JEE Main 2023 Paper 1 (B.E./B.Tech) Jan 25 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 25 January 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The equations of two sides of a variable triangle are $x = 0$ and $y = 3$, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is : [4 marks]

(A) $4y^2 - 18y - 3x - 18 = 0$

(B) $4y^2 + 18y + 3x + 18 = 0$

(C) $4y^2 - 18y + 3x + 18 = 0$

(D) $4y^2 - 18y - 3x + 18 = 0$

Correct Answer: (C) $4y^2 - 18y + 3x + 18 = 0$

Solution:

Concept:

- A tangent to $y^2 = 4ax$ with nonzero slope m is $y = mx + a/m$; here $a = 3/2$.
- The lines $x = 0$ and $y = 3$ meet at a right angle. The circumcentre of a right triangle is the midpoint of its hypotenuse.

Step 1: Find the endpoints of the hypotenuse The tangent is $y = mx + 3/(2m)$. It cuts the two fixed sides at $P = (0, 3/(2m))$ and $Q = ((3 - 3/(2m))/m, 3)$. The right-angle vertex is $(0, 3)$, so PQ is the hypotenuse. Take a nondegenerate triangle; a tangent through the right-angle vertex is excluded.

Step 2: Express the midpoint using one parameter Write $t = 1/m$ and let the circumcentre be (X, Y). The midpoint formula gives

$$X = \frac{3t}{2} - \frac{3t^2}{4}, \qquad Y = \frac{3}{2} + \frac{3t}{4}.$$

Thus $t = (4Y - 6)/3$. Both coordinates must use the same tangent parameter.

Step 3: Eliminate the tangent parameter Substitute this value of t into the expression for X:

$$\begin{aligned} 3X &= \frac{9}{2}t - \frac{9}{4}t^2 \\ &= (6Y - 9) - (4Y^2 - 12Y + 9) \\ &= -4Y^2 + 18Y - 18. \end{aligned}$$

Hence the coefficient of X and the constant term are both positive when the equation is moved to the left. The other three listed equations have different signs.

Step 4: Conclusion The locus has equation $4y^2 - 18y + 3x + 18 = 0$, with degenerate-triangle parameter values excluded. Hence, the correct option is (C).

Quick Tip to Solve: Recognise the right angle first: the midpoint of the two tangent intercepts gives the circumcentre without solving perpendicular bisectors.

2. The foot of perpendicular of the point (2, 0, 5) on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then, which of the following is NOT correct? [4 marks]

(A) $\frac{\alpha}{\beta} = -8$

(B) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$

(C) $\frac{\beta}{\gamma} = -5$

(D) $\frac{\gamma}{\alpha} = \frac{5}{8}$

Correct Answer: (C) $\frac{\beta}{\gamma} = -5$

Solution:

Concept:

- A point on the line can be written as $H = (-1+2t, 1+5t, -1-t)$, with direction vector $\mathbf{d} = (2, 5, -1)$.
- The foot from P satisfies $(H - P) \cdot \mathbf{d} = 0$. The question asks for the relation that is NOT correct.

Step 1: Impose perpendicularity For $P = (2, 0, 5)$,

$$H - P = (-3 + 2t, 1 + 5t, -6 - t).$$

Taking the dot product with the line direction gives

$$2(-3 + 2t) + 5(1 + 5t) - (-6 - t) = 0.$$

The constants add to 5, while the coefficient of t is $4 + 25 + 1 = 30$. Therefore $5 + 30t = 0$, or $t = -1/6$.

Step 2: Compute all three coordinates Substitution gives

$$\alpha = -\frac{4}{3}, \qquad \beta = \frac{1}{6}, \qquad \gamma = -\frac{5}{6}.$$

These coordinates belong to the given line, and their displacement from P is orthogonal to its direction, so they describe the required foot.

Step 3: Check each proposed relation The ratios and product are

$$\frac{\alpha}{\beta} = -8, \qquad \frac{\alpha\beta}{\gamma} = \frac{4}{15}, \qquad \frac{\beta}{\gamma} = -\frac{1}{5}, \qquad \frac{\gamma}{\alpha} = \frac{5}{8}.$$

The first, second and fourth statements are true. The third statement claims -5, which is the reciprocal of the actual ratio.

Step 4: Conclusion The incorrect relation is $\beta/\gamma = -5$, since actually $\beta/\gamma = -1/5$. Hence, the correct option is (C).

Quick Tip to Solve: After finding a projection, evaluate every listed ratio; a NOT-correct question often hides a reciprocal error.

3. The number of functions

$$f : \{1, 2, 3, 4\} \to \{a \in \mathbb{Z} : |a| \leq 8\}$$

satisfying $f(n) + \frac{1}{n} f(n+1) = 1$**, for all** $n \in \{1, 2, 3\}$**, is** [4 marks]

(A) 2

(B) 3

(C) 1

(D) 4

Correct Answer: (A) 2

Solution:

Concept:

- The recurrence $f(n) + f(n+1)/n = 1$ determines the later function values once $f(1)$ is chosen.
- Every value must be an integer in the codomain $\{a \in \mathbb{Z} : |a| \leq 8\}$. Count admissible initial values, not independent choices at four inputs.

Step 1: Express the function in terms of its first value Put $f(1) = t$, where t is an integer. Using the recurrence successively for $n = 1, 2, 3$ gives

$$\begin{aligned} f(2) &= 1 - t, \\ f(3) &= 2(1 - f(2)) = 2t, \\ f(4) &= 3(1 - f(3)) = 3 - 6t. \end{aligned}$$

Thus a possible function is completely specified by the ordered tuple $(t, 1 - t, 2t, 3 - 6t)$.

Step 2: Apply the codomain bounds The last value must satisfy

$$-8 \leq 3 - 6t \leq 8 \quad \Longrightarrow \quad -\frac{5}{6} \leq t \leq \frac{11}{6}.$$

Since t is an integer, only $t = 0$ and $t = 1$ remain. It is necessary to check the other three values too; the strongest-looking bound alone would not guarantee sufficiency.

Step 3: Verify the surviving functions The two tuples are $(0, 1, 0, 3)$ and $(1, 0, 2, -3)$. All entries lie between -8 and 8, and the recurrence is satisfied by construction. No other integer t survives the bound on $f(4)$, so counts one, three and four are ruled out.

Step 4: Conclusion Exactly two functions satisfy all the conditions. Hence, the correct option is (A).

Quick Tip to Solve: For a recurrence defining a function on a finite set, start with one parameter and use the tightest output bound before checking the remaining outputs.

4. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2, \sqrt{3N}, N+2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is : [4 marks]

(A) 8

(B) 16

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- Three positive numbers a, b, c are in geometric progression precisely when $b^2 = ac$.
- Two fair dice have 36 equally likely ordered outcomes. Different sums are not equally likely.

Step 1: Translate the geometric-progression condition The three terms are $N-2, \sqrt{3N}, N+2$, where a dice sum satisfies $2 \le N \le 12$. The middle-term condition gives

$$3N = (N-2)(N+2) = N^2 - 4.$$

Consequently

$$N^2 - 3N - 4 = (N-4)(N+1) = 0.$$

Only $N = 4$ belongs to the possible range of dice sums; $N = -1$ is impossible. For $N = 4$, the terms $2, \sqrt{12}, 6$ really do have equal positive successive ratios.

Step 2: Count ordered dice outcomes The outcomes with sum four are $(1,3), (2,2), (3,1)$. They give three favourable outcomes out of 36, so

$$\Pr(N = 4) = \frac{3}{36} = \frac{1}{12}.$$

The two orders $(1,3)$ and $(3,1)$ must both be counted because the dice are independent.

Step 3: Match the required probability form Equating the calculated probability with the form in the question,

$$\frac{k}{48} = \frac{1}{12} \quad \Longrightarrow \quad k = 4.$$

The other listed values would produce probabilities $1/6$, $1/3$ or $1/24$, none of which equals the outcome count.

Step 4: Conclusion The required value is $k = 4$. Hence, the correct option is (D).

Quick Tip to Solve: First reduce the progression condition to an equation for the dice sum, then count ordered outcomes rather than treating each possible sum as equally likely.

5. If the function $f(x)=\begin{cases}(1+|\cos x|)^{\frac{\lambda}{|\cos x|}} & , \quad 0<x<\frac{\pi}{2} \\ \mu & , \quad x=\frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}} & , \quad \frac{\pi}{2}<x<\pi\end{cases}$ **is continuous at** $x=\frac{\pi}{2}$**, then** $9\lambda+6\log_e\mu+\mu^6-e^{6\lambda}$ **is equal to** [4 marks]

(A) 11

(B) 10

(C) 8

(D) $2e^4 + 8$

Correct Answer: (B) 10

Solution:

Concept:

- Continuity at $x=\pi/2$ requires the left limit, right limit and the assigned value μ to be equal.
- The standard limits are $\log(1+u)/u \to 1$ as $u \to 0$ and $\cot v \sim 1/v$ as $v \to 0$.

Step 1: Find the left-hand limit As $x \to (\pi/2)^-$, put $u=|\cos x| \to 0^+$. Taking logarithms of the first branch gives

$$\log f(x)=\lambda\frac{\log(1+u)}{u} \longrightarrow \lambda.$$

Therefore the left limit is e^{λ}. The absolute value keeps the base positive, so this logarithmic calculation is valid.

Step 2: Find the right-hand limit Write $x=\pi/2+h$, with $h \to 0^+$. By periodicity, $\cot 6x=\cot 6h$ and $\cot 4x=\cot 4h$. Hence

$$\frac{\cot 6x}{\cot 4x}=\frac{\tan 4h}{\tan 6h} \longrightarrow \frac{4}{6}=\frac{2}{3}.$$

The right limit is thus $e^{2/3}$, not $e^{3/2}$.

Step 3: Use continuity and simplify It follows that $\mu=e^{2/3}$ and, because the real exponential is one-to-one, $\lambda=2/3$. Then

$$\begin{aligned}9\lambda+6\log_e\mu+\mu^6-e^{6\lambda} &= 6+4+e^4-e^4 \\ &= 10.\end{aligned}$$

The exponential terms cancel exactly. Retaining them or reversing the cotangent ratio would incorrectly suggest one of the other choices.

Step 4: Conclusion The expression equals 10. Hence, the correct option is (B).

Quick Tip to Solve: Near a common pole, replace a ratio of cotangents by the reciprocal ratio of their small arguments; use continuity before expanding the final expression.

6. Let A, B, C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements

(S1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric

(S2) $A^{26}C^{13} - C^{13}A^{26}$ is symmetric

Then, [4 marks]

(A) Only S2 is true

(B) Only S1 is true

(C) Both S1 and S2 are false

(D) Both S1 and S2 are true

Correct Answer: (A) Only S2 is true

Solution:

Concept:

- For a symmetric matrix A, every power satisfies $(A^n)^T = A^n$. For skew-symmetric B, $(B^n)^T = (-1)^n B^n$.
- Transposition reverses product order: $(UV)^T = V^T U^T$. A commutator must therefore be tested with the transpose of both factors.

Step 1: Test the first statement Put $P = A^{13}$ and $Q = B^{26}$. Both are symmetric, since the exponent of B is even. Thus

$$(PQ - QP)^T = QP - PQ = -(PQ - QP).$$

The expression in S1 is skew-symmetric in general, rather than symmetric. It could be both only when it vanishes, and commutation is not given.

Step 2: Check that the first expression need not vanish For example, take $A = \mathrm{diag}(1, 2, 3)$ and $B = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. Here $B^4 = -2B^2$, so $B^{26} = (-2)^{12}B^2$ has a nonzero entry in position $(2, 3)$. Since the corresponding diagonal entries of A^{13} differ, its commutator with B^{26} is nonzero. S1 is not a valid general assertion.

Step 3: Test the second statement Put $R = A^{26}$, which is symmetric, and $S = C^{13}$, which is skew-symmetric. Then

$$(RS - SR)^T = (-S)R - R(-S) = RS - SR.$$

So S2 is symmetric for every allowed pair of matrices. The choices asserting S1 or denying S2 are therefore ruled out.

Step 4: Conclusion Only S2 is true. Hence, the correct option is (A).

Quick Tip to Solve: An even power of a skew-symmetric matrix is symmetric; an odd power remains skew-symmetric. Then reverse the product order when transposing.

7. The shortest distance between the lines $x+1=2y=-12z$ and $x=y+2=6z-6$ is : [4 marks]

(A) 3

(B) $\frac{5}{2}$

(C) $\frac{3}{2}$

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- For lines through $\mathbf{a}$ and $\mathbf{b}$ with nonparallel directions $\mathbf{d}_1, \mathbf{d}_2$, the distance is

$$d = \frac{|(\mathbf{b}-\mathbf{a}) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)|}{|\mathbf{d}_1 \times \mathbf{d}_2|}.$$

- The scalar triple product projects the joining vector onto a common normal to both lines. Scaling a direction vector does not change the distance.

Step 1: Choose points and direction vectors For the first line, set $x+1=2y=-12z=t$, giving $(x,y,z)=(-1+t, t/2, -t/12)$. We may use $\mathbf{a}=(-1,0,0)$ and $\mathbf{d}_1=(12,6,-1)$. For the second line set $x=y+2=6z-6=s$, giving $(s, s-2, 1+s/6)$. Thus $\mathbf{b}=(0,-2,1)$ and $\mathbf{d}_2=(6,6,1)$.

Step 2: Calculate the common normal The cross product is

$$\mathbf{d}_1 \times \mathbf{d}_2 = (12,-18,36) = 6(2,-3,6).$$

Its magnitude is $6\sqrt{4+9+36}=42$, which is nonzero, so the nonparallel-line formula applies.

Step 3: Project the joining vector The joining vector is $\mathbf{b}-\mathbf{a}=(1,-2,1)$. Hence

$$(\mathbf{b}-\mathbf{a}) \cdot (\mathbf{d}_1 \times \mathbf{d}_2) = 12+36+36=84.$$

Therefore

$$d = \frac{|84|}{42} = 2.$$

This is positive, so the lines are skew rather than intersecting. The other three listed lengths disagree with the exact projection calculation.

Step 4: Conclusion The shortest distance is 2 units. Hence, the correct option is (D).

Quick Tip to Solve: Use integer multiples of fractional direction vectors to simplify the cross product, but keep the same scaling in its magnitude and scalar triple product.

8. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is : [4 marks]

(A) $4\sqrt{6} - \frac{28}{3}$
(B) $4\sqrt{6} - \frac{44}{3}$
(C) $4\sqrt{6} + \frac{28}{3}$
(D) $4\sqrt{6} + \frac{44}{3}$

Correct Answer: (C) $4\sqrt{6} + \frac{28}{3}$

Solution:

Concept:

- Completing squares identifies the centre and the axes of a hyperbola. For a horizontal hyperbola, the transverse axis is horizontal through its centre.
- The area between an upper curve and a lower curve is $\int (y_{\text{upper}} - y_{\text{lower}})\, dx$. The condition to the right of a vertical line determines the lower limit.

Step 1: Locate the two axes Complete the squares in the given equation:

$$16(x+2)^2 - (y-2)^2 = 16, \qquad (x+2)^2 - \frac{(y-2)^2}{16} = 1.$$

The centre is $(-2, 2)$, and the positive square is the horizontal one. Consequently the transverse axis T is $y = 2$, while the conjugate axis C is $x = -2$. These are axes through the centre, rather than the coordinate axes.

Step 2: Determine and integrate the bounded region The parabola is $y = x^2 - 4$. Below T and above this parabola requires $x^2 - 4 \le 2$, so $-\sqrt{6} \le x \le \sqrt{6}$. Being on the right of C further requires $x \ge -2$. Thus the exact interval is $[-2, \sqrt{6}]$, and

$$A = \int_{-2}^{\sqrt{6}} (6 - x^2)\, dx = \left[6x - \frac{x^3}{3}\right]_{-2}^{\sqrt{6}}.$$

The upper endpoint contributes $4\sqrt{6}$, and the lower endpoint contributes $-28/3$, giving $A = 4\sqrt{6} + 28/3$. Options A and B incorrectly subtract the left-hand contribution; D has the wrong rational term.

Step 3: Conclusion The required area is $\boxed{4\sqrt{6} + \frac{28}{3}}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: First translate the hyperbola to its centre; its axes supply the horizontal boundary and vertical cutoff for the area integral.

9. Let the function $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is [4 marks]

(A) $\left(-\frac{9}{2}, \frac{9}{2}\right)$

(B) $\left(\frac{9}{2}, \infty\right)$

(C) $\left(0, \frac{9}{2}\right)$

(D) $\left(-\infty, \frac{9}{2}\right)$

Correct Answer: (D) $\left(-\infty, \frac{9}{2}\right)$

Solution:

Concept:

- A differentiable polynomial has stationary points at the real roots of its derivative. A quadratic with positive leading coefficient is positive outside its two distinct roots and negative between them.
- Two real nonzero roots have opposite signs exactly when their product is negative. Vieta's formula expresses this product as the constant term divided by the leading coefficient.

Step 1: Differentiate and impose opposite signs The derivative is

$$f\prime(x) = 6x^2 + 2(2p-7)x + 3(2p-9).$$

Write its stationary roots as $r_1 < r_2$. The question needs $r_1 < 0 < r_2$. Their product must therefore satisfy

$$r_1 r_2 = \frac{3(2p-9)}{6} = \frac{2p-9}{2} < 0.$$

This gives $p < 9/2$. A negative root product also guarantees that the roots are distinct and real, so no further discriminant restriction is needed.

Step 2: Check the types of the stationary points For $p < 9/2$, the upward-opening derivative quadratic is positive before r_1, negative between r_1 and r_2, and positive after r_2. Hence f changes from increasing to decreasing at the negative root, giving a local maximum; at the positive root it changes from decreasing to increasing, giving a local minimum. At $p = 9/2$, one stationary point is zero and is excluded. Options A and C impose unnecessary lower bounds; for example $p = -10$ satisfies the negative-product test. Option B gives a positive product and cannot have roots of opposite signs.

Step 3: Conclusion All and only $\boxed{p \in (-\infty, 9/2)}$ satisfy both extremum conditions. Hence, the correct option is (D).

Quick Tip to Solve: When a cubic with positive leading coefficient needs one negative maximum and one positive minimum, test the product of the derivative roots first.

10. Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2, z \neq -i$. Then z lies on the circle of radius 2 and centre : [4 marks]

(A) (0, -2)

(B) (0, 0)

(C) (0, 2)

(D) (2, 0)

Correct Answer: (A) (0, -2)

Solution:

Concept:

- For $z = x + iy$, the modulus $|z - (a + ib)|$ is the Euclidean distance from (x, y) to (a, b).
- A fixed ratio of distances to two distinct points defines an Apollonius circle when the ratio is different from one. Squaring positive moduli gives its Cartesian equation.

Step 1: Convert the modulus condition into coordinates Let $z = x + iy$, where x, y are real. Since $z \neq -i$, multiply by the denominator modulus to get $|z - 2i| = 2|z + i|$. Both sides are nonnegative, so squaring is equivalent to the original condition:

$$x^2 + (y - 2)^2 = 4\{x^2 + (y + 1)^2\}.$$

The factor on the right is four because the distance ratio is two. Using only a factor two here would give the wrong circle.

Step 2: Complete the square and identify the centre Expanding and collecting terms gives

$$x^2 + y^2 - 4y + 4 = 4x^2 + 4y^2 + 8y + 4,$$

$$3x^2 + 3y^2 + 12y = 0, \qquad x^2 + (y + 2)^2 = 4.$$

This is a circle with centre $(0, -2)$ and radius 2. The excluded point $(0, -1)$ does not lie on this circle, since substitution gives left-hand side 1, not 4; hence no point of the circle is lost. Centres $(0, 0)$, $(0, 2)$, and $(2, 0)$ do not reproduce the linear term $4y$, ruling out B, C, and D.

Step 3: Conclusion The circle has radius 2 and centre $\boxed{(0, -2)}$. Hence, the correct option is (A).

Quick Tip to Solve: Square the distance ratio, expand once, and divide before completing the square; the sign inside the square is opposite to the centre coordinate.

11. The integral $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to [4 marks]

(A) $\frac{11}{12} + \log_e 4$

(B) $\frac{11}{6} + \log_e 4$

(C) $\frac{11}{12} - \log_e 4$

(D) $\frac{11}{6} - \log_e 4$

Correct Answer: (D) $\frac{11}{6} - \log_e 4$

Solution:

Concept:

- For an integral containing $x^{-3}\,dx$ and rational expressions in x^2, the substitution $u = x^{-2}$ converts it to a rational integral.
- Change the bounds with each substitution. Logarithmic terms from $\int dv/v$ are evaluated as differences, which combine into a logarithm of a ratio.

Step 1: Reduce the integral to a rational expression Let I denote the given integral and put $u = 1/x^2$. Then $du = -2x^{-3}\,dx$, and $x^2 + 2 = (1 + 2u)/u$. When $x = 1$, $u = 1$; when $x = 2$, $u = 1/4$. Reversing the limits removes the minus sign:

$$I = 8\int_{1/4}^{1} \frac{u^2}{(1+2u)^2}\,du.$$

Now set $v = 1 + 2u$. Since $u = (v - 1)/2$ and $du = dv/2$, the coefficient eight cancels the denominator eight, and the new bounds are $3/2$ and 3.

Step 2: Integrate and evaluate both endpoints The transformed integrand separates directly:

$$I = \int_{3/2}^{3} \left(1 - \frac{2}{v} + \frac{1}{v^2}\right) dv = \left[v - 2\ln v - \frac{1}{v}\right]_{3/2}^{3}.$$

Its rational part is

$$\left(3 - \frac{1}{3}\right) - \left(\frac{3}{2} - \frac{2}{3}\right) = \frac{11}{6},$$

and its logarithmic part is $-2\ln 3 + 2\ln(3/2) = -2\ln 2 = -\ln 4$. Thus the answer is $11/6 - \ln 4$, about 0.447. C is negative, impossible for a positive integrand; A and B have the wrong logarithm sign and do not match the evaluated endpoints.

Step 3: Conclusion The integral equals $\boxed{\frac{11}{6} - \log_e 4}$. Hence, the correct option is (D).

Quick Tip to Solve: With $x^{-3}\,dx$, try $u = 1/x^2$; after that, a linear shift often turns the denominator into simple inverse powers.

12. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m, such that the range of f is [0, 2]. Then the value of m is _____ [4 marks]

(A) 4

(B) 3

(C) 5

(D) 2

Correct Answer: (C) 5

Solution:

Concept:

- The sinusoidal expression $\sqrt{2}(\sin x - \cos x) = 2\sin(x - \pi/4)$ takes every value in $[-2, 2]$ for real x.
- A real logarithm requires a positive argument and a base greater than zero and different from one. A logarithm with base greater than one is strictly increasing.

Step 1: Use the domain to restrict the parameter The logarithm argument has exact range

$$m - 2 + [-2, 2] = [m - 4, m].$$

The function is defined for every real x, including values attaining the minimum. Therefore $m - 4 > 0$, so $m > 4$. This also ensures $\sqrt{m} > 1$, making the logarithm increasing. The endpoint cannot be zero: the logarithm of zero is undefined. These conditions already exclude the proposed values 4, 3, and 2.

Step 2: Match the range endpoints Because the logarithm is continuous and increasing on the positive interval, its range is

$$\left[\log_{\sqrt{m}}(m - 4), \log_{\sqrt{m}} m\right].$$

For every allowed m, the upper endpoint is two, since $(\sqrt{m})^2 = m$. For the lower endpoint to be zero, its argument must equal one:

$$\log_{\sqrt{m}}(m - 4) = 0 \quad \Longrightarrow \quad m - 4 = 1 \quad \Longrightarrow \quad m = 5.$$

Conversely, at $m = 5$ the argument ranges over $[1, 5]$, and logarithms to base $\sqrt{5}$ take exactly the values $[0, 2]$. This checks both domain and the complete required range.

Step 3: Conclusion The unique admissible value is $\boxed{m = 5}$. Hence, the correct option is (C).

Quick Tip to Solve: Before matching logarithmic endpoints, use the minimum argument to enforce the stated domain; here it immediately forces the base above one.

13. The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1, 3, 5, 7, 9 without repetition, is : [4 marks]

(A) 120

(B) 6

(C) 72

(D) 12

Correct Answer: (C) 72

Solution:

Concept:

- A number strictly between 5000 and 10000 has exactly four decimal digits. Its thousands digit decides whether it exceeds 5000.
- For ordered choices without repetition, multiply the available choices at successive positions. After choosing one digit from five, four digits remain for the next position.

Step 1: Choose the thousands digit The permitted digit set is $\{1, 3, 5, 7, 9\}$. A four-digit number beginning with 1 or 3 is below 5000, so these cannot occupy the thousands place. The thousands digit may be 5, 7, or 9, giving three choices. A number beginning with 5 is indeed strictly greater than 5000, because its remaining digits are selected from nonzero digits; equality with 5000 is impossible. Any such four-digit number is automatically below 10000.

Step 2: Fill the remaining places in order After fixing the thousands digit, four distinct digits remain. Choose the hundreds digit in four ways, the tens digit in three ways, and the units digit in two ways. Consequently

$$N = 3 \times 4 \times 3 \times 2 = 72.$$

Equivalently, each of the three allowed initial digits has ${}^4P_3 = 24$ completions. Counting ${}^5P_4 = 120$, as in A, incorrectly includes numbers starting with 1 or 3. The values 6 and 12 in B and D omit many ordered completions; for instance one allowed thousands digit alone already gives 24. No division by a factorial is appropriate, because reversing the last digits generally changes the number.

Step 3: Conclusion There are $\boxed{72}$ such numbers. Hence, the correct option is (C).

Quick Tip to Solve: Apply a numerical size restriction to the leading digit first, then use permutations for the remaining ordered positions.

14. Let $y = y(t)$ be a solution of the differential equation $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$ where, $\alpha > 0, \beta > 0$ and $\gamma > 0$. Then $\lim_{t\to\infty} y(t)$ [4 marks]

(A) is 0

(B) is 1

(C) is -1

(D) does not exist

Correct Answer: (A) is 0

Solution:

Concept:

- The integrating factor for $y\prime + \alpha y = \gamma e^{-\beta t}$ is $e^{\alpha t}$, so multiplication turns the left side into $(e^{\alpha t}y)\prime$.
- For a positive constant c, both e^{-ct} and te^{-ct} tend to zero as $t \to \infty$. The latter limit follows from exponential growth dominating a polynomial.

Step 1: Solve when the two decay rates differ Multiplying the equation by the integrating factor gives

$$\frac{d}{dt}(e^{\alpha t}y) = \gamma e^{(\alpha-\beta)t}.$$

If $\alpha \neq \beta$, integrate to obtain

$$e^{\alpha t}y = C + \frac{\gamma}{\alpha-\beta}e^{(\alpha-\beta)t}, \qquad y = Ce^{-\alpha t} + \frac{\gamma}{\alpha-\beta}e^{-\beta t}.$$

Both exponents have negative coefficients because $\alpha, \beta > 0$. Thus every solution in this case tends to zero, irrespective of the arbitrary constant C and the sign of $\alpha - \beta$.

Step 2: Include the equal-rate case and take the limit If $\alpha = \beta$, the transformed equation instead reads $(e^{\alpha t}y)\prime = \gamma$, so

$$y = (C + \gamma t)e^{-\alpha t}.$$

Here $Ce^{-\alpha t} \to 0$ and $te^{-\alpha t} = t/e^{\alpha t} \to 0$, for example by one application of L'Hopital's rule. Therefore the limit is zero in the equal-rate case too. Options B and C would require a nonzero persistent term, but every term decays. Option D is excluded because these explicit formulas give an existing limit for every permitted parameter choice.

Step 3: Conclusion For every solution and all the specified positive parameters, $\boxed{\lim_{t\to\infty} y(t) = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Always separate the case of equal exponential rates before dividing by their difference; the resulting factor of t still decays against the exponential.

15. Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$**, where** $i = \sqrt{-1}$**. If** $\mathrm{M} = \mathrm{A^T BA}$**, then the inverse of the matrix** $\mathrm{AM^{2023}A^T}$ **is** [4 marks]

(A) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

Correct Answer: (C) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Solution:

Concept:

- For a real orthogonal matrix, $A^{-1} = A^T$, so $AA^T = A^TA = I$. Its columns have unit length and zero mutual scalar product.
- If $B = I + N$ with $N^2 = 0$, the binomial expansion gives $B^m = I + mN$. Similarity preserves products and positive integer powers.

Step 1: Check orthogonality and simplify the power The two column norms squared of A are both $(1+9)/10 = 1$, while their scalar product is $(3-3)/10 = 0$. Thus $AA^T = I$. In products of $M = A^TBA$, every adjacent factor AA^T cancels. Consequently,

$$M^{2023} = A^TB^{2023}A, \qquad AM^{2023}A^T = B^{2023}.$$

This avoids raising a matrix with complicated entries directly to a large power.

Step 2: Compute the power and invert it Write $B = I+N$, where $N = \begin{pmatrix} 0 & -i \\ 0 & 0 \end{pmatrix}$. Direct multiplication gives $N^2 = 0$, so all binomial terms after the linear term vanish:

$$B^{2023} = I + 2023N = \begin{pmatrix} 1 & -2023i \\ 0 & 1 \end{pmatrix}.$$

Since $(I+2023N)(I-2023N) = I$, its inverse is $I - 2023N$. Option (A) is the matrix before inversion; (B) and (D) incorrectly move the nonzero off-diagonal entry to the lower-left corner.

Step 3: Conclusion The required inverse is $\begin{pmatrix} 1 & 2023i \\ 0 & 1 \end{pmatrix}$. Hence, the correct option is (C).

Quick Tip to Solve: Cancel the orthogonal similarity factors before taking a large power; a strictly upper triangular two-by-two matrix has square zero.

16. $\sum_{k=0}^{6} {}^{51-k}C_3$ **is equal to :** [4 marks]

(A) ${}^{51}\mathrm{C}_4 - {}^{45}\mathrm{C}_4$

(B) ${}^{51}\mathrm{C}_3 - {}^{45}\mathrm{C}_3$

(C) ${}^{52}\mathrm{C}_3 - {}^{45}\mathrm{C}_3$

(D) ${}^{52}\mathrm{C}_4 - {}^{45}\mathrm{C}_4$

Correct Answer: (D) ${}^{52}\mathrm{C}_4 - {}^{45}\mathrm{C}_4$

Solution:

Concept:

- Pascal's identity gives $\binom{r+1}{4} = \binom{r}{4} + \binom{r}{3}$, hence $\binom{r}{3} = \binom{r+1}{4} - \binom{r}{4}$.
- A sum of consecutive binomial coefficients can therefore telescope: all intermediate fourth-order coefficients cancel, leaving only the two boundary terms.

Step 1: Rewrite the summation limits As k increases from 0 to 6, the upper entry $51 - k$ decreases from 51 to 45. The order of addition does not affect the result, so the given sum is

$$S = \binom{45}{3} + \binom{46}{3} + \cdots + \binom{51}{3}.$$

There are seven terms, with both endpoints included. This endpoint check is essential because dropping the term with upper entry 51 would change the first boundary coefficient.

Step 2: Apply Pascal's identity term by term Using the identity for each upper entry gives

$$\begin{aligned} S &= \left[\binom{46}{4} - \binom{45}{4}\right] + \left[\binom{47}{4} - \binom{46}{4}\right] + \cdots + \left[\binom{52}{4} - \binom{51}{4}\right] \\ &= \binom{52}{4} - \binom{45}{4}. \end{aligned}$$

Option (A) omits the contribution corresponding to upper entry 51. Options (B) and (C) retain lower entry 3 rather than increasing it to 4, so they cannot be the telescoping boundary expression.

Step 3: Conclusion The sum is $\binom{52}{4} - \binom{45}{4}$. Hence, the correct option is (D).

Quick Tip to Solve: For consecutive upper entries with fixed lower entry, increase the lower entry by one and subtract the boundary coefficients.

17. Let $f(x) = 2x^n + \lambda, \lambda \in R, n \in N$, and $f(4) = 133, f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is [4 marks]

(A) 60

(B) 58

(C) 61

(D) 59

Correct Answer: (A) 60

Solution:

Concept:

- Subtracting two values of $f(x) = 2x^n + \lambda$ eliminates the additive constant λ.
- For a positive integer $N = p^a q^b$ with distinct primes, the sum of its positive divisors is $(1 + p + \cdots + p^a)(1 + q + \cdots + q^b)$.

Step 1: Determine the exponent from the two values The stated values imply

$$2(5^n - 4^n) = 255 - 133 = 122, \qquad 5^n - 4^n = 61.$$

For $n = 1, 2, 3$, the differences are 1, 9 and 61 respectively, so $n = 3$ works. It is also unique: if $d_n = 5^n - 4^n$, then

$$d_{n+1} = 5d_n + 4^n > d_n$$

for every positive integer n. Thus the sequence is strictly increasing and cannot take the value 61 at another natural exponent. Substituting back gives $\lambda = 133 - 2(4^3) = 5$, consistent with $f(5) = 255$.

Step 2: Factor the required difference and add its divisors The constant cancels again when computing the required integer:

$$f(3) - f(2) = 2(3^3 - 2^3) = 2(27 - 8) = 38 = 2 \cdot 19.$$

Since both prime factors occur to the first power, its positive divisors are exactly $1, 2, 19, 38$. Their sum is

$$1 + 2 + 19 + 38 = (1 + 2)(1 + 19) = 60.$$

The alternatives 58, 61 and 59 are not this sum; in particular, 61 is the earlier power difference, not the sum of divisors requested.

Step 3: Conclusion The sum of all positive integer divisors of $f(3) - f(2)$ is 60. Hence, the correct option is (A).

Quick Tip to Solve: Eliminate an additive constant by subtraction first; then factor the resulting integer before summing its positive divisors.

18. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(\mathrm{p} \rightarrow \mathrm{q})\Delta(\mathrm{p}\nabla\mathrm{q})$ is a tautology. Then [4 marks]

(A) $\Delta = \vee, \nabla = \vee$

(B) $\Delta = \vee, \nabla = \wedge$

(C) $\Delta = \wedge, \nabla = \wedge$

(D) $\Delta = \wedge, \nabla = \vee$

Correct Answer: (A) $\Delta = \vee, \nabla = \vee$

Solution:

Concept:

- The implication $p \rightarrow q$ is equivalent to $\neg p \vee q$; it is false only when p is true and q is false.
- A tautology must be true for every truth assignment. One counterexample eliminates a proposed pair of connectives.

Step 1: Determine the outer connective Take $p = \mathrm{T}$ and $q = \mathrm{F}$. Then $p \rightarrow q$ is false. If the outer connective Δ were conjunction, the entire expression would be false regardless of the second factor. A conjunction cannot be a tautology here, so $\Delta = \vee$. This eliminates both (C) and (D) immediately. We still need to choose the inner connective so that the second expression is true in the one case where the implication fails.

Step 2: Choose the inner connective and verify all cases For the same assignment $p = \mathrm{T}, q = \mathrm{F}$, the expression $p \wedge q$ is false but $p \vee q$ is true. Therefore $\nabla = \vee$; option (B) fails at this assignment. With both connectives disjunction, the expression simplifies to

$$\begin{aligned}(p \rightarrow q) \vee (p \vee q) &= (\neg p \vee q) \vee p \vee q \\ &= (\neg p \vee p) \vee q \\ &= \mathrm{T}.\end{aligned}$$

The complementary pair $\neg p, p$ guarantees truth independently of q, so this verifies every truth assignment rather than only the test assignment.

Step 3: Conclusion The required connectives are $\Delta = \vee$ and $\nabla = \vee$. Hence, the correct option is (A).

Quick Tip to Solve: Test an implication at its sole false assignment first; this often fixes both unknown connectives without a full truth table.

19. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to : [4 marks]

(A) $3\left(\hat{i} + \hat{j} + \hat{k}\right)$

(B) $3\left(\hat{i} - \hat{j} - \hat{k}\right)$

(C) $3\left(\hat{i} + \hat{j} - \hat{k}\right)$

(D) $3\left(\hat{i} - \hat{j} + \hat{k}\right)$

Correct Answer: (A) $3\left(\hat{i} + \hat{j} + \hat{k}\right)$

Solution:

Concept:

- For $\mathbf{a} = (-1, -1, 1)$ and $\mathbf{b} = (x, y, z)$, expand the scalar and vector products using their coordinate definitions.
- The scalar product fixes the component parallel to $\mathbf{a}$, while the vector product fixes the perpendicular component; together they determine $\mathbf{b}$.

Step 1: Translate the two vector conditions into equations Writing $\mathbf{b} = x\hat{i} + y\hat{j} + z\hat{k}$, the scalar product gives

$$-x - y + z = 1.$$

The coordinate expansion of the cross product is

$$\mathbf{a} \times \mathbf{b} = (-z - y)\hat{i} + (x + z)\hat{j} + (x - y)\hat{k}.$$

Comparing with $\hat{i} - \hat{j}$, we obtain $-y - z = 1$, $x + z = -1$ and $x - y = 0$. In particular, $y = x$ and $z = -1 - x$. These signs follow from the negative first and second coordinates of $\mathbf{a}$.

Step 2: Solve and form the requested vector Substitute $y = x$, $z = -1 - x$ into the scalar-product equation:

$$-2x - 1 - x = 1 \quad \Rightarrow \quad x = y = -\frac{2}{3}, \qquad z = -\frac{1}{3}.$$

Hence

$$\begin{aligned} \mathbf{a} - 6\mathbf{b} &= (-1, -1, 1) - 6\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\right) \\ &= (-1, -1, 1) + (4, 4, 2) = (3, 3, 3). \end{aligned}$$

Direct substitution also recovers both original products. Options (B), (C) and (D) each have a negative coordinate, whereas every coordinate of the computed vector is positive.

Step 3: Conclusion $\mathbf{a} - 6\mathbf{b} = 3(\hat{i} + \hat{j} + \hat{k})$. Hence, the correct option is (A).

Quick Tip to Solve: When both dot and cross products are specified, introduce three coordinates and compare components before forming the final vector.

20. If the four points, whose position vectors are $3\hat{i}-4\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -2\hat{i}-\hat{j}+3\hat{k}$ and $5\hat{i}-2\alpha\hat{j}+4\hat{k}$ are coplanar, then α is equal to : [4 marks]

(A) $\frac{73}{17}$
(B) $-\frac{73}{17}$
(C) $-\frac{107}{17}$
(D) $\frac{107}{17}$

Correct Answer: (A) $\frac{73}{17}$

Solution:

Concept:

- Four points P, Q, R, S are coplanar exactly when the scalar triple product $(\overrightarrow{PQ} \times \overrightarrow{PR}) \cdot \overrightarrow{PS}$ vanishes.
- Use three displacement vectors from one common point. Taking the scalar triple product of three position vectors alone would test a different plane through the origin.

Step 1: Form displacement vectors from the first point Let the position vectors be $P = (3, -4, 2)$, $Q = (1, 2, -1)$, $R = (-2, -1, 3)$ and $S = (5, -2\alpha, 4)$. Subtract the coordinates of P from each of the other points:

$$\overrightarrow{PQ} = (-2, 6, -3), \qquad \overrightarrow{PR} = (-5, 3, 1), \qquad \overrightarrow{PS} = (2, 4 - 2\alpha, 2).$$

The two fixed displacement vectors are not parallel, so they determine a plane with a nonzero normal vector.

Step 2: Calculate the normal and impose coplanarity The normal obtained from the cross product is

$$\begin{aligned}\overrightarrow{PQ} \times \overrightarrow{PR} &= (6 + 9,\ 15 + 2,\ -6 + 30) \\ &= (15, 17, 24).\end{aligned}$$

Therefore coplanarity requires

$$15(2) + 17(4 - 2\alpha) + 24(2) = 0.$$

Expanding carefully gives $30 + 68 - 34\alpha + 48 = 0$, or $146 - 34\alpha = 0$. Thus $\alpha = 146/34 = 73/17$. The negative alternatives give a nonzero positive scalar product; $107/17$ gives $146 - 214 = -68$, so the remaining positive alternative also fails.

Step 3: Conclusion The value is $\alpha = \dfrac{73}{17}$. Hence, the correct option is (A).

Quick Tip to Solve: Choose one point as the common origin of all three displacement vectors; then a single cross product gives the plane normal.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then the value of k is _____. [4 marks]

Solution:

Concept:

- Let S denote a smoker and C a person with lung cancer. The required probability is $P(S \mid C)$, not $P(C \mid S)$.
- Bayes' theorem combines the prior probabilities with the relative cancer risks: $P(S \mid C) = P(C \mid S)P(S)/P(C)$.

Step 1: Express the population shares and conditional risks The smoker and non-smoker proportions are $P(S) = 1/4$ and $P(S^c) = 3/4$. Let the cancer probability for a non-smoker be r. The stated relative risk gives $P(C \mid S) = 27r$. By the total-probability formula,

$$P(C) = 27r\left(\frac{1}{4}\right) + r\left(\frac{3}{4}\right) = \frac{30r}{4}.$$

The unknown underlying cancer rate cancels when conditioning on cancer; only the ratio of risks and the population shares matter.

Step 2: Apply Bayes' theorem and identify the integer The joint probability of being a smoker and having cancer is $27r/4$. Hence

$$P(S \mid C) = \frac{27r/4}{30r/4} = \frac{27}{30} = \frac{9}{10}.$$

Comparing this dimensionless probability with the stated form $k/10$ gives $k = 9$. This is a conditional probability of 90 percent, not the original 25 percent smoker share: cancer cases overrepresent smokers because their risk is much higher. The value 27 is a relative-risk multiplier, not itself a probability.

Step 3: Conclusion The required value is $k = 9$, a dimensionless integer.

Quick Tip to Solve: For a risk ratio, multiply each group's population share by its relative risk and normalize those two weights.

22. A triangle is formed by X-axis, Y-axis and the line $3x + 4y = 60$. Then the number of points P(a, b) which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is _____.

[4 marks]

Solution:

Concept:

- A point strictly inside the first-quadrant triangle satisfies $a > 0$, $b > 0$ and $3a + 4b < 60$. The inequality is strict because the boundary is excluded.
- Since a is a positive integer and b is a multiple of a, write $b = ka$, with k a positive integer. Each pair (a, k) determines exactly one point.

Step 1: Translate the geometric condition into an integer inequality Substitute $b = ka$ into the line inequality. Then

$$a(3 + 4k) < 60, \qquad k \geq 1.$$

The least possible value of $3 + 4k$ is 7. Consequently $7a < 60$, so only $a = 1, 2, \ldots, 8$ can contribute. Neither $a = 0$ nor $k = 0$ is allowed: those points lie on an axis rather than strictly inside the triangle.

Step 2: Count the allowed multiples for each abscissa For each fixed a, require $k < (60 - 3a)/(4a)$. The complete count is

a	1	2	3	4	5	6	7	8
allowed number of k	14	6	4	2	2	1	1	1

For example, when $a = 4$, the condition is $12 + 16k < 60$, so $k = 1, 2$; $k = 3$ gives equality and must be excluded. When $a = 5$, the values $k = 1, 2$ give $35 < 60$ and $55 < 60$, respectively.

Step 3: Add the disjoint counts Points with different first coordinates are distinct, so the counts may be added directly:

$$14 + 6 + 4 + 2 + 2 + 1 + 1 + 1 = 31.$$

All points counted have positive coordinates and satisfy the strict line inequality. The bound $a \leq 8$ proves that no further abscissae have been omitted.

Step 4: Conclusion The required number of points is $\boxed{31}$ (a dimensionless count).

Quick Tip to Solve: Write the multiple as b=ka first, and check equality cases separately: a point on the sloping side is not an interior point.

23. The remainder when $(2023)^{2023}$ is divided by 35 is _____.

[4 marks]

Solution:

Concept:

- If integers are congruent modulo 35, their positive integer powers are also congruent modulo 35. Thus the base can be reduced before exponentiation.
- Since $35 = 5 \cdot 7$, a remainder is also determined uniquely by its residues modulo 5 and modulo 7 in the range $0 \leq r < 35$.

Step 1: Reduce the base and establish the repeating residues Divide the base by 35:

$$2023 = 35 \cdot 57 + 28.$$

Hence $2023^{2023} \equiv 28^{2023} \pmod{35}$. Successive small powers give

$$28^2 \equiv 14, \quad 28^3 \equiv 7, \quad 28^4 \equiv 21, \quad 28^5 \equiv 28 \pmod{35}.$$

The residues cycle through $28, 14, 7, 21$; multiplying each by 28 produces the next, and multiplying 21 by 28 returns to 28. This directly establishes the cycle without assuming that the base is coprime to 35.

Step 2: Locate the exponent in the cycle The exponent satisfies $2023 = 4 \cdot 505 + 3$. It therefore occupies the third position of that four-term residue cycle, yielding

$$2023^{2023} \equiv 7 \pmod{35}.$$

The number 7 lies between 0 and 34, so it is the remainder itself, not merely another representative of its congruence class.

Step 3: Check the result using the two prime factors As an independent check, 2023 is divisible by 7, so the remainder must be divisible by 7. Modulo 5 the base is 3, and $3^4 \equiv 1 \pmod{5}$, giving $3^{2023} \equiv 3^3 \equiv 2 \pmod{5}$. Among $0, 7, 14, 21, 28$, only 7 has residue 2 modulo 5. This confirms the remainder.

Step 4: Conclusion The required remainder is $\boxed{7}$, a dimensionless integer.

Quick Tip to Solve: For a composite divisor, factor it and check residues modulo each coprime factor; avoid applying Euler's theorem when the base and divisor are not coprime.

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24. If $\int_{\frac{1}{3}}^{3} |\log_e x| dx = \frac{m}{n} \log_e \left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal to _____. [4 marks]

Solution:

Concept:

- The logarithm changes sign at $x = 1$: $|\ln x| = -\ln x$ on $(0, 1)$ and $|\ln x| = \ln x$ on $(1, \infty)$.
- Integration by parts gives $\int \ln x \, dx = x \ln x - x + C$. Evaluate the two pieces separately before combining them.

Step 1: Split the integral where the logarithm vanishes Both limits are positive, and the point $x = 1$ lies between them. Therefore

$$I = -\int_{1/3}^{1} \ln x \, dx + \int_{1}^{3} \ln x \, dx.$$

Dropping the absolute value over the whole interval would incorrectly subtract the positive area below the logarithmic curve on $(1/3, 1)$. The split ensures that each contribution is nonnegative.

Step 2: Evaluate both contributions exactly Using $F(x) = x\ln x - x$, the first contribution is

$$-\left[F(x)\right]_{1/3}^{1} = \frac{2}{3} - \frac{1}{3}\ln 3.$$

The second contribution is

$$\left[F(x)\right]_{1}^{3} = 3\ln 3 - 2.$$

Adding them yields

$$I = \frac{8}{3}\ln 3 - \frac{4}{3} = \frac{4}{3}\left(2\ln 3 - 1\right).$$

The sign in the first endpoint term follows from $\ln(1/3) = -\ln 3$.

Step 3: Match the prescribed logarithmic form Use $2\ln 3 = \ln 9$ and $1 = \ln e$ to write

$$I = \frac{4}{3}\ln\left(\frac{9}{e}\right) = \frac{4}{3}\ln\left(\frac{3^2}{e}\right).$$

Thus the requested coprime natural numbers are $m = 4$ and $n = 3$; they have no common factor greater than 1. Finally,

$$m^2 + n^2 - 5 = 16 + 9 - 5 = 20.$$

Step 4: Conclusion The requested value is $\boxed{20}$ (dimensionless).

Quick Tip to Solve: Locate the zeros of the expression inside an absolute value before integrating; here x=1 is the only point where the sign changes.

25. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is _____

[4 marks]

Solution:

Concept:

- The compulsory minima are one red apple, one white apple and two oranges, using four of the five fruits. Only one additional fruit remains to be assigned to a category.
- For individually distinguishable fruits, choose each category using combinations and multiply the independent selections. If only the numbers of each type matter, count category distributions instead.

Step 1: List every admissible category distribution Write the numbers of red apples, white apples and oranges as (r, w, o). They must satisfy

$$r + w + o = 5, \qquad r \geq 1, \quad w \geq 1, \quad o \geq 2.$$

After the minimum allocation $(1, 1, 2)$, the extra fruit can be red, white or orange. Hence the complete

list is

$$(2,1,2), \quad (1,2,2), \quad (1,1,3).$$

All three are feasible with the stated stock. No fourth case exists because only one fruit remains after satisfying the minima.

Step 2: Count selections when individual fruits are distinguished For the three disjoint distributions, the counts are respectively

$$\binom{7}{2}\binom{5}{1}\binom{8}{2} = 21 \cdot 5 \cdot 28 = 2940,$$
$$\binom{7}{1}\binom{5}{2}\binom{8}{2} = 7 \cdot 10 \cdot 28 = 1960,$$
$$\binom{7}{1}\binom{5}{1}\binom{8}{3} = 7 \cdot 5 \cdot 56 = 1960.$$

The fruits are selected, not arranged, so no ordering factor is introduced. Adding the counts gives

$$2940 + 1960 + 1960 = 6860.$$

Step 3: Account for the alternative interpretation accepted by the key If fruits of the same type are regarded as indistinguishable, each distribution listed in the first step contributes exactly one way, giving 3 ways. The question does not explicitly specify distinguishability. Accordingly the accepted key allows both 6860 and 3. The larger count uses the individual fruits in the basket, whereas the smaller count uses only the category composition of the gift.

Step 4: Conclusion The selection count is $\boxed{6860}$ when individual fruits are distinguished; $\boxed{3}$ category distributions are possible when same-type fruits are indistinguishable. The official key accepts both values; both are dimensionless counts.

Quick Tip to Solve: First list category distributions meeting all minima; only then decide whether each distribution needs binomial factors for individual fruit choices.

26. For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}, 10$ and $\frac{1}{b}$ are in an arithmetic progression, then $16a + 12b$ is equal to _____. [4 marks]

Solution:

Concept:

- For three consecutive terms of a geometric progression, the square of the middle term equals the product of the outer terms. Thus $b^2 = a/18$.
- For three consecutive terms of an arithmetic progression, twice the middle term equals the sum of the outer terms. The positivity of a, b permits multiplication and division by them.

Step 1: Express both progression conditions algebraically The geometric-progression condition gives

$$a = 18b^2.$$

From the arithmetic progression $1/a, 10, 1/b$,

$$\frac{1}{a} + \frac{1}{b} = 20.$$

Multiplying through by the positive product ab gives $a + b = 20ab$. These equations preserve the original conditions because neither variable can be zero.

Step 2: Solve for the positive value of b Substitute $a = 18b^2$ into $a + b = 20ab$:

$$18b^2 + b = 360b^3.$$

Divide by $b > 0$ and factor:

$$360b^2 - 18b - 1 = (12b - 1)(30b + 1) = 0.$$

The two algebraic possibilities are $b = 1/12$ and $b = -1/30$. The latter violates the stipulated positivity, so

$$b = \frac{1}{12}, \qquad a = 18\left(\frac{1}{12}\right)^2 = \frac{1}{8}.$$

Step 3: Check the progressions and evaluate the requested expression The three terms $1/8, 1/12, 1/18$ have common ratio $2/3$, confirming the geometric progression. Their required reciprocals are $8, 10, 12$, with common difference 2, confirming the arithmetic progression. Thus

$$16a + 12b = 16 \cdot \frac{1}{8} + 12 \cdot \frac{1}{12} = 2 + 1 = 3.$$

The negative root has been excluded before evaluating this expression, so the result corresponds to the unique allowed positive pair.

Step 4: Conclusion The required value is $\boxed{3}$, which is dimensionless.

Quick Tip to Solve: Convert GP to a middle-term square and AP to a middle-term average; use positivity before cancelling a variable or selecting a quadratic root.

27. If the shortest distance between the line joining the points (1, 2, 3) and (2, 3, 4), and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For lines through position vectors $\mathbf{p}, \mathbf{q}$ with nonparallel direction vectors $\mathbf{u}, \mathbf{v}$, the shortest distance is $d = |(\mathbf{q} - \mathbf{p}) \cdot (\mathbf{u} \times \mathbf{v})|/|\mathbf{u} \times \mathbf{v}|$.
- A zero denominator in the symmetric line notation means that coordinate is constant. Here $(z - 2)/0$ specifies $z = 2$, and the direction vector has zero third component.

Step 1: Read a point and a direction from each line The first line joins $P = (1, 2, 3)$ and $(2, 3, 4)$, so a

direction vector is

$$\mathbf{u} = (2,3,4) - (1,2,3) = (1,1,1).$$

For the second line, put the common parameter equal to t. Its equations are $x = 1 + 2t$, $y = -1 - t$, $z = 2$. Hence take $Q = (1,-1,2)$, $\mathbf{v} = (2,-1,0)$ and $\overrightarrow{PQ} = (0,-3,-1)$.

Step 2: Compute the scalar triple product The common perpendicular has direction

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = (1,2,-3).$$

Its magnitude is $\sqrt{1+4+9} = \sqrt{14}$, which is nonzero, so the nonparallel-lines formula applies. The numerator is

$$|(0,-3,-1) \cdot (1,2,-3)| = |-6+3| = 3.$$

The absolute value removes the orientation dependence of the scalar triple product.

Step 3: Square the distance before multiplying Therefore

$$\alpha = \frac{3}{\sqrt{14}}, \qquad \alpha^2 = \frac{9}{14}.$$

It is simpler to square directly than to rationalise the distance first. The requested expression is

$$28\alpha^2 = 28 \cdot \frac{9}{14} = 18.$$

The nonzero triple product also confirms that the two lines do not intersect. The distance is measured in the coordinate length unit; the reported numerical expression follows the units implicit in the given coordinates.

Step 4: Conclusion The required numerical value of $28\alpha^2$ is $\boxed{18}$.

Quick Tip to Solve: For skew lines, form the cross product once and then the scalar triple product; a zero direction component fixes a coordinate rather than invalidating the line.

28. Let $\alpha \in \mathbb{R}$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is _____. [4 marks]

Solution:

Concept:

- For the given monic quadratic, Vieta's relations give $\alpha + \beta = -60^{1/4}$ and $\alpha\beta = a$. Powers of roots can be expressed through their sum and product.
- If a quadratic with real linear coefficient has one real root and its other root is obtained by subtracting it from the real root sum, then the other root is real too. Fourth powers of real numbers are nonnegative.

Step 1: Check the printed real-root restriction The question explicitly states $\alpha \in \mathbb{R}$. Since

$$\beta = -60^{1/4} - \alpha,$$

the second root β is then real as well, regardless of how the constant term was initially specified. Consequently $\alpha^4 \geq 0$ and $\beta^4 \geq 0$, so

$$\alpha^4 + \beta^4 \geq 0.$$

This contradicts the printed condition $\alpha^4 + \beta^4 = -30$. Thus no pair of roots satisfies all the printed conditions.

Step 2: Derive the formal parameter equation underlying the key For completeness, temporarily retain only the algebraic root relations and the power-sum equation. Put $s = \alpha + \beta = -60^{1/4}$, so $s^2 = \sqrt{60}$ and $s^4 = 60$. Then

$$\alpha^4 + \beta^4 = (s^2 - 2a)^2 - 2a^2 = 60 - 4a\sqrt{60} + 2a^2.$$

Equating this to -30 gives

$$a^2 - 2\sqrt{60}\, a + 45 = 0.$$

The product of the two formal solutions is therefore 45 by Vieta's relation.

Step 3: Distinguish formal solutions from admissible solutions The formal values are $a = \sqrt{60} \pm \sqrt{15} = \sqrt{15}$ and $3\sqrt{15}$. For either value, the original quadratic has discriminant $\sqrt{60} - 4a = 2\sqrt{15} - 4a < 0$, so neither has a real root α. Thus the algebra producing 45 is correct only after the printed real-root requirement is discarded. It does not establish an admissible value for the literal question.

Step 4: Conclusion The printed real-root condition is inconsistent, so there are no admissible values of a and their requested product is not defined as stated. The official key value $\boxed{45}$ is the product of the two formal algebraic candidates obtained when the real-root restriction is ignored; neither candidate satisfies that restriction.

Quick Tip to Solve: Check sign restrictions before using a power-sum identity: a sum of real fourth powers cannot be negative, even if the resulting formal parameter equation has roots.

29. If m and n respectively are the numbers of positive and negative values of θ in the interval $[-\pi, \pi]$ that satisfy the equation $\cos 2\theta \cos \frac{\theta}{2} = \cos 3\theta \cos \frac{9\theta}{2}$, then mn is equal to _____. [4 marks]

Solution:

Concept:

- Product-to-sum gives $2\cos u \cos v = \cos(u+v) + \cos(u-v)$. Apply it to both sides before counting roots.
- The difference identity $\cos u - \cos v = -2\sin((u+v)/2)\sin((u-v)/2)$ factors the resulting equation. Count distinct roots, excluding zero from both requested counts.

Step 1: Reduce the products The two products share a cosine term. Multiplying by 2 gives

$$\cos\frac{5\theta}{2}+\cos\frac{3\theta}{2}=\cos\frac{15\theta}{2}+\cos\frac{3\theta}{2}.$$

Subtracting the common term is valid for every real angle; no division by a trigonometric function is required.

Step 2: Factor and count the roots Therefore

$$\cos\frac{5\theta}{2}-\cos\frac{15\theta}{2}=2\sin(5\theta)\sin\frac{5\theta}{2}=0.$$

The first factor gives $\theta=k\pi/5$; the second gives $\theta=2k\pi/5$, already included in the first family. In the closed interval, $k=-5,-4,\ldots,4,5$. There are five strictly positive roots and five strictly negative roots. The root zero belongs to neither category, while both endpoints are included.

Step 3: Conclusion Thus $m=5$, $n=5$, and the required dimensionless value is $\boxed{mn=25}$.

Quick Tip to Solve: After factoring a trigonometric equation, check whether one root family is contained in another before adding their counts.

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30. Points P(−3, 2), Q(9, 10) and R(α, 4) lie on a circle C and PR as its diameter. The tangents to C at the points Q and R intersect at the point S. If S lies on the line $2x-ky=1$, then k is equal to _____. [4 marks]

Solution:

Concept:

- An angle subtended by a diameter is a right angle: $(Q-P)\cdot(Q-R)=0$. The centre is the midpoint of the diameter.
- For a circle with centre (h,k_0) and radius r, the tangent at (x_1,y_1) is $(x_1-h)(x-h)+(y_1-k_0)(y-k_0)=r^2$.

Step 1: Find the diameter endpoint and centre Since PR is a diameter,

$$12(9-\alpha)+8(10-4)=0.$$

Hence $108-12\alpha+48=0$, giving $\alpha=13$. The centre is $O=(5,3)$, and $r^2=(13-5)^2+(4-3)^2=65$. This also equals $(9-5)^2+(10-3)^2$, checking that Q lies on this circle.

Step 2: Intersect the tangents and determine k The radius vectors at Q and R are $(4,7)$ and $(8,1)$. Their tangents are

$$4(x-5)+7(y-3)=65,\qquad 8(x-5)+(y-3)=65.$$

Thus $4x+7y=106$ and $8x+y=108$. Substituting $y=108-8x$ gives $-52x=-650$. Therefore $S=(25/2,8)$. Finally the given line requires

$$2\left(\frac{25}{2}\right)-8k=1,$$

so $8k = 24$.

Step 3: Conclusion The required dimensionless parameter is $\boxed{k = 3}$.

Quick Tip to Solve: Use the right angle at the third point to find an unknown diameter endpoint; this avoids solving a general circle equation.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The light rays from an object have been reflected towards an observer from a standard flat mirror, the image observed by the observer are :-

A. Real

B. Erect

C. Smaller in size then object

D. Laterally inverted

Choose the most appropriate answer from the options given below : [4 marks]

(A) B and D only

(B) A, C, and D only

(C) A and D only

(D) B and C only

Correct Answer: (A) B and D only

Solution:

Concept:

- A standard plane mirror forms a virtual image of a real object: the reflected rays diverge and their backward extensions meet behind the mirror.
- The image is erect, the same size as the object, and laterally inverted. Its distance behind the mirror equals the object distance in front.

Step 1: Assess the real and erect statements A real image requires the reflected rays themselves to converge. A plane mirror does not produce such a convergence for the ordinary object described here, so statement A is false. The image remains upright relative to the object, so statement B is true. The observer sees the virtual image by tracing the reflected rays backward.

Step 2: Assess size and lateral inversion The magnitude of linear magnification of a plane mirror is unity. Thus the image is the same size, rather than smaller, and statement C is false. The familiar left-right reversal when an object is compared with its mirror image is lateral inversion, making statement D true. All combinations containing A or C therefore contain a false statement and must be rejected.

Step 3: Conclusion Only statements B and D hold. Hence, the correct option is (A).

Quick Tip to Solve: For a plane mirror remember virtual, erect, equal size and lateral inversion; then test each listed statement independently.

32. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. If time taken by particle to go from $x = 0$ to $\frac{A}{2}$ is 2 s; then time taken by particle in going from $x = \frac{A}{2}$ to A is [4 marks]

(A) 4 s

(B) 1.5 s

(C) 3 s

(D) 2 s

Correct Answer: (A) 4 s

Solution:

Concept:

- For SHM starting from the mean position towards the positive extreme, write $x = A\sin\omega t$, with angular frequency ω.
- Equal displacement intervals do not take equal times: the speed decreases as the particle approaches the positive extreme. Use the corresponding phase angles.

Step 1: Find the angular frequency Take the first passage from the mean position towards $+A$. At $x = A/2$, $\sin\omega t = 1/2$, so the phase is $\pi/6$. The specified time is 2 s, and therefore

$$\omega(2) = \frac{\pi}{6}, \qquad \omega = \frac{\pi}{12}\,\mathrm{rad\,s^{-1}}.$$

This corresponds to a period $T = 24\,\mathrm{s}$.

Step 2: Calculate the remaining phase interval The extreme $x = A$ has phase $\pi/2$. Hence the phase change from $A/2$ to A is $\pi/2 - \pi/6 = \pi/3$, twice the initial phase change. The required time is

$$\Delta t = \frac{\pi/3}{\pi/12} = 4\,\mathrm{s}.$$

This exceeds the initial time because the particle slows down. The other listed times do not correspond to this phase difference; using equal times for equal half-amplitude distances would be incorrect.

Step 3: Conclusion The required time is $\boxed{4\,\mathrm{s}}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert positions to phase using the sine law; the last half of the amplitude takes twice as long as the first half.

33. The distance travelled by a particle is related to time t as $x = 4t^2$. The velocity of the particle at t=5s is :- [4 marks]

(A) $25\,\mathrm{m\,s^{-1}}$

(B) $20\,\mathrm{m\,s^{-1}}$

(C) $8\,\mathrm{m\,s^{-1}}$

(D) $40\,\mathrm{m\,s^{-1}}$

Correct Answer: (D) $40\,\mathrm{m\,s^{-1}}$

Solution:

Concept:

- Instantaneous velocity is the time derivative of displacement: $v = dx/dt$. Differentiate the given position law before substituting a time.
- For $x = 4t^2$, the coefficient has units $\mathrm{m\,s^{-2}}$, and the velocity varies linearly with time.

Step 1: Differentiate the position law Applying the power rule to the given relation gives

$$v(t) = \frac{d}{dt}(4t^2) = 8t.$$

This is the instantaneous rate of change, not the average rate over the complete elapsed interval. The motion for positive time is in the increasing-x direction, so the value obtained from this derivative is positive.

Step 2: Evaluate at the requested instant At $t = 5\,\mathrm{s}$,

$$v(5) = 8 \times 5 = 40\,\mathrm{m\,s^{-1}}.$$

The value $20\,\mathrm{m\,s^{-1}}$ would instead be $x(5)/5 = 100/5$, the average speed from rest over the first five seconds. The value 8 is the coefficient in the velocity law and is also the constant acceleration numerically, not the requested velocity. The value 25 comes from squaring the time without differentiating, so it has no basis as a speed here.

Step 3: Conclusion The instantaneous velocity is $\boxed{40\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: When position is proportional to the square of time, instantaneous speed at an instant is twice the average speed from time zero.

34. Match List I with List II

	List I		List II
A.	Isothermal Process	I.	Work done by the gas decreases internal energy
B.	Adiabatic Process	II.	No change in internal energy
C.	Isochoric Process	III.	The heat absorbed goes partly to increase internal energy and partly to do work
D.	Isobaric Process	IV.	No work is done on or by the gas

Choose the correct answer from the options given below : [4 marks]

(A) A-I, B-II, C-IV, D-III

(B) A-II, B-I, C-III, D-IV

(C) A-II, B-I, C-IV, D-III

(D) A-I, B-II, C-III, D-IV

Correct Answer: (C) A-II, B-I, C-IV, D-III

Solution:

Concept:

- Use the first law with work done by the gas positive: $Q = \Delta U + W$. For an ideal gas the internal energy depends only on temperature.
- The defining constraints are constant temperature (isothermal), zero heat transfer (adiabatic), constant volume (isochoric), and constant pressure (isobaric).

Step 1: Match the isothermal and adiabatic processes In an isothermal ideal-gas process, temperature is unchanged, hence $\Delta U = 0$. This gives A-II. In an adiabatic process $Q = 0$, so $\Delta U = -W$. Positive work done by the gas is supplied at the expense of its internal energy, giving B-I. These two matches already reject the combinations that put A-I or B-II.

Step 2: Match the constant-volume and constant-pressure processes In an isochoric process, $dV = 0$, hence $W = \int P\,dV = 0$: no expansion or compression work occurs. Thus C-IV. In an isobaric heating process, absorbed heat both raises internal energy and supplies expansion work, as expressed by $Q = \Delta U + P\Delta V$. Thus D-III. Interchanging III and IV would wrongly assign expansion work to a fixed-volume process. Only one listed combination contains all four correct matches.

Step 3: Conclusion The matching is A-II, B-I, C-IV, D-III. Hence, the correct option is (C).

Quick Tip to Solve: Write the first law once, then set the quantity constrained by each process to zero or constant before matching descriptions.

35. A wire of length 1m moving with velocity 8 m/s at right angles to a magnetic field of 2T. The magnitude of induced emf, between the ends of wire will be _____. [4 marks]

(A) 20 V

(B) 8 V

(C) 16 V

(D) 12 V

Correct Answer: (C) 16 V

Solution:

Concept:

- A conductor cutting a uniform magnetic field develops motional emf of magnitude $\mathcal{E} = B\ell v$ in the standard perpendicular arrangement.
- Magnetic force qvB separates charges along the moving wire; at equilibrium the electric force balances it, giving $E = vB$ and potential difference $E\ell$.

Step 1: Identify the perpendicular motional-emf relation The question gives the wire length $\ell = 1\,\mathrm{m}$, speed $v = 8\,\mathrm{m\,s^{-1}}$, and magnetic field $B = 2\,\mathrm{T}$. Since the motion is at right angles to the field, the sine factor in the motional emf is unity. The requested quantity is the magnitude of the potential difference, so no choice of terminal sign is needed.

Step 2: Substitute the given values The induced emf is

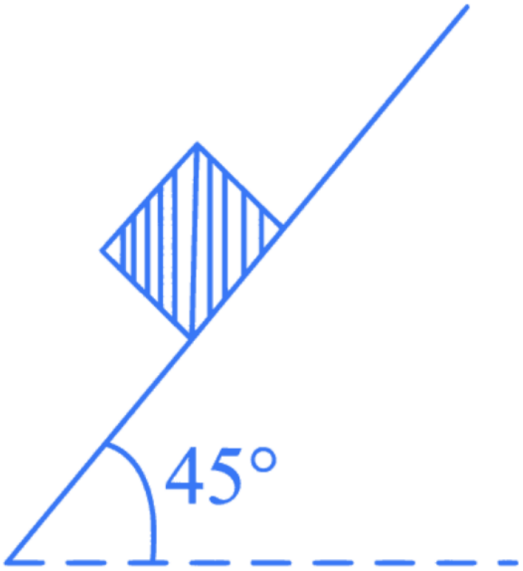
$$\mathcal{E} = B\ell v = 2 \times 1 \times 8 = 16\,\mathrm{V}.$$

The unit check is $\mathrm{T\,m^2\,s^{-1}} = \mathrm{Wb\,s^{-1}} = \mathrm{V}$. The value 8 V omits the field-strength factor, while 20 V and 12 V do not result from the product of the stated quantities. There is no resistance specified, and none is needed to calculate this open-circuit emf.

Step 3: Conclusion The magnitude of the induced emf is $\boxed{16\,\mathrm{V}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a straight moving wire, check perpendicularity and multiply field, active length and speed; resistance is needed only for current.

36. Consider a block kept on an inclined plane (incline at 45°) as shown in the figure. If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and inclined plane(μ) is equal to : [4 marks]



(A) 0.60

(B) 0.33

(C) 0.25

(D) 0.50

Correct Answer: (B) 0.33

Solution:

Concept:

- The component of weight down an incline is $mg\sin\theta$, while the normal reaction is $N = mg\cos\theta$ for a force applied along the plane.
- Limiting static friction has magnitude μN and opposes the direction in which the block is about to move. Its direction therefore reverses between the two limiting situations.

Step 1: Write the force needed for impending upward motion When the block is just about to move up, both gravity along the slope and friction act downwards. If the required upward force is F_u, equilibrium along the plane gives

$$F_u = mg\sin\theta + \mu mg\cos\theta.$$

The applied force is along the incline, so it does not change the normal reaction.

Step 2: Write the force needed to prevent downward motion For impending downward sliding, friction points up the plane. The additional upward holding force F_d is therefore

$$F_d = mg\sin\theta - \mu mg\cos\theta.$$

Use the given condition $F_u = 2F_d$:

$$\sin\theta + \mu\cos\theta = 2(\sin\theta - \mu\cos\theta).$$

Thus $3\mu\cos\theta = \sin\theta$, giving $\mu = \tan\theta/3$.

Step 3: Substitute the inclination and check the choices At $\theta = 45^\circ$, $\tan\theta = 1$, so

$$\mu = \frac{1}{3} \simeq 0.33.$$

As a check, the two forces become $4mg/(3\sqrt{2})$ and $2mg/(3\sqrt{2})$, whose ratio is exactly two. The values 0.60, 0.25 and 0.50 instead give ratios 4, $5/3$ and 3, respectively, and do not meet the condition.

Step 4: Conclusion The coefficient of friction is $\mu = 1/3 \approx 0.33$, a dimensionless quantity. Hence, the correct option is (B).

Quick Tip to Solve: For a parallel applied force, write the two thresholds as $mg(\sin\theta \pm \mu\cos\theta)$; reverse friction, not gravity.

37. Match List I with List II.

	List I		List II
A.	**Gauss's Law in Electrostatics**	**I.**	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$
B.	**Faraday's Law**	**II.**	$\oint \vec{B} \cdot d\vec{A} = 0$
C.	**Gauss's Law in Magnetism**	**III.**	$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$
D.	**Ampere-Maxwell Law**	**IV.**	$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-II, C-III, D-IV

(B) A-III, B-IV, C-I, D-II

(C) A-IV, B-I, C-II, D-III

(D) A-II, B-III, C-IV, D-I

Correct Answer: (C) A-IV, B-I, C-II, D-III

Solution:

Concept:

- Gauss's electric law relates electric flux through a closed surface to the enclosed charge: $\oint \vec{E} \cdot d\vec{S} = q/\varepsilon_0$.
- Faraday's law relates circulation of the electric field to changing magnetic flux. Magnetic Gauss law states that the net magnetic flux through every closed surface is zero.
- The Ampere-Maxwell law contains both conduction current and the displacement-current term $\varepsilon_0\, d\Phi_E/dt$.

Step 1: Match the electric and magnetic flux laws Gauss's law in electrostatics uses a closed-surface electric flux integral and an enclosed charge on the right. This is expression IV, so A matches IV.

Gauss's law in magnetism is

$$\oint \vec{B} \cdot d\vec{A} = 0,$$

which is expression II, so C matches II. The zero net magnetic flux represents the absence of isolated magnetic poles; it is not the electric Gauss law.

Step 2: Match the circulation laws Faraday's induction law is

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}.$$

The minus sign expresses Lenz's law. This identifies B with I. Finally, the magnetic circulation around a closed path obeys

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.$$

This is expression III, so D matches III.

Step 3: Assemble and compare the complete mapping The four matches are therefore A-IV, B-I, C-II and D-III. All four entries must be consistent, not just one pair. Option (A) incorrectly makes the electric Gauss law a circulation equation. Option (B) assigns it the magnetic circulation equation, and option (D) assigns it the zero magnetic-flux equation. Only option (C) keeps both surface-flux laws and both circulation laws correctly associated.

Step 4: Conclusion The correct matching is A-IV, B-I, C-II, D-III. Hence, the correct option is (C).

Quick Tip to Solve: First separate closed-surface flux integrals from closed-loop circulation integrals; then identify the field and the source on the right.

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38. Every planet revolves around the sun in an elliptical orbit :-

A. The force acting on a planet is inversely proportional to square of distance from sun.

B. Force acting on planet is inversely proportional to product of the masses of the planet and the sun.

C. The Centripetal force acting on the planet is directed away from the sun.

D. The square of time period of revolution of planet around sun is directly proportional to cube of semi-major axis of elliptical orbit.

Choose the correct answer from the options given below : [4 marks]

(A) C and D only

(B) B and C only

(C) A and D only

(D) A and C only

Correct Answer: (C) A and D only

Solution:

Concept:

- Newton's law gives the attractive gravitational-force magnitude $F = GMm/r^2$, with r the separation of the Sun and planet.
- For an elliptical orbit about the same central mass, Kepler's third law states $T^2 \propto a^3$, where a is the semi-major axis.

Step 1: Test the statements about the force magnitude From

$$F = \frac{GMm}{r^2},$$

the force varies inversely as the square of the instantaneous separation when the masses are fixed. Thus statement A is true, and the elliptical shape does not alter this inverse-square dependence. The same equation shows that the force is directly proportional to the product Mm, rather than inversely proportional to it. Therefore statement B is false.

Step 2: Test the direction and the period law Gravity is attractive, so the force on the planet is directed towards the Sun at a focus of the ellipse. Statement C, which says the force is away from the Sun, is false. For the two-body orbit,

$$T^2 = \frac{4\pi^2}{G(M+m)}a^3.$$

In the usual planetary approximation $m \ll M$, the coefficient is $4\pi^2/(GM)$. Thus the square of the orbital period is proportional to the cube of the semi-major axis, making D true.

Step 3: Choose the pair of true statements Exactly A and D are true. Option (A) includes the incorrect outward-force statement C. Option (B) includes both incorrect statements B and C. Option (D) again includes C, despite the attraction being towards the Sun. The only offered pair that contains both true statements and neither false statement is option (C).

Step 4: Conclusion Only statements A and D are correct. Hence, the correct option is (C).

Quick Tip to Solve: Check orbital-force statements directly against GMm/r^2; use the semi-major axis, not the instantaneous radius, in Kepler's period law.

39. Two objects are projected with same velocity 'u' however at different angles α and β with the horizontal. If $\alpha + \beta = 90°$, the ratio of horizontal range of the first object to the 2nd object will be : [4 marks]

(A) 1 : 1

(B) 2 : 1

(C) 1 : 2

(D) 4 : 1

Correct Answer: (A) 1 : 1

Solution:

Concept:

- For projectile motion returning to the launch level, with uniform gravity and negligible air resistance, the horizontal range is $R = u^2 \sin(2\theta)/g$.
- Complementary launch angles satisfy $\beta = 90° - \alpha$, so their doubled angles are supplementary and have equal sines.

Step 1: Obtain the range for each launch angle The horizontal speed remains $u\cos\theta$, while the time of flight is $2u\sin\theta/g$. Multiplying gives

$$R = (u\cos\theta)\frac{2u\sin\theta}{g} = \frac{u^2\sin 2\theta}{g}.$$

The two projectiles have the same initial speed and experience the same gravitational acceleration, so these factors cancel in their range ratio:

$$\frac{R_1}{R_2} = \frac{\sin 2\alpha}{\sin 2\beta}.$$

Step 2: Use the complementary-angle condition The relation $\alpha + \beta = 90°$ gives $2\beta = 180° - 2\alpha$. Hence

$$\sin 2\beta = \sin(180° - 2\alpha) = \sin 2\alpha.$$

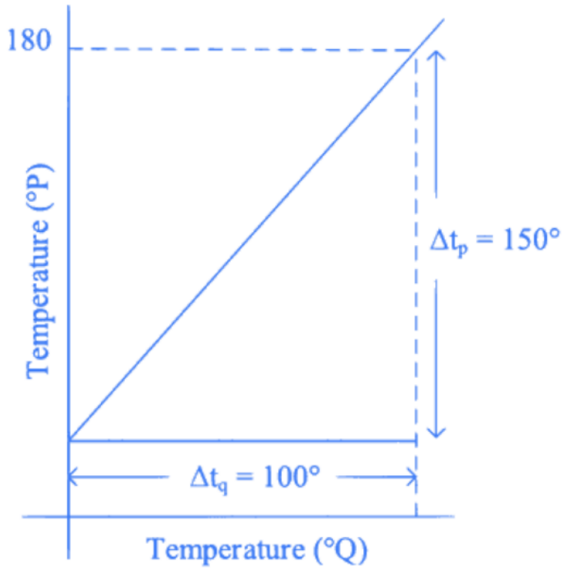
Consequently $R_1/R_2 = 1$. One object can have a longer flight time but then has a smaller horizontal speed; the two effects exactly compensate for complementary angles.

Step 3: Check the proposed ratios The equal-range result gives 1:1. Ratios 2:1, 1:2 and 4:1 would require unequal values of $u^2\sin 2\theta$, whereas both the common speed and equal doubled-angle sines have already been established. Thus none of these unequal ratios is consistent with the stated complementary angles.

Step 4: Conclusion The horizontal ranges are equal, so $R_1 : R_2 = 1 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: For equal-speed projectiles landing at launch height, complementary elevation angles always give the same horizontal range.

40. The graph between two temperature scales P and Q is shown in the figure. Between upper fixed point and lower fixed point there are 150 equal divisions of scale P and 100 divisions on scale Q. The relationship for conversion between the two scales is given by :- [4 marks]



(A) $\frac{t_P}{100} = \frac{t_Q - 180}{150}$
(B) $\frac{t_P}{180} - \frac{t_Q - 40}{100}$
(C) $\frac{t_Q}{150} = \frac{t_P - 180}{100}$
(D) $\frac{t_Q}{100} = \frac{t_P - 30}{150}$

Correct Answer: (D) $\frac{t_Q}{100} = \frac{t_P - 30}{150}$

Solution:

Concept:

- A linear temperature scale is calibrated by equal fractions of the interval between its lower and upper fixed points: $(t - L)/(U - L)$.
- The graph gives the upper reading on scale P as 180 and the P interval as 150, while the Q interval is 100 and its lower fixed point is zero.

Step 1: Read the two calibration intervals The vertical axis represents scale P, and the marked upper fixed point is $P_U = 180$. Since the vertical separation of the fixed points is 150 divisions, the lower P reading is

$$P_L = 180 - 150 = 30.$$

On the horizontal Q axis, the lower fixed point is at zero and the upper one is 100 divisions higher, so $Q_L = 0$ and $Q_U = 100$. The graph's intercept is therefore essential; scale P does not start at zero.

Step 2: Equate the fractional temperature rises The same physical temperature must occupy the same fractional position within the two fixed-point intervals. Therefore

$$\frac{t_P - P_L}{P_U - P_L} = \frac{t_Q - Q_L}{Q_U - Q_L}.$$

Substituting the readings just found gives

$$\frac{t_P - 30}{150} = \frac{t_Q}{100}.$$

Equivalently, $t_P = 30 + 1.5t_Q$, so the slope and intercept agree with the graph.

Step 3: Verify both endpoints and eliminate the other options At $t_Q = 0$, the equation yields $t_P = 30$, and at $t_Q = 100$, it yields $t_P = 180$. Both fixed points are reproduced. Option (A) offsets Q by 180 instead of offsetting P by 30. Option (C) interchanges the interval lengths and uses the upper P value as an offset. Option (B), as printed, is a difference rather than the required calibration equality and also uses the wrong interval and offset.

Step 4: Conclusion The conversion is $\frac{t_Q}{100} = \frac{t_P - 30}{150}$. Hence, the correct option is (D).

Quick Tip to Solve: Before writing a temperature-conversion ratio, extract each lower fixed point and each interval separately; the upper reading is not the interval length.

41. A point charge of 10 μC is placed at the origin. At what location on the X-axis should a point charge of 40 μC be placed so that the net electric field is zero at $x = 2$cm on the X-axis? [4 marks]

(A) $x = 6$ cm

(B) $x = 8$ cm

(C) $x = 4$ cm

(D) $x = -4$ cm

Correct Answer: (A) $x = 6$ cm

Solution:

Concept:

- The electric-field magnitude of a point charge is $E = k|q|/r^2$, and its direction is away from a positive charge.
- Zero net field requires equal field magnitudes in opposite directions at the specified point; equality of magnitudes alone does not suffice.

Step 1: Determine the necessary side of the second charge At the observation point $x_P = 2\,\text{cm}$, the positive $10\,\mu\text{C}$ charge at the origin produces a field along the positive X-axis. To cancel it, the positive $40\,\mu\text{C}$ charge must produce a field along the negative X-axis at P. It must therefore lie to the right of P, so its coordinate x must exceed $2\,\text{cm}$. A charge at $-4\,\text{cm}$ would reinforce the first field.

Step 2: Equate the two field magnitudes Use centimetres consistently in the distance ratio, since the common unit factor cancels. For $x > 2$, the distances from P are 2 and $x - 2$, respectively. Thus

$$\frac{k(10)}{2^2} = \frac{k(40)}{(x-2)^2}.$$

Cancelling the common factors gives

$$(x-2)^2 = 16, \qquad x - 2 = 4\,\text{cm},$$

where the positive separation is selected because the charge must lie to the right of P.

Step 3: Locate the charge and check the alternatives The required coordinate is $x = 2 + 4 = 6\,\text{cm}$. The larger charge needs twice the distance because its magnitude is four times larger. At $x = 8\,\text{cm}$, its distance is 6 cm and its field is too small. At $x = 4\,\text{cm}$, its distance is 2 cm and its field is four times too large. The negative-coordinate option gives the wrong field direction.

Step 4: Conclusion The $40\,\mu\text{C}$ charge must be placed at $x = 6\,\text{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: For cancellation by two positive charges, put the zero-field point between them and use distance ratio $r_2/r_1 = \sqrt{q_2/q_1}$.

42. The resistance of a wire is 5 Ω. It's new resistance in ohm if stretched to 5 times of it's original length will be : [4 marks]

(A) 25

(B) 625

(C) 5

(D) 125

Correct Answer: (D) 125

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area.
- On stretching the same wire at unchanged temperature, its volume is conserved and its material resistivity stays unchanged. Thus $AL = A\prime L\prime$.

Step 1: Find the cross-sectional area after stretching Let the original length and area be L and A, and the final values be $L\prime$ and $A\prime$. The stated length increase is $L\prime = 5L$. Conservation of the wire's material volume gives

$$A\prime L\prime = AL, \qquad A\prime = \frac{AL}{5L} = \frac{A}{5}.$$

Thus the wire becomes thinner as it lengthens; keeping the original area would not describe stretching the same wire.

Step 2: Calculate the resistance ratio With unchanged resistivity, divide the final resistance by the initial one:

$$\frac{R\prime}{R} = \frac{\rho L\prime/A\prime}{\rho L/A} = \frac{L\prime}{L}\frac{A}{A\prime} = 5 \times 5 = 25.$$

One factor of five comes from the longer path, and the other comes from the smaller area available

for current. Therefore

$$R\prime = 25(5\,\Omega) = 125\,\Omega.$$

Step 3: Compare the options with the physical change The value 25 ohms accounts for length increase alone and misses the decrease in area. A resistance of 5 ohms would require no net geometrical change in L/A, contrary to the stretching. The value 625 ohms overcounts the change: the correct multiplicative factor is 25, not 125. Only 125 ohms incorporates both the longer length and the reduced cross-section.

Step 4: Conclusion The new resistance is $125\,\Omega$. Hence, the correct option is (D).

Quick Tip to Solve: For the same wire stretched at constant volume and temperature, resistance varies as the square of its length: $R\prime/R = (L\prime/L)^2$.

43. Match List I with List II

	List I		List II
A.	**Young's Modulus (Y)**	**I.**	$[\mathrm{ML^{-1}T^{-1}}]$
B.	**Co-efficient of Viscosity (η)**	**II.**	$[\mathrm{ML^{2}T^{-1}}]$
C.	**Planck's Constant (h)**	**III.**	$[\mathrm{ML^{-1}T^{-2}}]$
D.	**Work function (φ)**	**IV.**	$[\mathrm{ML^{2}T^{-2}}]$

Choose the correct answer from the options given below : [4 marks]

(A) A-II, B-III, C-IV, D-I

(B) A-I, B-II, C-III, D-IV

(C) A-I, B-III, C-IV, D-II

(D) A-III, B-I, C-II, D-IV

Correct Answer: (D) A-III, B-I, C-II, D-IV

Solution:

Concept:

- Dimensions follow from the defining equation of a quantity. Mass, length and time have base dimensions M, L, T.
- Young's modulus is stress divided by strain; viscosity follows from shear stress $\tau = \eta\, dv/dy$.
- Planck's relation is $E = h\nu$, while a work function is an energy.

Step 1: Find the two mechanical dimensions Strain is a ratio of lengths and is dimensionless. Stress is force per area, so $[Y] = [F]/[A] = [MLT^{-2}]/[L^2] = [ML^{-1}T^{-2}]$. Thus A matches III. In the viscosity equation the velocity gradient has dimensions $[dv/dy] = [LT^{-1}]/[L] = [T^{-1}]$. Dividing shear stress by this gradient gives $[\eta] = [ML^{-1}T^{-2}]/[T^{-1}] = [ML^{-1}T^{-1}]$, so B matches I.

Step 2: Match the quantum quantities and compare options Frequency has dimensions T^{-1}, and energy has dimensions ML^2T^{-2}. Consequently $[h] = [E]/[\nu] = [ML^2T^{-1}]$, which is II. A work function is the minimum energy required to liberate an electron, so $[\varphi] = [ML^2T^{-2}]$, which is IV. Therefore C matches II and D matches IV. Options (A), (B) and (C) all give an incorrect match for

Young's modulus; only (D) also gets all the other entries right.

Step 3: Conclusion The matching is A-III, B-I, C-II, D-IV. Hence, the correct option is (D).

Quick Tip to Solve: Derive dimensions from definitions: modulus is pressure, viscosity is pressure times time, and Planck's constant is energy times time.

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44. Statement I : When a Si sample is doped with Boron, it becomes P type and when doped by Arsenic it becomes N-type semi conductor such that P-type has excess holes and N-type has excess electrons.

Statement II : When such P-type and N-type semi-conductors, are fused to make a junction, a current will automatically flow which can be detected with an externally connected ameter.

In the light of above statements, choose the most appropriate answer from the options given below

[4 marks]

(A) Statement I is incorrect but statement II is correct

(B) Both Statement I and statement II are correct

(C) Statement I is correct but statement II is incorrect

(D) Both Statement I and Statement II are incorrect

Correct Answer: (C) Statement I is correct but statement II is incorrect

Solution:

Concept:

- Trivalent acceptor impurities in silicon produce holes as majority carriers; pentavalent donor impurities produce electrons as majority carriers.
- At thermal equilibrium a p-n junction has equal and opposite diffusion and drift currents, giving zero net current without an external source.

Step 1: Test the doping statement Silicon has four valence electrons. A boron atom has three, so it cannot complete one of the four covalent bonds without accepting an electron. The resulting mobile hole makes the material p-type. Arsenic has five valence electrons: four form covalent bonds and the fifth is readily supplied as a conduction electron, making the material n-type. Thus Statement I correctly identifies both majority carriers. The doped solids remain electrically neutral overall; majority carriers do not mean a net bulk charge.

Step 2: Test the claim of an external current When the two regions meet, electrons and holes initially diffuse across the junction and recombine. Fixed ionised donors and acceptors create the depletion region and its built-in electric field. The field opposes further majority-carrier diffusion and produces a drift current. At equilibrium the two contributions cancel. The contact potential is an internal equilibrium potential and does not act as a battery supplying a sustained current through an external ammeter. Statement II is therefore false. This excludes (A), which reverses the truth values, (B), which accepts both, and (D), which rejects the true doping statement.

Step 3: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish a junction's built-in electric field from an externally applied bias: an unbiased junction in equilibrium supplies no net circuit current.

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45. According to law of equipartition of energy the molar specific heat of a diatomic gas at constant volume where the molecule has one additional vibrational mode is :- [4 marks]

(A) $\frac{9}{2}R$

(B) $\frac{5}{2}R$

(C) $\frac{3}{2}R$

(D) $\frac{7}{2}R$

Correct Answer: (D) $\frac{7}{2}R$

Solution:

Concept:

- Each active quadratic term in a molecule's energy contributes $\frac{1}{2}k_BT$ to its mean energy, or $\frac{1}{2}RT$ per mole.
- A diatomic molecule has three translational and two rotational terms. One active vibrational mode adds both kinetic and potential energy terms.

Step 1: Count all active energy contributions The three independent translational velocity components contribute $3RT/2$ per mole. Rotation of a linear diatomic molecule about its two perpendicular axes contributes another $2RT/2 = RT$. These five quadratic terms give the usual rigid-diatomic energy $5RT/2$. The question explicitly activates one additional vibrational mode, so it cannot be treated as a rigid molecule with vibration frozen out.

Step 2: Include vibration and differentiate the energy A single vibrational mode behaves as a harmonic oscillator. It has a quadratic kinetic-energy term and a quadratic elastic potential-energy term, each contributing $RT/2$ per mole. The extra contribution is therefore RT, rather than $RT/2$. Thus the molar internal energy and heat capacity are

$$U_m = \frac{3}{2}RT + RT + RT = \frac{7}{2}RT, \qquad C_{V,m} = \left(\frac{dU_m}{dT}\right)_V = \frac{7}{2}R.$$

Option (C) includes translation only; (B) includes translation and rotation but omits vibration; (A) would count two extra vibrational modes rather than the single one specified. Only (D) includes exactly seven quadratic terms.

Step 3: Conclusion The molar heat capacity at constant volume is $C_{V,m} = \frac{7}{2}R$. Hence, the correct option is (D).

Quick Tip to Solve: One vibrational mode adds R to molar C_V, because it stores both kinetic and potential energy; it does not add merely $R/2$.

46. A body of mass is taken from earth surface to the height h equal to twice the radius of earth (R_e), the increase in potential energy will be :

(g = acceleration due to gravity on the surface of Earth) [4 marks]

(A) $\frac{1}{2}mgR_e$

(B) $3\ mgR_e$

(C) $\frac{1}{3}mgR_e$

(D) $\frac{2}{3}mgR_e$

Correct Answer: (D) $\frac{2}{3}mgR_e$

Solution:

Concept:

- With zero potential energy at infinity, a body of mass m has $U(r) = -GM_em/r$ outside a spherical Earth of mass M_e.
- The surface gravitational acceleration is $g = GM_e/R_e^2$. Height is measured above the surface, whereas r is measured from Earth's centre.

Step 1: Identify the initial and final distances Initially the body is on the surface, so $r_i = R_e$. The stated height is $h = 2R_e$, which makes the final distance from the centre $r_f = R_e + h = 3R_e$. The field changes considerably over this distance, so the near-surface approximation $\Delta U = mgh$ cannot be used. We must subtract the exact gravitational potential energies at the two radii.

Step 2: Subtract the energies and express the answer using g Using the same zero of potential energy at both positions gives

$$\Delta U = U_f - U_i = -\frac{GM_em}{3R_e} + \frac{GM_em}{R_e}$$
$$= \frac{2GM_em}{3R_e} = \frac{2}{3}mgR_e.$$

The positive sign is appropriate: energy must be supplied to move the body farther from Earth. The increase is smaller than the escape energy $GM_em/R_e = mgR_e$, as the final radius is finite. Option (A) corresponds to a final radius of $2R_e$, confusing height with radius; (B) exceeds even the escape energy; (C) gives the wrong difference between the two reciprocal radii.

Step 3: Conclusion The increase in gravitational potential energy is $\Delta U = \frac{2}{3}mgR_e$. Hence, the correct option is (D).

Quick Tip to Solve: For a large height use $\Delta U = mgR_eh/(R_e + h)$, with surface g; always add Earth's radius to the stated height.

47. For a moving coil galvanometer, the deflection in the coil is 0.05 rad when a current of 10 mA is passes through it. If the torsional constant of suspension wire is $4.0 \times 10^{-5}\text{N m rad}^{-1}$, the magnetic field is 0.01T and the number of turns in the coil is 200, the area of each turn (in cm^2) is : [4 marks]

(A) 1.5

(B) 2.0

(C) 0.5

(D) 1.0

Correct Answer: (D) 1.0

Solution:

Concept:

- In the radial magnetic field of a moving-coil galvanometer, the magnetic torque is $\tau_m = NIAB$, where N is the turn count and A the area of each turn.
- At steady deflection the restoring torque $\kappa\theta$ balances the magnetic torque, giving $NIAB = \kappa\theta$.

Step 1: Convert the current and calculate the restoring torque The given current is $I = 10\,\text{mA} = 0.010\,\text{A}$; the deflection $\theta = 0.05\,\text{rad}$ is already in the angular unit used by the torsional constant. Therefore

$$\tau_r = \kappa\theta = (4.0 \times 10^{-5})(0.05) = 2.0 \times 10^{-6}\,\text{N m}.$$

This torque is the total restoring torque on the coil, and it balances the magnetic torque from all 200 turns.

Step 2: Solve for the area of one turn With $B = 0.01\,\text{T}$ and $N = 200$, rearranging the torque balance yields

$$A = \frac{\kappa\theta}{NIB} = \frac{2.0 \times 10^{-6}}{200 \times 0.010 \times 0.01} = 1.0 \times 10^{-4}\,\text{m}^2.$$

Since $1\,\text{m}^2 = 10^4\,\text{cm}^2$, the area is $1.0\,\text{cm}^2$. As a direct check, $200 \times 0.010 \times 10^{-4} \times 0.01 = 2.0 \times 10^{-6}\,\text{N m}$, exactly the required torque. The areas in (A), (B) and (C) would give respectively 1.5, 2 and 0.5 times this torque and hence the wrong deflection.

Step 3: Conclusion The area of each turn is $1.0\,\text{cm}^2$. Hence, the correct option is (D).

Quick Tip to Solve: Use $A = \kappa\theta/(NIB)$ with current in amperes; then multiply the answer in square metres by 10^4 for square centimetres.

48. Given below are two statements :

Statement I : Stopping potential in photoelectric effect does not depend on the power of the light source.

Statement II : For a given metal, the maximum kinetic energy of the photoelectron depends on the wavelength of the incident light.

In the light of above statements, choose the most appropriate answer from the options given below

[4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Statement I is correct but Statement II is incorrect

(C) Both Statement I and Statement II are correct

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (C) Both Statement I and Statement II are correct

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = hc/\lambda - \phi$, with work function ϕ fixed for a given metal.
- The stopping potential satisfies $eV_s = K_{\max}$. At fixed frequency, source power controls photon flux and photocurrent rather than the energy of a single photon.

Step 1: Test the statement about stopping potential Increasing the power of monochromatic incident light, while keeping its frequency fixed, increases the number of photons arriving per second. It can increase the number of emitted electrons and the saturation current. Each photon still has energy $h\nu$, so the maximum available kinetic energy of one photoelectron does not increase. Consequently the stopping potential needed to halt the fastest electrons is unchanged. Statement I is correct within the usual single-photon photoelectric regime.

Step 2: Test the wavelength dependence and compare options For the same metal, ϕ remains constant, and

$$K_{\max} = \frac{hc}{\lambda} - \phi, \qquad V_s = \frac{hc}{e\lambda} - \frac{\phi}{e}.$$

Thus, provided the wavelength is shorter than the threshold wavelength and emission occurs, decreasing λ increases the maximum kinetic energy. This establishes Statement II. If the wavelength exceeds the threshold, emission ceases, which is also consistent with dependence on photon energy. Options (A), (B) and (D) each call at least one of these true statements incorrect. No claim that Statement II must explain Statement I is required by the listed choices.

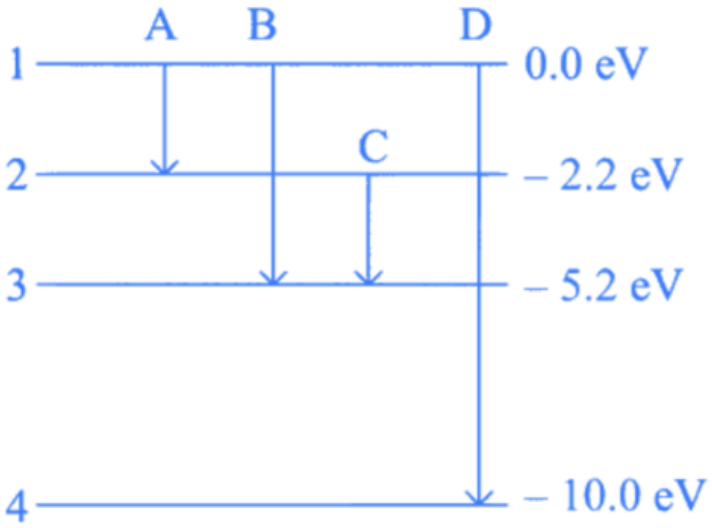
Step 3: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (C).

Quick Tip to Solve: At fixed wavelength, intensity changes the number of photoelectrons; changing wavelength changes the energy per photon and therefore the stopping potential.

49. The energy levels of an atom is shown in figure.

Which one of these transitions will result in the emission of a photon of wavelength 124.1 nm?

Given (h = 6.62 $\times$ 10^{-34} Js) [4 marks]



(A) C

(B) B

(C) A

(D) D

Correct Answer: (D) D

Solution:

Concept:

- An emitted photon carries the decrease in atomic energy: $E_\gamma = E_i - E_f = hc/\lambda$, with $E_i > E_f$.
- To compare with energy levels in electronvolts use $1\,\text{eV} = 1.6 \times 10^{-19}$ J. The downward arrows in the diagram indicate emission.

Step 1: Calculate the energy of the specified photon The wavelength is $\lambda = 124.1\,\text{nm} = 124.1 \times 10^{-9}\,\text{m}$. Taking $c = 3.0 \times 10^8\,\text{m s}^{-1}$ and the given Planck constant gives

$$E_\gamma = \frac{(6.62 \times 10^{-34})(3.0 \times 10^8)}{124.1 \times 10^{-9}} \simeq 1.6003 \times 10^{-18}\,\text{J} \simeq 10.0\,\text{eV}.$$

The small rounding difference is consistent with the supplied constants and wavelength. The transition must therefore have an energy separation of 10.0 eV.

Step 2: Compare all four downward transitions The diagram shows energy levels 0.0, -2.2, -5.2 and $-10.0\,\text{eV}$. Arrow A runs from 0.0 to $-2.2\,\text{eV}$, so its photon energy is 2.2 eV. Arrow B runs from 0.0 to $-5.2\,\text{eV}$, giving 5.2 eV. Arrow C runs from -2.2 to $-5.2\,\text{eV}$, giving 3.0 eV. Arrow D runs from 0.0 to $-10.0\,\text{eV}$, giving 10.0 eV. Only D matches the calculated photon energy; the transitions listed in (A), (B) and (C) all have smaller gaps and hence longer wavelengths.

Step 3: Conclusion Transition D emits the photon of wavelength $124.1\,\text{nm}$. Hence, the correct option

is (D).

Quick Tip to Solve: Convert the requested wavelength into a photon energy first, then compare the difference of the two levels at each arrow; compare gaps, not absolute energies.

50. Match List I with List II.

	List I		List II
A.	Troposphere	I.	Approximate 65–75 km over Earth's surface
B.	E-Part of Stratosphere	II.	Approximate 300 km over Earth's surface
C.	F_2-Part of Thermosphere	III.	Approximate 10 km over Earth's surface
D.	D-Part of Stratosphere	IV.	Approximate 100 km over Earth's surface

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-IV, C-II, D-I

(B) A-III, B-II, C-I, D-IV

(C) A-I, B-IV, C-III, D-II

(D) A-I, B-II, C-IV, D-III

Correct Answer: (A) A-III, B-IV, C-II, D-I

Solution:

Concept:

- The atmospheric regions used in radio-wave propagation are identified by their approximate heights above the surface of Earth.
- The D, E and F regions occur in increasing order of altitude; the troposphere is the lowest region listed here.

Step 1: Match the lowest region and the D region The troposphere extends approximately up to 10 km, so A matches III. The D region is approximately 65–75 km above Earth, so D matches I. These heights describe atmospheric layers rather than the distance travelled by a particular radio wave.

Step 2: Match the E and F regions The E region is approximately 100 km high, giving B–IV. The F_2 region is much higher, at approximately 300 km, giving C–II. Thus the complete sequence is A–III, B–IV, C–II, D–I. The second combination wrongly puts the E region at 300 km; the third and fourth wrongly put the troposphere at 65–75 km. These errors eliminate all the other listed combinations.

Step 3: Conclusion The matching is A–III, B–IV, C–II, D–I. Hence, the correct option is (A).

Quick Tip to Solve: Order the regions by height first: troposphere, D, E, then F_2. This quickly checks a matching sequence.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Two long parallel wires carrying currents 8A and 15A in opposite directions are placed at a distance of 7 cm from each other. A point P is at equidistant from both the wires such that the lines joining the point P to the wires are perpendicular to each other. The magnitude of magnetic field at P is _____ $\times 10^{-6}$ T.

(Given : $\sqrt{2} = 1.4$) [4 marks]

Solution:

Concept:

- For a long straight wire carrying current I, the field at perpendicular distance r is $B = \mu_0 I/(2\pi r)$, with $\mu_0/(2\pi) = 2 \times 10^{-7}$ in SI units.
- Magnetic fields add as vectors. A field is perpendicular to the radius from its wire, so perpendicular radii here produce perpendicular field vectors.

Step 1: Find the distance from each wire The two equal distances from P form the perpendicular sides of a right isosceles triangle, whose hypotenuse is the 7 cm separation of the wires. Therefore

$$r\sqrt{2} = 7\,\text{cm}, \qquad r = \frac{7}{1.4} = 5\,\text{cm} = 0.05\,\text{m}.$$

The given approximation for $\sqrt{2}$ is used as instructed.

Step 2: Add the perpendicular fields The individual magnitudes are

$$B_1 = \frac{2 \times 10^{-7} \times 8}{0.05} = 32 \times 10^{-6}\,\text{T}, \qquad B_2 = \frac{2 \times 10^{-7} \times 15}{0.05} = 60 \times 10^{-6}\,\text{T}.$$

Applying the right-hand rule gives perpendicular field directions at P; opposite currents do not make the fields collinear at this point. Thus

$$B = \sqrt{32^2 + 60^2} \times 10^{-6} = 68 \times 10^{-6}\,\text{T}.$$

Step 3: Conclusion The magnetic field magnitude is 68×10^{-6} T. The required numerical entry is 68.

Quick Tip to Solve: First determine the angle between the field vectors; opposite currents alone do not justify subtracting their magnitudes.

52. A capacitor has capacitance 5μF when it's parallel plates are separated by air medium of thickness d. A slab of material of dielectric constant 1.5 having area equal to that of plates but thickness $\frac{d}{2}$ is inserted between the plates. Capacitance of the capacitor in the presence of slab will be _____ μF. [4 marks]

Solution:

Concept:

- A dielectric slab filling the full plate area but only part of the separation forms two capacitive layers in series, since the electric field passes through both layers.
- For an air thickness d_a, dielectric thickness t and relative permittivity K, the effective separation is $d_a + t/K$, so $C = \varepsilon_0 A/(d_a + t/K)$.

Step 1: Separate the air and dielectric layers The slab occupies thickness $t = d/2$ and leaves air thickness $d_a = d/2$. The original air capacitor has $C_0 = \varepsilon_0 A/d = 5\,\mu\text{F}$. Since the slab covers the entire area, the layers are stacked along the field; they are not capacitors side by side in parallel.

Step 2: Calculate the effective separation Use $K = 1.5 = 3/2$. The dielectric contribution to the effective separation is smaller than its actual thickness:

$$d_{\text{eff}} = \frac{d}{2} + \frac{d/2}{3/2} = \frac{d}{2} + \frac{d}{3} = \frac{5d}{6}.$$

Hence

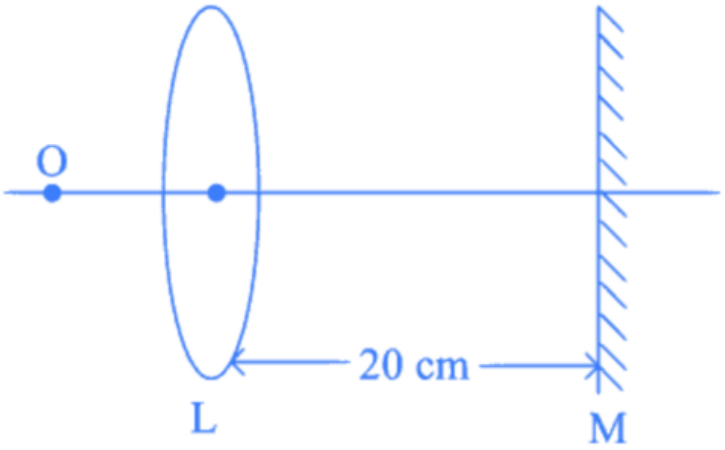
$$C = \frac{\varepsilon_0 A}{5d/6} = \frac{6}{5} C_0 = \frac{6}{5} \times 5 = 6\,\mu\text{F}.$$

The increase is smaller than multiplication by 1.5 because half of the gap still contains air.

Step 3: Conclusion The new capacitance is $6\,\mu$F, so the required numerical entry is 6.

Quick Tip to Solve: For a slab across the whole area, replace its thickness by t/K and add the remaining air thickness.

53. An object is placed on the principal axis of convex lens of focal length 10cm as shown. A plane mirror is placed on the other side of lens at a distance of 20 cm. The image produced by the plane mirror is 5cm inside the mirror. The distance of the object from the lens is _____ cm. [4 marks]



Solution:

Concept:

- A plane mirror forms its image as far behind the reflecting surface as its object lies in front of that surface.
- For the initial left-to-right passage through a thin lens, $1/f = 1/v - 1/u$. A real object to the left has $u < 0$, while a real image to the right has $v > 0$.

Step 1: Locate the image made by the lens The image produced by the mirror is 5 cm behind its surface. Therefore the object for the plane mirror, which is the image produced by the lens, must be 5 cm in front of the mirror. Since the mirror is 20 cm to the right of the lens, this intermediate image lies

$$v = 20 - 5 = 15\,\text{cm}$$

to the right of the lens. The given mirror image is used to locate this intermediate image before applying the lens formula.

Step 2: Apply the lens equation with signs The convex lens has positive focal length $f = 10\,\text{cm}$, and its intermediate image distance is $v = +15\,\text{cm}$. Thus

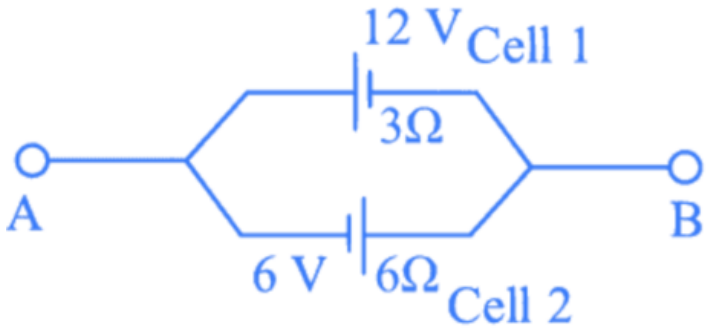
$$\frac{1}{10} = \frac{1}{15} - \frac{1}{u}, \qquad -\frac{1}{u} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}.$$

It follows that $u = -30\,\text{cm}$. The negative sign identifies the object side, while the question asks for the positive distance from the lens.

Step 3: Conclusion The object is $30\,\text{cm}$ from the lens. The required numerical entry is 30.

Quick Tip to Solve: Work backwards from the plane-mirror image: reflect its position in the mirror to find the lens image first.

54. Two cells are connected between points A and B as shown. Cell 1 has emf of 12 V and internal resistance of 3Ω. Cell 2 has emf of 6V and internal resistance of 6Ω. An external resistor R of 4Ω is connected across A and B. The current flowing through R will be _____ A. [4 marks]



Solution:

Concept:

- Kirchhoff's junction rule equates the current supplied at a node to the current leaving through the load.
- A cell with emf E and internal resistance r, whose positive terminal faces A, supplies $(E - V)/r$ when $V = V_A - V_B$. Reverse its emf sign if its positive terminal faces B.

Step 1: Read the cell polarities The longer plate of the 12 V cell is toward A, whereas the longer plate of the 6 V cell is toward B. Therefore the cells oppose one another. Let $V = V_A - V_B$; the current in the external 4-ohm resistor from A to B is $V/4$. The branch currents supplied toward A are $(12 - V)/3$ and $(-6 - V)/6$, respectively. A negative branch current simply means that its actual direction is opposite to this assigned direction.

Step 2: Solve the junction equation Adding the two branch contributions gives

$$\frac{12 - V}{3} + \frac{-6 - V}{6} = \frac{V}{4}.$$

Multiplying by 12 and simplifying,

$$4(12 - V) + 2(-6 - V) = 3V, \qquad 36 = 9V.$$

Thus $V = 4\,\text{V}$, and the load current is

$$I_R = \frac{V}{4} = 1\,\text{A}.$$

As a check, the 12 V branch supplies $8/3\,\text{A}$, while $5/3\,\text{A}$ flows through the other branch from A to B; their difference is the 1 A load current.

Step 3: Conclusion The current through R is $1\,\text{A}$, flowing from A to B. The required numerical entry is 1.

Quick Tip to Solve: Assign signed emfs from the longer cell plates before combining parallel cells; these two emfs oppose each other.

55. A series LCR circuit is connected to an AC source of 220 V, 50 Hz. The circuit contains a resistance R = 80Ω, an inductor of inductive reactance $X_L = 70\Omega$, and a capacitor of capacitive reactance $X_C = 130\Omega$. The power factor of circuit is $\frac{x}{10}$. The value of x is : [4 marks]

Solution:

Concept:

- For a series LCR circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where Z is the impedance magnitude.
- The power factor is $\cos\phi = R/Z$. The sign of $X_L - X_C$ determines whether current leads or lags, while its square determines the impedance.

Step 1: Find the net reactance and impedance The inductor and capacitor have opposite reactive effects, so their reactances are subtracted rather than added. With the stated values,

$$X = X_L - X_C = 70 - 130 = -60\,\Omega.$$

The negative result means the circuit is capacitive, so current leads the applied voltage. The impedance magnitude is

$$Z = \sqrt{80^2 + (-60)^2} = \sqrt{6400 + 3600} = 100\,\Omega.$$

Step 2: Calculate the power factor and compare Only the resistance dissipates average power; hence the ratio of resistance to impedance gives

$$\cos\phi = \frac{R}{Z} = \frac{80}{100} = 0.8 = \frac{8}{10}.$$

Comparing this with the given form $x/10$ yields $x = 8$. The supply voltage would set the rms current to $220/100 = 2.2\,\text{A}$, but is not needed for the power-factor ratio. Similarly, the supplied reactances already correspond to the stated frequency, so no separate inductance or capacitance calculation is necessary.

Step 3: Conclusion The power factor is 0.8 (leading), and the dimensionless numerical entry is $x = 8$.

Quick Tip to Solve: Use the 60–80–100 impedance triangle; subtract the reactances before forming the hypotenuse.

56. If a solid sphere of mass 5 kg and a disc of mass 4 kg have the same radius. Then the ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be $\frac{x}{7}$. The value of x is _____. [4 marks]

Solution:

Concept:

- The parallel-axis theorem is $I = I_{CM} + Md^2$, where the displaced axis must be parallel to the centre-of-mass axis.
- A uniform disc has moment of inertia $MR^2/4$ about a diameter in its plane; a solid sphere has $2MR^2/5$ about a diameter.

Step 1: Find the disc moment about the specified tangent Let the common radius be R. The tangent

to the disc lies in its plane, so its parallel central axis is a diameter, not the axis normal to the disc. For disc mass $M_d = 4\,\text{kg}$,

$$I_d = \frac{1}{4}M_dR^2 + M_dR^2 = \frac{5}{4}(4)R^2 = (5\,\text{kg})R^2.$$

Here the last expression has units of $\text{kg}\,\text{m}^2$ when R is measured in metres.

Step 2: Find the sphere moment and form the ratio A tangent axis of the sphere is parallel to a diameter and a distance R from it. With sphere mass $M_s = 5\,\text{kg}$,

$$I_s = \frac{2}{5}M_sR^2 + M_sR^2 = \frac{7}{5}(5)R^2 = (7\,\text{kg})R^2.$$

The common radius cancels in the requested ratio:

$$\frac{I_d}{I_s} = \frac{5R^2}{7R^2} = \frac{5}{7} = \frac{x}{7}.$$

Using the disc moment about its perpendicular central axis instead would answer a different tangent-axis question.

Step 3: Conclusion The ratio is $5/7$, so the dimensionless numerical entry is $x = 5$.

Quick Tip to Solve: For a disc tangent in its own plane, start from the diameter value $MR^2/4$, then add MR^2.

57. A body of mass 1 kg collides head on elastically with a stationary body of mass 3 kg. After collision, the smaller body reverses its direction of motion and moves with a speed of 2 m/s. The initial speed of the smaller body before collision is _____ ms^{-1}. [4 marks]

Solution:

Concept:

- In an isolated collision, total linear momentum is conserved. Choose the initial motion of the lighter body as positive.
- For a perfectly elastic collision, the relative speed of separation equals the relative speed of approach: $v_2 - v_1 = u_1 - u_2$.

Step 1: Assign signed velocities The masses are $m_1 = 1\,\text{kg}$ and $m_2 = 3\,\text{kg}$. Let the unknown initial speed of the first body be u. The second body is initially at rest, so $u_2 = 0$. Since the first body reverses direction after impact, its final velocity is $v_1 = -2\,\text{m}\,\text{s}^{-1}$, not positive 2. Writing velocities with their signs is essential when using momentum conservation.

Step 2: Apply momentum and elasticity Momentum conservation gives

$$1(u) + 3(0) = 1(-2) + 3v_2, \qquad u = -2 + 3v_2.$$

The elastic-collision condition gives

$$v_2 - (-2) = u - 0, \qquad v_2 = u - 2.$$

Substituting the second equation into the first,

$$u = -2 + 3(u - 2) = 3u - 8, \qquad 2u = 8, \qquad u = 4\,\mathrm{m\,s^{-1}}.$$

The heavier body therefore moves forward with $v_2 = 2\,\mathrm{m\,s^{-1}}$.

Step 3: Check the energy balance The initial kinetic energy is $\frac{1}{2}(1)(4^2) = 8\,\mathrm{J}$. The final kinetic energy is

$$\tfrac{1}{2}(1)(2^2) + \tfrac{1}{2}(3)(2^2) = 2 + 6 = 8\,\mathrm{J}.$$

Both the energy balance and momentum balance are satisfied, confirming that the reversal sign has been handled correctly.

Step 4: Conclusion The initial speed of the smaller body is $\boxed{4\,\mathrm{m\,s^{-1}}}$. The required numerical entry is $\boxed{4}$.

Quick Tip to Solve: For a stationary target, use $v_1 = (m_1 - m_2)u_1/(m_1 + m_2)$; keep the minus sign when the lighter body rebounds.

58. A spherical drop of liquid splits into 1000 identical spherical drops. If u_i is the surface energy of the original drop and u_f is the total surface energy of the resulting drops, the (ignoring evaporation), $\frac{u_f}{u_i} = \left(\frac{10}{x}\right)$. Then value of x is _____ : [4 marks]

Solution:

Concept:

- Surface energy of a liquid drop is $u = TA$, where T is surface tension and A is its surface area.
- With evaporation ignored, the total liquid volume is conserved. For n equal daughter drops, $R^3 = nr^3$, where R and r are the original and final radii.

Step 1: Use conservation of volume Let the original radius be R, and let each of the 1000 identical drops have radius r. Equating their combined volume to the initial volume,

$$\frac{4}{3}\pi R^3 = 1000\frac{4}{3}\pi r^3.$$

Therefore

$$r = \frac{R}{\sqrt[3]{1000}} = \frac{R}{10}.$$

The radius is reduced by a factor of ten, rather than by a factor of one thousand, because volume depends on the cube of radius.

Step 2: Compare total surface areas The initial surface energy is

$$u_i = T(4\pi R^2).$$

The daughter drops have a combined surface energy

$$u_f = 1000T(4\pi r^2) = 1000T\,4\pi\frac{R^2}{100} = 10T(4\pi R^2).$$

The surface tension is the same for the original and daughter drops, and cancels in the ratio. Thus

$$\frac{u_f}{u_i} = 10.$$

Fragmentation increases the total area and therefore increases the surface energy.

Step 3: Identify the requested parameter The question expresses the energy ratio as $10/x$. Equating this to the calculated value,

$$\frac{10}{x} = 10 \quad \Longrightarrow \quad x = 1.$$

The parameter x is dimensionless: it comes from a ratio of two energies, so no unit is attached to its numerical entry.

Step 4: Conclusion The required value is $\boxed{x = 1}$ (dimensionless).

Quick Tip to Solve: For breakup into n identical drops without volume loss, the total surface energy multiplies by $n^{1/3}$.

59. A nucleus disintegrates into two smaller parts, which have their velocities in the ratio 3 : 2. The ratio of their nuclear sizes will be $\left(\frac{x}{3}\right)^{\frac{1}{3}}$. The value of '$x$' is :- [4 marks]

Solution:

Concept:

- The radius of a nucleus is $R = R_0A^{1/3}$, where A is its mass number and R_0 is a common nuclear-radius constant.
- In a two-fragment disintegration of a nucleus initially at rest, the fragments have equal and opposite momenta: $m_1v_1 = m_2v_2$ in magnitude.

Step 1: Relate fragment masses to their speeds Label the fragments in the order used for the stated speed ratio, so that $v_1 : v_2 = 3 : 2$. Momentum conservation requires

$$m_1v_1 = m_2v_2 \quad \Longrightarrow \quad \frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{2}{3}.$$

The faster fragment is therefore lighter. Using the usual nuclear mass approximation $m \propto A$, this also gives $A_1/A_2 = 2/3$.

Step 2: Apply the nuclear radius relation Both fragments obey the nuclear-radius law, with the same proportionality constant. Dividing their radii eliminates R_0:

$$\frac{R_1}{R_2} = \frac{R_0A_1^{1/3}}{R_0A_2^{1/3}} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{2}{3}\right)^{1/3}.$$

The same ratio applies to their diameters. The term nuclear size here refers to linear size, rather than nuclear volume.

Step 3: Compare with the prescribed form The question gives the ratio of linear sizes as $(x/3)^{1/3}$. Hence

$$\left(\frac{x}{3}\right)^{1/3} = \left(\frac{2}{3}\right)^{1/3}.$$

Cubing both positive quantities yields $x/3 = 2/3$, so $x = 2$. Reversing the radius ratio would contradict the ordering of the specified velocities.

Step 4: Conclusion The required numerical value is $\boxed{x = 2}$, a dimensionless parameter.

Quick Tip to Solve: For two nuclear fragments, invert the speed ratio to obtain the mass ratio, then take its cube root for the radius ratio.

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60. A train blowing a whistle of frequency 320 Hz approaches an observer standing on the platform at a speed of 66 m/s. The frequency observed by the observer will be (given speed of sound = 330 ms^{-1}) _____ Hz. [4 marks]

Solution:

Concept:

- For sound in a stationary medium, a source approaching a stationary observer compresses the wavelength in front of it.
- If sound speed is v, source speed toward the observer is v_s, and emitted frequency is f, the observed frequency is $f_o = fv/(v - v_s)$.

Step 1: Identify the moving participant The train is the source and moves toward the platform observer at $v_s = 66\,\mathrm{m\,s^{-1}}$. The observer is stationary, so there is no observer-speed correction in the numerator. Use $f = 320\,\mathrm{Hz}$ and $v = 330\,\mathrm{m\,s^{-1}}$. Because the source is approaching, the effective denominator is $v - v_s$; using a plus sign would incorrectly describe recession.

Step 2: Calculate the compressed wavelength Between successive emissions separated by a period $1/f$, a wavefront travels v/f, while the train moves forward by v_s/f. Their difference is the wavelength in front of the train:

$$\lambda_{\text{front}} = \frac{v - v_s}{f} = \frac{330 - 66}{320} = \frac{264}{320} = 0.825\,\mathrm{m}.$$

The waves still travel through the air at $330\,\mathrm{m\,s^{-1}}$.

Step 3: Find the received frequency The stationary observer receives wavefronts at the rate

$$f_o = \frac{v}{\lambda_{\text{front}}} = \frac{330}{0.825} = 400\,\mathrm{Hz}.$$

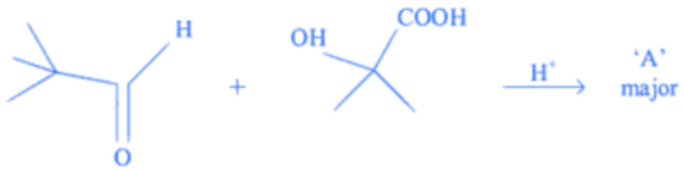
Equivalently, $f_o = 320 \times 330/264 = 320 \times 1.25$. The result exceeds the emitted frequency, as required for an approaching source; a value below 320 Hz would signal an incorrect direction sign.

Step 4: Conclusion The observed frequency is $\boxed{400\,\mathrm{Hz}}$. The numerical entry is $\boxed{400}$.

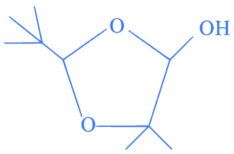
Quick Tip to Solve: For an approaching moving source, subtract its speed in the denominator; a stationary observer leaves the numerator equal to sound speed.

Chemistry – Section A: Multiple Choice Questions (80 marks)

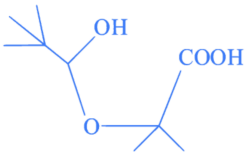
61. 'A' in the given reaction is [4 marks]



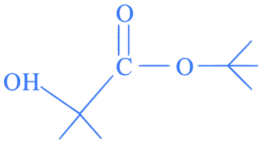
(A) 2-tert-Butyl-5,5-dimethyl-1,3-dioxolan-4-ol (cyclic hemiacetal)



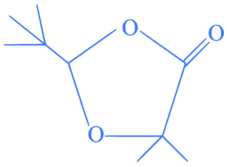
(B) 2-[(1-Hydroxy-2,2-dimethylpropyl)oxy]-2-methylpropanoic acid (open-chain hemiacetal)



(C) tert-Butyl 2-hydroxy-2-methylpropanoate (open-chain ester)

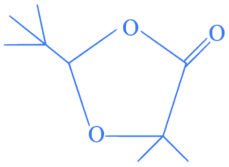


(D) 2-tert-Butyl-5,5-dimethyl-1,3-dioxolan-4-one (cyclic acetal ester)



Correct Answer: (D)

2-tert-Butyl-5,5-dimethyl-1,3-dioxolan-4-one (cyclic acetal ester)



Solution:

Concept:

- An aldehyde reacts with an alcohol under acid catalysis through a hemiacetal intermediate. A second oxygen nucleophile can replace the hydroxyl group after protonation and loss of water.
- An alpha-hydroxy carboxylic acid contains an alcoholic hydroxyl and a carboxylic hydroxyl on adjacent carbons. With an aldehyde, these can form a favourable five-membered cyclic acetal ester.

Step 1: Identify the two reactants The aldehyde is $(CH_3)_3C-CHO$. Its carbonyl carbon bears hydrogen and a tert-butyl group. The other reactant is the alpha-hydroxy acid $HO-C(CH_3)_2-COOH$. Its alcoholic oxygen and carboxylic hydroxyl oxygen are available to bond to the aldehyde carbon.

The acid carbonyl need not be reduced or removed during this condensation.

Step 2: Follow acid-catalysed ring closure Protonation activates the aldehyde carbonyl for attack by the alcoholic hydroxyl group. This produces the open-chain hemiacetal connectivity shown in option (B). Proton transfer and loss of water allow intramolecular capture by the carboxyl hydroxyl oxygen. The resulting ring contains, in sequence, the former aldehyde carbon, a carboxylic oxygen, the carboxyl carbonyl carbon, the dimethyl-substituted carbon and the alcoholic oxygen. This five-membered ring is a 1,3-dioxolan-4-one derivative.

Step 3: Compare all four structures Option (D) preserves the acid carbonyl, both methyl groups on the alpha carbon, and the tert-butyl group and hydrogen on the former aldehyde carbon. Option (B) stops at the open-chain hemiacetal rather than the cyclic condensation product. Option (A) changes the carboxyl carbonyl into a hydroxyl-bearing carbon, requiring a reduction absent from the conditions. Option (C) forms a tert-butyl ester and loses the aldehyde-carbon connectivity; acid-catalysed acetal formation does not produce that rearrangement.

Step 4: Conclusion The major product is the five-membered cyclic acetal ester shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: With an aldehyde and an alpha-hydroxy acid, track both hydroxyl oxygens: a five-membered ring can form while the acid carbonyl remains intact.

62. When the hydrogen ion concentration [H^+] changes by a factor of 1000, the value of pH of the solution _____ [4 marks]

(A) decreases by 2 units

(B) increases by 2 units

(C) decreases by 3 units

(D) increases by 1000 units

Correct Answer: (C) decreases by 3 units

Solution:

Concept:

- The pH is defined by $\mathrm{pH} = -\log_{10}[\mathrm{H}^+]$, with hydrogen-ion concentration expressed relative to the standard concentration.
- A multiplicative change in concentration becomes an additive change in pH: $\Delta\mathrm{pH} = -\log_{10}([\mathrm{H}^+]_f/[\mathrm{H}^+]_i)$.

Step 1: Represent the concentration change Let the initial hydrogen-ion concentration be c. Taking a change by a factor of 1000 to mean multiplication, the final concentration is $1000c$. The initial pH is $-\log_{10} c$, and the final pH is $-\log_{10}(1000c)$. Writing both values before subtracting avoids confusing a concentration factor with a pH difference.

Step 2: Evaluate the logarithmic difference Using $\log_{10}(ab) = \log_{10} a + \log_{10} b$,

$$\mathrm{pH}_f = -\log_{10} 1000 - \log_{10} c = \mathrm{pH}_i - 3.$$

Therefore

$$\Delta\mathrm{pH} = \mathrm{pH}_f - \mathrm{pH}_i = -3.$$

For example, an increase from 10^{-5} to $10^{-2}\,\mathrm{mol\,L^{-1}}$ changes pH from 5 to 2, confirming the decrease of three units.

Step 3: Rule out the other options A change of two pH units, whether upward or downward, corresponds to a concentration factor of 100, so neither (A) nor (B) fits a factor of 1000. Option (D) confuses the concentration multiplier with the logarithmic pH change. If the concentration were instead divided by 1000, pH would rise by three units, but that alternative is not among the listed choices; the listed response uses the multiplication interpretation.

Step 4: Conclusion On multiplying the hydrogen-ion concentration by 1000, pH decreases by 3 units. Hence, the correct option is (C).

Quick Tip to Solve: Count powers of ten in the hydrogen-ion factor: each tenfold increase lowers pH by one unit.

63. Match List I with List II

	List I (Amines)		List II ($\mathrm{pK_b}$)
A.	**Aniline**	**I.**	**3.25**
B.	**Ethanamine**	**II.**	**3.00**
C.	**N-Ethylethanamine**	**III.**	**9.38**
D.	**N, N-Diethylethanamine**	**IV.**	**3.29**

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-II, C-I, D-IV

(B) A-III, B-II, C-IV, D-I

(C) A-III, B-IV, C-II, D-I

(D) A-I, B-IV, C-II, D-III

Correct Answer: (C) A-III, B-IV, C-II, D-I

Solution:

Concept:

- Base strength is measured by K_b, while $\mathrm{p}K_b = -\log_{10} K_b$. A smaller $\mathrm{p}K_b$ means a stronger base.
- In water, amine basicity reflects both electron donation by alkyl groups and solvation of the conjugate acid. Aniline is much weaker because its nitrogen lone pair is delocalised into the benzene ring.

Step 1: Assign aniline first Aniline has a nitrogen lone pair conjugated with the aromatic ring. This resonance delocalisation reduces its availability for bonding to a proton. Its $\mathrm{p}K_b$ is therefore much larger than those of the aliphatic ethylamines in the list. The value 9.38 belongs to aniline, giving A–III. Option (D), which assigns aniline the value 3.25, is immediately excluded.

Step 2: Match the ethylamines The aqueous basicity order for these ethyl-substituted amines is diethylamine greater than triethylamine greater than ethylamine. N-Ethylethanamine is diethylamine,

the secondary amine, with $pK_b = 3.00$. N,N-Diethylethanamine is triethylamine, with $pK_b = 3.25$. Ethanamine is ethylamine, with $pK_b = 3.29$. Thus the required individual matches are

$$B - IV, \qquad C - II, \qquad D - I.$$

The small difference between the primary and tertiary amines is specific to their aqueous behaviour; an induction-only argument is insufficient.

Step 3: Select the complete matching set Combining the aromatic and aliphatic assignments gives A–III, B–IV, C–II, D–I. Option (A) wrongly assigns 3.00 to ethanamine and 3.25 to the secondary amine. Option (B) also assigns 3.00 to ethanamine and makes the secondary amine the weakest aliphatic base. Option (D) misidentifies aniline. Only option (C) retains all four tabulated values in their correct positions.

Step 4: Conclusion The correct matching is A–III, B–IV, C–II, D–I. Hence, the correct option is (C).

Quick Tip to Solve: Identify aniline by its large pKb first, then use the aqueous ethylamine order: secondary, tertiary, primary in decreasing basic strength.

64. What is the mass ratio of ethylene glycol ($C_2H_6O_2$, molar mass = 62 g/mol) required for making 500 g of 0.25 molal aqueous solution and 250 mL of 0.25 molar aqueous solution? [4 marks]

(A) 1 : 2

(B) 1 : 1

(C) 2 : 1

(D) 3 : 1

Correct Answer: (C) 2 : 1

Solution:

Concept:

- Molality is the number of moles of solute per kilogram of solvent; molarity is the number of moles per litre of solution.
- The mass of ethylene glycol is its amount in moles multiplied by its molar mass, $62\,\mathrm{g\,mol^{-1}}$. Solution mass includes both solvent and solute.

Step 1: Find the mass for the molal solution Let the ethylene glycol mass be x grams. The solvent mass is $(500 - x)$ grams, so the molality condition is

$$0.25 = \frac{x/62}{(500 - x)/1000}.$$

Therefore $1000x = 15.5(500 - x)$, giving

$$x = \frac{7750}{1015.5} = 7.6317\,\mathrm{g}.$$

The distinction between solution and solvent matters: 500 g is the solution mass, not exactly the mass

of water.

Step 2: Find the mass for the molar solution The volume of solution is $250\,\text{mL} = 0.250\,\text{L}$. Thus

$$n = MV = 0.25 \times 0.250 = 0.0625\,\text{mol},$$

and the ethylene glycol mass is

$$y = 0.0625 \times 62 = 3.875\,\text{g}.$$

No density is required because molarity already uses the final solution volume.

Step 3: Compare the two masses The exact ratio from the given definitions is

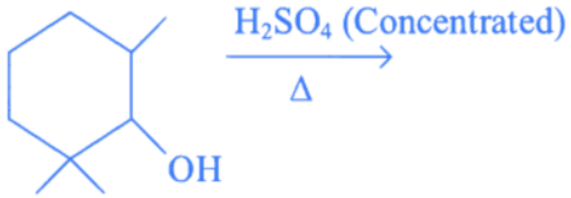
$$x : y = \frac{7.6317}{3.875} : 1 = 1.9695 : 1.$$

For this dilute solution, taking its water mass as approximately 500 g gives $x \approx 0.25 \times 0.500 \times 62 = 7.75\,\text{g}$, so the ratio is approximately $2 : 1$. The listed ratios $1 : 2$, $1 : 1$ and $3 : 1$ are inconsistent with the calculated relative masses.

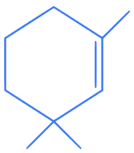
Step 4: Conclusion The masses are $7.6317\,\text{g}$ and $3.875\,\text{g}$; their ratio is $1.9695 : 1$, approximately $2 : 1$, the intended listed answer. Hence, the correct option is (C).

Quick Tip to Solve: Use solvent mass for molality and solution volume for molarity; check whether a simple option uses the dilute-solution approximation.

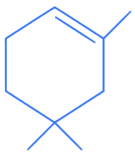
65. Find out the major product from the following reaction. [4 marks]



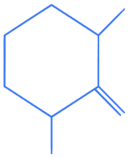
(A) 1,3,3-Trimethylcyclohex-1-ene



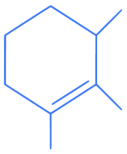
(B) 1,4,4-Trimethylcyclohex-1-ene



(C) 1,3-Dimethyl-2-methylidenecyclohexane

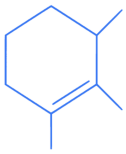


(D) 1,2,3-Trimethylcyclohex-1-ene



Correct Answer: (D)

1,2,3-Trimethylcyclohex-1-ene



Solution:

Concept:

- Concentrated H_2SO_4 and heat dehydrate an alcohol by protonation, loss of water, and elimination. A secondary alcohol can undergo an E1 pathway with carbocation rearrangement.
- A neighbouring alkyl group can migrate with its bonding electron pair in a 1,2-shift. Elimination from the resulting more stable carbocation preferentially produces the more substituted alkene.

Step 1: Generate the initial carbocation Name the alcohol-bearing ring carbon C1, the neighbouring gem-dimethyl carbon C2, and the other methyl-bearing neighbour C6. Protonation changes the poor leaving group $-OH$ into $-OH_2^+$; departure of water then leaves a secondary carbocation at C1. The reagent and heating favour dehydration rather than substitution, and the carbon skeleton need not remain unchanged.

Step 2: Rearrange to a tertiary carbocation C2 has two methyl groups and no hydrogen. A methyl group migrates from C2 to C1, carrying the electron pair of its old C2–methyl bond. The positive charge consequently moves to C2. C2 is now bonded to three carbon groups and is a tertiary carbocation. C1 now carries a methyl group and still has the hydrogen needed for the next elimination.

Step 3: Select the most substituted alkene Loss of the C1 hydrogen produces the C1=C2 double bond. Each double-bond carbon has a methyl substituent and a ring-carbon substituent, so this alkene is tetrasubstituted. It is the structure in option (D), with methyl groups on three consecutive ring carbons. Options (A) and (B) show less substituted endocyclic double bonds; option (C) has a terminal exocyclic methylene group and is also less substituted. These do not gain the same alkene stabilisation as the rearranged product.

Step 4: Conclusion The major product is the rearranged, tetrasubstituted alkene 1,2,3-trimethylcyclohex-1-ene shown in (D). Hence, the correct option is (D).

Quick Tip to Solve: Before eliminating from an alcohol-derived carbocation, inspect an adjacent quaternary carbon for a methyl shift that enables a more substituted alkene.

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66. The isomeric deuterated bromide with molecular formula C_4H_8DBr having two chiral carbon atoms is [4 marks]

(A) 2 - Bromo - 1 - deuterobutane

(B) 2 - Bromo - 1 - deutero - 2 - methylpropane

(C) 2 - Bromo - 3 - deuterobutane

(D) 2 - Bromo - 2 - deuterobutane

Correct Answer: (C) 2 - Bromo - 3 - deuterobutane

Solution:

Concept:

- A tetrahedral carbon is chiral when its four attached substituents are different. Isotopes count as different groups, so hydrogen H and deuterium D are distinguishable.
- Count stereogenic carbons separately: one chiral carbon does not make all neighbouring tetrahedral

carbons chiral. A carbon with two hydrogen atoms or two identical methyl groups is not a chiral centre.

Step 1: Inspect the proposed two-centre structure For 2-bromo-3-deuterobutane the condensed structure is

$$CH_3-CH(Br)-CH(D)-CH_3.$$

Carbon 2 bears Br, H, CH_3, and $CH(D)CH_3$: four different groups. Carbon 3 bears D, H, CH_3, and $CH(Br)CH_3$: again four different groups. Both central carbons are therefore chiral.

Step 2: Rule out the remaining structures In (A), $CH_2D-CH(Br)-CH_2-CH_3$, only carbon 2 is chiral; the terminal deuterated carbon still has two identical H atoms. In (B), the central carbon of the branched skeleton has two identical methyl substituents, while the terminal CH_2D is also achiral. In (D), $CH_3-C(Br)(D)-CH_2-CH_3$, carbon 2 is chiral but carbon 3 is CH_2, so there is only one chiral carbon. Thus neither mere isotopic substitution nor the presence of bromine guarantees two stereogenic centres.

Step 3: Conclusion Only 2-bromo-3-deuterobutane has two chiral carbon atoms, at C2 and C3. Hence, the correct option is (C).

Quick Tip to Solve: When counting chiral centres in deuterated molecules, treat H and D as different, then still check all four groups at every candidate carbon.

67. Match List I with List II

	List I Isomeric pairs		List II Type of isomers
A.	Propanamine and N-Methylethanamine	I.	Metamers
B.	Hexan-2-one and Hexan-3-one	II.	Positional isomers
C.	Ethanamide and Hydroxyethanimine	III.	Functional isomers
D.	o-nitrophenol and p-nitrophenol	IV.	Tautomers

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-III, B-IV, C-I, D-II

(C) A-II, B-III, C-I, D-IV

(D) A-IV, B-III, C-I, D-II

Correct Answer: (A) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Structural isomers have the same molecular formula but different connectivity. Functional isomers differ in functional-group type, whereas positional isomers retain the same type at different positions.
- Metamers differ in the distribution of alkyl groups around a polyvalent functional group. Tautomers

interconvert by proton transfer accompanied by a shift of a double bond.

Step 1: Classify the amines and ketones A: Propanamine is the primary amine $CH_3CH_2CH_2NH_2$, whereas N-methylethanamine is the secondary amine $CH_3CH_2NHCH_3$. In the classification used here, the primary and secondary amines are functional isomers: A–III. B: Hexan-2-one has methyl and butyl groups on the sides of CO; hexan-3-one has ethyl and propyl groups. Both are ketones with the same formula, but the carbon distribution around the carbonyl differs, giving the listed metameric pair: B–I.

Step 2: Classify the amide and substituted phenols C: Ethanamide and hydroxyethanimine form the amide–imidic acid tautomeric pair

$$CH_3C(=O)NH_2 \rightleftharpoons CH_3C(OH)=NH.$$

A hydrogen shifts from N to O while the double bond shifts from C=O to C=N, hence C–IV. D: The nitro group is at the ortho position in one phenol and the para position in the other; the functional groups remain the same. They are positional isomers, hence D–II.

Step 3: Match the complete code The combined assignment is A–III, B–I, C–IV, D–II. Option (B) incorrectly makes the ketones tautomers and the amide pair metamers. Option (C) wrongly assigns the amines as positional isomers and the phenols as tautomers. Option (D) wrongly makes the two amines tautomers. Only the first code preserves all four classifications.

Step 4: Conclusion The matching is A–III, B–I, C–IV, D–II. Hence, the correct option is (A).

Quick Tip to Solve: For matching questions, identify a distinctive tautomeric pair first; then distinguish position changes from redistribution of carbon groups about a functional group.

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68. A. Ammonium salts produce haze in atmosphere.

B. Ozone gets produced when atmospheric oxygen reacts with chlorine radicals.

C. Polychlorinated biphenyls act as cleansing solvents.

D. 'Blue baby' syndrome occurs due to the presence of excess of sulphate ions in water.

Choose the correct answer from the options given below : [4 marks]

(A) B and C only

(B) A and D only

(C) A, B and C only

(D) A and C only

Correct Answer: (D) A and C only

Solution:

Concept:

- Fine suspended particles scatter light and cause atmospheric haze; ammonium salts formed from atmospheric ammonia can contribute to these aerosols.
- Chlorine radicals participate in ozone destruction. Excess nitrate in drinking water is associated with methaemoglobinaemia, also called blue baby syndrome.

Step 1: Evaluate the atmospheric statements A is correct. Ammonia can react with acidic atmospheric species to give fine ammonium-salt particles, which scatter light and produce haze. B is incorrect: chlorine radicals do not generate ozone by reacting with atmospheric oxygen. They catalytically deplete ozone, for example

$$Cl^{\bullet} + O_3 \longrightarrow ClO^{\bullet} + O_2, \qquad ClO^{\bullet} + O \longrightarrow Cl^{\bullet} + O_2.$$

The regenerated chlorine radical can continue the destruction cycle.

Step 2: Evaluate the organic pollutant and water statements C is correct in the stated environmental-chemistry context: polychlorinated biphenyls have been used as cleansing solvents and are persistent organic pollutants. This historical application does not imply that they are harmless; their persistence allows accumulation in the environment. D is incorrect because blue baby syndrome is linked to excess nitrate, not sulphate. Nitrate can be reduced to nitrite, which oxidises haemoglobin to methaemoglobin and reduces oxygen transport.

Step 3: Select the statement combination The truth sequence is true, false, true, false. Thus exactly A and C are correct. Option (A) includes false B and omits true A; option (B) includes false D and omits true C; option (C) includes false B. Option (D) contains both true statements and neither false statement.

Step 4: Conclusion Only statements A and C are correct. Hence, the correct option is (D).

Quick Tip to Solve: Associate chlorine radicals with ozone depletion and nitrate-contaminated drinking water with blue baby syndrome; these two corrections settle the combination.

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69. Statement I : Dipole moment is a vector quantity and by convention it is depicted by a small arrow with tail on the negative centre and head pointing towards the positive centre.

Statement II : The crossed arrow of the dipole moment symbolizes the direction of the shift of charges in the molecules.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is correct but Statement II is incorrect

(B) Statement I is incorrect but Statement II is correct

(C) Both Statement I and Statement II are correct

(D) Both Statement I and Statement II are incorrect

Correct Answer: (B) Statement I is incorrect but Statement II is correct

Solution:

Concept:

- The electric dipole moment is a vector. In the chemical bond-polarity notation a crossed arrow points from the positive end towards the negative end, indicating displacement of electron density.
- The physics dipole vector $\boldsymbol{\mu} = q\boldsymbol{r}$ points from negative to positive; it must be distinguished from the crossed arrow convention used to depict bond polarity in chemistry.

Step 1: Assess statement I in the given convention The opening claim that dipole moment is a vector is correct. However, the chemical convention referred to in this pair of statements is a crossed arrow, with its crossed tail at the positive centre and its head at the negative centre. Statement I reverses that direction by putting its tail on the negative centre. As a complete statement about the chemical notation it is therefore incorrect. One should not combine the direction of the physical dipole vector with the crossed chemical arrow.

Step 2: Assess statement II and choose the option The crossed arrow in chemical structural notation shows the direction of the shift of electronic charge towards the more electronegative atom. The cross marks the electron-deficient, positive end; the arrowhead marks the electron-rich, negative end. Thus statement II correctly describes the purpose of this notation. For example, the chemical polarity arrow for HCl points from H towards Cl, with the cross at H. Since I is incorrect but II is correct, option (B) is selected; (A) reverses the verdicts, (C) accepts the incorrect direction in I, and (D) rejects the correct explanation in II.

Step 3: Conclusion In the crossed-arrow chemical convention, statement I is incorrect and statement II is correct. Hence, the correct option is (B).

Quick Tip to Solve: Keep the two conventions separate: the physical dipole vector points negative to positive; the crossed chemical polarity arrow points towards the negative end.

70. Match List I with List II

	List I Coordination entity		**List II Wavelength of light absorbed in nm**
A.	$[CoCl(NH_3)_5]^{2+}$	**I.**	**310**
B.	$[Co(NH_3)_6]^{3+}$	**II.**	**475**
C.	$[Co(CN)_6]^{3-}$	**III.**	**535**
D.	$[Cu(H_2O)_4]^{2+}$	**IV.**	**600**

Choose the correct answer from the options given below : [4 marks]

(A) A-IV, B-I, C-III, D-II

(B) A-III, B-II, C-I, D-IV

(C) A-III, B-I, C-II, D-IV

(D) A-II, B-III, C-IV, D-I

Correct Answer: (B) A-III, B-II, C-I, D-IV

Solution:

Concept:

- A d-electron transition absorbs a photon with energy matching the ligand-field splitting:

$$\Delta = \frac{hc}{\lambda}.$$

  Greater splitting therefore means shorter absorbed wavelength.
- For comparable cobalt(III) complexes, the spectrochemical series places Cl^- below NH_3, and CN^- above NH_3 in ligand-field strength. The metal identity and oxidation state also affect the splitting.

Step 1: Order the cobalt(III) complexes In A, $[CoCl(NH_3)_5]^{2+}$, cobalt is +3 because the chloride ligand contributes -1 and ammonia is neutral. B, $[Co(NH_3)_6]^{3+}$, and C, $[Co(CN)_6]^{3-}$, also contain cobalt(III). Replacing ammonia by weaker chloride lowers the splitting in A, while replacing ammonia by strong-field cyanide raises it in C. Therefore

$$\Delta_C > \Delta_B > \Delta_A, \qquad \lambda_C < \lambda_B < \lambda_A.$$

Step 2: Assign the supplied wavelengths The listed cobalt absorption values are assigned as C: $310\,nm$, B: $475\,nm$, and A: $535\,nm$, consistent with the inverse ordering. The aqua copper(II) complex D absorbs the longest listed wavelength, $600\,nm$. Thus A–III, B–II, C–I, D–IV. These numerical wavelengths are the supplied matching data; ligand-field reasoning identifies their consistent order rather than calculating them from ligand names alone.

Step 3: Check the alternatives Option (A) assigns the strongest-field cyanide complex a longer wavelength than the ammine complex, contradicting its larger splitting. Option (C) interchanges the assignments of B and C and has the same reversal. Option (D) gives the cyanide complex the longest wavelength and the aqua copper complex the shortest. Only option (B) gives the required cobalt ordering and the corresponding aqua-complex value.

Step 4: Conclusion The matching is A–III (535 nm), B–II (475 nm), C–I (310 nm), D–IV (600 nm). Hence, the correct option is (B).

Quick Tip to Solve: Convert ligand-field strength into wavelength only after remembering the inverse relation: stronger field means larger splitting and shorter absorbed wavelength.

71. A chloride salt solution acidified with dil.HNO_3 gives a curdy white precipitate, [A], on addition of $AgNO_3$. [A] on treatment with NH_4OH gives a clear solution B. A and B are respectively : [4 marks]

(A) $H[AgCl_3]$ & $(NH_4)[Ag(OH)_2]$

(B) $AgCl$ & $[Ag(NH_3)_2]Cl$

(C) $H[AgCl_3]$ & $[Ag(NH_3)_2]Cl$

(D) $AgCl$ & $(NH_4)[Ag(OH)_2]$

Correct Answer: (B) $AgCl$ & $[Ag(NH_3)_2]Cl$

Solution:

Concept:

- A curdy white precipitate with silver nitrate is characteristic of silver chloride, $AgCl$. Its formation is an ionic precipitation reaction.
- Ammonia acts as a ligand for Ag^+, forming the soluble diamminesilver(I) complex $[Ag(NH_3)_2]^+$. Complex formation can therefore dissolve a sparingly soluble salt.

Step 1: Identify the white precipitate The chloride salt supplies chloride ions and silver nitrate supplies silver ions. Their combination gives the observed insoluble curdy white solid:

$$Ag^+(aq) + Cl^-(aq) \longrightarrow AgCl(s).$$

Thus A is silver chloride, not the hydrogen trichloroargentate species shown in two of the options. Dilute nitric acid removes interfering basic anions without introducing additional chloride ions.

Step 2: Explain dissolution in aqueous ammonia Aqueous ammonium hydroxide supplies ammonia molecules. Two ammonia ligands bind to each silver(I) ion, lowering the concentration of free silver ions and drawing the dissolution equilibrium towards solution:

$$AgCl(s) + 2\,NH_3(aq) \rightleftharpoons [Ag(NH_3)_2]^+(aq) + Cl^-(aq).$$

The resulting clear solution can be represented as $[Ag(NH_3)_2]Cl$. The complex cation is diamminesilver(I); chloride remains its counter-ion.

Step 3: Check the proposed pairs Only the pair containing both $AgCl$ and $[Ag(NH_3)_2]Cl$ describes the two observations. Options A and C incorrectly identify the initial precipitate. Option D correctly names the precipitate but incorrectly attributes dissolution to a hydroxido complex rather than the ammine complex formed by ammonia.

Step 4: Conclusion A is $AgCl$, and B is $[Ag(NH_3)_2]Cl$ in solution. Hence, the correct option is (B).

Quick Tip to Solve: For a silver-halide test, use both colour and solubility: curdy white $AgCl$ dissolves in ammonia by ammine-complex formation.

72. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Butylated hydroxy anisole when added to butter increases its shelf life.

Reason R : Butylated hydroxy anisole is more reactive towards oxygen than food.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) A is correct but R is not correct

(B) Both A and R are correct but R is NOT the correct explanation of A

(C) Both A and R are correct and R is the correct explanation of A

(D) A is not correct but R is correct

Correct Answer: (C) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- Oxidation of fats and oils causes rancidity, producing undesirable odours and flavours and shortening the useful life of food.
- Antioxidants such as butylated hydroxy anisole (BHA) inhibit oxidative deterioration. In the school-level description they react preferentially with oxygen or interrupt the radical chain that oxidises food.

Step 1: Evaluate the assertion Butter contains fats that deteriorate on exposure to air. Adding a suitable small amount of BHA retards this oxidation and delays rancidity. Because the butter retains its acceptable quality for a longer time, its shelf life increases. Assertion A therefore correctly states the role of BHA as a food antioxidant; it is not being used as a sweetener or as a nutrient.

Step 2: Evaluate the reason BHA is more readily attacked in the oxidation process than the food components it protects. Its phenolic antioxidant action can interrupt the propagation of lipid oxidation by reacting with reactive radicals. This preferential protective reaction is the meaning of the statement that it is more reactive towards oxygen than the food. Reason R is therefore correct in the context of the question.

Step 3: Connect the reason with the assertion Preferential oxidation or radical interception by the antioxidant suppresses oxidation of the butter fat. Reduced oxidation delays rancidity, which directly explains the increased shelf life. Thus the reason supplies the causal link required for the assertion, rather than merely being another unrelated true property. Options A and D are excluded because neither statement is false. Option B is excluded because the two statements are causally connected.

Step 4: Conclusion Both A and R are correct, and R correctly explains A. Hence, the correct option is (C).

Quick Tip to Solve: In food-preservation assertion-reason questions, connect antioxidant action to slower fat oxidation and then to delayed rancidity.

73. Potassium dichromate acts as a strong oxidizing agent in acidic solution. During this process, the oxidation state changes from : [4 marks]

(A) + 2 to + 1

(B) + 6 to + 2

(C) + 6 to + 3

(D) + 3 to + 1

Correct Answer: (C) + 6 to + 3

Solution:

Concept:

- The sum of oxidation numbers in an ion equals its charge. Oxygen normally has oxidation number -2 in the dichromate ion $Cr_2O_7{}^{2-}$.
- An oxidising agent accepts electrons and is itself reduced. In acidic solution, dichromate is reduced to chromium(III), Cr^{3+}.

Step 1: Calculate the initial oxidation state Let the oxidation number of each chromium atom in $Cr_2O_7{}^{2-}$ be x. The seven oxygen atoms contribute -14, while the ion has net charge -2. Therefore

$$2x + 7(-2) = -2 \quad \Rightarrow \quad 2x = 12 \quad \Rightarrow \quad x = +6.$$

Potassium is a spectator ion in this oxidation process; the change occurs at chromium.

Step 2: Write the acidic reduction half-reaction The standard reduction half-reaction is

$$Cr_2O_7{}^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

Chromium ends in the $+3$ state. Each of the two chromium atoms accepts three electrons, giving six electrons for one dichromate ion. The oxygen and hydrogen atoms are balanced by water and acid, and the total charge on each side is $+6$.

Step 3: Compare all proposed changes The relevant change is thus $+6$ to $+3$, which is reduction and accounts for dichromate oxidising another substance. Options A and D have the wrong initial oxidation state for chromium in dichromate. Option B begins with the correct state but ends at $+2$, whereas the normal stable product of this acidic dichromate reduction is chromium(III), not chromium(II).

Step 4: Conclusion Chromium changes from $+6$ in dichromate to $+3$ in acidic solution. Hence, the correct option is (C).

Quick Tip to Solve: For acidic dichromate, remember the balanced half-reaction: two chromium atoms each gain three electrons, so six electrons are accepted overall.

74. Which one among the following metals is the weakest reducing agent? [4 marks]

(A) K

(B) Na

(C) Li

(D) Rb

Correct Answer: (B) Na

Solution:

Concept:

- A metal is a reducing agent when it loses an electron: $\mathrm{M} \longrightarrow \mathrm{M}^{+} + \mathrm{e}^{-}$. A more negative aqueous standard reduction potential for $\mathrm{M}^{+}/\mathrm{M}$ indicates a stronger reducing metal.
- The aqueous reducing strengths of alkali metals depend on atomisation, ionisation and hydration together. The isolated-atom ionisation-energy trend alone does not give their aqueous order.

Step 1: Use the appropriate electrochemical comparison Compare the standard reduction potentials for formation of the metals from their aqueous ions. Approximate values are

$$E^{\circ}(\mathrm{Li}^{+}/\mathrm{Li}) = -3.04\,\mathrm{V}, \quad E^{\circ}(\mathrm{Na}^{+}/\mathrm{Na}) = -2.71\,\mathrm{V},$$
$$E^{\circ}(\mathrm{K}^{+}/\mathrm{K}) = -2.93\,\mathrm{V}, \quad E^{\circ}(\mathrm{Rb}^{+}/\mathrm{Rb}) = -2.98\,\mathrm{V}.$$

These values include the energetic effects of forming and hydrating the aqueous ions.

Step 2: Identify the weakest electron donor Sodium has the least negative reduction potential of these four metals. Reversing the reduction reaction to describe metal oxidation shows that sodium has the smallest standard tendency to donate electrons among the listed metals under aqueous standard conditions. It is therefore the weakest reducing agent of the four, even though sodium itself is still a powerful reducing metal.

Step 3: Rule out the other metals Potassium and rubidium have more negative reduction potentials than sodium and are stronger reducing agents. Lithium is the strongest aqueous reducing agent here: the particularly large hydration enthalpy of its small ion outweighs its relatively high ionisation enthalpy. Thus choosing lithium merely because it is hardest to ionise as an isolated atom would ignore hydration. Options A, C and D are excluded by the same potential comparison.

Step 4: Conclusion Among the listed alkali metals, sodium is the weakest reducing agent in aqueous solution. Hence, the correct option is (B).

Quick Tip to Solve: For alkali-metal reducing power in aqueous solution, compare standard electrode potentials and include hydration; do not use ionisation energy alone.

75. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Carbon forms two important oxides - CO and CO_2. CO is neutral whereas CO_2 is acidic in nature

Reason R : CO_2 can combine with water in a limited way to form carbonic acid, while CO is sparingly soluble in water

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) A is not correct but R is correct

(B) Both A and R are correct but R is NOT the correct explanation of A

(C) Both A and R are correct and R is the correct explanation of A

(D) A is correct but R is not correct

Correct Answer: (C) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- An acidic oxide reacts with water to form an acid or with a base to form a salt. CO_2 is an acidic non-metal oxide; CO is a neutral oxide.
- The dissolution and hydration of carbon dioxide in water establish a limited equilibrium producing carbonic acid: $CO_2 + H_2O \rightleftharpoons H_2CO_3$.

Step 1: Judge the assertion about the two oxides Carbon monoxide and carbon dioxide are both important oxides of carbon. Carbon monoxide does not show the usual acidic or basic oxide behaviour under ordinary conditions and is classified as neutral. Carbon dioxide reacts with bases, for example

$$CO_2 + 2\,NaOH \longrightarrow Na_2CO_3 + H_2O,$$

so it is acidic. Both classifications in Assertion A are correct.

Step 2: Judge the reason about their behaviour in water Carbon dioxide combines with water to a limited extent to form carbonic acid:

$$CO_2 + H_2O \rightleftharpoons H_2CO_3.$$

The equilibrium is limited, but the acid that forms can ionise and gives the solution its acidic character. Carbon monoxide is sparingly soluble in water and does not form an acid by ordinary hydration. These are the contrasting observations stated in R, so the reason is correct.

Step 3: Test the explanatory link Formation of carbonic acid accounts for the acidic nature of carbon dioxide in water; carbon monoxide lacks the corresponding acid-forming behaviour. The stated contrast therefore explains the different classifications in A in the intended oxide-and-water context. Options A and D require one of the true statements to be false. Option B denies the relevant explanatory connection and is therefore also rejected.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (C).

Quick Tip to Solve: To classify these carbon oxides, connect CO_2 with carbonic acid and base neutralisation; CO has no analogous ordinary acid-forming reaction.

76. Match List I with List II

	List I (Name of polymer)		List II (Uses)
A.	Glyptal	I.	Flexible pipes
B.	Neoprene	II.	Synthetic wool
C.	Acrilan	III.	Paints and Lacquers
D.	LDP	IV.	Gaskets

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-III, B-IV, C-I, D-II

(C) A-III, B-II, C-IV, D-I

(D) A-III, B-IV, C-II, D-I

Correct Answer: (D) A-III, B-IV, C-II, D-I

Solution:

Concept:

- A polymer application follows its characteristic physical properties: resin-like film formation, elastomeric resistance, fibre formation or flexibility.
- Glyptal is an alkyd polyester used in coating formulations; neoprene is a synthetic rubber; Acrilan is an acrylic fibre; LDP denotes low-density polythene.

Step 1: Match the coating resin and the rubber Glyptal is formed from a polyhydric alcohol such as ethylene glycol and a dibasic acid such as phthalic acid. Its resinous coating behaviour makes it useful in paints and lacquers, so A matches III. Neoprene is obtained by polymerisation of chloroprene and is an elastomer with useful resistance to oils and chemicals. It is used for gaskets, seals and related rubber articles, so B matches IV.

Step 2: Match the fibre and the flexible thermoplastic Acrilan is an acrylic fibre associated with acrylonitrile-based polymers. Its wool-like properties allow its use as synthetic wool, so C matches II. Low-density polythene has a branched structure and reduced close packing of chains compared with high-density polythene. The resulting softer, flexible material is used for flexible pipes, so D matches I.

Step 3: Compare the complete matching codes The combined matching is

A-III, B-IV, C-II, D-I.

Option A incorrectly assigns flexible pipes to neoprene and gaskets to Acrilan. Option B correctly matches the first two polymers but interchanges the applications of Acrilan and LDP. Option C incorrectly assigns synthetic wool to neoprene and gaskets to Acrilan. Only option D preserves all four property-based matches.

Step 4: Conclusion The matching is A-III, B-IV, C-II, D-I. Hence, the correct option is (D).

Quick Tip to Solve: Fix the distinctive pairs first: Acrilan is synthetic wool and glyptal is a coating resin; then check neoprene gaskets and flexible LDP pipes.

77. Given below are two statements :

Statement I : In froth floatation method of rotating paddle agitates the mixture to drive air out of it.

Statement II : Iron pyrites and generally avoided for extraction of iron due to environmental reasons.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is true but Statement II is false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are true

(D) Both Statement I and Statement II are false

Correct Answer: (B) Statement I is false but Statement II is true

Solution:

Concept:

- Froth flotation concentrates suitable sulphide ores through preferential wetting: ore particles attach to air bubbles in the presence of frothing agents, whereas gangue is wetted by water.
- Roasting a sulphide ore such as iron pyrites, FeS_2, can release sulphur dioxide. Sulphur-oxide pollution is an important environmental disadvantage of sulphide-ore processing.

Step 1: Check the stated function of the paddle In froth flotation, the rotating paddle agitates the mixture and helps draw air into it, generating bubbles and froth. Hydrophobic ore particles become attached to bubbles and rise, while the more water-wetted gangue remains behind. The paddle does not operate to drive air out of the mixture as Statement I claims. Since it reverses the relevant role of air, Statement I is false.

Step 2: Check the environmental statement Iron pyrites contains sulphur as well as iron. Its roasting produces sulphur dioxide, illustrated by

$$4\,FeS_2 + 11\,O_2 \longrightarrow 2\,Fe_2O_3 + 8\,SO_2.$$

The gaseous product causes air pollution and contributes to acid rain if released untreated. Sulphur-related difficulties make pyrites generally unsuitable as the preferred raw material for extracting iron; iron oxide ores avoid this large sulphur load. Statement II is therefore true in the intended comparison.

Step 3: Select the true-false combination Statement I is false, whereas Statement II is true. Option A reverses both judgements. Option C wrongly accepts the claim that the paddle removes air, and option D wrongly rejects the environmental disadvantage of iron pyrites. The combination required is therefore the one that rejects I but accepts II; the two statements are judged independently rather than as an assertion-reason pair.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (B).

Quick Tip to Solve: In flotation questions, bubbles must be supplied to lift ore particles. For pyrites, track the sulphur into sulphur dioxide during roasting.

78. Match List I with List II

	List I		List II
A.	**Cobalt catalyst**	**I.**	**$(H_2 + Cl_2)$ production**
B.	**Syngas**	**II.**	**Water gas production**
C.	**Nickel catalyst**	**III.**	**Coal gasification**
D.	**Brine solution**	**IV.**	**Methanol production**

Choose the correct answer from the options given below : [4 marks]

(A) A-IV, B-III, C-I, D-II

(B) A-IV, B-III, C-II, D-I

(C) A-II, B-III, C-IV, D-I

(D) A-IV, B-I, C-II, D-III

Correct Answer: (B) A-IV, B-III, C-II, D-I

Solution:

Concept:

- Synthesis gas, or syngas, is a mixture of carbon monoxide and hydrogen. Its preparation and conversion are distinguished by their reactants and catalysts.
- Coal gasification produces syngas. Steam reforming of methane over nickel produces the carbon monoxide-hydrogen mixture also called water gas.
- Electrolysis of aqueous brine releases hydrogen at the cathode and chlorine at the anode; a catalyst used in methanol synthesis belongs to a different process.

Step 1: Match the catalyst entries The cobalt-catalysed synthesis referred to in the list is methanol production from carbon monoxide and hydrogen, so A matches IV. Nickel catalyses the reaction of methane with steam to produce water gas:

$$CH_4 + H_2O \xrightarrow{Ni} CO + 3\,H_2.$$

Thus C matches II. These matches must not be exchanged: nickel here identifies the gas-production step, rather than the subsequent synthesis of methanol.

Step 2: Match syngas and brine Passing steam over hot coal gives the gasification reaction

$$C + H_2O \longrightarrow CO + H_2.$$

Its product is syngas, giving B–III. During electrolysis of brine, the overall reaction is

$$2\,NaCl + 2\,H_2O \longrightarrow 2\,NaOH + H_2 + Cl_2.$$

Therefore D matches I. Water itself is reduced to hydrogen at the cathode, while chloride is oxidised at the anode.

Step 3: Compare the complete combinations The four matches together are A–IV, B–III, C–II and D–I. Option (A) interchanges the nickel and brine matches. Option (C) assigns cobalt to water gas and nickel to methanol. Option (D) wrongly connects syngas with hydrogen-chlorine production and

brine with coal gasification. Only option (B) preserves all four industrial associations.

Step 4: Conclusion The correct matching is A–IV, B–III, C–II, D–I. Hence, the correct option is (B).

Quick Tip to Solve: Fix brine first: electrolysis produces both H_2 and Cl_2. Then use nickel to identify steam reforming and water gas.

79. Which of the following represents the correct order of metallic character of the given elements?

[4 marks]

(A) K < Mg < Be < Si

(B) Si < Be < Mg < K

(C) Be < Si < Mg < K

(D) Be < Si < K < Mg

Correct Answer: (B) Si < Be < Mg < K

Solution:

Concept:

- Metallic character describes an atom's tendency to lose electrons and form positive ions. Lower ionisation energy generally favours this behaviour.
- Within a group metallic character increases downwards because the valence electron lies farther from the nucleus and is more shielded.
- Across a period from left to right, increasing effective nuclear charge makes electron removal harder, so metallic character generally decreases.

Step 1: Compare the alkaline-earth elements Beryllium and magnesium belong to Group 2, with magnesium below beryllium. Magnesium has an additional occupied shell and a larger atomic radius. Its valence electrons are held less strongly, so it loses electrons more readily. Therefore

$$\mathrm{Be} < \mathrm{Mg}$$

in metallic character. This comparison follows a group trend and does not depend on confusing atomic number with metallic nature.

Step 2: Place silicon and potassium Silicon is a metalloid in Group 14; it has a much smaller tendency to form positive ions than the s-block metals listed. It therefore lies below beryllium in metallic character. Potassium is a Group 1 metal in Period 4, with one outer electron that is very readily lost to form K^+. It is more metallic than magnesium. Combining these comparisons gives

$$\mathrm{Si} < \mathrm{Be} < \mathrm{Mg} < \mathrm{K}.$$

Step 3: Eliminate the inconsistent orders Option (A) reverses the entire sequence and incorrectly makes potassium the least metallic. Option (C) puts silicon above beryllium even though silicon is a metalloid and beryllium is a metal. Option (D) makes both that error and the additional error of putting magnesium above potassium. Option (B) alone agrees with the group trends and the classifications of all four elements.

Step 4: Conclusion The increasing order of metallic character is $\mathrm{Si} < \mathrm{Be} < \mathrm{Mg} < \mathrm{K}$. Hence, the correct option is (B).

Quick Tip to Solve: For metallic character, move down and left in the periodic table; place a metalloid such as silicon below the s-block metals in this list.

80. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : The alkali metals and their salts impart characteristic colour to reducing flame.

Reason R : Alkali metals can be detected using flame tests.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) A is not correct but R is correct

(B) A is correct but R is not correct

(C) Both A and R are correct and R is the correct explanation of A

(D) Both A and R are correct but R is NOT the correct explanation of A

Correct Answer: (A) A is not correct but R is correct

Solution:

Concept:

- Alkali metals are identified by characteristic flame colours in an oxidising, non-luminous flame. The specified flame conditions are part of the standard flame test.
- Heat excites the outer electrons to higher energy states. On returning to lower states, they emit radiation of characteristic frequencies, which can lie in the visible region.
- In an assertion-reason question, evaluate the exact wording of each statement. A familiar statement becomes incorrect when an essential experimental condition is changed.

Step 1: Check the flame condition in the assertion Assertion A specifies a reducing flame. The standard statement is that alkali metals and their salts impart characteristic colours to an oxidising flame. The test uses the non-luminous flame so that the characteristic emission can be observed clearly. Thus the assertion does not correctly describe the prescribed flame-test condition: the word reducing makes it incorrect. The general fact that these elements show characteristic emission must not lead us to overlook the stated condition.

Step 2: Evaluate the reason independently Alkali metals can be detected using flame tests. For example, lithium gives crimson red, sodium gives yellow, and potassium gives lilac colour. Their low ionisation enthalpies permit excitation of valence electrons by flame energy. Subsequent electronic transitions produce characteristic emission that distinguishes the elements. Reason R therefore correctly describes an analytical use of this emission; it does not specify the incorrect reducing-flame condition.

Step 3: Select the appropriate combination The two truth values are now fixed: Assertion A is incorrect and Reason R is correct. Option (A) gives precisely this combination. Option (B) reverses both truth values. Options (C) and (D) both treat the assertion as correct, so neither can be selected,

regardless of their claims about explanation. The decisive detail is the flame condition, rather than whether alkali metals exhibit flame colours at all.

Step 4: Conclusion A is not correct because the standard characteristic-colour test uses an oxidising flame; R is correct. Hence, the correct option is (A).

Quick Tip to Solve: Underline the experimental condition in an assertion: the standard alkali-metal flame-test statement says oxidising flame, not reducing flame.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. 28.0 L of CO_2 is produced on complete combustion of 16.8 L gaseous mixture of ethene and methane at 25°C and 1 atm. Heat evolved during the combustion process is _____ kJ.

Given : $\Delta H_c\ (CH_4) = -900\ \text{kJ mol}^{-1}$

$$\Delta H_c\ (C_2H_4) = -1400\ \text{kJ mol}^{-1}$$

[4 marks]

Solution:

Concept:

- At the same temperature and pressure, ideal-gas volumes are proportional to their mole numbers. One mole of methane forms one mole of carbon dioxide; one mole of ethene forms two.
- The heat of combustion is extensive: multiply the amount of each fuel by its molar combustion enthalpy, and add the contributions.
- At 25°C and $1\ \text{atm}$, convert volumes to moles with $n = PV/(RT)$; the standard-temperature molar volume is not applicable.

Step 1: Determine the two fuel volumes Let the ethene and methane volumes at the stated conditions be x and y litres. Complete combustion gives $C_2H_4 + 3\,O_2 \longrightarrow 2\,CO_2 + 2\,H_2O$ and $CH_4 + 2\,O_2 \longrightarrow CO_2 + 2\,H_2O$. The total fuel volume and carbon dioxide volume give

$$x + y = 16.8, \qquad 2x + y = 28.0.$$

Subtraction yields $x = 11.2\,\text{L}$ of ethene and $y = 5.6\,\text{L}$ of methane.

Step 2: Convert each volume to amount Use $T = 298\,\text{K}$, $P = 1\,\text{atm}$ and $R = 0.0821\,\text{L atm mol}^{-1}\,\text{K}^{-1}$. The common molar volume is

$$RT/P = 0.0821(298) = 24.4658\,\text{L mol}^{-1}.$$

Therefore $n(C_2H_4) = 11.2/24.4658$ mol and $n(CH_4) = 5.6/24.4658$ mol. Both volumes must be converted using the same actual temperature.

Step 3: Add the combustion energies The enthalpy change of the mixture is

$$\Delta H = -1400\left(\frac{11.2}{24.4658}\right) - 900\left(\frac{5.6}{24.4658}\right)$$
$$= -\frac{20720}{24.4658} \simeq -846.9\,\text{kJ}.$$

A negative enthalpy change means energy leaves the reacting system. The question asks for heat evolved, so report its positive magnitude, rounded to the nearest integer.

Step 4: Conclusion The heat evolved is approximately $847\,\text{kJ}$. The numerical entry is $\boxed{847}$.

Quick Tip to Solve: First solve the gas-volume equations using the number of carbon atoms; only then convert the fuel volumes to moles at the stated temperature.

---

82. The number of pairs of the solutions having the same value of the osmotic pressure from the following is _____.

(Assume 100% ionization)

A. 0.500 M C_2H_5OH (aq) **and 0.25** M KBr (aq)

B. 0.100 M $K_4[Fe(CN)_6]$ (aq) **and 0.100** M $FeSO_4(NH_4)_2SO_4$ (aq)

C. 0.05 M $K_4[Fe(CN)_6]$ (aq) **and 0.25** M NaCl (aq)

D. 0.15 M NaCl (aq) **and 0.1** M $BaCl_2$ (aq)

E. 0.02 M $KCl.MgCl_2.6H_2O$ (aq) **and 0.05** M KCl (aq) [4 marks]

Solution:

Concept:

- At a common temperature, osmotic pressure is $\pi = iCRT$, where C is molarity and i is the number of independently dissolved particles produced per formula unit.
- With complete ionisation, count all free ions, but keep an intact complex ion as one particle. Water of crystallisation joins the solvent and does not count as an extra solute ion.

Step 1: Count the particles in the salts Ethanol is a nonelectrolyte with $i = 1$. KBr, NaCl and KCl each give two ions, while $BaCl_2$ gives three. The complex salt $K_4[Fe(CN)_6]$ gives four K^+ ions and one intact complex anion, so $i = 5$. Mohr's salt $FeSO_4 \cdot (NH_4)_2SO_4$ gives one Fe^{2+}, two $NH_4{}^+$ and two $SO_4{}^{2-}$, also giving $i = 5$. Carnallite $KCl \cdot MgCl_2 \cdot 6\,H_2O$ gives one potassium ion, one magnesium ion and three chloride ions, hence $i = 5$.

Step 2: Compare the effective concentrations Equal iC values give equal osmotic pressures. For the

two solutions in each listed pair,

Pair	iC (first)	iC (second)	Equal?
A	$1(0.500) = 0.500$	$2(0.25) = 0.500$	Yes
B	$5(0.100) = 0.500$	$5(0.100) = 0.500$	Yes
C	$5(0.05) = 0.250$	$2(0.25) = 0.500$	No
D	$2(0.15) = 0.300$	$3(0.10) = 0.300$	Yes
E	$5(0.02) = 0.100$	$2(0.05) = 0.100$	Yes

The entries are effective molar concentrations in $\mathrm{mol\,L^{-1}}$.

Step 3: Count the matching pairs Pairs A, B, D and E each contain two solutions with the same total concentration of osmotically active particles. Pair C fails because the second solution has twice the effective particle concentration of the first. Thus four of the five listed pairs satisfy the equal-osmotic-pressure condition.

Step 4: Conclusion The number of listed pairs with equal osmotic pressure is $\boxed{4}$ (a dimensionless count).

Quick Tip to Solve: Compare iC, not molarity alone. Keep complex ions intact and exclude crystal water when counting independently dissolved solute particles.

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83. A first order reaction has the rate constant, $\mathrm{k = 4.6 \times 10^{-3}\ s^{-1}}$. The number of correct statement/s from the following is/are _____

Given : $\log 3 = 0.48$

A. Reaction completes in 1000 s.

B. The reaction has a half-life of 500 s.

C. The time required for 10% completion is 25 times the time required for 90% completion.

D. The degree of dissociation is equal to $(1 - \mathrm{e^{-kt}})$

E. The rate and the rate constant have the same unit. [4 marks]

Solution:

Concept:

- For a first-order reaction, $[A]_t = [A]_0 e^{-kt}$, so the reacted fraction is $\alpha = 1 - e^{-kt}$.
- The half-life is $t_{1/2} = \ln 2/k$, and the time for a reacted fraction f is $t = -\ln(1-f)/k$.
- The first-order rate constant has units $\mathrm{s^{-1}}$, while the reaction rate has concentration-per-time units.

Step 1: Check completion and half-life At $1000\,\mathrm{s}$, $kt = 4.6$, and the fraction remaining is $e^{-4.6} \approx 0.010$, not zero. The reaction is about 99 percent complete, rather than completely finished; statement A is false. The half-life is

$$t_{1/2} = \frac{0.693}{4.6 \times 10^{-3}} \approx 151\,\mathrm{s}.$$

It is not $500\,\mathrm{s}$, so statement B is false. Ideal first-order completion occurs only asymptotically.

Step 2: Compare the two percentage times The times for 10 percent and 90 percent completion are

$$t_{10} = \frac{\ln(10/9)}{k}, \qquad t_{90} = \frac{\ln 10}{k}.$$

Thus $t_{10}/t_{90} \approx 0.10536/2.30259 \approx 0.0458$. In particular, the time for less conversion is smaller, not 25 times larger. Using the supplied rounded $\log 3 = 0.48$ gives $\log 9 = 0.96$ and $t_{10}/t_{90} \approx 0.04 = 1/25$; this also directly contradicts statement C as written.

Step 3: Check dissociation and units The fraction dissociated equals one minus the fraction remaining: $\alpha = 1 - [A]_t/[A]_0 = 1 - e^{-kt}$. Therefore D is true. From rate $= k[A]$, rate has units $\mathrm{mol\,L^{-1}\,s^{-1}}$, whereas k has units $\mathrm{s^{-1}}$. They do not have the same unit, so E is false. Only statement D survives all the checks.

Step 4: Conclusion Only statement D is correct. The number of correct statements is $\boxed{1}$, a dimensionless count.

Quick Tip to Solve: For first-order percentage questions, write the fraction remaining first; this prevents reversing the times for 10 percent and 90 percent conversion.

84.

$$Pt(s)|H_2(g)(1\,bar)|H^+(aq)(1\,M)||M^{3+}(aq), M^+(aq)|Pt(s)$$

The $\mathrm{E_{cell}}$ for the given cell is 0.1115 V at 298 K when $\frac{[M^+(aq)]}{[M^{3+}(aq)]} = 10^a$
The value of a is _____
Given : $\mathrm{E^{\theta}_{M^{3+}/M^+}} = 0.2$ **V**

$$\frac{2.303RT}{F} = 0.059V$$

[4 marks]

Solution:

Concept:

- The left electrode is the standard hydrogen electrode under the stated conditions, so its reduction potential is zero.
- The right-hand reduction is $\mathrm{M^{3+} + 2\,e^- \longrightarrow M^+}$, involving two electrons. Its reaction quotient is $Q = [\mathrm{M^+}]/[\mathrm{M^{3+}}]$.
- At 298 K, the Nernst equation is $E = E^\circ - (0.059/n)\log_{10} Q$. The cell potential equals the right reduction potential minus the left reduction potential.

Step 1: Identify the cell reaction and electron number Hydrogen is oxidised at the left electrode, while $\mathrm{M^{3+}}$ is reduced at the right electrode. Adding the half-reactions gives

$$\mathrm{H_2 + M^{3+} \longrightarrow 2\,H^+ + M^+}.$$

The oxidation state of M falls from +3 to +1, so $n = 2$. The hydrogen pressure is one bar and the

hydrogen-ion concentration is one molar, leaving their standard-state factors equal to unity in the reaction quotient.

Step 2: Apply the Nernst equation The standard cell potential is $E^\circ_{\text{cell}} = 0.200 - 0 = 0.200\,\text{V}$. Since $[\text{M}^+]/[\text{M}^{3+}] = 10^a$, its base-ten logarithm is a. Therefore

$$0.1115 = 0.200 - \frac{0.059}{2}a.$$

The negative sign is required because increasing the product-to-reactant ratio lowers the reduction potential.

Step 3: Solve for the exponent and verify Rearranging yields

$$a = \frac{2(0.200 - 0.1115)}{0.059} = \frac{0.1770}{0.059} = 3.$$

Thus the concentration ratio is $10^3 = 1000$. Substitution checks the result: $0.200 - 0.0295(3) = 0.1115\,\text{V}$, exactly the given potential. The ratio exceeds one, consistently making the actual potential smaller than its standard value.

Step 4: Conclusion The required exponent is $\boxed{a = 3}$. It is dimensionless.

Quick Tip to Solve: Count the two transferred electrons from the oxidation-state change before inserting the concentration ratio into the Nernst equation.

85. The number of given orbitals which have electron density along the axis is _____

$$\text{p}_\text{x}, \text{p}_\text{y}, \text{p}_\text{z}, \text{d}_{\text{xy}}, \text{d}_{\text{yz}}, \text{d}_{\text{xz}}, \text{d}_{\text{z}^2}, \text{d}_{\text{x}^2-\text{y}^2}$$

[4 marks]

Solution:

Concept:

- Electron density is proportional to the square of the orbital wavefunction. An axis is a direction of nonzero density when the angular wavefunction does not vanish in that direction.
- The three p orbitals are oriented along the corresponding Cartesian axes; the orientation of a d orbital follows from its angular factor, such as xy or $x^2 - y^2$.

Step 1: Count the p orbitals The two lobes of p_x point along the positive and negative x-axis. Similarly, p_y has density along the y-axis and p_z along the z-axis. Each has a nodal plane perpendicular to its own axis, but that does not remove density along the axis on which its lobes lie. Thus all three listed p orbitals qualify.

Step 2: Separate the two kinds of d orbitals The orbitals d_{xy}, d_{yz} and d_{xz} have their lobes between the corresponding coordinate axes. Their angular factors xy, yz and xz vanish on every coordinate axis, so none contributes to the count. In contrast, $d_{x^2-y^2}$ has four lobes along the x- and y-axes. The d_{z^2} orbital has two principal lobes along the z-axis and also an equatorial torus, so it qualifies as well.

Step 3: Add the qualifying orbitals The qualifying set is

$$p_x, \quad p_y, \quad p_z, \quad d_{z^2}, \quad d_{x^2-y^2}.$$

There are three p orbitals and two d orbitals in this set; each orbital is counted once, even if its lobes occupy more than one axis.

Step 4: Conclusion The number of listed orbitals having electron density along a coordinate axis is $3 + 2 = \boxed{5}$. This is a dimensionless count.

Quick Tip to Solve: For d orbitals, mixed-coordinate subscripts xy, yz and xz indicate lobes between axes; the two squared-coordinate forms have lobes on axes.

---

86. Total number of moles of AgCl precipitated on addition of excess of $AgNO_3$ to one mole each of the following complexes $[Co(NH_3)_4Cl_2]Cl$, $[Ni(H_2O)_6]Cl_2$, $[Pt(NH_3)_2Cl_2]$ and $[Pd(NH_3)_4]Cl_2$ is _____.

[4 marks]

Solution:

Concept:

- Chloride ions outside the coordination sphere dissociate in water and precipitate with silver ions: $Ag^+ + Cl^- \longrightarrow AgCl(s)$. One mole of free chloride produces one mole of silver chloride.
- Chloride ligands inside square brackets are coordinated to the metal. They are not counted as immediately ionisable chloride in the standard silver-nitrate test.

Step 1: Count the counterions of each complex For $[Co(NH_3)_4Cl_2]Cl$, the single chloride outside the brackets is a counterion; the two chlorides inside the brackets remain coordinated. Hence one mole of this complex supplies one mole of precipitable chloride. For $[Ni(H_2O)_6]Cl_2$, both chlorides are outside the coordination sphere, so one mole supplies two moles of chloride.

Step 2: Check the neutral and palladium complexes The complex $[Pt(NH_3)_2Cl_2]$ is neutral and has both chloride ligands inside its coordination sphere. It therefore contributes no free chloride to this test. In $[Pd(NH_3)_4]Cl_2$, ammonia is the coordinated ligand and the two chlorides are counterions. Thus one mole of the palladium complex gives two moles of silver chloride. Excess silver nitrate ensures that all available counterion chloride is precipitated.

Step 3: Use the one-to-one precipitation ratio The stated sample contains one mole of each complex, so their contributions are added directly:

$$n(AgCl) = 1 + 2 + 0 + 2 = 5 \text{ mol}.$$

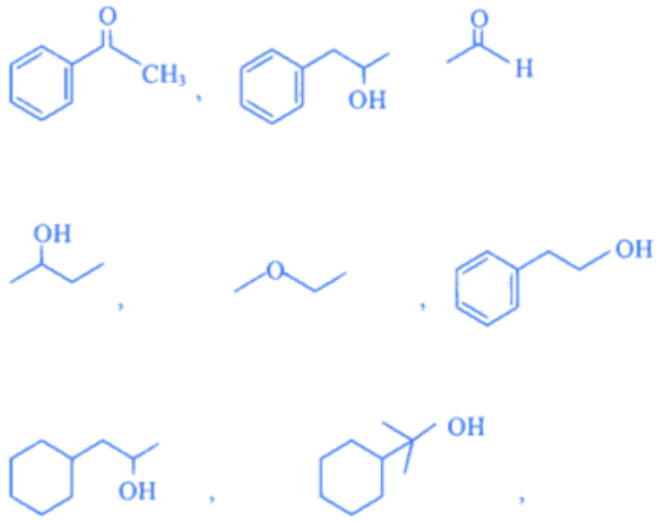
Counting all chlorine atoms would incorrectly include the cobalt- and platinum-bound ligands; only the five counterion chlorides are relevant.

Step 4: Conclusion The total precipitate is $\boxed{5 \text{ mol}}$ of $AgCl$; the required numerical entry is $\boxed{5}$.

Quick Tip to Solve: In a silver-nitrate test, first circle only the chloride ions outside the square brackets; each gives one AgCl formula unit.

---

87. Number of compounds giving (i) red colouration with ceric ammonium nitrate and also (ii) positive iodoform test from the following is _____ [4 marks]



Solution:

Concept:

- Ceric ammonium nitrate gives the characteristic red colour with an alcoholic hydroxyl group. A carbonyl compound without an alcoholic OH group does not satisfy this alcohol test.
- The iodoform test is positive for methyl ketones and for alcohols oxidisable to methyl ketones, especially $CH_3CH(OH)R$; ethanol and ethanal are familiar special cases. Both tests must be positive for a compound to be counted.

Step 1: Select compounds with an alcoholic hydroxyl group Read the structures from left to right, row by row. Acetophenone and ethanal have carbonyl groups but no alcoholic OH group, while methoxyethane is an ether. These three fail the ceric ammonium nitrate requirement, although acetophenone and ethanal can themselves give the iodoform test. The remaining five structures contain an alcoholic OH group and must be checked against the second requirement.

Step 2: Identify the methyl carbinol arrangement The alcohol $C_6H_5CH_2CH(OH)CH_3$, butan-2-ol $CH_3CH(OH)CH_2CH_3$, and the bottom-left alcohol $C_6H_{11}CH_2CH(OH)CH_3$ each contain the group $CH_3CH(OH)^-$. Oxidation converts each into a methyl ketone CH_3CO-R, which undergoes haloform cleavage to yellow CHI_3. Therefore all three satisfy both tests.

Step 3: Exclude the other two alcohols The primary alcohol $C_6H_5CH_2CH_2OH$ oxidises initially to $C_6H_5CH_2CHO$, which is not ethanal and is not a methyl ketone. The bottom-right alcohol has a tertiary OH-bearing carbon, with no hydrogen on that carbon; it cannot undergo the required oxidation to a methyl ketone. Thus neither of these two alcohols gives the positive iodoform result required here.

Step 4: Conclusion Exactly $\boxed{3}$ compounds give both the red ceric ammonium nitrate test and the positive iodoform test. The answer is a dimensionless count.

Quick Tip to Solve: For a combined test, take the intersection: first retain alcohols, then look specifically for an OH-bearing carbon bonded to both H and CH3.

88. Number of hydrogen atoms per molecule of a hydrocarbon A having 85.8% carbon is _____
(Given : Molar mass of A = 84 g mol^{-1}) [4 marks]

Solution:

Concept:

- A hydrocarbon contains only carbon and hydrogen. For C_xH_y, the molar mass is $12x + y$ using the usual atomic masses C = 12 and H = 1.
- The mass percentage of carbon gives the mass of carbon in one mole. Atom counts in a molecular formula must be whole numbers; a slight inconsistency between numerical data must be checked against this integer requirement.

Step 1: Find the carbon mass in one mole One mole of the hydrocarbon has mass 84 g. The quoted carbon percentage therefore gives

$$m_C = \frac{85.8}{100} \times 84 = 72.072 \text{ g}.$$

The number of moles of carbon atoms per mole of compound is

$$x = \frac{72.072}{12} = 6.006 \simeq 6.$$

The result is not exactly an integer: the printed percentage and molar mass are slightly inconsistent. Taking the nearest whole carbon count gives six carbon atoms per molecule.

Step 2: Use the remaining molecular mass for hydrogen Six carbon atoms contribute $6 \times 12 = 72$ to the molecular mass. Since the substance contains no other element, the remaining mass belongs entirely to hydrogen:

$$y = 84 - 72 = 12.$$

Equivalently, the given hydrogen percentage is $100 - 85.8 = 14.2\%$, corresponding to 11.928 g per mole, which likewise gives approximately twelve hydrogen atoms per molecule.

Step 3: Check the complete molecular formula The nearest-integer formula is C_6H_{12}, whose molar mass is

$$6(12) + 12(1) = 84 \text{ g mol}^{-1}.$$

It gives

$$\%C = \frac{72}{84} \times 100 = 85.714\ldots\%.$$

This rounds to 85.7%, not the printed 85.8%. Thus the composition has a small numerical inconsistency; it cannot be explained by ordinary rounding to one decimal place. Nevertheless, the exact molar mass and nearest integer carbon count identify C_6H_{12}, giving the required integer hydrogen count.

Step 4: Conclusion The hydrocarbon is C_6H_{12}, so each molecule contains $\boxed{12}$ hydrogen atoms. The numerical count has no physical unit.

Quick Tip to Solve: After determining the nearest integer carbon count from the percentage, use the total molar mass to obtain the exact integer hydrogen count.

89. Based on the given figure, the number of correct statement/s is/are _____.

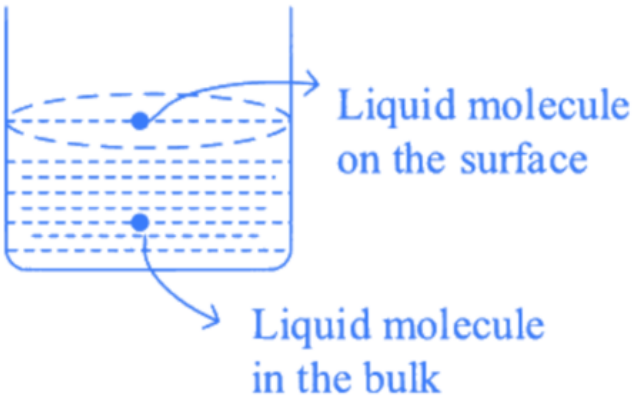
A. Surface tension is the outcome of equal attractive and repulsive forces acting on the liquid molecule in bulk.

B. Surface tension is due to uneven forces acting on the molecules present on the surface.

C. The molecule in the bulk can never come to the liquid surface.

D. The molecules on the surface are responsible for vapour pressure if the system is a closed system.

[4 marks]



Solution:

Concept:

- An interior liquid molecule is surrounded by neighbours in all directions, so its cohesive attractions balance on average. A surface molecule has fewer neighbours above it and experiences an inward resultant attraction.
- Liquid molecules move continuously and can exchange positions between the bulk and surface. Evaporation and condensation in a closed system lead to an equilibrium vapour pressure at fixed temperature.

Step 1: Judge statements A and B Statement A is incorrect. Surface tension is not caused by equal attractive and repulsive forces on bulk molecules. The bulk has no preferred direction of cohesive attraction. Statement B is correct: uneven forces on surface molecules make the surface contract and resist an increase in area.

Step 2: Judge statement C Statement C is incorrect because liquid molecules are not fixed. Thermal motion allows an interior molecule to reach the surface and a surface molecule to return to the bulk. The word never contradicts this continuous exchange of positions.

Step 3: Judge statement D and count the true statements Statement D is correct. Molecules escape from the liquid surface into the vapour phase. In the closed system, vapour accumulates and condensation also occurs. At equilibrium, evaporation and condensation rates are equal and the vapour exerts its equilibrium pressure. Thus surface molecules provide the route into the vapour phase. B and D are true, whereas A and C are false.

Step 4: Conclusion The number of correct statements is $\boxed{2}$, namely B and D. This is a dimensionless count.

Quick Tip to Solve: For surface-tension questions, distinguish balanced bulk cohesion from unbalanced surface cohesion, and remember that liquid molecules continually exchange positions.

90. The number of incorrect statement/s from the following is/are _____

A. Water vapours are adsorbed by anhydrous calcium chloride.

B. There is a decrease in surface energy during adsorption.

C. As the adsorption proceeds, ΔH becomes more and more negative.

D. Adsorption is accompanied by decrease in entropy of the system. [4 marks]

Solution:

Concept:

- Adsorption concentrates a substance at a surface; absorption takes it into the bulk. Anhydrous calcium chloride removes water through bulk uptake and hydration rather than the surface adsorption illustrated by silica gel.
- Adsorption usually lowers surface energy and is exothermic. The adsorbed particles have less freedom than in the gas phase, so their entropy decreases; the magnitude of heat released generally falls as coverage increases.

Step 1: Judge statements A and B Statement A is incorrect. The action of anhydrous $CaCl_2$ on water vapour is classified as absorption, with hydration in the bulk, not merely adsorption on its surface. Statement B is correct. Surface particles possess residual attractive forces; attachment of adsorbate reduces this unsatisfied attraction and lowers the surface energy. This energy reduction is why adsorption is generally accompanied by release of heat.

Step 2: Judge the trend in adsorption enthalpy Statement C is incorrect. The most favourable adsorption sites are occupied first. As the surface becomes covered, fewer highly energetic sites remain and attraction for further incoming particles is reduced; adsorbate interactions can also oppose further uptake. Consequently, the heat released per additional amount adsorbed generally decreases. The adsorption enthalpy becomes less negative as adsorption proceeds, rather than more and more negative as stated.

Step 3: Judge the entropy change and count the errors Statement D is correct for the adsorption process considered here. Gas molecules moving freely in three dimensions become localised at surface sites and lose translational freedom. Their arrangement is therefore more ordered and the entropy change of adsorption is negative. We have two correct statements, B and D, and two incorrect statements, A and C. The question asks for the latter count.

Step 4: Conclusion The incorrect statements are A and C, giving $\boxed{2}$ incorrect statements. The answer is a dimensionless count.

Quick Tip to Solve: Do not confuse adsorption with absorption: calcium chloride takes up water into its bulk, while an adsorbent such as silica gel holds it chiefly on its surface.