JEE Main 2023 Paper 1 (B.E./B.Tech) Jan 24 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 24 January 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $f(x)$ be a function such that $f(x+y) = f(x).f(y)$ for all $x, y \in \mathbb{N}$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is [4 marks]

(A) 9

(B) 7

(C) 6

(D) 8

Correct Answer: (B) 7

Solution:

Concept:

- The identity $f(x+y) = f(x)f(y)$, together with $f(1) = 3$, determines the values on positive integers recursively.
- A geometric progression with first term a and common ratio $r \neq 1$ has sum $S_n = a(r^n - 1)/(r-1)$. Here each successive term is three times its predecessor.

Step 1: Find the general term For any positive integer k, take $x = k$ and $y = 1$. Then $f(k+1) = f(k)f(1) = 3f(k)$. Starting with $f(1) = 3$, this gives $f(2) = 9$, $f(3) = 27$, and by induction

$$f(k) = 3^k.$$

The exponent is k, rather than $k-1$, because the first term itself equals 3.

Step 2: Sum the progression and solve for the exponent The given condition becomes $3+3^2+\cdots+3^n = 3279$. Applying the geometric-series formula and simplifying gives

$$\frac{3(3^n - 1)}{2} = 3279 \quad \Longrightarrow \quad 3^n - 1 = 2186.$$

Thus $3^n = 2187 = 3^7$, and therefore $n = 7$. As a direct check, $3(2187-1)/2 = 3279$, exactly the prescribed sum. The values 6, 8 and 9 cannot work because successive partial sums strictly increase: all the terms 3^k are positive. In particular, the sum through six terms is only 1092, whereas including an eighth term makes the sum exceed the target.

Step 3: Conclusion The required value is $\boxed{n = 7}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the first two terms before applying the sum formula: the sequence starts at 3, so its sum is $3(3^n - 1)/2$.

2. The number of real solutions of the equation $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$, is [4 marks]

(A) 3

(B) 4

(C) 0

(D) 2

Correct Answer: (C) 0

Solution:

Concept:

- The reciprocal terms require $x \neq 0$. For a real nonzero x, the quantity $t = x + 1/x$ is also real.
- The identity $x^2+1/x^2 = (x+1/x)^2-2 = t^2-2$ converts the equation into a quadratic. A quadratic with negative discriminant has no real root.

Step 1: Make the reciprocal substitution Put $t = x+1/x$, retaining the domain $x \in \mathbb{R}\setminus\{0\}$. Expanding t^2 produces the cross term 2, so the original equation is equivalent to

$$3(t^2-2)-2t+5=0 \quad \Longleftrightarrow \quad 3t^2-2t-1=0.$$

Factorising carefully, $3t^2-2t-1=(3t+1)(t-1)$. Thus the only candidate values are $t=1$ and $t=-1/3$.

Step 2: Check which candidates come from real values of x For real nonzero x, either $x>0$ and $x+1/x \geq 2$, or $x<0$ and $x+1/x \leq -2$. Neither 1 nor $-1/3$ satisfies this necessary condition. Equivalently, solving $x+1/x=t$ requires

$$x^2-tx+1=0, \qquad \Delta = t^2-4 \geq 0.$$

For $t=1$, $\Delta=-3$; for $t=-1/3$, $\Delta=-35/9$. Both are negative. Thus both algebraic candidates fail the real-domain test. In particular, counting the two roots in t as two roots in x would be incorrect. The counts 2, 3 and 4 in the other options are all excluded.

Step 3: Conclusion There are $\boxed{0}$ real solutions. Hence, the correct option is (C).

Quick Tip to Solve: After substituting $t = x+1/x$, always impose $|t| \geq 2$ for real nonzero x; roots of the transformed quadratic alone are insufficient.

3. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition is : [4 marks]

(A) 48

(B) 120

(C) 168

(D) 220

Correct Answer: (C) 168

Solution:

Concept:

- The multiplication principle counts independent successive digit choices. With no repetition, each chosen digit reduces the available choices by one.
- Numbers greater than 7000 can have either four or five digits here. Separate these disjoint cases, count each once, then add their totals.

Step 1: Count the eligible four-digit integers An integer with fewer than four digits is less than 7000. For a four-digit integer, its first digit must be 7 or 8, giving two choices. If it starts with 7, it still exceeds 7000 because all remaining available digits are positive. For either first digit, arrange any three of the four remaining digits in the last three positions:

$$N_4 = 2 \times 4 \times 3 \times 2 = 48.$$

The falling numbers 4, 3 and 2 enforce the no-repetition condition, rather than permitting any digit to be reused.

Step 2: Include all five-digit integers Every five-digit integer made from the five given digits is automatically greater than 7000, regardless of its first digit. All five digits must occur once, so

$$N_5 = 5! = 120.$$

The four-digit and five-digit collections have no common member. Therefore the required total is

$$N = N_4 + N_5 = 48 + 120 = 168.$$

Option (A) counts only four-digit numbers and misses all five-digit numbers. Option (B) counts only five-digit numbers and misses the 48 qualifying four-digit numbers. Option (D) exceeds the exhaustive sum of these two possible lengths, so it is also incorrect.

Step 3: Conclusion The required number of integers is $\boxed{168}$. Hence, the correct option is (C).

Quick Tip to Solve: When the number of digits is not specified, first list every possible length; here both four-digit and five-digit numbers can exceed 7000.

4. Let $\overrightarrow{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\overrightarrow{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\overrightarrow{\beta}_1$ be parallel to $\overrightarrow{\alpha}$ and $\overrightarrow{\beta}_2$ be perpendicular to $\overrightarrow{\alpha}$. If $\overrightarrow{\beta} = \overrightarrow{\beta}_1 + \overrightarrow{\beta}_2$, then the value of $5\overrightarrow{\beta}_2 . \left(\hat{i} + \hat{j} + \hat{k}\right)$ is : [4 marks]

(A) 9

(B) 7

(C) 6

(D) 11

Correct Answer: (B) 7

Solution:

Concept:

- The component of $\vec{\beta}$ parallel to a nonzero vector $\vec{\alpha}$ is $\vec{\beta}_1 = (\vec{\beta} \cdot \vec{\alpha})/(\vec{\alpha} \cdot \vec{\alpha})\,\vec{\alpha}$.
- The perpendicular component is the remainder $\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$. Taking its dot product with $\hat{i} + \hat{j} + \hat{k}$ adds its three Cartesian components.

Step 1: Calculate the parallel component Using the given vectors gives

$$\vec{\beta} \cdot \vec{\alpha} = 4 + 6 - 20 = -10, \qquad \vec{\alpha} \cdot \vec{\alpha} = 16 + 9 + 25 = 50.$$

Thus the coefficient of the parallel projection is $-10/50 = -1/5$, and

$$\vec{\beta}_1 = -\frac{1}{5}\vec{\alpha} = -\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j} - \hat{k}.$$

The negative coefficient means the parallel component points opposite to $\vec{\alpha}$; changing this sign would change the required answer.

Step 2: Find the perpendicular remainder and the requested scalar Subtract the parallel component from $\vec{\beta}$:

$$\vec{\beta}_2 = \frac{9}{5}\hat{i} + \frac{13}{5}\hat{j} - 3\hat{k}.$$

Its orthogonality can be checked directly: $\vec{\beta}_2 \cdot \vec{\alpha} = 36/5 + 39/5 - 15 = 0$. Finally,

$$5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k}) = 5\left(\frac{9}{5} + \frac{13}{5} - 3\right) = 22 - 15 = 7.$$

This also follows by writing $5\vec{\beta}_2 = 5\vec{\beta} + \vec{\alpha}$, whose components are $(9, 13, -15)$. Their sum is 7, ruling out 9, 6 and 11.

Step 3: Conclusion The requested scalar is $\boxed{7}$. Hence, the correct option is (B).

Quick Tip to Solve: Compute $5\vec{\beta}_2 = 5\vec{\beta} + \vec{\alpha}$ after finding the projection coefficient $-1/5$; this avoids fractional components in the final dot product.

5. If the system of equations

$$x + 2y + 3z = 3$$

$$4x + 3y - 4z = 4$$

$$8x + 4y - \lambda z = 9 + \mu$$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to : [4 marks]

(A) $\left(\frac{72}{5}, \frac{21}{5}\right)$
(B) $\left(-\frac{72}{5}, -\frac{21}{5}\right)$
(C) $\left(-\frac{72}{5}, \frac{21}{5}\right)$
(D) $\left(\frac{72}{5}, -\frac{21}{5}\right)$

Correct Answer: (D) $\left(\frac{72}{5}, -\frac{21}{5}\right)$

Solution:

Concept:

- Three linear equations in three unknowns have infinitely many solutions when their coefficient and augmented matrices have the same rank smaller than three.
- The first two coefficient rows are independent. Hence the third entire equation, including its right-hand side, must be a linear combination of the first two.

Step 1: Express the third row using the first two Let the third equation equal a times the first plus b times the second. Comparing the coefficients of x and y gives

$$a + 4b = 8, \qquad 2a + 3b = 4.$$

Subtracting the second equation from twice the first gives $5b = 12$, so $b = 12/5$. Substituting back gives $a = 8 - 48/5 = -8/5$. These coefficients are fixed uniquely because the determinant $1 \cdot 3 - 2 \cdot 4 = -5$ is nonzero.

Step 2: Match the z coefficient and constant term The same combination must also reproduce the third coefficient of z:

$$-\lambda = 3a - 4b = -\frac{24}{5} - \frac{48}{5} = -\frac{72}{5}.$$

Therefore $\lambda = 72/5$. Matching the right-hand side gives

$$9 + \mu = 3a + 4b = -\frac{24}{5} + \frac{48}{5} = \frac{24}{5}, \qquad \mu = -\frac{21}{5}.$$

With these values the third equation contributes no new restriction, and the first two independent equations leave one free variable, proving infinitely many solutions. The alternatives with negative λ fail the coefficient condition; the alternative with positive μ fails consistency of the constants.

Step 3: Conclusion The ordered pair is $\boxed{(\lambda, \mu) = (72/5, -21/5)}$. Hence, the correct option is (D).

Quick Tip to Solve: For infinitely many solutions, match the constants with the same row-combination coefficients used for x and y; a zero determinant alone does not guarantee consistency.

6. The set of all values of a for which $\lim_{x\to a}([x-5]-[2x+2])=0$, where $[\alpha]$ denotes the greatest integer less than or equal to α is equal to [4 marks]

(A) $[-7.5,-6.5]$

(B) $(-7.5,-6.5]$

(C) $[-7.5,-6.5)$

(D) $(-7.5,-6.5)$

Correct Answer: (D) $(-7.5,-6.5)$

Solution:

Concept:

- A difference of integer-valued functions can have limit zero only if it is zero throughout a sufficiently small deleted neighbourhood of the limiting point.
- The equality $\lfloor u\rfloor=\lfloor v\rfloor=k$ is equivalent to putting both u and v in the same interval $[k,k+1)$. Endpoints need separate one-sided checks.

Step 1: Locate where the two floor values agree Write their common value as k. Then the two inequalities are

$$x\in[k+5,k+6), \qquad x\in[(k-2)/2,(k-1)/2).$$

These intervals overlap only when $k+5<(k-1)/2$ and $(k-2)/2<k+6$, giving $-14<k<-11$. Since k is an integer, only $k=-13,-12$ are possible. Their intersections are respectively $[-7.5,-7)$ and $[-7,-6.5)$. Thus the difference is zero on $[-7.5,-6.5)$.

Step 2: Test the boundary points for the two-sided limit Every point strictly between -7.5 and -6.5, including -7, has a whole neighbourhood on which the difference equals zero. At the left endpoint, however, just below -7.5 the two floors are -13 and -14, giving difference 1, whereas just above it the difference is 0. At the right endpoint, just below -6.5 the difference is 0, whereas just above it the floors are -12 and -11, giving difference -1. Neither endpoint has the prescribed two-sided limit. Therefore all options containing either endpoint are excluded; the function value at an endpoint cannot replace the limit test.

Step 3: Conclusion The required set is $\boxed{(-7.5,-6.5)}$. Hence, the correct option is (D).

Quick Tip to Solve: First solve equality of the floor values, then remove any boundary where the one-sided limits differ; equality at a point does not determine its two-sided limit.

7. The locus of the mid points of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to : [4 marks]

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{3\pi}{4}$

(D) $\frac{\pi}{6}$

Correct Answer: (A) $\frac{\pi}{2}$

Solution:

Concept:

- The perpendicular from a circle centre to a chord bisects both the chord and the angle subtended by it at the centre.
- For a circle of radius R, a chord with central angle θ has midpoint at distance $R\cos(\theta/2)$ from the centre. Its midpoint locus therefore has radius $r = R\cos(\theta/2)$ for the usual minor central angle.

Step 1: Calculate the first and third locus radii The circle $(x-4)^2 + (y-5)^2 = 4$ has centre $(4, 5)$ and radius $R = 2$. In the right triangle formed by a centre, chord midpoint and endpoint, the hypotenuse is 2 and the angle at the centre is half the subtended angle. Thus

$$r_1^2 = 4\cos^2(\pi/6) = 3, \qquad r_3^2 = 4\cos^2(\pi/3) = 1.$$

The centre coordinates translate the locus but do not affect either radius.

Step 2: Use the given relation and recover the angle From $r_1^2 = r_2^2 + r_3^2$, we obtain $r_2^2 = 3 - 1 = 2$. Therefore

$$4\cos^2(\theta_2/2) = 2, \qquad \cos^2(\theta_2/2) = \frac{1}{2}.$$

For the minor central angle $0 \le \theta_2 \le \pi$, its half lies in $[0, \pi/2]$, so $\theta_2/2 = \pi/4$. It follows that $\theta_2 = \pi/2$. To check the alternatives, the angles $\pi/4$, $3\pi/4$ and $\pi/6$ give squared locus radii $2 + \sqrt{2}$, $2 - \sqrt{2}$ and $2 + \sqrt{3}$, respectively, none equal to the required 2.

Step 3: Conclusion The required angle is $\boxed{\theta_2 = \pi/2}$. Hence, the correct option is (A).

Quick Tip to Solve: The locus radius is the distance from the centre to the chord midpoint, so use cosine of the half-angle, rather than the chord half-length involving sine.

8. Let $y = y(x)$ be the solution of the differential equation $(x^2 - 3y^2)dx + 3xy\,dy = 0, y(1) = 1$. Then $6y^2(e)$ is equal to [4 marks]

(A) $\frac{3}{2}e^2$

(B) $3e^2$

(C) e^2

(D) $2e^2$

Correct Answer: (D) $2e^2$

Solution:

Concept:

- A differential equation involving y^2 and $y\,dy/dx$ becomes linear on putting $z = y^2$, since $z\prime = 2yy\prime$.
- For a linear equation $z\prime + P(x)z = Q(x)$, multiplication by the integrating factor $e^{\int P(x)dx}$ converts the left side into a product derivative.

Step 1: Convert the equation into a linear equation Divide the equation by dx and rearrange on the positive x-interval containing the initial point:

$$3xy\frac{dy}{dx} = 3y^2 - x^2.$$

With $z = y^2$, this becomes

$$\frac{3x}{2}\frac{dz}{dx} = 3z - x^2, \qquad \frac{dz}{dx} - \frac{2z}{x} = -\frac{2x}{3}.$$

The substitution keeps the requested square of the solution directly, so there is no need to choose a square-root sign at the final evaluation.

Step 2: Integrate and impose the initial value The integrating factor is $e^{-2\ln x} = x^{-2}$, giving

$$\frac{d}{dx}\left(\frac{z}{x^2}\right) = -\frac{2}{3x}.$$

Thus

$$\frac{z}{x^2} = C - \frac{2}{3}\ln x.$$

Since $y(1) = 1$, we have $z(1) = 1$, hence $C = 1$. Therefore

$$y^2 = x^2\left(1 - \frac{2}{3}\ln x\right).$$

This expression is positive from $x = 1$ through $x = e$, as required for this real solution.

Step 3: Evaluate the requested expression Use $\ln e = 1$:

$$y^2(e) = e^2\left(1 - \frac{2}{3}\right) = \frac{e^2}{3}, \qquad 6y^2(e) = 2e^2.$$

The other three choices have coefficients $3/2$, 3, and 1 instead of 2; none satisfies this substitution and initial condition.

Step 4: Conclusion The requested value is $2e^2$. Hence, the correct option is (D).

Quick Tip to Solve: When both y^2 and yy' occur, first try $z = y^2$; apply the initial condition before evaluating at e.

9. If $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in \mathbb{R}$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \ldots + f\left(\frac{2022}{2023}\right)$ is equal to [4 marks]

(A) 2011

(B) 2010

(C) 1010

(D) 1011

Correct Answer: (D) 1011

Solution:

Concept:

- Pair terms at complementary arguments: the arguments $k/2023$ and $(2023-k)/2023$ add to one.
- If $f(x) + f(1-x)$ is constant, a long finite sum can be evaluated by pairing its first and last terms.

Step 1: Establish the complementary-argument identity Put $t = 2^{2x}$, which is positive for every real x. Then

$$f(x) = \frac{t}{t+2}, \qquad 2^{2(1-x)} = \frac{4}{t}.$$

Consequently,

$$f(1-x) = \frac{4/t}{4/t+2} = \frac{2}{t+2}.$$

The denominators are nonzero, so adding these identities is valid for all the arguments in the sum:

$$f(x) + f(1-x) = \frac{t+2}{t+2} = 1.$$

The constant 2 in the original denominator is essential to this exact cancellation; it cannot be omitted.

Step 2: Pair all the terms without duplication For each integer k from 1 through 1011, pair

$$f\left(\frac{k}{2023}\right) + f\left(\frac{2023-k}{2023}\right) = 1.$$

The first arguments cover $1/2023$ through $1011/2023$; their partners cover $2022/2023$ down to $1012/2023$. Thus every one of the 2022 terms is included exactly once, and there is no unpaired middle term. The sum is therefore

$$S = 1011 \times 1 = 1011.$$

As a check, its average term is $1/2$, consistent with the complementary pairs. The choices 2011 and 2010 are too large, and 1010 misses one complete pair.

Step 3: Conclusion The required sum is 1011. Hence, the correct option is (D).

Quick Tip to Solve: Before summing fractional arguments individually, test $f(x) + f(1-x)$; then count the complementary pairs carefully.

10. The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is : [4 marks]

(A) 125

(B) 150

(C) 225

(D) 120

Correct Answer: (D) 120

Solution:

Concept:

- With entries restricted to 0 and 1, a row sum of 1 forces exactly one entry in that row to be 1.
- The column condition requires the chosen columns of these row-wise 1s to be distinct. This is a permutation of the five columns.

Step 1: Translate both sum conditions into placements Every row contains five binary entries. Its sum cannot equal 1 if it has no 1 or if it has two or more 1s. Hence each of the five rows contains exactly one 1, and its other four entries are 0. Applying the same reasoning to columns, each column must also contain exactly one 1. Therefore two different rows cannot select the same column: that column would then have sum at least 2. Once the locations of the five 1s are fixed, all remaining entries are forced, so there is no further choice of values.

Step 2: Count the admissible column choices Choose the column containing the 1 in the first row in five ways. For the second row only four columns remain available. The next three rows then have three, two, and one choice respectively. By the multiplication principle,

$$N = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120.$$

Every such placement satisfies both conditions: each row has its assigned 1, and the distinct chosen columns exhaust all five columns. Conversely, any matrix satisfying the conditions produces exactly this ordered list of columns, so none is omitted or counted twice. The rows and columns are labelled positions; we do not divide by $5!$ again. The other proposed values, 125, 150, and 225, do not equal the number of bijections between the five rows and five columns.

Step 3: Conclusion There are 120 such matrices. Hence, the correct option is (D).

Quick Tip to Solve: A binary square matrix with every row sum and column sum equal to 1 is a permutation matrix; its count at order n is $n!$.

11. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to : [4 marks]

(A) 220

(B) 210

(C) 105

(D) 200

Correct Answer: (B) 210

Solution:

Concept:

- Write the six terms of an arithmetic progression as $a, a+d, \ldots, a+5d$. Their mean is $a + 5d/2$.
- The variance of six observations is $\sigma^2 = \frac{1}{6}\sum_{j=1}^{6}(a_j - \bar{a})^2$; the divisor is 6, not the sample divisor 5.

Step 1: Determine the first term and common difference From $a_1 + a_3 = 10$,

$$a + (a + 2d) = 10 \quad \Longrightarrow \quad a + d = 5.$$

The given mean supplies the independent relation

$$a + \frac{5}{2}d = \frac{19}{2}.$$

Subtracting the first relation from the second gives $3d/2 = 9/2$, so $d = 3$ and $a = 2$. Thus the actual observations are

$$2,\ 5,\ 8,\ 11,\ 14,\ 17.$$

Their sum is 57, and dividing by 6 confirms the mean $19/2$. This check also ensures the first condition was interpreted as the sum of the first and third terms.

Step 2: Find the squared deviations and variance Subtract $19/2$ from each observation. The deviations are

$$-\frac{15}{2}, -\frac{9}{2}, -\frac{3}{2}, \frac{3}{2}, \frac{9}{2}, \frac{15}{2}.$$

They occur in opposite pairs, so their squared sum is

$$\sum(a_j - \bar{a})^2 = \frac{2(225 + 81 + 9)}{4} = \frac{315}{2}.$$

Consequently,

$$\sigma^2 = \frac{1}{6} \cdot \frac{315}{2} = \frac{105}{4}, \qquad 8\sigma^2 = 210.$$

The choice 105 drops a factor of 2 at the final multiplication; 220 and 200 disagree with the exact squared-deviation total. No rounding is involved.

Step 3: Conclusion $8\sigma^2 = 210$. Hence, the correct option is (B).

Quick Tip to Solve: For an even-length arithmetic progression, pair the equal squared deviations on opposite sides of the mean before adding them.

12. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is [4 marks]

(A) $-\frac{1}{2}\left(1-i\sqrt{3}\right)$
(B) $-\frac{1}{2}\left(\sqrt{3}-i\right)$
(C) $\frac{1}{2}\left(1-i\sqrt{3}\right)$
(D) $\frac{1}{2}\left(\sqrt{3}+i\right)$

Correct Answer: (B) $-\frac{1}{2}\left(\sqrt{3}-i\right)$

Solution:

Concept:

- For real a, b, conjugates $a+ib$ and $a-ib$ have equal moduli; their quotient has modulus 1 and twice the numerator argument.
- De Moivre's formula gives $(\cos\phi+i\sin\phi)^n=\cos(n\phi)+i\sin(n\phi)$.

Step 1: Express the numerator with a simple argument Let $\theta=2\pi/9$ and set $\alpha=\pi/4-\theta/2=5\pi/36$. The half-angle identity gives

$$\frac{\cos\theta}{1+\sin\theta}=\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)=\tan\alpha.$$

Here $\theta=40^\circ$, so both the real part $1+\sin\theta$ and imaginary part $\cos\theta$ are positive. The numerator is therefore in the first quadrant with argument exactly α, with no additional multiple of π. Its conjugate in the denominator has argument $-\alpha$, and is nonzero.

Step 2: Cube the quotient and evaluate its argument Cancelling the equal nonzero moduli of the conjugate numerator and denominator, their ratio is

$$\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}=\cos(2\alpha)+i\sin(2\alpha).$$

Cubing gives angle $6\alpha=5\pi/6$, hence

$$z^3=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}=-\frac{\sqrt{3}}{2}+\frac{i}{2}=-\frac{1}{2}(\sqrt{3}-i).$$

The result has negative real part and positive imaginary part. This excludes C and D by their real parts. Choice A has the correct quadrant but interchanges the magnitudes of the real and imaginary parts, corresponding to a different angle.

Step 3: Conclusion The value is $-\frac{1}{2}(\sqrt{3}-i)$. Hence, the correct option is (B).

Quick Tip to Solve: For a quotient of complex conjugates, cancel the moduli first and double the argument; check the quadrant before multiplying the angle.

13. If $f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3), x \in \mathbb{R}$, then [4 marks]

(A) $2f(0) - f(1) + f(3) = f(2)$

(B) $f(1) + f(2) + f(3) = f(0)$

(C) $f(3) - f(2) = f(1)$

(D) $3f(1) + f(2) = f(3)$

Correct Answer: (A) $2f(0) - f(1) + f(3) = f(2)$

Solution:

Concept:

- Although derivatives of f occur in the defining formula, the evaluated quantities $f\prime(1), f''(2), f'''(3)$ are constants with respect to x.
- Treat these derivative values as unknown coefficients, differentiate the polynomial, and then impose their defining evaluation conditions.

Step 1: Determine the derivative coefficients Set $A = f\prime(1)$, $B = f''(2)$, and $C = f'''(3)$. The given identity is

$$f(x) = x^3 - Ax^2 + Bx - C.$$

Its derivatives are

$$f\prime(x) = 3x^2 - 2Ax + B, \quad f''(x) = 6x - 2A, \quad f'''(x) = 6.$$

Thus $C = 6$, $A = 3 - 2A + B$, and $B = 12 - 2A$. Substitution gives $3A = 15 - 2A$, hence $A = 3$ and $B = 6$. Therefore

$$f(x) = x^3 - 3x^2 + 6x - 6.$$

Step 2: Evaluate the function and test each relation Direct substitution gives

$$f(0) = -6, \quad f(1) = -2, \quad f(2) = 2, \quad f(3) = 12.$$

For choice A, the left side is

$$2f(0) - f(1) + f(3) = -12 + 2 + 12 = 2 = f(2),$$

so the relation holds. The left side of choice B is $-2 + 2 + 12 = 12$, whereas its right side is -6. Choice C asserts $12 - 2 = -2$, which is false. Choice D asserts $3(-2) + 2 = 12$, but the left side is -4. Checking all four rules out accidental agreement with an untested expression. A final coefficient check gives $f\prime(1) = 3$, $f''(2) = 6$, and $f'''(3) = 6$, as used above.

Step 3: Conclusion The true relation is $2f(0) - f(1) + f(3) = f(2)$. Hence, the correct option is (A).

Quick Tip to Solve: An evaluated derivative such as $f\prime(1)$ is a constant coefficient: assign it a letter before differentiating the identity.

14. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx$ **is equal to :** [4 marks]

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) 2π

Correct Answer: (D) 2π

Solution:

Concept:

- The substitution $x = (3/2)\sin t$ converts $\sqrt{9-4x^2}$ into $3\cos t$ on an interval where $\cos t > 0$.
- When changing variables in a definite integral, transform both endpoints and the differential, including every constant factor.

Step 1: Transform the radical and endpoints Choose $x = (3/2)\sin t$ on $-\pi/2 < t < \pi/2$. Then

$$dx = \frac{3}{2}\cos t\, dt, \qquad \sqrt{9-4x^2} = 3\cos t.$$

At the lower endpoint $x = 3\sqrt{2}/4$, $\sin t = \sqrt{2}/2$, hence $t = \pi/4$. At the upper endpoint $x = 3\sqrt{3}/4$, $\sin t = \sqrt{3}/2$, hence $t = \pi/3$. Both angles lie in the chosen interval and have positive cosine, so the square-root simplification involves no sign change. In particular, the denominator never vanishes along the interval of integration.

Step 2: Integrate with the correct constant multiplier After substitution, the cosine factors cancel and the integral is

$$I = \int_{\pi/4}^{\pi/3} \frac{48}{3\cos t}\left(\frac{3}{2}\cos t\right)dt = 24\int_{\pi/4}^{\pi/3} dt.$$

Therefore

$$I = 24\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = 24\cdot\frac{\pi}{12} = 2\pi.$$

Equivalently, the antiderivative is $24\arcsin(2x/3)$; differentiation supplies a useful check on the factor 24. The options $\pi/2$, $\pi/3$, and $\pi/6$ all miss that multiplier or the transformed endpoint difference. The integrand is positive throughout, consistent with the positive final value.

Step 3: Conclusion The definite integral equals 2π. Hence, the correct option is (D).

Quick Tip to Solve: For $\sqrt{a^2-b^2x^2}$, set $bx = a\sin t$ and explicitly write dx; losing its factor is the main source of errors.

15. If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x+2y+z=0$ and $3y-z=3$ is (α,β,γ), then $\alpha+\beta+\gamma$ is equal to : [4 marks]

(A) 3

(B) 1

(C) -1

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- A direction vector of a line parallel to two planes is perpendicular to both plane normals; their cross product supplies such a vector.
- For a perpendicular foot $F = A + t\mathbf{d}$ from P, use $(P-F)\cdot\mathbf{d}=0$, or $t=(P-A)\cdot\mathbf{d}/(\mathbf{d}\cdot\mathbf{d})$.

Step 1: Find the direction of the line The normals of the given planes are $\mathbf{n}_1 = (1,2,1)$ and $\mathbf{n}_2 = (0,3,-1)$. Their constant terms do not affect the direction. Hence choose

$$\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 = (-5,1,3).$$

Indeed, $-5+2+3=0$ and $3-3=0$, so this vector is parallel to both planes.

Step 2: Apply the perpendicularity condition Let $A=(3,2,1)$ and $P=(1,9,7)$. A general point of the line is $F=(3-5t,2+t,1+3t)$. Since PF must be perpendicular to the line,

$$t=\frac{(-2,7,6)\cdot(-5,1,3)}{25+1+9}=\frac{10+7+18}{35}=1.$$

Thus $F=(-2,3,4)$. Checking directly, $P-F=(3,6,3)$ has zero scalar product with $(-5,1,3)$.

Step 3: Add the coordinates The required sum is $\alpha+\beta+\gamma=-2+3+4=5$. The other proposed sums, 3, 1 and -1, do not equal the coordinate sum of the uniquely determined perpendicular foot.

Step 4: Conclusion The foot is $(-2,3,4)$, and $\alpha+\beta+\gamma=5$. Hence, the correct option is (D).

Quick Tip to Solve: Use the cross product for the line direction, then one dot product to locate the perpendicular foot.

16. Let A be a 3 × 3 matrix such that $|\mathrm{adj}(\mathrm{adj}(\mathrm{adj}\ \mathrm{A}))| = 12^4$. Then $|\mathrm{A}^{-1}\ \mathrm{adj}\ \mathrm{A}|$ is equal to [4 marks]

(A) 12

(B) $2\sqrt{3}$

(C) 1

(D) $\sqrt{6}$

Correct Answer: The determinant can be $2\sqrt{3}$ or $-2\sqrt{3}$; only the positive value is listed as (B).

Solution:

Concept:

- For a 3×3 matrix M, $\det(\mathrm{adj}\,M) = (\det M)^2$. Repeated adjugates therefore square the determinant at each stage.
- The determinant is multiplicative: $\det(A^{-1}\,\mathrm{adj}\,A) = \det(A)^{-1}\det(\mathrm{adj}\,A)$. Determinants can be negative; vertical determinant bars are not scalar absolute-value bars.

Step 1: Reduce the given determinant Put $d = \det A$. Three adjugations give

$$\det(\mathrm{adj}(\mathrm{adj}(\mathrm{adj}\,A))) = d^8 = 12^4.$$

Since the right side is nonzero, A is invertible. Taking the positive fourth root of $(d^2)^4 = 12^4$ gives $d^2 = 12$, so $d = \pm2\sqrt{3}$. The even power does not determine the sign.

Step 2: Evaluate the requested expression and check uniqueness The requested determinant is

$$\det(A^{-1}\,\mathrm{adj}\,A) = \frac{d^2}{d} = d = \pm2\sqrt{3}.$$

Both signs are realizable: $A = \mathrm{diag}(2\sqrt{3}, 1, 1)$ and $A = \mathrm{diag}(-2\sqrt{3}, 1, 1)$ satisfy the given determinant condition. The latter gives $-2\sqrt{3}$, which is absent from the options. Among the listed positive values, only $2\sqrt{3}$ squares to 12; 12, 1 and $\sqrt{6}$ do not. Nevertheless, selecting the positive value alone would require an additional condition $\det A > 0$.

Step 3: Conclusion The literal conditions allow $\pm2\sqrt{3}$, so the value is not uniquely determined. The listed positive value is option (B), $2\sqrt{3}$, but the equally valid negative value is missing; this is a sign ambiguity in the printed question.

Quick Tip to Solve: Keep both signs when an even power fixes a determinant; matrix determinant bars do not assert positivity.

17. Let the plane containing the line of intersection of the planes

P_1 : $x + (\lambda + 4)y + z = 1$ **and**

P_2 : $2x + y + z = 2$

pass through the points (0, 1, 0) and (1, 0, 1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P_2 is : [4 marks]

(A) $2\sqrt{6}$

(B) $3\sqrt{6}$

(C) $4\sqrt{6}$

(D) $5\sqrt{6}$

Correct Answer: (B) $3\sqrt{6}$

Solution:

Concept:

- A plane through the intersection of $P_1 = 0$ and $P_2 = 0$ can be written as $P_1 + kP_2 = 0$, where k is a real parameter.
- The distance from (x_0, y_0, z_0) to $ax + by + cz + d = 0$ is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2 + b^2 + c^2}$.

Step 1: Use the two points to determine the parameters Write the required plane as

$$x + (\lambda + 4)y + z - 1 + k(2x + y + z - 2) = 0.$$

Substituting $(1, 0, 1)$ gives $1 + k = 0$, hence $k = -1$. Substituting $(0, 1, 0)$ gives $\lambda + 3 - k = 0$. Therefore $\lambda + 4 = 0$, so $\lambda = -4$. The resulting plane is $-x - y + 1 = 0$, which contains both specified points.

Step 2: Calculate the distance from the specified plane The point is now $(2\lambda, \lambda, -\lambda) = (-8, -4, 4)$. Its distance is required from P_2, not from the newly constructed plane. Using $P_2 : 2x + y + z - 2 = 0$,

$$d = \frac{|2(-8) - 4 + 4 - 2|}{\sqrt{2^2 + 1^2 + 1^2}} = \frac{18}{\sqrt{6}} = 3\sqrt{6}.$$

The numerator is 18 and the normal length is $\sqrt{6}$, ruling out the multiples $2\sqrt{6}, 4\sqrt{6}, 5\sqrt{6}$.

Step 3: Conclusion The required perpendicular distance from P_2 is $3\sqrt{6}$ units. Hence, the correct option is (B).

Quick Tip to Solve: Use the plane pencil to find the parameter, but apply the distance formula to the plane actually named.

18. The equations of the sides AB and AC of a triangle ABC are $(\lambda+1)x+\lambda y=4$ and $\lambda x+(1-\lambda)y+\lambda=0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2=6x$ in the first quadrant is : [4 marks]

(A) 4

(B) $2\sqrt{2}$

(C) 2

(D) $\sqrt{6}$

Correct Answer: (B) $2\sqrt{2}$

Solution:

Concept:

- An orthocentre lies on every altitude. Thus the line joining a vertex to the orthocentre is perpendicular to the opposite side.
- For $y^2=4ax$, a nonvertical tangent with slope $m\neq 0$ is $y=mx+a/m$, touching at $(a/m^2, 2a/m)$.

Step 1: Determine the vertex on the y-axis At the common vertex $A=(0,y_A)$, the two side equations imply $\lambda y_A=4$ and $(1-\lambda)y_A+\lambda=0$. Since $\lambda\neq 0$, eliminating y_A gives

$$4(1-\lambda)+\lambda^2=0, \qquad (\lambda-2)^2=0.$$

Therefore $\lambda=2$, $A=(0,2)$, $AB: 3x+2y=4$, and $AC: y=2x+2$.

Step 2: Find vertex C using its altitude The altitude from C passes through $H=(1,2)$ and is perpendicular to AB. Its slope is $2/3$, so it is $y-2=\frac{2}{3}(x-1)$. Intersecting this with AC gives

$$2x=\frac{2}{3}(x-1), \qquad x=-\frac{1}{2}, \qquad y=1.$$

Hence $C=(-\frac{1}{2},1)$.

Step 3: Select the tangent touching in the first quadrant Here $4a=6$, hence $a=3/2$. A tangent through C satisfies

$$1=-\frac{m}{2}+\frac{3}{2m}, \qquad m^2+2m-3=0.$$

Thus $m=1$ or -3. The first-quadrant contact requires positive $y=3/m$, so choose $m=1$. The contact is $T=(3/2,3)$, and

$$CT=\sqrt{(3/2+1/2)^2+(3-1)^2}=\sqrt{8}=2\sqrt{2}.$$

The negative-slope tangent touches below the x-axis. The other listed lengths do not equal $\sqrt{8}$.

Step 4: Conclusion The tangent segment to the first-quadrant part of the parabola has length $2\sqrt{2}$ units. Hence, the correct option is (B).

Quick Tip to Solve: The orthocentre locates C using an altitude; the contact point, rather than the slope alone, selects the quadrant.

19. Let p and q be two statements. Then $\sim (p \wedge (p \Rightarrow \sim q))$ is equivalent to [4 marks]

(A) $(\sim p) \vee q$

(B) $p \vee (p \wedge (\sim q))$

(C) $p \vee (p \wedge q)$

(D) $p \vee ((\sim p) \wedge q)$

Correct Answer: (A) $(\sim p) \vee q$

Solution:

Concept:

- An implication $p \Rightarrow r$ is equivalent to $(\sim p) \vee r$. De Morgan's laws interchange conjunction and disjunction under negation.
- Distributivity and complementarity give $p \wedge ((\sim p) \vee r) \equiv p \wedge r$, because $p \wedge (\sim p)$ is always false.

Step 1: Remove the implication and simplify the conjunction Here the consequent is $\sim q$. Therefore

$$p \Rightarrow \sim q \equiv (\sim p) \vee (\sim q).$$

Substituting into the inner expression,

$$\begin{aligned} p \wedge (p \Rightarrow \sim q) &\equiv p \wedge ((\sim p) \vee (\sim q)) \\ &\equiv (p \wedge \sim p) \vee (p \wedge \sim q) \\ &\equiv p \wedge \sim q. \end{aligned}$$

The first term vanishes because no statement and its negation can both be true.

Step 2: Negate the simplified expression and eliminate alternatives Apply De Morgan's law:

$$\sim (p \wedge \sim q) \equiv (\sim p) \vee \sim (\sim q) \equiv (\sim p) \vee q.$$

Options (B) and (C) simplify by absorption to p; option (D) simplifies to $p \vee q$. None is equivalent to the result. For example, when p and q are both false, the original expression is true, whereas each of those three alternatives is false. This single assignment independently rules all three out.

Step 3: Conclusion The equivalent statement is $(\sim p) \vee q$. Hence, the correct option is (A).

Quick Tip to Solve: First replace the implication, then simplify inside the outer negation; a truth assignment quickly rejects distractors.

20. If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \ldots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$ **then** α **is equal to :** [4 marks]

(A) 30

(B) 10

(C) 15

(D) 60

Correct Answer: (C) 15

Solution:

Concept:

- The binomial coefficients satisfy $\binom{n}{k} = \binom{n}{n-k}$. Pairing terms with complementary indices simplifies sums weighted by k.
- Vandermonde's identity gives $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$, by comparing the coefficient of x^n in $(1+x)^n(1+x)^n$.

Step 1: Pair the weighted sum with its reverse Denote the given sum by $S = \sum_{k=0}^{30} k\binom{30}{k}^2$. The added $k = 0$ term is zero, so the sum is unchanged. On replacing k with $30 - k$ and using binomial symmetry,

$$S = \sum_{k=0}^{30} (30-k)\binom{30}{k}^2.$$

Adding this equation to the original expression gives

$$2S = 30\sum_{k=0}^{30} \binom{30}{k}^2.$$

Thus the average weight is 15, not 30; forgetting to divide by two would double the result.

Step 2: Evaluate the unweighted sum The coefficient of x^{30} in $(1+x)^{30}(1+x)^{30} = (1+x)^{60}$ yields

$$\sum_{k=0}^{30} \binom{30}{k}\binom{30}{30-k} = \binom{60}{30} = \frac{60!}{(30!)^2}.$$

Since complementary binomial coefficients are equal,

$$S = 15\frac{60!}{(30!)^2}.$$

Comparison with the question gives $\alpha = 15$. The values 10 and 60 have no such coefficient identity, and 30 omits the factor of two in the pairing step.

Step 3: Conclusion The coefficient is $\alpha = 15$. Hence, the correct option is (C).

Quick Tip to Solve: For a sum of k times symmetric binomial squares, pair k with n-k to replace the weight by n/2.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is _____. [4 marks]

Solution:

Concept:

- An equivalence relation is reflexive, symmetric and transitive. Its equivalence classes partition the set, and every ordered pair within a class must belong to the relation.
- To obtain the smallest extension of a relation, merge exactly those elements that its existing pairs force into one equivalence class, then count the missing ordered pairs.

Step 1: Determine the equivalence classes forced by R The pair (a, b) forces a and b into the same class. The pairs (b, c) and (b, d) then force c and d into that same class as well. Thus all four elements belong to a single class. For instance symmetry supplies $(b, a), (c, b), (d, b)$, and transitivity then connects a to c and d and also connects c with d. No partition into smaller classes can contain the original three pairs.

Step 2: Count the required pairs and subtract the existing ones One class of four elements requires every ordered pair of the set with itself, including the four diagonal pairs. Its complete relation is $\{a, b, c, d\} \times \{a, b, c, d\}$, containing

$$4^2 = 16$$

distinct ordered pairs. The given three pairs are distinct and already present, so the number to add is

$$16 - 3 = 13.$$

This count is attainable: adding precisely the missing pairs produces the universal relation, which is reflexive, symmetric and transitive. It is minimal because any equivalence extension has already been forced to contain all 16 pairs.

Step 3: Conclusion The minimum number of ordered pairs to add is $\boxed{13}$. This is a dimensionless count of relation elements.

Quick Tip to Solve: Treat existing pairs as links between elements; each connected group must become a complete equivalence class.

22. If the shortest between the lines $\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5}$ is 6, then the square of sum of all possible values of λ is : [4 marks]

Solution:

Concept:

- The shortest distance between nonparallel lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$, with directions $\mathbf{b}_1, \mathbf{b}_2$, is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.

- The cross product is normal to both directions. Its scalar projection measures the separation perpendicular to both lines; the absolute value requires retaining both signs when solving for a parameter.

Step 1: Extract the points and directions The first line passes through $\mathbf{a}_1 = (-\sqrt{6}, \sqrt{6}, \sqrt{6})$, and the second passes through $\mathbf{a}_2 = (\lambda, 2\sqrt{6}, -2\sqrt{6})$. Their direction vectors are $(2, 3, 4)$ and $(3, 4, 5)$. Thus

$$\mathbf{b}_1 \times \mathbf{b}_2 = (-1, 2, -1), \qquad |\mathbf{b}_1 \times \mathbf{b}_2| = \sqrt{6}.$$

The nonzero cross product confirms that the nonparallel-line distance formula applies.

Step 2: Apply the prescribed distance The displacement between the chosen points is $(\lambda + \sqrt{6}, \sqrt{6}, -3\sqrt{6})$. Its dot product with the normal is

$$-(\lambda + \sqrt{6}) + 2\sqrt{6} + 3\sqrt{6} = -\lambda + 4\sqrt{6}.$$

The distance is given as 6, so

$$\frac{|4\sqrt{6} - \lambda|}{\sqrt{6}} = 6 \quad \Longrightarrow \quad \lambda = 4\sqrt{6} \pm 6\sqrt{6}.$$

Therefore the two values are $10\sqrt{6}$ and $-2\sqrt{6}$.

Step 3: Square the sum of the two values Both values satisfy the absolute-value equation, so neither may be discarded because of its sign. Their sum is $8\sqrt{6}$, giving

$$(\lambda_1 + \lambda_2)^2 = (8\sqrt{6})^2 = 64 \times 6 = 384.$$

The question asks for the square of the sum, rather than the sum of the squares.

Step 4: Conclusion The requested numerical value is $\boxed{384}$ (dimensionless).

Quick Tip to Solve: For a distance equation involving an absolute value, solve both signs before forming the sum of parameter values.

---

23. Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$, $\vec{a} \cdot \vec{c} = 7$, $2\vec{b} \cdot \vec{c} + 43 = 0$, $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$. Then $\left|\vec{a} \cdot \vec{b}\right|$ is equal to : [4 marks]

Solution:

Concept:

- Equality of the cross products gives $(\mathbf{a} - \mathbf{b}) \times \mathbf{c} = \mathbf{0}$. Since $\mathbf{c} \neq \mathbf{0}$, the vectors $\mathbf{c}$ and $\mathbf{a} - \mathbf{b}$ are parallel.
- Dot products can then be evaluated by writing $\mathbf{c} = k(\mathbf{a} - \mathbf{b})$, where k is a real scalar. The given scalar products determine λ^2.

Step 1: Express the unknown vector using the cross-product condition Here $\mathbf{a} - \mathbf{b} = (-2, 7, 2\lambda)$, which is nonzero for every real λ. Also $\mathbf{a} \cdot \mathbf{c} = 7$ rules out $\mathbf{c} = \mathbf{0}$. Therefore write $\mathbf{c} = k(\mathbf{a} - \mathbf{b})$.

Calculate

$$\mathbf{a}\cdot\mathbf{b} = 3 - 10 - \lambda^2 = -7 - \lambda^2,$$

$$|\mathbf{a}|^2 = 5 + \lambda^2, \qquad |\mathbf{b}|^2 = 34 + \lambda^2.$$

Step 2: Use the two prescribed scalar products Put $t = \lambda^2 \geq 0$. The first scalar-product condition becomes

$$k(12 + 2t) = 7.$$

The condition $2\mathbf{b}\cdot\mathbf{c} + 43 = 0$ gives $\mathbf{b}\cdot\mathbf{c} = -43/2$, hence

$$k(-41 - 2t) = -\frac{43}{2}.$$

Divide these equations; this is legitimate because their first right-hand side is nonzero. Thus

$$\frac{41 + 2t}{12 + 2t} = \frac{43}{14}.$$

Step 3: Solve for the squared parameter and take the magnitude Cross-multiplication gives

$$574 + 28t = 516 + 86t, \qquad 58t = 58, \qquad t = 1.$$

Consequently $\mathbf{a}\cdot\mathbf{b} = -7 - 1 = -8$. The requested vertical bars mean its absolute value, so the negative scalar product itself is not the final response. As a check, $k = 1/2$ satisfies both original dot-product conditions.

Step 4: Conclusion $|\mathbf{a}\cdot\mathbf{b}| = \boxed{8}$, a dimensionless numerical value.

Quick Tip to Solve: Equal cross products first give a parallel-vector relation; introduce one scalar multiple instead of three unknown components.

---

24. Let the sum of the coefficients of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n, x \neq 0$. $n \in \mathbb{N}$, be 376. Then the coefficient of x^4 is _____. [4 marks]

Solution:

Concept:

- The general term of $(x - 3/x^2)^n$, indexed from $r = 0$, is $\binom{n}{r}(-3)^r x^{n-3r}$. Each choice of the second term reduces the exponent of x by three.
- The first three coefficients, in expansion order, are $1, -3n, 9\binom{n}{2}$. Their signed sum determines the positive integer n.

Step 1: Determine the positive integer exponent The stated sum of the first three coefficients is

$$1 - 3n + \frac{9n(n-1)}{2} = 376.$$

Multiply by two and simplify to obtain

$$9n^2 - 15n - 750 = 0 \quad \Longrightarrow \quad 3n^2 - 5n - 250 = 0.$$

Factorising yields $(n-10)(3n+25) = 0$. Since $n \in \mathbb{N}$, the allowed root is $n = 10$; the negative root $-25/3$ cannot be the exponent in this finite binomial expansion.

Step 2: Locate the term whose power is four With $n = 10$, the general term is

$$T_{r+1} = \binom{10}{r}(-3)^r x^{10-3r}.$$

To obtain the requested x^4 term, equate the exponent to four:

$$10 - 3r = 4 \quad \Longrightarrow \quad r = 2.$$

This index lies between 0 and 10, so the term actually occurs. Notice that the exponent changes by three, rather than two, because replacing x by $-3/x^2$ changes the total power by -3.

Step 3: Evaluate its signed coefficient The coefficient at $r = 2$ is

$$\binom{10}{2}(-3)^2 = \frac{10 \times 9}{2} \times 9 = 45 \times 9 = 405.$$

Its sign is positive because the second-term selection occurs twice. A quick check on the condition is $1 - 30 + 405 = 376$, reproducing the supplied sum exactly.

Step 4: Conclusion The coefficient of x^4 is $\boxed{405}$ (dimensionless).

Quick Tip to Solve: Write the exponent as $n - 3r$ before finding the coefficient; the negative powers make the usual $n - r$ pattern misleading.

25. The equations of the sides AB, BC and CA of a triangle ABC are : $2x + y = 0, x + py = 21a, (a \pm 0)$ **and** $x - y = 3$ **respectively. Let P(2, a) be the centroid of** Δ**ABC. Then (BC)^2 is equal to _____.**

[4 marks]

Solution:

Concept:

- The centroid coordinates are the averages of the three vertex coordinates: $x_A + x_B + x_C = 3x_P$ and $y_A + y_B + y_C = 3y_P$.
- A vertex is the intersection of its two adjacent side lines. Once the endpoints of a side are known, its squared length is the sum of the squared coordinate differences.

Step 1: Find the common vertex of the two fixed sides The lines AB and CA are $2x + y = 0$ and $x - y = 3$. Substituting $y = -2x$ into the latter gives $3x = 3$, hence $A = (1, -2)$. Write $B = (u, -2u)$ and $C = (v, v - 3)$, using their respective fixed side equations. The centroid $(2, a)$ now gives

$$u + v = 5, \qquad -2u + v - 5 = 3a.$$

Substitute $v = 5 - u$ into the second equation to obtain $u = -a$ and $v = 5 + a$.

Step 2: Use the equation of the remaining side The preceding coordinates are

$$B = (-a, 2a), \qquad C = (5 + a, 2 + a).$$

Both lie on $x + py = 21a$. Substitution of B gives $-a + 2pa = 21a$. Since the problem excludes $a = 0$, division by a gives $p = 11$. Substitution of C then yields

$$5 + a + 11(2 + a) = 21a, \qquad 27 = 9a, \qquad a = 3.$$

Thus $B = (-3, 6)$ and $C = (8, 5)$.

Step 3: Compute and check the side length Using the distance formula without taking an unnecessary square root,

$$(BC)^2 = (8 + 3)^2 + (5 - 6)^2 = 121 + 1 = 122.$$

For verification, both B and C satisfy $x + 11y = 63$, and the average of A, B and C is $(2, 3)$, agreeing with the supplied centroid after $a = 3$. These checks also distinguish BC from the other two sides.

Step 4: Conclusion The required squared side length is $\boxed{122}$ square coordinate units.

Quick Tip to Solve: Use the centroid to express both unknown vertices in terms of a before substituting them into the variable side equation.

26. Let f be a differentiable function defined on $[0, \pi/2]$ such that $f(x) > 0$ and

$$f(x) + \int_0^x f(t)\sqrt{1 - (\log_e f(t))^2}\, dt = e, \qquad x \in [0, \pi/2].$$

Then $\left(6 \log_e f(\pi/6)\right)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The fundamental theorem of calculus turns the integral equation into a differential equation. Positivity permits the substitution $u(x) = \log_e f(x)$.
- The resulting equation is $u' = -\sqrt{1 - u^2}$, with $u(0) = 1$. At the endpoint $u = 1$, division by $\sqrt{1 - u^2}$ loses constant and delayed solutions; uniqueness must be checked.

Step 1: Differentiate and identify the conventional branch At $x = 0$, the integral is zero, so $f(0) = e$ and $u(0) = 1$. Differentiation and division by the positive $f(x)$ give

$$f'(x) + f(x)\sqrt{1 - (\log_e f(x))^2} = 0, \qquad u' = -\sqrt{1 - u^2}.$$

For the branch that begins decreasing immediately, $u(x) = \cos x$ on $[0, \pi/2]$. Indeed $-\sin x = -\sqrt{1 - \cos^2 x}$ there. This branch gives $36 \cos^2(\pi/6) = 27$, the official numerical key. The original integral is verified directly because $\sqrt{1 - \cos^2 t} = \sin t$ throughout the interval and $\int_0^x e^{\cos t} \sin t\, dt =$

$e - e^{\cos x}$.

Step 2: Check whether the printed assumptions force that branch They do not. The positive differentiable function $f(x) = e$ for every x in the stated interval also satisfies the original equation: $\log_e f = 1$, the entire integrand vanishes, and its left side is exactly e. For this valid solution,

$$(6 \log_e f(\pi/6))^2 = 6^2 = 36.$$

This is an exact counterexample to a uniquely determined value of 27; it does not involve a rounding or numerical approximation.

Step 3: Describe the family responsible for the ambiguity More generally, choose $c \in [0, \pi/2]$ and put $u_c(x) = 1$ for $x \le c$, and $u_c(x) = \cos(x - c)$ for $x \ge c$. The two values and first derivatives agree at c. Thus $f_c = e^{u_c}$ is positive and differentiable. Since $u_c' = -\sqrt{1 - u_c^2}$ on both pieces, f_c plus the printed integral has derivative zero and initial value e, proving the integral identity exactly.

Step 4: Conclusion The official key is $\boxed{27}$, obtained from $f(x) = e^{\cos x}$. However, the printed assumptions do not determine a unique answer: the equally valid $f(x) = e$ gives $\boxed{36}$. All these numerical values are dimensionless.

Quick Tip to Solve: Before separating variables at an initial endpoint where the square root vanishes, test constant solutions; division by that factor can remove them.

27. If the area of the region bounded by the curves $y^2 - 2y = -x, x + y = 0$ is A, then 8 A is equal to _____

[4 marks]

Solution:

Concept:

- When both boundaries are expressed as x in terms of y, horizontal strips give the enclosed area as $\int (x_{\text{right}} - x_{\text{left}})\, dy$.
- The parabola is $x = 2y - y^2$, while the line is $x = -y$. Their intersection ordinates set the integration limits; the sign of their difference identifies which curve is on the right.

Step 1: Find the two intersections Substitute the line equation into the parabola or directly equate their x-expressions:

$$2y - y^2 = -y \quad \Longrightarrow \quad 3y - y^2 = 0 \quad \Longrightarrow \quad y = 0, 3.$$

The corresponding points are $(0, 0)$ and $(-3, 3)$. These are the only intersection points, so the bounded region lies between the horizontal levels $y = 0$ and $y = 3$.

Step 2: Form the nonnegative strip width For $0 \le y \le 3$, the difference of the two abscissae is

$$(2y - y^2) - (-y) = 3y - y^2 = y(3 - y) \ge 0.$$

Therefore the parabola is the right boundary and the line is the left boundary. Integrating with respect to y avoids solving the parabola for two square-root branches in x, and it gives the entire bounded

region in one integral.

Step 3: Evaluate the area and the requested multiple The area is

$$A = \int_0^3 (3y - y^2)\, dy = \left[\frac{3}{2}y^2 - \frac{1}{3}y^3\right]_0^3 = \frac{27}{2} - 9 = \frac{9}{2}.$$

The question asks for $8A$, not A itself, so

$$8A = 8\left(\frac{9}{2}\right) = 36.$$

The positive answer agrees with integrating a nonnegative width. The intermediate value $9/2$ is the actual area of the enclosed region.

Step 4: Conclusion $A = 9/2$ square units, hence $\boxed{8A = 36}$ square units.

Quick Tip to Solve: For a sideways parabola and a line, horizontal strips often avoid splitting the area into multiple integrals.

28. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black; and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola, is : [4 marks]

Solution:

Concept:

- With equally likely urns, Bayes' theorem gives $P(C \mid R) = P(R \mid C)/(P(R \mid A) + P(R \mid B) + P(R \mid C))$, where R denotes drawing a red ball.
- In an equilateral triangle whose one vertex is the origin on $y^2 = \lambda x$, the other vertices must have equal distance from the origin. On this parabola that forces equal abscissae and opposite ordinates.

Step 1: Determine the red-ball probability in the third urn Put $r = \lambda/(\lambda + 4)$. The first two urns have red probabilities $4/10$ and $5/10$. Each urn is chosen with probability $1/3$, which cancels in the conditional ratio. Therefore

$$\frac{r}{4/10 + 5/10 + r} = \frac{2}{5}.$$

Solving gives $5r = 1.8 + 2r$, hence $r = 3/5$. Consequently

$$\frac{\lambda}{\lambda + 4} = \frac{3}{5} \quad \Longrightarrow \quad 5\lambda = 3\lambda + 12, \qquad \lambda = 6.$$

Step 2: Locate the other two vertices on the parabola Let the origin be O and the other vertices be P and Q. For a point (x, y) on the parabola, its squared distance from O is $x^2 + y^2 = x^2 + \lambda x$. This is strictly increasing for $x \geq 0$, since $\lambda = 6 > 0$. The equality OP=OQ thus forces both other vertices to have the same x-coordinate. Being distinct, they have opposite y-coordinates. Write $P = (h, k)$, $Q = (h, -k)$, with $h, k > 0$.

Step 3: Impose the equilateral condition and square the side Now $PQ = 2k$, while $OP^2 = h^2 + k^2$. Equating their squares gives $h^2 = 3k^2$. The parabola also gives $k^2 = \lambda h$. For the nondegenerate triangle, $h > 0$, so

$$h = 3\lambda, \qquad k^2 = 3\lambda^2.$$

The squared side length is therefore

$$PQ^2 = 4k^2 = 12\lambda^2 = 12(6)^2 = 432.$$

The equal-distance argument forces this nondegenerate triangle, so no larger competing equilateral triangle with the required origin vertex is omitted.

Step 4: Conclusion The requested squared side length is $\boxed{432}$ square coordinate units.

Quick Tip to Solve: Finish the Bayes calculation first, then use equal distances from the parabola vertex to establish the symmetry of the triangle.

29. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$**. Then** $\sum_{\theta\in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ **is equal to _____.**

[4 marks]

Solution:

Concept:

- For finite tangents, $\tan a + \tan b = \sin(a+b)/(\cos a \cos b)$. A zero numerator solves the equation only when both denominator factors are nonzero.
- The bound $-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$ limits the integer values arising from $\sin[\pi(\cos\theta + \sin\theta)] = 0$.

Step 1: Reduce the tangent equation The tangent functions in the question must both be defined. Thus we first require $\cos(\pi\cos\theta) \neq 0$ and $\cos(\pi\sin\theta) \neq 0$. Combining the tangents then gives

$$\sin[\pi(\cos\theta + \sin\theta)] = 0 \quad \Longrightarrow \quad \cos\theta + \sin\theta = k, \qquad k \in \mathbb{Z}.$$

The range bound restricts k to $-1, 0, 1$; values such as ± 2 are impossible.

Step 2: List and check every allowed angle For $k = 1$, the angles in the given half-open interval are $0, \pi/2$; for $k = -1$, they are $\pi, 3\pi/2$. For $k = 0$, they are $3\pi/4, 7\pi/4$. At the four axis angles the sine and cosine values are 0 or ± 1, so both tangents are finite. At the last two angles they are $\pm 1/\sqrt{2}$, not half-integers, so the tangents are again defined. No candidate is lost, and no extraneous pole has been included.

Step 3: Evaluate the sum over the set At each axis angle, $\sin^2(\theta + \pi/4) = 1/2$. At $3\pi/4$ and $7\pi/4$, the shifted angles are π and 2π, respectively, giving zero. Therefore

$$\sum_{\theta\in S} \sin^2(\theta + \pi/4) = 4\left(\frac{1}{2}\right) + 0 + 0 = 2.$$

Step 4: Conclusion The required numerical value is $\boxed{2}$, a dimensionless sum.

Quick Tip to Solve: Combine the tangents before solving, then check their poles. The range of sine plus cosine leaves only three integer cases.

30. If $\frac{1^3+2^3+3^3+\ldots \text{up to } n \text{ terms}}{1\cdot3+2\cdot5+3\cdot7+\ldots \text{up to } n \text{ terms}} = \frac{9}{5}$**, then the value of** n **is** [4 marks]

Solution:

Concept:

- The sum of the first n cubes is $[n(n+1)/2]^2$, where n is a positive integer.
- The denominator is $\sum_{r=1}^{n} r(2r+1) = 2\sum r^2 + \sum r$. Products such as $1 \cdot 3$ denote multiplication, not decimal numbers.

Step 1: Express the numerator and denominator The numerator contains the cubes of the consecutive integers from 1 through n. The denominator contains the terms $r(2r+1)$, so the standard finite-sum identities yield

$$N = \frac{n^2(n+1)^2}{4}, \qquad D = \frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2} = \frac{n(n+1)(4n+5)}{6}.$$

Since $n \geq 1$, both $n(n+1)$ and $4n+5$ are positive, making cancellation legitimate.

Step 2: Solve the resulting quadratic Substitute these expressions into the printed ratio and simplify:

$$\frac{N}{D} = \frac{3n(n+1)}{2(4n+5)} = \frac{9}{5}.$$

Cross-multiplication gives $15n(n+1) = 18(4n+5)$, and hence

$$5n^2 - 19n - 30 = 0, \qquad (n-5)(5n+6) = 0.$$

The algebraic roots are 5 and $-6/5$. The negative noninteger root cannot represent a number of terms, so only $n = 5$ is admissible.

Step 3: Check the admissible number of terms For five terms the cube sum is $15^2 = 225$. The denominator is

$$1\cdot3 + 2\cdot5 + 3\cdot7 + 4\cdot9 + 5\cdot11 = 3 + 10 + 21 + 36 + 55 = 125.$$

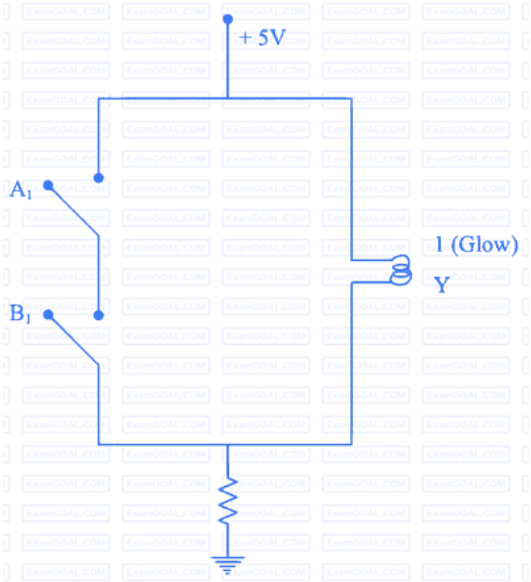
Their ratio is $225/125 = 9/5$, exactly as required. This check also confirms that the denominator pattern was interpreted correctly.

Step 4: Conclusion The number of terms is $\boxed{n = 5}$. A count of terms has no physical unit.

Quick Tip to Solve: Rewrite the denominator as a sum of quadratic polynomials before applying finite-sum formulae; reject roots that cannot count terms.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The logic gate equivalent to the given circuit diagram is : [4 marks]



(A) NOR

(B) OR

(C) NAND

(D) AND

Correct Answer: (C) NAND

Solution:

Concept:

- A closed switch represents input 1 and an open switch input 0. The stated output is 1 when the lamp glows and 0 when it does not.
- Two series switches form a conducting bypass only when both are closed. A NAND output is $Y = \overline{AB}$, the negation of the AND combination.

Step 1: Identify the role of the switch branch The two switches labelled A_1 and B_1 lie in series in the branch across the lamp. They do not lie in series with the lamp itself. If either switch is open, this bypass is interrupted, so current supplied through the external circuit passes through the lamp and it glows. If both switches close, they connect the two lamp terminals by an ideal conducting path; its potential difference then vanishes and it stops glowing. The resistor limits the current in this switched circuit.

Step 2: Construct the input-output table The four combinations are therefore

A	B	Y	lamp state
0	0	1	glowing
0	1	1	glowing
1	0	1	glowing
1	1	0	off

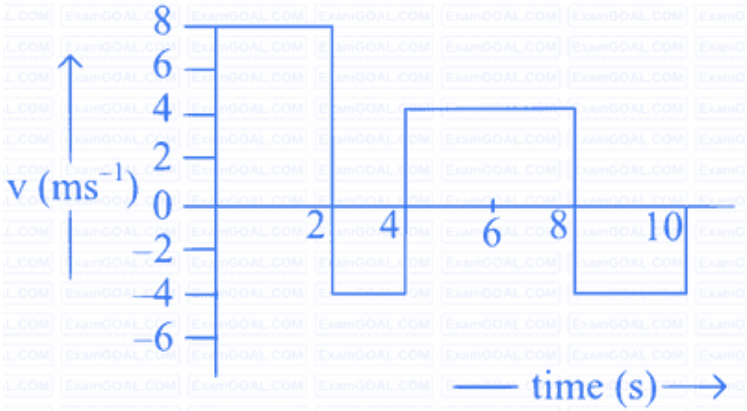
Only the simultaneous input $A = B = 1$ makes the output zero. In Boolean form this reads $Y = \overline{AB}$.

Step 3: Compare all four named gates A NAND gate has precisely the table just obtained. An AND gate instead gives output 1 only for $(1, 1)$, which reverses the observed lamp states. An OR gate gives output 0 at $(0, 0)$, although the lamp here glows then. A NOR gate also gives 0 for either mixed input, whereas the lamp glows whenever one switch remains open. Thus none of those three alternatives reproduces the full circuit behaviour.

Step 4: Conclusion The equivalent gate is $\boxed{\text{NAND}}$, with $Y = \overline{AB}$. Hence, the correct option is (C).

Quick Tip to Solve: First decide whether the switches feed the lamp or short it. A series bypass across a lamp gives NAND rather than AND behaviour.

32. The velocity time graph of a body moving in a straight line is shown in the figure.
The ratio of displacement to distance travelled by the body in time 0 to 10s is : [4 marks]



(A) 1 : 1
(B) 1 : 3
(C) 1 : 4
(D) 1 : 2

Correct Answer: (B) 1 : 3

Solution:

Concept:

- Displacement is the signed area under a velocity-time graph: $\Delta x = \int v\, dt$. Portions below the time axis contribute negatively.

- Distance travelled is $\int |v|\,dt$. Every rectangular area contributes its positive magnitude, even when the body moves backwards.

Step 1: Read the constant-velocity intervals The graph gives $8\,\mathrm{m\,s^{-1}}$ from 0 to 2 s, $-4\,\mathrm{m\,s^{-1}}$ from 2 to 4 s, $4\,\mathrm{m\,s^{-1}}$ from 4 to 8 s, and $-4\,\mathrm{m\,s^{-1}}$ from 8 to 10 s. The vertical jumps have zero time width, so they contribute no additional area. Notice that the positive middle interval lasts four seconds, whereas each of the other intervals lasts two seconds.

Step 2: Compute signed and unsigned areas The successive signed displacements are

$$8(2) = 16\,\mathrm{m}, \quad -4(2) = -8\,\mathrm{m}, \quad 4(4) = 16\,\mathrm{m}, \quad -4(2) = -8\,\mathrm{m}.$$

Adding with signs gives

$$\Delta x = 16 - 8 + 16 - 8 = 16\,\mathrm{m}.$$

For the distance, add their magnitudes instead:

$$s = 16 + 8 + 16 + 8 = 48\,\mathrm{m}.$$

Step 3: Reduce the ratio and compare the choices Both quantities refer to the same complete ten-second interval, so their ratio is

$$\Delta x : s = 16 : 48 = 1 : 3.$$

The ratio cannot be $1 : 1$, since the body reverses direction and loses net displacement while still covering distance. The alternatives $1 : 4$ and $1 : 2$ disagree with the measured rectangular areas. As a check, the positive area totals 32 m and the negative area has magnitude 16 m, giving the same signed total 16 m and distance 48 m.

Step 4: Conclusion The displacement-to-distance ratio is $\boxed{1 : 3}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the positive area P and the magnitude N below the axis; the required ratio is (P minus N) to (P plus N).

33. A long solenoid is formed by winding 70 turns cm^{-1}. If 2.0 A current flows, then the magnetic field produced inside the solenoid is _____ ($\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}) [4 marks]

(A) 88×10^{-4} T

(B) 1232×10^{-4} T

(C) 176×10^{-4} T

(D) 352×10^{-4} T

Correct Answer: (C) 176×10^{-4} T

Solution:

Concept:

- For an ideal long air-core solenoid, the magnetic field well inside it is $B = \mu_0 n I$, where n is turns per unit length and I is the current.
- The SI value of permeability supplied in the question requires n in m^{-1}, not turns per centimetre. The number of turns itself is dimensionless.

Step 1: Convert the winding density There are 70 turns in each centimetre of axial length. Since a metre contains 100 centimetres, the turns per metre are

$$n = 70 \times 100 = 7000\,\mathrm{m}^{-1}.$$

Equivalently, dividing 70 turns by $0.01\,\mathrm{m}$ gives the same density. The word long allows the standard uniform interior-field approximation; no radius or total winding length is required.

Step 2: Substitute the current and permeability Use the current $I = 2.0\,\mathrm{A}$ and the specified value of μ_0:

$$\begin{aligned} B &= (4\pi \times 10^{-7})(7000)(2.0)\,\mathrm{T} \\ &= 56000\pi \times 10^{-7}\,\mathrm{T} \\ &= 56\pi \times 10^{-4}\,\mathrm{T}. \end{aligned}$$

Taking $\pi \simeq 22/7$, as reflected by the rounded answer choices, gives $56\pi = 176$. Thus $B \simeq 176 \times 10^{-4}\,\mathrm{T} = 1.76 \times 10^{-2}\,\mathrm{T}$.

Step 3: Check the scale against the options The permeability has units $\mathrm{T\,m\,A}^{-1}$; multiplying by m^{-1} and amperes leaves tesla, as required. Option A is half the calculated field and would result from using 1 A rather than 2 A. Options B and D are respectively seven and two times the calculated value, so they do not follow from the specified winding density and current. Only option C matches both the coefficient and the power of ten.

Step 4: Conclusion The interior magnetic field is approximately $\boxed{176 \times 10^{-4}\,\mathrm{T}}$. Hence, the correct option is (C).

Quick Tip to Solve: Always change turns per centimetre into turns per metre before using permeability in SI units; multiply the stated density by 100.

34. A cell of emf 90 V is connected across series combination of two resistors each of 100Ω resistance. A voltmeter of resistance 400Ω is used to measure the potential difference across each resistor. The reading of the voltmeter will be : [4 marks]

(A) 40 V

(B) 90 V

(C) 45 V

(D) 80 V

Correct Answer: (A) 40 V

Solution:

Concept:

- A voltmeter is connected in parallel with the resistor whose voltage it measures. A finite voltmeter resistance changes that branch resistance and therefore changes the circuit current.
- For parallel resistances, $R_p = R_1R_2/(R_1 + R_2)$. Series branches share current, so the voltage division must use the loaded branch resistance.

Step 1: Find the resistance of the measured branch When the 400-ohm voltmeter is connected across either 100-ohm resistor, the pair is a parallel combination:

$$R_p = \frac{100 \times 400}{100 + 400} = 80\,\Omega.$$

The other 100-ohm resistor remains in series with this loaded branch. Because the original resistors are identical, moving the same voltmeter from one to the other gives an equivalent circuit and therefore the same reading. The question concerns one measurement at a time.

Step 2: Calculate the current and voltmeter reading The total circuit resistance during measurement is

$$R_{\text{total}} = 100 + 80 = 180\,\Omega.$$

With the given ideal 90 V source, the current supplied to the series combination is

$$I = \frac{90}{180} = 0.50\,\text{A}.$$

The voltmeter measures the potential difference across the 80-ohm equivalent branch, which is

$$V_{\text{meter}} = IR_p = 0.50 \times 80 = 40\,\text{V}.$$

The unmeasured resistor has a 50 V drop, and these drops add to the source emf of 90 V.

Step 3: Reject the unloaded-circuit answers The value 45 V would apply if an ideal voltmeter of infinite resistance were used; it ignores the specified loading. A 90 V reading would require connection across the full source rather than a single resistor. The 80 V choice mistakes the branch resistance value for a voltage and does not satisfy the voltage-divider calculation. The parallel meter current is $40/400 = 0.10\,\text{A}$, and the resistor current is $40/100 = 0.40\,\text{A}$; their sum checks the 0.50 A supply current.

Step 4: Conclusion The voltmeter reads $\boxed{40\,\text{V}}$ across either resistor when measured separately.

Hence, the correct option is (A).

Quick Tip to Solve: Replace the resistor and finite-resistance voltmeter by their parallel equivalent before applying the series voltage divider.

35. An α-particle, a proton and an electron have the same kinetic energy. Which one of the following is correct in case of their de-Broglie wavelength: [4 marks]

(A) $\lambda_\alpha > \lambda_p < \lambda_e$

(B) $\lambda_\alpha > \lambda_p > \lambda_e$

(C) $\lambda_\alpha = \lambda_p = \lambda_e$

(D) $\lambda_\alpha < \lambda_p < \lambda_e$

Correct Answer: (D) $\lambda_\alpha < \lambda_p < \lambda_e$

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is momentum.
- For nonrelativistic kinetic energy, $K = p^2/(2m)$, so at the same kinetic energy $\lambda = h/\sqrt{2mK} \propto m^{-1/2}$.

Step 1: Relate the wavelength to the mass Equal kinetic energies do not imply equal speeds or equal momenta. Eliminating the speed from $K = mv^2/2$ and $p = mv$ gives

$$p = \sqrt{2mK}, \qquad \lambda = \frac{h}{\sqrt{2mK}}.$$

The common factors h and $\sqrt{2K}$ are the same for all three particles, leaving the inverse square-root dependence on mass. Therefore the heavier particle has the shorter wavelength, even though all have the same kinetic energy.

Step 2: Order the particle masses and wavelengths An alpha particle contains two protons and two neutrons, so its mass is approximately four proton masses. A proton is in turn much heavier than an electron. Consequently,

$$m_\alpha > m_p > m_e \quad \Longrightarrow \quad \lambda_\alpha < \lambda_p < \lambda_e.$$

For a quantitative check, $\lambda_\alpha/\lambda_p \simeq 1/2$, whereas $\lambda_e/\lambda_p = \sqrt{m_p/m_e} \simeq \sqrt{1836}$, a value much greater than one.

Step 3: Compare the complete inequality with every choice Option A already fails its first comparison, since it makes the alpha wavelength larger than the proton wavelength. Option B reverses the mass-based wavelength order entirely. Option C would follow from equal momenta, not equal kinetic energies for unequal masses. Only option D places the alpha particle first with the shortest wavelength and the electron last with the longest. Electric charge does not enter this comparison because no accelerating voltage is prescribed; the given common quantity is kinetic energy.

Step 4: Conclusion The correct order is $\boxed{\lambda_\alpha < \lambda_p < \lambda_e}$. Hence, the correct option is (D).

Quick Tip to Solve: Identify what is held equal: equal energy gives wavelength inversely proportional to square root of mass, whereas equal momentum gives equal wavelengths.

36. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : A pendulum clock when taken to Mount Everest becomes fast.

Reason R : The value of g (acceleration due to gravity) is less at Mount Everest than its value on the surface of earth.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) Both A and R are correct but R is NOT the correct explanation of A

(B) A is correct but R is not correct

(C) Both A and R are correct and R is the correct explanation of A

(D) A is not correct but R is correct

Correct Answer: (D) A is not correct but R is correct

Solution:

Concept:

- The period of a simple pendulum for small oscillations is $T = 2\pi\sqrt{l/g}$, where l is its effective length and g the local gravitational acceleration.
- For a fixed clock mechanism, the indicated time depends on the number of oscillations completed. A larger actual period means fewer oscillations per real second, so the clock loses time.

Step 1: Evaluate the assertion Taking the clock to a higher altitude reduces the gravitational acceleration while its pendulum length is unchanged. From the period formula, $T \propto g^{-1/2}$. Hence its period increases; each swing takes longer and the clock becomes slow. Assertion A says it becomes fast, so A is false.

Step 2: Evaluate the reason At height h above the surface, neglecting the small corrections due to rotation and local geology, the acceleration is

$$g_h = \frac{GM}{(R+h)^2} = g_0\left(\frac{R}{R+h}\right)^2.$$

Since $R + h > R$, we have $g_h < g_0$. Thus the reason correctly states the decrease of gravity at Mount Everest.

Step 3: Match the truth values The reason predicts a longer pendulum period rather than the faster clock claimed in the assertion. Therefore the assertion is not correct but the reason is correct. Options (A) and (C) incorrectly declare both statements true, while option (B) reverses the truth values.

Step 4: Conclusion The clock loses time at Mount Everest: A is false and R is true. Hence, the correct option is (D).

Quick Tip to Solve: For an unchanged pendulum, lower g means a longer period and a slow clock; distinguish the period of one swing from the clock reading.

37. Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ_2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio, $\frac{\gamma_1}{\gamma_2}$ is : [4 marks]

(A) $\frac{35}{27}$
(B) $\frac{25}{21}$
(C) $\frac{21}{25}$
(D) $\frac{27}{35}$

Correct Answer: (B) $\frac{25}{21}$

Solution:

Concept:

- For an ideal gas with f active quadratic degrees of freedom, equipartition gives $C_V = fR/2$ per mole.
- Mayer's relation is $C_P - C_V = R$, so $\gamma = C_P/C_V = (f+2)/f$. A rigid diatomic molecule has rotational but no vibrational energy.

Step 1: Calculate the monoatomic ratio A monoatomic gas has three translational degrees of freedom, one along each spatial axis, and no molecular rotations that store energy in this model. Therefore

$$C_{V1} = \frac{3R}{2}, \qquad C_{P1} = \frac{5R}{2}, \qquad \gamma_1 = \frac{5}{3}.$$

The same molar gas constant occurs in numerator and denominator and cancels.

Step 2: Calculate the diatomic ratio For the diatomic gas treated as a rigid rotator, there are three translational and two rotational degrees of freedom. Rotation about the internuclear axis is neglected and vibration is excluded by the stated rigid approximation. Thus

$$f_2 = 5, \qquad C_{V2} = \frac{5R}{2}, \qquad C_{P2} = \frac{7R}{2}, \qquad \gamma_2 = \frac{7}{5}.$$

Step 3: Divide in the specified order The question asks for the monoatomic ratio divided by the diatomic ratio, not its reciprocal:

$$\frac{\gamma_1}{\gamma_2} = \frac{5/3}{7/5} = \frac{5}{3} \times \frac{5}{7} = \frac{25}{21}.$$

Option (C) gives the reciprocal. Options (A) and (D) do not follow from the stipulated degree-of-freedom counts. The answer must exceed one since $5/3 > 7/5$.

Step 4: Conclusion The required dimensionless ratio is $\gamma_1/\gamma_2 = 25/21$. Hence, the correct option is (B).

Quick Tip to Solve: Write $\gamma = 1 + 2/f$ first: use $f = 3$ for a monoatomic gas and $f = 5$ for a rigid diatomic gas, then preserve the requested ratio order.

38. When a beam of white light is allowed to pass through convex lens parallel to principal axis, the different colours of light converge at different point on the principle axis after refraction. This is called : [4 marks]

(A) Spherical aberration

(B) Scattering

(C) Polarisation

(D) Chromatic aberration

Correct Answer: (D) Chromatic aberration

Solution:

Concept:

- The refractive index of the lens material depends on wavelength. This dispersion makes a lens refract different colours by different amounts.
- For a thin lens in air, the lens-maker relation is $1/f = (n-1)(1/R_1 - 1/R_2)$, where n is the refractive index and the radii describe its fixed surfaces.

Step 1: Identify what changes between the rays The incident beam is white light, so it contains many wavelengths. Even though the rays enter parallel to the same principal axis, the glass has a different refractive index for each wavelength. Consequently the lens power depends on colour and each colour has its own focal length. Under normal dispersion violet light is refracted more strongly and comes to a focus closer to a convex lens than red light.

Step 2: Name the resulting defect The image-forming defect caused by different colours having different focal positions is chromatic aberration. The separation is along the principal axis in the situation described, so it is specifically longitudinal chromatic aberration. The defining observation is the dependence on colour, rather than merely the shape of the lens surface.

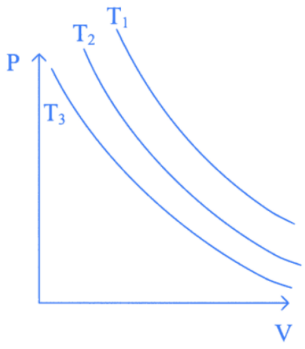
Step 3: Exclude the other optical effects Spherical aberration concerns rays of one wavelength at different distances from the principal axis focusing differently because of spherical surfaces. Scattering redirects light through interactions with particles or inhomogeneities; it does not describe the distinct lens foci here. Polarisation restricts the direction of transverse electric-field oscillations and likewise does not account for the colour-dependent focal length.

Step 4: Conclusion The different focal positions of different colours constitute chromatic aberration. Hence, the correct option is (D).

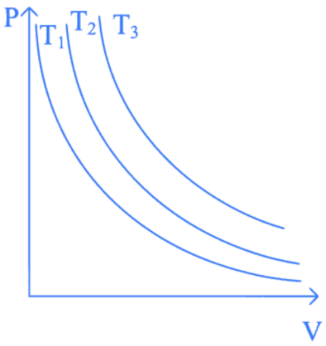
Quick Tip to Solve: When the focus changes with colour, think chromatic aberration; when it changes with distance from the axis for one colour, think spherical aberration.

39. In an Isothermal change, the change in pressure and volume of a gas can be represented for three different temperature; $T_3 > T_2 > T_1$ as : [4 marks]

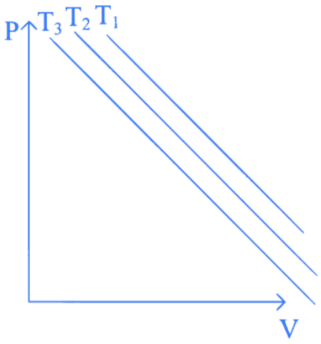
(A)



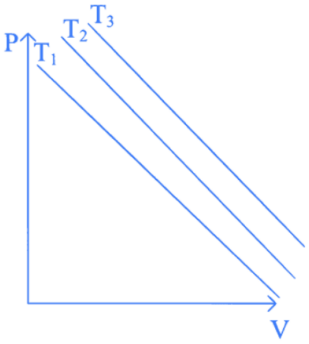
(B)



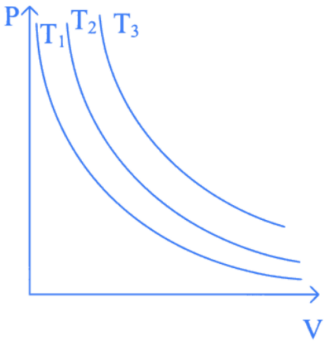
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- For a fixed amount n of ideal gas, $PV = nRT$. Along an isotherm the absolute temperature T is constant.
- On a pressure-versus-volume diagram, $P = nRT/V$; hence each isotherm is a decreasing rectangular hyperbola, not a straight line.

Step 1: Determine the shape of each isotherm Keeping T fixed makes the product PV constant. Increasing the volume decreases the pressure inversely. In particular, doubling the volume halves the pressure, so the curve is not a line with a constant negative slope. Its slope and curvature are

$$\frac{dP}{dV} = -\frac{nRT}{V^2} < 0, \qquad \frac{d^2P}{dV^2} = \frac{2nRT}{V^3} > 0.$$

Thus the required curves decrease and flatten as volume increases.

Step 2: Order the curves by temperature Compare the gases at the same positive volume. Since $P = nRT/V$, a larger temperature gives a larger pressure:

$$T_3 > T_2 > T_1 \quad \Longrightarrow \quad P_3 > P_2 > P_1.$$

Equivalently, at a common pressure, $V_3 > V_2 > V_1$. Therefore the hottest isotherm lies above and to the right of the colder isotherms.

Step 3: Check all four graphs Graph (B) has the hyperbolic shape and places T_3 on the outermost curve, followed by T_2 and T_1. Graph (A) reverses the temperature ordering. Graphs (C) and (D) are straight lines, which contradict $PV =$ constant even if the labels of one graph happen to have the correct order.

Step 4: Conclusion Graph (B) correctly represents the three isotherms with $T_3 > T_2 > T_1$. Hence, the correct option is (B).

Quick Tip to Solve: Apply two tests to an isotherm graph: it must be hyperbolic, and at the same volume the hotter gas must have the greater pressure.

40. The electric field and magnetic field components of an electromagnetic wave going through vacuum is described by

$$\mathrm{E_x = E_o \sin(kz - \omega t)}$$

$$\mathrm{B_y = B_o \sin(kz - \omega t)}$$

Then the correct relation between E_0 and B_0 is given by [4 marks]

(A) $\mathrm{E_o B_o = \omega k}$

(B) $\mathrm{E_0 = kB_0}$

(C) $\mathrm{kE_0 = \omega B_0}$

(D) $\mathrm{\omega E_0 = kB_0}$

Correct Answer: (C) $\mathrm{kE_0 = \omega B_0}$

Solution:

Concept:

- A plane electromagnetic wave in vacuum has phase speed $c = \omega/k$, where ω is angular frequency and k is angular wave number.
- The transverse electric and magnetic fields are perpendicular, in phase, and have amplitudes related by $E_0 = cB_0$. Their amplitude ratio has the dimensions of speed.

Step 1: Use the wave phase to obtain the speed For a point of fixed phase, $kz - \omega t$ is constant. Differentiating gives $k\,dz/dt - \omega = 0$, so the phase travels in the positive z direction with speed ω/k. Since the wave propagates through vacuum, this is the speed of light c.

Step 2: Relate the field amplitudes Faraday's law relates the spatial variation of the electric field to the time variation of the magnetic field. For the given components its relevant equation is

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}.$$

Substituting the sinusoidal fields gives

$$kE_0 \cos(kz - \omega t) = \omega B_0 \cos(kz - \omega t).$$

Equality at every position and time requires equality of the amplitudes of these cosine terms.

Step 3: Rearrange and compare the options The resulting relation is

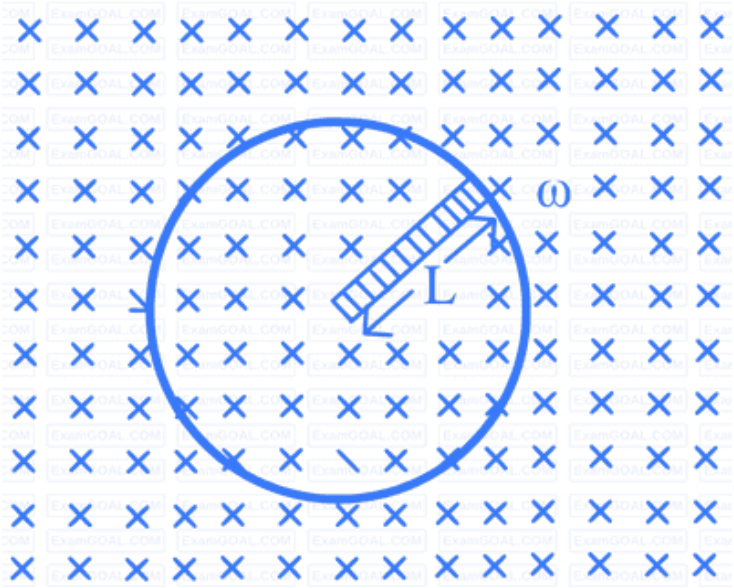
$$kE_0 = \omega B_0, \qquad \frac{E_0}{B_0} = \frac{\omega}{k} = c.$$

Option (D) interchanges ω and k and would give the inverse speed. Option (B) omits the frequency factor, while option (A) gives a product of amplitudes instead of the fixed amplitude ratio. Neither satisfies the wave equation above.

Step 4: Conclusion The amplitudes satisfy $kE_0 = \omega B_0$. Hence, the correct option is (C).

Quick Tip to Solve: Remember $E_0/B_0 = c = \omega/k$; cross-multiplication immediately gives the correct arrangement of the wave number and angular frequency.

41. A metallic rod of length 'L' is rotated with an angular speed of 'ω' normal to a uniform magnetic field 'B' about an axis passing through one end of rod as shown in figure. The induced emf will be : [4 marks]



(A) $\frac{1}{2}B^2L^2\omega$

(B) $\frac{1}{2}BL^2\omega$

(C) $\frac{1}{4}BL^2\omega$

(D) $\frac{1}{4}B^2L\omega$

Correct Answer: (B) $\frac{1}{2}BL^2\omega$

Solution:

Concept:

- A conductor moving through a magnetic field develops a motional emf. An element dr moving perpendicular to the field contributes $d\mathcal{E} = Bv\,dr$ in magnitude.
- For a rigid rod rotating with angular speed ω about one end, a point at distance r from the pivot has tangential speed $v(r) = \omega r$.

Step 1: Choose an element along the rod The field is normal to the plane of rotation, so the local velocity is perpendicular to the field. Consider an element of rod of length dr, located a distance r from the fixed end. Its velocity has magnitude ωr, and the magnetic force per unit charge has magnitude $B\omega r$ along the rod. The elemental potential difference is therefore

$$d\mathcal{E} = B\omega r\,dr.$$

Step 2: Add the contributions from pivot to tip The pivot is at $r = 0$ and the free end at $r = L$. Adding the elemental emfs gives

$$\mathcal{E} = \int_0^L B\omega r\,dr = B\omega\left[\frac{r^2}{2}\right]_0^L = \frac{1}{2}B\omega L^2.$$

Using the tip speed for the entire rod would incorrectly double the answer because the speed grows continuously from zero to ωL.

Step 3: Check the alternatives and dimensions The result is linear in the magnetic field, so the expressions involving B^2 in (A) and (D) cannot be correct. The integral fixes the numerical coefficient as $1/2$, excluding (C). Also $B\omega L^2$ has units of tesla times square metre per second, which are volts, as required for an emf.

Step 4: Conclusion The induced emf between the ends has magnitude $\mathcal{E} = \frac{1}{2}BL^2\omega$ volts when SI quantities are used. Hence, the correct option is (B).

Quick Tip to Solve: For a rod rotating about one end, integrate $B\omega r\, dr$, or use the mean speed $\omega L/2$; using the tip speed throughout loses the factor of two.

---

42. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Steel is used in the construction of buildings and bridges.

Reason R : Steel is more elastic and its elastic limit is high.

In the light of above statements, choose the most appropriate answer from the options given below

[4 marks]

(A) Both A and R are correct and R is the correct explanation of A

(B) A is correct but R is not correct

(C) Both A and R are correct but R is NOT the correct explanation of A

(D) A is not correct but R is correct

Correct Answer: (A) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- Young's modulus is $Y =$ longitudinal stress/longitudinal strain within the elastic range. A larger modulus means less strain under the same stress.
- The elastic limit is the greatest stress up to which a body returns to its original shape after removal of the load. Elasticity in this sense differs from ease of stretching.

Step 1: Evaluate the assertion Steel is a standard structural material in buildings and bridges. Members such as beams and girders must carry substantial loads while maintaining their designed shape. The assertion describes this established use and is correct. The question concerns the physical properties that make the material suitable, rather than a claim that it is the only material used.

Step 2: Evaluate the reason Steel has a large Young's modulus and a high elastic limit. For a specified longitudinal stress σ, its strain is

$$\varepsilon = \frac{\sigma}{Y}.$$

Thus a large Y gives small deformation, and a high elastic limit allows the material to withstand substantial stress while remaining able to recover its shape. In the textbook meaning of elasticity, steel is more elastic than a material such as rubber despite rubber stretching more easily.

Step 3: Test the explanatory connection Small deformation preserves the dimensions of the structure during use, while recovery after the load is removed avoids permanent distortion within the elastic range. These properties directly explain the suitability stated in A, so R is correct and explains A. Option (B) incorrectly rejects R, option (D) incorrectly rejects A, and option (C) incorrectly denies their causal connection.

Step 4: Conclusion Both statements are correct, and the high elasticity and elastic limit explain the structural use of steel. Hence, the correct option is (A).

Quick Tip to Solve: In elasticity questions, a material that strains less for the same stress has the larger modulus; being easy to stretch does not mean being more elastic.

---

43. If the distance of the earth from Sun is 1.5×10^6 km. Then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years is : [4 marks]

(A) 6×10^6 km

(B) 3×10^7 km

(C) 6×10^7 km

(D) 3×10^6 km

Correct Answer: (D) 3×10^6 km

Solution:

Concept:

- Kepler's third law for bodies orbiting the same Sun is $T^2 \propto r^3$, where T is orbital period and r is the orbital radius or semimajor axis.
- A ratio to Earth's orbit removes the common factor involving the Sun's mass: $r_p/r_e = (T_p/T_e)^{2/3}$. Use the numerical Earth–Sun distance supplied in the question.

Step 1: Form the period ratio For Earth, the orbital period is one year. The planet's period is given as 2.83 years. Since both periods use years, no conversion to seconds is necessary when taking their ratio. Thus

$$\left(\frac{r_p}{r_e}\right)^3 = \left(\frac{T_p}{T_e}\right)^2 = (2.83)^2 = 8.0089.$$

The common central body is essential: the proportionality constant is the same because both bodies revolve about the Sun.

Step 2: Calculate the distance and compare the options Taking the cube root gives

$$\frac{r_p}{r_e} = (8.0089)^{1/3} \simeq 2.0007.$$

With the printed value $r_e = 1.5 \times 10^6\,\text{km}$,

$$r_p \simeq 2.0007(1.5 \times 10^6) = 3.0011 \times 10^6\,\text{km}.$$

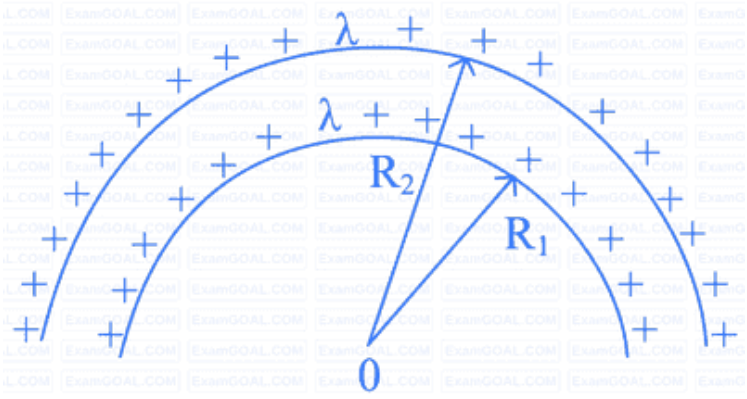
At the precision of the options this is $3 \times 10^6\,\text{km}$. Option A would mean four times Earth's radius and hence a period of eight years. Options B and C correspond to radius ratios twenty and forty,

respectively, and cannot give the stated period. The distance grows with period, but not linearly; the power is $2/3$.

Step 3: Conclusion The planet is approximately $3 \times 10^6\,\text{km}$ from the Sun using the distance specified. Hence, the correct option is (D).

Quick Tip to Solve: Recognise $2.83 \simeq \sqrt{8}$: squaring the period ratio gives eight, whose cube root is two.

44. The electric potential at the centre of two concentric half rings of radii R_1 and R_2, having same linear charge density λ is : [4 marks]



(A) $\frac{\lambda}{2\varepsilon_0}$
(B) $\frac{\lambda}{\varepsilon_0}$
(C) $\frac{2\lambda}{\varepsilon_0}$
(D) $\frac{\lambda}{4\varepsilon_0}$

Correct Answer: (A) $\frac{\lambda}{2\varepsilon_0}$

Solution:

Concept:

- Electric potential is a scalar. The contribution of charge dq at distance R is $dV = (4\pi\varepsilon_0)^{-1}dq/R$, taking zero potential at infinity.
- A uniformly charged semicircular wire has length πR and charge $Q = \lambda\pi R$. All its elements are equally distant from its centre.

Step 1: Find the contribution of either half ring For the half ring of radius R_1, each element is a distance R_1 from the common centre. Therefore the denominator in the potential integral is constant:

$$V_1 = \frac{1}{4\pi\varepsilon_0 R_1}\int dq = \frac{\lambda\pi R_1}{4\pi\varepsilon_0 R_1} = \frac{\lambda}{4\varepsilon_0}.$$

The radius cancels because increasing the radius increases the total charge in exactly the same proportion when the linear density is held fixed.

Step 2: Add the two scalar potentials For the second half ring, the same reasoning gives

$$V_2 = \frac{\lambda \pi R_2}{4\pi\varepsilon_0 R_2} = \frac{\lambda}{4\varepsilon_0}.$$

Both rings have the same charge density and sign. Their potentials add without resolving components:

$$V = V_1 + V_2 = \frac{\lambda}{2\varepsilon_0}.$$

Option D is the potential of only one half ring. Options B and C overcount the total contribution by factors of two and four. Cancellation arguments appropriate to vector electric fields do not apply to the scalar potential here. The result is independent of the unequal radii precisely because the specified quantity is linear charge density rather than equal total charge.

Step 3: Conclusion The electric potential at the common centre is $V = \lambda/(2\varepsilon_0)$. Hence, the correct option is (A).

Quick Tip to Solve: At the centre, every uniformly charged semicircle with the same λ contributes $\lambda/(4\varepsilon_0)$, regardless of its radius.

45. Given below are two statements:

Statement I : Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.

Statement II : Acceleration due to earth's gravity is same at a height 'h' and depth 'd' from earth's surface, if h = d.

In the light of above statements, choose the most appropriate answer from the options given below

[4 marks]

(A) Statement I is correct but statement II is incorect

(B) Both Statement I and II are correct

(C) Statement I is incorrect but statement II is correct

(D) Both Statement I and Statement II are incorrect

Correct Answer: (A) Statement I is correct but statement II is incorect

Solution:

Concept:

- Outside a spherical Earth, $g_h = GM/(R+h)^2 = g_0[R/(R+h)]^2$, where R is Earth's radius and $g_0 = GM/R^2$.
- In the usual uniform-density Earth model, the shell theorem gives $g_d = g_0(1 - d/R)$ at depth d. Near the surface, $g_h \simeq g_0(1 - 2h/R)$.

Step 1: Test Statement I above and below the surface For a positive height h, the factor $[R/(R+h)]^2$ is less than one, so gravity decreases as the body is taken upward. Inside Earth, only the enclosed mass contributes to the radial field. In the uniform-density model the enclosed mass falls as the cube of the distance from the centre, so the resulting gravity is proportional to that distance. Thus

$g_d = g_0(1 - d/R) < g_0$ for positive depth. Statement I is correct under the standard model used for the variation with depth.

Step 2: Compare equal small height and depth If $h = d$ close to the surface,

$$g_h \simeq g_0\left(1 - \frac{2h}{R}\right), \qquad g_d = g_0\left(1 - \frac{h}{R}\right).$$

The losses are different: the upward loss is approximately twice the downward loss at equal distances. Consequently equal height and depth do not in general give equal gravity; the approximate condition for equality is $d = 2h$, not $d = h$. Statement II is incorrect. A single possible special equality would not establish the general statement. Options B and C incorrectly accept Statement II, while D incorrectly rejects Statement I.

Step 3: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: For small distances remember the coefficients: height gives $2h/R$, while depth gives d/R; equal gravity requires approximately $d = 2h$.

46. A photon is emitted in transition from n = 4 to n = 1 level in hydrogen atom. The corresponding wavelength for this transition is (given, h = 4 $\times$ 10^{-15} eVs) : [4 marks]

(A) 99.3 nm

(B) 94.1 nm

(C) 974 nm

(D) 941 nm

Correct Answer: (B) 94.1 nm

Solution:

Concept:

- Hydrogen energy levels are $E_n = -13.6/n^2\,\text{eV}$. In an emission transition, the photon takes away the positive energy difference $E_i - E_f$.
- The photon satisfies $E_\gamma = h\nu = hc/\lambda$. With h expressed in $\text{eV}\,\text{s}$, the photon energy can be kept in electron volts.

Step 1: Evaluate the emitted photon energy The initial level is $n_i = 4$ and the final level is $n_f = 1$. Their energies are $E_4 = -13.6/16 = -0.85\,\text{eV}$ and $E_1 = -13.6\,\text{eV}$. Therefore

$$E_\gamma = E_4 - E_1 = -0.85 - (-13.6) = 12.75\,\text{eV}.$$

The energy is positive, as required for emission. Since the lower level is the ground state, this line lies in the Lyman series and should have an ultraviolet wavelength.

Step 2: Use the supplied Planck constant Taking $c = 3 \times 10^8\,\text{m}\,\text{s}^{-1}$ and using the given $h =$

$4 \times 10^{-15}\,\text{eV s}$,

$$\lambda = \frac{hc}{E_\gamma} = \frac{(4 \times 10^{-15})(3 \times 10^8)}{12.75}\,\text{m} = 9.4118 \times 10^{-8}\,\text{m}.$$

Since $1\,\text{nm} = 10^{-9}\,\text{m}$, this is $94.118\,\text{nm}$, or $94.1\,\text{nm}$. Option A does not match the energy and the supplied constant. Options C and D are about ten times too large and would correspond to photon energies an order of magnitude smaller. Using a more precise tabulated Planck constant instead of the stated value would change the numerical wavelength and is inappropriate for these options.

Step 3: Conclusion The required wavelength is approximately $94.1\,\text{nm}$. Hence, the correct option is (B).

Quick Tip to Solve: When h is supplied in eV s, keep the Bohr energy difference in eV and avoid an unnecessary joule conversion.

47. The frequency (ν) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as $\nu = r^a \rho^b s^c$. The values of a, b and c respectively are [4 marks]

(A) $\left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right)$
(B) $\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$
(C) $\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$
(D) $\left(-\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$

Correct Answer: (B) $\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$

Solution:

Concept:

- The dimensions of frequency, radius and density are $[\nu] = T^{-1}$, $[r] = L$, and $[\rho] = ML^{-3}$, respectively.
- Surface tension is force per unit length, so $[s] = (MLT^{-2})/L = MT^{-2}$. Equality of dimensions requires matching the powers of each fundamental dimension separately.

Step 1: Write the dimension equation Insert the dimensions into the stated dependence, allowing an implicit dimensionless multiplier which cannot affect the exponents:

$$T^{-1} = L^a (ML^{-3})^b (MT^{-2})^c = M^{b+c} L^{a-3b} T^{-2c}.$$

The left-hand side contains no mass or length factor. Hence their exponents on the right must each vanish, while the time exponent must be minus one. This gives three independent linear equations for the three unknown powers.

Step 2: Solve the powers and test the alternatives Matching time gives $-2c = -1$, so $c = 1/2$. Matching mass gives $b + c = 0$, so $b = -1/2$. Finally, matching length gives $a - 3b = 0$, leading to $a = -3/2$. Thus

$$\nu \propto \sqrt{\frac{s}{\rho r^3}}.$$

As a check, the dimensions inside the square root are $(MT^{-2})/[(ML^{-3})L^3] = T^{-2}$, giving frequency after taking the square root. Option A has the wrong time exponent because c is negative. Option C leaves a nonzero length power, and D leaves a nonzero mass power. Only B satisfies all three dimension balances simultaneously.

Step 3: Conclusion The exponents are $(a, b, c) = (-3/2, -1/2, 1/2)$. Hence, the correct option is (B).

Quick Tip to Solve: Determine c from time first, then b from mass and a from length; this avoids solving the three equations together.

48. If two vectors $\vec{P} = \hat{i} + 2m\hat{j} + m\hat{k}$ and $\vec{Q} = 4\hat{i} - 2\hat{j} + m\hat{k}$ are perpendicular to each other. Then, the value of m will be : [4 marks]

(A) -1

(B) 3

(C) 1

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- For two nonzero vectors, $\vec{P} \cdot \vec{Q} = |\vec{P}||\vec{Q}| \cos\theta$. Perpendicularity means $\theta = 90°$, hence the scalar product is zero.
- In an orthonormal Cartesian basis, multiply corresponding components and add: $\vec{P} \cdot \vec{Q} = P_xQ_x + P_yQ_y + P_zQ_z$. Cross terms between different unit vectors vanish.

Step 1: Apply the perpendicularity condition The components of the first vector are $(1, 2m, m)$, and those of the second are $(4, -2, m)$. Their scalar product is therefore

$$\vec{P} \cdot \vec{Q} = (1)(4) + (2m)(-2) + (m)(m) = 4 - 4m + m^2.$$

The negative sign in the second vector's y-component must be retained. Since perpendicular vectors have zero scalar product, the required equation is $m^2 - 4m + 4 = 0$, not an equation obtained by equating their magnitudes.

Step 2: Solve and verify the value Factor the quadratic:

$$m^2 - 4m + 4 = (m - 2)^2 = 0, \qquad m = 2.$$

At this value the vectors become $\vec{P} = (1, 4, 2)$ and $\vec{Q} = (4, -2, 2)$, both nonzero, and their scalar product is $4 - 8 + 4 = 0$. The perpendicularity criterion is therefore valid without any zero-vector ambiguity. To check the other options, $m = -1$ gives scalar product 9, while $m = 3$ and $m = 1$ each give 1. None of these is zero. The repeated root also shows that there is only one real value of the parameter satisfying the condition.

Step 3: Conclusion The parameter is $m = 2$. Hence, the correct option is (D).

Quick Tip to Solve: Take the dot product rather than the cross product for perpendicularity, and recognise $m^2 - 4m + 4$ as a perfect square.

49. A body of mass 200g is tied to a spring of spring constant 12.5 N/m, while the other end of spring is fixed at point O. If the body moves about O in a circular path on a smooth horizontal surface with constant angular speed 5 rad/s. Then the ratio of extension in the spring to its natural length will be : [4 marks]

(A) 1 : 2

(B) 2 : 3

(C) 2 : 5

(D) 1 : 1

Correct Answer: (B) 2 : 3

Solution:

Concept:

- A spring extended by x exerts tension kx, where k is the spring constant. On a smooth horizontal surface this radial force supplies the required centripetal force.
- Uniform circular motion at angular speed ω requires inward force $m\omega^2 r$. If the spring's natural length is l_0, the orbit radius is its stretched length $r = l_0 + x$.

Step 1: Express the centripetal balance Convert the body mass to SI units: $m = 200\,\text{g} = 0.200\,\text{kg}$. The vertical weight and normal reaction balance, while the spring force acts horizontally toward the fixed point. Thus

$$kx = m\omega^2(l_0 + x).$$

It is essential to use the total length $l_0 + x$ as the circular radius: the extension alone is not the distance from the pivot. The angular speed is constant, so no tangential acceleration is needed.

Step 2: Calculate the extension ratio Substitute $k = 12.5\,\text{N m}^{-1}$ and $\omega = 5\,\text{rad s}^{-1}$:

$$12.5x = (0.200)(25)(l_0 + x) = 5(l_0 + x).$$

Hence $7.5x = 5l_0$, which gives

$$\frac{x}{l_0} = \frac{5}{7.5} = \frac{2}{3}.$$

The positive denominator $k - m\omega^2 = 7.5\,\text{N m}^{-1}$ confirms a finite positive extension. Option C, $2/5$, is actually $m\omega^2/k$, the ratio of extension to the stretched length, not to the natural length. Options A and D do not satisfy the radial force balance when substituted. The sought ratio is dimensionless because both numerator and denominator are lengths.

Step 3: Conclusion The ratio of extension to natural length is $x : l_0 = 2 : 3$. Hence, the correct option is (B).

Quick Tip to Solve: Use $x/l_0 = m\omega^2/(k - m\omega^2)$; the often-confused quantity $m\omega^2/k$ equals $x/(l_0 + x)$.

50. Match List I with List II

	List-I		List-II
A.	AM Broadcast	I.	88-108 MHz
B.	FM Broadcast	II.	540-1600 kHz
C.	Television	III.	3.7-4.2 GHz
D.	Satellite Communication	IV.	54MHz - 890MHz

Choose the correct answer from the options given below : [4 marks]

(A) A-IV, B-III, C-I, D-II

(B) A-II, B-I, C-IV, D-III

(C) A-I, B-III, C-II, D-IV

(D) A-II, B-III, C-I, D-IV

Correct Answer: (B) A-II, B-I, C-IV, D-III

Solution:

Concept:

- Communication services occupy characteristic frequency bands. AM broadcasting uses medium-frequency radio waves, while FM broadcasting uses the very-high-frequency band.
- Terrestrial television occupies VHF and UHF bands; satellite links commonly use microwave frequencies. The unit conversions are $1\,\text{MHz} = 10^3\,\text{kHz}$ and $1\,\text{GHz} = 10^3\,\text{MHz}$.

Step 1: Match the radio broadcasting services AM broadcast is assigned the range 540–$1600\,\text{kHz}$, which is entry II. FM broadcast uses 88–$108\,\text{MHz}$, which is entry I. Thus the first two matches are A-II and B-I. These bands differ substantially: converting the AM band to MHz gives only 0.540–$1.600\,\text{MHz}$, so it cannot be interchanged with the FM range.

Step 2: Match television and satellite communication The broad terrestrial television range in the table is 54–$890\,\text{MHz}$, entry IV. The satellite communication range is 3.7–$4.2\,\text{GHz}$, entry III. Consequently the remaining matches are C-IV and D-III. The satellite band is at a much higher frequency than the television band, consistent with the use of microwaves for satellite links.

Step 3: Check the complete option Combining the four independent identifications gives A-II, B-I, C-IV, D-III. Option (B) contains all four matches. Option (A) incorrectly puts AM in the television band. Option (C) assigns AM to the FM band and television to the AM band. Option (D) starts with the correct AM match but puts FM in the satellite microwave band; therefore it fails even though its first match is correct.

Step 4: Conclusion The correct matching is $\boxed{\text{A-II, B-I, C-IV, D-III}}$. Hence, the correct option is (B).

Quick Tip to Solve: First identify AM in kHz and satellite links in GHz; then distinguish the FM and television ranges that are both quoted in MHz.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A convex lens of refractive index 1.5 and focal length 18cm in air is immersed in water. The change in focal length of the lens will be _____ cm.
(Given refractive index of water $= \frac{4}{3}$) [4 marks]

Solution:

Concept:

- For a thin lens in a surrounding medium, the lens maker formula uses the relative refractive index: $1/f = (n_\ell/n_m - 1)(1/R_1 - 1/R_2)$.
- Here n_ℓ and n_m are the lens and surrounding-medium indices. Immersion changes the optical contrast but leaves the radii and their geometric factor unchanged.

Step 1: Determine the index contrast in each medium For air, take $n_m = 1$, so the index factor is $n_\ell - 1 = 1.5 - 1 = 0.5$. In water the relative index must be calculated before subtracting unity:

$$\frac{n_\ell}{n_w} = \frac{3/2}{4/3} = \frac{9}{8}, \qquad \frac{n_\ell}{n_w} - 1 = \frac{1}{8}.$$

Since the relative index is still greater than one, the convex lens remains converging.

Step 2: Compare the focal lengths Write $K = 1/R_1 - 1/R_2$. The lens has the same K in both surroundings, so

$$\frac{1/f_w}{1/f_a} = \frac{(1/8)K}{(1/2)K} = \frac{1}{4}.$$

Hence $f_w = 4f_a$. Substituting the stated air focal length gives

$$f_w = 4 \times 18 = 72\,\text{cm}.$$

The reduction in refractive contrast reduces the optical power, which explains why the focal length becomes longer.

Step 3: Calculate the requested change The question asks for the change, rather than the new focal length. Subtract the initial value from the final value:

$$\Delta f = f_w - f_a = 72 - 18 = 54\,\text{cm}.$$

The positive sign means an increase. Using the absolute index 1.5 in the immersed-lens formula would incorrectly predict no change; the relative index is essential.

Step 4: Conclusion The focal length increases by $\boxed{54\,\text{cm}}$. The numerical entry is 54.

Quick Tip to Solve: In lens immersion problems, form the ratio of index contrasts first; after finding the new focal length, subtract the original if a change is requested.

52. A body of mass 1kg begins to move under the action of a time dependent force $\overrightarrow{F} = \left(t\hat{i} + 3t^2\hat{j}\right)$ N, where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The power developed by above force, at the time t = 2s, will be _____ W. [4 marks]

Solution:

Concept:

- Newton's second law gives $\vec{a} = \vec{F}/m$. Integrating each acceleration component with respect to time gives the corresponding velocity component.
- Instantaneous power is the scalar product $P = \vec{F} \cdot \vec{v} = F_x v_x + F_y v_y$; the force and velocity must be evaluated at the same instant.

Step 1: Integrate the force components The body begins to move under the given force, so its initial velocity is zero. With $m = 1\,\text{kg}$, the component accelerations are $a_x = t$ and $a_y = 3t^2$ in SI units. Therefore

$$v_x(t) = \int_0^t \tau\, d\tau = \frac{t^2}{2}, \qquad v_y(t) = \int_0^t 3\tau^2\, d\tau = t^3.$$

The lower limit enforces the initial condition and prevents an arbitrary integration constant from entering the velocity.

Step 2: Evaluate the vectors at two seconds At $t = 2\,\text{s}$, the force components and velocity components are

$$\vec{F} = 2\hat{i} + 12\hat{j}\ \text{N}, \qquad \vec{v} = 2\hat{i} + 8\hat{j}\ \text{m s}^{-1}.$$

The force direction varies with time; consequently its instantaneous direction need not coincide with the accumulated velocity direction. Multiplying the magnitudes without the angle would not give the correct power.

Step 3: Take the scalar product Multiply matching Cartesian components and add them:

$$P = F_x v_x + F_y v_y = (2)(2) + (12)(8) = 4 + 96 = 100\,\text{W}.$$

There are no cross terms because $\hat{i} \cdot \hat{j} = 0$. As a check, the kinetic energy is $K = \frac{1}{2}[(t^2/2)^2 + (t^3)^2] = t^4/8 + t^6/2$, and its derivative at $t = 2$ is also $100\,\text{W}$.

Step 4: Conclusion The instantaneous power at $2\,\text{s}$ is $\boxed{100\,\text{W}}$. The numerical entry is 100.

Quick Tip to Solve: For a time-dependent vector force, integrate components to obtain velocity and then use a dot product; do not replace it by a product of magnitudes.

53. The energy released per fission of nucleus of ^{240}X is 200 MeV. The energy released if all the atoms in 120g of pure ^{240}X undergo fission is _____ $\times 10^{25}$ MeV.

(Given $N_A = 6 \times 10^{23}$) [4 marks]

Solution:

Concept:

- The number of nuclei in a pure sample is $N = (m/M)N_A$, where m is its mass and M its molar mass in consistent mass units.
- If every nucleus undergoes one fission and releases energy ε, the total released energy is $E = N\varepsilon$. Keep MeV throughout when the requested energy is in MeV.

Step 1: Find the amount of the isotope The mass number 240 corresponds to the molar mass $240\,\mathrm{g\,mol^{-1}}$ for the calculation. The pure sample has mass $120\,\mathrm{g}$, so

$$n = \frac{120}{240} = \frac{1}{2}\,\mathrm{mol}.$$

This is half a mole of nuclei, rather than 120 separate nuclei. Purity means that the whole stated sample mass belongs to the specified fissionable isotope.

Step 2: Count the nuclei that undergo fission Using the supplied value $N_A = 6 \times 10^{23}\,\mathrm{mol^{-1}}$,

$$N = nN_A = \frac{1}{2}(6 \times 10^{23}) = 3 \times 10^{23}.$$

Since all atoms undergo fission, this is also the number of fission events. No half-life or decay-law factor is required, and the energy per event is applied once to each original nucleus.

Step 3: Multiply and express the requested coefficient Each event releases $200\,\mathrm{MeV}$. Therefore

$$E = (3 \times 10^{23})(200) = 600 \times 10^{23} = 6 \times 10^{25}\,\mathrm{MeV}.$$

The blank multiplies $10^{25}\,\mathrm{MeV}$, so its value is 6, not the full energy written in exponential form. Conversion to joules is unnecessary because the question already specifies the required unit.

Step 4: Conclusion The released energy is $\boxed{6 \times 10^{25}\,\mathrm{MeV}}$; the coefficient to enter is $\boxed{6}$.

Quick Tip to Solve: Count nuclei using mass divided by molar mass, then multiply by the energy per fission. Read the power of ten printed outside the answer blank.

54. A uniform solid cylinder with radius R and length L has moment of inertia I_1, about the axis of the cylinder. A concentric solid cylinder of radius $R' = \frac{R}{2}$ and length $L' = \frac{L}{2}$ is carved out of the original cylinder. If I_2 is the moment of inertia of the carved out portion of the cylinder then $\frac{I_1}{I_2} =$ _____. (Both I_1 and I_2 are about the axis of the cylinder) [4 marks]

Solution:

Concept:

- A uniform solid cylinder has axial moment of inertia $I = \frac{1}{2}MR^2$, where M is its mass and R its radius.
- For a uniform material, mass is proportional to volume $\pi R^2 L$. Reducing both the radius and the length therefore changes the mass as well as the explicit radius factor in the inertia.

Step 1: Find the mass of the removed cylinder Let the original cylinder have density ρ and mass $M = \rho\pi R^2 L$. The concentric carved-out portion has the same density, radius $R/2$, and length $L/2$. Thus

$$M_2 = \rho\pi\left(\frac{R}{2}\right)^2\frac{L}{2} = \frac{1}{8}\rho\pi R^2 L = \frac{M}{8}.$$

The radius reduction contributes a factor of one quarter to volume, and the length reduction supplies another factor of one half.

Step 2: Calculate both axial moments of inertia The original solid cylinder has

$$I_1 = \frac{1}{2}MR^2.$$

The question defines I_2 as the inertia of the carved-out portion itself, not of the material remaining after removal. Its axial inertia is

$$I_2 = \frac{1}{2}\left(\frac{M}{8}\right)\left(\frac{R}{2}\right)^2 = \frac{MR^2}{64}.$$

Both axes are the common cylinder axis, so no parallel-axis correction is needed.

Step 3: Form the dimensionless ratio Dividing the two moments gives

$$\frac{I_1}{I_2} = \frac{MR^2/2}{MR^2/64} = 32.$$

The same result follows from $I \propto R^4 L$ at fixed density: reducing the radius by two changes inertia by $1/16$, and reducing the length by two gives $1/32$ overall. Subtracting I_2 from I_1 would answer a different question about the remaining hollowed body.

Step 4: Conclusion The required ratio is $\boxed{I_1/I_2 = 32}$, a dimensionless number.

Quick Tip to Solve: At fixed density, axial cylinder inertia scales as radius to the fourth power times length. Distinguish the removed portion from the remaining body.

55. If a copper wire is stretched to increase its length by 20%. The percentage increase in resistance of the wire is _____%. [4 marks]

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area.
- During stretching, the volume AL remains constant and resistivity is taken unchanged. Consequently $R = \rho L^2/V$, so resistance is proportional to the square of length.

Step 1: Use the stated increase in length Let the original length and area be L and A. An increase of 20 percent gives

$$L\prime = \left(1 + \frac{20}{100}\right) L = 1.2L.$$

Stretching makes the wire longer and thinner. The change of area cannot be ignored: assuming a fixed area would predict only a 20 percent resistance increase, contrary to the constant-volume condition for a stretched wire.

Step 2: Apply conservation of volume The original and final volumes are equal, so

$$A\prime L\prime = AL, \qquad A\prime = \frac{AL}{1.2L} = \frac{A}{1.2}.$$

At unchanged temperature, take the copper resistivity to remain ρ. The resistance ratio is then

$$\frac{R\prime}{R} = \frac{\rho L\prime/A\prime}{\rho L/A} = \frac{1.2}{1/1.2} = (1.2)^2 = 1.44.$$

Both the longer path and the smaller conducting area increase resistance.

Step 3: Convert the ratio into a percentage increase The new resistance is 144 percent of its original value, but the increase is measured relative to the original resistance:

$$\frac{R\prime - R}{R} \times 100 = (1.44 - 1) \times 100 = 44\%.$$

Retaining the squared factor exactly is necessary here. The small-change estimate $2\Delta L/L$ would give 40 percent and omit the additional quadratic contribution of 4 percent.

Step 4: Conclusion The resistance increases by $\boxed{44\%}$. The numerical entry in the percentage blank is 44.

Quick Tip to Solve: For a stretched wire of fixed volume, square the length multiplier to get the resistance multiplier, then subtract one before converting to percent.

56. A parallel plate capacitor with air between the plate has a capacitance of 15pF. The separation between the plate becomes twice and the space between them is filled with a medium of dielectric constant 3.5. Then the capacitance becomes $\frac{x}{4}$ pF. The value of x is _____. [4 marks]

Solution:

Concept:

- A parallel plate capacitor completely filled by a dielectric has capacitance $C = K\varepsilon_0 A/d$, where K is the dielectric constant and d the plate separation.
- When the plate area is unchanged, capacitance is proportional to K/d. An increased dielectric constant raises capacitance, while an increased separation lowers it.

Step 1: Identify the original and final parameters Initially the medium is air, whose dielectric constant is taken as one, and

$$C_0 = \frac{\varepsilon_0 A}{d} = 15\,\text{pF}.$$

Finally the separation is $d\prime = 2d$, the dielectric fills the entire space, and $K = 3.5$. The area of overlap is unchanged. Since the dielectric completely fills the gap, there is no separate air layer that would require a series-capacitor calculation.

Step 2: Calculate the changed capacitance by a ratio Use the same area and initial separation in both expressions:

$$C\prime = \frac{3.5\varepsilon_0 A}{2d} = \frac{3.5}{2}C_0.$$

Substituting $C_0 = 15\,\text{pF}$,

$$C\prime = 1.75 \times 15 = 26.25\,\text{pF}.$$

The dielectric factor is larger than the separation factor, so the net capacitance increases. Multiplying by 3.5 alone would neglect the doubled gap; dividing by two alone would neglect the dielectric.

Step 3: Compare with the specified answer form The final capacitance is written in the question as $(x/4)\,\text{pF}$. Therefore

$$\frac{x}{4} = 26.25, \qquad x = 4 \times 26.25 = 105.$$

The number x is the numerical coefficient defined by that expression, rather than the capacitance itself. Equivalently, $26.25\,\text{pF} = (105/4)\,\text{pF}$, which verifies the substitution directly.

Step 4: Conclusion The new capacitance is $26.25\,\text{pF}$, giving $\boxed{x = 105}$.

Quick Tip to Solve: Use the multiplier dielectric constant divided by the separation multiplier; then match the result to the fraction specified in the question.

57. A single turn current loop in the shape of a right angle triangle with sides 5 cm, 12 cm, 13 cm is carrying a current of 2 A. The loop is in a uniform magnetic field of magnitude 0.75 T whose direction is parallel to the current in the 13 cm side of the loop. The magnitude of the magnetic force on the 5 cm side will be $\frac{x}{130}$ N. The value of x is _____. [4 marks]

Solution:

Concept:

- A straight wire of length l, carrying current I in a uniform magnetic field B, experiences force $F = IlB\sin\theta$, where θ is the angle between the current and the field.
- When the field is parallel to one side of a triangular loop, the angle for another side is obtained from the geometry of the triangle. Reversing a side direction changes the force direction but not its magnitude.

Step 1: Find the angle between the short side and the field The 13 cm side is the hypotenuse of the right triangle. Let θ be the acute angle between the 5 cm side and the hypotenuse. The side opposite this angle is 12 cm, so

$$\sin\theta = \frac{12}{13}.$$

The current may make the supplementary angle with the field, depending on its direction around the loop. Since supplementary angles have equal sine, the magnitude calculation is unchanged.

Step 2: Convert the length and substitute The wire segment of interest is the 5 cm side, whose length is $0.05\,\text{m}$. The supplied current is $2\,\text{A}$, and the field is $0.75\,\text{T}$. Thus

$$F = 2 \times 0.05 \times 0.75 \times \frac{12}{13} = \frac{0.9}{13} = \frac{9}{130}\,\text{N}.$$

The factor $12/13$ is essential: only the component of this wire perpendicular to the field produces the magnetic force.

Step 3: Compare with the prescribed form The question defines the magnitude as $x/130$ newtons. Comparing numerators in the same unit gives $x = 9$. The side parallel to the field has zero magnetic force, but this does not make the force on the 5 cm side zero.

Step 4: Conclusion The force magnitude on the 5 cm side is $\frac{9}{130}\,\text{N}$, hence $\boxed{x = 9}$ (dimensionless).

Quick Tip to Solve: Use the triangle to calculate the sine of the angle with the field; do not insert the hypotenuse as the wire length.

58. A Spherical ball of radius 1mm and density 10.5 g/cc is dropped in glycerine of coefficient of viscosity 9.8 poise and density 1.5 g/cc. Viscous force on the ball when it attains constant velocity is 3696×10^{-x} N. The value of x is _____.

(Given, g = 9.8 m/s^2 and $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- At terminal velocity the acceleration vanishes, so weight is balanced by buoyancy plus viscous drag: $F_v = V(\rho_s - \rho_f)g$.
- For a sphere, $V = 4\pi r^3/3$. All densities and lengths must be converted to SI units before calculating a force in newtons.

Step 1: Convert the radius and density difference The radius is $r = 1\,\text{mm} = 10^{-3}\,\text{m}$. Since $1\,\text{g cm}^{-3} = 10^3\,\text{kg m}^{-3}$, the two densities are

$$\rho_s = 10500\,\text{kg m}^{-3}, \qquad \rho_f = 1500\,\text{kg m}^{-3}.$$

The difference is $9000\,\text{kg m}^{-3}$, rather than the density of the ball alone. The displaced glycerine supports part of the weight.

Step 2: Balance the forces at constant speed The ball moves downward; both buoyancy and viscous drag act upward. With zero net force, the drag equals the effective weight:

$$\begin{aligned} F_v &= \frac{4}{3}\pi r^3(\rho_s - \rho_f)g \\ &= \frac{4}{3} \times \frac{22}{7} \times 10^{-9} \times 9000 \times 9.8 \\ &= 3.696 \times 10^{-4}\,\text{N}. \end{aligned}$$

The viscosity would be needed to obtain the terminal speed using Stokes' law. It is unnecessary here because the force at that speed follows directly from force balance.

Step 3: Match the power of ten Rewrite the computed force using the coefficient given in the question:

$$3.696 \times 10^{-4} = 3696 \times 10^{-7}.$$

Therefore the exponent in $3696 \times 10^{-x}\,\text{N}$ is $x = 7$. Keeping the coefficient as 3696, rather than 3.696, changes the exponent by three.

Step 4: Conclusion The terminal viscous force is $3696 \times 10^{-7}\,\text{N}$; therefore $\boxed{x = 7}$ (dimensionless).

Quick Tip to Solve: At terminal speed use drag = weight minus buoyancy; calculate the speed only if the question actually asks for it.

59. A mass m attached to free end of a spring executes SHM with a period of 1s. If the mass is increased by 3 kg the period of the oscillation increases by one second, the value of mass m is _____ kg. [4 marks]

Solution:

Concept:

- For a mass m oscillating on an ideal spring of force constant k, $T = 2\pi\sqrt{m/k}$. For the same spring, T^2 is proportional to the attached mass.
- A stated increase in period is added to the initial period before taking the ratio. The mass increment and the initial mass must use the same units.

Step 1: Identify the initial and final periods Initially the spring carries mass m and its period is $T_1 = 1\,\text{s}$. Adding $3\,\text{kg}$ changes the attached mass to $m + 3$, with m expressed in kilograms. The period increases by one second, so

$$T_2 = 1 + 1 = 2\,\text{s}.$$

The final period is twice the initial period. It is not an increase of a factor of one; the distinction matters because the period depends on the square root of mass.

Step 2: Eliminate the spring constant using a ratio The two period equations are

$$T_1^2 = \frac{4\pi^2 m}{k}, \qquad T_2^2 = \frac{4\pi^2(m+3)}{k}.$$

The spring constant does not change when the load is increased. Dividing the equations gives

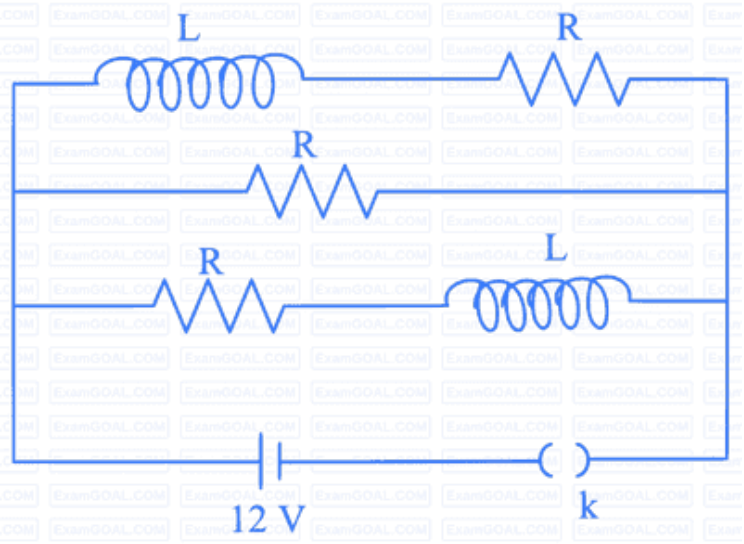
$$\frac{T_2^2}{T_1^2} = \frac{m+3}{m} = \frac{2^2}{1^2} = 4.$$

Step 3: Solve and check the mass Multiplying by the positive mass m yields $m + 3 = 4m$, hence $3m = 3$ and $m = 1\,\text{kg}$. The new mass is then $4\,\text{kg}$, four times the original. Since the period varies as the square root of mass, this indeed doubles the period from 1 s to 2 s. No value of k needs to be calculated.

Step 4: Conclusion The original mass is $\boxed{m = 1\,\text{kg}}$, so the numerical entry is 1.

Quick Tip to Solve: Square the period ratio first: a doubling of period requires four times the total attached mass.

60. Three identical resistors with resistance R = 12Ω and two identical inductors with self inductance L = 5 mH are connected to an ideal battery with emf of 12 V as shown in figure. The current through the battery long after the switch has been closed will be _____ A. [4 marks]



Solution:

Concept:

- For an ideal inductor, the potential difference is $V_L = L\,di/dt$. Long after a DC switch is closed, the currents are constant and each ideal inductor behaves as a wire.
- Parallel branches have a common potential difference. Their currents add at the battery, and each branch current follows Ohm's law after its steady-state resistance is identified.

Step 1: Replace the inductors by their steady-state equivalents The diagram has three branches between the left and right battery rails. The upper branch contains an inductor and a resistor in series; the middle branch contains a resistor; the lower branch contains a resistor and an inductor in series. Long after closing the switch, $di/dt = 0$, so the voltage across either ideal inductor is zero. Each outer branch therefore retains only its resistor as an effective resistance.

Step 2: Calculate the equivalent resistance Each branch has resistance $R = 12\,\Omega$, and all three branches join the same two rails. Consequently they are in parallel, not in series:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{1}{4}, \qquad R_{\text{eq}} = 4\,\Omega.$$

The inductance $5\,\text{mH}$ controls the transient response after switching. It does not contribute a resistance in the final DC state.

Step 3: Find and check the battery current Using the ideal battery voltage of 12 V gives

$$I_{\text{battery}} = \frac{12}{4} = 3\,\text{A}.$$

As a check, each 12-ohm branch is across the full 12 V battery and carries 1 A. Their three currents sum to 3 A. Treating the inductors as open circuits would instead describe the instant just after switching, not the long-time condition asked here.

Step 4: Conclusion The current through the battery long after closing the switch is $\boxed{3\,\text{A}}$.

Quick Tip to Solve: For ideal inductors under steady DC, replace each by a wire before deciding which resistors are in parallel.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The number of s-electrons present in an ion with 55 protons in its unipositive state is [4 marks]

(A) 10

(B) 9

(C) 12

(D) 8

Correct Answer: (A) 10

Solution:

Concept:

- The number of protons is the atomic number Z. An ion of charge $+1$ has one fewer electron than its neutral atom.
- Every occupied s subshell accommodates two electrons at most. Count electrons in all occupied s subshells, rather than counting only the outermost shell.

Step 1: Identify the atom and its unipositive ion An atom with 55 protons is caesium, Cs, whose atomic number is 55. Its neutral ground-state configuration is $[\text{Xe}]6s^1$. Formation of Cs^+ removes the outermost $6s$ electron. The resulting ion has 54 electrons and the closed-shell xenon configuration. Its proton count remains 55 because ion formation does not alter the nucleus.

Step 2: List and count the occupied s subshells The configuration of the ion can be written in full as

$$\begin{aligned}\text{Cs}^+ :\quad & 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\\ & 4s^2\,3d^{10}\,4p^6\,5s^2\,4d^{10}\,5p^6.\end{aligned}$$

The occupied s subshells are $1s, 2s, 3s, 4s, 5s$; each is filled with two electrons. Thus

$$N_s = 2+2+2+2+2 = 10.$$

Step 3: Rule out the alternative counts The count 9 would require an incompletely occupied s subshell among these five filled core subshells. The count 12 incorrectly fills the $6s$ subshell of the ion, which is empty. The count 8 omits one of the five occupied s subshells. Hence none of those alternatives represents the complete xenon core. The removed electron is the neutral atom's sole $6s$ electron; core electrons are retained.

Step 4: Conclusion There are $\boxed{10}$ s-electrons in Cs^+. Hence, the correct option is (A).

Quick Tip to Solve: Write the neutral configuration first, remove the outermost electron, and then count all s superscripts in the ion.

62. Find out the major products from the following reactions. [4 marks]



(A) $A = CH_3CH_2CH(CH_3)CH_2OH, \quad B = CH_3CH_2CH(CH_3)CH_2OH$

(B) $A = CH_3CH_2CH(CH_3)CH_2OH, \quad B = CH_3CH_2C(OH)(CH_3)_2$

(C) $A = CH_3CH_2C(OH)(CH_3)_2, \quad B = CH_3CH_2CH(CH_3)CH_2OH$

(D) $A = CH_3CH_2C(OH)(CH_3)_2, \quad B = CH_3CH_2C(OH)(CH_3)_2$

Correct Answer: (B) $A = CH_3CH_2CH(CH_3)CH_2OH, \quad B = CH_3CH_2C(OH)(CH_3)_2$

Solution:

Concept:

- Hydroboration followed by alkaline peroxide oxidation adds water across an alkene in the anti-Markovnikov orientation: OH ends up on the less substituted double-bond carbon.
- Oxymercuration followed by demercuration gives Markovnikov hydration without carbocation rearrangement: OH attaches to the more substituted double-bond carbon.

Step 1: Identify the alkene and the two reaction paths The drawn alkene is $CH_2{=}C(CH_3)CH_2CH_3$, or 2-methylbut-1-ene. Its terminal double-bond carbon is a CH_2 group, whereas the other double-bond carbon bears methyl and ethyl groups. The right-hand sequence, BH_3 in THF followed by H_2O_2/OH^-, forms product A. The left-hand sequence, $Hg(OAc)_2$ and water followed by $NaBH_4$, forms product B.

Step 2: Assign the hydroxyl position in each product For A, boron initially attaches preferentially to the less substituted terminal carbon; oxidation replaces the carbon-boron bond with a carbon-oxygen bond. Therefore

$$A = HOCH_2CH(CH_3)CH_2CH_3,$$

which is 2-methylbutan-1-ol. For B, Markovnikov hydration puts OH on the carbon bearing the methyl and ethyl substituents:

$$B = CH_3C(OH)(CH_3)CH_2CH_3,$$

which is 2-methylbutan-2-ol. Both products retain all five carbon atoms of the reactant.

Step 3: Compare both assignments with the choices Option (B) gives precisely the primary alcohol for A and the tertiary alcohol for B. Option (A) incorrectly gives anti-Markovnikov hydration for both routes. Option (C) interchanges the two product assignments. Option (D) incorrectly gives Markovnikov hydration for both routes. The product letters A and B label reaction products and must not be confused with the option letters.

Step 4: Conclusion Product A is 2-methylbutan-1-ol and product B is 2-methylbutan-2-ol. Hence, the correct option is (B).

Quick Tip to Solve: Mark the less substituted alkene carbon for hydroboration and the more substituted carbon for oxymercuration before matching product letters.

63. What is the number of unpaired electron(s) in the highest occupied molecular orbital of the following species : $N_2; N_2^+; O_2; O_2^+$? [4 marks]

(A) 0, 1, 0, 1

(B) 2, 1, 0, 1

(C) 0, 1, 2, 1

(D) 2, 1, 2, 1

Correct Answer: (C) 0, 1, 2, 1

Solution:

Concept:

- Electrons fill molecular orbitals in increasing energy order. Hund's rule puts electrons singly with parallel spins into a degenerate pair before pairing them.
- For N_2 the highest occupied orbital is bonding σ_{2p_z}; for O_2 the highest occupied set consists of the degenerate antibonding π^*_{2p} orbitals. Ionisation removes an electron from the highest occupied level.

Step 1: Examine nitrogen and its cation In N_2, after the lower orbitals are filled, the valence ordering has $\pi^2_{2p_x}\pi^2_{2p_y}\sigma^2_{2p_z}$. Its highest occupied σ_{2p_z} orbital contains a paired pair, so there are zero unpaired electrons there. Removing one electron to form ${N_2}^+$ leaves $\sigma^1_{2p_z}$, with exactly one unpaired electron. The first two entries must therefore be $0, 1$.

Step 2: Examine oxygen and its cation Oxygen has two more electrons than nitrogen. In O_2, the highest occupied degenerate antibonding orbitals have occupation

$$\pi^*_{2p_x}{}^1\,\pi^*_{2p_y}{}^1.$$

By Hund's rule they contain two unpaired electrons in total. Ionisation removes one of these highest-energy electrons, so ${O_2}^+$ has just one electron in that degenerate set and hence one unpaired electron.

Step 3: Assemble the sequence and reject the distractors In the printed order $N_2, {N_2}^+, O_2, {O_2}^+$, the required counts are $0, 1, 2, 1$. Options (B) and (D) incorrectly assign two unpaired electrons to the closed-shell nitrogen molecule. Option (A) incorrectly pairs both of oxygen's highest-energy electrons and assigns zero to O_2, contrary to Hund's rule. The degenerate oxygen HOMO set must be treated together.

Step 4: Conclusion The unpaired-electron sequence is $\boxed{0, 1, 2, 1}$. Hence, the correct option is (C).

Quick Tip to Solve: Remove a cation electron from the HOMO, and apply Hund's rule separately to the two degenerate oxygen antibonding orbitals.

64. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Benzene is more stable than hypothetical cyclohexatriene

Reason R : The delocalised π electron cloud is attracted more strongly by nuclei of carbon atoms.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) A is true but R is false

(B) Both A and R are correct and R is the correct explanation of A

(C) Both A and R are correct but R is NOT the correct explanation of A

(D) A is false but R is true

Correct Answer: (B) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- Benzene is aromatic: its planar cyclic system has six delocalised π electrons, satisfying the $4n + 2$ rule with $n = 1$.
- Resonance delocalisation lowers the electronic energy relative to a hypothetical ring containing three isolated, localised double bonds. Greater effective attraction of the delocalised electron cloud to the carbon nuclei contributes to this stabilisation.

Step 1: Evaluate the assertion Hypothetical cyclohexatriene would have three independent double bonds. Its expected heat of hydrogenation would therefore be about three times that of cyclohexene. Benzene releases substantially less heat on hydrogenation, demonstrating that its initial state is lower in energy. Thus benzene is more stable than the hypothetical localised structure, so Assertion A is true.

Step 2: Evaluate the reason The six p orbitals overlap continuously around the ring, distributing the π electrons over all six carbon atoms. The delocalised cloud experiences stabilising attraction to the nuclei throughout the conjugated framework. This distribution lowers the electronic energy compared with three isolated double bonds. Thus Reason R is also true.

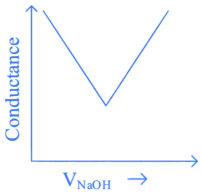
Step 3: Test whether the reason explains the assertion The reason states the energetic benefit of the same delocalisation that makes benzene more stable than hypothetical cyclohexatriene. It therefore supplies a cause for the assertion, rather than merely an unrelated structural observation. Options (A) and (D), which make one statement false, are excluded; (C) incorrectly denies the explanatory connection between the two true statements.

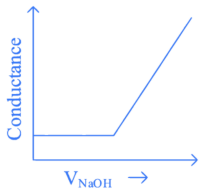
Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (B).

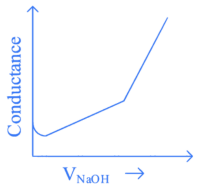
Quick Tip to Solve: For aromatic-stability assertions, connect delocalisation to a lower-energy starting state; the smaller heat of hydrogenation is evidence of that stabilisation.

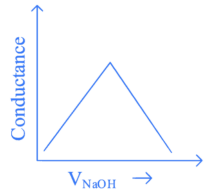
65. Choose the correct representation of conductometric titration of benzoic acid vs sodium hydroxide.

[4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- Benzoic acid, C_6H_5COOH, is a weak electrolyte; sodium hydroxide and sodium benzoate are strong electrolytes.
- Conductance depends on both the concentration and mobility of ions. H^+ and OH^- have much higher ionic mobilities than ordinary ions such as Na^+.

Step 1: Follow the conductance before equivalence The acid initially supplies a small concentration of highly mobile H^+. The first additions of base remove these ions and replace their contribution with that of slower Na^+, giving a small initial decrease. Neutralisation then produces increasingly more ionised sodium benzoate, so conductance rises gradually. The reaction is

$$C_6H_5COOH + NaOH \longrightarrow C_6H_5COONa + H_2O.$$

The gradual rising segment, rather than a long flat segment, describes the formation of the conducting salt.

Step 2: Identify the change after equivalence At equivalence essentially all the acid has become benzoate salt. Additional sodium hydroxide now leaves excess, highly mobile OH^- ions in solution. Conductance therefore rises more steeply. Option (C) shows the slight initial dip followed by a gentle increase and then a steeper increase. The pronounced V-shaped curve in (A) instead represents a strong acid against a strong base. The flat initial portion in (B) misses salt formation, while the decreasing final segment in (D) contradicts the accumulation of excess hydroxide.

Step 3: Conclusion The required conductometric curve is the graph in option (C). Hence, the correct option is (C).

Quick Tip to Solve: For a weak acid versus a strong base, look for salt formation before equivalence and a steeper rise from excess OH^- after it.

66. Given below are two statements:

Statement I: $H_2NCOCH_2CH_2COCH_3$ **under Clemmensen reduction conditions will give**

$$HOOCCH_2CH_2CH_2CH_3.$$

Statement II: $(CH_3)_2CHCH_2CH(Cl)COCH_3$ **under Wolff–Kishner reduction conditions will give**

$$(CH_3)_2CHCH_2CH(Cl)CH_2CH_3.$$

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is false but Statement II is true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Clemmensen reduction uses $Zn(Hg)$ and concentrated HCl to convert an aldehyde or ketone carbonyl group to a methylene group.
- Wolff–Kishner reduction uses hydrazine and strong base on heating. Other functional groups must be checked for their stability under these strongly acidic or basic conditions.

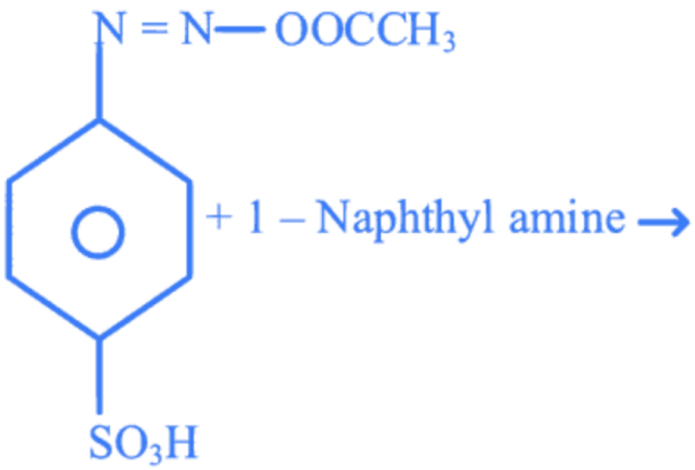
Step 1: Check Statement I under acidic conditions The substrate in Statement I contains both a ketone and a primary amide. Under Clemmensen conditions the ketone carbonyl is reduced to $-CH_2-$, while the amide undergoes acid hydrolysis to a carboxylic acid. Thus the displayed conversion of $H_2NCOCH_2CH_2COCH_3$ into $HOOCCH_2CH_2CH_2CH_3$ is consistent with the reaction conditions. The carboxylic acid group is not reduced by Clemmensen reagent. Statement I is true.

Step 2: Check Statement II under basic conditions The second substrate is an alpha-chloro ketone. Although Wolff–Kishner conditions normally reduce a carbonyl group to methylene, the strongly basic, heated medium also allows elimination involving the chlorine-bearing carbon and an adjacent hydrogen. The displayed product retains chlorine while representing only the carbonyl reduction; that is not the correct outcome for this base-sensitive substrate. Hence Statement II is false. Options (A) and (B) incorrectly regard Statement II as true, while option (D) incorrectly rejects the first conversion.

Step 3: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Before applying a carbonyl-reduction rule, check other functional groups against the medium: amides hydrolyse in acid and halo compounds can eliminate in hot base.

67. Choose the correct colour of the product for the following reaction. [4 marks]



(A) White

(B) Yellow

(C) Red

(D) Blue

Correct Answer: (C) Red

Solution:

Concept:

- Aromatic diazonium compounds undergo coupling with activated aromatic rings to form azo compounds containing the $-N=N-$ linkage.
- Diazotised sulphanilic acid couples with 1-naphthylamine to produce a strongly conjugated red azo dye; this is the colour-forming reaction of the Griess test.

Step 1: Identify the reacting aromatic systems The diazo component shown is derived from sulphanilic acid: the aromatic ring carries a sulphonic acid group and the diazonium-derived group. The second component is 1-naphthylamine, whose amino group activates the naphthalene ring toward electrophilic coupling. The coupling joins the aromatic systems through the azo linkage, rather than producing a simple substitution product with no extended conjugation.

Step 2: Assign the observed colour The product has an extended conjugated chromophore involving the benzene ring, the azo linkage and the naphthalene ring. In this particular sulphanilic-acid/1-naphthylamine coupling reaction the characteristic product is a red azo dye. Thus the white option is inconsistent with formation of this strongly coloured chromophore. Yellow and blue are not the characteristic colours of this named reagent combination; one should identify the actual coupling system rather than assume that every azo compound has the same colour.

Step 3: Conclusion The product is red. Hence, the correct option is (C).

Quick Tip to Solve: Recognise diazotised sulphanilic acid plus 1-naphthylamine as the red azo-dye reaction used in the Griess test.

68. Which one amongst the following are good oxidizing agents?

A. Sm^{2+}

B. Ce^{2+}

C. Ce^{4+}

D. Tb^{4+}

Choose the most appropriate answer from the options given below : [4 marks]

(A) C and D only

(B) A and B only

(C) D only

(D) C only

Correct Answer: (A) C and D only

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. Most lanthanoids preferentially form the $+3$ oxidation state.
- High oxidation states such as Ce^{4+} and Tb^{4+} tend to gain an electron to form their trivalent ions; divalent lanthanoid ions generally tend to lose an electron instead.

Step 1: Classify the two divalent ions For Sm^{2+}, oxidation to the common trivalent state is favoured, so it behaves as a reducing agent rather than a good oxidising agent. The listed Ce^{2+} is likewise a low oxidation-state species and has a strong tendency to lose an electron to approach Ce^{3+}. Neither A nor B is therefore included in the requested oxidising-agent pair.

Step 2: Classify the tetravalent ions The relevant reductions are

$$Ce^{4+} + e^- \longrightarrow Ce^{3+}$$

and

$$Tb^{4+} + e^- \longrightarrow Tb^{3+}.$$

Both tetravalent ions can oxidise another species while returning to the more usual trivalent state. Therefore C and D are the good oxidising agents in the given list. Option (B) selects the low-valent species and reverses the redox tendency. Options (C) and (D) each omit one of the two valid high-valent oxidants, so neither is complete.

Step 3: Conclusion Ce^{4+} and Tb^{4+}, namely C and D only, are good oxidising agents. Hence, the correct option is (A).

Quick Tip to Solve: For lanthanoid redox questions, first compare the listed oxidation state with $+3$: $+4$ ions commonly oxidise, whereas $+2$ ions commonly reduce.

69. A student has studied the decomposition of a gas AB_3 at 25°C. He obtained the following data.

p (mm Hg)	50	100	200	400
relative $t_{1/2}$ (s)	4	2	1	0.5

The order of the reaction is [4 marks]

(A) 2

(B) 0.5

(C) 1

(D) 0 (zero)

Correct Answer: (A) 2

Solution:

Concept:

- At fixed temperature, the initial concentration of an ideal gas is proportional to its initial pressure: $[A]_0 = p_0/(RT)$.
- For an nth-order reaction with $n \neq 1$, the half-life varies as $t_{1/2} \propto [A]_0^{1-n}$. For first order it is independent of initial concentration.

Step 1: Find the pressure dependence of half-life When pressure doubles from 50 to $100\,\text{mm Hg}$, the half-life decreases from 4 to $2\,\text{s}$. The next two doublings give the same factor. Equivalently, all four products of pressure and half-life are equal:

$$50(4) = 100(2) = 200(1) = 400(0.5) = 200.$$

Thus the observations establish $t_{1/2} \propto p_0^{-1}$, and therefore $t_{1/2} \propto [A]_0^{-1}$.

Step 2: Determine and check the order Comparing the measured exponent with the general half-life relation gives

$$1 - n = -1 \quad \Rightarrow \quad n = 2.$$

This agrees with the integrated second-order result $t_{1/2} = 1/(k[A]_0)$. A first-order reaction would have constant half-life, so option (C) is excluded. A zero-order reaction would have half-life proportional to pressure, opposite to the observed trend. An order of 0.5 would give $t_{1/2} \propto p_0^{0.5}$, also increasing rather than decreasing.

Step 3: Conclusion The reaction is of second order. Hence, the correct option is (A).

Quick Tip to Solve: Check the product $p_0 t_{1/2}$: if it is constant at fixed temperature, the reaction follows second-order half-life behaviour.

70. The hybridization and magnetic behaviour of cobalt ion in $[Co(NH_3)_6]^{3+}$ complex, respectively is : [4 marks]

(A) sp^3d^2 and paramagnetic

(B) sp^3d^2 and diamagnetic

(C) d^2sp^3 and diamagnetic

(D) d^2sp^3 and paramagnetic

Correct Answer: (C) d^2sp^3 and diamagnetic

Solution:

Concept:

- Neutral NH_3 ligands leave cobalt in the $+3$ oxidation state in $[Co(NH_3)_6]^{3+}$. The cobalt(III) ion has a $3d^6$ configuration.
- Six ligands produce an octahedral field. For this cobalt(III) ammine complex the splitting is sufficient for pairing, giving a low-spin $t_{2g}^6e_g^0$ arrangement.

Step 1: Find the metal configuration The charge balance is $x + 6(0) = +3$, so $x = +3$. Neutral cobalt has atomic configuration $[Ar]3d^74s^2$; loss of the two $4s$ electrons and one $3d$ electron gives Co^{3+} as $[Ar]3d^6$. The high charge on cobalt increases the octahedral splitting, so the six electrons pair in the lower-energy t_{2g} orbitals in this ammine complex.

Step 2: Infer hybridisation and magnetic behaviour Pairing leaves two inner $3d$ orbitals available for bonding. In valence-bond terminology they combine with one $4s$ and three $4p$ orbitals to give d^2sp^3 hybridisation and octahedral geometry. All six d electrons are paired, so the number of unpaired electrons is zero and the complex is diamagnetic. Options (A) and (B) incorrectly assign outer-orbital sp^3d^2 hybridisation. Option (D) has the correct hybridisation but incorrectly calls the paired complex paramagnetic.

Step 3: Conclusion The cobalt ion has d^2sp^3 hybridisation and diamagnetic behaviour. Hence, the correct option is (C).

Quick Tip to Solve: For Co^{3+} ammine complexes, count d^6 and check the low-spin arrangement before choosing inner-orbital hybridisation and magnetism.

71. $K_2Cr_2O_7$ **paper acidified with dilute** H_2SO_4 **turns green when exposed to :** [4 marks]

(A) Sulphur dioxide

(B) Sulphur trioxide

(C) Hydrogen sulphide

(D) Carbon dioxide

Correct Answer: Both SO2 and H2S reduce acidified dichromate to green Cr(III); A is the intended key but C is also chemically defensible.

Solution:

Concept:

- Acidified dichromate contains chromium in oxidation state +6. Reduction produces green aqueous Cr^{3+}, so the colour change detects a reducing substance.
- Sulphur dioxide is the conventional gas tested using orange acidified potassium dichromate paper. However, hydrogen sulphide is also a reducing gas; the literal options therefore require care.

Step 1: Identify the intended gas Sulphur dioxide is oxidised from sulphur(+4) to sulphur(+6), while dichromate is reduced to chromium(III). The balanced ionic equation is

$$Cr_2O_7{}^{2-} + 3\,SO_2 + 2\,H^+ \longrightarrow 2\,Cr^{3+} + 3\,SO_4{}^{2-} + H_2O.$$

Thus option (A) gives the familiar orange-to-green test and is the keyed answer.

Step 2: Check the competing reducing gas Hydrogen sulphide also reduces acidified dichromate. Sulphur changes from -2 to 0 in the reaction

$$Cr_2O_7{}^{2-} + 3\,H_2S + 8\,H^+ \longrightarrow 2\,Cr^{3+} + 3\,S + 7\,H_2O.$$

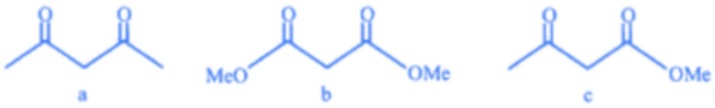
This reaction also forms green chromium(III), although sulphur may be deposited. Consequently option (C) cannot be excluded by the stated colour change alone.

Step 3: Reject the nonreducing gases In sulphur trioxide, sulphur already has its usual maximum oxidation state +6; it does not provide the required reducing action. Carbon dioxide similarly contains carbon(+4) and does not reduce dichromate under this test. Options (B) and (D) therefore do not account for green chromium(III).

Step 4: Conclusion The intended keyed answer is sulphur dioxide, option (A). Literally, hydrogen sulphide, option (C), can also turn acidified dichromate green, as the balanced reaction shows; the stated test does not uniquely distinguish (A) from (C).

Quick Tip to Solve: For a dichromate colour test, track chromium from +6 to +3 and check whether each gas can donate electrons.

72. Which will undergo deprotonation most readily in the basic medium? [4 marks]



(A) a only

(B) Both a and c

(C) c only

(D) b only

Correct Answer: (A) a only

Solution:

Concept:

- Deprotonation is easiest for the strongest acid, whose conjugate base is most stable. A methylene group between two carbonyl groups gives an enolate with charge delocalised onto both oxygen atoms.
- An ester alkoxy group donates electron density to its carbonyl by resonance. This makes an ester carbonyl less effective than a ketone carbonyl at stabilising an adjacent enolate.

Step 1: Identify the three methylene environments Compound a is acetylacetone, a beta-diketone. Compound b is dimethyl malonate, a diester, while compound c is methyl acetoacetate, a beta-keto ester. In each, the central CH_2 group is flanked by carbonyl groups; removal of a proton here gives the most resonance-stabilised conjugate base available in that molecule.

Step 2: Compare conjugate-base stabilisation For a, both carbonyl groups are ketones and efficiently delocalise the negative charge. In c, one of these is replaced by an ester, whose $-\mathrm{OMe}$ group donates by resonance and weakens this stabilisation. In b, both carbonyls are esters. The qualitative acidity order is therefore

$$\text{a (diketone)} > \text{c (keto ester)} > \text{b (diester)}.$$

Typical central-methylene pKa values are approximately 9, 11 and 13 respectively, consistent with this electronic comparison.

Step 3: Apply the word most All three central methylene groups can lose a proton in a sufficiently basic medium, but the question asks which does so most readily. Only a has the greatest acidity. The presence of an active methylene in c does not put it at the same acidity as a. Hence neither c alone nor both a and c is the best choice, and the diester b is the least acidic of the three.

Step 4: Conclusion Compound a undergoes deprotonation most readily. Hence, the correct option is (A).

Quick Tip to Solve: For active methylene acidity, count ketone carbonyls: diketone is more acidic than keto ester, which is more acidic than diester.

73. Which of the following cannot be explained by crystal field theory? [4 marks]

(A) The order of spectrochemical series

(B) Stability of metal complexes

(C) Magnetic properties of transition metal complexes

(D) Colour of metal complexes

Correct Answer: (A) The order of spectrochemical series

Solution:

Concept:

- Crystal field theory treats ligands as point charges or dipoles and metal-ligand interactions as electrostatic. Their approach splits the initially degenerate metal d orbitals.
- The resulting splitting and d-electron arrangement explain crystal field stabilisation, unpaired electrons and many d–d absorption colours. The ligand field-strength order itself is empirical in this theory.

Step 1: Use the splitting predicted by the model In an octahedral field, the d orbitals separate into lower t_{2g} and upper e_g levels with separation Δ_o. Placing the metal d electrons in these levels gives the crystal field stabilisation energy. For example, each t_{2g} electron contributes $-0.4\Delta_o$, while each e_g electron contributes $+0.6\Delta_o$. This gives a basis for comparing the relative stabilisation of complexes, so option (B) is not the limitation asked for.

Step 2: Explain magnetism and colour Comparing the splitting with pairing energy determines whether electrons pair in the lower levels or occupy higher levels. The predicted number of unpaired electrons accounts for magnetic behaviour, rejecting option (C). Absorption of suitable visible light promotes a d electron across the split levels; the remaining transmitted or reflected light gives the observed colour. Thus the model also explains the origin of colour for many transition-metal complexes, rejecting option (D).

Step 3: Identify what remains unexplained The spectrochemical series orders ligands by their observed splitting strength. A purely electrostatic model cannot derive this complete order or explain why ligands such as carbon monoxide produce unusually strong fields. Such details require covalent bonding and metal-ligand orbital interactions. Crystal field theory uses ligand strength as input rather than predicting the empirical sequence from first principles.

Step 4: Conclusion The order of the spectrochemical series cannot be explained by crystal field theory. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish using a measured splitting to predict properties from deriving why one ligand produces more splitting than another.

74. Given below are two statements:

Statement I : Pure Aniline and other arylamines are usually colourless.

Statement II : Arylamines get coloured on storage due to atmospheric reduction.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct

(B) Statement I is correct but Statement II is incorrect

(C) Both Statement I and Statement II are incorrect

(D) Statement I is incorrect but Statement I is correct

Correct Answer: (B) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- Arylamines contain an amino group bonded directly to an aromatic ring. Freshly purified aniline and many simple arylamines are usually colourless substances.
- Exposure to atmospheric oxygen can oxidise arylamines during storage. Coloured oxidation products are responsible for the darkening of an old sample.

Step 1: Evaluate Statement I Pure aniline is a colourless oily liquid, and the statement that pure aniline and other arylamines are usually colourless is the standard description of this class. A coloured bottle of stored aniline does not disprove it, because such a sample may contain oxidation products rather than only the freshly purified compound. The word usually also avoids claiming that every substituted aromatic amine must be colourless. Statement I is therefore correct.

Step 2: Evaluate Statement II The observation that arylamines can acquire colour during storage is correct, but the proposed process is incorrect. Atmospheric oxygen acts as an oxidising agent. It oxidises some of the amine to products with extended conjugation that absorb visible light, giving a yellow, brown or darker appearance. This change is atmospheric oxidation, not atmospheric reduction. Therefore Statement II, which explicitly says reduction, is false.

Step 3: Compare the offered combinations Option (A) wrongly accepts the reduction explanation. Option (C) wrongly rejects the usual colourless nature of freshly pure aniline. The printed option (D) says Statement I is incorrect and then Statement I is correct; regardless of that contradictory wording, it does not represent the established result. Only option (B) gives Statement I true and Statement II false. No reaction formula for a single unique storage product is needed, since atmospheric oxidation can give a mixture.

Step 4: Conclusion Statement I is correct but Statement II is incorrect: stored arylamines become coloured through atmospheric oxidation. Hence, the correct option is (B).

Quick Tip to Solve: In statement questions, test the named process separately from the observation: correct darkening does not make reduction correct.

75. In which of the following reactions the hydrogen peroxide acts as a reducing agent? [4 marks]

(A) $Mn^{2+} + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$

(B) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(C) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$

(D) $2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$

Correct Answer: (B) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

Solution:

Concept:

- Oxygen has oxidation state -1 in H_2O_2. Peroxide can be reduced to oxygen(-2) or oxidised to molecular oxygen(0), depending on the other reactant.
- A reducing agent is itself oxidised: it donates electrons while the other substance is reduced. Follow the peroxide oxygen, rather than just looking for oxygen-containing products.

Step 1: Find the reaction producing molecular oxygen In option (B), hydrogen peroxide gives O_2. Its oxygen oxidation state increases from -1 to 0, which is oxidation. The oxidation half-reaction can be written

$$H_2O_2 \longrightarrow O_2 + 2\,H^+ + 2\,e^-.$$

The released electrons reduce chlorine in $HOCl$ from +1 to -1 in Cl^-. Thus peroxide acts as the reducing agent in this reaction.

Step 2: Check the metal-ion reactions In option (A), manganese changes from +2 to +4, so manganese is oxidised. Peroxide accepts electrons and supplies oxygen(-2) in hydroxide; it acts as an oxidising agent. In option (D), iron changes from +2 to +3. Again peroxide accepts the electrons lost by the metal ions, and its oxygen becomes hydroxide oxygen(-2). Therefore neither (A) nor (D) represents reducing action by peroxide.

Step 3: Check oxidation of sulphide In option (C), sulphur in PbS changes from -2 to +6 in $PbSO_4$, an increase of eight units. Four molecules of peroxide accept the corresponding eight electrons as their oxygen is reduced from -1 to -2. Peroxide therefore oxidises sulphide to sulphate. This excludes (C) and confirms that only (B) shows oxidation of the peroxide itself. The oxidation-state argument does not depend on inferring electron transfer merely from the direction of the printed arrows.

Step 4: Conclusion Hydrogen peroxide is oxidised to oxygen and therefore acts as a reducing agent in the reaction with hypochlorous acid. Hence, the correct option is (B).

Quick Tip to Solve: For peroxide, formation of O2 indicates reducing action; formation of water or hydroxide usually indicates oxidising action.

76. The metal which is extracted by oxidation and subsequent reduction from its ore is : [4 marks]

(A) Al

(B) Cu

(C) Ag

(D) Fe

Correct Answer: C is the intended cyanide-leaching answer, but B also satisfies oxidation followed by reduction in copper sulphide metallurgy.

Solution:

Concept:

- Extraction of silver by cyanide involves oxidative dissolution of the metal into a soluble complex, followed by reduction and displacement using zinc.
- The wording oxidation and subsequent reduction alone is broad: copper sulphide metallurgy also includes roasting followed by self-reduction. Distinguishing the intended process requires noting this overlap.

Step 1: Describe the keyed silver process In aerated cyanide solution, oxygen permits oxidative dissolution of silver into the dicyanidoargentate(I) complex. The metal changes from oxidation state 0 to +1. A representative equation is

$$4\,\mathrm{Ag} + 8\,\mathrm{CN^-} + \mathrm{O_2} + 2\,\mathrm{H_2O} \longrightarrow 4\,[\mathrm{Ag(CN)_2}]^- + 4\,\mathrm{OH^-}.$$

Silver-bearing ores are treated by cyanide leaching; the soluble silver complex is then the species from which the metal is recovered.

Step 2: Recover silver by reduction Zinc displaces silver from the complex:

$$2\,[\mathrm{Ag(CN)_2}]^- + \mathrm{Zn} \longrightarrow [\mathrm{Zn(CN)_4}]^{2-} + 2\,\mathrm{Ag}.$$

Silver(I) gains electrons and becomes silver metal. This oxidation followed by reduction is the standard cyanide-extraction description intended by option (C). Aluminium is normally obtained by electrolytic reduction of alumina, while iron is obtained through reduction of its oxide ore by carbon monoxide and related furnace reactions; these are not the particular silver leaching sequence.

Step 3: Check the scope of the literal wording Copper sulphide extraction also includes oxidation during roasting and subsequent self-reduction, for example

$$2\,\mathrm{Cu_2O} + \mathrm{Cu_2S} \longrightarrow 6\,\mathrm{Cu} + \mathrm{SO_2}.$$

Copper(I) is reduced to copper(0) in this step. Since the question does not specify cyanide leaching or the oxidation of metal during dissolution, its broad phrase can also describe option (B). The keyed silver answer identifies the intended textbook process, but the literal wording does not exclude the copper route.

Step 4: Conclusion The intended keyed answer is Ag, option (C), through oxidative cyanide leaching followed by zinc reduction. As written, Cu, option (B), is also defensible because its extraction includes oxidation followed by self-reduction; the process wording alone is not unique.

Quick Tip to Solve: For silver, connect aerated cyanide leaching with zinc displacement; for copper sulphide, distinguish roasting from self-reduction.

77. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Beryllium has less negative value of reduction potential compared to the other alkaline earth metals.

Reason R : Beryllium has large hydration energy due to small size of Be^{2+} but relatively large value of atomization enthalpy

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both A and R are correct but R is NOT the correct explanation of A

(B) Both A and R are correct and R is the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

Correct Answer: (B) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- The reduction potential for $\mathrm{M}^{2+}(\mathrm{aq}) + 2\,\mathrm{e}^- \longrightarrow \mathrm{M}(\mathrm{s})$ reflects the combined energetics of atomisation, ionisation and hydration, rather than ionic size alone.
- Beryllium is very small and has strong metallic bonding. Its small ion has a large hydration enthalpy in magnitude, while formation of gaseous atoms and ions requires substantial energy.

Step 1: Evaluate the assertion Beryllium has a less negative standard reduction potential than the other alkaline earth metals. Its value is approximately $-1.85\,\mathrm{V}$, compared with about $-2.37\,\mathrm{V}$ for magnesium and still more negative values for calcium, strontium and barium. Thus the reduction of aqueous beryllium ions to the metal is relatively less unfavourable on this potential scale. Assertion A is correct.

Step 2: Evaluate the reason The Be^{2+} ion is very small and is strongly hydrated, so its hydration enthalpy has a large negative magnitude. At the same time, converting solid beryllium into free gaseous atoms requires relatively large atomisation enthalpy; its ionisation enthalpies are also large. The reason correctly contrasts strong hydration with the substantial energetic cost associated with the compact metal. Reason R is therefore correct.

Step 3: Connect the energetics with the potential For oxidation of the metal to its aqueous divalent ion, atomisation and ionisation are energy-consuming contributions while hydration is energy-releasing. The large energy costs for beryllium are not fully compensated by its large hydration energy. Oxidation is consequently less favourable than a hydration-only argument would suggest, and the reverse reduction potential is less negative. This is the competition stated in R, so R explains A. Option (A) incorrectly denies the explanatory link; options (C) and (D) incorrectly reject one of the two true statements.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: Never infer aqueous reduction potentials from hydration alone: compare atomisation and ionisation costs with hydration gain.

78. Correct statement is : [4 marks]

(A) An average human being consumes more food than air

(B) An average human being consumes 100 times more air than food

(C) An average human being consumes nearly 15 times more air than food

(D) An average human being consumes equal amount of food and air

Correct Answer: (C) An average human being consumes nearly 15 times more air than food

Solution:

Concept:

- Air enters the human body continuously through respiration, whereas food is consumed intermittently. The relevant comparison is the total daily intake, rather than the number of meals or breaths.
- The standard environmental-chemistry estimate is that the mass of air consumed by an average adult is nearly fifteen times the mass of food consumed. This illustrates the importance of maintaining clean ambient air.

Step 1: Identify the comparison being asked The statement concerns an average human being, so it asks for the usual approximate daily consumption ratio. It does not ask for an exact physiological constant for every individual. Breathing takes place throughout the day and night; consequently, the daily mass of inhaled air greatly exceeds the daily mass of food eaten. We therefore require an air-to-food ratio substantially larger than unity.

Step 2: Test all four statements The accepted comparison is

$$\frac{\text{daily mass of air consumed}}{\text{daily mass of food consumed}} \approx 15.$$

Thus statement (C) gives the required approximate factor. Option (A) reverses the comparison by making food consumption greater. Option (D) incorrectly makes the two masses equal. Option (B) has the right direction of comparison but replaces the approximate factor fifteen by one hundred, so it substantially exaggerates the ratio. The word nearly is appropriate because body size, activity and food intake vary between people.

Step 3: Conclusion An average human being consumes nearly 15 times more air than food. Hence, the correct option is (C).

Quick Tip to Solve: For this NCERT environmental comparison, remember the approximate mass ratio air : food = 15 : 1; do not confuse it with an exact breathing-volume measurement.

79. Identify the correct statements about alkali metals.

A. The order of standard reduction potential (M^+ | M) for alkali metal ions is Na > Rb > Li.

B. CsI is highly soluble in water.

C. Lithium carbonate is highly stable to heat.

D. Potassium dissolved in concentrated liquid ammonia is blue in colour and paramagnetic.

E. All the alkali metal hydrides are ionic solids.

Choose the correct answer from the options given below : [4 marks]

(A) C and E only

(B) A and E only

(C) A, B, D only

(D) A, B and E only

Correct Answer: (B) A and E only

Solution:

Concept:

- The aqueous reduction potential of an alkali metal depends on atomisation, ionisation and hydration enthalpies. Lithium has an exceptionally negative potential because its small ion is strongly hydrated.
- The solubility of an ionic salt reflects the balance between lattice enthalpy and hydration enthalpy. Solutions of alkali metals in ammonia change properties on passing from dilute to concentrated conditions.

Step 1: Check statements A and B For the indicated aqueous reduction couples, approximate standard potentials are

$$E^\circ_{Na^+/Na} = -2.71\,V, \quad E^\circ_{Rb^+/Rb} = -2.98\,V, \quad E^\circ_{Li^+/Li} = -3.04\,V.$$

Hence their numerical order is Na > Rb > Li, and A is correct. CsI is an exception to the general high solubility of alkali-metal salts in the standard comparison: both of its large ions have relatively low hydration enthalpies. Thus B, which calls CsI highly soluble, is not the accepted statement here.

Step 2: Check C, D and E and select the combination Lithium carbonate is comparatively unstable to heating and decomposes:

$$Li_2CO_3 \xrightarrow{\Delta} Li_2O + CO_2.$$

Therefore C is false. A dilute metal-ammonia solution is blue and paramagnetic because of solvated electrons; at high concentration it becomes bronze-coloured and diamagnetic through electron pairing. Since D explicitly specifies concentrated solution, it is false. Alkali metals form ionic saline hydrides containing the hydride ion, H^-, so E is correct. The correct pair is therefore A and E. Option (A) contains false C, while options (C) and (D) contain false B; option (C) also contains false D.

Step 3: Conclusion Only statements A and E are correct. Hence, the correct option is (B).

Quick Tip to Solve: In metal-ammonia questions, underline dilute or concentrated first: the familiar blue, paramagnetic description applies to dilute solutions.

80. Match List I with List II.

	List-I: Type		List-II: Name
A.	Antifertility drug	i)	Norethindrone
B.	Tranquilizer	ii)	Meprobomate
C.	Antihistamine	iii)	Seldane
D.	Antibiotic	iv)	Ampicillin

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-III, C-II, D-IV

(B) A-I, B-II, C-III, D-IV

(C) A-II, B-I, C-III, D-IV

(D) A-IV, B-III, C-II, D-I

Correct Answer: (B) A-I, B-II, C-III, D-IV

Solution:

Concept:

- Drugs are classified by therapeutic action. Antifertility drugs regulate reproductive processes, tranquilizers reduce anxiety, antihistamines oppose histamine action, and antibiotics act against susceptible microorganisms.
- A matching question must be solved by identifying the use of each named compound independently; similar-looking drug names are not a reliable guide to their therapeutic class.

Step 1: Match the hormone and nervous-system drug Norethindrone is a synthetic progestogen used in oral contraceptive formulations. It belongs to the antifertility category, so A matches (i). Meprobomate, the name printed in List II for the familiar tranquilizing drug meprobamate, belongs to the tranquilizer category. It is used for its calming action rather than as an antihistamine or antibiotic. Therefore B matches (ii). These two matches already exclude option (C), which exchanges the first two classes, and option (D), which assigns the antifertility category to ampicillin.

Step 2: Match the remaining drugs and eliminate the alternatives Seldane is an antihistamine, associated with the control of allergic responses mediated by histamine, so C matches (iii). Ampicillin is a penicillin-family antibiotic used against susceptible bacterial infections, so D matches (iv). The complete correspondence is

$$\mathrm{A}-\mathrm{I}, \quad \mathrm{B}-\mathrm{II}, \quad \mathrm{C}-\mathrm{III}, \quad \mathrm{D}-\mathrm{IV}.$$

Option (A) exchanges the tranquilizer and antihistamine matches, while option (D) also exchanges the antifertility and antibiotic matches. Only option (B) preserves all four therapeutic classifications. None of the matches depends on a dosage or on a particular commercial formulation.

Step 3: Conclusion The matching is A-I, B-II, C-III and D-IV. Hence, the correct option is (B).

Quick Tip to Solve: Anchor the matching with norethindrone as the progestogen and ampicillin as the antibiotic, then check the tranquilizer and antihistamine pair separately.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Total number of tripeptides possible by mixing of valine and proline is _____ [4 marks]

Solution:

Concept:

- A tripeptide is an ordered chain of three amino-acid residues joined by two peptide bonds. The sequence is read from the amino terminus to the carboxyl terminus.
- When each position can independently contain either of two amino acids and repetition is allowed, the multiplication principle gives $2 \times 2 \times 2$ sequences. Reversing an unsymmetrical sequence generally gives a different peptide.

Step 1: Assign the possibilities to the three positions Write V for valine and P for proline. The first residue may be V or P; independently the second may be V or P; and the third again may be V or P. The statement does not require both types to occur in each peptide, so the two homotripeptides VVV and PPP must be included. Nor does it restrict the supply of either amino acid. Formation of two peptide bonds fixes the chain length at three residues; it does not remove the distinction between different residue orders.

Step 2: Count and check the ordered sequences The multiplication principle gives

$$N = 2^3 = 8.$$

An explicit check is obtained by listing

$$\mathrm{VVV},\ \mathrm{VVP},\ \mathrm{VPV},\ \mathrm{PVV},\ \mathrm{VPP},\ \mathrm{PVP},\ \mathrm{PPV},\ \mathrm{PPP}.$$

There is one sequence with three V residues, three with two V and one P, three with one V and two P, and one with three P residues. Thus the grouped count is also $1+3+3+1=8$. For example VVP and PVV have different amino-terminal residues and are distinct linear peptides; treating them as a single arrangement would undercount. No extra stereoisomer count is requested beyond the sequences of the supplied amino acids.

Step 3: Conclusion The total number of tripeptides is $\boxed{8}$. This is a dimensionless count.

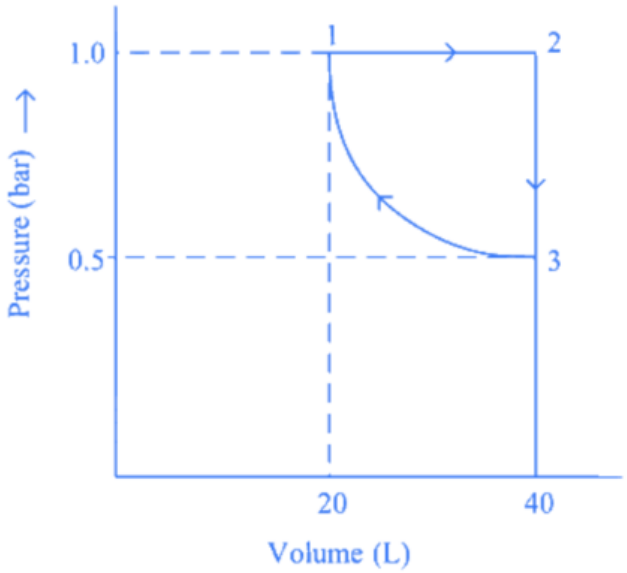
Quick Tip to Solve: Count peptide residues as ordered positions from the N-terminus; with two available amino acids, a chain of length three gives 2 cubed sequences.

82. One mole of an ideal monoatomic gas is subjected to changes as shown in the graph. The magnitude of the work done (by the system or on the system) is _____ J (nearest integer)

Given ; $\log 2 = 0.3$

$$\ln 10 = 2.3$$

[4 marks]



Solution:

Concept:

- For a quasistatic gas process the work done by the gas is $W = \int P\,dV$. It is positive on expansion, negative on compression, and zero at constant volume.
- On an isothermal branch of an ideal gas, $PV = nRT$ is constant and $W = nRT\ln(V_f/V_i)$. In the graph the curved return branch joins points with equal $PV = 20\,\text{L bar}$.

Step 1: Evaluate the straight branches The cycle is $1 \to 2 \to 3 \to 1$. From 1 to 2 the pressure is $1.0\,\text{bar}$ and volume increases from 20 to $40\,\text{L}$. Therefore

$$W_{12} = 1.0(40 - 20) = 20\,\text{L bar} = 2000\,\text{J},$$

using $1\,\text{L bar} = 100\,\text{J}$. From 2 to 3 volume remains $40\,\text{L}$, so $dV = 0$ and $W_{23} = 0$. A pressure decrease alone does not perform boundary work when there is no displacement of the boundary.

Step 2: Evaluate the curved return and sum the work For the isothermal compression from 3 to 1, the endpoint products are $0.5 \times 40 = 1.0 \times 20 = 20\,\text{L bar}$, so

$$W_{31} = 2000\ln\left(\frac{20}{40}\right) = -2000\ln 2.$$

The given approximations must be used:

$$\ln 2 = (\log 2)\ln 10 = 0.3 \times 2.3 = 0.69.$$

Thus $W_{31} = -1380\,\text{J}$, and the net work by the system is

$$W = 2000 + 0 - 1380 = 620\,\text{J}.$$

Its positive sign agrees with the clockwise cycle. The work-on-system convention would reverse the sign, but the requested magnitude would remain unchanged.

Step 3: Conclusion The magnitude of net work is $\boxed{620\,\text{J}}$, so the required nearest integer is $\boxed{620}$.

Quick Tip to Solve: Keep L bar until the last conversion and use the supplied logarithm values; replacing them with a calculator value changes this numerical answer.

83. Maximum number of isomeric monochloro derivatives which can be obtained from 2,2,5,5-tetramethylhexane by chlorination is _____

[4 marks]

Solution:

Concept:

- In monochlorination one hydrogen atom is replaced by chlorine. Symmetry identifies equivalent hydrogen positions, while a newly created stereogenic carbon can produce two enantiomeric derivatives.
- The word maximum requires counting stereoisomers as well as constitutional derivatives. A tetrahedral carbon is stereogenic only when its four attached groups are different.

Step 1: Find the symmetry-distinct hydrogen sites The parent skeleton can be written in condensed form as

$$(CH_3)_3C{-}CH_2{-}CH_2{-}C(CH_3)_3.$$

The two ends are equivalent. All six methyl groups are symmetry-equivalent, and the two central methylene groups are also equivalent. The two quaternary carbon atoms bear no hydrogen, so chlorine cannot replace a hydrogen there. Thus there are only two constitutional substitution types: replacement in a methyl group, or replacement in a central methylene group. Counting the six methyl groups separately would count the same product repeatedly.

Step 2: Include the stereoisomers of each product Methyl substitution gives

$$ClCH_2{-}C(CH_3)_2{-}CH_2{-}CH_2{-}C(CH_3)_3.$$

The substituted carbon still has two identical hydrogen atoms, and the adjacent quaternary carbon has two identical methyl groups, so this product has no stereogenic centre. It contributes one derivative. Methylene substitution gives

$$(CH_3)_3C{-}CH(Cl){-}CH_2{-}C(CH_3)_3.$$

The $CH(Cl)$ carbon is attached to H, Cl, a tert-butyl group, and a CH_2-tert-butyl group. These are four different groups, so it gives an R/S enantiomeric pair. The total maximum is consequently $1 + 2 = 3$.

There is no additional meso form because this product has only one stereogenic centre.

Step 3: Conclusion The maximum number of isomeric monochloro derivatives is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: First count inequivalent C-H sites using symmetry, then test each substitution product for a newly created stereogenic centre.

---

84. Following figure shows spectrum of an ideal black body at four different temperatures. The number of correct statement/s from the following is _____.

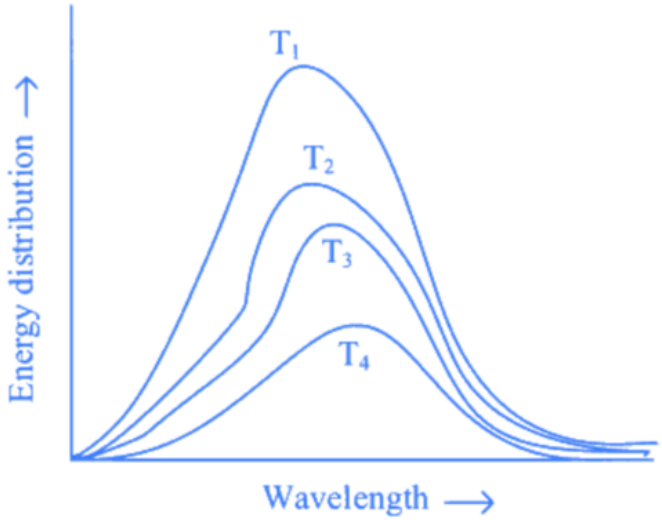
A. $T_4 > T_3 > T_2 > T_1$

B. The black body consists of particles performing simple harmonic motion.

C. The peak of the spectrum shifts to shorter wavelength as temperature increases.

D. $\frac{T_1}{v_1} = \frac{T_2}{v_2} = \frac{T_3}{v_3} \neq$ **constant**

E. The given spectrum could be explained using quantisation of energy. [4 marks]



Solution:

Concept:

- For an ideal black body, increasing temperature raises the emitted intensity and shifts the wavelength of maximum emission towards shorter wavelengths: $\lambda_{\max} T = b$, Wien's displacement law.
- Planck's quantum explanation allows discrete energy exchanges $E = nh\nu$. A defining property of a black body is perfect absorption; the definition does not require all its particles to execute simple harmonic motion.

Step 1: Use the plotted spectra to assess A, C and D The curve labelled T_1 has the greatest intensity and its maximum lies at the shortest wavelength; T_4 has the lowest intensity and longest peak

wavelength. Thus the temperature order is

$$T_1 > T_2 > T_3 > T_4.$$

Statement A reverses this order and is false. Wien's displacement law directly supports C: as temperature rises, $\lambda_{\max}$ falls. If ν_i denotes the frequency associated with the peak, $\nu_i = c/\lambda_{\max,i}$, so

$$\frac{T_i}{\nu_i} = \frac{\lambda_{\max,i} T_i}{c} = \frac{b}{c},$$

a constant. Statement D explicitly says the ratio is not constant, and is therefore false.

Step 2: Assess B and E and count the true statements Statement B makes a general claim about all particles of a black body. That is not a necessary property of an ideal black body: black-body behaviour describes absorption and thermal radiation. Harmonic oscillators are a model used in developing radiation theory, not a requirement that every constituent particle of every black body perform simple harmonic motion. Hence B is not accepted as a general statement. Statement E is correct: quantisation of energy exchange yields Planck's distribution and explains the observed spectrum, including the short-wavelength fall that classical equipartition fails to explain. The true statements are exactly C and E, giving a count of two.

Step 3: Conclusion Only statements C and E are correct. The required number is $\boxed{2}$, a dimensionless count.

Quick Tip to Solve: Read the temperature order from both peak height and peak wavelength, and remember that Wien's law makes peak wavelength times temperature constant.

85. Sum of π-bonds present in peroxodisulphuric acid and pyrosulphuric acid is _____ [4 marks]

Solution:

Concept:

- Count each conventional sulphur–oxygen double bond as one π-bond: a double bond consists of one σ-bond and one π-bond.
- Peroxodisulphuric acid contains an $O-O$ peroxide bridge, whereas pyrosulphuric acid contains an $S-O-S$ bridge. Both bridges contain only single bonds.

Step 1: Write the structure of peroxodisulphuric acid Peroxodisulphuric acid is $H_2S_2O_8$, with the connectivity $HO-S(=O)_2-O-O-S(=O)_2-OH$. Each sulphur is bonded to two terminal oxygen atoms by double bonds. Thus its two sulphur centres together contribute $2 + 2 = 4$ π-bonds. The peroxide linkage is an oxygen–oxygen single bond and adds no π-bond. The hydroxyl groups likewise contain only single bonds.

Step 2: Count the bonds in pyrosulphuric acid Pyrosulphuric acid, or disulphuric acid, is $H_2S_2O_7$. Its connectivity is $HO-S(=O)_2-O-S(=O)_2-OH$. Once again, each sulphur has two terminal $S=O$ bonds, giving four π-bonds. Its bridging oxygen makes two sulphur–oxygen single bonds; these are

not additional double bonds. Consequently, the total is

$$N_\pi = 4 + 4 = 8.$$

Step 3: Conclusion The sum of the numbers of π-bonds is $\boxed{8}$. This is a dimensionless bond count.

Quick Tip to Solve: Write the peroxide or oxygen bridge first, then count only the terminal $\mathrm{S{=}O}$ double bonds; an $\mathrm{O{-}O}$ link is not a π-bond.

86. The number of units, which are used to express concentration of solutions from the following is _____

Mass percent, Mole, Mole fraction, Molarity, ppm, Molality [4 marks]

Solution:

Concept:

- A concentration measure specifies how much solute is present relative to a specified amount of solution or solvent.
- A mole measures the amount of a substance. Without a denominator describing the mixture, it does not give a concentration.

Step 1: Classify the concentration measures Mass percent is 100 times the mass of solute divided by the mass of solution. Mole fraction is the number of moles of a component divided by the total moles in the mixture. Both compare a component with the whole solution and therefore specify composition. Molarity is the amount of solute, in moles, per litre of solution, so it is also a concentration measure.

Step 2: Check the remaining entries Parts per million, ppm, expresses a component-to-mixture ratio multiplied by 10^6, with the relevant mass or volume basis understood. Molality is the amount of solute in moles per kilogram of solvent. These are also concentration measures. In contrast, mole alone is the SI unit of amount of substance: one mole of solute can be placed in many different volumes and therefore correspond to many concentrations. The five qualifying entries are mass percent, mole fraction, molarity, ppm and molality.

Step 3: Conclusion The number of listed concentration measures is $\boxed{5}$, a dimensionless count.

Quick Tip to Solve: Distinguish amount from concentration: molarity and molality specify a denominator, whereas mole by itself does not.

87. The total pressure observed by mixing two liquids A and B is 350 mm Hg when their mole fractions are 0.7 and 0.3 respectively.

The total pressure becomes 410 mm Hg if the mole fractions are changed to 0.2 and 0.8 respectively for A and B. The vapour pressure of pure A is _____ mm Hg. (Nearest integer)

Consider the liquids and solutions behave ideally. [4 marks]

Solution:

Concept:

- For an ideal binary liquid solution, Raoult's law gives $P = x_A P_A^\circ + x_B P_B^\circ$, where P_i° is the vapour pressure of pure component i at the same temperature.
- The two mixtures provide simultaneous linear equations in the same two pure-component vapour pressures.

Step 1: Set up the two pressure equations Let $a = P_A^\circ$ and $b = P_B^\circ$, both in $\mathrm{mm\,Hg}$. Substituting the first pair of mole fractions gives

$$0.7a + 0.3b = 350.$$

For the second mixture, the equation is

$$0.2a + 0.8b = 410.$$

These use liquid-phase mole fractions, as required by Raoult's law. The pure-component pressures do not change when only the composition changes.

Step 2: Eliminate one variable and verify the result Multiply both equations by ten, then eliminate a:

$$\begin{aligned} 7a + 3b &= 3500, \\ 2a + 8b &= 4100, \\ 49b &= 21700. \end{aligned}$$

Therefore $b = 442.857\,\mathrm{mm\,Hg}$. Substitution into the first equation gives

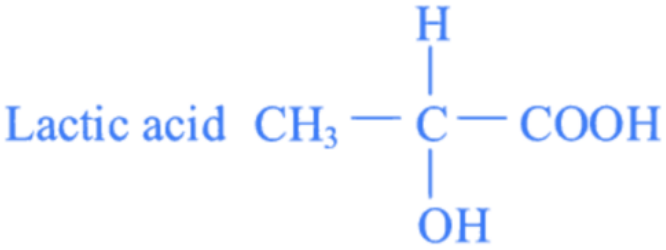
$$a = \frac{350 - 0.3(442.857)}{0.7} = 314.286\,\mathrm{mm\,Hg}.$$

As a check, the second composition gives $0.2(314.286) + 0.8(442.857) = 410\,\mathrm{mm\,Hg}$, as required. The component whose fraction increases is more volatile, consistent with the increase in total pressure.

Step 3: Conclusion The vapour pressure of pure A is $314.286\,\mathrm{mm\,Hg}$, or $\boxed{314\,\mathrm{mm\,Hg}}$ to the nearest integer. The numerical entry is 314.

Quick Tip to Solve: Subtract the two mixture equations first to obtain $P_B^\circ - P_A^\circ$; then substitute into either total-pressure equation.

88. If the pKa of lactic acid is 5, then the pH of 0.005 M calcium lactate solution at 25°C is _____ $\times 10^{-1}$ (Nearest integer) [4 marks]



Solution:

Concept:

- Calcium lactate is a salt of a strong base and a weak acid. Lactate, L^-, hydrolyses according to $L^- + H_2O \rightleftharpoons HL + OH^-$.
- For the conjugate base, $K_b = K_w/K_a$. At 25°C, $K_w = 10^{-14}$, and $pH + pOH = 14$.

Step 1: Find the concentration of lactate ions Each formula unit of calcium lactate provides one Ca^{2+} ion and two lactate ions. Thus a $0.005\,mol\,L^{-1}$ salt solution has formal lactate concentration

$$C_L = 2(0.005) = 0.010\,mol\,L^{-1}.$$

The factor of two must be included before applying the weak-base hydrolysis relation. From $pK_a = 5$, we have $K_a = 10^{-5}$, so

$$K_b = \frac{10^{-14}}{10^{-5}} = 10^{-9}.$$

Step 2: Calculate hydroxide concentration and pH Let $x = [OH^-]$ formed by hydrolysis. Since K_b is small, $x \ll C_L$, and

$$x \simeq \sqrt{K_b C_L} = \sqrt{10^{-9} \times 10^{-2}} = 10^{-5.5}\,mol\,L^{-1}.$$

The approximation is consistent because $x/C_L \simeq 3.2 \times 10^{-4}$. Therefore

$$pOH = 5.5, \qquad pH = 14 - 5.5 = 8.5.$$

The solution is basic, as expected for hydrolysis of the conjugate base of a weak acid. If the required blank is n, then $n \times 10^{-1} = 8.5$, so $n = 85$.

Step 3: Conclusion The pH is $8.5 = \boxed{85} \times 10^{-1}$. Enter 85 in the blank. Both pH and the requested numerical coefficient are dimensionless.

Quick Tip to Solve: For calcium salts of monobasic acids, double the salt concentration to obtain the anion concentration before using $[OH^-] \simeq \sqrt{K_b C}$.

89. The number of statement/s which are the characteristics of physisorption is _____

A. It is highly specific in nature

B. Enthalpy of adsorption is high

C. It decreases with increase in temperature

D. It results into unimolecular layer

E. No activation energy is needed [4 marks]

Solution:

Concept:

- Physisorption is caused mainly by weak intermolecular van der Waals forces between adsorbate and adsorbent. It differs from chemisorption, which involves formation of chemical bonds.
- Physical adsorption is exothermic and usually reversible; it can form multiple layers and generally does not require appreciable activation energy.

Step 1: Test specificity and adsorption enthalpy Statement A is false: physisorption is not highly specific because van der Waals attraction can act between many kinds of molecules and surfaces. High specificity is more characteristic of chemical bond formation in chemisorption. Statement B is also false: the magnitude of the enthalpy of physisorption is relatively low, consistent with weak intermolecular binding. A high adsorption enthalpy instead suggests chemisorption.

Step 2: Test temperature, layers and activation energy Statement C is true. Since adsorption releases heat, increasing temperature favours desorption and reduces the equilibrium extent of physisorption. Statement D is false: physisorption can produce multimolecular layers because an adsorbed molecule can attract further molecules; restriction to a unimolecular layer is characteristic of chemisorption. Statement E is true: physisorption ordinarily needs no appreciable activation energy, since no new chemical bond must be formed. Exactly C and E satisfy the stated characteristics.

Step 3: Conclusion The correct statements are C and E, so the required number is $\boxed{2}$, a dimensionless count.

Quick Tip to Solve: Associate physisorption with weak forces, low temperature and multiple layers; specificity and high bond energies point toward chemisorption.

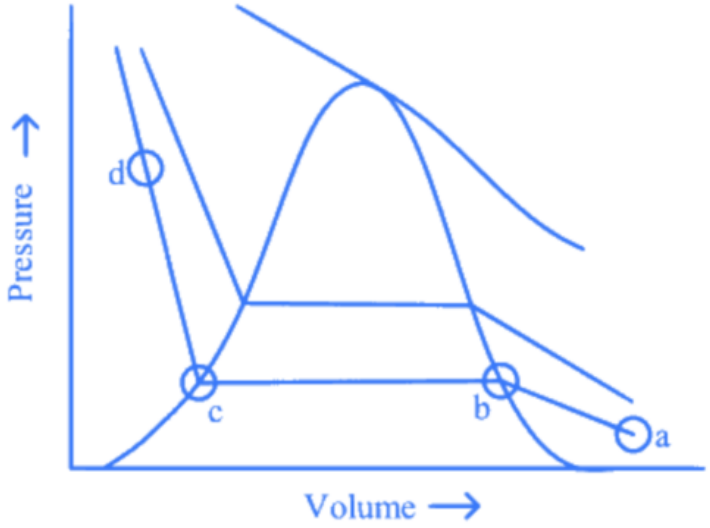
90. The number of statement/s, which are correct with respect to the compression of carbon dioxide from point (a) in the Andrews isotherm from the following is _____

A. Carbon dioxide remains as a gas upto point (b)

B. Liquid carbon dioxide appears at point (c)

C. Liquid and gaseous carbon dioxide coexist between points (b) and (c)

D. As the volume decreases from (b) and (c), the amount of liquid decreases [4 marks]



Solution:

Concept:

- On a subcritical Andrews isotherm, compression first compresses the gas, then liquefies it at nearly constant pressure, and finally compresses the liquid.
- The horizontal segment from b to c represents gas–liquid coexistence. Compression proceeds toward smaller volume, from the right-hand end b to the left-hand end c.

Step 1: Locate the onset and completion of liquefaction From a toward b, carbon dioxide is gaseous and its pressure rises as its volume decreases. Thus A is true: it remains a gas up to the onset of condensation at b. The first liquid appears at b, not at c. At c, liquefaction is complete and the last gas disappears. Therefore B, which places the appearance of liquid at c, is false. Further compression beyond c causes a steep pressure rise because a liquid is only slightly compressible.

Step 2: Follow the phase amounts along the horizontal segment Between b and c, both liquid and gaseous carbon dioxide are present, so C is true. As compression decreases the volume along this horizontal segment, more gas condenses. The amount of liquid therefore increases, while the amount of gas decreases. The pressure stays approximately constant at the saturation vapour pressure for that temperature. Consequently D is false, since it states that the amount of liquid decreases. The two correct statements are A and C; the positions of b and c must be read in the direction of decreasing volume.

Step 3: Conclusion Statements A and C are correct. The required number is $\boxed{2}$, a dimensionless count.

Quick Tip to Solve: Read compression from right to left on a pressure–volume graph: the first liquid appears at the right end of the plateau, and all gas is gone at the left end.