JEE Main 2023 Paper 1 (B.E./B.Tech) Jan 29 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 29 January 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation [4 marks]

(A) $7x^2 - 245x + 250 = 0$

(B) $49x^2 - 245x + 250 = 0$

(C) $49x^2 + 245x + 250 = 0$

(D) $7x^2 + 245x - 250 = 0$

Correct Answer: (B) $49x^2 - 245x + 250 = 0$

Solution:

Concept:

- For a quadratic equation $px^2 + qx + r = 0$, the sum of its roots is $-q/p$ and their product is r/p. These relations avoid solving both quadratics separately.
- The common root satisfies both equations. Eliminate λ first; since $\lambda \neq 0$, neither root of either equation can be zero.

Step 1: Eliminate the parameter using the common root Substituting α into both equations gives

$$14\alpha^2 - 31\alpha + 3\lambda = 0, \qquad 35\alpha^2 - 53\alpha + 4\lambda = 0.$$

Multiply the first equality by 4 and the second by 3 and subtract. This removes the same term 12λ:

$$-49\alpha^2 + 35\alpha = 0.$$

The root $\alpha = 0$ would imply $\lambda = 0$, which is excluded. Therefore $\alpha = 5/7$. Substitution in the first equation then gives $3\lambda = 15$, so $\lambda = 5$.

Step 2: Find the other roots and form the required quadratic Use the sums of roots of the original quadratics:

$$\beta = \frac{31}{14} - \frac{5}{7} = \frac{3}{2}, \qquad \gamma = \frac{53}{35} - \frac{5}{7} = \frac{4}{5}.$$

The new roots are consequently

$$u = \frac{3\alpha}{\beta} = \frac{10}{7}, \qquad v = \frac{4\alpha}{\gamma} = \frac{25}{7}.$$

Their sum is 5 and their product is $250/49$. Thus the monic equation is $x^2 - 5x + 250/49 = 0$. Multiplication by 49 clears the denominator. Option A has the wrong root sum and product, C has a negative root sum, and D has a negative product, so none can represent these two positive roots.

Step 3: Conclusion The equation is $49x^2 - 245x + 250 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: When two quadratics share a root and a parameter, eliminate the parameter before using the sum and product of roots.

2. Let B and C be the two points on the line $y + x = 0$ such that B and C are symmetric with respect to the origin. Suppose A is a point on $y - 2x = 2$ such that $\triangle ABC$ is an equilateral triangle. Then, the area of the $\triangle ABC$ is : [4 marks]

(A) $\frac{10}{\sqrt{3}}$

(B) $2\sqrt{3}$

(C) $3\sqrt{3}$

(D) $\frac{8}{\sqrt{3}}$

Correct Answer: (D) $\frac{8}{\sqrt{3}}$

Solution:

Concept:

- The segment joining two points symmetric about the origin has midpoint $O = (0, 0)$. In an equilateral triangle, the median from the opposite vertex is also its perpendicular altitude.
- If an equilateral triangle has side s and altitude h, then $h = \sqrt{3}s/2$ and its area is $\sqrt{3}s^2/4 = h^2/\sqrt{3}$.

Step 1: Locate the vertex on the perpendicular bisector Since both B and C lie on $y = -x$, their common side has slope -1. Its midpoint is the origin by the symmetry condition. Every point equidistant from B and C lies on the perpendicular bisector of this segment. In particular, the equilateral vertex A lies on the line through the origin with slope 1, namely $y = x$. The problem also places A on $y - 2x = 2$. Solving these two linear equations,

$$x - 2x = 2 \quad \Longrightarrow \quad x = -2, \qquad y = -2.$$

Thus $A = (-2, -2)$; choosing an arbitrary point on the given line instead would not satisfy the equilateral condition.

Step 2: Use the altitude to calculate the area The perpendicular from A to the base BC ends at its midpoint O, so its length is

$$h = AO = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}.$$

Consequently $s = 2h/\sqrt{3} = 4\sqrt{2/3}$, which also verifies that a nondegenerate equilateral triangle can be constructed on the base line. Its area is

$$\Delta = \frac{1}{2}sh = \frac{h^2}{\sqrt{3}} = \frac{8}{\sqrt{3}}.$$

This is the same as $8\sqrt{3}/3$. The other options correspond to different squared altitudes: 10, 6, and 9, respectively, whereas the actual squared altitude is 8.

Step 3: Conclusion The area is $\frac{8}{\sqrt{3}}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: For an equilateral triangle whose base midpoint is known, find the opposite vertex on the perpendicular bisector and use area $= h^2/\sqrt{3}$.

3. Three rotten apples are mixed accidently with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent mean and variance of X, respectively, then $10(\mu^2+\sigma^2)$ is equal to : [4 marks]

(A) 20

(B) 30

(C) 250

(D) 25

Correct Answer: (A) 20

Solution:

Concept:

- Sampling without replacement from a finite population produces a hypergeometric random variable. Here the population size is $N = 10$, the number of rotten apples is $K = 3$, and the sample size is $n = 4$.
- For this distribution, $\mu = nK/N$ and $\sigma^2 = n(K/N)(1 - K/N)(N - n)/(N - 1)$. The final factor accounts for the dependence created by sampling without replacement.

Step 1: Calculate the mean number of rotten apples Each draw has marginal probability $3/10$ of being rotten. Although the draws are dependent, linearity of expectation still applies. If I_j is 1 when draw j is rotten and 0 otherwise, then $X = I_1 + I_2 + I_3 + I_4$. Thus

$$\mu = E(X) = 4\left(\frac{3}{10}\right) = \frac{6}{5}.$$

The seven good apples are the complementary category; they do not give a separate term in the count of rotten apples. Also, the mean need not be an integer even though each possible observed count is an integer.

Step 2: Apply the finite-population correction and evaluate Use the variance formula with the same population and sample sizes:

$$\sigma^2 = 4 \cdot \frac{3}{10} \cdot \frac{7}{10} \cdot \frac{10-4}{10-1} = \frac{14}{25}.$$

A binomial approximation would omit $6/9$ and give an incorrect result, since drawing four of only ten apples changes the composition appreciably. Now

$$10(\mu^2+\sigma^2) = 10\left(\frac{36}{25} + \frac{14}{25}\right) = 10 \cdot 2 = 20.$$

As a check, $E[X(X-1)] = 4 \cdot 3 \cdot (3 \cdot 2)/(10 \cdot 9) = 4/5$, so $E(X^2) = 4/5 + 6/5 = 2$, agreeing with $\mu^2 + \sigma^2$. The values 30, 250 and 25 therefore do not match the required second moment.

Step 3: Conclusion The required value is 20. Hence, the correct option is (A).

Quick Tip to Solve: For sampling without replacement, retain the factor $(N - n)/(N - 1)$ in the variance; here $\mu^2 + \sigma^2$ is simply $E(X^2)$.

4. Let $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$ **and** $S = \left\{\theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$**. If** $4\beta = \sum_{\theta \in S} \theta$**, then** $f(\beta)$ **is equal to** [4 marks]

(A) $\frac{9}{8}$

(B) $\frac{3}{2}$

(C) $\frac{5}{4}$

(D) $\frac{11}{8}$

Correct Answer: (C) $\frac{5}{4}$

Solution:

Concept:

- Use periodicity to reduce the shifted sines: $\sin(3\pi/2 - \theta) = -\cos\theta$ and $\sin(3\pi + \theta) = -\sin\theta$. Their fourth powers remove the signs.
- The identity $\sin^4\theta + \cos^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$ simplifies the function before differentiation. All solutions must lie in the stated closed interval.

Step 1: Simplify and differentiate the function Replacing the shifted fourth powers by $\cos^4\theta$ and $\sin^4\theta$, we get

$$f(\theta) = 3\left(1 - \frac{1}{2}\sin^2 2\theta\right) - 2 + 2\sin^2 2\theta$$
$$= 1 + \frac{1}{2}\sin^2 2\theta.$$

Apply the chain rule carefully: the inner angle is 2θ, so its derivative contributes a factor 2. Therefore

$$f'(\theta) = 2\sin 2\theta\cos 2\theta = \sin 4\theta.$$

The equation defining S becomes $\sin 4\theta = -\sqrt{3}/2$. Neither endpoint of $[0, \pi]$ satisfies it.

Step 2: List all angles and evaluate at their average Since $0 \le 4\theta \le 4\pi$, both negative-sine solutions occur twice:

$$4\theta = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}.$$

Thus $S = \{\pi/3, 5\pi/12, 5\pi/6, 11\pi/12\}$, whose sum is $5\pi/2$. The defining relation $4\beta = \sum_{\theta \in S}\theta$ gives $\beta = 5\pi/8$. Consequently

$$f(\beta) = 1 + \frac{1}{2}\sin^2\frac{5\pi}{4} = 1 + \frac{1}{4} = \frac{5}{4}.$$

The square of the sine is $1/2$, not 1; this rules out option B. Options A and D would require squared sines $1/4$ and $3/4$, inconsistent with the computed angle.

Step 3: Conclusion We obtain $f(\beta) = \frac{5}{4}$. Hence, the correct option is (C).

Quick Tip to Solve: Simplify fourth powers before differentiating, and solve for the multiple angle over its full interval before dividing the solutions.

5. Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is : [4 marks]

(A) $\frac{1}{6}$
(B) $\frac{2}{15}$
(C) $\frac{5}{24}$
(D) 0.08

Correct Answer: (D) 0.08

Solution:

Concept:

- A random distribution of the 15 distinct shirts is a uniformly random permutation. A player receiving the correct shirt is a fixed point of that permutation.
- The derangement number $D_m = m!\sum_{j=0}^{m}(-1)^j/j!$ counts permutations of m objects with no fixed points. Exactly k correct shirts occur in $\binom{15}{k}D_{15-k}$ permutations.

Step 1: Count the complement of the requested event Let X be the number of correct shirts. The event required is $X \geq 3$; it is simpler to subtract the mutually exclusive events $X = 0, 1, 2$. Among the $15!$ equally likely assignments,

$$P(X = k) = \frac{\binom{15}{k}D_{15-k}}{15!}.$$

Choose the k players whose shirts are correct first, then derange all remaining shirts among the remaining players. A general permutation of the remaining shirts would overcount, because it could produce additional fixed points. Therefore

$$P(X \geq 3) = 1 - \frac{D_{15} + 15D_{14} + 105D_{13}}{15!}.$$

Step 2: Evaluate the probability and compare the options The derangement recurrence $D_m = (m-1)(D_{m-1} + D_{m-2})$, starting from $D_0 = 1, D_1 = 0$, gives

$$D_{13} = 2290792932, \quad D_{14} = 32071101049, \quad D_{15} = 481066515734.$$

Hence

$$P(X \geq 3) = \frac{105008078671}{1307674368000} = 0.080301397\ldots.$$

This is approximately 0.08, the decimal option D. For a quick cross-check, the fixed-point count is very close to a Poisson variable of mean 1; this yields $1 - e^{-1}(1 + 1 + 1/2) \approx 0.08030$. The other listed numbers are approximately $0.1667, 0.1333, 0.2083$, all substantially larger. The decimal choice is understood to the precision displayed; the exact finite-permutation probability is given above.

Step 3: Conclusion The probability is $0.080301397\ldots$, which rounds to 0.08. Hence, the correct option is (D).

Quick Tip to Solve: For at least three fixed points, subtract the counts for zero, one and two fixed points; the remaining objects must be deranged.

6. Let the tangents at the points $A(4,-11)$ and $B(8,-5)$ on the circle $x^2+y^2-3x+10y-15=0$, intersect at the point C. Then the radius of the circle, whose centre is C and the line joining A and B is its tangent, is equal to : [4 marks]

(A) $\frac{2\sqrt{13}}{3}$

(B) $\frac{3\sqrt{3}}{4}$

(C) $\sqrt{13}$

(D) $2\sqrt{13}$

Correct Answer: (A) $\frac{2\sqrt{13}}{3}$

Solution:

Concept:

- The tangent at a point of a circle is perpendicular to the radius at that point. Completing squares or reading coefficients gives the centre of $x^2+y^2+2gx+2fy+c=0$ as $(-g,-f)$.
- A circle tangent to a line has radius equal to the perpendicular distance of its centre from that line: $|ax_0+by_0+c|/\sqrt{a^2+b^2}$ for $ax+by+c=0$.

Step 1: Find the intersection of the two tangents The given circle has centre $O=(3/2,-5)$. At $A=(4,-11)$, the radius direction is $(5/2,-6)$, so the tangent is

$$\frac{5}{2}(x-4)-6(y+11)=0, \quad \text{or} \quad 5x-12y-152=0.$$

At $B=(8,-5)$, the radius is horizontal, since B and O have the same ordinate. Its tangent is consequently vertical: $x=8$. Substituting this into the first tangent gives $40-12y-152=0$, and therefore

$$C=\left(8,-\frac{28}{3}\right).$$

These equations use the actual radius directions, so no assumption about the slope of chord AB enters the tangent construction.

Step 2: Calculate the distance from the new centre to the chord line The slope of AB is $(-5+11)/(8-4)=3/2$, so its equation is

$$y+11=\frac{3}{2}(x-4), \quad \text{or} \quad 3x-2y-34=0.$$

The required new circle is centred at C and tangent to this line, not to either of the original tangent lines. Its radius is therefore

$$r=\frac{|3(8)-2(-28/3)-34|}{\sqrt{3^2+(-2)^2}}=\frac{26}{3\sqrt{13}}=\frac{2\sqrt{13}}{3}.$$

This positive perpendicular distance uniquely determines the radius. Options C and D are larger than this distance by factors $3/2$ and 3; option B has a different numerical value as well.

Step 3: Conclusion The required radius is $\frac{2\sqrt{13}}{3}$ units. Hence, the correct option is (A).

Quick Tip to Solve: Find the intersection of the original tangents first; the second circle radius is its centre-to-chord distance, not its distance to a chord endpoint.

7. Let $f(x) = x + \frac{a}{\pi^2-4}\sin x + \frac{b}{\pi^2-4}\cos x, x \in R$ be a function which satisfies $f(x) = x + \int_0^{\pi/2} \sin(x+y)f(y)dy$. then $(a+b)$ is equal to [4 marks]

(A) $-2\pi(\pi+2)$

(B) $-\pi(\pi-2)$

(C) $-\pi(\pi+2)$

(D) $-2\pi(\pi-2)$

Correct Answer: (A) $-2\pi(\pi+2)$

Solution:

Concept:

- Expand $\sin(x+y) = \sin x\cos y + \cos x \sin y$. Integration with respect to y leaves the coefficients of $\sin x$ and $\cos x$ as constants.
- Write $A = a/(\pi^2-4)$ and $B = b/(\pi^2-4)$. Comparing coefficients with $f(x) = x + A\sin x + B\cos x$ converts the integral equation into two linear equations.

Step 1: Evaluate the coefficient integrals The expanded equation implies

$$A = \int_0^{\pi/2} f(y)\cos y\, dy, \qquad B = \int_0^{\pi/2} f(y)\sin y\, dy.$$

Integration by parts gives

$$\int_0^{\pi/2} y\cos y\, dy = \frac{\pi}{2} - 1, \qquad \int_0^{\pi/2} y\sin y\, dy = 1.$$

Also, $\int_0^{\pi/2} \sin y\cos y\, dy = 1/2$, while each of the sine-square and cosine-square integrals is $\pi/4$. Substituting $f(y) = y + A\sin y + B\cos y$, taking care to retain which trigonometric factor multiplies each coefficient, yields

$$A = \frac{\pi}{2} - 1 + \frac{A}{2} + \frac{\pi B}{4}, \qquad B = 1 + \frac{\pi A}{4} + \frac{B}{2}.$$

Step 2: Add the equations and restore the original parameters Only the sum is asked for, so there is no need to solve individually for A and B. Let $T = A + B$. Adding the two equations gives

$$T = \frac{\pi}{2} + \left(\frac{1}{2} + \frac{\pi}{4}\right)T.$$

Thus $(2-\pi)T = 2\pi$, and $T = 2\pi/(2-\pi)$. Since $a + b = (\pi^2-4)T$,

$$a + b = (\pi-2)(\pi+2)\frac{2\pi}{2-\pi} = -2\pi(\pi+2).$$

The sign is negative because $2 - \pi < 0$. Option D has the wrong factor $\pi - 2$, while B and C miss the factor 2, with B additionally using the wrong binomial.

Step 3: Conclusion The sum is $a + b = -2\pi(\pi+2)$. Hence, the correct option is (A).

Quick Tip to Solve: When an integral kernel separates into sine and cosine terms, compare coefficients; if only their sum is required, add the equations immediately.

8. Let α and β be real numbers. Consider a 3 $\times$ 3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then [4 marks]

(A) $\alpha = 1$

(B) $\alpha = 4$

(C) $\beta = 8$

(D) $\beta = -8$

Correct Answer: No option is forced for all allowed matrices; the zero matrix gives alpha = beta = 0. The intended nonscalar answer is D.

Solution:

Concept:

- Powers of a matrix satisfying $A^2 = 3A + \alpha I$ can be reduced to linear combinations of A and I.
- Coefficients of A and I can be compared separately only when these matrices are linearly independent, that is, when A is not a scalar matrix.

Step 1: Reduce the fourth power Squaring and substituting the given relation gives

$$\begin{aligned} A^4 &= 9A^2 + 6\alpha A + \alpha^2 I \\ &= (27 + 6\alpha)A + (9\alpha + \alpha^2)I. \end{aligned}$$

Hence $(6\alpha + 6)A + (\alpha^2 + 9\alpha - \beta)I = 0$. If A is not scalar, independence forces $\alpha = -1$ and then $\beta = 1 - 9 = -8$. This is the usual intended calculation.

Step 2: Check the omitted scalar case The statement does not exclude scalar matrices. In particular, take the valid 3×3 zero matrix $A = 0$. The first equation gives $\alpha = 0$, and the second gives $\beta = 0$. Both equations are satisfied, but none of the four listed equalities holds. Thus the stated hypotheses do not force any listed choice. The dimension being three does not establish independence of A and I.

Step 3: Conclusion The intended answer is $\beta = -8$, option (D), under the additional assumption that A is not scalar. Literally, the question has no uniquely forced option: $A = 0, \alpha = \beta = 0$ is a counterexample.

Quick Tip to Solve: Before comparing matrix coefficients, check that the matrices used as basis elements are linearly independent.

9. A light ray emits from the origin making an angle 30° with the positive x-axis. After getting reflected by the line $x + y = 1$, if this ray intersects x-axis at Q, then the abscissa of Q is : [4 marks]

(A) $\frac{2}{(\sqrt{3}-1)}$

(B) $\frac{2}{3-\sqrt{3}}$

(C) $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$

(D) $\frac{2}{3+\sqrt{3}}$

Correct Answer: (D) $\frac{2}{3+\sqrt{3}}$

Solution:

Concept:

- The incident ray has slope $\tan 30^\circ = 1/\sqrt{3}$. Its point of incidence satisfies both its equation and the mirror equation.
- Reflection in $x + y = 1$, whose normal is parallel to $(1, 1)$, changes a direction (u, v) into $(-v, -u)$.

Step 1: Locate the incidence point The incident ray is $y = x/\sqrt{3}$, with $x \geq 0$. Combining it with $x + y = 1$ gives

$$P = \left(\frac{\sqrt{3}}{\sqrt{3}+1}, \frac{1}{\sqrt{3}+1} \right).$$

Both coordinates are positive, as required for the incident ray. A unit incident direction is $(\sqrt{3}/2, 1/2)$.

Step 2: Find the reflected intercept The reflected direction is $(-1/2, -\sqrt{3}/2)$, so the reflected line through P has slope $\sqrt{3}$. Setting its ordinate to zero gives

$$x_Q = x_P - \frac{y_P}{\sqrt{3}} = \frac{\sqrt{3} - 1/\sqrt{3}}{\sqrt{3}+1} = \frac{2}{3+\sqrt{3}}.$$

The parameter along the reflected direction is positive, so this intersection lies on the reflected ray, not its backward extension. The other three expressions do not equal this intercept.

Step 3: Conclusion The abscissa is $\boxed{2/(3+\sqrt{3})}$. Hence, the correct option is (D).

Quick Tip to Solve: For a mirror of the form x+y=constant, reflect the direction by swapping its components and reversing both signs.

10. For two non-zero complex numbers z_1 and z_2, if $\mathrm{Re}\,(z_1 z_2) = 0$ and $\mathrm{Re}\,(z_1 + z_2) = 0$, then which of the following are possible?

A. $\mathrm{Im}\,(z_1) > 0$ **and** $\mathrm{Im}\,(z_2) > 0$

B. $\mathrm{Im}\,(z_1) < 0$ **and** $\mathrm{Im}\,(z_2) > 0$

C. $\mathrm{Im}\,(z_1) > 0$ **and** $\mathrm{Im}\,(z_2) < 0$

D. $\mathrm{Im}\,(z_1) < 0$ **and** $\mathrm{Im}\,(z_2) < 0$

Choose the correct answer from the options given below : [4 marks]

(A) A and C

(B) A and B

(C) B and D

(D) B and C

Correct Answer: (D) B and C

Solution:

Concept:

- Write $z_1 = a + ib$ and use $\mathrm{Re}(z_1 + z_2) = 0$ to write $z_2 = -a + id$, with real a, b, d.
- The real part of the product is $-a^2 - bd$. Thus the hypotheses impose $bd = -a^2 \leq 0$.

Step 1: Exclude equal signs Multiplication gives

$$(a + ib)(-a + id) = -a^2 - bd + ia(d - b).$$

Its real part is zero precisely when $bd = -a^2$. If both imaginary parts were strictly positive, or both strictly negative, their product would be positive. This contradicts $bd \leq 0$. Therefore statements A and D in the question are impossible.

Step 2: Exhibit both opposite-sign cases The pair $z_1 = 1 - i,\ z_2 = -1 + i$ has imaginary parts negative and positive respectively. Their sum is zero and their product is $2i$, so statement B is possible. Interchanging the two numbers gives imaginary parts positive and negative respectively, while preserving both hypotheses, proving statement C is possible. The numbers in these examples are nonzero as stipulated. Every other answer choice includes one of the impossible same-sign statements.

Step 3: Conclusion Exactly statements B and C are possible. Hence, the correct option is (D).

Quick Tip to Solve: Convert real-part constraints to real variables, then use the sign of the product of the imaginary components.

11. Let $y = f(x)$ be the solution of the differential equation $y(x+1)dx - x^2dy = 0, y(1) = e$. Then $\lim_{x\to 0^+} f(x)$ is equal to [4 marks]

(A) e^2

(B) 0

(C) $\frac{1}{e^2}$

(D) $\frac{1}{e}$

Correct Answer: (B) 0

Solution:

Concept:

- For $x > 0$, the equation becomes $dy/y = (x^{-1} + x^{-2})\,dx$ on the nonzero solution selected by $y(1) = e$.
- Integrating x^{-2} produces $-x^{-1}$, so the exponential factor decays strongly as $x \to 0^+$.

Step 1: Integrate and apply the initial value Separate the variables to obtain

$$\ln|y| = \ln x - \frac{1}{x} + C, \qquad y = Kxe^{-1/x}.$$

The initial condition gives $e = Ke^{-1}$, hence $K = e^2$. Consequently $f(x) = e^2xe^{-1/x}$ on the positive interval containing $x = 1$. Direct logarithmic differentiation gives $f'/f = 1/x + 1/x^2$, confirming the differential equation.

Step 2: Evaluate the one-sided limit For $x > 0$, $0 < e^{-1/x} < 1$. Therefore

$$0 < f(x) = e^2xe^{-1/x} < e^2x.$$

The upper bound tends to zero as $x \to 0^+$, so the squeeze theorem gives the required limit. The constants e^2, e^{-2} and e^{-1} cannot be the limit; the multiplicative constant in a solution is not itself its limiting value.

Step 3: Conclusion $\lim_{x\to 0^+} f(x) = \boxed{0}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the minus sign in the integral of x to the power minus two; it decides growth versus decay near zero.

12. Let Δ be the area of the region $\{(x, y) \in R^2 : x^2 + y^2 \leq 21, y^2 \leq 4x, x \geq 1\}$. Then $\frac{1}{2}\left(\Delta - 21\sin^{-1}\frac{2}{\sqrt{7}}\right)$ is equal to [4 marks]

(A) $2\sqrt{3} - \frac{1}{3}$

(B) $2\sqrt{3} - \frac{2}{3}$

(C) $\sqrt{3} - \frac{4}{3}$

(D) $\sqrt{3} - \frac{2}{3}$

Correct Answer: (C) $\sqrt{3} - \frac{4}{3}$

Solution:

Concept:

- The region is symmetric about the x-axis. Its upper boundary is the smaller of $2\sqrt{x}$ and $\sqrt{21 - x^2}$.
- The curves intersect where $4x = 21 - x^2$, giving the relevant intersection $x = 3$ and $y = 2\sqrt{3}$.

Step 1: Split the upper-half area Because $x \geq 1$, the upper-half area is

$$\frac{\Delta}{2} = \int_1^3 2\sqrt{x}\,dx + \int_3^{\sqrt{21}} \sqrt{21 - x^2}\,dx.$$

The parabola controls the height before $x = 3$; the circle controls it afterward. The first integral is $4\sqrt{3} - 4/3$. The negative intersection $x = -7$ is outside both the stated range and the relevant circle boundary.

Step 2: Evaluate and simplify Use the circle antiderivative

$$\int \sqrt{21 - x^2}\,dx = \frac{x\sqrt{21 - x^2}}{2} + \frac{21}{2}\sin^{-1}\frac{x}{\sqrt{21}}.$$

The second integral equals $21\pi/4 - 3\sqrt{3} - (21/2)\sin^{-1}(3/\sqrt{21})$. Since the two acute angles satisfy

$$\sin^{-1}\frac{3}{\sqrt{21}} + \sin^{-1}\frac{2}{\sqrt{7}} = \frac{\pi}{2},$$

we obtain $\Delta/2 = \sqrt{3} - 4/3 + (21/2)\sin^{-1}(2/\sqrt{7})$. Subtracting the requested inverse-sine term leaves only the algebraic part. This differs from the other three listed values.

Step 3: Conclusion $\frac{1}{2}\left(\Delta - 21\sin^{-1}\frac{2}{\sqrt{7}}\right) = \boxed{\sqrt{3} - \frac{4}{3}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use symmetry first and split at the intersection: the smaller positive boundary determines each vertical strip.

13. The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log_e x}-(2x+3)}, x \in \mathbb{R}$ is [4 marks]

(A) $(-1, \infty) - \{3\}$

(B) $\mathbb{R} - \{-1, 3)$

(C) $(2, \infty) - \{3\}$

(D) $\mathbb{R} - \{3\}$

Correct Answer: (C) $(2, \infty) - \{3\}$

Solution:

Concept:

- A real logarithm requires a positive argument, a positive base and a base different from one.
- Every denominator must be nonzero; simplifying an expression does not remove restrictions inherited from its logarithms.

Step 1: Apply the logarithmic restrictions For $\log_{x+1}(x-2)$, require $x-2>0$, $x+1>0$, and $x+1 \neq 1$. The denominator also contains $\log_e x$, requiring $x>0$. Their intersection is simply $x>2$; in that interval the base exceeds three, so its separate restrictions are automatically satisfied. In particular, values between minus one and two cannot belong to the domain.

Step 2: Remove denominator zeros Only on the allowed range simplify

$$e^{2\log_e x} = x^2, \qquad x^2-(2x+3) = (x-3)(x+1).$$

The denominator vanishes at $x=3$ and $x=-1$. Of these, only three lies in $x>2$, so remove it. At $x=3$ the numerator is zero as well, but the original expression is still undefined: a zero denominator cannot be included by cancellation or by a limiting value. The other options retain forbidden logarithm arguments or negative x-values.

Step 3: Conclusion The domain is $\boxed{(2,\infty)\setminus\{3\}}$. Hence, the correct option is (C).

Quick Tip to Solve: Intersect all logarithm restrictions before factoring the denominator, then exclude its zeros from that intersection.

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14. Let $f : R \to R$ be a function such that $f(x) = \frac{x^2+2x+1}{x^2+1}$. Then [4 marks]

(A) $f(x)$ is many-one in $(-\infty, -1)$

(B) $f(x)$ is one-one in $(-\infty, \infty)$

(C) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$

(D) $f(x)$ is many-one in $(1, \infty)$

Correct Answer: (C) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$

Solution:

Concept:

- Rewrite $f(x) = 1 + 2x/(x^2+1)$; its denominator is positive for every real input.
- A strictly monotonic function on an interval is one-one there. A repeated value at distinct inputs

disproves global injectivity.

Step 1: Check monotonicity on the stated intervals Differentiation yields

$$f'(x) = \frac{2(1-x^2)}{(x^2+1)^2}.$$

Thus the derivative is negative on $(1, \infty)$ and on $(-\infty, -1)$. The function is strictly decreasing on each of these intervals, so it is one-one on each. It remains strictly decreasing on $[1, \infty)$: the zero derivative at the endpoint $x = 1$ does not cause equal values at two distinct points. This rules out the many-one claims on the two outer intervals.

Step 2: Disprove injectivity on all real numbers For nonzero x, the expression $2x/(1+x^2)$ is unchanged by replacing x with $1/x$. For instance,

$$f(2) = \frac{9}{5} = f(1/2), \qquad 2 \neq \frac{1}{2}.$$

Therefore f is not one-one on the whole real line. The interval claim in the third option is true while its global failure is demonstrated by this explicit pair; the claim of global injectivity is false.

Step 3: Conclusion The function is one-one on $[1, \infty)$, but not on $\mathbb{R}$. Hence, the correct option is (C).

Quick Tip to Solve: For expressions involving 2x divided by 1+x squared, test reciprocal inputs to find repeated values quickly.

15. Let $[x]$ denote the greatest integer $\leq x$. Consider the function $f(x) = \max\{x^2, 1 + [x]\}$. Then the value of the integral $\int_0^2 f(x)dx$ is [4 marks]

(A) $\frac{5+4\sqrt{2}}{3}$

(B) $\frac{4+5\sqrt{2}}{3}$

(C) $\frac{8+4\sqrt{2}}{3}$

(D) $\frac{1+5\sqrt{2}}{3}$

Correct Answer: (A) $\frac{5+4\sqrt{2}}{3}$

Solution:

Concept:

- For an integer n, the greatest integer function satisfies $[x] = n$ throughout $n \leq x < n+1$. Thus its value changes at integers, and the maximum must be compared separately on each interval.
- The integral of a piecewise function is the sum over its intervals. Changing the value at an isolated endpoint does not change a definite integral. For the polynomial part, $\int x^2\,dx = x^3/3$.

Step 1: Find the active expression on each interval For $0 \leq x < 1$, we have $1 + [x] = 1$ and $x^2 \leq 1$, so $f(x) = 1$. For $1 \leq x < 2$, the competing constant is 2. The equality $x^2 = 2$ occurs at $x = \sqrt{2}$. Therefore $f(x) = 2$ from 1 to $\sqrt{2}$, and $f(x) = x^2$ from $\sqrt{2}$ to 2. At $x = 2$, the maximum is 4, which is also consistent with the last expression.

Step 2: Integrate and combine the contributions Use all three intervals; using only the integer break at 1 would miss the change in the maximum inside the second interval.

$$\begin{aligned}\int_0^2 f(x)\,dx &= \int_0^1 1\,dx + \int_1^{\sqrt{2}} 2\,dx + \int_{\sqrt{2}}^2 x^2\,dx \\ &= 1 + 2(\sqrt{2}-1) + \frac{8-2\sqrt{2}}{3} \\ &= \frac{5+4\sqrt{2}}{3}.\end{aligned}$$

The coefficient of $\sqrt{2}$ is $2 - 2/3 = 4/3$, and the constant is $1 - 2 + 8/3 = 5/3$. This independently rules out each of the other three expressions.

Step 3: Conclusion The required integral is $\boxed{(5+4\sqrt{2})/3}$. Hence, the correct option is (A).

Quick Tip to Solve: For a maximum involving $[x]$, split at integers first, then solve the equality of the competing expressions inside every interval.

16. Let $A = \left\{(x,y) \in \mathbb{R}^2 : y \geq 0, 2x \leq y \leq \sqrt{4-(x-1)^2}\right\}$ **and**

$B = \left\{(x,y) \in \mathbb{R} \times \mathbb{R} : 0 \leq y \leq \min\left\{2x, \sqrt{4-(x-1)^2}\right\}\right\}$. .

Then the ratio of the area of A to the area of B is [4 marks]

(A) $\frac{\pi}{\pi+1}$

(B) $\frac{\pi-1}{\pi+1}$

(C) $\frac{\pi}{\pi-1}$

(D) $\frac{\pi+1}{\pi-1}$

Correct Answer: (B) $\frac{\pi-1}{\pi+1}$

Solution:

Concept:

- The curve $y = \sqrt{4-(x-1)^2}$ is the upper semicircle of radius 2 centred at $(1,0)$, with domain $-1 \leq x \leq 3$. Its area is 2π, and symmetry about $x = 1$ divides that area into two equal parts of area π.
- An area described by inequalities must respect all of them. In particular, $y \geq 0$ remains a lower bound even where the line $y = 2x$ is negative. Region B exists only where its upper bound is nonnegative.

Step 1: Locate the relevant intersection For the line to meet the upper semicircle, $x \geq 0$ is necessary. Squaring the equality gives

$$4x^2 = 4 - (x-1)^2, \qquad 5x^2 - 2x - 3 = 0.$$

The roots are 1 and $-3/5$, but the negative root is extraneous because $2x < 0$ there. The actual intersection is $(1,2)$. The area below $y = 2x$ from 0 to 1 is a triangle with base 1 and height 2, hence

area 1.

Step 2: Calculate each region and take the ratio For $-1 \le x < 0$, all points below the semicircle and above the axis belong to A. For $0 \le x \le 1$, remove the triangle below the line. Hence A is the left half of the semicircle with that triangle removed: $|A| = \pi - 1$. Region B consists of the same triangle on $[0, 1]$, together with the complete right half of the semicircle on $[1, 3]$. Thus $|B| = 1 + \pi$. The two regions partition the semicircle apart from their common boundary, confirming $|A| + |B| = 2\pi$. Therefore

$$\frac{|A|}{|B|} = \frac{\pi - 1}{\pi + 1}.$$

The numerator is smaller than the denominator, which also rejects the reciprocal in option D; neither A nor C has the required removed triangle in the numerator.

Step 3: Conclusion The ratio is $\boxed{(\pi - 1)/(\pi + 1)}$. Hence, the correct option is (B).

Quick Tip to Solve: After squaring a line–semicircle intersection equation, reject roots where the line is negative; then exploit symmetry before integrating.

17. Consider the following system of equations

$$\alpha x + 2y + z = 1$$

$$2\alpha x + 3y + z = 1$$

$$3x + \alpha y + 2z = \beta$$

for some $\alpha, \beta \in \mathbb{R}$. Then which of the following is NOT correct. [4 marks]

(A) It has a solution for all $\alpha \neq -1$ and $\beta = 2$

(B) It has no solution if $\alpha = -1$ and $\beta \neq 2$

(C) It has no solution for $\alpha = -1$ and for all $\beta \in \mathbb{R}$

(D) It has no solution for $\alpha = 3$ and for all $\beta \neq 2$

Correct Answer: (C) It has no solution for $\alpha = -1$ and for all $\beta \in \mathbb{R}$

Solution:

Concept:

- Elimination preserves a linear system's solution set. An equation $0 = \beta - 2$ is consistent only when $\beta = 2$; a nonzero coefficient determines the variable uniquely.
- The question asks for the statement that is NOT correct. Thus we must establish the parameter cases, including singular values, and then test the four statements rather than selecting a true consistency condition.

Step 1: Eliminate y and z in terms of x Subtract equation 1 from equation 2 to obtain $\alpha x + y = 0$. Hence $y = -\alpha x$, and equation 1 then gives $z = 1 + \alpha x$. No division by α was used, so these relations

hold for every real parameter. Substitution into equation 3 yields

$$3x - \alpha^2 x + 2 + 2\alpha x = \beta,$$

or

$$(3 - \alpha)(\alpha + 1)x = \beta - 2.$$

Step 2: Classify the parameters and test the statements When $\alpha \notin \{-1, 3\}$, the coefficient is nonzero and every β gives a unique solution. When $\alpha = -1$ or 3, the system has no solution for $\beta \neq 2$ and infinitely many solutions for $\beta = 2$, with x free and y,z determined above. Statement A is true: for $\beta = 2$ every $\alpha \neq -1$ is consistent, including 3. Statements B and D correctly describe the inconsistent singular cases. Statement C is false because at $\alpha = -1, \beta = 2$, taking $x = 0$ gives $y = 0, z = 1$, which satisfies all three equations. Indeed every x with $y = x, z = 1 - x$ works in that case.

Step 3: Conclusion Statement C is false. Hence, the correct option is (C).

Quick Tip to Solve: At a singular parameter value, check the right-hand side after elimination; a zero coefficient can give either no solutions or infinitely many.

18. If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to : [4 marks]

(A) 24

(B) 0

(C) 18

(D) 6

Correct Answer: (A) 24

Solution:

Concept:

- Three vectors are coplanar exactly when their scalar triple product vanishes: $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$. This supplies a linear relation between the unknown components.
- The length of the projection of $\vec{a}$ on $\vec{b}$ is $|\vec{a} \cdot \vec{b}|/|\vec{b}|$. The absolute value permits both orientations along the same line. A positive signed scalar projection uses the positive case.

Step 1: Apply the coplanarity condition The cross product is

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 10\hat{i} - 2\hat{j} - 14\hat{k}.$$

Therefore

$$10\lambda - 2\mu - 56 = 0, \qquad \mu = 5\lambda - 28.$$

Also $|\vec{b}| = \sqrt{4 + 16 + 4} = \sqrt{24} = 2\sqrt{6}$, while $\vec{a} \cdot \vec{b} = -2\lambda + 4\mu - 8 = 18\lambda - 120$.

Step 2: Use the projection and sum the admissible values The specified projection length gives

$$|18\lambda - 120| = \sqrt{54}\sqrt{24} = 36.$$

For $18\lambda - 120 = 36$, $\lambda = 26/3$ and $\mu = 46/3$, so $\lambda + \mu = 24$. For $18\lambda - 120 = -36$, $\lambda = 14/3$ and $\mu = -14/3$, so $\lambda + \mu = 0$. Thus the sum of all possible values is $24 + 0 = 24$. If projection is interpreted as the positive signed scalar projection, only the first pair is retained, and the requested sum remains 24. The negative-orientation case alone would give option B, but it omits the other allowed pair; 18 and 6 satisfy neither complete result.

Step 3: Conclusion The sum is $\boxed{24}$. Hence, the correct option is (A).

Quick Tip to Solve: Use coplanarity to eliminate one unknown before imposing the projection; when projection means length, include both signs of the dot product.

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19. Let $x = 2$ be a root of the equation $x^2 + px + q = 0$ and $f(x) = \begin{cases} \frac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4}, & x \neq 2p \\ 0, & x = 2p \end{cases}$

Then $\lim\limits_{x \to 2p^+} [f(x)]$, where $[.]$ denotes greatest integer function, is [4 marks]

(A) 2

(B) 1

(C) 0

(D) -1

Correct Answer: (C) 0

Solution:

Concept:

- If 2 is a root of $x^2 + px + q = 0$, substitution gives $4 + 2p + q = 0$. This algebraic constraint should be used before attempting the trigonometric limit.
- The greatest integer function is discontinuous at integers. It is safe to determine its local value by bounding the inner expression in an interval between consecutive integers, rather than interchanging a limit and the floor operation without justification.

Step 1: Simplify the argument using the root condition The root condition gives $q + 4 = -2p$. Consequently

$$q^2 + 8q + 16 = (q + 4)^2 = 4p^2$$

and

$$x^2 - 4px + q^2 + 8q + 16 = (x - 2p)^2.$$

Put $t = x - 2p$. As $x \to 2p^+$, we have $t \to 0^+$, and at all nearby points the nonzero branch is

$$f(x) = \frac{1 - \cos(t^2)}{t^4}.$$

The assigned value at $t = 0$ does not affect this one-sided limit.

Step 2: Bound the expression before taking its floor Using $1-\cos u = 2\sin^2(u/2)$, obtain

$$f(x) = \frac{1}{2}\left(\frac{\sin(t^2/2)}{t^2/2}\right)^2 \longrightarrow \frac{1}{2}.$$

For sufficiently small positive t, the sine is positive and strictly smaller than its positive argument. Hence $0 < f(x) < 1/2 < 1$ throughout a punctured right neighbourhood. Its greatest integer is therefore identically 0 there, so its limit is 0. Option B would require values at least 1, option A at least 2, and option D negative values; all contradict the established local bound. The special definition $f(2p) = 0$ is consistent with the floor result but was not used to infer the limit.

Step 3: Conclusion The required limit is $\boxed{0}$. Hence, the correct option is (C).

Quick Tip to Solve: Before taking a greatest-integer limit, place the inner function strictly between two consecutive integers near the approach point.

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20. If p, q and r are three propositions, then which of the following combination of truth values of p, q and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false? [4 marks]

(A) $p = F, q = T, r = F$

(B) $p = T, q = T, r = F$

(C) $p = T, q = F, r = T$

(D) $p = T, q = F, r = F$

Correct Answer: (A) $p = F, q = T, r = F$

Solution:

Concept:

- An implication $U \rightarrow V$ is false exactly when its antecedent U is true and its consequent V is false. If U is false, the implication is true regardless of V.
- A disjunction is false only if both its members are false, while a conjunction is true only if both members are true. These rules let us work backwards from the required false conclusion without constructing a full eight-row truth table.

Step 1: Force the consequent to be false The consequent is $(\sim q) \vee r$. For it to be false we must have

$$\sim q = F, \qquad r = F,$$

so $q = T$ and $r = F$. These restrictions already exclude options C and D, where q is false. Both options A and B meet the consequent restriction, so their antecedents must now be checked; a false consequent alone does not make an implication false.

Step 2: Require a true antecedent and check the options With $q = T, r = F$, the antecedent becomes

$$(p \vee T) \wedge ((\sim p) \vee F) = T \wedge (\sim p) = \sim p.$$

It is true precisely for $p = F$. Thus $(p, q, r) = (F, T, F)$ gives $U = T, V = F$, and the implication is

false. In option B, p is true, so the second conjunct is false and the implication is $F \to F$, which is true. In C the consequent is true because r is true, and in D it is true because $\sim q$ is true. Therefore each of B, C and D makes the complete expression true, leaving only A.

Step 3: Conclusion The required assignment is $\boxed{p = F,\ q = T,\ r = F}$. Hence, the correct option is (A).

Quick Tip to Solve: To falsify an implication, first force its consequent false, then use those restrictions to make the antecedent true.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $a_1, a_2, a_3, \ldots$ be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to _____.

[4 marks]

Solution:

Concept:

- In a geometric progression with first term a and common ratio r, $a_n = ar^{n-1}$. Since the terms are positive and increasing, $a > 0$ and $r > 1$; positivity fixes the sign when a square root is taken.
- Products of equally spaced GP terms equal the square of their middle term. More generally, add the exponents of r in a product; this often avoids calculating a and r separately.

Step 1: Determine the fifth and seventh terms The given product is

$$a_4a_6 = (ar^3)(ar^5) = a^2r^8 = (ar^4)^2 = a_5^2 = 9.$$

Every term is positive, so $a_5 = 3$, not -3. The second condition now gives

$$a_5 + a_7 = 24, \qquad a_7 = 21.$$

Thus $r^2 = a_7/a_5 = 7$, and $r = \sqrt{7} > 1$, consistent with increasing terms. The conditions therefore do describe an admissible positive GP.

Step 2: Rewrite the requested products around the fifth term For the first product, the term indices 1 and 9 are equidistant from 5:

$$a_1a_9 = a(ar^8) = a^2r^8 = a_5^2 = 9.$$

For the three-term product, add the exponents carefully:

$$a_2a_4a_9 = (ar)(ar^3)(ar^8) = a^3r^{12} = (ar^4)^3 = a_5^3 = 27.$$

Although the indices add to 15, the exponents add to 12 because each term has exponent one less than its index. Finally the remaining pair is already known from the data, so

$$a_1a_9 + a_2a_4a_9 + a_5 + a_7 = 9 + 27 + 24 = 60.$$

There is no need to evaluate the first term explicitly; doing so would introduce avoidable fractions and radicals without changing these symmetric products.

Step 3: Conclusion The requested numerical value is $\boxed{60}$.

Quick Tip to Solve: For GP products, compare the average of the term indices with a known middle term; here the products reduce to powers of the fifth term.

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22. Suppose f is a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in N$ and $f(1) = \frac{1}{5}$. If $\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then m is equal to _____. [4 marks]

Solution:

Concept:

- On positive integers, additivity gives $f(n) = nf(1)$ by repeated addition. No continuity assumption is needed because only integer arguments are involved.
- A rational summand with consecutive linear factors often telescopes after partial fractions: $1/((n+1)(n+2)) = 1/(n+1) - 1/(n+2)$.

Step 1: Determine the function on natural numbers Apply the relation with $x = n$ and $y = 1$. Since $f(1) = 1/5$, we have $f(n+1) = f(n) + 1/5$. Starting with $n = 1$, induction therefore gives

$$f(n) = \frac{n}{5} \quad (n \in \mathbb{N}).$$

The factor n in the numerator cancels the first factor of the denominator in the required sum.

Step 2: Evaluate the finite sum Substitute the function value and split each term:

$$\begin{aligned}\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} &= \frac{1}{5}\sum_{n=1}^{m} \frac{1}{(n+1)(n+2)} \\ &= \frac{1}{5}\sum_{n=1}^{m}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) \\ &= \frac{1}{5}\left(\frac{1}{2} - \frac{1}{m+2}\right).\end{aligned}$$

All intermediate fractions cancel, leaving the first positive term and the last negative term.

Step 3: Solve for the upper limit Equating this expression to the stated value gives

$$\frac{1}{2} - \frac{1}{m+2} = \frac{5}{12}, \qquad \frac{1}{m+2} = \frac{1}{12}.$$

Thus $m + 2 = 12$, so $m = 10$. Substitution yields $(1/5)(1/2 - 1/12) = 1/12$, confirming the endpoint and avoiding an off-by-one error.

Step 4: Conclusion The required positive integer is $\boxed{m = 10}$.

Quick Tip to Solve: Cancel the factor supplied by additivity before decomposing the fraction; the surviving telescoping endpoints are $1/2$ and $1/(m+2)$.

23. Let the coefficients of three consecutive terms in the binomial expansion of $(1+2x)^n$ be in the ratio 2 : 5 : 8. Then the coefficient of the term, which is in the middle of those three terms, is _____. [4 marks]

Solution:

Concept:

- The coefficient of x^r in $(1+2x)^n$ is $a_r = \binom{n}{r}2^r$. The factor 2^r must be included; these are not just the coefficients of $(1+x)^n$.
- Adjacent coefficients satisfy $a_{r+1}/a_r = 2(n-r)/(r+1)$, obtained by cancelling factorials.

Step 1: Express the first adjacent ratio Let the three consecutive powers be x^r, x^{r+1}, x^{r+2}. Their coefficient ratio is $2:5:8$, so

$$\frac{2(n-r)}{r+1} = \frac{5}{2}.$$

Cross multiplication gives $4(n-r) = 5(r+1)$, or

$$4n = 9r + 5.$$

The index r denotes the first of the three terms, not the middle one.

Step 2: Use the second adjacent ratio For the next pair, the required ratio is $8/5$, giving

$$\frac{2(n-r-1)}{r+2} = \frac{8}{5}.$$

Hence $10(n-r-1) = 8(r+2)$, or $10n = 18r + 26$. Combining this with $4n = 9r + 5$ yields

$$5(9r+5) = 2(18r+26), \qquad 9r = 27.$$

Thus $r = 3$ and $n = 8$.

Step 3: Calculate and check the middle coefficient The middle power is $x^{r+1} = x^4$, so its coefficient is

$$\binom{8}{4}2^4 = 70 \times 16 = 1120.$$

For a direct check, the adjacent coefficients are $\binom{8}{3}2^3 = 448$ and $\binom{8}{5}2^5 = 1792$. Dividing $448 : 1120 : 1792$ by 224 gives exactly $2:5:8$.

Step 4: Conclusion The coefficient of the middle of the three terms is $\boxed{1120}$.

Quick Tip to Solve: Use two adjacent ratios to find both the binomial exponent and the term index; include the power of 2 in each coefficient.

24. If the co-efficient of x^9 in $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x - \frac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- In a binomial expansion, the general term $\binom{n}{k}A^{n-k}B^k$ determines both the coefficient and the exponent of the variable.
- For Laurent expressions, negative powers are treated by the same exponent laws. Match the required power first, then compare its coefficient, keeping the sign from the second term.

Step 1: Locate the coefficient of the positive power Choose k factors of $1/(\beta x)$ in the first expansion. Its general term is

$$\binom{11}{k}\alpha^{11-k}\beta^{-k}x^{33-4k}.$$

For the power x^9, $33 - 4k = 9$, so $k = 6$. The desired coefficient is therefore

$$\binom{11}{6}\frac{\alpha^5}{\beta^6}.$$

The denominator in the original expression requires $\beta \neq 0$.

Step 2: Locate the coefficient of the negative power In the second expansion, taking k factors of $-1/(\beta x^3)$ gives

$$\binom{11}{k}(-1)^k\alpha^{11-k}\beta^{-k}x^{11-4k}.$$

Set $11 - 4k = -9$, which gives $k = 5$. The coefficient is

$$-\binom{11}{5}\frac{\alpha^6}{\beta^5}.$$

The minus sign remains because five negative factors have been selected.

Step 3: Equate and simplify the coefficients By symmetry, $\binom{11}{6} = \binom{11}{5}$. For the nonzero coefficient case, cancellation in the stated equality gives

$$\frac{\alpha^5}{\beta^6} = -\frac{\alpha^6}{\beta^5} \quad \Longrightarrow \quad 1 = -\alpha\beta.$$

Thus $(\alpha\beta)^2 = 1$. Strictly, if $\alpha = 0$ is allowed, both specified coefficients vanish and equality also holds, giving the value 0. The standard nonzero-coefficient interpretation selects 1.

Step 4: Conclusion For the intended nonzero coefficients, $\boxed{(\alpha\beta)^2 = 1}$. If $\alpha = 0$ is admitted, the wording also permits 0; the stated key uses the nonzero case.

Quick Tip to Solve: Find the exponent of x before simplifying coefficients, and preserve the odd-power minus sign; cancellation of α^5 requires a nonzero coefficient.

25. If all the six digit numbers $x_1\,x_2\,x_3\,x_4\,x_5\,x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72^{th} number is _____. [4 marks]

Solution:

Concept:

- Strictly increasing digits are distinct and belong to $\{1, 2, \ldots, 9\}$. Each choice of six digits produces exactly one valid number.
- For fixed-length decimal numbers, numerical order equals lexicographic order. A fixed prefix is followed by all increasing selections from the remaining larger digits.

Step 1: Count the numbers starting with 1 After the first digit 1, select five digits from $2, 3, \ldots, 9$. This gives

$$\binom{8}{5} = 56$$

numbers. They occupy positions 1 through 56. Thus position 72 lies among numbers starting with 2, and its position within that block is

$$72 - 56 = 16.$$

The entire block starting with 2 has $\binom{7}{5} = 21$ members, so this conclusion is consistent.

Step 2: Determine the next digit Within the prefix 2, the smallest possible second digit is 3. For prefix 23, the remaining four digits are chosen from $4, 5, 6, 7, 8, 9$, giving

$$\binom{6}{4} = 15$$

numbers. These are the first 15 in the block. The required member is the next one, so it has prefix 24 and is the first number with that prefix. Its overall position is $56 + 15 + 1 = 72$.

Step 3: Complete the smallest suffix and add the digits The first number with prefix 24 uses the four smallest available digits larger than 4: they are 5, 6, 7 and 8. Hence the required number is

$$245678.$$

It satisfies every strict inequality and has six nonzero digits. Its digit sum is

$$2 + 4 + 5 + 6 + 7 + 8 = 32.$$

Choosing 245679 instead would advance to the next number and would therefore correspond to position 73.

Step 4: Conclusion The 72nd number is 245678, whose digit sum is $\boxed{32}$.

Quick Tip to Solve: Count complete prefix blocks with combinations, subtract them from the rank, and remember that a residual rank of 1 means the smallest possible suffix.

26. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function that satisfies the relation $f(x+y) = f(x)+f(y)-1, \forall x, y \in \mathbb{R}$. If $f'(0) = 2$, then $|f(-2)|$ is equal to _____. [4 marks]

Solution:

Concept:

- Substitution of zero in a functional equation often determines its constant term. Here $f(x + y) = f(x) + f(y) - 1$ is additive after subtracting 1.
- Differentiability allows the increment quotient to be used directly: the relation makes the derivative at every point equal to the derivative at zero.

Step 1: Determine the value at zero Put $x = y = 0$ in the given identity:

$$f(0) = 2f(0) - 1.$$

It follows that $f(0) = 1$. Therefore the equation with $y = h$ can be rearranged as

$$f(x + h) - f(x) = f(h) - 1 = f(h) - f(0).$$

The increment depends only on h, which is the central consequence of the functional equation.

Step 2: Differentiate using the increment quotient For $h \neq 0$, divide the last relation by h and let $h \to 0$. Differentiability at zero gives

$$\begin{aligned} f\prime(x) &= \lim_{h\to 0} \frac{f(x + h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{f(h) - f(0)}{h} = f\prime(0) = 2. \end{aligned}$$

Consequently $f(x) = 2x + C$ throughout the real line. Using $f(0) = 1$ fixes $C = 1$.

Step 3: Evaluate the required absolute value The function is $f(x) = 2x + 1$. Substituting $x = -2$ gives

$$f(-2) = 2(-2) + 1 = -3, \qquad |f(-2)| = 3.$$

As a check, $f(x + y) = 2x + 2y + 1$, while $f(x) + f(y) - 1$ gives the same expression. The derivative of this function is indeed 2, so both given conditions are satisfied.

Step 4: Conclusion The required absolute value is $\boxed{|f(-2)| = 3}$.

Quick Tip to Solve: First find $f(0)$, then form the increment quotient; the constant shift disappears and the derivative becomes constant.

27. Five digit numbers are formed using the digits 1, 2, 3, 5, 7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1. Then the serial number of 35337 is _____. [4 marks]

Solution:

Concept:

- All numbers have five digits chosen independently from the ordered alphabet $1 < 2 < 3 < 5 < 7$. Repetition gives five choices at every unfilled position.
- In descending order, a number precedes the target precisely when its first differing digit is larger. Counting by that first differing position prevents double counting.

Step 1: Count larger first and second digits For the target 35337, the first digit is 3. Larger permitted first digits are 5 and 7; each allows any four following digits. They contribute

$$2 \cdot 5^4 = 1250.$$

With the first digit fixed as 3, the second target digit is 5. Only 7 is larger, leaving three arbitrary positions and contributing

$$1 \cdot 5^3 = 125.$$

Every number in these two groups is larger than the target.

Step 2: Count larger digits at the remaining positions Fix the prefix 35. The third target digit is 3, and the two larger choices 5 and 7 contribute

$$2 \cdot 5^2 = 50.$$

Fix the prefix 353. The fourth digit is again 3, so the larger choices contribute

$$2 \cdot 5 = 10.$$

Finally, with prefix 3533, the last target digit is 7, the largest allowed digit; it contributes zero preceding numbers. The groups are disjoint because each has a different first differing position.

Step 3: Convert the number of predecessors into a serial number Adding the counts gives

$$1250 + 125 + 50 + 10 + 0 = 1435$$

numbers strictly larger than 35337. The first number has no predecessor but is assigned serial number 1. Hence the serial number is one more than the predecessor count:

$$1435 + 1 = 1436.$$

This final addition is necessary because the count excludes the target itself.

Step 4: Conclusion The serial number of 35337 in descending order is $\boxed{1436}$.

Quick Tip to Solve: At each position count allowed digits larger than the target digit, multiply by $5^{\text{remaining positions}}$, and add 1 after summing.

28. Let the co-ordinates of one vertex of ΔABC be $A(0, 2, \alpha)$ and the other two vertices lie on the line $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. For $\alpha \in \mathbb{Z}$, if the area of ΔABC is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to _____. [4 marks]

Solution:

Concept:

- The area of a triangle is half the product of a base and its perpendicular height. Since both B and C lie on the given line, the height is the distance of A from that line.
- For a line through P with direction vector $\mathbf{d}$, the squared perpendicular distance is $h^2 = |\mathbf{v}|^2 - (\mathbf{v} \cdot \mathbf{d})^2/|\mathbf{d}|^2$, where $\mathbf{v} = \overrightarrow{PA}$.

Step 1: Find the required perpendicular distance Take BC as the base. From the given area and length,

$$21 = \frac{1}{2}(2\sqrt{21})h \quad \Longrightarrow \quad h = \sqrt{21}.$$

The line can be written $(x, y, z) = (-\alpha, 1, -4) + t(5, 2, 3)$. Thus choose $P = (-\alpha, 1, -4)$ and direction vector $\mathbf{d} = (5, 2, 3)$.

Step 2: Apply the point-to-line distance formula Using $A = (0, 2, \alpha)$,

$$\mathbf{v} = (\alpha, 1, \alpha + 4), \quad |\mathbf{d}|^2 = 38.$$

The other required quantities are

$$|\mathbf{v}|^2 = 2\alpha^2 + 8\alpha + 17, \qquad \mathbf{v} \cdot \mathbf{d} = 8\alpha + 14.$$

Since $h^2 = 21$, substitution gives

$$2\alpha^2 + 8\alpha + 17 - \frac{(8\alpha + 14)^2}{38} = 21.$$

The distance is to the entire line, which is the supporting line of the base.

Step 3: Use the integer restriction Multiply by 38 and simplify:

$$12\alpha^2 + 80\alpha - 348 = 0, \qquad 3\alpha^2 + 20\alpha - 87 = 0.$$

Factorisation gives

$$(\alpha - 3)(3\alpha + 29) = 0.$$

Hence $\alpha = 3$ or $-29/3$. Only 3 belongs to $\mathbb{Z}$, so the required square is $3^2 = 9$. For a check at $\alpha = 3$, the distance squared is $59 - 38 = 21$, as required.

Step 4: Conclusion The integer coordinate parameter is $\alpha = 3$, and hence $\boxed{\alpha^2 = 9}$.

Quick Tip to Solve: Translate area and base length into a point-to-line distance first; then use the integer condition to discard the nonintegral root.

29. Let the equation of the plane P containing the line $x + 10 = \frac{8-y}{2} = z$ be $ax + by + 3z = 2(a + b)$ and the distance of the plane P from the point (1, 27, 7) be c. Then $a^2 + b^2 + c^2$ is equal to _____.

[4 marks]

Solution:

Concept:

- A line lies in a plane when every point of its parametric representation satisfies the plane equation. Consequently, the constant term and the coefficient of the line parameter must vanish separately.
- The perpendicular distance of a point (x_0, y_0, z_0) from $Ax + By + Cz + D = 0$ is $|Ax_0 + By_0 + Cz_0 + D|/\sqrt{A^2 + B^2 + C^2}$.

Step 1: Parameterise the given line Put the common ratio equal to t. Then every point on the line has coordinates $x = t - 10$, $y = 8 - 2t$, and $z = t$. Substitution into the specified plane gives

$$a(t - 10) + b(8 - 2t) + 3t = 2(a + b).$$

Collecting the parameter and constant terms,

$$(a - 2b + 3)t - 12a + 6b = 0.$$

This must hold for every real t, rather than for just one intersection point.

Step 2: Determine the plane coefficients Equate both coefficients to zero:

$$a - 2b + 3 = 0, \qquad -12a + 6b = 0.$$

The second relation gives $b = 2a$. Substituting this into the first yields $-3a + 3 = 0$, hence $a = 1$ and $b = 2$. Thus the plane is $x + 2y + 3z - 6 = 0$. As a check, both its value at $(-10, 8, 0)$ and its normal dot the direction $(1, -2, 1)$ are zero.

Step 3: Calculate the distance and required sum Using the given point, the numerator in the distance formula is $|1 + 2(27) + 3(7) - 6| = 70$. Therefore

$$c = \frac{70}{\sqrt{1 + 4 + 9}} = \frac{70}{\sqrt{14}}, \qquad c^2 = 350.$$

The requested quantity is a sum of squares, so no approximation to the square root is necessary:

$$a^2 + b^2 + c^2 = 1 + 4 + 350 = 355.$$

Step 4: Conclusion The required numerical value is $\boxed{355}$.

Quick Tip to Solve: For a plane containing a parametric line, equate the parameter coefficient and constant term separately; square the distance directly when only its square is needed.

30. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a} - \vec{b} + \vec{c}, \lambda\vec{a} - 3\vec{b} + 4\vec{c}, -\vec{a} + 2\vec{b} - 3\vec{c}$ and $2\vec{a} - 4\vec{b} + 6\vec{c}$ respectively. If $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, then λ is equal to _____. [4 marks]

Solution:

Concept:

- Non-coplanar vectors $\vec{a}, \vec{b}, \vec{c}$ are linearly independent, so their scalar triple product is nonzero and they can be used as a basis.
- Three displacement vectors are coplanar precisely when their scalar triple product vanishes. In an independent basis this is equivalent to a zero determinant of their coefficient rows.

Step 1: Form the displacement vectors Subtract the position vector of A from each of the other position vectors. This is necessary because the condition concerns $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$, not the original position vectors:

$$\overrightarrow{AB} = (\lambda - 1)\vec{a} - 2\vec{b} + 3\vec{c},$$
$$\overrightarrow{AC} = -2\vec{a} + 3\vec{b} - 4\vec{c},$$
$$\overrightarrow{AD} = \vec{a} - 3\vec{b} + 5\vec{c}.$$

All three displacements now have a common initial point.

Step 2: Impose coplanarity using the determinant The scalar triple product of these vectors is their coefficient determinant multiplied by $[\vec{a}, \vec{b}, \vec{c}]$. Since the latter is nonzero, coplanarity requires

$$\begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0.$$

Expanding along the first row gives

$$\begin{aligned} 0 &= (\lambda - 1)(15 - 12) + 2(-10 + 4) + 3(6 - 3) \\ &= 3(\lambda - 1) - 12 + 9 \\ &= 3\lambda - 6. \end{aligned}$$

Hence the unique possible value is $\lambda = 2$.

Step 3: Verify the resulting dependence For $\lambda = 2$, the displacement coefficients are $(1, -2, 3)$, $(-2, 3, -4)$, and $(1, -3, 5)$. They obey

$$\overrightarrow{AD} = 3\overrightarrow{AB} + \overrightarrow{AC}.$$

Indeed, the right side has coefficients $(3 - 2, -6 + 3, 9 - 4) = (1, -3, 5)$. Thus the third displacement lies in the plane spanned by the first two, confirming the determinant result without assuming the basis vectors are orthogonal.

Step 4: Conclusion The required value is $\boxed{\lambda = 2}$.

Quick Tip to Solve: Always subtract the same base point before testing coplanarity of joining vectors; independence of the original three vectors then reduces the problem to a coefficient determinant.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Two particles of equal mass 'm' move in a circle of radius 'r' under the action of their mutual gravitational attraction. The speed of each particle will be : [4 marks]

(A) $\sqrt{\frac{Gm}{4r}}$

(B) $\sqrt{\frac{Gm}{2r}}$

(C) $\sqrt{\frac{Gm}{r}}$

(D) $\sqrt{\frac{4Gm}{r}}$

Correct Answer: (A) $\sqrt{\frac{Gm}{4r}}$

Solution:

Concept:

- For two equal masses orbiting only under mutual attraction, the centre of mass is their midpoint. Each particle follows a circle about that fixed midpoint.
- The gravitational attraction $F = Gm^2/d^2$, where d is the separation of the particles, supplies the centripetal force mv^2/r for each orbit.

Step 1: Distinguish orbital radius from separation Each particle has orbital radius r, measured from the common centre of mass. Since the masses are equal, they occupy diametrically opposite ends of a line through that centre, so their instantaneous separation is

$$d = r + r = 2r.$$

The gravitational force must therefore be evaluated with $2r$, while the centripetal acceleration uses the individual orbital radius r. Confusing these two distances changes the numerical factor in the speed.

Step 2: Balance the inward force for either mass The force exerted by one particle on the other points towards the centre of mass and has magnitude

$$F = \frac{Gm^2}{(2r)^2} = \frac{Gm^2}{4r^2}.$$

For either mass separately, circular motion requires

$$\frac{mv^2}{r} = \frac{Gm^2}{4r^2}.$$

Cancel m and multiply by r to obtain

$$v^2 = \frac{Gm}{4r}, \qquad v = \sqrt{\frac{Gm}{4r}}.$$

The positive root is taken because speed is a magnitude.

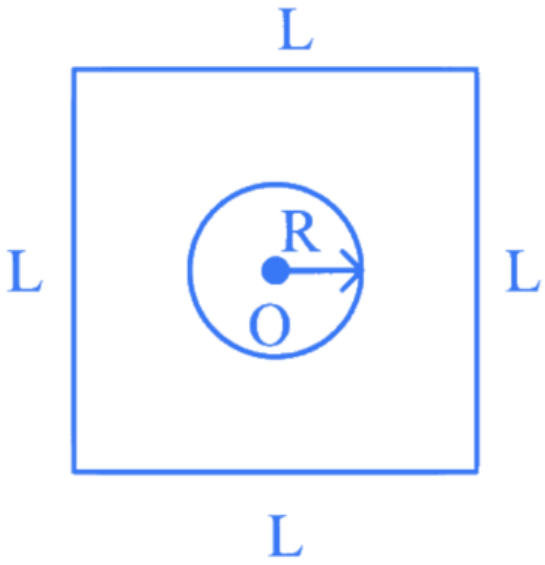
Step 3: Check the alternatives and physical scaling The expression has the dimensions of speed

because Gm/r has units $\mathrm{m^2\,s^{-2}}$. Increasing the orbital radius reduces the speed as $r^{-1/2}$, as expected for gravitational circular motion. Option (B) has twice the required squared speed, option (C) has four times, and option (D) has sixteen times. None balances the actual gravitational force at separation $2r$.

Step 4: Conclusion The speed of each particle is $\boxed{\sqrt{Gm/(4r)}}$. Hence, the correct option is (A).

Quick Tip to Solve: In an equal-mass binary orbit, use twice the orbital radius in the inverse-square gravitational force, but the single orbital radius in the centripetal force.

32. Find the mutual inductance in the arrangement, when a small circular loop of wire of radius 'R' is placed inside a large square loop of wire of side L ($L \gg R$). The loops are coplanar and their centres coincide : [4 marks]



(A) $M = \frac{\sqrt{2}\mu_0 R}{L^2}$

(B) $M = \frac{2\sqrt{2}\mu_0 R}{L^2}$

(C) $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$

(D) $M = \frac{\sqrt{2}\mu_0 R^2}{L}$

Correct Answer: (C) $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$

Solution:

Concept:

- Mutual inductance is the flux through one single-turn loop per unit current in the other: $M = \Phi/I$. Reciprocity allows us to choose the loop whose field is simpler to calculate.
- A straight finite wire at perpendicular distance d produces $B = \mu_0 I(\sin\theta_1 + \sin\theta_2)/(4\pi d)$. For $R \ll L$, the square loop field is approximately uniform over the small circle.

Step 1: Find the field at the centre of the square Pass a current I through the square loop. At its centre, the perpendicular distance from each side is $L/2$, and the lines joining the centre to the

endpoints of that side make angles 45° with its perpendicular bisector. Thus one side contributes

$$B_1 = \frac{\mu_0 I}{4\pi(L/2)}(2\sin 45^\circ) = \frac{\sqrt{2}\mu_0 I}{2\pi L}.$$

By the right-hand rule, the fields from all four sides have the same normal direction, so they add.

Step 2: Calculate the linked flux The total central field is

$$B = 4B_1 = \frac{2\sqrt{2}\mu_0 I}{\pi L}.$$

Since the small circle is concentric and $L \gg R$, variation of this field across its area is negligible to the approximation used in the options. Its area is πR^2, and its normal is parallel to the magnetic field. Therefore

$$\Phi \simeq B\pi R^2 = \frac{2\sqrt{2}\mu_0 I R^2}{L}.$$

There is no cosine reduction because the two loops are coplanar.

Step 3: Divide by the source current and compare The flux-to-current ratio gives

$$M = \frac{\Phi}{I} = \frac{2\sqrt{2}\mu_0 R^2}{L}.$$

Options (A) and (B) scale as R/L^2, rather than the area R^2, and do not have inductance dimensions. Option (D) has the right dimensions but misses a factor of two from the square-loop field. The result is the leading approximation for a small circular loop inside the much larger square.

Step 4: Conclusion $\boxed{M = 2\sqrt{2}\mu_0 R^2/L}$. Hence, the correct option is (C).

Quick Tip to Solve: Choose current in the large square, add the four finite-wire fields at its centre, then multiply by the small loop area before dividing by current.

33. In a cuboid of dimension $2\ \mathrm{L} \times 2\ \mathrm{L} \times \mathrm{L}$, a charge q is placed at the center of the surface 'S' having area of $4\ \mathrm{L}^2$. The flux through the opposite surface to 'S' is given by [4 marks]

(A) $\frac{q}{2\epsilon_0}$
(B) $\frac{q}{3\epsilon_0}$
(C) $\frac{q}{12\epsilon_0}$
(D) $\frac{q}{6\epsilon_0}$

Correct Answer: (D) $\frac{q}{6\epsilon_0}$

Solution:

Concept:

- Gauss law gives total outward electric flux q/ϵ_0 through any closed surface enclosing a point charge. Equal flux through individual faces additionally requires geometrical symmetry.
- Reflecting the given cuboid through the face carrying the charge forms a cube of side $2L$, with the

charge at its centre. This construction gives six symmetry-equivalent square outer faces.

Step 1: Construct a symmetric closed surface The original cuboid has a square face S of side $2L$ and depth L. Place an identical reflected cuboid on the other side of S. Together they form a cube of dimensions $2L \times 2L \times 2L$. The charge, initially at the centre of their shared face, is now at the geometric centre of this larger cube. The face opposite S in the original cuboid becomes one complete outer face of the cube, without any change to its position relative to the charge.

Step 2: Apply Gauss law and cube symmetry The enlarged cube is a closed surface with the charge strictly inside, so

$$\Phi_{\text{cube}} = \frac{q}{\epsilon_0}.$$

The electric field of a central point charge has cubic symmetry: rotations interchange the six faces while leaving the charge and field configuration unchanged. Each face therefore carries an equal share of the total flux:

$$\Phi_{\text{one face}} = \frac{1}{6}\Phi_{\text{cube}} = \frac{q}{6\epsilon_0}.$$

This is exactly the original cuboid face whose flux was requested.

Step 3: Check why other fractions do not apply The entire original cuboid receives half of the total flux, namely $q/(2\epsilon_0)$, but this flux is shared by its five outer faces and is not the flux of the opposite face alone. Those five faces are not all equivalent, so dividing that half equally among them is invalid. The opposite square is a full cube face, not half of one, so $q/(12\epsilon_0)$ is also incorrect. The fraction $1/3$ likewise does not follow from the six-face symmetry.

Step 4: Conclusion The flux through the surface opposite S is $\boxed{q/(6\epsilon_0)}$. Hence, the correct option is (D).

Quick Tip to Solve: When a charge lies at the centre of a cuboid face, reflect the cuboid across that face and check whether the combined solid becomes a cube.

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34. Surface tension of a soap bubble is $2.0 \times 10^{-2}\text{Nm}^{-1}$. Work done to increase the radius of soap bubble from 3.5 cm to 7 cm will be:

Take $\left[\pi = \frac{22}{7}\right]$ [4 marks]

(A) 18.48×10^{-4} J

(B) 5.76×10^{-4} J

(C) 0.72×10^{-4} J

(D) 9.24×10^{-4} J

Correct Answer: (A) 18.48×10^{-4} J

Solution:

Concept:

- The work required for a quasistatic increase of area at constant surface tension T is the increase in surface energy, $W = T\Delta A$.
- A soap bubble has two liquid-air surfaces, an inner and an outer surface. For a thin film of radius r,

the combined area is $A = 2(4\pi r^2) = 8\pi r^2$.

Step 1: Convert the radii and identify both surfaces The initial and final radii in SI units are

$$r_1 = 3.5 \times 10^{-2}\,\text{m}, \qquad r_2 = 7.0 \times 10^{-2}\,\text{m}.$$

Because this is a soap bubble, both surfaces of the film increase in area. We therefore use $8\pi r^2$, rather than the single-surface area $4\pi r^2$ appropriate to a liquid drop. With surface tension assumed constant, the work is stored as additional surface energy.

Step 2: Calculate the increase in area and energy The difference of squared radii is

$$r_2^2 - r_1^2 = 0.004900 - 0.001225 = 0.003675\,\text{m}^2.$$

Using the specified value of π, the combined area increase is

$$\Delta A = 8 \times \frac{22}{7} \times 0.003675 = 0.0924\,\text{m}^2.$$

Consequently,

$$W = T\Delta A = (2.0 \times 10^{-2})(0.0924) = 0.001848\,\text{J} = 18.48 \times 10^{-4}\,\text{J}.$$

The work is positive because the area increases against surface tension.

Step 3: Check the numerical alternatives Option (D), 9.24×10^{-4} J, is exactly half of the required result and would follow from counting only one surface. Options (B) and (C) do not equal the energy change obtained from the squared radii and the given surface tension. A useful independent scaling check is that doubling the radius quadruples the area, so the area increase equals three times the initial total area, which yields the same work.

Step 4: Conclusion The required work is $\boxed{18.48 \times 10^{-4}\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: Count two surfaces for a soap bubble and use the difference of squared radii; doubling the radius makes the area increase three times its initial value.

35. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: If dQ and dW represent the heat supplied to the system and the work done on the system respectively. Then according to the first law of thermodynamics $dQ = dU - dW$.

Reason R: First law of thermodynamics is based on law of conservation of energy.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both A and R are correct but R is not the correct explanation of A

(B) Both A and R are correct and R is the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

Correct Answer: (B) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- For a closed system, energy conservation states that the increase in internal energy equals heat supplied to the system plus work done on it.
- The sign of the work term depends on the definition. If dW denotes work done on the system, then $dU = dQ + dW$; if it denotes work done by the system, its sign is reversed.

Step 1: Evaluate the assertion with the stated convention The assertion explicitly defines dQ as heat supplied to the system and dW as work done on the system. Both are energy inputs when positive. Therefore the internal-energy change is

$$dU = dQ + dW.$$

Rearranging this equality yields

$$dQ = dU - dW,$$

which is exactly the equation in Assertion A. The assertion is therefore correct. Using $dQ = dU + dW$ here would incorrectly treat the same symbol as work done by the system.

Step 2: Evaluate the reason independently The first law of thermodynamics applies the law of conservation of energy to thermodynamic processes. Heat and work are modes of energy transfer, whereas internal energy is a property of the system. The law states that the transferred energy is accounted for by the change in internal energy; it is not created or destroyed in the process. Reason R is thus correct in its own right.

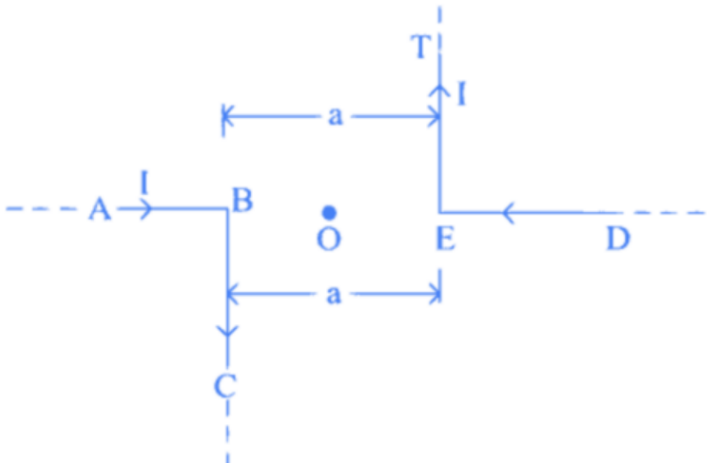
Step 3: Test the explanatory connection Conservation of energy is precisely what gives the balance $dU = dQ + dW$ under the definitions stated in A. Rearranging that balance proves the asserted form, so R explains A rather than merely making an unrelated true statement. For example, in adiabatic compression, $dQ = 0$ and positive work on the gas gives $dU = dW > 0$, consistent with the asserted equation. Since both statements are true and the reason explains the assertion, options (A), (C), and (D) are eliminated.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct

option is (B).

Quick Tip to Solve: Before applying the first law, underline whether work is done on or by the system; writing the energy-input balance first prevents sign errors.

36. The magnitude of magnetic induction at mid point O due to current arrangement as shown in Fig will be [4 marks]



(A) $\frac{\mu_0 I}{\pi a}$
(B) $\frac{\mu_0 I}{4\pi a}$
(C) $\frac{\mu_0 I}{2\pi a}$
(D) 0

Correct Answer: (A) $\frac{\mu_0 I}{\pi a}$

Solution:

Concept:

- Biot–Savart law gives zero field when the current element and the vector from it to the observation point are parallel. Thus a wire contributes no field on its own straight-line extension.
- At perpendicular distance r from the end of a semi-infinite straight wire, the magnetic field is $B = \mu_0 I/(4\pi r)$. Directions must be found using the right-hand rule before adding the fields.

Step 1: Identify the contributing portions The point O lies on the line through the horizontal portions AB and DE. For every element of either horizontal portion, $\mathrm{d}\boldsymbol{\ell}$ is parallel or antiparallel to the displacement toward O. Their cross products vanish, so these portions contribute zero. The two vertical portions start at the level of O and extend away from it, making each a semi-infinite wire.

Step 2: Find the distances and directions The separation of the two vertical wires is a, and O is their midpoint. Each perpendicular distance is therefore $a/2$, not a. The current in the left wire is downward and the current in the right wire is upward. At the point between them both fields point out of the page, so they reinforce rather than cancel.

Step 3: Add the two fields For each vertical portion,

$$B_1 = B_2 = \frac{\mu_0 I}{4\pi(a/2)} = \frac{\mu_0 I}{2\pi a}.$$

Hence $B = B_1 + B_2 = \mu_0 I/(\pi a)$. Option C counts just one vertical portion; option B misses both the midpoint distance and the second contribution. Zero would require cancellation, which the current directions do not produce.

Step 4: Conclusion The magnetic induction has magnitude $\boxed{\mu_0 I/(\pi a)}$. Hence, the correct option is (A).

Quick Tip to Solve: First discard collinear wire portions, then use the actual perpendicular distance to each remaining portion and check directions before adding.

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37. A stone is projected at angle 30° to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be - [4 marks]

(A) 1 : 4

(B) 1 : 2

(C) 4 : 3

(D) 4 : 1

Correct Answer: (C) 4 : 3

Solution:

Concept:

- For projectile motion without air resistance, gravity changes only the vertical component of velocity. The horizontal component remains $u\cos\theta$ throughout the flight.
- Kinetic energy is $K = \frac{1}{2}mv^2$. At the highest point the vertical component is zero, but the horizontal component is still nonzero.

Step 1: Resolve the launch velocity Let the speed at projection be u and the mass be m. At $\theta = 30^\circ$, the velocity components are $u_x = u\cos 30^\circ = \sqrt{3}u/2$ and $u_y = u\sin 30^\circ = u/2$. Their squared sum is u^2, so the initial kinetic energy is $K_0 = \frac{1}{2}mu^2$. There is no need to know either the mass or the launch speed individually.

Step 2: Evaluate the kinetic energy at the top At maximum height, the upward velocity has decreased to zero under gravity. This does not mean the stone is at rest: it continues moving horizontally. Thus

$$v_{\text{top}} = u\cos 30^\circ, \qquad K_{\text{top}} = \frac{1}{2}mu^2\cos^2 30^\circ = \frac{3}{4}K_0.$$

The lost quarter of the initial kinetic energy has become gravitational potential energy.

Step 3: Form the requested ratio The question asks for initial energy divided by top energy, in that order. Therefore

$$\frac{K_0}{K_{\text{top}}} = \frac{1}{\cos^2 30^\circ} = \frac{4}{3}.$$

The ratios in A and B would make the initial energy smaller than that at the top, contrary to the energy conversion. A ratio of 4:1 would use the initial vertical fraction $\sin^2 30^\circ$ as the remaining energy, whereas the remaining fraction is horizontal.

Step 4: Conclusion The required ratio is $\boxed{4:3}$. Hence, the correct option is (C).

Quick Tip to Solve: At the top of a projectile path keep the horizontal kinetic energy: the remaining fraction is $\cos^2\theta$, not $\sin^2\theta$.

38. A block of mass m slides down the plane inclined at angle 30° with an acceleration $\frac{g}{4}$. The value of coefficient of kinetic friction will be: [4 marks]

(A) $\frac{2\sqrt{3}-1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2\sqrt{3}}$

(D) $\frac{2\sqrt{3}+1}{2}$

Correct Answer: (C) $\frac{1}{2\sqrt{3}}$

Solution:

Concept:

- Resolve the weight into $mg\sin\theta$ down the plane and $mg\cos\theta$ normal to it. With no normal acceleration, the normal reaction is $N = mg\cos\theta$.
- Kinetic friction has magnitude $f_k = \mu_k N$ and opposes the sliding motion. Newton's second law along the plane gives $mg\sin\theta - f_k = ma$ for downward motion.

Step 1: Set up the force balance Choose down the incline as positive because that is the direction of sliding and of the stated acceleration. Gravity contributes $mg\sin 30^\circ$ in this direction, while kinetic friction acts up the plane. The normal reaction does not enter the tangential equation directly. It determines friction through $f_k = \mu_k mg\cos 30^\circ$. Consequently,

$$mg\sin 30^\circ - \mu_k mg\cos 30^\circ = m\frac{g}{4}.$$

Step 2: Solve for the coefficient Cancel the common nonzero factor mg, since the acceleration and the coefficient do not depend on the block's mass. Substituting the exact trigonometric values gives

$$\frac{1}{2} - \mu_k\frac{\sqrt{3}}{2} = \frac{1}{4}.$$

Thus $\mu_k\sqrt{3}/2 = 1/4$, and

$$\mu_k = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}.$$

Step 3: Check the result against the motion The frictional deceleration is now $\mu_k g\cos 30^\circ = g/4$. Subtracting this from the gravity component $g/2$ leaves exactly $g/4$, as given. The coefficient is dimensionless and smaller than $\tan 30^\circ$, which is required for a positive downward acceleration. The

other three options exceed that threshold and would not give the specified downward acceleration.

Step 4: Conclusion The coefficient of kinetic friction is $\boxed{1/(2\sqrt{3})}$. Hence, the correct option is (C).

Quick Tip to Solve: For a block sliding down an incline use $\mu_k = (g\sin\theta - a)/(g\cos\theta)$, with friction opposing the motion.

39. Match List I with List II :

	List I (Physical Quantity)		List II (Dimensional Formula)
A.	**Pressure gradient**	**I.**	$[\mathrm{M^{\circ}L^{2}\ T^{-2}}]$
B.	**Energy density**	**II.**	$[\mathrm{M^{1}L^{-1}\ T^{-2}}]$
C.	**Electric Field**	**III.**	$[\mathrm{M^{1}L^{-2}\ T^{-2}}]$
D.	**Latent heat**	**IV.**	$[\mathrm{M^{1}\ L^{1}\ T^{-3}\ A^{-1}}]$

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-II, C-IV, D-I

(B) A-III, B-II, C-I, D-IV

(C) A-II, B-III, C-IV, D-I

(D) A-II, B-III, C-I, D-IV

Correct Answer: (A) A-III, B-II, C-IV, D-I

Solution:

Concept:

- Dimensions follow from defining equations. Pressure is force per area; a pressure gradient is pressure change per unit distance, so an extra inverse length is essential.
- Energy density is energy per volume, electric field is force per charge, and specific latent heat is energy per mass. Electric charge has dimensions $[\mathrm{AT}]$.

Step 1: Match pressure gradient and energy density Force has dimensions $[\mathrm{MLT^{-2}}]$. Hence pressure has $[\mathrm{ML^{-1}T^{-2}}]$, and its spatial gradient has

$$[\nabla p] = [\mathrm{ML^{-2}T^{-2}}],$$

which is III. Energy density is

$$[u] = \frac{[\mathrm{ML^{2}T^{-2}}]}{[\mathrm{L^{3}}]} = [\mathrm{ML^{-1}T^{-2}}],$$

which is II. Thus A-III and B-II are fixed before considering the remaining quantities.

Step 2: Match electric field and latent heat From $E = F/q$, with $[q] = [\mathrm{AT}]$,

$$[E] = [\mathrm{MLT^{-3}A^{-1}}],$$

which is IV. Here latent heat denotes the material's specific latent heat, defined by $Q = mL$. Therefore

$$[L] = [Q]/[m] = [\mathrm{M^{0}L^{2}T^{-2}}],$$

which is I. This also has the dimensions of speed squared, since the mass factor cancels.

Step 3: Assemble and check the matching The complete matching is A-III, B-II, C-IV, D-I. Option B incorrectly assigns electric field a formula without current dimension. Options C and D interchange pressure gradient and energy density, missing the extra spatial derivative in the former. Option D also exchanges electric field and specific latent heat. Only the first option respects all four defining relations.

Step 4: Conclusion The matching is A-III, B-II, C-IV, D-I. Hence, the correct option is (A).

Quick Tip to Solve: Write each quantity as a ratio of familiar quantities; distinguish pressure from its gradient by one extra inverse length.

40. A car is moving on a horizontal curved road with radius 50 m. The approximate maximum speed of car will be, if friction between tyres and road is 0.34. [take g = 10 ms^{-2}] [4 marks]

(A) 3.4 ms^{-1}

(B) 13 ms^{-1}

(C) 22.4 ms^{-1}

(D) 17 ms^{-1}

Correct Answer: (B) 13 ms^{-1}

Solution:

Concept:

- On an unbanked horizontal road, the normal reaction balances weight, $N = mg$. Static friction between tyres and road supplies the inward centripetal force.
- The maximum available static friction is μN. At the limiting speed, $mv_{\max}^2/r = \mu mg$, so $v_{\max} = \sqrt{\mu r g}$.

Step 1: Identify the force that turns the car The weight and normal force act vertically and cancel because there is no vertical acceleration. The horizontal force required to change the direction of velocity must therefore be supplied by the tyre-road friction. Below the limiting speed, static friction adjusts to the needed value; it is not necessarily equal to μN. At the maximum speed it reaches this limiting value.

Step 2: Apply the limiting condition With radius $r = 50\,\text{m}$, coefficient $\mu = 0.34$, and the given $g = 10\,\text{m s}^{-2}$,

$$\frac{mv_{\max}^2}{50} = 0.34\,m \times 10.$$

The mass cancels. Therefore

$$v_{\max}^2 = 0.34 \times 50 \times 10 = 170\,\text{m}^2\,\text{s}^{-2}, \qquad v_{\max} = \sqrt{170} \simeq 13.04\,\text{m s}^{-1}.$$

Step 3: Compare the possible speeds The approximate value is $13\,\text{m s}^{-1}$. The number 3.4 is μg, an acceleration rather than a speed. The values 17 and $22.4\,\text{m s}^{-1}$ would demand centripetal accelerations 5.78 and about $10.04\,\text{m s}^{-2}$, both exceeding the available $3.4\,\text{m s}^{-2}$. Thus they would cause outward

slipping on this road.

Step 4: Conclusion The maximum speed is approximately $\boxed{13\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (B).

Quick Tip to Solve: On a flat curve, set the largest friction equal to the centripetal force and take the square root only after evaluating μrg.

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41. Which of the following are true?

A. Speed of light in vacuum is dependent on the direction of propagation.

B. Speed of light in a medium is independent of the wavelength of light.

C. The speed of light is independent of the motion of the source.

D. The speed of light in a medium is independent of intensity.

Choose the correct answer from the options given below: [4 marks]

(A) B and D only

(B) B and C only

(C) A and C only

(D) C and D only

Correct Answer: (D) C and D only

Solution:

Concept:

- In vacuum, light has the same speed c in every direction and that speed does not depend on the motion of its source. This is a property of electromagnetic propagation in vacuum.
- In an ordinary linear medium, the speed is $v = c/n$. The refractive index may depend on wavelength, producing dispersion, while it is independent of intensity within the usual linear-optics model.

Step 1: Evaluate the direction and wavelength claims Statement A is false: vacuum is isotropic, so changing the direction of propagation does not change c. Statement B is also false as a general statement about a medium. In a dispersive medium the refractive index is $n(\lambda)$, and hence $v(\lambda) = c/n(\lambda)$. Different colours can therefore travel with different speeds, which is why a prism separates white light into its component colours.

Step 2: Evaluate source motion and intensity Statement C is true. The speed of light is not obtained by adding the source velocity to c; moving the source can change the observed frequency through the Doppler effect without changing the vacuum propagation speed. Statement D is true in the standard treatment of transparent linear media used here. Increasing intensity changes the wave amplitude and energy flow, not the refractive index or propagation speed. Nonlinear high-intensity effects lie outside this ordinary propagation model.

Step 3: Select the set containing only true statements The true pair is C and D. Option A includes the false wavelength-independence claim B, and option B makes the same error. Option C includes the false direction-dependence claim A. Option D alone includes both established true statements and excludes the two false statements. Distinguishing vacuum invariance from dispersion in a material is

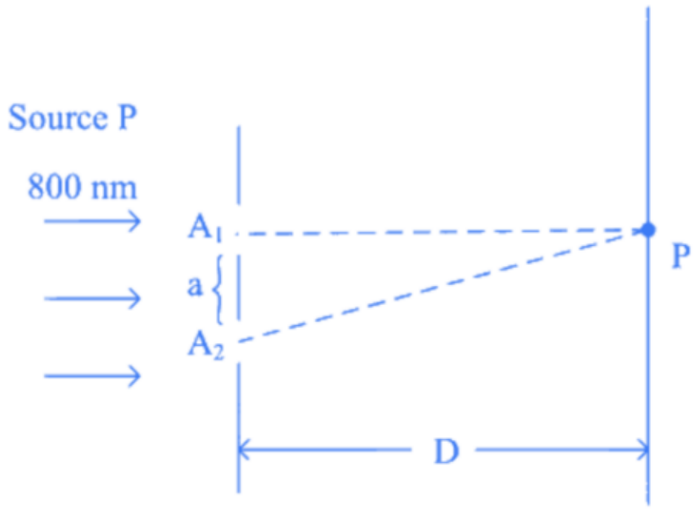
the deciding point.

Step 4: Conclusion Only statements C and D are true. Hence, the correct option is (D).

Quick Tip to Solve: Keep vacuum invariance separate from material dispersion: wavelength can alter the refractive index, while ordinary intensity changes only the amplitude.

42. In a Young's double slit experiment, two slits are illuminated with a light of wavelength 800 nm. The line joining A_1P is perpendicular to A_1A_2 as shown in the figure. If the first minimum is detected at P, the value of slits separation 'a' will be:

The distance of screen from slits D = 5 cm [4 marks]



(A) 0.2 mm

(B) 0.5 mm

(C) 0.4 mm

(D) 0.1 mm

Correct Answer: (A) 0.2 mm

Solution:

Concept:

- For two coherent slits illuminated in phase, a first intensity minimum occurs when the optical path difference is $\lambda/2$. The geometry, rather than the usual central-screen formula alone, determines that path difference.
- Here A_1P is perpendicular to the slit separation. Thus $A_1P = D$ and $A_2P = \sqrt{D^2 + a^2}$, where a is the slit separation.

Step 1: Write the exact path difference Both slits receive the same incident phase from the normally incident illumination shown. The observation point is directly opposite the upper slit, not opposite the midpoint of the two slits. Hence the two outgoing path lengths are unequal even though the upper

path is horizontal. Their difference is

$$\Delta = \sqrt{D^2 + a^2} - D.$$

For the first destructive fringe, set this equal to $\lambda/2$.

Step 2: Solve for slit separation The equation becomes $\sqrt{D^2 + a^2} = D + \lambda/2$. Squaring and subtracting D^2,

$$a^2 = D\lambda + \frac{\lambda^2}{4}.$$

Insert $D = 5\,\text{cm} = 0.05\,\text{m}$ and $\lambda = 800\,\text{nm} = 8.00 \times 10^{-7}\,\text{m}$. This gives $D\lambda = 4.00 \times 10^{-8}\,\text{m}^2$, while $\lambda^2/4 = 1.60 \times 10^{-13}\,\text{m}^2$, negligible at the precision of the options.

Step 3: Evaluate and check the options Therefore

$$a \simeq \sqrt{4.00 \times 10^{-8}} = 2.00 \times 10^{-4}\,\text{m} = 0.200\,\text{mm}.$$

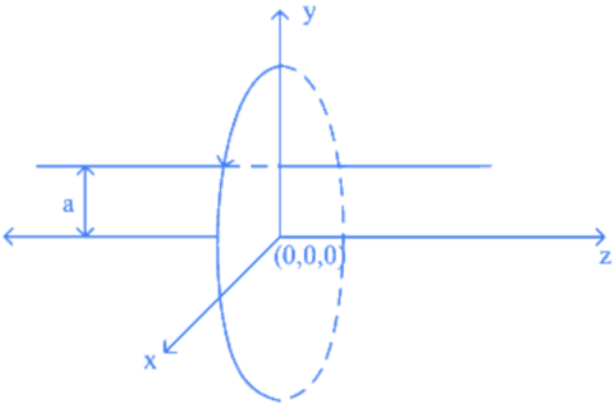
As a check, $a \ll D$, so using $\Delta \simeq a^2/(2D)$ reproduces $\lambda/2$. The other proposed separations give different path differences and do not satisfy this first-minimum condition. The exact tiny correction above does not affect the selected approximate value.

Step 4: Conclusion The slit separation is $\boxed{0.2\,\text{mm}}$. Hence, the correct option is (A).

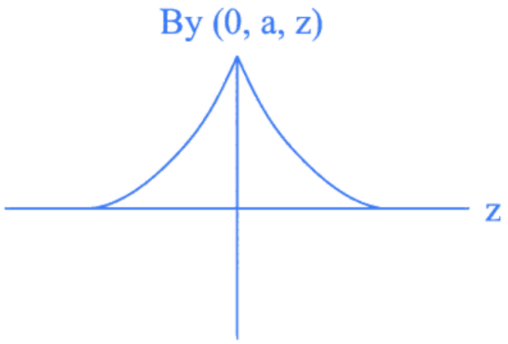
Quick Tip to Solve: When the observation point is level with one slit, use $\sqrt{D^2 + a^2} - D$; the first-minimum approximation gives $a = \sqrt{D\lambda}$.

43. A single current carrying loop of wire carrying current I flowing in anticlockwise direction seen from +ve z direction and lying in xy plane is shown in figure. The plot of $\hat{j}$ component of magnetic field (By) at a distance 'a' (less than radius of the coil) and on yz plane vs z coordinate looks like

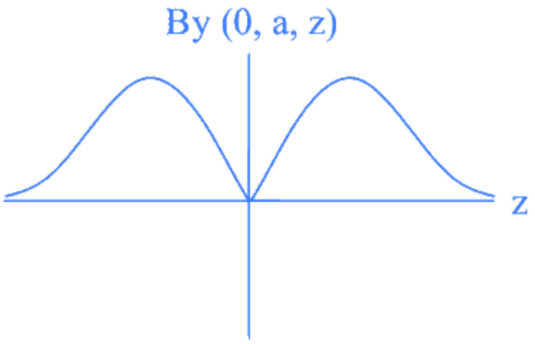
[4 marks]



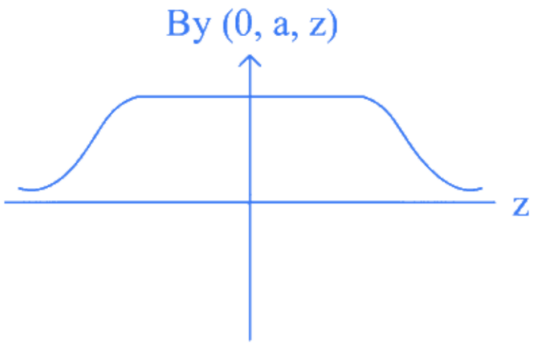
(A)



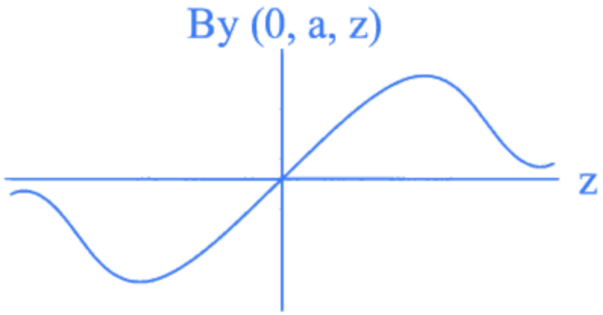
(B)



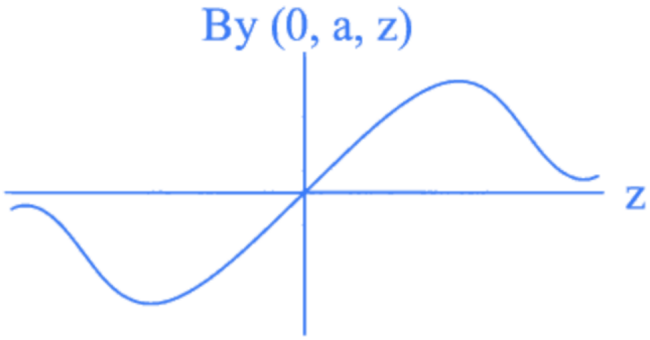
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- The right-hand rule gives a magnetic moment along positive z for the stated anticlockwise current.
- At the point $(0, a, z)$, reflection in the plane of the loop reverses the radial component B_y, while leaving the axial component unchanged.

Step 1: Use symmetry at the loop plane At $z = 0$, both a current element and the displacement from it to the observation point lie in the xy plane. Their cross product is perpendicular to that plane. Thus every contribution has zero y component, and $B_y(0, a, 0) = 0$. The point lies inside the loop, since $a < R$, so there is no wire singularity there. Any graph with a nonzero value at the origin is excluded.

Step 2: Determine the sign above and below the loop Parameterise the loop by $(R\cos\phi, R\sin\phi, 0)$. Biot–Savart gives the radial component in the form

$$B_y = \frac{\mu_0 I R z}{4\pi} \int_0^{2\pi} \frac{\sin\phi \, d\phi}{(R^2 + a^2 + z^2 - 2aR\sin\phi)^{3/2}}.$$

Pair elements with opposite values of $\sin\phi$. The positive-sine element is closer to $y = a > 0$, so its contribution outweighs the negative-sine one. The integral is positive. Hence B_y is positive for positive z and negative for negative z.

Step 3: Check the remote field and select the graph The denominator depends on z^2, so $B_y(-z) = -B_y(z)$. Far from the finite loop the magnetic field tends to zero. The curve must therefore have a negative lobe on the left, cross zero at the origin, and have a positive lobe on the right before approaching zero. Options A, B and C are nonnegative on both sides and violate this odd symmetry.

Step 4: Conclusion The required curve is the odd curve with $B_y < 0$ for $z < 0$ and $B_y > 0$ for $z > 0$. Hence, the correct option is (D).

Quick Tip to Solve: For an off-axis point above a current loop, separate the radial and axial field components: the radial component is odd in the height, whereas the axial component is even.

44. Which one of the following statement is not correct in the case of light emitting diodes?

A. It is a heavily doped p-n junction.

B. It emits light only when it is forward biased.

C. It emits light only when it is reverse biased.

D. The energy of the light emitted is equal to or slightly less than the energy gap of the semiconductor used.

Choose the correct answer from the options given below: [4 marks]

(A) A

(B) B

(C) C and D

(D) C

Correct Answer: (D) C

Solution:

Concept:

- An LED is a heavily doped semiconductor p-n junction designed for radiative electron–hole recombination.
- Forward bias injects majority carriers across the junction; recombination releases photons with energies approximately equal to the semiconductor band gap.

Step 1: Check doping and the operating bias Statement A is correct: heavy doping permits substantial carrier injection in the active junction region. Statement B is also correct for normal LED operation. A forward bias lowers the junction barrier, allowing electrons and holes to reach the active region and recombine. The emitted radiation is therefore associated with forward current, rather than normal reverse leakage current.

Step 2: Test the reverse-bias and photon-energy statements Statement C says that light is emitted only under reverse bias. This reverses the operating condition of an LED and is incorrect. Under ordinary reverse bias the carrier injection needed for useful electroluminescence is absent; excessive reverse voltage may instead damage the device. Statement D is correct: in radiative recombination a conduction-band electron falls to a lower-energy state associated with the valence band, and $h\nu$ is equal to or slightly less than the band-gap energy E_g.

Step 3: Match the incorrect statement to the option The question asks for the statement that is not correct, rather than all correct statements. Only C fails. The answer option labelled D contains the statement C alone. Option A wrongly rejects heavy doping, option B wrongly rejects forward-bias emission, and option C wrongly includes the valid energy statement D along with C.

Step 4: Conclusion Only statement C is incorrect: an LED emits light in forward bias. Hence, the correct option is (D).

Quick Tip to Solve: Keep statement letters separate from answer-option letters: here statement C is selected by option D. For LED questions, first identify forward injection and radiative recombination.

45. A bicycle tyre is filled with air having pressure of $270\ \mathrm{kPa}$ at $27^\circ\mathrm{C}$. The approximate pressure of the air in the tyre when the temperature increases to $36^\circ\mathrm{C}$ is [4 marks]

(A) 262 kPa

(B) 360 kPa

(C) 270 kPa

(D) 278 kPa

Correct Answer: (D) 278 kPa

Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nRT$. If the tyre volume is approximately constant, P/T is constant.
- Temperature in a gas-law ratio must be absolute temperature in kelvin, with $T = \theta + 273$ approximately.

Step 1: Identify the quantities kept fixed The tyre is already filled, so the amount of air does not change during heating, provided there is no leak. Its expansion is neglected in this approximate calculation. With both n and V fixed, the ideal-gas equation gives

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}.$$

Heating increases molecular kinetic energy and the momentum transferred to the tyre wall, so the final pressure must be slightly greater than the initial pressure.

Step 2: Convert the temperatures and calculate The initial and final absolute temperatures are

$$T_1 = 27 + 273 = 300\,\mathrm{K}, \qquad T_2 = 36 + 273 = 309\,\mathrm{K}.$$

Therefore

$$P_2 = P_1\frac{T_2}{T_1} = 270\frac{309}{300}\,\mathrm{kPa} = 278.1\,\mathrm{kPa}.$$

The temperature rises by only $9/300 = 0.03$, so pressure rises by about three percent. The increase is $8.1\,\mathrm{kPa}$, which confirms the calculation.

Step 3: Compare the available approximations The nearest given pressure is $278\,\mathrm{kPa}$. A value of $262\,\mathrm{kPa}$ would represent a decrease on heating. The unchanged value $270\,\mathrm{kPa}$ ignores the temperature increase, while $360\,\mathrm{kPa}$ results from using the Celsius ratio $36/27$, which is invalid for the ideal-gas law.

Step 4: Conclusion The approximate final pressure is $278\,\mathrm{kPa}$. Hence, the correct option is (D).

Quick Tip to Solve: At constant volume, use the fractional check $\Delta P/P = \Delta T/T$, with the denominator in kelvin. A small temperature rise should give a comparably small pressure rise.

46. The threshold wavelength for photoelectric emission from a material is 5500 $\overset{o}{A}$. Photoelectrons will be emitted, when this material is illuminated with monochromatic radiation from a

A. 75 W infra-red lamp

B. 10 W infra-red lamp

C. 75 W ultra-violet lamp

D. 10 W ultra-violet lamp

Choose the correct answer from the options given below : [4 marks]

(A) C only

(B) A and D only

(C) C and D only

(D) B and C only

Correct Answer: (C) C and D only

Solution:

Concept:

- Photoelectric emission requires the photon energy to exceed or equal the work function: $h\nu \geq \phi = hc/\lambda_0$.
- Wavelength determines the energy of an individual photon. Lamp power controls the rate of photon delivery and cannot compensate for an insufficient photon energy.

Step 1: Express the threshold as a wavelength condition The threshold wavelength is $\lambda_0 = 5500\,\text{Å} = 550\,\text{nm}$. Since photon energy is $E = hc/\lambda$, emission is possible for $\lambda \leq 550\,\text{nm}$. Equivalently, the threshold photon energy is about $1240/550 = 2.25\,\text{eV}$. A photon must supply this minimum energy to release one electron from the material.

Step 2: Evaluate the two spectral regions Infrared radiation has wavelengths longer than visible red light, hence longer than the given $550\,\text{nm}$ threshold. Its photons are insufficiently energetic, so neither the 75 W infrared lamp A nor the 10 W infrared lamp B causes emission in the usual single-photon photoelectric effect. Ultraviolet radiation has shorter wavelengths than visible light and therefore shorter than $550\,\text{nm}$. Its photons exceed the threshold, so both ultraviolet sources C and D produce photoelectrons.

Step 3: Check the role of lamp power and the options The 10 W ultraviolet lamp can produce emission even though its power is less than that of the 75 W infrared lamp. Higher sub-threshold intensity still does not meet the energy requirement per photon. Option A wrongly excludes D; options B and D include an infrared source. Only the option containing C and D, without A or B, agrees with the threshold test.

Step 4: Conclusion Both the 75 W and 10 W ultraviolet lamps produce photoelectric emission, so C and D only are selected. Hence, the correct option is (C).

Quick Tip to Solve: For photoelectric emission, compare wavelength with the threshold before looking at watts. Power changes how many suitable photons arrive, while wavelength decides whether each photon can eject an electron.

47. Ratio of thermal energy released in two resistors R and 3R connected in parallel in an electric circuit is : [4 marks]

(A) 1 : 1

(B) 1 : 27

(C) 1 : 3

(D) 3 : 1

Correct Answer: (D) 3 : 1

Solution:

Concept:

- Joule heat generated in a resistor is $H = VIt = I^2Rt = V^2t/R$, when the electrical energy is converted to heat.
- Parallel branches have the same potential difference. For a common observation time, their heating energies are inversely proportional to their resistances.

Step 1: Choose the expression suited to parallel branches Both resistors are connected between the same two circuit nodes, so the voltage across R is the same as that across $3R$. Let this common voltage be V, and consider the thermal energy released by each over the same time t. The useful form of Joule's law is

$$H = \frac{V^2 t}{R}.$$

Using I^2Rt without first accounting for the different branch currents would incorrectly treat the circuit as a series connection.

Step 2: Compute each energy and form the ratio The energies released are

$$H_R = \frac{V^2 t}{R}, \qquad H_{3R} = \frac{V^2 t}{3R}.$$

Dividing in the order asked gives

$$\frac{H_R}{H_{3R}} = \frac{V^2 t/R}{V^2 t/(3R)} = 3.$$

The resistor R draws three times the current of the resistor $3R$, because $I = V/R$. Since both currents flow through the same voltage, its power and heat in a fixed time are also three times as large.

Step 3: Reject the other ratios The ratio $1 : 1$ ignores the unequal resistances. The ratio $1 : 3$ would follow for equal currents, as in a series circuit, whereas the branches here have equal voltage. The ratio $1 : 27$ introduces an unsupported cubic dependence on resistance; Joule heating at fixed voltage has an inverse first-power dependence.

Step 4: Conclusion The thermal energies released in R and $3R$, respectively, are in the ratio $3 : 1$. Hence, the correct option is (D).

Quick Tip to Solve: Choose Joule's-law variables to match the connection: use V^2t/R for parallel resistors and I^2Rt for series resistors, over the same interval.

48. If a radioactive element having half-life of $30\ \text{min}$ is undergoing beta decay, the fraction of radioactive element remains undecayed after $90\ \text{min}$. will be [4 marks]

(A) $\frac{1}{16}$
(B) $\frac{1}{4}$
(C) $\frac{1}{8}$
(D) $\frac{1}{2}$

Correct Answer: (C) $\frac{1}{8}$

Solution:

Concept:

- The number of undecayed parent nuclei follows $N = N_0 e^{-\lambda t}$, where λ is the decay constant.
- With half-life $T_{1/2}$, the remaining fraction is $N/N_0 = (1/2)^{t/T_{1/2}}$. This relation applies to beta decay as well as other radioactive decay modes.

Step 1: Count the elapsed half-lives The given half-life is $30\,\text{min}$, and the elapsed interval is $90\,\text{min}$. The number of half-lives is therefore

$$n = \frac{t}{T_{1/2}} = \frac{90}{30} = 3.$$

Both times use minutes, so no conversion to seconds is needed in this ratio. The half-life refers to the parent radioactive species; daughter nuclei formed by beta decay are not counted as undecayed parent nuclei.

Step 2: Apply successive halving After the first 30 minutes the remaining number is $N_0/2$. After 60 minutes it is half of that, namely $N_0/4$. After 90 minutes it is $N_0/8$. In formula form,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Thus 12.5 percent of the original parent nuclei remain undecayed, while 87.5 percent have decayed. The question asks for the remaining fraction, so the latter percentage must not be used as the answer.

Step 3: Check the option values against time The value $1/2$ corresponds to one half-life, or 30 minutes. The value $1/4$ corresponds to two half-lives, or 60 minutes. The value $1/16$ would require four half-lives, or 120 minutes. Only $1/8$ corresponds to the stated 90 minutes, regardless of the fact that the specified decay mode is beta decay.

Step 4: Conclusion The fraction remaining undecayed after $90\,\text{min}$ is $1/8$. Hence, the correct option is (C).

Quick Tip to Solve: First divide elapsed time by half-life, then raise one-half to that power. Keep the fraction remaining separate from the fraction that has already decayed.

49. A person observes two moving trains, 'A' reaching the station and 'B' leaving the station with equal speed of $30\ \mathrm{m/s}$. If both trains emit sounds with frequency $300\ \mathrm{Hz}$, (Speed of sound: $330\ \mathrm{m/s}$) approximate difference of frequencies heard by the person will be: [4 marks]

(A) 10 Hz

(B) 55 Hz

(C) 80 Hz

(D) 33 Hz

Correct Answer: (B) 55 Hz

Solution:

Concept:

- For a stationary observer and a source approaching with speed u, the observed frequency is $f_{\text{app}} = fv/(v-u)$.
- For a receding source, $f_{\text{rec}} = fv/(v+u)$. Here v is sound speed in the medium, and source motion changes the wavelength reaching the observer.

Step 1: Calculate the approaching-train frequency Train A approaches the station, where the observer is taken to be stationary. Its source speed is $u = 30\,\mathrm{m\,s^{-1}}$, emitted frequency is $f = 300\,\mathrm{Hz}$, and sound speed is $v = 330\,\mathrm{m\,s^{-1}}$. The wavefront spacing ahead of the source is reduced, so

$$f_A = 300\frac{330}{330-30} = 330\,\mathrm{Hz}.$$

The denominator must decrease for approach, giving a frequency above the emitted value.

Step 2: Calculate the receding-train frequency Train B moves away at the same speed. The wavefront spacing reaching the station increases, giving

$$f_B = 300\frac{330}{330+30} = 275\,\mathrm{Hz}.$$

The frequency heard is below 300 Hz, as expected for recession. The observer does not move with either train, so the observer-motion formula, in which the speed correction appears in the numerator, does not apply here.

Step 3: Subtract the observed frequencies The requested difference is

$$\Delta f = f_A - f_B = 330 - 275 = 55\,\mathrm{Hz}.$$

One can check directly that

$$\Delta f = \frac{2fvu}{v^2-u^2} = \frac{2(300)(330)(30)}{330^2-30^2} = 55\,\mathrm{Hz}.$$

Neither the source speed nor the upward shift of one source alone is the required frequency difference. The other offered values 10 Hz, 80 Hz and 33 Hz do not equal the separation of these two received frequencies.

Step 4: Conclusion The approximate difference in the two frequencies heard is $55\,\mathrm{Hz}$. Hence, the

correct option is (B).

Quick Tip to Solve: For equally fast approaching and receding sources, calculate both received frequencies before subtracting. Their shifts are not exactly equal because the two denominators differ.

50. If the height of transmitting and receiving antennas are 80 m each, the maximum line of sight distance will be:

Given : Earth's radius $= 6.4 \times 10^6$ **m** [4 marks]

(A) 28 km

(B) 64 km

(C) 32 km

(D) 36 km

Correct Answer: (B) 64 km

Solution:

Concept:

- The line of sight of an antenna ends at the tangent to the spherical Earth. For an antenna of height $h \ll R$, its horizon distance is $d \simeq \sqrt{2Rh}$, where R is Earth's radius.
- Two elevated antennas can communicate directly when the transmitter and receiver horizon distances add: $d_{\max} = \sqrt{2Rh_t} + \sqrt{2Rh_r}$. Heights and radius must first be in the same unit.

Step 1: Find the horizon distance of one antenna The heights are equal, so both antennas have the same horizon distance. Using the right triangle formed by the Earth's centre, the antenna top and the tangent point gives $d^2 = (R+h)^2 - R^2 = 2Rh + h^2$. Here h/R is tiny, so the h^2 term is negligible. Thus

$$d = \sqrt{2(6.4 \times 10^6)(80)} = \sqrt{1.024 \times 10^9} = 32000\,\text{m} = 32\,\text{km}.$$

Step 2: Add the two horizon distances The most distant direct path has its tangent point between the two antennas. Consequently each antenna contributes its own horizon distance, rather than only the transmitting antenna contributing.

$$d_{\max} = 32 + 32 = 64\,\text{km}.$$

The value 32 km in option (C) counts only one antenna. The values 28 km and 36 km do not follow from the specified radius and heights; they are not alternative unit conversions of the result.

Step 3: Conclusion The maximum line of sight distance is $\boxed{64\,\text{km}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal antenna heights, calculate one horizon distance and double it; do not put the sum of the heights inside a single square root.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Two simple harmonic waves having equal amplitudes of 8 cm and equal frequency of 10 Hz are moving along the same direction. The resultant amplitude is also 8 cm. The phase difference between the individual waves is _____ degree. [4 marks]

Solution:

Concept:

- For two coherent waves of amplitudes a_1, a_2 and phase difference ϕ, the resultant amplitude obeys $A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$. This follows by adding their displacement phasors.
- Equal frequency allows a constant phase difference. For equal amplitudes a, the formula becomes $A^2 = 2a^2(1+\cos\phi)$; the usual phase difference is chosen between 0° and 180°.

Step 1: Substitute the amplitudes Both individual amplitudes are $a = 8\,\text{cm}$, and their resultant is also $A = 8\,\text{cm}$. All three amplitudes use the same unit, so the centimetre factor cancels in their ratio.

$$8^2 = 8^2 + 8^2 + 2(8)(8)\cos\phi.$$

Dividing by 64 gives $1 = 2 + 2\cos\phi$. The frequency 10 Hz is not needed numerically: it establishes the common oscillation frequency, rather than changing this amplitude relation.

Step 2: Determine the relative phase Rearranging the phasor relation gives

$$\cos\phi = -\frac{1}{2}, \qquad \phi = 120^\circ.$$

This value lies between complete constructive interference at 0°, which would give 16 cm, and complete destructive interference at 180°, which would give zero. Equivalently, $A = 2a|\cos(\phi/2)|$ gives $|\cos(\phi/2)| = 1/2$. The principal phase separation is therefore 120°; reversing which wave leads changes the sign, not the physical separation.

Step 3: Conclusion The phase difference between the two waves is $\boxed{120^\circ}$. The numerical entry in degrees is 120.

Quick Tip to Solve: When two equal-amplitude waves produce that same amplitude, set the resultant-to-individual amplitude ratio to one; their principal phase difference is 120 degrees.

52. A solid sphere of mass 2 kg is making pure rolling on a horizontal surface with kinetic energy 2240 J. The velocity of centre of mass of the sphere will be _____ ms^{-1}. [4 marks]

Solution:

Concept:

- Pure rolling combines translation of the centre of mass and rotation about that centre. The total kinetic energy is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

- For a uniform solid sphere, $I = \frac{2}{5}mr^2$. The no-slip rolling condition is $v = r\omega$, so the unknown radius cancels from the total energy.

Step 1: Express the total energy in terms of speed Substitute the moment of inertia and the rolling relation before inserting numbers:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2$$
$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2.$$

The given 2240 J is the kinetic energy of the rolling sphere, so it includes both contributions. Using just translational energy would omit the rotational motion and overestimate the centre-of-mass speed.

Step 2: Solve for the centre-of-mass speed With $m = 2\,\text{kg}$, the energy equation becomes

$$2240 = \frac{7}{10}(2)v^2 = 1.4v^2.$$

Hence

$$v^2 = \frac{2240}{1.4} = 1600\,\text{m}^2\,\text{s}^{-2}, \qquad v = 40\,\text{m}\,\text{s}^{-1}.$$

The positive root is used because the question asks for the magnitude of velocity. As a check, the translational part is $1600\,\text{J}$ and the rotational part is $640\,\text{J}$, whose sum is precisely 2240 J. No work or energy loss needs to be introduced for the specified pure rolling state.

Step 3: Conclusion The velocity magnitude of the centre of mass is $\boxed{40\,\text{m}\,\text{s}^{-1}}$. The numerical entry is 40.

Quick Tip to Solve: For a solid sphere rolling without slipping, use the total energy factor 7/10 multiplying mv squared; radius is unnecessary.

53. In a metre bridge experiment the balance point is obtained if the gaps are closed by 2Ω and 3Ω. A shunt of X Ω is added to 3Ω resistor to shift the balancing point by 22.5 cm. The value of X is _____.

[4 marks]

Solution:

Concept:

- At balance in a uniform metre bridge, the ratio of the two gap resistances equals the ratio of wire lengths: $R_L/R_R = l/(100 - l)$, with lengths in centimetres.
- A shunt is connected in parallel. A resistor X shunting $3\,\Omega$ gives an equivalent resistance $R = 3X/(3 + X)$, smaller than $3\,\Omega$.

Step 1: Find the initial and final balance lengths Take the $2\,\Omega$ resistance in the left gap and $3\,\Omega$ in the right gap; choosing the opposite orientation gives the same shunt. Initially,

$$\frac{2}{3} = \frac{l}{100 - l} \quad \Rightarrow \quad l = 40\,\text{cm}.$$

Shunting the right gap reduces its resistance, increasing the ratio $2/R$. Therefore the left balance length increases, rather than decreases. The new balance length is $l\prime = 40 + 22.5 = 62.5\,\text{cm}$, and the other wire segment is $37.5\,\text{cm}$.

Step 2: Determine the parallel resistance and shunt Apply the balance equation again:

$$\frac{2}{R} = \frac{62.5}{37.5} = \frac{5}{3} \quad \Rightarrow \quad R = \frac{6}{5}\,\Omega.$$

Now equate this with the parallel combination:

$$\frac{3X}{3+X} = \frac{6}{5} \quad \Rightarrow \quad 15X = 18 + 6X \quad \Rightarrow \quad X = 2\,\Omega.$$

The parallel combination of 3 and 2 ohms is indeed 1.2 ohms, consistent with the required reduction. A decreased left balance length would wrongly imply that adding a parallel path increased the right gap resistance.

Step 3: Conclusion The shunt resistance is $\boxed{X = 2\,\Omega}$. The numerical entry is 2.

Quick Tip to Solve: Before using the stated balance-point shift, decide its direction: shunting a gap reduces its resistance and changes the bridge ratio accordingly.

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54. A point charge $q_1 = 4q_0$ is placed at origin. Another point charge $q_2 = -q_0$ is placed at $x = 12$ cm. Charge of proton is q_0. The proton is placed on x axis so that the electrostatic force on the proton is zero. In this situation, the position of the proton from the origin is _____ cm. [4 marks]

Solution:

Concept:

- The electrostatic force on a proton is $F = q_0E$, so it vanishes where the net electric field of the two fixed charges is zero. The proton charge cancels from the balance equation.
- The field of a positive point charge points away from it, whereas that of a negative point charge points towards it. Cancellation requires opposite directions as well as equal magnitudes.

Step 1: Identify the possible interval Between $x = 0$ and $x = 12\,\text{cm}$, both fields point towards positive x: away from $+4q_0$ and towards $-q_0$. They cannot cancel there. For $x < 0$, the stronger charge $+4q_0$ is also nearer than the negative charge. Its field magnitude exceeds the opposing field, so no cancellation is possible on that side either. The only possible location is beyond the smaller charge, $x > 12\,\text{cm}$. In this region the positive charge field points right and the negative charge field points left.

Step 2: Balance the two opposing fields Using distances in centimetres is valid in their ratio, as the common conversion factor cancels. For $x > 12$,

$$\frac{k(4q_0)}{x^2} = \frac{kq_0}{(x-12)^2}.$$

Cancel kq_0 and take positive square roots because both distances are positive:

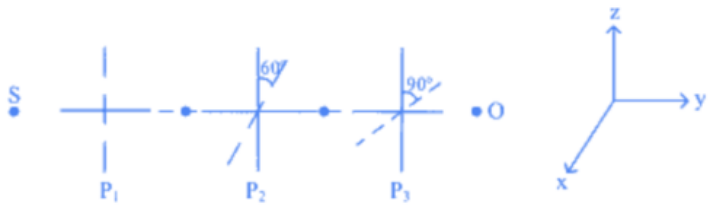
$$\frac{2}{x} = \frac{1}{x-12} \quad \Rightarrow \quad 2(x-12) = x \quad \Rightarrow \quad x = 24\,\text{cm}.$$

At this point the distances are 24 cm and 12 cm. The field of the charge four times as large is evaluated at twice the distance, so their magnitudes are equal by the inverse-square law.

Step 3: Conclusion The proton must be at $\boxed{x = +24\,\text{cm}}$, or 24 cm from the origin on the positive x axis. The numerical entry is 24.

Quick Tip to Solve: For unlike charges, the zero-field point lies outside their interval on the side of the smaller magnitude charge; then balance field magnitudes.

55. As shown in the figure, three identical polaroids P_1, P_2 and P_3 are placed one after another. The pass axis of P_2 and P_3 are inclined at angle of 60° and 90° with respect to axis of P_1. The source S has an intensity of 256 $\frac{W}{m^2}$. The intensity of light at point O is _____ $\frac{W}{m^2}$. [4 marks]



Solution:

Concept:

- An ideal polaroid transmits half the intensity of incident unpolarized light. After the first polaroid the beam is linearly polarized along its transmission axis.
- Malus' law gives $I_{\text{out}} = I_{\text{in}} \cos^2\theta$, where θ is the angle between the incident polarization direction and the next polaroid's pass axis. Each transmitted beam acquires the latest axis direction.

Step 1: Apply the first and second polaroids The incident light from S is unpolarized, so the intensity after P_1 is

$$I_1 = \frac{256}{2} = 128\,\text{W m}^{-2}.$$

The pass axis of P_2 is at 60° to that of P_1. Therefore

$$I_2 = I_1 \cos^2 60^\circ = 128\left(\frac{1}{2}\right)^2 = 32\,\text{W m}^{-2}.$$

The light leaving P_2 is polarized along P_2, so this axis, not the original P_1 axis, determines its transmission through the third polaroid.

Step 2: Use the relative angle at the third polaroid The third pass axis is 90° from P_1, while the

second is 60° from P_1, as shown. Their mutual angle is

$$\theta_{23} = 90^\circ - 60^\circ = 30^\circ.$$

Thus the final intensity at O is

$$I_O = I_2 \cos^2 30^\circ = 32 \left(\frac{\sqrt{3}}{2}\right)^2 = 24\,\mathrm{W\,m^{-2}}.$$

Although P_1 and P_3 are crossed, the intermediate polaroid changes the polarization direction, allowing a nonzero transmitted intensity. Applying a factor of $\cos^2 90^\circ$ directly after P_2 would therefore be incorrect.

Step 3: Conclusion The intensity at O is $\boxed{24\,\mathrm{W\,m^{-2}}}$. The numerical entry is 24.

Quick Tip to Solve: In a chain of polaroids, use angles between consecutive pass axes. Halve intensity only at the first polaroid when the incident light is unpolarized.

56. A 0.4 kg mass takes 8s to reach ground when dropped from a certain height 'P' above surface of earth. The loss of potential energy in the last second of fall is _____ J.
(Take g = 10 m/s^2) [4 marks]

Solution:

Concept:

- A body dropped from rest under constant gravitational acceleration has downward displacement $s(t) = \frac{1}{2}gt^2$. Distances during an interval are found by subtracting cumulative displacements.
- The decrease in gravitational potential energy over a downward distance Δs is $mg\Delta s$. The last second of an eight-second fall is the interval from 7 s to 8 s, not the first second.

Step 1: Find the distance fallen during the last second Use the stated constant acceleration $g = 10\,\mathrm{m\,s^{-2}}$ and zero initial velocity. The distances below the release point at the two ends of the final second are

$$s(8) = \frac{1}{2}(10)(8^2) = 320\,\mathrm{m}, \qquad s(7) = \frac{1}{2}(10)(7^2) = 245\,\mathrm{m}.$$

Hence the downward distance covered during that interval is

$$\Delta s = 320 - 245 = 75\,\mathrm{m}.$$

The height 320 m is the total drop; using it would calculate the energy lost over the whole fall, rather than the requested final second.

Step 2: Calculate the potential-energy loss The magnitude of the potential-energy decrease is

$$U(7) - U(8) = mg\Delta s = (0.4)(10)(75) = 300\,\mathrm{J}.$$

The actual change $U(8) - U(7)$ is negative because the mass moves down, but the question asks for the

loss, a positive quantity. As an independent check, the velocities at 7 s and 8 s are 70 and 80 metres per second. Their kinetic-energy difference is $\frac{1}{2}(0.4)(80^2 - 70^2) = 300\,\text{J}$, equal to this potential-energy decrease when air resistance is neglected.

Step 3: Conclusion The loss of potential energy during the last second is $\boxed{300\,\text{J}}$. The numerical entry is 300.

Quick Tip to Solve: For the final second of a fall lasting T seconds, subtract the distances at T and T minus one; multiplying total height by mg gives the wrong interval.

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57. A certain elastic conducting material is stretched into a circular loop. It is placed with its plane perpendicular to a uniform magnetic field B = 0.8 T. When released the radius of the loop starts shrinking at a constant rate of 2 cms^{-1}. The induced emf in the loop at an instant when the radius of the loop is 10 cm will be _____ mV. [4 marks]

Solution:

Concept:

- Faraday's law gives the magnitude of the induced emf as $\mathcal{E} = |d\Phi/dt|$, where Φ is the magnetic flux through the single conducting loop.
- With the magnetic field perpendicular to the loop, $\Phi = B\pi r^2$. Both the field strength and its orientation are constant, so only the area changes.

Step 1: Convert the radius and contraction rate The instantaneous radius is $r = 10\,\text{cm} = 0.10\,\text{m}$. The radius decreases at $2\,\text{cm}\,\text{s}^{-1}$, so its signed rate is $dr/dt = -0.020\,\text{m}\,\text{s}^{-1}$. The minus sign describes contraction; it will not make the requested emf magnitude negative. Use the radius at the stated instant rather than integrating over the entire contraction.

Step 2: Differentiate the changing magnetic flux The area changes at twice the radius times its rate, multiplied by π:

$$\frac{d\Phi}{dt} = 2\pi B r \frac{dr}{dt}.$$

Consequently,

$$\mathcal{E} = 2\pi(0.8)(0.10)(0.020) = 0.0032\pi\,\text{V}.$$

This derivative accounts for the changing circumference automatically. Taking $B\pi r^2$ itself as an emf would incorrectly use flux instead of its rate of change.

Step 3: Express the induced emf in millivolts Multiply the result in volts by 10^3:

$$\mathcal{E} = 3.2\pi\,\text{mV} \simeq 10.05\,\text{mV}.$$

Thus the numerical entry rounded to the nearest integer is 10. By Lenz's law, the induced current produces a field in the original field direction to oppose the decreasing flux; the emf magnitude is positive.

Step 4: Conclusion The induced emf is $3.2\pi\,\text{mV} \approx 10.05\,\text{mV}$; the required integer entry is $\boxed{10}$.

Quick Tip to Solve: For a shrinking circular loop in a constant normal field, use $2\pi B|dr/dt|$ and convert both centimetre quantities before substituting.

58. A tennis ball is dropped on to the floor from a height of 9.8 m. It rebounds to a height 5.0 m. Ball comes in contact with the floor for 0.2s. The average acceleration during contact is _____ ms^{-2}. (Given g = 10 ms^{-2}) [4 marks]

Solution:

Concept:

- Average acceleration during contact is $a_{\text{av}} = (v_f - v_i)/\Delta t$. Velocity is signed, so the reversal of direction must be included.
- For the fall and rebound, $v^2 = 2gh$ relates the speed immediately adjacent to the collision to the corresponding height. The contact interval itself is treated separately.

Step 1: Determine the velocity just before impact Choose upward as the positive direction. The ball is dropped from rest, so its speed after falling $9.8\,\text{m}$ is

$$|v_i| = \sqrt{2(10)(9.8)} = \sqrt{196} = 14\,\text{m}\,\text{s}^{-1}.$$

It is travelling downward immediately before contact. Its velocity is therefore $v_i = -14\,\text{m}\,\text{s}^{-1}$, not positive 14. The given value $g = 10\,\text{m}\,\text{s}^{-2}$ is used throughout.

Step 2: Determine the velocity just after contact At the highest point of the rebound the speed is zero. Applying the same kinematic relation to the upward journey through $5.0\,\text{m}$ gives

$$v_f = +\sqrt{2(10)(5.0)} = +10\,\text{m}\,\text{s}^{-1}.$$

The plus sign is essential because the ball leaves the floor upward. These two speeds refer to the beginning and end of the contact interval, rather than to the top of either trajectory.

Step 3: Calculate the average acceleration during impact The velocity change is

$$\Delta v = 10 - (-14) = 24\,\text{m}\,\text{s}^{-1}.$$

Divide by the stated contact time:

$$a_{\text{av}} = \frac{24}{0.2} = 120\,\text{m}\,\text{s}^{-2}.$$

This is the average net acceleration of the ball. Adding another g would instead be relevant to finding the floor force per unit mass, which is not requested.

Step 4: Conclusion The average acceleration is $\boxed{120\,\text{m}\,\text{s}^{-2}}$, directed upward. The numerical entry is 120.

Quick Tip to Solve: In a rebound, the change in velocity equals the sum of the incoming and outgoing speeds because their directions are opposite.

59. A body cools from 60°C to 40°C in 6 minutes. If, temperature of surroundings is 10°C. Then, after the next 6 minutes, its temperature will be _____°C. [4 marks]

Solution:

Concept:

- Newton's cooling law states $d(T - T_s)/dt = -k(T - T_s)$, with constant surroundings temperature T_s and cooling constant $k > 0$.
- Its integrated form is $T - T_s = (T_0 - T_s)e^{-kt}$. Equal time intervals therefore multiply the excess temperature by the same factor.

Step 1: Work with the temperature above the surroundings The surroundings remain at $T_s = 10°\text{C}$. Initially the excess temperature is

$$\theta_0 = 60 - 10 = 50°\text{C}.$$

After the first six minutes it is

$$\theta_1 = 40 - 10 = 30°\text{C}.$$

The cooling rate depends on these excess temperatures, rather than on the Celsius temperature measured from zero. Temperature differences in Celsius and kelvin have equal numerical values.

Step 2: Find the factor for a six-minute interval From the exponential cooling solution,

$$e^{-6k} = \frac{\theta_1}{\theta_0} = \frac{30}{50} = \frac{3}{5}.$$

There is no need to evaluate the logarithm or determine k separately. The same body under the same conditions has the same value of k, so another six-minute interval gives exactly the same multiplicative factor.

Step 3: Apply the factor and restore the ambient temperature The excess temperature after the next six minutes is

$$\theta_2 = \theta_1\frac{3}{5} = 30\frac{3}{5} = 18°\text{C}.$$

The actual body temperature is therefore

$$T_2 = T_s + \theta_2 = 10 + 18 = 28°\text{C}.$$

Subtracting another fixed $20°\text{C}$ would assume a constant cooling rate. That is incorrect: as the body approaches its surroundings, its rate of cooling decreases.

Step 4: Conclusion After the next six minutes, the temperature is $\boxed{28°\text{C}}$. The numerical entry is 28.

Quick Tip to Solve: For equal cooling intervals, form a geometric progression of $T - T_s$, then add the surroundings temperature back at the end.

60. A radioactive element $^{242}_{92}$X emits two α-particles, one electron and two positrons. The product nucleus is represented by $^{234}_{\mathrm{P}}$Y. The value of P is _____. [4 marks]

Solution:

Concept:

- An alpha particle is a $^{4}_{2}\mathrm{He}$ nucleus. Each alpha emission decreases the parent mass number by 4 and atomic number by 2.
- Beta-minus emission changes a neutron into a proton, increasing atomic number by 1. Positron emission changes a proton into a neutron, decreasing atomic number by 1. Neither changes mass number.

Step 1: Account for both alpha particles The starting nucleus has mass number $A = 242$ and atomic number $Z = 92$. After the two alpha emissions,

$$A_1 = 242 - 2(4) = 234, \qquad Z_1 = 92 - 2(2) = 88.$$

This already reproduces the mass number 234 printed for the product. The remaining emissions can change its charge number but cannot change its number of nucleons.

Step 2: Account for the emitted electron The electron is a beta-minus particle, $^{0}_{-1}e$. Charge conservation in a nuclear emission gives

$$Z_{\text{before}} = Z_{\text{after}} + (-1).$$

Thus the daughter atomic number rises by one. Following the alpha emissions it becomes $88 + 1 = 89$. It is important not to subtract one merely because the emitted electron has negative charge: the daughter must become more positive to conserve total charge.

Step 3: Account for both emitted positrons Each positron has charge number $+1$, so its emission reduces the daughter atomic number by one. Two positrons therefore give

$$P = 89 - 2 = 87.$$

The complete bookkeeping can also be written directly as

$$92 = P + 2(2) - 1 + 2(1),$$

which again yields $P = 87$. The mass-number equation is $242 = 234 + 2(4)$, since electrons and positrons carry zero nucleon number.

Step 4: Conclusion The product nucleus is $^{234}_{87}$Y in the question's notation, so $\boxed{P = 87}$.

Quick Tip to Solve: Check mass number and charge number separately: alpha changes both, while electron and positron emissions change only the charge number.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The major product P for the sequence starting from $C_6H_5CO(CH_2)_3CONH_2$ is to be identified. The successive reagents are (1) Zn/Hg, HCl; (2) $LiAlH_4$; (3) H_3O^+. [4 marks]

(A) $C_6H_5CH(OH)(CH_2)_3CH(OH)NH_2$

(B) $C_6H_5(CH_2)_5NH_2$

(C) $C_6H_5CH(OH)(CH_2)_4NH_2$

(D) $C_6H_5CH(OH)(CH_2)_4OH$

Correct Answer: (B) $C_6H_5(CH_2)_5NH_2$

Solution:

Concept:

- Clemmensen reduction, using Zn/Hg and HCl, converts a ketone carbonyl group $R{-}CO{-}R'$ into a methylene group $R{-}CH_2{-}R'$.
- Lithium aluminium hydride reduces a primary amide $RCONH_2$ to the primary amine RCH_2NH_2, retaining the carbonyl carbon in the chain.

Step 1: Identify the two carbonyl functions The starting compound is $C_6H_5CO(CH_2)_3CONH_2$. Its carbonyl next to the phenyl group is a ketone, whereas the carbonyl bonded to NH_2 belongs to an amide. Although both contain oxygen doubly bonded to carbon, their responses to the successive reagents differ. The three intervening methylene groups form an unchanged carbon skeleton.

Step 2: Carry out the Clemmensen reduction first The first reagent removes the ketone oxygen and replaces the carbonyl by CH_2:

$$C_6H_5CO(CH_2)_3CONH_2 \xrightarrow{Zn/Hg,\ HCl} C_6H_5CH_2(CH_2)_3CONH_2.$$

This is a deoxygenation to a methylene group, not reduction to a secondary alcohol. Consequently any proposed final structure retaining $C_6H_5CH(OH)^-$ fails to represent the first step correctly.

Step 3: Reduce the amide and compare the products The subsequent $LiAlH_4$ treatment, followed by acidic work-up, reduces the amide:

$$C_6H_5(CH_2)_4CONH_2 \xrightarrow[H_3O^+]{LiAlH_4} C_6H_5(CH_2)_5NH_2.$$

The amide carbon becomes the fifth methylene carbon; it is not eliminated. Option A retains two hydroxyl groups, option C retains the ketone-derived hydroxyl group, and option D additionally replaces the nitrogen terminus by an alcohol. Only B has the fully reduced ketone and the amide-derived primary amine.

Step 4: Conclusion The major product is $C_6H_5(CH_2)_5NH_2$, 5-phenylpentan-1-amine. Hence, the correct option is (B).

Quick Tip to Solve: Apply the reagents in their given order: Clemmensen removes ketone oxygen, and $LiAlH_4$ changes $-CONH_2$ into $-CH_2NH_2$ without losing carbon.

62. The magnetic behaviour of Li_2O, Na_2O_2 and KO_2, respectively, are : [4 marks]

(A) paramagnetic, paramagnetic and diamagnetic

(B) paramagnetic, diamagnetic and paramagnetic

(C) diamagnetic, paramagnetic and diamagnetic

(D) diamagnetic, diamagnetic and paramagnetic

Correct Answer: (D) diamagnetic, diamagnetic and paramagnetic

Solution:

Concept:

- A species is paramagnetic if it has unpaired electrons; an all-paired electronic configuration is diamagnetic. The oxygen-containing anion determines the distinction here.
- Alkali-metal cations Li^+, Na^+ and K^+ have closed-shell configurations and are diamagnetic. Their neutral-atom unpaired electrons must not be counted in these ionic compounds.

Step 1: Identify the oxide in lithium oxide Lithium oxide is Li_2O, comprising two Li^+ ions and one oxide ion O^{2-}. The oxide ion has ten electrons, the closed-shell configuration of neon. Its electrons are all paired, as are those of the lithium ions. Hence Li_2O is diamagnetic. This immediately rules out A and B, both of which classify it as paramagnetic.

Step 2: Identify the peroxide ion in sodium peroxide Sodium peroxide contains O_2^{2-}, not isolated neutral oxygen atoms. In molecular-orbital terms the two antibonding π^* orbitals are fully occupied by four paired electrons. There are no unpaired electrons in the peroxide ion. With closed-shell Na^+ ions as well, Na_2O_2 is diamagnetic. Option C is therefore also excluded.

Step 3: Identify the superoxide ion in potassium superoxide Potassium superoxide contains K^+ and O_2^-. Adding one electron to molecular oxygen leaves three electrons in the two degenerate antibonding π^* orbitals. One orbital contains a pair and the other contains a single electron. Thus the superoxide ion has one unpaired electron and is paramagnetic. The resulting sequence is diamagnetic, diamagnetic, paramagnetic, matching D alone.

Step 4: Conclusion Li_2O and Na_2O_2 are diamagnetic, whereas KO_2 is paramagnetic. Hence, the correct option is (D).

Quick Tip to Solve: First identify oxide, peroxide or superoxide: O^{2-} and O_2^{2-} have paired electrons, while O_2^- has one unpaired electron.

63. Compound that will give positive Lassaigne's test for both nitrogen and halogen is : [4 marks]

(A) $NH_2OH.HCl$

(B) $N_2H_4.HCl$

(C) $CH_3NH_2.HCl$

(D) NH_4Cl

Correct Answer: (C) $CH_3NH_2.HCl$

Solution:

Concept:

- In sodium fusion, carbon and nitrogen in an organic compound form sodium cyanide. The cyanide is converted to ferrocyanide and subsequently gives the characteristic Prussian-blue nitrogen test.
- Halogens are converted into sodium halides during fusion; after appropriate acidification, chloride gives a white precipitate of $AgCl$ with silver nitrate.

Step 1: Check the requirement for the nitrogen test A nitrogen atom alone does not ensure a positive Lassaigne nitrogen test. Formation of $NaCN$ during fusion also needs carbon in the sample. The alternatives $NH_2OH \cdot HCl$, $N_2H_4 \cdot HCl$ and NH_4Cl contain nitrogen and chlorine but contain no carbon. They therefore cannot furnish the cyanide required for the characteristic nitrogen test under the usual procedure.

Step 2: Identify the compound containing carbon and nitrogen Methylamine hydrochloride, $CH_3NH_2 \cdot HCl$, contains carbon in its methyl group and nitrogen in the amine group. Fusion with sodium converts the relevant elements into water-soluble ionic products, including $NaCN$. The cyanide then reacts with ferrous ions to form the ferrocyanide complex. On the usual subsequent treatment with ferric ions, a Prussian-blue product confirms nitrogen.

Step 3: Confirm the halogen test and eliminate the alternatives The same methylamine hydrochloride sample contains chloride. Chloride remains available in the fusion extract as soluble chloride and, after the prescribed treatment to remove interfering cyanide species, gives

$$Ag^+ + Cl^- \longrightarrow AgCl\downarrow.$$

Thus C satisfies both tests. A, B and D can supply chloride but fail the essential carbon requirement for cyanide formation. Their nitrogen content must not be mistaken for a guaranteed positive Lassaigne nitrogen result.

Step 4: Conclusion The compound giving positive tests for both nitrogen and halogen is $CH_3NH_2 \cdot HCl$. Hence, the correct option is (C).

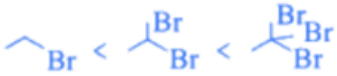
Quick Tip to Solve: For Lassaigne's nitrogen test, look for carbon as well as nitrogen; a carbon-free nitrogen salt cannot form the required cyanide.

64. Identify the correct order for the given property for following compounds.

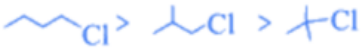
Choose the correct answer from the option given below : [4 marks]

(A) Boiling Point: 

(B) Density: 

(C) Boiling Point: 

(D) Density: 

(E) Boiling Point: 

(A) (A), (B) and (E) only

(B) (A), (C) and (D) only

(C) (A), (C) and (E) only

(D) (B), (C) and (D) only

Correct Answer: (C) (A), (C) and (E) only

Solution:

Concept:

- Boiling point rises with molecular size and polarizability when the structures are comparable; stronger London forces require more energy to overcome.
- For isomeric alkyl halides, branching reduces exposed surface area and generally lowers the boiling point. Density trends must be distinguished from boiling-point trends.

Step 1: Check the boiling-point statements In (A), the increasing carbon-chain length increases intermolecular dispersion forces, so the displayed order is correct. In (C), replacing more hydrogen atoms by bromine increases mass and polarizability, so the displayed boiling-point order is also correct. In (E), the straight-chain butyl chloride has the largest contact area, whereas tert-butyl chloride is the most compact. Thus the descending order from the straight-chain to the most branched isomer is correct.

Step 2: Reject the density statements For the same alkyl group, the usual density order is alkyl chloride less than alkyl bromide less than alkyl iodide. Statement (B) puts the bromide before the chloride in an increasing order and is incorrect. In (D), the displayed dihalogenated compounds put the iodine-containing member below the bromine-containing and chlorine-containing members. This reverses the increase expected from the heavier halogen, so (D) is incorrect. The three correct statements are therefore (A), (C) and (E); each other option contains (B) or (D).

Step 3: Conclusion The correct statements are A, C and E only. Hence, the correct option is (C).

Quick Tip to Solve: For boiling points compare chain length first, then branching; for density compare the halogen masses separately.

65. The bond dissociation energy is highest for [4 marks]

(A) Cl_2

(B) I_2

(C) Br_2

(D) F_2

Correct Answer: (A) Cl_2

Solution:

Concept:

- Bond dissociation energy is the energy required to break one mole of gaseous diatomic molecules into gaseous atoms.
- Halogen bond strength depends on orbital overlap and repulsion between nonbonding electron pairs; the smallest atom need not give the strongest bond.

Step 1: Account for the fluorine exception Each fluorine atom has three lone pairs in a very small valence shell. In F_2, the close approach of the two atoms causes appreciable lone-pair repulsion, weakening the F–F bond despite its short length. Its dissociation energy is about $159\,\mathrm{kJ\,mol^{-1}}$, so option (D) cannot be chosen merely from the small atomic radius.

Step 2: Compare the remaining halogens The Cl–Cl bond has effective overlap with less severe lone-pair crowding and a dissociation energy of about $243\,\mathrm{kJ\,mol^{-1}}$. Down the group, the larger, more diffuse orbitals and greater internuclear distances reduce bond strength: Br–Br is about $193\,\mathrm{kJ\,mol^{-1}}$ and I–I about $151\,\mathrm{kJ\,mol^{-1}}$. Hence the order is $Cl_2 > Br_2 > F_2 > I_2$. Neither bromine nor iodine exceeds chlorine, ruling out options (C) and (B).

Step 3: Conclusion Cl_2 has the highest bond dissociation energy. Hence, the correct option is (A).

Quick Tip to Solve: Remember the F–F lone-pair exception before applying a simple down-the-group trend.

66. Number of cyclic tripeptides formed with 2 amino acids A and B is : [4 marks]

(A) 2

(B) 3

(C) 5

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- A cyclic tripeptide contains three amino-acid residues joined by three peptide bonds, with no distinct starting or ending residue.
- Sequences related by a cyclic rotation describe the same cyclic peptide. Count distinct residue arrangements rather than all linear sequences.

Step 1: List the possible compositions Each of the three positions can contain A or B. Classify the

products by the number of B residues: zero, one, two or three. The corresponding representatives are AAA, AAB, ABB and BBB. These four compositions cannot become one another by rotation because rotation preserves the number of each residue.

Step 2: Remove repetitions caused by ring rotation For the composition with two A residues and one B residue, AAB, ABA and BAA are rotations of the same ring, so they give only one cyclic tripeptide. Similarly, ABB, BAB and BBA all represent one ring with one A residue and two B residues. AAA and BBB each supply one more possibility. Consequently the total is $1 + 1 + 1 + 1 = 4$. Counting all eight linear strings overcounts the mixed rings; counting only two omits the mixed compositions. Three or five does not match the four distinct composition classes.

Step 3: Conclusion The number of cyclic tripeptides is 4. Hence, the correct option is (D).

Quick Tip to Solve: For a three-residue ring made from two residue types, group sequences by composition and identify cyclic rotations.

67. The shortest wavelength of hydrogen atom in Lyman series is λ. The longest wavelength is Balmer series of He^+ is [4 marks]

(A) $\frac{36\lambda}{5}$
(B) $\frac{5}{9\lambda}$
(C) $\frac{9\lambda}{5}$
(D) $\frac{5\lambda}{9}$

Correct Answer: (C) $\frac{9\lambda}{5}$

Solution:

Concept:

- For a hydrogen-like ion, the Rydberg relation is $1/\lambda_{if} = RZ^2(1/n_f^2 - 1/n_i^2)$, where Z is nuclear charge.
- The shortest wavelength in a series is its limit $n_i \to \infty$; the longest is the transition from the first allowed upper level.

Step 1: Express the hydrogen Lyman limit For hydrogen, $Z = 1$, and the Lyman series terminates at $n_f = 1$. Its shortest wavelength occurs at the greatest energy difference, namely an initial level tending to infinity. Therefore

$$\frac{1}{\lambda} = R\,(1 - 0) = R.$$

This identifies the given λ as $1/R$, rather than as the wavelength of the first Lyman line.

Step 2: Find the first Balmer line of helium ion For He^+, $Z = 2$, and the Balmer series ends at $n_f = 2$. The smallest energy gap within that series is the $3 \to 2$ transition, hence it has the longest wavelength λ_B. Thus

$$\frac{1}{\lambda_B} = 4R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{9}, \qquad \lambda_B = \frac{9}{5R} = \frac{9\lambda}{5}.$$

Option (A) misses the factor $Z^2 = 4$; (D) reverses the wavelength ratio, while (B) has reciprocal-length dimensions.

Step 3: Conclusion The longest wavelength is $9\lambda/5$. Hence, the correct option is (C).

Quick Tip to Solve: Choose the first line for a longest wavelength, the series limit for a shortest wavelength, and retain the factor Z squared.

68. Match List I with List II

	List I		List II
	Reaction		**Reagents**
(A)	**Hoffmann Degradation**	**(I)**	$\text{Conc.KOH}, \Delta$
(B)	**Clemenson reduction**	**(II)**	$\text{CHCl}_3, \text{NaOH}/\text{H}_3\text{O}^{\oplus}$
(C)	**Cannizaro reaction**	**(III)**	Br_2, NaOH
(D)	**Reimer-Tiemann Reaction**	**(IV)**	$\text{Zn} - \text{Hg}/\text{HCl}$

Choose the correct answer from the options given below : [4 marks]

(A) (A) - III, (B) - IV, (C) - I, (D) - II

(B) (A) - II, (B) - IV, (C) - I, (D) - III

(C) (A) - III, (B) - IV, (C) - II, (D) - I

(D) (A) - II, (B) - I, (C) - III, (D) - IV

Correct Answer: (A) (A) - III, (B) - IV, (C) - I, (D) - II

Solution:

Concept:

- Named reactions can be matched by identifying the functional-group transformation and the characteristic reagent rather than by similar-sounding names.
- Hofmann degradation removes the carbonyl carbon of a primary amide; Clemmensen reduction converts a carbonyl group to methylene.

Step 1: Match degradation and reduction Hofmann degradation of a primary amide uses bromine in aqueous sodium hydroxide. The product is a primary amine with one carbon fewer:

$$\text{RCONH}_2 + \text{Br}_2 + 4\,\text{NaOH} \longrightarrow \text{RNH}_2 + \text{Na}_2\text{CO}_3 + 2\,\text{NaBr} + 2\,\text{H}_2\text{O}.$$

Therefore (A) matches (III). Clemmensen reduction uses zinc amalgam and hydrochloric acid, $\text{Zn}-\text{Hg}/\text{HCl}$, to reduce an aldehyde or ketone carbonyl group to CH_2. Hence (B) matches (IV), rather than concentrated alkali.

Step 2: Match disproportionation and formylation Cannizzaro reaction occurs for an aldehyde lacking an alpha hydrogen under concentrated strong alkali. One molecule is oxidised to carboxylate and another reduced to alcohol, so (C) matches concentrated KOH and heat, (I). Reimer–Tiemann formylation of phenol uses CHCl_3 and aqueous NaOH, followed by acid work-up, giving principally salicylaldehyde. Thus (D) matches (II). The resulting complete mapping is A–III, B–IV, C–I, D–II; every other option assigns at least one incompatible reagent.

Step 3: Conclusion The matching is A–III, B–IV, C–I, D–II. Hence, the correct option is (A).

Quick Tip to Solve: Associate bromine plus alkali with Hofmann, zinc amalgam plus acid with Clemmensen, and chloroform plus alkali with Reimer–Tiemann.

69. The increasing order of $\mathrm{pK_a}$ for the following phenols is

(A) 2, 4 - Dinitrophenol

(B) 4 - Nitrophenol

(C) 2, 4, 5 - Trimethylphenol

(D) Phenol

(E) 3-Chlorophenol

Choose the correct answer from the option given below : [4 marks]

(A) (A), (E), (B), (D), (C)

(B) (C), (E), (D), (B), (A)

(C) (C), (D), (E), (B), (A)

(D) (A), (B), (E), (D), (C)

Correct Answer: (D) (A), (B), (E), (D), (C)

Solution:

Concept:

- $\mathrm{p}K_a = -\log K_a$: a stronger acid has a lower $\mathrm{p}K_a$. Rank conjugate-base stability before reversing to the requested scale.
- Electron-withdrawing substituents stabilise phenoxide, while electron-donating alkyl groups destabilise it relative to phenol.

Step 1: Compare the electron-withdrawing substituents In 2,4-dinitrophenol (A), two nitro groups strongly withdraw electrons by induction and, at the ortho and para positions, by resonance. Its phenoxide is therefore the most stabilised of the listed ions and (A) has the lowest pKa. A single para nitro group in (B) also stabilises phenoxide strongly by resonance, so (B) follows (A). The meta chlorine in (E) withdraws inductively, but its stabilising effect is weaker than that of a para nitro group. Thus the increasing pKa order among these is A, then B, then E.

Step 2: Place phenol and the methyl-substituted phenol Unsubstituted phenol (D) lacks the electron-withdrawing substituent of (E), so it is less acidic and has a larger pKa than 3-chlorophenol. In 2,4,5-trimethylphenol (C), the methyl groups donate electron density and reduce the relative stability of the phenoxide ion; (C) is consequently the weakest acid and has the greatest pKa. Combining both comparisons gives

$$\mathrm{p}K_a(A) < \mathrm{p}K_a(B) < \mathrm{p}K_a(E) < \mathrm{p}K_a(D) < \mathrm{p}K_a(C).$$

Option (A) interchanges nitrophenol and chlorophenol; (B) and (C) start with the least acidic member and reverse the main trend.

Step 3: Conclusion The increasing order is A, B, E, D, C. Hence, the correct option is (D).

Quick Tip to Solve: Write strongest acid first for increasing pKa: nitro stabilisation dominates meta-halogen induction, and methyl groups raise pKa.

70. Chiral complex from the following is :

Here en = ethylene diamine [4 marks]

(A) cis – $[PtCl_2(en)_2]^{2+}$

(B) trans – $[Co(NH_3)_4Cl_2]^{+}$

(C) trans – $[PtCl_2(en)_2]^{2+}$

(D) cis – $[PtCl_2(NH_3)_2]$

Correct Answer: (A) cis – $[PtCl_2(en)_2]^{2+}$

Solution:

Concept:

- A chiral complex is not superimposable on its mirror image. A plane of symmetry or an inversion centre makes a complex achiral.
- Ethylenediamine, en, is bidentate. Two en ligands and two chloride ligands give an octahedral six-coordinate complex, whereas two ammonia and two chloride ligands around Pt(II) give a square-planar complex.

Step 1: Test the two chelate complexes In cis-$[PtCl_2(en)_2]^{2+}$, the adjacent chloride ligands and the two chelate rings produce a twisted arrangement with two non-superimposable mirror forms, conventionally called Δ and Λ. It has no mirror plane that superimposes those two forms, so option (A) is chiral. In the trans isomer, the chlorides occupy opposite positions; the resulting symmetric arrangement has a symmetry element that makes the mirror form superimposable. Thus option (C) is achiral despite containing the same ligands.

Step 2: Eliminate the remaining structures Trans-$[Co(NH_3)_4Cl_2]^{+}$ has four identical ammonia ligands in an equatorial plane and opposite chloride ligands. Its planes of symmetry preclude optical isomerism, eliminating (B). Cis-$[PtCl_2(NH_3)_2]$ is square planar: the molecular plane itself is a mirror plane, so the complex is achiral even though it shows geometrical cis–trans isomerism. This eliminates (D). Geometrical isomerism alone is therefore not a test of chirality; the cis octahedral bis-chelate arrangement is decisive here.

Step 3: Conclusion The chiral complex is cis-$[PtCl_2(en)_2]^{2+}$. Hence, the correct option is (A).

Quick Tip to Solve: Count donor atoms to identify geometry, then check symmetry; cis octahedral bis-chelates can be optically active while square-planar complexes are achiral.

71. The standard electrode potential (M^{3+}/M^{2+}) for V, Cr, Mn & Co are -0.26 V, -0.41 V, + 1.57 V and + 1.97 V, respectively. The metal ions which can liberate H_2 from a dilute acid are : [4 marks]

(A) Mn^{2+} and Co^{2+}

(B) V^{2+} and Mn^{2+}

(C) V^{2+} and Cr^{2+}

(D) Cr^{2+} and Co^{2+}

Correct Answer: (C) V^{2+} and Cr^{2+}

Solution:

Concept:

- The standard hydrogen electrode has reduction potential $E^{\circ}(H^{+}/H_2) = 0\,V$. Hydrogen liberation requires reduction of acid protons.
- An ion M^{2+} acts as the reducing agent by oxidation to M^{3+}. For this coupling, $E^{\circ}_{cell} = 0 - E^{\circ}(M^{3+}/M^{2+})$; a positive value favours the reaction.

Step 1: Write the electron-transfer reaction Two divalent metal ions each supply one electron, while two hydrogen ions accept them. The balanced net reaction is

$$2\,M^{2+} + 2\,H^{+} \longrightarrow 2\,M^{3+} + H_2.$$

The tabulated values are reduction potentials, although the metal ion undergoes the reverse, oxidation process. Therefore their signs must be reversed for the anode.

Step 2: Compare the four cell potentials For vanadium and chromium, the resulting potentials are

$$E^{\circ}_{cell}(V^{2+}) = +0.26\,V, \qquad E^{\circ}_{cell}(Cr^{2+}) = +0.41\,V.$$

Both ions can reduce protons. For manganese and cobalt the cell potentials are -1.57 V and -1.97 V, respectively; these ions do not liberate hydrogen by the proposed oxidation. Each other listed pair includes at least one of these unsuitable ions.

Step 3: Conclusion V^{2+} and Cr^{2+} can liberate H_2 from dilute acid. Hence, the correct option is (C).

Quick Tip to Solve: For hydrogen liberation by a lower oxidation-state ion, compare its higher/lower reduction potential with zero; a negative value identifies the stronger reductant.

72. Match List I with List II.

	List I		List II
	Antimicrobials		**Names**
(A)	**Narrow Spectrum Antibiotic**	**(I)**	**Furacin**
(B)	**Antiseptic**	**(II)**	**Sulphur dioxide**
(C)	**Disinfectants**	**(III)**	**Penicillin G**
(D)	**Broad spectrum antibiotic**	**(IV)**	**Chloramphenicol**

Choose the correct answer from the options given below : [4 marks]

(A) (A) - III, (B) - I, (C) - II, (D) - IV

(B) (A) - II, (B) - I, (C) - IV, (D) - III

(C) (A) - I, (B) - II, (C) - IV, (D) - III

(D) (A) - III, (B) - I, (C) - IV, (D) - II

Correct Answer: (A) (A) - III, (B) - I, (C) - II, (D) - IV

Solution:

Concept:

- Antibiotics inhibit the growth of microorganisms or destroy them at suitable low concentrations. Narrow-spectrum and broad-spectrum refer to the range of susceptible organisms.
- Antiseptics are suitable for application to living tissues, whereas disinfectants are used to destroy microorganisms on non-living objects and surfaces.

Step 1: Match the two antibiotics Penicillin G is the narrow-spectrum antibiotic in the list, so (A) matches (III). Chloramphenicol acts against a broad range of bacteria and is a broad-spectrum antibiotic, so (D) matches (IV). These two identifications already distinguish the correct mapping from alternatives that interchange the drug categories.

Step 2: Match the antiseptic and disinfectant Furacin is used as an antiseptic and is matched with (B), giving (B)–(I). Sulphur dioxide is used as a disinfectant, giving (C)–(II). The full matching is therefore (A)–(III), (B)–(I), (C)–(II), (D)–(IV). The remaining proposed mappings wrongly classify sulphur dioxide or interchange chloramphenicol and penicillin G, or interchange the disinfectant and broad-spectrum antibiotic.

Step 3: Conclusion Penicillin G, Furacin, sulphur dioxide and chloramphenicol match the four categories in order. Hence, the correct option is (A).

Quick Tip to Solve: Fix the two familiar antibiotic examples first: penicillin G is narrow-spectrum and chloramphenicol is broad-spectrum; then match the two remaining categories.

73. The correct order of hydration enthalpies is

(A) K^+

(B) Rb^+

(C) Mg^{2+}

(D) Cs^+

(E) Ca^{2+}

Choose the correct answer from the options given below : [4 marks]

(A) E > C > A > B > D

(B) C > A > E > B > D

(C) C > E > A > D > B

(D) C > E > A > B > D

Correct Answer: (D) C > E > A > B > D

Solution:

Concept:

- Hydration enthalpy is the enthalpy change when gaseous ions become hydrated by water. Hydration releases energy, so its signed value is negative.
- The strength, or magnitude, of hydration grows with ionic charge density. Qualitatively, the magnitude increases with z^2/r, where z is ionic charge and r is ionic radius.

Step 1: Compare the divalent ions The doubly charged ions attract water dipoles much more strongly than the large singly charged alkali-metal ions. Magnesium and calcium therefore precede potassium, rubidium and caesium in the order of hydration-enthalpy magnitude. Within Group 2, magnesium is smaller than calcium and has the stronger ion–dipole attraction. Thus (C) precedes (E).

Step 2: Order the alkali-metal ions and assemble the sequence Down Group 1, ionic size increases from K^+ through Rb^+ to Cs^+. Their charge is unchanged, so hydration becomes weaker in that order. Combining the comparisons gives

$$|\Delta H_{\text{hyd}}|: \quad Mg^{2+} > Ca^{2+} > K^+ > Rb^+ > Cs^+.$$

Options (A) and (B) violate the divalent-ion comparisons, while (C) incorrectly puts caesium above rubidium. The displayed ordering uses hydration strength, as intended by the options, rather than the algebraic ordering of negative enthalpies.

Step 3: Conclusion The order of hydration-enthalpy magnitudes is $C > E > A > B > D$. Hence, the correct option is (D).

Quick Tip to Solve: For hydration trends, rank charge first and radius second; explicitly distinguish the magnitude released from the negative signed enthalpy.

74. Correct statement about smog is : [4 marks]

(A) NO_2 is present in classical smog

(B) Both NO_2 and SO_2 are present in classical smog

(C) Photochemical smog has high concentration of oxidizing agents

(D) Classical smog also has high concentration of oxidizing agents

Correct Answer: (C) Photochemical smog has high concentration of oxidizing agents

Solution:

Concept:

- Classical smog forms in cool, humid conditions from smoke, fog and sulphur-containing pollutants. It is conventionally described as reducing smog.
- Photochemical smog forms when sunlight acts on nitrogen oxides and hydrocarbons. Secondary pollutants such as ozone and peroxyacetyl nitrate are strong oxidizing agents.

Step 1: Identify the oxidizing type of smog In sunlight, nitrogen dioxide undergoes photodissociation. The oxygen atom produced combines with molecular oxygen to produce ozone:

$$NO_2 \xrightarrow{\text{light}} NO + O, \qquad O + O_2 \longrightarrow O_3.$$

Hydrocarbon oxidation also produces reactive species and peroxyacetyl nitrate. The resulting mixture contains high concentrations of oxidants and is therefore called oxidizing smog.

Step 2: Reject the classical-smog descriptions Classical smog is associated especially with SO_2, smoke and fog; it is not characterized by the photochemical nitrogen-dioxide/ozone system. Thus the statements assigning NO_2, alone or together with SO_2, as the defining classical-smog composition are not the accepted descriptions. Classical smog is reducing rather than strongly oxidizing, so the claim that it too has a high concentration of oxidizing agents is incorrect.

Step 3: Conclusion Photochemical smog has a high concentration of oxidizing agents. Hence, the correct option is (C).

Quick Tip to Solve: Associate cool, humid sulphur smog with reducing conditions and sunny nitrogen-oxide/hydrocarbon smog with ozone and other oxidants.

75. During the borax bead test with $CuSO_4$, a blue green colour of the bead was observed in oxidising flame due to the formation of : [4 marks]

(A) CuO

(B) $Cu(BO_2)_2$

(C) Cu_3B_2

(D) Cu

Correct Answer: (B) $Cu(BO_2)_2$

Solution:

Concept:

- On heating, borax first loses its water of crystallization, then forms sodium metaborate and boric anhydride. The molten borate mixture forms a glassy bead.
- Boric anhydride reacts with metal oxides to form characteristic coloured metal metaborates. The colour of the dissolved metal compound identifies the metal in the borax bead test.

Step 1: Form the reactive borate bead Heating borax produces the metaborate/boric-anhydride mixture:

$$Na_2B_4O_7 \cdot 10\,H_2O \longrightarrow Na_2B_4O_7 + 10\,H_2O,$$

$$Na_2B_4O_7 \longrightarrow 2\,NaBO_2 + B_2O_3.$$

Copper sulphate supplies copper(II) oxide on heating. The oxidising flame maintains copper in the oxidized copper(II) state rather than reducing it to the metal.

Step 2: Identify the coloured copper compound The copper oxide combines with boric anhydride according to

$$CuO + B_2O_3 \longrightarrow Cu(BO_2)_2.$$

The copper(II) metaborate dissolved in the glassy bead causes the observed blue-green colour. Copper oxide is a precursor, not the metaborate responsible for the bead colour. Copper boride is not produced by this oxide–boric-anhydride reaction, and metallic copper would correspond to reducing rather than oxidising conditions.

Step 3: Conclusion The coloured compound is copper(II) metaborate, $Cu(BO_2)_2$. Hence, the correct option is (B).

Quick Tip to Solve: In a borax bead test, identify the metal metaborate formed from the metal oxide and boric anhydride; do not stop at the oxide precursor.

76. Which of the given compounds can enhance the efficiency of hydrogen storage tank? [4 marks]

(A) SiH_4

(B) Li/P_4

(C) $NaNi_5$

(D) Di-isobutylaluminium hydride

Correct Answer: No scientifically defensible listed formula as written: the familiar storage alloy is $LaNi_5$, whereas intended/key option (C) prints $NaNi_5$.

Solution:

Concept:

- A useful solid hydrogen-storage material can take up hydrogen and release it under controlled conditions. Reversible storage improves the usable capacity and operation of a hydrogen tank.
- Nickel-containing intermetallic hydrogen absorbers are distinguished from molecular hydrides or hydride reagents used mainly for chemical synthesis.

Step 1: Identify the established storage material The standard textbook example is lanthanum pentanickel, $LaNi_5$, an intermetallic alloy that absorbs hydrogen reversibly. Hydrogen can enter sites in the alloy framework and subsequently be released when suitable conditions are restored. This reversible absorption is the relevant property for improving a hydrogen-storage tank; it is distinct from simply containing hydrogen atoms in a chemical formula.

Step 2: Compare the literal options with the intended answer Option (C) prints $NaNi_5$, which is not the formula $LaNi_5$. The printed formula cannot be assigned the established storage properties of lanthanum pentanickel merely by choosing the closest-looking entry. SiH_4 is a volatile covalent hydride, di-isobutylaluminium hydride is a reactive synthetic reducing reagent, and Li/P_4 is not the stated textbook reversible hydrogen-storage alloy. Thus the intended/key selection is (C), but the scientifically correct familiar alloy is absent among the literal formulae. The distinction is a formula error, not a chemical equivalence between sodium and lanthanum.

Step 3: Conclusion As written, no listed formula is a defensible answer for the standard alloy. The intended/key option is (C), printed $NaNi_5$; the scientifically correct textbook hydrogen-storage alloy is $LaNi_5$.

Quick Tip to Solve: For reversible hydrogen storage, look for an intermetallic alloy that absorbs hydrogen; distinguish it from a covalent hydride or a synthetic reducing reagent.

77. "A" obtained by Ostwald's method involving air oxidation of NH_3, upon further air oxidation produces "B", "B" on hydration forms an oxoacid of Nitrogen along with evolution of "A". The oxoacid also produces "A" and gives positive brown ring test.

Identify A and B, respectively : [4 marks]

(A) N_2O_3, NO_2

(B) NO_2, N_2O_5

(C) NO_2, N_2O_4

(D) NO, NO_2

Correct Answer: (D) NO, NO_2

Solution:

Concept:

- In the Ostwald process, catalytic air oxidation of ammonia produces nitric oxide, NO, which is oxidized further to nitrogen dioxide, NO_2.
- Absorption of nitrogen dioxide in water forms nitric acid and regenerates nitric oxide. Nitrate is identified by the brown-ring test through formation of a nitrosyl complex.

Step 1: Trace the first two oxidation steps The initial catalytic oxidation is

$$4\,NH_3 + 5\,O_2 \xrightarrow{Pt/Rh} 4\,NO + 6\,H_2O.$$

Hence A is NO. Further air oxidation gives

$$2\,NO + O_2 \longrightarrow 2\,NO_2,$$

so B is NO_2. This already singles out the pair containing nitric oxide first and nitrogen dioxide second.

Step 2: Check the hydration and brown-ring clues Nitrogen dioxide reacts with water to give the oxoacid nitric acid while regenerating A:

$$3\,NO_2 + H_2O \longrightarrow 2\,HNO_3 + NO.$$

Dilute nitric acid can produce nitric oxide on reduction, for example in reaction with copper:

$$3\,Cu + 8\,HNO_3 \longrightarrow 3\,Cu(NO_3)_2 + 2\,NO + 4\,H_2O.$$

In the brown-ring test, nitrate is reduced by iron(II) to nitric oxide, which forms the brown nitrosyl complex $[Fe(H_2O)_5(NO)]^{2+}$. All clues therefore agree with A as nitric oxide; the other options start with a different oxide and fail the first oxidation step.

Step 3: Conclusion A is NO and B is NO_2. Hence, the correct option is (D).

Quick Tip to Solve: Trace the Ostwald sequence as ammonia to nitric oxide to nitrogen dioxide to nitric acid; nitric oxide is regenerated when nitrogen dioxide is absorbed in water.

78. Which of the following salt solutions would coagulate the colloid solution formed when $FeCl_3$ is added to $NaOH$ solution, at the fastest rate? [4 marks]

(A) 10 mL of 0.15 mol dm^{-3} $CaCl_2$

(B) 10 mL of 0.2 mol dm^{-3} $AlCl_3$

(C) 10 mL of 0.1 mol dm^{-3} Na_2SO_4

(D) 10 mL of 0.1 mol dm^{-3} $Ca_3(PO_4)_2$

Correct Answer: (B) 10 mL of 0.2 mol dm^{-3} $AlCl_3$

Solution:

Concept:

- A freshly formed hydroxide sol preferentially adsorbs an ion from the reagent present in excess. Adding $FeCl_3$ to NaOH gives a negatively charged $Fe(OH)_3$ sol through adsorption of OH^-.
- The Hardy–Schulze rule compares the valency of the counter-ion: for a negative sol, a cation of greater positive charge has greater coagulating power.

Step 1: Identify the charge of the sol The precipitation reaction is

$$FeCl_3 + 3\,NaOH \longrightarrow Fe(OH)_3 + 3\,NaCl.$$

Since ferric chloride is added to sodium hydroxide solution, hydroxide is the surrounding excess electrolyte. Adsorbed OH^- makes the colloidal particles negative; therefore, compare the cations rather than the anions in the proposed salts.

Step 2: Compare the effective counter-ions $CaCl_2$ and $Ca_3(PO_4)_2$ supply Ca^{2+}, Na_2SO_4 supplies Na^+, and $AlCl_3$ supplies Al^{3+}. Their charge-based coagulating order is

$$Al^{3+} > Ca^{2+} > Na^+.$$

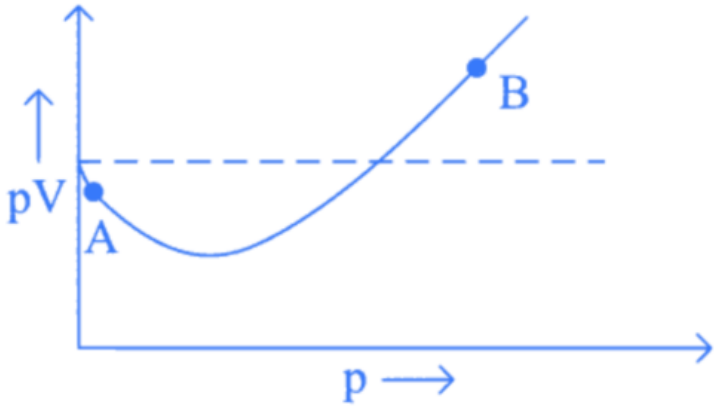
The aluminium chloride solution is also the most concentrated salt solution listed, so its stated amount does not reverse the valency comparison. Phosphate has a large negative charge, but it is a co-ion for this negatively charged sol and is not the relevant ion for the Hardy–Schulze comparison.

Step 3: Conclusion The fastest coagulation is produced by 10 mL of $0.2\,mol\,dm^{-3}$ $AlCl_3$. Hence, the correct option is (B).

Quick Tip to Solve: First identify which reagent is the medium in excess. Reversing the order of addition can reverse the sol charge and change the relevant coagulating ion.

79. For 1 mol of gas, the plot of pV vs. p is shown below. p is the pressure and V is the volume of the gas

What is the value of compressibility factor at point A ? [4 marks]



(A) $1 + \frac{b}{V}$

(B) $1 - \frac{a}{RTV}$

(C) $1 + \frac{a}{RTV}$

(D) $1 - \frac{b}{V}$

Correct Answer: (B) $1 - \frac{a}{RTV}$

Solution:

Concept:

- For one mole, the compressibility factor is $Z = pV/(RT)$. Values below one indicate that attractive interactions lower the pressure-volume product below its ideal value.
- The van der Waals equation is $(p + a/V^2)(V - b) = RT$, where a represents attractions and b accounts for excluded volume.

Step 1: Interpret point A Point A is on the low-pressure side of the curve, below the horizontal ideal-gas reference. Hence the attraction correction dominates and $Z < 1$. At comparatively low pressure the gas occupies a large volume, so the excluded volume b is neglected relative to V in this approximation.

Step 2: Reduce the equation in the attraction-dominated region Putting $V - b \simeq V$ gives

$$\left(p + \frac{a}{V^2}\right) V = RT.$$

Thus

$$pV = RT - \frac{a}{V}, \qquad Z = 1 - \frac{a}{RTV}.$$

The positive corrections in options (A) and (C) would give $Z > 1$, appropriate to positive deviation rather than A. Option (D) incorrectly uses a negative excluded-volume correction: repulsive or volume effects increase the pressure-volume product, whereas the negative correction here comes from attraction. The selected expression is the usual low-pressure approximation, not the full van der Waals expression at arbitrary density.

Step 3: Conclusion At point A, $Z = 1 - \frac{a}{RTV}$ in the attraction-dominated approximation. Hence, the correct option is (B).

Quick Tip to Solve: On a pV versus pressure graph, compare the point with the ideal horizontal line before choosing the sign and the van der Waals parameter.

80. The reaction representing the Mond process for metal refining is _____. [4 marks]

(A) $ZnO + C \xrightarrow{\Delta} Zn + CO$

(B) $2K[Au(CN)_2] + Zn \xrightarrow{\Delta} K_2[Zn(CN)_4] + 2Au$

(C) $Zr + 2I_2 \xrightarrow{\Delta} ZrI_4$

(D) $Ni + 4CO \xrightarrow{\Delta} Ni(CO)_4$

Correct Answer: (D) $Ni + 4CO \xrightarrow{\Delta} Ni(CO)_4$

Solution:

Concept:

- The Mond process purifies nickel by converting it into volatile nickel tetracarbonyl, $Ni(CO)_4$, and then decomposing that compound to recover pure nickel.
- Vapour-phase refining requires formation of a volatile compound under one set of conditions and its decomposition under another, leaving nonvolatile impurities behind.

Step 1: Identify the volatile intermediate Impure nickel reacts with carbon monoxide at about 330–350 K to form nickel tetracarbonyl:

$$Ni + 4\,CO \longrightarrow Ni(CO)_4.$$

The product is volatile and is separated from impurities that do not undergo the same selective carbonyl formation. Nickel remains associated with four carbon monoxide ligands in the transported species.

Step 2: Distinguish the other refining and extraction reactions Heating the separated carbonyl to about 450–470 K decomposes it:

$$Ni(CO)_4 \longrightarrow Ni + 4\,CO.$$

The deposited nickel is pure. Option (A) is reduction of zinc oxide by carbon, an extraction step. Option (B) is displacement of gold from its cyanide complex by zinc. Option (C), formation of volatile zirconium iodide, belongs to the van Arkel method, which uses an iodide rather than a carbonyl. Only option (D) contains the characteristic nickel–carbon monoxide conversion of the Mond process.

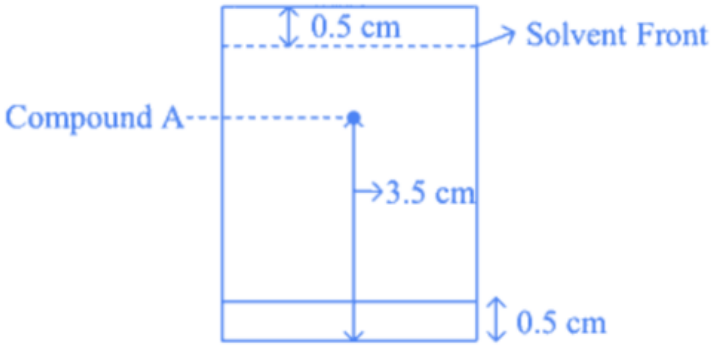
Step 3: Conclusion The reaction representing the Mond process is $Ni + 4\,CO \longrightarrow Ni(CO)_4$. Hence, the correct option is (D).

Quick Tip to Solve: Associate Mond with nickel carbonyl and van Arkel with volatile iodides of titanium or zirconium; the volatile compound identifies the process.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Following chromatogram was developed by adsorption of compound 'A' on a 6 cm TLC glass plate. Retardation factor of the compound 'A' is _____ $\times 10^{-1}$. [4 marks]



Solution:

Concept:

- The retardation factor is $R_f = d_A/d_s$, where both distances are measured from the original baseline: d_A is the spot travel and d_s is the solvent-front travel.
- The ratio is dimensionless and normally lies between zero and one. Plate length is not automatically the distance moved by the solvent.

Step 1: Read the reference lines in the chromatogram The plate is $6\,\text{cm}$ long. The starting baseline lies $0.5\,\text{cm}$ above its lower edge, and the solvent front lies $0.5\,\text{cm}$ below its upper edge. Thus the solvent travels

$$d_s = 6 - 0.5 - 0.5 = 5.0\,\text{cm}.$$

The compound spot is shown $3.5\,\text{cm}$ above the lower edge, so its distance from the starting line is

$$d_A = 3.5 - 0.5 = 3.0\,\text{cm}.$$

Step 2: Calculate the ratio and extract the requested coefficient Use the distances measured from the same starting line:

$$R_f = \frac{3.0}{5.0} = 0.60 = 6 \times 10^{-1}.$$

Dividing 3.5 by 6 would mix distances from plate edges with actual migration distances. Likewise, subtracting only one edge margin from the solvent distance would overestimate its travel. The spot remains behind the solvent front, as required by the ratio being less than one; this provides a useful consistency check on the reference-line reading.

Step 3: Conclusion The retardation factor is $0.60 = 6 \times 10^{-1}$. The number to enter is $\boxed{6}$.

Quick Tip to Solve: Draw an imaginary zero at the spotting baseline, then subtract its height from both measured positions before forming the TLC ratio.

82. Millimoles of calcium hydroxide required to produce 100 mL of the aqueous solution of pH 12 is $x \times 10^{-1}$. The value of x is _____ (Nearest integer).

Assume complete dissociation. [4 marks]

Solution:

Concept:

- For an aqueous solution at the standard temperature used for $pK_w = 14$, $\text{pH} + \text{pOH} = 14$ and $[\text{OH}^-] = 10^{-\text{pOH}}$.
- Complete dissociation of calcium hydroxide gives two hydroxide ions per formula unit: $\text{Ca(OH)}_2 \longrightarrow \text{Ca}^{2+} + 2\,\text{OH}^-$.

Step 1: Convert pH into hydroxide concentration The solution has $\text{pH} = 12$, hence

$$\text{pOH} = 14 - 12 = 2, \qquad [\text{OH}^-] = 10^{-2}\,\text{mol}\,\text{L}^{-1}.$$

Convert the specified volume before using concentration:

$$100\,\text{mL} = 0.100\,\text{L}.$$

The amount of hydroxide required is

$$n(\text{OH}^-) = 10^{-2} \times 0.100 = 1.00 \times 10^{-3}\,\text{mol}.$$

Step 2: Apply the dissociation ratio and requested scaling Each mole of Ca(OH)_2 produces two moles of hydroxide ions, so

$$n(\text{Ca(OH)}_2) = \frac{1.00 \times 10^{-3}}{2} = 5.00 \times 10^{-4}\,\text{mol}.$$

Converting moles to millimoles gives

$$n = 0.500\,\text{mmol} = 5 \times 10^{-1}\,\text{mmol}.$$

Therefore the coefficient x, rather than the amount expressed as a decimal, is the requested numerical answer. Treating the base as monobasic would incorrectly double the required amount; the factor of two must come from its dissociation equation.

Step 3: Conclusion The required amount is $0.5\,\text{mmol}$, so $x = \boxed{5}$ (nearest integer).

Quick Tip to Solve: Find hydroxide moles first, divide by the number of hydroxide ions released per formula unit, and only then convert to millimoles.

83. For certain chemical reaction $X \rightarrow Y$, the rate of formation of product is plotted against the time as shown in the figure. The number of correct statement/s from the following is _____.

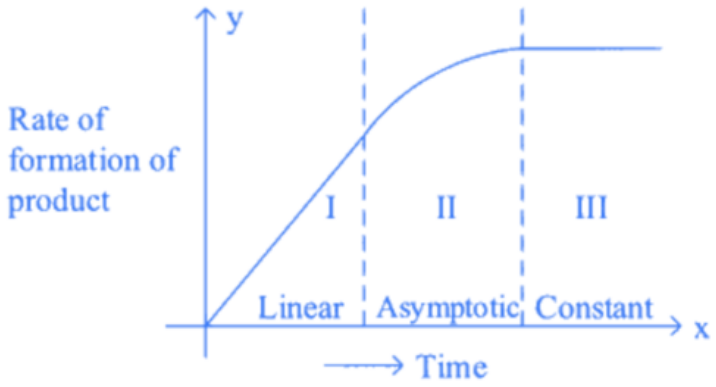
(A) Over all order of this reaction is one.

(B) Order of this reaction can't be determined.

(C) In region I and III, the reaction is of first and zero order respectively.

(D) In region-II, the reaction is of first order.

(E) In region-II, the order of reaction is in the range of 0.1 to 0.9. [4 marks]



Solution:

Concept:

- Reaction order is defined through concentration dependence, for example $r = k[X]^n$. It is not defined by the slope or curvature of a rate-versus-time graph.
- A change in rate with time can reflect changing concentrations, intermediate concentrations or a reaction mechanism. Establishing order requires an appropriate concentration-rate relation or tested integrated law.

Step 1: Identify what the plotted axes establish The vertical axis is the rate of formation of product and the horizontal axis is time. Region I shows rate increasing linearly with time, region II shows a curved approach to a plateau, and region III shows approximately constant rate. These describe temporal behaviour only. In particular, a straight line of rate against time is not the defining first-order graph; first-order decay is established by the logarithm of reactant concentration against time under the relevant conditions.

Step 2: Evaluate all five statements (A) is unsupported because the graph does not specify a concentration-dependent rate law. (B) is correct: the order cannot be determined from this graph alone. (C) is not justified: a linear rise in rate does not prove first order, and a constant rate segment by itself does not establish the complete reaction law. (D) is unsupported because curvature against time is not a first-order criterion. (E) likewise has no basis: the numerical interval 0.1–0.9 for order cannot be inferred from the curvature shown. Thus none of the proposed order assignments follows from the supplied information.

Step 3: Conclusion Only statement (B) is correct. The number of correct statements is $\boxed{1}$.

Quick Tip to Solve: Always read the axes before assigning reaction order: rate versus concentration and rate versus time are different plots with different implications.

84. The number of molecules or ions from the following, which do not have odd number of electrons are _____.

(A) NO_2

(B) ICl_4^-

(C) BrF_3

(D) ClO_2

(E) NO_2^+

(F) NO

[4 marks]

Solution:

Concept:

- The total electron count of a neutral species is the sum of atomic numbers. Add one electron for each unit negative charge and subtract one for each unit positive charge.
- The same odd/even classification follows from valence-electron counting, because the inner-shell electron counts of the atoms here are even.

Step 1: Count the electrons in each species Using atomic numbers $N = 7$, $O = 8$, $Cl = 17$, $Br = 35$, $F = 9$ and $I = 53$:

$$\begin{aligned} NO_2 &: 7 + 2(8) = 23 \quad \text{(odd)}, \\ ICl_4^- &: 53 + 4(17) + 1 = 122 \quad \text{(even)}, \\ BrF_3 &: 35 + 3(9) = 62 \quad \text{(even)}, \\ ClO_2 &: 17 + 2(8) = 33 \quad \text{(odd)}, \\ NO_2^+ &: 7 + 2(8) - 1 = 22 \quad \text{(even)}, \\ NO &: 7 + 8 = 15 \quad \text{(odd)}. \end{aligned}$$

Step 2: Select the species satisfying the negative wording The question asks for species that do not have an odd number of electrons, so select the even counts rather than the odd ones. These are ICl_4^-, BrF_3 and NO_2^+. There are three such entries. The charge correction is essential: adding an electron to neutral ICl_4 changes an odd count to even, and removing an electron from NO_2 does the same. No molecular geometry or hybridisation assignment is needed to decide this electron-counting question.

Step 3: Conclusion The qualifying species are (B), (C) and (E). Their number is $\boxed{3}$.

Quick Tip to Solve: Count total electrons and apply the ionic charge before checking parity; the phrase "do not have odd means select even-electron species.

85. Following figure shows dependence of molar conductance of two electrolytes on concentration. $\Lambda \overset{o}{m}$ is the limiting molar conductivity.

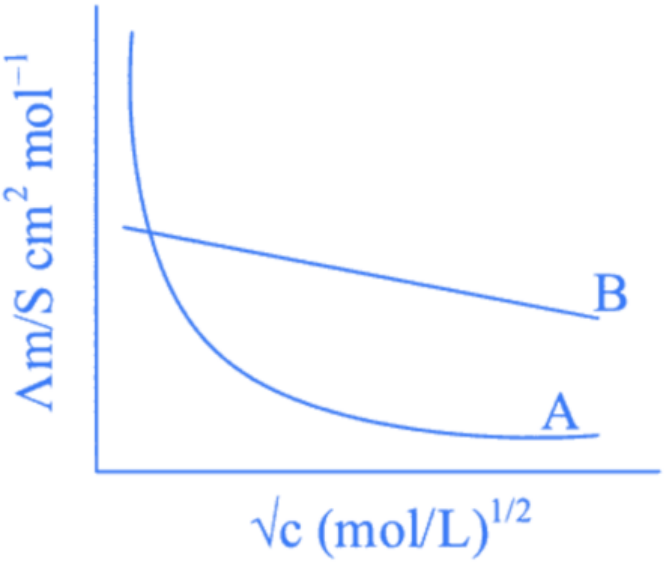
The number of incorrect statement(s) from the following is _____

(A) $\Lambda \overset{o}{m}$ for electrolyte A is obtained by extrapolation

(B) For electrolyte B, Λm vs $\sqrt{c}$ graph is a straight line with intercept equal to $\Lambda \overset{o}{m}$

(C) At infinite dilution, the value of degree of dissociation approaches zero for electrolyte B.

(D) $\Lambda \overset{o}{m}$ for any electrolyte A and B can be calculated using λ° for individual ions [4 marks]



Solution:

Concept:

- A strong electrolyte is substantially dissociated throughout the dilute concentration range. Its molar conductivity follows $\Lambda_m = \Lambda_m^\circ - K\sqrt{c}$, where K is positive.
- A weak electrolyte dissociates increasingly on dilution, so its molar conductivity changes nonlinearly. Kohlrausch's law gives $\Lambda_m^\circ = \nu_+\lambda_+^\circ + \nu_-\lambda_-^\circ$ for either type.

Step 1: Identify the two electrolytes The nearly straight descending line B is characteristic of a strong electrolyte: its conductivity changes mainly because interionic effects diminish on dilution. The strongly curved line A represents a weak electrolyte, for which the degree of dissociation changes markedly. The horizontal coordinate is $\sqrt{c}$, not concentration itself; the distinction matters when interpreting a straight-line extrapolation.

Step 2: Check statements A and B Statement A is incorrect. The curved plot for weak electrolyte A cannot be extrapolated reliably to obtain its limiting molar conductivity; Kohlrausch's law is used instead. Statement B is correct: the dilute-solution straight line for strong electrolyte B has the ordinate intercept Λ_m° at $\sqrt{c} = 0$. Its slope is negative, but the limiting intercept is positive.

Step 3: Check statements C and D Statement C is incorrect. At infinite dilution, dissociation is complete, so the degree of dissociation approaches unity, not zero. Statement D is correct: at infinite dilution ions migrate independently, allowing limiting ionic conductivities to be added with their stoichiometric coefficients for both A and B. This law does not require a linear experimental plot for

the weak electrolyte.

Step 4: Conclusion The incorrect statements are A and C. Therefore, the required number is $\boxed{2}$.

Quick Tip to Solve: First identify a weak electrolyte by its curved plot; straight-line extrapolation applies to the strong electrolyte, while limiting ionic conductivities work for both.

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86. Solid Lead nitrate is dissolved in 1 litre of water. The solution was found to boil at 100.15°C. When 0.2 mol of NaCl is added to the resulting solution, it was observed that the solution froze at $-0.8°$ C. The solubility product of $PbCl_2$ formed is _____ $\times 10^{-6}$ at 298 K. (Nearest integer)

Given : $K_b = 0.5$ K kg mol^{-1} and $K_f = 1.8$ K kg mol^{-1}. Assume molality to the equal to molarity in all cases. [4 marks]

Solution:

Concept:

- Boiling-point elevation and freezing-point depression count dissolved particles: $\Delta T_b = K_b m_{\text{particles}}$ and $\Delta T_f = K_f m_{\text{particles}}$. The stated approximation permits molarity to replace molality.
- For $PbCl_2(s) \rightleftharpoons Pb^{2+}(aq) + 2\,Cl^-(aq)$, $K_{sp} = [Pb^{2+}][Cl^-]^2$. Precipitated ions no longer contribute to the colligative effect.

Step 1: Calculate the original lead nitrate concentration The initial elevation is $100.15 - 100 = 0.15\,K$. Lead nitrate dissociates into one lead ion and two nitrate ions, giving three particles per formula unit. Thus, if its concentration is c,

$$3c = \frac{0.15}{0.5} = 0.30, \qquad c = 0.10\,\text{mol}\,L^{-1}.$$

Before adding salt, the solution therefore contains 0.10 mol lead ions and 0.20 mol nitrate ions in the stated one-litre basis.

Step 2: Use the freezing point to determine precipitation Adding 0.20 mol $NaCl$ supplies 0.20 mol each of sodium and chloride ions. Before precipitation the total particle concentration would be $0.30 + 0.40 = 0.70$. Let x mol of $PbCl_2$ precipitate. Each mole removes three moles of dissolved ions, so

$$0.70 - 3x = \frac{0.80}{1.8} = \frac{4}{9}.$$

$$x = \frac{23}{270} = 0.085185\,\text{mol}.$$

Nitrate and sodium are spectator ions, and their amounts remain unchanged.

Step 3: Evaluate the remaining ion product The equilibrium concentrations on the same volume basis are

$$[Pb^{2+}] = 0.10 - x = \frac{2}{135}, \qquad [Cl^-] = 0.20 - 2x = \frac{4}{135} \quad \text{mol}\,L^{-1}.$$

Consequently,

$$K_{sp} = \frac{2}{135}\left(\frac{4}{135}\right)^2 = 1.3006 \times 10^{-5} = 13.006 \times 10^{-6}.$$

Both residual concentrations are positive, consistent with partial precipitation and a saturated solution.

Step 4: Conclusion $K_{sp} \simeq 13.006 \times 10^{-6}$. The required nearest integer is $\boxed{13}$.

Quick Tip to Solve: Count every dissolved ion, including spectators. Precipitating one formula unit of lead chloride removes three particles from the solution.

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87. Water decomposes at 2300 K

$$H_2O(g) \rightarrow H_2(g) + \frac{1}{2}O_2(g)$$

The percent of water decomposing at 2300 K and 1 bar is _____ (Nearest integer). Equilibrium constant for the reaction is 2×10^{-3} at 2300 K. [4 marks]

Solution:

Concept:

- The degree of dissociation α is the fraction of the original water decomposed. Equilibrium partial pressures are mole fractions multiplied by the total pressure.
- For $H_2O(g) \rightleftharpoons H_2(g) + \frac{1}{2}O_2(g)$, the dimensionless pressure equilibrium constant is

$$K_p = \frac{(p_{H_2}/p^\circ)(p_{O_2}/p^\circ)^{1/2}}{p_{H_2O}/p^\circ}$$

, with $p^\circ = 1\,\text{bar}$.

Step 1: Express the equilibrium composition Start with one mole of water vapour. When a fraction α decomposes, the equilibrium mole amounts of water, hydrogen and oxygen are respectively $1-\alpha$, α and $\alpha/2$. The total is $1+\alpha/2$. Hence the partial pressures at total pressure P are

$$p_{H_2O} = \frac{1-\alpha}{1+\alpha/2}P, \quad p_{H_2} = \frac{\alpha}{1+\alpha/2}P, \quad p_{O_2} = \frac{\alpha/2}{1+\alpha/2}P.$$

The change in total moles must be included even though the conversion is small.

Step 2: Substitute in the equilibrium constant At $P = p^\circ = 1\,\text{bar}$, substitution and cancellation give

$$2 \times 10^{-3} = \frac{\alpha}{1-\alpha}\sqrt{\frac{\alpha}{2+\alpha}}.$$

Since the equilibrium constant is small, an initial estimate takes $1-\alpha \simeq 1$ and $2+\alpha \simeq 2$. This yields

$$\alpha \simeq \left(2(2 \times 10^{-3})^2\right)^{1/3} = 0.0200.$$

The estimate justifies the small-dissociation assumption.

Step 3: Refine and round the percentage Keeping the original denominators, the squared equation is

$$4 \times 10^{-6}(1-\alpha)^2(2+\alpha) = \alpha^3.$$

Its physically admissible root is approximately $\alpha = 0.01980$. Thus the conversion is about 1.98%, very close to the initial estimate. The other mathematical possibilities are irrelevant unless they give a real fraction between zero and one.

Step 4: Conclusion The percentage of water decomposed is approximately 1.98%. Rounded to the nearest integer, the answer is $\boxed{2}$.

Quick Tip to Solve: Use a one-mole basis and include the $\alpha/2$ increase in total moles; check that the small-dissociation approximation leaves the required rounding unchanged.

88. Consider the following reaction approaching equilibrium at 27°C and 1 atm pressure

$$\mathrm{A} + \mathrm{B} \underset{k_r=10^2}{\overset{k_f=10^3}{\rightleftharpoons}} \mathrm{C} + \mathrm{D}$$

The standard Gibb's energy change $(\Delta_r G^\theta)$ at 27°C is (−) _____ kJ mol^{-1} (Nearest integer).

(Given : $\mathrm{R} = 8.3\ \mathrm{J\ K^{-1}\ mol^{-1}}$ and $\ln 10 = 2.3$) [4 marks]

Solution:

Concept:

- For an elementary reversible reaction at equilibrium, equality of forward and reverse rates gives $K = k_f/k_r$. Both directions here have the same overall concentration order.
- Standard reaction Gibbs energy is related to the equilibrium constant by $\Delta_r G^\circ = -RT \ln K$. Temperature must be absolute and joules must be converted to kilojoules.

Step 1: Find the equilibrium constant from the rates For the reaction $\mathrm{A} + \mathrm{B} \rightleftharpoons \mathrm{C} + \mathrm{D}$, the forward rate is $k_f[\mathrm{A}][\mathrm{B}]$ and the reverse rate is $k_r[\mathrm{C}][\mathrm{D}]$. At equilibrium these rates are equal, even though neither rate must vanish. Rearrangement gives

$$K = \frac{[\mathrm{C}][\mathrm{D}]}{[\mathrm{A}][\mathrm{B}]} = \frac{k_f}{k_r} = \frac{10^3}{10^2} = 10.$$

Because the stoichiometric mole totals on both sides are equal, no extra pressure-power conversion is required here.

Step 2: Calculate the standard Gibbs energy The stated temperature is $27^\circ\mathrm{C} = 300\,\mathrm{K}$. Use the supplied values rather than replacing them with more precise constants:

$$\Delta_r G^\circ = -8.3 \times 300 \times \ln 10 \quad \mathrm{J\,mol^{-1}}.$$

$$\Delta_r G^\circ = -8.3 \times 300 \times 2.3 = -5727\,\mathrm{J\,mol^{-1}}.$$

The negative sign is expected because $K > 1$: the standard-state reaction favours products.

Step 3: Convert units and interpret the requested blank Divide by 1000 to change joules to kilojoules:

$$\Delta_r G^\circ = -5.727\,\mathrm{kJ\,mol^{-1}}.$$

The question already prints a negative sign before the blank, so the numerical entry is the positive

magnitude. Rounding 5.727 to the nearest integer gives 6. The actual Gibbs energy change at equilibrium is zero; this calculation concerns the standard reaction Gibbs energy, which need not be zero.

Step 4: Conclusion $\Delta_r G^\circ \simeq -5.727\,\text{kJ mol}^{-1}$. With the minus sign already supplied, the blank is $\boxed{6}$.

Quick Tip to Solve: Form k_f/k_r before taking the logarithm, and distinguish the standard Gibbs energy from the zero actual Gibbs energy at equilibrium.

89. The sum of bridging carbonyls in W(CO)_6 and $\text{Mn}_2\text{(CO)}_{10}$ is _____. [4 marks]

Solution:

Concept:

- A terminal carbonyl ligand binds through its carbon atom to one metal centre. A bridging carbonyl binds between two or more metal centres and is conventionally denoted μ-CO.
- A metal–metal bond can complete the electron count in a polynuclear carbonyl without requiring any bridging carbonyl ligands. Count ligand attachments from the actual molecular structure.

Step 1: Examine tungsten hexacarbonyl W(CO)_6 is a mononuclear octahedral complex. Its six carbonyl ligands are all terminal and each donates a lone pair from carbon to the same tungsten atom. Since the molecule contains only one metal centre, none of its carbonyls bridges between metal atoms. Tungsten contributes six valence electrons, and the six terminal carbonyls contribute twelve, giving the familiar eighteen-electron total. Its number of bridging carbonyls is therefore zero.

Step 2: Examine dimanganese decacarbonyl $\text{Mn}_2\text{(CO)}_{10}$ consists of two Mn(CO)_5 units joined by a manganese–manganese bond. Each manganese has five terminal carbonyl ligands; the compound does not contain bridging CO groups. At each metal, manganese supplies seven valence electrons, the five terminal carbonyls supply ten, and the metal–metal bond supplies one electron to its electron count. The resulting total is eighteen. Thus a bridge is not needed to complete either metal's valence shell.

Step 3: Add the two structural counts The question asks for the sum of bridging carbonyls, not the total number of carbonyl ligands or the number of metal atoms. The six ligands in the tungsten compound and the ten in the manganese compound are all terminal. Consequently,

$$N_{\text{bridging}} = N_{\text{W(CO)}_6} + N_{\text{Mn}_2\text{(CO)}_{10}} = 0 + 0 = 0.$$

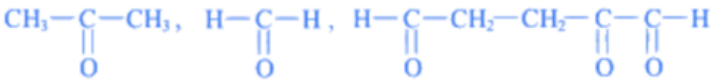
A direct metal–metal bond in the second compound must not be mistaken for a carbonyl bridge.

Step 4: Conclusion Neither compound has a bridging carbonyl ligand. The required sum is $\boxed{0}$.

Quick Tip to Solve: Do not infer bridging CO merely from the presence of two metal atoms: dimanganese decacarbonyl has a direct metal–metal bond and ten terminal CO ligands.

90. 17 mg of a hydrocarbon (M.F. $C_{10}H_{16}$) takes up 8.40 mL of the H_2 gas measured at 0°C and 760 mm of Hg. Ozonolysis of the same hydrocarbon yields

The number of double bond/s present in the hydrocarbon is _____. [4 marks]



Solution:

Concept:

- Complete catalytic hydrogenation consumes one mole of H_2 per mole of carbon–carbon double bonds. At the stated standard conditions, one mole of gas occupies $22.4\,\text{L}$.
- The degree of unsaturation for a hydrocarbon C_cH_h is $(2c + 2 - h)/2$. Each double bond or ring counts one, whereas a triple bond counts two.

Step 1: Calculate moles of hydrocarbon The molecular formula is $C_{10}H_{16}$, so the molar mass using the usual atomic masses is

$$M = 10(12) + 16(1) = 136\,\text{g mol}^{-1}.$$

Convert the sample mass from milligrams to grams before dividing:

$$n_{\text{hydrocarbon}} = \frac{17 \times 10^{-3}}{136} = 1.25 \times 10^{-4}\,\text{mol}.$$

This is the number of molecules available to absorb the measured hydrogen; the number of carbon atoms does not multiply this mole amount.

Step 2: Calculate hydrogen uptake per molecule The hydrogen volume is $8.40\,\text{mL} = 8.40 \times 10^{-3}\,\text{L}$ at $0°\text{C}$ and $760\,\text{mm Hg}$, so

$$n_{H_2} = \frac{8.40 \times 10^{-3}}{22.4} = 3.75 \times 10^{-4}\,\text{mol}.$$

The mole ratio is

$$\frac{n_{H_2}}{n_{\text{hydrocarbon}}} = \frac{3.75 \times 10^{-4}}{1.25 \times 10^{-4}} = 3.$$

Thus hydrogenation removes three equivalents of carbon–carbon unsaturation per hydrocarbon molecule.

Step 3: Use ozonolysis and the formula to identify double bonds The displayed ozonolysis products are carbonyl compounds, the characteristic cleavage products of carbon–carbon double bonds. Hence the hydrogen uptake is assigned to alkene double bonds rather than to a triple bond. As a consistency check, the molecular formula gives

$$\text{DBE} = \frac{2(10) + 2 - 16}{2} = 3.$$

Three double bonds account for the entire degree of unsaturation, leaving no additional ring or triple bond. Each such bond takes up one molecule of hydrogen on saturation.

Step 4: Conclusion The hydrocarbon contains $\boxed{3}$ carbon–carbon double bonds.

Quick Tip to Solve: Divide hydrogen moles by hydrocarbon moles first; then use the ozonolysis products to distinguish alkene bonds from other sources of unsaturation.