JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 11 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 11 April 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let a, b, c and d be positive real numbers such that $a+b+c+d=11$. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is [4 marks]

(A) 110

(B) 108

(C) 90

(D) 55

Correct Answer: (C) 90

Solution:

Concept:

- For positive numbers, the arithmetic mean is at least their geometric mean. Equality holds exactly when all numbers being averaged are equal.
- To maximise a product with exponents $5, 3, 2, 1$ and a fixed sum, split the variables into respectively five, three, two and one equal pieces. The total number of pieces is 11.

Step 1: Apply AM-GM to eleven positive quantities Use five copies of $a/5$, three copies of $b/3$, two copies of $c/2$, and one copy of d. Their sum is the given sum $a+b+c+d=11$. Hence their arithmetic mean is 1, and AM-GM gives

$$\left(\frac{a}{5}\right)^5\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2 d \le 1^{11}=1.$$

Multiplying by the positive constant $5^5 3^3 2^2$ therefore bounds the requested product.

Step 2: Check equality and calculate the parameter The bound is attained when $a/5=b/3=c/2=d=1$, namely at $(a,b,c,d)=(5,3,2,1)$. These positive values do satisfy the sum constraint, so this is a maximum, rather than merely an upper bound. Its value is

$$5^5\cdot 3^3\cdot 2^2 = 3125\cdot 27\cdot 4 = 337500.$$

Consequently $3750\beta = 337500$, giving $\beta = 90$. The alternatives $110, 108, 55$ would give different products and do not equal the attained maximum divided by 3750.

Step 3: Conclusion The maximum is 337500, so $\boxed{\beta=90}$. Hence, the correct option is (C).

Quick Tip to Solve: In a fixed-sum product, distribute the available sum in the ratio of the positive exponents; here that ratio is $5:3:2:1$.

2. The sum of the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$, which are in the ratio $1:3:5$, is equal to : [4 marks]

(A) 63

(B) 92

(C) 25

(D) 41

Correct Answer: (A) 63

Solution:

Concept:

- In $(1+x)^N$, the coefficient of x^r is $\binom{N}{r}$, and consecutive coefficients satisfy $\binom{N}{r+1}/\binom{N}{r} = (N-r)/(r+1)$.
- The given ratio $1:3:5$ means that the second coefficient divided by the first is 3, while the third divided by the second is $5/3$. Both conditions must hold for the same expansion.

Step 1: Express both coefficient ratios Put $N = n+2$, and let the first of the three consecutive coefficients be $\binom{N}{r}$. Then

$$\frac{N-r}{r+1} = 3, \qquad \frac{N-r-1}{r+2} = \frac{5}{3}.$$

The first equation gives $N = 4r+3$. The denominators are positive because r is a nonnegative integer and all three coefficients are present in the expansion.

Step 2: Determine the exponent and add the coefficients Substitute $N = 4r+3$ in the second equation:

$$3(3r+2) = 5(r+2) \quad \Longrightarrow \quad 9r+6 = 5r+10.$$

Thus $r = 1$, $N = 7$, and $n = 5$. The coefficients are therefore

$$\binom{7}{1} = 7, \qquad \binom{7}{2} = 21, \qquad \binom{7}{3} = 35.$$

They indeed have the required ratio $1:3:5$, and their sum is $7+21+35 = 63$. The other proposed totals $92, 25, 41$ cannot arise from this uniquely determined triple.

Step 3: Conclusion The required sum of the three coefficients is $\boxed{63}$. Hence, the correct option is (A).

Quick Tip to Solve: Use adjacent coefficient ratios to find the exponent and position first; knowing only the ratio does not determine the common multiplier.

3. If the system of linear equations

$$7x + 11y + \alpha z = 13$$

$$5x + 4y + 7z = \beta$$

$$175x + 194y + 57z = 361$$

has infinitely many solutions, then $\alpha + \beta + 2$ is equal to : [4 marks]

(A) 6

(B) 4

(C) 5

(D) 3

Correct Answer: (B) 4

Solution:

Concept:

- A system of three linear equations in three unknowns has infinitely many solutions when its coefficient and augmented matrices have the same rank, smaller than three.
- The first two coefficient rows are independent: their x, y minor is $7 \cdot 4 - 5 \cdot 11 = -27 \neq 0$. Thus the third complete equation must be a linear combination of the first two.

Step 1: Find the necessary row combination Let the third coefficient row equal p times the first plus q times the second. Comparing the coefficients of x and y gives

$$7p + 5q = 175, \qquad 11p + 4q = 194.$$

Multiplying these equations by 4 and 5, respectively, and subtracting gives $27p = 270$. Hence $p = 10$, and then $q = 21$. These multipliers are unique because the minor above is nonzero.

Step 2: Match the remaining coefficient and constant The same row combination must also reproduce the coefficient of z and the right-hand side:

$$10\alpha + 21 \cdot 7 = 57, \qquad 10 \cdot 13 + 21\beta = 361.$$

Therefore $10\alpha = -90$, so $\alpha = -9$, while $21\beta = 231$, so $\beta = 11$. With these values the third equation is exactly $10E_1 + 21E_2$; the two independent equations leave one free variable, verifying infinitely many solutions. Finally $\alpha + \beta + 2 = -9 + 11 + 2 = 4$. The values $6, 5, 3$ do not equal this sum.

Step 3: Conclusion $\alpha = -9$ and $\beta = 11$, giving $\boxed{\alpha + \beta + 2 = 4}$. Hence, the correct option is (B).

Quick Tip to Solve: When two coefficient columns already give an independent pair of rows, find the row multipliers from those columns before matching the parameter and constant.

4. For $a \in \mathbb{C}$, let $A = \{z \in \mathbb{C} : \text{Re}(a + \bar{z}) > \text{Im}(\bar{a} + z)\}$ and $B = \{z \in \mathbb{C} : \text{Re}(a + \bar{z}) < \text{Im}(\bar{a} + z)\}$. Then among the two statements :

(S1): If $\text{Re}(a), \text{Im}(a) > 0$, then the set A contains all the real numbers

(S2) : If $\text{Re}(a), \text{Im}(a) < 0$, then the set B contains all the real numbers, [4 marks]

(A) both are false

(B) only (S1) is true

(C) only (S2) is true

(D) both are true

Correct Answer: (A) both are false

Solution:

Concept:

- Write $a = u + iv$ and $z = x + iy$, where u, v, x, y are real. Conjugation changes the sign of the imaginary part and leaves the real part unchanged.
- A claim that a set contains every real number requires its defining inequality to hold for every $x \in \mathbb{R}$ with $y = 0$, including arbitrarily large positive and negative values.

Step 1: Reduce the defining inequalities From $\bar{z} = x - iy$ and $\bar{a} = u - iv$, we obtain

$$\text{Re}(a + \bar{z}) = u + x, \qquad \text{Im}(\bar{a} + z) = y - v.$$

Thus A is described by $x - y > -u - v$, whereas B is described by $x - y < -u - v$. On the real axis these become $x > -u - v$ and $x < -u - v$, respectively. Neither inequality holds for all real values of x.

Step 2: Test each statement with an explicit real number For (S1), even if $u, v > 0$, the real number $x = -u - v - 1$ fails $x > -u - v$, since it is one less than the boundary. Therefore (S1) is false. For (S2), even if $u, v < 0$, the real number $x = -u - v + 1$ fails $x < -u - v$, since it is one greater than the boundary. Therefore (S2) is also false. The signs of u, v only move the boundary; they cannot make either open half-plane contain the whole real axis. Options claiming either statement, or both statements, to be true are consequently excluded.

Step 3: Conclusion Both (S1) and (S2) are false. Hence, the correct option is (A).

Quick Tip to Solve: For a claim involving all real numbers, substitute $z = x$ and test values on both sides of the resulting boundary; a sign condition on the boundary is insufficient.

5. If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial numbers, then the serial number of the word THAMS is : [4 marks]

(A) 103

(B) 104

(C) 102

(D) 101

Correct Answer: (A) 103

Solution:

Concept:

- Dictionary order compares letters from left to right using their alphabetical order. The five distinct letters are ordered $A < H < M < S < T$.
- If a smaller letter is placed at a position while the earlier prefix stays fixed, every permutation of the remaining letters precedes the target. After counting preceding words, add one for the target itself.

Step 1: Count words preceding the initial T The target word is THAMS. Any word beginning with A, H, M, or S comes before every word beginning with T. For each of these four first letters, the other four distinct letters can be arranged in $4! = 24$ ways. Thus

$$4 \cdot 4! = 4 \cdot 24 = 96$$

words precede the entire block beginning with T. We now restrict attention to that block, keeping its first letter fixed.

Step 2: Count within the T block and convert to rank After fixing T, the remaining letters are A, H, M, S. The target second letter is H. Only A is smaller, so the block with prefix TA contributes $3! = 6$ further preceding words. With prefix TH fixed, the remaining letters are A, M, S, and the target suffix AMS is already their first alphabetical arrangement. There are no additional preceding words at the third, fourth or fifth positions. Hence the number of words before THAMS is $96 + 6 = 102$, and its serial number is $102 + 1 = 103$. Option 102 counts predecessors only; 101 and 104 are inconsistent with this exact block count.

Step 3: Conclusion The serial number of THAMS is $\boxed{103}$. Hence, the correct option is (A).

Quick Tip to Solve: Count complete smaller-prefix blocks with factorials and add one only at the end; the number of earlier words is one less than the dictionary rank.

6. Let $\mathrm{A} = \{1,3,4,6,9\}$ and $\mathrm{B} = \{2,4,5,8,10\}$. Let R be a relation defined on $\mathrm{A} \times \mathrm{B}$ such that $\mathrm{R} = \{((a_1,b_1),(a_2,b_2)) : a_1 \le b_2$ and $b_1 \le a_2\}$. Then the number of elements in the set R is : [4 marks]

(A) 180

(B) 26

(C) 52

(D) 160

Correct Answer: (D) 160

Solution:

Concept:

- An element of the relation is an ordered pair $((a_1,b_1),(a_2,b_2))$. It is specified by four entries with $a_1, a_2 \in A$ and $b_1, b_2 \in B$.
- The condition $a_1 \le b_2$ uses only (a_1, b_2), and $b_1 \le a_2$ uses only (b_1, a_2). Since these two choices use disjoint entries, their counts multiply.

Step 1: Count choices satisfying the first inequality For each $a_1 \in \{1,3,4,6,9\}$, count the elements of $B = \{2,4,5,8,10\}$ that are at least a_1. The respective counts are $5,4,4,2,1$: every element works for $a_1 = 1$; only $8, 10$ work for $a_1 = 6$; and only 10 works for $a_1 = 9$. Therefore

$$\#\{(a_1,b_2) : a_1 \le b_2\} = 5+4+4+2+1 = 16.$$

Step 2: Count the independent second choices For $b_1 = 2,4,5,8,10$, the numbers of elements $a_2 \in A$ satisfying $b_1 \le a_2$ are respectively $4,3,2,1,0$. Their sum is 10. Every one of the 16 first choices combines uniquely with each of these 10 second choices, giving

$$|R| = 16 \cdot 10 = 160.$$

The value 26 merely adds independent choice counts instead of multiplying them. The value 52 and the value 180 likewise fail the explicit enumeration; ordered relation elements must retain all four entries.

Step 3: Conclusion The number of elements in R is $\boxed{160}$. Hence, the correct option is (D).

Quick Tip to Solve: Group each inequality by the entries it actually uses. Independent constraints on disjoint pairs allow a product of two much smaller counts.

7. If the 1011^{th} term from the end in the binominal expansion of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^{2022}$ is 1024 times 1011^{th} R term from the beginning, then $|x|$ is equal to [4 marks]

(A) $\frac{5}{16}$

(B) 8

(C) 12

(D) 15

Correct Answer: (A) $\frac{5}{16}$

Solution:

Concept:

- For $(A+B)^N$, the kth term from the beginning is $T_k=\binom{N}{k-1}A^{N-k+1}B^{k-1}$. The kth term from the end is T_{N-k+2}.
- Here $N=2022$, $A=4x/5$, and $B=-5/(2x)$, with $x\neq 0$. The two terms to compare have even powers of the negative quantity B, so their ratio is positive.

Step 1: Identify the two terms and cancel coefficients The 1011th term from the end is $T_{2022-1011+2}=T_{1013}$, whereas the specified term from the beginning is T_{1011}. Hence

$$T_{1013}=\binom{2022}{1012}A^{1010}B^{1012},\qquad T_{1011}=\binom{2022}{1010}A^{1012}B^{1010}.$$

The binomial coefficients are equal because $2022-1012=1010$. Consequently the ratio simplifies to B^2/A^2, avoiding any large coefficient calculation.

Step 2: Use the given ratio to determine the magnitude The condition now gives

$$1024=\frac{B^2}{A^2}=\left(\frac{-5/(2x)}{4x/5}\right)^2=\frac{625}{64x^4}.$$

Therefore

$$x^4=\frac{625}{65536}=\left(\frac{5}{16}\right)^4.$$

For real nonzero x, both signs are possible but the magnitude is fixed: $|x|=5/16$. Substitution into the ratio returns exactly 1024. The other offered magnitudes $8, 12, 15$ give much smaller positive ratios and cannot satisfy the equation.

Step 3: Conclusion The required magnitude is $\boxed{|x|=\frac{5}{16}}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert an end-position to a beginning-position before writing the term; symmetry of the binomial coefficients often makes the ratio collapse.

8. $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$, **then** $\lambda, \frac{\lambda}{3}$ **are the roots of the equation :** [4 marks]

(A) $4x^2+24x-27=0$

(B) $4x^2-24x+27=0$

(C) $4x^2-24x-27=0$

(D) $4x^2+24x+27=0$

Correct Answer: (B) $4x^2-24x+27=0$

Solution:

Concept:

- The determinant is a polynomial in x. Its diagonal entries differ from the common off-diagonal entry by $1, \lambda, \lambda^2$, so subtracting rows makes the expansion short.
- For roots r, s, the monic quadratic is $t^2-(r+s)t+rs=0$. Match both the sum and product rather than choosing by a sign alone.

Step 1: Expand the determinant Subtract row 1 from rows 2 and 3. These operations preserve the determinant and give

$$D=\begin{vmatrix} x+1 & x & x \\ -1 & \lambda & 0 \\ -1 & 0 & \lambda^2 \end{vmatrix} = \lambda^3+x(\lambda+\lambda^2+\lambda^3).$$

In particular, terms of degree two and three in x cancel; expanding the original matrix without using its repeated entries would obscure this cancellation.

Step 2: Compare the constant terms The displayed equality is an identity in x. Equating its constant terms gives

$$\lambda^3=\frac{9\cdot 81}{8}=\frac{729}{8}, \qquad \lambda=\frac{9}{2}.$$

As a check, the coefficient of x is

$$\frac{9}{2}+\frac{81}{4}+\frac{729}{8}=\frac{927}{8}=\frac{9\cdot 103}{8},$$

so the linear coefficient agrees as well.

Step 3: Form the equation with the specified roots The two roots are $9/2$ and $3/2$. Therefore their sum is 6 and their product is $27/4$. The required equation is

$$t^2-6t+\frac{27}{4}=0, \quad \text{or} \quad 4t^2-24t+27=0.$$

Options (A) and (D) give a negative sum, while (C) gives a negative product. Neither agrees with these two positive roots.

Step 4: Conclusion The equation is $4x^2-24x+27=0$. Hence, the correct option is (B).

Quick Tip to Solve: For a matrix with one common off-diagonal entry, subtract one row from the others before expanding; then compare the constant term first.

9. Let the mean of 6 observations $1, 2, 4, 5, x$ and y be 5 and their variance be 10 . Then their mean deviation about the mean is equal to : [4 marks]

(A) $\frac{10}{3}$

(B) $\frac{8}{3}$

(C) $\frac{7}{3}$

(D) 3

Correct Answer: (B) $\frac{8}{3}$

Solution:

Concept:

- The mean fixes the sum of the observations: $\sum x_i = n\bar{x}$. The population variance is $\sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$.
- Mean deviation about the mean uses absolute distances, $\frac{1}{n}\sum |x_i - \bar{x}|$; it is different from variance and standard deviation.

Step 1: Recover the unknown sum There are six observations and the mean is 5, so their sum is 30. The four given observations have sum $1 + 2 + 4 + 5 = 12$. Consequently,

$$x + y = 30 - 12 = 18.$$

We need the individual observations because the absolute deviations depend on their positions relative to the mean.

Step 2: Use the variance to find both observations Using the stated variance,

$$10 = \frac{1^2 + 2^2 + 4^2 + 5^2 + x^2 + y^2}{6} - 25.$$

Hence the sum of all squares is 210, and

$$x^2 + y^2 = 210 - 46 = 164.$$

Now $(x + y)^2 = x^2 + y^2 + 2xy$, so

$$324 = 164 + 2xy, \qquad xy = 80.$$

Thus x, y are the roots of $t^2 - 18t + 80 = (t - 8)(t - 10)$: the unknown observations are $8, 10$, in either order.

Step 3: Average the absolute distances The absolute deviations from 5 are $4, 3, 1, 0, 3, 5$. Their sum is 16, giving

$$\text{MD}_{\bar{x}} = \frac{4 + 3 + 1 + 0 + 3 + 5}{6} = \frac{16}{6} = \frac{8}{3}.$$

This calculation uses all six values, including the zero deviation of the observation 5. The other proposed values $10/3, 7/3, 3$ do not equal this average; taking a square root of the variance would calculate a different statistic.

Step 4: Conclusion The mean deviation about the mean is $8/3$. Hence, the correct option is (B).

Quick Tip to Solve: Use the mean and variance to recover the sum and sum of squares first; calculate absolute deviations only after finding the missing values.

10. If $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying $\int_0^{\frac{\pi}{2}} f(\sin 2x) \sin x dx + \alpha \int_0^{\frac{\pi}{4}} f(\cos 2x) \cos x dx = 0$, then the value of α is : [4 marks]

(A) $-\sqrt{3}$

(B) $\sqrt{2}$

(C) $-\sqrt{2}$

(D) $\sqrt{3}$

Correct Answer: (C) $-\sqrt{2}$

Solution:

Concept:

- The reflection identity $\int_a^b h(x)\,dx = \int_a^b h(a+b-x)\,dx$ can pair complementary sine and cosine factors.
- The shifts $\sin(\pi/2 - 2t) = \cos 2t$ and $\sin(\pi/4 - t) + \cos(\pi/4 - t) = \sqrt{2}\cos t$ convert the first integral to the second.

Step 1: Split and reflect the first integral Write

$$I = \int_0^{\pi/2} f(\sin 2x) \sin x\,dx, \qquad J = \int_0^{\pi/4} f(\cos 2x) \cos x\,dx.$$

Split I at $\pi/4$. In its upper half put $u = \pi/2 - x$. The limits reverse, $\sin 2x = \sin 2u$, and $\sin x = \cos u$. Therefore

$$I = \int_0^{\pi/4} f(\sin 2x)(\sin x + \cos x)\,dx.$$

The continuity of f ensures that these changes of variable in the definite integrals are valid.

Step 2: Shift the quarter-period interval In this expression put $t = \pi/4 - x$. On reversing the limits again,

$$\sin 2x = \cos 2t,$$
$$\sin x + \cos x = \sqrt{2}\cos t.$$

It follows that

$$I = \sqrt{2}\int_0^{\pi/4} f(\cos 2t) \cos t\,dt = \sqrt{2}J.$$

This is an integral identity independent of the particular continuous function: no special formula for f is needed.

Step 3: Match the coefficient The required relation is $I + \alpha J = 0$. Substituting the identity gives

$$(\sqrt{2} + \alpha)J = 0.$$

Thus the coefficient making the relation hold identically is $\alpha = -\sqrt{2}$. The positive value $\sqrt{2}$ adds the two contributions instead of cancelling them, and the magnitudes $\sqrt{3}$ cannot arise from the quarter-period shift. The factor comes from adding sine and cosine at an angle of $\pi/4$.

Step 4: Conclusion The required coefficient is $\alpha = -\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: When an integrand contains $\sin 2x$ on a half-period, split at $\pi/4$, reflect the upper half, and then use a quarter-period shift.

11. Let f and g be two functions defined by

$$f(x) = \begin{cases} x+1, & x < 0 \\ |x-1|, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

Then $(g \circ f)(x)$ is : [4 marks]

(A) continuous everywhere but not differentiable at $x = 1$

(B) differentiable everywhere

(C) not continuous at $x = -1$

(D) continuous everywhere but not differentiable exactly at one point

Correct Answer: (D) continuous everywhere but not differentiable exactly at one point

Solution:

Concept:

- To evaluate $g(f(x))$, the branch of g is chosen from the sign of $f(x)$, not directly from the sign of x.
- A piecewise linear function is continuous at a join if its limiting values agree, and differentiable there only if its one-sided slopes agree.

Step 1: Find where the inner function is negative For $x < 0$, the inner value is $f(x) = x + 1$. This value is negative exactly when $x < -1$. For $-1 \leq x < 0$, it is nonnegative. For every $x \geq 0$, the inner value $|x - 1|$ is also nonnegative. Hence the negative-input branch of g is used only on $(-\infty, -1)$. The change in the absolute value at $x = 1$ does not itself create a change of branch in the composite.

Step 2: Write the composite explicitly If $x < -1$, then $f(x) < 0$ and $g(f(x)) = f(x) + 1 = x + 2$. Everywhere else $f(x) \geq 0$, so the second branch of g gives the constant 1. Thus

$$(g \circ f)(x) = \begin{cases} x+2, & x < -1, \\ 1, & x \geq -1. \end{cases}$$

This formula also covers $x = 0$ and $x = 1$; neither needs an additional piece. In particular, the corner of the original absolute-value function is removed by the constant branch of g.

Step 3: Check the only join At $x = -1$, the left limit is $-1 + 2 = 1$, equal to the right limit and the function value. Therefore the composite is continuous there and hence everywhere. Its left derivative is 1, whereas its right derivative is 0, so it is not differentiable at -1. At all other points it is linear or constant and is differentiable. Option (A) locates the corner incorrectly at 1, (B) misses the corner, and (C) incorrectly asserts a discontinuity.

Step 4: Conclusion The composite is continuous everywhere and not differentiable exactly at the single point $x = -1$. Hence, the correct option is (D).

Quick Tip to Solve: For a composite piecewise function, first solve the condition on the inner output that selects each outer branch; apparent corners can disappear.

12. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{5}{x(x^5+1)}y = \frac{(x^5+1)^2}{x^7}, x > 0$. If $y(1) = 2$, then $y(2)$ is equal to : [4 marks]

(A) $\frac{693}{128}$
(B) $\frac{697}{128}$
(C) $\frac{637}{128}$
(D) $\frac{679}{128}$

Correct Answer: (A) $\frac{693}{128}$

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, an integrating factor is $\mu(x) = e^{\int P(x)\,dx}$, and $(\mu y)' = \mu Q$.
- The domain $x > 0$ permits logarithms of x, and the initial value determines the integration constant before evaluating the required point.

Step 1: Find the integrating factor Decompose the coefficient as

$$\frac{5}{x(x^5+1)} = \frac{5}{x} - \frac{5x^4}{x^5+1}.$$

An antiderivative is $5\log x - \log(x^5+1)$. Therefore a positive integrating factor on the stated domain is

$$\mu(x) = \frac{x^5}{x^5+1}.$$

Multiplying the full differential equation by this factor makes the left side the derivative of μy, rather than a product of separate derivatives.

Step 2: Integrate the transformed equation The right side simplifies to

$$\mu Q = \frac{x^5}{x^5+1}\frac{(x^5+1)^2}{x^7} = \frac{x^5+1}{x^2} = x^3 + x^{-2}.$$

Consequently,

$$\frac{x^5 y}{x^5+1} = \int (x^3 + x^{-2})\,dx = \frac{x^4}{4} - \frac{1}{x} + C.$$

The negative sign in the antiderivative of x^{-2} is essential: differentiating $-1/x$ gives $+x^{-2}$.

Step 3: Apply the initial value and evaluate At $x = 1$, $y = 2$, the left side is 1, so

$$1 = \frac{1}{4} - 1 + C, \qquad C = \frac{7}{4}.$$

At $x = 2$, this gives

$$\frac{32}{33}y(2) = 4 - \frac{1}{2} + \frac{7}{4} = \frac{21}{4}, \qquad y(2) = \frac{33}{32}\frac{21}{4} = \frac{693}{128}.$$

The remaining numerators $697, 637, 679$ cannot satisfy this exact transformed value $21/4$; they result from neither the initial condition nor the integrated equation.

Step 4: Conclusion $y(2) = 693/128$. Hence, the correct option is (A).

Quick Tip to Solve: For a coefficient containing $x(x^n+1)$, try writing it as $n/x - nx^{n-1}/(x^n+1)$ to reveal the integrating factor.

13. Let the function $f : [0, 2] \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} e^{\min\{x^2, x-[x]\}}, & x \in [0, 1) \\ e^{[x-\log_e x]}, & x \in [1, 2] \end{cases}$$

where $[t]$ denotes the greatest integer less than or equal to t. Then the value of the integral $\int_0^2 xf(x)dx$ is : [4 marks]

(A) $2e - 1$

(B) $2e - \frac{1}{2}$

(C) $1 + \frac{3e}{2}$

(D) $(e - 1)\left(e^2 + \frac{1}{2}\right)$

Correct Answer: (B) $2e - \frac{1}{2}$

Solution:

Concept:

- The greatest integer $[t]$ is constant between successive integers. Simplify it on each interval before integrating.
- Values at isolated endpoints do not change a definite integral. The substitution $u = x^2$ evaluates $\int xe^{x^2}\,dx$.

Step 1: Simplify the first interval For $0 \le x < 1$, $[x] = 0$, so $x - [x] = x$. Since $x^2 \le x$ on this interval, the minimum in the exponent is x^2. Thus $f(x) = e^{x^2}$ there, and

$$I_1 = \int_0^1 xe^{x^2}\,dx = \frac{1}{2}\left[e^{x^2}\right]_0^1 = \frac{e-1}{2}.$$

Although the definition changes at 1, its single endpoint value has no effect on this integral.

Step 2: Locate the second exponent between integers Put $h(x) = x - \log_e x$. On $[1, 2]$,

$$h\prime(x) = 1 - \frac{1}{x} \ge 0.$$

Hence h increases from $h(1) = 1$ to $h(2) = 2 - \log_e 2 < 2$. Therefore $1 \le h(x) < 2$ throughout this interval, and its greatest integer is exactly 1. The exponential in the second branch is consequently the constant e, not e^x/x: the brackets in the exponent must be applied before exponentiating.

Step 3: Add the two integrals The second contribution is

$$I_2 = \int_1^2 xe\,dx = e\left[\frac{x^2}{2}\right]_1^2 = \frac{3e}{2}.$$

Combining the pieces,

$$\int_0^2 xf(x)\,dx = \frac{e-1}{2} + \frac{3e}{2} = 2e - \frac{1}{2}.$$

Option (A) subtracts 1 rather than $1/2$, and (C) and (D) do not equal the two contributions obtained

after the floor and minimum operations. The endpoint behaviour cannot supply an extra term to convert this value to any of those choices.

Step 4: Conclusion The integral equals $2e - \frac{1}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Handle the minimum and floor operations first; only then integrate. A monotonicity check often proves that a floor stays constant over an entire interval.

14. The domain of the function $f(x) = \frac{1}{\sqrt{[x]^2-3[x]-10}}$ is : (where [x] denotes the greatest integer less than or equal to x) [4 marks]

(A) $(-\infty, -2) \cup [6, \infty)$

(B) $(-\infty, -3] \cup [6, \infty)$

(C) $(-\infty, -2) \cup (5, \infty)$

(D) $(-\infty, -3] \cup (5, \infty)$

Correct Answer: (A) $(-\infty, -2) \cup [6, \infty)$

Solution:

Concept:

- A square root in a denominator requires its radicand to be strictly positive: zero would leave the denominator zero.
- If $n = [x]$, then n is an integer and $n \le x < n + 1$. Convert an inequality on n to intervals in x only after imposing integrality.

Step 1: Solve the radicand inequality Write $n = [x]$. For the function to have a real, finite value, we require

$$n^2 - 3n - 10 = (n - 5)(n + 2) > 0.$$

This upward-opening quadratic is positive outside its roots. Thus $n < -2$ or $n > 5$. Equality at either root is excluded because the square root is in the denominator. Treating the inequality as non-strict would incorrectly include a zero denominator.

Step 2: Use the integer restriction Because n is an integer, these conditions become $n \le -3$ or $n \ge 6$. The first means exactly $x < -2$: for every such real number the floor is at most -3, whereas at $x = -2$ the floor becomes -2. The second means exactly $x \ge 6$: at $x = 6$ the floor is 6, already valid. Therefore

$$x \in (-\infty, -2) \cup [6, \infty).$$

In particular, all numbers strictly between -3 and -2 are allowed, since their floor is -3.

Step 3: Check the proposed endpoints The value $x = 6$ gives radicand $36 - 18 - 10 = 8 > 0$, so the right endpoint is included. Every x in $[5, 6)$ has floor 5 and gives radicand zero, so using $(5, \infty)$ incorrectly admits these numbers. This rules out (C) and (D). Option (B) wrongly omits the valid interval $(-3, -2)$. These endpoint tests distinguish the choices without ignoring the discontinuities of the floor function.

Step 4: Conclusion The domain is $(-\infty, -2) \cup [6, \infty)$. Hence, the correct option is (A).

Quick Tip to Solve: Solve the quadratic for the integer $[x]$, then translate the allowed integer values into real intervals; test both boundary points explicitly.

15. If the radius of the largest circle with centre (2,0) inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to : [4 marks]

(A) 72

(B) 92

(C) 115

(D) 69

Correct Answer: (B) 92

Solution:

Concept:

- The largest circle with a fixed centre inside an ellipse touches its boundary at a nearest boundary point. Its radius is the minimum Euclidean distance from that centre to the ellipse.
- Use the ellipse equation to eliminate y^2. Minimising the square of a nonnegative distance gives the same point as minimising the distance. The admissible interval here is $-6 \le x \le 6$.

Step 1: Express the distance using one coordinate Let $T(x, y)$ be a boundary point and $C = (2, 0)$. Since $x^2 + 4y^2 = 36$, we have $y^2 = 9 - x^2/4$. Therefore the squared distance is

$$CT^2 = (x - 2)^2 + y^2 = \frac{3}{4}x^2 - 4x + 13.$$

The centre is inside the ellipse because $2^2 < 36$. Thus the first boundary reached as the circle expands determines its largest allowable radius.

Step 2: Minimise the quadratic on the ellipse Complete the square rather than comparing only the horizontal and vertical endpoints:

$$CT^2 = \frac{3}{4}\left(x - \frac{8}{3}\right)^2 + \frac{23}{3}.$$

The minimum occurs at $x = 8/3$, which lies in $[-6, 6]$, and the corresponding $y^2 = 65/9$ is nonnegative. Hence this is an actual boundary point, not merely an algebraic minimum outside the ellipse. Consequently $r^2 = 23/3$ and

$$12r^2 = 12 \cdot \frac{23}{3} = 92.$$

The other proposed numbers, 72, 115 and 69, do not equal this minimum-distance calculation.

Step 3: Conclusion The required value is $12r^2 = 92$. Hence, the correct option is (B).

Quick Tip to Solve: For a circle with prescribed centre inside an ellipse, minimise squared distance to the boundary; the nearest point need not be an axis endpoint.

16. The converse of $((\sim p) \wedge q) \Rightarrow r$ is [4 marks]

(A) $((\sim p) \vee q) \Rightarrow r$

(B) $(\sim \mathrm{r}) \Rightarrow \mathrm{p} \wedge \mathrm{q}$

(C) $(\mathrm{p} \vee (\sim \mathrm{q})) \Rightarrow (\sim \mathrm{r})$

(D) $(\sim \mathrm{r}) \Rightarrow ((\sim \mathrm{p}) \wedge \mathrm{q})$

Correct Answer: (C) $(\mathrm{p} \vee (\sim \mathrm{q})) \Rightarrow (\sim \mathrm{r})$

Solution:

Concept:

- For an implication $A \Rightarrow B$, the converse is $B \Rightarrow A$. The complete antecedent and consequent exchange places without negating either.
- Every implication is logically equivalent to its contrapositive: $X \Rightarrow Y$ is equivalent to $(\sim Y) \Rightarrow (\sim X)$. De Morgan's law gives $\sim ((\sim p) \wedge q) = p \vee (\sim q)$.

Step 1: Form the converse in direct form Take the entire conjunction as the antecedent:

$$A = ((\sim p) \wedge q), \qquad B = r.$$

The given statement is $A \Rightarrow B$, so its converse is

$$r \Rightarrow ((\sim p) \wedge q).$$

This direct form does not appear verbatim among the options. We must therefore check whether an alternative is logically equivalent to it, rather than rejecting all four merely because they look different.

Step 2: Take the contrapositive of the converse The contrapositive of $r \Rightarrow A$ is $(\sim A) \Rightarrow (\sim r)$. Negate the conjunction by De Morgan's law:

$$\sim A = \sim ((\sim p) \wedge q) = p \vee (\sim q).$$

Consequently the converse can be written equivalently as

$$(p \vee (\sim q)) \Rightarrow (\sim r),$$

which is option (C). As an algebraic check, this implication simplifies to

$$\sim (p \vee (\sim q)) \vee (\sim r) = ((\sim p) \wedge q) \vee (\sim r),$$

exactly the disjunction obtained from $r \Rightarrow A$.

Step 3: Eliminate the other alternatives Option (A) changes the original conjunction to a disjunction and leaves r on the right. Option (B) has the wrong antecedent and consequent, while (D) negates r before implying A. For a common counterexample, take p false, q false and r false. The converse is true because r is false. Yet (A), (B) and (D) are all false at these truth values. Thus these options are not equivalent to the converse.

Step 4: Conclusion The converse $r \Rightarrow ((\sim p) \wedge q)$ is equivalent to $(p \vee (\sim q)) \Rightarrow (\sim r)$, its contrapositive. Hence, the correct option is (C).

Quick Tip to Solve: If the direct converse is absent from the choices, take its contrapositive and apply De Morgan's law; equivalent statements can have different appearances.

17. If four distinct points with position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar, then $[\vec{a}\ \vec{b}\ \vec{c}]$ is equal to :

[4 marks]

(A) $[\vec{d}\ \ \vec{b}\ \ \vec{a}] + [\vec{a}\ \ \vec{c}\ \ \vec{d}] + [\vec{d}\ \ \vec{b}\ \ \vec{c}]$
(B) $[\vec{b}\ \ \vec{c}\ \ \vec{d}] + [\vec{d}\ \ \vec{a}\ \ \vec{c}] + [\vec{d}\ \ \vec{b}\ \ \vec{a}]$
(C) $[\vec{a}\ \ \vec{d}\ \ \vec{b}] + [\vec{d}\ \ \vec{c}\ \ \vec{a}] + [\vec{d}\ \ \vec{b}\ \ \vec{c}]$
(D) $[\vec{d}\ \ \vec{c}\ \ \vec{a}] + [\vec{b}\ \ \vec{d}\ \ \vec{a}] + [\vec{c}\ \ \vec{d}\ \ \vec{b}]$

Correct Answer: (D) $[\vec{d}\ \ \vec{c}\ \ \vec{a}] + [\vec{b}\ \ \vec{d}\ \ \vec{a}] + [\vec{c}\ \ \vec{d}\ \ \vec{b}]$

Solution:

Concept:

- Four points are coplanar precisely when the three displacement vectors from any one of them have zero scalar triple product. Use $[\vec{u}\,\vec{v}\,\vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$.
- A scalar triple product is linear in each position. A cyclic rearrangement leaves it unchanged, while interchanging any two positions reverses its sign. A product with two equal vectors is zero.

Step 1: Expand the coplanarity condition Take the point with position vector $\vec{d}$ as reference. Coplanarity gives

$$[\vec{a} - \vec{d}\,\vec{b} - \vec{d}\,\vec{c} - \vec{d}] = 0.$$

Expansion eliminates all terms containing two copies of $\vec{d}$, leaving

$$[\vec{a}\,\vec{b}\,\vec{c}] - [\vec{d}\,\vec{b}\,\vec{c}] - [\vec{a}\,\vec{d}\,\vec{c}] - [\vec{a}\,\vec{b}\,\vec{d}] = 0.$$

This condition concerns differences of position vectors, so it does not require the plane itself to pass through the origin.

Step 2: Rearrange into the printed order Move the last three terms to the other side and apply cyclic invariance:

$$\begin{aligned}[\vec{a}\,\vec{b}\,\vec{c}] &= [\vec{d}\,\vec{b}\,\vec{c}] + [\vec{a}\,\vec{d}\,\vec{c}] + [\vec{a}\,\vec{b}\,\vec{d}] \\ &= [\vec{c}\,\vec{d}\,\vec{b}] + [\vec{d}\,\vec{c}\,\vec{a}] + [\vec{b}\,\vec{d}\,\vec{a}].\end{aligned}$$

These are exactly the three terms in (D). In contrast, (A) reverses the needed signs of the terms involving $[\vec{a}\,\vec{b}\,\vec{d}]$ and $[\vec{a}\,\vec{c}\,\vec{d}]$; (B) has the same sign errors. Option (C) reverses the sign of the needed $[\vec{a}\,\vec{b}\,\vec{d}]$ term. They are not general coplanarity identities.

Step 3: Conclusion $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{d}\,\vec{c}\,\vec{a}] + [\vec{b}\,\vec{d}\,\vec{a}] + [\vec{c}\,\vec{d}\,\vec{b}]$. Hence, the correct option is (D).

Quick Tip to Solve: Expand the triple product of three displacement vectors first, then convert terms to cyclic order. Track a minus sign for every single swap.

18. Let the line passing through the points $\mathrm{P}(2,-1,2)$ and $\mathrm{Q}(5,3,4)$ meet the plane $x-y+z=4$ at the point R. Then the distance of the point R from the plane $x+2y+3z+2=0$ measured parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$ is equal to : [4 marks]

(A) $\sqrt{31}$

(B) $\sqrt{189}$

(C) $\sqrt{61}$

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- A line through P and Q has vector form $P+t(Q-P)$. Substitute it into the plane equation to locate the intersection.
- A distance measured parallel to a specified line is the length along that direction to the target plane. It is generally different from the shortest, perpendicular distance to the plane.

Step 1: Find the intersection point R The direction vector from $P(2,-1,2)$ to $Q(5,3,4)$ is $(3,4,2)$. Hence points on the line satisfy

$$x=2+3t, \qquad y=-1+4t, \qquad z=2+2t.$$

Substituting into $x-y+z=4$ gives

$$2+3t-(-1+4t)+2+2t=4 \quad \Rightarrow \quad 5+t=4.$$

Thus $t=-1$ and $R=(-1,-5,0)$. Checking the first plane gives $-1-(-5)+0=4$, as required.

Step 2: Intersect the parallel line with the second plane The specified symmetric line has direction $(2,2,1)$. The line through R parallel to it is

$$(x,y,z)=(-1,-5,0)+\lambda(2,2,1).$$

At its intersection with $x+2y+3z+2=0$,

$$(-1+2\lambda)+2(-5+2\lambda)+3\lambda+2=0,$$

so $-9+9\lambda=0$, giving $\lambda=1$. The displacement is $(2,2,1)$ and its length is

$$|\lambda|\sqrt{2^2+2^2+1^2}=3.$$

The values $\sqrt{31}, \sqrt{189}, \sqrt{61}$ do not equal this length; they cannot be obtained by the required parallel displacement.

Step 3: Conclusion The distance from R to the second plane measured in the prescribed direction is 3. Hence, the correct option is (D).

Quick Tip to Solve: For distance along a direction, intersect a line through the point parallel to that direction with the plane, then take the displacement length.

19. Let P be the plane passing through the points $(5,3,0),(13,3,-2)$ and $(1,6,2)$. For $\alpha \in \mathbb{N}$, if the distances of the points $A(3,4,\alpha)$ and $B(2,\alpha,a)$ from the plane P are 2 and 3 respectively, then the positive value of a is : [4 marks]

(A) 6

(B) 4

(C) 5

(D) 3

Correct Answer: (B) 4

Solution:

Concept:

- Two nonparallel displacement vectors in a plane have a cross product normal to the plane. The plane through X_0 is $\vec{n} \cdot (\vec{x} - \vec{x}_0) = 0$.
- The perpendicular distance of (x_1, y_1, z_1) from $Ax + By + Cz + D = 0$ is $|Ax_1 + By_1 + Cz_1 + D|/\sqrt{A^2 + B^2 + C^2}$. Both signs of each absolute-value equation must be considered.

Step 1: Determine the plane equation Using $(5,3,0)$ as a base point, the two displacement vectors are

$$\vec{u} = (8,0,-2), \qquad \vec{v} = (-4,3,2).$$

Their cross product is $(6,-8,24) = 2(3,-4,12)$, so a normal is $(3,-4,12)$. Therefore the plane is

$$3(x-5) - 4(y-3) + 12z = 0 \quad \Rightarrow \quad 3x - 4y + 12z - 3 = 0.$$

The normal has magnitude $\sqrt{9+16+144} = 13$. All three given points satisfy this equation.

Step 2: Use the first distance to find alpha For $A = (3,4,\alpha)$, the distance condition gives

$$\frac{|9 - 16 + 12\alpha - 3|}{13} = 2 \quad \Rightarrow \quad |12\alpha - 10| = 26.$$

Hence $\alpha = 3$ or $\alpha = -4/3$. The restriction $\alpha \in \mathbb{N}$ retains only $\alpha = 3$.

Step 3: Apply the second distance and retain positive a Now $B = (2,3,a)$. Its distance from the plane is 3, so

$$\frac{|6 - 12 + 12a - 3|}{13} = 3 \quad \Rightarrow \quad |12a - 9| = 39.$$

The two solutions are $a = 4$ and $a = -5/2$; the positive one is 4. For comparison, the other displayed positive numbers 6, 5 and 3 give distances $63/13, 51/13, 27/13$, respectively, rather than 3.

Step 4: Conclusion The positive value is $a = 4$. Hence, the correct option is (B).

Quick Tip to Solve: Keep both signs in a point-plane distance equation, then use the given natural-number or positivity restriction to discard invalid roots.

20. The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due west of the tower is 30°. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to [4 marks]

(A) 10

(B) $\frac{5}{2}\sqrt{5}$

(C) $5\sqrt{5}$

(D) 5

Correct Answer: (A) 10

Solution:

Concept:

- For an angle of elevation θ, a vertical tower of height h and a horizontal observer distance d satisfy $\tan\theta = h/d$. The angle is measured from the horizontal at the observer.
- Due south and due west are perpendicular horizontal directions from the tower foot. The two observer distances are therefore perpendicular legs of a right triangle, not distances along the same straight line.

Step 1: Find each observer distance from the tower Let O be the tower foot, S the person due south and W the person due west. The vertical tower has height $OP = 5\,\mathrm{m}$. For the person at S,

$$\tan 45^\circ = \frac{OP}{OS} = \frac{5}{OS} \quad \Rightarrow \quad OS = 5\,\mathrm{m}.$$

For the person at W,

$$\tan 30^\circ = \frac{OP}{OW} = \frac{5}{OW} \quad \Rightarrow \quad OW = 5\sqrt{3}\,\mathrm{m}.$$

These distances are along the ground. The sloping sight lines to the top are not the sides needed for the distance between the people.

Step 2: Combine the perpendicular horizontal distances Since the south and west directions meet at a right angle, $\angle SOW = 90^\circ$. Apply Pythagoras to the ground triangle:

$$SW^2 = OS^2 + OW^2 = 5^2 + (5\sqrt{3})^2 = 25 + 75 = 100.$$

As a distance is nonnegative, $SW = 10\,\mathrm{m}$. This also exceeds each individual leg, as a hypotenuse must. The value $5\,\mathrm{m}$ is just the southern distance, $(5/2)\sqrt{5}$ is too short, and $5\sqrt{5}$ has square 125 instead of the required 100. Thus none of the other three options matches the geometry.

Step 3: Conclusion The distance between the two persons is $10\,\mathrm{m}$. Hence, the correct option is (A).

Quick Tip to Solve: First find the horizontal distance from each elevation angle. When observers stand in perpendicular compass directions, combine those distances by Pythagoras.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = 11, \vec{b} \cdot (\vec{a} \times \vec{c}) = 27$ and $\vec{b} \cdot \vec{c} = -\sqrt{3}|\vec{b}|$, then $|\vec{a} \times \vec{c}|^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Write the unknown vector in Cartesian components, $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$. Each given dot product then becomes a linear equation in x, y, z.
- The cyclic identity $\vec{b} \cdot (\vec{a} \times \vec{c}) = (\vec{b} \times \vec{a}) \cdot \vec{c}$ determines the correct sign. The squared magnitude of a vector is the sum of squares of its Cartesian components.

Step 1: Translate the constraints into linear equations Here $|\vec{b}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$, so the last condition is $\vec{b} \cdot \vec{c} = -3$. Also

$$\vec{b} \times \vec{a} = (1, 1, -1) \times (1, 2, 3) = (5, -4, 1).$$

Thus the three equations are

$$\begin{aligned} x + 2y + 3z &= 11, \\ 5x - 4y + z &= 27, \\ x + y - z &= -3. \end{aligned}$$

Using $\vec{a} \times \vec{b}$ in the second equation would reverse its sign, so the order of the cross product matters.

Step 2: Solve for the unknown vector From the third equation, $x = -3 - y + z$. Substitution into the first yields $y + 4z = 14$, hence

$$y = 14 - 4z, \qquad x = -17 + 5z.$$

The second equation becomes

$$5(-17 + 5z) - 4(14 - 4z) + z = 27,$$

which simplifies to $42z = 168$. Therefore $z = 4, y = -2, x = 3$, giving $\vec{c} = (3, -2, 4)$. This vector satisfies $\vec{a} \cdot \vec{c} = 3 - 4 + 12 = 11$ and $\vec{b} \cdot \vec{c} = 3 - 2 - 4 = -3$.

Step 3: Evaluate and check the cross product Now calculate

$$\vec{a} \times \vec{c} = (1, 2, 3) \times (3, -2, 4) = (14, 5, -8).$$

Its dot product with $\vec{b}$ is $14 + 5 + 8 = 27$, verifying the remaining constraint. Finally,

$$|\vec{a} \times \vec{c}|^2 = 14^2 + 5^2 + (-8)^2 = 196 + 25 + 64 = 285.$$

The question asks for the squared magnitude, so there is no final square root. All vectors here are given without physical units.

Step 4: Conclusion The required squared magnitude is $\boxed{285}$.

Quick Tip to Solve: Convert the scalar triple product to a dot product with a known cross product; solve the linear system before taking the final vector magnitude.

22. Let $\mathrm{A} = \{1, 2, 3, 4, 5\}$ and $\mathrm{B} = \{1, 2, 3, 4, 5, 6\}$. Then the number of functions $f : \mathrm{A} \to \mathrm{B}$ satisfying $f(1) + f(2) = f(4) - 1$ is equal to _____. [4 marks]

Solution:

Concept:

- A function assigns exactly one element of the codomain to each element of its domain. Different inputs are allowed to have the same image; injectivity is not required here.
- Use the multiplication principle after counting the ordered pairs $(f(1), f(2))$. The equation fixes $f(4)$, whereas $f(3)$ and $f(5)$ remain unrestricted.

Step 1: Express the dependent image Put $a = f(1)$, $b = f(2)$ and $c = f(4)$. All three belong to $\{1, 2, 3, 4, 5, 6\}$, and the given condition becomes

$$c = a + b + 1.$$

Since $a, b \geq 1$, the lower bound on c is automatic. Its upper bound $c \leq 6$ requires $a + b \leq 5$.

Step 2: Count the permissible ordered pairs For sums $2, 3, 4, 5$, the numbers of positive ordered pairs are respectively $1, 2, 3, 4$: for a sum s, choose a from 1 through $s - 1$, after which $b = s - a$. Thus

$$1 + 2 + 3 + 4 = 10$$

pairs are possible. All their entries lie in the codomain. Order matters because the images of inputs 1 and 2 are distinguishable. Each pair gives exactly one permissible value of c.

Step 3: Assign the remaining inputs Neither $f(3)$ nor $f(5)$ occurs in the condition, so each has six independent choices. These choices may equal any of the other images. Therefore the total number of complete functions is

$$10 \times 6 \times 6 = 360.$$

There is no extra factor for $f(4)$, because its image was already fixed by the chosen pair.

Step 4: Conclusion The required number of functions is $\boxed{360}$.

Quick Tip to Solve: Count the restricted images first; multiply by one codomain-size factor for each input that is still free.

23. Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64\mathrm{x}^2+5\mathrm{Nx}+1=0$ has no real root is $\frac{\mathrm{p}}{\mathrm{q}}$, where p and q are coprime, then $q-p$ is equal to _____. [4 marks]

Solution:

Concept:

- For a real quadratic $ax^2+bx+c=0$, no real root means $b^2-4ac<0$. Equality would give a repeated real root and must be excluded.
- Tosses are independent. With head probability $1/4$, the probability of no head in the first m tosses is $(3/4)^m$. The first-head time N takes positive integer values.

Step 1: Find the permitted first-head times For the equation in the question,

$$\Delta=(5N)^2-4(64)(1)=25N^2-256.$$

Consequently no real root occurs exactly when

$$25N^2<256, \qquad N<\frac{16}{5}.$$

Since N is a positive integer, this event is $N\in\{1,2,3\}$. In particular, $N=4$ is already outside the event; the threshold is not rounded up.

Step 2: Compute the event probability The event $N\le 3$ means that at least one head appears within the first three tosses. Its complement is three consecutive tails. Hence

$$\Pr(N\le 3)=1-\left(\frac{3}{4}\right)^3=1-\frac{27}{64}=\frac{37}{64}.$$

Equivalently, add the disjoint probabilities $1/4+3/16+9/64$ for the first head on tosses 1, 2 and 3. Both methods account for the stopping rule without including any tosses after the first head.

Step 3: Reduce the fraction and form the difference The numerator 37 is prime and does not divide $64=2^6$, so the probability is already in lowest terms. We therefore identify $p=37$ and $q=64$, giving

$$q-p=64-37=27.$$

The probability is not $27/64$; that is the complementary event of three tails.

Step 4: Conclusion The requested numerical value is $\boxed{27}$.

Quick Tip to Solve: Convert the discriminant condition into an integer range for the first-head time, then use the all-tails complement.

24. If A is the area in the first quadrant enclosed by the curve $\mathrm{C}: 2\mathrm{x}^2 - \mathrm{y} + 1 = 0$, the tangent to C at the point $(1, 3)$ and the line $\mathrm{x} + \mathrm{y} = 1$, then the value of $60\ \mathrm{A}$ is _____. [4 marks]

Solution:

Concept:

- The area enclosed between curves is the integral of the upper ordinate minus the lower ordinate over the appropriate interval.
- A tangent at (x_0, y_0) has equation $y - y_0 = y\prime(x_0)(x - x_0)$. The lower boundary may change at the intersection of two lines, requiring a split integral.

Step 1: Find the tangent and boundary intersections The parabola is $y = 2x^2 + 1$, so $dy/dx = 4x$. At $(1, 3)$, the slope is 4 and the tangent is

$$y - 3 = 4(x - 1), \qquad y = 4x - 1.$$

The other line is $y = 1 - x$. These lines meet at $4x - 1 = 1 - x$, or $(x, y) = (2/5, 3/5)$. The parabola meets $y = 1 - x$ when $2x^2 + x = 0$; the first-quadrant boundary point is $(0, 1)$. The tangent meets it at $(1, 3)$.

Step 2: Identify the upper and lower boundaries For $0 \le x \le 2/5$, the enclosed region lies between the parabola above and $1 - x$ below. For $2/5 \le x \le 1$, its lower boundary is instead $4x - 1$. Both pieces lie in the first quadrant. Thus

$$A = \int_0^{2/5} (2x^2 + x)\, dx + \int_{2/5}^{1} (2x^2 - 4x + 2)\, dx.$$

The second integrand is $2(x - 1)^2$, consistent with tangency at the right endpoint.

Step 3: Evaluate the two pieces The contributions are

$$\left[\frac{2x^3}{3} + \frac{x^2}{2}\right]_0^{2/5} = \frac{46}{375}, \qquad \left[\frac{2(x-1)^3}{3}\right]_{2/5}^{1} = \frac{18}{125}.$$

Adding them gives $A = 100/375 = 4/15$ square units. Therefore

$$60A = 60\left(\frac{4}{15}\right) = 16.$$

Splitting at the line-line intersection prevents either line from being incorrectly used across the entire region.

Step 4: Conclusion The area is $4/15$ square units, and $\boxed{60A = 16}$.

Quick Tip to Solve: Find the intersection of the two straight boundaries before integrating; it is where the lower ordinate changes.

25. Let $S = \left\{z \in \mathbb{C} - \{i, 2i\} : \frac{z^2+8iz-15}{z^2-3iz-2} \in \mathbb{R}\right\}$. **If** $\alpha - \frac{13}{11}i \in S, \alpha \in \mathbb{R} - \{0\}$, **then** $242\alpha^2$ **is equal to** _____. [4 marks]

Solution:

Concept:

- For $D \neq 0$, the quotient $N/D = N\overline{D}/|D|^2$ is real exactly when the imaginary part of $N\overline{D}$ is zero.
- Write $z = a + ib$, with real a, b. Retain the nonzero-real-part condition before dividing by a, and check the excluded denominator zeros.

Step 1: Separate numerator and denominator For $z = a + ib$, let the numerator be $U + iV$ and denominator be $R + iT$. Direct expansion gives

$$U = a^2 - b^2 - 8b - 15, \quad V = a(2b + 8),$$
$$R = a^2 - b^2 + 3b - 2, \quad T = a(2b - 3).$$

The denominator factors as $(z - i)(z - 2i)$, so its zeros are exactly the excluded points stated in the question.

Step 2: Impose the reality condition The imaginary part after multiplying by the conjugate denominator is $VR - UT$. Thus

$$\begin{aligned} 0 &= a\{(2b + 8)(a^2 - b^2 + 3b - 2) \\ &\qquad - (2b - 3)(a^2 - b^2 - 8b - 15)\} \\ &= a(11a^2 + 11b^2 + 26b - 61). \end{aligned}$$

For the specified point, $a = \alpha \neq 0$, so division by this factor is justified. It leaves a real quadratic relation in a and b.

Step 3: Substitute the given imaginary part Use $b = -13/11$. Since

$$11b^2 + 26b = \frac{169}{11} - \frac{338}{11} = -\frac{169}{11},$$

the relation becomes

$$11\alpha^2 - \frac{169}{11} - 61 = 0, \qquad \alpha^2 = \frac{840}{121}.$$

These real parts are nonzero, so neither resulting point equals i or $2i$. Finally,

$$242\alpha^2 = 242\left(\frac{840}{121}\right) = 1680.$$

Both possible signs of α give the same requested value.

Step 4: Conclusion The required value is $\boxed{1680}$.

Quick Tip to Solve: Test reality by multiplying by the conjugate denominator; this avoids dividing complex expressions into lengthy fractions.

26. The number of points, where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in \mathbb{R}$ cuts x-axis, is equal to _____. [4 marks]

Solution:

Concept:

- The substitution $t = e^{2x}$ maps the real line bijectively onto $t > 0$. Therefore distinct positive roots in t correspond to distinct real intercepts.
- A reciprocal quartic simplifies after division by t^2 and the substitution $u = t + 1/t$. For positive t, $u \geq 2$, with equality only at $t = 1$.

Step 1: Convert the exponential equation The equation $f(x) = 0$ becomes

$$t^4 - t^3 - 3t^2 - t + 1 = 0, \qquad t = e^{2x} > 0.$$

Divide by t^2, which is nonzero, to get

$$\left(t^2 + \frac{1}{t^2}\right) - \left(t + \frac{1}{t}\right) - 3 = 0.$$

The reciprocal coefficients indicate that positive roots, if present, occur in reciprocal pairs.

Step 2: Solve the reduced quadratic with its domain With $u = t + 1/t$, use $t^2 + t^{-2} = u^2 - 2$. Hence

$$u^2 - u - 5 = 0, \qquad u = \frac{1 \pm \sqrt{21}}{2}.$$

The negative-sign value is negative and cannot satisfy $u \geq 2$. The positive-sign value $u_0 = (1 + \sqrt{21})/2$ is strictly greater than 2, because $\sqrt{21} > 3$. It is therefore admissible.

Step 3: Count the positive roots and confirm crossing The equation $t + 1/t = u_0$ is $t^2 - u_0 t + 1 = 0$. Its discriminant is $u_0^2 - 4 > 0$; its sum and product show that it has two distinct positive roots, one below 1 and the other above 1. Each gives one real $x = (\log t)/2$. These are crossings rather than tangencies: in the factorization $f = t^2(u^2 - u - 5)$, the derivative of the quadratic at u_0 is $2u_0 - 1 \neq 0$, and $du/dt = 1 - t^{-2} \neq 0$ at both roots. Thus both zeros are simple, and the continuous curve changes sign there.

Step 4: Conclusion The curve cuts the x-axis at exactly $\boxed{2}$ points.

Quick Tip to Solve: For a reciprocal exponential quartic, use $t = e^{2x} > 0$ and then $t + 1/t$; always impose the bound $t + 1/t \geq 2$.

27. Let the line $l : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$ meet the plane $P : x + 2y + 3z = 4$ at the point (α, β, γ). If the angle between the line l and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to _____.

[4 marks]

Solution:

Concept:

- If θ is the acute angle between a line of direction $\mathbf{d}$ and a plane of normal $\mathbf{n}$, then $\sin\theta = |\mathbf{d} \cdot \mathbf{n}|/(|\mathbf{d}||\mathbf{n}|)$.
- Use the common parameter in the symmetric line equation to obtain coordinates, then impose the plane equation. The line-plane angle is complementary to the line-normal angle.

Step 1: Parametrize the line and identify directions Let the common value of the three expressions be t. Then

$$x = t, \qquad y = 1 + 2t, \qquad z = 3 + \lambda t.$$

Notice that $1 - y = -2t$ gives a positive $2t$ term in y. A direction vector is $\mathbf{d} = (1, 2, \lambda)$, while the plane normal is $\mathbf{n} = (1, 2, 3)$. Their dot product is $5 + 3\lambda$, and their squared lengths are $5 + \lambda^2$ and 14.

Step 2: Use the given angle to determine the parameter Because $\cos\theta = \sqrt{5/14}$, we have $\sin^2\theta = 9/14$. The line-plane formula gives

$$\frac{(5 + 3\lambda)^2}{14(5 + \lambda^2)} = \frac{9}{14}.$$

Therefore

$$25 + 30\lambda + 9\lambda^2 = 45 + 9\lambda^2, \qquad \lambda = \frac{2}{3}.$$

Squaring is valid because the angle formula uses the absolute dot product; the value obtained also satisfies the original positive sine ratio.

Step 3: Locate the intersection and evaluate the expression Substitute the line coordinates into the plane:

$$t + 2(1 + 2t) + 3\left(3 + \frac{2}{3}t\right) = 4.$$

Thus $7t + 11 = 4$, so $t = -1$ and

$$(\alpha, \beta, \gamma) = \left(-1, -1, \frac{7}{3}\right).$$

These coordinates satisfy the plane: $-1 - 2 + 7 = 4$. Finally,

$$\alpha + 2\beta + 6\gamma = -1 - 2 + 14 = 11.$$

The coefficient of the parameter is nonzero, so the intersection is unique.

Step 4: Conclusion The requested value is $\boxed{11}$.

Quick Tip to Solve: Use sine for a line-plane angle, and handle the minus sign in $(1 - y)/(-2)$ before writing the direction vector.

28. For $k \in \mathbb{N}$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots$ is 10 , then the value of k is _____.

[4 marks]

Solution:

Concept:

- For $|r| < 1$, $\sum r^n = 1/(1-r)$, $\sum nr^n = r/(1-r)^2$, and $\sum n^2 r^n = r(1+r)/(1-r)^3$, with all sums starting at $n = 0$.
- The numerators $1, 4, 8, 13, 19, \ldots$ have successive differences $3, 4, 5, 6, \ldots$, so they form a quadratic sequence. A convergent series requires $k > 1$.

Step 1: Express the general coefficient Index the first coefficient by $n = 0$. The coefficient at index n is

$$a_n = 1 + \sum_{j=1}^{n} (j+2) = \frac{n^2 + 5n + 2}{2}.$$

This gives $a_0 = 1$, $a_1 = 4$, $a_2 = 8$, as required. Since k is natural, $k = 1$ gives terms that grow and hence cannot produce a finite sum. Set $r = 1/k$, so the possible convergent cases have $0 < r < 1$.

Step 2: Sum the three standard series Using the quadratic expression for a_n,

$$\begin{aligned} S &= \frac{1}{2} \sum_{n=0}^{\infty} (n^2 + 5n + 2) r^n \\ &= \frac{r(1+r)}{2(1-r)^3} + \frac{5r}{2(1-r)^2} + \frac{1}{1-r} \\ &= \frac{1 + r - r^2}{(1-r)^3}. \end{aligned}$$

For $k = 2$, we have $r = 1/2$, and consequently

$$S = \frac{1 + 1/2 - 1/4}{(1 - 1/2)^3} = \frac{5/4}{1/8} = 10.$$

Thus this natural value satisfies the condition exactly.

Step 3: Exclude all other natural values Each coefficient a_n is positive. For every $n \geq 1$, the term a_n/k^n strictly decreases as positive k increases. Therefore for any integer $k > 2$, the sum is strictly less than its value 10 at $k = 2$; termwise comparison is justified because both positive series converge. There is no natural integer strictly between 1 and 2, and $k = 1$ was already excluded. Hence the solution in the specified domain is unique.

Step 4: Conclusion The required natural number is $\boxed{k = 2}$.

Quick Tip to Solve: Use first differences to identify a quadratic coefficient, then sum with the three geometric-series identities.

29. Let the tangent to the parabola $y^2 = 12x$ at the point $(3, \alpha)$ be perpendicular to the line $2x + 2y = 3$. Then the square of distance of the point $(6, -4)$ from the normal to the hyperbola $\alpha^2 x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to _____. [4 marks]

Solution:

Concept:

- For a curve defined implicitly, differentiate with respect to x to find the tangent slope. The normal slope is the negative reciprocal of a nonzero tangent slope.
- The perpendicular distance from (x_0, y_0) to $Ax + By + C = 0$ is $d = |Ax_0 + By_0 + C|/\sqrt{A^2 + B^2}$. Squaring removes the absolute-value sign.

Step 1: Find the parameter from the tangent direction The line $2x + 2y = 3$ has slope -1, so a perpendicular tangent has slope 1. Differentiating the parabola gives

$$2y\frac{dy}{dx} = 12, \qquad \frac{dy}{dx} = \frac{6}{y}.$$

At $(3, \alpha)$, therefore $6/\alpha = 1$, giving $\alpha = 6$. This also satisfies the parabola because $6^2 = 12 \cdot 3$; the negative ordinate would give the wrong tangent slope.

Step 2: Write the required hyperbola normal Substitute $\alpha = 6$ into the hyperbola to obtain $4x^2 - y^2 = 36$. The specified point becomes $(5, 8)$, and indeed $4(25) - 64 = 36$. Implicit differentiation yields

$$8x - 2y\frac{dy}{dx} = 0, \qquad \left.\frac{dy}{dx}\right|_{(5,8)} = \frac{4(5)}{8} = \frac{5}{2}.$$

The normal slope is consequently $-2/5$, so its equation is

$$y - 8 = -\frac{2}{5}(x - 5) \quad \Longrightarrow \quad 2x + 5y - 50 = 0.$$

Step 3: Evaluate the squared distance Using $(x_0, y_0) = (6, -4)$,

$$d^2 = \frac{[2(6) + 5(-4) - 50]^2}{2^2 + 5^2} = \frac{(-58)^2}{29} = 116.$$

The denominator uses the coefficients of the normal itself, rather than those of the tangent.

Step 4: Conclusion The required square of the distance is $\boxed{116}$ square coordinate units.

Quick Tip to Solve: Determine the sign of the parabola ordinate from the tangent slope before substituting into the second conic; then use the squared point-to-line formula directly.

30. If the line $l_1 : 3y-2x = 3$ is the angular bisector of the lines $l_2 : x-y+1 = 0$ and $l_3 : \alpha x+\beta y+17 = 0$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- An angle bisector reflects either of the two intersecting lines onto the other. Thus the two lines share the intersection point on the bisector, and their direction vectors are reflections in it.
- For a direction vector v along a mirror line, reflection of a vector u is $u_{\mathrm{r}} = 2\frac{u\cdot v}{v\cdot v}v - u$. The equation of a line is unchanged when multiplied throughout by a nonzero constant.

Step 1: Locate the common intersection The known line is $y = x + 1$. At its intersection with the bisector $3y - 2x = 3$,

$$3(x + 1) - 2x = 3 \quad \Longrightarrow \quad x = 0,\ y = 1.$$

The second line must pass through this same point. Substituting into $\alpha x+\beta y+17 = 0$ gives $\beta+17 = 0$, hence $\beta = -17$.

Step 2: Reflect the direction of the known line A direction vector for $x - y + 1 = 0$ is $u = (1, 1)$, and a direction vector for the bisector is $v = (3, 2)$. Since $u \cdot v = 5$ and $v \cdot v = 13$, its reflected direction is

$$u_{\mathrm{r}} = \frac{10}{13}(3, 2) - (1, 1) = \left(\frac{17}{13}, \frac{7}{13}\right).$$

The reflected line therefore has slope $7/17$. Through $(0, 1)$, its equation is

$$y - 1 = \frac{7}{17}x \quad \Longrightarrow \quad 7x - 17y + 17 = 0.$$

The prescribed constant term is already 17, fixing $\alpha = 7$ and $\beta = -17$ without any scale ambiguity.

Step 3: Substitute both coefficients Keeping the negative sign of β in the linear term,

$$\begin{aligned}\alpha^2 + \beta^2 - \alpha - \beta &= 7^2 + (-17)^2 - 7 - (-17)\\ &= 49 + 289 - 7 + 17 = 348.\end{aligned}$$

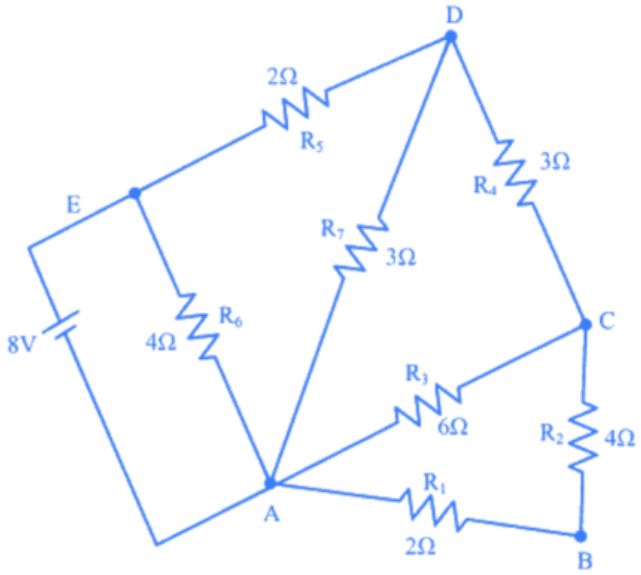
Reflection automatically handles either choice of angular bisector because reflecting across perpendicular bisectors produces the same unoriented line.

Step 4: Conclusion The required value is $\boxed{348}$.

Quick Tip to Solve: Find the common vertex first; it immediately fixes one coefficient. Reflect a direction vector across the bisector to avoid guessing the signs in the angle-bisector formula.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The current flowing through R_2 is : [4 marks]



(A) $\frac{1}{2}$ A

(B) $\frac{1}{3}$ A

(C) $\frac{1}{4}$ A

(D) $\frac{2}{3}$ A

Correct Answer: (B) $\frac{1}{3}$ A

Solution:

Concept:

- For ideal wires and an ideal voltage source, all points on one uninterrupted wire have the same potential. Series resistances add, while parallel branches share the same voltage.
- Ohm's law is $I = V/R$. A resistor directly across an ideal source does not change the voltage available to the other parallel branches.

Step 1: Reduce the network between A and C The path $A \to B \to C$ contains $R_1 = 2\,\Omega$ and $R_2 = 4\,\Omega$ in series, because no extra branch joins their midpoint B. Its resistance is $6\,\Omega$. This path is parallel to $R_3 = 6\,\Omega$, so

$$R_{AC} = \frac{6 \times 6}{6 + 6} = 3\,\Omega.$$

Both branches carry equal current, but the requested current through R_2 is only the current in the A-B-C branch.

Step 2: Reduce the network between D and A The path through C has resistance $R_4 + R_{AC} = 3 + 3 = 6\,\Omega$. It is parallel to $R_7 = 3\,\Omega$, giving

$$R_{DA} = \frac{3 \times 6}{3 + 6} = 2\,\Omega.$$

This is in series with $R_5 = 2\,\Omega$, making the E-D-A branch $4\,\Omega$. The $8\,\text{V}$ source fixes $V_{EA} = 8\,\text{V}$, so the branch current is $8/4 = 2\,\text{A}$. The separate R_6 branch draws extra source current but does not alter this voltage.

Step 3: Recover the current through R2 The voltage across D-A is $2 \times 2 = 4\,\text{V}$. The D-C-A path thus carries $4/6 = 2/3\,\text{A}$. Its two equal $3\,\Omega$ sections split the voltage equally, leaving $2\,\text{V}$ across C-A. Therefore

$$I_{R_2} = \frac{V_{CA}}{R_1 + R_2} = \frac{2}{2+4} = \frac{1}{3}\,\text{A}.$$

The values $1/2$ and $1/4\,\text{A}$ do not follow this branch voltage; $2/3\,\text{A}$ is the current before it divides between the two A-C paths.

Step 4: Conclusion The magnitude of current through R_2 is $\boxed{1/3\,\text{A}}$. Hence, the correct option is (B).

Quick Tip to Solve: Reduce the network from its far-right series pair backwards, then undo the reductions to recover the requested branch current rather than the total source current.

32. A plane electromagnetic wave of frequency $20\ \text{MHz}$ propagates in free space along x-direction. At a particular space and time, $\overrightarrow{\text{E}} = 6.6\hat{j}\ \text{V/m}$. What is $\overrightarrow{\text{B}}$ at this point? [4 marks]

(A) $-2.2 \times 10^{-8}\hat{i}T$

(B) $2.2 \times 10^{-8}\hat{i}T$

(C) $2.2 \times 10^{-8}\hat{k}T$

(D) $-2.2 \times 10^{-8}\hat{k}T$

Correct Answer: (C) $2.2 \times 10^{-8}\hat{k}T$

Solution:

Concept:

- In a plane electromagnetic wave in vacuum, the electric field, magnetic field and propagation direction are mutually perpendicular. Their instantaneous magnitudes satisfy $E = cB$, with $c = 3.0 \times 10^8\,\text{m s}^{-1}$.
- Energy travels in the direction $\vec{E} \times \vec{B}$. For propagation along the positive x axis, $\vec{B} = \frac{1}{c}\hat{i} \times \vec{E}$. Electric and magnetic fields oscillate in phase.

Step 1: Calculate the instantaneous magnetic-field magnitude The supplied electric field at the specified event has magnitude $6.6\,\text{V m}^{-1}$. The vacuum field relation applies to instantaneous values as well as amplitudes, so

$$B = \frac{E}{c} = \frac{6.6}{3.0 \times 10^8} = 2.2 \times 10^{-8}\,\text{T}.$$

The frequency $20\,\text{MHz}$ fixes the oscillation rate and wavelength, but is unnecessary for this local amplitude ratio. There is no factor of frequency in E/B in vacuum.

Step 2: Fix the vector direction by the right-hand rule The propagation direction is $+\hat{i}$ and $\vec{E}$ points along $+\hat{j}$. Hence

$$\vec{B} = \frac{1}{c}\hat{i} \times (6.6\hat{j}) = 2.2 \times 10^{-8}\hat{k}\,\text{T}.$$

As a check, $\hat{j} \times \hat{k} = \hat{i}$, which recovers the stated propagation direction. A magnetic field along either $+\hat{i}$ or $-\hat{i}$ would be longitudinal, contradicting the transverse nature of the wave. A field along $-\hat{k}$ would make $\vec{E} \times \vec{B}$ point towards negative x. Thus options (A), (B) and (D) fail on direction even though they contain the same magnitude.

Step 3: Conclusion At the specified point, $\boxed{\vec{B} = 2.2 \times 10^{-8}\hat{k}\,\mathrm{T}}$. Hence, the correct option is (C).

Quick Tip to Solve: Compute the magnitude from E/c and then use propagation direction crossed with electric-field direction to get the magnetic-field direction; do not reverse that cross product.

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33. An electron is allowed to move with constant velocity along the axis of current carrying straight solenoid.

A. The electron will experience magnetic force along the axis of the solenoid.

B. The electron will not experience magnetic force.

C. The electron will continue to move along the axis of the solenoid.

D. The electron will be accelerated along the axis of the solenoid.

E. The electron will follow parabolic path-inside the solenoid.

Choose the correct answer from the options given below: [4 marks]

(A) B, C and D only

(B) B and C only

(C) A and D only

(D) B and E only

Correct Answer: (B) B and C only

Solution:

Concept:

- The magnetic force on a moving charge is $\vec{F}_B = q\vec{v} \times \vec{B}$, with magnitude $|q|vB\sin\theta$, where θ is the angle between velocity and magnetic field.
- On the axis of a straight current-carrying solenoid the magnetic field is directed along that axis. A zero magnetic force produces no magnetic acceleration or change in the direction of motion.

Step 1: Evaluate the force for axial motion The electron moves along the solenoid axis, and the solenoid field at the axial points is also axial. The velocity is therefore parallel or antiparallel to the magnetic field, depending on current direction and the direction of electron motion. In either case $\theta = 0$ or π, so

$$F_B = |e|vB\sin\theta = 0.$$

The negative charge changes the direction of a nonzero magnetic force, but cannot turn a zero cross product into a force. Statement B is therefore true, while statement A, which proposes an axial magnetic force, is false.

Step 2: Determine the subsequent trajectory With no magnetic force, the magnetic acceleration is

zero:

$$\vec{a}_B = \frac{\vec{F}_B}{m_e} = 0.$$

Under the stated conditions the electron retains its velocity and continues along the axis. Thus statement C is true. Statement D is false because an axial acceleration would require an axial force, which is absent. Statement E is false because a parabolic trajectory would need a nonzero transverse acceleration; no such acceleration is produced here.

Step 3: Match the true statements to the options Exactly B and C are correct. Option (A) includes the false statement D. Option (C) contains A and D, both false. Option (D) combines true B with false E. Only option (B) lists both true statements without adding a false one. The magnitude of the solenoid field is not needed, since its direction alone makes the magnetic cross product vanish.

Step 4: Conclusion The correct statements are B and C only. Hence, the correct option is (B).

Quick Tip to Solve: Check the angle between velocity and field before thinking about circular or helical motion. An axial velocity has no perpendicular component, so there is no magnetic deflection.

34. The Thermodynamic process, in which internal energy of the system remains constant is [4 marks]

(A) Isobaric

(B) Isochoric

(C) Adiabatic

(D) Isothermal

Correct Answer: (D) Isothermal

Solution:

Concept:

- For an ideal gas of fixed amount, internal energy depends only on temperature: $U = U(T)$. For a temperature interval with constant molar heat capacity, $\Delta U = nC_V \Delta T$.
- An isothermal process has constant temperature. The first law, with work done by the gas taken positive, is $\Delta U = Q - W$; unchanged internal energy need not mean that heat and work each vanish.

Step 1: Apply the temperature dependence of internal energy In the ideal-gas thermodynamics used for this question, keeping internal energy constant requires the temperature to remain unchanged. An isothermal process has $T_{\text{final}} = T_{\text{initial}}$, hence

$$\Delta T = 0 \quad \Longrightarrow \quad \Delta U = nC_V(0) = 0.$$

The gas can still expand or compress while exchanging heat with its surroundings. For such a process the first law gives $Q = W$, so supplied heat compensates the work rather than raising the internal energy.

Step 2: Compare the other thermodynamic constraints An isobaric process fixes pressure, not temperature; a gas heated at fixed pressure generally expands and increases its internal energy. An

isochoric process fixes volume, so $W = 0$, but heat supplied at fixed volume changes temperature and gives $\Delta U = Q$. An adiabatic process fixes $Q = 0$, so $\Delta U = -W$; expansion cools the gas and compression heats it. None of these three constraints alone ensures constant internal energy. The constant-temperature condition is the one that does so for an ideal gas.

Step 3: Identify the applicable process The thermodynamic name corresponding to constant temperature is isothermal, making (D) the intended ideal-gas answer. The dependence $U = U(T)$ is essential to this reasoning; constant temperature does not by itself guarantee constant internal energy for every possible nonideal material or changing phase. Within the standard ideal-gas model the result follows directly without needing the pressure-volume path.

Step 4: Conclusion The process is isothermal in the standard ideal-gas model. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish the quantity held fixed by each name: isothermal fixes temperature, isobaric pressure, isochoric volume, and adiabatic heat exchange.

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35. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: A bar magnet dropped through a metallic cylindrical pipe takes more time to come down compared to a non-magnetic bar with same geometry and mass.

Reason R: For the magnetic bar, Eddy currents are produced in the metallic pipe which oppose the motion of the magnetic bar.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) A is true but R is false

(B) A is false but R is true

(C) Both A and R are true but R is NOT the correct explanation of A

(D) Both A and R are true and R is the correct explanation of A

Correct Answer: (D) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Faraday's law gives the induced emf in a conducting loop as $\mathcal{E} = -d\Phi_B/dt$. A moving magnet changes the magnetic flux through loops within a surrounding metal pipe.
- Lenz's law states that the induced current produces a magnetic effect opposing the change that causes it. Eddy currents circulate within the bulk conductor and dissipate electrical energy as Joule heat.

Step 1: Assess the assertion about the falling time As the magnetic bar falls through the pipe, the magnetic flux through successive cross sections of the conducting wall changes. This induces eddy currents whose interaction with the moving magnet creates a retarding magnetic force. Its downward acceleration is then smaller than it would be without this braking effect, and it can approach a terminal speed if the retarding force balances its weight. A non-magnetic bar of the same geometry

and mass does not produce the comparable changing magnetic flux, so it lacks this magnetic braking. The magnetic bar consequently takes longer to descend; Assertion A is true.

Step 2: Assess the reason independently The metal pipe provides closed conducting paths around its circumference. Faraday's law predicts currents because the passing magnetic field changes the flux, and Lenz's law fixes their direction. Ahead of the magnet their field resists its approach; behind it their field resists its departure. The resultant interaction opposes the downward motion. Reason R therefore correctly describes the induced eddy currents and their braking action, and is true.

Step 3: Check whether the reason explains the assertion The effect named in R is exactly the physical cause of the increased falling time in A. Mechanical energy lost by the falling magnetic bar is converted into thermal energy in the pipe, consistent with a force opposing motion. No direct contact or change in the magnet's mass is needed. Options (A) and (B) incorrectly declare one statement false, while (C) denies a causal link that follows from electromagnetic induction and Lenz's law.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (D).

Quick Tip to Solve: In eddy-current braking, trace the chain: changing flux induces current, Lenz's law fixes its direction, and the resulting magnetic interaction opposes the motion.

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36. The energy of He^+ ion in its first excited state is, (The ground state energy for the Hydrogen atom is -13.6 eV) : [4 marks]

(A) -13.6 eV

(B) -27.2 eV

(C) -3.4 eV

(D) -54.4 eV

Correct Answer: (A) -13.6 eV

Solution:

Concept:

- A hydrogen-like ion has one electron. Its bound-state energy is $E_n = -13.6Z^2/n^2\,\mathrm{eV}$, where Z is the nuclear charge and n is the principal quantum number.
- The ground state has $n = 1$; the first excited state has $n = 2$. A negative energy means the electron is bound relative to a free electron at infinity.

Step 1: Identify the ion and the level Helium has atomic number two. Removing one electron leaves the one-electron ion He^+, so the hydrogen-like formula applies with $Z = 2$. The words first excited state specify $n = 2$, not the ground state. Thus both the nuclear-charge factor and the level factor must be included.

Step 2: Substitute the quantum numbers Using the given hydrogen ground-state energy as the energy scale,

$$E_2(\mathrm{He}^+) = (-13.6)\frac{2^2}{2^2}\,\mathrm{eV} = -13.6\,\mathrm{eV}.$$

The factor of four from the nuclear charge is exactly cancelled by the factor of four from the excited level. This also checks the sign: an electron in the second bound level still has negative energy.

Step 3: Check the alternatives The value $-54.4\,\text{eV}$ is the helium-ion ground-state energy and therefore corresponds to the wrong level. The value $-3.4\,\text{eV}$ is the first excited energy of hydrogen, so it omits the nuclear-charge factor. The value $-27.2\,\text{eV}$ uses neither the required Z^2/n^2 ratio nor a correct integer level for this ion.

Step 4: Conclusion The first excited-state energy of He^+ is $\boxed{-13.6\,\text{eV}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a hydrogen-like ion compare Z/n with one: when $Z = n$, its energy equals the hydrogen ground-state energy.

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37. When one light ray is reflected from a plane mirror with 30° angle of reflection, the angle of deviation of the ray after reflection is : [4 marks]

(A) 140°

(B) 130°

(C) 120°

(D) 110°

Correct Answer: (C) 120°

Solution:

Concept:

- The angle of incidence and angle of reflection are measured from the normal, and the law of reflection gives $i = r$.
- The deviation is the angle between the incident ray continued straight ahead and the actual reflected ray. For a plane mirror it is $\delta = 180^\circ - 2i$.

Step 1: Apply the law of reflection The given reflection angle is $r = 30^\circ$, measured between the reflected ray and the mirror normal. Therefore $i = 30^\circ$ as well. It would be incorrect to treat the stated angle as one measured from the mirror surface; that complementary angle is 60°. The normal is the reference line for the reflection law.

Step 2: Calculate the turning angle The incident and reflected ray segments make an angle $i + r = 60^\circ$ on the reflecting side of the mirror. However, the unreflected straight continuation of the incident ray points in the opposite direction to the incident segment drawn away from the point of incidence. Hence the actual turning of its direction is the supplement:

$$\delta = 180^\circ - (i + r) = 180^\circ - 60^\circ = 120^\circ.$$

This is consistent with normal incidence: a ray then reverses its direction and has deviation 180°.

Step 3: Compare the listed angles Only 120° satisfies the deviation formula for an incidence angle of 30°. Conversely, deviations 140°, 130°, and 110° would require incidence angles 20°, 25°, and 35°, respectively. None agrees with the given reflection angle.

Step 4: Conclusion The angle of deviation is $\boxed{120^\circ}$. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish the angle between the two ray segments from the turning of the light direction: mirror deviation is the supplement of $2i$.

38. A capacitor of capacitance C is charged to a potential V. The flux of the electric field through a closed surface enclosing the positive plate of the capacitor is : [4 marks]

(A) Zero

(B) $\frac{CV}{\varepsilon_0}$

(C) $\frac{CV}{2\varepsilon_0}$

(D) $\frac{2CV}{\varepsilon_0}$

Correct Answer: (B) $\frac{CV}{\varepsilon_0}$

Solution:

Concept:

- Capacitance relates the magnitude of either plate charge to the potential difference: $Q = CV$. The plates carry equal and opposite charges $+Q$ and $-Q$.
- Gauss's law gives the net outward electric flux through any closed surface as $\Phi_E = Q_{\text{enclosed}}/\varepsilon_0$. Charges outside the surface do not contribute to its net flux.

Step 1: Find the enclosed charge The charged capacitor has a positive plate carrying $+Q$ and a negative plate carrying $-Q$. Since its capacitance is C and the potential difference is V,

$$Q = CV.$$

The surface specified in the question encloses the positive plate, so its enclosed charge is $+CV$. It does not enclose both plates. The neutrality of the capacitor as a whole is therefore not the relevant enclosed-charge condition.

Step 2: Apply Gauss law to the closed surface Take the area vectors to point outward from the closed surface. Gauss's law then gives

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{CV}{\varepsilon_0}.$$

The result is positive, as expected for net positive charge enclosed. The negative plate may change the local field on portions of the surface, but because it is outside, its total signed contribution to the closed-surface flux is zero.

Step 3: Reject the incorrect factors Zero would apply to a surface enclosing both plates and hence zero net charge. The factor $1/2$ belongs to some single-sheet field expressions, not the total flux fixed by Gauss's law. A factor of two would incorrectly count two positive charges. Thus neither $CV/(2\varepsilon_0)$ nor $2CV/\varepsilon_0$ is appropriate.

Step 4: Conclusion The net outward flux is $\boxed{\Phi_E = CV/\varepsilon_0}$, in $\mathrm{N\,m^2/C}$. Hence, the correct option is (B).

Quick Tip to Solve: For a closed-surface flux question, first list only the charges inside the surface; use the capacitor relation $Q = CV$ afterwards.

39. The ratio of the de-Broglie wavelengths of proton and electron having same Kinetic energy : (Assume $m_p = m_e \times 1849$) [4 marks]

(A) 1:43

(B) 1:62

(C) 2:43

(D) 1:30

Correct Answer: (A) 1:43

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of momentum.
- For nonrelativistic motion, $K = p^2/(2m)$, giving $\lambda = h/\sqrt{2mK}$. At fixed kinetic energy the wavelength is inversely proportional to the square root of mass.

Step 1: Express both wavelengths at the same energy Let the common kinetic energy of the proton and electron be K. Their momenta are different even though their energies are equal:

$$p_p = \sqrt{2m_pK}, \qquad p_e = \sqrt{2m_eK}.$$

The heavier proton has the larger momentum. Since wavelength is inversely proportional to momentum, its wavelength must be smaller. This gives a useful direction check before evaluating the ratio.

Step 2: Form the proton-to-electron ratio Using the de Broglie relation for each particle,

$$\frac{\lambda_p}{\lambda_e} = \frac{h/\sqrt{2m_pK}}{h/\sqrt{2m_eK}} = \sqrt{\frac{m_e}{m_p}}.$$

The common energy and Planck's constant cancel. Substitute the specified mass relation $m_p = 1849m_e$:

$$\frac{\lambda_p}{\lambda_e} = \frac{1}{\sqrt{1849}} = \frac{1}{43},$$

because $43^2 = 1849$.

Step 3: Check the other ratios The ratio is dimensionless and independent of the particular common kinetic energy. The ratios $1:62$, $2:43$, and $1:30$ do not give the exact reciprocal square root of the stated mass ratio. Using a direct inverse mass ratio would also be wrong: that dependence applies at fixed speed, whereas the question fixes kinetic energy.

Step 4: Conclusion The required ratio is $\boxed{\lambda_p : \lambda_e = 1:43}$. Hence, the correct option is (A).

Quick Tip to Solve: At equal kinetic energy use $\lambda \propto m^{-1/2}$; at equal speed use $\lambda \propto m^{-1}$. Identify what is fixed first.

40. Eight equal drops of water are falling through air with a steady speed of $10\ \mathrm{cm/s}$. If the drops coalesce, the new velocity is:- [4 marks]

(A) $40\ \mathrm{cm/s}$

(B) $16\ \mathrm{cm/s}$

(C) $10\ \mathrm{cm/s}$

(D) $5\ \mathrm{cm/s}$

Correct Answer: (A) $40\ \mathrm{cm/s}$

Solution:

Concept:

- For a small spherical drop in the Stokes regime, viscous drag is $6\pi\eta r v$. Balancing it with the effective weight gives $v_t = 2(\rho - \rho_{\text{air}})gr^2/(9\eta)$.
- For the same liquid and surrounding air, terminal speed is proportional to the square of the radius. On coalescence, the total volume of the liquid is conserved.

Step 1: Find the radius after coalescence Let each original spherical drop have radius r and the combined drop have radius R. Conservation of the total volume of eight equal drops gives

$$\frac{4}{3}\pi R^3 = 8\left(\frac{4}{3}\pi r^3\right).$$

Therefore $R^3 = 8r^3$, so $R = 2r$. The radius doubles rather than becoming eight times as large because volume, not radius, is additive.

Step 2: Use the terminal-speed scaling The original steady speed is the terminal speed $v_t = 10\,\mathrm{cm/s}$. For the combined drop in the same air, all density, viscosity and gravity factors are unchanged. Hence

$$\frac{V_t}{v_t} = \frac{R^2}{r^2} = 2^2 = 4, \qquad V_t = 4(10) = 40\,\mathrm{cm/s}.$$

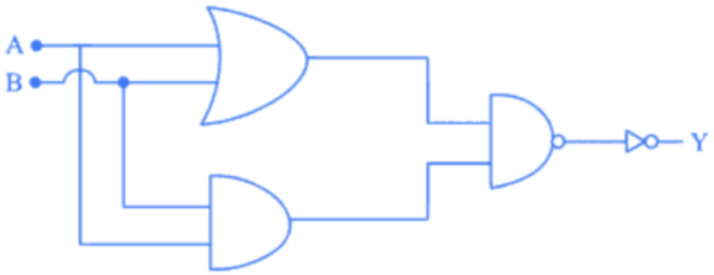
This is the new steady falling speed after the merged drop adjusts to the balance between gravity, buoyancy and viscous drag.

Step 3: Check the alternatives and interpretation The value $16\,\mathrm{cm/s}$ does not follow from either volume conservation or the square-radius scaling. Keeping the speed at $10\,\mathrm{cm/s}$ ignores the change of radius, while $5\,\mathrm{cm/s}$ reverses the trend: a larger drop has greater effective weight relative to its Stokes drag. The steady-speed interpretation, rather than the instant of collision, determines the requested value.

Step 4: Conclusion The new terminal velocity has magnitude $\boxed{40\,\mathrm{cm/s}}$, directed downward. Hence, the correct option is (A).

Quick Tip to Solve: When n identical drops merge in the Stokes regime, radius grows by $n^{1/3}$ and terminal speed by $n^{2/3}$.

41. The logic operations performed by the given digital circuit is equivalent to: [4 marks]



(A) NOR

(B) AND

(C) NAND

(D) OR

Correct Answer: (B) AND

Solution:

Concept:

- An OR gate gives $A + B$, an AND gate gives AB, and an inversion bubble complements its input or output. Here $+$ means Boolean OR rather than arithmetic addition.
- Two successive inversions cancel: $\overline{\overline{X}} = X$. The Boolean absorption identity $AB(A + B) = AB$ simplifies this circuit.

Step 1: Write the two intermediate outputs The upper gate is an OR gate with inputs A and B, giving $P = A + B$. The lower gate is an AND gate receiving the same two inputs, giving $Q = AB$. The small crossover on a wire indicates that crossing wires are not joined at that location; the marked junctions supply the intended branches. Thus the two parallel operations must both be included.

Step 2: Account for the output inversions The next gate has an AND-shaped body and a bubble at its output, so it is NAND. Its output is $\overline{PQ}$. The final triangular gate with an output bubble is a NOT gate. It complements that NAND output once again:

$$Y = \overline{\overline{PQ}} = PQ = (A + B)AB.$$

Ignoring either bubble would incorrectly leave a complemented result and suggest NAND.

Step 3: Simplify and test the truth values Expand with Boolean algebra:

$$Y = AAB + ABB = AB + AB = AB,$$

using $A^2 = A$, $B^2 = B$, and $X + X = X$. Equivalently the outputs for input pairs $00, 01, 10, 11$ are $0, 0, 0, 1$. This is exactly the AND truth table. OR would give one for 01 and 10; NOR would give one for 00; NAND would give zero for 11. Each therefore disagrees with the given circuit.

Step 4: Conclusion The circuit performs $\boxed{Y = AB}$, which is the AND operation. Hence, the correct option is (B).

Quick Tip to Solve: Count every inversion bubble before simplifying a gate network: NAND followed by NOT restores AND of the intermediate inputs.

42. A projectile is projected at 30° from horizontal with initial velocity $40\ \text{ms}^{-1}$. The velocity of the projectile at $\text{t} = 2\ \text{s}$ from the start will be : (Given $g = 10\ \text{m/s}^2$) [4 marks]

(A) $20\sqrt{3}\ \text{ms}^{-1}$

(B) Zero

(C) $20\ \text{ms}^{-1}$

(D) $40\sqrt{3}\ \text{ms}^{-1}$

Correct Answer: (A) $20\sqrt{3}\ \text{ms}^{-1}$

Solution:

Concept:

- For projectile motion with negligible air resistance, the horizontal velocity is constant: $v_x = u\cos\theta$. The vertical velocity is $v_y = u\sin\theta - gt$.
- The velocity vector is $\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$, and its magnitude is $\sqrt{v_x^2 + v_y^2}$. At the highest point only the vertical component vanishes.

Step 1: Resolve the launch velocity Take the positive horizontal direction along the launch and the positive vertical direction upward. With $u = 40\,\text{m/s}$ and $\theta = 30^\circ$,

$$u_x = 40\cos 30^\circ = 20\sqrt{3}\,\text{m/s}, \qquad u_y = 40\sin 30^\circ = 20\,\text{m/s}.$$

Gravity acts downward, so it changes only the vertical component. No horizontal force is present in the ideal projectile model.

Step 2: Evaluate both components after two seconds At $t = 2\,\text{s}$, the horizontal component is still $v_x = 20\sqrt{3}\,\text{m/s}$. The vertical component is

$$v_y = 20 - 10(2) = 0.$$

The projectile is at its highest point because the upward velocity has just been reduced to zero. It has not stopped: its horizontal component remains nonzero.

Step 3: Find the velocity magnitude and compare options The velocity vector at that instant is $\mathbf{v} = 20\sqrt{3}\,\hat{\mathbf{i}}\,\text{m/s}$, so

$$|\mathbf{v}| = \sqrt{(20\sqrt{3})^2 + 0^2} = 20\sqrt{3}\,\text{m/s}.$$

Zero would confuse vertical velocity with total velocity. The value $20\,\text{m/s}$ is the initial vertical component, not the speed at the top. The value $40\sqrt{3}\,\text{m/s}$ doubles the conserved horizontal component and exceeds the initial speed, which is impossible while the projectile rises.

Step 4: Conclusion The velocity is horizontal with magnitude $\boxed{20\sqrt{3}\,\text{m/s}}$. Hence, the correct option is (A).

Quick Tip to Solve: Calculate the time to the top, $u\sin\theta/g$, first; if the given time matches it, the speed is simply $u\cos\theta$.

43. A body of mass $500\ \mathrm{g}$ moves along x-axis such that it's velocity varies with displacement x according to the relation $v = 10\sqrt{x}\ \mathrm{m/s}$ the force acting on the body is:- [4 marks]

(A) 166 N

(B) 5 N

(C) 25 N

(D) 125 N

Correct Answer: (C) 25 N

Solution:

Concept:

- Newton's second law gives the net force as $F = ma$, with mass in kilograms and acceleration in $\mathrm{m\,s^{-2}}$.
- When velocity is given as a function of position, the chain rule gives $a = \dfrac{dv}{dt} = \dfrac{dv}{dx}\dfrac{dx}{dt} = v\dfrac{dv}{dx}$. This avoids finding position explicitly as a function of time.

Step 1: Convert the mass and identify the given relation The mass is $500\,\mathrm{g} = 0.500\,\mathrm{kg}$, and the velocity along the positive x-axis is $v = 10\sqrt{x}\,\mathrm{m\,s^{-1}}$, with x measured in metres. Velocity is changing as the body moves, so its acceleration must be obtained by differentiation. The coefficient 10 is used with the stated SI position and velocity units.

Step 2: Calculate the acceleration using position Differentiate the velocity with respect to displacement, then multiply by the velocity itself:

$$\frac{dv}{dx} = \frac{5}{\sqrt{x}}, \qquad a = v\frac{dv}{dx} = (10\sqrt{x})\frac{5}{\sqrt{x}} = 50\,\mathrm{m\,s^{-2}}.$$

For $x > 0$, the position factors cancel. Thus the speed increases with position but the acceleration remains constant; dv/dx alone is not the acceleration.

Step 3: Apply the force law and compare the options Using the mass in kilograms,

$$F = ma = (0.500)(50) = 25\,\mathrm{N}.$$

The force is in the positive x-direction because the calculated acceleration is positive. Options (A), (B) and (D) give 166 N, 5 N and 125 N respectively; none equals the product of the stated mass and the derived acceleration. Treating grams as kilograms would also give an incorrect force.

Step 4: Conclusion The net force acting on the body is $25\,\mathrm{N}$, directed along the positive x-axis. Hence, the correct option is (C).

Quick Tip to Solve: For a relation $v^2 = 100x$, differentiate directly: $2a = d(v^2)/dx = 100$, so $a = 50\,\mathrm{m\,s^{-2}}$.

44. A space ship of mass 2×10^4 kg is launched into a circular orbit close to the earth surface. The additional velocity to be imparted to the space ship in the orbit to overcome the gravitational pull will be (if $g = 10$ m/s^2 and radius of earth $= 6400$ km): [4 marks]

(A) $7.9(\sqrt{2}-1)$km/s

(B) $11.2(\sqrt{2}-1)$km/s

(C) $7.4(\sqrt{2}-1)$km/s

(D) $8(\sqrt{2}-1)$km/s

Correct Answer: (D) $8(\sqrt{2}-1)$km/s

Solution:

Concept:

- For a near-surface circular orbit, $v_o = \sqrt{GM/R} = \sqrt{gR}$, where R is the Earth's radius and g is its surface gravitational acceleration.
- The escape speed at the same radius is $v_e = \sqrt{2GM/R} = \sqrt{2}\,v_o$. The smallest extra speed is obtained by increasing the existing tangential velocity in its own direction.

Step 1: Calculate the speed already possessed in orbit Convert the given radius into SI units: $R = 6400\,\text{km} = 6.4 \times 10^6\,\text{m}$. The circular orbital speed follows by equating gravity to the centripetal force:

$$\frac{mv_o^2}{R} = mg, \qquad v_o = \sqrt{gR} = \sqrt{10(6.4 \times 10^6)} = 8000\,\text{m s}^{-1}.$$

Thus the spacecraft is already moving at $8\,\text{km s}^{-1}$; it does not start the escape manoeuvre from rest.

Step 2: Find the additional speed needed for escape At the threshold of escape the final total mechanical energy is zero. At the orbital radius this requires $\frac{1}{2}mv_e^2 - GMm/R = 0$, hence $v_e = \sqrt{2gR}$. The required tangential increment is therefore

$$\Delta v = v_e - v_o = (\sqrt{2}-1)\sqrt{gR} = 8(\sqrt{2}-1)\,\text{km s}^{-1}.$$

The spacecraft mass cancels from both orbital and escape conditions, so the given 2×10^4 kg does not enter this speed difference.

Step 3: Check the numerical coefficient The coefficient must be 8 because the problem explicitly specifies $g = 10\,\text{m s}^{-2}$ and $R = 6400\,\text{km}$. Option (A) uses the familiar rounded orbital speed 7.9 km/s from different constants; (B) incorrectly places the escape-speed coefficient 11.2 in the difference, and (C) uses 7.4. They do not follow from the supplied values.

Step 4: Conclusion The additional velocity is $8(\sqrt{2}-1)\,\text{km s}^{-1}$, approximately $3.31\,\text{km s}^{-1}$, tangentially in the direction of motion. Hence, the correct option is (D).

Quick Tip to Solve: An orbiting body already has speed $\sqrt{gR}$. Subtract that from $\sqrt{2gR}$; do not supply the full escape speed again.

45. When vector $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ is subtracted from vector $\overrightarrow{\mathrm{B}}$, it gives a vector equal to $2\hat{j}$. Then the magnitude of vector $\overrightarrow{\mathrm{B}}$ will be : [4 marks]

(A) 3

(B) $\sqrt{33}$

(C) $\sqrt{6}$

(D) $\sqrt{5}$

Correct Answer: (B) $\sqrt{33}$

Solution:

Concept:

- Vector subtraction is ordered: subtracting $\vec{A}$ from $\vec{B}$ means $\vec{B} - \vec{A}$, not $\vec{A} - \vec{B}$.
- The Cartesian unit vectors are mutually perpendicular. For $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, its magnitude is $|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$.

Step 1: Translate the statement into a vector equation The stated result of subtracting the given vector from $\vec{B}$ is $2\hat{j}$. Consequently,

$$\vec{B} - \vec{A} = 2\hat{j}, \qquad \vec{B} = \vec{A} + 2\hat{j}.$$

The added vector has only a y-component, so the x- and z-components of $\vec{B}$ remain equal to the corresponding components of $\vec{A}$. This sign is fixed by the wording of the subtraction.

Step 2: Find the components of the unknown vector Insert $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$:

$$\vec{B} = 2\hat{i} + 3\hat{j} + 2\hat{k} + 2\hat{j} = 2\hat{i} + 5\hat{j} + 2\hat{k}.$$

As a check, subtracting $\vec{A}$ from this expression gives $(2-2)\hat{i} + (5-3)\hat{j} + (2-2)\hat{k} = 2\hat{j}$, exactly as required. There is no extra x- or z-component in the difference.

Step 3: Compute the magnitude and reject the other choices Use the sum of squared components rather than their algebraic sum:

$$|\vec{B}| = \sqrt{2^2 + 5^2 + 2^2} = \sqrt{4 + 25 + 4} = \sqrt{33}.$$

The magnitude must exceed 5 because its y-component alone is 5 and its other components are nonzero. This immediately rules out 3, $\sqrt{6}$ and $\sqrt{5}$, which are all smaller than 5. Adding component magnitudes directly would not give the length of a three-dimensional vector.

Step 4: Conclusion The unknown vector is $\vec{B} = 2\hat{i} + 5\hat{j} + 2\hat{k}$, with magnitude $\sqrt{33}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the subtraction equation before changing signs. Then check that the resulting vector reproduces the given difference component by component.

46. If force (F), velocity (V) and time (T) are considered as fundamental physical quantity, then dimensional formula of density will be : [4 marks]

(A) $FV^{-2}\ T^{2}$

(B) $FV^{4}\ T^{-6}$

(C) $F^{2}\ V^{-2}\ T^{6}$

(D) $FV^{-4}\ T^{-2}$

Correct Answer: (D) $FV^{-4}\ T^{-2}$

Solution:

Concept:

- Mass density has ordinary dimensions $[\rho] = ML^{-3}$. A change of the chosen fundamental quantities changes its dimensional representation, not the physical density.
- Use $[F] = MLT^{-2}$ and $[V] = LT^{-1}$. Equate powers of M, L and T independently to express density using force, velocity and time.

Step 1: Assume powers of the new fundamental quantities Let the required dimensional expression be $[\rho] = [F]^a[V]^b[T]^c$. Here a, b and c are numbers to be determined, and T denotes time. Substitute the conventional dimensional formulae:

$$ML^{-3} = (MLT^{-2})^a(LT^{-1})^bT^c = M^aL^{a+b}T^{-2a-b+c}.$$

The exponents must agree for any choice of mass, length and time units; numerical constants cannot correct a dimensional mismatch.

Step 2: Solve the equations for the exponents Matching the mass exponent gives $a = 1$. The length exponent then gives $a + b = -3$, hence $b = -4$. Finally the time exponent on the left is zero, so

$$-2a - b + c = 0 \quad \Rightarrow \quad -2 + 4 + c = 0 \quad \Rightarrow \quad c = -2.$$

Therefore the expression contains the first power of force, the inverse fourth power of velocity and the inverse second power of time.

Step 3: Verify the result against the alternatives Direct substitution checks the result:

$$[FV^{-4}T^{-2}] = (MLT^{-2})(L^{-4}T^{4})T^{-2} = ML^{-3}.$$

Option (A) instead gives $ML^{-1}T^{2}$, and option (B) gives $ML^{5}T^{-12}$. Option (C) has mass dimension M^{2}, which already excludes density; its complete dimensions are $M^{2}T^{4}$. Only the expression in (D) has all three required exponents.

Step 4: Conclusion The dimensional formula of density in the force-velocity-time system is $[\rho] = FV^{-4}T^{-2}$. Hence, the correct option is (D).

Quick Tip to Solve: First use $L = VT$ and $M = FT/V$. Then $M/L^{3} = (FT/V)/(V^{3}T^{3}) = FV^{-4}T^{-2}$.

47. If V is the gravitational potential due to sphere of uniform density on it's surface, then it's value at the center of sphere will be:- [4 marks]

(A) $\frac{3\,V}{2}$

(B) $\frac{V}{2}$

(C) $\frac{4}{3}$ V

(D) V

Correct Answer: (A) $\frac{3\,V}{2}$

Solution:

Concept:

- Taking gravitational potential as zero at infinity, a uniform solid sphere of mass M and radius R has surface potential $V_s = -GM/R$. Potential is negative for an attractive gravitational field.
- Inside the sphere the enclosed mass is proportional to r^3, so the inward field magnitude is $g(r) = GMr/R^3$. The potential at an interior point includes the contribution of the entire sphere, not just the enclosed mass.

Step 1: Identify the given surface potential The symbol V in the question denotes the surface value, so $V = -GM/R$. The body has uniform volume density; its interior field varies linearly with the distance r from the centre. In particular, the gravitational field is zero at the centre, but this does not imply that the gravitational potential there is zero. Field describes the gradient of potential, rather than its absolute value.

Step 2: Calculate the potential change from the surface to the centre For the signed outward radial coordinate, $d\phi/dr = GMr/R^3$ inside the sphere. Integrating between R and 0 gives

$$\phi(0) - \phi(R) = \int_R^0 \frac{GMr}{R^3}\,dr = -\frac{GM}{2R}.$$

Adding this to the surface potential,

$$\phi(0) = -\frac{GM}{R} - \frac{GM}{2R} = -\frac{3GM}{2R}.$$

Thus the central potential is more negative than the surface value, as expected on moving deeper into the attractive field.

Step 3: Express the central value in terms of V Since $V = -GM/R$, the preceding result becomes $\phi(0) = 3V/2$. The ratio is positive even though both potentials themselves are negative. Option (B), V/2, would make the potential less negative on moving inward. Option (C), 4V/3, gives the wrong integrated potential difference, and option (D), V, would apply to the constant potential inside a thin spherical shell rather than a uniform solid sphere.

Step 4: Conclusion The gravitational potential at the centre is $\frac{3V}{2}$, where V is the signed surface potential. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish a solid sphere from a spherical shell: the solid sphere has central potential $-3GM/(2R)$, while a shell has $-GM/R$ everywhere inside.

48. The root mean square speed of molecules of nitrogen gas at $27^{\circ}\mathrm{C}$ is approximately : (Given mass of a nitrogen molecule $= 4.6 \times 10^{-26}$ kg and take Boltzmann constant $\mathrm{k_B} = 1.4 \times 10^{-23}\mathrm{JK}^{-1}$) [4 marks]

(A) 91 m/s

(B) 1260 m/s

(C) 27.4 m/s

(D) 523 m/s

Correct Answer: (D) 523 m/s

Solution:

Concept:

- For an ideal gas molecule of mass m, the mean translational kinetic energy is $\frac{1}{2}mv_{\mathrm{rms}}^2 = \frac{3}{2}k_{\mathrm{B}}T$. Therefore $v_{\mathrm{rms}} = \sqrt{3k_{\mathrm{B}}T/m}$.
- The temperature must be on the absolute Kelvin scale. Since the mass of one molecule is supplied, use Boltzmann's constant rather than the molar gas constant R.

Step 1: Convert the temperature and collect the supplied values The absolute temperature is $T = 27 + 273 = 300\,\mathrm{K}$. Use the stated approximate values $k_{\mathrm{B}} = 1.4 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ and $m = 4.6 \times 10^{-26}\,\mathrm{kg}$ for a nitrogen molecule. It is the mass of the whole molecule that enters the kinetic-energy expression, so there is no factor of two to insert for the two atoms of nitrogen.

Step 2: Calculate the mean square speed Substitution into the kinetic-theory relation gives

$$v_{\mathrm{rms}}^2 = \frac{3(1.4 \times 10^{-23})(300)}{4.6 \times 10^{-26}} = \frac{1.26 \times 10^{-20}}{4.6 \times 10^{-26}} \approx 2.739 \times 10^5\,\mathrm{m^2\,s^{-2}}.$$

The unit check is $\mathrm{J/kg} = \mathrm{m^2\,s^{-2}}$, so taking the square root yields a speed. Keeping the powers of ten separate avoids confusing the squared speed with the speed itself.

Step 3: Take the square root and compare the options The required speed is

$$v_{\mathrm{rms}} = \sqrt{2.739 \times 10^5} \approx 523.4\,\mathrm{m\,s^{-1}}.$$

Thus the closest listed value is 523 m/s. The values 91 m/s and 27.4 m/s are much too small, while 1260 m/s is too large; none has a square close to $2.739 \times 10^5\,\mathrm{m^2\,s^{-2}}$. Using 27 instead of 300 for temperature would also underestimate the speed.

Step 4: Conclusion The approximate root mean square speed is $523\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use molecular mass with k_{B}, or molar mass with R. Convert Celsius to Kelvin before applying the square-root temperature dependence.

49. A car P travelling at $20\ \text{ms}^{-1}$ sounds its horn at a frequency of $400\ \text{Hz}$. Another car Q is travelling behind the first car in the same direction with a velocity $40\ \text{ms}^{-1}$. The frequency heard by the passenger of the car Q is approximately [Take, velocity of sound $= 360\ \text{ms}^{-1}$] [4 marks]

(A) 485 Hz

(B) 514 Hz

(C) 421 Hz

(D) 471 Hz

Correct Answer: (C) 421 Hz

Solution:

Concept:

- Doppler shifts depend on source and observer velocities relative to the medium and on the direction in which the observed wave travels. A following observer can hear a higher frequency if it catches up with the receding source.
- Behind a source moving away at speed v_s, successive wavefronts have separation $\lambda = (v + v_s)/f$. An observer approaching these backward-travelling waves at speed v_o meets them at relative speed $v + v_o$.

Step 1: Choose the wave direction and the correct signs Car Q is behind car P, so sound reaching Q travels backward, opposite to the cars' direction of motion. The source P moves forward, away from the region behind it; this increases the wavelength behind P. Meanwhile Q moves forward into the backward-travelling wavefronts, increasing the rate at which it encounters them. Both speeds therefore enter with plus signs in the relevant formula.

Step 2: Find the wavelength behind the source In one period $1/f$, a wavefront travels backward a distance v/f, while P moves forward by v_s/f before emitting the next corresponding wavefront. Their separation behind the source is

$$\lambda = \frac{v + v_s}{f} = \frac{360 + 20}{400} = 0.95\,\text{m}.$$

The source frequency remains 400 Hz; motion changes the wavelength in the air, not the emission period of the horn.

Step 3: Calculate the frequency detected by Q The wavefronts and Q approach each other at $v + v_o = 360 + 40 = 400\,\text{m s}^{-1}$. Hence

$$f_Q = \frac{v + v_o}{\lambda} = 400\frac{360 + 40}{360 + 20} = \frac{160000}{380} \approx 421.05\,\text{Hz}.$$

This is greater than 400 Hz because Q closes the distance to P. The other values 485 Hz, 514 Hz and 471 Hz do not satisfy the ratio $(v + 40)/(v + 20)$; using the same sign rule without checking the backward wave direction would produce an incorrect shift.

Step 4: Conclusion The passenger in car Q hears approximately $421\,\text{Hz}$. Hence, the correct option is (C).

Quick Tip to Solve: For two cars moving forward with the observer behind, use $f_Q = f(v + v_o)/(v + v_s)$. Check that equal car speeds would give no shift.

50. In satellite communication, the uplink frequency band used is : [4 marks]

(A) $3.7 - 4.2$ GHz

(B) $5.925 - 6.425$ GHz

(C) $76 - 88$ MHz

(D) $420 - 890$ MHz

Correct Answer: (B) $5.925 - 6.425$ GHz

Solution:

Concept:

- An uplink carries a signal from an Earth station to the satellite; a downlink carries it back from the satellite to an Earth station.
- In the conventional C-band satellite system the uplink uses the band around $6\,\text{GHz}$, whereas the downlink uses the band around $4\,\text{GHz}$.

Step 1: Identify the direction of transmission The word uplink selects the Earth-to-satellite carrier band. The satellite transponder receives this signal, changes its carrier frequency and retransmits it towards Earth. Distinct receive and transmit frequencies prevent the strong transmitted signal from interfering with the received signal.

Step 2: Compare the frequency ranges The interval 5.925–$6.425\,\text{GHz}$ is the conventional C-band uplink allocation. The interval 3.7–$4.2\,\text{GHz}$ is the corresponding downlink band, so option A reverses the required direction. The MHz ranges in options C and D belong to much lower terrestrial broadcast frequencies; neither is the specified satellite uplink band.

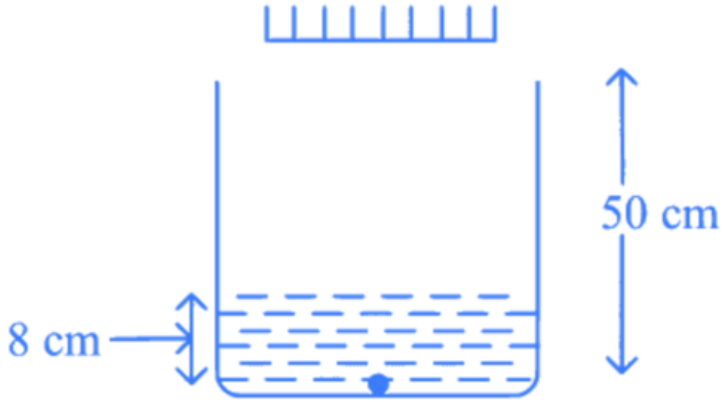
Step 3: Conclusion The uplink frequency band is 5.925–$6.425\,\text{GHz}$. Hence, the correct option is (B).

Quick Tip to Solve: For the conventional C-band satellite link, remember the pair: about 6 GHz going up and about 4 GHz coming down.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. As shown in the figure, a plane mirror is fixed at a height of 50 cm from the bottom of tank containing water $\left(\mu = \frac{4}{3}\right)$. The height of water in the tank is 8 cm. A small bulb is placed at the bottom of the water tank. The distance of image of the bulb formed by mirror from the bottom of the tank is _____ cm. [4 marks]



Solution:

Concept:

- For an object in water viewed nearly normally from air, apparent depth is $d_{\text{app}} = d/\mu$, where d is real depth and μ is the refractive index of water relative to air.
- A plane mirror forms a virtual image as far behind its plane as the effective object is in front of it. Measure both positions from one fixed reference, the tank bottom.

Step 1: Locate the apparent bulb The surface is $8\,\text{cm}$ above the bottom. Refraction makes the bulb appear closer to this surface:

$$d_{\text{app}} = \frac{8}{4/3} = 6\,\text{cm}.$$

Thus its effective position for the mirror is $8 - 6 = 2\,\text{cm}$ above the bottom, rather than at the actual bottom.

Step 2: Reflect the effective position in the mirror The mirror is $50\,\text{cm}$ above the bottom. Its object distance is therefore $50 - 2 = 48\,\text{cm}$. The image lies $48\,\text{cm}$ above the mirror, giving

$$h_{\text{image}} = 50 + 48 = 98\,\text{cm}.$$

The requested distance is from the bottom, so the mirror height must be added, not omitted.

Step 3: Conclusion The image is $98\,\text{cm}$ from the tank bottom. The numerical entry is $\boxed{98}$.

Quick Tip to Solve: Find the apparent position relative to the bottom first; then use image height = twice mirror height minus effective object height.

52. A circular plate is rotating in horizontal plane, about an axis passing through its center and perpendicular to the plate, with an angular velocity ω. A person sits at the center having two dumbbells in his hands. When he stretches out his hands, the moment of inertia of the system becomes triple. If E be the initial Kinetic energy of the system, then final Kinetic energy will be $\frac{E}{x}$. The value of x is [4 marks]

Solution:

Concept:

- When the net external torque about the rotation axis is zero, angular momentum is conserved: $I_i\omega_i = I_f\omega_f$. The person, dumbbells and plate must be treated as a single system.
- Rotational kinetic energy is $K = \frac{1}{2}I\omega^2 = L^2/(2I)$. At fixed angular momentum, increasing moment of inertia lowers the kinetic energy.

Step 1: Apply angular momentum conservation Let the initial moment of inertia be I. Stretching the arms changes it to $3I$, while the internal forces do not supply an external torque about the axis. Hence

$$I\omega = 3I\omega_f \quad \Rightarrow \quad \omega_f = \frac{\omega}{3}.$$

The angular speed cannot remain unchanged during this redistribution.

Step 2: Compare the rotational energies The initial energy is $E = \frac{1}{2}I\omega^2$. Substitute both the new inertia and the new speed:

$$K_f = \frac{1}{2}(3I)\left(\frac{\omega}{3}\right)^2 = \frac{1}{3}\left(\frac{1}{2}I\omega^2\right) = \frac{E}{3}.$$

Kinetic energy itself is not the conserved quantity here; work associated with the internal motion accounts for its change.

Step 3: Conclusion Comparing $K_f = E/3$ with E/x gives $\boxed{x = 3}$. The ratio parameter is dimensionless.

Quick Tip to Solve: For a rotating system with no external torque, use K proportional to 1/I, not K proportional to I.

---

53. The surface tension of soap solution is 3.5×10^{-2} Nm^{-1}. The amount of work done required to increase the radius of soap bubble from 10 cm to 20 cm is _____ $\times 10^{-4}$ J.

$$(\text{take } \pi = 22/7)$$

[4 marks]

Solution:

Concept:

- Surface tension T is the increase in surface energy per unit increase in area. The quasistatic work required is $W = T\,\Delta A_{\text{total}}$.

- A soap bubble has two liquid-air surfaces. For a thin film of radius r, its total area is $2(4\pi r^2) = 8\pi r^2$; both surfaces must be included.

Step 1: Calculate the increase in total area Convert the radii before substituting: $r_1 = 0.10\,\text{m}$, $r_2 = 0.20\,\text{m}$. Therefore

$$\Delta A_{\text{total}} = 8\pi(r_2^2 - r_1^2) = 8\pi(0.0400 - 0.0100) = 0.24\pi\,\text{m}^2.$$

Using only $4\pi r^2$ would describe one interface and give half the required work.

Step 2: Multiply by the surface tension With the specified value of π,

$$\begin{aligned} W &= (3.5 \times 10^{-2})(0.24)\frac{22}{7}\,\text{J} \\ &= 0.0264\,\text{J} = 264 \times 10^{-4}\,\text{J}. \end{aligned}$$

The unit is joule because $(\text{N/m})(\text{m}^2) = \text{N m}$. The positive sign agrees with the increase of surface energy during expansion.

Step 3: Conclusion The required work is 264×10^{-4} J, so the numerical entry is $\boxed{264}$.

Quick Tip to Solve: Count the interfaces before using surface energy: a soap bubble has two, whereas an ordinary liquid drop has one.

---

54. A wire of density $8 \times 10^3\ \text{kg/m}^3$ is stretched between two clamps $0.5\ \text{m}$ apart. The extension developed in the wire is $3.2 \times 10^{-4}\ \text{m}$. If $Y = 8 \times 10^{10}\ \text{N/m}^2$, the fundamental frequency of vibration in the wire will be _____ Hz. [4 marks]

Solution:

Concept:

- Young's modulus relates tensile stress to strain: $Y = (F/A)/(\Delta L/L)$, so the tension is $F = YA\Delta L/L$. Here A is the wire cross-sectional area.
- The wave speed is $v = \sqrt{F/\lambda}$, where linear mass density is $\lambda = \rho A$. A wire fixed at both ends has fundamental frequency $f_1 = v/(2L)$.

Step 1: Eliminate the unknown cross-sectional area Combining the elastic tension with the linear mass density gives

$$v^2 = \frac{YA\Delta L/L}{\rho A} = \frac{Y\Delta L}{\rho L}.$$

Thus the wire area is not needed. The stated separation $L = 0.5\,\text{m}$ is the vibrating length, and the given small extension produces the tension.

Step 2: Evaluate the speed and fundamental frequency Substitute the values in SI units:

$$v^2 = \frac{(8 \times 10^{10})(3.2 \times 10^{-4})}{(8 \times 10^3)(0.5)} = 6400\,\text{m}^2\text{s}^{-2}.$$

Hence $v = 80\,\mathrm{m\,s^{-1}}$. The fundamental contains one half-wavelength between the clamps, so

$$f_1 = \frac{80}{2(0.5)} = 80\,\mathrm{Hz}.$$

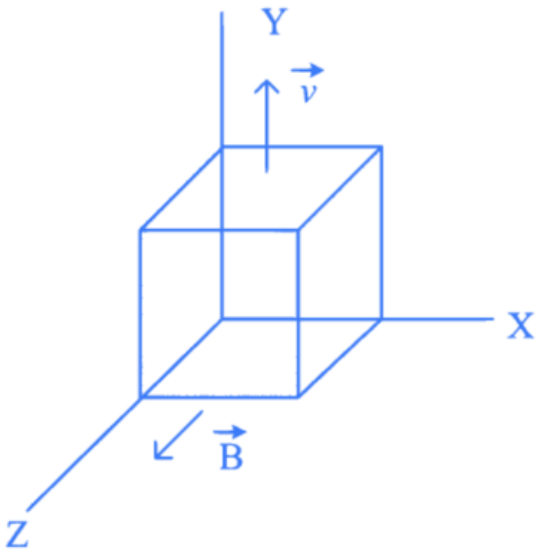
Using a full wavelength in the length would incorrectly double this result.

Step 3: Conclusion The fundamental frequency is $\boxed{80\,\mathrm{Hz}}$. The numerical entry is 80.

Quick Tip to Solve: When extension and density are given, combine tension with linear density first: the unknown wire area cancels.

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55. A metallic cube of side $15\ \mathrm{cm}$ moving along y-axis at a uniform velocity of $2\ \mathrm{ms^{-1}}$. In a region of uniform magnetic field of magnitude $0.5\ \mathrm{T}$ directed along z-axis. In equilibrium the potential difference between the faces of higher and lower potential developed because of the motion through the field will be _____ mV. [4 marks]



Solution:

Concept:

- Charges in a conductor moving with velocity $\boldsymbol{v}$ through magnetic field $\boldsymbol{B}$ experience magnetic force $q\boldsymbol{v} \times \boldsymbol{B}$. Charge separation creates an opposing electric field.
- In equilibrium, $\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B}$. For mutually perpendicular velocity and magnetic field, the magnitude is $E = vB$, and the potential difference over separation l is $\Delta V = El$.

Step 1: Identify the relevant pair of faces The motion is along the positive y-axis and the field is along the positive z-axis, so $\boldsymbol{v} \times \boldsymbol{B}$ is along positive x. Positive charge is driven towards the positive-x face and electrons towards the opposite face. The positive-x face is therefore at higher potential. These faces are separated by the cube side.

Step 2: Calculate the equilibrium voltage Convert the side to metres: $l = 15\,\text{cm} = 0.15\,\text{m}$. Then

$$E = vB = (2)(0.5) = 1\,\text{V m}^{-1}, \qquad \Delta V = El = (1)(0.15) = 0.15\,\text{V}.$$

The question asks for the difference between higher and lower potentials, so its magnitude is positive. Converting volts to millivolts gives $0.15 \times 1000 = 150\,\text{mV}$.

Step 3: Conclusion The potential difference is $\boxed{150\,\text{mV}}$, so the numerical entry is 150.

Quick Tip to Solve: Use the dimension along v cross B as the separation in Blv; the length along the direction of motion does not set the voltage.

56. A nucleus disintegrates into two nuclear parts, in such a way that ratio of their nuclear sizes is $1 : 2^{1/3}$. Their respective speed have a ratio of $n : 1$. The value of n is _____. [4 marks]

Solution:

Concept:

- Nuclear radius follows $R = R_0 A^{1/3}$, where A is mass number. Thus nuclear mass, approximately proportional to A, is proportional to R^3.
- For disintegration of a nucleus initially at rest, the two fragments have equal and opposite momenta: $m_1 v_1 = m_2 v_2$. Their speed ratio is the inverse of their mass ratio.

Step 1: Convert the radius ratio into a mass ratio The given nuclear sizes represent radii. Cubing the radius ratio gives

$$\frac{m_1}{m_2} = \frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{1}{2^{1/3}}\right)^3 = \frac{1}{2}.$$

It would be incorrect to use the radius ratio itself as the mass ratio, since nuclear volume varies as the cube of radius.

Step 2: Use the recoil momentum relation The disintegration forces are internal, so the total momentum remains zero in the original nucleus rest frame. The fragments move in opposite directions with

$$\frac{v_1}{v_2} = \frac{m_2}{m_1} = 2.$$

The smaller fragment consequently moves faster. Comparing this result with the stated respective speed ratio $n : 1$ fixes n without any knowledge of the energy released; kinetic energies need not be equal.

Step 3: Conclusion The respective speeds have ratio $2 : 1$, so $\boxed{n = 2}$. This ratio parameter is dimensionless.

Quick Tip to Solve: Cube a nuclear radius ratio to get the mass ratio, then invert that ratio to get recoil speeds.

57. A block of mass $5\ \text{kg}$ starting from rest pulled up on a smooth incline plane making an angle of 30° with horizontal with an affective acceleration of $1\ \text{ms}^{-2}$. The power delivered by the pulling force at $t = 10\ \text{s}$ from the start is _____ W.

[use $\text{g} = 10\ \text{ms}^{-2}$]

(calculate the nearest integer value) [4 marks]

Solution:

Concept:

- For motion along a smooth incline, resolve forces parallel to the plane. The component of weight down the incline is $mg\sin\theta$, and the normal reaction has no component along the motion.
- Instantaneous mechanical power is $P = \mathbf{F}\cdot\mathbf{v} = Fv$ when the pulling force and velocity point up the incline. For constant acceleration from rest, $v = at$.

Step 1: Find the pulling force Take the direction up the incline as positive. The acceleration is the net acceleration, so the applied force must both oppose the downhill component of gravity and produce the stated acceleration. Newton's second law gives

$$F - mg\sin\theta = ma.$$

With $m = 5\,\text{kg}$, $g = 10\,\text{m s}^{-2}$, $\theta = 30^\circ$, and $a = 1\,\text{m s}^{-2}$,

$$F = 5(1) + 5(10)\left(\frac{1}{2}\right) = 30\,\text{N}.$$

Step 2: Calculate the speed and power at ten seconds The block starts from rest and its acceleration remains constant, hence

$$v = 0 + at = 1(10) = 10\,\text{m s}^{-1}.$$

The pulling force is parallel to the displacement, so no extra cosine factor is needed in the power expression:

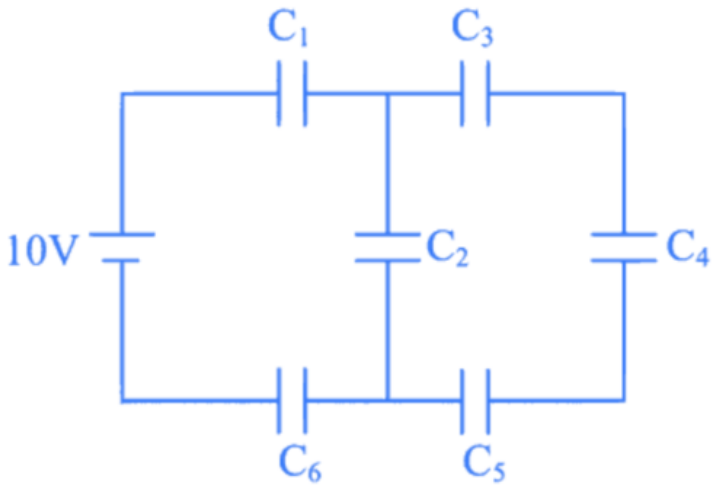
$$P = Fv = 30(10) = 300\,\text{W}.$$

Using ma alone would give only the rate of increase of kinetic energy and would omit the simultaneous increase in gravitational potential energy.

Step 3: Conclusion The instantaneous power delivered by the pulling force at $t = 10\,\text{s}$ is $\boxed{300\,\text{W}}$. The required nearest integer is 300.

Quick Tip to Solve: On a smooth incline use $F = m(a + g\sin\theta)$, then multiply by the instantaneous speed, not by the average speed.

58. In the given circuit, $\mathrm{C}_1 = 2\mu\mathrm{F}, \mathrm{C}_2 = 0.2\mu\mathrm{F}, \mathrm{C}_3 = 2\mu\mathrm{F}, \mathrm{C}_4 = 4\mu\mathrm{F}, \mathrm{C}_5 = 2\mu\mathrm{F}, \mathrm{C}_6 = 2\mu\mathrm{F}$, The charge stored on capacitor C_4 is _____ μC. [4 marks]



Solution:

Concept:

- Capacitors in series carry equal charge; their equivalent capacitance obeys $1/C_s = \sum_i 1/C_i$. Parallel branches share a potential difference and their capacitances add.
- For a capacitor, $Q = CV$. When C is expressed in μF and V in volts, the numerical charge is in μC. Initially uncharged, isolated junctions retain zero net charge.

Step 1: Reduce the central network Label the junction between C_1 and C_3 as X, and the junction between C_6 and C_5 as Y. The right-hand path from X to Y contains C_3, C_4, C_5 in series. Its equivalent is

$$\frac{1}{C_s} = \frac{1}{2} + \frac{1}{4} + \frac{1}{2} = \frac{5}{4}, \qquad C_s = 0.8\,\mu\mathrm{F}.$$

This branch is parallel to $C_2 = 0.2\,\mu\mathrm{F}$. Therefore the complete capacitance between X and Y is

$$C_{XY} = 0.8 + 0.2 = 1.0\,\mu\mathrm{F}.$$

Step 2: Find the voltage across the central network The circuit now consists of $C_1 = 2\,\mu\mathrm{F}$, $C_{XY} = 1\,\mu\mathrm{F}$, and $C_6 = 2\,\mu\mathrm{F}$ in series across $10\,\mathrm{V}$. Thus

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{2} + 1 + \frac{1}{2} = 2, \qquad C_{\mathrm{eq}} = 0.5\,\mu\mathrm{F}.$$

The charge in this reduced series chain is $Q = 0.5(10) = 5\,\mu\mathrm{C}$. Hence $V_{XY} = Q/C_{XY} = 5\,\mathrm{V}$.

Step 3: Recover the charge on the specified capacitor The right-hand series branch has capacitance $0.8\,\mu\mathrm{F}$ and potential difference $5\,\mathrm{V}$. Its charge is

$$Q_s = C_s V_{XY} = 0.8(5) = 4\,\mu\mathrm{C}.$$

Every capacitor in this branch, including C_4, carries this charge magnitude. The total central-network

charge is $5\,\mu\text{C}$, but $1\,\mu\text{C}$ resides on the parallel capacitor C_2.

Step 4: Conclusion The charge stored on C_4 is $\boxed{4\,\mu\text{C}}$, so the required numerical entry is 4.

Quick Tip to Solve: First combine the right-hand three-capacitor series path; it lies in parallel with the middle capacitor, not in series with it.

59. A coil has an inductance of 2H and resistance of $4\ \Omega$. A $10\ \text{V}$ is applied across the coil. The energy stored in the magnetic field after the current has built up to its equilibrium value will be _____ $\times 10^{-2}$ J. [4 marks]

Solution:

Concept:

- In a series resistance-inductance circuit under a constant applied voltage, $V = RI + L\,dI/dt$. At equilibrium the current is constant, so the inductive voltage vanishes.
- The magnetic energy stored by an inductor carrying current I is $U = \frac{1}{2}LI^2$. Resistance fixes the equilibrium current but does not itself store magnetic energy.

Step 1: Find the final steady current The supply is a constant $10\,\text{V}$, the coil resistance is $4\,\Omega$, and its inductance is $2\,\text{H}$. Initially the inductance opposes the rise of current, but after equilibrium is reached $dI/dt = 0$. Consequently the full applied voltage balances the resistive drop:

$$10 = 4I_\infty, \qquad I_\infty = \frac{10}{4} = 2.5\,\text{A}.$$

The current is not zero in the steady state; only the induced emf opposing a change of current becomes zero.

Step 2: Evaluate and express the stored energy Substitute the final current into the energy formula:

$$U = \frac{1}{2}(2)(2.5)^2 = 6.25\,\text{J}.$$

The question requests a coefficient multiplying $10^{-2}\,\text{J}$, rather than energy directly in joules. Since $10^{-2}\,\text{J} = 0.01\,\text{J}$,

$$6.25\,\text{J} = 625 \times 10^{-2}\,\text{J}.$$

This energy remains in the magnetic field while the steady current flows. The continuing resistive power loss is a separate transfer of energy to heat.

Step 3: Conclusion The magnetic energy is $\boxed{6.25\,\text{J} = 625 \times 10^{-2}\,\text{J}}$. The required numerical entry is 625.

Quick Tip to Solve: For the final state of a DC coil first use $I = V/R$; only then substitute in $\frac{1}{2}LI^2$ and convert to the requested power of ten.

60. Two identical cells each of emf 1.5 V are connected in series across a $10\ \Omega$ resistance. An ideal voltmeter connected across $10\ \Omega$ resistance reads 1.5 V. The internal resistance of each cell is _____ Ω. [4 marks]

Solution:

Concept:

- Identical cells connected in series aiding each other have total emf $E_{\text{tot}} = 2E$ and total internal resistance $r_{\text{tot}} = 2r$.
- An ideal voltmeter draws no current. Its reading across the external resistor is the loaded terminal voltage $V_R = IR$, which differs from the open-circuit emf when the cells have internal resistance.

Step 1: Calculate the current through the resistor The voltmeter is connected across the $10\,\Omega$ external resistor and reads $1.5\,\text{V}$. Because it is ideal, it does not provide a parallel conducting path. Ohm's law gives

$$I = \frac{V_R}{R} = \frac{1.5}{10} = 0.15\,\text{A}.$$

The same current passes through both series cells, since there is a single conducting loop.

Step 2: Apply the complete circuit equation The two cells provide a combined emf of $1.5+1.5 = 3.0\,\text{V}$. The voltage lost across their internal resistances is the difference between this emf and the measured external voltage:

$$I(2r) = 3.0 - 1.5 = 1.5\,\text{V}.$$

Therefore

$$2r = \frac{1.5}{0.15} = 10\,\Omega, \qquad r = 5\,\Omega.$$

As a check, the total circuit resistance is $10 + 2(5) = 20\,\Omega$, giving $I = 3/20 = 0.15\,\text{A}$ and the observed terminal voltage $IR = 1.5\,\text{V}$. The $10\,\Omega$ internal resistance belongs to the pair, not to each cell.

Step 3: Conclusion The internal resistance of each cell is $\boxed{5\,\Omega}$. The required numerical entry is 5.

Quick Tip to Solve: Use the terminal voltage to find current, then divide the remaining emf drop by that current; finally divide by the number of identical cells.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Which one of the following pairs is an example of polar molecular solids? [4 marks]

(A) $HCl(s), AlN(s)$

(B) $MgO(s), SO_2(s)$

(C) $SO_2(s), NH_3(s)$

(D) $SO_2(s), CO_2(s)$

Correct Answer: (C) $SO_2(s), NH_3(s)$

Solution:

Concept:

- Molecular solids consist of discrete molecules held together by intermolecular forces. Polar molecular solids contain molecules with permanent electric dipole moments.
- Bond polarity alone does not establish molecular polarity: the molecular geometry determines whether the bond dipoles cancel. Ionic and extended covalent solids are not molecular solids.

Step 1: Identify the two polar molecules In SO_2, the molecule is bent because of the lone pair on sulfur. Its polar sulfur-oxygen bonds therefore have a nonzero resultant dipole. Solid sulfur dioxide is consequently a polar molecular solid. In NH_3, the molecule is trigonal pyramidal, and the nitrogen-hydrogen bond dipoles also do not cancel. Ammonia forms a molecular solid with permanent molecular dipoles; its molecules additionally exhibit hydrogen bonding. Thus both substances in the pair $SO_2(s)$, $NH_3(s)$ satisfy the question.

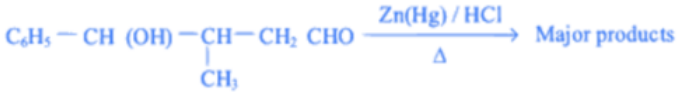
Step 2: Reject the other pairs Option (A) includes AlN, which forms an extended solid rather than a lattice of separate polar molecules; the presence of polar HCl does not make the whole pair correct. Option (B) contains MgO, an ionic solid built from Mg^{2+} and O^{2-}, so it is also excluded. In option (D), CO_2 is a linear symmetric molecule. Its two carbon-oxygen bond dipoles cancel, so solid carbon dioxide is a nonpolar molecular solid. Only option (C) contains two polar molecular substances.

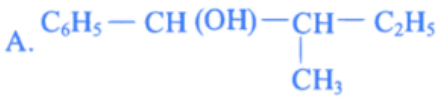
Step 3: Conclusion Both $SO_2(s)$ and $NH_3(s)$ are polar molecular solids. Hence, the correct option is (C).

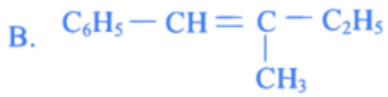
Quick Tip to Solve: Check two things for each member of a pair: does it consist of molecules, and do the bond dipoles cancel in the actual geometry?

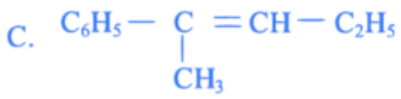
62. The major product formed in the following reaction is

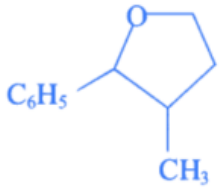
Choose the correct answer from the options given below : [4 marks]



A. 

B. 

C. 

D. 

(A) D only

(B) B only

(C) A only

(D) C only

Correct Answer: (B) B only

Solution:

Concept:

- Clemmensen reduction, using $Zn(Hg)/HCl$, converts an aldehyde carbonyl group $-CHO$ to $-CH_3$. The reaction medium is strongly acidic.
- A benzylic alcohol can undergo acid-catalysed dehydration on heating. Loss of water followed by removal of a neighbouring hydrogen produces an alkene; conjugation with the benzene ring stabilises the product.

Step 1: Reduce the terminal aldehyde The starting compound is

$$C_6H_5-CH(OH)-CH(CH_3)-CH_2-CHO.$$

The aldehyde group at the end of the chain is reduced to a methyl group by amalgamated zinc in hydrochloric acid. Thus the carbon skeleton becomes

$$C_6H_5-CH(OH)-CH(CH_3)-CH_2-CH_3.$$

This structure corresponds to product A in the displayed list. However, it retains a benzylic hydroxyl

group and is not the final major product under the strongly acidic, heated conditions shown.

Step 2: Account for dehydration and compare products Protonation of the benzylic hydroxyl group makes water a leaving group. Its departure gives a resonance-stabilised benzylic carbocation. Loss of the adjacent hydrogen then forms the double bond between the benzylic carbon and the carbon bearing the methyl substituent:

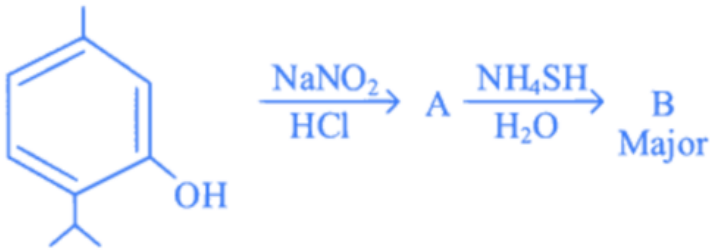
$$C_6H_5-CH=C(CH_3)-CH_2-CH_3.$$

This conjugated alkene is the structure labelled B in the product list. Structure A retains the alcohol; structure C places the methyl group on a different carbon and would require a rearranged carbon skeleton; structure D is a cyclic ether rather than the expected reduction and dehydration product. Thus B alone is selected. The structure label B corresponds to answer option (B), whereas the other answer options refer to D, A and C respectively.

Step 3: Conclusion The major product is $C_6H_5-CH=C(CH_3)-C_2H_5$, labelled B in the figure. Hence, the correct option is (B).

Quick Tip to Solve: With hot $Zn(Hg)/HCl$, reduce the carbonyl but also check whether an acid-sensitive benzylic alcohol can dehydrate.

63. Compound 'B' is [4 marks]



(A) 4-Amino-2-isopropyl-5-methylphenol

(B) 2-Isopropyl-5-methylbenzenethiol

(C) 2-Isopropyl-4-mercapto-5-methylphenol

(D) 1-Isopropyl-4-methylbenzene

Correct Answer: (A) 4-Amino-2-isopropyl-5-methylphenol

Solution:

Concept:

- Phenolic $-OH$ strongly activates the benzene ring and directs electrophilic substitution to its ortho and para positions. Nitrous acid is generated in situ from $NaNO_2/HCl$.
- Phenols undergo ring nitrosation with nitrous acid. The nitroso substituent $-NO$ can then be reduced to $-NH_2$ by ammonium hydrosulfide, NH_4SH, without replacing the phenolic hydroxyl group.

Step 1: Locate the nitrosation position Number the carbon bearing the phenolic hydroxyl group as position 1. In the given starting compound the isopropyl group is at position 2 and the methyl group is at position 5. One ortho position is already occupied by the isopropyl group. Nitrosation occurs mainly at the available para position, carbon 4, giving compound A as 2-isopropyl-5-methyl-4-nitrosophenol. The carbon skeleton and the original phenolic hydroxyl group remain intact during this electrophilic substitution.

Step 2: Reduce the introduced nitroso group Treatment with NH_4SH/H_2O reduces the newly introduced nitroso group to an amino group. It does not replace the phenolic $-OH$ by $-SH$, nor introduce a sulfhydryl group on the ring. Compound B is therefore 4-amino-2-isopropyl-5-methylphenol, the structure shown in option (A). Option (B) incorrectly replaces the original hydroxyl group by $-SH$. Option (C) introduces a ring $-SH$ group instead of reducing the nitroso group to $-NH_2$. Option (D) removes the hydroxyl group and introduces no amino group, so it also fails to describe the two reactions. The sulfur-containing reagent acts here as a reducing agent rather than as a source of a new ring substituent.

Step 3: Conclusion Compound B is 4-amino-2-isopropyl-5-methylphenol: the amino group is para to the retained phenolic hydroxyl group. Hence, the correct option is (A).

Quick Tip to Solve: In this sequence, track the group first installed by nitrosation: $-NO$ becomes $-NH_2$; sulfur from the reducing reagent is not installed on the ring.

---

64. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : $[CoCl(NH_3)_5]^{2+}$ **absorbs at lower wavelength of light with respect to** $[Co(NH_3)_5(H_2O)]^{3+}$

Reason R : It is because the wavelength of the light absorbed depends on the oxidation state of the metal ion.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) A is false but R is true

(B) Both A and R are true but R is NOT the correct explanation of A

(C) Both A and R are true and R is the correct explanation of A

(D) A is true but R is false

Correct Answer: (A) A is false but R is true

Solution:

Concept:

- In an octahedral complex, absorption promotes an electron between crystal-field levels separated by Δ_o. The absorbed photon obeys $\Delta_o = hc/\lambda$, so a smaller splitting means a longer wavelength.
- The splitting depends on the metal, its oxidation state and the ligands. In the spectrochemical series, Cl^- is a weaker-field ligand than H_2O.

Step 1: Find the metal oxidation states Let the oxidation state of cobalt be x. Ammonia and water

are neutral, whereas chloride has charge -1. Thus the first complex gives $x - 1 = +2$, and the second gives $x = +3$. Cobalt is therefore in oxidation state $+3$ in both complexes. The different overall charges do not imply different cobalt oxidation states.

Step 2: Evaluate Assertion A Five ammonia ligands are common to the two complexes. Replacing the sixth ligand, chloride, by the stronger-field water ligand increases the splitting. Consequently,

$$\Delta_o(\text{chloro complex}) < \Delta_o(\text{aqua complex})$$

and hence

$$\lambda(\text{chloro complex}) > \lambda(\text{aqua complex}).$$

The assertion claims a lower wavelength for the chloro complex, so it is false.

Step 3: Evaluate Reason R and the choices For a given metal and comparable ligand environment, changing the oxidation state changes metal-ligand interactions and the crystal-field splitting, and thus can change the wavelength absorbed. Reason R is therefore a true general statement. It does not describe the actual difference here, since both cobalt centres are $+3$. Choices (B) and (C) incorrectly declare A true; choice (D) also incorrectly declares R false.

Step 4: Conclusion Assertion A is false, while Reason R is true. Hence, the correct option is (A).

Quick Tip to Solve: Calculate the oxidation state before comparing colours: the charge outside the brackets is the complex charge, not necessarily the metal oxidation state.

65. If Ni^{2+} is replaced by Pt^{2+} in the complex $[NiCl_2Br_2]^{2-}$, which of the following properties are expected to get changed ?

A. Geometry

B. Geometrical isomerism

C. Optical isomerism

D. Magnetic properties [4 marks]

(A) A and D

(B) A, B and D

(C) A, B and C

(D) B and C

Correct Answer: (B) A, B and D

Solution:

Concept:

- Both Ni^{2+} and Pt^{2+} are d^8 ions, but their four-coordinate complexes need not have the same geometry. Weak-field halides favour tetrahedral nickel(II), whereas platinum(II) strongly favours square-planar geometry.
- Geometrical and optical isomerism are different tests: cis/trans arrangements can exist in a square plane without producing a pair of nonsuperimposable mirror images.

Step 1: Compare the geometries and spins For $[NiCl_2Br_2]^{2-}$, the four halide ligands give the usual tetrahedral nickel(II) complex. Its d^8 configuration has two unpaired electrons, so it is paramagnetic. The corresponding platinum(II) complex, $[PtCl_2Br_2]^{2-}$, is square planar and has all electrons paired, so it is diamagnetic. Thus both A (geometry) and D (magnetic properties) change.

Step 2: Test the two kinds of isomerism In a tetrahedral MA_2B_2 complex, every pair of vertices has the same geometrical relationship, so there is no distinct cis/trans pair. In square-planar MA_2B_2, like ligands may be adjacent (cis) or opposite (trans). Geometrical isomerism therefore changes, establishing B. Neither tetrahedral MA_2B_2 nor either square-planar arrangement is optically active: the required chirality is absent. Hence C does not change.

Step 3: Select the complete set The changed properties are A, B and D. Option (A) leaves out the new geometrical isomerism, option (C) incorrectly includes optical isomerism and omits the spin change, and option (D) both omits geometry and magnetism and incorrectly includes optical isomerism.

Step 4: Conclusion Geometry, geometrical isomerism and magnetic properties change; optical isomerism does not. Hence, the correct option is (B).

Quick Tip to Solve: For a four-coordinate d^8 halide complex, first distinguish tetrahedral Ni(II) from square-planar Pt(II); then test cis/trans and spin separately.

---

66. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : A solution of the product obtained by heating a mole of glycine with a mole of chlorine in presence of red phosphorous generates chiral carbon atom.

Reason R : A molecule with 2 chiral carbons is always optically active.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Correct Answer: (C) A is true but R is false

Solution:

Concept:

- The Hell-Volhard-Zelinsky reaction uses a halogen and red phosphorus to replace an alpha hydrogen of a carboxylic acid by halogen. A tetrahedral carbon is chiral when its four substituents are different.
- The presence of chiral centres alone does not guarantee optical activity. A meso compound has stereogenic centres but is achiral because of internal symmetry; a racemic mixture also shows no net rotation.

Step 1: Evaluate Assertion A Glycine is NH_2-CH_2-COOH. Chlorination at its alpha carbon replaces

one of the two alpha hydrogens, producing $NH_2-CHCl-COOH$:

$$NH_2-CH_2-COOH + Cl_2 \xrightarrow{\text{red P}} NH_2-CHCl-COOH + HCl.$$

The alpha carbon in the product is bonded to NH_2, H, Cl and COOH. These are four different groups, so a chiral carbon is generated. Assertion A is true.

Step 2: Evaluate Reason R The word "always" makes the reason false. A molecule with two stereogenic carbons can be a meso compound; meso-tartaric acid is a familiar example. Its internal symmetry makes the molecule superimposable on its mirror image, so it is not optically active. Moreover, the assertion concerns formation of a chiral centre, not the net optical rotation of the reaction mixture: an achiral starting material can yield equal quantities of two enantiomers.

Step 3: Compare the answer statements Since A is true and R is false, neither statement saying both are true can be selected. The statement saying A is false and R true reverses both assessments. The correct classification depends on the four groups at the product carbon and the meso counterexample, rather than merely counting stereogenic centres.

Step 4: Conclusion The chlorinated glycine product contains a chiral carbon, but two chiral carbons do not always ensure optical activity. Hence, the correct option is (C).

Quick Tip to Solve: When a statement says chiral centres "always" imply optical activity, test a meso structure before accepting it.

67. Match List I with List II.

	LIST I: Complex		LIST II: Colour
A.	$Mg(NH_4)PO_4$	I.	brown
B.	$K_3[Co(NO_2)_6]$	II.	white
C.	$MnO(OH)_2$	III.	yellow
D.	$Fe_4[Fe(CN)_6]_3$	IV.	blue

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-IV, C-I, D-III

(B) A-II, B-III, C-I, D-IV

(C) A-III, B-IV, C-II, D-I

(D) A-II, B-III, C-IV, D-I

Correct Answer: (B) A-II, B-III, C-I, D-IV

Solution:

Concept:

- Qualitative inorganic analysis identifies compounds using characteristic precipitate colours. The colour must be associated with the whole compound, rather than guessed from one ion in isolation.
- The standard analytical compounds here are magnesium ammonium phosphate, potassium hexanitritocobaltate(III), hydrated manganese dioxide written as $MnO(OH)_2$, and Prussian blue.

Step 1: Match the white and yellow compounds Magnesium ammonium phosphate, $Mg(NH_4)PO_4$, is a white precipitate formed in the phosphate test with magnesia mixture. Therefore A matches II. Potassium hexanitritocobaltate(III), $K_3[Co(NO_2)_6]$, is yellow and is associated with the cobaltinitrite test for potassium. Therefore B matches III. These two familiar precipitates already rule out the assignments putting A with yellow or B with blue.

Step 2: Match the brown and blue compounds The compound $MnO(OH)_2$, often represented as hydrated manganese dioxide in qualitative chemistry, is brown. Hence C matches I. Ferric ferrocyanide, $Fe_4[Fe(CN)_6]_3$, is the intensely blue compound called Prussian blue, giving D matches IV. Do not exchange these colours merely because both compounds contain transition metals: their electronic structures and compositions are different.

Step 3: Check every proposed matching The complete mapping is

$$A-II,\quad B-III,\quad C-I,\quad D-IV.$$

Option (A) wrongly interchanges the colours of the cobalt and iron compounds. Option (C) misassigns all four familiar analytical colours, while option (D) swaps the brown manganese compound and blue iron compound. Only option (B) preserves all four correct identifications.

Step 4: Conclusion The colours are white, yellow, brown and blue for A, B, C and D respectively. Hence, the correct option is (B).

Quick Tip to Solve: Anchor the matching with Prussian blue $Fe_4[Fe(CN)_6]_3$ and white magnesium ammonium phosphate, then check the remaining two colours.

---

68. What weight of glucose must be dissolved in $100\ g$ of water to lower the vapour pressure by $0.20\ mm\ Hg$?

(Assume dilute solution is being formed)

Given : Vapour pressure of pure water is $54.2\ mm\ Hg$ at room temperature. Molar mass of glucose is $180\ g\ mol^{-1}$ [4 marks]

(A) 3.69 g

(B) 2.59 g

(C) 3.59 g

(D) 4.69 g

Correct Answer: (A) 3.69 g

Solution:

Concept:

- For a nonvolatile solute, Raoult's law gives $p = x_w p^0$, hence $(p^0 - p)/p^0 = x_g$, where x_g is the glucose mole fraction.
- For the dilute solution specified, glucose moles are much smaller than water moles, so $x_g \simeq n_g/n_w$. Glucose is a nonelectrolyte and does not require an ionic particle multiplier.

Step 1: Calculate the solvent moles and relative lowering Take the molar mass of water as $18\,g\,mol^{-1}$.

The solvent amount and the dimensionless relative lowering are

$$n_w = \frac{100}{18}\,\text{mol}, \qquad \frac{\Delta p}{p^0} = \frac{0.20}{54.2}.$$

The numerator is the decrease 0.20, not the remaining solution pressure 54.0. Both pressures are in the same units, which cancel in the ratio.

Step 2: Apply the dilute-solution approximation If m grams of glucose are dissolved, then $n_g = m/180$. Therefore

$$\frac{0.20}{54.2} \simeq \frac{m/180}{100/18}.$$

Solving for the solute mass gives

$$m = \frac{0.20}{54.2} \times \frac{100}{18} \times 180 = \frac{200}{54.2} \simeq 3.69\,\text{g}.$$

This corresponds to only about 0.0205 mol glucose against 5.56 mol water, consistent with the stated dilute approximation.

Step 3: Check the alternatives and approximation The calculated mass matches 3.69 g. The other listed masses, 2.59 g, 3.59 g and 4.69 g, do not satisfy the specified relative lowering with this approximation. If the full mole-fraction denominator were retained, the result would be about 3.70 g; the question explicitly requests the dilute treatment, explaining the listed 3.69 g choice.

Step 4: Conclusion The required glucose mass under the stated dilute-solution assumption is $3.69\,\text{g}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the vapour-pressure decrease divided by the pure-solvent pressure; in a dilute nonelectrolyte solution this equals solute moles divided by solvent moles.

69. A solution is prepared by adding 2 g of "X" to 1 mole of water. Mass percent of "X" in the solution is : [4 marks]

(A) 20%

(B) 10%

(C) 2%

(D) 5%

Correct Answer: (B) 10%

Solution:

Concept:

- Mass percentage of a solute is $100\,m_{\text{solute}}/m_{\text{solution}}$. Its denominator is the mass of the complete solution, including both solute and solvent.
- A mole specifies an amount of substance, not a mass in grams. Convert the given mole of water using its molar mass, $18\,\text{g mol}^{-1}$, before using the mass-percentage formula.

Step 1: Convert water amount to mass The solvent amount is one mole of water. Using $M(\text{H}_2\text{O}) =$

$2(1) + 16 = 18\,\text{g mol}^{-1}$, its mass is

$$m_{\text{water}} = nM = 1 \times 18 = 18\,\text{g}.$$

The solute mass is given directly as 2 g. No molar mass of X is required, since the requested quantity is mass percent, not mole fraction or molarity.

Step 2: Find the total solution mass On dissolving X, the solution contains both the 2 g solute and the 18 g water. Thus

$$m_{\text{solution}} = 2 + 18 = 20\,\text{g}.$$

Using 18 g alone would calculate solute mass per solvent mass rather than the mass fraction of the solution. Volumes and any contraction on mixing are irrelevant, because the definition uses mass.

Step 3: Calculate the percentage and test the choices Substitute into the definition:

$$\text{mass percent of X} = \frac{2}{20} \times 100 = 10\%.$$

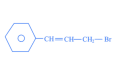
This means every 100 g of the same composition would contain 10 g X and 90 g water. The choices 20%, 2% and 5% correspond to solute fractions 0.20, 0.02 and 0.05 respectively, and none equals the actual fraction $2/20 = 0.10$. The sum of the solute and solvent mass percentages, 10% and 90%, is 100%, providing a check.

Step 4: Conclusion The mass percentage of X in the solution is 10%. Hence, the correct option is (B).

Quick Tip to Solve: For mass percent, turn every given mole quantity into grams and add the solute to the solvent before forming the denominator.

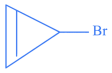
70. Compound from the following that will not produce precipitate on reaction with $AgNO_3$ is :

[4 marks]

(A) $C_6H_5-CH=CH-CH_2Br$ 

(B) 1-bromocyclohex-1-ene 

(C) $C_6H_5-CH_2Br$ 

(D) 3-bromocycloprop-1-ene 

Correct Answer: (B) 1-bromocyclohex-1-ene 

Solution:

Concept:

- Silver nitrate gives a silver halide precipitate when an organic halide can release halide ion under the test conditions. Here the precipitate would be pale-yellow $AgBr$.
- Ionisation is facilitated by stabilisation of the resulting carbocation. Allylic and benzylic cations are resonance-stabilised, whereas forming a vinyl carbocation is strongly unfavourable.

Step 1: Classify the carbon bonded to bromine In option (A), $C_6H_5-CH=CH-CH_2Br$, bromine is on an allylic carbon. In option (B), 1-bromocyclohex-1-ene, bromine is attached directly to a carbon

of the double bond, so it is vinylic. In option (C), $C_6H_5-CH_2Br$, the carbon bonded to bromine is benzylic. In option (D), bromine is on the saturated vertex of a cyclopropene ring, adjacent to the double bond.

Step 2: Compare the products of ionisation Loss of bromide from (A) forms an allylic cation with extended resonance stabilisation, and loss from (C) forms the resonance-stabilised benzyl cation. Loss from (D) produces the cyclopropenyl cation: its three p orbitals form a cyclic conjugated system containing two pi electrons, satisfying $4n + 2$ for $n = 0$. This aromatic stabilisation favours ionisation. Each can therefore supply bromide for precipitation with silver ion.

Step 3: Identify the resistant vinyl bromide Option (B) has a relatively strong carbon-bromine bond at an sp^2 carbon, with partial double-bond character arising from conjugation. Ionisation would generate a very unstable vinyl carbocation. It therefore resists the usual silver-nitrate substitution test and does not give the bromide precipitate under these conditions. The other three choices have cation stabilisation unavailable to the vinyl compound, so they are excluded.

Step 4: Conclusion 1-Bromocyclohex-1-ene, the vinylic bromide in option (B), does not produce the precipitate. Hence, the correct option is (B).

Quick Tip to Solve: Locate the carbon carrying Br: allylic, benzylic and aromatic-cation-forming halides react readily, but Br directly on a double-bond carbon resists the silver nitrate test.

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71. The magnetic moment is measured in Bohr Magneton (BM).

Spin only magnetic moment of Fe in $[Fe(H_2O)_6]^{3+}$ and $[Fe(CN)_6]^{3-}$ complexes respectively is :

[4 marks]

(A) 3.87 B.M. and 1.732 B.M.

(B) 5.92 B.M. and 1.732 B.M.

(C) 6.92 B.M. in both

(D) 4.89 B.M. and 6.92 B.M.

Correct Answer: (B) 5.92 B.M. and 1.732 B.M.

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons. The ligand field determines whether pairing occurs.
- For iron, $Z = 26$. Neutral iron is $[Ar]3d^6 4s^2$; removal of the two outer $4s$ electrons and one $3d$ electron gives Fe^{3+}, a $3d^5$ ion.

Step 1: Find the oxidation state in both complexes Water is neutral, so the charge of $[Fe(H_2O)_6]^{3+}$ makes iron $+3$. Each cyanide ligand has charge -1; hence $x - 6 = -3$, giving $x = +3$ in $[Fe(CN)_6]^{3-}$ also. Both therefore contain five d electrons, but this does not mean that their spin states must be identical.

Step 2: Count unpaired electrons in the aqua complex Water is a weak-field ligand. Its octahedral splitting is too small to cause pairing before the higher orbitals are occupied. The high-spin

arrangement is $t_{2g}^3 e_g^2$, with one electron in each of the five d orbitals. Thus $n = 5$, and

$$\mu_{\text{aqua}} = \sqrt{5(5+2)} = \sqrt{35} = 5.92\,\text{BM}.$$

Step 3: Count unpaired electrons in the cyanide complex Cyanide is a strong-field ligand. Electrons pair in the lower t_{2g} set, producing the low-spin arrangement $t_{2g}^5 e_g^0$. Two of these orbitals are doubly occupied and one is singly occupied, so $n = 1$:

$$\mu_{\text{cyanide}} = \sqrt{1(1+2)} = \sqrt{3} = 1.732\,\text{BM}.$$

Option A incorrectly gives the first ion only three unpaired electrons. Options C and D use incompatible moments; in particular, 6.92 BM would require six unpaired electrons, impossible for a d-five ion.

Step 4: Conclusion The moments are $5.92\,\text{BM}$ and $1.732\,\text{BM}$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Find oxidation state before counting spins. For octahedral d-five iron, weak-field water gives five unpaired electrons whereas strong-field cyanide gives one.

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72. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: $CH_3CH(CH_3)CH(Cl)COCH_3$ **can be subjected to Wolff-Kishner reduction to give** $CH_3CH(CH_3)CH(Cl)CH_2CH_3$**.**

Reason R: Wolff-Kishner reduction is used to convert $> C = O$ **into** $> CH_2$**.**

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) A is false but R is true

(C) A is true but R is false

(D) Both A and R are true but R is NOT the correct explanation of A

Correct Answer: (B) A is false but R is true

Solution:

Concept:

- Wolff-Kishner reduction converts an aldehyde or ketone carbonyl group to a methylene group using hydrazine, strong base and heat.
- The reaction conditions matter as much as the formal carbonyl change. A substrate bearing an eliminable halogen can undergo dehydrohalogenation under strongly basic conditions.

Step 1: Test the carbonyl statement in the reason A ketone first forms a hydrazone with hydrazine. Under hot alkaline conditions it loses nitrogen, and the carbonyl carbon becomes a methylene carbon.

In general the net functional-group transformation is

$$\mathrm{RCOR'} \longrightarrow \mathrm{RCH_2R'}.$$

Consequently the transformation stated in Reason R is correct. This immediately rules out option C, which calls the reason false.

Step 2: Check the other functional group in the assertion The substrate in Assertion A is $\mathrm{CH_3CH(CH_3)CH(Cl)COCH_3}$. Besides its ketone group, it contains chlorine on the adjacent carbon and an available hydrogen on the neighbouring carbon. In the strongly basic, heated Wolff-Kishner medium, loss of hydrogen chloride can form a carbon-carbon double bond:

$$\mathrm{CH_3CH(CH_3)CH(Cl)COCH_3} \longrightarrow \mathrm{(CH_3)_2C{=}CHCOCH_3 + HCl}.$$

This equation identifies the eliminable groups; in the actual alkaline medium the released acid is neutralised. The conditions do not preserve the carbon-chlorine bond as the displayed target requires.

Step 3: Combine the two truth values The assertion claims formation of the saturated product with chlorine retained, merely replacing the carbonyl oxygen by two hydrogens. Because the base-sensitive chloro group undergoes elimination, that claimed selective conversion is false. The general carbonyl-to-methylene statement remains true. Options A and D both wrongly classify the assertion as true, while option C reverses the actual truth values.

Step 4: Conclusion Assertion A is false but Reason R is true. Hence, the correct option is (B).

Quick Tip to Solve: For a named reduction, inspect every functional group under its actual reagents. Hot strong base can eliminate a halide even when the named reaction concerns a carbonyl.

73. For a chemical reaction $\mathrm{A + B \longrightarrow Product}$, the order is 1 with respect to A and B.

Rate $(\mathrm{mol\,L^{-1}\,s^{-1}})$	**[A]** $(\mathrm{mol\,L^{-1}})$	**[B]** $(\mathrm{mol\,L^{-1}})$
0.10	**20**	**0.5**
0.40	x	**0.5**
0.80	**40**	y

What is the value of x and y? [4 marks]

(A) 80 and 4

(B) 160 and 4

(C) 80 and 2

(D) 40 and 4

Correct Answer: (C) 80 and 2

Solution:

Concept:

- A first-order dependence on each reactant gives the rate law $r = k[\mathrm{A}][\mathrm{B}]$. The overall reaction order is two, and k is constant for the experiments at the same temperature.

- Taking a ratio of rates cancels the unknown rate constant. If one concentration stays fixed, the rate ratio directly equals the ratio of the other concentration.

Step 1: Find the unknown concentration x Compare the first and second rows. The concentration of B is $0.5\,\text{mol}\,\text{L}^{-1}$ in both, so it cancels in their rate ratio:

$$\frac{0.40}{0.10} = \frac{kx(0.5)}{k(20)(0.5)} = \frac{x}{20}.$$

The left-hand side equals 4. Therefore

$$x = 4 \times 20 = 80\,\text{mol}\,\text{L}^{-1}.$$

A quadrupled rate at constant B requires a quadrupled A concentration, because the order in A is one.

Step 2: Find the unknown concentration y Compare the third and first rows. The rate increases by a factor of 8 while A increases from 20 to 40, a factor of 2. Thus B must increase by the remaining factor of 4:

$$\frac{0.80}{0.10} = \frac{40}{20}\frac{y}{0.5}, \qquad 8 = 2\frac{y}{0.5}.$$

Solving gives $y = 2\,\text{mol}\,\text{L}^{-1}$. Equivalently, the first row gives $k = 0.10/(20 \times 0.5) = 0.01\,\text{L}\,\text{mol}^{-1}\,\text{s}^{-1}$, and the third requires $0.80 = 0.01 \times 40 \times y$.

Step 3: Check the options against both rows The pair $(80, 2)$ reproduces the second rate as $0.01 \times 80 \times 0.5 = 0.40$ and the third as $0.01 \times 40 \times 2 = 0.80$. Option A has the correct x but doubles y incorrectly. Option B makes both concentrations too large. Option D fails the fixed-B comparison and also has the wrong y.

Step 4: Conclusion $x = 80\,\text{mol}\,\text{L}^{-1}$ and $y = 2\,\text{mol}\,\text{L}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: In rate-law tables, first compare rows where one concentration is unchanged; then divide out its contribution when comparing rows where both concentrations change.

74.

$$CH_3CH_2CH(OH)CH_3 \xrightarrow[\text{(ii) Mg, dry ether; (iii) } D_2O]{\text{(i) NaI, } H_3PO_4} [X]$$

Product [X] formed in the above reaction is: [4 marks]

(A) $CH_3CH_2CH(OH)CH_3$

(B) $CH_3CH_2CH(D)CH_3$

(C) $CH_3CH{=}CHCH_3$

(D) $CH_3CH_2CH{=}CH_2$

Correct Answer: (B) $CH_3CH_2CH(D)CH_3$

Solution:

Concept:

- An alcohol can be converted to an alkyl iodide using hydrogen iodide generated from sodium iodide and phosphoric acid.
- Magnesium in dry ether inserts into a carbon-iodine bond, producing a Grignard reagent. Quenching that reagent with heavy water replaces the carbon-magnesium bond by carbon-deuterium.

Step 1: Convert the alcohol to its iodide The starting compound is butan-2-ol, $CH_3CH_2CH(OH)CH_3$. The first reagent combination supplies iodide in an acidic medium, so the hydroxyl group is replaced by iodine at the same secondary carbon:

$$CH_3CH_2CH(OH)CH_3 \xrightarrow{NaI,\, H_3PO_4} CH_3CH_2CH(I)CH_3.$$

The carbon skeleton remains a four-carbon chain. This substitution prepares the carbon-halogen bond needed in the next stage.

Step 2: Form the organomagnesium compound In dry ether, magnesium inserts into the C-I bond to give sec-butylmagnesium iodide:

$$CH_3CH_2CH(I)CH_3 + Mg \longrightarrow CH_3CH_2CH(MgI)CH_3.$$

Dry conditions are essential: ordinary water would prematurely protonate the reagent and destroy its ability to receive deuterium in the final controlled quench.

Step 3: Quench with heavy water and compare products The carbon attached to magnesium behaves as the basic carbon centre and abstracts deuterium from D_2O:

$$CH_3CH_2CH(MgI)CH_3 + D_2O \longrightarrow CH_3CH_2CH(D)CH_3 + MgI(OD).$$

The original C-H bond at this carbon remains; D occupies the position formerly occupied by OH, then I and MgI. Option A retains OH and therefore ignores the substitution and Grignard steps. Options C and D are alkenes; neither represents the deuterolysis of a Grignard reagent.

Step 4: Conclusion Product X is 2-deuteriobutane, $CH_3CH_2CHDCH_3$. Hence, the correct option is (B).

Quick Tip to Solve: Track the same carbon through OH, halogen and MgX. A final heavy-water quench places D exactly where MgX was bonded, without adding a double bond.

75. Alkali metal from the following with least melting point is : [4 marks]

(A) Rb

(B) Na

(C) K

(D) Cs

Correct Answer: (D) Cs

Solution:

Concept:

- Alkali metals contain one valence electron per atom, which participates in metallic bonding. Their melting points reflect the cohesion of the metallic lattice.
- Down Group 1, atomic size increases. The attraction between the positive metal cores and delocalised valence electrons becomes weaker, so the metallic lattice generally melts more easily.

Step 1: Arrange the metals down the group The four choices belong to Group 1. Their order of increasing atomic number and atomic size is

$$\mathrm{Na} < \mathrm{K} < \mathrm{Rb} < \mathrm{Cs}.$$

Each has the same number of outermost electrons, so the main comparison is the increase in distance between the positive cores and the delocalised electrons. A larger atom does not mean stronger metallic bonding in this group.

Step 2: Apply the trend in lattice cohesion As the atoms become larger, their charge density decreases and the metallic attraction weakens. Less thermal energy is needed to disrupt the lattice. Hence the melting-point order for these metals is

$$T_m(\mathrm{Na}) > T_m(\mathrm{K}) > T_m(\mathrm{Rb}) > T_m(\mathrm{Cs}).$$

The approximate temperatures illustrate the trend: sodium melts near $98\,^\circ\mathrm{C}$, potassium near $64\,^\circ\mathrm{C}$, rubidium near $39\,^\circ\mathrm{C}$, and caesium near $29\,^\circ\mathrm{C}$. These values are consistent with the qualitative metallic-bonding argument.

Step 3: Eliminate the alternatives Rubidium is a tempting choice because it also melts at a low temperature, but caesium lies below it and melts still lower. Sodium and potassium are higher in the group and have stronger metallic cohesion, so neither can be the least-melting member among the four. The question asks for the minimum melting point, not the highest reactivity or the smallest atomic size.

Step 4: Conclusion Caesium, Cs, has the least melting point among the listed metals. Hence, the correct option is (D).

Quick Tip to Solve: For Group 1 melting points, move downward to find the lower value: the single valence electron binds increasingly large metal cores less strongly.

76. Given below are two statements :

Statement I : In the metallurgy process, sulphide ore is converted to oxide before reduction.

Statement II : Oxide ores in general are easier to reduce.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- In the usual extraction route for a sulphide ore, roasting in excess air converts the sulphide into an oxide and releases sulphur dioxide.
- Metal oxides are generally more convenient to reduce to metals than their corresponding sulphides. The reduction method depends on the particular metal and the stability of its oxide.

Step 1: Judge Statement I using the roasting step A concentrated sulphide ore is commonly heated strongly in excess oxygen before the reduction stage. For example, zinc sulphide undergoes roasting:

$$2\,\mathrm{ZnS} + 3\,\mathrm{O_2} \longrightarrow 2\,\mathrm{ZnO} + 2\,\mathrm{SO_2}.$$

Sulphur leaves as gaseous sulphur dioxide and zinc is retained as its oxide. This is the standard preparatory change described in Statement I, so the statement is correct in the conventional sulphide-ore extraction sequence being tested.

Step 2: Judge Statement II using the reduction step The oxide can then be reduced with a suitable reducing agent. In the zinc example, carbon reduces zinc oxide at a high temperature:

$$\mathrm{ZnO} + \mathrm{C} \longrightarrow \mathrm{Zn} + \mathrm{CO}.$$

Thus converting sulphide to oxide provides a practical route to the metal. Statement II correctly says that oxides are, in general, easier to reduce in this metallurgical context. The wording does not claim that all oxides can be reduced with carbon; highly reactive metals require other methods, such as electrolysis.

Step 3: Select the matching combination Statement I describes the conversion before reduction, and Statement II gives the general chemical reason for preferring the oxide stage. Both are correct. Option B rejects both valid statements. Option C wrongly rejects the usefulness of oxide reduction, while option D wrongly rejects the roasting conversion. The example of zinc shows both steps within one coherent extraction route.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: Remember the sequence for a typical sulphide ore: concentrate, roast to the oxide, then reduce. Do not confuse roasting in air with calcination in limited or no air.

77. Which of the following compounds is an example of Freon? [4 marks]

(A) $C_2H_2F_2$

(B) C_2HF_3

(C) C_2F_4

(D) $C_2Cl_2F_2$

Correct Answer: (D) $C_2Cl_2F_2$

Solution:

Concept:

- In the environmental-chemistry classification used here, Freons are chlorofluorocarbon compounds associated with refrigeration and atmospheric ozone depletion.
- The relevant elemental signature is carbon combined with chlorine and fluorine. This distinguishes the chlorofluorocarbon choice from compounds containing only hydrogen and fluorine, or only fluorine.

Step 1: Identify the characteristic elements The question asks for an example of the Freon class. The usual school-level examples are chlorofluorocarbon refrigerants. Inspecting the molecular formula is enough to distinguish that class among the listed choices: the selected compound must contain carbon, chlorine and fluorine together. This is a classification question; it does not require computing molecular mass or identifying a specific commercial refrigerant number.

Step 2: Compare every printed formula Option A, $C_2H_2F_2$, has carbon, hydrogen and fluorine but no chlorine. Option B, C_2HF_3, likewise lacks chlorine. Option C, C_2F_4, contains only carbon and fluorine. These three do not have the chlorofluorocarbon composition tested here. Option D, $C_2Cl_2F_2$, is the sole formula containing both chlorine and fluorine attached within a carbon compound, and therefore is the intended Freon-type choice among these alternatives.

Step 3: Connect the classification with environmental chemistry Chlorofluorocarbon compounds are discussed because sufficiently stable molecules can reach the stratosphere, where ultraviolet radiation can release chlorine-containing reactive species. Chlorine radicals can then participate in catalytic ozone destruction. This context explains why the presence of chlorine matters in distinguishing the selected option. The printed $C_2Cl_2F_2$ has one degree of unsaturation and is not the usual saturated Freon-12, whose formula is CCl_2F_2. Thus the formula as printed is anomalous for the familiar refrigerant example. Nevertheless, D is the intended chlorofluorocarbon selection among the supplied options; the other three contain no chlorine.

Step 4: Conclusion Among the printed options, the Freon-type chlorofluorocarbon choice is $C_2Cl_2F_2$. Hence, the correct option is (D).

Quick Tip to Solve: For this classification MCQ, scan first for the compound containing both Cl and F. Keep its printed formula distinct from the formula of a named refrigerant such as Freon-12.

78. Which hydride among the following is less stable? [4 marks]

(A) NH_3

(B) HF

(C) LiH

(D) BeH_2

Correct Answer: (D) BeH_2

Solution:

Concept:

- The stability of a hydride depends on the nature and strength of its bonding. Ionic lattice stabilization, strong covalent bonds and electron deficiency must be distinguished.
- LiH is an ionic hydride, whereas BeH_2 is an electron-deficient covalent hydride with bridging hydrogen atoms in its polymeric structure.

Step 1: Identify the stable bonding arrangements LiH contains Li^+ and H^- ions and benefits from strong electrostatic attraction in its crystal lattice. HF has a particularly strong H–F bond, and NH_3 has ordinary stable N–H covalent bonds. Thus neither ionic character alone nor the number of hydrogen atoms is a reason to choose one of these as the least stable.

Step 2: Compare with the electron-deficient hydride Beryllium has only two valence electrons. In BeH_2, simple localized two-centre bonding cannot provide an octet at Be; it forms electron-deficient bridge bonds in the solid. Among the four choices, this covalent electron-deficient hydride is comparatively less thermally stable. The strong H–F bond excludes HF, lattice stabilization excludes LiH, and stable N–H bonding excludes ammonia.

Step 3: Conclusion The less stable hydride is BeH_2. Hence, the correct option is (D).

Quick Tip to Solve: First separate the ionic hydride LiH from electron-deficient BeH2; their stability cannot be ranked from molecular mass alone.

79. Given below are two statements :

Statement I : Ethene at 333 to $343\ K$ and 6-7 atm pressure in the presence of $AlEt_3$ and $TiCl_4$ undergoes addition polymerization to give LDP.

Statement II : Caprolactam at 533-543 K in H_2O through step growth polymerizes to give Nylon 6.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true but Statement II is false

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are false

Correct Answer: (C) Statement I is false but Statement II is true

Solution:

Concept:

- Ethene polymerization with the Ziegler–Natta catalyst $Al(C_2H_5)_3/TiCl_4$ produces relatively linear, high-density polyethene (HDP).
- Nylon 6 is a polyamide obtained from caprolactam by heating with water; ring opening gives amino-acid units that form amide links during chain growth.

Step 1: Test Statement I against the catalyst and conditions The stated temperature, 333–343 K, moderate pressure and triethylaluminium/titanium tetrachloride catalyst identify the preparation of high-density polyethene. Its chains have little branching and pack closely. Low-density polyethene instead results from the high-pressure free-radical process and is extensively branched. Therefore the product named LDP in Statement I is incorrect, so Statement I is false.

Step 2: Test Statement II and select the combination Caprolactam is heated at 533–543 K with water to prepare Nylon 6. Water opens the lactam ring, and the resulting amino and carboxyl functionalities form the repeating polyamide chain by the step-growth treatment used in the syllabus. The repeat unit is $[-NH-(CH_2)_5-CO-]_n$. Thus Statement II is true. A and B wrongly accept Statement I, while D wrongly rejects Statement II.

Step 3: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (C).

Quick Tip to Solve: Use the ethene catalyst as the diagnostic clue: Ziegler–Natta conditions give HDP, while high-pressure radical conditions give LDP.

80. One mole of P_4 reacts with 8 moles of $SOCl_2$ to give 4 moles of A, x mole of SO_2 and 2 moles of B. A, B and x respectively are : [4 marks]

(A) $POCl_3$, S_2Cl_2 and 4

(B) PCl_3, S_2Cl_2 and 4

(C) $POCl_3$, S_2Cl_2 and 2

(D) PCl_3, S_2Cl_2 and 2

Correct Answer: (B) PCl_3, S_2Cl_2 and 4

Solution:

Concept:

- Chemical equations conserve the number of atoms of every element. The specified coefficients permit the products to be identified without guessing.
- Thionyl chloride supplies chlorine and oxygen; in this reaction phosphorus forms PCl_3, while sulfur appears as SO_2 and S_2Cl_2.

Step 1: Balance sulfur and oxygen for the proposed products Eight moles of $SOCl_2$ supply 8 mol of S atoms, 8 mol of O atoms and 16 mol of Cl atoms. Two moles of S_2Cl_2 account for 4 mol of S atoms. The remaining 4 mol of S atoms must therefore give 4 mol of SO_2, so $x = 4$. This also uses all 8 mol of oxygen atoms.

Step 2: Use chlorine conservation to determine A Two moles of S_2Cl_2 contain 4 mol of Cl atoms, leaving 12 mol of Cl atoms for four moles of the phosphorus compound. Consequently each phosphorus atom is bonded to three chlorine atoms, giving PCl_3. The balanced equation is

$$P_4 + 8\,SOCl_2 \longrightarrow 4\,PCl_3 + 4\,SO_2 + 2\,S_2Cl_2.$$

Option A would require additional oxygen for $POCl_3$; options C and D have the wrong sulfur dioxide coefficient and do not conserve sulfur.

Step 3: Conclusion $A = PCl_3$, $B = S_2Cl_2$, and $x = 4$. Hence, the correct option is (B).

Quick Tip to Solve: Balance sulfur first using the fixed amount of S2Cl2, then check oxygen and chlorine to distinguish PCl3 from POCl3.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The number of possible isomeric products formed when 3-chloro-1-butene reacts with HCl through carbocation formation is _____. [4 marks]

Solution:

Concept:

- Electrophilic addition of HCl to an unsymmetrical alkene proceeds through a carbocation; chloride capture can occur directly or after a favourable rearrangement.

- Count distinct constitutional products and their stereoisomers. A meso form counts once, and a pair of enantiomers counts as two isomeric products.

Step 1: Form the carbocation and enumerate constitutions The starting alkene is

$$CH_2{=}CH{-}CH(Cl){-}CH_3.$$

Protonation at the terminal carbon gives the secondary carbocation

$$CH_3{-}CH^+{-}CH(Cl){-}CH_3.$$

Direct attack by chloride produces 2,3-dichlorobutane. A 1,2-hydride shift from the adjacent chlorine-bearing carbon gives

$$CH_3{-}CH_2{-}C^+(Cl){-}CH_3.$$

Chloride attack then gives 2,2-dichlorobutane. The positive centre adjacent to chlorine is stabilized by chlorine lone-pair donation.

Step 2: Count the stereoisomers of each constitution 2,3-Dichlorobutane has two stereogenic centres. Since its two ends are identical, the configurations $(2R, 3S)$ and $(2S, 3R)$ describe the same meso compound. Together with the enantiomeric pair $(2R, 3R)$ and $(2S, 3S)$, it gives three stereoisomers. In 2,2-dichlorobutane the carbon bearing chlorine has two identical Cl substituents and is not stereogenic, so this constitution gives only one product. Hence the total is $3 + 1 = 4$.

Step 3: Conclusion The number of possible isomeric products is $\boxed{4}$ (a dimensionless count).

Quick Tip to Solve: After listing the direct-addition and hydride-shift products, apply symmetry before using the naive two-stereocentre count.

82. The maximum number of lone pairs of electrons on the central atom from the following species is _____.

ClO_3^-, XeF_4, SF_4 **and** I_3^- [4 marks]

Solution:

Concept:

- A lone pair is a nonbonding electron pair on the central atom. Count central-atom valence electrons, including any negative charge, after assigning electrons to bonding.
- In VSEPR notation AX_mE_n, m is the number of bonded atoms and n is the number of central-atom lone pairs; multiple bonds count as one region.

Step 1: Count lone pairs in the chlorine, xenon and sulfur species For ClO_3^-, chlorine has seven valence electrons and the negative charge adds one. Its Lewis structures have three Cl–O bonding regions and one lone pair on Cl, giving AX_3E. In XeF_4, xenon has eight valence electrons: four are used in four Xe–F bonds and the remaining four make two lone pairs, so it is AX_4E_2. In SF_4, sulfur has six valence electrons; four form S–F bonds and two remain as one lone pair, giving AX_4E.

Step 2: Evaluate the triiodide ion and compare For the central iodine in I_3^-, seven valence electrons

plus one from the negative charge give eight. Two are assigned to the two I–I bonds, leaving six electrons as three lone pairs. Thus I_3^- is AX_2E_3: its three equatorial lone pairs leave the two bonds axial, producing a linear ion. The central-atom lone-pair counts in the listed order are $1, 2, 1, 3$, whose maximum is 3.

Step 3: Conclusion The maximum is $\boxed{3}$ lone pairs, on the central iodine of I_3^-. This is a dimensionless count.

Quick Tip to Solve: For XeF4, SF4 and I3-, use one central electron per single bond and pair the electrons left on the central atom.

83. The total number of intensive properties from the following is _____

Volume, Molar heat capacity, Molarity, E^θ cell, Gibbs free energy change, Molar mass, Mole [4 marks]

Solution:

Concept:

- An intensive property is independent of the amount of material or size of the system. An extensive property changes when the amount of the same system is multiplied.
- A molar quantity is normalized per mole and is generally intensive; ratios such as concentration are also unchanged when an unchanged homogeneous system is scaled.

Step 1: Identify the four intensive entries Molar heat capacity is heat capacity per mole, so its value is unchanged by doubling the sample. Molarity is moles of solute per litre of solution: doubling both numerator and denominator leaves it unchanged. The standard cell potential E°_{cell} is a potential difference determined by the electrodes and conditions, rather than electrode size. Molar mass is mass per mole and is a characteristic of the substance. These four entries are therefore intensive.

Step 2: Exclude the extensive entries and count Volume doubles when the size of the system is doubled at the same conditions, so it is extensive. Gibbs free energy change for a reaction is proportional to the extent of reaction: doubling the amount reacting doubles ΔG, unlike cell potential. The final entry, mole, denotes amount of substance and also scales with the sample; it is extensive. Thus the intensive entries are molar heat capacity, molarity, standard cell potential and molar mass, giving a total of four.

Step 3: Conclusion The total number of intensive properties is $\boxed{4}$ (a dimensionless count).

Quick Tip to Solve: Apply the doubling test: if doubling the same sample doubles the property, it is extensive; per-mole values and concentration remain unchanged.

84. Number of compounds from the following which will not produce orange red precipitate with Benedict solution is _____.

Glucose, maltose, sucrose, ribose, 2-deoxyribose, amylose, lactose [4 marks]

Solution:

Concept:

- Benedict solution contains alkaline copper(II) ions. Reducing carbohydrates convert them into the orange-red precipitate of copper(I) oxide, Cu_2O.
- A free anomeric hemiacetal group can open to a reducing carbonyl form. Locking the anomeric centres in a glycosidic bond prevents this normal reducing-sugar test.

Step 1: Classify the reducing sugars in the list Glucose, ribose and 2-deoxyribose are aldoses with a free anomeric centre and can open to their aldehydic forms. The absence of an OH group at carbon 2 in 2-deoxyribose does not remove its aldehyde-forming anomeric centre. Maltose and lactose each retain one free anomeric centre even though another centre participates in a glycosidic bond. These five carbohydrates therefore reduce Benedict solution and give the orange-red precipitate.

Step 2: Identify the negative tests and total them In sucrose, the glycosidic linkage joins the anomeric carbon of glucose to the anomeric carbon of fructose. Neither centre is free, so sucrose is nonreducing. Amylose is the starch polysaccharide built mainly from $\alpha(1 \rightarrow 4)$-linked glucose units and does not give the ordinary Benedict test; the single end group in a long macromolecule does not yield the characteristic reducing-sugar response. Therefore sucrose and amylose are the two listed compounds that do not produce the precipitate.

Step 3: Conclusion The compounds are sucrose and amylose, so the required number is $\boxed{2}$ (a dimensionless count).

Quick Tip to Solve: Check for a free anomeric centre: maltose and lactose have one, sucrose has none, and amylose is treated as a nonreducing polysaccharide in this test.

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85. The number of correct statements from the following is _____.

A. For $1s$ orbital, the probability density is maximum at the nucleus

B. For $2s$ orbital, the probability density first increases to maximum and then decreases sharply to zero.

C. Boundary surface diagrams of the orbitals encloses a region of 100% probability of finding the electron.

D. p and d-orbitals have 1 and 2 angular nodes respectively.

E. probability density of p-orbital is zero at the nucleus [4 marks]

Solution:

Concept:

- Probability density at a point is $|\psi|^2$. It is different from radial probability, which also includes the volume of a spherical shell and is proportional to $r^2|R(r)|^2$.
- The number of angular nodes is the azimuthal quantum number l: it is zero for s, one for p and

two for d orbitals. A boundary surface represents a large, specified probability, usually about 90%, rather than certainty.

Step 1: Test the statements about s orbitals Statement A is correct: the $1s$ wavefunction has its largest magnitude at $r = 0$, and its square therefore gives maximum probability density at the nucleus. Statement B is incorrect. For $2s$, probability density is already nonzero and maximal at the nucleus; it initially decreases to a radial node, where it is zero, and then has a smaller outer maximum before tending to zero at large distance. The claim that it first increases from the nucleus confuses probability density with radial probability. The latter vanishes at the origin because of the spherical-shell factor r^2.

Step 2: Check the boundary surface and angular nodes Statement C is incorrect: an orbital's wavefunction extends beyond any finite boundary surface, so a finite diagram does not enclose 100% probability. Statement D is correct because p orbitals have $l = 1$ and d orbitals have $l = 2$, giving one and two angular nodes respectively. Statement E is also correct: the radial factor of a p orbital vanishes at $r = 0$, so its probability density at the nucleus is zero. Thus A, D and E are accepted, whereas B and C are rejected.

Step 3: Conclusion The correct statements are A, D and E. Hence the number to be entered is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: Before judging an orbital graph, identify whether its vertical axis is probability density or radial probability; only the latter necessarily vanishes at the nucleus for every orbital.

86. 4.5 moles each of hydrogen and iodine is heated in a sealed ten litre vessel. At equilibrium, 3 moles of HI were found. The equilibrium constant for $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ is _____. [4 marks]

Solution:

Concept:

- For $H_2(g) + I_2(g) \rightleftharpoons 2\,HI\,(g)$, the concentration equilibrium constant is $K_c = [HI]^2/([H_2][I_2])$. Equilibrium concentrations, rather than the initial amounts, must be substituted.
- Reaction coefficients determine the changes in mole numbers. Forming two moles of hydrogen iodide consumes one mole each of hydrogen and iodine; equal initial reactant amounts therefore remain equal at equilibrium.

Step 1: Determine the equilibrium mole numbers Initially the vessel contains $4.5\,\text{mol}$ each of H_2 and I_2, with no hydrogen iodide specified. Let the reaction extent be $x\,\text{mol}$. Then the equilibrium amounts are $4.5 - x$, $4.5 - x$ and $2x$ moles, respectively. Since the amount of HI found at equilibrium is $3\,\text{mol}$,

$$2x = 3 \quad \Rightarrow \quad x = 1.5.$$

Therefore each reactant has $4.5 - 1.5 = 3.0\,\text{mol}$ left. The three species have equal mole numbers at equilibrium, although their initial mole numbers were not all equal.

Step 2: Substitute the concentrations in the equilibrium expression Divide each equilibrium amount

by the common vessel volume of $10\,\mathrm{L}$: all three concentrations equal $0.30\,\mathrm{mol\,L^{-1}}$. Hence

$$K_c = \frac{(0.30)^2}{(0.30)(0.30)} = 1.$$

The concentration factors cancel because the total gaseous stoichiometric coefficient is two on both sides. Equivalently, using equilibrium mole numbers directly gives $3^2/(3 \times 3) = 1$. Also $\Delta n_g = 2 - 1 - 1 = 0$, so $K_p = K_c(RT)^0 = K_c$; the unspecified choice between these constants does not change the answer.

Step 3: Conclusion The equilibrium constant is $\boxed{1}$, dimensionless in the activity convention. The same numerical value is obtained for K_c and K_p.

Quick Tip to Solve: In this reaction, halve the amount of HI formed to find the amount of each reactant consumed. When $\Delta n_g = 0$, the common volume cancels from the equilibrium quotient.

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87. The number of correct statements from the following is _____

A. $\mathrm{E_{cell}}$ **is an intensive parameter**

B. A negative $\mathrm{E^{\ominus}}$ **means that the redox couple is a stronger reducing agent than the** $\mathrm{H^+/H_2}$ **couple.**

C. The amount of electricity required for oxidation or reduction depends on the stoichiometry of the electrode reaction.

D. The amount of chemical reaction which occurs at any electrode during electrolysis by a current is proportional to the quantity of electricity passed through the electrolyte. [4 marks]

Solution:

Concept:

- Cell potential is the electrical work available per unit charge. It obeys $\Delta G = -nFE_{\text{cell}}$, where n is the number of electrons transferred per reaction as written and F is the Faraday constant.
- Standard electrode potentials are reduction potentials measured relative to the standard hydrogen electrode, whose potential is zero. Faraday's laws relate chemical change to the charge $Q = It$ passed.

Step 1: Evaluate the two statements about potential A is correct. If a cell reaction is multiplied by any factor, both ΔG and n are multiplied by that factor, so their ratio $-\Delta G/(nF)$ and hence E_{cell} stay unchanged. Potential is therefore intensive. B is also correct. A negative standard reduction potential indicates a smaller tendency to accept electrons than the $\mathrm{H^+/H_2}$ reference couple. Conversely, the reduced member of that couple has a greater tendency to donate electrons. It is thus a stronger reducing agent than hydrogen under standard conditions. The sign is interpreted for a reduction potential, rather than for an arbitrarily reversed half-reaction.

Step 2: Apply electron stoichiometry and Faraday's law C is correct: reducing one mole of $\mathrm{Ag^+}$ requires one mole of electrons, whereas reducing one mole of $\mathrm{Cu^{2+}}$ requires two. In general, for one mole of a species requiring z electrons, the charge required is zF; therefore the electrode equation's stoichiometry matters. D is correct as well. For a fixed electrode reaction with molar mass M, the

mass transformed is

$$m = \frac{M}{zF}Q = \frac{M}{zF}It.$$

Thus the amount of chemical change is directly proportional to the quantity of electricity passed, as stated by Faraday's first law.

Step 3: Conclusion All four statements A, B, C and D are correct. The required dimensionless number is $\boxed{4}$.

Quick Tip to Solve: Potential does not scale when a reaction is multiplied, but the electron count and required charge do. Use this distinction to separate intensive potential from extensive electrical work.

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88. The volume of hydrogen liberated at STP by treating 2.4 g of magnesium with excess of hydrochloric acid is _____ $\times 10^{-2}$ L

Given : Molar volume of gas is 22.4 L at STP.

Molar mass of magnesium is 24 g mol^{-1} [4 marks]

Solution:

Concept:

- A balanced chemical equation gives the mole ratio between reactants and products. The amount of a solid in moles is its mass divided by its molar mass, $n = m/M$.
- The volume of a gas at the stated STP is $V = nV_m$, using the given molar volume $V_m = 22.4\,\text{L mol}^{-1}$. A value requested as a multiple of 10^{-2} L is the coefficient in that representation, not the gas volume in litres itself.

Step 1: Establish the limiting reactant and mole ratio Magnesium displaces hydrogen from hydrochloric acid according to

$$\text{Mg} + 2\,\text{HCl} \longrightarrow \text{MgCl}_2 + \text{H}_2.$$

The balanced equation shows that one mole of magnesium produces one mole of hydrogen, provided sufficient acid is available. Since hydrochloric acid is explicitly in excess, magnesium is the limiting reactant and is completely consumed. The coefficient two on HCl is the acid requirement; it does not imply two moles of hydrogen per mole of magnesium.

Step 2: Calculate the hydrogen volume at STP Using the stated molar mass rather than a more precise tabulated value,

$$n(\text{Mg}) = \frac{2.4\,\text{g}}{24\,\text{g mol}^{-1}} = 0.10\,\text{mol}.$$

Hence $n(\text{H}_2) = 0.10\,\text{mol}$, and

$$V(\text{H}_2) = (0.10)(22.4) = 2.24\,\text{L}.$$

This uses the gas molar volume at the STP specified in the problem, so no additional pressure or temperature conversion is necessary.

Step 3: Express the result in the requested scale Let the blank be a. The question asks for $a \times 10^{-2}$ L, so

$$a \times 10^{-2} = 2.24 \quad \Rightarrow \quad a = 224.$$

Multiplying the coefficient by the stated scale returns $224 \times 0.01 = 2.24\,\text{L}$, which checks the conversion.

Step 4: Conclusion The hydrogen volume is $\boxed{2.24\,\text{L} = 224 \times 10^{-2}\,\text{L}}$ at STP. Enter $\boxed{224}$ in the blank.

Quick Tip to Solve: Use the balanced equation to obtain moles of gas first; convert to volume second, and only then divide by the power of ten printed beside the answer blank.

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89. The number of correct statements about modern adsorption theory of heterogeneous catalysis from the following is _____

A. The catalyst is diffused over the surface of reactants.

B. Reactants are adsorbed on the surface of the catalyst.

C. Occurrence of chemical reaction on the catalyst's surface through formation of an intermediate.

D. It is a combination of intermediate compound formation theory and the old adsorption theory.

E. It explains the action of the catalyst as well as those of catalytic promoters and poisons. [4 marks]

Solution:

Concept:

- Heterogeneous catalysis occurs at the interface between a catalyst and reactants in another phase. The modern adsorption theory combines adsorption of reactants with formation and decomposition of a surface intermediate.
- Its reaction sequence is diffusion of reactants to the surface, adsorption, surface reaction, desorption of products and diffusion of products away. The catalyst offers a pathway of lower activation energy and is regenerated.

Step 1: Identify which substance reaches and occupies the surface A is incorrect because it reverses the roles of catalyst and reactants. Reactant molecules diffuse towards the catalyst surface; the catalyst does not diffuse over the surface of reactants. B is correct: the reactants are adsorbed at active sites on the catalyst. Adsorption brings the reacting species close together and can weaken their bonds, facilitating the subsequent chemical transformation. Adsorption must be sufficiently strong to activate reactants but not so strong that products cannot leave the surface.

Step 2: Examine the role of the surface intermediate C is correct. The adsorbed species react on the surface through an intermediate, which subsequently forms products. Products desorb and release the occupied sites for another catalytic cycle. D is also correct: this intermediate stage is the feature of intermediate compound formation theory, while concentration and activation of reactants at the surface come from adsorption theory. The modern description combines these two aspects rather than treating adsorption alone as the complete reaction.

Step 3: Check the claimed scope of the theory and count E is incorrect in the stated description of this theory. The basic adsorption and surface-intermediate sequence explains catalytic action, but does not by itself provide a general explanation of the effects of catalytic promoters and poisons. Such effects require additional information about how a particular additive changes or blocks the active surface. Therefore the statement claiming that the theory explains all these actions is not accepted. The true statements are B, C and D; A and E are false.

Step 4: Conclusion There are $\boxed{3}$ correct statements: B, C and D. The answer is a dimensionless count.

Quick Tip to Solve: Trace the five surface steps in order. Reactants move to and adsorb on the catalyst; products then desorb, leaving the catalyst surface available again.

90. $\mathrm{Mg\,(NO_3)_2 \cdot XH_2O}$ **and** $\mathrm{Ba\,(NO_3)_2 \cdot YH_2O}$**, represent formula of the crystalline forms of nitrate salts. Sum of** X **and** Y **is _____.** [4 marks]

Solution:

Concept:

- Water of crystallisation is water present in a fixed stoichiometric amount in the crystalline salt. Its coefficient is written after a centred dot and is distinct from the subscripts within the nitrate ion.
- Magnesium nitrate crystallises as the hexahydrate, $\mathrm{Mg(NO_3)_2 \cdot 6\,H_2O}$, whereas barium nitrate crystallises in its anhydrous form, $\mathrm{Ba(NO_3)_2}$. The smaller magnesium ion has a substantially greater tendency to hydration than the larger barium ion.

Step 1: Identify the hydration number of magnesium nitrate The crystalline magnesium nitrate named in the question is magnesium nitrate hexahydrate. Its formula may be written as $\mathrm{Mg(NO_3)_2 \cdot 6\,H_2O}$; the prefix hexa indicates six water molecules per formula unit of the salt. Comparing this with the given form $\mathrm{Mg(NO_3)_2}.X\mathrm{H_2O}$, we obtain

$$X = 6.$$

The two nitrate ions balance the charge of $\mathrm{Mg^{2+}}$. Their coefficient two belongs to the salt composition and must not be added to, or multiplied into, the number of water molecules.

Step 2: Identify the hydration number of barium nitrate Barium nitrate crystallises without water of crystallisation in the standard crystalline form considered here. Consequently its formula is simply $\mathrm{Ba(NO_3)_2}$, corresponding to $\mathrm{Ba(NO_3)_2}.0\mathrm{H_2O}$ when expressed in the question's notation. Hence

$$Y = 0.$$

Being soluble in water does not require a salt to contain water in its crystal formula. Water surrounding dissolved ions and fixed water of crystallisation are different concepts, so a hydration number must not be assigned merely because the compound forms an aqueous solution.

Step 3: Add the two coefficients The question asks for the sum of the water coefficients, rather than the number of atoms or the number of nitrate ions. Therefore

$$X + Y = 6 + 0 = 6.$$

Step 4: Conclusion The coefficients are $X = 6$ and $Y = 0$. Their sum is $\boxed{6}$, a dimensionless number.

Quick Tip to Solve: Remember the contrast: magnesium nitrate is a hexahydrate, while barium nitrate is anhydrous. Count only the waters after the dot when a question asks for hydration coefficients.