JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 11 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 11 April 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let R be a rectangle given by the lines $x = 0, x = 2, y = 0$ and $y = 5$. Let A$(\alpha, 0)$ and B$(0, \beta), \alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4 : 1. Then, the mid-point of AB lies on a : [4 marks]

(A) hyperbola

(B) straight line

(C) parabola

(D) circle

Correct Answer: (A) hyperbola

Solution:

Concept:

- The area of a rectangle is the product of its side lengths. Here the given rectangle has area $2 \times 5 = 10$.
- An intercept segment from $(\alpha, 0)$ to $(0, \beta)$ cuts off a right triangle of area $\alpha\beta/2$ at the origin. The midpoint coordinates are $X = \alpha/2$ and $Y = \beta/2$.

Step 1: Identify the smaller region The intercept bounds imply $0 \le \alpha\beta \le 10$, so the triangle at the origin has area at most 5. The division ratio $4 : 1$ gives region areas 8 and 2, because their sum is 10. Therefore the triangular region must be the region of area 2: it cannot be the region of area 8. This observation removes the apparent ambiguity in which side of AB has the larger area.

Step 2: Express the area condition The area of triangle OAB uses perpendicular sides along the coordinate axes. Consequently,

$$\frac{\alpha\beta}{2} = 2 \quad \Longrightarrow \quad \alpha\beta = 4.$$

The endpoints are nonzero because a zero intercept would give no enclosed triangular area. Thus the midpoint lies in the first quadrant.

Step 3: Find and classify the midpoint locus Writing the midpoint as (X, Y), substitute $\alpha = 2X$ and $\beta = 2Y$ into the area condition:

$$4XY = 4 \quad \Longrightarrow \quad XY = 1.$$

This is a rectangular hyperbola with coordinate axes as asymptotes. The intercept restrictions select only a portion of its first-quadrant branch; they do not change the type of conic. A straight line has a linear equation, while a circle and a parabola do not have this nonzero constant-product equation.

Step 4: Conclusion The midpoint satisfies $XY = 1$, so it lies on a hyperbola. Hence, the correct option is (A).

Quick Tip to Solve: For a fixed area cut off by an intercept segment, first find the product of the intercepts; the midpoint then has a fixed coordinate product.

2. Let $S = \{M = [a_{ij}], a_{ij} \in \{0, 1, 2\}, 1 \le i, j \le 2\}$ be a sample space and $A = \{M \in S : M$ is invertible $\}$ be an event. Then $P(A)$ is equal to : [4 marks]

(A) $\frac{47}{81}$
(B) $\frac{49}{81}$
(C) $\frac{50}{81}$
(D) $\frac{16}{27}$

Correct Answer: (C) $\frac{50}{81}$

Solution:

Concept:

- There are $3^4 = 81$ equally likely matrices, since each of the four entries is chosen independently from $\{0, 1, 2\}$.
- A matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is singular precisely when its determinant is zero, that is, $ad = bc$. Count the complement to obtain the probability of invertibility.

Step 1: Tabulate the possible ordered products Consider one ordered pair (a, d). Its product is zero for the five pairs $(0, 0), (0, 1), (0, 2), (1, 0), (2, 0)$. Product 1 occurs only for $(1, 1)$. Product 2 occurs for $(1, 2)$ and $(2, 1)$, and product 4 occurs only for $(2, 2)$. Thus the frequencies for products $0, 1, 2, 4$ are respectively $5, 1, 2, 1$; they sum to 9 as required.

Step 2: Count the zero determinants The ordered pair (b, c) has exactly the same product frequencies. For each fixed product value, any pair producing that value in ad can be combined with any pair producing it in bc. The resulting sets are disjoint, so the number of singular matrices is

$$5^2 + 1^2 + 2^2 + 1^2 = 25 + 1 + 4 + 1 = 31.$$

This count includes matrices with zero rows or columns automatically and counts the zero matrix only once.

Step 3: Take the complement and compare the options The number of matrices with nonzero determinant is $81 - 31 = 50$. Therefore

$$P(A) = \frac{50}{81}.$$

The proposed probabilities $47/81$ and $49/81$ would require 34 and 32 singular matrices respectively, contradicting the product count. Also $16/27 = 48/81$, which would require 33 singular matrices. Only the numerator 50 agrees with the exhaustive count.

Step 4: Conclusion The probability of an invertible matrix is $50/81$. Hence, the correct option is (C).

Quick Tip to Solve: For a small-entry two-by-two matrix, tabulate ordered-pair product frequencies; their squared frequencies count zero determinants directly.

3. Let $x_1, x_2, \ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200 . If $y_i = i\,(x_i - i)\,, 1 \le i \le 100$, then the mean of $y_1, y_2, \ldots, y_{100}$ is : [4 marks]

(A) 10051.50

(B) 10049.50

(C) 10100

(D) 10101.50

Correct Answer: (B) 10049.50

Solution:

Concept:

- For an arithmetic progression with first term a, common difference d and n terms, the mean is $a + (n-1)d/2$.
- The identities $\sum_{i=1}^{n} i = n(n+1)/2$ and $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$ evaluate the mean of a polynomial in the index.

Step 1: Determine the common difference The first term is 2, and there are 100 terms with mean 200. Therefore

$$2 + \frac{99d}{2} = 200, \qquad d = 4.$$

Equivalently the last term is 398, whose average with the first term is 200. The general term is consequently

$$x_i = 2 + 4(i-1) = 4i - 2.$$

Keeping the constant term -2 is essential when we form the new sequence.

Step 2: Write the transformed term as a polynomial Use the definition given in the question, rather than assuming that the transformed sequence remains arithmetic:

$$y_i = i(x_i - i) = i(4i - 2 - i) = 3i^2 - 2i.$$

This is quadratic in i, so the mean cannot be obtained by merely averaging its first and last terms. We must sum the square and linear contributions separately.

Step 3: Evaluate the mean and identify its value The required mean is

$$\begin{aligned}\overline{y} &= \frac{1}{100}\left(3\frac{100 \cdot 101 \cdot 201}{6} - 2\frac{100 \cdot 101}{2}\right) \\ &= \frac{3 \cdot 101 \cdot 201}{6} - 101 \\ &= 10150.5 - 101 = 10049.5.\end{aligned}$$

The options 10051.50 and 10101.50 are not this result. The value 10100 would follow from omitting part of the linear correction, so it also fails the exact sum. The answer is 10049.50, with the trailing zero only indicating the displayed decimal format.

Step 4: Conclusion The mean is 10049.50. Hence, the correct option is (B).

Quick Tip to Solve: After finding the AP difference from its mean, expand the transformed term in powers of the index before applying standard summation formulae.

4. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to : [4 marks]

(A) $-\pi + \tan^{-1}\frac{8}{9}$

(B) $-\pi + \tan^{-1}\frac{33}{5}$

(C) $\pi - \tan^{-1}\frac{8}{9}$

(D) $\pi - \tan^{-1}\frac{33}{5}$

Correct Answer: (C) $\pi - \tan^{-1}\frac{8}{9}$

Solution:

Concept:

- Multiplication by i rotates a complex number through 90° anticlockwise about the origin, while multiplication by $-i$ rotates it clockwise.
- The principal argument lies in $(-\pi, \pi]$. The signs of the real and imaginary parts determine the quadrant before an inverse tangent is used.

Step 1: Apply each rotation in its stated direction For the first point, the anticlockwise rotation gives

$$w_1 = i(5 + 4i) = 5i + 4i^2 = -4 + 5i.$$

For the second point, the clockwise rotation gives

$$w_2 = -i(3 + 5i) = -3i - 5i^2 = 5 - 3i.$$

The rotations have opposite signs; applying the same multiplier to both points would change the required difference.

Step 2: Subtract the rotated points The question asks for the argument of the difference, not the difference of the arguments. Subtracting the real and imaginary parts gives

$$w_1 - w_2 = (-4 + 5i) - (5 - 3i) = -9 + 8i.$$

Its real part is negative and its imaginary part is positive, so this point lies in the second quadrant. Its modulus is nonzero, namely $\sqrt{81 + 64} = \sqrt{145}$, so its argument is well defined.

Step 3: Choose the second-quadrant principal angle The acute reference angle is $\phi = \tan^{-1}(8/9)$. A second-quadrant angle is therefore

$$\text{Arg}(w_1 - w_2) = \pi - \tan^{-1}\frac{8}{9}.$$

This lies between $\pi/2$ and π, within the principal range. Options (A) and (B) describe negative angles in the third-quadrant principal representation, inconsistent with a positive imaginary part. Option (D) is in the second quadrant but uses the wrong reference ratio $33/5$, rather than the actual ratio $8/9$.

Step 4: Conclusion The principal argument is $\pi - \tan^{-1}(8/9)$. Hence, the correct option is (C).

Quick Tip to Solve: Perform rotations by multiplying by the appropriate signed imaginary unit, subtract first, and only then use the quadrant to select the principal argument.

5. Let $y = y(x)$ be a solution curve of the differential equation.

$\left(1 - x^2y^2\right) dx = ydx + xdy$**.**

If the line $x = 1$ intersects the curve $y = y(x)$ at $y = 2$ and the line $x = 2$ intersects the curve $y = y(x)$ at $y = \alpha$, then a value of α is : [4 marks]

(A) $\frac{1+3e^2}{2(3e^2-1)}$

(B) $\frac{3e^2}{2(3e^2-1)}$

(C) $\frac{1-3e^2}{2(3e^2+1)}$

(D) $\frac{3e^2}{2(3e^2+1)}$

Correct Answer: (A) $\frac{1+3e^2}{2(3e^2-1)}$

Solution:

Concept:

- The product differential is $d(xy) = y\,dx + x\,dy$. This identifies the substitution $u = xy$ in the given differential equation.
- After the substitution, the equation becomes separable: $du/(1-u^2) = dx$. Partial fractions integrate the left side.

Step 1: Replace the product by a new dependent variable Let $u = xy$. The given equation becomes

$$(1 - u^2)\,dx = du.$$

The initial intersection $(x, y) = (1, 2)$ gives $u(1) = 2$. This excludes the constant solutions $u = 1$ and $u = -1$, which cannot pass through that initial value. On the branch containing the initial point, $u > 1$.

Step 2: Integrate and use the initial value Separating variables and using partial fractions gives

$$\int \frac{du}{1-u^2} = \frac{1}{2}\log\left|\frac{1+u}{1-u}\right| = x + C.$$

Because $u > 1$, the absolute-value ratio equals $(u+1)/(u-1)$. At $x = 1, u = 2$, this ratio is 3. Subtract the initial relation to eliminate the integration constant:

$$\log\frac{u+1}{u-1} - \log 3 = 2(x-1).$$

Thus $(u+1)/(u-1) = 3e^{2(x-1)}$.

Step 3: Find the intersection at x equal to two At $x = 2$, put $K = 3e^2 > 1$. Solving $(u+1)/(u-1) = K$ gives

$$u + 1 = Ku - K, \qquad u = \frac{K+1}{K-1}.$$

Since $y = u/x$, we obtain

$$\alpha = \frac{1+3e^2}{2(3e^2-1)}.$$

Option (B) omits the required 1 in the numerator. Option (C) is negative, inconsistent with this

positive branch; option (D) uses the wrong sign in the denominator and also omits the 1. The branch has no singularity between 1 and 2 because $3e^{2(x-1)} > 1$ there.

Step 4: Conclusion A value of α is $\dfrac{1+3e^2}{2(3e^2-1)}$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever the right side contains $y\,dx + x\,dy$, try $u = xy$ before attempting to solve directly for the derivative of y.

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6. The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x \left(\log_e\left(e^x + \sqrt{1+e^{2x}}\right)\right) dx$ is equal to : [4 marks]

(A) $\log_e\left(\frac{(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) + \frac{\sqrt{5}}{2}$

(B) $\log_e\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$

(C) $\log_e\left(\frac{2(2+\sqrt{5})}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$

(D) $\log_e\left(\frac{\sqrt{2}(3-\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) + \frac{\sqrt{5}}{2}$

Correct Answer: (B) $\log_e\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$

Solution:

Concept:

- If the integrand contains $e^x\,dx$ and a function of e^x, the substitution $t = e^x$ gives $dt = e^x\,dx$.
- The derivative of $\log(t + \sqrt{1+t^2})$ is $1/\sqrt{1+t^2}$. Integration by parts then gives an elementary antiderivative.

Step 1: Transform the bounds and the integral Write the required value as I. With $t = e^x$, the bounds $-\log 2$ and $\log 2$ become $1/2$ and 2 respectively, so

$$I = \int_{1/2}^{2} \log(t + \sqrt{1+t^2})\,dt.$$

Both bounds are positive, and the logarithm has a positive argument throughout this interval. No absolute-value or branch ambiguity arises.

Step 2: Integrate by parts Take the logarithm as the first factor and dt as the differential to integrate. Its derivative is $1/\sqrt{1+t^2}$, and hence

$$\int \log(t + \sqrt{1+t^2})\,dt = t\log(t + \sqrt{1+t^2}) - \sqrt{1+t^2}.$$

Differentiation checks this directly: the extra term $t/\sqrt{1+t^2}$ from the first product cancels the derivative of the square root.

Step 3: Evaluate and combine the logarithms At $t = 2$, the square root is $\sqrt{5}$, while at $t = 1/2$ it is

$\sqrt{5}/2$. Therefore

$$I = 2\log(2+\sqrt{5}) - \frac{1}{2}\log\frac{1+\sqrt{5}}{2} - \frac{\sqrt{5}}{2}$$
$$= \log\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}.$$

The lower-endpoint factor 2 produces $\sqrt{2}$ in the numerator. Thus (A) has both an incorrect radical constant sign and a missing factor, (C) loses the square and has the wrong coefficient, and (D) has the wrong logarithm and constant sign.

Step 4: Conclusion The value is $\log\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the exponential differential to a new variable first; after integration by parts, retain the lower-endpoint logarithm before combining factors.

7. Let sets A and B have 5 elements each. Let the mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is _____. [4 marks]

(A) 36

(B) 40

(C) 38

(D) 32

Correct Answer: (C) 38

Solution:

Concept:

- Adding a constant shifts the mean by that constant but leaves the variance unchanged. Thus the two transformed groups have means 2 and 10 and variances 12 and 20.
- For two equally sized groups, the combined variance is the average within-group variance plus the average squared displacement of their means from the combined mean.

Step 1: Find the transformed means and the combined mean The five elements coming from A each decrease by 3, so their mean becomes $5-3=2$. The five from B each increase by 2, so their mean becomes $8+2=10$. Their variances remain 12 and 20. Since the group sizes are equal, the ten-element mean is

$$\mu_C = \frac{5\cdot 2 + 5\cdot 10}{10} = 6.$$

The question treats the transformed values as ten observations as stated.

Step 2: Include the separation between the group means For an observation in the first group, its deviation from 6 equals its deviation from its own mean 2, followed by a shift of -4. The average cross term is zero because deviations from a group mean sum to zero. The same reasoning applies to

the other group, whose mean is 4 above 6. Therefore

$$\sigma_C^2 = \frac{5\{12 + (2-6)^2\} + 5\{20 + (10-6)^2\}}{10}$$
$$= \frac{28+36}{2} = 32.$$

Simply averaging 12 and 20 would give 16 and would omit the additional dispersion caused by the separated group centres.

Step 3: Add the requested statistics The requested quantity is the sum, rather than either statistic separately:

$$\mu_C + \sigma_C^2 = 6 + 32 = 38.$$

Option (D), 32, is the variance alone and omits the mean. The alternatives 36 and 40 do not equal the result of the combined-variance calculation. Including both the within-group and between-group contributions selects 38 uniquely.

Step 4: Conclusion The sum of the mean and variance is 38. Hence, the correct option is (C).

Quick Tip to Solve: When combining shifted groups, preserve each variance and add the squared distance between each shifted group mean and the overall mean.

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8. For any vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10\,|a_i| < 1, i = 1, 2, 3$, consider the following statements :

(A): $\max\{|a_1|, |a_2|, |a_3|\} \leq |\vec{a}|$

(B) : $|\vec{a}| \leq 3\max\{|a_1|, |a_2|, |a_3|\}$ [4 marks]

(A) Only (B) is true

(B) Only (A) is true

(C) Neither (A) nor (B) is true

(D) Both (A) and (B) are true

Correct Answer: (D) Both (A) and (B) are true

Solution:

Concept:

- The magnitude of a vector is its Euclidean norm: $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$. The square of each real component is nonnegative.
- If $M = \max\{|a_1|, |a_2|, |a_3|\}$, each component has absolute value at most M. Bounds on the sum of squares compare the largest component with the magnitude.

Step 1: Check statement (A) Choose a component whose absolute value is M. Its square is one of the three nonnegative terms defining the squared magnitude, so

$$|\vec{a}|^2 = a_1^2 + a_2^2 + a_3^2 \geq M^2.$$

Taking nonnegative square roots gives $M \leq |\vec{a}|$. Thus statement (A) is true. Equality can occur when the other two components vanish, so the non-strict inequality includes this boundary case.

Step 2: Check statement (B) Since $a_i^2 \le M^2$ for all three components,

$$|\vec{a}|^2 \le 3M^2, \qquad |\vec{a}| \le \sqrt{3}\,M \le 3M.$$

The last comparison follows from $M \ge 0$ and $\sqrt{3} < 3$. Therefore statement (B) is also true, although its factor 3 is a weaker upper bound than necessary. The restrictions $10|a_i| < 1$ do not invalidate either argument; these bounds hold for every real vector, including the zero vector.

Step 3: Eliminate the remaining choices Option (A) incorrectly denies the first inequality, while option (B) incorrectly denies the second. Option (C) denies both proved inequalities. Only the choice asserting that both statements are true survives the direct sum-of-squares comparison.

Step 4: Conclusion Both given statements are true. Hence, the correct option is (D).

Quick Tip to Solve: Compare a largest squared component with the sum of all squared components: $M \le |\vec{a}| \le \sqrt{3}M$.

9. Let A be a 2×2 matrix with real entries such that $\mathrm{A}' = \alpha\mathrm{A} + \mathrm{I}$, where $\alpha \in \mathbb{R} - \{-1, 1\}$. If $\det\left(A^2 - A\right) = 4$, then the sum of all possible values of α is equal to : [4 marks]

(A) 2

(B) $\frac{3}{2}$

(C) 0

(D) $\frac{5}{2}$

Correct Answer: (D) $\frac{5}{2}$

Solution:

Concept:

- The prime denotes transpose. Transposing twice returns the original matrix, and $(\alpha A + I)^{\mathsf{T}} = \alpha A^{\mathsf{T}} + I$ for real α.
- For a scalar c, the determinant of the 2×2 scalar matrix cI is c^2. Both signs must be considered when recovering a scalar from its squared value.

Step 1: Use the transpose relation twice From $A^{\mathsf{T}} = \alpha A + I$, transpose once more and substitute the original relation:

$$A = \alpha A^{\mathsf{T}} + I = \alpha^2 A + (\alpha + 1)I.$$

Since $\alpha \neq \pm 1$, division by $1 - \alpha^2$ is valid. Hence

$$A = \frac{\alpha + 1}{1 - \alpha^2} I = \frac{1}{1 - \alpha} I.$$

This reduces the matrix condition to a scalar equation and also ensures that the proposed matrix is real whenever α is real and allowed.

Step 2: Apply the determinant condition Compute the difference before taking its determinant:

$$A^2 - A = \left(\frac{1}{(1-\alpha)^2} - \frac{1}{1-\alpha}\right) I = \frac{\alpha}{(1-\alpha)^2} I.$$

Consequently

$$\left(\frac{\alpha}{(1-\alpha)^2}\right)^2 = 4, \qquad \frac{\alpha}{(1-\alpha)^2} = \pm 2.$$

For the positive sign, $2\alpha^2 - 5\alpha + 2 = 0$, giving $\alpha = 2, \frac{1}{2}$. For the negative sign, $2\alpha^2 - 3\alpha + 2 = 0$; its discriminant is $9 - 16 = -7$, so it gives no real value.

Step 3: Check admissibility and sum the roots Both real roots differ from -1 and 1 and give matrices satisfying the original transpose relation. Their sum is

$$2 + \frac{1}{2} = \frac{5}{2}.$$

The value 2 in option (A) counts just one root; $3/2$ is their difference rather than their sum, and zero is inconsistent with the two positive admissible roots.

Step 4: Conclusion The required sum is $\frac{5}{2}$. Hence, the correct option is (D).

Quick Tip to Solve: Transpose the relation again first; excluding $\alpha = \pm 1$ then forces A to be a scalar matrix.

10. Let $f(x) = [x^2 - x] + |-x + [x]|$, where $x \in \mathbb{R}$ and $[t]$ denotes the greatest integer less than or equal to t. Then, f is : [4 marks]

(A) continuous at $x = 0$, but not continuous at $x = 1$

(B) continuous at $x = 0$ and $x = 1$

(C) continuous at $x = 1$, but not continuous at $x = 0$

(D) not continuous at $x = 0$ and $x = 1$

Correct Answer: (C) continuous at $x = 1$, but not continuous at $x = 0$

Solution:

Concept:

- Continuity at c requires both one-sided limits to exist and equal $f(c)$. The floor function changes value when its argument crosses an integer.
- The signs of $x^2 - x = x(x-1)$ near 0 and 1 determine its floor. The other term is $|[x] - x|$, which must be evaluated on the same side.

Step 1: Compute the limits at zero At zero, $f(0) = 0$. For $x < 0$ sufficiently close to zero, $[x] = -1$ and $0 < x^2 - x < 1$, so

$$f(x) = 0 + |-x - 1| = 1 + x.$$

Thus $\lim_{x \to 0^-} f(x) = 1$. For $0 < x < 1$ close to zero, $[x] = 0$ and $-1 < x^2 - x < 0$. Therefore

$$f(x) = -1 + |-x| = -1 + x,$$

and $\lim_{x \to 0^+} f(x) = -1$. The limits disagree with each other and with $f(0)$; the function is not continuous at zero.

Step 2: Compute the limits at one At one, both terms are zero, so $f(1) = 0$. For $x < 1$ sufficiently

close to one, $[x] = 0$ and $[x^2 - x] = -1$. It follows that

$$f(x) = -1 + x \longrightarrow 0.$$

For $x > 1$ sufficiently close to one, $[x] = 1$ and $0 < x^2 - x < 1$, giving

$$f(x) = 0 + |1 - x| = x - 1 \longrightarrow 0.$$

Both limits are equal to $f(1)$, establishing continuity at one despite the separate floor-function jumps.

Step 3: Compare the descriptions Options (A) and (B) incorrectly claim continuity at zero, where the one-sided limits are 1 and -1. Option (D) incorrectly rejects continuity at one. The remaining description precisely matches the two independent limit tests.

Step 4: Conclusion The function is continuous at $x = 1$, but not continuous at $x = 0$. Hence, the correct option is (C).

Quick Tip to Solve: Write the floor values separately on each side; jumps in different terms may cancel at one point but reinforce at another.

11. Consider ellipses $\mathrm{E}_k : kx^2 + k^2y^2 = 1, k = 1, 2, \ldots, 20$. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k. If r_k is the radius of the circle C_k, then the value of $\sum_{k=1}^{20} \frac{1}{r_k^2}$ is : [4 marks]

(A) 2870

(B) 3210

(C) 3320

(D) 3080

Correct Answer: (D) 3080

Solution:

Concept:

- For the ellipse $x^2/a^2 + y^2/b^2 = 1$, adjacent axis endpoints are $(a, 0)$ and $(0, b)$. Their joining chord lies on $x/a + y/b = 1$.
- The four such chords form a rhombus symmetric about both coordinate axes. Its inscribed circle is centered at the origin, and its radius is the perpendicular distance from the origin to any side.

Step 1: Find the semiaxes and the chord equation Rewrite the given ellipse as

$$\frac{x^2}{1/k} + \frac{y^2}{1/k^2} = 1.$$

Thus $a = 1/\sqrt{k}$ and $b = 1/k$. For $k > 1$, these are the major and minor semiaxes respectively; at $k = 1$ the same construction gives a square. The chord in the first quadrant is

$$\sqrt{k}\,x + ky = 1.$$

Reflections across the axes produce the other three sides at the same perpendicular distance from the origin.

Step 2: Calculate the circle radius Use the distance from $(0,0)$ to $\sqrt{k}\,x + ky - 1 = 0$:

$$r_k = \frac{|-1|}{\sqrt{k+k^2}} = \frac{1}{\sqrt{k+k^2}}.$$

The perpendicular foot lies on the chord segment, so the circle touches the side itself rather than just its extension. Symmetry guarantees tangency to all four chords. Hence

$$\frac{1}{r_k^2} = k + k^2.$$

Keeping both contributions is essential: the reciprocal squared radius is not merely k^2.

Step 3: Sum the reciprocal squares Apply the standard integer and square sums:

$$\begin{aligned}\sum_{k=1}^{20} \frac{1}{r_k^2} &= \sum_{k=1}^{20} k + \sum_{k=1}^{20} k^2 \\ &= \frac{20 \cdot 21}{2} + \frac{20 \cdot 21 \cdot 41}{6} \\ &= 210 + 2870 = 3080.\end{aligned}$$

Option (A) omits the linear sum. The values 3210 and 3320 do not equal the sum of the two required contributions.

Step 4: Conclusion The required value is 3080. Hence, the correct option is (D).

Quick Tip to Solve: For the chord joining the axis endpoints, use the origin-to-line distance; it gives $r^{-2} = a^{-2} + b^{-2}$ immediately.

12. Area of the region $\{(x,y) : x^2 + (y-2)^2 \le 4, x^2 \ge 2y\}$ is [4 marks]

(A) $2\pi + \frac{16}{3}$

(B) $\pi - \frac{8}{3}$

(C) $\pi + \frac{8}{3}$

(D) $2\pi - \frac{16}{3}$

Correct Answer: (D) $2\pi - \frac{16}{3}$

Solution:

Concept:

- The circle $x^2 + (y-2)^2 \le 4$ has center $(0,2)$ and radius 2. The condition $x^2 \ge 2y$ selects points on or below the parabola $y = x^2/2$.
- The required region is symmetric about the y-axis. With vertical strips, its area is twice the integral of upper boundary minus lower boundary over $0 \le x \le 2$.

Step 1: Locate the boundary intersections Substitute $y = x^2/2$ into the circle equation:

$$x^2 + \left(\frac{x^2}{2} - 2\right)^2 = 4 \quad \Longrightarrow \quad \frac{x^4}{4} - x^2 = 0.$$

Therefore $x = 0, \pm 2$, with intersection points $(0, 0)$ and $(\pm 2, 2)$. On the right half of the circle, its lower branch is $y = 2 - \sqrt{4 - x^2}$. Between 0 and 2 the parabola lies above this branch and below the upper branch; thus it supplies the upper edge of the selected region.

Step 2: Integrate the vertical-strip height The area is

$$A = 2\int_0^2 \left[\frac{x^2}{2} - 2 + \sqrt{4 - x^2}\right] dx$$
$$= 2\left[\frac{x^3}{6} - 2x\right]_0^2 + 2\int_0^2 \sqrt{4 - x^2}\, dx.$$

The remaining integral is the area of a quarter-circle of radius 2, so it equals π. Substituting the polynomial endpoints gives

$$A = 2\left(\frac{4}{3} - 4\right) + 2\pi = 2\pi - \frac{16}{3}.$$

Both reflected lobes have been included by the factor 2.

Step 3: Check the sign and the choices The result is positive, approximately 0.950 square units, consistent with the two narrow regions near the bottom of the circle. Option (B) is exactly half the required area. The plus signs in options (A) and (C) add rather than subtract the area between the lower circle branch and the parabola.

Step 4: Conclusion The area is $2\pi - \frac{16}{3}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: For the condition $x^2 \geq 2y$, retain the portion below the parabola and integrate from the lower circle branch upward.

13. Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}, \hat{j} - \hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a} \cdot \vec{b} = 6$, then the ordered pair $(\theta, |\vec{a} \times \vec{b}|)$ is equal to : [4 marks]

(A) $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$
(B) $\left(\frac{\pi}{3}, 6\right)$
(C) $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$
(D) $\left(\frac{\pi}{4}, 6\right)$

Correct Answer: (D) $\left(\frac{\pi}{4}, 6\right)$

Solution:

Concept:

- A plane spanned by two nonparallel vectors has normal equal to their cross product. A line common to two planes is perpendicular to both normals.

- For vectors $\vec{a}, \vec{b}$, $\cos\theta = (\vec{a}\cdot\vec{b})/(|\vec{a}||\vec{b}|)$ and $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$.

Step 1: Find a direction along the intersection The first plane is spanned by $(1,1,0)$ and $(1,0,1)$, whose cross product is $\vec{n}_1 = (1,-1,-1)$. The second is spanned by $(1,-1,0)$ and $(0,1,-1)$, giving $\vec{n}_2 = (1,1,1)$. A common direction is

$$\vec{n}_1 \times \vec{n}_2 = (0,-2,2).$$

Rescaling this direction, write $\vec{a} = t(0,-1,1)$, where $t \neq 0$. This direction is orthogonal to both normals, as required for a line lying in both planes.

Step 2: Use the given dot product The vector $\vec{b} = (2,-2,1)$ has magnitude 3. Therefore

$$\vec{a}\cdot\vec{b} = t(0+2+1) = 3t = 6,$$

so $t = 2$, $\vec{a} = (0,-2,2)$, and $|\vec{a}| = 2\sqrt{2}$. The positive dot product fixes the orientation of the parallel vector and makes the required angle acute. Compute

$$\cos\theta = \frac{6}{(2\sqrt{2})(3)} = \frac{1}{\sqrt{2}}, \qquad \theta = \frac{\pi}{4}.$$

Step 3: Calculate the cross-product magnitude Use the magnitude identity:

$$|\vec{a}\times\vec{b}| = \sqrt{(2\sqrt{2})^2(3)^2 - 6^2} = \sqrt{72-36} = 6.$$

The angle $\pi/3$ in options (A) and (B) fails the dot-product calculation. Option (C) has the correct angle but the wrong cross-product magnitude; substituting $3\sqrt{6}$ would give square 54 instead of 36.

Step 4: Conclusion The ordered pair is $\left(\frac{\pi}{4}, 6\right)$. Hence, the correct option is (D).

Quick Tip to Solve: Cross the spanning vectors to obtain each normal, then cross the normals; use the dot product to fix the scale and direction.

14. The number of triplets (x, y, z), where x, y, z are distinct non negative integers satisfying $x+y+z = 15$, is : [4 marks]

(A) 136

(B) 80

(C) 92

(D) 114

Correct Answer: (D) 114

Solution:

Concept:

- Stars and bars counts ordered nonnegative integer solutions of $x+y+z = n$: there are $\binom{n+2}{2}$ solutions before enforcing distinctness.

- Use inclusion-exclusion to remove triples in which at least one pair of coordinates is equal. Intersections of two pair-equality conditions force all three coordinates to be equal.

Step 1: Count all nonnegative triples With no restriction on equality, distribute 15 identical units among the three labeled variables:

$$N_{\text{all}} = \binom{15+3-1}{3-1} = \binom{17}{2} = 136.$$

The word triplets means ordered triples (x, y, z), so permutations of distinct entries are different solutions. The count 136 therefore matches option (A) only before removing the prohibited equal-coordinate cases.

Step 2: Count the triples with repeated coordinates Let E_1 mean $x = y$, E_2 mean $y = z$, and E_3 mean $z = x$. In E_1, put $x = y = t$; then $z = 15 - 2t \geq 0$, giving $t = 0, 1, \ldots, 7$. Hence each equality event has 8 triples. Any two equality events force $x = y = z$, and the sum fixes the unique triple $(5, 5, 5)$. The intersection of all three events is the same single triple. Thus

$$|E_1 \cup E_2 \cup E_3| = 3 \cdot 8 - 3 \cdot 1 + 1 = 22.$$

The correction is necessary because $(5, 5, 5)$ was included once in every equality event.

Step 3: Remove all repetitions The number with pairwise distinct coordinates is

$$136 - 22 = 114.$$

Equivalently, subtracting the 24 pair-equality counts first removes the all-equal triple three times; adding 2 back restores the correct exclusion once. Option (A) retains repetitions, while 80 and 92 are inconsistent with the exact union count. Since each distinct unordered choice has six permutations, $114 = 6 \cdot 19$ also passes the ordering check.

Step 4: Conclusion The number of distinct ordered triples is 114. Hence, the correct option is (D).

Quick Tip to Solve: For three variables, pair-equality events overlap only at the all-equal triple; count that overlap before subtracting repeated cases.

15. The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is : [4 marks]

(A) 8

(B) 7

(C) 5

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- A real logarithm $\log_b u$ requires $b > 0$, $b \neq 1$, and $u > 0$. Its sign depends on whether the base is above or below one.
- For $b > 1$, $\log_b u \geq 0$ means $u \geq 1$; for $0 < b < 1$, it means $0 < u \leq 1$. These cases must be handled separately.

Step 1: Apply the domain restrictions Here the base is $b = x + 7/2$. Thus $x > -7/2$ and $x \neq -5/2$. The argument is a square, but it must still be nonzero and defined, so $x \neq 7$ and $x \neq 3/2$. The integral candidates therefore start at -3. The only integer giving a base between zero and one is $x = -3$; all integers $x \geq -2$ give a base greater than one.

Step 2: Test the base below one At $x = -3$, the argument equals $(-10/-9)^2 = 100/81 > 1$. Since the base is $1/2 < 1$, its logarithm is negative. Hence this candidate does not satisfy the inequality. This separate check prevents an incorrect count obtained by treating the logarithm as increasing for every admissible base.

Step 3: Solve and count for the base above one For $x \geq -2$, multiply by the positive squared denominator to obtain

$$(x-7)^2 \geq (2x-3)^2.$$

Factoring the difference gives

$$(-x-4)(3x-10) \geq 0 \quad \Longleftrightarrow \quad (x+4)(3x-10) \leq 0.$$

Thus $-4 \leq x \leq 10/3$. Intersecting with $x \geq -2$ gives exactly $-2, -1, 0, 1, 2, 3$; none violates the argument restrictions. The counts 8, 7 and 5 do not equal this explicitly enumerated set.

Step 4: Conclusion There are $\boxed{6}$ integral solutions. Hence, the correct option is (D).

Quick Tip to Solve: List the integers for which the base is below one first; only then solve the usual argument-at-least-one inequality for the remaining bases.

16. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events? [4 marks]

(A) 10

(B) 15

(C) 21

(D) 9

Correct Answer: (C) 21

Solution:

Concept:

- When recipients can win in several events, the sum of event totals counts a person once for every event in which that person wins.
- Let n_1, n_2, n_3 denote the numbers receiving medals in exactly one, exactly two and exactly three events. These categories are disjoint and exhaust the recipients.

Step 1: Count the people and the medal incidences The total number of distinct men is 60, while exactly five received medals in all three events. Therefore

$$n_1 + n_2 + n_3 = 60, \qquad n_3 = 5.$$

Adding the three event counts counts all person-event incidences:

$$48 + 25 + 18 = 91.$$

A man in exactly two events contributes two to this sum, whereas a man in all three contributes three.

Step 2: Remove the single count for every recipient The incidence count is consequently

$$n_1 + 2n_2 + 3n_3 = 91.$$

Subtracting the distinct-person equation eliminates n_1:

$$n_2 + 2n_3 = 91 - 60 = 31.$$

Each triple winner produces two extra counts beyond the one already included in 60. Thus the five triple winners account for ten of these 31 extra counts.

Step 3: Find and verify the exact-two count Substitute $n_3 = 5$:

$$n_2 = 31 - 2(5) = 21.$$

The corresponding exact-one count is $n_1 = 60 - 21 - 5 = 34$. Checking gives $34 + 2(21) + 3(5) = 91$, so both the recipient total and the event total agree. The alternatives 10, 15 and 9 fail the equation $n_2 + 10 = 31$. In particular, 31 itself would be the excess count, not the number of men receiving exactly two medals.

Step 4: Conclusion Exactly $\boxed{21}$ men received medals in exactly two of the three events. Hence, the

correct option is (C).

Quick Tip to Solve: Subtract the union size from the sum of event sizes: exact-two winners contribute one excess count and exact-three winners contribute two.

17. Let (α, β, γ) be the image of the point $\mathrm{P}(2, 3, 5)$ in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to : [4 marks]

(A) 10

(B) 9

(C) 5

(D) 12

Correct Answer: (A) 10

Solution:

Concept:

- The image of a point in a plane lies on the normal through that point. The perpendicular foot is the midpoint of the point and its reflected image.
- For the plane $ax + by + cz + d = 0$, the reflected point is $P - 2\frac{ax_P + by_P + cz_P + d}{a^2 + b^2 + c^2}(a, b, c)$.

Step 1: Identify the plane normal and signed numerator Write the plane as $2x + y - 3z - 6 = 0$. Its normal is $\mathbf{n} = (2, 1, -3)$, whose squared length is

$$\mathbf{n} \cdot \mathbf{n} = 4 + 1 + 9 = 14.$$

At $P = (2, 3, 5)$, the plane expression equals

$$2(2) + 3 - 3(5) - 6 = -14.$$

This sign determines which direction along the normal leads toward the plane.

Step 2: Obtain the reflected coordinates Using the reflection formula,

$$P' = (2, 3, 5) - 2\left(\frac{-14}{14}\right)(2, 1, -3).$$

Therefore

$$(\alpha, \beta, \gamma) = (2, 3, 5) + (4, 2, -6) = (6, 5, -1).$$

The midpoint is $(4, 4, 2)$, and $2(4) + 4 - 3(2) = 6$, confirming that it lies in the mirror plane. Also $P' - P = (4, 2, -6)$ is parallel to the plane normal, as reflection requires.

Step 3: Compute the requested sum Adding the coordinates gives

$$\alpha + \beta + \gamma = 6 + 5 - 1 = 10.$$

There is a useful independent check: the sum of the components of the normal is $2 + 1 - 3 = 0$. Any displacement parallel to this normal leaves the coordinate sum unchanged, so the image must have

the same sum as P, namely $2+3+5=10$. Thus 9, 5 and 12 cannot be the required sum.

Step 4: Conclusion $\alpha+\beta+\gamma=\boxed{10}$. Hence, the correct option is (A).

Quick Tip to Solve: Before calculating a reflection, check the normal-component sum: if it is zero, reflection preserves the sum of the point coordinates.

---

18. If equation of the plane that contains the point $(-2,3,5)$ and is perpendicular to each of the planes $2x+4y+5z=8$ and $3x-2y+3z=5$ is $\alpha x+\beta y+\gamma z+97=0$ then $\alpha+\beta+\gamma=$ [4 marks]

(A) 15

(B) 16

(C) 17

(D) 18

Correct Answer: (A) 15

Solution:

Concept:

- Two planes are perpendicular when their normal vectors have zero dot product. The required normal must therefore be perpendicular to both given normal vectors.
- The cross product of two nonparallel normal vectors gives a suitable direction for the required normal. A point on the plane then fixes its constant term.

Step 1: Calculate a normal to the required plane The normals of the two given planes are $\mathbf{n}_1=(2,4,5)$ and $\mathbf{n}_2=(3,-2,3)$. Their cross product is

$$\begin{aligned}\mathbf{n}_1\times\mathbf{n}_2&=(4\cdot3-5(-2),\ 5\cdot3-2\cdot3,\ 2(-2)-4\cdot3)\\&=(22,9,-16).\end{aligned}$$

The vector is nonzero, so it determines a plane normal uniquely up to a scalar multiplier.

Step 2: Use the given point to write the plane The plane through $(-2,3,5)$ with this normal is

$$22(x+2)+9(y-3)-16(z-5)=0.$$

On expanding, its equation becomes

$$22x+9y-16z+44-27+80=0,$$

or $22x+9y-16z+97=0$. Its constant is already the required $+97$; hence no additional scaling or reversal of signs is needed. Thus $\alpha=22$, $\beta=9$ and $\gamma=-16$.

Step 3: Check perpendicularity and add the coefficients The dot products verify both conditions:

$$22(2)+9(4)-16(5)=44+36-80=0,$$

$$22(3)+9(-2)-16(3)=66-18-48=0.$$

The requested sum is therefore

$$\alpha + \beta + \gamma = 22 + 9 - 16 = 15.$$

The other offered totals, 16, 17 and 18, cannot follow from the normal with its scale fixed by the prescribed constant term.

Step 4: Conclusion $\alpha + \beta + \gamma = \boxed{15}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the cross product for the normal, then compare the constant term before reading off coefficients; a normal vector alone has an arbitrary scale.

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19. Let $f : [2, 4] \to \mathbb{R}$ be a differentiable function such that $(x \log_e x) f'(x) + (\log_e x) f(x) + f(x) \geq 1, x \in [2, 4]$ with $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$.

Consider the following two statements :

(A) : $f(x) \leq 1$, for all $x \in [2, 4]$

(B) : $f(x) \geq \frac{1}{8}$, for all $x \in [2, 4]$

Then, [4 marks]

(A) Neither statement (A) nor statement (B) is true

(B) Only statement (A) is true

(C) Only statement (B) is true

(D) Both the statements (A) and (B) are true

Correct Answer: (D) Both the statements (A) and (B) are true

Solution:

Concept:

- The product rule gives $\frac{d}{dx}[x \ln x\, f(x)] = x \ln x\, f'(x) + (\ln x + 1) f(x)$. Recognising this product converts the differential inequality to monotonicity.
- For a nondecreasing function h on $[2, 4]$, its values lie between $h(2)$ and $h(4)$. Division by $x \ln x > 0$ preserves inequalities on this interval.

Step 1: Form a monotone auxiliary function Set $h(x) = x \ln x\, f(x)$. The displayed hypothesis says $h'(x) \geq 1 > 0$, which in particular implies that h is nondecreasing. The endpoint data give

$$h(2) = 2 \ln 2 \cdot \tfrac{1}{2} = \ln 2, \qquad h(4) = 4 \ln 4 \cdot \tfrac{1}{4} = \ln 4.$$

Consequently the conditional bounds are $\ln 2 \leq h(x) \leq \ln 4$.

Step 2: Deduce the two stated bounds On $[2, 4]$, $2 \ln 2 \leq x \ln x \leq 4 \ln 4$. All quantities are positive, and hence

$$f(x) = \frac{h(x)}{x \ln x} \leq \frac{\ln 4}{2 \ln 2} = 1,$$

$$f(x) \geq \frac{\ln 2}{4 \ln 4} = \frac{1}{8}.$$

Thus both (A) and (B) follow from the hypotheses as conditional statements. Options asserting that only one bound holds, or that neither holds, do not reflect these two deductions.

Step 3: Check the compatibility of the printed hypotheses There is a further issue in the literal inequality: the mean value theorem applied to h would give some $\xi \in (2,4)$ with

$$h'(\xi) = \frac{h(4) - h(2)}{4 - 2} = \frac{\ln 2}{2} < 1,$$

contradicting $h' \geq 1$. Thus no differentiable function actually satisfies all the printed conditions. The option selection above is the conditional implication requested by the statement question; it does not establish existence of such a function. Both bounds remain consequences of the weaker monotonicity condition.

Step 4: Conclusion Conditionally, both stated bounds follow, giving option (D). The stronger printed derivative bound is inconsistent with the two endpoint values, so an actual function with all these properties does not exist. Hence, the correct option is (D).

Quick Tip to Solve: Look for a product derivative, but also compare its endpoint secant slope with any lower bound imposed on its derivative.

20. The number of elements in the set

$S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\}$ **is :** [4 marks]

(A) 9

(B) 8

(C) 12

(D) 10

Correct Answer: (A) 9

Solution:

Concept:

- Use $\sin^2\theta = 1 - \cos^2\theta$ to express an even-power trigonometric equation as a polynomial in $u = \cos^2\theta$, with $0 \leq u \leq 1$.
- Count angles only after finding the allowed squared-cosine values. On the closed interval $[0, 2\pi]$, both endpoints are included and are distinct elements.

Step 1: Reduce the expression to a cubic polynomial Put $u = \cos^2\theta$, so that $\sin^6\theta = (1-u)^3$. The given equation becomes

$$3u^2 - 5u - 2(1-u)^3 + 2 = 0.$$

Expanding the cube and combining like terms gives

$$3u^2 - 5u - 2 + 6u - 6u^2 + 2u^3 + 2 = 0,$$

hence $2u^3 - 3u^2 + u = 0$. Factoring,

$$u(2u-1)(u-1) = 0.$$

All three resulting values $0, 1/2, 1$ belong to the permitted interval for $\cos^2\theta$.

Step 2: List the angles for each root For $u = 0$, the angles are $\pi/2, 3\pi/2$, giving two elements. For $u = 1/2$, the angles are $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$, giving four elements. For $u = 1$, the angles are $0, \pi, 2\pi$, giving three elements. These three groups are disjoint because they have different values of $\cos^2\theta$.

Step 3: Add the distinct solutions The total number of elements is

$$2 + 4 + 3 = 9.$$

Each listed angle makes one of the three factors zero, and the factorisation exhausts the original equation, so there are no additional solutions. The answer 8 would result from losing one endpoint, although 0 and 2π are different real numbers in the set. The totals 10 and 12 overcount angles for the extreme squared-cosine values.

Step 4: Conclusion The set has $\boxed{9}$ elements. Hence, the correct option is (A).

Quick Tip to Solve: For equations in squared sine or cosine, count the extreme values 0 and 1 separately; closed endpoints can change the count by one.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$**, where** $a, c \in \mathbb{R}$**. If** $A^3 = A$ **and the positive value of** a **belongs to the interval** $(n-1, n]$**, where** $n \in \mathbb{N}$**, then** n **is equal to _____.** [4 marks]

Solution:

Concept:

- A matrix identity holds entry by entry. Multiplying the specific matrix is often simpler than finding its eigenvalues when only two entries are unknown.
- Use an off-diagonal entry and a diagonal entry of $A^3 = A$ to obtain scalar equations for a and c, then verify that they satisfy the complete matrix identity.

Step 1: Compute the square of the matrix Direct row-by-column multiplication gives

$$A^2 = \begin{pmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & 1 & 2+3c \end{pmatrix}.$$

For example, the top-right entry is $0\cdot 2 + 1\cdot 3 + 2\cdot 0 = 3$, while the bottom-left entry is $1\cdot 0 + c\cdot a + 0\cdot 1 = ac$.

These entries retain both unknowns without assuming either is nonzero.

Step 2: Use the cube identity to obtain two equations Multiplying once more gives

$$A^3 = \begin{pmatrix} 2ac+3 & a+3c+2 & 2(a+3c+2) \\ a(a+3c+2) & 2ac+3 & 3(a+3c+2) \\ a+3c+2 & c(a+3c+2) & 2ac+3 \end{pmatrix}.$$

Comparison with A requires

$$2ac+3=0, \qquad a+3c+2=1.$$

These two conditions are also sufficient: substituting them in the displayed cube reproduces every entry of A. Thus no further matrix constraint is being omitted.

Step 3: Find the positive parameter and its integer interval From $a+3c=-1$, obtain $c=(-1-a)/3$. Substitute this in $2ac+3=0$:

$$2a\frac{-1-a}{3}+3=0 \quad \Longrightarrow \quad 2a^2+2a-9=0.$$

Therefore

$$a=\frac{-1\pm\sqrt{19}}{2}.$$

Only the plus sign gives a positive number. Since $3<\sqrt{19}<5$, this positive value satisfies $1<a<2$, so it lies in $(2-1,2]$. The interval convention $(n-1,n]$ determines a unique natural number n.

Step 4: Conclusion The positive value is $a=(\sqrt{19}-1)/2\in(1,2]$. Therefore the required dimensionless integer is $\boxed{n=2}$.

Quick Tip to Solve: In a matrix polynomial condition, first inspect a simple entry such as the top-middle entry; then use a diagonal entry to close the scalar system.

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22. Let $\mathrm{H_n}: \frac{x^2}{1+n}-\frac{y^2}{3+n}=1, n\in N$. Let k be the smallest even value of n such that the eccentricity of $\mathrm{H_k}$ is a rational number. If l is the length of the latus rectum of $\mathrm{H_k}$, then $21l$ is equal to _____.

[4 marks]

Solution:

Concept:

- For a hyperbola $x^2/a^2-y^2/b^2=1$, the eccentricity satisfies $e^2=1+b^2/a^2$, and its latus rectum has length $l=2b^2/a$. Here $a>0$ is the semi-transverse axis.
- A positive rational number in lowest terms has a rational square root precisely when both its numerator and denominator are perfect squares. This criterion lets us test the eccentricity without guessing its decimal value.

Step 1: Express the eccentricity in terms of an odd integer Put $d=n+1$. Since n is even, d is odd.

Using $a^2 = n+1$ and $b^2 = n+3$, we obtain

$$e^2 = 1 + \frac{n+3}{n+1} = \frac{2d+2}{d}.$$

Moreover, $\gcd(2d+2, d) = \gcd(2, d) = 1$, because d is odd. Thus this fraction is already in lowest terms. Rationality of e requires $d = v^2$ and $2v^2 + 2 = u^2$ for positive odd v.

Step 2: Find the first admissible even value The positive even values of n start at 2, so $d > 1$ and hence $v > 1$. It is enough to examine the odd values $v = 3, 5, 7$ in order:

$$2(3)^2 + 2 = 20, \qquad 2(5)^2 + 2 = 52, \qquad 2(7)^2 + 2 = 100.$$

The first two are not squares, whereas $100 = 10^2$. Consequently the smallest possible denominator is $d = 49$, giving $k = 48$. No smaller odd denominator can work, because a denominator that is not a square has already been excluded by the reduced-fraction criterion.

Step 3: Calculate the requested multiple of the latus rectum At $k = 48$, the hyperbola has $a^2 = 49$, $b^2 = 51$, and $e = 10/7$, which is indeed rational. Therefore

$$l = \frac{2(51)}{7} = \frac{102}{7}, \qquad 21l = 21 \cdot \frac{102}{7} = 306.$$

We use the full latus rectum length, rather than its half-length b^2/a.

Step 4: Conclusion The smallest even parameter is $k = 48$, the latus rectum length is $102/7$, and the required numerical value is $\boxed{306}$.

Quick Tip to Solve: For even n, first reduce $e^2 = (2n+4)/(n+1)$. Its odd denominator must be a square, greatly shortening the search.

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23. For $m, n > 0$, let $\alpha(m, n) = \int_0^2 t^m (1+3t)^n dt$. If $11\alpha(10, 6) + 18\alpha(11, 5) = p(14)^6$, then p is equal to _____. [4 marks]

Solution:

Concept:

- Linearity of integration combines integrals with the same limits into one integral. The coefficients 11 and 18 suggest differentiating a product involving t^{11} and $(1+3t)^6$.
- The product rule gives $(uv)' = u'v + uv'$; the chain rule contributes a factor of 3 when differentiating a power of $1+3t$. The fundamental theorem then evaluates the integral using endpoint values.

Step 1: Combine the two given integrals By the definition of α, the expression on the left is

$$11\alpha(10, 6) + 18\alpha(11, 5) = \int_0^2 \left[11t^{10}(1+3t)^6 + 18t^{11}(1+3t)^5\right] dt.$$

Both terms have exactly the same interval. Their exponents differ by one in the pattern produced by differentiating a product, so there is no need to expand the sixth power or compute two large sums

separately.

Step 2: Identify an exact derivative Differentiate the candidate product, keeping both product-rule terms:

$$\frac{d}{dt}\left[t^{11}(1+3t)^6\right] = 11t^{10}(1+3t)^6 + t^{11}\cdot 6(1+3t)^5\cdot 3$$
$$= 11t^{10}(1+3t)^6 + 18t^{11}(1+3t)^5.$$

This is exactly the combined integrand, including the coefficient 18. All factors are polynomials on the integration interval, so the endpoint evaluation is valid without any singularity or domain restriction.

Step 3: Evaluate and isolate the coefficient The lower endpoint contributes zero because of the factor t^{11}. Thus

$$11\alpha(10,6) + 18\alpha(11,5) = \left[t^{11}(1+3t)^6\right]_0^2 = 2^{11}7^6.$$

Now $14^6 = 2^6 7^6$. Comparing with $p14^6$ and cancelling the positive common factor gives

$$p = \frac{2^{11}7^6}{2^6 7^6} = 2^5 = 32.$$

Step 4: Conclusion The required dimensionless coefficient is $\boxed{p = 32}$.

Quick Tip to Solve: When two definite integrals have adjacent exponents, check whether their weighted sum is the derivative of a product before expanding either integrand.

24. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits on the allotted seat, is _____. [4 marks]

Solution:

Concept:

- A derangement is a permutation with no fixed point: here no student occupies the seat bearing that student's roll number. Students and seats are distinct, so there are $5!$ unrestricted arrangements.
- Inclusion-exclusion counts the complement of the union of the events that particular students occupy their own seats. If a chosen set of j students is fixed, the remaining students can be arranged in $(5-j)!$ ways.

Step 1: Define the forbidden events Let A_i be the event that student i sits in the allotted seat, for $i = 1, \ldots, 5$. We need arrangements outside $A_1 \cup \cdots \cup A_5$. For any specified j students, fixing their seats leaves precisely $(5-j)!$ bijections between the remaining students and remaining seats. There are $\binom{5}{j}$ ways to choose that set of students. These counts include arrangements with additional fixed students, which is why alternating corrections are necessary.

Step 2: Apply inclusion-exclusion Subtract the arrangements having a specified fixed student, add

the pairwise intersections back, and continue:

$$\begin{aligned} D_5 &= 5! - \binom{5}{1}4! + \binom{5}{2}3! - \binom{5}{3}2! \\ &\quad + \binom{5}{4}1! - \binom{5}{5}0! \\ &= 120 - 5(24) + 10(6) - 10(2) + 5(1) - 1(1) \\ &= 120 - 120 + 60 - 20 + 5 - 1 = 44. \end{aligned}$$

The final term represents the unique arrangement with all five students in their own seats; it must be subtracted according to the alternating-sign rule.

Step 3: Check the count with the recurrence The standard derangement recurrence is $D_n = (n-1)(D_{n-1} + D_{n-2})$, with $D_1 = 0$ and $D_2 = 1$. It yields $D_3 = 2$, $D_4 = 9$, and

$$D_5 = 4(D_4 + D_3) = 4(9 + 2) = 44.$$

This agrees with inclusion-exclusion. Requiring that none sits correctly is stronger than merely excluding the single arrangement in which everyone sits correctly; the count $5! - 1$ would therefore be inappropriate.

Step 4: Conclusion The number of permissible seating arrangements is $\boxed{44}$.

Quick Tip to Solve: For a seating problem with no one in the assigned place, use derangements: subtracting only the completely correct arrangement does not enforce the condition.

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25. Let a line l pass through the origin and be perpendicular to the lines
$l_1 : \vec{r} = (\hat{i} - 11\hat{j} - 7\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in \mathbb{R}$ **and**
$l_2 : \vec{r} = (-\hat{i} + \hat{\mathrm{k}}) + \mu(2\hat{i} + 2\hat{j} + \hat{\mathrm{k}}), \mu \in \mathbb{R}$**.**
If P **is the point of intersection of** l **and** l_1**, and** $\mathrm{Q}(\propto, \beta, \gamma)$ **is the foot of perpendicular from P on** l_2**, then** $9(\alpha + \beta + \gamma)$ **is equal to _____.** [4 marks]

Solution:

Concept:

- A direction vector perpendicular to two nonparallel directions is their cross product. Once this direction is found, a line through the origin has position vector equal to a scalar multiple of it.
- If $Q = A + \mu\mathbf{v}$ is the perpendicular foot of P on a line with direction $\mathbf{v}$, then $(P - Q) \cdot \mathbf{v} = 0$. Hence $\mu = ((P - A) \cdot \mathbf{v})/(\mathbf{v} \cdot \mathbf{v})$.

Step 1: Find the direction of the line through the origin The two given direction vectors are $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (2, 2, 1)$. Their cross product is

$$\mathbf{u} \times \mathbf{v} = (2 - 6, 6 - 1, 2 - 4) = (-4, 5, -2).$$

It is nonzero and has zero dot product with each given direction. Thus a point on l has coordinates

$(-4t, 5t, -2t)$. Either sign of this direction describes the same line.

Step 2: Locate the intersection with the first line A point on l_1 is $(1+\lambda, -11+2\lambda, -7+3\lambda)$. Equating its first two coordinates to those on l gives

$$1+\lambda = -4t, \qquad -11+2\lambda = 5t.$$

Substituting $\lambda = -4t-1$ into the second equation gives $-13 = 13t$, so $t=-1$ and $\lambda = 3$. The third coordinate also agrees: $-7+3(3) = 2 = -2(-1)$. Therefore $P = (4, -5, 2)$.

Step 3: Project the intersection onto the second line For l_2, take $A = (-1, 0, 1)$ and $\mathbf{v} = (2, 2, 1)$. Then

$$\mu = \frac{(5,-5,1)\cdot(2,2,1)}{2^2+2^2+1^2} = \frac{10-10+1}{9} = \frac{1}{9}.$$

Consequently

$$Q = \left(-\frac{7}{9}, \frac{2}{9}, \frac{10}{9}\right), \qquad \alpha+\beta+\gamma = \frac{5}{9}.$$

As a direct check, $P - Q = (43, -47, 8)/9$, whose dot product with $(2, 2, 1)$ is $(86-94+8)/9 = 0$. The computed point is therefore the perpendicular foot, not just an arbitrary point on the line.

Step 4: Conclusion The required value is $9(\alpha+\beta+\gamma) = 9(5/9) = \boxed{5}$.

Quick Tip to Solve: Use a cross product for the common perpendicular direction, then a dot-product projection for the foot; the two perpendicularity conditions play different roles.

26. The mean of the coefficients of $x, x^2, \ldots, x^7$ in the binomial expansion of $(2+x)^9$ is _____.

[4 marks]

Solution:

Concept:

- In $(2+x)^9$, the coefficient of x^r is $\binom{9}{r}2^{9-r}$. Evaluating the polynomial at $x=1$ gives the sum of all its coefficients, including the constant and highest-degree terms.
- The arithmetic mean of the coefficients of $x, x^2, \ldots, x^7$ is their sum divided by 7. The exponents label seven terms; they are not weights in the required average.

Step 1: Obtain the sum of all coefficients Write $(2+x)^9 = \sum_{r=0}^{9} c_r x^r$, where $c_r = \binom{9}{r}2^{9-r}$. Substitution of $x=1$ gives

$$\sum_{r=0}^{9} c_r = (2+1)^9 = 3^9 = 19683.$$

This includes every coefficient exactly once. The requested list stops at degree 7 and omits the constant, so we must remove degrees 0, 8 and 9 before averaging. Evaluating at another value of x would weight the coefficients unequally.

Step 2: Remove the three unwanted coefficients The constant coefficient is $c_0 = 2^9 = 512$. The

coefficient of x^8 is $c_8 = \binom{9}{8}2^1 = 18$, and the leading coefficient is $c_9 = \binom{9}{9}2^0 = 1$. Hence

$$c_1 + c_2 + \cdots + c_7 = 19683 - 512 - 18 - 1 = 19152.$$

The binomial coefficient in c_8 is essential: taking it to be merely 2 would incorrectly omit the nine choices for the position of the remaining factor 2.

Step 3: Calculate and verify the mean There are seven entries in the requested list, so

$$\text{Mean} = \frac{19152}{7} = 2736.$$

For a numerical check, the seven coefficients are

$$2304,\ 4608,\ 5376,\ 4032,\ 2016,\ 672,\ 144.$$

Their sum is 19152, agreeing with the subtraction method. This also checks that the coefficient of x^7 has been included and those of x^8, x^9 have not. The average is a pure number, since the question concerns polynomial coefficients rather than a physical quantity.

Step 4: Conclusion The required arithmetic mean is $\boxed{2736}$.

Quick Tip to Solve: For the sum of a long consecutive group of binomial coefficients, evaluate the polynomial at 1 and subtract only the coefficients outside the requested group.

27. If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}$ is equal to _____. [4 marks]

Solution:

Concept:

- Vieta's relations for $x^2 - 7x - 1 = 0$ give $a + b = 7$ and $ab = -1$. Therefore both roots are nonzero, and reciprocal powers can be related to powers of the other root.
- Instead of computing powers as large as 21, find a polynomial identity satisfied by each root. Multiplying that identity by an appropriate power preserves it and matches the numerator.

Step 1: Derive an identity of degree four Let r denote either root. Its defining equation gives $r^2 - 1 = 7r$. Squaring both sides produces

$$r^4 - 2r^2 + 1 = 49r^2, \qquad r^4 + 1 = 51r^2.$$

This identity is valid separately for $r = a$ and $r = b$. The squaring is used only to derive a true consequence for the known roots, rather than to introduce additional possible roots into the problem.

Step 2: Match the high powers in the numerator Multiply the identity by r^{17}:

$$r^{21} + r^{17} = 51r^{19}.$$

Writing this first for a, then for b, and adding gives

$$a^{21} + b^{21} + a^{17} + b^{17} = 51(a^{19} + b^{19}).$$

Thus the full numerator is a constant multiple of the denominator. There is no need to approximate either irrational root, so no rounding error enters the calculation.

Step 3: Confirm that division is legitimate The roots are $a, b = (7 \pm \sqrt{53})/2$, in either order. The positive root $A = (7 + \sqrt{53})/2$ exceeds 1 and the negative root is $-1/A$, since the product is -1. Therefore

$$a^{19} + b^{19} = A^{19} - A^{-19} > 0.$$

The denominator cannot vanish. Dividing the identity obtained above by it is valid and yields exactly 51. As another check, $a^2 + b^2 = (a + b)^2 - 2ab = 49 + 2 = 51$, the same constant appearing in the fourth-power identity.

Step 4: Conclusion The value of the given ratio is $\boxed{51}$.

Quick Tip to Solve: When powers differ symmetrically by 2, divide or rearrange the quadratic relation to obtain a fourth-power identity instead of expanding the individual roots.

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28. The number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to _____. [4 marks]

Solution:

Concept:

- The general term in the expansion of $(\sqrt{3} + \sqrt[4]{5})^{680}$ is $T_{r+1} = \binom{680}{r} 3^{(680-r)/2} 5^{r/4}$, where r ranges from 0 to 680 inclusive.
- An integer has integral exponents in its prime factorisation. Multiplying by the integer binomial coefficient can add integer prime exponents, but it cannot remove a fractional part in an exponent of 3 or 5.

Step 1: Write the criterion for an individual term In the term T_{r+1}, the exponent of 3 is $(680 - r)/2$ and that of 5 is $r/4$. Because 3 and 5 are distinct primes, their irrational factors cannot cancel each other. More formally, take fourth powers: an integral term would have fourth-power prime valuations

$$v_3(T_{r+1}^4) = 4v_3\left(\binom{680}{r}\right) + 2(680 - r),$$

$$v_5(T_{r+1}^4) = 4v_5\left(\binom{680}{r}\right) + r.$$

Both must be divisible by 4. In particular, the second condition forces r to be divisible by 4.

Step 2: Show the condition is sufficient If $r = 4j$, then $r/4 = j$ is an integer and

$$\frac{680 - r}{2} = 340 - 2j$$

is also an integer. Both exponents are nonnegative for the admissible range $0 \le j \le 170$. The binomial coefficient is an integer as well, so every such term is an integer. Conversely, the prime-valuation argument excludes every r not divisible by 4, even if its binomial coefficient happens to contain additional factors of 3 or 5.

Step 3: Count all admissible indices including endpoints The possible values are

$$r = 0, 4, 8, \ldots, 680.$$

This arithmetic progression has

$$\frac{680 - 0}{4} + 1 = 170 + 1 = 171$$

entries. Both endpoints must be included: $r = 0$ gives 3^{340}, while $r = 680$ gives 5^{170}, and each is an integer. We are counting individual terms before any grouping, as the question asks, rather than attempting to test the rationality of the entire expanded sum.

Step 4: Conclusion The number of integral terms is $\boxed{171}$.

Quick Tip to Solve: For binomial terms containing radicals of distinct primes, require each prime exponent to be integral; count the allowed indices inclusively from 0 to the full power.

29. The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is True, is equal to _____. [4 marks]

Solution:

Concept:

- A proposition $A \Rightarrow B$ is false only when its antecedent A is true and its consequent B is false. In all other cases it is true.
- Three independently chosen propositions p, q, r have $2^3 = 8$ ordered truth assignments. The ordered nature means that changing which proposition is true gives a distinct assignment.

Step 1: Identify when the consequent is false Put $A = (p \vee q) \wedge (p \vee r)$ and $B = q \vee r$. A disjunction is false only if both of its constituents are false. Consequently,

$$B = \mathrm{F} \quad \Longleftrightarrow \quad q = \mathrm{F}, \quad r = \mathrm{F}.$$

If either q or r is true, the consequent is already true, so the implication is true irrespective of the antecedent. There are three such ordered pairs (q, r), and each allows either value of p, giving six true assignments.

Step 2: Check the remaining two assignments For $q = r = \mathrm{F}$, the antecedent simplifies without any ambiguity:

$$(p \vee \mathrm{F}) \wedge (p \vee \mathrm{F}) = p \wedge p = p.$$

Thus $(p, q, r) = (\mathrm{T}, \mathrm{F}, \mathrm{F})$ makes the antecedent true and the consequent false; it is the unique false

assignment. The assignment $(\mathrm{F}, \mathrm{F}, \mathrm{F})$ makes both false, and a false antecedent gives a true implication. Therefore the required count is

$$6 + 1 = 8 - 1 = 7.$$

This also checks the often-missed case in which the consequent is false but the implication remains true because its antecedent is false.

Step 3: Conclusion There are $\boxed{7}$ ordered triplets for which the given statement is true. This is a dimensionless count.

Quick Tip to Solve: For counting implications, count the false cases first: require a true antecedent and a false consequent, then subtract from the total.

30. Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \ldots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $\left(16S - (25)^{-54}\right)$ is equal to _____.

[4 marks]

Solution:

Concept:

- For an arithmetic-geometric sum, multiply by the common geometric ratio and subtract. Adjacent arithmetic coefficients then differ by one, leaving a geometric sum.
- The finite geometric identity is $r + r^2 + \cdots + r^n = r(1 - r^n)/(1 - r)$, for $r \neq 1$. Retain the endpoint term until the requested expression is formed.

Step 1: Shift the series by its geometric ratio Write $r = 1/5$, so that

$$S = 109 + 108r + 107r^2 + \cdots + 2r^{107} + r^{108}.$$

Multiplication by r shifts every power up by one. Subtracting the shifted series from the original reduces each coefficient after the constant term by one, and introduces a final negative term:

$$(1 - r)S = 109 - (r + r^2 + \cdots + r^{108}) - r^{109}.$$

In particular, the last term $-r^{109}$ cannot be dropped even though it is very small; the expression in the question is designed to cancel the exact finite-series remainder.

Step 2: Evaluate the finite remainder exactly Since $r/(1 - r) = 1/4$, the geometric sum is

$$r + r^2 + \cdots + r^{108} = \frac{1 - r^{108}}{4}.$$

Also $r^{109} = r^{108}/5$. Hence

$$\begin{aligned} \frac{4}{5}S &= 109 - \frac{1}{4} + \frac{r^{108}}{4} - \frac{r^{108}}{5} \\ &= \frac{435}{4} + \frac{r^{108}}{20}. \end{aligned}$$

Multiply by 20 to obtain $16S = 2175 + r^{108}$. The subtracted quantity in the question is

$$25^{-54} = (5^2)^{-54} = 5^{-108} = r^{108}.$$

The finite remainder therefore cancels exactly, rather than approximately. No rounding of the original sum is required.

Step 3: Conclusion The requested value is $16S - 25^{-54} = \boxed{2175}$. It is a dimensionless number.

Quick Tip to Solve: When a finite arithmetic-geometric sum is paired with a tiny power, keep its endpoint correction: that power often cancels the remainder exactly.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Given below are two statements :

Statements I : Astronomical unit (Au), Parsec (Pc) and Light year (ly) are units for measuring astronomical distances.

Statements II : $\mathrm{Au} < \mathrm{Parsec(Pc)} < \mathrm{ly}$

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Both Statements I and Statements II are incorrect.

(B) Both Statements I and Statements II are correct,

(C) Statements I is incorrect but Statements II is correct.

(D) Statements I is correct but Statements II is incorrect.

Correct Answer: (D) Statements I is correct but Statements II is incorrect.

Solution:

Concept:

- An astronomical unit is a distance comparable to the mean Earth–Sun separation; a light year is the distance light travels in one year. Both are units of length, not time.
- A parsec is the distance at which one astronomical unit subtends one arcsecond. The conversion $1\,\mathrm{pc} \approx 3.26\,\mathrm{ly}$ fixes the order of the astronomical units.

Step 1: Judge Statement I using the meaning of the units Astronomical distances are so large that ordinary metres are inconvenient. The astronomical unit, light year and parsec provide progressively convenient scales for Solar System and stellar distances. Their approximate SI magnitudes are

$$1\,\mathrm{Au} \approx 1.496 \times 10^{11}\,\mathrm{m},$$
$$1\,\mathrm{ly} \approx 9.46 \times 10^{15}\,\mathrm{m},$$
$$1\,\mathrm{pc} \approx 3.086 \times 10^{16}\,\mathrm{m}.$$

All three represent lengths. In particular, the word "year" in light year does not make it a time unit: it specifies the travel time used in defining a distance. Statement I is therefore correct.

Step 2: Test the inequality in Statement II and eliminate the choices Comparing the numerical

magnitudes shows that the correct increasing order is

$$\text{Au} < \text{ly} < \text{pc}.$$

Equivalently, one parsec is about 3.26 light years, so it exceeds one light year. Statement II instead places the parsec between the astronomical unit and light year and is incorrect. Option (A) incorrectly rejects Statement I; option (B) incorrectly accepts the stated order; option (C) both rejects the correct first statement and accepts the incorrect second statement. Only option (D) has the required combination.

Step 3: Conclusion Statement I is correct and Statement II is incorrect. Hence, the correct option is (D).

Quick Tip to Solve: Remember $1\,\text{pc} \approx 3.26\,\text{ly}$: the parsec is larger than the light year, despite their names.

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32. $1\ \text{kg}$ **of water at** 100°C **is converted into steam at** 100°C **by boiling at atmospheric pressure. The volume of water changes from** $1.00 \times 10^{-3}\ \text{m}^3$ **as a liquid to** $1.671\ \text{m}^3$ **as steam. The change in internal energy of the system during the process will be**

(Given latent heat of vaporisaiton $= 2257\ \text{kJ/kg}$**, Atmospheric pressure =** $1 \times 10^5\ \text{Pa}$**)** [4 marks]

(A) + 2090 kJ

(B) − 2426 kJ

(C) + 2476 kJ

(D) − 2090 kJ

Correct Answer: (A) + 2090 kJ

Solution:

Concept:

- Using work done by the system as positive, the first law is $Q = \Delta U + W$. Heat supplied during vaporisation need not all become internal energy.
- For boiling at constant atmospheric pressure, $Q = mL$ and $W = P(V_f - V_i)$. Pressure times volume has units of joules, so convert it to kilojoules before subtracting.

Step 1: Find the heat supplied and the expansion The water changes phase at 100°C, so use the latent heat directly; no sensible-heat term $mc\Delta T$ is needed. For the given mass,

$$Q = mL = (1\,\text{kg})(2257\,\text{kJ kg}^{-1}) = 2257\,\text{kJ}.$$

The change in volume includes the initial liquid volume:

$$\Delta V = 1.671 - 1.00 \times 10^{-3} = 1.670\,\text{m}^3.$$

This volume increase is positive, meaning the steam pushes back the atmosphere and the system performs positive work on its surroundings.

Step 2: Subtract the work from the supplied heat At the specified constant pressure,

$$W = P\Delta V = (1 \times 10^5)(1.670)\,\text{J}$$
$$= 1.670 \times 10^5\,\text{J} = 167\,\text{kJ}.$$

Therefore the first law gives

$$\Delta U = Q - W = 2257 - 167 = +2090\,\text{kJ}.$$

The positive sign means that the internal energy increases during vaporisation, even though the temperature is constant. The latent heat accounts for molecular rearrangement and external expansion work. Options (B) and (D) have an incorrect negative sign. Option (C) is also inconsistent with the first law because it exceeds the heat absorbed while the system is doing positive work.

Step 3: Conclusion The change in internal energy is $\boxed{+2090\,\text{kJ}}$. Hence, the correct option is (A).

Quick Tip to Solve: At constant-pressure boiling, start with mL and subtract $P\Delta V$; constant temperature does not imply constant internal energy during a phase change.

33. The critical angle for a denser-rarer interface is 45°. The speed of light in rarer medium is 3×10^8 m/s. The speed of light in the denser medium is: [4 marks]

(A) 2.12×10^8 m/s

(B) 5×10^7 m/s

(C) $\sqrt{2} \times 10^8$ m/s

(D) 3.12×10^7 m/s

Correct Answer: (A) 2.12×10^8 m/s

Solution:

Concept:

- At the critical angle C, a ray travels from the denser medium into the rarer medium with the refracted angle equal to 90°. Snell's law then gives $n_d \sin C = n_r$.
- The refractive index satisfies $n = c/v$, where c is the speed in vacuum. Thus $\sin C = n_r/n_d = v_d/v_r$, linking critical angle directly to the speeds.

Step 1: Express the critical-angle condition in terms of speeds Let v_d denote the speed in the denser medium and v_r that in the rarer medium. The critical condition is

$$\frac{c}{v_d}\sin C = \frac{c}{v_r}.$$

Cancelling c and rearranging gives

$$v_d = v_r \sin C.$$

This relation also checks the direction of the ratio: since $0 < \sin C < 1$, light must travel more slowly in the optically denser medium. Dividing the rarer-medium speed by $\sin C$ would give the opposite

physical behaviour.

Step 2: Substitute the angle and compare the options The given values are $C = 45^\circ$ and $v_r = 3 \times 10^8\,\mathrm{m\,s^{-1}}$. Therefore

$$\begin{aligned} v_d &= (3 \times 10^8)\frac{1}{\sqrt{2}}\,\mathrm{m\,s^{-1}} \\ &= \frac{3}{\sqrt{2}} \times 10^8\,\mathrm{m\,s^{-1}} \\ &\approx 2.12 \times 10^8\,\mathrm{m\,s^{-1}}. \end{aligned}$$

Option (A) agrees with this value. Option (C), although it contains $\sqrt{2}$, has the wrong numerical coefficient and does not give $v_d/v_r = \sin 45^\circ$. Options (B) and (D) are much smaller and would correspond to different critical angles. The required speed is below the rarer-medium speed, as expected.

Step 3: Conclusion The speed in the denser medium is $\boxed{2.12 \times 10^8\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a critical angle, use $v_d/v_r = \sin C$. The denser-medium speed must be the smaller of the two speeds.

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34. On a temperature scale 'X', the boiling point of water is 65°X and the freezing point is -15°X. Assume that the X scale is linear. The equivalent temperature corresponding to -95°X on the Farenheit scale would be: [4 marks]

(A) -148°F

(B) -48°F

(C) -63°F

(D) -112°F

Correct Answer: (A) -148°F

Solution:

Concept:

- For linear temperature scales, the fraction of the interval between two fixed points is the same on either scale. A temperature below the lower fixed point has a negative fraction.
- The freezing and boiling points of water are -15°X and 65°X on the given scale, and 32°F and 212°F on Fahrenheit.

Step 1: Set up the fixed-point conversion Let X and F be numerical temperature readings in the two scales. Since the relation is linear and both fixed points are known,

$$\frac{X - (-15)}{65 - (-15)} = \frac{F - 32}{212 - 32}.$$

The interval between freezing and boiling is 80 degrees on the X scale and 180 degrees on Fahrenheit.

Thus

$$\frac{X+15}{80} = \frac{F-32}{180}.$$

The offset $+15$ must be included: comparing $X/80$ with $F/180$ would wrongly assume that the zero readings represent the same physical temperature.

Step 2: Evaluate the Fahrenheit reading and check its position At the stated temperature $X = -95$, the fixed-point fraction is

$$\frac{-95+15}{80} = \frac{-80}{80} = -1.$$

The reading lies one full fixed-point interval below freezing. Consequently,

$$F - 32 = -180,$$
$$F = 32 - 180 = -148.$$

As a check, this position corresponds to $-100°\text{C}$, and $F = (9/5)(-100) + 32 = -148$. The other listed readings, -48, -63 and -112, fail the same fixed-point ratio and therefore cannot represent $-95°\text{X}$. The negative extrapolation is valid because the question explicitly states that the scale is linear.

Step 3: Conclusion The equivalent temperature is $\boxed{-148°\text{F}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use differences from a fixed point, not the raw readings: here -95 is exactly one 80-degree interval below the freezing point -15.

35. The radii of two planets 'A' and 'B' are 'R' and '4R' and their densities are ρ and $\rho/3$ respectively. The ratio of acceleration due to gravity at their surfaces $(g_A : g_B)$ will be: [4 marks]

(A) 3 : 16

(B) 4 : 3

(C) 1 : 16

(D) 3 : 4

Correct Answer: (D) 3 : 4

Solution:

Concept:

- For a spherical planet, the acceleration due to gravity at its surface is $g = GM/R^2$, where M is the planet's mass and R its radius.
- The mass in terms of mean density is $M = (4\pi/3)\rho R^3$. Substitution shows that surface gravity varies as the product ρR, rather than as density alone.

Step 1: Express surface gravity using density and radius Combine the mass formula with Newton's gravitational law:

$$g = \frac{G}{R^2}\left(\frac{4}{3}\pi\rho R^3\right) = \frac{4}{3}\pi G\rho R.$$

The common factors $4\pi G/3$ cancel when comparing the two planets. Although planet B has a lower density, its larger volume gives it a greater total mass. It is therefore necessary to include both density

and radius before deciding which planet has stronger surface gravity.

Step 2: Insert both planets' parameters and simplify For planet A, the product controlling surface gravity is ρR. For planet B, use the given radius $4R$ and density $\rho/3$:

$$\frac{g_A}{g_B} = \frac{\rho R}{(\rho/3)(4R)} = \frac{3}{4}.$$

An independent mass-ratio check gives $M_B/M_A = (1/3)4^3 = 64/3$. Dividing this by the radius-squared ratio 16 yields $g_B/g_A = 4/3$, in agreement with the previous result. Option (B) is the reversed ratio; options (A) and (C) do not correctly combine the cubic change in mass with the inverse-square radius dependence. The required order in the question is A to B.

Step 3: Conclusion The ratio of surface accelerations is $\boxed{g_A : g_B = 3 : 4}$. Hence, the correct option is (D).

Quick Tip to Solve: When a planet's density and radius are given, first reduce g to $(4\pi G/3)\rho R$; this avoids separate mass calculations.

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36. Three vessels of equal volume contain gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and third contains uranium hexafloride (polyatomic). Arrange these on the basis of their root mean square speed (v_{rms}) and choose the correct answer from the options given below: [4 marks]

(A) $\text{v}_{\text{rms}}(\text{ mono }) = \text{v}_{\text{rms}}(\text{ dia }) = \text{v}_{\text{rms}}(\text{ poly })$

(B) v_{rms} (mono) $> \text{v}_{\text{rms}}(\text{ dia }) > \text{v}_{\text{rms}}$ (poly)

(C) v_{rms} (dia) $< \text{v}_{\text{rms}}$ (poly) $< \text{v}_{\text{rms}}$ (mono)

(D) v_{rms} (mono) $< \text{v}_{\text{rms}}$ (dia) $< \text{v}_{\text{rms}}$ (poly)

Correct Answer: (B) v_{rms} (mono) $> \text{v}_{\text{rms}}(\text{ dia }) > \text{v}_{\text{rms}}$ (poly)

Solution:

Concept:

- The root mean square translational speed of an ideal gas is $v_{\text{rms}} = \sqrt{3RT/M}$, where T is its absolute temperature and M its molar mass in kg mol^{-1}.
- At a common temperature, the mean translational kinetic energy per molecule is the same for all gases. The speed therefore varies inversely as the square root of molecular mass; it is not fixed by the number of internal degrees of freedom.

Step 1: Compare the masses of the three gases The gases must be identified chemically rather than merely classified as monoatomic, diatomic and polyatomic. Neon has molar mass approximately $20.2\,\text{g mol}^{-1}$, chlorine gas Cl_2 approximately $70.9\,\text{g mol}^{-1}$, and uranium hexafluoride UF_6 approximately $238 + 6(19) = 352\,\text{g mol}^{-1}$. Thus their molecular masses increase in the stated order. Equal pressure and volume do not make these masses equal.

Step 2: Apply the inverse square root dependence All three vessels have the same temperature, so

the numerator $3RT$ is common. The molar masses satisfy

$$M_{\mathrm{Ne}} < M_{\mathrm{Cl_2}} < M_{\mathrm{UF_6}}.$$

Since $v_{\mathrm{rms}} \propto 1/\sqrt{M}$, the first gas is fastest and the last gas slowest:

$$v_{\mathrm{Ne}} > v_{\mathrm{Cl_2}} > v_{\mathrm{UF_6}}.$$

Equal speeds in (A) would require equal molar masses. Option (C) incorrectly makes the heavier polyatomic gas faster than chlorine; (D) reverses the complete mass dependence.

Step 3: Conclusion The RMS speed ordering for these gases is

$$v_{\mathrm{rms}}(\text{mono}) > v_{\mathrm{rms}}(\text{dia}) > v_{\mathrm{rms}}(\text{poly}).$$

Hence, the correct option is (B).

Quick Tip to Solve: At equal temperature, first compare actual molar masses, then reverse their order for RMS speeds. Atomicity alone is not a general speed rule.

37. The logic performed by the circuit shown in figure is equivalent to : [4 marks]



(A) NAND

(B) AND

(C) OR

(D) NOR

Correct Answer: (B) AND

Solution:

Concept:

- A NOR gate produces the complement of the Boolean sum: $Y = \overline{A + B}$. A small circle at a gate output denotes logical inversion.
- If both inputs of a NOR gate are tied to one input a, its output is $\overline{a + a} = \overline{a}$. Boolean addition obeys $a + a = a$, and De Morgan's law gives $\overline{\overline{a} + \overline{b}} = ab$.

Step 1: Reduce the two input gates The first two gate symbols have the OR shape and an output bubble, so they are NOR gates. Each has both of its inputs joined together: the upper one receives a at both terminals and the lower one receives b at both terminals. Their intermediate outputs are

therefore

$$P = \overline{a + a} = \overline{a}, \qquad Q = \overline{b + b} = \overline{b}.$$

These stages act as two separate inverters; the drawing does not combine a and b until the last gate.

Step 2: Apply the final NOR operation and check the truth table The last gate is also NOR. Substituting the intermediate outputs and applying De Morgan's law gives

$$Y = \overline{P + Q} = \overline{\overline{a} + \overline{b}} = ab.$$

For input pairs $(a, b) = (0, 0), (0, 1), (1, 0), (1, 1)$, the output sequence is $0, 0, 0, 1$. This is the AND truth table. NAND instead gives $1, 1, 1, 0$, OR gives $0, 1, 1, 1$, and NOR gives $1, 0, 0, 0$; each differs from the circuit for at least one input pair.

Step 3: Conclusion The circuit performs $Y = ab$, which is the AND operation. Hence, the correct option is (B).

Quick Tip to Solve: A NOR gate with its inputs joined acts as NOT. Reduce those gates first, then use De Morgan's law on the final gate.

---

38. A coin placed on a rotating table just slips when it is placed at a distance of 1 cm from the center. If the angular velocity of the table in halved, it will just slip when placed at a distance of _____ from the centre : [4 marks]

(A) 1 cm

(B) 8 cm

(C) 4 cm

(D) 2 cm

Correct Answer: (C) 4 cm

Solution:

Concept:

- A coin rotating without sliding needs centripetal force $m\omega^2 r$, directed toward the centre. Static friction between the coin and the horizontal table supplies this force.
- At the threshold of slipping, the required force equals limiting static friction: $m\omega^2 r = \mu_s mg$. The normal reaction is mg, and the same coin and surfaces have the same μ_s.

Step 1: Write the original limiting condition Let the original angular speed be ω and the original limiting radius be $r_1 = 1\,\text{cm}$. The vertical forces balance, while the friction force acts horizontally inward. Consequently

$$m\omega^2 r_1 = \mu_s mg, \qquad \omega^2 r_1 = \mu_s g.$$

The mass cancels. The friction coefficient is not needed numerically because it does not change when the table speed is changed.

Step 2: Compare the new radius at half the angular speed Write the new speed as $\omega_2 = \omega/2$ and the

new limiting radius as r_2. At this second slipping threshold,

$$\left(\frac{\omega}{2}\right)^2 r_2 = \mu_s g = \omega^2 r_1.$$

Thus

$$r_2 = 4r_1 = 4\,\text{cm}.$$

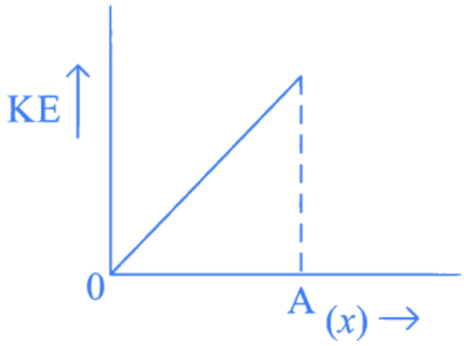
At a fixed radius, halving angular speed reduces the required centripetal force to one quarter, so the coin may be placed four times farther out before slipping. The values $1\,\text{cm}$ and $2\,\text{cm}$ do not restore the original limiting force, while $8\,\text{cm}$ would require twice the available friction.

Step 3: Conclusion The new distance from the centre is $4\,\text{cm}$. Hence, the correct option is (C).

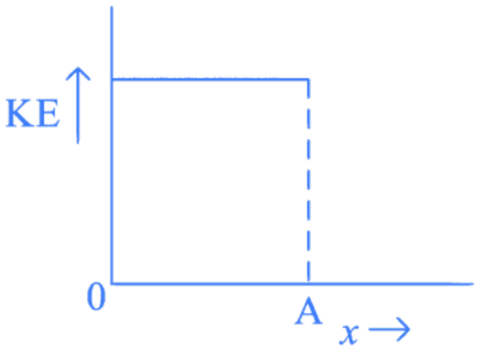
Quick Tip to Solve: For slipping on the same horizontal rotating table, $\omega^2 r$ is constant. A factor of two in angular speed becomes a factor of four in radius.

39. The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by [4 marks]

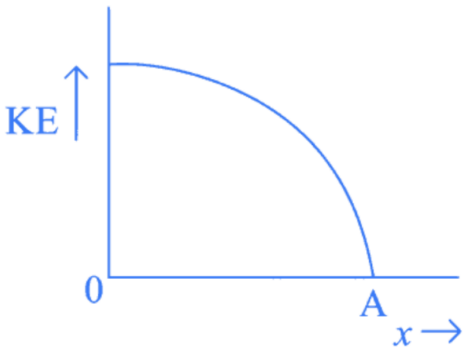
(A)



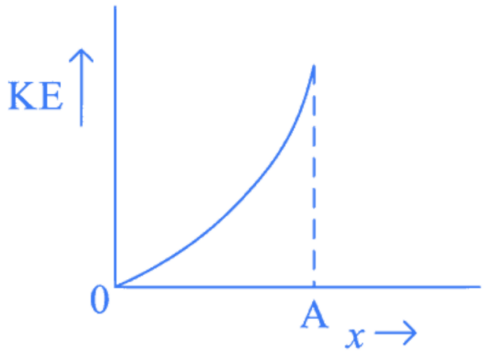
(B)



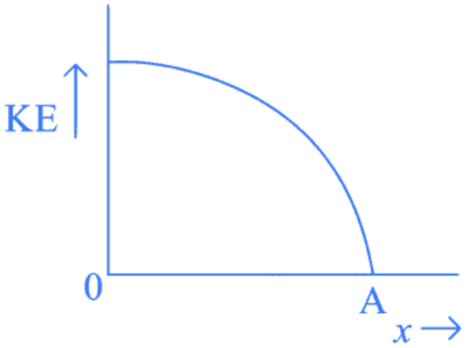
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- For a particle of mass m, angular frequency ω and amplitude A, total SHM energy is $E = \frac{1}{2}m\omega^2A^2$. Its potential energy at displacement x is $U = \frac{1}{2}m\omega^2x^2$.
- Mechanical energy conservation gives kinetic energy $K = E - U = \frac{1}{2}m\omega^2(A^2 - x^2)$. This relation is a quadratic function of displacement, not a linear function of it.

Step 1: Check the values at the two ends of the interval The motion starts at the mean position, where $x = 0$, potential energy is minimum and speed is maximum. Therefore

$$K(0) = \frac{1}{2}m\omega^2A^2.$$

At the extreme position $x = A$, the particle momentarily stops before reversing direction, so

$$K(A) = \frac{1}{2}m\omega^2(A^2 - A^2) = 0.$$

The required graph must start with positive kinetic energy and end at zero. This immediately rejects the increasing graphs (A) and (D), as well as the constant positive graph (B).

Step 2: Determine the shape between the endpoints The complete relation may be expressed as

$$\frac{K}{E} = 1 - \frac{x^2}{A^2}, \qquad 0 \le x \le A.$$

Its slope and curvature are

$$\frac{dK}{dx} = -m\omega^2 x, \qquad \frac{d^2K}{dx^2} = -m\omega^2 < 0.$$

Thus the curve begins with a horizontal tangent and falls with increasingly negative slope. It is the first-quadrant segment of a downward-opening parabola, matching the decreasing curved graph in (C).

Step 3: Conclusion Kinetic energy decreases parabolically from its maximum at $x = 0$ to zero at $x = A$, as shown in graph (C). Hence, the correct option is (C).

Quick Tip to Solve: For any SHM energy graph, test the mean and extreme positions first. Then use $K/E = 1 - (x/A)^2$ to distinguish a parabola from a straight line.

---

40. A parallel plate capacitor of capacitance $2\ \mathrm{F}$ is charged to a potential V, The energy stored in the capacitor is E_1. The capacitor is now connected to another uncharged identical capacitor in parallel combination. The energy stored in the combination is E_2. The ratio $\mathrm{E}_2/\mathrm{E}_1$ is : [4 marks]

(A) 1 : 2

(B) 2 : 3

(C) 2 : 1

(D) 1 : 4

Correct Answer: (A) 1 : 2

Solution:

Concept:

- A capacitor of capacitance C at voltage V stores charge $Q = CV$ and energy $U = \frac{1}{2}CV^2$.
- When an isolated charged capacitor is connected in parallel to an identical uncharged capacitor, total charge is conserved. Their final voltages are equal, but their total electrostatic energy need not be conserved during the transient.

Step 1: Find the common voltage after connection Initially, only the first capacitor is charged, so the total charge available is $Q_{\text{total}} = CV$, with $C = 2\,\mathrm{F}$. Two identical capacitors in parallel have total capacitance $C_{\text{eq}} = 2C$. If the final common voltage is V_f, charge conservation requires

$$2CV_f = CV, \qquad V_f = \frac{V}{2}.$$

Each capacitor finally receives half the original charge. The original capacitance value cancels from this voltage ratio.

Step 2: Calculate the total final energy and compare The original stored energy and the energy of the final two-capacitor combination are

$$E_1 = \frac{1}{2}CV^2,$$

$$E_2 = 2\left[\frac{1}{2}C\left(\frac{V}{2}\right)^2\right] = \frac{1}{4}CV^2.$$

Consequently

$$\frac{E_2}{E_1} = \frac{CV^2/4}{CV^2/2} = \frac{1}{2}.$$

Option (D), $1:4$, is the energy ratio for just one final capacitor, not for the entire combination. Options (B) and (C) do not follow from the conserved charge; (C) would even imply an increase in stored energy without an external source. The missing energy is released during charge redistribution through transient heating and electromagnetic radiation.

Step 3: Conclusion For the complete parallel combination, $E_2 : E_1 = 1:2$. Hence, the correct option is (A).

Quick Tip to Solve: Charge is conserved in capacitor sharing, but electrostatic energy is not. After finding the common voltage, include the energy of both final capacitors.

41. The electric field in an electromagnetic wave is given as

$$\overrightarrow{\mathrm{E}} = 20\sin\omega\left(\mathrm{t} - \frac{x}{\mathrm{c}}\right)\overrightarrow{\mathrm{j}}\,\mathrm{NC}^{-1}$$

where ω and c are angular frequency and velocity of electromagnetic wave respectively. The energy contained in a volume of $5 \times 10^{-4}\ \mathrm{m}^3$ will be

(Given $\varepsilon_0 = 8.85 \times 10^{-12}\mathrm{C}^2/\mathrm{Nm}^2$) [4 marks]

(A) $17 \cdot 7 \times 10^{-13}$ J

(B) $28 \cdot 5 \times 10^{-13}$ J

(C) $8 \cdot 85 \times 10^{-13}$ J

(D) $88 \cdot 5 \times 10^{-13}$ J

Correct Answer: (C) $8 \cdot 85 \times 10^{-13}$ J

Solution:

Concept:

- In a plane electromagnetic wave, electric and magnetic fields contribute equally to energy density. The instantaneous total density is $u = \varepsilon_0 E^2$.
- For a sinusoidal wave of electric amplitude E_0, $\langle E^2\rangle = E_0^2/2$. Therefore its cycle-averaged total energy density is $\langle u\rangle = \frac{1}{2}\varepsilon_0 E_0^2$, and the average energy in volume $\mathcal{V}$ is $\langle U\rangle = \langle u\rangle\mathcal{V}$.

Step 1: Identify the amplitude and the appropriate energy density The field is $\vec{E} = 20\sin[\omega(t - x/c)]\,\hat{j}\,\mathrm{N\,C^{-1}}$, so the amplitude is $E_0 = 20\,\mathrm{N\,C^{-1}}$. Since no particular phase or time is specified, the energy asked for is the usual time-averaged wave energy. Both electric and magnetic contributions are included; averaging only the electric contribution would omit half the wave energy. Thus

$$\langle u\rangle = \frac{1}{2}(8.85 \times 10^{-12})(20)^2 = 1.77 \times 10^{-9}\,\mathrm{J\,m^{-3}}.$$

Step 2: Multiply by the given volume The specified volume is $\mathcal{V} = 5 \times 10^{-4}\,\mathrm{m}^3$. Hence

$$\begin{aligned}\langle U \rangle &= (1.77 \times 10^{-9})(5 \times 10^{-4}) \\ &= 8.85 \times 10^{-13}\,\mathrm{J}.\end{aligned}$$

The units $\mathrm{J\,m^{-3}}$ times m^3 reduce to joules. Option (A) is twice this value and results from using the peak total energy density instead of the cycle average. Options (B) and (D) agree with neither the squared field amplitude nor the given volume; (D) is ten times the calculated average.

Step 3: Conclusion The energy contained in the volume, averaged over an oscillation, is $8.85 \times 10^{-13}\,\mathrm{J}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the peak field in $\langle u \rangle = \varepsilon_0 E_0^2/2$. This already includes both fields; do not double it again or replace the amplitude by its square incorrectly.

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42. An average force of $125\ \mathrm{N}$ is applied on a machine gun firing bullets each of mass $10\ \mathrm{g}$ at the speed of $250\ \mathrm{m/s}$ to keep it in position. The number of bullets fired per second by the machine gun is : [4 marks]

(A) 25

(B) 50

(C) 5

(D) 100

Correct Answer: (B) 50

Solution:

Concept:

- Average force equals the rate of change of momentum: $F = \Delta p/\Delta t$. A bullet initially at rest gains momentum mv when fired at speed v.
- If n identical bullets are fired per second, their forward momentum gain per second is nmv. The gun receives an equal backward momentum change, so the external holding force has magnitude $F = nmv$.

Step 1: Find the momentum supplied to one bullet Convert the bullet mass to SI units before multiplying by its speed:

$$m = 10\,\mathrm{g} = 10 \times 10^{-3}\,\mathrm{kg} = 0.010\,\mathrm{kg}.$$

The momentum of one outgoing bullet is therefore

$$p = mv = (0.010)(250) = 2.50\,\mathrm{kg\,m\,s^{-1}}.$$

The recoil impulse on the gun has the same magnitude and the opposite direction. Because the gun is kept in position, the applied force must balance the cumulative recoil averaged over time.

Step 2: Use the required holding force to obtain the firing rate Let n denote the number of bullets fired each second. In one second their total forward momentum is $n(2.50)\,\mathrm{kg\,m\,s^{-1}}$. Equating its rate

to the given force yields

$$125 = n(2.50), \qquad n = \frac{125}{2.50} = 50\,\mathrm{s}^{-1}.$$

Checking the alternatives directly, rates 25, 5 and $100\,\mathrm{s}^{-1}$ require forces 62.5, 12.5 and 250 N, respectively. Only $50\,\mathrm{s}^{-1}$ produces the specified 125 N. The calculation uses momentum per second, rather than the kinetic energy carried by the bullets.

Step 3: Conclusion The machine gun fires 50 bullets per second. Hence, the correct option is (B).

Quick Tip to Solve: For steady recoil questions, divide the holding force by the momentum of one bullet: $n = F/(mv)$. Always convert grams to kilograms first.

43. A metallic surface is illuminated with radiation of wavelength λ, the stopping potential is V_0. If the same surface is illuminated with radiation of wavelength 2λ, the stopping potential becomes $\frac{V_0}{4}$. The threshold wavelength for this metallic surface will be [4 marks]

(A) 3λ

(B) 4λ

(C) $\frac{3}{2}\lambda$

(D) $\frac{\lambda}{4}$

Correct Answer: (A) 3λ

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = hc/\lambda - \phi$, where V_s is stopping potential, ϕ is work function and e is the elementary charge; h is Planck's constant and c is the speed of light.
- The work function belongs to the metallic surface and remains unchanged when the incident wavelength changes. At threshold, emitted electrons have zero maximum kinetic energy, so $\phi = hc/\lambda_0$.

Step 1: Write the two energy equations For the first illumination, the incident photon energy is hc/λ, and the maximum kinetic energy is eV_0. For the second illumination, doubling the wavelength halves the photon energy. Hence

$$eV_0 = \frac{hc}{\lambda} - \phi, \qquad \frac{eV_0}{4} = \frac{hc}{2\lambda} - \phi.$$

Step 2: Eliminate the stopping potential Multiply the second equation by four and equate the two expressions for eV_0:

$$\frac{hc}{\lambda} - \phi = \frac{2hc}{\lambda} - 4\phi.$$

Therefore $3\phi = hc/\lambda$, giving $\phi = hc/(3\lambda)$. This positive work function is consistent with emission at both incident wavelengths.

Step 3: Convert the work function into threshold wavelength Using $hc/\lambda_0 = hc/(3\lambda)$, cancel the common factor hc to obtain $\lambda_0 = 3\lambda$. As a check, this makes the two kinetic energies $2hc/(3\lambda)$ and $hc/(6\lambda)$, whose ratio is indeed four. The proposed 4λ gives a different potential ratio; $3\lambda/2$ and $\lambda/4$

would not even allow emission at incident wavelength 2λ.

Step 4: Conclusion The threshold wavelength is $\boxed{3\lambda}$. Hence, the correct option is (A).

Quick Tip to Solve: Write both photoelectric equations with the same work function; doubling wavelength halves photon energy, not the stopping potential.

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44. The free space inside a current carrying toroid is filled with a material of susceptibility 2×10^{-2}. The percentage increase in the value of magnetic field inside the toroid will be [4 marks]

(A) 0.1%

(B) 1%

(C) 2%

(D) 0.2%

Correct Answer: (C) 2%

Solution:

Concept:

- Magnetic susceptibility χ_m relates magnetisation to magnetising field: $M = \chi_m H$. For a linear material, $B = \mu_0(H + M) = \mu_0(1 + \chi_m)H$.
- For a toroid with fixed turns, geometry and current, $H = NI/(2\pi r)$ remains the same. The relative permeability is $\mu_r = 1 + \chi_m$.

Step 1: Express the field before and after filling Let B_0 be the magnetic field when the toroid contains free space. At a point inside the winding, $B_0 = \mu_0 H$. Filling the toroid changes the permeability, so the new field is

$$B = \mu_0(1 + \chi_m)H = (1 + \chi_m)B_0.$$

The positive susceptibility means that the magnetisation reinforces the original magnetic field.

Step 2: Calculate the fractional increase The increase must be compared with the original field, not with the new field:

$$\frac{B - B_0}{B_0} = \frac{(1 + \chi_m)B_0 - B_0}{B_0} = \chi_m.$$

With $\chi_m = 2 \times 10^{-2} = 0.02$, this is a fractional increase of 0.02. The factor 1.02 is the ratio of final to original field, whereas 0.02 is the increase itself.

Step 3: Convert the fraction into a percentage Therefore

$$\text{percentage increase} = 100\chi_m = 100(2 \times 10^{-2}) = 2\%.$$

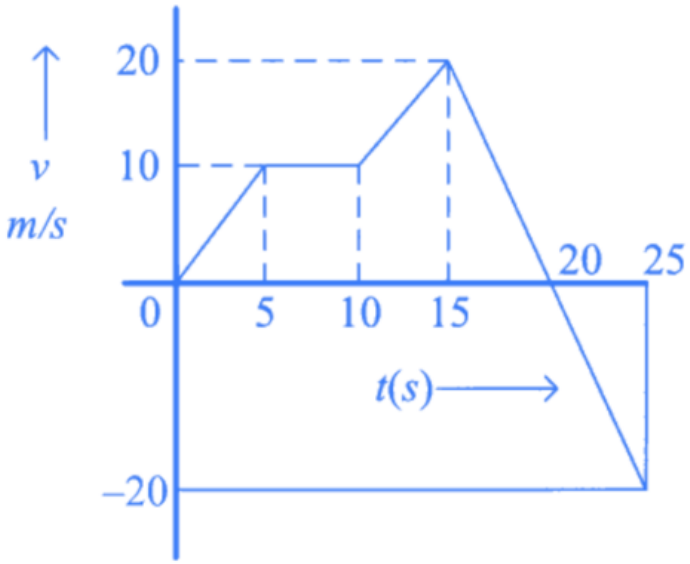
The current and the toroid dimensions cancel from the ratio, so no numerical values for them are needed. The alternatives 0.1%, 1% and 0.2% would require susceptibilities 0.001, 0.01 and 0.002, respectively, inconsistent with the given value.

Step 4: Conclusion The magnetic field increases by $\boxed{2\%}$. Hence, the correct option is (C).

Quick Tip to Solve: At fixed magnetising current, the fractional increase in magnetic field is χ_m; multiply it by 100 to obtain percent.

---

45. From the $v - t$ graph shown, the ratio of distance to displacement in 25 s of motion is: [4 marks]



(A) 1

(B) $\frac{3}{5}$

(C) $\frac{1}{2}$

(D) $\frac{5}{3}$

Correct Answer: (D) $\frac{5}{3}$

Solution:

Concept:

- Displacement is the signed area under a velocity-time graph: $\Delta x = \int v\,dt$. Areas below the time axis contribute negatively.
- Distance is $\int |v|\,dt$, so add the magnitudes of all areas. A triangular area is half base times height, and a trapezium uses the mean of its parallel sides.

Step 1: Find the area above the time axis The velocity is nonnegative from 0 to $20\,\text{s}$. Separate this region at 5, 10 and $15\,\text{s}$, where the slope changes. The four contributions are

$$A_1 = \tfrac{1}{2}(5)(10) = 25\,\text{m},$$
$$A_2 = (5)(10) = 50\,\text{m},$$
$$A_3 = \tfrac{1}{2}(10+20)(5) = 75\,\text{m},$$
$$A_4 = \tfrac{1}{2}(5)(20) = 50\,\text{m}.$$

Thus the total positive area is $A_+ = 25 + 50 + 75 + 50 = 200\,\text{m}$.

Step 2: Include the reversal of motion From 20 to $25\,\text{s}$, velocity is negative and reaches $-20\,\text{m s}^{-1}$. The triangular area has magnitude

$$A_- = \tfrac{1}{2}(25-20)(20) = 50\,\text{m}.$$

This is an additional distance travelled in the reverse direction, but it subtracts from the net displacement. Hence distance is $200 + 50 = 250\,\text{m}$, while displacement is $200 - 50 = 150\,\text{m}$.

Step 3: Form the requested ratio Both quantities have units of length, so their ratio is dimensionless:

$$\frac{\text{distance}}{\text{displacement}} = \frac{250}{150} = \frac{5}{3}.$$

A ratio of one would ignore the reverse motion; $3/5$ is the reciprocal of the requested ratio. The option $1/2$, like $3/5$, is also below one and is impossible here because the positive net displacement cannot exceed the distance.

Step 4: Conclusion The ratio of distance to displacement is $\boxed{5/3}$. Hence, the correct option is (D).

Quick Tip to Solve: Split the graph at every slope change and zero crossing. Add absolute areas for distance and signed areas for displacement.

46. Two identical heater filaments are connected first in parallel and then in series. At the same applied voltage, the ratio of heat produced in same time for parallel to series will be: [4 marks]

(A) 4 : 1

(B) 1 : 4

(C) 2 : 1

(D) 1 : 2

Correct Answer: (A) 4 : 1

Solution:

Concept:

- At a fixed applied voltage V, the heat produced in time t by an equivalent resistance R_{eq} is $H = V^2 t/R_{\text{eq}}$.
- Two identical resistances R have equivalent resistance $R/2$ in parallel and $2R$ in series. The total heat is compared for the complete pair in each connection.

Step 1: Calculate the heat in the parallel connection Let the resistance of each heater filament be R. In parallel each filament receives the full applied voltage, and the two branches together have

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}, \qquad R_p = \frac{R}{2}.$$

The total heat released by the pair during time t is therefore

$$H_p = \frac{V^2 t}{R/2} = \frac{2V^2 t}{R}.$$

Step 2: Calculate the heat in the series connection In series the same current passes through both

filaments, and their resistances add:

$$R_s = R + R = 2R, \qquad H_s = \frac{V^2 t}{2R}.$$

The applied voltage across the complete pair remains V; each identical filament now receives $V/2$. This reduced voltage is why series operation produces substantially less heat than parallel operation.

Step 3: Compare the total heat outputs Cancel the common voltage, time and filament-resistance factors:

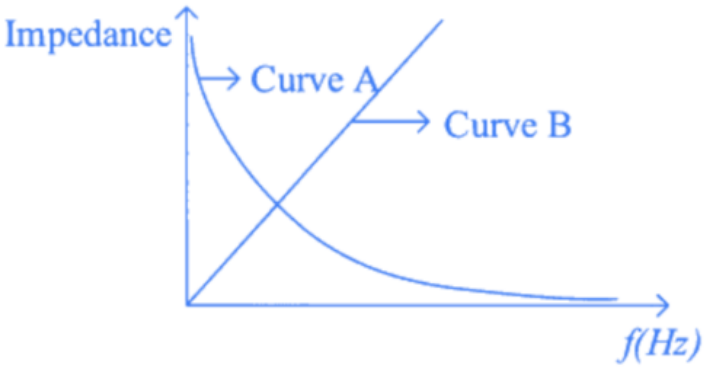
$$\frac{H_p}{H_s} = \frac{2V^2 t/R}{V^2 t/(2R)} = 4.$$

Thus the parallel connection releases four times as much heat in the stated time. The option $1:4$ reverses the required order. Ratios $2:1$ and $1:2$ do not account for the full factor of four between the equivalent resistances. Using I^2Rt with the same current in both arrangements would be incorrect, because the supply holds voltage constant.

Step 4: Conclusion The heat ratio, parallel to series, is $\boxed{4:1}$. Hence, the correct option is (A).

Quick Tip to Solve: For the same supply voltage and heating time, heat varies inversely with equivalent resistance; compare $2R$ with $R/2$.

47. As per the given graph, choose the correct representation for curve A and curve B.
Where X_C = reactance of pure capacitive circuit connected with A.C. source
X_L = reactance of pure inductive circuit connected with A.C. source
R = impedance of pure resistive circuit connected with A.C. source.
Z = Impedance of the LCR series circuit } [4 marks]



(A) $A = X_L, B = R$
(B) $A = X_L,\ B = Z$
(C) $A = X_C, B = X_L$
(D) $A = X_C, B = R$
Correct Answer: (C) $A = X_C, B = X_L$

Solution:

Concept:

- For an ideal capacitor, $X_C = 1/(\omega C) = 1/(2\pi f C)$; its reactance falls inversely with frequency f.
- For an ideal inductor, $X_L = \omega L = 2\pi f L$, a straight-line increase through the origin. An ideal resistor has frequency-independent impedance R.

Step 1: Identify the decreasing curve Curve A begins at high impedance for small frequency and approaches zero as the frequency increases. This is the characteristic inverse dependence

$$X_C(f) = \frac{1}{2\pi f C}.$$

In the low-frequency limit a capacitor strongly opposes current; increasing frequency makes its reactance smaller. Thus curve A represents capacitive reactance, not inductive reactance.

Step 2: Identify the straight rising curve Curve B is a straight line starting at the origin. Its impedance is proportional to frequency, matching

$$X_L(f) = 2\pi f L.$$

For fixed inductance L, the slope is the constant $2\pi L$. A resistor would instead give a horizontal line at height R, so it cannot represent curve B. The shapes, rather than the point where the curves intersect, identify the elements.

Step 3: Check all the proposed assignments Options (A) and (B) incorrectly label the decreasing curve as X_L, which must increase with frequency. Option (D) correctly identifies curve A but labels the rising line as resistance, contradicting the constant impedance of a pure resistor. For comparison, the impedance of a series LCR circuit is

$$Z = \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2},$$

which has a minimum at resonance and is not the straight line B. Only option (C) assigns both curves their correct frequency dependences.

Step 4: Conclusion Curve A is X_C and curve B is X_L. Hence, the correct option is (C).

Quick Tip to Solve: On an impedance-versus-frequency plot, a falling hyperbola signals a capacitor, a rising straight line an inductor, and a horizontal line a resistor.

48. The current sensitivity of moving coil galvanometer is increased by 25%. This increase is achieved only by changing in the number of turns of coils and area of cross section of the wire while keeping the resistance of galvanometer coil constant. The percentage change in the voltage sensitivity will be: [4 marks]

(A) $+25\%$

(B) -50%

(C) -25%

(D) Zero

Correct Answer: (A) $+25\%$

Solution:

Concept:

- For a moving-coil galvanometer, current sensitivity is $S_I = \theta/I = NBA/k$, where k is the torsional constant and A is the coil area.
- Voltage sensitivity is $S_V = \theta/V$. Since $V = IR_g$ for coil resistance R_g, the two sensitivities satisfy $S_V = S_I/R_g$.

Step 1: Relate voltage and current sensitivities The magnetic torque $NBIA$ is balanced by restoring torque $k\theta$, giving $\theta = (NBA/k)I$. When the applied quantity is voltage, substitute $I = V/R_g$:

$$\theta = \frac{NBA}{kR_g}V, \qquad S_V = \frac{\theta}{V} = \frac{S_I}{R_g}.$$

The cross-sectional area of the wire influences resistance, but the problem explicitly states that the final coil resistance is kept constant.

Step 2: Apply the stated change at constant resistance An increase of 25% means the new current sensitivity is $S_I' = 1.25S_I$. As $R_g' = R_g$,

$$\frac{S_V'}{S_V} = \frac{S_I'/R_g'}{S_I/R_g} = \frac{1.25S_I}{S_I} = 1.25.$$

Thus the voltage sensitivity changes by the same multiplicative factor as the current sensitivity. No separate values of turn count or wire dimensions are needed once the resistance constraint is imposed.

Step 3: Express and check the percentage change The percentage change is measured relative to the old voltage sensitivity:

$$\frac{S_V' - S_V}{S_V} \times 100 = (1.25 - 1)100 = +25\%.$$

A decrease of 50% or 25% contradicts proportionality at fixed resistance. Zero change would require the resistance to rise by the same factor as current sensitivity, whereas the question specifically excludes such a change.

Step 4: Conclusion The voltage sensitivity increases by $\boxed{25\%}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $S_V = S_I/R_g$ before tracking coil changes. If resistance is fixed, current and voltage sensitivities have identical percentage changes.

49. Two radioactive elements A and B initially have same number of atoms. The half life of A is same as the average life of B. If λ_A and λ_B are decay constants of A and B respectively, then choose the correct relation from the given options. [4 marks]

(A) $\lambda_A = \lambda_B \ln 2$

(B) $\lambda_A \ln 2 = \lambda_B$

(C) $\lambda_A = 2\lambda_B$

(D) $\lambda_A = \lambda_B$

Correct Answer: (A) $\lambda_A = \lambda_B \ln 2$

Solution:

Concept:

- Radioactive decay follows $N = N_0 e^{-\lambda t}$, where λ is the decay constant with dimensions of inverse time.
- The half-life is $T_{1/2} = \ln 2/\lambda$, whereas mean or average life is $\tau = 1/\lambda$. These are different measures of the same exponential decay law.

Step 1: Write the two lifetimes with their own constants For element A, the half-life is the time when $N_A = N_{0A}/2$. Applying the exponential law gives

$$\frac{1}{2} = e^{-\lambda_A T_{1/2,A}} \quad \Rightarrow \quad T_{1/2,A} = \frac{\ln 2}{\lambda_A}.$$

For element B, the average life is $\tau_B = 1/\lambda_B$. The identical initial atom numbers do not change either lifetime because both quantities depend on the decay constant alone.

Step 2: Impose the equality given in the question The half-life of A equals the average life of B, so

$$\frac{\ln 2}{\lambda_A} = \frac{1}{\lambda_B}.$$

Multiplying by the positive product $\lambda_A \lambda_B$ gives

$$\lambda_B \ln 2 = \lambda_A.$$

Keep the factor $\ln 2$ with λ_B after cross multiplication; moving it to the wrong side produces the reciprocal relation.

Step 3: Check the magnitude and eliminate alternatives Since $\ln 2 \approx 0.693 < 1$, the result requires $\lambda_A < \lambda_B$. Option (B) would instead imply $\lambda_A = \lambda_B/\ln 2 > \lambda_B$; option (C) also gives a larger λ_A. Option (D) assumes equal decay constants, which would make A's half-life smaller than B's mean life by the factor $\ln 2$, rather than equal. Thus only the relation in option (A) satisfies the two specified lifetime definitions.

Step 4: Conclusion The decay constants satisfy $\boxed{\lambda_A = \lambda_B \ln 2}$. Hence, the correct option is (A).

Quick Tip to Solve: Write $T_{1/2} = \ln 2/\lambda$ and $\tau = 1/\lambda$ separately with species subscripts before equating lifetimes.

50. A transmitting antenna is kept on the surface of the earth. The minimum height of receiving antenna required to receive the signal in line of sight at 4 km distance from it is $x \times 10^{-2}$ m. The value of x is _____.

(Let, radius of earth $\mathrm{R} = 6400$ km) [4 marks]

(A) 1.25

(B) 125

(C) 1250

(D) 12.5

Correct Answer: (B) 125

Solution:

Concept:

- Line-of-sight communication is limited by the curvature of Earth. A ray just touching the surface is tangent to the spherical Earth at the horizon.
- For an antenna of height $h \ll R$, its horizon distance is $d \simeq \sqrt{2Rh}$. With the transmitter at ground level, only the receiving antenna contributes a horizon distance.

Step 1: Identify the limiting geometry At the minimum receiving height, the straight signal path from the ground transmitter just grazes Earth. The radius at the transmitting point is perpendicular to that tangent. Thus the right triangle gives $(R+h)^2 = R^2 + d^2$. Since the required height is tiny compared with Earth's radius, h^2 can be neglected, leaving $d^2 = 2Rh$. A transmitting height of zero contributes no extra horizon range.

Step 2: Calculate the receiving height Use consistent SI units: $R = 6400\,\mathrm{km} = 6.4 \times 10^6\,\mathrm{m}$ and $d = 4\,\mathrm{km} = 4.0 \times 10^3\,\mathrm{m}$. Therefore

$$h = \frac{d^2}{2R} = \frac{(4.0 \times 10^3)^2}{2(6.4 \times 10^6)} = 1.25\,\mathrm{m}.$$

This small height verifies the approximation $h \ll R$.

Step 3: Extract the requested coefficient The question asks for x, not the height in metres. Comparing $h = 1.25\,\mathrm{m}$ with $h = x \times 10^{-2}\,\mathrm{m}$, we obtain $x = 1.25 \times 10^2 = 125$. Option (A) is the height itself rather than its coefficient. Options (C) and (D) would give heights of $12.5\,\mathrm{m}$ and $0.125\,\mathrm{m}$, neither of which matches the limiting distance.

Step 4: Conclusion The minimum receiving height is $1.25\,\mathrm{m} = 125 \times 10^{-2}\,\mathrm{m}$, so $\boxed{x = 125}$. Hence, the correct option is (B).

Quick Tip to Solve: Calculate the height with both distances in the same unit, then separately convert it into the coefficient multiplying the printed power of ten.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The equation of wave is given by

$$\mathrm{Y} = 10^{-2} \sin 2\pi(160t - 0.5x + \pi/4)$$

where x and Y are in m and t in s. The speed of the wave is _____ $\mathrm{km\ h^{-1}}$. [4 marks]

Solution:

Concept:

- A travelling sinusoidal wave has the form $Y = A\sin(\omega t - kx + \phi)$; its phase speed is $v = \omega/k$. The amplitude and constant phase do not affect its speed.
- When the phase is written as $2\pi(ft - x/\lambda + \text{constant})$, the time coefficient is the frequency f and the magnitude of the position coefficient is $1/\lambda$.

Step 1: Identify the temporal and spatial frequencies Compare $Y = 10^{-2}\sin[2\pi(160t - 0.5x + \pi/4)]$ with the standard travelling-wave form. The frequency is $f = 160\,\mathrm{Hz}$, while $1/\lambda = 0.5\,\mathrm{m^{-1}}$, so $\lambda = 2\,\mathrm{m}$. Equivalently, $\omega = 2\pi(160)\,\mathrm{rad\,s^{-1}}$ and $k = 2\pi(0.5)\,\mathrm{rad\,m^{-1}}$. The outer factor 2π multiplies both coefficients.

Step 2: Find the phase speed in SI units For a point of constant phase, $160t - 0.5x + \pi/4$ remains constant. Differentiating gives $160 - 0.5\,dx/dt = 0$, hence the phase travels along positive x with speed

$$v = \frac{160}{0.5} = 320\,\mathrm{m\,s^{-1}}.$$

The same value follows from $f\lambda = 160 \times 2$. Neither the amplitude $10^{-2}\,\mathrm{m}$ nor the fixed phase term enters this ratio.

Step 3: Convert to the requested unit One metre per second is $3.6\,\mathrm{km\,h^{-1}}$. Therefore

$$v = 320 \times 3.6 = 1152\,\mathrm{km\,h^{-1}}.$$

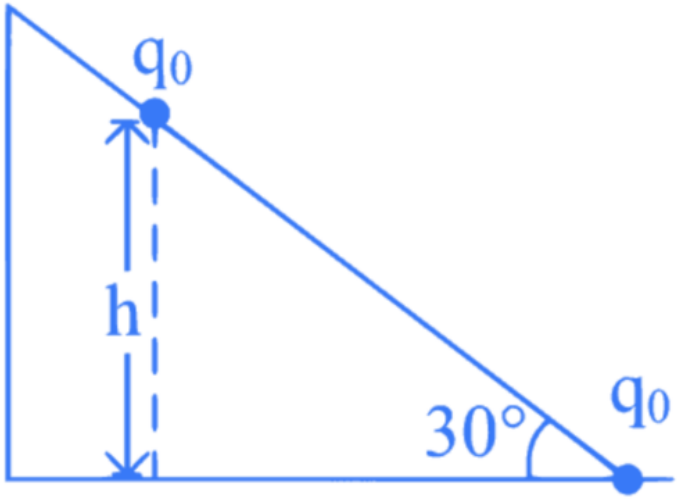
Multiplying by 3.6, rather than dividing, is necessary because there are 3600 seconds in an hour and 1000 metres in a kilometre.

Step 4: Conclusion The wave speed is $\boxed{1152\,\mathrm{km\,h^{-1}}}$; the numerical entry is 1152.

Quick Tip to Solve: Take the ratio of the coefficients of time and position inside the same phase bracket; a common factor such as 2π cancels.

52. As shown in the figure, a configuration of two equal point charges $(q_0 = +2\mu\text{C})$ is placed on an inclined plane. Mass of each point charge is 20 g. Assume that there is no friction between charge and plane. For the system of two point charges to be in equilibrium (at rest) the height $\text{h} = x \times 10^{-3}$ m. The value of x is _____.

(Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^{2}C^{-2}, $g = 10$ m s^{-2}) [4 marks]



Solution:

Concept:

- Like point charges repel along their joining line with magnitude $F_e = kq_0^2/r^2$, where r is their separation and $k = 1/(4\pi\varepsilon_0)$.
- On a smooth incline of angle θ, the component of weight down the incline is $mg\sin\theta$. The normal reaction balances the perpendicular component and supplies no tangential force.

Step 1: Balance forces along the incline The upper charge is repelled up the plane by the lower positive charge. Both charges lie along the incline, so the full Coulomb force acts along the slope. For the upper charge to remain at rest, this upward force must equal the component of its weight down the plane:

$$\frac{kq_0^2}{r^2} = mg\sin 30^\circ.$$

The lower charge is at the foot of the incline as shown; the limiting separation is obtained from equilibrium of the upper one. The required height is vertical, not the distance along the slope.

Step 2: Calculate the charge separation Convert the given quantities: $q_0 = 2 \times 10^{-6}\,\text{C}$, $m = 20\,\text{g} = 0.020\,\text{kg}$, and $\sin 30^\circ = 1/2$. Thus

$$r^2 = \frac{(9 \times 10^9)(2 \times 10^{-6})^2}{(0.020)(10)(1/2)} = \frac{0.036}{0.10} = 0.36\,\text{m}^2.$$

Since separation is positive, $r = 0.60\,\text{m}$.

Step 3: Convert the separation into vertical height From the right triangle in the figure, $h = r\sin 30^\circ$.

Hence

$$h = 0.60 \times \frac{1}{2} = 0.30\,\text{m} = 300 \times 10^{-3}\,\text{m}.$$

Comparing with the stated form $h = x \times 10^{-3}\,\text{m}$ gives the required coefficient. Using r directly as h would miss the geometrical factor of one half.

Step 4: Conclusion The equilibrium height is $\boxed{h = 0.300\,\text{m}}$, giving $\boxed{x = 300}$.

Quick Tip to Solve: First balance forces parallel to the incline to find the charge separation; only then project that separation vertically to obtain h.

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53. A projectile fired at 30° to the ground is observed to be at same height at time 3 s and 5 s after projection, during its flight. The speed of projection of the projectile is _____ m s^{-1}.
(Given $g = 10\ \text{ms}^{-2}$) [4 marks]

Solution:

Concept:

- The vertical displacement of a projectile is $y = u_y t - \frac{1}{2}gt^2$, with $u_y = u\sin\theta$, when air resistance is neglected.
- Two distinct times at the same height are symmetric about the instant of maximum height. Their sum is $2u_y/g$, independent of that common height.

Step 1: Equate the heights at the two stated times Write the vertical displacement from the launch level at each time. At $t_1 = 3\,\text{s}$, $y_1 = 3u_y - \frac{1}{2}g(3)^2$, and at $t_2 = 5\,\text{s}$, $y_2 = 5u_y - \frac{1}{2}g(5)^2$. The heights are equal even though the projectile is ascending at the earlier time and descending at the later time. Thus

$$u_y(t_2 - t_1) = \frac{g}{2}(t_2^2 - t_1^2).$$

Step 2: Find the initial vertical component Factor $t_2^2 - t_1^2 = (t_2 - t_1)(t_2 + t_1)$. Since the times are distinct, cancel $t_2 - t_1$, obtaining

$$u_y = \frac{g}{2}(t_1 + t_2) = \frac{10}{2}(3 + 5) = 40\,\text{m s}^{-1}.$$

The highest point occurs at $u_y/g = 4\,\text{s}$, precisely midway between 3 s and 5 s, which checks the time symmetry.

Step 3: Recover the full launch speed The launch angle is 30°, so $u_y = u\sin 30^\circ = u/2$. Therefore

$$u = \frac{40}{1/2} = 80\,\text{m s}^{-1}.$$

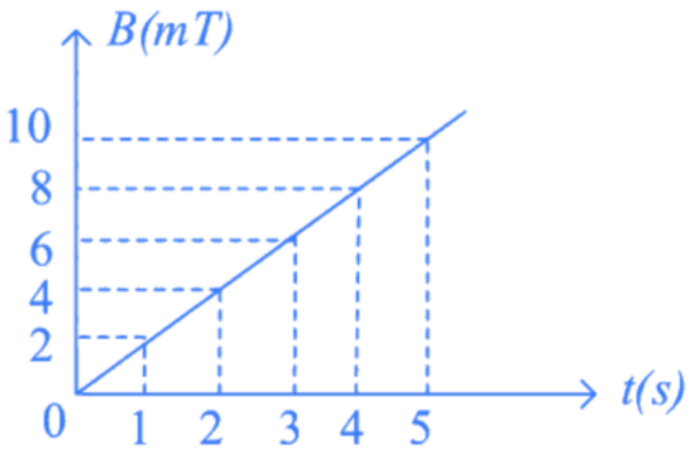
As a further check, the common height is $40(3) - 5(9) = 75\,\text{m}$ and also $40(5) - 5(25) = 75\,\text{m}$. The total flight time is $2u_y/g = 8\,\text{s}$, so both observation times lie within the flight.

Step 4: Conclusion The speed of projection is $\boxed{80\,\text{m s}^{-1}}$; the numerical entry is 80.

Quick Tip to Solve: For two observations at equal height, average their times to locate the highest point; then use $u\sin\theta = g\,t_{\text{top}}$.

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54. The magnetic field B crossing normally a square metallic plate of area $4\ \mathrm{m}^2$ is changing with time as shown in figure. The magnitude of induced emf in the plate during $\mathrm{t} = 2s$ to $\mathrm{t} = 4s$, is _____ mV. [4 marks]



Solution:

Concept:

- Faraday's law gives the induced emf around a conducting boundary as $\varepsilon = -d\Phi_B/dt$. The minus sign expresses Lenz's law; only the magnitude is required here.
- For a uniform magnetic field normal to a stationary plate, the magnetic flux through its boundary is $\Phi_B = BA$, so $|\varepsilon| = A|dB/dt|$.

Step 1: Read the magnetic field at the stated times The vertical axis is labelled B in millitesla and the horizontal axis is time in seconds. From the straight-line graph, $B(2\,\mathrm{s}) = 4\,\mathrm{mT}$ and $B(4\,\mathrm{s}) = 8\,\mathrm{mT}$. Therefore the increase over the required interval is $\Delta B = 4\,\mathrm{mT}$ in $\Delta t = 2\,\mathrm{s}$. The graph is linear throughout this interval, so its average slope is also its instantaneous slope.

Step 2: Calculate the rate of change of flux Convert the field change before using SI units:

$$\frac{dB}{dt} = \frac{(8-4)\times 10^{-3}}{4-2} = 2\times 10^{-3}\,\mathrm{T\,s^{-1}}.$$

The plate area is fixed at $A = 4\,\mathrm{m}^2$, and the field crosses it normally, so the flux factor is $\cos 0^\circ = 1$. Hence

$$\left|\frac{d\Phi_B}{dt}\right| = 4(2\times 10^{-3}) = 8\times 10^{-3}\,\mathrm{Wb\,s^{-1}}.$$

Step 3: Express the emf in millivolts One weber per second equals one volt. Thus the magnitude of the induced emf is $8\times 10^{-3}\,\mathrm{V} = 8\,\mathrm{mV}$. The square shape does not require multiplying by the perimeter: magnetic flux depends on the enclosed area. There is a single plate boundary, so no additional number-of-turns factor is introduced. Lenz's law determines circulation direction but does not change the requested positive magnitude.

Step 4: Conclusion The magnitude of the induced emf from 2 s to 4 s is $\boxed{8\,\mathrm{mV}}$; the numerical entry is 8.

Quick Tip to Solve: For a straight B-versus-t graph, multiply its slope by the area; keep track of the milli prefix on both field and final voltage.

55. A force $\vec{F} = (2 + 3x)\hat{i}$ acts on a particle in the x direction where F is in newton and x is in meter. The work done by this force during a displacement from $x = 0$ to $x = 4$ m, is _____ J. [4 marks]

Solution:

Concept:

- Work is the line integral $W = \int \vec{F} \cdot d\vec{r}$. For force and displacement along the x-axis, this becomes $W = \int_{x_i}^{x_f} F_x(x)\, dx$.
- The force here varies linearly with position, so either direct integration or the average of its endpoint values gives the exact work. Force must not be treated as constant at one endpoint.

Step 1: Set up the definite integral The particle moves from $x_i = 0$ to $x_f = 4\,\text{m}$, and the force is $\vec{F} = (2 + 3x)\hat{i}$. Since $d\vec{r} = dx\,\hat{i}$, the dot product is $(2 + 3x)dx$. The force remains positive over the interval, so it acts in the direction of displacement and does positive work. The required expression is

$$W = \int_0^4 (2 + 3x)\, dx.$$

The coefficients carry the units needed to make the force a value in newtons.

Step 2: Integrate and evaluate the limits Integrating the constant and position-dependent contributions separately gives

$$W = \left[2x + \frac{3}{2}x^2\right]_0^4.$$

At the upper limit the two terms are $2(4) = 8$ and $\frac{3}{2}(4)^2 = 24$; at the lower limit both vanish. Hence

$$W = 8 + 24 = 32\,\text{J}.$$

The unit follows from newtons times metres, which is a joule.

Step 3: Check using the average force The endpoint forces are $F(0) = 2\,\text{N}$ and $F(4) = 14\,\text{N}$. Because the variation is linear, their arithmetic mean is the exact mean force: $F_{\text{av}} = (2 + 14)/2 = 8\,\text{N}$. Multiplying by the displacement gives $8 \times 4 = 32\,\text{J}$, agreeing with the integral. Using the final force alone would overestimate the work, while using the initial force alone would underestimate it.

Step 4: Conclusion The work done by the force is $\boxed{32\,\text{J}}$; the numerical entry is 32.

Quick Tip to Solve: For a force linear in displacement, the area under the force graph is a trapezium: use the average endpoint force times the displacement.

56. A solid sphere of mass $500\ \mathrm{g}$ and radius $5\ \mathrm{cm}$ is rotated about one of its diameter with angular speed of $10\ \mathrm{rad\ s^{-1}}$. If the moment of inertia of the sphere about its tangent is $x \times 10^{-2}$ times its angular momentum about the diameter. Then the value of x will be _____. [4 marks]

Solution:

Concept:

- For a uniform solid sphere, the moment of inertia about a diameter is $I_d = \frac{2}{5}MR^2$. Its angular momentum about that diameter is $L_d = I_d\omega$.
- A tangent axis is parallel to a diameter and displaced from the centre by R. The parallel-axis theorem gives $I_t = I_d + MR^2 = \frac{7}{5}MR^2$.

Step 1: Calculate the diameter-axis angular momentum Convert the quantities to SI units: $M = 500\,\mathrm{g} = 0.500\,\mathrm{kg}$, $R = 5\,\mathrm{cm} = 0.050\,\mathrm{m}$, and $\omega = 10\,\mathrm{rad\,s^{-1}}$. The diameter moment of inertia is

$$I_d = \frac{2}{5}(0.500)(0.050)^2 = 5.0 \times 10^{-4}\,\mathrm{kg\,m^2}.$$

Therefore

$$L_d = I_d\omega = 5.0 \times 10^{-3}\,\mathrm{kg\,m^2\,s^{-1}}.$$

Use the diameter axis here because the stated angular momentum refers to that axis.

Step 2: Find the tangent-axis moment of inertia The shortest distance between the tangent axis and its parallel diameter is one radius. Applying the parallel-axis theorem gives

$$I_t = I_d + MR^2 = 5.0 \times 10^{-4} + (0.500)(0.050)^2 = 1.75 \times 10^{-3}\,\mathrm{kg\,m^2}.$$

Equivalently, the tangent moment is $(7/5)MR^2$. The angular speed is not used in the moment of inertia; it enters only the angular momentum.

Step 3: Compare the two numerical quantities The ratio in SI units is

$$\frac{I_t}{L_d} = \frac{1.75 \times 10^{-3}}{5.0 \times 10^{-3}} = 0.35\,\mathrm{s} = 35 \times 10^{-2}\,\mathrm{s}.$$

The same result follows directly from $I_t/L_d = [(7/5)MR^2]/[(2/5)MR^2\omega] = 7/(2\omega)$. The printed numerical comparison therefore has coefficient $x = 35$; dimensionally this ratio has the unit of time, since it compares moment of inertia with angular momentum.

Step 4: Conclusion In the stated SI numerical comparison, $I_t/L_d = 0.35\,\mathrm{s}$, so $\boxed{x = 35}$.

Quick Tip to Solve: Form the ratio before substituting mass and radius: $I_t/L_d = 7/(2\omega)$, so the common factor MR^2 cancels.

57. The length of a wire becomes l_1 and l_2 when 100 N and 120 N tensions are applied respectively. If $10\,l_2 = 11\,l_1$, the natural length of wire will be $\frac{1}{x}\,l_1$. Here the value of x is _____. [4 marks]

Solution:

Concept:

- For the same uniform wire within its elastic limit, the extension is proportional to the applied tension: $F = k(l - L)$, where L is the natural length and $k = YA/L$ is constant.
- The natural length is the length at zero tension. Applied force is proportional to extension, not to the total stretched length; the unstretched portion must be subtracted first.

Step 1: Express the two extensions using one elastic constant Let the natural length be L. For tensions of $100\,\text{N}$ and $120\,\text{N}$, Hooke's law gives

$$100 = k(l_1 - L), \qquad 120 = k(l_2 - L).$$

Taking the ratio eliminates the unknown Young modulus and cross-sectional area:

$$\frac{l_2 - L}{l_1 - L} = \frac{120}{100} = \frac{6}{5}.$$

The larger tension must produce the larger length, consistent with the given relation.

Step 2: Insert the given length ratio and find the natural length The condition $10l_2 = 11l_1$ gives $l_2 = 11l_1/10$. Cross-multiplying the extension ratio,

$$5(l_2 - L) = 6(l_1 - L) \quad \Longrightarrow \quad L = 6l_1 - 5l_2.$$

Therefore,

$$L = 6l_1 - 5\left(\frac{11}{10}l_1\right) = \frac{1}{2}l_1.$$

As a check, the extensions are $l_1/2$ and $3l_1/5$, whose ratio is $5 : 6$, exactly the ratio of the two tensions. Both are positive.

Step 3: Conclusion The natural length is $L = l_1/2$. Comparing with $L = l_1/x$, the requested dimensionless value is $\boxed{x = 2}$.

Quick Tip to Solve: Use tension ratios only for extensions. With two measured lengths, eliminate the elastic constant before substituting the length relation.

58. The radius of curvature of each surface of a convex lens having refractive index 1.8 is $20\ \text{cm}$. The lens is now immersed in a liquid of refractive index 1.5 . The ratio of power of lens in air to its power in the liquid will be $x : 1$. The value of x is _____. [4 marks]

Solution:

Concept:

- For a thin lens of index n_g surrounded by a medium of index n_m, its focal power is $P = (n_g/n_m - 1)(1/R_1 - 1/R_2)$. The refractive index must be measured relative to the surrounding medium.
- For a biconvex lens with light travelling left to right, $R_1 = +20\,\text{cm}$ and $R_2 = -20\,\text{cm}$. The curvature factor remains unchanged on immersion.

Step 1: Calculate the power of the lens in air Taking the refractive index of air as unity and converting radii to metres,

$$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{0.20} - \frac{1}{-0.20} = 10\,\text{m}^{-1}.$$

Hence the power in air is

$$P_{\text{air}} = (1.8 - 1) \times 10 = 8\,\text{D}.$$

The positive value is consistent with a converging biconvex lens whose material is optically denser than its surroundings.

Step 2: Use the relative refractive index in the liquid Inside the liquid, the relevant index is not 1.8 itself but

$$n_{\text{rel}} = \frac{1.8}{1.5} = 1.2.$$

Consequently,

$$P_{\text{liquid}} = (1.2 - 1) \times 10 = 2\,\text{D}.$$

The lens still converges, but more weakly because the index contrast has decreased. Forming the ratio cancels the geometrical factor:

$$\frac{P_{\text{air}}}{P_{\text{liquid}}} = \frac{0.8}{0.2} = 4.$$

Subtracting the two absolute indices without dividing by the medium index would give an incorrect ratio.

Step 3: Conclusion The power ratio is $P_{\text{air}} : P_{\text{liquid}} = 4 : 1$. Thus the required dimensionless value is $\boxed{x = 4}$.

Quick Tip to Solve: When the same lens is immersed, cancel its curvature factor and compare $n_g/n_m - 1$ in the two media.

59. A monochromatic light is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of six different wavelengths. The frequency of incident light is $x \times 10^{15}$ Hz. The value of x is _____.

(Given h $= 4.25 \times 10^{-15}$ eVs) [4 marks]

Solution:

Concept:

- The hydrogen energy levels are $E_n = -13.6/n^2\,\text{eV}$. Absorption from the ground state to level n requires one photon with energy $E_n - E_1 = h\nu$.
- If all transitions among levels $1, 2, \ldots, n$ occur in an excited sample, the maximum number of different emission wavelengths is $N = n(n-1)/2$, counting each pair of levels once.

Step 1: Find the highest excited level from the number of lines The sample emits six distinct wavelengths. For the standard hydrogen cascade, this means

$$\frac{n(n-1)}{2} = 6 \quad \Longrightarrow \quad n^2 - n - 12 = 0.$$

Factoring gives $(n-4)(n+3) = 0$. The physically admissible principal quantum number is $n = 4$, since a quantum number cannot be negative. The possible downward transitions are $4 \to 3, 4 \to 2, 4 \to 1, 3 \to 2, 3 \to 1$ and $2 \to 1$, giving the stated six lines.

Step 2: Calculate the energy and frequency of the absorbed photon The incident monochromatic photon excites an atom from $n = 1$ to $n = 4$, rather than supplying the energy of just one of the later cascade transitions. Its energy is

$$\Delta E = 13.6\left(1 - \frac{1}{16}\right) = 13.6\frac{15}{16} = 12.75\,\text{eV}.$$

Using the supplied Planck constant in electron-volt seconds keeps the units consistent:

$$\nu = \frac{12.75\,\text{eV}}{4.25 \times 10^{-15}\,\text{eV s}} = 3.00 \times 10^{15}\,\text{Hz}.$$

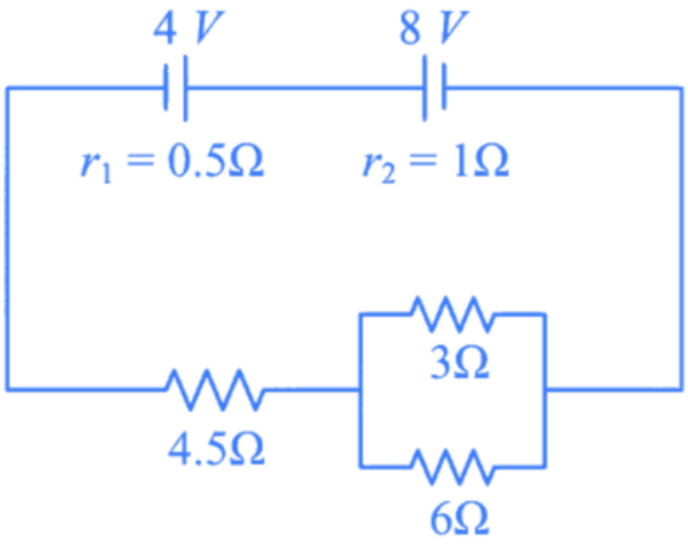
No extra conversion to joules is needed because the electron-volt unit cancels.

Step 3: Conclusion The incident frequency is $\boxed{3.00 \times 10^{15}\,\text{Hz}}$. Therefore its dimensionless coefficient is $\boxed{x = 3}$.

Quick Tip to Solve: Use the line count to identify the excited level first; then use the ground-to-excited energy difference, not a cascade energy, for the incident photon.

60. In the circuit diagram shown in figure given below, the current flowing through resistance $3\ \Omega$ is $\frac{x}{3}A$.

The value of x is _____ [4 marks]



Solution:

Concept:

- In a single loop, opposing cell emfs subtract, while all internal resistances add to the external series resistance. The long line of a cell symbol denotes its positive terminal.
- Parallel resistors share one potential difference. For resistors R_a, R_b, $R_p = R_aR_b/(R_a + R_b)$, and the current in an individual branch is its common voltage divided by its resistance.

Step 1: Reduce the parallel pair and identify the net emf The $3\,\Omega$ and $6\,\Omega$ resistors connect between the same two nodes, so

$$R_p = \frac{3 \times 6}{3 + 6} = 2\,\Omega.$$

The cell symbols show the $4\,\mathrm{V}$ and $8\,\mathrm{V}$ emfs opposing one another. Thus the larger cell drives the circuit with net emf

$$\mathcal{E}_{\text{net}} = 8 - 4 = 4\,\mathrm{V}.$$

Their internal resistances remain in series, even though their emfs oppose; resistance does not acquire a minus sign.

Step 2: Calculate the loop current and the current in the smaller branch The total resistance seen by the net emf includes the $4.5\,\Omega$ resistor, both internal resistances and the parallel equivalent:

$$R_{\text{total}} = 4.5 + 0.5 + 1 + 2 = 8\,\Omega.$$

The current delivered to the parallel combination is therefore

$$I = \frac{4}{8} = \frac{1}{2}\,\mathrm{A}.$$

Its voltage is $V_p = IR_p = (1/2) \times 2 = 1\,\text{V}$. Hence

$$I_{3\Omega} = \frac{V_p}{3} = \frac{1}{3}\,\text{A}.$$

The other branch carries $1/6\,\text{A}$, and their sum $1/3 + 1/6 = 1/2\,\text{A}$ verifies current conservation at the junction.

Step 3: Conclusion The current through the $3\,\Omega$ resistance has magnitude $\boxed{1/3\,\text{A}}$. Comparing with $x/3\,\text{A}$ gives $\boxed{x = 1}$.

Quick Tip to Solve: Check cell polarities before adding emfs. After finding the loop current, use the common parallel voltage to obtain the current through the required resistor.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Find out the correct statement from the options given below for the above 2 reactions. [4 marks]



(A) Reaction (I) is of 1^{st} order and reaction (II) is of 2^{nd} order

(B) Reaction (I) is of 2^{nd} order and reaction (II) is of 1^{st} order

(C) Reactions (I) and (II) both are of 2^{nd} order

(D) Reactions (I) and (II) both are of 1^{st} order

Correct Answer: (A) Reaction (I) is of 1^{st} order and reaction (II) is of 2^{nd} order

Solution:

Concept:

- An S_N1 reaction is controlled by unimolecular ionisation to a carbocation, so its rate is $v = k[\text{substrate}]$ and its overall order is one.
- An S_N2 reaction is a concerted nucleophilic displacement with rate $v = k[\text{substrate}][\text{nucleophile}]$; its overall order is two. Substituents affect the stability of a possible benzyl carbocation.

Step 1: Assess the methoxy-substituted benzyl chloride in reaction I In reaction (I), the para-methoxy group donates electron density into the aromatic ring by resonance. The benzyl carbocation formed by cleavage of the carbon–chlorine bond can delocalise its positive charge over that ring. Methoxy donation further stabilises this cation, lowering the barrier for ionisation.

Accordingly, this substrate favours the S_N1 route in the comparison shown. The slow step contains only the benzyl chloride molecule; nucleophilic capture follows the formation of the stabilised cation.

Its rate is first order in the substrate, rather than second order in substrate and nucleophile.

Step 2: Assess the nitro-substituted substrate and compare all choices In reaction (II), the para-nitro group withdraws electron density through its inductive and resonance effects. It destabilises a benzyl carbocation, so unimolecular ionisation is disfavoured relative to direct nucleophilic displacement at the primary benzylic carbon. The favoured substitution is therefore S_N2, with both substrate and nucleophile participating in the rate-controlling event.

Reaction (II) is second order. Option (B) reverses these two mechanistic assignments. Option (C) incorrectly assigns the methoxy-stabilised first reaction a second-order route, while option (D) incorrectly assigns the nitro-substituted second reaction a first-order route.

Step 3: Conclusion Reaction (I) is first order and reaction (II) is second order. Hence, the correct option is (A).

Quick Tip to Solve: For a benzylic substrate, ask whether the ring substituent stabilises or destabilises a benzyl carbocation before assigning the substitution order.

62. 'A' and 'B' in the above reactions are: [4 marks]



(A) $A = \mathrm{RCH(CO_2H)(CH_2)_3CHO}, \quad B = \mathrm{RCH(CO_2H)(CH_2)_3CH_3}$

(B) $A = \mathrm{RCO(CH_2)_4CO_2H}, \quad B = \mathrm{R(CH_2)_5CO_2H}$

(C) $A = \mathrm{RCH(CO_2H)(CH_2)_3CO_2H}, \quad B = \mathrm{RCH(CONHNH_2)(CH_2)_3CONHNH_2}$

(D) $A = \mathrm{RCO(CH_2)_3CHO}, \quad B = \mathrm{R(CH_2)_5CH_3}$

Correct Answer: (B) $A = \mathrm{RCO(CH_2)_4CO_2H}, \quad B = \mathrm{R(CH_2)_5CO_2H}$

Solution:

Concept:

- Vigorous oxidation of a substituted cyclic ketone by $\mathrm{KMnO_4}$ cleaves a carbon–carbon bond adjacent to its carbonyl group and opens the ring. For the shown 2-alkylcyclohexanone, the major product follows the more substituted alpha-carbon cleavage.
- $\mathrm{NH_2NH_2}$ with KOH performs Wolff–Kishner reduction: a ketone carbonyl $\mathrm{R_2C{=}O}$ becomes $\mathrm{R_2CH_2}$. A carboxylic acid group survives the reduction and is regenerated on acidic work-up.

Step 1: Identify the ring-opening oxidation product A Label the original ring carbonyl carbon C_1 and the adjacent alkyl-bearing carbon C_2. Oxidative cleavage of their connecting bond opens the six-membered ring. The original C_1 terminus gives a carboxyl group, while oxidation at C_2, which is attached to alkyl group R, gives a ketone. The four remaining ring carbons stay as four consecutive methylene groups between these termini:

$$A = \mathrm{RCO(CH_2)_4CO_2H}.$$

This is a keto acid. Strong permanganate oxidation does not stop at the aldehyde products proposed in options (A) and (D), since aldehydes are readily oxidised further.

Step 2: Reduce the ketone while retaining the carboxyl group Hydrazine forms the corresponding hydrazone at the ketone carbonyl. Under the strongly basic Wolff–Kishner conditions, loss of nitrogen replaces that carbonyl oxygen by two hydrogens. The acid group is present as carboxylate during the basic stage and returns to CO_2H on the stated H_3O^+ work-up:

$$RCO(CH_2)_4CO_2H \xrightarrow[H_3O^+]{NH_2NH_2,\,KOH} RCH_2(CH_2)_4CO_2H.$$

Thus $B = R(CH_2)_5CO_2H$. Option (C) gives hydrazide groups instead of the ketone-to-methylene reduction, while option (D) wrongly removes the carboxyl functionality altogether. Only option (B) gives both the correct oxidised intermediate and final acid.

Step 3: Conclusion $\boxed{A = RCO(CH_2)_4CO_2H, \quad B = R(CH_2)_5CO_2H}$. Hence, the correct option is (B).

Quick Tip to Solve: After ring cleavage, count all six original ring carbons. In the second stage replace only the ketone CO by CH_2, keeping CO_2H.

63. The complex that dissolves in water is : [4 marks]

(A) $Fe_4\,[Fe(CN)_6]_3$

(B) $(NH_4)_3\,[As\,(Mo_3O_{10})_4]$

(C) $K_3\,[Co\,(NO_2)_6]$

(D) $[Fe_3(OH)_2(OAc)_6]\,Cl$

Correct Answer: (D) $[Fe_3(OH)_2(OAc)_6]\,Cl$

Solution:

Concept:

- Formation of a coordination compound does not automatically make it water-soluble. The solubility depends on the particular complex salt and its lattice and hydration energies; familiar qualitative-analysis precipitates provide useful comparisons.
- The basic ferric acetate complex $[Fe_3(OH)_2(OAc)_6]Cl$, where OAc denotes acetate, forms a water-soluble complex salt. Its chloride is outside the coordination entity and acts as a counterion.

Step 1: Exclude the characteristic insoluble complex salts The first compound, $Fe_4[Fe(CN)_6]_3$, is ferric ferrocyanide, commonly called Prussian blue. It is obtained as a deeply coloured insoluble precipitate, so option (A) does not fit.

The ammonium heteropoly compound $(NH_4)_3[As(Mo_3O_{10})_4]$ is ammonium arsenomolybdate, associated with the yellow precipitate formed by arsenate and ammonium molybdate in acidic solution. It is not the water-soluble choice, excluding option (B).

Similarly, potassium hexanitritocobaltate(III), $K_3[Co(NO_2)_6]$, is the sparingly soluble yellow potassium salt used in qualitative identification of potassium ions. Its containing potassium does not make this particular complex salt freely soluble, so option (C) is excluded.

Step 2: Identify the soluble basic ferric acetate complex In option (D), the three iron centres form the basic acetate complex. Counting formal charges with iron in oxidation state $+3$, the two hydroxide ligands and six acetate ligands contribute

$$3(+3) + 2(-1) + 6(-1) = +1.$$

The coordination entity is therefore a complex cation of charge $+1$, paired with one external chloride ion. The basic ferric acetate complex dissolves in water, unlike the three characteristic precipitates above. The charge calculation checks the written chloride stoichiometry, while the known solubility of this specific complex supplies the deciding chemical property.

Step 3: Conclusion The water-soluble compound is $\boxed{[Fe_3(OH)_2(OAc)_6]Cl}$. Hence, the correct option is (D).

Quick Tip to Solve: Recognise Prussian blue, ammonium arsenomolybdate and potassium cobaltinitrite as precipitates; avoid applying simple potassium or ammonium salt rules blindly to complex salts.

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64. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : In the photoelectric effect, the electrons are ejected from the metal surface as soon as the beam of light of frequency greater than threshold frequency strikes the surface.

Reason R : When the photon of any energy strikes an electron in the atom, transfer of energy from the photon to the electron takes place.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both A and R are correct and R is the correct explanation of A

(B) A is not correct but R is correct

(C) A is correct but R is not correct

(D) Both A and R are correct but R is NOT the correct explanation of A

Correct Answer: (C) A is correct but R is not correct

Solution:

Concept:

- A photon has energy $E = h\nu$, where h is Planck's constant and ν is its frequency. Photoemission requires this energy to exceed the metal's work function ϕ.
- The threshold frequency is $\nu_0 = \phi/h$. A photon transfers energy in one interaction; electrons do not accumulate the energies of many low-frequency photons before emission.

Step 1: Assess the assertion using the threshold condition For light of frequency greater than the threshold frequency, one photon supplies enough energy to overcome the surface binding energy. Einstein's equation gives

$$K_{\max} = h\nu - \phi = h(\nu - \nu_0) > 0.$$

The energy transfer takes place in an individual interaction. There is therefore no measurable waiting

time for the electron to gather energy from the beam. Assertion A correctly describes the immediate emission observed when suitable light reaches the surface.

Step 2: Assess the unrestricted energy claim in the reason Reason R says that a photon of *any* energy transfers its energy to an electron in an atom. This unrestricted claim is false. A bound atom absorbs photons only when the energy is compatible with an allowed transition, or when sufficient energy is available for ionisation. Merely striking the atom does not guarantee absorption. In photoemission, a photon with energy less than the work function cannot eject the electron. Thus the reason omits the essential energy condition and cannot be accepted as stated.

Step 3: Compare the assertion–reason combinations Since A is true and R is false, options (A) and (D), which treat both statements as correct, are excluded. Option (B) reverses their truth values and is also excluded. The instantaneous nature of emission does not make an arbitrary photon energy sufficient for absorption or escape.

Step 4: Conclusion Assertion A is correct, but Reason R is not correct. Hence, the correct option is (C).

Quick Tip to Solve: In a photon assertion–reason problem, check words such as "any: immediate energy transfer does not remove the threshold or allowed-transition condition.

65. Match List - I with List - II:

	List - I Species		List - II Geometry/Shape
A.	H_3O^+	**I.**	**Tetrahedral**
B.	**Acetylide anion**	**II.**	**Linear**
C.	NH_4^+	**III.**	**Pyramidal**
D.	ClO_2^-	**IV.**	**Bent**

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-II, D-IV

(B) A-III, B-II, C-I, D-IV

(C) A-III, B-IV, C-I, D-II

(D) A-III, B-IV, C-II, D-I

Correct Answer: (B) A-III, B-II, C-I, D-IV

Solution:

Concept:

- VSEPR theory counts regions of electron density around a central atom. A multiple bond counts as one region when determining the arrangement of the bonded atoms.
- Molecular shape describes the positions of atoms, whereas electron-pair geometry also includes lone pairs. Lone pairs can therefore change a tetrahedral electron-pair arrangement into a pyramidal or bent molecular shape.

Step 1: Assign the shapes of hydronium and ammonium In H_3O^+, oxygen has three bond pairs and one lone pair. The four electron domains have a tetrahedral arrangement, but the three hydrogen

atoms form a trigonal-pyramidal molecular shape. Hence A matches III. In NH_4^+, nitrogen has four bonds and no lone pair. Both the electron-domain geometry and the molecular shape are tetrahedral. Consequently C matches I. The positive charge on these ions must be included when counting the electrons; it does not make their shapes identical.

Step 2: Assign the shapes of acetylide and chlorite The carbon centres in the acetylide anion are sp-hybridised. The carbon–carbon triple bond and the remaining bond or lone-pair direction define a linear arrangement along the carbon axis. Therefore B matches II. In ClO_2^-, the central chlorine has two bonding domains and two lone pairs. Although the electron-domain arrangement is tetrahedral, the two oxygen atoms define a bent shape. Thus D matches IV. Treating a double bond as two separate directions would incorrectly change this VSEPR count.

Step 3: Check the complete matching against every option Combining the assignments gives A-III, B-II, C-I and D-IV. Option (A) incorrectly gives tetrahedral acetylide and linear ammonium. Option (C) assigns bent geometry to acetylide and linear geometry to chlorite. Option (D) also assigns acetylide a bent shape and makes both ammonium and chlorite incorrect. Only one printed combination preserves all four independent assignments.

Step 4: Conclusion The matching is A-III, B-II, C-I, D-IV. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish electron-pair geometry from molecular shape: four domains give tetrahedral ammonium, pyramidal hydronium and bent chlorite according to the number of lone pairs.

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66. For elements B, C, N, Li, Be, O and F, the correct order of first ionization enthalpy is [4 marks]

(A) $Li < Be < B < C < O < N < F$

(B) $B > Li > Be > C > N > O > F$

(C) $Li < Be < B < C < N < O < F$

(D) $Li < B < Be < C < O < N < F$

Correct Answer: (D) $Li < B < Be < C < O < N < F$

Solution:

Concept:

- First ionisation enthalpy is the energy required to remove the most loosely bound electron from an isolated gaseous atom. Across period 2, increasing effective nuclear charge generally raises this energy.
- Two important exceptions are $B < Be$, because boron loses a higher-energy $2p$ electron, and $O < N$, because oxygen loses one electron from a paired $2p$ orbital.

Step 1: Apply the general trend across the period The listed elements occur in period 2 in the sequence Li, Be, B, C, N, O, F. As nuclear charge increases, the valence electrons are generally held more strongly because electrons are added to the same main shell and shielding does not fully offset the extra nuclear attraction. Lithium consequently has the lowest first ionisation enthalpy of the listed atoms, while fluorine has the highest. A simple left-to-right ordering is a useful starting point, but it must be corrected for subshell and pairing effects.

Step 2: Insert the two electronic-configuration exceptions Beryllium has configuration $1s^2 2s^2$, whereas boron has $1s^2 2s^2 2p^1$. The boron electron is removed from a less penetrating, higher-energy $2p$ orbital, making its removal easier than removal of a beryllium $2s$ electron. Thus B precedes Be in an increasing-energy order. Nitrogen has three singly occupied $2p$ orbitals, while oxygen has one paired $2p$ orbital. Repulsion in that pair makes oxygen's first electron easier to remove, so O precedes N.

Step 3: Arrange the elements and reject the other orders Keeping the overall increasing trend and applying both exceptions gives

$$\mathrm{Li} < \mathrm{B} < \mathrm{Be} < \mathrm{C} < \mathrm{O} < \mathrm{N} < \mathrm{F}.$$

Option (A) misses the Be/B exception. Option (C) misses both exceptions. Option (B) places fluorine at the low end and is incompatible with the general trend as well as the measured exception pattern.

Step 4: Conclusion The increasing order is $\mathrm{Li} < \mathrm{B} < \mathrm{Be} < \mathrm{C} < \mathrm{O} < \mathrm{N} < \mathrm{F}$. Hence, the correct option is (D).

Quick Tip to Solve: For period 2 ionisation energies, write the broad increasing trend first, then exchange Be with B and N with O.

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67. $25\ \mathrm{mL}$ **of silver nitrate solution (1M) is added dropwise to** $25\ \mathrm{mL}$ **of potassium iodide** $(1.05\mathrm{M})$ **solution. The ion(s) present in very small quantity in the solution is/are :** [4 marks]

(A) I^- only

(B) K^+ only

(C) Ag^+ and I^- both

(D) NO_3^- only

Correct Answer: (C) Ag^+ and I^- both

Solution:

Concept:

- Silver ions react with iodide ions in a 1:1 ratio to form sparingly soluble AgI: $\mathrm{Ag}^+ + \mathrm{I}^- \longrightarrow \mathrm{AgI(s)}$. Potassium and nitrate remain spectator ions.
- Use $n = CV$, with volume in litres, to determine the limiting reagent. After precipitation, excess iodide is reduced but not completely removed; dissolved silver is additionally suppressed by the common-ion effect.

Step 1: Compare the initial amounts of the reacting ions The moles supplied by the two solutions are

$$n(\mathrm{Ag}^+) = 1.00(0.025) = 0.0250\ \mathrm{mol},$$

$$n(\mathrm{I}^-) = 1.05(0.025) = 0.02625\ \mathrm{mol}.$$

Thus silver ion is the limiting reagent. Addition of the silver nitrate dropwise precipitates nearly all the added silver as AgI, consuming the same amount of iodide. Iodide is in slight excess, rather than exactly equivalent to the silver.

Step 2: Find what remains in the combined solution Ignoring the extremely small amount from dissolution of the precipitate, the residual iodide is

$$0.02625 - 0.0250 = 0.00125 \text{ mol}.$$

The total solution volume is 0.050 L, so its concentration is about 0.025 mol L^{-1}. In contrast, potassium remains at $0.02625/0.050 = 0.525 \text{ mol L}^{-1}$, and nitrate at $0.0250/0.050 = 0.500 \text{ mol L}^{-1}$. Dissolved silver is much smaller still: $[\text{Ag}^+] = K_{\text{sp}}/[\text{I}^-]$. The excess iodide therefore suppresses free silver strongly.

Step 3: Interpret the small-quantity comparison in the options Both reacting ions have been depleted relative to the abundant spectator ions: silver is only a trace, and residual iodide is about one-twentieth of the spectator-ion concentrations. Iodide is not absent and is much more abundant than silver, but both are present in small quantities in the comparison asked. Options (B) and (D) wrongly name spectators. Option (A) omits the even smaller quantity of silver ion. The combined choice accounts for both depleted reacting ions.

Step 4: Conclusion The ions present in small quantities relative to the spectator ions are Ag^+ and I^-, with silver far more depleted. Hence, the correct option is (C).

Quick Tip to Solve: After precipitation, calculate the excess reactant rather than setting both concentrations to zero; compare it with the unchanged spectator-ion concentrations.

68. When a solution of mixture having two inorganic salts was treated with freshly prepared ferrous sulphate in acidic medium, a dark brown ring was formed whereas on treatment with neutral FeCl_3, it gave deep red colour which disppeared on boiling and a brown red ppt was formed. The mixture contains : [4 marks]

(A) SO_3^{2-} & $\text{C}_2\text{O}_4^{2-}$

(B) CH_3COO^- & NO_3^-

(C) $\text{C}_2\text{O}_4{}^{2-}$ & NO_3^-

(D) SO_3^{2-} & CH_3COO^-

Correct Answer: (B) CH_3COO^- & NO_3^-

Solution:

Concept:

- The brown-ring test uses freshly prepared FeSO_4 in strongly acidic medium. Nitrate is reduced to nitric oxide, which forms a brown nitrosyl complex with ferrous ions.
- Acetate gives a deep-red ferric acetate colour with neutral FeCl_3. On boiling, hydrolysis produces a brown-red basic ferric acetate precipitate and the soluble red colour disappears.

Step 1: Identify the anion responsible for the brown ring In acidic solution, ferrous ions reduce nitrate to nitric oxide:

$$\text{NO}_3{}^- + 3\,\text{Fe}^{2+} + 4\,\text{H}^+ \longrightarrow \text{NO} + 3\,\text{Fe}^{3+} + 2\,\text{H}_2\text{O}.$$

The liberated nitric oxide binds to another ferrous ion to form the brown nitrosyl species, commonly represented in the hydrated complex $[Fe(H_2O)_5(NO)]^{2+}$. The characteristic dark-brown ring therefore identifies nitrate in the mixture. Fresh ferrous sulphate is used because the test requires Fe^{2+}, rather than a solution largely oxidised to ferric ions.

Step 2: Use the neutral ferric chloride observation The deep-red colour with neutral ferric chloride, followed by loss of colour and a brown-red precipitate on boiling, is the characteristic acetate test. Ferric acetate forms initially and hydrolyses on heating. A representative equation for basic ferric acetate formation is

$$Fe(CH_3COO)_3 + 2\,H_2O \longrightarrow Fe(OH)_2(CH_3COO)\,(s) + 2\,CH_3COOH.$$

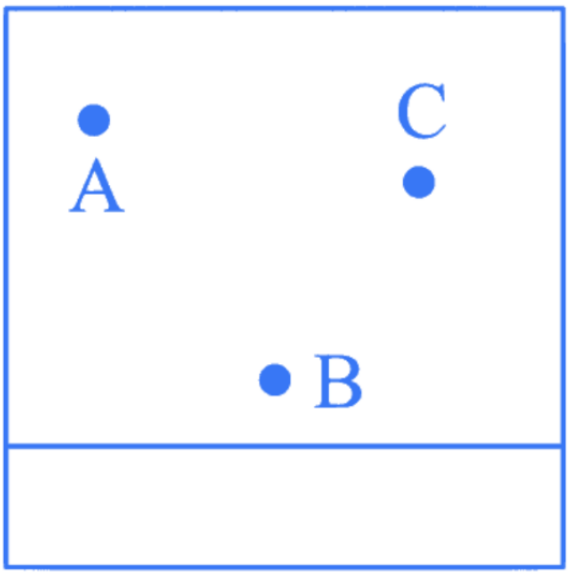
The complete sequence, including the change on boiling, distinguishes this observation from a colour caused by another ligand. Thus the second anion is CH_3COO^-.

Step 3: Check the two-anion combinations The mixture must contain both nitrate and acetate. Option (A) contains neither required pair. Option (C) contains nitrate but replaces acetate with oxalate, which does not explain the specified ferric acetate hydrolysis observation. Option (D) contains acetate but lacks nitrate, so it cannot account for the brown-ring test. Only option (B) explains both separate qualitative observations.

Step 4: Conclusion The mixture contains CH_3COO^- and $NO_3{}^-$. Hence, the correct option is (B).

Quick Tip to Solve: Read qualitative tests as a sequence: the red ferric colour together with its brown-red precipitate on boiling is stronger evidence for acetate than colour alone.

69. Thin layer chromatography of a mixture shows the following observation:
The correct order of elution in the silica gel column chromatography is [4 marks]



(A) B, A, C
(B) C, A, B
(C) A, C, B
(D) B, C, A
Correct Answer: (C) A, C, B

Solution:

Concept:

- Silica gel is a polar adsorbent used as the stationary phase in normal-phase chromatography. Stronger adsorption retards migration in TLC and delays elution in a silica column.
- The retention factor is R_f = distance travelled by solute/distance travelled by solvent front. Under comparable normal-phase conditions, a larger R_f indicates a component that elutes earlier.

Step 1: Read the relative migration distances from the TLC plate All three components start at the horizontal baseline shown near the bottom of the plate. The spot labelled A is highest above that line, the spot labelled C is intermediate, and the spot labelled B is closest to the baseline. The solvent-front distance is common to the three spots, so the same ordering holds for their retention factors:

$$R_f(A) > R_f(C) > R_f(B).$$

No numerical scale is required: their vertical positions alone determine the ordering. The horizontal positions merely distinguish the spots and do not measure their migration.

Step 2: Translate migration into adsorption and column elution A travels the farthest because it is retained least strongly by the silica surface relative to its transport in the mobile phase. C has intermediate retention, while B is most strongly adsorbed. In the corresponding silica gel column, the least retained component spends more time moving with the solvent and emerges first. The most strongly retained component spends more time attached to the stationary phase and emerges last. Therefore the elution order follows decreasing TLC migration, not increasing migration.

Step 3: Compare the predicted sequence with the printed choices The first, second and last components are consequently A, C and B. Option (A) puts the most strongly retained component B first and C last. Option (B) incorrectly places C before A even though A has the larger R_f. Option (D) again starts with B and reverses the observed mobility pattern. Only option (C) preserves the full decreasing-R_f sequence. This comparison uses the same polar stationary phase specified in the question.

Step 4: Conclusion The correct order of elution is A, C, B. Hence, the correct option is (C).

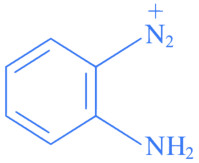
Quick Tip to Solve: For silica TLC and a silica column, the spot farthest above the baseline normally emerges first from the column; do not reverse the order just because the column is vertical. Compare migration only for the same stationary phase and comparable solvent conditions.

70.

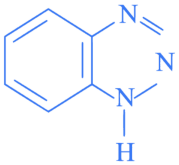
o-Phenylenediamine $\xrightarrow{HNO_2}$ X **(Major Product)**

'X' is [4 marks]

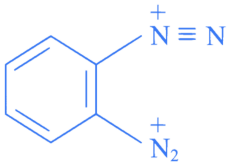
(A) *o*-Aminobenzenediazonium ion



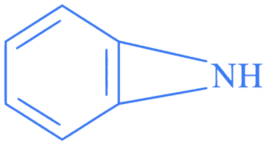
(B) 1H-Benzotriazole



(C) Benzene-1,2-bis(diazonium) ion

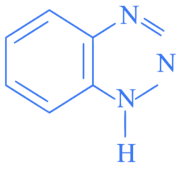


(D) Benzene-fused three-membered NH ring



Correct Answer: (B)

1H-Benzotriazole



Solution:

Concept:

- A primary aromatic amino group reacts with nitrous acid to form a diazonium group. In an ortho-diamine, the neighbouring amino group can react intramolecularly with this diazonium functionality.
- Intramolecular cyclisation is favoured when it produces a stable five-membered heterocycle. Here the two original amino nitrogens and one nitrogen introduced by nitrous acid form the three-nitrogen ring of benzotriazole.

Step 1: Form the initial ortho-amino diazonium intermediate The starting compound is *o*-phenylenediamine, which has two adjacent NH_2 groups on a benzene ring. Nitrous acid diazotises one amino group. This creates an ortho-amino arenediazonium intermediate, corresponding to the general connectivity shown in option (A). However, the remaining amino group is located beside the diazonium group and can therefore participate immediately in a further intramolecular reaction. Stopping at the open diazonium intermediate would overlook this especially favourable neighbouring-group reaction.

Step 2: Close the nitrogen-containing ring The neighbouring amino nitrogen attacks the terminal nitrogen of the diazonium group. Formation of the new nitrogen–nitrogen bond closes a five-membered ring fused to benzene. Proton transfer and deprotonation give neutral 1H-benzotriazole. The product contains three consecutive nitrogen atoms in that five-membered ring, one of which bears hydrogen. The net stoichiometry is

$$C_6H_4(NH_2)_2 + HNO_2 \longrightarrow C_6H_5N_3 + 2\,H_2O.$$

This formula also checks the atom balance and shows why one extra nitrogen from nitrous acid is incorporated in the cyclic product.

Step 3: Identify the major product among the structures Option (B) is the fused five-membered benzotriazole ring and matches this cyclisation. Option (A) is only the uncyclised intermediate. Option (C), containing two diazonium groups, does not describe the favoured ring closure of the adjacent

amino group. Option (D) has a fused three-membered ring with only one nitrogen; it has neither the nitrogen count nor the five-membered connectivity required by the reaction. Thus the benzotriazole structure is selected.

Step 4: Conclusion The major product X is 1H-benzotriazole, the benzene-fused five-membered ring containing three nitrogen atoms. Hence, the correct option is (B).

Quick Tip to Solve: With an ortho-diamine and nitrous acid, look beyond initial diazotisation: the adjacent amino group traps the terminal diazonium nitrogen to make the five-membered benzotriazole ring.

71. For compound having the formula $GaAlCl_4$, the correct option from the following is : [4 marks]

(A) Cl forms bond with both Al and Ga in $GaAlCl_4$

(B) Oxidation state of Ga in the salt $GaAlCl_4$ is $+3$.

(C) Ga is coordinated with Cl in $GaAlCl_4$

(D) Ga is more electronegative than Al and is present as a cationic part of the salt $GaAlCl_4$

Correct Answer: (D) Ga is more electronegative than Al and is present as a cationic part of the salt $GaAlCl_4$

Solution:

Concept:

- The formula of an ionic salt must be resolved into its cation and anion before assigning oxidation numbers. Chlorine has oxidation number -1 in a chloride.
- Gallium can exhibit the $+1$ oxidation state because its $4s^2$ electron pair may remain uninvolved in bonding. Aluminium commonly exhibits $+3$ and forms the tetrahedral tetrachloroaluminate ion, $[AlCl_4]^-$.

Step 1: Separate the ions The compound is represented as $Ga^+[AlCl_4]^-$. For the complex anion, charge balance gives

$$x + 4(-1) = -1 \quad \Longrightarrow \quad x = +3.$$

Thus aluminium is the central atom in the anion and the four chloride ligands are coordinated to it. The gallium ion supplies the balancing positive charge; it is not the central atom of a chlorido complex in this formulation.

Step 2: Determine the oxidation state of gallium The one tetrachloroaluminate anion has charge -1, so the gallium cation must have charge $+1$. Hence the oxidation state of gallium is $+1$, not $+3$. Option (B) therefore violates the ionic charge balance.

Step 3: Compare the remaining statements Gallium is more electronegative than aluminium, but electronegativity alone does not decide which atom is the cation in this complex salt. The stable chlorido complex here is the aluminium-containing anion, while gallium occurs as the cation. Options (A) and (C) incorrectly extend the aluminium–chloride coordination to gallium; there are no chloride ligands coordinated to the gallium cation in the stated salt formulation.

Step 4: Conclusion $GaAlCl_4$ is $Ga^+[AlCl_4]^-$: gallium is more electronegative than aluminium and occurs in the cationic part. Hence, the correct option is (D).

Quick Tip to Solve: Write $GaAlCl_4$ as $Ga^+[AlCl_4]^-$ first; this immediately distinguishes oxidation state from coordination.

72. The set which does not have ambidentate ligand(s) is : [4 marks]

(A) $C_2O_4^{2-}, NO_2^-, NCS^-$

(B) $C_2O_4^{2-}$, ethylene diammine, H_2O

(C) $NO_2^-, C_2O_4^{2-}$, EDTA $^{4-}$

(D) $EDTA^{4-}, NCS^-, C_2O_4^{2-}$

Correct Answer: (B) $C_2O_4^{2-}$, ethylene diammine, H_2O

Solution:

Concept:

- An ambidentate ligand has two different possible donor atoms, but normally binds through only one of them in a given coordination bond. Denticity instead counts donor atoms attached simultaneously to a metal.
- NO_2^- can coordinate through nitrogen or oxygen, while NCS^- can coordinate through nitrogen or sulfur. These are standard ambidentate ligands.

Step 1: Classify the ligands with alternative donor atoms The nitrite ion can form nitro (N-bonded) or nitrito (O-bonded) complexes. Similarly, the thiocyanate ion can form an N-bonded or S-bonded linkage. Therefore any set containing NO_2^- or NCS^- fails the requirement that no ambidentate ligand be present. This test depends on the identity of the donor atom, not simply on whether a formula contains several heteroatoms.

Step 2: Inspect every member of the second set Oxalate, $C_2O_4^{2-}$, is a bidentate ligand: two oxygen donors bind the same metal and form a chelate ring. Ethylenediamine is also bidentate, using its two nitrogen donors simultaneously. Water is monodentate and coordinates through its oxygen atom. None is classified as an ambidentate ligand. EDTA, appearing in the other sets, is normally hexadentate rather than ambidentate.

Step 3: Eliminate the other three sets Option (A) contains both nitrite and thiocyanate. Option (C) contains nitrite, and option (D) contains thiocyanate. Each therefore contains at least one ambidentate ligand. In contrast, the set in option (B) contains two chelating bidentate ligands and the ordinary monodentate ligand water, so it alone satisfies the negative condition in the question.

Step 4: Conclusion The set $C_2O_4^{2-}$, ethylenediamine and H_2O has no ambidentate ligand. Hence, the correct option is (B).

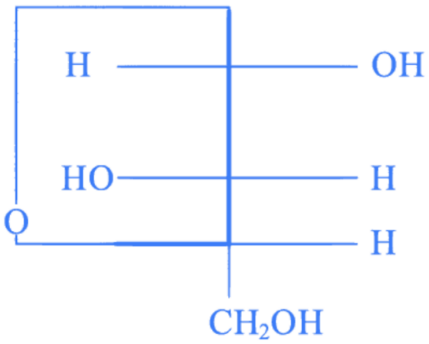
Quick Tip to Solve: Distinguish two donors used together (bidentate) from two alternative donor atoms used one at a time (ambidentate).

73. L-isomer of tetrose $X(C_4H_8O_4)$ gives positive Schiff's test and has two chiral carbons. On acetylation, 'X' yields triacetate. 'X' also undergoes following reactions:

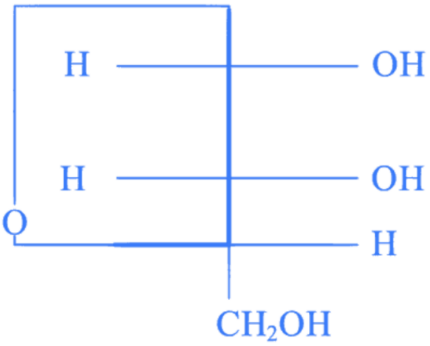
$$A \xleftarrow{HNO_3} X \xrightarrow{NaBH_4} B \quad \textbf{(chiral compound)}$$

'X' is [4 marks]

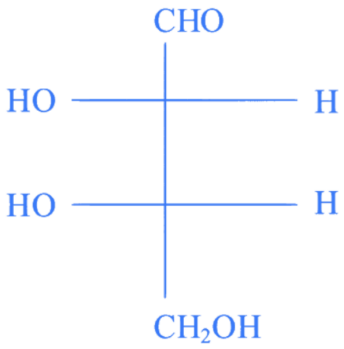
(A)



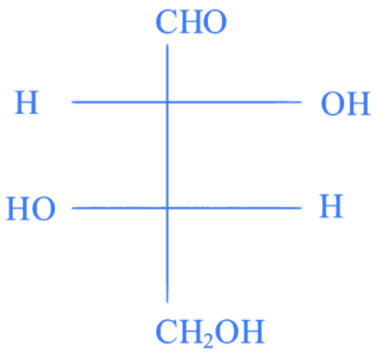
(B)



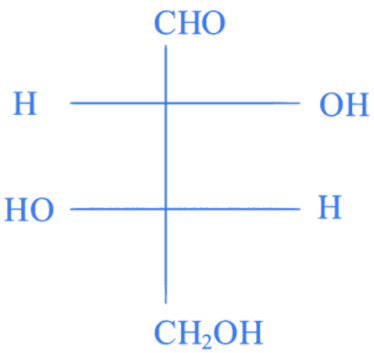
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- A positive Schiff test indicates a free aldehyde group. An open-chain aldotetrose has the structure $CHO-CHOH-CHOH-CH_2OH$, with two stereogenic carbon atoms and three hydroxyl groups.
- In a Fischer projection with CHO at the top, an L-sugar has its lowest stereogenic carbon hydroxyl on the left. Reduction converts the aldehyde end into CH_2OH without changing the two existing configurations.

Step 1: Use the functional-group tests The formula $C_4H_8O_4$, aldehyde test and two chiral carbons identify an aldotetrose rather than a ketotetrose. Its two secondary alcohol groups and one primary alcohol group account for the triacetate on acetylation. The aldehyde remains a carbonyl group during ordinary hydroxyl acetylation. Thus the relevant open-chain choices are the Fischer projections in (C) and (D).

Step 2: Test the stereochemistry of the reduction product In (C), the two hydroxyl groups lie on the same side of the Fischer projection. Reducing its CHO group gives erythritol, whose identical CH_2OH ends and internal symmetry make it meso and achiral. This contradicts the stated chirality of B. In (D), the hydroxyl groups are on opposite sides, so reduction gives threitol. The two stereogenic centres then have the same absolute configuration and the product has no internal mirror plane: it is chiral.

Step 3: Check the L-configuration and oxidation In option (D), the hydroxyl on the lower stereogenic carbon is on the left, confirming the L-series. The sugar is L-threose. Nitric acid oxidises the aldehyde and terminal primary alcohol to carboxyl groups, forming the corresponding tartaric acid while retaining the two central configurations. The cyclic alternatives (A) and (B) do not display the required open-chain aldotetrose arrangement; neither accounts for the complete aldehyde, triacetate and chiral-alditol evidence as the L-threose projection does.

Step 4: Conclusion X is L-threose, with the upper hydroxyl on the right and lower hydroxyl on the left in the displayed open-chain Fischer projection. Its reduction gives chiral threitol. Hence, the correct option is (D).

Quick Tip to Solve: For an aldotetrose, reduce both ends mentally to CH_2OH: erythrose gives meso erythritol, while threose gives chiral threitol.

74. Which of the following complex has a possibility to exist as meridional isomer? [4 marks]

(A) $[Pt(NH_3)_2Cl_2]$

(B) $[Co(en)_2Cl_2]$

(C) $[Co(en)_3]$

(D) $[Co\,(NH_3)_3\,(NO_2)_3]$

Correct Answer: (D) $[Co\,(NH_3)_3\,(NO_2)_3]$

Solution:

Concept:

- Facial and meridional isomerism occurs in octahedral complexes with three donor positions of one kind and three of another, classically $[MA_3B_3]$. It is a geometrical, rather than optical, distinction.
- In the facial arrangement the three identical ligands occupy one triangular face and are mutually cis. In the meridional arrangement they lie along a meridian, so two of them are trans.

Step 1: Recognise the required ligand count For $[Co(NH_3)_3(NO_2)_3]$, both ammonia and nitrite are monodentate ligands in the stated complex. Three of each give six donor atoms and the octahedral MA_3B_3 pattern. One pair of ammonia ligands may occupy opposite positions, with the third in the perpendicular plane; this is the meridional arrangement. A second arrangement puts all three ammonia ligands on the same face.

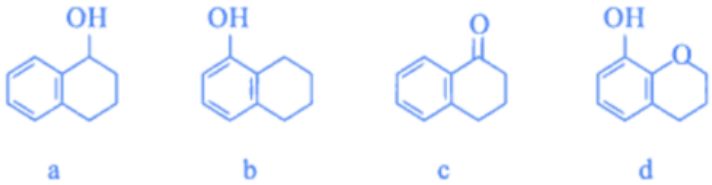
Step 2: Check the platinum and bis-chelate choices $[Pt(NH_3)_2Cl_2]$ is a four-coordinate square-planar MA_2B_2 complex. It shows cis/trans isomerism, not fac/mer isomerism. In $[Co(en)_2Cl_2]$, the two ethylenediamine ligands each occupy two adjacent sites, leaving two chlorido positions. The geometrical alternatives are again described as cis and trans for the chlorido pair. It is not a three-and-three set of monodentate ligands.

Step 3: Check the tris-chelate choice $[Co(en)_3]$ has three identical bidentate ligands and may show optical isomerism through its two helical arrangements. There are no two sets of three monodentate ligands to distribute in facial and meridional positions. Thus options (A), (B) and (C) test other kinds of stereoisomerism, whereas option (D) has exactly the ligand pattern required here.

Step 4: Conclusion $[Co(NH_3)_3(NO_2)_3]$ can exist as facial and meridional geometrical isomers. Hence, the correct option is (D).

Quick Tip to Solve: Before looking for fac/mer isomers, count donor positions: the standard pattern is octahedral MA_3B_3.

75. Arrange the following compounds in increasing order of rate of aromatic electrophilic substitution reaction [4 marks]



(A) d, b, a, c

(B) b, c, a, d

(C) c, a, b, d

(D) d, b, c, a

Correct Answer: (C) c, a, b, d

Solution:

Concept:

- Electrophilic aromatic substitution is faster when substituents increase the electron density of the aromatic ring and stabilise its sigma-complex intermediate. Resonance donation by an oxygen directly bonded to the ring is especially effective.
- A carbonyl group attached directly to the aromatic ring withdraws electrons by resonance and induction. An alcohol group on a saturated carbon cannot donate its lone pair directly into the aromatic pi system.

Step 1: Identify the least and moderately reactive rings In structure c, the carbonyl carbon is directly attached to the benzene ring. The carbonyl withdraws electron density by its negative resonance effect, destabilising the cationic substitution intermediate and giving the slowest aromatic substitution. In a, the hydroxyl group is attached to a saturated carbon of the fused ring. Its lone pair is therefore not conjugated with the aromatic ring. The saturated carbon framework is much less deactivating than the directly attached carbonyl, so a reacts faster than c.

Step 2: Compare the two phenolic compounds In b, the hydroxyl group is directly bonded to an aromatic carbon. Its oxygen lone pair donates into the ring by resonance, increasing electron density and stabilising ortho/para attack intermediates. Compound b is therefore more reactive than a. Compound d also has a phenolic hydroxyl group, and additionally the ether oxygen of the fused saturated ring is directly bonded to an aromatic carbon. This second oxygen supplies an additional resonance-donating effect, so d is the most activated of the four rings.

Step 3: Assemble and test the increasing order Combining the electron-withdrawing carbonyl, nonconjugated alcohol, phenolic donation and two oxygen-donor effects gives

$$\boxed{c < a < b < d}.$$

Option (A) reverses this trend by placing the strongly activated d first. Option (B) incorrectly places b below the carbonyl-containing c. Option (D) again puts d first and places c above b. Only option (C) follows the increasing electron density and intermediate stabilisation.

Step 4: Conclusion The increasing order of aromatic electrophilic substitution rate is c, a, b, d. Hence, the correct option is (C).

Quick Tip to Solve: Check whether oxygen is directly attached to the aromatic ring: phenolic and ether oxygen donate by resonance, but a benzylic alcohol does not.

76. The polymer X - consists of linear molecules and is closely packed. It is prepared in the presence of triethylaluminium and titanium tetrachloride under low pressure. The polymer X is- [4 marks]

(A) High density polythene

(B) Polytetrafluoroethane

(C) Low density polythene

(D) Polyacrylonitrile

Correct Answer: (A) High density polythene

Solution:

Concept:

- Polymer properties depend on chain architecture. Nearly unbranched chains pack closely and have stronger intermolecular attractions than heavily branched chains, producing greater density and crystallinity.
- Triethylaluminium with titanium tetrachloride is a Ziegler–Natta catalyst system. It enables coordination polymerisation of ethene at relatively low pressure to give predominantly linear polythene.

Step 1: Identify the catalyst-controlled polymerisation The monomer corresponding to polythene is ethene. Under the stated catalyst conditions its double bonds join to form the saturated carbon backbone:

$$n\,CH_2{=}CH_2 \longrightarrow [-CH_2-CH_2-]_n.$$

Coordination polymerisation suppresses the extensive branching associated with the high-pressure free-radical route. Consequently, the molecules produced here are substantially linear rather than highly branched.

Step 2: Connect chain shape with density Linear chains can approach one another closely and form more ordered, crystalline regions. The closer packing strengthens dispersion interactions and gives a harder, denser material. These are the defining structural features of high-density polythene (HDPE). Thus the morphological clue and the catalyst clue independently identify the same polymer; neither clue requires confusing density with the mass of one monomer unit.

Step 3: Exclude the other polymers Low-density polythene is made through the high-pressure radical route and has substantial branching, preventing close packing, so (C) is inconsistent. The fluorinated polymer in (B) has fluorine substituents and is based on tetrafluoroethene rather than ethene; it is not the product of the stated polyethylene route. Polyacrylonitrile in (D) is formed from acrylonitrile and contains nitrile substituents. It likewise does not match the given monomer/catalyst description.

Step 4: Conclusion X is high-density polythene (HDPE), whose largely linear chains pack closely.

Hence, the correct option is (A).

Quick Tip to Solve: Pair low pressure plus Ziegler–Natta catalyst with HDPE; pair high-pressure radical polymerisation and branching with LDPE.

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77. Given below are two statements:

Statement-I : Methane and steam passed over a heated Ni catalyst produces hydrogen gas.

Statement-II : Sodium nitrite reacts with $\mathrm{NH_4Cl}$ to give $\mathrm{H_2O}, \mathrm{N_2}$ and NaCl.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is correct but Statement II is incorrect

(B) Both the statements I and II are incorrect

(C) Both the statements I and II are correct

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (C) Both the statements I and II are correct

Solution:

Concept:

- Steam reforming of methane is an industrial source of hydrogen. A heated nickel catalyst promotes reaction of methane with steam to form carbon monoxide and hydrogen.
- Ammonium nitrite decomposes on warming to nitrogen and water. It can be generated in situ by reaction of sodium nitrite with ammonium chloride, leaving sodium chloride in solution.

Step 1: Verify Statement I by balancing steam reforming Methane reacts with water vapour over heated nickel according to

$$\mathrm{CH_4 + H_2O \xrightarrow{Ni,\ heat} CO + 3\,H_2}.$$

There is one carbon atom and one oxygen atom on each side. The six hydrogen atoms on the left appear as three hydrogen molecules on the right. Hence hydrogen gas is indeed produced, and Statement I is correct. Carbon monoxide in the product does not invalidate the statement, which does not claim that hydrogen is the only product.

Step 2: Verify Statement II through ammonium nitrite The ions first give ammonium nitrite, which decomposes readily on warming:

$$\mathrm{NH_4NO_2 \longrightarrow N_2 + 2\,H_2O}.$$

Including the sodium and chloride spectator ions gives the overall reaction

$$\mathrm{NaNO_2 + NH_4Cl \longrightarrow NaCl + N_2 + 2\,H_2O}.$$

Both nitrogen atoms become molecular nitrogen, both oxygen atoms become water, and all four hydrogen atoms of ammonium are accounted for. Therefore Statement II also names the correct products.

Step 3: Choose the combination of truth values Statement I is true because methane steam reforming yields hydrogen; Statement II is true because nitrite with ammonium chloride gives nitrogen, water and sodium chloride. Option (A) wrongly rejects the second balanced reaction. Option (B) rejects both valid preparations, while option (D) wrongly rejects steam reforming. The two statements concern distinct gas preparations and do not need to describe the same process.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (C).

Quick Tip to Solve: Balance the two preparations separately: steam reforming gives H_2, whereas ammonium nitrite decomposition gives N_2.

78. In the extraction process of copper, the product obtained after carrying out the reactions

(i) $2Cu_2\,S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

(ii) $2Cu_2O + Cu_2\,S \rightarrow 6Cu + SO_2$ **is called** [4 marks]

(A) Copper matte

(B) Blister copper

(C) Reduced copper

(D) Copper scrap

Correct Answer: (B) Blister copper

Solution:

Concept:

- Copper extraction includes partial oxidation of copper(I) sulphide followed by self-reduction: an oxide and a sulphide react without adding an external reducing agent.
- The name of the metallic product depends on the stage of extraction. Escape of gaseous SO_2 gives the freshly obtained copper its characteristic blistered surface.

Step 1: Identify the partial oxidation The first reaction converts part of the sulphide into copper(I) oxide while sulphur is oxidised to sulphur dioxide. It is not yet the final metal-producing step.

$$2\,Cu_2S + 3\,O_2 \longrightarrow 2\,Cu_2O + 2\,SO_2$$

Copper matte is the sulphide-rich molten mixture, principally containing Cu_2S and FeS, obtained earlier in smelting; it is not the copper metal obtained after both reactions.

Step 2: Identify the metal formed by self-reduction The remaining sulphide reduces the oxide to metallic copper:

$$2\,Cu_2O + Cu_2S \longrightarrow 6\,Cu + SO_2$$

The sulphur dioxide produced escapes from the hot metal. As the copper cools, this escaping gas produces a blistered appearance, giving the product the name blister copper. The term describes this impure copper obtained in the extraction stage, before refining. Copper scrap means discarded copper material, so it does not name this reaction product. Although reduction occurs, reduced copper is not the conventional extraction-stage name asked for here.

Step 3: Conclusion The product after the given oxidation and self-reduction reactions is **blister copper**. Hence, the correct option is (B).

Quick Tip to Solve: Link blister copper with the escape of SO_2 during self-reduction; matte is the preceding sulphide mixture.

79. Given below are two statements:

Statement I : If BOD is 4 ppm and dissolved oxygen is 8 ppm, then it is a good quality water.

Statement II : If the concentration of zinc and nitrate salts are 5 ppm each, then it can be a good quality water.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is incorrect but Statement II is correct

(B) Both the statements I and II are incorrect

(C) Both the statements I and II are correct

(D) Statement I is correct but Statement II is incorrect

Correct Answer: (C) Both the statements I and II are correct

Solution:

Concept:

- Biochemical oxygen demand (BOD) measures the oxygen required by microorganisms to oxidise biodegradable organic matter. Lower BOD indicates less organic pollution.
- Water quality also depends on dissolved oxygen and dissolved substances. Typical drinking-water limits used here are $5\,\text{ppm}$ for zinc and $50\,\text{ppm}$ for nitrate.

Step 1: Evaluate Statement I A BOD below about $5\,\text{ppm}$ is associated with clean water; highly polluted water has a much larger BOD. The stated BOD is only $4\,\text{ppm}$. The dissolved oxygen concentration is $8\,\text{ppm}$, which is sufficiently high to support aquatic life and is consistent with good quality water. High BOD would consume the available oxygen, whereas this sample combines low demand with a high oxygen content. Statement I is therefore correct.

Step 2: Evaluate Statement II and combine the results The zinc concentration is $5\,\text{ppm}$, at the stated desirable limit, and the nitrate concentration is $5\,\text{ppm}$, well below the $50\,\text{ppm}$ limit. Thus these two concentrations do not, by themselves, rule out good quality water; the wording says that it can be good quality water. Statement II is also correct. Option (A) incorrectly rejects Statement I, option (B) rejects both true statements, and option (D) incorrectly rejects Statement II. The option accepting both statements is consequently selected.

Step 3: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (C).

Quick Tip to Solve: For water-quality statements, check BOD and dissolved oxygen separately, then compare each dissolved impurity with its own limit.

80. Match List-I with List-II:

	List - I		List - II
A.	K	I.	Thermonuclear reactions
B.	KCl	II.	Fertilizer
C.	KOH	III.	Sodium potassium pump
D.	Li	IV.	Absorbent of CO_2

Choose the correct answer from the options given below : [4 marks]

(A) A-IV, B-III, C-I, D-II

(B) A-III, B-II, C-IV, D-I

(C) A-III, B-IV, C-II, D-I

(D) A-IV, B-I, C-III, D-II

Correct Answer: (B) A-III, B-II, C-IV, D-I

Solution:

Concept:

- Applications of alkali metals and their compounds depend on their chemical and biological properties; distinguish elemental potassium from its chloride and hydroxide.
- Potassium ions participate in the sodium-potassium pump, potassium chloride supplies a plant nutrient, and potassium hydroxide absorbs acidic carbon dioxide.

Step 1: Match potassium and potassium chloride Entry A, potassium, matches III, the sodium-potassium pump. This transport mechanism maintains sodium and potassium ion concentration gradients across cell membranes. Entry B, KCl, matches II, fertilizer: potassium chloride supplies the potassium nutrient required by plants. These matches already exclude options (A) and (D), which associate A with absorption of carbon dioxide rather than the biological pump. The pump involves potassium ions; it is not a use of caustic potash as a gas absorbent.

Step 2: Match potassium hydroxide and lithium Entry C, KOH, matches IV, absorbent of CO_2. It is a strong base and reacts with the acidic oxide; for example,

$$2\,KOH + CO_2 \longrightarrow K_2CO_3 + H_2O$$

Entry D, lithium, matches I, thermonuclear reactions. Lithium isotopes are used in producing the tritium needed for such reactions. The complete matching is therefore A-III, B-II, C-IV, D-I. Option (C) reverses the applications of KCl and KOH, so it fails despite matching A and D correctly. Only option (B) gives all four associations.

Step 3: Conclusion The correct matching is A-III, B-II, C-IV, D-I. Hence, the correct option is (B).

Quick Tip to Solve: Fix the easy pair KOH-carbon dioxide first, then use KCl-fertilizer to distinguish the remaining choices.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. A solution of sugar is obtained by mixing $200\ \text{g}$ of its 25% solution and $500\ \text{g}$ of its 40% solution (both by mass). The mass percentage of the resulting sugar solution is _____ (Nearest integer)

[4 marks]

Solution:

Concept:

- Mass percentage of sugar is $100 m_{\text{sugar}}/m_{\text{solution}}$. The denominator is the mass of the entire solution, including solvent.
- When two solutions are mixed without loss or reaction, both the masses of solute and the total masses of solution are additive. The final percentage is a mass-weighted average.

Step 1: Calculate the sugar contributed by each solution The first solution has a mass of $200\,\text{g}$, of which 25% is sugar. Thus

$$m_1 = \frac{25}{100} \times 200 = 50\,\text{g}.$$

The second solution has a mass of $500\,\text{g}$, of which 40% is sugar:

$$m_2 = \frac{40}{100} \times 500 = 200\,\text{g}.$$

The different amounts of solution must be included. Taking the simple arithmetic mean of 25 and 40 would incorrectly assign equal weights to the two solutions.

Step 2: Combine the masses and round only the final percentage The total sugar mass and total solution mass are

$$m_{\text{sugar}} = 50 + 200 = 250\,\text{g}, \qquad m_{\text{solution}} = 200 + 500 = 700\,\text{g}.$$

Consequently,

$$w = \frac{250}{700} \times 100 = 35.714285\ldots\%.$$

The solvent mass is $700 - 250 = 450\,\text{g}$, but that is not the denominator for mass percentage. The question requests the nearest integer, so round 35.714285 to 36. As a check, the percentage lies between 25 and 40 and is closer to 40 because more of the concentrated solution was used.

Step 3: Conclusion The resulting solution contains approximately $\boxed{36\%}$ sugar by mass; the requested numerical entry is $\boxed{36}$.

Quick Tip to Solve: Find grams of solute in each solution first; divide their sum by the total solution mass, and round at the end.

82. 0.004 M K_2SO_4 solution is isotonic with 0.01 M glucose solution. Percentage dissociation of K_2SO_4 is _____ (Nearest integer) [4 marks]

Solution:

Concept:

- For dilute solutions, osmotic pressure is $\pi = iCRT$, where i is the van't Hoff factor and C is molarity. Isotonic solutions at the same temperature have equal iC.
- If one formula unit dissociates into ν ions with degree of dissociation α, particle counting gives $i = 1 + (\nu - 1)\alpha$. Glucose does not dissociate, so its factor is 1.

Step 1: Calculate the electrolyte particle factor The two osmotic pressures are equal at their common temperature, so R and T cancel. For the potassium sulphate solution, let the particle factor be i. Then

$$i(0.004) = 1(0.01), \qquad i = \frac{0.01}{0.004} = 2.5.$$

This factor is larger than 1 because the electrolyte generates ions, but less than the complete-dissociation value of 3. It therefore corresponds to partial dissociation rather than a fully dissociated salt.

Step 2: Convert the factor into percentage dissociation The dissociation is

$$K_2SO_4 \longrightarrow 2\,K^+ + {SO_4}^{2-}.$$

Start with one mole of salt. After dissociation, there are $1 - \alpha$ mole of undissociated salt, 2α mole of potassium ions and α mole of sulphate ions. The total is $1 + 2\alpha$, hence

$$2.5 = 1 + 2\alpha, \qquad \alpha = \frac{1.5}{2} = 0.75.$$

The percentage dissociation is $100\alpha = 75\%$, already an integer. Using $i = 1 + \alpha$ would be appropriate for a two-ion electrolyte, not for potassium sulphate.

Step 3: Conclusion The percentage dissociation of K_2SO_4 is $\boxed{75\%}$; the requested nearest-integer entry is $\boxed{75}$.

Quick Tip to Solve: Count all ions per formula unit: K_2SO_4 gives three, so use $i = 1 + 2\alpha$, not $1 + \alpha$.

83. The ratio of spin-only magnetic moment values μ_{eff} $[Cr(CN)_6]^{3-}$ / μ_{eff} $[Cr\,(H_2O)_6]^{3+}$ is _____. [4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu_{\text{eff}} = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons and μ_B is the Bohr magneton.
- Both complexes are octahedral chromium(III) species. An octahedral d^3 ion has configuration $t_{2g}^3 e_g^0$ with three unpaired electrons, for both strong- and weak-field ligands.

Step 1: Find the chromium oxidation state and electron count Each cyanide ligand has charge -1. In $[Cr(CN)_6]^{3-}$, the oxidation state x therefore satisfies $x - 6 = -3$, giving $x = +3$. Water is neutral, so chromium is also $+3$ in $[Cr(H_2O)_6]^{3+}$. Neutral chromium has configuration $[Ar]3d^5 4s^1$; removing three electrons gives Cr^{3+} with configuration $[Ar]3d^3$. Thus the metal ion has the same number of d electrons in both complexes.

Step 2: Count unpaired electrons and form the ratio The three d electrons occupy the three separate lower-energy t_{2g} orbitals. There is no need to occupy an e_g orbital or pair electrons within a t_{2g} orbital. Although cyanide is a stronger-field ligand than water, this does not alter the unpaired-electron count of an octahedral d^3 ion. For each complex,

$$n = 3, \qquad \mu_{\text{eff}} = \sqrt{3(3+2)}\,\mu_B = \sqrt{15}\,\mu_B.$$

The desired ratio is therefore $\sqrt{15}/\sqrt{15} = 1$. The Bohr-magneton units cancel, leaving a dimensionless ratio.

Step 3: Conclusion The ratio of the two spin-only magnetic moments is $\boxed{1}$ (dimensionless).

Quick Tip to Solve: For octahedral d^3, both weak- and strong-field ligands leave three unpaired electrons; ligand strength does not change this ratio.

84.

$$KClO_3 + 6FeSO_4 + 3H_2SO_4 \rightarrow KCl + 3Fe_2(SO_4)_3 + 3H_2O$$

The above reaction was studied at $300\ K$ **by monitoring the concentration of** $FeSO_4$ **in which initial concentration was** $10M$ **and after half an hour became 8.8 M. The rate of production of** $Fe_2(SO_4)_3$ **is** _____ $\times 10^{-6}$ mol L s^{-1} **(Nearest integer)** [4 marks]

Solution:

Concept:

- Average disappearance rate is the positive decrease in concentration divided by the elapsed time. Convert the time to seconds when the required rate is per second.
- Stoichiometric coefficients relate changes in concentration. Here six moles of $FeSO_4$ consumed produce three moles of $Fe_2(SO_4)_3$, so the product formation rate is one-half the reactant disappearance rate.

Step 1: Calculate the average disappearance rate of ferrous sulphate The concentration decreases from $10.0\,\text{mol}\,L^{-1}$ to $8.8\,\text{mol}\,L^{-1}$. Over the stated interval,

$$-\Delta[FeSO_4] = 10.0 - 8.8 = 1.2\,\text{mol}\,L^{-1}, \qquad \Delta t = 30 \times 60 = 1800\,s.$$

Therefore,

$$-\frac{\Delta[FeSO_4]}{\Delta t} = \frac{1.2}{1800} = 6.666\ldots \times 10^{-4}\,\text{mol}\,L^{-1}\,s^{-1}.$$

This is an average rate over half an hour; the data do not specify an instantaneous rate law.

Step 2: Apply the coefficient ratio to the product Using the coefficients 6 for ferrous sulphate and 3

for ferric sulphate gives

$$\frac{\Delta[Fe_2(SO_4)_3]}{\Delta t} = \frac{3}{6}\left(-\frac{\Delta[FeSO_4]}{\Delta t}\right).$$

Hence the formation rate is

$$\frac{1}{2} \times 6.666\ldots \times 10^{-4} = 3.333\ldots \times 10^{-4} = 333.333\ldots \times 10^{-6}\,\text{mol}\,\text{L}^{-1}\,\text{s}^{-1}.$$

The rate is positive because product concentration increases. Round the coefficient multiplying 10^{-6}, rather than rounding the small rate itself to zero.

Step 3: Conclusion The average product formation rate is $3.333\ldots \times 10^{-4}\,\text{mol}\,\text{L}^{-1}\,\text{s}^{-1}$. The requested nearest-integer coefficient is $\boxed{333}$.

Quick Tip to Solve: Use the product-to-reactant coefficient ratio $3/6$ after converting 30 minutes to 1800 seconds.

85. The number of hyperconjugation structures involved to stabilize carbocation formed in the above reaction is _____. [4 marks]



Solution:

Concept:

- In acidic substitution, protonation of an alcohol makes water a leaving group. The resulting carbocation can undergo successive one-carbon alkyl and hydride shifts before bromide traps it.
- A no-bond hyperconjugative structure is counted for each hydrogen on a carbon directly bonded to the positively charged carbon. Count these alpha hydrogens in the rearranged carbocation, rather than in the original alcohol.

Step 1: Identify the starting cation and label the skeleton Label the methyl-bearing upper ring junction P, the lower junction Q, and the carbon bearing the alcohol R. Let S be the methylene of the left-hand six-membered ring directly bonded to P. Protonation followed by loss of water gives a secondary carbocation at R, adjacent to the quaternary carbon P. This first cation is not the final one used for the hyperconjugation count.

Step 2: Follow the shifts to a spiro skeleton First the methyl group migrates from P to R, transferring the positive charge to P. Next a hydride shifts from Q to P, placing the charge on Q. The $P-S$ bond then migrates to form $Q-S$: its electron pair bonds to the electron-deficient Q, leaving the charge on P. The former six-membered ring is now a five-membered ring attached through Q; together with the other five-membered ring, the skeleton is spirocyclic, with Q the common atom.

Step 3: Place the charge on the methyl-bearing carbon A final hydride shift from R to the adjacent cationic P makes P a methylene and puts the positive charge on R. This is a tertiary carbocation. Its three carbon neighbours are the migrated methyl group, the methylene P, and the original methylene on the other side of R in the right-hand ring. The spiro atom Q is not directly bonded to the positive centre and contributes no alpha hydrogen.

Step 4: Count the contributing alpha hydrogens The methyl neighbour supplies three alpha hydrogens, and each of the two methylene neighbours supplies two. Each such carbon-hydrogen bond can donate into the vacant orbital at the carbocation centre, giving one no-bond structure:

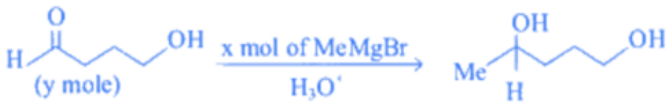
$$N_{\text{hyperconjugative}} = 3 + 2 + 2 = 7.$$

Counting the hydrogens before completing the rearrangements would refer to a different, less stable intermediate.

Step 5: Conclusion The rearranged tertiary carbocation has seven alpha hydrogens, so the number of hyperconjugative structures is $\boxed{7}$. This is a dimensionless count.

Quick Tip to Solve: Complete alkyl and hydride shifts first; then count only the C-H bonds on the three carbons directly attached to the final positively charged carbon.

86. The ratio x/y on completion of the above reaction is _____. [4 marks]



Solution:

Concept:

- A Grignard reagent reacts with an acidic alcohol proton before it can act as a carbon nucleophile. Each alcoholic $\mathrm{O-H}$ group consumes one mole of MeMgBr by proton transfer.
- An aldehyde consumes another mole of Grignard reagent per mole of carbonyl group. Addition of $\mathrm{CH_3}^-$, followed by acidic hydrolysis, converts $-\mathrm{CHO}$ to $-\mathrm{CH(OH)CH_3}$.

Step 1: Identify both reactive sites The starting molecule is a hydroxy aldehyde: it contains one terminal alcohol group and one aldehyde group. The product has a new methyl group on the former carbonyl carbon, while the terminal alcohol is restored during the acidic work-up. Thus the amount of reagent must account for both sites; counting only the carbonyl addition omits an unavoidable acid-base reaction.

Step 2: Consume the first equivalent by proton transfer For each mole of substrate, the alcohol proton reacts with one mole of methylmagnesium bromide:

$$\mathrm{ROH + CH_3MgBr \longrightarrow ROMgBr + CH_4}.$$

The methyl group of this equivalent becomes methane. It is therefore not available for forming the

new carbon-carbon bond. The magnesium alkoxide remains until the subsequent H_3O^+ treatment.

Step 3: Add the second equivalent and perform work-up A second mole of $MeMgBr$ adds to the aldehyde, producing the secondary alkoxide. Acidic hydrolysis protonates both alkoxide groups, giving the two alcohol groups shown. Therefore y moles of substrate require y moles for deprotonation and y moles for carbonyl addition:

$$x = y + y = 2y, \qquad \frac{x}{y} = 2.$$

Step 4: Conclusion The required molar ratio is $\boxed{x/y = 2}$. This is a dimensionless ratio.

Quick Tip to Solve: Count acidic protons first, then count Grignard additions to carbonyl groups; an alcohol consumes reagent without adding a methyl group to the product.

---

87. A mixture of 1 mole of H_2O and 1 mole of CO is taken in a 10 litre container and heated to 725 K. At equilibrium 40% of water by mass reacts with carbon monoxide according to the equation :

$CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$.

The equilibrium constant $K_c \times 10^2$ for the reaction is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- For the reaction $CO + H_2O \rightleftharpoons CO_2 + H_2$, all stoichiometric coefficients are one. Thus

$$K_c = \frac{[CO_2][H_2]}{[CO][H_2O]}.$$

- For a given pure substance, the fraction of its initial mass consumed equals the fraction of its initial number of moles consumed. The common container volume cancels because both sides contain two gaseous species.

Step 1: Convert the mass percentage into reaction extent Initially there is $1\,mol$ of water. Its initial mass is $18\,g$; forty percent corresponds to $7.2\,g$, or $0.40\,mol$. Since water and carbon monoxide react in a one-to-one ratio, the reaction extent is $0.40\,mol$, and the same amount of carbon monoxide is consumed.

Step 2: Write the equilibrium amounts and concentrations The equilibrium amounts are $0.60\,mol$ each of CO and H_2O, and $0.40\,mol$ each of CO_2 and H_2. In the $10\,L$ container these give reactant concentrations $0.060\,mol\,L^{-1}$ each and product concentrations $0.040\,mol\,L^{-1}$ each. No product was initially specified.

Step 3: Evaluate the requested scaled constant Substitute the equilibrium concentrations, rather than the initial ones:

$$K_c = \frac{(0.040)(0.040)}{(0.060)(0.060)} = \frac{4}{9}.$$

The question asks for the constant multiplied by 10^2:

$$K_c \times 10^2 = \frac{400}{9} = 44.444\ldots.$$

Rounding this value, not K_c itself, to the nearest integer gives 44.

Step 4: Conclusion $K_c = 4/9$, so the requested nearest integer is $\boxed{44}$. The constant is dimensionless for this reaction because the concentration powers cancel.

Quick Tip to Solve: For equal initial amounts in a one-to-one reaction with conversion α, use $K_c = [\alpha/(1-\alpha)]^2$; apply any numerical scaling before rounding.

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88. Solid fuel used in rocket is a mixture of Fe_2O_3 and Al (in ratio 1 : 2). The heat evolved (kJ) per gram of the mixture is _____. (Nearest integer)

Given: $\Delta H_f^{\theta}\,(Al_2O_3) = -1700\ \text{kJ mol}^{-1}$

$$\Delta H_f^{\theta}\,(Fe_2O_3) = -840\ \text{kJ mol}^{-1}$$

Molar mass of Fe, Al and O are 56, 27 and 16 g mol^{-1} respectively. [4 marks]

Solution:

Concept:

- Hess law gives $\Delta H^\circ_{\text{rxn}} = \sum \nu \Delta H^\circ_f(\text{products}) - \sum \nu \Delta H^\circ_f(\text{reactants})$. Elements in their standard states have zero standard enthalpy of formation.
- Heat evolved is reported as a positive magnitude. To obtain energy per gram, divide the heat released for a stoichiometric reaction batch by the combined mass of all reactants in that batch.

Step 1: Write the thermite reaction Aluminium reduces iron(III) oxide according to

$$Fe_2O_3(s) + 2\,Al(s) \longrightarrow Al_2O_3(s) + 2\,Fe(s).$$

The stated one-to-two ratio is the stoichiometric mole ratio in this reaction. One mole of iron(III) oxide and two moles of aluminium constitute a complete reacting batch, with no excess reactant left over.

Step 2: Calculate the heat released by one batch Only the oxide formation enthalpies contribute, since $Al(s)$ and $Fe(s)$ are elements in their standard states. Hence

$$\Delta H^\circ_{\text{rxn}} = (-1700) - (-840) = -860\,\text{kJ}.$$

The negative sign means the mixture releases heat; the heat evolved by the batch is therefore $860\,\text{kJ}$, rather than a negative reported release.

Step 3: Divide by the total reactant mass The supplied atomic masses give

$$M(Fe_2O_3) = 2(56) + 3(16) = 160\,\text{g mol}^{-1}.$$

The two moles of aluminium weigh $2(27) = 54\,\mathrm{g}$, so the batch mass is $160 + 54 = 214\,\mathrm{g}$. The specific heat release is

$$\frac{860}{214} = 4.0187\ldots\ \mathrm{kJ\,g^{-1}}.$$

Its nearest integer is 4; using only the oxide mass would incorrectly neglect the aluminium in the fuel mixture.

Step 4: Conclusion The heat evolved per gram of mixture is approximately $4.02\,\mathrm{kJ\,g^{-1}}$, giving the required nearest integer $\boxed{4}$.

Quick Tip to Solve: In a thermite heat-per-gram calculation, include both the oxide and the metal in the denominator, and report heat evolved as a positive magnitude.

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89. In an electrochemical reaction of lead, at standard temperature, if $\mathrm{E^0\left(Pb^{2+}/Pb\right) = m}$ Volt and $\mathrm{E^0\left(Pb^{4+}/Pb\right) = n}$ Volt, then the value of $\mathrm{E^0\left(Pb^{2+}/Pb^{4+}\right)}$ is given by $\mathrm{m - xn}$. The value of x is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- Standard electrode potentials are intensive quantities and cannot be added directly. Convert each half-reaction to standard Gibbs energy using $\Delta G^\circ = -n_e F E^\circ$, where n_e is the number of electrons.
- Gibbs energies add when reactions are added. Reversing a half-reaction reverses both its Gibbs energy and its electrode potential; multiplying a reaction changes its Gibbs energy but not its potential.

Step 1: Convert the two reductions into Gibbs energies The given half-reactions are

$$\mathrm{Pb^{2+} + 2\,e^- \longrightarrow Pb}, \qquad \Delta G_1^\circ = -2Fm,$$

$$\mathrm{Pb^{4+} + 4\,e^- \longrightarrow Pb}, \qquad \Delta G_2^\circ = -4Fn.$$

Here F is the Faraday constant. The factor of four for the second reduction is essential because its oxidation-state change is from $+4$ to zero, not from $+4$ to $+2$.

Step 2: Form the requested oxidation half-reaction The requested notation $\mathrm{Pb^{2+}/Pb^{4+}}$ represents $\mathrm{Pb^{2+} \longrightarrow Pb^{4+} + 2\,e^-}$. Add the first reduction to the reverse of the second reduction; metallic lead cancels, as do two of the electrons. Thus

$$\Delta G^\circ = (-2Fm) + 4Fn = 2F(2n - m).$$

The net half-reaction transfers two electrons.

Step 3: Recover its potential and compare coefficients For the resulting half-reaction, use the same signed relation between Gibbs energy and electrode potential:

$$E^\circ = -\frac{\Delta G^\circ}{2F} = -\frac{2F(2n - m)}{2F} = m - 2n.$$

Comparing this with the stated expression $m - xn$, the coefficient is $x = 2$. This also shows why a simple subtraction of m and n, without electron weighting, would be incorrect.

Step 4: Conclusion $E^\circ(\mathrm{Pb}^{2+}/\mathrm{Pb}^{4+}) = m - 2n\,\mathrm{V}$. Therefore the dimensionless coefficient is $\boxed{x = 2}$.

Quick Tip to Solve: Weight each potential by its electron count through ΔG°; add those energies, then divide by the electron count of the final half-reaction.

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90. An atomic substance A of molar mass $12\ \mathrm{g\ mol^{-1}}$ has a cubic crystal structure with edge length of $300\ \mathrm{pm}$. The no. of atoms present in one unit cell of A is _____. (Nearest integer)
Given the density of A is $3.0\ \mathrm{g\ mL^{-1}}$ and $\mathrm{N_A} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$ [4 marks]

Solution:

Concept:

- The mass of a unit cell containing Z atoms is ZM/N_A, where M is molar mass and N_A is Avogadro constant.
- A cubic cell with edge a has volume a^3. Combining mass and volume gives $\rho = ZM/(N_\mathrm{A}a^3)$, or $Z = \rho a^3 N_\mathrm{A}/M$. All length and density units must be compatible.

Step 1: Convert the edge length into centimetres The given density is $3.0\,\mathrm{g\,mL^{-1}} = 3.0\,\mathrm{g\,cm^{-3}}$. Since $1\,\mathrm{pm} = 10^{-10}\,\mathrm{cm}$, the edge is

$$a = 300 \times 10^{-10} = 3.00 \times 10^{-8}\,\mathrm{cm}.$$

Its cube is

$$a^3 = 27.0 \times 10^{-24} = 2.70 \times 10^{-23}\,\mathrm{cm^3}.$$

Cubing the length conversion is necessary because a volume is required.

Step 2: Calculate the mass and atom count of one cell The density gives the cell mass as

$$m_\mathrm{cell} = \rho a^3 = 3.0(2.70 \times 10^{-23}) = 8.10 \times 10^{-23}\,\mathrm{g}.$$

One atom has mass M/N_A, so dividing the cell mass by this atomic mass gives

$$Z = \frac{(8.10 \times 10^{-23})(6.02 \times 10^{23})}{12} = 4.0635.$$

The exponents cancel, leaving a count rather than a mass.

Step 3: Apply the requested integer rounding The experimental inputs have limited precision, so the numerical calculation need not produce an exact integer before rounding. The nearest whole number to 4.0635 is 4. A face-centred cubic arrangement is consistent with this count, but the density calculation determines the answer without needing to assume the type of cubic lattice in advance.

Step 4: Conclusion The unit cell contains $\boxed{4}$ atoms, to the nearest integer. This is a dimensionless count per unit cell.

Quick Tip to Solve: Convert picometres to centimetres before cubing when density is in $\mathrm{g\,cm^{-3}}$, then use $Z = \rho a^3 N_\mathrm{A}/M$.