JEE Main 2023 Paper 1 (B.E./B.Tech) Jan 29 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 29 January 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $y = y(x)$ be the solution of the differential equation $x\log_e x\frac{dy}{dx} + y = x^2\log_e x, (x > 1)$. If $y(2) = 2$, then $y(e)$ is equal to [4 marks]

(A) $\frac{1+e^2}{2}$

(B) $\frac{1+e^2}{4}$

(C) $\frac{2+e^2}{2}$

(D) $\frac{4+e^2}{4}$

Correct Answer: (D) $\frac{4+e^2}{4}$

Solution:

Concept:

- A first-order equation $y' + P(x)y = Q(x)$ has integrating factor $\exp(\int P(x)\,dx)$. Multiplication by this factor makes the left side an exact product derivative.
- Here $x > 1$, so $\ln x > 0$ and division by $x \ln x$ is valid. Integration by parts evaluates $\int x \ln x\,dx$.

Step 1: Put the equation in linear form Divide the given equation by $x \ln x$:

$$y' + \frac{y}{x \ln x} = x.$$

The integrating factor is therefore

$$I(x) = \exp\left(\int \frac{dx}{x \ln x}\right) = \exp(\ln(\ln x)) = \ln x.$$

We retain this factor, rather than x, because the coefficient of y contains both x and its logarithm.

Step 2: Integrate the product derivative Multiplying by $\ln x$ gives

$$\frac{d}{dx}(y \ln x) = x \ln x.$$

Integrate by parts with $u = \ln x$ and $dv = x\,dx$:

$$y \ln x = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C.$$

The negative second term is essential: differentiating the right side gives precisely $x \ln x$.

Step 3: Use the initial value and evaluate At $x = 2$, the condition $y(2) = 2$ gives

$$2 \ln 2 = 2 \ln 2 - 1 + C, \qquad C = 1.$$

Now $\ln e = 1$, so

$$y(e) = \frac{e^2}{2} - \frac{e^2}{4} + 1 = \frac{e^2 + 4}{4}.$$

The other expressions give a different coefficient of e^2 or a different constant, and do not match this uniquely determined solution.

Step 4: Conclusion The value at $x = e$ is $\frac{4+e^2}{4}$. Hence, the correct option is (D).

Quick Tip to Solve: After finding the integrating factor $\ln x$, determine the integration constant before substituting $x = e$; the initial condition contributes the final $+1$.

2. Let R be a relation defined on $\mathbb{N}$ as $a\mathrm{R}b$ if $2a + 3b$ is a multiple of $5, a, b \in \mathbb{N}$. Then R is [4 marks]

(A) an equivalence relation

(B) non reflexive

(C) symmetric but not transitive

(D) transitive but not symmetric

Correct Answer: (A) an equivalence relation

Solution:

Concept:

- A relation is an equivalence relation exactly when it is reflexive, symmetric and transitive on its domain.
- Divisibility by 5 can be tested by congruences. Since $3 \equiv -2 \pmod 5$ and 2 is invertible modulo 5, the condition simplifies to equality of remainders.

Step 1: Simplify the defining condition For arbitrary $a, b \in \mathbb{N}$,

$$2a + 3b \equiv 2(a - b) \pmod 5.$$

As $\gcd(2, 5) = 1$, divisibility of $2(a - b)$ by 5 is equivalent to divisibility of $a - b$. Thus

$$aRb \iff a \equiv b \pmod 5.$$

This description concerns all natural numbers, rather than a few examples of related pairs.

Step 2: Verify reflexivity and symmetry For any a, we have $a - a = 0$, a multiple of 5. Equivalently, $2a + 3a = 5a$, so aRa; hence R is reflexive. If aRb, then $a - b$ is divisible by 5, and its negative $b - a$ is also divisible by 5. Therefore bRa, which proves symmetry. This rules out both the non-reflexive description and the description that denies symmetry.

Step 3: Verify transitivity If aRb and bRc, write $a - b = 5m$ and $b - c = 5n$ for integers m, n. Adding gives

$$a - c = 5(m + n).$$

Thus $a \equiv c \pmod 5$, so aRc. This proves transitivity and also rules out the option that claims symmetry without transitivity. All three properties hold on the entire specified domain.

Step 4: Conclusion R is an equivalence relation, namely congruence modulo 5 on $\mathbb{N}$. Hence, the correct option is (A).

Quick Tip to Solve: For a divisibility relation, reduce the coefficients modulo the divisor first. Here $2a + 3b$ becomes $2(a - b)$, revealing the familiar congruence relation.

3. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is : [4 marks]

(A) 79

(B) 84

(C) 89

(D) 86

Correct Answer: (C) 89

Solution:

Concept:

- In dictionary order, compare words from left to right using alphabetical order. Each smaller available letter at a position contributes the factorial of the number of remaining positions.
- The five letters are distinct and have order $G < H < O < T < U$. The serial number is one plus the number of words preceding TOUGH.

Step 1: Count words with a smaller first letter The target is TOUGH. The letters smaller than its first letter T are G, H and O. For each such choice, the remaining four distinct letters can be arranged in $4!$ ways. Therefore the number of preceding words whose first letter differs from T is

$$3 \cdot 4! = 3 \cdot 24 = 72.$$

Words starting with U come after the target and contribute nothing.

Step 2: Fix T and count the next positions After fixing T, the available letters are G, H, O and U. The second letter of TOUGH is O, and the smaller choices G and H each allow $3!$ completions:

$$2 \cdot 3! = 12.$$

After fixing TO, the remaining letters are G, H and U. The third letter U has two smaller available choices, giving

$$2 \cdot 2! = 4.$$

These blocks are disjoint because their first position of difference from the target is different.

Step 3: Finish the count and convert it to a rank After fixing TOU, only G and H remain. The fourth letter G is the smaller one, so there is no earlier word in this final block; H is then forced. Altogether

$$N_{\text{before}} = 72 + 12 + 4 = 88.$$

A serial number starts at one, not zero. Consequently

$$\text{rank}(\text{TOUGH}) = 88 + 1 = 89.$$

The other offered values omit a block or fail to count the target itself correctly.

Step 4: Conclusion The serial number of TOUGH is 89. Hence, the correct option is (C).

Quick Tip to Solve: Write the alphabetical order first, count only unused smaller letters at each position, and add one at the end to turn a count of predecessors into a serial number.

4. If $\vec{a} = \hat{i} + 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$. Then $\vec{r} \cdot \vec{c}$ is equal to : [4 marks]

(A) 36

(B) 30

(C) 34

(D) 32

Correct Answer: (C) 34

Solution:

Concept:

- The cross product is antisymmetric: $\vec{b} \times \vec{c} = -\vec{c} \times \vec{b}$. A vector with zero cross product with a nonzero vector is parallel to it.
- The condition $\vec{r} \cdot \vec{a} = 0$ fixes the scalar in the resulting parallel-vector expression. The required value then follows from the ordinary dot product.

Step 1: Convert the cross-product condition to parallelism Using antisymmetry, the given equation becomes

$$\vec{r} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}, \qquad (\vec{r} - \vec{c}) \times \vec{b} = \vec{0}.$$

Since $\vec{b} = (1, 1, 1) \neq \vec{0}$, there is a real number t such that

$$\vec{r} = \vec{c} + t\vec{b} = (7 + t, -3 + t, 4 + t).$$

The order of the cross products matters; changing its sign would produce a different and incorrect vector.

Step 2: Use the perpendicularity condition The vector $\vec{a} = (1, 0, 2)$. Hence

$$0 = \vec{r} \cdot \vec{a} = (7 + t) + 2(4 + t) = 15 + 3t.$$

It follows that $t = -5$, and therefore

$$\vec{r} = (2, -8, -1).$$

As a check, $\vec{r} - \vec{c} = (-5, -5, -5) = -5\vec{b}$, so the cross-product equation is satisfied; also $2 + 2(-1) = 0$, confirming the second condition.

Step 3: Evaluate the requested scalar product With $\vec{c} = (7, -3, 4)$, calculate component by component:

$$\vec{r} \cdot \vec{c} = 2(7) + (-8)(-3) + (-1)(4) = 14 + 24 - 4 = 34.$$

Both negative middle components produce a positive contribution. The alternatives 36, 30 and 32 differ from this scalar product and cannot satisfy the uniquely determined value.

Step 4: Conclusion $\vec{r} \cdot \vec{c} = 34$. Hence, the correct option is (C).

Quick Tip to Solve: Collect the two cross products into $(\vec{r} - \vec{c}) \times \vec{b}$ before expanding components. One scalar parameter then replaces three unknown components.

5. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is :

[4 marks]

(A) 400

(B) 472

(C) 507

(D) 432

Correct Answer: (D) 432

Solution:

Concept:

- Three-digit positive integers run from 100 through 999, including both endpoints. The number divisible by d is $\lfloor 999/d \rfloor - \lfloor 99/d \rfloor$.
- For divisibility by 3 or 4, use inclusion and exclusion with $\text{lcm}(3, 4) = 12$. Every multiple of 48 already belongs to that union and is then removed once.

Step 1: Count the multiples of each divisor Let A contain the three-digit multiples of 3 and B the three-digit multiples of 4. Their sizes are

$$|A| = 333 - 33 = 300, \qquad |B| = 249 - 24 = 225.$$

Numbers divisible by both are multiples of 12, so

$$|A \cap B| = 83 - 8 = 75.$$

The subtraction at the lower end excludes all one-digit and two-digit multiples; it must use 99 rather than 100.

Step 2: Form the union without double counting The word either here counts numbers divisible by at least one of the two divisors. A multiple of 12 was included in both separate counts, so subtract it once:

$$|A \cup B| = 300 + 225 - 75 = 450.$$

Subtracting twice would incorrectly remove numbers divisible by both 3 and 4, although the question excludes only multiples of 48.

Step 3: Remove the forbidden multiples of 48 As 48 is divisible by both 3 and 4, every three-digit multiple of 48 lies in the union just counted. There are

$$\left\lfloor \frac{999}{48} \right\rfloor - \left\lfloor \frac{99}{48} \right\rfloor = 20 - 2 = 18$$

such numbers, from 144 through 960. Thus the required count is

$$450 - 18 = 432.$$

The options 400, 472 and 507 do not equal the union count after this explicit exclusion.

Step 4: Conclusion There are 432 such three-digit numbers. Hence, the correct option is (D).

Quick Tip to Solve: First count the union of multiples of 3 and 4, using multiples of 12 for the overlap. Only then remove multiples of 48, since all of them are already in that union.

6. Let $S = \{w_1, w_2, ...\}$ be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}, n \geq 2$. Let $A = \{2k + 3l : k, l \in N\}$ and $B = \{w_n : n \in A\}$. Then P(B) is equal to : [4 marks]

(A) $\frac{3}{32}$

(B) $\frac{1}{32}$

(C) $\frac{1}{16}$

(D) $\frac{3}{64}$

Correct Answer: (D) $\frac{3}{64}$

Solution:

Concept:

- The probabilities of all elementary outcomes sum to one. A constant ratio of $1/2$ makes their sequence geometric.
- Here $k, l \in \mathbb{N}$ are positive integers. To find the event B, identify the distinct possible indices $2k + 3l$; different representations of an index do not create additional outcomes.

Step 1: Normalize the probability sequence Let $p = P(w_1)$. The recurrence gives $P(w_n) = p/2^{n-1}$. Using the infinite geometric sum,

$$1 = \sum_{n=1}^{\infty} P(w_n) = \frac{p}{1 - 1/2} = 2p,$$

so $p = 1/2$ and

$$P(w_n) = \frac{1}{2^n}.$$

Normalization is necessary before summing the probability of the specified event.

Step 2: Determine exactly which indices occur The smallest value of $2k + 3l$ is 5, obtained from $k = l = 1$. The next possible values are 7 and 8, from $(k, l) = (2, 1)$ and $(1, 2)$. Increasing k by one adds 2, so 7 generates every odd integer at least 7, and 8 generates every even integer at least 8. The value 6 is impossible: with $l = 1$, it would require $2k = 3$; with $l \geq 2$, the value is at least 8. Thus

$$A = \{5\} \cup \{7, 8, 9, \ldots\}.$$

Step 3: Sum the probabilities of these outcomes There is one outcome w_n for each distinct index, regardless of how many pairs (k, l) represent it. Therefore

$$P(B) = \frac{1}{2^5} + \sum_{n=7}^{\infty} \frac{1}{2^n} = \frac{1}{32} + \frac{1/128}{1 - 1/2} = \frac{1}{32} + \frac{1}{64} = \frac{3}{64}.$$

The value $1/32$ counts only w_5, while $1/16$ would include the impossible index 6. The remaining option also does not match the exact sum.

Step 4: Conclusion The required probability is $\frac{3}{64}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the possible indices before summing. With positive k, l, the set starts at 5 but skips 6; use the isolated term at 5 plus the full geometric tail from 7.

7. Consider a function $f : \mathbb{N} \to \mathbb{R}$, satisfying $f(1) + 2f(2) + 3f(3) + ... + xf(x) = x(x+1)f(x); x \geq 2$ with $f(1) = 1$. Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to [4 marks]

(A) 8000

(B) 8400

(C) 8100

(D) 8200

Correct Answer: (C) 8100

Solution:

Concept:

- Two consecutive identities for a partial sum can be subtracted to isolate its last term. The given identity is valid only for integers $x \geq 2$.
- The first applicable identity must be used separately to find $f(2)$; the resulting recurrence then applies from $x = 3$ onward and telescopes.

Step 1: Find the first value allowed by the identity Insert $x = 2$ into the given equation and use $f(1) = 1$:

$$1 + 2f(2) = 2(3)f(2).$$

Hence $4f(2) = 1$, so $f(2) = 1/4$. We do not insert $x = 1$ into the stated identity, since its domain of validity explicitly begins at 2.

Step 2: Subtract two successive partial sums For an integer $n \geq 3$, the identities at n and $n - 1$ are both valid. Their left sides differ by $nf(n)$, so

$$nf(n) = n(n+1)f(n) - (n-1)nf(n-1).$$

After rearranging and dividing by n,

$$nf(n) = (n-1)f(n-1), \qquad f(n) = \frac{n-1}{n} f(n-1).$$

Thus $nf(n)$ is constant for $n \geq 2$. Its value is $2f(2) = 1/2$, giving

$$f(n) = \frac{1}{2n} \quad (n \geq 2).$$

Step 3: Compute the requested reciprocals Both 2022 and 2028 are at least 2, so the formula applies and gives

$$\frac{1}{f(2022)} + \frac{1}{f(2028)} = 2(2022) + 2(2028) = 4044 + 4056 = 8100.$$

As a check, substituting the formula into the original sum yields $1 + (n-1)/2 = (n+1)/2$, exactly equal to $n(n+1)/(2n)$. This also confirms that the exceptional value $f(1) = 1$ is handled correctly. The other options are inconsistent with these reciprocals.

Step 4: Conclusion The required value is 8100. Hence, the correct option is (C).

Quick Tip to Solve: For identities involving $\sum kf(k)$, subtract adjacent sums. Check the first allowed index separately: here the recurrence starts at 3, and $f(2) = 1/4$ fixes its constant.

8. The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solution x, is : [4 marks]

(A) $[-2, -1]$
(B) $\left[-\frac{3}{2}, -1\right]$
(C) $\left[-2, -\frac{3}{2}\right]$
(D) $\left[-1, -\frac{1}{2}\right]$

Correct Answer: (B) $\left[-\frac{3}{2}, -1\right]$

Solution:

Concept:

- Use $\cos 2x = 1 - 2\sin^2 x$ and $\cos^2 x = 1 - \sin^2 x$ to express the entire left side in one variable.
- For real x, the substitution $u = \sin^2 x$ has exactly the range $[0, 1]$. Completing the square gives the full attainable range, including its endpoints.

Step 1: Reduce the trigonometric expression Set $u = \sin^2 x$. It is important to retain the square on $\cos 2x$, rather than replacing it by $\cos^2 x$. Then

$$\begin{aligned}\cos^2 2x - 2\sin^4 x - 2\cos^2 x &= (1-2u)^2 - 2u^2 - 2(1-u)\\ &= 2u^2 - 2u - 1\\ &= 2\left(u - \frac{1}{2}\right)^2 - \frac{3}{2}.\end{aligned}$$

The original equation therefore asks which values this quadratic takes on the closed interval $0 \le u \le 1$.

Step 2: Find and check both extreme values On this interval, $0 \le (u - 1/2)^2 \le 1/4$. Consequently the minimum is $-3/2$, attained at $u = 1/2$, and the maximum is -1, attained at $u = 0$ or $u = 1$. These values correspond, for example, to $x = \pi/4$ and $x = 0$, respectively. Continuity ensures that every value between the extremes occurs. Option (A) includes impossible values below $-3/2$; option (C) misses all larger attainable values; option (D) includes values above -1.

Step 3: Conclusion The required set is $\boxed{[-3/2, -1]}$. Hence, the correct option is (B).

Quick Tip to Solve: For a range involving even powers of sine and cosine, set $u = \sin^2 x$ and write its constraint $0 \le u \le 1$ before finding the quadratic extrema.

9. The area of the region $A = \{(x, y) : |\cos x - \sin x| \le y \le \sin x, 0 \le x \le \frac{\pi}{2}\}$ is [4 marks]

(A) $\sqrt{5} + 2\sqrt{2} - 4.5$

(B) $1 - \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{5}}$

(C) $\sqrt{5} - 2\sqrt{2} + 1$

(D) $\frac{3}{\sqrt{5}} - \frac{3}{\sqrt{2}} + 1$

Correct Answer: (C) $\sqrt{5} - 2\sqrt{2} + 1$

Solution:

Concept:

- The vertical thickness of the region is upper boundary minus lower boundary, provided that this difference is nonnegative.
- The absolute value $|\cos x - \sin x|$ changes its expression at $x = \pi/4$; the region begins only where this lower boundary meets $\sin x$.

Step 1: Determine the actual interval of the region For $0 \le x \le \pi/4$, the lower boundary is $\cos x - \sin x$. The condition that it lie below $\sin x$ is $\tan x \ge 1/2$. Thus put $\alpha = \tan^{-1}(1/2)$, so the first piece runs from α to $\pi/4$. For $\pi/4 \le x \le \pi/2$, the lower boundary is $\sin x - \cos x$, which always lies below the upper boundary. No area arises from $0 \le x < \alpha$, because no ordinate satisfies both inequalities there.

Step 2: Integrate the two positive thicknesses The desired area is therefore

$$\mathcal{A} = \int_{\alpha}^{\pi/4} (2\sin x - \cos x)\, dx + \int_{\pi/4}^{\pi/2} \cos x\, dx$$
$$= [-2\cos x - \sin x]_{\alpha}^{\pi/4} + [\sin x]_{\pi/4}^{\pi/2}.$$

Since $\sin\alpha = 1/\sqrt{5}$ and $\cos\alpha = 2/\sqrt{5}$, the first contribution is $\sqrt{5} - 3/\sqrt{2}$ and the second is $1 - 1/\sqrt{2}$. Their sum is $\sqrt{5} - 2\sqrt{2} + 1$, approximately 0.408. The other options do not equal this sum: they either use an incorrect starting point or an incorrect coefficient after splitting the absolute value.

Step 3: Conclusion The area is $\boxed{\sqrt{5} - 2\sqrt{2} + 1}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: First solve lower boundary $\le$ upper boundary; integrating from the stated left endpoint without this check can count an interval where the region is empty.

10. Let $\vec{a} = 4\hat{i}+3\hat{j}$ and $\vec{b} = 3\hat{i}-4\hat{j}+5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c}.\left(\vec{a} \times \vec{b}\right)+25 = 0$, $\vec{c}.(\hat{i}+\hat{j}+\hat{k}) = 4$, and projection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{b}$ equals : [4 marks]

(A) $\frac{3}{\sqrt{2}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{5}$

(D) $\frac{5}{\sqrt{2}}$

Correct Answer: (D) $\frac{5}{\sqrt{2}}$

Solution:

Concept:

- The scalar projection of $\vec{c}$ on a nonzero vector $\vec{a}$ is $\vec{c}\cdot\vec{a}/|\vec{a}|$; it is not merely the dot product.
- Write $\vec{c} = x\hat{i}+y\hat{j}+z\hat{k}$. Each given dot product becomes a linear equation for its three components.

Step 1: Translate the constraints into equations Here $|\vec{a}| = \sqrt{4^2+3^2} = 5$, so the projection condition gives $4x+3y = 5$. Also

$$\vec{a}\times\vec{b} = 15\hat{i}-20\hat{j}-25\hat{k}.$$

The remaining two conditions therefore give

$$15x-20y-25z = -25, \qquad x+y+z = 4.$$

The minus sign on the right of the first equation comes from moving the printed $+25$ to the other side. All three conditions are necessary to determine the requested projection.

Step 2: Solve for the vector and project it Substitute $z = 4-x-y$ into the cross-product equation. This yields $40x+5y = 75$, or $8x+y = 15$. Combining this with $4x+3y = 5$ gives $x = 2$, $y = -1$, and $z = 3$. Direct substitution verifies both $\vec{c}\cdot(\vec{a}\times\vec{b}) = -25$ and the component sum 4. Finally,

$$\vec{c}\cdot\vec{b} = 2(3)+(-1)(-4)+3(5) = 25,$$
$$|\vec{b}| = \sqrt{3^2+(-4)^2+5^2} = 5\sqrt{2},$$
$$\text{proj}_{\vec{b}}\vec{c} = \frac{25}{5\sqrt{2}} = \frac{5}{\sqrt{2}}.$$

The other options give different scalar projections and fail this dot-product computation; in particular $1/5$ cannot result from dividing 25 by $5\sqrt{2}$.

Step 3: Conclusion The required scalar projection is $\boxed{5/\sqrt{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: Normalize by the length of the vector onto which you project. Check the recovered components in all three constraints before taking the final dot product.

11. The value of the integral $\int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dt$ is [4 marks]

(A) $\tan^{-1}\frac{1}{2} - \frac{1}{3}\tan^{-1}8 + \frac{\pi}{3}$
(B) $\tan^{-1}2 - \frac{1}{3}\tan^{-1}8 + \frac{\pi}{3}$
(C) $\tan^{-1}2 + \frac{1}{3}\tan^{-1}8 - \frac{\pi}{3}$
(D) $\tan^{-1}\frac{1}{2} + \frac{1}{3}\tan^{-1}8 - \frac{\pi}{3}$

Correct Answer: (C) $\tan^{-1}2 + \frac{1}{3}\tan^{-1}8 - \frac{\pi}{3}$

Solution:

Concept:

- The identity $t^6 + 1 = (t^2 + 1)(t^4 - t^2 + 1)$ helps split the rational integrand into inverse-tangent derivatives.
- Use $d(\tan^{-1} t)/dt = 1/(1 + t^2)$ and $d(\tan^{-1}(t^3))/dt = 3t^2/(1 + t^6)$, with the chain rule.

Step 1: Find a useful exact decomposition The numerator can be recovered by adding two simple rational functions:

$$\frac{1}{1+t^2} + \frac{t^2}{1+t^6} = \frac{t^4 - t^2 + 1 + t^2}{t^6 + 1}$$
$$= \frac{t^4 + 1}{t^6 + 1}.$$

This equality holds throughout $[1, 2]$, where both denominators are positive. It is more direct than a full partial-fraction decomposition into the quadratic factors. In particular the second term has the precise t^2 factor needed for the substitution $u = t^3$.

Step 2: Integrate and substitute the endpoints An antiderivative is

$$F(t) = \tan^{-1} t + \frac{1}{3}\tan^{-1}(t^3).$$

Differentiating this expression reproduces the original integrand exactly, including the factor $1/3$. Thus

$$I = F(2) - F(1)$$
$$= \tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \left(\frac{\pi}{4} + \frac{1}{3}\frac{\pi}{4}\right)$$
$$= \tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{3}.$$

All inverse tangents here are their principal values. Options (A) and (B) have the wrong sign for the second endpoint contribution, while (D) substitutes $1/2$ for the actual upper endpoint 2 in the first inverse tangent.

Step 3: Conclusion The integral is $\boxed{\tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \pi/3}$. Hence, the correct option is (C).

Quick Tip to Solve: When the denominator is $1 + t^6$, look for a t^2 term that integrates through t^3; verify the proposed decomposition before evaluating limits.

12. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}K}{a}=\frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is equal to [4 marks]

(A) (50, 101)

(B) (50, 51)

(C) (51, 101)

(D) (51, 99)

Correct Answer: (A) (50, 101)

Solution:

Concept:

- For $(1+x)^n$, subtracting the value at $x=-1$ from that at $x=1$ isolates twice the sum of odd-power coefficients.
- There are 201 terms in a power-200 binomial expansion, so its unique middle term is term 101, corresponding to index $r=100$.

Step 1: Evaluate the odd-coefficient sum and middle term Let E and K be the sums of the even- and odd-power coefficients in $(1+x)^{99}$. Substitution gives $E+K=2^{99}$ and $E-K=0$, hence $K=2^{98}$. For the second expansion, the general term has the form

$$T_{r+1}=\binom{200}{r}2^{200-r}\left(\frac{1}{\sqrt{2}}\right)^r.$$

At $r=100$, its middle term is

$$a=\binom{200}{100}2^{100}2^{-50}=\binom{200}{100}2^{50}.$$

The factor 2^{-50}, rather than 2^{-100}, is essential because the denominator contains a square root.

Step 2: Reduce the ratio and isolate its power of two Use the adjacent-binomial-coefficient ratio

$$\frac{\binom{200}{99}}{\binom{200}{100}}=\frac{100}{101}.$$

It follows that

$$\begin{aligned}\frac{\binom{200}{99}K}{a}&=\frac{100}{101}2^{98-50}\\&=\frac{100\cdot 2^{48}}{101}=\frac{25\cdot 2^{50}}{101}.\end{aligned}$$

Both 25 and 101 are odd, and they are coprime, so this is already the required reduced odd-over-odd form. Thus $l=50$ and $n=101$. Options (B) and (D) have the wrong odd denominator, while option (C) assigns one extra factor of 2 to the numerator.

Step 3: Conclusion The ordered pair is $\boxed{(l,n)=(50,101)}$. Hence, the correct option is (A).

Quick Tip to Solve: Cancel adjacent binomial coefficients before using factorials, and remove every factor of 2 from the remaining integer numerator to identify l.

13. The value of the integral $\int_{1/2}^{2} \frac{\tan^{-1}x}{x}dx$ is equal to : [4 marks]

(A) $\frac{\pi}{2}\log_e 2$

(B) $\frac{\pi}{4}\log_e 2$

(C) $\frac{1}{2}\log_e 2$

(D) $\pi\log_e 2$

Correct Answer: (A) $\frac{\pi}{2}\log_e 2$

Solution:

Concept:

- For positive x, the principal inverse tangents satisfy $\tan^{-1} x + \tan^{-1}(1/x) = \pi/2$.
- On an interval whose endpoints are reciprocals, the substitution $x = 1/t$ transforms dx/x into $-dt/t$, enabling the transformed and original integrals to be added.

Step 1: Apply the reciprocal substitution carefully Denote the given integral by I. Put $x = 1/t$, so that $dx/x = -dt/t$. The lower limit $x = 1/2$ becomes $t = 2$, and the upper limit $x = 2$ becomes $t = 1/2$. Reversing the transformed limits absorbs the minus sign, giving

$$I = \int_{1/2}^{2} \frac{\tan^{-1}(1/t)}{t}\,dt.$$

The integrand is continuous because the entire interval is positive. The reciprocal substitution therefore introduces no singular point and preserves the same numerical integral.

Step 2: Add the two representations and evaluate Rename the dummy variable t as x, then add the transformed equation to the original one:

$$\begin{aligned} 2I &= \int_{1/2}^{2} \frac{\tan^{-1} x + \tan^{-1}(1/x)}{x}\,dx \\ &= \frac{\pi}{2}\int_{1/2}^{2} \frac{dx}{x} \\ &= \frac{\pi}{2}\,[\ln x]_{1/2}^{2} = \pi \ln 2. \end{aligned}$$

Dividing by 2 yields $I = (\pi/2)\ln 2$. Option (D) is the intermediate value $2I$, not the requested integral. Option (B) divides by 2 once too often, and option (C) omits the inverse-tangent angle factor π.

Step 3: Conclusion The value is $\boxed{\frac{\pi}{2}\ln 2}$. Hence, the correct option is (A).

Quick Tip to Solve: For an inverse-tangent integral with dx/x and reciprocal positive limits, try $x \mapsto 1/x$ and add the two forms before attempting an antiderivative.

14. The shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ is : [4 marks]

(A) $2\sqrt{3}$

(B) $3\sqrt{3}$

(C) $4\sqrt{3}$

(D) $5\sqrt{3}$

Correct Answer: (C) $4\sqrt{3}$

Solution:

Concept:

- For nonparallel lines $\vec{r}=\vec{a}_1+s\vec{b}_1$ and $\vec{r}=\vec{a}_2+t\vec{b}_2$, their shortest distance is $|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|/|\vec{b}_1\times\vec{b}_2|$.
- The cross product is perpendicular to both line directions. Taking the scalar projection of a joining vector onto this common normal gives the distance, with absolute value ensuring a nonnegative result.

Step 1: Read points and direction vectors from the lines From the symmetric equations, select

$$\vec{a}_1=(1,-8,4),\quad \vec{b}_1=(2,-7,5),$$
$$\vec{a}_2=(1,2,6),\quad \vec{b}_2=(2,1,-3).$$

In particular $y+8=y-(-8)$, so the first point has second coordinate -8. Hence the joining vector is $\vec{a}_2-\vec{a}_1=(0,10,2)$. The two direction vectors are not proportional, so the nonparallel-line distance formula applies. One must not use the distance between these two arbitrarily selected points as the distance between the lines.

Step 2: Compute the common normal and scalar projection The cross product is

$$\vec{b}_1\times\vec{b}_2=\big(21-5,\ 10+6,\ 2+14\big)$$
$$=(16,16,16).$$

Its magnitude is $16\sqrt{3}$. The scalar triple product with the joining vector is $0(16)+10(16)+2(16)=192$. Therefore

$$d=\frac{|192|}{16\sqrt{3}}=\frac{12}{\sqrt{3}}=4\sqrt{3}.$$

The nonzero numerator also confirms that the lines do not intersect. The values $2\sqrt{3}$, $3\sqrt{3}$, and $5\sqrt{3}$ do not equal this projection; none can satisfy the correctly computed scalar-triple-product formula.

Step 3: Conclusion The shortest distance is $\boxed{4\sqrt{3}}$ units. Hence, the correct option is (C).

Quick Tip to Solve: You may replace the cross product $(16,16,16)$ by the parallel normal $(1,1,1)$; using the same normal in numerator and denominator cancels its scale.

15. Let f and g be the twice differentiable functions on $\mathbb{R}$ such that

$$f''(x) = g''(x) + 6x$$

$$f'(1) = 4g'(1) - 3 = 9$$

$f(2) = 3g(2) = 12$.

Then which of the following is NOT true? [4 marks]

(A) $g(-2) - f(-2) = 20$

(B) There exists $x_0 \in (1, 3/2)$ such that $f(x_0) = g(x_0)$

(C) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

(D) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

Correct Answer: (D) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

Solution:

Concept:

- Set $h(x) = f(x) - g(x)$. Subtracting the derivatives converts both unknown functions into one polynomial difference.
- Integrate $h''(x) = 6x$ twice, using the given derivative and function values to determine both constants.

Step 1: Determine the difference of the functions The chained equality gives $f'(1) = 9$ and $g'(1) = 3$, hence $h'(1) = 6$. Also $f(2) = 12$ and $g(2) = 4$, so $h(2) = 8$. Integrating gives

$$h'(x) = 3x^2 + C, \qquad 6 = 3 + C \Rightarrow C = 3.$$

$$h(x) = x^3 + 3x + D, \qquad 8 = 8 + 6 + D \Rightarrow D = -6.$$

Step 2: Check the first three assertions Thus $h(x) = x^3 + 3x - 6$. At -2, $h(-2) = -20$, so $g(-2) - f(-2) = 20$ and (A) is true. Since $h(1) = -2$ and $h(3/2) = 15/8 > 0$, continuity gives a zero strictly between these arguments; (B) is true. Finally $h'(x) = 3x^2 + 3 > 0$, and

$$|f'(x) - g'(x)| < 6 \iff 3x^2 + 3 < 6 \iff -1 < x < 1.$$

This proves (C), including the strict endpoints.

Step 3: Test the remaining bound To disprove (D), one counterexample in its stated interval suffices. Take $x = -3/4$, which lies strictly between -1 and 2. Then

$$h(-3/4) = -\frac{27}{64} - \frac{9}{4} - 6 = -\frac{555}{64}, \qquad |h(-3/4)| = \frac{555}{64} > 8.$$

The failure concerns the absolute value, despite the fact that the polynomial is increasing. Its value approaches -10 as x approaches -1 from the right.

Step 4: Conclusion The assertion in (D) is not true. Hence, the correct option is (D).

Quick Tip to Solve: For two unknown functions linked through their derivatives, solve for their difference first. Disprove a universal inequality by choosing one interior point.

16. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to : [4 marks]

(A) $B \Rightarrow ((\sim A) \Rightarrow B)$

(B) $A \Rightarrow (A \Leftrightarrow B)$

(C) $A \Rightarrow ((\sim A) \Rightarrow B)$

(D) $B \Rightarrow (A \Rightarrow B)$

Correct Answer: (A) $B \Rightarrow ((\sim A) \Rightarrow B)$

Solution:

Concept:

- An implication $P \Rightarrow Q$ is equivalent to $(\sim P) \vee Q$, and a statement containing $P \vee (\sim P)$ is a tautology.
- Two propositions are equivalent when their truth values agree for every assignment, rather than just one assignment.

Step 1: Simplify the statement in the question Let $S = B \Rightarrow ((\sim A) \vee B)$. Removing the outer implication gives

$$S \equiv (\sim B) \vee (\sim A) \vee B \equiv \text{True}.$$

Indeed, if B is false the implication is automatically true; if B is true its consequent already contains the true disjunct B. Thus an equivalent option must also always be true.

Step 2: Check each proposed equivalence For (A), the inner implication is $(\sim A) \Rightarrow B \equiv A \vee B$, so the whole option is $(\sim B) \vee A \vee B$, a tautology. For (C), the result is $(\sim A) \vee A \vee B$, also a tautology. For (D), it is $(\sim B) \vee (\sim A) \vee B$, again a tautology. In contrast, (B) is false when A is true and B is false: then $A \Leftrightarrow B$ is false while the outer antecedent is true. Since the original proposition stays true in this case, (B) cannot be equivalent.

Step 3: Identify the accepted alternatives The three tautologies have identical truth values to the original statement for all four possible assignments of A and B. Consequently this printed question has several correct alternatives, and selecting only one does not make the other two incorrect. Option (A) is a valid representative answer; options (C) and (D) satisfy exactly the same equivalence test.

Step 4: Conclusion Options (A), (C) and (D) are equivalent to the given statement; the official answer key accepts (A), (C) and (D). Hence, the correct option is (A), with (C) and (D) also accepted.

Quick Tip to Solve: Expand each implication into a disjunction. A variable and its negation in that disjunction immediately establish a tautology.

17. The plane $2x - y + z = 4$ intersects the line segment joining the points A $(a, -2, 4)$ and B $(2, b, -3)$ at the point C in the ratio 2 : 1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point $(a - b, b, 2b - a)$ then CP^2 is equal to : [4 marks]

(A) $\frac{17}{3}$
(B) $\frac{97}{3}$
(C) $\frac{16}{3}$
(D) $\frac{73}{3}$

Correct Answer: (A) $\frac{17}{3}$

Solution:

Concept:

- Internal division in the ratio $AC : CB = 2 : 1$ gives $C = (A + 2B)/3$.
- A point on a plane satisfies its equation; its squared distance from the origin is the sum of the squares of its coordinates.

Step 1: Express the intersection coordinates Using the section formula in the stated order,

$$C = \left(\frac{a+4}{3}, \frac{2b-2}{3}, -\frac{2}{3}\right).$$

Substitute these coordinates in $2x - y + z = 4$:

$$\frac{2a + 8 - 2b + 2 - 2}{3} = 4 \Rightarrow a - b = 2.$$

Therefore $a = b + 2$; this relation also fixes the first coordinate of P.

Step 2: Use the distance and sign restrictions Since $OC = \sqrt{5}$, the sum of the squared coordinates is 5. On replacing a by $b + 2$,

$$(b + 6)^2 + (2b - 2)^2 + 4 = 45.$$

$$5b^2 + 4b - 1 = 0 \Rightarrow (5b - 1)(b + 1) = 0.$$

The pair $b = 1/5, a = 11/5$ has positive product and must be rejected. The required pair is $b = -1, a = 1$, whose product is negative. Both roots satisfy the distance equation, so this sign check is essential.

Step 3: Compute the required squared length The admissible parameters give

$$C = \left(\frac{5}{3}, -\frac{4}{3}, -\frac{2}{3}\right), \qquad P = (2, -1, -3).$$

$$CP^2 = \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(-\frac{7}{3}\right)^2 = \frac{51}{9} = \frac{17}{3}.$$

As a check, $OC^2 = (25 + 16 + 4)/9 = 5$, and $2(5/3) - (-4/3) - 2/3 = 4$. The other proposed squared lengths do not equal the coordinate calculation; no square root is taken because the question asks for the square of CP.

Step 4: Conclusion $CP^2 = 17/3$ square units. Hence, the correct option is (A).

Quick Tip to Solve: Use the sign condition only after solving the plane and distance equations; it selects the admissible parameter pair without guessing.

18. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is : [4 marks]

(A) $\frac{9}{\sqrt{5}}$

(B) $3\sqrt{5}$

(C) $5\sqrt{3}$

(D) $\frac{3}{2}\sqrt{5}$

Correct Answer: (B) $3\sqrt{5}$

Solution:

Concept:

- Implicit differentiation gives the tangent slopes $3/(2y)$ for $y^2 = 3x$ and $-x/(4y)$ for $x^2/4 + y^2 = 1$.
- For a triangle, area is one half of a chosen base times its perpendicular altitude.

Step 1: Find the point on the parabola The line $x + 2y = 1$ has slope $-1/2$. Since the tangent at P is parallel to it,

$$\frac{3}{2y_P} = -\frac{1}{2} \Rightarrow y_P = -3, \qquad x_P = \frac{y_P^2}{3} = 3.$$

Thus $P = (3, -3)$. The negative sign of the tangent slope fixes the lower half of the parabola; choosing positive y would give the wrong tangent direction.

Step 2: Find the two points on the ellipse The line $x - y = 2$ has slope 1, so the required perpendicular tangents have slope -1. Their contact points obey

$$-\frac{x}{4y} = -1 \Rightarrow x = 4y.$$

Substitution in the ellipse yields $5y^2 = 1$. Consequently

$$Q = \left(\frac{4}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \qquad R = \left(-\frac{4}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right).$$

These are opposite endpoints of a diameter on the line $x - 4y = 0$.

Step 3: Calculate the base and altitude The length of that diameter and the distance of P from its line are

$$QR = 2\sqrt{\frac{16+1}{5}} = 2\sqrt{\frac{17}{5}}, \qquad h = \frac{|3 - 4(-3)|}{\sqrt{1+16}} = \frac{15}{\sqrt{17}}.$$

Therefore

$$[PQR] = \frac{1}{2} \cdot 2\sqrt{\frac{17}{5}} \cdot \frac{15}{\sqrt{17}} = \frac{15}{\sqrt{5}} = 3\sqrt{5}.$$

The factor one half is necessary even though QR is a diameter. This value differs from (A), (C) and (D), so none of those areas fits the coordinates.

Step 4: Conclusion The area of triangle PQR is $3\sqrt{5}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: When two tangent points are opposite on a central conic, use their joining diameter as the base and a point-to-line distance for the height.

19. If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane $z=a$ is : [4 marks]

(A) 28

(B) 22

(C) 10

(D) 16

Correct Answer: (A) 28

Solution:

Concept:

- Each symmetric line equation can be written parametrically by setting its three ratios equal to one parameter.
- At an intersection, the corresponding x, y and z coordinates agree. Distance from a point to $z=a$ is $|z-a|$.

Step 1: Parametrize the lines separately Use parameter t on the first line and s on the second. Their coordinates are

$$L_1 : (x, y, z) = (1+t, 2+2t, -3+t),$$

$$L_2 : (x, y, z) = (a+2s, -2+3s, 3+s).$$

The parameters should not be identified with one another: the direction vectors and the base points differ, even though the two lines meet at the same point.

Step 2: Solve the intersection equations Equating the z coordinates gives $t-s=6$, while the y coordinates give $2t-3s=-4$. Substituting $t=s+6$ into the second equation gives

$$2s+12-3s=-4 \Rightarrow s=16, \qquad t=22.$$

The x coordinates then imply

$$1+22=a+2(16) \Rightarrow a=-9.$$

The point of intersection is $P=(23,46,19)$. Substituting in the second line independently gives $(-9+32,-2+48,3+16)=(23,46,19)$, confirming that all three coordinates agree.

Step 3: Measure the perpendicular distance The requested plane is $z=a=-9$, parallel to the xy-plane. Its normal vector is $(0,0,1)$, of unit length. Thus only the z-coordinate separation contributes:

$$d=\frac{|0(23)+0(46)+19-a|}{\sqrt{0^2+0^2+1^2}}=|19-(-9)|=28.$$

The x and y coordinates do not enter this perpendicular distance. The alternatives 22, 10 and 16 are not the separation from the plane; in particular, 22 and 16 are the line parameters, not the distance.

Step 4: Conclusion The distance of P from the plane is 28 units. Hence, the correct option is (A).

Quick Tip to Solve: For a plane with one fixed coordinate, first find that coordinate of the point and then subtract the plane value inside an absolute value.

20. The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$

is invertible, is : [4 marks]

(A) $\{k\pi, k \in \mathbb{Z}\}$

(B) $\mathbb{R}$

(C) $\left\{(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\right\}$

(D) $\left\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\right\}$

Correct Answer: (B) $\mathbb{R}$

Solution:

Concept:

- A square matrix is invertible exactly when its determinant is nonzero.
- Column factors may be taken outside a determinant, and adding a multiple of one row to another does not change the determinant.

Step 1: Remove the exponential column factors Write $u = \sin t$ and $v = \cos t$. Factor e^t, e^{-t}, e^{-t} from the first, second and third columns, respectively. The determinant becomes

$$\det M = e^{-t}\begin{vmatrix} 1 & u-2v & -2u-v \\ 1 & 2u+v & u-2v \\ 1 & v & u \end{vmatrix}.$$

The prefactor e^{-t} is strictly positive for every finite real t, so it cannot introduce a zero or restrict the domain.

Step 2: Reduce to a quadratic expression Apply $R_1 \leftarrow R_1 - R_3$ and $R_2 \leftarrow R_2 - R_3$. Expanding along the first column leaves

$$\begin{aligned} \det M &= e^{-t}\begin{vmatrix} u-3v & -2u-2v \\ 2u & -2v \end{vmatrix} \\ &= e^{-t}\{-2v(u-3v) + 4u(u+v)\} \\ &= e^{-t}(4u^2 + 2uv + 6v^2). \end{aligned}$$

The cofactor sign is positive because the surviving entry is in row 3, column 1. Retaining that sign and the mixed product prevents a spurious trigonometric zero.

Step 3: Prove positivity for every real parameter Since $u^2 + v^2 = 1$, the last factor can be bounded without solving any trigonometric equation:

$$4u^2 + 2uv + 6v^2 = 4 + 2v^2 + 2uv \geq 4 - 1 = 3 > 0.$$

Here $|uv| \leq 1/2$, and $v^2 \geq 0$. Therefore $\det M > 0$ for every real t. In particular, no isolated progression such as those listed in (A), (C) or (D) describes the full invertibility set: parameters outside those progressions also yield nonzero determinants.

Step 4: Conclusion The matrix is invertible for every $t \in \mathbb{R}$. Hence, the correct option is (B).

Quick Tip to Solve: Factor exponentials from columns before expanding. Then use $\sin^2 t + \cos^2 t = 1$ to prove a quadratic expression stays positive.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is _____.

[4 marks]

Solution:

Concept:

- The prime factorisation is $54 = 2 \cdot 3^3$. To have greatest common divisor exactly 2, an integer must be even and must not be divisible by 3.
- The number of multiples of a positive integer d in an interval $[L, U]$ is $\lfloor U/d \rfloor - \lfloor (L-1)/d \rfloor$.

Step 1: Translate the greatest common divisor condition An integer with $\gcd(n, 54) = 2$ must contain the prime factor 2. It must not contain any factor 3, since then its greatest common divisor with 54 would be at least 6. Extra factors of 2 in n are allowed: 54 contains only one factor of 2, so even a multiple of 4 can still have greatest common divisor 2. Thus the admissible numbers are precisely the even four-digit integers with the multiples of 6 removed.

Step 2: Count the even four-digit integers The four-digit interval is $1000 \le n \le 9999$. It contains

$$N_2 = \left\lfloor \frac{9999}{2} \right\rfloor - \left\lfloor \frac{999}{2} \right\rfloor = 4999 - 499 = 4500$$

even integers. This calculation includes the first allowed even integer 1000 and the last allowed one 9998; it excludes all three-digit and five-digit numbers.

Step 3: Remove the integers having a factor of three Among the counted even integers, exactly the multiples of 6 are also divisible by 3. Their number is

$$N_6 = \left\lfloor \frac{9999}{6} \right\rfloor - \left\lfloor \frac{999}{6} \right\rfloor = 1666 - 166 = 1500.$$

The first such multiple is 1002 and the last is 9996. Each is excluded once, regardless of how many factors of 3 it contains. There is no further exclusion for multiples of 9 or 27, because they have already been removed whenever they are even. Subtracting gives

$$N = N_2 - N_6 = 4500 - 1500 = 3000.$$

Step 4: Conclusion The total number of four-digit integers whose greatest common divisor with 54 is 2 is $\boxed{3000}$.

Quick Tip to Solve: For gcd conditions, factor the fixed integer first. Here the entire condition reduces to counting even integers and subtracting multiples of 6.

22. Let $X = \{11, 12, 13, ..., 40, 41\}$ and $Y = \{61, 62, 63, ..., 90, 91\}$ be the two sets of observations. If $\overline{x}$ and $\overline{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $\left|\overline{x} + \overline{y} - \sigma^2\right|$ is equal to _____. [4 marks]

Solution:

Concept:

- The mean of consecutive integers from a to b is $(a + b)/2$. Both groups contain the same number of observations.
- For n consecutive integers with unit spacing, the population variance is $(n^2 - 1)/12$. Combined variance includes the variance within each group and the squared displacement of its mean from the combined mean.

Step 1: Find the two means Each set has $41 - 11 + 1 = 31$ observations. Their means are

$$\overline{x} = \frac{11 + 41}{2} = 26, \qquad \overline{y} = \frac{61 + 91}{2} = 76.$$

Since the groups have equal sizes, the mean of all 62 observations is

$$\mu = \frac{26 + 76}{2} = 51.$$

The union has no repeated observations, so these equal weights are applicable.

Step 2: Combine the within-group and between-group contributions Translating a set does not change its variance. Thus both groups have within-group variance

$$v = \frac{31^2 - 1}{12} = 80.$$

For any observation, write its displacement from 51 as its displacement from its own group mean plus the displacement of that group mean from 51. The cross term sums to zero in each group. Both group means lie 25 units from 51, so

$$\sigma^2 = \frac{(80 + 25^2) + (80 + 25^2)}{2} = 705.$$

Step 3: Evaluate the requested absolute value Substitute the two group means and the variance, retaining the absolute value until the last step:

$$\left|\overline{x} + \overline{y} - \sigma^2\right| = |26 + 76 - 705| = |-603| = 603.$$

Using 80 alone would omit the large separation between the two groups.

Step 4: Conclusion The required numerical value is $\boxed{603}$.

Quick Tip to Solve: For two equally large translated groups, add the common within-group variance to the square of half the distance between their means.

23. Let $\alpha = 8-14i, A = \left\{z \in c : \frac{\alpha z - \overline{\alpha}\overline{z}}{z^2-(\overline{z})^2-112i} = 1\right\}$ **and** $B = \{z \in c : |z+3i| = 4\}$**. Then** $\sum_{z \in A \cap B} (\mathrm{Re}z - \mathrm{Im}z)$ **is equal to _____.** [4 marks]

Solution:

Concept:

- Write $z = x + iy$, so $\overline{z} = x - iy$, $\mathrm{Re}\, z = x$, and $\mathrm{Im}\, z = y$. An equation involving conjugates can then be converted into real coordinates.
- The modulus condition $|z + 3i| = 4$ represents the circle $x^2 + (y+3)^2 = 16$. A rational equation is valid only where its denominator is nonzero.

Step 1: Convert the rational equation to a locus For $\alpha = 8 - 14i$, direct multiplication gives

$$\alpha z - \overline{\alpha}\,\overline{z} = 2i(8y - 14x), \qquad z^2 - \overline{z}^{\,2} - 112i = 4i(xy - 28).$$

Consequently, subject to $xy \neq 28$, equality of the numerator and denominator gives

$$16y - 28x = 4xy - 112.$$

Rearranging and factoring yields

$$xy + 7x - 4y - 28 = (x-4)(y+7) = 0.$$

Thus a point of A lies on $x = 4$ or $y = -7$, with the denominator restriction retained.

Step 2: Intersect both lines with the circle On $x = 4$, the circle equation gives $16 + (y+3)^2 = 16$, hence $y = -3$. On $y = -7$, it gives $x^2 + 16 = 16$, hence $x = 0$. The intersections are therefore $(4, -3)$ and $(0, -7)$. Their products xy are -12 and 0, respectively, so neither makes the original denominator zero. Each line is tangent to this circle and contributes one distinct point.

Step 3: Add the real-minus-imaginary values The corresponding complex numbers are $4 - 3i$ and $-7i$. Their contributions to the requested sum are

$$4 - (-3) = 7, \qquad 0 - (-7) = 7.$$

Adding over the two distinct members of $A \cap B$ gives $7 + 7 = 14$.

Step 4: Conclusion The required sum is $\boxed{14}$.

Quick Tip to Solve: After cross-multiplying a complex rational equation, record excluded denominator values before intersecting the resulting loci.

24. Let $\alpha_1, \alpha_2, ..., \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq \ ... \ \geq |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to _____. [4 marks]

Solution:

Concept:

- An odd polynomial with only odd powers has a factor x; replacing x^2 by a new variable reduces the remaining equation to a cubic.
- Order complex roots by their moduli, rather than by real-number order. Within a tied pair, its product is unchanged when the two roots exchange positions.

Step 1: Factor the seventh-degree equation First separate the zero root:

$$x^7 + 3x^5 - 13x^3 - 15x = x\big(x^6 + 3x^4 - 13x^2 - 15\big).$$

Put $t = x^2$. The cubic factor satisfies

$$t^3 + 3t^2 - 13t - 15 = (t - 3)(t + 1)(t + 5).$$

The factorisation can be checked by multiplying $(t + 1)(t + 5) = t^2 + 6t + 5$ by $t - 3$. Hence

$$x(x^2 - 3)(x^2 + 1)(x^2 + 5) = 0.$$

Step 2: List and order all seven roots The roots are

$$0, \quad \sqrt{3}, \quad -\sqrt{3}, \quad i, \quad -i, \quad i\sqrt{5}, \quad -i\sqrt{5}.$$

Their moduli occur in descending groups $\sqrt{5}, \sqrt{3}, 1, 0$. Therefore α_1, α_2 are the two roots $\pm i\sqrt{5}$, α_3, α_4 are $\pm\sqrt{3}$, and α_5, α_6 are $\pm i$. The remaining root α_7 is zero. No ambiguity from equal moduli affects these paired products.

Step 3: Evaluate the signed combination Using $i^2 = -1$, the products are

$$\alpha_1\alpha_2 = (i\sqrt{5})(-i\sqrt{5}) = 5,$$

$$\alpha_3\alpha_4 = (\sqrt{3})(-\sqrt{3}) = -3, \qquad \alpha_5\alpha_6 = i(-i) = 1.$$

The minus sign in front of the middle product must be retained, so the expression is

$$5 - (-3) + 1 = 9.$$

Step 4: Conclusion The required value is $\boxed{9}$.

Quick Tip to Solve: For opposite real roots the product is negative, while opposite purely imaginary roots have a positive product; pair the roots only after sorting their moduli.

25. Let $\{a_k\}$ and $\{b_k\}, k \in N$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in N$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to _____. [4 marks]

Solution:

Concept:

- For a geometric progression with first term 4 and ratio r, its k-th term is $4r^{k-1}$. If $|r| < 1$, its infinite sum is $4/(1-r)$.
- The sum and the sum of squares of two numbers determine their product through $(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$.

Step 1: Determine the two ratios The supplied second and third terms of the combined sequence imply

$$4(r_1 + r_2) = 5, \qquad 4(r_1^2 + r_2^2) = \frac{13}{4}.$$

Thus $r_1 + r_2 = 5/4$, $r_1^2 + r_2^2 = 13/16$, and

$$r_1r_2 = \frac{(5/4)^2 - 13/16}{2} = \frac{3}{8}.$$

The ratios are the roots of

$$t^2 - \frac{5}{4}t + \frac{3}{8} = 0, \qquad (2t-1)(4t-3) = 0.$$

Since $r_1 < r_2$, take $r_1 = 1/2$ and $r_2 = 3/4$.

Step 2: Sum the combined sequence Both ratios have absolute value less than one, so both geometric series converge. Addition of the convergent series gives

$$\sum_{k=1}^{\infty} c_k = \sum_{k=1}^{\infty} a_k + \sum_{k=1}^{\infty} b_k = \frac{4}{1-1/2} + \frac{4}{1-3/4} = 8 + 16 = 24.$$

The subtraction outside the summation is applied once, after evaluating this infinite sum.

Step 3: Evaluate the correction terms Use the exponent $k-1$ for each requested term:

$$a_6 = 4\left(\frac{1}{2}\right)^5 = \frac{1}{8}, \qquad b_4 = 4\left(\frac{3}{4}\right)^3 = \frac{27}{16}.$$

Hence

$$12a_6 + 8b_4 = \frac{12}{8} + \frac{8 \cdot 27}{16} = \frac{3}{2} + \frac{27}{2} = 15.$$

The required difference is therefore $24 - 15 = 9$.

Step 4: Conclusion The required numerical value is $\boxed{9}$.

Quick Tip to Solve: Use the inequality between the ratios to assign the smaller root to the first sequence before calculating its later terms.

26. Let A be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to _____. [4 marks]

Solution:

Concept:

- A real symmetric matrix of order two can be written as $A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$. Its determinant is $ac - b^2$ and its diagonal sum is $a + c$.
- If one row of the left multiplier is a scalar multiple of another, the corresponding rows of the product have the same relation.

Step 1: Read off the second row of the product The second row $(3, 3/2)$ of the multiplier is $3/2$ times its first row $(2, 1)$. Therefore the second row of the product must be $3/2$ times $(1, 2)$. This immediately gives

$$\alpha = \frac{3}{2}, \qquad \beta = 3.$$

The multiplier is singular, so attempting to invert it would be invalid; the symmetry and determinant of A provide the remaining information.

Step 2: Solve for the symmetric matrix Write $A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$. Multiplication by the first row gives

$$2a + b = 1, \qquad 2b + c = 2.$$

Thus $a = (1 - b)/2$ and $c = 2 - 2b$. Insert these into the stated determinant condition:

$$ac - b^2 = \frac{1 - b}{2}(2 - 2b) - b^2 = (1 - b)^2 - b^2 = 1 - 2b = 2.$$

It follows that $b = -1/2$, $a = 3/4$, and $c = 3$. These values satisfy both first-row equations and $ac - b^2 = 9/4 - 1/4 = 2$.

Step 3: Calculate the requested quotient The diagonal sum is

$$s = a + c = \frac{3}{4} + 3 = \frac{15}{4}.$$

Consequently,

$$\frac{\beta s}{\alpha^2} = \frac{3(15/4)}{(3/2)^2} = \frac{45/4}{9/4} = 5.$$

The square in the denominator applies to all of α.

Step 4: Conclusion The required value is $\boxed{5}$.

Quick Tip to Solve: Check row dependence before using matrix inverses: here it supplies both unknown product entries immediately, despite the singular multiplier.

27. A circle with centre (2, 3) and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to _____. [4 marks]

Solution:

Concept:

- For the circle $(x-h)^2 + (y-k)^2 = R^2$, the chord of contact from an external point (α, β) has equation $(\alpha - h)(x - h) + (\beta - k)(y - k) = R^2$.
- The tangents at the endpoints of a chord meet at its pole. This allows their intersection to be found without separately calculating the endpoints.

Step 1: Shift the origin to the circle centre Let $u = x - 2$, $v = y - 3$. The circle becomes

$$u^2 + v^2 = 16.$$

The line through the points of contact becomes

$$(u + 2) + (v + 3) = 3, \qquad u + v = -2.$$

Its distance from the new origin is $2/\sqrt{2} = \sqrt{2} < 4$, confirming that it meets the circle at two distinct points, as required.

Step 2: Identify the pole of the chord Write the tangent intersection in shifted coordinates as $(a, b) = (\alpha - 2, \beta - 3)$. If the contact points are (u_1, v_1) and (u_2, v_2), the two tangent equations evaluated at (a, b) give $au_j + bv_j = 16$. Thus both contact points lie on

$$au + bv = 16.$$

Their chord already has equation $u + v = -2$, equivalently

$$-8u - 8v = 16.$$

Comparing the same line with the same constant term gives $a = b = -8$.

Step 3: Return to the original coordinates Translating back to the original origin gives

$$\alpha = a + 2 = -6, \qquad \beta = b + 3 = -5.$$

Now evaluate exactly the combination requested:

$$4\alpha - 7\beta = 4(-6) - 7(-5) = -24 + 35 = 11.$$

The pole lies at distance $\sqrt{(-8)^2 + (-8)^2} = 8\sqrt{2} > 4$ from the centre, consistent with the existence of two real tangents.

Step 4: Conclusion The tangents meet at $S = (-6, -5)$, and the required value is $\boxed{11}$.

Quick Tip to Solve: Put a chord equation in the form matching the circle radius squared before comparing its coefficients with the chord-of-contact formula.

28. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to _____. [4 marks]

Solution:

Concept:

- The tangent slope follows from implicit differentiation. For a circle, the tangent is perpendicular to the radius at the point of contact.
- If a triangle has side lengths a, b, c, area Δ, and circumradius r, then $r = abc/(4\Delta)$.

Step 1: Find the two tangents For the parabola $y^2 = 2x$, implicit differentiation gives $2y\,dy/dx = 2$. At $(2, 2)$, the slope is $1/2$, so its tangent is

$$y - 2 = \frac{1}{2}(x - 2), \qquad x - 2y + 2 = 0.$$

The circle is $(x - 2)^2 + y^2 = 4$, with centre $(2, 0)$. Its radius to $(2, 2)$ is vertical; hence its tangent is the horizontal line $y = 2$.

Step 2: Locate all vertices of the triangle The two tangents meet at $A = (2, 2)$. The line $x + y + 2 = 0$ meets $y = 2$ at $B = (-4, 2)$. To meet the parabola tangent, substitute $x = 2y - 2$ into $x + y + 2 = 0$, giving $3y = 0$. Thus the third vertex is $C = (-2, 0)$. The vertices are distinct, so they form the stated triangle.

Step 3: Compute its area and circumradius The horizontal base has length $AB = 6$, and the perpendicular height from C to $y = 2$ is 2. Therefore

$$\Delta = \frac{1}{2}(6)(2) = 6.$$

The other side lengths are

$$AC = \sqrt{4^2 + 2^2} = \sqrt{20}, \qquad BC = \sqrt{2^2 + (-2)^2} = \sqrt{8}.$$

Using the circumradius formula gives

$$r^2 = \frac{AB^2\, AC^2\, BC^2}{16\Delta^2} = \frac{36 \cdot 20 \cdot 8}{16 \cdot 36} = 10.$$

Step 4: Conclusion The square of the circumradius is $\boxed{10}$ square coordinate units.

Quick Tip to Solve: When one tangent is horizontal, use it as the base for the area and calculate the square of the circumradius directly to avoid unnecessary radicals.

29. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1), b_n = b_{n-1} + a_{n-1}, \forall n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to _____. [4 marks]

Solution:

Concept:

- The recurrence for a_n adds the successive integers: $a_n = 1 + \sum_{k=1}^{n-1} k = 1 + n(n-1)/2$.
- The recurrence $b_n = b_{n-1} + a_{n-1}$ must use the preceding value of a, not the value with index n. Finite sums can be evaluated exactly with powers of two as common denominators.

Step 1: Generate the two sequences Starting with $a_1 = b_1 = 1$, use each recurrence in its stated order. For example, $a_3 = 2 + 2 = 4$, while $b_3 = b_2 + a_2 = 2 + 2 = 4$. Continuing gives

n	1	2	3	4	5	6	7	8	9	10
a_n	1	2	4	7	11	16	22	29	37	46
b_n	1	2	4	8	15	26	42	64	93	130

As a check on the last entry, $b_{10} = b_9 + a_9 = 93 + 37 = 130$; using a_{10} would incorrectly shift the recurrence.

Step 2: Evaluate the finite sums Write the terms of S over $2^{10} = 1024$. Their numerators are $512, 512, 512, 512, 480, 416, 336, 256, 186, 130$, respectively. Thus

$$S = \frac{3852}{1024} = \frac{963}{256}.$$

For T, the common denominator is $2^7 = 128$. The numerators are $128, 128, 96, 64, 40, 24, 14, 8$, so

$$T = \frac{502}{128} = \frac{251}{64}.$$

These are finite sums: no infinite-series tail may be included.

Step 3: Combine before multiplying Use the same denominator before taking the difference:

$$2S - T = \frac{1926}{256} - \frac{1004}{256} = \frac{922}{256} = \frac{461}{128}.$$

The prescribed multiplier is $2^7 = 128$, which cancels the denominator exactly; rounding either sum prematurely is unnecessary.

Step 4: Conclusion The required numerical value is $\boxed{2^7(2S - T) = 461}$.

Quick Tip to Solve: For a short recurrence, tabulate both sequences side by side and check that the second recurrence uses the previous column.

30. If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point (1, −3) is $x - 4y = 13$, then the value of $a + b$ is equal to _____. [4 marks]

Solution:

Concept:

- A point on a curve satisfies its equation, so substituting $(1, -3)$ gives one relation between a and b. The denominator must be nonzero there.
- For finite nonzero slopes, the tangent and normal satisfy $m_t m_n = -1$. Differentiate the rational function using the quotient rule, not by differentiating numerator and denominator separately.

Step 1: Use the given point and normal The normal can be written $y = x/4 - 13/4$, so its slope is $1/4$. Therefore the tangent slope at $x = 1$ is -4. Substituting the point into the curve gives

$$-3 = \frac{1-a}{(1+b)(1-2)} = \frac{a-1}{b+1}, \qquad a = -3b - 2.$$

Here $b \neq -1$, since otherwise the stated point would not be on the rational curve. Notice that the factor $1 - 2 = -1$ is essential to the sign of this relation.

Step 2: Differentiate and impose the slope Set $D(x) = (x + b)(x - 2)$, so $D'(x) = 2x + b - 2$. The quotient rule yields

$$y' = \frac{D(x) - (x-a)D'(x)}{D(x)^2}.$$

At $x = 1$, we have $D(1) = -(b + 1)$ and $D'(1) = b$, hence

$$-4 = \frac{-(b+1) - (1-a)b}{(b+1)^2} = \frac{ab - 2b - 1}{(b+1)^2}.$$

Insert $a = -3b - 2$ and factor the numerator:

$$-4 = -\frac{3b^2 + 4b + 1}{(b+1)^2} = -\frac{3b+1}{b+1}.$$

Cancellation is valid because $b \neq -1$. Thus $3b + 1 = 4b + 4$, giving $b = -3$ and $a = 7$.

Step 3: Check the point and the line With these constants, $y(1) = (-6)/2 = -3$ and the derivative at that point is -4, as required. Its normal has slope $1/4$, so

$$y + 3 = \frac{1}{4}(x - 1) \quad \Longleftrightarrow \quad x - 4y = 13.$$

Both the position and the prescribed normal are therefore satisfied.

Step 4: Conclusion The constants are $a = 7$ and $b = -3$, so $\boxed{a + b = 4}$.

Quick Tip to Solve: Use the point equation first to eliminate one constant; then the normal slope gives a single equation for the other.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The ratio of de-Broglie wavelength of an α particle and a proton accelerated from rest by the same potential is $\frac{1}{\sqrt{m}}$, the value of m is - [4 marks]

(A) 2

(B) 16

(C) 8

(D) 4

Correct Answer: (C) 8

Solution:

Concept:

- A particle accelerated from rest through potential difference V gains kinetic energy qV. In the nonrelativistic de Broglie relation, $\lambda = h/p = h/\sqrt{2MqV}$.
- An alpha particle has approximately four times the proton mass and twice its charge: $M_\alpha = 4m_p$, $q_\alpha = 2e$. Both factors must be included when the potential differences are equal.

Step 1: Compare the acquired momenta The potential difference is the same, but the kinetic energies are not the same: the alpha particle gains $2eV$, whereas the proton gains eV. Consequently,

$$p_p = \sqrt{2m_p eV}, \qquad p_\alpha = \sqrt{2(4m_p)(2e)V} = \sqrt{8}\, p_p.$$

The larger mass and larger charge both increase the alpha particle momentum. Using the mass ratio alone would miss the additional factor of two in kinetic energy.

Step 2: Invert the momentum ratio Since h is common, wavelength varies inversely with momentum:

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{p_p}{p_\alpha} = \frac{1}{\sqrt{8}}.$$

Compare this with the supplied expression $1/\sqrt{m}$; the quantity under the radical is 8, not the wavelength ratio itself.

Step 3: Eliminate the other choices The choice $m = 4$ follows from considering only the fourfold mass ratio and ignoring alpha charge. The choice $m = 2$ ignores the mass ratio, while $m = 16$ would require a momentum ratio of four and is inconsistent with $p^2 = 2MqV$. The product of the actual mass and charge ratios is uniquely $4 \times 2 = 8$.

Step 4: Conclusion Thus $\boxed{m = 8}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal accelerating potentials, compare the products of particle mass and charge before taking the inverse square root.

32. A force acts for 20 s on a body of mass 20 kg, starting from rest, after which the force ceases and then body describes 50 m in the next 10 s. The value of force will be: [4 marks]

(A) 40 N

(B) 20 N

(C) 5 N

(D) 10 N

Correct Answer: (C) 5 N

Solution:

Concept:

- After the applied force ceases, a body with no resultant force moves with constant velocity. Its speed in that interval is distance divided by time.
- During the force interval, impulse equals change of momentum: $F\Delta t = M(v-u)$. The body starts from rest, so $u = 0$.

Step 1: Find the speed when the force ceases The body travels $50\,\mathrm{m}$ in the next $10\,\mathrm{s}$, when the applied force no longer acts. Its velocity is therefore constant during those ten seconds:

$$v = \frac{50}{10} = 5\,\mathrm{m\,s^{-1}}.$$

This is also its speed at the end of the preceding twenty seconds because velocity does not jump when the force is switched off. The stated $50\,\mathrm{m}$ belongs to the coasting interval, not to the accelerated interval.

Step 2: Apply Newton's second law during the force interval Starting from rest, the speed increases to $5\,\mathrm{m\,s^{-1}}$ in $20\,\mathrm{s}$. Thus

$$a = \frac{5-0}{20} = 0.25\,\mathrm{m\,s^{-2}}, \qquad F = Ma = 20(0.25) = 5\,\mathrm{N}.$$

Equivalently, the impulse is $F(20) = 20(5)$, which gives the same result. The mass is $20\,\mathrm{kg}$, and the duration used for the impulse is the time the force acts.

Step 3: Check the alternatives against the motion For the stated mass and force duration, $v = F\Delta t/M = F$ numerically in SI units. Hence 40, 20, and $10\,\mathrm{N}$ would produce coasting distances of 400, 200, and $100\,\mathrm{m}$, respectively, during the next ten seconds. Only $5\,\mathrm{N}$ reproduces $50\,\mathrm{m}$.

Step 4: Conclusion The applied force is $\boxed{5\,\mathrm{N}}$. Hence, the correct option is (C).

Quick Tip to Solve: Split motion at the instant the force stops: the later uniform-motion interval reveals the final speed of the earlier interval.

33. The equation of a circle is given by $x^2 + y^2 = a^2$, where a is the radius. If the equation is modified to change the origin other than (0, 0), then find out the correct dimensions of A and B in a new equation : $(x - At)^2 + \left(y - \frac{t}{B}\right)^2 = a^2$. The dimensions of t is given as $[T^{-1}]$. [4 marks]

(A) $A = [L^{-1}T^{-1}], B = [LT^{-1}]$

(B) $A = [L^{-1}T^{-1}], B = [LT]$

(C) $A = [LT], B = [L^{-1}T^{-1}]$

(D) $A = [L^{-1}T], B = [LT^{-1}]$

Correct Answer: (C) $A = [LT], B = [L^{-1}T^{-1}]$

Solution:

Concept:

- Quantities added or subtracted must have identical dimensions. Since x, y, a are lengths, both At and t/B must be lengths.
- The question explicitly assigns $[t] = T^{-1}$; the symbol is not an ordinary time variable here. Use the stated dimension rather than its usual interpretation.

Step 1: Determine the dimension of A The first squared bracket is $(x - At)^2$, so its inner terms must each have length dimension. Therefore

$$[A][t] = L.$$

Using the given $[t] = T^{-1}$ gives

$$[A] = \frac{L}{T^{-1}} = LT.$$

Squaring does not change this requirement: the whole bracket then has dimension L^2, matching a^2.

Step 2: Determine the dimension of B Similarly, the second bracket is $y - t/B$, where y is a length. Thus

$$\frac{[t]}{[B]} = L \quad \Longrightarrow \quad [B] = \frac{[t]}{L} = L^{-1}T^{-1}.$$

The displacement of the origin changes the centre coordinates, but introduces no permission to add unlike physical dimensions.

Step 3: Check both shifts and all choices With the pair just found,

$$[At] = (LT)(T^{-1}) = L, \qquad [t/B] = \frac{T^{-1}}{L^{-1}T^{-1}} = L.$$

Both squared terms and the right-hand side are therefore L^2. Options (A), (B), and (D) all give A a dimension different from LT, so they already fail the first subtraction; (C) satisfies both independently.

Step 4: Conclusion The dimensions are $\boxed{[A] = LT,\ [B] = L^{-1}T^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a shifted coordinate, set each shift equal in dimension to length, and use the dimension explicitly assigned to its parameter.

34. A square loop of area 25 cm^2 has a resistance of 10 Ω. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 sec, will be [4 marks]

(A) 1.0×10^{-3} J

(B) 5×10^{-3} J

(C) 2.5×10^{-3} J

(D) 1.0×10^{-4} J

Correct Answer: (A) 1.0×10^{-3} J

Solution:

Concept:

- Faraday's law gives induced emf magnitude $\mathcal{E} = |d\Phi/dt|$. A square loop pulled uniformly across a straight field boundary loses linked area uniformly.
- For slow uniform withdrawal, the external work is dissipated as Joule heat: $W = I^2 R\tau = \mathcal{E}^2\tau/R$. Initial and final kinetic energies are equal.

Step 1: Calculate the flux change and emf The field is perpendicular to the plane, so the initial flux is BA, with no additional sine or cosine factor. Convert the area carefully:

$$A = 25\,\text{cm}^2 = 25 \times 10^{-4}\,\text{m}^2 = 2.5 \times 10^{-3}\,\text{m}^2.$$

The initial flux is $40.0(2.5 \times 10^{-3}) = 0.100\,\text{Wb}$, and the final flux is zero. In the stated uniform withdrawal over $\tau = 1.0\,\text{s}$,

$$\mathcal{E} = \frac{BA}{\tau} = 0.100\,\text{V}.$$

Step 2: Convert induced current into dissipated energy The resistance is $R = 10\,\Omega$, hence

$$I = \frac{\mathcal{E}}{R} = 0.0100\,\text{A}.$$

The induced current opposes withdrawal according to Lenz's law. The pulling force supplies the electrical energy lost as heat, giving

$$W = I^2 R\tau = (0.0100)^2(10)(1.0) = 1.0 \times 10^{-3}\,\text{J}.$$

Equivalently, $W = B^2A^2/(R\tau)$. The work is positive because it is done against the magnetic drag.

Step 3: Check the scale of the alternatives The induced electrical power is $\mathcal{E}^2/R = 0.0100/10 = 0.00100\,\text{W}$. Over one second it gives $0.00100\,\text{J}$. The other listed values would require a different flux-change rate or resistance and do not follow from the supplied quantities. In particular, area must be converted as a squared length.

Step 4: Conclusion The work done is $\boxed{1.0 \times 10^{-3}\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: For uniform withdrawal use W = flux-change squared divided by resistance and withdrawal time, after converting area to SI units.

35. Heat energy of 184 kJ is given to ice of mass 600 g at $-12^{\circ}\mathrm{C}$. Specific heat of ice is $2222.3\ \mathrm{J\ kg^{-1^{\circ}}\ C^{-1}}$ and latent heat of ice in 336 $\mathrm{kJ/kg^{-1}}$

A. Final temperature of system will be 0°C.

B. Final temperature of the system will be greater than 0°C.

C. The final system will have a mixture of ice and water in the ratio of 5 : 1.

D. The final system will have a mixture of ice and water in the ratio of 1 : 5.

E. The final system will have water only.

Choose the correct answer from the options given below : [4 marks]

(A) A and E only

(B) B and D only

(C) A and C only

(D) A and D only

Correct Answer: (D) A and D only

Solution:

Concept:

- Heat first raises the ice to $0^{\circ}\mathrm{C}$: $Q_1 = mc_{\text{ice}}\Delta T$. Only the remaining heat is available for melting.
- During melting at atmospheric pressure, temperature remains $0^{\circ}\mathrm{C}$. The melted mass is $m_w = (Q - Q_1)/L_f$; any unmelted mass remains ice.

Step 1: Warm the ice to its melting point The initial mass is $m = 600\,\mathrm{g} = 0.600\,\mathrm{kg}$, and the temperature rise needed is $12^{\circ}\mathrm{C}$. Thus

$$Q_1 = (0.600)(2222.3)(12)\,\mathrm{J} = 16000.56\,\mathrm{J} \simeq 16.0\,\mathrm{kJ}.$$

Out of the $184\,\mathrm{kJ}$ supplied, approximately $168\,\mathrm{kJ}$ therefore remains after the entire initial ice sample reaches zero degrees. Latent heat must not be applied to the full input before this warming stage.

Step 2: Find the masses of water and remaining ice The latent heat of fusion is $L_f = 336\,\mathrm{kJ\,kg^{-1}}$. Melting all $0.600\,\mathrm{kg}$ would need $201.6\,\mathrm{kJ}$ beyond the warming heat, more than is available. The melted mass is

$$m_w = \frac{184 - 16.00056}{336}\,\mathrm{kg} \simeq 0.500\,\mathrm{kg}.$$

Therefore the ice remaining is $0.600 - 0.500 = 0.100\,\mathrm{kg}$, and

$$m_{\text{ice}} : m_{\text{water}} = 0.100 : 0.500 = 1 : 5.$$

The negligible departure from exact values results from the rounded printed specific heat.

Step 3: Evaluate the statements Ice and water coexist at equilibrium, so the final temperature is $0^{\circ}\mathrm{C}$: statement A is true and B is false. The ice-to-water mass ratio is $1 : 5$, so D is true and C reverses the ratio. E is false because insufficient heat has been supplied to melt all the ice. Only A and D describe the final state.

Step 4: Conclusion The final system is ice and water at $0^{\circ}\mathrm{C}$, in the mass ratio $1 : 5$. Hence, the correct option is (D).

Quick Tip to Solve: In ice calorimetry, subtract the warming heat first, divide the remainder by latent heat, then subtract melted mass from the original mass.

36. Identify the correct statements from the following :

A. Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is negative.

B. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.

C. Work done by friction on a body sliding down an inclined plane is positive.

D. Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity is zero.

E. Work done by the air resistance on an oscillating pendulum is negative.

Choose the correct answer from the options given below : [4 marks]

(A) A and C only

(B) B and D only

(C) B, D and E only

(D) B and E only

Correct Answer: (D) B and E only

Solution:

Concept:

- Work by a constant force is $W = Fs\cos\theta$, where θ is the angle between force and displacement. The sign depends on this angle, not merely on whether the body moves.
- The work-energy theorem concerns the net work: $W_{\text{net}} = \Delta K$. Zero net work does not require the work by every individual force to vanish.

Step 1: Check the two lifting statements The bucket moves upward. The man exerts an upward tension through the rope, so his force has the same direction as the displacement and does positive work: statement A is false. Gravity acts downward while the bucket rises, giving $W_g = -mgh < 0$. Statement B is therefore true; lifting at constant speed would make these two works cancel, without making either work zero.

Step 2: Check friction and the applied force Kinetic friction on a body sliding down a stationary incline acts up the incline, opposite its displacement. Its work is $W_f = -f_k s$, so statement C is false. On a rough horizontal plane, uniform velocity requires the applied driving force to balance friction. The applied force still does positive work Fs, while friction does equal negative work. Only their sum is zero, so statement D is false.

Step 3: Check the resistive force on the pendulum Air resistance opposes the instantaneous velocity of the bob. Over a small displacement, $dW = \mathbf{F}_{\text{air}} \cdot d\mathbf{r} < 0$; equivalently its power is $\mathbf{F}_{\text{air}} \cdot \mathbf{v} < 0$. It removes mechanical energy during both halves of the oscillation. Thus statement E is true. The correct set is B and E; options containing A, C or D must be rejected.

Step 4: Conclusion Only statements B and E are correct. Hence, the correct option is (D).

Quick Tip to Solve: For each force separately, compare its direction with displacement. Constant speed implies zero total work, not zero applied-force work.

37. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option) [4 marks]

(A) Decrease the focal length of the eye piece.

(B) Increase the wave length of the light

(C) Increase the refractive index of the medium between the object and objective lens

(D) Decrease the diameter of the objective lens

Correct Answer: (C) Increase the refractive index of the medium between the object and objective lens

Solution:

Concept:

- The numerical aperture of an objective is $\mathrm{NA} = n \sin\alpha$, where n is the refractive index of the object-side medium and α is the half-angle of the accepted cone of light.
- The limiting distance resolved by a microscope is approximately $d_{\min} = 0.61\lambda/(n \sin\alpha)$. Resolving power is proportional to $1/d_{\min}$, so increasing numerical aperture improves resolution.

Step 1: Identify what must improve To analyse the bacteria better, the instrument must distinguish two nearby features as separate. It must therefore reduce $d_{\min}$, rather than merely make an unresolved image look larger. For a fixed objective aperture and wavelength, increasing n increases the denominator of the resolution formula. The smallest distinguishable separation then falls, and the resolving power rises. This is the principle behind immersion objectives.

Step 2: Compare the four proposed changes Increasing the refractive index of the medium between object and objective increases numerical aperture and is the required change. Decreasing the eyepiece focal length mainly raises angular magnification; it cannot restore detail already lost through diffraction at the objective. Increasing wavelength raises $d_{\min}$, making resolution worse. Decreasing objective diameter reduces its angular aperture for the same arrangement and therefore also makes resolution worse. Thus the eyepiece change is not the best choice, and the wavelength and diameter changes act in the wrong direction.

Step 3: Conclusion Increase the refractive index of the medium between the object and objective lens. Hence, the correct option is (C).

Quick Tip to Solve: Use the objective numerical aperture to judge resolution. Eyepiece magnification enlarges the available image detail but does not set the diffraction limit.

38. Given below are two statements :

Statement I : Electromagnetic waves are not deflected by electric and magnetic field.

Statement II : The amplitude of electric field and the magnetic field in electromagnetic waves are related to each other as $E_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} B_0$.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true and Statement II is false

(C) Both Statement I and Statement II are false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Statement I is true and Statement II is false

Solution:

Concept:

- An electromagnetic wave in vacuum consists of transverse electric and magnetic fields. A static external electric or magnetic field does not bend its path as it bends the trajectory of a charged particle.
- For a plane electromagnetic wave in vacuum, $E_0/B_0 = c$, where $c = 1/\sqrt{\mu_0\varepsilon_0}$. The ratio of electric field to magnetic field intensity is instead the wave impedance.

Step 1: Evaluate Statement I Electromagnetic radiation carries energy and momentum but does not carry a net electric charge. In the usual vacuum setting of the question, external electric and magnetic fields do not exert a charged-particle Lorentz force that deflects the radiation. Linear superposition of fields does not itself bend the propagation direction. Statement I is true. This conclusion concerns deflection by the stated fields and does not assert that light cannot be refracted by a material medium.

Step 2: Evaluate Statement II The correct amplitude relation is

$$E_0 = cB_0 = \frac{B_0}{\sqrt{\mu_0\varepsilon_0}}.$$

The printed factor $\sqrt{\mu_0/\varepsilon_0}$ is the vacuum impedance, not the speed of light. It relates E_0 to H_0, since $B_0 = \mu_0 H_0$. Using it directly as the ratio E_0/B_0 confuses magnetic flux density with magnetic field intensity. Statement II is false. Consequently the choices that call both statements true, both false, or I false and II true are all excluded.

Step 3: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: Remember the distinction: $E/B = c$, whereas E/H is wave impedance. Their units also distinguish speed from impedance.

39. The time taken by an object to slide down 45° rough inclined plane is n times as it takes to slide down a perfectly smooth 45° incline plane. The coefficient of kinetic friction between the object and the incline plane is : [4 marks]

(A) $1-\frac{1}{n^2}$

(B) $1+\frac{1}{n^2}$

(C) $\sqrt{1-\frac{1}{n^2}}$

(D) $\sqrt{\frac{1}{1-n^2}}$

Correct Answer: (A) $1-\frac{1}{n^2}$

Solution:

Concept:

- For a body sliding down an incline at angle θ, the normal force is $N = mg\cos\theta$; kinetic friction $\mu_k N$ acts up the slope.
- For motion from rest through the same distance s with constant acceleration, $s = \frac{1}{2}at^2$. Hence the travel time is inversely proportional to the square root of acceleration.

Step 1: Find the two accelerations For the smooth plane, only the component of weight acts along the slope:

$$a_s = g\sin 45^\circ = \frac{g}{\sqrt{2}}.$$

For the rough plane, subtract the uphill friction from the downhill component of weight:

$$a_r = g(\sin 45^\circ - \mu_k \cos 45^\circ) = \frac{g}{\sqrt{2}}(1-\mu_k).$$

The mass cancels, as expected for this ideal sliding problem. Both bodies traverse the same incline distance from rest.

Step 2: Use the given ratio of times Writing $s = \frac{1}{2}a_s t_s^2 = \frac{1}{2}a_r t_r^2$, and using $t_r = nt_s$, gives

$$a_s = a_r n^2, \qquad \frac{a_r}{a_s} = \frac{1}{n^2}.$$

But the force calculation gave $a_r/a_s = 1-\mu_k$. Therefore

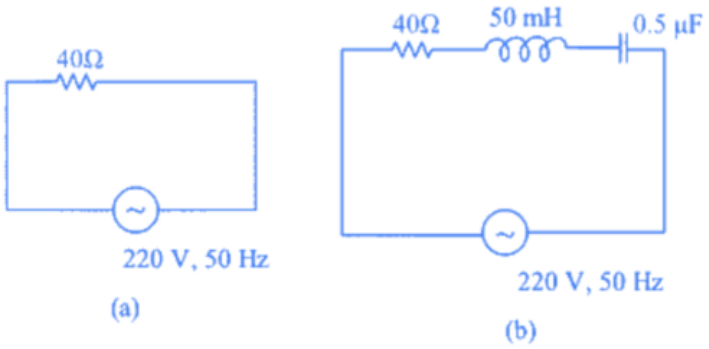
$$1-\mu_k = \frac{1}{n^2} \quad \Rightarrow \quad \mu_k = 1-\frac{1}{n^2}.$$

The coefficient is dimensionless. For $n = 1$ it vanishes, while an increased delay gives a positive coefficient below one. The plus-sign expression fails the frictionless limit, and the square-root alternatives do not follow from the squared-time relation.

Step 3: Conclusion The coefficient of kinetic friction is $\boxed{\mu_k = 1-1/n^2}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal distances from rest, square the time ratio before comparing accelerations: $t_r/t_s = \sqrt{a_s/a_r}$.

40. For the given figures, choose the correct options : [4 marks]



(A) The rms current in circuit (b) can be larger than that in (a)

(B) The rms current in figure (a) is always equal to that in figure (b)

(C) The rms current in circuit (b) can never be larger than that in (a)

(D) At resonance, current in (b) is less than that in (a)

Correct Answer: (C) The rms current in circuit (b) can never be larger than that in (a)

Solution:

Concept:

- For a purely resistive AC circuit, $I_{\text{rms}} = V_{\text{rms}}/R$. For a series RLC circuit, $I_{\text{rms}} = V_{\text{rms}}/Z$.
- The impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L$, $X_C = 1/(\omega C)$, and $\omega = 2\pi f$. Thus $Z \geq R$.

Step 1: Compare the impedances Both circuits have the same $40\,\Omega$ resistance and the same $220\,\text{V}$ rms source. Circuit (a) has no reactance, so

$$I_a = \frac{220}{40} = 5.5\,\text{A}.$$

Circuit (b) has an inductor and capacitor in series. Their net reactance enters the impedance as a square:

$$Z_b = \sqrt{40^2 + (\omega L - 1/(\omega C))^2} \geq 40\,\Omega.$$

It follows directly that $I_b = 220/Z_b \leq 5.5\,\text{A} = I_a$. Reactive cancellation can reduce the impedance to the resistance, but never below it.

Step 2: Check resonance and the listed claims At resonance, $X_L = X_C$, so $Z_b = R$ and the currents are equal. Away from resonance, the nonzero squared term makes $I_b < I_a$. For the displayed values at $50\,\text{Hz}$, $X_L \approx 15.7\,\Omega$ and $X_C \approx 6366\,\Omega$, so this particular circuit is far from resonance and its current is much smaller. Option A violates $Z_b \geq R$; option B incorrectly demands equality at every frequency; option D incorrectly makes the resonant current smaller. Only the assertion that the current in (b) can never exceed that in (a) is valid.

Step 3: Conclusion The rms current in the series RLC circuit can never exceed that in the purely resistive circuit. Hence, the correct option is (C).

Quick Tip to Solve: Compare impedance before calculating reactances. A series RLC circuit with fixed R has maximum current at resonance, equal to V_{rms}/R.

41. An object moves at a constant speed along a circular path in a horizontal plane with center at the origin. When the object is at $x = +2$ m, its velocity is $-4\hat{j}$ m/s. The object's velocity (v) and acceleration (a) at $x = -2$ m will be [4 marks]

(A) $v = 4\hat{i}\ \text{m/s}, a = 8\hat{j}\ \text{m/s}^2$

(B) $v = 4\hat{j}\ \text{m/s}, a = 8\hat{i}\ \text{m/s}^2$

(C) $v = -4\hat{i}\ \text{m/s}, a = -8\hat{j}\ \text{m/s}^2$

(D) $v = -4\hat{j}\ \text{m/s}, a = 8\hat{i}\ \text{m/s}^2$

Correct Answer: (B) $v = 4\hat{j}\ \text{m/s}, a = 8\hat{i}\ \text{m/s}^2$

Solution:

Concept:

- In uniform circular motion, velocity is tangential to the circle and has constant magnitude, while centripetal acceleration points toward the centre.
- The acceleration magnitude is $a = v^2/r$. At diametrically opposite points both the velocity vector and the position vector reverse direction.

Step 1: Determine the radius and sense of rotation At the positive-x point, the velocity is $-4\hat{\mathbf{j}}\,\text{m s}^{-1}$, vertically downward. Since a circular-path tangent is perpendicular to the radius, the corresponding radius lies along the x-axis. With the centre at the origin and $x = +2\,\text{m}$, the radius is $r = 2\,\text{m}$. Downward motion at the rightmost point means clockwise rotation. The speed everywhere is $4\,\text{m s}^{-1}$.

Step 2: Find velocity and acceleration at the opposite point At $x = -2\,\text{m}$, the body is at the leftmost point. Clockwise motion there is vertically upward, so

$$\mathbf{v} = 4\hat{\mathbf{j}}\,\text{m s}^{-1}.$$

The centre lies to its right, so the acceleration is along the positive x-axis. Its magnitude is

$$a = \frac{v^2}{r} = \frac{4^2}{2} = 8\,\text{m s}^{-2}, \qquad \mathbf{a} = 8\hat{\mathbf{i}}\,\text{m s}^{-2}.$$

Options A and C wrongly make velocity radial and acceleration tangential. Option D has the correct inward acceleration but retains the downward velocity of the opposite point, contrary to the continuous clockwise motion.

Step 3: Conclusion At $x = -2\,\text{m}$, $\mathbf{v} = 4\hat{\mathbf{j}}\,\text{m s}^{-1}$ and $\mathbf{a} = 8\hat{\mathbf{i}}\,\text{m s}^{-2}$. Hence, the correct option is (B).

Quick Tip to Solve: At opposite ends of a diameter, reverse the velocity direction and point the acceleration toward the centre; keep their magnitudes unchanged.

42. The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become.

[4 marks]

(A) 12 hours

(B) 3 hours

(C) 6 hours

(D) 4 hours

Correct Answer: (B) 3 hours

Solution:

Concept:

- For a satellite orbiting the same central body, Kepler's third law gives $T^2 \propto r^3$, or $T \propto r^{3/2}$, where r is the orbital separation from the centre.
- For a circular Earth orbit, $T = 2\pi\sqrt{r^3/(GM)}$. In a ratio, the common factors 2π, G and Earth's mass M cancel.

Step 1: Relate the original and new orbits Let the original separation and period be r_1 and $T_1 = 24\,\text{h}$. The new separation is $r_2 = r_1/4$. Apply the period relation to the same central body:

$$\frac{T_2^2}{T_1^2} = \left(\frac{r_2}{r_1}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}.$$

The shorter orbital radius has a stronger gravitational acceleration and a smaller circumference, so a shorter period is physically expected.

Step 2: Take the positive square root and calculate An orbital period is positive, hence

$$\frac{T_2}{T_1} = \frac{1}{8}, \qquad T_2 = \frac{24}{8} = 3\,\text{h}.$$

The period decreases by a factor of eight, rather than four, because the radius enters with power $3/2$. The 12-hour choice corresponds to a factor of two and the 6-hour choice to direct proportionality with radius; neither obeys Kepler's law. The 4-hour choice similarly gives the wrong ratio. The separation in the question is the orbital separation; one must not replace it by height above the surface or subtract Earth's radius, since no altitude is specified.

Step 3: Conclusion The new time period is $\boxed{3\,\text{hours}}$. Hence, the correct option is (B).

Quick Tip to Solve: If orbital radius is divided by k, divide the period by $k^{3/2}$. Here $4^{3/2} = 8$, so no orbital constants are needed.

43. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its centre. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the centre of the coil by the same current will be : [4 marks]

(A) 2 T

(B) 4 T

(C) 8 T

(D) 16 T

Correct Answer: (A) 2 T

Solution:

Concept:

- The magnetic induction at the centre of a circular coil of N turns and radius r, carrying current I, is $B = \mu_0 NI/(2r)$. The fields of the turns add in the same direction.
- Unwinding and rewinding the same wire preserves its total length: $L = N(2\pi r)$. Changing the turn count therefore also changes the radius; the radius cannot be held fixed.

Step 1: Relate the radii before and after rewinding Let the initial four-turn coil have radius r, and let the final single turn have radius R. Equating the length of the same wire gives

$$4(2\pi r) = 2\pi R \quad \Rightarrow \quad R = 4r.$$

Thus the final turn is four times as large in radius as each original turn. The current is specified to remain the same in both arrangements.

Step 2: Compare the magnetic inductions Initially, $B_i = \mu_0(4)I/(2r)$. Finally, $B_f = \mu_0 I/(2R) = \mu_0 I/(8r)$. Taking the ratio cancels both the current and the magnetic constant:

$$\frac{B_f}{B_i} = \frac{1}{4}\frac{r}{R} = \frac{1}{16}.$$

Hence $B_f = 32/16 = 2\,\mathrm{T}$. The value $8\,\mathrm{T}$ would incorrectly account only for the reduction in turn count while ignoring the increase in radius. The other proposed values also fail this wire-length constraint.

Step 3: Conclusion The final magnetic induction is $\boxed{2\,\mathrm{T}}$. Hence, the correct option is (A).

Quick Tip to Solve: For the same wire and current, substitute $r = L/(2\pi N)$ into the field formula: the central field is proportional to N^2.

44. A fully loaded boeing aircraft has a mass of 5.4×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 1080 km/h. If the density of air ρ is 1.2 kg m^{-3}, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be. ($g = 10$ m/s^2) [4 marks]

(A) 16

(B) 8

(C) 6

(D) 10

Correct Answer: (D) 10

Solution:

Concept:

- In level flight the upward lift equals the aircraft weight: $(P_l - P_u)A = mg$, where the subscripts denote the lower and upper wing surfaces.
- Bernoulli's equation at approximately equal heights gives $P_l - P_u = \frac{1}{2}\rho(v_u^2 - v_l^2)$. Faster flow over the upper surface has lower pressure.

Step 1: Calculate the pressure difference needed for level flight Convert the given aircraft speed to SI units, taking it as the lower-surface relative air speed:

$$v_l = 1080\frac{1000}{3600} = 300\,\mathrm{m\,s^{-1}}.$$

The wing area is the total area, so the necessary pressure difference is

$$\Delta P = \frac{mg}{A} = \frac{(5.4 \times 10^5)(10)}{500} = 1.08 \times 10^4\,\mathrm{Pa}.$$

The pressure below must exceed that above to supply an upward force.

Step 2: Obtain the fractional speed increase Use Bernoulli's equation with $\rho = 1.2\,\mathrm{kg\,m^{-3}}$:

$$v_u^2 = 300^2 + \frac{2(1.08 \times 10^4)}{1.2} = 108000.$$

Consequently, $v_u = 300\sqrt{1.2}\,\mathrm{m\,s^{-1}}$, and the percentage increase is

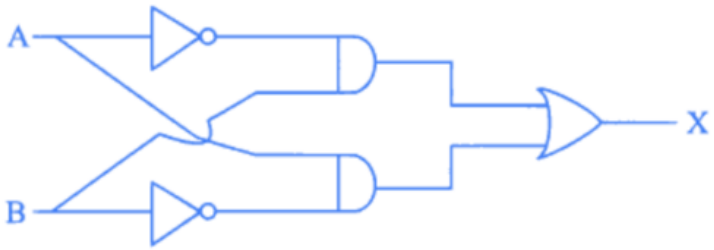
$$100\frac{v_u - v_l}{v_l} = 100(\sqrt{1.2} - 1) \simeq 9.54\%.$$

This rounds to the listed choice 10%. Equivalently, for a small increase Δv, neglecting $(\Delta v)^2$ gives $\Delta P \simeq \rho v_l \Delta v$, hence $\Delta v/v_l \simeq 0.10$. Values 16, 8 and 6 percent do not give the required lift in this approximation.

Step 3: Conclusion The fractional speed increase is approximately 10% (the exact Bernoulli value is 9.54%). Hence, the correct option is (D).

Quick Tip to Solve: Convert km/h to m/s before applying Bernoulli. For a small speed increase, use $\Delta P \simeq \rho v \Delta v$.

45. For the given logic gates combination, the correct truth table will be [4 marks]



(A)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(B)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

(C)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

(D)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Correct Answer: (D)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Solution:

Concept:

- A NOT gate complements its input, an AND gate gives 1 only when both inputs are 1, and an OR gate gives 1 when at least one input is 1.
- In this circuit the upper AND gate receives $\overline{A}$ and B, while the lower AND gate receives A and $\overline{B}$. Their outputs enter the final OR gate.

Step 1: Write the Boolean expression from the circuit The diagonal wires cross without an electrical junction, so they carry the original inputs to the opposite AND gates. The two intermediate outputs

are

$$Y_1 = \overline{A}B, \qquad Y_2 = A\overline{B}.$$

The final OR operation therefore gives

$$X = Y_1 + Y_2 = \overline{A}B + A\overline{B}.$$

This is the exclusive-OR operation: one, and only one, of the original inputs must be high.

Step 2: Evaluate all four input combinations For $(A, B) = (0, 0)$, both products vanish and $X = 0$. For $(0, 1)$, the first product is 1 and $X = 1$. For $(1, 0)$, the second product is 1 and $X = 1$. For $(1, 1)$, both complemented factors are 0 and $X = 0$. The complete output sequence is thus

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Option A is the ordinary OR table and incorrectly gives 1 for two high inputs. Option B is the NOR table and fails for unequal inputs. Option C reduces to the complement of B and even gives 1 for two low inputs. Only option D agrees for every row.

Step 3: Conclusion The combination implements $X = A \oplus B$, with output sequence $0, 1, 1, 0$. Hence, the correct option is (D).

Quick Tip to Solve: Trace each AND gate separately before combining its output. The expression $\overline{A}B + A\overline{B}$ immediately identifies an XOR gate.

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46. At 300 K, the rms speed of oxygen molecules is $\sqrt{\frac{\alpha+5}{\alpha}}$ times to that of its average speed in the gas. Then, the value of α will be

(used $\pi = \frac{22}{7}$) [4 marks]

(A) 27

(B) 28

(C) 24

(D) 32

Correct Answer: (B) 28

Solution:

Concept:

- For a gas of molar mass M at absolute temperature T, the root-mean-square speed is $v_{\text{rms}} = \sqrt{3RT/M}$. It is the square root of the mean of the squared molecular speeds.
- The average molecular speed is $\overline{v} = \sqrt{8RT/(\pi M)}$. Both speeds have the same temperature and molar-mass dependence, so their ratio depends only on π.

Step 1: Form the ratio of the two speeds Use the same oxygen gas at the same temperature in the two speed formulae. The factors R, T and M cancel:

$$\frac{v_{\text{rms}}}{\overline{v}} = \sqrt{\frac{3RT/M}{8RT/(\pi M)}} = \sqrt{\frac{3\pi}{8}}.$$

There is no need to insert the molar mass of oxygen or the given temperature of $300\,\text{K}$. The ratio is greater than one, as expected because squaring gives greater weight to faster molecules.

Step 2: Compare with the stated expression and solve Equate the squared ratio to the one given in the question, using precisely $\pi = 22/7$:

$$\frac{\alpha + 5}{\alpha} = \frac{3\pi}{8} = \frac{3 \times 22}{8 \times 7} = \frac{33}{28}.$$

Thus

$$1 + \frac{5}{\alpha} = 1 + \frac{5}{28} \quad \Rightarrow \quad \alpha = 28.$$

A direct check gives $(28 + 5)/28 = 33/28$. The alternatives 27, 24 and 32 produce different squared ratios and therefore cannot reproduce the required rms-to-average speed ratio.

Step 3: Conclusion The required value is $\boxed{\alpha = 28}$. Hence, the correct option is (B).

Quick Tip to Solve: Cancel temperature and molar mass by taking the speed ratio first, then square it to remove both radicals.

47. A point charge 2×10^{-2} C is moved from P to S in a uniform electric field of $30\ \text{NC}^{-1}$ directed along positive x-axis. If coordinates of P and S are (1, 2, 0) m and (0, 0, 0) m respectively, the work done by electric field will be [4 marks]

(A) 600 mJ

(B) -1200 mJ

(C) 1200 mJ

(D) -600 mJ

Correct Answer: (D) -600 mJ

Solution:

Concept:

- The force on a charge q in a uniform electric field is $\vec{F} = q\vec{E}$. Work done by that field is the dot product $W = q\vec{E} \cdot (\vec{r}_f - \vec{r}_i)$.
- Only the displacement parallel to the field contributes to the work. A positive charge moving opposite the field has negative work done on it by the electric field.

Step 1: Find the displacement component along the field The initial position is $P = (1, 2, 0)\,\text{m}$, and the final position is $S = (0, 0, 0)\,\text{m}$. Therefore

$$\Delta\vec{r} = \vec{r}_S - \vec{r}_P = -\hat{\imath} - 2\hat{\jmath} \quad \text{m}.$$

The field is $\vec{E} = 30\hat{\imath}\,\mathrm{N\,C^{-1}}$. Since it has no y-component, the displacement of $-2\,\mathrm{m}$ along the y-axis contributes no work, regardless of the path taken.

Step 2: Evaluate the signed work and convert units Take the dot product before multiplying the charge:

$$W = (2 \times 10^{-2})(30)(-1) = -0.60\,\mathrm{J}.$$

Since $1\,\mathrm{J} = 1000\,\mathrm{mJ}$, this is

$$W = -600\,\mathrm{mJ}.$$

The negative sign follows physically because the positive charge moves through a net displacement opposite the force. The positive options confuse work done by the electric field with work required from an external agent. The magnitude $1200\,\mathrm{mJ}$ incorrectly uses the transverse coordinate difference or an extra factor of two. As a second check, $V_S - V_P = -E(x_S - x_P) = 30\,\mathrm{V}$, so $W = -q\Delta V = -0.60\,\mathrm{J}$.

Step 3: Conclusion The work done by the electric field is $\boxed{-600\,\mathrm{mJ}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a field along the x-axis, use only $x_f - x_i$. Keep its sign; a distance or total displacement magnitude loses the work direction.

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48. With the help of potentiometer, we can determine the value of emf of a given cell. The sensitivity of the potentiometer is

(A) directly proportional to the length of the potentiometer wire

(B) directly proportional to the potential gradient of the wire

(C) inversely proportional to the potential gradient of the wire

(D) inversely proportional to the length of the potentiometer wire

Choose the correct option for the above statements : [4 marks]

(A) A only

(B) B and D only

(C) A and C only

(D) inversely C only

Correct Answer: (C) A and C only

Solution:

Concept:

- A potentiometer balances a cell emf against the potential drop along a uniform wire: $\mathcal{E} = k\ell$, where k is the potential gradient and ℓ is the balancing length.
- Greater sensitivity means a small change in emf produces a larger measurable change in balancing length. Thus $d\ell/d\mathcal{E} = 1/k$: the sensitivity increases when the potential gradient decreases.

Step 1: Test the potential-gradient statements For a small emf change $\Delta\mathcal{E}$, the balance point shifts through

$$\Delta\ell = \frac{\Delta\mathcal{E}}{k}.$$

At a smaller gradient, the same small emf difference corresponds to a larger displacement, which is easier to detect with the scale. Therefore sensitivity is inversely proportional to potential gradient. Statement C is correct, while statement B reverses this dependence and is false.

Step 2: Relate the gradient to the wire length If a fixed potential difference V is maintained across the full wire of length L, its potential gradient is $k = V/L$. Hence

$$\text{sensitivity} \propto \frac{1}{k} = \frac{L}{V}.$$

With the driving potential difference held fixed, increasing the wire length reduces the potential drop per unit length and improves sensitivity. Statement A is therefore correct and statement D is false. The relevant comparison keeps the other operating conditions fixed; merely increasing current would instead increase the gradient and reduce sensitivity. Option A omits the true statement C, option B combines two false statements, and the final option omits the true length dependence. The pair A and C is the complete correct selection.

Step 3: Conclusion Statements A and C only are correct. Hence, the correct option is (C).

Quick Tip to Solve: Start from $\mathcal{E} = k\ell$: a smaller k spreads a small voltage difference over a longer measurable length.

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49. Substance A has atomic mass number 16 and half life of 1 day. Another substance B has atomic mass number 32 and half life of $\frac{1}{2}$ day. If both A and B simultaneously start undergo radio activity at the same time with initial mass 320 g each, how many total atoms of A and B combined would be left after 2 days. [4 marks]

(A) 1.69×10^{24}

(B) 3.38×10^{24}

(C) 6.76×10^{23}

(D) 6.76×10^{24}

Correct Answer: (B) 3.38×10^{24}

Solution:

Concept:

- After time t, the number of undecayed nuclei is $N = N_0(1/2)^{t/T_{1/2}}$, where $T_{1/2}$ is the half-life of that substance.
- Convert a mass to the number of atoms using $N_0 = (m/M)N_A$, with M the molar mass in $\mathrm{g\,mol^{-1}}$. Equal initial masses do not imply equal initial atom counts when the mass numbers differ.

Step 1: Find the surviving atoms of substance A For A, use molar mass $16\,\mathrm{g\,mol^{-1}}$ and initial mass $320\,\mathrm{g}$. Its initial amount is $320/16 = 20$ moles. In two days it undergoes $2/1 = 2$ half-lives, leaving the fraction $1/4$:

$$N_A^{\text{remaining}} = 20N_A\left(\frac{1}{2}\right)^2 = 5N_A.$$

Here N_A without a superscript denotes Avogadro's constant, not the name of substance A. This is the

number of undecayed parent atoms, as required by the decay question.

Step 2: Find the surviving atoms of B and add For B, the initial amount is $320/32 = 10$ moles. Its half-life is $1/2$ day, so two days contain four half-lives:

$$N_B^{\text{remaining}} = 10N_A \left(\frac{1}{2}\right)^4 = \frac{10}{16}N_A = 0.625N_A.$$

The combined number is therefore

$$N_{\text{total}} = (5 + 0.625)N_A = 5.625(6.022 \times 10^{23}) \simeq 3.38 \times 10^{24}.$$

One must add the surviving numbers of atoms, rather than add the remaining masses and divide by a single molar mass. The other listed values do not equal this independently calculated sum; using the same decay fraction for both materials would also be incorrect because their half-lives differ.

Step 3: Conclusion The combined number of surviving atoms of A and B is $\boxed{3.38 \times 10^{24}}$. Hence, the correct option is (B).

Quick Tip to Solve: For each isotope, divide the initial mass by its own molar mass, apply its own half-life factor, and only then add the atom counts.

50. The modulation index for an A.M. wave having maximum and minimum peak-to-peak voltages of 14 mV and 6 mV respectively is- [4 marks]

(A) 0.4

(B) 0.6

(C) 1.4

(D) 0.2

Correct Answer: (A) 0.4

Solution:

Concept:

- The modulation index is the ratio of the message amplitude to the unmodulated carrier amplitude. For ordinary amplitude modulation, the envelope extremes are $E_{\max} = E_c(1 + m)$ and $E_{\min} = E_c(1 - m)$.
- Consequently, $m = (E_{\max} - E_{\min})/(E_{\max} + E_{\min})$. Multiplying both measured extreme voltages by the same factor does not change this dimensionless ratio.

Step 1: Use the envelope extremes The maximum and minimum voltages are both specified using the same peak-to-peak convention. Each is twice the corresponding peak amplitude, so the common factor of two cancels in the envelope formula. We therefore use 14 mV and 6 mV directly; converting them to volts would also multiply numerator and denominator by the same factor.

Step 2: Calculate and check the index Adding the extreme values gives twice the carrier contribution,

while subtracting gives twice the message contribution. Thus

$$m = \frac{14 - 6}{14 + 6} = \frac{8}{20} = 0.4.$$

The result is less than unity, consistent with a positive minimum envelope and no overmodulation. The value 0.6 does not equal the difference-to-sum ratio, 1.4 would imply overmodulation, and 0.2 results from introducing an unnecessary factor of two after the common factor has already cancelled.

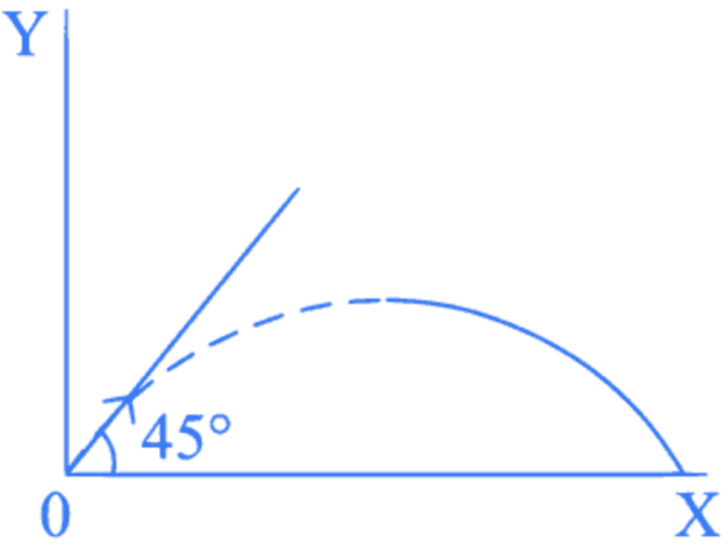
Step 3: Conclusion The modulation index is $\boxed{0.4}$, or 40 percent. Hence, the correct option is (A).

Quick Tip to Solve: Use the difference divided by the sum of envelope extremes; peak and peak-to-peak readings give the same ratio when used consistently.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A particle of mass 100 g is projected at time t = 0 with a speed 20 ms^{-1} at an angle 45° to the horizontal as given in the figure. The magnitude of the angular momentum of the particle about the starting point at time t = 2s is found to be $\sqrt{\text{K}}$ kg m^2/s. The value of K is _____.
(Take g = 10 ms^{-2}) [4 marks]



Solution:

Concept:

- Angular momentum about the launch point is $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$. For motion in the vertical plane, its signed component is $L_z = m(xv_y - yv_x)$.
- For projectile motion without air resistance, $x = u_x t$, $y = u_y t - gt^2/2$, $v_x = u_x$ and $v_y = u_y - gt$. Gravity changes angular momentum about the launch point because its line of action generally does not pass through that point.

Step 1: Resolve the initial velocity and find the motion Convert the mass before using SI units: $m = 100\,\mathrm{g} = 0.100\,\mathrm{kg}$. The launch components are

$$u_x = 20\cos 45^\circ = 10\sqrt{2}\,\mathrm{m\,s^{-1}}, \qquad u_y = 10\sqrt{2}\,\mathrm{m\,s^{-1}}.$$

At $t = 2\,\mathrm{s}$, the coordinates are $x = 20\sqrt{2}\,\mathrm{m}$, $y = (20\sqrt{2} - 20)\,\mathrm{m}$, and the velocity components are $v_x = 10\sqrt{2}\,\mathrm{m\,s^{-1}}$, $v_y = (10\sqrt{2} - 20)\,\mathrm{m\,s^{-1}}$.

Step 2: Evaluate the cross product and compare with the required form Keeping the cross-product order avoids adding the two contributions:

$$\begin{aligned} L_z &= m\{(u_x t)(u_y - gt) - (u_y t - gt^2/2)u_x\} \\ &= -\tfrac{1}{2}mu_x gt^2 \\ &= -\tfrac{1}{2}(0.100)(10\sqrt{2})(10)(2)^2 \\ &= -20\sqrt{2}\,\mathrm{kg\,m^2\,s^{-1}}. \end{aligned}$$

The negative sign indicates angular momentum into the page when the upward direction is positive. The question asks for its magnitude, so we square $20\sqrt{2}$ to identify K.

Step 3: Conclusion $|\mathbf{L}| = 20\sqrt{2} = \sqrt{800}\,\mathrm{kg\,m^2\,s^{-1}}$. Therefore, $\boxed{K = 800}$.

Quick Tip to Solve: For a projectile launched from the origin, the initial vertical-velocity terms cancel: use $|L| = mu_x gt^2/2$.

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52. In an experiment of measuring the refractive index of a glass slab using travelling microscope in physics lab, a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab as 5.00 mm. Travelling microscope has 20 divisions in one cm on main scale and 20 divisions on vernier scale is equal to 49 divisions on main scale. The estimated uncertainty in the measurement of refractive index of the slab is $\frac{x}{10} \times 10^{-3}$, where x is _____ [4 marks]

Solution:

Concept:

- For real thickness a and apparent thickness b, the refractive index is $\mu = a/b$. First-order maximum-error propagation gives $\Delta\mu = \mu(\Delta a/a + \Delta b/b)$; the error terms add rather than subtract.
- For an ordinary direct vernier, the least count is $|1\,\mathrm{MSD} - 1\,\mathrm{VSD}|$. The stated scale relationship must be checked before assigning an instrumental uncertainty.

Step 1: Calculate the index and inspect the stated scale The measured index is $\mu = 5.25/5.00 = 1.05$. Since 20 main-scale divisions occupy 1 cm, one main-scale division is $s = 0.50\,\mathrm{mm}$. The printed relationship gives

$$1\,\mathrm{VSD} = \frac{49}{20}s = 1.225\,\mathrm{mm}.$$

This is not the usual travelling-microscope direct vernier whose vernier spacing is slightly smaller than a main-scale division. In particular, it does not imply a least count of $0.01\,\mathrm{mm}$.

Step 2: Propagate the literal direct-vernier difference Using the ordinary difference convention on

the printed numbers gives $\delta = |0.500 - 1.225| = 0.725\,\text{mm}$. If each stated thickness is assigned that uncertainty, the first-order estimate is

$$\Delta\mu = 1.05\left(\frac{0.725}{5.25} + \frac{0.725}{5.00}\right)$$
$$= 0.29725.$$

Thus $\Delta\mu = x \times 10^{-4}$ would give $x = 2972.5$, not 41. The large uncertainty also makes the first-order approximation much less accurate than it would be for a normal fine vernier.

Step 3: Explain the discrepancy with the answer key The final key lists 41. That value follows if the scale relationship is instead 50 vernier divisions equal to 49 main-scale divisions: then $\delta = 0.50 - 49(0.50)/50 = 0.01\,\text{mm}$, and

$$\Delta\mu = 1.05\left(\frac{0.01}{5.25} + \frac{0.01}{5.00}\right) = 0.0041 = \frac{41}{10} \times 10^{-3}.$$

The printed relationship says 20, however, so this correction cannot be silently assumed. A different generalized vernier calibration would require its reading convention to be specified; it would not justify inserting the key's least count without explanation.

Step 4: Conclusion The official key gives $\boxed{x = 41}$, corresponding to a $0.01\,\text{mm}$ least count. As printed, the scale relationship is inconsistent with that result: the ordinary direct-vernier difference gives $x = 2972.5$. Therefore the printed data do not support the keyed value without correcting or clarifying the vernier specification.

Quick Tip to Solve: Check the main-scale and vernier spacing first. A plausible refractive index alone does not establish the instrument's least count.

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53. An inductor of inductance 2 μH is connected in series with a resistance, a variable capacitor and an AC source of frequency 7 kHz. The value of capacitance for which maximum current is drawn into the circuit $\frac{1}{x}$F, where the value of x is _____.
(Take $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- In a series LCR circuit, the current amplitude is $I = V/\sqrt{R^2 + (\omega L - 1/(\omega C))^2}$. At fixed source voltage and frequency it is maximum when the impedance is minimum.
- Resonance occurs when inductive and capacitive reactances cancel: $\omega L = 1/(\omega C)$, giving $C = 1/(\omega^2 L)$, where $\omega = 2\pi f$.

Step 1: Identify the resonance condition The resistor limits the largest current but does not change the capacitance at which the reactive terms cancel in this ideal series circuit. Thus, for maximum current,

$$\omega L - \frac{1}{\omega C} = 0 \quad \Rightarrow \quad C = \frac{1}{4\pi^2 f^2 L}.$$

The problem expresses the capacitance as $1/x$ farad, so the quantity to calculate is $x = 4\pi^2 f^2 L$ with L in henry and f in hertz.

Step 2: Convert units and calculate the denominator The supplied values are $L = 2 \times 10^{-6}\,\text{H}$ and $f = 7 \times 10^3\,\text{Hz}$. Using the specified value of pi gives

$$\omega = 2\left(\frac{22}{7}\right)(7 \times 10^3) = 44000\,\text{rad}\,\text{s}^{-1}.$$

Therefore,

$$x = \omega^2 L = (44000)^2(2 \times 10^{-6}) = 3872.$$

At this capacitance the reactances are equal in magnitude and opposite in sign, so the total impedance reduces to R, confirming that it gives maximum rather than minimum current.

Step 3: Conclusion The resonant capacitance is $C = \dfrac{1}{3872}\,\text{F} \approx 258.3\,\mu\text{F}$. Hence, $\boxed{x = 3872}$.

Quick Tip to Solve: When the answer is written as $C = 1/x$, compute $x = \omega^2 L$ directly and convert microhenry to henry first.

54. A particle of mass 250 g executes a simple harmonic motion under a periodic force $\text{F} = (-25\ x)\text{N}$. The particle attains a maximum speed of 4 m/s during its oscillation. The amplitude of the motion is _____ cm. [4 marks]

Solution:

Concept:

- A restoring force proportional to displacement has the SHM form $F = -kx$. Newton's second law gives $a = -(k/m)x = -\omega^2 x$, so $\omega = \sqrt{k/m}$.
- In SHM, the maximum speed is attained at the mean position and equals $v_{\max} = \omega A$, where A is the displacement amplitude.

Step 1: Extract the angular frequency from the force law The printed force is $F = -25x$ newton, so the force constant is $k = 25\,\text{N}\,\text{m}^{-1}$ when displacement is in metres. The negative sign indicates that the force points towards equilibrium; it does not make the angular frequency negative. Convert the particle's mass to $m = 250\,\text{g} = 0.250\,\text{kg}$. Hence,

$$\omega = \sqrt{\frac{25}{0.250}} = \sqrt{100} = 10\,\text{rad}\,\text{s}^{-1}.$$

Step 2: Use the maximum speed and convert the amplitude The speed-displacement relation is $v^2 = \omega^2(A^2 - x^2)$. Its largest value occurs at $x = 0$, so the given maximum speed fixes the amplitude without requiring the initial phase:

$$A = \frac{v_{\max}}{\omega} = \frac{4}{10} = 0.40\,\text{m} = 40\,\text{cm}.$$

As an independent energy check, the maximum kinetic energy is $\frac{1}{2}(0.250)(4)^2 = 2\,\text{J}$, while the

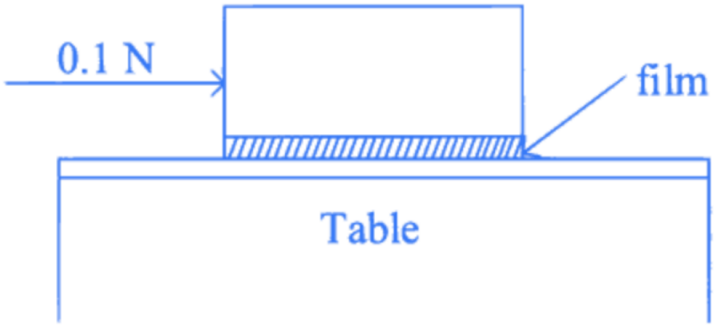
maximum elastic potential energy is $\frac{1}{2}(25)(0.40)^2 = 2\,\text{J}$, in agreement.

Step 3: Conclusion The amplitude of the motion is $\boxed{40\,\text{cm}}$; the numerical entry is $\boxed{40}$.

Quick Tip to Solve: Identify k from the coefficient of $-x$, then use $A = v_{\max}\sqrt{m/k}$; convert the final metre value to centimetres.

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55. A metal block of base area 0.20 m^2 is placed on a table, as shown in figure. A liquid film of thickness 0.25 mm is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed. IF the viscosity of the liquid is 5.0×10^{-3} Pl, the speed of block is _____ $\times 10^{-3}$ m/s. [4 marks]



Solution:

Concept:

- For laminar shear of a Newtonian liquid, the tangential viscous force is $F_v = \eta A\, dv/dy$, where η is dynamic viscosity and A is the area in contact.
- With no slipping at the surfaces, a stationary table and a block moving at speed v across a thin film of thickness d give a velocity gradient v/d. Constant block speed requires the applied force to balance the viscous resistance.

Step 1: Write the force balance across the film The lower liquid surface is at rest with the table, while the upper surface moves with the block. For a uniform thin film, the velocity changes linearly through its thickness, giving

$$F = F_v = \eta A\frac{v}{d} \quad \Rightarrow \quad v = \frac{Fd}{\eta A}.$$

The vertical weight and normal forces balance independently and are not included in this horizontal force balance. Since acceleration is zero, the net horizontal force must also vanish.

Step 2: Substitute SI values and express the requested coefficient Use $A = 0.20\,\text{m}^2$, $F = 0.1\,\text{N}$, $\eta = 5.0 \times 10^{-3}\,\text{Pa s}$, and

$$d = 0.25\,\text{mm} = 2.5 \times 10^{-4}\,\text{m}.$$

The poiseuille, Pl, is the SI unit Pa s; no additional factor is needed. Therefore,

$$v = \frac{(0.1)(2.5 \times 10^{-4})}{(5.0 \times 10^{-3})(0.20)} = 2.5 \times 10^{-2}\,\text{m s}^{-1}.$$

The film thickness belongs in the numerator: a thicker film has a smaller velocity gradient at a given speed and hence less viscous resistance.

Step 3: Conclusion The speed is $v = 25 \times 10^{-3}\,\mathrm{m\,s^{-1}}$. Thus, the required numerical coefficient is $\boxed{25}$.

Quick Tip to Solve: For a moving plate over a thin liquid film, use $v = Fd/(\eta A)$; remember that Pl already equals $\mathrm{Pa\,s}$.

56. A car is moving on a circular path of radius 600 m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete first quarter of revolution, if it is moving with an initial speed of 54 km/hr is $t(1 - e^{-\pi/2})s$. The value of t is _____. [4 marks]

Solution:

Concept:

- The tangential acceleration is $a_t = dv/dt$, while the centripetal acceleration is $a_c = v^2/R$. For increasing speed, equality gives $dv/dt = v^2/R$.
- Arc length and angle satisfy $ds = R\,d\theta$, and $v = ds/dt$. Changing the independent variable from time to angle makes the acceleration condition directly integrable.

Step 1: Find speed as a function of angular position The initial speed is $54\,\mathrm{km\,h^{-1}} = 15\,\mathrm{m\,s^{-1}}$. With increasing speed,

$$\frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = \frac{v}{R}\frac{dv}{d\theta} = \frac{v^2}{R}.$$

As the initial speed is positive, division by v/R is valid. Therefore, $dv/d\theta = v$, and integrating from the starting point gives

$$\ln\frac{v}{v_0} = \theta \quad \Rightarrow \quad v = v_0 e^{\theta}.$$

Step 2: Integrate the time over a quarter turn The angular speed is $d\theta/dt = v/R$. Thus the elapsed time for the first quarter revolution is

$$\begin{aligned} T &= \int_0^{\pi/2} \frac{R}{v_0 e^{\theta}}\,d\theta \\ &= \frac{600}{15}\left[-e^{-\theta}\right]_0^{\pi/2} \\ &= 40\left(1 - e^{-\pi/2}\right)\ \mathrm{s}. \end{aligned}$$

This matches the decreasing exponential in the expression supplied by the question. Equality of acceleration magnitudes alone also allows slowing motion; that sign would give $40(e^{\pi/2} - 1)\,\mathrm{s}$, so the specified answer form selects the increasing-speed case used here.

Step 3: Conclusion Comparing $T = 40(1 - e^{-\pi/2})\,\mathrm{s}$ with the given form gives $\boxed{t = 40}$.

Quick Tip to Solve: For equal positive tangential and centripetal accelerations, integrate $dv/d\theta = v$ first; then integrate $dt = (R/v)\,d\theta$.

57. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8 (medium -1) and 6.8 (medium -2), respectively. To satisfy the condition, so that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be $\tan^{-1}\left(1+\frac{10}{\theta}\right)^{\frac{1}{2}}$ the value of θ is _____.

(Given for dielectric media, $\mu_r = 1$) [4 marks]

Solution:

Concept:

- At the polarising angle i_B, reflected and refracted rays are perpendicular. Brewster's law is $\tan i_B = n_2/n_1$, for incidence from medium 1 into medium 2.
- For a dielectric with relative permeability $\mu_r = 1$, its refractive index is $n = \sqrt{\varepsilon_r}$. Thus the ratio of indices depends on the ratio of dielectric constants, not their difference.

Step 1: Express the polarising angle using the two media The incident ray is in medium 1, with dielectric constant 2.8, and the transmitted ray is in medium 2, with dielectric constant 6.8. Combining Snell's law with $i_B + r = 90^\circ$ gives $n_1 \sin i_B = n_2 \cos i_B$. Hence,

$$\tan i_B = \frac{n_2}{n_1} = \sqrt{\frac{6.8}{2.8}} = \sqrt{\frac{17}{7}}.$$

The higher index in the second medium makes this angle greater than 45°, consistent with this ratio.

Step 2: Compare the expression inside the square root The stated form is $i_B = \tan^{-1}\sqrt{1+10/\theta}$. Since both expressions describe the same incidence angle, equate their squared tangents:

$$1 + \frac{10}{\theta} = \frac{17}{7} = 1 + \frac{10}{7}.$$

Subtracting 1 gives $10/\theta = 10/7$, so $\theta = 7$. The parameter is dimensionless, since it occurs in a sum with 1 inside a trigonometric expression.

Step 3: Conclusion The required numerical value is $\boxed{\theta = 7}$.

Quick Tip to Solve: For nonmagnetic dielectrics, square Brewster's law first: $\tan^2 i_B = \varepsilon_{r2}/\varepsilon_{r1}$. Then compare directly with the given form.

58. For a charged spherical ball, electrostatic potential inside the ball varies with r as $\mathrm{V} = 2ar^2 + b$. Here, a and b are constant and r is the distance from the center. The volume charge density inside the ball is $-\lambda a\varepsilon$. The value of λ is _____.

ε = permittivity of the medium [4 marks]

Solution:

Concept:

- The radial electric field in a spherically symmetric charge distribution is $E_r = -dV/dr$; the minus sign makes the field point toward decreasing potential.
- Gauss's law in a uniform medium of permittivity ε is $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\varepsilon$. A spherical Gaussian

surface converts this to $4\pi r^2 E_r = Q_{\text{enc}}/\varepsilon$.

Step 1: Differentiate the prescribed potential For any radius inside the ball, use the given potential $V(r) = 2ar^2 + b$. The constant offset b changes the reference of potential but contributes no electric field. Therefore,

$$E_r = -\frac{d}{dr}(2ar^2 + b) = -4ar.$$

This is the signed radial component: if $a > 0$, the field is directed inward. Its linear dependence on radius is the characteristic internal field of a uniform spherical charge distribution.

Step 2: Obtain the enclosed charge and charge density The flux through a sphere of radius r is $(-4ar)(4\pi r^2)$. Gauss's law therefore gives

$$Q_{\text{enc}}(r) = -16\pi a\varepsilon r^3.$$

Differentiate with respect to radius and divide by the shell volume per unit radius:

$$\rho = \frac{1}{4\pi r^2}\frac{dQ_{\text{enc}}}{dr} = \frac{-48\pi a\varepsilon r^2}{4\pi r^2} = -12a\varepsilon.$$

This is independent of radius. Comparing with the specified $\rho = -\lambda a\varepsilon$ determines the positive coefficient.

Step 3: Conclusion The volume charge density is $-12a\varepsilon$, so $\boxed{\lambda = 12}$.

Quick Tip to Solve: For a potential proportional to r^2, the field is linear in r. Use a spherical Gaussian surface; a one-dimensional derivative of the field alone misses the geometric factor of three.

59. When two resistance R_1 and R_2 connected in series and introduced into the left gap of a meter bridge and a resistance of 10 Ω is introduced into the right gap, a null point is found at 60 cm from left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3 Ω is introduced into the right gap to get null point at 40 cm from left end. The product of R_1 R_2 is _____ Ω^2 [4 marks]

Solution:

Concept:

- For a uniform metre bridge at balance, the ratio of left-gap to right-gap resistances equals $l/(100-l)$, where l is the balancing length in centimetres measured from the left end.
- Two resistors in series have resistance $S = R_1 + R_2$; in parallel their resistance is $P = R_1R_2/(R_1 + R_2)$. Consequently their product is $R_1R_2 = SP$.

Step 1: Find the sum using the series arrangement In the first arrangement the series combination occupies the left gap and the $10\,\Omega$ resistor occupies the right gap. The left and right wire segments are 60 cm and 40 cm. Thus

$$\frac{R_1 + R_2}{10} = \frac{60}{40}, \qquad R_1 + R_2 = 15\,\Omega.$$

The series resistance is larger than the right-gap resistance, as expected because the left balancing

segment is longer. This also fixes which ratio must be used.

Step 2: Use the parallel arrangement to obtain the product In the second arrangement the left-gap parallel resistance balances against $3\,\Omega$, with wire segments 40 cm and 60 cm. Hence

$$\frac{P}{3} = \frac{40}{60}, \qquad P = 2\,\Omega.$$

Now substitute $P = R_1R_2/(R_1 + R_2)$ and the sum found above:

$$R_1R_2 = P(R_1 + R_2) = (2\,\Omega)(15\,\Omega) = 30\,\Omega^2.$$

There is no need to solve separately for the two individual resistances, since the question requests only their product.

Step 3: Conclusion The required product is $\boxed{R_1R_2 = 30\,\Omega^2}$, so the numerical entry is $\boxed{30}$.

Quick Tip to Solve: Find the series sum and parallel equivalent separately, then multiply them. Keep the left/right resistance ratio in the same order as the wire lengths.

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60. A null point is found at 200 cm in potentiometer when cell in secondary circuit is shunted by 5Ω. When a resistance of 15Ω is used for shunting, null point moves to 300 cm. The internal resistance of the cell is _____Ω. [4 marks]

Solution:

Concept:

- A potentiometer with fixed potential gradient k measures a terminal potential difference as $V = kl$, where l is the balancing length.
- When a cell of emf $\mathcal{E}$ and internal resistance r is shunted by an external load R, its terminal voltage is $V = \mathcal{E}R/(R + r)$. The null detector draws no current at balance.

Step 1: Write the voltage for each shunting resistance The shunt is connected across the cell, so it acts as the external load and causes current to flow through the internal resistance. For the $5\,\Omega$ shunt and the $15\,\Omega$ shunt respectively,

$$k(200) = \frac{5\mathcal{E}}{5 + r}, \qquad k(300) = \frac{15\mathcal{E}}{15 + r}.$$

Both measurements use the same primary potentiometer circuit, so k is unchanged. A larger load resistance draws less current and gives the larger terminal voltage and balancing length.

Step 2: Eliminate the emf and potential gradient Divide the first balance equation by the second, retaining the same order on both sides:

$$\frac{200}{300} = \frac{5/(5 + r)}{15/(15 + r)} = \frac{15 + r}{3(5 + r)}.$$

Simplifying,

$$\frac{2}{3} = \frac{15+r}{3(5+r)}, \qquad 2(5+r) = 15+r.$$

Thus $10+2r = 15+r$, giving $r = 5\,\Omega$. As a check, the two terminal voltages are then $\mathcal{E}/2$ and $3\mathcal{E}/4$; their ratio is $2/3$, exactly the observed length ratio.

Step 3: Conclusion The internal resistance is $\boxed{5\,\Omega}$; the required numerical entry is $\boxed{5}$.

Quick Tip to Solve: When both balancing lengths are measured with loads connected, use the ratio of $R/(R+r)$. Do not treat either loaded length as the open-circuit emf length.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Given below are two statements :

Statement I : The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga.

Statement II : The d orbitals in Ga are completely filled.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) Statement I is correct but statement II is incorrect

(B) Statement I is incorrect but statement II is correct

(C) Both the statements I and II are correct

(D) Both the statements I and II are incorrect

Correct Answer: (C) Both the statements I and II are correct

Solution:

Concept:

- First ionisation enthalpy is the energy required to remove the outermost electron from an isolated gaseous atom. It depends on atomic size, shielding and effective nuclear attraction.
- Boron, aluminium and gallium have outer configurations ns^2np^1. The filled $3d^{10}$ subshell of gallium shields the valence electrons poorly, producing the d-block contraction effect.

Step 1: Assess the comparison of ionisation enthalpies From B to Al, the electron removed changes from the compact second shell to the more extended third shell. Increased size and shielding cause a substantial reduction in first ionisation enthalpy. Approximate values are $801\,\text{kJ mol}^{-1}$ for B and $578\,\text{kJ mol}^{-1}$ for Al. Ga is very close to Al, about $579\,\text{kJ mol}^{-1}$, since its intervening d electrons shield poorly. Thus the pronounced B-to-Al fall is much larger than the almost absent fall from Al to Ga; in fact Ga is slightly higher. Statement I expresses this anomalously small change and is correct.

Step 2: Check gallium's electronic configuration and select the pair Gallium has atomic number 31 and configuration

$$\text{Ga}: \ [\text{Ar}]\,3d^{10}4s^2 4p^1.$$

Its occupied d subshell has ten electrons, so all five d orbitals are completely filled. Statement II is

therefore correct. Full occupancy does not imply strong shielding: filled d electrons still shield the outer electron less effectively than s and p electrons. Options A and B each incorrectly reject one true statement, while option D incorrectly rejects both.

Step 3: Conclusion Both statements I and II are correct. Hence, the correct option is (C).

Quick Tip to Solve: For Al and Ga, do not apply an unqualified down-group decrease: poor shielding by $3d^{10}$ makes Ga's first ionisation enthalpy nearly equal to, and slightly above, Al's.

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62. Given below are two statements :

Statement I : Nickel is being used as the catalyst for producing syn gas and edible fats.

Statement II : Silicon forms both electron rich and electron deficient hydrides.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is correct but statement II is incorrect

(B) Both the statements I and II are correct

(C) Both the statements I and II are incorrect

(D) Statement I is incorrect but statement II is correct

Correct Answer: (A) Statement I is correct but statement II is incorrect

Solution:

Concept:

- Nickel is an industrial hydrogenation catalyst and is also used in steam reforming of hydrocarbons to produce synthesis gas, a mixture chiefly of carbon monoxide and hydrogen.
- Covalent hydrides are electron deficient, electron precise or electron rich according to whether the central atom has too few electrons, exactly enough for ordinary bonds, or lone pairs beyond its bonding requirements.

Step 1: Evaluate the two catalytic applications of nickel In steam reforming, methane reacts with steam over a nickel catalyst to form synthesis gas:

$$CH_4 + H_2O \xrightarrow{Ni} CO + 3\,H_2.$$

Nickel also catalyses addition of hydrogen across carbon-carbon double bonds in unsaturated vegetable oils, converting them into more saturated fats used as edible fats. These are two distinct reactions sharing nickel as a suitable catalyst. Thus both uses named in statement I are valid, and statement I is correct.

Step 2: Classify silicon hydrides and eliminate the alternatives Silicon is a group 14 element with four valence electrons. In silane, SiH_4, it makes four conventional two-centre two-electron bonds and has an octet with no lone pair on the central atom. The ordinary silicon hydrides are therefore electron precise. Electron-deficient hydrides are characteristic of group 13, while electron-rich hydrides have central-atom lone pairs, as in group 15 to 17 examples. The claim that silicon forms both electron-rich and electron-deficient hydrides is false. Hence statement II is incorrect. B wrongly accepts it; C and D

wrongly reject the true nickel statement.

Step 3: Conclusion Statement I is correct but statement II is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: Count the central atom's valence electrons before classifying a covalent hydride: group 14 hydrides are electron precise, while lone pairs indicate electron-rich hydrides.

63. Which of the following relations are correct?

(A) $\Delta \mathrm{U} = \mathrm{q} + \mathrm{p}\Delta \mathrm{V}$

(B) $\Delta \mathrm{G} = \Delta \mathrm{H} - \mathrm{T}\Delta \mathrm{S}$

(C) $\Delta \mathrm{S} = \frac{q_{rev}}{T}$

(D) $\Delta \mathrm{H} = \Delta \mathrm{U} - \Delta \mathrm{nRT}$

Choose the most appropriate answer from the options given below : [4 marks]

(A) A and B only

(B) B and C only

(C) C and D only

(D) B and D only

Correct Answer: (B) B and C only

Solution:

Concept:

- With the chemistry sign convention, $\Delta U = q + w$. Expansion work done on the system is $w = -p_{\text{ext}}\Delta V$ for constant external pressure.
- Gibbs energy is $G = H - TS$, entropy change for a reversible isothermal transfer is $\Delta S = q_{\text{rev}}/T$, and ideal-gas enthalpy satisfies $H = U + nRT$.

Step 1: Test the energy and enthalpy relations Relation A writes a plus sign before $p\Delta V$. For expansion, however, the system does work on its surroundings, and work on the system is negative. The correct constant-pressure expansion formula is

$$\Delta U = q - p_{\text{ext}}\Delta V.$$

Thus A is incorrect. For an ideal-gas reaction at fixed temperature, the change in pV is $\Delta n_g RT$, giving

$$\Delta H = \Delta U + \Delta n_g RT.$$

Relation D has the opposite sign and is also incorrect. Here Δn_g means gaseous product moles minus gaseous reactant moles.

Step 2: Confirm the Gibbs-energy and entropy relations At the constant temperature used for the usual Gibbs-energy change relation, subtracting the initial from the final value of $G = H - TS$ yields

$$\Delta G = \Delta H - T\Delta S.$$

Hence B is correct. Entropy is defined through reversible heat transfer as $dS = \delta q_{\text{rev}}/T$; for an isothermal reversible process this integrates to $\Delta S = q_{\text{rev}}/T$, so C is correct in the stated standard relation. The heat must be reversible heat, not arbitrary actual heat. Therefore B and C form the correct pair. Options A, C and D each include at least one of the incorrect sign relations.

Step 3: Conclusion Relations B and C only are correct. Hence, the correct option is (B).

Quick Tip to Solve: Check the signs first: expansion work on a system is negative, whereas the ideal-gas correction from internal energy to enthalpy is $+\Delta n_g RT$.

64. A solution of CrO_5 in amyl alcohol has a _____ colour. [4 marks]

(A) Yellow

(B) Orange-Red

(C) Green

(D) Blue

Correct Answer: (D) Blue

Solution:

Concept:

- Chromium(VI) gives a characteristic deep-blue peroxide, CrO_5, when an acidified chromate or dichromate solution reacts with hydrogen peroxide.
- The colour of a compound depends on its chemical species as well as its environment. Extraction into an organic solvent such as amyl alcohol stabilises the blue chromium peroxide species long enough for identification.

Step 1: Identify the compound responsible for the test The formula given is CrO_5, chromium peroxide. Its usual representation contains one terminal oxo group and two peroxide groups. The peroxide oxygens have oxidation number -1, whereas the oxo oxygen has oxidation number -2; chromium therefore remains in oxidation state $+6$. This distinguishes the peroxide from ordinary chromium(III) salts.

Step 2: Apply the characteristic colour observation Chromium peroxide gives a deep-blue solution in amyl alcohol. This is the blue colour used in the qualitative peroxide test for chromium. A representative formation reaction is

$$Cr_2O_7^{2-} + 4\,H_2O_2 + 2\,H^+ \longrightarrow 2\,CrO_5 + 5\,H_2O.$$

The organic layer is examined for the blue product; it is not assigned the colour of the starting dichromate.

Step 3: Exclude the alternative colours Yellow is associated with chromate, while orange or orange-red is associated with dichromate rather than this peroxide. Green commonly indicates chromium(III) species after reduction or decomposition. None describes the freshly formed CrO_5 solution in amyl alcohol. The formula and solvent together identify the blue test species.

Step 4: Conclusion The solution of CrO_5 in amyl alcohol is blue. Hence, the correct option is (D).

Quick Tip to Solve: Associate the chromium peroxide test with a blue organic layer; do not transfer the yellow or orange colour of chromate or dichromate to the peroxide.

65. According to MO theory the bond orders for O_2^{2-}, CO and NO^+ respectively, are [4 marks]

(A) 1, 3 and 3

(B) 2, 3 and 3

(C) 1, 2 and 3

(D) 1, 3 and 2

Correct Answer: (A) 1, 3 and 3

Solution:

Concept:

- The molecular-orbital bond order is $\mathrm{BO} = (N_b - N_a)/2$, where N_b and N_a are the numbers of electrons in bonding and antibonding orbitals.
- Adding electrons to antibonding orbitals lowers bond order. Isoelectronic diatomic species with the same relevant orbital occupation have the same bond order.

Step 1: Find the bond order of peroxide Neutral O_2 has twelve valence electrons and bond order 2. Forming ${O_2}^{2-}$ adds two electrons to the antibonding π^* orbitals, one in each previously singly occupied orbital. The valence bonding and antibonding counts become eight and six, respectively:

$$\mathrm{BO}({O_2}^{2-}) = \frac{8-6}{2} = 1.$$

Thus the first entry cannot be 2.

Step 2: Count electrons in carbon monoxide and nitrosonium Carbon contributes four valence electrons and oxygen six, so CO has ten valence electrons. In NO^+, nitrogen contributes five and oxygen six, but one electron is removed for the positive charge:

$$N_v(\mathrm{CO}) = 4 + 6 = 10, \qquad N_v(\mathrm{NO}^+) = 5 + 6 - 1 = 10.$$

Both are isoelectronic with N_2. Their occupied valence orbitals contain eight bonding electrons and two antibonding electrons.

Step 3: Calculate and compare the ordered triplet For each ten-valence-electron species,

$$\mathrm{BO}(\mathrm{CO}) = \mathrm{BO}(\mathrm{NO}^+) = \frac{8-2}{2} = 3.$$

Closed core orbitals, if included, add equal bonding and antibonding contributions and do not change these results. The ordered triplet is therefore $(1, 3, 3)$. Option B overestimates the peroxide bond order; option C underestimates that of carbon monoxide; option D underestimates that of nitrosonium.

Step 4: Conclusion The bond orders of ${O_2}^{2-}$, CO and NO^+ are 1, 3 and 3, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Use the ten-valence-electron shortcut for CO and NO^+, and remember that the two extra electrons in peroxide enter antibonding orbitals.

66. When a hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A? [4 marks]

(A) C_9H_6

(B) C_6H_6

(C) C_8H_6

(D) C_9H_9

Correct Answer: (C) C_8H_6

Solution:

Concept:

- Complete combustion of one mole of C_xH_y follows

$$C_xH_y + (x + y/4)O_2 \longrightarrow xCO_2 + (y/2)H_2O.$$

- The water coefficient fixes the hydrogen count, while the oxygen coefficient then fixes the carbon count. The word equivalents here describes the stoichiometric amounts relative to one amount of hydrocarbon.

Step 1: Determine the hydrogen content from water Let the molecular formula be C_xH_y. Every water molecule contains two hydrogen atoms, and complete combustion transfers all the hydrocarbon hydrogen into water. Since three corresponding amounts of water form per amount of the hydrocarbon,

$$\frac{y}{2} = 3 \quad \Rightarrow \quad y = 6.$$

The required formula therefore has six hydrogen atoms; C_9H_9 already fails this condition.

Step 2: Determine the carbon content from oxygen demand The oxygen consumption is $x + y/4$. Substituting the stated value 9.5 and the hydrogen count just found gives

$$x + \frac{6}{4} = 9.5, \qquad x = 9.5 - 1.5 = 8.$$

Thus carbon contributes eight molecules of carbon dioxide, and the candidate formula is C_8H_6. The oxygen used to form water must be subtracted before assigning the carbon count.

Step 3: Check the balanced reaction and alternatives The complete equation is

$$C_8H_6 + \tfrac{19}{2}O_2 \longrightarrow 8\,CO_2 + 3\,H_2O.$$

There are nineteen oxygen atoms on each side: sixteen in carbon dioxide and three in water. In comparison, C_9H_6 would need 10.5 amounts of oxygen and C_6H_6 would need 7.5, even though both produce three amounts of water. Only the eight-carbon formula satisfies both observations simultaneously.

Step 4: Conclusion The molecular formula of hydrocarbon A is C_8H_6. Hence, the correct option is (C).

Quick Tip to Solve: For C_xH_y, read y directly from twice the water coefficient, then use $x =$ oxygen coefficient $- y/4$.

67. The set of correct statements is :

(i) Manganese exhibits +7 oxidation state in its oxide.

(ii) Ruthenium and Osmium exhibit +8 oxidation in their oxides.

(iii) Sc shows +4 oxidation state which is oxidizing in nature.

(iv) Cr shows oxidising nature in +6 oxidation state. [4 marks]

(A) (ii), (iii) and (iv)

(B) (ii) and (iii)

(C) (i) and (iii)

(D) (i), (ii) and (iv)

Correct Answer: (D) (i), (ii) and (iv)

Solution:

Concept:

- Oxidation states in ordinary binary oxides follow charge neutrality, taking oxygen as -2. High oxidation states of transition metals are often stabilised by oxygen.
- An oxidising species accepts electrons and is itself reduced. Chromium(VI) commonly acts as an oxidising agent, whereas scandium strongly favours the $+3$ state.

Step 1: Test the manganese and heavier transition-metal statements Manganese forms Mn_2O_7. If its oxidation state is x, charge neutrality gives

$$2x + 7(-2) = 0 \quad \Rightarrow \quad x = +7.$$

Statement (i) is correct. Ruthenium and osmium form the tetroxides RuO_4 and OsO_4. In each case,

$$x + 4(-2) = 0 \quad \Rightarrow \quad x = +8.$$

Hence statement (ii) is also correct.

Step 2: Check the assertion about scandium Scandium has ground-state configuration $[Ar]3d^1 4s^2$. Loss of these three outer electrons produces the particularly stable argon-core ion Sc^{3+}. The usual scandium chemistry is consequently dominated by $+3$, not the claimed $+4$ oxidising state. Statement (iii) is false. It must be excluded rather than accepted by analogy with other early transition metals.

Step 3: Check chromium and select the full set Chromium in the $+6$ state, as in acidified dichromate, accepts electrons to form Cr^{3+}:

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

This reduction illustrates its oxidising nature, so statement (iv) is correct. The required set is exactly (i), (ii) and (iv). Each of options A, B and C includes the false scandium statement, which rules all three out.

Step 4: Conclusion Statements (i), (ii) and (iv) are correct. Hence, the correct option is (D).

Quick Tip to Solve: Use Mn_2O_7, RuO_4 and OsO_4 as oxidation-state checks, and treat scandium as a characteristic $+3$ element.

68. Correct order of spin only magnetic moment of the following complex ions is :
(Given At.no. Fe : 26, Co : 27) [4 marks]

(A) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$
(B) $[FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$
(C) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$
(D) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{3-}$

Correct Answer: (C) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons and μ_B is the Bohr magneton.
- In octahedral complexes, weak-field fluoride gives high-spin configurations. For cobalt(III), oxalate produces the low-spin d^6 complex in this comparison.

Step 1: Identify the metal oxidation states and electron counts In $[FeF_6]^{3-}$ and $[CoF_6]^{3-}$, six fluoride ligands contribute charge -6; the metal is therefore $+3$. Three oxalate ligands also contribute -6, so cobalt is $+3$ in $[Co(C_2O_4)_3]^{3-}$. Removing the outer $4s$ electrons before $3d$ gives $Fe^{3+} : d^5$ and $Co^{3+} : d^6$.

Step 2: Count unpaired electrons using the ligand fields Fluoride is weak-field. The high-spin iron(III) complex has $t_{2g}^3 e_g^2$, with five unpaired electrons. The high-spin cobalt(III) fluoride complex has $t_{2g}^4 e_g^2$, with four unpaired electrons. In the cobalt(III) oxalate complex the electrons pair into the lower set, giving $t_{2g}^6 e_g^0$ and no unpaired electrons. Thus the relevant counts are 5, 4 and 0, respectively.

Step 3: Calculate the moments and rank them Applying the formula separately gives

$$\mu_{FeF_6} = \sqrt{35}\,\mu_B \approx 5.92\,\mu_B,$$

$$\mu_{CoF_6} = \sqrt{24}\,\mu_B \approx 4.90\,\mu_B, \qquad \mu_{Co(ox)_3} = 0.$$

The iron complex must come first and the paired oxalate complex last. Option A reverses the two fluoride complexes; B puts the diamagnetic oxalate above cobalt fluoride; D puts it first. Those orders contradict the unpaired-electron counts.

Step 4: Conclusion The order is $[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$. Hence, the correct option is (C).

Quick Tip to Solve: Find oxidation state before d-electron count, then distinguish high-spin d^5, high-spin d^6 and low-spin d^6: their unpaired counts are 5, 4 and 0.

69. Reaction of propanamide with $Br_2/KOH(aq)$ produces : [4 marks]

(A) Propanenitrile

(B) Propylamine

(C) Ethylnitrile

(D) Ethylamine

Correct Answer: (D) Ethylamine

Solution:

Concept:

- A primary amide reacts with bromine and aqueous alkali in the Hofmann bromamide reaction to form a primary amine containing one carbon atom fewer than the amide.
- The amide carbonyl carbon is lost as carbonate under the alkaline conditions. The group attached to that carbonyl carbon becomes the group attached to the amino nitrogen.

Step 1: Write the substrate and identify the retained group Propanamide has structure $CH_3CH_2CONH_2$. In the general notation $RCONH_2$, its R group is ethyl, $CH_3CH_2{}^-$, not propyl. The reagents Br_2 and aqueous KOH identify a Hofmann rearrangement rather than reduction of the carbonyl group or dehydration to a nitrile.

Step 2: Apply the one-carbon degradation The overall balanced transformation is

$$CH_3CH_2CONH_2 + Br_2 + 4\,KOH \longrightarrow CH_3CH_2NH_2 + K_2CO_3 + 2\,KBr + 2\,H_2O.$$

Bromination at nitrogen is followed by rearrangement to an isocyanate. Hydrolysis and loss of the carbonyl carbon then give the amine. The ethyl group remains intact throughout, so the organic product contains two carbons even though the starting amide contains three.

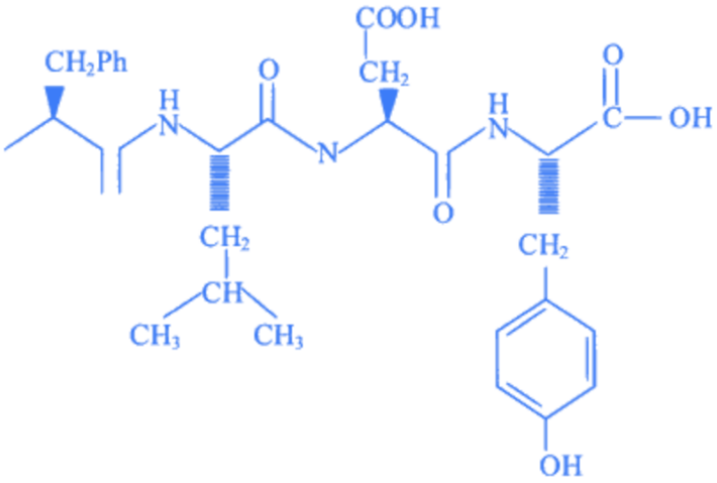
Step 3: Identify the product and eliminate the distractors The product $CH_3CH_2NH_2$ is ethylamine, also called ethanamine, a primary amine. Propylamine would retain all three carbons and is associated with reduction of propanamide, not this degradation. Propanenitrile and the nitrile named in option C contain a carbon-nitrogen triple bond, whereas the present reaction gives an amino group. A dehydrating reagent would be needed to convert a primary amide directly into a nitrile. Thus none of those alternatives describes this reagent combination.

Step 4: Conclusion Propanamide gives ethylamine, $CH_3CH_2NH_2$, with $Br_2/KOH\,(aq)$. Hence, the correct option is (D).

Quick Tip to Solve: In a Hofmann bromamide problem, delete the amide carbonyl carbon and replace $-CONH_2$ by $-NH_2$ on the remaining carbon group.

70. Following tetrapeptide can be represented as

(F, L, D, Y, I, Q, P are one letter codes for amino acids) [4 marks]



(A) YQLF

(B) FIQY

(C) PLDY

(D) FLDY

Correct Answer: (D) FLDY

Solution:

Concept:

- Peptide sequences are written from the amino-terminal end towards the carboxyl-terminal end. Follow the repeating backbone rather than reading side chains in an arbitrary order.
- The side chains identify the residues: benzyl is phenylalanine (F), isobutyl is leucine (L), carboxymethyl is aspartic acid (D), and para-hydroxybenzyl is tyrosine (Y).

Step 1: Choose the direction of the peptide sequence The structure has the terminal carboxyl group at the right-hand end. Therefore the residue sequence is read along the backbone from left to right, towards that carboxyl terminus. Each peptide linkage $-CO-NH-$ connects successive amino-acid residues. The carbon bearing each side chain identifies one residue; the side-chain carboxyl group must not be mistaken for the terminal backbone carboxyl group.

Step 2: Identify the first two residues from their side chains The leftmost residue bears $-CH_2C_6H_5$, a benzyl group, so it is phenylalanine, code F. The next residue bears $-CH_2CH(CH_3)_2$, the isobutyl side chain of leucine, code L. Isoleucine, code I, instead has a sec-butyl side chain whose branching begins at the carbon directly next to the alpha carbon. Proline, code P, would have its side chain closed back onto the backbone nitrogen, which is not shown.

Step 3: Identify the last two residues and check the alternatives The third side chain is $-CH_2COOH$, identifying aspartic acid, code D. Glutamine, code Q, would contain $-CH_2CH_2CONH_2$, with an extra methylene and a terminal amide instead of the shown acid. The fourth side chain is $-CH_2C_6H_4OH$, identifying tyrosine, code Y. Consequently the sequence is F-L-D-Y. YQLF reverses and misidentifies residues, FIQY wrongly assigns both middle residues, and PLDY wrongly assigns the first residue.

Step 4: Conclusion The tetrapeptide is phenylalanine-leucine-aspartic acid-tyrosine, represented by FLDY. Hence, the correct option is (D).

Quick Tip to Solve: Locate the terminal backbone $-COOH$ first, then read towards it and identify residues from side chains; distinguish leucine from isoleucine by the position of branching.

71. Match List I with List II

	List I		List II
A.	**van't Hoff factor, i**	**I.**	**Cryoscopic constant**
B.	k_f	**II.**	**Isotonic solutions**
C.	**Solutions with same osmotic pressure**	**III.**	$\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$
D.	**Azeotropes**	**IV.**	**Solutions with same composition of vapour above it**

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-III, B-I, C-II, D-IV

(C) A-III, B-II, C-I, D-IV

(D) A-I, B-III, C-II, D-IV

Correct Answer: (B) A-III, B-I, C-II, D-IV

Solution:

Concept:

- The van't Hoff factor is the ratio of the observed colligative effect to its value for an undissociated solute; it corrects the number of solute particles.
- For molar masses calculated from a colligative property, $i = M_{\text{normal}}/M_{\text{abnormal}}$. Cryoscopic constant K_f is the molal freezing-point depression constant.
- Isotonic solutions have equal osmotic pressures at the same temperature. An azeotrope has the same composition in its equilibrium liquid and vapour phases.

Step 1: Match the colligative-property terms For a fixed mass of solute, the inferred particle count is inversely proportional to the molar mass. Dissociation therefore gives a smaller apparent molar mass and $i > 1$, while association gives a larger apparent molar mass and $i < 1$. Thus A matches III. The symbol K_f denotes the cryoscopic constant, so B matches I. It is not a name for a class of solutions.

Step 2: Match the solution descriptions Solutions having equal osmotic pressure are isotonic, giving C-II. An azeotrope boils without a change in composition because its vapour and liquid have identical compositions, giving D-IV. Hence the complete sequence is A-III, B-I, C-II, D-IV. Option A interchanges the last two definitions; option C misidentifies K_f, and option D interchanges the first two terms.

Step 3: Conclusion The matching is $\boxed{\text{A-III, B-I, C-II, D-IV}}$. Hence, the correct option is (B).

Quick Tip to Solve: Match the uniquely named cryoscopic constant and isotonic solutions first; then check the molar-mass ratio and azeotrope definition.

72. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement) [4 marks]

(A) 2-Bromo-3,3-dimethylpentane

(B) 2-Bromopropane

(C) 1-Bromo-2-methylbutane

(D) 2-Bromopentane

Correct Answer: (D) 2-Bromopentane

Solution:

Concept:

- Dehydrohalogenation removes a halogen from the alpha-carbon and a hydrogen from an adjacent beta-carbon, forming a double bond between them.
- Count distinct constitutional alkenes and then include geometrical isomers where each double-bond carbon has two different substituents. Rearrangements are excluded here.

Step 1: Count the products of the first three compounds In 2-bromo-3,3-dimethylpentane, carbon 3 has no hydrogen; elimination is possible only toward carbon 1, giving one terminal alkene. In 2-bromopropane, the two methyl groups are equivalent and both give propene, again one product. In 1-bromo-2-methylbutane, the only adjacent carbon bearing a beta-hydrogen is carbon 2, so only 2-methylbut-1-ene is obtained. Each terminal alkene has a CH_2 end and cannot show geometrical isomerism.

Step 2: Count the products of 2-bromopentane For $CH_3CH(Br)CH_2CH_2CH_3$, removal of a hydrogen from carbon 1 gives pent-1-ene; removal from carbon 3 gives pent-2-ene. The internal alkene has two different groups on each double-bond carbon, so it occurs as E- and Z-pent-2-ene. The total is therefore one terminal alkene plus two geometrical isomers, or three distinct alkenes. None of options A, B or C has this combination of two elimination directions and an internal alkene with geometrical isomerism.

Step 3: Conclusion 2-Bromopentane gives the maximum: pent-1-ene, E-pent-2-ene and Z-pent-2-ene, a total of $\boxed{3}$ isomeric alkenes. Hence, the correct option is (D).

Quick Tip to Solve: First mark all beta-carbons with hydrogens; after listing distinct double-bond positions, count E/Z forms separately.

73. An indicator 'X' is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with H_2O_2 at room temp. The indicator 'X' forms blue coloured complex with compound 'A' present in the solution. The indicator 'X' and compound 'A' respectively are : [4 marks]

(A) Methyl orange and H_2O_2

(B) Methyl orange and iodine

(C) Starch and H_2O_2

(D) Starch and iodine

Correct Answer: (D) Starch and iodine

Solution:

Concept:

- Hydrogen peroxide oxidises iodide ions to iodine in acidic solution: $H_2O_2 + 2\,I^- + 2\,H^+ \longrightarrow I_2 + 2\,H_2O$.
- Starch gives an intense blue complex with iodine in the presence of iodide; the coloured species involves polyiodide held within the starch helix.

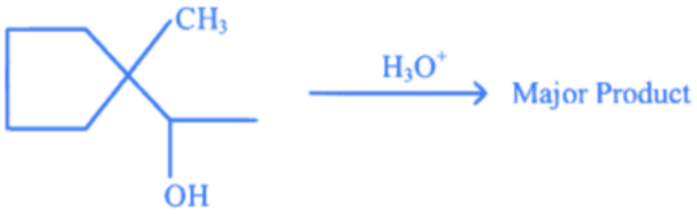
Step 1: Identify the coloured reaction product The reaction being monitored consumes iodide and hydrogen peroxide and produces iodine. As the iodide concentration changes, the rate of iodine formation changes. Iodine combines with excess iodide to establish $I_2 + I^- \rightleftharpoons I_3^-$; iodine/polyiodide species can then interact with starch. Thus the compound called A in the question is iodine, rather than hydrogen peroxide, which is a reactant and does not produce the characteristic blue starch complex.

Step 2: Identify the indicator and eliminate the distractors Starch is the indicator used to reveal iodine by a deep blue colour. The colour provides a convenient visual endpoint in the kinetic experiment, allowing a time interval to be associated with a defined extent of reaction. Methyl orange is an acid-base indicator and changes colour with pH; it does not give the characteristic blue complex with iodine. Hence options A and B have the wrong indicator, and option C names the right indicator but the wrong compound. Only option D gives both required substances correctly.

Step 3: Conclusion X is starch and A is iodine. Hence, the correct option is (D).

Quick Tip to Solve: In iodide-peroxide rate experiments, associate the blue colour with the starch-iodine complex, not with the oxidising reagent.

74. Find out the major product for the following reaction. [4 marks]



(A) 1-Methyl-1-vinylcyclopentane

(B) 1,6-Dimethylcyclohex-1-ene

(C) 1,2-Dimethylcyclohex-1-ene

(D) 1-Methylene-2-methylcyclohexane

Correct Answer: (C) 1,2-Dimethylcyclohex-1-ene

Solution:

Concept:

- Acid-catalysed dehydration of a secondary alcohol proceeds by protonation, loss of water and elimination through a carbocation intermediate.
- A neighbouring cyclopentane carbon-carbon bond can migrate to a carbocation centre, expanding a five-membered ring to a six-membered ring. Subsequent elimination favours the more substituted alkene.

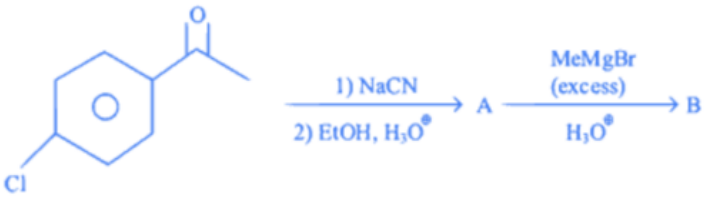
Step 1: Generate the carbocation The starting alcohol is $CH_3CH(OH)^-$ attached to a cyclopentane carbon that also bears a methyl group. Protonation changes the hydroxyl group into a good leaving group, and loss of water generates a secondary carbocation on the external $CH(CH_3)$ carbon. Direct removal of a hydrogen from the terminal methyl group would produce the less substituted vinyl product in option A; it does not exploit the favourable rearrangement pathway.

Step 2: Expand the ring and eliminate Migration of an adjacent ring carbon-carbon bond into the electron-deficient carbon inserts that carbon into the ring. The product of this 1,2-shift is a six-membered ring with the positive charge on the former substituted ring carbon. That centre is tertiary, so the shift also improves carbocation stability. Removal of the hydrogen on the neighbouring newly incorporated carbon forms the double bond between the two methyl-bearing ring carbons. This gives a tetrasubstituted endocyclic alkene. The alkene in option B has a less substituted double bond, while option D has a less stable exocyclic double bond.

Step 3: Conclusion The major product is $\boxed{\text{1,2-dimethylcyclohex-1-ene}}$, obtained by ring expansion followed by elimination. Hence, the correct option is (C).

Quick Tip to Solve: For an alcohol beside a substituted cyclopentane carbon, check ring expansion before applying the more-substituted-alkene rule.

75. Find out the major products from the following reaction sequence. [4 marks]



(A) $A = (\text{p-ClC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{CO}_2\text{H}$
$B = (\text{p-ClC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{C(OH)(CH}_3)_2$.
(B) $A = (\text{p-ClC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{CO}_2\text{Et}$
$B = (\text{p-ClC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{C(OH)(CH}_3)_2$.
(C) $A = (\text{p-NCC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{CN}$
$B = (\text{p-CH(OH)(CH}_3)\text{C}_6\text{H}_4)\text{C(OH)(CH}_3)\text{CH(OH)(CH}_3)$.
(D) $A = (\text{p-ClC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{CN}$
$B = (\text{p-ClC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{C(NH}_2)(\text{CH}_3)_2$.
Correct Answer: (B) $A = (\text{p-ClC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{CO}_2\text{Et}$
$B = (\text{p-ClC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{C(OH)(CH}_3)_2$.

Solution:

Concept:

- Cyanide adds to a ketone carbonyl to form a cyanohydrin after protonation; the original carbonyl oxygen becomes a hydroxyl group.
- An alcoholic acidic treatment converts the nitrile-derived group into an ester. An ester reacts with two equivalents of a Grignard reagent, followed by hydrolysis, to give a tertiary alcohol.

Step 1: Form the cyanohydrin and identify A Write $\text{Ar} = p\text{-ClC}_6\text{H}_4$. The starting ketone is ArCOCH_3. Cyanide attacks its carbonyl carbon, giving $\text{ArC(OH)(CH}_3)\text{CN}$ on protonation. The aryl chlorine remains attached to the ring: ordinary cyanohydrin formation is nucleophilic addition at the ketone, not substitution of this unactivated aryl chloride. The next treatment is ethanol in acid, so the nitrile-derived group becomes the ethyl ester. Thus $A = \text{ArC(OH)(CH}_3)\text{CO}_2\text{Et}$, not the free acid in option A or the unchanged nitriles in C and D.

Step 2: Apply excess methylmagnesium bromide The excess Grignard reagent first consumes reagent by deprotonating the existing hydroxyl group. At the ester carbonyl, methyl addition and loss of ethoxide give a methyl ketone, which immediately receives a second methyl group. Acidic work-up protonates both alkoxide groups, giving

$$B = \text{ArC(OH)(CH}_3)\text{C(OH)(CH}_3)_2.$$

The product is therefore a vicinal diol with two added methyl groups at the former ester carbonyl carbon. The aryl chlorine is retained throughout. This pair of structures is the pair in option B; the alternatives either retain the nitrile, change the aryl substituent or propose the wrong functional group.

Step 3: Conclusion The products are

$$A = (\text{p-ClC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{CO}_2\text{Et},$$

$$B = (\text{p-ClC}_6\text{H}_4)\text{C(OH)(CH}_3)\text{C(OH)(CH}_3)_2.$$

Hence, the correct option is (B).

Quick Tip to Solve: Track the carbon of the added cyanide: it becomes the ester carbonyl carbon, then the new tertiary alcohol carbon after two methyl additions.

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76. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease? [4 marks]

(A) Stomach ulcers

(B) Depression and hypertension

(C) Anxiety and stress

(D) Hyperacidity

Correct Answer: (B) Depression and hypertension

Solution:

Concept:

- Equanil is a tranquilliser named in the textbook treatment of drugs acting on the central nervous system. It is used for controlling depression and hypertension.
- This question asks for the specific therapeutic use associated with Equanil in that classification. Antacids and anti-ulcer medicines belong to a different therapeutic category.

Step 1: Identify the specific textbook indication Equanil is a tranquillising medicine. In the prescribed classification, its stated use is controlling depression and hypertension. The two symptoms in option B therefore reproduce the indication associated specifically with this named drug. The question is a drug-use identification item, so the specific association is more decisive than the general observation that tranquilisers can have calming and anxiety-relieving effects.

Step 2: Distinguish the competing descriptions Stomach ulcers and hyperacidity are treated with medicines affecting gastric acid secretion or neutralising acid; these are not the indication being tested for Equanil. This rules out options A and D. Anxiety and stress are broad indications associated with the tranquilliser class, which makes option C tempting, but the drug-specific textbook pairing for Equanil is depression and hypertension. Therefore B is the intended answer among the listed pairs. This identifies the examination classification and does not infer an individual medical diagnosis merely from a prescription.

Step 3: Conclusion The drug-specific textbook indication is $\boxed{\text{depression and hypertension}}$. Hence, the correct option is (B).

Quick Tip to Solve: When several choices resemble a drug class's general effects, use the specific drug-indication pair given in the textbook: Equanil is linked with depression and hypertension.

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77. **Match List I and List II**

	List I		List II
A.	Osmosis	I.	Solvent molecules pass through semi permeable membrane towards solvent side.
B.	Reverse osmosis	II.	Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes.
C.	Electro osmosis	III.	Solvent molecules pass through semi permeable membrane towards solution side.
D.	Electrophoresis	IV.	Dispersion medium moves in an electric field.

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-III, B-I, C-II, D-IV

(C) A-I, B-III, C-IV, D-II

(D) A-I, B-III, C-II, D-IV

Correct Answer: (A) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Osmosis is movement of solvent through a semipermeable membrane from the pure solvent or dilute side toward the more concentrated solution.
- Reverse osmosis uses an applied pressure exceeding the osmotic pressure to force solvent out of a solution toward the pure-solvent side.
- Electrophoresis is movement of charged colloidal particles in an electric field; electro-osmosis is movement of the dispersion medium when the particles are restrained.

Step 1: Match osmosis and reverse osmosis In ordinary osmosis the membrane allows solvent, but not solute, to pass. The net solvent flow is toward the solution side, matching description III, so A-III. In reverse osmosis pressure is applied to the solution side to oppose and reverse this natural flow. Solvent then passes toward the solvent side, matching description I, so B-I. Options C and D interchange these two processes and are therefore eliminated before considering the electrical phenomena.

Step 2: Distinguish the two electrical motions When charged colloidal particles move under an applied potential, they migrate toward the oppositely charged electrode. This is electrophoresis, giving D-II. When the particles cannot move and the dispersion medium itself moves under the electric field, the process is electro-osmosis, giving C-IV. The complete sequence is thus A-III, B-I, C-IV, D-II. Option B incorrectly assigns the particle-motion description to electro-osmosis and the medium-motion description to electrophoresis; option A keeps both distinctions correct.

Step 3: Conclusion The correct matching is $\boxed{\text{A-III, B-I, C-IV, D-II}}$. Hence, the correct option is (A).

Quick Tip to Solve: For the electrical pair ask what moves: particles means electrophoresis; dispersion medium means electro-osmosis.

78. The major component of which of the following ore is sulphide based mineral? [4 marks]

(A) Sphalerite

(B) Siderite

(C) Malachite

(D) Calamine

Correct Answer: (A) Sphalerite

Solution:

Concept:

- An ore is identified by its major mineral constituent; a sulphide mineral contains sulphur combined with a metal as sulphide.
- Common zinc ores include sphalerite, ZnS, and calamine, $ZnCO_3$. Similar metal content does not mean the ores belong to the same chemical class.

Step 1: Identify the composition of sphalerite Sphalerite, also called zinc blende, consists predominantly of zinc sulphide, ZnS. Its anion is sulphide, S^{2-}, so it directly satisfies the requirement. The question asks for the chemical nature of the mineral rather than the metal extracted from it.

Step 2: Check the remaining ore names Siderite is iron(II) carbonate, $FeCO_3$, and is therefore a carbonate ore. Malachite is basic copper carbonate, conventionally written $CuCO_3 \cdot Cu(OH)_2$, so it contains carbonate and hydroxide rather than sulphide. Calamine is zinc carbonate, $ZnCO_3$, and is also a carbonate ore. Although sphalerite and calamine both supply zinc, only sphalerite has sulphide as its principal mineral component. Thus options (B), (C) and (D) fail the specific sulphide criterion.

Step 3: Conclusion Sphalerite has ZnS as its major component. Hence, the correct option is (A).

Quick Tip to Solve: Recall zinc blende as the sulphide ore of zinc and calamine as its carbonate ore; the shared metal can otherwise be misleading.

79. The concentration of dissolved Oxygen in water for growth of fish should be more than X ppm and Biochemical Oxygen Demand in clean water should be less than Y ppm. X and Y in ppm are, respectively. [4 marks]

(A)

X	Y
4	15

(B)

X	Y
4	8

(C)

X	Y
6	12

(D)

X	Y
6	5

Correct Answer: (D)

X	Y
6	5

Solution:

Concept:

- Dissolved oxygen (DO) is the oxygen available in water for aquatic respiration. An adequate DO level supports fish and other aquatic organisms.
- Biochemical oxygen demand (BOD) measures the oxygen consumed by microorganisms while decomposing biodegradable organic matter under the specified test conditions. Lower BOD indicates cleaner water.

Step 1: Assign the dissolved-oxygen threshold For the growth of fish, the concentration of dissolved oxygen should exceed $6\,\text{ppm}$. Thus the first blank is $X = 6$. This refers to oxygen already present in the water, not oxygen demanded by decomposing waste. The threshold of $4\,\text{ppm}$ in options (A) and (B) does not give the required value.

Step 2: Assign the clean-water BOD threshold Clean water has BOD below $5\,\text{ppm}$, so $Y = 5$. A high BOD means microorganisms consume a greater amount of oxygen, leaving less available for aquatic life. The alternatives 15, 8 and $12\,\text{ppm}$ therefore do not supply the specified clean-water limit. Combining the two independent criteria gives the ordered pair $(X, Y) = (6, 5)$, which occurs only in option (D).

Step 3: Conclusion The required thresholds are $X = 6\,\text{ppm}$ and $Y = 5\,\text{ppm}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the directions separate: fish require high dissolved oxygen, whereas clean water is associated with low biochemical oxygen demand.

80. Match List I with List II

	List I		List II
A.	Elastomeric polymer	I.	Urea formaldehyde resin
B.	Fibre Polymer	II.	Polystyrene
C.	Thermosetting Polymer	III.	Polyester
D.	Thermoplastic Polymer	IV.	Neoprene

Choose the correct answer from the options given below : [4 marks]

(A) A-II, B-I, C-IV, D-III

(B) A-IV, B-I, C-III, D-II

(C) A-IV, B-III, C-I, D-II

(D) A-II, B-III, C-I, D-IV

Correct Answer: (C) A-IV, B-III, C-I, D-II

Solution:

Concept:

- Polymer classes reflect intermolecular forces and chain structure: elastomers stretch reversibly, fibres have strong interchain attraction, and thermosets have extensive cross-linking.
- Thermoplastics consist mainly of linear or slightly branched chains and can soften on heating; permanently cross-linked thermosets cannot be remoulded in this way.

Step 1: Match the elastomer and fibre Neoprene is a synthetic rubber obtained from chloroprene. Its rubber-like elasticity identifies it as the elastomer, giving A–IV. Polyester chains contain polar ester groups and can be formed into strong fibres, exemplified by terylene. It is therefore the fibre polymer in this list, giving B–III. These two assignments already rule out options (A), (B) and (D), but the remaining matches should also be checked.

Step 2: Match the thermosetting and thermoplastic materials Urea formaldehyde resin forms a highly cross-linked three-dimensional network on curing. The rigid network makes it a thermosetting polymer, so C–I. Polystyrene is a familiar thermoplastic: its chains can move relative to one another on heating and the material can be remoulded, so D–II. The complete set is therefore A–IV, B–III, C–I and D–II. Each listed example now corresponds to its appropriate class without repeating a match.

Step 3: Conclusion The correct matching is A–IV, B–III, C–I, D–II. Hence, the correct option is (C).

Quick Tip to Solve: Use neoprene as the rubber anchor and urea formaldehyde as the cross-linked resin anchor; then verify polyester and polystyrene.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The volume of HCl, containing 73 g L^{-1}, required to completely neutralise NaOH obtained by reacting 0.69 g of metallic sodium with water, is _____ mL. (Nearest Integer)
(Given : molar masses of Na, Cl, O, H, are 23, 35.5, 16 and 1 g mol^{-1} respectively.) [4 marks]

Solution:

Concept:

- Sodium reacts with water to form sodium hydroxide: $2\,Na + 2\,H_2O \longrightarrow 2\,NaOH + H_2$. Thus one mole of sodium produces one mole of sodium hydroxide.
- Neutralisation is $HCl + NaOH \longrightarrow NaCl + H_2O$. The acid and base react in a 1:1 mole ratio, and solution volume follows $V = n/c$.

Step 1: Find the amount of sodium hydroxide formed Using the given atomic mass of sodium, the number of moles of sodium is

$$n(\mathrm{Na}) = \frac{0.69\,\mathrm{g}}{23\,\mathrm{g\,mol^{-1}}} = 0.030\,\mathrm{mol}.$$

The balanced sodium-water reaction produces two moles of sodium hydroxide from two moles of sodium. Therefore $n(\mathrm{NaOH}) = 0.030\,\mathrm{mol}$, and complete neutralisation requires $0.030\,\mathrm{mol}$ of hydrogen chloride. The coefficient 2 in the first equation does not double this amount because the sodium coefficient is also 2.

Step 2: Convert acid concentration and calculate its volume The molar mass of hydrogen chloride is $1 + 35.5 = 36.5\,\mathrm{g\,mol^{-1}}$. Its concentration is therefore

$$c(\mathrm{HCl}) = \frac{73}{36.5} = 2.0\,\mathrm{mol\,L^{-1}}.$$

The acid volume needed is

$$V = \frac{0.030}{2.0} = 0.015\,\mathrm{L} = 15\,\mathrm{mL}.$$

This value is already an integer, so rounding does not change it.

Step 3: Conclusion The required volume of the hydrogen chloride solution is $\boxed{15\,\mathrm{mL}}$. The numerical entry is $\boxed{15}$.

Quick Tip to Solve: Trace the mole ratio from sodium to sodium hydroxide to acid before converting the given mass concentration into molarity.

82. The equilibrium constant for the reaction

$Zn(s) + Sn^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Sn(s)$ **is** 1×10^{20} **at 298 K. The magnitude of standard electrode potential of** Sn/Sn^{2+} **if** $E^{\Theta}_{Zn^{2+}/Zn} = -0.76$ V **is _____** $\times 10^{-2}$ **V. (Nearest integer)**

Given : $\frac{2.303RT}{F} = 0.059$ V [4 marks]

Solution:

Concept:

- For a cell reaction transferring n electrons, $E^{\circ}_{cell} = (0.059/n)\log_{10} K$ at the stated temperature and with the supplied constant.
- Using reduction potentials, $E^{\circ}_{cell} = E^{\circ}_{cathode} - E^{\circ}_{anode}$. Reversing a half-reaction changes the sign of its electrode potential.

Step 1: Calculate the standard cell potential In the given reaction, zinc is oxidised to Zn^{2+}, while Sn^{2+} is reduced to tin. Both changes involve two electrons, so $n = 2$. Since $K = 10^{20}$,

$$E^{\circ}_{cell} = \frac{0.059}{2}\log_{10}(10^{20}) = 0.590\,\text{V}.$$

The large equilibrium constant gives a positive standard cell potential, consistent with the reaction being favourable in its written direction.

Step 2: Obtain the tin electrode potential and its magnitude Tin is the cathode and zinc is the anode, so

$$0.590 = E^{\circ}_{Sn^{2+}/Sn} - (-0.760).$$

Hence

$$E^{\circ}_{Sn^{2+}/Sn} = -0.170\,\text{V}.$$

The potential for the reverse tin oxidation half-reaction is $+0.170$ V. The question asks for the magnitude, which is the same for either convention. Writing $0.170\,\text{V} = 17 \times 10^{-2}\,\text{V}$ gives the requested coefficient.

Step 3: Conclusion The electrode-potential magnitude is $0.17\,\text{V} = \boxed{17} \times 10^{-2}\,\text{V}$. The numerical entry is $\boxed{17}$.

Quick Tip to Solve: Use cathode minus anode with reduction potentials throughout; take the magnitude only after finding the signed tin potential.

83. At 298 K

$$N_2\ (g) + 3H_2\ (g) \rightleftharpoons\ 2NH_3\ (g),\ K_1 = 4 \times 10^5$$

$$N_2\ (g) + O_2\ (g) \rightleftharpoons\ 2NO\ (g),\ K_2 = 1.6 \times 10^{12}$$

$$H_2\ (g) + \frac{1}{2}O_2\ (g) \rightleftharpoons\ H_2O\ (g),\ K_3 = 1.0 \times 10^{-13}$$

Based on above equilibria, then equilibrium constant of the reaction,

$$2NH_3(g) + \frac{5}{2}O_2\ (g) \rightleftharpoons\ 2NO\ (g) + 3H_2O\ (g)$$

is _____ $\times 10^{-33}$ (Nearest integer). [4 marks]

Solution:

Concept:

- Reversing an equilibrium replaces its constant by its reciprocal. Multiplying every reaction coefficient by a factor m raises the constant to the power m.
- Adding balanced reactions multiplies their equilibrium constants. Species cancelled while adding equations must be cancelled on opposite sides in matching amounts.

Step 1: Construct the required overall reaction Reverse the first equilibrium to obtain $2\,NH_3(g) \rightleftharpoons N_2(g) + 3\,H_2(g)$, with constant $1/K_1$. Add the second equilibrium, $N_2(g) + O_2(g) \rightleftharpoons 2\,NO(g)$, with constant K_2. Finally add three times the water-formation equilibrium, $3\,H_2(g) + \frac{3}{2}\,O_2(g) \rightleftharpoons 3\,H_2O(g)$, with constant K_3^3. The intermediate N_2 and H_2 cancel. Oxygen consumption becomes $1 + 3/2 = 5/2$, giving exactly the reaction requested.

Step 2: Evaluate the product of constants The required equilibrium constant is therefore

$$K = \frac{K_2 K_3^3}{K_1}.$$

Substitute the values with the printed powers of ten unchanged:

$$\begin{aligned} K &= \frac{(1.6 \times 10^{12})(1.0 \times 10^{-13})^3}{4 \times 10^5} \\ &= \frac{1.6}{4} \times 10^{12-39-5} \\ &= 0.4 \times 10^{-32} = 4 \times 10^{-33}. \end{aligned}$$

The negative exponent in the water-formation constant is used as given; it is cubed because three moles of water are formed.

Step 3: Conclusion The required constant is $K = 4 \times 10^{-33}$. Thus the nearest-integer coefficient is $\boxed{4}$.

Quick Tip to Solve: Build the target reaction first, then multiply constants: reversed ammonia formation contributes a reciprocal and tripled water formation a cube.

84. The denticity of the ligand present in the Fehling's reagent is _____. [4 marks]

Solution:

Concept:

- Denticity is the number of donor atoms of one ligand that can coordinate to a metal centre; it is not the charge on the ligand or the coordination number of the entire complex.
- Fehling's reagent contains a copper(II) complex of tartrate in alkaline solution. The tartrate framework contains two carboxylate groups and two adjacent hydroxyl groups.

Step 1: Identify the ligand in the reagent Fehling's reagent is prepared using copper(II) sulphate and alkaline sodium potassium tartrate. Tartrate acts as the complexing ligand and holds copper(II) in solution under alkaline conditions. The donor framework can be represented as $^{-}OOC-CH(OH)-CH(OH)-COO^{-}$. Sodium and potassium are counterions rather than the atoms responsible for the tartrate ligand's donor character.

Step 2: Count the coordinating oxygen sites For the denticity asked here, tartrate supplies four oxygen donor sites: one from each of the two carboxylate groups and one from each of the two hydroxyl groups. Thus the count is $2 + 2 = 4$, and the ligand is classified as tetradentate in this context. The two negative charges of tartrate do not imply bidenticity; formal charge and number of coordinating donor atoms describe different features. Similarly, counting all six oxygen atoms in the formula would overcount the donor set used for this classification.

Step 3: Conclusion The ligand in Fehling's reagent is tetradentate, so its denticity is $\boxed{4}$.

Quick Tip to Solve: Identify tartrate first and count its participating donor oxygens; do not equate the ligand's charge with its denticity.

85. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of compound is _____ g mol^{-1} (Nearest integer). [4 marks]

Solution:

Concept:

- Complete combustion converts every carbon atom of the organic compound into one molecule of CO_2; hence the moles of carbon equal the moles of carbon dioxide produced.
- The mass percentage of carbon is $100m_C/m_{sample}$. The molar mass of the compound is $M = m_{sample}/n_{compound}$.

Step 1: Find the carbon present in the sample The molar mass of CO_2 is $12 + 2(16) = 44\,g\,mol^{-1}$. Thus the amount of carbon dioxide produced is

$$n(CO_2) = \frac{4.4}{44} = 0.10\,mol.$$

Each mole of CO_2 contains one mole of carbon atoms, so the sample originally contained $0.10\,mol$ of carbon. Its mass is therefore

$$m_C = 0.10 \times 12 = 1.20\,g.$$

The oxygen in carbon dioxide is not part of this carbon mass and must not be counted towards the stated carbon percentage.

Step 2: Use the mass percentage to find the sample mass Carbon constitutes 60% of the original sample by mass. Consequently,

$$0.60 m_{\text{sample}} = 1.20\,\text{g}, \qquad m_{\text{sample}} = 2.00\,\text{g}.$$

This is the mass of the given $0.01\,\text{mol}$ of the organic compound, not the mass of one mole.

Step 3: Calculate the molar mass Divide the mass of compound by its amount:

$$M = \frac{2.00\,\text{g}}{0.01\,\text{mol}} = 200\,\text{g mol}^{-1}.$$

As a check, one mole of the compound contains $0.60(200) = 120\,\text{g}$ of carbon, corresponding to ten carbon atoms per molecule. Its $0.01\,\text{mol}$ therefore yields $0.10\,\text{mol}$ of carbon dioxide, as required.

Step 4: Conclusion The molar mass is $\boxed{200\,\text{g mol}^{-1}}$; the required nearest-integer entry is $\boxed{200}$.

Quick Tip to Solve: First convert the carbon dioxide mass to carbon mass using the factor $12/44$, then divide by the carbon mass fraction and the moles of compound.

86. Total number of acidic oxides among

$N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O$ **and** NO **is _____.** [4 marks]

Solution:

Concept:

- Acidic oxides react with bases to give salts, often with water. Many non-metal oxides are acidic, but some low-oxidation-state oxides are neutral.
- An oxide is classified by its acid-base behaviour, not simply by the presence of oxygen or by whether the element is a metal.

Step 1: Identify the acidic oxides Among the listed species, N_2O_3, NO_2, Cl_2O_7 and SO_2 are acidic oxides. Their reactions with water illustrate the corresponding acidic products:

$$N_2O_3 + H_2O \longrightarrow 2\,HNO_2$$

$$2\,NO_2 + H_2O \longrightarrow HNO_2 + HNO_3$$

$$Cl_2O_7 + H_2O \longrightarrow 2\,HClO_4$$

$$SO_2 + H_2O \longrightarrow H_2SO_3$$

The last equation represents the usual school-level description of aqueous sulfur dioxide. Nitrogen dioxide is included even though it gives a mixture of two acids, rather than just one acid.

Step 2: Exclude neutral and basic oxides N_2O, CO and NO are neutral oxides; they do not show the ordinary acid-base behaviour needed for classification as acidic oxides. In particular, carbon monoxide

must not be confused with acidic carbon dioxide. The metal oxides CaO and Na_2O are basic and yield hydroxides with water:

$$CaO + H_2O \longrightarrow Ca(OH)_2$$

$$Na_2O + H_2O \longrightarrow 2\,NaOH$$

These five excluded oxides account for the remainder of the nine substances, so none is left unclassified.

Step 3: Count the required species The acidic set is precisely $\{N_2O_3, NO_2, Cl_2O_7, SO_2\}$. Each formula represents one listed oxide, so the number required is four; the number of acids produced in any one reaction does not change this count.

Step 4: Conclusion There are $\boxed{4}$ acidic oxides.

Quick Tip to Solve: Remember the common neutral oxides CO, NO and N_2O; excluding them prevents the mistaken assumption that every non-metal oxide is acidic.

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87. Assume that the radius of the first Bohr orbit of hydrogen atom is 0.6 $\mathring{A}$. The radius of the third Bohr orbit of He^+ is _____ picometer. (Nearest Integer) [4 marks]

Solution:

Concept:

- For a one-electron atom or ion, the Bohr-orbit radius is $r_n = a_0 n^2/Z$, where n is the principal quantum number, Z is the nuclear charge and a_0 is the first hydrogen-orbit radius.
- For He^+, the nucleus has $Z = 2$ while only one electron remains. Use the value of a_0 explicitly supplied in the question.

Step 1: Identify the orbit and nuclear charge The requested orbit is the third orbit, so $n = 3$. The ion He^+ has a helium nucleus of charge $+2e$, hence $Z = 2$. Its ionic charge of $+1$ does not mean $Z = 1$; that charge only tells us that one of the two electrons has been removed. Therefore the one-electron Bohr formula is applicable.

Step 2: Scale the supplied hydrogen radius The question gives $a_0 = 0.6\,\mathring{A}$. Relative to the first hydrogen orbit, the third orbit grows by $3^2 = 9$, while the doubled nuclear charge contracts it by a factor of two:

$$r_3(He^+) = 0.6 \times \frac{3^2}{2} = 2.7\,\mathring{A}.$$

This combines both effects. Using only n^2 would incorrectly give the hydrogen third-orbit radius; dividing by the ionic charge would likewise give the wrong result.

Step 3: Convert the radius into picometres An angstrom is 10^{-10} m, and a picometre is 10^{-12} m. Thus

$$1\,\mathring{A} = 100\,pm, \qquad r_3 = 2.7(100) = 270\,pm.$$

The number is already an integer. There is no need to substitute the more usual measured hydrogen Bohr radius, because the problem specifically assumes $0.6\,\mathring{A}$.

Step 4: Conclusion The radius is $\boxed{270\,pm}$, so the nearest-integer entry is $\boxed{270}$.

Quick Tip to Solve: For any hydrogen-like ion, first multiply the given hydrogen radius by n^2/Z; convert angstroms to picometres only after that scaling.

88. For conversion of compound A $\rightarrow$ B, the rate constant of the reaction was found to be 4.6×10^{-5} L mol^{-1} s^{-1}. The order of the reaction is _____. [4 marks]

Solution:

Concept:

- For a reaction with overall order n, the rate law can be written as $v = k[A]^n$ for this dimensional analysis; for several reactants, n is the sum of their concentration exponents.
- The rate has units of concentration per time, $\text{mol}\,\text{L}^{-1}\,\text{s}^{-1}$. Therefore the units of k depend on the overall order.

Step 1: Write the dimensional relation Let the concentration unit be $C = \text{mol}\,\text{L}^{-1}$. Since $v = k[A]^n$, dividing rate units by concentration raised to n gives

$$[k] = \frac{C\,\text{s}^{-1}}{C^n} = C^{1-n}\,\text{s}^{-1}.$$

Equivalently, in litre and mole units this is

$$[k] = \text{L}^{n-1}\text{mol}^{1-n}\text{s}^{-1}.$$

This relation is independent of the numerical magnitude of the rate constant.

Step 2: Compare with the given units The reported rate constant is $4.6 \times 10^{-5}\,\text{L}\,\text{mol}^{-1}\,\text{s}^{-1}$. Its concentration factor is $\text{L}\,\text{mol}^{-1} = C^{-1}$. Comparing concentration exponents gives

$$1 - n = -1 \quad \Longrightarrow \quad n = 2.$$

The litre exponent provides the same check: $n - 1 = 1$, again giving $n = 2$.

Step 3: Check the inferred order For a second-order rate law, multiplying k by concentration squared gives

$$\text{L}\,\text{mol}^{-1}\,\text{s}^{-1} \times (\text{mol}\,\text{L}^{-1})^2 = \text{mol}\,\text{L}^{-1}\,\text{s}^{-1},$$

which is exactly a rate unit. A first-order constant would have only s^{-1}, while a zero-order constant would have rate units themselves. The conversion notation $A \rightarrow B$ alone does not specify the reaction order; it is the supplied units that determine it.

Step 4: Conclusion The reaction is second order, and the required numerical answer is $\boxed{2}$.

Quick Tip to Solve: Rewrite the units of k as concentration to a power times s^{-1}; the concentration exponent is always $1 - n$.

89. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is _____ $\times 10^{21}$ (Nearest integer).

(Given $N_A = 6.02 \times 10^{23}$) [4 marks]

Solution:

Concept:

- In a close-packed arrangement of N atoms, there are N octahedral voids and $2N$ tetrahedral voids, giving $3N$ interstitial voids in total.
- These counting relations apply to both hexagonal close packing and cubic close packing. The atom count follows from $N = nN_A$.

Step 1: Calculate the number of metal atoms The metal sample contains $0.02\,\text{mol}$. Using the supplied Avogadro constant,

$$N = 0.02(6.02 \times 10^{23}) = 1.204 \times 10^{22}$$

atoms. This is a count of constituent atoms, not a count of unit cells; therefore no extra division by the number of atoms per hexagonal unit cell is required.

Step 2: Count both kinds of interstitial void The close-packed atoms provide one octahedral and two tetrahedral voids per atom. Consequently,

$$N_{\text{oct}} = N = 1.204 \times 10^{22},$$

$$N_{\text{tet}} = 2N = 2.408 \times 10^{22}.$$

The question asks for total voids, so both categories must be added. Coordination number twelve describes neighbouring atoms and is not a multiplier for counting these voids.

Step 3: Add and express the result in the requested scale Adding the two populations gives

$$N_{\text{voids}} = 3N = 3.612 \times 10^{22} = 36.12 \times 10^{21}.$$

The blank is the coefficient multiplying 10^{21}, rather than the full number of voids. Rounding that coefficient, 36.12, to the nearest integer gives 36. The separate contributions are 12.04×10^{21} and 24.08×10^{21}, whose sum checks the result.

Step 4: Conclusion The total is 36.12×10^{21} voids; the required nearest-integer entry is $\boxed{36}$.

Quick Tip to Solve: For either hcp or ccp, total interstitial voids equal three times the number of atoms. Round only the final coefficient in the requested power-of-ten scale.

90. On heating, $LiNO_3$ gives how many compounds among the following _____

$$Li_2O, N_2, O_2, LiNO_2, NO_2$$

[4 marks]

Solution:

Concept:

- Lithium nitrate is exceptional among alkali-metal nitrates: on strong heating it forms the metal oxide, nitrogen dioxide and oxygen, rather than stopping at a nitrite.
- Lithium resembles magnesium in several properties because its small cation has relatively high polarising power. This helps distinguish its nitrate decomposition from that of sodium or potassium nitrate.

Step 1: Write the correct decomposition reaction For lithium nitrate, the balanced equation is

$$4\,LiNO_3 \xrightarrow{\Delta} 2\,Li_2O + 4\,NO_2 + O_2.$$

The residue is lithium oxide, while nitrogen dioxide and oxygen are evolved. In contrast, the usual heavier alkali-metal nitrate pattern gives a nitrite and oxygen; applying that pattern to lithium would incorrectly select $LiNO_2$.

Step 2: Verify the balanced equation On the left there are four lithium atoms, four nitrogen atoms and twelve oxygen atoms. On the right, $2\,Li_2O$ supplies four lithium and two oxygen atoms, $4\,NO_2$ supplies four nitrogen and eight oxygen atoms, and O_2 supplies two more oxygen atoms. The atom totals therefore agree. Nitrogen is released as NO_2, not as elemental N_2.

Step 3: Match the products to the list From the five entries, the substances actually formed are Li_2O, O_2 and NO_2. Neither N_2 nor $LiNO_2$ occurs in the decomposition equation. Thus three entries in the given list are products. Although the question uses the word compounds, its list includes elemental oxygen and nitrogen; its requested product count includes oxygen as one of the listed substances. It is the number of different products, not their stoichiometric coefficients, that must be counted.

Step 4: Conclusion Three listed substances are formed, so the required answer is $\boxed{3}$.

Quick Tip to Solve: Remember the lithium exception: heating its nitrate gives oxide plus NO_2 and O_2. Count distinct products, not the coefficients in the equation.