JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 12 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 12 April 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Two dice A and B are rolled. Let the numbers obtained on A and B be α and β respectively. If the variance of $\alpha - \beta$ is $\frac{p}{q}$, where p and q are co-prime, then the sum of the positive divisors of p is equal to : [4 marks]

(A) 48

(B) 31

(C) 72

(D) 36

Correct Answer: (A) 48

Solution:

Concept:

- For independent random variables, variances add even when their difference is taken: $\text{Var}(\alpha - \beta) = \text{Var}(\alpha) + \text{Var}(\beta)$. The minus sign disappears on squaring.
- For a fair die, all six outcomes have probability $1/6$, and $\text{Var}(X) = E(X^2) - [E(X)]^2$. The numerator must be taken after reducing the fraction to lowest terms.

Step 1: Calculate the variance of one die For either die, the expected value and second moment are

$$E(X) = \frac{1+2+3+4+5+6}{6} = \frac{7}{2}, \qquad E(X^2) = \frac{1+4+9+16+25+36}{6} = \frac{91}{6}.$$

Hence

$$\text{Var}(X) = \frac{91}{6} - \frac{49}{4} = \frac{182-147}{12} = \frac{35}{12}.$$

Both dice have this same variance because their outcome distributions are identical.

Step 2: Add the variances and factor the numerator The dice rolls are independent, so their covariance is zero. Therefore

$$\text{Var}(\alpha - \beta) = \frac{35}{12} + \frac{35}{12} = \frac{35}{6}.$$

Since 35 and 6 are coprime, $p = 35 = 5 \cdot 7$. Its positive divisors are exactly $1, 5, 7, 35$. Their sum is

$$1 + 5 + 7 + 35 = (1+5)(1+7) = 48.$$

The alternatives 31, 72 and 36 do not equal this divisor sum; neither the denominator nor the variance itself is the quantity requested.

Step 3: Conclusion The sum of the positive divisors of p is 48. Hence, the correct option is (A).

Quick Tip to Solve: For independent dice, add variances for both a sum and a difference; reduce the resulting fraction before finding the numerator and its divisor sum.

2. Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to [4 marks]

(A) 50

(B) 75

(C) 100

(D) 125

Correct Answer: (C) 100

Solution:

Concept:

- If $B = MAM^{-1}$, then $B^n = MA^nM^{-1}$: the adjacent factors $M^{-1}M$ cancel in every product.
- Write $A = I + E/51$, where $E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. Since $E^2 = 0$, the binomial expansion terminates: $A^n = I + nE/51$.

Step 1: Identify the inverse and the nilpotent part Let

$$M = \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}, \qquad M^{-1} = \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}.$$

Their product is the identity, so the displayed definition is indeed a similarity transformation. Direct multiplication gives

$$N = MEM^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}.$$

Consequently $B = I + N/51$. Also $N^2 = ME^2M^{-1} = 0$, which means the same terminating binomial argument applies directly to powers of B.

Step 2: Sum the powers and then their entries For each positive integer n,

$$B^n = I + \frac{n}{51}N.$$

Using $1 + 2 + \cdots + 50 = 50 \cdot 51/2 = 1275$, we obtain

$$\sum_{n=1}^{50} B^n = 50I + 25N = \begin{pmatrix} 75 & 25 \\ -25 & 25 \end{pmatrix}.$$

The sum of its four entries is $75 + 25 - 25 + 25 = 100$. In particular, 75 is only the top-left entry, while 50 and 125 are not the requested total. The sum of the entries of N is zero, so the same total follows immediately from the $50I$ term.

Step 3: Conclusion The sum of all elements of $\sum_{n=1}^{50} B^n$ is 100. Hence, the correct option is (C).

Quick Tip to Solve: When a matrix is similar to an upper triangular matrix with equal diagonal entries, isolate its square-zero part before taking powers; it avoids repeated multiplication.

3. Let $y = y(x), y > 0$, be a solution curve of the differential equation $\left(1+x^2\right) \mathrm{d}y = y(x-y)\mathrm{d}x$. If $y(0) = 1$ and $y(2\sqrt{2}) = \beta$, then [4 marks]

(A) $e^{\beta^{-1}} = e^{-2}(3+2\sqrt{2})$

(B) $e^{3\beta^{-1}} = e(5+\sqrt{2})$

(C) $e^{3\beta^{-1}} = e(3+2\sqrt{2})$

(D) $e^{\beta^{-1}} = e^{-2}(5+\sqrt{2})$

Correct Answer: (C) $e^{3\beta^{-1}} = e(3+2\sqrt{2})$

Solution:

Concept:

- The differential equation is a Bernoulli equation. Because $y > 0$, the reciprocal substitution $u = 1/y$ is valid and turns it into a first-order linear equation.
- For $u\prime + P(x)u = Q(x)$, multiplication by $e^{\int P(x)\,dx}$ makes the left side an exact product derivative. Here the integrating factor is $\sqrt{1+x^2}$.

Step 1: Convert to a linear equation Divide the equation by $(1+x^2)y^2$ and use $u\prime = -y\prime/y^2$. This gives

$$u\prime + \frac{x}{1+x^2}u = \frac{1}{1+x^2}.$$

Since

$$e^{\int x/(1+x^2)\,dx} = \sqrt{1+x^2},$$

the product rule yields

$$\frac{d}{dx}\left(u\sqrt{1+x^2}\right) = \frac{1}{\sqrt{1+x^2}}.$$

Step 2: Apply the initial value and evaluate the solution Integration gives

$$u\sqrt{1+x^2} = \ln\left(x+\sqrt{1+x^2}\right) + C.$$

At $x = 0$, the given value $y(0) = 1$ implies $u(0) = 1$, so $C = 1$. The logarithm is well-defined for every real x, and the resulting solution has positive denominator on the interval from 0 to $2\sqrt{2}$. At the required endpoint, $\sqrt{1+x^2} = 3$; hence

$$\frac{3}{\beta} = 1 + \ln(3+2\sqrt{2}).$$

Exponentiating both sides gives $e^{3\beta^{-1}} = e(3+2\sqrt{2})$. The factor 3 multiplying β^{-1} comes from the integrating factor. Thus (A) and (D) have an incorrect exponent and constant; (B) has the wrong logarithm argument.

Step 3: Conclusion The required relation is $e^{3\beta^{-1}} = e(3+2\sqrt{2})$. Hence, the correct option is (C).

Quick Tip to Solve: For an equation containing both y and y^2, try the reciprocal $1/y$; remember to carry the integrating factor into the final value of the reciprocal.

4. Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point $\mathrm{P}(a, b, c)$ on l_1 is nearest to the point $\mathrm{Q}(-4, -3, 2)$, then $|a| + |b| + |c|$ is equal to [4 marks]

(A) 12

(B) 14

(C) 10

(D) 8

Correct Answer: (C) 10

Solution:

Concept:

- Two lines with direction vectors $\mathbf{v}$ and $\mathbf{w}$, through points R and S, are coplanar when $(S - R) \cdot (\mathbf{v} \times \mathbf{w}) = 0$.
- The closest point on a line to an external point is the perpendicular projection. For $P = R + t\mathbf{v}$, its parameter is $t = (Q - R) \cdot \mathbf{v}/(\mathbf{v} \cdot \mathbf{v})$.

Step 1: Use coplanarity to determine the first line For l_1, choose $R = (-5, -4, \alpha)$ and $\mathbf{v} = (3, 1, -2)$. The normals of the two planes defining l_2 are $(3, 2, 1)$ and $(1, -3, 2)$, so

$$\mathbf{w} = (3, 2, 1) \times (1, -3, 2) = (7, -5, -11).$$

Setting $z = 0$ in those plane equations gives the point $S = (32/11, -37/11, 0)$ on l_2. Now

$$\mathbf{v} \times \mathbf{w} = (-21, 19, -22).$$

The coplanarity condition is

$$-21\frac{87}{11} + 19\frac{7}{11} + 22\alpha = 0,$$

which simplifies to $-154 + 22\alpha = 0$, giving $\alpha = 7$.

Step 2: Find the perpendicular foot and the requested sum Thus $R = (-5, -4, 7)$, while $Q - R = (1, 1, -5)$. Its projection parameter is

$$t = \frac{3 + 1 + 10}{9 + 1 + 4} = 1.$$

Therefore

$$P = R + \mathbf{v} = (-2, -3, 5).$$

As a check, $P - Q = (2, 0, 3)$ is perpendicular to $\mathbf{v}$, since $6 - 6 = 0$. Finally,

$$|a| + |b| + |c| = 2 + 3 + 5 = 10.$$

The other proposed totals 12, 14 and 8 do not equal the sum of the absolute coordinates; using signed coordinates would instead give zero and would miss the absolute-value requirement.

Step 3: Conclusion The nearest point is $(-2, -3, 5)$, and $|a| + |b| + |c| = 10$. Hence, the correct option is (C).

Quick Tip to Solve: Use a scalar triple product to settle coplanarity first, then project onto the now fixed line; verify the foot with a zero dot product.

5. The number of five digit numbers, greater than 40000 and divisible by 5 , which can be formed using the digits $0, 1, 3, 5, 7$ and 9 without repetition, is equal to : [4 marks]

(A) 132

(B) 72

(C) 120

(D) 96

Correct Answer: (C) 120

Solution:

Concept:

- A number divisible by 5 ends in 0 or 5. These two cases are disjoint, so their counts can be added.
- The condition that a five-digit number exceeds 40000 restricts its first digit to 5, 7 or 9. With no repetition, each fixed pair of end digits leaves four available digits for the three middle positions.

Step 1: Count numbers with final digit zero If the units digit is 0, the first digit may be 5, 7 or 9, giving three choices. After fixing those two positions, four of the original six digits remain. Fill the three ordered middle positions without repetition in

$$^4P_3 = 4 \cdot 3 \cdot 2 = 24$$

ways. It is a permutation, rather than a combination, because changing the order of these digits changes the number. Thus this case contributes

$$3 \cdot 24 = 72$$

valid five-digit numbers.

Step 2: Count numbers with final digit five and add If the units digit is 5, that digit cannot also be used first. The first digit must therefore be 7 or 9, giving two choices. Again four digits remain for the three middle positions, contributing 24 arrangements for each first digit. Zero is allowed in a middle position and creates no leading-zero problem. Hence this case contributes

$$2 \cdot 24 = 48.$$

The total is

$$72 + 48 = 120.$$

The value 72 counts only the final-zero case, and 96 does not account correctly for the different first-digit counts in the two cases. Neither 132 nor 96 equals the complete disjoint count; no number is counted twice because the last digits differ.

Step 3: Conclusion There are 120 such numbers. Hence, the correct option is (C).

Quick Tip to Solve: For divisibility by 5, split by the last digit first; then impose the lower bound on the leading digit before permuting the remaining positions.

6. Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1+3i)$ and radius $r = 1$. Let $z_1 = 1 + i$ and the complex number z_2 be outside the circle C such that $|z_1 - z_0|\,|z_2 - z_0| = 1$. If z_0, z_1 and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to : [4 marks]

(A) $\frac{3}{2}$
(B) $\frac{5}{2}$
(C) $\frac{13}{2}$
(D) $\frac{7}{2}$

Correct Answer: (B) $\frac{5}{2}$

Solution:

Concept:

- The modulus $|z_1 - z_0|$ is the distance between the two corresponding points. Collinearity means $z_2 - z_0 = t(z_1 - z_0)$ for a real scalar t.
- The distance-product condition fixes $|t|$, but not its sign. Both positions on the line must be checked before choosing the smaller value of $|z_2|^2$.

Step 1: Find the allowed displacement from the centre With $z_0 = \frac{1}{2} + \frac{3}{2}i$ and $z_1 = 1 + i$,

$$z_1 - z_0 = \frac{1}{2} - \frac{1}{2}i, \qquad |z_1 - z_0|^2 = \frac{1}{2}.$$

Write $z_2 - z_0 = t(z_1 - z_0)$, where t is real. The given product becomes

$$|z_1 - z_0|\,|t|\,|z_1 - z_0| = 1.$$

Thus $|t|/2 = 1$, so $t = 2$ or $t = -2$. In either case $|z_2 - z_0| = \sqrt{2} > 1$, satisfying the requirement that z_2 lies outside the circle.

Step 2: Evaluate both squared moduli For $t = 2$,

$$z_2 = \left(\frac{1}{2} + \frac{3}{2}i\right) + (1 - i) = \frac{3}{2} + \frac{1}{2}i,$$

so $|z_2|^2 = 9/4 + 1/4 = 5/2$. For $t = -2$,

$$z_2 = \left(\frac{1}{2} + \frac{3}{2}i\right) - (1 - i) = -\frac{1}{2} + \frac{5}{2}i,$$

so $|z_2|^2 = 1/4 + 25/4 = 13/2$. Therefore the smaller value is $5/2$. The value $13/2$ in (C) is the other valid value, rather than the smaller one. Neither $3/2$ nor $7/2$ arises from either possible point. Distances from the centre must not be confused with distances from the origin.

Step 3: Conclusion The smaller squared modulus is $\frac{5}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: When three complex points are collinear and a modulus fixes a distance, use a real multiplier and retain both signs until the final comparison.

7. If the point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on the curve traced by the mid-points of the line segments of the lines $x\cos\theta + y\sin\theta = 7, \theta \in \left(0, \frac{\pi}{2}\right)$ between the co-ordinates axes, then α is equal to : [4 marks]

(A) -7

(B) 7

(C) $-7\sqrt{3}$

(D) $7\sqrt{3}$

Correct Answer: (B) 7

Solution:

Concept:

- For the line $x\cos\theta + y\sin\theta = 7$, the intercepts on the axes are found by setting the other coordinate to zero. The midpoint is the coordinate-wise average of those intercept points.
- The parameter lies strictly in the first quadrant, so $\sin\theta$ and $\cos\theta$ are both positive. This sign restriction determines the positive horizontal coordinate uniquely.

Step 1: Express the midpoint coordinates in terms of the angle The two axis intercept points are

$$A = \left(\frac{7}{\cos\theta}, 0\right), \qquad B = \left(0, \frac{7}{\sin\theta}\right).$$

Consequently the midpoint $M = (X, Y)$ satisfies

$$X = \frac{7}{2\cos\theta}, \qquad Y = \frac{7}{2\sin\theta}.$$

Equivalently, eliminating the angle gives $49/(4X^2) + 49/(4Y^2) = 1$, with $X, Y > 0$. These are midpoint coordinates, not the original intercepts; the factor 2 is essential.

Step 2: Use the given vertical coordinate to find the horizontal one Substituting $Y = 7\sqrt{3}/3$ into the midpoint expression yields

$$\sin\theta = \frac{7}{2Y} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}.$$

The allowed interval $0 < \theta < \pi/2$ gives $\theta = \pi/3$ and $\cos\theta = 1/2$. Thus

$$\alpha = X = \frac{7}{2(1/2)} = 7.$$

The negative options (A) and (C) violate the first-quadrant restriction. The positive alternative $7\sqrt{3}$ in (D) does not satisfy the midpoint formula for this same angle, so it is also excluded. The line has x-intercept 14 at this angle, whose half is exactly the required 7.

Step 3: Conclusion The horizontal coordinate is $\alpha = 7$. Hence, the correct option is (B).

Quick Tip to Solve: Write the axis intercept points first and halve their nonzero coordinates for the midpoint; use the parameter interval to remove spurious signs.

8. Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$ is equal to : [4 marks]

(A) 72

(B) 9

(C) 729

(D) 81

Correct Answer: (D) 81

Solution:

Concept:

- The quadratic formula gives conjugate roots. In polar form, a complex number raised to an integer power has its modulus raised to that power and its argument multiplied by the integer.
- For conjugate roots $\alpha = re^{i\theta}$, $\beta = re^{-i\theta}$, their power sum is $S_n = \alpha^n + \beta^n = 2r^n \cos(n\theta)$. This converts the large exponents into simple trigonometric values.

Step 1: Express the two roots in polar form The discriminant is $6 - 12 = -6$, so the roots are

$$\alpha = \frac{-\sqrt{6} + i\sqrt{6}}{2}, \qquad \beta = \frac{-\sqrt{6} - i\sqrt{6}}{2}.$$

Their common modulus is $\sqrt{3}$, and their arguments are $3\pi/4$ and $-3\pi/4$. Consequently,

$$\alpha^8 = \beta^8 = (\sqrt{3})^8 e^{\pm 6\pi i} = 81.$$

This repeated eighth power is useful because the largest exponents in the numerator and denominator differ by eight.

Step 2: Simplify the power sums before dividing Since $\cos(21\pi/2) = \cos(15\pi/2) = 0$, we obtain

$$S_{14} = 0, \qquad S_{10} = 0.$$

Also, the identity for the eighth powers gives $S_{23} = 81S_{15}$. The denominator is nonzero, because

$$S_{15} = 2(\sqrt{3})^{15} \cos(45\pi/4) = -\sqrt{2}(\sqrt{3})^{15} \neq 0.$$

Therefore the required quotient is

$$\frac{S_{23} + S_{14}}{S_{15} + S_{10}} = \frac{81S_{15}}{S_{15}} = 81.$$

The alternatives 72, 9 and 729 do not have the required eighth-power multiplier; in particular, the modulus must be raised to the eighth power, not the fourth or twelfth power.

Step 3: Conclusion The value of the expression is 81. Hence, the correct option is (D).

Quick Tip to Solve: Before expanding large powers of quadratic roots, test whether a small common power of both roots is the same real number.

9. Let $\mathrm{P}\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right), \mathrm{Q}, \mathrm{R}$ and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$, where p and q are coprime, then $p+q$ is equal to : [4 marks]

(A) 143

(B) 147

(C) 137

(D) 157

Correct Answer: (D) 157

Solution:

Concept:

- A chord through the centre of an ellipse is a diameter, so its endpoints are opposite points. If its half-length is r in a direction making angle θ with the positive x-axis, its full length is $2r$.
- For $x^2/a^2 + y^2/b^2 = 1$, substitution of $(r\cos\theta, r\sin\theta)$ gives $1/r^2 = \cos^2\theta/a^2 + \sin^2\theta/b^2$. Perpendicular directions interchange the squared sine and cosine.

Step 1: Write the reciprocal squared lengths in perpendicular directions The given ellipse is

$$\frac{x^2}{4} + \frac{y^2}{9} = 1,$$

so $a^2 = 4$ and $b^2 = 9$. Let the half-lengths of the diameters PQ and RS be r_1, r_2. If PQ makes angle θ with the x-axis, RS makes angle $\theta + \pi/2$. Thus

$$\frac{1}{r_1^2} = \frac{\cos^2\theta}{4} + \frac{\sin^2\theta}{9},$$
$$\frac{1}{r_2^2} = \frac{\sin^2\theta}{4} + \frac{\cos^2\theta}{9}.$$

The stated coordinates of P specify one such diameter, but the sum is independent of its direction.

Step 2: Add the two expressions and reduce the fraction Adding cancels the dependence on θ:

$$\frac{1}{r_1^2} + \frac{1}{r_2^2} = \frac{1}{4} + \frac{1}{9} = \frac{13}{36}.$$

Since the question uses full diameter lengths, $PQ = 2r_1$ and $RS = 2r_2$. Hence

$$\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{1}{4}\left(\frac{13}{36}\right) = \frac{13}{144}.$$

The integers 13 and 144 are coprime. Therefore $p = 13$, $q = 144$, and $p+q = 157$. The other proposed totals, 143, 147 and 137, do not correspond to this reduced fraction. Omitting the factor of four would confuse radii with complete chords.

Step 3: Conclusion We obtain $p + q = 13 + 144 = 157$. Hence, the correct option is (D).

Quick Tip to Solve: For perpendicular diameters of a central ellipse, add reciprocal squared half-lengths first, then divide by four for full lengths.

10. If the local maximum value of the function $f(x) = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$, $x \in \left(0, \frac{\pi}{2}\right)$, is $\frac{k}{e}$, then $\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$ is equal to [4 marks]

(A) $e^3 + e^6 + e^{10}$

(B) $e^3 + e^5 + e^{11}$

(C) $e^3 + e^6 + e^{11}$

(D) $e^5 + e^6 + e^{11}$

Correct Answer: (C) $e^3 + e^6 + e^{11}$

Solution:

Concept:

- The function is positive on the stated interval. Because the logarithm is strictly increasing, maximizing f is equivalent to maximizing $\log f$.
- The substitution $t = \sin^2 x$ maps $0 < x < \pi/2$ monotonically onto $0 < t < 1$. It turns the variable exponent into the differentiable expression $h(t) = \frac{t}{2}\log(3e/(4t))$.

Step 1: Locate and classify the stationary point Taking logarithms gives

$$h(t) = \log f(x) = \frac{t}{2}\log\left(\frac{3e}{4t}\right).$$

Differentiate with respect to t:

$$h'(t) = \frac{1}{2}\left[\log\left(\frac{3e}{4t}\right) - 1\right] = \frac{1}{2}\log\left(\frac{3}{4t}\right).$$

Thus the only stationary point in $(0, 1)$ is $t = 3/4$. Moreover,

$$h''(t) = -\frac{1}{2t} < 0.$$

This confirms a strict local maximum. Since $dt/dx = 2\sin x \cos x > 0$ throughout the given interval, the classification is also valid for the original variable x.

Step 2: Evaluate the maximum and the requested powers At $t = 3/4$,

$$h(3/4) = \frac{3}{8}\log e = \frac{3}{8}, \qquad f_{\max} = e^{3/8}.$$

The question states that this maximum equals k/e, so $k = e^{11/8}$. The three terms therefore become

$$\left(\frac{k}{e}\right)^8 = e^3, \qquad \frac{k^8}{e^5} = e^6, \qquad k^8 = e^{11}.$$

Their sum is $e^3 + e^6 + e^{11}$. Option A has the wrong final exponent 10; option B has the wrong middle exponent 5; option D has the wrong first exponent 5. The multiplier e in $k = e f_{\max}$ must be included before taking the eighth power.

Step 3: Conclusion The required value is $e^3 + e^6 + e^{11}$. Hence, the correct option is (C).

Quick Tip to Solve: For a positive variable-base power, maximize its logarithm; substitute the repeated trigonometric square before differentiating.

11. The area of the region enclosed by the curve $y = x^3$ and its tangent at the point $(-1, -1)$ is :

[4 marks]

(A) $\frac{23}{4}$
(B) $\frac{19}{4}$
(C) $\frac{27}{4}$
(D) $\frac{31}{4}$

Correct Answer: (C) $\frac{27}{4}$

Solution:

Concept:

- The tangent to $y = f(x)$ at $(x_0, f(x_0))$ has slope $f\prime(x_0)$. Finding every intersection with the tangent determines the limits of the enclosed region.
- Area is the integral of the upper curve minus the lower curve. A double root at a tangency point does not mean that the two curves cannot enclose a region with a further intersection.

Step 1: Find the tangent and the intersection limits For $y = x^3$, the derivative is $3x^2$. At $x = -1$, the slope is 3, giving

$$y + 1 = 3(x + 1), \qquad y = 3x + 2.$$

Intersections of the curve and line satisfy

$$x^3 - 3x - 2 = 0 = (x + 1)^2(x - 2).$$

Thus they meet at $x = -1$, the tangency point, and $x = 2$. Between these limits,

$$(3x + 2) - x^3 = (2 - x)(x + 1)^2 \geq 0.$$

The line is therefore the upper boundary over the whole enclosed region, so the absolute value is unnecessary after this sign check.

Step 2: Integrate the vertical separation The required area is

$$A = \int_{-1}^{2} (3x + 2 - x^3)\, dx.$$

An antiderivative is $\frac{3}{2}x^2 + 2x - \frac{1}{4}x^4$, hence

$$\begin{aligned} A &= \left[\frac{3}{2}x^2 + 2x - \frac{1}{4}x^4\right]_{-1}^{2} \\ &= (6 + 4 - 4) - \left(\frac{3}{2} - 2 - \frac{1}{4}\right) \\ &= 6 - \left(-\frac{3}{4}\right) = \frac{27}{4}. \end{aligned}$$

This positive value is the geometric area, measured in square units. Values $23/4$, $19/4$, and $31/4$ are excluded by the endpoint evaluation; retaining the negative lower-limit value is essential. There is only one bounded region because the difference has no other real roots.

Step 3: Conclusion The enclosed area is $\frac{27}{4}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Factor the curve-minus-tangent polynomial; the tangency root repeats, and its remaining real root supplies the other area limit.

12. Let D be the domain of the function $f(x) = \sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g : D \to \mathbb{R}$ defined by $g(x) = x - [x]$, ($[x]$ is the greatest integer function), is (α, β), then $\alpha^2 + \frac{5}{\beta}$ is equal to [4 marks]

(A) 45

(B) 136

(C) 46

(D) nearly 135

Correct Answer: No exact option: $\alpha^2 + 5/\beta \approx 136.5009831$. The final key accepts (D), nearly 135, as a coarse approximation.

Solution:

Concept:

- For a real logarithm, its base must be positive and different from 1, and its argument must be positive. For a real inverse sine, its input must lie in the closed interval $[-1, 1]$.
- When $0 < b < 1$, the logarithm is decreasing. Hence $-1 \le \log_b A \le 1$ is equivalent to $b \le A \le b^{-1}$. The fractional part equals x whenever $0 < x < 1$.

Step 1: Apply the domain conditions with the printed base Put $t = \log_3 x$. Positivity of the logarithm argument $A = -(6 + 2t)/(5x)$ requires $x > 0$ and $t < -3$, so $0 < x < 1/27$. In particular the printed base $3x$ lies between zero and one. The inverse-sine condition therefore becomes

$$3x \le -\frac{6 + 2t}{5x} \le \frac{1}{3x}.$$

Multiplication by the positive number $5x$ and rearrangement give

$$-\frac{23}{6} \le t, \qquad t + 3 + \frac{15}{2}x^2 \le 0.$$

Let $a = 3^{-23/6}$, and let b denote the positive root of

$$\log_3 b + 3 + \frac{15}{2}b^2 = 0.$$

The left-hand side is strictly increasing, with derivative $1/(x \log 3) + 15x > 0$. It is negative at a and positive at $1/27$; hence b exists uniquely and the domain is $[a, b]$. Both endpoints are included because inverse sine permits input 1 and -1.

Step 2: Evaluate the fractional-part range and assess the alternatives Throughout this domain $0 < x < 1$, so $[x] = 0$ and $g(x) = x$. The range is consequently $[a, b]$, a closed interval rather than the

open interval printed in the question. Solving the monotone endpoint equation numerically gives

$$a \approx 0.01482638216, \qquad b \approx 0.03662983180.$$

Using the endpoints as α, β, the requested quantity is

$$a^2 + \frac{5}{b} \approx 136.5009831.$$

It is neither 45 nor 46 nor exactly 136. The final answer key accepts option (D), 'nearly 135'. That rough value results from neglecting the small quadratic term in the endpoint equation: then $b \approx 1/27$, so $5/b \approx 135$. The full printed equation, however, gives the numerical value above. Thus (D) is the key's intended coarse approximation, while an exact interpretation of the question has no matching option and its open-range premise is also inaccurate.

Step 3: Conclusion As printed, the actual range is $[3^{-23/6}, b]$, where $\log_3 b + 3 + 15b^2/2 = 0$, and the endpoint expression is approximately 136.5009831. The final key accepts (D), nearly 135, as a coarse approximation; the exact question has no correct numerical option and incorrectly specifies an open range.

Quick Tip to Solve: For an inverse sine of a logarithm, impose the logarithm conditions first; reverse exponential inequalities when its base is below one.

13. If $\frac{1}{n+1}{}^{n}C_n + \frac{1}{n}{}^{n}C_{n-1} + \ldots + \frac{1}{2}{}^{n}C_1 + {}^{n}C_0 = \frac{1023}{10}$ **then** n **is equal to :** [4 marks]

(A) 9

(B) 6

(C) 7

(D) 8

Correct Answer: (A) 9

Solution:

Concept:

- The binomial theorem states $(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$. Integrating each term from 0 to 1 produces the denominator $r+1$.
- For a nonnegative integer n, the sum is finite and can be integrated term by term. The resulting identity is $\sum_{r=0}^{n} \binom{n}{r}/(r+1) = (2^{n+1}-1)/(n+1)$.

Step 1: Match the given sum to the integrated expansion Although the terms are printed in descending order of the lower binomial index, the sum is exactly

$$S = \sum_{r=0}^{n} \frac{\binom{n}{r}}{r+1}.$$

For example, the term at $r = n$ is $\binom{n}{n}/(n+1)$, and the term at $r = 0$ is $\binom{n}{0}$. Therefore

$$S = \int_0^1 (1+x)^n \, dx$$
$$= \left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1 = \frac{2^{n+1}-1}{n+1}.$$

The subtraction of 1 arises from the lower endpoint, and must not be omitted.

Step 2: Check the four possible integers The equation becomes

$$\frac{2^{n+1}-1}{n+1} = \frac{1023}{10}.$$

Since $1023 = 1024 - 1 = 2^{10} - 1$, $n + 1 = 10$, or $n = 9$, satisfies it exactly. The other choices give

$$n = 6 : \ \frac{127}{7}, \qquad n = 7 : \ \frac{255}{8}, \qquad n = 8 : \ \frac{511}{9}.$$

None equals $1023/10$. These explicit substitutions distinguish the printed alternatives without assuming that matching the numerator alone always proves an equation. The integrated binomial expression also increases with n, because $(1+x)^n$ increases on $0 < x < 1$, so there cannot be a second nonnegative-integer solution.

Step 3: Conclusion The required integer is $n = 9$. Hence, the correct option is (A).

Quick Tip to Solve: A binomial sum with denominator r+1 often comes directly from integrating the binomial expansion between zero and one.

14. Let $\langle a_n \rangle$ be a sequence such that $a_1 + a_2 + \ldots + a_n = \frac{n^2+3n}{(n+1)(n+2)}$. If $28 \sum_{k=1}^{10} \frac{1}{a_k} = p_1 p_2 p_3 \ldots p_m$, where $p_1, p_2, \ldots, p_m$ are the first m prime numbers, then m is equal to [4 marks]

(A) 5

(B) 7

(C) 6

(D) 8

Correct Answer: (C) 6

Solution:

Concept:

- If $S_n = a_1 + \cdots + a_n$, then $a_n = S_n - S_{n-1}$, with $S_0 = 0$. This difference recovers the individual terms from the given partial-sum formula.
- The sum of the consecutive products satisfies $\sum_{k=1}^{n} k(k+1)(k+2) = n(n+1)(n+2)(n+3)/4$, as follows by subtracting successive products of four consecutive factors.

Step 1: Find the individual term of the sequence Rewrite the partial sum as

$$S_n = \frac{n^2+3n}{(n+1)(n+2)} = 1 - \frac{2}{(n+1)(n+2)}.$$

The same formula gives $S_0 = 0$, so it also works when obtaining the first term. Subtracting consecutive partial sums gives

$$\begin{aligned} a_n &= \frac{2}{n(n+1)} - \frac{2}{(n+1)(n+2)} \\ &= \frac{4}{n(n+1)(n+2)}. \end{aligned}$$

In particular all the terms involved are nonzero, so their reciprocals can be summed.

Step 2: Sum the reciprocals and factor the result We have

$$28\sum_{k=1}^{10}\frac{1}{a_k} = 7\sum_{k=1}^{10}k(k+1)(k+2).$$

To evaluate the sum, observe that the difference between $k(k+1)(k+2)(k+3)$ and $(k-1)k(k+1)(k+2)$ is $4k(k+1)(k+2)$. Telescoping therefore yields

$$7\sum_{k=1}^{10}k(k+1)(k+2) = \frac{7\cdot 10\cdot 11\cdot 12\cdot 13}{4} = 30030.$$

Its prime factorization is

$$30030 = 2\cdot 3\cdot 5\cdot 7\cdot 11\cdot 13.$$

These are precisely the first six primes, each appearing once. Five primes would omit 13, while seven or eight primes would introduce 17 or further primes that do not divide 30030.

Step 3: Conclusion The product contains the first six prime numbers, so $m = 6$. Hence, the correct option is (C).

Quick Tip to Solve: Use the difference of partial sums before reciprocating; consecutive three-factor products can then be summed by a four-factor telescoping identity.

15. Among the two statements

$(S1) : (p \Rightarrow q) \wedge (p \wedge (\sim q))$ **is a contradiction and**

$(S2) : (p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$ **is a tautology** [4 marks]

(A) both are false.

(B) only (S1) is true.

(C) both are true.

(D) only (S2) is true.

Correct Answer: (C) both are true.

Solution:

Concept:

- A contradiction has truth value false for every assignment of its variables; a tautology has truth value true for every assignment.
- The implication $p \Rightarrow q$ is equivalent to $(\sim p) \vee q$. Use distributive laws and $p \vee (\sim p) = T$, $p \wedge (\sim p) = F$.

Step 1: Simplify the first expression The second factor $p \wedge (\sim q)$ can be true only when $p = T$ and $q = F$, exactly the assignment at which $p \Rightarrow q$ is false. Algebraically,

$$[(\sim p) \vee q] \wedge p \wedge (\sim q) = [(\sim p) \wedge p \wedge (\sim q)] \vee [q \wedge p \wedge (\sim q)] = F.$$

Thus the expression never becomes true. The statement (S1), which calls it a contradiction, is true.

Step 2: Group the four alternatives in the second expression The first two terms have common factor q, while the final two have common factor $\sim q$. Therefore the whole expression is

$$q \wedge [p \vee (\sim p)] \ \vee \ (\sim q) \wedge [p \vee (\sim p)] = q \vee (\sim q) = T.$$

The four original terms also correspond to all four possible assignments of p, q; precisely one of them is true at each assignment. Hence (S2) correctly identifies a tautology.

Step 3: Compare the listed choices Both classifications have been established independently. Option (A) rejects both correct classifications; (B) rejects the tautology in (S2); and (D) rejects the contradiction in (S1). None of those choices can describe the two expressions.

Step 4: Conclusion Both (S1) and (S2) are true. Hence, the correct option is (C).

Quick Tip to Solve: For an implication conjoined with its counterexample, look for an immediate contradiction; a disjunction listing all truth assignments is a tautology.

16. Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to : [4 marks]

(A) -2

(B) 1

(C) -1

(D) 2

Correct Answer: (B) 1

Solution:

Concept:

- Three vectors in three dimensions are coplanar exactly when their scalar triple product, or the determinant of their components, is zero.
- Shift each diagonal entry by writing $x = a - 1$, $y = b - 1$, $z = c - 1$. The hypothesis that none of a, b, c equals one makes $xyz \neq 0$.

Step 1: Write and expand the coplanarity condition The determinant of the three given vectors must vanish:

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = a(bc-1) - (c-1) + (1-b) = abc - a - b - c + 2 = 0.$$

This uses the actual component order and does not require any pair of the vectors to be perpendicular.

Step 2: Shift the variables and factor Substituting $a = 1 + x$, $b = 1 + y$, $c = 1 + z$, the constant and linear terms cancel, leaving

$$xyz + xy + xz + yz = 0.$$

Since x, y, z are nonzero, divide by their product to obtain

$$1 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0.$$

Thus the sum of these three reciprocals is -1.

Step 3: Convert to the requested denominators Each requested denominator has the opposite sign: $1 - a = -x$, $1 - b = -y$, $1 - c = -z$. Consequently

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = -\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 1.$$

The value -1 would arise from forgetting this final sign change. The alternatives -2 and 2 do not satisfy the determinant identity.

Step 4: Conclusion The required sum is 1. Hence, the correct option is (B).

Quick Tip to Solve: For a matrix with ones off the diagonal, shifting the diagonal variables by one often turns the determinant into a simple reciprocal identity.

17. Let $\lambda \in \mathbb{Z}, \vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{c}$ be a vector such that $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = \overrightarrow{0}, \vec{a} \cdot \vec{c} = -17$ and $\vec{b} \cdot \vec{c} = -20$. Then $|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})|^2$ is equal to : [4 marks]

(A) 53

(B) 62

(C) 49

(D) 46

Correct Answer: (D) 46

Solution:

Concept:

- A zero cross product implies that two nonzero vectors are parallel. Here $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = (\vec{a} + \vec{b}) \times \vec{c}$.
- Use the two scalar products to find the proportionality constant and the integer λ; then evaluate the requested cross product in components.

Step 1: Express the unknown vector using parallelism Since $\vec{a} + \vec{b} = (\lambda + 3, 0, 1)$ is nonzero, write $\vec{c} = t(\lambda + 3, 0, 1)$. The scalar products give

$$t(\lambda^2 + 3\lambda - 1) = -17, \qquad t(3\lambda + 11) = -20.$$

In particular, $t \neq 0$ and $3\lambda + 11 \neq 0$, so eliminating t is legitimate.

Step 2: Use the integer condition Equating the two expressions for t gives

$$20(\lambda^2 + 3\lambda - 1) = 17(3\lambda + 11),$$

$$20\lambda^2 + 9\lambda - 207 = (\lambda - 3)(20\lambda + 69) = 0.$$

Its roots are 3 and $-69/20$. Only the first belongs to $\mathbb{Z}$, hence $\lambda = 3$. Now $t = -20/(9 + 11) = -1$, and $\vec{c} = (-6, 0, -1)$. Direct checks give $\vec{a} \cdot \vec{c} = -18 + 1 = -17$ and $\vec{b} \cdot \vec{c} = -18 - 2 = -20$.

Step 3: Compute the cross product and its squared length The second vector in the requested product is $(3, 1, 1)$, with a positive last component. Therefore

$$\vec{c} \times (3, 1, 1) = (1, 3, -6), \qquad |\vec{c} \times (3, 1, 1)|^2 = 1^2 + 3^2 + (-6)^2 = 46.$$

This fixes the squared magnitude uniquely; the other offered values $53, 62, 49$ disagree with the sum of the component squares.

Step 4: Conclusion The squared magnitude is 46. Hence, the correct option is (D).

Quick Tip to Solve: Cancel the self-cross-product first, and retain the integer restriction when eliminating parameters; it removes the extraneous noninteger root.

18. The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1-x)^{100}$, is equal to [4 marks]

(A) ${}^{99}C_{49}$

(B) ${}^{101}C_{50}$

(C) $-{}^{99}C_{49}$

(D) $-{}^{101}C_{50}$

Correct Answer: (C) $-{}^{99}C_{49}$

Solution:

Concept:

- The coefficient of x^r in $(1-x)^{100}$ is $(-1)^r\binom{100}{r}$, where the constant term corresponds to $r=0$.
- Pascal's identity $\binom{n}{r}=\binom{n-1}{r}+\binom{n-1}{r-1}$ makes a partial alternating binomial sum telescope.

Step 1: Identify the exact summation limits The first fifty terms run from the constant term through the term containing x^{49}, not x^{50}. Thus the requested coefficient sum is

$$S=\sum_{r=0}^{49}(-1)^r\binom{100}{r}.$$

Substitution of $x=1$ in the full expansion would include another fifty-one terms, so it cannot directly evaluate this partial sum.

Step 2: Apply Pascal's identity and cancel adjacent terms Using Pascal's identity in every summand,

$$S=\sum_{r=0}^{49}(-1)^r\binom{99}{r}+\sum_{r=1}^{49}(-1)^r\binom{99}{r-1}.$$

In the second sum put $j=r-1$. It becomes $-\sum_{j=0}^{48}(-1)^j\binom{99}{j}$, which cancels the first forty-nine entries of the first sum. Only its last entry survives:

$$S=(-1)^{49}\binom{99}{49}=-\binom{99}{49}.$$

Step 3: Check the sign and the offered coefficients The last index is odd, so the surviving term is negative. This rules out the two positive choices (A) and (B). The surviving coefficient has upper index 99 and lower index 49, not 101 and 50, ruling out (D). The argument also explains why replacing a partial sum by the full zero sum would be incorrect.

Step 4: Conclusion The sum is $-\binom{99}{49}$. Hence, the correct option is (C).

Quick Tip to Solve: For the first $m+1$ coefficients of $(1-x)^n$, telescope with Pascal's identity to obtain $(-1)^m\binom{n-1}{m}$.

19. Let the plane P: $4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is equal to : [4 marks]

(A) 126

(B) 105

(C) 85

(D) 90

Correct Answer: (A) 126

Solution:

Concept:

- Every plane through the intersection of $P = 0$ and $Q = 0$ can be written $P + \mu Q = 0$, with the possible limiting plane $Q = 0$.
- A rotation through a right angle about a line in the original plane produces a plane through that line perpendicular to the original plane. Their normal vectors therefore have zero dot product.

Step 1: Write the family of planes through the rotation axis Use $P = 4x-y+z-10$ and $Q = x+y-z-4$. The new plane has equation

$$(4 + \mu)x + (-1 + \mu)y + (1 - \mu)z - 10 - 4\mu = 0.$$

Its normal is $\vec{n} = (4 + \mu, -1 + \mu, 1 - \mu)$. The original normal is $\vec{n}_0 = (4, -1, 1)$. The plane $Q = 0$ alone is not perpendicular to the original, since its normal has dot product 2 with $\vec{n}_0$.

Step 2: Impose the ninety-degree rotation Perpendicularity gives

$$\vec{n} \cdot \vec{n}_0 = 4(4 + \mu) - (-1 + \mu) + (1 - \mu) = 18 + 2\mu = 0.$$

Therefore $\mu = -9$, and the rotated plane is

$$-5x - 10y + 10z + 26 = 0.$$

Both possible signs of the right-angle rotation give this same unoriented plane.

Step 3: Find the distance and the scaled result The distance formula uses the absolute value of the plane expression divided by the length of its normal. At $(2, 3, -4)$,

$$\alpha = \frac{|-5(2) - 10(3) + 10(-4) + 26|}{\sqrt{25 + 100 + 100}} = \frac{54}{15} = \frac{18}{5}.$$

Hence $35\alpha = 35(18/5) = 126$. The other values $105, 85, 90$ do not equal this distance multiplied by 35.

Step 4: Conclusion The requested value is $35\alpha = 126$. Hence, the correct option is (A).

Quick Tip to Solve: For rotation about the intersection of two planes, use the plane pencil and impose the angle condition on the normals before applying the point-to-plane distance formula.

20. In a triangle ABC, if $\cos \mathrm{A} + 2\cos \mathrm{B} + \cos C = 2$ and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then $\cos \mathrm{A} - \cos \mathrm{C}$ is equal to [4 marks]

(A) $\frac{3}{7}$

(B) $\frac{9}{7}$

(C) $\frac{10}{7}$

(D) $\frac{5}{7}$

Correct Answer: (C) $\frac{10}{7}$

Solution:

Concept:

- In a triangle $A+C=\pi-B$. The sum-to-product identity gives $\cos A+\cos C=2\sin(B/2)\cos((A-C)/2)$.
- The sine rule relates side lengths to angles, and the cosine rule then gives individual angle cosines without calculating any inverse trigonometric functions.

Step 1: Simplify the given cosine relation Write $s=\sin(B/2)>0$. Since $\cos B=1-2s^2$, the given condition becomes

$$2s\cos\frac{A-C}{2}+2(1-2s^2)=2.$$

Dividing by $2s$, which is nonzero in a nondegenerate triangle, gives

$$\cos\frac{A-C}{2}=2\sin\frac{B}{2}.$$

Step 2: Convert this identity into a relation between sides Let R be the circumradius and a,b,c the opposite side lengths. The sine rule and sum-to-product formula yield

$$a+c=2R(\sin A+\sin C)=4R\cos\frac{B}{2}\cos\frac{A-C}{2}.$$

Substitution from the previous step gives

$$a+c=8R\cos\frac{B}{2}\sin\frac{B}{2}=4R\sin B=2b.$$

With $a=3, c=7$, we obtain $b=5$, which satisfies the triangle inequalities.

Step 3: Evaluate the two cosine values Apply the cosine rule to the sides $3,5,7$:

$$\cos A=\frac{5^2+7^2-3^2}{2(5)(7)}=\frac{13}{14},\qquad \cos C=\frac{3^2+5^2-7^2}{2(3)(5)}=-\frac{1}{2}.$$

Hence $\cos A-\cos C=13/14+1/2=10/7$. In particular the larger opposite side gives an obtuse angle C; retaining its negative cosine is essential. The other three fractions do not equal this difference.

Step 4: Conclusion The difference is $\frac{10}{7}$. Hence, the correct option is (C).

Quick Tip to Solve: Use half-angle and sum-to-product identities to translate unusual cosine sums into side relations, then apply the cosine rule to the resulting three side lengths.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. **If** $\sum_{k=1}^{n} D_k = 96$, **then** n **is equal to** _____. [4 marks]

Solution:

Concept:

- Replacing one row of a determinant by itself minus another row leaves its value unchanged. A row with a zero first entry simplifies expansion.
- Here n is the positive integer upper limit of the sum. Use $\sum_{k=1}^{n} k = n(n+1)/2$ after expressing the determinant as a linear function of k.

Step 1: Simplify the determinant by a row operation Replace the second row by $R_2 - R_3$. The determinant becomes

$$D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ 0 & 2 & -n-2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}.$$

The middle row is now simple, and only the first row depends on k, so the determinant must be linear in k.

Step 2: Expand and collect constant and linear terms Expansion along the first row gives

$$\begin{aligned} D_k &= 2(n^2+n+2) + (n+2)(n^2+n) \\ &\quad - 2kn(n+2) - 2n(2k-1) \\ &= n^3 + 5n^2 + 6n + 4 - 2kn(n+4). \end{aligned}$$

The negative sign in the middle cofactor and the factor $2k-1$ in the final cofactor are both needed for the cancellation in the sum.

Step 3: Sum and solve for the positive integer Treat n as fixed while summing over k:

$$\begin{aligned} \sum_{k=1}^{n} D_k &= n(n^3+5n^2+6n+4) - 2n(n+4)\frac{n(n+1)}{2} \\ &= 2n^2 + 4n = 2n(n+2). \end{aligned}$$

Thus $2n(n+2) = 96$, or $(n-6)(n+8) = 0$. The root -8 cannot be a positive upper limit of this sum, leaving $n = 6$. As a check, $2(6)(8) = 96$, exactly the given total.

Step 4: Conclusion The required positive integer is $n = 6$, a dimensionless value.

Quick Tip to Solve: Before expanding a parameter-dependent determinant, subtract nearly equal rows; then sum the linear dependence on the running index using the arithmetic-series formula.

22. Let the digits a, b, c be in A. P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed? [4 marks]

Solution:

Concept:

- For three distinct equally spaced digits, write their values as $a, a+d, a+2d$, with $d \neq 0$. A consecutive triple is in A.P. precisely when its first and last values sum to twice its middle value.
- The forbidden triples when counting the complement are therefore aaa, bbb, ccc, abc, cba. Equal consecutive digits form an A.P. with common difference zero. All arrangements of the multiset number $9!/(3!)^3 = 1680$, assuming the digits are nonzero as in the usual interpretation.

Step 1: Count arrangements with no A.P. triple Use an exact recurrence retaining the last two digits. Encode a, b, c by $0, 1, 2$. Let $F(i, j, k; u, v)$ count prefixes using i, j, k copies of these digits, ending in u, v, with no A.P. triple. Start with the empty prefix of weight one, and form the first two positions freely. Appending digit z is allowed only if its count is below three and $u + z \neq 2v$; add the old count to the state with that digit count increased and last pair (v, z). Every legal prefix has one unique predecessor, so this recurrence neither omits nor duplicates a number.

Step 2: Evaluate the recurrence and subtract the complement Summing the recurrence over all states at each length gives the following checkable finite table. The cap of three copies of each digit is applied at every extension.

Length	0	1	2	3	4	5	6	7	8	9
No A.P. triple	1	3	9	22	54	126	260	438	584	472

At length nine the three counts necessarily all equal three. Hence the required complement subtraction is

$$1680 - 472 = 1208.$$

The expression $2 \cdot 7!/(2!)^3 = 1260$ instead counts arrangements with a distinguished block abc or cba. An arrangement can contain several such blocks, so it is counted repeatedly; this also ignores constant triples.

Step 3: Conclusion Under the literal definition of an A.P. (including common difference zero), the answer is $\boxed{1208}$, which differs from the listed key value 1260 for the counting reasons above. Even if only nonconstant triples abc, cba are allowed, the exact count is 944, rather than 1260. The digits must be distinct and nonzero for the usual nine-digit multiset interpretation; the question does not specify these restrictions explicitly.

Quick Tip to Solve: Before treating a consecutive pattern as a single block, check whether one arrangement can contain that pattern more than once; marked occurrences are not distinct arrangements.

23. Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line $x + y = 2$. Then $r_1^2 + r_2^2 - r_1 r_2$ is equal to _____. [4 marks]

Solution:

Concept:

- A circle in the first quadrant touching both axes has centre (r, r) and radius $r > 0$. The distance from this centre to a line is found using the point-to-line distance formula.
- A chord of length $2l$, at perpendicular distance d from the centre of a circle, satisfies $r^2 = d^2 + l^2$. This follows from the right triangle formed by the radius and half of the chord.

Step 1: Express the line distance in terms of the radius For the line $x + y - 2 = 0$, the perpendicular distance from (r, r) is

$$d = \frac{|2r - 2|}{\sqrt{2}} = \sqrt{2}|r - 1|.$$

The stated intercept is the chord cut out on this line. Its length is 2, so its half-length is 1. The perpendicular from the centre bisects the chord; this is why the right triangle uses 1 rather than 2.

Step 2: Obtain the equation satisfied by both radii Using the radius as the hypotenuse gives

$$r^2 = 2(r - 1)^2 + 1.$$

Expand and rearrange:

$$r^2 = 2r^2 - 4r + 3 \quad \Longrightarrow \quad r^2 - 4r + 3 = 0.$$

Thus $(r - 1)(r - 3) = 0$, giving the two positive radii 1 and 3. Both satisfy the original distance-and-chord equation, so neither root is extraneous.

Step 3: Evaluate the symmetric expression Equivalently, the quadratic gives $r_1 + r_2 = 4$ and $r_1 r_2 = 3$. Therefore

$$r_1^2 + r_2^2 - r_1 r_2 = (r_1 + r_2)^2 - 3r_1 r_2 = 16 - 9 = 7.$$

A direct substitution of radii 1 and 3 also gives $1 + 9 - 3 = 7$. The absolute value in the distance formula has caused no sign ambiguity because the distance was squared.

Step 4: Conclusion The requested value is $\boxed{7}$ square units.

Quick Tip to Solve: A fixed chord length usually leads to a quadratic in the radius. Use its root sum and product to evaluate a symmetric expression without solving separately.

24. A fair $n(n > 1)$ faces die is rolled repeatedly until a number less than n appears. If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to _____. [4 marks]

Solution:

Concept:

- Each roll has success probability $p = (n - 1)/n$, because precisely the faces $1, 2, \ldots, n - 1$ are less than n. The remaining face n is a failure.

- For independent repeated trials stopped at the first success, the number T of trials, including the successful trial, has mean $E(T) = 1/p$. The event in the problem concerns this total number of rolls.

Step 1: Write the stopping-time distribution For the process to stop on roll t, the first $t-1$ rolls must all show n, and the next must show a smaller number. Thus

$$P(T=t) = \left(\frac{1}{n}\right)^{t-1} \frac{n-1}{n}, \qquad t = 1, 2, \ldots.$$

Since $n > 1$, the failure probability is strictly less than one and the process terminates with probability one. The probabilities form a geometric series with sum one.

Step 2: Calculate the mean and use the given value The geometric mean gives

$$E(T) = \frac{1}{(n-1)/n} = \frac{n}{n-1}.$$

For a direct check, write $q = 1/n$ and use $\sum_{t=1}^{\infty} tq^{t-1} = 1/(1-q)^2$; multiplication by $1-q$ gives the same result. Equate this to the stated mean:

$$\frac{n}{n-1} = \frac{n}{9}.$$

Because $n > 1$, division by n is legitimate, so $n - 1 = 9$ and $n = 10$.

Step 3: Check the stopping rule With ten faces, nine are successful outcomes and only face 10 requires another roll. Consequently the mean number of rolls is $10/9$, exactly the required value $n/9$. Counting failures alone would have mean $1/9$, which is a different random variable and would not match the wording.

Step 4: Conclusion The die has $\boxed{10}$ faces, so $n = 10$.

Quick Tip to Solve: For a first-success problem, identify whether the successful trial is included. Its inclusion changes the mean from $(1-p)/p$ to $1/p$.

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25. If $\int_{-0.15}^{0.15} \left|100x^2 - 1\right| \, dx = \frac{k}{3000}$, then k is equal to _____. [4 marks]

Solution:

Concept:

- The function $|100x^2 - 1|$ is even, so the integral on $[-a, a]$ equals twice the integral on $[0, a]$. This halves the interval that must be analysed.
- To remove an absolute value, first locate every zero of its argument. Here $100x^2 - 1 = 0$ gives $x = \pm 1/10$; the argument is negative between these roots and positive outside.

Step 1: Split the positive half at its sign change Write $0.15 = 3/20$ exactly. On $[0, 1/10]$, the integrand

is $1 - 100x^2$; on $[1/10, 3/20]$, it is $100x^2 - 1$. Therefore

$$I = 2\left[\int_0^{1/10} (1 - 100x^2)\,dx + \int_{1/10}^{3/20} (100x^2 - 1)\,dx\right].$$

The first piece and the second piece are both nonnegative, as an absolute-value integral must be.

Step 2: Evaluate both elementary integrals The first integral is

$$\left[x - \frac{100x^3}{3}\right]_0^{1/10} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}.$$

For the second use the antiderivative $100x^3/3 - x$. At $x = 3/20$ its value is $-3/80$, and at $x = 1/10$ its value is $-1/15$. Hence this piece is

$$-\frac{3}{80} + \frac{1}{15} = \frac{7}{240}.$$

Keeping fractions avoids rounding the final numerical value.

Step 3: Compare with the specified denominator Combining both parts and restoring the evenness factor gives

$$I = 2\left(\frac{1}{15} + \frac{7}{240}\right) = 2 \cdot \frac{23}{240} = \frac{23}{120}.$$

The question defines $I = k/3000$. Consequently

$$k = 3000 \cdot \frac{23}{120} = 25 \cdot 23 = 575.$$

The central interval contributes positive area after its sign is reversed; integrating the original quadratic without the split would subtract that area incorrectly.

Step 4: Conclusion The required integer is $\boxed{k = 575}$.

Quick Tip to Solve: For an even absolute-value integrand, combine symmetry with its positive-side zeros before integrating; convert decimal bounds into fractions for exact arithmetic.

26. The number of relations, on the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 3)$, which are reflexive and transitive but not symmetric, is _____. [4 marks]

Solution:

Concept:

- Reflexivity requires $(1, 1), (2, 2), (3, 3)$. Transitivity requires (x, z) whenever both (x, y) and (y, z) belong to the relation.
- The required pairs $(1, 2), (2, 3)$ therefore force $(1, 3)$. Only the three reverse pairs $(2, 1), (3, 2), (3, 1)$ remain optional; every possible relation can be decided by these three choices.

Step 1: Form the mandatory relation Let

$$R_0 = \{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)\}.$$

This is already reflexive and transitive: it is the relation $x \le y$ on the three-element set. It is not symmetric because, for example, it contains $(1,2)$ without $(2,1)$. Thus choosing no reverse pair gives one admissible relation.

Step 2: Test the optional reverse pairs Adding $(2,1)$ alone remains transitive: 1 and 2 become mutually related, and both are related to 3. This gives $R_0 \cup \{(2,1)\}$. Likewise adding $(3,2)$ alone gives $R_0 \cup \{(3,2)\}$, another transitive nonsymmetric relation. Their failure of symmetry is witnessed by $(2,3)$ and $(1,2)$, respectively, whose reverse pairs are missing.

Step 3: Exclude all remaining choices If $(3,1)$ is included, composition with $(1,2)$ forces $(3,2)$, and composition of $(2,3)$ with $(3,1)$ forces $(2,1)$. All reverse pairs would then be present. Also including both $(2,1)$ and $(3,2)$ forces $(3,1)$ by transitivity. These choices produce the universal relation, which is symmetric and hence excluded. Thus the only possibilities are the base relation and the two single additions just listed.

Step 4: Conclusion The number of reflexive, transitive, but nonsymmetric relations is $\boxed{3}$.

Quick Tip to Solve: First take the transitive closure of the mandatory pairs; then inspect only the remaining pairs and the further pairs each would force.

---

27. Let $[x]$ be the greatest integer $\le x$. Then the number of points in the interval $(-2,1)$, where the function $f(x) = |[x]| + \sqrt{x - [x]}$ is discontinuous, is _____. [4 marks]

Solution:

Concept:

- Between consecutive integers the greatest-integer function is constant. On $(k, k+1)$, the given function equals $|k| + \sqrt{x-k}$, which is continuous throughout that open interval.
- Only integers inside the specified interval can be discontinuities. At an integer k, compare the left limit, right limit and actual value separately, because the fractional part resets from nearly one to zero.

Step 1: Identify the possible exceptional points The integers lying strictly inside $(-2,1)$ are -1 and 0. The endpoints -2 and 1 do not belong to the requested interval and cannot be counted. Every noninteger point has a neighbourhood avoiding an integer, on which $[x]$ stays fixed; the square root then has a positive argument and varies continuously.

Step 2: Calculate one-sided limits at an integer As $x \to k^-$, we have $[x] = k-1$ and $x - [x] \to 1$, so

$$\lim_{x \to k^-} f(x) = |k-1| + 1.$$

At $x = k$, the fractional part is zero; from the right it tends to zero as well. Thus

$$f(k) = \lim_{x \to k^+} f(x) = |k|.$$

This shows that the function is right-continuous, but left-continuity still needs checking.

Step 3: Check each interior integer At $k = -1$, the left limit is $|-2| + 1 = 3$, whereas the value and right limit equal 1. At $k = 0$, the left limit is $|-1| + 1 = 2$, whereas the value and right limit equal 0. Both points therefore have jumps and are discontinuities. The earlier interval analysis establishes that there are no other points to count; merely noting that greatest-integer functions jump does not replace checking whether the added square-root term cancels a jump.

Step 4: Conclusion The discontinuities in $(-2, 1)$ occur at -1 and 0, giving $\boxed{2}$ points.

Quick Tip to Solve: At an integer k, the fractional part tends to 1 from the left and to 0 from the right. Use those two values before deciding whether a jump cancels.

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28. Let the positive numbers a_1, a_2, a_3, a_4 and a_5 be in a G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$ respectively, where m and n are co-prime. If the mean of their reciprocals is $\frac{31}{40}$ and $a_3 + a_4 + a_5 = 14$, then $m + n$ is equal to _____. [4 marks]

Solution:

Concept:

- For five positive terms in a G.P., write them as $t/r^2, t/r, t, tr, tr^2$, where $t = a_3 > 0$ and $r > 0$. The symmetric representation relates the sum of the terms directly to the sum of their reciprocals.
- The variance of five observations is $\sigma^2 = \frac{1}{5}\sum a_i^2 - \bar{a}^2$. The mean is already given; finding the ratio and middle term supplies the sum of squares.

Step 1: Find the middle term from the two means The given means imply

$$S = \sum a_i = 5 \cdot \frac{31}{10} = \frac{31}{2}, \qquad H = \sum \frac{1}{a_i} = 5 \cdot \frac{31}{40} = \frac{31}{8}.$$

Put $B = r^{-2} + r^{-1} + 1 + r + r^2$. Then $S = tB$ and $H = B/t$, so

$$t^2 = \frac{S}{H} = 4.$$

Positivity selects $t = 2$; the negative root cannot represent the middle positive term.

Step 2: Use the last three terms to determine the ratio The condition $a_3 + a_4 + a_5 = 14$ becomes

$$2 + 2r + 2r^2 = 14, \quad r^2 + r - 6 = 0.$$

Thus $(r - 2)(r + 3) = 0$. Since all terms are positive, the common ratio is positive, giving $r = 2$. The five terms are therefore

$$\frac{1}{2}, \quad 1, \quad 2, \quad 4, \quad 8.$$

Their sum is $31/2$, checking the original mean; their reciprocals sum to $31/8$, checking the second condition.

Step 3: Evaluate and reduce the variance The sum of squares is

$$\sum a_i^2 = \frac{1}{4} + 1 + 4 + 16 + 64 = \frac{341}{4}.$$

Consequently

$$\sigma^2 = \frac{341}{20} - \left(\frac{31}{10}\right)^2 = \frac{1705 - 961}{100} = \frac{744}{100} = \frac{186}{25}.$$

Since 186 and 25 have no common prime factor, the required coprime numerator and denominator are $m = 186$, $n = 25$. Their sum is $186 + 25 = 211$.

Step 4: Conclusion The variance is $186/25$, and the requested value is $\boxed{m + n = 211}$.

Quick Tip to Solve: For an odd-length G.P., divide the sum of its terms by the sum of their reciprocals: the symmetric representation immediately gives the square of the middle term.

---

29. Let $I(x) = \int \sqrt{\frac{x+7}{x}}\, dx$ and $I(9) = 12 + 7\log_e 7$. If $I(1) = \alpha + 7\log_e(1 + 2\sqrt{2})$, then α^4 is equal to _____.

[4 marks]

Solution:

Concept:

- For positive x, rewrite the integrand as $\sqrt{x+7}/\sqrt{x}$. A substitution $x = t^2$, with $t > 0$, removes the denominator radical.
- An indefinite integral represents a family of antiderivatives. The specified value $I(9)$ fixes the additive constant, which must be retained until that condition has been used.

Step 1: Remove the radical in the denominator Put $x = t^2$, so that $dx = 2t\,dt$. Since the two evaluation points are positive, use the positive square root. Then

$$I(x) = 2\int \sqrt{t^2 + 7}\, dt.$$

The standard integral of a quadratic radical gives

$$I(x) = t\sqrt{t^2 + 7} + 7\ln\left(t + \sqrt{t^2 + 7}\right) + C.$$

Returning to the original variable,

$$I(x) = \sqrt{x(x+7)} + 7\ln\left(\sqrt{x} + \sqrt{x+7}\right) + C.$$

Step 2: Apply the given value at nine At $x = 9$, the radical product is $\sqrt{9 \cdot 16} = 12$, while the logarithm has argument $3 + 4 = 7$. Thus

$$I(9) = 12 + 7\ln 7 + C.$$

Comparing with the prescribed value $12 + 7\ln 7$ gives $C = 0$. The condition fixes the constant without any change in the logarithm convention.

Step 3: Evaluate at one and identify the coefficient Substitute $x = 1$ into the same antiderivative:

$$I(1) = \sqrt{8} + 7\ln(1 + \sqrt{8}) = 2\sqrt{2} + 7\ln(1 + 2\sqrt{2}).$$

The logarithmic term already has exactly the required form, so the remaining term is $\alpha = 2\sqrt{2}$. Therefore

$$\alpha^4 = (2\sqrt{2})^4 = 16 \cdot 4 = 64.$$

Step 4: Conclusion The value is $\alpha = 2\sqrt{2}$, and hence the required dimensionless numerical answer is $\boxed{64}$.

Quick Tip to Solve: For $\sqrt{(x+a)/x}$, try $x = t^2$; fix the integration constant at the given point before comparing the requested expression.

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30. Let the plane $x + 3y - 2z + 6 = 0$ meet the co-ordinate axes at the points A, B, C. If the orthocenter of the triangle ABC is $\left(\alpha, \beta, \frac{6}{7}\right)$, then $98(\alpha + \beta)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The intercepts of a plane are found by setting the other two coordinates to zero. The orthocentre lies in the plane of the triangle and on each altitude.
- An altitude from a vertex is perpendicular to the opposite side. Thus, if H is the orthocentre, $(H - A) \cdot (C - B) = 0$ and $(H - B) \cdot (C - A) = 0$.

Step 1: Find the three axis intercepts For the plane $x + 3y - 2z + 6 = 0$, setting $y = z = 0$ gives $x = -6$; setting $x = z = 0$ gives $y = -2$; setting $x = y = 0$ gives $z = 3$. Consequently,

$$A = (-6, 0, 0), \quad B = (0, -2, 0), \quad C = (0, 0, 3).$$

The signs matter because the first two intercepts are on the negative axes.

Step 2: Use the altitude through A Write $H = (\alpha, \beta, 6/7)$, as stated. Since $C - B = (0, 2, 3)$, perpendicularity of the altitude through A gives

$$(\alpha + 6, \beta, 6/7) \cdot (0, 2, 3) = 0.$$

Hence

$$2\beta + \frac{18}{7} = 0 \quad \Rightarrow \quad \beta = -\frac{9}{7}.$$

This condition uses a three-dimensional dot product and does not require a projected diagram.

Step 3: Use the altitude through B and check the plane Here $C - A = (6, 0, 3)$, so

$$(\alpha, \beta + 2, 6/7) \cdot (6, 0, 3) = 0.$$

It follows that $6\alpha + 18/7 = 0$, giving $\alpha = -3/7$. These coordinates satisfy the plane equation because

$$-\frac{3}{7} + 3\left(-\frac{9}{7}\right) - 2\left(\frac{6}{7}\right) + 6 = 0.$$

Thus the two perpendicularity conditions locate the required point within the triangle plane.

Step 4: Calculate the requested expression Add the first two coordinates, then square:

$$\alpha + \beta = -\frac{12}{7}, \qquad 98(\alpha+\beta)^2 = 98 \cdot \frac{144}{49} = 288.$$

The square makes the requested result positive although both coordinates are negative.

Step 5: Conclusion The orthocentre is $(-3/7, -9/7, 6/7)$, and the required dimensionless numerical answer is $\boxed{288}$.

Quick Tip to Solve: For a triangle given by axis intercepts, apply altitude dot products directly in three dimensions; check that the resulting orthocentre still lies in the original plane.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A particle is executing simple harmonic motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be [4 marks]

(A) 1 : 1

(B) 1 : 4

(C) 2 : 1

(D) 1 : 3

Correct Answer: (D) 1 : 3

Solution:

Concept:

- For SHM of amplitude A and force constant k, the total mechanical energy is $E = \frac{1}{2}kA^2$, while the potential energy at displacement x is $U = \frac{1}{2}kx^2$.
- The kinetic energy is the remainder $K = E - U = \frac{1}{2}k(A^2 - x^2)$. The requested ratio is potential energy divided by kinetic energy, not potential energy divided by total energy.

Step 1: Evaluate the potential energy at the specified displacement The particle is at $x = A/2$. Squaring this displacement gives $x^2 = A^2/4$, so

$$U = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{8}kA^2 = \frac{E}{4}.$$

The sign of the displacement would not affect the result because potential energy depends on its square. Thus the particle has one quarter of its total energy as potential energy at either of these two positions.

Step 2: Find the kinetic energy by conservation The total energy remains E throughout an ideal SHM cycle. Subtract the potential energy just found:

$$K = E - \frac{E}{4} = \frac{3E}{4}.$$

Equivalently, $K = 3kA^2/8$. Both energies have the same dimensions and contain the same constant factors, which will cancel in their ratio.

Step 3: Form the ratio and compare the options Therefore

$$\frac{U}{K} = \frac{E/4}{3E/4} = \frac{1}{3}, \qquad U : K = 1 : 3.$$

The option $1 : 4$ confuses U/E with U/K. The option $1 : 1$ would require equal energies, occurring at $|x| = A/\sqrt{2}$, not $A/2$. The option $2 : 1$ would make potential energy larger than kinetic energy, contrary to the fractions obtained here.

Step 4: Conclusion The potential energy to kinetic energy ratio is $\boxed{1 : 3}$. Hence, the correct option is (D).

Quick Tip to Solve: Use $U : K = x^2 : (A^2 - x^2)$; substitute the fractional displacement before simplifying the ratio.

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32. Given below are two statements:

Statement I : A truck and a car moving with same kinetic energy are brought to rest by applying breaks which provide equal retarding forces. Both come to rest in equal distance.

Statement II : A car moving towards east takes a turn and moves towards north, the speed remains unchanged. The acceleration of the car is zero.

In the light of given statements, choose the most appropriate answer from the options given below

[4 marks]

(A) Statement I is incorrect but Statement II is correct.

(B) Statement I is correct but Statement II is incorrect.

(C) Both Statement I and Statement II are correct.

(D) Both Statement I and Statement II are incorrect.

Correct Answer: (B) Statement I is correct but Statement II is incorrect.

Solution:

Concept:

- The work-energy theorem gives $W = \Delta K$. A constant braking force of magnitude F, opposing motion over a stopping distance s, does work $-Fs$.
- Acceleration is the rate of change of the velocity vector. Constant speed makes the magnitude of velocity constant, but changing direction still changes velocity and produces acceleration.

Step 1: Test Statement I with the stopping-work equation Let the common initial kinetic energy

be K_0 and the common retarding force magnitude be F. When either vehicle stops, its final kinetic energy is zero. Hence

$$-Fs = 0 - K_0 \quad \Rightarrow \quad s = \frac{K_0}{F}.$$

Both quantities on the right are identical, so the stopping distances are equal. Different masses and initial speeds do not affect this energy comparison. Therefore Statement I is correct.

Step 2: Test Statement II using the direction of velocity Choose east as the positive x-direction and north as the positive y-direction. For unchanged speed v,

$$\vec{v}_i = v\hat{\mathbf{i}}, \qquad \vec{v}_f = v\hat{\mathbf{j}}.$$

Consequently,

$$\Delta\vec{v} = v(\hat{\mathbf{j}} - \hat{\mathbf{i}}) \neq \vec{0}.$$

The average acceleration during the turn is nonzero, and the car has a normal acceleration while turning. Zero tangential acceleration does not imply zero total acceleration. Thus Statement II is incorrect.

Step 3: Select the matching combination The results are I true and II false. Option (A) reverses both truth values. Option (C) incorrectly treats constant speed as constant velocity. Option (D) incorrectly rejects the work-energy result for the braking vehicles. Only option (B) has the correct pair.

Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: For stopping distance compare energy divided by braking force; for turning motion check the velocity direction as well as its magnitude.

33. Match List I with List II

	LIST I		LIST II
A.	**Spring constant**	**I.**	$[\mathrm{T}^{-1}]$
B.	**Angular speed**	**II.**	$[\mathrm{MT}^{-2}]$
C.	**Angular momentum**	**III.**	$[\mathrm{ML}^2]$
D.	**Moment of inertia**	**IV.**	$[\mathrm{ML}^2\mathrm{T}^{-1}]$

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-I, C-III, D-II

(B) A-II, B-III, C-I, D-IV

(C) A-II, B-I, C-IV, D-III

(D) A-I, B-III, C-II, D-IV

Correct Answer: (C) A-II, B-I, C-IV, D-III

Solution:

Concept:

- Dimensions are expressed using mass M, length L and time T. A defining equation is more reliable than memorising a list of units.
- Use $F = kx$ for a spring, $\omega = \mathrm{d}\theta/\mathrm{d}t$ for angular speed, $\vec{L} = \vec{r} \times \vec{p}$ for angular momentum, and $I = \sum mr^2$ for moment of inertia.

Step 1: Match spring constant and angular speed From Hooke's law, the magnitude of the spring constant is force divided by extension. Therefore

$$[k] = \frac{[F]}{[x]} = \frac{MLT^{-2}}{L} = MT^{-2}.$$

This is entry II. Angle is dimensionless in dimensional analysis, so angular speed has dimension

$$[\omega] = T^{-1},$$

which is entry I. The first two matches are consequently A-II and B-I.

Step 2: Match angular momentum and moment of inertia Linear momentum has dimension $[p] = MLT^{-1}$. Multiplying by the perpendicular distance in angular momentum gives

$$[L] = [r][p] = L \cdot MLT^{-1} = ML^2T^{-1}.$$

Thus C matches IV. For moment of inertia, each term is a mass times a squared distance:

$$[I] = ML^2.$$

Thus D matches III. Angular momentum contains a time factor, while moment of inertia does not; interchanging them is a common error.

Step 3: Combine all four results and eliminate the alternatives The complete mapping is

$$\boxed{\mathrm{A-II,\ B-I,\ C-IV,\ D-III}}.$$

Option (A) assigns angular-momentum dimensions to the spring constant and swaps the last matches. Option (B) incorrectly gives angular speed the dimensions of moment of inertia. Option (D) assigns reciprocal time to the spring constant. Each fails a defining equation, whereas option (C) satisfies all four.

Step 4: Conclusion The correct matching is A-II, B-I, C-IV, D-III. Hence, the correct option is (C).

Quick Tip to Solve: Recover dimensions from defining equations: spring stiffness is force per length, and angular momentum equals moment of inertia times angular speed.

34. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : EM waves used for optical communication have longer wavelengths than that of microwave, employed in Radar technology.

Reason R : Infrared EM waves are more energetic than microwaves, (used in Radar)

In the light of given statements, choose the correct answer from the options given below. [4 marks]

(A) Both A and R are true but R is NOT the correct explanation of A

(B) A is true but R is false

(C) Both A and R are true and r is the correct explanation of A

(D) A is false but R is true

Correct Answer: (D) A is false but R is true

Solution:

Concept:

- Electromagnetic radiation is ordered by wavelength and frequency through $c = \nu\lambda$. Infrared radiation has a shorter wavelength and a higher frequency than microwaves.
- The energy of one photon is $E = h\nu = hc/\lambda$. Therefore a shorter wavelength implies a larger photon energy when comparing two electromagnetic bands.

Step 1: Evaluate the assertion about wavelengths Optical communication commonly uses near-infrared radiation in optical fibres. Radar uses microwaves. Near-infrared wavelengths are of the order of micrometres, whereas radar wavelengths are much longer. Therefore

$$\lambda_{\text{optical}} < \lambda_{\text{microwave}}.$$

The assertion claims the opposite ordering, namely longer wavelengths for optical communication, so Assertion A is false. The name optical communication does not move its radiation into the long-wavelength radio part of the spectrum.

Step 2: Evaluate the reason about energy The shorter infrared wavelength corresponds to a higher frequency by $\nu = c/\lambda$. Consequently,

$$\frac{E_{\text{IR}}}{E_{\text{micro}}} = \frac{\lambda_{\text{micro}}}{\lambda_{\text{IR}}} > 1.$$

Thus an infrared photon is more energetic than a microwave photon. Reason R is true. This comparison concerns energy per photon; a comparison of total beam powers would also require the photon flux.

Step 3: Compare the assertion-reason combinations Since A is false and R is true, the explanatory relationship offered in options (A) and (C) cannot apply: both incorrectly state that A is true. Option (B) has the reverse truth values. The true energy statement is consistent with shorter infrared wavelengths and directly highlights why the wavelength ordering in the assertion is wrong.

Step 4: Conclusion Assertion A is false but Reason R is true. Hence, the correct option is (D).

Quick Tip to Solve: Order the electromagnetic bands by wavelength first, then reverse that order for photon energy using $E = hc/\lambda$.

35. The ratio of escape velocity of a planet to the escape velocity of earth will be:-

Given: Mass of the planet is 16 times mass of earth and radius of the planet is 4 times the radius of earth. [4 marks]

(A) $1:4$

(B) $1:\sqrt{2}$

(C) $4:1$

(D) $2:1$

Correct Answer: (D) $2:1$

Solution:

Concept:

- For a spherical planet of mass M and radius R, escape speed from its surface is $v_e = \sqrt{2GM/R}$, neglecting atmosphere and planetary rotation.
- Escape means reaching infinity with zero remaining speed. Equating initial kinetic energy to the magnitude of the surface gravitational potential energy makes the escaping object's mass cancel.

Step 1: Express the escape-speed ratio For the planet and Earth, respectively,

$$v_p = \sqrt{\frac{2GM_p}{R_p}}, \qquad v_E = \sqrt{\frac{2GM_E}{R_E}}.$$

Divide these formulae before substituting any numerical gravitational constants:

$$\frac{v_p}{v_E} = \sqrt{\frac{M_p}{M_E}\frac{R_E}{R_p}}.$$

The universal constant and factor two are identical for both bodies and cancel exactly. This ratio compares speeds, so it is dimensionless.

Step 2: Insert the mass and radius multipliers The given relations are $M_p = 16M_E$ and $R_p = 4R_E$. Hence

$$\frac{v_p}{v_E} = \sqrt{16 \cdot \frac{1}{4}} = \sqrt{4} = 2.$$

The positive square root is required because escape speed is a nonnegative magnitude. Although the planet has a larger radius, its mass has increased by a still larger factor, so the escape speed increases.

Step 3: Check the proposed ratios The required ratio of planet to Earth is $2:1$. Option (C), $4:1$, is the ratio of the squared speeds, obtained by forgetting the final square root. Option (A), $1:4$, incorrectly reverses the mass-to-radius combination and omits the root. Option (B), $1:\sqrt{2}$, would mean a lower escape speed on the planet and does not follow from the stated factors. The direct substitution uniquely selects option (D).

Step 4: Conclusion The escape-speed ratio is $\boxed{v_p : v_E = 2:1}$. Hence, the correct option is (D).

Quick Tip to Solve: For two spherical bodies, take the square root of the mass ratio divided by the radius ratio; do not confuse the speed ratio with its square.

36. Two satellites A and B move round the earth in the same orbit. The mass of A is twice the mass of B. The quantity which is same for the two satellites will be [4 marks]

(A) Potential energy

(B) Kinetic energy

(C) Total energy

(D) Speed

Correct Answer: (D) Speed

Solution:

Concept:

- For a circular orbit of radius r, gravity supplies the centripetal force: $GMm/r^2 = mv^2/r$. Here M is the mass of the earth and m is the satellite mass.
- The orbital energies are $U = -GMm/r$, $K = GMm/(2r)$, and $E = -GMm/(2r)$. The reference for potential energy is zero at infinity.

Step 1: Compare the orbital speeds The two satellites move in the same orbit, so the orbital radius and the attracting mass of the earth are the same. Cancelling the satellite mass from the force equation gives

$$v = \sqrt{\frac{GM}{r}}.$$

No satellite mass occurs in this expression. Consequently, doubling the satellite mass does not change the speed required for this orbit: $v_A = v_B$. This cancellation is why a heavier satellite does not need a larger orbital speed at the same altitude.

Step 2: Check the three energy choices Write $m_B = m$, so that $m_A = 2m$. At the common orbital radius,

$$\frac{U_A}{U_B} = \frac{K_A}{K_B} = \frac{E_A}{E_B} = 2.$$

Both potential and total energies are negative, but their magnitudes double with mass; their values therefore are not equal. Kinetic energy is positive and doubles as well. Thus none of the three energy choices is independent of satellite mass, whereas the speed is. The comparison concerns total satellite energies, not energies per unit mass.

Step 3: Conclusion The speed is the same for the two satellites. Hence, the correct option is (D).

Quick Tip to Solve: At a fixed orbital radius, distinguish total energy, which is proportional to satellite mass, from orbital speed, in which that mass cancels.

37. Three forces $F_1 = 10\ \text{N}, F_2 = 8\ \text{N},\ \text{F}_3 = 6\ \text{N}$ are acting on a particle of mass $5\ \text{kg}$. The forces F_2 and F_3 are applied perpendicularly so that particle remains at rest. If the force F_1 is removed, then the acceleration of the particle is: [4 marks]

(A) $4.8\ \text{ms}^{-2}$

(B) $7\ \text{ms}^{-2}$

(C) $2\ \text{ms}^{-2}$

(D) $0.5\ \text{ms}^{-2}$

Correct Answer: (C) $2\ \text{ms}^{-2}$

Solution:

Concept:

- For equilibrium, the vector sum of all forces is zero: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$. Removing one force leaves a resultant equal and opposite to that force.
- Perpendicular forces combine by Pythagoras, and Newton's second law gives the acceleration magnitude as $a = F_{\text{net}}/m$.

Step 1: Find the remaining resultant The particle is initially at rest in equilibrium. The force of $10\,\text{N}$ balances the other two forces; it does not add to them in magnitude. Once $\vec{F}_1$ is removed, the only forces left are $\vec{F}_2$ and $\vec{F}_3$. Because these are perpendicular,

$$F_{\text{net}} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10\,\text{N}.$$

Equivalently, their vector resultant is $-\vec{F}_1$, which immediately confirms the magnitude and shows that its direction is opposite to the removed force.

Step 2: Apply the mass and compare the choices The mass remains $5\,\text{kg}$; removal of a force does not remove any mass. Therefore,

$$a = \frac{F_{\text{net}}}{m} = \frac{10}{5} = 2\,\text{m}\,\text{s}^{-2}.$$

The perpendicular forces cannot be combined by simply adding $8+6$, nor should the removed force be included in a scalar sum of all three. The proposed accelerations 4.8, 7, and $0.5\,\text{m}\,\text{s}^{-2}$ would require resultants of 24, 35, and $2.5\,\text{N}$, respectively, none equal to the actual remaining resultant.

Step 3: Conclusion The particle accelerates at $2\,\text{m}\,\text{s}^{-2}$, opposite to the direction of the removed force. Hence, the correct option is (C).

Quick Tip to Solve: In a balanced force system, removing one force leaves a resultant exactly opposite to it; divide its magnitude by the mass.

38. An ice cube has a bubble inside. When viewed from one side the apparent distance of the bubble is 12 cm. When viewed from the opposite side, the apparent distance of the bubble is observed as 4 cm. If the side of the ice cube is 24 cm, the refractive index of the ice cube is [4 marks]

(A) $\frac{3}{2}$
(B) $\frac{4}{3}$
(C) $\frac{2}{3}$
(D) $\frac{6}{5}$

Correct Answer: (A) $\frac{3}{2}$

Solution:

Concept:

- For an object inside a transparent medium viewed approximately normally from air, μ = real depth/apparent depth, where μ is the medium's refractive index relative to air.
- The real distances of one internal point from two opposite parallel faces add to the thickness of the medium; the two apparent distances do not add to that thickness.

Step 1: Express the two real depths Let the real depth of the bubble from the first face be x cm. Its real depth from the opposite face is then $(24 - x)$ cm. Each observation is made through one plane ice-air surface. The apparent depths therefore give

$$\frac{x}{\mu} = 12, \qquad \frac{24 - x}{\mu} = 4.$$

The first equation gives $x = 12\mu$, and the second gives $24 - x = 4\mu$. Both equations must use the same refractive index, since the material is the same when viewed from either side.

Step 2: Add the depths and verify the result Adding the two equations eliminates the unknown bubble position:

$$24 = 12\mu + 4\mu = 16\mu, \qquad \mu = \frac{24}{16} = \frac{3}{2}.$$

As a check, the two real depths are 18 cm and 6 cm. Dividing each by $3/2$ reproduces the observed 12 cm and 4 cm. The value $2/3$ is the reciprocal ratio, not the refractive index of ice relative to air. The values $4/3$ and $6/5$ would imply total real thicknesses $64/3$ cm and $96/5$ cm, which disagree with 24 cm.

Step 3: Conclusion The refractive index of the ice cube is $\mu = 3/2$. Hence, the correct option is (A).

Quick Tip to Solve: For opposite-side apparent depths through a slab, divide its thickness by the sum of the two apparent depths.

39. A proton and an α-particle are accelerated from rest by 2 V and 4 V potentials, respectively. The ratio of their de-Broglie wavelength is : [4 marks]

(A) 4 : 1

(B) 2 : 1

(C) 8 : 1

(D) 16 : 1

Correct Answer: (A) 4 : 1

Solution:

Concept:

- A particle of charge magnitude q accelerated from rest through potential difference V gains kinetic energy $K = qV$. At these low voltages the motion is nonrelativistic.
- The de Broglie relation is $\lambda = h/p = h/\sqrt{2mqV}$. An alpha particle has charge $2e$ and approximately four times the proton mass.

Step 1: Compare the acquired kinetic energies Let the proton mass be m and its charge be e. The proton is accelerated through $2\,\text{V}$, while the alpha particle has mass $4m$, charge $2e$, and accelerating potential $4\,\text{V}$. Thus

$$K_p = e(2\,\text{V}) = 2\,\text{eV}, \qquad K_\alpha = 2e(4\,\text{V}) = 8\,\text{eV}.$$

The alpha particle receives four times the proton's kinetic energy, not twice, because its charge as well as the potential difference is larger.

Step 2: Form the wavelength ratio Wavelength is inversely proportional to momentum, so the required proton-to-alpha ratio is

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p} = \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}} = \sqrt{4 \times 4} = 4.$$

The common factors h and 2 cancel when the ratio is taken. A ratio of $2 : 1$ would neglect one of the two factors of four under the square root. A ratio of $16 : 1$ would omit the square root, while $8 : 1$ does not follow from the momentum relation. These voltages are far below the particle rest energies, so the classical momentum expression used here is consistent.

Step 3: Conclusion The ratio is $\lambda_p : \lambda_\alpha = 4 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: For particles accelerated from rest, compare mqV under a square root and invert the momentum ratio to obtain the wavelength ratio.

40. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : If an electric dipole of dipole moment 30×10^{-5} C m is enclosed by a closed surface, the net flux coming out of the surface will be zero.

Reason R : Electric dipole consists of two equal and opposite charges.

In the light of above, statements, choose the correct answer from the options given below. [4 marks]

(A) A is true but R is false

(B) A is false but R is true

(C) Both A and R are true but R is NOT the correct explanation of A

(D) Both A and R are true and R is the correct explanation of A

Correct Answer: (D) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Gauss's law relates the net outward electric flux through any closed surface to the algebraic enclosed charge: $\Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\varepsilon_0$.
- An electric dipole consists of two equal and opposite charges separated by a finite distance. Its dipole moment can be nonzero even though its net charge is zero.

Step 1: Evaluate the assertion The entire electric dipole is enclosed, so both charges lie inside the closed surface. If their values are $+q$ and $-q$, the net enclosed charge is

$$Q_{\text{enc}} = q - q = 0.$$

Gauss's law then gives

$$\Phi_E = \frac{0}{\varepsilon_0} = 0.$$

Thus Assertion A is true. The stated dipole moment $30 \times 10^{-5}\,\text{C m}$ does not change this result: dipole moment measures the charge separation, whereas the closed-surface flux depends on net charge.

Step 2: Evaluate the reason Reason R is true because equal and opposite charges are precisely the charge pair defining an electric dipole. Their equality and opposite signs make the enclosed charge vanish, which directly explains the zero net flux using Gauss's law.

Step 3: Check the explanatory link Zero net flux does not mean the field is zero everywhere on the surface: field lines can enter and leave, with equal algebraic contributions overall. Options (A) and (B) incorrectly declare one statement false, and option (C) incorrectly denies the explanatory link. Both statements are true, and the reason supplies the cancellation needed in the assertion.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (D).

Quick Tip to Solve: For a closed-surface flux question, count the algebraic enclosed charge first; a nonzero dipole moment does not imply nonzero net flux.

41. Given below are two statements:

Statement I : The diamagnetic property depends on temperature.

Statement II : The induced magnetic dipole moment in a diamagnetic sample is always opposite to the magnetizing field.

In the light of given statements, choose the correct answer from the options given below. [4 marks]

(A) Statement I is incorrect but Statement II is true.

(B) Statement I is correct but Statement II is false.

(C) Both Statement I and Statement II are False.

(D) Both Statement I and Statement II are true.

Correct Answer: (A) Statement I is incorrect but Statement II is true.

Solution:

Concept:

- Diamagnetism is the weak magnetic response induced by an applied field in matter without a permanent net atomic magnetic moment. Its susceptibility is small and negative.
- In the usual treatment of diamagnetic materials, the susceptibility is essentially independent of temperature; it does not follow the inverse-temperature law of paramagnetism.

Step 1: Test the temperature statement Statement I claims that the diamagnetic property depends on temperature. In the standard magnetic classification, a diamagnetic response comes from changes in the orbital motion of electrons caused by the applied magnetic field. It does not depend on thermal alignment of pre-existing permanent moments. Hence diamagnetic susceptibility is taken to be independent of temperature, and Statement I is incorrect. Confusing this response with paramagnetism would lead to applying Curie's temperature dependence to the wrong class of materials.

Step 2: Test the induced-moment statement For a linear isotropic diamagnetic sample, the magnetization obeys

$$\vec{M} = \chi_m \vec{H}, \qquad \chi_m < 0.$$

The negative sign means that the induced magnetization, and thus the induced magnetic dipole moment of the sample, is opposite to the magnetizing field. This induced response opposes the applied field rather than reinforcing it. Statement II is therefore true. Option (B) reverses both truth values. Option (C) wrongly rejects the opposite-direction response, and option (D) wrongly accepts the temperature-dependence claim. Only the combination of false Statement I and true Statement II matches both physical facts.

Step 3: Conclusion Statement I is incorrect, but Statement II is true. Hence, the correct option is (A).

Quick Tip to Solve: Pair diamagnetism with negative, nearly temperature-independent susceptibility; pair the familiar Curie law with paramagnetism.

42. A wire of resistance $160\ \Omega$ is melted and drawn in a wire of one-fourth of its length. The new resistance of the wire will be [4 marks]

(A) $640\ \Omega$

(B) $40\ \Omega$

(C) $16\ \Omega$

(D) $10\ \Omega$

Correct Answer: (D) $10\ \Omega$

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/A$, where ρ is resistivity, L is length, and A is cross-sectional area.
- When the same material is melted and reshaped without loss, its volume AL is conserved. Comparing the wires at the same temperature keeps resistivity unchanged.

Step 1: Use conservation of volume Let the original length and cross-sectional area be L and A. The new wire has length $L' = L/4$. Since the amount of metal is unchanged,

$$AL = A'L' = A'\frac{L}{4}, \qquad A' = 4A.$$

The shorter wire must therefore be thicker. It would be incorrect to change only the length in the resistance formula while retaining the original area. The reshaping changes both geometric factors, each reducing the resistance.

Step 2: Compute the new resistance and reject the alternatives With the same resistivity, the resistance ratio is

$$\frac{R'}{R} = \frac{L'}{L}\frac{A}{A'} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}.$$

Consequently,

$$R' = \frac{160}{16} = 10\,\Omega.$$

The choice $40\,\Omega$ accounts for the shorter length but overlooks the fourfold area increase. The choice $640\,\Omega$ changes resistance in the wrong direction for a shorter, thicker wire. The value $16\,\Omega$ mistakes the geometric reduction factor for an actual resistance; the original $160\,\Omega$ must be divided by that factor. Equivalently, substituting $A = \text{volume}/L$ shows directly that resistance varies as L^2 for fixed volume.

Step 3: Conclusion The new resistance is $10\,\Omega$. Hence, the correct option is (D).

Quick Tip to Solve: For a reshaped wire of unchanged volume and material, use $R \propto L^2$; one-fourth the length gives one-sixteenth the resistance.

43. A ball is thrown vertically upward with an initial velocity of $150\ \mathrm{m/s}$. The ratio of velocity after $3\ \mathrm{s}$ and $5\ \mathrm{s}$ is $\frac{x+1}{x}$. The value of x is _____.

{ **take,** $g = 10\ \mathrm{m/s^2}$} [4 marks]

(A) -5

(B) 10

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Concept:

- Choose upward as positive. With air resistance neglected, the ball has constant acceleration $a = -g$, so its velocity at time t is $v = u - gt$.
- A ratio of velocities must use the velocities at the two stated instants in the stated order. Both instants occur before the highest point, so both velocities are positive.

Step 1: Find the two velocities The initial velocity is $u = 150\,\mathrm{m\,s^{-1}}$ and the downward gravitational acceleration has magnitude $10\,\mathrm{m\,s^{-2}}$. Substituting the two times separately gives

$$v_3 = 150 - 10(3) = 120\,\mathrm{m\,s^{-1}}, \qquad v_5 = 150 - 10(5) = 100\,\mathrm{m\,s^{-1}}.$$

The ball reaches its highest point only at $u/g = 15\,\mathrm{s}$; hence there is no reversal of direction between the instants being compared.

Step 2: Equate and solve the ratio The required ratio is therefore

$$\frac{v_3}{v_5} = \frac{120}{100} = \frac{6}{5} = \frac{x+1}{x}.$$

Cross-multiplication gives $6x = 5x + 5$, and hence $x = 5$. This is dimensionless, as it occurs in a ratio. Checking the alternatives, $x = -5$ gives $4/5$, $x = 10$ gives $11/10$, and $x = 6$ gives $7/6$; none equals $6/5$.

Step 3: Conclusion The value is $\boxed{x = 5}$. Hence, the correct option is (C).

Quick Tip to Solve: For successive-time velocity ratios, calculate both signed velocities first; using distances or reversing the ratio changes the value of the parameter.

44. If the r. m.s speed of chlorine molecule is $490\ \mathrm{m/s}$ at $27^\circ\mathrm{C}$, the r. m. s speed of argon molecules at the same temperature will be (Atomic mass of argon $= 39.9\mathrm{u}$, molecular mass of chlorine $= 70.9\mathrm{u}$) [4 marks]

(A) $451.7\ \mathrm{m/s}$

(B) $751.7\ \mathrm{m/s}$

(C) $551.7\ \mathrm{m/s}$

(D) $651.7\ \mathrm{m/s}$

Correct Answer: (D) $651.7\ \mathrm{m/s}$

Solution:

Concept:

- For a gas at absolute temperature T, the root mean square molecular speed is $v_{\mathrm{rms}} = \sqrt{3RT/M}$, where M is the molar mass.
- At the same temperature the factor $3RT$ cancels in a speed ratio, giving $v_1/v_2 = \sqrt{M_2/M_1}$. A lighter gas therefore has a greater rms speed.

Step 1: Form the speed ratio Chlorine is present as diatomic molecules, whereas argon is monatomic. We must therefore compare the given molecular mass $70.9\,\mathrm{u}$ for chlorine with the atomic mass $39.9\,\mathrm{u}$ for argon, rather than halving the chlorine value. Since both gases are at $27^\circ\mathrm{C}$, their common absolute temperature is approximately $300\,\mathrm{K}$, although its value cancels:

$$\frac{v_{\mathrm{Ar}}}{v_{\mathrm{Cl_2}}} = \sqrt{\frac{70.9}{39.9}}.$$

Step 2: Evaluate and check the alternatives Using the given chlorine speed,

$$v_{\mathrm{Ar}} = 490\sqrt{\frac{70.9}{39.9}} = 490(1.33302\ldots) \simeq 653.2\,\mathrm{m\,s^{-1}}.$$

The result exceeds $490\,\mathrm{m\,s^{-1}}$, as expected for the lighter particles. No printed option equals this computed value exactly. Option (D), $651.7\,\mathrm{m\,s^{-1}}$, is uniquely closest, differing by only about $1.5\,\mathrm{m\,s^{-1}}$ or 0.23%. The other values, 451.7, 751.7, and $551.7\,\mathrm{m\,s^{-1}}$, differ by roughly 201.5, 98.5, and $101.5\,\mathrm{m\,s^{-1}}$, respectively. Thus the approximate option data point to (D), while the given numbers themselves yield $653.2\,\mathrm{m\,s^{-1}}$.

Step 3: Conclusion The given data yield $\boxed{v_{\mathrm{Ar}} \simeq 653.2\,\mathrm{m\,s^{-1}}}$. Among the printed approximate answers, $651.7\,\mathrm{m\,s^{-1}}$ is uniquely closest. Hence, the correct option is (D).

Quick Tip to Solve: At a common temperature, scale a known rms speed by the square root of the inverse mass ratio, using molecular mass for a molecular gas.

45. A $12.5\ \text{eV}$ electron beam is used to bombard gaseous hydrogen at room temperature. The number of spectral lines emitted will be: [4 marks]

(A) 2

(B) 4

(C) 3

(D) 1

Correct Answer: (C) 3

Solution:

Concept:

- The hydrogen energy levels are $E_n = -13.6/n^2\ \text{eV}$. Exciting a ground-state atom to level n requires $\Delta E = 13.6(1 - 1/n^2)\ \text{eV}$.
- At room temperature the hydrogen atoms are initially essentially in their ground state. An incident electron can transfer only up to its kinetic energy in a collision; the atom absorbs an allowed excitation energy, not an arbitrary amount.

Step 1: Find the highest accessible level Compare the incident energy $12.5\,\text{eV}$ with the first excitation thresholds:

$$\Delta E_{1\to2} = 10.2\,\text{eV}, \qquad \Delta E_{1\to3} = 13.6\left(\frac{8}{9}\right) \simeq 12.09\,\text{eV}.$$

$$\Delta E_{1\to4} = 13.6\left(\frac{15}{16}\right) = 12.75\,\text{eV}.$$

Thus excitation to $n = 3$ is possible, but excitation to $n = 4$ is not. The beam also lies below the $13.6\,\text{eV}$ ground-state ionisation threshold.

Step 2: Count distinct downward transitions Across the excited gas, radiative decay can produce the transitions $3 \to 2$, $3 \to 1$, and $2 \to 1$. Their energy differences are distinct, so they give three spectral lines. Equivalently, for levels 1 through 3, the number of pairs is

$$N = \frac{3(3-1)}{2} = 3.$$

Counting just one direct transition misses cascade emission; counting two misses the direct $3 \to 1$ transition, while four would require an additional accessible level.

Step 3: Conclusion The emitted spectrum contains $\boxed{3}$ distinct lines. Hence, the correct option is (C).

Quick Tip to Solve: First compare the beam energy with discrete excitation thresholds; only then use the number of accessible levels to count all possible spectral lines.

46. Given below are two statements:

Statement I : When the frequency of an a.c source in a series LCR circuit increases, the current in the circuit first increases, attains a maximum value and then decreases.

Statement II : In a series LCR circuit, the value of power factor at resonance is one.

In the light of given statements, choose the most appropriate answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are False.

(B) Statement I is incorrect but Statement II is true.

(C) Both Statement I and Statement II are true.

(D) Statement I is correct but Statement II is false.

Correct Answer: (C) Both Statement I and Statement II are true.

Solution:

Concept:

- For a series LCR circuit driven at angular frequency ω, the impedance is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$, and the current amplitude is $I_0 = V_0/Z$.
- The power factor is $\cos\phi = R/Z$. Resonance occurs when the inductive and capacitive reactances cancel, at $\omega_0 = 1/\sqrt{LC}$.

Step 1: Test Statement I using the impedance For a fixed applied voltage amplitude, at very low frequency the capacitive reactance $1/(\omega C)$ is large, so the current is small. Increasing frequency toward resonance reduces the magnitude of the net reactance and hence reduces Z. At resonance the impedance has its minimum value R, giving the maximum current V_0/R. Further increasing frequency makes ωL dominate, increasing the impedance and decreasing the current. Thus the stated rise, maximum and fall are the resonance response of a series LCR circuit, and Statement I is true.

Step 2: Test Statement II and select the pair At resonance, $\omega_0 L = 1/(\omega_0 C)$, so the circuit is effectively resistive:

$$Z_{\text{res}} = R, \qquad \cos\phi_{\text{res}} = \frac{R}{R} = 1.$$

Voltage and current are in phase, making Statement II true. Option (A) rejects both correct statements, option (B) wrongly rejects the frequency response, and option (D) wrongly rejects the unity power factor. The pair of true statements is uniquely represented by (C).

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (C).

Quick Tip to Solve: In a series resonance question, start with cancellation of the two reactances: it immediately gives minimum impedance, maximum current and unity power factor.

47. In an n-p-n common emitter (CE) transistor the collector current changes from 5 mA to 16 mA for the change in base current from 100 $\mu\mathrm{A}$ and 200 $\mu\mathrm{A}$, respectively. The current gain of transistor is _____. [4 marks]

(A) 210

(B) 0.9

(C) 9

(D) 110

Correct Answer: (D) 110

Solution:

Concept:

- The small-signal current gain of a common-emitter transistor is $\beta_{\text{ac}} = \Delta I_C/\Delta I_B$, the ratio of changes in collector and base current.
- Both current changes must be expressed in the same unit before division. The gain is dimensionless; endpoint ratios such as I_C/I_B are not the requested change ratio.

Step 1: Calculate the collector and base current changes The collector current rises from $5\,\mathrm{mA}$ to $16\,\mathrm{mA}$, so

$$\Delta I_C = 16 - 5 = 11\,\mathrm{mA}.$$

The corresponding base current rises from $100\,\mu\mathrm{A}$ to $200\,\mu\mathrm{A}$, so

$$\Delta I_B = 200 - 100 = 100\,\mu\mathrm{A} = 0.100\,\mathrm{mA}.$$

Both changes are positive, consistent with amplification in the transistor active region.

Step 2: Divide the changes and compare the options The common-emitter current gain is

$$\beta_{\text{ac}} = \frac{11\,\mathrm{mA}}{0.100\,\mathrm{mA}} = 110.$$

The identical current units cancel, leaving a pure number. A value such as 210 can result from incorrectly combining endpoint currents instead of subtracting them. Neither 0.9 nor 9 equals the measured ratio of changes. Directly using the numerical values 11 and 100 without converting the units would also give a spurious result; the microampere-to-milliampere conversion is essential.

Step 3: Conclusion The current gain is $\boxed{\beta_{\text{ac}} = 110}$. Hence, the correct option is (D).

Quick Tip to Solve: When two operating points are supplied, subtract first and then divide: CE current gain uses the change in collector current divided by the change in base current.

48. An engine operating between the boiling and freezing points of water will have

A. efficiency more than 27%.

B. efficiency less than the efficiency of a Carnot engine operating between the same two temperatures.

C. efficiency equal to 27%

D. efficiency less than 27%

Choose the correct answer from the options given below: [4 marks]

(A) B and C only

(B) B and D only

(C) B, C and D only

(D) A and B only

Correct Answer: (B) B and D only

Solution:

Concept:

- The maximum efficiency for a heat engine between reservoirs at absolute temperatures T_H and T_C is the Carnot efficiency $\eta_C = 1 - T_C/T_H$.
- A practical irreversible engine has efficiency below the reversible Carnot limit between the same two reservoirs. Temperatures must be in kelvin; Celsius values cannot be used in the ratio.

Step 1: Calculate the reversible upper limit The boiling and freezing points of water correspond approximately to $T_H = 373\,\text{K}$ and $T_C = 273\,\text{K}$. Hence

$$\eta_C = 1 - \frac{273}{373} = \frac{100}{373} \simeq 0.2681,$$

$$100\eta_C \simeq 26.81\%.$$

Thus even the ideal reversible limit is slightly below 27%; a real engine cannot have an efficiency above this limit.

Step 2: Evaluate the listed claims For the practical engine meant in this comparison, irreversibility makes its efficiency smaller than the efficiency of the Carnot engine operating between the same temperatures, so B is correct. Its efficiency must also be less than 27%, making D correct. A contradicts the maximum-efficiency bound and is false. C treats the rounded Carnot limit as an exact efficiency of the engine and is false. Consequently the combinations in options (A) and (C), which include C, are excluded; option (D) includes the impossible claim A. Only the combination B and D remains.

Step 3: Conclusion The correct statements are B and D only. Hence, the correct option is (B).

Quick Tip to Solve: Calculate the Carnot limit in kelvin before judging efficiency statements, and distinguish the reversible upper bound from the efficiency of a practical engine.

49. A body cools from $80^\circ C$ to $60^\circ C$ in 5 minutes. The temperature of the surrounding is $20^\circ C$. The time it takes to cool from $60^\circ C$ to $40^\circ C$ is : [4 marks]

(A) 500 s

(B) $\frac{25}{3}$ S

(C) 450 s

(D) 420 s

Correct Answer: (A) 500 s

Solution:

Concept:

- Newton's law of cooling states that the cooling rate is proportional to the excess temperature above the surroundings: $-dT/dt = k(T - T_s)$.
- For the finite temperature intervals used in the standard approximate calculation, take the rate at the mean body temperature: $(T_1 - T_2)/t \simeq k((T_1 + T_2)/2 - T_s)$. The same constant k applies to both stages.

Step 1: Determine the cooling constant approximately In the first interval the temperature falls by $20^\circ C$ in $5\,\text{min}$, while its mean temperature is $(80 + 60)/2 = 70^\circ C$. The mean excess above the surroundings is therefore $70 - 20 = 50^\circ C$. The finite-interval form gives

$$\frac{80 - 60}{5} = k(70 - 20), \qquad k \simeq \frac{4}{50}\,\text{min}^{-1}.$$

Only temperature differences appear, so using Celsius differences here is valid.

Step 2: Apply the same approximation to the second interval The second interval also has a $20^\circ C$ drop, but the mean temperature is $50^\circ C$, with excess $30^\circ C$. Thus

$$\frac{20}{t} = \frac{4}{50}(30), \qquad t = \frac{25}{3}\,\text{min} = 500\,\text{s}.$$

The later interval takes longer because the temperature excess is smaller. Option (B) has the numerical value in minutes but labels it in seconds; (C) and (D) do not satisfy the mean-rate ratio. Integrating the differential law exactly would give $300 \ln 2/\ln(3/2) \simeq 513\,\text{s}$; the listed answer uses the finite-interval approximation.

Step 3: Conclusion Using the mean-temperature approximation implicit in the options, $\boxed{t = 500\,\text{s}}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal temperature drops in the approximate cooling formula, the times are inversely proportional to the mean excess temperatures; convert minutes to seconds at the end.

50. The amplitude of $15\sin(1000\pi t)$ is modulated by $10\sin(4\pi t)$ signal. The amplitude modulated signal contains frequencies of

A. 500 Hz

B. 2 Hz

C. 250 Hz

D. 498 Hz

E. 502 Hz

Choose the correct answer from the options given below: [4 marks]

(A) A and C only

(B) A and B only

(C) A and D only

(D) A, D and E only

Correct Answer: (D) A, D and E only

Solution:

Concept:

- An amplitude-modulated carrier contains the carrier frequency f_c and two sidebands at $f_c - f_m$ and $f_c + f_m$, where f_m is the signal frequency.
- For a sinusoid written as $A\sin(\omega t)$, the frequency in hertz is $f = \omega/(2\pi)$. The amplitude controls sideband strength rather than their frequencies.

Step 1: Identify the carrier and signal frequencies The rapidly oscillating wave is the carrier. Its angular frequency is $1000\pi\,\text{rad}\,\text{s}^{-1}$, whereas that of the modulating signal is $4\pi\,\text{rad}\,\text{s}^{-1}$. Therefore,

$$f_c = \frac{1000\pi}{2\pi} = 500\,\text{Hz}, \qquad f_m = \frac{4\pi}{2\pi} = 2\,\text{Hz}.$$

The modulation index is $10/15 = 2/3$, so ordinary amplitude modulation applies without overmodulation.

Step 2: Locate both sidebands and test the listed frequencies Writing the amplitude-varying wave as $[15 + 10\sin(4\pi t)]\sin(1000\pi t)$, the product of the sinusoids produces sum and difference frequencies. Thus,

$$f_- = 500 - 2 = 498\,\text{Hz}, \qquad f_+ = 500 + 2 = 502\,\text{Hz}.$$

The original carrier at $500\,\text{Hz}$ remains present. Thus statements A, D and E are correct. The $2\,\text{Hz}$ signal is not a separate component of the modulated high-frequency wave, and there is no component at $250\,\text{Hz}$. Options (A) and (B) include absent frequencies; option (C) omits the upper sideband.

Step 3: Conclusion The frequencies are $498\,\text{Hz}$, $500\,\text{Hz}$, and $502\,\text{Hz}$, corresponding to A, D and E only. Hence, the correct option is (D).

Quick Tip to Solve: Convert angular frequencies to hertz first, then place two sidebands symmetrically about the carrier; do not include the message frequency itself.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. To maintain a speed of 80 km/h by a bus of mass 500 kg on a plane rough road for 4 km distance, the work done by the engine of the bus will be _____ KJ. [The coefficient of friction between tyre of bus and road is 0.04.] [4 marks]

Solution:

Concept:

- At constant speed on a horizontal road, the change in kinetic energy is zero. Hence the engine balances the opposing frictional force.
- The normal reaction is $N = mg$, the resistance in the given model is $f = \mu N$, and work over distance s is $W = fs$.

Step 1: Find the resisting force The bus moves on a plane road, so there is no change in gravitational potential energy. Its speed stays at $80\,\mathrm{km\,h^{-1}}$, so its initial and final kinetic energies are equal. The engine therefore supplies energy only to overcome the given resistance. Using $m = 500\,\mathrm{kg}$, $\mu = 0.04$, and $g = 9.8\,\mathrm{m\,s^{-2}}$,

$$N = 500 \times 9.8 = 4900\,\mathrm{N},$$

$$f = \mu mg = 0.04 \times 4900 = 196\,\mathrm{N}.$$

The forward engine force must have this same magnitude because the acceleration is zero.

Step 2: Calculate the work over the given distance Convert the distance before multiplying by the force:

$$s = 4\,\mathrm{km} = 4000\,\mathrm{m}.$$

The engine force acts along the displacement, so its work is positive:

$$W = fs = 196 \times 4000 = 784000\,\mathrm{J}.$$

Since $1\,\mathrm{kJ} = 1000\,\mathrm{J}$,

$$W = 784\,\mathrm{kJ}.$$

Friction does equal negative work, leaving the net work zero, consistently with the constant speed. The stated speed would be needed to calculate power or travel time; it does not enter the work for a specified distance.

Step 3: Conclusion The work done by the engine is $784\,\mathrm{kJ}$. The numerical entry is $\boxed{784}$.

Quick Tip to Solve: For steady motion over a specified distance on a level rough road, calculate μmgs; the speed is unnecessary unless power is asked for.

52. 64 identical drops each charged upto potential of $10\ \mathrm{mV}$ are combined to form a bigger drop. The potential of the bigger drop will be _____ mV. [4 marks]

Solution:

Concept:

- The potential of an isolated spherical conducting drop of radius r and charge q is $V = kq/r$, where $k = 1/(4\pi\varepsilon_0)$.
- When identical drops combine, both total charge and total liquid volume are conserved. Potential is not additive because the final radius changes.

Step 1: Find the radius and charge of the combined drop Let each small drop have radius r, charge q, and potential $V = 10\,\mathrm{mV}$. For $n = 64$ drops, conservation of volume gives

$$\frac{4\pi R^3}{3} = 64\frac{4\pi r^3}{3}, \qquad R = 64^{1/3}r = 4r.$$

Conservation of charge gives $Q = 64q$. The large drop therefore has sixty-four times the charge but only four times the radius. This distinction is essential: adding the initial potentials would ignore the changing geometry.

Step 2: Compare the final and initial potentials Apply the spherical-conductor potential formula to the final drop:

$$V_{\text{big}} = \frac{kQ}{R} = \frac{k(64q)}{4r} = 16\frac{kq}{r} = 16V.$$

Substituting the given potential in millivolts directly,

$$V_{\text{big}} = 16 \times 10 = 160\,\mathrm{mV}.$$

Equivalently, the general result for n identical drops is $V_{\text{big}} = n^{2/3}V$. The capacitance becomes four times larger while the charge becomes sixty-four times larger, giving the same factor of sixteen in potential. No conversion to volts is needed because the required answer is also in millivolts.

Step 3: Conclusion The potential of the bigger drop is $160\,\mathrm{mV}$. The numerical entry is $\boxed{160}$.

Quick Tip to Solve: For n identical charged drops, use $R = n^{1/3}r$ and $Q = nq$; hence the potential scales as $n^{2/3}$, not as n.

53. For a rolling spherical shell, the ratio of rotational kinetic energy and total kinetic energy is $\frac{x}{5}$. The value of x is _____. [4 marks]

Solution:

Concept:

- For pure rolling without slipping, the centre-of-mass speed and angular speed satisfy $v = \omega R$. Total kinetic energy is the sum of translation and rotation.
- A thin spherical shell has moment of inertia $I = \frac{2}{3}MR^2$ about a diameter. This differs from the moment of inertia of a solid sphere.

Step 1: Write the rotational and translational energies Let the mass and radius of the shell be M and R, and let its centre move at speed v. The translational energy is

$$K_{\text{tr}} = \frac{1}{2}Mv^2.$$

Using $I = \frac{2}{3}MR^2$ and the rolling relation $\omega = v/R$, the rotational energy about the centre is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\frac{v^2}{R^2} = \frac{1}{3}Mv^2.$$

The radius cancels because a smaller angular speed exactly compensates for the larger moment of inertia at fixed linear speed.

Step 2: Form the required fraction of total energy Add the two contributions rather than comparing rotation with translation alone:

$$K_{\text{total}} = \frac{1}{2}Mv^2 + \frac{1}{3}Mv^2 = \frac{5}{6}Mv^2.$$

Consequently,

$$\frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{\frac{1}{3}Mv^2}{\frac{5}{6}Mv^2} = \frac{2}{5}.$$

The question gives this ratio as $x/5$, so matching the numerators yields $x = 2$. As a check, the translational fraction is $3/5$, and the two fractions add to one. Using the solid-sphere inertia or placing only translational energy in the denominator would give a different, inappropriate ratio.

Step 3: Conclusion The rotational fraction is $2/5$, so $\boxed{x = 2}$. This numerical value is dimensionless.

Quick Tip to Solve: For a rolling body with $I = \beta MR^2$, the rotational fraction of its total kinetic energy is $\beta/(1+\beta)$; use $\beta = 2/3$ for a spherical shell.

---

54. The current flowing through a conductor connected across a source is 2 A and 1.2 A at $0°$C and $100°$C respectively. The current flowing through the conductor at $50°$C will be _____ $\times 10^2$ mA. [4 marks]

Solution:

Concept:

- For a metal conductor over the stated range, $R_T = R_0(1 + \alpha T)$, with T measured from $0°$C. Resistance varies linearly with temperature.
- With the same applied voltage V, Ohm's law gives $R = V/I$. It is resistance, or reciprocal current, that is linear in temperature, not current itself.

Step 1: Use the endpoint currents to compare resistances Let the fixed source voltage be V. At the two temperatures,

$$R_0 = \frac{V}{2}, \qquad R_{100} = \frac{V}{1.2}.$$

Their ratio eliminates the unknown voltage:

$$\frac{R_{100}}{R_0} = \frac{2}{1.2} = \frac{5}{3} = 1 + 100\alpha.$$

Hence $100\alpha = 2/3$ and $\alpha = 1/150\,^\circ\mathrm{C}^{-1}$. The positive coefficient is consistent with the current falling as the conductor heats up. The endpoint currents must not simply be averaged, because current is inversely proportional to resistance.

Step 2: Calculate the current at the midpoint temperature At $50^\circ\mathrm{C}$, the resistance is

$$R_{50} = R_0\left(1 + \frac{50}{150}\right) = \frac{4}{3}R_0.$$

Thus the current is

$$I_{50} = \frac{V}{R_{50}} = \frac{3}{4}\frac{V}{R_0} = \frac{3}{4} \times 2 = 1.5\,\mathrm{A}.$$

Converting to the form required in the question,

$$1.5\,\mathrm{A} = 1500\,\mathrm{mA} = 15 \times 10^2\,\mathrm{mA}.$$

Alternatively, the midpoint resistance is the arithmetic mean of the endpoint resistances, so the midpoint current is their harmonic mean, $2(2)(1.2)/(2 + 1.2) = 1.5\,\mathrm{A}$.

Step 3: Conclusion The current is $1.5\,\mathrm{A} = 15 \times 10^2\,\mathrm{mA}$. Therefore the numerical entry is $\boxed{15}$.

Quick Tip to Solve: At the midpoint temperature under a constant voltage, average resistances or take the harmonic mean of currents; averaging the currents would give the wrong value.

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55. Glycerin of density $1.25 \times 10^3\ \mathrm{kg\ m^{-3}}$ is flowing through the conical section of pipe The area of cross-section of the pipe at its ends are $10\ \mathrm{cm^2}$ and $5\ \mathrm{cm^2}$ and pressure drop across its length is $3\ \mathrm{Nm^{-2}}$. The rate of flow of glycerin through the pipe is $x \times 10^{-5}\ \mathrm{m^3\ s^{-1}}$. The value of x is _____. [4 marks]

Solution:

Concept:

- For steady incompressible flow, the volume flow rate is $Q = A_1 v_1 = A_2 v_2$, where A is the local cross-sectional area.
- Between the ends of a horizontal pipe, Bernoulli's equation gives $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$. The narrower end has the greater speed and lower pressure.

Step 1: Convert the areas and relate the speeds Take end 1 as the wider end. The given cross-sectional areas in SI units are

$$A_1 = 10 \times 10^{-4} = 1.0 \times 10^{-3}\,\mathrm{m^2},$$

$$A_2 = 5 \times 10^{-4}\,\mathrm{m^2}.$$

Since the same volume passes each section per second,

$$A_1 v_1 = A_2 v_2 \quad \Longrightarrow \quad v_2 = 2v_1.$$

The pressure drop is $P_1 - P_2 = 3\,\mathrm{N\,m^{-2}} = 3\,\mathrm{Pa}$. The density is $1.25 \times 10^3\,\mathrm{kg\,m^{-3}}$. Use the usual ideal-flow relation for the stated pressure and area data.

Step 2: Apply Bernoulli and calculate the volume flow rate At equal heights the gravitational terms cancel. Substituting the continuity relation into Bernoulli's equation gives

$$3 = \frac{1}{2}(1.25 \times 10^3)\big((2v_1)^2 - v_1^2\big) = 1875 v_1^2.$$

Therefore,

$$v_1^2 = \frac{3}{1875} = 1.6 \times 10^{-3}\,\mathrm{m^2\,s^{-2}}, \qquad v_1 = 0.040\,\mathrm{m\,s^{-1}}.$$

The volume flow rate follows using the wider area:

$$Q = A_1 v_1 = (1.0 \times 10^{-3})(0.040) = 4.0 \times 10^{-5}\,\mathrm{m^3\,s^{-1}}.$$

The narrower-end check gives $A_2 v_2 = (5 \times 10^{-4})(0.080)$, the same flow rate. Thus the requested coefficient is four.

Step 3: Conclusion The flow rate is $4 \times 10^{-5}\,\mathrm{m^3\,s^{-1}}$, giving $\boxed{x = 4}$.

Quick Tip to Solve: Use continuity before Bernoulli: halving the area doubles the speed, so the difference of squared speeds is $3v_1^2$. Convert square centimetres using 10^{-4}.

56. Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. The image of the distant object formed by the combination is at _____ cm from the first lens. [4 marks]

Solution:

Concept:

- A distant object sends parallel rays onto the first convex lens, which forms its image at its principal focus.
- For each thin lens use the Cartesian formula $1/f = 1/v - 1/u$. The image made by the first lens serves as the object for the second lens, with distances measured from that second lens.

Step 1: Find the intermediate image and its distance from the second lens Take light to travel from left to right and place the first lens at coordinate $x = 0$. Its focal length is $f_1 = +20\,\mathrm{cm}$. For the distant object, $u_1 \to -\infty$, so

$$\frac{1}{v_1} = \frac{1}{f_1}, \qquad v_1 = 20\,\mathrm{cm}.$$

The intermediate real image is therefore at $x = 20\,\mathrm{cm}$. The second lens is at $x = 60\,\mathrm{cm}$, so this point lies $60 - 20 = 40\,\mathrm{cm}$ to its left. Thus the second lens has a real object with $u_2 = -40\,\mathrm{cm}$.

Step 2: Locate the final image and refer it to the first lens For the second convex lens, $f_2 = +20\,\mathrm{cm}$.

Apply the lens equation with the object distance just found:

$$\frac{1}{20} = \frac{1}{v_2} - \frac{1}{-40} = \frac{1}{v_2} + \frac{1}{40}.$$

Hence,

$$\frac{1}{v_2} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}, \qquad v_2 = +40\,\text{cm}.$$

The final image lies forty centimetres beyond the second lens. The requested distance is from the first lens, so add the separation:

$$x_{\text{final}} = 60 + 40 = 100\,\text{cm}.$$

A useful check is that the intermediate object is at twice the second lens's focal length, so its image must also be at twice that focal length on the other side.

Step 3: Conclusion The final image is $100\,\text{cm}$ from the first lens, on the far side of the second lens. The numerical entry is $\boxed{100}$.

Quick Tip to Solve: After solving for the image distance from the second lens, translate back to the reference lens named in the question; here add the $60\,\text{cm}$ separation.

57. For a certain organ pipe, the first three resonance frequencies are in the ratio of $1 : 3 : 5$ respectively. If the frequency of fifth harmonic is 405 Hz and the speed of sound in air is $324\ \text{ms}^{-1}$ the length of the organ pipe is _____ m. [4 marks]

Solution:

Concept:

- An air column closed at one end supports only odd harmonics: $f_n = nv/(4L)$, where $n = 1, 3, 5, \ldots$, v is the speed of sound and L is the pipe length.
- The first three resonance frequencies of such a pipe are $f_1, 3f_1, 5f_1$. An open pipe instead supports consecutive harmonics with frequency ratio $1 : 2 : 3$.

Step 1: Identify the end condition The given resonance ratio $1 : 3 : 5$ shows that the even harmonics are absent. Therefore the organ pipe behaves as an air column closed at one end. At its closed end there is a displacement node, while its open end is a displacement antinode. This gives a fundamental wavelength equal to four times the pipe length.

Step 2: Find the fundamental frequency The fifth harmonic means five times the fundamental frequency; it does not mean the fifth allowed resonance. Thus

$$5f_1 = 405\,\text{Hz}, \qquad f_1 = 81\,\text{Hz}.$$

The third allowed resonance is the fifth harmonic, consistently with the three frequencies listed in the question.

Step 3: Calculate the length Use the closed-pipe fundamental relation and the stated speed of sound:

$$L = \frac{v}{4f_1} = \frac{324}{4 \times 81}\,\mathrm{m} = 1\,\mathrm{m}.$$

Equivalently, direct substitution in $405 = 5(324)/(4L)$ gives the same result. The units are metres because speed divided by frequency has dimensions of length.

Step 4: Conclusion The length of the organ pipe is $\boxed{1\,\mathrm{m}}$. The numerical entry is $\boxed{1}$.

Quick Tip to Solve: Identify the pipe from its allowed frequency ratio before substituting. In a closed pipe, the fifth harmonic is the third resonance, not the fifth resonance.

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58. A common example of alpha decay is $^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}_2\mathrm{He}^4 + \mathrm{Q}$

Given :

$^{238}_{92}\mathrm{U} = 238.05060$ u**,**

$^{234}_{90}\mathrm{Th} = 234.04360$ u**,**

$^{4}_{2}\mathrm{He} = 4.00260$ u **and**

$$1\mathrm{u} = 931.5\frac{\mathrm{MeV}}{c^2}$$

The energy released (Q) **during the alpha decay of** $^{238}_{92}\mathrm{U}$ **is _____ MeV** [4 marks]

Solution:

Concept:

- The energy released in a nuclear decay is $Q = (m_{\text{initial}} - m_{\text{final}})c^2$. A positive mass decrease means that energy is available to the decay products.
- The conversion $1\,\mathrm{u}\,c^2 = 931.5\,\mathrm{MeV}$ converts an atomic-mass-unit difference directly into energy.

Step 1: Add the masses of the products The daughter thorium atom and the helium atom together account for all products whose rest masses are specified. Their sum is

$$m_{\text{final}} = 234.04360 + 4.00260 = 238.04620\,\mathrm{u}.$$

The atomic numbers satisfy $92 = 90 + 2$, so electron rest masses cancel when these consistent atomic masses are used. No extra electron mass should be subtracted.

Step 2: Find the mass converted into energy Subtract the combined product mass from the uranium mass, retaining all given decimal places:

$$\Delta m = 238.05060 - 238.04620 = 0.00440\,\mathrm{u}.$$

The result is positive, so the initial state has the greater rest energy and the decay releases energy. Adding the masses instead, or reversing the subtraction, would not give the released energy.

Step 3: Convert the mass decrease into MeV Multiply by the given energy equivalent of one atomic

mass unit:

$$Q = 0.00440 \times 931.5\,\text{MeV} = 4.0986\,\text{MeV}.$$

This is the total released energy, shared principally between the alpha particle and the recoiling daughter; it is not necessary to calculate their separate kinetic energies. For the numerical integer response, this value rounds to 4.

Step 4: Conclusion The energy release is $Q = 4.0986\,\text{MeV} \approx \boxed{4\,\text{MeV}}$; the numerical entry is $\boxed{4}$.

Quick Tip to Solve: First add both product masses, then subtract from the parent. Keep the mass difference unrounded until after multiplication by 931.5.

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59. A conducting circular loop is placed in a uniform magnetic field of 0.4 T with its plane perpendicular to the field. Somehow, the radius of the loop starts expanding at a constant rate of 1 mm/s. The magnitude of induced emf in the loop at an instant when the radius of the loop is 2 cm will be _____ μV. [4 marks]

Solution:

Concept:

- Faraday's law gives the magnitude of induced emf as $|\mathcal{E}| = |d\Phi_B/dt|$. The flux is $\Phi_B = BA\cos\theta$, with θ measured from the area normal.
- For a circular loop of changing radius, $A = \pi r^2$ and $dA/dt = 2\pi r\, dr/dt$. The magnetic field here is uniform and constant.

Step 1: Express the magnetic flux The plane of the loop is perpendicular to the magnetic field, so the area normal is parallel to the field. Hence $\theta = 0$, not 90°, in the flux formula. The flux magnitude is therefore

$$\Phi_B = B\pi r^2.$$

Although the magnetic field does not change, the expanding area increases the flux, and that change produces the emf.

Step 2: Differentiate before substituting Apply the chain rule to the radius-dependent area:

$$|\mathcal{E}| = B\frac{d(\pi r^2)}{dt} = 2\pi B r\frac{dr}{dt}.$$

Convert the two length quantities independently:

$$r = 2\,\text{cm} = 2 \times 10^{-2}\,\text{m}, \qquad \frac{dr}{dt} = 1\,\text{mm}\,\text{s}^{-1} = 10^{-3}\,\text{m}\,\text{s}^{-1}.$$

The factor of two arises from differentiating the square of the radius.

Step 3: Evaluate and convert to microvolts Substitute the instantaneous radius and constant expansion rate:

$$|\mathcal{E}| = 2\pi(0.4)(2 \times 10^{-2})(10^{-3})\,\text{V} = 1.6\pi \times 10^{-5}\,\text{V}.$$

Since $1\,\mu\text{V} = 10^{-6}\,\text{V}$, this is $16\pi\,\mu\text{V} \approx 50.27\,\mu\text{V}$. The direction would oppose the increasing flux by

Lenz's law, but the question asks only for magnitude.

Step 4: Conclusion The induced emf is $16\pi\ \mu\text{V} \approx \boxed{50\ \mu\text{V}}$ to the nearest integer. The numerical entry is $\boxed{50}$.

Quick Tip to Solve: For an expanding loop use $dA/dt = 2\pi r\dot{r}$. Convert centimetres and millimetres to metres before evaluating the emf in volts.

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60. A compass needle oscillates 20 times per minute at a place where the dip is 30° and 30 times per minute where the dip is 60°. The ratio of total magnetic field due to the earth at two places respectively is $\frac{4}{\sqrt{x}}$. The value of x is [4 marks]

Solution:

Concept:

- A compass needle oscillating in the horizontal plane has $f = (2\pi)^{-1}\sqrt{mB_H/I}$, where m is its magnetic moment, I its moment of inertia and B_H the horizontal field.
- The horizontal component of Earth's total field is $B_H = B\cos\delta$, where δ is the angle of dip. For the same needle, $B_H \propto f^2$.

Step 1: Relate the horizontal fields to the frequencies Label the first and second places by 1 and 2. The same needle is used, so its magnetic moment and moment of inertia cancel in the frequency ratio. Since both counts refer to one minute, there is no need to convert them separately to hertz:

$$\frac{B_{H1}}{B_{H2}} = \left(\frac{f_1}{f_2}\right)^2 = \left(\frac{20}{30}\right)^2 = \frac{4}{9}.$$

A greater oscillation frequency indicates a greater horizontal restoring field.

Step 2: Recover the ratio of total fields The requested ratio concerns the total field, rather than its horizontal component. Use the two different dip angles explicitly:

$$\frac{B_1}{B_2} = \frac{B_{H1}}{B_{H2}}\frac{\cos 60^\circ}{\cos 30^\circ} = \frac{4}{9}\frac{1/2}{\sqrt{3}/2} = \frac{4}{9\sqrt{3}}.$$

Taking only the square of the frequency ratio would miss this dip correction.

Step 3: Compare the specified form The question writes the same ratio as $4/\sqrt{x}$. Therefore

$$\sqrt{x} = 9\sqrt{3}, \qquad x = (9\sqrt{3})^2 = 81 \times 3 = 243.$$

Both x and the field ratio are dimensionless. The ordering of the places has been kept unchanged throughout, so no reciprocal is required.

Step 4: Conclusion The value is $\boxed{x = 243}$, a dimensionless number.

Quick Tip to Solve: Compass oscillations measure the horizontal field. Square the frequency ratio first, then divide each horizontal field by the cosine of its own dip angle.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Given below are two statements :

Statement I : Boron is extremely hard indicating its high lattice energy.

Statement II : Boron has highest melting and boiling point compared to its other group members.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Statement I is incorrect but Statement II is correct

(C) Statement I is correct but Statement II is incorrect

(D) Both statement I and Statement II are correct

Correct Answer: (D) Both statement I and Statement II are correct

Solution:

Concept:

- Boron forms an extended covalently bonded solid rather than the simple metallic structures characteristic of the heavier Group 13 elements. Strong bonding throughout its lattice accounts for its hardness.
- A large energy requirement for disrupting the solid network is reflected in very high melting and boiling temperatures. These physical properties must be compared within the stated group.

Step 1: Evaluate Statement I Boron is an extremely hard solid. Its atoms are held in a strongly bonded lattice, so substantial energy is required to disrupt the structure. The statement uses high lattice energy in the sense of strong cohesion in the solid, not as a claim that elemental boron is an ionic crystal. Thus the physical property and the inference about the strong lattice are consistent, and Statement I is correct.

Step 2: Evaluate Statement II Among the Group 13 members considered here, boron has the highest melting point and the highest boiling point. Its strong covalent framework requires a particularly large energy input to overcome the solid-state bonding. The heavier members are metallic and do not retain this same strongly linked boron network. Therefore Statement II is also correct; the unusual behaviour of boron does not make either comparison false.

Step 3: Choose the matching combination The two statements are assessed independently before selecting an option. Option (A) declares both incorrect, option (B) rejects the first and option (C) rejects the second; each contradicts at least one of the evaluations above. Option (D) alone retains both correct statements. No assertion about Statement II being the unique explanation of Statement I is required by these options.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (D).

Quick Tip to Solve: For boron, connect hardness and high thermal transition temperatures to its covalent network. Do not treat the word lattice as implying an ionic solid in every context.

62. Match List I with List II

	LIST IComplex		LIST IICFSE (Δ_0)
A.	$[Cu(NH_3)_6]^{2+}$	**I.**	-0.6
B.	$[Ti(H_2O)_6]^{3+}$	**II.**	-2.0
C.	$[Fe(CN)_6]^{3-}$	**III.**	-1.2
D.	$[NiF_6]^{4-}$	**IV.**	-0.4

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-IV, C-II, D-III

(B) A-III, B-IV, C-I, D-II

(C) A-II, B-III, C-I, D-IV

(D) A-I, B-II, C-IV, D-III

Correct Answer: (A) A-I, B-IV, C-II, D-III

Solution:

Concept:

- In an octahedral field each t_{2g} electron contributes $-0.4\Delta_0$, while each e_g electron contributes $+0.6\Delta_0$. Thus $\text{CFSE} = (-0.4n_t + 0.6n_e)\Delta_0$.
- Find the metal oxidation state and then the d-electron count. Strong-field cyanide gives low-spin iron(III); the stated CFSE values describe orbital stabilization, without an additional pairing-energy term.

Step 1: Calculate the copper and titanium entries In $[Cu(NH_3)_6]^{2+}$, ammonia is neutral, so copper is $+2$ and has d^9. The occupancy is $t_{2g}^6 e_g^3$, giving

$$\text{CFSE}_A = [6(-0.4) + 3(0.6)]\Delta_0 = -0.6\Delta_0.$$

Thus A matches I. In $[Ti(H_2O)_6]^{3+}$, titanium is $+3$, or d^1, and its single electron occupies t_{2g}. Hence $\text{CFSE}_B = -0.4\Delta_0$, so B matches IV.

Step 2: Calculate the iron and nickel entries For $[Fe(CN)_6]^{3-}$, six cyanide ions require iron to be $+3$, or d^5. Cyanide is strong field, giving low-spin $t_{2g}^5 e_g^0$ and $\text{CFSE}_C = 5(-0.4)\Delta_0 = -2.0\Delta_0$, so C matches II. For $[NiF_6]^{4-}$, nickel is $+2$, or d^8. Its octahedral occupancy is $t_{2g}^6 e_g^2$, giving

$$\text{CFSE}_D = [6(-0.4) + 2(0.6)]\Delta_0 = -1.2\Delta_0.$$

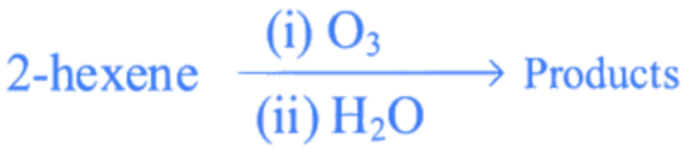
Therefore D matches III.

Step 3: Match all four results The complete sequence is A-I, B-IV, C-II, D-III. Option (B) incorrectly gives copper $-1.2\Delta_0$; option (C) incorrectly gives titanium $-1.2\Delta_0$; option (D) incorrectly gives titanium $-2.0\Delta_0$. These alternatives already fail before considering their other mismatches. Option (A) agrees with all four independently calculated entries.

Step 4: Conclusion The matching is $\boxed{\text{A} - \text{I},\ \text{B} - \text{IV},\ \text{C} - \text{II},\ \text{D} - \text{III}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use oxidation state before counting d electrons. For low-spin d^5 cyanide all five electrons occupy t_{2g}; for octahedral d^8, the occupancy is always $t_{2g}^6 e_g^2$.

63. The two products formed in above reaction are - [4 marks]



(A) Butanal and acetaldehyde

(B) Butanoic acid and acetic acid

(C) Butanal and acetic acid

(D) Butanoic acid and acetaldehyde

Correct Answer: (B) Butanoic acid and acetic acid

Solution:

Concept:

- Ozonolysis cleaves the carbon-carbon double bond, converting each alkene carbon into a carbonyl carbon. The carbon skeleton on each side of the original double bond determines the product chain length.
- With ozone followed by water and no reductive reagent, the aldehydes initially formed are oxidized to carboxylic acids by the oxidizing species generated in the work-up. A reductive work-up such as zinc would preserve the aldehydes.

Step 1: Locate the double bond and divide the skeleton Write 2-hexene in condensed form as $CH_3CH{=}CHCH_2CH_2CH_3$. Cleave between carbons 2 and 3. The left alkene carbon has a methyl group and hydrogen; the right has a propyl group and hydrogen. The first carbonyl fragments therefore have two and four carbons, respectively. No carbon is lost in this cleavage, so the product carbon counts must add to six.

Step 2: Apply the stated work-up The reaction shows O_3, followed by H_2O, without zinc or another reducing agent. Cleavage initially corresponds to ethanal CH_3CHO and butanal $CH_3CH_2CH_2CHO$. Under the oxidative conditions, each aldehydic group becomes a carboxylic acid group. Consequently the final products are CH_3COOH and $CH_3CH_2CH_2COOH$, namely acetic acid and butanoic acid.

Step 3: Exclude the aldehyde-containing choices Option (A) would describe preservation of both aldehydes under a reductive work-up. Options (C) and (D) leave only one of the aldehydes oxidized, although both cleavage sites produce aldehydes susceptible to the stated conditions. There is no reagent in the scheme that selectively preserves just one of them. Thus the acid-acid combination, option (B), is the consistent final product pair.

Step 4: Conclusion The products are butanoic acid and acetic acid. Hence, the correct option is (B).

Quick Tip to Solve: After cleaving an alkene, check the work-up. Ozone followed by water without a reducing agent gives acids from aldehyde fragments; ozone followed by zinc preserves the aldehydes.

64. The bond order and magnetic property of acetylide ion are same as that of [4 marks]

(A) O_2^+

(B) O_2^-

(C) N_2^+

(D) NO^+

Correct Answer: (D) NO^+

Solution:

Concept:

- Acetylide is ${C_2}^{2-}$. Adding two electrons to neutral C_2 gives ten valence electrons, the same number as N_2.
- Molecular-orbital bond order is $\text{BO} = (N_b - N_a)/2$, where N_b and N_a count bonding and antibonding electrons. A species is diamagnetic when all its electrons are paired.

Step 1: Count the electrons in acetylide Each carbon contributes four valence electrons and the dianionic charge contributes two more. Thus the valence-electron count is $2(4) + 2 = 10$. For the ordering appropriate to this species, the occupied valence orbitals are $(\sigma 2s)^2(\sigma^* 2s)^2(\pi 2p_x)^2(\pi 2p_y)^2(\sigma 2p_z)^2$. All these occupied orbitals contain electron pairs.

Step 2: Compute its bond order There are eight bonding and two antibonding valence electrons. Therefore

$$\text{BO}({C_2}^{2-}) = \frac{8-2}{2} = 3.$$

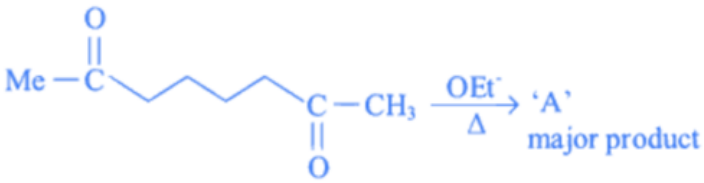
The paired occupation makes the ion diamagnetic. Filled inner-shell bonding and antibonding orbitals cancel in this calculation.

Step 3: Compare every option NO^+ has $5+6-1 = 10$ valence electrons, so it is isoelectronic with N_2: it has bond order 3 and no unpaired electron. In contrast, ${O_2}^+$ has bond order 2.5 and one unpaired electron, ${O_2}^-$ has bond order 1.5 and one unpaired electron, and ${N_2}^+$ has bond order 2.5 and one unpaired electron. None of these three matches either required combination.

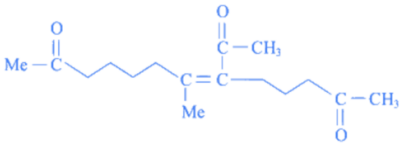
Step 4: Conclusion Acetylide and NO^+ both have bond order 3 and are diamagnetic. Hence, the correct option is (D).

Quick Tip to Solve: Count valence electrons including the ionic charge first: ten-electron diatomic species commonly match the triple-bonded, paired-electron N_2 pattern.

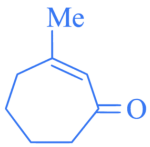
65. 'A' in the above reaction is: [4 marks]



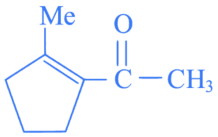
(A) Acyclic condensation product: $CH_3CO(CH_2)_4C(CH_3)=C(COCH_3)(CH_2)_3COCH_3$.



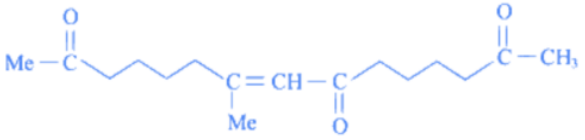
(B) 3-Methylcyclohept-2-en-1-one.



(C) 1-(2-Methylcyclopent-1-en-1-yl)ethanone.

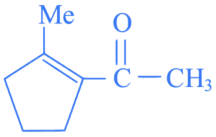


(D) Acyclic condensation product: $CH_3CO(CH_2)_4C(CH_3)=CHCO(CH_2)_4COCH_3$.



Correct Answer: (C)

1-(2-Methylcyclopent-1-en-1-yl)ethanone.



Solution:

Concept:

- A diketone with alpha hydrogens forms an enolate in ethoxide base. Intramolecular attack on its other carbonyl group can produce a cyclic aldol addition product.
- Five- and six-membered rings are generally favoured. Heating dehydrates a beta-hydroxy carbonyl compound to a conjugated alpha,beta-unsaturated carbonyl compound.

Step 1: Identify the competing enolate sites The substrate is $CH_3CO(CH_2)_4COCH_3$, an eight-carbon diketone. Its terminal methyl groups and its chain methylene groups adjacent to a carbonyl have alpha hydrogens. Removing a chain alpha hydrogen gives an enolate that can attack the other ketone within the same molecule; the symmetry means either end leads to the same product.

Step 2: Count the ring atoms Number the eight-carbon chain consecutively, starting at a terminal methyl group. The carbonyl carbons are C2 and C7, while C3 to C6 are methylene groups. Attack of

the enolate carbon C3 on carbonyl carbon C7 creates the C3–C7 bond. The ring is C3, C4, C5, C6, C7: five atoms. C3 retains the acetyl substituent C2–C1, while C7 retains its methyl substituent C8. Protonation initially gives a cyclic beta-hydroxy ketone.

Step 3: Dehydrate and select the major product Under heating, loss of water establishes a double bond between C3 and C7, conjugated with the retained acetyl carbonyl. The result is 1-(2-methylcyclopent-1-en-1-yl)ethanone, the five-membered enone shown in (C). Option (B) instead requires closure from a terminal methyl enolate, giving a less favourable seven-membered ring. Options (A) and (D) are intermolecular dimeric condensation products; the favourable intramolecular five-membered closure is preferred.

Step 4: Conclusion The major product is the acetyl-substituted methylcyclopentene shown in (C), formed by intramolecular aldol condensation followed by dehydration. Hence, the correct option is (C).

Quick Tip to Solve: For a tethered diketone, draw the proposed new enolate-carbonyl bond and count every ring atom before choosing between a cyclic and a dimeric aldol product.

---

66. Given below are two statement: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: $5\mathrm{f}$ **electrons can participate in bonding to a far greater extent than** $4\mathrm{f}$ **electrons**

Reason R: $5\mathrm{f}$ **orbitals are not as buried as** $4\mathrm{f}$ **orbitals**

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Correct Answer: (A) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Bond formation requires suitable orbital energies and spatial overlap. More radially available valence orbitals can overlap with ligand orbitals more effectively.
- The $4f$ orbitals of lanthanoids are comparatively core-like and strongly shielded by outer shells; the $5f$ orbitals of actinoids, particularly early members, are more spatially available for bonding.

Step 1: Evaluate the assertion The assertion compares participation of the two f subshells, not simply their occupancy. Lanthanoid $4f$ electrons are largely localized near the core and contribute little direct covalent overlap. Actinoid $5f$ orbitals are more extended and can mix with ligand orbitals to a greater extent. The assertion that $5f$ electrons participate more extensively in bonding is therefore true.

Step 2: Evaluate the reason The reason states that $5f$ orbitals are not as buried as $4f$ orbitals. This is the standard comparison: the actinoid $5f$ orbitals are less effectively hidden inside filled outer shells and are more accessible to approaching ligands. Thus the reason is true. This does not mean every $5f$

electron in every actinoid compound must form a covalent bond; the statement concerns their greater general availability.

Step 3: Test the explanatory link Less burial permits greater spatial overlap between the metal $5f$ orbitals and ligand orbitals. Greater overlap directly accounts for their increased bonding participation, so the reason explains the assertion. Option (B) incorrectly denies this link. Option (C) incorrectly calls the reason false, while option (D) incorrectly calls the assertion false.

Step 4: Conclusion Both the assertion and the reason are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

Quick Tip to Solve: In an Assertion–Reason item, check the two truth values separately, then ask whether the reason supplies the physical mechanism behind the assertion.

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67. A metal chloride contains 55.0% of chlorine by weight . $100\ \mathrm{mL}$ vapours of the metal chloride at STP weigh $0.57\ \mathrm{g}$. The molecular formula of the metal chloride is

(Given: Atomic mass of chlorine is $35.5\mathrm{u}$) [4 marks]

(A) MCl_2

(B) MCl_4

(C) MCl_3

(D) MCl

Correct Answer: (A) MCl_2

Solution:

Concept:

- At STP one mole of an ideal gas occupies approximately $22.4\,\mathrm{L}$. Hence molar mass equals gas density multiplied by molar volume.
- If the formula is MCl_x, one mole contains $35.5x$ grams of chlorine. Its chlorine mass fraction is therefore $35.5x/M_r$, with M_r the numerical molar mass.

Step 1: Calculate the molar mass The stated volume is $100\,\mathrm{mL} = 0.100\,\mathrm{L}$, and its mass is $0.57\,\mathrm{g}$. Therefore the vapour density at STP is

$$\rho = \frac{0.57}{0.100} = 5.7\,\mathrm{g\,L^{-1}}.$$

Multiplication by the molar volume gives

$$M = 5.7 \times 22.4 = 127.68\,\mathrm{g\,mol^{-1}}.$$

This is the mass of one mole of complete chloride molecules, rather than the atomic mass of the metal.

Step 2: Find the number of chlorine atoms Chlorine accounts for 55.0 percent of the molecular mass, so its contribution per mole of chloride is

$$0.550 \times 127.68 = 70.224\,\mathrm{g\,mol^{-1}}.$$

Each chlorine atom contributes $35.5\,\mathrm{g\,mol^{-1}}$. Consequently

$$x = \frac{70.224}{35.5} = 1.978 \simeq 2.$$

The small departure from an integer reflects the rounded experimental volume, mass and percentage data.

Step 3: Check the alternative formulae Two chlorine atoms contribute about $71\,\mathrm{g}$ per mole, consistent with the measured chlorine fraction. One chlorine would contribute only $35.5\,\mathrm{g}$, while three and four would contribute 106.5 and $142\,\mathrm{g}$, respectively. These do not match the approximately $70.2\,\mathrm{g}$ of chlorine required per mole, so the other three formulae are excluded.

Step 4: Conclusion There are two chlorine atoms in each molecule, so the formula is $\mathrm{MCl_2}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the molar mass before using the percentage: chlorine atoms per molecule equal the chlorine mass fraction times molar mass divided by 35.5.

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68. Correct statements for the given reaction are :

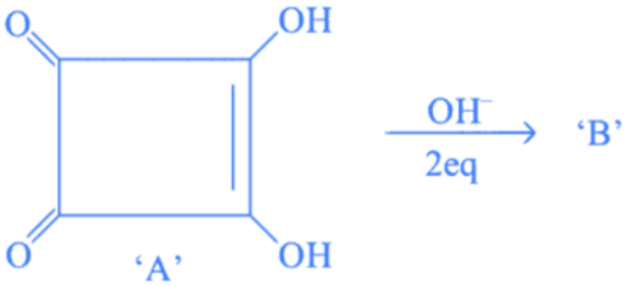
A. Compound 'B' is aromatic

B. The completion of above reaction is very slow

C. 'A' shows tautomerism

D. The bond lengths of C-C in compound B are found to be same

Choose the correct answer from the options given below: [4 marks]



(A) B, C and D only

(B) A, C and D only

(C) A, B and D only

(D) A, B and C only

Correct Answer: (B) A, C and D only

Solution:

Concept:

- The depicted compound is squaric acid, a cyclic conjugated dihydroxy dione. Two equivalents of hydroxide remove its two acidic hydroxyl protons, producing the squarate dianion.

- A planar cyclic conjugated system with $4n + 2$ delocalized pi electrons is aromatic. Resonance can make symmetry-related carbon–carbon bonds equivalent.

Step 1: Identify the product and its electron system Compound A reacts with two equivalents of hydroxide by acid–base neutralization:

$$H_2C_4O_4 + 2\,OH^- \longrightarrow C_4O_4{}^{2-} + 2\,H_2O.$$

The product B is the squarate dianion. The four ring carbons remain conjugated, and resonance spreads negative charge over the oxygen-bearing framework. Its aromatic description involves six delocalized pi electrons, satisfying $4n + 2$ with $n = 1$. Statement A is therefore correct.

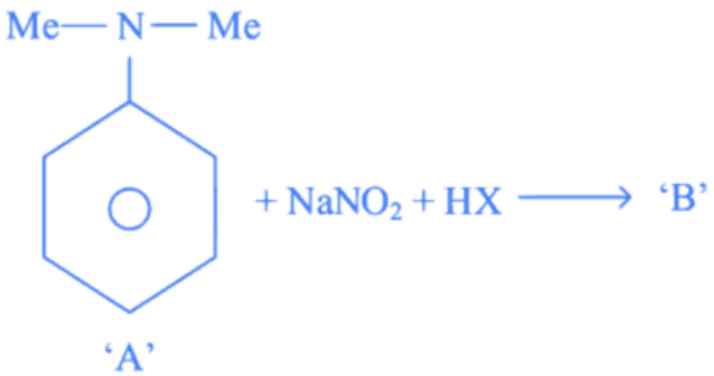
Step 2: Judge the rate and tautomerism statements The formation of B is chiefly proton transfer from acidic hydroxyl groups to hydroxide. Such neutralization is rapid; no difficult skeletal rearrangement is required. Thus the claim in statement B that completion is very slow is false. Compound A contains transferable hydroxyl hydrogens adjacent to the conjugated carbonyl system. Proton transfer accompanied by movement of pi bonds gives keto–enol tautomeric forms, so statement C is true.

Step 3: Use symmetry to assess the ring bonds The dianion has resonance contributors related by the symmetry of its four-membered carbon framework. The localized single and double bonds of any one contributor do not represent the actual separate bond lengths. Delocalization makes the four ring C–C bonds equivalent, so statement D is true. Thus the required set is A, C and D. Options (A), (C) and (D) all include the false slow-reaction statement B and are rejected.

Step 4: Conclusion Statements A, C and D are correct; statement B is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the rate of acid–base deprotonation from structural tautomerism, and use the symmetry of a delocalized ion to judge bond-length equality.

69. The incorrect statement regarding the reaction given below is [4 marks]



(A) 'B' is N-nitroso ammonium compound

(B) The product 'B' formed in the above reaction is p-nitroso compound at low temperature

(C) The electrophile involved in the reaction is NO^+

(D) The reaction occurs at low temperature

Correct Answer: (A) 'B' is N-nitroso ammonium compound

Solution:

Concept:

- Acidified nitrite forms the nitrosonium electrophile NO^+. An electron-rich aromatic ring can undergo electrophilic substitution by this species.
- The dimethylamino group in N,N-dimethylaniline activates its ring and directs substitution mainly to the para position when that position is free. A tertiary nitrogen has no N–H bond.

Step 1: Identify the amine and electrophile The given substrate is N,N-dimethylaniline, $C_6H_5N(CH_3)_2$, a tertiary aromatic amine. Its nitrogen is attached to two methyl groups and a phenyl group. Acidification of $NaNO_2$ generates nitrous acid, from which NO^+ is formed under acidic conditions. Thus option (C), which identifies the electrophile, is a correct statement and cannot be the requested incorrect one.

Step 2: Locate the nitroso group in the product The amino-substituted aromatic ring reacts with NO^+ by electrophilic aromatic substitution. Attack occurs preferentially at the unoccupied para position; loss of a ring proton restores aromaticity. The isolated product is para-nitroso-N,N-dimethylaniline, with the $-N{=}O$ group bonded to a ring carbon. It is therefore a C-nitroso compound, agreeing with option (B). The usual reaction is performed cold, so the low-temperature statement in option (D) is also correct.

Step 3: Reject the nitrogen-bound product description A stable N-nitrosamine is characteristically obtained by nitrosation of a secondary amine, which has an N–H bond. The present tertiary aromatic amine instead gives ring nitrosation as its product. Calling B an N-nitroso ammonium compound incorrectly places the nitroso group on the amine nitrogen rather than on the para carbon of the benzene ring. Hence option (A) is the only incorrect description among the four.

Step 4: Conclusion The incorrect statement is that B is an N-nitroso ammonium compound; B is the para-C-nitroso aromatic product. Hence, the correct option is (A).

Quick Tip to Solve: First classify the amine: a tertiary aromatic amine with a free para position typically gives ring nitrosation, while secondary amines give N-nitrosamines.

70. For lead storage battery pick the correct statements

A. During charging of battery, $PbSO_4$ on anode is converted into PbO_2

B. During charging of battery, $PbSO_4$ on cathode is converted into PbO_2

C. Lead storage battery consists of grid of lead packed with PbO_2 as anode

D. Lead storage battery has $\sim 38\%$ solution of sulphuric acid as an electrolyte

Choose the correct answer from the options given below: [4 marks]

(A) A, B, D only

(B) B, C only

(C) B, C, D only

(D) B, D only

Correct Answer: (D) B, D only

Solution:

Concept:

- During discharge, the negative plate is lead (the anode) and the positive plate is lead dioxide (the cathode). Both plates become coated with $PbSO_4$.
- Charging reverses the discharge reactions. In the customary battery description, the names anode plate and cathode plate refer to their discharge roles even while describing their regeneration on charging.

Step 1: Recall the discharge electrodes The discharge half-reactions are

$$Pb + {SO_4}^{2-} \longrightarrow PbSO_4 + 2\,e^-$$

at the lead anode, and

$$PbO_2 + {SO_4}^{2-} + 4\,H^+ + 2\,e^- \longrightarrow PbSO_4 + 2\,H_2O$$

at the lead dioxide cathode. Thus the plate originally containing lead dioxide is the cathode plate, not the anode plate. Statement C reverses this construction and is false.

Step 2: Reverse the reactions on charging At the original lead plate, charging reduces $PbSO_4$ back to Pb. At the original lead dioxide plate, charging oxidizes $PbSO_4$ back to PbO_2. With plate names retained from discharge, statement B correctly describes regeneration of the cathode plate, while statement A incorrectly assigns formation of lead dioxide to the original anode plate. Strict instantaneous electrochemical roles reverse during charging: the positive plate is then the oxidation electrode. Keeping the battery plate naming convention explicit avoids confusing these two usages.

Step 3: Check the electrolyte and combinations The electrolyte is an aqueous sulphuric acid solution of about 38 percent concentration, so statement D is true. The correct statements are therefore B and D only. Option (A) includes the false statement A. Options (B) and (C) include the false statement C, which wrongly puts lead dioxide at the discharge anode. Only option (D) retains both correct statements and excludes both incorrect ones.

Step 4: Conclusion Using the battery electrodes named by their discharge roles, B and D alone are correct. Hence, the correct option is (D).

Quick Tip to Solve: Write the discharge reactions first, then reverse them for charging; keep track of whether a question names a physical battery plate or its instantaneous oxidation/reduction role.

71. Given below are two statements :

Statement I : $SbCl_5$ **is more covalent than** $SbCl_3$

Statement II: The higher oxides of halogens also tend to be more stable than the lower ones.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (A) Both statement I and Statement II are correct

Solution:

Concept:

- Fajans' rule relates covalent character to polarization: a cation with greater charge and smaller effective size distorts the anion electron cloud more strongly.
- The stability of a halogen oxide depends on its bonding and oxidation state. Within the usual halogen-oxide trend, higher oxides tend to be more stable than lower oxides.

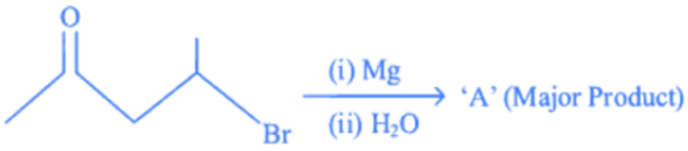
Step 1: Evaluate the antimony chlorides In $SbCl_5$, antimony has oxidation state $+5$, compared with $+3$ in $SbCl_3$. The higher oxidation state increases the polarizing power of the antimony centre toward the chloride ligands. Greater distortion of the chloride electron cloud gives greater sharing of electron density and hence greater covalent character. Thus Statement I is correct. This comparison is about relative bonding character; it does not mean that either chloride must be treated as a completely ionic salt.

Step 2: Evaluate the halogen-oxide trend Statement II states the standard trend that higher oxides of halogens tend to be more stable than their lower oxides. The wording is a tendency, rather than a claim that every oxide of every halogen is stable under all conditions. For example, the higher chlorine oxides illustrate the increased stability associated with the higher oxidation states. Hence Statement II is also correct. Options (B) and (D) wrongly reject Statement I, while option (C) wrongly rejects Statement II.

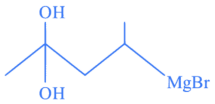
Step 3: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

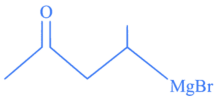
Quick Tip to Solve: For two statements, test the bonding argument and the stability trend independently; do not infer the truth of one from the other.

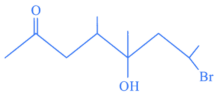
72. In the following reaction, 'A' is [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- Magnesium inserts into a carbon-bromine bond to produce a Grignard reagent, whose carbon bonded to magnesium is nucleophilic.
- A Grignard carbon adds to the electrophilic carbon of a ketone. When both groups belong to the same molecule, a new intramolecular carbon-carbon bond closes a ring; water subsequently protonates the alkoxide.

Step 1: Locate the nucleophilic carbon The starting material is $CH_3COCH_2CH(Br)CH_3$. In the first operation magnesium changes the $CH(Br)$ group into $CH(MgBr)$. Label the carbonyl carbon C1, the intervening methylene carbon C2, and the magnesium-bearing carbon C3. The existing chain already contains the bonds C1–C2 and C2–C3.

Step 2: Close the ring and protonate the oxygen C3 attacks C1, and the carbonyl pi electrons move onto oxygen. The new C3–C1 bond therefore completes a three-membered ring, rather than adding any carbon atoms to the molecule. The carbonyl carbon retains its methyl substituent and becomes an alkoxide carbon; C3 retains the other methyl substituent. The second operation, treatment with water, converts the magnesium alkoxide into an alcohol. Thus the product is 1,2-dimethylcyclopropan-1-ol, exactly the structure drawn in option (A).

Step 3: Reject the incomplete and intermolecular products Option (B) has a geminal diol and still contains a carbon-magnesium bond; it is not the hydrolysed intramolecular addition product. Option (C) is only the Grignard intermediate, with its ketone still present, so it ignores both ring closure and aqueous work-up. Option (D) joins two starting skeletons and retains bromine, whereas the printed sequence gives the intramolecular cyclic alcohol represented by (A).

Step 4: Conclusion The major product is 1,2-dimethylcyclopropan-1-ol. Hence, the correct option is

(A). The listed key selects the uncyclised Grignard intermediate, which is not the product after the two printed operations; the reaction therefore establishes (A).

Quick Tip to Solve: Count the atoms along the existing path between the nucleophilic carbon and carbonyl carbon: closing their new bond here gives a three-membered ring. Apply the water work-up before selecting the product.

73. The major product 'P' formed in the following sequence of reactions is

$$C_6H_5-CH=CH-CH_2-COOH \xrightarrow[\text{(ii) } RNH_2]{\text{(i) } SOCl_2} \xrightarrow[\text{(iv) } H_3O^+]{\text{(iii) } LiAlH_4} P.$$

[4 marks]

(A) $C_6H_5-CH=CH-CH_2-CH(OH)-NHR$

(B) $C_6H_5-CH(NHR)-CH(Cl)-CH_2-CH_2OH$

(C) $C_6H_5-CH_2-CH_2-CH_2-C(=O)-NHR$

(D) $C_6H_5-CH=CH-CH_2-CH_2-NHR$

Correct Answer: (D) $C_6H_5-CH=CH-CH_2-CH_2-NHR$

Solution:

Concept:

- Thionyl chloride converts a carboxylic acid to its acyl chloride; a primary amine then gives an N-substituted amide.
- Lithium aluminium hydride reduces the amide carbonyl group to a methylene group, retaining the carbon chain and the carbon-nitrogen bond.

Step 1: Form the amide intermediate Write C_6H_5 for the phenyl group. The starting acid is $C_6H_5-CH=CH-CH_2-COOH$. Thionyl chloride replaces the carboxylic hydroxyl group with chlorine, without changing the carbon-carbon double bond. Reaction with RNH_2 replaces the acyl chloride chlorine with NHR:

$$C_6H_5-CH=CH-CH_2-COOH \xrightarrow{SOCl_2} C_6H_5-CH=CH-CH_2-COCl.$$

$$C_6H_5-CH=CH-CH_2-COCl \xrightarrow{RNH_2} C_6H_5-CH=CH-CH_2-CONHR.$$

Step 2: Reduce the amide and compare the products $LiAlH_4$ reduces $-CONHR$ to $-CH_2NHR$. Acidic work-up completes the reduction sequence. The isolated carbon-carbon double bond remains, so the product is

$$C_6H_5-CH=CH-CH_2-CH_2-NHR.$$

Option (A) is a hydroxyamine rather than the fully reduced amine. Option (B) incorrectly introduces chlorine onto the carbon skeleton and changes the double bond. Option (C) retains an amide carbonyl and removes the original double bond. None represents amide reduction by lithium aluminium hydride. Option (D) retains the alkene and converts only the amide carbonyl into methylene.

Step 3: Conclusion The major product is $C_6H_5-CH=CH-CH_2-CH_2-NHR$, the secondary amine

shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: For an amide treated with lithium aluminium hydride, replace the carbonyl oxygen by two hydrogens and retain the C–N bond.

74. The density of alkali metals is in the order : [4 marks]

(A) $\mathrm{Na} < \mathrm{K} < \mathrm{Cs} < \mathrm{Rb}$

(B) $\mathrm{K} < \mathrm{Na} < \mathrm{Rb} < \mathrm{Cs}$

(C) $\mathrm{K} < \mathrm{Cs} < \mathrm{Na} < \mathrm{Rb}$

(D) $\mathrm{Na} < \mathrm{Rb} < \mathrm{K} < \mathrm{Cs}$

Correct Answer: (B) $\mathrm{K} < \mathrm{Na} < \mathrm{Rb} < \mathrm{Cs}$

Solution:

Concept:

- Density is mass per unit volume: $\rho = M/V_m$, where M is molar mass and V_m is molar volume.
- Down the alkali-metal group, density generally increases, but potassium is less dense than sodium because the volume increase outweighs the mass increase in this step.

Step 1: Apply the sodium-potassium exception One must not rank alkali-metal densities from atomic mass alone. Potassium has a larger molar mass than sodium, but its larger atomic size also gives a sufficiently larger molar volume. Consequently potassium is less dense. Representative room-temperature densities are approximately $0.86\,\mathrm{g\,cm^{-3}}$ for potassium and $0.97\,\mathrm{g\,cm^{-3}}$ for sodium. Thus the required order must begin $\mathrm{K} < \mathrm{Na}$. This immediately rejects options (A) and (D), both of which place sodium below potassium.

Step 2: Compare rubidium and caesium Further down the group, the increasing mass produces higher densities: rubidium is about $1.53\,\mathrm{g\,cm^{-3}}$, and caesium about $1.9\,\mathrm{g\,cm^{-3}}$. Both are denser than sodium, and caesium is denser than rubidium. Combining these comparisons gives

$$\rho(\mathrm{K}) < \rho(\mathrm{Na}) < \rho(\mathrm{Rb}) < \rho(\mathrm{Cs}).$$

Option (C) incorrectly places caesium below sodium and rubidium. Option (A) also reverses the rubidium-caesium pair. Only option (B) respects both the potassium exception and the subsequent increase in density.

Step 3: Conclusion The density order is $\mathrm{K} < \mathrm{Na} < \mathrm{Rb} < \mathrm{Cs}$. Hence, the correct option is (B).

Quick Tip to Solve: Remember the density exception $\mathrm{K} < \mathrm{Na}$ first, then continue with the ordinary increase from Na to Rb to Cs.

75. Match List I with List II

	LIST IType of Hydride		LIST IIExample
A.	**Electron deficient hydride**	**I.**	MgH_2
B.	**Electron rich hydride**	**II.**	HF
C.	**Electron precise hydride**	**III.**	B_2H_6
D.	**Saline hydride**	**IV.**	CH_4

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-II, C-I, D-IV

(B) A-II, B-III, C-I, D-IV

(C) A-III, B-II, C-IV, D-I

(D) A-II, B-III, C-IV, D-I

Correct Answer: (C) A-III, B-II, C-IV, D-I

Solution:

Concept:

- Electron-deficient covalent hydrides have too few electrons for all ordinary two-centre bonds; electron-rich hydrides possess lone pairs on the central atom.
- Electron-precise hydrides have exactly the electrons needed for ordinary covalent bonds, whereas saline hydrides contain hydrogen as hydride ion, H^-.

Step 1: Match the covalent hydrides Diborane, B_2H_6, is electron deficient: its bridging hydrogen atoms take part in three-centre two-electron bonds. Therefore A matches III. In hydrogen fluoride, fluorine has three lone pairs in addition to the H–F bond; it is electron rich, so B matches II. Methane, CH_4, has four ordinary carbon-hydrogen bonds and no electron deficiency or lone pair on carbon. It is electron precise, giving C-IV. These classifications concern the bonding electron count, rather than merely the presence of hydrogen in the formula.

Step 2: Identify the saline hydride and eliminate mismatches Magnesium hydride, MgH_2, is the saline hydride among the listed examples and is classified using hydride ions associated with the metal. Hence D-I. The complete matching is

A-III, B-II, C-IV, D-I.

Option (A) interchanges methane and magnesium hydride, incorrectly making the metal hydride electron precise. Options (B) and (D) interchange hydrogen fluoride and diborane: HF has lone pairs, while diborane is electron deficient. Only option (C) assigns each of the four examples to its appropriate hydride class.

Step 3: Conclusion The matching is A-III, B-II, C-IV, D-I. Hence, the correct option is (C).

Quick Tip to Solve: Use one diagnostic feature per class: bridges in diborane, lone pairs in HF, four ordinary bonds in methane, and hydride ion in the metal hydride.

76. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : In the Ellingham diagram, a sharp change in slope of the line is observed for $\mathrm{Mg} \rightarrow \mathrm{MgO}$ **at** $\sim 1120^{\circ}\mathrm{C}$

Reason R: There is a large change of entropy associated with the change of state

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) **A** is false but **R** is true

(C) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(D) **A** is true but **R** is false

Correct Answer: (A) Both **A** and **R** are true and **R** is the correct explanation of **A**

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy of oxide formation against temperature: $\Delta G^{\circ} = \Delta H^{\circ} - T\Delta S^{\circ}$.
- Within a fixed phase range, the line slope is approximately $-\Delta S^{\circ}$. A phase transition changes the reaction entropy and therefore changes the slope.

Step 1: Test the assertion using the phase transition In the neighbourhood of $1120^{\circ}\mathrm{C}$, magnesium undergoes the liquid-to-vapour transition, at its boiling point. Below the transition, the oxidation reaction consumes condensed magnesium; above it, it consumes magnesium vapour. The standard entropy of the metal is much larger in the gaseous state, so the entropy change for oxide formation changes appreciably. As a result, the $\mathrm{Mg} \longrightarrow \mathrm{MgO}$ line has a sharp change in slope. Assertion A is therefore true. The event is a physical phase change of magnesium, rather than the creation of a different magnesium oxide.

Step 2: Test the reason and its connection to the slope Vaporization produces a large entropy change, so Reason R is true. Since the Ellingham slope is $-\Delta S^{\circ}$, the change in reaction entropy directly explains the kink. More specifically, gaseous magnesium raises the reactant entropy; oxide formation then has a more negative entropy change and a more positive slope. The reason therefore explains the assertion rather than merely stating an unrelated fact. Option (B) wrongly rejects the assertion, (D) wrongly rejects the reason, and (C) wrongly denies their causal connection.

Step 3: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: When an Ellingham line changes slope suddenly, check whether a reactant or product melts or boils at that temperature.

77. Match List I with List II

	LIST I		LIST II
A.	Nitrogen oxides in air	I.	Eutrophication
B.	Methane in air	II.	pH of rain water becomes 5.6
C.	Carbon dioxide	III.	Global warming
D.	Phosphate fertilisers in water	IV.	Acid rain

Choose the correct answer from the options given below : [4 marks]

(A) A-II, B-III, C-I, D-IV

(B) A-I, B-II, C-III, D-IV

(C) A-IV, B-II, C-III, D-I

(D) A-IV, B-III, C-II, D-I

Correct Answer: (D) A-IV, B-III, C-II, D-I

Solution:

Concept:

- Acid rain arises when atmospheric pollutants generate strong acids; normal rain is mildly acidic because dissolved carbon dioxide forms carbonic acid.
- Methane is a greenhouse gas, while excess phosphate nutrients stimulate algal growth and eutrophication in water bodies.

Step 1: Match the atmospheric substances Nitrogen oxides undergo atmospheric oxidation and reaction with water to form nitric acid, so A matches IV, acid rain. Methane absorbs outgoing infrared radiation and contributes to the greenhouse effect, so B matches III, global warming. Carbon dioxide dissolves in rainwater and forms weak carbonic acid:

$$CO_2 + H_2O \rightleftharpoons H_2CO_3.$$

This accounts for the normal rainwater pH of about 5.6, giving C-II. It is necessary to distinguish this natural weak acidity from acid rain caused by additional strong acids. Although carbon dioxide also contributes to warming, the specific pH statement fixes its match here.

Step 2: Match phosphate and select the complete combination Phosphate fertilizers entering water supply excess nutrients to algae and aquatic plants. The resulting blooms and subsequent decomposition deplete dissolved oxygen; this process is eutrophication. Thus D-I. Putting the four matches together gives

A-IV, B-III, C-II, D-I.

Option (A) incorrectly associates carbon dioxide with eutrophication and phosphate with acid rain. Option (B) places nitrogen oxides under eutrophication. Option (C) assigns the normal-rain pH to methane. Only (D) preserves all four chemically appropriate associations.

Step 3: Conclusion The correct matching is A-IV, B-III, C-II, D-I. Hence, the correct option is (D).

Quick Tip to Solve: Use the specific clue pH 5.6 for dissolved carbon dioxide; reserve acid rain for nitrogen oxides and eutrophication for phosphate nutrients.

78. Four gases A, B, C and D have critical temperatures $5.3, 33.2, 126.0$ and 154.3 K respectively. For their adsorption on a fixed amount of charcoal, the correct order is: [4 marks]

(A) D > C > B > A

(B) C > B > D > A

(C) D > C > A > B

(D) C > D > B > A

Correct Answer: (A) D > C > B > A

Solution:

Concept:

- Physical adsorption of a gas on charcoal is caused mainly by intermolecular van der Waals attractions between the gas and the adsorbent surface.
- Critical temperature is the highest temperature at which pressure can liquefy a gas. A higher critical temperature indicates stronger intermolecular attractions and easier liquefaction.

Step 1: Identify the comparison required The amount and nature of the adsorbent are fixed: every gas is tested on charcoal. Under comparable temperature and pressure conditions, the gas that liquefies more readily is also adsorbed more strongly. We therefore compare critical temperatures rather than molecular masses or the alphabetical names of the gases. This is a comparison of physical adsorption, for which condensability is the relevant trend.

Step 2: Arrange the critical temperatures The supplied temperatures give

$$T_c(D) = 154.3\,\text{K} > T_c(C) = 126.0\,\text{K} > T_c(B) = 33.2\,\text{K} > T_c(A) = 5.3\,\text{K}.$$

Thus D has the strongest tendency to condense and accumulate on the charcoal surface, while A has the weakest. The adsorption order follows the same descending order.

Step 3: Check the competing sequences Option (B) incorrectly places C above D and D below B. Option (C) correctly puts D and C first but reverses A and B. Option (D) reverses C and D. Only option (A) preserves every comparison in the supplied critical-temperature sequence.

Step 4: Conclusion The correct adsorption order is $D > C > B > A$. Hence, the correct option is (A).

Quick Tip to Solve: For gases adsorbed physically on the same solid under comparable conditions, rank their critical temperatures: the more readily liquefied gas is adsorbed more strongly.

79. Match List I with List II

	LIST I (Examples)		LIST II (Type)
A.	2-chloro-1,3- butadiene	I.	Biodegradable polymer
B.	Nylon 2-nylon 6	II.	Synthetic Rubber
C.	Polyacrylonitrile	III.	Polyester
D.	Dacron	IV.	Addition Polymer

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-IV, C-I, D-III

(B) A-IV, B-I, C-III, D-II

(C) A-II, B-I, C-IV, D-III

(D) A-IV, B-III, C-I, D-II

Correct Answer: (C) A-II, B-I, C-IV, D-III

Solution:

Concept:

- A polymer can be classified by its monomer, linkage, mode of polymerisation or biodegradability. These descriptions must be matched to the particular examples given.
- Addition polymers form by joining unsaturated monomers without eliminating small molecules; polyesters contain ester linkages in their main chain.

Step 1: Match the rubber and biodegradable polymer 2-Chloro-1,3-butadiene is chloroprene. Its polymerisation gives neoprene, a synthetic rubber, so A matches II. The entry is the monomer used to make the rubber, rather than the name of a finished polymer. Nylon 2-nylon 6 is a biodegradable copolyamide made from glycine and 6-aminohexanoic acid; its peptide-like amide linkages can undergo biological degradation. Therefore B matches I.

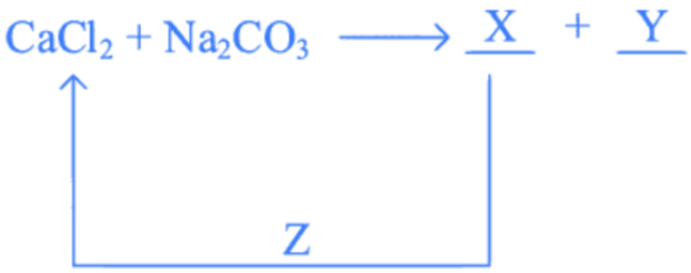
Step 2: Match the addition polymer and polyester Polyacrylonitrile is produced by addition polymerisation of acrylonitrile, $CH_2{=}CHCN$. Opening the carbon-carbon double bond creates the carbon backbone without loss of a small molecule, so C matches IV. Dacron is the polyester formed from ethylene glycol and terephthalic acid. Its repeating ester linkages identify D with III.

Step 3: Select the complete combination Collecting these assignments gives A-II, B-I, C-IV and D-III. Option (A) swaps the classifications of nylon 2-nylon 6 and polyacrylonitrile. Options (B) and (D) assign Dacron to synthetic rubber, which its ester-linked structure rules out. Option (C) is the only combination that satisfies all four matches simultaneously.

Step 4: Conclusion The matching is A-II, B-I, C-IV, D-III. Hence, the correct option is (C).

Quick Tip to Solve: Use distinctive pairs first: Dacron is a polyester and nylon 2-nylon 6 is biodegradable; then identify chloroprene as the monomer of neoprene.

80. In the given reaction cycle, X, Y and Z respectively are: [4 marks]



(A) $X = CaCO_3, \quad Y = NaCl, \quad Z = KCl$

(B) $X = CaO, \quad Y = NaCl + CO_2, \quad Z = NaCl$

(C) $X = CaCO_3, \quad Y = NaCl, \quad Z = HCl$

(D) $X = CaO, \quad Y = NaCl + CO_2, \quad Z = KCl$

Correct Answer: (C) $X = CaCO_3, \quad Y = NaCl, \quad Z = HCl$

Solution:

Concept:

- Mixing soluble calcium chloride and sodium carbonate causes a precipitation reaction: insoluble calcium carbonate separates, while sodium chloride remains dissolved.
- A carbonate reacts with an acid to form a salt, carbon dioxide and water. Hydrochloric acid supplies the chloride ions needed to regenerate calcium chloride.

Step 1: Identify the products of the first reaction Exchange the cations of the two aqueous salts. Calcium ions combine with carbonate ions to form the sparingly soluble carbonate, and sodium ions remain with chloride ions. The balanced equation is

$$CaCl_2(aq) + Na_2CO_3(aq) \longrightarrow CaCO_3(s) + 2\,NaCl(aq).$$

Consequently, the calcium-containing product X is $CaCO_3$, while Y is $NaCl$. The coefficient 2 balances sodium and chlorine; it does not change the chemical identity requested for Y. Formation of calcium oxide would require decomposition of the carbonate under heating, which is not the precipitation step shown.

Step 2: Complete the cycle back to calcium chloride To turn X into calcium chloride, use hydrochloric acid. Its hydrogen ions consume carbonate, while chloride becomes the counter-ion of calcium:

$$CaCO_3(s) + 2\,HCl(aq) \longrightarrow CaCl_2(aq) + CO_2(g) + H_2O\,(l).$$

Each element and the overall charge balance, and escaping carbon dioxide helps the reaction proceed. Thus Z is HCl. Potassium chloride and sodium chloride are neutral salts; they supply no acid to consume carbonate and cannot perform the conversion shown.

Step 3: Compare all four triples Option (A) identifies X and Y but incorrectly chooses KCl for Z. Options (B) and (D) incorrectly identify X as CaO and include carbon dioxide in the first reaction. Option (C) alone matches both balanced equations, including the acid needed for the return arrow.

Step 4: Conclusion $X = \mathrm{CaCO_3}$, $Y = \mathrm{NaCl}$, and $Z = \mathrm{HCl}$. Hence, the correct option is (C).

Quick Tip to Solve: For a salt reaction cycle, first identify the insoluble precipitate, then choose an acid whose anion matches the salt required on the return arrow.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. In an oligopeptide named Alanylglycylphenyl alanyl isoleucine, the number of $\mathrm{sp^2}$ hybridised carbons is _____. [4 marks]

Solution:

Concept:

- A peptide is formed by condensation of amino acids through amide or peptide links, $-\mathrm{CO}-\mathrm{NH}-$. Every carbonyl carbon is trigonal planar and sp^2 hybridised.
- Each carbon of a benzene ring is sp^2 hybridised. Saturated alkyl and ordinary amino-acid alpha carbons have four sigma bonds and are sp^3 hybridised.

Step 1: Read the amino-acid sequence Alanylglycylphenylalanylisoleucine contains four residues: alanine, glycine, phenylalanine and isoleucine, in that order. Joining four amino acids makes three peptide links and leaves one terminal carboxyl group. Condensation removes water but does not remove any of the original carboxyl carbon atoms. Consequently, all four residues still contribute one carbonyl carbon each.

Step 2: Count the carbonyl and aromatic carbons The three peptide carbonyls contribute three sp^2 carbons and the terminal carboxyl carbon contributes one more. Phenylalanine has a benzyl side chain, $-\mathrm{CH_2}-\mathrm{C_6H_5}$, whose benzene ring contributes six additional sp^2 carbons. Its linking methylene carbon is saturated and is not counted.

$$N(sp^2) = 3 + 1 + 6 = 10.$$

Step 3: Exclude the remaining carbons The alanine methyl group, glycine methylene group, isoleucine alkyl side chain and all alpha carbons are saturated. None contains a carbon-carbon double bond or an aromatic carbon, so none adds to the count. In particular, the number of peptide bonds alone would miss the terminal carbonyl and the aromatic ring.

Step 4: Conclusion The oligopeptide contains $\boxed{10}$ sp^2-hybridised carbon atoms. The requested count is dimensionless.

Quick Tip to Solve: For an ordinary linear peptide, count one carbonyl carbon per residue, then add aromatic and other unsaturated side-chain carbons separately.

82. 80 mole percent of $MgCl_2$ is dissociated in aqueous solution. The vapour pressure of 1.0 molal aqueous solution of $MgCl_2$ at $38^\circ C$ is _____ mm Hg. (Nearest integer)

Given : Vapour pressure of water at $38^\circ C$ is 50 mm Hg [4 marks]

Solution:

Concept:

- For a nonvolatile solute, Raoult's law gives $p = x_w p^0$, where x_w is the mole fraction of water and p^0 is the vapour pressure of pure water.
- If one formula unit dissociates into ν particles with fractional dissociation α, the van't Hoff factor is $i = 1 + (\nu - 1)\alpha$. Effective solute-particle moles must be used in the mole fraction.

Step 1: Choose a one-kilogram water basis A 1.0 molal solution contains 1.0 mol of $MgCl_2$ per 1000 g of water. Thus

$$n_w = \frac{1000}{18} = 55.556\,\text{mol}.$$

The dissociation fraction is 80 percent, or $\alpha = 0.80$; this is a fraction of the solute formula units, not the mole fraction of solute in the entire solution.

Step 2: Count the dissolved particles The dissociation is

$$MgCl_2 \longrightarrow Mg^{2+} + 2\,Cl^-.$$

Each dissociated unit gives three ions, so $i = 1 + 2(0.80) = 2.60$. Equivalently, there are 0.20 mol of undissociated salt, 0.80 mol of magnesium ions and 1.60 mol of chloride ions. The total is therefore 2.60 mol of solute particles.

Step 3: Apply Raoult's law and round The water mole fraction and the solution vapour pressure are

$$x_w = \frac{55.556}{55.556 + 2.60} = 0.95529,$$

$$p = 50(0.95529) = 47.76\,\text{mm Hg}.$$

This is below the pure-water value, as expected for a nonvolatile solute. Rounding the calculated pressure to the nearest integer, rather than rounding the mole fraction first, gives 48.

Step 4: Conclusion The vapour pressure is $\boxed{48\,\text{mm Hg}}$ to the nearest integer.

Quick Tip to Solve: Convert molality to a 1000 g solvent basis, multiply solute moles by the dissociation factor, and include these particles in the mole-fraction denominator.

83. Values of work function (W_0) for a few metals are given below

Metal	Li	Na	K	Mg	Cu	Ag
W_0/eV	2.42	2.3	2.25	3.7	4.8	4.3

The number of metals which will show photoelectric effect when light of wavelength $400\ \text{nm}$ falls on it is _____

Given: $\text{h} = 6.6 \times 10^{-34}\ \text{J s}$

$$c = 3 \times 10^{8}\ \text{ms}^{-1}$$

$$e = 1.6 \times 10^{-19}\text{C}$$

[4 marks]

Solution:

Concept:

- One photon carries energy $E = hc/\lambda$. A metal emits photoelectrons only if this energy is at least its work function W_0.
- The work function is the minimum energy needed to eject an electron. Photon energy must be expressed in electronvolts to compare directly with the given table.

Step 1: Calculate the incident photon energy The wavelength is $400\,\text{nm} = 4.00 \times 10^{-7}\,\text{m}$. Using the stated constants,

$$E = \frac{(6.6 \times 10^{-34})(3.0 \times 10^{8})}{4.00 \times 10^{-7}} = 4.95 \times 10^{-19}\,\text{J}.$$

This is the energy of an individual photon; increasing the number of photons would not change the threshold comparison for a given wavelength.

Step 2: Convert the energy and test each metal Since $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$,

$$E = \frac{4.95 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.09375\,\text{eV}.$$

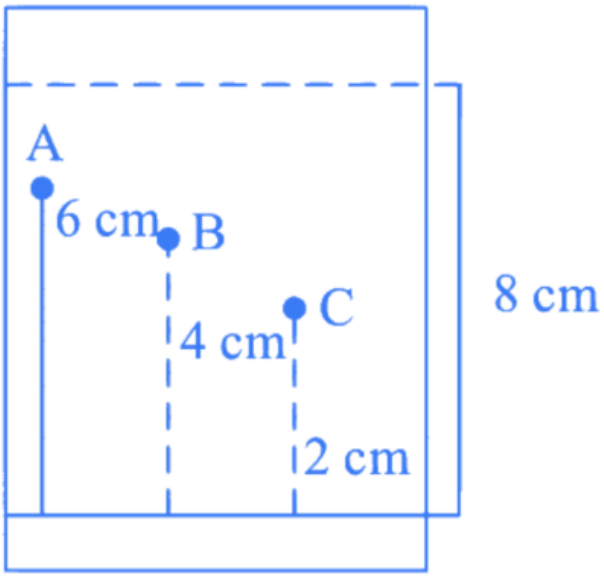
Lithium, sodium and potassium have work functions 2.42, 2.30 and 2.25 eV respectively, all smaller than 3.09375 eV. Their electrons can therefore escape with positive maximum kinetic energy.

Step 3: Count the allowed metals Magnesium requires 3.7 eV, copper 4.8 eV and silver 4.3 eV. Each of these exceeds the incident photon energy, so none of these three emits photoelectrons at 400 nm. The condition concerns the work function rather than the atomic mass or position of the metal in the table. Exactly the first three metals pass the threshold test.

Step 4: Conclusion The metals showing the photoelectric effect are Li, Na and K. Their number is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: Calculate one photon energy in eV, then count every work function at or below that energy; no separate kinetic-energy calculation is needed.

84. Three organic compounds A, B and C were allowed to run in thin layer chromatography using hexane and gave the following result (see figure). The R_f value of the most polar compound is _____ $\times 10^{-2}$ [4 marks]



Solution:

Concept:

- The retention factor in thin-layer chromatography is $R_f = d_s/d_f$, where d_s is the distance travelled by a solute spot from the origin and d_f is the distance travelled by the solvent front.
- On the usual polar TLC adsorbent with nonpolar hexane as mobile phase, a more polar compound interacts more strongly with the stationary phase and travels a shorter distance.

Step 1: Read all distances from the origin The figure gives travel distances of 6 cm for A, 4 cm for B and 2 cm for C, measured from the starting baseline. The solvent front is 8 cm above that same baseline. The distance to the top edge of the plate is irrelevant: the solvent front, shown by the dashed horizontal line, sets the denominator of every retention factor.

Step 2: Identify the most strongly retained compound Hexane is nonpolar. The more polar compound is held more strongly by the polar stationary surface, so it moves less far with the solvent. The observed distance order is $d_A > d_B > d_C$; hence C is the most polar of the three. Selecting the farthest-travelling spot would reverse the stationary-phase interaction trend.

Step 3: Calculate the requested retention factor For compound C,

$$R_f(C) = \frac{2\,\text{cm}}{8\,\text{cm}} = \frac{1}{4} = 0.25.$$

The centimetres cancel, so the retention factor has no unit. The question writes the answer as a coefficient multiplied by 10^{-2}. Expressing the decimal this way gives

$$0.25 = 25 \times 10^{-2}.$$

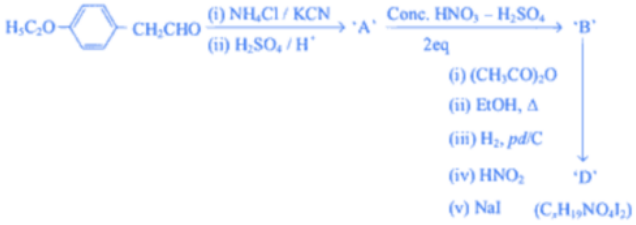
For comparison, A and B would have retention factors 0.75 and 0.50; neither corresponds to the most

polar compound.

Step 4: Conclusion The most polar compound is C and its R_f is $0.25 = \boxed{25} \times 10^{-2}$. The required entry is 25 (dimensionless).

Quick Tip to Solve: With a polar stationary phase and hexane, choose the shortest-travelling spot first, then divide its distance by the solvent-front distance.

85. The value of x in compound 'D' is _____. [4 marks]



Solution:

Concept:

- The Strecker synthesis converts an aldehyde into an alpha-amino acid: the cyanide carbon becomes the carboxyl carbon, so the carbon chain gains one carbon.
- Nitration, reduction of nitro groups and replacement of diazonium groups by iodine change functional groups without adding or removing carbon atoms. Acetylation and ethyl ester formation each add two carbons.

Step 1: Count the carbons in amino acid A The starting compound is a para-ethoxy-substituted phenylacetaldehyde. Its benzene ring contains six carbons, the ethoxy group contains two, and the side chain $-CH_2CHO$ contains two. Thus it starts with ten carbons. Strecker synthesis with NH_4Cl/KCN, followed by acidic hydrolysis, gives $Ar{-}CH_2{-}CH(NH_2){-}COOH$. The cyanide carbon is retained as the carboxyl carbon, so A has $10 + 1 = 11$ carbons.

Step 2: Track the carbon changes through the remaining reagents Two equivalents of the nitrating mixture introduce two nitro groups on the aromatic ring; no carbon is introduced. Acetic anhydride acetylates the amino group of A, adding an acetyl unit, $-COCH_3$, containing two carbons. Heating with ethanol esterifies the carboxyl group to $-COOC_2H_5$, retaining the original carboxyl carbon and adding two ethyl carbons. Consequently the carbon count becomes

$$11 + 2 + 2 = 15.$$

Hydrogen over Pd/C reduces the aromatic nitro groups to amino groups. Nitrous acid diazotizes those groups and sodium iodide replaces them with iodine. Neither operation affects the carbon skeleton; the protected amino group remains as an amide.

Step 3: Conclusion Compound D has 15 carbon atoms. Therefore the requested dimensionless integer is $\boxed{x = 15}$.

Quick Tip to Solve: For a long reaction sequence asking only a molecular carbon count, record the carbon added at each step: cyanide adds one, acetylation adds two and ethyl esterification adds two.

86. The mass of NH_3 produced when 131.8 kg of cyclohexanecarbaldehyde undergoes Tollen's test is _____ kg. (Nearest Integer)

Molar Mass of

C = 12g/mol

N = 14g/mol

O = 16g/mol [4 marks]

Solution:

Concept:

- Tollens reagent oxidizes an aldehyde to an ammonium carboxylate while two silver(I) ions are reduced to metallic silver. The balanced reaction determines the ammonia yield.
- For a fixed reaction, mole ratios apply to the balanced equation. Cyclohexanecarbaldehyde is $C_6H_{11}CHO$, or $C_7H_{12}O$, with molar mass $112\,\mathrm{g\,mol^{-1}}$.

Step 1: Balance the Tollens reaction Let R denote the cyclohexyl group. With the diamminesilver hydroxide form of Tollens reagent, the balanced equation is

$$\mathrm{RCHO} + 2\,[\mathrm{Ag(NH_3)_2}]\mathrm{OH} \longrightarrow \mathrm{RCOONH_4} + 2\,\mathrm{Ag} + 3\,\mathrm{NH_3} + \mathrm{H_2O}.$$

The four coordinated ammonia ligands do not all emerge as free ammonia: one nitrogen is retained in the ammonium ion of the carboxylate. Thus one mole of aldehyde yields three moles of free NH_3, not four.

Step 2: Calculate the mass and round at the end Cyclohexanecarbaldehyde has seven carbon atoms, twelve hydrogen atoms and one oxygen atom. Using the given atomic masses and the usual $H = 1$,

$$M = 7(12) + 12(1) + 16 = 112\,\mathrm{g\,mol^{-1}}.$$

Hence

$$n_{\text{aldehyde}} = \frac{131800}{112} = 1176.7857\,\mathrm{mol}.$$

The ammonia molar mass is $14 + 3 = 17\,\mathrm{g\,mol^{-1}}$, so

$$m_{\mathrm{NH_3}} = \frac{131800}{112} \times 3 \times 17 = 60016.1\,\mathrm{g} = 60.0161\,\mathrm{kg}.$$

The unrounded mass is close to 60 kg; round only after this calculation.

Step 3: Conclusion The mass of ammonia produced, to the nearest integer, is $\boxed{60\,\mathrm{kg}}$.

Quick Tip to Solve: Balance the ammonium carboxylate form of the Tollens equation before counting ammonia: one of the four ligand nitrogen atoms stays in the ammonium salt.

87. The reaction $2NO + Br_2 \rightarrow 2NOBr$

takes places through the mechanism given below:

$NO + Br_2 \Leftrightarrow NOBr_2$ **(fast)**

$NOBr_2 + NO \rightarrow 2NOBr$ **(slow)**

The overall order of the reaction is _____. [4 marks]

Solution:

Concept:

- The rate of a multistep mechanism is controlled by its slow elementary step. Its concentration dependence follows that elementary step, not the coefficients of the overall equation.
- A fast reversible step preceding the slow step can be treated as a pre-equilibrium. Its equilibrium constant expresses the intermediate concentration in terms of reactant concentrations.

Step 1: Write the rate law for the slow step The slow elementary reaction is

$$NOBr_2 + NO \longrightarrow 2\,NOBr.$$

Therefore its reaction rate has the form

$$r = k_2[NOBr_2][NO].$$

It is first order in each species participating in this elementary collision. However, $NOBr_2$ is an intermediate; leaving it in this expression would not give the requested overall order in the original reactants.

Step 2: Eliminate the intermediate using the fast equilibrium The preceding fast step is

$$NO + Br_2 \rightleftharpoons NOBr_2.$$

Its equilibrium constant is

$$K = \frac{[NOBr_2]}{[NO][Br_2]}.$$

Rearranging and substituting into the slow-step rate law gives

$$[NOBr_2] = K[NO][Br_2],$$
$$r = k_2K[NO]^2[Br_2] = k[NO]^2[Br_2].$$

The powers of the reactant concentrations are two for nitric oxide and one for bromine. A constant factor caused by the convention for product formation rate would only change k, not these powers.

Step 3: Conclusion The reaction is second order in NO and first order in Br_2. Its overall order is the dimensionless integer $\boxed{2 + 1 = 3}$.

Quick Tip to Solve: Use the slow elementary step first, then substitute the fast-equilibrium expression for any intermediate; add the final concentration exponents to obtain overall order.

88. An analyst wants to convert 1 L HCl of $pH = 1$ to a solution of HCl of pH 2. The volume of water needed to do this dilution is _____ mL. (Nearest integer) [4 marks]

Solution:

Concept:

- For a dilute strong monoprotic acid such as hydrochloric acid, complete dissociation gives $[H^+] \simeq c_{HCl}$. The pH definition is $pH = -\log_{10}[H^+]$.
- Dilution conserves the moles of dissolved acid: $c_i V_i = c_f V_f$. The water added is the final solution volume minus the initial solution volume.

Step 1: Convert both pH values into concentrations The initial solution has pH 1, hence

$$[H^+]_i = 10^{-1} = 0.10\,\text{mol}\,L^{-1}.$$

The target solution has pH 2, hence

$$[H^+]_f = 10^{-2} = 0.010\,\text{mol}\,L^{-1}.$$

Thus the acid concentration must decrease by a factor of ten. At these concentrations the contribution from the self-ionization of water is negligible, so the strong-acid approximation is appropriate.

Step 2: Find the final volume and subtract the starting volume The original volume is $1.0\,L$, containing $0.10 \times 1.0 = 0.10$ mol of hydrochloric acid. No acid is removed during dilution, so

$$V_f = \frac{c_i V_i}{c_f} = \frac{0.10 \times 1.0}{0.010} = 10.0\,L.$$

This is the final volume of solution, not the volume of added water. Assuming the additive-volume approximation used for this dilution calculation,

$$V_{water} = V_f - V_i = 10.0 - 1.0 = 9.0\,L.$$

Converting litres to millilitres gives $9.0 \times 1000 = 9000\,mL$.

Step 3: Conclusion The required water volume, to the nearest integer, is $\boxed{9000\,mL}$.

Quick Tip to Solve: An increase of one pH unit requires tenfold dilution of a strong acid. Calculate the final volume first, then subtract the original acid volume to get the water added.

89. One mole of an ideal gas at $350\ \mathrm{K}$ is in a $2.0\ \mathrm{L}$ vessel of thermally conducting walls, which are in contact with the surroundings. It undergoes isothermal reversible expansion from 2.0 L to $3.0\ \mathrm{L}$ against a constant pressure of $4\ \mathrm{atm}$. The change in entropy of the surroundings ($\Delta \mathrm{S}$) is _____ JK^{-1} (Nearest integer)

Given: $\mathrm{R} = 8.314\ \mathrm{J\ K^{-1}\ mol^{-1}}$. [4 marks]

Solution:

Concept:

- For an isothermal reversible expansion of an ideal gas, $\Delta S_{\text{gas}} = nR\ln(V_f/V_i)$. Reversibility makes the total entropy change zero, so $\Delta S_{\text{surr}} = -\Delta S_{\text{gas}}$.
- The surroundings supply heat during an isothermal expansion and therefore lose entropy. A constant external pressure describes a different, irreversible path; it cannot remain equal to the changing ideal-gas pressure throughout this expansion.

Step 1: Apply the explicitly stated reversible condition Here $n = 1\,\text{mol}$, $V_i = 2.0\,\text{L}$, $V_f = 3.0\,\text{L}$ and $R = 8.314\,\text{J K}^{-1}\,\text{mol}^{-1}$. Hence

$$\Delta S_{\text{gas}} = 1(8.314)\ln\left(\frac{3}{2}\right) = 3.371\,\text{J K}^{-1}.$$

For the reversible path,

$$\Delta S_{\text{surr}} = -3.371\,\text{J K}^{-1},$$

which rounds to $-3\,\text{J K}^{-1}$. Its positive magnitude is 3, but a magnitude is not the signed entropy change of the surroundings.

Step 2: Check the conflicting constant-pressure statement The printed phrase "against a constant pressure of 4 atm" is incompatible with a reversible ideal-gas expansion between these volumes: the gas pressure decreases continuously. If 4 atm is instead taken as the constant external pressure for an irreversible path, then

$$q_{\text{gas}} = P_{\text{ext}}(V_f - V_i) = 4(1)\,\text{L atm} = 405.3\,\text{J}.$$

An isothermal surroundings reservoir at 350 K would then have

$$\Delta S_{\text{surr}} = -\frac{405.3}{350} = -1.158\,\text{J K}^{-1},$$

rounding to $-1\,\text{J K}^{-1}$. This alternative confirms that the printed path descriptions cannot both hold.

Step 3: Conclusion Using the stated reversible condition, the signed answer is $\boxed{-3\,\text{J K}^{-1}}$. The answer key gives $+3$, which equals the rounded magnitude (or the gas entropy increase), not the surroundings entropy change. The contradictory constant-pressure clause also prevents a unique answer satisfying all printed conditions.

Quick Tip to Solve: Always attach an entropy change to its named subsystem: during reversible isothermal gas expansion, the gas gains entropy while the surroundings lose exactly the same amount.

90. At 600 K, the root mean square (rms) speed of gas X (molar mass = 40) is equal to the most probable speed of gas Y at 90 K. The molar mass of the gas Y is _____ g mol^{-1}. (Nearest integer) [4 marks]

Solution:

Concept:

- The root mean square speed is $v_{\text{rms}} = \sqrt{3RT/M}$, whereas the most probable speed is $v_{\text{mp}} = \sqrt{2RT/M}$. The molar mass M must use consistent units.
- When two characteristic speeds are equal, square the equality before solving for the unknown molar mass. The common gas constant cancels, so a ratio calculation can use both molar masses in grams per mole.

Step 1: Express the two equal speeds with their correct factors For gas X, $T_X = 600\,\text{K}$ and $M_X = 40\,\text{g mol}^{-1}$. For gas Y, $T_Y = 90\,\text{K}$, while M_Y is unknown. The equality given in the question is

$$\sqrt{\frac{3R(600)}{M_X}} = \sqrt{\frac{2R(90)}{M_Y}}.$$

The factor 3 belongs to the rms speed of X and the factor 2 belongs to the most probable speed of Y. They must not be interchanged or treated as equal characteristic speeds at the two temperatures.

Step 2: Square, rearrange and evaluate the molar mass Squaring both sides and cancelling the positive constant R gives

$$\frac{3(600)}{M_X} = \frac{2(90)}{M_Y}.$$

Cross-multiplication yields

$$M_Y = \frac{2(90)M_X}{3(600)}.$$

Substituting $M_X = 40\,\text{g mol}^{-1}$,

$$M_Y = \frac{180 \times 40}{1800} = 4.0\,\text{g mol}^{-1}.$$

As a direct check, using grams per mole consistently in the ratio gives $3(600)/40 = 45$ and $2(90)/4 = 45$. The two squared-speed expressions therefore agree; SI conversion would multiply both by the same factor.

Step 3: Conclusion The molar mass of gas Y, to the nearest integer, is $\boxed{4\,\text{g mol}^{-1}}$.

Quick Tip to Solve: Keep the numerical factors attached to the named speeds: rms uses 3 and most probable uses 2. Squaring an equality of speeds makes the temperature-to-molar-mass comparison immediate.