JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 13 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 13 April 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The area of the region enclosed by the curve $f(x) = \max\{\sin x, \cos x\}, -\pi \le x \le \pi$ and the x-axis is [4 marks]

(A) $2\sqrt{2}(\sqrt{2}+1)$

(B) 4

(C) $2(\sqrt{2}+1)$

(D) $4(\sqrt{2})$

Correct Answer: (B) 4

Solution:

Concept:

- The required geometric area is $\int_{-\pi}^{\pi} |f(x)|\,dx$. An interval where the selected function is negative contributes the negative of its signed integral.
- The larger of $\sin x$ and $\cos x$ changes where $\sin x = \cos x$, namely at $-3\pi/4$ and $\pi/4$ in the given interval.

Step 1: Identify the selected function and its sign Comparing the two functions on each interval gives

$$f(x) = \begin{cases} \sin x, & -\pi \le x \le -3\pi/4, \\ \cos x, & -3\pi/4 \le x \le \pi/4, \\ \sin x, & \pi/4 \le x \le \pi. \end{cases}$$

The first sine portion is below the axis. The cosine portion is below the axis up to $-\pi/2$, and above it thereafter. The final sine portion is nonnegative. Thus the absolute value must also split the cosine interval at $-\pi/2$.

Step 2: Integrate the four pieces Using $\int \sin x\,dx = -\cos x$ and $\int \cos x\,dx = \sin x$, the total area is

$$\begin{aligned} A = &-\int_{-\pi}^{-3\pi/4} \sin x\,dx - \int_{-3\pi/4}^{-\pi/2} \cos x\,dx \\ &+ \int_{-\pi/2}^{\pi/4} \cos x\,dx + \int_{\pi/4}^{\pi} \sin x\,dx \\ = &\left(1 - \frac{1}{\sqrt{2}}\right) + \left(1 - \frac{1}{\sqrt{2}}\right) + \left(1 + \frac{1}{\sqrt{2}}\right) + \left(1 + \frac{1}{\sqrt{2}}\right) = 4. \end{aligned}$$

The radical terms cancel only after the negative portions have been counted positively. Options (A), (C) and (D) have different values and do not equal this area.

Step 3: Conclusion The enclosed area is 4 square units. Hence, the correct option is (B).

Quick Tip to Solve: For an area under a maximum function, locate both the switching points and the zeros of the selected pieces before integrating.

2. Let PQ be a focal chord of the parabola $y^2 = 36x$ of length 100 , making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that PM : MQ = 3 : 1. Then which of the following points does NOT lie on the line passing through M and perpendicular to the line PQ? [4 marks]

(A) $(6, 29)$

(B) $(-3, 43)$

(C) $(3, 33)$

(D) $(-6, 45)$

Correct Answer: (B) $(-3, 43)$

Solution:

Concept:

- For $y^2 = 4ax$, a focal chord inclined at θ has length $4a\csc^2\theta$. Here $a = 9$, and the focus is $(9, 0)$.
- Internal division in the ratio $PM : MQ = 3 : 1$ gives $M = (P + 3Q)/4$. Perpendicular slopes have product -1.

Step 1: Find the focal chord and its endpoints The chord-length formula gives

$$100 = 36\csc^2\theta, \qquad \sin\theta = \frac{3}{5}.$$

The angle is acute, so $\cos\theta = 4/5$ and $\tan\theta = 3/4$. Since the chord passes through the focus, its equation is

$$y = \frac{3}{4}(x - 9), \qquad x = 9 + \frac{4}{3}y.$$

Substituting in the parabola gives

$$y^2 = 324 + 48y, \qquad (y - 54)(y + 6) = 0.$$

The positive ordinate identifies $P = (81, 54)$, leaving $Q = (1, -6)$. As a check, their distance is $\sqrt{80^2 + 60^2} = 100$, as required.

Step 2: Construct the perpendicular and test the options The section formula yields

$$M = \left(\frac{81 + 3}{4}, \frac{54 - 18}{4}\right) = (21, 9).$$

The line through M perpendicular to the chord has slope $-4/3$, so

$$y - 9 = -\frac{4}{3}(x - 21), \qquad 4x + 3y = 111.$$

For (A), $4(6) + 3(29) = 111$; for (C), $4(3) + 3(33) = 111$; for (D), $4(-6) + 3(45) = 111$. These three points lie on the required line. For (B), however, $4(-3) + 3(43) = 117$, which is not 111. The question asks for this nonmember.

Step 3: Conclusion The point $(-3, 43)$ does not lie on the perpendicular through M. Hence, the correct option is (B).

Quick Tip to Solve: Use the sign of the ordinate to label the chord endpoints before applying the division ratio; reversing them changes the point M.

3. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{d} \cdot \vec{a} = 24$, then $|\vec{d}|^2$ is equal to : [4 marks]

(A) 313

(B) 413

(C) 423

(D) 323

Correct Answer: (B) 413

Solution:

Concept:

- If $(\vec{d} - \vec{c}) \times \vec{b} = \vec{0}$ and $\vec{b} \neq \vec{0}$, the difference is parallel to $\vec{b}$. Hence $\vec{d} = \vec{c} + t\vec{b}$ for a real scalar t.
- The dot product is linear: $(\vec{c} + t\vec{b}) \cdot \vec{a} = \vec{c} \cdot \vec{a} + t(\vec{b} \cdot \vec{a})$. The squared magnitude is the sum of the squared components.

Step 1: Reduce the cross-product condition to one unknown Subtracting the two given cross products gives

$$(\vec{d} - \vec{c}) \times \vec{b} = \vec{0}.$$

Since $\vec{b} = (3, -2, 7)$ is nonzero, every vector satisfying this condition has the form

$$\vec{d} = (2, -1, 4) + t(3, -2, 7).$$

This is a complete parameterisation, including $t = 0$, rather than an assumption that $\vec{d}$ itself is parallel to $\vec{b}$. The given dot product now determines the remaining scalar.

Step 2: Determine the scalar and calculate the norm With $\vec{a} = (1, 4, 2)$, calculate

$$\vec{c} \cdot \vec{a} = 2 - 4 + 8 = 6, \qquad \vec{b} \cdot \vec{a} = 3 - 8 + 14 = 9.$$

Therefore

$$6 + 9t = 24, \qquad t = 2, \qquad \vec{d} = (8, -5, 18).$$

The required square, rather than the magnitude itself, is

$$|\vec{d}|^2 = 8^2 + (-5)^2 + 18^2 = 64 + 25 + 324 = 413.$$

Indeed, $\vec{d} \cdot \vec{a} = 8 - 20 + 36 = 24$, confirming the scalar. The difference $\vec{d} - \vec{c} = 2\vec{b}$ also verifies the cross-product constraint directly. Thus 313, 423 and 323 in the other options cannot be the squared norm of the uniquely determined vector.

Step 3: Conclusion $|\vec{d}|^2 = 413$. Hence, the correct option is (B).

Quick Tip to Solve: Equal cross products with the same nonzero vector determine the difference only up to a parallel vector; use the dot product to fix its coefficient.

4. For the system of linear equations

$$2x + 4y + 2az = b$$

$$x + 2y + 3z = 4$$

$$2x - 5y + 2z = 8$$

which of the following is NOT correct? [4 marks]

(A) It has infinitely many solutions if $a = 3, b = 8$

(B) It has infinitely many solutions if $a = 3, b = 6$

(C) It has unique solution if $a = b = 8$

(D) It has unique solution if $a = b = 6$

Correct Answer: (B) It has infinitely many solutions if $a = 3, b = 6$

Solution:

Concept:

- Elementary row operations preserve the solution set. Subtracting twice the second equation from the first isolates the parameter-dependent equation for z.
- A reduced equation $0 = c \neq 0$ is inconsistent. A redundant equation can leave infinitely many solutions, while three independent equations give a unique solution.

Step 1: Eliminate the common terms Let the equations in their printed order be E_1, E_2, E_3. The operation $E_1 - 2E_2$ gives

$$2(a - 3)z = b - 8.$$

The operation $E_3 - 2E_2$ independently gives

$$-9y - 4z = 0.$$

If $a \neq 3$, the first reduced equation fixes z, the second fixes y, and then E_2 fixes x. Consequently the system has a unique solution for every b whenever $a \neq 3$. This already verifies both unique-solution options, because their values of a are 8 and 6.

Step 2: Check the singular cases and the negative wording For $a = 3$, the first reduced equation becomes $0 = b - 8$. If $b = 8$, it is an identity, and the other equations give

$$z = t, \qquad y = -\frac{4}{9}t, \qquad x = 4 - \frac{19}{9}t, \qquad t \in \mathbb{R}.$$

There are infinitely many solutions, so (A) is true. If $b = 6$, the same equation reads $0 = -2$, which is impossible: there is no solution. Thus (B) is false. Options (C) and (D) are true by the nonsingular argument above; they do not satisfy the request to select the statement that is NOT correct.

Step 3: Conclusion For $a = 3, b = 6$, the system is inconsistent, so the assertion of infinitely many solutions is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: When two coefficient rows become proportional, compare their right-hand sides too; a zero determinant alone does not establish infinitely many solutions.

5. Let $y = y_1(x)$ and $y = y_2(x)$ be the solution curves of the differential equation $\frac{dy}{dx} = y + 7$ with initial conditions $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then the curves $y = y_1(x)$ and $y = y_2(x)$ intersect at [4 marks]

(A) no point

(B) two points

(C) infinite number of points

(D) one point

Correct Answer: (A) no point

Solution:

Concept:

- Introducing $u = y + 7$ turns the differential equation into $u\prime = u$, whose solution is $u = Ce^x$. Thus $y = Ce^x - 7$.
- Two solution curves intersect at a real abscissa only if their ordinates agree there. The real exponential e^x is strictly positive for every finite real x.

Step 1: Apply each initial condition separately Writing $u = y + 7$ gives $du/dx = u$. Integrating the nonzero solution yields $u = Ce^x$, and the same formula includes the zero solution when $C = 0$. Therefore

$$y = Ce^x - 7.$$

For the first curve, $y_1(0) = 0$ gives $C_1 - 7 = 0$, hence $C_1 = 7$. For the second, $y_2(0) = 1$ gives $C_2 - 7 = 1$, hence $C_2 = 8$. The two particular solutions are

$$y_1(x) = 7e^x - 7, \qquad y_2(x) = 8e^x - 7.$$

Differentiating either expression recovers $dy/dx = y + 7$, so both satisfy the original differential equation as well as their own initial data.

Step 2: Compare the ordinates for all real abscissae Their difference is especially simple:

$$y_2(x) - y_1(x) = (8 - 7)e^x = e^x > 0 \quad \text{for every } x \in \mathbb{R}.$$

The second curve therefore lies strictly above the first at every finite real abscissa. An intersection would require $e^x = 0$, which has no real solution. Although the difference tends to zero as $x \to -\infty$, this limiting approach is not an intersection at a point of the plane. Hence there cannot be one point, two points, or infinitely many intersection points, ruling out (D), (B), and (C), respectively.

Step 3: Conclusion The solution curves intersect at no point. Hence, the correct option is (A).

Quick Tip to Solve: For two particular solutions of this equation, subtract their formulas; a nonzero constant times e to the x can never vanish at a finite x.

6. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is : [4 marks]

(A) $\frac{81}{64}$
(B) $\frac{37}{16}$
(C) $\frac{21}{16}$
(D) $\frac{15}{16}$

Correct Answer: (C) $\frac{21}{16}$

Solution:

Concept:

- If a head is three times as likely as a tail, $P(H) = 3/4$ and $P(T) = 1/4$, since the two probabilities sum to 1.
- The stopping rule permits $X = 1, 2, 3$ only. Its mean is $E(X) = \sum_x xP(X = x)$, using all terminal sequences that produce each value.

Step 1: Build the distribution using the stopping rule The experiment ends on the first head, unless three tails have already occurred. Thus it ends after one toss precisely on H, and after two tosses precisely on TH. To reach the third toss, the first two must be TT; either outcome on the third then ends the experiment. Assuming independent tosses of this coin,

$$\begin{aligned} P(X = 1) &= \frac{3}{4}, \\ P(X = 2) &= \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16}, \\ P(X = 3) &= P(TTH) + P(TTT) \\ &= \frac{3}{64} + \frac{1}{64} = \frac{1}{16}. \end{aligned}$$

These probabilities sum to $12/16 + 3/16 + 1/16 = 1$. In particular, the last probability includes both terminal outcomes; counting only three tails would omit TTH.

Step 2: Calculate the weighted mean and check the options Multiply each possible number of tosses by its probability:

$$E(X) = 1\left(\frac{3}{4}\right) + 2\left(\frac{3}{16}\right) + 3\left(\frac{1}{16}\right) = \frac{12 + 6 + 3}{16} = \frac{21}{16}.$$

A separate check uses the survival probabilities:

$$E(X) = P(X \geq 1) + P(X \geq 2) + P(X \geq 3) = 1 + \frac{1}{4} + \frac{1}{16} = \frac{21}{16}.$$

Option (D) is below 1 and cannot be the mean of a variable at least 1. Options (A) and (B) differ from both exact computations. The common value is option (C).

Step 3: Conclusion The mean number of tosses is $\frac{21}{16}$. Hence, the correct option is (C).

Quick Tip to Solve: For a positive integer stopping time capped at three tosses, add the probabilities of reaching tosses 1, 2 and 3 to get the mean directly.

7. The set of all $a \in \mathbb{R}$ for which the equation $x|x-1|+|x+2|+a=0$ has exactly one real root, is : [4 marks]

(A) $(-\infty, \infty)$

(B) $(-6, \infty)$

(C) $(-\infty, -3)$

(D) $(-6, -3)$

Correct Answer: (A) $(-\infty, \infty)$

Solution:

Concept:

- The modulus expressions change form at $x=-2$ and $x=1$. On each resulting interval, remove their signs according to the sign of their arguments.
- The equation is $g(x)=-a$, where $g(x)=x|x-1|+|x+2|$. A continuous, strictly increasing function with range $\mathbb{R}$ meets every horizontal line exactly once.

Step 1: Expand the three pieces without losing the factor x For $x<-2$, $|x-1|=1-x$ and $|x+2|=-x-2$, so $g(x)=-x^2-2$. For $-2 \le x<1$, the second modulus is $x+2$ while the first remains $1-x$. For $x \ge 1$, both inner expressions are nonnegative. Hence

$$g(x)=\begin{cases} -x^2-2, & x<-2, \\ -x^2+2x+2, & -2 \le x<1, \\ x^2+2, & x \ge 1. \end{cases}$$

At -2, both neighbouring formulas give -6; at 1, both give 3. Thus the pieces join continuously, with no jump or duplicate value caused by the interval endpoints.

Step 2: Establish monotonicity and count intersections The derivatives on the open pieces are

$$g'(x)=-2x>0 \ (x<-2), \qquad 2-2x>0 \ (-2<x<1), \qquad 2x>0 \ (x>1).$$

The function is strictly increasing on each piece and across the continuous joins. Its three successive ranges are $(-\infty,-6)$, $[-6,3)$, and $[3,\infty)$, which together cover every real number exactly once. Consequently every real target $-a$ has one and only one preimage. The boundary targets -6 and 3 also give single roots, at -2 and 1 respectively. There is therefore no restriction to the smaller intervals in (B), (C), or (D).

Step 3: Conclusion The equation has exactly one real root for every $a \in \mathbb{R}$, so the required set is $(-\infty, \infty)$. Hence, the correct option is (A).

Quick Tip to Solve: Count roots by the ranges of the modulus pieces; checking how their endpoint values join can establish a unique root for every parameter at once.

8.

$$\int_0^{\infty} \frac{6}{e^{3x}+6e^{2x}+11e^{x}+6}\,dx =$$

[4 marks]

(A) $\log_e(256/81)$

(B) $\log_e(64/27)$

(C) $\log_e(32/27)$

(D) $\log_e(512/81)$

Correct Answer: (C) $\log_e(32/27)$

Solution:

Concept:

- For an integral involving powers of e^x, the substitution $t = e^x$ gives $dx = dt/t$, with $x = 0$ corresponding to $t = 1$ and $x \to \infty$ to $t \to \infty$.
- The cubic factors as $t^3 + 6t^2 + 11t + 6 = (t+1)(t+2)(t+3)$. Partial fractions therefore reduce the integral to logarithms; their combined limit must be taken before evaluating infinity.

Step 1: Transform the integral Writing I for the required value, substitute $t = e^x$ and factor the denominator:

$$I = \int_1^{\infty} \frac{6}{t(t+1)(t+2)(t+3)}\,dt.$$

All four factors are positive for $t \geq 1$, so the integrand is positive. Its decay as $6/t^4$ also establishes convergence, which is needed before using the improper upper limit.

Step 2: Resolve into elementary fractions Multiply by the denominator and substitute $t = 0, -1, -2, -3$ to obtain the respective coefficients $1, -3, 3, -1$. Thus

$$\frac{6}{t(t+1)(t+2)(t+3)} = \frac{1}{t} - \frac{3}{t+1} + \frac{3}{t+2} - \frac{1}{t+3}.$$

An antiderivative is

$$F(t) = \log \frac{t(t+2)^3}{(t+1)^3(t+3)}.$$

This identity can be checked directly by differentiating the logarithm.

Step 3: Evaluate both limits and compare options The numerator and denominator inside the logarithm both have degree four and leading coefficient one, so $F(t) \to 0$ as $t \to \infty$. At the lower endpoint,

$$F(1) = \log \frac{1 \cdot 27}{8 \cdot 4} = \log \frac{27}{32}.$$

Hence

$$I = 0 - \log \frac{27}{32} = \log \frac{32}{27}.$$

The other three options contain different positive logarithm arguments and therefore cannot equal this exact value.

Step 4: Conclusion The integral equals $\boxed{\log_e(32/27)}$. Hence, the correct option is (C).

Quick Tip to Solve: After substituting an exponential, include the extra factor $1/t$ from dx. Combine logarithms before taking an infinite limit to avoid separate divergent terms.

9. Let $s_1, s_2, s_3, \ldots, s_{10}$ respectively be the sum to 12 terms of 10 A.P. s whose first terms are $1, 2, 3, \ldots .10$ and the common differences are $1, 3, 5, \ldots \ldots, 19$ respectively. Then $\sum_{i=1}^{10} s_i$ is equal to : [4 marks]

(A) 7360

(B) 7220

(C) 7260

(D) 7380

Correct Answer: (C) 7260

Solution:

Concept:

- The sum of the first n terms of an arithmetic progression with first term a and common difference d is $S_n = \frac{n}{2}[2a + (n-1)d]$.
- The i-th progression has first term $a_i = i$ and common difference $d_i = 2i - 1$, for $1 \leq i \leq 10$. The required total adds ten different progression sums, each containing twelve terms.

Step 1: Express the sum for a general progression Apply the arithmetic-progression formula separately to the i-th progression. Here $n = 12$, so there are eleven common-difference increments from its first term to its twelfth term:

$$s_i = \frac{12}{2}\{2i + 11(2i - 1)\}.$$

Expanding the bracket gives $2i + 22i - 11 = 24i - 11$, and therefore

$$s_i = 144i - 66.$$

For instance, $s_1 = 78$, agreeing with $1 + 2 + \cdots + 12$.

Step 2: Add the ten progression sums Use the linear expression for s_i, keeping the constant term once for each of the ten progressions:

$$\begin{aligned}\sum_{i=1}^{10} s_i &= 144\sum_{i=1}^{10} i - 66\sum_{i=1}^{10} 1 \\ &= 144\left(\frac{10 \cdot 11}{2}\right) - 66 \cdot 10 \\ &= 7920 - 660 = 7260.\end{aligned}$$

The factor ten multiplying 66 is essential: the constant is not subtracted just once.

Step 3: Verify the result against the alternatives The sequence of sums s_i itself forms an arithmetic progression with common difference 144. Its first term is 78, and its tenth term is $1440 - 66 = 1374$. Thus a second calculation gives $10(78 + 1374)/2 = 7260$. Only the third option matches; 7360, 7220, and 7380 fail both independent sum evaluations.

Step 4: Conclusion $\sum_{i=1}^{10} s_i = \boxed{7260}$. Hence, the correct option is (C).

Quick Tip to Solve: When many arithmetic progressions have linearly varying first terms and differences, calculate s_i first; the resulting sums often form another arithmetic progression.

10. Let $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$, $\alpha > 2$, be the adjoint of a matrix A, and $|A| = 2$. Then

$$\begin{bmatrix} \alpha & -2\alpha & \alpha \end{bmatrix} B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$$

is equal to: [4 marks]

(A) 32

(B) -16

(C) 0

(D) 16

Correct Answer: (B) -16

Solution:

Concept:

- For a square matrix of order n, $\det(\operatorname{adj} A) = (\det A)^{n-1}$. Here $B = \operatorname{adj} A$ has order three, so $\det B = 2^2 = 4$.
- The requested expression is a scalar quadratic form $v^T B v$, with $v = \alpha(1, -2, 1)^T$. Matrix multiplication must preserve the given row and column order; symmetry of B is not required.

Step 1: Determine the admissible parameter Expand the determinant of B along the first row:

$$\begin{aligned} \det B &= (8 - 3\alpha) - 3(4 - 3\alpha) + \alpha(\alpha - 2\alpha) \\ &= -\alpha^2 + 6\alpha - 4. \end{aligned}$$

Since $\det B = 4$,

$$\alpha^2 - 6\alpha + 8 = 0, \qquad (\alpha - 2)(\alpha - 4) = 0.$$

The printed restriction $\alpha > 2$ rules out $\alpha = 2$, leaving $\alpha = 4$.

Step 2: Multiply the matrix by the column Keep the factor α outside the column and perform each row product:

$$B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} 1 - 6 + \alpha \\ 1 - 4 + 3 \\ \alpha - 2\alpha + 4 \end{bmatrix} = \alpha \begin{bmatrix} \alpha - 5 \\ 0 \\ 4 - \alpha \end{bmatrix}.$$

In particular, the middle component is zero for every α. This provides a useful check against a mistaken middle-row multiplication.

Step 3: Complete the scalar product Premultiply by the row vector in the question:

$$\begin{aligned} v^T B v &= \alpha^2\{(\alpha - 5) - 2 \cdot 0 + (4 - \alpha)\} \\ &= -\alpha^2 = -16. \end{aligned}$$

With $\alpha = 4$, the column product is $(-4, 0, 0)^T$, which gives the same scalar immediately. The positive

values 32 and 16, and zero, contradict this direct product; only the negative alternative remains.

Step 4: Conclusion The listed key value of 32 does not agree with the printed matrix: $\det B = 4$ forces $\alpha = 4$, and direct multiplication gives -16, not 32. The required scalar is $\boxed{-16}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the determinant of the adjoint to find the parameter first. For a quadratic form, multiply the matrix by the column before taking the final dot product.

11. Fractional part of the number $\frac{4^{2022}}{15}$ is equal to [4 marks]

(A) $\frac{8}{15}$
(B) $\frac{4}{15}$
(C) $\frac{1}{15}$
(D) $\frac{14}{15}$

Correct Answer: (C) $\frac{1}{15}$

Solution:

Concept:

- For a positive integer N, write $N = 15q + r$, where q is an integer and $0 \le r < 15$. The fractional part of $N/15$ is exactly $r/15$.
- The relation $4^2 = 16 \equiv 1 \pmod{15}$ means that every even power of four has remainder one modulo fifteen. This avoids expanding the large power.

Step 1: Use the parity of the exponent The exponent 2022 is even, so express the numerator as

$$4^{2022} = (4^2)^{1011} = 16^{1011} = (15+1)^{1011}.$$

In the binomial expansion, every term other than the constant term includes a factor 15. Thus the entire numerator is one more than a multiple of fifteen, with no need to calculate the large integer itself.

Step 2: Separate the integer and fractional parts There exists an integer q satisfying

$$4^{2022} = 15q + 1.$$

Dividing by 15 gives

$$\frac{4^{2022}}{15} = q + \frac{1}{15}.$$

Since $0 < 1/15 < 1$, the integer part is precisely q, and the remaining fraction is already in the standard interval $[0, 1)$. No further subtraction of an integer is necessary.

Step 3: Check all proposed remainders The options $8/15$, $4/15$, and $14/15$ would require the numerator to have remainders 8, 4, and 14, respectively. But the powers of four alternate between remainders 4 for an odd exponent and 1 for an even exponent. Consequently, none of those three remainders is possible for exponent 2022. This modular check independently confirms the selected

fraction.

Step 4: Conclusion The fractional part is $\boxed{1/15}$. Hence, the correct option is (C).

Quick Tip to Solve: To find the fractional part of a huge integer divided by a small integer, compute only its remainder. Here squaring four produces one modulo fifteen.

12. For $x \in \mathbb{R}$, two real valued functions $f(x)$ and $g(x)$ are such that, $g(x) = \sqrt{x} + 1$ and $f \circ g(x) = x + 3 - \sqrt{x}$. Then $f(0)$ is equal to [4 marks]

(A) 5

(B) 0

(C) -3

(D) 1

Correct Answer: The printed conditions do not determine f(0); 5 requires an additional assumption.

Solution:

Concept:

- Knowing $f \circ g$ determines f only on the range of g. Values of f outside that range require additional information about f.
- For real square roots, $\sqrt{x}$ requires $x \geq 0$, and $g(x) = \sqrt{x} + 1$ has range $[1, \infty)$. In particular, no permitted input satisfies $g(x) = 0$.

Step 1: Determine what the composition actually fixes Set $t = \sqrt{x} + 1$. Then $t \geq 1$, $\sqrt{x} = t - 1$, and $x = (t-1)^2$. Substituting in the printed composition yields

$$\begin{aligned} f(t) &= (t-1)^2 + 3 - (t-1) \\ &= t^2 - 3t + 5, \qquad t \geq 1. \end{aligned}$$

The restriction $t \geq 1$ is essential. Squaring does not extend the range of the substitution to negative square-root values.

Step 2: Test whether zero is covered To obtain $f(0)$ directly from the composition, one would need $\sqrt{x} + 1 = 0$, or $\sqrt{x} = -1$. This has no real solution. Consequently, the equation places no condition on $f(0)$. For any real constant c, one can define

$$f_c(t) = \begin{cases} t^2 - 3t + 5, & t \geq 1, \\ c, & t < 1. \end{cases}$$

Every such function satisfies the printed composition for all allowed x, while $f_c(0) = c$.

Step 3: Explain the tempting polynomial extrapolation If an additional condition required f to be a polynomial, equality with $t^2 - 3t + 5$ throughout $[1, \infty)$ would determine that polynomial globally and give $f(0) = 5$, option (A). The question states only that the functions are real valued; it does not impose that extra condition. In fact, the examples above allow $f(0) = 5, 0, -3, 1$, so none of the four displayed values is uniquely implied. Also, the phrase $x \in \mathbb{R}$ cannot make the displayed square root

real for negative inputs.

Step 4: Conclusion As printed, $\boxed{f(0) \text{ is not determined}}$. The value 5, option (A), follows only with an extra polynomial-extension assumption. All four listed values are compatible with the composition on its real domain, so there is no uniquely justified option.

Quick Tip to Solve: Before recovering an outer function from a composition, find the inner function's range. Never evaluate the recovered formula outside that range without an additional hypothesis.

13. For the differentiable function $f : \mathbb{R} - \{0\} \to \mathbb{R}$, let $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$, then $\left|f(3) + f'\left(\frac{1}{4}\right)\right|$ is equal to [4 marks]

(A) 13

(B) $\frac{29}{5}$

(C) $\frac{33}{5}$

(D) 7

Correct Answer: (A) 13

Solution:

Concept:

- Replace x by $1/x$ in a reciprocal functional equation to obtain a second linear equation in $f(x)$ and $f(1/x)$. This is valid because $x \neq 0$.
- Eliminating the reciprocal value gives an explicit formula for $f(x)$, which can then be differentiated on each interval of its domain. The derivative of x^{-1} is $-x^{-2}$.

Step 1: Eliminate the reciprocal function value The given equation and its version at $1/x$ are

$$3f(x) + 2f(1/x) = 1/x - 10,$$
$$2f(x) + 3f(1/x) = x - 10.$$

Multiply the first equation by three and the second by two, then subtract:

$$5f(x) = \frac{3}{x} - 2x - 10.$$

Hence

$$f(x) = \frac{3}{5x} - \frac{2x}{5} - 2.$$

The coefficient determinant is $9 - 4 = 5 \neq 0$, so this determines the function uniquely for every nonzero input.

Step 2: Evaluate the required function and derivative values At $x = 3$,

$$f(3) = \frac{1}{5} - \frac{6}{5} - 2 = -3.$$

Differentiate the explicit expression:

$$f'(x) = -\frac{3}{5x^2} - \frac{2}{5}.$$

Thus

$$f'(1/4) = -\frac{3 \cdot 16}{5} - \frac{2}{5} = -10.$$

The derivative is evaluated at $1/4$, not at 4; this distinction changes the reciprocal-square factor.

Step 3: Apply the absolute value and reject the distractors The expression inside the absolute-value bars is $-3 - 10 = -13$, so

$$\left|f(3) + f'(1/4)\right| = |-13| = 13.$$

A direct substitution of the derived $f(x)$ back into the original equation gives $1/x - 10$, checking the elimination. The alternatives $29/5$, $33/5$, and 7 differ from this exact sum and cannot satisfy the requested calculation.

Step 4: Conclusion The required absolute value is $\boxed{13}$. Hence, the correct option is (A).

Quick Tip to Solve: For an equation coupling $f(x)$ and $f(1/x)$, substitute $1/x$ once and eliminate. Differentiate the resulting explicit formula, rather than the original coupled equation.

14.

$$\max_{0 \le x \le \pi} \left\{ x - 2\sin x \cos x + \frac{1}{3}\sin 3x \right\} =$$

[4 marks]

(A) $\frac{5\pi+2+3\sqrt{3}}{6}$

(B) 0

(C) $\frac{\pi+2-3\sqrt{3}}{6}$

(D) π

Correct Answer: (A) $\frac{5\pi+2+3\sqrt{3}}{6}$

Solution:

Concept:

- A continuous function on a closed interval attains its global extrema at an endpoint or an interior stationary point. All candidate values must therefore be compared.
- Use $2\sin x \cos x = \sin 2x$, $\cos 2x = 2\cos^2 x - 1$, and $\cos 3x = 4\cos^3 x - 3\cos x$ to factor the derivative algebraically.

Step 1: Find all stationary points Let $h(x) = x - \sin 2x + \frac{1}{3}\sin 3x$. Then

$$h'(x) = 1 - 2\cos 2x + \cos 3x.$$

On writing $c = \cos x$, this becomes

$$4c^3 - 4c^2 - 3c + 3 = (c-1)(4c^2 - 3).$$

Since $0 \le x \le \pi$, the interior stationary points are $x = \pi/6$ and $x = 5\pi/6$. The root $c = 1$ gives the endpoint $x = 0$.

Step 2: Determine the nature and values of the candidates For $0 < x < \pi$, $c - 1 < 0$. The derivative is negative on $(0, \pi/6)$, positive on $(\pi/6, 5\pi/6)$, and negative on $(5\pi/6, \pi)$. Thus the second stationary point is a local maximum. The endpoint and stationary values are

$$h(0) = 0, \qquad h(\pi) = \pi,$$
$$h(\pi/6) = \frac{\pi + 2 - 3\sqrt{3}}{6},$$
$$h(5\pi/6) = \frac{5\pi + 2 + 3\sqrt{3}}{6}.$$

Here $\sin(5\pi/3) = -\sqrt{3}/2$ and $\sin(5\pi/2) = 1$.

Step 3: Select the global maximum The function increases from its local minimum to $5\pi/6$, then decreases strictly to its endpoint value π. Hence its value at $5\pi/6$ exceeds π; it also exceeds the other endpoint and the local minimum. Equivalently, the difference from π is $(2 + 3\sqrt{3} - \pi)/6 > 0$. Options (B) and (D) give endpoints, and option (C) gives the local minimum, so only option (A) is the maximum.

Step 4: Conclusion The global maximum is $\boxed{(5\pi + 2 + 3\sqrt{3})/6}$, attained at $x = 5\pi/6$. Hence, the correct option is (A).

Quick Tip to Solve: Factor a trigonometric derivative in $\cos x$, then compare both endpoints with every interior stationary value; a local maximum alone does not establish the global result.

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15. The number of symmetric matrices of order 3, with all the entries from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is : [4 marks]

(A) 10^9

(B) 9^{10}

(C) 10^6

(D) 6^{10}

Correct Answer: (C) 10^6

Solution:

Concept:

- A real square matrix is symmetric when $A^T = A$. Thus each entry below the main diagonal equals the corresponding entry above it.
- The multiplication principle counts independent choices. For an order-n symmetric matrix, the independent entries are the n diagonal entries and the $n(n-1)/2$ entries above the diagonal.

Step 1: Write the general symmetric matrix For order three, write the matrix as

$$A = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}.$$

The six symbols a, b, c, d, e, f may be selected independently. The entries in positions $(2,1), (3,1), (3,2)$ are then forced by the chosen values of b, c, e. No additional condition relates distinct diagonal entries.

Step 2: Count the choices independently The given set contains ten elements, including zero. Each of the three diagonal entries has ten choices, and each of the three independent off-diagonal entries also has ten choices. Therefore

$$N = 10^3 \cdot 10^3 = 10^6.$$

Repeated values are allowed: the condition only says that every entry belongs to the set. There is no requirement of distinctness, invertibility or a nonzero determinant.

Step 3: Check the alternatives Option (A), 10^9, counts arbitrary order-three matrices and ignores symmetry. Option (B), 9^{10}, incorrectly changes both the number of choices and the number of free entries. Option (D), 6^{10}, interchanges the six independent positions with the ten available values. Only option (C) uses ten choices in each of six free positions.

Step 4: Conclusion The required number is $\boxed{10^6}$. Hence, the correct option is (C).

Quick Tip to Solve: For symmetric matrices, count the diagonal and just one triangular half: there are $n(n+1)/2$ free entries.

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16. Let the equation of plane passing through the line of intersection of the planes $x + 2y + az = 2$ and $x - y + z = 3$ be $5x - 11y + bz = 6a - 1$. For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to : [4 marks]

(A) -2

(B) 4

(C) 2

(D) -4

Correct Answer: (D) -4

Solution:

Concept:

- Every plane containing the intersection of two distinct planes can be written as a linear combination of their equations. Match coefficients using the stated normalization.
- The perpendicular distance of (x_0, y_0, z_0) from $Ax + By + Cz = D$ is $|Ax_0 + By_0 + Cz_0 - D|/\sqrt{A^2 + B^2 + C^2}$.

Step 1: Determine the plane parameters Use coefficients u, v so that

$$u(x + 2y + az - 2) + v(x - y + z - 3) = 0.$$

Comparison with the given plane yields $u+v=5$ and $2u-v=-11$. Adding gives $3u=-6$, hence $u=-2$ and $v=7$. Its constant is $2u+3v=17$, so $6a-1=17$, giving $a=3$. The coefficient of z is $b=ua+v=-6+7=1$. Thus the plane is $5x-11y+z=17$.

Step 2: Apply the distance condition At the point $(3,-c,c)$, substitution in the plane expression gives

$$5(3)-11(-c)+c-17=12c-2.$$

The normal has length $\sqrt{25+121+1}=7\sqrt{3}$. Therefore

$$\frac{|12c-2|}{7\sqrt{3}}=\frac{2}{\sqrt{3}} \quad \Longrightarrow \quad |12c-2|=14.$$

The two algebraic possibilities are $c=4/3$ and $c=-1$. The problem requires $c\in\mathbb{Z}$, so only $c=-1$ is admissible.

Step 3: Evaluate and compare Substitute the accepted values without dropping the sign of c:

$$\frac{a+b}{c}=\frac{3+1}{-1}=-4.$$

The values $-2,4,2$ in the other options do not equal this ratio. In particular, $+4$ would result from incorrectly using $|c|$ in the requested expression.

Step 4: Conclusion $\boxed{(a+b)/c=-4}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep both signs after removing an absolute value, then impose the integer condition before evaluating the ratio.

17. The negation of the statement $((A\wedge(B\vee C))\Rightarrow(A\vee B))\Rightarrow A$ is [4 marks]

(A) equivalent to $B\vee\sim C$

(B) equivalent to $\sim A$

(C) equivalent to $\sim C$

(D) a fallacy

Correct Answer: (B) equivalent to $\sim A$

Solution:

Concept:

- An implication $P\Rightarrow Q$ is false only when P is true and Q is false. Its negation is $P\wedge\neg Q$.
- A tautology is true for every assignment. In particular, if the antecedent of an implication logically forces its consequent, that implication is a tautology.

Step 1: Simplify the inner implication Let

$$T=(A\wedge(B\vee C))\Rightarrow(A\vee B).$$

If its antecedent is true, then A is true, so $A\vee B$ must also be true. If the antecedent is false, the

implication is true by definition. Thus T is true in all cases, independently of the values of B and C. This conclusion does not require B itself to be true.

Step 2: Negate the full statement The original statement is $T \Rightarrow A$. Since T is always true, this is equivalent to A. Alternatively, directly negate the outer implication:

$$\neg(T \Rightarrow A) = T \wedge \neg A = \text{True} \wedge \neg A = \neg A.$$

Only the final consequent is negated in this identity; negating the two sides separately would not give the negation of an implication.

Step 3: Exclude the other descriptions The negation depends only on A, so it cannot be equivalent to either $B \vee \neg C$ or $\neg C$ for all truth assignments. For example, varying A while fixing B, C changes $\neg A$ but leaves those expressions unchanged. It is also not a fallacy, because it is true whenever A is false and false whenever A is true.

Step 4: Conclusion The negation is equivalent to $\boxed{\neg A}$. Hence, the correct option is (B).

Quick Tip to Solve: Before expanding a nested implication, check whether its inner implication is automatically true from the antecedent.

18. Among

(S1): $\lim_{n\to\infty} \frac{1}{n^2}(2+4+6+\ldots\ldots+2n) = 1$

(S2) : $\lim_{n\to\infty} \frac{1}{n^{16}}\left(1^{15}+2^{15}+3^{15}+\ldots\ldots+n^{15}\right) = \frac{1}{16}$ [4 marks]

(A) Only (S1) is true

(B) Both (S1) and (S2) are true

(C) Both (S1) and (S2) are false

(D) Only (S2) is true

Correct Answer: (B) Both (S1) and (S2) are true

Solution:

Concept:

- The sum of the first n natural numbers is $n(n+1)/2$; this evaluates the sum of the first n even numbers exactly.
- For a continuous function f on $[0,1]$, the Riemann sum $\frac{1}{n}\sum_{k=1}^{n} f(k/n)$ tends to $\int_0^1 f(x)\,dx$.

Step 1: Evaluate the first limit The arithmetic progression contains exactly n terms. Its sum is

$$2+4+\cdots+2n = 2\frac{n(n+1)}{2} = n(n+1).$$

Consequently,

$$\frac{2+4+\cdots+2n}{n^2} = 1+\frac{1}{n} \longrightarrow 1.$$

Thus statement (S1) is true. The lower-order contribution $1/n$ tends to zero and does not alter the limiting value.

Step 2: Convert the second limit to an integral Factor the denominator so that the power and the

mesh width appear separately:

$$\frac{1^{15}+2^{15}+\cdots+n^{15}}{n^{16}}=\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^{15}.$$

Here the interval width is $1/n$, and the right endpoints are k/n. The function x^{15} is continuous on the entire interval, so this sum converges to

$$\int_0^1 x^{15}\,dx=\left[\frac{x^{16}}{16}\right]_0^1=\frac{1}{16}.$$

Thus statement (S2) is true as well.

Step 3: Combine the truth values Both limiting expressions have exactly the values claimed. Therefore neither statement is false. Option (A) incorrectly rejects (S2), option (D) incorrectly rejects (S1), and option (C) rejects both verified limits. The common mistake in (S2) is to overlook the extra factor $1/n$, which is essential for its Riemann-sum interpretation.

Step 4: Conclusion Both (S1) and (S2) are true. Hence, the correct option is (B).

Quick Tip to Solve: For sums of powers, rewrite k^p/n^{p+1} as $(1/n)(k/n)^p$ and integrate x^p on $[0,1]$.

19. Let the tangent and normal at the point $(3\sqrt{3},1)$ on the ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$ meet the y-axis at the points A and B respectively. Let the circle C be drawn taking AB as a diameter and the line $x=2\sqrt{5}$ intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point (α,β), then $\alpha^2-\beta^2$ is equal to : [4 marks]

(A) 61

(B) $\frac{304}{5}$

(C) 60

(D) $\frac{314}{5}$

Correct Answer: (B) $\frac{304}{5}$

Solution:

Concept:

- The tangent to $x^2/a^2+y^2/b^2=1$ at (x_1,y_1) is $xx_1/a^2+yy_1/b^2=1$; the normal has slope perpendicular to the tangent.
- For a circle with centre $(0,k)$ and radius R, the chord of contact from (α,β) has equation $\alpha x+(\beta-k)(y-k)=R^2$.

Step 1: Find the endpoints of the diameter The tangent at $(3\sqrt{3},1)$ is

$$\frac{3\sqrt{3}x}{36}+\frac{y}{4}=1.$$

Putting $x=0$ gives $A=(0,4)$. Differentiating the ellipse gives $y'=-x/(9y)$, whose value at the point

is $-1/\sqrt{3}$. The normal slope is therefore $\sqrt{3}$, giving

$$y - 1 = \sqrt{3}(x - 3\sqrt{3}), \qquad y = \sqrt{3}x - 8.$$

Thus $B = (0, -8)$.

Step 2: Write the circle and use its contact chord The midpoint of AB is $(0, -2)$, and its half-length is 6. The circle is consequently

$$x^2 + (y + 2)^2 = 36.$$

The line $x = 2\sqrt{5}$ cuts this circle since $20 < 36$; its intersection points are $(2\sqrt{5}, 2)$ and $(2\sqrt{5}, -6)$. Their tangent intersection (α, β) has this vertical line as its chord of contact. Comparing

$$\alpha x + (\beta + 2)(y + 2) = 36$$

with $x = 2\sqrt{5}$ yields $\beta = -2$ and $\alpha = 36/(2\sqrt{5}) = 18/\sqrt{5}$.

Step 3: Calculate the requested difference Therefore

$$\alpha^2 - \beta^2 = \frac{324}{5} - 4 = \frac{304}{5}.$$

The result is 60.8, so the integer choices 61 and 60 are not exact. The remaining fraction $314/5$ also differs; retaining the centre ordinate and subtracting its square is necessary.

Step 4: Conclusion $\boxed{\alpha^2 - \beta^2 = 304/5}$. Hence, the correct option is (B).

Quick Tip to Solve: For two tangents meeting outside a circle, use their contact chord; a vertical contact chord immediately fixes the intersection at the centre height.

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20. The distance of the point $(-1, 2, 3)$ from the plane $\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 10$ parallel to the line of the shortest distance between the lines $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$ is : [4 marks]

(A) $3\sqrt{6}$

(B) $3\sqrt{5}$

(C) $2\sqrt{6}$

(D) $2\sqrt{5}$

Correct Answer: (C) $2\sqrt{6}$

Solution:

Concept:

- The line of shortest distance between skew lines is perpendicular to both line directions. Its direction is therefore their cross product.
- To measure distance to a plane along a prescribed direction $\mathbf{d}$, intersect $\mathbf{r} = \mathbf{r}_0 + t\mathbf{d}$ with the plane and use the length $|t|\|\mathbf{d}\|$. This need not be the perpendicular distance.

Step 1: Find the specified direction The given lines have direction vectors $\mathbf{u} = (2, 0, 1)$ and $\mathbf{v} =$

$(1, -1, 1)$. Their cross product is

$$\mathbf{d} = \mathbf{u} \times \mathbf{v} = (1, -1, -2).$$

Indeed, $\mathbf{d} \cdot \mathbf{u} = 2 - 2 = 0$ and $\mathbf{d} \cdot \mathbf{v} = 1 + 1 - 2 = 0$. Hence this is a valid direction parallel to their common perpendicular. Its magnitude is $\sqrt{1 + 1 + 4} = \sqrt{6}$.

Step 2: Intersect the parallel line with the plane The required path through $(-1, 2, 3)$ is

$$(x, y, z) = (-1 + t,\ 2 - t,\ 3 - 2t).$$

The plane has Cartesian equation $x - 2y + 3z = 10$. Substituting the path gives

$$(-1 + t) - 2(2 - t) + 3(3 - 2t) = 10, \qquad 4 - 3t = 10.$$

Thus $t = -2$, and the intersection point is $(-3, 4, 7)$. It satisfies the plane because $-3 - 8 + 21 = 10$.

Step 3: Measure the segment and compare the options The displacement from the original point is $(-2, 2, 4) = -2\mathbf{d}$. Its length is

$$\sqrt{(-2)^2 + 2^2 + 4^2} = \sqrt{24} = 2\sqrt{6}.$$

The options involving $\sqrt{5}$ use an incorrect direction-vector norm, while $3\sqrt{6}$ uses an incorrect parameter magnitude. The ordinary perpendicular distance to the plane would answer a different question.

Step 4: Conclusion The required distance is $\boxed{2\sqrt{6}}$ units. Hence, the correct option is (C).

Quick Tip to Solve: For distance along a direction, substitute a parametric line into the plane first; divide by the direction length only when normalizing it.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let the mean of the data

x	1	3	5	7	9
Frequency (f)	4	24	28	α	8

be 5. If m and σ^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3\alpha}{m+\sigma^2}$ is equal to _____ [4 marks]

Solution:

Concept:

- For frequencies f_i, the mean is $\bar{x} = \sum f_i x_i / \sum f_i$. Use the stated mean to find the unknown frequency before calculating dispersion.
- The mean deviation about the mean is $m = \sum f_i |x_i - \bar{x}|/N$, whereas the variance is $\sigma^2 = \sum f_i (x_i - \bar{x})^2/N$, with $N = \sum f_i$.

Step 1: Determine the unknown frequency The total frequency and weighted sum are

$$N = 4 + 24 + 28 + \alpha + 8 = 64 + \alpha,$$

$$\sum fx = 4 + 72 + 140 + 7\alpha + 72 = 288 + 7\alpha.$$

Since the mean is 5, we get $288 + 7\alpha = 5(64 + \alpha)$. Hence $2\alpha = 32$, giving $\alpha = 16$ and $N = 80$. This value is positive, as a frequency must be.

Step 2: Calculate the mean deviation The absolute deviations from 5, in the given order of values, are $4, 2, 0, 2, 4$. Weighting by their frequencies yields

$$\sum f|x - 5| = 4(4) + 24(2) + 28(0) + 16(2) + 8(4) = 128.$$

Therefore $m = 128/80 = 8/5$. The observations at $x = 5$ contribute zero to the numerator but still count in the total frequency in the denominator.

Step 3: Calculate the variance and the final ratio The squared deviations are $16, 4, 0, 4, 16$, so

$$\sum f(x - 5)^2 = 4(16) + 24(4) + 28(0) + 16(4) + 8(16) = 352.$$

Thus $\sigma^2 = 352/80 = 22/5$. Add the two quantities before forming the requested ratio:

$$m + \sigma^2 = \frac{8}{5} + \frac{22}{5} = 6, \qquad \frac{3\alpha}{m + \sigma^2} = \frac{48}{6} = 8.$$

No square root is taken, since the expression asks for variance rather than standard deviation.

Step 4: Conclusion The required numerical value is $\boxed{8}$ (dimensionless).

Quick Tip to Solve: Use the same deviation row twice: absolute values give mean deviation, and squares give variance; both use the full frequency total.

22. Let $w = z\bar{z} + k_1 z + k_2 iz + \lambda(1 + i), k_1, k_2 \in \mathbb{R}$. Let $\mathrm{Re}(w) = 0$ be the circle C of radius 1 in the first quadrant touching the line $y = 1$ and the y-axis. If the curve $\mathrm{Im}(w) = 0$ intersects C at A and B, then $30(AB)^2$ is equal to _____ [4 marks]

Solution:

Concept:

- Write $z = x + iy$. Then $z\bar{z} = x^2 + y^2$, while the real and imaginary parts of the remaining linear terms determine a circle and a line.
- A chord cut by a line at perpendicular distance d from the centre of a circle of radius r has squared length $4(r^2 - d^2)$. The radius to the chord midpoint is perpendicular to the chord.

Step 1: Locate the circle and determine its coefficients The circle lies in the first quadrant and touches the y-axis, so its centre has x-coordinate 1. Tangency to $y = 1$ gives possible centre ordinates 0 and 2.

Ordinate 0 would put part of the circle below the first quadrant, so the centre is $(1, 2)$.

$$(x-1)^2 + (y-2)^2 = 1 \quad \Longrightarrow \quad x^2 + y^2 - 2x - 4y + 4 = 0.$$

On the other hand, $\mathrm{Re}(w) = x^2 + y^2 + k_1 x - k_2 y + \lambda$. Comparing coefficients yields $k_1 = -2$, $k_2 = 4$ and $\lambda = 4$.

Step 2: Find the line and the chord length The imaginary part is $k_1 y + k_2 x + \lambda$. Thus its zero set is the straight line

$$-2y + 4x + 4 = 0, \qquad 2x - y + 2 = 0.$$

Its distance from $(1, 2)$ is

$$d = \frac{|2(1) - 2 + 2|}{\sqrt{2^2 + (-1)^2}} = \frac{2}{\sqrt{5}}.$$

Since $d < 1$, there are indeed two intersections. The right triangle formed by the centre, chord midpoint and an endpoint gives

$$(AB)^2 = 4\left(1 - \frac{4}{5}\right) = \frac{4}{5}.$$

Step 3: Conclusion Therefore $30(AB)^2 = 30 \cdot \frac{4}{5} = \boxed{24}$, a dimensionless value.

Quick Tip to Solve: Find the centre from tangencies first, then use the distance-to-line chord formula instead of solving both intersection coordinates.

23. The number of seven digit positive integers formed using the digits $1, 2, 3$ and 4 only and sum of the digits equal to 12 is _____. [4 marks]

Solution:

Concept:

- Each of the seven positions independently contains a digit from $\{1, 2, 3, 4\}$. Because none is zero, every valid ordered seven-tuple gives one seven-digit positive integer.
- Subtracting 1 from each digit converts the condition into a bounded nonnegative integer sum. Stars and bars counts unrestricted sums, and inclusion-exclusion removes entries exceeding the upper bound.

Step 1: Shift the digit sum to nonnegative variables Let the digits be $d_1, \ldots, d_7$, and set $u_i = d_i - 1$. Then $0 \le u_i \le 3$, and

$$u_1 + u_2 + \cdots + u_7 = 12 - 7 = 5.$$

Without the upper bounds, the number of nonnegative integer solutions is

$$\binom{5+7-1}{7-1} = \binom{11}{6} = 462.$$

These are ordered positions, so no further division by factorials is needed. Repeated digits are allowed and are already counted correctly by this formulation.

Step 2: Exclude values that would produce an invalid digit A forbidden solution has some $u_i \geq 4$, corresponding to a digit at least 5. For a specified position, replace u_i by $v_i = u_i - 4$. The seven resulting nonnegative variables sum to $5 - 4 = 1$, giving

$$\binom{1+7-1}{7-1} = \binom{7}{6} = 7$$

solutions for that position. There are seven possible positions, so subtract $7 \times 7 = 49$. Two positions cannot both violate the bound, because they would require a total of at least 8, greater than 5. Hence there is no overlap correction to add back.

$$N = 462 - 49 = 413.$$

Step 3: Conclusion The required number of seven-digit positive integers is $\boxed{413}$. This is a count, so it has no physical unit.

Quick Tip to Solve: After subtracting the minimum digit, compare the excess total with twice the first forbidden value; here 5 is below 8, so only one exclusion term is needed.

24. If S is the set

$$S = \left\{ x \in \mathbb{R} : \sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4} \right\},$$

then

$$\sum_{x \in S} \left(\sin\left((x^2+x+5)\frac{\pi}{2}\right) - \cos\left((x^2+x+5)\pi\right) \right)$$

is equal to _____. [4 marks]

Solution:

Concept:

- For every real t, $\sin^{-1}(t/\sqrt{1+t^2}) = \tan^{-1} t$. Both sides lie in $(-\pi/2, \pi/2)$, so this identity respects the principal branch.
- When taking tangents of an angle equation, retain the original principal-angle condition and check the resulting candidates. Trigonometric values at integer multiples of $\pi/2$ then evaluate the required sum.

Step 1: Replace the inverse sines by principal arctangents Since $x^2 + 2x + 2 = (x+1)^2 + 1$, the defining equation becomes

$$\tan^{-1}(x+1) - \tan^{-1} x = \frac{\pi}{4}.$$

The difference is positive because arctangent is increasing. Taking the tangent gives

$$\frac{(x+1) - x}{1 + x(x+1)} = 1.$$

The denominator is always positive: $x^2+x+1=(x+1/2)^2+3/4>0$. Therefore no zero denominator is introduced, and the equality reduces to $x^2+x=0$. Its candidates are $x=0$ and $x=-1$.

Step 2: Check both roots and evaluate the requested terms For $x=0$, the original difference is $\tan^{-1}1-\tan^{-1}0=\pi/4$. For $x=-1$, it is $\tan^{-1}0-\tan^{-1}(-1)=\pi/4$. Thus $S=\{-1,0\}$, with neither extraneous nor missing roots. For each of these values, $x^2+x=0$, so $x^2+x+5=5$. Each summand is consequently

$$\sin\frac{5\pi}{2}-\cos(5\pi)=1-(-1)=2.$$

There are two elements in S; hence the sum is $2+2=4$.

Step 3: Conclusion The required dimensionless sum is $\boxed{4}$.

Quick Tip to Solve: Use the arctangent identity before algebra; once the equation gives the same value of x^2+x at both roots, evaluate the summand only once.

25. Let for $x\in\mathbb{R}, S_0(x)=x, S_k(x)=C_kx+k\int_0^x S_{k-1}(t)dt$, where $C_0=1, C_k=1-\int_0^1 S_{k-1}(x)dx, k=1,2,3,\ldots$ Then $S_2(3)+6C_3$ is equal to _____. [4 marks]

Solution:

Concept:

- The recurrence constructs S_k from the preceding polynomial, so only S_1, S_2 and the first three constants are needed. Integrate each polynomial term by term.
- Compute C_k from S_{k-1} before using it in S_k. The definite integrals over $[0,1]$ must keep their exact rational values to avoid rounding errors.

Step 1: Construct the first polynomial and the second constant Starting with $S_0(x)=x$, the first constant is

$$C_1=1-\int_0^1 x\,dx=\frac{1}{2}.$$

The recurrence at $k=1$ gives

$$S_1(x)=\frac{x}{2}+\int_0^x t\,dt=\frac{x}{2}+\frac{x^2}{2}.$$

Now use this entire polynomial, including both terms, in the next constant:

$$C_2=1-\int_0^1\left(\frac{x}{2}+\frac{x^2}{2}\right)dx=1-\frac{1}{4}-\frac{1}{6}=\frac{7}{12}.$$

Step 2: Construct the second polynomial and the third constant The factor $k=2$ multiplies the integral in the second recurrence. Thus

$$\begin{aligned}S_2(x)&=\frac{7}{12}x+2\int_0^x\left(\frac{t}{2}+\frac{t^2}{2}\right)dt\\&=\frac{7}{12}x+\frac{x^2}{2}+\frac{x^3}{3}.\end{aligned}$$

It follows that

$$C_3 = 1 - \left(\frac{7}{24} + \frac{1}{6} + \frac{1}{12}\right) = \frac{11}{24}.$$

Also, $S_2(3) = 7/4 + 9/2 + 9 = 61/4$. Hence

$$S_2(3) + 6C_3 = \frac{61}{4} + \frac{11}{4} = 18.$$

All integrations used the preceding polynomial, rather than an unknown later term.

Step 3: Conclusion The required dimensionless value is $S_2(3) + 6C_3 = \boxed{18}$.

Quick Tip to Solve: Write each newly found polynomial before computing the next constant, and keep the multiplier k outside the integral until integrating is complete.

---

26. Let $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a} = \vec{b} \times \vec{c}$ and $|\vec{b}|^2 = 50$, then $|72 - |\vec{b} + \vec{c}|^2|$ is equal to _____. [4 marks]

Solution:

Concept:

- Lagrange's identity relates a cross product to a dot product: $|\vec{b} \times \vec{c}|^2 = |\vec{b}|^2|\vec{c}|^2 - (\vec{b} \cdot \vec{c})^2$.
- The squared length of a sum is $|\vec{b} + \vec{c}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c}$. Retain both possible signs of the dot product until applying the outer absolute value.

Step 1: Find the magnitude of the scalar product From the given components,

$$|\vec{a}|^2 = 3^2 + 1^2 + (-1)^2 = 11, \qquad |\vec{c}|^2 = 2^2 + (-3)^2 + 3^2 = 22.$$

The vectors are compatible with the specified cross product because $\vec{a} \cdot \vec{c} = 6 - 3 - 3 = 0$. Since $\vec{a} = \vec{b} \times \vec{c}$ and $|\vec{b}|^2 = 50$, Lagrange's identity gives

$$11 = 50 \cdot 22 - (\vec{b} \cdot \vec{c})^2.$$

Therefore $(\vec{b} \cdot \vec{c})^2 = 1089 = 33^2$, so $\vec{b} \cdot \vec{c} = \pm 33$. Neither sign can be discarded using the magnitude condition alone.

Step 2: Evaluate the length of the sum and its absolute deviation Substitute the scalar product into the squared-norm formula:

$$|\vec{b} + \vec{c}|^2 = 50 + 22 + 2(\pm 33) = 72 \pm 66.$$

Thus the possible squared lengths are 138 and 6. The question asks for the absolute difference from 72, not for either squared length itself:

$$\left|72 - |\vec{b} + \vec{c}|^2\right| = |72 - (72 \pm 66)| = 66.$$

Both possible vectors therefore give the same requested value. It is unnecessary to solve for the three

components of $\vec{b}$.

Step 3: Conclusion The required value is $\boxed{66}$. The vectors have no assigned physical units, so this is dimensionless.

Quick Tip to Solve: When a cross product and both norms are known, use Lagrange's identity; an outer absolute value often removes the remaining sign ambiguity.

---

27. Let α be the constant term in the binomial expansion of $\left(\sqrt{x}-\frac{6}{x^{\frac{3}{2}}}\right)^{n}, n \leq 15$. If the sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is $\lambda\alpha$, then λ is equal to _____. [4 marks]

Solution:

Concept:

- The general term with index r is $\binom{n}{r}(-6)^r x^{n/2-2r}$, for $r = 0, 1, \ldots, n$. A constant term requires a zero exponent of x.
- The sum of all coefficients is the value at $x = 1$, namely $(-5)^n$. Subtract the constant coefficient to obtain the sum of all remaining coefficients.

Step 1: Determine the possible exponents and select the correct one The constant term occurs when $n/2 - 2r = 0$, or $r = n/4$. Hence n must be divisible by 4. For positive integral $n \leq 15$, the possibilities are 4, 8 and 12. Their constant coefficients are

n	$\alpha = \binom{n}{n/4}(-6)^{n/4}$	$(-5)^n - \alpha$
4	-24	649
8	1008	389617
12	-47520	244188145

Only $n = 4$ gives the prescribed remaining coefficient sum 649. If exponent zero were allowed, its expansion would have no remaining terms and would give zero instead, so it is excluded too.

Step 2: Extract the coefficient of the specified negative power With $n = 4$, the general exponent is $2 - 2r$. The requested power is $x^{-n} = x^{-4}$, so

$$2 - 2r = -4 \quad \Longrightarrow \quad r = 3.$$

Its coefficient is

$$\binom{4}{3}(-6)^3 = 4(-216) = -864.$$

The constant coefficient is $\alpha = -24$. By the definition of λ, we have $\lambda\alpha = -864$, and therefore

$$\lambda = \frac{-864}{-24} = 36.$$

The signs of the two coefficients cancel; using their unsigned binomial factors alone would lose the powers of 6.

Step 3: Conclusion The required dimensionless ratio is $\lambda = \boxed{36}$.

Quick Tip to Solve: Set the general exponent equal to zero and then to the target power; these identify the relevant binomial indices before any expansion is needed.

28. Let m_1 and m_2 be the slopes of the tangents drawn from the point $\mathrm{P}(4,1)$ to the hyperbola $H : \frac{y^2}{25} - \frac{x^2}{16} = 1$. If Q is the point from which the tangents drawn to H have slopes $|m_1|$ and $|m_2|$ and they make positive intercepts α and β on the x-axis, then $\frac{(PQ)^2}{\alpha\beta}$ is equal to _____. [4 marks]

Solution:

Concept:

- For the hyperbola $y^2/25 - x^2/16 = 1$, a nonvertical line $y = mx + c$ is tangent when $c^2 = 25 - 16m^2$. This follows by setting the quadratic intersection discriminant to zero.
- A line of positive slope m has positive x-intercept $-c/m$ only when its y-intercept c is negative. This sign condition selects the tangents defining Q.

Step 1: Find the two tangent slopes from the given point A line through $P(4,1)$ has equation $y = mx + 1 - 4m$. Applying the tangency condition gives

$$(1 - 4m)^2 = 25 - 16m^2 \quad \Longrightarrow \quad 4m^2 - m - 3 = 0.$$

Factoring yields $(m - 1)(4m + 3) = 0$, so the two slopes are 1 and $-3/4$. The required slopes for the new pair are therefore 1 and $3/4$, both positive. For these slopes, the permitted intercept magnitudes are respectively $\sqrt{25 - 16} = 3$ and $\sqrt{25 - 9} = 4$.

Step 2: Select the new tangents and compute their intersection Positive x-intercepts require the negative choices of the y-intercepts. The two new tangents are consequently

$$y = x - 3, \qquad y = \frac{3}{4}x - 4.$$

Their intersection satisfies $x - 3 = 3x/4 - 4$, so $x = -4$ and $y = -7$. Thus $Q = (-4, -7)$. The two positive intercepts on the x-axis are

$$\alpha = 3, \qquad \beta = \frac{16}{3}, \qquad \alpha\beta = 16.$$

The distance calculation gives

$$(PQ)^2 = (4 + 4)^2 + (1 + 7)^2 = 128.$$

Hence $(PQ)^2/(\alpha\beta) = 128/16 = 8$. Selecting a different sign for an intercept would violate the question's positivity condition.

Step 3: Conclusion The required dimensionless ratio is $\dfrac{(PQ)^2}{\alpha\beta} = \boxed{8}$.

Quick Tip to Solve: After taking absolute values of the slopes, choose tangent intercept signs again from the given positivity condition; do not simply reflect the original intersection.

29. The sum to 20 terms of the series $2\cdot 2^2-3^2+2\cdot 4^2-5^2+2\cdot 6^2-\ldots\ldots$ is equal to _____. [4 marks]

Solution:

Concept:

- Pair consecutive terms before summing an alternating series. Here each pair contains twice an even square followed by the negative of the next odd square.
- The standard finite sums are $\sum_{r=1}^{n} r = n(n+1)/2$ and $\sum_{r=1}^{n} r^2 = n(n+1)(2n+1)/6$.

Step 1: Count the complete pairs The first pair is $2(2)^2 - 3^2$, the second is $2(4)^2 - 5^2$, and so on. Each pair contributes two terms of the stated series, so twenty terms give exactly ten pairs. The last pair is $2(20)^2 - 21^2$; stopping at the square of 20 would omit the twentieth term.

Step 2: Express a general pair For the pair numbered r, the even integer is $2r$ and the following odd integer is $2r+1$. Hence its contribution simplifies to

$$2(2r)^2 - (2r+1)^2 = 8r^2 - (4r^2 + 4r + 1) = 4r^2 - 4r - 1.$$

The negative constant must be retained for each of the ten pairs.

Step 3: Sum the quadratic expression Use the two finite-sum formulae with $n = 10$:

$$\sum_{r=1}^{10} r = 55, \qquad \sum_{r=1}^{10} r^2 = 385.$$

Therefore

$$S_{20} = 4(385) - 4(55) - 10 = 1540 - 220 - 10 = 1310.$$

This also agrees with the first pair, whose value is -1, obtained by setting $r = 1$ in the general expression.

Step 4: Conclusion The required sum of the first twenty terms is $\boxed{1310}$.

Quick Tip to Solve: Count pairs rather than the largest integer: twenty terms here mean ten even-square/odd-square pairs.

---

30. Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point $Q(6, -2, \alpha), \alpha > 0$, from P is 13 , then α is equal to _____. [4 marks]

Solution:

Concept:

- For a plane $ax + by + cz + d = 0$, reflect a point R using $P = R - 2f(R)(a, b, c)/(a^2 + b^2 + c^2)$, where $f(R) = ax_R + by_R + cz_R + d$.
- The segment from a point to its reflected image is parallel to the plane normal, and the midpoint lies on the plane. Distances in three dimensions follow the sum of three squared coordinate differences.

Step 1: Reflect the given point Here the plane normal is $(1, -2, 1)$, whose squared length is 6. For

$R = (5/3, 5/3, 8/3)$,

$$f(R) = \frac{5}{3} - 2\frac{5}{3} + \frac{8}{3} - 2 = -1.$$

Consequently

$$P = \left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right) + \frac{1}{3}(1, -2, 1) = (2, 1, 3).$$

The sign is positive because the signed plane expression is negative.

Step 2: Apply the distance condition The squared distance between $Q = (6, -2, \alpha)$ and $P = (2, 1, 3)$ is

$$QP^2 = (6-2)^2 + (-2-1)^2 + (\alpha - 3)^2.$$

Since $QP = 13$,

$$16 + 9 + (\alpha - 3)^2 = 169 \quad \Rightarrow \quad (\alpha - 3)^2 = 144.$$

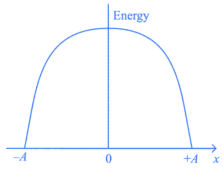
Thus $\alpha = 3 + 12 = 15$ or $\alpha = 3 - 12 = -9$.

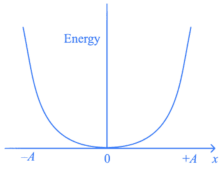
Step 3: Enforce the stated sign The problem explicitly imposes $\alpha > 0$, so the negative root is inadmissible. At $\alpha = 15$, the coordinate differences are $4, -3, 12$, and $\sqrt{16 + 9 + 144} = 13$, confirming the positive root.

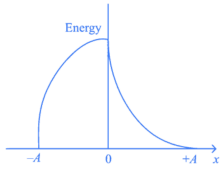
Step 4: Conclusion The required positive value is $\boxed{\alpha = 15}$.

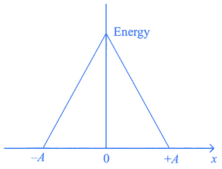
Quick Tip to Solve: Find the reflected point before using the distance formula, and apply the positivity condition only after finding both roots.

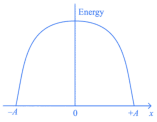
31. Which graph represents the difference between total energy and potential energy of a particle executing SHM vs it's distance from mean position ? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- For SHM with spring constant $k = m\omega^2$ and amplitude A, the total mechanical energy is $E = \frac{1}{2}kA^2$, independent of position.
- The potential energy measured from the mean position is $U(x) = \frac{1}{2}kx^2$. Therefore the difference $E - U$ is the kinetic energy, which cannot be negative over $-A \le x \le A$.

Step 1: Subtract the position-dependent energy The requested quantity is not potential energy itself. Subtract it from the constant total energy:

$$E - U(x) = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}k(A^2 - x^2).$$

This is a quadratic function of displacement with a negative coefficient of x^2, so its graph is a downward-opening parabola.

Step 2: Locate the maximum and zeros At the mean position $x = 0$, the difference equals $\frac{1}{2}kA^2$, its maximum value. At either extreme, $x = \pm A$, it is zero because the particle is instantaneously at rest and all the energy is potential. The expression is even in x, so the two halves of the graph must be mirror images.

Step 3: Compare the four graphs Graph (A) has a maximum at the origin and vanishes at both extremes, matching the downward quadratic. Graph (B) instead has a minimum at the mean position, as potential energy does. Graph (C) is asymmetric despite the dependence on x^2. Graph (D) has straight sides and a sharp vertex, so it would describe a piecewise linear dependence rather than the smooth quadratic required here.

Step 4: Conclusion The energy difference is $\frac{1}{2}k(A^2 - x^2)$, represented by graph (A). Hence, the correct option is (A).

Quick Tip to Solve: Use the three anchor points first: kinetic energy is maximum at the mean position and zero at both turning points.

32. A bullet of $10\ \mathrm{g}$ leaves the barrel of gun with a velocity of $600\ \mathrm{m/s}$. If the barrel of gun is $50\ \mathrm{cm}$ long and mass of gun is $3\ \mathrm{kg}$, then value of impulse supplied to the gun will be : [4 marks]

(A) 12 Ns

(B) 3 Ns

(C) 6 Ns

(D) 36 Ns

Correct Answer: (C) 6 Ns

Solution:

Concept:

- Impulse is the change in momentum: $\vec{J} = \Delta\vec{p}$. Its SI unit is $\mathrm{N\,s} = \mathrm{kg\,m\,s^{-1}}$.
- During the short firing interval, the force of the gun on the bullet and the force of the bullet on the gun are an action-reaction pair. Their impulses are equal in magnitude and opposite in direction.

Step 1: Calculate the bullet momentum change The bullet mass must first be converted to SI units: $m = 10\,\mathrm{g} = 0.010\,\mathrm{kg}$. Initially the bullet and gun are at rest. With the stated bullet speed $v = 600\,\mathrm{m\,s^{-1}}$,

$$\Delta p_{\text{bullet}} = m(v - 0) = 0.010 \times 600 = 6\,\mathrm{kg\,m\,s^{-1}}.$$

This is the forward impulse delivered to the bullet.

Step 2: Transfer the impulse to the gun Newton's third law makes the impulse on the gun the negative of that on the bullet. Its magnitude is therefore $6\,\mathrm{N\,s}$, directed backward. As a check, the gun recoil speed is $6/3 = 2\,\mathrm{m\,s^{-1}}$, giving recoil momentum $3 \times 2 = 6\,\mathrm{kg\,m\,s^{-1}}$.

Step 3: Reject the distractors The barrel length would be needed to infer a force or acceleration under an additional assumption about the motion, but impulse needs only the momentum change. The alternatives 12, 3, and $36\,\mathrm{N\,s}$ do not equal mv. Doubling the momentum change or multiplying by the gun mass again would count an unrelated factor.

Step 4: Conclusion The impulse supplied to the gun has magnitude $\boxed{6\,\mathrm{N\,s}}$, opposite the bullet motion. Hence, the correct option is (C).

Quick Tip to Solve: For firing impulse, calculate bullet momentum first; the gun receives the same magnitude with the opposite direction.

33. A planet having mass $9\mathrm{Me}$ and radius $4\mathrm{R_e}$, where Me and Re are mass and radius of earth respectively, has escape velocity in $\mathrm{km/s}$ given by:

(Given escape velocity on earth $\mathrm{V_e} = 11.2 \times 10^3\ \mathrm{m/s}$) [4 marks]

(A) 33.6

(B) 11.2

(C) 16.8

(D) 67.2

Correct Answer: (C) 16.8

Solution:

Concept:

- Escape speed from the surface of a spherical planet is $v_{\text{esc}} = \sqrt{2GM/R}$, obtained by equating initial kinetic energy to the energy required to reach infinity.
- When comparing two bodies, the common constant $2G$ cancels: $v_p/v_e = \sqrt{(M_p/M_e)/(R_p/R_e)}$. The speed depends on the square root of the mass-to-radius ratio.

Step 1: Form the planet-to-Earth ratio The planet has $M_p = 9M_e$ and $R_p = 4R_e$. Therefore

$$\frac{v_p}{v_e} = \sqrt{\frac{9M_e/(4R_e)}{M_e/R_e}} = \sqrt{\frac{9}{4}} = \frac{3}{2}.$$

The radius belongs in the denominator before taking the square root; using the mass ratio alone would give an incorrect factor of three.

Step 2: Convert and substitute the reference speed The supplied Earth value is $11.2 \times 10^3\,\mathrm{m\,s^{-1}}$, which equals $11.2\,\mathrm{km\,s^{-1}}$. Multiplying by the dimensionless ratio gives

$$v_p = \frac{3}{2}(11.2) = 16.8\,\mathrm{km\,s^{-1}}.$$

The larger radius weakens the effect of the larger mass, so the result exceeds Earth's escape speed by only fifty percent.

Step 3: Compare the available values The value 33.6 would keep the mass scaling but neglect the fourfold radius. The value 11.2 would mean no change in M/R, although that ratio increases by $9/4$. The value 67.2 has an even larger incorrect multiplier. Only 16.8 corresponds to the required factor $3/2$.

Step 4: Conclusion The planet's escape velocity is $\boxed{16.8\,\mathrm{km\,s^{-1}}}$. Hence, the correct option is (C).

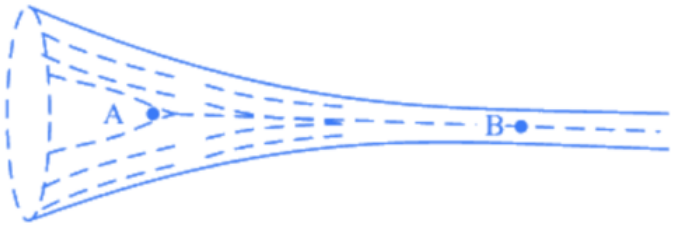
Quick Tip to Solve: Compare escape speeds using the square root of the mass ratio divided by the radius ratio; avoid recalculating G.

34. The figure shows a liquid of given density flowing steadily in horizontal tube of varying cross - section. Cross sectional areas at A is $1.5\ \mathrm{cm}^2$, and B is $25\ \mathrm{mm}^2$, if the speed of liquid at B is $60\ \mathrm{cm/s}$ then $(\mathrm{P_A} - \mathrm{P_B})$ is :

(Given $\mathrm{P_A}$ and $\mathrm{P_B}$ are liquid pressures at A and B% points.

density $\rho = 1000\ \mathrm{kg\ m^{-3}}$

A and B are on the axis of tube [4 marks]



(A) 27 Pa

(B) 175 Pa

(C) 135 Pa

(D) 36 Pa

Correct Answer: (B) 175 Pa

Solution:

Concept:

- For steady incompressible flow, volume conservation gives $A_A v_A = A_B v_B$, where A is cross-sectional area and v is flow speed.
- Along a horizontal streamline, Bernoulli's equation is $P + \frac{1}{2}\rho v^2 =$ constant. The gravitational terms cancel because the two points are at the same height.

Step 1: Use consistent area and speed units Convert $A_A = 1.5\,\mathrm{cm}^2$ to $150\,\mathrm{mm}^2$, so $A_B/A_A = 25/150 = 1/6$. The speed at the narrower part is $v_B = 60\,\mathrm{cm\,s^{-1}} = 0.60\,\mathrm{m\,s^{-1}}$. By continuity,

$$v_A = \frac{A_B}{A_A} v_B = \frac{1}{6}(0.60) = 0.10\,\mathrm{m\,s^{-1}}.$$

Step 2: Subtract the two Bernoulli equations Both points lie on the axis of the horizontal tube, so their elevations are equal. Rearranging Bernoulli's equation gives

$$P_A - P_B = \frac{1}{2}\rho(v_B^2 - v_A^2).$$

With $\rho = 1000\,\mathrm{kg\,m^{-3}}$,

$$P_A - P_B = \frac{1}{2}(1000)(0.60^2 - 0.10^2) = 500(0.36 - 0.01) = 175\,\mathrm{Pa}.$$

Step 3: Check the sign and options The wider section A has lower flow speed, so its static pressure must be higher. Thus the positive sign of the result is physically consistent. None of 27, 135, or 36 Pa equals the kinetic-energy-density difference calculated with the area ratio of six. In particular,

centimetres and millimetres must not be mixed in forming the area ratio.

Step 4: Conclusion The pressure difference is $\boxed{P_A - P_B = 175\,\text{Pa}}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the second speed by continuity first; then use the difference of squared speeds in Bernoulli's equation.

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35. A vessel of depth 'd' is half filled with oil of refractive index n_1 and the other half is filled with water of refractive index n_2. The apparent depth of this vessel when viewed from above will be- [4 marks]

(A) $\frac{2d(n_1+n_2)}{n_1n_2}$

(B) $\frac{d(n_1+n_2)}{2n_1n_2}$

(C) $\frac{dn_1n_2}{2(n_1+n_2)}$

(D) $\frac{dn_1n_2}{(n_1+n_2)}$

Correct Answer: (B) $\frac{d(n_1+n_2)}{2n_1n_2}$

Solution:

Concept:

- For near-normal viewing from air, a layer of real thickness t and refractive index n contributes an apparent thickness t/n.
- For plane parallel layers viewed normally, the apparent depth of the bottom is the sum of these reduced thicknesses. Here both oil and water have the same real thickness $d/2$.

Step 1: Find the contribution from each half The vessel has total depth d, so the oil thickness and water thickness are each $d/2$. The oil layer contributes $d/(2n_1)$ to the optical apparent depth measured from the upper surface. The water layer contributes $d/(2n_2)$. The indices enter separately because light traverses two different media.

Step 2: Add the reduced thicknesses The apparent depth is therefore

$$d_{\text{app}} = \frac{d}{2n_1} + \frac{d}{2n_2} = \frac{d}{2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right) = \frac{d(n_1+n_2)}{2n_1n_2}.$$

This result is independent of which liquid is above the other for the paraxial apparent-depth calculation, since interchanging the two contributions leaves their sum unchanged.

Step 3: Test the expression against limiting cases If $n_1 = n_2 = n$, the two liquids act optically as one layer and the formula reduces to d/n, as expected. If both indices equal unity, it reduces to the actual depth d. Option (A) is four times the required expression. Options (C) and (D) fail the equal-index test, giving $dn/4$ and $dn/2$, respectively, instead of d/n.

Step 4: Conclusion The apparent depth is $\boxed{\frac{d(n_1+n_2)}{2n_1n_2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For layered apparent-depth questions, add thickness divided by refractive index for each layer instead of averaging the indices.

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36. The difference between threshold wavelengths for two metal surfaces A and B having work function $\phi_A = 9$ eV and $\phi_B = 4 \cdot 5$ eV in nm is:

{ **Given, hc** $= 1242$ eVnm} [4 marks]

(A) 264

(B) 138

(C) 540

(D) 276

Correct Answer: (B) 138

Solution:

Concept:

- The work function ϕ is the minimum energy required to remove an electron from a metal surface. At the threshold, the emitted electron has zero kinetic energy.
- The threshold wavelength obeys $hc/\lambda_0 = \phi$, hence $\lambda_0 = hc/\phi$. With hc in eV nm and ϕ in eV, the result is directly in nm.

Step 1: Calculate each threshold wavelength Use the same supplied value of hc for both metals. For surface A,

$$\lambda_A = \frac{1242}{9} = 138\,\text{nm}.$$

For surface B,

$$\lambda_B = \frac{1242}{4.5} = 276\,\text{nm}.$$

Halving the work function doubles the largest wavelength capable of producing photoemission, consistent with these results.

Step 2: Take the difference and compare the choices The question asks for the difference of the wavelengths, rather than either individual threshold. Since B has the lower work function, its threshold wavelength is larger:

$$\Delta\lambda = \lambda_B - \lambda_A = 276 - 138 = 138\,\text{nm}.$$

The value 276 nm is the threshold for B alone, so option (D) does not answer the requested difference. Neither 264 nm nor 540 nm follows from the given threshold relation; thus options (A) and (C) are excluded.

Step 3: Conclusion The difference between the threshold wavelengths is $\boxed{138\,\text{nm}}$. Hence, the correct option is (B).

Quick Tip to Solve: When one work function is half the other, the threshold wavelengths are in the ratio 2:1; their difference equals the smaller threshold wavelength.

37. The rms speed of oxygen molecule in a vessel at particular temperature is $\left(1+\frac{5}{x}\right)^{\frac{1}{2}} v$, where v is the average speed of the molecule. The value of x will be:

(**Take** $\pi=\frac{22}{7}$) [4 marks]

(A) 4

(B) 8

(C) 28

(D) 27

Correct Answer: (C) 28

Solution:

Concept:

- For a gas molecule of mass m at absolute temperature T, the rms speed is $v_{\text{rms}}=\sqrt{3k_BT/m}$.
- The average speed is $v=\sqrt{8k_BT/(\pi m)}$. Their ratio is independent of the gas mass and temperature when both speeds describe the same equilibrium gas.

Step 1: Form the ratio of the two speeds Divide the rms-speed expression by the average-speed expression and square the result:

$$\left(\frac{v_{\text{rms}}}{v}\right)^2=\frac{3k_BT/m}{8k_BT/(\pi m)}=\frac{3\pi}{8}.$$

All factors containing oxygen molecular mass and the particular temperature cancel. No numerical value of either quantity is required.

Step 2: Compare with the stated expression The problem gives $v_{\text{rms}}=\sqrt{1+5/x}\,v$. Therefore, using the specified approximation for π,

$$1+\frac{5}{x}=\frac{3\pi}{8}=\frac{3}{8}\frac{22}{7}=\frac{33}{28}.$$

Subtract unity before solving:

$$\frac{5}{x}=\frac{33-28}{28}=\frac{5}{28},\qquad x=28.$$

For the other proposed values 4, 8 and 27, the quantity $1+5/x$ is respectively $9/4$, $13/8$ and $32/27$; none equals $33/28$. This also checks the choice directly.

Step 3: Conclusion The required dimensionless value is $\boxed{x=28}$. Hence, the correct option is (C).

Quick Tip to Solve: For rms speed versus average speed, square the ratio first: $v_{\text{rms}}^2/v_{\text{avg}}^2=3\pi/8$; the gas and temperature then disappear.

38. The source of time varying magnetic field may be

(A) a permanent magnet

(B) an electric field changing linearly with time

(C) direct current

(D) a decelerating charge particle

(E) an antenna fed with a digital signal

Choose the correct answer from the options given below: [4 marks]

(A) (D) only

(B) (A) only

(C) (B) and (D) only

(D) (C) and (E) only

Correct Answer: Statements (D) and (E) are valid; their combination is absent from the printed options.

Solution:

Concept:

- A stationary permanent magnet and a steady direct current produce time-independent magnetic fields at fixed observation points. A changing charge velocity or changing current can produce a time-dependent field.
- The Maxwell displacement-current term is proportional to $\partial\vec{E}/\partial t$. A linearly changing electric field has a constant derivative, so this term alone does not establish a time-varying magnetic field.

Step 1: Evaluate statements A, B and C A permanent magnet at rest has a static magnetic field, so statement (A) is not a time-varying source as described. Direct current means a steady current, making (C) false for this purpose. For statement (B), write $\vec{E} = \vec{E}_0 + \vec{a}t$. Its displacement-current contribution is constant because $\partial\vec{E}/\partial t = \vec{a}$; a magnetic field changing with time does not follow from a constant displacement current.

Step 2: Evaluate the changing-charge and antenna sources A decelerating charged particle has a changing velocity, so its magnetic field generally changes with time: statement (D) is valid. An antenna fed with a digital signal has a current that changes during the signal transitions and consequently produces time-varying electromagnetic fields: statement (E) is also valid. The physically supported combination is therefore (D) and (E).

Step 3: Check every printed alternative The printed alternatives are (D) only, (A) only, (B) and (D) only, and (C) and (E) only. The first omits the valid antenna source; the second selects a static source; the third includes (B) and omits (E); the fourth includes direct current and omits (D). Thus none gives the valid combination. This follows from the literal printed wording, without changing either a statement or an alternative.

Step 4: Conclusion The valid sources are statements (D) and (E), but this combination is absent from the printed alternatives. The question-sheet key selects option (A), meaning (D) only; it excludes the time-varying field produced during digital-signal transitions in the antenna. Consequently there is no fully correct printed option for the literal question.

Quick Tip to Solve: Distinguish a field from its time derivative: a linearly changing electric field has constant displacement current, whereas digital antenna transitions entail changing currents.

39. Two charges each of magnitude 0.01 C and separated by a distance of 0.4 mm constitute an electric dipole. If the dipole is placed in an uniform electric field '$\vec{E}$' of 10 dyne/C making 30° angle with $\vec{E}$, the magnitude of torque acting on dipole is: [4 marks]

(A) $4 \cdot 0 \times 10^{-10}$ Nm

(B) 1.5×10^{-9} Nm

(C) 1.0×10^{-8} Nm

(D) 2.0×10^{-10} Nm

Correct Answer: (D) 2.0×10^{-10} Nm

Solution:

Concept:

- For two equal and opposite charges separated by the full distance d, the electric dipole moment has magnitude $p = qd$, directed from the negative charge to the positive charge.
- The torque magnitude in a uniform electric field is $\tau = pE \sin\theta$, where θ is the angle between the dipole moment and the field. All quantities must be in consistent units.

Step 1: Convert the separation and field to SI units The quoted separation is the full charge-to-charge distance; it must not be doubled. Since $1\,\text{mm} = 10^{-3}\,\text{m}$,

$$d = 0.4\,\text{mm} = 4 \times 10^{-4}\,\text{m}.$$

Also, $1\,\text{dyne} = 10^{-5}\,\text{N}$, so

$$E = 10\,\text{dyne/C} = 10^{-4}\,\text{N/C}.$$

The dipole moment is therefore

$$p = (0.01)(4 \times 10^{-4}) = 4 \times 10^{-6}\,\text{C m}.$$

Step 2: Apply the torque formula and check the options The stated angle gives $\sin 30^\circ = 1/2$. Thus

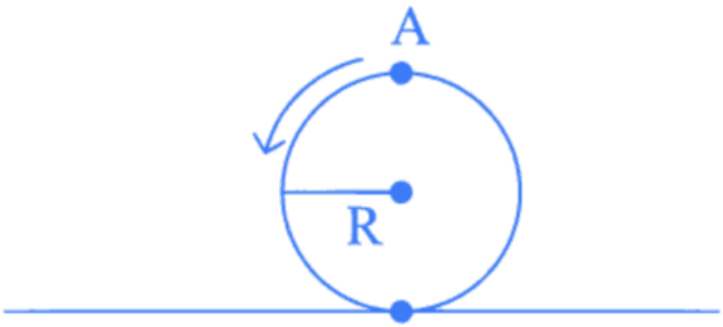
$$\tau = (4 \times 10^{-6})(10^{-4})\frac{1}{2}$$
$$= 2.0 \times 10^{-10}\,\text{N m}.$$

Option (A) is twice this result and would incorrectly omit the sine factor. Options (B) and (C) have neither the magnitude given by the SI substitution nor the proper angular reduction. The unit check is $(\text{C m})(\text{N/C}) = \text{N m}$.

Step 3: Conclusion The magnitude of torque is $\boxed{2.0 \times 10^{-10}\,\text{N m}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the full stated charge separation in $p = qd$, and convert dyne to newton before applying $pE \sin\theta$.

40. A disc is rolling without slipping on a surface. The radius of the disc is R. At $t = 0$, the top most point on the disc is A as shown in figure. When the disc completes half of its rotation, the displacement of point A from its initial position is [4 marks]



(A) $R\sqrt{(\pi^2+1)}$

(B) $2R$

(C) $R\sqrt{(\pi^2+4)}$

(D) $2R\sqrt{(1+4\pi^2)}$

Correct Answer: (C) $R\sqrt{(\pi^2+4)}$

Solution:

Concept:

- In rolling without slipping, a rotation through angle θ translates the centre of the disc by a horizontal distance $R\theta$. The centre remains at height R.
- Displacement is the straight-line vector between the initial and final positions of the marked point. It is different from the curved distance travelled by that point along its cycloidal path.

Step 1: Locate the marked point before and after half a turn Take the initial centre as $(0, R)$, with the horizontal surface as $y = 0$. Point A begins at the top, so its initial position is $(0, 2R)$. After half a rotation, $\theta = \pi$, and the centre has moved horizontally by πR. The marked point, initially at the top, is now at the bottom of the disc. Hence its final position is $(\pi R, 0)$ for rightward rolling; leftward rolling reverses only the horizontal sign.

Step 2: Calculate the displacement magnitude The horizontal and vertical changes in position have magnitudes πR and $2R$, respectively. They are perpendicular, giving

$$|\Delta\vec{r}| = \sqrt{(\pi R)^2 + (2R)^2}$$
$$= R\sqrt{\pi^2 + 4}.$$

Option (A) uses only R for the vertical change although A drops a full diameter. Option (B) counts the vertical component alone and misses translation. Option (D) does not correspond to the two endpoint-coordinate changes. The result is the same for either rolling direction.

Step 3: Conclusion The displacement magnitude of point A is $\boxed{R\sqrt{\pi^2+4}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a marked rolling point, locate the two endpoints: half a turn moves the centre by πR, while a point starting at the top drops by $2R$.

41.

$$^{238}_{92}A \rightarrow ^{234}_{90}B + ^{4}_{2}D + Q$$

In the given nuclear reaction, the approximate amount of energy released will be:
[Given, mass of $^{238}_{92}$ A $= 238.05079 \times 931.5$ MeV/c^2,
mass of $^{234}_{90}B = 234 \cdot 04363 \times 931 \cdot 5$ MeV/c^2,
mass of $^{4}_{2}D = 4 \cdot 00260 \times 931 \cdot 5$ MeV/c^2] [4 marks]

(A) 2.12 MeV

(B) 4.25 MeV

(C) 3.82 MeV

(D) 5.9 MeV

Correct Answer: (B) 4.25 MeV

Solution:

Concept:

- The energy released in a nuclear reaction is the decrease in total rest energy: $Q = (m_{\text{initial}} - m_{\text{final}})c^2$. A positive mass defect means that energy is released.
- The supplied masses are numerical mass values multiplied by $931.5\,\text{MeV}/c^2$. Subtract the numerical values first, then multiply the mass defect by 931.5 to obtain MeV.

Step 1: Find the total final mass and mass defect The reaction conserves the mass number, $238 = 234 + 4$, and the charge number, $92 = 90 + 2$. Both product masses must be subtracted from the parent mass. In the supplied common mass unit,

$$m_{\text{final}} = 234.04363 + 4.00260 = 238.04623.$$

Hence the numerical mass defect is

$$\Delta m = 238.05079 - 238.04623 = 0.00456.$$

The positive sign confirms an exothermic decay rather than an energy-absorbing reaction.

Step 2: Convert the mass defect into released energy Use the given conversion factor once:

$$\begin{aligned} Q &= (0.00456)(931.5)\,\text{MeV} \\ &= 4.24764\,\text{MeV} \\ &\simeq 4.25\,\text{MeV}. \end{aligned}$$

The factor c^2 cancels the c^{-2} in the stated mass units. Options 2.12, 3.82 and 5.9 MeV do not follow from the supplied mass difference. In particular, there is no reason to halve the total release; its division between product kinetic energies does not change the reaction Q-value.

Step 3: Conclusion The approximate energy released is $\boxed{4.25\,\text{MeV}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a decay Q-value, subtract the sum of all product masses before converting to energy; do not round either large mass early.

42. Under isothermal condition, the pressure of a gas is given by $\mathrm{P} = a\ \mathrm{V}^{-3}$, where a is a constant and V is the volume of the gas. The bulk modulus at constant temperature is equal to [4 marks]

(A) $\frac{P}{2}$

(B) 2 P

(C) 3 P

(D) P

Correct Answer: (C) 3 P

Solution:

Concept:

- Bulk modulus measures the pressure rise produced by a small fractional volume decrease: $B_T = -V(\partial P/\partial V)_T$. The minus sign gives a positive modulus for a stable material whose pressure falls when its volume increases.
- The stated isothermal equation of state is $P = aV^{-3}$, with a constant. Differentiate this specific relation at fixed temperature rather than replacing it by the ideal-gas law.

Step 1: Differentiate pressure with respect to volume The power rule applied to the given expression gives

$$\left(\frac{\partial P}{\partial V}\right)_T = -3aV^{-4}.$$

The derivative is negative, as expected: increasing volume lowers the pressure. Holding temperature fixed is already built into the supplied relation, so no adiabatic exponent or additional gas-law assumption is needed.

Step 2: Insert the derivative into the modulus definition Multiply the derivative by $-V$:

$$\begin{aligned} B_T &= -V(-3aV^{-4}) \\ &= 3aV^{-3} \\ &= 3P. \end{aligned}$$

The result has pressure units, since volume times pressure-per-volume is pressure. Option (D), P, would apply to an ideal-gas isotherm $P \propto V^{-1}$, but that is not the relation printed here. Options (A) and (B) correspond to coefficients $1/2$ and 2 rather than the exponent 3 obtained by differentiation.

Step 3: Conclusion The bulk modulus at constant temperature is $\boxed{B_T = 3P}$. Hence, the correct option is (C).

Quick Tip to Solve: For any equation of state $P \propto V^{-n}$, direct differentiation gives the bulk modulus $B = nP$ along that specified process.

43. Two trains 'A' and 'B' of length 'l' and '$4l$' are travelling into a tunnel of length 'L' in parallel tracks from opposite directions with velocities $108\ \mathrm{km/h}$ and $72\ \mathrm{km/h}$, respectively. If train 'A' takes $35\ \mathrm{s}$ less time than train 'B' to cross the tunnel then. length 'L' of tunnel is :

(Given $\mathrm{L} = 60l$) [4 marks]

(A) 900 m

(B) 1200 m

(C) 1800 m

(D) 2700 m

Correct Answer: (C) 1800 m

Solution:

Concept:

- A train clears a tunnel only after its rear end exits. Its front therefore travels the tunnel length plus the train length, so crossing time is total distance divided by speed.
- Each train crosses the stationary tunnel separately. Opposite travel directions do not require relative speed here; the measured quantities are the two individual crossing times.

Step 1: Convert the two speeds Use SI units consistently before comparing the times. The speeds of A and B are respectively

$$v_A = 108\frac{5}{18} = 30\,\mathrm{m\,s^{-1}}, \qquad v_B = 72\frac{5}{18} = 20\,\mathrm{m\,s^{-1}}.$$

Train A has length l, whereas train B has length $4l$. Thus the longer train is also the slower one, consistent with B taking more time.

Step 2: Express the difference of crossing times The given tunnel length is $L = 60l$. Hence the required distances and times are

$$t_A = \frac{60l + l}{30} = \frac{61l}{30}, \qquad t_B = \frac{60l + 4l}{20} = \frac{64l}{20}.$$

The fact that A takes 35 seconds less means $t_B - t_A = 35$, with this order of subtraction.

Step 3: Determine the tunnel length and check it Substituting the times gives

$$\left(\frac{64}{20} - \frac{61}{30}\right) l = 35 \quad \Rightarrow \quad \frac{7l}{6} = 35.$$

Therefore $l = 30\,\mathrm{m}$, so $L = 60(30) = 1800\,\mathrm{m}$. For this value, $t_A = 61\,\mathrm{s}$ and $t_B = 96\,\mathrm{s}$, whose difference is exactly 35 seconds. The other listed lengths do not satisfy the time difference.

Step 4: Conclusion The tunnel is $\boxed{1800\,\mathrm{m}}$ long. Hence, the correct option is (C).

Quick Tip to Solve: For complete passage through a tunnel, add the train length to the tunnel length. Use relative speed only when comparing the trains crossing each other.

44. Which of the following Maxwell's equation is valid for time varying conditions but not valid for static conditions : [4 marks]

(A) $\oint \overrightarrow{\mathrm{E}} \cdot \overrightarrow{dl} = 0$

(B) $\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I$

(C) $\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{\partial \phi_B}{\partial t}$

(D) $\oint \vec{D} \cdot \overrightarrow{dA} = Q$

Correct Answer: (C) $\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{\partial \phi_B}{\partial t}$

Solution:

Concept:

- Faraday's induction law relates circulation of electric field around a closed loop to the rate of change of magnetic flux: $\oint \vec{E} \cdot d\vec{l} = -d\Phi_B/dt$. A changing magnetic flux creates a non-conservative electric field.
- In electrostatics the electric field is conservative and its closed-loop integral vanishes. Gauss's law, however, connects enclosed charge to electric displacement flux in both static and time-dependent situations.

Step 1: Identify the equation describing a time-dependent effect When magnetic flux through a fixed loop varies, Faraday's law gives

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}.$$

The nonzero right-hand side describes induced emf. Its minus sign expresses Lenz's law: the induced response opposes the change of flux that produces it. This is precisely the equation displayed in option (C).

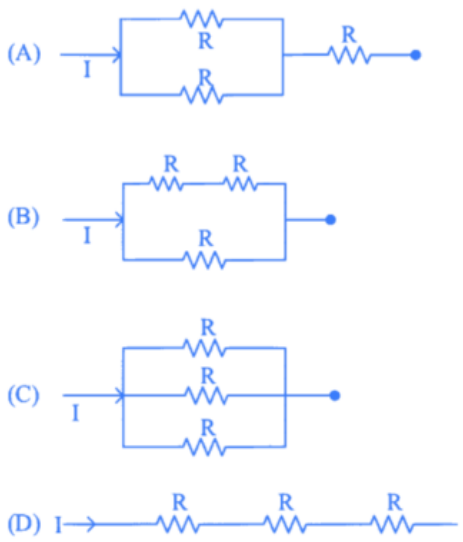
Step 2: Compare the remaining expressions Option (A) is the zero-circulation result used in electrostatics and cannot describe a general induced electric field. Option (B), $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$, is the magnetostatic form of Ampere's law: a general time-varying case requires the displacement-current term. Option (D) is Gauss's law for $\vec{D}$, which remains applicable to static fields and does not isolate the induction effect.

Step 3: Interpret the static limit carefully For static magnetic flux, $\partial \Phi_B/\partial t = 0$, so Faraday's law reduces to $\oint \vec{E} \cdot d\vec{l} = 0$. Thus the general law remains mathematically valid in the static limit as well. The wording is understood here as asking which displayed equation explicitly describes time-varying electromagnetic induction rather than a specifically static relation.

Step 4: Conclusion Faraday's law is the intended time-varying equation, $\oint \vec{E} \cdot d\vec{l} = -\partial \Phi_B/\partial t$. Hence, the correct option is (C). Strictly, this general law also holds in the static limit, when both sides vanish.

Quick Tip to Solve: Spot a time derivative of magnetic flux to identify Faraday induction; remember that Ampere's time-dependent law also needs displacement current.

45. Different combination of 3 resistors of equal resistance R are shown in the figures. The increasing order for power dissipation is: [4 marks]



(A) $P_B < P_C < P_D < P_A$

(B) $P_C < P_B < P_A < P_D$

(C) $P_C < P_D < P_A < P_B$

(D) $P_A < P_B < P_C < P_D$

Correct Answer: (B) $P_C < P_B < P_A < P_D$

Solution:

Concept:

- The arrows in all four networks specify the same total input current I. Their total powers are therefore $P = I^2 R_{\text{eq}}$, so increasing equivalent resistance means increasing dissipation.
- Resistances add directly in series. In parallel, their reciprocals add; two equal resistors give $R/2$ and three equal resistors give $R/3$.

Step 1: Reduce networks A and B In network A, the two resistors in the rectangular loop are parallel and the right-hand resistor is in series with that pair. Therefore

$$R_A = (R \parallel R) + R = \frac{R}{2} + R = \frac{3R}{2}.$$

In B the upper branch contains two series resistors, while the lower branch contains one:

$$R_B = (2R) \parallel R = \frac{(2R)R}{2R + R} = \frac{2R}{3}.$$

Use the full input current for each equivalent resistance, not the current in just one branch.

Step 2: Reduce networks C and D Network C has three individual resistors connected between the same two junctions. Thus

$$\frac{1}{R_C} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R}, \qquad R_C = \frac{R}{3}.$$

Network D has all three resistors on one unbranched path, so $R_D = 3R$. These equivalents include the dissipation in every resistor of their respective networks.

Step 3: Compare the total powers Since I^2 is common and positive,

$$\frac{R}{3} < \frac{2R}{3} < \frac{3R}{2} < 3R \quad \Rightarrow \quad P_C < P_B < P_A < P_D.$$

This agrees only with (B). Option (A) incorrectly puts B below C, option (C) incorrectly places D below A, and option (D) incorrectly makes A the least dissipative. Ranking by V^2/R would reverse the order only for equal applied voltages, which the arrows do not specify.

Step 4: Conclusion The increasing power order is $\boxed{P_C < P_B < P_A < P_D}$. Hence, the correct option is (B).

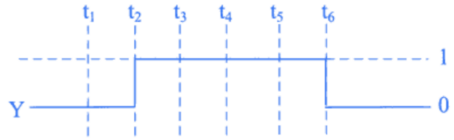
Quick Tip to Solve: First identify what is held equal: for equal current rank power with resistance; for equal voltage rank it in the reverse order.

46. For the following circuit and given inputs A and B, choose the correct option for output 'Y'

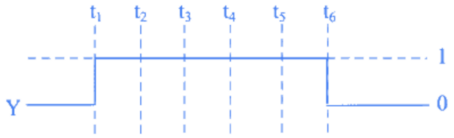
[4 marks]



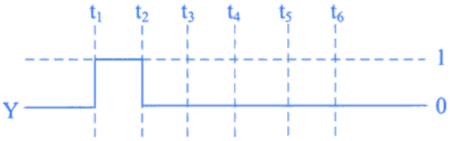
(A)



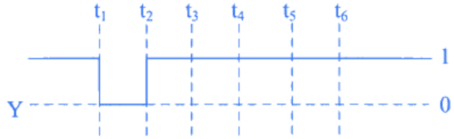
(B)



(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- Tying both inputs of a NOR gate to A gives $\overline{A+A}=\overline{A}$. Tying both inputs of an OR gate to B gives $B+B=B$.
- The last gate is NAND. Hence the circuit output is $Y=\overline{\overline{A}\,B}=A+\overline{B}$ by De Morgan's law. It is low only for $A=0, B=1$.

Step 1: Simplify the circuit before tracing the signals The small bubble after the upper OR-shaped gate denotes inversion, whereas the lower OR-shaped gate has no output bubble. The two signals entering the final NAND gate are consequently $\overline{A}$ and B. Thus

$$Y=\overline{\overline{A}\cdot B}=A+\overline{B}.$$

The truth table is $(A,B;Y)=(0,0;1),(0,1;0),(1,0;1),(1,1;1)$. Only one input combination produces a low output.

Step 2: Read each interval of the supplied input traces Before t_1, both inputs are low and $Y=1$. Between t_1 and t_2, B is high while A remains low, so $Y=0$. Between t_2 and t_3, both are high and $Y=1$. Between t_3 and t_4, both are low, again giving $Y=1$. Between t_4 and t_5, both are high; between t_5 and t_6, A is high and B low. Both intervals also give $Y=1$. After t_6, both return low and Y stays high.

Step 3: Match the output with the offered waveform The output has a single low pulse extending from t_1 to t_2, with a high level everywhere else. That is waveform (D). Waveform (C) is its complement and is therefore wrong. Waveforms (A) and (B) start low and end low, contradicting the NAND output when both external inputs are zero. Ideal gates are used here, so propagation delays are not added.

Step 4: Conclusion The output is high except for $t_1<t<t_2$, when it is low. Hence, the correct option is (D).

Quick Tip to Solve: Simplify tied-input gates first, then find the single input combination that forces the final output low; this is faster than redrawing every signal.

47. A body of mass (5 ± 0.5) kg is moving with a velocity of (20 ± 0.4) m/s. Its kinetic energy will be [4 marks]

(A) (1000 ± 140) J

(B) (500 ± 0.14) J

(C) (1000 ± 0.14) J

(D) (500 ± 140) J

Correct Answer: (A) (1000 ± 140) J

Solution:

Concept:

- For a product of measured quantities raised to powers, the maximum fractional uncertainty is the sum of the absolute powers times the corresponding fractional uncertainties.
- For $K = \frac{1}{2}mv^2$, the exact factor $1/2$ contributes no uncertainty, and $\Delta K/K = \Delta m/m + 2\Delta v/v$ to first order.

Step 1: Calculate the nominal kinetic energy The central measured values are $m = 5\,\text{kg}$ and $v = 20\,\text{m}\,\text{s}^{-1}$. Therefore

$$K = \frac{1}{2}(5)(20)^2 = \frac{5}{2}(400) = 1000\,\text{J}.$$

The square on velocity is essential: a central value of 500 J would omit a factor of two. The uncertainties are then propagated about this central energy.

Step 2: Add the maximum fractional contributions The given absolute uncertainties are $\Delta m = 0.5\,\text{kg}$ and $\Delta v = 0.4\,\text{m}\,\text{s}^{-1}$. Consequently

$$\frac{\Delta m}{m} = \frac{0.5}{5} = 0.10, \qquad \frac{\Delta v}{v} = \frac{0.4}{20} = 0.02.$$

Because energy contains v^2, velocity contributes twice its fractional uncertainty:

$$\frac{\Delta K}{K} = 0.10 + 2(0.02) = 0.14.$$

This is the usual first-order maximum-error estimate, not a root-sum-square statistical uncertainty.

Step 3: Convert the fractional error to joules The number 0.14 is a dimensionless fractional error, equivalent to 14 percent. It is not the absolute error in joules. Multiplication by the nominal energy yields

$$\Delta K = (1000)(0.14) = 140\,\text{J}.$$

Hence options (B) and (D) have the wrong central value, while (C) confuses a fractional uncertainty with an absolute one. The specified options make the intended first-order treatment explicit.

Step 4: Conclusion The kinetic energy is $\boxed{(1000 \pm 140)\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: Keep fractional error dimensionless until the final multiplication by the measured quantity; a squared speed doubles its fractional error.

48. The ratio of powers of two motors is $\frac{3\sqrt{x}}{\sqrt{x}+1}$, that are capable of raising $300\ \text{kg}$ water in 5 minutes and $50\ \text{kg}$ water in 2 minutes respectively from a well of $100\ \text{m}$ deep. The value of x will be [4 marks]

(A) 16

(B) 4

(C) 2

(D) 2.4

Correct Answer: (A) 16

Solution:

Concept:

- To lift water of mass m through a vertical height h, the required gravitational work is $W = mgh$. Average useful lifting power is $P = W/t = mgh/t$.
- For two motors lifting water through the same height, g and h cancel in the power ratio. Compare their mass-lifting rates using the same time unit.

Step 1: Find the physical power ratio The first motor lifts 300 kg in 5 minutes and the second lifts 50 kg in 2 minutes. Both lift through 100 m, so

$$\frac{P_1}{P_2} = \frac{300g(100)/5}{50g(100)/2} = \frac{300 \times 2}{5 \times 50} = \frac{12}{5}.$$

The common conversion from minutes to seconds cancels, provided minutes are used for both. The work ratio alone would be 6, but power also accounts for the different durations.

Step 2: Solve the equation involving the square root Equate this value to the given expression and set $u = \sqrt{x}$, with $u \geq 0$:

$$\frac{3u}{u+1} = \frac{12}{5}.$$

Since $u + 1 > 0$, cross-multiplication is valid. It gives

$$15u = 12u + 12, \qquad 3u = 12, \qquad u = 4.$$

Therefore $x = u^2 = 16$, rather than 4; the latter is the square root of the required variable.

Step 3: Check the result and distinguish the distractors For $x = 16$, the printed expression is $3(4)/(4+1) = 12/5$, which equals the physical power ratio. The value 4 would give $3(2)/(2+1) = 2$, not 2.4. The other choices 2 and 2.4 likewise cannot have square root 4. In particular, 2.4 is the ratio of powers itself, not the value of x. No numerical value of g is needed because it cancels.

Step 4: Conclusion The required value is $\boxed{x = 16}$, a dimensionless number. Hence, the correct option is (A).

Quick Tip to Solve: For lifting through equal heights, compare m/t, then introduce $u = \sqrt{x}$ so the algebra becomes linear; square only at the end.

49. Two bodies are having kinetic energies in the ratio 16 : 9. If they have same linear momentum, the ratio of their masses respectively is : [4 marks]

(A) $3 : 4$

(B) $4 : 3$

(C) $9 : 16$

(D) $16 : 9$

Correct Answer: (C) $9 : 16$

Solution:

Concept:

- For nonrelativistic motion, linear momentum is $p = mv$ and kinetic energy is $K = \frac{1}{2}mv^2$. Eliminating velocity gives $K = p^2/(2m)$.
- At a fixed nonzero momentum, kinetic energy is inversely proportional to mass. Thus the body with greater kinetic energy must have the smaller mass.

Step 1: Express both energies using the common momentum Let the two masses be m_1, m_2, with the same momentum magnitude p. Their velocities need not be equal: $v_1 = p/m_1$ and $v_2 = p/m_2$. Substitution into kinetic energy gives

$$K_1 = \frac{p^2}{2m_1}, \qquad K_2 = \frac{p^2}{2m_2}.$$

This step is necessary because direct proportionality between energy and mass applies at fixed velocity, not at fixed momentum.

Step 2: Use the given energy ratio Dividing the two expressions cancels the common factor $p^2/2$:

$$\frac{K_1}{K_2} = \frac{p^2/(2m_1)}{p^2/(2m_2)} = \frac{m_2}{m_1} = \frac{16}{9}.$$

Taking the reciprocal gives

$$\frac{m_1}{m_2} = \frac{9}{16}.$$

The masses must be reported in the same order as the energies, so the answer is 9:16 rather than 16:9.

Step 3: Check the ordering and other choices The first body has more kinetic energy, so its mass must indeed be smaller for the same momentum. As a check, choose masses $9m_0$ and $16m_0$: their energies are proportional to $1/9$ and $1/16$, giving ratio 16:9 as required. Options (A) and (B) use the square-root ratio, which would arise from a different condition such as equal masses when comparing momenta. Option (D) reverses the inverse dependence on mass.

Step 4: Conclusion The masses are in the ratio $\boxed{m_1 : m_2 = 9 : 16}$. Hence, the correct option is (C).

Quick Tip to Solve: Whenever equal momentum is specified, rewrite energy as $p^2/(2m)$; the mass ratio is the reciprocal of the kinetic-energy ratio.

50. Match List I with List II.

	List I (Layer of atmosphere)		List II (Approximate height)
(A)	F_1**-layer**	**(I)**	10 km
(B)	**D-layer**	**(II)**	170–190 km
(C)	**Troposphere**	**(III)**	100 km
(D)	**E-layer**	**(IV)**	65–75 km

Heights are above the earth's surface. Choose the correct answer from the options given below:

[4 marks]

(A) A - II, B - IV, C - III, D - I

(B) A - II, B-I, C - IV, D - III

(C) A - II, B - IV, C - I, D - III

(D) A - III, B - IV, C - I, D - II

Correct Answer: (C) A - II, B - IV, C - I, D - III

Solution:

Concept:

- The troposphere is the lowest atmospheric region and extends approximately 10 km above the surface in this classification.
- The ionospheric regions are arranged in increasing height as D, E and F. Their approximate heights distinguish the layers that affect radio propagation.

Step 1: Match the lowest two layers Start with the layer nearest the earth. The troposphere occupies the lowest region, corresponding to the listed height of 10 km. Thus C matches I. The D-layer lies above the troposphere and is the lowest of the listed ionospheric layers. Its approximate range is 65–75 km, so B matches IV. This immediately excludes option (A), which puts the troposphere at 100 km, and option (B), which places the D-layer at 10 km.

Step 2: Match the E and F layers The E-layer is approximately 100 km above the surface, so D matches III. The higher F_1-layer is at approximately 170–190 km, giving A with II. Therefore the complete correspondence is

$$A:\text{II}, \quad B:\text{IV}, \quad C:\text{I}, \quad D:\text{III}.$$

Option (D) reverses the E and F layer heights and is therefore incorrect. Option (C) alone preserves all four layer-height associations.

Step 3: Conclusion The correct matching is A–II, B–IV, C–I, D–III. Hence, the correct option is (C).

Quick Tip to Solve: Remember the height order: troposphere, D, E, then F. Match the lowest layer first to eliminate distractors quickly.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. When a resistance of $5\ \Omega$ is shunted with a moving coil galvanometer, it shows a full scale deflection for a current of $250\ \text{mA}$, however when $1050\ \Omega$ resistance is connected with it in series, it gives full scale deflection for 25 volt. The resistance of galvanometer is _____ Ω. [4 marks]

Solution:

Concept:

- At full-scale deflection, the galvanometer carries its fixed full-scale current I_g, independent of the external conversion circuit.
- A shunt shares the galvanometer voltage, whereas a series resistance shares its current. Use $V = IR$ with each connection separately.

Step 1: Write the shunt equation Let the galvanometer resistance be G ohms. The ammeter reads the total current $I = 250\,\text{mA} = 0.250\,\text{A}$. Hence the current in the $5\,\Omega$ shunt is $0.250 - I_g$, not the whole ammeter current. Equal voltages across the two parallel branches give

$$I_g G = 5(0.250 - I_g), \qquad I_g(G + 5) = 1.25.$$

Consequently, $I_g = 1.25/(G + 5)$. This relation includes the small current still passing through the galvanometer and avoids treating the shunt as the complete circuit.

Step 2: Use the series connection and solve For the voltmeter connection, the galvanometer and the $1050\,\Omega$ resistance carry the same full-scale current. The applied voltage is $25\,\text{V}$, so

$$I_g(G + 1050) = 25.$$

Eliminate I_g between the two equations:

$$\frac{G + 1050}{G + 5} = \frac{25}{1.25} = 20, \qquad G + 1050 = 20G + 100.$$

Thus $19G = 950$, giving $G = 50\,\Omega$. As a check, $I_g = 25/1100 = 1/44\,\text{A}$; the corresponding shunt current is $0.250 - 1/44 = 10/44\,\text{A}$. Their voltage drops are both $50/44\,\text{V}$, consistent with the parallel connection.

Step 3: Conclusion The resistance of the galvanometer is $\boxed{50\,\Omega}$. The required numerical entry is 50.

Quick Tip to Solve: Write one full-scale equation for each conversion and eliminate the unchanged galvanometer current; do not equate it to the ammeter reading.

52. A fish rising vertically upward with a uniform velocity of $8\ \text{ms}^{-1}$, observes that a bird is diving vertically downward towards the fish with the velocity of $12\ \text{ms}^{-1}$. If the refractive index of water is $\frac{4}{3}$, then the actual velocity of the diving bird to pick the fish, will be _____ ms^{-1}. [4 marks]

Solution:

Concept:

- For an object in air observed from water through a plane surface, its apparent height is $h_{\text{app}} = (n_w/n_a)h$, with $n_w/n_a = 4/3$.
- The approach speed is the rate of decrease of separation. Oppositely directed velocities add in magnitude when forming the relative speed.

Step 1: Find the apparent motion of the bird Let the actual downward speed of the bird be $v\,\text{m s}^{-1}$, and measure heights relative to the stationary water surface. Its height h in air appears to the fish as

$$h_{\text{app}} = \frac{4}{3}h.$$

Because this magnification factor is constant, differentiating with respect to time gives the apparent downward speed $4v/3$. The bird therefore appears farther from the surface than it really is and its apparent vertical speed relative to that surface is also magnified. The reciprocal factor $3/4$ would instead apply to an underwater object observed from air.

Step 2: Include the upward motion of the fish The fish rises at $8\,\text{m s}^{-1}$ while the apparent bird image moves downward. Therefore their separation decreases at

$$v_{\text{relative}} = 8 + \frac{4}{3}v.$$

The stated observed approach speed is $12\,\text{m s}^{-1}$, so

$$12 = 8 + \frac{4}{3}v, \qquad \frac{4}{3}v = 4, \qquad v = 3\,\text{m s}^{-1}.$$

Checking the interpretation, a bird moving downward at this speed has apparent downward speed $4\,\text{m s}^{-1}$. Together with the fish's upward speed, this gives the specified $12\,\text{m s}^{-1}$. Merely subtracting 8 from 12 gives the apparent speed, which still needs the refraction correction.

Step 3: Conclusion The bird actually dives downward at $\boxed{3\,\text{m s}^{-1}}$. The required numerical entry is 3.

Quick Tip to Solve: First subtract the fish speed to obtain the image speed relative to the surface; then divide by the apparent-height magnification $4/3$.

53. The radius of 2^{nd} orbit of He^+ of Bohr's model is r_1 and that of fourth orbit of Be^{3+} is represented as r_2. Now the ratio $\frac{r_2}{r_1}$ is $x : 1$. The value of x is _____. [4 marks]

Solution:

Concept:

- A one-electron ion has Bohr-orbit radius $r_n = a_0 n^2/Z$, where a_0 is the Bohr radius, n the orbit number and Z the nuclear charge number.
- For ratios of radii, the common factor a_0 cancels. Use the atomic number for Z, rather than the net ionic charge.

Step 1: Find the radius for helium The ion He^+ contains one electron around a nucleus with atomic number $Z_1 = 2$. It therefore obeys the hydrogen-like Bohr formula. For its second orbit, $n_1 = 2$, giving

$$r_1 = a_0 \frac{2^2}{2} = 2a_0.$$

The superscript plus sign denotes the removal of one electron; it does not make the nuclear charge number equal to one. The electron still feels the attraction of a nucleus with two protons. Using $Z = 1$ here would miss the stronger attraction and overestimate the radius.

Step 2: Calculate the beryllium radius and compare Beryllium has atomic number four. Removing three electrons gives the one-electron ion Be^{3+}, so $Z_2 = 4$, while the stated fourth orbit has $n_2 = 4$. Hence

$$r_2 = a_0 \frac{4^2}{4} = 4a_0.$$

The required ratio is consequently

$$\frac{r_2}{r_1} = \frac{4a_0}{2a_0} = 2, \qquad r_2 : r_1 = 2 : 1.$$

This also follows directly as $(n_2/n_1)^2(Z_1/Z_2) = 4 \times (1/2) = 2$. Although the beryllium nucleus attracts the electron more strongly, the larger orbit number contributes a factor of four to the radius, leaving the net factor of two. The ratio is dimensionless, as required for comparison with $x : 1$.

Step 3: Conclusion The ratio is $r_2 : r_1 = 2 : 1$, so $\boxed{x = 2}$.

Quick Tip to Solve: For two hydrogen-like ions, use $r_2/r_1 = (n_2/n_1)^2(Z_1/Z_2)$; the ionic charge is not the nuclear charge number.

54. A potential V_0 is applied across a uniform wire of resistance R. The power dissipation is P_1. The wire is then cut into two equal halves and a potential of V_0 is applied across the length of each half. The total power dissipation across two wires is P_2. The ratio $P_2 : P_1$ is $\sqrt{x} : 1$. The value of x is _____.

[4 marks]

Solution:

Concept:

- A uniform wire has resistance $R = \rho l/A$. Cutting it into equal lengths halves its resistance without changing its material or cross-sectional area.
- When the voltage across a resistor is specified, its dissipated power is $P = V^2/R$. Total power is the sum over both pieces.

Step 1: Calculate the initial power and piece resistance The original wire has resistance R and voltage V_0 across its ends. Its original dissipated power is therefore

$$P_1 = \frac{V_0^2}{R}.$$

Each half has length $l/2$ but the same resistivity ρ and area A. Thus its resistance is

$$R_{\text{half}} = \frac{\rho(l/2)}{A} = \frac{R}{2}.$$

The operation cuts the wire across its length; it does not stretch the wire or halve its cross-sectional area. Consequently no additional area factor enters the resistance.

Step 2: Add the power of both halves The problem specifies the full voltage V_0 across each half separately. The power of one half is consequently

$$P_{\text{half}} = \frac{V_0^2}{R/2} = 2P_1.$$

Both equal pieces dissipate this amount, so the total is

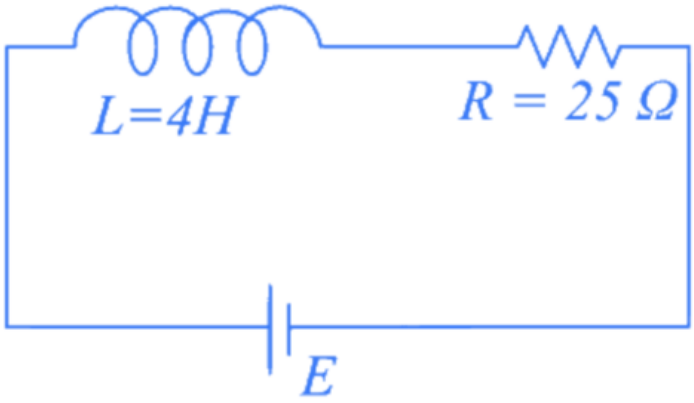
$$P_2 = 2P_{\text{half}} = 4P_1, \qquad P_2 : P_1 = 4 : 1.$$

The given representation is $\sqrt{x} : 1$. Comparing its first term with the calculated power ratio gives $\sqrt{x} = 4$, hence $x = 16$. Keeping only the factor of two for the resistance reduction would omit the second wire. Treating the pieces as series-connected under a single shared voltage would also contradict the condition that each piece has V_0 across it.

Step 3: Conclusion The total power ratio is $4 : 1 = \sqrt{16} : 1$; therefore $\boxed{x = 16}$.

Quick Tip to Solve: At fixed voltage, halving a wire doubles its individual power. Then count both halves before converting the ratio into $\sqrt{x} : 1$.

55. In the given figure, an inductor and a resistor are connected in series with a battery of emf E volt. $\frac{E^a}{2b}$ J/s represents the maximum rate at which the energy is stored in the magnetic field (inductor). The numerical value of $\frac{b}{a}$ will be _____. [4 marks]



Solution:

Concept:

- An ideal inductor stores magnetic energy $U = \frac{1}{2}Li^2$. Its storage rate is $dU/dt = Li\,di/dt$, rather than the total battery power.
- Kirchhoff's law for the series RL circuit is $L\,di/dt = E - Ri$, with the figure giving $R = 25\,\Omega$ and $L = 4\,\text{H}$.

Step 1: Express the storage rate in terms of current During current growth after connection, the battery supplies power Ei. The resistor converts Ri^2 into heat, while the remaining power increases the inductor's magnetic energy. Using the loop equation gives

$$\frac{dU}{dt} = Li\frac{di}{dt} = i(E - Ri) = Ei - Ri^2.$$

The current increases from zero towards E/R, so the current that maximises this quadratic must lie within that accessible interval. The inductance controls how quickly the circuit reaches a particular current, but it cancels from the instantaneous storage rate written as a function of current.

Step 2: Maximise the rate and identify the exponents Complete the square to find the maximum without solving explicitly for time:

$$Ei - Ri^2 = \frac{E^2}{4R} - R\left(i - \frac{E}{2R}\right)^2.$$

Thus the maximum is attained at $i = E/(2R)$, halfway to the final current. With $R = 25\,\Omega$,

$$\left(\frac{dU}{dt}\right)_{\text{max}} = \frac{E^2}{100}\,\text{J s}^{-1}.$$

Compare with the printed form $E^a/(2b)\,\text{J s}^{-1}$. The power of emf is $a = 2$, and the denominator satisfies $2b = 100$, giving $b = 50$. Therefore $b/a = 50/2 = 25$. At the final steady current the storage rate falls to zero because the current is no longer changing.

Step 3: Conclusion The required numerical value is $\boxed{b/a = 25}$.

Quick Tip to Solve: Maximise $Ei - Ri^2$, the battery power minus resistor heating; its peak occurs at half the final current.

56. At a given point of time the value of displacement of a simple harmonic oscillator is given as $\mathrm{y} = \mathrm{A}\cos(30^\circ)$**. If amplitude is** 40 cm **and kinetic energy at that time is** 200 J**, the value of force constant is** 1.0×10^x Nm^{-1}**. The value of** x **is _____.** [4 marks]

Solution:

Concept:

- A harmonic oscillator with force constant k, amplitude A, and displacement y has total energy $\frac{1}{2}kA^2$ and potential energy $\frac{1}{2}ky^2$.
- The kinetic energy is their difference: $K = \frac{1}{2}k(A^2 - y^2)$. The amplitude and displacement must be in metres when energy is in joules.

Step 1: Find the displacement factor in the kinetic energy The given instantaneous displacement is $y = A\cos 30^\circ$. Since $\cos 30^\circ = \sqrt{3}/2$,

$$y^2 = \frac{3}{4}A^2, \qquad A^2 - y^2 = \frac{1}{4}A^2.$$

Substituting this relation into the kinetic energy expression yields

$$K = \frac{1}{2}k\left(\frac{1}{4}A^2\right) = \frac{kA^2}{8}.$$

The oscillator is relatively close to its extreme position, so only one quarter of its total energy is kinetic at this instant. This agrees with the squared cosine displacement and provides a useful check on the energy partition.

Step 2: Substitute the SI amplitude and determine the exponent Convert the amplitude before squaring: $A = 40\,\mathrm{cm} = 0.40\,\mathrm{m}$. With the specified kinetic energy $200\,\mathrm{J}$,

$$200 = \frac{k(0.40)^2}{8} = 0.020k, \qquad k = \frac{200}{0.020} = 10000\,\mathrm{N\,m^{-1}}.$$

Write this in the requested scientific-notation form:

$$k = 1.0 \times 10^4\,\mathrm{N\,m^{-1}}.$$

Comparing with $1.0 \times 10^x\,\mathrm{N\,m^{-1}}$ gives $x = 4$. Checking the result, the total energy is $\frac{1}{2}(10000)(0.16) = 800\,\mathrm{J}$, and the potential energy is three quarters of it, $600\,\mathrm{J}$. Their difference is the given kinetic energy of $200\,\mathrm{J}$.

Step 3: Conclusion The force constant is $1.0 \times 10^4\,\mathrm{N\,m^{-1}}$, so $\boxed{x = 4}$.

Quick Tip to Solve: Use $K = \frac{1}{2}kA^2\sin^2\theta$ when $y = A\cos\theta$, and convert centimetres to metres before squaring.

57. A thin infinite sheet charge and an infinite line charge of respective charge densities $+\sigma$ and $+\lambda$ are placed parallel at 5 m distance from each other. Points 'P' and 'Q' are at $\frac{3}{\pi}$ m and $\frac{4}{\pi}$ m perpendicular distances from line charge towards sheet charge, respectively. 'E_P' and 'E_Q' are the magnitudes of resultant electric field intensities at point 'P' and 'Q', respectively. If $\frac{E_p}{E_0} = \frac{4}{a}$ for $2|\sigma| = |\lambda|$, then the value of a is _____. [4 marks]

Solution:

Concept:

- An infinite positive sheet produces a uniform field of magnitude $E_s = \sigma/(2\varepsilon_0)$, directed away from the sheet. A positive infinite line produces $E_l = \lambda/(2\pi\varepsilon_0 r)$, directed radially away from the line.
- Between the sheet and the parallel line, their electric fields oppose. Superposition therefore requires subtracting their magnitudes, rather than adding them. Here both points lie between the sources because their distances from the line are less than $5\,\mathrm{m}$.

Step 1: Identify the fields at each point Take the positive direction from the line towards the sheet. The line field is positive and the sheet field is negative. At the two stated perpendicular distances,

$$E_{lP} = \frac{\lambda}{2\pi\varepsilon_0(3/\pi)} = \frac{\lambda}{6\varepsilon_0}, \qquad E_{lQ} = \frac{\lambda}{8\varepsilon_0}.$$

The sources carry positive densities and satisfy $\lambda = 2\sigma$, so these fields are respectively $\sigma/(3\varepsilon_0)$ and $\sigma/(4\varepsilon_0)$.

Step 2: Subtract and form the magnitude ratio The sheet field exceeds the line field at both points, so both resultants point towards the line. Their magnitudes are

$$E_P = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{3\varepsilon_0} = \frac{\sigma}{6\varepsilon_0}, \qquad E_Q = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{4\varepsilon_0} = \frac{\sigma}{4\varepsilon_0}.$$

Thus $E_P/E_Q = 2/3$. The denominator printed as E_0 is understood as the field E_Q defined in the preceding sentence; no separate field E_0 is defined. Equating $2/3 = 4/a$ gives $2a = 12$.

Step 3: Conclusion The field ratio is $E_P/E_Q = 2/3$, and hence $\boxed{a = 6}$. This is a dimensionless numerical answer.

Quick Tip to Solve: First determine the directions between positive sources; subtract the sheet and line fields before taking their ratio.

58. A solid sphere is rolling on a horizontal plane without slipping. If the ratio of angular momentum about axis of rotation of the sphere to the total energy of moving sphere is $\pi : 22$ then, the value of its angular speed will be _____ rad/s. [4 marks]

Solution:

Concept:

- For a solid sphere, the moment of inertia about its central rotation axis is $I = \frac{2}{5}MR^2$, where M is its mass and R its radius. Its spin angular momentum about this axis is $L = I\omega$.
- Rolling without slipping relates the centre-of-mass speed to angular speed by $v = R\omega$. The total

kinetic energy includes both translation and rotation: $K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$.

Step 1: Express the total energy in terms of angular speed Substitute the rolling condition into the translational term, and use the central moment of inertia in the rotational term:

$$K = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{7}{10}MR^2\omega^2.$$

The translational contribution cannot be omitted, even though the numerator is the sphere's spin angular momentum. That numerator is $L = \frac{2}{5}MR^2\omega$, evaluated about the central rotation axis.

Step 2: Use the stated ratio Dividing cancels the common mass and squared radius:

$$\frac{L}{K} = \frac{(2/5)MR^2\omega}{(7/10)MR^2\omega^2} = \frac{4}{7\omega}.$$

The given ratio is $\pi/22$, so

$$\frac{4}{7\omega} = \frac{\pi}{22} \quad \Longrightarrow \quad \omega = \frac{88}{7\pi}\,\mathrm{rad\,s^{-1}}.$$

Numerically this is approximately $4.002\,\mathrm{rad\,s^{-1}}$; using the usual numerical approximation $\pi = 22/7$ gives exactly $4\,\mathrm{rad\,s^{-1}}$.

Step 3: Conclusion The required angular speed is $88/(7\pi)\,\mathrm{rad\,s^{-1}}$, giving the numerical entry $\boxed{4}$ in $\mathrm{rad\,s^{-1}}$.

Quick Tip to Solve: Write both energy terms in ω before dividing; the ratio L/K is proportional to $1/\omega$, not to ω.

59. The elastic potential energy stored in a steel wire of length 20 m stretched through 2 cm is 80 J. The cross sectional area of the wire is _____ $\mathrm{mm^2}$.

(**Given,** $y = 2.0 \times 10^{11}\mathrm{Nm^{-2}}$) [4 marks]

Solution:

Concept:

- Within the elastic limit, a wire behaves like a spring with force $F = (YA/L)\Delta L$. Here Y is Young's modulus, A the cross-sectional area, and L the original length.
- Because the tensile force rises linearly from zero, the stored elastic energy is $U = \frac{1}{2}F\Delta L = YA(\Delta L)^2/(2L)$. The extension must be expressed in metres when Y is in $\mathrm{N\,m^{-2}}$.

Step 1: Rearrange the energy formula and convert the extension The stated extension is $\Delta L = 2\,\mathrm{cm} = 2 \times 10^{-2}\,\mathrm{m}$, and the original length is $L = 20\,\mathrm{m}$. Solving the elastic-energy expression for the unknown area gives

$$A = \frac{2LU}{Y(\Delta L)^2}.$$

The factor of two is essential: using the final force multiplied by the extension would overestimate the work because the force is not constant during stretching.

Step 2: Substitute and convert the area unit Use $U = 80\,\text{J}$ and $Y = 2.0 \times 10^{11}\,\text{N m}^{-2}$:

$$A = \frac{2(20)(80)}{(2.0 \times 10^{11})(2 \times 10^{-2})^2} = \frac{3200}{8.0 \times 10^{7}} = 4.0 \times 10^{-5}\,\text{m}^2.$$

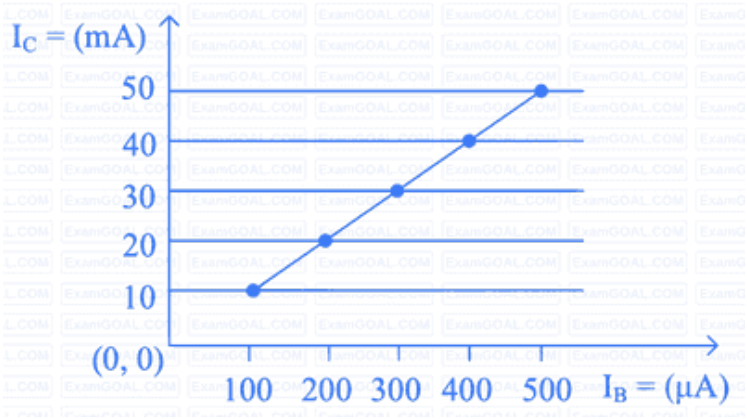
Since $1\,\text{m}^2 = 10^6\,\text{mm}^2$, this becomes $40\,\text{mm}^2$. As a check, the final force is $YA\Delta L/L = 8000\,\text{N}$, and $\frac{1}{2}(8000)(0.02) = 80\,\text{J}$, reproducing the given energy.

Step 3: Conclusion The cross-sectional area is $\boxed{40\,\text{mm}^2}$; therefore the numerical entry is $\boxed{40}$.

Quick Tip to Solve: Square the extension only after converting centimetres to metres, then multiply the final area in m^2 by 10^6 to obtain mm^2.

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60. From the given transfer characteristic of a transistor in CE configuration, the value of power gain of this configuration is 10^x, for $R_B = 10\,\text{k}\Omega$, and $R_C = 1\,\text{k}\Omega$. The value of x is _____. [4 marks]



Solution:

Concept:

- The small-signal current gain of a transistor in common-emitter configuration is $\beta = \Delta I_C/\Delta I_B$, the slope of its transfer characteristic. Collector and base currents must first be put in the same units.
- For input resistance R_B and collector load R_C, the magnitude of voltage gain is $A_v = \beta R_C/R_B$. Power gain is the product of current gain and voltage-gain magnitude: $A_P = \beta^2 R_C/R_B$.

Step 1: Read the current gain from the straight line The given plot has I_C in milliamperes and I_B in microamperes. Choose the well-separated points $(100\,\mu\text{A}, 10\,\text{mA})$ and $(500\,\mu\text{A}, 50\,\text{mA})$. Their differences give

$$\beta = \frac{40\,\text{mA}}{400\,\mu\text{A}} = \frac{40 \times 10^{-3}}{400 \times 10^{-6}} = 100.$$

Using the numerical slope $40/400$ without accounting for the axis units would give the wrong current gain by a factor of one thousand.

Step 2: Calculate the voltage and power gains The resistance ratio is $R_C/R_B = (1\,\text{k}\Omega)/(10\,\text{k}\Omega) = 0.1$.

Thus the voltage-gain magnitude is $A_v = 100(0.1) = 10$, while

$$A_P = \beta A_v = 100(10) = 1000 = 10^3.$$

The common-emitter voltage signal is inverted, but that phase reversal does not make the power gain negative. Equivalently, the ratio of output to input powers is $(I_C^2 R_C)/(I_B^2 R_B) = \beta^2 R_C/R_B$.

Step 3: Conclusion The power gain is 10^3, so $\boxed{x = 3}$. The exponent x and the power gain are dimensionless.

Quick Tip to Solve: Check whether the two current axes use mA and microampere units; calculate β first, then use $A_P = \beta^2 R_C/R_B$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. In which of the following processes, the bond order increases and paramagnetic character changes to diamagnetic one ? [4 marks]

(A) $O_2 \rightarrow O_2^{2-}$

(B) $N_2 \rightarrow N_2^{+}$

(C) $NO \rightarrow NO^{+}$

(D) $O_2 \rightarrow O_2^{+}$

Correct Answer: (C) $NO \rightarrow NO^{+}$

Solution:

Concept:

- Molecular-orbital bond order is $(N_b - N_a)/2$, where N_b and N_a count bonding and antibonding electrons. Removing one antibonding electron raises bond order by one half.
- A paramagnetic species contains at least one unpaired electron. A diamagnetic species has all electrons paired. A process must satisfy both the bond-order and magnetism conditions to qualify.

Step 1: Test the nitric oxide process Neutral NO has eleven valence electrons. Its last electron occupies an antibonding π^* orbital singly, giving bond order 2.5 and paramagnetism. Ionization removes this unpaired antibonding electron:

$$NO \longrightarrow NO^{+} + e^{-}.$$

The resulting NO^{+} is isoelectronic with N_2, with all electrons paired and bond order 3. Thus option (C) increases the bond order and makes the species diamagnetic.

Step 2: Reject the other processes using both criteria In (A), adding two electrons to O_2 fills its antibonding π^* orbitals. Although $O_2{}^{2-}$ is diamagnetic, its bond order falls from 2 to 1, so the first requirement fails. In (B), neutral N_2 is already diamagnetic, and removing a bonding electron gives paramagnetic $N_2{}^{+}$, reducing bond order from 3 to 2.5. In (D), removal of an antibonding electron raises the bond order of O_2 from 2 to 2.5, but $O_2{}^{+}$ still has one unpaired electron and remains paramagnetic. Neither process fulfils both requirements.

Step 3: Conclusion Only $NO \longrightarrow NO^+$ raises bond order from 2.5 to 3 while changing a paramagnetic species into a diamagnetic one. Hence, the correct option is (C).

Quick Tip to Solve: After changing the electron count, check the unpaired electrons separately; a higher bond order alone does not guarantee diamagnetism.

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62. Which of the following statements are not correct?

A. The electron gain enthalpy of F is more negative than that of Cl.

B. Ionization enthalpy decreases in a group of periodic table.

C. The electronegativity of an atom depends upon the atoms bonded to it.

D. Al_2O_3 and NO are examples of amphoteric oxides.

Choose the most appropriate answer from the options given below : [4 marks]

(A) A, B, C and D

(B) A, C and D Only

(C) A, B and D Only

(D) B and D Only

Correct Answer: No listed combination is correct: A and D alone are incorrect; statement C is a valid dependence on bonding environment.

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change when an isolated gaseous atom accepts an electron. Chlorine has a more negative electron gain enthalpy than fluorine because its larger valence shell reduces electron-electron repulsion.
- Ionization enthalpy generally decreases down a group as radius and shielding increase. Electronegativity is a bonded-atom property affected by the chemical environment; amphoteric oxides react with both acids and bases.

Step 1: Evaluate statements A and B Statement A is incorrect. The added electron in fluorine enters a compact $2p$ shell and experiences stronger repulsion than in chlorine's $3p$ shell. Consequently chlorine's electron gain enthalpy is more negative. Statement B expresses the usual down-group trend and is correct: the outer electron becomes more distant and better shielded, so it is generally easier to remove. No direction opposite to the conventional down-group comparison is specified.

Step 2: Evaluate C and D and compare the choices Statement C is correct: an atom's electronegativity depends on its bonding environment, including the atoms bonded to it; it is not a directly measured, fixed isolated-atom energy. Statement D is incorrect. Al_2O_3 is amphoteric, but NO is a neutral oxide. Therefore the incorrect statements are A and D only. None of the supplied combinations is A and D only. Option (A) incorrectly includes B and C; (B) incorrectly includes C; (C) incorrectly includes B; and (D) includes the correct statement B while omitting the incorrect A.

Step 3: Conclusion For the statements as printed, the incorrect pair is $\boxed{\text{A and D only}}$, which is absent from the options. The listed key selects (B), but that includes the correct statement C; therefore no

supplied option is fully correct.

Quick Tip to Solve: Evaluate each statement independently before comparing combinations; remember that NO is neutral and chlorine has the more negative electron gain enthalpy.

63. 2-Methyl propyl bromide reacts with $C_2H_5O^-$ and gives 'A' whereas on reaction with C_2H_5OH it gives 'B'. The mechanism followed in these reactions and the products 'A' and 'B' respectively are : [4 marks]

(A) $S_N1, A =$ tert-butyl ethyl ether; $S_N2, B =$ iso-butyl ethyl ether

(B) $S_N1,\ A =$ tert-butyl ethyl ether; $S_N1,\ B = 2$-butyl ethyl ether

(C) $S_N2,\ A =$ iso-butyl ethyl ether; $S_N1,\ B =$ tert-butyl ethyl ether

(D) $S_N2,\ A = 2$-butyl ethyl ether; $S_N2,\ B =$ iso-butyl ethyl ether

Correct Answer: (C) $S_N2,\ A =$ iso-butyl ethyl ether; $S_N1,\ B =$ tert-butyl ethyl ether

Solution:

Concept:

- 2-Methylpropyl bromide is isobutyl bromide, $(CH_3)_2CHCH_2Br$, a primary halide. A strong nucleophile such as ethoxide attacks its primary carbon through a concerted S_N2 displacement.
- In the ionization pathway associated with ethanol, rearrangement to the much more stable tertiary carbocation can precede nucleophilic capture. The resulting ether then has a tert-butyl rather than an isobutyl group.

Step 1: Find A with ethoxide Ethoxide, $C_2H_5O^-$, carries a negative charge and is a much stronger nucleophile than neutral ethanol. Attack at the carbon bearing bromine displaces bromide directly:

$$(CH_3)_2CHCH_2Br + C_2H_5O^- \longrightarrow (CH_3)_2CHCH_2OC_2H_5 + Br^-.$$

The carbon skeleton is retained because the concerted S_N2 route does not generate a free carbocation. Product A is therefore isobutyl ethyl ether, not a tert-butyl ether.

Step 2: Find B through the rearranged ionization pathway With neutral ethanol, the pathway represented in the options is ionization followed by a neighbouring hydride shift. The initially developing primary isobutyl cation rearranges to the tertiary tert-butyl cation:

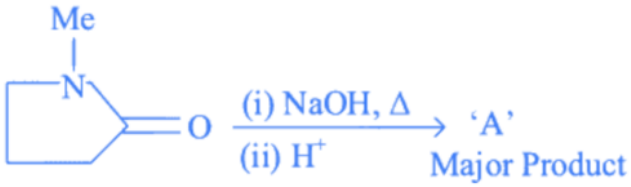
$$(CH_3)_2CHCH_2^+ \longrightarrow (CH_3)_3C^+.$$

Ethanol captures this cation, and loss of a proton produces $(CH_3)_3COC_2H_5$, tert-butyl ethyl ether. This rearranged product identifies an S_N1 pathway. Options (A) and (B) assign S_N1 to ethoxide and give the wrong A. Option (D) assigns a second S_N2 route and does not give the rearranged tert-butyl product for B. The direct substitution with strong nucleophile and the rearranged solvolysis pairing is precisely option (C).

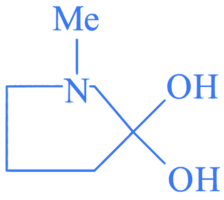
Step 3: Conclusion A is isobutyl ethyl ether by S_N2, and B is tert-butyl ethyl ether by the rearranged S_N1 pathway. Hence, the correct option is (C).

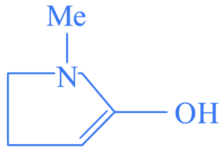
Quick Tip to Solve: A rearranged ether skeleton signals a carbocation pathway; direct ethoxide displacement on a primary carbon retains the original carbon skeleton.

64. In the reaction given below, 'A' is [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- A lactam is a cyclic amide. Heating with aqueous alkali hydrolyses its acyl carbon–nitrogen bond, opening the ring to an amino carboxylate.
- The subsequent acid work-up converts the carboxylate group into a carboxylic acid. Hydrolysis changes the amide linkage without reducing the carbonyl carbon.

Step 1: Identify the functional group The starting five-membered ring contains a carbonyl carbon directly bonded to nitrogen, and nitrogen bears a methyl group. It is therefore an N-methyl lactam, rather than a cyclic ketone. Between nitrogen and the carbonyl carbon along the other side of the ring there are three methylene groups. Opening the acyl–nitrogen bond must preserve this three-carbon segment and the nitrogen–methyl bond.

Step 2: Apply alkaline hydrolysis Hydroxide attacks the electrophilic amide carbonyl. The tetrahedral intermediate undergoes acyl substitution, with cleavage of the bond between nitrogen and the carbonyl carbon. Proton transfers give the open-chain amino carboxylate. Acidification then gives the amino acid constitution:

$$CH_3NH(CH_2)_3COO^- \xrightarrow{H^+} CH_3NH(CH_2)_3COOH.$$

Depending on the medium the amino group can be protonated, but this does not change the connectivity represented by option (D).

Step 3: Reject the other structures Option (A) is an amino aldehyde and would require reduction of the amide carbonyl; neither sodium hydroxide nor the acid work-up is a reducing reagent. Option (B) retains the ring as a geminal diol, which is not the stable hydrolysis product of an amide under heating. Option (C) is a cyclic enol and also retains the acyl–nitrogen ring bond that hydrolysis cleaves.

Step 4: Conclusion The major product is $CH_3NH(CH_2)_3COOH$, the ring-opened N-methyl amino

acid shown in (D). Hence, the correct option is (D).

Quick Tip to Solve: For a lactam under hot aqueous alkali, break the carbonyl C–N bond and place carboxyl and amino groups at the two ends; count the intervening methylene groups.

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65. The mismatched combinations are

A. Chlorophyll - Co

B. Water hardness - EDTA

C. Photography $-\,[Ag(CN)_2]^-$

D. Wilkinson catalyst $-\,[(Ph_3P)_3\,RhCl]$

E. Chelating ligand - D-Penicillamine

Choose the correct answer from the options given below : [4 marks]

(A) A and E Only

(B) D and E Only

(C) A and C Only

(D) A, C, and E Only

Correct Answer: (C) A and C Only

Solution:

Concept:

- Metal complexes have characteristic biological and industrial applications. A matching question requires checking both the metal or ligand and the application named.
- Chelating ligands bind through more than one donor atom; their uses include complexometric titration and removal of toxic metals.

Step 1: Check the metal and photographic complex Chlorophyll contains magnesium, not cobalt, at the centre of its porphyrin-like ring. Thus combination A is mismatched. Cobalt is associated with vitamin B12, which explains the tempting confusion. In photographic fixing, unreacted silver bromide dissolves by forming a soluble silver thiosulphate complex, conventionally written $[Ag(S_2O_3)_2]^{3-}$. The dicyanoargentate complex $[Ag(CN)_2]^-$ is associated with cyanide chemistry such as metal extraction or plating, rather than the usual photographic fixing process. Hence C is also mismatched.

Step 2: Verify the remaining applications EDTA forms stable chelates with calcium and magnesium ions and is used in estimating water hardness, so B is correctly matched. Wilkinson's catalyst is chlorotris(triphenylphosphine)rhodium(I), $RhCl(PPh_3)_3$, a homogeneous hydrogenation catalyst; D is correct. D-Penicillamine has donor groups capable of binding a metal ion to form a chelate and is used as a chelating agent, for example in copper removal. Thus E is correct.

Step 3: Choose the combination of mismatches The complete set of mismatches is A and C only. Option (A) incorrectly includes E and omits C. Option (B) incorrectly rejects the correctly named rhodium catalyst as well as the chelating ligand. Option (D) includes E in addition to the two genuine errors. Checking all five entries avoids selecting a combination merely because it contains the obvious chlorophyll error.

Step 4: Conclusion Only A (chlorophyll–Co) and C (photography–dicyanoargentate) are mismatched. Hence, the correct option is (C).

Quick Tip to Solve: Remember the distinctive pairs: chlorophyll–Mg, vitamin B12–Co, photographic fixing–thiosulphate, and water-hardness titration–EDTA.

66. ClF_5 **at room temperature is a:** [4 marks]

(A) Colourless liquid with square pyramidal geometry

(B) Colourless gas with square pyramidal geometry

(C) Colourless gas with trigonal bipyramidal geometry.

(D) Colourless liquid with trigonal bipyramidal geometry

Correct Answer: (B) Colourless gas with square pyramidal geometry

Solution:

Concept:

- In ClF_5, chlorine has seven valence electrons. Five chlorine–fluorine bonds leave one lone pair on the central atom.
- Six electron domains have an octahedral electron-pair arrangement. With one lone pair, the molecular shape is square pyramidal, written AX_5E.

Step 1: Count the central electron domains Chlorine contributes seven valence electrons. Forming five single bonds to fluorine uses five of these in bonding, leaving two electrons as one lone pair. Thus the steric number is

$$5 \text{ bonding domains} + 1 \text{ lone pair} = 6.$$

The five fluorine atoms occupy five vertices of an octahedral arrangement; one vertex is occupied by the lone pair. The four atoms in a plane form the square base, and the fifth atom is at the apex.

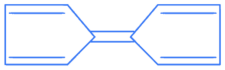
Step 2: Identify the physical state Chlorine pentafluoride is a colourless gas at ordinary room temperature and atmospheric pressure. Its normal boiling point is below room temperature (approximately $-13\,^{\circ}C$), so it does not remain liquid at room temperature. The state must be checked separately from the VSEPR shape: counting electron pairs cannot establish whether a molecular compound is a gas or a liquid.

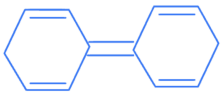
Step 3: Eliminate the inconsistent choices Option (A) gives the correct square pyramidal shape but the wrong liquid state. Option (C) has the correct gaseous state but a trigonal bipyramidal shape, which would correspond to five electron domains without this lone pair. Option (D) is wrong on both the liquid state and molecular geometry. Only (B) combines the physical property with the correct five-bond, one-lone-pair molecular structure.

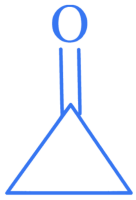
Step 4: Conclusion At room temperature, ClF_5 is a colourless gas with square pyramidal geometry. Hence, the correct option is (B). The supplied key gives a liquid-state description, but this conflicts with the normal boiling point below room temperature; the printed compound therefore gives (B).

Quick Tip to Solve: Distinguish molecular shape from electron geometry: AX_5E has an octahedral domain arrangement but a square pyramidal molecular shape.

67. Among the following compounds, the one which shows highest dipole moment is [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- The molecular dipole moment reflects the separation of positive and negative charge. Resonance forms with well-stabilised separated charges can make a major contribution.
- A planar cyclic conjugated ring is aromatic when it contains $4n + 2$ pi electrons. Cyclopropenyl cations have two, while cyclopentadienyl anions have six pi electrons.

Step 1: Use symmetry to compare the first two compounds Options (A) and (B) contain identical hydrocarbon ring systems on the two sides of a central double bond. There is no directional donor–acceptor asymmetry that favours a permanent separation of charge from one end to the other. Symmetry-related polar contributions cancel. Thus their conjugation alone does not make either a strongly polar molecule; a large conjugated system need not possess a large net dipole.

Step 2: Compare the polar resonance forms of C and D Cyclopropanone in (C) has a polar carbonyl bond and hence a nonzero dipole moment. However, option (D), calicene, has unlike three- and five-membered rings linked by an exocyclic double bond. Moving the linking pi electrons toward the five-membered ring gives a cyclopropenyl cation on the three-membered ring and a cyclopentadienyl anion on the five-membered ring. The former has two cyclic pi electrons, and the latter has six. Both rings are aromatic in this charge-separated contributor, strongly stabilising the polar resonance form.

Step 3: Select the largest permanent dipole The simultaneous aromatic stabilisation of both charged rings gives option (D) substantial charge separation across the inter-ring bond. It is therefore much more polar than the symmetrical hydrocarbons (A) and (B), and its exceptionally stabilised zwitterionic contributor gives a larger dipole than the ordinary carbonyl polarity of (C). The decision depends on aromatic stabilisation of separated charges, rather than on the presence of oxygen alone.

Step 4: Conclusion The compound in (D), with linked three- and five-membered rings, has the highest dipole moment. Hence, the correct option is (D).

Quick Tip to Solve: For unlike rings joined by an exocyclic double bond, test whether charge separation makes both rings aromatic; a 3-membered cation and 5-membered anion is a particularly favourable pair.

68. 2-Methyl propyl bromide reacts with $C_2H_5O^-$ and gives 'A' whereas on reaction with C_2H_5OH it gives 'B'. The mechanism followed in these reactions and the products 'A' and 'B' respectively are: [4 marks]

(A) S_N1, A = tert-butyl ethyl ether; S_N2, B = iso-butyl ethyl ether

(B) S_N1, A = tert-butyl ethyl ether; S_N1, B = 2-butyl ethyl ether

(C) S_N2, A = iso-butyl ethyl ether; S_N1, B = tert-butyl ethyl ether

(D) S_N2, A = 2-butyl ethyl ether; S_N2, B = iso-butyl ethyl ether

Correct Answer: (C) S_N2, A = iso-butyl ethyl ether; S_N1, B = tert-butyl ethyl ether

Solution:

Concept:

- A strong nucleophile can attack a primary alkyl halide by concerted S_N2 displacement, preserving the carbon skeleton.
- In the solvolysis pathway represented here, a weak neutral nucleophile reacts through a carbocation pathway; rearrangement to a more stable carbocation can precede capture.

Step 1: React the bromide with ethoxide 2-Methylpropyl bromide is isobutyl bromide, $(CH_3)_2CHCH_2Br$. The carbon bearing bromine is primary. Ethoxide, $C_2H_5O^-$, is a strong nucleophile and attacks this carbon directly, displacing bromide in one step. No free carbocation forms, so no hydride migration occurs:

$$(CH_3)_2CHCH_2Br + C_2H_5O^- \longrightarrow (CH_3)_2CHCH_2OC_2H_5 + Br^-.$$

Product A is consequently iso-butyl ethyl ether, with the original carbon skeleton unchanged.

Step 2: Follow the ethanol pathway Ethanol is a much weaker, neutral nucleophile. In the carbocation solvolysis pathway described by the answer choices, ionisation and rearrangement give the more stable tertiary tert-butyl cation. A 1,2-hydride shift from the adjacent carbon converts the initially developing primary cation into $(CH_3)_3C^+$. Ethanol attacks this tertiary centre, and loss of a proton from the oxonium species gives tert-butyl ethyl ether, $(CH_3)_3COC_2H_5$. This is the rearranged product B of the S_N1 pathway.

Step 3: Match both mechanisms and products Only (C) assigns direct S_N2 formation of iso-butyl ethyl ether to A and rearranged S_N1 formation of tert-butyl ethyl ether to B. Options (A) and (B) wrongly assign a carbocation pathway and rearranged product to the strong-nucleophile reaction. Option (D) gives the wrong unrearranged ether skeleton for A and does not account for rearrangement in the ethanol reaction.

Step 4: Conclusion A is iso-butyl ethyl ether by S_N2; B is tert-butyl ethyl ether by the rearranged S_N1 pathway. Hence, the correct option is (C).

Quick Tip to Solve: First locate the carbon bearing the leaving group. A concerted substitution retains the skeleton; a carbocation route can undergo a hydride shift before the nucleophile attacks.

69. The pair of lanthanides in which both elements have high third - ionization energy is : [4 marks]

(A) Dy, Gd

(B) Lu, Yb

(C) Eu, Yb

(D) Eu, Gd

Correct Answer: (C) Eu, Yb

Solution:

Concept:

- The third ionisation energy removes an electron from M^{2+} to form M^{3+}. Its size depends especially on the configuration of the divalent ion.
- Half-filled $4f^7$ and completely filled $4f^{14}$ subshells are unusually stable. Removing an electron from either configuration is energetically costly.

Step 1: Write the configurations before the third ionisation Europium has neutral configuration $[Xe]4f^7 6s^2$. Removal of the two outer $6s$ electrons produces Eu^{2+} with the stable half-filled $4f^7$ configuration. Similarly ytterbium is $[Xe]4f^{14}6s^2$; its divalent ion Yb^{2+} has the stable filled $4f^{14}$ configuration. The third electron must therefore be removed from a particularly stable f subshell in both cases.

Step 2: Relate stability to the energy change The relevant changes are

$$Eu^{2+}(4f^7) \longrightarrow Eu^{3+}(4f^6) + e^-$$

$$Yb^{2+}(4f^{14}) \longrightarrow Yb^{3+}(4f^{13}) + e^-.$$

Both changes destroy a favourable half-filled or filled subshell. Consequently both Eu and Yb have unusually high third ionisation energies relative to the smooth lanthanoid trend. Notice that it is the stability of the ion before ionisation, not merely that of the product, which raises the energy required.

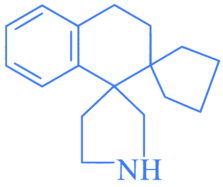
Step 3: Reject pairs containing Gd or Lu Gadolinium can form Gd^{3+} with the stable $4f^7$ configuration, whereas lutetium forms Lu^{3+} with $4f^{14}$. Their third ionisations remove a non-f electron and lead toward these stable trivalent arrangements, rather than destroying the special divalent configurations of Eu and Yb. Dysprosium does not supply the needed half-filled or filled divalent configuration either. Thus (A), (B) and (D) do not contain two elements meeting the criterion.

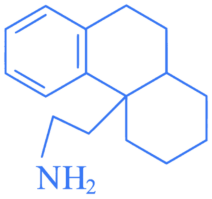
Step 4: Conclusion The required pair is europium and ytterbium, Eu and Yb. Hence, the correct option is (C).

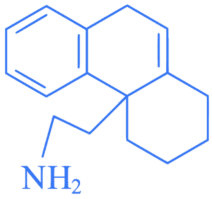
Quick Tip to Solve: For the third ionisation energy, write the divalent ion first: $Eu^{2+} = 4f^7$ and $Yb^{2+} = 4f^{14}$ are the two especially stable cases.

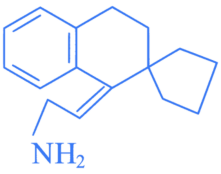
70. In the reaction given below, 'B' is [4 marks]

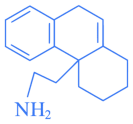


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- Protonation of an alcohol converts its hydroxyl group into water, a good leaving group. Loss of water can generate a carbocation that undergoes skeletal rearrangement.
- A neighbouring carbon–carbon bond may migrate to an adjacent cationic centre, expanding a small ring. Subsequent base treatment can give an alkene by elimination.

Step 1: Generate the cation at the alcohol carbon Label the carbon bearing hydroxyl as P and the neighbouring spiro carbon of the cyclopentane ring as Q. Hydrochloric acid protonates the hydroxyl group, enabling loss of water at P. This centre is benzylic, so formation of a cation there is facilitated. The amino group is protonated in the acidic medium, greatly reducing its nucleophilicity; it therefore does not simply attack to give the nitrogen-containing spiro ring in option (A).

Step 2: Expand the five-membered carbon ring A carbon–carbon bond of the cyclopentane ring attached to Q migrates toward the electron-deficient P centre. The migrating bond retains its electron pair, makes a new bond to P, and transfers the positive charge to Q. The five-membered ring thereby expands to a six-membered ring incorporating P. This rearrangement relieves the less favourable small-ring arrangement and places the positive centre at the neighbouring carbon. The aminoethyl chain remains attached to P throughout; no carbon atoms are removed.

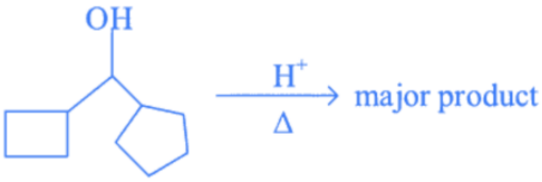
Step 3: Eliminate to obtain the final alkene On treatment with potassium hydroxide, elimination gives the alkene shown in (C), between Q and its adjacent carbon in the non-aromatic ring. The other adjacent centre P is fully substituted after migration and has no hydrogen available for the corresponding elimination. Option (B) has the expanded ring but is saturated and fails to account for

the final elimination. Option (D) retains the original five-membered spiro ring and places the double bond toward the side chain, omitting the favourable ring expansion. Thus only (C) contains both the rearranged carbon skeleton and the appropriate unsaturation.

Step 4: Conclusion B is the ring-expanded alkene carrying the unchanged aminoethyl group, as drawn in (C). Hence, the correct option is (C).

Quick Tip to Solve: In a spiro alcohol rearrangement, label the alcohol carbon and adjacent spiro carbon, migrate one small-ring bond, and then check which neighbour of the resulting cation actually has a removable hydrogen.

71. In the above reaction, left hand side and right hand side rings are named as 'A' and 'B' respectively. They undergo ring expansion. The correct statement for this process is: [4 marks]



(A) Finally both rings will become six membered each.

(B) Only A will become 6 membered.

(C) Ring expansion can go upto seven membered rings

(D) Finally both rings will become five membered each.

Correct Answer: (A) Finally both rings will become six membered each.

Solution:

Concept:

- Acid-catalysed dehydration begins by protonating the alcohol group. Water is a good leaving group, so its loss produces a carbocation next to the two rings.
- A carbocation can rearrange through a neighbouring carbon–carbon bond migration. Ring expansion relieves the severe angle strain of a four-membered ring and can ultimately produce a stable six-membered ring skeleton.

Step 1: Generate the rearranging cation The left ring A initially contains four carbon atoms and the right ring B contains five. The carbon bearing the hydroxyl group lies between them. Under H^+ and heat, protonation changes $-OH$ into $-OH_2{}^+$; departure of water leaves an electron-deficient centre. A neighbouring ring bond can migrate to that centre, carrying its bonding electron pair and relocating the positive charge.

Step 2: Follow the strain-relieving expansions Call the alcohol carbon X, the adjacent cyclobutane carbon A1, and the adjacent cyclopentane carbon B1. A cyclobutane bond migrates to X, incorporating X into that ring and making it five-membered; the charge moves to A1. A hydride shift from X to A1 restores the cation at X. Next a bond of ring B migrates to X, expanding B from five to six members. The resulting intermediate has a five-membered and a six-membered ring meeting at X, with the

charge at B1. Migration of an adjacent bond from X in the five-membered ring to B1 expands that ring to six members too. The final rings are fused and share the X–B1 bond. Deprotonation gives the alkene. Thus ten skeletal carbons suffice for two six-membered rings sharing two atoms.

Step 3: Eliminate the alternative descriptions Both rings reach six members, so the claim that only A becomes six-membered is incomplete. The product is not the intermediate framework with two five-membered rings. Expansion to a seven-membered ring is not the preferred endpoint: the favourable six-membered framework is reached without creating that less favourable ring size.

Step 4: Conclusion Finally, both A and B become six-membered rings. Hence, the correct option is (A).

Quick Tip to Solve: Count the starting rings first, then follow all favourable carbocation rearrangements rather than stopping at the first strain-relieving shift.

72. The products formed in the above reaction are [4 marks]



(A) Two optically active products

(B) One optically active and one meso product

(C) Two optically inactive products

(D) One optically inactive and one meso product.

Correct Answer: (B) One optically active and one meso product

Solution:

Concept:

- Addition of HCN to a planar aldehyde carbonyl creates a new stereogenic centre. A pre-existing stereocentre makes the two addition products diastereomers.
- Hydrolysis converts the nitrile into $-COOH$, and oxidation with HNO_3 converts the terminal $-CH_2OH$ into a second carboxyl group. The resulting tartaric acids must be tested for internal symmetry.

Step 1: Track the carbon chain D-(+)-glyceraldehyde is $CHO-CH(OH)-CH_2OH$. Cyanide attacks either face of the aldehyde group, giving two cyanohydrins, $NC-CH(OH)-CH(OH)-CH_2OH$. The configuration at the original stereocentre is retained while the new centre can have either configuration. Hydrolysis of $-CN$ extends the chain by one carbon and produces $HOOC-CH(OH)-CH(OH)-CH_2OH$.

Step 2: Examine the two oxidation products Nitric acid oxidizes the primary alcohol end, giving $HOOC-CH(OH)-CH(OH)-COOH$. In a Fischer projection the two possible relative arrangements are the erythro and threo arrangements inherited from the two cyanohydrins. One arrangement is

mesotartaric acid, with equivalent ends and an internal symmetry element; it is optically inactive despite having two stereocentres. The other is one member of the chiral tartaric-acid enantiomeric pair and is optically active.

Step 3: Check the offered combinations The original chiral starting material fixes one stereocentre, so the chiral product is not formed as an equal pair of enantiomers. Thus the products are not two optically active compounds and not two optically inactive compounds. In particular, the non-meso product is active, which excludes the last option.

Step 4: Conclusion The reaction gives one optically active tartaric acid and one meso product. Hence, the correct option is (B).

Quick Tip to Solve: After oxidizing both ends of a carbohydrate derivative to identical groups, check for meso symmetry before counting optically active products.

73. The energy of an electron in the first Bohr orbit of hydrogen atom is -2.18×10^{-18} J. Its energy in the third Bohr orbit is _____. [4 marks]

(A) One third of this value

(B) Three times of this value

(C) $\frac{1}{9}$ th of this value

(D) $\frac{1}{27}$ of this value

Correct Answer: (C) $\frac{1}{9}$ th of this value

Solution:

Concept:

- For hydrogen in the Bohr model, $E_n = E_1/n^2$, where n is the principal quantum number and the energy zero is the separated electron and proton.
- Bound-state energies are negative. Increasing n makes the energy less negative, although its absolute magnitude decreases.

Step 1: Apply the inverse-square dependence The given first-orbit energy is $E_1 = -2.18 \times 10^{-18}$ J. The third orbit has $n = 3$, so the ratio depends on the square of three, not on three itself:

$$\frac{E_3}{E_1} = \frac{1}{3^2} = \frac{1}{9}.$$

The same hydrogen atom is considered in both cases, so no change in nuclear charge or other constant enters the ratio.

Step 2: Substitute the first-orbit energy Using the supplied value gives

$$E_3 = \frac{-2.18 \times 10^{-18}}{9} = -2.42 \times 10^{-19}\ \text{J}.$$

The sign remains negative because the electron is still bound. This value is closer to zero than E_1, as expected for a more highly excited orbit. The energy itself has increased even though the magnitude

is only one ninth of its original magnitude.

Step 3: Reject the other scaling laws One third would require an inverse-first-power dependence on n, which Bohr energies do not have. Three times would make the state more tightly bound at larger orbit number and contradict the model. One twenty-seventh is an inverse-cube factor; it is not the energy-level law. The inverse-square factor uniquely selects one ninth.

Step 4: Conclusion The third-orbit energy is $E_3 = E_1/9 = -2.42 \times 10^{-19}\,\text{J}$, one ninth of the given value. Hence, the correct option is (C).

Quick Tip to Solve: For two hydrogen energy levels, take their ratio first: $E_m/E_n = n^2/m^2$, retaining the negative sign of each bound-state energy.

74. The radical which mainly causes ozone depletion in the presence of UV radiations is : [4 marks]

(A) $\dot{N}O$

(B) $\dot{C}H_3$

(C) $\dot{C}l$

(D) $\dot{O}H$

Correct Answer: (C) $\dot{C}l$

Solution:

Concept:

- Ultraviolet radiation can break carbon–chlorine bonds in chlorofluorocarbons, releasing chlorine radicals in the stratosphere.
- A radical catalyst can destroy ozone while being regenerated. It is therefore the catalytic cycle, rather than a single consumption reaction, that explains extensive ozone loss.

Step 1: Identify the radical released by ultraviolet light Chlorofluorocarbons are sufficiently persistent to reach the stratosphere. There, energetic ultraviolet light breaks a carbon–chlorine bond homolytically, producing $Cl^{\bullet}$. This chlorine atom has an unpaired electron and reacts readily with ozone. Its formation connects the ultraviolet radiation in the question with the ozone-depletion mechanism.

Step 2: Write the ozone-destruction cycle The chlorine radical reacts with ozone to form chlorine monoxide radical and molecular oxygen:

$$Cl^{\bullet} + O_3 \longrightarrow ClO^{\bullet} + O_2.$$

The chlorine monoxide radical then reacts with atomic oxygen:

$$ClO^{\bullet} + O \longrightarrow Cl^{\bullet} + O_2.$$

Adding these equations cancels both chlorine-containing intermediates and gives $O_3 + O \longrightarrow 2\,O_2$. The chlorine radical is regenerated, so one chlorine atom can participate in many ozone-destruction events.

Step 3: Distinguish the other radicals $NO^{\bullet}$ and $OH^{\bullet}$ also participate in atmospheric radical chemistry,

but they are not the chlorine radical released from chlorofluorocarbons in the standard ultraviolet-driven ozone-depletion mechanism tested here. The methyl radical $CH_3^{\bullet}$ is likewise not the principal species in that mechanism. The chlorine option directly matches the catalytic cycle above.

Step 4: Conclusion The radical chiefly associated with this ozone-depletion process is $Cl^{\bullet}$. Hence, the correct option is (C).

Quick Tip to Solve: For ozone depletion linked to CFCs and ultraviolet light, write the two chlorine-cycle reactions and check that the chlorine radical is regenerated.

75. What happens when a lyophilic sol is added to a lyophobic sol? [4 marks]

(A) Film of lyophobic sol is formed over lyophilic sol.

(B) Lyophilic sol is dispersed in lyophobic sol.

(C) Film of lyophilic sol is formed over lyophobic sol.

(D) Lyophobic sol is coagulated

Correct Answer: (C) Film of lyophilic sol is formed over lyophobic sol.

Solution:

Concept:

- Lyophilic colloidal particles have a strong affinity for the dispersion medium and possess a solvated protective layer.
- A lyophilic colloid can adsorb around lyophobic particles. This protective coating reduces the tendency of the lyophobic particles to aggregate and coagulate.

Step 1: Compare the two kinds of sol A lyophobic sol has relatively little affinity between its dispersed particles and the liquid medium. Its stability depends strongly on surface charge, and adding a suitable electrolyte can cause coagulation. A lyophilic sol, such as gelatin or starch in water, is strongly hydrated and more resistant to precipitation. Its solvated material can therefore act as a protective colloid when added to a lyophobic sol.

Step 2: Describe the protective film The lyophilic material becomes adsorbed on the surfaces of the lyophobic particles. Its affinity for the medium gives these coated particles an effective solvated envelope. Thus the film is formed by the lyophilic sol over the lyophobic sol, rather than the other way round. The coating impedes close particle contact and helps prevent aggregation when the sol is disturbed or electrolytes are introduced.

Step 3: Check the proposed outcomes The first option reverses the identity of the coating and the particle being protected. Saying merely that one sol is dispersed in the other misses the specific surface adsorption responsible for protection. Coagulation is also the opposite of the usual protective action of an added lyophilic colloid. The description that explicitly places a lyophilic film over lyophobic particles is therefore the correct one.

Step 4: Conclusion A film of lyophilic sol forms over the lyophobic sol particles, producing protective action. Hence, the correct option is (C).

Quick Tip to Solve: Remember the direction of protection: the solvent-loving lyophilic material coats the solvent-repelling lyophobic particles.

76. Which one of the following is most likely a mismatch ? [4 marks]

(A) Titanium - van Arkel Method

(B) Zinc - Liquation

(C) Nickel - Mond process

(D) Copper - Electrolysis

Correct Answer: (B) Zinc - Liquation

Solution:

Concept:

- A refining method is selected from differences in volatility, melting behaviour, electrochemical behaviour, or formation of a volatile compound.
- Liquation separates a low-melting metal from higher-melting impurities. Distillation instead separates a volatile metal from less volatile impurities.

Step 1: Check titanium and nickel The van Arkel method is correctly associated with titanium: crude titanium is converted into volatile TiI_4, which decomposes on a hot filament to deposit pure metal. Nickel is correctly associated with the Mond process. It forms volatile $Ni(CO)_4$ with carbon monoxide at a suitable lower temperature; heating the carbonyl then decomposes it to deposit purified nickel and regenerate carbon monoxide.

Step 2: Check copper and zinc Copper is commonly refined electrolytically, with impure copper as the anode and a thin sheet of pure copper as the cathode. Dissolution at the anode and deposition at the cathode transfer copper while separating impurities. Zinc, on the other hand, is conventionally refined by distillation because it is comparatively volatile. The metal is vaporized and condensed away from nonvolatile impurities.

Step 3: Locate the mismatch Liquation is the standard association for easily fusible metals such as tin or lead when the impurities have higher melting points. It does not give the expected refining-method pairing for zinc in this list. The titanium, nickel and copper pairs each match their characteristic processes, leaving zinc–liquation as the single incorrect association.

Step 4: Conclusion The mismatch is zinc–liquation; zinc is characteristically refined by distillation. Hence, the correct option is (B).

Quick Tip to Solve: Keep volatility and fusibility separate: zinc points to distillation, while tin or lead commonly points to liquation.

77. The incorrect statement from the following for borazine is : [4 marks]

(A) It can react with water.

(B) It is a cyclic compound.

(C) It has electronic delocalization.

(D) It contains banana bonds.

Correct Answer: (D) It contains banana bonds.

Solution:

Concept:

- Borazine, $B_3N_3H_6$, contains a six-membered ring with alternating boron and nitrogen atoms. It is commonly called inorganic benzene.
- Nitrogen lone-pair donation into vacant boron orbitals gives electronic delocalization, but the polar B–N bonds make borazine more reactive toward water than benzene.

Step 1: Check the ring and delocalization statements The borazine skeleton is a cyclic sequence of three boron and three nitrogen atoms, each bearing hydrogen. Thus the statement that it is cyclic is true. The approximately planar ring permits interaction between the nitrogen lone pairs and the vacant orbitals on adjacent boron atoms. This produces delocalized electronic interaction around the ring, although its bonding and aromatic stabilization are not identical to those of benzene.

Step 2: Check its reaction with water Boron is electron deficient and the B–N bonds are polar, so water can attack and hydrolyse borazine. Its overall hydrolysis may be represented as

$$B_3N_3H_6 + 9\,H_2O \longrightarrow 3\,B(OH)_3 + 3\,NH_3 + 3\,H_2.$$

This accounts for the true statement that borazine can react with water. Merely resembling benzene in ring shape does not give it benzene's resistance to hydrolysis.

Step 3: Identify the incorrect bond description Banana bonds refer to the bent three-centre two-electron bridge bonds characteristic of diborane, especially its bridging B–H–B units. Borazine does not contain those hydrogen bridges; its hydrogen atoms are terminal and its ring is built from B–N bonds. Therefore the claim that it contains banana bonds confuses borazine with an electron-deficient borane.

Step 4: Conclusion The incorrect statement is that borazine contains banana bonds. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish the two boron compounds: borazine has an alternating B–N ring; diborane has bridging B–H–B banana bonds.

78. Match the following

	Column A		Column B
(a)	Nylon 6	(I)	Natural Rubber
(b)	Vulcanized Rubber	(II)	Cross Linked
(c)	cis-1, 4-polyisoprene	(III)	Caprolactam
(d)	Polychloroprene	(IV)	Neoprene

Choose the correct answer from options given below: [4 marks]

(A) $a \rightarrow III, b \rightarrow II, c \rightarrow I, d \rightarrow IV$

(B) $a \rightarrow II, b \rightarrow III, c \rightarrow IV, d \rightarrow I$

(C) $a \rightarrow IV, b \rightarrow III, c \rightarrow I, d \rightarrow II$

(D) $a \rightarrow III, b \rightarrow IV, c \rightarrow I, d \rightarrow II$

Correct Answer: (A) $a \rightarrow III, b \rightarrow II, c \rightarrow I, d \rightarrow IV$

Solution:

Concept:

- Nylon 6 is a polyamide obtained by ring-opening polymerisation of caprolactam. The number 6 describes the carbon count of its monomer unit.
- Vulcanisation introduces sulphur cross-links between rubber chains. Natural rubber and neoprene have different monomers even though both are elastomers.

Step 1: Match nylon 6 and vulcanised rubber Caprolactam opens its ring to form the repeating polyamide chain of nylon 6. Consequently, entry (a) matches (III). Vulcanised rubber is produced by heating rubber with sulphur, which connects neighbouring chains and improves strength and elasticity. Its identifying description is cross-linked, so entry (b) matches (II). This does not identify vulcanised rubber with the separate named polymer neoprene.

Step 2: Identify the two named elastomers Natural rubber is cis-1,4-polyisoprene: its isoprene units have the cis arrangement that gives the chain its characteristic flexibility. Thus entry (c) matches (I). Chloroprene, or 2-chlorobuta-1,3-diene, polymerises to polychloroprene, whose common name is neoprene. Therefore entry (d) matches (IV). The complete mapping is

$$a \rightarrow \text{III}, \quad b \rightarrow \text{II}, \quad c \rightarrow \text{I}, \quad d \rightarrow \text{IV}.$$

Step 3: Eliminate the remaining combinations Option (B) incorrectly matches nylon 6 with cross-linking and natural rubber with neoprene. Option (C) incorrectly identifies nylon 6 as neoprene. Option (D) gives the right nylon and natural-rubber matches but exchanges vulcanised rubber and polychloroprene. Only option (A) preserves all four chemical identities.

Step 4: Conclusion The complete correspondence is nylon 6–caprolactam, vulcanised rubber–cross-linked, cis-polyisoprene–natural rubber and polychloroprene–neoprene. Hence, the correct option is (A).

Quick Tip to Solve: Anchor the matching with nylon 6–caprolactam and polychloroprene–neoprene; then distinguish natural rubber from sulphur-cross-linked rubber.

79. $Be(OH)_2$ **reacts with** $Sr(OH)_2$ **to yield an ionic salt.**

Choose the incorrect option related to this reaction from the following : [4 marks]

(A) Be is tetrahedrally coordinated in the ionic salt.

(B) The element Be is present in the cationic part of the ionic salt.

(C) Both Sr and Be elements are present in the ionic salt.

(D) The reaction is an example of acid - base neutralization reaction.

Correct Answer: (B) The element Be is present in the cationic part of the ionic salt.

Solution:

Concept:

- $Be(OH)_2$ is amphoteric: it behaves as an acid towards a sufficiently strong base and forms a hydroxo complex.
- In the tetrahydroxoberyllate ion $[Be(OH)_4]^{2-}$, four hydroxide ligands surround beryllium tetrahedrally; the entire complex is an anion.

Step 1: Write the salt formed with the strong base Strontium hydroxide supplies hydroxide ions and Sr^{2+}. Beryllium hydroxide accepts two additional hydroxide ions to give the complex anion:

$$Be(OH)_2 + 2\,OH^- \longrightarrow [Be(OH)_4]^{2-}.$$

The ions combine to form strontium tetrahydroxoberyllate, $Sr[Be(OH)_4]$. An overall way to represent the reaction is

$$Be(OH)_2 + Sr(OH)_2 \longrightarrow Sr[Be(OH)_4].$$

Step 2: Determine where each metal is present The cation is Sr^{2+}, whereas Be is located inside $[Be(OH)_4]^{2-}$. Thus Be is in the anionic part, not in the cationic part. The charge check gives $+2 + 4(-1) = -2$ for the complex. Four oxygen donor atoms bond to Be, accounting for the tetrahedral coordination stated in option (A). Both metal elements occur in the salt, so option (C) is also true.

Step 3: Check the acid-base description and select the incorrect claim The amphoteric hydroxide takes the acidic role and the strongly basic strontium hydroxide takes the basic role in salt formation. This is the acid-base reaction described in option (D). Options (A), (C) and (D) therefore describe the product or reaction correctly. The claim in option (B) reverses the actual ionic location of Be and is the sole incorrect statement.

Step 4: Conclusion Be belongs to the anion $[Be(OH)_4]^{2-}$, while Sr forms the cation. Hence, the correct option is (B).

Quick Tip to Solve: For an amphoteric hydroxide reacting with excess base, write the hydroxo complex first; its charge immediately identifies the anionic metal-containing part.

80. Given below are two statements :

Statement I : Permutit process is more efficient compared to the synthetic resin method for the softening of water.

Statement II : Synthetic resin method results in the formation of soluble sodium salts.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Statement I is correct but Statement II is incorrect

(B) Both the Statements I and II are correct

(C) Both the Statements I and II are incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (C) Both the Statements I and II are incorrect

Solution:

Concept:

- Permutit (sodium zeolite) exchanges its Na^+ ions for hardness-causing Ca^{2+} and Mg^{2+}; the other ions remain in the water.
- The synthetic-resin deionisation method uses a hydrogen-form cation exchanger and a hydroxide-form anion exchanger. The released H^+ and OH^- combine to form water.

Step 1: Evaluate Statement I The permutit process softens water by replacing calcium and magnesium with sodium. It does not remove all dissolved ions, so the treated water still contains soluble salts. By comparison, the synthetic-resin method can remove both cations and anions and produce demineralised water. It is therefore the more efficient method in the comparison made here. The assertion that permutit is more efficient reverses this order, making Statement I incorrect.

Step 2: Evaluate Statement II In the cation-exchange column, an illustrative exchange is

$$2\,\mathrm{RH} + \mathrm{Ca}^{2+} \longrightarrow \mathrm{R_2Ca} + 2\,\mathrm{H}^{+}.$$

In the anion-exchange column, chloride can be removed by

$$\mathrm{R\!\!\diagup OH} + \mathrm{Cl}^{-} \longrightarrow \mathrm{R\!\!\diagup Cl} + \mathrm{OH}^{-}.$$

The released ions then give $H^+ + OH^- \longrightarrow H_2O$. Thus this complete synthetic-resin treatment removes dissolved salts rather than leaving soluble sodium salts as its characteristic products. Formation of such sodium salts describes the sodium-zeolite softening route. Statement II is incorrect.

Step 3: Compare the truth values with the options Both statements have failed their separate tests. Option (A) would require the first statement to be true, option (B) requires both true, and option (D) requires the second true. None of those combinations applies. The pair of incorrect statements is precisely the combination listed in option (C).

Step 4: Conclusion Both Statement I and Statement II are incorrect. Hence, the correct option is (C).

Quick Tip to Solve: Remember the products: sodium zeolite leaves sodium salts, whereas the two-column resin method replaces ions by hydrogen and hydroxide, producing water.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. $KMnO_4$ **is titrated with ferrous ammonium sulphate hexahydrate in presence of dilute** H_2SO_4**. Number of water molecules produced for 2 molecules of** $KMnO_4$ **is _____.** [4 marks]

Solution:

Concept:

- In acid solution, one permanganate ion accepts five electrons and oxidises five Fe^{2+} ions to Fe^{3+}.
- Ferrous ammonium sulphate hexahydrate (Mohr's salt) is $(NH_4)_2Fe(SO_4)_2 \cdot 6\,H_2O$. Each formula unit supplies one ferrous ion and six water molecules of crystallisation.

Step 1: Balance the redox water contribution The reduction and oxidation half-reactions are

$$MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O,$$

$$Fe^{2+} \longrightarrow Fe^{3+} + e^-.$$

Combining them for two permanganate ions gives

$$2\,MnO_4^- + 16\,H^+ + 10\,Fe^{2+} \longrightarrow 2\,Mn^{2+} + 10\,Fe^{3+} + 8\,H_2O.$$

Thus the redox transformation produces eight water molecules. The two potassium permanganate formula units supply the required two permanganate ions.

Step 2: Include the hexahydrate contribution Ten ferrous ions require ten formula units of Mohr's salt. Since each one contains six waters of crystallisation, these ten formula units contribute

$$N_{\text{crystal water}} = 10 \times 6 = 60.$$

When the complete reaction is written with the explicitly specified hexahydrate as reactant and the soluble sulphate products without their hydration waters, these sixty waters appear on the product side along with the eight redox waters. Counting only the net ionic equation would discard the hydration information supplied in the question.

Step 3: Add the two water counts The requested molecular-equation count is therefore

$$N_{\text{water}} = N_{\text{redox water}} + N_{\text{crystal water}} = 8 + 60 = 68.$$

The sulphate and ammonium ions are spectators in the electron balance, but the water attached to the salt must still be included in this total.

Step 4: Conclusion The number of water molecules for two molecules of $KMnO_4$, including the hexahydrate water, is $\boxed{68}$.

Quick Tip to Solve: First balance the ionic redox equation, then add six crystal waters for each of the ten Mohr-salt units; eight alone misses the hexahydrate contribution.

82. An organic compound gives 0.220 g of CO_2 and 0.126 g of H_2O on complete combustion. If the $\%$ of carbon is 24 then the $\%$ of hydrogen is _____ $\times 10^{-1}$. (Nearest integer) [4 marks]

Solution:

Concept:

- Complete combustion transfers all carbon atoms to CO_2 and all hydrogen atoms to H_2O. Their masses follow from the atomic-mass fractions in those products.
- The elemental percentage equals elemental mass divided by the original sample mass, multiplied by 100. The stated carbon percentage fixes that sample mass.

Step 1: Obtain the mass of carbon and the sample mass Using molar masses $M(CO_2) = 44\,\text{g mol}^{-1}$ and atomic mass of carbon 12, the carbon recovered is

$$m_C = 0.220 \times \frac{12}{44} = 0.060\,\text{g}.$$

Carbon accounts for 24 percent of the original compound, so

$$m_{\text{sample}} = \frac{0.060}{24/100} = 0.250\,\text{g}.$$

The carbon dioxide mass itself is not the carbon mass: its oxygen was supplied during combustion and must be excluded.

Step 2: Calculate the hydrogen mass and percentage The hydrogen mass fraction in water is $2/18$, because two hydrogen atoms occur in each water molecule. Therefore

$$m_H = 0.126 \times \frac{2}{18} = 0.014\,\text{g}.$$

Relative to the same original sample, this gives

$$\%H = \frac{0.014}{0.250} \times 100 = 5.6.$$

The carbon and hydrogen data are internally consistent because they refer to a single 0.250 g portion. No assumption that the compound contains only carbon and hydrogen is required.

Step 3: Express the percentage in the requested scaled form The blank multiplies 10^{-1}, so denote it by x and solve

$$x \times 10^{-1} = 5.6, \qquad x = 56.$$

The nearest-integer instruction leaves this value unchanged. Entering 5.6 in the blank would introduce an extra factor of ten and would not reproduce the percentage.

Step 4: Conclusion The hydrogen content is $5.6\% = 56 \times 10^{-1}\%$. The required nearest-integer entry is $\boxed{56}$.

Quick Tip to Solve: Use carbon dioxide to obtain the sample mass from the known carbon percentage; use water for hydrogen, then apply the power of ten printed after the blank.

83. $t_{87.5}$ **is the time required for the reaction to undergo** 87.5% **completion and** t_{50} **is the time required for the reaction to undergo** 50% **completion. The relation between** $t_{87.5}$ **and** t_{50} **for a first order reaction is** $t_{87.5} = x \times t_{50}$ **The value of** x **is _____. (Nearest integer)** [4 marks]

Solution:

Concept:

- For a first-order reaction with initial concentration $[A]_0$, the concentration at time t is $[A]_t = [A]_0 e^{-kt}$.
- Consequently $t = k^{-1} \ln([A]_0/[A]_t)$. Completion percentage must first be converted into the fraction of reactant remaining.

Step 1: Write the time for 50 percent completion When the reaction has undergone 50 percent completion, the fraction remaining is $1 - 0.50 = 0.50$. Substituting this fraction into the integrated rate law yields

$$t_{50} = \frac{1}{k} \ln \frac{1}{0.50} = \frac{\ln 2}{k}.$$

This is the first-order half-life. Its value does not depend on the starting concentration, because that concentration cancels from the ratio in the logarithm.

Step 2: Write the time for 87.5 percent completion The fraction left after 87.5 percent completion is $1 - 0.875 = 0.125 = 1/8$. Thus

$$t_{87.5} = \frac{1}{k} \ln \frac{1}{1/8} = \frac{\ln 8}{k} = \frac{3 \ln 2}{k}.$$

The logarithm contains the reciprocal of the remaining fraction, not the completed fraction. Physically, the remaining concentration successively falls to one-half, one-quarter and one-eighth of its original value after three equal half-lives.

Step 3: Divide the two expressions The required constant follows directly:

$$x = \frac{t_{87.5}}{t_{50}} = \frac{3 \ln 2/k}{\ln 2/k} = 3.$$

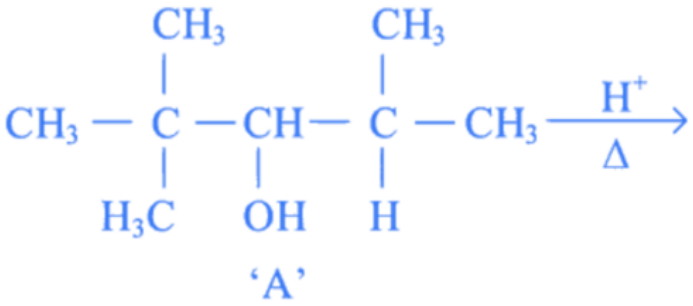
Both the rate constant and the initial concentration cancel. Since this is an exact integer, rounding to the nearest integer does not change it. The factor is dimensionless because it is a ratio of times measured in the same unit.

Step 4: Conclusion $t_{87.5} = 3t_{50}$, so the requested nearest-integer value is $\boxed{x = 3}$ (dimensionless).

Quick Tip to Solve: Convert percentage completed to percentage left: 12.5 percent is one-eighth, so three successive halvings are needed.

84. For the given reaction

The total number of possible products formed by tertiary carbocation of A is _____. [4 marks]



'A'

Solution:

Concept:

- Acid-catalysed dehydration protonates the hydroxyl group and eliminates water, initially producing a carbocation at the alcohol carbon.
- A neighbouring hydride or methyl group can shift to produce a more stable tertiary carbocation. Deprotonation of its distinct beta positions then gives the possible alkene products.

Step 1: Form the initial carbocation and identify rearrangements The alcohol is

$$(CH_3)_3C-CH(OH)-CH(CH_3)_2.$$

Water loss gives the central secondary ion

$$(CH_3)_3C-CH^+-CH(CH_3)_2.$$

The left neighbouring carbon is quaternary and has no hydrogen; it can donate a methyl group. The right neighbouring carbon has a hydrogen and can donate a hydride. Each of these shifts places the positive charge on a carbon attached to three carbon groups, giving a tertiary ion.

Step 2: Count products from the hydride-shift ion Hydride migration from the right gives

$$(CH_3)_3C-CH_2-C^+(CH_3)_2.$$

Loss of a beta hydrogen from the middle methylene forms

$$(CH_3)_3C-CH=C(CH_3)_2.$$

Loss from either of the two equivalent methyl groups instead gives

$$(CH_3)_3C-CH_2-C(CH_3)=CH_2.$$

The two methyl groups yield the same constitution, so they contribute one product, not two. Neither alkene has geometrical isomerism: one double-bond carbon bears either two identical methyl groups or two hydrogens.

Step 3: Count products from the methyl-shift ion Methyl migration from the left gives

$$(CH_3)_2C^+-CH(CH_3)-CH(CH_3)_2.$$

Removing a beta hydrogen from either equivalent methyl group gives

$$CH_2=C(CH_3)-CH(CH_3)-CH(CH_3)_2.$$

Removing the beta hydrogen from the central methine gives

$$(CH_3)_2C=C(CH_3)-CH(CH_3)_2.$$

These are two further distinct alkene constitutions. The terminal product has no geometrical pair, and the internal product has identical methyl substituents on one double-bond carbon. Counting the distinct alkene products through the tertiary ions therefore gives $2 + 2 = 4$.

Step 4: Conclusion The two rearranged tertiary carbocations furnish two distinct alkenes each. The required total number of products is $\boxed{4}$.

Quick Tip to Solve: For each tertiary ion, group equivalent beta carbons before eliminating hydrogen; equivalent methyl groups must not be counted twice.

85. $20\ \text{mL}$ **of calcium hydroxide was consumed when it was reacted with** $10\ \text{mL}$ **of unknown solution of** H_2SO_4**. Also** $20\ \text{mL}$ **standard solution of** $0.5\ \text{M}\ HCl$ **containing 2 drops of phenolphthalein was titrated with calcium hydroxide, the mixture showed pink colour when burette displayed the value of** $35.5\ \text{mL}$ **whereas the burette showed** $25.5\ \text{mL}$ **initially. The concentration of** H_2SO_4 **is _____ M. (Nearest integer)** [4 marks]

Solution:

Concept:

- At the acid-base equivalence point, the number of acid equivalents equals the number of base equivalents. One mole of $Ca(OH)_2$ supplies two moles of hydroxide ions.
- Both $Ca(OH)_2$ and H_2SO_4 have two equivalents per mole, so their molar amounts are equal when they neutralise one another completely.

Step 1: Calculate the volume of calcium hydroxide delivered The volume delivered by a burette is its final reading minus its initial reading, not its final reading alone. Thus the standardisation uses

$$V_b = 35.5 - 25.5 = 10.0\,\text{mL} = 0.0100\,\text{L}.$$

The standard acid contains

$$n(HCl) = 0.5 \times 0.0200 = 0.0100\,\text{mol}.$$

The pink phenolphthalein colour marks the first persistent slight excess of base.

Step 2: Standardise the calcium hydroxide solution The balanced reaction is

$$\mathrm{Ca(OH)_2 + 2\,HCl \longrightarrow CaCl_2 + 2\,H_2O.}$$

Therefore the delivered base contains half as many moles as the acid:

$$n(\mathrm{Ca(OH)_2}) = \frac{0.0100}{2} = 0.00500\,\mathrm{mol}.$$

Its concentration is

$$M_b = \frac{0.00500}{0.0100} = 0.500\,\mathrm{mol\,L^{-1}}.$$

Step 3: Use the sulfuric acid neutralisation The second reaction is

$$\mathrm{Ca(OH)_2 + H_2SO_4 \longrightarrow CaSO_4 + 2\,H_2O.}$$

The mole ratio is now one to one. The stated 20 mL base contains $0.500 \times 0.0200 = 0.0100$ mol, so the 10 mL acid sample also contains 0.0100 mol. Hence

$$M_a = \frac{0.0100}{0.0100} = 1.00\,\mathrm{mol\,L^{-1}}.$$

Step 4: Conclusion The concentration of $\mathrm{H_2SO_4}$ is $1.00\,\mathrm{M}$. The requested nearest integer is $\boxed{1}$.

Quick Tip to Solve: First subtract the burette readings, then write each balanced neutralisation: the HCl-to-base ratio is 2:1, but the sulfuric-acid-to-base ratio is 1:1.

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86. Solution of $12\ \mathrm{g}$ of non-electrolyte (A) prepared by dissolving it in $1000\ \mathrm{mL}$ of water exerts the same osmotic pressure as that of $0.05\ \mathrm{M}$ glucose solution at the same temperature. The empirical formula of A is $\mathrm{CH_2O}$. The molecular mass of A is _____ g. (Nearest integer) [4 marks]

Solution:

Concept:

- For a dilute non-electrolyte solution, osmotic pressure follows $\pi = CRT$, where C is molar concentration, R is the gas constant and T is absolute temperature.
- Isotonic non-electrolyte solutions at the same temperature have equal molar concentrations. Neither glucose nor A dissociates, so both have van't Hoff factor $i = 1$.

Step 1: Equate the molar concentrations The two solutions exert equal osmotic pressure at the same temperature. Consequently,

$$C_A RT = C_{\text{glucose}} RT \quad \Rightarrow \quad C_A = 0.050\,\mathrm{mol\,L^{-1}}.$$

The relevant equality is the number of solute molecules per unit volume; equal osmotic pressure does not mean equal masses of different solutes.

Step 2: Calculate the molecular mass from the dissolved mass Using the dilute-solution volume of

$1000\,\text{mL} = 1.000\,\text{L}$, the solution contains

$$n_A = C_A V = 0.050 \times 1.000 = 0.050\,\text{mol}.$$

The mass dissolved is 12 g, so the molar mass is

$$M_A = \frac{m_A}{n_A} = \frac{12}{0.050} = 240\,\text{g mol}^{-1}.$$

The stated water volume supplies the usual dilute-solution volume approximation in this calculation.

Step 3: Check against the empirical formula The empirical formula CH_2O has formula mass

$$12 + 2(1) + 16 = 30.$$

The ratio of molecular to empirical formula mass is

$$\frac{240}{30} = 8.$$

This is a whole number, as required, and corresponds to molecular formula $C_8H_{16}O_8$. The empirical mass alone would give 30, but osmotic pressure shows that eight empirical units occur in a molecule.

Step 4: Conclusion The molecular mass of A is $\boxed{240}$ (equivalently a molar mass of $240\,\text{g mol}^{-1}$); enter 240 to the nearest integer.

Quick Tip to Solve: For isotonic non-electrolytes at one temperature, cancel RT immediately. Find molar mass from mass divided by CV, then check that it is an integer multiple of the empirical formula mass.

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87. A metal surface of $100\ \text{cm}^2$ area has to be coated with nickel layer of thickness $0.001\ \text{mm}$. A current of $2\ \text{A}$ was passed through a solution of $\text{Ni}(\text{NO}_3)_2$ for 'x' seconds to coat the desired layer. The value of x is _____. (Nearest integer) (ρ_{Ni} (density of Nickel) is $10\ \text{g mL}$, Molar mass of Nickel is $60\ \text{g mol}^{-1}$ $\text{F} = 96500\ \text{C mol}^{-1}$) [4 marks]

Solution:

Concept:

- For a uniform metal coating, deposited volume equals coated area multiplied by thickness. Consistent length units must be used before multiplying.
- Faraday's law gives $m = MIT/(zF)$, where M is the molar mass, I the current, T the time, and z the number of electrons per metal ion. Nickel(II) requires $z = 2$.

Step 1: Convert thickness and calculate the deposited mass The thickness is

$$0.001\,\text{mm} = 0.0001\,\text{cm} = 10^{-4}\,\text{cm}.$$

Thus the nickel layer occupies

$$V = 100\,\text{cm}^2 \times 10^{-4}\,\text{cm} = 0.0100\,\text{cm}^3.$$

Using the given nickel density value of $10\,\mathrm{g}$ per mL, and $1\,\mathrm{cm}^3 = 1\,\mathrm{mL}$, the deposited mass is

$$m = 10 \times 0.0100 = 0.100\,\mathrm{g}.$$

Step 2: Calculate the electron requirement and charge Nickel nitrate provides divalent nickel ions. At the cathode the deposition reaction is

$$\mathrm{Ni}^{2+} + 2\,\mathrm{e}^- \longrightarrow \mathrm{Ni}.$$

The nickel amount and electron amount are therefore

$$n_{\mathrm{Ni}} = \frac{0.100}{60} = \frac{1}{600}\,\mathrm{mol}, \qquad n_e = \frac{2}{600}\,\mathrm{mol}.$$

The required charge is

$$Q = n_e F = \frac{2(96500)}{600} = 321.667\,\mathrm{C}.$$

Step 3: Convert charge to deposition time For a steady current, $Q = IT$. With $I = 2\,\mathrm{A} = 2\,\mathrm{C\,s^{-1}}$,

$$T = \frac{Q}{I} = \frac{321.667}{2} = 160.833\,\mathrm{s}.$$

This is also obtained directly from $T = mzF/(MI)$. Keep the unrounded value through the calculation, then round only the final time.

Step 4: Conclusion The required deposition time is approximately $160.833\,\mathrm{s}$, so $\boxed{x = 161\,\mathrm{s}}$ to the nearest integer.

Quick Tip to Solve: Convert millimetres to centimetres before multiplying by area. Then use two electrons per nickel atom; using one electron would incorrectly halve the deposition time.

88.

$$\mathrm{A_2} + \mathrm{B_2} \rightarrow 2\mathrm{AB}.\Delta H_f^0 = -200\ \mathrm{kJ\ mol^{-1}}$$

$\mathrm{AB}, \mathrm{A_2}$ and $\mathrm{B_2}$ are diatomic molecules. If the bond enthalpies of $\mathrm{A_2}$, $\mathrm{B_2}$ and AB are in the ratio $1 : 0.5 : 1$, then the bond enthalpy of $\mathrm{A_2}$ is _____ $\mathrm{kJ\ mol^{-1}}$ (Nearest integer) [4 marks]

Solution:

Concept:

- Bond enthalpy is the energy required to break one mole of the specified gaseous bonds. Bond breaking absorbs energy and bond formation releases energy.
- The reaction enthalpy is $\Delta H = \sum D(\text{bonds broken}) - \sum D(\text{bonds formed})$. The stoichiometric coefficient of each diatomic molecule counts its single bond.

Step 1: Express all three bond enthalpies using one variable Let the bond enthalpy of $\mathrm{A_2}$ be D,

measured in $\mathrm{kJ\,mol^{-1}}$. The given ratio

$$D(\mathrm{A_2}) : D(\mathrm{B_2}) : D(\mathrm{AB}) = 1 : 0.5 : 1$$

then gives

$$D(\mathrm{B_2}) = 0.5D, \qquad D(\mathrm{AB}) = D.$$

The ratio refers to each mole of bonds; it must not be confused with the coefficients in the reaction.

Step 2: Count bonds broken and bonds formed For the reaction as written,

$$\mathrm{A_2 + B_2 \longrightarrow 2\,AB},$$

one A–A bond and one B–B bond are broken. This requires $D + 0.5D = 1.5D$. Two A–B bonds are formed because two AB molecules are produced. Formation releases $2D$. Consequently,

$$\Delta H = 1.5D - 2D = -0.5D.$$

The negative sign agrees with the stated exothermic reaction.

Step 3: Insert the stated reaction enthalpy and solve Using the enthalpy value attached to the complete written equation,

$$-200 = -0.5D \quad \Rightarrow \quad D = \frac{200}{0.5} = 400\,\mathrm{kJ\,mol^{-1}}.$$

As a check, the broken bonds require $400 + 200 = 600\,\mathrm{kJ}$, while the two new bonds release $800\,\mathrm{kJ}$. Their difference is $-200\,\mathrm{kJ}$, exactly the supplied value.

Step 4: Conclusion The bond enthalpy of $\mathrm{A_2}$ is $\boxed{400\,\mathrm{kJ\,mol^{-1}}}$. The nearest-integer entry is 400.

Quick Tip to Solve: Count both AB bonds on the product side before applying broken minus formed. The factor 2 in 2AB is essential even though the bond-enthalpy ratio for AB itself is 1.

89. $25.0\ \mathrm{mL}$ **of** $0.050\ \mathrm{M\ Ba(NO_3)_2}$ **is mixed with** $25.0\ \mathrm{mL}$ **of** $0.020\ \mathrm{M\ NaF}$. $\mathrm{K_{Sp}}$ **of** $\mathrm{BaF_2}$ **is** 0.5×10^{-6} **at** $298\ \mathrm{K}$. **The ratio of** $\left[\mathrm{Ba^{2+}}\right]\left[\mathrm{F^-}\right]^2$ **and** $\mathrm{K_{sp}}$ **is _____.**

(Nearest integer) [4 marks]

Solution:

Concept:

- Immediately on mixing, each ion concentration is its number of moles divided by the total mixed volume. Dilution must be applied to both reacting ions.
- For $\mathrm{BaF_2}$, the ionic product is $Q = [\mathrm{Ba^{2+}}][\mathrm{F^-}]^2$. Comparing the initial ionic product with K_{sp} predicts whether precipitation begins.

Step 1: Calculate the total volume and barium ion concentration The two 25.0 mL solutions give a total volume of

$$V_{\mathrm{total}} = 25.0 + 25.0 = 50.0\,\mathrm{mL} = 0.0500\,\mathrm{L}.$$

Barium nitrate dissociates to give one barium ion per formula unit. Its initial amount is

$$n_{Ba^{2+}} = 0.050 \times 0.0250 = 0.00125\,\text{mol}.$$

Therefore, before precipitation,

$$[Ba^{2+}] = \frac{0.00125}{0.0500} = 0.0250\,\text{M}.$$

Step 2: Calculate the diluted fluoride concentration and ionic product Sodium fluoride supplies one fluoride ion per formula unit. Thus

$$n_{F^-} = 0.020 \times 0.0250 = 0.000500\,\text{mol},$$

and its concentration in the mixed solution is

$$[F^-] = \frac{0.000500}{0.0500} = 0.0100\,\text{M}.$$

The initial ionic product is consequently

$$Q = (0.0250)(0.0100)^2 = 2.50 \times 10^{-6}.$$

The fluoride concentration is squared because two fluoride ions occur in the dissolution stoichiometry.

Step 3: Compare the ionic product with the solubility product At 298 K the given value is $K_{sp} = 0.5 \times 10^{-6}$. Hence

$$\frac{Q}{K_{sp}} = \frac{2.50 \times 10^{-6}}{0.50 \times 10^{-6}} = 5.$$

This ratio refers to the initial mixture used to test precipitation. Since it exceeds one, BaF_2 precipitates; after solid-liquid equilibrium is reached, the remaining ion concentrations instead satisfy $Q/K_{sp} = 1$.

Step 4: Conclusion The requested initial ionic-product-to-solubility-product ratio is $\boxed{5}$, a dimensionless number. It predicts precipitation of BaF_2.

Quick Tip to Solve: Equal solution volumes halve each original concentration. Square the diluted fluoride concentration, then compare Q with Ksp before allowing precipitation to change the concentrations.

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90. A certain quantity of real gas occupies a volume of $0.15\ \text{dm}^3$ at $100\ \text{atm}$ and $500\ \text{K}$ when its compressibility factor is 1.07 . Its volume at 300 atm and $300\ \text{K}$ (When its compressibility factor is 1.4) is _____ $\times 10^{-4}\ \text{dm}^3$ (Nearest integer) [4 marks]

Solution:

Concept:

- The compressibility factor is defined by $Z = PV/(nRT)$. Thus a real gas satisfies $PV = ZnRT$, rather than necessarily obeying the ideal-gas equation.
- For a fixed amount of gas, $PV/(ZT) = nR$ remains constant. Both the temperature change and

the change in Z must be retained when comparing the two states.

Step 1: Write the relation between the initial and final states The quantity of gas is unchanged, so eliminate nR from the two equations:

$$\frac{P_1V_1}{Z_1T_1} = \frac{P_2V_2}{Z_2T_2}.$$

Solving for the required volume gives

$$V_2 = V_1\frac{P_1}{P_2}\frac{T_2}{T_1}\frac{Z_2}{Z_1}.$$

The compressibility ratio is final divided by initial. Reversing that ratio would contradict $V = ZnRT/P$ at fixed pressure and temperature.

Step 2: Substitute every supplied value Use $V_1 = 0.15\,\mathrm{dm}^3$, $P_1 = 100\,\mathrm{atm}$, $P_2 = 300\,\mathrm{atm}$, $T_1 = 500\,\mathrm{K}$, $T_2 = 300\,\mathrm{K}$, $Z_1 = 1.07$, and $Z_2 = 1.4$. Then

$$V_2 = 0.15\left(\frac{100}{300}\right)\left(\frac{300}{500}\right)\left(\frac{1.4}{1.07}\right)\,\mathrm{dm}^3.$$

Pressures may remain in atmospheres because they appear only as a ratio; temperatures must be absolute.

Step 3: Evaluate and convert to the requested scale The pressure and temperature factors alone give $0.15 \times \frac{1}{3} \times 0.6 = 0.0300$. Including the real-gas correction,

$$V_2 = 0.0300 \times \frac{1.4}{1.07} = 0.0392523\,\mathrm{dm}^3.$$

In the requested form this is

$$V_2 = 392.523 \times 10^{-4}\,\mathrm{dm}^3.$$

The coefficient, rather than the volume in unscaled cubic decimetres, is what must be rounded to the nearest integer.

Step 4: Conclusion From the values printed, $V_2 = 392.523 \times 10^{-4}\,\mathrm{dm}^3$, giving $\boxed{393}$ to the nearest integer. The supplied key value 392 differs: rounding the stated numbers gives 393, whereas 392 results from truncation.

Quick Tip to Solve: For a fixed amount of real gas, use PV/(ZT), then convert the volume into the requested power-of-ten units before rounding. Keep the 1.07 denominator unrounded.