JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 10 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 10 April 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If $I(x) = \int e^{\sin^2 x}(\cos x \sin 2x - \sin x)dx$ and $I(0) = 1$, then $I\left(\frac{\pi}{3}\right)$ is equal to : [4 marks]

(A) $-e^{\frac{3}{4}}$

(B) $-\frac{1}{2}e^{\frac{3}{4}}$

(C) $e^{\frac{3}{4}}$

(D) $\frac{1}{2}e^{\frac{3}{4}}$

Correct Answer: (D) $\frac{1}{2}e^{\frac{3}{4}}$

Solution:

Concept:

- The product rule gives $(uv)' = u'v + uv'$. Recognising both terms of a derivative avoids unnecessary integration by parts.
- For $v = e^{\sin^2 x}$, the chain rule gives $v' = e^{\sin^2 x} \sin 2x$. The condition on $I(0)$ determines the additive constant.

Step 1: Recognise the complete derivative Differentiate the product of the exponential and cosine. The cosine derivative supplies the negative sine term, while differentiating the exponential supplies the other term:

$$\frac{d}{dx}\left(e^{\sin^2 x}\cos x\right) = e^{\sin^2 x}(\cos x \sin 2x - \sin x).$$

Therefore the entire integrand, including its minus sign, is exactly this derivative, and

$$I(x) = e^{\sin^2 x}\cos x + C.$$

Step 2: Apply the initial condition and evaluate At zero, the exponential is one and the cosine is one. Thus $1 = 1 + C$, so $C = 0$. At the requested angle, $\sin^2(\pi/3) = 3/4$ and $\cos(\pi/3) = 1/2$. Consequently

$$I(\pi/3) = e^{3/4} \cdot \frac{1}{2}.$$

The value is positive, excluding (A) and (B). Option (C) omits the cosine factor and is twice the required value.

Step 3: Conclusion $I(\pi/3) = \frac{1}{2}e^{3/4}$. Hence, the correct option is (D).

Quick Tip to Solve: When an exponential of a trigonometric expression multiplies a sum, test the product of that exponential with the remaining trigonometric factor before integrating.

2. The shortest distance between the lines $\frac{x+2}{1} = \frac{y}{-2} = \frac{z-5}{2}$ and $\frac{x-4}{1} = \frac{y-1}{2} = \frac{z+3}{0}$ is : [4 marks]

(A) 8

(B) 7

(C) 6

(D) 9

Correct Answer: (D) 9

Solution:

Concept:

- For nonparallel lines with points $\mathbf{a}_1, \mathbf{a}_2$ and directions $\mathbf{b}_1, \mathbf{b}_2$, their shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.
- The cross product is normal to both lines. Projecting a joining vector onto its unit direction gives the separation of the lines; the absolute value makes the distance nonnegative.

Step 1: Read the points and direction vectors The first line passes through $A = (-2, 0, 5)$ with direction $\mathbf{b}_1 = (1, -2, 2)$. The second passes through $B = (4, 1, -3)$ with direction $\mathbf{b}_2 = (1, 2, 0)$. A zero direction component means $z = -3$ on the second line; it is not a division operation. The joining vector is

$$\overrightarrow{AB} = (6, 1, -8).$$

Step 2: Compute the normal projection Taking the cross product and its magnitude gives

$$\mathbf{b}_1 \times \mathbf{b}_2 = (-4, 2, 4), \qquad |\mathbf{b}_1 \times \mathbf{b}_2| = \sqrt{16 + 4 + 16} = 6.$$

Its scalar product with the joining vector is

$$(6, 1, -8) \cdot (-4, 2, 4) = -24 + 2 - 32 = -54.$$

Hence $d = 54/6 = 9$ units. The nonzero scalar triple product also confirms that the lines are skew. The values 8, 7 and 6 do not equal this perpendicular projection.

Step 3: Conclusion The shortest distance is 9 units. Hence, the correct option is (D).

Quick Tip to Solve: Keep the signs of the reference-point coordinates separate from the printed numerators; then use the scalar triple product and take its absolute value only at the end.

3. Let the ellipse $E : x^2 + 9y^2 = 9$ intersect the positive x and y-axes at the points A and B respectively. Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle with vertices A, P and the origin O is $\frac{m}{n}$, where m and n are coprime, then $m - n$ is equal to : [4 marks]

(A) 15

(B) 16

(C) 17

(D) 18

Correct Answer: (C) 17

Solution:

Concept:

- The ellipse $x^2/9 + y^2 = 1$ has semiaxes 3 and 1. Its major-axis endpoints determine a circle of radius 3 centred at the origin.
- A triangle with a horizontal base of length b and opposite vertex at height h has area $bh/2$. Find the second line-circle intersection before calculating this height.

Step 1: Form the circle and secant equations The positive intercepts of the ellipse are $A = (3, 0)$ and $B = (0, 1)$. Its major axis joins $(-3, 0)$ to $(3, 0)$, so the specified circle is

$$x^2 + y^2 = 9.$$

The line through the two intercepts is

$$\frac{x}{3} + y = 1, \qquad y = 1 - \frac{x}{3}.$$

Since A is already on the circle, P is its other intersection with this secant.

Step 2: Find the second point and the area Substituting the line into the circle and simplifying gives

$$x^2 + (1 - x/3)^2 = 9 \quad \Longrightarrow \quad 5x^2 - 3x - 36 = 0.$$

$$(x - 3)(5x + 12) = 0.$$

The root $x = 3$ gives A; the other gives $P = (-12/5, 9/5)$. With $OA = 3$ as base, the perpendicular height is $9/5$, even though P lies to the left of the origin. Thus

$$[OAP] = \frac{1}{2} \cdot 3 \cdot \frac{9}{5} = \frac{27}{10}.$$

The integers 27 and 10 are coprime, so their difference is 17. Options 15, 16 and 18 do not arise from this reduced area.

Step 3: Conclusion $m = 27$, $n = 10$, and $m - n = 17$. Hence, the correct option is (C).

Quick Tip to Solve: If one secant intersection is already known, factor out its root when solving the line-circle equation; use the other point for the triangle height.

4. Let O be the origin and the position vector of the point P be $-\hat{i}-2\hat{j}+3\hat{k}$. If the position vectors of the points A, B and C are $-2\hat{i}+\hat{j}-3\hat{k}, 2\hat{i}+4\hat{j}-2\hat{k}$ and $-4\hat{i}+2\hat{j}-\hat{k}$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is : [4 marks]

(A) $\frac{7}{3}$

(B) 3

(C) $\frac{10}{3}$

(D) $\frac{8}{3}$

Correct Answer: (B) 3

Solution:

Concept:

- The vector $\overrightarrow{AB} \times \overrightarrow{AC}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$. A scalar projection onto a unit normal is the dot product with that unit normal.
- Divide the cross product by its magnitude before projecting. The positive projection length is $|\mathbf{p} \cdot \mathbf{n}|/|\mathbf{n}|$, independent of which orientation of the normal is chosen.

Step 1: Construct the two displacements and a normal Subtract the position vector of A from each of the other two vectors:

$$\overrightarrow{AB} = (4, 3, 1), \qquad \overrightarrow{AC} = (-2, 1, 2).$$

Their cross product is

$$\mathbf{n} = (3 \cdot 2 - 1 \cdot 1,\ 1 \cdot (-2) - 4 \cdot 2,\ 4 \cdot 1 - 3 \cdot (-2)) = (5, -10, 10).$$

The dot products $\mathbf{n} \cdot (4, 3, 1) = 0$ and $\mathbf{n} \cdot (-2, 1, 2) = 0$ verify the required perpendicularity.

Step 2: Normalise and project the given vector The normal has magnitude $\sqrt{25 + 100 + 100} = 15$, hence its unit vector is $(1, -2, 2)/3$. With $\overrightarrow{OP} = (-1, -2, 3)$, the projection along this chosen normal is

$$\frac{(-1)(1) + (-2)(-2) + 3(2)}{3} = \frac{-1 + 4 + 6}{3} = 3.$$

Choosing the opposite normal changes only the signed component, not its length. Values $7/3$, $10/3$ and $8/3$ disagree with the verified dot product.

Step 3: Conclusion The required projection length is 3. Hence, the correct option is (B).

Quick Tip to Solve: Cancel any common factor in the cross product before normalising; here the simpler normal $(1, -2, 2)$ has magnitude 3.

5. If $f(x)=\frac{(\tan 1^\circ)x+\log_e(123)}{x\log_e(1234)-(\tan 1^\circ)}, x>0$, then the least value of $f(f(x))+f\left(f\left(\frac{4}{x}\right)\right)$ is : [4 marks]

(A) 2

(B) 4

(C) 0

(D) 8

Correct Answer: (B) 4

Solution:

Concept:

- A fractional linear function $f(t)=(at+b)/(ct-a)$ is an involution when the compositions are defined: direct substitution yields $f(f(t))=t$, provided $a^2+bc\neq 0$.
- For positive x, the arithmetic-geometric mean inequality gives $x+4/x\geq 2\sqrt{4}=4$, with equality exactly at $x=2$.

Step 1: Compute the double composition Write $a=\tan 1^\circ$, $b=\ln 123$, and $c=\ln 1234$, all positive. Substitution, followed by multiplication by $ct-a$, gives

$$f(f(t))=\frac{a(at+b)+b(ct-a)}{c(at+b)-a(ct-a)}=\frac{(a^2+bc)t}{a^2+bc}=t.$$

This identity applies on the domain where the inner and outer function values exist. Since the constant a^2+bc is positive, no further zero factor is introduced in this simplification.

Step 2: Minimise on the admissible positive domain The required expression reduces to

$$x+\frac{4}{x}=4+\frac{(x-2)^2}{x}\geq 4 \qquad (x>0).$$

At $x=2$, both inputs x and $4/x$ equal 2, and $2c-a>0$, so the inner functions are defined and positive. The outer denominators are also nonzero by the composition calculation. Therefore equality is actually attained, rather than being merely an infimum. Options 2 and 0 violate the lower bound, while 8 is larger than the attained minimum.

Step 3: Conclusion The least value is 4, attained at $x=2$. Hence, the correct option is (B).

Quick Tip to Solve: For a function of the form $(ax+b)/(cx-a)$, simplify its self-composition before inserting numerical logarithms or trigonometric constants.

6. If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal, then a^4b^4 is equal to : [4 marks]

(A) 22

(B) 33

(C) 44

(D) 11

Correct Answer: (A) 22

Solution:

Concept:

- The term obtained by selecting the second summand r times has power $13 - 3r$. Its full expression is

$$T_{r+1} = \binom{13}{r} a^{13-r}(\pm 1)^r b^{-r} x^{13-3r}.$$

- To extract a coefficient, first equate the exponent $13 - 3r$ to the desired power. Signs depend on the parity of r in the expansion with a minus sign.

Step 1: Locate the two required terms For the first expansion, $13 - 3r = 7$ gives $r = 2$. The minus sign is raised to an even power, so the coefficient is positive:

$$[x^7]\left(ax - \frac{1}{bx^2}\right)^{13} = \binom{13}{2}\frac{a^{11}}{b^2}.$$

For the second expansion, $13 - 3r = -5$ gives $r = 6$, and

$$[x^{-5}]\left(ax + \frac{1}{bx^2}\right)^{13} = \binom{13}{6}\frac{a^7}{b^6}.$$

Here $b \neq 0$ is required for the printed expressions to exist.

Step 2: Equate and simplify the coefficient ratio Equating these coefficients and cancelling the common nonzero factors in the nonzero-coefficient case gives

$$a^4b^4 = \frac{\binom{13}{6}}{\binom{13}{2}} = \frac{1716}{78} = 22.$$

The binomial coefficients can be checked directly: $\binom{13}{2} = 13 \cdot 12/2 = 78$, and $\binom{13}{6} = 1716$. Thus 33, 44 and 11 do not satisfy the coefficient ratio. If zero coefficients were allowed with $a = 0$, both would vanish; the listed options select the nondegenerate value determined above.

Step 3: Conclusion For the nonzero coefficient comparison, $a^4b^4 = 22$. Hence, the correct option is (A).

Quick Tip to Solve: In this expansion the powers fall by 3, not 1: replacing one ax with $1/(bx^2)$ changes the exponent from 1 to -2.

7. An arc PQ of a circle subtends a right angle at its centre O. The mid point of the arc PQ is R. If $\overrightarrow{OP} = \vec{u}, \overrightarrow{OR} = \vec{v}$, and $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2 are the roots of the equation : [4 marks]

(A) $x^2 + x - 2 = 0$

(B) $3x^2 + 2x - 1 = 0$

(C) $3x^2 - 2x - 1 = 0$

(D) $x^2 - x - 2 = 0$

Correct Answer: (D) $x^2 - x - 2 = 0$

Solution:

Concept:

- All radius vectors of a circle have the same magnitude. The radius to the midpoint of the minor arc bisects the angle between the endpoint radii.
- For perpendicular radius vectors of magnitude r, their sum has magnitude $r\sqrt{2}$ and points along the internal angle bisector. A monic quadratic with roots p, q is $x^2 - (p+q)x + pq = 0$.

Step 1: Express the arc-midpoint radius Let $\mathbf{w} = \overrightarrow{OQ}$. Since the arc subtends a right angle, $\mathbf{u} \cdot \mathbf{w} = 0$, and both vectors have length r. Thus

$$|\mathbf{u} + \mathbf{w}| = \sqrt{r^2 + r^2} = r\sqrt{2}.$$

The vector $\mathbf{v} = \overrightarrow{OR}$ has length r and points in the same direction as this sum, so

$$\mathbf{v} = \frac{\mathbf{u} + \mathbf{w}}{\sqrt{2}}, \qquad \mathbf{w} = -\mathbf{u} + \sqrt{2}\,\mathbf{v}.$$

Step 2: Form the quadratic from the two roots Comparing with $\mathbf{w} = \alpha\mathbf{u} + \beta\mathbf{v}$ gives $\alpha = -1$ and $\beta = \sqrt{2}$. The requested roots are α and β^2, namely -1 and 2, not -1 and $\sqrt{2}$. Their sum is 1 and product is -2. Hence

$$(x+1)(x-2) = x^2 - x - 2.$$

Option (A) has the wrong root sum. Options (B) and (C), after dividing by 3, have product $-1/3$, so neither can have these roots.

Step 3: Conclusion The required equation is $x^2 - x - 2 = 0$. Hence, the correct option is (D).

Quick Tip to Solve: Use the sum of equal-length endpoint radius vectors to find the arc-bisector direction; remember to square β before forming the quadratic.

8. Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are coprime, then $4m - 3n$ is equal to : [4 marks]

(A) 12

(B) 6

(C) 8

(D) 10

Correct Answer: (C) 8

Solution:

Concept:

- Two fair dice produce 36 equally likely ordered outcomes. Different values of their sum are not equally likely.
- To compare 2^N with $N!$, check the first admissible sums and then use the ratio between consecutive factorials and powers.

Step 1: Find the sums satisfying the inequality For the smallest possible sums,

$$2^2 = 4 > 2! = 2, \qquad 2^3 = 8 > 3! = 6, \qquad 2^4 = 16 < 4! = 24.$$

Once $2^k < k!$ holds for $k \geq 4$, multiplying by 2 gives $2^{k+1} < 2k! < (k+1)!$, because $k + 1 > 2$. Hence the required event is exactly $N \geq 4$, not merely a selection of individual larger sums.

Step 2: Count the complementary outcomes The sum 2 occurs only for $(1, 1)$, and the sum 3 occurs for $(1, 2)$ and $(2, 1)$. Thus exactly three of the 36 outcomes fail the inequality. Therefore

$$P(2^N < N!) = 1 - \frac{3}{36} = \frac{33}{36} = \frac{11}{12}.$$

The fraction is already in lowest terms, so $m = 11$ and $n = 12$.

Step 3: Evaluate the requested combination Substitute the reduced numerator and denominator:

$$4m - 3n = 4(11) - 3(12) = 44 - 36 = 8.$$

The alternatives 12, 6 and 10 do not equal this value; using 33 and 36 instead would violate the coprime condition.

Step 4: Conclusion The requested value is $\boxed{8}$. Hence, the correct option is (C).

Quick Tip to Solve: Count the few excluded dice outcomes, and reduce the probability before using its numerator and denominator.

9. A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm^2) is equal to : [4 marks]

(A) 1025

(B) 900

(C) 800

(D) 675

Correct Answer: (C) 800

Solution:

Concept:

- If corner squares have side x, the open box has height x and a square base of side $30 - 2x$, with $0 < x < 15$.
- Maximise the volume first. Its surface area is the area of tin retained after removing the four corner squares, since folding does not change area.

Step 1: Express and differentiate the volume The volume is

$$V(x) = x(30 - 2x)^2.$$

Using the product rule and then factoring,

$$V'(x) = (30 - 2x)^2 - 4x(30 - 2x) = (30 - 2x)(30 - 6x).$$

The stationary values are $x = 5$ and $x = 15$. The latter gives a zero-width base and lies at the excluded degenerate boundary, so it cannot describe the required box.

Step 2: Confirm the maximum Throughout $0 < x < 15$, the factor $30 - 2x$ is positive. Therefore $V'(x) > 0$ for $x < 5$, and $V'(x) < 0$ for $x > 5$. This proves a maximum at $x = 5$ cm. The resulting box has base $20\,\text{cm} \times 20\,\text{cm}$ and height $5\,\text{cm}$; the volume tends to zero at both degenerate ends.

Step 3: Calculate the open surface area The base and four side walls have total area

$$S = 20^2 + 4(20)(5) = 400 + 400 = 800\,\text{cm}^2.$$

Equivalently, subtract the removed pieces: $30^2 - 4(5^2) = 800$. The value 900 ignores the cuts; 1025 exceeds the original sheet area, and 675 does not correspond to the volume-maximising cut.

Step 4: Conclusion The surface area is $\boxed{800\,\text{cm}^2}$. Hence, the correct option is (C).

Quick Tip to Solve: For a folded open box, retained sheet area equals surface area; optimise the volume before substituting the cut size.

10. $96 \cos\frac{\pi}{33} \cos\frac{2\pi}{33} \cos\frac{4\pi}{33} \cos\frac{8\pi}{33} \cos\frac{16\pi}{33}$ **is equal to :** [4 marks]

(A) 4

(B) 2

(C) 1

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- The double-angle identity $2\sin t\cos t = \sin 2t$ collapses a product whose cosine arguments repeatedly double.
- After five doublings, $2^5 \sin t \prod_{j=0}^{4} \cos(2^j t) = \sin 32t$. Here $t = \pi/33$, so $\sin t \neq 0$.

Step 1: Collapse the doubling product Let $t = \pi/33$, and denote the five-cosine product by P. Start with $2\sin t\cos t = \sin 2t$, multiply by $2\cos 2t$, and repeat for $4t, 8t, 16t$. The intermediate sine factors cancel successively, giving

$$32 \sin t\, P = \sin 32t, \qquad P = \frac{\sin 32t}{32 \sin t}.$$

There are five cosine factors, so the multiplier is 32, rather than 16 or 64.

Step 2: Use supplementary angles Since $33t = \pi$, we have $32t = \pi - t$. The sine of supplementary angles is the same:

$$\sin 32t = \sin(\pi - t) = \sin t.$$

Consequently $P = 1/32$. This cancellation is valid because $0 < t < \pi$, ensuring that the denominator is nonzero.

Step 3: Apply the numerical coefficient The expression asked for is therefore

$$96P = \frac{96}{32} = 3.$$

The alternatives 4, 2 and 1 would require a different net denominator, and none follows from the five doublings. All the cosine factors are positive here, consistent with the positive result.

Step 4: Conclusion The expression equals $\boxed{3}$. Hence, the correct option is (D).

Quick Tip to Solve: For a cosine product at angles t, 2t, 4t and so on, multiply by sine t and repeatedly use the double-angle identity.

11. For the system of linear equations

$$2x - y + 3z = 5$$

$$3x + 2y - z = 7$$

$4x + 5y + \alpha z = \beta$,

which of the following is NOT correct? [4 marks]

(A) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

(B) The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$

(C) The system is inconsistent for $\alpha = -5$ and $\beta = 8$

(D) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

Correct Answer: (A) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

Solution:

Concept:

- Elimination distinguishes a unique solution, a free parameter and an inconsistent equation. A relation $0 = c \neq 0$ means inconsistency.
- The first two equations are independent in x, y, since their coefficient determinant is $2(2) - (-1)(3) = 7 \neq 0$.

Step 1: Eliminate the first two variables Write the left sides of the first two equations as $L_1 = 2x - y + 3z$ and $L_2 = 3x + 2y - z$. Their combination is

$$-L_1 + 2L_2 = 4x + 5y - 5z.$$

For any solution of those equations, its value must be $-5 + 2(7) = 9$. Comparing this with the third equation gives the decisive condition

$$(\alpha + 5)z = \beta - 9.$$

Step 2: Classify every parameter case If $\alpha \neq -5$, this equation determines exactly one z, and the independent first two equations then give exactly one pair (x, y). If $\alpha = -5$, the condition becomes $0 = \beta - 9$. Thus $\beta \neq 9$ is inconsistent, whereas $\beta = 9$ leaves z free and gives infinitely many solutions. This establishes the complete classification without assuming a special value of β.

Step 3: Identify the false statement At $\alpha = -6, \beta = 9$, the condition is $-z = 0$, so $z = 0$. Solving $2x - y = 5$ and $3x + 2y = 7$ gives $x = 17/7, y = -1/7$: a unique solution, not infinitely many. Therefore (A) is false. Statement (B) follows from $\alpha \neq -5$; (C) gives the contradiction $0 = -1$; and (D) gives an identity and a free parameter. Those three statements are correct.

Step 4: Conclusion The statement that is NOT correct is (A). Hence, the correct option is (A).

Quick Tip to Solve: Look for a linear combination of two equations matching the remaining equation before expanding a full determinant.

12. A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius : [4 marks]

(A) $\frac{2}{3}\lambda$

(B) $\frac{3}{5}\lambda$

(C) $\frac{\sqrt{19}}{7}\lambda$

(D) $\frac{\sqrt{19}}{5}\lambda$

Correct Answer: (D) $\frac{\sqrt{19}}{5}\lambda$

Solution:

Concept:

- The perpendicular from the centre of a circle to a chord bisects the chord. For radius λ and chord length λ, its midpoint distance follows from Pythagoras.
- A point dividing a fixed-length chord internally in a fixed ratio has a fixed offset from the chord midpoint, so its distance from the centre is constant.

Step 1: Locate the chord midpoint Let O be the centre and M the midpoint of AB. Since $OA = \lambda$ and $AM = \lambda/2$, the right triangle OMA gives

$$OM^2 = OA^2 - AM^2 = \lambda^2 - \frac{\lambda^2}{4} = \frac{3\lambda^2}{4}.$$

The fact that the chord and radius have the same length must not be confused with the chord being a diameter.

Step 2: Locate the dividing point Let P satisfy $AP : PB = 2 : 3$. Since $AB = \lambda$, we obtain $AP = 2\lambda/5$. Therefore the distance from the midpoint is

$$MP = \left|\frac{\lambda}{2} - \frac{2\lambda}{5}\right| = \frac{\lambda}{10}.$$

The line OM is perpendicular to the whole chord, hence also to MP. Interchanging the direction of the ratio changes the side of M, but not this distance.

Step 3: Calculate and identify the locus Apply Pythagoras to OMP:

$$OP^2 = OM^2 + MP^2 = \frac{3\lambda^2}{4} + \frac{\lambda^2}{100} = \frac{19\lambda^2}{25}.$$

Thus $OP = \sqrt{19}\lambda/5$. Rotating the chord through all orientations rotates P about O at this constant distance, giving the full circle. The squared radius is $19\lambda^2/25$, which differs from the squared radii in (A), (B) and (C).

Step 4: Conclusion The locus has radius $\boxed{\frac{\sqrt{19}}{5}\lambda}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the chord midpoint: the centre-to-midpoint distance and the division-point offset are perpendicular, so their squares add.

13. Let f be a differentiable function such that $x^2 f(x) - x = 4\int_0^x t f(t)dt$, $f(1) = \frac{2}{3}$. Then $18f(3)$ is equal to : [4 marks]

(A) 160

(B) 210

(C) 150

(D) 180

Correct Answer: (A) 160

Solution:

Concept:

- The fundamental theorem of calculus gives $\frac{d}{dx}\int_0^x t f(t)\,dt = x f(x)$ on the positive interval of interest.
- After differentiation, solve the first-order linear differential equation using an integrating factor, and use $f(1) = 2/3$ to determine its constant.

Step 1: Differentiate the integral equation Differentiate both sides of the given relation, using the product rule on $x^2 f(x)$:

$$2xf(x) + x^2 f'(x) - 1 = 4xf(x).$$

For $x > 0$, rearrange and divide by x^2:

$$f'(x) - \frac{2}{x}f(x) = \frac{1}{x^2}.$$

The integrating factor is $e^{\int -2/x\,dx} = x^{-2}$ on this interval.

Step 2: Integrate and apply the specified value Multiplication by the integrating factor turns the left side into a product derivative:

$$\frac{d}{dx}\left(\frac{f(x)}{x^2}\right) = x^{-4}.$$

Integrating gives

$$\frac{f(x)}{x^2} = -\frac{1}{3x^3} + C, \qquad f(x) = Cx^2 - \frac{1}{3x}.$$

At $x = 1$, $2/3 = C - 1/3$, so $C = 1$. Thus the required positive-argument solution is $f(x) = x^2 - 1/(3x)$.

Step 3: Check the relation and evaluate For this solution, $tf(t) = t^3 - 1/3$ for $t > 0$, which has a finite extension at the integration endpoint. Thus

$$4\int_0^x t f(t)\,dt = x^4 - \frac{4x}{3} = x^2 f(x) - x.$$

This verifies the original relation, rather than only its derivative. Finally,

$$18f(3) = 18\left(9 - \frac{1}{9}\right) = 18 \cdot \frac{80}{9} = 160.$$

The other choices 210, 150 and 180 disagree with this substitution; omitting the reciprocal correction

would instead give 162, which is not a listed choice.

Step 4: Conclusion The requested value is $\boxed{160}$. Hence, the correct option is (A).

Quick Tip to Solve: After differentiating an integral equation, substitute the final function back into the original relation to check any lost constant.

14. Let the first term α and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three terms is 33033, then the sum of these three terms is equal to [4 marks]

(A) 241

(B) 231

(C) 220

(D) 210

Correct Answer: (B) 231

Solution:

Concept:

- The first three GP terms are $\alpha, \alpha r, \alpha r^2$, so their squared sum is $\alpha^2(1 + r^2 + r^4)$.
- Both α and r are positive integers; use square divisors of the given total to restrict α, rather than testing arbitrary progressions.

Step 1: Factor the given squared sum The condition is

$$\alpha^2(1 + r^2 + r^4) = 33033 = 3 \cdot 7 \cdot 11^2 \cdot 13.$$

Since α^2 divides this integer, prime factorisation permits only $\alpha = 1$ or $\alpha = 11$. None of the primes 3, 7 or 13 can divide α, since its square would then require two copies of that prime.

Step 2: Determine the integer common ratio If $\alpha = 1$, we would require $1 + r^2 + r^4 = 33033$. The left side strictly increases with positive r, but

$$1 + 13^2 + 13^4 = 28731 < 33033 < 38613 = 1 + 14^2 + 14^4.$$

Thus there is no integer r in that case. For $\alpha = 11$, divide by 121 to obtain

$$1 + r^2 + r^4 = 273.$$

The positive integer $r = 4$ works because $1 + 16 + 256 = 273$; strict increase makes it the only positive solution.

Step 3: Add the three terms The terms are $11, 44, 176$, and their sum is

$$S = 11(1 + 4 + 16) = 11 \cdot 21 = 231.$$

As a check, $11^2 + 44^2 + 176^2 = 121 + 1936 + 30976 = 33033$. The alternatives 241, 220 and 210 do not equal the sum of these uniquely determined terms.

Step 4: Conclusion The sum is $\boxed{231}$. Hence, the correct option is (B).

Quick Tip to Solve: When a GP has integer first term and ratio, factor the squared-sum total first: its square divisors sharply restrict the first term.

15. Let the complex number $z = x + iy$ be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to : [4 marks]

(A) $\frac{4}{3}$
(B) $\frac{3}{2}$
(C) $\frac{3}{4}$
(D) $\frac{2}{3}$

Correct Answer: (C) $\frac{3}{4}$

Solution:

Concept:

- A complex quotient is purely imaginary precisely when its real part is zero, provided its denominator is nonzero. Multiply by the conjugate of the denominator to expose that real part.
- The relation $x + y^2 = 0$ gives $x = -y^2$, and therefore $x^2 = y^4$. This directly connects the required fourth-power expression to the real-part condition.

Step 1: Expand using the conjugate Write $2z - 3i = 2x + i(2y - 3)$ and $2z + i = 2x + i(2y + 1)$. Consequently,

$$\operatorname{Re}\frac{2z-3i}{2z+i} = \frac{4x^2 + (2y-3)(2y+1)}{4x^2 + (2y+1)^2}.$$

The denominator is a squared modulus, so it is positive for every admissible quotient. The numerator, rather than the imaginary part, must vanish. Expanding its product gives $4x^2 + 4y^2 - 4y - 3 = 0$.

Step 2: Use the second condition Divide the last identity by four:

$$x^2 + y^2 - y = \frac{3}{4}.$$

Now substitute $x = -y^2$, without needing to solve separately for the possible values of y:

$$y^4 + y^2 - y = \frac{3}{4}.$$

The excluded denominator point would have $x = 0, y = -1/2$; it does not satisfy $x + y^2 = 0$. Thus this exclusion does not remove a solution of the two given conditions. Options (A), (B), and (D) have values different from the forced invariant $3/4$, so none can be correct.

Step 3: Conclusion The required value is $\boxed{3/4}$. Hence, the correct option is (C).

Quick Tip to Solve: After taking the real part, look for the requested expression as an invariant; do not solve the resulting quartic for y.

16. Let two vertices of a triangle ABC be (2, 4, 6) and (0, −2, −5), and its centroid be (2, 1, −1). If the image of the third vertex in the plane $x + 2y + 4z = 11$ is (α, β, γ), then $\alpha\beta + \beta\gamma + \gamma\alpha$ is equal to : [4 marks]

(A) 72

(B) 74

(C) 76

(D) 70

Correct Answer: (B) 74

Solution:

Concept:

- The centroid of a triangle is the coordinate-wise average of its three vertices: $G = (A + B + C)/3$. Hence the missing vertex is $C = 3G - A - B$.
- Reflection of a point X in $\boldsymbol{n} \cdot X = d$ is $X' = X - 2(\boldsymbol{n} \cdot X - d)\boldsymbol{n}/|\boldsymbol{n}|^2$. The displacement is along the plane normal.

Step 1: Find the third vertex Let the unknown vertex be (a, b, c). The centroid equations give

$$a = 3(2) - 2 - 0 = 4, \qquad b = 3(1) - 4 - (-2) = 1,$$

$$c = 3(-1) - 6 - (-5) = -4.$$

Thus the third vertex is $C = (4, 1, -4)$. Using the centroid itself in the reflection formula would reflect the wrong point; the averaging step is essential.

Step 2: Reflect and evaluate the expression For the plane, $\boldsymbol{n} = (1, 2, 4)$, so $|\boldsymbol{n}|^2 = 1 + 4 + 16 = 21$. The signed plane expression at C is

$$4 + 2(1) + 4(-4) - 11 = -21.$$

Therefore

$$C' = (4, 1, -4) - \frac{2(-21)}{21}(1, 2, 4) = (6, 5, 4).$$

As a check, its midpoint with C is $(5, 3, 0)$, which satisfies $5 + 2(3) + 4(0) = 11$. The displacement $(2, 4, 8)$ is normal to the plane. Finally,

$$\alpha\beta + \beta\gamma + \gamma\alpha = 6(5) + 5(4) + 4(6) = 30 + 20 + 24 = 74.$$

This excludes the other proposed totals 72, 76, and 70.

Step 3: Conclusion The reflected vertex is $(6, 5, 4)$, and the required value is $\boxed{74}$. Hence, the correct option is (B).

Quick Tip to Solve: Check a reflected point by testing that its midpoint with the original point lies on the mirror plane.

17. The negation of the statement $(p \vee q) \wedge (q \vee (\sim r))$ is : [4 marks]

(A) $((\sim p) \vee r)) \wedge (\sim q)$

(B) $(p \vee r) \wedge (\sim q)$

(C) $((\sim p) \vee (\sim q)) \vee (\sim r)$

(D) $((\sim p) \vee (\sim q)) \wedge (\sim r)$

Correct Answer: (A) $((\sim p) \vee r)) \wedge (\sim q)$

Solution:

Concept:

- De Morgan's laws are $\neg(A \wedge B) = (\neg A) \vee (\neg B)$ and $\neg(A \vee B) = (\neg A) \wedge (\neg B)$. Double negation restores the original statement.
- The distributive identity $(u \wedge v) \vee (w \wedge v) = (u \vee w) \wedge v$ allows the common proposition to be factored.

Step 1: Negate the outer conjunction first Set $S = (p \vee q) \wedge (q \vee \neg r)$. Applying the outer De Morgan law before simplifying the inner statements gives

$$\neg S = \neg(p \vee q) \vee \neg(q \vee \neg r).$$

Apply the second law independently to each disjunction:

$$\neg S = (\neg p \wedge \neg q) \vee (\neg q \wedge r).$$

Notice that r is positive in the second term: negating the already negated r removes the negation. Also, the central connective is now a disjunction, since the original central connective was a conjunction.

Step 2: Factor and compare all alternatives Both terms contain $\neg q$, so the distributive law yields

$$\neg S = (\neg p \vee r) \wedge \neg q.$$

This is exactly option (A). To check the other forms rather than relying only on appearance, choose p false, q false, r false. The required negation is true, but (B) is false. To reject (D), choose p true, q false, and r false: the required negation is false, whereas (D) is true. To reject (C), choose p true, q true, r false: the original statement is true, so its negation is false, whereas (C) is true. These assignments establish that the alternatives are not equivalent to the requested negation.

Step 3: Conclusion The negation is $\boxed{((\sim p) \vee r) \wedge (\sim q)}$. Hence, the correct option is (A).

Quick Tip to Solve: Negate the outermost connective first, and simplify double negations before trying to match the options.

18. Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ and the plane $x + y + z = 2$. If the distance of the point P from the plane $3x - 4y + 12z = 32$ is q, then q and 2q are the roots of the equation : [4 marks]

(A) $x^2 + 18x - 72 = 0$

(B) $x^2 - 18x - 72 = 0$

(C) $x^2 + 18x + 72 = 0$

(D) $x^2 - 18x + 72 = 0$

Correct Answer: (D) $x^2 - 18x + 72 = 0$

Solution:

Concept:

- Equate each symmetric ratio to a parameter and substitute the resulting coordinates into the plane equation to find the point of intersection.
- The distance from (x_0, y_0, z_0) to $ax + by + cz = d$ is $|ax_0 + by_0 + cz_0 - d|/\sqrt{a^2 + b^2 + c^2}$. A monic quadratic with roots u, v is $X^2 - (u + v)X + uv = 0$.

Step 1: Locate the intersection point Let the common ratio equal t. Be careful with the reversed numerator $1 - z$: it gives a negative coefficient of t in z. Thus

$$x = 3t - 3, \qquad y = t - 2, \qquad z = 1 - 2t.$$

Substitute these into $x + y + z = 2$:

$$(3t - 3) + (t - 2) + (1 - 2t) = 2, \qquad 2t - 4 = 2.$$

It follows that $t = 3$, and consequently $P = (6, 1, -5)$. Checking the first plane gives $6 + 1 - 5 = 2$, as required.

Step 2: Calculate the distance and form the quadratic For the second plane, the normal has length $\sqrt{3^2 + (-4)^2 + 12^2} = 13$. Hence

$$q = \frac{|3(6) - 4(1) + 12(-5) - 32|}{13} = \frac{|-78|}{13} = 6.$$

The two roots are therefore 6 and 12, whose sum is 18 and product is 72. Using a fresh symbol X for the polynomial variable,

$$(X - 6)(X - 12) = X^2 - 18X + 72 = 0.$$

Both roots are positive distances. A positive linear coefficient would give a negative root sum, excluding (A) and (C); a negative constant would give opposite-sign roots, excluding (B). Only (D) has both the correct sum and product.

Step 3: Conclusion The equation is $\boxed{x^2 - 18x + 72 = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: In a symmetric line equation, preserve a reversed numerator such as 1-z; then check distance roots using their sum and product.

19. The slope of tangent at any point (x, y) on a curve $y = y(x)$ is $\frac{x^2+y^2}{2xy}, x > 0$. If $y(2) = 0$, then a value of $y(8)$ is : [4 marks]

(A) $-4\sqrt{2}$

(B) $2\sqrt{3}$

(C) $4\sqrt{3}$

(D) $-2\sqrt{3}$

Correct Answer: (C) $4\sqrt{3}$

Solution:

Concept:

- The slope condition gives $dy/dx = (x^2 + y^2)/(2xy)$. Multiplication by $2y$ suggests the substitution $u = y^2$, since $du/dx = 2y\, dy/dx$.
- The resulting first-order linear equation is solved with an integrating factor. The condition $y(2) = 0$ fixes the integration constant on the curve.

Step 1: Reduce to a linear equation Away from $y = 0$, multiply the slope equation by $2y$, and put $u = y^2$:

$$\frac{du}{dx} = x + \frac{u}{x}, \qquad \frac{du}{dx} - \frac{u}{x} = x.$$

Because $x > 0$, the integrating factor is $e^{-\int dx/x} = 1/x$. Therefore

$$\frac{d}{dx}\left(\frac{u}{x}\right) = 1, \qquad \frac{u}{x} = x + C.$$

This yields the implicit curve $y^2 = x^2 + Cx$. The calculation also follows directly by differentiating y^2/x, which avoids the more lengthy homogeneous substitution.

Step 2: Apply the curve condition and select a value Use the point $(2, 0)$ in the implicit curve:

$$0 = 4 + 2C, \qquad C = -2.$$

Thus $y^2 = x^2 - 2x$. At $x = 8$,

$$y(8)^2 = 64 - 16 = 48, \qquad y(8) = \pm 4\sqrt{3}.$$

The question asks for a value, so the positive value $4\sqrt{3}$ appearing among the options is sufficient. The other options have squares 32, 12, and 12, respectively, and cannot satisfy the curve equation at $x = 8$. At the initial point the displayed slope quotient is undefined; the curve passes through it with a vertical tangent, and the initial condition is applied to its implicit equation. On either branch for $x > 2$, differentiating the implicit equation reproduces the given slope.

Step 3: Conclusion A listed value is $\boxed{4\sqrt{3}}$. Hence, the correct option is (C).

Quick Tip to Solve: When the slope contains y in its denominator and y squared in its numerator, try u=y squared before a homogeneous substitution.

20. If A is a 3 × 3 matrix and $|A| = 2$, then $|3\,adj\,(|3A|A^2)|$ is equal to : [4 marks]

(A) $3^{12} \cdot 6^{10}$

(B) $3^{11} \cdot 6^{10}$

(C) $3^{12} \cdot 6^{11}$

(D) $3^{10} \cdot 6^{11}$

Correct Answer: (B) $3^{11} \cdot 6^{10}$

Solution:

Concept:

- For a matrix of order three, $|kM| = k^3|M|$, $|M^2| = |M|^2$, and $|\operatorname{adj} M| = |M|^2$. Scalars and matrices must be distinguished inside nested determinants.
- The inner quantity $|3A|$ is a scalar, while $|3A|A^2$ is a matrix. The outer multiplier 3 multiplies the entire adjugate matrix.

Step 1: Evaluate the inner matrix determinant The given determinant is $|A| = 2$. As the matrix order is three,

$$|3A| = 3^3|A| = 27(2) = 54.$$

Put $B = |3A|A^2 = 54A^2$. Its determinant is

$$|B| = 54^3|A^2| = 54^3|A|^2 = 4 \cdot 54^3.$$

The factor 54 is cubed because it scales each of three rows. Squaring it here instead would incorrectly treat the matrix as being of order two.

Step 2: Apply the adjugate and outer scalar rules Use the adjugate determinant rule, followed by the scalar rule for the outer 3:

$$|3\operatorname{adj} B| = 3^3|\operatorname{adj} B| = 27|B|^2 = 27(4 \cdot 54^3)^2.$$

Since $54 = 2 \cdot 3^3$, the prime powers simplify to

$$27 \cdot 16 \cdot 54^6 = 3^3 \cdot 2^4 \cdot 2^6 \cdot 3^{18} = 2^{10}3^{21} = 3^{11}6^{10}.$$

This agrees with (B). In prime-power form, (A) has $2^{10}3^{22}$, (C) has $2^{11}3^{23}$, and (D) has $2^{11}3^{21}$, so each differs from the calculated determinant. The positivity is also consistent with the square appearing in the adjugate rule.

Step 3: Conclusion The determinant is $\boxed{3^{11} \cdot 6^{10}}$. Hence, the correct option is (B).

Quick Tip to Solve: Name the inner matrix first; then apply scalar scaling, the determinant of a power, and the adjugate rule one layer at a time.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $f:(-2,2)\to R$ be defined by $f(x)=\begin{cases} x[x], & -2<x<0 \\ (x-1)[x], & 0\le x\le 2 \end{cases}$ where $[x]$ denotes the greatest integer function. If m and n respectively are the number of points in $(-2,2)$ at which $y=|f(x)|$ is not continuous and not differentiable, then $m+n$ is equal to _____. [4 marks]

Solution:

Concept:

- The greatest integer function is constant on each interval between consecutive integers. Therefore the formula for $g(x)=|f(x)|$ can be written explicitly on each such interval.
- Differentiability requires continuity. At a continuous joining point, compare the left and right derivatives; endpoints outside the open domain $(-2,2)$ are not counted.

Step 1: Write the absolute-value function interval by interval For $-2<x<-1$, $[x]=-2$, and for $-1\le x<0$, $[x]=-1$. Both products $x[x]$ are positive because $x<0$. For $0\le x<1$, the greatest integer is zero, while for $1\le x<2$ it is one. Thus

$$g(x)=\begin{cases} -2x, & -2<x<-1, \\ -x, & -1\le x<0, \\ 0, & 0\le x<1, \\ x-1, & 1\le x<2. \end{cases}$$

Every open interval in this display has a linear, hence continuous and differentiable, formula. Only the interior junctions $-1, 0, 1$ need checking.

Step 2: Count discontinuities and failures of differentiability At $x=-1$, the left limit is 2 while the right limit and the function value are 1. This is a jump, so g is neither continuous nor differentiable there. At $x=0$, both one-sided limits and the value equal zero, but the one-sided slopes are -1 and 0. It is continuous and not differentiable. At $x=1$, both limits and the value again equal zero, but the slopes are 0 and 1. It too is continuous and not differentiable. Consequently,

$$m=1, \qquad n=3, \qquad m+n=4.$$

The jump point must be included in the nondifferentiability count as well; that count is not restricted to corners of a continuous graph. The point 2 is excluded by the stated domain, even though the second displayed branch includes its endpoint.

Step 3: Conclusion There is one discontinuity and three points of nondifferentiability. The required dimensionless integer is $\boxed{4}$.

Quick Tip to Solve: Include every discontinuity in the nondifferentiability count, then add continuous corners where the one-sided slopes differ.

22. **Let $y = p(x)$ be the parabola passing through the points $(-1, 0), (0, 1)$ and $(1, 0)$. If the area of the region $\{(x, y) : (x + 1)^2 + (y - 1)^2 \leq 1, y \leq p(x)\}$ is A, then $12(\pi - 4A)$ is equal to _____.** [4 marks]

Solution:

Concept:

- The parabola through the three given points is $p(x) = 1 - x^2$. The circle has centre $(-1, 1)$ and radius one.
- Integrate the difference between the parabola and the lower circular boundary over the interval where that difference is nonnegative.

Step 1: Find the common boundaries Inside the circle, $-2 \leq x \leq 0$, and its lower boundary is $y = 1 - \sqrt{1 - (x + 1)^2}$. For a point below the parabola to exist, we require $\sqrt{1 - (x + 1)^2} \geq x^2$. Both sides are nonnegative, so squaring gives

$$x^4 + x^2 + 2x \leq 0, \qquad x(x + 1)(x^2 - x + 2) \leq 0.$$

Since $x^2 - x + 2 > 0$, the allowed interval is exactly $[-1, 0]$.

Step 2: Integrate the vertical height The parabola lies below the upper circular boundary on this interval. Thus the full height of the required region is the parabola minus the lower circular arc:

$$A = \int_{-1}^{0} \left[\sqrt{1 - (x + 1)^2} - x^2\right] dx.$$

With $u = x + 1$, the first integral is the area of a quarter unit circle, $\pi/4$. The second integral equals $[x^3/3]_{-1}^{0} = 1/3$. Therefore

$$A = \frac{\pi}{4} - \frac{1}{3}.$$

Step 3: Evaluate the required expression Substitute the area rather than rounding its numerical value:

$$12(\pi - 4A) = 12\left[\pi - 4\left(\frac{\pi}{4} - \frac{1}{3}\right)\right] = 12 \cdot \frac{4}{3} = 16.$$

The circular terms cancel exactly; the result is dimensionless.

Step 4: Conclusion The required value is $\boxed{16}$.

Quick Tip to Solve: Use the lower circular arc and determine the overlap interval first; integrating over the entire disk would count points above the parabola.

23. **The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ is _____.** [4 marks]

Solution:

Concept:

- An absolute-value inequality $|u| < 6$ is equivalent to $-6 < u < 6$. Strict inequalities exclude endpoint integers.
- Complete the square: $n^2 - 10n + 19 = (n - 5)^2 - 6$, which makes the symmetry around 5 explicit.

Step 1: Remove the absolute value The condition becomes

$$-6 < (n-5)^2 - 6 < 6.$$

Adding six throughout yields

$$0 < (n-5)^2 < 12.$$

The lower bound is strict: $n = 5$ gives an absolute value of six and must be excluded, even though it is at the centre of the range.

Step 2: List all possible integer distances Since $n - 5$ is an integer, its square is a perfect square. The positive perfect squares less than twelve are exactly $1, 4, 9$. Therefore

$$n - 5 \in \{-3, -2, -1, 1, 2, 3\},$$

so

$$n \in \{2, 3, 4, 6, 7, 8\}.$$

No larger distance works: distance four already gives $(n-5)^2 = 16$, which violates the upper bound.

Step 3: Count and check the candidates The six candidates form three symmetric pairs around five. Their original polynomial values are respectively $3, -2, -5, -5, -2, 3$, whose absolute values are all strictly below six. The omitted centre gives -6, so there are no further elements. This direct check also confirms that neither bound has accidentally been treated as inclusive.

Step 4: Conclusion The number of elements is $\boxed{6}$.

Quick Tip to Solve: After completing the square, count nonzero integer distances from the centre; remember that a strict lower bound can remove the centre itself.

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24. If the mean of the frequency distribution

Class:	0–10	10–20	20–30	30–40	40–50
Frequency:	2	3	x	5	4

is 28, then its variance is _____. [4 marks]

Solution:

Concept:

- Use the class midpoints m_i to calculate the mean and variance of the grouped distribution.
- For frequencies f_i, $\bar{m} = \sum f_i m_i / \sum f_i$ and $\sigma^2 = \sum f_i m_i^2 / \sum f_i - \bar{m}^2$.

Step 1: Find the unknown frequency The midpoints are $5, 15, 25, 35, 45$, so the total frequency is $N = 14 + x$. The first moment is

$$\sum f_i m_i = 2(5) + 3(15) + x(25) + 5(35) + 4(45) = 410 + 25x.$$

Using the given mean,

$$\frac{410+25x}{14+x}=28 \quad \Longrightarrow \quad 410+25x=392+28x.$$

Hence $3x=18$, giving $x=6$ and $N=20$. This is a nonnegative integer frequency, as required.

Step 2: Calculate the second moment Weight the squares of the same class midpoints by their frequencies:

$$\begin{aligned}\sum f_i m_i^2 &= 2(5^2)+3(15^2)+6(25^2)+5(35^2)+4(45^2)\\ &= 50+675+3750+6125+8100\\ &= 18700.\end{aligned}$$

Thus the mean squared midpoint is $18700/20=935$. The class boundaries themselves are not the representative values for this calculation.

Step 3: Subtract the square of the mean The variance is the second moment about zero minus the squared mean:

$$\sigma^2=935-28^2=935-784=151.$$

As an independent check, the weighted squared deviations from 28 sum to $2(529)+3(169)+6(9)+5(49)+4(289)=3020$, and $3020/20=151$.

Step 4: Conclusion The variance is $\boxed{151}$ in squared units of the measured variable.

Quick Tip to Solve: Find the missing frequency from the mean first, then use the shortcut variance formula with class midpoints, not class limits.

25. The number of permutations, of the digits 1, 2, 3, ..., 7 without repetition, which neither contain the string 153 nor the string 2467, is _____. [4 marks]

Solution:

Concept:

- There are 7! permutations of seven different digits when no restrictions are imposed.
- A prescribed consecutive string can be treated as one block with its internal order fixed. Use inclusion-exclusion for the two forbidden strings.

Step 1: Count each forbidden event Let E mean that the string 153 occurs and F mean that the string 2467 occurs. For E, treat 153 as a single object along with 2, 4, 6 and 7. These five different objects can be ordered in $5!=120$ ways. For F, treat 2467 as one object along with 1, 3 and 5, giving $4!=24$ arrangements. There is no internal factorial: each forbidden string has one specified order.

Step 2: Restore the overlap once The two strings use disjoint digits and together use all seven digits. A permutation containing both consists of the block 153 and the block 2467. The two blocks can appear in either order, so

$$|E\cap F|=2!=2.$$

These two permutations were subtracted twice in the separate forbidden-event counts, so inclusion-

exclusion adds them back once.

Step 3: Count permutations avoiding both strings Neither forbidden string may occur. Therefore the required complement has size

$$\begin{aligned} 7! - |E| - |F| + |E \cap F| &= 5040 - 120 - 24 + 2 \\ &= 4898. \end{aligned}$$

The word string requires consecutiveness; merely having the three or four digits appear in their relative order would be a different restriction and would not justify this block count.

Step 4: Conclusion The required number of permutations is $\boxed{4898}$.

Quick Tip to Solve: For forbidden disjoint strings, collapse each into a block and add back arrangements containing both; their fixed internal orders contribute no extra factorial.

26. The coefficient of x^7 in $(1 - x + 2x^3)^{10}$ is _____. [4 marks]

Solution:

Concept:

- In a product of ten identical trinomial factors, select one term from each factor. A multinomial coefficient counts each selection pattern.
- If $-x$ is selected b times and $2x^3$ is selected c times, the resulting exponent is $b + 3c$.

Step 1: List every possible selection pattern To obtain x^7, require $b + 3c = 7$, with b, c nonnegative integers and $b + c \leq 10$. Thus the only choices are

$$(b, c) = (7, 0), (4, 1), (1, 2).$$

The remaining $a = 10 - b - c$ selections are the constant 1. The corresponding triples (a, b, c) are $(3, 7, 0), (5, 4, 1), (7, 1, 2)$. Since $c \geq 3$ would already contribute degree nine, this list is complete.

Step 2: Evaluate each multinomial contribution For a triple (a, b, c), its contribution is

$$\frac{10!}{a!b!c!}(-1)^b 2^c.$$

For the three triples this gives

$$\begin{aligned} (3, 7, 0) &: -\binom{10}{7} = -120, \\ (5, 4, 1) &: \frac{10!}{5!4!1!}\, 2 = 2520, \\ (7, 1, 2) &: \frac{10!}{7!1!2!}(-1)\, 4 = -1440. \end{aligned}$$

In particular, the last term is negative because one factor supplies $-x$.

Step 3: Add the contributions The coefficient is the sum of all valid patterns, including their signs:

$$-120 + 2520 - 1440 = 960.$$

The numerical factors two from the cubic terms have already been included, and no other term of the expansion can have degree seven.

Step 4: Conclusion The coefficient of x^7 is $\boxed{960}$.

Quick Tip to Solve: First solve the degree equation for the counts of nonconstant selections; then attach the multinomial coefficient, sign and power of two to each pattern.

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27. Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple played in a match, is 840, then the total number of persons, who participated in the tournament, is _____. [4 marks]

Solution:

Concept:

- A mixed doubles match uses two men and two women, paired into two opposing mixed teams. A match is unchanged by interchanging the names of its two teams.
- The condition that no couple plays in a match excludes both spouses of any participating married pair, so the four players must come from four different couples.

Step 1: Choose the players without choosing spouses Let n couples participate, giving n men and n women. Choose the two male players in $\binom{n}{2}$ ways. Their two wives cannot be selected, so the two female players must be chosen from the other $n-2$ women, in $\binom{n-2}{2}$ ways. Each selected set of players contains exactly two men and two women, with no married couple among them.

Step 2: Count the team pairings For fixed men M_1, M_2 and women W_1, W_2, there are exactly two mixed team arrangements: (M_1W_1) versus (M_2W_2), or (M_1W_2) versus (M_2W_1). The opposing teams are not ordered, so there is no extra factor for reversing the match. Therefore

$$2\binom{n}{2}\binom{n-2}{2} = 840.$$

Step 3: Solve for the number of couples Expanding gives

$$\frac{n(n-1)(n-2)(n-3)}{2} = 840,$$

or $n(n-1)(n-2)(n-3) = 1680$. The four consecutive factors are $8 \cdot 7 \cdot 6 \cdot 5 = 1680$, so $n = 8$. For integers $n \geq 4$, this product increases strictly with n, establishing uniqueness. The requested quantity is people, not couples, so it is $2n = 16$.

Step 4: Conclusion The total number of participating persons is $\boxed{16}$.

Quick Tip to Solve: Choose the men first, exclude their wives, then choose the women; distinguish the two mixed team pairings without ordering the opposing teams.

28. Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$. Then, 6a + 5bc is equal to _____. [4 marks]

Solution:

Concept:

- All bases are positive, so taking natural logarithms preserves the given equalities. Put $A = \ln a$, $B = \ln b$, $C = \ln c$, and $L = \ln 2$.
- The exponential equations then become polynomial equations in logarithms. Retain every factor when solving; positivity does not imply that a logarithm is positive.

Step 1: Transform both equations exactly Taking logarithms of the two printed equations yields

$$A(L + A) = B(B + C), \qquad BL = AC.$$

If $A = 0$, the second equation gives $B = 0$, so $a = b = 1$, contradicting distinctness. Hence $A \neq 0$ and $C = BL/A$. Substituting this into the first equation and multiplying by A gives

$$A^2(L + A) = B^2(A + L),$$

so

$$(A + L)(A - B)(A + B) = 0.$$

Step 2: Identify the branch giving the keyed value The factor $A - B = 0$ is excluded because it makes $a = b$. If $A = -L$, then $a = 1/2$ and the second equation gives $C = -B$, hence $bc = 1$. On this branch,

$$6a + 5bc = 6\left(\frac{1}{2}\right) + 5 = 8.$$

This explains the value eight in the answer key, but the remaining factor must also be checked.

Step 3: Check the additional admissible branch If $A = -B$, then $ab = 1$ and $C = -L$, giving $c = 1/2$. These values need not violate distinctness. For example, $(a, b, c) = (3, 1/3, 1/2)$ consists of three distinct positive numbers and satisfies both logged equations, hence both original equations. For this example,

$$6a + 5bc = 18 + \frac{5}{6} = \frac{113}{6} \neq 8.$$

Thus the printed hypotheses allow more than one value; excluding the reciprocal branch would require an additional condition not present in the question.

Step 4: Conclusion As printed, the expression has no unique value. The answer key gives 8, which follows on the branch $a = 1/2, bc = 1$, but the valid counterexample $(3, 1/3, 1/2)$ gives $113/6$.

Quick Tip to Solve: When a factorization contains $A^2 - B^2$, check both $A = B$ and $A = -B$; distinct positive bases only exclude the first.

29. Let a common tangent to the curves $y^2 = 4x$ and $(x-4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For the parabola $y^2 = 4ax$, a tangent of nonzero slope m is $y = mx + a/m$, touching it at $(a/m^2, 2a/m)$. Here $a = 1$.
- A line is tangent to a circle when its perpendicular distance from the centre equals the radius. The contact point on the circle is the perpendicular foot from the centre.

Step 1: Find the common tangent slopes The circle has centre $O = (4, 0)$ and radius 4. Substitute the parabola tangent $mx - y + 1/m = 0$ in the distance condition:

$$\frac{|4m + 1/m|}{\sqrt{m^2+1}} = 4.$$

Squaring and multiplying by m^2 gives

$$16m^4 + 8m^2 + 1 = 16m^4 + 16m^2, \qquad m^2 = \frac{1}{8}.$$

The vertical tangent $x = 0$ to the parabola is not a distinct common tangent with two separated contact points: it touches both curves at the origin. The nonvertical common tangents give the intended separated points.

Step 2: Calculate the two contact points Take $m = 1/(2\sqrt{2})$; reflection in the x-axis gives the same distance for the negative slope. The parabola contact is

$$P = (8, 4\sqrt{2}).$$

For the foot from $(4, 0)$, note that $4m + 1/m = 12m$ and $1 + m^2 = 9/8$. Thus

$$Q = \left(4 - \frac{12m^2}{1+m^2}, \frac{12m}{1+m^2}\right) = \left(\frac{8}{3}, \frac{8\sqrt{2}}{3}\right).$$

Step 3: Evaluate the squared distance The coordinate differences are $16/3$ and $4\sqrt{2}/3$. Therefore

$$(PQ)^2 = \left(\frac{16}{3}\right)^2 + \left(\frac{4\sqrt{2}}{3}\right)^2 = \frac{256 + 32}{9} = 32.$$

The second oblique tangent gives the reflected contact points and the same value.

Step 4: Conclusion For either oblique common tangent, $\boxed{(PQ)^2 = 32}$ square coordinate units; the required numerical entry is 32.

Quick Tip to Solve: Use the slope form of a parabola tangent, then impose the centre-to-line distance condition for the circle. Reflection avoids solving both slopes separately.

30. The sum of all those terms, of the arithmetic progression 3, 8, 13, ..., 373, which are not divisible by 3, is equal to _____. [4 marks]

Solution:

Concept:

- The sum of an arithmetic progression with n terms, first term a and last term l, is $S = n(a+l)/2$.
- To sum terms satisfying an exclusion condition, subtract the sum of the excluded subsequence from the sum of the full progression. Congruences identify which indices are excluded.

Step 1: Count and sum every term The progression has first term 3, common difference 5, and final term 373. Hence

$$373 = 3 + (n-1)5, \qquad n = 75.$$

Its full sum is

$$S_{\text{all}} = \frac{75}{2}(3+373) = 75 \times 188 = 14100.$$

This total includes both the multiples of three and the terms requested in the question.

Step 2: Identify the excluded terms Write the term as $a_n = 5n - 2$. Modulo three, $5n - 2 \equiv 2n + 1$, so divisibility by three requires $n \equiv 1 \pmod 3$. Thus the excluded indices are $1, 4, 7, \ldots, 73$, a total of 25. Their terms form

$$3, 18, 33, \ldots, 363,$$

an arithmetic progression of difference 15. The next multiple in this subsequence would be 378, which lies beyond the stated last term and must not be included.

Step 3: Subtract the divisible subsequence The sum of the excluded multiples is

$$S_{\text{div}} = \frac{25}{2}(3+363) = 25 \times 183 = 4575.$$

Therefore the sum of all terms not divisible by three is

$$S_{\text{required}} = 14100 - 4575 = 9525.$$

As a count check, $75 - 25 = 50$ terms remain; the final term 373 is one of them.

Step 4: Conclusion The required sum is $\boxed{9525}$, so the numerical entry is 9525.

Quick Tip to Solve: Find the index residue class first: every third term is divisible by three, but the excluded subsequence begins with the very first term.

31. Given below are two statements:

Statement I : If the number of turns in the coil of a moving coil galvanometer is doubled then the current sensitivity becomes double.

Statement II : Increasing current sensitivity of a moving coil galvanometer by only increasing the number of turns in the coil will also increase its voltage sensitivity in the same ratio

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is true but Statement II is false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are false

(D) Both Statement I and Statement II are true

Correct Answer: (A) Statement I is true but Statement II is false

Solution:

Concept:

- At equilibrium, magnetic torque $NBIA$ equals restoring torque $k\theta$, where N is the number of turns, B the field, A the coil area and k the torsional constant.
- Current sensitivity is $S_I = \theta/I = NBA/k$. Voltage sensitivity is $S_V = \theta/V = S_I/R$, where R is the coil resistance.

Step 1: Evaluate Statement I Keeping the magnetic field, coil area and suspension unchanged, the deflection per unit current is proportional to N. Therefore

$$\frac{S_I(2N)}{S_I(N)} = \frac{2NBA/k}{NBA/k} = 2.$$

Doubling the turns doubles the current sensitivity. Statement I is therefore true. This refers to angular deflection for a given current, rather than the current produced by a given voltage.

Step 2: Evaluate Statement II If more turns are wound using the same wire and coil dimensions, the wire length increases in proportion to N. Since $R = \rho L/a$, the resistance also increases in proportion to N. On doubling the turns,

$$S_V' = \frac{S_I'}{R'} = \frac{2S_I}{2R} = S_V.$$

The increased torque sensitivity is offset by the smaller current that a given voltage drives through the higher coil resistance. Thus increasing turns alone does not increase voltage sensitivity in the same ratio. Statement II is false.

Step 3: Select the matching combination The truth values are true for Statement I and false for Statement II. Option (A) states exactly this combination. Options (B) and (C) incorrectly reject Statement I, while option (D) incorrectly accepts Statement II. Distinguishing θ/I from θ/V is essential because the coil resistance enters only the latter.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: When turns change, check both torque per ampere and coil resistance before comparing voltage sensitivity.

32. A physical quantity P is given as

$$P = \frac{a^2 b^3}{c\sqrt{d}}$$

The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4% respectively. The percentage error in the measurement of quantity P will be [4 marks]

(A) 12%

(B) 13%

(C) 16%

(D) 14%

Correct Answer: (B) 13%

Solution:

Concept:

- For $P = a^{\alpha} b^{\beta} c^{\gamma} d^{\delta}$, the maximum fractional error is the sum of the fractional input errors multiplied by the absolute values of the powers.
- For multiplication or division, maximum percentage uncertainties add; a denominator contributes a positive uncertainty even though its exponent is negative. A square root has exponent $1/2$.

Step 1: Express the powers explicitly Rewrite the quantity as

$$P = a^2 b^3 c^{-1} d^{-1/2}.$$

Its logarithmic differential is

$$\frac{dP}{P} = 2\frac{da}{a} + 3\frac{db}{b} - \frac{dc}{c} - \frac{1}{2}\frac{dd}{d}.$$

This signed relation describes small changes. To obtain the largest possible measurement error, allow each input error to have the sign that increases the magnitude of the output error.

Step 2: Add the maximum percentage contributions Taking absolute contributions gives

$$\frac{\Delta P}{P} \times 100 = 2(1\%) + 3(2\%) + 1(3\%) + \frac{1}{2}(4\%).$$

The contributions are respectively 2%, 6%, 3% and 2%. Therefore

$$\text{percentage error in } P = 2 + 6 + 3 + 2 = 13\%.$$

The square root halves the contribution from d; it does not double it. The denominator errors must not be subtracted in a maximum-error calculation.

Step 3: Check the offered values Only option (B) equals the weighted sum 13%. The values 12%, 16% and 14% cannot be obtained when the absolute powers $2, 3, 1, 1/2$ are correctly applied to the four given errors. Adding the unweighted errors or using signed cancellation would answer a different uncertainty question.

Step 4: Conclusion The maximum percentage error is $\boxed{13\%}$. Hence, the correct option is (B).

Quick Tip to Solve: First turn every factor into a power, then multiply its percentage error by the absolute exponent and add.

33. Given below are two statements:

Statement I : Pressure in a reservoir of water is same at all points at the same level of water.

Statement II : The pressure applied to enclosed water is transmitted in all directions equally.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is false but Statement II is true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are true

Correct Answer: (D) Both Statement I and Statement II are true

Solution:

Concept:

- For a liquid at rest under uniform gravity, $p = p_0 + \rho g h$, where h is depth below a common free surface and ρ is the liquid density.
- Pascal's law states that an externally applied pressure change in a confined fluid is transmitted undiminished throughout the fluid and to its containing walls.

Step 1: Test the equal-level statement At any two points of the same horizontal level in a connected reservoir, the depths below the free surface are equal. The water has the same density and the surface pressure p_0 is common. Therefore

$$p_1 = p_0 + \rho g h = p_2.$$

This is hydrostatic equilibrium. If a horizontal pressure difference persisted at the same level, water would accelerate horizontally, contrary to the reservoir being at rest. Statement I is true.

Step 2: Test the pressure-transmission statement For enclosed water, an applied pressure increment Δp is transmitted equally in all directions. Each point gains that same pressure increment, while any existing depth-dependent hydrostatic difference can remain. This is precisely Pascal's law, the basis of a hydraulic press or hydraulic brake. The pressure, rather than the applied force itself, is transmitted equally: different piston areas can experience different forces $F = \Delta p\, A$. Statement II is also true.

Step 3: Match the two truth values Both statements are individually correct in their stated settings. Option (D) is the only choice declaring both true. Option (A) rejects both valid laws, option (B) incorrectly rejects hydrostatic equality, and option (C) incorrectly rejects Pascal's law. No claim that Statement II explains Statement I is needed because this is a two-statement truth-value question.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (D).

Quick Tip to Solve: Equal depth determines the hydrostatic pressure; an applied increment in a confined fluid is governed by Pascal's law.

34. The energy of an electromagnetic wave contained in a small volume oscillates with [4 marks]

(A) double the frequency of the wave

(B) the frequency of the wave

(C) half the frequency of the wave

(D) zero frequency

Correct Answer: (A) double the frequency of the wave

Solution:

Concept:

- The instantaneous electric and magnetic energy densities are $u_E = \varepsilon_0 E^2/2$ and $u_B = B^2/(2\mu_0)$. For a plane wave in vacuum, $B = E/c$.
- The electric and magnetic fields in a travelling electromagnetic wave are in phase. Their energy densities add, and both are proportional to the square of the sinusoidal field.

Step 1: Write the energy in the fixed small volume At a fixed position, let $E = E_0 \cos(\omega t - kx)$. Since $c^2 = 1/(\mu_0 \varepsilon_0)$, the magnetic contribution equals the electric contribution:

$$u_B = \frac{E^2}{2\mu_0 c^2} = \frac{1}{2}\varepsilon_0 E^2 = u_E.$$

Thus the total energy density is

$$u = \varepsilon_0 E_0^2 \cos^2(\omega t - kx).$$

Multiplication by a sufficiently small fixed volume ΔV gives its instantaneous contained energy $U = u\Delta V$.

Step 2: Identify its oscillation frequency Use the identity $\cos^2\phi = (1 + \cos 2\phi)/2$. Then

$$U = \frac{\varepsilon_0 E_0^2 \Delta V}{2}\left[1 + \cos(2\omega t - 2kx)\right].$$

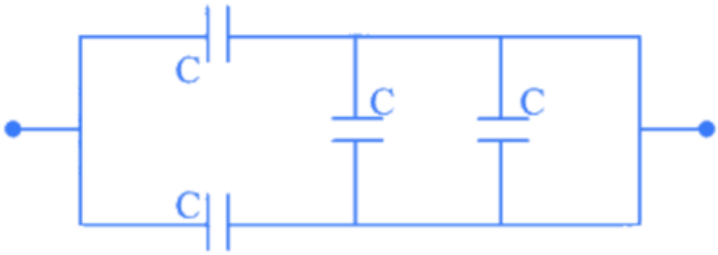
The constant term is the mean energy, and the varying term has angular frequency 2ω. If the wave frequency is $f = \omega/(2\pi)$, its energy oscillation frequency is $2f$. A reversal of the field changes its sign but not its squared magnitude, so two energy cycles occur per field cycle.

Step 3: Reject the remaining possibilities Option (B) describes the field frequency, not the frequency of its squared magnitude. Option (C) would imply a longer energy period, contrary to squaring a sinusoid. Option (D) applies to the time-averaged energy only; the question asks about the instantaneous energy in a small volume, which varies in time.

Step 4: Conclusion The energy oscillates at twice the wave frequency, $\boxed{2f}$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever an observable is proportional to the square of a sinusoidal field, expand the square to find a constant part and a double-frequency part.

35. The equivalent capacitance of the combination shown is : [4 marks]



(A) $\frac{C}{2}$

(B) 4C

(C) $\frac{5}{3}$C

(D) 2C

Correct Answer: (D) 2C

Solution:

Concept:

- Capacitors in parallel share the same two electrical nodes and have equivalent capacitance equal to the sum of their capacitances.
- An ideal connecting wire has zero potential difference along it. A capacitor whose two plates are linked by such a wire is short-circuited and contributes no terminal capacitance.

Step 1: Identify the actual circuit nodes Call the left terminal L and the right terminal R. The wire on the left joins the left plates of the upper and lower capacitors, so both those plates belong to node L. The upper rail after its capacitor and the lower rail after its capacitor are joined by the uninterrupted wire at the far right. They therefore form one node, R, even though they are drawn at different heights.

Step 2: Remove the two shorted middle capacitors Each vertical capacitor is connected between the upper and lower rails on the right of the two leftmost capacitors. Both its plates consequently belong to node R. Hence

$$\Delta V_{\text{vertical}} = V_R - V_R = 0, \qquad Q_{\text{vertical}} = C\Delta V_{\text{vertical}} = 0.$$

These two elements do not affect the charge drawn from the external terminals. This conclusion follows from the right-hand wire, rather than from the visual symmetry of the drawing.

Step 3: Combine the remaining parallel pair The upper-left and lower-left capacitors each connect directly from L to R. For an applied terminal voltage V, each stores charge CV, so

$$Q_{\text{total}} = CV + CV = 2CV, \qquad C_{\text{eq}} = \frac{Q_{\text{total}}}{V} = 2C.$$

Option (A) would describe two series capacitors. Option (B) incorrectly counts all four as active parallel elements, while option (C) retains a branch that the wire shorts out.

Step 4: Conclusion The equivalent capacitance is $\boxed{2C}$. Hence, the correct option is (D).

Quick Tip to Solve: Label connected wires as nodes before applying series or parallel rules; a capacitor connected from a node back to itself is shorted.

36. Consider two containers A and B containing monoatomic gases at the same Pressure (P), Volume (V) and Temperature (T). The gas in A is compressed isothermally to $\frac{1}{8}$ of its original volume while the gas in B is compressed adiabatically to $\frac{1}{8}$ of its original volume. The ratio of final pressure of gas in B to that of gas in A is [4 marks]

(A) $\frac{1}{8}$

(B) $8^{\frac{3}{2}}$

(C) 4

(D) 8

Correct Answer: (C) 4

Solution:

Concept:

- For a fixed amount of ideal gas, an isothermal process satisfies $PV =$ constant, since its temperature is fixed.
- A reversible adiabatic process satisfies $PV^{\gamma} =$ constant. For a monoatomic gas, $\gamma = C_P/C_V = 5/3$.

Step 1: Find the isothermal final pressure Both containers initially have pressure P and volume V. In container A the final volume is $V/8$, so Boyle's law gives

$$P_A \frac{V}{8} = PV \quad \Rightarrow \quad P_A = 8P.$$

The compression increases pressure, but heat leaves A to keep its temperature fixed.

Step 2: Find the adiabatic final pressure For container B no heat is exchanged. Work done on the gas raises its internal energy and temperature, making its pressure rise more than in A. Using the monoatomic value of γ,

$$P_B \left(\frac{V}{8}\right)^{5/3} = PV^{5/3}, \qquad P_B = P\,8^{5/3} = 32P.$$

Step 3: Compare the pressures Take the ratio in the requested order, B to A:

$$\frac{P_B}{P_A} = \frac{32P}{8P} = 4.$$

The value 8 is the pressure multiplication factor for A, not the ratio. The value $1/8$ is the volume fraction, and $8^{3/2}$ uses an incorrect exponent for the required pressure ratio.

Step 4: Conclusion The final pressure in B is four times that in A: $P_B : P_A = 4 : 1$. Hence, the correct option is (C).

Quick Tip to Solve: For equal compression ratios, divide the two process laws directly: $P_B/P_A = (V_i/V_f)^{\gamma-1}$.

37. The range of the projectile projected at an angle of 15° with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45° with horizontal, then its range will be [4 marks]

(A) $50\sqrt{2}$ m

(B) 100 m

(C) $100\sqrt{2}$ m

(D) 50 m

Correct Answer: (B) 100 m

Solution:

Concept:

- For a projectile returning to its launch level, neglecting air resistance, the horizontal range is $R = u^2 \sin(2\theta)/g$.
- At fixed initial speed u, the range is proportional to $\sin(2\theta)$, and its maximum occurs at $45°$.

Step 1: Use the given range to determine the common factor At the initial launch angle $15°$, the double angle is $30°$. Hence,

$$50 = \frac{u^2}{g} \sin 30° = \frac{u^2}{2g}.$$

Therefore $u^2/g = 100\,\text{m}$. The quantity u^2/g has dimensions of length; the numerical value of g is unnecessary because it is unchanged between launches.

Step 2: Calculate the range at the new angle At $45°$, the double angle is $90°$, for which the sine is unity. Keeping the same initial speed gives

$$R_{45} = \frac{u^2}{g} \sin 90° = 100 \times 1 = 100\,\text{m}.$$

Equivalently, $R_{45}/R_{15} = 1/(1/2) = 2$, so the range doubles. It does not stay at 50 m, since the angle has changed. The factors $\sqrt{2}$ in the other choices arise from using single-angle components rather than the double-angle range formula.

Step 3: Conclusion The new horizontal range is $\boxed{100\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Compare ranges using $\sin 2\theta$; the same speed and gravitational acceleration cancel before substitution.

38. A particle executes S.H.M. of amplitude A along x-axis. At t = 0, the position of the particle is $x = \frac{A}{2}$ and it moves along positive x-axis. The displacement of particle in time t is $x = A\sin(wt + \delta)$, then the value of δ will be [4 marks]

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

Correct Answer: (D) $\frac{\pi}{6}$

Solution:

Concept:

- For $x = A\sin(\omega t + \delta)$, the initial displacement gives $\sin\delta = x(0)/A$.
- The velocity is $v = A\omega\cos(\omega t + \delta)$. Its initial sign selects the correct phase quadrant.

Step 1: Apply the initial displacement Substitute $t = 0$ and the given $x(0) = A/2$ into the displacement equation:

$$\frac{A}{2} = A\sin\delta \quad \Rightarrow \quad \sin\delta = \frac{1}{2}.$$

Within one full cycle, this gives $\delta = \pi/6$ or $5\pi/6$. Initial position alone cannot distinguish between these phases: a particle crosses the same position in opposite directions during a cycle.

Step 2: Use the direction of motion The particle initially moves in the positive x-direction, so its initial velocity is positive. Differentiating the specified sine form,

$$v(0) = A\omega\cos\delta > 0.$$

Thus $\cos\delta > 0$. Together with $\sin\delta = 1/2 > 0$, this places the phase in the first quadrant and selects $\delta = \pi/6$, modulo 2π. At $\pi/2$, $\pi/3$ and $\pi/4$, the initial displacements would instead be A, $\sqrt{3}A/2$ and $A/\sqrt{2}$, respectively, so none matches the stated position.

Step 3: Conclusion The phase constant is $\boxed{\delta = \pi/6}$ radians (modulo 2π). Hence, the correct option is (D).

Quick Tip to Solve: In SHM, use initial displacement to find candidate phases and initial velocity to choose the quadrant.

39. The de Broglie wavelength of a molecule in a gas at room temperature (300 K) is λ_1. If the temperature of the gas is increased to 600 K, then the de Broglie wavelength of the same gas molecule becomes [4 marks]

(A) 2 λ_1

(B) $\frac{1}{2}\lambda_1$

(C) $\frac{1}{\sqrt{2}}\lambda_1$

(D) $\sqrt{2}\,\lambda_1$

Correct Answer: (C) $\frac{1}{\sqrt{2}}\lambda_1$

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where p is the magnitude of molecular momentum.
- For thermal motion of a fixed molecular species, characteristic kinetic energy is proportional to absolute temperature: $p^2/(2m) \propto T$. Thus $\lambda \propto T^{-1/2}$.

Step 1: Relate thermal momentum to temperature For the same gas molecule the mass m is unchanged. Using the rms thermal momentum as a representative value,

$$\frac{p_{\text{rms}}^2}{2m} = \frac{3}{2}k_B T, \qquad p_{\text{rms}} = \sqrt{3mk_B T}.$$

Consequently the characteristic de Broglie wavelength is $h/\sqrt{3mk_B T}$. Any consistent thermal speed convention gives the same inverse square-root dependence, because its numerical constant cancels in the ratio.

Step 2: Take the wavelength ratio The temperature rises from $T_1 = 300\,\text{K}$ to $T_2 = 600\,\text{K}$. Therefore,

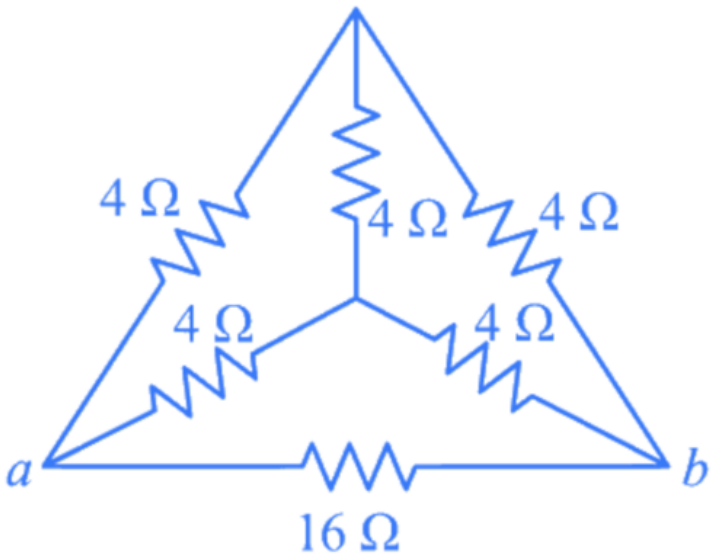
$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{300}{600}} = \frac{1}{\sqrt{2}}.$$

The wavelength decreases because molecular momentum increases. Thus $2\lambda_1$ and $\sqrt{2}\lambda_1$ have the wrong trend. The choice $\lambda_1/2$ incorrectly treats momentum as directly proportional to temperature instead of its square root.

Step 3: Conclusion The wavelength becomes $\boxed{\lambda_1/\sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a fixed gas species, doubling absolute temperature multiplies thermal momentum by $\sqrt{2}$ and divides wavelength by $\sqrt{2}$.

40. The equivalent resistance of the circuit shown below between points a and b is : [4 marks]



(A) 16Ω

(B) 20Ω

(C) 3.2Ω

(D) 24Ω

Correct Answer: (C) 3.2Ω

Solution:

Concept:

- In a balanced Wheatstone bridge, the two intermediate junctions are at equal potential, so the resistor connecting them carries no current.
- Resistances add in series along each independent branch; for parallel branches, $1/R_{\text{eq}} = \sum_i 1/R_i$.

Step 1: Identify the balanced bridge Call the top vertex c and the central junction d. Along the path a-c-b the two outer resistors are each $4\,\Omega$. Along the path a-d-b the two inner resistors are also each $4\,\Omega$. Their arm ratios are equal:

$$\frac{R_{ac}}{R_{cb}} = \frac{4}{4} = \frac{R_{ad}}{R_{db}}.$$

Thus c and d are at the same potential when a voltage is applied between a and b. No current flows through the central $4\,\Omega$ resistor c-d; it may be omitted for this equivalent-resistance calculation.

Step 2: Combine the remaining branches The upper two resistors give $4 + 4 = 8\,\Omega$; the two resistors through d give another $8\,\Omega$. The bottom $16\,\Omega$ resistor forms a third branch directly between a and b. Hence,

$$\frac{1}{R_{ab}} = \frac{1}{8} + \frac{1}{8} + \frac{1}{16} = \frac{5}{16}, \qquad R_{ab} = \frac{16}{5} = 3.2\,\Omega.$$

The equivalent resistance must be smaller than the smallest conducting branch resistance, $8\,\Omega$. The choices 16, 20 and 24 ohms violate this parallel-network check and cannot be correct.

Step 3: Conclusion The equivalent resistance between a and b is $\boxed{3.2\,\Omega}$. Hence, the correct option is (C).

Quick Tip to Solve: Check equal arm ratios before adding resistors: a balanced bridge removes the central branch from the current calculation.

41. An object is placed at a distance of 12 cm in front of a plane mirror. The virtual and erect image is formed by the mirror. Now the mirror is moved by 4 cm towards the stationary object. The distance by which the position of image would be shifted, will be [4 marks]

(A) 4 cm towards mirror

(B) 2 cm towards mirror

(C) 8 cm away from mirror

(D) 8 cm towards mirror

Correct Answer: (D) 8 cm towards mirror

Solution:

Concept:

- A plane mirror forms an image as far behind the mirror as the object is in front. With coordinates along the normal, $x_i = 2x_m - x_o$.
- For a stationary object, a mirror displacement Δx_m produces an image displacement $\Delta x_i = 2\Delta x_m$ in the same direction.

Step 1: Set up fixed coordinates Place the stationary object at $x_o = 0$ and the initial mirror at $x_m = 12\,\text{cm}$, with positive x pointing from the object through the mirror. The initial image lies 12 cm behind that mirror:

$$x_{i1} = 2(12) - 0 = 24\,\text{cm}.$$

Using a fixed coordinate frame is essential: the question asks how far the image position shifts, rather than only how its distance from the moving mirror changes.

Step 2: Find the new image position and displacement The mirror moves 4 cm toward the object, so its new coordinate is $x_{m2} = 12 - 4 = 8\,\text{cm}$. The image is now 8 cm behind the new mirror:

$$x_{i2} = 2(8) = 16\,\text{cm}, \qquad \Delta x_i = 16 - 24 = -8\,\text{cm}.$$

The image shifts toward the mirror/object side by 8 cm, in the same direction as the mirror motion. The magnitude is twice the mirror displacement, eliminating 4 cm and 2 cm. The choice describing 8 cm away from the mirror reverses the direction of the image shift.

Step 3: Conclusion The image shifts $\boxed{8\,\text{cm}}$ toward the mirror. Hence, the correct option is (D).

Quick Tip to Solve: With the object fixed, use $x_i = 2x_m - x_o$; the image moves twice as far as the mirror in the same direction.

42. Two satellites of masses m and 3m revolve around the earth in circular orbits of radii r & 3r respectively. The ratio of orbital speeds of the satellites respectively is [4 marks]

(A) 3 : 1

(B) $\sqrt{3}$: 1

(C) 1 : 1

(D) 9 : 1

Correct Answer: (B) $\sqrt{3}$: 1

Solution:

Concept:

- In a circular orbit, gravitational attraction supplies centripetal force: $GMm_s/r^2 = m_s v^2/r$.
- The resulting speed $v = \sqrt{GM/r}$ depends on the orbital radius and central mass, but not the satellite mass m_s.

Step 1: Derive the radius dependence For a satellite of any mass m_s orbiting Earth, equate Newton's gravitational force to the required centripetal force:

$$\frac{GMm_s}{r^2} = \frac{m_s v^2}{r} \quad \Rightarrow \quad v^2 = \frac{GM}{r}.$$

The satellite mass cancels exactly. Thus the factor of three between the two satellite masses cannot be inserted into the orbital-speed ratio. Both satellites orbit the same Earth, so G and M are common.

Step 2: Compare the two orbital speeds For the first satellite the orbit radius is r, whereas for the second it is $3r$. Hence,

$$v_1 = \sqrt{\frac{GM}{r}}, \qquad v_2 = \sqrt{\frac{GM}{3r}}, \qquad \frac{v_1}{v_2} = \sqrt{3}.$$

The smaller orbit has the greater speed. A ratio 3:1 would incorrectly use inverse radius rather than inverse square-root radius; 9:1 confuses force dependence with speed, while 1:1 ignores the different radii.

Step 3: Conclusion The orbital-speed ratio, in the stated order, is $\boxed{\sqrt{3} : 1}$. Hence, the correct option is (B).

Quick Tip to Solve: Cancel the satellite mass first; for circular orbits around the same body, compare speeds using $v \propto 1/\sqrt{r}$.

43. The position-time graphs for two students A and B returning from the school to their homes are shown in figure.

(A) A lives closer to the school

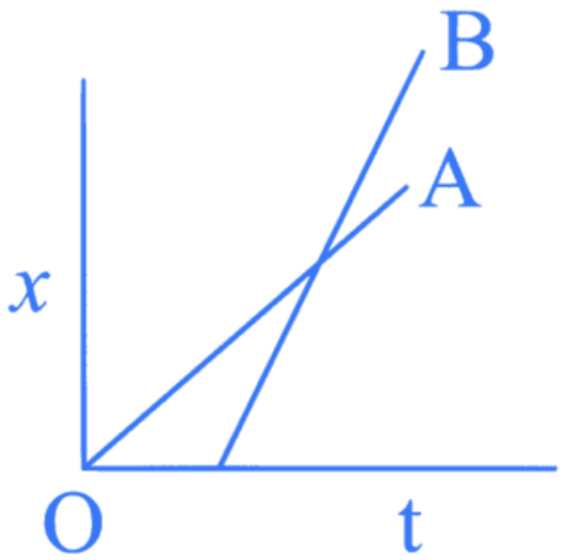
(B) B lives closer to the school

(C) A takes lesser time to reach home

(D) A travels faster than B

(E) B travels faster than A

Choose the correct answer from the options given below : [4 marks]



(A) (A), (C) and (D) only

(B) (A) and (E) only

(C) (A), (C) and (E) only

(D) (B) and (E) only

Correct Answer: (B) (A) and (E) only

Solution:

Concept:

- The slope of a position-time graph is velocity: $v = \Delta x/\Delta t$. A steeper rising straight line means a greater constant speed.
- The final vertical coordinate gives the distance of the home from the school; journey duration is the horizontal interval between departure and arrival.

Step 1: Compare the distances of the homes The school is at the origin of position. The endpoint of line A lies below the endpoint of line B, so the final position of A is smaller. Therefore A lives closer to school: statement (A) is true and statement (B) is false. The intersection of the two lines represents equal positions at one instant; it does not identify either home.

Step 2: Compare speeds and journey durations Line B is steeper than line A, so $v_B > v_A$. Thus statement (E) is true and statement (D) is false. Student A starts at the origin of time, whereas B

departs later. Their arrivals are shown at approximately the same time, so B has the shorter journey duration. A does not take less time, and statement (C) is false. One must compare elapsed times from the separate departure points, rather than simply compare the final time coordinates.

Step 3: Select the matching combination Only statements (A) and (E) survive both tests. The first combination incorrectly includes (C) and (D); the third incorrectly includes (C); the fourth incorrectly claims that B lives closer. The second combination alone gives the smaller final distance for A together with the greater slope for B.

Step 4: Conclusion Only the closer-home statement for A and the faster-motion statement for B hold. Hence, the correct option is (B).

Quick Tip to Solve: Use height for distance from school, slope for speed, and horizontal journey length for travel time.

44. The angular momentum for the electron in Bohr's orbit is L. If the electron is assumed to revolve in second orbit of hydrogen atom, then the change in angular momentum will be [4 marks]

(A) L

(B) $\frac{L}{2}$

(C) zero

(D) 2 L

Correct Answer: (A) L

Solution:

Concept:

- In a stationary Bohr orbit, the magnitude of orbital angular momentum is quantised: $L_n = n\hbar = nh/(2\pi)$.
- The Coulomb force is central, so its torque about the nucleus vanishes: $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = 0$.

Step 1: Express the allowed angular momenta Bohr quantisation gives $L_n = n\hbar$, where $\hbar = h/(2\pi)$. If the stated reference angular momentum L denotes the first-orbit value, then

$$L_1 = \hbar = L.$$

The second orbit has $n = 2$, so its allowed magnitude is $L_2 = 2\hbar = 2L$. This is the interpretation leading to the intended answer. However, the printed question says only Bohr orbit, without explicitly identifying the initial orbit as the first.

Step 2: Subtract the reference value from the second-orbit value Under the first-orbit interpretation, the increase to the second orbit is

$$\Delta L = L_2 - L_1 = 2L - L = L.$$

More generally, if the initial orbit is n, then $L = n\hbar$ and the change in magnitude on reaching the

second orbit is

$$\Delta L = (2-n)\hbar = \left(\frac{2}{n} - 1\right) L.$$

Thus an initial quantum number is required to specify a transition uniquely. The reference value cannot be identified as the fundamental value from quantisation alone.

Step 3: Distinguish a value from a change If instead the question asks for change while the electron revolves in the fixed second orbit, the central Coulomb force has zero torque and the angular momentum remains constant, giving zero change. The value $2L$ is the second-orbit magnitude only under the first-orbit reference convention; it is not the increase. Similarly $L/2$ is not the first-to-second increase. Hence the intended option follows only after supplying the usual first-orbit interpretation; the literal wording does not give a unique transition.

Step 4: Conclusion If L denotes the first-orbit value, the increase to the second orbit is L. Hence, the intended correct option is (A). The initial orbit is unspecified in the wording; motion within a fixed second orbit gives zero change.

Quick Tip to Solve: Use $L_n = n\hbar$; calculate the second-orbit value first, then subtract the reference first-orbit value.

45. Given below are two statements:

Statement I : Maximum power is dissipated in a circuit containing an inductor, a capacitor and a resistor connected in series with an AC source, when resonance occurs

Statement II : Maximum power is dissipated in a circuit containing pure resistor due to zero phase difference between current and voltage.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are true

Correct Answer: (D) Both Statement I and Statement II are true

Solution:

Concept:

- The mean power supplied to a series AC circuit is $P = V_{\text{rms}} I_{\text{rms}} \cos\phi = I_{\text{rms}}^2 R$. Ideal inductors and capacitors store and return energy.
- For a series LCR circuit, $Z^2 = R^2 + (X_L - X_C)^2$, where $X_L = \omega L$ and $X_C = 1/(\omega C)$.

Step 1: Test Statement I using resonance For a fixed source rms voltage and fixed resistance, the current is $I_{\text{rms}} = V_{\text{rms}}/Z$. Consequently the average dissipated power is

$$P = \frac{V_{\text{rms}}^2 R}{R^2 + (X_L - X_C)^2}.$$

The denominator has its smallest value when $X_L = X_C$, which is the series resonance condition. At that point $Z = R$, the current is greatest, and $P_{\max} = V_{\text{rms}}^2/R$. Statement I is therefore true.

Step 2: Test Statement II using the phase factor A pure resistance obeys $v(t) = Ri(t)$, so voltage and current reach their maxima together. Thus $\phi = 0$ and $\cos\phi = 1$, the largest possible power factor. For the given rms voltage and current,

$$P = V_{\text{rms}} I_{\text{rms}}.$$

There is no reactive part returning energy to the source. This is the sense in which the dissipated power is maximum, so Statement II is true as well.

Step 3: Match the truth values to the options The first and third choices each reject one of the established statements, while the second rejects both. The fourth choice alone says both statements are true. Resonance makes the series LCR combination effectively resistive, which is consistent with the unity power factor of a pure resistor.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (D).

Quick Tip to Solve: For series resonance, first cancel the reactances; the remaining impedance is R and the power factor is one.

46. Match List I with List II :

	List I		List II
(A)	3 Translational degrees of freedom	(I)	Monoatomic gases
(B)	3 Translational, 2 rotational degrees of freedoms	(II)	Polyatomic gases
(C)	3 Translational, 2 rotational and 1 vibrational degrees of freedom	(III)	Rigid diatomic gases
(D)	3 Translational, 3 rotational and more than one vibrational degrees of freedom	(IV)	Nonrigid diatomic gases

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

(B) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(C) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(D) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

Correct Answer: (A) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

Solution:

Concept:

- Every freely moving molecule has three translational degrees of freedom, corresponding to motion along three mutually perpendicular directions.
- A rigid linear diatomic molecule has two active rotational degrees of freedom. Nonrigid molecules additionally possess vibrational modes; a nonlinear polyatomic molecule has three rotational modes.

Step 1: Match the translation-only and rigid cases A monatomic gas is modelled as particles without molecular rotational or vibrational modes contributing in the usual ideal-gas treatment. Its three degrees of freedom are translational, so (A) matches (I). A rigid diatomic molecule can rotate about two axes perpendicular to its internuclear line. Rotation about that line has negligible moment of inertia in this model. It therefore has three translational and two rotational modes, giving (B) matched to (III).

Step 2: Match the vibrational cases If the bond length of a diatomic molecule may change, its atoms can vibrate relative to one another. This is the nonrigid diatomic case. The one vibrational mode stated in row (C), together with three translations and two rotations, therefore matches (IV). Each vibrational mode has both kinetic and potential energy terms in equipartition; the row is counting modes, not asserting that a vibrational mode has only one energy term. Row (D) describes the nonlinear polyatomic case, with three rotational modes and several vibrational modes, so (D) matches (II).

Step 3: Check the full ordered matching The complete correspondence is

$$(A)\to(I), \quad (B)\to(III), \quad (C)\to(IV), \quad (D)\to(II).$$

The second and third options wrongly assign the translation-only case to a nonrigid diatomic gas. The fourth swaps the rigid and nonrigid diatomic assignments, so it also fails.

Step 4: Conclusion The ordered match is I, III, IV, II for the four rows respectively. Hence, the correct option is (A).

Quick Tip to Solve: Match translation-only to monatomic first; then distinguish rigid diatomic from nonrigid diatomic by vibration.

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47. Assuming the earth to be a sphere of uniform mass density, the weight of a body at a depth $d = \frac{R}{2}$ from the surface of earth, if its weight on the surface of earth is 200 N, will be:

(Given R = radius of earth) [4 marks]

(A) 100 N

(B) 400 N

(C) 300 N

(D) 500 N

Correct Answer: (A) 100 N

Solution:

Concept:

- Inside a spherically symmetric uniform Earth, only the mass enclosed within the radial distance contributes to the net gravitational attraction.
- At depth d, the distance from the centre is $r = R - d$, and the gravitational acceleration is $g(r) = g_s r/R = g_s(1 - d/R)$.

Step 1: Find the mass enclosed at the new radius Let the density be ρ and total mass of Earth be M.

Uniform density gives

$$M = \frac{4\pi}{3}\rho R^3, \qquad M_r = M\frac{r^3}{R^3}.$$

By the shell theorem, the spherical shells outside radius r produce zero resultant gravitational field at the body. We must therefore use M_r, rather than the entire mass M, in the gravitational-force formula inside Earth.

Step 2: Derive and apply the weight ratio For the same body of mass m,

$$W_r = \frac{GM_r m}{r^2} = \frac{GMm}{R^3}r.$$

At the surface $W_s = GMm/R^2$, so

$$\frac{W_r}{W_s} = \frac{r}{R} = 1 - \frac{d}{R}.$$

Here $d = R/2$, hence $r = R/2$, and

$$W_r = 200\left(1 - \frac{1}{2}\right) = 100\,\text{N}.$$

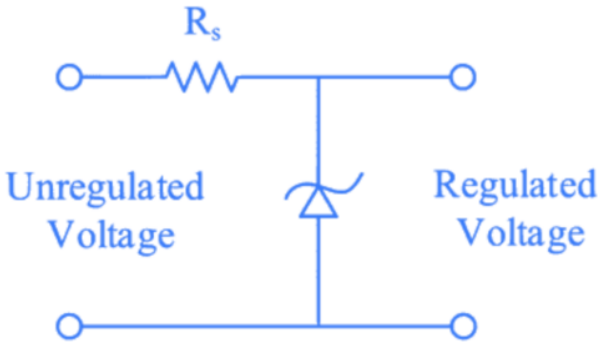
The body mass itself has not changed; the smaller gravitational acceleration is responsible for the reduced weight.

Step 3: Check the direction of change The field of a uniform sphere falls linearly from its surface value to zero at its centre. A weight greater than 200 N at this depth would contradict that dependence. Therefore 400 N, 300 N and 500 N are all excluded. Using the exterior inverse-square law with the full Earth mass at $r = R/2$ would be inappropriate, because half-radius encloses only one eighth of the mass.

Step 4: Conclusion The weight at depth $R/2$ is $100\,\text{N}$. Hence, the correct option is (A).

Quick Tip to Solve: Inside a uniform Earth use W proportional to distance from the centre, not the exterior inverse-square formula.

48. A zener diode of power rating 1.6 W is to be used as voltage regulator. If the zener diode has a breakdown of 8V and it has to regulate voltage fluctuating between 3 V and 10 V. The value of resistance R_s for safe operation of diode will be [4 marks]



(A) 13 Ω

(B) 13.3 Ω

(C) 10 Ω

(D) 12 Ω

Correct Answer: (C) 10 Ω

Solution:

Concept:

- The maximum safe Zener current follows from its power rating: $P_{Z,\max} = V_Z I_{Z,\max}$.
- When the diode is in breakdown, the series resistor carries the excess input voltage: $I_Z = (V_{\text{in}} - V_Z)/R_s$ for the shown unloaded regulator.

Step 1: Convert the power rating into a current limit The breakdown voltage is $V_Z = 8\,\text{V}$ and the maximum permitted dissipation is $1.6\,\text{W}$. Therefore

$$I_{Z,\max} = \frac{1.6}{8} = 0.20\,\text{A}.$$

This limit is on the diode current in reverse breakdown. The series resistance must prevent a larger current at the greatest input voltage, since that is where the voltage drop across the resistor is largest.

Step 2: Use the worst-case input voltage At the upper input voltage $V_{\text{in,max}} = 10\,\text{V}$, the resistor must drop

$$V_{R_s} = 10 - 8 = 2\,\text{V}.$$

Requiring $I_Z \le 0.20\,\text{A}$ gives

$$R_s \ge \frac{2}{0.20} = 10\,\Omega.$$

Thus the limiting design value is $10\,\Omega$, at which the diode dissipates exactly its rated power at the upper input.

Step 3: Interpret the lower input and the offered values When the input is below 8 V, including at 3 V, the diode is not in Zener breakdown and cannot hold the output at 8 V. That does not set the maximum-current design condition. The offered 12, 13 and 13.3 ohm resistors also limit current

safely, but exceed the minimum limiting resistance calculated from the stated rating. The standard requested design value is the threshold 10 ohm; the circuit does not provide regulation throughout the stated range below breakdown.

Step 4: Conclusion The minimum limiting series resistance for safe operation is $R_s = 10\,\Omega$. Hence, the correct option is (C).

Quick Tip to Solve: Use maximum input voltage for Zener safety: first find P divided by breakdown voltage, then the excess voltage divided by that current.

49. A particle of mass m moving with velocity v collides with a stationary particle of mass 2m. After collision, they stick together and continue to move together with velocity [4 marks]

(A) v

(B) $\frac{v}{3}$

(C) $\frac{v}{4}$

(D) $\frac{v}{2}$

Correct Answer: (B) $\frac{v}{3}$

Solution:

Concept:

- In a collision with negligible external impulse, total linear momentum of the two-particle system is conserved.
- If the bodies stick together, they share one final velocity and form a combined mass. Kinetic energy need not be conserved in this perfectly inelastic collision.

Step 1: Write the initial and final momenta Take the initial direction of the moving particle as positive. The first particle has mass m and velocity v, while the second has mass $2m$ and zero initial velocity. The initial total momentum is therefore

$$p_i = mv + (2m)(0) = mv.$$

After sticking, the total moving mass is $m + 2m = 3m$. If its common velocity is u, the final momentum is $p_f = 3mu$.

Step 2: Apply momentum conservation Equating the system momenta before and after impact gives

$$mv = 3mu \quad \Longrightarrow \quad u = \frac{v}{3}.$$

The final velocity has the same direction as the original velocity, since the initial momentum is positive and there is no other initial momentum to reverse it. The final mass is three times the initially moving mass, explaining the factor of three in the denominator.

Step 3: Check the energy change and other options The initial kinetic energy is $K_i = mv^2/2$, while

$$K_f = \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \frac{mv^2}{6}.$$

Thus $K_f = K_i/3$. The remaining energy becomes deformation, heat and internal energy, which is consistent with sticking. A final speed v, $v/4$, or $v/2$ would give final momentum $3mv$, $3mv/4$, or $3mv/2$, respectively. None equals the initial momentum mv.

Step 4: Conclusion The common velocity is $v/3$ in the original direction of motion. Hence, the correct option is (B).

Quick Tip to Solve: For sticking collisions, divide the initial total momentum by the combined mass; do not conserve kinetic energy.

50. A carrier wave of amplitude 15 V is modulated by a sinusoidal base band signal of magnitude 3 V. The ratio of maximum amplitude to minimum amplitude in an amplitude modulated wave is [4 marks]

(A) 1

(B) 5

(C) 2

(D) $\frac{3}{2}$

Correct Answer: (D) $\frac{3}{2}$

Solution:

Concept:

- In amplitude modulation, the instantaneous envelope amplitude is the carrier amplitude plus the varying baseband signal. Its extreme values are $A_{\max} = A_c + A_m$ and $A_{\min} = A_c - A_m$.
- The modulation index is $\mu = A_m/A_c$. For $\mu < 1$, the envelope never reverses sign and the ratio of its extreme amplitudes is $(1+\mu)/(1-\mu)$.

Step 1: Identify the two amplitudes The carrier amplitude is $A_c = 15\,\text{V}$, while the amplitude of the sinusoidal modulating signal is $A_m = 3\,\text{V}$. Therefore $\mu = 3/15 = 0.2$. This is less than unity, so ordinary amplitude modulation without overmodulation applies. The given signal amplitude is not itself the minimum envelope amplitude; it is the amount by which the carrier envelope increases or decreases.

Step 2: Calculate the envelope extremes At the positive peak of the modulating signal, its contribution adds to the carrier envelope. At the negative peak, the same contribution subtracts. Hence

$$A_{\max} = 15 + 3 = 18\,\text{V}, \qquad A_{\min} = 15 - 3 = 12\,\text{V}.$$

Both are positive, as expected from the small modulation index.

Step 3: Compare the options The required ratio is

$$\frac{A_{\max}}{A_{\min}} = \frac{18}{12} = \frac{3}{2}.$$

A ratio of 1 would describe an unmodulated carrier, whereas the given signal changes its envelope. The value 5 is the carrier-to-signal amplitude ratio, not the envelope-extremes ratio. A ratio of 2 would require a modulation index of $1/3$, which is not the given value.

Step 4: Conclusion The maximum-to-minimum envelope amplitude ratio is $\boxed{3/2}$. Hence, the correct option is (D).

Quick Tip to Solve: Calculate the envelope peaks by adding and subtracting the modulating amplitude; do not divide the carrier amplitude by the signal amplitude.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A transverse harmonic wave on a string is given by

$$y(x,t) = 5\sin(6t + 0.003x)$$

where x and y are in cm and t in sec. The wave velocity is _____ ms^{-1}. [4 marks]

Solution:

Concept:

- A travelling sinusoidal wave has phase $\omega t \pm kx$, where ω is angular frequency and k is angular wave number. Its speed has magnitude $v = \omega/k$.
- A point of constant phase moves in the direction of propagation. A plus sign between the time and space terms denotes propagation towards decreasing x. Units of the spatial coordinate must be retained when reading k.

Step 1: Read the frequency and wave number Compare $y = 5\sin(6t+0.003x)$ with $y = A\sin(\omega t+kx)$. Since time is in seconds, $\omega = 6\,\text{rad}\,\text{s}^{-1}$. Since x is measured in centimetres, $k = 0.003\,\text{rad}\,\text{cm}^{-1}$, not radians per metre. The amplitude $5\,\text{cm}$ sets the transverse displacement and does not enter the propagation speed.

Step 2: Follow a point of constant phase Let $6t + 0.003x$ remain constant. Differentiating with respect to time gives

$$6 + 0.003\frac{dx}{dt} = 0, \qquad \frac{dx}{dt} = -\frac{6}{0.003} = -2000\,\text{cm}\,\text{s}^{-1}.$$

The negative sign specifies motion along the negative x-direction. It does not make the speed, which is a magnitude, negative.

Step 3: Convert the result to SI units Use $100\,\text{cm} = 1\,\text{m}$. Thus

$$|v| = \frac{2000}{100} = 20\,\text{m}\,\text{s}^{-1}.$$

Equivalently, express the original wave number as $0.3\,\text{rad}\,\text{m}^{-1}$, giving $6/0.3 = 20\,\text{m}\,\text{s}^{-1}$. Both methods keep the units consistent and lead to the same numerical entry.

Step 4: Conclusion The required numerical value is $\boxed{20}$, corresponding to a speed of $20\,\text{m}\,\text{s}^{-1}$. The signed propagation velocity is $-20\,\text{m}\,\text{s}^{-1}$, along negative x.

Quick Tip to Solve: Before using omega divided by k, check whether x is in metres or centimetres. Use the sign of the phase to state the propagation direction.

52. 10 resistors each of resistance 10 Ω can be connected in such as to get maximum and minimum equivalent resistance. The ratio of maximum and minimum equivalent resistance will be _____.

[4 marks]

Solution:

Concept:

- Resistors in series carry the same current, and their resistances add: $R_s = \sum_i R_i$. This arrangement gives the greatest equivalent resistance using all identical positive resistors.
- Resistors in parallel share the same potential difference, and their conductances add: $1/R_p = \sum_i 1/R_i$. For n identical resistors of resistance R, $R_s = nR$ and $R_p = R/n$.

Step 1: Construct the maximum resistance To oppose the current as much as possible, connect all ten resistors end to end in one series path. Every unit of charge must then pass through all ten resistors. Since each resistance is $10\,\Omega$,

$$R_{\max} = 10 \times 10 = 100\,\Omega.$$

A parallel branch would offer an additional route for current and reduce the equivalent resistance below this series value.

Step 2: Construct the minimum resistance For the minimum, connect each of the ten resistors directly across the same pair of terminals. The total conductance is

$$\frac{1}{R_{\min}} = 10\left(\frac{1}{10\,\Omega}\right) = 1\,\Omega^{-1}.$$

Therefore $R_{\min} = 1\,\Omega$. This uses all ten resistors; it does not replace any resistor with an ideal short circuit. Mixed series-parallel arrangements cannot exceed the all-series upper extreme or go below the all-parallel lower extreme.

Step 3: Form the dimensionless ratio The question asks for maximum divided by minimum, rather than their difference or the reverse ratio. Hence

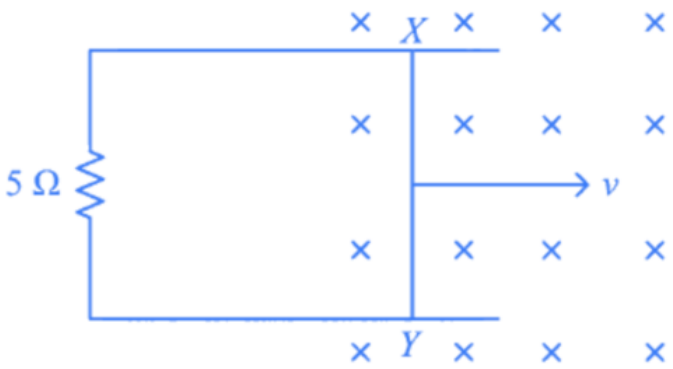
$$\frac{R_{\max}}{R_{\min}} = \frac{100\,\Omega}{1\,\Omega} = 100.$$

The unit cancels. More generally, the resistance of each identical resistor cancels in $(nR)/(R/n) = n^2$, giving the same result for $n = 10$.

Step 4: Conclusion The ratio of maximum to minimum equivalent resistance is $\boxed{100}$, a dimensionless quantity.

Quick Tip to Solve: For n identical resistors, the extreme configurations are all series and all parallel; their maximum-to-minimum ratio is always n squared.

53. A 1 m long metal rod XY completes the circuit as shown in figure. The plane of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the circuit is 5Ω, the force needed to move the rod in direction, as indicated, with a constant speed of 4 m/s will be _____ 10^{-3} N. [4 marks]



Solution:

Concept:

- A conducting rod of length l moving at speed v perpendicular to a uniform magnetic field B develops motional emf $\mathcal{E} = Blv$. In a closed circuit of resistance R, the induced current is $I = \mathcal{E}/R$.
- The magnetic force on the current-carrying rod has magnitude $F_B = BIl$. By Lenz's law it opposes the motion, so a constant speed requires an equal applied force in the direction of motion.

Step 1: Calculate the induced emf The diagram shows the rod moving right, with the magnetic field into the plane of the circuit. Thus the rod length, velocity and magnetic field are mutually perpendicular. Substitute $B = 0.15\,\text{T}$, $l = 1\,\text{m}$, and $v = 4\,\text{m s}^{-1}$:

$$\mathcal{E} = Blv = 0.15 \times 1 \times 4 = 0.60\,\text{V}.$$

The growing flux through the circuit induces a current opposing that increase.

Step 2: Find the current and resisting force The total circuit resistance is $5\,\Omega$, so Ohm's law gives

$$I = \frac{0.60}{5} = 0.12\,\text{A}.$$

The magnetic force on the rod is then

$$F_B = BIl = 0.15 \times 0.12 \times 1 = 0.018\,\text{N}.$$

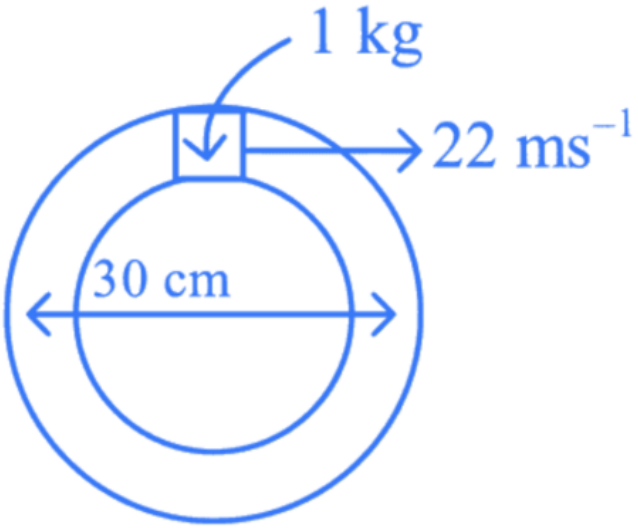
Its direction is leftwards, opposing the rod's rightward motion. Therefore the external agent must pull to the right with the same force.

Step 3: Check the energy balance and requested scale The mechanical power supplied is $Fv = 0.018 \times 4 = 0.072\,\text{W}$. The Joule heating is $I^2R = (0.12)^2 \times 5 = 0.072\,\text{W}$, confirming the result. At constant speed the kinetic energy does not change. Finally, $0.018\,\text{N} = 18 \times 10^{-3}\,\text{N}$, so the coefficient in the blank is 18.

Step 4: Conclusion The applied force required is $\boxed{18 \times 10^{-3}\,\text{N}}$ to the right. The numerical entry is $\boxed{18}$.

Quick Tip to Solve: Combine motional emf and magnetic force to obtain F equal to B squared l squared v divided by R, then convert to the scale printed beside the blank.

54. A closed circular tube of average radius 15 cm, whose inner walls are rough, is kept in vertical plane. A block of mass 1 kg just fit inside the tube. The speed of block is 22 m/s, when it is introduced at the top of tube. After completing five oscillations, the block stops at the bottom region of tube. The work done by the tube on the block is _____ J. (Given g = 10 m/s^2). [4 marks]



Solution:

Concept:

- The work-energy theorem gives $W_g + W_{\text{tube}} = K_f - K_i$. Work by gravity depends only on the initial and final heights, even when the path includes many complete turns or oscillations.
- The stationary tube exerts a normal reaction and friction. The normal reaction is perpendicular to instantaneous motion and does no work; friction removes mechanical energy. Work done by the tube on the block is therefore negative.

Step 1: Choose the initial and final states The block starts at the top with mass $m = 1\,\text{kg}$ and speed $u = 22\,\text{m}\,\text{s}^{-1}$. Its initial kinetic energy is

$$K_i = \tfrac{1}{2}mu^2 = \tfrac{1}{2} \times 1 \times 22^2 = 242\,\text{J}.$$

At the final state it is at rest at the bottom, so $K_f = 0$. The average radius is $r = 0.15\,\text{m}$, giving a net downward displacement of $2r = 0.30\,\text{m}$.

Step 2: Evaluate gravity and tube work Gravity does positive work because the final position is lower:

$$W_g = mg(2r) = 1 \times 10 \times 0.30 = 3\,\text{J}.$$

Applying the work-energy theorem to the whole journey gives

$$W_{\text{tube}} = K_f - K_i - W_g = 0 - 242 - 3 = -245\,\text{J}.$$

The stated number of oscillations is unnecessary because only the endpoints and initial speed determine the total mechanical-energy loss.

Step 3: Interpret the sign and numerical convention The negative sign is physically essential: the rough tube opposes motion and dissipates energy. The block initially has $242+3 = 245\,\text{J}$ of mechanical energy relative to the bottom, all of which is lost before it stops. Thus the magnitude of the dissipated energy is $245\,\text{J}$. A positive work of that size by the tube would increase the block's mechanical energy, contrary to the described stopping.

Step 4: Conclusion The tube does $\boxed{-245\,\text{J}}$ of work on the block; its magnitude is $245\,\text{J}$. The supplied key value 245 represents the magnitude of energy dissipated, whereas literal signed work is negative.

Quick Tip to Solve: For frictional motion over many cycles, compare initial and final mechanical energies. Distinguish negative frictional work from the positive magnitude of energy dissipated.

55. Unpolarised light of intensity 32 Wm^{-2} passes through the combination of three polaroids such that the pass axis of the last polaroid is perpendicular to that of the pass axis of first polaroid. If intensity of emerging light is 3 Wm^{-2}, then the angle between pass axis of first two polaroids is _____ °. [4 marks]

Solution:

Concept:

- An ideal first polaroid transmits half the intensity of unpolarised light. For already plane-polarised light, Malus's law gives $I_{\text{out}} = I_{\text{in}} \cos^2 \alpha$, where α is the angle between successive pass axes.
- If the first and third pass axes are perpendicular and the middle axis makes angle θ with the first, its angle with the third can be taken as $90^\circ - \theta$. The product of transmission factors is symmetric under $\theta \leftrightarrow 90^\circ - \theta$.

Step 1: Find the intensity after the first two polaroids The incident unpolarised intensity is $I_0 = 32\,\text{W m}^{-2}$. The first polaroid produces plane-polarised light of intensity

$$I_1 = I_0/2 = 16\,\text{W m}^{-2}.$$

If the first two pass axes are separated by θ, the second transmits $I_2 = 16\cos^2\theta$. The factor one-half is used only at the first polaroid; the light reaching the later polaroids is already polarised.

Step 2: Apply Malus law at the last polaroid The last axis is perpendicular to the first. Its relative

angle with the middle axis is $90^\circ - \theta$, so

$$I_3 = 16\cos^2\theta\cos^2(90^\circ - \theta) = 16\cos^2\theta\sin^2\theta.$$

Using $\sin 2\theta = 2\sin\theta\cos\theta$, this becomes $I_3 = 4\sin^2 2\theta$. The specified emerging intensity therefore gives $3 = 4\sin^2 2\theta$.

Step 3: Retain both permitted angles For the usual smaller angle between pass axes, $0 \le \theta \le 90^\circ$. Hence $0 \le 2\theta \le 180^\circ$, and $\sin^2 2\theta = 3/4$ gives $2\theta = 60^\circ$ or 120°. Dividing by two yields $\theta = 30^\circ$ or 60°. Both satisfy the intensity condition: the factors $3/4$ and $1/4$ simply exchange places.

Step 4: Conclusion The angle between the first two pass axes is $\boxed{30^\circ \text{ or } 60^\circ}$. Both numerical entries 30 and 60 are valid.

Quick Tip to Solve: With crossed first and last polaroids, the transmitted intensity is unchanged when the middle angle is replaced by its complement; check both angles.

56. Three concentric spherical metallic shells X, Y and Z of radius a, b and c respectively [a < b < c] have surface charge densities $\sigma, -\sigma$ and σ respectively. The shells X and Z are at same potential. If the radii of X & Y are 2 cm and 3 cm, respectively. The radius of shell Z is _____ cm. [4 marks]

Solution:

Concept:

- The potential due to a spherical shell of charge Q and radius R is kQ/R everywhere inside it and kQ/r outside it, where $k = 1/(4\pi\varepsilon_0)$. Potentials from different shells add algebraically.
- The given surface charge densities determine total shell charges through surface area: $Q_X = 4\pi a^2\sigma$, $Q_Y = -4\pi b^2\sigma$, and $Q_Z = 4\pi c^2\sigma$. Equal density magnitudes do not mean equal charge magnitudes.

Step 1: Write the potential of the inner shell At shell X, the contribution from X itself is kQ_X/a. Since this position lies inside Y and Z, their contributions are their constant interior potentials. Consequently

$$V_X = k\left(\frac{Q_X}{a} + \frac{Q_Y}{b} + \frac{Q_Z}{c}\right) = 4\pi k\sigma(a - b + c).$$

The negative sign of Y's charge is included explicitly. All potentials use the same zero at infinity.

Step 2: Write the potential of the outer shell At the surface of Z, all three contributions have radial denominator c. Therefore

$$V_Z = \frac{k}{c}(Q_X + Q_Y + Q_Z) = 4\pi k\sigma\frac{a^2 - b^2 + c^2}{c}.$$

Equate this to V_X, since X and Z are at the same potential. Cancel the common factor $4\pi k\sigma$ and multiply by c:

$$c(a - b + c) = a^2 - b^2 + c^2.$$

Step 3: Solve for the unknown radius The terms c^2 cancel on the two sides, leaving

$$c(a-b) = a^2 - b^2 = (a-b)(a+b).$$

Since $a < b$, the factor $a - b$ is nonzero and can be cancelled. This gives $c = a + b$. The radii may now be inserted in centimetres because the relation is homogeneous in length: $c = 2 + 3 = 5\,\text{cm}$. This also satisfies the required ordering $a < b < c$.

Step 4: Conclusion The radius of shell Z is $\boxed{5\,\text{cm}}$. The required numerical entry is $\boxed{5}$.

Quick Tip to Solve: Convert surface densities to total charges before calculating potentials. At an inner shell, use each outer shell radius in its constant interior potential.

57. If the earth suddenly shrinks to $\frac{1}{64}$th of its original volume with its mass remaining the same, the period of rotation of earth becomes $\frac{24}{x}$h. The value of x is _____. [4 marks]

Solution:

Concept:

- With no external torque, the angular momentum of Earth remains constant: $I\omega = I\,2\pi/T$. Here I is its moment of inertia and T its rotation period.
- For a spherical body whose mass and relative mass distribution remain unchanged during uniform contraction, $I = kMR^2$; the same dimensionless factor k applies before and after shrinking.

Step 1: Convert the volume ratio into a radius ratio Volume is proportional to the cube of the radius, rather than to the radius itself. Thus the given contraction gives

$$\frac{V_f}{V_i} = \left(\frac{R_f}{R_i}\right)^3 = \frac{1}{64}, \qquad \frac{R_f}{R_i} = \frac{1}{4}.$$

The mass remains fixed, so the moment of inertia becomes $I_f/I_i = (1/4)^2 = 1/16$. Taking the moment of inertia proportional to volume would give an incorrect period.

Step 2: Use conservation to obtain the new period Equating the initial and final angular momenta gives

$$\frac{I_i}{T_i} = \frac{I_f}{T_f} \quad \Longrightarrow \quad T_f = \frac{I_f}{I_i}T_i = \frac{24}{16}\,\text{h} = 1.5\,\text{h}.$$

The smaller moment of inertia makes the angular speed sixteen times larger. Since period is inversely proportional to angular speed, the period is divided by sixteen. Comparing with the specified form $T_f = 24/x\,\text{h}$ determines the required dimensionless number.

Step 3: Conclusion The new rotation period is $1.5\,\text{h} = 24/16\,\text{h}$. Therefore $\boxed{x = 16}$.

Quick Tip to Solve: First take the cube root of a volume ratio, then square the radius ratio for the moment of inertia.

58. The current required to be passed through a solenoid of 15 cm length and 60 turns in order of demagnetise a bar magnet of magnetic intensity $2.4 \times 10^3\ Am^{-1}$ is _____ A. [4 marks]

Solution:

Concept:

- For a long solenoid with N turns in length l, its magnetic field intensity is $H = nI = NI/l$. It has units $\mathrm{A\,m^{-1}}$.
- To demagnetise the magnet, apply an opposing field whose intensity equals the stated demagnetising requirement. The current magnitude follows from $I = Hl/N$, without a factor of μ_0.

Step 1: Express the solenoid data in SI units The length is $l = 15\,\mathrm{cm} = 0.15\,\mathrm{m}$, while the total number of turns is $N = 60$. Hence the number of turns per unit length is

$$n = \frac{60}{0.15} = 400\,\mathrm{m^{-1}}.$$

The specified magnetic intensity is $H = 2.4 \times 10^3\,\mathrm{A\,m^{-1}}$. It is an intensity, not a magnetic flux density measured in tesla, so the relation needed here is directly $H = nI$.

Step 2: Find the opposing current magnitude Rearranging the solenoid relation and substituting gives

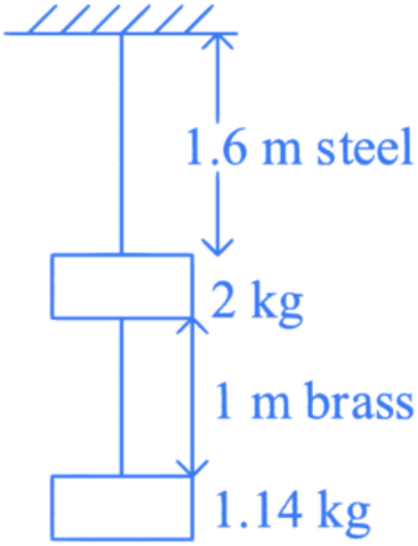
$$I = \frac{H}{n} = \frac{2.4 \times 10^3}{400} = 6\,\mathrm{A}.$$

Equivalently, $I = (2.4 \times 10^3)(0.15)/60$. The current direction must make the solenoid field oppose the magnetisation; reversing that direction would reinforce the magnet instead. As a numerical check, a current of $6\,\mathrm{A}$ in a winding with 400 turns per metre produces $2400\,\mathrm{A\,m^{-1}}$, exactly the stated intensity.

Step 3: Conclusion The required current magnitude is $\boxed{6\,\mathrm{A}}$, directed to produce the opposing magnetic field.

Quick Tip to Solve: Distinguish field intensity H from flux density B: H = nI, whereas B includes the permeability.

59. Two wires each of radius 0.2 cm and negligible mass, one made of steel and the other made of brass are loaded as shown in the figure. The elongation of the steel wire is _____ $\times 10^{-6}$ m. [Young's modulus for steel = 2×10^{11} Nm^{-2} and g = 10 ms^{-2}] [4 marks]



Solution:

Concept:

- The extension of a uniform elastic wire is $\Delta L = FL/(AY)$, where F is tension, L its original length, A its cross-sectional area and Y Young's modulus.
- The upper steel wire supports both suspended masses. The brass wire supports only the lower mass; its own extension does not change the static load carried by the steel wire.

Step 1: Identify the load and area of the steel wire The diagram shows a $2\,\text{kg}$ mass at the steel-brass junction and a $1.14\,\text{kg}$ mass below the brass wire. Since both wires have negligible mass, equilibrium gives the tension in steel as

$$F = (2 + 1.14)g = 3.14(10) = 31.4\,\text{N}.$$

The steel length is $1.6\,\text{m}$. Its radius is $0.2\,\text{cm} = 2 \times 10^{-3}\,\text{m}$, so

$$A = \pi r^2 = 4\pi \times 10^{-6}\,\text{m}^2.$$

The supplied dimension is a radius, so it must not be halved.

Step 2: Calculate the elastic extension Using $Y = 2 \times 10^{11}\,\text{N m}^{-2}$,

$$\Delta L = \frac{31.4(1.6)}{(4\pi \times 10^{-6})(2 \times 10^{11})} = \frac{6.28}{\pi} \times 10^{-5}\,\text{m}.$$

With $\pi \simeq 3.14$, this is $2.0 \times 10^{-5}\,\text{m} = 20 \times 10^{-6}\,\text{m}$. The brass modulus is unnecessary because the requested quantity is the extension of steel alone. Using only the junction mass would underestimate the steel tension.

Step 3: Conclusion The steel wire elongates by $\boxed{20 \times 10^{-6}\,\text{m}}$. The number to enter is $\boxed{20}$.

Quick Tip to Solve: For an upper wire in a hanging series arrangement, include every mass suspended below that wire.

60. The decay constant for a radioactive nuclide is 1.5 $\times$ 10^{-5} s^{-1}. Atomic weight of the substance is 60 g mole^{-1}, ($N_A = 6 \times 10^{23}$). The activity of 1.0 μg of the substance is _____ $\times$ 10^{10} Bq. [4 marks]

Solution:

Concept:

- The activity is $A = \lambda N$, where λ is the decay constant and N the number of undecayed nuclei. One becquerel is one decay per second.
- For a pure sample of mass m and molar mass M, the number of nuclei is $N = (m/M)N_A$. Mass and molar mass must use the same mass unit.

Step 1: Convert the sample mass and count the nuclei The given sample has mass $1.0\,\mu\text{g} = 1.0 \times 10^{-6}\,\text{g}$. It is not $10^{-3}\,\text{g}$, which would be one milligram. With the molar mass $60\,\text{g}\,\text{mol}^{-1}$, the amount and number of nuclei are

$$n = \frac{10^{-6}}{60}\,\text{mol}, \qquad N = \frac{10^{-6}}{60}(6 \times 10^{23}) = 10^{16}.$$

The factor of sixty in the denominator cancels the six in Avogadro's number with an additional factor of ten.

Step 2: Apply the decay law and express the requested coefficient Multiply the number of nuclei by the given decay constant:

$$A = (1.5 \times 10^{-5}\,\text{s}^{-1})(10^{16}) = 1.5 \times 10^{11}\,\text{s}^{-1}.$$

Thus the sample undergoes 1.5×10^{11} decays each second. The answer blank multiplies $10^{10}\,\text{Bq}$, so write the same result as $15 \times 10^{10}\,\text{Bq}$. A coefficient of 1.5 would be appropriate only if the printed power were 10^{11}, rather than the actual 10^{10}.

Step 3: Conclusion The activity is $\boxed{15 \times 10^{10}\,\text{Bq}}$, so the required numerical entry is $\boxed{15}$.

Quick Tip to Solve: After calculating the activity, rewrite it using the power of ten printed beside the answer blank.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The compound which does not exist is [4 marks]

(A) $(NH_4)_2BeF_4$

(B) $PbEt_4$

(C) BeH_2

(D) NaO_2

Correct Answer: (D) NaO_2

Solution:

Concept:

- The stability of alkali-metal oxides, peroxides and superoxides depends on the sizes of their cations and anions. Larger alkali cations stabilise the relatively large superoxide ion more effectively.
- Beryllium forms covalent hydride and tetrafluoroberyllate complexes; lead forms organometallic compounds such as tetraethyllead. These provide checks on the other proposed compounds.

Step 1: Check the beryllium and lead compounds In $(NH_4)_2BeF_4$, two NH_4^+ ions balance the tetrafluoroberyllate anion $[BeF_4]^{2-}$. Beryllium can attain a four-coordinate environment through bonding to fluoride ligands, so option (A) is a recognised complex salt. Option (B), $PbEt_4$, is tetraethyllead, where Et denotes an ethyl group C_2H_5; it is a known organolead compound. Option (C), BeH_2, is also known and has an electron-deficient polymeric structure with bridging hydrogens. Electron deficiency alone therefore does not make the hydride nonexistent.

Step 2: Apply the textbook oxide stability trend In the usual NCERT treatment, sodium forms the peroxide Na_2O_2, while the heavier alkali metals potassium, rubidium and caesium readily form superoxides MO_2. The relatively small sodium cation does not provide the usual size match for stabilising the large superoxide ion in this standard comparison. Thus NaO_2 is the compound excluded by the conventional alkali-metal oxide trend tested here. The other three options are not excluded: each has a standard recognised bonding description or established compound identity.

Step 3: Conclusion Sodium superoxide is known under specialised conditions, so the literal phrase "does not exist" is too broad. Using the standard alkali-metal oxide stability trend intended by this question, the required compound is $\boxed{NaO_2}$. Hence, the correct option is (D).

Quick Tip to Solve: In the standard alkali-metal trend, remember lithium oxide, sodium peroxide, and heavier-metal superoxides.

62. The number of molecules and moles in 2.8375 litres of O_2 at STP are respectively [4 marks]

(A) 1.505×10^{23} and 0.250 mol

(B) 7.527×10^{22} and 0.250 mol

(C) 7.527×10^{23} and 0.125 mol

(D) 7.527×10^{22} and 0.125 mol

Correct Answer: (D) 7.527×10^{22} and 0.125 mol

Solution:

Concept:

- For the STP convention used here, the standard pressure is $1\,\text{bar}$ and temperature is $273.15\,\text{K}$; the ideal-gas molar volume is approximately $22.7\,\text{L mol}^{-1}$.
- The amount of gas is $n = V/V_m$, and the number of molecules is $N = nN_A$. Molecular oxygen remains O_2; counting atoms instead would incorrectly double the molecular count.

Step 1: Find the amount of oxygen gas The stated volume is $V = 2.8375\,\text{L}$. Using the standard-state molar volume of $22.7\,\text{L mol}^{-1}$,

$$n = \frac{2.8375}{22.7} = 0.125\,\text{mol}.$$

This is one eighth of a mole. The value $0.250\,\text{mol}$ is twice as large, so both options (A) and (B) fail the volume-to-moles calculation. Using the older one-atmosphere molar volume of $22.4\,\text{L mol}^{-1}$ would not reproduce the deliberately selected volume in this question.

Step 2: Convert moles into molecules and compare the remaining options With $N_A = 6.022 \times 10^{23}\,\text{mol}^{-1}$,

$$N = (0.125)(6.022 \times 10^{23}) = 7.5275 \times 10^{22}.$$

To the precision of the printed options, this is 7.527×10^{22} molecules. Option (C) has the correct amount of gas but a molecular count ten times too large; it incorrectly uses the exponent 23. Option (D) is the only pair with both the appropriate molecular count and $0.125\,\text{mol}$, in the order requested.

Step 3: Conclusion The sample contains approximately $\boxed{7.527 \times 10^{22}}$ molecules and $\boxed{0.125\,\text{mol}}$. Hence, the correct option is (D).

Quick Tip to Solve: Check both entries of a paired option: calculate moles first, then multiply by Avogadro's constant.

63. The one that does not stabilize 2° and 3° structures of proteins is [4 marks]

(A) $-O-O-$linkage

(B) $-S-S-$linkage

(C) van der Waals forces

(D) H-bonding

Correct Answer: (A) $-O-O-$linkage

Solution:

Concept:

- Secondary structure is principally stabilised by hydrogen bonds between peptide backbone groups. Tertiary structure also depends on hydrogen bonding, van der Waals contacts, hydrophobic interactions and, where present, disulfide bridges.
- A disulfide linkage is $-S-S-$, formed between cysteine residues. A peroxide linkage $-O-O-$ is chemically different and is not a normal structural cross-link of proteins.

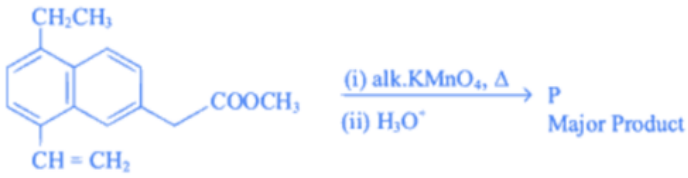
Step 1: Identify the recognised protein stabilising interactions Backbone carbonyl oxygen and amide hydrogen groups form hydrogen bonds that help maintain alpha helices and beta sheets. This makes option (D) a genuine stabilising interaction, especially for secondary structure. Van der Waals forces arise between closely packed atoms and side chains, contributing to the stability of a folded protein, so option (C) is also recognised. Covalent $-S-S-$ bridges connect cysteine side chains and can lock parts of a polypeptide into a stable three-dimensional arrangement. Thus option (B) contributes particularly to tertiary structure; it is not the requested excluded linkage.

Step 2: Distinguish peroxide from disulfide The $-O-O-$ group in option (A) is a peroxide linkage. It must not be confused with the sulfur-sulfur bond formed from cysteine thiol groups. Peroxide bonds are not among the normal interactions listed for maintaining the secondary and tertiary conformations of proteins. The question asks for the interaction absent from this stabilisation scheme, rather than an interaction that must operate identically at both structural levels. The hydrogen-bond, disulfide and van der Waals options all belong to the recognised stabilising set, leaving the oxygen-oxygen linkage as the exception.

Step 3: Conclusion The linkage not responsible for the normal stabilisation of these protein structures is $\boxed{-O-O-}$. Hence, the correct option is (A).

Quick Tip to Solve: Associate cysteine cross-links with sulfur-sulfur bonds; an oxygen-oxygen bond is a peroxide, not a disulfide.

64. The major product 'P' formed in the given reaction is [4 marks]



(A) Naphthalene bearing COOH at all three original side-chain attachment sites.

(B) Naphthalene bearing upper-left CH_2COOH, lower-left $CH(OH)CH_2OH$, and right CH_2COOH.

(C) Naphthalene bearing upper-left and lower-left COOH, and right CH_2COOH.

(D) Naphthalene bearing upper-left and right CH_2COOH, and lower-left $COCH_3$.

Correct Answer: (A) Naphthalene bearing COOH at all three original side-chain attachment sites.

Solution:

Concept:

- Hot alkaline $KMnO_4$, followed by acidification, converts an aromatic side chain having a benzylic hydrogen into COOH.
- The carbon directly bonded to the aromatic ring is retained as the carboxyl carbon; additional side-chain carbons are removed during exhaustive oxidation.

Step 1: Inspect each attachment site The upper-left ethyl group is $Ar-CH_2CH_3$, the lower-left vinyl group is $Ar-CH=CH_2$, and the right-hand ester-containing chain is $Ar-CH_2COOCH_3$. Each has at least one hydrogen on the carbon directly attached to the aromatic system. Thus all three sites are susceptible to vigorous permanganate oxidation.

Step 2: Apply exhaustive oxidation The ethyl side chain is shortened to $Ar-COOH$. Oxidative cleavage of the vinyl substituent also gives $Ar-COOH$; this is not the cold, dilute permanganate condition that stops at a vicinal diol. At the right-hand site the benzylic carbon is oxidized to the carboxyl level, so the product is directly attached COOH, rather than retained CH_2COOH. Acidification converts carboxylate salts into their free acids.

Step 3: Compare the products Option (A) has carboxyl groups at all three original attachment sites and satisfies these transformations. Option (B) incorrectly retains extended side chains and a glycol. Option (C) retains a CH_2COOH group at the right-hand site. Option (D) retains extended chains and a ketone. Neither those chains nor that partial oxidation pattern represents the stated vigorous conditions.

Step 4: Conclusion The major product is the naphthalene tricarboxylic acid shown in (A). Hence, the correct option is (A).

Quick Tip to Solve: For hot alkaline permanganate, first circle each benzylic hydrogen; do not confuse exhaustive side-chain oxidation with cold alkene dihydroxylation.

65. Prolonged heating is avoided during the preparation of ferrous ammonium sulphate to : [4 marks]

(A) prevent hydrolysis

(B) prevent breaking

(C) prevent oxidation

(D) prevent reduction

Correct Answer: (C) prevent oxidation

Solution:

Concept:

- Mohr's salt is ferrous ammonium sulphate, $(NH_4)_2Fe(SO_4)_2 \cdot 6\,H_2O$; its iron must remain in the $+2$ oxidation state.
- Ferrous ions can be oxidized by atmospheric oxygen to ferric ions, particularly when a solution is heated for an unnecessarily long time.

Step 1: Identify the ion that must be preserved The preparation combines ferrous sulphate and ammonium sulphate in the required ratio, followed by crystallization of their double salt. The product is useful as a source of Fe^{2+}, so conversion of this ion into Fe^{3+} contaminates the preparation and changes its composition. Gentle heating helps dissolve the salts, but prolonged exposure of the hot solution to air is undesirable.

Step 2: Relate heating to oxidation The relevant unwanted process is oxidation of iron, not reduction. In acidic solution it may be represented by $4\,Fe^{2+} + O_2 + 4\,H^+ \longrightarrow 4\,Fe^{3+} + 2\,H_2O$. Avoiding prolonged heating limits this oxidation while allowing the solution to be concentrated sufficiently for crystallization. The added dilute sulphuric acid also helps maintain suitable acidic conditions.

Step 3: Eliminate the remaining choices Hydrolysis is controlled principally by the acidic medium, rather than being the reason specified for avoiding prolonged heating. "Breaking" does not identify the relevant chemical change of the ferrous ion. Reduction would lower the oxidation state, whereas the problem during hot exposure to air is its increase from $+2$ to $+3$.

Step 4: Conclusion Prolonged heating is avoided to prevent oxidation of Fe^{2+} into Fe^{3+}. Hence, the correct option is (C).

Quick Tip to Solve: In a Mohr's salt preparation, distinguish the roles: acid suppresses hydrolysis, and limited heating helps preserve ferrous iron.

66. Which of the following statements are correct?

(A) The M^{3+}/M^{2+} reduction potential for iron is greater than manganese.

(B) The higher oxidation states of first row d-block elements get stabilized by oxide ion.

(C) Aqueous solution of Cr^{2+} can liberate hydrogen from dilute acid.

(D) Magnetic moment of V^{2+} is observed between 4.4 - 5.2 BM.

Choose the correct answer from the options given below : [4 marks]

(A) (B), (C) only

(B) (A), (B) only

(C) (A), (B), (D) only

(D) (C), (D) only

Correct Answer: (A) (B), (C) only

Solution:

Concept:

- Reduction potentials reflect the relative stability of oxidation states, including the extra stability of half-filled d^5 configurations.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}$ BM, where n is the number of unpaired electrons.

Step 1: Test statements A and B The Mn^{3+}/Mn^{2+} potential is about +1.51 V, whereas Fe^{3+}/Fe^{2+} is about +0.77 V. Manganese therefore has the larger reduction potential; the stability of Mn^{2+} with its half-filled d^5 arrangement contributes to this. Statement (A) is false. Oxygen stabilizes high oxidation states through strong metal–oxygen bonding, as illustrated by chromate and permanganate. Statement (B) is true.

Step 2: Test statement C Chromium(II) is a strong reducing agent and readily becomes chromium(III). The standard reduction potential of Cr^{3+}/Cr^{2+} is negative, approximately -0.41 V. Coupling its reverse with hydrogen-ion reduction gives a positive cell potential. Thus $2\,Cr^{2+} + 2\,H^{+} \longrightarrow 2\,Cr^{3+} + H_2$ is feasible, and statement (C) is true.

Step 3: Test statement D and select the combination Vanadium has atomic number 23, so V^{2+} is $3d^3$, with three unpaired electrons. Its spin-only moment is $\sqrt{3(3+2)} = \sqrt{15} \approx 3.87$ BM, not the quoted range 4.4–5.2 BM. Statement (D) is false. Only (B) and (C) survive; all other offered combinations include false statement (A) or (D).

Step 4: Conclusion The oxide-stabilization and chromium(II) statements alone are correct. Hence, the correct option is (A).

Quick Tip to Solve: For transition-metal statement questions, determine the d-electron count before evaluating a magnetic-moment range.

67. The octahedral diamagnetic low spin complex among the following is : [4 marks]

(A) $[Co(NH_3)_6]^{3+}$

(B) $[NiCl_4]^{2-}$

(C) $[CoCl_6]^{3-}$

(D) $[CoF_6]^{3-}$

Correct Answer: (A) $[Co(NH_3)_6]^{3+}$

Solution:

Concept:

- An octahedral d^6 complex is low spin when Δ_o exceeds the pairing energy, producing $t_{2g}^6 e_g^0$.
- Diamagnetism requires all electrons to be paired; coordination number and ligand strength must both be checked.

Step 1: Determine the cobalt configuration In $[Co(NH_3)_6]^{3+}$, ammonia is neutral, so cobalt is $+3$. Neutral cobalt is $[Ar]3d^7 4s^2$, and Co^{3+} is d^6. The six donor atoms give an octahedral complex. The relatively large splitting for cobalt(III) with ammonia favours electron pairing in the lower-energy orbitals.

Step 2: Fill the octahedral levels The low-spin arrangement is $t_{2g}^6 e_g^0$: each of the three lower-energy orbitals contains a pair and the two higher-energy orbitals are empty. Hence there are no unpaired electrons, its spin-only moment is zero, and the complex is diamagnetic. It meets all three stated requirements: octahedral geometry, a low-spin arrangement, and diamagnetism.

Step 3: Reject the other complexes In $[NiCl_4]^{2-}$, nickel is $+2$ and d^8; the chloride complex is tetrahedral and paramagnetic, so it already fails the geometry requirement. Both $[CoCl_6]^{3-}$ and $[CoF_6]^{3-}$ contain cobalt(III), but chloride and fluoride are weak-field ligands. Their octahedral d^6 arrangements are high spin, $t_{2g}^4 e_g^2$, with four unpaired electrons rather than none.

Step 4: Conclusion The octahedral, low-spin, diamagnetic complex is $[Co(NH_3)_6]^{3+}$. Hence, the correct option is (A).

Quick Tip to Solve: Do not choose a complex from oxidation state alone: check coordination number, ligand strength, and the resulting unpaired-electron count.

68. Using column chromatography, mixture of two compounds 'A' and 'B' was separated. 'A' eluted first, this indicates 'B' has [4 marks]

(A) high R_f, weaker adsorption

(B) high R_f, stronger adsorption

(C) low R_f, weaker adsorption

(D) low R_f, stronger adsorption

Correct Answer: (D) low R_f, stronger adsorption

Solution:

Concept:

- In adsorption column chromatography, a compound more strongly retained on the stationary phase travels more slowly and elutes later.
- For a comparable thin-layer system, R_f is the solute travel distance divided by the solvent-front travel distance; stronger adsorption lowers it.

Step 1: Interpret the elution order Both compounds experience a competition between adsorption on the stationary phase and transport by the mobile phase. Since A emerges first under the same column conditions, it spends a smaller fraction of its time adsorbed. B remains on the stationary phase longer and therefore has stronger adsorption than A. The conclusion concerns the relative affinities in the specified separation system.

Step 2: Translate retention into an Rf comparison With the same type of adsorbent and a comparable solvent system, strongly held B travels a shorter distance while the solvent front moves forward. Its retardation factor $R_f = d_{\text{solute}}/d_{\text{solvent front}}$ is consequently lower. The low value and late elution are two manifestations of the same stronger stationary-phase retention.

Step 3: Check both parts of every option Option (A) describes weak retention and high mobility, the opposite of B. Option (B) combines a high R_f with strong adsorption, which is inconsistent for the comparison being made. Option (C) has the correct low R_f but the wrong adsorption strength. Only option (D) joins low R_f to stronger adsorption and explains why B comes out after A.

Step 4: Conclusion Compound B has a low R_f and stronger adsorption. Hence, the correct option is (D).

Quick Tip to Solve: Link the three observations directly: stronger adsorption means slower column elution and a smaller Rf in a comparable planar system.

69. Given the thermochemical equations

$$2\,\mathrm{CO\,(g)} + \mathrm{O_2(g)} \longrightarrow 2\,\mathrm{CO_2(g)}, \quad \Delta H_1^\circ = -x\,\mathrm{kJ\,mol^{-1}},$$
$$\mathrm{C(graphite)} + \mathrm{O_2(g)} \longrightarrow \mathrm{CO_2(g)}, \quad \Delta H_2^\circ = -y\,\mathrm{kJ\,mol^{-1}}.$$

The standard enthalpy change for $\mathrm{C(graphite)} + \frac{1}{2}\,\mathrm{O_2(g)} \longrightarrow \mathrm{CO\,(g)}$ **is:** [4 marks]

(A) $\frac{x-2y}{2}$

(B) $\frac{x+2y}{2}$

(C) $2y - x$

(D) $\frac{2x-y}{2}$

Correct Answer: (A) $\frac{x-2y}{2}$

Solution:

Concept:

- Enthalpy is a state function, so thermochemical equations may be added algebraically to obtain a target reaction.
- Multiplying an equation multiplies its enthalpy; reversing an equation changes the sign of its enthalpy.

Step 1: Normalize carbon monoxide combustion The first equation burns two moles of carbon monoxide. Divide both the equation and its enthalpy by two:

$$\mathrm{CO\,(g)} + \frac{1}{2}\,\mathrm{O_2(g)} \longrightarrow \mathrm{CO_2(g)}, \qquad \Delta H = -\frac{x}{2}\,\mathrm{kJ\,mol^{-1}}.$$

The factor one-half is essential because the target reaction produces only one mole of CO.

Step 2: Reverse and add the equations Reverse the normalized equation to place CO on the product side:

$$\mathrm{CO_2(g)} \longrightarrow \mathrm{CO\,(g)} + \frac{1}{2}\,\mathrm{O_2(g)}, \qquad \Delta H = +\frac{x}{2}\,\mathrm{kJ\,mol^{-1}}.$$

Add it to $\mathrm{C(graphite)} + \mathrm{O_2(g)} \longrightarrow \mathrm{CO_2(g)}$, whose enthalpy is $-y$. Carbon dioxide cancels and half a mole of oxygen cancels, leaving exactly $\mathrm{C(graphite)} + \frac{1}{2}\,\mathrm{O_2(g)} \longrightarrow \mathrm{CO\,(g)}$.

Step 3: Simplify and compare The enthalpies add to

$$\Delta H^\circ = -y + \frac{x}{2} = \frac{x-2y}{2}\,\mathrm{kJ\,mol^{-1}}.$$

The sum $(x + 2y)/2$ gives the wrong sign to the graphite-combustion contribution. The expression $2y - x$ has the wrong overall sign and scale, while $(2x - y)/2$ mishandles the stoichiometric factors. Thus only the first expression follows the balanced target reaction.

Step 4: Conclusion $\Delta H^\circ = \frac{x-2y}{2}\,\mathrm{kJ\,mol^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the target reaction first, then cancel species explicitly; reverse a thermochemical equation only together with its enthalpy sign.

70. Isomeric amines with molecular formula $C_8H_{11}N$ give the following tests

Isomer (P) $\Rightarrow$ Can be prepared by Gabriel phthalimide synthesis

Isomer (Q) $\Rightarrow$ Reacts with Hinsberg's reagent to give solid insoluble in NaOH

Isomer (R) $\Rightarrow$ Reacts with HONO followed by β-naphthol in NaOH to give red dye.

Isomers (P), (Q) and (R) respectively are [4 marks]

(A) P: 2-ethylaniline; Q: $C_6H_5CH_2CH_2NH_2$; R: $C_6H_5NHCH_2CH_3$.

(B) P: $C_6H_5CH_2NHCH_3$; Q: $C_6H_5CH_2CH_2NH_2$; R: 2-methylaniline.

(C) P: $C_6H_5CH_2CH_2NH_2$; Q: 2-$CH_3C_6H_4CH_2NHCH_3$; R: 4-ethylaniline.

(D) P: $C_6H_5CH_2CH_2NH_2$; Q: $C_6H_5CH_2N(CH_3)_2$; R: 2-ethylaniline.

Correct Answer: (C) P: $C_6H_5CH_2CH_2NH_2$; Q: 2-$CH_3C_6H_4CH_2NHCH_3$; R: 4-ethylaniline.

Solution:

Concept:

- Gabriel synthesis gives primary aliphatic amines; an aromatic ring may be present provided nitrogen is attached to an alkyl carbon.
- A secondary amine gives a Hinsberg sulphonamide without an N–H bond; this product is insoluble in aqueous alkali.
- A primary aromatic amine forms an arenediazonium salt, which couples with alkaline beta-naphthol to form an azo dye.

Step 1: Identify P from Gabriel synthesis P must have an alkyl group attached to NH_2. The $C_6H_5CH_2CH_2NH_2$ structure qualifies: its nitrogen is attached to a saturated side-chain carbon, not directly to the ring. An aryl amine cannot be obtained by the usual Gabriel substitution, and a secondary amine is not its product. This excludes the P assignments in options (A) and (B).

Step 2: Identify Q from the Hinsberg test A secondary amine R_2NH reacts with benzenesulphonyl chloride to give $C_6H_5SO_2NR_2$. The product has no acidic N–H proton and does not form a soluble salt with NaOH. A primary amine instead gives an alkali-soluble sulphonamide; a tertiary amine has no N–H and does not give that sulphonamide. Thus the secondary Q in (C) matches, whereas tertiary Q in (D) does not.

Step 3: Identify R and inspect the printed formula The ring-bound primary amino group of 4-ethylaniline gives a diazonium salt with cold nitrous acid and subsequently an azo dye with beta-naphthol. Therefore the functional classes P, Q, R in (C) satisfy all three tests. However, the drawn Q contains a ring methyl, a benzylic carbon, and an N-methyl group: its formula is $C_9H_{13}N$, not the stated $C_8H_{11}N$. The printed structures therefore contain a formula inconsistency; it must not be hidden by deleting a carbon.

Step 4: Conclusion Option (C) is the combination selected by the three diagnostic tests. The drawn Q has the formula inconsistency noted above, so the structures are not literally all isomers of $C_8H_{11}N$. Hence, the correct option is (C).

Quick Tip to Solve: Apply Gabriel, Hinsberg, and diazotization tests in that order, and independently count carbon atoms when a common molecular formula is stated.

71. Given below are two statements:

Statement I : Aqueous solution of $K_2Cr_2O_7$ is preferred as a primary standard in volumetric analysis over $Na_2Cr_2O_7$ aqueous solution.

Statement II : $K_2Cr_2O_7$ has a higher solubility in water than $Na_2Cr_2O_7$.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are false

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are true

Correct Answer: (A) Statement I is true but Statement II is false

Solution:

Concept:

- A primary standard must be available in a pure, stable composition so that weighing a known mass establishes a reliable amount of substance.
- Potassium dichromate, $K_2Cr_2O_7$, is a stable, non-hygroscopic crystalline oxidising agent. Sodium dichromate, $Na_2Cr_2O_7$, is more soluble and hygroscopic.

Step 1: Evaluate the primary-standard preference Potassium dichromate can be obtained in pure form and does not readily absorb moisture from air. Its weighed mass therefore represents a reproducible amount of dichromate. Sodium dichromate absorbs moisture, so the mass measured may include a variable amount of water. This makes direct preparation of a solution of accurately known concentration less convenient. Thus Statement I is true; its truth is about reliable composition, rather than the largest possible solubility.

Step 2: Compare the solubilities The actual order is that sodium dichromate is more soluble in water than potassium dichromate. Potassium dichromate is sufficiently soluble to prepare a standard solution, but it does not have the higher solubility claimed in Statement II. Hence Statement II is false. High solubility alone would not establish suitability as a primary standard, because stability and the ability to weigh a definite composition are also essential.

Step 3: Select the matching statement combination The required combination is true for I and false for II. Option (B) wrongly rejects I, option (C) reverses both truth values, and option (D) incorrectly accepts the solubility claim. Only option (A) matches both independent checks.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: For a primary standard, check purity and resistance to moisture before comparing solubilities.

72. Suitable reaction condition for preparation of Methyl phenyl ether is [4 marks]

(A) Benzene, MeBr

(B) $PhO^{\ominus}Na^{\oplus}$, MeBr

(C) Ph - Br, $MeO^{\ominus}Na^{\oplus}$

(D) $PhO^{\ominus}Na^{\oplus}$, MeOH

Correct Answer: (B) $PhO^{\ominus}Na^{\oplus}$, MeBr

Solution:

Concept:

- Williamson synthesis joins an alkoxide or phenoxide nucleophile to an alkyl halide by an S_N2 reaction.
- For an aryl alkyl ether, retain the aryl–oxygen bond in phenoxide and form the oxygen–alkyl bond using a primary or methyl halide.

Step 1: Choose the bond to form Methyl phenyl ether is anisole, $C_6H_5OCH_3$. Its oxygen is attached to a phenyl group on one side and a methyl group on the other. Choose sodium phenoxide as the oxygen nucleophile and methyl bromide as the electrophile. The methyl carbon is unhindered and its bromide group can leave during backside attack; this is a particularly suitable substrate for Williamson synthesis.

Step 2: Write the substitution The phenoxide ion attacks methyl bromide and forms the ether while bromide leaves.

$$C_6H_5ONa + CH_3Br \longrightarrow C_6H_5OCH_3 + NaBr$$

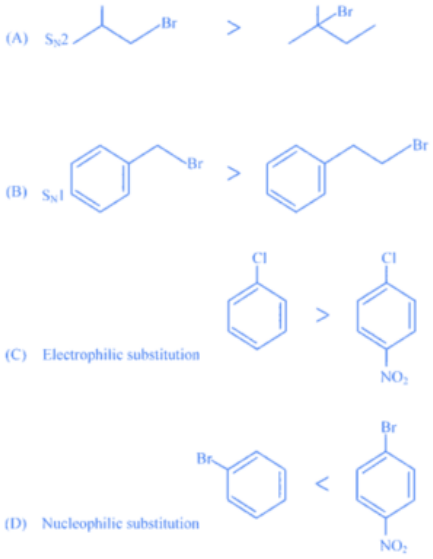
The reaction forms the required oxygen–methyl bond directly. Sodium is the counter-ion and does not become part of the ether molecule. The phenyl–oxygen linkage already present in phenoxide remains intact.

Step 3: Reject the alternative reagent pairs Benzene with methyl bromide supplies no oxygen atom and cannot directly give anisole. Bromobenzene with sodium methoxide would require ordinary substitution at an unactivated aryl carbon: its carbon–bromine bond has partial double-bond character and does not undergo the normal Williamson mechanism. Phenoxide with methanol lacks a suitable leaving group on the methyl carbon; hydroxide would be a poor leaving group under these basic conditions. Thus only the phenoxide–methyl bromide pair gives the intended straightforward preparation.

Step 4: Conclusion Sodium phenoxide and methyl bromide prepare methyl phenyl ether. Hence, the correct option is (B).

Quick Tip to Solve: For anisole, use phenoxide plus a methyl halide; avoid trying ordinary substitution on bromobenzene.

73. Identify the correct order of reactivity for the following pairs towards the respective mechanism Choose the correct answer from the options given below: [4 marks]



(A) (A), (B) and (D) only

(B) (A), (B), (C) and (D)

(C) (B), (C) and (D) only

(D) (A), (C) and (D) only

Correct Answer: (B) (A), (B), (C) and (D)

Solution:

Concept:

- S_N2 substitution is favoured by an unhindered electrophilic carbon, whereas S_N1 substitution is favoured by a stable carbocation.
- An electron-withdrawing nitro group deactivates a ring towards electrophilic substitution but activates an ortho- or para-haloarene towards addition–elimination nucleophilic substitution.

Step 1: Check the two aliphatic substitution comparisons In pair (A), the left bromide has a primary carbon bearing bromine, whereas the right bromide is tertiary at that carbon. Backside attack in an S_N2 process is much easier at the primary centre, so the displayed left-greater-than-right order is correct. In pair (B), benzyl bromide forms a resonance-stabilised benzyl carbocation. The other compound has bromine on a side-chain carbon separated from the ring by an additional carbon; simple ionisation does not directly produce the benzylic cation. Therefore the shown S_N1 order is also correct.

Step 2: Check the aromatic substitution comparisons In pair (C), chlorobenzene reacts more readily in electrophilic substitution than para-nitrochlorobenzene. The additional nitro group withdraws electron density and makes attack by an electrophile less favourable, so (C) is correct. In pair (D), para-nitrobromobenzene reacts more readily than bromobenzene in nucleophilic aromatic substitution. The para nitro group stabilises the negatively charged addition intermediate by resonance, so the

displayed less-than sign is correct.

Step 3: Combine the four verdicts All four comparisons, (A), (B), (C) and (D), agree with their stated mechanisms. The same substituent can have opposite effects in different mechanisms, so the nitro-group comparisons are not contradictory. Options (A), (C) and (D) each omit one correct comparison; the option listing all four is required.

Step 4: Conclusion All four displayed reactivity comparisons are correct. Hence, the correct option is (B).

Quick Tip to Solve: Rank each pair using its printed mechanism: compare steric hindrance for bimolecular substitution and carbocation stability for unimolecular substitution; then check the nitro-group effect on each aromatic pathway.

74. The pair from the following pairs having both compounds with net non-zero dipole moment is [4 marks]

(A) cis-butene, trans-butene

(B) CH_2Cl_2, $CHCl_3$

(C) Benzene, anisidine

(D) 1,4-Dichlorobenzene, 1,3-Dichlorobenzene

Correct Answer: (B) CH_2Cl_2, $CHCl_3$

Solution:

Concept:

- The molecular dipole moment is the vector sum of all bond dipoles. Polar bonds can cancel when the molecular geometry has the appropriate symmetry.
- A pair qualifies only when both molecules have a non-zero resultant; one polar member cannot compensate for a non-polar partner.

Step 1: Test the chloromethane pair Both dichloromethane, CH_2Cl_2, and chloroform, $CHCl_3$, have tetrahedral carbon centres. Their substituents are not all identical, so their bond-dipole vectors do not cancel. In dichloromethane, the two carbon–chlorine bond vectors are at a tetrahedral angle, not opposite directions. In chloroform, the resultant of three carbon–chlorine vectors is not cancelled by the different carbon–hydrogen bond. Thus each molecule has a non-zero dipole moment.

Step 2: Check the alkene and benzene pairs For the usual cis- and trans-but-2-ene comparison, cis-butene has a resultant dipole, but the trans isomer has opposing bond contributions that cancel by symmetry. Therefore option (A) fails the condition on both members. Benzene is a symmetric planar ring and its resultant dipole is zero. Hence option (C) fails regardless of the polar substituents present in anisidine.

Step 3: Check the dichlorobenzenes In 1,4-dichlorobenzene the identical carbon–chlorine bond dipoles are opposite and cancel. In 1,3-dichlorobenzene their directions are not opposite, so a resultant remains. Thus option (D) also contains a zero-dipole member. The chloromethane pair is the only listed pair in which neither molecular dipole vanishes.

Step 4: Conclusion Both CH_2Cl_2 and $CHCl_3$ have non-zero dipole moments. Hence, the correct option is (B).

Quick Tip to Solve: Treat dipoles as vectors: tetrahedral chloromethanes and para-disubstituted benzenes have different cancellation patterns.

75. The enthalpy change for the adsorption process and micelle formation respectively are [4 marks]

(A) $\Delta H_{\text{ads}} > 0$ and $\Delta H_{\text{mic}} < 0$

(B) $\Delta H_{\text{ads}} > 0$ and $\Delta H_{\text{mic}} > 0$

(C) $\Delta H_{\text{ads}} < 0$ and $\Delta H_{\text{mic}} < 0$

(D) $\Delta H_{\text{ads}} < 0$ and $\Delta H_{\text{mic}} > 0$

Correct Answer: (D) $\Delta H_{\text{ads}} < 0$ and $\Delta H_{\text{mic}} > 0$

Solution:

Concept:

- Adsorption binds particles at a surface and releases energy as attractive interactions are established; the usual adsorption enthalpy is negative.
- In the standard surface-chemistry treatment used here, micelle formation is endothermic and entropy-driven: $\Delta H_{\text{mic}} > 0$, with a favourable entropy contribution to $\Delta G = \Delta H - T\Delta S$.

Step 1: Determine the adsorption sign When gas or dissolved particles become adsorbed, attractive interactions with the adsorbent lower the energy of the system. Heat is released to the surroundings, so

$$\Delta H_{\text{ads}} < 0.$$

Adsorption also restricts the motion of the adsorbate, usually lowering its entropy. Its exothermic enthalpy contribution is therefore central to its thermodynamic favourability. This immediately eliminates options (A) and (B), both of which assign a positive adsorption enthalpy.

Step 2: Determine the micellisation sign In the convention tested by this question, association of surfactant ions or molecules into micelles has a positive enthalpy change. Micellisation can nevertheless occur spontaneously because hydrophobic aggregation releases ordered water from around the separate hydrocarbon tails, producing a favourable entropy contribution. Above the critical micelle concentration and under the appropriate temperature conditions, that entropy gain can outweigh the positive enthalpy term. Thus spontaneity does not require a negative micellisation enthalpy.

Step 3: Match both inequalities The required combination is negative for adsorption and positive for micelle formation. Option (C) gets the adsorption sign right but assigns a negative micellisation enthalpy; it is therefore not the combination requested in the standard treatment. Option (D) contains both required signs.

Step 4: Conclusion The enthalpy changes are $\Delta H_{\text{ads}} < 0$ and $\Delta H_{\text{mic}} > 0$. Hence, the correct option is (D).

Quick Tip to Solve: Do not infer the sign of enthalpy from spontaneity alone: the entropy term can drive micellisation.

76. Lime reacts exothermally with water to give 'A' which has low solubility in water. Aqueous solution of 'A' is often used for the test of CO_2, a test in which insoluble B is formed. If B is further reacted with CO_2 then soluble compound is formed, 'A' is : [4 marks]

(A) Quick lime

(B) White lime

(C) Lime water

(D) Slaked lime

Correct Answer: (D) Slaked lime

Solution:

Concept:

- Quicklime is calcium oxide, CaO; slaked lime is calcium hydroxide, $Ca(OH)_2$. Their reaction with water is called slaking.
- Limewater is the clear aqueous solution of calcium hydroxide. Carbon dioxide first precipitates calcium carbonate and then dissolves it as hydrogen carbonate in excess.

Step 1: Identify the hydration product The exothermic hydration of lime is

$$CaO + H_2O \longrightarrow Ca(OH)_2.$$

Therefore compound A is calcium hydroxide. It is only sparingly soluble in water, exactly as the question states. The chemical name of the solid product is slaked lime. Quicklime is the starting oxide, so it cannot also be the product A of this hydration reaction.

Step 2: Verify the carbon dioxide test The aqueous solution of A reacts with carbon dioxide to produce insoluble calcium carbonate B:

$$Ca(OH)_2 + CO_2 \longrightarrow CaCO_3\downarrow + H_2O.$$

The white precipitate makes limewater milky. This reaction checks the identity independently: the calcium hydroxide solution supplies the calcium ions and hydroxide needed for the familiar laboratory test.

Step 3: Verify the excess-gas reaction and select the name On passing excess carbon dioxide through the suspension, the precipitate dissolves:

$$CaCO_3 + CO_2 + H_2O \longrightarrow Ca(HCO_3)_2.$$

Calcium hydrogen carbonate is soluble, explaining the final observation. Limewater names the solution used for the test, whereas A is the sparingly soluble compound whose solution is used. White lime is not the precise chemical name identifying the hydration product in the options. Slaked lime is the unambiguous match to all three chemical changes.

Step 4: Conclusion Compound A is slaked lime, $Ca(OH)_2$. Hence, the correct option is (D).

Quick Tip to Solve: Follow the sequence: quicklime to slaked lime to carbonate, then soluble hydrogen carbonate in excess carbon dioxide.

77. Which of the following is used as a stabilizer during the concentration of sulphide ores? [4 marks]

(A) Cresols

(B) Xanthates

(C) Fatty acids

(D) Pine oils

Correct Answer: (A) Cresols

Solution:

Concept:

- Sulphide ores are commonly concentrated by froth flotation: selectively wetted mineral particles attach to air bubbles and rise as a froth.
- A collector promotes attachment of the valuable mineral to bubbles; a froth stabiliser helps the resulting froth persist long enough for removal.

Step 1: Identify the role asked for The question asks specifically for a stabiliser during concentration, rather than asking for any reagent used in flotation. During froth flotation the mineral-bearing bubbles must survive their rise to the surface and the subsequent skimming operation. A stabiliser helps maintain that froth and prevents it from collapsing too quickly, so the concentrated sulphide particles can be collected efficiently.

Step 2: Recall the stabilising reagent Cresols are used as froth stabilisers in the standard description of the froth-flotation process. Their role is to help sustain the mineral-bearing froth. This differs from making the sulphide mineral surface water-repellent in the first place. Keeping these two functions distinct is essential because the options include several reagents associated with flotation, but they are not all classified under the function requested.

Step 3: Distinguish the remaining choices Xanthates and fatty acids are associated with the collector function: they promote the hydrophobic character needed for selective attachment of ore particles to air bubbles. Pine oil is commonly listed among flotation collectors or frothing reagents, but the named froth stabiliser in the conventional ore-concentration classification is cresol. Thus the presence of familiar flotation reagents in options (B), (C) and (D) does not make them the answer to this role-specific question. Select the option that explicitly matches stabilisation.

Step 4: Conclusion Cresols are used as froth stabilisers during the concentration of sulphide ores. Hence, the correct option is (A).

Quick Tip to Solve: In flotation questions, identify the requested function first: collector, stabiliser and depressant are different roles.

78. Given below are two reactions, involved in the commercial production of dihydrogen (H_2). The two reactions are carried out at temperature "T_1" and "T_2", respectively

$$C(s) + H_2O(g) \xrightarrow{T_1} CO(g) + H_2(g)$$

$$CO(g) + H_2O(g) \xrightarrow[Catalyst]{T_2} CO_2(g) + H_2(g)$$

The temperatures T_1 and T_2 are correctly related as : [4 marks]

(A) $T_1 < T_2$

(B) $T_1 = T_2$

(C) $T_1 = 100$ K, $T_2 = 1270$ K

(D) $T_1 > T_2$

Correct Answer: (D) $T_1 > T_2$

Solution:

Concept:

- The first reaction is the steam-carbon reaction: steam passes over red-hot coke to form water gas, a mixture of CO and H_2.
- The second reaction is the water-gas shift reaction, which converts carbon monoxide and steam into carbon dioxide and additional hydrogen. Its operating temperature is lower than that of steam-carbon gasification.

Step 1: Identify the high-temperature gasification step The reaction $C(s)+H_2O(g) \longrightarrow CO(g)+H_2(g)$ is endothermic. Steam is passed over red-hot coke, typically near $1270\,K$, so substantial heating is needed to generate water gas. This identifies T_1 as the higher operating temperature. A temperature of $100\,K$ would not provide steam or a red-hot carbon surface and therefore cannot describe this process.

Step 2: Compare the catalytic shift step In $CO(g) + H_2O(g) \longrightarrow CO_2(g) + H_2(g)$, the water-gas mixture reacts with more steam over a suitable catalyst, commonly iron chromate, around $673\,K$. The shift is exothermic, so this lower temperature is favourable for hydrogen production while the catalyst gives a useful rate. Thus the two steps are not operated at the same temperature. The relations $T_1 < T_2$ and $T_1 = T_2$ both contradict these operating conditions; the stated $100\,K$ value is also impossible for the first step.

Step 3: Conclusion The steam-carbon step requires a higher temperature than the catalytic water-gas shift step: $\boxed{T_1 > T_2}$. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish red-hot coke gasification from the lower-temperature catalytic shift: they are consecutive hydrogen-producing steps with different heat requirements.

79. Match List I with List II

	List I Industry		List II Waste Generated
(A)	Steel plants	(I)	Gypsum
(B)	Thermal power plants	(II)	Fly ash
(C)	Fertilizer industries	(III)	Slag
(D)	Paper mills	(IV)	Bio-degradable wastes

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(C) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(D) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Correct Answer: (A) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Solution:

Concept:

- Match each industrial process with its characteristic waste rather than with a material merely used somewhere in that industry.
- Slag is associated with metallurgical extraction, fly ash with coal combustion, gypsum with phosphate-fertilizer manufacture, and biodegradable organic matter with paper processing.

Step 1: Match the mineral and combustion wastes Steel plants generate slag, which contains the gangue and other impurities removed during metallurgical processing. Therefore (A) matches (III). Coal-based thermal power plants produce fine mineral particles carried in flue gases; these are collected as fly ash, so (B) matches (II). These are different waste streams: slag is a fused metallurgical by-product, whereas fly ash is particulate combustion residue.

Step 2: Match the remaining industries and eliminate alternatives Fertilizer industries, especially those involving phosphate-rock processing, generate gypsum as a characteristic by-product; hence (C) matches (I). Paper mills generate organic residues containing cellulose and related substances, classified here as biodegradable wastes, so (D) matches (IV). Combining all four gives (A)-(III), (B)-(II), (C)-(I), (D)-(IV). Option (B) incorrectly gives thermal power plants biodegradable waste and paper mills fly ash. Option (C) gives steel plants biodegradable waste and thermal plants gypsum. Option (D) interchanges the characteristic waste categories even more extensively; none preserves the four established matches.

Step 3: Conclusion The four matches obtained above give the complete correct pairing. Hence, the correct option is (A).

Quick Tip to Solve: Anchor slag to steel and fly ash to thermal power first; the fertilizer–gypsum and paper–biodegradable matches then complete the list.

80. Match List I with List II

	List I Polymer		List II Type/Class
(A)	Nylon-2-Nylon-6	(I)	Thermosetting polymer
(B)	Buna-N	(II)	Biodegradable polymer
(C)	Urea-formaldehyde resin	(III)	Synthetic rubber
(D)	Dacron	(IV)	Polyester

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(B) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(C) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(D) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Correct Answer: (C) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Solution:

Concept:

- A polymer can be classified by its chemical linkage, physical behaviour or susceptibility to biological degradation; use the characteristic requested in the matching list.
- Nylon-2-nylon-6 is biodegradable, Buna-N is an elastomer, urea-formaldehyde is a thermosetting resin, and Dacron contains ester linkages.

Step 1: Identify the biodegradable polymer and synthetic rubber Nylon-2-nylon-6 is a biodegradable copolyamide prepared from amino-acid monomers such as glycine and aminocaproic acid. Thus (A) matches biodegradable polymer (II), rather than polyester (IV). Buna-N is a copolymer of 1,3-butadiene and acrylonitrile. It has rubber-like elastic behaviour and is a synthetic rubber, giving (B)-(III); it is not the thermosetting urea-formaldehyde resin.

Step 2: Match the resin and polyester and test every option Urea-formaldehyde resin forms a cross-linked network on curing. Such a thermosetting polymer cannot simply be melted and reshaped repeatedly, so (C)-(I). Dacron, also called polyethylene terephthalate, is a condensation polyester made from ethylene glycol and terephthalic acid; hence (D)-(IV). The resulting sequence is (II), (III), (I), (IV). Option (A) places nylon-2-nylon-6 in the polyester class and Buna-N in the thermosetting class, both incorrectly. Option (B) misclassifies Buna-N, the resin and Dacron. Option (D) has the correct Buna-N and resin matches but interchanges the biodegradable polymer and polyester. Only option (C) gives every classification correctly.

Step 3: Conclusion The four matches obtained above give the complete correct pairing. Hence, the correct option is (C).

Quick Tip to Solve: Use the distinctive pair Buna-N–synthetic rubber and Dacron–polyester to narrow a polymer matching question quickly.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. In potassium ferrocyanide, there are _____ pairs of electrons in the t_{2g} set of orbitals. [4 marks]

Solution:

Concept:

- Potassium ferrocyanide is $K_4[Fe(CN)_6]$; the complex anion has charge -4, and each cyanide ligand has charge -1.
- In an octahedral field the five metal d orbitals split into lower-energy t_{2g} and higher-energy e_g sets. Cyanide is a strong-field ligand and favours a low-spin configuration.

Step 1: Find the oxidation state and d-electron count Let the oxidation state of iron be x. Charge balance in the complex gives

$$x + 6(-1) = -4, \qquad x = +2.$$

Neutral iron has configuration $[Ar]\,3d^6 4s^2$. Removing its two outer $4s$ electrons gives Fe^{2+} with a d^6 configuration. The four potassium ions are counterions and do not add electrons to the metal orbital occupancy.

Step 2: Fill the lower octahedral orbitals There are six cyanide ligands, giving octahedral coordination. Because cyanide produces a large crystal-field splitting, pairing within the lower t_{2g} set is energetically preferred to promoting electrons into e_g. The configuration is therefore

$$(t_{2g})^6 (e_g)^0.$$

The t_{2g} set contains three orbitals, and all six electrons occupy them as two electrons per orbital. Consequently each orbital contains one pair, for three pairs in total. Counting six electrons as six pairs would double-count; a pair always contains two electrons.

Step 3: Conclusion The t_{2g} set contains six electrons arranged in $\boxed{3}$ electron pairs. The numerical answer is 3 (a dimensionless count).

Quick Tip to Solve: Determine the metal oxidation state before filling orbitals; for strong-field octahedral d-six, all three lower orbitals are doubly occupied.

82. The number of bent-shaped molecule/s from the following is _____

N_3^-, NO_2^-, I_3^-, O_3, SO_2 [4 marks]

Solution:

Concept:

- VSEPR theory predicts molecular geometry from the number of bonded atoms and lone pairs around the central atom. Molecular shape describes atom positions, not the positions of lone pairs.
- An AX_2E species is bent, whereas AX_2 and the usual AX_2E_3 arrangement are linear.

Step 1: Separate the linear ions The azide ion N_3^- has a linear arrangement of its three nitrogen atoms; the central nitrogen has two bonding directions and no stereochemically active lone pair that bends the skeleton. The triiodide ion I_3^- has five electron domains around the central iodine: two bond pairs and three lone pairs. The three lone pairs occupy equatorial positions in a trigonal-bipyramidal electron-domain arrangement, leaving the two bonds axial and opposite. Thus these two ions are linear, even though their electronic descriptions differ.

Step 2: Count the bent species explicitly In nitrite, NO_2^-, the central nitrogen has two bonding directions and one lone pair, so its molecular shape is bent. Ozone, O_3, similarly has two bonded terminal oxygens and a lone pair on the central oxygen, giving a bent structure. Sulfur dioxide, SO_2, has two sulfur–oxygen bonding domains and one central lone pair; double bonds count as single domains for VSEPR, so it too is bent. Resonance changes the bond description but does not turn any of these three into a linear molecule. The counted bent species are therefore nitrite, ozone and sulfur dioxide.

Step 3: Conclusion The bent species are NO_2^-, O_3 and SO_2. Their number is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: For triatomic species, distinguish AX-two-E from AX-two-E-three: the former bends, while three equatorial lone pairs leave the latter linear.

83. The number of correct statement/s involving equilibria in physical processes from the following is _____

(A) Equilibrium is possible only in a closed system at a given temperature.

(B) Both the opposing processes occur at the same rate.

(C) When equilibrium is attained at a given temperature, the value of all its parameters became equal.

(D) For dissolution of solids in liquids, the solubility is constant at a given temperature. [4 marks]

Solution:

Concept:

- At physical equilibrium in a closed system at fixed temperature, opposing processes continue at equal rates; equilibrium is dynamic rather than a complete cessation of change.
- Macroscopic properties remain constant with time at equilibrium, but different properties do not become numerically equal to one another.

Step 1: Test the system and rate statements Statement (A) is correct in the stated physical-equilibrium setting: a closed system prevents continuing loss or gain of matter, and fixed temperature is needed to specify a reproducible equilibrium state. For example, evaporation and condensation can balance in a closed vessel at a given temperature. Statement (B) is also correct: at equilibrium the rates of the forward and reverse physical processes are equal. Equal opposing rates produce no net macroscopic change, although molecules still undergo both processes.

Step 2: Distinguish constant parameters from equal parameters Statement (C) is incorrect. At equilibrium, quantities such as pressure, concentration and phase amounts become constant with time under fixed conditions; they do not all become equal. They may even have different physical

dimensions, so an assertion that every parameter becomes equal has no general meaning. Statement (D) is correct: the solubility of a particular solid in a specified liquid has a definite value at a fixed temperature, corresponding to its saturated solution at equilibrium. This does not claim that every solid has the same solubility or that solubility remains unchanged when temperature is varied. Therefore (A), (B) and (D) are correct, while (C) is false.

Step 3: Conclusion Three statements, (A), (B) and (D), are correct. The numerical answer is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: In equilibrium statements, replace "all quantities become equal" with "macroscopic quantities become constant"; only the two opposing rates must be equal.

84. The sum of lone pairs present on the central atom of the interhalogen IF_5 and IF_7 is _____

[4 marks]

Solution:

Concept:

- Iodine has seven valence electrons. Each single bond to a terminal fluorine uses one iodine valence electron; electrons remaining on the central iodine form lone pairs.
- Count only lone pairs on the central atom as asked. The lone pairs on the fluorine atoms do not enter this total.

Step 1: Count the central lone pair in iodine pentafluoride In IF_5, iodine forms five iodine–fluorine single bonds. Of its seven valence electrons, five participate singly in these bonds, leaving two on iodine:

$$7 - 5 = 2 \text{ nonbonding electrons}, \qquad \frac{2}{2} = 1 \text{ lone pair}.$$

This gives six electron domains in total, corresponding to an octahedral electron-domain arrangement and a square-pyramidal molecular shape. The lone pair occupies the sixth position, so it must be included in the requested central-atom count.

Step 2: Count the heptafluoride and add In IF_7, iodine forms seven single bonds with seven fluorine atoms. All seven iodine valence electrons participate in bonding, leaving

$$7 - 7 = 0 \text{ nonbonding electrons}, \qquad 0 \text{ central lone pairs}.$$

The molecule has a pentagonal-bipyramidal arrangement. Although every fluorine also carries nonbonding electrons, those are terminal-atom lone pairs and are outside the question. Adding the two central-atom counts gives

$$1 + 0 = 1.$$

The result is a count of pairs rather than a count of individual electrons.

Step 3: Conclusion The sum of lone pairs on the central iodine atoms in IF_5 and IF_7 is $\boxed{1}$, a dimensionless count.

Quick Tip to Solve: For a neutral iodine fluoride with single bonds, subtract the number of fluorines from seven and divide the remaining central electrons by two.

85. The number of incorrect statement/s about the black body from the following is _____

(A) Emit or absorb energy in the form of electromagnetic radiation.

(B) Frequency distribution of the emitted radiation depends on temperature.

(C) At a given temperature, intensity vs frequency curve passes through a maximum value.

(D) The maximum of the intensity vs frequency curve is at a higher frequency at higher temperature compared to that at lower temperature. [4 marks]

Solution:

Concept:

- A black body is an ideal absorber of all incident electromagnetic radiation and, at thermal equilibrium, an ideal thermal emitter.
- Its spectral intensity is a function of both frequency and absolute temperature. The frequency at which the spectrum peaks increases with temperature.

Step 1: Check the nature of the energy exchange Statement (A) is correct. Thermal radiation consists of electromagnetic waves, so the energy emitted or absorbed is electromagnetic energy. A black body does not require a material medium to exchange this energy; being an ideal absorber does not prevent it from emitting at a nonzero temperature.

Step 2: Check the spectrum at a fixed temperature Statement (B) is correct: changing the temperature changes the distribution of emitted energy over frequency. Statement (C) is also correct: at any fixed positive temperature the spectral intensity first rises with frequency, reaches a finite peak, and then falls at high frequencies. Thus the intensity is not constant over frequency.

Step 3: Check the shift and count the incorrect statements Statement (D) is correct. For a spectrum expressed per unit frequency, the peak frequency is proportional to absolute temperature, so a hotter body has its peak at a higher frequency. This is consistent with the shift towards shorter characteristic wavelengths on heating. All four statements are correct; the question asks for incorrect statements, not correct ones.

Step 4: Conclusion The number of incorrect statements is $\boxed{0}$ (a dimensionless count).

Quick Tip to Solve: Mark each statement true or false first; when the question asks for incorrect statements, count only the false entries.

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86. A molecule undergoes two independent first order reactions whose respective half lives are 12 min and 3 min. If both the reactions are occurring then the time taken for the 50% consumption of the reactant is _____ min. (Nearest integer) [4 marks]

Solution:

Concept:

- For independent first-order pathways consuming the same reactant, their disappearance rates add: $-d[A]/dt = (k_1 + k_2)[A]$.
- For a first-order reaction, $t_{1/2} = \ln 2/k$. The observed half-life therefore uses the sum of the two rate constants.

Step 1: Find the rate constant of each pathway Let the reactant be A and let the two independent pathways form different products. At a given instant both pathways act on the same remaining concentration. From the given half-lives,

$$k_1 = \frac{\ln 2}{12}\,\text{min}^{-1}, \qquad k_2 = \frac{\ln 2}{3}\,\text{min}^{-1}.$$

The shorter half-life corresponds to the faster pathway; it is not appropriate to average the two half-lives.

Step 2: Combine the disappearance rates The total first-order rate constant is

$$k_{\text{eff}} = k_1 + k_2 = \ln 2\left(\frac{1}{12} + \frac{1}{3}\right) = \frac{5\ln 2}{12}\,\text{min}^{-1}.$$

Integration gives $[A]_t = [A]_0 e^{-k_{\text{eff}}t}$. For 50% consumption, the remaining fraction is $[A]_t/[A]_0 = 1/2$, so $k_{\text{eff}}t = \ln 2$.

Step 3: Calculate and round the required time Therefore

$$t = \frac{\ln 2}{5\ln 2/12} = \frac{12}{5} = 2.4\,\text{min}.$$

This is less than either individual half-life, as expected because the reactant is being consumed through both pathways simultaneously. Rounding only the final value to the nearest integer gives 2.

Step 4: Conclusion The required time is $2.4\,\text{min}$, or $\boxed{2\,\text{min}}$ to the nearest integer.

Quick Tip to Solve: For competing first-order paths, add the rate constants, or use $1/t_{1/2,\text{eff}} = 1/t_{1/2,1} + 1/t_{1/2,2}$.

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87. In the following reactions, the total number of oxygen atoms in X and Y is _____.

$Na_2O + H_2O \rightarrow 2X$

$Cl_2O_7 + H_2O \rightarrow 2Y$ [4 marks]

Solution:

Concept:

- A basic metal oxide reacts with water to form the corresponding hydroxide; an acidic nonmetal oxide gives the corresponding oxyacid.
- Identify X and Y from balanced equations before counting atoms. A coefficient counts molecules, whereas a subscript counts atoms in one formula unit.

Step 1: Identify the product of sodium oxide with water Sodium oxide is a basic oxide. It reacts with water to form sodium hydroxide:

$$\text{Na}_2\text{O} + \text{H}_2\text{O} \longrightarrow 2\,\text{NaOH}.$$

Comparison with the first equation shows that $X = \text{NaOH}$. One formula unit of X contains one

oxygen atom. The coefficient 2 balances sodium and hydrogen; it is not part of the formula of X.

Step 2: Identify the product of dichlorine heptoxide with water Dichlorine heptoxide is the acid anhydride of perchloric acid, with chlorine in oxidation state +7. Its reaction with water is

$$\mathrm{Cl_2O_7 + H_2O \longrightarrow 2\,HClO_4}.$$

Thus $Y = \mathrm{HClO_4}$, which contains four oxygen atoms in one molecule. The equation balances eight oxygen atoms in total on each side.

Step 3: Add the oxygen counts in X and Y The question asks for the oxygen atoms in the chemical species X and Y, rather than in the stoichiometric amounts 2X and 2Y. Therefore

$$N_{\mathrm{O}}(X) + N_{\mathrm{O}}(Y) = 1 + 4 = 5.$$

Multiplying both counts by 2 would instead count the oxygen atoms in all the product molecules shown by the two equations and would answer a different question.

Step 4: Conclusion The total number of oxygen atoms in X and Y is $\boxed{5}$ (a dimensionless count).

Quick Tip to Solve: When products are denoted by X and Y, determine their molecular formulae first; do not include the reaction coefficients in the requested atom count.

88.

$$\mathrm{FeO_4^{2-} \xrightarrow{+2.2V} Fe^{3+} \xrightarrow{+0.70V} Fe^{2+} \xrightarrow{-0.45V} Fe^{0}}$$

$E^{\theta}_{FeO_4^{2-}/Fe^{2+}}$ **is** $x \times 10^{-3}$ **V. The value of** x **is** _____ [4 marks]

Solution:

Concept:

- Standard reduction potentials cannot be added directly. Combine the corresponding Gibbs energies using $\Delta G^{\circ} = -nFE^{\circ}$.
- For consecutive reductions, the net potential is the electron-weighted mean: $E^{\circ}_{\mathrm{net}} = (n_1E^{\circ}_1 + n_2E^{\circ}_2)/(n_1 + n_2)$.

Step 1: Determine the electron changes In $\mathrm{FeO_4^{2-}}$, oxygen contributes -8 and the ion has charge -2, so iron is in oxidation state +6. Its reduction to $\mathrm{Fe^{3+}}$ consumes three electrons per iron atom. The next reduction, $\mathrm{Fe^{3+} \longrightarrow Fe^{2+}}$, consumes one electron. The required overall reduction therefore consumes four electrons.

Step 2: Add the Gibbs-energy changes The two relevant potentials are +2.2 V and +0.70 V. Their Gibbs-energy changes add as

$$\Delta G^{\circ}_{\mathrm{net}} = -3F(2.2) - F(0.70) = -7.30F.$$

The potential -0.45 V for the final reduction to iron metal is excluded, because the requested final species is $\mathrm{Fe^{2+}}$, not Fe.

Step 3: Convert the net potential to the requested form For four electrons,

$$-4FE^{\circ}_{\text{net}} = -7.30F, \qquad E^{\circ}_{\text{net}} = \frac{7.30}{4} = 1.825\,\text{V}.$$

Writing this as $x \times 10^{-3}$ V gives $x = 1825$. The value lies between 0.70 V and 2.2 V, as expected for a weighted mean, and is nearer 2.2 V because that step involves three electrons.

Step 4: Conclusion $E^{\circ}_{FeO_4^{2-}/Fe^{2+}} = 1825 \times 10^{-3}$ V, so $\boxed{x = 1825}$.

Quick Tip to Solve: In a Latimer sequence, weight each potential by its electron count; stop at the oxidation state requested in the question.

89. If the degree of dissociation of aqueous solution of weak monobasic acid is determined to be 0.3, then the observed freezing point will be _____% higher than the expected/theoretical freezing point. (Nearest integer) [4 marks]

Solution:

Concept:

- A weak monobasic acid dissociates as $HA \rightleftharpoons H^+ + A^-$. For degree of dissociation α, its van't Hoff factor is $i = 1 + \alpha$.
- The freezing-point depression is $\Delta T_f = iK_f m$; increased particle number increases the depression and lowers the actual freezing temperature.

Step 1: Count the particles after dissociation Start with one mole of HA. If the dissociated fraction is $\alpha = 0.3$, the solution contains $1 - \alpha = 0.7$ mole of undissociated acid, 0.3 mole of hydrogen ions and 0.3 mole of conjugate-base ions. The total is

$$0.7 + 0.3 + 0.3 = 1.3 \text{ moles of solute particles.}$$

Hence $i = 1.3$, compared with the theoretical value 1 for an undissociated solute.

Step 2: Compare the depressions at the same molality For the same solvent and analytical molality,

$$\Delta T_{f,\text{theory}} = K_f m, \qquad \Delta T_{f,\text{obs}} = 1.3K_f m.$$

Thus the percentage increase in the depression is

$$\frac{1.3K_f m - K_f m}{K_f m} \times 100 = 30\%.$$

This calculation uses a temperature difference, not an absolute freezing temperature.

Step 3: Interpret the wording carefully The numerical result 30 refers to the increase in freezing-point depression, which is the intended colligative quantity. The wording says the freezing point is higher, but dissociation actually lowers the freezing temperature: $T_{f,\text{obs}} = T_f^0 - 1.3K_f m$, whereas $T_{f,\text{theory}} = T_f^0 - K_f m$. A percentage increase in the absolute freezing point cannot be inferred from the degree of dissociation alone.

Step 4: Conclusion The intended numerical answer is $\boxed{30}$: the freezing-point depression is 30% higher. The actual freezing point is lower, so the printed phrase freezing point higher must be understood as referring to the depression.

Quick Tip to Solve: Distinguish T_f from ΔT_f: dissociation raises the depression but lowers the freezing temperature.

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90. At constant temperature, a gas is at a pressure of 940.3 mm Hg. The pressure at which its volume decreases by 40% is _____ mm Hg. (Nearest integer) [4 marks]

Solution:

Concept:

- For a fixed amount of gas at constant temperature, Boyle's law gives $P_1V_1 = P_2V_2$.
- A decrease of 40% leaves 60% of the original volume, so $V_2 = 0.60V_1$. Pressure and volume vary inversely under these conditions.

Step 1: Express the final volume correctly Let the initial volume be V_1. The change in volume is $0.40V_1$, so the final volume is

$$V_2 = V_1 - 0.40V_1 = 0.60V_1.$$

The value 40% describes the amount lost, not the amount remaining. Using $V_2 = 0.40V_1$ would represent a 60% decrease and would give a different pressure.

Step 2: Apply the constant-temperature gas relation The temperature and amount of gas are unchanged, so

$$P_2 = \frac{P_1V_1}{V_2} = \frac{940.3V_1}{0.60V_1} = 1567.166\ldots \text{ mm Hg}.$$

The unknown initial volume cancels, so no volume unit or numerical volume is needed. The pressure unit remains mm Hg because the volume ratio is dimensionless.

Step 3: Check the direction and round The final volume is smaller, so the final pressure must exceed the initial value 940.3 mm Hg. The calculated pressure is $5/3$ times the initial pressure, consistent with the inverse volume ratio $1/0.60$. Keep the unrounded quotient during calculation and round the final pressure to the nearest integer; the fractional part is less than 0.5.

Step 4: Conclusion The pressure is $\boxed{1567\,\text{mm Hg}}$ to the nearest integer.

Quick Tip to Solve: For a volume decrease of $p\%$, use the remaining fraction $1 - p/100$ in the denominator of Boyle's law.