current I produces central field $\mu_0 I/(2r)$, perpendicular to its plane.
- For tightly and uniformly wound turns between radii r_1 and r_2, the turn density along the radius is $n/(r_2 - r_1)$. Contributions from the turns have the same axial direction.

Step 1: Replace closely spaced turns by radial elements The radial width of the winding is $r_2 - r_1$. A narrow strip of width dr, situated at radius r, contains

$$dN = \frac{n}{r_2 - r_1}\,dr.$$

Since the winding is tight, each small part of the spiral may be treated as a circular turn for its contribution at the centre. Its central field is

$$dB = \frac{\mu_0 I}{2r}\,dN = \frac{\mu_0 nI}{2(r_2 - r_1)}\frac{dr}{r}.$$

All these fields add rather than cancel, because the current circulates around the centre in the same sense throughout the winding.

Step 2: Integrate over the winding width and test the result Integrating from the inner radius to the outer radius yields

$$B = \frac{\mu_0 nI}{2(r_2 - r_1)}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\mu_0 nI}{2(r_2 - r_1)}\ln\left(\frac{r_2}{r_1}\right).$$

The logarithm is dimensionless and positive for $r_2 > r_1 > 0$. Option A omits this radial integration

factor; option B substitutes only the outer radius and has the wrong coefficient. Option C reverses the logarithm and lacks the factor of two. In the narrow-winding limit, $\ln(r_2/r_1) \simeq (r_2 - r_1)/r_1$, so the result approaches $\mu_0 nI/(2r_1)$, the expected field of n nearly equal circular turns.

Step 3: Conclusion The field magnitude is $\boxed{\frac{\mu_0 nI}{2(r_2 - r_1)} \ln\left(\frac{r_2}{r_1}\right)}$. Hence, the correct option is (D).

Quick Tip to Solve: For a spiral coil, count turns in a radial strip and integrate the single-loop field $1/r$; the logarithm comes from the changing radius.

---

40. C_o is the capacitance of a parallel plate capacitor with air as a medium between the plates (as shown in Fig. 1). If half space between the plates is filled with a dielectric of relative permittivity ε_r (as shown in Fig. 2), the new capacitance of the capacitor will be : [4 marks]

(A) $\frac{C_o}{2}(1 + \varepsilon_r)$

(B) $C_o + \varepsilon_r$

(C) $\frac{C_o \varepsilon_r}{2}$

(D) $C_o(1 + \varepsilon_r)$

Correct Answer: (A) $\frac{C_o}{2}(1 + \varepsilon_r)$

Solution:

Concept:

- A parallel-plate capacitor has capacitance $C = \varepsilon A/d$, with plate area A, separation d, and medium permittivity ε.
- In Fig. 2 the dielectric fills half the plate area through the full separation. The air-filled and dielectric-filled regions share the same two plates and voltage, so they act in parallel.

Step 1: Assign the two side-by-side capacitances Write the original air capacitance as $C_0 = \varepsilon_0 A/d$. The air region after insertion has area $A/2$ and the original separation d, hence

$$C_{\text{air}} = \frac{\varepsilon_0 (A/2)}{d} = \frac{C_0}{2}.$$

The dielectric region also has area $A/2$ and separation d, but permittivity $\varepsilon_0 \varepsilon_r$. Thus

$$C_{\text{dielectric}} = \frac{\varepsilon_0 \varepsilon_r (A/2)}{d} = \frac{\varepsilon_r C_0}{2}.$$

This interpretation follows the side-by-side shading in the figure; the dielectric is not a layer occupying

half the plate separation.

Step 2: Add the parallel contributions and check limits The two regions are connected across the same plate potential difference, so their charges add. Therefore

$$C_{\text{new}} = C_{\text{air}} + C_{\text{dielectric}} = \frac{C_0}{2}(1 + \varepsilon_r).$$

At $\varepsilon_r = 1$, insertion makes no change of medium and the expression correctly gives C_0. Option B adds a dimensionless relative permittivity to a capacitance and is dimensionally invalid. Option C counts only the dielectric half, discarding the air contribution. Option D gives twice the original capacitance even when $\varepsilon_r = 1$, which fails the no-change test. Fringing at the boundary between the two media is neglected in the parallel-plate model.

Step 3: Conclusion The new capacitance is $\boxed{\frac{C_0}{2}(1 + \varepsilon_r)}$. Hence, the correct option is (A).

Quick Tip to Solve: Classify the dielectric geometry first: side-by-side regions spanning the full plate gap are parallel capacitors, while layers along the gap are in series.

---

41. A sample of monoatomic gas is taken at initial pressure of 75 kPa. The volume of the gas is then compressed from 1200 cm^3 to 150 cm^3 adiabatically. In this process, the value of workdone on the gas will be : [4 marks]

(A) 79 J

(B) 405 J

(C) 4050 J

(D) 9590 J

Correct Answer: (B) 405 J

Solution:

Concept:

- For an ideal monoatomic gas, $\gamma = C_P/C_V = 5/3$. A reversible adiabatic change satisfies $PV^\gamma =$ constant.
- The work done on the gas in adiabatic compression is its increase in internal energy, $W_{\text{on}} = (P_2V_2 - P_1V_1)/(\gamma - 1)$. Compression makes this quantity positive.

Step 1: Find the final pressure-volume product The volume ratio is

$$\frac{V_1}{V_2} = \frac{1200}{150} = 8.$$

Using the adiabatic law,

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{5/3} = 75 \times 8^{5/3} = 2400\,\text{kPa}.$$

For the energy calculation, convert volumes to SI units: $V_1 = 1.2 \times 10^{-3}\,\text{m}^3$ and $V_2 = 1.5 \times 10^{-4}\,\text{m}^3$.

Consequently,

$$P_1V_1 = 75000(1.2 \times 10^{-3}) = 90\,\text{J},$$

$$P_2V_2 = 2400000(1.5 \times 10^{-4}) = 360\,\text{J}.$$

Step 2: Calculate work with the compression sign convention No heat is exchanged in the adiabatic process. With work done on the gas taken positive, the first law gives $W_{\text{on}} = \Delta U$. Therefore

$$W_{\text{on}} = \frac{360 - 90}{5/3 - 1} = \frac{270}{2/3} = 405\,\text{J}.$$

The value is positive because compression raises the temperature and internal energy. The values 4050 J and 9590 J do not follow the stated volume ratio and SI pressure-volume products; 79 J is also inconsistent with the internal-energy increase. Using the initial pressure times the volume decrease would be wrong because the gas pressure rises strongly during adiabatic compression rather than remaining constant.

Step 3: Conclusion The work done on the gas is $\boxed{405\,\text{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: For monoatomic adiabatic compression, use $\gamma = 5/3$; calculate both PV products in joules and take final minus initial for work on the gas.

42. Which of the following equations correctly represents a travelling wave having wavelength λ = 4.0 cm, frequency v = 100 Hz and travelling in positive x-axis direction? [4 marks]

(A) $y = A\sin[(0.50\,\pi\,cm^{-1})x - (100\,\pi\,s^{-1})t]$

(B) $y = A\sin\ 2\pi[(0.25\ cm^{-1})x - (50\,s^{-1})t]$

(C) $y = A\sin\left[\left(\frac{2\pi}{4}\,cm^{-1}\right)x - \left(\frac{2\pi}{100}\,s^{-1}\right)t\right]$

(D) $y = A\sin\ \pi[(0.5\ cm^{-1})x - (200\ s^{-1})t]$

Correct Answer: (D) $y = A\sin\ \pi[(0.5\ cm^{-1})x - (200\ s^{-1})t]$

Solution:

Concept:

- A sinusoidal wave travelling along positive x can be written $y = A\sin(kx - \omega t + \phi)$, with $k = 2\pi/\lambda$ and $\omega = 2\pi f$.
- The coefficients of position and time in the complete phase determine wavelength and frequency. Any factor outside a square bracket must multiply both terms inside it.

Step 1: Calculate the required phase coefficients Keep position in centimetres since the given wavelength is $4.0\,\text{cm}$. The required wave number and angular frequency are

$$k = \frac{2\pi}{4.0} = 0.5\pi\,\text{cm}^{-1},$$

$$\omega = 2\pi(100) = 200\pi\,\text{s}^{-1}.$$

Thus, choosing zero initial phase, the required expression is

$$y = A\sin\left[(0.5\pi\ \text{cm}^{-1})x - (200\pi\ \text{s}^{-1})t\right].$$

The minus sign is essential for travel in the positive x direction: keeping the phase constant gives $dx/dt = \omega/k > 0$.

Step 2: Expand the options before comparing Option D has phase $\pi[(0.5\,\text{cm}^{-1})x - (200\,\text{s}^{-1})t]$, exactly the required two coefficients. Option A has the right wave number but angular frequency $100\pi\,\text{s}^{-1}$, corresponding to only 50 Hz. Option B also has angular frequency $2\pi \times 50 = 100\pi\,\text{s}^{-1}$, again giving 50 Hz. Option C has angular frequency $2\pi/100\,\text{s}^{-1}$, which corresponds to frequency 0.01 Hz rather than 100 Hz. As an extra check, the required phase speed is $f\lambda = 400\,\text{cm}\,\text{s}^{-1}$, equal to ω/k for option D. The sine argument is dimensionless after multiplying each coefficient by its corresponding coordinate.

Step 3: Conclusion The required equation is $\boxed{y = A\sin\{\pi[(0.5\,\text{cm}^{-1})x - (200\,\text{s}^{-1})t]\}}$. Hence, the correct option is (D).

Quick Tip to Solve: Expand every factor in the phase, then compare the x coefficient with $2\pi/\lambda$ and the t coefficient with $2\pi f$.

43. An expression for oscillating electric field in a plane electromagnetic wave is given as E_z = 300 sin($5\pi \times 10^3$x − $3\pi \times 10^{11}$t) Vm^{-1}

Then, the value of magnetic field amplitude will be :

(Given : speed of light in Vacuum c = 3×10^8 ms^{-1}) [4 marks]

(A) 1×10^{-6} T

(B) 5×10^{-6} T

(C) 18×10^9 T

(D) 21×10^9 T

Correct Answer: (B) 5×10^{-6} T

Solution:

Concept:

- For a plane wave $E_z = E_0\sin(kx - \omega t)$, the phase speed is $v = \omega/k$. The phase coefficients determine the actual propagation speed.
- Faraday's law relates the amplitudes by $E_0 = vB_0$. The vacuum value c may replace v only for a wave propagating in vacuum.

Step 1: Extract the propagation speed The coefficient outside the sine gives $E_0 = 300\,\text{V}\,\text{m}^{-1}$. The coefficients inside the phase give $k = 5\pi \times 10^3\,\text{m}^{-1}$ and $\omega = 3\pi \times 10^{11}\,\text{s}^{-1}$. Consequently,

$$v = \frac{\omega}{k} = \frac{3\pi \times 10^{11}}{5\pi \times 10^3} = 6 \times 10^7\,\text{m}\,\text{s}^{-1}.$$

This is one fifth of the vacuum speed supplied in the question. The expression therefore describes a wave with that phase speed, rather than a vacuum wave; its refractive index would be $c/v = 5$.

Step 2: Use the amplitude relation at this speed For the given plane wave, Faraday's law yields $kE_0 = \omega B_0$, so

$$B_0 = \frac{kE_0}{\omega} = \frac{E_0}{v} = \frac{300}{6 \times 10^7} = 5 \times 10^{-6}\,\mathrm{T}.$$

Option (A) would follow from dividing by the vacuum speed, which is not the speed encoded in this wave's phase. Merely being given the value of c does not establish that propagation occurs in vacuum. The enormous field amplitudes in options (C) and (D) fail the same amplitude relation by many orders of magnitude. Using both the spatial and temporal coefficients avoids discarding information supplied by the field expression.

Step 3: Conclusion The magnetic-field amplitude is $5 \times 10^{-6}\,\mathrm{T}$, using the phase speed $\omega/k = 6 \times 10^7\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a specified wave equation, calculate ω/k before using $B_0 = E_0/v$; the vacuum speed is applicable only when the wave actually propagates in vacuum.

44. An electric cable of copper has just one wire of radius 9 mm. Its resistance is 14 Ω. If this single copper wire of the cable is replaced by seven identical well insulated copper wires each of radius 3 mm connected in parallel, then the new resistance of the combination will be : [4 marks]

(A) 9 Ω

(B) 18 Ω

(C) 28 Ω

(D) 126 Ω

Correct Answer: (B) 18 Ω

Solution:

Concept:

- A uniform wire has resistance $R = \rho l/A = \rho l/(\pi r^2)$, where ρ is resistivity and l its length.
- For n identical resistors of resistance R_w connected in parallel, the equivalent resistance is R_w/n.

Step 1: Find the resistance of each thinner wire The replacement wires span the same cable length and are made of the same copper, so length and resistivity cancel in the resistance ratio. Reducing the radius from 9 mm to 3 mm reduces the area by a factor of nine, not three. Each thinner wire therefore has

$$R_w = 14\left(\frac{9}{3}\right)^2 = 14 \times 9 = 126\,\Omega.$$

The seven wires are insulated from each other along their lengths, and meet only at the parallel connections at their ends.

Step 2: Combine the seven parallel branches All seven branches have the same potential difference. Their currents add, so their conductances add:

$$\frac{1}{R_{\mathrm{eq}}} = \frac{7}{126}, \qquad R_{\mathrm{eq}} = \frac{126}{7} = 18\,\Omega.$$

One can also compare total copper cross-sections: the original area is proportional to 81 while the new total is proportional to $7 \times 9 = 63$, so the new resistance is larger by $81/63 = 9/7$. Option (D) is only the resistance of one thin wire; (A) and (C) do not account correctly for both the squared-radius change and the seven parallel branches.

Step 3: Conclusion The new resistance is $18\,\Omega$. Hence, the correct option is (B).

Quick Tip to Solve: For equal-length parallel wires of one material, add their cross-sectional areas; the equivalent resistance is inversely proportional to that total.

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45. In series RLC resonator, if the self inductance and capacitance become double, the new resonant frequency (f_2) and new quality factor (Q_2) will be :

(f_1 = original resonant frequency, Q_1 = original quality factor) [4 marks]

(A) $f_2 = \frac{f_1}{2}$ and $Q_2 = Q_1$

(B) $f_2 = f_1$ and $Q_2 = \frac{Q_1}{Q_2}$

(C) $f_2 = 2f_1$ and $Q_2 = Q_1$

(D) $f_2 = f_1$ and $Q_2 = 2Q_1$

Correct Answer: (A) $f_2 = \frac{f_1}{2}$ and $Q_2 = Q_1$

Solution:

Concept:

- The resonant frequency of a series RLC circuit is $f_0 = 1/(2\pi\sqrt{LC})$, obtained by equating inductive and capacitive reactances.
- The series-circuit quality factor is $Q = \omega_0 L/R = (1/R)\sqrt{L/C}$. The resistance is unchanged here.

Step 1: Calculate the new resonant frequency Let the original inductance and capacitance be L and C. After the change, $L_2 = 2L$ and $C_2 = 2C$. Their product becomes four times its initial value. Thus

$$f_2 = \frac{1}{2\pi\sqrt{(2L)(2C)}} = \frac{1}{4\pi\sqrt{LC}} = \frac{f_1}{2}.$$

Both changes lower the resonant frequency, so it cannot stay fixed or increase. The square root is essential: doubling both components doubles the denominator rather than quadrupling it.

Step 2: Calculate the new quality factor The quality factor depends on the ratio of inductance to capacitance as well as the resistance. Since both reactive components double, their ratio does not change:

$$Q_2 = \frac{1}{R}\sqrt{\frac{2L}{2C}} = \frac{1}{R}\sqrt{\frac{L}{C}} = Q_1.$$

Equivalently, $\omega_2 = \omega_1/2$ and $L_2 = 2L$, so the product $\omega_2 L_2$ is unchanged. Only option (A) gives both results. Options (B) and (D) incorrectly keep the frequency unchanged, while (C) predicts a frequency increase even though the LC product increases.

Step 3: Conclusion The new values are $f_2 = f_1/2$ and $Q_2 = Q_1$. Hence, the correct option is (A).

Quick Tip to Solve: For RLC scaling, track the product LC for frequency and the ratio L/C for quality factor, keeping the resistance fixed.

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46. Find the ratio of maximum intensity to the minimum intensity in the interference pattern if the widths of the two slits in Young's experiment are in the ratio of 9 : 16. (Assuming intensity of light is directly proportional to the width of slits) [4 marks]

(A) 3 : 4

(B) 4 : 3

(C) 7 : 1

(D) 49 : 1

Correct Answer: (D) 49 : 1

Solution:

Concept:

- Light intensity is proportional to the square of wave amplitude: $I \propto a^2$. The question additionally states that slit intensity is proportional to slit width.
- For coherent waves, $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$.

Step 1: Convert the intensity ratio to an amplitude ratio The slit widths are in the ratio $9 : 16$. By the stated assumption, the individual intensities have this same ratio. However, interference adds amplitudes, so we first take square roots:

$$\frac{I_1}{I_2} = \frac{9}{16}, \qquad \frac{a_1}{a_2} = \sqrt{\frac{9}{16}} = \frac{3}{4}.$$

Write the amplitudes as $3a$ and $4a$. Their unequal values mean that destructive interference cannot make the resultant intensity zero.

Step 2: Form the bright-to-dark intensity ratio At constructive interference the resultant amplitude is $3a + 4a = 7a$. At destructive interference its magnitude is $|3a - 4a| = a$. Squaring both amplitudes gives

$$\frac{I_{\max}}{I_{\min}} = \frac{(3a + 4a)^2}{(3a - 4a)^2} = \frac{49a^2}{a^2} = 49.$$

The common proportionality factor cancels. Ratios $3 : 4$ and $4 : 3$ describe the individual amplitudes or their inverse, rather than fringe intensities. The ratio $7 : 1$ is the resultant amplitude ratio and must still be squared to obtain the intensity ratio.

Step 3: Conclusion The maximum-to-minimum intensity ratio is $49 : 1$. Hence, the correct option is (D).

Quick Tip to Solve: In unequal-slit interference, take square roots of the individual intensities before adding and subtracting, then square the resulting ratio.

47. A source of monochromatic light liberates 9×10^{20} photon per second with wavelength 600 nm when operated at 400 W. The number of photons emitted per second with wavelength of 800 nm by the source of monochromatic light operating at same power will be : [4 marks]

(A) 12×10^{20}

(B) 6×10^{20}

(C) 9×10^{20}

(D) 24×10^{20}

Correct Answer: (A) 12×10^{20}

Solution:

Concept:

- The energy of a photon of wavelength λ is $\varepsilon = hc/\lambda$. A longer wavelength gives a smaller energy per photon.
- For fixed emitted optical power, $P = \dot{N}hc/\lambda$, where $\dot{N}$ is the number of photons emitted per second. Hence $\dot{N} \propto \lambda$.

Step 1: Write the equal-power relation Let the initial and final photon emission rates be $\dot{N}_1$ and $\dot{N}_2$. Applying the same-power condition to the two wavelengths gives

$$\dot{N}_1 \frac{hc}{\lambda_1} = \dot{N}_2 \frac{hc}{\lambda_2}, \qquad \frac{\dot{N}_2}{\dot{N}_1} = \frac{\lambda_2}{\lambda_1}.$$

The Planck constant, speed of light and common power cancel. We therefore use the stated initial photon rate directly instead of estimating a new source efficiency from the operating power.

Step 2: Substitute the two wavelengths The wavelengths are 600 nm and 800 nm; using the same wavelength unit in numerator and denominator makes conversion unnecessary:

$$\dot{N}_2 = 9 \times 10^{20} \times \frac{800}{600} = 12 \times 10^{20}\,\mathrm{s}^{-1}.$$

The rate increases because each 800 nm photon carries only three quarters as much energy as a 600 nm photon. Option (B) decreases the rate in the wrong direction. Option (C) ignores the change in photon energy. Option (D) doubles the required rate and therefore would not preserve the same emitted power.

Step 3: Conclusion The source emits 12×10^{20} photons per second at 800 nm. Hence, the correct option is (A).

Quick Tip to Solve: At unchanged optical power, photon rate scales with wavelength, so longer-wavelength light requires more photons each second.

48. A hydrogen atom in ground state absorbs 12.09 eV of energy. The orbital angular momentum of the electron is increased by : [4 marks]

(A) 1.05×10^{-34} Js

(B) 2.11×10^{-34} Js

(C) 3.16×10^{-34} Js

(D) 4.22×10^{-34} Js

Correct Answer: (B) 2.11×10^{-34} Js

Solution:

Concept:

- In the Bohr model of hydrogen, $E_n = -13.6/n^2\,\text{eV}$, with the ground state at $n = 1$.
- The allowed orbital angular momentum is $L_n = n\hbar = nh/(2\pi)$. An increase in angular momentum requires subtracting the initial value from the final value.

Step 1: Identify the excited energy level The electron starts with energy $-13.6\,\text{eV}$. Absorbing the specified energy changes it to

$$E_f = -13.6 + 12.09 = -1.51\,\text{eV}.$$

This is the third Bohr level because

$$E_3 = -\frac{13.6}{9}\,\text{eV} \simeq -1.511\,\text{eV}.$$

Equivalently, $13.6(1 - 1/9) = 12.0889\,\text{eV}$, which rounds to the absorbed energy given. The atom remains bound; the supplied energy is below the ground-state ionisation energy.

Step 2: Subtract the initial angular momentum The initial orbital angular momentum is $L_1 = \hbar$, while the final Bohr value is $L_3 = 3\hbar$. Thus

$$\Delta L = L_3 - L_1 = 2\hbar = \frac{2h}{2\pi} \simeq 2.11 \times 10^{-34}\,\text{J s}.$$

Option (A) is approximately one unit of $\hbar$, which would correspond to a smaller change. Option (C) gives the final angular momentum itself, not its increase. Option (D) is approximately four units of $\hbar$, inconsistent with a transition from the first to third orbit.

Step 3: Conclusion The orbital angular momentum increases by $2.11 \times 10^{-34}\,\text{J s}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the final principal quantum number from energy first; for a Bohr-orbit increase, use $(n_f - n_i)\hbar$, rather than $n_f\hbar$.

49. A cyclotron is working at a frequency of 10 MHz. If the radius of its dees is 60 cm. The maximum kinetic energy of accelerated proton will be :

(Take : e = 1.6 $\times$ 10^{-19} C, m_p = 1.67 $\times$ 10^{-27} kg) [4 marks]

(A) 7.4 MeV

(B) 14.86 MeV

(C) 7.4 GeV

(D) 704 GeV

Correct Answer: (A) 7.4 MeV

Solution:

Concept:

- For circular motion, the orbital frequency satisfies $v = 2\pi fr$. The maximum orbit has radius equal to the usable dee radius.
- The nonrelativistic kinetic energy is $K = mv^2/2$. Convert joules to electronvolts using $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.

Step 1: Find the maximum proton speed The stated cyclotron frequency is $10\,\text{MHz} = 1.0 \times 10^7\,\text{s}^{-1}$, and the dee radius is $60\,\text{cm} = 0.60\,\text{m}$. At the largest orbit, the speed is

$$v_{\text{max}} = 2\pi fr = 2\pi(10^7)(0.60) \simeq 3.77 \times 10^7\,\text{m s}^{-1}.$$

The radius is not a diameter: using 0.30 m would lower the energy by a factor of four. The problem uses the usual nonrelativistic cyclotron relation with the given proton mass.

Step 2: Calculate and convert the kinetic energy Substitute the maximum speed into the kinetic-energy expression:

$$K_{\text{max}} = \frac{1}{2}(1.67 \times 10^{-27})(3.77 \times 10^7)^2 \simeq 1.187 \times 10^{-12}\,\text{J}.$$

Converting the result to megaelectronvolts gives

$$K_{\text{max}} = \frac{1.187 \times 10^{-12}}{1.6 \times 10^{-13}}\,\text{MeV} \simeq 7.42\,\text{MeV}.$$

Option (B) is approximately twice this energy and results from omitting the factor one half. Options (C) and (D) have energies in GeV, far above the energy associated with the calculated speed for a proton of this mass.

Step 3: Conclusion The maximum kinetic energy is approximately $7.4\,\text{MeV}$. Hence, the correct option is (A).

Quick Tip to Solve: When cyclotron frequency and maximum radius are given, use $v = 2\pi fr$ directly; no separate magnetic-field calculation is needed.

50. A speech signal given by 11 sin(2200 πt) V is used for amplitude modulation with a carrier signal given by 44 sin(6600 πt) V. The minimum amplitude of modulated wave will be : [4 marks]

(A) 33 V

(B) 55 V

(C) 8.25 V

(D) 13.75 V

Correct Answer: (A) 33 V

Solution:

Concept:

- In ordinary amplitude modulation, the carrier amplitude varies as $A(t) = A_c + A_m \sin \omega_m t$, where A_c is the carrier amplitude and A_m is the message amplitude.
- The modulation index is $m = A_m/A_c$. For $m < 1$, the envelope never changes sign, and its extrema are $A_c(1 + m)$ and $A_c(1 - m)$.

Step 1: Identify the amplitudes The coefficients of the sine functions, rather than their angular frequencies, give the amplitudes. Thus the message amplitude is $A_m = 11\,\text{V}$, while the unmodulated carrier amplitude is $A_c = 44\,\text{V}$. Their angular frequencies are 2200π and $6600\pi\,\text{rad}\,\text{s}^{-1}$, respectively; these determine the rate of oscillation but do not enter the envelope minimum.

Step 2: Calculate the modulation index For the given signal,

$$m = \frac{11}{44} = 0.25.$$

Since this is below unity, there is no overmodulation. The instantaneous envelope is $44[1 + 0.25\sin(2200\pi t)]\,\text{V}$. The smallest envelope occurs when the message sine equals -1, not when the rapidly oscillating carrier passes through zero.

Step 3: Find and compare the envelope extrema The minimum amplitude is

$$A_{\text{min}} = 44(1 - 0.25) = 44 - 11 = 33\,\text{V}.$$

The maximum is $44 + 11 = 55\,\text{V}$, so option (B) represents the maximum rather than the minimum. Options (C) and (D) incorrectly multiply the envelope extrema by the modulation index; there is no such extra factor in the amplitude-modulation formula.

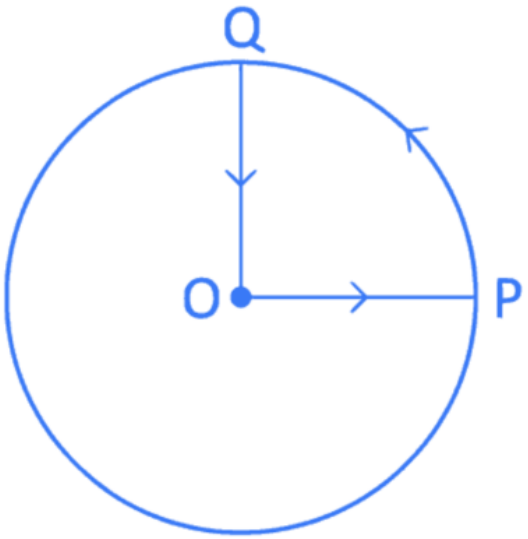
Step 4: Conclusion The minimum amplitude of the modulated wave is $\boxed{33\,\text{V}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the sine coefficients as amplitudes: the AM envelope minimum is carrier amplitude minus message amplitude, provided the modulation index is below one.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A person starts his journey from centre 'O' of the park and comes back to the same position following path OPQO as shown in the figure. The radius of path taken by the person is 200 m and he takes 3 min 58 sec to complete his journey. The average speed of the person is _____ ms^{-1}. (take π = 3.14) [4 marks]



Solution:

Concept:

- Average speed is total distance travelled divided by total elapsed time: $v_{\text{av}} = s/t$. It is a scalar and depends on the actual path.
- The length of a circular arc subtending θ radians is $r\theta$. The diagram shows two radii and a quarter-circle arc.

Step 1: Read the complete path The person travels from the centre O along the radius to P, then counterclockwise along the short arc to Q, and finally along a radius back to O. The angle POQ is a right angle, so the curved portion is one quarter of the circumference. Each of the straight portions has length $r = 200\,\text{m}$. Returning to the start makes the displacement zero, but it does not make the distance zero.

Step 2: Add the three path lengths The quarter-circle arc has length

$$s_{PQ} = \frac{1}{4}(2\pi r) = \frac{3.14 \times 200}{2} = 314\,\text{m}.$$

Consequently,

$$s = OP + s_{PQ} + QO = 200 + 314 + 200 = 714\,\text{m}.$$

Using a full circumference here would count a journey that the arrows do not show.

Step 3: Convert time and divide The total time must be in seconds because the requested speed is in

metres per second:

$$t = 3(60) + 58 = 238\,\text{s}.$$

Therefore,

$$v_{\text{av}} = \frac{714}{238} = 3\,\text{m s}^{-1}.$$

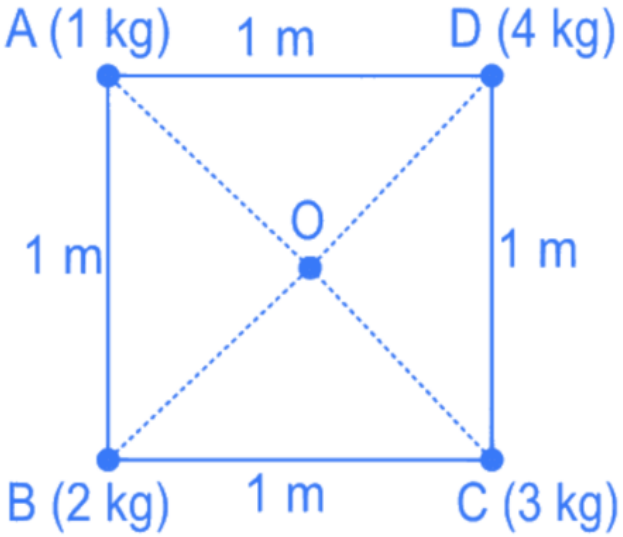
The average velocity would instead be zero because the final and initial positions coincide; the question specifically asks for average speed.

Step 4: Conclusion The average speed is $\boxed{3\,\text{m s}^{-1}}$. The number to enter is 3.

Quick Tip to Solve: For a closed journey, keep distance and displacement separate. Add both radii and only the circular arc actually traversed before dividing by the total time.

---

52. Four particles with a mass of 1 kg, 2 kg, 3 kg and 4 kg are situated at the corners of a square with side 1 m (as shown in the figure). The moment of inertia of the system, about an axis passing through the point O and perpendicular to the plane of the square, is _____ kg m^2. [4 marks]



Solution:

Concept:

- For point particles, the moment of inertia is $I = \sum_i m_i r_i^2$, where r_i is the perpendicular distance from the rotation axis.
- The centre of a square bisects each diagonal. A square of side a has diagonal $a\sqrt{2}$, so every corner is a distance $a/\sqrt{2}$ from its centre.

Step 1: Locate the axis and distances The figure places O at the intersection of the diagonals of the square. The required axis passes through O perpendicular to the square. Thus the perpendicular distance of each particle from the axis equals its distance from O in the plane. Although the masses differ, all four distances are identical by the symmetry of the square.

Step 2: Calculate the squared corner distance Each corner lies half a side horizontally and half a side vertically from the centre. Pythagoras therefore gives

$$r^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}\,\mathrm{m}^2.$$

Equivalently, half of the diagonal is $\sqrt{2}/2\,\mathrm{m}$. The distance is not the side length $1\,\mathrm{m}$, because the axis is through the centre rather than a corner.

Step 3: Add the four contributions The total moment of inertia is

$$I = 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right) + 4\left(\frac{1}{2}\right) = 5\,\mathrm{kg\,m}^2.$$

Since the common squared distance factors out, this is also $I = (1+2+3+4)(1/2)$. Every contribution is positive; the positions on opposite sides of the axis do not cancel.

Step 4: Conclusion The moment of inertia is $\boxed{5\,\mathrm{kg\,m}^2}$. The number to enter is 5.

Quick Tip to Solve: For particles at the corners of a square about its central perpendicular axis, use the common squared distance $a^2/2$ and multiply it by the sum of the masses.

---

53. The excess pressure inside a liquid drop is 500 Nm^{-2}. If the radius of the drop is 2 mm, the surface tension of liquid is x $\times$ 10^{-3} Nm^{-1}. The value of x is _____. [4 marks]

Solution:

Concept:

- For a spherical liquid drop in air, excess internal pressure is $\Delta P = 2T/r$, where T is surface tension and r is the radius.
- A liquid drop has one liquid-air interface. The formula $4T/r$ applies to a soap bubble with two interfaces, so it must not be used here.

Step 1: Select the drop relation The stated pressure is the excess pressure, which means the internal pressure minus the external pressure. No atmospheric-pressure term is to be added. For one spherical interface, the pressure force and surface-tension force balance to give $\Delta P = 2T/r$. Rearranging for the quantity requested,

$$T = \frac{\Delta P\, r}{2}.$$

Step 2: Convert the radius and substitute The given radius must first be expressed in metres:

$$r = 2\,\mathrm{mm} = 2 \times 10^{-3}\,\mathrm{m}.$$

With $\Delta P = 500\,\mathrm{N\,m}^{-2}$,

$$T = \frac{500 \times 2 \times 10^{-3}}{2} = 0.5\,\mathrm{N\,m}^{-1}.$$

The units are consistent because pressure multiplied by length has units $\mathrm{N\,m}^{-1}$. The stated dimension

is a radius, so no extra halving is required.

Step 3: Express the result in the requested scale The question writes the surface tension as $x \times 10^{-3}\,\mathrm{N\,m^{-1}}$. Rewrite the calculated result in that form:

$$0.5\,\mathrm{N\,m^{-1}} = 500 \times 10^{-3}\,\mathrm{N\,m^{-1}}.$$

Matching the coefficients gives $x = 500$. The requested entry is the coefficient 500, rather than the surface tension expressed as the decimal 0.5.

Step 4: Conclusion The surface tension is $0.5\,\mathrm{N\,m^{-1}}$, so $\boxed{x = 500}$.

Quick Tip to Solve: Distinguish a one-surface liquid drop from a two-surface soap bubble, and convert the final physical value into the power-of-ten scale specified in the blank.

54. Eight similar drops of mercury are maintained at 12 V each. All these spherical drops combine into a single big drop. The potential energy of bigger drop will be _____ E. Where E is the potential energy of a single smaller drop. [4 marks]

Solution:

Concept:

- An isolated conducting sphere of radius r has capacitance $C = 4\pi\varepsilon_0 r$, potential $V = q/C$, and electrostatic energy $U = q^2/(2C)$.
- When identical liquid drops merge, their volumes and total charge are conserved. Their potentials and electrostatic energies are not generally conserved.

Step 1: Find the radius of the larger drop Let each small drop have radius r, charge q, and capacitance C. If the radius after eight drops merge is R, conservation of mercury volume gives

$$\frac{4}{3}\pi R^3 = 8\left(\frac{4}{3}\pi r^3\right), \qquad R = 2r.$$

Capacitance is proportional to radius, not to volume, so the larger drop has capacitance $C_{\text{big}} = 2C$.

Step 2: Conserve charge and calculate the new potential Each initial drop is at $12\,\mathrm{V}$, so its charge is $q = 12C$. With no charge loss, the larger drop has charge $Q = 8q$. Thus

$$V_{\text{big}} = \frac{8q}{2C} = 4\frac{q}{C} = 48\,\mathrm{V}.$$

The potential therefore rises on coalescence; retaining $12\,\mathrm{V}$ for the final drop would require an external voltage source that is not part of the stated situation.

Step 3: Compare energies with one small drop The energy of a single smaller drop is $E = q^2/(2C)$. The energy of the bigger drop is

$$U_{\text{big}} = \frac{(8q)^2}{2(2C)} = 32\frac{q^2}{2C} = 32E.$$

The comparison is with one small drop, not with the combined initial energy $8E$. Relative to the combined initial energy the factor would be four, which is a different ratio.

Step 4: Conclusion The bigger drop has electrostatic potential energy $\boxed{32E}$. The required coefficient is 32.

Quick Tip to Solve: For n identical charged drops merging without charge loss, the radius scales as $n^{1/3}$, the charge as n, and the energy relative to one drop as $n^{5/3}$.

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55. A series LCR circuit with $R = \frac{250}{11}\,\Omega$ and $X_L = \frac{70}{11}\,\Omega$ is connected across a 220 V, 50 Hz supply. The value of capacitance needed to maximize the average power of the circuit will be _____ μF. (Take : $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- The impedance of a series LCR circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $X_C = 1/(\omega C)$ and $\omega = 2\pi f$.
- For fixed rms voltage and resistance, the average dissipated power is $P = V_{\text{rms}}^2 R/[R^2 + (X_L - X_C)^2]$. It is maximized when the reactances cancel.

Step 1: Impose the maximum-power condition The numerator of the power expression is fixed as capacitance varies. Its denominator is smallest when the nonnegative squared term vanishes. Thus the maximum occurs at series resonance:

$$X_C = X_L = \frac{70}{11}\,\Omega.$$

At this point the impedance equals the resistance, the power factor is unity, and the current is largest. The given resistance and voltage determine the maximum power but are not needed to locate the resonant capacitance.

Step 2: Calculate the angular frequency Using the supplied value of π and the frequency $f = 50\,\text{Hz}$,

$$\omega = 2\pi f = 2\left(\frac{22}{7}\right)50 = \frac{2200}{7}\,\text{rad}\,\text{s}^{-1}.$$

The reactance specified in the question already applies at this supply frequency, so it is unnecessary to calculate the inductance separately.

Step 3: Solve for the capacitance From $X_C = 1/(\omega C) = X_L$,

$$C = \frac{1}{\omega X_L} = \frac{1}{(2200/7)(70/11)}.$$

The denominator simplifies exactly to 2000, giving

$$C = \frac{1}{2000}\,\text{F} = 5 \times 10^{-4}\,\text{F} = 500\,\mu\text{F}.$$

The conversion uses $1\,\mu\text{F} = 10^{-6}\,\text{F}$; the answer requested in microfarads is therefore 500.

Step 4: Conclusion The required capacitance is $\boxed{500\,\mu\text{F}}$. The number to enter is 500.

Quick Tip to Solve: For maximum power in a series LCR circuit at fixed voltage and frequency, first set $X_C = X_L$; then use $C = 1/(2\pi f X_L)$.

56. The refractive index of an equilateral prism is $\sqrt{2}$. The angle of emergence under minimum deviation position of prism, in degree, is _____. [4 marks]

Solution:

Concept:

- At minimum deviation, a prism has a symmetric ray path: the angle of incidence equals the angle of emergence, and the two internal refraction angles are equal.
- The prism relation is $r_1 + r_2 = A$. At minimum deviation this gives $r_1 = r_2 = A/2$, while Snell's law at the exit face gives $\sin e = \mu \sin r_2$ for emergence into air.

Step 1: Determine the prism angle An equilateral prism has an equilateral triangular cross-section, so its refracting angle is $A = 60^\circ$. In the minimum-deviation position the internal path is symmetric. The two internal refraction angles therefore satisfy

$$r_1 = r_2 = \frac{A}{2} = 30^\circ.$$

These angles are measured relative to the normals to the prism faces, as are the external angles of incidence and emergence.

Step 2: Apply refraction at the second face The light travels from the prism of refractive index $\mu = \sqrt{2}$ into air, whose refractive index is taken as unity. Snell's law gives

$$\mu \sin r_2 = \sin e.$$

Substitute the internal angle:

$$\sin e = \sqrt{2} \sin 30^\circ = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$

This value is below unity, so a transmitted emerging ray exists; there is no total internal reflection for this path.

Step 3: Choose the physical emergence angle The angle of emergence lies between zero and 90°, because the ray emerges into the air on the outward side of the face. The physical solution of $\sin e = 1/\sqrt{2}$ is therefore $e = 45^\circ$. Symmetry also gives $i = 45^\circ$. As a check, the corresponding minimum deviation is $i + e - A = 45^\circ + 45^\circ - 60^\circ = 30^\circ$, distinct from the emergence angle asked for.

Step 4: Conclusion The angle of emergence at minimum deviation is $\boxed{45^\circ}$. The number to enter is 45.

Quick Tip to Solve: At minimum deviation, halve the prism angle before applying Snell's law. Do not confuse the resulting emergence angle with the minimum deviation itself.

57. A hydrogen atom in its first excited state absorbs a photon of energy x $\times$ 10^{-2} eV and excited to a higher energy state where the potential energy of electron is -1.08 eV. The value of x is _____. [4 marks]

Solution:

Concept:

- For a hydrogen atom in the Bohr model, the total energy is $E_n = -13.6/n^2\,\text{eV}$, where n is its principal quantum number.
- For an electron bound by the Coulomb force, potential energy and total energy obey $U_n = 2E_n$. The absorbed photon supplies the increase in total energy, not the increase in potential energy.

Step 1: Find the initial total energy The ground state has $n = 1$, so the first excited state is $n = 2$. Its total energy is therefore

$$E_i = -\frac{13.6}{2^2} = -3.40\,\text{eV}.$$

The negative sign denotes a bound electron, with the zero of energy at infinite separation. Excitation makes the total energy less negative.

Step 2: Convert the final potential energy to total energy The question gives the potential energy in the final state as $U_f = -1.08\,\text{eV}$. Hence

$$E_f = \frac{U_f}{2} = -0.54\,\text{eV}.$$

Using the given potential energy directly as the final total energy would omit the positive kinetic energy. The given value is rounded; it is consistent with excitation approximately to $n = 5$.

Step 3: Calculate the photon energy and extract the coefficient Energy conservation during absorption gives

$$E_\gamma = E_f - E_i = -0.54 - (-3.40) = 2.86\,\text{eV}.$$

In the requested notation,

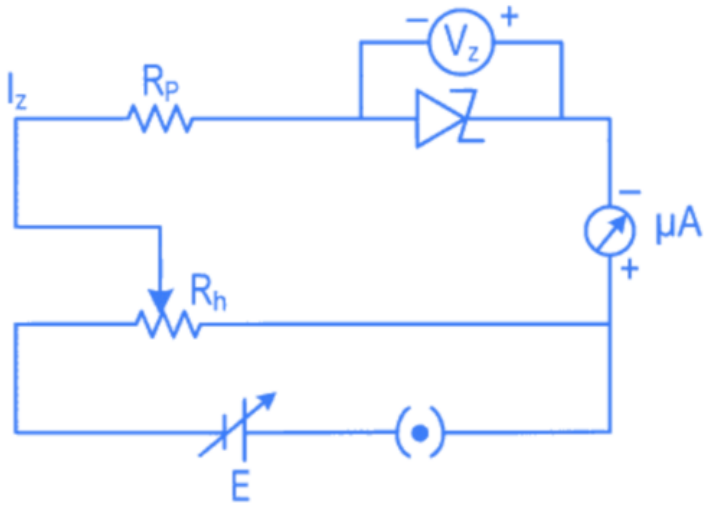
$$2.86\,\text{eV} = 286 \times 10^{-2}\,\text{eV}.$$

The photon energy is positive, as expected for absorption into a higher bound state.

Step 4: Conclusion The absorbed photon has energy $2.86\,\text{eV}$, so the requested dimensionless coefficient is $\boxed{x = 286}$.

Quick Tip to Solve: When a Bohr-model question supplies potential energy, divide it by two before calculating a photon energy from the difference of total energies.

58. The circuit diagram used to study the characteristic curve of a zener diode is connected to variable power supply (0 – 15 V) as shown in figure. A zener diode with maximum potential V_z = 10 V and maximum power dissipation of 0.4 W is connected across a potential divider arrangement. The value of resistance R_P connected in series with the zener diode to protect it from the damage is _____ Ω. [4 marks]



Solution:

Concept:

- A reverse-biased Zener diode in breakdown maintains an approximately constant voltage V_Z. Its safe current is limited by its power rating: $P_{\max} = V_Z I_{Z,\max}$.
- The series protecting resistance carries the Zener current and drops the excess supply voltage. The largest current occurs at the maximum applied voltage.

Step 1: Calculate the maximum safe Zener current The diode can dissipate at most $0.4\,\mathrm{W}$ while maintaining $10\,\mathrm{V}$ across itself. Thus

$$I_{Z,\max} = \frac{P_{\max}}{V_Z} = \frac{0.4}{10} = 0.040\,\mathrm{A} = 40\,\mathrm{mA}.$$

The power rating refers to the diode itself, so it must be divided by the diode voltage rather than the full supply voltage.

Step 2: Find the largest voltage across the protecting resistor The potential-divider arrangement can apply up to the full supply of $15\,\mathrm{V}$. In breakdown the diode takes $10\,\mathrm{V}$, leaving

$$V_{R_P} = V_{\max} - V_Z = 15 - 10 = 5\,\mathrm{V}.$$

The meter is treated as ideal for this calculation. Lower applied voltages do not produce a larger current, so checking this upper limit protects the diode throughout the supply range.

Step 3: Apply Ohm's law at the current limit The smallest series resistance that limits the current to the safe value is

$$R_P = \frac{V_{R_P}}{I_{Z,\max}} = \frac{5}{0.040} = 125\,\Omega.$$

At this resistance the maximum diode power is $10 \times 0.040 = 0.4\,\text{W}$, exactly its stated rating. A smaller resistance would permit excessive breakdown current at the upper supply setting.

Step 4: Conclusion The required protecting series resistance is $\boxed{R_P = 125\,\Omega}$.

Quick Tip to Solve: First convert the Zener power rating to a maximum current, then divide the excess supply voltage by that current.

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59. An employee of a factory moving away from his workplace by a car listens to the siren of the factory. He drives the car at the speed of 72 kmh^{-1} in the direction of wind which is blowing at 72 kmh^{-1} speed. Frequency of siren is 720 Hz. The employee hears an apparent frequency of _____ Hz.

(Assume speed of sound to be 340 ms^{-1}) [4 marks]

Solution:

Concept:

- The speed of sound v is specified relative to the air. With wind of speed w along propagation, the wavefront speed relative to the ground is $v + w$.
- For a stationary source of frequency f, the ground-frame wavelength is $\lambda = (v + w)/f$. A receiver receding at ground speed u encounters wavefronts at frequency $f\prime = (v + w - u)/\lambda$.

Step 1: Express both speeds in SI units The employee travels away from the factory in the same direction as the wind, so the sound reaching the car propagates downwind. Both the car speed and the wind speed are

$$u = w = 72 \times \frac{1000}{3600} = 20\,\text{m s}^{-1}.$$

The speed $340\,\text{m s}^{-1}$ is the speed of sound in the air, not the speed of a wavefront relative to the ground.

Step 2: Calculate the downwind wavelength The factory siren remains at rest on the ground and emits 720 successive compressions each second. These travel downwind at

$$v + w = 340 + 20 = 360\,\text{m s}^{-1}.$$

Therefore their separation is

$$\lambda = \frac{360}{720} = 0.50\,\text{m}.$$

Wind changes this wavelength even though the source frequency is unchanged.

Step 3: Find the rate at which the car meets wavefronts The car moves in the same direction as the wavefronts, so their speed relative to the car is

$$360 - 20 = 340\,\text{m s}^{-1}.$$

Hence

$$f\prime = \frac{340}{0.50} = 680\,\text{Hz}.$$

This is lower than 720 Hz, consistent with the receiver moving away from the source. Although the car is stationary relative to the air here, the factory is not; that distinction prevents incorrectly concluding that no shift occurs.

Step 4: Conclusion The employee hears an apparent frequency of $\boxed{680\,\text{Hz}}$.

Quick Tip to Solve: With wind, compute the ground-frame wave speed and wavelength first; then subtract the receiver speed to count arriving wavefronts.

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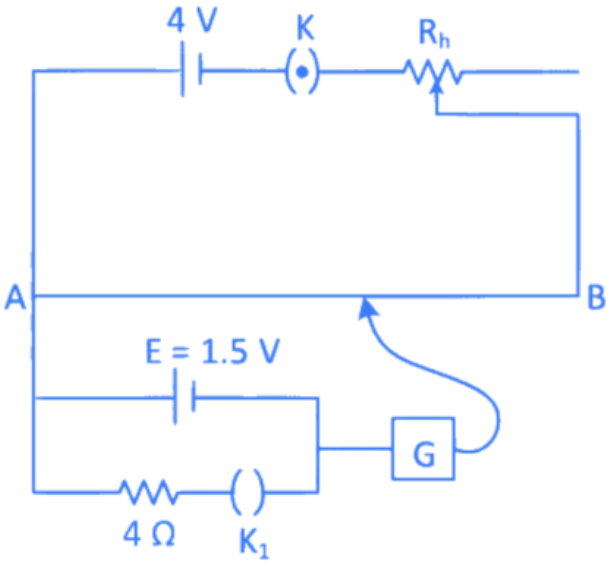
60. The circuit diagram of potentiometer used to measure the internal resistance of a cell (E) is shown in figure. The key 'K' is kept closed so as to send constant current through potentiometer wire. When key 'K_1' is kept open the null point is found to be at 120 cm on the potentiometer wire. When the key 'K_1' is closed the null point is shifted at 80 cm at the potentiometer wire. The internal resistance of the given cell is _____ Ω. [4 marks]



Solution:

Concept:

- For a potentiometer wire with constant potential gradient k, the measured potential difference is $V = k\ell$, where ℓ is the balancing length.
- A cell with emf E, internal resistance r, and external load R has terminal voltage $V = ER/(R+r)$. Thus $r = R(E/V - 1)$.

Step 1: Identify the open-circuit emf and loaded terminal voltage With K_1 open, no current is drawn from the test cell at balance. Its emf balances at $\ell_1 = 120\,\text{cm}$, giving $E = k\ell_1$. Closing K_1 connects the $4\,\Omega$ resistor shown in the circuit across the test cell. The shorter balancing length $\ell_2 = 80\,\text{cm}$ now measures its terminal voltage $V = k\ell_2$.

Step 2: Eliminate the unchanged potential gradient The primary key K remains closed and the wire

current is held constant, so the same k applies to both readings. Dividing gives

$$\frac{E}{V} = \frac{k\ell_1}{k\ell_2} = \frac{120}{80} = \frac{3}{2}.$$

There is no need to find k from the primary supply voltage. The centimetre units cancel because the two lengths are expressed in the same unit.

Step 3: Use the load resistance to calculate the internal resistance The current through the loaded cell is $I = V/R$, and its internal voltage drop is $E - V = Ir$. Hence

$$r = R\left(\frac{E}{V} - 1\right) = 4\left(\frac{120}{80} - 1\right) = 4 \times \frac{1}{2} = 2\,\Omega.$$

As a check, the depicted $1.5\,\text{V}$ emf gives $V = 1.0\,\text{V}$, $I = 0.25\,\text{A}$, and an internal drop $0.50\,\text{V}$, consistent with $r = 2\,\Omega$.

Step 4: Conclusion The internal resistance of the given cell is $\boxed{2\,\Omega}$.

Quick Tip to Solve: Use the external load resistance from the circuit and the ratio of balancing lengths: $r = R(\ell_1 - \ell_2)/\ell_2$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The number of radial nodes and total number of nodes in 4p orbital respectively are : [4 marks]

(A) 2 and 3

(B) 2 and 2

(C) 3 and 4

(D) 4 and 4

Correct Answer: (A) 2 and 3

Solution:

Concept:

- An orbital has $n - 1$ total nodes, where n is its principal quantum number. A node is a surface on which the orbital wavefunction vanishes.
- The number of angular nodes is ℓ, and the number of radial nodes is $n - \ell - 1$, where ℓ is the azimuthal quantum number.

Step 1: Assign the quantum numbers of the orbital The numeral in $4p$ specifies $n = 4$. The orbital letter p corresponds to $\ell = 1$, since the sequence s, p, d, f corresponds to $\ell = 0, 1, 2, 3$. Different orientations of a p orbital have the same node counts; the magnetic quantum number does not change this calculation.

Step 2: Count radial nodes and total nodes separately The radial wavefunction therefore has

$$N_{\text{radial}} = n - \ell - 1 = 4 - 1 - 1 = 2$$

radial nodes. The total count is

$$N_{\text{total}} = n - 1 = 4 - 1 = 3.$$

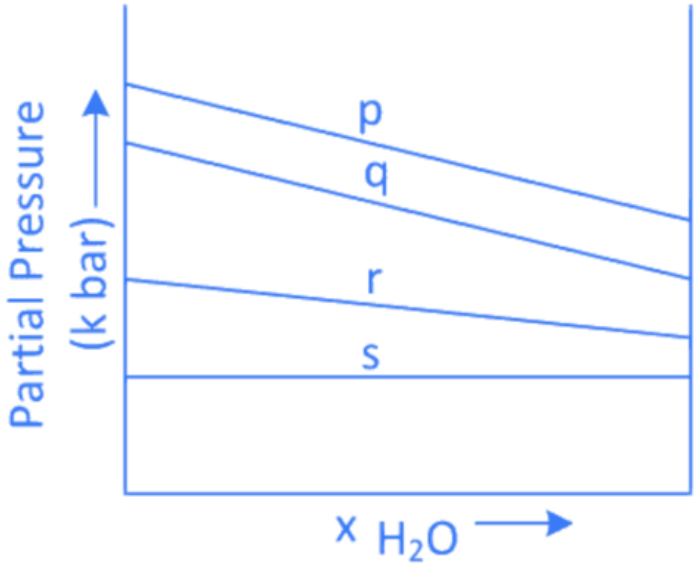
Equivalently, adding the one angular nodal plane of a p orbital to its two radial nodal surfaces gives $2 + 1 = 3$. Do not confuse the principal quantum number with the number of nodes.

Step 3: Compare the ordered pair with the options The question asks for radial nodes first and total nodes second, so the required ordered pair is $(2, 3)$. Option (B), $(2, 2)$, counts radial nodes correctly but omits the angular node from the total. Option (C), $(3, 4)$, overcounts both quantities, and option (D), $(4, 4)$, incorrectly treats n itself as a node count.

Step 4: Conclusion There are $\boxed{2 \text{ radial nodes and } 3 \text{ total nodes}}$ in a $4p$ orbital. Hence, the correct option is (A).

Quick Tip to Solve: For any orbital, calculate $n - \ell - 1$ first and check that adding ℓ angular nodes gives $n - 1$ total nodes.

62. For a solution of the gases A, B, C and D in water at 298 K, the values of Henry's law constant (K_H) are 30.40, 2.34, 1.56×10^{-5} and 0.513 k bar respectively. In the given graph, the lines marked as 'p' and 's' correspond respectively to : [4 marks]



(A) A and C

(B) B and A

(C) D and A

(D) C and D

Correct Answer: (A) A and C

Solution:

Concept:

- Henry's law for a gas dissolved in water is $P = K_H x_{\text{gas}}$, where P is its partial pressure and x_{gas} is its mole fraction in solution.
- For the gas-water solution, $x_{\text{gas}} = 1 - x_{\text{H}_2\text{O}}$. Thus $P = K_H(1 - x_{\text{H}_2\text{O}})$, a line with slope $-K_H$.

Step 1: Read the axes and relate each line to its Henry constant The vertical axis shows partial pressure and the horizontal axis shows the mole fraction of water, rather than of the dissolved gas. Increasing the water fraction decreases the gas fraction, so the negative slopes agree with Henry's law. At any common water mole fraction less than one, the gas with larger K_H has larger partial pressure. Its line also has a greater downward slope magnitude.

Step 2: Order the supplied constants All four constants are supplied in the same pressure unit, so they can be compared directly:

$$K_H(A) = 30.40 > K_H(B) = 2.34 > K_H(D) = 0.513 > K_H(C) = 1.56 \times 10^{-5}.$$

Gas A therefore corresponds to the uppermost and steepest line p, while gas C has the smallest pressure and a nearly horizontal line s. The intermediate lines q and r correspond to B and D respectively.

Step 3: Check the ordered assignments against the options The requested order is p followed by s, giving A followed by C. Option (B) incorrectly places A on the lowest line; option (C) also makes that error and places D on the highest line. Option (D) reverses the extreme constant ordering by putting C on the uppermost line. Only option (A) respects the pressure ordering at a fixed composition.

Step 4: Conclusion Lines p and s represent gases $\boxed{A \text{ and } C}$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Check whether the horizontal axis is gas or solvent mole fraction before using the slope: with water fraction, the slope is $-K_H$.

63. The equilibrium constant for the reversible reaction

$2A(g) \rightleftharpoons 2B(g) + C(g)$ is K_1

$\frac{3}{2}A(g) \rightleftharpoons \frac{3}{2}B(g) + \frac{3}{4}C(g)$ is K_2.

K_1 and K_2 are related as : [4 marks]

(A) $K_1 = \sqrt{K_2}$

(B) $K_2 = \sqrt{K_1}$

(C) $K_2 = K_1^{3/4}$

(D) $K_1 = K_2^{3/4}$

Correct Answer: (C) $K_2 = K_1^{3/4}$

Solution:

Concept:

- At a fixed temperature, multiplying every stoichiometric coefficient in a reaction by a factor a raises its equilibrium constant to that power: $K_{\text{new}} = K_{\text{old}}^{a}$.
- The equilibrium-constant expression contains product activities divided by reactant activities, each raised to its stoichiometric coefficient. Scaling coefficients therefore scales every exponent equally.

Step 1: Find the common stoichiometric scale factor The first reaction is $2A(g) \rightleftharpoons 2B(g) + C(g)$. To obtain the second, compare its coefficient $3/2$ for A with the original coefficient 2:

$$a = \frac{3/2}{2} = \frac{3}{4}.$$

The coefficient of B changes by the same factor, and that of C becomes $1 \times 3/4 = 3/4$, confirming that the entire reaction is multiplied by $3/4$. It has not been reversed.

Step 2: Apply the exponent rule explicitly Using activities a_A, a_B, a_C, the first constant is

$$K_1 = \frac{a_B^2 a_C}{a_A^2}.$$

For the second reaction,

$$K_2 = \frac{a_B^{3/2} a_C^{3/4}}{a_A^{3/2}} = \left(\frac{a_B^2 a_C}{a_A^2}\right)^{3/4} = K_1^{3/4}.$$

This demonstration also applies to the usual concentration or partial-pressure form when each is used consistently.

Step 3: Eliminate the incorrect relations Option (A) would imply $K_2 = K_1^2$, associated with doubling the reaction, not multiplying it by $3/4$. Option (B) would correspond to halving all coefficients. Option (D) interchanges the direction of the power relation; solving our result for K_1 gives $K_1 = K_2^{4/3}$, not $K_2^{3/4}$. Thus only option (C) uses the correct common multiplier.

Step 4: Conclusion The required relationship is $\boxed{K_2 = K_1^{3/4}}$. Hence, the correct option is (C).

Quick Tip to Solve: Compute the coefficient ratio for one species, verify it for every species, and use that ratio as the exponent of the original equilibrium constant.

64. In which of the following half cells, electrochemical reaction is pH dependent? [4 marks]

(A) $Pt\,|\,Fe^{3+},\,Fe^{2+}$

(B) $MnO_4^-\,|Mn^{2+}$

(C) $Ag\,|\,AgCl\,|Cl^{-1}$

(D) $\frac{1}{2}F_2\,|F^-$

Correct Answer: (B) $MnO_4^-\,|Mn^{2+}$

Solution:

Concept:

- The Nernst equation relates an electrode potential to its reaction quotient: $E = E^\circ - (RT/nF)\ln Q$, where n is the number of electrons transferred.
- A half-cell potential depends explicitly on pH when $\mathrm{H^+}$ or $\mathrm{OH^-}$ occurs in its balanced electrode reaction. Since $\mathrm{pH} = -\log a_{\mathrm{H^+}}$, changing hydrogen-ion activity then changes Q.

Step 1: Write the permanganate reduction In acidic solution, manganese is reduced from oxidation state $+7$ in permanganate to $+2$. Balancing oxygen with water, hydrogen with protons and charge with electrons gives

$$\mathrm{MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O}.$$

The reaction quotient, omitting liquid water, is

$$Q = \frac{a_{\mathrm{Mn^{2+}}}}{a_{\mathrm{MnO_4^-}}\,a_{\mathrm{H^+}}^{8}}.$$

Thus protons enter the potential expression directly.

Step 2: Compare all four half-cells At $298\,\mathrm{K}$, holding the manganese-species activities fixed,

$$E = E^\circ - \frac{0.0591}{5}\log\frac{a_{\mathrm{Mn^{2+}}}}{a_{\mathrm{MnO_4^-}}} - \frac{8(0.0591)}{5}\mathrm{pH}.$$

By contrast, the iron electrode has $\mathrm{Fe^{3+} + e^- \longrightarrow Fe^{2+}}$, the silver chloride electrode has $\mathrm{AgCl(s) + e^- \longrightarrow Ag(s) + Cl^-}$, and the fluorine electrode has $\mathrm{F_2 + 2e^- \longrightarrow 2\,F^-}$. None of these written electrode reactions contains protons or hydroxide ions. Their quotients therefore have no explicit pH term under the stated half-cell conditions.

Step 3: Conclusion The $\mathrm{MnO_4^-\,|\,Mn^{2+}}$ half-cell is pH dependent because its reduction consumes hydrogen ions. Hence, the correct option is (B).

Quick Tip to Solve: Balance the half-reaction first: the presence of $\mathrm{H^+}$ or $\mathrm{OH^-}$ immediately identifies an explicit pH dependence.

65. The correct order of electron gain enthalpy (− ve value) is : [4 marks]

(A) O > S > Se > Te

(B) O < S < Se < Te

(C) O < S > Se > Te

(D) O < S > Se < Te

Correct Answer: (C) O < S > Se > Te

Solution:

Concept:

- Electron gain enthalpy describes $X(g) + e^- \longrightarrow X^-(g)$. A more exothermic addition has a larger magnitude of its negative enthalpy.
- Down Group 16, increasing size and shielding generally reduce the attraction for an added electron, but oxygen is anomalous because its compact 2p shell produces strong electron-electron repulsion.

Step 1: Recognise the oxygen anomaly Oxygen receives the added electron in its small 2p subshell, which already contains paired electrons. The electron experiences appreciable repulsion in this compact region. Sulfur receives the electron in the more spacious 3p subshell, where the repulsive penalty is lower. Consequently, sulfur releases more energy than oxygen on gaining an electron; oxygen cannot be placed above sulfur in the order of negative magnitudes.

Step 2: Order the heavier chalcogens From sulfur to selenium to tellurium, the valence shell becomes larger and more shielded. The added electron is held less strongly, so the energy released decreases. Approximate electron gain enthalpies, in $\mathrm{kJ\,mol^{-1}}$, are -141, -200, -195 and -190 for O, S, Se and Te respectively. Thus the negative magnitudes follow

$$\mathrm{O} < \mathrm{S} > \mathrm{Se} > \mathrm{Te}.$$

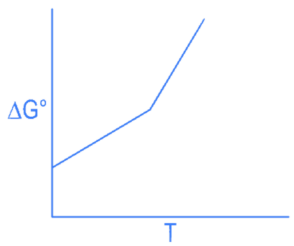
Option (A) misses the oxygen anomaly. Option (B) wrongly increases the release steadily down the group. Option (D) incorrectly makes tellurium exceed selenium. Here the parenthetical negative-value wording refers to the amount of energy released, rather than reversing the comparison as algebraic signed numbers.

Step 3: Conclusion The required order is $\mathrm{O} < \mathrm{S} > \mathrm{Se} > \mathrm{Te}$. Hence, the correct option is (C).

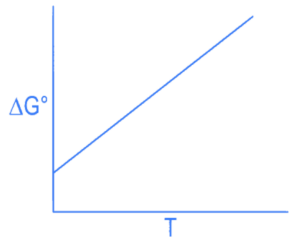
Quick Tip to Solve: Remember that sulfur, rather than oxygen, has the most negative electron gain enthalpy in Group 16; then apply the usual decrease in magnitude down the group.

66. ΔG° vs T plot for the formation of MgO, involving reaction $2Mg + O_2 \rightarrow 2MgO$, will look like : [4 marks]

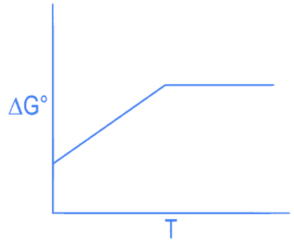
(A)



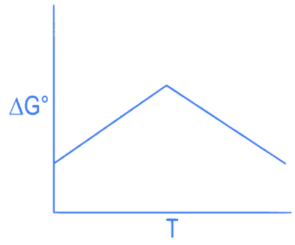
(B)



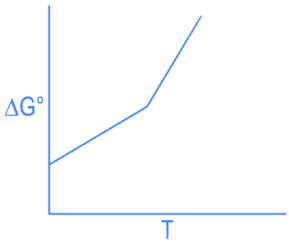
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- For oxide formation, $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$. In a temperature interval without a phase change, the slope of its Ellingham line is $-\Delta S^\circ$.
- Consuming gaseous oxygen to produce a solid oxide decreases entropy, so the oxide-formation line has positive slope. A phase change in a reactant changes the reaction entropy and hence the slope.

Step 1: Determine the initial slope The reaction is $2\,\mathrm{Mg} + \mathrm{O_2} \longrightarrow 2\,\mathrm{MgO}$. At lower temperatures magnesium and its oxide are solids, while oxygen is a gas. Removing one mole of gas produces a negative reaction entropy. Therefore

$$\Delta S^\circ < 0, \qquad \frac{d\Delta G^\circ}{dT} = -\Delta S^\circ > 0.$$

The graph must initially rise as temperature increases; a falling initial segment would not represent this gas-consuming reaction.

Step 2: Account for the break in the line At a phase transition of magnesium, such as melting, the

entropy of the magnesium reactant increases. With the oxide still in its corresponding solid region, the reaction entropy becomes more negative because reactant entropy is subtracted from product entropy. Consequently $-\Delta S^\circ$ increases and the plot acquires a steeper positive slope after the transition. The same direction of slope change follows if magnesium vaporisation is considered at higher temperature. For a reactant phase transition, the liquid or gaseous state has higher entropy than the preceding condensed state. Subtracting this larger reactant entropy from the product entropy increases the positive slope. The important feature is an upward kink, not the absolute vertical position of the schematic curves. Option (B) omits the slope change, option (C) makes the later slope zero, and option (D) changes it to negative; each conflicts with the increase in reactant entropy.

Step 3: Conclusion The line rises and becomes steeper after the magnesium phase transition, as in plot (A). Hence, the correct option is (A).

Quick Tip to Solve: For an Ellingham kink, ask which side changes phase: increasing reactant entropy makes ΔS° more negative and steepens the upward slope.

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67. **A** $\xrightarrow{573\,K}$ **Red phosphorus** $\underset{under\ pressure}{\xrightarrow{heat\,;\,803\,K}}$ **B**

Red phosphorus is obtained by heating "A" at 573 K, and can be converted to "B" by heating at 803 K under pressure.

A and B, respectively, are [4 marks]

(A) β-black phosphorus and white phosphorus.

(B) white phosphorus and β-black phosphorus.

(C) α-black phosphorus and white phosphorus.

(D) white phosphorus and α-black phosphorus.

Correct Answer: (D) white phosphorus and α-black phosphorus.

Solution:

Concept:

- White, red and black phosphorus are allotropes: their phosphorus atoms are arranged differently, giving different physical properties and stabilities.
- Allotropic conversion depends on the starting allotrope and the stated thermal conditions. White phosphorus gives red phosphorus on heating in an inert atmosphere; heating red phosphorus in a sealed tube at about $803\,\mathrm{K}$ produces alpha-black phosphorus.

Step 1: Identify the starting allotrope White phosphorus consists of discrete $\mathrm{P_4}$ tetrahedra. Heating it at approximately $573\,\mathrm{K}$ in an inert atmosphere converts it to polymeric red phosphorus. No new element or chemical reagent is needed: the change rearranges phosphorus bonding. Therefore the substance marked A must be white phosphorus. This already excludes options (A) and (C), which place a black allotrope before red phosphorus under these conditions.

Step 2: Identify B from the second conversion The second conversion starts from red phosphorus and specifies heating at $803\,\mathrm{K}$. These are the conditions for preparing alpha-black phosphorus by heating red phosphorus in a sealed tube. The pressure annotation on this arrow does not change the

identity associated with this starting allotrope and temperature. Beta-black phosphorus is instead prepared by heating white phosphorus at about $473\,\mathrm{K}$ under high pressure. Thus beta-black must not be selected merely because pressure is mentioned. Option (B) has the correct starting substance but the wrong black allotrope. Option (D) correctly identifies both A as white phosphorus and B as alpha-black phosphorus. Keeping track of the material entering each arrow avoids confusing the two distinct routes for making black phosphorus.

Step 3: Conclusion A is white phosphorus and B is α-black phosphorus. Hence, the correct option is (D).

Quick Tip to Solve: Pair each black-phosphorus route with its starting allotrope: red phosphorus at $803\,\mathrm{K}$ gives alpha-black, while white phosphorus at $473\,\mathrm{K}$ under high pressure gives beta-black.

68. Correct formula of the compound which gives a white precipitate with $BaCl_2$ solution, but not with $AgNO_3$ solution, is : [4 marks]

(A) $[Co(NH_3)_5Br]SO_4$

(B) $[Co(NH_3)_5SO_4]Br$

(C) $[Pt(NH_3)_4Cl_2]Br_2$

(D) $[Pt(NH_3)_4Br_2]Cl_2$

Correct Answer: (A) $[Co(NH_3)_5Br]SO_4$

Solution:

Concept:

- An ion outside the coordination brackets dissociates readily in water and undergoes ordinary precipitation tests. A ligand coordinated inside the brackets is not present as a free counter-ion.
- Free sulfate gives white $BaSO_4$ with Ba^{2+}, whereas free chloride or bromide precipitates with Ag^{+} as a silver halide.

Step 1: Apply the barium chloride test A white precipitate with barium chloride indicates free sulfate ions:

$$Ba^{2+} + {SO_4}^{2-} \longrightarrow BaSO_4(s).$$

In $[Co(NH_3)_5Br]SO_4$, sulfate lies outside the coordination sphere. Dissociation is

$$[Co(NH_3)_5Br]SO_4 \longrightarrow [Co(NH_3)_5Br]^{2+} + {SO_4}^{2-}.$$

Thus this compound supplies the sulfate required for the white precipitate. In option (B), sulfate is coordinated to cobalt and bromide is the external ion, so it does not satisfy the stated sulfate test.

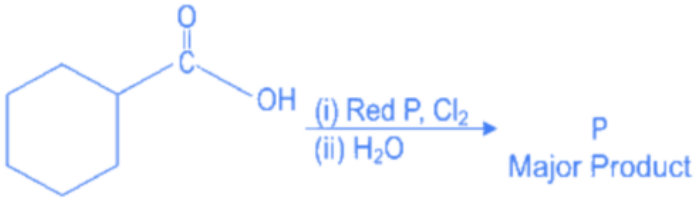
Step 2: Check the absence of a silver precipitate In option (A), bromide is bonded inside the coordination sphere; it is not released as free bromide by ordinary dissolution. The external sulfate is not the halide needed for the usual silver-halide test, so no characteristic precipitate with silver nitrate is obtained under these qualitative-test conditions. In option (B), external Br^{-} gives $AgBr$. Option (C) likewise has two external bromide ions and gives $AgBr$, while option (D) has external chloride and gives $AgCl$. The latter two also lack sulfate entirely. Only option (A) simultaneously

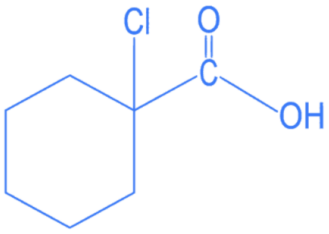
supplies free sulfate and keeps its halide coordinated.

Step 3: Conclusion The compound is $[Co(NH_3)_5Br]SO_4$. Hence, the correct option is (A).

Quick Tip to Solve: For precipitation tests on coordination salts, read outside the brackets first: counter-ions respond immediately, whereas coordinated ligands do not behave as free ions.

69. Consider the given chemical reaction. Identify the product P (major product). [4 marks]



(A)


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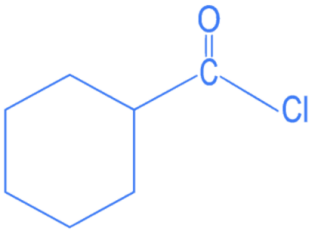
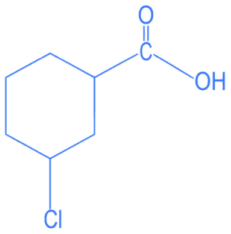
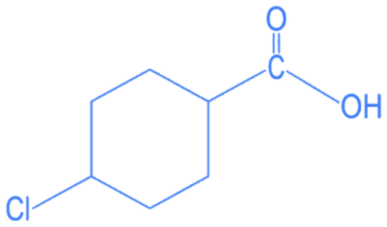
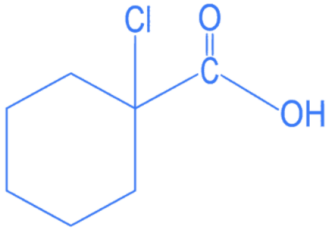


(C)


(D)


Correct Answer: (A)



Solution:

Concept:

- Carboxylic acids containing an alpha hydrogen undergo alpha-halogenation with halogen and red phosphorus, followed by hydrolysis: this is the Hell-Volhard-Zelinsky reaction.
- The alpha carbon is directly bonded to the carboxyl carbon. In cyclohexanecarboxylic acid, it is the ring carbon bearing the COOH substituent.

Step 1: Locate the reactive hydrogen The starting structure is cyclohexanecarboxylic acid. Its carboxyl group is attached directly to a ring carbon, and that carbon bears one hydrogen. It is therefore the only alpha carbon available for the required enolisation. Red phosphorus and chlorine generate phosphorus chlorides in situ, converting the acid into its acyl chloride and enabling halogenation through its enol. Chlorine replaces the alpha hydrogen, rather than substituting randomly at a remote ring position.

Step 2: Follow the hydrolysis and compare the products After alpha chlorination, the second step with water hydrolyses the acyl chloride back to a carboxylic acid. The ring retains its six-carbon skeleton; both chlorine and the carboxyl group are now attached to carbon 1. Hence the product is 1-chlorocyclohexane-1-carboxylic acid, the compound shown in option (A). Option (B) is the unhalogenated acyl chloride, an intermediate-type structure that cannot be the final product after water. Options (C) and (D) place chlorine at ring positions remote from the carboxyl-bearing carbon; neither represents alpha halogenation. Only the hydrogen on the carbon next to the carbonyl group is replaced in the indicated reaction sequence.

Step 3: Conclusion P is 1-chlorocyclohexane-1-carboxylic acid, with Cl and COOH on the same ring carbon. Hence, the correct option is (A).

Quick Tip to Solve: In an HVZ reaction on a cyclic carboxylic acid, mark the ring carbon carrying COOH; its hydrogen is the alpha hydrogen that is replaced.

70. Choose the reaction which is not possible: [4 marks]

(A) $\mathrm{C_6H_5X + 2\,Na \xrightarrow{Ether} C_6H_5{-}C_6H_5}$

(B) $\mathrm{p\text{-}ClC_6H_4NO_2 + NaOH \xrightarrow[\text{then HCl}]{\Delta} p\text{-}HOC_6H_4NO_2}$

(C) $\mathrm{C_6H_5CH_2CHO + [Ag(NH_3)_2]^+ + OH^- \xrightarrow{\Delta} C_6H_5CH_2COO^-}$

(D) $\mathrm{C_6H_5CHO + Cu^{2+} + OH^- \longrightarrow C_6H_5COO^-}$

Correct Answer: (D) $\mathrm{C_6H_5CHO + Cu^{2+} + OH^- \longrightarrow C_6H_5COO^-}$

Solution:

Concept:

- Tollens reagent oxidises aldehydes, including aromatic aldehydes, to carboxylates. Fehling-type alkaline copper(II) reagent generally oxidises aliphatic aldehydes but does not give the characteristic reaction with benzaldehyde.
- Aryl halides can undergo Fittig coupling with sodium in dry ether. An electron-withdrawing nitro group ortho or para to a halogen facilitates nucleophilic aromatic substitution.

Step 1: Verify the first three transformations Option (A) is Fittig coupling: two aryl-halide molecules couple with sodium in dry ether to form biphenyl. Its balanced form is

$$2\,C_6H_5X + 2\,Na \longrightarrow C_6H_5{-}C_6H_5 + 2\,NaX.$$

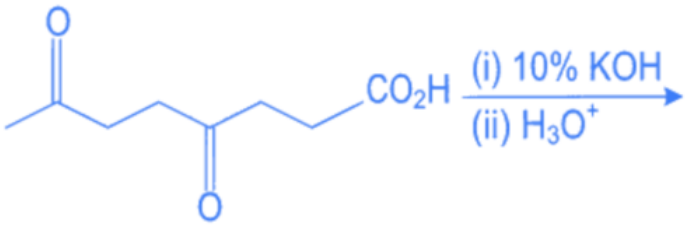
In (B), the para nitro group activates the carbon bearing chlorine towards hydroxide substitution. Heating with sodium hydroxide gives the nitrophenoxide, and acidification yields para-nitrophenol. In (C), phenylacetaldehyde, $C_6H_5CH_2CHO$, is oxidised by Tollens reagent to phenylacetate, $C_6H_5CH_2COO^-$. The aldehyde carbon is retained and converted into a carboxylate group; the intervening methylene group separates the aldehyde from the aromatic ring, so this substrate behaves as an aliphatic aldehyde.

Step 2: Identify the incompatible reagent in the last reaction Option (D) shows benzaldehyde, C_6H_5CHO, being converted to benzoate using alkaline Cu^{2+}. Benzaldehyde does not reduce Fehling-type alkaline copper(II) reagent, so this is not the indicated feasible oxidation. The distinction is between phenylacetaldehyde in (C), whose aldehyde group is on a side chain, and benzaldehyde in (D), whose aldehyde carbon is bonded directly to the aromatic ring. Benzaldehyde can be oxidised by Tollens reagent or stronger oxidants, but replacing the actual reagent by one of those would change the reaction. Therefore (A), (B) and (C) are possible under their indicated conditions, while (D) is the exception.

Step 3: Conclusion The benzaldehyde-to-benzoate conversion with alkaline copper(II) reagent is not possible as shown. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish C_6H_5CHO from $C_6H_5CH_2CHO$: direct aromatic attachment changes the response to Fehling-type reagent, although both aldehydes respond to Tollens reagent.

71. Which among the following will be the major product of the given reaction? [4 marks]



(A) Cyclopent-2-en-1-one with a $-CH_2CH_2CO_2H$ group at C3.

(B) Cyclopent-2-en-1-one with $-CH_2CO_2H$ at C2 and a methyl group at C3.

(C) Cyclohex-3-en-1-one with $-CO_2H$ at C3 and a methyl group at C4.

(D) A cyclopentanone ring and a five-membered lactone ring sharing one spiro carbon: 1-oxaspiro[4.4]nonane-2,7-dione.

Correct Answer: (D) A cyclopentanone ring and a five-membered lactone ring sharing one spiro carbon: 1-oxaspiro[4.4]nonane-2,7-dione.

Solution:

Concept:

- Base removes an alpha hydrogen from a ketone to generate an enolate. Intramolecular attack is favoured when it produces a five-membered ring.
- A gamma-hydroxy carboxylic acid can form a five-membered cyclic ester, or lactone, on acidification. The hydroxyl oxygen becomes the ester oxygen.

Step 1: Identify the ring-forming enolate The substrate is $CH_3COCH_2CH_2COCH_2CH_2CO_2H$. Number the carbon chain from the methyl end: the ketone carbonyls are at C2 and C5, and the acid carbon is C8. Hydroxide generates the C1 enolate. Its attack on C5 forms a new C1–C5 bond and a ring containing C1, C2, C3, C4 and C5. The C2 carbonyl remains, while the C5 carbonyl oxygen becomes an alkoxide. Attack between closer alpha-carbon positions would require an unfavourably small ring.

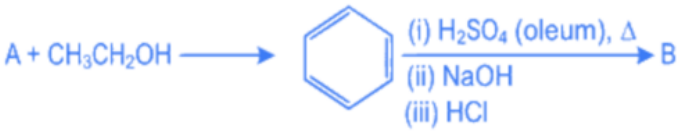
Step 2: Follow acidification and cyclic ester formation Protonation gives a cyclopentanone bearing both an OH group and a $-CH_2CH_2CO_2H$ chain on C5. Along this side chain the OH is gamma to the acid group. During acidic work-up it reacts intramolecularly with that carboxylic acid, eliminating water and forming a five-membered lactone. The lactone ring consists of the hydroxyl oxygen, the acid carbonyl carbon, the two side-chain methylene carbons and C5. The two rings have only C5 in common, so the product is spirocyclic. This traps the aldol-addition product rather than simply giving its dehydrated enone.

Step 3: Compare the four proposed products Option D retains the original C2 ketone and converts the acid into the correctly sized lactone. Option A is the dehydrated cyclopentenone and leaves the acid uncyclised. Option B puts the acid side chain and methyl group on the wrong ring positions for the new C1–C5 bond. Option C proposes a six-membered unsaturated ring and does not follow the favoured five-membered aldol closure. Count all eight substrate carbon atoms: the spiro product retains them, with one ketone and one ester carbonyl.

Step 4: Conclusion The major product is the spirocyclic ketone–lactone, 1-oxaspiro[4.4]nonane-2,7-dione. Hence, the correct option is (D).

Quick Tip to Solve: After an intramolecular aldol addition, inspect any pendant carboxylic acid: a newly formed gamma-OH can trap the product as a five-membered lactone.

72. Consider the above reaction sequence. Identify the component A and component B : [4 marks]



(A) $A = C_6H_5NH_2$; $B = C_6H_5Cl$.

(B) $A = C_6H_5N_2{}^+Cl^-$; $B = C_6H_5OH$.

(C) $A = C_6H_5Cl$; $B = C_6H_5NH_2$.

(D) $A = C_6H_5OH$; $B = C_6H_5N_2{}^+Cl^-$.

Correct Answer: (B) $A = C_6H_5N_2{}^+Cl^-$; $B = C_6H_5OH$.

Solution:

Concept:

- Ethanol reduces an aryl diazonium salt to the corresponding arene, with loss of nitrogen; ethanol is oxidised to ethanal.
- Benzene gives benzenesulphonic acid with oleum. Alkali fusion replaces the sulphonate group by phenoxide, and acidification gives phenol.

Step 1: Identify the precursor that gives benzene The first transformation removes the substituent from A and replaces it with hydrogen. Benzenediazonium chloride undergoes exactly this reduction with ethanol:

$$C_6H_5N_2Cl + CH_3CH_2OH \longrightarrow C_6H_6 + N_2 + CH_3CHO + HCl.$$

Nitrogen is a stable gaseous leaving product. Thus $A = C_6H_5N_2{}^+Cl^-$. Aniline does not become benzene merely by adding ethanol: it would first need diazotisation. Chlorobenzene is not reduced to benzene by ethanol under these conditions, and phenol does not lose its OH group on simple treatment with ethanol.

Step 2: Track the benzene through sulphonation and alkali treatment Oleum supplies the sulphonating species, introducing $-SO_3H$ on benzene:

$$C_6H_6 \xrightarrow{\text{oleum, } \Delta} C_6H_5SO_3H.$$

The sodium salt undergoes alkali fusion to sodium phenoxide. The relevant substitution can be represented as

$$C_6H_5SO_3Na + 2\,NaOH \longrightarrow C_6H_5ONa + Na_2SO_3 + H_2O.$$

The acidification shown at the end protonates phenoxide:

$$C_6H_5ONa + HCl \longrightarrow C_6H_5OH + NaCl.$$

Therefore B is phenol; HCl is the work-up acid and does not insert chlorine into the aromatic ring.

Step 3: Match both components simultaneously The required pair is the diazonium chloride for A and phenol for B. Option A wrongly starts from aniline and ends at chlorobenzene. Option C starts from chlorobenzene and requires formation of an amino group even though the sequence provides no

nitrogen reagent. Option D reverses phenol and the diazonium salt; the later steps cannot create a diazonium group. Only option B satisfies both independently established identities.

Step 4: Conclusion $A = C_6H_5N_2{}^+Cl^-$ and $B = C_6H_5OH$. Hence, the correct option is (B).

Quick Tip to Solve: Ethanol converts an aryl diazonium group into H; oleum followed by alkali fusion and acidification converts benzene into phenol.

73. The sugar produced after complete hydrolysis of DNA is [4 marks]

(A) a pentose sugar.

(B) a hexose sugar.

(C) a polysaccharide.

(D) a disaccharide.

Correct Answer: (A) a pentose sugar.

Solution:

Concept:

- DNA is a polynucleotide. Its nucleotide units contain a nitrogenous base, phosphate and the sugar 2-deoxy-D-ribose.
- A pentose has five carbon atoms. The term deoxy describes replacement of an OH group by H; it does not change the carbon count.

Step 1: Identify the sugar part of DNA The name deoxyribonucleic acid identifies its carbohydrate as deoxyribose. Hydrolysis cleaves the linkages joining the molecular components, so the sugar obtained from the DNA nucleotide framework is 2-deoxy-D-ribose. The phosphate groups are part of the sugar–phosphate backbone, while the bases are adenine, guanine, cytosine and thymine. Neither a phosphate group nor a nitrogenous base changes the classification of the sugar released. RNA instead contains ribose, which also has five carbon atoms.

Step 2: Classify the released monosaccharide Deoxyribose has five carbon atoms and molecular formula $C_5H_{10}O_4$. Its carbon atoms are conventionally labelled $1'$ through $5'$. It is therefore a pentose monosaccharide. The ring form commonly drawn in a nucleotide has four carbon atoms and one oxygen atom in the ring, with the fifth carbon outside the ring; counting only ring atoms would give the wrong classification. Classification as a pentose depends on all carbon atoms in the molecule, rather than the number of atoms forming the ring.

Step 3: Reject the other carbohydrate classes Option B would require a six-carbon sugar such as glucose, which is absent from the DNA nucleotide backbone. A polysaccharide is made of many linked monosaccharide units, so option C describes a carbohydrate polymer rather than the individual sugar released. A disaccharide has two monosaccharide units linked together, whereas the DNA sugar is a single deoxyribose unit. Complete hydrolysis does not join sugar molecules into a disaccharide or polysaccharide, so options C and D fail.

Step 4: Conclusion The sugar is 2-deoxy-D-ribose, a pentose sugar. Hence, the correct option is (A).

Quick Tip to Solve: Both ribose and deoxyribose contain five carbons; deoxy changes the oxygen content, not the pentose classification.

74. The reagent neutral ferric chloride is used to detect the presence of _____ [4 marks]

(A) sulphide ion and alcoholic $-OH$ group.

(B) acetate ion and phenolic $-OH$ group.

(C) sulphide ion and phenolic $-OH$ group.

(D) acetate ion and alcoholic $-OH$ group.

Correct Answer: (B) acetate ion and phenolic $-OH$ group.

Solution:

Concept:

- Neutral ferric chloride gives characteristic coloured complexes with many phenols through coordination of phenoxide-type oxygen donors to ferric ions.
- Acetate ions give a red ferric acetate colour with neutral ferric chloride. Ordinary alcohols do not give the characteristic phenol test.

Step 1: Apply the reagent to the phenolic group Phenols have an OH group attached directly to an aromatic carbon. Their interaction with neutral $FeCl_3$ forms coloured iron(III) complexes, commonly violet, blue or green depending on the phenol. The colour is a coordination-compound response and should not be confused with the colour of the original ferric chloride solution. An ordinary alcohol has its OH group attached to an aliphatic carbon and does not give this characteristic phenolic colour test. Thus both choices containing an alcoholic OH group are unsuitable.

Step 2: Identify the inorganic ion that also responds Acetate ions react with neutral ferric chloride to give the red coloration associated with ferric acetate. A simple composition equation is

$$3\,CH_3COONa + FeCl_3 \longrightarrow Fe(CH_3COO)_3 + 3\,NaCl.$$

The useful analytical observation is the red colour, which supports the presence of acetate under the stated test conditions. Neutral conditions are specified because excessive acidity can suppress the characteristic complex-forming responses. Sulphide is ordinarily tested through characteristic metal sulphide formation, such as black lead sulphide with lead acetate, rather than this neutral ferric chloride identification test.

Step 3: Select the pair and eliminate the alternatives The correct combination is acetate ion plus the phenolic OH group. Option A contains sulphide and an alcoholic OH group, so neither member gives the intended pair of responses. Option C contains the correct phenolic group but substitutes sulphide for acetate. Option D contains acetate but changes the phenolic OH to alcoholic OH. A pair question requires both entries to be correct; recognising only the phenolic test without checking the ion can lead to selecting the wrong option.

Step 4: Conclusion Neutral ferric chloride detects acetate ion and a phenolic $-OH$ group. Hence, the correct option is (B).

Quick Tip to Solve: Associate neutral ferric chloride with coloured phenol complexes and the red acetate test; an ordinary alcoholic OH does not give the phenol response.

75. An element X has a body centred cubic (bcc) structure with a cell edge of 200 pm. The density of the element is 5 g cm^{-3}. The number of atoms present in 300 g of the element X is _____.
Given : Avogadro constant, $N_A = 6.0 \times 10^{23}$ mol^{-1}. [4 marks]

(A) 5 N_A

(B) 6 N_A

(C) 15 N_A

(D) 25 N_A

Correct Answer: (D) 25 N_A

Solution:

Concept:

- A body-centred cubic cell contains $8(1/8) + 1 = 2$ atoms: eight corner contributions and one whole body-centre atom.
- Density connects the bulk sample to the cell volume: $m_{\text{cell}} = \rho a^3$. Dividing sample mass by cell mass gives the number of cells.

Step 1: Convert the edge and find the cell mass The edge is $a = 200\,\text{pm}$. Since $1\,\text{pm} = 10^{-10}\,\text{cm}$,

$$a = 2.00 \times 10^{-8}\,\text{cm}, \qquad a^3 = 8.00 \times 10^{-24}\,\text{cm}^3.$$

Multiplying by the given density gives

$$m_{\text{cell}} = 5 \times 8.00 \times 10^{-24} = 4.00 \times 10^{-23}\,\text{g}.$$

Cubing the conversion factor is essential: using an edge in metres with a density in grams per cubic centimetre would introduce a large error.

Step 2: Count the unit cells and their atoms The 300 g sample contains

$$N_{\text{cells}} = \frac{300}{4.00 \times 10^{-23}} = 7.50 \times 10^{24}.$$

Each cell has two atoms, rather than the nine geometric positions drawn in a bcc sketch, because each corner atom is shared among eight cells. Therefore

$$N_{\text{atoms}} = 2 \times 7.50 \times 10^{24} = 1.50 \times 10^{25}.$$

This method counts atoms directly and does not need the chemical identity of element X. It uses the density and lattice geometry consistently.

Step 3: Express the answer using the supplied Avogadro constant With $N_A = 6.0 \times 10^{23}\,\text{mol}^{-1}$, the numerical ratio is

$$\frac{1.50 \times 10^{25}}{6.0 \times 10^{23}} = 25.$$

Thus the atom count is $25N_A$, conventionally meaning the number of atoms in 25 mol. As a cross-check, $M = \rho a^3 N_A/2 = 12\,\text{g mol}^{-1}$, giving $300/12 = 25$ mol. The alternatives $5N_A$, $6N_A$ and $15N_A$ would correspond to incorrect cell masses or atom counts; none equals the direct lattice calculation.

Step 4: Conclusion The sample contains 1.50×10^{25} atoms, or $25N_A$ atoms in the option notation. Hence, the correct option is (D).

Quick Tip to Solve: Count atoms directly with $N = z\,m/(\rho a^3)$; for bcc use $z = 2$ and convert the edge into centimetres before cubing.

76. Match List - I with List - II :

	List-I		List-II
(a)	Sodium hydride	(i)	Lewis acid
(b)	Silane	(ii)	Saline hydride
(c)	Vanadium hydride	(iii)	Molecular hydride
(d)	Aluminium hydride	(iv)	Non-stoichiometric hydride

Correct answer is : [4 marks]

(A) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(B) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

(C) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

(D) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

Correct Answer: (A) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

Solution:

Concept:

- Ionic or saline hydrides contain hydride ions, while molecular hydrides contain predominantly covalent element–hydrogen bonds.
- Transition-metal hydrides can have variable hydrogen content and are non-stoichiometric. An electron-deficient hydride can accept an electron pair and act as a Lewis acid.

Step 1: Classify sodium hydride and silane Sodium is a highly electropositive alkali metal. Its hydride NaH is represented as $\mathrm{Na^+H^-}$, making it a saline hydride. Thus (a) matches (ii). Silane, $\mathrm{SiH_4}$, is a discrete covalent molecule. Silicon and hydrogen form covalent bonds, so it belongs to the molecular hydride category: (b) matches (iii). Do not infer that every solid hydride is ionic or that every hydrogen compound of a metallic-looking element has the same bonding type.

Step 2: Classify vanadium hydride and aluminium hydride Vanadium is a transition metal. Hydrogen can occupy interstitial positions in its metallic lattice, and the proportion of hydrogen can vary. Vanadium hydride is therefore the non-stoichiometric hydride in the list: (c) matches (iv). Aluminium hydride, $\mathrm{AlH_3}$, is electron deficient. A simple monomeric Lewis description places only six electrons around aluminium, allowing it to accept an electron pair from a donor. That electron-pair acceptance is the defining property of a Lewis acid, so (d) matches (i). Its association or polymerisation does not remove the relevance of its electron-deficient aluminium centres.

Step 3: Assemble the complete matching sequence The four matches are

$$(a) \to (ii), \quad (b) \to (iii), \quad (c) \to (iv), \quad (d) \to (i).$$

Option A reproduces every match. Option B exchanges the identities of silane and vanadium hydride, confusing molecular and non-stoichiometric hydrides. Option C wrongly makes sodium hydride molecular and silane saline, and also misclassifies the other two. Option D correctly recognises silane and vanadium hydride but incorrectly assigns Lewis acidity to sodium hydride and a saline character to aluminium hydride. All four positions must be checked together.

Step 4: Conclusion The sequence is sodium hydride–saline, silane–molecular, vanadium hydride–non-stoichiometric, and aluminium hydride–Lewis acid. Hence, the correct option is (A).

Quick Tip to Solve: Use one representative per class: NaH is saline, silane is molecular, transition-metal hydrides are often interstitial, and electron-deficient AlH3 is a Lewis acid.

77. Correct statement about alkali metal oxides is : [4 marks]

(A) peroxides are colored.

(B) superoxides are paramagnetic.

(C) oxides are paramagnetic.

(D) peroxides are both colored and paramagnetic.

Correct Answer: (B) superoxides are paramagnetic.

Solution:

Concept:

- A species with one or more unpaired electrons is paramagnetic; a species with all electrons paired is diamagnetic.
- The anions in alkali-metal oxides, peroxides and superoxides are respectively O^{2-}, ${O_2}^{2-}$ and ${O_2}^{-}$. Their electron counts determine their magnetic behaviour.

Step 1: Compare the electron arrangements of the oxygen species The oxide ion O^{2-} has ten electrons and the closed-shell electronic configuration of neon. All its electrons are paired. For peroxide, the two additional electrons relative to O_2 fill the two antibonding π^* orbitals as pairs, so ${O_2}^{2-}$ is also diamagnetic. Superoxide has only one additional electron relative to O_2: its two π^* orbitals together contain three electrons, leaving one unpaired. The alkali-metal cation is closed shell, so it does not remove this unpaired-electron contribution.

Step 2: Identify the magnetic statement that must be true The unpaired electron in ${O_2}^{-}$ makes alkali-metal superoxides paramagnetic. For example, a formula such as KO_2 contains the superoxide anion, whereas K_2O_2 contains peroxide and K_2O contains oxide. Keeping these formulas distinct prevents attributing the magnetism of one oxygen species to another. Magnetism follows the electronic structure of the anion, rather than simply the presence of oxygen or the metallic character of the counter-ion.

Step 3: Eliminate the remaining statements Option C claims that ordinary oxides are paramagnetic, but their oxide ions have closed shells, so it is false. Option D requires peroxides to be paramagnetic, which conflicts with their paired electrons and is false regardless of the colour claim. The blanket colour statement in option A is not the correct characteristic of alkali-metal peroxides in this classification; the characteristic coloured, paramagnetic species are the superoxides. Colour alone is a less reliable

discriminator than the definite molecular-orbital electron count. Option B states the robust correct magnetic property.

Step 4: Conclusion Alkali-metal superoxides are paramagnetic because O_2^- contains one unpaired electron. Hence, the correct option is (B).

Quick Tip to Solve: Remember the sequence: oxide is closed shell, peroxide has paired antibonding electrons, and superoxide has one unpaired electron.

78. Which one of the following chemicals has not been used for the dry cleaning of clothes? [4 marks]

(A) Tetrachloroethene

(B) Liquid CO_2

(C) Ethanol

(D) H_2O_2

Correct Answer: (C) Ethanol

Solution:

Concept:

- Dry cleaning removes oily dirt using a suitable cleaning medium instead of ordinary aqueous washing. The conventional solvent is tetrachloroethene.
- Green chemistry replaces hazardous cleaning materials with safer alternatives. Liquid carbon dioxide with a suitable detergent is an alternative cleaning system, and hydrogen peroxide is used in improved laundering and bleaching.

Step 1: Identify the established cleaning chemicals Tetrachloroethene, also called perchloroethylene, has been widely used as a dry-cleaning solvent because it dissolves grease. Thus option (A) cannot answer the negative question. Liquid CO_2 is used with an appropriate detergent as an environmentally preferable alternative, so option (B) is also excluded. These choices refer to different cleaning technologies rather than to ingredients mixed in one bath.

Step 2: Distinguish ethanol from the peroxide application Hydrogen peroxide, H_2O_2, replaces chlorine-containing bleaching chemicals in improved laundry practice and is included among the cleaning chemicals in this context. It therefore does not supply the intended exception. Ethanol is an organic solvent used in many other applications, but is not the dry-cleaning chemical in the listed conventional and green cleaning methods. Hence (C), rather than (D), is the required negative choice.

Step 3: Conclusion Ethanol has not been used for the dry cleaning of clothes in the cleaning methods considered here. Hence, the correct option is (C).

Quick Tip to Solve: For a negative green-chemistry question, first recall the conventional solvent and its carbon-dioxide alternative, then identify the exception.

79. TLC analysis of a mixture having 3 components (A, B, C) using silica gel as the stationary phase gave following R_f values; for A = 0.72, B = 0.48, C = 0.20.

Regarding the above observations, which one of the following statements is not correct for column chromatography of the mixture? [4 marks]

(A) A is the most non-polar component

(B) C is the most soluble component in mobile phase

(C) A will be eluted first

(D) C is the most strongly absorbed component

Correct Answer: (B) C is the most soluble component in mobile phase

Solution:

Concept:

- For thin-layer chromatography, R_f is the distance travelled by the component divided by the distance travelled by the solvent front.
- Silica gel is a polar stationary phase. A component retained strongly by silica travels less, has a smaller R_f, and generally elutes later in normal-phase column chromatography with the same mobile phase.

Step 1: Order the components by mobility The given values satisfy $R_f(A) = 0.72 > R_f(B) = 0.48 > R_f(C) = 0.20$. Therefore A spends relatively more time travelling with the mobile phase and less time retained on silica. C shows the greatest retention and the lowest relative mobility. Under the same normal-phase conditions, the elution order is A, followed by B, followed by C.

Step 2: Test each statement against retention A is the least polar of these components in the stated silica-gel system, consistent with its largest R_f, so (A) is correct. The weakly retained A emerges from the column first, so (C) is correct. C interacts most strongly with the stationary phase; the word "absorbed" in (D) refers here to adsorption and retention on silica. Thus (D) expresses the expected retention order. Statement (B) reverses the observation: C is not the component with the greatest relative preference for the mobile phase, because it has travelled the least.

Step 3: Conclusion The incorrect statement is "C is the most soluble component in mobile phase." Hence, the correct option is (B).

Quick Tip to Solve: On polar silica, the largest TLC R_f normally predicts the earliest column elution; the smallest predicts the strongest retention.

80. Match List-I with List-II.

	List-I: Polymer		List-II: Repeat unit
(a)	**Acrilan**	**(i)**	$[-CH_2-C(Cl)=CH-CH_2-]_n$
(b)	**Neoprene**	**(ii)**	$[-CH_2-CH(C_6H_5)-]_n$
(c)	**Polystyrene**	**(iii)**	$[-CH_2-CH(CN)-]_n$
(d)	**Buna-N**	**(iv)**	$[-CH_2-CH=CH-CH_2-CH_2-CH(CN)-]_n$

Choose the correct match from the options given below: [4 marks]

(A) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(B) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

(C) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(D) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

Correct Answer: (C) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

Solution:

Concept:

- Addition polymerisation preserves characteristic substituents of the monomer while joining its carbon skeleton into the polymer chain.
- Acrilan is polyacrylonitrile; neoprene is polychloroprene; polystyrene comes from styrene; Buna-N is the copolymer of butadiene and acrylonitrile.

Step 1: Identify the two substituted ethene polymers Acrylonitrile has the monomer formula $CH_2=CH-CN$. Its polymer contains the repeat unit $-CH_2-CH(CN)-$, which is structure (iii). Therefore (a) matches (iii). Styrene has a phenyl group attached to the vinyl carbon, $C_6H_5-CH=CH_2$, and produces $-CH_2-CH(C_6H_5)-$, which is structure (ii). Hence (c) matches (ii). The nitrile and phenyl substituents are the quickest way to distinguish these two structures.

Step 2: Identify the rubber repeat units and eliminate options Neoprene is formed from chloroprene, $CH_2=C(Cl)-CH=CH_2$. Its 1,4-addition repeat unit retains chlorine and a carbon-carbon double bond, $-CH_2-C(Cl)=CH-CH_2-$, shown in (i). Buna-N contains both butadiene-derived and acrylonitrile-derived segments, so it matches the longer nitrile-containing structure (iv). Option (A) exchanges the two rubber structures; (B) exchanges neoprene and polystyrene; (D) assigns Acrilan to the phenyl-containing structure. Only (C) matches all four polymers.

Step 3: Conclusion The matching is Acrilan–(iii), Neoprene–(i), Polystyrene–(ii), Buna-N–(iv). Hence, the correct option is (C).

Quick Tip to Solve: Identify the diagnostic side group first: nitrile for Acrilan, phenyl for polystyrene, and chlorine for neoprene.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Blister copper is produced by reaction of copper oxide with copper sulphide.

$2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$

When 2.86×10^3 g of Cu_2O and 4.77×10^3 g of Cu_2S are used for reaction, the mass of copper produced is _____ g. (nearest integer)

(Atomic mass of Cu = 63.5 a.m. u, S = 32.0 a.m. u, O = 16.0 a.m. u) [4 marks]

Solution:

Concept:

- The balanced reaction is $2\,Cu_2O + Cu_2S \longrightarrow 6\,Cu + SO_2$. Two moles of copper(I) oxide consume one mole of copper(I) sulphide and yield six moles of copper.
- The limiting reactant is found by comparing available moles divided by their stoichiometric coefficients. Only that reactant determines the maximum copper yield.

Step 1: Convert the given reactant masses to moles Using the specified atomic masses, the molar masses are

$$M(Cu_2O) = 2(63.5) + 16 = 143\ \text{g mol}^{-1}, \qquad M(Cu_2S) = 2(63.5) + 32 = 159\ \text{g mol}^{-1}.$$

Thus

$$n(Cu_2O) = \frac{2860}{143} = 20\ \text{mol}, \qquad n(Cu_2S) = \frac{4770}{159} = 30\ \text{mol}.$$

These amounts must be compared in the reaction ratio, rather than by mass alone.

Step 2: Find the limiting reagent and calculate copper The reaction extents supported by the reactants are $20/2 = 10$ mol from the oxide and $30/1 = 30$ mol from the sulphide. Therefore Cu_2O is limiting. Only 10 mol of sulphide are needed to consume all 20 mol of oxide, leaving sulphide in excess. The copper formed is

$$n(Cu) = 20 \times \frac{6}{2} = 60\ \text{mol}.$$

Its mass is

$$m(Cu) = 60(63.5) = 3810\ \text{g}.$$

The calculation counts copper contributed by both reactants through the balanced equation.

Step 3: Conclusion The mass of copper produced is $\boxed{3810\ \text{g}}$, so the nearest-integer entry is 3810.

Quick Tip to Solve: Compare moles divided by coefficients to choose the limiting reagent; using the larger mass or the smaller mole count alone can mislead.

82. Amongst the following, the number of molecule/(s) having net resultant dipole moment is _____.

NF_3, BF_3, BeF_2, $CHCl_3$, H_2S, SiF_4, CCl_4, PF_5 [4 marks]

Solution:

Concept:

- The molecular dipole moment is the vector sum of bond dipoles. Polar bonds do not guarantee a polar molecule: molecular symmetry can cancel their contributions.
- VSEPR geometry distinguishes symmetric linear, trigonal-planar, tetrahedral and trigonal-bipyramidal molecules from bent and trigonal-pyramidal molecules.

Step 1: Identify the molecules with nonzero resultant dipoles NF_3 is trigonal pyramidal because nitrogen has a lone pair. Its bond dipoles do not cancel completely, so its dipole moment is small but nonzero. $CHCl_3$ is tetrahedral with three chlorine atoms and one hydrogen atom; unequal substituents remove complete tetrahedral cancellation, giving a net dipole. H_2S is bent because sulfur has two lone pairs, and its two S–H bond dipoles have a nonzero resultant. These are three polar molecules.

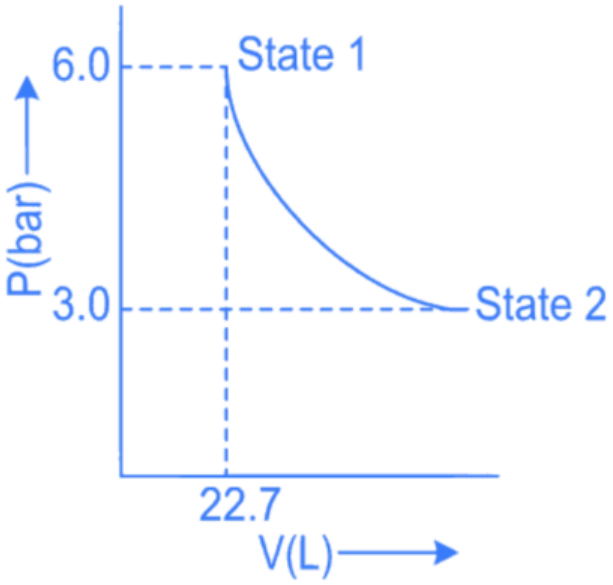
Step 2: Check cancellation in the remaining five molecules BF_3 is trigonal planar with three identical B–F bonds whose vectors cancel. BeF_2 is linear with equal opposite bond dipoles. Both SiF_4 and CCl_4 are regular tetrahedral molecules with four identical outer atoms, so their bond dipoles cancel. In trigonal-bipyramidal PF_5, the two axial dipoles cancel each other, while the three equatorial dipoles cancel in their plane. Hence none of these five adds to the polar-molecule count.

Step 3: Conclusion The molecules with net dipole moments are NF_3, $CHCl_3$, and H_2S. Their number is $\boxed{3}$ (a dimensionless count).

Quick Tip to Solve: Count a molecule as polar only after checking its geometry and identical substituents; a lone pair can prevent cancellation, but bond polarity alone is insufficient.

83. 1.0 mol of monoatomic ideal gas is expanded from state 1 to state 2 as shown in the figure. The magnitude of the work done for the expansion of gas from state 1 to state 2 at 300 K is _____ J. (Nearest integer)

(Given : R = 8.3 J K^{-1} mol^{-1}, ln10 = 2.3, log2 = 0.30) [4 marks]



Solution:

Concept:

- For a reversible isothermal expansion of an ideal gas, the magnitude of work is $|w| = nRT\ln(V_2/V_1)$. Temperature remains constant along the isothermal path.
- At a fixed temperature, Boyle's law gives $P_1V_1 = P_2V_2$, so the volume ratio can be obtained directly from the pressure ratio without converting litres to cubic metres.

Step 1: Obtain the volume ratio from the graph The plotted states have pressures $P_1 = 6.0$ bar and $P_2 = 3.0$ bar. For the isothermal expansion represented by the falling curve,

$$\frac{V_2}{V_1} = \frac{P_1}{P_2} = \frac{6.0}{3.0} = 2.$$

Thus the gas doubles its volume. Only this ratio is required for the logarithmic work formula; the initial volume label does not need to be inserted separately. The pressure decreases as the gas expands, so work is found by integration, not by multiplying the final pressure by the volume change.

Step 2: Evaluate the logarithm and work magnitude The given common logarithm must be converted to a natural logarithm:

$$\ln 2 = (\log_{10} 2)(\ln 10) = (0.30)(2.3) = 0.69.$$

With one mole of gas at 300 K,

$$|w| = (1.0)(8.3)(300)(0.69) = 1718.1 \text{ J}.$$

In the chemistry convention, work on the expanding system is negative, $w = -1718.1$ J, but the question explicitly asks for its magnitude. The monoatomic nature is not needed for this isothermal-

work expression.

Step 3: Conclusion The magnitude of the expansion work is $\boxed{1718\ \text{J}}$ to the nearest integer.

Quick Tip to Solve: For isothermal work, replace the volume ratio by the inverse pressure ratio and convert a supplied common logarithm using $\ln x = 2.303 \log_{10} x$.

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84. For the reaction P $\rightarrow$ B, the values of frequency factor A and activation energy E_A are 4×10^{13} s^{-1} and 8.3 kJ mol^{-1} respectively. If the reaction is of first order, the temperature at which the rate constant is 2×10^{-6} s^{-1} is _____ $\times 10^{-1}$ K.

(Given : ln 10 = 2.3, R = 8.3 J K^{-1} mol^{-1}, log2 = 0.30) [4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_A/(RT)]$, where k is the rate constant, A is the frequency factor and E_A is the activation energy.
- Taking natural logarithms gives $\ln(A/k) = E_A/(RT)$. Activation energy must be in joules per mole when R is in $\text{J}\,\text{K}^{-1}\,\text{mol}^{-1}$.

Step 1: Calculate the logarithmic factor The activation energy is $8.3\ \text{kJ}\,\text{mol}^{-1} = 8300\ \text{J}\,\text{mol}^{-1}$. Both A and k have units of s^{-1}, so their ratio is dimensionless:

$$\frac{A}{k} = \frac{4 \times 10^{13}}{2 \times 10^{-6}} = 2 \times 10^{19}.$$

The logarithm is therefore

$$\ln(A/k) = \ln 2 + 19 \ln 10 = (0.30)(2.3) + 19(2.3) = 44.39.$$

The power is positive 19 because division by 10^{-6} raises the power by six.

Step 2: Solve for temperature and convert to the requested entry Rearranging the logarithmic Arrhenius equation gives

$$T = \frac{E_A}{R\ln(A/k)} = \frac{8300}{8.3(44.39)} = \frac{1000}{44.39} = 22.5276\ \text{K}.$$

The blank multiplies 10^{-1} K. If that blank is x, then $T = 0.1x$ K, and therefore

$$x = 10(22.5276) = 225.276\ldots.$$

Rounding this coefficient to the nearest integer yields 225; reporting 23 would round the temperature rather than the quantity asked for.

Step 3: Conclusion The temperature is approximately 22.53 K, or 225.3×10^{-1} K. The required nearest-integer entry is $\boxed{225}$.

Quick Tip to Solve: After solving Arrhenius' equation, inspect the unit multiplier beside the blank: a value expressed in 10^{-1} K requires ten times the numerical kelvin value.

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85. In the following brown complex, the oxidation state of iron is +_____.

$$[Fe(H_2O)_6]^{2+} + NO \rightarrow [Fe(H_2O)_5(NO)]^{2+} + H_2O$$

Brown complex

[4 marks]

Solution:

Concept:

- Oxidation state is formal electron bookkeeping: the sum of the metal oxidation state and ligand charges equals the charge on the coordination entity.
- In the brown-ring nitrosyl complex, the nitrosyl ligand is assigned as NO^+. Each coordinated water molecule is electrically neutral.

Step 1: Identify the charged coordination entity The product is $[Fe(H_2O)_5(NO)]^{2+}$, so the complete coordination entity has charge $+2$. The five water ligands contribute zero to the charge sum. Water displaced during substitution is neutral as well and is outside the product coordination sphere.

Step 2: Assign the nitrosyl ligand charge Use the conventional nitrosyl description NO^+ for this complex. Nitric oxide is a non-innocent ligand: its formal charge in a metal complex cannot be taken to be zero merely because the incoming free molecule is neutral. In this oxidation-state assignment, the coordinated nitrosyl contributes $+1$.

Step 3: Apply charge balance Let the oxidation state of iron be x. Then the ligand charges and metal charge must reproduce the ionic charge:

$$x + 5(0) + (+1) = +2.$$

$$x = +1.$$

The initial aqua ion contains iron in the $+2$ state, but carrying that initial value unchanged into the nitrosyl product would neglect the formal electronic redistribution used to describe the nitrosyl bond.

Step 4: Conclusion The oxidation state of iron in the brown complex is $\boxed{+1}$. Thus the number to enter after the printed plus sign is $\boxed{1}$.

Quick Tip to Solve: For a nitrosyl complex, assign the nitrosyl charge before balancing the metal oxidation state; neutral incoming NO does not automatically mean a neutral coordinated ligand.

86. Spin only magnetic moment (μ_s) of $K_3[Fe(CN)_6]$ is _____ B.M.
(Nearest integer)

[4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu_s = \sqrt{n(n+2)}\,\text{B.M.}$, where n is the number of unpaired electrons.
- Cyanide is a strong-field ligand. In an octahedral iron(III) cyanide complex, electron pairing gives

a low-spin d^5 configuration.

Step 1: Find the oxidation state and electron count Three K^+ ions balance the complex anion $[Fe(CN)_6]^{3-}$. Each cyanide ligand has charge -1. If the oxidation state of iron is x,

$$x + 6(-1) = -3 \quad \Rightarrow \quad x = +3.$$

Iron has atomic number 26, so its neutral configuration is $[Ar]3d^6 4s^2$; removal of three electrons gives Fe^{3+} with $3d^5$.

Step 2: Count the unpaired electrons in the ligand field The six ligands produce an octahedral field. Since CN^- is strong-field, the five d electrons occupy the lower t_{2g} orbitals before the higher e_g orbitals. Thus the configuration is $t_{2g}^5 e_g^0$: two lower orbitals are doubly occupied and the third has one electron. Therefore $n = 1$, rather than the five unpaired electrons of a high-spin d^5 ion.

Step 3: Calculate and round the moment Substitute the unpaired-electron count into the spin-only expression:

$$\mu_s = \sqrt{1(1+2)} = \sqrt{3} = 1.732\,\text{B.M.}$$

The question asks for the nearest integer, so round the final magnetic moment only after evaluating the square root.

Step 4: Conclusion The spin-only moment is approximately $1.73\,\text{B.M.}$, whose nearest integer is $\boxed{2}\,\text{B.M.}$. Enter $\boxed{2}$.

Quick Tip to Solve: Find the metal oxidation state first, then use ligand strength to count unpaired electrons; the free-ion count is incorrect for strong-field cyanide.

87. An organic compound with 51.6% sulfur is heated in a Carius tube. The amount of this compound which will form 0.752 g of barium sulphate is _____ $\times 10^{-1}$ g.
(Given molar mass of barium sulphate 233 g mol^{-1}) (Nearest integer). [4 marks]

Solution:

Concept:

- In the Carius method, sulfur in the organic compound is oxidised to sulfate and measured by precipitation as $BaSO_4$. One mole of precipitate contains one mole of sulfur.
- The mass fraction of sulfur is $w_S = 0.516$, so the sample mass is the recovered sulfur mass divided by 0.516.

Step 1: Calculate the amount of barium sulfate The precipitate has mass $0.752\,\text{g}$ and molar mass $233\,\text{g mol}^{-1}$. Therefore

$$n(BaSO_4) = \frac{0.752}{233} = 3.2275 \times 10^{-3}\,\text{mol}.$$

Every formula unit of barium sulfate contains one sulfur atom; the sulfate oxygen and barium are not part of the sulfur originally present in the organic sample.

Step 2: Convert the precipitate to sulfur mass Using the sulfur molar mass $32\,\mathrm{g\,mol^{-1}}$,

$$m_{\mathrm{S}} = \frac{0.752}{233} \times 32 = 0.10328\,\mathrm{g}.$$

This is the sulfur recovered from the sample, not the mass of the complete organic compound. The remaining elements account for the other 48.4% of its mass.

Step 3: Use the percentage and requested scale If m is the sample mass, its sulfur mass is $0.516m$. Hence

$$m = \frac{0.10328}{0.516} = 0.20015\,\mathrm{g}.$$

Write this in the form requested:

$$m = 2.0015 \times 10^{-1}\,\mathrm{g}.$$

The blank is the coefficient of 10^{-1}, so its nearest integer is obtained by rounding 2.0015, not by rounding the mass in grams.

Step 4: Conclusion The compound mass is approximately $0.200\,\mathrm{g} = 2.0015 \times 10^{-1}\,\mathrm{g}$. The required nearest-integer coefficient is $\boxed{2}$.

Quick Tip to Solve: Convert BaSO4 to sulfur first using the mass ratio $32/233$; then divide by the sulfur fraction and apply the power of ten printed beside the blank.

88. A hydrocarbon 'X' is found to have molar mass of 80. A 10.0 mg of compound 'X' on hydrogenation consumed 8.40 mL of H_2 gas (measured at STP). Ozonolysis of compound 'X' yields only formaldehyde and dialdehyde. The total number of fragments/molecules produced from the ozonolysis of compound 'X' is _____. [4 marks]

Solution:

Concept:

- One mole of hydrogen is consumed per carbon-carbon double bond on complete hydrogenation. At STP, one mole of gas occupies $22.4\,\mathrm{L}$.
- Ozonolysis cleaves each alkene double bond into carbonyl groups. Terminal $\mathrm{CH_2}{=}$ groups yield formaldehyde; an acyclic chain cut at three different double bonds produces four fragments.

Step 1: Find the hydrogen consumed per molecule The hydrocarbon sample has mass $10.0\,\mathrm{mg} = 0.0100\,\mathrm{g}$. Its amount is

$$n_X = \frac{0.0100}{80} = 1.25 \times 10^{-4}\,\mathrm{mol}.$$

The hydrogen amount at STP is

$$n_{\mathrm{H_2}} = \frac{8.40 \times 10^{-3}}{22.4} = 3.75 \times 10^{-4}\,\mathrm{mol}.$$

Thus $n_{\mathrm{H_2}}/n_X = 3$: each hydrocarbon molecule consumes three hydrogen molecules.

Step 2: Identify the unsaturated carbon skeleton The hydrocarbon of molar mass 80 with three

hydrogenatable alkene bonds is C_6H_8, since $6(12) + 8 = 80$. Its degree of unsaturation is $(2 \times 6 + 2 - 8)/2 = 3$, leaving no ring after the three double bonds are counted. The stated ozonolysis products identify the chain as $CH_2{=}CH{-}CH{=}CH{-}CH{=}CH_2$. A triple bond would generate a different carbonyl-product pattern and would not match the stated formaldehyde and dialdehyde products.

Step 3: Count all molecules after cleavage Cleaving the three double bonds of hexa-1,3,5-triene releases one formaldehyde molecule from each terminal carbon. The two internal two-carbon segments each become ethanedial (glyoxal):

$$CH_2{=}CH{-}CH{=}CH{-}CH{=}CH_2 \xrightarrow[\text{work-up}]{O_3} 2\,HCHO + 2\,OHC{-}CHO.$$

There are two molecules of formaldehyde and two molecules of the dialdehyde, even though there are only two distinct product types. Hence the total fragment count is $2 + 2 = 4$.

Step 4: Conclusion Ozonolysis produces two formaldehyde molecules and two ethanedial molecules per molecule of X. The total number of fragments is $\boxed{4}$, a dimensionless count.

Quick Tip to Solve: Distinguish the number of product types from the number of product molecules: two types can arise as four fragments when three double bonds are cleaved.

89. 100 mL of 0.3 M acetic acid is shaken with 0.8 g of wood charcoal. The final concentration of acetic acid in the solution after adsorption is 0.125 M. The mass of acetic acid adsorbed per gram of carbon is _____ $\times$ 10^{-4} g.

(Given : Molar mass of acetic acid = 60 g mol^{-1}) [4 marks]

Solution:

Concept:

- The amount adsorbed is the decrease in the amount of solute remaining in solution, assuming the stated solution volume remains $100\,\text{mL}$.
- Adsorption per unit adsorbent mass is x/m, where x is the mass of acetic acid removed from solution and m is the charcoal mass.

Step 1: Calculate the decrease in dissolved acid Convert the solution volume to litres: $100\,\text{mL} = 0.100\,\text{L}$. The initial amount of acetic acid is

$$n_i = 0.300 \times 0.100 = 0.0300\,\text{mol}.$$

After adsorption the dissolved amount is

$$n_f = 0.125 \times 0.100 = 0.0125\,\text{mol}.$$

The difference represents acid attached to the charcoal rather than acid still in the liquid.

Step 2: Convert the adsorbed amount to mass The amount adsorbed is

$$\Delta n = n_i - n_f = 0.0300 - 0.0125 = 0.0175\,\text{mol}.$$

With molar mass $60\,\text{g mol}^{-1}$, the adsorbed acid mass is

$$x = 0.0175 \times 60 = 1.050\,\text{g}.$$

Using the final concentration alone would calculate the unadsorbed acid and would answer a different question.

Step 3: Express adsorption per gram of charcoal The experiment uses $0.8\,\text{g}$ of charcoal. Hence

$$\frac{x}{m} = \frac{1.050}{0.8} = 1.3125\,\text{g g}^{-1}.$$

For one gram of charcoal this corresponds to $1.3125\,\text{g}$ of acid. The requested coefficient is found from

$$1.3125 = 13125 \times 10^{-4}.$$

Keep this power-of-ten conversion separate from the division by the actual charcoal mass.

Step 4: Conclusion The adsorbed acid mass is $1.3125\,\text{g}$ per gram of charcoal, or $13125 \times 10^{-4}\,\text{g}$ for one gram of carbon. Enter $\boxed{13125}$.

Quick Tip to Solve: Use the concentration drop, not the final concentration; multiply by the solution volume and molar mass before dividing by the charcoal mass.

90. How many of the following drugs belong to the tranquilizer class _____ ?
Veronal, Luminal, Propanal, Seconal [4 marks]

Solution:

Concept:

- Tranquilizers are drugs used to relieve anxiety, stress or related disturbances; the school classification includes barbiturates with sedative and hypnotic actions.
- Veronal, Luminal and Seconal are barbiturate drugs. Propanal is a simple aldehyde and is not a tranquilizer.

Step 1: Classify the first two named substances Veronal is the trade name of barbital, a barbiturate sedative. It belongs to the tranquilizer class in the prescribed classification. Luminal is phenobarbital, another barbiturate; it is also included in that class. Their therapeutic identities, rather than a similarity of spelling, determine their inclusion. Thus the first two entries both contribute to the required count.

Step 2: Check the remaining substances individually Propanal has the condensed structural formula CH_3CH_2CHO. It is an ordinary three-carbon aldehyde, not a member of the barbiturate drug family and not a listed tranquilizer. Seconal is the trade name of secobarbital, which is a barbiturate sedative-

hypnotic; it belongs to the tranquilizer class. The similar endings of Propanal and Seconal do not imply that they have the same chemical or therapeutic classification.

Step 3: Count only the qualifying drugs The included names are Veronal, Luminal and Seconal. Propanal is excluded. Therefore

$$N_{\text{tranquilizers}} = 1 + 1 + 0 + 1 = 3.$$

The question asks for the number among the four listed substances, so no concentration, dose or physical unit enters the result. Counting all four merely because the sentence calls them drugs would overlook the deliberately included aldehyde.

Step 4: Conclusion Veronal, Luminal and Seconal belong to the tranquilizer class, whereas Propanal does not. The required number is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: Recognise the barbiturate trade names Veronal, Luminal and Seconal; Propanal is identified instead by the aldehyde group in CH_3CH_2CHO.