JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 10 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 10 April 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $S = \left\{z = x + iy : \frac{2z-3i}{4z+2i} \text{ is a real number}\right\}$. Then which of the following is NOT correct? [4 marks]

(A) $y + x^2 + y^2 \neq -\frac{1}{4}$

(B) $(x, y) = \left(0, -\frac{1}{2}\right)$

(C) $x = 0$

(D) $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

Correct Answer: (B) $(x, y) = \left(0, -\frac{1}{2}\right)$

Solution:

Concept:

- A complex quotient is real exactly when its imaginary part is zero, provided its denominator is nonzero. Rationalise the denominator before imposing this condition.
- For $z = x + iy$, the forbidden denominator $4z + 2i = 0$ corresponds to $x = 0,\ y = -\frac{1}{2}$. This exclusion remains necessary even after simplifying an expression.

Step 1: Rationalise the quotient Write the numerator as $2x + i(2y - 3)$ and the denominator as $4x + i(4y + 2)$. Multiplying by the conjugate of the denominator gives

$$\frac{2z - 3i}{4z + 2i} = \frac{8x^2 + (2y - 3)(4y + 2) - 16ix}{16x^2 + (4y + 2)^2}.$$

The denominator is positive at every point where the quotient is defined. Therefore the imaginary part vanishes precisely when $-16x = 0$, so the locus lies on the imaginary axis.

Step 2: Apply the domain restriction and test the statements Thus the full locus is $x = 0$, with $y \neq -\frac{1}{2}$. This proves (C) and (D). Moreover,

$$x^2 + y^2 + y = \left(y + \frac{1}{2}\right)^2 - \frac{1}{4} > -\frac{1}{4}$$

on the locus, proving the inequality in (A). The point proposed in (B) is exactly the excluded point: it makes the denominator zero while the numerator is $-4i$. It cannot be restored by cancellation or by the reality condition. Since the question asks which statement is NOT correct, choose this excluded point rather than either true description of the locus.

Step 3: Conclusion The point $(0, -\frac{1}{2})$ is not in S, so statement (B) is NOT correct. Hence, the correct option is (B).

Quick Tip to Solve: Before equating the imaginary part to zero, record all points at which the original complex quotient is undefined.

2. Eight persons are to be transported from city A to city B in three cars of different makes. If each car can accommodate at most three persons, then the number of ways, in which they can be transported, is : [4 marks]

(A) 560

(B) 1680

(C) 3360

(D) 1120

Correct Answer: (B) 1680

Solution:

Concept:

- The persons are distinct, and cars of different makes are distinguishable. Assigning groups to labelled cars is counted by combinations; arrangements of passengers within a car are not requested.
- With capacities $3, 3, 3$, carrying eight people forces the occupancy pattern $(3, 3, 2)$. No car can carry fewer than two because the other two together carry at most six.

Step 1: Fix the occupancy pattern Total capacity is nine, only one more than the number of passengers. Hence exactly one car has two passengers and the other two have three each. There are three choices for the car with two passengers. Once this car is specified, the other two cars remain distinguishable by their makes; we must assign a definite group to each. Patterns such as $(4, 2, 2)$ violate capacity, whereas leaving a car empty cannot carry all eight.

Step 2: Assign passengers to the labelled cars For each chosen two-person car, choose its passengers in $\binom{8}{2}$ ways. Choose the three passengers for one specified remaining car in $\binom{6}{3}$ ways. The final three go to the last car without a further choice. Thus

$$N = 3\binom{8}{2}\binom{6}{3} = 3 \times 28 \times 20 = 1680.$$

There is no division by $2!$ for the equal-size groups, because interchanging these groups between different makes produces a new transportation arrangement. Conversely, multiplying by seat permutations would overcount: only which car carries each person matters. The other numerical options fail this labelled-group count, so the required total is uniquely 1680.

Step 3: Conclusion The number of transportation arrangements is 1680. Hence, the correct option is (B).

Quick Tip to Solve: With eight passengers and total capacity nine, identify the single vacant seat first; then count groups assigned to labelled cars.

3. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to : [4 marks]

(A) 13

(B) 10

(C) 20

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- An integer can be replaced by its residue before taking a positive integer power. Compute residues separately for each divisor, keeping the remainders in the prescribed ranges.
- For divisor m, the standard remainder lies between 0 and $m-1$. An even power of -1 equals 1, which is useful for the computation modulo seven.

Step 1: Find the remainder modulo three Let $N = 22^{2022} + 2022^{22}$. Since $22 \equiv 1 \pmod 3$ and $2022 \equiv 0 \pmod 3$,

$$N \equiv 1^{2022} + 0^{22} = 1 \pmod 3.$$

The exponent on the second term is positive, so that term contributes zero, not one. The permitted remainders on division by three are zero, one and two; therefore the value $\alpha = 1$ is already in the required range. There is no need to evaluate either enormous power.

Step 2: Find the remainder modulo seven and combine Now $22 = 3 \times 7 + 1$ and $2022 = 289 \times 7 - 1$. Consequently

$$N \equiv 1^{2022} + (-1)^{22} = 1 + 1 = 2 \pmod 7.$$

The exponent 22 is even, so the second term has positive residue one. Thus $\beta = 2$, which is between zero and six and hence is the actual remainder. Substitute the two independently obtained remainders:

$$\alpha^2 + \beta^2 = 1^2 + 2^2 = 5.$$

The values 13, 10 and 20 would require different remainders and do not satisfy these congruences. In particular, reducing the exponents without first reducing the bases is unnecessary and can obscure the simple residues here.

Step 3: Conclusion We obtain $\alpha = 1$, $\beta = 2$, and $\alpha^2 + \beta^2 = 5$. Hence, the correct option is (D).

Quick Tip to Solve: Look first for bases congruent to zero, one or minus one; their powers settle this remainder problem immediately.

4. Let $g(x) = f(x) + f(1-x)$ and $f''(x) > 0, x \in (0,1)$. If g is decreasing in the interval $(0,\alpha)$ and increasing in the interval $(\alpha,1)$, then $\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{1}{\alpha}\right) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to : [4 marks]

(A) $\frac{3\pi}{4}$

(B) π

(C) $\frac{5\pi}{4}$

(D) $\frac{3\pi}{2}$

Correct Answer: (B) π

Solution:

Concept:

- The inequality $f'' > 0$ makes f' strictly increasing. Compare the arguments x and $1-x$ to determine the sign of the derivative of their symmetric sum.
- For principal inverse tangents, each positive argument gives an angle between zero and $\pi/2$. The tangent addition identity must be accompanied by a quadrant check.

Step 1: Determine the change from decrease to increase Differentiating the given expression and using the chain rule gives

$$g'(x) = f'(x) - f'(1-x).$$

For $0 < x < \frac{1}{2}$, we have $x < 1-x$, so strict increase of f' gives $g'(x) < 0$. For $\frac{1}{2} < x < 1$, the inequality reverses and $g'(x) > 0$. At $x = \frac{1}{2}$, the derivative is zero. Thus the stated transition point between decreasing and increasing intervals is $\alpha = \frac{1}{2}$. This also follows from the symmetry $g(x) = g(1-x)$, but the derivative establishes the required signs.

Step 2: Evaluate the inverse-tangent sum with its correct branch After substitution the expression becomes

$$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3.$$

Put $u = \tan^{-1} 2$ and $v = \tan^{-1} 3$. Then

$$\tan(u+v) = \frac{2+3}{1-2\times 3} = -1.$$

Both u and v exceed $\pi/4$, and each is less than $\pi/2$. Therefore $\pi/2 < u+v < \pi$, selecting $u+v = 3\pi/4$, not $-\pi/4$. Adding $\tan^{-1} 1 = \pi/4$ gives π. The other listed angles do not equal this principal-angle sum.

Step 3: Conclusion The transition is $\alpha = \frac{1}{2}$, and the required sum equals π. Hence, the correct option is (B).

Quick Tip to Solve: For a symmetric sum $f(x) + f(1-x)$, compare derivatives around one half; always check the quadrant when adding inverse tangents.

5. If the coefficients of x and x^2 in $(1+x)^p(1-x)^q$ are 4 and -5 respectively, then $2p+3q$ is equal to : [4 marks]

(A) 66

(B) 60

(C) 69

(D) 63

Correct Answer: (D) 63

Solution:

Concept:

- In a product of binomial expansions, the coefficient of x^2 includes both individual quadratic terms and the product of the two linear terms.
- The first two coefficients of $(1+x)^p$ are p and $p(p-1)/2$; replacing x by $-x$ reverses the linear sign but leaves the quadratic sign unchanged.

Step 1: Translate the two coefficient conditions The expansions through the quadratic terms are

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2}x^2 + \cdots,$$
$$(1-x)^q = 1 - qx + \frac{q(q-1)}{2}x^2 + \cdots.$$

The coefficient of x in the product is $p-q$. Hence $p-q=4$. The coefficient of x^2 is

$$\frac{p(p-1)}{2} + \frac{q(q-1)}{2} - pq = -5.$$

The term $-pq$ is essential: it comes from multiplying px and $-qx$, rather than from either binomial alone.

Step 2: Solve for the sum and difference Rearranging the quadratic coefficient gives

$$\frac{(p-q)^2 - (p+q)}{2} = -5.$$

Since $p-q=4$, we obtain $16-(p+q)=-10$, so $p+q=26$. Solving the sum-and-difference equations yields

$$p = \frac{26+4}{2} = 15, \qquad q = \frac{26-4}{2} = 11.$$

A direct check of the quadratic coefficient gives $105+55-165=-5$, and the linear coefficient is $15-11=4$, confirming both conditions. Finally,

$$2p+3q = 2(15)+3(11) = 30+33 = 63.$$

The alternatives 66, 60 and 69 do not follow from this uniquely determined pair, so they are eliminated.

Step 3: Conclusion The parameters are $p=15$ and $q=11$, giving $2p+3q=63$. Hence, the correct option is (D).

Quick Tip to Solve: For this product, write its quadratic coefficient as $\big((p-q)^2-(p+q)\big)/2$; the linear coefficient already gives the difference.

6. Let f be a continuous function satisfying $\int_0^{t^2}\left(f(x)+x^2\right)dx=\frac{4}{3}t^3, \forall t>0$. Then $f\left(\frac{\pi^2}{4}\right)$ is equal to : [4 marks]

(A) $-\pi\left(1+\frac{\pi^3}{16}\right)$

(B) $\pi\left(1-\frac{\pi^3}{16}\right)$

(C) $-\pi^2\left(1+\frac{\pi^2}{16}\right)$

(D) $\pi^2\left(1-\frac{\pi^2}{16}\right)$

Correct Answer: (B) $\pi\left(1-\frac{\pi^3}{16}\right)$

Solution:

Concept:

- For continuous h, the chain rule and the fundamental theorem give $\frac{d}{dt}\int_0^{u(t)}h(x)\,dx=h(u(t))u'(t)$. The derivative of the upper limit must be retained.
- Here the upper limit is t^2 and $t>0$. This permits division by $2t$ and determines the positive square root when a particular value of t^2 is specified.

Step 1: Differentiate the identity with respect to the parameter Apply the fundamental theorem to the entire integrand, remembering that its second term becomes $(t^2)^2=t^4$ at the upper endpoint:

$$2t\left[f(t^2)+t^4\right]=4t^2.$$

As $t>0$, divide by $2t$ to obtain

$$f(t^2)+t^4=2t, \qquad f(t^2)=2t-t^4.$$

In particular, the derivative of $\frac{4}{3}t^3$ is $4t^2$, not $4t$. Omitting the upper-limit derivative would produce a different function and the wrong powers in the answer.

Step 2: Substitute the required argument and check the function For $t^2=\pi^2/4$, the stipulated positive parameter is $t=\pi/2$. Hence

$$f\left(\frac{\pi^2}{4}\right)=\pi-\frac{\pi^4}{16}=\pi\left(1-\frac{\pi^3}{16}\right).$$

As a check, the relation found for positive arguments is $f(x)=2\sqrt{x}-x^2$. Therefore $f(x)+x^2=2\sqrt{x}$, whose integral from zero to t^2 is $\frac{4}{3}(t^2)^{3/2}=\frac{4}{3}t^3$, exactly the supplied identity. Options (A), (C) and (D) have incorrect signs or powers compared with this verified value.

Step 3: Conclusion The requested value is $\pi\left(1-\frac{\pi^3}{16}\right)$. Hence, the correct option is (B).

Quick Tip to Solve: Differentiate the integral identity before substituting: evaluate every integrand term at the upper limit and multiply by its derivative.

7. For $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if $\int \left(\left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x} \right) \log_e x dx = \frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x} - \frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x} + C$, where $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ and C is constant of integration, then $\alpha + 2\beta + 3\gamma - 4\delta$ is equal to : [4 marks]

(A) -8

(B) -4

(C) 1

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- For $x > 0$, write $(x/e)^{kx} = \exp\{kx(\log x - 1)\}$. Logarithmic differentiation then gives its derivative as $k \log x \, (x/e)^{kx}$.
- The reciprocal power $(e/x)^{kx} = \exp\{kx(1 - \log x)\}$ instead has derivative $-k \log x \, (e/x)^{kx}$. These opposite signs determine the signs of their antiderivatives.

Step 1: Recognise each term as a derivative Differentiate the two powers with exponent $2x$:

$$\frac{d}{dx}\left(\frac{x}{e}\right)^{2x} = 2\log x \left(\frac{x}{e}\right)^{2x},$$
$$\frac{d}{dx}\left(\frac{e}{x}\right)^{2x} = -2\log x \left(\frac{e}{x}\right)^{2x}.$$

For the first exponent, differentiating $2x(\log x - 1)$ gives $2\log x$: the additional constants cancel. For the second, differentiating $2x(1 - \log x)$ gives $-2\log x$. The real-valued logarithm requires $x > 0$, which is also the domain used for these powers.

Step 2: Integrate and compare the four parameters It follows that the given antiderivative is

$$\frac{1}{2}\left(\frac{x}{e}\right)^{2x} - \frac{1}{2}\left(\frac{e}{x}\right)^{2x} + C.$$

Differentiating this expression recovers the sum of both positive integrand terms multiplied by $\log x$, so the minus sign before the second term is necessary. Comparing with the specified form gives $\alpha = \gamma = 2$ and $\beta = \delta = 2$, all natural numbers. Therefore

$$\alpha + 2\beta + 3\gamma - 4\delta = 2 + 4 + 6 - 8 = 4.$$

The alternative numbers $-8, -4, 1$ do not equal this parameter combination; they would require changing the verified coefficient or exponent.

Step 3: Conclusion The four parameters all equal 2, and the requested expression is 4. Hence, the correct option is (D).

Quick Tip to Solve: Convert a variable power to an exponential first; differentiating $x(\log x - 1)$ leaves exactly $\log x$.

8. Let μ be the mean and σ be the standard deviation of the distribution

x_i	0	1	2	3	4	5
f_i	$k+2$	$2k$	k^2-1	k^2-1	k^2+1	$k-3$

where $\sum f_i = 62$. If $[x]$ denotes the greatest integer $\leq x$, then $\left[\mu^2+\sigma^2\right]$ is equal to : [4 marks]

(A) 9

(B) 8

(C) 6

(D) 7

Correct Answer: (B) 8

Solution:

Concept:

- For a frequency distribution, $N = \sum f_i$, $\mu = \sum f_i x_i / N$, and $\sigma^2 = \sum f_i x_i^2 / N - \mu^2$. Here the standard deviation is that of the complete distribution, so the denominator is N.
- Thus $\mu^2 + \sigma^2$ is simply the second raw moment $\sum f_i x_i^2 / N$. Frequencies must be nonnegative, and the greatest-integer function is applied only after calculating this moment.

Step 1: Determine the admissible value of the parameter Adding all six frequencies gives

$$(k+2) + 2k + (k^2-1) + (k^2-1) + (k^2+1) + (k-3) = 62.$$

Therefore $3k^2 + 4k - 64 = 0$, or $(k-4)(3k+16) = 0$. The roots are 4 and $-16/3$. The negative root makes $2k$ negative and cannot describe frequencies. Hence $k = 4$, and the frequencies in increasing order of x_i are $6, 8, 15, 15, 17, 1$. Their sum is indeed 62.

Step 2: Calculate the second raw moment and take its floor Use squared observations, rather than the observations themselves:

$$\begin{aligned}\sum f_i x_i^2 &= 6(0)^2 + 8(1)^2 + 15(2)^2 \\ &\quad + 15(3)^2 + 17(4)^2 + 1(5)^2 \\ &= 8 + 60 + 135 + 272 + 25 = 500.\end{aligned}$$

Consequently $\mu^2 + \sigma^2 = 500/62 = 250/31$. Since $8 = 248/31 < 250/31 < 279/31 = 9$, its greatest integer is 8, not the rounded-up value 9. Values 6 and 7 fail the same bounding test.

Step 3: Conclusion The required value is $\boxed{8}$. Hence, the correct option is (B).

Quick Tip to Solve: When mean squared and variance appear together, replace their sum by the second raw moment before doing any arithmetic.

9. Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 5\hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 12$. Then $(-\hat{i} + \hat{j} - \hat{k}) \cdot (\vec{c} \times \vec{d})$ is equal to : [4 marks]

(A) 24

(B) 42

(C) 44

(D) 48

Correct Answer: (C) 44

Solution:

Concept:

- A vector perpendicular to two nonparallel vectors is a scalar multiple of their cross product. Write $\vec{d} = \lambda(\vec{a} \times \vec{b})$, and use the given dot product to determine both its magnitude and sign.
- The cross product is ordered: $\vec{c} \times \vec{d}$ is the negative of $\vec{d} \times \vec{c}$. A scalar triple product is evaluated by taking the final dot product with the indicated vector.

Step 1: Find the perpendicular vector with the prescribed dot product Using the component formula gives

$$\vec{a} \times \vec{b} = (7 \cdot 5 - (-1)0,\ (-1)3 - 2 \cdot 5,\ 2 \cdot 0 - 7 \cdot 3) = (35, -13, -21).$$

For $\vec{c} = (1, -1, 2)$, its dot product with this vector is $35 + 13 - 42 = 6$. The constraint therefore reads $6\lambda = 12$, giving $\lambda = 2$ and $\vec{d} = (70, -26, -42)$. As a check, $2(70) + 7(-26) - (-42) = 0$, and $3(70) + 5(-42) = 0$, so both perpendicularity conditions hold.

Step 2: Evaluate the ordered cross product and final scalar Keep $\vec{c}$ first in the cross product:

$$\begin{aligned}\vec{c} \times \vec{d} &= \big((-1)(-42) - 2(-26),\ 2(70) - 1(-42),\ 1(-26) - (-1)(70)\big) \\ &= (94, 182, 44).\end{aligned}$$

Now take the dot product with $(-1, 1, -1)$:

$$(-1, 1, -1) \cdot (94, 182, 44) = -94 + 182 - 44 = 44.$$

The values $24, 42, 48$ do not satisfy this complete component calculation; choosing a single component of the cross product would not evaluate the requested scalar.

Step 3: Conclusion The required scalar is $\boxed{44}$. Hence, the correct option is (C).

Quick Tip to Solve: After scaling a perpendicular cross-product vector, check both zero dot products before evaluating the final triple product.

10. If the points P and Q are respectively the circumcenter and the orthocentre of a $\triangle \mathrm{ABC}$, then $\overrightarrow{\mathrm{PA}}+\overrightarrow{\mathrm{PB}}+\overrightarrow{\mathrm{PC}}$ is equal to : [4 marks]

(A) $\overrightarrow{QP}$

(B) $\overrightarrow{PQ}$

(C) $2\overrightarrow{PQ}$

(D) $2\overrightarrow{QP}$

Correct Answer: (B) $\overrightarrow{PQ}$

Solution:

Concept:

- Take the circumcentre P as the origin. The position vectors $\vec{a}=\overrightarrow{PA}$, $\vec{b}=\overrightarrow{PB}$, and $\vec{c}=\overrightarrow{PC}$ then have equal length R, the circumradius.
- To identify the orthocentre, it is enough to show that a candidate point lies on two altitudes. A vector is perpendicular to a side precisely when its dot product with a vector along that side is zero.

Step 1: Construct a candidate for the orthocentre Let H be the point whose position vector from P is $\vec{h}=\vec{a}+\vec{b}+\vec{c}$. The displacement from A to H is $\vec{h}-\vec{a}=\vec{b}+\vec{c}$, whereas a vector along BC is $\vec{c}-\vec{b}$. Their scalar product is

$$(\vec{b}+\vec{c})\cdot(\vec{c}-\vec{b})=|\vec{c}|^2-|\vec{b}|^2=R^2-R^2=0.$$

Thus H lies on the altitude from A. This argument uses equality of distances from the circumcentre, not equality of the side lengths of the triangle.

Step 2: Identify the point and preserve the direction Similarly, the displacement from B to H is $\vec{a}+\vec{c}$. Its dot product with a vector $\vec{c}-\vec{a}$ along AC is $|\vec{c}|^2-|\vec{a}|^2=0$. Hence H is also on the altitude from B, so it is the orthocentre Q. Therefore

$$\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{h}=\overrightarrow{PQ}.$$

Reversing the arrow would negate this vector. Neither doubled vector agrees with the derived coefficient of one; thus the other three proposed expressions fail in a general triangle.

Step 3: Conclusion The sum is $\boxed{\overrightarrow{PQ}}$. Hence, the correct option is (B).

Quick Tip to Solve: With the circumcentre as origin, equal vertex-vector lengths make the altitude test collapse to a difference of equal squares.

11. Let A be the point $(1,2)$ and B be any point on the curve $x^2+y^2=16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio $3:2$ is the point C(α,β), then the length of the line segment AC is : [4 marks]

(A) $\frac{3\sqrt{5}}{5}$

(B) $\frac{6\sqrt{5}}{5}$

(C) $\frac{2\sqrt{5}}{5}$

(D) $\frac{4\sqrt{5}}{5}$

Correct Answer: (A) $\frac{3\sqrt{5}}{5}$

Solution:

Concept:

- For internal division $AP:PB=m:n$, the section formula is $P=(nA+mB)/(m+n)$. The coefficient of B is therefore $m/(m+n)$, not $n/(m+n)$.
- A fixed translation followed by a fixed scaling maps a circle to another circle. Substituting the coordinates of the moving point gives the locus and determines its centre without having to sketch the circle.

Step 1: Express the moving point in terms of the circle point Write $B=(u,v)$, where $u^2+v^2=16$, and let $P=(x,y)$. Since $AP:PB=3:2$,

$$x=\frac{2+3u}{5},\qquad y=\frac{4+3v}{5}.$$

In vector form this is $P=\frac{2}{5}A+\frac{3}{5}B$. Thus the moving circle is scaled by $3/5$, while its centre at the origin is translated to $\frac{2}{5}A$. For a direct algebraic check, invert the two equations to obtain $u=(5x-2)/3$ and $v=(5y-4)/3$.

Step 2: Find the centre and its distance from the fixed point Substitution into the equation of the original circle yields

$$\left(x-\frac{2}{5}\right)^2+\left(y-\frac{4}{5}\right)^2=\frac{144}{25}.$$

Hence $C=(\alpha,\beta)=(2/5,4/5)$. The distance sought is

$$AC=\sqrt{\left(1-\frac{2}{5}\right)^2+\left(2-\frac{4}{5}\right)^2}=\sqrt{\frac{9}{25}+\frac{36}{25}}=\frac{3\sqrt{5}}{5}.$$

The radius $12/5$ is a different quantity and must not be confused with AC. Reversing the ratio would produce $2\sqrt{5}/5$, explaining one distractor; the other multiples also fail the section formula.

Step 3: Conclusion The required length is $\boxed{3\sqrt{5}/5}$. Hence, the correct option is (A).

Quick Tip to Solve: For a fixed point and a moving circle point, apply the same section formula to the circle centre to locate the new centre.

12. Let $\mathrm{A} = \{2,3,4\}$ **and** $\mathrm{B} = \{8,9,12\}$**. Then the number of elements in the relation** $\mathrm{R} = \{((a_1,\ \mathrm{b}_1),(a_2,\ \mathrm{b}_2)) \in (A \times B, A \times B) : a_1$ **divides** b_2 **and** a_2 **divides** $\mathrm{b}_1\}$ **is :** [4 marks]

(A) 18

(B) 24

(C) 36

(D) 12

Correct Answer: (C) 36

Solution:

Concept:

- An element of this relation is an ordered pair of ordered pairs, $((a_1,b_1),(a_2,b_2))$. All four entries matter, and the two divisibility conditions connect the crossed pairs (a_1,b_2) and (a_2,b_1).
- Independent choices multiply. Count the admissible pairs $(a,b) \in A \times B$ with $a \mid b$, then make one such choice for each crossed pair. There is no requirement that the two crossed pairs be different.

Step 1: Count each admissible divisor and multiple pair For $a = 2$, the allowed values of b are $8, 12$, giving two choices. For $a = 3$, they are $9, 12$, again two choices. For $a = 4$, they are $8, 12$, again two. Thus the auxiliary set

$$D = \{(2,8),(2,12),(3,9),(3,12),(4,8),(4,12)\}$$

has 6 elements. The value 9 is not divisible by 2 or 4, and 8 is not divisible by 3; including such pairs would overcount the relation.

Step 2: Choose the crossed pairs independently Choose $(a_1,b_2) \in D$ in six ways, and independently choose $(a_2,b_1) \in D$ in six ways. These choices specify exactly one element $((a_1,b_1),(a_2,b_2))$ of R. Conversely, every element of R recovers exactly these two choices, so this construction neither omits nor duplicates any element. Consequently

$$|R| = |D|^2 = 6 \times 6 = 36.$$

Do not divide by two: the relation consists of ordered pairs, and interchanging the outer entries generally gives a different element. The answers $18, 24, 12$ would require extra restrictions not present in the definition.

Step 3: Conclusion The relation contains $\boxed{36}$ elements. Hence, the correct option is (C).

Quick Tip to Solve: When divisibility conditions cross the indices, first count one allowed divisor-multiple pair; independence then gives the square.

13. The statement $\sim [p \vee (\sim (p \wedge q))]$ is equivalent to : [4 marks]

(A) $(\sim (p \wedge q)) \wedge q$

(B) $\sim (p \vee q)$

(C) $(p \wedge q) \wedge (\sim p)$

(D) $\sim (p \wedge q)$

Correct Answer: (C) $(p \wedge q) \wedge (\sim p)$

Solution:

Concept:

- De Morgan laws state $\sim (X \vee Y) = (\sim X) \wedge (\sim Y)$ and $\sim (X \wedge Y) = (\sim X) \vee (\sim Y)$. Double negation gives $\sim (\sim X) = X$.
- Two statements are equivalent when their truth values agree for every assignment of p, q. A contradiction is false for every assignment, so another expression for the same contradiction is equivalent even if it looks different.

Step 1: Push the outer negation through the disjunction Apply De Morgan law to the complete bracket, treating $\sim (p \wedge q)$ as the second statement:

$$\begin{aligned} \sim [p \vee (\sim (p \wedge q))] &= (\sim p) \wedge \sim (\sim (p \wedge q)) \\ &= (\sim p) \wedge (p \wedge q) \\ &= (p \wedge q) \wedge (\sim p). \end{aligned}$$

The last rearrangement uses associativity and commutativity of conjunction. It is exactly the expression in option (C). It contains both p and its negation, so it is always false, regardless of q.

Step 2: Check the other expressions against the contradiction The other options are not identically false. For option (A), choose p false and q true; then $(\sim (p \wedge q)) \wedge q$ is true. For option (B), choose both p and q false; then $\sim (p \vee q)$ is true. For option (D), the same assignment makes $\sim (p \wedge q)$ true. Each therefore differs from the given statement for at least one assignment. Another useful check is that the inner disjunction simplifies to $p \vee \sim p \vee \sim q$, a tautology, whose negation is indeed a contradiction.

Step 3: Conclusion The equivalent statement is $\boxed{(p \wedge q) \wedge (\sim p)}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep each bracket intact when applying De Morgan law, and use a single counterexample assignment to reject a false equivalence.

14. If $S_n = 4 + 11 + 21 + 34 + 50 + \ldots$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to : [4 marks]

(A) 227

(B) 226

(C) 220

(D) 223

Correct Answer: (D) 223

Solution:

Concept:

- If consecutive first differences form an arithmetic progression, the original sequence has a constant second difference and its general term is quadratic in the term number.
- The difference of partial sums $S_{29} - S_9$ contains terms numbered 10 through 29, inclusive. Use $\sum_{n=1}^{N} n = N(N+1)/2$ and $\sum_{n=1}^{N} n^2 = N(N+1)(2N+1)/6$.

Step 1: Derive the general term from the differences The terms $4, 11, 21, 34, 50$ have first differences $7, 10, 13, 16$, with common difference 3. Thus, for $n \geq 2$, the increment from term $n-1$ to term n is $3n+1$. Starting with $a_1 = 4$,

$$\begin{aligned} a_n &= 4 + \sum_{j=2}^{n} (3j+1) \\ &= 4 + 3\left(\frac{n(n+1)}{2} - 1\right) + (n-1) \\ &= \frac{3n^2 + 5n}{2}. \end{aligned}$$

This gives $a_1 = 4$ and $a_5 = 50$, checking both ends of the displayed initial terms.

Step 2: Sum precisely the required range The necessary sums over the twenty retained terms are

$$\sum_{n=10}^{29} n = \frac{29 \cdot 30 - 9 \cdot 10}{2} = 390,$$

$$\sum_{n=10}^{29} n^2 = \frac{29 \cdot 30 \cdot 59 - 9 \cdot 10 \cdot 19}{6} = 8270.$$

Hence

$$S_{29} - S_9 = \frac{3(8270) + 5(390)}{2} = 13380, \qquad \frac{S_{29} - S_9}{60} = 223.$$

Starting at term 9 or stopping at term 28 would change the sum and is not the stated partial-sum difference. The values $227, 226, 220$ do not match the exact quadratic sum.

Step 3: Conclusion The required value is $\boxed{223}$. Hence, the correct option is (D).

Quick Tip to Solve: For a sequence with constant second differences, derive its quadratic term and subtract prefix sums with the endpoints written explicitly.

15. Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to : [4 marks]

(A) 15

(B) 60

(C) 30

(D) 90

Correct Answer: (B) 60

Solution:

Concept:

- Each fair-die roll has odd and even probabilities both equal to $1/2$. The rolls are independent, so the number of odd outcomes has distribution $\mathrm{Bin}(n, 1/2)$.
- The probability of exactly r specified-parity outcomes is $\binom{n}{r}2^{-n}$. Equal probabilities therefore give an equality between binomial coefficients.

Step 1: Determine the number of rolls The equality in the question is $\binom{n}{7}/2^n = \binom{n}{9}/2^n$. Both events require $n \geq 9$. Cancel the common denominator and use the ratio of the coefficients:

$$\frac{\binom{n}{9}}{\binom{n}{7}} = \frac{(n-7)(n-8)}{8 \cdot 9} = 1.$$

Thus $n^2 - 15n - 16 = 0$, or $(n-16)(n+1) = 0$. A negative number of trials is impossible, hence $n = 16$. This also agrees with symmetry, since seven and nine are equally spaced from eight.

Step 2: Evaluate the even-outcome probability Exactly two even outcomes among sixteen rolls can occur in $\binom{16}{2} = 120$ positions. Every fixed sequence of parity outcomes has probability 2^{-16}. Consequently

$$P(\text{exactly two even outcomes}) = \frac{120}{2^{16}} = \frac{60}{2^{15}}.$$

Comparing this with the form specified in the question gives $k = 60$. The coefficient 120 cannot be used directly because its denominator is 2^{16}, whereas the requested denominator is 2^{15}. Values 15, 30 and 90 do not reproduce this probability.

Step 3: Conclusion There are 16 rolls, and the required numerator is $\boxed{k = 60}$. Hence, the correct option is (B).

Quick Tip to Solve: Use symmetry of binomial coefficients to find n, but always convert the probability to the denominator asked for before reading off k.

16. Let the image of the point $\mathrm{P}(1,2,6)$ in the plane passing through the points $\mathrm{A}(1,2,0), \mathrm{B}(1,4,1)$ and $\mathrm{C}(0,5,1)$ be $\mathrm{Q}(\alpha,\beta,\gamma)$. Then $\left(\alpha^2+\beta^2+\gamma^2\right)$ is equal to : [4 marks]

(A) 76

(B) 62

(C) 70

(D) 65

Correct Answer: (D) 65

Solution:

Concept:

- A plane through three non-collinear points has a normal perpendicular to both joining vectors. Its equation follows by taking their cross product.
- The reflection of a point P in $\mathbf{n}\cdot\mathbf{r}+d=0$ is $Q=P-2\frac{\mathbf{n}\cdot P+d}{\|\mathbf{n}\|^2}\mathbf{n}$. The midpoint of PQ lies in the plane.

Step 1: Find the plane through the three points The two vectors within the plane are

$$\overrightarrow{AB}=(0,2,1), \qquad \overrightarrow{AC}=(-1,3,1).$$

Their cross product is $(-1,-1,2)$, so we may choose the opposite normal $\mathbf{n}=(1,1,-2)$. Using $A=(1,2,0)$, the equation is

$$x+y-2z-3=0.$$

Substitution of B gives $1+4-2-3=0$, and substitution of C gives $0+5-2-3=0$, verifying the plane.

Step 2: Reflect P and calculate the requested sum At $P=(1,2,6)$, the signed plane expression is $1+2-12-3=-12$, and the squared normal length is $1+1+4=6$. Therefore

$$Q=(1,2,6)-2\frac{-12}{6}(1,1,-2)=(5,6,-2).$$

As a check, the midpoint $(3,4,2)$ satisfies $3+4-4-3=0$, and $Q-P=(4,4,-8)$ is normal to the plane. Thus

$$\alpha^2+\beta^2+\gamma^2=25+36+4=65.$$

The other proposed totals 76, 62 and 70 do not equal the sum obtained from the reflected coordinates.

Step 3: Conclusion The image is $Q=(5,6,-2)$, giving $\boxed{65}$. Hence, the correct option is (D).

Quick Tip to Solve: After applying the reflection formula, check both that PQ is normal to the plane and that its midpoint lies on the plane.

17. Let the line $\frac{x}{1} = \frac{6-y}{2} = \frac{z+8}{5}$ intersect the lines $\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1}$ and $\frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x - 2y + z = 14$ is : [4 marks]

(A) 3

(B) $\frac{10}{3}$

(C) 4

(D) $\frac{11}{3}$

Correct Answer: (C) 4

Solution:

Concept:

- A symmetric line equation can be converted to parametric coordinates using one common parameter. Watch negative signs in numerators such as $6 - y$.
- The distance of (x_0, y_0, z_0) from $ax + by + cz + d = 0$ is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2 + b^2 + c^2}$.

Step 1: Find both intersection points Put the common parameter on the first line equal to t. Then

$$(x, y, z) = (t, 6 - 2t, 5t - 8).$$

On the second line write $(x, y, z) = (5 + 4u, 7 + 3u, -2 + u)$. The x and y equations give $t - 4u = 5$ and $-2t - 3u = 1$, so $u = -1, t = 1$. The z equation also gives -3 on both lines. Hence $A = (1, 4, -3)$. For the third line use $(x, y, z) = (-3 + 6v, 3 - 3v, 6 + v)$. Now $t - 6v = -3$, $-2t + 3v = -3$, which give $v = 1, t = 3$. Both z coordinates are 7, hence $B = (3, 0, 7)$.

Step 2: Use the midpoint in the plane-distance formula The required midpoint is

$$M = \left(\frac{1+3}{2}, \frac{4+0}{2}, \frac{-3+7}{2}\right) = (2, 2, 2).$$

Write the plane with all terms on the left: $2x - 2y + z - 14 = 0$. Its normal has length $\sqrt{4 + 4 + 1} = 3$. Therefore

$$d = \frac{|2(2) - 2(2) + 2 - 14|}{3} = \frac{12}{3} = 4.$$

The absolute value ensures a positive geometric distance. Omitting the plane constant or changing the sign of y produces incorrect results; none of 3, 10/3 and 11/3 equals this distance.

Step 3: Conclusion The distance is $\boxed{4}$ units. Hence, the correct option is (C).

Quick Tip to Solve: Check each intersection in all three coordinates, then take the midpoint before applying the plane-distance formula.

18. Let $S = \left\{x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10\right\}$ and $\beta = \sum_{x \in S} \tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta - 14)^2$ is equal to :

[4 marks]

(A) 16

(B) 32

(C) 8

(D) 64

Correct Answer: (B) 32

Solution:

Concept:

- An exponential equation involving 9^u and 9^{1-u} reduces to a quadratic in the positive variable $t = 9^u$.
- On $(-\pi/2, \pi/2)$, tangent is one-to-one, so each prescribed tangent value produces a unique angle. The identity $\tan(\pi/12) = 2 - \sqrt{3}$ follows from the tangent subtraction formula.

Step 1: Find every member of S Let $t = 9^{\tan^2 x} > 0$. The first exponential is $9/t$, so

$$t + \frac{9}{t} = 10 \quad \Longrightarrow \quad t^2 - 10t + 9 = 0.$$

The roots are $t = 1$ and $t = 9$, yielding $\tan^2 x = 0$ or 1. Within the specified open interval this gives exactly

$$S = \{0, -\pi/4, \pi/4\}.$$

No periodic copies of these angles lie in the allowed interval. In particular, the zero-angle solution must be retained, although its contribution to the sum is zero.

Step 2: Compute beta and simplify the requested expression Using the evenness of the square of tangent,

$$\beta = 0 + 2\tan^2(\pi/12).$$

The subtraction formula gives

$$\tan(\pi/12) = \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} = 2 - \sqrt{3}.$$

Thus

$$\beta = 2(2 - \sqrt{3})^2 = 14 - 8\sqrt{3}.$$

Finally

$$\frac{(\beta - 14)^2}{6} = \frac{(-8\sqrt{3})^2}{6} = \frac{192}{6} = 32.$$

The alternatives 16, 8 and 64 do not agree with this exact squared value. Counting just one of the two nonzero angles would incorrectly halve beta before taking the square.

Step 3: Conclusion The requested value is $\boxed{32}$. Hence, the correct option is (B).

Quick Tip to Solve: List all angles in the stated interval before evaluating a sum over solutions; opposite angles contribute equally to an even function.

19. Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle : [4 marks]

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{12}$

Correct Answer: (B) $\frac{\pi}{3}$

Solution:

Concept:

- Dividing the ellipse by 285 gives $x^2/19 + y^2/15 = 1$; its minor axis is therefore the y-axis.
- A line $y = mx + c$ is tangent to this ellipse when $c^2 = 19m^2 + 15$. Its distance from the centre is $|c|/\sqrt{1+m^2}$, which must equal the circle radius 4.

Step 1: Impose both tangency conditions The ellipse and circle are concentric at the origin. A nonvertical line tangent to the ellipse has

$$c^2 = 19m^2 + 15.$$

Tangency to the circle $x^2 + y^2 = 16$ requires that the perpendicular distance from the origin to the line $mx - y + c = 0$ be exactly 4:

$$\frac{c^2}{m^2+1} = 16.$$

Equating the two forms of c^2 gives

$$19m^2 + 15 = 16m^2 + 16, \qquad 3m^2 = 1.$$

Hence $|m| = 1/\sqrt{3}$. A vertical line could be tangent to the ellipse only at $x = \pm\sqrt{19}$, but circle tangents are $x = \pm 4$; there is no overlooked vertical common tangent.

Step 2: Measure the angle from the minor axis The acute inclination of a common tangent to the x-axis is $\arctan|m| = \pi/6$. The question asks for its angle with the minor axis, which is the y-axis, so take the complementary angle:

$$\theta = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}.$$

Reflection in either coordinate axis changes signs of m or c but leaves this acute angle unchanged. In particular, the option $\pi/6$ corresponds to measuring from the major axis instead. The other alternatives $\pi/4$ and $\pi/12$ would have slopes inconsistent with the simultaneous tangency equations.

Step 3: Conclusion The common tangents make an acute angle $\boxed{\pi/3}$ with the minor axis. Hence, the correct option is (B).

Quick Tip to Solve: Identify which axis is minor before converting a tangent slope into the requested angle; a slope naturally gives the angle from the x-axis.

20. If $\mathrm{A} = \frac{1}{5!6!7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, **then** $|\operatorname{adj}(\operatorname{adj}(2\ \mathrm{A}))|$ **is equal to :** [4 marks]

(A) 2^{12}

(B) 2^{20}

(C) 2^{8}

(D) 2^{16}

Correct Answer: None of the printed options equals the result for the stated matrix.

Solution:

Concept:

- For a three-by-three matrix M and scalar c, $\det(cM) = c^3 \det M$. A scalar outside a matrix multiplies every entry, unlike a scalar outside a determinant.
- For a matrix B of order three, $\det(\operatorname{adj} B) = (\det B)^2$. Applying this twice gives $\det(\operatorname{adj}(\operatorname{adj} B)) = (\det B)^4$.

Step 1: Evaluate the determinant of the factorial matrix Let $P = 5!6!7!$ and denote the printed factorial matrix by M, so the stated matrix is $A = M/P$. Factoring 5!, 6! and 7! from its three rows respectively gives

$$\det M = P\begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}.$$

Subtract the first row from the second and third rows. The remaining two-by-two determinant is

$$\begin{vmatrix} 1 & 14 \\ 2 & 30 \end{vmatrix} = 30 - 28 = 2.$$

Therefore $\det M = 2P$. The scalar $1/P$ in the question is outside a matrix with square brackets; it consequently contributes $1/P^3$ to the determinant, giving $\det A = 2/P^2$.

Step 2: Apply the adjugate identity to the literal matrix For $B = 2A$, its determinant is

$$\det B = 2^3 \det A = \frac{16}{P^2}.$$

Applying the adjugate determinant identity twice yields

$$|\operatorname{adj}(\operatorname{adj}(2A))| = \left(\frac{16}{P^2}\right)^4 = \frac{2^{16}}{(5!6!7!)^8}.$$

This number is smaller than 1, whereas all four printed options are positive powers of 2 greater than 1. Thus none is correct for the matrix as stated. The value 2^{16} would follow from $\det A = 2$, but that requires treating the scalar as a determinant normalization instead of the printed matrix scaling; those are mathematically different operations.

Step 3: Conclusion For the matrix as printed, the result is $\boxed{2^{16}/(5!6!7!)^8}$. None of the listed options is valid; no unique correct option can be selected.

Quick Tip to Solve: For an n by n matrix, move a scalar through a determinant as its nth power. Do not confuse scalar matrix multiplication with row-by-row normalization.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let the tangent at any point P on a curve passing through the points (1, 1) and $\left(\frac{1}{10}, 100\right)$, intersect positive x-axis and y-axis at the points A and B respectively. If $\mathrm{PA} : \mathrm{PB} = 1 : k$ and $y = y(x)$ is the solution of the differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2}, y(0) = k$, then $4y(1) - 6\log_e 3$ is equal to _____.

[4 marks]

Solution:

Concept:

- The intercept form of a tangent with positive intercepts a and b is $x/a + y/b = 1$, and its slope is $-b/a$. Internal division of AB relates those intercepts to the coordinates of P.
- For a positive logarithm argument, $e^{y'} = g(x)$ implies $y' = \ln g(x)$. Integrate this derivative and then impose the specified initial value.

Step 1: Determine k from the first curve Write $P = (x, y)$, $A = (a, 0)$, $B = (0, b)$. Since $PA : PB = 1 : k$, the internal-division formula gives

$$x = \frac{ka}{k+1}, \qquad y = \frac{b}{k+1}.$$

Hence the tangent slope satisfies

$$\frac{dy}{dx} = -\frac{b}{a} = -\frac{ky}{x}.$$

Separating variables yields $\ln y = -k \ln x + C$, so $y = C_1 x^{-k}$ on the positive branch. The point (1,1) gives $C_1 = 1$, and the second point gives $100 = (1/10)^{-k} = 10^k$. Therefore $k = 2$. This constant belongs to the intercept ratio; the subsequent initial-value problem defines a new function y.

Step 2: Integrate the specified differential equation With $k = 2$, the new equation becomes $e^{y'} = 2x+1$, giving $y' = \ln(2x + 1)$ for $x > -1/2$. Use the initial value $y(0) = 2$ and a definite integral:

$$y(1) = 2 + \int_0^1 \ln(2x + 1)\, dx.$$

Substitute $u = 2x + 1$ to obtain

$$\int_0^1 \ln(2x + 1)\, dx = \frac{1}{2}[u \ln u - u]_1^3 = \frac{3 \ln 3 - 2}{2}.$$

Thus $y(1) = 1 + \frac{3}{2} \ln 3$, and the requested combination is

$$4y(1) - 6 \ln 3 = 4 + 6 \ln 3 - 6 \ln 3 = 4.$$

The logarithm remains real throughout the integration interval.

Step 3: Conclusion The intercept ratio gives $k = 2$, and the required numerical answer is $\boxed{4}$.

Quick Tip to Solve: Use separate stages for the curve defining k and the new initial-value problem; the definite integral from zero automatically preserves the initial value.

22. If the area of the region $\{(x, y) : |x^2 - 2| \le y \le x\}$ is A, then $6A + 16\sqrt{2}$ is equal to _____.

[4 marks]

Solution:

Concept:

- The area between an upper curve $y = u(x)$ and lower curve $y = l(x)$ is $\int [u(x) - l(x)]\,dx$, over precisely the interval where the upper curve is above the lower curve.
- The absolute value $|x^2 - 2|$ changes its algebraic expression at $x = \sqrt{2}$ on the nonnegative axis. Thus the area integral must be split there.

Step 1: Find the admissible interval Since $0 \le |x^2 - 2| \le x$, necessarily $x \ge 0$. The inequality is equivalent to

$$-x \le x^2 - 2 \le x.$$

The left inequality gives $(x + 2)(x - 1) \ge 0$, hence $x \ge 1$. The right gives $(x - 2)(x + 1) \le 0$, hence $x \le 2$. Their intersection is $[1, 2]$, not the whole interval from the origin to 2.

Step 2: Integrate the two vertical heights For $1 \le x \le \sqrt{2}$, the lower curve is $2 - x^2$. For $\sqrt{2} \le x \le 2$, it is $x^2 - 2$. Therefore

$$\begin{aligned} A &= \int_1^{\sqrt{2}} (x + x^2 - 2)\,dx + \int_{\sqrt{2}}^2 (x - x^2 + 2)\,dx \\ &= \left[\frac{x^2}{2} + \frac{x^3}{3} - 2x\right]_1^{\sqrt{2}} + \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{\sqrt{2}}^2 \\ &= \left(\frac{13}{6} - \frac{4\sqrt{2}}{3}\right) + \left(\frac{7}{3} - \frac{4\sqrt{2}}{3}\right) \\ &= \frac{9}{2} - \frac{8\sqrt{2}}{3}. \end{aligned}$$

Both integrands are nonnegative on their own intervals, as required for an area.

Step 3: Conclusion The requested quantity is $6A + 16\sqrt{2} = 27 - 16\sqrt{2} + 16\sqrt{2} = \boxed{27}$.

Quick Tip to Solve: First solve both inequalities hidden in the absolute value; then split the integral at the zero of the expression inside it.

23. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to _____.

[4 marks]

Solution:

Concept:

- When arranging a multiset, divide the factorial of the number of positions by the factorial of each repeated-digit multiplicity. The two copies of 2 are indistinguishable.
- The sum of numbers can be obtained by adding the total digit contribution to each place. Symmetry makes that digit sum the same in the thousands, hundreds, tens and units places.

Step 1: Count the distinct arrangements Every number uses the digits 1, 2, 2 and 3 exactly once. As

none is zero, every arrangement is a valid four-digit number. There are

$$\frac{4!}{2!} = 12$$

distinct numbers. Counting $4!$ would distinguish the two identical copies of 2 and count each displayed number twice. We need the sum over distinct numbers, so this repetition correction is essential.

Step 2: Count each digit at a fixed place Fix any one place, such as the units place. If its digit is 1, the remaining multiset is $2, 2, 3$, giving $3!/2! = 3$ arrangements. If its digit is 3, the same argument gives 3 arrangements. If its digit is 2, the remaining digits $1, 2, 3$ are all different, giving $3! = 6$ arrangements. These counts total $3 + 3 + 6 = 12$, checking the enumeration. The sum of the digits appearing in this one place over all numbers is

$$1(3) + 2(6) + 3(3) = 24.$$

The argument applies equally to each of the other three places: fixing a position does not change the permutations of the remaining three positions.

Step 3: Combine the place values Each occurrence contributes its digit times the place value. Thus the total sum is

$$24(1000 + 100 + 10 + 1) = 24(1111) = 26664.$$

As a cross-check, the mean digit in every place is $24/12 = 2$, so the mean number is 2222, and twelve such mean values give the same total.

Step 4: Conclusion The sum of all distinct four-digit numbers is $\boxed{26664}$.

Quick Tip to Solve: For a repeated-digit permutation sum, count digit frequencies in one position and multiply their weighted sum by the repunit 1111.

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24. Let S be the set of values of λ, for which the system of equations

$6\lambda x - 3y + 3z = 4\lambda^2$,

$2x + 6\lambda y + 4z = 1$,

$3x + 2y + 3\lambda z = \lambda$ **has no solution. Then** $12 \sum_{i \in S} |\lambda|$ **is equal to _____.** [4 marks]

Solution:

Concept:

- An invertible coefficient matrix gives a unique solution for every right-hand side. A system can have no solution only when its determinant vanishes.
- For a singular matrix, a linear combination of coefficient rows that is zero must give the same zero combination of right-hand sides. A nonzero result proves inconsistency.

Step 1: Find all singular parameter values The coefficient matrix and right-hand side are

$$M = \begin{pmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{pmatrix}, \qquad b = \begin{pmatrix} 4\lambda^2 \\ 1 \\ \lambda \end{pmatrix}.$$

Expanding the determinant along the first row gives

$$\begin{aligned} \det M &= 6\lambda(18\lambda^2 - 8) + 3(6\lambda - 12) + 3(4 - 18\lambda) \\ &= 108\lambda^3 - 84\lambda - 24 \\ &= 12(\lambda - 1)(3\lambda + 1)(3\lambda + 2). \end{aligned}$$

Thus only $1, -1/3, -2/3$ require a consistency check. Every other value gives a nonzero determinant and therefore cannot belong to S.

Step 2: Test the right-hand sides at each root Write R_1, R_2, R_3 for the coefficient rows. At $\lambda = 1$, the relation $-2R_1 - 3R_2 + 6R_3 = 0$ gives the right-hand-side combination $-8 - 3 + 6 = -5 \neq 0$. Thus no solution exists. At $\lambda = -1/3$, the relation $2R_1 - R_2 + 2R_3 = 0$ gives

$$2\left(\frac{4}{9}\right) - 1 + 2\left(-\frac{1}{3}\right) = -\frac{7}{9} \neq 0.$$

At $\lambda = -2/3$, the relation $16R_1 - R_2 + 22R_3 = 0$ gives

$$16\left(\frac{16}{9}\right) - 1 + 22\left(-\frac{2}{3}\right) = \frac{115}{9} \neq 0.$$

Each is a contradiction of the form $0 = c$, with $c \neq 0$; all three singular values are inconsistent.

Step 3: Conclusion We have $S = \{1, -1/3, -2/3\}$. Therefore $12\sum_{\lambda \in S} |\lambda| = 12(1 + 1/3 + 2/3) = \boxed{24}$.

Quick Tip to Solve: A zero determinant alone does not prove no solution: check a coefficient-row dependence against the constants at every singular value.

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25. If the domain of the function $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha, \beta) \cup (\gamma, \delta]$, then $|3\alpha + 10(\beta + \gamma) + 21\delta|$ is equal to _____. [4 marks]

Solution:

Concept:

- The real inverse secant is defined for inputs in $(-\infty, -1] \cup [1, \infty)$. Consequently its argument must have absolute value at least one.
- A rational argument must also have a nonzero denominator. Squaring an absolute-value inequality is valid here, but it does not restore a point where the original fraction is undefined.

Step 1: Impose the real inverse-secant condition The argument $2x/(5x + 3)$ exists only if $x \neq -3/5$.

For all such x, the domain condition is

$$\left|\frac{2x}{5x+3}\right| \geq 1.$$

The denominator squared is strictly positive on this restricted domain, so squaring and multiplying by it preserves the inequality:

$$4x^2 \geq (5x+3)^2,$$
$$21x^2 + 30x + 9 \leq 0,$$
$$3(7x+3)(x+1) \leq 0.$$

The zeros are -1 and $-3/7$. The quadratic has a positive leading coefficient, so the inequality holds between these roots, including both roots.

Step 2: Remove the pole and identify the endpoints The excluded point $-3/5$ lies strictly between -1 and $-3/7$. Hence the domain is

$$\left[-1, -\frac{3}{5}\right) \cup \left(-\frac{3}{5}, -\frac{3}{7}\right].$$

At $x = -1$, the argument equals 1, and at $x = -3/7$, it equals -1, so both outer endpoints really are included. The two sides of the pole remain open. Comparison with the stated interval notation gives

$$\alpha = -1, \qquad \beta = \gamma = -\frac{3}{5}, \qquad \delta = -\frac{3}{7}.$$

In particular the repeated boundary value must be included twice in the sum $\beta + \gamma$; the two open endpoints represent the same excluded point.

Step 3: Evaluate the required expression Substitution gives

$$|3\alpha + 10(\beta+\gamma) + 21\delta| = \left|-3 + 10\left(-\frac{6}{5}\right) - 9\right| = |-24| = 24.$$

Step 4: Conclusion The value requested is $\boxed{24}$.

Quick Tip to Solve: For inverse secant of a rational function, solve the squared argument inequality and separately delete every pole of the original fraction.

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26. Let the equations of two adjacent sides of a parallelogram ABCD be $2x-3y=-23$ and $5x+4y=23$. If the equation of its one diagonal AC is $3x+7y=23$ and the distance of A from the other diagonal is d, then $50\ \mathrm{d}^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Adjacent side lines meet at a vertex. A diagonal meets those lines at the two vertices on the opposite diagonal pair, allowing the three consecutive vertices to be determined.
- The diagonals of a parallelogram bisect one another, equivalently $A+C = B+D$. The perpendicular distance from (x_0, y_0) to $ax+by+c=0$ is $|ax_0+by_0+c|/\sqrt{a^2+b^2}$.

Step 1: Find the vertices from the three given lines The intersection of the adjacent side lines

$$2x - 3y = -23, \qquad 5x + 4y = 23$$

is $B = (-1, 7)$. It cannot be A or C because it does not lie on the stated diagonal $3x + 7y = 23$. We may call the intersection of the first side with AC the vertex A; reversing A and C will give the same perpendicular distance to BD. Solving $2x - 3y = -23$ with $3x + 7y = 23$ gives

$$A = (-4, 5).$$

Solving $5x + 4y = 23$ with that same diagonal gives

$$C = (3, 2).$$

Substitution checks each intersection in both of its defining equations. Thus the three points are vertices of the given parallelogram, rather than arbitrary points on the lines.

Step 2: Locate the other diagonal Using the parallelogram identity,

$$D = A + C - B = (-4 + 3 + 1,\ 5 + 2 - 7) = (0, 0).$$

The other diagonal joins $B = (-1, 7)$ to the origin. Its direction vector is $(-1, 7)$, so a normal vector is $(7, 1)$, giving

$$BD: \quad 7x + y = 0.$$

As a check, both D and B satisfy this equation. Their midpoint $(-1/2, 7/2)$ is also the midpoint of AC, confirming the diagonal construction.

Step 3: Calculate the perpendicular distance Apply the point-to-line formula to A and BD:

$$d = \frac{|7(-4) + 5|}{\sqrt{7^2 + 1^2}} = \frac{23}{\sqrt{50}}.$$

The distance from C gives $|21 + 2|/\sqrt{50}$, the same value, so the allowed interchange of A and C has no effect on the answer.

Step 4: Conclusion Thus $50d^2 = 50 \cdot 23^2/50 = \boxed{529}$.

Quick Tip to Solve: Use the midpoint identity to find the fourth vertex; then the second diagonal is simply the line through that vertex and the common side intersection.

27. Let the foot of perpendicular from the point $\mathrm{A}(4,3,1)$ on the plane $\mathrm{P}: x-y+2z+3=0$ be N. If $\mathrm{B}(5,\alpha,\beta), \alpha,\beta \in \mathbb{Z}$ is a point on plane P such that the area of the triangle ABN is $3\sqrt{2}$, then $\alpha^2+\beta^2+\alpha\beta$ is equal to _____. [4 marks]

Solution:

Concept:

- The vector normal to the plane $x-y+2z+3=0$ is $n=(1,-1,2)$, with $n\cdot n=6$. The perpendicular projection subtracts the signed normal component from the point.
- Since N and B lie in the plane and AN is perpendicular to it, triangle ABN is right angled at N. Its area is $AN \cdot BN/2$, so its area fixes the length BN.

Step 1: Compute the foot and the perpendicular length At $A=(4,3,1)$, the plane expression is

$$4-3+2(1)+3=6.$$

Thus the foot is

$$N=A-\frac{6}{6}(1,-1,2)=(3,4,-1).$$

This lies on the plane because $3-4-2+3=0$, and $A-N=(1,-1,2)$ is parallel to its normal. Consequently $AN=\sqrt{6}$. These two checks ensure we have the actual perpendicular foot, not merely a point in the plane.

Step 2: Translate the area and plane conditions The area is $3\sqrt{2}$, so

$$\frac{1}{2}\sqrt{6}\,BN=3\sqrt{2} \quad\Longrightarrow\quad BN=2\sqrt{3}.$$

For $B=(5,\alpha,\beta)$, the distance from N therefore gives

$$4+(\alpha-4)^2+(\beta+1)^2=12.$$

The requirement that B lies on the plane is

$$5-\alpha+2\beta+3=0, \qquad \alpha=8+2\beta.$$

Substitution eliminates α:

$$\begin{aligned}(4+2\beta)^2+(\beta+1)^2&=8,\\ 5\beta^2+18\beta+9&=0,\\ (5\beta+3)(\beta+3)&=0.\end{aligned}$$

Step 3: Use the integer restriction The possible real values are $\beta=-3$ and $\beta=-3/5$. Since both parameters must be integers, the second value is inadmissible. Hence $\beta=-3$ and $\alpha=8-6=2$. In fact, B becomes $(5,2,-3)$, for which $BN^2=4+4+4=12$, confirming the prescribed area.

Step 4: Conclusion The required number is $\alpha^2+\beta^2+\alpha\beta=4+9-6=\boxed{7}$.

Quick Tip to Solve: Project A onto the plane first: the area then becomes the area of a right triangle and supplies an ordinary distance equation in the plane.

28. Suppose $a_1, a_2, 2, a_3, a_4$ be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to _____. [4 marks]

Solution:

Concept:

- An arithmetico-geometric progression has terms $(a + (n-1)d)r^{n-1}$, where the linear factor is an arithmetic progression and the powers form a geometric progression.
- The corresponding common ratio is $r = 2$. It is not the common ratio of the full sequence, because its arithmetic factors generally change from term to term.

Step 1: Express the five terms correctly Absorb any initial geometric factor into a and d. The five terms can then be written

$$a, \quad 2(a+d), \quad 4(a+2d), \quad 8(a+3d), \quad 16(a+4d).$$

The third term is given as 2, so

$$4(a+2d) = 2, \qquad a + 2d = \frac{1}{2}.$$

This equation uses the central term of the actual arithmetico-geometric progression. Treating the entire sequence as a geometric progression would ignore the equally spaced arithmetic factors and need not satisfy the stated sum.

Step 2: Apply the total-sum condition Adding the five expressions, collect separately the coefficients of a and d:

$$\begin{aligned} \text{sum} &= (1+2+4+8+16)a \\ &\quad + (2+8+24+64)d \\ &= 31a + 98d = \frac{49}{2}. \end{aligned}$$

From the central-term condition, $a = 1/2 - 2d$. Therefore

$$31\left(\frac{1}{2} - 2d\right) + 98d = \frac{49}{2},$$

which simplifies to $36d = 9$. Thus $d = 1/4$ and $a = 0$. A zero first arithmetic factor is allowed; the geometric component still has common ratio 2.

Step 3: Evaluate and check the final term The last term, denoted by a_4 in the question, is the fifth term of our indexed formula. Hence

$$a_4 = 16(a+4d) = 16(0+1) = 16.$$

The full five terms are $0, 1/2, 2, 6, 16$. They sum to $49/2$, while their arithmetic factors $0, 1/4, 1/2, 3/4, 1$ have constant difference $1/4$. This verifies both defining conditions.

Step 4: Conclusion The requested last term is $\boxed{16}$.

Quick Tip to Solve: Write each term as a linear factor times a power of 2, and use the known middle term before expanding the sum.

29. Let the quadratic curve passing through the point $(-1, 0)$ and touching the line $y = x$ at $(1, 1)$ be $y = f(x)$. Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha + 1)$ in the first quadrant is _____. [4 marks]

Solution:

Concept:

- A quadratic function has the form $f(x) = ax^2 + bx + c$. Passing through a point fixes its function value, while touching a given line fixes both the value and the derivative at the contact point.
- The normal is perpendicular to the tangent. If the tangent slope is nonzero, the normal slope is $-1/f'(x)$. Its x-intercept is found by setting $y = 0$ in its equation.

Step 1: Determine the quadratic The point $(-1, 0)$ gives $a - b + c = 0$. Contact with $y = x$ at $(1, 1)$ gives $a + b + c = 1$ and $2a + b = 1$. Subtracting the first value equation from the second yields $2b = 1$, hence $b = 1/2$. Then $a = 1/4$ and $c = 1/4$. Therefore

$$f(x) = \frac{x^2 + 2x + 1}{4} = \frac{(x + 1)^2}{4}.$$

Step 2: Locate the first-quadrant point The specified point must satisfy both the curve and $y = x + 1$, so

$$\frac{(\alpha + 1)^2}{4} = \alpha + 1 \quad \Longrightarrow \quad (\alpha + 1)(\alpha - 3) = 0.$$

The root $\alpha = -1$ gives $(-1, 0)$, which is not in the first quadrant. Thus $\alpha = 3$ and the required point is $(3, 4)$.

Step 3: Find the normal intercept Differentiate: $f'(x) = (x + 1)/2$. At $x = 3$, the tangent slope is 2, so the normal slope is $-1/2$. Its point-slope equation is

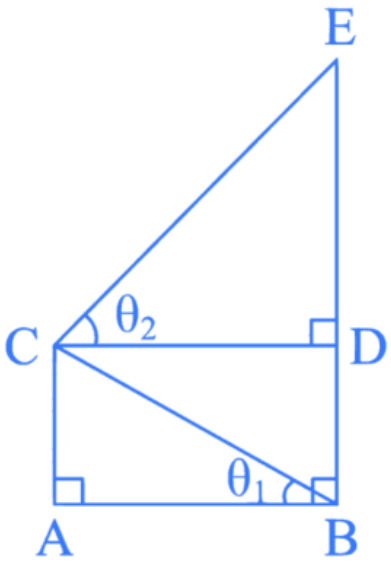
$$y - 4 = -\frac{1}{2}(x - 3).$$

At the x-axis, $y = 0$, and hence $-4 = -(x - 3)/2$, giving $x = 11$. Substitution back into the normal equation verifies the intercept.

Step 4: Conclusion The normal meets the x-axis at $(11, 0)$. The required numerical answer is $\boxed{11}$.

Quick Tip to Solve: Use the tangent slope as the third condition on the quadratic, then discard any intersection outside the stated quadrant before finding the normal.

30. In the figure, $\theta_1 + \theta_2 = \frac{\pi}{2}$ and $\sqrt{3}(\mathrm{BE}) = 4(\mathrm{AB})$. If the area of $\triangle\mathrm{CAB}$ is $2\sqrt{3} - 3$ unit ^{2}, when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter (in unit) of $\triangle\mathrm{CED}$ is equal to _____. [4 marks]



Solution:

Concept:

- For the right triangle CAB, let $AB = x > 0$ and $AC = y > 0$. Then $\tan\theta_1 = y/x$, and its area is $xy/2$. The rectangle in the figure gives $CD = AB = x$ and $BD = AC = y$.
- Complementary acute angles satisfy $\tan\theta_2 = 1/\tan\theta_1$. The two possible angle pairs must both be examined before using the condition that θ_2/θ_1 is largest.

Step 1: Relate the heights and angles Since $\sqrt{3}\,BE = 4AB$, the total right-side height is $BE = 4x/\sqrt{3}$. Therefore $ED = 4x/\sqrt{3} - y$. Write $t = y/x = \tan\theta_1 > 0$. In right triangle CED,

$$\tan\theta_2 = \frac{ED}{CD} = \frac{4}{\sqrt{3}} - t.$$

Using complementarity, $t + 1/t = 4/\sqrt{3}$.

Step 2: Choose the required angle pair The preceding equation gives

$$\sqrt{3}t^2 - 4t + \sqrt{3} = 0, \qquad t = \sqrt{3} \text{ or } \frac{1}{\sqrt{3}}.$$

These correspond respectively to $(\theta_1, \theta_2) = (\pi/3, \pi/6)$ and $(\pi/6, \pi/3)$. Their ratios are $1/2$ and 2. The largest ratio is therefore obtained for $y = x/\sqrt{3}$, giving $ED = \sqrt{3}x$.

Step 3: Use the area and add the sides The area condition becomes

$$\frac{x^2}{2\sqrt{3}} = 2\sqrt{3} - 3, \qquad x^2 = 12 - 6\sqrt{3} = (3 - \sqrt{3})^2.$$

Lengths are positive, so $x = 3 - \sqrt{3}$. Triangle CED has legs x and $\sqrt{3}x$, so its hypotenuse is $2x$. Its

perimeter is

$$x + \sqrt{3}x + 2x = (3 + \sqrt{3})(3 - \sqrt{3}) = 6.$$

Step 4: Conclusion The required perimeter of $\triangle CED$ is $\boxed{6}$ units.

Quick Tip to Solve: For complementary angles, replace the two tangents by t and $1/t$. Compare both angle pairs before applying the area condition.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A person travels x distance with velocity v_1 and then x distance with velocity v_2 in the same direction. The average velocity of the person is v, then the relation between v, v_1 and v_2 will be.

[4 marks]

(A) $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$
(B) $V = \frac{v_1 + V_2}{2}$
(C) $\frac{1}{\mathrm{v}} = \frac{1}{\mathrm{v}_1} + \frac{1}{\mathrm{v}_2}$
(D) $\frac{2}{\mathrm{V}} = \frac{1}{\mathrm{v}_1} + \frac{1}{\mathrm{v}_2}$
Correct Answer: (D) $\frac{2}{\mathrm{V}} = \frac{1}{\mathrm{v}_1} + \frac{1}{\mathrm{v}_2}$

Solution:

Concept:

- Average velocity is total displacement divided by total elapsed time, not an average of the two speed values unless the corresponding times are equal.
- For travel over a distance x at constant positive velocity v_i, the elapsed time is x/v_i. Here the two displacements are in the same direction, so they add without cancellation.

Step 1: Find the time for each equal distance The first leg takes $t_1 = x/v_1$ and the second takes $t_2 = x/v_2$. Thus

$$t_{\text{total}} = \frac{x}{v_1} + \frac{x}{v_2}.$$

Although the distances are equal, these times generally differ. The slower leg takes longer and consequently has greater weight in the time average.

Step 2: Divide total displacement by total time The total displacement is $2x$. Hence

$$v = \frac{2x}{x/v_1 + x/v_2} = \frac{2v_1v_2}{v_1 + v_2}.$$

Taking the reciprocal and rearranging gives

$$\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}.$$

The common distance cancels, so the result depends only on the two velocities.

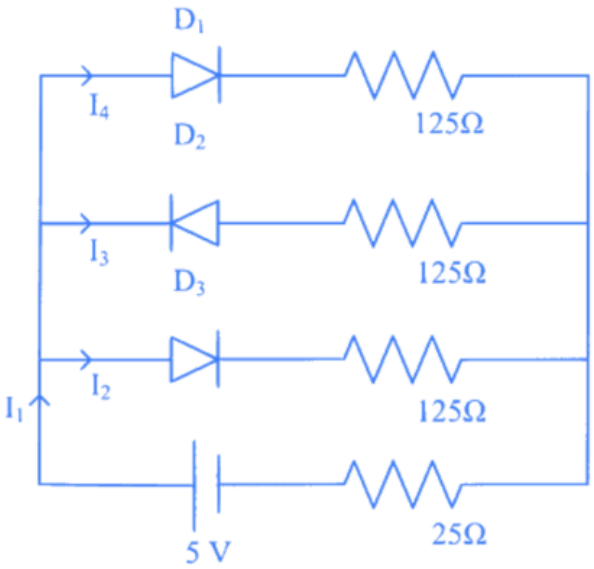
Step 3: Check the alternatives Option (D) is precisely the derived relation. Option (A) incorrectly

adds velocities from separate legs. Option (B) is the arithmetic mean, which applies to equal travel times rather than equal travel distances. Option (C) misses the factor of two arising from total displacement $2x$. As a simple check, if $v_1 = v_2 = u$, the average must be u; (A) gives $2u$ and (C) gives $u/2$. The harmonic mean also correctly lies between the two positive velocities.

Step 4: Conclusion The required relation is $\boxed{2/v = 1/v_1 + 1/v_2}$. Hence, the correct option is (D).

Quick Tip to Solve: For equal distances use the harmonic mean; for equal times use the arithmetic mean. Always start from total displacement divided by total time.

32. If each diode has a forward bias resistance of $25\ \Omega$ in the below circuit,
Which of the following options is correct : [4 marks]



(A) $\frac{I_3}{I_4} = 1$
(B) $\frac{I_1}{I_2} = 2$
(C) $\frac{I_2}{I_3} = 1$
(D) $\frac{I_1}{I_2} = 1$
Correct Answer: (B) $\frac{I_1}{I_2} = 2$

Solution:

Concept:

- A forward-biased diode conducts through its stated forward resistance, while a reverse-biased diode is treated as an open circuit in this model. The battery polarity determines which branches conduct.
- Parallel conducting branches have the same terminal voltage. Equal branch resistances therefore carry equal currents, and Kirchhoff's junction law gives the current entering their junction.

Step 1: Identify the conducting branches The long battery plate is on the left, so the left rail is

at higher potential. Diodes D_1 and D_3 are oriented to conduct from left to right and are forward biased. Diode D_2 faces the opposite way and is reverse biased. Therefore $I_3 = 0$. The forward-biased branches carry currents I_4 and I_2, respectively.

Step 2: Find the branch and total currents Each conducting branch has resistance

$$R_b = 125 + 25 = 150\,\Omega.$$

The two equal branches are in parallel, giving $R_p = 75\,\Omega$. Including the bottom $25\,\Omega$ series resistor, the battery sees $100\,\Omega$. Therefore

$$I_1 = \frac{5}{100} = 0.050\,\text{A}, \qquad I_2 = I_4 = 0.025\,\text{A}.$$

The parallel-network voltage is $I_1 R_p = 3.75\,\text{V}$, which confirms that the assumed rail polarity is consistent.

Step 3: Compare the proposed ratios At the left junction, $I_1 = I_2 + I_3 + I_4 = 2I_2$. Consequently $I_1/I_2 = 2$, as in option (B). Option (A) is false because $I_3/I_4 = 0$. Option (C) divides a nonzero I_2 by the zero reverse-branch current and cannot equal one. Option (D) would require all the entering current to use only one branch, contradicting the two identical conducting paths.

Step 4: Conclusion $\boxed{I_1/I_2 = 2}$. Hence, the correct option is (B).

Quick Tip to Solve: First mark each diode as conducting or open from the battery polarity. Only then combine the resistors; equal surviving parallel branches split the total current equally.

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33. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : An electric fan continues to rotate for some time after the current is switched off.

Reason R : Fan continues to rotate due to inertia of motion.

In the light of above statements, choose the most appropriate answer from the options given below.

[4 marks]

(A) A is not correct but R is correct

(B) A is correct but R is not correct

(C) Both A and R are correct and R is the correct explanation of A

(D) Both A and R are correct but R is NOT the correct explanation of A

Correct Answer: (C) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- A rotating body has angular momentum $L = I\omega$, where I is its moment of inertia and ω is its angular speed. Its rotation changes in response to external torque: $\tau_{\text{net}} = I\alpha$ for fixed I.
- Inertia is the tendency to retain the current state of motion. Switching off a motor removes its driving torque but does not instantly remove the rotating blades'angular velocity.

Step 1: Evaluate the assertion Immediately before switching off, the fan blades have nonzero angular velocity. On disconnecting the current, the electromagnetic driving torque disappears. There is no impulsive braking torque that could bring the blades instantly to rest. They therefore keep rotating for a while. Friction in the bearings and air resistance slow them gradually. Thus Assertion A is correct.

Step 2: Evaluate the reason The blades and rotor possess rotational inertia. Their existing rotational motion persists even after the source supplying energy is removed, while resisting torques progressively reduce their angular momentum. This is the rotational form of inertia of motion, so Reason R is also correct. Continued rotation does not imply that electrical current is still flowing or that a fresh driving force is required at every instant.

Step 3: Check whether the reason explains the assertion The reason identifies exactly why the fan does not stop immediately: its inertia resists an abrupt change in its rotational state. Resistive torque explains why it ultimately stops, and does not invalidate inertia as the explanation for temporary continued rotation. Hence the reason directly explains the assertion. Options (A) and (B) each incorrectly deny one true statement, while option (D) incorrectly denies this explanatory connection.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish removal of driving torque from application of a stopping impulse. A fan coasts through inertia, then slows because friction and air resistance still act.

34. In a metallic conductor, under the effect of applied electric field, the free electrons of the conductor

[4 marks]

(A) move in the straight line paths in the same direction

(B) move with the uniform velocity throughout from lower potential to higher potential

(C) drift from higher potential to lower potential.

(D) move in the curved paths from lower potential to higher potential

Correct Answer: (D) move in the curved paths from lower potential to higher potential

Solution:

Concept:

- An electric field points from higher electric potential to lower electric potential. An electron has charge $-e$, so its electric force $\mathbf{F} = -e\mathbf{E}$ is opposite to the field.
- Conduction electrons retain rapid random thermal motion and undergo collisions with the lattice. The field adds a small mean drift velocity, $\mathbf{v}_d = -e\tau\mathbf{E}/m$, opposite to $\mathbf{E}$, where τ is the mean relaxation time.

Step 1: Find the net direction of electron motion Since the electric field points towards decreasing potential and the electron force points against the field, the average electron drift is from lower potential towards higher potential. The direction of conventional current is opposite to this electron drift. Therefore the higher-to-lower potential drift proposed in option (C) has the wrong sign.

Step 2: Describe the actual trajectories Individual electrons are not carried in one straight ordered

stream. Between successive collisions their velocities change under the electric force, while collisions repeatedly scatter them into different directions. Their motion combines random thermal motion with directed drift. The resulting paths are irregular, with accelerated segments generally curved in the field, and an overall drift from lower to higher potential. Option (D) expresses this qualitative description.

Step 3: Eliminate the remaining options Option (A) ignores both the random initial velocity directions and repeated scattering. Option (B) has the correct overall potential direction but incorrectly claims uniform velocity throughout: each individual electron accelerates between collisions and changes velocity in collisions. A steady average drift velocity is a statistical average, not an unchanging velocity of every electron. Thus neither ordered straight-line motion nor uniform individual motion is the correct microscopic picture.

Step 4: Conclusion The electrons follow curved, scattered paths with net drift from lower potential to higher potential. Hence, the correct option is (D).

Quick Tip to Solve: Use the electron's negative charge to fix the drift direction, then distinguish its small average drift from its much faster random microscopic motion.

35. A gas mixture consists of 2 moles of oxygen and 4 moles of neon at temperature T. Neglecting all vibrational modes, the total internal energy of the system will be, [4 marks]

(A) 4RT

(B) 16RT

(C) 8RT

(D) 11RT

Correct Answer: (D) 11RT

Solution:

Concept:

- For an ideal gas with f active quadratic degrees of freedom, equipartition gives molar internal energy $fRT/2$, so n moles have $U = nfRT/2$. Internal energies of gas components add.
- Oxygen is diatomic: with vibrational modes excluded it has three translational and two rotational degrees of freedom, hence $f_{O_2} = 5$. Neon is monatomic and has only three translational degrees of freedom, hence $f_{Ne} = 3$.

Step 1: Compute the oxygen contribution The mixture contains 2 moles of oxygen. Each molecule has the five active degrees specified by the non-vibrating diatomic model. Its total internal energy is therefore

$$U_{O_2} = 2\left(\frac{5}{2}RT\right) = 5RT.$$

Counting oxygen as monatomic would omit its two rotational modes. Including vibration would add energy that the question explicitly instructs us to neglect.

Step 2: Compute the neon contribution Neon atoms contribute three independent translational kinetic-energy terms per atom. Their monatomic nature means that no molecular rotational or

vibrational contribution is counted. For the 4 moles present,

$$U_{\text{Ne}} = 4\left(\frac{3}{2}RT\right) = 6RT.$$

Both components have the same temperature T, so the same factor RT appears in their energies.

Step 3: Add the component energies Internal energy is extensive and additive for this ideal-gas mixture:

$$U = U_{\text{O}_2} + U_{\text{Ne}} = 5RT + 6RT = 11RT.$$

Thus option (D) matches. None of the coefficients 4, 16 or 8 in the other options equals the required mole-weighted sum $(2 \times 5 + 4 \times 3)/2 = 11$. One must weight degrees of freedom by the number of moles rather than using a single unweighted molecular value for the whole mixture.

Step 4: Conclusion The total internal energy is $\boxed{11RT}$. Hence, the correct option is (D).

Quick Tip to Solve: For a gas mixture, calculate $n_i f_i RT/2$ for each component separately and add. Here use five active modes for oxygen and three for neon.

36. A bar magnet is released from rest along the axis of a very long vertical copper tube. After some time the magnet will [4 marks]

(A) move down with almost constant speed

(B) move down with an acceleration equal to g

(C) move down with an acceleration greater than g

(D) oscillate inside the tube

Correct Answer: (A) move down with almost constant speed

Solution:

Concept:

- A changing magnetic flux through a conducting loop induces an emf: $\mathcal{E} = -d\Phi_B/dt$. In the copper wall this produces closed eddy-current loops.
- Lenz's law states that the induced currents oppose the change producing them. Thus the force on a moving magnet opposes its velocity; electrical resistance converts mechanical energy into heat.

Step 1: Identify the induced currents As the magnet descends, the magnetic flux through different circular portions of the copper tube changes. The tube is a conductor even though copper is not ferromagnetic. Eddy currents therefore appear in its wall. Their fields oppose the approaching magnet below it and the receding magnet above it, producing a net upward magnetic drag. This is electromagnetic braking, not attraction to a magnetic material.

Step 2: Balance the forces after the initial transient Initially the magnet is at rest and its induced drag is zero. Gravity starts its downward motion. As its speed grows, flux changes more quickly and

the induced current and opposing force grow. Taking downward as positive gives

$$m\frac{dv}{dt} = mg - F_{\text{drag}}(v).$$

In a very long uniform tube, the magnet eventually reaches a terminal speed v_t for which $F_{\text{drag}}(v_t) = mg$. Its acceleration is then almost zero while its speed remains nonzero. Gravitational potential energy continues to become Joule heat in the copper.

Step 3: Reject the other proposed motions Acceleration equal to g would require no magnetic drag, contrary to the induced currents. Acceleration greater than g would require an extra downward force, whereas the induced force points upward. There is no restoring equilibrium that would sustain oscillation inside the uniform tube; resistance dissipates energy.

Step 4: Conclusion The magnet moves downward with an almost constant terminal speed after some time. Hence, the correct option is (A).

Quick Tip to Solve: For a magnet falling through a conducting tube, first use Lenz's law to find drag, then set that drag equal to weight at terminal speed.

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37. Two projectiles are projected at 30° and 60° with the horizontal with the same speed. The ratio of the maximum height attained by the two projectiles respectively is: [4 marks]

(A) $1 : \sqrt{3}$

(B) $\sqrt{3} : 1$

(C) $1 : 3$

(D) $2 : \sqrt{3}$

Correct Answer: (C) $1 : 3$

Solution:

Concept:

- For a projectile launched with speed u at angle θ, the initial vertical component is $u_y = u\sin\theta$. Gravity changes only this component when air resistance is neglected.
- At the highest point the vertical speed vanishes. The constant-acceleration relation gives $H = u_y^2/(2g) = u^2\sin^2\theta/(2g)$, measured above the launch point.

Step 1: Obtain the height for each launch angle Apply $v_y^2 = u_y^2 - 2gH$ between launch and the highest point. Since $v_y = 0$, the height depends on the square of the vertical component, not on the horizontal range. For the first projectile,

$$H_{30} = \frac{u^2\sin^2 30^\circ}{2g} = \frac{u^2}{8g}.$$

For the second projectile,

$$H_{60} = \frac{u^2\sin^2 60^\circ}{2g} = \frac{3u^2}{8g}.$$

Both projectiles have the same launch speed and experience the same gravitational acceleration.

Step 2: Form the ordered ratio The question asks for the heights at 30° and 60°, respectively. Divide in that order:

$$\frac{H_{30}}{H_{60}} = \frac{u^2/(8g)}{3u^2/(8g)} = \frac{1}{3}.$$

The larger launch angle has the larger vertical component and therefore the larger height, which provides a useful physical check on the ordering.

Step 3: Check the distractors The ratio $1 : \sqrt{3}$ compares the vertical launch speeds rather than their squared values. The ratio $\sqrt{3} : 1$ additionally reverses the relative sizes. The value $2 : \sqrt{3}$ does not follow from either vertical component or maximum-height relation. Complementary angles give equal horizontal ranges, but they do not give equal maximum heights.

Step 4: Conclusion The required maximum-height ratio is $H_{30} : H_{60} = 1 : 3$. Hence, the correct option is (C).

Quick Tip to Solve: At a common launch speed, maximum heights are proportional to $\sin^2\theta$; square the trigonometric ratio before comparing heights.

38. A gas is compressed adiabatically, which one of the following statement is NOT true. [4 marks]

(A) There is no heat supplied to the system

(B) The temperature of the gas increases.

(C) There is no change in the internal energy

(D) The change in the internal energy is equal to the work done on the gas.

Correct Answer: (C) There is no change in the internal energy

Solution:

Concept:

- An adiabatic process has no heat exchange: $Q = 0$. This condition concerns energy transfer as heat and does not imply constant internal energy.
- Using work done on the gas, the first law is $\Delta U = Q + W_{\text{on}}$. Compression gives positive work on the gas and therefore raises its internal energy.

Step 1: Apply the adiabatic condition The gas is compressed without heat crossing its boundary. Thus option (A), saying that no heat is supplied to the system, is true. Energy may still enter by mechanical work: a piston moving inward exerts a force through a displacement on the gas. Adiabatic and isothermal are different conditions, so the absence of heat transfer is not evidence that temperature or internal energy stays fixed.

Step 2: Use the work sign consistently For compression the volume decreases. The work done by the gas is $W_{\text{by}} = \int P\,dV < 0$, while $W_{\text{on}} = -W_{\text{by}} > 0$. The first law then gives

$$\Delta U = Q - W_{\text{by}} = W_{\text{on}} > 0.$$

Therefore option (D), equating the internal-energy increase to work done on the gas, is true. Option (C), claiming no internal-energy change, contradicts this energy balance.

Step 3: Check the temperature statement For the usual ideal-gas description, $\Delta U = nC_V\Delta T$, with $C_V > 0$. The positive internal-energy change implies $\Delta T > 0$. Equivalently, a reversible adiabatic compression obeys $TV^{\gamma-1} =$ constant, so reducing V raises T. Thus option (B) is true. The question asks for the statement that is NOT true, so the false internal-energy claim must be selected.

Step 4: Conclusion Adiabatic compression increases internal energy through work done on the gas. The statement that there is no change in internal energy is false. Hence, the correct option is (C).

Quick Tip to Solve: In an adiabatic problem set $Q = 0$, then apply the first law; never replace zero heat exchange by zero internal-energy change.

39. For a periodic motion represented by the equation

$$y = \sin\omega t + \cos\omega t$$

the amplitude of the motion is [4 marks]

(A) 1

(B) $\sqrt{2}$

(C) 0.5

(D) 2

Correct Answer: (B) $\sqrt{2}$

Solution:

Concept:

- A simple harmonic displacement can be written as $y = A\sin(\omega t+\phi)$. Its amplitude A is the greatest displacement magnitude, independent of the phase constant ϕ.
- For $y = a\sin\omega t + b\cos\omega t$, the two terms are in phase quadrature. Comparing coefficients gives $A = \sqrt{a^2+b^2}$.

Step 1: Combine the two trigonometric terms Expand a single sinusoid using the angle-addition identity:

$$A\sin(\omega t+\phi) = A\cos\phi\sin\omega t + A\sin\phi\cos\omega t.$$

Comparing with the stated displacement, both coefficients are one. Hence $A\cos\phi = 1$ and $A\sin\phi = 1$. Squaring and adding eliminates the unknown phase because $\sin^2\phi + \cos^2\phi = 1$.

Step 2: Compute and verify the amplitude The coefficient equations give

$$A^2 = 1^2 + 1^2 = 2, \qquad A = \sqrt{2}.$$

The phase can be chosen as $\phi = \pi/4$, so the full displacement becomes

$$y = \sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right).$$

Since the sine ranges from -1 to 1, the displacement ranges from $-\sqrt{2}$ to $\sqrt{2}$. This directly confirms

the amplitude.

Step 3: Compare with the proposed values The value one is only the amplitude of either separate term, not their resultant. Directly adding amplitudes to get two would require the two oscillations to be in phase; sine and cosine of the same argument differ in phase by $\pi/2$. The value 0.5 is also inconsistent with the actual maximum: at $\omega t = \pi/4$, each term is $1/\sqrt{2}$, so their sum is $\sqrt{2}$.

Step 4: Conclusion The motion has amplitude $\sqrt{2}$ in the displacement units used. Hence, the correct option is (B).

Quick Tip to Solve: For a sine term plus a cosine term of the same frequency, use $A = \sqrt{a^2 + b^2}$, not the arithmetic sum of coefficients.

40. The amplitude of magnetic field in an electromagnetic wave propagating along y-axis is 6.0×10^{-7} T. The maximum value of electric field in the electromagnetic wave is [4 marks]

(A) 6.0×10^{-7} Vm^{-1}

(B) 5×10^{14} Vm^{-1}

(C) 180 Vm^{-1}

(D) 2×10^{15} Vm^{-1}

Correct Answer: (C) 180 Vm^{-1}

Solution:

Concept:

- For an electromagnetic wave in vacuum or air, the electric and magnetic fields oscillate in phase and their amplitudes obey $E_0 = cB_0$. Here $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$.
- The maximum field value is its amplitude. The propagation direction fixes the orientation of the transverse fields but does not change the scalar amplitude relation.

Step 1: Identify the amplitude relation The specified magnetic-field amplitude is $B_0 = 6.0 \times 10^{-7}\,\mathrm{T}$. We require the maximum electric field, so use E_0, not an rms value. For a sinusoidal electromagnetic wave, $E(t)/B(t) = c$ where the ratio is defined, and the same ratio holds between the amplitudes. No factor of $\sqrt{2}$ enters when converting one amplitude to the other.

Step 2: Substitute the magnetic amplitude Multiply the magnetic amplitude by the propagation speed:

$$E_0 = (3.0 \times 10^8)(6.0 \times 10^{-7})\,\mathrm{V\,m^{-1}}.$$

Combine the numerical factors and the powers separately:

$$E_0 = 18.0 \times 10^1\,\mathrm{V\,m^{-1}} = 180\,\mathrm{V\,m^{-1}}.$$

The unit check follows from $1\,\mathrm{T} = 1\,\mathrm{V\,s\,m^{-2}}$, so multiplying by metres per second gives volts per metre.

Step 3: Check direction and alternative values Propagation along the y-axis means both fields are perpendicular to that axis and to each other. It introduces no change in their amplitude ratio. Option

(A) simply copies the magnetic numerical value into electric units and omits the factor c. Options (B) and (D) have enormous positive powers inconsistent with multiplying 10^{-7} by 10^{8}; that product has power 10^{1}.

Step 4: Conclusion The maximum electric field is $180\,\mathrm{V\,m^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $E_0 = cB_0$ for amplitudes; combine the powers of ten before the numerical factors to avoid an exponent error.

41. The time period of a satellite, revolving above earth's surface at a height equal to R will be (Given $g = \pi^2\ \mathrm{m/s^2}, \mathrm{R} =$ radius of earth) [4 marks]

(A) $\sqrt{32R}$

(B) $\sqrt{4\mathrm{R}}$

(C) $\sqrt{8R}$

(D) $\sqrt{2R}$

Correct Answer: (A) $\sqrt{32R}$

Solution:

Concept:

- For a circular orbit of radius r, gravity provides centripetal force: $GMm/r^2 = mv^2/r$. Consequently $T = 2\pi\sqrt{r^3/(GM)}$.
- The surface gravitational acceleration is $g = GM/R^2$, where R is Earth's radius. The orbital radius is measured from Earth's centre, so $r = R + h$.

Step 1: Convert the given altitude into orbital radius The altitude above the surface is $h = R$. Therefore the distance of the satellite from the centre of Earth is

$$r = R + h = 2R.$$

It would be incorrect to put $r = R$, since that would describe an orbit at the surface instead of the stated height. Also, the supplied g is the surface value; it is used to express $GM = gR^2$, not as the gravitational acceleration at the satellite.

Step 2: Derive the period at this radius Substitute the radius and the surface-gravity relation into the circular-orbit formula:

$$T = 2\pi\sqrt{\frac{(2R)^3}{gR^2}} = 2\pi\sqrt{\frac{8R}{g}}.$$

Equivalently, the orbital gravitational acceleration is $g/4$, and the condition $v^2/(2R) = g/4$ gives the same speed and period. The satellite mass cancels because gravitational and inertial masses occur in the same proportion.

Step 3: Insert the prescribed numerical value of gravity Using $g = \pi^2\,\mathrm{m\,s^{-2}}$, with R expressed in

metres, the period in seconds has numerical value

$$T = 2\pi\sqrt{\frac{8R}{\pi^2}} = 2\sqrt{8R} = \sqrt{32R}.$$

This matches the first expression. The other options have incorrect numerical factors, arising from confusing altitude with orbital radius or omitting part of the 2π period factor.

Step 4: Conclusion For R entered in metres, the period is $T = \sqrt{32R}$ seconds. Hence, the correct option is (A).

Quick Tip to Solve: For a satellite, start with $r = R + h$, then replace GM by gR^2; surface gravity is not the gravity at orbital height.

42. Given below are two statements:

Statement I : For diamagnetic substance, $-1 \leq \chi < 0$, where χ is the magnetic susceptibility.

Statement II : Diamagnetic substances when placed in an external magnetic field, tend to move from stronger to weaker part of the field.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is correct but Statement II is false

(C) Both Statement I and Statement II are False

(D) Statement I is incorrect but Statement II is true.

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Magnetic susceptibility is defined by $\mathbf{M} = \chi\mathbf{H}$ for a linear isotropic material. A diamagnetic response has magnetisation opposite to the applied field, so $\chi < 0$.
- In SI units $\mu_r = 1 + \chi$. Ordinary diamagnets have small negative susceptibility; the ideal complete-flux-expulsion limit corresponds to $\chi = -1$.

Step 1: Evaluate the susceptibility statement An applied field induces magnetic moments opposing the field in a diamagnetic substance. Its susceptibility is therefore negative rather than positive. The relative permeability is less than one and is nonnegative in the elementary static description. Thus $\mu_r = 1 + \chi$ gives

$$-1 \leq \chi < 0.$$

Most ordinary diamagnetic substances have $|\chi| \ll 1$, while the lower endpoint represents perfect diamagnetism. Statement I correctly includes this limiting value.

Step 2: Evaluate motion in a nonuniform field A diamagnetic specimen is repelled from stronger field regions. For a small linear specimen of volume V in a weakly varying field, its magnetic potential

energy may be written approximately as

$$U = -\frac{\chi V}{2\mu_0} B^2.$$

With $\chi < 0$, this energy increases with B^2. The force $\mathbf{F} = -\nabla U$ therefore points toward smaller field magnitude, which is the weaker part of the field. Statement II is true. In a spatially uniform field there is no such net translational force; the statement concerns the stronger and weaker regions specified.

Step 3: Match the pair of truth values Both the negative susceptibility range and the direction of migration agree with diamagnetic behaviour. The options making either statement false conflict with these properties. A tendency to move toward stronger field would instead describe a positive-susceptibility response such as paramagnetism. The question asks only whether each statement is true, rather than whether one is an explanation of the other.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: For diamagnets remember the pair: $\chi < 0$ and motion toward weaker field; $\mu_r = 1 + \chi$ supplies the susceptibility bound.

43. The ratio of intensities at two points P and Q on the screen in a Young's double slit experiment where phase difference between two waves of same amplitude are $\pi/3$ and $\pi/2$, respectively are

[4 marks]

(A) 2 : 3

(B) 1 : 3

(C) 3 : 1

(D) 3 : 2

Correct Answer: (D) 3 : 2

Solution:

Concept:

- For two coherent waves of equal individual intensity I_0, their resultant intensity is $I = 2I_0(1 + \cos\phi) = 4I_0\cos^2(\phi/2)$, where ϕ is their phase difference.
- The individual amplitudes are the same at the two points, so the same factor I_0 occurs in both intensities and cancels in their ratio.

Step 1: Evaluate the intensity at P At P the phase difference is $\phi_P = \pi/3$, and $\cos(\pi/3) = 1/2$. Thus the interference term is positive and enhances the individual intensities:

$$I_P = 2I_0\left(1 + \frac{1}{2}\right) = 3I_0.$$

This is less than the maximum $4I_0$, as expected because the phase difference is not zero.

Step 2: Evaluate the intensity at Q and compare At Q, $\phi_Q = \pi/2$, so $\cos\phi_Q = 0$. Therefore

$$I_Q = 2I_0(1+0) = 2I_0, \qquad \frac{I_P}{I_Q} = \frac{3I_0}{2I_0} = \frac{3}{2}.$$

The ratio must be larger than one because the smaller phase difference at P gives stronger constructive interference. Thus $2:3$ reverses the requested order, while $1:3$ and $3:1$ do not follow from the evaluated intensities.

Step 3: Conclusion The intensity ratio in the order P to Q is $I_P : I_Q = 3:2$. Hence, the correct option is (D).

Quick Tip to Solve: For equal amplitudes, compare $1+\cos\phi$ at the two points; do not take the ratio of phase differences.

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44. Young's moduli of the material of wires A and B are in the ratio of $1:4$, while its area of cross sections are in the ratio of $1:3$. If the same amount of load is applied to both the wires, the amount of elongation produced in the wires A and B will be in the ratio of

[Assume length of wires A and B are same] [4 marks]

(A) 1 : 12

(B) 1 : 36

(C) 12 : 1

(D) 36 : 1

Correct Answer: (C) 12 : 1

Solution:

Concept:

- Young's modulus is stress divided by strain: $Y = (F/A)/(\Delta L/L)$. Hence $\Delta L = FL/(AY)$, with force F, original length L, area A and extension ΔL.
- For equal loads and equal original lengths, extension varies inversely as the product of cross-sectional area and Young's modulus.

Step 1: Write the extension ratio Use the elongation formula separately for the wires and divide. The same force and original length cancel:

$$\frac{\Delta L_A}{\Delta L_B} = \frac{FL/(A_A Y_A)}{FL/(A_B Y_B)} = \frac{A_B Y_B}{A_A Y_A}.$$

Both the smaller area and smaller modulus of wire A make it stretch more.

Step 2: Substitute the two given ratios The modulus ratio gives $Y_B/Y_A = 4$, while the area ratio gives $A_B/A_A = 3$. Consequently,

$$\frac{\Delta L_A}{\Delta L_B} = 3 \times 4 = 12.$$

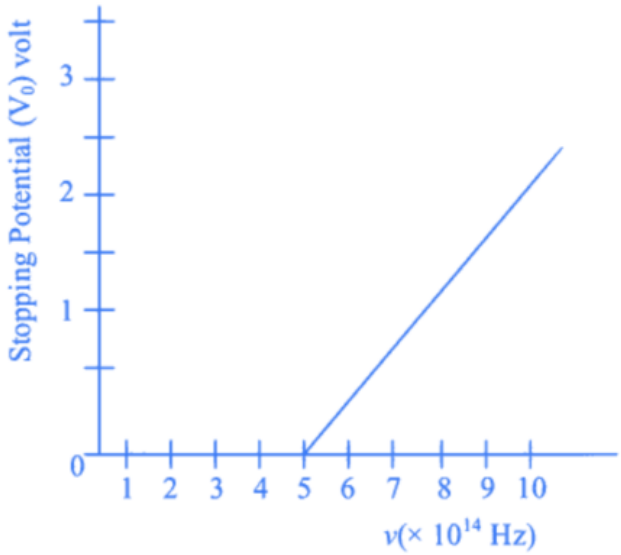
The inverse ratio $1:12$ would incorrectly make the stiffer, thicker wire stretch more. The values $36:1$ and $1:36$ incorrectly square the supplied area ratio; the areas themselves, rather than radii, were

given.

Step 3: Conclusion The extensions are in the ratio $\Delta L_A : \Delta L_B = 12 : 1$. Hence, the correct option is (C).

Quick Tip to Solve: For equal force and length, multiply the inverse modulus ratio by the inverse area ratio; area is already given, so do not square it.

45. The variation of stopping potential (V_0) as a function of the frequency (v) of the incident light for a metal is shown in figure. The work function of the surface is [4 marks]



(A) 1.36 eV

(B) 18.6 eV

(C) 2.98 eV

(D) 2.07 eV

Correct Answer: (D) 2.07 eV

Solution:

Concept:

- Einstein's photoelectric equation is $eV_0 = h\nu - W$, where W is the work function and V_0 is the stopping potential.
- The frequency-axis intercept occurs at $V_0 = 0$, so it is the threshold frequency ν_0, satisfying $W = h\nu_0$.

Step 1: Read the threshold frequency The plotted straight line meets the horizontal frequency axis at the tick labelled 5. The axis scale is 10^{14} Hz, so

$$\nu_0 = 5 \times 10^{14}\,\text{Hz}.$$

At this intercept the maximum photoelectron kinetic energy is zero; the photon energy is used entirely to overcome the work function.

Step 2: Convert the work function to electronvolts Using $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$ and $1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$,

$$W = (6.626 \times 10^{-34})(5 \times 10^{14}) = 3.313 \times 10^{-19}\,\mathrm{J}.$$

Therefore

$$W = \frac{3.313 \times 10^{-19}}{1.602 \times 10^{-19}}\,\mathrm{eV} \simeq 2.07\,\mathrm{eV}.$$

The other listed energies would correspond to different threshold frequencies and therefore disagree with the horizontal intercept. The vertical coordinates of other points are stopping potentials, not the work function itself.

Step 3: Conclusion The surface has work function $W \simeq 2.07\,\mathrm{eV}$. Hence, the correct option is (D).

Quick Tip to Solve: Read the frequency-axis intercept first; multiply it by $h = 4.136 \times 10^{-15}\,\mathrm{eV\,s}$ to obtain the work function directly in eV.

46. Given below are two statements:

Statement I : Rotation of the earth shows effect on the value of acceleration due to gravity (g)

Statement II : The effect of rotation of the earth on the value of 'g' at the equator is minimum and that at the pole is maximum.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Statement I is false but statement II is true

(B) Statement I is true but statement II is false

(C) Both Statement I and Statement II are true

(D) Both Statement I and Statement II are false

Correct Answer: (B) Statement I is true but statement II is false

Solution:

Concept:

- An observer rotating with Earth experiences a centrifugal effect. Its outward radial component reduces the effective acceleration due to gravity measured relative to Earth.
- In a spherical-Earth model, the radial reduction at latitude λ is $\omega^2 R\cos^2\lambda$, where R is Earth's radius and ω its angular speed.

Step 1: Evaluate Statement I Rotation changes apparent weight because a surface object must undergo centripetal acceleration about the rotation axis. The supporting force is therefore smaller than it would be on a nonrotating Earth. At the equator,

$$g_{\mathrm{eff,eq}} = g - \omega^2 R.$$

This explicitly shows that rotation affects the measured gravitational acceleration, so Statement I is true.

Step 2: Compare the equator with the poles At the equator the distance from the rotation axis is greatest, and the centrifugal acceleration is directed radially outward. The rotational reduction is therefore maximum there. At a pole the point lies on the rotation axis, so its distance from that axis is zero and the rotational correction vanishes. Equivalently, $\cos^2 \lambda$ is one at the equator and zero at the poles. Statement II reverses these two facts, so it is false. Options calling Statement I false or Statement II true are consequently excluded.

Step 3: Conclusion Statement I is true, but Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: Judge the rotational effect using distance from the rotation axis: maximum at the equator and zero at the poles.

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47. In an experiment with vernier callipers of least count 0.1 mm, when two jaws are joined together the zero of vernier scale lies right to the zero of the main scale and 6th division of vernier scale coincides with the main scale division. While measuring the diameter of a spherical bob, the zero of vernier scale lies in between 3.2 cm and 3.3 cm marks, and 4th division of vernier scale coincides with the main scale division. The diameter of bob is measured as [4 marks]

(A) 3.18 cm

(B) 3.22 cm

(C) 3.26 cm

(D) 3.25 cm

Correct Answer: (A) 3.18 cm

Solution:

Concept:

- The observed reading of a vernier caliper is main-scale reading plus the coinciding vernier division multiplied by the least count.
- A vernier zero to the right of the main-scale zero when the jaws are closed gives a positive zero error. Subtract this error from the observed measurement.

Step 1: Find the zero error The least count must be put in the same units as the main-scale reading:

$$\text{LC} = 0.1\,\text{mm} = 0.01\,\text{cm}.$$

With closed jaws, the sixth vernier division coincides and the vernier zero lies to the right. Thus

$$\text{zero error} = +6(0.01) = +0.06\,\text{cm}.$$

The instrument overreads, so its zero correction is negative.

Step 2: Correct the observed diameter The vernier zero lies after the $3.2\,\text{cm}$ mark, so the main-scale reading is $3.2\,\text{cm}$. The fourth vernier coincidence contributes $4 \times 0.01 = 0.04\,\text{cm}$:

$$d_{\text{obs}} = 3.2 + 0.04 = 3.24\,\text{cm}.$$

Subtracting the positive error gives

$$d = 3.24 - 0.06 = 3.18\,\text{cm}.$$

Adding the error would increase an already excessive reading. None of the other listed diameters results from the main-scale reading, the fourth coincidence and the required negative correction together.

Step 3: Conclusion The corrected bob diameter is $3.18\,\text{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the signed zero error separately. A right-shifted vernier zero gives positive error, which must be subtracted from the measured reading.

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48. The distance between two plates of a capacitor is d and its capacitance is C_1, when air is the medium between the plates. If a metal sheet of thickness $\frac{2d}{3}$ and of the same area as plate is introduced between the plates, the capacitance of the capacitor becomes C_2. The ratio $\frac{\mathrm{C}_2}{\mathrm{C}_1}$ is [4 marks]

(A) 1 : 1

(B) 3 : 1

(C) 2 : 1

(D) 4 : 1

Correct Answer: (B) 3 : 1

Solution:

Concept:

- An air-filled parallel-plate capacitor of plate area A and separation d has capacitance $C_1 = \varepsilon_0 A/d$, neglecting edge effects.
- The electric field inside a metal slab in electrostatic equilibrium is zero. A slab of thickness t therefore removes that thickness from the region across which the potential drops.

Step 1: Find the effective air separation When the isolated metal sheet is inserted without shorting the plates, the remaining two air gaps have total thickness $d - t$. Although the two gap widths can differ, their potential drops add, and their series combination depends only on their sum. Thus

$$C_2 = \frac{\varepsilon_0 A}{d - t}.$$

No potential drop occurs across the conducting sheet itself.

Step 2: Substitute the slab thickness and compare Here $t = 2d/3$, giving

$$d - t = d - \frac{2d}{3} = \frac{d}{3}.$$

Therefore

$$C_2 = \frac{\varepsilon_0 A}{d/3} = \frac{3\varepsilon_0 A}{d} = 3C_1, \qquad \frac{C_2}{C_1} = 3.$$

The capacitance increases because the field-bearing separation is reduced. A ratio of one would ignore

the slab; ratios two or four would require different remaining air thicknesses. The metal should not be treated as an ordinary finite-permittivity dielectric.

Step 3: Conclusion The capacitance ratio is $C_2 : C_1 = 3 : 1$. Hence, the correct option is (B).

Quick Tip to Solve: For a metal slab between capacitor plates, replace d by $d - t$; only the total air-gap thickness contributes to the potential difference.

49. The half life of a radioactive substance is T. The time taken, for disintegrating $\frac{7}{8}$th part of its original mass will be: [4 marks]

(A) 8T

(B) 3T

(C) T

(D) 2T

Correct Answer: (B) 3T

Solution:

Concept:

- For radioactive decay, the undecayed population after time t is $N = N_0(1/2)^{t/T}$, where T is the half-life.
- The mass of the original radioactive species is proportional to its undecayed population. The remaining fraction, rather than the disintegrated fraction, belongs in the decay law.

Step 1: Calculate the fraction remaining The question asks for the time when $7/8$ of the original radioactive mass has disintegrated. The fraction of the original nuclide still present is therefore

$$\frac{N}{N_0} = 1 - \frac{7}{8} = \frac{1}{8}.$$

This does not mean the whole specimen has vanished; the decay law tracks the original radioactive nuclei, including those that have transformed into daughter nuclei.

Step 2: Count the required half-lives Each half-life halves the amount still undecayed:

$$1 \longrightarrow \frac{1}{2} \longrightarrow \frac{1}{4} \longrightarrow \frac{1}{8}.$$

Equivalently,

$$\left(\frac{1}{2}\right)^{t/T} = \frac{1}{8} = \left(\frac{1}{2}\right)^{3} \quad \Rightarrow \quad t = 3T.$$

One half-life would disintegrate only half the original amount, and two would disintegrate $3/4$, not $7/8$. Eight half-lives would leave only $1/256$. Thus the time must be three half-lives.

Step 3: Conclusion The time required for $7/8$ of the original radioactive mass to disintegrate is $3T$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the decayed fraction to the remaining fraction first; $1/8 = 2^{-3}$ means exactly three half-lives.

50. A message signal of frequency 3 kHz is used to modulate a carrier signal of frequency 1.5 MHz. The bandwidth of the amplitude modulated wave is [4 marks]

(A) 6 MHz

(B) 6 kHz

(C) 3 MHz

(D) 3 kHz

Correct Answer: (B) 6 kHz

Solution:

Concept:

- In ordinary amplitude modulation, a sinusoidal message of frequency f_m produces a lower sideband at $f_c - f_m$ and an upper sideband at $f_c + f_m$, where f_c is the carrier frequency.
- The transmission bandwidth is the difference between the highest and lowest sideband frequencies: $B = (f_c + f_m) - (f_c - f_m) = 2f_m$. The carrier sets the location of the spectrum, while the message sets its width.

Step 1: Express the frequencies in the same unit The message frequency is $f_m = 3\,\text{kHz}$. Convert the carrier frequency before forming its sidebands:

$$f_c = 1.5\,\text{MHz} = 1500\,\text{kHz}.$$

It is much larger than the message frequency, as expected for a radio-frequency carrier. There is no need to double the carrier frequency to determine the occupied bandwidth.

Step 2: Find both sidebands and subtract The sideband frequencies are

$$f_{\text{L}} = 1500 - 3 = 1497\,\text{kHz}, \qquad f_{\text{U}} = 1500 + 3 = 1503\,\text{kHz}.$$

Therefore,

$$B = 1503 - 1497 = 6\,\text{kHz}.$$

Option (D) counts only one sideband displacement from the carrier and is too small by a factor of two. Options (A) and (C) incorrectly give a bandwidth on the megahertz scale; that scale belongs to the carrier frequency rather than the message bandwidth.

Step 3: Conclusion The amplitude-modulated wave has bandwidth $\boxed{6\,\text{kHz}}$. Hence, the correct option is (B).

Quick Tip to Solve: For conventional double-sideband AM, double the highest message frequency; the carrier frequency cancels from the bandwidth.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A force of $-\mathrm{P}\hat{\mathrm{k}}$ acts on the origin of the coordinate system. The torque about the point $(2, -3)$ is $\mathrm{P}(a\hat{i} + b\hat{j})$, The ratio of $\frac{a}{b}$ is $\frac{x}{2}$. The value of x is - [4 marks]

Solution:

Concept:

- Torque about a specified point is $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ runs from that reference point to the point of application of the force.
- The right-handed Cartesian products are $\hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$ and $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$. The reference point must be subtracted from the force application point before taking the product.

Step 1: Form the displacement from the reference point The force is applied at the origin $O = (0, 0, 0)$, while the point about which torque is required is $A = (2, -3, 0)$. Thus

$$\mathbf{r} = \overrightarrow{AO} = (0 - 2)\hat{\mathbf{i}} + [0 - (-3)]\hat{\mathbf{j}} = -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}.$$

The given force is $\mathbf{F} = -P\hat{\mathbf{k}}$. Taking $\overrightarrow{OA}$ instead would reverse both torque components; the required position vector is always directed towards the point where the force acts.

Step 2: Evaluate the cross product and compare coefficients Using the vector products gives

$$\begin{aligned}\boldsymbol{\tau} &= (-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (-P\hat{\mathbf{k}}) \\ &= 2P(-\hat{\mathbf{j}}) - 3P\hat{\mathbf{i}} \\ &= P(-3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}).\end{aligned}$$

Comparing with $P(a\hat{\mathbf{i}} + b\hat{\mathbf{j}})$, we obtain $a = -3$ and $b = -2$. Consequently

$$\frac{a}{b} = \frac{-3}{-2} = \frac{3}{2} = \frac{x}{2}, \qquad x = 3.$$

The negative signs in both components cancel in the ratio; the ratio and the requested number are dimensionless.

Step 3: Conclusion The torque is $P(-3\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$, so the requested value is $\boxed{x = 3}$.

Quick Tip to Solve: Write the reference-to-application displacement first. For a force parallel to the z-axis, the torque lies entirely in the xy-plane.

52. A straight wire carrying a current of 14 A is bent into a semi-circular arc of radius 2.2 cm as shown in the figure. The magnetic field produced by the current at the centre (O) of the arc. is _____ $\times 10^{-4}$ T [4 marks]



Solution:

Concept:

- By the Biot–Savart law, a circular arc of radius R, angle θ in radians and current I produces a field $B = \mu_0 I\theta/(4\pi R)$ at its centre.
- A straight wire segment along a line through the observation point contributes zero there: each current element has $d\ell$ parallel or antiparallel to its displacement vector, so their cross product vanishes.

Step 1: Identify which parts of the wire contribute The two straight leads in the figure lie on the diameter line through O. Their contributions at O are zero by the Biot–Savart cross product. The curved portion is a semicircle, so its central angle is $\theta = \pi$. Therefore the complete wire produces the same field at the centre as the semicircular arc alone:

$$B = \frac{\mu_0 I\pi}{4\pi R} = \frac{\mu_0 I}{4R}.$$

A half-circle gives half the central field of a complete circular turn; using the full-circle expression would double the answer.

Step 2: Convert the radius and calculate the field The radius is $R = 2.2\,\text{cm} = 2.2 \times 10^{-2}\,\text{m}$, and the current is $I = 14\,\text{A}$. With $\mu_0 = 4\pi \times 10^{-7}\,\text{T m A}^{-1}$,

$$B = \frac{4\pi \times 10^{-7} \times 14}{4 \times 2.2 \times 10^{-2}}$$
$$\simeq 2.00 \times 10^{-4}\,\text{T}.$$

The current travels clockwise along the upper arc from the left lead to the right lead. The right-hand rule therefore places the field into the plane of the figure. The question asks for its magnitude in the specified power-of-ten unit.

Step 3: Conclusion The field magnitude is $\boxed{2 \times 10^{-4}\,\text{T}}$, so the required numerical entry is $\boxed{2}$.

Quick Tip to Solve: Check straight leads before adding their fields: a lead directed along the radius through the observation point contributes zero there.

53. A rectangular block of mass $5\ \text{kg}$ attached to a horizontal spiral spring executes simple harmonic motion of amplitude $1\ \text{m}$ and time period $3.14\ \text{s}$. The maximum force exerted by spring on block is _____ N [4 marks]

Solution:

Concept:

- A mass on an ideal spring obeys $F = -kx = -m\omega^2 x$, where k is the spring constant, x is displacement from equilibrium and ω is angular frequency.
- The period is $T = 2\pi/\omega$. Because the restoring force magnitude increases with $|x|$, its largest value occurs at an extreme displacement $|x| = A$, giving $F_{\max} = m\omega^2 A$.

Step 1: Find the angular frequency from the period The given mass, amplitude and period are $m = 5\,\text{kg}$, $A = 1\,\text{m}$ and $T = 3.14\,\text{s}$. The angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.14} \simeq 2\,\text{rad}\,\text{s}^{-1}.$$

The period is the time for a complete oscillation, so using $1/T$ as the angular frequency would omit the essential factor 2π. Equivalently, the spring constant is $k = m\omega^2 \simeq 20\,\text{N}\,\text{m}^{-1}$.

Step 2: Evaluate the maximum restoring force At either end of the oscillation, the displacement magnitude equals the amplitude. Hence

$$\begin{aligned} F_{\max} &= m\left(\frac{2\pi}{T}\right)^2 A \\ &= 5\left(\frac{2\pi}{3.14}\right)^2 (1) \\ &\simeq 20\,\text{N}. \end{aligned}$$

The sign of the instantaneous force is towards equilibrium and reverses between the two extremes, but its maximum magnitude is positive. At the centre the force is zero even though the speed is greatest, so maximum speed must not be confused with maximum force. The precision of the supplied period supports the numerical value 20 N.

Step 3: Conclusion The maximum spring-force magnitude is $\boxed{20\,\text{N}}$. The numerical entry is $\boxed{20}$.

Quick Tip to Solve: For a spring oscillator use $F_{\max} = 4\pi^2 mA/T^2$; maximum force occurs at an extreme, where the speed is zero.

54. A square loop of side $2.0\ \text{cm}$ is placed inside a long solenoid that has 50 turns per centimetre and carries a sinusoidally varying current of amplitude $2.5\ \text{A}$ and angular frequency $700\ \text{rad s}^{-1}$. The central axes of the loop and solenoid coincide. The amplitude of the emf induced in the loop is $x \times 10^{-4}\ \text{V}$. The value of x is _____.

$$\textbf{(Take, } \pi = \frac{22}{7}\textbf{)}$$

[4 marks]

Solution:

Concept:

- The magnetic field inside a long solenoid is $B = \mu_0 n i$, where n is the number of turns per metre and i is the solenoid current.
- For one loop whose area vector is parallel to the solenoid axis, the flux is $\Phi = BA$. Faraday's law gives $\mathcal{E} = -d\Phi/dt$, so a sinusoidal current of amplitude I_0 and angular frequency ω produces emf amplitude $\mathcal{E}_0 = \mu_0 n A I_0 \omega$.

Step 1: Convert the turn density and determine the loop area The square side is $a = 2.0\,\text{cm} = 2.0 \times 10^{-2}\,\text{m}$, so

$$A = a^2 = 4.0 \times 10^{-4}\,\text{m}^2.$$

The turn density is $n = 50\,\text{cm}^{-1} = 5000\,\text{m}^{-1}$. The loop and solenoid have coincident central axes, hence the loop area vector is along the magnetic field and the flux factor is $\cos 0 = 1$. The receiving square is a single loop; the solenoid turn density is already included in its magnetic field.

Step 2: Differentiate the sinusoidal flux and substitute Write $i = I_0 \sin \omega t$. Then $\Phi = \mu_0 n A I_0 \sin \omega t$, and its derivative has amplitude

$$\begin{aligned}
\mathcal{E}_0 &= \mu_0 n A I_0 \omega \\
&= (4\pi \times 10^{-7})(5000)(4 \times 10^{-4})(2.5)(700) \\
&= 1.4\pi \times 10^{-3}\,\text{V} \\
&= 4.4 \times 10^{-3}\,\text{V} \quad (\pi = 22/7) \\
&= 44 \times 10^{-4}\,\text{V}.
\end{aligned}$$

The minus sign in Faraday's law specifies the opposing direction of the induced emf, while its amplitude is a positive magnitude.

Step 3: Conclusion The induced emf amplitude is $\boxed{44 \times 10^{-4}\,\text{V}}$, giving $\boxed{x = 44}$.

Quick Tip to Solve: Convert turns per centimetre to turns per metre, and square the side only after converting it to metres; both errors strongly affect the emf.

55. If the maximum load carried by an elevator is $1400\ \text{kg}$ ($600\ \text{kg}$ - Passengers + 800 kg - elevator), which is moving up with a uniform speed of $3\ \text{m s}^{-1}$ and the frictional force acting on it is $2000\ \text{N}$, then the maximum power used by the motor is _____ kW $\left(\text{g} = 10\ \text{m/s}^2\right)$ [4 marks]

Solution:

Concept:

- At constant velocity, the net force on an elevator is zero. The upward driving force must balance both the weight and any frictional force opposing the upward motion.
- Mechanical power delivered by a force along the direction of motion is $\mathcal{P} = Fv$. For an elevator ascending uniformly, $\mathcal{P} = (mg + f)v$, where m includes the elevator and passengers and f is the friction magnitude.

Step 1: Find the total resisting force at the maximum load The stated load of $1400\,\text{kg}$ already includes the $600\,\text{kg}$ of passengers and the $800\,\text{kg}$ elevator. Thus use $m = 1400\,\text{kg}$, rather than adding the elevator mass again. Its weight is

$$mg = 1400 \times 10 = 14000\,\text{N}.$$

While the elevator rises, friction of magnitude $2000\,\text{N}$ acts downwards together with gravity. Uniform speed means zero acceleration, so the motor must provide

$$F = 14000 + 2000 = 16000\,\text{N}.$$

Zero resultant force does not imply zero motor force or zero work.

Step 2: Multiply the driving force by the upward speed For $v = 3\,\text{m s}^{-1}$, the power delivered to the elevator is

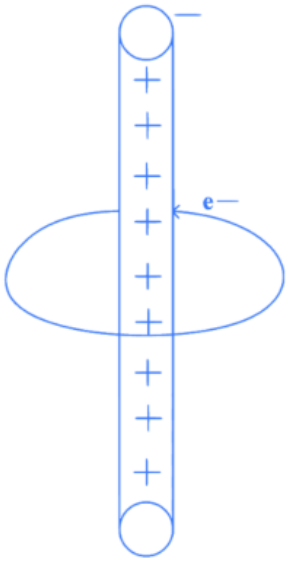
$$\begin{aligned}\mathcal{P} = Fv &= 16000 \times 3 \\ &= 48000\,\text{J s}^{-1} \\ &= 48\,\text{kW}.\end{aligned}$$

The same result separates into $mgv = 42\,\text{kW}$ to increase gravitational potential energy and $fv = 6\,\text{kW}$ to overcome friction. No acceleration term is required because the velocity is uniform. No motor efficiency is specified, so the requested power follows directly from the mechanical force and speed.

Step 3: Conclusion The required motor power at the maximum load is $\boxed{48\,\text{kW}}$, so the numerical entry is $\boxed{48}$.

Quick Tip to Solve: At uniform upward speed, add weight and downward friction before multiplying by speed; count the elevator mass only once.

56. An electron revolves around an infinite cylindrical wire having uniform linear charge density $2 \times 10^{-8} \mathrm{Cm}^{-1}$ in circular path under the influence of attractive electrostatic field as shown in the figure. The velocity of electron with which it is revolving is _____ $\times 10^6$ m s^{-1}. Given mass of electron $= 9 \times 10^{-31}$ kg [4 marks]



Solution:

Concept:

- Outside an infinitely long uniformly charged cylindrical wire, Gauss's law gives $E = \lambda/(2\pi\varepsilon_0 r)$, where λ is charge per unit length and r is the distance from its axis.
- The positive wire attracts an electron with force magnitude eE. For circular motion this electrostatic force supplies the centripetal force: $mv^2/r = eE$. Here e is the positive magnitude of the electron charge.

Step 1: Equate the electric and centripetal forces Choose a coaxial Gaussian cylinder of radius r and length L. Its curved-surface flux is $E(2\pi rL)$, while its enclosed charge is λL. Thus

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}.$$

The electron revolves outside the charged wire, and the radial attraction points towards the axis, exactly in the direction required for circular motion. Therefore

$$\frac{mv^2}{r} = \frac{e\lambda}{2\pi\varepsilon_0 r} \quad \Rightarrow \quad v^2 = \frac{e\lambda}{2\pi\varepsilon_0 m}.$$

The orbital radius cancels because both the field force and the needed centripetal force contain the same factor $1/r$.

Step 2: Insert the charge density and electron constants Use $\lambda = 2 \times 10^{-8}\,\mathrm{C\,m^{-1}}$, $m = 9 \times 10^{-31}\,\mathrm{kg}$,

$e = 1.6 \times 10^{-19}\,\mathrm{C}$, and $1/(4\pi\varepsilon_0) = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$. Since $1/(2\pi\varepsilon_0) = 18 \times 10^9$,

$$\begin{aligned} v^2 &= \frac{(18 \times 10^9)(1.6 \times 10^{-19})(2 \times 10^{-8})}{9 \times 10^{-31}} \\ &= 6.4 \times 10^{13}\,\mathrm{m^2\,s^{-2}}, \\ v &= 8.0 \times 10^6\,\mathrm{m\,s^{-1}}. \end{aligned}$$

Take the positive square root because the question requests the speed.

Step 3: Conclusion The electron speed is $\boxed{8 \times 10^6\,\mathrm{m\,s^{-1}}}$, giving the numerical entry $\boxed{8}$.

Quick Tip to Solve: For an infinite charged wire, the field varies as $1/r$; equating electric and centripetal forces cancels the orbital radius.

57. A rectangular parallelopiped is measured as $1\ \mathrm{cm} \times 1\ \mathrm{cm} \times 100\ \mathrm{cm}$. If its specific resistance is $3 \times 10^{-7}\ \Omega\mathrm{m}$, then the resistance between its two opposite rectangular faces will be _____ $\times 10^{-7}\ \Omega$. [4 marks]

Solution:

Concept:

- The resistance of a uniform material is $R = \rho L/A$, where L is the separation of the electrodes and A is the area normal to the current.
- Opposite rectangular faces are the long $1\,\mathrm{cm} \times 100\,\mathrm{cm}$ faces. The square end faces would give a different resistance.

Step 1: Identify the current path The dimensions are $1\,\mathrm{cm}$, $1\,\mathrm{cm}$, and $100\,\mathrm{cm}$. Each long rectangular face has area $100\,\mathrm{cm^2}$, and its opposite face is separated from it by the remaining short dimension, $1\,\mathrm{cm}$. Thus the current crosses the short dimension, rather than travelling along the entire length. Either pair of opposite long faces gives the same result because the two short dimensions are equal.

Step 2: Convert the dimensions to SI units The resistivity is given in $\Omega\,\mathrm{m}$, so use metres throughout:

$$L = 1 \times 10^{-2}\,\mathrm{m}, \qquad A = (1 \times 10^{-2})(100 \times 10^{-2}) = 10^{-2}\,\mathrm{m^2}.$$

Notice that $100\,\mathrm{cm^2} = 10^{-2}\,\mathrm{m^2}$, not $1\,\mathrm{m^2}$.

Step 3: Substitute into the resistance formula With electrodes covering these opposite faces, the cross-sectional area is constant along the path. Therefore

$$R = \frac{(3 \times 10^{-7})(10^{-2})}{10^{-2}} = 3 \times 10^{-7}\,\Omega.$$

The length-to-area ratio is $1\,\mathrm{m^{-1}}$; multiplying it by resistivity produces resistance in ohms. The question asks for the coefficient of $10^{-7}\,\Omega$, not the resistance between the square ends.

Step 4: Conclusion The resistance is $3 \times 10^{-7}\,\Omega$. The required numerical entry is $\boxed{3}$.

Quick Tip to Solve: In a cuboid resistance question, first circle the two electrode faces: their area gives A, while their separation gives L.

58. Figure below shows a liquid being pushed out of the tube by a piston having area of cross section $2.0\ \text{cm}^2$. The area of cross section at the outlet is $10\ \text{mm}^2$. If the piston is pushed at a speed of $4\ \text{cm s}^{-1}$, the speed of outgoing fluid is _____ cms^{-1} [4 marks]



Solution:

Concept:

- For steady incompressible flow, conservation of volume gives $A_1 v_1 = A_2 v_2$.
- The piston displaces a volume $A_1 v_1$ each second. With no leakage or compression, this same volume leaves the outlet each second.

Step 1: Write the piston volume flow rate The piston has area $A_1 = 2.0\,\text{cm}^2$ and speed $v_1 = 4\,\text{cm s}^{-1}$. In one second it advances $4\,\text{cm}$, displacing a cylinder of liquid whose volume equals area times advance:

$$Q = A_1 v_1 = (2.0)(4) = 8.0\,\text{cm}^3\,\text{s}^{-1}.$$

This is a volume flow rate, not a speed. Every second the outlet must deliver the same $8.0\,\text{cm}^3$ of liquid.

Step 2: Express both areas in the same units The outlet area is $10\,\text{mm}^2$. Since $1\,\text{cm} = 10\,\text{mm}$, squaring the conversion gives

$$1\,\text{cm}^2 = 100\,\text{mm}^2, \qquad A_2 = \frac{10}{100} = 0.10\,\text{cm}^2.$$

The outlet area is therefore one twentieth of the piston area. A smaller outlet requires a proportionally greater fluid speed.

Step 3: Apply continuity at the outlet Conservation of the liquid volume gives

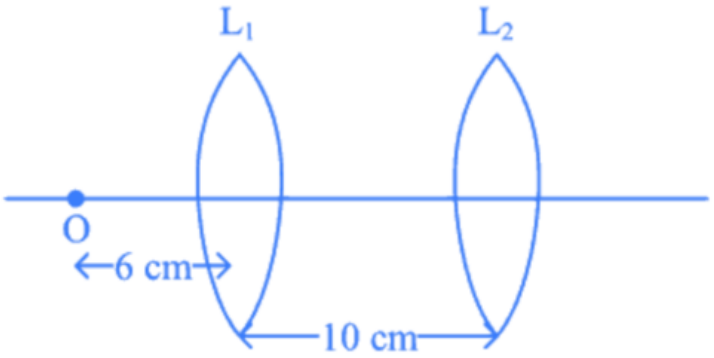
$$v_2 = \frac{Q}{A_2} = \frac{8.0\,\text{cm}^3\,\text{s}^{-1}}{0.10\,\text{cm}^2} = 80\,\text{cm s}^{-1}.$$

Equivalently, $v_2 = (A_1/A_2)v_1 = 20 \times 4$. The area units cancel to leave centimetres per second, which is the unit requested. No pressure or density data are needed because the piston speed directly specifies the volume flow rate.

Step 4: Conclusion The outgoing liquid speed is $\boxed{80\,\text{cm s}^{-1}}$, so the numerical entry is 80.

Quick Tip to Solve: Square the length conversion when converting areas; $10\,\text{mm}^2$ is $0.10\,\text{cm}^2$.

59. A point object, 'O' is placed in front of two thin symmetrical coaxial convex lenses L_1 and L_2 with focal length 24 cm and 9 cm respectively. The distance between two lenses is 10 cm and the object is placed 6 cm away from lens L_1 as shown in the figure. The distance between the object and the image formed by the system of two lenses is _____ cm. [4 marks]



Solution:

Concept:

- For each lens use the Cartesian sign convention and $1/v - 1/u = 1/f$. Convex lenses have positive focal lengths.
- The first lens image acts as the object for the second lens. Distances must be measured afresh from the second lens, not from the original object.

Step 1: Locate the first lens image Take light to travel from left to right. Relative to L_1, the object distance is $u_1 = -6\,\text{cm}$, while $f_1 = +24\,\text{cm}$. Hence

$$\frac{1}{v_1} = \frac{1}{24} + \frac{1}{-6} = -\frac{1}{8}, \qquad v_1 = -8\,\text{cm}.$$

This negative distance is a virtual image $8\,\text{cm}$ to the left of L_1. This is consistent with the original object being inside the first convex lens focal length.

Step 2: Find the second lens object distance Place L_1 at coordinate $x = 0$ and L_2 at $x = 10\,\text{cm}$. The first image is at $x = -8\,\text{cm}$. The rays incident on the second lens diverge as though coming from that point, which is $18\,\text{cm}$ to its left. Therefore

$$u_2 = -18\,\text{cm}, \qquad f_2 = +9\,\text{cm}.$$

The lens separation adds to the magnitude $8\,\text{cm}$; it does not subtract from it.

Step 3: Locate the final image and measure from the object For the second lens,

$$\frac{1}{v_2} = \frac{1}{9} - \frac{1}{18} = \frac{1}{18}, \qquad v_2 = 18\,\text{cm}.$$

The final real image is at $x = 10 + 18 = 28\,\text{cm}$. The original object is at $x = -6\,\text{cm}$, so their separation is

$$28 - (-6) = 34\,\text{cm}.$$

The intermediate image distance and the distance from the final lens are not the object-to-final-image separation requested.

Step 4: Conclusion The distance between the original object and the final image is $\boxed{34\,\text{cm}}$.

Quick Tip to Solve: Use a common coordinate axis after finding each image; it prevents confusing lens-relative image distance with the final object-image separation.

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60. If 917 $\overset{o}{A}$ be the lowest wavelength of Lyman series then the lowest wavelength of Balmer series will be _____ $\overset{o}{A}$. [4 marks]

Solution:

Concept:

- The hydrogen Rydberg formula is $1/\lambda = R(1/n_f^2 - 1/n_i^2)$, with $n_i > n_f$.
- The shortest wavelength in a series is its series limit, obtained when $n_i \to \infty$. Lyman transitions end at $n_f = 1$, and Balmer transitions end at $n_f = 2$.

Step 1: Identify the limiting transition Within either spectral series, increasing the upper energy level increases the photon energy and decreases its wavelength. The smallest possible wavelength is consequently obtained at the upper-level limit $n_i \to \infty$, not from the first visible line of the series. In this limit the term $1/n_i^2$ vanishes, leaving

$$\frac{1}{\lambda_{\min}} = \frac{R}{n_f^2}, \qquad \lambda_{\min} = \frac{n_f^2}{R}.$$

The same Rydberg constant applies to both series of the same atom.

Step 2: Compare the Lyman and Balmer limits For Lyman, $n_f = 1$, so $\lambda_{\text{L}} = 1/R$. For Balmer, $n_f = 2$, so $\lambda_{\text{B}} = 4/R$. Dividing the second expression by the first cancels the Rydberg constant exactly:

$$\frac{\lambda_{\text{B}}}{\lambda_{\text{L}}} = 4.$$

Thus the Balmer limit is longer, in agreement with its smaller limiting transition energy.

Step 3: Use the given Lyman wavelength The problem gives the Lyman limit as $917\,\text{Å}$. Retain this specified value rather than replacing it by a tabulated approximation. Therefore

$$\lambda_{\text{B}} = 4(917) = 3668\,\text{Å}.$$

Multiplication by a dimensionless ratio preserves the angstrom unit. No conversion to metres is needed because the requested answer uses the same wavelength unit as the data.

Step 4: Conclusion The lowest Balmer-series wavelength is $\boxed{3668\,\text{Å}}$. The numerical entry is 3668.

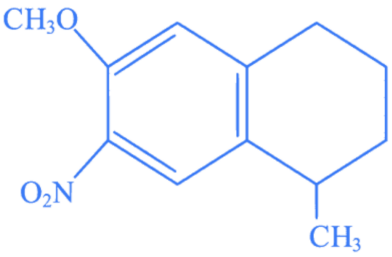
Quick Tip to Solve: At a hydrogen series limit, wavelength is proportional to the square of the final quantum number: compare n_f^2 directly.

Chemistry – Section A: Multiple Choice Questions (80 marks)

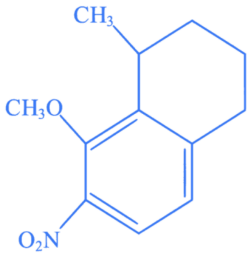
61. The major product 'P' formed in the given reaction is: [4 marks]



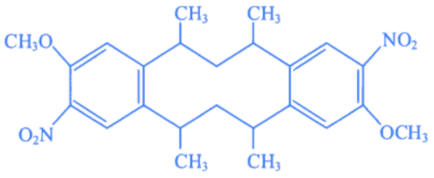
(A)



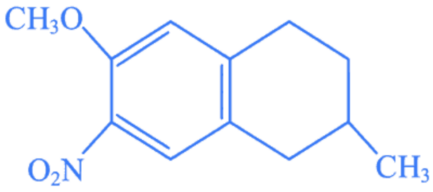
(B)



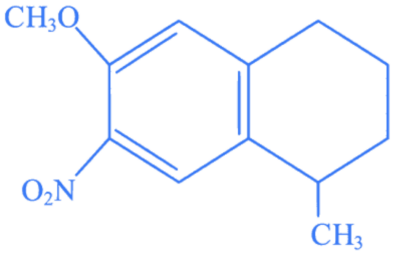
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- Anhydrous $AlCl_3$ activates the alkyl chloride, making the carbon bearing chlorine electrophilic. An arene can capture it intramolecularly.
- A methoxy group directs electrophilic substitution to its ortho and para positions. A nitro group deactivates its ortho and para positions and favours attack meta to itself.

Step 1: Identify the electrophilic carbon and ring size The side chain attached to the aromatic ring is $-CH_2CH_2CH_2CH(Cl)CH_3$. Coordination of chlorine to the Lewis acid facilitates formation of an electrophilic centre at $CH(Cl)$. Attack by a neighbouring aromatic carbon links that carbon to this centre. The ring thereby formed contains the two adjacent aromatic carbons and the four side-chain carbons between them: it is a six-membered ring fused to the benzene ring. The terminal methyl group remains attached to the newly bonded saturated carbon.

Step 2: Choose the favoured aromatic position The two aromatic positions adjacent to the side-chain attachment can each form a six-membered ring. One is ortho to methoxy and the other is para to methoxy; both are meta to nitro. The para position is less crowded because it avoids attack immediately next to the methoxy group. Intramolecular attack there, followed by loss of a proton to restore aromaticity, therefore gives the major product drawn in option (A).

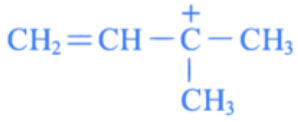
Step 3: Compare the alternative products Option (B) corresponds to closure at the more hindered position ortho to methoxy. Option (C) joins two substrate units into a much larger framework rather than the favourable single-molecule six-membered closure. Option (D) places the methyl substituent at a different saturated ring carbon and does not preserve the direct bond formation at the original chlorine-bearing carbon. The connectivity in (A) both preserves the side-chain skeleton and matches the preferred substitution site.

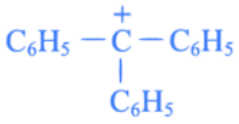
Step 4: Conclusion The major product is the fused six-membered ring shown in option (A), formed by cyclisation para to methoxy. Hence, the correct option is (A).

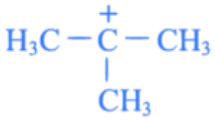
Quick Tip to Solve: For intramolecular Friedel–Crafts reactions, count the new ring atoms first, then compare directing effects and steric crowding at the possible arene attack sites.

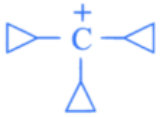
62. The decreasing order of hydride affinity for following carbocations is:

Choose the correct answer from the options given below: [4 marks]

A. 

B. 

C. 

D. 

(A) A, C, D, B

(B) C, A, B, D

(C) A, C, B, D

(D) C, A, D, B

Correct Answer: (B) C, A, B, D

Solution:

Concept:

- Hydride affinity measures the tendency of a carbocation to accept H^-. A less stabilized carbocation has a greater driving force for hydride capture.
- Compare resonance and sigma conjugation before counting alkyl groups: delocalization can outweigh ordinary inductive and hyperconjugative stabilization.

Step 1: Rank the ordinary and allylic carbocations C is the tert-butyl cation. Its three methyl groups stabilize the positive centre through the positive inductive effect and hyperconjugation, but its charge remains comparatively localized. A is a tertiary allylic carbocation: its positive centre is adjacent to a carbon-carbon double bond, so the charge is additionally delocalized by allylic resonance. Thus A is more stable than C, and C has the larger hydride affinity. This already rules out the options beginning with A.

Step 2: Compare the extensively delocalized cations B is the triphenylmethyl cation, whose positive charge is distributed into three phenyl rings by resonance. It is more stabilized than A. D is the tris(cyclopropyl)methyl cation. The bent carbon-carbon bonds of the cyclopropyl groups overlap effectively with the vacant p orbital at the positively charged carbon. This strong sigma conjugation, available from all three cyclopropyl groups, makes D still more stabilized than B. The stability order is consequently

$$D > B > A > C.$$

Step 3: Reverse the stability order for hydride affinity Greater stabilization of the starting cation reduces the energy released by neutralizing its charge with hydride. Therefore the decreasing hydride-affinity sequence is the reverse of the cation-stability sequence:

$$C > A > B > D.$$

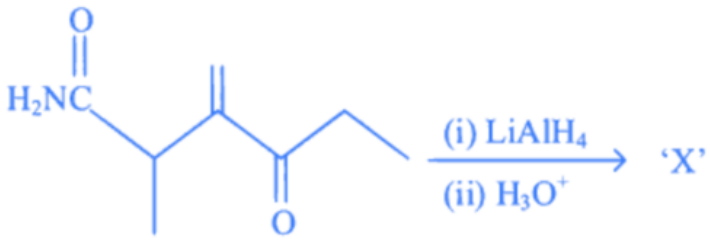
Option (B) gives precisely this order. Option (D) correctly starts with C and A but incorrectly interchanges B and D; options (A) and (C) wrongly put the resonance-stabilized A ahead of localized C in hydride affinity.

Step 4: Conclusion The decreasing hydride-affinity order is $\boxed{C > A > B > D}$. Hence, the correct option is (B).

Quick Tip to Solve: Do not copy the carbocation stability order into a hydride-affinity answer: stabilize the cations first, then reverse the order.

63. In the reaction given below:

The product 'X' is: [4 marks]



(A) $H_2NCH_2CH(CH_3)C(=CH_2)CH(OH)CH_2CH_3$

(B) $H_2NCH(OH)CH(CH_3)C(=CH_2)CH(OH)CH_2CH_3$

(C) $H_2NCH_2CH(CH_3)CH(CH_3)CH(OH)CH_2CH_3$

(D) $H_2NCH(OH)CH(CH_3)CH(CH_3)CH(OH)CH_2CH_3$

Correct Answer: (A) $H_2NCH_2CH(CH_3)C(=CH_2)CH(OH)CH_2CH_3$

Solution:

Concept:

- $LiAlH_4$ reduces a primary amide $RCONH_2$ to RCH_2NH_2, replacing the carbonyl oxygen while retaining the carbon-nitrogen bond.
- It reduces a ketone R_2CO to a secondary alcohol after acidic work-up. Under the usual conditions the carbon-carbon double bond remains unchanged.

Step 1: Identify the three functional features The substrate can be written in condensed form as

$$H_2NC(=O)CH(CH_3)C(=CH_2)C(=O)CH_2CH_3.$$

It contains a primary amide on the left, a ketone on the right, and an exocyclic methylene group

between them. Apply the reagent independently to each carbonyl group while retaining the carbon skeleton. The later H_3O^+ treatment protonates the intermediate oxygen and nitrogen species; it does not supply a separate alkene hydrogenation reagent.

Step 2: Reduce the amide and ketone The amide end $-C(=O)NH_2$ becomes $-CH_2NH_2$, so no oxygen remains on that terminal carbon. At the ketone carbon, hydride addition produces an alkoxide that becomes $-CH(OH)-$ on work-up. The intervening $C{=}CH_2$ unit is retained. The complete product is therefore

$$H_2NCH_2CH(CH_3)C(=CH_2)CH(OH)CH_2CH_3.$$

This connectivity matches the first structure.

Step 3: Eliminate the other structures Option (A) has both the terminal aminomethyl group and the secondary alcohol, with the carbon-carbon double bond intact. Option (B) incorrectly retains an OH group on the amide-derived carbon as an amino alcohol; amide reduction proceeds beyond that intermediate to the amine. Option (C) wrongly reduces the carbon-carbon double bond while otherwise giving the carbonyl reduction products. Option (D) contains both of these errors. Hence only (A) has all functional-group changes correct.

Step 4: Conclusion The product is $H_2NCH_2CH(CH_3)C(=CH_2)CH(OH)CH_2CH_3$, represented by option (A). Hence, the correct option is (A).

Quick Tip to Solve: For $LiAlH_4$, mark every carbonyl first: amide gives an amine, ketone gives an alcohol, and an ordinary carbon-carbon double bond survives.

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64. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : The energy required to form Mg^{2+} from Mg is much higher than that required to produce Mg^+

Reason R : Mg^{2+} is small ion and carry more charge than Mg^+

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) A is false but R is true

(B) Both A and R are correct but R is NOT the correct explanation of A

(C) Both A and R are correct and R is the correct explanation of A

(D) A is true but R is false

Correct Answer: (C) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- Ionization enthalpy is the energy needed to remove an electron from an isolated gaseous species. Successive removal becomes more difficult as positive charge increases and the ionic radius contracts.
- Magnesium has configuration $[Ne]3s^2$; its first and second ionizations remove the two valence $3s$

electrons.

Step 1: Evaluate the assertion The first step is $\mathrm{Mg\,(g)} \longrightarrow \mathrm{Mg^{+}(g) + e^{-}}$, requiring the first ionization enthalpy. Formation of $\mathrm{Mg^{2+}}$ from neutral magnesium requires both the first and second ionization energies. The second electron is also held more strongly after one electron has already been removed. Thus the total energy for forming the doubly charged ion is much higher, and assertion A is true.

Step 2: Evaluate the reason The nuclear charge remains the same during ionization while the electron count falls. Reduced screening and electron-electron repulsion allow the remaining electron cloud to contract. Therefore $\mathrm{Mg^{2+}}$ is smaller than $\mathrm{Mg^{+}}$, and its positive charge is larger. Both comparisons in reason R are correct.

Step 3: Connect the two statements The progression toward a smaller, more positively charged ion accompanies stronger effective attraction on the electrons retained during successive ionization. Additional energy is required to reach that state. In the successive-ionization comparison intended here, R accounts for the higher energy requirement in A. Options declaring either statement false are eliminated; the option denying this relation is also excluded.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (C).

Quick Tip to Solve: For successive ionization, track the charge and contraction of the ion; distinguish total energy from one individual ionization step.

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65. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : $3.1500\ \mathrm{g}$ **of hydrated oxalic acid dissolved in water to make** $250.0\ \mathrm{mL}$ **solution will result in** $0.1\ \mathrm{M}$ **oxalic acid solution.**

Reason R : Molar mass of hydrated oxalic acid is $126\ \mathrm{g\ mol^{-1}}$

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both A and R are true but R is NOT the correct explanation of A

(B) A is true but R is false

(C) Both A and R are true and R is the correct explanation of A

(D) A is false but R is true

Correct Answer: (C) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Molarity is moles of solute per litre of final solution: $M = n/V$. The final solution volume, rather than the volume of water added, enters this formula.
- Hydrated oxalic acid is the dihydrate, $\mathrm{H_2C_2O_4 \cdot 2\,H_2O}$, so its water of crystallization contributes to its molar mass.

Step 1: Evaluate the molar-mass statement Using the usual atomic masses, anhydrous oxalic acid

has molar mass $2(1) + 2(12) + 4(16) = 90\,\text{g mol}^{-1}$. The two water molecules add $2(18) = 36\,\text{g mol}^{-1}$. Therefore the hydrated acid has molar mass $126\,\text{g mol}^{-1}$, and reason R is true.

Step 2: Calculate the concentration in the assertion The number of moles represented by the weighed hydrate is

$$n = \frac{3.1500}{126} = 0.02500\,\text{mol}.$$

Each mole of the hydrate supplies one mole of oxalic acid. The solution volume is $250.0\,\text{mL} = 0.2500\,\text{L}$. Consequently,

$$M = \frac{0.02500}{0.2500} = 0.1000\,\text{mol L}^{-1}.$$

This is the stated $0.1\,\text{M}$ concentration, so assertion A is true.

Step 3: Check the explanatory connection The molar mass stated in R is exactly the conversion factor needed to turn the weighed mass into the amount used in A. Along with the given final volume, it directly gives the asserted concentration. Thus R explains A, rather than merely being an unrelated true fact. The alternatives making either statement false, or denying the explanation, do not fit the calculation.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (C).

Quick Tip to Solve: Include waters of crystallization in molar mass, then convert the final solution volume to litres before calculating molarity.

66. The correct order of the number of unpaired electrons in the given complexes is

A. $[\text{Fe(CN)}_6]^{3-}$

B. $[\text{FeF}_6]^{3-}$

C. $[\text{CoF}_6]^{3-}$

D. $[\text{Cr}$ **(oxalate)**$_3]^{3-}$

E. $[\text{Ni(CO)}_4]$

Choose the correct answer from the options given below: [4 marks]

(A) E < A < B < D < C

(B) A < E < D < C < B

(C) A < E < C < B < D

(D) E < A < D < C < B

Correct Answer: (D) E < A < D < C < B

Solution:

Concept:

- Find the metal oxidation state and its d-electron count before filling the split orbitals. Octahedral strong-field ligands can cause pairing; weak-field ligands usually give high-spin arrangements.
- For first-row ions, CN^- is a strong-field ligand, while F^- is weak-field. The tetrahedral carbonyl

$Ni(CO)_4$ contains neutral, d^{10} nickel.

Step 1: Count the electrons in A and B Both iron complexes contain Fe^{3+}, which is d^5. In A, cyanide gives low-spin $t_{2g}^5 e_g^0$, with one unpaired electron. In B, fluoride gives high-spin $t_{2g}^3 e_g^2$, with five unpaired electrons. Thus the same oxidation state does not imply the same magnetic behaviour: ligand strength matters.

Step 2: Count the electrons in C, D and E In C, Co^{3+} is d^6; the weak-field fluoride complex has high-spin $t_{2g}^4 e_g^2$, leaving four electrons unpaired. In D, three oxalate ligands contribute charge -6, so chromium is $+3$, or d^3. Its octahedral configuration t_{2g}^3 has three unpaired electrons. In E, CO is neutral, so nickel is in oxidation state zero with d^{10}; all electrons are paired.

Step 3: Arrange the counts in increasing order The unpaired-electron counts for A, B, C, D and E are respectively $1, 5, 4, 3, 0$. Hence

$$0 < 1 < 3 < 4 < 5 \quad \Longrightarrow \quad E < A < D < C < B.$$

Orders placing A before E contradict the diamagnetism of d^{10} nickel, while placing B before D or C contradicts its five unpaired electrons.

Step 4: Conclusion The required order is $E < A < D < C < B$. Hence, the correct option is (D).

Quick Tip to Solve: Write oxidation state, d count and spin state in three separate columns before ordering magnetic moments or unpaired electrons.

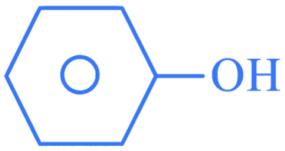
67. The correct order for acidity of the following hydroxyl compound is :

A. CH_3OH

B. $(CH_3)_3\,COH$

Choose the correct answer from the options given below: [4 marks]

C.



D.



E.



(A) E > C > D > A > B

(B) D > E > C > A > B

(C) E > D > C > B > A

(D) C > E > D > B > A

Correct Answer: (A) E > C > D > A > B

Solution:

Concept:

- Acidity increases when the conjugate base formed by loss of H^+ is stabilized. Phenoxide ions are resonance-stabilized, whereas the negative charge in an ordinary alkoxide is localized mainly on oxygen.
- An electron-withdrawing substituent stabilizes phenoxide; an electron-donating substituent destabilizes it. In aqueous solution, alkyl donation and poorer solvation reduce the acidity of more substituted alcohols.

Step 1: Identify the aromatic hydroxyl compounds The structures labelled C, D and E are phenol, para-methoxyphenol and para-nitrophenol respectively. They all form phenoxide-type conjugate bases. Compared with methanol A and tert-butyl alcohol B, their conjugate bases delocalize negative charge into the aromatic system, making these phenols substantially more acidic.

Step 2: Compare the three phenols The nitro group in E withdraws electrons by inductive and resonance effects, stabilizing the conjugate base and increasing acidity. The para-methoxy group in D donates by resonance; this donation is stronger than its opposing inductive withdrawal for this comparison and destabilizes the phenoxide ion. Unsubstituted phenol C lies between them. Therefore

$$E > C > D.$$

Step 3: Compare the two alcohols and combine the orders Methanol A has no electron-releasing alkyl group attached to its oxygen-bearing carbon beyond the methyl framework. Tert-butyl alcohol B has three methyl groups that donate electron density and gives a bulky alkoxide less effectively solvated by water. Thus methanol is more acidic than tert-butyl alcohol: $A > B$. Combining this with the phenol comparison gives $E > C > D > A > B$. The other choices either put methoxyphenol above nitrophenol or reverse the alcohol order.

Step 4: Conclusion The decreasing acidity order is $E > C > D > A > B$. Hence, the correct option is (A).

Quick Tip to Solve: Compare conjugate bases first: nitro stabilizes phenoxide, methoxy donates into it, and increased alkyl substitution weakens aqueous alcohol acidity.

68. Incorrect method of preparation for alcohols from the following is: [4 marks]

(A) Hydroboration-oxidation of alkene.

(B) Reaction of Ketone with RMgBr followed by hydrolysis.

(C) Ozonolysis of alkene.

(D) Reaction of alkyl halide with aqueous NaOH.

Correct Answer: (C) Ozonolysis of alkene.

Solution:

Concept:

- An alcohol contains a hydroxyl group bonded to a saturated carbon. Identify the actual functional group produced by each reaction, rather than treating every oxygen-adding reaction as alcohol formation.
- Hydroboration followed by oxidation hydrates an alkene overall. Grignard addition to a ketone followed by hydrolysis forms a tertiary alcohol, while aqueous hydroxide can substitute a halide group.

Step 1: Check the three alcohol-forming routes In hydroboration-oxidation, boron first adds across the double bond and oxidation replaces the carbon-boron bond by a carbon-oxygen bond, giving an alcohol. A ketone reacts with RMgBr by nucleophilic addition at its carbonyl carbon; hydrolysis of the resulting magnesium alkoxide gives a tertiary alcohol. An alkyl halide can undergo substitution with aqueous NaOH, replacing halogen by OH. Thus A, B and D are standard preparation routes under their suitable conditions.

Step 2: Determine what ozonolysis produces Ozone cleaves a carbon-carbon double bond. With the usual reductive work-up the resulting fragments are aldehydes or ketones; oxidative work-up may further oxidize aldehydes to acids. The carbonyl groups are not converted directly to alcohol groups by ozonolysis. An additional reduction would be a separate reaction, absent from the listed method.

Step 3: Apply the negative wording of the question The question asks for the incorrect preparation method. Although ozonolysis introduces oxygen into the fragments, oxygen incorporation alone does not establish the alcohol functional group. The three valid alcohol routes must therefore be rejected as answers, leaving the double-bond-cleavage reaction as the required choice.

Step 4: Conclusion Ozonolysis of an alkene is not a direct method of alcohol preparation. Hence, the correct option is (C).

Quick Tip to Solve: For an incorrect-method question, identify the product functional group of every route; ozonolysis gives carbonyl fragments, not alcohols directly.

69. Match List I with List II.

	List I: Complex		List II: Crystal Field splitting energy (Δ_o)
A.	$[Ti(H_2O)_6]^{2+}$	**I.**	-1.2
B.	$[V(H_2O)_6]^{2+}$	**II.**	-0.6
C.	$[Mn(H_2O)_6]^{3+}$	**III.**	0
D.	$[Fe(H_2O)_6]^{3+}$	**IV.**	-0.8

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-I, C-II, D-III

(B) A-II, B-IV, C-III, D-I

(C) A-II, B-IV, C-I, D-III

(D) A-IV, B-I, C-III, D-II

Correct Answer: (A) A-IV, B-I, C-II, D-III

Solution:

Concept:

- In an octahedral field each t_{2g} electron contributes $-0.4\Delta_o$, and each e_g electron contributes $+0.6\Delta_o$. Thus $\text{CFSE} = (-0.4n_t + 0.6n_e)\Delta_o$.
- Water is a weak-field ligand for the ions listed, so the d^4 and d^5 configurations here are high spin. The listed negative coefficients represent stabilization energies in units of Δ_o.

Step 1: Evaluate the titanium and vanadium complexes Water is neutral. Titanium in A is $+2$, hence d^2; both electrons occupy t_{2g}, giving $-0.8\Delta_o$. Thus A matches IV. Vanadium in B is also $+2$, hence d^3; its three electrons occupy the three lower orbitals singly, giving $-1.2\Delta_o$. Thus B matches I.

Step 2: Evaluate the manganese and iron complexes Manganese in C is $+3$, or d^4. Its high-spin arrangement is $t_{2g}^3 e_g^1$, so

$$\text{CFSE}_C = [3(-0.4) + 1(0.6)]\Delta_o = -0.6\Delta_o.$$

Hence C matches II. Iron in D is $+3$, or d^5. The high-spin arrangement is $t_{2g}^3 e_g^2$, giving

$$\text{CFSE}_D = [3(-0.4) + 2(0.6)]\Delta_o = 0.$$

Thus D matches III.

Step 3: Select the complete matching code The four assignments are A-IV, B-I, C-II and D-III. Only one option includes all four together. The alternatives assigning zero stabilization to the manganese d^4 complex or negative stabilization to the high-spin iron d^5 complex fail immediately. No pairing-energy term is needed for these high-spin fillings because there are no paired electrons in the five d orbitals.

Step 4: Conclusion The matching is A-IV, B-I, C-II, D-III. Hence, the correct option is (A).

Quick Tip to Solve: For high-spin octahedral d counts 1 through 5, use CFSE coefficients -0.4, -0.8, -1.2, -0.6 and 0, in units of the splitting.

70. In Carius tube, an organic compound 'X' is treated with sodium peroxide to form a mineral acid 'Y'.

The solution of $BaCl_2$ is added to 'Y' to form a precipitate 'Z'. 'Z' is used for the quantitative estimation of an extra element. 'X' could be [4 marks]

(A) Cytosine

(B) Methionine

(C) Chloroxylenol

(D) A nucleotide

Correct Answer: (B) Methionine

Solution:

Concept:

- In quantitative elemental analysis, sulfur in an organic compound is oxidized to sulfate and ultimately weighed as insoluble $BaSO_4$. The precipitate must have a definite composition for gravimetric estimation.
- Addition of $BaCl_2$ to an appropriate sulfate solution gives a white precipitate: $Ba^{2+} + {SO_4}^{2-} \longrightarrow BaSO_4(s)$.

Step 1: Identify the mineral acid and precipitate The oxidation converts sulfur from its organic form into sulfate. On appropriate acidification, the mineral acid Y is sulfuric acid, H_2SO_4. Barium chloride then supplies Ba^{2+}, forming Z, barium sulfate:

$$H_2SO_4 + BaCl_2 \longrightarrow BaSO_4(s) + 2\,HCl.$$

It is the sulfate precipitation that identifies the extra element being estimated as sulfur.

Step 2: Look for sulfur in the proposed compounds Methionine is a sulfur-containing amino acid. Its side chain is $-CH_2CH_2SCH_3$, so it provides sulfur that can be oxidized to sulfate. Cytosine is a nitrogen-containing nucleobase without sulfur. Chloroxylenol contains chlorine, but chlorine is not estimated as barium sulfate. A normal nucleotide contains a nitrogenous base, sugar and phosphate group; its characteristic extra element is phosphorus rather than sulfur.

Step 3: Connect the choice to gravimetric estimation Each mole of sulfur converted quantitatively into sulfate yields one mole of $BaSO_4$. Thus a measured precipitate mass can be converted to sulfur mass by the molar-mass ratio $M(S)/M(BaSO_4)$. This one-to-one relation explains why the stated precipitate is useful and confirms that the required starting compound must contain sulfur.

Step 4: Conclusion Compound X could be methionine, whose sulfur gives sulfate and then $BaSO_4$. Hence, the correct option is (B).

Quick Tip to Solve: Use the weighed precipitate to identify the element: barium sulfate signals sulfur, whereas silver halides signal halogens.

71. Match List I with List II

	List - I		List - II (Δ_0)
A.	**16 g of** CH_4 (g)	**I.**	**Weighs 28 g**
B.	**1 g of** H_2 (g)	**II.**	60.2×10^{23} **electrons**
C.	**1 mole of** N_2 (g)	**III.**	**Weighs 32 g**
D.	**0.5 mol of** SO_2 (g)	**IV.**	**Occupies 11.4 L volume of STP**

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-III, C-IV, D-I

(B) A-II, B-IV, C-I, D-III

(C) A-I, B-III, C-II, D-IV

(D) A-II, B-IV, C-III, D-I

Correct Answer: (B) A-II, B-IV, C-I, D-III

Solution:

Concept:

- The amount of a substance is $n = m/M$, where m is mass and M is molar mass. One mole contains $N_A = 6.02 \times 10^{23}$ molecules.
- For a neutral molecule, the electron count is the sum of the atomic numbers of its atoms. Gas volume at standard conditions is proportional to the number of moles.

Step 1: Match methane with its electron count Methane has molar mass $12 + 4(1) = 16\,\text{g mol}^{-1}$, so 16 g is one mole. Each CH_4 molecule has $6 + 4 = 10$ electrons. Thus the sample contains

$$10N_A = 60.2 \times 10^{23} \text{ electrons},$$

giving A-II. This counts electrons in all atoms, rather than merely the four bonding pairs.

Step 2: Convert hydrogen mass to its standard volume The molar mass of H_2 is $2\,\text{g mol}^{-1}$, so 1 g corresponds to 0.5 mol. Half a mole occupies half the standard molar gas volume. Using the 1-bar standard molar volume of about $22.7\,\text{L mol}^{-1}$, this is approximately $11.4\,\text{L}$, matching IV. Thus B-IV.

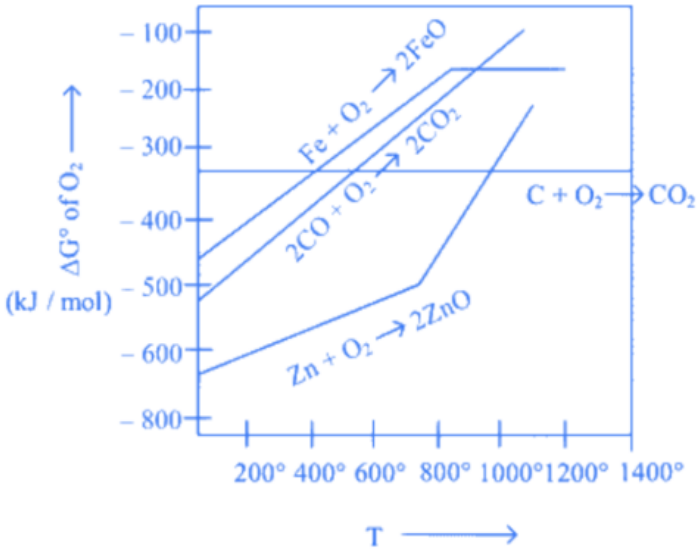
Step 3: Match the two remaining masses One mole of N_2 weighs $2(14) = 28\,\text{g}$, giving C-I. The molar mass of SO_2 is $32 + 2(16) = 64\,\text{g mol}^{-1}$, so 0.5 mol weighs $32\,\text{g}$, giving D-III. Option A wrongly assigns 32 g to hydrogen; C fails already for methane; D interchanges the nitrogen and sulphur dioxide mass matches.

Step 4: Conclusion The complete matching is A-II, B-IV, C-I and D-III. Hence, the correct option is (B).

Quick Tip to Solve: First match the two straightforward molar masses, then count ten electrons per methane molecule to fix the complete matching.

72. Gibbs energy vs T plot for the formation of oxides is given below.

For the given diagram, the correct statement is - [4 marks]



(A) At $600^{\circ}C$, C can reduce FeO

(B) At $600^{\circ}C$, C can reduce ZnO

(C) At $600^{\circ}C$, CO can reduce ZnO

(D) At $600^{\circ}C$, CO cannot reduce FeO

Correct Answer: (A) At $600^{\circ}C$, C can reduce FeO

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy of oxide formation against temperature. Lower lines correspond to more favourable oxygen uptake at the same temperature.
- For a reduction, reverse the metal-oxidation reaction and add the reductant-oxidation reaction on the same oxygen basis. Reduction is feasible when the resulting ΔG° is negative.

Step 1: Compare carbon oxidation with iron oxidation At the marked temperature $600^{\circ}C$, the horizontal $C + O_2 \longrightarrow CO_2$ line lies below the $2\,Fe + O_2 \longrightarrow 2\,FeO$ line. Subtracting iron-oxide formation from carbon oxidation therefore gives a negative Gibbs energy change:

$$2\,FeO + C \longrightarrow 2\,Fe + CO_2.$$

Carbon can consequently reduce FeO at this temperature, as stated in option A.

Step 2: Check the alternatives using the zinc line The zinc-oxide formation line lies well below both the carbon-to-carbon-dioxide line and the carbon-monoxide oxidation line at $600^{\circ}C$. Neither of these oxidation reactions supplies a sufficiently negative Gibbs energy to compensate for reversing zinc-oxide formation. Thus carbon cannot reduce ZnO under the stated diagram conditions, rejecting B, and CO cannot reduce it either, rejecting C.

Step 3: Test the carbon monoxide claim for iron The $2\,CO + O_2 \longrightarrow 2\,CO_2$ line lies below the iron

oxidation line at $600^\circ C$. Accordingly, $FeO + CO \longrightarrow Fe + CO_2$ is feasible. Option D says that carbon monoxide cannot perform this reduction, so its claim is false. The comparison must be made at the specified temperature rather than from the relative slopes alone.

Step 4: Conclusion At $600^\circ C$, carbon can reduce FeO according to the given plot. Hence, the correct option is (A).

Quick Tip to Solve: Draw a mental vertical at the required temperature: a reductant works when its oxidation line is below the metal-oxide formation line.

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73. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Physical properties of isotopes of hydrogen are different.

Reason R : Mass difference between isotopes of hydrogen is very large.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) **A** is true but **R** is false

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) **A** is false but **R** is true

(D) Both **A** and **R** are true and R is the correct explanation of **A**

Correct Answer: (D) Both **A** and **R** are true and R is the correct explanation of **A**

Solution:

Concept:

- Protium, deuterium and tritium have the same atomic number, 1, but approximate mass numbers 1, 2 and 3, respectively.
- Mass-dependent physical properties include diffusion rates and molecular vibrational frequencies. Identical electronic configurations do not require identical physical properties.

Step 1: Evaluate the assertion The isotopes have different masses even though their electronic structures are alike. Their molecules therefore differ in density, diffusion rate, zero-point energy and related physical properties. For example, heavier hydrogen molecules diffuse more slowly under the same conditions. Thus the assertion that physical properties of hydrogen isotopes differ is true.

Step 2: Evaluate the reason The mass differences are exceptionally large in relative terms: deuterium is approximately twice as heavy as protium and tritium approximately three times as heavy. A difference of one atomic mass unit is a substantial fraction of the mass of hydrogen, whereas it is a much smaller fractional change for isotopes of a heavy element. The reason is therefore true in the intended relative-mass sense.

Step 3: Connect the reason to the observed isotope effect For molecular motion at a fixed temperature, characteristic speeds vary as the inverse square root of mass. For bond vibrations, the frequency varies inversely with the square root of reduced mass. Consequently, the large fractional mass changes directly produce sizeable differences in physical behaviour. The reason explains the assertion, so the

option saying both are true without an explanatory connection is excluded. The options declaring either statement false are excluded as well.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (D).

Quick Tip to Solve: For isotope questions, separate unchanged electronic structure from mass-dependent motion; hydrogen has the largest fractional isotope-mass changes.

74. The correct order of metallic character is : [4 marks]

(A) K $>$ Be $>$ Ca

(B) Ca $>$ K $>$ Be

(C) K $>$ Ca $>$ Be

(D) Be $>$ Ca $>$ K

Correct Answer: (C) K $>$ Ca $>$ Be

Solution:

Concept:

- Metallic character is the tendency of an atom to lose electrons and form positive ions. Lower ionisation energy generally favours this behaviour.
- Metallic character increases down a group as atomic size and shielding increase, and generally decreases from left to right across a period as effective nuclear attraction increases.

Step 1: Compare potassium with calcium in the same period Potassium and calcium both lie in period 4. Potassium is in group 1 and calcium is in group 2. Potassium has the configuration $[\mathrm{Ar}]4s^1$, whereas calcium has $[\mathrm{Ar}]4s^2$. The more weakly held outer electron of potassium is removed more readily. Moving from potassium to calcium increases the nuclear attraction on the valence shell, so potassium is more metallic than calcium:

$$\mathrm{K} > \mathrm{Ca}.$$

Step 2: Compare calcium with beryllium down group 2 Beryllium is in period 2 of group 2, above calcium. Calcium has more occupied shells and greater shielding, so its valence electrons are farther from the nucleus and are lost more readily. Beryllium has a small radius and relatively high ionisation energy; its compounds also show appreciable covalent character. Hence

$$\mathrm{Ca} > \mathrm{Be}.$$

Step 3: Assemble and check the entire order Combining the two comparisons gives $\mathrm{K} > \mathrm{Ca} > \mathrm{Be}$. Option A incorrectly places beryllium above calcium. Option B reverses the potassium-calcium comparison within period 4. Option D places beryllium first and potassium last, contradicting both the group and period trends. The result concerns metallic character of the elements, rather than simply their positions in a list of atomic masses.

Step 4: Conclusion The correct order is $\mathrm{K} > \mathrm{Ca} > \mathrm{Be}$. Hence, the correct option is (C).

Quick Tip to Solve: Compare K with Ca across period 4 first, then compare Ca with Be down group 2; combine those two unambiguous inequalities.

75. Buna-S can be represented as: [4 marks]

(A) $[-CH_2-CH=CH-CH_2-CH(C_6H_5)-CH_2-]_n$

(B) $[-CH_2-CH=C(C_6H_5)-CH=CH-CH_2-]_n$

(C) $[-CH_2-CH=CH-CH=C(C_6H_5)-CH_2-]_n$

(D) $[-CH=CH-CH=CH-CH(C_6H_5)-CH_2-]_n$

Correct Answer: (A) $[-CH_2-CH=CH-CH_2-CH(C_6H_5)-CH_2-]_n$

Solution:

Concept:

- Buna-S, or styrene-butadiene rubber, is an addition copolymer of 1,3-butadiene and styrene. The first monomer supplies a four-carbon chain segment; the second supplies a two-carbon segment with a phenyl side group.
- In the usual 1,4-addition of butadiene, one double bond remains per incorporated butadiene unit. Addition of styrene consumes its vinyl double bond and leaves a phenyl substituent on a saturated backbone carbon.

Step 1: Identify the butadiene-derived segment When the terminal carbon atoms of butadiene join the polymer chain, the four-carbon segment is represented as $-CH_2-CH=CH-CH_2-$. The central double bond remains, while the terminal carbon atoms become methylene groups. A correct simplified Buna-S repeat representation must contain this segment; it must not preserve the original two terminal double bonds of the free monomer.

Step 2: Identify the styrene-derived segment The vinyl group of styrene undergoes addition to give the two-carbon segment $-CH(C_6H_5)-CH_2-$, equivalently written in the opposite orientation. Its phenyl ring stays attached to a saturated CH carbon. Joining the two monomer-derived portions gives

$$[-CH_2-CH=CH-CH_2-CH(C_6H_5)-CH_2-]_n .$$

This is the structure shown in option A.

Step 3: Reject the other structural formulae Options B and C put the phenyl substituent on a double-bonded backbone carbon and contain an additional backbone double bond in the six-carbon displayed segment. Option D also retains two backbone double bonds instead of the one expected after addition of the two monomer units. These structures do not show the standard butadiene-styrene addition pattern. The displayed segment is a conventional simplified representation of the copolymer, rather than a requirement of a perfectly alternating commercial sequence.

Step 4: Conclusion Buna-S contains the butadiene segment joined to a saturated styrene-derived segment with a phenyl side group. Hence, the correct option is (A).

Quick Tip to Solve: Locate the phenyl-bearing carbon: in Buna-S it belongs to a saturated styrene segment, while the butadiene segment retains one double bond.

76. Ferric chloride is applied to stop bleeding because - [4 marks]

(A) $FeCl_3$ reacts with the constituents of blood which is a positively charged sol.

(B) Blood absorbs $FeCl_3$ and forms a complex.

(C) Cl^- ions cause coagulation of blood.

(D) Fe^{3+} ions coagulate blood which is a negatively charged sol.

Correct Answer: (D) Fe^{3+} ions coagulate blood which is a negatively charged sol.

Solution:

Concept:

- Colloidal particles remain dispersed partly because their like charges produce mutual electrostatic repulsion. Neutralising this charge promotes aggregation or coagulation.
- The effective coagulating ion carries a charge opposite to that of the sol. The Hardy-Schulze rule associates greater counter-ion valency with stronger coagulating power.

Step 1: Identify the charged colloid and its counter-ion In the standard colloidal description relevant here, blood is treated as a negatively charged sol. A positively charged ion is therefore needed to reduce the electrical stabilisation of its dispersed particles. Ferric chloride provides Fe^{3+} and Cl^- ions in aqueous solution:

$$FeCl_3 \longrightarrow Fe^{3+} + 3\,Cl^-.$$

The ferric ion has the required opposite charge.

Step 2: Explain why ferric ions help stop bleeding The trivalent ferric ions neutralise the negative charge on colloidal blood particles and facilitate their coagulation at the application site. Once electrostatic repulsion is sufficiently reduced, the particles can aggregate and form a clot-like coagulum that checks blood flow. Its high positive charge makes Fe^{3+} an effective coagulant according to the counter-ion valency rule.

Step 3: Check the competing explanations Option A incorrectly describes blood as a positively charged sol, so it identifies the sign of the dispersed particles wrongly. Option B attributes the effect simply to absorption and complex formation, whereas the tested colloidal mechanism is charge neutralisation and coagulation. Option C selects chloride, a negatively charged ion; chloride is not the counter-ion required to coagulate a negative sol. Option D correctly identifies both the negative sol and the positively charged ferric coagulating ion.

Step 4: Conclusion Fe^{3+} ions coagulate blood, treated as a negatively charged sol. Hence, the correct option is (D).

Quick Tip to Solve: For a coagulation question, determine the sign of the sol first and select the oppositely charged ion; then compare its valency.

77. The correct relationships between unit cell edge length 'a' and radius of sphere 'r' for face-centred and body-centred cubic structures respectively are : [4 marks]

(A) $r = 2\sqrt{2}a$ and $\sqrt{3}r = 4a$

(B) $r = 2\sqrt{2}a$ and $4r = \sqrt{3}a$

(C) $2\sqrt{2}r = a$ and $\sqrt{3}r = 4a$

(D) $2\sqrt{2}r = a$ and $4r = \sqrt{3}a$

Correct Answer: (D) $2\sqrt{2}r = a$ and $4r = \sqrt{3}a$

Solution:

Concept:

- Atoms are modelled as touching hard spheres of radius r, and a denotes the edge length of the cubic unit cell. The relevant diagonal follows from Pythagoras.
- In a face-centred cubic lattice the contact direction is a face diagonal; in a body-centred cubic lattice it is the body diagonal. Each such complete contact diagonal spans four radii.

Step 1: Find the face-centred cubic relation The diagonal of a square face of side a has length $\sqrt{2}a$. Along this diagonal, a corner atom touches the face-centred atom, which in turn touches the opposite corner atom. The centre-to-centre distance from one corner to the other is therefore $r + 2r + r = 4r$. Consequently,

$$\sqrt{2}a = 4r \quad \Rightarrow \quad a = 2\sqrt{2}r.$$

This is the first relation required by the question.

Step 2: Find the body-centred cubic relation The space diagonal of a cube has length $\sqrt{a^2 + a^2 + a^2} = \sqrt{3}a$. It passes through the body-centred atom and the two opposite corner atoms. These three atoms touch along the body diagonal, again spanning $4r$. Hence

$$\sqrt{3}a = 4r.$$

The body diagonal, rather than the edge or face diagonal, controls the radius relation in this lattice.

Step 3: Compare the pair with every option The face-centred result eliminates A and B because they incorrectly state $r = 2\sqrt{2}a$, reversing the relation between the radius and edge length. Options C and D have the correct face-centred relation, but C reverses the body-centred equation by stating $\sqrt{3}r = 4a$. Only D gives both $2\sqrt{2}r = a$ and $4r = \sqrt{3}a$ in the requested face-centred, body-centred order.

Step 4: Conclusion For FCC, $a = 2\sqrt{2}r$; for BCC, $4r = \sqrt{3}a$. Hence, the correct option is (D).

Quick Tip to Solve: Remember the contact directions: FCC uses a face diagonal and BCC a body diagonal; in each case the full diagonal spans four radii.

78. Number of water molecules in washing soda and soda ash respectively are : [4 marks]

(A) 1 and 0

(B) 10 and 1

(C) 1 and 10

(D) 10 and 0

Correct Answer: (D) 10 and 0

Solution:

Concept:

- Water of crystallisation is the fixed number of water molecules associated with one formula unit of a crystalline hydrated salt. It is written after a dot in the chemical formula.
- Washing soda is sodium carbonate decahydrate, $Na_2CO_3 \cdot 10\,H_2O$, whereas soda ash is anhydrous sodium carbonate, Na_2CO_3.

Step 1: Identify washing soda The common name washing soda refers specifically to the decahydrate, not to an arbitrary solution of sodium carbonate in water. Its formula $Na_2CO_3 \cdot 10\,H_2O$ contains ten water molecules per formula unit. Thus the first number requested is 10.

Step 2: Identify soda ash Soda ash is the anhydrous form of sodium carbonate. Its formula is simply Na_2CO_3, with no water of crystallisation. The oxygen atoms in the carbonate ion are part of CO_3^{2-}; they must not be counted as water molecules. Thus the second number is 0.

Step 3: Compare the ordered pairs The question asks for washing soda first and soda ash second, so the required pair is $(10, 0)$. Option A assigns only one water molecule to washing soda, and option C reverses the identity and hydration count. Option B correctly starts with 10 but incorrectly gives one water molecule to anhydrous soda ash. Only option D matches both formulae.

Step 4: Conclusion The numbers of water molecules in washing soda and soda ash are 10 and 0, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Write the formulae before counting: washing soda is the decahydrate, while the word ash identifies the anhydrous carbonate.

79. The reaction used for preparation of soap from fat is : [4 marks]

(A) an addition reaction

(B) alkaline hydrolysis reaction

(C) reduction reaction

(D) an oxidation reaction

Correct Answer: (B) alkaline hydrolysis reaction

Solution:

Concept:

- Fats are triesters of glycerol with long-chain fatty acids. An ester undergoes hydrolysis when its ester linkage is cleaved by reaction with water.

- Saponification is the alkaline hydrolysis of a fat using NaOH or KOH. It produces glycerol and sodium or potassium salts of long-chain fatty acids, which are soaps.

Step 1: Recognise the functional group in a fat A triglyceride contains three ester linkages. The preparation of soap breaks these linkages and converts each fatty-acid portion into its carboxylate salt. The glycerol portion is released as an alcohol. The reaction therefore belongs to hydrolysis of esters rather than an addition across a double bond.

Step 2: Write the alkaline hydrolysis reaction For a fat represented by $\mathrm{C_3H_5(OOCR)_3}$, where R is a long hydrocarbon chain, the reaction is

$$\mathrm{C_3H_5(OOCR)_3 + 3\,NaOH \longrightarrow C_3H_5(OH)_3 + 3\,RCOONa.}$$

The products are glycerol and the sodium salt of the fatty acid. Alkali supplies the hydroxide needed for cleavage and leaves the fatty acid in its salt form.

Step 3: Eliminate the other reaction classes Addition would attach groups across a multiple bond and would not describe cleavage of the ester groups. Reduction would require a reducing process, and oxidation would require an oxidising process. Neither is responsible for producing soap from fat here. Hydrogenation of an unsaturated fat is an addition reaction, but it produces a more saturated fat rather than soap.

Step 4: Conclusion Soap is prepared from fat by alkaline hydrolysis, called saponification. Hence, the correct option is (B).

Quick Tip to Solve: When a fat reacts with hot alkali to give glycerol and fatty-acid salts, identify the process immediately as saponification.

80. The delicate balance of $\mathrm{CO_2}$ and $\mathrm{O_2}$ is NOT disturbed by : [4 marks]

(A) Burning of Coal

(B) Burning of petroleum

(C) Respiration

(D) Deforestation

Correct Answer: (C) Respiration

Solution:

Concept:

- The natural carbon dioxide–oxygen balance is maintained through linked biological processes. Photosynthesis consumes $\mathrm{CO_2}$ and releases $\mathrm{O_2}$, while respiration uses oxygen and releases carbon dioxide.
- Combustion of fossil fuels adds carbon dioxide while consuming oxygen. Removal of forests also reduces the photosynthetic uptake of carbon dioxide.

Step 1: Assess the two combustion processes Coal and petroleum are fossil fuels. Burning them converts their carbon into carbon dioxide and consumes atmospheric oxygen. For carbon, the

representative equation is

$$C + O_2 \longrightarrow CO_2.$$

Large-scale fossil-fuel combustion adds carbon stored underground to the active atmospheric cycle. Thus both burning coal and burning petroleum can disturb the balance; options A and B are excluded.

Step 2: Assess deforestation Green plants take up carbon dioxide during photosynthesis and release oxygen. Deforestation reduces this natural sink for carbon dioxide and the corresponding biological source of oxygen. It therefore disturbs the balance rather than maintaining it, so option D is excluded.

Step 3: Place respiration in the natural cycle Respiration is a normal biological process whose carbon dioxide production is balanced by photosynthetic uptake in the natural cycle. A representative equation is

$$C_6H_{12}O_6 + 6\,O_2 \longrightarrow 6\,CO_2 + 6\,H_2O.$$

Although respiration exchanges the two gases, this exchange is part of their normal recycling. Among the listed activities it is the natural balancing process, rather than the added disturbance associated with fossil-fuel burning or forest removal.

Step 4: Conclusion Respiration belongs to the natural carbon dioxide–oxygen cycle and is the requested activity. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish a normal exchange within the biological cycle from an extra fossil-carbon input or the removal of a photosynthetic sink.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. An aqueous solution of volume $300\ \text{cm}^3$ contains $0.63\ \text{g}$ of protein. The osmotic pressure of the solution at $300\ \text{K}$ is 1.29 mbar. The molar mass of the protein is _____ g mol^{-1}

Given : R = 0.083 L bar K^{-1} mol^{-1} [4 marks]

Solution:

Concept:

- For a dilute solution of a non-electrolyte, osmotic pressure obeys $\pi V = nRT$, where π is osmotic pressure, V is solution volume and n is the amount of solute.
- If mass w of a solute has molar mass M, then $n = w/M$. Hence $M = wRT/(\pi V)$. Pressure and volume units must match the given gas constant.

Step 1: Convert the data to compatible units The gas constant is $R = 0.083\,\text{L bar}\,\text{K}^{-1}\,\text{mol}^{-1}$, so use litres for volume and bar for pressure:

$$V = 300\,\text{cm}^3 = 0.300\,\text{L}, \qquad \pi = 1.29\,\text{mbar} = 1.29 \times 10^{-3}\,\text{bar}.$$

The protein mass is $w = 0.63\,\text{g}$ and temperature is $T = 300\,\text{K}$. A protein is treated as a non-dissociating solute in this dilute-solution calculation.

Step 2: Rearrange and substitute in the osmotic-pressure relation Insert $n = w/M$ into $\pi V = nRT$ and isolate the molar mass:

$$M = \frac{wRT}{\pi V} = \frac{0.63 \times 0.083 \times 300}{(1.29 \times 10^{-3}) \times 0.300}\,\mathrm{g\,mol^{-1}}.$$

The numerator is 15.687 and the denominator is 0.000387. Therefore

$$M = 40534.8837\ldots\,\mathrm{g\,mol^{-1}}.$$

Step 3: Check the unit and report the value The factors litre and bar cancel between RT and πV, and kelvin cancels against the temperature, leaving grams per mole. A high molar mass is consistent with a protein producing only a small osmotic pressure at the stated mass concentration. Reporting the numerical value to the nearest integer gives 40535.

Step 4: Conclusion The molar mass of the protein is $\boxed{40535\,\mathrm{g\,mol^{-1}}}$, to the nearest integer.

Quick Tip to Solve: With R in litre-bar units, convert both cubic centimetres to litres and millibar to bar before substituting.

82. The number of molecules from the following which contain only two lone pair of electrons is _____

$$\mathrm{H_2O, N_2, CO, XeF_4, NH_3, NO, CO_2,\ F_2}$$

[4 marks]

Solution:

Concept:

- A lone pair is a pair of valence electrons not shared in a bond. Count lone pairs over the whole molecule, including terminal atoms, when the question says the molecule contains only two lone pairs.
- Use Lewis structures and the total valence-electron count. A singly occupied orbital in an odd-electron species is an unpaired electron, not a lone pair.

Step 1: Identify the three molecules with two lone pairs In $\mathrm{H_2O}$, oxygen makes two single bonds and retains two lone pairs; hydrogen has none. In $\mathrm{N_2}$, the nitrogen atoms share a triple bond and each nitrogen retains one lone pair, giving two in total. In CO, the carbon–oxygen triple bond is accompanied by one lone pair on carbon and one on oxygen, again giving two.

Step 2: Count the lone pairs in the remaining closed-shell molecules In $\mathrm{XeF_4}$, xenon has two lone pairs but each of the four fluorine atoms also has three. Its molecular total is therefore $2 + 4(3) = 14$, not two. $\mathrm{NH_3}$ has one lone pair on nitrogen. Each oxygen atom in $\mathrm{CO_2}$ has two lone pairs, so the molecular total is four. Each fluorine atom in $\mathrm{F_2}$ has three lone pairs, giving six in total.

Step 3: Check the odd-electron molecule and total the matches NO has $5 + 6 = 11$ valence electrons. Its usual Lewis representation has a double bond, three lone pairs in total and one unpaired electron.

The unpaired electron must not be counted as a pair. Thus NO is also excluded. Exactly H_2O, N_2 and CO satisfy the condition, so the count is $1 + 1 + 1 = 3$.

Step 4: Conclusion The molecules containing exactly two lone pairs in total are H_2O, N_2 and CO. The required number is $\boxed{3}$.

Quick Tip to Solve: Do not count only the central atom: XeF4 has two lone pairs on xenon but fourteen lone pairs in the whole molecule.

83. For a metal ion, the calculated magnetic moment is 4.90 BM. This metal ion has _____ number of unpaired electrons. [4 marks]

Solution:

Concept:

- The calculated spin-only magnetic moment of a transition-metal ion is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons and BM denotes Bohr magneton.
- Only unpaired electrons contribute to the spin-only moment because paired electrons have opposite spins whose magnetic contributions cancel. The required count is a non-negative integer.

Step 1: Apply the spin-only formula The given calculated magnetic moment is $4.90\,\text{BM}$. Substitute it in the spin-only expression and square both sides:

$$\sqrt{n(n+2)} = 4.90, \qquad n(n+2) = 4.90^2 = 24.01.$$

Because the magnetic moment is rounded to two decimal places, the small difference from 24 is a rounding effect. The electron count itself is an integer.

Step 2: Solve for the admissible electron count Using $n(n+2) \simeq 24$ gives

$$n^2 + 2n - 24 = 0, \qquad (n-4)(n+6) = 0.$$

The possible algebraic roots are 4 and -6. A negative number of unpaired electrons has no physical meaning, so select $n = 4$. Equivalently, from the rounded input one obtains $n = -1 + \sqrt{25.01} \simeq 4.001$, confirming the same integer.

Step 3: Verify the magnetic moment for that count For four unpaired electrons, the predicted moment is

$$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.89898\ldots\ \text{BM},$$

which rounds exactly to $4.90\,\text{BM}$. For comparison, three and five unpaired electrons would give $\sqrt{15} \simeq 3.87$ and $\sqrt{35} \simeq 5.92\,\text{BM}$, respectively, so they do not fit.

Step 4: Conclusion The metal ion has $\boxed{4}$ unpaired electrons.

Quick Tip to Solve: Remember the common spin-only values: four unpaired electrons give square root of 24, approximately 4.90 BM.

84. In alkaline medium, the reduction of permanganate anion involves a gain of _____ electrons.

[4 marks]

Solution:

Concept:

- In the permanganate ion MnO_4^-, oxygen has oxidation number -2. Therefore manganese has oxidation number +7 because $x + 4(-2) = -1$.
- In the usual neutral or alkaline permanganate reduction, the product is MnO_2, where manganese has oxidation number +4. A decrease in oxidation number corresponds to a gain of electrons.

Step 1: Find the initial and final oxidation states For the reactant MnO_4^-, the four oxygen atoms contribute -8, so manganese must be +7 to give the net charge -1. The usual manganese product in alkaline permanganate redox reactions is MnO_2. Its two oxygen atoms contribute -4, so manganese is +4.

Step 2: Calculate the electron gain The change in manganese oxidation number is from +7 to +4. Reduction therefore requires three electrons per manganese atom:

$$Mn^{VII} + 3\,e^- \longrightarrow Mn^{IV}.$$

There is one manganese atom per permanganate ion, so one permanganate ion gains three electrons in this conversion. The count refers to an ion, rather than to an arbitrarily scaled overall chemical equation.

Step 3: Check with the balanced alkaline half-reaction The corresponding half-reaction is

$$MnO_4^- + 2\,H_2O + 3\,e^- \longrightarrow MnO_2 + 4\,OH^-.$$

Both sides contain one manganese atom, six oxygen atoms and four hydrogen atoms. The charge on each side is -4, confirming the three-electron count. The medium-dependent product matters: reduction in acid to Mn^{2+} would require five electrons, while specially controlled strongly alkaline reduction to manganate would be a different, one-electron conversion.

Step 4: Conclusion For the usual alkaline reduction of permanganate to manganese dioxide, the gain is $\boxed{3}$ electrons per permanganate ion.

Quick Tip to Solve: Identify the manganese product before counting electrons: Mn(VII) to Mn(IV) requires three electrons; the acidic Mn(II) product would give five.

85. The number of incorrect statement/s from the following is _____

A. The successive half lives of zero order reactions decreases with time.

B. A substance appearing as reactant in the chemical equation may not affect the rate of reaction

C. Order and molecularity of a chemical reaction can be a fractional number

D. The rate constant units of zero and second order reaction are $\mathrm{mol\ L^{-1}\ s^{-1}}$ and $\mathrm{mol^{-1}\ L\ s^{-1}}$ respectively

[4 marks]

Solution:

Concept:

- For a zero-order reaction, the concentration falls linearly: $[A]_t = [A]_0 - kt$. The time required to halve a concentration is proportional to the concentration at the start of that interval.
- The order is the sum of experimentally determined exponents in the rate law. Molecularity counts the reacting species in an elementary step and is always a positive integer.
- From $r = k[A]^m$, the units of k are $(\mathrm{mol\,L^{-1}})^{1-m}\mathrm{s^{-1}}$.

Step 1: Check the statements about half-life and rate dependence In statement A, the first halving takes $[A]_0/(2k)$, the next takes $[A]_0/(4k)$, and the next takes $[A]_0/(8k)$. Thus successive half-lives decrease, so A is correct. In statement B, a reactant can have zero order in the experimentally observed rate law. Its concentration then has exponent zero and changing it need not change the rate under those conditions. The balanced equation alone does not determine the rate law. Hence B is also correct.

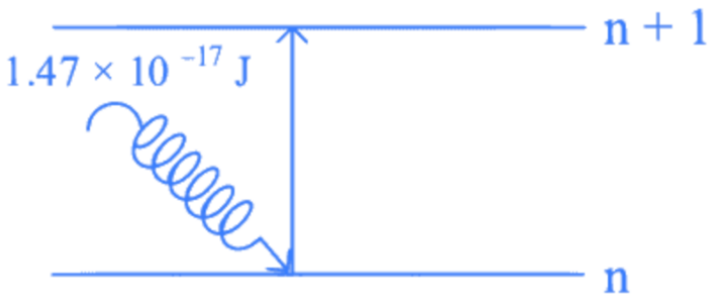
Step 2: Check fractional values and the stated units Statement C incorrectly allows fractional molecularity. Reaction order may be fractional, but a count of participating species in an elementary event cannot be fractional. Therefore C is incorrect. For a zero-order reaction, $k = r$, so its units are $\mathrm{mol\,L^{-1}\,s^{-1}}$. For second order, dividing the rate units by concentration squared gives $\mathrm{L\,mol^{-1}\,s^{-1}}$. Both units in D are correct. There are no additional incorrect statements among A, B and D.

Step 3: Conclusion Only statement C is incorrect. Therefore the number of incorrect statements is $\boxed{1}$.

Quick Tip to Solve: Distinguish an experimental exponent (order) from a count of species (molecularity); only the former can be fractional.

86. The electron in the n^{th} orbit of Li^{2+} is excited to $(n+1)$ orbit using the radiation of energy 1.47×10^{-17} J (as shown in the diagram). The value of n is _____

Given: $R_H = 2.18 \times 10^{-18}$ J [4 marks]



Solution:

Concept:

- The one-electron ion Li^{2+} has nuclear charge $Z = 3$. Its Bohr energy in orbit n is $E_n = -R_H Z^2/n^2$, where $R_H = 2.18 \times 10^{-18}$ J.
- Excitation to the next orbit requires the positive difference $\Delta E = R_H Z^2[1/n^2 - 1/(n+1)^2]$. The orbit number must be a positive integer.

Step 1: Form the adjacent-orbit energy difference The electron absorbs radiation, so the final orbit has a higher, less negative energy. Substitute the given nuclear charge and photon energy into the energy difference, rather than equating the photon energy to the magnitude of a single level:

$$1.47 \times 10^{-17} = 9(2.18 \times 10^{-18}) \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right).$$

Thus the bracket is approximately $1.47/1.962 = 0.7492$. The small departure from a simple fraction reflects the rounded photon energy supplied in the question.

Step 2: Identify and check the positive integer orbit For $n = 1$, the bracket is $1 - 1/4 = 3/4$. The corresponding excitation energy is

$$\Delta E = 1.962 \times 10^{-17} \left(\frac{3}{4} \right) = 1.4715 \times 10^{-17} \text{ J},$$

which rounds to the given 1.47×10^{-17} J. The adjacent-level gap decreases as n increases: it equals the integral of $2/x^3$ from n to $n+1$. Already at $n = 2$, the bracket is only $1/4 - 1/9 = 5/36$. Therefore no larger positive integer gives the required energy.

Step 3: Conclusion The initial orbit is the ground orbit, so $\boxed{n = 1}$. The excitation is from orbit 1 to orbit 2.

Quick Tip to Solve: For hydrogen-like ions include the factor Z^2, and use an energy difference for a transition rather than a single level energy.

87. The number of endothermic process/es from the following is _____.

A. $I_2(\ g) \rightarrow 2I(g)$

B. $HCl(g) \rightarrow H(g) + Cl(g)$

C. $H_2O(l) \rightarrow H_2O(g)$

D. $C(s) + O_2(\ g) \rightarrow CO_2(\ g)$

E. Dissolution of ammonium chloride in water [4 marks]

Solution:

Concept:

- An endothermic process absorbs heat and has $\Delta H > 0$. Bond dissociation and conversion of a liquid to a gas require energy to overcome attractive interactions.
- The enthalpy of a process is its overall heat change. Dissolution includes both disruption of existing interactions and formation of new solute–solvent interactions; its sign depends on their balance.

Step 1: Classify the two bond dissociations and vaporisation In A, $I_2(g) \longrightarrow 2\,I(g)$, the iodine–iodine bond is broken to form separate gaseous atoms. Breaking this bond requires energy, so A is endothermic. In B, $HCl(g) \longrightarrow H(g) + Cl(g)$, the hydrogen–chlorine bond is likewise broken without forming a replacement bond. This is also endothermic. In C, $H_2O(l) \longrightarrow H_2O(g)$, molecules move from the liquid into the gas. Energy is required to overcome intermolecular attractions, including hydrogen bonding; the covalent O–H bonds remain intact. Thus C is endothermic too.

Step 2: Classify combustion and ammonium chloride dissolution Process D, $C(s)+O_2(g) \longrightarrow CO_2(g)$, is combustion of carbon. Formation of the stable carbon dioxide product releases more energy than is needed to disrupt the initial reactants, so its net enthalpy is negative. D is exothermic and must not be counted. Dissolution of NH_4Cl in water is endothermic: the energy required for disruption of the ionic lattice and other initial interactions exceeds the energy released on hydration. The solution therefore absorbs heat from its surroundings. Hence E is included.

Step 3: Conclusion The endothermic processes are A, B, C and E. Their number is $\boxed{4}$.

Quick Tip to Solve: Count bond breaking and vaporisation as endothermic, but classify a complete reaction by its net enthalpy rather than by bond breaking alone.

88.

$$A(g) \rightleftharpoons 2\,B(g) + C(g)$$

For the given reaction, if the initial pressure is 450 mm Hg and the pressure at time t is 720 mm Hg at a constant temperature T and constant volume V. The fraction of $A(g)$ decomposed under these conditions is $x \times 10^{-1}$. The value of x is _____ (nearest integer) [4 marks]

Solution:

Concept:

- At constant temperature and volume, the ideal-gas relation $P = nRT/V$ makes total pressure proportional to total gas moles.

- For $\mathrm{A(g)} \rightleftharpoons 2\,\mathrm{B(g)} + \mathrm{C(g)}$, each mole of A decomposed increases the total gas amount by two moles. The fraction decomposed is denoted by α.

Step 1: Express total gas moles after decomposition Let the initial amount of pure A be n_0 mol. If the fraction decomposed at the specified time is α, the amounts are $n_A = n_0(1-\alpha)$, $n_B = 2n_0\alpha$, and $n_C = n_0\alpha$. Adding them,

$$n_{\text{total}} = n_0(1 - \alpha + 2\alpha + \alpha) = n_0(1 + 2\alpha).$$

The coefficient is 2 because the reaction turns one gas molecule into three; the total rises by two, not by three. No equilibrium constant is needed because the pressure at the stated time is supplied directly.

Step 2: Use the pressure ratio and convert to the requested form The temperature and volume cancel between the initial and final ideal-gas equations, leaving

$$\frac{720}{450} = 1 + 2\alpha.$$

Both pressures have the same unit, so conversion from millimetres of mercury is unnecessary. Hence

$$1.6 = 1 + 2\alpha, \qquad \alpha = \frac{0.6}{2} = 0.30.$$

This lies between zero and one as a fraction must. As a check, the pressure contributions are 315 mm Hg from A, 270 mm Hg from B and 135 mm Hg from C; their sum is 720 mm Hg. Finally $0.30 = 3 \times 10^{-1}$.

Step 3: Conclusion The fraction decomposed is 0.30. In the requested form $x \times 10^{-1}$, the nearest integer value is $\boxed{x = 3}$.

Quick Tip to Solve: For gas decomposition at fixed temperature and volume use $P_t/P_0 = 1 + \alpha\Delta\nu$, counting the net increase of gas moles per mole of reactant.

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89. The specific conductance of 0.0025 M acetic acid is 5×10^{-5} S cm^{-1} at a certain temperature. The dissociation constant of acetic acid is _____ $\times 10^{-7}$ (Nearest integer)
Consider limiting molar conductivity of CH_3COOH as 400 S cm^2 mol^{-1} [4 marks]

Solution:

Concept:

- Molar conductivity is $\Lambda_m = 1000\kappa/c$ when κ is in $\mathrm{S\,cm^{-1}}$ and c is in $\mathrm{mol\,L^{-1}}$. The factor 1000 converts litres to cubic centimetres.
- For a weak monoprotic acid, $\alpha = \Lambda_m/\Lambda_m^\circ$. Applying the concentration form of the dissociation equilibrium gives $K_a = c\alpha^2/(1-\alpha)$.

Step 1: Calculate molar conductivity and the dissociated fraction Use the given specific conductance

and concentration:

$$\Lambda_m = \frac{1000(5 \times 10^{-5})}{0.0025} = 20\,\mathrm{S\,cm^2\,mol^{-1}}.$$

The limiting molar conductivity is $400\,\mathrm{S\,cm^2\,mol^{-1}}$, so

$$\alpha = \frac{20}{400} = 0.050.$$

Thus five percent of the acid is dissociated. The conductivity is not itself the dissociated fraction; it must first be expressed per mole and compared with the limiting value.

Step 2: Apply the dissociation equilibrium without dropping the denominator For $CH_3COOH \rightleftharpoons H^+ + CH_3COO^-$, the undissociated acid concentration is $c(1-\alpha)$, while each ion has concentration $c\alpha$. Therefore

$$K_a = \frac{(c\alpha)^2}{c(1-\alpha)} = \frac{0.0025(0.050)^2}{0.950} = 6.578947 \times 10^{-6}.$$

In the concentration convention its unit is $\mathrm{mol\,L^{-1}}$. To express it using the factor printed in the question,

$$K_a = 65.78947 \times 10^{-7}.$$

Keep $1-\alpha = 0.95$ rather than replacing it by 1: the latter would change the integer requested after rounding.

Step 3: Conclusion The dissociation constant is approximately $6.58 \times 10^{-6}\,\mathrm{mol\,L^{-1}}$. The nearest integer multiplying 10^{-7} is $\boxed{66}$.

Quick Tip to Solve: Keep the $1-\alpha$ denominator until the final rounding; even a small approximation can change a numerical-value answer.

90. The difference in the oxidation state of Xe between the oxidised product of Xe formed on complete hydrolysis of XeF_4 is _____ [4 marks]

Solution:

Concept:

- Fluorine has oxidation state -1 in its compounds. Electrical neutrality therefore assigns xenon the oxidation state $+4$ in XeF_4.
- Complete hydrolysis of xenon tetrafluoride produces both elemental xenon and xenon trioxide. The oxidised xenon product is XeO_3, in which oxygen has oxidation state -2.

Step 1: Identify the oxidised xenon product of complete hydrolysis The overall balanced hydrolysis is

$$6\,XeF_4 + 12\,H_2O \longrightarrow 2\,XeO_3 + 4\,Xe + 24\,HF + 3\,O_2.$$

Both sides contain six xenon atoms, twenty-four fluorine atoms, twenty-four hydrogen atoms and twelve oxygen atoms. The two xenon-containing products have different oxidation states. Elemental xenon is at zero and is the reduced xenon product. Xenon in XeO_3 is at a higher oxidation state than in XeF_4, so it is the oxidised xenon product relevant to the question. Do not compare the original

compound with the reduced elemental xenon.

Step 2: Calculate the change from the tetrafluoride to the oxidised product Let the oxidation state of xenon in the original tetrafluoride be a. Neutrality gives

$$a + 4(-1) = 0 \quad \Rightarrow \quad a = +4.$$

For the trioxide, let the xenon oxidation state be b. Since this is an ordinary oxide without peroxide bonds, oxygen is assigned -2:

$$b + 3(-2) = 0 \quad \Rightarrow \quad b = +6.$$

The increase in xenon oxidation state on forming the oxidised hydrolysis product is therefore $(+6) - (+4) = 2$. Oxidation numbers are dimensionless, so no physical unit accompanies this difference.

Step 3: Conclusion Xenon changes from $+4$ in XeF_4 to $+6$ in the oxidised product XeO_3. The required difference is $\boxed{2}$.

Quick Tip to Solve: For hydrolysis of XeF_4, identify which xenon product is oxidised before comparing oxidation states; elemental xenon is the reduced product.