JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 13 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 13 April 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let a_1, a_2, a_3, ... be a G.P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6(a_2+a_4)(a_4+a_6)$ is equal to [4 marks]

(A) $2\sqrt{2}$

(B) 2

(C) $3\sqrt{3}$

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- For a geometric progression with first term a and common ratio r, the nth term is $a_n = ar^{n-1}$. Positivity and strict increase imply $a > 0$ and $r > 1$.
- Products of equidistant terms simplify to a square: $a_3a_5 = a^2r^6 = (ar^3)^2$. Use this identity to avoid solving separately for every term.

Step 1: Use the product condition The given product yields

$$a^2r^6 = \frac{1}{9}.$$

Since both factors are positive, taking the positive square root gives

$$ar^3 = \frac{1}{3}.$$

This is precisely the fourth term of the progression; the negative square root is excluded by the stated positivity.

Step 2: Determine the ratio The sum condition is

$$ar^5 + ar^7 = ar^3r^2(1+r^2) = 2.$$

Substituting the known fourth term and writing $t = r^2$, we obtain

$$t(1+t) = 6, \qquad (t-2)(t+3) = 0.$$

Because $t > 1$, only $t = 2$ is admissible. Thus $r^2 = 2$; the root -3 cannot be a square of a real positive ratio.

Step 3: Evaluate the requested expression Factor each sum before multiplying:

$$\begin{aligned} 6(a_2+a_4)(a_4+a_6) &= 6[ar(1+r^2)][ar^3(1+r^2)] \\ &= 6a^2r^4(1+r^2)^2 \\ &= 6\frac{a^2r^6}{r^2}(1+r^2)^2 \\ &= 6\cdot\frac{1}{18}\cdot 9 = 3. \end{aligned}$$

The exact value is rational; $2\sqrt{2}$ and $3\sqrt{3}$ do not satisfy this calculation, and the remaining rational alternative 2 is also excluded.

Step 4: Conclusion The expression equals $\boxed{3}$. Hence, the correct option is (D).

Quick Tip to Solve: When GP terms have symmetric indices, first replace their product by the square of the middle term; then express the remaining conditions in r^2.

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2. Let $|\vec{a}| = 2, |\vec{b}| = 3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then $|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})|^2$ is equal to : [4 marks]

(A) 441

(B) 482

(C) 841

(D) 882

Correct Answer: (D) 882

Solution:

Concept:

- The cross product is bilinear and antisymmetric: $\vec{a}\times\vec{a} = \vec{0}$, $\vec{b}\times\vec{b} = \vec{0}$, and $\vec{b}\times\vec{a} = -\vec{a}\times\vec{b}$.
- The magnitude is $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$, where θ is the angle between the original vectors. Square the entire scalar multiplier when a squared magnitude is asked.

Step 1: Expand the cross product Keep the order of the factors while distributing:

$$\begin{aligned} (\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b}) &= 2\vec{a}\times\vec{a} - 3\vec{a}\times\vec{b} \\ &\quad + 4\vec{b}\times\vec{a} - 6\vec{b}\times\vec{b} \\ &= -3\vec{a}\times\vec{b} - 4\vec{a}\times\vec{b} \\ &= -7(\vec{a}\times\vec{b}). \end{aligned}$$

The two mixed terms reinforce one another. Treating the cross product as commutative would instead give an incorrect coefficient.

Step 2: Compute the squared magnitude For the given lengths and angle,

$$|\vec{a}\times\vec{b}|^2 = 2^2\cdot 3^2\sin^2\frac{\pi}{4} = 36\cdot\frac{1}{2} = 18.$$

Consequently,

$$|-7(\vec{a}\times\vec{b})|^2 = 49\cdot 18 = 882.$$

The minus sign determines direction but disappears on taking a squared magnitude. Equivalently the unsquared magnitude is $21\sqrt{2}$, whose square is again 882.

Step 3: Check the listed alternatives The value 441 would lose the factor of two in $(21\sqrt{2})^2$. The value 841 is 29^2, unrelated to the coefficient 7 and the given sine factor. The remaining value 482 likewise differs from the exact product $49\cdot 18$. Only 882 respects both the bilinear expansion and the given angle.

Step 4: Conclusion The squared magnitude is $\boxed{882}$. Hence, the correct option is (D).

Quick Tip to Solve: For $(p\vec{a}+q\vec{b})\times(r\vec{a}+s\vec{b})$, the coefficient of $\vec{a}\times\vec{b}$ is $ps-qr$; here it is -7.

3. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is : [4 marks]

(A) 324

(B) 328

(C) 326

(D) 327

Correct Answer: (D) 327

Solution:

Concept:

- Dictionary order compares letters from left to right. At each position, count unused letters smaller than the target letter while keeping the earlier prefix fixed.
- All six letters of MONDAY are distinct. A fixed prefix leaving k unused letters has $k!$ completions. The rank is one plus the number of words that precede the target.

Step 1: Count words before the first two letters The alphabetical list of letters is A, D, M, N, O, Y. A word preceding MONDAY may first differ at its initial letter: either A or D is smaller than M. These contribute

$$2\cdot 5! = 240.$$

Now fix the first letter M. Of the remaining letters A, D, N, O, Y, the three letters A, D, N precede O. Their blocks contribute

$$3\cdot 4! = 72.$$

These blocks do not overlap with those already counted, because they begin with M.

Step 2: Count words sharing the prefix MO After fixing MO, the unused letters are A, D, N, Y. At the third position, A and D are smaller than N, giving

$$2\cdot 3! = 12.$$

Fixing MON, only A is smaller than the fourth letter D, so there are

$$1 \cdot 2! = 2$$

additional words. With $MOND$ fixed, the fifth letter A is already the smallest available letter. The final letter is then forced, so neither position adds any preceding words.

Step 3: Convert the count to a serial number The number of preceding words is

$$240 + 72 + 12 + 2 = 326.$$

Serial numbers start at one, so MONDAY itself has number

$$326 + 1 = 327.$$

The alternative 326 counts only the predecessors; 328 adds one word too many, and 324 misses part of the fixed-prefix count.

Step 4: Conclusion The serial number is $\boxed{327}$. Hence, the correct option is (D).

Quick Tip to Solve: Add the fixed-prefix factorial blocks to count predecessors, then add exactly one for the target word itself.

4. Let $S = \{z \in \mathbb{C} : \bar{z} = i\left(z^2 + \mathrm{Re}(\bar{z})\right)\}$. Then $\sum_{z \in S} |z|^2$ is equal to : [4 marks]

(A) $\frac{7}{2}$

(B) 4

(C) 3

(D) $\frac{5}{2}$

Correct Answer: (B) 4

Solution:

Concept:

- Write $z = x + iy$ with real x, y; then $\bar{z} = x - iy$, $\mathrm{Re}(\bar{z}) = x$, and $|z|^2 = x^2 + y^2$.
- An equality of complex numbers requires equality of both real and imaginary parts. Factor the resulting equations before dividing so that solutions with zero real part are retained.

Step 1: Equate the real and imaginary parts Expand the right side of the defining equation:

$$i(z^2 + x) = -2xy + i(x^2 - y^2 + x).$$

Therefore

$$x = -2xy, \qquad -y = x^2 - y^2 + x.$$

The first equation becomes $x(1 + 2y) = 0$. It divides the solution set into $x = 0$ and $y = -\frac{1}{2}$, without losing any root through division by x.

Step 2: Solve both cases completely For $x = 0$, the second equation gives $-y = -y^2$, so $y = 0$ or $y = 1$. These yield $z = 0$ and $z = i$. For $y = -\frac{1}{2}$, substitute into the second equation:

$$\frac{1}{2} = x^2 - \frac{1}{4} + x, \qquad x^2 + x - \frac{3}{4} = 0.$$

Factoring gives $(x - \frac{1}{2})(x + \frac{3}{2}) = 0$. Thus the other two numbers are $\frac{1}{2} - \frac{i}{2}$ and $-\frac{3}{2} - \frac{i}{2}$. The cases are disjoint, so these four numbers constitute the whole set.

Step 3: Sum the squared moduli Their squared moduli are respectively

$$0, \quad 1, \quad \frac{1}{4} + \frac{1}{4} = \frac{1}{2}, \quad \frac{9}{4} + \frac{1}{4} = \frac{5}{2}.$$

Hence

$$\sum_{z \in S} |z|^2 = 0 + 1 + \frac{1}{2} + \frac{5}{2} = 4.$$

The alternative 3 omits the root i; $\frac{5}{2}$ is just one root's contribution. The value $\frac{7}{2}$ also fails to include the complete sum of all four distinct solutions.

Step 4: Conclusion The required sum is $\boxed{4}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the factor $x = 0$ when separating real and imaginary parts: dividing by x would discard two members of this set.

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5. If $\lim\limits_{x \to 0} \frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{1 - \cos(2x)} = 17$**, then** $5a^2 + b^2$ **is equal to** [4 marks]

(A) 64

(B) 68

(C) 72

(D) 76

Correct Answer: (B) 68

Solution:

Concept:

- Near zero, use $e^{kx} = 1 + kx + \frac{1}{2}k^2x^2 + O(x^3)$ and $\cos(kx) = 1 - \frac{1}{2}k^2x^2 + O(x^4)$.
- The denominator $1 - \cos(2x)$ has leading term $2x^2$. A finite limit therefore forces the linear term of the numerator to vanish before its quadratic coefficient can be compared.

Step 1: Expand the numerator through quadratic terms The separate expansions are

$$e^{ax} = 1 + ax + \frac{a^2x^2}{2} + O(x^3),$$
$$\cos(bx) = 1 - \frac{b^2x^2}{2} + O(x^4),$$
$$\frac{cxe^{-cx}}{2} = \frac{c}{2}x - \frac{c^2}{2}x^2 + O(x^3).$$

Notice the negative quadratic term in the last expression. Since that entire expression is subtracted, it contributes a positive $c^2x^2/2$ to the numerator.

Step 2: Impose finiteness and evaluate the limit Combining the terms gives

$$e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2} = \left(a - \frac{c}{2}\right)x + \frac{a^2+b^2+c^2}{2}x^2 + O(x^3).$$

Meanwhile $1 - \cos(2x) = 2x^2 + O(x^4)$. Unless $a - c/2 = 0$, the quotient contains a nonzero multiple of $1/x$, which cannot approach the finite value 17. Hence $c = 2a$, and the limit is

$$\frac{a^2+b^2+c^2}{4} = 17.$$

Step 3: Identify the required combination Substituting $c^2 = 4a^2$, we obtain

$$5a^2 + b^2 = 68.$$

No separate value of a or b is needed: the limiting equation fixes exactly the combination requested. The other alternatives 64, 72, and 76 would produce limits 16, 18, and 19, respectively, after the necessary cancellation, so none can give 17.

Step 4: Conclusion The required value is $\boxed{68}$. Hence, the correct option is (B).

Quick Tip to Solve: If the denominator starts with x^2, first cancel every linear numerator term; only then compare quadratic coefficients.

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6. The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, is : [4 marks]

(A) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

(B) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

(C) $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

(D) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

Correct Answer: (B) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

Solution:

Concept:

- Two lines through points P, Q, with direction vectors $\vec{d}, \vec{e}$, are coplanar precisely when $(Q - P) \cdot (\vec{d} \times \vec{e}) = 0$.
- In a symmetric equation $(x - x_0)/l = (y - y_0)/m = (z - z_0)/n$, the point is (x_0, y_0, z_0) and the direction vector is (l, m, n). A numerator $x + 1$ therefore means $x_0 = -1$.

Step 1: Read the data for the original line The given line passes through

$$P = (-3, 1, 5)$$

with direction vector

$$\vec{d} = (-3, 1, 5).$$

Alternatives A, B and D pass through $Q = (-1, 2, 5)$, so their joining vector is $Q - P = (2, 1, 0)$. Alternative C instead passes through $(1, 2, 5)$, giving the joining vector $(4, 1, 0)$. These point differences must be kept distinct even though several direction entries coincide.

Step 2: Test the first two alternatives For a direction vector $\vec{e} = (-1, 2, k)$,

$$\vec{d} \times \vec{e} = (k - 10, 3k - 5, -5).$$

For the common joining vector $(2, 1, 0)$, the scalar triple product is

$$2(k - 10) + (3k - 5) = 5k - 25.$$

In A, $k = 4$, so the result is $-5 \neq 0$: the lines are skew. In B, $k = 5$, so the result is 0: these two lines lie in a plane.

Step 3: Exclude the remaining alternatives For C, the direction is $(-1, 2, 5)$, so the cross product is $(-5, 10, -5)$. Its dot product with $(4, 1, 0)$ is

$$-20 + 10 = -10 \neq 0.$$

For D, the direction is $(1, 2, 5)$, and

$$\vec{d} \times \vec{e} = (-5, 20, -7).$$

Its dot product with $(2, 1, 0)$ is $-10 + 20 = 10 \neq 0$. Thus neither C nor D is coplanar with the original line; B is the unique admissible alternative.

Step 4: Conclusion The required line is $\boxed{\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}}$. Hence, the correct option is (B).

Quick Tip to Solve: Read both the point and the direction from each option; changing $x + 1$ to $x - 1$ changes the joining vector and can change coplanarity.

7. The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is : [4 marks]

(A) $\frac{26}{3}$

(B) $\frac{80}{9}$

(C) 9

(D) 8

Correct Answer: (B) $\frac{80}{9}$

Solution:

Concept:

- The term selecting the second summand r times from $(A + B)^n$ is $\binom{n}{r} A^{n-r} B^r$, with integer $0 \leq r \leq n$.

- When the two summands contain different powers of x, first equate the total exponent to the requested exponent. Only afterward evaluate the numerical coefficient, including its sign.

Step 1: Write the general term with its exponent Here $A = 2x^3$, $B = -1/(3x^2)$, and $n = 5$. Selecting the second term r times gives

$$\begin{aligned} T_{r+1} &= \binom{5}{r}(2x^3)^{5-r}\left(-\frac{1}{3x^2}\right)^r \\ &= \binom{5}{r}\frac{2^{5-r}(-1)^r}{3^r}x^{3(5-r)-2r} \\ &= \binom{5}{r}\frac{2^{5-r}(-1)^r}{3^r}x^{15-5r}. \end{aligned}$$

The negative exponent in the second summand must be included; it lowers the total power by two for every selection.

Step 2: Select the term containing the fifth power The required exponent condition is

$$15 - 5r = 5, \qquad r = 2.$$

This lies in the permitted integer range, so the requested term exists. Moreover, the exponent decreases by five whenever r increases by one, so no second term can also contribute to x^5. The relevant term is the third term, since the general term is numbered $r + 1$.

Step 3: Evaluate the numerical coefficient Substitute $r = 2$ into the coefficient:

$$\binom{5}{2}\frac{2^3(-1)^2}{3^2} = 10 \cdot \frac{8}{9} = \frac{80}{9}.$$

The sign is positive because the second summand is chosen twice. The value 8 would omit both the binomial multiplier and the denominator, while 9 and $26/3$ do not equal the exact product $10 \cdot 8/9$. There is no need to expand all six terms of the binomial.

Step 4: Conclusion The coefficient of x^5 is $\boxed{\frac{80}{9}}$. Hence, the correct option is (B).

Quick Tip to Solve: For $(ax^p + bx^q)^n$, solve $p(n - r) + qr = k$ to locate the coefficient of x^k, then evaluate only that term.

8. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to [4 marks]

(A) -64

(B) $-64\sqrt{2}$

(C) $-128\sqrt{2}$

(D) -128

Correct Answer: (D) -128

Solution:

Concept:

- For roots of a real quadratic, the quadratic formula gives a conjugate pair. Write each nonzero root as $r(\cos\theta + i\sin\theta)$ before taking a large power.
- De Moivre's theorem gives $\alpha^n + \beta^n = 2r^n\cos(n\theta)$ for conjugates with arguments θ and $-\theta$. The radius must be raised to the same power as the roots.

Step 1: Find the roots in polar form The discriminant is $2 - 8 = -6$, so the roots are not real. The quadratic formula gives

$$\alpha, \beta = \frac{\sqrt{2} \pm i\sqrt{6}}{2}.$$

Their common modulus is $\sqrt{(\sqrt{2}/2)^2 + (\sqrt{6}/2)^2} = \sqrt{2}$. Dividing their real and imaginary parts by this modulus gives $\cos\theta = 1/2$ and $\sin\theta = \sqrt{3}/2$. Thus we may take their arguments as $\pi/3$ and $-\pi/3$, respectively.

Step 2: Evaluate the fourteenth powers The imaginary parts cancel when the two powers are added:

$$\alpha^{14} + \beta^{14} = 2(\sqrt{2})^{14}\cos\left(\frac{14\pi}{3}\right).$$

Since $14\pi/3 = 4\pi + 2\pi/3$, the cosine equals $-1/2$. Also $(\sqrt{2})^{14} = 2^7 = 128$. Consequently

$$\alpha^{14} + \beta^{14} = 2 \cdot 128 \cdot (-1/2) = -128.$$

The answer is real, as expected for a sum of conjugate powers. Options (B) and (C) incorrectly retain a factor $\sqrt{2}$; option (A) loses a factor of two in the power calculation.

Step 3: Conclusion The required value is $\boxed{-128}$. Hence, the correct option is (D).

Quick Tip to Solve: For a conjugate pair, add the powers using twice the real part; reduce the angle modulo 2π before evaluating the cosine.

9. Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82, 6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation : [4 marks]

(A) $x^2 - 7x + 12 = 0$

(B) $x^2 - 13x + 42 = 0$

(C) $x^2 - 14x + 48 = 0$

(D) $x^2 - 10x + 25 = 0$

Correct Answer: (B) $x^2 - 13x + 42 = 0$

Solution:

Concept:

- The vertices of a triangle formed by three lines are their pairwise intersections. Each vertex must satisfy the two equations used to find it.
- The centroid is the arithmetic mean of the three vertex coordinates. If the desired roots are r and s, their monic quadratic is $x^2 - (r + s)x + rs = 0$.

Step 1: Find the three vertices Intersect $15x - y = 82$ with $6x - 5y = -4$. Substituting $y = 15x - 82$ gives $-69x = -414$, hence the first vertex is $(6, 8)$. Intersect the first line with $9x + 4y = 17$: substitution gives $69x = 345$, hence the second vertex is $(5, -7)$. For the remaining pair, multiply $6x - 5y = -4$ by 4 and $9x + 4y = 17$ by 5. Adding gives $69x = 69$, so $x = 1$ and $y = 2$. The third vertex is therefore $(1, 2)$.

Step 2: Calculate the centroid and the requested roots Taking coordinate averages,

$$\alpha = \frac{6 + 5 + 1}{3} = 4, \qquad \beta = \frac{8 - 7 + 2}{3} = 1.$$

The two requested numbers are consequently

$$r = \alpha + 2\beta = 6, \qquad s = 2\alpha - \beta = 7.$$

Their sum is 13 and their product is 42. Therefore the required equation is

$$x^2 - 13x + 42 = (x - 6)(x - 7) = 0.$$

Option (A) has roots 3 and 4, option (C) has roots 6 and 8, and option (D) has the repeated root 5; none gives the required pair 6 and 7.

Step 3: Conclusion The equation is $\boxed{x^2 - 13x + 42 = 0}$. Hence, the correct option is (B).

Quick Tip to Solve: Find all three pairwise intersections before averaging; the centroid uses vertex coordinates, not the coefficients of the three lines.

10. If the system of equations

$$2x + y - z = 5$$

$$2x - 5y + \lambda z = \mu$$

$$x + 2y - 5z = 7$$

has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to [4 marks]

(A) 916

(B) 912

(C) 920

(D) 904

Correct Answer: (A) 916

Solution:

Concept:

- Infinitely many solutions require the coefficient matrix and augmented matrix to have the same rank, less than the number of unknowns.
- If two rows are independent, the remaining equation must be a linear combination of them, including its constant term. A zero determinant alone does not establish consistency.

Step 1: Express the second row through the other rows The first and third coefficient rows, $(2, 1, -1)$ and $(1, 2, -5)$, are independent because their x, y minor is $4 - 1 = 3 \neq 0$. For the rank to remain two, let the second row be a times the first plus b times the third. The x and y coefficients require

$$2a + b = 2, \qquad a + 2b = -5.$$

From $b = 2 - 2a$, substitution gives $-3a = -9$. Thus $a = 3$ and $b = -4$.

Step 2: Enforce the same relation on the constants The z coefficient and constant must follow that same combination:

$$\lambda = 3(-1) - 4(-5) = 17, \qquad \mu = 3(5) - 4(7) = -13.$$

With these values the second equation is exactly three times the first minus four times the third. Hence it imposes no extra restriction, and the two independent equations in three unknowns indeed leave infinitely many solutions. Now

$$(\lambda + \mu)^2 + (\lambda - \mu)^2 = 4^2 + 30^2 = 16 + 900 = 916.$$

Equivalently, $2(\lambda^2 + \mu^2) = 2(289 + 169) = 916$. The other offered values, 912, 920 and 904, fail this exact substitution.

Step 3: Conclusion The required value is $\boxed{916}$. Hence, the correct option is (A).

Quick Tip to Solve: For infinitely many solutions, apply the coefficient-row dependence to the constants too; otherwise the singular system may have no solution.

11. The area of the region $\{(x, y) : x^2 \le y \le |x^2 - 4|, y \ge 1\}$ is [4 marks]

(A) $\frac{4}{3}(4\sqrt{2} + 1)$

(B) $\frac{3}{4}(4\sqrt{2} + 1)$

(C) $\frac{4}{3}(4\sqrt{2} - 1)$

(D) $\frac{3}{4}(4\sqrt{2} - 1)$

Correct Answer: (C) $\frac{4}{3}(4\sqrt{2} - 1)$

Solution:

Concept:

- For vertical strips, area is the integral of the upper boundary minus the lower boundary wherever the upper boundary is at least the lower boundary.
- The conditions $y \ge x^2$ and $y \ge 1$ combine to give lower boundary $\max(x^2, 1)$. Both bounds are even functions, so the region is symmetric about the y-axis.

Step 1: Identify the admissible interval and split points If $|x| \ge 2$, then $|x^2 - 4| = x^2 - 4 < x^2$, so no points are possible. For $|x| < 2$, the upper bound is $4 - x^2$. Requiring $x^2 \le 4 - x^2$ gives $|x| \le \sqrt{2}$. In this interval the upper curve is also at least 1. For $0 \le x \le 1$, the lower boundary is 1, while for $1 \le x \le \sqrt{2}$, it is x^2. The split at 1 is essential because of the extra horizontal constraint.

Step 2: Integrate the two strips and double By symmetry the area, in square units, is

$$A = 2\left[\int_0^1 (3 - x^2)\,dx + \int_1^{\sqrt{2}} (4 - 2x^2)\,dx\right].$$

The first integral is $3 - 1/3 = 8/3$. For the second,

$$\left[4x - \frac{2x^3}{3}\right]_1^{\sqrt{2}} = \frac{8\sqrt{2}}{3} - \frac{10}{3}.$$

Combining gives

$$A = 2\left(\frac{8}{3} + \frac{8\sqrt{2} - 10}{3}\right) = \frac{4}{3}(4\sqrt{2} - 1).$$

The plus sign in option (A) would overcount the strip removed by $y \ge 1$. Options (B) and (D) also have an incorrect prefactor; direct integration fixes it as $4/3$.

Step 3: Conclusion The area is $\boxed{\frac{4}{3}(4\sqrt{2} - 1)}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: When several inequalities give lower bounds for y, use their maximum and split the integral where those lower bounds intersect.

12. The value of $\dfrac{e^{-\frac{\pi}{4}}+\int_0^{\frac{\pi}{4}} e^{-x}\tan^{50}x\,dx}{\int_0^{\frac{\pi}{4}} e^{-x}(\tan^{49}x+\tan^{51}x)\,dx}$ **is** [4 marks]

(A) 51

(B) 50

(C) 25

(D) 49

Correct Answer: (B) 50

Solution:

Concept:

- The adjacent odd powers combine as $\tan^{49}x+\tan^{51}x=\tan^{49}x\sec^2x$. This is proportional to the derivative of $\tan^{50}x$.
- Integration by parts, or the product derivative of $e^{-x}\tan^{50}x$, links the numerator integral to the denominator. The endpoint term must be retained.

Step 1: Differentiate the matching product Write the numerator integral and denominator integral separately as

$$I=\int_0^{\pi/4} e^{-x}\tan^{50}x\,dx,$$

$$J=\int_0^{\pi/4} e^{-x}(\tan^{49}x+\tan^{51}x)\,dx.$$

The product and chain rules give

$$\frac{d}{dx}(e^{-x}\tan^{50}x)=-e^{-x}\tan^{50}x+50e^{-x}\tan^{49}x\sec^2x.$$

Using $\sec^2x=1+\tan^2x$, the last term is exactly 50 times the integrand defining J. All functions are continuous on the integration interval, so the fundamental theorem applies directly.

Step 2: Integrate and evaluate the endpoint term Integrate the derivative from 0 to $\pi/4$. Since $\tan 0=0$ and $\tan(\pi/4)=1$,

$$e^{-\pi/4}=-I+50J.$$

Therefore $e^{-\pi/4}+I=50J$, and the given quotient is $(50J)/J=50$. The cancellation is valid because the integrand of J is positive on $(0,\pi/4]$, so $J>0$. Options 49 and 51 confuse the powers in the integrand with the derivative coefficient. Option 25 loses the factor arising from differentiating $\tan^{50}x$; the power rule supplies 50, not 25.

Step 3: Conclusion The value of the quotient is $\boxed{50}$. Hence, the correct option is (B).

Quick Tip to Solve: When a denominator contains $\tan^{n-1}x+\tan^{n+1}x$, factor out $\tan^{n-1}x$ and look for the derivative of $\tan^n x$.

13. The range of $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is [4 marks]

(A) $[0, 2\pi]$

(B) $[0, 2\pi)$

(C) $[0, \pi)$

(D) $[0, \pi]$

Correct Answer: (B) $[0, 2\pi)$

Solution:

Concept:

- The principal inverse sine $\sin^{-1} t$ is increasing on $[-1, 1]$, with values in $[-\pi/2, \pi/2]$. First determine the actual range of its argument.
- An endpoint belongs to a range only when some finite domain value attains it. A limit as $|x| \to \infty$ alone does not include that endpoint.

Step 1: Determine the range of the inner expression For every real x, $x^2 + 1 > 0$, and

$$t = \frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}.$$

Thus $0 \le t < 1$. The lower endpoint occurs at $x = 0$; the upper endpoint cannot occur because $x^2 = x^2 + 1$ is impossible. Conversely, for every $0 \le t < 1$, choosing $x^2 = t/(1-t)$ produces a real x with precisely that value of t. Hence there are no missing interior values.

Step 2: Apply inverse sine and the scale factor Because inverse sine is continuous and strictly increasing on this argument interval,

$$0 \le \sin^{-1} t < \frac{\pi}{2}.$$

Multiplying by the positive constant 4 preserves both inequalities:

$$0 \le f(x) < 2\pi.$$

At $x = 0$, $f(x) = 0$, whereas as $|x| \to \infty$ the function approaches 2π without reaching it. Option (A) incorrectly includes this unattainable upper endpoint. Options (C) and (D) stop at π, even though values of t close to 1 give function values arbitrarily close to 2π.

Step 3: Conclusion The range is $\boxed{[0, 2\pi)}$. Hence, the correct option is (B).

Quick Tip to Solve: Rewrite $x^2/(x^2+1)$ as $1 - 1/(x^2+1)$; this immediately reveals which endpoint is excluded before applying inverse sine.

14. Let for a triangle ABC,

$$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$$

$$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

$$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB} \cdot \overrightarrow{CA}$ is equal to [4 marks]

(A) 60

(B) 54

(C) 120

(D) 108

Correct Answer: (A) 60

Solution:

Concept:

- Triangle area is half the magnitude of the cross product of two side vectors. Reversing a side changes the cross-product sign but not its magnitude.
- The directed-side relation is $\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB}$. After using the area to determine the unknown component, apply the dot-product formula component by component.

Step 1: Use the area to determine delta Put $u = \overrightarrow{AB} = (-2, 1, 3)$ and $v = \overrightarrow{CA} = (4, 3, \delta)$. Although these directions are not both based at the same vertex, $|u \times v|$ is still twice the area because $\overrightarrow{AC} = -v$. Computing the cross product gives

$$u \times v = (\delta - 9,\ 12 + 2\delta,\ -10).$$

The given area therefore implies

$$(\delta - 9)^2 + (12 + 2\delta)^2 + 100 = (10\sqrt{6})^2 = 600.$$

Expanding and simplifying yields $5\delta^2 + 30\delta + 325 = 600$, or $\delta^2 + 6\delta - 55 = 0$.

Step 2: Find the positive root and evaluate the dot product Factor the quadratic:

$$(\delta - 5)(\delta + 11) = 0.$$

The printed restriction $\delta > 0$ selects $\delta = 5$. Thus $\overrightarrow{CA} = (4, 3, 5)$. Following the route from C through A to B gives

$$\overrightarrow{CB} = (4, 3, 5) + (-2, 1, 3) = (2, 4, 8).$$

Consequently

$$\overrightarrow{CB} \cdot \overrightarrow{CA} = 2 \cdot 4 + 4 \cdot 3 + 8 \cdot 5 = 8 + 12 + 40 = 60.$$

This is a dot product, not twice the area or its square. The other numbers 54, 120 and 108 do not equal the component-wise scalar product; 120 is twice the required result.

Step 3: Conclusion The scalar product is $\boxed{60}$. Hence, the correct option is (A).

Quick Tip to Solve: Keep track of arrow directions: $\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB}$. Use the positive-component condition only after solving the area equation.

15. Let the centre of a circle C be (α, β) and its radius $r < 8$. Let $3x + 4y = 24$ and $3x - 4y = 32$ be two tangents and $4x + 3y = 1$ be a normal to C. Then $(\alpha - \beta + r)$ is equal to : [4 marks]

(A) 7

(B) 9

(C) 5

(D) 6

Correct Answer: (A) 7

Solution:

Concept:

- A normal to a circle passes through its centre. Thus the given normal imposes $4\alpha + 3\beta = 1$.
- The perpendicular distances of the centre from both tangents equal the same positive radius. For $ax + by + c = 0$, this distance is $|a\alpha + b\beta + c|/\sqrt{a^2 + b^2}$.

Step 1: Find both angle bisectors Both tangent normals have length 5, so equality of their distances gives

$$|3\alpha + 4\beta - 24| = |3\alpha - 4\beta - 32|.$$

The expressions inside the absolute values may be equal or opposite. Equality yields $8\beta = -8$, hence $\beta = -1$. Opposite signs yield $6\alpha = 56$, hence $\alpha = 28/3$. These are the two possible loci of centres; retaining just one without checking the radius would lose a valid algebraic possibility.

Step 2: Apply the normal and radius restriction For $\beta = -1$, the normal gives $4\alpha - 3 = 1$, so $\alpha = 1$. Its distance from the first tangent is

$$r = \frac{|3 - 4 - 24|}{5} = 5.$$

For $\alpha = 28/3$, the normal gives $\beta = -109/9$. This centre has

$$r = \frac{|28 - 436/9 - 24|}{5} = \frac{80}{9} > 8.$$

The latter case violates the strict radius bound. For the retained centre the second tangent also has distance $|3 + 4 - 32|/5 = 5$, confirming both tangencies.

Step 3: Evaluate the required combination Use the admissible values only:

$$\alpha - \beta + r = 1 - (-1) + 5 = 7.$$

The alternatives $9, 5, 6$ are not this combination; in particular, 5 is only the radius and does not include the centre coordinates.

Step 4: Conclusion The centre is $(1, -1)$, the radius is 5, and $\alpha - \beta + r = \boxed{7}$. Hence, the correct option is (A).

Quick Tip to Solve: Use equal distances to get both angle bisectors, then use the normal and the radius bound to select the centre.

16. The plane, passing through the points $(0, -1, 2)$ and $(-1, 2, 1)$ and parallel to the line passing through $(5, 1, -7)$ and $(1, -1, -1)$, also passes through the point : [4 marks]

(A) $(0, 5, -2)$

(B) $(2, 0, 1)$

(C) $(1, -2, 1)$

(D) $(-2, 5, 0)$

Correct Answer: (D) $(-2, 5, 0)$

Solution:

Concept:

- Two nonparallel directions lying in a plane determine a normal through their cross product.
- A plane through A with normal $\mathbf{n}$ satisfies $\mathbf{n} \cdot (\mathbf{r} - \mathbf{A}) = 0$. A line parallel to the plane supplies a direction perpendicular to that normal.

Step 1: Form the two directions Let $A = (0, -1, 2)$ and $B = (-1, 2, 1)$. The vector joining the two points of the plane is

$$\overrightarrow{AB} = (-1, 3, -1).$$

The direction of the given parallel line is

$$(1, -1, -1) - (5, 1, -7) = (-4, -2, 6).$$

Scaling a direction by a nonzero scalar does not change parallelism, so use $\mathbf{v} = (-2, -1, 3)$. These vectors are not proportional, so they specify a unique plane through A.

Step 2: Find the normal and plane equation Their cross product is

$$(-1, 3, -1) \times (-2, -1, 3) = (8, 5, 7).$$

Indeed, its dot products with the two directions are $-8 + 15 - 7 = 0$ and $-16 - 5 + 21 = 0$. The required plane is therefore

$$8x + 5(y + 1) + 7(z - 2) = 0,$$

which simplifies to $8x + 5y + 7z - 9 = 0$. Substitution of B gives $-8 + 10 + 7 - 9 = 0$, checking that the second prescribed point is also on it.

Step 3: Test every listed point Evaluate $8x+5y+7z-9$ for the options. For $(0, 5, -2)$ it is $25-14-9 = 2$.

For $(2,0,1)$ it is $16+7-9=14$. For $(1,-2,1)$ it is $8-10+7-9=-4$. Finally, for $(-2,5,0)$ it is $-16+25-9=0$. Only the last point satisfies the plane equation.

Step 4: Conclusion The required plane passes through $\boxed{(-2,5,0)}$. Hence, the correct option is (D).

Quick Tip to Solve: Take a cross product of the joining vector and the parallel-line direction, then test the options in the resulting plane equation.

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17. Let for $A=\begin{bmatrix}1 & 2 & 3\\ \alpha & 3 & 1\\ 1 & 1 & 2\end{bmatrix}$, $|A|=2$. If $|2\,\mathrm{adj}\,(2\,\mathrm{adj}\,(2\mathrm{A}))|=32^{\mathrm{n}}$, then $3n+\alpha$ is equal to [4 marks]

(A) 11

(B) 9

(C) 12

(D) 10

Correct Answer: (A) 11

Solution:

Concept:

- For a matrix M of order three, $\det(cM)=c^3\det M$ and $\det(\mathrm{adj}\,M)=(\det M)^2$.
- Apply the determinant identities to the innermost matrix first. The repeated scalar factors must each be cubed because the matrix order remains three.

Step 1: Determine the parameter in the matrix Expand $\det A$ along its first row:

$$\begin{aligned}\det A &= 1(6-1)-2(2\alpha-1)+3(\alpha-3)\\ &= 5-4\alpha+2+3\alpha-9=-\alpha-2.\end{aligned}$$

The given determinant is 2, so $-\alpha-2=2$, giving $\alpha=-4$. This computation uses the alternating cofactor signs; overlooking the minus sign on the middle entry would change the parameter.

Step 2: Evaluate the nested determinants First, the scalar multiple inside the first adjugate has determinant

$$\det(2A)=2^3\det A=8\cdot 2=16=2^4.$$

Set $B=2\,\mathrm{adj}(2A)$. Its determinant is

$$\det B=2^3\{\det(2A)\}^2=2^3(2^4)^2=2^{11}.$$

The determinant asked for is then

$$\det(2\,\mathrm{adj}\,B)=2^3(\det B)^2=2^3(2^{11})^2=2^{25}.$$

Every factor of 2 outside an adjugate is a scalar multiplying the full matrix, rather than a scalar multiplying its determinant directly.

Step 3: Compare powers and form the answer Since $32^n = (2^5)^n = 2^{5n}$, the equality $2^{25} = 32^n$ implies $n = 5$. Hence

$$3n + \alpha = 3(5) - 4 = 11.$$

The other choices $9, 12, 10$ do not agree with the independently determined pair $(n, \alpha) = (5, -4)$.

Step 4: Conclusion We obtain $n = 5$ and $\alpha = -4$, so $3n + \alpha = \boxed{11}$. Hence, the correct option is (A).

Quick Tip to Solve: For order three, each transformation M to 2 adj(M) changes a determinant d into 8d squared; apply that rule from the inside outward.

---

18. Let N be the foot of perpendicular from the point $P(1, -2, 3)$ on the line passing through the points $(4, 5, 8)$ and $(1, -7, 5)$. Then the distance of N from the plane $2x - 2y + z + 5 = 0$ is : [4 marks]

(A) 7

(B) 6

(C) 9

(D) 8

Correct Answer: (A) 7

Solution:

Concept:

- The perpendicular foot N on a line satisfies $(\mathbf{P} - \mathbf{N}) \cdot \mathbf{d} = 0$, where $\mathbf{d}$ is the line direction.
- The distance from (x_0, y_0, z_0) to $ax + by + cz + d = 0$ is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2 + b^2 + c^2}$.

Step 1: Parameterise the line The difference of the supplied points is

$$(4, 5, 8) - (1, -7, 5) = (3, 12, 3) = 3(1, 4, 1).$$

Use the direction $\mathbf{d} = (1, 4, 1)$ and take the point $A = (4, 5, 8)$ on the line. Any point of the line can be written

$$N = (4 + t, 5 + 4t, 8 + t).$$

This point must additionally satisfy perpendicularity to PN; merely using a point on the line would not identify the required foot.

Step 2: Impose perpendicularity to locate the foot For $P = (1, -2, 3)$, the vector from N to P is

$$P - N = (-3 - t, -7 - 4t, -5 - t).$$

Its dot product with the direction is zero:

$$(-3 - t) + 4(-7 - 4t) + (-5 - t) = -36 - 18t = 0.$$

Thus $t = -2$ and $N = (2, -3, 6)$. As a direct check, $P - N = (-1, 1, -3)$, whose dot product with $(1, 4, 1)$ is $-1 + 4 - 3 = 0$.

Step 3: Calculate the distance from the plane Substitute this foot into the plane expression, keeping

the constant $+5$:

$$d = \frac{|2(2) - 2(-3) + 6 + 5|}{\sqrt{2^2 + (-2)^2 + 1^2}} = \frac{|4 + 6 + 6 + 5|}{3} = 7.$$

The absolute value ensures a nonnegative distance and the denominator normalises the plane normal. The alternative values $6, 9, 8$ do not equal this perpendicular distance; 6 is the foot's third coordinate, rather than the distance.

Step 4: Conclusion The foot is $N = (2, -3, 6)$ and its distance from the given plane is $\boxed{7}$ units. Hence, the correct option is (A).

Quick Tip to Solve: Find the foot by a zero dot product before applying the point-to-plane distance formula; the original point P is not the point whose distance is asked.

19. The statement $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ is equivalent to _____. [4 marks]

(A) $(\sim p) \vee (\sim q)$

(B) $p \vee (\sim q)$

(C) $\mathrm{p} \vee \mathrm{q}$

(D) $(\sim p) \vee q$

Correct Answer: (A) $(\sim p) \vee (\sim q)$

Solution:

Concept:

- The distributive law gives $(a \wedge b) \vee (a \wedge c) = a \wedge (b \vee c)$.
- The complement law is $q \vee \neg q = \mathrm{T}$, and the absorption identity is $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$.

Step 1: Combine the two terms containing not p Write S for the entire given statement. The last two conjunctions have $\neg p$ in common, so

$$\begin{aligned} S &= (p \wedge \neg q) \vee [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \\ &= (p \wedge \neg q) \vee [\neg p \wedge (q \vee \neg q)] \\ &= (p \wedge \neg q) \vee \neg p. \end{aligned}$$

This grouping is permitted by associativity of disjunction. The bracket is true exactly when p is false, regardless of the truth value of q.

Step 2: Distribute to remove the remaining conjunction Apply the distributive identity to the remaining expression:

$$\begin{aligned} \neg p \vee (p \wedge \neg q) &= (\neg p \vee p) \wedge (\neg p \vee \neg q) \\ &= \mathrm{T} \wedge (\neg p \vee \neg q) \\ &= \neg p \vee \neg q. \end{aligned}$$

Thus the original statement is false only when both p and q are true. Equivalently, it is the negation of $p \wedge q$, by De Morgan's law.

Step 3: Distinguish the other options At $(p, q) = (\mathrm{T}, \mathrm{T})$, the original statement is false, whereas both

$p \vee \neg q$ and $p \vee q$ are true. This rejects (B) and (C). At $(p, q) = (\mathrm{T}, \mathrm{F})$, the original first conjunction is true, but $\neg p \vee q$ is false, rejecting (D). Only (A) agrees for every assignment.

Step 4: Conclusion The statement is equivalent to $\boxed{(\sim p) \vee (\sim q)}$. Hence, the correct option is (A).

Quick Tip to Solve: The three conjunctions list every truth assignment except p and q both true; their disjunction is the negation of p AND q.

20. The random variable X follows binomial distribution B(n, p), for which the difference of the mean and the variance is 1 . If $2\mathrm{P}(\mathrm{X}=2)=3\mathrm{P}(\mathrm{X}=1)$, then $n^2\mathrm{P}(\mathrm{X}>1)$ is equal to : [4 marks]

(A) 15

(B) 12

(C) 11

(D) 16

Correct Answer: (C) 11

Solution:

Concept:

- For $X \sim B(n, p)$, the mean is np, the variance is $np(1-p)$, and $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$.
- For $0 < p < 1$, adjacent probabilities obey $P(X=2)/P(X=1) = (n-1)p/[2(1-p)]$. The complement gives $P(X>1) = 1 - P(X=0) - P(X=1)$.

Step 1: Translate the mean and variance condition Their difference is

$$np - np(1-p) = np^2 = 1.$$

Hence $n = 1/p^2$. The endpoint $p = 0$ is impossible. If $p = 1$, this equation forces $n = 1$, but then $P(X=1) = 1$ and $P(X=2) = 0$, contradicting the stated probability relation. Consequently $0 < p < 1$, so division by $P(X=1)$ is legitimate.

Step 2: Use the probability ratio to identify n and p The relation in the question gives

$$\frac{P(X=2)}{P(X=1)} = \frac{3}{2} = \frac{(n-1)p}{2(1-p)}.$$

Thus $(n-1)p = 3(1-p)$, or $(n+2)p = 3$. Substituting $p = 3/(n+2)$ in $np^2 = 1$ yields

$$9n = (n+2)^2, \qquad n^2 - 5n + 4 = 0.$$

The possibilities are $n = 1$ or $n = 4$. The former gives the excluded endpoint $p = 1$; hence $n = 4$ and $p = 1/2$.

Step 3: Evaluate the complementary probability For this distribution,

$$P(X=0) = \frac{1}{16}, \qquad P(X=1) = 4\frac{1}{16} = \frac{4}{16}.$$

Therefore

$$P(X>1)=1-\frac{1}{16}-\frac{4}{16}=\frac{11}{16},$$

and $n^2P(X>1)=16(11/16)=11$. The other choices $15, 12, 16$ do not remove both the zero-success and one-success probabilities correctly.

Step 4: Conclusion The parameters are $n=4$, $p=1/2$, and $n^2P(X>1)=\boxed{11}$. Hence, the correct option is (C).

Quick Tip to Solve: First use mean minus variance to get np squared; then use the adjacent-probability ratio and reject any endpoint introduced by cancellation.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is _____ [4 marks]

Solution:

Concept:

- For N observations, the mean is $\bar{x}=(\sum x)/N$ and the variance is $\sigma^2=(\sum x^2)/N-\bar{x}^2$.
- Correcting a wrongly entered observation requires correcting both its first-power contribution to the sum and its squared contribution to the sum of squares.

Step 1: Recover the totals from the reported statistics There are 10 observations. The initially calculated mean and standard deviation were 50 and 12, respectively. Thus the incorrect total was

$$S=10\cdot 50=500.$$

Since the initially calculated variance is $12^2=144$, the incorrect sum of squares was

$$Q=10(144+50^2)=10(2644)=26440.$$

We use the population-variance formula for the listed group, rather than a sample estimate with denominator 9.

Step 2: Replace the two erroneous observations The actual observations 20 and 25 were entered as 45 and 50. Remove the entered values and insert the correct ones:

$$S_c=500-45-50+20+25=450, \qquad \bar{x}_c=45.$$

Correct the squared total separately:

$$\begin{aligned} Q_c &= 26440 - 45^2 - 50^2 + 20^2 + 25^2 \\ &= 26440 - 2025 - 2500 + 400 + 625 \\ &= 22940. \end{aligned}$$

The change in the ordinary sum is -50, while the squared total changes by -3500; those corrections cannot be interchanged.

Step 3: Recompute the variance around the corrected mean The corrected mean square is $Q_c/10 = 2294$. Subtract the square of the corrected mean:

$$\sigma_c^2 = \frac{22940}{10} - 45^2 = 2294 - 2025 = 269.$$

Although the two observations are both reduced, the variance can increase: dispersion is measured about the newly corrected mean, not the old mean of 50. Both statistics therefore had to be corrected together.

Step 4: Conclusion The correct variance is $\boxed{269}$ squared mark units. The required numerical entry is 269.

Quick Tip to Solve: Recover sum and sum of squares first, correct both totals, and only then subtract the square of the new mean.

22. Let $\mathrm{A} = \{-4, -3, -2, 0, 1, 3, 4\}$ and $\mathrm{R} = \{(a, b) \in \mathrm{A} \times \mathrm{A} : b = |a| \text{ or } b^2 = a + 1\}$ be a relation on A. Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is _____ [4 marks]

Solution:

Concept:

- A relation on A is reflexive when every diagonal pair (a, a), for $a \in A$, belongs to it.
- Symmetry requires the reverse (b, a) of every pair (a, b). The smallest required relation is the union of R, its inverse, and the diagonal.

Step 1: List the pairs from each condition For $b = |a|$, both coordinates must belong to A. This gives

$$(-4, 4), (-3, 3), (0, 0), (1, 1), (3, 3), (4, 4).$$

The value $|-2| = 2$ is excluded because $2 \notin A$. For $b^2 = a + 1$, write $a = b^2 - 1$ and test each $b \in A$. The only valid pairs are

$$(3, -2), (0, 1).$$

In particular, $b = 0$ would require $a = -1$, which is outside A; values $b = -3, 3$ require $a = 8$, while $b = -4, 4$ require $a = 15$, also outside A.

Step 2: Complete the diagonal The existing diagonal pairs are $(0, 0), (1, 1), (3, 3), (4, 4)$. The remaining

elements of A are $-4, -3, -2$, so reflexivity forces exactly three new pairs:

$$(-4,-4), (-3,-3), (-2,-2).$$

These pairs are their own reverses and do not introduce additional symmetry requirements.

Step 3: Add the missing reverse pairs The four off-diagonal pairs originally present are

$$(-4,4), (-3,3), (3,-2), (0,1).$$

None has its reverse in R. Symmetry therefore forces

$$(4,-4), (3,-3), (-2,3), (1,0).$$

These four are distinct and none is a diagonal pair. After adding these and the three diagonals, every original pair has its reverse, every added off-diagonal pair has its original reverse, and all diagonal pairs are present. No further additions are needed.

Step 4: Conclusion The minimum number of pairs to add is $3 + 4 = \boxed{7}$. This is a count of ordered pairs and has no physical unit.

Quick Tip to Solve: Form the missing diagonal pairs first, then reverse only the off-diagonal pairs; this avoids double-counting additions.

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23. Let $f_n = \int_0^{\frac{\pi}{2}} \left(\sum_{k=1}^{n} \sin^{k-1} x\right)\left(\sum_{k=1}^{n} (2k-1)\sin^{k-1} x\right)\cos x\,dx, n \in \mathbb{N}$**. Then** $f_{21} - f_{20}$ **is equal to** _____

[4 marks]

Solution:

Concept:

- The substitution $t = \sin x$ gives $dt = \cos x\,dx$ and transforms the limits $0, \pi/2$ into $0, 1$.
- For the polynomial $S_n(t) = \sum_{k=1}^{n} t^{k-1}$, differentiation shows that the weighted sum equals $S_n(t) + 2tS_n'(t)$.

Step 1: Transform both finite sums Using the substitution, the integral becomes

$$f_n = \int_0^1 S_n(t)\left(\sum_{k=1}^{n}(2k-1)t^{k-1}\right)dt.$$

The sums are finite, so differentiating and rearranging terms involves no convergence issue. In particular,

$$2tS_n'(t) = \sum_{k=1}^{n} 2(k-1)t^{k-1}.$$

Adding $S_n(t)$ changes each coefficient to $2k-1$, exactly the coefficient required in the second factor.

Step 2: Recognise the complete derivative The transformed integrand is $S_n^2 + 2tS_nS_n'$. By the product

and chain rules,

$$\frac{d}{dt}\left[tS_n(t)^2\right] = S_n(t)^2 + 2tS_n(t)S_n'(t).$$

Therefore the fundamental theorem of calculus gives

$$f_n = \left[tS_n(t)^2\right]_0^1.$$

Working with the polynomial rather than a rational geometric-sum formula also avoids an apparent singularity at $t = 1$.

Step 3: Evaluate the endpoints and the difference At $t = 1$, every one of the n summands in S_n equals one, so $S_n(1) = n$. At $t = 0$, the polynomial is finite, and the outside factor t makes the boundary term zero. Hence

$$f_n = n^2, \qquad f_{21} - f_{20} = 21^2 - 20^2 = (21 - 20)(21 + 20).$$

The difference can thus be obtained without expanding either of the large products.

Step 4: Conclusion The requested difference is $\boxed{41}$, a dimensionless value.

Quick Tip to Solve: When two polynomial sums differ by the weights $2k - 1$, test the derivative of t times the square of the unweighted sum.

24. The remainder, when 7^{103} is divided by 17, is _____ [4 marks]

Solution:

Concept:

- To find the remainder on division by 17, replace integers by congruent residues modulo 17; multiplication and exponentiation preserve congruence.
- Repeated squaring computes large powers efficiently. A power congruent to -1 immediately supplies a short period.

Step 1: Find a useful small power of seven Start with $7^2 = 49$. Since $49 = 2 \cdot 17 + 15$,

$$7^2 \equiv 15 \equiv -2 \pmod{17}.$$

Squaring this congruence yields

$$7^4 \equiv (-2)^2 = 4 \pmod{17}.$$

One more square gives

$$7^8 \equiv 4^2 = 16 \equiv -1 \pmod{17}.$$

Thus $7^{16} \equiv 1 \pmod{17}$. The negative representative is helpful because its square is immediately one.

Step 2: Reduce the exponent and evaluate Divide the exponent by the period:

$$103 = 6 \cdot 16 + 7.$$

It follows that

$$7^{103} = (7^{16})^6\, 7^7 \equiv 7^7 \pmod{17}.$$

To calculate the remaining power without large integers, use $7^7 = 7^4\, 7^2\, 7$. Therefore

$$7^7 \equiv 4(-2)7 = -56 \pmod{17}.$$

Adding $4 \cdot 17 = 68$ gives the representative 12.

Step 3: Choose the ordinary division remainder A congruence class contains many integers, but the division algorithm requires the unique remainder r with $0 \le r < 17$. Our residue satisfies $-56 + 68 = 12$, and 12 lies in this required interval. Thus the congruence already supplies the actual remainder, rather than only an equivalent negative residue. As a check, multiplying 12 by 7 gives $84 \equiv -1$, consistent with the earlier result for 7^8.

Step 4: Conclusion The remainder when 7^{103} is divided by 17 is $\boxed{12}$. It is a dimensionless integer.

Quick Tip to Solve: Look for a small power congruent to 1 or -1, reduce the exponent, then bring the final residue into the interval from zero to divisor minus one.

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25. Let $f(x) = \sum_{k=1}^{10} kx^k, x \in \mathbb{R}$. If $2f(2) + f'(2) = 119(2)^{\mathrm{n}} + 1$ then n is equal to _____ [4 marks]

Solution:

Concept:

- A finite geometric sum can be differentiated to obtain a weighted sum: $\sum_{k=1}^{10} kx^k = x\frac{d}{dx}\sum_{k=0}^{10} x^k$.
- Rather than differentiate a rational expression twice, multiply by $(x-1)^2$; its derivative at $x = 2$ produces the combination $2f(2) + f'(2)$.

Step 1: Express the weighted sum as a rational function For $x \neq 1$, the finite geometric identity is

$$\sum_{k=0}^{10} x^k = \frac{x^{11} - 1}{x - 1}.$$

Differentiating and multiplying by x gives

$$f(x) = \frac{10x^{12} - 11x^{11} + x}{(x-1)^2}.$$

The formula is valid near the evaluation point $x = 2$, so there is no concern about the denominator vanishing in the following calculation.

Step 2: Differentiate the cleared identity First clear the denominator:

$$(x-1)^2 f(x) = 10x^{12} - 11x^{11} + x.$$

Apply the product rule to the left side and ordinary power differentiation to the right side:

$$2(x-1)f(x)+(x-1)^2f'(x)=120x^{11}-121x^{10}+1.$$

This identity retains the two terms together, exactly as needed in the question, and avoids separately evaluating two long sums.

Step 3: Substitute the evaluation point and compare At $x=2$, the factors $x-1$ and $(x-1)^2$ are both one. Thus

$$\begin{aligned}2f(2)+f'(2)&=120\cdot 2^{11}-121\cdot 2^{10}+1\\&=(240-121)2^{10}+1\\&=119\cdot 2^{10}+1.\end{aligned}$$

Comparing with the given value $119\cdot 2^n+1$, subtract one and divide by the nonzero coefficient 119. This yields $2^n=2^{10}$. The exponential with base two is injective for real exponents, so the exponent is uniquely determined.

Step 4: Conclusion The required exponent is $n=\boxed{10}$, a dimensionless value.

Quick Tip to Solve: When a problem combines a finite weighted sum and its derivative, clear the geometric-sum denominator before differentiating.

26. For $x\in(-1,1]$, the number of solutions of the equation $\sin^{-1}x=2\tan^{-1}x$ is equal to _____.

[4 marks]

Solution:

Concept:

- For $x\in[-1,1]$, $\sin(\sin^{-1}x)=x$, and the principal value $\sin^{-1}x$ lies in $[-\pi/2,\pi/2]$.
- The identity $\sin(2\tan^{-1}x)=2x/(1+x^2)$ gives a necessary algebraic condition; every resulting candidate must still satisfy the original equation and domain.

Step 1: Obtain an algebraic condition Apply sine to both sides of the printed equation. Every solution must satisfy

$$x=\sin(2\tan^{-1}x)=\frac{2x}{1+x^2}.$$

The denominator $1+x^2$ is strictly positive for all real x, so multiplication does not change the set of solutions to this necessary equation. It gives

$$x(1+x^2)=2x\quad\Longrightarrow\quad x(x^2-1)=0.$$

The only algebraic candidates are $x=-1,0,1$. Do not cancel x prematurely, because that would lose the zero solution.

Step 2: Apply the stated domain The allowed interval is $(-1,1]$. Its left endpoint is excluded, whereas its right endpoint is included. Consequently $x=-1$ cannot be counted, regardless of whether it solves the unrestricted inverse-trigonometric equation. The remaining candidates are $x=0$ and $x=1$. There are no other possible candidates because the cubic equation has already been completely

factored.

Step 3: Check the original principal-value equation Taking sine can sometimes identify unequal angles, so verify the remaining candidates directly. At zero,

$$\sin^{-1} 0 = 0 = 2\tan^{-1} 0.$$

At one, using principal values,

$$\sin^{-1} 1 = \frac{\pi}{2}, \qquad 2\tan^{-1} 1 = 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}.$$

Both satisfy the original equation. Hence every retained algebraic candidate is valid, and the necessary condition has not introduced any extra counted solution.

Step 4: Conclusion The solution set in $(-1, 1]$ is $\{0, 1\}$. Therefore the required number of solutions is $\boxed{2}$, a dimensionless count.

Quick Tip to Solve: After applying a trigonometric function to an inverse-trigonometric equation, check candidates in the original equation and inspect open endpoints carefully.

27. If $y = y(x)$ is the solution of the differential equation
$\frac{dy}{dx} + \frac{4x}{(x^2-1)}y = \frac{x+2}{(x^2-1)^{\frac{5}{2}}}, x > 1$ **such that**
$y(2) = \frac{2}{9}\log_e(2+\sqrt{3})$ **and** $y(\sqrt{2}) = \alpha\log_e(\sqrt{\alpha}+\beta)+\beta-\sqrt{\gamma}, \alpha, \beta, \gamma \in \mathbb{N}$**, then** $\alpha\beta\gamma$ **is equal to :** [4 marks]

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, multiplication by the integrating factor $I(x) = e^{\int P(x)dx}$ makes the left side $(Iy)'$.
- On $x > 1$, $\int dx/\sqrt{x^2-1} = \log(x + \sqrt{x^2-1})$, and all square roots and logarithms below are real.

Step 1: Find and apply the integrating factor Here $P(x) = 4x/(x^2-1)$, so

$$\int P(x)dx = 2\log(x^2-1), \qquad I(x) = (x^2-1)^2.$$

The positivity of $x^2 - 1$ follows from $x > 1$. Multiplying the differential equation by $I(x)$ gives

$$\frac{d}{dx}\left[(x^2-1)^2 y\right] = \frac{x+2}{\sqrt{x^2-1}}.$$

The two numerator terms now have standard antiderivatives.

Step 2: Integrate and use the initial value Integrating gives

$$(x^2-1)^2 y = \sqrt{x^2-1} + 2\log(x + \sqrt{x^2-1}) + C.$$

Indeed, differentiating the square-root term produces $x/\sqrt{x^2-1}$, and differentiating the logarithm

contributes $1/\sqrt{x^2-1}$. At $x=2$, the prescribed value makes the left side $9y(2) = 2\log(2+\sqrt{3})$. Thus

$$2\log(2+\sqrt{3}) = \sqrt{3} + 2\log(2+\sqrt{3}) + C,$$

and hence $C = -\sqrt{3}$.

Step 3: Evaluate at the second point and identify the integers At $x=\sqrt{2}$, one has $x^2-1=1$, so the denominator $(x^2-1)^2$ also equals one. The solution therefore gives

$$y(\sqrt{2}) = 2\log(\sqrt{2}+1) + 1 - \sqrt{3}.$$

Comparing the displayed form with $\alpha\log(\sqrt{\alpha}+\beta)+\beta-\sqrt{\gamma}$, we obtain $\alpha=2$, $\beta=1$, and $\gamma=3$. The same value $\alpha=2$ occurs in the logarithm coefficient and inside its square root, and $\beta=1$ occurs both inside and outside the logarithm, so all printed positions agree.

Step 4: Conclusion The required product is $\alpha\beta\gamma = 2\cdot 1\cdot 3 = \boxed{6}$, a dimensionless integer.

Quick Tip to Solve: For a linear equation, multiply through by the integrating factor before integrating; use the initial condition on the undivided expression to simplify arithmetic.

28. Let $[\alpha]$ denote the greatest integer $\leq \alpha$. Then $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+\ldots+[\sqrt{120}]$ is equal to _____

[4 marks]

Solution:

Concept:

- For an integer $k \geq 1$, $\lfloor\sqrt{n}\rfloor = k$ exactly when $k^2 \leq n < (k+1)^2$.
- The number of integers in such a complete block is $(k+1)^2 - k^2 = 2k+1$. Grouping equal floor values converts the sum into standard polynomial sums.

Step 1: Locate the complete square blocks The upper endpoint is $120 = 11^2 - 1$. Therefore the blocks with floor values $k = 1, 2, \ldots, 10$ are all complete. For a fixed k, the integers run from k^2 through $(k+1)^2 - 1$, inclusive, giving

$$(k+1)^2 - 1 - k^2 + 1 = 2k+1$$

terms. Each of these terms contributes the value k. For example, $n = 1, 2, 3$ contribute three ones, and $n = 4, 5, 6, 7, 8$ contribute five twos.

Step 2: Convert the original sum into two standard sums Adding the contributions of all the blocks gives

$$\sum_{n=1}^{120}\lfloor\sqrt{n}\rfloor = \sum_{k=1}^{10} k(2k+1) = 2\sum_{k=1}^{10} k^2 + \sum_{k=1}^{10} k.$$

No partial final block is needed because the last integer is exactly one less than 11^2. As a count check, $\sum_{k=1}^{10}(2k+1) = 110 + 10 = 120$, so the grouping accounts for every original term once.

Step 3: Evaluate and combine The familiar formulas for the first m positive integers and their squares

give

$$\sum_{k=1}^{10} k = \frac{10 \cdot 11}{2} = 55,$$

and

$$\sum_{k=1}^{10} k^2 = \frac{10 \cdot 11 \cdot 21}{6} = 385.$$

Substituting these values into the grouped expression yields

$$2 \cdot 385 + 55 = 770 + 55 = 825.$$

This value also has the expected scale: the floor values range from one to ten, with more integers occupying the larger square blocks, so the average contribution is above the midpoint of that range.

Step 4: Conclusion The required sum is $\boxed{825}$, a dimensionless integer.

Quick Tip to Solve: For sums of $\lfloor \sqrt{n} \rfloor$, group integers between consecutive squares; a complete block at level k contains $2k + 1$ terms.

29. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits $1, 2, 3, 4, 5$ with repetition, is _____. [4 marks]

Solution:

Concept:

- A positive integer is divisible by 6 exactly when it is divisible by both 2 and 3. Thus its units digit must be even and its digit sum must be a multiple of three.
- Repetition is allowed, so the hundreds and tens positions are independent choices. Group the available digits by their residues modulo three: $R_0 = \{3\}$, $R_1 = \{1, 4\}$, $R_2 = \{2, 5\}$.

Step 1: Fix the possible units digits The only even available digits are 2 and 4. Every choice of hundreds digit is nonzero, so it produces a genuine three-digit number. Count the two units-digit cases separately; they cannot overlap.

Step 2: Count numbers ending in 2 The first two digits must have residue sum 1, because the last digit contributes residue 2. Their ordered residue pairs are $(0, 1), (1, 0), (2, 2)$. Hence the number of choices is

$$1 \cdot 2 + 2 \cdot 1 + 2 \cdot 2 = 8.$$

Order matters because the hundreds and tens positions are distinct.

Step 3: Count numbers ending in 4 Now the last digit contributes residue 1, so the first two digits must have residue sum 2. The ordered pairs are $(0, 2), (2, 0), (1, 1)$. Their count is

$$1 \cdot 2 + 2 \cdot 1 + 2 \cdot 2 = 8.$$

The allowed repetition is already included in the products, for example the pair $(1, 1)$ includes equal digits.

Step 4: Conclusion The required total is $8 + 8 = \boxed{16}$ three-digit numbers.

Quick Tip to Solve: For divisibility by six, first restrict the last digit, then count ordered residue pairs for divisibility by three.

30. The foci of a hyperbola are $(\pm 2, 0)$ and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line $2x + 3y = 6$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x - and y-axes are a and b respectively, then $|6a| + |5b|$ is equal to _____ [4 marks]

Solution:

Concept:

- For the hyperbola $x^2/A^2 - y^2/B^2 = 1$, the focal distance satisfies $c^2 = A^2 + B^2$, and eccentricity $e = c/A$. Capital letters distinguish its parameters from the requested intercepts.
- A tangent of slope m has equation $y = mx + k$, with $k^2 = A^2m^2 - B^2$. Perpendicular nonvertical lines have slopes whose product is -1.

Step 1: Determine the hyperbola The foci give $c = 2$, and $e = 3/2$. Therefore

$$A = \frac{c}{e} = \frac{4}{3}, \qquad A^2 = \frac{16}{9}, \qquad B^2 = 4 - \frac{16}{9} = \frac{20}{9}.$$

The transverse axis is the x-axis, as required by the positions of the foci.

Step 2: Choose the tangent in the first quadrant The given line has slope $-2/3$, so the required tangent has slope $m = 3/2$. Consequently

$$k^2 = \frac{16}{9}\frac{9}{4} - \frac{20}{9} = \frac{16}{9}.$$

Thus $k = \pm 4/3$. The tangent at a first-quadrant point has positive x-coordinate and positive y-coordinate and therefore negative y-intercept. Choose $y = 3x/2 - 4/3$. Indeed its contact point is $(2, 5/3)$, which lies in the first quadrant and satisfies the hyperbola.

Step 3: Calculate the signed intercepts Setting $y = 0$ gives $a = 8/9$; setting $x = 0$ gives $b = -4/3$. Keep these signs until taking the absolute values:

$$|6a| + |5b| = \left|\frac{16}{3}\right| + \left|-\frac{20}{3}\right| = 12.$$

Step 4: Conclusion The required value is $\boxed{12}$.

Quick Tip to Solve: Find the tangent slope first, and use the quadrant of contact to select the sign of its constant term.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The distance travelled by an object in time t is given by $s = (2.5)t^2$. The instantaneous speed of the object at $\mathrm{t} = 5\ \mathrm{s}$ will be: [4 marks]

(A) $5\ \mathrm{ms}^{-1}$

(B) $12.5\ \mathrm{ms}^{-1}$

(C) $62.5\ \mathrm{ms}^{-1}$

(D) $25\ \mathrm{ms}^{-1}$

Correct Answer: (D) $25\ \mathrm{ms}^{-1}$

Solution:

Concept:

- Instantaneous speed is the derivative of distance travelled with respect to time: $v = ds/dt$, provided the distance function is differentiable.
- For a power law $s = kt^n$, differentiation gives $v = nkt^{n-1}$. Average speed s/t is generally different from instantaneous speed.

Step 1: Differentiate the distance function The given distance is $s = 2.5t^2$. Differentiating with respect to t, while keeping the numerical coefficient fixed, gives

$$v(t) = \frac{d}{dt}(2.5t^2) = 2(2.5)t = 5t.$$

The resulting quantity is in metres per second when distance and time are in SI units. Its sign is positive for the specified positive time, so its magnitude equals the instantaneous speed.

Step 2: Substitute the specified time At $t = 5\,\mathrm{s}$,

$$v(5) = 5 \times 5 = 25\,\mathrm{m\,s^{-1}}.$$

One can also identify the motion as uniformly accelerated from rest with acceleration $5\,\mathrm{m\,s^{-2}}$, which gives the same speed from $v = at$.

Step 3: Check the competing values The value $12.5\,\mathrm{m\,s^{-1}}$ is $s(5)/5$, the average speed over the first five seconds. The number 62.5 is the distance $s(5)$ in metres, rather than a speed. The number 5 is the acceleration coefficient in the differentiated velocity law, and is not the velocity at five seconds. Therefore none of options (A), (B) or (C) gives the required instantaneous speed.

Step 4: Conclusion The instantaneous speed is $\boxed{25\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Differentiate a distance-time formula before inserting the time; dividing distance by time gives an average.

32. Given below are two statements:

Statement I : For a planet, if the ratio of mass of the planet to its radius increases, the escape velocity from the planet also increases.

Statement II : Escape velocity is independent of the radius of the planet.

In the light of above statements, choose the most appropriate answer form the options given below

[4 marks]

(A) Both Statement I and Statement II are correct

(B) Statement I is correct but statement II is incorrect

(C) Both Statement I and Statement II are incorrect

(D) Statement I is incorrect but statement II is correct

Correct Answer: (B) Statement I is correct but statement II is incorrect

Solution:

Concept:

- Escape speed from the surface of a spherical planet is $v_e = \sqrt{2GM/R}$, where M and R are its mass and radius and G is the gravitational constant.
- This follows by setting the launch kinetic energy equal to the magnitude of the gravitational potential energy, with the object just reaching infinity at zero speed. The escaping object's mass cancels.

Step 1: Relate escape speed to the mass-radius ratio Write the formula as

$$v_e = \sqrt{2G}\sqrt{\frac{M}{R}}.$$

Since G is fixed and the square-root function is increasing for positive arguments, an increase of M/R necessarily increases the escape speed. For example, multiplying this ratio by four doubles the escape speed. Statement I is therefore correct.

Step 2: Test the claimed independence of radius At a fixed planetary mass,

$$v_e \propto R^{-1/2}.$$

Changing the radius changes the gravitational potential at the surface and hence the minimum kinetic energy needed to escape. Doubling the radius at fixed mass reduces the escape speed by a factor $\sqrt{2}$. Thus Statement II is incorrect. Independence from the escaping body's mass must not be confused with independence from the planet's radius.

Step 3: Compare the logical combinations Option (A) incorrectly accepts Statement II. Option (C) incorrectly rejects Statement I, and option (D) reverses both truth values. The combination in option (B) agrees with the direct consequences of the escape-speed formula. No assumption about a relation between different planets' densities is supplied or needed.

Step 4: Conclusion Statement I is correct and Statement II is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: Keep planetary mass, planetary radius and the escaping body's mass distinct when reading escape-speed statements.

33. In the equation $\left[X + \frac{a}{Y^2}\right][Y - b] = \mathrm{R}T$, X is pressure, Y is volume, R is universal gas constant and T is temperature. The physical quantity equivalent to the ratio $\frac{a}{b}$ is: [4 marks]

(A) Impulse

(B) Energy

(C) Pressure gradient

(D) Coefficient of viscosity

Correct Answer: (B) Energy

Solution:

Concept:

- Only quantities having the same dimensions can be added or subtracted. Hence each bracket determines the dimensions of its correction parameter.
- Pressure has dimensions $[P] = ML^{-1}T^{-2}$, volume has dimensions $[V] = L^3$, and pressure multiplied by volume has the dimensions of energy.

Step 1: Determine the dimensions of a and b In the first bracket, a/Y^2 is added to pressure X. Therefore

$$\left[\frac{a}{Y^2}\right] = [P], \qquad [a] = [P][V]^2.$$

In the second bracket, b is subtracted from volume Y, so

$$[b] = [V].$$

These conclusions use dimensional homogeneity and do not require numerical values for either parameter.

Step 2: Form the requested ratio Dividing the two results gives

$$\left[\frac{a}{b}\right] = \frac{[P][V]^2}{[V]} = [P][V].$$

In fundamental dimensions this is

$$(ML^{-1}T^{-2})(L^3) = ML^2T^{-2}.$$

The right side RT for a molar gas equation also carries energy units, providing a consistency check on the pressure-volume product.

Step 3: Eliminate the other quantities Impulse has dimensions MLT^{-1}, whereas a pressure gradient has dimensions $ML^{-2}T^{-2}$. The coefficient of dynamic viscosity has dimensions $ML^{-1}T^{-1}$. None matches ML^2T^{-2}. Energy, including mechanical work, has exactly the required dimensions; dimensional equivalence does not require interpreting a/b as a particular microscopic energy.

Step 4: Conclusion The ratio a/b is dimensionally equivalent to energy. Hence, the correct option is (B).

Quick Tip to Solve: Use the additions inside the brackets first; after finding each parameter, simplify their ratio to a familiar product.

34. The initial pressure and volume of an ideal gas are P_0 and V_0. The final pressure of the gas when the gas is suddenly compressed to volume $\frac{V_0}{4}$ will be :

(Given γ = ratio of specific heats at constant pressure and at constant volume) [4 marks]

(A) $P_0(4)^{\frac{1}{\gamma}}$

(B) P_0

(C) $4P_0$

(D) $P_0(4)^{\gamma}$

Correct Answer: (D) $P_0(4)^{\gamma}$

Solution:

Concept:

- A sudden compression is treated as adiabatic in this ideal-gas question because there is negligible time for heat transfer to the surroundings.
- For the standard ideal-gas adiabatic relation, $PV^{\gamma} =$ constant, where $\gamma = C_P/C_V$. Compression increases the temperature as work is done on the gas.

Step 1: Identify the thermodynamic relation The initial state is (P_0, V_0), and the final volume is $V_f = V_0/4$. With the adiabatic model intended by the question,

$$P_0 V_0^{\gamma} = P_f V_f^{\gamma}.$$

The phrase sudden compression rules out maintaining thermal equilibrium with a reservoir throughout the compression; using the isothermal relation $PV =$ constant would miss the temperature increase.

Step 2: Calculate the final pressure Substitute the final volume and solve for pressure:

$$P_f = P_0 \left(\frac{V_0}{V_f}\right)^{\gamma} = P_0 \left(\frac{V_0}{V_0/4}\right)^{\gamma} = P_0\, 4^{\gamma}.$$

The volume ratio is dimensionless, so the answer has the same pressure units as P_0. For an ideal gas $\gamma > 1$, making the adiabatic pressure rise larger than the isothermal fourfold rise.

Step 3: Check the choices Option (C), $4P_0$, would describe isothermal compression. Option (B) would keep the pressure unchanged despite the specified compression. Option (A) uses the reciprocal exponent, which would arise from an incorrect rearrangement of the adiabatic relation. Thus only option (D) contains the correct volume ratio raised to γ.

Step 4: Conclusion The final pressure is $\boxed{P_f = P_0\, 4^{\gamma}}$. Hence, the correct option is (D).

Quick Tip to Solve: In the standard rapid-compression model, use the volume ratio to the power gamma, rather than the isothermal first power.

35. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : A spherical body of radius $(5 \pm 0.1)\text{mm}$ having a particular density is falling through a liquid of constant density. The percentage error in the calculation of its terminal velocity is 4%.

Reason R : The terminal velocity of the spherical body falling through the liquid is inversely proportional to its radius.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) A is false but **R** is true

(B) A is true but **R** is false

(C) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(D) Both **A** and **R** are true and **R** is the correct explanation of **A**

Correct Answer: (B) A is true but **R** is false

Solution:

Concept:

- For a small sphere falling through a viscous liquid in the Stokes regime, $v_t = 2r^2(\rho_s - \rho_l)g/(9\eta)$, where r is the radius and η the dynamic viscosity.
- When all other quantities are fixed, $v_t \propto r^2$. For a measured quantity raised to a power, the first-order maximum fractional error is multiplied by the magnitude of that power.

Step 1: Evaluate the assertion The measured radius is $(5 \pm 0.1)\,\text{mm}$. Its fractional error is

$$\frac{\Delta r}{r} = \frac{0.1}{5} = 0.02.$$

Since terminal velocity depends on r^2, its percentage error, using the usual first-order propagation rule, is

$$\frac{\Delta v_t}{v_t} \times 100 = 2\frac{\Delta r}{r} \times 100 = 2(0.02)(100) = 4\%.$$

The common millimetre unit cancels in the ratio; converting to metres would give the same result. Assertion A is true.

Step 2: Evaluate the reason independently Stokes drag is $6\pi\eta r v_t$. At terminal speed it balances the effective downward weight, which is proportional to the sphere volume r^3. Hence $rv_t \propto r^3$, giving $v_t \propto r^2$, not r^{-1}. The stated inverse dependence is therefore false.

Step 3: Select the logical outcome Because the assertion is true and the reason is false, option (B) is the required combination. Option (A) reverses the truth values. Both options (C) and (D) incorrectly accept the reason as true, so neither can apply; the reason cannot explain the assertion when its claimed radius dependence is itself incorrect.

Step 4: Conclusion The percentage error is 4%, so A is true while R is false. Hence, the correct option is (B).

Quick Tip to Solve: Derive the radius power in terminal velocity before propagating its error; Stokes drag alone does not give the final dependence.

36. A vehicle of mass $200\ \mathrm{kg}$ is moving along a levelled curved road of radius $70\ \mathrm{m}$ with angular velocity of $0.2\ \mathrm{rad/s}$. The centripetal force acting on the vehicle is: [4 marks]

(A) $560\ \mathrm{N}$

(B) $14\ \mathrm{N}$

(C) $2800\ \mathrm{N}$

(D) $2240\ \mathrm{N}$

Correct Answer: (A) $560\ \mathrm{N}$

Solution:

Concept:

- An object moving at angular speed ω on a circle of radius r has tangential speed $v = r\omega$. Its velocity changes direction even if its speed is constant.
- The net inward force required is $F_c = mv^2/r = mr\omega^2$, where m is the mass. On a level road the inward horizontal force is supplied by tyre-road friction.

Step 1: Identify the given angular speed The mass is $m = 200\,\mathrm{kg}$, the radius is $r = 70\,\mathrm{m}$, and the angular speed is $\omega = 0.2\,\mathrm{rad\,s^{-1}}$. The given quantity is angular speed, so first convert it to linear speed:

$$v = r\omega = 70(0.2) = 14\,\mathrm{m\,s^{-1}}.$$

The value 14 is a speed, not the force requested.

Step 2: Calculate the inward acceleration and force The acceleration points toward the centre of curvature. Its magnitude and the corresponding resultant force are

$$a_c = r\omega^2 = 70(0.2)^2 = 2.8\,\mathrm{m\,s^{-2}},$$

$$F_c = ma_c = 200(2.8) = 560\,\mathrm{N}.$$

Equivalently, $200(14)^2/70$ gives the same force. Weight and the normal reaction balance vertically and do not contribute to this horizontal resultant.

Step 3: Check the proposed alternatives The unit of $mr\omega^2$ is $\mathrm{kg\,m\,s^{-2}}$, as required for force. Option (B) uses the numerical value of the speed and has the wrong physical interpretation. Values 2800 N and 2240 N would require inward accelerations of 14 and 11.2 $\mathrm{m\,s^{-2}}$, respectively, neither of which equals $r\omega^2$.

Step 4: Conclusion The centripetal force is $\boxed{560\,\mathrm{N}}$, directed toward the centre of the curve. Hence, the correct option is (A).

Quick Tip to Solve: When angular speed is supplied, use $F_c = mr\omega^2$ directly; remember to square the angular speed, not just multiply by it.

37. Two planets A and B of radii R and 1.5 R have densities ρ and $\rho/2$ respectively. The ratio of acceleration due to gravity at the surface of B to A is: [4 marks]

(A) 2 : 1

(B) 2 : 3

(C) 4 : 3

(D) 3 : 4

Correct Answer: (D) 3 : 4

Solution:

Concept:

- The gravitational acceleration on the surface of a spherical planet is $g = GM/R^2$, where M and R are its mass and radius.
- Using the mean density ρ, its mass is $M = (4\pi/3)\rho R^3$. Consequently surface gravity is proportional to ρR, rather than to density alone.

Step 1: Express gravity in terms of density Substitute the volume-density expression for mass into the surface-gravity formula:

$$g = \frac{G}{R^2}\left(\frac{4\pi}{3}\rho R^3\right) = \frac{4\pi G}{3}\rho R.$$

The constants are identical for both planets, so they cancel in a ratio. The density here is the mean density; no assumption about the two planets having equal mass is appropriate.

Step 2: Insert the radius and density ratios For planet A the parameters are $R_A = R$ and $\rho_A = \rho$. For planet B they are $R_B = 1.5R$ and $\rho_B = \rho/2$. Thus

$$\frac{g_B}{g_A} = \frac{\rho_B R_B}{\rho_A R_A} = \frac{(\rho/2)(1.5R)}{\rho R} = \frac{3}{4}.$$

The larger radius increases the density-based gravity factor by 1.5, while the halved density reduces it by two.

Step 3: Check the direction of the requested ratio The question asks for B to A, so the ratio is $3 : 4$. Option (C), $4 : 3$, reverses the order. Option (A) ignores the radius and reverses the density effect, while option (B) does not use the product of density and radius. As a further check, $M_B/M_A = (1/2)(1.5)^3 = 27/16$; dividing this by $(R_B/R_A)^2 = 9/4$ again yields $3/4$.

Step 4: Conclusion The requested ratio is $\boxed{g_B : g_A = 3 : 4}$. Hence, the correct option is (D).

Quick Tip to Solve: For surface gravity comparisons with density given, reduce GM/R^2 to $(4\pi G/3)\rho R$ before substituting the ratios.

38. In an electromagnetic wave, at an instant and at particular position, the electric field is along the negative z-axis and magnetic field is along the positive x-axis. Then the direction of propagation of electromagnetic wave is: [4 marks]

(A) at 45° angle from positive y-axis

(B) positive y-axis

(C) negative y-axis

(D) positive z-axis

Correct Answer: (C) negative y-axis

Solution:

Concept:

- The energy-flow direction of an electromagnetic wave is the direction of its Poynting vector, $\mathbf{S} = (1/\mu_0)\mathbf{E} \times \mathbf{B}$ in vacuum.
- The ordered unit-vector products are $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$, and $\hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$. Reversing the order changes the sign.

Step 1: Write the field vectors with their signs The electric field lies along the negative z-axis, whereas the magnetic field lies along the positive x-axis. With positive magnitudes E and B, write

$$\mathbf{E} = -E\hat{\mathbf{k}}, \qquad \mathbf{B} = B\hat{\mathbf{i}}.$$

The minus sign on the electric field must be retained when taking the vector product. Neither field direction alone gives the direction of energy transport.

Step 2: Calculate the ordered cross product Use electric field crossed with magnetic field, in that order:

$$\mathbf{E} \times \mathbf{B} = (-E\hat{\mathbf{k}}) \times (B\hat{\mathbf{i}}) = -EB(\hat{\mathbf{k}} \times \hat{\mathbf{i}}) = -EB\hat{\mathbf{j}}.$$

Since $1/\mu_0$ is positive, multiplication by it does not change this direction. The propagation direction is therefore the negative y-axis.

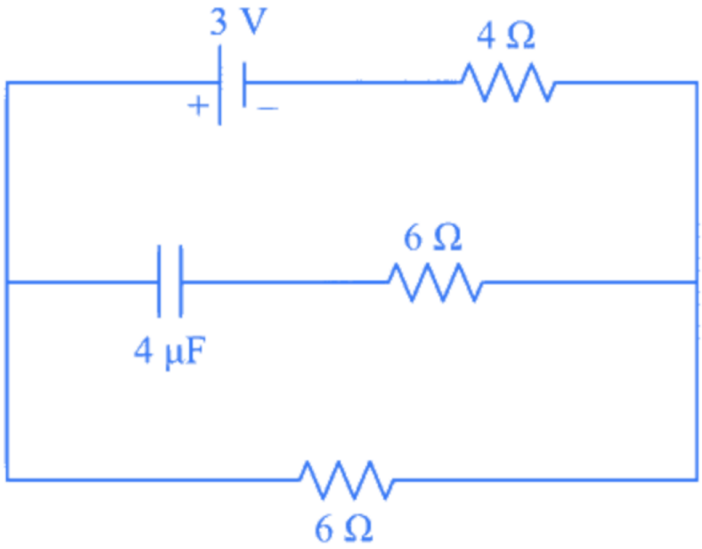
Step 3: Check perpendicularity and reject other directions A transverse electromagnetic wave propagates perpendicular to both fields. A direction tilted by 45° from the y-axis would have a component in the xz plane, so it cannot satisfy that condition here. Propagation along positive z is parallel to the electric field and is also excluded. Positive y is perpendicular to both fields but has the wrong handedness: it would result from reversing the cross-product order or losing the negative sign on $\mathbf{E}$.

Step 4: Conclusion The wave travels along the $\boxed{\text{negative y-axis}}$. Hence, the correct option is (C).

Quick Tip to Solve: For wave direction, always take $\mathbf{E} \times \mathbf{B}$; write signed unit vectors first so a negative-axis field is handled correctly.

39. In the network shown below, the charge accumulated in the capacitor in steady state will be:

[4 marks]



(A) 10.3 μC

(B) 7.2 μC

(C) 4.8 μC

(D) 12 μC

Correct Answer: (B) 7.2 μC

Solution:

Concept:

- At steady state an ideal capacitor in a direct-current circuit carries no current. Its branch is open for current calculations, but its final voltage need not vanish.
- The magnitude of charge stored is $Q = CV_C$. Any resistor carrying zero current has zero voltage drop, by Ohm's law.

Step 1: Identify the conducting loop at steady state The middle branch contains the $4\,\mu\text{F}$ capacitor in series with a $6\,\Omega$ resistor. Once charging is complete, current through this whole branch is zero. The remaining closed loop contains the 3 V source, the top $4\,\Omega$ resistor and the bottom $6\,\Omega$ resistor. These two resistors are in series in the conducting loop, even though the drawing has three horizontal branches.

Step 2: Find the potential difference between the side rails The loop resistance and the current magnitude are

$$R_{\text{loop}} = 4 + 6 = 10\,\Omega, \qquad I = \frac{3}{10} = 0.3\,\text{A}.$$

Therefore the potential difference between the left and right rails equals the voltage drop across the bottom resistor:

$$V_{LR} = I(6) = 0.3(6) = 1.8\,\text{V}.$$

This also follows from the top branch: the source rise is 3 V and the resistor drop is 1.2 V, leaving 1.8

V across the rails.

Step 3: Use the capacitor voltage to calculate charge The middle resistor has zero current, so its voltage drop is zero. The capacitor therefore has the full rail-to-rail potential difference across it:

$$V_C = 1.8\,\text{V}, \qquad Q = CV_C = (4\,\mu\text{F})(1.8\,\text{V}) = 7.2\,\mu\text{C}.$$

Option (D), 12 microcoulombs, incorrectly puts the whole source voltage across the capacitor and ignores the top resistor drop. The values in (A) and (C) likewise do not satisfy the steady-state rail voltage and $Q = CV$.

Step 4: Conclusion The accumulated charge has magnitude $\boxed{7.2\,\mu\text{C}}$. Hence, the correct option is (B).

Quick Tip to Solve: Open the capacitor branch for a steady DC current calculation, then restore it to read the voltage between its terminals; zero current does not imply zero charge.

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40. In a Young's double slits experiment, the ratio of amplitude of light coming from slits is $2:1$. The ratio of the maximum to minimum intensity in the interference pattern is: [4 marks]

(A) 25 : 9

(B) 9 : 1

(C) 9 : 4

(D) 2 : 1

Correct Answer: (B) 9 : 1

Solution:

Concept:

- In two-wave interference, constructive interference gives resultant amplitude $a_1 + a_2$, while destructive interference gives magnitude $|a_1 - a_2|$.
- Intensity is proportional to the square of the resultant amplitude. Thus $I_{\max}/I_{\min} = (a_1+a_2)^2/(a_1-a_2)^2$ for coherent waves of the same frequency.

Step 1: Choose amplitudes with the specified ratio The stated ratio is an amplitude ratio, not an intensity ratio. Let the amplitudes reaching the observation region from the two slits be $a_1 = 2a$ and $a_2 = a$, where $a > 0$. The individual intensities would then be in the ratio $4:1$, because intensity depends on amplitude squared. Keeping this distinction prevents an incorrect substitution into the interference formula.

Step 2: Calculate the extreme resultant intensities At a bright fringe the waves arrive in phase, so the amplitude is $2a + a = 3a$. At a dark fringe the phase difference is π, so the amplitude magnitude is $2a - a = a$. For the same positive proportionality constant K,

$$I_{\max} = K(3a)^2 = 9Ka^2, \qquad I_{\min} = K(a)^2 = Ka^2.$$

Unequal amplitudes therefore leave a nonzero minimum intensity; cancellation is not complete.

Step 3: Form the ratio and compare the choices The common factor Ka^2 cancels:

$$\frac{I_{\max}}{I_{\min}} = \frac{9Ka^2}{Ka^2} = 9.$$

Hence the ratio is $9 : 1$. The choice $25 : 9$ would follow from treating 4 and 1 as the amplitudes, confusing the individual intensity ratio with the given amplitude ratio. The choices $9 : 4$ and $2 : 1$ do not use the squared sum and squared difference required for maximum and minimum interference intensities.

Step 4: Conclusion The maximum-to-minimum intensity ratio is $\boxed{9 : 1}$. Hence, the correct option is (B).

Quick Tip to Solve: For a given amplitude ratio $p : q$, write the interference contrast immediately as $(p+q)^2 : (p-q)^2$, keeping amplitudes distinct from intensities.

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41. The mean free path of molecules of a certain gas at STP is 1500 d, where d is the diameter of the gas molecules. While maintaining the standard pressure, the mean free path of the molecules at 373 K is approximately: [4 marks]

(A) 750 d

(B) 1500 d

(C) 2049 d

(D) 1098 d

Correct Answer: (C) 2049 d

Solution:

Concept:

- For molecules modelled as hard spheres of diameter d, the mean free path is $\lambda = 1/(\sqrt{2}\pi d^2 n)$, where n is molecular number density.
- The ideal-gas relation gives $n = P/(k_B T)$, so $\lambda = k_B T/(\sqrt{2}\pi d^2 P)$. At fixed pressure and unchanged molecular diameter, mean free path is proportional to absolute temperature.

Step 1: Identify the temperature and the quantity held fixed At STP the initial absolute temperature is approximately $T_1 = 273\,\text{K}$, and the initial mean free path is $\lambda_1 = 1500d$. The gas is taken to $T_2 = 373\,\text{K}$ while pressure remains standard. The collision diameter belongs to the same molecules and is unchanged. Temperature must be used in kelvin in the proportionality; a Celsius ratio would have no physical meaning.

Step 2: Apply the constant-pressure scaling At constant pressure the number density decreases as temperature rises, because $n = P/(k_B T)$. Molecules consequently travel farther on average between collisions. Dividing the expressions for the two mean free paths gives

$$\frac{\lambda_2}{\lambda_1} = \frac{T_2}{T_1}, \qquad \lambda_2 = 1500d\left(\frac{373}{273}\right).$$

Numerically, $373/273 \simeq 1.3663$, so

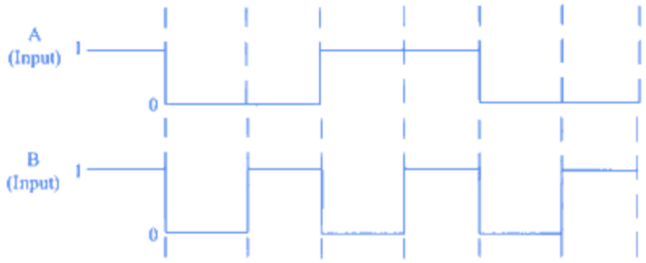
$$\lambda_2 \simeq 2049.45d \simeq 2049d.$$

Step 3: Check the trend and the alternatives The temperature increase must increase the mean free path at constant pressure. This immediately rejects 750d and 1098d, both smaller than the original 1500d. The unchanged value 1500d would be appropriate if the number density were unchanged, such as fixed volume for a fixed quantity of gas, but that is not the stated condition. The available value 2049d has both the correct trend and the calculated magnitude.

Step 4: Conclusion The new mean free path is approximately $\boxed{2049d}$. Hence, the correct option is (C).

Quick Tip to Solve: Before scaling mean free path, identify the constraint: $\lambda \propto T/P$, so at constant pressure use kelvin temperatures in a direct ratio.

42. The output from NAND gate having inputs A and B given below will be, [4 marks]



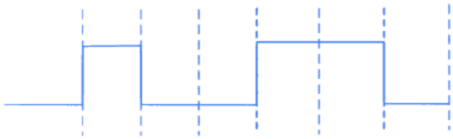
(A)



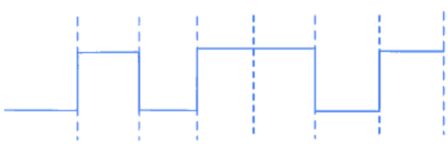
(B)



(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- A NAND gate produces the complement of the AND result: $Y = \overline{AB}$. Its output is low only when both inputs are high.
- To compare pulse diagrams, divide time at every input transition and determine the logical state separately in each open interval. Dashed vertical lines mark these transitions, not additional pulses.

Step 1: Read the two input states interval by interval Label the dashed transition times from left to right as $t_1, t_2, \ldots, t_7$. Before t_1, both A and B are high. Between successive lines, the input pairs (A, B) are

$$(0,0),\ (0,1),\ (1,0),\ (1,1),\ (0,0),\ (0,1).$$

In particular, A stays low from t_1 to t_3, then high from t_3 to t_5, while B alternates at every successive marked line. The initial interval must also be checked because it distinguishes the starting output level.

Step 2: Apply the NAND truth table The four input combinations give $00 \mapsto 1$, $01 \mapsto 1$, $10 \mapsto 1$, and $11 \mapsto 0$. Therefore, before t_1 the output is 0, and for the six successive intervals it is

$$Y: \quad 1,\ 1,\ 1,\ 0,\ 1,\ 1.$$

Equivalently the output rises at t_1, remains high through t_4, falls for $t_4 < t < t_5$, and rises again at t_5. Input changes that do not change AB do not create an output transition.

Step 3: Match the pulse train to the printed diagrams Option (B) begins low, has a long high stretch after the first transition, has its single interior low pulse where A and B are simultaneously high, and then returns high. Option (A) starts high and goes low while A is low, contradicting NAND behaviour. Options (C) and (D) also show low intervals when at least one input is low, so they fail the truth table. No propagation delay is specified, and the ideal logical waveform is intended.

Step 4: Conclusion The NAND output is low only during the intervals with $A = B = 1$; this is the waveform in (B). Hence, the correct option is (B).

Quick Tip to Solve: For NAND pulse questions, mark just the overlap of the two high inputs; shade those intervals low and keep the output high everywhere else.

43. A $10\ \mu C$ charge is divided into two parts and placed at $1\ cm$ distance so that the repulsive force between them is maximum. The charges of the two parts are: [4 marks]

(A) $9\ \mu C, 1\ \mu C$

(B) $5\ \mu C, 5\ \mu C$

(C) $8\ \mu C, 2\ \mu C$

(D) $7\ \mu C, 3\ \mu C$

Correct Answer: (B) $5\ \mu C, 5\ \mu C$

Solution:

Concept:

- Coulomb's law gives the repulsion between two positive point charges as $F = kq_1q_2/r^2$, where $k = 1/(4\pi\varepsilon_0)$ and r is their separation.
- The total charge is conserved: $q_1 + q_2 = Q = 10\,\mu C$. At fixed separation, maximizing force means maximizing the product of the charges.

Step 1: Express the force using one charge Let the first part be q and the other be $Q - q$, with $0 \le q \le Q$. Since the charges are placed at the fixed separation $r = 1\,cm = 10^{-2}\,m$, the factor k/r^2 is positive and constant. Thus

$$F(q) = \frac{k}{r^2} q(Q - q).$$

The separation affects the value of the force, but cannot affect which division maximizes it.

Step 2: Maximize the charge product Complete the square rather than guessing from the choices:

$$q(Q - q) = \frac{Q^2}{4} - \left(q - \frac{Q}{2}\right)^2.$$

The subtracted square is nonnegative, so the maximum occurs uniquely at $q = Q/2$. Both parts are therefore $5\,\mu C$. Equivalently, the first derivative is $Q - 2q$, and the second derivative is -2, confirming a maximum.

Step 3: Compare all the divisions In units of $(\mu C)^2$, the products in options (A), (B), (C) and (D) are respectively 9, 25, 16 and 21. Only the equal division gives the largest product; each unequal division gives smaller repulsion at the same distance.

Step 4: Conclusion The two charges are $5\,\mu C$ and $5\,\mu C$. Hence, the correct option is (B).

Quick Tip to Solve: For a fixed positive sum, the product is greatest when the two parts are equal; use this before substituting the separation.

44. Given below are two statements:

Statement I : An AC circuit undergoes electrical resonance if it contains either a capacitor or an inductor.

Statement II : An AC circuit containing a pure capacitor or a pure inductor consumes high power due to its non-zero power factor.

In the light of above statements, choose the correct answer form the options given below: [4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is true but statement II is false

(C) Statement I is false but statement II is true

(D) Both Statement I and Statement II are true

Correct Answer: (A) Both Statement I and Statement II are false

Solution:

Concept:

- Electrical resonance in a series LCR circuit occurs when inductive and capacitive reactances cancel: $X_L = \omega L = X_C = 1/(\omega C)$.
- The average AC power is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$, where ϕ is the phase difference between voltage and current.

Step 1: Test the resonance statement A circuit containing only an ideal capacitor or only an ideal inductor cannot cancel its own reactance. Resonance requires both kinds of reactive energy storage, with energy exchanged between the electric field of the capacitor and the magnetic field of the inductor. For a series LCR circuit, the condition is

$$\omega L - \frac{1}{\omega C} = 0, \qquad \omega_0 = \frac{1}{\sqrt{LC}}.$$

Merely including either component alone does not establish resonance. Statement I is false.

Step 2: Evaluate the power factor In a pure inductor current lags voltage by $\pi/2$; in a pure capacitor it leads voltage by $\pi/2$. In either case,

$$\cos\phi = \cos(\pi/2) = 0, \qquad P_{\text{av}} = 0.$$

Energy is temporarily stored and returned to the source each cycle, rather than consumed as net average power. Instantaneous power need not vanish, but its positive and negative contributions cancel. Statement II is therefore false.

Step 3: Match the truth values to the choices Both statements have been independently found false. Option (B) incorrectly accepts Statement I, option (C) incorrectly accepts Statement II, and option (D) accepts both false claims. The option describing two false statements is (A).

Step 4: Conclusion Statement I and Statement II are both false. Hence, the correct option is (A).

Quick Tip to Solve: For an ideal pure inductor or capacitor, remember the quarter-cycle phase difference: its cosine and the average consumed power are zero.

45. Given below are two statements:

Statement I : Out of microwaves, infrared rays and ultraviolet rays, ultraviolet rays are the most effective for the emission of electrons from a metallic surface.

Statement II : Above the threshold frequency, the maximum kinetic energy of photoelectrons is inversely proportional to the frequency of the incident light.

In the light of above statements, choose the correct answer form the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true but statement II is false

(C) Statement I is false but statement II is true

(D) Both Statement I and Statement II are false

Correct Answer: (B) Statement I is true but statement II is false

Solution:

Concept:

- A photon has energy $E = h\nu$. Photoelectric emission requires $h\nu \geq \phi$, where $\phi = h\nu_0$ is the work function and ν_0 the threshold frequency.
- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = h(\nu - \nu_0)$ for frequencies above threshold.

Step 1: Compare the incident radiations The frequency ordering is microwaves, infrared, then ultraviolet in increasing order. Since photon energy is proportional to frequency, ultraviolet photons carry the greatest energy among these radiations. They are therefore the most effective of the three for overcoming the work function and releasing electrons from a metallic surface. Increasing the intensity of subthreshold microwave or infrared light cannot compensate for inadequate energy of each individual photon. Statement I is true.

Step 2: Check how maximum kinetic energy varies For a fixed metal, the work function is constant. Applying energy conservation to a single photon gives

$$h\nu = \phi + K_{\max}, \qquad K_{\max} = h\nu - \phi.$$

Thus the graph of maximum kinetic energy against frequency is a straight line of positive slope h, beginning at ν_0. It increases linearly with frequency above threshold; it is not inversely proportional to frequency. Statement II is false.

Step 3: Eliminate the other combinations Option (A) wrongly treats the inverse dependence as true. Options (C) and (D) both reject the correct frequency comparison in Statement I. The combination of a true Statement I and false Statement II is precisely option (B).

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: Write $K_{\max} = h(\nu - \nu_0)$ first: its positive slope immediately rules out an inverse dependence on frequency.

46. An electron is moving along the positive x-axis. If the uniform magnetic field is applied parallel to the negative z-axis, then

A. The electron will experience magnetic force along positive y-axis

B. The electron will experience magnetic force along negative y-axis

C. The electron will not experience any force in magnetic field

D. The electron will continue to move along the positive x-axis

E. The electron will move along circular path in magnetic field

Choose the correct answer from the options given below: [4 marks]

(A) A and E only

(B) B and D only

(C) B and E only

(D) C and D only

Correct Answer: (C) B and E only

Solution:

Concept:

- The magnetic Lorentz force is $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. The electron has negative charge $q = -e$, so its force is opposite to the cross-product direction.
- A uniform magnetic field exerts no work because its force is perpendicular to velocity. Motion initially perpendicular to the field is circular at constant speed.

Step 1: Find the cross-product direction The initial velocity is $\mathbf{v} = v\hat{\mathbf{i}}$, while the field is $\mathbf{B} = -B\hat{\mathbf{k}}$, with $v, B > 0$. Using $\hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$, we obtain

$$\mathbf{v} \times \mathbf{B} = vB\hat{\mathbf{j}}.$$

This is the direction for a positive charge, not yet the force on the electron.

Step 2: Apply the electron charge and determine the path Multiplying by $-e$ reverses the direction:

$$\mathbf{F} = -evB\hat{\mathbf{j}}.$$

The initial magnetic force is therefore along the negative y-axis, so statement B is true and A is false. Its magnitude is nonzero because the velocity and field are perpendicular, making C false. As the force continually turns the velocity without changing its magnitude, the electron follows a circle in the plane perpendicular to the field, namely the xy-plane. The force supplies the centripetal acceleration, with $r = mv/(eB)$ in the nonrelativistic form. Thus E is true and D is false.

Step 3: Select the correct pair The true statements are B and E only. Option (A) has the wrong initial force direction; option (B) incorrectly retains straight-line motion; option (D) incorrectly asserts both zero force and straight-line motion.

Step 4: Conclusion The electron initially experiences force along negative y and follows a circular path: B and E only. Hence, the correct option is (C).

Quick Tip to Solve: Find $\mathbf{v} \times \mathbf{B}$ for a positive charge first, then reverse it for an electron; perpendicular motion in a uniform field is circular.

47. A particle executes SHM of amplitude A. The distance from the mean position when its's kinetic energy becomes equal to its potential energy is : [4 marks]

(A) $\frac{1}{\sqrt{2}}A$

(B) $\frac{1}{2}A$

(C) 2 A

(D) $\sqrt{2A}$

Correct Answer: (A) $\frac{1}{\sqrt{2}}A$

Solution:

Concept:

- For SHM of amplitude A, the total mechanical energy is $E = \frac{1}{2}kA^2$, where $k = m\omega^2$ is the effective restoring-force constant.
- At displacement x from equilibrium, $U = \frac{1}{2}kx^2$ and $K = \frac{1}{2}k(A^2 - x^2)$, taking potential energy zero at the mean position.

Step 1: Translate equal energies into an equation At the mean position the potential energy is zero and kinetic energy is maximum; at either extreme all the mechanical energy is potential. Equal energies must therefore occur at an intermediate displacement. Set the standard SHM expressions equal:

$$\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2.$$

The same positive factor $k/2$ occurs on both sides and cancels. There is no need to know the mass, frequency or spring constant.

Step 2: Solve for the distance from the mean Rearranging the remaining equation gives

$$A^2 - x^2 = x^2, \qquad x^2 = \frac{A^2}{2}.$$

The possible signed displacements are $x = \pm A/\sqrt{2}$. The question asks for a distance, so use the positive magnitude $|x| = A/\sqrt{2}$. Each energy is then one-half of the conserved total energy.

Step 3: Check the other options At $|x| = A/2$, potential energy is only $E/4$, while kinetic energy is $3E/4$, so option (B) fails. A displacement $2A$ exceeds the amplitude and is inaccessible, ruling out (C). The expression $\sqrt{2A}$ does not have the dimensions of length when A is a length, ruling out (D).

Step 4: Conclusion The distance is $A/\sqrt{2}$. Hence, the correct option is (A).

Quick Tip to Solve: In SHM, use $U/E = x^2/A^2$. Equal kinetic and potential energies mean $U/E = 1/2$, not $|x|/A = 1/2$.

48. A passenger sitting in a train A moving at $90\ \mathrm{km/h}$ observes another train B moving in the opposite direction for $8\ \mathrm{s}$. If the velocity of the train B is $54\ \mathrm{km/h}$, then length of train B is: [4 marks]

(A) 80 m

(B) 200 m

(C) 120 m

(D) 320 m

Correct Answer: (D) 320 m

Solution:

Concept:

- The velocity of train B relative to a passenger in A is $v_{B/A} = v_B - v_A$, using signed velocities along a chosen axis.
- The passenger is a point observer. During the complete passing interval, the distance swept past that observer equals the length of train B: $L_B = v_{\text{rel}}t$.

Step 1: Convert and assign the velocities Choose the direction of train A as positive. Then train A has velocity $+90\,\mathrm{km\,h^{-1}}$ and train B has velocity $-54\,\mathrm{km\,h^{-1}}$, because the trains move in opposite directions. Converting to SI units,

$$v_A = 90\frac{5}{18} = 25\,\mathrm{m\,s^{-1}}, \qquad v_B = -54\frac{5}{18} = -15\,\mathrm{m\,s^{-1}}.$$

The magnitudes must be added when computing the relative speed of oppositely moving trains.

Step 2: Calculate the relative speed and passing distance In the frame of the passenger, the velocity of B is

$$v_{B/A} = -15 - 25 = -40\,\mathrm{m\,s^{-1}}.$$

Its speed past the passenger is therefore $40\,\mathrm{m\,s^{-1}}$. From the instant the front of B reaches the passenger until its rear passes, B travels its own length relative to that observer. Hence

$$L_B = |v_{B/A}|t = 40 \times 8 = 320\,\mathrm{m}.$$

The length of train A is irrelevant because the observer is a passenger at one location, not the entire train.

Step 3: Exclude the other distances Using the difference of the speed magnitudes would give $80\,\mathrm{m}$, option (A), which is appropriate to neither direction here. Multiplying the individual speeds by time gives $200\,\mathrm{m}$ and $120\,\mathrm{m}$, options (B) and (C); neither is the relative distance needed.

Step 4: Conclusion Train B is $320\,\mathrm{m}$ long. Hence, the correct option is (D).

Quick Tip to Solve: When a whole train passes a passenger, use that train's length and the relative speed; opposite directions require adding speed magnitudes.

49. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : The binding energy per nucleon is practically independent of the atomic number for nuclei of mass number in the range 30 to 170 .

Reason R : Nuclear force is short ranged.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) A is false but **R** is true

(B) A is true but **R** is false

(C) Both **A** and **R** are true and **R** is the correct explanation of **A**

(D) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Correct Answer: (C) Both **A** and **R** are true and **R** is the correct explanation of **A**

Solution:

Concept:

- The binding energy per nucleon is B/A, the energy required on average to separate a nucleon from a nucleus; here A denotes mass number.
- Nuclear forces have a short range of a few femtometres and show saturation: a nucleon interacts strongly with only its nearby neighbours.

Step 1: Evaluate the assertion For intermediate-mass nuclei, approximately in the range $30 \lesssim A \lesssim 170$, the binding-energy-per-nucleon curve is nearly flat, with values of order $8\,\mathrm{MeV}$. It is not exactly constant: there is a broad maximum near the iron region and a gradual decrease for heavier nuclei. The word practically allows these modest variations. Consequently, the assertion that the binding energy per nucleon is practically independent of atomic number in this mass range is true.

Step 2: Evaluate the reason The attractive nuclear interaction becomes ineffective beyond a few femtometres. A nucleon therefore does not bind equally strongly to every other nucleon throughout a large nucleus. Instead, only a limited number of nearby nucleons contribute substantially to its strong-interaction binding. This short-range property of the nuclear force is well established, so Reason R is true.

Step 3: Connect short range to nearly constant binding per nucleon Once a nucleus is large enough, most nucleons have a similar local environment. Adding nucleons adds roughly the same amount of binding for each additional nucleon, making the dominant total binding approximately proportional to mass number: $B \propto A$. Thus B/A remains approximately constant. Surface and Coulomb effects account for the departures from perfect constancy. Short range, through saturation, therefore explains the observed assertion rather than being an unrelated true statement. Options (A) and (B) wrongly reject a true statement, and (D) wrongly denies this explanatory connection.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (C).

Quick Tip to Solve: Connect short range to saturation: each interior nucleon has only a limited set of strongly interacting neighbours, so total binding grows roughly as A.

50. To radiate EM signal of wavelength λ with high efficiency, the antennas should have a minimum size equal to: [4 marks]

(A) $\frac{\lambda}{2}$

(B) λ

(C) $\frac{\lambda}{4}$

(D) 2λ

Correct Answer: (C) $\frac{\lambda}{4}$

Solution:

Concept:

- An antenna converts an electrical oscillation into an outgoing electromagnetic wave. Efficient radiation requires its length to be comparable with the wavelength rather than very small compared with it.
- The minimum practical resonant antenna length in the standard communication-system model is a quarter wavelength: $l_{\min} = \lambda/4$. A half-wave dipole has two quarter-wave arms.

Step 1: Identify the relevant wavelength scale The signal has wavelength λ, so the antenna dimension must be a significant fraction of λ. If an antenna is much shorter than this scale, its radiation resistance becomes very small and it radiates inefficiently. This is why audio-frequency signals are modulated onto a higher-frequency carrier: increasing frequency decreases wavelength and permits a practical antenna size.

Step 2: Select the minimum efficient length The smallest conventional resonant radiator is a quarter-wave antenna, for which the standing-wave pattern has a current antinode at the feed and a current node at the free end. Thus

$$l_{\min} = \frac{\lambda}{4}.$$

The length $\lambda/2$ is associated with a complete half-wave dipole and is larger than the required minimum. The lengths λ and 2λ are still larger; they cannot be selected when the question asks for the minimum size.

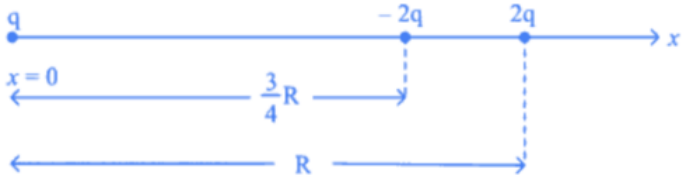
Step 3: Conclusion The minimum antenna size is $\boxed{\lambda/4}$. Hence, the correct option is (C).

Quick Tip to Solve: For the minimum antenna length use a quarter wavelength; distinguish it from the total length of a half-wave dipole.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Three point charges $\mathrm{q}, -2\mathrm{q}$ and $2\mathrm{q}$ are placed on x-axis at a distance $x = 0, x = \frac{3}{4}R$ and $x = R$ respectively from origin as shown. If $\mathrm{q} = 2 \times 10^{-6}\mathrm{C}$ and $\mathrm{R} = 2\,\mathrm{cm}$, the magnitude of net force experienced by the charge $-2q$ is _____ N. [4 marks]



Solution:

Concept:

- Coulomb's law gives the magnitude of the force between two point charges: $F = k|q_1 q_2|/r^2$, where $k = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$.
- Unlike charges attract. The two positive charges lie on opposite sides of $-2q$, so their forces on it are opposite and must be subtracted.

Step 1: Find the two separations The negative charge is at $3R/4$. Its distance from the charge at the origin is $r_L = 3R/4$, while its distance from the charge at R is $r_R = R/4$. With $R = 2\,\mathrm{cm} = 0.020\,\mathrm{m}$,

$$r_L = 0.015\,\mathrm{m}, \qquad r_R = 0.005\,\mathrm{m}.$$

The left positive charge attracts it towards the origin; the right positive charge attracts it in the positive x direction.

Step 2: Calculate and combine the forces Using $q = 2 \times 10^{-6}\,\mathrm{C}$, the leftward force is

$$F_L = \frac{k(2q^2)}{r_L^2} = \frac{9 \times 10^9 \times 8 \times 10^{-12}}{(0.015)^2} = 320\,\mathrm{N}.$$

The rightward force is

$$F_R = \frac{k(4q^2)}{r_R^2} = \frac{9 \times 10^9 \times 16 \times 10^{-12}}{(0.005)^2} = 5760\,\mathrm{N}.$$

Therefore

$$F_{\text{net}} = 5760 - 320 = 5440\,\mathrm{N}.$$

The nearer right-hand charge produces the larger force, consistent with its greater charge and smaller separation.

Step 3: Conclusion The required magnitude is $\boxed{5440\,\mathrm{N}}$, directed along the positive x-axis.

Quick Tip to Solve: Compute the separations from the target charge first; forces from charges on opposite sides need a direction check before addition.

52. An atom absorbs a photon of wavelength $500\ \text{nm}$ and emits another photon of wavelength $600\ \text{nm}$. The net energy absorbed by the atom in this process is $n \times 10^{-4}\ \text{eV}$. The value of n is _____. [Assume the atom to be stationary during the absorption and emission process] (Take $\text{h} = 6.6 \times 10^{-34}\ \text{Js}$ and $\text{c} = 3 \times 10^{8}\ \text{m/s}$) [4 marks]

Solution:

Concept:

- A photon of wavelength λ carries energy $E = hc/\lambda$, with the wavelength expressed in metres when SI constants are used.
- For a stationary atom the net internal energy gained is the absorbed photon energy minus the emitted photon energy. Convert joules to electronvolts using $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.

Step 1: Subtract the photon energies The incoming wavelength is shorter, so its photon energy is greater and the atom gains energy overall. With $\lambda_a = 500 \times 10^{-9}\,\text{m}$ and $\lambda_e = 600 \times 10^{-9}\,\text{m}$,

$$\Delta E = hc\left(\frac{1}{\lambda_a} - \frac{1}{\lambda_e}\right).$$

The reciprocal-wavelength difference is

$$\frac{1}{500 \times 10^{-9}} - \frac{1}{600 \times 10^{-9}} = \frac{10^6}{3}\,\text{m}^{-1}.$$

We subtract energies, not the wavelengths themselves.

Step 2: Convert to the requested coefficient Substitute the supplied constants:

$$\Delta E = (6.6 \times 10^{-34})(3 \times 10^{8})\frac{10^6}{3} = 6.6 \times 10^{-20}\,\text{J}.$$

In electronvolts this is

$$\Delta E = \frac{6.6 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.4125\,\text{eV}.$$

The question writes the same energy as $n \times 10^{-4}\,\text{eV}$, hence

$$n = \frac{0.4125}{10^{-4}} = 4125.$$

The assumption of a stationary atom allows us to omit any recoil kinetic-energy contribution.

Step 3: Conclusion The net absorbed energy is $0.4125\,\text{eV} = 4125 \times 10^{-4}\,\text{eV}$, so $\boxed{n = 4125}$ (dimensionless).

Quick Tip to Solve: Find the difference of reciprocal wavelengths first, then convert the energy and extract the coefficient of the stated power of ten.

53. A light rope is wound around a hollow cylinder of mass 5 kg and radius 70 cm. The rope is pulled with a force of 52.5 N. The angular acceleration of the cylinder will be _____ rad s^{-2}. [4 marks]

Solution:

Concept:

- For a tangential force F applied at radius R, the torque about the cylinder axis is $\tau = FR$.
- A thin hollow cylinder has axial moment of inertia $I = MR^2$. Rotational Newton's law is $\tau = I\alpha$, where α is angular acceleration.

Step 1: Calculate the torque about the axis The rope is light, so its pull supplies a tangential force of $52.5\,\mathrm{N}$ at the rim. Convert the radius before substituting:

$$R = 70\,\mathrm{cm} = 0.70\,\mathrm{m}.$$

The torque magnitude is therefore

$$\tau = FR = 52.5 \times 0.70 = 36.75\,\mathrm{N\,m}.$$

The radius is perpendicular to the tangential rope force, so the lever arm is the full radius and no additional sine factor is needed.

Step 2: Use the hollow-cylinder moment of inertia For the hollow cylinder of mass $5\,\mathrm{kg}$,

$$I = MR^2 = 5(0.70)^2 = 2.45\,\mathrm{kg\,m^2}.$$

Applying the rotational equation gives

$$\alpha = \frac{\tau}{I} = \frac{36.75}{2.45} = 15\,\mathrm{rad\,s^{-2}}.$$

Equivalently, cancellation of one radius in $FR/(MR^2)$ gives $\alpha = F/(MR) = 52.5/(5 \times 0.70)$. A solid-cylinder expression $I = MR^2/2$ would give a different result and is inappropriate for the stated hollow cylinder. The acceleration is in the rotational sense of the applied pull.

Step 3: Conclusion The angular acceleration is $\boxed{15\,\mathrm{rad\,s^{-2}}}$.

Quick Tip to Solve: Choose the moment of inertia from the mass distribution first: a hollow cylinder uses MR^2, whereas a solid cylinder uses $MR^2/2$.

54. A straight wire AB of mass 40 g and length 50 cm is suspended by a pair of flexible leads in uniform magnetic field of magnitude 0.40 T as shown in the figure. The magnitude of the current required in the wire to remove the tension in the supporting leads is _____ A.

(**Take** $g = 10\ \text{ms}^{-2}$ **).** [4 marks]



Solution:

Concept:

- A straight wire of length L carrying current I in a uniform magnetic field has force $F_B = BIL\sin\theta$. Here the wire is perpendicular to the field, so $\sin\theta = 1$.
- At the threshold of zero supporting tension, the upward magnetic force alone balances the downward weight: $BIL = mg$.

Step 1: Set the force-balance condition The figure shows a horizontal wire and a magnetic field into the page. To eliminate tension in both flexible leads, the magnetic force must point upwards and equal the entire weight. Thus

$$T_1 + T_2 + BIL = mg.$$

Setting $T_1 = T_2 = 0$ yields

$$I = \frac{mg}{BL}.$$

A current from A to B produces the required upward force by $\boldsymbol{F} = I\boldsymbol{L} \times \boldsymbol{B}$. The opposite direction would increase the supporting tension rather than remove it.

Step 2: Substitute SI values Convert the mass and length:

$$m = 40\,\text{g} = 0.040\,\text{kg}, \qquad L = 50\,\text{cm} = 0.50\,\text{m}.$$

The weight is $mg = 0.040 \times 10 = 0.40\,\text{N}$, while $BL = 0.40 \times 0.50 = 0.20\,\text{T m}$. Therefore

$$I = \frac{0.40}{0.20} = 2.0\,\text{A}.$$

This is the current in the wire itself; it is not divided by two merely because there are two leads. The

two tensions both vanish under the single balance of weight and magnetic force.

Step 3: Conclusion The required current magnitude is $\boxed{2\,\text{A}}$, flowing from A to B for the field shown.

Quick Tip to Solve: At zero tension use $BIL = mg$ for the whole wire, and use the cross-product direction to ensure the magnetic force lifts it.

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55. Two plates A and B have thermal conductivities $84\ \text{Wm}^{-1}\ \text{K}^{-1}$ and $126\ \text{Wm}^{-1}\ \text{K}^{-1}$ respectively. They have same surface area and same thickness. They are placed in contact along their surfaces. If the temperatures of the outer surfaces of A and B are kept at $100°\text{C}$ and $0°\text{C}$ respectively, then the temperature of the surface of contact in steady state is _____ °C. [4 marks]

Solution:

Concept:

- In steady one-dimensional conduction the same heat current passes through plates arranged in series; no heat accumulates at their common interface.
- Fourier's law gives $\dot{Q} = kA\Delta T/d$, where k is thermal conductivity, A is area and d is thickness.

Step 1: Equate the heat currents through both plates Let the interface temperature be T in degrees Celsius. Plate A lies between $100°\text{C}$ and T, and plate B lies between T and $0°\text{C}$. Because both plates have the same area and thickness,

$$\frac{84A(100-T)}{d} = \frac{126A(T-0)}{d}.$$

Cancel the common positive factor A/d. Temperature differences have the same numerical values in Celsius and kelvin, so no temperature-origin conversion is required for this equation.

Step 2: Solve for the interface temperature The balance becomes

$$84(100-T) = 126T.$$

Expanding and collecting the temperature terms,

$$8400 = 210T, \qquad T = 40°\text{C}.$$

As a physical check, plate A has the smaller conductivity and therefore the larger thermal resistance. Its temperature drop must be larger: $100-40 = 60°\text{C}$, compared with $40-0 = 40°\text{C}$ across B. Indeed, $84\times 60 = 126\times 40 = 5040$, confirming equal heat currents. A simple arithmetic mean would be justified only for equal thermal resistances, which is not the case here.

Step 3: Conclusion The steady-state contact temperature is $\boxed{40°\text{C}}$.

Quick Tip to Solve: For equal area and thickness, equate $k_1\Delta T_1 = k_2\Delta T_2$; the less conductive plate has the larger temperature drop.

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56. In an experiment with sonometer when a mass of $180\ \mathrm{g}$ is attached to the string, it vibrates with fundamental frequency of $30\ \mathrm{Hz}$. When a mass m is attached, the string vibrates with fundamental frequency of $50\ \mathrm{Hz}$. The value of m is _____ g. [4 marks]

Solution:

Concept:

- The fundamental frequency of a stretched string is $f = (1/2L)\sqrt{T/\mu}$, where L is vibrating length and μ is mass per unit length.
- For the same sonometer string and vibrating length, only the tension changes. With a hanging mass m, $T = mg$, so f^2 is proportional to m.

Step 1: Form a frequency ratio The initial load is $m_1 = 180\,\mathrm{g}$ and its fundamental frequency is $f_1 = 30\,\mathrm{Hz}$. With the new load m, the frequency is $f_2 = 50\,\mathrm{Hz}$. Since the length and linear density are unchanged, their common factors cancel:

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{mg}{m_1 g}} = \sqrt{\frac{m}{m_1}}.$$

The acceleration due to gravity cancels too; no numerical value of g is needed.

Step 2: Square the ratio and determine the load Squaring gives

$$\frac{m}{180} = \left(\frac{50}{30}\right)^2 = \frac{25}{9}.$$

Thus

$$m = 180 \times \frac{25}{9} = 20 \times 25 = 500\,\mathrm{g}.$$

It is valid to use grams for both masses because only their dimensionless ratio occurs. As a check, increasing the mass from 180 to $500\,\mathrm{g}$ increases the tension by $25/9$, whose square root is $5/3$; multiplying $30\,\mathrm{Hz}$ by $5/3$ gives exactly $50\,\mathrm{Hz}$. Using a direct proportionality between frequency and load would miss the square-root dependence.

Step 3: Conclusion The required hanging mass is $\boxed{500\,\mathrm{g}}$.

Quick Tip to Solve: For an unchanged sonometer string, square the frequency ratio to obtain the load ratio; frequency itself varies as the square root of load.

57. An insulated copper wire of 100 turns is wrapped around a wooden cylindrical core of the cross-sectional area $24\ \text{cm}^2$. The two ends of the wire are connected to a resistor. The total resistance in the circuit is $12\ \Omega$. If an externally applied uniform magnetic field in the core along its axis changes from $1.5\ \text{T}$ in one direction to $1.5\ \text{T}$ in the opposite direction, the charge flowing through a point in the circuit during the change of magnetic field will be _____ mC. [4 marks]

Solution:

Concept:

- Faraday's law relates the induced emf to the rate of change of magnetic flux linkage: $\mathcal{E} = -N\, d\Phi/dt$, with $\Phi = BA$ for a field normal to each turn.
- For a purely resistive circuit, integrating $I = \mathcal{E}/R$ over time gives the magnitude of transferred charge $Q = NA|\Delta B|/R$.

Step 1: Assign signs to the initial and final fields Take the initial field direction as positive. The initial field is $B_i = +1.5\,\text{T}$, while the final field is $B_f = -1.5\,\text{T}$. Since the field reverses direction, its signed change is $\Delta B = -1.5 - 1.5 = -3.0\,\text{T}$. Its magnitude is therefore $3.0\,\text{T}$, rather than zero or $1.5\,\text{T}$.

Step 2: Convert the coil area and integrate the current The area linked by each turn is $A = 24\times10^{-4} = 2.4 \times 10^{-3}\,\text{m}^2$. All $N = 100$ turns link this same flux. With constant total resistance $R = 12\,\Omega$,

$$Q = \left|\int I\, dt\right| = \frac{N}{R}\left|\int A\, dB\right| = \frac{NA|B_f - B_i|}{R}.$$

The time taken for the change is unnecessary because time integration cancels the time derivative in Faraday's law.

Step 3: Evaluate the charge in the requested unit Substitute the total resistance, not a resistance per turn:

$$Q = \frac{100(2.4 \times 10^{-3})(3.0)}{12} = 6.0 \times 10^{-2}\,\text{C}.$$

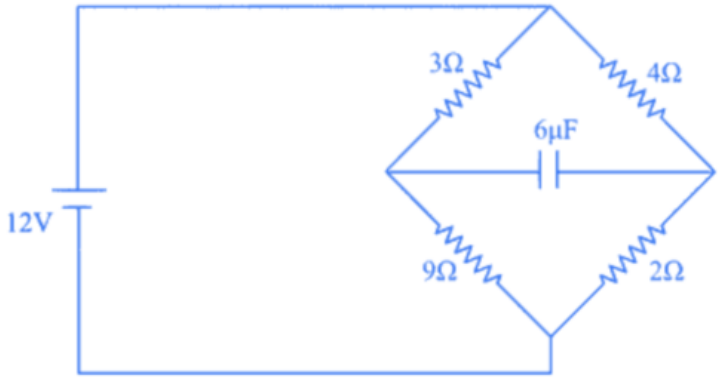
Since $1\,\text{C} = 10^3\,\text{mC}$, this is $60\,\text{mC}$. The sign of the induced current is set by Lenz's law; the question requests the amount of charge, so its magnitude is reported.

Step 4: Conclusion The charge transferred is $60\,\text{mC}$. The required numerical value is $\boxed{60}$.

Quick Tip to Solve: On field reversal, subtract signed fields first; the magnitudes add. Integrated induced charge depends on the flux change, not the duration.

58. In the circuit shown, the energy stored in the capacitor is $n\ \mu$J. The value of n is _____

[4 marks]



Solution:

Concept:

- A capacitor carries no steady current in a DC circuit after charging. Its two terminals can therefore be treated as open when finding their node potentials.
- The energy stored across a potential difference V_C is $U = \frac{1}{2}CV_C^2$. The node potentials are determined by resistive voltage division.

Step 1: Find the left midpoint potential Take the lower battery terminal as zero and the upper terminal as $12\,\text{V}$. In steady state the capacitor does not connect the two resistor branches by a conducting path. The left branch contains $3\,\Omega$ and $9\,\Omega$ in series, so its current is

$$I_L = \frac{12}{3+9} = 1\,\text{A}.$$

The potential at the junction of these resistors is the voltage across the lower $9\,\Omega$ resistor: $V_L = I_L(9) = 9\,\text{V}$.

Step 2: Find the right midpoint and capacitor voltage The right branch has $4\,\Omega$ above its junction and $2\,\Omega$ below it. Its total resistance is $6\,\Omega$, and its steady current is

$$I_R = \frac{12}{4+2} = 2\,\text{A}.$$

Thus its midpoint potential is $V_R = I_R(2) = 4\,\text{V}$. The capacitor lies between the two midpoints, not across the battery, so

$$|V_C| = |V_L - V_R| = |9-4| = 5\,\text{V}.$$

Its left plate is at the higher potential.

Step 3: Calculate the electrostatic energy The capacitance shown is $C = 6\,\mu\text{F} = 6\times10^{-6}\,\text{F}$. Hence

$$U = \frac{1}{2}(6\times10^{-6})(5)^2 = 75\times10^{-6}\,\text{J} = 75\,\mu\text{J}.$$

Using $12\,\text{V}$ here would incorrectly place the capacitor directly across the source. The resistor currents determine its final voltage; they do not enter the stored-energy expression separately.

Step 4: Conclusion The stored energy is $\boxed{75\,\mu\text{J}}$, so the dimensionless coefficient is $\boxed{n = 75}$.

Quick Tip to Solve: For a capacitor bridging two resistor junctions, first find each junction voltage relative to the same node, then square their difference in the energy formula.

59. A bi convex lens of focal length 10 cm is cut in two identical parts along a plane perpendicular to the principal axis. The power of each lens after cut is _____ D. [4 marks]

Solution:

Concept:

- The power of a thin lens in air is $P = 1/f$, with focal length f in metres. Positive focal length corresponds to a converging lens.
- The lens maker relation is $P = (\mu - 1)(1/R_1 - 1/R_2)$. A plane surface has infinite radius and contributes zero curvature.

Step 1: Calculate the original lens power The original biconvex lens has focal length $f = 10\,\text{cm} = 0.10\,\text{m}$. Its power is therefore

$$P = \frac{1}{0.10} = +10\,\text{D}.$$

The cut is made perpendicular to the principal axis, so each piece has one original curved face and a new plane face. It does not merely reduce the aperture while preserving both original refracting surfaces.

Step 2: Compare the curvatures before and after cutting The two pieces are identical; hence the original biconvex lens has equal-magnitude curved radii. Let these be $+R$ and $-R$ for light travelling from left to right. The original power is

$$P = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(\mu - 1)}{R}.$$

For either plano-convex piece the plane surface has $1/R_{\text{plane}} = 0$, so its power is $P_{\text{piece}} = (\mu - 1)/R$. Flipping a thin lens in air does not alter this converging power.

Step 3: Obtain the power and check its sign The power of each piece is half the original power:

$$P_{\text{piece}} = \frac{P}{2} = \frac{10}{2} = +5\,\text{D}.$$

Equivalently each new focal length is $f_{\text{piece}} = 0.20\,\text{m} = 20\,\text{cm}$. The piece remains thicker at its centre than at its edge, so it still converges light and the power remains positive. The two identical pieces have the same result.

Step 4: Conclusion Each cut lens has power $\boxed{+5\,\text{D}}$. The required numerical entry is $\boxed{5}$.

Quick Tip to Solve: A perpendicular central cut of an equiconvex lens removes one curved surface from each piece: the power halves and the focal length doubles.

60. A car accelerates from rest to u m/s. The energy spent in this process is E J. The energy required to accelerate the car from u m/s to 2um/s is nE J. The value of n is _____. [4 marks]

Solution:

Concept:

- The translational kinetic energy of a car of constant mass m, moving at speed v, is $K = \frac{1}{2}mv^2$.
- The work required to change its speed equals the change in kinetic energy: $W = K_f - K_i$. Compare the two specified speed intervals using the same mass.

Step 1: Express the energy for the first acceleration Initially the car is at rest, so its initial kinetic energy is zero. When it reaches speed u, its kinetic energy becomes $\frac{1}{2}mu^2$. Thus the given energy is

$$E = \frac{1}{2}mu^2 - 0 = \frac{1}{2}mu^2.$$

This identifies the reference energy in terms of the mass and speed without needing either of their numerical values. The energy is associated with a change of speed, rather than simply the final speed itself.

Step 2: Calculate the additional energy from u to 2u The car already possesses kinetic energy E at the start of the second interval. At speed $2u$, its kinetic energy is

$$K(2u) = \frac{1}{2}m(2u)^2 = 4\left(\frac{1}{2}mu^2\right) = 4E.$$

Consequently the energy added during the second acceleration is

$$W_2 = K(2u) - K(u) = 4E - E = 3E.$$

The starting kinetic energy must be subtracted; $4E$ would be the total energy gained from rest to $2u$, not the additional energy requested.

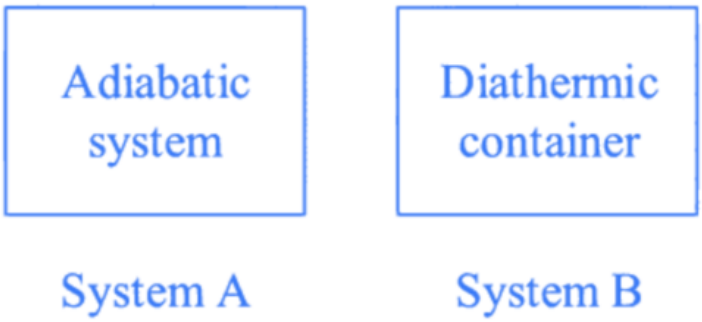
Step 3: Match the coefficient in the question The required second energy is written as nE J. Since the calculated energy is $3E$ J, dividing the two expressions by the common nonzero energy scale E gives $n = 3$. The coefficient is dimensionless. A factor of two would wrongly assume kinetic energy is proportional to speed; its quadratic dependence is essential here.

Step 4: Conclusion The required additional energy is $3E$ J, and hence $\boxed{n = 3}$.

Quick Tip to Solve: For equal mass, form the ratio of squared-speed differences: the intervals 0 to u and u to 2u require energies in the ratio 1 to 3.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. What happens when methane undergoes combustion in systems A and B respectively? [4 marks]



(A) System A: Temperature falls; System B: Temperature rises

(B) System A: Temperature rises; System B: Temperature remains same

(C) System A: Temperature falls; System B: Temperature remains same

(D) System A: Temperature remains same; System B: Temperature rises

Correct Answer: (B) System A: Temperature rises; System B: Temperature remains same

Solution:

Concept:

- Methane combustion is exothermic: chemical energy is released as CH_4 reacts with oxygen to form CO_2 and water.
- An adiabatic boundary prevents heat exchange. A diathermic boundary permits heat exchange with the surroundings and allows the reaction heat to be removed under the usual thermal-reservoir conditions.

Step 1: Analyse combustion in system A System A is explicitly labelled adiabatic. Its boundary does not allow the released energy to leave as heat, so $q = 0$ across the boundary. This does not mean that combustion releases no energy or that the temperature is fixed. Instead, the chemical-energy decrease increases the thermal energy of the reacting material and products. For this exothermic reaction the temperature of the adiabatic system rises.

Step 2: Analyse the diathermic system B System B has a diathermic container, allowing the heat of combustion to pass to the surroundings. In the comparison represented by these systems, the surroundings maintain thermal contact and remove the released heat, so the system temperature remains the same. The distinction is the ability to exchange heat through the boundary. An adiabatic wall retains the reaction energy; a diathermic wall permits its transfer to the thermal surroundings.

Step 3: Compare the complete option pairs The required pair is therefore: temperature rises in A and temperature remains the same in B. Option A incorrectly predicts cooling in A, despite exothermic combustion in an insulated system, and also predicts a rise in B. Option C again predicts cooling in A. Option D interchanges the expected behaviours by making the insulated system remain at the same temperature and the diathermic one rise. Only option B has both entries correct.

Step 4: Conclusion System A becomes hotter, while system B remains at the same temperature.

Hence, the correct option is (B).

Quick Tip to Solve: Adiabatic means no heat crosses the wall, not constant temperature: an exothermic reaction heats an insulated system.

62. In the wet tests for detection of various cations by precipitation, Ba^{2+} cations are detected by obtaining precipitate of [4 marks]

(A) $BaSO_4$

(B) $Ba(OAc)_2$

(C) $BaCO_3$

(D) $Ba(ox)$: Barium oxalate

Correct Answer: (C) $BaCO_3$

Solution:

Concept:

- Wet qualitative analysis separates cations into analytical groups by precipitation using selected group reagents under controlled conditions.
- Barium belongs to the carbonate group with calcium and strontium. Ammonium carbonate precipitates these cations as their carbonates in the presence of ammonium chloride and ammonium hydroxide.

Step 1: Identify the analytical group of barium In systematic qualitative analysis, Ba^{2+}, Sr^{2+} and Ca^{2+} are grouped together because they form insoluble carbonates under the prescribed wet-test conditions. The reagent is $(NH_4)_2CO_3$, used in a medium containing NH_4Cl and NH_4OH. For the barium ion named in the question, this procedure gives barium carbonate as the white precipitate.

Step 2: Write the precipitation equation The carbonate reagent supplies carbonate ions, which combine with barium ions in a 1:1 ratio:

$$Ba^{2+}(aq) + {CO_3}^{2-}(aq) \longrightarrow BaCO_3(s).$$

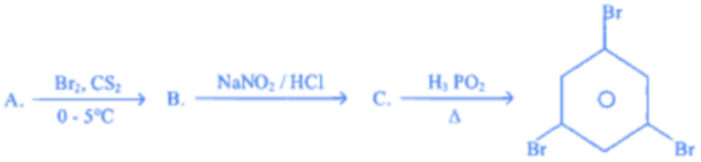
The charges $+2$ and -2 balance, giving the neutral formula $BaCO_3$. Formation of the sparingly soluble solid removes barium ions from solution. Thus the formula required by this group-based wet detection test is the carbonate, rather than a soluble acetate salt.

Step 3: Distinguish the other proposed salts Option A, $BaSO_4$, is also an insoluble white barium salt and is used in a separate sulfate confirmatory test. It must not be confused with the carbonate precipitation step used for the analytical cation group in this question. Option B, barium acetate, is soluble and is not the precipitate. Option D is barium oxalate; oxalate is not the group reagent used here and is associated particularly with the confirmatory precipitation of calcium. Therefore the carbonate in option C is the required answer.

Step 4: Conclusion The wet cation-group precipitation produces $BaCO_3$. Hence, the correct option is (C).

Quick Tip to Solve: For group analysis of Ba, Sr and Ca, remember ammonium carbonate as the group reagent; distinguish it from later individual confirmatory tests.

63. Compound A from the following reaction sequence is: [4 marks]



(A) Benzoic acid

(B) Phenol

(C) Salicylic acid

(D) Aniline

Correct Answer: (D) Aniline

Solution:

Concept:

- An amino group strongly activates a benzene ring and directs electrophilic substitution to its two ortho positions and its para position.
- An aromatic primary amine forms a diazonium salt with $NaNO_2/HCl$; hypophosphorous acid, H_3PO_2, then replaces the diazonium group by hydrogen.

Step 1: Work backwards from the last reagent The final product shown is 1,3,5-tribromobenzene. The last reagent is heated H_3PO_2, which reduces an aryl diazonium group to hydrogen with release of nitrogen. Thus the preceding intermediate C must be a tribromobenzenediazonium salt, with the diazonium group occupying the ring site that becomes hydrogen in the last step. The three bromine atoms stay on the aromatic ring during this reduction.

Step 2: Identify the group required for diazotisation The reagent $NaNO_2/HCl$ generates nitrous acid, which converts a primary aromatic amino group into a diazonium chloride. Therefore intermediate B must retain an $-NH_2$ group. Bromination of aniline places bromine at the 2, 4 and 6 positions relative to this strongly activating group. The sequence is consequently aniline to 2,4,6-tribromoaniline, then the corresponding diazonium salt, and finally replacement of the diazonium group by hydrogen.

Step 3: Check the ring substitution and all alternatives Removing the group at position 1 of the 2,4,6-tribrominated intermediate leaves three alternating bromine substituents, which are renumbered as 1,3,5 in the product. Benzoic acid and salicylic acid contain carboxyl groups and cannot give the required primary-amine diazotisation sequence. Phenol activates bromination but has no primary amino group to form this diazonium intermediate under the stated reagents. Aniline alone provides both the activating group and the diazotisation step.

Step 4: Conclusion Compound A is aniline, $C_6H_5NH_2$. Hence, the correct option is (D).

Quick Tip to Solve: The pair NaNO2/HCl followed by H3PO2 identifies an aromatic amino group that is first diazotised and then replaced by hydrogen.

64. The total number of stereoisomers for the complex $[\mathrm{Cr}(ox)_2\mathrm{ClBr}]^{3-}$ (where ox = oxalate) is :

[4 marks]

(A) 1

(B) 3

(C) 2

(D) 4

Correct Answer: (B) 3

Solution:

Concept:

- Oxalate, ox^{2-}, is a symmetrical bidentate ligand. Two oxalates occupy four coordination sites, and chloride and bromide occupy the remaining two sites of an octahedron.
- Stereoisomers have the same connectivity but differ in spatial arrangement. Count geometrical arrangements first, then count the optical forms of each arrangement separately.

Step 1: Identify the cis and trans arrangements The two different monodentate ligands can be adjacent, giving the cis arrangement, or opposite, giving the trans arrangement. Rotation of an entire complex does not produce a new isomer. Interchanging the equivalent ends of one oxalate also does not produce a new species, because the two donor atoms of this symmetrical chelate are equivalent.

Step 2: Check each geometry for optical activity In the cis arrangement the two chelate rings have two possible handed dispositions. These are the non-superimposable mirror-image forms Δ and Λ, so cis contributes two stereoisomers. In the trans arrangement a symmetry plane containing the two opposite monodentate bonds relates the chelate arrangement to its mirror image; it is optically inactive and contributes one form.

Step 3: Add distinct stereoisomers The count is therefore

$$N = 2_{\text{cis}} + 1_{\text{trans}} = 3.$$

Counting only the two geometries gives option (C), but misses optical isomerism. Option (A) misses geometrical isomerism, while option (D) incorrectly treats the trans form as an enantiomeric pair.

Step 4: Conclusion There are three stereoisomers: cis-Δ, cis-Λ, and trans. Hence, the correct option is (B).

Quick Tip to Solve: For octahedral complexes with two identical bidentate ligands, count cis/trans forms first and test each for a mirror plane before doubling it.

65. The major product for the following reaction is: [4 marks]



(A) $HSCH_2CH_2OCH_2CH_2CN$

(B) $HOCH_2CH_2SCH_2CH_2CN$

(C) $CH_2{=}CHC(=NH)OCH_2CH_2S$

(D) $CH_2{=}CHC(=NH)SCH_2CH_2OH$

Correct Answer: (B) $HOCH_2CH_2SCH_2CH_2CN$

Solution:

Concept:

- Acrylonitrile, $CH_2{=}CHCN$, contains an electron-withdrawing nitrile group conjugated with its double bond. This makes the terminal, beta carbon susceptible to nucleophilic conjugate addition.
- In 2-mercaptoethanol, $HOCH_2CH_2SH$, sulfur is the more effective nucleophilic site for this addition. The sulfur atom forms a bond to the beta carbon while the hydroxy group remains unchanged.

Step 1: Choose the attacking functional group The reagent has both an alcohol and a thiol group. Sulfur is larger and more polarizable than oxygen, and its thiolate is a particularly effective nucleophile toward the activated alkene. Thus the new linkage is a thioether, rather than the ether linkage formed by attack through oxygen. The alcohol group remains available at the end of the reagent chain.

Step 2: Add across the activated double bond Sulfur bonds to the terminal carbon of acrylonitrile; proton transfer supplies hydrogen to the carbon adjacent to the nitrile. The net addition is

$$HOCH_2CH_2SH + CH_2{=}CHCN \longrightarrow HOCH_2CH_2SCH_2CH_2CN.$$

The carbon skeleton has two methylene groups between sulfur and the nitrile carbon. Neither the carbon-nitrogen triple bond nor the hydroxy group needs to be consumed.

Step 3: Compare the four products Option (B) has exactly the required thioether linkage and retains both the hydroxy and nitrile groups. Option (A) joins the chains through oxygen instead. Options (C) and (D) retain the carbon-carbon double bond and convert the nitrile into an imino derivative; those are not the major conjugate-addition product under the conditions shown.

Step 4: Conclusion The major product is $HOCH_2CH_2SCH_2CH_2CN$, formed by sulfur conjugate addition to acrylonitrile. Hence, the correct option is (B).

Quick Tip to Solve: For a thiol adding to an alkene activated by a nitrile, place sulfur at the beta carbon and keep the nitrile group intact.

66. The naturally occurring amino acid that contains only one basic functional group in its chemical structure is [4 marks]

(A) asparagine

(B) arginine

(C) histidine

(D) lysine

Correct Answer: (A) asparagine

Solution:

Concept:

- A basic functional group has an electron pair available to accept a proton. Every amino acid listed has an alpha-amino group, but some also have a basic group in the side chain.
- An amide nitrogen is not counted as an ordinary basic amino group: its lone pair is delocalized into the adjacent carbonyl group, strongly suppressing its basicity.

Step 1: Inspect the side chain of asparagine Asparagine has the structure $H_2NCH(CH_2CONH_2)COOH$. Its alpha-amino group is basic. The side-chain $-CONH_2$ group is a carboxamide, not a second free amino group. Resonance between nitrogen and the carbonyl group makes its nitrogen lone pair unavailable in the way it is in an amine. Consequently asparagine has only one basic functional group in the classification used here.

Step 2: Eliminate the other amino acids Lysine has an additional terminal side-chain amino group, $-(CH_2)_4NH_2$, so it has two basic sites of the relevant type. Arginine has a strongly basic guanidino side chain in addition to its alpha-amino group. Histidine has a basic imidazole nitrogen in its side chain as well as the alpha-amino group; the pyridine-like ring nitrogen can accept a proton.

Step 3: Separate nitrogen count from basic-group count Asparagine contains two nitrogen atoms, but counting nitrogen atoms alone would give the wrong classification. One is amide nitrogen, whose electron pair participates in resonance, and the other is amino nitrogen. Options (B), (C), and (D) all contain a genuinely basic side-chain group and therefore fail the condition of only one basic functional group.

Step 4: Conclusion Asparagine has one basic amino group and a nonbasic side-chain amide group. Hence, the correct option is (A).

Quick Tip to Solve: When counting basic groups in an amino acid, distinguish an amide $-CONH_2$ from a free amine $-NH_2$.

67. Which of the following complexes will exhibit maximum attraction to an applied magnetic field?

[4 marks]

(A) $[\mathrm{Ni}\,(\mathrm{H_2O})_6]^{2+}$

(B) $[\mathrm{Co}\,(\mathrm{H_2O})_6]^{2+}$

(C) $[\mathrm{Co(en)_3}]^{3+}$

(D) $[\mathrm{Zn}\,(\mathrm{H_2O})_6]^{2+}$

Correct Answer: (B) $[\mathrm{Co}\,(\mathrm{H_2O})_6]^{2+}$

Solution:

Concept:

- Paramagnetic complexes are attracted by an applied magnetic field because they contain unpaired electrons. For the same spin-only comparison, larger n gives a larger moment $\mu = \sqrt{n(n+2)}\,\mu_{\mathrm{B}}$.
- Find the metal oxidation state and d-electron count before filling the octahedral levels. Water produces high-spin cobalt(II), whereas cobalt(III) with ethylenediamine is low spin.

Step 1: Count unpaired electrons in the aqua complexes Water is neutral, so nickel and cobalt are both in the $+2$ oxidation state. Ni^{2+} is d^8, giving the octahedral distribution $t_{2g}^6 e_g^2$, with two unpaired electrons. Co^{2+} is d^7; with water it is high spin, $t_{2g}^5 e_g^2$, with three unpaired electrons.

Step 2: Check the remaining two complexes Ethylenediamine is neutral and cobalt in $[\mathrm{Co(en)_3}]^{3+}$ is $+3$, hence d^6. Its octahedral ligand-field splitting is sufficient for the low-spin configuration $t_{2g}^6 e_g^0$, with all electrons paired. Zinc(II) is d^{10}, giving $t_{2g}^6 e_g^4$, also with all electrons paired. Thus neither option (C) nor option (D) shows the largest paramagnetic attraction.

Step 3: Compare the magnetic moments The spin-only moments of the two paramagnetic candidates are

$$\mu_{\mathrm{Ni}} = \sqrt{2(2+2)} = \sqrt{8}\,\mu_{\mathrm{B}}, \qquad \mu_{\mathrm{Co}} = \sqrt{3(3+2)} = \sqrt{15}\,\mu_{\mathrm{B}}.$$

The high-spin cobalt(II) aqua complex therefore exceeds nickel(II), while the other candidates have zero spin-only moment.

Step 4: Conclusion $[\mathrm{Co(H_2O)_6}]^{2+}$, with three unpaired electrons, exhibits maximum attraction. Hence, the correct option is (B).

Quick Tip to Solve: Compare unpaired electrons only after checking both the metal oxidation state and whether the octahedral complex is high spin or low spin.

68. Given below are two statements :

Statement I : SO_2 and H_2O both possess V-shaped structure.

Statement II : The bond angle of SO_2 is less than that of H_2O.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Both Statement I and Statement II are correct

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (C) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- VSEPR theory distinguishes electron-domain geometry from molecular shape. Lone pairs affect the arrangement but do not appear as vertices in the named molecular shape.
- SO_2 has three electron domains about sulfur, whereas H_2O has four about oxygen. A multiple bond is counted as one electron domain for this purpose.

Step 1: Evaluate the molecular-shape statement Sulfur dioxide has two bonding domains and one lone pair on sulfur: an AX_2E arrangement. Its electron-domain geometry is trigonal planar, but the positions of its atoms give a bent or V-shaped molecule. Water has two bonding domains and two lone pairs on oxygen, an AX_2E_2 arrangement. Its electron-domain geometry is tetrahedral and its molecular shape is also bent. Statement I is therefore correct.

Step 2: Compare the bond angles The $O-S-O$ angle in sulfur dioxide is approximately 119°, close to the trigonal-planar value of 120°. In water, the two lone pairs compress the $H-O-H$ angle from the tetrahedral value of 109.5° to about 104.5°. Thus

$$\angle OSO > \angle HOH.$$

Statement II reverses the actual inequality and is incorrect.

Step 3: Match the truth values to the options The required combination is a correct first statement and an incorrect second statement. Option (A) wrongly rejects the bent shapes; option (B) accepts the false angle comparison; and option (D) reverses both truth values. The fact that two molecules have the same shape name does not require their bond angles to be equal or ordered as the statement suggests.

Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: For bent molecules, count lone pairs and identify the parent electron geometry before comparing angles; V-shaped does not mean a fixed angle.

69. The covalency and oxidation state respectively of boron in $[BF_4]^-$, are : [4 marks]

(A) 3 and 4

(B) 4 and 3

(C) 4 and 4

(D) 3 and 5

Correct Answer: (B) 4 and 3

Solution:

Concept:

- Covalency counts the covalent bonds formed by an atom; oxidation state is a formal electron-bookkeeping quantity determined by assigning bonding electrons to the more electronegative atom.
- In $BF_4{}^-$, four fluorine atoms are bonded to boron. Fluorine has oxidation state -1, and the sum of all oxidation states equals the overall ion charge.

Step 1: Count the bonds at boron Tetrafluoroborate has four B–F bonds in a tetrahedral arrangement. Therefore boron has covalency 4. One may describe formation as $BF_3 + F^- \longrightarrow BF_4{}^-$, with fluoride donating a lone pair to electron-deficient boron. Once the ion is formed, all four B–F bonds count toward covalency; the coordinate origin of one bond does not reduce the count to three.

Step 2: Calculate the oxidation state Let the oxidation state of boron be x. Each of the four fluorines contributes -1, while the charge on the complete ion is -1. Hence

$$x + 4(-1) = -1 \quad \Rightarrow \quad x = +3.$$

The ion charge must be included: setting the total to zero would incorrectly give $+4$.

Step 3: Choose the ordered pair The question asks for covalency first and oxidation state second, so the ordered pair is $(4, +3)$. Option (A) interchanges the ideas and uses an incorrect oxidation state. Option (C) confuses the number of bonds with oxidation state, while option (D) supplies neither the actual covalency nor the oxidation state. Formal charge, coordination number, and oxidation state are related bookkeeping concepts but must not be substituted for one another.

Step 4: Conclusion The covalency of boron is 4 and its oxidation state is $+3$. Hence, the correct option is (B).

Quick Tip to Solve: Count bonds for covalency, then write a separate charge-balance equation for oxidation state; include the negative charge of $BF_4{}^-$.

70. Given below are two statements :

Statement I : Tropolone is an aromatic compound and has 8π electrons.

Statement II : π electrons of $> C = O$ group in tropolone is involved in aromaticity.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- An aromatic circuit must be cyclic, planar, continuously conjugated, and contain $(4n+2)$ delocalized pi electrons. Count electrons in the aromatic ring circuit, rather than adding every pi bond in the molecule.
- Tropolone has an aromatic seven-membered ring described by resonance contributions with tropylium-like ring character. The aromatic ring circuit contains six pi electrons; the exocyclic carbonyl pi pair is not an additional pair in that circuit.

Step 1: Identify the aromatic ring circuit Tropolone is 2-hydroxycyclohepta-2,4,6-trien-1-one. Donation of the carbonyl pi pair toward oxygen gives a charge-separated resonance contribution with negative charge on oxygen and tropylium-like positive charge distributed over the ring. The ring has three double-bond electron pairs, corresponding to six delocalized pi electrons, and satisfies $4n+2=6$ for $n=1$.

Step 2: Evaluate Statement I carefully The assertion combines a correct classification, aromaticity, with an incorrect count of eight electrons for its aromatic system. The six ring pi electrons constitute the aromatic circuit. Simply adding the two electrons of the exocyclic carbonyl pi bond gives eight pi electrons in the entire bond inventory, but this is not the aromatic electron count. Thus Statement I, as a statement about the aromatic pi system, is false.

Step 3: Evaluate Statement II and select the option The carbonyl pi pair is associated with the exocyclic oxygen in the charge-separated description; it is not counted as part of the closed six-electron aromatic ring path. Carbonyl polarization and resonance help describe the molecule, but do not make that pair an extra pair circulating around the ring. Statement II is therefore false in the aromatic electron-counting sense. Options (A) and (D) accept Statement II, and option (C) accepts the incorrect aromatic count in Statement I.

Step 4: Conclusion Both Statement I and Statement II are false: tropolone has a six-pi-electron aromatic ring circuit, without the exocyclic carbonyl pi pair in that count. Hence, the correct option is (B).

Quick Tip to Solve: For a ring bearing an exocyclic carbonyl, use a charge-separated resonance form and count the electrons in the closed ring path, not every pi electron in the molecule.

71. Match List I with List II

1 - Bromopropane is reacted with reagents in List I to give product in List II

	LIST I - Reagent		LIST II - Product
A.	KOH **(alc)**	**I.**	**Nitrile**
B.	KCN **(alc)**	**II.**	**Ester**
C.	$AgNO_2$	**III.**	**Alkene**
D.	$H_3CCOOAg$	**IV.**	**Nitroalkane**

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-II, C-III, D-IV

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-I, C-IV, D-II

(D) A-I, B-III, C-IV, D-II

Correct Answer: (C) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Alcoholic KOH favours elimination of a hydrogen halide from an alkyl halide, producing an alkene.
- For ambident nucleophiles the attacking atom matters: ionic KCN gives a nitrile, while $AgNO_2$ gives a nitroalkane.

Step 1: Identify the elimination product In 1-bromopropane the bromine is on the terminal carbon. Alcoholic KOH removes a beta hydrogen as bromide leaves. The double bond is therefore formed between the first two carbons:

$$CH_3CH_2CH_2Br + KOH \longrightarrow CH_3CH{=}CH_2 + KBr + H_2O.$$

Thus reagent A produces propene, the alkene in entry III.

Step 2: Match the substitution products With alcoholic KCN, carbon attack replaces bromide by $-CN$, giving $CH_3CH_2CH_2CN$, a nitrile: B-I. Silver nitrite favours nitrogen attack and gives $CH_3CH_2CH_2NO_2$, a nitroalkane: C-IV. Silver acetate replaces bromide by the acetate group, giving $CH_3COOCH_2CH_2CH_3$, propyl ethanoate, an ester: D-II. These substitutions distinguish cyanide from nitrite and from carboxylate; they are not all the same functional-group conversion.

Step 3: Eliminate the incorrect combinations Option A incorrectly calls the alcoholic-base product a nitrile and the cyanide product an ester. Option B incorrectly calls the base product a nitroalkane. Option D has the last two matches right but interchanges the alkene and nitrile matches. Only option C contains all four correct assignments.

Step 4: Conclusion A-III, B-I, C-IV, D-II. Hence, the correct option is (C).

Quick Tip to Solve: First lock in alcoholic KOH as elimination; then distinguish KCN (nitrile) from $AgNO_2$ (nitroalkane).

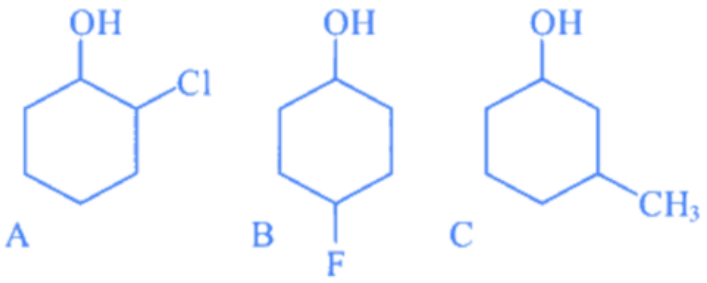
72. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Order of acidic nature of the following compounds is A > B > C.

Reason R : Fluoro is a stronger electron withdrawing group than Chloro group.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]



(A) Both A and R are correct but R is NOT the correct explanation of A

(B) A is false but R is true

(C) A is true but R is false

(D) Both A and R are correct and R is the correct explanation of A

Correct Answer: (A) Both A and R are correct but R is NOT the correct explanation of A

Solution:

Concept:

- An electron-withdrawing substituent stabilises an alkoxide conjugate base by its negative inductive effect, increasing alcohol acidity.
- The inductive effect falls rapidly with the number of sigma bonds between the substituent and the acidic group; alkyl groups exert a positive inductive effect.

Step 1: Evaluate the assertion using the structures Compound A has chlorine on the carbon adjacent to the carbon bearing OH. Compound B has fluorine farther away, at the opposite carbon of the cyclohexane ring. Compound C has a methyl substituent, which donates electron density. After removal of the hydroxyl proton, nearby chlorine stabilises the negative charge of the alkoxide more effectively than the distant fluorine does. Methyl donation destabilises that negative charge rather than stabilising it. The printed order A greater than B greater than C is consequently correct.

Step 2: Evaluate the reason independently Fluorine is more electronegative than chlorine, and at comparable positions its negative inductive effect is stronger. Thus the statement that fluoro is a stronger electron-withdrawing group than chloro is itself true. This statement compares substituents at comparable distances; it does not remove the strong distance dependence of induction.

Step 3: Test whether the reason explains the order The reason alone would suggest the fluoro compound should be more acidic if the positions were identical. Here the positions are different, so the closer chloro group has the stronger effective influence on the alkoxide. Distance is the missing explanation. The false-assertion and false-reason options B and C are excluded because both statements are true; option D is excluded because it claims the given reason explains the order.

Step 4: Conclusion Both Assertion A and Reason R are correct, but R is not the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: For inductive acidity comparisons, count the bonds from the substituent to OH before comparing electronegativities.

73. The correct group of halide ions which can be oxidised by oxygen in acidic medium is : [4 marks]

(A) Br^- only

(B) Cl^-, Br^- and I^- only

(C) Br^- and I^- only

(D) I^- only

Correct Answer: (C) Br^- and I^- only

Solution:

Concept:

- A redox process is thermodynamically favourable when $E^\circ_{\text{cell}} = E^\circ_{\text{oxidant}} - E^\circ_{\text{halogen/halide}} > 0$.
- In acid the oxygen/water reduction potential is approximately $1.23\,\text{V}$; larger halogen reduction potentials mean their halide ions are harder to oxidise.

Step 1: Write the two coupled half reactions Oxygen is reduced in acid according to

$$O_2 + 4\,H^+ + 4\,e^- \longrightarrow 2\,H_2O, \qquad E^\circ = 1.23\,\text{V}.$$

Oxidation of a halide is $2\,X^- \longrightarrow X_2 + 2\,e^-$. The tabulated halogen potentials refer to the reverse reduction, $X_2 + 2\,e^- \longrightarrow 2\,X^-$. Subtract that reduction potential from the oxygen value; do not add two reduction potentials.

Step 2: Compare chloride, bromide and iodide The approximate reduction potentials of Cl_2/Cl^-, Br_2/Br^- and I_2/I^- are respectively 1.36, 1.09 and $0.54\,\text{V}$. Thus

$$E^\circ_{\text{cell}}(Cl^-) = 1.23 - 1.36 = -0.13\,\text{V},$$
$$E^\circ_{\text{cell}}(Br^-) = 1.23 - 1.09 = +0.14\,\text{V},$$
$$E^\circ_{\text{cell}}(I^-) = 1.23 - 0.54 = +0.69\,\text{V}.$$

Bromide and iodide satisfy the positive-potential test; chloride does not.

Step 3: Select the complete group Option A omits iodide although its oxidation is favourable. Option D omits bromide. Option B wrongly includes chloride, whose calculated cell potential is negative. Therefore the complete permitted pair is bromide and iodide only.

Step 4: Conclusion The supplied key selects iodide alone, but the printed question gives no kinetic restriction: the positive standard cell potentials above show that both Br^- and I^- can be oxidised thermodynamically. Hence, the correct option is (C).

Quick Tip to Solve: Place the oxidant potential between the halogen potentials: species below acidic oxygen can have their halide ions oxidised.

74. **Match List I with List II**

	LIST I		LIST II
A.	Weak intermolecular forces of attraction	I.	Hexamethylenediamine + adipic acid
B.	Hydrogen bonding	II.	$AlEt_3 + TiCl_4$
C.	Heavily branched polymer	III.	2 - chloro - 1, 3 - butadiene
D.	High density polymer	IV.	Phenol + formaldehyde

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-IV, B-I, C-III, D-II

(C) A-II, B-IV, C-I, D-III

(D) A-IV, B-II, C-III, D-I

Correct Answer: (A) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Elastomers have relatively weak intermolecular attractions, whereas polyamides gain strong intermolecular hydrogen bonding from their amide groups.
- Phenol-formaldehyde produces a highly branched or cross-linked resin; Ziegler-Natta catalysis produces linear high-density polyethylene.

Step 1: Identify the elastomer and the polyamide Entry III is 2-chloro-1,3-butadiene, chloroprene. Its addition polymer is neoprene, an elastomer associated with weak intermolecular forces and elastic behaviour, so A-III. Entry I combines hexamethylenediamine with adipic acid to form nylon-6,6 by condensation. Its repeated $-CONH-$ groups provide hydrogen-bond donors and acceptors between chains, so B-I. Hydrogen bonding is a property of the resulting polyamide, not of every addition polymer.

Step 2: Identify the branched resin and high-density polymer Phenol and formaldehyde, entry IV, form phenol-formaldehyde resin. Further reaction at available ring positions links chains into a heavily branched or cross-linked structure; therefore C-IV. The $AlEt_3/TiCl_4$ combination in entry II is a Ziegler-Natta catalyst system. Polymerisation of ethene with this system gives relatively linear polyethylene chains which pack efficiently and have higher density. Hence D-II.

Step 3: Check the answer combinations The resulting sequence is III, I, IV, II. Option B incorrectly assigns the weak-force elastomer to phenol-formaldehyde and the heavily branched resin to chloroprene. Option C mismatches both the polyamide and the high-density polymer. Option D assigns the catalyst to hydrogen bonding and nylon monomers to high density, so neither provides the required four matches.

Step 4: Conclusion A-III, B-I, C-IV, D-II. Hence, the correct option is (A).

Quick Tip to Solve: Anchor nylon-6,6 to hydrogen bonding and Ziegler-Natta catalyst to HDPE; then match neoprene and phenol-formaldehyde.

75. Better method for preparation of BeF_2, among the following is : [4 marks]

(A) $BeH_2 + F_2 \xrightarrow{\Delta} BeF_2$

(B) $(NH_4)_2\,BeF_4 \xrightarrow{\Delta} BeF_2$

(C) $Be + F_2 \xrightarrow{\Delta} BeF_2$

(D) $BeO + C + F_2 \xrightarrow{\Delta} BeF_2$

Correct Answer: (B) $(NH_4)_2\,BeF_4 \xrightarrow{\Delta} BeF_2$

Solution:

Concept:

- Thermal decomposition of ammonium tetrafluoroberyllate gives beryllium fluoride while volatile ammonia and hydrogen fluoride are removed.
- A preferred preparative route is judged by controlled conversion and separation of products, not merely by whether a reaction with fluorine can be written.

Step 1: Identify the suitable fluoride precursor The complex salt $(NH_4)_2BeF_4$ already contains beryllium and fluoride in a suitable combination. On heating, it decomposes to the desired fluoride. The balanced overall decomposition is

$$(NH_4)_2BeF_4 \xrightarrow{\Delta} BeF_2 + 2\,NH_3 + 2\,HF.$$

The atom balance is one beryllium, four fluorines, two nitrogens and eight hydrogens on each side. The gaseous products can leave the reaction mixture, leaving BeF_2 as the solid product.

Step 2: Compare the routes involving elemental fluorine Direct reaction of beryllium with fluorine is highly vigorous, so writing $Be + F_2 \longrightarrow BeF_2$ does not make it the better controlled preparation. Fluorination of beryllium hydride likewise introduces a reactive hydride and a highly reactive fluorinating agent, with extra hydrogen-containing products to manage. The oxide-carbon-fluorine mixture is not the preferred simple preparation of this fluoride; it combines an oxide reduction scheme with aggressive fluorination rather than the clean decomposition of a fluoride precursor.

Step 3: Choose the preparative method Option B is the controlled thermal-decomposition route. Options A and C both require direct use of elemental fluorine, while D is not the standard preferred method among those listed. The question asks for the better method, so merely noting that fluorine can form a fluoride is insufficient.

Step 4: Conclusion Heat $(NH_4)_2BeF_4$ to obtain BeF_2. Hence, the correct option is (B).

Quick Tip to Solve: For BeF_2, remember the ammonium tetrafluoroberyllate precursor: heating removes NH_3 and HF.

76. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Isotopes of hydrogen have almost same chemical properties, but difference in their rates of reaction.

Reason R : Isotopes of hydrogen have different enthalpy of bond dissociation.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) A is not correct but R is correct

(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is correct but R is not correct

(D) Both A and R are correct and R is the correct explanation of A

Correct Answer: (D) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- Isotopes have the same nuclear charge and electronic configuration, which makes their general chemical behaviour almost identical.
- Changing isotopic mass changes vibrational energies, zero-point energy and effective bond-dissociation enthalpies, leading to different reaction rates.

Step 1: Judge the assertion Protium, deuterium and tritium all have atomic number one. Their neutral atoms have one electron and the same valence electronic configuration. They therefore form analogous compounds and show almost the same chemical properties. Their nuclear masses, however, differ substantially in relative terms. Reactions involving cleavage or formation of bonds to these isotopes need not have identical rate constants. Thus the assertion correctly distinguishes chemical similarity from equality of reaction rates.

Step 2: Judge the reason For a bond described approximately as a harmonic oscillator, the vibrational frequency varies as $\nu \propto 1/\sqrt{\mu}$, where μ is the reduced mass. A heavier isotope lowers the frequency and hence lowers the zero-point vibrational energy. The energy needed to dissociate a molecule from its lowest vibrational level therefore changes with isotope. For example, the effective bond-dissociation enthalpy of D_2 is greater than that of H_2. The reason is consequently true.

Step 3: Connect the reason to the rate difference Where a hydrogen-containing bond is broken during the rate-determining process, the different bond energies alter the activation energy and thus the reaction rate. This is the kinetic isotope effect. It explains why similar electronic chemistry does not imply the same rates. Option A wrongly rejects the assertion; C wrongly rejects the reason; B overlooks the causal link between isotope-dependent bond energies and rates.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (D).

Quick Tip to Solve: Separate electronic configuration, which controls chemical similarity, from isotopic mass, which changes bond vibration and reaction rate.

77. Which of the following are the Green house gases?

A. Water vapour

B. Ozone

C. I_2

D. Molecular hydrogen

Choose the most appropriate answer from the options given below : [4 marks]

(A) A and D only

(B) A and B only

(C) B and C only

(D) C and D only

Correct Answer: (B) A and B only

Solution:

Concept:

- A greenhouse gas absorbs outgoing terrestrial infrared radiation and re-emits it, reducing the rate at which the lower atmosphere loses heat.
- A vibration must change the molecular dipole moment to absorb infrared radiation strongly; homonuclear diatomic molecules lack ordinary dipole-active vibrations.

Step 1: Assess water vapour and ozone Water is a bent polar molecule. Its rotational and vibrational transitions absorb infrared radiation strongly, so water vapour, entry A, is a major natural greenhouse gas. Ozone is also bent and has infrared-active vibrational modes. It absorbs in the terrestrial infrared region and contributes to the greenhouse effect, so entry B also qualifies. Ozone can both protect against ultraviolet radiation and act as a greenhouse gas; those two roles are not mutually exclusive.

Step 2: Assess iodine and molecular hydrogen Iodine, I_2, and hydrogen, H_2, are homonuclear diatomic molecules. Ordinary stretching of either molecule does not produce a changing dipole moment, so these are not classified as greenhouse gases in this list. In particular, atmospheric hydrogen should not be confused with water vapour, which contains hydrogen but has a very different molecular geometry and infrared spectrum. The entries refer to the molecules as listed, rather than to indirect effects of later atmospheric reactions.

Step 3: Check the proposed pairs The qualifying entries are A and B. Option A wrongly includes molecular hydrogen and omits ozone. Option C wrongly includes iodine and omits water vapour. Option D includes neither of the qualifying gases. Only option B lists both water vapour and ozone without adding a non-qualifying species.

Step 4: Conclusion Water vapour and ozone, A and B only, are the greenhouse gases listed. Hence, the correct option is (B).

Quick Tip to Solve: Ask whether the molecule has infrared-active motions; do not classify every atmospheric gas or every hydrogen-containing gas as a greenhouse gas.

78. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The diameter of colloidal particles in solution should not be much smaller than wavelength of light to show Tyndall effect.

Reason R : The light scatters in all directions when the size of particles is large enough.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both A and R are correct and R is the correct explanation of A

(B) A is true but R is false

(C) Both A and R are correct but R is NOT the correct explanation of A

(D) A is false but R is true

Correct Answer: (A) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- The Tyndall effect is the scattering of an incident light beam by dispersed colloidal particles; scattered light makes the path visible from the side.
- For an appreciable Tyndall effect, particle size must not be very small compared with the wavelength of the incident light. A difference in refractive index between the dispersed phase and medium also favours visibility.

Step 1: Evaluate the assertion Assertion A states a condition on particle size relative to the wavelength, rather than claiming that every colloidal particle must be larger than the wavelength. When particles are extremely small compared with visible wavelengths, their scattering becomes too weak for the beam path to be readily visible. Thus the stated requirement that the diameter should not be much smaller than the wavelength is correct.

Step 2: Evaluate the reason Reason R correctly describes the origin of the effect: particles of sufficient size scatter incident light into different directions. An observer looking sideways therefore receives light from the beam path. This is scattering, rather than production of new light or a chemical reaction.

Step 3: Connect the reason to the assertion The size dependence of appreciable scattering explains why the assertion imposes its size condition. Both statements are true and R explains A. Option B incorrectly rejects R; option D incorrectly rejects A; option C accepts both statements but denies the causal connection that produces the visible beam.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: For Tyndall-effect assertions, connect visibility of the beam to sideways scattering and compare particle size with the light wavelength.

79. Given below are two statements related to Ellingham diagram:

Statement I : Ellingham diagrams can be constructed for formation of oxides, sulfides and halides of metals.

Statement II : It consists of plots of $\Delta_f H^0$ vs T for formation of oxides of elements.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (C) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy change of formation, $\Delta_f G^\circ$, against absolute temperature T.
- The thermodynamic relation is $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$. For an approximately linear segment, its intercept is related to enthalpy and its slope to the negative entropy change.

Step 1: Assess the range of compounds Statement I is correct. Although oxide formation diagrams are especially common in extraction metallurgy, the same Gibbs-energy-versus-temperature construction applies to the formation of metal sulfides and metal halides. The construction is a thermodynamic comparison and is not restricted to reactions with oxygen.

Step 2: Check the quantity on the vertical axis Statement II explicitly names standard enthalpy of formation, $\Delta_f H^\circ$, as the plotted ordinate. This is incorrect: the ordinate is standard Gibbs energy of formation. Gibbs energy determines thermodynamic feasibility at the specified temperature and includes the entropy contribution. An enthalpy-versus-temperature plot would not be the Ellingham diagram described here.

Step 3: Select the matching combination The required truth values are therefore I true and II false. Option A wrongly accepts the enthalpy ordinate. Option B wrongly rejects the valid extension to sulfides and halides. Option D reverses both truth values. Only option C has the correct combination of the two statements.

Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: Remember the Ellingham ordinate as Gibbs energy: the relation $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$ explains its temperature dependence.

80. Identify the correct order of standard enthalpy of formation of sodium halides : [4 marks]

(A) $\mathrm{NaI} < \mathrm{NaBr} < \mathrm{NaCl} < \mathrm{NaF}$

(B) $\mathrm{NaI} < \mathrm{NaBr} < \mathrm{NaF} < \mathrm{NaCl}$

(C) $\mathrm{NaF} < \mathrm{NaCl} < \mathrm{NaBr} < \mathrm{NaI}$

(D) $\mathrm{NaCl} < \mathrm{NaF} < \mathrm{NaBr} < \mathrm{NaI}$

Correct Answer: (C) $\mathrm{NaF} < \mathrm{NaCl} < \mathrm{NaBr} < \mathrm{NaI}$

Solution:

Concept:

- The standard enthalpy of formation is the enthalpy change for forming one mole of a compound from its elements in their standard states. Signed values, rather than absolute magnitudes, must be ordered.
- For the solid sodium halides, formation becomes less exothermic from fluoride to iodide. The decreasing lattice stabilization down the halide group is central to this overall trend.

Step 1: Compare the signed formation enthalpies The standard formation enthalpies of solid sodium fluoride, chloride, bromide and iodide are all negative. Representative rounded values, in $\mathrm{kJ\,mol^{-1}}$, are about -574, -411, -361 and -288, respectively. These establish the order of the actual signed quantities without treating a larger heat release as a numerically larger enthalpy.

Step 2: Write the increasing order The fluoride has the most negative formation enthalpy and is consequently the smallest in the requested ordering. Moving towards iodide makes the value less negative. Therefore

$$\Delta_{\mathrm{f}}H^{\circ}(\mathrm{NaF}) < \Delta_{\mathrm{f}}H^{\circ}(\mathrm{NaCl}) < \Delta_{\mathrm{f}}H^{\circ}(\mathrm{NaBr}) < \Delta_{\mathrm{f}}H^{\circ}(\mathrm{NaI}).$$

Step 3: Eliminate the distractors Option A reverses the signed ordering and would instead resemble an ordering of positive magnitudes of heat released. Option B additionally interchanges fluoride and chloride in that reversed pattern. Option D incorrectly places chloride below fluoride. Option C alone follows the increasingly less negative formation enthalpies.

Step 4: Conclusion The correct order is $\mathrm{NaF} < \mathrm{NaCl} < \mathrm{NaBr} < \mathrm{NaI}$ for the signed standard formation enthalpies. The supplied key selects the reverse ordering, which reverses the order of these negative signed enthalpies; the printed question asks for enthalpy of formation, so the signed values prove (C). Hence, the correct option is (C).

Quick Tip to Solve: For exothermic formation, first keep the minus signs: a more exothermic compound has a smaller, more negative formation enthalpy.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. A(g) → 2B(g) + C(g) is a first order reaction. The initial pressure of the system was found to be 800 mm Hg which increased to 1600 mm Hg after 10 min. The total pressure of the system after 30 min will be _____ mm Hg. (Nearest integer) [4 marks]

Solution:

Concept:

- At fixed temperature and volume, partial pressures are proportional to numbers of moles. For $\mathrm{A(g)} \longrightarrow 2\,\mathrm{B(g)} + \mathrm{C(g)}$, each mole of reactant consumed increases the total amount by two moles.
- For first-order disappearance, the remaining fraction is e^{-kt}; equal time intervals multiply that fraction by the same factor.

Step 1: Express total pressure in terms of decomposition Let x, measured in mm Hg, denote the partial-pressure equivalent of the amount of A consumed. Initially only A is present at $P_0 = 800\,\mathrm{mm\,Hg}$. Its pressure becomes $800 - x$, while the product partial pressures are $2x$ and x. Therefore

$$P_t = (800 - x) + 2x + x = 800 + 2x.$$

Complete reaction would give $2400\,\mathrm{mm\,Hg}$, which provides a useful upper bound.

Step 2: Find the fraction left after ten minutes Use the measured total pressure at ten minutes:

$$1600 = 800 + 2x \quad \Rightarrow \quad x = 400\,\mathrm{mm\,Hg}.$$

The reactant partial pressure is then $800 - 400 = 400\,\mathrm{mm\,Hg}$, so half the original A remains. Thus ten minutes is the half-life; the doubling of total pressure itself must not be used as a first-order reactant-pressure ratio.

Step 3: Advance through three half-lives Thirty minutes corresponds to three ten-minute half-lives. The remaining reactant pressure is

$$P_A(30) = 800\left(\frac{1}{2}\right)^3 = 100\,\mathrm{mm\,Hg}.$$

Thus $x = 800 - 100 = 700\,\mathrm{mm\,Hg}$, and

$$P_{30} = 800 + 2(700) = 2200\,\mathrm{mm\,Hg}.$$

The result lies between the ten-minute pressure and the complete-conversion limit, as required.

Step 4: Conclusion The total pressure after 30 min is $\boxed{2200\,\mathrm{mm\,Hg}}$. The requested nearest-integer entry is 2200.

Quick Tip to Solve: Convert total pressure to the amount of reactant left before applying first-order kinetics; the total gas amount changes during decomposition.

82. 0.400 g **of an organic compound** (X) **gave** 0.376 g **of** AgBr **in Carius method for estimation of bromine.** % **of bromine in the compound** (X) **is _____.**

(Given: Molar mass $AgBr = 188$ g mol^{-1}

$Br = 80$ g mol^{-1}**)** [4 marks]

Solution:

Concept:

- In the Carius method, bromine in the organic sample is quantitatively converted to silver bromide, AgBr. One mole of the precipitate contains one mole of bromine atoms.
- The bromine mass fraction in AgBr is $M(\text{Br})/M(\text{AgBr})$. Percentage bromine in the original compound is its recovered bromine mass divided by sample mass, multiplied by 100.

Step 1: Find the amount of silver bromide The collected precipitate has mass $0.376\,\text{g}$ and its supplied molar mass is $188\,\text{g mol}^{-1}$. Hence

$$n(\text{AgBr}) = \frac{0.376}{188} = 0.00200\,\text{mol}.$$

The silver in the precipitate comes from the analytical reagent, so the entire precipitate mass is not the bromine mass in the original sample.

Step 2: Convert the precipitate amount to bromine mass Each formula unit of silver bromide has one bromine atom. Therefore

$$n(\text{Br}) = n(\text{AgBr}) = 0.00200\,\text{mol},$$

$$m(\text{Br}) = 0.00200 \times 80 = 0.160\,\text{g}.$$

This calculation uses the given atomic molar mass of bromine; there is no extra factor of two for elemental Br_2, because the atom count in AgBr is already being used.

Step 3: Calculate the mass percentage in the sample The original organic compound weighed $0.400\,\text{g}$. Thus

$$\%\text{Br} = \frac{0.160}{0.400} \times 100 = 40.0\%.$$

Equivalently, the full calculation is $(0.376/0.400)(80/188) \times 100$. The recovered bromine mass is smaller than both the sample mass and the precipitate mass, consistent with the interpretation.

Step 4: Conclusion The bromine content of compound X is $\boxed{40\%}$ by mass; the numerical entry is 40.

Quick Tip to Solve: In Carius calculations, multiply the precipitate mass by the halogen-to-silver-halide molar-mass ratio before dividing by the sample mass.

83. See the following chemical reaction:

$$Cr_2O_7^{2-} + XH^+ + 6\,Fe^{2+} \longrightarrow YCr^{3+} + 6\,Fe^{3+} + ZH_2O$$

The sum of X, Y and Z is _____. [4 marks]

Solution:

Concept:

- Balance oxygen with water and hydrogen with H^+ in acidic medium. Balance charge with electrons in each half-reaction before combining them.
- Chromium changes from oxidation state $+6$ in $Cr_2O_7^{2-}$ to $+3$, while each Fe^{2+} loses one electron to form Fe^{3+}.

Step 1: Balance chromium, oxygen and hydrogen One dichromate ion contains two chromium atoms, so its reduction produces two Cr^{3+} ions: $Y = 2$. Its seven oxygen atoms are balanced with seven water molecules on the product side, giving $Z = 7$. These water molecules contain fourteen hydrogen atoms, requiring fourteen hydrogen ions on the reactant side, so $X = 14$.

Step 2: Check electron and charge balance The acidic reduction half-reaction is

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

Two chromium atoms each gain three electrons, giving a total gain of six. The oxidation half-reaction is

$$6\,Fe^{2+} \longrightarrow 6\,Fe^{3+} + 6\,e^-.$$

Adding them cancels the six electrons and reproduces the given iron coefficients.

Step 3: Verify the equation and sum the coefficients The balanced overall reaction is

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,Fe^{2+} \longrightarrow 2\,Cr^{3+} + 6\,Fe^{3+} + 7\,H_2O.$$

The left charge is $-2 + 14 + 12 = 24$, equal to the right charge $6 + 18 = 24$. Chromium, iron, oxygen and hydrogen are all conserved. Finally,

$$X + Y + Z = 14 + 2 + 7 = 23.$$

The six iron ions are fixed coefficients and are not included in the sum requested.

Step 4: Conclusion $X = 14$, $Y = 2$ and $Z = 7$, so the required dimensionless sum is $\boxed{23}$.

Quick Tip to Solve: For acidic dichromate reduction, remember two chromium ions, seven waters and fourteen hydrogen ions, then verify the six-electron balance.

84. $1\ \text{g}$ **of a carbonate** (M_2CO_3) **on treatment with excess** HCl **produces** $0.01\ \text{mol}$ **of** CO_2**. The molar mass of** M_2CO_3 **is _____** g mol^{-1}**. (Nearest integer)** [4 marks]

Solution:

Concept:

- A carbonate ion reacts with excess acid to give one molecule of carbon dioxide per carbonate ion. A mole of M_2CO_3 therefore yields one mole of CO_2.
- Molar mass is $M = m/n$, where m is sample mass and n is the amount in moles. The subscript two on M counts metal atoms and does not change the carbonate-to-carbon-dioxide ratio.

Step 1: Write the acid-carbonate reaction The stoichiometric equation is

$$M_2CO_3 + 2\,HCl \longrightarrow 2\,MCl + H_2O + CO_2.$$

Both sides contain two metal atoms, one carbon atom and three oxygen atoms. Because hydrochloric acid is in excess, the amount of carbonate limits the carbon dioxide formed. There is no need to determine the quantity of acid or the identity of the metal.

Step 2: Determine the amount of carbonate consumed The coefficients of M_2CO_3 and CO_2 are both one. Hence

$$n(M_2CO_3) = n(CO_2) = 0.010\,\text{mol}.$$

One must not divide this by two: the two metal atoms and the two acid molecules do not imply production of two carbon dioxide molecules. Each carbonate formula unit contains just one carbon atom.

Step 3: Divide mass by amount The sample mass is $1\,\text{g}$, so

$$M(M_2CO_3) = \frac{1\,\text{g}}{0.010\,\text{mol}} = 100\,\text{g mol}^{-1}.$$

As a consistency check, $0.010\,\text{mol}$ of a material of this molar mass weighs exactly $1\,\text{g}$. The requested value is already an integer, so rounding does not change it.

Step 4: Conclusion The molar mass of M_2CO_3 is $\boxed{100\,\text{g mol}^{-1}}$. The nearest-integer entry is 100.

Quick Tip to Solve: For a simple carbonate with one CO_3^{2-} group per formula unit, moles of salt equal moles of carbon dioxide released by excess acid.

85. The orbital angular momentum of an electron in $3\ \text{s}$ **orbital is** $\frac{xh}{2\pi}$**. The value of** x **is _____ (nearest integer)** [4 marks]

Solution:

Concept:

- The magnitude of orbital angular momentum is $L = \sqrt{\ell(\ell+1)}\hbar$, where $\hbar = h/(2\pi)$ and ℓ is the orbital angular momentum quantum number.
- The subshell letters s, p, d and f correspond to $\ell = 0, 1, 2, 3$, respectively. The principal quantum

number determines the shell but does not replace ℓ in the angular momentum formula.

Step 1: Identify the quantum numbers The electron occupies a $3s$ orbital. The leading number gives the principal quantum number $n = 3$, while the letter s fixes $\ell = 0$. Although the third shell permits several subshells, the electron is specifically in its s subshell, so no p- or d-subshell value of ℓ is applicable.

Step 2: Evaluate the orbital angular momentum Substitute the s-subshell value into the quantum mechanical expression, rather than using the Bohr expression for a circular orbit:

$$L = \sqrt{0(0+1)}\frac{h}{2\pi} = 0.$$

The vanishing result refers to orbital angular momentum. It does not imply that the electron has no spin, nor does it depend on the electron being in the first or third shell.

Step 3: Compare with the requested coefficient The question writes this same magnitude as $xh/(2\pi)$. Equating the coefficients of the nonzero constant $h/(2\pi)$ gives

$$x = \sqrt{\ell(\ell+1)} = 0.$$

This is already an exact integer, so rounding does not change it.

Step 4: Conclusion The orbital angular momentum of the $3s$ electron is zero. Thus the required dimensionless nearest-integer value is $\boxed{x = 0}$.

Quick Tip to Solve: Read the subshell letter first: every s orbital has $\ell = 0$, irrespective of its principal quantum number.

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86. Sea water contains 29.25% $NaCl$ and 19% $MgCl_2$ by weight of solution. The normal boiling point of the sea water is _____ °C (Nearest integer)

Assume 100% ionization for both $NaCl$ and $MgCl_2$

Given : $K_b\,(H_2O) = 0.52\ \text{K kg mol}^{-1}$

Molar mass of $NaCl$ and $MgCl_2$ is 58.5 and 95 g mol^{-1} respectively. [4 marks]

Solution:

Concept:

- Boiling-point elevation is $\Delta T_b = K_b \sum i_j m_j$. Each molality uses the mass of the common solvent in kilograms, not the total mass of solution.
- Complete ionization gives $NaCl \longrightarrow Na^+ + Cl^-$, with $i = 2$, and $MgCl_2 \longrightarrow Mg^{2+} + 2\,Cl^-$, with $i = 3$.

Step 1: Choose a solution mass and find the water mass Take $100\,\text{g}$ of sea water. The stated mass percentages then give $29.25\,\text{g}$ of $NaCl$ and $19.00\,\text{g}$ of $MgCl_2$. The remaining mass is the solvent:

$$m_{\text{water}} = 100 - 29.25 - 19.00 = 51.75\,\text{g} = 0.05175\,\text{kg}.$$

Subtracting both solutes is essential because the percentages refer to the solution.

Step 2: Count the dissolved particles Convert each salt mass to moles using its given molar mass:

$$n_{\mathrm{NaCl}} = \frac{29.25}{58.5} = 0.500\,\mathrm{mol}, \qquad n_{\mathrm{MgCl_2}} = \frac{19}{95} = 0.200\,\mathrm{mol}.$$

Under the stipulated complete ionization, the effective amount of solute particles is

$$n_{\mathrm{particles}} = 2(0.500) + 3(0.200) = 1.600\,\mathrm{mol}.$$

Both salts contribute to the elevation in the same water sample.

Step 3: Calculate the elevation and final temperature The total particle molality and elevation are

$$m_{\mathrm{particles}} = \frac{1.600}{0.05175} = 30.9179\,\mathrm{mol\,kg^{-1}},$$

$$\Delta T_b = 0.52(30.9179) = 16.0773\,\mathrm{K}.$$

A temperature difference of one kelvin equals a difference of one degree Celsius. At normal pressure pure water boils at $100\,^\circ\mathrm{C}$, so

$$T_b = 100 + 16.0773 = 116.0773\,^\circ\mathrm{C}.$$

Step 4: Conclusion The normal boiling point is approximately $116.08\,^\circ\mathrm{C}$; the required nearest integer is $\boxed{116}$.

Quick Tip to Solve: For several electrolytes, add $i \times n$ for all solutes and divide once by the mass of the common solvent.

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87. At 298 K, the standard reduction potential for Cu^{2+}/Cu electrode is 0.34 V.
Given : $K_{sp}Cu(OH)_2 = 1 \times 10^{-20}$
Take $\frac{2.303RT}{F} = 0.059$ V
The reduction potential at pH = 14 for the above couple is $(-)x \times 10^{-2}$ V. The value of x is _____

[4 marks]

Solution:

Concept:

- For $Cu^{2+} + 2e^- \longrightarrow Cu(s)$, the Nernst equation is $E = E^\circ + (0.059/2)\log[Cu^{2+}]$ at 298 K, with solid copper having unit activity.
- The dissolved copper concentration in equilibrium with $Cu(OH)_2$ is fixed by $K_{sp} = [Cu^{2+}][OH^-]^2$. At 298 K, pH + pOH = 14.

Step 1: Determine the hydroxide concentration The specified pH = 14 implies

$$\mathrm{pOH} = 14 - 14 = 0, \qquad [OH^-] = 10^0 = 1\,\mathrm{mol\,L^{-1}}.$$

Use the hydroxide concentration in the solubility product; the hydrogen-ion concentration is not the factor multiplying the copper concentration in that expression.

Step 2: Use the solubility product For the equilibrium $\mathrm{Cu(OH)_2(s)} \rightleftharpoons \mathrm{Cu^{2+}(aq) + 2\,OH^-(aq)}$, solve for the copper-ion concentration:

$$[\mathrm{Cu^{2+}}] = \frac{K_{\mathrm{sp}}}{[\mathrm{OH^-}]^2} = \frac{10^{-20}}{1^2} = 10^{-20}\,\mathrm{mol\,L^{-1}}.$$

The high hydroxide concentration therefore makes the dissolved copper concentration extremely small; one must not use unit copper concentration merely because a standard potential is supplied.

Step 3: Apply the Nernst equation and extract x Two electrons are transferred per copper ion, so the logarithmic coefficient is $0.059/2$:

$$E = 0.34 + \frac{0.059}{2}\log(10^{-20}) = 0.34 - 0.590 = -0.250\,\mathrm{V}.$$

The plus sign before $\log[\mathrm{Cu^{2+}}]$ follows from the inverse copper concentration in the reaction quotient. Comparing with $-x \times 10^{-2}\,\mathrm{V}$,

$$x = \frac{0.250}{0.01} = 25.$$

Step 4: Conclusion The electrode reduction potential is $-0.25\,\mathrm{V} = -25 \times 10^{-2}\,\mathrm{V}$. Hence the required dimensionless coefficient is $\boxed{x = 25}$.

Quick Tip to Solve: First use pH and the solubility product to find the free metal-ion concentration; only then substitute into the Nernst equation.

---

88. 20 mL of 0.1 M NaOH is added to 50 mL of 0.1 M acetic acid solution. The pH of the resulting solution is _____ $\times 10^{-2}$ (Nearest integer)

Given : $\mathrm{pKa\,(CH_3COOH)} = 4.76$

$$\log 2 = 0.30$$

$$\log 3 = 0.48$$

[4 marks]

Solution:

Concept:

- A strong base first neutralizes the weak acid quantitatively: $\mathrm{CH_3COOH + OH^- \longrightarrow CH_3COO^- + H_2O}$. The remaining acid and produced acetate form a buffer.
- For an acid buffer, $\mathrm{pH} = \mathrm{p}K_a + \log([\text{acetate}]/[\text{acetic acid}])$. Because both species share the final volume, their mole ratio equals their concentration ratio.

Step 1: Calculate the amounts before mixing Use molarity times volume in litres:

$$n_{\text{acid}} = 0.1(0.050) = 0.0050\,\text{mol}, \qquad n_{\text{OH}^-} = 0.1(0.020) = 0.0020\,\text{mol}.$$

The base supplies fewer moles than the acid, so it is completely consumed. This is partial neutralization rather than an excess-strong-base problem; calculating pH from leftover hydroxide would be inappropriate.

Step 2: Find the buffer composition after reaction The reaction consumes acid and produces acetate in a one-to-one ratio. Thus

$$n_{\text{acid, remaining}} = 0.0050 - 0.0020 = 0.0030\,\text{mol},$$

$$n_{\text{acetate}} = 0.0020\,\text{mol}.$$

Both quantities occupy the same final volume, $70\,\text{mL}$, which cancels in their ratio. The salt-to-acid ratio is therefore $2/3$, not $2/5$; the latter would incorrectly use the acid amount before neutralization.

Step 3: Evaluate the pH and requested scaled number Apply the buffer equation using the supplied logarithms:

$$\begin{aligned}\text{pH} &= 4.76 + \log\frac{2}{3} \\ &= 4.76 + \log 2 - \log 3 \\ &= 4.76 + 0.30 - 0.48 = 4.58.\end{aligned}$$

The blank multiplies 10^{-2}, so the numerical entry is obtained from

$$4.58 = x \times 10^{-2}, \qquad x = 458.$$

pH is dimensionless and no concentration unit is attached to this number.

Step 4: Conclusion The resulting solution has $\text{pH} = 4.58 = 458 \times 10^{-2}$. The requested nearest-integer entry is $\boxed{458}$.

Quick Tip to Solve: Finish the neutralization table before using the buffer equation: salt formed goes above acid remaining in the logarithm.

89. If the formula of Borax is $Na_2B_4O_x(OH)_y \cdot zH_2O$, then $x + y + z =$ _____ [4 marks]

Solution:

Concept:

- Ordinary borax has the conventional hydrated formula $Na_2B_4O_7 \cdot 10\,H_2O$. Its structural formulation is $Na_2[B_4O_5(OH)_4] \cdot 8\,H_2O$.
- Water written outside the borate unit is water of crystallization. Hydroxyl groups inside that unit must be counted separately from those water molecules.

Step 1: Write borax in the requested form The question explicitly separates oxide oxygen, hydroxyl

groups and external water in $Na_2B_4O_x(OH)_y \cdot zH_2O$. The borate ion in borax is $[B_4O_5(OH)_4]^{2-}$, accompanied by two sodium ions and eight water molecules. Thus the relevant full formula is

$$Na_2B_4O_5(OH)_4 \cdot 8\,H_2O.$$

The familiar decahydrate formula is an equivalent overall composition, but its seven oxide oxygens and ten waters cannot simply be copied into this hydroxyl-explicit expression.

Step 2: Check equivalence by counting atoms In the hydroxyl-explicit formula, the oxygen count is $5 + 4 + 8 = 17$, and the hydrogen count is $4 + 2(8) = 20$. In $Na_2B_4O_7 \cdot 10\,H_2O$, the corresponding totals are $7 + 10 = 17$ oxygen atoms and $2(10) = 20$ hydrogen atoms. Both forms also contain two sodium and four boron atoms. This confirms that moving hydrogen and oxygen into four hydroxyl groups has preserved the composition rather than changed the hydrate.

Step 3: Read off and add the coefficients Compare each group directly with the requested notation:

$$x = 5, \qquad y = 4, \qquad z = 8.$$

Here x counts the oxygen atoms written outside hydroxyl groups within the tetraborate unit, y counts hydroxyl groups, and z counts external water molecules. Their sum is

$$x + y + z = 5 + 4 + 8 = 17.$$

Step 4: Conclusion Using $Na_2B_4O_5(OH)_4 \cdot 8\,H_2O$, the required dimensionless sum is $\boxed{x + y + z = 17}$.

Quick Tip to Solve: Distinguish the structural hydrate notation from the conventional decahydrate formula: borax has four explicit hydroxyl groups and eight external waters.

---

90. Sodium metal crystallizes in a body centred cubic lattice with unit cell edge length of $4\ \overset{o}{A}$. The radius of sodium atom is _____ $\times 10^{-1}\ \overset{o}{A}$ (Nearest integer) [4 marks]

Solution:

Concept:

- In a body-centred cubic lattice, atoms touch along the body diagonal. The diagonal passes through a corner atom, the central atom and the opposite corner atom.
- For cubic edge length a and atomic radius r, the BCC contact relation is $\sqrt{3}a = 4r$. Edge contact $a = 2r$ belongs to a simple cubic lattice and must not be used here.

Step 1: Identify the contact direction A cube with edge length a has face diagonal $\sqrt{2}a$ and body diagonal $\sqrt{3}a$, obtained by applying Pythagoras twice. In BCC packing the central atom lies halfway along that body diagonal. The distance from one corner centre to the central atom centre is $2r$, and the distance onward to the opposite corner centre is another $2r$. Therefore the full diagonal is $4r$.

Step 2: Substitute the supplied unit-cell length Use $a = 4\,\text{Å}$ in the BCC relation:

$$r = \frac{\sqrt{3}}{4}a = \frac{\sqrt{3}}{4}(4\,\text{Å}) = \sqrt{3}\,\text{Å}.$$

Numerically,

$$r \approx 1.73205\,\text{Å}.$$

The radius has the same length unit as the supplied edge. The body-diagonal expression is larger than the face diagonal, but dividing by four yields a radius less than half the edge, consistent with BCC geometry.

Step 3: Convert to the requested scale and round The blank is the coefficient of 10^{-1} Å, so do not round the radius itself to an integer angstrom. Write

$$1.73205\,\text{Å} = 17.3205 \times 10^{-1}\,\text{Å}.$$

Rounding the coefficient 17.3205 to its nearest integer gives 17. In SI units the unrounded radius is $1.73205 \times 10^{-10}\,\text{m}$, providing a consistent unit check.

Step 4: Conclusion The atomic radius is approximately $1.732\,\text{Å}$, or $17.3205 \times 10^{-1}\,\text{Å}$. Thus the required nearest-integer entry is $\boxed{17}$.

Quick Tip to Solve: For BCC use the body diagonal, $4r = \sqrt{3}a$, then convert the radius to the stated power-of-ten unit before rounding.