JEE Main 2023 Paper 1 (B.E./B.Tech) Jan 24 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 24 January 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $\mathrm{p}, \mathrm{q} \in \mathbb{R}$ and $\left(1-\sqrt{3}i\right)^{200} = 2^{199}(p+iq), i = \sqrt{-1}$ then $\mathrm{p}+\mathrm{q}+\mathrm{q}^2$ and $\mathrm{p}-\mathrm{q}+\mathrm{q}^2$ are roots of the equation. [4 marks]

(A) $x^2 + 4x - 1 = 0$

(B) $x^2 - 4x + 1 = 0$

(C) $x^2 + 4x + 1 = 0$

(D) $x^2 - 4x - 1 = 0$

Correct Answer: (B) $x^2 - 4x + 1 = 0$

Solution:

Concept:

- De Moivre theorem gives $(r(\cos\theta + i\sin\theta))^m = r^m(\cos m\theta + i\sin m\theta)$. Here the modulus and argument determine the required real numbers.
- If a monic quadratic has roots α, β, its equation is $x^2 - (\alpha+\beta)x + \alpha\beta = 0$. Compute both symmetric expressions to distinguish the signs of the options.

Step 1: Write the complex number in polar form The number $1 - \sqrt{3}i$ has modulus 2 and lies in the fourth quadrant, so its argument is $-\pi/3$. Therefore

$$(1-\sqrt{3}i)^{200} = 2^{200}\left(\cos\frac{200\pi}{3} - i\sin\frac{200\pi}{3}\right).$$

Since $200 = 6(33) + 2$, the angle reduces modulo 2π to $2\pi/3$. It follows that

$$(1-\sqrt{3}i)^{200} = 2^{200}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 2^{199}(-1-\sqrt{3}i).$$

Comparing real and imaginary parts with the given expression yields $p = -1$ and $q = -\sqrt{3}$.

Step 2: Form the quadratic from the two roots We have $q^2 = 3$, so the required roots are

$$\alpha = p + q + q^2 = 2 - \sqrt{3}, \qquad \beta = p - q + q^2 = 2 + \sqrt{3}.$$

Their sum is 4, and their product is $4 - 3 = 1$. Substitution into the monic quadratic formula gives

$$x^2 - 4x + 1 = 0.$$

Option (A) has the wrong root sum and product; option (C) has the wrong root sum; option (D) has the wrong product. Thus the signs of both coefficients are essential, even though the roots are obtained from conjugate expressions.

Step 3: Conclusion The required equation is $x^2 - 4x + 1 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Reduce the power modulo six before evaluating this complex number, then use the sum and product of the conjugate roots.

2. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$$x + y + z = 1$$

$$2x + \mathrm{N}y + 2z = 2$$

$$3x + 3y + \mathrm{N}z = 3$$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is : [4 marks]

(A) 18

(B) 21

(C) 20

(D) 19

Correct Answer: (C) 20

Solution:

Concept:

- A square linear system has a unique solution precisely when the determinant of its coefficient matrix is nonzero. The right-hand constants do not affect this criterion.
- For a fair die, each of the six outcomes has probability $1/6$. Count only the values satisfying the determinant condition when finding the probability and the requested sum.

Step 1: Find the determinant condition The coefficient matrix is

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{pmatrix}.$$

Apply $R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - 3R_1$. These operations preserve the determinant and give

$$\det M = \begin{vmatrix} 1 & 1 & 1 \\ 0 & N-2 & 0 \\ 0 & 0 & N-3 \end{vmatrix} = (N-2)(N-3).$$

Hence a unique solution exists exactly when $N \neq 2$ and $N \neq 3$. At either excluded value the coefficient matrix is singular; a consistent singular system still cannot have a unique solution.

Step 2: Count and add the admissible outcomes The admissible die outcomes are $1, 4, 5, 6$, giving four favourable outcomes out of six equally likely outcomes. Thus

$$P(\text{unique solution}) = \frac{4}{6} = \frac{k}{6}, \qquad k = 4.$$

The sum of the possible values of N for which the stated unique-solution event occurs is $1+4+5+6 =$

16. The required combined sum is therefore

$$k + 1 + 4 + 5 + 6 = 4 + 16 = 20.$$

Summing all six die outcomes instead would include the two singular cases and would not answer the conditional requirement. The other listed numbers 18, 21 and 19 do not equal the result obtained from the determinant and the four valid outcomes.

Step 3: Conclusion The required sum is 20. Hence, the correct option is (C).

Quick Tip to Solve: Subtract multiples of the first row before expanding the determinant; its two forbidden die outcomes then appear immediately.

3. The relation $\mathrm{R} = \{(\mathrm{a}, \mathrm{b}) : \gcd(\mathrm{a}, \mathrm{b}) = 1, 2\mathrm{a} \neq \mathrm{b}, \mathrm{a}, \mathrm{b} \in \mathbb{Z}\}$ **is :** [4 marks]

(A) reflexive but not symmetric

(B) transitive but not reflexive

(C) symmetric but not transitive

(D) neither symmetric nor transitive

Correct Answer: (D) neither symmetric nor transitive

Solution:

Concept:

- A relation is symmetric if every related pair (a, b) also gives (b, a). One counterexample disproves symmetry, even when part of the defining condition is symmetric.
- Transitivity requires that $(a, b) \in R$ and $(b, c) \in R$ imply $(a, c) \in R$. Reflexivity requires $(a, a) \in R$ for every integer a.

Step 1: Test symmetry using the extra restriction The condition $\gcd(a, b) = 1$ is symmetric, but the additional condition $2a \neq b$ need not remain true after interchanging the entries. Choose $a = 2, b = 1$. Then

$$\gcd(2, 1) = 1, \qquad 2(2) = 4 \neq 1,$$

so $(2, 1) \in R$. For the reverse pair $(1, 2)$, the gcd condition still holds but $2(1) = 2$, which violates the required inequality. Thus $(1, 2) \notin R$, and the relation is not symmetric. Merely observing the symmetry of the gcd would overlook a defining restriction.

Step 2: Test transitivity and reflexivity explicitly Take the successive pairs $(2, 3)$ and $(3, 4)$. They belong to the relation because

$$\gcd(2, 3) = \gcd(3, 4) = 1, \qquad 4 \neq 3, \quad 6 \neq 4.$$

However, $(2, 4) \notin R$: its gcd is 2, and it also violates $2a \neq b$. This is a direct counterexample to transitivity. The relation is also not reflexive, since $(2, 2) \notin R$ as its gcd is 2. Therefore option (A) fails its reflexivity claim, option (B) fails its transitivity claim, and option (C) fails its symmetry claim. All these checks use integers in the stated domain and retain both conditions of the relation.

Step 3: Conclusion The relation is neither symmetric nor transitive. Hence, the correct option is (D).

Quick Tip to Solve: For a relation defined by several conditions, test all conditions after reversing a pair; a symmetric gcd condition alone is insufficient.

---

4. $\lim_{t\to 0}\left(1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+\ldots+n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ **is equal to** [4 marks]

(A) $\frac{n(n+1)}{2}$

(B) n

(C) n^2 + n

(D) n^2

Correct Answer: (B) n

Solution:

Concept:

- For nonnegative numbers, the largest term is at most their sum, which is at most the number of terms times the largest term. This estimate isolates the dominant base.
- The squeeze theorem states that if lower and upper bounds tend to the same value, the bounded expression has that limit. Here $\sin^2 t \to 0^+$.

Step 1: Bound the sum by its largest term Write $u = \sin^2 t$, so $u > 0$ for sufficiently small nonzero t, and $u \to 0^+$. Set $S = 1^{1/u} + 2^{1/u} + \cdots + n^{1/u}$, where n is the fixed positive integer in the question. Since each term is positive and no term exceeds $n^{1/u}$,

$$n^{1/u} \le S \le n\, n^{1/u}.$$

Raising the entire inequality to the positive power u preserves its direction. We obtain

$$n \le S^u \le n^{1+u}.$$

This approach is valid from both sides of $t = 0$, because the squared sine is positive on both sides.

Step 2: Take limits of both bounds The lower bound is the constant n. For the upper bound, continuity of the exponential function gives

$$\lim_{u\to 0^+} n^{1+u} = n \lim_{u\to 0^+} e^{u\log n} = n.$$

Hence the squeeze theorem gives $\lim S^u = n$. In particular, the growing exponent inside the sum cannot be replaced by zero or by one; the largest base determines the result after the final small power is taken. The sum $n(n+1)/2$ would arise from adding first powers instead, and the expressions $n^2 + n$ and n^2 do not satisfy the squeeze bounds for general n. For $n = 1$, the same estimate gives the limit directly as 1.

Step 3: Conclusion The given limit equals n. Hence, the correct option is (B).

Quick Tip to Solve: For a sum of large positive powers followed by the reciprocal power, bound the sum between its largest term and the term count times that term.

---

5. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ **is equal to :** [4 marks]

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

Correct Answer: (B) $\frac{\pi}{3}$

Solution:

Concept:

- Inverse trigonometric values must lie in their principal ranges. For a positive tangent argument the arctangent lies in $(0,\pi/2)$; a secant argument greater than one also gives an acute principal angle.
- The standard values $\tan(\pi/6)=1/\sqrt{3}$ and $\sec(\pi/6)=2/\sqrt{3}$ identify the angles after the radicals are simplified exactly.

Step 1: Simplify the argument of the arctangent Observe that $3+\sqrt{3}=\sqrt{3}(1+\sqrt{3})$. Cancelling the nonzero common factor yields

$$\frac{1+\sqrt{3}}{3+\sqrt{3}}=\frac{1}{\sqrt{3}}.$$

Since $\pi/6$ lies within the principal arctangent range $(-\pi/2,\pi/2)$,

$$\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)=\frac{\pi}{6}.$$

The branch check matters: adding a multiple of π would retain the tangent value but would not be the inverse function specified in the question.

Step 2: Simplify the secant argument and add The second fraction has a common positive factor $2+\sqrt{3}$:

$$\frac{8+4\sqrt{3}}{6+3\sqrt{3}}=\frac{4(2+\sqrt{3})}{3(2+\sqrt{3})}=\frac{4}{3}.$$

Its positive square root is $2/\sqrt{3}$, which is greater than one. Thus the principal inverse secant gives

$$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=\frac{\pi}{6}.$$

Adding the two terms gives $\pi/6+\pi/6=\pi/3$. Option (C) counts only one of the two equal angles; option (A) is too large and option (D) is not the sum of the identified principal values. No approximate decimal evaluation or inverse-angle addition formula is necessary once the common factors are recognized.

Step 3: Conclusion The expression equals $\pi/3$. Hence, the correct option is (B).

Quick Tip to Solve: Factor the numerator and denominator before using inverse trigonometric identities; both arguments here reduce to standard values.

6. Let $y = y(x)$ be the solution of the differential equation $x^3dy + (xy - 1)dx = 0, x > 0, y\left(\frac{1}{2}\right) = 3 - e$. Then y (1) is equal to [4 marks]

(A) $2 - e$

(B) 3

(C) 1

(D) e

Correct Answer: (C) 1

Solution:

Concept:

- A first-order equation $y\prime + P(x)y = Q(x)$ has integrating factor $I = e^{\int P(x)dx}$, making $(Iy)\prime = IQ(x)$.
- The initial condition fixes the integration constant. The restriction $x > 0$ allows division by x^3 throughout the interval containing both required arguments.

Step 1: Apply the integrating factor Dividing the equation by x^3dx gives

$$y\prime + \frac{y}{x^2} = \frac{1}{x^3}.$$

The integrating factor is $I = e^{-1/x}$, because $\int x^{-2}dx = -1/x$. Consequently

$$\frac{d}{dx}\left(ye^{-1/x}\right) = \frac{e^{-1/x}}{x^3}.$$

Put $u = 1/x$, so $du = -dx/x^2$. The integral on the right becomes

$$-\int ue^{-u}du = (u + 1)e^{-u} + C.$$

Hence

$$ye^{-1/x} = \left(1 + \frac{1}{x}\right)e^{-1/x} + C, \qquad y = 1 + \frac{1}{x} + Ce^{1/x}.$$

The positive exponential in the last term results from dividing by the integrating factor, not changing its sign arbitrarily.

Step 2: Use the initial value and evaluate at one At $x = 1/2$, the formula yields

$$3 - e = 1 + 2 + Ce^2, \qquad C = -\frac{1}{e}.$$

Therefore

$$y(x) = 1 + \frac{1}{x} - e^{1/x-1}, \qquad y(1) = 2 - 1 = 1.$$

As a check, differentiating gives $y\prime = -x^{-2} + e^{1/x-1}x^{-2}$. Adding y/x^2 cancels the exponential and constant terms, leaving exactly x^{-3}. The initial value also checks as $3 - e$. Option (A) does not equal the evaluation of this solution, while options (B) and (D) likewise fail the unique value obtained after fixing C.

Step 3: Conclusion The required value is $y(1) = 1$. Hence, the correct option is (C).

Quick Tip to Solve: When the integrating factor contains $e^{-1/x}$, use $u = 1/x$ to integrate the remaining rational-exponential product.

7. Let Ω be the sample space and $A \subseteq \Omega$ be an event.

Given below are two statements :

(S1) : If P(A) = 0, then A = ϕ

(S2) : If P(A) = 1, then A = Ω

Then : [4 marks]

(A) both (S1) and (S2) are true

(B) both (S1) and (S2) are false

(C) only (S2) is true

(D) only (S1) is true

Correct Answer: (B) both (S1) and (S2) are false

Solution:

Concept:

- Probability measures assign $P(\varnothing) = 0$ and $P(\Omega) = 1$, but the converse implications need not hold on a general sample space.
- For a variable uniformly distributed on $[0, 1]$, intervals have probability equal to length and a singleton has probability zero. Removing such a singleton does not change total probability.

Step 1: Test the first statement with a nonempty null event The question does not restrict the sample space to finitely many equally likely outcomes. Consider choosing a real number uniformly from $\Omega = [0, 1]$, and let $A = \{1/2\}$. This is a nonempty event. Nevertheless, for every $\varepsilon > 0$, its probability is bounded by the probability of an interval about $1/2$ of length at most 2ε. Thus $0 \leq P(A) \leq 2\varepsilon$ for arbitrarily small positive ε, giving $P(A) = 0$. The event is still not the empty set, so (S1) is false.

Step 2: Test the second statement by taking a complement In the same sample space, take $B = [0, 1] \setminus \{1/2\}$. Since the omitted singleton has probability zero,

$$P(B) = 1 - P(\{1/2\}) = 1.$$

Yet $B \neq \Omega$, because it does not contain $1/2$. This disproves (S2). An event with probability one is called almost sure; it can exclude outcomes of zero probability. In a finite equiprobable experiment the two statements would hold, but that additional hypothesis is absent here and cannot be imposed. Thus option (A), and both mixed alternatives (C) and (D), each incorrectly asserts at least one universal implication. The counterexamples distinguish statements about set equality from statements about probability.

Step 3: Conclusion Both (S1) and (S2) are false in general. Hence, the correct option is (B).

Quick Tip to Solve: To test a claimed converse involving probability zero or one, use a singleton in a continuous uniform sample space and then take its complement.

8. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is : [4 marks]

(A) $\frac{22}{3}$

(B) 9

(C) $\frac{23}{3}$

(D) $\frac{25}{3}$

Correct Answer: (B) 9

Solution:

Concept:

- For curves expressed as $x = x(y)$, the enclosed area is $\int (x_{\text{right}} - x_{\text{left}})\, dy$. Horizontal strips avoid splitting the region into separate integrals.
- Intersection values give the integration limits. An area is nonnegative, so the order of the two boundary functions must be checked inside the interval.

Step 1: Find the intersection ordinates The parabola and line can be written as $x = 1 - y^2/4$ and $x = y/2 - 1$, respectively. Equating their abscissae gives

$$1 - \frac{y^2}{4} = \frac{y}{2} - 1 \quad \Longrightarrow \quad y^2 + 2y - 8 = (y+4)(y-2) = 0.$$

Thus the ordinates of intersection are -4 and 2. The corresponding points are $(-3, -4)$ and $(0, 2)$.

Step 2: Integrate the horizontal width At $y = 0$, the parabola has $x = 1$, while the line has $x = -1$; hence the parabola forms the right boundary. Moreover, the width is $-(y+4)(y-2)/4$, which is nonnegative throughout $[-4, 2]$. Therefore

$$\begin{aligned} A &= \int_{-4}^{2} \left(2 - \frac{y}{2} - \frac{y^2}{4}\right) dy \\ &= \left[2y - \frac{y^2}{4} - \frac{y^3}{12}\right]_{-4}^{2} \\ &= \frac{7}{3} - \left(-\frac{20}{3}\right) = 9. \end{aligned}$$

The other three proposed fractions do not equal this definite integral; retaining both the negative lower limit and the cubic term is essential.

Step 3: Conclusion The enclosed area is 9 square units. Hence, the correct option is (B).

Quick Tip to Solve: When both boundaries are simple functions of y, integrate right minus left with respect to y to avoid unnecessary subdivisions.

9. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has : [4 marks]

(A) exactly two solutions in $(-\infty, \infty)$

(B) no solution

(C) a unique solution in $(-\infty, \infty)$

(D) a unique solution in $(-\infty, 1)$

Correct Answer: (C) a unique solution in $(-\infty, \infty)$

Solution:

Concept:

- On the interval $[n, n+1)$, the greatest integer $[x]$ is the fixed integer n. Every algebraic root obtained in that interval must satisfy this interval restriction.
- Factoring after replacing the greatest integer by an integer can separate a root independent of the interval from a root that contradicts the definition of the greatest integer.

Step 1: Factor with the integer part fixed Let $n = [x]$. Then $n \le x < n+1$, and the given equation becomes

$$x^2 - (n+4)x + (n+3) = 0.$$

The coefficient pattern permits the exact factorisation

$$(x-1)(x-n-3) = 0.$$

Consequently every possible solution must satisfy either $x = 1$ or $x = n+3$. This factorisation is valid for every integer n, including negative integers, so it does not discard any range of the real line.

Step 2: Check each root against its integer part For $x = 1$, the integer part is $n = 1$, so the restriction $n \le x < n+1$ becomes $1 \le 1 < 2$, which holds. Direct substitution also gives $1 - 4 + 1 + 3 = 1 = 1 \cdot 1$. For the second candidate, $x = n+3$, the required inequality $x < n+1$ would give $n+3 < n+1$, or $3 < 1$, an impossibility. Thus no second root can belong to the interval from which its quadratic was formed. In particular there is no root below 1, since the only surviving root is the endpoint 1 itself. This rules out the claim of exactly two roots, the claim of no roots, and the claim of a unique root in the open interval $(-\infty, 1)$.

Step 3: Conclusion The equation has exactly one real solution, $x = 1$, in $(-\infty, \infty)$. Hence, the correct option is (C).

Quick Tip to Solve: After solving a greatest-integer equation with $n = [x]$, impose $n \le x < n+1$ on every candidate; algebraic roots alone are insufficient.

10. If A and B are two non-zero n $\times$ n matrices such that $\mathrm{A}^2 + \mathrm{B} = \mathrm{A}^2\mathrm{B}$, then : [4 marks]

(A) $\mathrm{A}^2\mathrm{B} = \mathrm{I}$

(B) $\mathrm{A}^2 = \mathrm{I}$ or $\mathrm{B} = \mathrm{I}$

(C) $\mathrm{A}^2\mathrm{B} = \mathrm{B}\mathrm{A}^2$

(D) $\mathrm{AB} = \mathrm{I}$

Correct Answer: (C) $\mathrm{A}^2\mathrm{B} = \mathrm{B}\mathrm{A}^2$

Solution:

Concept:

- Matrix multiplication is associative but generally not commutative. A rearrangement must therefore preserve the order of the factors until commutation is established.
- A square matrix with a right inverse is invertible. In particular, $XY = I$ for square matrices implies $YX = I$, for example by taking determinants and using inverse uniqueness.

Step 1: Produce a product equal to the identity Write $X = A^2$. The hypothesis is $X + B = XB$. Add the identity matrix to the difference $XB - X - B$ to obtain

$$(X - I)(B - I) = XB - X - B + I = I.$$

Both factors are square matrices of the same order. Taking determinants shows that neither factor has zero determinant, since their determinant product is 1. Hence $B - I = (X - I)^{-1}$, and the reversed product also equals the identity:

$$(B - I)(X - I) = I.$$

No cancellation by the possibly singular matrices A or B has been used.

Step 2: Expand the reversed product and compare Expanding the last identity in its actual factor order yields

$$BX - B - X + I = I \quad \Longrightarrow \quad BX = B + X.$$

The original equation gives $XB = X + B$; addition of matrices is commutative, so these right sides coincide. Therefore $XB = BX$, which is $A^2B = BA^2$. To check that the remaining conclusions are not forced, take $A = 2I$ and $B = \frac{4}{3}I$. They are nonzero and satisfy $A^2 + B = \frac{16}{3}I = A^2B$, but $A^2B \neq I$, neither A^2 nor B equals I, and $AB = \frac{8}{3}I \neq I$. This single admissible example rejects options (A), (B), and (D).

Step 3: Conclusion The relation necessarily implies $A^2B = BA^2$. Hence, the correct option is (C).

Quick Tip to Solve: Turn $X + B = XB$ into $(X - I)(B - I) = I$; invertibility of these shifted matrices proves the required commutation safely.

11. For three positive integers p, q, r, $x^{pq^2} = y^{qr} = z^{p^2r}$ and r = pq + 1 such that 3, 3 $\log_y x$, 3 $\log_z y$, 7 $\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then r-p-q is equal to [4 marks]

(A) 12

(B) -6

(C) 6

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- An arithmetic progression beginning at 3 with common difference $1/2$ has successive terms $3, 7/2, 4, 9/2$. Equate these terms with the logarithmic expressions in their printed order.
- For positive bases different from 1, $\log_y x = \ln x / \ln y$. Equality of positive powers becomes a linear relation between their logarithms.

Step 1: Extract the logarithmic ratios The four given expressions occupy consecutive places in the progression. Hence

$$3\log_y x = \frac{7}{2}, \qquad 3\log_z y = 4, \qquad 7\log_x z = \frac{9}{2}.$$

Dividing by their coefficients gives

$$\frac{\ln x}{\ln y} = \frac{7}{6}, \qquad \frac{\ln y}{\ln z} = \frac{4}{3}, \qquad \frac{\ln z}{\ln x} = \frac{9}{14}.$$

Their product is 1, as required by the cyclic change-of-base identity, so these ratios are mutually consistent.

Step 2: Use the equal powers to find the integers From $x^{pq^2} = y^{qr}$, taking logarithms and cancelling the positive integer q gives $pq \ln x = r \ln y$. Therefore $r/(pq) = 7/6$, or $6r = 7pq$. With $r = pq + 1$, this becomes $6pq + 6 = 7pq$, so $pq = 6$ and $r = 7$. Next, $y^{qr} = z^{p^2r}$ gives $q \ln y = p^2 \ln z$, whence

$$\frac{p^2}{q} = \frac{4}{3} \quad \Longrightarrow \quad 3p^2 = 4q.$$

Substitute $q = 6/p$: $3p^3 = 24$, so the positive integer solution is $p = 2$, $q = 3$. The requested expression must be formed only after finding all three integers; the alternative numerical choices do not equal $7 - 2 - 3$.

Step 3: Conclusion We obtain $r - p - q = 7 - 2 - 3 = 2$. Hence, the correct option is (D).

Quick Tip to Solve: Write the numerical A.P. terms first, then convert the equal powers into logarithmic ratios; the relation $r = pq + 1$ immediately fixes pq.

12. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{Area(\Delta PQR)}{Area(\Delta ABC)}$ is equal to : [4 marks]

(A) $\frac{5}{2}$

(B) 4

(C) 2

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- The section formula locates a point dividing a segment in a given ratio. A point with $QA : AR = 1 : 2$ lies one third of the way from Q to R.
- An invertible affine coordinate change multiplies all planar areas by the same factor, so ratios of areas are unchanged. This permits convenient coordinates for the original triangle.

Step 1: Choose coordinates and locate the three points Set $P = (0, 0)$, $Q = (1, 0)$, and $R = (0, 1)$. Any nondegenerate triangle can be mapped to this one by an affine transformation, without changing the required area ratio or the side-division ratios. The given divisions then give

$$A = \frac{2Q + R}{3} = \left(\frac{2}{3}, \frac{1}{3}\right), \qquad B = \frac{2R + P}{3} = \left(0, \frac{2}{3}\right),$$

and

$$C = \frac{2P + Q}{3} = \left(\frac{1}{3}, 0\right).$$

In particular, the weight attached to the far endpoint is one third; reversing this unintentionally can obscure the cyclic arrangement.

Step 2: Compute the determinant area The original triangle has area $1/2$. For the inner triangle, use the vectors based at A:

$$B - A = \left(-\frac{2}{3}, \frac{1}{3}\right), \qquad C - A = \left(-\frac{1}{3}, -\frac{1}{3}\right).$$

Its area is half the absolute determinant:

$$\begin{aligned}[ABC] &= \frac{1}{2}\left|\left(-\frac{2}{3}\right)\left(-\frac{1}{3}\right) - \left(\frac{1}{3}\right)\left(-\frac{1}{3}\right)\right| \\ &= \frac{1}{2}\left(\frac{2}{9} + \frac{1}{9}\right) = \frac{1}{6}.\end{aligned}$$

Consequently $[PQR]/[ABC] = (1/2)/(1/6) = 3$. This exact ratio excludes $5/2$, 4, and 2. Both determinants would acquire the same area scale under the inverse affine transformation, so the result applies to the given triangle of arbitrary shape.

Step 3: Conclusion The required ratio is $\dfrac{\text{Area}(\triangle PQR)}{\text{Area}(\triangle ABC)} = 3$. Hence, the correct option is (D).

Quick Tip to Solve: For ratios of areas formed by fixed side divisions, use the reference triangle $(0, 0), (1, 0), (0, 1)$; affine invariance removes unnecessary geometry.

13. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

Then at $x = 0$ [4 marks]

(A) f is continuous but f' is not continuous

(B) f and f' both are continuous

(C) f is continuous but not differentiable

(D) f' is continuous but not differentiable

Correct Answer: (A) f is continuous but f' is not continuous

Solution:

Concept:

- Continuity at zero requires $\lim_{x\to 0} f(x) = f(0)$. The bound $|\sin t| \leq 1$ allows the squeeze theorem to handle the oscillatory factor.
- The derivative at zero must be found from its defining difference quotient. Continuity of the derivative is a separate condition requiring $\lim_{x\to 0} f'(x) = f'(0)$.

Step 1: Check continuity and the derivative at zero For $x \neq 0$, $|x^2 \sin(1/x)| \leq x^2$. Since $x^2 \to 0$, the squeeze theorem gives $\lim_{x\to 0} f(x) = 0 = f(0)$, so f is continuous at zero. Its derivative there is

$$f'(0) = \lim_{h\to 0} \frac{h^2 \sin(1/h) - 0}{h} = \lim_{h\to 0} h \sin(1/h) = 0,$$

again because the absolute value of the final expression is at most $|h|$. Thus the function is differentiable as well as continuous at the specified point.

Step 2: Test continuity of the derivative For $x \neq 0$, the product and chain rules give

$$f'(x) = 2x \sin(1/x) - \cos(1/x).$$

The first term tends to zero, but the cosine term continues oscillating with amplitude 1. More explicitly, for $x_n = 1/(2\pi n)$, the derivative equals -1; for $y_n = 1/((2n+1)\pi)$, it equals 1. Both sequences tend to zero, and their derivative values have different limits. Hence f' has no limit at zero and cannot be continuous there. Options (B) and (D) incorrectly assert continuity of the derivative, while option (C) incorrectly denies the derivative of f at zero.

Step 3: Conclusion At $x = 0$, f is continuous and differentiable, but f' is not continuous. Hence, the correct option is (A).

Quick Tip to Solve: For $x^2 \sin(1/x)$, compute the derivative at zero from the limit first; differentiating away from zero alone cannot determine it.

14. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the : [4 marks]

(A) length of latus rectum 2

(B) directrix 4x = -3

(C) directrix 4x = 3

(D) length of latus rectum $\frac{3}{2}$

Correct Answer: (C) directrix 4x = 3

Solution:

Concept:

- The slope form of a tangent to $y^2 = 4ax$ is $y = mx + a/m$, for $m \neq 0$. Here $4a = 24$, so $a = 6$.
- The midpoint coordinates of intersections can be found from sums of roots, without solving separately for the two intersection points. For $y^2 = 4ax$, the directrix is $x = -a$ and the latus rectum has length $|4a|$.

Step 1: Find the midpoint in terms of the tangent slope A nonvertical tangent to the given parabola has equation $y = mx + 6/m$. Substituting into $xy = 2$ gives

$$mx^2 + \frac{6}{m}x - 2 = 0.$$

If the intersection abscissae are x_1, x_2, their sum is $-6/m^2$. Therefore the midpoint abscissa is $h = (x_1 + x_2)/2 = -3/m^2$. Since the midpoint also lies on the line, its ordinate is

$$k = mh + \frac{6}{m} = -\frac{3}{m} + \frac{6}{m} = \frac{3}{m}.$$

The vertical tangent $x = 0$ has no intersection with $xy = 2$, so it supplies no additional segment of the stated kind.

Step 2: Eliminate the slope and identify the directrix Squaring the ordinate relation gives $k^2 = 9/m^2$. Combining this with $h = -3/m^2$ yields

$$k^2 = -3h.$$

Thus the midpoints lie on the parabola $y^2 = -3x$. Comparing with $y^2 = 4ax$ gives the signed parameter $a = -3/4$. Its directrix is consequently $x = -a = 3/4$, or $4x = 3$, and its latus rectum length is 3. The proposed lengths 2 and $3/2$ are therefore incorrect. The other proposed directrix has the wrong sign: the locus opens towards negative x, placing its directrix on the positive side of the vertex. Real distinct intersections merely restrict which points of this parabola arise; they do not change its directrix.

Step 3: Conclusion The containing parabola is $y^2 = -3x$, whose directrix is $4x = 3$. Hence, the correct option is (C).

Quick Tip to Solve: Use Vieta's sum of roots to find the midpoint before eliminating the tangent slope; keep the signed parabola parameter when locating the directrix.

15. The value of $\sum_{r=0}^{22} {}^{22}C_r {}^{23}C_r$ is [4 marks]

(A) ${}^{44}C_{23}$

(B) ${}^{45}C_{23}$

(C) ${}^{44}C_{22}$

(D) ${}^{45}C_{24}$

Correct Answer: (B) ${}^{45}C_{23}$

Solution:

Concept:

- Vandermonde's identity is obtained by equating a coefficient in $(1+x)^m(1+x)^n = (1+x)^{m+n}$. It states $\sum_r \binom{m}{r}\binom{n}{k-r} = \binom{m+n}{k}$.
- The symmetry identity $\binom{n}{r} = \binom{n}{n-r}$ converts equal lower indices into complementary indices, which are needed for coefficient multiplication.

Step 1: Rewrite the second binomial coefficient Use symmetry in the factor with upper index 23. For every integer r between 0 and 22,

$$\binom{23}{r} = \binom{23}{23-r}.$$

Thus the required sum is $\sum_{r=0}^{22} \binom{22}{r}\binom{23}{23-r}$. The two lower indices in each term now add to 23.

Step 2: Extract the coefficient of the required power In the product $(1+x)^{22}(1+x)^{23}$, choose x^r from the first factor and x^{23-r} from the second. Their product contributes to x^{23}, and summing these contributions gives exactly the stated sum. On the other hand,

$$(1+x)^{22}(1+x)^{23} = (1+x)^{45}.$$

The coefficient of x^{23} on the right is $\binom{45}{23}$. No extra term with $r = 23$ occurs, because the first polynomial has degree 22.

Step 3: Compare the options The upper index must be $22 + 23 = 45$, so the two options with upper index 44 are excluded. The coefficient sought has lower index 23, rather than 24. Also, $\binom{45}{23} = \binom{45}{22}$, whereas $\binom{45}{24} = \binom{45}{21}$ is different.

Step 4: Conclusion The value of the sum is $\boxed{\binom{45}{23}}$. Hence, the correct option is (B).

Quick Tip to Solve: When two binomial factors have the same lower index, apply symmetry to one factor before using Vandermonde's identity.

16. The distance of the point (7, −3, −4) from the plane passing through the points (2, −3, 1), (−1, 1, −2) and (3, −4, 2) is : [4 marks]

(A) $4\sqrt{2}$

(B) 4

(C) 5

(D) $5\sqrt{2}$

Correct Answer: (D) $5\sqrt{2}$

Solution:

Concept:

- Two nonparallel vectors lying in a plane determine a normal vector through their cross product. A normal (a, b, c) gives the plane equation $ax + by + cz + d = 0$.
- The perpendicular distance of (x_0, y_0, z_0) from that plane is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2 + b^2 + c^2}$. The absolute value makes the distance nonnegative.

Step 1: Find two directions in the plane Call the three given points $A = (2, -3, 1)$, $B = (-1, 1, -2)$, and $C = (3, -4, 2)$. Their displacement vectors are

$$\overrightarrow{AB} = (-3, 4, -3), \qquad \overrightarrow{AC} = (1, -1, 1).$$

The cross product is

$$\overrightarrow{AB} \times \overrightarrow{AC} = (1, 0, -1).$$

It is nonzero, so the three points determine a unique plane with this normal.

Step 2: Write and verify the plane equation Using the normal $(1, 0, -1)$ and point A, the equation is

$$(x - 2) - (z - 1) = 0, \qquad x - z - 1 = 0.$$

The other two points satisfy it: $-1 - (-2) - 1 = 0$ and $3 - 2 - 1 = 0$. This substitution is a useful check on both the normal and constant term.

Step 3: Substitute the external point For $P = (7, -3, -4)$, the signed numerator is $7 - (-4) - 1 = 10$. Hence

$$d = \frac{|10|}{\sqrt{1^2 + 0^2 + (-1)^2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}.$$

The coordinate $y = -3$ does not appear because the plane is parallel to the y-axis. The alternatives 4, 5 and $4\sqrt{2}$ do not equal this normalized distance.

Step 4: Conclusion The perpendicular distance is $\boxed{5\sqrt{2}}$ units. Hence, the correct option is (D).

Quick Tip to Solve: After finding a plane normal, check every given point in the equation before applying the distance formula.

17. The compound statement $(\sim (P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to [4 marks]

(A) $((\sim P) \vee Q) \wedge (\sim Q)$

(B) $(\sim Q) \vee P$

(C) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

(D) $(\sim P) \vee Q$

Correct Answer: (C) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

Solution:

Concept:

- An implication $A \Rightarrow B$ is equivalent to $(\sim A) \vee B$. It fails only when its antecedent is true and its consequent is false.
- De Morgan's law gives $\sim (P \wedge Q) = (\sim P) \vee (\sim Q)$. Absorption gives $X \vee (X \wedge Y) = X$.

Step 1: Simplify the antecedent Let $A = \sim (P \wedge Q) \vee ((\sim P) \wedge Q)$. Applying De Morgan's law,

$$A = (\sim P) \vee (\sim Q) \vee ((\sim P) \wedge Q).$$

The final conjunction already implies $\sim P$, so absorption removes it. Therefore $A = (\sim P) \vee (\sim Q)$.

Step 2: Eliminate the implication The consequent is $B = (\sim P) \wedge (\sim Q)$. Thus

$$\begin{aligned} A \Rightarrow B &= (\sim A) \vee B \\ &= (P \wedge Q) \vee ((\sim P) \wedge (\sim Q)). \end{aligned}$$

This expression is true exactly when P and Q have the same truth value: both true or both false.

Step 3: Identify the equivalent option Option (C) is the conjunction $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$, namely $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$. It too is true exactly when the two truth values agree. Option (A) fails when both are true. Option (B) is true at P true, Q false, and option (D) is true at P false, Q true; the original expression is false in both these unequal cases.

Step 4: Conclusion The statement is equivalent to $\boxed{((\sim P) \vee Q) \wedge ((\sim Q) \vee P)}$. Hence, the correct option is (C).

Quick Tip to Solve: Simplify the antecedent before replacing an implication; absorption often removes an apparently extra term.

18. The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is : [4 marks]

(A) $13\sqrt{2}$

(B) 26

(C) $20\sqrt{2}$

(D) 31

Correct Answer: (B) 26

Solution:

Concept:

- A distance measured parallel to a specified line is the length from the point to the plane along that direction, rather than the perpendicular point-to-plane distance.
- If a line through P has direction vector $\mathbf{d}$, its points are $P + t\mathbf{d}$, and its displacement length is $|t|\,|\mathbf{d}|$.

Step 1: Read the direction with the correct sign Set the three ratios of the given line equal to a parameter s. Then $x = -4+3s$, $y = 2-4s$, and $z = 3+12s$. Thus a parallel direction is $\mathbf{d} = (3, -4, 12)$, with magnitude

$$|\mathbf{d}| = \sqrt{9 + 16 + 144} = 13.$$

The minus sign in $2 - y$ is essential: the second direction component is negative.

Step 2: Intersect the parallel line with the plane The required line through $P = (-1, 9, -16)$ has coordinates

$$x = -1 + 3t, \quad y = 9 - 4t, \quad z = -16 + 12t.$$

Substitute in $2x + 3y - z = 5$:

$$2(-1 + 3t) + 3(9 - 4t) - (-16 + 12t) = 5.$$

This reduces to $41 - 18t = 5$, hence $t = 2$. The intersection is $(5, 1, 8)$, which satisfies $10 + 3 - 8 = 5$.

Step 3: Calculate the actual length The displacement is $2(3, -4, 12) = (6, -8, 24)$, so its length is

$$\sqrt{6^2 + (-8)^2 + 24^2} = 2\sqrt{169} = 26.$$

The parameter itself is not the distance because the chosen direction is not a unit vector. The perpendicular-distance formula would answer a different question; the other listed numerical values do not equal the length of this displacement.

Step 4: Conclusion The specified distance is $\boxed{26}$ units. Hence, the correct option is (B).

Quick Tip to Solve: For a distance measured along a line, solve for its intersection parameter and multiply its absolute value by the direction-vector magnitude.

19. Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{v} \cdot \vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda \vec{v}$. Then $\vec{u} \cdot \vec{w}$ is equal to : [4 marks]

(A) $-\frac{2}{3}$

(B) $\frac{3}{2}$

(C) 2

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- A cross product $\mathbf{v} \times \mathbf{w}$ is perpendicular to each of $\mathbf{v}$ and $\mathbf{w}$; its dot product with either vector is zero.
- The dot product is distributive. These perpendicularity relations determine the required scalar without finding the three components of $\mathbf{w}$.

Step 1: Evaluate the known dot products From the given components,

$$\mathbf{u} \cdot \mathbf{v} = 1(2) + (-1)(1) + (-2)(-1) = 3, \qquad \mathbf{v} \cdot \mathbf{v} = 2^2 + 1^2 + (-1)^2 = 6.$$

The supplied scalar product is $\mathbf{v} \cdot \mathbf{w} = 2$. Keep these three values separate: the first two follow from the known vectors, and the third is a condition on the unknown vector.

Step 2: Determine the scalar parameter Dot $\mathbf{v} \times \mathbf{w} = \mathbf{u} + \lambda \mathbf{v}$ with $\mathbf{v}$. The left side vanishes by perpendicularity. Consequently

$$0 = \mathbf{u} \cdot \mathbf{v} + \lambda(\mathbf{v} \cdot \mathbf{v}) = 3 + 6\lambda, \qquad \lambda = -\frac{1}{2}.$$

This also checks that the right-hand vector is indeed perpendicular to $\mathbf{v}$.

Step 3: Use perpendicularity to the second vector Now dot the same vector equation with $\mathbf{w}$. Again the left side is zero, giving

$$0 = \mathbf{u} \cdot \mathbf{w} + \lambda(\mathbf{v} \cdot \mathbf{w}) = \mathbf{u} \cdot \mathbf{w} - 1.$$

Therefore $\mathbf{u} \cdot \mathbf{w} = 1$. The alternatives $-2/3$, $3/2$, and 2 fail this exact dot-product condition, so none can be selected.

Step 4: Conclusion We obtain $\boxed{\mathbf{u} \cdot \mathbf{w} = 1}$. Hence, the correct option is (D).

Quick Tip to Solve: When a cross product equals a linear combination, dot with its two input vectors before attempting component equations.

20. Let α be a root of the equation $(a-c)x^2+(b-a)x+(c-b)=0$ where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$ is singular. Then, the value of $\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$ is

[4 marks]

(A) 3

(B) 6

(C) 12

(D) 9

Correct Answer: (A) 3

Solution:

Concept:

- For three numbers p, q, r, the identity $p^3+q^3+r^3-3pqr=(p+q+r)(p^2+q^2+r^2-pq-qr-rp)$ is useful when their sum is zero.
- The distinctness of a, b, c makes each of $a-c$, $b-a$, and $c-b$ nonzero, so a common denominator formed from them is valid.

Step 1: Introduce the cyclic differences Put $p=a-c$, $q=b-a$, and $r=c-b$. These definitions give

$$p+q+r=(a-c)+(b-a)+(c-b)=0.$$

Since the original three numbers are distinct, none of these differences vanishes. The required expression becomes $p^2/(qr)+q^2/(pr)+r^2/(pq)$.

Step 2: Combine the three fractions Use the nonzero common denominator pqr. We obtain

$$\frac{p^2}{qr}+\frac{q^2}{pr}+\frac{r^2}{pq}=\frac{p^3+q^3+r^3}{pqr}.$$

Because $p+q+r=0$, the sum-of-cubes identity gives $p^3+q^3+r^3=3pqr$. Thus the quotient is exactly 3, with no dependence on the particular values of the differences.

Step 3: Check the role of the additional conditions The root and singular-matrix conditions restrict the possible triples but are unnecessary for evaluating this expression: the zero-sum relation of the cyclic differences already determines it for every distinct triple. In particular, there is no need to divide by $\alpha-1$, which might be zero. The proposed values 6, 12 and 9 contradict the identity; only 3 survives the simplification.

Step 4: Conclusion The required value is $\boxed{3}$. Hence, the correct option is (A).

Quick Tip to Solve: For fractions built from cyclic differences, name the differences first; their sum is often zero and unlocks a standard identity.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set S = {$x + \lambda : x$ is an integer solution of E} is _____ [4 marks]

Solution:

Concept:

- For an integer x, its absolute value $k = |x|$ is a nonnegative integer. The equation can therefore be classified by the possible values of k.
- An absolute value is nonnegative. Hence $|\lambda - 3| = 2k - k^2 = k(2 - k)$ forces $0 \le k \le 2$. For each allowed k, both signs of x must be considered.

Step 1: List all possible absolute values Rearrange the equation to obtain

$$|\lambda - 3| = 2|x| - |x|^2 = k(2 - k).$$

If $k > 2$, the right side is negative, impossible for an absolute value. Since k is an integer, the complete list is $k = 0, 1, 2$. This argument excludes every other integer solution without a search over real λ.

Step 2: Determine the permitted parameter values For $k = 0$, we have $x = 0$ and $|\lambda - 3| = 0$, so $\lambda = 3$ and $x + \lambda = 3$. For $k = 1$, $|\lambda - 3| = 1$, giving $\lambda = 2$ or 4 and $x = 1$ or -1. The resulting sums are 3, 1, 5 and 3. For $k = 2$, the absolute value is again zero, so $\lambda = 3$ and $x = 2$ or -2; the sums are 5 and 1.

Step 3: Compare and verify the maximum Collecting all admissible values gives $S = \{1, 3, 5\}$. In particular, the maximum is attained by $x = 2, \lambda = 3$: direct substitution gives $4 - 4 + 0 = 0$. It is also attained by $x = 1, \lambda = 4$, since $1 - 2 + 1 = 0$. No larger value is possible because all three nonnegative integer possibilities for $|x|$, and all parameter values for each, have been exhausted.

Step 4: Conclusion The largest element is $\boxed{5}$. This is a dimensionless numerical value.

Quick Tip to Solve: Isolate the parameter absolute value first; its nonnegativity gives a finite list of possible integer absolute values.

22. The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is equal to _____ [4 marks]

Solution:

Concept:

- The shortest distance between two nonparallel lines through points A, B, with direction vectors $\mathbf{u}, \mathbf{v}$, is $d = \dfrac{|(B - A) \cdot (\mathbf{u} \times \mathbf{v})|}{|\mathbf{u} \times \mathbf{v}|}$.
- The cross product is perpendicular to both lines. Projecting the displacement between any chosen points onto its unit direction gives their perpendicular separation.

Step 1: Identify the points and directions The first line passes through $A = (2, -1, 6)$ and has direction

$\mathbf{u} = (3, 2, 2)$. In the second line, $(1 - y)/2$ gives a negative y-direction, so take $B = (6, 1, -8)$ and $\mathbf{v} = (3, -2, 0)$. The zero denominator means $z = -8$, not an undefined line. Thus $B - A = (4, 2, -14)$.

Step 2: Evaluate the normal projection Compute the cross product and its length:

$$\mathbf{u} \times \mathbf{v} = (4, 6, -12), \qquad |\mathbf{u} \times \mathbf{v}| = \sqrt{16 + 36 + 144} = 14.$$

It is nonzero, so the lines are not parallel. The required scalar triple product is

$$(4, 2, -14) \cdot (4, 6, -12) = 16 + 12 + 168 = 196.$$

Hence the perpendicular separation is $196/14 = 14$. The nonzero triple product also confirms that the lines are skew rather than intersecting.

Step 3: Conclusion The shortest distance between the two lines is $\boxed{14}$ coordinate units. The numerical entry is $\boxed{14}$.

Quick Tip to Solve: Rewrite a numerator such as $1 - y$ before reading a direction vector: its coefficient of y reverses the sign of that component.

23. The 4$^{\text{th}}$ term of GP is 500 and its common ratio is $\frac{1}{m}, m \in \mathbb{N}$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is _____

[4 marks]

Solution:

Concept:

- For a GP with first term a and ratio r, its nth term is $T_n = ar^{n-1}$. Consecutive partial sums satisfy $S_n - S_{n-1} = T_n$.
- Here $r = 1/m > 0$, so multiplying an inequality by a power of m preserves its direction. Strict inequalities must exclude equality at an endpoint.

Step 1: Express the added terms using the fourth term The fourth term is $ar^3 = 500$. It is more efficient to move forward from this known term than to determine the first term. Two further factors of r give

$$T_6 = 500r^2 = \frac{500}{m^2},$$

and three further factors give

$$T_7 = 500r^3 = \frac{500}{m^3}.$$

The first condition says the sixth term exceeds 1; the second says the seventh term is less than one-half.

Step 2: Solve and count the integer possibilities Using the partial-sum differences, the two conditions become

$$\frac{500}{m^2} > 1 \implies m^2 < 500, \qquad \frac{500}{m^3} < \frac{1}{2} \implies m^3 > 1000.$$

Since $22^2 = 484 < 500 < 529 = 23^2$, the first condition gives $m \leq 22$. Since $10^3 = 1000$, the strict second condition requires $m \geq 11$. Thus the complete list is $11, 12, \ldots, 22$; every member meets both

inequalities. The count is $22 - 11 + 1 = 12$.

Step 3: Conclusion There are $\boxed{12}$ possible natural-number values of m.

Quick Tip to Solve: When two inequalities involve consecutive partial sums, first replace each difference by its single GP term; no sum formula is needed.

---

24. Let C be the largest circle centred at (2, 0) and inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If (1, α) lies on C, then 10 α^2 is equal to _____ [4 marks]

Solution:

Concept:

- For a circle with a fixed centre inside a convex ellipse, the largest admissible radius equals the minimum distance from that centre to the ellipse boundary.
- On $x^2/36 + y^2/16 = 1$, eliminate y^2 to minimize the squared distance. Squared distance has the same minimizer as distance and avoids radicals.

Step 1: Minimize distance to the ellipse boundary Let $P = (x, y)$ be any boundary point, so $-6 \le x \le 6$ and $y^2 = 16 - 4x^2/9$. Its squared distance from the fixed centre $O = (2, 0)$ is

$$OP^2 = (x-2)^2 + y^2 = \frac{5}{9}x^2 - 4x + 20.$$

Complete the square to obtain

$$OP^2 = \frac{5}{9}\left(x - \frac{18}{5}\right)^2 + \frac{64}{5}.$$

The vertex $x = 18/5$ lies in the permitted interval, and the corresponding y^2 is nonnegative. Therefore this minimum is attained on the ellipse.

Step 2: Use the point on the circle The centre lies inside the ellipse because $2^2/36 < 1$. A disk about this centre stays inside until it reaches the nearest boundary, so its largest squared radius is $R^2 = 64/5$. Its equation is consequently

$$(x-2)^2 + y^2 = \frac{64}{5}.$$

Substituting the stated point $(1, \alpha)$ gives

$$1 + \alpha^2 = \frac{64}{5}, \qquad \alpha^2 = \frac{59}{5}.$$

Both possible signs of α give the same requested value.

Step 3: Conclusion Thus $10\alpha^2 = 10 \cdot \frac{59}{5} = \boxed{118}$.

Quick Tip to Solve: For a largest circle with a prescribed centre, minimize distance to the entire ellipse boundary; the nearest point need not lie on an axis.

---

25. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is _____ [4 marks]

Solution:

Concept:

- A selection of k objects from n distinct objects is counted by $\binom{n}{k}$; order does not matter when choosing courses.
- Split the possibilities by the number of language courses. The product rule counts choices within a case, and the sum rule combines disjoint cases.

Step 1: Separate language and other courses Of the 12 available courses, 5 are language courses, leaving $12 - 5 = 7$ non-language courses. At most two language courses means that their number k can be 0, 1 or 2. For a fixed k, choose k out of the 5 languages and independently choose the remaining $5 - k$ out of the other 7 courses. Therefore the number for that case is $\binom{5}{k}\binom{7}{5-k}$. These cases cannot overlap because they have different numbers of languages.

Step 2: Evaluate all allowed cases For no language course, the count is

$$\binom{5}{0}\binom{7}{5} = 1 \cdot 21 = 21.$$

For exactly one language course, it is

$$\binom{5}{1}\binom{7}{4} = 5 \cdot 35 = 175.$$

For exactly two language courses, it is

$$\binom{5}{2}\binom{7}{3} = 10 \cdot 35 = 350.$$

All three cases are feasible with the available seven non-language courses. Adding them gives $21 + 175 + 350 = 546$. Using permutations instead would count each chosen set repeatedly in different orders.

Step 3: Conclusion The boy can choose his five courses in $\boxed{546}$ ways.

Quick Tip to Solve: Translate "at most two" into the disjoint cases 0, 1 and 2 before writing combinations, and ensure that each case still contains five courses overall.

26. The value of $12\int_0^3 \left|x^2-3x+2\right| dx$ is _____ [4 marks]

Solution:

Concept:

- An absolute value satisfies $|f(x)| = f(x)$ where $f(x) \geq 0$, and $|f(x)| = -f(x)$ where $f(x) < 0$. Split a definite integral at the sign-changing zeros.
- For the polynomial $f(x) = x^2 - 3x + 2$, an antiderivative is $F(x) = x^3/3 - 3x^2/2 + 2x$. Evaluate each signed piece with its own limits.

Step 1: Locate the zeros and signs Factor the quadratic as

$$x^2 - 3x + 2 = (x-1)(x-2).$$

Both roots, 1 and 2, lie in the integration interval $[0, 3]$. Since its leading coefficient is positive, the quadratic is nonnegative on $[0, 1]$ and $[2, 3]$, but nonpositive on $[1, 2]$. Thus

$$\int_0^3 |f(x)|\, dx = \int_0^1 f(x)\, dx - \int_1^2 f(x)\, dx + \int_2^3 f(x)\, dx.$$

The minus sign on the middle integral converts its negative signed area to positive area.

Step 2: Evaluate the pieces and apply the multiplier For the antiderivative stated above,

$$F(0) = 0, \quad F(1) = \frac{5}{6}, \quad F(2) = \frac{2}{3}, \quad F(3) = \frac{3}{2}.$$

Hence the three positive contributions are

$$\frac{5}{6}, \qquad -\left(\frac{2}{3} - \frac{5}{6}\right) = \frac{1}{6}, \qquad \frac{3}{2} - \frac{2}{3} = \frac{5}{6}.$$

Their sum is $11/6$. The factor 12 is outside the whole integral, so it multiplies this complete sum, not just one of the intervals.

Step 3: Conclusion The required value is $12 \cdot \frac{11}{6} = \boxed{22}$.

Quick Tip to Solve: For an absolute polynomial integral, mark every real root inside the limits and make a sign chart before integrating.

27. Suppose $\sum_{r=0}^{2023} r^2\, {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is _____ [4 marks]

Solution:

Concept:

- The binomial theorem gives $(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$. Differentiation produces sums weighted by r and by $r(r-1)$.

- The identity $r^2 = r(r-1) + r$ combines the first two differentiated sums, avoiding the error of treating r^2 as $r(r-1)$.

Step 1: Derive the two weighted binomial sums Differentiate the binomial expansion once and evaluate at $x = 1$:

$$\sum_{r=0}^{n} r\binom{n}{r} = n2^{n-1}.$$

Differentiate it twice and again put $x = 1$:

$$\sum_{r=0}^{n} r(r-1)\binom{n}{r} = n(n-1)2^{n-2}.$$

The expansion is a finite polynomial, so termwise differentiation is valid. The r=0 term vanishes in both sums, and the r=1 term vanishes in the second, so retaining the original summation limits causes no extra contribution.

Step 2: Combine the sums and compare coefficients Add these identities using $r^2 = r(r-1) + r$. This gives

$$\begin{aligned}\sum_{r=0}^{n} r^2\binom{n}{r} &= n(n-1)2^{n-2} + n2^{n-1} \\ &= n(n+1)2^{n-2}.\end{aligned}$$

For $n = 2023$, the sum is $2023 \cdot 2024 \cdot 2^{2021}$. The question writes the same number as $2023\alpha 2^{2022}$. Cancel the common positive factor $2023 \cdot 2^{2021}$, obtaining $2024 = 2\alpha$.

Step 3: Conclusion Therefore $\alpha = 2024/2 = \boxed{1012}$.

Quick Tip to Solve: Express a squared summation index as $r(r-1) + r$, then use second and first derivatives of the binomial theorem.

28. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is _____. [4 marks]

Solution:

Concept:

- Permutations of a multiset with multiplicities $n_1, n_2, \ldots$ are counted by $n!/(n_1!n_2!\cdots)$; exchanging identical digits does not create a new number.
- Positions are numbered from the left. A 9-digit number has four even positions, 2, 4, 6 and 8, and five odd positions, 1, 3, 5, 7 and 9.

Step 1: Identify the forced sets of positions The digits of 123412341 consist of three 1s, two 2s, two 3s and two 4s. There are exactly four even digits: two 2s and two 4s. The restriction says every even digit must occupy an even position. Since there are also exactly four even positions, these positions are completely filled by those even digits. Consequently the remaining five odd positions must contain the three 1s and two 3s; no further positional restriction remains.

Step 2: Arrange the digits within each set of positions Arrange the two 2s and two 4s among the four even positions in

$$\frac{4!}{2!2!} = 6$$

distinct ways. Independently arrange the three 1s and two 3s among the five odd positions in

$$\frac{5!}{3!2!} = 10$$

distinct ways. Each pair of arrangements determines exactly one number, and each valid number gives exactly one such pair, so multiplication counts without omission or repetition. There is no leading-zero issue because none of the available digits is zero. Thus the count is $6 \cdot 10 = 60$.

Step 3: Conclusion The number of valid 9-digit numbers is $\boxed{60}$.

Quick Tip to Solve: Count even digits and even positions first: when these counts agree, all odd digits are automatically forced into the odd positions.

29. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is _____ [4 marks]

Solution:

Concept:

- The ellipse is $x^2/16 + y^2/9 = 1$, with semiaxes $a = 4$ and $b = 3$. At the parametric point $(4\cos t, 3\sin t)$, its tangent is $x\cos t/4 + y\sin t/3 = 1$.
- If a line meets the axes at $(p, 0)$ and $(0, q)$, the distance between the intercepts is $\sqrt{p^2 + q^2}$. Minimise its square, since the square root is increasing.

Step 1: Find the intercepts The tangent gives $A = (4/\cos t, 0)$ and $B = (0, 3/\sin t)$. Both intercepts are finite only when neither sine nor cosine vanishes. The signs of the intercepts do not affect their squared distance. Thus, on putting $u = \cos^2 t$, where $0 < u < 1$,

$$AB^2 = 16\sec^2 t + 9\csc^2 t = \frac{16}{u} + \frac{9}{1-u}.$$

This accounts for tangents in every quadrant, rather than restricting the result to positive intercepts.

Step 2: Minimise the squared length By Cauchy–Schwarz,

$$\left(\frac{16}{u} + \frac{9}{1-u}\right)(u + 1 - u) \geq (4 + 3)^2 = 49.$$

Equality requires $4/u = 3/(1-u)$, which gives $u = 4/7$. This lies in the allowed interval, so the lower bound is attained by an actual tangent. For example, choose $\cos t = 2/\sqrt{7}$ and $\sin t = \sqrt{3/7}$; then the squared intercepts are 28 and 21, whose sum is 49.

Step 3: Conclusion The minimum length is $AB_{\min} = \sqrt{49} = 7$. Therefore, the required numerical answer is $\boxed{7}$.

Quick Tip to Solve: For an ellipse with semiaxes a and b, write the tangent intercepts parametrically; Cauchy gives the minimum intercept segment as a+b.

30. The value of $\frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ is _____ [4 marks]

Solution:

Concept:

- For an integrable function on $[0, a]$, reflection gives $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. This substitution reverses the limits and its negative differential restores their order.
- On $[0, \pi/2]$, sine and cosine are nonnegative and never vanish simultaneously. Hence their positive powers have a strictly positive sum throughout this interval.

Step 1: Reflect the integral Denote the unscaled integral by

$$I = \int_0^{\pi/2} \frac{\cos^{2023} x}{\sin^{2023} x + \cos^{2023} x}\,dx.$$

Under $x = \pi/2 - t$, cosine becomes sine and sine becomes cosine. Therefore, after renaming the variable,

$$I = \int_0^{\pi/2} \frac{\sin^{2023} x}{\cos^{2023} x + \sin^{2023} x}\,dx.$$

There is no endpoint singularity: the original integrand is 1 at zero and 0 at $\pi/2$, and its denominator is positive between them.

Step 2: Add the complementary forms Add the two expressions for the same integral. Their denominators agree, and their numerators add to that denominator:

$$2I = \int_0^{\pi/2} \frac{\cos^{2023} x + \sin^{2023} x}{\sin^{2023} x + \cos^{2023} x}\,dx = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2}.$$

Thus $I = \pi/4$. The large exponent need never be expanded or integrated separately; the reflection symmetry decides the integral exactly. Finally apply the prefactor printed outside the integral: $(8/\pi)(\pi/4) = 2$.

Step 3: Conclusion The specified value is $\boxed{2}$.

Quick Tip to Solve: When sine and cosine appear symmetrically in a denominator, pair f(x) with f(pi/2-x) before attempting any expansion.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Photodiodes are preferably operated in reverse bias condition for light intensity measurement.

Reason R : The current in the forward bias is more than the current in the reverse bias for a $p-n$ junction diode.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) A is false but R is true

(C) A is true but R is false

(D) Both A and R are true but R is NOT the correct explanation of A

Correct Answer: (D) Both A and R are true but R is NOT the correct explanation of A

Solution:

Concept:

- A photodiode detects light through electron–hole pairs created by absorbed photons. The depletion-region electric field separates these carriers, producing a photocurrent that depends on incident optical intensity.
- Reverse bias widens the depletion region and permits measurement of the small light-generated current against a small dark-current background. Forward bias instead produces a large injection current.

Step 1: Evaluate the assertion In reverse bias, the electric field sweeps photoelectrons and holes across the junction. Increasing the illumination increases the number of generated carriers and consequently the reverse photocurrent. The dark current is comparatively small, so a change caused by light is easier to identify and measure. A photodiode is therefore preferably operated in reverse bias for intensity measurements. Assertion A is true; it is not necessary for the diode to carry a large total current to make a sensitive light detector.

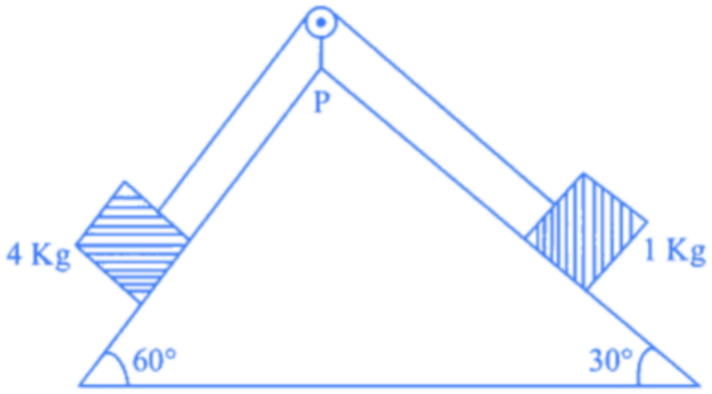
Step 2: Evaluate the reason For an ordinary p–n junction at comparable operating biases, the forward current is indeed much larger than its reverse saturation current. Forward bias lowers the junction barrier and injects majority carriers, while reverse bias suppresses that majority-carrier flow. Thus Reason R is also true. This statement describes the usual current asymmetry of the diode, but does not state the intensity dependence of photocurrent or the improved sensitivity in reverse bias.

Step 3: Test the explanatory link and options The assertion is explained by collection of light-generated minority carriers and the measurable change in reverse current, rather than merely by the forward current exceeding the reverse current. Hence R is not the correct explanation of A. Option A incorrectly asserts that explanatory link; B incorrectly calls A false, and C incorrectly calls R false.

Step 4: Conclusion Both A and R are true, but R is not the correct explanation of A. Hence, the correct option is (D).

Quick Tip to Solve: In photodiode questions, distinguish a true diode characteristic from the carrier-generation mechanism that explains light sensitivity.

32. As per given figure, a weightless pulley P is attached on a double inclined frictionless surfaces. The tension in the string (massless) will be (if g = 10 m/s^2) [4 marks]



(A) $\left(4\sqrt{3}-1\right) N$
(B) $\left(4\sqrt{3}+1\right) N$
(C) $4\left(\sqrt{3}-1\right) N$
(D) $4\left(\sqrt{3}+1\right) N$

Correct Answer: (D) $4\left(\sqrt{3}+1\right) N$

Solution:

Concept:

- A massless string passing over an ideal weightless pulley has the same tension throughout. Its two connected masses have equal acceleration magnitudes along their respective inclines.
- For a block on a smooth incline of angle θ, gravity has a component $mg\sin\theta$ parallel to the plane. The normal reaction balances the perpendicular component and does not enter the motion along the plane.

Step 1: Choose the direction of motion The 4 kg block lies on the 60° slope and the 1 kg block on the 30° slope. Their downward components along the planes are

$$F_1 = 4(10)\sin 60^\circ = 20\sqrt{3}\,\mathrm{N}, \qquad F_2 = 1(10)\sin 30^\circ = 5\,\mathrm{N}.$$

Since $20\sqrt{3} > 5$, the heavier block accelerates down its plane and the lighter block up its plane. Tension opposes the former motion and assists the latter.

Step 2: Apply Newton's law to both blocks Writing each equation positive along the chosen motion gives

$$20\sqrt{3} - T = 4a, \qquad T - 5 = a.$$

Add the equations to eliminate the internal tension:

$$5a = 20\sqrt{3} - 5 \quad \Rightarrow \quad a = 4\sqrt{3} - 1\,\mathrm{m\,s^{-2}}.$$

Substitute in the lighter block equation:

$$T = 5 + a = 4\sqrt{3} + 4 = 4(\sqrt{3} + 1)\,\text{N}.$$

The term $4\sqrt{3} - 1$ is the acceleration magnitude, not the tension; selecting A would confuse these quantities.

Step 3: Check the result against the options The result exceeds 5 N, as required to accelerate the 1 kg block uphill, and is less than $20\sqrt{3}$ N, as required for the 4 kg block to accelerate downhill. Only D equals the computed tension. B has the wrong constant term, and C incorrectly subtracts rather than adds the 5 N gravity component when solving the lighter-block equation.

Step 4: Conclusion The string tension is $\boxed{4(\sqrt{3}+1)\,\text{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: Add the two incline equations to find acceleration, then use the lighter block to find tension; do not mistake the net force for tension.

33. Given below are two statements :

Statement I : If the Brewster's angle for the light propagating from air to glass is θ_{B}, then the Brewster's angle for the light propagating from glass to air is $\frac{\pi}{2} - \theta_B$

Statement II : The Brewster's angle for the light propagating from glass to air is $\tan^{-1}(\mu_{\text{g}})$ where μ_{g} is the refractive index of glass.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are false

(B) Both Statement I and Statement II are true

(C) Statement I is false but Statement II is true

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- Brewster's law for propagation from a medium of refractive index n_1 to one of index n_2 is $\tan i_B = n_2/n_1$. The index in the formula is relative to the incident medium.
- For a positive number z, $\tan^{-1} z + \tan^{-1}(1/z) = \pi/2$. Both Brewster angles are acute, so the complementary-angle identity applies without a branch ambiguity.

Step 1: Test Statement I Take the refractive index of air as 1 and that of glass as μ_g. For air to glass, Brewster's law gives

$$\tan\theta_B = \mu_g.$$

For the reversed propagation from glass to air, denote the Brewster angle by θ'_B. The incident medium

is now glass, so

$$\tan\theta_B' = \frac{1}{\mu_g} = \cot\theta_B = \tan\left(\frac{\pi}{2} - \theta_B\right).$$

Both angles lie between zero and $\pi/2$, hence $\theta_B' = \pi/2 - \theta_B$. Statement I is true.

Step 2: Test Statement II and eliminate the alternatives The reversed angle is $\tan^{-1}(1/\mu_g)$, not $\tan^{-1}\mu_g$. Statement II uses the relative refractive index in the wrong direction and is false. For ordinary glass $\mu_g > 1$, the air-to-glass Brewster angle exceeds 45°, while the glass-to-air angle is smaller than 45°; the proposed expression in II would incorrectly make both angles exceed 45°. A incorrectly rejects Statement I; B incorrectly accepts Statement II; C reverses both truth values.

Step 3: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: Write the incident and transmitted media before using Brewster's law; reversing them replaces the relative refractive index by its reciprocal.

34. A 100 m long wire having cross-sectional area 6.25×10^{-4} m^2 and Young's modulus is 10^{10} Nm^{-2} is subjected to a load of 250 N, then the elongation in the wire will be : [4 marks]

(A) 6.25×10^{-6} m

(B) 4×10^{-3} m

(C) 4×10^{-4} m

(D) 6.25×10^{-3} m

Correct Answer: (B) 4×10^{-3} m

Solution:

Concept:

- Young's modulus is the ratio of longitudinal stress to longitudinal strain within the elastic regime: $Y = (F/A)/(\Delta L/L)$. Here F is the tensile force, A the area and L the original length.
- Rearranging gives $\Delta L = FL/(AY)$. The load specified in newtons is already a force; it must not be multiplied by the acceleration due to gravity.

Step 1: Substitute the data in SI units Use $F = 250\,\text{N}$, $L = 100\,\text{m}$, $A = 6.25 \times 10^{-4}\,\text{m}^2$ and $Y = 10^{10}\,\text{N m}^{-2}$. The product of area and modulus is

$$AY = (6.25 \times 10^{-4})(10^{10}) = 6.25 \times 10^{6}\,\text{N}.$$

The numerator is $FL = 250 \times 100 = 2.5 \times 10^{4}\,\text{N m}$. Retaining these powers of ten separately prevents an error in the final exponent.

Step 2: Calculate and check the extension The extension is

$$\Delta L = \frac{2.5 \times 10^{4}}{6.25 \times 10^{6}}\,\text{m} = 0.4 \times 10^{-2}\,\text{m} = 4 \times 10^{-3}\,\text{m}.$$

The units reduce from N m/N to metres, as required. As a further check, the stress is $F/A = 4 \times 10^{5}\,\text{N m}^{-2}$; dividing by Y gives strain 4×10^{-5}, and multiplying by 100 m again gives 0.004 m. A

and D retain an incorrect numerical factor; C is too small by a factor of ten.

Step 3: Conclusion The elongation is $\boxed{4 \times 10^{-3}\,\mathrm{m}}$, or 4 mm. Hence, the correct option is (B).

Quick Tip to Solve: Compute AY first and check the extension again as original length times strain; the given load in newtons needs no extra factor g.

---

35. The weight of a body at the surface of earth is 18 N. The weight of the body at an altitude of 3200 km above the earth's surface is (given, radius of earth $\mathrm{R_e} = 6400$ km) : [4 marks]

(A) 9.8 N

(B) 4.9 N

(C) 19.6 N

(D) 8 N

Correct Answer: (D) 8 N

Solution:

Concept:

- Outside a spherical Earth, gravitational acceleration at distance r from its centre is $g(r) = GM/r^2$. A body's weight is $W(r) = mGM/r^2$, where its mass m remains constant.
- At altitude h, the distance from Earth's centre is $R_e + h$, so $W_h/W_0 = [R_e/(R_e + h)]^2$. Altitude is measured from the surface, while the inverse-square law uses distance from the centre.

Step 1: Find the correct radial distance The given altitude is $h = 3200\,\mathrm{km}$ and Earth's radius is $R_e = 6400\,\mathrm{km}$. Thus $h = R_e/2$, and the orbital position has radial distance

$$r = R_e + h = 9600\,\mathrm{km} = \frac{3}{2}R_e.$$

It is incorrect to put 3200 km directly into the denominator of the gravitational formula: that would measure distance from the surface rather than the source centre. Since the same units appear in the ratio, conversion of kilometres into metres is unnecessary here.

Step 2: Apply the inverse-square ratio Use the known surface weight $W_0 = 18\,\mathrm{N}$:

$$W_h = 18\left(\frac{6400}{9600}\right)^2\,\mathrm{N} = 18\left(\frac{2}{3}\right)^2\,\mathrm{N} = 18\frac{4}{9}\,\mathrm{N} = 8\,\mathrm{N}.$$

This is less than the surface value, consistent with weaker gravity at greater radial distance. The fractional-altitude approximation $g_h \simeq g_0(1 - 2h/R_e)$ is unsuitable because $h/R_e = 1/2$ is not small. Options A and B do not follow the squared radial ratio, and C incorrectly gives a weight larger than the surface weight.

Step 3: Conclusion The weight at the specified altitude is $\boxed{8\,\mathrm{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use radius plus altitude in the inverse-square law; if the altitude is comparable to Earth's radius, keep the exact ratio rather than a small-height approximation.

---

36. From the photoelectric effect experiment, following observations are made. Identify which of these are correct.

A. The stopping potential depends only on the work function of the metal.

B. The saturation current increases as the intensity of incident light increases.

C. The maximum kinetic energy of a photo electron depends on the intensity of the incident light.

D. Photoelectric effect can be explained using wave theory of light.

Choose the correct answer from the options given below : [4 marks]

(A) A, B, D only

(B) A, C, D only

(C) B, C only

(D) B only

Correct Answer: (D) B only

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = eV_s$, where ϕ is the metal work function and V_s is the stopping potential.
- For a fixed frequency above threshold, intensity controls the photon arrival rate and hence the saturation current, while frequency controls the maximum electron energy.

Step 1: Test the statements about electron energy Statement A is false: $V_s = (h\nu - \phi)/e$ depends on the incident frequency as well as the work function. Changing frequency for the same metal changes the stopping potential. Statement C is also false. At a fixed frequency, each photon has the same energy $h\nu$, so increasing the number of photons cannot increase the maximum kinetic energy gained by an individual photoelectron.

Step 2: Test the statements about current and the light model Statement B is correct. Increasing the incident intensity at a fixed frequency above threshold provides more photons per second. More electrons can then be emitted and collected per second, so the saturation current increases. Statement D is false: classical wave theory cannot account for the threshold frequency, effectively instantaneous emission, and the observed frequency dependence of maximum kinetic energy. The photon model explains these observations.

Step 3: Compare the proposed combinations Only statement B survives all the tests. Options (A) and (B) include the false claims A and D, while option (C) incorrectly includes C. Thus none of these three combinations describes the experimental observations.

Step 4: Conclusion Only B is correct. Hence, the correct option is (D).

Quick Tip to Solve: Separate the roles of intensity and frequency: intensity changes the electron count, whereas frequency changes the maximum energy.

37. Two long straight wires P and Q carrying equal current 10A each were kept parallel to each other at 5 cm distance. Magnitude of magnetic force experienced by 10 cm length of wire P is F_1. If distance between wires is halved and currents on them are doubled, force F_2 on 10 cm length of wire P will be: [4 marks]

(A) $\frac{F_1}{8}$

(B) 10 F_1

(C) $\frac{F_1}{10}$

(D) 8 F_1

Correct Answer: (D) 8 F_1

Solution:

Concept:

- The magnitude of force between long parallel wires is $F = \mu_0 I_P I_Q L/(2\pi d)$, where d is their separation and L is the interacting length.
- For an unchanged wire length, the force scales with the product of the two currents and inversely with the separation; the current directions determine attraction or repulsion, not this magnitude ratio.

Step 1: Express the initial magnetic force The field produced by wire Q at wire P is $B_Q = \mu_0 I_Q/(2\pi d)$. The force on the stated length of P is $I_P L B_Q$. With $I_P = I_Q = 10\,\text{A}$, $L = 0.10\,\text{m}$, and $d = 0.05\,\text{m}$, this gives

$$F_1 = \frac{(4\pi \times 10^{-7})(10)(10)(0.10)}{2\pi(0.05)} = 4 \times 10^{-5}\,\text{N}.$$

Step 2: Apply both changes to the force law Each current is doubled, so their product increases by $2 \times 2 = 4$. Halving the separation adds another factor of 2. Therefore

$$\frac{F_2}{F_1} = \frac{(2I_P)(2I_Q)}{I_P I_Q}\frac{d}{d/2} = 4 \times 2 = 8.$$

Numerically, $F_2 = 3.2 \times 10^{-4}\,\text{N}$. The length remains 10 cm and cancels in the ratio.

Step 3: Eliminate the remaining ratios The force must increase because both changes strengthen the interaction. This rules out the fractional values in options (A) and (C). The multiplication of the two current factors and the inverse-distance factor gives exactly 8, so the factor 10 in option (B) is inconsistent with the force law.

Step 4: Conclusion $F_2 = 8F_1$. Hence, the correct option is (D).

Quick Tip to Solve: For two wires, include both current changes: doubling both currents multiplies the force by four before changing the distance.

38. A circular loop of radius r is carrying current I A. The ratio of magnetic field at the center of circular loop and at a distance r from the center of the loop on its axis is : [4 marks]

(A) $3\sqrt{2} : 2$

(B) $1 : 3\sqrt{2}$

(C) $2\sqrt{2} : 1$

(D) $1 : \sqrt{2}$

Correct Answer: (C) $2\sqrt{2} : 1$

Solution:

Concept:

- For one circular current loop of radius r, the axial magnetic field a distance x from the centre is $B(x) = \mu_0 I r^2/[2(r^2 + x^2)^{3/2}]$.
- At the centre, $x = 0$, so $B(0) = \mu_0 I/(2r)$. Ratios at two positions on the same loop cancel the common current and permeability factors.

Step 1: Evaluate the field at the centre Substitute $x = 0$ into the axial-field expression. Since $(r^2)^{3/2} = r^3$ for the positive radius,

$$B_c = \frac{\mu_0 I r^2}{2r^3} = \frac{\mu_0 I}{2r}.$$

This is the first quantity in the requested ratio; maintaining its order avoids obtaining the reciprocal answer.

Step 2: Evaluate the field one radius along the axis At the second point $x = r$, the denominator contains $r^2 + r^2 = 2r^2$. Hence

$$B_r = \frac{\mu_0 I r^2}{2(2r^2)^{3/2}} = \frac{\mu_0 I}{2^{5/2} r}.$$

Divide the centre field by this value:

$$\frac{B_c}{B_r} = \frac{\mu_0 I}{2r} \frac{2^{5/2} r}{\mu_0 I} = 2^{3/2} = 2\sqrt{2}.$$

Step 3: Check the ordering and options The field decreases as one moves away from the centre along the axis, so the requested first-to-second ratio must exceed one. Options (B) and (D) have the opposite trend. Option (A) gives $3\sqrt{2}/2$, which does not equal the exact factor $2\sqrt{2}$ obtained from the power $3/2$.

Step 4: Conclusion The magnetic-field ratio is $B_c : B_r = 2\sqrt{2} : 1$. Hence, the correct option is (C).

Quick Tip to Solve: Use the dimensionless form $B(x)/B(0) = [1 + (x/r)^2]^{-3/2}$ to calculate axial-field ratios directly.

39. If two charges q_1 and q_2 are separated with distance 'd' and placed in a medium of dielectric constant K. What will be the equivalent distance between charges in air for the same electrostatic force? [4 marks]

(A) $d\sqrt{k}$

(B) $1.5d\sqrt{k}$

(C) $k\sqrt{d}$

(D) $2d\sqrt{k}$

Correct Answer: (A) $d\sqrt{k}$

Solution:

Concept:

- In a medium of relative permittivity K, Coulomb's force magnitude is $F_m = |q_1 q_2|/(4\pi\varepsilon_0 K d^2)$.
- Air is treated as having relative permittivity one. Changing the medium changes the force prefactor, whereas changing distance changes its inverse-square factor.

Step 1: Write the force at the two separations Let D denote the required separation in air, keeping the charges unchanged. The force in air is

$$F_a = \frac{|q_1 q_2|}{4\pi\varepsilon_0 D^2}.$$

The force in the given dielectric at separation d is

$$F_m = \frac{|q_1 q_2|}{4\pi\varepsilon_0 K d^2}.$$

Equating these expressions implements the stated requirement of the same electrostatic force.

Step 2: Solve the inverse-square relation Cancel the common charge product and Coulomb constant. This leaves

$$\frac{1}{D^2} = \frac{1}{Kd^2}, \qquad D^2 = Kd^2.$$

Since a separation is positive, choose the positive root:

$$D = d\sqrt{K}.$$

The nature of the interaction, attractive or repulsive, is unchanged by this distance adjustment; the calculation concerns its magnitude.

Step 3: Check dimensions and limiting behaviour For $K > 1$, the dielectric weakens the force at the same distance, so the separation in air must be greater than d. At $K = 1$, it must reduce exactly to d. Options (B) and (D) introduce unjustified factors 1.5 and 2 and fail the latter check. Option (C), $K\sqrt{d}$, has the wrong dimensions for a distance.

Step 4: Conclusion The equivalent separation in air is $d\sqrt{K}$. Hence, the correct option is (A).

Quick Tip to Solve: Equate Kd^2 with the squared air separation; the dielectric factor enters the distance through a square root.

40. A conducting circular loop of radius $\frac{10}{\sqrt{\pi}}$ cm is placed perpendicular to a uniform magnetic field of 0.5 T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25 s is : [4 marks]

(A) emf = 10 mV

(B) emf = 5 mV

(C) emf = 100 mV

(D) emf = 1 mV

Correct Answer: (A) emf = 10 mV

Solution:

Concept:

- For a fixed one-turn loop, the induced emf magnitude is $|\mathcal{E}| = |d\Phi_B/dt|$, with flux $\Phi_B = BA\cos\theta$.
- The plane of the loop is perpendicular to the field, so its area normal is parallel to the field and $\cos\theta = 1$. A steady rate of field change produces a constant emf.

Step 1: Calculate the loop area in SI units The radius is $r = (10/\sqrt{\pi})\,\text{cm} = (0.10/\sqrt{\pi})\,\text{m}$. Squaring the radius before multiplying by π gives

$$A = \pi r^2 = \pi\frac{(0.10)^2}{\pi} = 0.010\,\text{m}^2.$$

Converting centimetres to metres before evaluating the area prevents a factor-of-ten or factor-of-one-hundred error.

Step 2: Find the constant rate of field change The field decreases uniformly from $0.5\,\text{T}$ to zero in $0.5\,\text{s}$, so

$$\frac{dB}{dt} = \frac{0 - 0.5}{0.5} = -1\,\text{T}\,\text{s}^{-1}.$$

Thus throughout this interval, including at $t = 0.25\,\text{s}$,

$$|\mathcal{E}| = A\left|\frac{dB}{dt}\right| = 0.010(1) = 0.010\,\text{V} = 10\,\text{mV}.$$

Step 3: Distinguish the field value from its rate At the halfway time the field itself is $0.25\,\text{T}$, but the emf depends on the slope of the field-time graph, which remains unchanged. Therefore halving the emf to 5 mV, as in (B), is incorrect. The values 100 mV and 1 mV in (C) and (D) differ by an extra factor of ten from the correctly converted flux-change rate.

Step 4: Conclusion The induced emf magnitude at $0.25\,\text{s}$ is $10\,\text{mV}$. Hence, the correct option is (A).

Quick Tip to Solve: For a linearly changing magnetic field, use the constant slope $\Delta B/\Delta t$; do not substitute the instantaneous field into Faraday's law.

41. 1 g of a liquid is converted to vapour at 3×10^5 Pa pressure. If 10% of the heat supplied is used for increasing the volume by 1600 cm^3 during this phase change, then the increase in internal energy in the process will be : [4 marks]

(A) 4800 J

(B) 4320 J

(C) 432000 J

(D) 4.32×10^8 J

Correct Answer: (B) 4320 J

Solution:

Concept:

- For heat Q supplied to a system and work W done by it, the first law gives $\Delta U = Q - W$.
- Expansion against constant pressure requires work $W = P\Delta V$. Here this mechanical work is 10 percent of the supplied heat, so the remaining 90 percent increases internal energy.

Step 1: Evaluate the expansion work The volume increase is

$$\Delta V = 1600\,\text{cm}^3 = 1600 \times 10^{-6}\,\text{m}^3 = 1.6 \times 10^{-3}\,\text{m}^3.$$

At the given pressure, the work done by the vaporising liquid is

$$W = P\Delta V = (3 \times 10^5)(1.6 \times 10^{-3}) = 480\,\text{J}.$$

This work is positive because the system expands and transfers mechanical energy to its surroundings.

Step 2: Recover the total heat and internal-energy change The problem states that only 10 percent of the supplied heat performs the expansion work. Therefore

$$0.10Q = 480\,\text{J}, \qquad Q = 4800\,\text{J}.$$

Applying the first law now gives

$$\Delta U = Q - W = 4800 - 480 = 4320\,\text{J}.$$

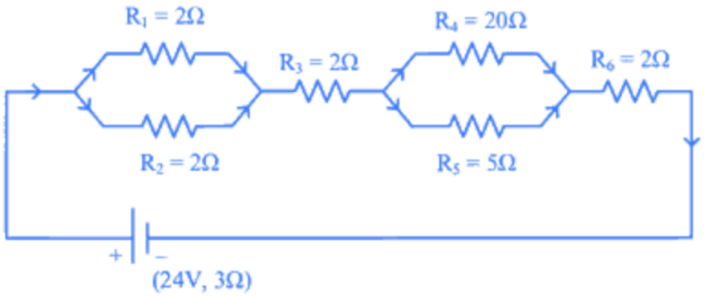
Equivalently, $\Delta U = 0.90Q$. The given mass identifies the amount vaporised but is not needed again once the total volume change is supplied.

Step 3: Compare all energy choices Option (A), 4800 J, is the total heat input, not the internal-energy increase, because some heat leaves as expansion work. Options (C) and (D) are much too large and do not satisfy the stated 10 percent energy split. A positive change of 4320 J conserves energy: adding it to the 480 J of work recovers exactly the supplied heat.

Step 4: Conclusion The increase in internal energy is $4320\,\text{J}$. Hence, the correct option is (B).

Quick Tip to Solve: When expansion work is 10 percent of the heat, $\Delta U = 9W$; first calculate $P\Delta V$ with the volume in cubic metres.

42. As shown in the figure, a network of resistors is connected to a battery of 24V with an internal resistance of 3 Ω. The currents through the resistors R_4 and R_5 are I_4 and I_5 respectively. The values of I_4 and I_5 are : [4 marks]



(A) $I_4 = \frac{8}{5}A$ and $I_5 = \frac{2}{5}A$
(B) $I_4 = \frac{6}{5}A$ and $I_5 = \frac{24}{5}A$
(C) $I_4 = \frac{2}{5}A$ and $I_5 = \frac{8}{5}A$
(D) $I_4 = \frac{24}{5}A$ and $I_5 = \frac{6}{5}A$

Correct Answer: (C) $I_4 = \frac{2}{5}A$ and $I_5 = \frac{8}{5}A$

Solution:

Concept:

- Resistors sharing both endpoints are parallel, with $R_p = R_aR_b/(R_a + R_b)$. Series resistances, including the battery internal resistance, add directly.
- The two resistors in a parallel pair have the same potential difference. Their branch currents obey $I_a = V_p/R_a$ and $I_b = V_p/R_b$, so current is larger in the smaller resistance.

Step 1: Reduce the external resistor network The left-hand parallel pair has $R_1 = R_2 = 2\,\Omega$, hence

$$R_{12} = \frac{2 \times 2}{2 + 2} = 1\,\Omega.$$

The right-hand parallel pair is $R_4 = 20\,\Omega$ and $R_5 = 5\,\Omega$, giving

$$R_{45} = \frac{20 \times 5}{20 + 5} = 4\,\Omega.$$

These two equivalents are in series with $R_3 = 2\,\Omega$ and $R_6 = 2\,\Omega$, so $R_{\text{ext}} = 1 + 2 + 4 + 2 = 9\,\Omega$.

Step 2: Include the internal resistance and find the common current The battery internal resistance is $3\,\Omega$, which is also in series with the external circuit. Thus

$$R_{\text{total}} = 9 + 3 = 12\,\Omega, \qquad I = \frac{24}{12} = 2\,\text{A}.$$

This current enters the R_4, R_5 parallel combination. Its voltage is

$$V_{45} = IR_{45} = 2 \times 4 = 8\,\text{V}.$$

Step 3: Calculate and check both branch currents Apply the same 8 V across each branch:

$$I_4 = \frac{8}{20} = \frac{2}{5}\,\text{A}, \qquad I_5 = \frac{8}{5}\,\text{A}.$$

Their sum is 2 A, matching the incoming current, and $I_5/I_4 = 4 = R_4/R_5$. Option (A) reverses the two currents. Options (B) and (D) give branch-current sums of 6 A, which exceed the available series current and violate the network calculation.

Step 4: Conclusion $I_4 = \frac{2}{5}$ A and $I_5 = \frac{8}{5}$ A. Hence, the correct option is (C).

Quick Tip to Solve: Include the battery internal resistance before splitting the current; then use the common voltage across the parallel pair.

43. A travelling wave is described by the equation

$y(x,t) = [0.05\sin(8x - 4t)]$ **m**

The velocity of the wave is : [all the quantities are in SI unit] [4 marks]

(A) $4\ \text{ms}^{-1}$

(B) $2\ \text{ms}^{-1}$

(C) $8\ \text{ms}^{-1}$

(D) $0.5\ \text{ms}^{-1}$

Correct Answer: (D) $0.5\ \text{ms}^{-1}$

Solution:

Concept:

- A progressive sinusoidal wave has the form $y = A\sin(kx - \omega t + \phi)$, where k is the angular wave number and ω is the angular frequency.
- The phase velocity is $v = \omega/k = f\lambda$. The amplitude controls the maximum displacement, not the speed.

Step 1: Identify the wave parameters Compare the given expression $y = 0.05\sin(8x - 4t)$ with the standard form. The coefficient of x is $k = 8\,\text{rad}\,\text{m}^{-1}$; the coefficient of t is $\omega = 4\,\text{rad}\,\text{s}^{-1}$. The amplitude is $0.05\,\text{m}$, and there is no additional phase constant. Do not interchange the spatial and temporal coefficients.

Step 2: Follow a fixed phase A crest corresponds to a fixed value of the phase. Thus $8x - 4t =$ constant. Differentiating this condition gives

$$8\frac{dx}{dt} - 4 = 0, \qquad \frac{dx}{dt} = \frac{4}{8} = 0.5\,\text{m}\,\text{s}^{-1}.$$

The positive sign shows travel along increasing x. Equivalently, $\lambda = 2\pi/8$ and $f = 4/(2\pi)$, whose product is the same speed.

Step 3: Reject the other speeds The value 4 is the angular frequency, not the wave speed, so (A) is incorrect. The value 8 is the wave number, excluding (C). Dividing k by ω gives $2\,\text{s}\,\text{m}^{-1}$, the reciprocal

of speed rather than a velocity; hence (B) is also incorrect.

Step 4: Conclusion The wave travels in the positive x-direction at $0.5\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: For $\sin(kx - \omega t)$, divide the coefficient of time by the coefficient of position and use the minus sign to identify positive propagation.

44. The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance upto which he can throw the same ball is : [4 marks]

(A) 136 m

(B) 272 m

(C) 68 m

(D) 192 m

Correct Answer: (B) 272 m

Solution:

Concept:

- For a vertical throw with initial speed u, the maximum height is $H = u^2/(2g)$, because the speed becomes zero at the highest point.
- For a projectile returning to its launch level, $R = u^2 \sin 2\theta/g$. The maximum range occurs at $\theta = 45°$.

Step 1: Use the vertical throw to fix the available speed The maximum vertical height corresponds to directing the full launch speed upward. Applying $v^2 = u^2 - 2gH$ with $v = 0$ gives

$$u^2 = 2gH = 2g(136\,\mathrm{m}).$$

The same man throwing the same ball is taken to provide this same maximum initial speed when choosing another launch angle. Air resistance is neglected, as in the standard projectile model.

Step 2: Maximise the horizontal range At a launch angle θ, the horizontal component is $u\cos\theta$ and the flight time back to the same level is $2u\sin\theta/g$. Their product gives

$$R = \frac{u^2 \sin 2\theta}{g}.$$

Since $\sin 2\theta \le 1$, the largest range is

$$R_{\max} = \frac{u^2}{g} = 2H = 2(136) = 272\,\mathrm{m}.$$

Step 3: Compare the proposed distances Option (A) equals the vertical height but omits the factor of two between maximum range and vertical height for a fixed launch speed. Option (C) is half the height, which has no maximum-range basis. Option (D), $192\,\mathrm{m}$, can be smaller than the maximum but is not the maximum obtained when $\sin 2\theta = 1$.

Step 4: Conclusion The maximum horizontal distance is $272\,\mathrm{m}$, obtained at 45°. Hence, the correct option is (B).

Quick Tip to Solve: For the same launch speed on level ground, maximum horizontal range is exactly twice the height of a purely vertical throw.

45. Given below are two statements :

Statement I : The temperature of a gas is -73°C. When the gas is heated to 527°C, the root mean square speed of the molecules is doubled.

Statement II : The product of pressure and volume of an ideal gas will be equal to translational kinetic energy of the molecules.

In the light of the above statements, choose the correct answer from the option given below :

[4 marks]

(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are true

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are false

Correct Answer: (A) Statement I is true but Statement II is false

Solution:

Concept:

- For molecules of fixed mass m, the root mean square speed is $v_{\mathrm{rms}} = \sqrt{3k_{\mathrm{B}}T/m}$, where T must be in kelvin.
- The total translational kinetic energy of an ideal gas is $U_{\mathrm{tr}} = \frac{3}{2}Nk_{\mathrm{B}}T = \frac{3}{2}PV$.

Step 1: Convert the temperatures and test Statement I The initial absolute temperature is $T_1 = -73 + 273 = 200\,\mathrm{K}$, and the final temperature is $T_2 = 527 + 273 = 800\,\mathrm{K}$. Because the gas and molecular mass do not change,

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{800}{200}} = 2.$$

Thus heating increases the mean squared speed by a factor of four and the root mean square speed by a factor of two. Statement I is true. Ratios of Celsius temperatures would be physically incorrect.

Step 2: Relate pressure-volume product to kinetic energy Kinetic theory gives $PV = \frac{1}{3}Nm\langle v^2\rangle$. The translational kinetic energy of all the molecules is $U_{\mathrm{tr}} = \frac{1}{2}Nm\langle v^2\rangle$. Comparing the two expressions gives

$$PV = \frac{2}{3}U_{\mathrm{tr}}, \qquad U_{\mathrm{tr}} = \frac{3}{2}PV.$$

Consequently the two quantities have the same dimensions but are not equal; the numerical factor matters. Statement II is false.

Step 3: Select the true-false combination The required combination is true for Statement I and false for Statement II. Option (B) wrongly accepts the equality in Statement II. Options (C) and (D) incorrectly reject the speed doubling demonstrated by the absolute-temperature ratio.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: Convert Celsius to kelvin before taking a speed ratio, and remember that total translational kinetic energy is $3PV/2$, not PV.

46. Match List I with List II

	List-I		List-II
A.	**Planck's constant (h)**	**I.**	$[\mathrm{M}^1\,\mathrm{L}^2\,\mathrm{T}^{-2}]$
B.	**Stopping potential (Vs)**	**II.**	$[\mathrm{M}^1\,\mathrm{L}^1\,\mathrm{T}^{-1}]$
C.	**Work function (ϕ)**	**III.**	$[\mathrm{M}^1\,\mathrm{L}^2\,\mathrm{T}^{-1}]$
D.	**Momentum (p)**	**IV.**	$[\mathrm{M}^1\,\mathrm{L}^2\,\mathrm{T}^{-3}\,\mathrm{A}^{-1}]$

Choose the correct answer from the options given below : [4 marks]

(A) A-I, B-III, C-IV, D-II

(B) A-III, B-IV, C-I, D-II

(C) A-II, B-IV, C-III, D-I

(D) A-III, B-I, C-II, D-IV

Correct Answer: (B) A-III, B-IV, C-I, D-II

Solution:

Concept:

- Energy has dimensions $[ML^2T^{-2}]$, while momentum has dimensions $[MLT^{-1}]$.
- Use $E = h\nu$ for Planck's constant and $V = E/q$ for potential, with $[q] = [AT]$.

Step 1: Find the dimensions of Planck's constant and potential Frequency has dimensions $[T^{-1}]$. From $E = h\nu$,

$$[h] = \frac{[E]}{[\nu]} = [ML^2T^{-1}],$$

so entry A matches III. Stopping potential is a potential difference; the energy imparted to a charge is qV_s. Since charge equals current multiplied by time,

$$[V_s] = \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}].$$

Thus B matches IV. The symbol A in the dimensional expression denotes electric current, rather than the list label.

Step 2: Find the dimensions of work function and momentum The work function is the minimum energy required to remove an electron from a surface, so

$$[\phi] = [ML^2T^{-2}],$$

which matches I. Momentum is mass times velocity, hence

$$[p] = [M][LT^{-1}] = [MLT^{-1}],$$

which matches II. The full association is therefore A-III, B-IV, C-I, D-II.

Step 3: Eliminate the alternative combinations Option (A) assigns energy dimensions to Planck's constant and includes incorrect potential dimensions. Option (C) assigns momentum dimensions to Planck's constant and action dimensions to work function. Option (D) correctly starts with A-III but swaps the physical dimensions of potential, energy and momentum. Only option (B) preserves all four relations.

Step 4: Conclusion The matching is A-III, B-IV, C-I, D-II. Hence, the correct option is (B).

Quick Tip to Solve: Use the defining equations $E = h\nu$, $E = qV$ and $p = mv$; they give all four dimensions without memorising a separate table.

---

47. Given below are two statements :

Statement I : An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

Statement II : Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are false

(B) Both Statement I and Statement II are true

(C) Statement I is false but Statement II is true

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- Uniform speed along a straight vertical path means zero acceleration. Newton's second law then requires the net force to vanish.
- The normal force N from the floor is the apparent weight of a person. With upward positive, $N - mg = ma_y$.

Step 1: Check the uniform-speed statement Let M denote the supported mass of the elevator system. If cable tension T balances its weight Mg, then

$$T - Mg = Ma_y = 0.$$

The acceleration is zero. The elevator can therefore remain at rest or move with a constant velocity, either upward or downward. A balanced force does not require zero velocity; it requires no change in velocity. Statement I is true.

Step 2: Check the downward acceleration statement Going downward with increasing speed means that the acceleration is downward. If its magnitude is $a > 0$, the upward-positive acceleration is $-a$.

For a person of mass m, the vertical equation is

$$N - mg = -ma, \qquad N = m(g - a).$$

While the person remains in contact with the floor, this normal force is less than mg. The person feels lighter, not heavier. In the limiting free-fall case $a = g$, the normal force becomes zero.

Step 3: Choose the correct combination Statement II claims a force greater than the weight, which would instead occur for upward acceleration. It is false here. The combination is therefore true for I and false for II. Options (A) and (C) reject the correct first statement, while option (B) accepts the incorrect second statement.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: For apparent weight, use the direction of acceleration rather than the direction of motion: downward acceleration reduces the normal force.

48. In $\overrightarrow{E}$ and $\overrightarrow{K}$ represent electric field and propagation vectors of the EM waves in vacuum, then magnetic field vector is given by :

(ω - angular frequency) : [4 marks]

(A) $\frac{1}{\omega}\left(\overline{K} \times \overline{E}\right)$

(B) $\overline{K} \times \overline{E}$

(C) $\omega\left(\overline{K} \times \overline{E}\right)$

(D) $\omega\left(\overline{E} \times \overline{K}\right)$

Correct Answer: (A) $\frac{1}{\omega}\left(\overline{K} \times \overline{E}\right)$

Solution:

Concept:

- For a plane electromagnetic wave in vacuum, $\vec{E}$, $\vec{B}$, and the propagation direction are mutually perpendicular, with $E = cB$.
- The wave vector has magnitude $K = \omega/c$. Maxwell's Faraday law is $\nabla \times \vec{E} = -\partial\vec{B}/\partial t$.

Step 1: Apply Faraday's law to a plane wave Take the common phase to be $\vec{K} \cdot \vec{r} - \omega t$, and write the fields as their constant vector amplitudes times the cosine of this phase. The spatial curl of the electric field is $-\vec{K} \times \vec{E}_0 \sin(\vec{K} \cdot \vec{r} - \omega t)$. The negative time derivative of the magnetic field is $-\omega\vec{B}_0 \sin(\vec{K} \cdot \vec{r} - \omega t)$. Equating the coefficients gives

$$\vec{K} \times \vec{E}_0 = \omega\vec{B}_0.$$

Step 2: Solve for the magnetic vector and check the magnitude The same relation holds between the instantaneous field vectors because their phases agree:

$$\vec{B} = \frac{1}{\omega}(\vec{K} \times \vec{E}).$$

Since $\vec{K} \perp \vec{E}$, the magnitude is $B = KE/\omega = E/c$, as required. Its direction also gives $\vec{E} \times \vec{B}$ along the propagation vector through the right-hand rule.

Step 3: Reject the incorrect factors and orientation Option (B) lacks division by angular frequency and therefore has the wrong dimensions. Options (C) and (D) multiply by frequency rather than dividing, so neither satisfies $E = cB$. In addition, reversing the cross product in (D) changes the sign of the vector direction.

Step 4: Conclusion The magnetic field is $\vec{B} = (\vec{K} \times \vec{E})/\omega$. Hence, the correct option is (A).

Quick Tip to Solve: Check both size and direction: $B = E/c$, and $\vec{E} \times \vec{B}$ must point along the direction of wave travel.

---

49. A modulating signal is a square wave, as shown in the figure.
If the carrier wave is given as $c(t) = 2\sin(8\pi t)$ volts, the modulation index is : [4 marks]



(A) $\frac{1}{3}$
(B) 1
(C) $\frac{1}{4}$
(D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Concept:

- The amplitude modulation index is $\mu = A_m/A_c$, where A_m is the peak modulating-signal amplitude and A_c is the unmodulated carrier amplitude.
- For a signal centred on zero, the peak amplitude is its largest displacement from the zero line, not its peak-to-peak value.

Step 1: Read the two signal amplitudes The square-wave graph has its positive level at $1\,\text{V}$ above the zero line and its negative level equally below zero. Its peak amplitude is consequently $A_m = 1\,\text{V}$. The time markings specify when the level changes, but do not change the amplitude. In $c(t) = 2\sin(8\pi t)$, the coefficient of the sine is the carrier amplitude, so $A_c = 2\,\text{V}$. The factor 8π is the angular frequency.

Step 2: Calculate the modulation index Apply the amplitude ratio directly:

$$\mu = \frac{A_m}{A_c} = \frac{1\,\text{V}}{2\,\text{V}} = \frac{1}{2}.$$

The result is dimensionless and corresponds to 50 percent modulation. The associated envelope levels

are proportional to $A_c + A_m = 3\,\text{V}$ and $A_c - A_m = 1\,\text{V}$; they also give $(3-1)/(3+1) = 1/2$. Since the index is below unity, there is no overmodulation.

Step 3: Rule out the other proposed ratios Option (A), $1/3$, divides the signal amplitude by the maximum envelope level rather than the carrier amplitude. Option (B) would result from using the peak-to-peak signal height $2\,\text{V}$ as its amplitude. Option (C), $1/4$, incorrectly halves the peak signal amplitude or doubles the carrier amplitude.

Step 4: Conclusion The modulation index is $\mu = 1/2 = 0.50$, or 50 percent. Hence, the correct option is (D).

Quick Tip to Solve: Read amplitude from the zero line to one peak. Do not use the full peak-to-peak height of the square wave in A_m/A_c.

---

50. Consider the following radioactive decay process

$$^{218}_{84}A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\gamma} A_3 \xrightarrow{\alpha} A_4 \xrightarrow{\beta^+} A_5 \xrightarrow{\gamma} A_6$$

The mass number and the atomic number of A_6 are given by : [4 marks]

(A) 210 and 84

(B) 210 and 80

(C) 211 and 80

(D) 210 and 82

Correct Answer: (B) 210 and 80

Solution:

Concept:

- An alpha particle has mass number 4 and atomic number 2. Alpha emission changes the parent numbers as $(A, Z) \mapsto (A-4, Z-2)$.
- In beta-minus decay a neutron changes into a proton, so mass number remains fixed and atomic number increases by 1. Beta-plus decay reverses this nuclear change and decreases atomic number by 1.
- Gamma emission removes excitation energy only; neither mass number nor atomic number changes. These rules apply to the nucleus remaining after each emission.

Step 1: Follow the first three emissions Start with mass number 218 and atomic number 84. The first alpha emission produces a nucleus with four fewer nucleons and two fewer protons. The following beta-minus emission restores one proton, while the gamma emission leaves both numbers unchanged:

$$(218, 84) \xrightarrow{\alpha} (214, 82) \xrightarrow{\beta^-} (214, 83) \xrightarrow{\gamma} (214, 83).$$

Thus the third intermediate nucleus has mass number 214 and atomic number 83.

Step 2: Complete the chain and compare the options The second alpha emission changes the pair to $(210, 81)$. The beta-plus emission then reduces the proton number by one, and the final gamma

emission changes neither number:

$$(214, 83) \xrightarrow{\alpha} (210, 81) \xrightarrow{\beta^+} (210, 80) \xrightarrow{\gamma} (210, 80).$$

The two beta changes cancel each other overall, while the two alpha emissions reduce the mass number by 8 and atomic number by 4. Hence 210 and 84 misses the proton loss, 211 and 80 has the wrong nucleon count, and 210 and 82 does not include the complete atomic-number change.

Step 3: Conclusion The mass number and atomic number of A_6 are respectively $\boxed{210 \text{ and } 80}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep a two-column tally of mass number and atomic number; gamma never changes either column.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A hollow cylindrical conductor has length of 3.14 m, while its inner and outer diameters are 4 mm and 8 mm respectively. The resistance of the conductor is $n \times 10^{-3}\Omega$. If the resistivity of the material is $2.4 \times 10^{-8}\Omega$m. The value of n is _____. [4 marks]

Solution:

Concept:

- For a uniform conductor, $R = \rho L/A$, where ρ is resistivity, L is length and A is the cross-sectional area perpendicular to current.
- For a hollow circular conductor the conducting area is the outer disc area minus the inner disc area: $A = \pi(r_o^2 - r_i^2)$. The hollow centre carries no current.

Step 1: Calculate the metal cross-sectional area The given quantities are diameters, so first divide them by two. In SI units, $r_o = 4 \times 10^{-3}\,\text{m}$ and $r_i = 2 \times 10^{-3}\,\text{m}$. Therefore

$$A = \pi\left[(4 \times 10^{-3})^2 - (2 \times 10^{-3})^2\right] = 12\pi \times 10^{-6}\,\text{m}^2.$$

Subtracting the radii before squaring would incorrectly replace an annulus by a smaller solid disc. Using the actual annular area accounts for all the metal through which charge flows.

Step 2: Substitute in the resistance formula Use the length $L = 3.14\,\text{m}$ and resistivity $\rho = 2.4 \times 10^{-8}\,\Omega\,\text{m}$:

$$R = \frac{(2.4 \times 10^{-8})(3.14)}{12\pi \times 10^{-6}} = \frac{2.4}{12}\frac{3.14}{\pi} \times 10^{-2}\,\Omega.$$

With $\pi \simeq 3.14$, this is $2.0 \times 10^{-3}\,\Omega$. The resistivity unit times length divided by area indeed gives ohms. The small positive resistance is consistent with the large conducting cross section of a good conductor.

Step 3: Identify the requested coefficient The problem writes the resistance in the form $R = n \times 10^{-3}\,\Omega$. Compare this directly with the calculated value, keeping the power of ten separate from its numerical coefficient. Thus $n \times 10^{-3} = 2 \times 10^{-3}$, giving $n = 2$. The coefficient is dimensionless; the physical

resistance retains the unit ohm.

Step 4: Conclusion The resistance is $\boxed{2 \times 10^{-3}\,\Omega}$, so the required dimensionless value is $\boxed{n = 2}$.

Quick Tip to Solve: For a hollow wire use the difference of squared radii, and convert millimetres to metres before squaring.

---

52. A block of a mass 2 kg is attached with two identical springs of spring constant 20 N/m each. The block is placed on a frictionless surface and the ends of the springs are attached to rigid supports (see figure). When the mass is displaced from its equilibrium position, it executes a simple harmonic motion. The time period of oscillation is $\frac{\pi}{\sqrt{x}}$ in SI unit. The value of x is _____. [4 marks]



Solution:

Concept:

- For a small displacement y from equilibrium, SHM obeys $F = -k_{\text{eff}}y$ and $T = 2\pi\sqrt{m/k_{\text{eff}}}$.
- With the block between two fixed springs, one spring stretches and the other compresses by the same displacement. Both changes produce restoring forces toward equilibrium.

Step 1: Add the two restoring forces Displace the block a distance y to the right. The spring on the left stretches by y and pulls the block left with force ky. The spring on the right compresses by y and also pushes it left with force ky. Thus

$$F = -ky - ky = -2ky.$$

Comparing with the SHM form gives $k_{\text{eff}} = 2k = 40\,\text{N m}^{-1}$. The springs do not behave as a series pair here: the moving mass is between them and each spring has a fixed outer end.

Step 2: Calculate the time period The mass is $m = 2\,\text{kg}$, so Newton's law can be written $\ddot{y} + (40/2)y = 0$. Hence the angular frequency is $\omega = \sqrt{20}\,\text{s}^{-1}$. The period is consequently

$$T = \frac{2\pi}{\sqrt{20}} = \frac{\pi}{\sqrt{5}}\,\text{s}.$$

The frictionless surface ensures that no damping force must be included; the equilibrium weight and normal reaction do not contribute to horizontal oscillation.

Step 3: Compare with the stated form The question gives $T = \pi/\sqrt{x}$ in SI units. Comparing the positive denominators yields $\sqrt{x} = \sqrt{5}$, and therefore $x = 5$. As a check, the period is about $1.40\,\text{s}$, shorter by a factor $\sqrt{2}$ than for the same mass attached to only one spring. This is expected because the two restoring contributions increase stiffness.

Step 4: Conclusion The period is $\boxed{T = \pi/\sqrt{5}\,\text{s}}$. The requested dimensionless numerical value is

$\boxed{x = 5}$.

Quick Tip to Solve: Test a displacement explicitly: if both springs push or pull the block back, their spring constants add.

---

53. Vectors $a\hat{i} + b\hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$, the ratio of a to b is $\frac{x}{2}$. The value of x is _____. [4 marks]

Solution:

Concept:

- Two nonzero perpendicular vectors have zero scalar product. In Cartesian components, $(u_x, u_y, u_z) \cdot (v_x, v_y, v_z) = u_x v_x + u_y v_y + u_z v_z$.
- The perpendicularity condition and the separately supplied relation provide two simultaneous linear equations for a and b. The requested coefficient is found only after obtaining their ratio.

Step 1: Translate perpendicularity into an equation The vectors have components $(a, b, 1)$ and $(2, -3, 4)$. Their scalar product must vanish:

$$2a - 3b + 4 = 0, \qquad 2a - 3b = -4.$$

The minus sign in the second vector's middle component is essential. The unit-vector cross products do not enter a scalar product because distinct Cartesian unit vectors are mutually perpendicular.

Step 2: Solve with the given relation The second equation is $3a+2b = 7$. Multiply the perpendicularity equation by 2 and the given equation by 3:

$$4a - 6b = -8,$$
$$9a + 6b = 21.$$

Adding eliminates b and gives $13a = 13$, so $a = 1$. Substitution in $3a + 2b = 7$ gives $3 + 2b = 7$, hence $b = 2$. Both parameters are positive even though their weighted scalar-product contributions have different signs.

Step 3: Calculate the ratio and check both conditions The ratio is $a/b = 1/2$. Comparing with the problem's expression $x/2$ gives $x = 1$. To verify the solution directly, the dot product is $2(1)-3(2)+4 = 0$, and the other condition gives $3(1) + 2(2) = 7$. Thus the same pair satisfies both independent conditions. No vector magnitude or angle calculation is necessary once perpendicularity has supplied the zero-dot-product equation.

Step 4: Conclusion We obtain $a = 1$, $b = 2$ and $a/b = 1/2$. Therefore the required dimensionless value is $\boxed{x = 1}$.

Quick Tip to Solve: Use a zero dot product before doing any angle calculation, then verify the simultaneous-equation solution in both conditions.

---

54. A hole is drilled in a metal sheet. At $27^\circ C$, the diameter of hole is 5 cm. When the sheet is heated to $177^\circ C$, the change in the diameter of hole is $d \times 10^{-3}$ cm. The value of d will be _____ if coefficient of linear expansion of the metal is $1.6 \times 10^{-5}/^\circ C$. [4 marks]

Solution:

Concept:

- A hole in a uniformly heated sheet expands as if it were filled with the same material. Its diameter is a linear dimension and follows $\Delta D = \alpha D_0 \Delta T$.
- The coefficient α is the coefficient of linear expansion. Use the temperature increase, not the final temperature; a Celsius temperature interval equals the corresponding kelvin interval.

Step 1: Find the temperature increase and expansion rule The sheet is heated from $27^\circ C$ to $177^\circ C$, so

$$\Delta T = 177 - 27 = 150^\circ C.$$

Imagine filling the opening with a disc of the same metal. Under uniform heating both the sheet and this imaginary disc expand together, so removing the disc again leaves a larger opening. Consequently the diameter increases rather than decreases. The area-expansion coefficient is unnecessary because the question asks for a change in diameter.

Step 2: Calculate the diameter increase Substitute $D_0 = 5\,\text{cm}$ and $\alpha = 1.6 \times 10^{-5}\,^\circ C^{-1}$:

$$\begin{aligned} \Delta D &= (1.6 \times 10^{-5})(5)(150)\,\text{cm} \\ &= 1200 \times 10^{-5}\,\text{cm} \\ &= 0.012\,\text{cm} = 12 \times 10^{-3}\,\text{cm}. \end{aligned}$$

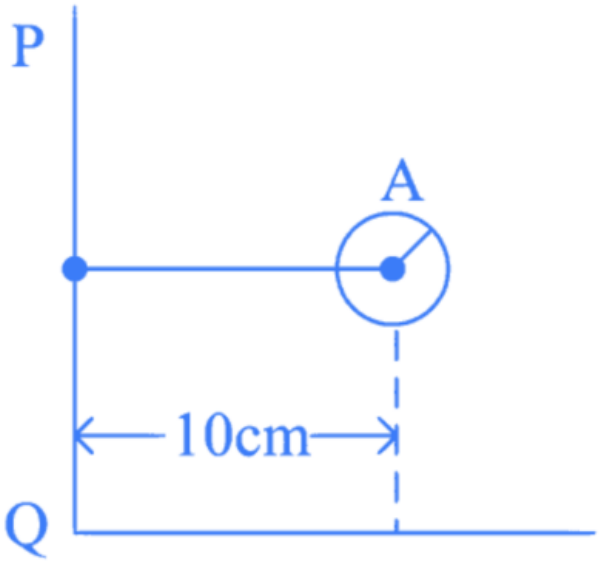
The expansion factor is $\alpha \Delta T = 0.0024$, a small fraction, consistent with the usual linear approximation. Multiplying by the original diameter gives the dimensional change rather than the final diameter.

Step 3: Extract the requested coefficient The stated form of the increase is $d \times 10^{-3}\,\text{cm}$. Comparing with $12 \times 10^{-3}\,\text{cm}$ gives $d = 12$. As an additional check, the new hole diameter is $5 + 0.012 = 5.012\,\text{cm}$, which is slightly larger than before and therefore consistent with positive thermal expansion.

Step 4: Conclusion The diameter increases by $\boxed{0.012\,\text{cm}}$, so the required dimensionless coefficient is $\boxed{d = 12}$.

Quick Tip to Solve: A heated hole grows with the sheet: use the linear coefficient for its diameter and subtract the two temperatures first.

55. Solid sphere A is rotating about an axis PQ. If the radius of the sphere is 5 cm then its radius of gyration about PQ will be $\sqrt{x}$ cm. The value of x is _____. [4 marks]



Solution:

Concept:

- The moment of inertia of a uniform solid sphere about any diameter is $I_{\text{cm}} = (2/5)MR^2$, where M is mass and R is its radius.
- For a parallel axis a perpendicular distance d from the centre, $I = I_{\text{cm}} + Md^2$. The radius of gyration K is defined by $I = MK^2$.

Step 1: Identify the axis separation from the figure The line PQ lies outside the sphere. The horizontal marked distance from PQ to the sphere's centre is $10\,\text{cm}$, while the sphere radius is $5\,\text{cm}$. Thus use $d = 10\,\text{cm}$ in the parallel-axis theorem. A diameter through the centre parallel to PQ is a suitable reference axis because the sphere is rotationally symmetric. The marked distance is not a distance to the surface; the diagram's dashed line passes through the centre.

Step 2: Apply the parallel-axis theorem Using centimetres throughout is convenient because the final radius is requested in centimetres. Substitute the radius and offset:

$$\begin{aligned} I_{PQ} &= \frac{2}{5}M(5\,\text{cm})^2 + M(10\,\text{cm})^2 \\ &= M(10 + 100)\,\text{cm}^2 \\ &= 110M\,\text{cm}^2. \end{aligned}$$

Both contributions are positive; moving the rotation axis away from the centre increases the moment of inertia. The mass need not be known numerically because it cancels in the definition of the radius of gyration.

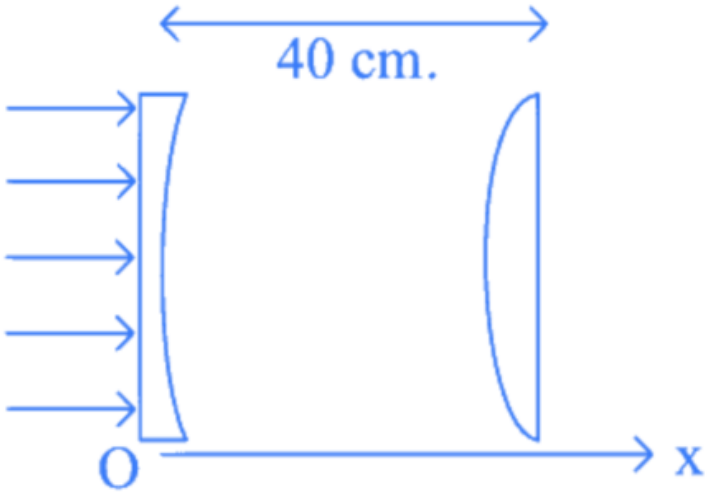
Step 3: Convert moment of inertia to radius of gyration By definition, $MK^2 = I_{PQ}$, so $K^2 = 110\,\text{cm}^2$ and $K = \sqrt{110}\,\text{cm}$. Comparing this result with the supplied expression $\sqrt{x}\,\text{cm}$ gives $x = 110$. A useful check is that $K > 10\,\text{cm}$: the sphere's own finite size adds a positive contribution beyond treating all its mass as concentrated at its centre.

Step 4: Conclusion The radius of gyration about PQ is $\boxed{\sqrt{110}\,\text{cm}}$, and the required dimensionless numerical value is $\boxed{x = 110}$.

Quick Tip to Solve: Use the perpendicular distance from the axis to the centre, then add its square to the central radius-of-gyration squared.

---

56. As shown in the figure, a combination of a thin plano concave lens and a thin plano convex lens is used to image an object placed at infinity. The radius of curvature of both the lenses is 30 cm and refraction index of the material for both the lenses is 1.75. Both the lenses are placed at distance of 40 cm from each other. Due to the combination, the image of the object is formed at distance $x =$ _____ cm, from concave lens. [4 marks]



Solution:

Concept:

- For a plano lens in air, the lens-maker formula gives focal-length magnitude $|f| = R/(\mu - 1)$. A plano-concave lens has negative focal length; a plano-convex lens has positive focal length.
- Apply $1/v - 1/u = 1/f$ successively using the Cartesian convention with incident light travelling left to right. The first image becomes the object for the second lens.

Step 1: Find the focal lengths and the first image The radius magnitude is $R = 30\,\text{cm}$ and $\mu = 1.75$, giving $|f| = 30/0.75 = 40\,\text{cm}$. Therefore $f_1 = -40\,\text{cm}$ for the first, concave lens and $f_2 = +40\,\text{cm}$ for the second, convex lens. Parallel incident rays from infinity form a virtual image at the first lens's focal point, so $v_1 = -40\,\text{cm}$, on its left. This sign records a virtual image; it does not mean that light travels backwards.

Step 2: Use the first image as the second object The convex lens is $40\,\text{cm}$ to the right of the concave lens. The point $40\,\text{cm}$ to the left of the first lens is therefore $80\,\text{cm}$ to the left of the second lens. Thus

its object distance is $u_2 = -80\,\text{cm}$. Applying the thin-lens equation gives

$$\frac{1}{v_2} - \frac{1}{-80} = \frac{1}{40}, \qquad \frac{1}{v_2} = \frac{1}{40} - \frac{1}{80} = \frac{1}{80}.$$

Hence $v_2 = 80\,\text{cm}$, a real image on the right of the convex lens.

Step 3: Measure the final distance from the concave lens The required reference point is the concave lens, not the convex lens. Add the separation between lenses to the image distance beyond the second lens:

$$x = 40 + 80 = 120\,\text{cm}.$$

Treating the lenses as being in contact would miss their separation and give an incorrect result; their equal and opposite powers cancel only in the contact arrangement. Here the diverging first lens makes the incident beam on the second lens diverge from a finite point.

Step 4: Conclusion The final image lies $80\,\text{cm}$ beyond the convex lens, or $\boxed{x = 120\,\text{cm}}$ to the right of the concave lens. The required numerical entry in centimetres is $\boxed{120}$.

Quick Tip to Solve: After solving for the last lens, translate the image position back to the reference lens named in the question.

57. A stream of a positively charged particles having $\frac{q}{m} = 2 \times 10^{11}\,\frac{C}{kg}$ and velocity $\vec{v}_0 = 3 \times 10^7 \hat{i}\,m/s$ is deflected by an electric field $1.8\hat{j}$ kV/m. The electric field exists in a region of 10 cm along x direction. Due to the electric field, the deflection of the charge particles in the y direction is _____ mm.

[4 marks]

Solution:

Concept:

- The electric force on a positive charge is $\vec{F} = q\vec{E}$, so the acceleration is $\vec{a} = (q/m)\vec{E}$. It acts in the positive y direction here.
- The field has no x component. Hence the horizontal velocity remains $v_x = v_0$, and the time inside a field region of length l is $t = l/v_0$.

Step 1: Find the transit time Convert the length before using the speed: $l = 10\,\text{cm} = 0.10\,\text{m}$. The particles enter with velocity entirely along x, so their initial vertical velocity is zero. Horizontal motion is uniform even though the total velocity changes. Thus

$$t = \frac{0.10}{3 \times 10^7} = \frac{1}{3} \times 10^{-8}\,\text{s}.$$

This is the time available for the transverse force to deflect a particle; the y motion does not change the distance to the exit plane.

Step 2: Calculate the vertical displacement The field magnitude is $1.8\,\text{kV/m} = 1.8 \times 10^3\,\text{V/m}$. Consequently

$$a_y = \frac{q}{m}E = (2 \times 10^{11})(1.8 \times 10^3) = 3.6 \times 10^{14}\,\text{m/s}^2.$$

Using the zero initial y velocity,

$$y = \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}(3.6 \times 10^{14})\frac{10^{-16}}{9} = 2.0 \times 10^{-3}\,\text{m}.$$

The sign is positive because both charge and electric field are positive along y.

Step 3: Conclusion The transverse deflection at the exit is $y = 2.0\,\text{mm}$ in the positive y direction. The requested numerical entry is $\boxed{2}$.

Quick Tip to Solve: Use the unchanged horizontal speed to find the time first; then apply uniformly accelerated motion only in the transverse direction.

58. A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5^{th} second. The force acted on the body is _____ N. [4 marks]

Solution:

Concept:

- For a body of mass m moving with speed v, its translational kinetic energy is $K = \frac{1}{2}mv^2$. The speed follows from energy, not from energy divided by time.
- For the force acting throughout the stated motion, use the constant acceleration relation $v = u + at$, followed by $F = ma$. The body starts from rest, so $u = 0$.

Step 1: Determine the speed after five seconds The mass is $m = 2\,\text{kg}$ and the kinetic energy at the end of the fifth second is $K = 10000\,\text{J}$. This describes the total kinetic energy at $t = 5\,\text{s}$, rather than an energy gained during just the fifth second. Therefore

$$10000 = \frac{1}{2}(2)v^2, \qquad v^2 = 10000, \qquad v = 100\,\text{m/s}.$$

Take the positive root for the speed, since kinetic energy does not specify a signed component of velocity.

Step 2: Obtain acceleration and force The speed has increased from zero to $100\,\text{m/s}$ in $5\,\text{s}$. For the applied constant force,

$$a = \frac{v - u}{t} = \frac{100 - 0}{5} = 20\,\text{m/s}^2.$$

Newton's second law then gives

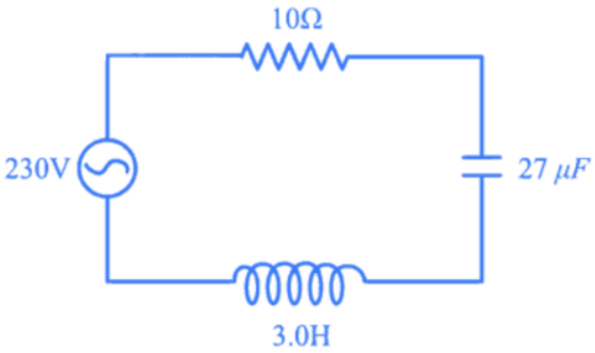
$$F = ma = 2 \times 20 = 40\,\text{N}.$$

As a check, the distance travelled is $s = \frac{1}{2}at^2 = 250\,\text{m}$, so the work $Fs = 40 \times 250 = 10000\,\text{J}$ equals the kinetic energy gained. No radius or rotational energy is involved in this translational calculation.

Step 3: Conclusion The force acting on the body is $\boxed{40\,\text{N}}$, so the numerical entry is 40.

Quick Tip to Solve: Find speed by taking the square root of twice the kinetic energy per unit mass; then divide its change by the full elapsed time.

59. In the circuit shown in the figure, the ratio of the quality factor and the band width is _____ s. [4 marks]



Solution:

Concept:

- For a series RLC circuit, resonance occurs at angular frequency $\omega_0 = 1/\sqrt{LC}$, where the inductive and capacitive reactances cancel.
- The quality factor is $Q = \omega_0 L/R$. The angular-frequency bandwidth between the half-power points is $\Delta\omega = R/L$, giving the ratio in seconds.

Step 1: Read the circuit and evaluate the resonance frequency The series circuit has resistance $R = 10\,\Omega$, inductance $L = 3.0\,\text{H}$, and capacitance $C = 27\,\mu\text{F} = 27 \times 10^{-6}\,\text{F}$. Thus

$$LC = 3(27 \times 10^{-6}) = 81 \times 10^{-6}\,\text{s}^2,$$

$$\omega_0 = \frac{1}{\sqrt{81 \times 10^{-6}}} = \frac{1}{9 \times 10^{-3}} = \frac{1000}{9}\,\text{rad/s}.$$

The source voltage amplitude does not affect the quality factor or bandwidth of this linear circuit; these depend only on R, L and C.

Step 2: Divide the quality factor by the bandwidth Use the same angular-frequency convention in both expressions. We obtain

$$Q = \frac{\omega_0 L}{R} = \frac{1000}{9}\frac{3}{10} = \frac{100}{3}, \qquad \Delta\omega = \frac{R}{L} = \frac{10}{3}\,\text{rad/s}.$$

Therefore

$$\frac{Q}{\Delta\omega} = \frac{100/3}{10/3} = 10\,\text{s}.$$

Quality factor is dimensionless and radians are dimensionless, so the resulting unit is seconds. Introducing a factor of 2π would instead mean using bandwidth in hertz, which is a different convention from the angular bandwidth used here.

Step 3: Conclusion The ratio of quality factor to angular-frequency bandwidth is $\boxed{10\,\text{s}}$. The numerical entry is 10.

Quick Tip to Solve: Keep the bandwidth convention consistent: R/L is an angular-frequency bandwidth; only divide by 2π when converting it to hertz.

60. Assume that protons and neutrons have equal masses. Mass of a nucleon is 1.6×10^{-27} kg and radius of nucleus is $1.5 \times 10^{-15}\ \mathrm{A}^{1/3}$ m. The approximate ratio of the nuclear density and water density is $n \times 10^{13}$. The value of n is _____. [4 marks]

Solution:

Concept:

- A nucleus containing A nucleons has mass $M = Am_n$, with m_n the common proton and neutron mass specified in the question.
- For a spherical nucleus, $V = 4\pi R^3/3$. With $R = r_0 A^{1/3}$, nuclear density $\rho = M/V$ is independent of A because the same factor A occurs in mass and volume.

Step 1: Express the density using the radius constant The given values are $m_n = 1.6 \times 10^{-27}\,\mathrm{kg}$ and $r_0 = 1.5 \times 10^{-15}\,\mathrm{m}$. Cube the entire radius expression, including its power of A:

$$R^3 = (1.5 \times 10^{-15})^3 A = 3.375 \times 10^{-45} A\,\mathrm{m}^3.$$

The density is consequently

$$\rho_N = \frac{A(1.6 \times 10^{-27})}{(4\pi/3)(3.375 \times 10^{-45})A} = \frac{1.6 \times 10^{-27}}{4.5\pi \times 10^{-45}}.$$

The cancellation of A is the familiar nearly constant nuclear density result.

Step 2: Compare with the density of water Evaluating the expression gives

$$\rho_N \simeq 1.132 \times 10^{17}\,\mathrm{kg/m}^3.$$

Use water density $\rho_w \simeq 10^3\,\mathrm{kg/m}^3$, in the same SI units. Their dimensionless ratio is

$$\frac{\rho_N}{\rho_w} \simeq 1.132 \times 10^{14} = 11.32 \times 10^{13}.$$

Matching this with $n \times 10^{13}$ gives $n \simeq 11.32$, which is approximately 11 for the integer numerical entry. Dividing by water density reduces the exponent by three; it does not leave the numerical nuclear density unchanged.

Step 3: Conclusion The ratio is approximately 11×10^{13}, so $\boxed{n = 11}$ (dimensionless, to the nearest integer).

Quick Tip to Solve: Cancel the mass number before substituting; then express the density ratio with exactly the power of ten requested in the question.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The primary and secondary valencies of cobalt respectively in $[Co(NH_3)_5Cl]Cl_2$ are : [4 marks]

(A) 2 and 8

(B) 2 and 6

(C) 3 and 5

(D) 3 and 6

Correct Answer: (D) 3 and 6

Solution:

Concept:

- In Werner's description, primary valency is the oxidation state of the metal. Secondary valency is its coordination number, the number of donor atoms directly bonded to it.
- An ammonia ligand is neutral and monodentate, whereas a chlorido ligand carries charge -1 and occupies one coordination site. Counterions outside square brackets are not coordinated ligands.

Step 1: Find the primary valency from charge balance In $[Co(NH_3)_5Cl]Cl_2$, the two chloride ions outside the brackets each have charge -1. Since the whole compound is neutral, the complex ion has charge $+2$. If x is the oxidation state of cobalt, the five ammonia ligands contribute zero and the coordinated chloride contributes -1:

$$x + 5(0) - 1 = +2, \qquad x = +3.$$

Thus cobalt has primary valency 3. Counting only the two external chlorides would give the charge of the complex ion, not the oxidation state of cobalt.

Step 2: Count the directly coordinated donor atoms The bracketed coordination sphere contains five ammonia ligands, each attached through one nitrogen donor atom, and one coordinated chloride donor. Hence

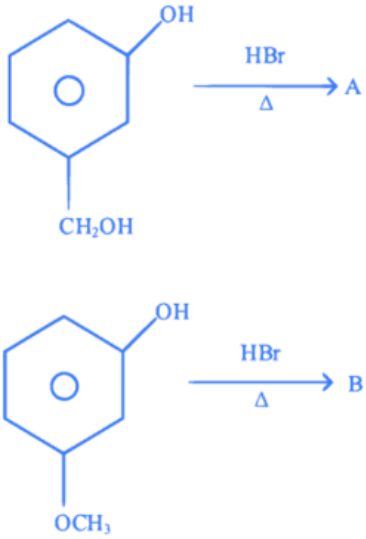
$$\text{coordination number} = 5 + 1 = 6.$$

The external two chlorides balance charge but do not occupy coordination sites in the displayed formula. Therefore secondary valency is 6 rather than 5 or 8. Options (A) and (B) incorrectly use primary valency 2; (C) has the correct primary value but omits the coordinated chloride when counting secondary valency. Only (D) combines both correct values.

Step 3: Conclusion The primary and secondary valencies of cobalt are $\boxed{3 \text{ and } 6}$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: For a coordination formula, compute oxidation state by charge balance, then count donor atoms inside the brackets separately.

62. 'A' and 'B' formed in the following set of reactions are : [4 marks]



(A) A: 3-bromobenzyl alcohol; B: 1,3-dibromobenzene.

(B) A: 3-(bromomethyl)phenol; B: 1-bromo-3-methoxybenzene.

(C) A: 3-(bromomethyl)phenol; B: benzene-1,3-diol.

(D) A: 3-(bromomethyl)phenol; B: 3-bromophenol.

Correct Answer: (C) A: 3-(bromomethyl)phenol; B: benzene-1,3-diol.

Solution:

Concept:

- Phenolic $\mathrm{Ar-OH}$ bonds resist substitution by HBr: resonance gives the aryl carbon–oxygen bond partial double-bond character, and ordinary substitution at an sp2 aryl carbon does not occur.
- A benzylic alcohol is converted to a benzyl bromide by HBr. An aryl methyl ether undergoes cleavage at its methyl–oxygen bond, producing the phenol and $\mathrm{CH_3Br}$.

Step 1: Identify the product A from the benzylic alcohol The first substrate contains a phenolic OH and a $\mathrm{CH_2OH}$ group meta to it. HBr protonates the alcoholic oxygen, converting OH into a better leaving group. Bromide replaces the benzylic hydroxy group, giving $\mathrm{HO-C_6H_4-CH_2Br}$, with the substituents still meta. The ring-bound phenolic OH remains intact. Thus A is 3-(bromomethyl)phenol, rather than an aryl bromide. There is no reaction here that moves either substituent to a different ring position.

Step 2: Identify B by cleavage of the methyl ether The second substrate is 3-methoxyphenol. Protonation of its ether oxygen makes the methyl group susceptible to attack by bromide. Cleavage takes place at the methyl carbon, not at the aromatic carbon:

$$\mathrm{HO-C_6H_4-OCH_3 + HBr \longrightarrow HO-C_6H_4-OH + CH_3Br.}$$

Consequently B is benzene-1,3-diol (resorcinol). Option (A) replaces phenolic OH by Br, (B) retains methoxy and replaces phenolic OH, and (D) puts Br directly on the ring; all contradict aryl–oxygen

bond resistance under these conditions. Option (C) alone shows A with CH_2Br and B with two phenolic OH groups.

Step 3: Conclusion A is 3-(bromomethyl)phenol and B is benzene-1,3-diol (resorcinol). Hence, the correct option is (C).

Quick Tip to Solve: With hot HBr, distinguish benzylic alcohol, aryl OH and aryl methyl ether: substitution occurs at benzyl or methyl carbon, not at aryl carbon.

---

63. Decreasing order of the hydrogen bonding in following forms of water is correctly represented by

A. Liquid water

B. Ice

C. Impure water

Choose the correct answer from the options given below : [4 marks]

(A) A = B > C

(B) B > A > C

(C) A > B > C

(D) C > B > A

Correct Answer: (B) B > A > C

Solution:

Concept:

- Water molecules form intermolecular hydrogen bonds through O–H groups and lone pairs on oxygen. The extent of this association depends on the molecular arrangement and disruption of the network.
- Ice has an ordered, open hydrogen-bonded lattice. Liquid water retains hydrogen bonding, but thermal motion breaks part of that extended network; impurities further disturb the water–water association in the comparison given.

Step 1: Compare ice with pure liquid water In ice, each water molecule participates in the regular tetrahedral hydrogen-bonded network of the solid. On melting, some hydrogen bonds break and the long-range arrangement collapses. The liquid can pack more closely even though its water–water hydrogen-bond network is less extensive than in ice. Therefore B (ice) has more hydrogen bonding than A (liquid water):

$$B > A.$$

The lower density of ice is a consequence of its open lattice and should not be mistaken for weaker or less extensive hydrogen bonding.

Step 2: Place impure water and test the options Impurities interrupt the regular association among water molecules, so the intended comparison puts C (impure water) below pure liquid water. Combining this with the solid–liquid comparison gives

$$B > A > C.$$

Option (A) incorrectly treats liquid water and ice as having equal hydrogen bonding. Option (C) reverses their order by placing the liquid above ice. Option (D) places impure water above the ordered ice network, which is inconsistent with this comparison. Option (B) alone has ice first, liquid water next and impure water last. This ordering compares the extent of the water hydrogen-bond network, rather than molecular closeness alone.

Step 3: Conclusion The decreasing order is $\boxed{\text{Ice} > \text{Liquid water} > \text{Impure water}}$, or $B > A > C$. Hence, the correct option is (B).

Quick Tip to Solve: Remember the open tetrahedral lattice of ice: stronger network formation and lower density can occur together.

---

64. Assertion A : Hydrolysis of an alkyl chloride is a slow reaction but in the presence of NaI, the rate of the hydrolysis increases.

Reason R : I^- is a good nucleophile as well as a good leaving group.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) A is true but R is false

(B) A is false but R is true

(C) Both A and R are true and R is the correct explanation of A

(D) Both A and R are true but R is NOT the correct explanation of A

Correct Answer: (C) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Hydrolysis replaces the halogen atom of an alkyl halide with a hydroxyl group by nucleophilic substitution.
- Iodide is a polarizable nucleophile and a good leaving group. Conversion of an alkyl chloride into an alkyl iodide supplies a more reactive intermediate.

Step 1: Evaluate the assertion An alkyl chloride has a relatively strong carbon–chlorine bond. Its substitution by water or hydroxide can therefore be slow. Adding sodium iodide introduces iodide ions, which can attack the carbon bearing chlorine and replace chloride. The resulting alkyl iodide undergoes hydrolysis more readily because its carbon–iodine bond is weaker. Thus a pathway through the iodide accelerates the overall hydrolysis, and assertion A is true.

Step 2: Evaluate the reason The large iodide ion is readily polarized and is a good nucleophile in the medium relevant to this reaction. It is also a good leaving group because the departing negative charge is accommodated on a large ion. These properties play different roles: nucleophilicity helps form the alkyl iodide, whereas leaving-group ability helps its subsequent hydrolysis. Hence reason R is true.

Step 3: Connect both statements The substitution sequence can be summarized as

$$\mathrm{RCl} + \mathrm{I}^- \longrightarrow \mathrm{RI} + \mathrm{Cl}^-, \qquad \mathrm{RI} + \mathrm{OH}^- \longrightarrow \mathrm{ROH} + \mathrm{I}^-.$$

Iodide is regenerated in the second step. Its ability to enter and then leave provides the catalytic route, so R explains the increased rate in A. Options (A) and (B) incorrectly deny one statement; option (D) misses the direct causal connection.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (C).

Quick Tip to Solve: Track iodide through both substitutions: it first enters as a nucleophile and then leaves, so it is regenerated.

65. An ammoniacal metal salt solution gives a brilliant red precipitate on addition of dimethylglyoxime. The metal ion is : [4 marks]

(A) Co^{2+}

(B) Fe^{2+}

(C) Ni^{2+}

(D) Cu^{2+}

Correct Answer: (C) Ni^{2+}

Solution:

Concept:

- Dimethylglyoxime (DMG) is a selective qualitative reagent for nickel(II) in ammoniacal solution.
- The ligand loses a proton on complex formation; two dimethylglyoximate ligands chelate one Ni^{2+} ion to form a sparingly soluble red complex.

Step 1: Identify the characteristic observation The decisive clues are the reagent dimethylglyoxime, the ammoniacal medium and the brilliant red precipitate. Together these describe the confirmatory test for nickel(II), rather than a general colour test for every transition metal. The precipitate is commonly called nickel dimethylglyoxime and is represented as $Ni(HDMG)_2$, where HDMG denotes the singly deprotonated ligand.

Step 2: Explain the role of the medium Ammonia makes the solution mildly alkaline and facilitates removal of the oxime proton during chelation. Each ligand binds through its nitrogen donor atoms, and the resulting nickel chelate is sufficiently insoluble to separate as the conspicuous red solid. A schematic ionic equation is

$$Ni^{2+} + 2\,H_2DMG + 2\,OH^- \longrightarrow Ni(HDMG)_2(s) + 2\,H_2O.$$

This equation shows the two-ligand-to-one-metal ratio and the acid–base role of the alkaline medium.

Step 3: Eliminate the other metal ions Cobalt(II), iron(II) and copper(II) do not give this characteristic brilliant red nickel chelate under the stated test conditions. Copper(II), for example, is familiar for its deep-blue ammine solution, which is a different observation. Therefore the reagent and precipitate together distinguish nickel from all three distractors; the choice is not based merely on the original colour of a metal salt.

Step 4: Conclusion The metal ion is Ni^{2+}. Hence, the correct option is (C).

Quick Tip to Solve: Remember DMG plus ammoniacal medium plus a brilliant red precipitate as the confirmatory combination for nickel(II).

66. In the following given reaction, 'A' is [4 marks]



(A) 1-Bromo-2-methylcyclohexane

(B) 1-Bromo-1,2-dimethylcyclopentane

(C) 1-Bromo-1-methylcyclohexane

(D) 2-Bromo-1,1-dimethylcyclopentane

Correct Answer: (B) 1-Bromo-1,2-dimethylcyclopentane

Solution:

Concept:

- Addition of HBr without peroxide proceeds through protonation of the alkene and a carbocation, followed by bromide capture.
- A cyclobutylcarbinyl cation can undergo ring expansion to relieve four-membered-ring strain; a further methyl shift can give a more stable tertiary cation.

Step 1: Protonate the double bond The substrate is cyclobutyl-substituted $C(CH_3)=CH_2$. Proton adds to its terminal CH_2 carbon, making that end a methyl group. The positive charge is placed on the more substituted carbon adjacent to the cyclobutane ring. This carbon is now attached to two methyl groups and the ring carbon. The cation is therefore situated where migration of a neighbouring cyclobutane bond can expand the ring.

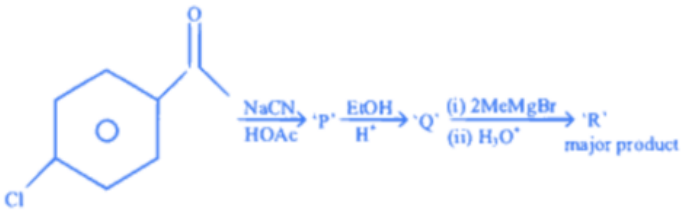
Step 2: Expand the strained ring One carbon–carbon bond of the cyclobutane migrates to the adjacent cationic centre. This includes the formerly exocyclic carbon in a five-membered ring and relieves ring strain. Immediately after migration, the two methyl groups are still on the newly incorporated ring carbon, while the positive charge is on the adjacent carbon. Merely trapping that intermediate would give the gem-dimethyl bromide shown in option (D), but the cation can rearrange further.

Step 3: Form and trap the more stable cation The carbon bearing the two methyl groups has no hydrogen available for a hydride shift. The favourable rearrangement is a methyl shift from this adjacent quaternary carbon to the cationic carbon. This distributes the methyl groups onto adjacent ring carbons and places the positive charge on a tertiary carbon. Bromide then attacks that tertiary site. The result is 1-bromo-1,2-dimethylcyclopentane. The six-membered-ring options (A) and (C) do not follow the one-carbon ring expansion, and option (D) retains the less stable cation-trapping connectivity.

Step 4: Conclusion The major product is 1-bromo-1,2-dimethylcyclopentane, the structure in option (B). Hence, the correct option is (B).

Quick Tip to Solve: For a cyclobutylcarbinyl cation, check ring expansion first and then ask whether a methyl shift creates a tertiary cation.

67. 'R' formed in the following sequence of reactions is : [4 marks]



(A) p-NC$-C_6H_4-C(OH)(CH_3)-C(CH_3)_3$

(B) p-NC$-C_6H_4-C(OH)(CH_3)_2$

(C) p-Cl$-C_6H_4-C(OH)(CH_3)-C(OH)(CH_3)_2$

(D) p-Cl$-C_6H_4-C(OH)(CH_3)_2$

Correct Answer: (C) p-Cl$-C_6H_4-C(OH)(CH_3)-C(OH)(CH_3)_2$

Solution:

Concept:

- Cyanide adds to a ketone carbonyl to give a cyanohydrin: the carbonyl carbon acquires OH and CN.
- Acidic ethanol converts the nitrile group into an ethyl ester. An ester reacts with methylmagnesium bromide by two carbon additions to give a tertiary alcohol after hydrolysis.

Step 1: Identify the cyanohydrin P Write Ar for the p-chlorophenyl group, p-Cl$-C_6H_4$. The starting ketone is $ArCOCH_3$. Cyanide attacks its electrophilic carbonyl carbon, and protonation gives

$$ArCOCH_3 \longrightarrow ArC(OH)(CH_3)CN.$$

The aromatic chlorine is not displaced by this mild cyanohydrin-forming reagent. Both its position on the ring and the original methyl substituent remain unchanged.

Step 2: Convert the nitrile into ester Q In ethanol with acid, the nitrile is converted into the corresponding ethyl ester. Thus the side chain becomes

$$ArC(OH)(CH_3)COOC_2H_5.$$

The carbon originally introduced by cyanide is now the ester carbonyl carbon. This carbon bookkeeping is essential: the reagent does not simply restore the original ketone, and the nitrile is not introduced onto the benzene ring.

Step 3: Apply the Grignard additions and compare products The ester carbonyl undergoes two methyl additions, with elimination of ethoxide after the first addition and addition to the intermediate ketone in the second. Acidic work-up gives

$$ArC(OH)(CH_3)C(OH)(CH_3)_2.$$

The hydroxyl on the original ketone carbon is retained, giving a vicinal diol. Free hydroxyl groups consume Grignard reagent by acid–base reaction, so sufficient reagent is understood for the indicated

carbonyl conversion. Options (A) and (B) incorrectly replace ring chlorine by nitrile, while (D) loses the extra carbon introduced by cyanide.

Step 4: Conclusion The product is p-$Cl-C_6H_4-C(OH)(CH_3)-C(OH)(CH_3)_2$, the vicinal diol shown in (C). Hence, the correct option is (C).

Quick Tip to Solve: Label the cyanide carbon mentally: it becomes the ester carbonyl carbon and finally the new tertiary-alcohol carbon.

68. The magnetic moment of a transition metal compound has been calculated to be 3.87 B.M. The metal ion is [4 marks]

(A) Ti^{2+}

(B) V^{2+}

(C) Cr^{2+}

(D) Mn^{2+}

Correct Answer: (B) V^{2+}

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$, where n is the number of unpaired electrons.
- For first-row transition-metal cations, remove the $4s$ electrons before the $3d$ electrons when obtaining the ionic configuration.

Step 1: Find the required unpaired-electron count The stated moment is $3.87\,\text{B.M.}$. Squaring it gives approximately 14.98, which is effectively 15 at the precision used. Hence

$$n(n+2) = 15 \quad \Rightarrow \quad n^2 + 2n - 15 = 0.$$

Factoring gives $(n-3)(n+5) = 0$. Since an electron count cannot be negative, the physical solution is $n = 3$. We therefore seek the ion with three unpaired electrons.

Step 2: Write the configurations of the options After removing two electrons, the relevant configurations are Ti^{2+} : $[Ar]3d^2$, V^{2+} : $[Ar]3d^3$, Cr^{2+} : $[Ar]3d^4$ and Mn^{2+} : $[Ar]3d^5$. In the usual unpaired-electron comparison for these ions, they have respectively two, three, four and five unpaired electrons. The exceptional neutral configuration of chromium must not be mistaken for the configuration of its divalent ion.

Step 3: Check the moment and reject distractors For V^{2+},

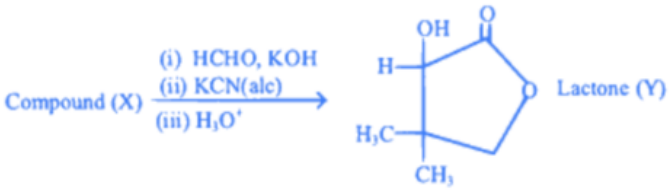
$$\mu = \sqrt{3(3+2)} = \sqrt{15} = 3.873\,\text{B.M.},$$

matching the given value. The other counts give $\sqrt{8} = 2.83$, $\sqrt{24} = 4.90$ and $\sqrt{35} = 5.92\,\text{B.M.}$, respectively. Thus none of the other listed divalent ions has the required spin-only moment.

Step 4: Conclusion The ion is V^{2+}, with three unpaired electrons. Hence, the correct option is (B).

Quick Tip to Solve: The familiar moments 2.83, 3.87, 4.90 and 5.92 B.M. correspond to two, three, four and five unpaired electrons.

69. Compound (X) undergoes following sequence of reactions to give the Lactone (Y). [4 marks]



(A) $(CH_3)_2CHCHO$

(B) $HOCH_2CH_2CH_2CH_2CHO$

(C) $HOCH_2C(CH_3)_2CHO$

(D) $CH_3CH_2CH_2CHO$

Correct Answer: (A) $(CH_3)_2CHCHO$

Solution:

Concept:

- An aldehyde with an alpha hydrogen adds to formaldehyde under alkaline conditions, introducing a hydroxymethyl group at its alpha carbon.
- Cyanohydrin formation adds one carbon, which becomes a carboxyl group on acidic hydrolysis. A suitably placed hydroxyl group can then form a five-membered cyclic ester (lactone).

Step 1: Work backwards from the lactone Open the ester linkage of the pictured five-membered lactone without changing its carbon skeleton. The corresponding hydroxy acid is

$$HOCH_2-C(CH_3)_2-CH(OH)-COOH.$$

Its terminal hydroxymethyl oxygen forms the ester bond to the carboxyl carbon. The adjacent carbon bears two methyl groups, and the carbon next to the acid bears the hydroxyl generated during cyanohydrin formation. These substituent positions are more diagnostic than the mere presence of a lactone ring.

Step 2: Undo cyanohydrin formation Before cyanide addition and hydrolysis, the aldehyde must have been

$$HOCH_2-C(CH_3)_2-CHO.$$

Addition of cyanide to this aldehyde gives $HOCH_2-C(CH_3)_2-CH(OH)-CN$. Acid hydrolysis converts its nitrile into the acid above, retaining the newly formed hydroxyl. This intermediate corresponds to the product after the first reagent, rather than the initial compound X.

Step 3: Identify X and reject the alternatives Isobutyraldehyde, $(CH_3)_2CHCHO$, has one alpha hydrogen. Its enolate attacks formaldehyde and, after protonation, gives $HOCH_2-C(CH_3)_2-CHO$. This supplies exactly the gem-dimethyl carbon and hydroxymethyl arm needed for the observed lactone. Option (C) already has this arm and no alpha hydrogen; it is the first-step product, not X. Option (B) has the wrong chain and lacks the methyl groups. Butanal in (D) likewise cannot yield the shown gem-dimethyl skeleton by this sequence.

Step 4: Conclusion Compound X is 2-methylpropanal, $(CH_3)_2CHCHO$. Hence, the correct option is

(A).

Quick Tip to Solve: Open a lactone at its ester bond, then reverse nitrile hydrolysis to recover the aldehyde before tracing the formaldehyde addition.

70. In the depression of freezing point experiment

A. Vapour pressure of the solution is less than that of pure solvent

B. Vapour pressure of the solution is more than that of pure solvent

C. Only solute molecules solidify at the freezing point

D. Only solvent molecules solidify at the freezing point

Choose the most appropriate answer from the options given below : [4 marks]

(A) A and C only

(B) A only

(C) A and D only

(D) B and C only

Correct Answer: (C) A and D only

Solution:

Concept:

- A nonvolatile solute lowers the vapour pressure of its solvent: for an ideal solution, $p_1 = x_1 p_1^\circ$, with $x_1 < 1$.
- At the freezing point of a dilute solution, pure solid solvent is in equilibrium with the liquid solution; the solute remains primarily in the liquid phase.

Step 1: Evaluate the vapour-pressure statements For a solution containing a nonvolatile solute, some surface positions and mole fraction are occupied by solute. The escaping tendency of solvent is reduced. Raoultś law gives

$$p_1 = x_1 p_1^\circ < p_1^\circ.$$

Consequently statement A is true and statement B, which claims an increase of vapour pressure, is false. The comparison must be made at the same temperature, as in the standard freezing-point experiment.

Step 2: Evaluate which substance freezes On cooling the solution to its freezing point, solvent crystals begin to separate. In the dilute-solution treatment, the solid phase is pure solvent and the dissolved solute is excluded from the solvent crystal lattice. Therefore D is true and C is false. As more solvent freezes, the remaining solution becomes more concentrated in solute, which can further lower its equilibrium freezing temperature.

Step 3: Connect the two effects and select the pair The freezing point occurs where the chemical potential of solid solvent equals that of liquid solvent. Dissolving the solute lowers the chemical potential of liquid solvent, so a lower temperature is required to reach equilibrium with the solid. This accounts for $\Delta T_f = K_f m$ in dilute nonelectrolyte solutions. The true statements are A and D only. Option (A) incorrectly includes C; option (B) omits the true statement D; and option (D) includes

both false statements B and C.

Step 4: Conclusion A and D are correct: vapour pressure decreases and solvent alone initially solidifies. Hence, the correct option is (C).

Quick Tip to Solve: In a freezing-point experiment, distinguish the liquid solution from the solid phase: the solid that first separates is solvent.

71. Order of Covalent bond:

A. $KF > KI;\ LiF > KF$

B. $KF < KI;\ LiF > KF$

C. $SnCl_4 > SnCl_2;\ CuCl > NaCl$

D. $LiF > KF;\ CuCl < NaCl$

E. $KF < KI;\ CuCl > NaCl$

Choose the correct answer from the options given below: [4 marks]

(A) B, C only

(B) A, B only

(C) C, E only

(D) B, C, E only

Correct Answer: (D) B, C, E only

Solution:

Concept:

- Covalent character increases when a cation polarises the electron cloud of an anion more strongly. Small, highly charged cations have greater polarising power.
- A large anion is more polarisable. For cations of comparable size and charge, a pseudo-noble-gas configuration can produce stronger polarisation than a noble-gas configuration.

Step 1: Compare the potassium halides and fluorides For the same cation, I^- is much larger and more polarisable than F^-. Thus KI is more covalent than KF, so the order $KF < KI$ is correct. For the same anion, the smaller Li^+ polarises it more than K^+. Therefore $LiF > KF$ in covalent character. This makes statement A false and statement B true.

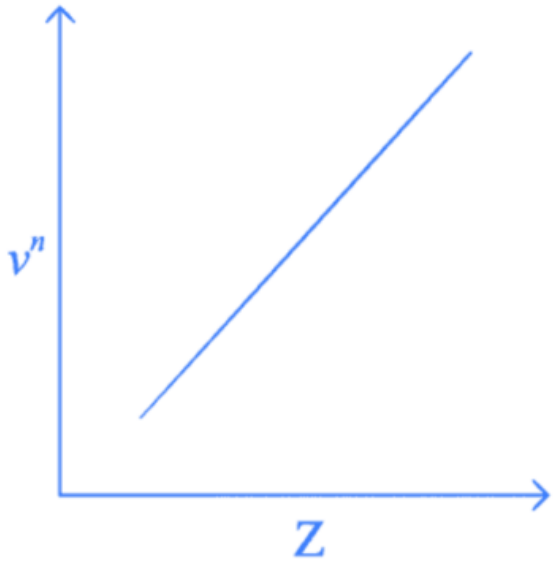
Step 2: Compare the chlorides In $SnCl_4$, tin has the higher oxidation state and greater polarising power than in $SnCl_2$. Hence $SnCl_4 > SnCl_2$. Also, Cu^+, with its filled d shell, polarises chloride more effectively than Na^+, so $CuCl > NaCl$. Statement C is true; statement D fails because its copper/sodium comparison is reversed.

Step 3: Select the complete set Statement E combines two comparisons already established: $KF < KI$ and $CuCl > NaCl$, so it is true. The true statements are B, C and E. Option A omits E, option B includes false statement A, and option C omits B.

Step 4: Conclusion B, C and E only are correct. Hence, the correct option is (D).

Quick Tip to Solve: Keep the ion held fixed in each comparison: larger anion or smaller, more strongly polarising cation means more covalent character.

72. It is observed that characteristic X-ray spectra of elements show regularity. When frequency to the power "n" i.e. v^n of X-rays emitted is plotted against atomic number "Z", following graph is obtained. The value of 'n' is : [4 marks]



(A) 1

(B) $\frac{1}{2}$

(C) 2

(D) 3

Correct Answer: (B) $\frac{1}{2}$

Solution:

Concept:

- Moseley related the frequency ν of a characteristic X-ray line to atomic number Z, rather than atomic mass.
- For a fixed spectral series, $\sqrt{\nu} = a(Z - b)$, where a is a constant for that series and b is its screening constant.

Step 1: Read the graph The vertical coordinate is ν^n and the horizontal coordinate is Z. The graph is a straight line with positive slope. Consequently, the power of frequency represented on the vertical axis must be the one which is a linear function of atomic number. The extrapolated line need not pass through the origin because inner electrons screen the nuclear charge.

Step 2: Apply Moseleys relation For a characteristic transition, the energy separation is proportional to the square of the effective nuclear charge. Using $E = h\nu$, write

$$\nu = C(Z - b)^2.$$

Taking the positive square root gives

$$\nu^{1/2} = \sqrt{C}\, Z - \sqrt{C}\, b.$$

This is exactly a straight-line equation of the form $y = mZ + c$, with $y = \nu^{1/2}$.

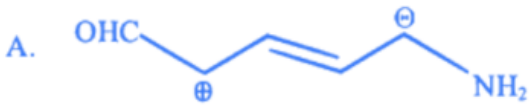
Step 3: Eliminate other powers With $n = 1$, frequency itself varies quadratically with $Z - b$, so the plot is not a straight line. Choosing $n = 2$ gives a fourth-power dependence, and $n = 3$ gives a sixth-power dependence. Neither agrees with the indicated line. The fractional power $1/2$ alone removes the square in Moseleyś frequency law.

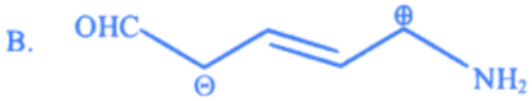
Step 4: Conclusion $n = \frac{1}{2}$, a dimensionless exponent. Hence, the correct option is (B).

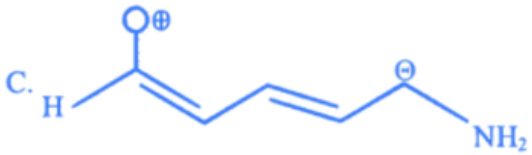
Quick Tip to Solve: For characteristic X-rays remember that it is the square root of frequency, rather than frequency itself, that is linear in atomic number.

73. Increasing order of stability of the resonance structures is :

Choose the correct answer from the options given below : [4 marks]

A. 

B. 

C. 

D. 

(A) D, C, B, A

(B) C, D, B, A

(C) C, A, B, D

(D) D, C, A, B

Correct Answer: (C) C, A, B, D

Solution:

Concept:

- Resonance contributors retain the atomic skeleton; only electrons and formal charges change. Their stability depends on octet completion and favourable charge placement.
- Complete octets strongly stabilise a contributor. For comparable electron counts, a negative charge is favoured next to an electron-withdrawing group, while a positive charge is stabilised by an electron-donating group.

Step 1: Identify the least favourable structure In C the oxygen bears a positive charge and the carbon adjacent to the amino end bears a negative charge. A positive charge on highly electronegative oxygen is especially unfavourable. The drawing also contains electron-deficient charge separation. The unfavourable positive charge on oxygen makes C the least stable contributor among those listed.

Step 2: Compare the two carbon-charge contributors In A the negative charge lies near the amino group and the positive charge near the aldehyde group. These placements oppose the usual stabilising effects: amino is electron donating and aldehyde is electron withdrawing. In B these placements are reversed. The carbanionic end next to the aldehyde group is stabilised, while the positive carbon next to the amino group benefits from electron donation. Thus B is more stable than A.

Step 3: Recognise the octet-complete contributor In D the amino nitrogen forms an additional bond and bears positive charge. Although this contributor has charge separation, the positively charged nitrogen has a complete octet, unlike the electron-deficient carbocation contributors. The negative charge is at the aldehyde end, where it is stabilised. These features make D the most stable. Combining the comparisons gives $C < A < B < D$. Options A and B wrongly put A above D; option D wrongly makes D least stable.

Step 4: Conclusion The increasing stability order is $C < A < B < D$. Hence, the correct option is (C).

Quick Tip to Solve: Check octet completion first, then compare where positive and negative charges sit relative to electron-donating and withdrawing groups.

74. Which of the following is true about freons? [4 marks]

(A) These are radicals of chlorine and chlorine monoxide

(B) These are chemicals causing skin cancer

(C) All radicals are called freons

(D) These are chlorofluorocarbon compounds

Correct Answer: (D) These are chlorofluorocarbon compounds

Solution:

Concept:

- Freons are chlorofluorocarbon compounds, commonly abbreviated CFCs. Their carbon framework carries chlorine and fluorine substituents.
- In the stratosphere, ultraviolet radiation can split a carbon-chlorine bond in a CFC and release a chlorine radical, which participates in catalytic ozone destruction.

Step 1: Identify the chemical class A familiar member is dichlorodifluoromethane, CCl_2F_2, also called freon-12. Such compounds were used as refrigerants and propellants because they are relatively inert in the lower atmosphere. The term freon names the halocarbon compound; it does not name every species produced when that compound decomposes.

Step 2: Distinguish the compound from its radicals Ultraviolet light can initiate a reaction such as

$$CCl_2F_2 \xrightarrow{h\nu} CClF_2^{\bullet} + Cl^{\bullet}.$$

The liberated chlorine radical can react with ozone to form chlorine monoxide radical. Chlorine and chlorine monoxide radicals are reactive intermediates in ozone depletion; they are not themselves freons. Hence option A confuses a parent compound with its reaction products, while option C wrongly applies the name to all radicals.

Step 3: Check the environmental statement Ozone depletion allows more harmful ultraviolet radiation to reach Earth and can increase skin-cancer risk. That is an indirect environmental consequence of CFC release, not the chemical definition of freons. Option B therefore does not identify the compounds correctly. Option D explicitly states their correct chemical class and is the appropriate answer.

Step 4: Conclusion Freons are chlorofluorocarbon compounds. Hence, the correct option is (D).

Quick Tip to Solve: Separate the CFC molecule from the chlorine radicals it releases: the molecule is a freon, while the radicals catalyse ozone depletion.

75. Match List I with List II

	List-I		List-II
A.	**Chlorophyll**	**I.**	Na_2CO_3
B.	**Soda ash**	**II.**	$CaSO_4$
C.	**Dentistry, Ornamental work**	**III.**	Mg^{2+}
D.	**Used in white washing**	**IV.**	$Ca(OH)_2$

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-I, C-II, D-IV

(B) A-II, B-III, C-IV, D-I

(C) A-II, B-I, C-III, D-IV

(D) A-III, B-IV, C-I, D-II

Correct Answer: (A) A-III, B-I, C-II, D-IV

Solution:

Concept:

- Magnesium occurs as the central metal ion in the chlorophyll pigment. Common sodium and calcium compounds are identified by their standard names and uses.
- Soda ash is sodium carbonate, plaster materials are based on calcium sulphate, and slaked lime is calcium hydroxide.

Step 1: Match chlorophyll and soda ash Chlorophyll contains a magnesium ion coordinated in its porphyrin-like ring system. Hence A, chlorophyll, matches III, Mg^{2+}. Soda ash is the common name for anhydrous sodium carbonate, Na_2CO_3. It is used, for example, in glass manufacture and water softening. Thus B matches I, not calcium hydroxide or calcium sulphate.

Step 2: Match the calcium compounds to their uses Calcium sulphate and its hydrated forms are the basis of gypsum and plaster of Paris. Plaster is useful for dental casts, moulds and ornamental work because it sets into a hard solid after mixing with water. In the supplied list this identifies C with II, $CaSO_4$. Whitewashing uses slaked lime, $Ca(OH)_2$, so D matches IV. The coating gradually forms calcium carbonate on exposure to carbon dioxide:

$$Ca(OH)_2 + CO_2 \longrightarrow CaCO_3 + H_2O.$$

Step 3: Assemble and check the matching The complete matching is A-III, B-I, C-II, D-IV. Option A contains all four associations. Option B assigns calcium sulphate to chlorophyll and magnesium to soda ash; option C also misidentifies chlorophyll and dental material. Option D wrongly gives calcium hydroxide for soda ash and sodium carbonate for dental work. These contradictions eliminate the other choices.

Step 4: Conclusion The matching is A-III, B-I, C-II, D-IV. Hence, the correct option is (A).

Quick Tip to Solve: Use the common names as anchors: soda ash is sodium carbonate and slaked lime for whitewashing is calcium hydroxide.

---

76. Statement I : For colloidal particles, the values of colligative properties are of small order as compared to values shown by true solutions at same concentration.

Statement II : For colloidal particles, the potential difference between the fixed layer and the diffused layer of same charges is called the electrokinetic potential or zeta potential.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is true and Statement II is false

(C) Both Statement I and Statement II are true

(D) Statement I is false but Statement II is true

Correct Answer: (B) Statement I is true and Statement II is false

Solution:

Concept:

- Colligative properties depend on the number of independently dispersed particles, rather than their chemical identity. At the same mass concentration, colloids contain far fewer particles than true solutions.
- A charged colloidal particle is accompanied by an electrical double layer: an attached layer and a diffuse layer of counterions. The electrokinetic potential is associated with this interfacial charge arrangement.

Step 1: Evaluate Statement I A colloidal particle is an aggregate containing many molecules or ions. If the same mass of solute is dispersed as large aggregates rather than individual molecular particles, the number of independently moving solute particles decreases greatly. For example, osmotic pressure follows $\pi = cRT$, where c is the molar concentration of dispersed particles. Small particle-

number concentration produces small osmotic pressure, freezing-point depression and related effects. Statement I is therefore true.

Step 2: Evaluate Statement II The charged colloidal surface attracts oppositely charged ions from the dispersion medium. Some form a relatively fixed layer and others form a diffuse counterion atmosphere. Zeta potential is the potential at the slipping plane relative to the bulk liquid, often described at this level using the fixed and diffuse double layers. The statement is incorrect in calling these layers layers of the same charges. The electrical double layer arises from opposite charges, not two layers carrying the same sign.

Step 3: Choose the statement combination The truth values are I true and II false. Option A incorrectly rejects Statement I. Option C incorrectly accepts the same-charge description in Statement II. Option D reverses both required truth values. Thus only option B agrees with the particle-count argument and the electrical-double-layer description.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: For colloids count dispersed aggregates for colligative properties, and check the sign of the counterions when judging a double-layer statement.

---

77. Match List I with List II

	List-I		List-II
A.	Reverberatory furnace	I.	Pig Iron
B.	Electrolytic cell	II.	Aluminium
C.	Blast furnace	III.	Silicon
D.	Zone Refining furnace	IV.	Copper

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-IV, C-I, D-II

(B) A-IV, B-II, C-I, D-III

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-III, C-II, D-IV

Correct Answer: (B) A-IV, B-II, C-I, D-III

Solution:

Concept:

- Metallurgical equipment is chosen according to the chemical reactivity of the metal and the physical or chemical principle used in extraction or purification.
- Aluminium is extracted electrolytically, iron is reduced in a blast furnace, copper metallurgy uses a reverberatory furnace, and semiconductor-grade silicon is purified by zone refining.

Step 1: Match the furnaces used for copper and iron A reverberatory furnace is used in copper metallurgy for smelting the concentrated and roasted ore with flux. The charge is heated mainly by radiation and reflected heat rather than by direct contact with the fuel. Thus A matches IV, copper. In a blast furnace, iron oxides are reduced mainly by carbon monoxide and the liquid iron contains

dissolved carbon and other impurities. This product is pig iron, so C matches I.

Step 2: Match electrolysis and zone refining Aluminium oxide is too stable for the ordinary carbon-reduction route used for less reactive metals. Aluminium is therefore obtained by electrolysis of alumina dissolved in molten cryolite, identifying B with II. Zone refining relies on impurities being more soluble in the melt than in the solid. A moving molten zone carries impurities toward one end of a rod and is useful in obtaining very pure silicon for semiconductor applications. Therefore D matches III.

Step 3: Check the full code The four matches give A-IV, B-II, C-I, D-III, exactly option B. Option A incorrectly assigns the reverberatory furnace to silicon and the electrolytic cell to copper in this list. Option C assigns blast-furnace extraction to aluminium, contradicting its electrolytic extraction. Option D also gives the electrolytic cell to silicon and the zone-refining furnace to copper. Only B satisfies all four standard associations.

Step 4: Conclusion The correct matching is A-IV, B-II, C-I, D-III. Hence, the correct option is (B).

Quick Tip to Solve: Anchor the distinctive methods first: aluminium needs molten electrolysis, while semiconductor silicon points to zone refining.

78. Reaction of BeO with ammonia and hydrogen fluoride gives A which on thermal decomposition gives BeF_2 and NH_4F. What is 'A'? [4 marks]

(A) $(NH_4)Be_2F_5$

(B) $(NH_4)_2BeF_4$

(C) $(NH_4)BeF_3$

(D) H_3NBeF_3

Correct Answer: (B) $(NH_4)_2BeF_4$

Solution:

Concept:

- Beryllium oxide is converted into ammonium tetrafluoroberyllate, $(NH_4)_2BeF_4$, by treatment with ammonia and hydrogen fluoride. The fluoride ions coordinate to the small Be^{2+} centre.
- The tetrafluoroberyllate ion is $[BeF_4]^{2-}$. Two ammonium ions are therefore needed to make the electrically neutral salt. Heating this salt gives beryllium fluoride and ammonium fluoride.

Step 1: Identify the fluoride complex Beryllium has a small ionic size and forms a tetrahedral complex with four fluoride ligands. Its charge follows from $+2 + 4(-1) = -2$. The counterion supplied by ammonia in the acidic fluoride medium is NH_4^+, so the product has two ammonium ions per complex ion.

$$BeO + 2\,NH_3 + 4\,HF \longrightarrow (NH_4)_2BeF_4 + H_2O$$

This equation conserves beryllium, fluorine, nitrogen, oxygen and hydrogen.

Step 2: Check the thermal decomposition and the alternatives The expected decomposition is

$$(NH_4)_2BeF_4 \xrightarrow{\Delta} BeF_2 + 2\,NH_4F.$$

Thus the product gives exactly the two named fluorides. Option (A) has two beryllium atoms and is not the tetrafluoroberyllate salt. Option (C) describes a trifluoro formulation rather than the usual four-coordinate complex produced in this preparation. Option (D) is not the ammonium salt of the tetrafluoroberyllate ion; its hydrogen and nitrogen composition also differs from the required salt.

Step 3: Conclusion The compound A is ammonium tetrafluoroberyllate, $(NH_4)_2BeF_4$. Hence, the correct option is (B).

Quick Tip to Solve: For this beryllium preparation, first write $[BeF_4]^{2-}$, then balance its charge with two ammonium ions.

79. Given below are two statements:

Statement I : Noradrenaline is a neurotransmitter.

Statement II : Low level of noradrenaline is not the cause of depression in human.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Statement I is correct but Statement II is incorrect

(C) Both Statement I and Statement II are correct

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (B) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- Noradrenaline is a neurotransmitter, a chemical messenger that carries signals between nerve cells. Its availability influences the signalling associated with mood.
- In the NCERT description of antidepressant action, a low level of noradrenaline is associated with depression. Drugs that inhibit its breakdown can increase its availability at the receptor.

Step 1: Evaluate Statement I Noradrenaline is released in nervous-system signalling and interacts with receptors on the receiving cell. This is the role of a neurotransmitter. It is therefore correct to classify noradrenaline as a neurotransmitter, and Statement I is true. The fact that noradrenaline can also have hormonal actions does not invalidate its neurotransmitter role.

Step 2: Evaluate Statement II and select the pair Statement II contains the negative wording **not the cause**. In the textbook account relevant to this question, low noradrenaline activity contributes to depression. Some antidepressants inhibit enzymes responsible for degrading noradrenaline, slowing its removal and increasing the time for which it can stimulate receptors. Therefore the assertion that low noradrenaline is not related to the cause of depression is false in this context. Option (A) incorrectly rejects Statement I; option (C) incorrectly accepts Statement II; and option (D) reverses both truth values. Only the true–false combination in option (B) remains.

Step 3: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: Underline the word **not** before judging Statement II: the textbook links reduced noradrenaline availability with depression.

80. Which of the Phosphorus oxoacid can create silver mirror from $AgNO_3$ solution? [4 marks]

(A) $H_4P_2O_5$

(B) $H_4P_2O_7$

(C) $(HPO_3)_n$

(D) $H_4P_2O_6$

Correct Answer: (A) $H_4P_2O_5$

Solution:

Concept:

- A silver mirror is formed when Ag^+ is reduced to metallic Ag. The phosphorus oxoacid must therefore act as a reducing agent and itself undergo oxidation.
- For the usual phosphorus oxoacids, reducing character is associated with a phosphorus–hydrogen bond. Hydrogens attached to oxygen account for acidity, rather than this reducing action.

Step 1: Identify the oxoacid with P–H bonds Pyrophosphorous acid, $H_4P_2O_5$, is obtained by condensation of two molecules of phosphorous acid:

$$2\,H_3PO_3 \longrightarrow H_4P_2O_5 + H_2O.$$

Its connectivity can be written as $(HO)(H)P(=O)-O-P(=O)(H)(OH)$. Thus each phosphorus retains a P–H bond. Phosphorus is in the $+3$ oxidation state, and oxidation to $+5$ can supply electrons to reduce silver ions.

Step 2: Eliminate the nonreducing alternatives Pyrophosphoric acid, $H_4P_2O_7$, and metaphosphoric acid, $(HPO_3)_n$, have phosphorus in the $+5$ oxidation state and no P–H bonds. Their hydrogens are attached to oxygen, so they do not show the required silver-ion reduction. Hypophosphoric acid, $H_4P_2O_6$, contains a P–P bond and four O–H groups, but no P–H bond; it is not the silver-mirror reagent among the listed acids. Merely having hydrogen in the molecular formula is therefore insufficient: its point of attachment must be examined.

Step 3: Conclusion $H_4P_2O_5$ reduces silver ions and can produce a silver mirror. Hence, the correct option is (A).

Quick Tip to Solve: For phosphorus oxoacids, inspect P–H bonds rather than counting total hydrogen atoms to identify reducing character.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. For independent process at 300 K

Process	$\Delta H/\text{kJ mol}^{-1}$	$\Delta S/\text{J K}^{-1}$
A	-25	-80
B	-22	**40**
C	**25**	-50
D	**22**	**20**

The number of non-spontaneous process from the following is _____ [4 marks]

Solution:

Concept:

- At constant temperature and pressure, the Gibbs energy change is $\Delta G = \Delta H - T\Delta S$. A process is spontaneous in the stated direction when $\Delta G < 0$, and non-spontaneous when $\Delta G > 0$.
- The enthalpy data are in kilojoules, whereas the entropy data use joules per kelvin. Convert entropy to kilojoules per kelvin before multiplying by $T = 300\,\text{K}$, so that both terms in the difference have consistent units.

Step 1: Calculate the Gibbs changes for A and B For process A, the entropy decrease makes the term $-T\Delta S$ positive:

$$\Delta G_A = -25 - 300(-80/1000) = -1\,\text{kJ mol}^{-1}.$$

The small negative result means A remains spontaneous at 300 K. For process B, the favourable negative enthalpy and positive entropy both promote spontaneity:

$$\Delta G_B = -22 - 300(40/1000) = -34\,\text{kJ mol}^{-1}.$$

Thus neither A nor B belongs in the non-spontaneous count.

Step 2: Calculate C and D and count positive values For C the enthalpy is positive and entropy is negative, so both contributions oppose the forward process:

$$\Delta G_C = 25 - 300(-50/1000) = 40\,\text{kJ mol}^{-1}.$$

For D the entropy increase partly offsets the positive enthalpy, but does not overcome it at 300 K:

$$\Delta G_D = 22 - 300(20/1000) = 16\,\text{kJ mol}^{-1}.$$

Both C and D have positive Gibbs changes. It would be incorrect to count processes using only the sign of enthalpy or only the sign of entropy; the numerical balance at the stated temperature decides the result.

Step 3: Conclusion The non-spontaneous processes at 300 K are C and D. Their number is $\boxed{2}$ (a dimensionless count).

Quick Tip to Solve: Convert entropy from joules to kilojoules first; A is close to the sign boundary and is easily misclassified.

82. When $Fe_{0.93}O$ is heated in presence of oxygen, it converts to Fe_2O_3. The number of correct statement/s from the following is _____

A. The equivalent weight of $Fe_{0.93}O$ is $\frac{\text{Molecular weight}}{0.79}$

B. The number of moles of Fe^{2+} and Fe^{3+} in 1 mole of $Fe_{0.93}O$ is 0.79 and 0.14 respectively

C. $Fe_{0.93}O$ is metal deficient with lattice comprising of cubic closed packed arrangement of O^{2-} ions

D. The % composition of Fe^{2+} and Fe^{3+} in $Fe_{0.93}O$ is 85% and 15% respectively [4 marks]

Solution:

Concept:

- In $Fe_{0.93}O$, iron exists in both $+2$ and $+3$ oxidation states. The total positive charge per formula unit must balance the charge -2 of one oxide ion.
- The equivalent weight in an oxidation reaction equals the formula mass divided by the number of moles of electrons lost per mole of the substance. Here only the Fe^{2+} fraction is oxidised further to Fe^{3+}.

Step 1: Find the amounts of the two iron ions Let x and y be the moles of Fe^{2+} and Fe^{3+} in one mole of the oxide. Atom count and charge neutrality give

$$x + y = 0.93, \qquad 2x + 3y = 2.$$

Subtracting twice the first equation from the second gives $y = 2 - 1.86 = 0.14$, and hence $x = 0.93 - 0.14 = 0.79$. Statement B is correct. Oxidation to Fe_2O_3 converts these 0.79 moles of divalent iron into trivalent iron, releasing 0.79 moles of electrons. Therefore $n = 0.79$ and equivalent weight $= M/0.79$, where M is the formula mass. Statement A is also correct.

Step 2: Check the defect and the percentages The oxygen sublattice retains the cubic close-packed arrangement characteristic of the rock-salt type oxide. Some iron sites are vacant, while some of the remaining iron is trivalent to maintain neutrality. Since the iron-to-oxygen ratio is below one, this is a metal-deficiency defect; Statement C is correct. The percentages must be calculated among occupied iron sites:

$$\%Fe^{2+} = \frac{0.79}{0.93} \times 100 = 84.95\% \simeq 85\%,$$

$$\%Fe^{3+} = \frac{0.14}{0.93} \times 100 = 15.05\% \simeq 15\%.$$

Thus Statement D is correct to the stated whole-percent precision. Dividing by one instead of 0.93 would confuse formula-unit amounts with the distribution of iron ions.

Step 3: Conclusion All four statements A, B, C and D are correct. The required number is $\boxed{4}$ (dimensionless).

Quick Tip to Solve: Use both atom count and charge balance, then divide each iron-ion amount by 0.93 when calculating its percentage among iron ions.

83. 5 g of NaOH was dissolved in deionized water to prepare a 450 mL stock solution. What volume (in mL) of this solution would be required to prepare 500 mL of 0.1 M solution? _____

Given : Molar Mass of Na, O and H is 23, 16 and 1 g mol^{-1} respectively [4 marks]

Solution:

Concept:

- Molarity is moles of solute per litre of solution: $M = n/V$. The molar mass of sodium hydroxide is the sum of the given atomic molar masses.
- Adding water changes the concentration but conserves the amount of solute. Thus the moles in the stock aliquot must equal those required in the final solution: $M_1V_1 = M_2V_2$, with both volumes expressed in the same unit.

Step 1: Calculate the stock concentration The given atomic masses give

$$M_{\mathrm{NaOH}} = 23 + 16 + 1 = 40\,\mathrm{g\,mol^{-1}}.$$

The amount dissolved is

$$n_{\mathrm{stock}} = \frac{5\,\mathrm{g}}{40\,\mathrm{g\,mol^{-1}}} = 0.125\,\mathrm{mol}.$$

The stated 450 mL is the volume of the prepared stock solution, so

$$M_1 = \frac{0.125}{0.450} = \frac{5}{18}\,\mathrm{mol\,L^{-1}}.$$

Using 450 as litres would introduce a factor of one thousand; converting the volume before applying the molarity definition avoids that error.

Step 2: Find the required aliquot by conserving solute The final 500 mL solution at 0.1 M must contain

$$n_{\mathrm{required}} = 0.1 \times 0.500 = 0.0500\,\mathrm{mol}.$$

Therefore the volume of stock that supplies this quantity is

$$V_1 = \frac{0.0500}{5/18} = 0.180\,\mathrm{L} = 180\,\mathrm{mL}.$$

Equivalently, this aliquot is $0.0500/0.125 = 0.4$ of the total stock, and $0.4 \times 450 = 180$ mL. This cross-check gives the same result. The aliquot is diluted with water until the total solution volume reaches 500 mL; 500 mL of water is not added to it.

Step 3: Conclusion The required stock-solution volume is $\boxed{180\,\mathrm{mL}}$.

Quick Tip to Solve: Calculate the required moles first: 0.0500 mol is 40 percent of the stock solute, so take 40 percent of its volume.

84. Uracil is a base present in RNA with the following structure. % of N in uracil is _____

Given:

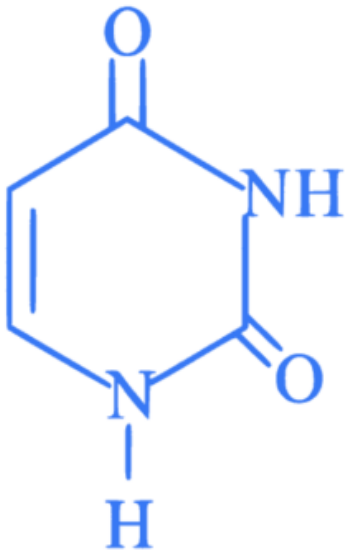
Molar mass N = 14 g mol^{-1}

O = 16 g mol^{-1}

C = 12 g mol^{-1}

H = 1 g mol^{-1}

[4 marks]



Solution:

Concept:

- The mass percentage of an element is its total mass in one mole of the compound divided by the molar mass of the compound, multiplied by 100.
- In a skeletal structural formula, unlabelled vertices are carbon atoms and their implicit hydrogens complete carbon valency four. Explicit N–H hydrogens must also be counted when obtaining the molecular formula.

Step 1: Count the atoms in the uracil structure The six-membered ring contains four carbon atoms and two nitrogen atoms. Two of the carbon atoms are carbonyl carbons, each bonded to one oxygen atom. Each nitrogen bears one hydrogen as shown. The two remaining ring carbon atoms form the carbon–carbon double bond, and each carries one hydrogen to complete its valency. Hence the molecule contains four hydrogens in total and its formula is $C_4H_4N_2O_2$. Counting only the two explicitly drawn N–H hydrogens would omit the two implicit C–H hydrogens.

Step 2: Calculate the molar mass and nitrogen percentage Using the given atomic molar masses,

$$M(C_4H_4N_2O_2) = 4(12) + 4(1) + 2(14) + 2(16).$$

$$M = 48 + 4 + 28 + 32 = 112\,\text{g mol}^{-1}.$$

The two nitrogen atoms contribute $2 \times 14 = 28\,\mathrm{g}$ to one mole of uracil. Consequently,

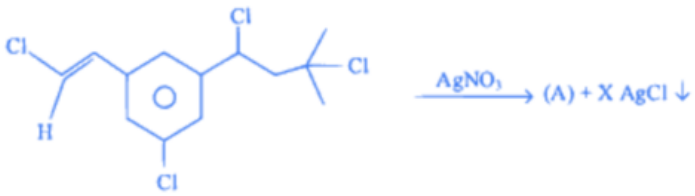
$$\%\mathrm{N} = \frac{28}{112} \times 100 = 25\%.$$

The common mass unit cancels in the fraction, giving a percentage rather than a molar quantity. As a check, nitrogen contributes exactly one quarter of the total molar mass, and one quarter corresponds to 25 percent.

Step 3: Conclusion The nitrogen content of uracil is $\boxed{25\%}$; the numerical entry is $\boxed{25}$.

Quick Tip to Solve: Include implicit C–H hydrogens when reading the ring: uracil is $\mathrm{C_4H_4N_2O_2}$, with molar mass 112 and nitrogen mass 28.

85. Number of moles of AgCl formed in the following reaction is _____ [4 marks]



Solution:

Concept:

- Silver nitrate detects chloride ions released by an organic chloride: $\mathrm{Ag^+ + Cl^- \longrightarrow AgCl\downarrow}$. Each mole of released chloride produces one mole of precipitate.
- In the usual silver-nitrate test, alkyl chlorides undergo ionisation or substitution, whereas aryl and vinylic carbon–chlorine bonds resist this reaction. The carbon bearing chlorine must therefore be identified before counting chlorine atoms.

Step 1: Classify the four chlorine atoms The chlorine at the left end is attached directly to a carbon of the carbon–carbon double bond; it is a vinylic chloride. The chlorine at the bottom of the benzene ring is an aryl chloride. In contrast, the chlorine on the side chain nearer the ring is on a saturated secondary carbon, and the chlorine at the right end is on a tertiary saturated carbon. Thus the structure contains two unreactive unsaturated-carbon chlorides and two reactive alkyl chlorides.

Step 2: Count only the chlorides that precipitate The aryl bond has partial double-bond character through resonance with the ring, and the vinylic bond is a strong bond to an sp^2-hybridised carbon. Neither releases chloride in the ordinary silver-nitrate test. The secondary and tertiary alkyl chlorides can release chloride; silver ion removes it as insoluble AgCl, favouring substitution. For one mole of the depicted substrate, the two reactive sites provide two moles of chloride ions.

$$n(\mathrm{AgCl}) = 2 \times 1 = 2\ \mathrm{mol}.$$

Counting all four chlorine atoms would wrongly include the aryl and vinylic sites.

Step 3: Conclusion Two alkyl chlorine atoms react, so the number of moles of AgCl formed is $\boxed{2}$ per

mole of the given compound.

Quick Tip to Solve: Before counting precipitated halide, mark each C–Cl bond as alkyl, aryl or vinylic; only the two alkyl sites respond here.

86. The number of correct statement/s from the following is _____

A. Larger the activation energy, smaller is the value of the rate constant.

B. The higher is the activation energy, higher is the value of the temperature coefficient.

C. At lower temperatures, increase in temperature causes more change in the value of k than at higher temperature

D. A plot of $\ln k$ vs $\frac{1}{T}$ is a straight line with slope equal to $-\frac{E_a}{R}$ [4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where A is the frequency factor, E_a the activation energy, R the gas constant and T the absolute temperature.
- The temperature coefficient is a ratio such as $k(T+10)/k(T)$; a ratio or fractional change must not be confused with the absolute change in the rate constant.

Step 1: Test statements A and B For the comparison in A, keep A and T fixed. Raising E_a makes the exponent more negative, so it lowers k: A is correct. For a fixed temperature interval ΔT,

$$\ln\frac{k(T+\Delta T)}{k(T)} = \frac{E_a\Delta T}{RT(T+\Delta T)}.$$

The right side increases with activation energy. Hence the ratio of rate constants, and therefore the temperature coefficient, is larger for higher activation energy: B is correct.

Step 2: Distinguish absolute and relative changes in C Differentiation gives

$$\frac{1}{k}\frac{dk}{dT} = \frac{E_a}{RT^2}, \qquad \frac{dk}{dT} = \frac{AE_a}{RT^2}e^{-E_a/(RT)}.$$

The fractional sensitivity is greater at lower temperature, but C claims more change in the value of k itself. Its absolute change also contains the strongly temperature-dependent factor k, so that claim is not generally valid. In ordinary activated reactions the absolute change can be larger at higher temperature. Thus C is false as stated.

Step 3: Read the slope in D and count Taking natural logarithms gives

$$\ln k = \ln A - \frac{E_a}{R}\frac{1}{T}.$$

This is the straight-line form $y = c + mx$, with $y = \ln k$, $x = 1/T$ and slope $-E_a/R$. Therefore D is correct. The correct statements are A, B and D.

Step 4: Conclusion There are $\boxed{3}$ correct statements: A, B and D. Statement C confuses absolute change with fractional temperature sensitivity.

Quick Tip to Solve: When comparing temperature effects, first decide whether the statement concerns the ratio of rate constants or their absolute difference.

87. The dissociation constant of acetic acid is $x \times 10^{-5}$. When 25 mL of 0.2 $\mathrm{M\ CH_3COONa}$ solution is mixed with 25 mL of 0.02 $\mathrm{M\ CH_3COOH}$ solution, the pH of the resultant solution is found to be equal to 5. The value of x is _____ [4 marks]

Solution:

Concept:

- Acetic acid and sodium acetate form an acidic buffer. The equilibrium is $\mathrm{CH_3COOH} \rightleftharpoons \mathrm{H^+} + \mathrm{CH_3COO^-}$, with $K_a = [\mathrm{H^+}][\mathrm{CH_3COO^-}]/[\mathrm{CH_3COOH}]$.
- The Henderson–Hasselbalch equation is $\mathrm{pH} = \mathrm{p}K_a + \log_{10}([\text{salt}]/[\text{acid}])$. The same final volume divides both concentrations, so their ratio equals the ratio of their moles.

Step 1: Find the amounts present after mixing The sodium acetate supplies acetate ions, while acetic acid supplies the weak-acid component. Their initial amounts are

$$n_{\text{salt}} = 0.2 \times 0.025 = 0.005 \text{ mol},$$

$$n_{\text{acid}} = 0.02 \times 0.025 = 0.0005 \text{ mol}.$$

The total volume is 0.050 L, so the formal salt and acid concentrations are $0.100\ \mathrm{mol\,L^{-1}}$ and $0.0100\ \mathrm{mol\,L^{-1}}$, respectively. The salt-to-acid ratio is therefore 10. Equal volumes do not mean equal numbers of moles because the two starting concentrations differ.

Step 2: Use the measured pH to obtain the constant Substitute the concentration ratio and the measured pH in the buffer equation:

$$5 = \mathrm{p}K_a + \log_{10}(10) = \mathrm{p}K_a + 1.$$

Thus $\mathrm{p}K_a = 4$, giving

$$K_a = 10^{-4} = 10 \times 10^{-5}.$$

Equivalently, pH 5 means $[\mathrm{H^+}] = 10^{-5}\ \mathrm{mol\,L^{-1}}$; multiplying this by the salt-to-acid ratio 10 gives the same value of K_a. As usual for this buffer calculation, the small additional dissociation of the weak acid is negligible compared with the analytical buffer concentrations.

Step 3: Conclusion Comparing $K_a = 10 \times 10^{-5}$ with $x \times 10^{-5}$ gives $\boxed{x = 10}$. The requested coefficient is dimensionless.

Quick Tip to Solve: For a buffer formed by mixing solutions, use the salt-to-acid mole ratio; the common final volume cancels automatically.

88. At 298 K, a 1 litre solution containing 10 mmol of $Cr_2O_7^{2-}$ and 100 mmol of Cr^{3+} shows a pH of 3.0.

Given : $Cr_2O_7^{2-} \rightarrow Cr^{3+}$; $E^\circ = 1.330$**V**

and $\frac{2.303\,RT}{F} = 0.059$ **V**

The potential for the half cell reaction is $x \times 10^{-3}$ **V. The value of** x **is _____** [4 marks]

Solution:

Concept:

- The balanced reduction half-reaction is $Cr_2O_7{}^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O$. Its electron transfer number is $n = 6$.
- At 298 K, the Nernst equation is $E = E^\circ - (0.059/n)\log_{10} Q$. Pure water is omitted from Q; the exponents of dissolved species come from the balanced reaction.

Step 1: Convert the given amounts and pH The volume is one litre. Therefore 10 mmol of dichromate gives $[Cr_2O_7{}^{2-}] = 0.010\ \text{mol}\,L^{-1}$, and 100 mmol of chromium(III) gives $[Cr^{3+}] = 0.100\ \text{mol}\,L^{-1}$. The stated pH 3.0 gives $[H^+] = 10^{-3}\ \text{mol}\,L^{-1}$. The formal concentrations are used in the usual ideal-solution approximation; no extra concentration factor arises from the one-litre volume.

Step 2: Form the reaction quotient with its powers Two chromium(III) ions are formed per dichromate ion, while fourteen hydrogen ions are consumed. Consequently,

$$Q = \frac{[Cr^{3+}]^2}{[Cr_2O_7{}^{2-}][H^+]^{14}}$$

$$= \frac{(10^{-1})^2}{10^{-2}(10^{-3})^{14}} = 10^{42}.$$

Hence $\log_{10} Q = 42$. Using only seven hydrogen ions or only three electrons would describe an unbalanced half-reaction and give the wrong pH dependence.

Step 3: Evaluate the potential and requested coefficient Insert the given standard potential and the supplied logarithmic factor:

$$E = 1.330 - \frac{0.059}{6}(42)$$

$$= 1.330 - 0.413 = 0.917\ \text{V}.$$

The positive correction term is subtracted because $Q > 1$; the reduction potential is lower than its standard value. Finally, express the result in millivolts, as requested rather than returning the value in volts.

Step 4: Conclusion The half-cell potential is $0.917\ \text{V} = 917 \times 10^{-3}\ \text{V}$. Therefore $\boxed{x = 917}$.

Quick Tip to Solve: Balance the acidic dichromate half-reaction first: the fourteen powers of hydrogen-ion concentration dominate the Nernst correction.

89. The d-electronic configuration of $[CoCl_4]^{2-}$ in tetrahedral crystal field in $e^m t_2^n$. Sum of "m" and "number of unpaired electrons" is _____ [4 marks]

Solution:

Concept:

- In $[CoCl_4]^{2-}$, each chloride ligand has charge -1. Oxidation-state counting identifies the metal ion before its d-electron configuration is determined.
- In a tetrahedral field the two e orbitals lie below the three t_2 orbitals. The small tetrahedral splitting and weak-field chloride ligands give a high-spin complex, so Hund's rule must be applied before pairing electrons.

Step 1: Determine the d-electron count of cobalt Let the cobalt oxidation state be z. Charge balance gives

$$z + 4(-1) = -2 \quad \Rightarrow \quad z = +2.$$

Neutral cobalt has atomic number 27 and configuration $[Ar]3d^7 4s^2$. Removing the two outer 4s electrons gives Co^{2+} with configuration $[Ar]3d^7$. Thus seven d electrons, not nine, must be distributed in the tetrahedral levels.

Step 2: Fill the tetrahedral levels in the high-spin order Place one electron in each of the two lower e orbitals, then one in each of the three upper t_2 orbitals before pairing. This uses five electrons. The sixth and seventh electrons pair in the lower e set, producing

$$e^4 t_2^3.$$

Each of the two e orbitals now contains a pair, while each of the three t_2 orbitals contains one electron. Therefore the e exponent is $m = 4$, and the number of unpaired electrons is 3. The exponent $n = 3$ happens to equal the unpaired count here, but these are different quantities and should be counted separately.

Step 3: Add the two requested numbers The problem asks for the exponent of e plus the number of unpaired electrons; it does not ask for $m + n$ as a general rule. Here the required calculation is

$$m + N_{\text{unpaired}} = 4 + 3 = 7.$$

The nonzero unpaired count also confirms that the ion is paramagnetic, consistent with the high-spin tetrahedral configuration.

Step 4: Conclusion The configuration is $e^4 t_2^3$, with three unpaired electrons. Hence the required sum is $\boxed{7}$.

Quick Tip to Solve: For tetrahedral complexes, put e below t2 and determine the metal oxidation state first; count unpaired electrons from orbital occupancy.

90. If wavelength of the first line of the Paschen series of hydrogen atom is 720 nm, then the wavelength of the second line of this series is _____ nm. (Nearest integer) [4 marks]

Solution:

Concept:

- For a hydrogen emission line, the Rydberg relation is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$ and R_H is the hydrogen Rydberg constant.
- The Paschen series has final level $n_f = 3$. Its first line is the transition $4 \to 3$, and its second line is $5 \to 3$. Taking a ratio cancels R_H, so the supplied first wavelength must be used directly.

Step 1: Write the factors for the two lines For the first line, let the wavelength be $\lambda_1 = 720$ nm. Its wavenumber is

$$\frac{1}{\lambda_1} = R_H\left(\frac{1}{9} - \frac{1}{16}\right) = R_H\frac{7}{144}.$$

The second line begins at the next higher initial level, $n_i = 5$, and has wavelength λ_2:

$$\frac{1}{\lambda_2} = R_H\left(\frac{1}{9} - \frac{1}{25}\right) = R_H\frac{16}{225}.$$

Both transitions end at the same level; moving the final level to 4 would describe another series rather than the second Paschen line.

Step 2: Take the wavelength ratio and round only at the end Because wavelength is the reciprocal of wavenumber, the ratio of the wavelengths is the inverse of the ratio of these transition factors:

$$\frac{\lambda_2}{\lambda_1} = \frac{7/144}{16/225} = \frac{175}{256}.$$

Therefore

$$\lambda_2 = 720 \times \frac{175}{256} = 492.1875 \text{ nm}.$$

The second transition releases more energy than the first and should have a shorter wavelength; $492.1875 < 720$ confirms this direction. The question gives a particular first-line wavelength, so replacing it by a memorised laboratory Paschen wavelength would ignore the supplied data. Rounding the calculated result to the nearest integer yields 492.

Step 3: Conclusion The wavelength of the second line is 492.1875 nm, or $\boxed{492 \text{ nm}}$ to the nearest integer.

Quick Tip to Solve: Within a named hydrogen series, fix the final level and increase the initial level for successive lines; use wavelength ratios to cancel Rydberg's constant.