JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 6 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 6 April 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $A = \{x \in R : [x+3] + [x+4] \le 3\}$,

$B = \left\{x \in R : 3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right\}$, **where [t] denotes greatest integer function. Then,** [4 marks]

(A) $B \subset C, A \neq B$

(B) $A \subset B, A \neq B$

(C) $A = B$

(D) $A \cap B = \phi$

Correct Answer: (C) $A = B$

Solution:

Concept:

- The greatest integer function satisfies $\lfloor t+1 \rfloor = \lfloor t \rfloor + 1$. An inequality involving an integer must first be reduced to a bound on that integer.
- An infinite geometric series with first term a and ratio r, where $|r| < 1$, sums to $a/(1-r)$. Powers with base greater than one preserve the direction of an inequality.

Step 1: Reduce the condition defining A Write $[x+4] = [x+3] + 1$. The inequality defining the first set becomes

$$2[x+3] + 1 \le 3 \quad \Longrightarrow \quad [x+3] \le 1.$$

Since the floor is at most 1 precisely when its argument is less than 2, this gives

$$x + 3 < 2 \quad \Longrightarrow \quad A = (-\infty, -1).$$

The endpoint is excluded: at $x = -1$, the two floors are 2 and 3, whose sum exceeds 3.

Step 2: Sum the series and reduce the condition defining B The ratio of consecutive terms is $1/10$, so

$$\sum_{r=1}^{\infty} \frac{3}{10^r} = \frac{3/10}{1 - 1/10} = \frac{1}{3}.$$

Consequently the expression on the left of the inequality defining B is constant:

$$3^x (1/3)^{x-3} = 3^x 3^{3-x} = 3^3.$$

As $3 > 1$, the inequality $3^3 < 3^{-3x}$ is equivalent to $3 < -3x$, or $x < -1$. Thus $B = (-\infty, -1)$ as well. The unequal-set alternatives are excluded because the intervals coincide, and the empty-intersection alternative fails because, for example, -2 belongs to both sets.

Step 3: Conclusion Both sets are $(-\infty, -1)$, so $A = B$. Hence, the correct option is (C).

Quick Tip to Solve: Use the integer shift inside the floor before solving its inequality; for the second set, combine the powers only after summing the geometric series.

2. If the system of equations

$$x + y + az = b$$

$$2x + 5y + 2z = 6$$

$$x + 2y + 3z = 3$$

has infinitely many solutions, then $2a + 3b$ is equal to : [4 marks]

(A) 28

(B) 25

(C) 20

(D) 23

Correct Answer: (D) 23

Solution:

Concept:

- Infinitely many solutions require a consistent system with at least one free variable. After elimination, a dependent equation must reduce to an identity, rather than determine that variable.
- Elementary row operations preserve the solution set. A relation between the coefficient rows must also hold between the corresponding right-hand sides.

Step 1: Eliminate x using the fixed equations Subtract twice the third equation from the second equation:

$$(2x + 5y + 2z) - 2(x + 2y + 3z) = 6 - 6.$$

This simplifies to $y - 4z = 0$, giving $y = 4z$. Substitution in the third equation then gives

$$x + 2(4z) + 3z = 3 \quad \Longrightarrow \quad x = 3 - 11z.$$

Thus the second and third equations describe a line of candidate solutions, parametrised by the real variable z. Their coefficient rows are independent, so this line has exactly one free parameter.

Step 2: Make the remaining equation an identity Insert $x = 3 - 11z$ and $y = 4z$ into the first equation:

$$3 - 11z + 4z + az = b \quad \Longrightarrow \quad (a - 7)z = b - 3.$$

If $a \neq 7$, this equation fixes one value of z, so the solution would be unique. If $a = 7$ but $b \neq 3$, it is inconsistent. Therefore infinitely many solutions occur precisely when

$$a = 7, \qquad b = 3.$$

For these values every real z satisfies all three equations, confirming sufficiency. Evaluate the requested expression:

$$2a + 3b = 2(7) + 3(3) = 14 + 9 = 23.$$

The other listed numbers 28, 25 and 20 do not equal this value and hence are excluded.

Step 3: Conclusion $2a + 3b = 23$. Hence, the correct option is (D).

Quick Tip to Solve: Use the equations without unknown parameters to express the variables in terms of one parameter, then require the last equation to hold identically.

3. The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line L : $9x + 5y = 45$ between the axes. If m_1 and m_2 are the slopes of the lines l_1 and l_2, then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on : [4 marks]

(A) $6x - y = 15$

(B) $6x + y = 10$

(C) $y - x = 5$

(D) $y - 2x = 5$

Correct Answer: (C) $y - x = 5$

Solution:

Concept:

- The intercepts of a line on the coordinate axes are obtained by setting the other coordinate to zero. Its trisection points divide the intercept segment internally in the ratios $1 : 2$ and $2 : 1$.
- The slope of a line through the origin and a point (u, v), with $u \neq 0$, is v/u. A point lies on a proposed line only if its coordinates satisfy that equation.

Step 1: Find the trisection points and slopes The intercepts of $9x + 5y = 45$ are $X = (5, 0)$ and $Y = (0, 9)$. Taking one-third and two-thirds of the displacement from X towards Y gives

$$T_1 = (10/3, 3), \qquad T_2 = (5/3, 6).$$

The lines from the origin through these points therefore have slopes

$$m_1 = \frac{3}{10/3} = \frac{9}{10}, \qquad m_2 = \frac{6}{5/3} = \frac{18}{5}.$$

The order in which the two trisection points are named does not affect their sum:

$$m_1 + m_2 = \frac{9}{10} + \frac{36}{10} = \frac{9}{2}.$$

Step 2: Locate the required intersection and test the choices The new line is $y = 9x/2$. Substitute this into the original line to obtain

$$9x + 5(9x/2) = 45 \quad \Longrightarrow \quad \frac{63x}{2} = 45.$$

Hence the intersection is

$$(x, y) = \left(\frac{10}{7}, \frac{45}{7}\right).$$

At this point, $y - x = (45 - 10)/7 = 5$, which verifies option (C). For the other options, the left-hand sides are

$$6x - y = 15/7, \quad 6x + y = 15, \quad y - 2x = 25/7.$$

These are respectively unequal to 15, 10 and 5, excluding (A), (B) and (D).

Step 3: Conclusion The required intersection lies on $y - x = 5$. Hence, the correct option is (C).

Quick Tip to Solve: Parametrise the intercept segment as $(5, 0) + t(-5, 9)$; setting $t = 1/3, 2/3$ gives both trisection points without memorising a ratio formula.

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4. One vertex of a rectangular parallelopiped is at the origin O and the lengths of its edges along x, y and z axes are $3, 4$ and 5 units respectively. Let P be the vertex $(3, 4, 5)$. Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is : [4 marks]

(A) $\frac{12}{\sqrt{5}}$

(B) $12\sqrt{5}$

(C) $\frac{12}{5}$

(D) $\frac{12}{5\sqrt{5}}$

Correct Answer: (C) $\frac{12}{5}$

Solution:

Concept:

- For lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$ with nonparallel directions $\mathbf{d}_1, \mathbf{d}_2$, the shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)|/|\mathbf{d}_1 \times \mathbf{d}_2|$.
- The common perpendicular is normal to both directions. For finite segments, the closest points on the supporting lines must also lie within the segments.

Step 1: Choose a permitted vertical edge The diagonal has parametrisation $(3t, 4t, 5t)$, $0 \le t \le 1$, and direction $\mathbf{d}_1 = (3, 4, 5)$. The two vertical edges not through O or P have fixed horizontal coordinates $(3, 0)$ and $(0, 4)$. Choose the edge through $Q = (3, 0, 0)$, whose direction is $\mathbf{d}_2 = (0, 0, 1)$. Then

$$\mathbf{d}_1 \times \mathbf{d}_2 = (4, -3, 0), \qquad |\mathbf{d}_1 \times \mathbf{d}_2| = 5.$$

The connecting vector from the origin to Q is $(3, 0, 0)$, so its scalar product with this normal is 12.

Step 2: Compute the distance and check the finite edges The skew-line formula gives

$$d = \frac{|(3, 0, 0) \cdot (4, -3, 0)|}{5} = \frac{12}{5}.$$

For the other permitted edge, use $Q = (0, 4, 0)$; the scalar product becomes -12, giving the same distance. To check the segments, minimise the squared horizontal separation from $(3, 0)$:

$$(3t - 3)^2 + (4t)^2 = 25t^2 - 18t + 9.$$

Its minimum is at $t = 9/25$, within $[0, 1]$, and the matching vertical coordinate is $5t = 9/5$, within the

edge. For $(0, 4)$, the minimum occurs at $t = 16/25$, also admissible. Thus the line distance is attained by the actual segments. Numerically it is 2.4, distinct from all three other listed expressions.

Step 3: Conclusion The shortest distance is $12/5$ units. Hence, the correct option is (C).

Quick Tip to Solve: For an edge parallel to the vertical axis, first minimise the horizontal separation; then check that the corresponding heights lie on both finite segments.

5. The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13 , then σ^2 is equal to : [4 marks]

(A) 12

(B) 11

(C) 10

(D) 9

Correct Answer: (C) 10

Solution:

Concept:

- For a population with mean μ and variance v, the mean of the squared observations is $E(X^2) = v + \mu^2$.
- For equal-sized groups, the combined mean and the combined second moment are arithmetic averages of the two group values. Variance is the second moment minus the square of the mean.

Step 1: Find the combined mean and second moment Both groups contain 15 numbers, so each contributes equal weight to the combined population. The combined mean is

$$\mu = \frac{15(12) + 15(14)}{30} = 13.$$

For the first group, the average square is $14+12^2 = 158$. For the second group it is $\sigma^2+14^2 = \sigma^2+196$. Therefore the average square across all 30 observations is

$$E(X^2) = \frac{158 + \sigma^2 + 196}{2} = 177 + \frac{\sigma^2}{2}.$$

These are population variances of the given sets, so no sample correction or denominator of 14 is appropriate.

Step 2: Use the specified combined variance The total variance is 13, hence

$$13 = 177 + \frac{\sigma^2}{2} - 13^2 = 8 + \frac{\sigma^2}{2}.$$

Solving yields $\sigma^2/2 = 5$, or $\sigma^2 = 10$. As a separate check, the combined variance is the average

within-group variance plus the variance of the group means:

$$\frac{14+10}{2}+\frac{(12-13)^2+(14-13)^2}{2}=12+1=13.$$

This confirms the required value. Substituting the other choices 12, 11 and 9 instead would produce combined variances 14, 13.5 and 12.5 respectively, so none satisfies the data.

Step 3: Conclusion The variance of the second group is $\sigma^2=10$. Hence, the correct option is (C).

Quick Tip to Solve: When the group sizes are equal, add the average of their variances to one-quarter of the squared difference between their means.

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6. Let $\mathrm{A}=[\mathrm{a_{ij}}]_{2\times2}$, where $\mathrm{a_{ij}}\neq0$ for all $\mathrm{i,j}$ and $\mathrm{A}^2=\mathrm{I}$. Let a be the sum of all diagonal elements of A and $\mathrm{b}=|\mathrm{A}|$. Then $3a^2+4b^2$ is equal to : [4 marks]

(A) 4

(B) 3

(C) 14

(D) 7

Correct Answer: (A) 4

Solution:

Concept:

- An involutory matrix satisfies $A^2=I$. Equating entries in this product gives relations among the entries of A.
- For a two-by-two matrix $\begin{pmatrix}p & q\\ r & s\end{pmatrix}$, its trace is $p+s$, and its determinant is $ps-qr$. Nonzero off-diagonal entries impose useful restrictions on the trace.

Step 1: Use the off-diagonal entries of the square Write

$$A=\begin{pmatrix}p & q\\ r & s\end{pmatrix},\qquad p,q,r,s\neq0.$$

Multiplication gives

$$A^2=\begin{pmatrix}p^2+qr & q(p+s)\\ r(p+s) & s^2+rq\end{pmatrix}.$$

Since this is the identity matrix, both off-diagonal entries are zero. Because $q\neq0$, the equation $q(p+s)=0$ forces $p+s=0$. Thus the sum of the diagonal elements is $a=0$, and $s=-p$. The nonzero-entry condition rules out the exceptional scalar matrices I and $-I$, whose traces would not be zero.

Step 2: Compute the determinant and requested expression The upper-left entry of $A^2=I$ also gives

$p^2 + qr = 1$. Therefore

$$b = |A| = ps - qr = -p^2 - qr = -(p^2 + qr) = -1.$$

It follows that

$$3a^2 + 4b^2 = 3(0)^2 + 4(-1)^2 = 4.$$

The determinant check $|A|^2 = |A^2| = |I| = 1$ agrees with this result; the entry relations fix its sign as negative. The other proposed values 3, 14 and 7 cannot arise because the trace term vanishes and the determinant-squared term is exactly 4. No particular numerical matrix needs to be chosen, since the conditions establish the answer for every admissible matrix.

Step 3: Conclusion $3a^2 + 4b^2 = 4$. Hence, the correct option is (A).

Quick Tip to Solve: For a two-by-two involution with nonzero off-diagonal entries, read the trace from the off-diagonal entries of its square before using determinants.

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7. Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$. Then $18\int_1^2 f(x)\,dx$ is equal to: [4 marks]

(A) $10\log_e 2 + 6$

(B) $5\log_e 2 - 3$

(C) $10\log_e 2 - 6$

(D) $5\log_e 2 + 3$

Correct Answer: (C) $10\log_e 2 - 6$

Solution:

Concept:

- If a functional equation relates $f(x)$ and $f(1/x)$ for $x > 0$, replacing x by $1/x$ provides a second valid linear equation in the same two unknown function values.
- Once the function is isolated, integrate term by term. On positive arguments, an antiderivative of $1/x$ is $\log_e x$, while an antiderivative of x is $x^2/2$.

Step 1: Isolate $f(x)$ by elimination The given equation and its version with reciprocal argument are

$$5f(x) + 4f(1/x) = 1/x + 3,$$

$$4f(x) + 5f(1/x) = x + 3.$$

Multiply the first by 5 and the second by 4, then subtract to cancel the reciprocal value. This gives

$$9f(x) = 5/x + 15 - 4x - 12 = 5/x - 4x + 3.$$

Thus

$$f(x) = \frac{1}{9}\left(\frac{5}{x} - 4x + 3\right), \qquad x > 0.$$

The elimination is legitimate because the coefficient determinant is $25 - 16 = 9 \neq 0$; the function

value is uniquely determined at each positive argument.

Step 2: Integrate on the specified positive interval Every term is continuous on $[1, 2]$, so the definite integral is evaluated directly:

$$18\int_1^2 f(x)\,dx = 2\int_1^2 \left(\frac{5}{x} - 4x + 3\right) dx.$$

Using an antiderivative yields

$$2\left[5\log_e x - 2x^2 + 3x\right]_1^2 = 2\left(5\log_e 2 - 2 - 1\right) = 10\log_e 2 - 6.$$

Here the upper endpoint contributes $5\log_e 2 - 2$, and the lower contributes 1. Options (A) and (D) have incorrect constant terms; (B) is only half the required expression, because it misses the factor 2 remaining after eliminating the function.

Step 3: Conclusion $18\int_1^2 f(x)\,dx = 10\log_e 2 - 6$. Hence, the correct option is (C).

Quick Tip to Solve: Create the reciprocal-argument equation first; solve the two linear equations before integrating to avoid carrying two unknown integrals.

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8. Let $I(x) = \int \frac{x^2\left(x\sec^2 x + \tan x\right)}{(x\tan x + 1)^2}dx$. If $I(0) = 0$, then $I\left(\frac{\pi}{4}\right)$ is equal to : [4 marks]

(A) $\log_e \frac{(\pi+4)^2}{32} - \frac{\pi^2}{4(\pi+4)}$

(B) $\log_e \frac{(\pi+4)^2}{16} - \frac{\pi^2}{4(\pi+4)}$

(C) $\log_e \frac{(\pi+4)^2}{16} + \frac{\pi^2}{4(\pi+4)}$

(D) $\log_e \frac{(\pi+4)^2}{32} + \frac{\pi^2}{4(\pi+4)}$

Correct Answer: (A) $\log_e \frac{(\pi+4)^2}{32} - \frac{\pi^2}{4(\pi+4)}$

Solution:

Concept:

- If $u = x\tan x + 1$, then $u\prime = x\sec^2 x + \tan x$. Thus the fraction containing $u\prime/u^2$ suggests differentiating $-1/u$, rather than expanding the trigonometric functions.
- Integration by parts gives $\int x^2 u\prime/u^2\,dx = -x^2/u + \int 2x/u\,dx$. A logarithmic derivative can evaluate the remaining integral.

Step 1: Recognise the derivative in the numerator Write $u = x\tan x + 1$. Since $d(-1/u)/dx = u\prime/u^2$, integration by parts, with the other factor x^2, gives

$$I(x) = -\frac{x^2}{x\tan x + 1} + 2\int \frac{x}{x\tan x + 1}\,dx.$$

The negative sign in the rational term follows from the derivative of a reciprocal; it must not be replaced by a positive sign.

Step 2: Evaluate the remaining logarithmic integral Multiplying numerator and denominator by

$\cos x$, we obtain

$$\frac{x}{x\tan x+1}=\frac{x\cos x}{x\sin x+\cos x}.$$

The derivative of $x\sin x+\cos x$ is $x\cos x$, because the two $\sin x$ terms cancel. Therefore, on the interval from 0 to $\pi/4$, where the logarithm argument is positive,

$$I(x)=-\frac{x^2}{x\tan x+1}+2\ln(x\sin x+\cos x)+C.$$

At $x=0$, both displayed terms vanish. The condition $I(0)=0$ consequently gives $C=0$.

Step 3: Substitute the upper argument and compare options At $x=\pi/4$, the rational term is $-\pi^2/[4(\pi+4)]$, and

$$x\sin x+\cos x=\frac{\pi+4}{4\sqrt{2}}.$$

Hence the logarithmic term becomes $\ln((\pi+4)^2/32)$. The denominator 32 excludes (B) and (C); the negative rational term excludes (D).

Step 4: Conclusion $I(\pi/4)=\ln\frac{(\pi+4)^2}{32}-\frac{\pi^2}{4(\pi+4)}$. Hence, the correct option is (A).

Quick Tip to Solve: For a numerator containing the derivative of a squared denominator base, integrate its reciprocal first; the leftover integral may be a logarithmic derivative.

9. Let $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k},\vec{b}=\hat{i}-2\hat{j}-2\hat{k}$ and $\vec{c}=-\hat{i}+4\hat{j}+3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a}\cdot\vec{d}=18$, then $|\vec{a}\times\vec{d}|^2$ is equal to : [4 marks]

(A) 680

(B) 720

(C) 760

(D) 640

Correct Answer: (B) 720

Solution:

Concept:

- A vector perpendicular to two nonparallel vectors is a scalar multiple of their cross product. Write $\vec{d}=\lambda(\vec{b}\times\vec{c})$, where the scalar λ is fixed by the given dot product.
- The identity $|\vec{a}\times\vec{d}|^2=|\vec{a}|^2|\vec{d}|^2-(\vec{a}\cdot\vec{d})^2$ avoids another lengthy determinant once $\vec{d}$ is known.

Step 1: Find the common perpendicular direction For $\vec{b}=(1,-2,-2)$ and $\vec{c}=(-1,4,3)$, direct component expansion gives

$$\vec{b}\times\vec{c}=(2,-1,2).$$

For example, the first component is $(-2)(3)-(-2)(4)=2$, the second is $(-2)(-1)-(1)(3)=-1$, and the third is $(1)(4)-(-2)(-1)=2$. Its dot product with each of $\vec{b}$ and $\vec{c}$ is zero, confirming the required direction. Therefore $\vec{d}=\lambda(2,-1,2)$.

Step 2: Use the prescribed scalar product The vector $\vec{a} = (2, 3, 4)$ gives

$$\vec{a} \cdot \vec{d} = \lambda(4 - 3 + 8) = 9\lambda = 18.$$

Thus $\lambda = 2$ and $\vec{d} = (4, -2, 4)$. A reversed cross-product direction would only reverse the intermediate scalar and give the same final $\vec{d}$. The relevant squared lengths are

$$|\vec{a}|^2 = 4 + 9 + 16 = 29, \qquad |\vec{d}|^2 = 16 + 4 + 16 = 36.$$

Step 3: Calculate the squared cross-product magnitude Substitute the squared lengths and the supplied scalar product into the identity:

$$|\vec{a} \times \vec{d}|^2 = 29 \cdot 36 - 18^2 = 1044 - 324 = 720.$$

As a direct arithmetic check, $\vec{a} \times \vec{d} = (20, 8, -16)$, whose squared length is $400 + 64 + 256 = 720$. This excludes 680, 760 and 640 in the other three options.

Step 4: Conclusion The required squared magnitude is $\boxed{720}$. Hence, the correct option is (B).

Quick Tip to Solve: After finding a common perpendicular through a cross product, fix its scale with the given dot product and use the squared-magnitude identity as an independent check.

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10. Let $a_1, a_2, a_3, \ldots, a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then

$\lim_{n\to\infty} \sqrt{\frac{d}{n}} \left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots\ldots\ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} \right)$ **is** [4 marks]

(A) $\frac{1}{\sqrt{d}}$

(B) 1

(C) 0

(D) $\sqrt{d}$

Correct Answer: (B) 1

Solution:

Concept:

- Consecutive terms of the arithmetic progression satisfy $a_{k+1} - a_k = d$, with fixed $d > 0$. Positivity permits rationalising each sum of square roots.
- The identity $1/(\sqrt{u} + \sqrt{v}) = (\sqrt{v} - \sqrt{u})/(v - u)$ turns the entire sum into a telescoping sum; only its first and last square roots survive.

Step 1: Rationalise a typical summand For every $k = 1, \ldots, n-1$, multiply numerator and denominator by $\sqrt{a_{k+1}} - \sqrt{a_k}$. Because the common difference is nonzero,

$$\frac{1}{\sqrt{a_k} + \sqrt{a_{k+1}}} = \frac{\sqrt{a_{k+1}} - \sqrt{a_k}}{d}.$$

This is an exact identity, not an approximation for large k. It is therefore preferable to estimating individual square roots before summing them.

Step 2: Cancel the intermediate terms Let S_n denote the sum inside parentheses. Substituting the rationalised expressions gives

$$S_n = \frac{(\sqrt{a_2} - \sqrt{a_1}) + \cdots + (\sqrt{a_n} - \sqrt{a_{n-1}})}{d} = \frac{\sqrt{a_n} - \sqrt{a_1}}{d}.$$

All interior square roots cancel once with each sign. Since $a_n = a_1 + (n-1)d$, the expression whose limit is required becomes

$$\sqrt{\frac{d}{n}}\, S_n = \sqrt{\frac{a_1 + (n-1)d}{dn}} - \sqrt{\frac{a_1}{dn}}.$$

Step 3: Take the limit and check dependence on the common difference The argument of the first square root is $1 + (a_1 - d)/(dn)$, which approaches 1. The second square root approaches zero, because a_1 and d are fixed positive constants. Thus the limit is $1 - 0 = 1$. In particular the factor d cancels in the limit, so the answers $1/\sqrt{d}$ and $\sqrt{d}$ cannot be the general result; the surviving first term also rules out zero.

Step 4: Conclusion The stated limit equals $\boxed{1}$. Hence, the correct option is (B).

Quick Tip to Solve: Rationalise reciprocal sums of consecutive square roots before taking a limit: a constant difference in the radicands usually makes the sum telescope.

11. The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is : [4 marks]

(A) $11 + \sqrt{3}$

(B) $9 + \sqrt{3}$

(C) $9 - \sqrt{3}$

(D) $11 - \sqrt{3}$

Correct Answer: (B) $9 + \sqrt{3}$

Solution:

Concept:

- The quadratic inside the absolute value factors as $x^2 - 8x + 15 = (x-3)(x-5)$. Its sign changes at 3 and 5, so the equation must be solved separately outside and inside that interval.
- An absolute value is nonnegative. Rewriting the equation as $|x^2 - 8x + 15| = 2x - 7$ first imposes $x \geq 7/2$, which is a useful check on candidate roots.

Step 1: Solve where the quadratic is nonnegative For $x \leq 3$ or $x \geq 5$, remove the absolute value with a positive sign:

$$x^2 - 8x + 15 - 2x + 7 = 0 \quad \Longrightarrow \quad x^2 - 10x + 22 = 0.$$

The quadratic formula yields $x = 5 \pm \sqrt{3}$. The root $5 + \sqrt{3}$ belongs to $x \geq 5$, so it is admissible. However, $5 - \sqrt{3}$ lies strictly between 3 and 5 and does not belong to this sign case. It must be rejected, even though it solves the auxiliary quadratic.

Step 2: Solve where the quadratic is nonpositive For $3 \le x \le 5$, the absolute value is the negative of the quadratic. Hence

$$-x^2 + 8x - 15 - 2x + 7 = 0 \quad \Longrightarrow \quad x^2 - 6x + 8 = 0.$$

Factoring gives $(x-2)(x-4) = 0$, so the candidates are 2 and 4. Only 4 lies in the present interval; 2 also violates the earlier condition $x \ge 7/2$. The interval endpoints 3 and 5 do not satisfy the original equation, so they contribute no additional roots.

Step 3: Add only the admissible roots The complete set of real solutions is $\{4, 5+\sqrt{3}\}$. Direct substitution of 4 gives $|16 - 32 + 15| - 8 + 7 = 1 - 1 = 0$, verifying the interior root. Their sum is

$$4 + (5 + \sqrt{3}) = 9 + \sqrt{3}.$$

The expressions with 11 include an inadmissible candidate, while changing the sign of the radical selects the rejected outer-case candidate. Thus none of the other listed sums is valid.

Step 4: Conclusion The sum of all the roots is $\boxed{9+\sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: When solving an absolute-value quadratic equation, write the interval beside each auxiliary quadratic and reject every candidate outside its own interval before adding roots.

12. If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6} : 1$, then the third term from the beginning is : [4 marks]

(A) $30\sqrt{2}$

(B) $60\sqrt{3}$

(C) $60\sqrt{2}$

(D) $30\sqrt{3}$

Correct Answer: (B) $60\sqrt{3}$

Solution:

Concept:

- For $(a+b)^n$, the term numbered $r+1$ from the beginning is $\binom{n}{r}a^{n-r}b^r$. Here $a = 2^{1/4}$ and $b = 3^{-1/4}$, both positive.
- The fifth term from the end has four powers of a, namely $\binom{n}{4}a^4b^{n-4}$. Its binomial coefficient equals that of the fifth term from the beginning, so the coefficients cancel in their ratio.

Step 1: Express the stated ratio using the general term The fifth terms from the beginning and end are respectively

$$T_5 = \binom{n}{4}a^{n-4}b^4, \qquad T_{n-3} = \binom{n}{4}a^4b^{n-4}.$$

Consequently,

$$\frac{T_5}{T_{n-3}} = \left(\frac{a}{b}\right)^{n-8}.$$

The exponent is $n-8$, because four powers are removed from each end. Counting the fifth term as exponent five instead of four would give an incorrect value of n.

Step 2: Determine the exponent of the expansion Since $a/b = 2^{1/4}3^{1/4} = 6^{1/4}$, the given ratio $\sqrt{6}:1$ implies

$$6^{(n-8)/4} = 6^{1/2}.$$

The base 6 is positive and different from 1, so equality requires $(n-8)/4 = 1/2$. Therefore $n = 10$. This also confirms that both fifth terms exist. No approximation to the fourth roots is needed.

Step 3: Evaluate the third term and compare the choices The third term corresponds to $r = 2$. With $n = 10$,

$$T_3 = \binom{10}{2} a^8 b^2 = 45 \cdot 2^2 \cdot 3^{-1/2} = \frac{180}{\sqrt{3}} = 60\sqrt{3}.$$

The radical is $\sqrt{3}$, rather than $\sqrt{2}$, because the remaining denominator is $\sqrt{3}$. Rationalising gives coefficient 60, excluding the choice $30\sqrt{3}$ as well as both choices involving $\sqrt{2}$.

Step 4: Conclusion The third term from the beginning is $\boxed{60\sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equally numbered terms from opposite ends of a binomial expansion, cancel the equal binomial coefficients first and compare just the powers of the ratio of the two bases.

13. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at $(2,2)$ is equal to : [4 marks]

(A) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$

(B) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

(C) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$

(D) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

Correct Answer: (B) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

Solution:

Concept:

- For positive x and y, logarithmic differentiation gives $d(x^y)/dx = x^y(y' \ln x + y/x)$. Both the base and the exponent depend on x.
- Similarly, $d(y^x)/dx = y^x(\ln y + xy'/y)$. Treating either exponent as constant would omit a necessary term and change the slope.

Step 1: Differentiate the implicit equation correctly The point $(2,2)$ satisfies the equation because $2(2^2) + 3(2^2) = 8 + 12 = 20$. In a positive neighbourhood of this point, differentiating $2x^y + 3y^x = 20$ gives

$$2x^y\left(y' \ln x + \frac{y}{x}\right) + 3y^x\left(\ln y + \frac{x}{y}y'\right) = 0.$$

The logarithms are defined here because both coordinates are positive. The constant right-hand side differentiates to zero.

Step 2: Insert the coordinates and collect slope terms At $x = y = 2$, both powers x^y and y^x equal 4, and both ratios y/x and x/y equal 1. Thus

$$8(y' \ln 2 + 1) + 12(\ln 2 + y') = 0.$$

Collecting the terms involving y' on one side yields

$$(8 \ln 2 + 12)y' = -(8 + 12 \ln 2).$$

The coefficient $8 \ln 2 + 12$ is strictly positive, so division is valid and the slope is uniquely determined.

Step 3: Simplify into the printed logarithmic form Dividing numerator and denominator by 4 gives

$$y' = -\frac{2 + 3 \ln 2}{3 + 2 \ln 2} = -\frac{2 + \ln 8}{3 + \ln 4}.$$

This is the expression in (B). In particular, the numerator contains the contribution 2 from differentiating the first base and $\ln 8$ from the second exponent. Interchanging these contributions or using $\ln 16$ produces the other listed ratios, which do not satisfy the collected derivative equation.

Step 4: Conclusion At $(2, 2)$, $\frac{dy}{dx} = -\frac{2 + \ln 8}{3 + \ln 4}$. Hence, the correct option is (B).

Quick Tip to Solve: For a variable raised to a variable, rewrite mentally as an exponential of a product with a logarithm; differentiate both factors before substituting the point.

14. The sum of the first 20 terms of the series $5 + 11 + 19 + 29 + 41 + \ldots$ is : [4 marks]

(A) 3420

(B) 3450

(C) 3250

(D) 3520

Correct Answer: (D) 3520

Solution:

Concept:

- The successive differences of the sequence are $6, 8, 10, 12, \ldots$, which form an arithmetic progression. A sequence with constant second difference has a quadratic general term.
- The standard finite sums are $\sum_{n=1}^{N} n = N(N+1)/2$ and $\sum_{n=1}^{N} n^2 = N(N+1)(2N+1)/6$. Apply them after identifying the actual term formula.

Step 1: Find a formula for the individual terms Let t_n denote the n-th term, with $t_1 = 5$. The observed difference is

$$t_{k+1} - t_k = 2k + 4 \qquad (k \geq 1).$$

Summing these differences from $k = 1$ to $n - 1$ gives

$$t_n = 5 + \sum_{k=1}^{n-1}(2k + 4) = 5 + n(n - 1) + 4(n - 1) = n^2 + 3n + 1.$$

Checking $n = 1, 2, 3$ returns 5, 11 and 19, confirming the indexing. The original terms themselves are not an arithmetic progression, so the constant-difference sum formula cannot be applied directly to them.

Step 2: Separate the required total into standard sums Using the general term, the sum of the first twenty terms is

$$S_{20} = \sum_{n=1}^{20}(n^2 + 3n + 1) = \sum_{n=1}^{20} n^2 + 3\sum_{n=1}^{20} n + 20.$$

Now evaluate the two nonconstant sums:

$$\sum_{n=1}^{20} n^2 = \frac{20 \cdot 21 \cdot 41}{6} = 2870, \qquad \sum_{n=1}^{20} n = \frac{20 \cdot 21}{2} = 210.$$

The constant 1 contributes twenty copies, not one.

Step 3: Combine and check the listed totals Therefore,

$$S_{20} = 2870 + 3(210) + 20 = 2870 + 630 + 20 = 3520.$$

As an additional term check, $t_{20} = 400 + 60 + 1 = 461$, consistent with the increasing even first differences. The three other values 3420, 3450 and 3250 differ from the exact standard-sum calculation, so they cannot represent the requested first twenty terms.

Step 4: Conclusion The sum of the first 20 terms is $\boxed{3520}$. Hence, the correct option is (D).

Quick Tip to Solve: When first differences form an arithmetic progression, derive the quadratic term by summing those differences; then sum powers of the index instead of guessing a total.

15. A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If the probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to : [4 marks]

(A) 82

(B) 164

(C) 123

(D) 75

Correct Answer: (C) 123

Solution:

Concept:

- A total of 5 on two fair dice has four ordered outcomes among 36 equally likely outcomes. Thus the success probability in one throw is $p = 1/9$, and the failure probability is $1 - p = 8/9$.

- For five independent repetitions, the number X of successes is binomial: $P(X=r)=\binom{5}{r}p^r(1-p)^{5-r}$. At least four means exactly four or exactly five, two disjoint events.

Step 1: Count the favourable outcomes The ordered pairs giving a sum of 5 are $(1,4),(2,3),(3,2),(4,1)$. The order matters because the dice are distinguishable. Therefore

$$p=\frac{4}{36}=\frac{1}{9}.$$

A sum of 5 is not one of eleven equally likely totals: the totals themselves have different numbers of underlying outcomes.

Step 2: Add the two required binomial terms For four successes there are five choices for the single unsuccessful throw; for five successes there is just one pattern. Hence

$$\begin{aligned}P(X\geq 4)&=\binom{5}{4}\left(\frac{1}{9}\right)^4\frac{8}{9}+\left(\frac{1}{9}\right)^5\\&=\frac{40+1}{9^5}=\frac{41}{3^{10}}=\frac{123}{3^{11}}.\end{aligned}$$

Comparing numerators with the specified denominator gives a unique value. The alternatives 82, 164 and 75 would give different probabilities and do not equal this binomial sum.

Step 3: Conclusion The required numerator is $\boxed{k=123}$. Hence, the correct option is (C).

Quick Tip to Solve: First count ordered dice outcomes; then interpret "at least four" as the sum of the four-success and five-success terms.

16. If the equation of the plane passing through the line of intersection of the planes $2x-y+z=3, 4x-3y+5z+9=0$ and parallel to the line $\frac{x+1}{-2}=\frac{y+3}{4}=\frac{z-2}{5}$ is $ax+by+cz+6=0$, then $a+b+c$ is equal to : [4 marks]

(A) 13

(B) 15

(C) 14

(D) 12

Correct Answer: (C) 14

Solution:

Concept:

- A plane through the intersection of $F_1=0$ and $F_2=0$ can be written as $F_1+tF_2=0$. Here $F_1=2x-y+z-3$ and $F_2=4x-3y+5z+9$.
- A plane is parallel to a line when its normal vector is perpendicular to the direction vector of that line. The given symmetric equation supplies the direction vector $(-2,4,5)$.

Step 1: Impose the parallel condition The normal of the plane in the family is $(2+4t,-1-3t,1+5t)$.

Its scalar product with the given line direction must vanish:

$$-2(2+4t)+4(-1-3t)+5(1+5t)=0,$$
$$-3+5t=0.$$

Therefore $t=3/5$. The coefficient of F_1 is nonzero here; the separate member $F_2=0$ is not parallel to the line, since its normal has scalar product 5 with the direction.

Step 2: Normalize the constant term Substitute the parameter and clear the denominator:

$$5F_1+3F_2=0,$$
$$22x-14y+20z+12=0,$$
$$11x-7y+10z+6=0.$$

The last division is essential because the requested equation has constant term 6. Thus $a=11$, $b=-7$, and $c=10$. Their sum is 14, excluding the alternatives 13, 15 and 12. As a check, $-22-28+50=0$, so the resulting plane really is parallel to the line.

Step 3: Conclusion $\boxed{a+b+c=14}$. Hence, the correct option is (C).

Quick Tip to Solve: After finding a plane from the intersection family, scale its entire equation to the requested constant before reading its coefficients.

17. Let the position vectors of the points A, B, C and D be $5\hat{i}+5\hat{j}+2\lambda\hat{k}, \hat{i}+2\hat{j}+3\hat{k}, -2\hat{i}+\lambda\hat{j}+4\hat{k}$ and $-\hat{i}+5\hat{j}+6\hat{k}$. Let the set $S=\{\lambda\in\mathbb{R}$: the points A, B, C and D are coplanar $\}$.

Then $\sum_{\lambda\in S}(\lambda+2)^2$ is equal to : [4 marks]

(A) $\frac{37}{2}$

(B) 25

(C) 13

(D) 41

Correct Answer: (D) 41

Solution:

Concept:

- Four points are coplanar precisely when three vectors joining one of them to the others have zero scalar triple product. Using B as the common origin keeps the calculation small.
- If the scalar triple product factors into a polynomial in λ, every real root contributes once to the set S. The required expression is the sum over those distinct values.

Step 1: Form the three displacement vectors Subtract the position vector of B from those of A, C and

D:

$$\overrightarrow{BA} = (4, 3, 2\lambda - 3),$$
$$\overrightarrow{BC} = (-3, \lambda - 2, 1),$$
$$\overrightarrow{BD} = (-2, 3, 3).$$

Compute the cross product of the last two vectors:

$$\overrightarrow{BC} \times \overrightarrow{BD} = (3\lambda - 9, 7, 2\lambda - 13).$$

These are displacement vectors, so the common origin of the given position vectors does not need to lie in the plane.

Step 2: Enforce coplanarity and sum the values Taking the scalar product with $\overrightarrow{BA}$ gives

$$\begin{aligned} 0 &= 4(3\lambda - 9) + 3(7) + (2\lambda - 3)(2\lambda - 13) \\ &= 4\lambda^2 - 20\lambda + 24 \\ &= 4(\lambda - 2)(\lambda - 3). \end{aligned}$$

Both roots are real, so $S = \{2, 3\}$. Consequently

$$\sum_{\lambda \in S} (\lambda + 2)^2 = (2 + 2)^2 + (3 + 2)^2 = 16 + 25 = 41.$$

The value 25 counts only the second root, while 13 and $37/2$ do not equal the sum over the full set.

Step 3: Conclusion The required sum is $\boxed{41}$. Hence, the correct option is (D).

Quick Tip to Solve: For four-point coplanarity, choose one point as the common base of all three vectors; do not take the triple product of three unshifted position vectors.

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18. From the top A of a vertical wall AB of height 30 m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60° respectively, B and Q are on the same horizontal level. If C is a point on AB such that CB = PQ, then the area (in m^2) of the quadrilateral BCPQ is equal to : [4 marks]

(A) $200(3 - \sqrt{3})$

(B) $300(\sqrt{3} - 1)$

(C) $300(\sqrt{3} + 1)$

(D) $600(\sqrt{3} - 1)$

Correct Answer: (D) $600(\sqrt{3} - 1)$

Solution:

Concept:

- The angle of depression from A equals the corresponding angle of elevation at the observed point. For a horizontal distance $d = BQ$, the tangent is the vertical height difference divided by d.
- Since $BC = PQ$ and both are vertical above the same horizontal base, C and P have the same

height. Thus BCPQ is a rectangle of width BQ and height PQ, so its area is $BQ \cdot PQ$.

Step 1: Find the horizontal separation Let $d = BQ$ and $h = PQ$, with lengths in metres. The bottom Q is at the level of B, so the vertical drop from A to Q is 30. The depression angle to Q yields

$$\tan 60^\circ = \frac{30}{d}, \qquad d = \frac{30}{\sqrt{3}} = 10\sqrt{3}.$$

The top P lies below A because its depression angle is positive.

Step 2: Calculate the tower height The vertical drop to P is $30 - h$, not h. Using $\tan 15^\circ = 2 - \sqrt{3}$,

$$\begin{aligned} 30 - h &= d \tan 15^\circ \\ &= 10\sqrt{3}(2 - \sqrt{3}) = 20\sqrt{3} - 30, \\ h &= 60 - 20\sqrt{3}. \end{aligned}$$

This is positive and less than 30, agreeing with the stated depression angles.

Step 3: Evaluate the rectangular area Therefore

$$\begin{aligned} [BCPQ] &= dh \\ &= 10\sqrt{3}(60 - 20\sqrt{3}) \\ &= 600\sqrt{3} - 600 = 600(\sqrt{3} - 1)\ \text{m}^2. \end{aligned}$$

Option B is half this area; option A also has the wrong factor, and option C has both a different factor and a plus sign. None equals the width-height product derived above.

Step 4: Conclusion The area is $\boxed{600(\sqrt{3} - 1)\ \text{m}^2}$. Hence, the correct option is (D).

Quick Tip to Solve: An angle of depression to the tower top uses the difference of heights; the angle to its foot uses the full wall height.

19. Statement $(\text{P} \Rightarrow \text{Q}) \wedge (\text{R} \Rightarrow \text{Q})$ is logically equivalent to : [4 marks]

(A) $(P \Rightarrow R) \wedge (Q \Rightarrow R)$

(B) $(P \Rightarrow R) \vee (Q \Rightarrow R)$

(C) $(P \wedge R) \Rightarrow Q$

(D) $(P \vee R) \Rightarrow Q$

Correct Answer: (D) $(P \vee R) \Rightarrow Q$

Solution:

Concept:

- An implication $P \Rightarrow Q$ is equivalent to $\neg P \vee Q$. It is false only when its antecedent is true and its consequent is false.
- Distributivity gives $(A \vee C) \wedge (B \vee C) = (A \wedge B) \vee C$, and De Morgan's law gives $\neg P \wedge \neg R = \neg(P \vee R)$.

Step 1: Eliminate the implications Apply the implication identity separately to the two factors:

$$(P \Rightarrow Q) \wedge (R \Rightarrow Q) = (\neg P \vee Q) \wedge (\neg R \vee Q)$$
$$= (\neg P \wedge \neg R) \vee Q.$$

The same consequent Q appears in both clauses, which is why the common term can be factored in this way.

Step 2: Restore a single implication Use De Morgan's law and then the implication identity in reverse:

$$(\neg P \wedge \neg R) \vee Q = \neg(P \vee R) \vee Q$$
$$= (P \vee R) \Rightarrow Q.$$

Thus whenever either P or R is true, Q must be true. Replacing the antecedent by $P \wedge R$ would impose a weaker condition.

Step 3: Rule out the other statements Take $P = \mathrm{T}, Q = \mathrm{T}, R = \mathrm{F}$. The original statement is true, but option A is false. For $P = \mathrm{T}, Q = \mathrm{F}, R = \mathrm{F}$, the original is false, whereas options B and C are true: B has $Q \Rightarrow R$ true, and C has a false antecedent. These counterexamples exclude all three alternatives without requiring a full truth table.

Step 4: Conclusion The equivalent statement is $\boxed{(P \vee R) \Rightarrow Q}$. Hence, the correct option is (D).

Quick Tip to Solve: When two implications have the same consequent and are joined by AND, combine their antecedents with OR, then verify using implication elimination.

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20. If $^{2n}C_3 : {}^{n}C_3 = 10 : 1$, then the ratio $\left(n^2 + 3n\right) : \left(n^2 - 3n + 4\right)$ is : [4 marks]

(A) $27 : 11$

(B) $2 : 1$

(C) $35 : 16$

(D) $65 : 37$

Correct Answer: (B) $2 : 1$

Solution:

Concept:

- For an integer $n \geq 3$, $\binom{n}{3} = n(n-1)(n-2)/6$. The restriction ensures that the denominator coefficient in the given ratio is defined and nonzero.
- The common factors in $\binom{2n}{3}/\binom{n}{3}$ should be cancelled before solving for n. Only after determining that integer should it be substituted into the requested ratio.

Step 1: Simplify the given coefficient ratio Expand both coefficients and cancel the common factors:

$$\frac{\binom{2n}{3}}{\binom{n}{3}} = \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)}$$
$$= \frac{4(2n-1)}{n-2} = 10.$$

Here $2n-2 = 2(n-1)$, which provides the second factor of 2 in the numerator. Cancelling is legitimate because $n \geq 3$.

Step 2: Solve for the admissible integer Cross-multiply the simplified equation:

$$4(2n-1) = 10(n-2),$$
$$8n - 4 = 10n - 20,$$
$$n = 8.$$

This value satisfies the restriction. Directly, $\binom{16}{3} = 560$ and $\binom{8}{3} = 56$, confirming the given ratio.

Step 3: Substitute into the requested expression For $n = 8$,

$$n^2 + 3n = 64 + 24 = 88, \qquad n^2 - 3n + 4 = 64 - 24 + 4 = 44.$$

Hence the ratio is $88 : 44 = 2 : 1$. The alternatives $27 : 11$, $35 : 16$ and $65 : 37$ have different quotients, so none satisfies the requested expression at the uniquely determined integer.

Step 4: Conclusion The required ratio is $\boxed{2:1}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $2n - 2 = 2(n-1)$ when cancelling the two binomial coefficients; missing this factor changes the equation for n.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $y = y(x)$ be a solution of the differential equation $(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0, 0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to _____. [4 marks]

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, an integrating factor is $e^{\int P(x)\,dx}$. On $0 < x < \pi/2$, both x and $\cos x$ are positive, so division by $x\cos x$ is valid.
- The product $u = xy$ satisfies $u'' = xy'' + 2y'$. Introducing this product avoids separately differentiating a quotient twice.

Step 1: Solve with an integrating factor Divide the equation by $x\cos x$ to obtain

$$y' + \left(\tan x + \frac{1}{x}\right)y = \frac{\sec x}{x}.$$

The integrating factor is

$$e^{\int (\tan x + 1/x)\,dx} = x \sec x.$$

Multiplication then gives

$$\frac{d}{dx}(xy \sec x) = \sec^2 x,$$

so $xy \sec x = \tan x + C$, or equivalently

$$xy = \sin x + C \cos x.$$

Step 2: Use the initial value At $x = \pi/3$, the left-hand side is exactly the product supplied in the question. Thus

$$\sqrt{3} = \sin \frac{\pi}{3} + C \cos \frac{\pi}{3} = \frac{\sqrt{3}}{2} + \frac{C}{2}.$$

It follows that $C = \sqrt{3}$, and therefore $u(x) = xy(x) = \sin x + \sqrt{3} \cos x$. This expression solves the equation throughout the stated interval.

Step 3: Evaluate the required derivative combination Differentiate u twice:

$$u'' = -\sin x - \sqrt{3} \cos x = -u.$$

Since $u'' = xy'' + 2y'$, the requested signed quantity is

$$\frac{\pi}{6} y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right) = -\frac{1}{2} - \sqrt{3}\frac{\sqrt{3}}{2} = -2.$$

The outer absolute value changes this to 2; it must be applied after the signed expression has been evaluated.

Step 4: Conclusion The required numerical value is $\boxed{2}$, which is dimensionless.

Quick Tip to Solve: If the target contains $xy'' + 2y'$, recognize it immediately as the second derivative of the product xy.

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22. If the area of the region $S = \{(x, y) : 2y - y^2 \le x^2 \le 2y, x \ge y\}$ is equal to $\frac{n+2}{n+1} - \frac{\pi}{n-1}$, then the natural number n is equal to _____. [4 marks]

Solution:

Concept:

- The inequality $x^2 \le 2y$ forces $y \ge 0$. Since $x \ge y$, the relevant branch is $x \ge 0$, so horizontal strips are convenient.
- At each height, the strip width is its right endpoint minus its left endpoint. The curve $x^2 = 2y - y^2$ is the circle $x^2 + (y - 1)^2 = 1$.

Step 1: Find the vertical limits The conditions $x \ge y \ge 0$ and $x^2 \le 2y$ imply $y^2 \le 2y$, hence $0 \le y \le 2$. Throughout this interval, the right endpoint is $x = \sqrt{2y}$. The left endpoint must satisfy both $x \ge y$

and $x \geq \sqrt{2y - y^2}$. Compare these two nonnegative quantities by squaring:

$$2y - y^2 \geq y^2 \quad \Longleftrightarrow \quad 0 \leq y \leq 1.$$

Thus the circle supplies the left boundary below $y = 1$, and the straight line supplies it above $y = 1$.

Step 2: Integrate the two strip widths Split at the boundary intersection $y = 1$:

$$\begin{aligned}\mathcal{A} &= \int_0^1 \left(\sqrt{2y} - \sqrt{2y - y^2}\right) dy + \int_1^2 \left(\sqrt{2y} - y\right) dy \\ &= \int_0^2 \sqrt{2y}\, dy - \int_0^1 \sqrt{1 - (y-1)^2}\, dy - \int_1^2 y\, dy.\end{aligned}$$

The middle integral is the area of a quarter of a unit circle. The other two integrals are $8/3$ and $3/2$, respectively. Therefore

$$\mathcal{A} = \frac{8}{3} - \frac{\pi}{4} - \frac{3}{2} = \frac{7}{6} - \frac{\pi}{4}.$$

Step 3: Identify the natural number For $n = 5$, the prescribed expression gives

$$\frac{n+2}{n+1} - \frac{\pi}{n-1} = \frac{7}{6} - \frac{\pi}{4}.$$

This is the required area. The expression is undefined at $n = 1$; for real $n > 1$, its derivative $\pi/(n-1)^2 - 1/(n+1)^2$ is positive, so no other admissible natural number gives this area.

Step 4: Conclusion The bounded area is $\frac{7}{6} - \frac{\pi}{4}$ square units. Hence the required numerical answer is $\boxed{5}$.

Quick Tip to Solve: Use horizontal strips and compare the two left boundaries first; they exchange roles at $y = 1$. The circle integral then becomes a quarter-circle area.

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23. Let the point $(p, p+1)$ lie inside the region $E = \left\{(x, y) : 3 - x \leq y \leq \sqrt{9 - x^2}, 0 \leq x \leq 3\right\}$. If the set of all values of p is the interval (a, b), then $b^2 + b - a^2$ is equal to _____. [4 marks]

Solution:

Concept:

- An interior point satisfies the boundary inequalities strictly. Here it must lie above $y = 3 - x$ and inside the circle $x^2 + y^2 = 9$, with $0 < x < 3$.
- After substituting $(x, y) = (p, p+1)$, retain all inequalities together. Squaring the upper-bound inequality is valid only when $p + 1$ is nonnegative.

Step 1: Impose the lower straight-line boundary The coordinate restriction requires $0 < p < 3$. Being above the line gives

$$p + 1 > 3 - p \quad \Longrightarrow \quad 2p > 2 \quad \Longrightarrow \quad p > 1.$$

This also ensures $p + 1 > 0$, which will justify squaring the inequality involving the nonnegative square root. Equality at $p = 1$ would put the point on the straight boundary, so it is excluded by the word

inside.

Step 2: Impose the circular boundary The upper condition for an interior point is

$$p + 1 < \sqrt{9 - p^2}.$$

Since both sides are nonnegative for any admissible point, square without reversing the inequality:

$$\begin{aligned} (p+1)^2 &< 9 - p^2, \\ 2p^2 + 2p - 8 &< 0, \\ p^2 + p - 4 &< 0. \end{aligned}$$

Its roots are $(-1 - \sqrt{17})/2$ and $(-1 + \sqrt{17})/2$. The upward-opening quadratic is negative between them. Intersecting that interval with $1 < p < 3$ gives

$$1 < p < \frac{\sqrt{17} - 1}{2}.$$

The upper endpoint is less than 3 and greater than 1, so this interval is nonempty and satisfies every coordinate restriction.

Step 3: Evaluate the requested combination Consequently $a = 1$ and $b = (\sqrt{17} - 1)/2$. Instead of expanding radicals, use the equation defining the positive root:

$$b^2 + b - 4 = 0 \quad \Longrightarrow \quad b^2 + b = 4.$$

It follows directly that

$$b^2 + b - a^2 = 4 - 1 = 3.$$

Both endpoints are excluded because one lies on the line and the other lies on the circle.

Step 4: Conclusion The interval is $\left(1, \frac{\sqrt{17}-1}{2}\right)$, and the required dimensionless value is $\boxed{3}$.

Quick Tip to Solve: When an endpoint is a quadratic root, use its quadratic equation to evaluate expressions such as $b^2 + b$, avoiding unnecessary radical expansion.

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24. Let $a \in \mathbb{Z}$ and $[t]$ be the greatest integer $\leq t$. Then the number of points, where the function $f(x) = [a + 13\sin x], x \in (0, \pi)$ is not differentiable, is _____. [4 marks]

Solution:

Concept:

- For an integer a, $\lfloor a + t \rfloor = a + \lfloor t \rfloor$. The constant shift changes function values but cannot change differentiability points.
- The floor function is locally constant away from integer thresholds. At a threshold crossing it jumps; a threshold attained only at a maximum must also be checked separately.

Step 1: Locate the possible thresholds On $(0, \pi)$, the quantity $13\sin x$ has range $(0, 13]$. Hence only

the integers $1, 2, \ldots, 13$ can cause a failure of local constancy. At any point where $13\sin x$ is not an integer, continuity of sine gives a small neighbourhood remaining between the same two consecutive integers. There f is constant and its derivative exists and equals zero. The endpoint level 0 does not occur in the open domain.

Step 2: Count the levels crossed twice For each $k = 1, 2, \ldots, 12$, the equation

$$13\sin x = k$$

has exactly two distinct solutions in $(0, \pi)$:

$$x = \sin^{-1}(k/13), \qquad x = \pi - \sin^{-1}(k/13).$$

At the first point sine increases through the level, and at the second it decreases through it. Since $\cos x \neq 0$ at these points, each is a genuine crossing. The floor has unequal one-sided limits, differing by one; f is therefore discontinuous and cannot be differentiable there. These levels contribute $2 \times 12 = 24$ points.

Step 3: Check the maximum level separately The level 13 is attained only at $x = \pi/2$. At that point

$$f(\pi/2) = a + 13.$$

For sufficiently close but unequal x, the value $13\sin x$ is strictly between 12 and 13, so $f(x) = a + 12$. Thus the two-sided limit is $a + 12$, while the function value is $a + 13$. This isolated discontinuity is also a point of non-differentiability, even though the graph does not cross the level 13.

Step 4: Conclusion There are $24 + 1 = \boxed{25}$ points of non-differentiability, independently of the integer a.

Quick Tip to Solve: For floor-of-sine problems, count twice-attained integer levels, then inspect the peak separately: touching an integer at a maximum can still create an isolated discontinuity.

25. Let $\mathrm{A} = \{1, 2, 3, 4, \ldots., 10\}$ and $\mathrm{B} = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a, b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$ is _____. [4 marks]

Solution:

Concept:

- Because $a, b \in \{1, \ldots, 10\}$, their difference $d = a - b$ is an integer between -9 and 9. Membership in the relation depends only on this difference.
- For a fixed integer difference d with $|d| \leq 9$, exactly $10 - |d|$ ordered pairs satisfy $a - b = d$. Different differences give disjoint sets of pairs.

Step 1: Reduce the membership test to an integer quadratic Write the defining expression as

$$2(a-b)^2 + 3(a-b) = 2d^2 + 3d = d(2d+3).$$

We require this value to be one of $0, 1, 2, 3, 4$. For $d \geq 1$,

$$2d^2 + 3d \geq 2 + 3 = 5,$$

so no positive difference works. The value at $d = 0$ is 0, which is allowed. At $d = -1$, the value is $2 - 3 = -1$, outside B; a negative value must not be counted merely because its magnitude is small.

Step 2: Determine all negative differences Put $d = -m$, where m is a positive integer. Then

$$2d^2 + 3d = 2m^2 - 3m = m(2m - 3).$$

For $m = 2$, this is 2, so $d = -2$ works. For $m \geq 3$, the factors obey $m \geq 3$ and $2m - 3 \geq 3$, giving a product of at least 9, which is larger than every element of B. Together with the separate check at $m = 1$, this proves that the only admissible differences are $d = 0$ and $d = -2$.

Step 3: Count the ordered pairs When $d = 0$, the ten diagonal pairs $(1, 1), \ldots, (10, 10)$ all belong to R. When $d = -2$, we have $b = a + 2$, so a can be any of $1, 2, \ldots, 8$, giving eight pairs:

$$(1, 3), (2, 4), \ldots, (8, 10).$$

These two families are disjoint. Their reversed off-diagonal pairs are not added: they have difference 2 and fail the membership condition. Therefore the total is $10 + 8 = 18$.

Step 4: Conclusion The relation contains $\boxed{18}$ ordered pairs.

Quick Tip to Solve: Test integer differences before counting pairs. For a permitted difference d, use $10 - |d|$; do not assume that a relation is symmetric.

---

26. A circle passing through the point $P(\alpha, \beta)$ in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then the value of $\alpha\beta$ is _____.

[4 marks]

Solution:

Concept:

- A circle touching both coordinate axes and passing through a first-quadrant point has centre (r, r) and radius $r > 0$. Its contact points are $(r, 0)$ and $(0, r)$.
- The perpendicular distance from (α, β) to $x + y - r = 0$ is $|\alpha + \beta - r|/\sqrt{2}$. Combining this with the circle equation eliminates the unknown radius.

Step 1: Write the circle and contact chord Tangency to both axes makes the absolute values of the two centre coordinates equal to the radius. A circle in any of the other three quadrants cannot contain a point having both coordinates strictly positive, so its centre is (r, r). Thus

$$(x - r)^2 + (y - r)^2 = r^2.$$

The axis contact points $A(r,0)$ and $B(0,r)$ determine the line

$$AB: \quad x+y=r.$$

Since P lies above this line, $\alpha+\beta-r>0$; the perpendicular foot specified in the question lies on its segment.

Step 2: Use the given perpendicular length Apply the point-to-line distance formula to $P(\alpha,\beta)$:

$$PQ=\frac{\alpha+\beta-r}{\sqrt{2}}=11.$$

Squaring gives

$$(\alpha+\beta-r)^2=2(11)^2=242.$$

The positive sign is consistent with the given position of P. No value of r needs to be found, because the circle equation relates this same expression directly to the product being asked for.

Step 3: Eliminate the radius using incidence Substitute P into the circle:

$$\alpha^2+\beta^2-2r(\alpha+\beta)+r^2=0.$$

Now expand the squared sum and group the expression from the circle equation:

$$\begin{aligned}(\alpha+\beta-r)^2&=\alpha^2+\beta^2+2\alpha\beta-2r(\alpha+\beta)+r^2\\&=2\alpha\beta.\end{aligned}$$

Comparing with 242 yields $2\alpha\beta=242$, so $\alpha\beta=121$. This also establishes the useful identity $PQ^2=\alpha\beta$ for this contact chord.

Step 4: Conclusion The required product is $\boxed{\alpha\beta=121}$ square units; the numerical entry is $\boxed{121}$.

Quick Tip to Solve: For a first-quadrant circle tangent to both axes, expand $(x+y-r)^2$ using the circle equation. It becomes $2xy$, so the squared distance to the contact chord is xy.

---

27. The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is _____. [4 marks]

Solution:

Concept:

- Each distinct orange must be assigned to one of three labelled children. Such an assignment is a function from a 20-element set to a 3-element set.
- The requirement that every child receives at least one orange means the function is onto. Inclusion-exclusion removes assignments in which one or more children receive nothing.

Step 1: Count unrestricted assignments For each of the 20 distinct oranges there are three independent

choices of recipient. Hence the unrestricted number of distributions is

$$3^{20} = 3,486,784,401.$$

The oranges are distinct, so distributions with the same three pile sizes can still be different. The children are distinct recipients, so no division by 3! is appropriate. Counting only positive integer triples of pile sizes would ignore the identities of the oranges.

Step 2: Exclude empty-child assignments Let E_i denote the assignments in which child i receives no orange. With that child excluded, each orange has two recipient choices, giving $|E_i| = 2^{20}$. There are three choices of excluded child. If two specified children are both empty, all oranges must go to the third child, giving exactly $1^{20} = 1$ assignment. There are $\binom{3}{2} = 3$ such intersections. The intersection in which all three children are empty has no assignments because 20 oranges must be distributed. Therefore inclusion-exclusion gives

$$N = 3^{20} - \binom{3}{1}2^{20} + \binom{3}{2}1^{20}.$$

The pair intersections are added back because an assignment with only one occupied child was subtracted twice in the preceding sum.

Step 3: Evaluate the integer count Use $2^{20} = 1,048,576$, so

$$\begin{aligned} N &= 3,486,784,401 - 3(1,048,576) + 3 \\ &= 3,486,784,401 - 3,145,728 + 3 \\ &= 3,483,638,676. \end{aligned}$$

This counts every assignment having all three children represented exactly once. There is no ordering within a child's collection and no further factor to insert.

Step 4: Conclusion The number of distributions is $\boxed{3483638676}$. This is a count of assignments and has no physical unit.

Quick Tip to Solve: Distinct objects distributed to labelled recipients suggest functions. Enforce at least one per recipient by inclusion-exclusion: $3^m - 3 \cdot 2^m + 3$ for $m > 0$.

---

28. Let the tangent to the curve $x^2 + 2x - 4y + 9 = 0$ at the point $\mathrm{P}(1, 3)$ on it meet the y-axis at A. Let the line passing through P and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at B. If B lies on the line $2x - 3y = 8$, then $(\mathrm{AB})^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Implicit differentiation gives the slope of the tangent to a curve. A line parallel to $x - 3y = 6$ has slope $1/3$.
- The required point B must satisfy the parallel line, the parabola and the additional straight-line condition. The latter selects the appropriate parabola intersection.

Step 1: Find the tangent intercept A Differentiate the curve equation with respect to x:

$$2x + 2 - 4\frac{dy}{dx} = 0 \quad \Longrightarrow \quad \frac{dy}{dx} = \frac{x+1}{2}.$$

At $P(1,3)$, the tangent slope is 1. Thus its equation is

$$y - 3 = x - 1 \quad \Longrightarrow \quad y = x + 2.$$

On the y-axis, $x = 0$, giving $A = (0,2)$. Substituting P into the original curve also confirms $1 + 2 - 12 + 9 = 0$, so the specified tangent point is valid.

Step 2: Determine the selected point B The line through P parallel to $x - 3y = 6$ is

$$y - 3 = \frac{x-1}{3} \quad \Longrightarrow \quad x - 3y = -8.$$

Since B also lies on $2x - 3y = 8$, subtracting the parallel-line equation from this one gives $x = 16$. Substituting back yields $16 - 3y = -8$, hence $y = 8$. Thus the only candidate satisfying both lines is $B = (16,8)$. Check the parabola explicitly:

$$y^2 = 8^2 = 64 = 4(16) = 4x.$$

Therefore the candidate is indeed the required parabola intersection; the extra condition has removed any ambiguity about which intersection to choose.

Step 3: Calculate the squared distance The difference in the x-coordinates is 16, and the difference in the y-coordinates is $8 - 2 = 6$. Use the squared distance formula directly:

$$(AB)^2 = (16-0)^2 + (8-2)^2 = 256 + 36 = 292.$$

There is no need to take a square root, since the question asks for the square of the length. Both coordinates refer to the same Cartesian system, so this is in square units.

Step 4: Conclusion The points are $A = (0,2)$ and $B = (16,8)$, giving the requested value $\boxed{(AB)^2 = 292}$.

Quick Tip to Solve: Use the extra line condition to locate B by two linear equations, then verify the parabola. This is quicker than solving a quadratic and testing both roots.

29. The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is _____. [4 marks]

Solution:

Concept:

- For a nonnegative integer n, the binomial theorem gives $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$. The index r counts the factors chosen from the second term.
- The coefficient is the constant multiplier of the required power of x. Multiplying powers adds their

exponents; the negative sign contributes the factor $(-1)^r$.

Step 1: Write the general term Choose r copies of $-x^{-3}$ and $15-r$ copies of x^4. Their contribution, including the number of ways to choose those copies, is

$$T_{r+1} = \binom{15}{r}(x^4)^{15-r}(-x^{-3})^r = \binom{15}{r}(-1)^r x^{60-7r}.$$

The exponent falls by seven each time the second term replaces the first: a power of four is replaced by a power of minus three.

Step 2: Match the required exponent To obtain x^{18}, equate the exponent of the general term to eighteen:

$$60-7r=18 \quad\Longrightarrow\quad 7r=42 \quad\Longrightarrow\quad r=6.$$

This integer lies between zero and fifteen, so the required term exists. Since the exponents change strictly with r, no other term contributes to this same power. Its sign is positive because six is even.

Step 3: Evaluate the coefficient The required coefficient is therefore

$$(-1)^6\binom{15}{6} = \frac{15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = 5005.$$

As a check, the selected powers are $(x^4)^9 = x^{36}$ and $(x^{-3})^6 = x^{-18}$, whose product is exactly x^{18}. The question asks only for the coefficient, so the power of x is not included in the numerical response.

Step 4: Conclusion The coefficient of x^{18} is $\boxed{5005}$. This is a dimensionless numerical coefficient.

Quick Tip to Solve: First solve the exponent equation $4(15-r)-3r=18$; only then evaluate the binomial coefficient and check the parity of r.

---

30. Let the image of the point $\mathrm{P}(1,2,3)$ in the plane $2x-y+z=9$ be Q. If the coordinates of the point R are $(6,10,7)$, then the square of the area of the triangle PQR is _____. [4 marks]

Solution:

Concept:

- The normal vector to $ax+by+cz=d$ is $\mathbf{n}=(a,b,c)$. Reflection across the plane moves a point only along this normal, with the plane halfway between the point and its image.
- For triangle vertices P,Q,R, its area Δ satisfies $\Delta = \frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|$. Thus its squared area is one quarter of the squared cross-product magnitude.

Step 1: Find the reflection parameter Here $\mathbf{n}=(2,-1,1)$, with $|\mathbf{n}|^2=6$. At $P=(1,2,3)$, the left side of the plane equation is $2-2+3=3$, six below nine. Write the reflected point as $Q=P+t\mathbf{n}$. The midpoint $P+\frac{t}{2}\mathbf{n}$ must lie on the plane, hence

$$3+\frac{t}{2}(6)=9 \quad\Longrightarrow\quad t=2.$$

Therefore $Q=(1,2,3)+2(2,-1,1)=(5,0,5)$. The midpoint is $(3,1,4)$, which indeed satisfies

$6 - 1 + 4 = 9$.

Step 2: Form the two sides from the same vertex Use the given point $R = (6, 10, 7)$ to obtain

$$\overrightarrow{PQ} = (4, -2, 2), \qquad \overrightarrow{PR} = (5, 8, 4).$$

Both vectors start at P, as required by the triangle-area formula. Notice that $\overrightarrow{PQ} = 2\mathbf{n}$, providing an independent directional check on the reflection.

Step 3: Compute the squared area The cross product is

$$\overrightarrow{PQ} \times \overrightarrow{PR} = ((-2)4 - 2(8),\ 2(5) - 4(4),\ 4(8) - (-2)5) = (-24, -6, 42).$$

Consequently,

$$\Delta^2 = \frac{1}{4}\left(24^2 + 6^2 + 42^2\right) = \frac{576 + 36 + 1764}{4} = 594.$$

The cross product gives the parallelogram area, so squaring the triangle-area factor introduces the divisor four, rather than two.

Step 4: Conclusion The square of the area of $\triangle PQR$ is $\boxed{594}$ in fourth powers of the coordinate unit.

Quick Tip to Solve: Use the midpoint condition to find the image, and calculate Δ^2 directly; taking a square root and then squaring it wastes work.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A particle is moving with constant speed in a circular path. When the particle turns by an angle $90°$, the ratio of instantaneous velocity to its average velocity is $\pi : x\sqrt{2}$. The value of x will be - [4 marks]

(A) 1

(B) 7

(C) 5

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- Instantaneous velocity in uniform circular motion has constant magnitude v, although its tangent direction changes continuously.
- Average velocity is displacement divided by elapsed time. Its magnitude depends on the straight chord, whereas the elapsed time is found from the travelled circular arc.

Step 1: Find the time for the quarter turn Let the radius be r. A turn of $90° = \pi/2$ radians covers a quarter-circle arc, so the distance travelled is $s = r\pi/2$. The speed is constant; therefore

$$\Delta t = \frac{s}{v} = \frac{\pi r}{2v}.$$

This time uses arc length, not chord length, because the particle actually moves along the circle.

Step 2: Calculate the average-velocity magnitude The initial and final radii are perpendicular. The displacement joins the endpoints of those radii, giving a right triangle with both perpendicular sides equal to r. Thus

$$|\Delta \mathbf{r}| = \sqrt{r^2 + r^2} = r\sqrt{2}.$$

The magnitude of average velocity is consequently

$$|\mathbf{v}_{\text{av}}| = \frac{r\sqrt{2}}{\pi r/(2v)} = \frac{2\sqrt{2}}{\pi} v.$$

The radius cancels, so the requested ratio is independent of the chosen circular path.

Step 3: Compare the two ratios Using the instantaneous magnitude v,

$$v : |\mathbf{v}_{\text{av}}| = v : \frac{2\sqrt{2}}{\pi} v = \pi : 2\sqrt{2}.$$

Comparison with $\pi : x\sqrt{2}$ gives $x = 2$. The values one, seven and five would give different average-velocity magnitudes and do not satisfy the chord-over-time calculation. A ratio of velocity vectors is not a scalar division; the question uses their magnitudes, as its numerical ratio indicates.

Step 4: Conclusion The value is $\boxed{x = 2}$. Hence, the correct option is (D).

Quick Tip to Solve: For a circular turn through θ radians, compare arc and chord: $v/|\mathbf{v}_{\text{av}}| = \theta/[2\sin(\theta/2)]$.

32. The induced emf can be produced in a coil by

A. moving the coil with uniform speed inside uniform magnetic field

B. moving the coil with non uniform speed inside uniform magnetic field

C. rotating the coil inside the uniform magnetic field

D. changing the area of the coil inside the uniform magnetic field

Choose the correct answer from the options given below: [4 marks]

(A) A and C only

(B) C and D only

(C) B and D only

(D) B and C only

Correct Answer: (B) C and D only

Solution:

Concept:

- Faraday's law states $\mathcal{E} = -N\, d\Phi_B/dt$, where N is the number of turns and Φ_B is magnetic flux through one turn.
- For a planar coil in a uniform magnetic field, $\Phi_B = BA\cos\theta$, where A is its enclosed area and θ is the angle between its normal and the magnetic field.

Step 1: Test translation in the uniform field In statements A and B the coil moves bodily inside a uniform magnetic field. Translation does not change its area or orientation, and the field has the same value everywhere inside that region. Hence B, A, and θ remain constant, so

$$\frac{d\Phi_B}{dt} = 0.$$

A changing translational speed in B does not, by itself, change the magnetic flux. Neither uniform nor nonuniform translation of the complete rigid coil wholly within this field produces a net emf around it.

Step 2: Test rotation and change of area For C, suitable rotation changes the angle θ. A coil rotating with angular speed ω, for example, has

$$\Phi_B = BA\cos\omega t, \qquad \mathcal{E} = NBA\omega\sin\omega t.$$

Thus rotation can produce an induced emf. For D, changing the enclosed area with nonzero normal field changes flux even at fixed angle:

$$\mathcal{E} = -NB\cos\theta\,\frac{dA}{dt}.$$

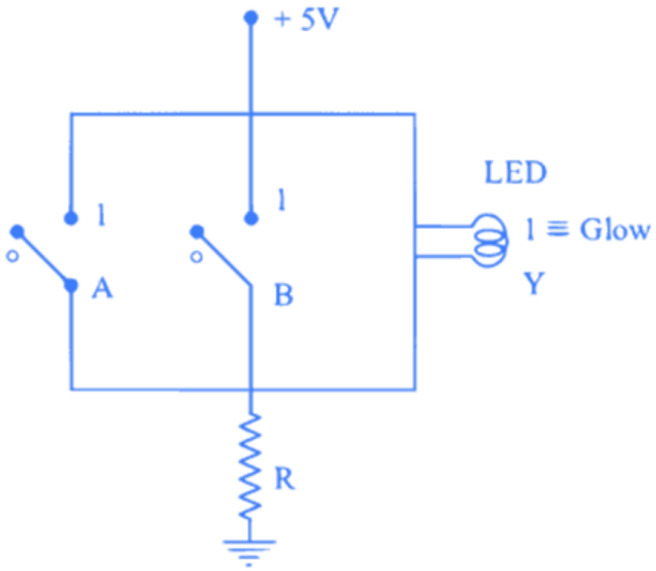
The word "can" permits these arrangements; rotation about a special axis or an instant of zero emf does not invalidate C.

Step 3: Choose the combination Only statements C and D describe mechanisms that can change the flux under the stated conditions. Option A incorrectly includes translation A, while options C and D incorrectly include translation B. Option B contains precisely C and D. Motion matters through its effect on flux, rather than through the mere existence of speed or acceleration.

Step 4: Conclusion An induced emf can be produced by C and D only. Hence, the correct option is (B).

Quick Tip to Solve: Before deciding whether motion induces emf, identify which factor in $BA\cos\theta$ changes; acceleration alone is not a flux change.

33. Name the logic gate equivalent to the diagram attached [4 marks]



(A) NOR

(B) NAND

(C) AND

(D) OR

Correct Answer: (A) NOR

Solution:

Concept:

- A logic output is one when the indicated LED glows, and zero when it does not. Each input is read from the corresponding switch position: one denotes its closed conducting position.
- A NOR gate implements the negation of the OR operation: $Y = \overline{A + B}$. It has output one only when both inputs are zero.

Step 1: Examine the state with both switches open With $A = 0$ and $B = 0$, neither switch provides a conducting shunt across the indicator. The supply can drive the indicator branch through the resistor, so the LED glows. Since the diagram explicitly identifies glow as output one, this state gives $Y = 1$. The resistor is part of the current path; it does not represent an additional logic input.

Step 2: Close either input switch The switches are parallel alternatives for establishing the shunt connection. Closing A, closing B, or closing both provides the low-resistance bypass. The potential difference driving the indicator is then removed, so it does not glow and the output is zero. This yields the complete table:

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

It is essential to use the stated glow convention: interchanging bright and dark would complement

the entire truth table.

Step 3: Identify the gate and rule out the others The table is the complement of OR: $A+B$ is zero only in the first row, and its negation is one only there. NAND would remain one for the two mixed-input rows, contrary to this circuit. AND would be one only for two closed switches, while OR would be zero for two open switches. Neither matches the observed switching table. The correct function is therefore NOR.

Step 4: Conclusion The equivalent gate is $\boxed{\text{NOR}}$, with $Y = \overline{A+B}$. Hence, the correct option is (A).

Quick Tip to Solve: Build the four-row truth table using the LED's glow convention before naming the gate; parallel shunts turn an OR-like switching condition into NOR.

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34. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : When a body is projected at an angle 45°, it's range is maximum.

Reason R : For maximum range, the value of $\sin 2\theta$ should be equal to one.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(B) **A** is true but **R** is false

(C) **A** is false but **R** is true

(D) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**

Correct Answer: (A) Both **A** and **R** are correct and **R** is the correct explanation of **A**

Solution:

Concept:

- For projection and landing at the same height, neglecting air resistance and with fixed initial speed u, the horizontal range is $R = u^2 \sin 2\theta/g$.
- The maximum value of $\sin 2\theta$ is one. For an upward projection with $0 \le \theta \le 90^\circ$, this occurs when $2\theta = 90^\circ$.

Step 1: Evaluate Assertion A Use the standard level-ground projectile model: the horizontal speed is $u\cos\theta$, and the flight time is $T = 2u\sin\theta/g$. Multiplying them gives

$$R = (u\cos\theta)\frac{2u\sin\theta}{g} = \frac{u^2}{g}\sin 2\theta.$$

At $\theta = 45^\circ$, the sine factor is $\sin 90^\circ = 1$, its greatest possible value. Therefore the assertion that the range is maximum at this angle is correct under these standard conditions.

Step 2: Evaluate Reason R For a fixed launch speed at the same location, u^2/g is positive and constant. Maximizing the range therefore amounts exactly to maximizing $\sin 2\theta$. Since

$$-1 \le \sin 2\theta \le 1,$$

its maximum is one, as the reason states. Restricting to ordinary upward launch angles gives $\sin 2\theta \geq 0$, but the upper bound remains one. Hence R is also correct.

Step 3: Check the explanatory link The condition in R leads directly to

$$\sin 2\theta = 1 \quad \Longrightarrow \quad 2\theta = 90^\circ \quad \Longrightarrow \quad \theta = 45^\circ.$$

Thus R explains A rather than merely being a separate true fact. Option B wrongly declares the reason false; option C wrongly declares the assertion false; option D denies the direct mathematical implication just obtained. If launch and landing heights differ, the range formula changes, but that is outside the standard situation tested here.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: For assertion-reason questions, verify both statements first, then explicitly show the equation that connects the reason to the assertion.

35. A source supplies heat to a system at the rate of 1000 W. If the system performs work at a rate of 200 W. The rate at which internal energy of the system increases is [4 marks]

(A) 600 W

(B) 1200 W

(C) 500 W

(D) 800 W

Correct Answer: (D) 800 W

Solution:

Concept:

- The first law of thermodynamics is $dU = \delta Q - \delta W$ when heat supplied to the system and work performed by the system are defined as positive.
- Dividing the first law by the elapsed time gives a power balance: $dU/dt = \dot{Q} - \dot{W}$. A watt is one joule per second.

Step 1: Assign the correct signs The source supplies heat to the system, so the incoming thermal power is $\dot{Q} = +1000\,\text{W}$. The system itself performs work on its surroundings, so its outgoing work power is $\dot{W} = +200\,\text{W}$ in the work-by-system convention. This outgoing energy cannot also remain stored inside the system. The minus sign in the first law accounts for that loss.

Step 2: Apply the first law per unit time Substitute the two rates into the energy balance:

$$\frac{dU}{dt} = \dot{Q} - \dot{W} = 1000 - 200 = 800\,\text{J}\,\text{s}^{-1}.$$

The positive result means that the internal energy increases rather than decreases. In each second, the system receives $1000\,\text{J}$, transfers $200\,\text{J}$ out as work, and stores the remaining $800\,\text{J}$. This accounting checks energy conservation directly.

Step 3: Compare with the listed choices The resulting rate is $800\,\mathrm{W}$, which matches option D. The value $1200\,\mathrm{W}$ would result from incorrectly adding the work done by the system to the incoming heat; that addition would be appropriate only if the stated work were done on it. The values $600\,\mathrm{W}$ and $500\,\mathrm{W}$ do not follow from the two given rates. No duration is specified or required, because the question asks for a rate rather than a total change in energy.

Step 4: Conclusion The internal energy increases at $\boxed{800\,\mathrm{W}}$. Hence, the correct option is (D).

Quick Tip to Solve: Write "heat in minus work out" before inserting numbers; this prevents the common sign error of adding the two power values.

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36. A small ball of mass M and density ρ is dropped in a viscous liquid of density ρ_0. After some time, the ball falls with a constant velocity. What is the viscous force on the ball ? [4 marks]

(A) $\mathrm{F} = \mathrm{Mg}\left(1 - \frac{\rho_O}{\rho}\right)$

(B) $\mathrm{F} = \mathrm{Mg}\left(1 + \frac{\rho}{P_o}\right)$

(C) $\mathrm{F} = \mathrm{Mg}\left(1 + \frac{\rho_o}{\rho}\right)$

(D) $F = Mg\,(1 \pm \rho\rho_0)$

Correct Answer: (A) $\mathrm{F} = \mathrm{Mg}\left(1 - \frac{\rho_O}{\rho}\right)$

Solution:

Concept:

- At terminal velocity the acceleration is zero, so the resultant force on the ball vanishes. The downward weight is balanced by the upward buoyant force and viscous drag.
- Archimedes' principle gives the buoyant force as the weight of displaced liquid: $B = \rho_0 V g$, where V is the ball volume. Its mass is $M = \rho V$.

Step 1: Identify the forces The ball moves downward through the liquid. Its weight Mg acts downward, while both liquid buoyancy and viscous resistance act upward. Constant velocity does not mean that drag or weight is zero; it means their vector sum, including buoyancy, is zero. The magnitude of the upward drag is denoted by F.

Step 2: Express buoyancy in terms of the given mass The volume is obtained from the density of the ball, not the liquid density. Thus

$$V = \frac{M}{\rho}, \qquad B = \rho_0 g \frac{M}{\rho}.$$

The ratio ρ_0/ρ is dimensionless, as required for comparing buoyancy with weight.

Step 3: Apply the terminal force balance Choosing downward as positive gives

$$Mg - B - F = 0.$$

Consequently,

$$F = Mg - B = Mg\left(1 - \frac{\rho_0}{\rho}\right).$$

For a sinking ball $\rho > \rho_0$, so this drag is positive and smaller than the weight. Option (C) would add buoyancy rather than subtract it, and (B) also has an incorrect density dependence. The density product in (D) cannot be added to the dimensionless number 1.

Step 4: Conclusion The viscous force acts upward with magnitude $\boxed{F = Mg(1 - \rho_0/\rho)}$. Hence, the correct option is (A).

Quick Tip to Solve: At terminal speed first subtract the buoyant force from the weight; the remaining effective weight equals the viscous drag. The liquid viscosity sets the speed, not this force balance.

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37. A small block of mass $100\ \mathrm{g}$ is tied to a spring of spring constant $7.5\ \mathrm{N/m}$ and length $20\ \mathrm{cm}$. The other end of spring is fixed at a particular point A. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity $5\ \mathrm{rad/s}$ about point A, then tension in the spring is - [4 marks]

(A) 0.50 N

(B) 1.5 N

(C) 0.75 N

(D) 0.25 N

Correct Answer: (C) 0.75 N

Solution:

Concept:

- The spring supplies the radial force for uniform circular motion: $T = m\omega^2 r$, where r is the stretched spring length.
- Hooke's law is $T = kx = k(r - l_0)$, where l_0 is the unstretched length. The radius must include the extension.

Step 1: Convert the quantities and define the extension Use SI units: $m = 0.100\,\mathrm{kg}$, $k = 7.5\,\mathrm{N\,m^{-1}}$, $l_0 = 0.20\,\mathrm{m}$, and $\omega = 5\,\mathrm{rad\,s^{-1}}$. If the spring stretches by x, the circular orbit radius is $r = 0.20 + x$. The normal force balances the weight, so neither contributes to the horizontal radial equation.

Step 2: Equate the elastic and centripetal forces The same tension must satisfy both laws:

$$kx = m\omega^2(l_0 + x).$$

Substitute the data before solving:

$$7.5x = (0.100)(5)^2(0.20 + x) = 2.5(0.20 + x).$$

Hence $5x = 0.50$, giving $x = 0.10\,\mathrm{m}$. The positive solution is consistent with a stretched spring.

Step 3: Calculate and check the tension The tension follows directly from the extension:

$$T = kx = 7.5(0.10) = 0.75\,\mathrm{N}.$$

The actual radius is $0.30\,\mathrm{m}$; independently, $m\omega^2 r = 0.100(25)(0.30) = 0.75\,\mathrm{N}$. Option (A), $0.50\,\mathrm{N}$,

incorrectly uses the unstretched length as the orbit radius. Values $1.5\,\mathrm{N}$ and $0.25\,\mathrm{N}$ do not satisfy both force equations with the given spring.

Step 4: Conclusion The extension is $0.10\,\mathrm{m}$ and the tension is $\boxed{0.75\,\mathrm{N}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a rotating spring write the orbit radius as natural length plus extension. Solve $kx = m\omega^2(l_0 + x)$ before evaluating tension.

38. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Earth has atmosphere whereas moon doesn't have any atmosphere.

Reason R : The escape velocity on moon is very small as compared to that on earth.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) **A** is false but **R** is true

(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**

(C) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(D) **A** is true but **R** is false

Correct Answer: (C) Both **A** and **R** are correct and **R** is the correct explanation of **A**

Solution:

Concept:

- Escape speed at the surface of a spherical body is $v_e = \sqrt{2GM/R}$, where M and R are its mass and radius.
- Atmospheric molecules have thermal motion. A body retains gases more effectively when their molecular speeds are well below its escape speed.

Step 1: Evaluate Assertion A Earth has a substantial atmosphere, whereas the Moon has no appreciable atmosphere in the usual school-level sense. The statement contrasts an atmosphere such as Earth's with the Moon's inability to sustain a substantial gaseous envelope; a very tenuous lunar exosphere does not alter that comparison. Thus Assertion A is correct in the context of atmospheric retention.

Step 2: Evaluate Reason R The surface escape speed depends on mass relative to radius, not on radius alone. Earth has an escape speed of about $11.2\,\mathrm{km\,s^{-1}}$, compared with about $2.4\,\mathrm{km\,s^{-1}}$ for the Moon. The Moon therefore has a much smaller escape speed, making Reason R correct. These values illustrate the size of the difference; the qualitative conclusion also follows from $\sqrt{M/R}$.

Step 3: Test the explanatory connection Gas molecules continually move with a distribution of thermal speeds. With the Moon's lower escape threshold, molecules can escape much more readily, so a substantial atmosphere is not retained over long times. Earth's stronger gravitational binding permits much better retention. Reason R therefore supplies the physical explanation for Assertion A. Option (B) wrongly denies this causal relation, while (A) and (D) wrongly declare one of the individually correct statements false.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (C).

Quick Tip to Solve: In an atmosphere-retention assertion, compare molecular thermal speeds with escape speed. Checking that both statements are true is only the first part; also test the causal link.

39. The kinetic energy of an electron, α-particle and a proton are given as $4\ \mathrm{K}, 2\ \mathrm{K}$ and K respectively. The de-Broglie wavelength associated with electron $(\lambda \mathrm{e}), \alpha$-particle $((\lambda \alpha)$ and the proton (λp) are as follows: [4 marks]

(A) $\lambda\alpha < \lambda p < \lambda e$

(B) $\lambda\alpha > \lambda p > \lambda e$

(C) $\lambda\alpha = \lambda p < \lambda e$

(D) $\lambda\alpha = \lambda p > \lambda e$

Correct Answer: (A) $\lambda\alpha < \lambda p < \lambda e$

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$. For nonrelativistic kinetic energy E, momentum is $p = \sqrt{2mE}$, giving $\lambda = h/\sqrt{2mE}$.
- An alpha particle has mass approximately $4m_p$, while the proton is much heavier than the electron: $m_p \simeq 1836m_e$.

Step 1: Compare the mass-energy products The energies are unequal, so comparing particle masses alone is insufficient. With the common positive energy unit K, the products determining momentum squared are

$$m_eE_e = 4m_eK, \qquad m_pE_p = m_pK, \qquad m_\alpha E_\alpha \simeq 8m_pK.$$

Since m_p greatly exceeds $4m_e$, their increasing order is $4m_eK < m_pK < 8m_pK$.

Step 2: Reverse the order for wavelength The wavelength is inversely proportional to the square root of each mass-energy product. In particular,

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{4m_e}} > 1, \qquad \frac{\lambda_\alpha}{\lambda_p} = \frac{1}{\sqrt{8}} < 1.$$

Thus the alpha particle has the greatest momentum and the shortest wavelength, while the electron has the smallest momentum and the longest wavelength. The electron's larger kinetic energy does not compensate for its much smaller mass.

Step 3: Reject the other proposed orderings Option (B) reverses the actual momentum-based order. Options (C) and (D) claim equal alpha and proton wavelengths, which would require equal momenta and hence equal values of mE. Here the alpha particle's mE is approximately eight times that of the proton, so neither equality is possible.

Step 4: Conclusion The required order is $\boxed{\lambda_\alpha < \lambda_p < \lambda_e}$. Hence, the correct option is (A).

Quick Tip to Solve: When particle energies differ, rank mE, then reverse the ranking for de Broglie wavelengths. Ranking masses or kinetic energies separately can give the wrong result.

40. A monochromatic light wave with wavelength λ_1 and frequency v_1 in air enters another medium. If the angle of incidence and angle of refraction at the interface are 45° and 30° respectively, then the wavelength λ_2 and frequency v_2 of the refracted wave are: [4 marks]

(A) $\lambda_2 = \lambda_1, v_2 = \frac{1}{\sqrt{2}} v_1$

(B) $\lambda_2 = \lambda_1, v_2 = \sqrt{2} v_1$

(C) $\lambda_2 = \sqrt{2}\lambda_1, v_2 = v_1$

(D) $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1, v_2 = v_1$

Correct Answer: (D) $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1, v_2 = v_1$

Solution:

Concept:

- Snell's law gives the relative refractive index $n_{21} = \sin i / \sin r = v_{\text{air}}/v_{\text{medium}}$, using the phase speeds of light.
- At a stationary interface the frequency of a monochromatic wave is unchanged. Since speed equals frequency times wavelength, a speed change produces a wavelength change.

Step 1: Find the relative refractive index Use the given angles, measured with respect to the normal:

$$n_{21} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}.$$

The refracted ray is closer to the normal, consistent with entry into an optically denser medium. Therefore the wave speed in the second medium is $1/\sqrt{2}$ times its speed in air.

Step 2: Keep the oscillation frequency unchanged The incident source fixes the temporal frequency. A stationary boundary cannot continually accumulate wave crests, so transmitted and incident oscillations have the same frequency:

$$v_2 = v_1.$$

Here v_1 and v_2 are the frequency symbols used in the question, rather than the propagation speeds. Refraction changes propagation speed without changing that frequency.

Step 3: Calculate the new wavelength Writing the propagation speeds as u_1 and u_2 avoids confusion with the printed frequency notation. From $u = \lambda v$,

$$\frac{\lambda_2}{\lambda_1} = \frac{u_2}{u_1}\frac{v_1}{v_2} = \frac{1}{\sqrt{2}}.$$

Options (A) and (B) incorrectly change frequency and leave wavelength unchanged. Option (C) keeps frequency correctly but increases wavelength, whereas the speed decreases in this medium.

Step 4: Conclusion The refracted wave has $\boxed{\lambda_2 = \lambda_1/\sqrt{2}, \quad v_2 = v_1}$. Hence, the correct option is (D).

Quick Tip to Solve: At a stationary optical boundary, preserve frequency first. Obtain the relative refractive index from the angle ratio and divide the original wavelength by it.

41. A long straight wire of circular cross-section (radius a) is carrying steady current I. The current I is uniformly distributed across this cross-section. The magnetic field is [4 marks]

(A) uniform in the region $r < a$ and inversely proportional to distance r from the axis, in the region $r > a$

(B) zero in the region $r < a$ and inversely proportional to r in the region $r > a$

(C) directly proportional to r in the region $r < a$ and inversely proportional to r in the region $r > a$

(D) inversely proportional to r in the region $r < a$ and uniform throughout in the region $r > a$

Correct Answer: (C) directly proportional to r in the region $r < a$ and inversely proportional to r in the region $r > a$

Solution:

Concept:

- Ampere's circuital law is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$. Circular symmetry makes the field tangential and constant in magnitude along a circle coaxial with a long wire.
- Uniform volume current distribution gives current density $J = I/(\pi a^2)$. The enclosed current depends on the area of wire inside the chosen Amperian loop.

Step 1: Find the field inside the wire For a circular loop of radius $r < a$, only the current through area πr^2 is enclosed:

$$I_{\text{enc}} = J\pi r^2 = I\frac{r^2}{a^2}.$$

Ampere's law therefore gives

$$B(2\pi r) = \mu_0 I\frac{r^2}{a^2}, \qquad B = \frac{\mu_0 I}{2\pi a^2}r.$$

Thus the interior field increases linearly with distance from the axis and vanishes only at the axis.

Step 2: Find the field outside the wire For $r > a$, the loop encloses the whole current I, regardless of the loop radius. Consequently,

$$B(2\pi r) = \mu_0 I, \qquad B = \frac{\mu_0 I}{2\pi r}.$$

The exterior field is inversely proportional to r. At $r = a$, both expressions yield $\mu_0 I/(2\pi a)$, so the result is continuous at the surface.

Step 3: Compare the options Option (A) incorrectly makes the inside field uniform. Option (B) confuses a uniformly conducting solid wire with a current confined to a hollow cylindrical shell; the solid wire encloses nonzero current for any nonzero interior radius. Option (D) incorrectly reverses the interior dependence and makes the outside field constant. Only option (C) has both the correct interior and exterior laws.

Step 4: Conclusion The field is proportional to r for $r < a$, and proportional to $1/r$ for $r > a$. Hence, the correct option is (C).

Quick Tip to Solve: For a solid current-carrying wire, calculate enclosed current before applying Ampere's law. Inside it grows as r^2; outside it has already reached the total current.

42. A planet has double the mass of the earth. Its average density is equal to that of the earth. An object weighing W on earth will weigh on that planet: [4 marks]

(A) $2^{2/3}$ W

(B) W

(C) 2 W

(D) $2^{1/3}$ W

Correct Answer: (D) $2^{1/3}$ W

Solution:

Concept:

- The surface gravitational acceleration of a spherical planet is $g = GM/R^2$. An object of unchanged mass m has weight $W = mg$.
- Equal mean density means $M = (4/3)\pi\rho R^3$, so mass scales as the cube of radius rather than as radius itself.

Step 1: Use equal density to find the radius ratio Let Earth have mass M_E and radius R_E, and let the planet have mass $M_P = 2M_E$ and radius R_P. Since their average densities are equal,

$$\frac{M_P}{M_E} = \left(\frac{R_P}{R_E}\right)^3 = 2.$$

Hence $R_P/R_E = 2^{1/3}$. The larger planet cannot be assigned Earth's radius while retaining the stated equal density.

Step 2: Calculate the gravity and weight ratios Taking the ratio of surface gravitational accelerations eliminates G:

$$\frac{g_P}{g_E} = \frac{M_P}{M_E}\left(\frac{R_E}{R_P}\right)^2 = \frac{2}{2^{2/3}} = 2^{1/3}.$$

The object's mass is unchanged, so its weight has exactly this same ratio:

$$W_P = mg_P = 2^{1/3}mg_E = 2^{1/3}W.$$

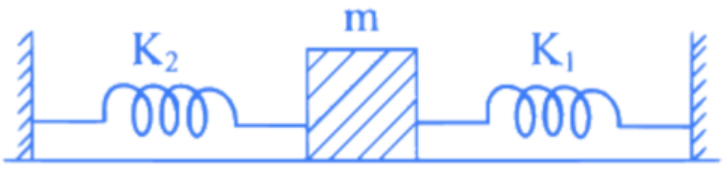
The result is greater than W, but smaller than $2W$, because the increase in radius partly offsets the doubled mass.

Step 3: Check the alternative scalings Option (C), $2W$, neglects the change in radius. Option (B), W, would require the radius ratio to be $\sqrt{2}$, inconsistent with equal density. Option (A), $2^{2/3}W$, uses the radius-squared factor without combining it correctly with the factor of two in mass. At fixed density, the direct relation $g = (4/3)\pi G\rho R$ also confirms the cube-root weight factor.

Step 4: Conclusion The object weighs $\boxed{2^{1/3}W}$ on the planet. Hence, the correct option is (D).

Quick Tip to Solve: At equal mean density use $R \propto M^{1/3}$ and $g \propto R$. Doubling mass then multiplies surface gravity and weight by $2^{1/3}$.

43. A mass m is attached to two strings as shown in figure. The spring constants of two springs are K_1 and K_2. For the frictionless surface, the time period of oscillation of mass m is : [4 marks]



(A) $2\pi\sqrt{\frac{m}{K_1+K_2}}$

(B) $2\pi\sqrt{\frac{m}{K_1-K_2}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{K_1-K_2}{m}}$

Correct Answer: (A) $2\pi\sqrt{\frac{m}{K_1+K_2}}$

Solution:

Concept:

- For a spring obeying Hooke's law, a displacement x from equilibrium produces a restoring force proportional to x. The restoring force opposes the displacement.
- A mass in simple harmonic motion satisfies $m\ddot{x} = -K_{\text{eff}}x$. Its angular frequency is $\omega = \sqrt{K_{\text{eff}}/m}$, and its period is $T = 2\pi/\omega$.

Step 1: Find the force for a small displacement Displace the block a distance x to the right of its equilibrium position. The left spring is stretched by x, so it pulls the block to the left with additional force K_2x. The right spring is compressed by x, so it also pushes the block to the left with additional force K_1x. Thus the two restoring forces act in the same direction even though the springs lie on opposite sides of the mass. Any equilibrium forces cancel when the displacement-dependent force is considered.

Step 2: Add the restoring forces Taking the rightward direction as positive, Newton's second law gives

$$m\ddot{x} = -K_1x - K_2x = -(K_1 + K_2)x.$$

Hence $K_{\text{eff}} = K_1 + K_2$. This is not the series-spring formula: the moving mass changes the length of each spring by the same magnitude x.

Step 3: Identify the period and test the options Comparing with $\ddot{x} = -\omega^2x$,

$$\omega^2 = \frac{K_1 + K_2}{m}, \qquad T = 2\pi\sqrt{\frac{m}{K_1 + K_2}}.$$

The difference $K_1 - K_2$ in (B) and (D) is incorrect because the restoring forces add. Option (C) is the ordinary frequency $f = 1/T$, with units of inverse seconds, rather than the time period.

Step 4: Conclusion The time period is $\boxed{T = 2\pi\sqrt{m/(K_1 + K_2)}}$. Hence, the correct option is (A).

Quick Tip to Solve: Displace the mass once and draw the directions of both restoring forces mentally: opposite-side springs can both restore in the same direction, so their constants add.

44. Two resistances are given as $\mathrm{R}_1 = (10 \pm 0.5)\Omega$ and $\mathrm{R}_2 = (15 \pm 0.5)\Omega$. The percentage error in the measurement of equivalent resistance when they are connected in parallel is - [4 marks]

(A) 2.33

(B) 5.33

(C) 4.33

(D) 6.33

Correct Answer: (C) 4.33

Solution:

Concept:

- For two parallel resistors, $R = R_1R_2/(R_1 + R_2)$. The same measured quantity occurring twice in a formula must not be treated as independent occurrences.
- The maximum first-order absolute error is $\Delta R = |\partial R/\partial R_1|\Delta R_1 + |\partial R/\partial R_2|\Delta R_2$. Percentage error is $100\Delta R/R$.

Step 1: Calculate the nominal equivalent resistance Use the central measured values $R_1 = 10\,\Omega$ and $R_2 = 15\,\Omega$. Their parallel equivalent is

$$R = \frac{10 \times 15}{10 + 15} = 6\,\Omega.$$

Both absolute uncertainties are $0.5\,\Omega$. Their individual fractional errors are different, so simply averaging their percentage errors would not propagate the measurement correctly.

Step 2: Differentiate the parallel-resistance expression The sensitivity coefficients are

$$\frac{\partial R}{\partial R_1} = \frac{R_2^2}{(R_1 + R_2)^2}, \qquad \frac{\partial R}{\partial R_2} = \frac{R_1^2}{(R_1 + R_2)^2}.$$

Both are positive; increasing either resistor increases the equivalent resistance. Adding the absolute contributions for the limiting error gives

$$\Delta R = \frac{15^2(0.5) + 10^2(0.5)}{25^2} = \frac{162.5}{625} = 0.26\,\Omega.$$

Step 3: Convert the absolute error into a percentage Therefore,

$$\frac{\Delta R}{R} \times 100 = \frac{0.26}{6} \times 100 = 4.333\ldots\%.$$

The rounded value is 4.33%, which matches (C). The values 2.33%, 5.33% and 6.33% do not follow from the correct weighted sensitivities. Adding the errors of the numerator and denominator independently would count the same resistance measurements twice.

Step 4: Conclusion The maximum percentage error is $\boxed{4.33\%}$. Hence, the correct option is (C).

Quick Tip to Solve: For parallel resistors use $\Delta R/R = \frac{R_2}{R_1+R_2}\frac{\Delta R_1}{R_1} + \frac{R_1}{R_1+R_2}\frac{\Delta R_2}{R_2}$; the weights prevent double-counting.

45. For the plane electromagnetic wave given by $E = E_0 \sin(\omega t - kx)$ and $B = B_0 \sin(\omega t - kx)$, the ratio of average electric energy density to average magnetic energy density is [4 marks]

(A) 1

(B) 4

(C) 2

(D) 1/2

Correct Answer: (A) 1

Solution:

Concept:

- The instantaneous electric and magnetic energy densities in vacuum are $u_E = \frac{1}{2}\varepsilon_0 E^2$ and $u_B = B^2/(2\mu_0)$, respectively.
- For a plane electromagnetic wave, $E_0 = cB_0$ and $c^2 = 1/(\mu_0\varepsilon_0)$. Over a complete cycle, $\langle \sin^2(\omega t - kx)\rangle = 1/2$.

Step 1: Average the electric energy density Insert the specified sinusoidal electric field into its energy density:

$$u_E = \frac{1}{2}\varepsilon_0 E_0^2 \sin^2(\omega t - kx).$$

At any fixed position, averaging over one complete oscillation replaces the squared sine by $1/2$. Therefore,

$$\langle u_E \rangle = \frac{1}{4}\varepsilon_0 E_0^2.$$

The energy depends on the square of the field; averaging the field itself would incorrectly give zero.

Step 2: Average the magnetic energy density The magnetic field has the same phase and frequency, so exactly the same averaging factor applies:

$$u_B = \frac{B_0^2}{2\mu_0}\sin^2(\omega t - kx), \qquad \langle u_B \rangle = \frac{B_0^2}{4\mu_0}.$$

Use the wave-amplitude relation $E_0 = cB_0$, rather than equating the numerical field amplitudes, which have different units.

Step 3: Form and simplify the ratio Taking the ratio of the two averages gives

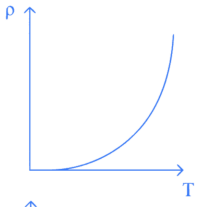
$$\frac{\langle u_E \rangle}{\langle u_B \rangle} = \mu_0\varepsilon_0 \frac{E_0^2}{B_0^2} = \mu_0\varepsilon_0 c^2 = 1.$$

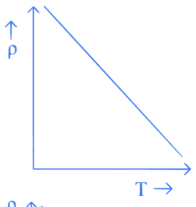
The electric and magnetic fields carry equal shares of the wave energy. Options 4 and 2 incorrectly give the electric part a larger share; option $1/2$ incorrectly gives it a smaller share. The common time-averaging factor cannot produce any of those unequal ratios.

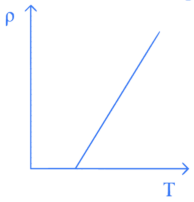
Step 4: Conclusion The ratio of average electric to average magnetic energy density is $\boxed{1:1}$. Hence, the correct option is (A).

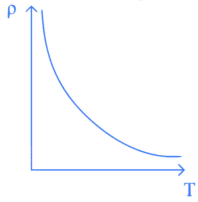
Quick Tip to Solve: For a plane electromagnetic wave, electric and magnetic energy densities are equal even instantaneously; averaging both over the same interval preserves their equality.

46. The resistivity (ρ) of semiconductor varies with temperature. Which of the following curve represents the correct behaviour : [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- A semiconductor has a negative temperature coefficient of resistivity in the usual intrinsic regime: heating creates many additional mobile electron-hole pairs.
- The conductivity is $\sigma = e(n\mu_e + p\mu_h)$, where n, p are carrier densities and μ_e, μ_h are mobilities. Resistivity is $\rho = 1/\sigma$.

Step 1: Identify the dominant effect of heating When temperature increases, thermal energy excites more electrons across the band gap from the valence band to the conduction band. Each excitation also creates a hole, so both kinds of charge carrier become more abundant. Although increased lattice scattering tends to reduce their mobilities, the strong increase in carrier concentration dominates the usual intrinsic-semiconductor behaviour tested here. Thus conductivity rises and resistivity falls with temperature.

Step 2: Determine the qualitative curve The intrinsic carrier concentration has a strong activated dependence, approximately

$$n_i \propto T^{3/2} \exp\left(-\frac{E_g}{2k_{\mathrm{B}}T}\right),$$

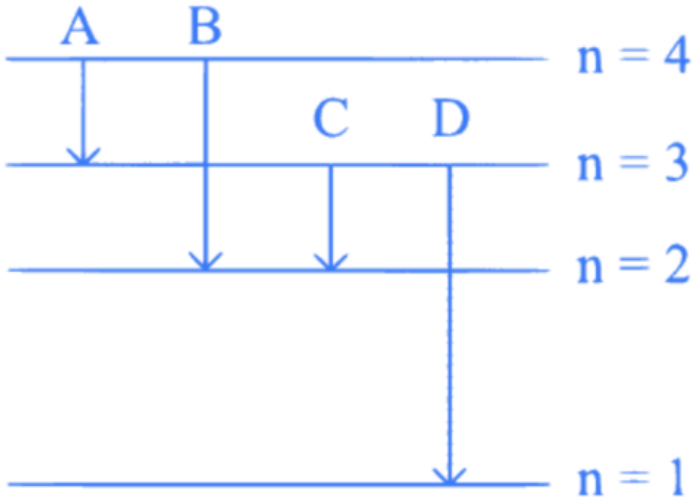
where E_g is the band gap and k_{B} is Boltzmann's constant. Hence the decrease of resistivity is nonlinear over a substantial temperature interval. Starting from a high resistivity at low temperature, the curve drops rapidly and then flattens over the range drawn. It is not a straight line with a fixed negative slope.

Step 3: Compare all four graphical options Options (A) and (C) show resistivity increasing as temperature rises, so both have the wrong sign of temperature coefficient. Option (B) shows a straight decreasing line and misses the characteristic curved temperature dependence. Option (D) shows the required nonlinear decrease, steep initially and progressively flatter. The horizontal axis is temperature and the vertical axis is resistivity, so an increasing conductivity must be read as a decreasing ordinate here.

Step 4: Conclusion The nonlinear decreasing resistivity curve is the graph in option (D). Hence, the correct option is (D).

Quick Tip to Solve: First read the vertical-axis quantity: heating a semiconductor increases conductivity, so its reciprocal, resistivity, follows a decreasing nonlinear curve.

47. The energy levels of an hydrogen atom are shown below. The transition corresponding to emission of shortest wavelength is : [4 marks]



(A) A

(B) C

(C) B

(D) D

Correct Answer: (D) D

Solution:

Concept:

- Hydrogen has energy levels $E_n = -13.6/n^2\,\text{eV}$. A downward transition emits a photon with energy equal to the loss of atomic energy.
- The photon relation is $\Delta E = hc/\lambda$. Therefore the shortest emitted wavelength corresponds to the largest positive energy difference, not merely the largest starting quantum number.

Step 1: Read the four transitions from the diagram The arrows show A as $4 \to 3$, B as $4 \to 2$, C as $3 \to 2$, and D as $3 \to 1$. All four arrows point downward, so each represents emission. The diagram gives the participating quantum numbers; use the hydrogen energy formula to compare the actual energy gaps, since equal-looking spacings in a schematic need not represent equal energies.

Step 2: Calculate the emitted photon energies For a transition $n_i \to n_f$,

$$\Delta E = 13.6\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\text{eV}.$$

Consequently the four energy losses are

$$\Delta E_A = 13.6\left(\frac{1}{9}-\frac{1}{16}\right) \simeq 0.661\,\text{eV},$$
$$\Delta E_B = 13.6\left(\frac{1}{4}-\frac{1}{16}\right) = 2.55\,\text{eV},$$
$$\Delta E_C = 13.6\left(\frac{1}{4}-\frac{1}{9}\right) \simeq 1.889\,\text{eV},$$
$$\Delta E_D = 13.6\left(1-\frac{1}{9}\right) \simeq 12.089\,\text{eV}.$$

Step 3: Select the largest gap and the corresponding option Transition D releases the largest photon energy by a wide margin, so λ_D is the smallest. The alternatives listing A, C and B correspond to smaller gaps and hence longer wavelengths. In particular, starting at $n = 4$ does not make B shorter than D: D ends at the much more deeply bound ground state. The printed option that names transition D is option (D).

Step 4: Conclusion Transition $\boxed{D:\ n = 3 \to n = 1}$ emits the shortest wavelength. Hence, the correct option is (D).

Quick Tip to Solve: Compare $1/n_f^2 - 1/n_i^2$, rather than the number of levels crossed; transitions ending at the ground state often have much larger energy gaps.

48. The number of air molecules per cm^3 increased from 3×10^{19} to 12×10^{19}. The ratio of collision frequency of air molecules before and after the increase in number respectively is: [4 marks]

(A) 1.25

(B) 0.25

(C) 0.50

(D) 0.75

Correct Answer: (B) 0.25

Solution:

Concept:

- The collision frequency of one gas molecule is the average number of collisions it experiences per unit time. It equals $z = \bar{v}/\ell$, where $\bar{v}$ is its mean speed and ℓ is the mean free path.
- For molecular diameter d and number density n, $\ell = 1/(\sqrt{2}\pi d^2 n)$. For the same gas at the same temperature, d and $\bar{v}$ remain unchanged, giving $z \propto n$.

Step 1: Relate collision frequency to number density Combining the mean-free-path expression with the definition of collision frequency gives

$$z = \sqrt{2}\pi d^2 n \bar{v}.$$

The gas species remains air, and only its number of molecules per unit volume is changed. With no

change in temperature specified, compare the frequencies at the same mean molecular speed. A larger number density shortens the average distance travelled between collisions and increases the collision frequency.

Step 2: Insert the initial and final densities The two number densities are

$$n_1 = 3 \times 10^{19}\,\mathrm{cm}^{-3}, \qquad n_2 = 12 \times 10^{19}\,\mathrm{cm}^{-3} = 4n_1.$$

Taking the ratio cancels the collision cross-section, the speed and the common density unit:

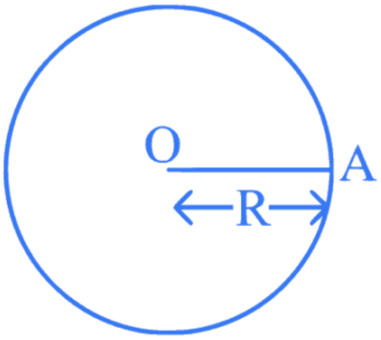
$$\frac{z_1}{z_2} = \frac{n_1}{n_2} = \frac{3 \times 10^{19}}{12 \times 10^{19}} = \frac{1}{4} = 0.25.$$

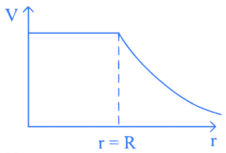
Step 3: Check the order of the requested ratio The problem asks for before divided by after. The final collision frequency is four times the initial value, so the requested ratio must be less than one. Option (A), 1.25, has the wrong trend, while 0.50 and 0.75 correspond to smaller increases than the stated fourfold density change. Do not square the density ratio: that would concern the count of all binary collision events per unit volume, rather than the collision frequency experienced by a molecule.

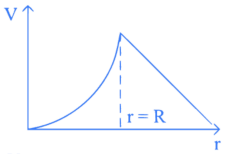
Step 4: Conclusion The collision-frequency ratio before to after is $\boxed{0.25}$, or $1 : 4$. Hence, the correct option is (B).

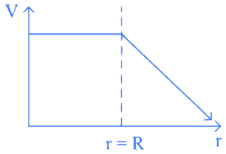
Quick Tip to Solve: For collision frequency per molecule use $z \propto n$; for a before/after ratio, put the initial density in the numerator before simplifying.

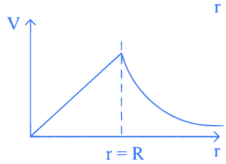
49. For a uniformly charged thin spherical shell, the electric potential (V) radially away from the centre (O) of shell can be graphically represented as - [4 marks]

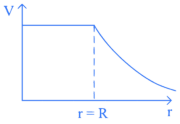


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- For a uniformly charged thin spherical shell of radius R and total charge Q, Gauss's law gives zero electric field inside and $E = kQ/r^2$ outside, where $k = 1/(4\pi\varepsilon_0)$.
- With potential zero at infinity, $V(r) = kQ/r$ for $r \geq R$. Since $E_r = -dV/dr$, zero field inside means constant potential there, equal to the surface value.

Step 1: Determine the potential outside the shell Outside a spherically symmetric charge distribution, the field and potential are those of a point charge at its centre. Thus,

$$V(r) = \frac{kQ}{r} \qquad (r \geq R).$$

For the positive potential drawn in the options, this decreases as an inverse-distance curve and approaches zero only as $r \to \infty$. It does not fall linearly to zero at a finite distance.

Step 2: Determine the potential in the interior For $r < R$, a concentric Gaussian surface encloses no shell charge and the electric field vanishes. Therefore,

$$V(r) - V(R) = -\int_R^r E_r\, dr = 0, \qquad V(r) = \frac{kQ}{R} \quad (0 \leq r \leq R).$$

Zero interior field does not mean zero interior potential: all interior points have the same nonzero potential measured relative to infinity. Potential is continuous at $r = R$, although its slope changes there.

Step 3: Match the full graph and exclude the distractors The correct plot is horizontal from the centre to $r = R$, then follows a decreasing curved $1/r$ tail. This is exactly graph (A). Graphs (B) and (D) incorrectly make potential increase inside the shell, implying a nonzero interior field. Graph (C) correctly has a constant interior portion but incorrectly makes the exterior decrease along a straight line. Both the interior plateau and the exterior inverse-distance dependence are needed.

Step 4: Conclusion The potential is kQ/R inside and kQ/r outside, matching graph (A). Hence, the correct option is (A).

Quick Tip to Solve: For a charged shell remember: field is zero inside, but potential is a constant equal to its surface value; outside, the potential has a curved $1/r$ tail.

50. By what percentage will the transmission range of a TV tower be affected when the height of the tower is increased by 21% ? [4 marks]

(A) 12%

(B) 15%

(C) 10%

(D) 14%

Correct Answer: (C) 10%

Solution:

Concept:

- For a transmitting antenna of height h above a spherical Earth of radius R, the line-of-sight range is $d = \sqrt{2Rh}$, using $h \ll R$.
- For the same Earth, the transmission range is proportional to $\sqrt{h}$. A percentage change of height must therefore be applied inside a square root, rather than directly to the range.

Step 1: Express the increased height Let the initial tower height be h. An increase of 21% makes the new height $h_2 = h + 0.21h = 1.21h$. The radius of Earth remains unchanged, and no separate receiving-antenna height is specified. Thus the question concerns the horizon range associated with this tower.

Step 2: Compare the two ranges Take the ratio so the common factor $2R$ cancels:

$$\frac{d_2}{d_1} = \sqrt{\frac{h_2}{h}} = \sqrt{1.21} = 1.10.$$

Therefore $d_2 - d_1 = 0.10d_1$. The new range is 110 percent of the original range, so the increase is 10 percent, not 110 percent.

Step 3: Check the offered percentages The percentage change is

$$\frac{d_2 - d_1}{d_1} \times 100 = 0.10 \times 100 = 10\%.$$

Options (A), (B) and (D) would imply height factors $1.12^2 = 1.2544$, $1.15^2 = 1.3225$ and $1.14^2 = 1.2996$, respectively. None equals the stated factor 1.21; only option (C) does.

Step 4: Conclusion The transmission range increases by $\boxed{10\%}$. Hence, the correct option is (C).

Quick Tip to Solve: For a finite height increase of $p\%$, use $100[\sqrt{1 + p/100} - 1]\%$; the half-percentage shortcut is only a small-change approximation.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The radius of fifth orbit of the Li^{++} is _____ $\times 10^{-12}$ m.

Take: radius of hydrogen atom $= 0.51\ \overset{o}{A}$ [4 marks]

Solution:

Concept:

- A hydrogen-like ion has only one electron, and its Bohr-orbit radius is $r_n = a_0 n^2/Z$, where Z is its atomic number and a_0 is the ground-state hydrogen radius.
- Lithium has nuclear charge $+3e$. Removing two electrons produces Li^{++}, leaving a single electron attracted by the full nuclear charge, so $Z = 3$.

Step 1: Identify the relevant quantum number The fifth orbit means the principal quantum number is $n = 5$. The charge state of the ion changes the nuclear attraction compared with hydrogen, so merely multiplying the hydrogen radius by 25 would overestimate the answer by a factor of three. Use the given radius of hydrogen rather than replacing it with another rounded value.

Step 2: Convert the given reference radius The stated reference radius is $a_0 = 0.51\,\text{Å}$. Since one angstrom equals $10^{-10}\,\text{m}$,

$$a_0 = 0.51 \times 10^{-10}\,\text{m} = 51 \times 10^{-12}\,\text{m}.$$

This is the radius of the first hydrogen orbit, which is the reference length in the Bohr formula.

Step 3: Calculate the fifth-orbit radius Substitute $n = 5$ and $Z = 3$:

$$r_5 = \frac{5^2}{3}(51 \times 10^{-12})\,\text{m} = \frac{1275}{3} \times 10^{-12}\,\text{m} = 425 \times 10^{-12}\,\text{m}.$$

The requested blank is the coefficient of $10^{-12}\,\text{m}$, not the radius expressed in angstroms. The result is positive, as required for a radius.

Step 4: Conclusion The fifth-orbit radius is $\boxed{425 \times 10^{-12}\,\text{m}}$, so the numerical entry is $\boxed{425}$.

Quick Tip to Solve: For a one-electron ion, remember both factors: radius grows as n^2 and shrinks as Z. Convert the given reference radius to the requested power of ten before substituting.

52. Two identical solid spheres each of mass $2\ \text{kg}$ and radii $10\ \text{cm}$ are fixed at the ends of a light rod. The separation between the centres of the spheres is $40\ \text{cm}$. The moment of inertia of the system about an axis perpendicular to the rod passing through its middle point is _____ $\times 10^{-3}\ \text{kg m}^2$ [4 marks]

Solution:

Concept:

- A uniform solid sphere of mass M and radius r has moment of inertia $I_{cm} = 2Mr^2/5$ about any diameter.
- For a parallel axis a perpendicular distance d from its centre, the parallel-axis theorem gives

$I = I_{\text{cm}} + Md^2$. Moments of inertia of the two spheres add about the common rotation axis.

Step 1: Locate the rotation axis The axis is perpendicular to the light rod and passes through the midpoint between the sphere centres. The centre-to-centre separation is $0.40\,\text{m}$, so each sphere centre is $d = 0.20\,\text{m}$ from this axis. The sphere radius is $r = 0.10\,\text{m}$, and each mass is $M = 2\,\text{kg}$. The light rod contributes negligible rotational inertia.

Step 2: Calculate one sphere's contribution The actual sphere is extended, so include its rotation about its own centre as well as its offset:

$$I_{\text{one}} = \frac{2}{5}(2)(0.10)^2 + 2(0.20)^2.$$

The first term is $0.008\,\text{kg m}^2$, and the second is $0.080\,\text{kg m}^2$. Hence $I_{\text{one}} = 0.088\,\text{kg m}^2$. Treating the sphere as a point would omit the first contribution.

Step 3: Add both identical contributions Both spheres have the same mass and the same offset from the common axis. Therefore

$$I_{\text{total}} = 2(0.088) = 0.176\,\text{kg m}^2 = 176 \times 10^{-3}\,\text{kg m}^2.$$

The centre offsets are half the separation, not the full 40 cm. Every term contains mass times length squared, consistent with the dimensions of moment of inertia.

Step 4: Conclusion The moment of inertia is $\boxed{176 \times 10^{-3}\,\text{kg m}^2}$. The numerical entry is $\boxed{176}$.

Quick Tip to Solve: For two equal extended bodies on a light rod, calculate $2(I_{\text{cm}} + Md^2)$, with d measured from each centre to the rotation axis.

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53. A steel rod has a radius of $20\ \text{mm}$ and a length of $2.0\ \text{m}$. A force of $62.8\ \text{kN}$ stretches it along its length. Young's modulus of steel is $2.0 \times 10^{11}\ \text{N/m}^2$. The longitudinal strain produced in the wire is _____ $\times 10^{-5}$ [4 marks]

Solution:

Concept:

- Young's modulus is the ratio of longitudinal stress to longitudinal strain: $Y = (F/A)/(\Delta L/L)$. Thus the strain is $\epsilon = F/(AY)$.
- For a circular rod of radius r, the cross-sectional area is $A = \pi r^2$. Strain is a ratio of lengths and is dimensionless.

Step 1: Convert the force and radius The applied tensile force is $F = 62.8\,\text{kN} = 6.28 \times 10^4\,\text{N}$. The radius is $r = 20\,\text{mm} = 2.0 \times 10^{-2}\,\text{m}$, not the diameter. The Young's modulus is already in SI units: $Y = 2.0 \times 10^{11}\,\text{N m}^{-2}$. The rod length would be needed to find its absolute extension, but cancels when finding strain directly.

Step 2: Find the tensile stress The area normal to the stretching force is

$$A = \pi(2.0 \times 10^{-2})^2 = 4\pi \times 10^{-4}\,\text{m}^2.$$

Using $\pi \simeq 3.14$, this is $1.256 \times 10^{-3}\,\mathrm{m}^2$. Consequently

$$\sigma = \frac{6.28 \times 10^4}{1.256 \times 10^{-3}} = 5.0 \times 10^7\,\mathrm{N\,m^{-2}}.$$

Step 3: Divide the stress by the modulus The longitudinal strain is

$$\epsilon = \frac{\sigma}{Y} = \frac{5.0 \times 10^7}{2.0 \times 10^{11}} = 2.5 \times 10^{-4} = 25 \times 10^{-5}.$$

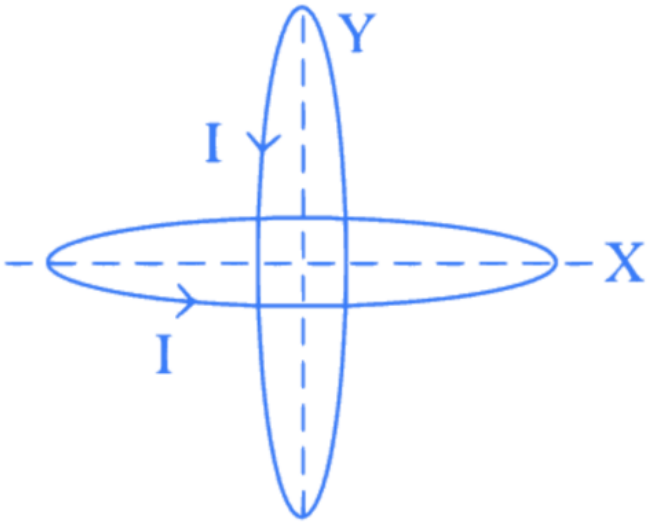
The units of stress and Young's modulus cancel. As a consistency check, the extension of the 2.0 m rod would be $\epsilon L = 5.0 \times 10^{-4}\,\mathrm{m}$, a small extension consistent with elastic stretching. Both the applied tensile stress and the resulting longitudinal strain are positive.

Step 4: Conclusion The longitudinal strain is $\boxed{25 \times 10^{-5}}$, dimensionless, and the numerical entry is $\boxed{25}$.

Quick Tip to Solve: When only strain is asked, use $F/(\pi r^2 Y)$ directly. Keep radius and diameter distinct: substituting the diameter as the radius changes the answer by a factor of four.

54. Two identical circular wires of radius $20\ \mathrm{cm}$ and carrying current $\sqrt{2}\ \mathrm{A}$ are placed in perpendicular planes as shown in figure. The net magnetic field at the centre of the circular wires is _____ $\times 10^{-8}$ T.

(Take $\pi = 3.14$) [4 marks]



Solution:

Concept:

- A single circular loop of radius R, carrying current I, produces a central magnetic field of magnitude $B_0 = \mu_0 I/(2R)$. Its direction is normal to the loop plane by the right-hand rule.

- The loop planes are perpendicular, so their normals and their central magnetic fields are perpendicular. Magnetic fields add as vectors, giving $B_{\text{net}} = \sqrt{B_1^2 + B_2^2}$.

Step 1: Identify the field directions The two wires have the same radius and carry the same current, so the magnitudes of their central fields are equal. The figure places the loop planes at right angles. The current arrows select the signs of the corresponding normal directions, but cannot make these perpendicular fields parallel or antiparallel. Therefore neither their scalar sum nor their difference gives the resultant magnitude.

Step 2: Calculate the field of either wire Convert the radius to SI units: $R = 20\,\text{cm} = 0.20\,\text{m}$. With $I = \sqrt{2}\,\text{A}$ and $\mu_0 = 4\pi \times 10^{-7}\,\text{T m A}^{-1}$,

$$B_0 = \frac{(4\pi \times 10^{-7})\sqrt{2}}{2(0.20)} = \pi\sqrt{2} \times 10^{-6}\,\text{T}.$$

This is the contribution of one single-turn wire, not both wires together.

Step 3: Combine the equal perpendicular fields Apply the Pythagorean rule to the vector components:

$$B_{\text{net}} = \sqrt{2}B_0 = \sqrt{2}(\pi\sqrt{2} \times 10^{-6}) = 2\pi \times 10^{-6}\,\text{T}.$$

Using the specified $\pi = 3.14$,

$$B_{\text{net}} = 6.28 \times 10^{-6}\,\text{T} = 628 \times 10^{-8}\,\text{T}.$$

Expressing the answer with the requested exponent leaves 628 as the numerical coefficient.

Step 4: Conclusion The net central magnetic field has magnitude $\boxed{628 \times 10^{-8}\,\text{T}}$. The numerical entry is $\boxed{628}$.

Quick Tip to Solve: Find the angle between loop normals before adding central fields. Equal fields at right angles have resultant $\sqrt{2}B_0$, whereas the scalar sum $2B_0$ applies only to parallel fields.

55. A pole is vertically submerged in swimming pool, such that it gives a length of shadow $2.15\ \text{m}$ within water when sunlight is incident at angle of 30° with the surface of water. If swimming pool is filled to a height of $1.5\ \text{m}$, then the height of the pole above the water surface in centimeters is $(n_w = 4/3)$ _____. [4 marks]

Solution:

Concept:

- Snell's law at the air-water interface is $\sin i = n_w \sin r$, taking the refractive index of air as one. Both angles are measured from the surface normal.
- The shadow distance on the pool floor equals the horizontal travel in air plus the horizontal travel in water. For a vertical depth d, the underwater contribution is $d \tan r$.

Step 1: Find the underwater ray angle The sunlight makes 30° with the horizontal water surface,

hence $i = 60^\circ$ with its vertical normal. For $n_w = 4/3$,

$$\sin r = \frac{\sin 60^\circ}{4/3} = \frac{3\sqrt{3}}{8}.$$

Therefore $\cos r = \sqrt{37}/8$, so $\tan r = 3\sqrt{3}/\sqrt{37} \simeq 0.85424$. The ray bends towards the normal on entering water.

Step 2: Separate the two horizontal displacements Let h be the pole height above the surface. The shadow-forming ray from the top travels horizontally $h \cot 30^\circ = \sqrt{3}h$ before reaching the water. It then travels horizontally $1.5 \tan r$ through the water depth. Hence the total floor shadow measured from the vertical pole is

$$2.15 = \sqrt{3}h + 1.5\left(\frac{3\sqrt{3}}{\sqrt{37}}\right).$$

The two distances add because the ray continues forwards across the interface.

Step 3: Solve for the exposed pole height The underwater contribution is approximately $1.28136\,\mathrm{m}$. Thus

$$h = \frac{2.15 - 1.28136}{\sqrt{3}}\,\mathrm{m} \simeq 0.50151\,\mathrm{m} = 50.151\,\mathrm{cm}.$$

Expressed as the integer numerical entry, this gives 50 cm. Omitting refraction would assign the entire 2.15 m shadow to the air angle and produce an incorrect exposed height.

Step 4: Conclusion The height above the water is approximately $50.15\,\mathrm{cm}$, giving the numerical entry $\boxed{50}$ in centimetres.

Quick Tip to Solve: Convert an angle given with the water surface to its complementary angle before using Snell's law; then add the horizontal ray displacements in air and water.

56. A particle of mass 10 g moves in a straight line with retardation $2x$, where x is the displacement in SI units. Its loss of kinetic energy for above displacement is $\left(\frac{10}{x}\right)^{-n}$ J. The value of n will be _____

[4 marks]

Solution:

Concept:

- For straight-line motion, the work-energy theorem gives $dK = F\,dx = ma(x)\,dx$, where $a(x)$ is the signed acceleration.
- A retardation of magnitude $2x$ means $a(x) = -2x$ along a positive displacement. The positive loss of kinetic energy is the negative of the work of this retarding force.

Step 1: Write the position-dependent retarding force The particle mass is $m = 10\,\mathrm{g} = 0.010\,\mathrm{kg}$. At a displacement s from the starting point, the retarding force is

$$F(s) = m(-2s) = -0.020s.$$

Here s is a dummy position variable, so it can be integrated from zero to the stated displacement x.

The numerical acceleration law uses SI units as specified in the question.

Step 2: Integrate the work over the displacement The change in kinetic energy is

$$K(x) - K(0) = \int_0^x (-0.020s)\, ds = -0.010x^2 \text{ J}.$$

Therefore the loss is

$$K(0) - K(x) = 0.010x^2 \text{ J}.$$

Because the force varies linearly with position, multiplying its final value by the full displacement would be twice the actual work. The integral correctly accounts for the increasing retardation.

Step 3: Compare with the specified power form The stated positive loss is $(10/x)^{-n}$ J. Rewrite it as $(x/10)^n$ J. The calculated loss can be written

$$0.010x^2 = \frac{x^2}{100} = \left(\frac{x}{10}\right)^2 = \left(\frac{10}{x}\right)^{-2}.$$

Both the power of x and the coefficient agree for $n = 2$. The positive exponent is consistent with an increasing energy loss as displacement grows.

Step 4: Conclusion The kinetic-energy loss is $x^2/100$ J, so $\boxed{n = 2}$. The requested exponent is dimensionless.

Quick Tip to Solve: For retardation proportional to displacement, integrate $m|a(s)|$ over distance. The loss then varies as displacement squared, rather than linearly.

57. The length of a metallic wire is increased by 20% and its area of cross section is reduced by 4%. The percentage change in resistance of the metallic wire is _____. [4 marks]

Solution:

Concept:

- For a uniform wire, $R = \rho L/A$, where ρ is its resistivity, L its length and A its cross-sectional area.
- For the same metal at unchanged temperature, resistivity is constant. Finite percentage changes must be applied as multiplicative factors; a differential approximation is unnecessary here.

Step 1: Express the changed dimensions Let the initial dimensions be L and A. Increasing length by twenty percent and reducing area by four percent gives

$$L' = 1.20L, \qquad A' = 0.96A.$$

The reduction in area increases resistance, so both stated changes act in the same direction. We use the independently supplied dimensional changes rather than imposing a separate volume-conservation condition.

Step 2: Form the exact resistance ratio Apply the resistance formula before and after the changes.

The unchanged resistivity cancels:

$$\frac{R'}{R} = \frac{\rho L'/A'}{\rho L/A} = \frac{1.20}{0.96} = 1.25.$$

Thus the new resistance is 1.25 times the initial resistance. Simply adding the two percentages would give only an approximate change and would miss the denominator effect.

Step 3: Calculate the percentage increase The fractional change is measured relative to the original resistance:

$$\frac{R' - R}{R} \times 100 = (1.25 - 1) \times 100 = 25\%.$$

The sign is positive because the wire is both longer and thinner.

Step 4: Conclusion The resistance increases by 25%. The required numerical entry is $\boxed{25}$.

Quick Tip to Solve: For finite changes in length and area, use $R'/R = (L'/L)/(A'/A)$ before converting to a percentage.

---

58. A parallel plate capacitor with plate area A and plate separation d is filled with a dielectric material of dielectric constant $K = 4$. The thickness of the dielectric material is x, where $x < d$. Let C_1 and C_2 be the capacitance of the system for $\chi = \frac{1}{3}d$ and $\mathcal{X} = \frac{2d}{3}$, respectively. If $C_1 = 2\mu F$ the value of C_2 is _____ μF [4 marks]



Solution:

Concept:

- A slab covering the entire plate area but only part of the separation creates two layers along the electric field. Their capacitances combine in series.
- The effective electrical separation is $d_{\text{eff}} = (d - x) + x/K$, so $C = \varepsilon_0 A/[d - x + x/K]$. Here $K = 4$ is the relative permittivity of the slab.

Step 1: Write the capacitance as a function of slab thickness The air layer has thickness $d - x$ and the dielectric has thickness x. The same charge crosses both layers, while their potential differences add:

$$\frac{1}{C} = \frac{d - x}{\varepsilon_0 A} + \frac{x}{K\varepsilon_0 A}.$$

With $C_0 = \varepsilon_0 A/d$ and $K = 4$, this becomes

$$C = \frac{C_0}{1 - 3x/(4d)}.$$

The two layers are in series, not parallel, because they lie successively between the plates.

Step 2: Evaluate the two thicknesses For the first thickness, $x = d/3$, the denominator is $1-1/4 = 3/4$. For the second, $x = 2d/3$, it is $1 - 1/2 = 1/2$. Therefore

$$C_1 = \frac{4}{3}C_0, \qquad C_2 = 2C_0.$$

Increasing the dielectric thickness decreases the effective separation, so the larger second capacitance is physically consistent.

Step 3: Use the given first capacitance Taking the ratio eliminates the unknown area and separation:

$$\frac{C_2}{C_1} = \frac{2}{4/3} = \frac{3}{2}.$$

Consequently,

$$C_2 = \frac{3}{2}(2\,\mu\text{F}) = 3\,\mu\text{F}.$$

No numerical value of the original plate geometry is required.

Step 4: Conclusion The second capacitance is $\boxed{C_2 = 3\,\mu\text{F}}$; the numerical entry in microfarads is $\boxed{3}$.

Quick Tip to Solve: For a dielectric slab inserted along the plate separation, replace its physical thickness x by the electrical thickness x/K.

---

59. An ideal transformer with purely resistive load operates at $12\ \text{kV}$ on the primary side. It supplies electrical energy to a number of nearby houses at $120\ \text{V}$. The average rate of energy consumption in the houses served by the transformer is 60 kW. The value of resistive load (Rs) required in the secondary circuit will be _____ $\text{m}\Omega$. [4 marks]

Solution:

Concept:

- For a purely resistive AC load, average power is $P = V_{\text{rms}} I_{\text{rms}} = V_{\text{rms}}^2/R$. The power factor is unity.
- An ideal transformer conserves power between primary and secondary. The load resistance must be calculated using the voltage on the same side as that load.

Step 1: Identify the secondary voltage and load power The houses form the secondary load. The secondary voltage is $V_s = 120\,\text{V}$, and their total average consumption is

$$P_s = 60\,\text{kW} = 6.0 \times 10^4\,\text{W}.$$

The primary voltage of $12\,\text{kV}$ specifies the input side; substituting it into the secondary resistance formula would calculate a resistance referred to the wrong winding.

Step 2: Calculate the equivalent resistive load All the houses can be replaced by one equivalent

resistance at the secondary terminals. Rearrange the resistive power equation:

$$R_s = \frac{V_s^2}{P_s} = \frac{(120)^2}{60000} = \frac{14400}{60000} = 0.24\,\Omega.$$

As a check, the secondary current is $I_s = P_s/V_s = 500\,\text{A}$, and $V_s/I_s = 120/500 = 0.24\,\Omega$, giving the same result.

Step 3: Convert the resistance to the requested unit Since $1\,\Omega = 1000\,\text{m}\Omega$,

$$R_s = 0.24 \times 1000 = 240\,\text{m}\Omega.$$

The ideal transformer input current would be $60000/12000 = 5\,\text{A}$, consistent with the hundredfold reduction in voltage and hundredfold increase in current.

Step 4: Conclusion The required secondary resistive load is $\boxed{240\,\text{m}\Omega}$. The numerical entry is $\boxed{240}$.

Quick Tip to Solve: Always pair load power with the voltage across that load; the transformer primary voltage does not belong in $R_s = V_s^2/P_s$.

---

60. A person driving car at a constant speed of $15\ \text{m/s}$ is approaching a vertical wall. The person notices a change of $40\ \text{Hz}$ in the frequency of his car's horn upon reflection from the wall. The frequency of horn is _____ Hz.

(Given: Speed of sound : $330\ \text{m/s}$) [4 marks]

Solution:

Concept:

- A reflected sound undergoes two Doppler transformations: the stationary wall first receives sound from a moving source, and the moving driver then receives the returning wave.
- For sound speed v, source-observer speed u towards a stationary wall and horn frequency f, the received echo frequency is $f_e = f(v+u)/(v-u)$.

Step 1: Find the frequency reaching the stationary wall The horn moves towards the wall at $u = 15\,\text{m}\,\text{s}^{-1}$. The wavefront spacing ahead of the horn is shortened, so the frequency received by the wall is

$$f_w = f\frac{v}{v-u}.$$

The wall is stationary and reflection does not change the frequency in its own frame. Hence the reflected wave in the air has this same frequency.

Step 2: Include the motion of the driver towards the echo The driver moves towards the returning reflected wave. Treating the wall as the stationary source of the echo gives

$$f_e = f_w\frac{v+u}{v} = f\frac{v+u}{v-u}.$$

Therefore the increase relative to the original horn frequency is

$$\Delta f = f_e - f = f\frac{2u}{v-u}.$$

Using only one Doppler shift would omit one part of the reflection process.

Step 3: Substitute the measured frequency change With $\Delta f = 40\,\text{Hz}$, $v = 330\,\text{m s}^{-1}$, and $u = 15\,\text{m s}^{-1}$,

$$40 = f\frac{30}{315}, \qquad f = 40\frac{315}{30} = 420\,\text{Hz}.$$

Indeed the echo is $420(345/315) = 460\,\text{Hz}$, exactly forty hertz above the horn.

Step 4: Conclusion The horn frequency is $\boxed{420\,\text{Hz}}$. The required numerical entry is $\boxed{420}$.

Quick Tip to Solve: For an echo from a stationary wall, apply Doppler twice and simplify to $\Delta f = 2uf/(v-u)$; use the exact denominator here.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: The spin only magnetic moment value for $[Fe(CN)_6]^{3-}$ **is** $1.74BM$**, whereas for** $[Fe\,(H_2O)_6]^{3+}$ **is** $5.92BM$**.**

Reason R : In both complexes, Fe **is present in +3 oxidation state.**

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) A is true but R is false

(C) A is false but R is true

(D) Both A and R are true but R is NOT the correct explanation of A

Correct Answer: (D) Both A and R are true but R is NOT the correct explanation of A

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons.
- For octahedral d^5 ions, strong-field ligands can produce a low-spin arrangement, while weak-field ligands produce a high-spin arrangement. Oxidation state alone does not specify the spin state.

Step 1: Check the oxidation state in the reason In $[Fe(CN)_6]^{3-}$, six cyanide ligands contribute a total charge of minus six. Thus $z - 6 = -3$, giving $z = +3$. In $[Fe(H_2O)_6]^{3+}$, all water ligands are neutral, so iron is again in oxidation state $+3$. Both ions therefore contain Fe^{3+}, with a $3d^5$ configuration. Reason R is true.

Step 2: Check the magnetic moments in the assertion Cyanide is a strong-field ligand and gives the

low-spin configuration $t_{2g}^5 e_g^0$, with one unpaired electron. Hence

$$\mu_{\text{CN}} = \sqrt{1(1+2)} = \sqrt{3} \simeq 1.73\,\text{BM},$$

consistent with the quoted approximate value $1.74\,\text{BM}$. Water is weak-field and gives $t_{2g}^3 e_g^2$, with five unpaired electrons:

$$\mu_{\text{aqua}} = \sqrt{5(5+2)} = \sqrt{35} \simeq 5.92\,\text{BM}.$$

Thus Assertion A is true.

Step 3: Decide whether the reason explains the assertion The common oxidation state establishes the same total number of d electrons, but cannot explain their different pairing. The different ligand-field strengths, and hence different numbers of unpaired electrons, explain the magnetic moments. Option A incorrectly treats R as a sufficient explanation, while B and C incorrectly reject one of the true statements.

Step 4: Conclusion Both A and R are true, but R is not the correct explanation of A. Hence, the correct option is (D).

Quick Tip to Solve: In magnetic-moment assertion questions, determine oxidation state first, then compare ligand strength before counting unpaired electrons.

62. The standard electrode potential of M^+/M in aqueous solution does not depend on [4 marks]

(A) Ionisation of a gaseous metal atom

(B) Sublimation of a solid metal

(C) Ionisation of a solid metal atom

(D) Hydration of a gaseous metal ion

Correct Answer: (C) Ionisation of a solid metal atom

Solution:

Concept:

- The standard potential for the $\text{M}^+(\text{aq})/\text{M}(\text{s})$ electrode is linked to the free-energy change for electron transfer between the solid metal and its hydrated aqueous ion.
- A useful thermodynamic cycle resolves metal oxidation into atomisation or sublimation, gaseous-atom ionisation, and hydration of the resulting gaseous ion.

Step 1: Write the steps connecting the physical states The metal begins as a solid, whereas the final cation is hydrated in water. The conventional cycle is

$$\begin{aligned} \text{M}(\text{s}) &\longrightarrow \text{M}(\text{g}), \\ \text{M}(\text{g}) &\longrightarrow \text{M}^+(\text{g}) + \text{e}^-, \\ \text{M}^+(\text{g}) &\longrightarrow \text{M}^+(\text{aq}). \end{aligned}$$

These steps represent sublimation, ionisation of a gaseous atom, and hydration of a gaseous metal ion, respectively. Their combined free-energy contribution governs the tendency to form the aqueous ion.

Step 2: Assess the listed dependencies Option A is a genuine contribution: removing an electron from the gaseous atom requires ionisation energy. Option B is also relevant because atoms must be separated from the solid metal lattice. Option D is relevant because stabilisation by water determines the energy of the aqueous cation. Consequently none of A, B or D is the requested independent process.

Step 3: Identify the process outside this cycle Option C refers to ionisation of a solid metal atom. This is not the gaseous-atom ionisation step used in the standard electrode-potential cycle. The energetic cost of leaving the solid is already represented separately by sublimation, followed by ionisation in the gaseous state. A separate solid-atom ionisation term is therefore not one of the stated dependencies.

Step 4: Conclusion The standard electrode potential does not depend on the separately stated ionisation of a solid metal atom. Hence, the correct option is (C).

Quick Tip to Solve: Remember the sequence solid metal to gaseous atom to gaseous ion to hydrated ion; match each option to one of these steps.

63. For the reaction

The correct statement is [4 marks]

$$RCH_2Br + I \xrightarrow{\text{Acetone}} \underset{\text{major}}{RCH_2I} + Br$$

(A) The solvent used in the reaction solvates the ions formed in rate determining step.

(B) Br^- can act as competing nucleophile.

(C) The reaction can occur in acetic acid also.

(D) The transition state formed in the above reaction is less polar than the localised anion.

Correct Answer: (D) The transition state formed in the above reaction is less polar than the localised anion.

Solution:

Concept:

- The displacement of bromide from a primary alkyl bromide by iodide in acetone proceeds by the concerted S_N2 mechanism.
- An anionic nucleophile has a localised negative charge before reaction. In the S_N2 transition state, this charge is distributed over the entering and leaving groups, reducing charge localisation.

Step 1: Identify the mechanism and transition state The displayed conversion is $RCH_2Br + I^- \longrightarrow RCH_2I + Br^-$ in acetone, with the iodide product major. Iodide attacks the carbon as the carbon-bromine bond breaks. The transition state may be represented schematically as

$$\left[I \cdots CH_2R \cdots Br\right]^{-\ddagger}.$$

There is no separate carbocation intermediate or ion-forming slow step, which rules out the description in A.

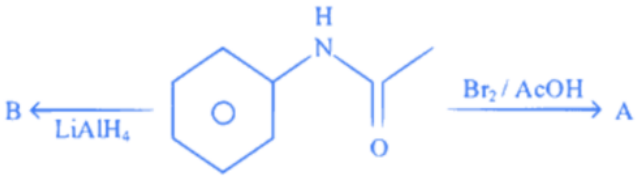
Step 2: Compare solvation and charge distribution Acetone is a polar aprotic solvent: it does not strongly hydrogen-bond to and stabilise the attacking anion in the manner of a protic solvent. The initial iodide ion carries a concentrated negative charge. In the transition state that negative charge is spread over two halogen centres, so the transition state is less polar in the stated sense than the localised anion. This is the comparison made in D.

Step 3: Exclude the remaining proposed descriptions In the standard acetone halide-exchange conditions, formation of insoluble bromide salt removes bromide from solution and drives the major iodide product. Bromide is consequently not an effective competing nucleophile in this reaction mixture, so B is not the correct description. Acetic acid is protic and strongly solvates anions; it does not provide the stated acetone conditions for this halide exchange, so C is rejected. The decisive mechanistic statement is the reduced charge localisation in the transition state.

Step 4: Conclusion The transition state is less polar than the localised anion because its negative charge is dispersed. Hence, the correct option is (D).

Quick Tip to Solve: For an anion reacting with a neutral alkyl halide by S_N2, compare localised reactant charge with the dispersed transition-state charge.

64. The major products A and B from the following reactions are: [4 marks]



(A) A: 2,4,6-tribromoacetanilide; B: $C_6H_5NHCH(OH)CH_3$.

(B) A: 4-bromoacetanilide; B: $C_6H_5NHCH_2CH_3$.

(C) A: 2,4,6-tribromoacetanilide; B: aniline, $C_6H_5NH_2$.

(D) A: 4-bromoacetanilide; B: $C_6H_5NHCH(OH)CH_3$.

Correct Answer: (B) A: 4-bromoacetanilide; B: $C_6H_5NHCH_2CH_3$.

Solution:

Concept:

- Acetylation moderates the activating effect of the amino group: the $-NHCOCH_3$ group remains ortho/para directing, but its lone pair is partly delocalised into the carbonyl.
- $LiAlH_4$ reduces an amide carbonyl to a methylene group while retaining its carbon atom and the carbon–nitrogen bond.

Step 1: Identify the electrophilic substitution product The substrate is acetanilide, $C_6H_5NHCOCH_3$. Bromine in acetic acid brominates its aromatic ring. The acetamido group directs substitution to the ortho and para positions; the less crowded para position gives the major product. Thus A is 4-bromoacetanilide, p-$BrC_6H_4NHCOCH_3$. Unlike free aniline in bromine water, the protected amino group does not give the displayed 2,4,6-tribromo product as the major product under these conditions.

Step 2: Apply complete amide reduction The separate reduction replaces the carbonyl oxygen by two hydrogens; it does not cleave off the acetyl carbon. Consequently the two-carbon substituent remains attached to nitrogen.

$$C_6H_5NHCOCH_3 \xrightarrow{LiAlH_4} C_6H_5NHCH_2CH_3$$

B is N-ethylaniline, a secondary amine. A carbon bearing both OH and NH would describe a partially reduced addition product rather than the isolated product of complete amide reduction.

Step 3: Eliminate the other product pairs Option A is wrong in both its tribromination pattern and its alcohol-containing B. Option C incorrectly combines tribromination with aniline; making aniline would discard the two-carbon acyl fragment instead of reducing it. Option D gets the para-brominated A right but stops the reduction at an alcohol-containing structure. Only option B combines the correct regioselectivity with the correct complete reduction product.

Step 4: Conclusion A is 4-bromoacetanilide and B is N-ethylaniline, $C_6H_5NHCH_2CH_3$. Hence, the correct option is (B).

Quick Tip to Solve: For amide reduction, mentally replace $C{=}O$ by CH_2 without deleting its carbon; acetanilide therefore gives an N-ethyl group.

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65. Which of the following options are correct for the reaction

$$2\,[Au(CN)_2]^-\,(aq) + Zn(s) \rightarrow 2Au(s) + [Zn(CN)_4]^{2-}\,(aq)$$

A. Redox reaction

B. Displacement reaction

C. Decomposition reaction

D. Combination reaction

Choose the correct answer from the options given below: [4 marks]

(A) A and B only

(B) C and D only

(C) A only

(D) A and D only

Correct Answer: (A) A and B only

Solution:

Concept:

- A redox reaction involves a change in oxidation states, with oxidation and reduction occurring together.
- A displacement reaction replaces a less reactive metal in its compound by a more reactive metal; the ligand can remain in a complex ion.

Step 1: Calculate the metal oxidation states Each cyanide ligand carries charge -1. In $[Au(CN)_2]^-$, if the gold oxidation state is x, then $x - 2 = -1$, giving $x = +1$. Product gold is the free element, with

oxidation state zero. Gold is therefore reduced. Zinc starts as the elemental metal, oxidation state zero. In $[Zn(CN)_4]^{2-}$, its oxidation state satisfies $y - 4 = -2$, so $y = +2$. Zinc is oxidised.

Step 2: Check electron transfer and displacement The changes can be represented by

$$Zn \longrightarrow Zn^{2+} + 2\,e^-$$

$$2\,Au^+ + 2\,e^- \longrightarrow 2\,Au$$

These equations track oxidation states; the actual ions remain coordinated to cyanide in the full reaction. Zinc supplies the two electrons required to reduce two gold centres. Thus statement A is correct. Zinc also replaces gold from its cyanide complex and forms its own cyanide complex, so statement B is correct.

Step 3: Exclude decomposition and combination A decomposition reaction has one reactant breaking into simpler products, which is not the pattern here: a complex ion and zinc react together. A combination reaction joins reactants into a single product; here metallic gold and a zinc complex are both formed. Therefore C and D are false. Option B contains two false classifications; option C omits the valid displacement classification; option D includes the false combination classification.

Step 4: Conclusion The reaction is both redox and displacement: A and B only. Hence, the correct option is (A).

Quick Tip to Solve: First track the oxidation states of the metals, then inspect the reaction pattern; redox and displacement are compatible descriptions of one reaction.

66. For a concentrated solution of a weak electrolyte (K_{eq} = equilibrium constant) A_2B_3 of concentration 'c', the degree of dissociation 'α' is :

[4 marks]

(A) $\left(\frac{K_{eq}}{25c^2}\right)^{\frac{1}{5}}$

(B) $\left(\frac{K_{eq}}{108c^4}\right)^{\frac{1}{5}}$

(C) $\left(\frac{K_{eq}}{5c^4}\right)^{\frac{1}{5}}$

(D) $\left(\frac{K_{eq}}{6c^5}\right)^{\frac{1}{5}}$

Correct Answer: (B) $\left(\frac{K_{eq}}{108c^4}\right)^{\frac{1}{5}}$

Solution:

Concept:

- The law of mass action uses the equilibrium concentration of each ion raised to its stoichiometric coefficient.
- For a concentrated solution of a weak electrolyte, the degree of dissociation α is small, so $1 - \alpha \simeq 1$.

Step 1: Write the dissociation and concentration balance For a neutral formula unit A_2B_3, the

stoichiometric dissociation is

$$A_2B_3 \rightleftharpoons 2\,A^{3+} + 3\,B^{2-}.$$

The charges ensure neutrality, although the equilibrium calculation depends on the coefficients 2 and 3. Start with concentration c and negligible free ions. If the fraction dissociated is α, the equilibrium concentrations are

$$[A_2B_3] = c(1-\alpha), \quad [A^{3+}] = 2c\alpha, \quad [B^{2-}] = 3c\alpha.$$

Step 2: Build and simplify the equilibrium expression Using the stoichiometric powers,

$$K_{eq} = \frac{(2c\alpha)^2(3c\alpha)^3}{c(1-\alpha)} = \frac{108c^4\alpha^5}{1-\alpha}.$$

The coefficient is $2^2 \times 3^3 = 108$, not the sum or product of the two ion coefficients. The concentration power is $2 + 3 - 1 = 4$, because the undissociated electrolyte appears in the denominator.

Step 3: Use weak dissociation and compare options For the stated concentrated weak-electrolyte solution, neglect α relative to 1 in the denominator. Then

$$K_{eq} \simeq 108c^4\alpha^5, \qquad \alpha \simeq \left(\frac{K_{eq}}{108c^4}\right)^{1/5}.$$

Option A uses the wrong coefficient and concentration power. Option C has the right powers but replaces the stoichiometric coefficient 108 by 5. Option D has both the wrong coefficient and a concentration power of 5 instead of 4.

Step 4: Conclusion

$$\boxed{\alpha = \left(\frac{K_{eq}}{108c^4}\right)^{1/5}}$$

under the weak-dissociation approximation. Hence, the correct option is (B).

Quick Tip to Solve: For A_pB_q, obtain the prefactor as p^pq^q; the concentration power is $p + q - 1$, while the power of α is $p + q$.

67. Match List I with List II

	LIST I Oxide		LIST II Type of bond
A.	N_2O_4	**I.**	**1 N = O bond**
B.	NO_2	**II.**	**1 N − O − N bond**
C.	N_2O_5	**III.**	**1 N − N bond**
D.	N_2O	**IV.**	**1 N=N / N ≡ N bond**

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-I, C-III, D-IV

(B) A-III, B-I, C-II, D-IV

(C) A-III, B-I, C-IV, D-II

(D) A-II, B-IV, C-III, D-I

Correct Answer: (B) A-III, B-I, C-II, D-IV

Solution:

Concept:

- Bond connectivity distinguishes nitrogen oxides more reliably than oxidation number alone; distinguish an N–N bond from an N–O–N bridge.
- Resonance redistributes bond order, but the underlying atom connectivity remains fixed. The list identifies characteristic bonds in the conventional molecular structures.

Step 1: Match dinitrogen tetroxide and nitrogen dioxide N_2O_4 is formed by pairing two NO_2 radicals through a direct nitrogen–nitrogen single bond. Its skeleton is $O_2N{-}NO_2$, so A corresponds to III. It does not contain an oxygen bridge between the nitrogen atoms. In the conventional resonance forms of NO_2, one N–O link is drawn as an N=O double bond and the other as an N–O bond, with the unpaired electron accounted for. Thus the listed feature for B is I.

Step 2: Match dinitrogen pentoxide and nitrous oxide The molecular form of N_2O_5 has the skeleton $O_2N{-}O{-}NO_2$. The central oxygen joins the two nitrogen atoms, giving one N–O–N bridge. Hence C corresponds to II. This is the molecular structure intended by the bond-connectivity list. Nitrous oxide has an N–N–O skeleton; its resonance description includes $N{\equiv}N^{+}{-}O^{-}$ and a form with N=N. Therefore D corresponds to IV.

Step 3: Compare the full sequence The four matches are A–III, B–I, C–II and D–IV. Option A swaps the direct N–N bond of N_2O_4 with the oxygen bridge of N_2O_5. Option C assigns the nitrogen–nitrogen multiple-bond entry to N_2O_5, which instead has an oxygen bridge. Option D also assigns the wrong entries to both NO_2 and N_2O. Only option B preserves all four connectivities.

Step 4: Conclusion A–III, B–I, C–II, D–IV. Hence, the correct option is (B).

Quick Tip to Solve: Remember the contrasting skeletons $O_2N{-}NO_2$ and $O_2N{-}O{-}NO_2$: the extra oxygen in the latter is the bridge.

68. The difference between electron gain enthalpies will be maximum between : [4 marks]

(A) Ar and Cl

(B) Ne and Cl

(C) Ne and F

(D) Ar and F

Correct Answer: (B) Ne and Cl

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change for adding an electron to an isolated gaseous atom; exothermic gain has a negative value.
- Closed-shell noble gases have positive electron gain enthalpies, whereas halogens have strongly negative values. The greatest signed separation pairs the most positive and most negative entries.

Step 1: Compare fluorine and chlorine Although fluorine is above chlorine in the periodic table, chlorine has the more negative electron gain enthalpy. The electron added to fluorine enters its compact 2p shell and experiences stronger electron–electron repulsion. In chlorine it enters the larger 3p shell with less crowding. Representative values are approximately

$$\Delta_{\text{eg}}H(\text{F}) = -328, \qquad \Delta_{\text{eg}}H(\text{Cl}) = -349\ \text{kJ mol}^{-1}.$$

Thus chlorine provides the more negative endpoint.

Step 2: Compare the noble gases Neon and argon have stable filled valence shells, so an incoming electron must occupy a higher-energy shell and electron addition is endothermic. Neon has the more positive electron gain enthalpy of these two noble gases: approximately $+116\ \text{kJ mol}^{-1}$, compared with $+96\ \text{kJ mol}^{-1}$ for argon. The smaller closed-shell neon atom is therefore the positive endpoint for the largest difference.

Step 3: Check all four pairwise differences Using these conventional approximate values, the magnitudes of the separations are

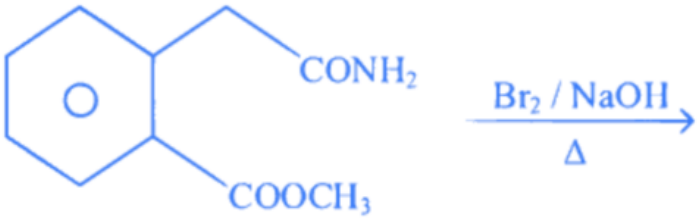
$$\begin{aligned} |H(\text{Ar}) - H(\text{Cl})| &\simeq 445, \\ |H(\text{Ne}) - H(\text{Cl})| &\simeq 465, \\ |H(\text{Ne}) - H(\text{F})| &\simeq 444, \\ |H(\text{Ar}) - H(\text{F})| &\simeq 424 \end{aligned} \quad \text{kJ mol}^{-1}.$$

The Ne–Cl separation is largest. Choosing F solely because it is more electronegative confuses electronegativity with the enthalpy of electron addition to an isolated atom.

Step 4: Conclusion The maximum difference is between Ne and Cl. Hence, the correct option is (B).

Quick Tip to Solve: Keep the signs: compare a positive noble-gas value with a negative halogen value, then remember that chlorine has a more negative electron gain enthalpy than fluorine.

69. The major product formed in the following reaction is [4 marks]



(A) Fused six-membered diketone ring: o-$C_6H_4(-CH_2-CO-CH_2-CO-)$.

(B) Fused cyclic anhydride: o-$C_6H_4(-CH_2-CO-O-CO-)$.

(C) Fused cyclic imide: o-$C_6H_4(-CH_2-CO-NH-CO-)$.

(D) Isoindolin-1-one, the five-membered lactam: o-$C_6H_4(-CH_2-NH-CO-)$.

Correct Answer: (D) Isoindolin-1-one, the five-membered lactam: o-$C_6H_4(-CH_2-NH-CO-)$.

Solution:

Concept:

- Hofmann bromamide rearrangement converts a primary amide $RCONH_2$ into RNH_2, with loss of the original amide carbonyl carbon.
- An amino group and an ester in neighbouring substituents can undergo intramolecular acyl substitution, forming a stable five-membered cyclic amide (lactam).

Step 1: Identify the group that undergoes rearrangement The substrate has adjacent aromatic substituents $-CH_2CONH_2$ and $-COOCH_3$. Bromine with sodium hydroxide and heat gives Hofmann rearrangement of the primary amide. The group migrating from the amide carbonyl to nitrogen is the whole substituted benzyl group, so the side-chain conversion is

$$Ar-CH_2-CONH_2 \xrightarrow{Br_2/NaOH} Ar-CH_2-NH_2.$$

Only the amide carbonyl carbon is removed; the benzylic CH_2 remains.

Step 2: Close the five-membered lactam ring The newly formed $-CH_2NH_2$ group is adjacent to the ester. Its nucleophilic nitrogen attacks the ester carbonyl intramolecularly, and loss of methanol gives the cyclic amide. The ring path is aromatic carbon–CH_2–NH–CO–adjacent aromatic carbon. It therefore contains five atoms, and it is fused to the benzene ring. The retained carbonyl comes from the ester, while the amide carbonyl has already been removed in the rearrangement. The product is isoindolin-1-one.

Step 3: Eliminate products with the wrong atom count Option A is a six-membered carbon ring with two carbonyls and no nitrogen. Option B is a cyclic anhydride, with an oxygen in place of the required nitrogen. Option C is a six-membered imide retaining two carbonyl groups; it would not account for loss of the original amide carbonyl carbon. Option D alone shows the required five-membered ring, one NH and one retained carbonyl, with the unchanged benzylic methylene.

Step 4: Conclusion The major product is isoindolin-1-one, a five-membered lactam with one NH and one carbonyl group. Hence, the correct option is (D).

Quick Tip to Solve: In Hofmann rearrangement, cross out the primary-amide carbonyl carbon first; then check whether the resulting amine can cyclise onto a neighbouring ester.

70. Match List I with List II

	LIST I Enzymatic reaction		LIST II Enzyme
A.	Sucrose $\rightarrow$ Glocuse and Fructose	I.	Zymase
B.	Glucose $\rightarrow$ ethyl alcohol and CO_2	II.	Pepsin
C.	Starch $\rightarrow$ Maltose	III.	Invertase
D.	Proteins $\rightarrow$ Amino acids	IV.	Diastase

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-II, D-IV

(B) A-III, B-I, C-IV, D-II

(C) A-I, B-IV, C-III, D-II

(D) A-I, B-II, C-IV, D-III

Correct Answer: (B) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Enzymes are selective biological catalysts. In matching questions, identify the substrate and the transformation before comparing answer codes.
- Invertase acts on sucrose, zymase brings about alcoholic fermentation, diastase acts on starch, and pepsin digests proteins.

Step 1: Match sucrose hydrolysis and fermentation Invertase catalyses hydrolysis of sucrose to the monosaccharides glucose and fructose. Thus A matches III. Its name recalls the inversion of optical rotation observed on hydrolysis of sucrose; it is not the enzyme mixture responsible for fermentation. Zymase, the enzyme complex in yeast, brings about conversion of glucose into ethyl alcohol and carbon dioxide. The overall reaction is

$$C_6H_{12}O_6 \longrightarrow 2\,C_2H_5OH + 2\,CO_2.$$

Therefore B matches I.

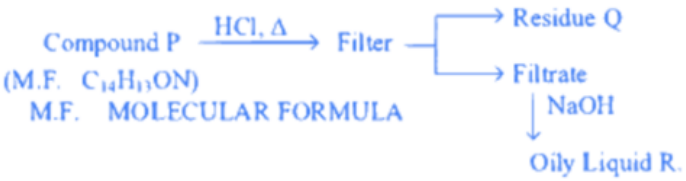
Step 2: Match starch and protein digestion Diastase, an amylolytic enzyme preparation, catalyses conversion of starch into maltose, so C matches IV. Pepsin is the proteolytic enzyme among the listed choices, giving D–II for the stated protein-digestion entry. Strictly, pepsin first forms smaller peptides, and further peptidases complete digestion to amino acids; the table uses the overall protein-to-amino-acids description. This does not change which of the four named enzymes belongs to the protein entry.

Step 3: Compare every proposed sequence The complete sequence is A–III, B–I, C–IV, D–II. Option A correctly matches the first two entries but incorrectly exchanges diastase and pepsin for starch and protein digestion. Option C assigns zymase to sucrose and invertase to starch, and option D similarly mismatches sucrose, fermentation and protein digestion. Option B alone places each substrate with the appropriate enzyme or enzyme preparation. The pepsin simplification in the question should not be taken to mean that diastase can digest proteins.

Step 4: Conclusion A–III, B–I, C–IV, D–II. Hence, the correct option is (B).

Quick Tip to Solve: Anchor the two distinctive pairs first: sucrose–invertase and fermentation–zymase; then starch–diastase leaves proteins–pepsin.

71. Compound P is neutral, Q gives effervescence with $NaHCO_3$ while R reacts with Hinsbergs reagent to give solid soluble in $NaOH$. Compound P is [4 marks]



(A) $C_6H_5CON(CH_3)C_6H_5$ (N-methyl-N-phenylbenzamide)

(B) $4\text{-}CH_3C_6H_4CONHC_6H_5$ (N-phenyl-4-methylbenzamide)

(C) $C_6H_5COC_6H_4NHCH_3$ (the methylamino group is meta to the carbonyl group)

(D) 3-Methyl-5-phenylbenzamide: $C_6H_3(CH_3)(C_6H_5)CONH_2$

Correct Answer: (B) $4\text{-}CH_3C_6H_4CONHC_6H_5$ (N-phenyl-4-methylbenzamide)

Solution:

Concept:

- An amide is essentially neutral because the nitrogen lone pair is delocalised into the carbonyl group. On acidic hydrolysis it yields a carboxylic acid and the ammonium salt corresponding to its nitrogen substituent.
- A carboxylic acid liberates CO_2 from $NaHCO_3$. A primary amine gives a sulfonamide containing an acidic N–H bond in the Hinsberg test; this product dissolves in aqueous $NaOH$.

Step 1: Identify the hydrolysis products The residue Q must be a carboxylic acid, since its reaction is

$$RCOOH + NaHCO_3 \longrightarrow RCOONa + CO_2 + H_2O.$$

The filtrate contains the protonated amine formed in hot hydrochloric acid. Adding sodium hydroxide releases the free amine R, observed as an oily liquid. These observations are characteristic of hydrolysis of a substituted amide.

Step 2: Use the Hinsberg observation The solid obtained from R with benzenesulfonyl chloride must retain an N–H bond, allowing deprotonation by alkali. Thus R is a primary amine. Aniline, $C_6H_5NH_2$, meets this condition: its sulfonamide $C_6H_5SO_2NHC_6H_5$ forms a soluble sodium salt. A secondary amine instead gives a sulfonamide with no N–H and it is insoluble in alkali.

Step 3: Test all four structures Option B hydrolyses to 4-methylbenzoic acid and anilinium chloride; base releases aniline. Its overall neutral hydrolysis is

$$4\text{-}CH_3C_6H_4CONHC_6H_5 + H_2O \longrightarrow 4\text{-}CH_3C_6H_4COOH + C_6H_5NH_2.$$

Option A yields N-methylaniline, a secondary amine, and fails the Hinsberg solubility test. Option C contains a ketone and a secondary amine rather than a hydrolysable amide, and is not the neutral precursor required. Option D is a primary carboxamide and gives ammonium salt rather than the required oily primary aromatic amine.

Step 4: Conclusion P is N-phenyl-4-methylbenzamide, $4\text{-}CH_3C_6H_4CONHC_6H_5$; Q is 4-methylbenzoic acid and R is aniline. Hence, the correct option is (B).

Quick Tip to Solve: Trace the amide C–N bond: the acyl side becomes the acid, while the nitrogen side identifies the amine tested by Hinsberg reagent.

72. Strong reducing and oxidizing agents among the following, respectively, are : [4 marks]

(A) Ce^{4+} and Eu^{2+}

(B) Eu^{2+} and Ce^{4+}

(C) Ce^{4+} and Tb^{4+}

(D) Ce^{3+} and Ce^{4+}

Correct Answer: (B) Eu^{2+} and Ce^{4+}

Solution:

Concept:

- A reducing agent donates an electron and is oxidised; an oxidising agent accepts an electron and is reduced. The order asked is reducing agent first, oxidising agent second.
- The +3 oxidation state is usual for lanthanoids. The ease of conversion between +2, +3 and +4 states determines the redox behaviour of the listed ions.

Step 1: Identify the electron donor Europium(II) is a strong reducing agent because it can donate an electron and become europium(III):

$$Eu^{2+} \longrightarrow Eu^{3+} + e^{-}.$$

Although Eu^{2+} has a half-filled $4f^7$ configuration, in aqueous redox chemistry conversion to the common +3 state makes it a powerful electron donor. A half-filled configuration alone does not make an ion incapable of oxidation.

Step 2: Identify the electron acceptor Cerium(IV) readily accepts an electron and becomes cerium(III):

$$Ce^{4+} + e^{-} \longrightarrow Ce^{3+}.$$

Therefore Ce^{4+} acts as a strong oxidising agent. Its high positive charge and favourable reduction to the trivalent ion account for this behaviour; the initial ion is reduced while it oxidises the other reactant.

Step 3: Check the order in each option The required pair is therefore Eu^{2+} followed by Ce^{4+}, exactly option B. Option A reverses their roles. Option C incorrectly puts Ce^{4+}, an oxidising ion, in the reducing-agent position. Option D correctly places Ce^{4+} second but replaces the strong reducing agent Eu^{2+} with Ce^{3+}. The two desired half-reactions combine as $Eu^{2+} + Ce^{4+} \longrightarrow Eu^{3+} + Ce^{3+}$.

Step 4: Conclusion The strong reducing and oxidising agents, respectively, are Eu^{2+} and Ce^{4+}. Hence, the correct option is (B).

Quick Tip to Solve: Write the electron on the right for the reducing agent and on the left for the oxidising agent before checking the order of the pair.

73. Match List I with List II

	LIST I Element detected		LIST II Reagent used / Product formed
A.	**Nitrogen**	**I.**	$Na_2[Fe(CN)_5NO]$
B.	**Sulphur**	**II.**	$AgNO_3$
C.	**Phosphorous**	**III.**	$Fe_4[Fe(CN)_6]_3$
D.	**Halogen**	**IV.**	$(NH_4)_2MoO_4$

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-II, B-I, C-IV, D-III

(C) A-IV, B-II, C-I, D-III

(D) A-II, B-IV, C-I, D-III

Correct Answer: (A) A-III, B-I, C-IV, D-II

Solution:

Concept:

- In Lassaigne analysis, fusion with sodium converts covalently bound nitrogen, sulphur and halogens into ionic species that can be tested in the aqueous extract.
- The matching must distinguish a reagent from the coloured product of a test: Prussian blue is a product for nitrogen, whereas sodium nitroprusside, silver nitrate and ammonium molybdate are reagents.

Step 1: Match nitrogen and sulphur Nitrogen forms sodium cyanide on sodium fusion. With ferrous ions it gives ferrocyanide; ferric ions then yield the Prussian-blue compound $Fe_4[Fe(CN)_6]_3$. Thus nitrogen A matches III. Sulphur forms sodium sulphide in the fusion extract, which gives a violet colour with sodium nitroprusside, $Na_2[Fe(CN)_5NO]$. Hence sulphur B matches I. The two tests use different iron complexes and must not be interchanged.

Step 2: Match phosphorus and halogens Phosphorus is oxidised to phosphate, which gives a yellow ammonium phosphomolybdate precipitate on treatment with ammonium molybdate in nitric acid. Thus phosphorus C matches IV, $(NH_4)_2MoO_4$. Halogens in the sodium extract are detected with silver nitrate after acidification with nitric acid. The corresponding silver halide precipitates according to $Ag^+ + X^- \longrightarrow AgX$, giving D-II. Acidification removes interfering cyanide and sulphide species.

Step 3: Compare the complete matching The four independently obtained links are A-III, B-I, C-IV and D-II. Option A contains all four. Option B incorrectly assigns silver nitrate to nitrogen and Prussian blue to halogen. Option C incorrectly associates nitrogen with ammonium molybdate and sulphur with silver nitrate. Option D also assigns silver nitrate to nitrogen and ammonium molybdate to sulphur. Therefore no alternative has the full correct correspondence.

Step 4: Conclusion The correct matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (A).

Quick Tip to Solve: Remember the diagnostic colours: nitrogen gives Prussian blue, sulphur gives violet with nitroprusside, and phosphate gives a yellow molybdate precipitate.

74. Polymer used in orlon is : [4 marks]

(A) Polyethene

(B) Polyacrylonitrile

(C) Polycarbonate

(D) Polyamide

Correct Answer: (B) Polyacrylonitrile

Solution:

Concept:

- Orlon is the trade name of an acrylic synthetic fibre whose polymer is polyacrylonitrile, commonly abbreviated PAN.
- Addition polymerisation opens the carbon–carbon double bond of acrylonitrile, $CH_2{=}CHCN$, while retaining its pendant nitrile group.

Step 1: Identify the monomer and repeat unit Acrylonitrile contains both a polymerisable alkene bond and a nitrile substituent. Successive monomer molecules join by addition across the alkene bond. The polymer chain therefore contains the repeating structural unit

$$[-CH_2 - CH(CN)-]_n .$$

This is polyacrylonitrile. The nitrile group is retained on every second carbon of the chain; it is not converted into an amide linkage during this polymerisation.

Step 2: Connect the polymer to the fibre Polyacrylonitrile can be spun into durable acrylic fibres sold under the trade name Orlon. Its fibres have a wool-like feel and are used as synthetic substitutes for wool in clothing and blankets. The question asks for the polymer identity, so the presence of an acrylonitrile-derived carbon backbone bearing nitrile groups is the decisive feature rather than the general fact that several polymers can form useful fibres.

Step 3: Eliminate the other polymers Polyethene is produced from ethene and has the repeat unit $[-CH_2 - CH_2-]_n$; it contains no nitrile substituent and is not Orlon. Polycarbonate contains carbonate linkages in its chain and belongs to a different polymer class. Polyamides, such as nylons, contain recurring $-CO-NH-$ linkages. They must not be confused with polyacrylonitrile, which has pendant $-CN$ groups rather than amide bonds. Only option B names the acrylic polymer required.

Step 4: Conclusion The polymer used in Orlon is polyacrylonitrile. Hence, the correct option is (B).

Quick Tip to Solve: Associate Orlon with acrylic fibre: its monomer is acrylonitrile and its repeating chain retains the nitrile group, $-CN$.

75. Match List I with List II

	LIST I Vitamin		LIST II Deficiency disease
A.	Vitamin A	I.	Beri-Beri
B.	Thiamine	II.	Cheilosis
C.	Ascorbic acid	III.	Xeropthalmia
D.	Riboflavin	IV.	Scurvy

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-IV, B-I, C-III, D-II

(C) A-IV, B-II, C-III, D-I

(D) A-III, B-II, C-IV, D-I

Correct Answer: (A) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Vitamins are required in small amounts for normal physiological functions, and their deficiencies lead to characteristic disorders. Matching uses the specific vitamin, including its common chemical name.
- Thiamine is vitamin B1, riboflavin is vitamin B2, and ascorbic acid is vitamin C. Vitamin A is associated with healthy vision and epithelial tissues.

Step 1: Match vitamin A and thiamine Vitamin A deficiency causes xerophthalmia, in which the eye surface becomes abnormally dry; visual disturbances can also occur. Thus A matches III. Thiamine is vitamin B1, whose deficiency causes beri-beri, involving nervous-system and cardiovascular disturbances. Therefore B matches I. The disease beri-beri should not be assigned to riboflavin merely because both thiamine and riboflavin belong to the B-vitamin group.

Step 2: Match ascorbic acid and riboflavin Ascorbic acid is vitamin C. Deficiency impairs normal collagen formation and causes scurvy, commonly associated with bleeding gums and poor wound healing. Thus C matches IV. Riboflavin is vitamin B2; its deficiency causes cheilosis, with cracking at the corners of the mouth, and related mucosal changes. Hence D matches II. These clinical associations distinguish the two water-soluble vitamins despite their shared need in the diet.

Step 3: Verify each proposed set Combining the links gives A-III, B-I, C-IV and D-II, which is option A. Option B swaps the vitamin A and vitamin C disorders. Option C also assigns scurvy to vitamin A and xerophthalmia to vitamin C, while mixing up the B-vitamin diseases. Option D correctly matches A and C but reverses thiamine and riboflavin. Checking all four links prevents accepting an option that contains only some familiar correct pairs.

Step 4: Conclusion The complete matching is vitamin A–xerophthalmia, thiamine–beri-beri, ascorbic acid–scurvy and riboflavin–cheilosis: A-III, B-I, C-IV, D-II. Hence, the correct option is (A).

Quick Tip to Solve: Use B1–beri-beri, B2–cheilosis and C–scurvy as fixed pairs, then match vitamin A to the eye disorder.

76. The setting time of Cement is increased by adding : [4 marks]

(A) Gypsum

(B) Limestone

(C) Clay

(D) Silica

Correct Answer: (A) Gypsum

Solution:

Concept:

- The setting of Portland cement involves hydration reactions of its components. Tricalcium aluminate reacts especially rapidly with water and can cause undesirable flash setting.
- Gypsum, $CaSO_4 \cdot 2\,H_2O$, is added in a small proportion during cement manufacture to retard the rapid aluminate hydration and provide sufficient working time.

Step 1: Interpret an increase in setting time Increasing the setting time means delaying the point at which fresh cement paste becomes rigid. It does not mean increasing the initial hydration rate. A useful additive must therefore control the fast-setting component so that cement can be mixed, transported and placed before appreciable stiffening. Tricalcium aluminate, conventionally written $3\,CaO \cdot Al_2O_3$, is the component whose very rapid hydration needs to be moderated.

Step 2: Identify the retarding additive Gypsum supplies sulphate ions that react with the aluminate phase to form calcium sulphoaluminate hydrate, commonly called ettringite. Formation of this sulphate-containing hydration product controls the initial rapid reaction of tricalcium aluminate and prevents flash setting. Consequently the cement paste remains workable for a longer period. The additive fulfilling the stated purpose is therefore gypsum rather than a principal raw material used to produce the clinker.

Step 3: Compare the remaining choices Limestone is primarily the calcium carbonate raw material that supplies lime during manufacture; it is not the standard setting-time regulator asked here. Clay supplies silica and alumina to the raw mixture. Silica is a constituent involved in forming calcium silicates, which contribute to cement strength, rather than the small sulphate additive used to control flash set. Thus options B, C and D name raw materials or constituents, whereas option A names the intended retarder.

Step 4: Conclusion Gypsum is added to increase the setting time of cement by controlling rapid aluminate hydration. Hence, the correct option is (A).

Quick Tip to Solve: For cement, distinguish the clinker raw materials from the setting regulator: gypsum delays flash setting; limestone and clay supply the clinker constituents.

77. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Loss of electron from hydrogen atom results in nucleus of $\sim 1.5 \times 10^{-3}$pm size.

Reason R : Proton (H^+) always exists in combined form.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(B) A is correct but R is not correct

(C) A is not correct but R is correct

(D) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**

Correct Answer: (D) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**

Solution:

Concept:

- An ordinary hydrogen atom has one proton in its nucleus and one electron outside it. Removing its only electron leaves the proton, H^+, rather than an atom-sized species.
- Assertion–Reason questions require separate truth checks followed by a causal check: two true statements need not stand in an explanation relationship.

Step 1: Evaluate Assertion A For the usual protium atom, ionisation is

$$H \longrightarrow H^+ + e^-.$$

The remaining species is its bare nucleus, a proton. Its nuclear size is of the order quoted, $1.5 \times 10^{-3}\,\text{pm} = 1.5 \times 10^{-15}\,\text{m}$, enormously smaller than the electronic dimensions of a hydrogen atom. Thus Assertion A is correct. Loss of the electron removes the electron cloud; it does not compress an atom into a new kind of nucleus.

Step 2: Evaluate Reason R in the chemical context Because a proton has extremely small size and hence very high charge density, it has a strong tendency to associate with electron-rich species. In ordinary chemical systems it occurs in combined or solvated forms rather than as an isolated free proton. In water, for example, it is represented as H_3O^+ and more extensively hydrated ions. This is the standard chemical meaning of the statement that H^+ always exists in combined form. Reason R is therefore correct in this context.

Step 3: Test whether the reason explains the assertion Combined existence describes how a proton behaves after it is produced in a chemical environment. It does not explain why removing the only electron from hydrogen leaves a nucleus of nuclear dimensions. The latter follows from the structure of the hydrogen atom. Indeed, the small size and large charge density help explain the tendency to combine, so the proposed causal direction is reversed. Option A incorrectly asserts the explanation, while B and C incorrectly reject one of the true statements.

Step 4: Conclusion Both A and R are correct, but R is not the correct explanation of A. Hence, the correct option is (D).

Quick Tip to Solve: After checking both statements, ask which direction the explanation runs: the tiny proton size explains its strong association, not the reverse.

78. Match List I with List II.

	List I: Name of reaction		List II: Reagent used
A.	Hell-Volhard-Zelinsky reaction	I.	$NaOH + I_2$
B.	Iodoform reaction	II.	(i) CrO_2Cl_2, CS_2; (ii) H_2O
C.	Etard reaction	III.	(i) Br_2/red phosphorus; (ii) H_2O
D.	Gatterman-Koch reaction	IV.	CO, HCl, anhydrous $AlCl_3$

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-III, B-I, C-II, D-IV

(C) A-I, B-II, C-III, D-IV

(D) A-III, B-II, C-I, D-IV

Correct Answer: (B) A-III, B-I, C-II, D-IV

Solution:

Concept:

- The reagents of a named reaction identify the functional-group transformation. Hell-Volhard-Zelinsky (HVZ) is alpha-halogenation of a carboxylic acid containing an alpha-hydrogen.
- Iodoform formation uses iodine in alkali; Etard oxidation uses chromyl chloride; Gatterman-Koch formylation introduces a formyl group into an aromatic ring.

Step 1: Match the halogenation reactions In the HVZ reaction, bromine and red phosphorus first bring about substitution at the alpha-carbon of a carboxylic acid. Hydrolysis gives the alpha-bromo acid, so A matches III. The iodoform reaction requires I_2 and NaOH; a methyl ketone, or a suitable compound oxidisable to it, yields yellow CHI_3. Thus B matches I. These are different processes: an alpha-bromo carboxylic acid is not the iodoform product.

Step 2: Match oxidation and formylation Etard oxidation converts the methyl group of an aromatic methyl compound into an aldehyde using CrO_2Cl_2 in CS_2, followed by hydrolysis. Hence C matches II. Gatterman-Koch formylation uses CO and HCl with anhydrous $AlCl_3$ (conventionally with a copper(I) chloride promoter), so D matches IV. The reagent list printed for IV therefore belongs to Gatterman-Koch, not Etard.

Step 3: Check the complete code Combining the four independent identifications gives

$$\mathrm{A-III},\quad \mathrm{B-I},\quad \mathrm{C-II},\quad \mathrm{D-IV}.$$

Option (A) interchanges the Etard and Gatterman-Koch reagents. Option (C) assigns alkaline iodine to HVZ and the HVZ reagents to Etard. Option (D) assigns chromyl chloride to iodoform and alkaline iodine to Etard; both assignments are incorrect.

Step 4: Conclusion The correct matching is A-III, B-I, C-II, D-IV. Hence, the correct option is (B).

Quick Tip to Solve: Anchor the match with CrO_2Cl_2 for Etard and I_2/NaOH for iodoform; these distinctive pairs quickly eliminate the wrong codes.

79. A compound is formed by two elements X and Y. The element Y forms cubic close packed arrangement and those of element X occupy one third of the tetrahedral voids. What is the formula of the compound? [4 marks]

(A) XY_3

(B) X_3Y_2

(C) X_3Y

(D) X_2Y_3

Correct Answer: (D) X_2Y_3

Solution:

Concept:

- A close-packed array containing N particles has $2N$ tetrahedral voids and N octahedral voids. Cubic close packing and face-centred cubic packing describe the same arrangement.
- The empirical formula uses the simplest whole-number ratio of the actual numbers of X and Y particles, not the ratio of occupied voids to all voids.

Step 1: Count sites in the close-packed host Let the number of Y atoms in the cubic close-packed host be N. The number of available tetrahedral sites is then $2N$. This factor of two is essential: one-third occupancy refers to one-third of the tetrahedral sites, not to one-third of the Y atoms. Only tetrahedral occupancy is specified; there is no reason to include octahedral sites.

Step 2: Apply the fractional occupancy Every occupied tetrahedral site contains an X atom. Since one-third of those sites are occupied,

$$N_X = \frac{1}{3}(2N) = \frac{2N}{3}, \qquad N_Y = N.$$

Therefore,

$$N_X : N_Y = \frac{2N}{3} : N = 2 : 3.$$

One can also take three conventional fcc cells: they contain twelve Y atoms and twenty-four tetrahedral sites. Filling one-third of these sites gives eight X atoms, and $8 : 12 = 2 : 3$.

Step 3: Compare the formulae Writing X first, the ratio obtained is X_2Y_3. The other candidates have X:Y ratios $1 : 3$, $3 : 2$, and $3 : 1$, respectively, so none reproduces the required occupancy. In particular, X_3Y_2 incorrectly reverses the ratio. Fractional atom counts in one unit cell are permissible averages, but the final formula is always expressed with whole-number subscripts.

Step 4: Conclusion The compound has the formula X_2Y_3. Hence, the correct option is (D).

Quick Tip to Solve: For tetrahedral occupancy fraction f, write X:Y $= 2f : 1$, then clear fractions; octahedral occupancy would instead give $f : 1$.

80. The possibility of photochemical smog formation is more at : [4 marks]

(A) Industrial areas

(B) Himalayan villages in winter

(C) The places with healthy vegetation

(D) Marshy lands

Correct Answer: (A) Industrial areas

Solution:

Concept:

- Photochemical smog is oxidising smog produced when nitrogen oxides and volatile organic compounds react under sunlight. Its secondary pollutants include ozone and peroxyacyl nitrates (PAN).
- Formation is favoured by strong precursor emissions together with sunlight and suitable atmospheric conditions. The question asks for the location most favourable among the choices, rather than an absolute guarantee of smog.

Step 1: Identify the required pollutants and light Industrial and urban activity releases nitrogen oxides and unburnt hydrocarbons, providing the precursor mixture for photochemical smog. Sunlight photodissociates NO_2; the oxygen atom generated can form ozone:

$$NO_2 \xrightarrow{\text{light}} NO + O, \qquad O + O_2 \longrightarrow O_3.$$

Organic radical chemistry helps ozone and other oxidants accumulate. Thus a polluted industrial region has a much greater supply of reactants than an unpolluted rural or naturally vegetated site.

Step 2: Assess each setting Industrial areas are the appropriate choice because combustion and related emissions supply both nitrogen oxides and organic pollutants. Himalayan villages in winter are relatively cold and are not the characteristic warm, sunny, highly polluted setting for photochemical smog. Healthy vegetation is not evidence of a high industrial precursor burden; high oxidant levels can actually injure plants. Marshy lands are associated with anaerobic decomposition and gases such as methane, not specifically with the nitrogen-oxide and hydrocarbon pollution mixture needed here.

Step 3: Distinguish the two smog types Cold, humid conditions are commonly associated with classical reducing smog containing smoke and sulphur dioxide, whereas photochemical smog is an oxidising mixture driven by sunlight. Merely finding fog, cold weather, or natural organic matter does not establish photochemical smog. Of the four listed environments, industrial areas best supply the necessary pollution precursors.

Step 4: Conclusion The possibility is greatest in industrial areas. Hence, the correct option is (A).

Quick Tip to Solve: Associate photochemical smog with sunlight plus NO_x and hydrocarbons; associate cold, sulphurous smoke-fog with classical smog.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Number of bromo derivatives obtained on treating ethane with excess of Br_2 in diffused sunlight is _____ [4 marks]

Solution:

Concept:

- Diffused sunlight initiates radical substitution of hydrogen by bromine. With excess bromine, replacement need not stop after monobromination and can continue until all six hydrogens of ethane are replaced.
- Each carbon can carry zero to three bromine atoms. The two carbon atoms are equivalent, so an arrangement (a, b) and its reversal (b, a) represent the same constitutional derivative.

Step 1: Represent every derivative without double counting Let a and b be the numbers of bromine atoms attached to the two carbons. Then $0 \le a, b \le 3$, and at least one bromine must be present. Choose $a \le b$ so that turning the molecule around does not count the same product twice. Ethane itself, $(0, 0)$, is not a bromo derivative. Bromine substitution preserves the carbon-carbon single bond.

Step 2: Enumerate by total bromine substitution For one bromine, the distribution is $(0, 1)$: one derivative. For two bromines, $(0, 2)$ and $(1, 1)$: two derivatives. For three, $(0, 3)$ and $(1, 2)$: two derivatives. For four, $(1, 3)$ and $(2, 2)$: two derivatives. For five, only $(2, 3)$: one derivative. For six, only $(3, 3)$: one derivative. These correspond to the different positions of substitution, including the completely brominated compound $CBr_3{-}CBr_3$.

Step 3: Add the distinct constitutional products The total is

$$1 + 2 + 2 + 2 + 1 + 1 = 9.$$

As an independent count, the unordered pairs allowing repetition from $0, 1, 2, 3$ number $\binom{5}{2} = 10$. Excluding $(0, 0)$ again gives nine. Rotation about the single bond creates conformations, not additional derivatives; no separate conformational count is required. No carbon here can have four different substituents, so there is no extra enantiomer count either.

Step 4: Conclusion The number of distinct bromo derivatives is $\boxed{9}$ (a dimensionless count).

Quick Tip to Solve: Count unordered pairs of substitution numbers on equivalent carbons; exclude the unreacted molecule and avoid counting a reversed pair twice.

82. The wavelength of an electron of kinetic energy 4.50×10^{-29} J is _____ $\times 10^{-5}$ m. (Nearest integer)

Given : mass of electron is 9×10^{-31} kg, h $= 6.6 \times 10^{-34}$ J s [4 marks]

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the electron momentum.
- For nonrelativistic kinetic energy K, $K = p^2/(2m)$, so $\lambda = h/\sqrt{2mK}$. The extremely small energy given is safely in the nonrelativistic regime.

Step 1: Calculate the momentum from the given energy Use the values in the question without replacing them by more precise physical constants:

$$\begin{aligned} 2mK &= 2(9 \times 10^{-31})(4.50 \times 10^{-29}) \\ &= 81.0 \times 10^{-60} \\ &= 8.10 \times 10^{-59}\ \mathrm{kg^2\,m^2\,s^{-2}}. \end{aligned}$$

Taking the square root is easiest in the first exponential form:

$$p = \sqrt{81.0 \times 10^{-60}} = 9.0 \times 10^{-30}\ \mathrm{kg\,m\,s^{-1}}.$$

The exponent is halved only after the numerical factor is arranged consistently.

Step 2: Find the wavelength and convert the requested scale Substituting this momentum in the wavelength formula gives

$$\lambda = \frac{6.6 \times 10^{-34}}{9.0 \times 10^{-30}} = 0.7333\ldots \times 10^{-4}\ \mathrm{m} = 7.3333\ldots \times 10^{-5}\ \mathrm{m}.$$

The unit is correct because $\mathrm{J\,s/(kg\,m\,s^{-1})} = \mathrm{m}$. The blank asks for the coefficient multiplying 10^{-5} m, so we round $7.3333\ldots$, not the wavelength expressed directly in metres.

Step 3: Apply the nearest-integer instruction The coefficient lies between seven and eight and is closer to seven. Thus the numerical response is 7. Retain the unrounded wavelength during the calculation; rounding the tiny energy or momentum too early can obscure the power of ten. The large wavelength relative to ordinary atomic dimensions is consistent with the exceptionally small electron kinetic energy supplied.

Step 4: Conclusion The wavelength is $7.3333\ldots \times 10^{-5}$ m; the required nearest-integer coefficient is $\boxed{7}$.

Quick Tip to Solve: Before taking a square root, rewrite the momentum square with an even exponent; here 81×10^{-60} avoids an awkward half-integer power.

83. The number of species from the following which have square pyramidal structure is _____

$$PF_5, BrF_4^-, IF_5, BrF_5, XeOF_4, ICl_4^-$$

[4 marks]

Solution:

Concept:

- VSEPR predicts shape from the steric number: the number of sigma-bonding domains plus lone pairs on the central atom. A multiple bond counts as one geometrical domain.
- An AX_5E species has six electron domains, an octahedral electron-pair arrangement, and a square pyramidal molecular shape. AX_4E_2 is square planar, whereas AX_5 is trigonal bipyramidal.

Step 1: Classify phosphorus and the tetrahalide anions In PF_5, phosphorus has five bonding domains and no lone pair, giving AX_5, which is trigonal bipyramidal. In $BrF_4{}^-$, the bromine valence count including the negative charge leaves two lone pairs after forming four bonds. It is AX_4E_2, with the lone pairs opposite one another, so the molecular shape is square planar. $ICl_4{}^-$ has the same domain count and is also square planar. These three species therefore do not contribute to the requested total.

Step 2: Identify the five-coordinate species with one lone pair For both IF_5 and BrF_5, the central halogen has seven valence electrons. Five are used in the five single bonds and the remaining two form one lone pair. Both are AX_5E, hence square pyramidal. In $XeOF_4$, xenon forms four Xe-F bonds and one Xe=O bond, and retains one lone pair. The double bond to oxygen is one domain, so the molecule again has five bonding domains and one lone pair. It too is square pyramidal.

Step 3: Count only the qualifying molecular shapes The qualifying set is

$$IF_5, \qquad BrF_5, \qquad XeOF_4.$$

There are exactly three members. The octahedral electron-domain geometry must not be confused with the observable molecular shape: omission of the lone-pair vertex leaves the square pyramid. Conversely, two opposite lone pairs remove two vertices and leave a square planar shape rather than a pyramid.

Step 4: Conclusion The number of square pyramidal species is $\boxed{3}$ (a dimensionless count).

Quick Tip to Solve: Look for six electron domains with exactly one lone pair; remember that Xe=O occupies one domain when counting the shape of $XeOF_4$.

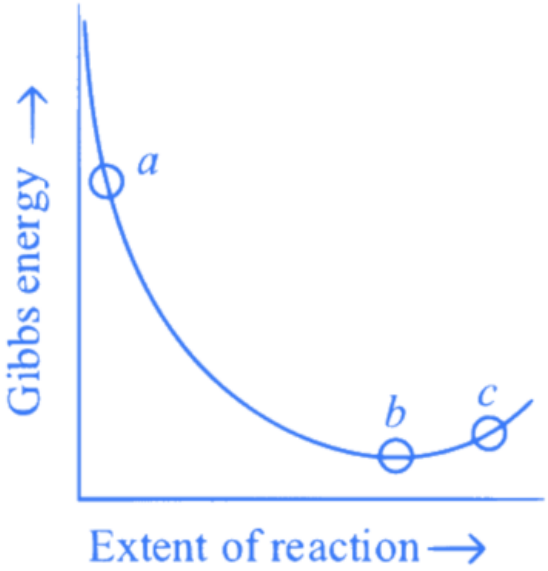
84. Consider the graph of Gibbs free energy G vs Extent of reaction. The number of statement/s from the following which are true with respect to points (a), (b) and (c) is _____

A. Reaction is spontaneous at (a) and (b)

B. Reaction is at equilibrium at point (b) and non-spontaneous at point (c)

C. Reaction is spontaneous at (a) and non-spontaneous at (c)

D. Reaction is non-spontaneous at (a) and (b) [4 marks]



Solution:

Concept:

- At constant temperature and pressure, spontaneous change proceeds in the direction that lowers the total Gibbs energy G. Equilibrium occurs at a minimum of the curve of G against reaction extent ξ.
- The slope $(\partial G/\partial \xi)_{T,p} = \Delta_r G$ determines the forward direction: negative slope favours forward reaction, zero slope identifies equilibrium, and positive slope makes forward reaction non-spontaneous.

Step 1: Read the slope at the marked points At (a), the plotted curve falls as reaction extent increases. Its slope is negative, so a small forward reaction lowers Gibbs energy and is spontaneous. At (b), the curve reaches its minimum. Its slope is zero, and the reaction is at equilibrium rather than undergoing a net spontaneous forward change. At (c), the curve rises with increasing extent; the positive slope means that moving farther forward would raise Gibbs energy. Forward reaction is therefore non-spontaneous there, while reverse reaction would lower Gibbs energy.

Step 2: Test each printed statement Statement A says the reaction is spontaneous at both (a) and (b). It is false because (b) is equilibrium. Statement B correctly identifies equilibrium at (b) and non-spontaneity of the forward reaction at (c), so B is true. Statement C correctly identifies forward spontaneity at (a) and forward non-spontaneity at (c), so C is true. Statement D is false because the forward reaction at (a) is spontaneous and (b) is an equilibrium point, not a non-spontaneous forward region.

Step 3: Count the true statements Only B and C survive the slope test. Hence the number is two. Do not decide spontaneity from the absolute vertical value of G: it is the change in G with an infinitesimal increase of reaction extent that matters. Also, the label non-spontaneous at (c) refers to the forward direction shown by increasing extent, and does not mean that the system cannot react in the reverse direction toward the minimum.

Step 4: Conclusion Statements B and C are true. The required number is $\boxed{2}$ (a dimensionless count).

Quick Tip to Solve: On a Gibbs-energy versus extent graph, read the local slope: downhill is spontaneous forward change, the minimum is equilibrium, and uphill favours reversal.

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85. Mass of Urea (NH_2CONH_2) required to be dissolved in 1000 g of water in order to reduce the vapour pressure of water by 25% is _____ g. (Nearest integer)

Given: Molar mass of N, C, O and H are $14, 12, 16$ and 1 g mol^{-1} respectively [4 marks]

Solution:

Concept:

- For a nonvolatile solute such as urea, Raoult's law gives $p = x_w p^0$, where p^0 is the vapour pressure of pure water and x_w is its mole fraction in the solution.
- The relative lowering is the solute mole fraction: $(p^0 - p)/p^0 = x_u = n_u/(n_u + n_w)$. Use this exact expression when the lowering is as large as 25 percent.

Step 1: Find the amount of water and the urea molar mass The solvent is water, with molar mass $18\,\text{g mol}^{-1}$. Its amount is therefore

$$n_w = \frac{1000}{18} = \frac{500}{9}\,\text{mol}.$$

From the given atomic masses, the molar mass of NH_2CONH_2 is

$$M_u = 2(14) + 12 + 16 + 4(1) = 60\,\text{g mol}^{-1}.$$

Urea is a molecular, nonvolatile solute, so no ionic dissociation factor is required.

Step 2: Apply the specified relative lowering A reduction by 25 percent means $p = 0.75p^0$, not $p = 0.25p^0$. Thus

$$\frac{n_u}{n_u + 500/9} = \frac{1}{4}.$$

On multiplying through,

$$4n_u = n_u + \frac{500}{9}, \qquad n_u = \frac{500}{27}\,\text{mol}.$$

The required mass is

$$m_u = \frac{500}{27} \times 60 = 1111.11\ldots\,\text{g}.$$

The solute amount is one third of the solvent amount; omitting it from the mole-fraction denominator would give an incorrect smaller mass.

Step 3: Conclusion The required mass of urea is $1111.11\ldots$ g, so the nearest-integer answer is $\boxed{1111\,\text{g}}$.

Quick Tip to Solve: For a fractional vapour-pressure lowering f, solve $n_u/n_w = f/(1-f)$. Do not use the dilute approximation for a 25 percent lowering.

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86. The value of $\log \text{K}$ for the reaction $\text{A} \rightleftharpoons \text{B}$ at 298 K is _____. (Nearest integer)

Given: $\Delta \text{H}^\circ = -54.07\ \text{kJ mol}^{-1}$

$$\Delta \text{S}^\circ = 10\ \text{J K}^{-1}\ \text{mol}^{-1}$$

(Take $2.303 \times 8.314 \times 298 = 5705$) [4 marks]

Solution:

Concept:

- At temperature T, the standard Gibbs energy change is $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$. Enthalpy and the entropy term must use the same energy unit.
- The thermodynamic equilibrium constant obeys $\Delta G^\circ = -RT \ln K = -2.303RT \log K$. Here log denotes the common logarithm and K is dimensionless.

Step 1: Calculate the standard Gibbs energy change Convert the enthalpy change from kilojoules to joules before using the entropy in joules:

$$\Delta H^\circ = -54070\,\text{J mol}^{-1}, \qquad T\Delta S^\circ = 298(10) = 2980\,\text{J mol}^{-1}.$$

Hence

$$\Delta G^\circ = -54070 - 2980 = -57050\,\text{J mol}^{-1}.$$

The positive entropy change makes the Gibbs energy still more negative at this temperature. This sign implies that products are strongly favoured in the standard equilibrium relation.

Step 2: Convert the Gibbs energy into the common logarithm Rearrange the equilibrium relation before substitution:

$$\log K = -\frac{\Delta G^\circ}{2.303RT}.$$

The question supplies the entire denominator as 5705, so there is no need to approximate its factors separately. Substituting gives

$$\log K = -\frac{-57050}{5705} = 10.$$

Both minus signs are essential: a large negative standard Gibbs energy must give a large positive logarithm, and hence an equilibrium constant much greater than unity. As a check, the result corresponds to $K = 10^{10}$, not $K = 10$.

Step 3: Conclusion At 298 K, $\boxed{\log K = 10}$. The requested nearest integer is 10, and the logarithm has no unit.

Quick Tip to Solve: Calculate ΔG° in joules first, then use the supplied $2.303RT$. A negative ΔG° requires $\log K > 0$.

87. Number of ambidentate ligands in a representative metal complex $[M(en)(SCN)_4]$ is _____. [en = ethylenediamine] [4 marks]

Solution:

Concept:

- An ambidentate ligand has two different possible donor atoms but normally attaches to a given metal through only one of them at a time. Its alternative modes of attachment can produce linkage isomerism.
- Denticity counts donor atoms simultaneously bonded to the same metal. Ethylenediamine, en, is a bidentate chelating ligand; thiocyanate, SCN^-, is an ambidentate ligand.

Step 1: Classify each kind of ligand Ethylenediamine has the structure $H_2NCH_2CH_2NH_2$. Its two nitrogen donor atoms can bind the same metal and form a chelate ring. These two equivalent nitrogen donors make en bidentate, rather than ambidentate. By contrast, thiocyanate can coordinate through its sulfur atom, giving a thiocyanato-S linkage, or through its nitrogen atom, giving a thiocyanato-N linkage. These are alternative donor choices for the same ion; they do not make each thiocyanate a bidentate ligand.

Step 2: Count the ambidentate ligand units in the formula The representative complex is $[M(en)(SCN)_4]$. The subscript 4 applies to the entire thiocyanate ligand, so it contains four separate thiocyanate ligand units. Every one belongs to the ambidentate class, whereas the single en ligand does not. Therefore

$$N_{\text{ambidentate}} = 4 \times 1 + 1 \times 0 = 4.$$

Do not answer 2 by counting the possible donor types of thiocyanate, or 6 by counting all the metal-ligand donor bonds. Neither of those is the requested number of ambidentate ligands.

Step 3: Conclusion There are $\boxed{4}$ ambidentate ligands in the complex. This is a count and has no physical unit.

Quick Tip to Solve: First classify each ligand, then multiply by its subscript. Ambidentate means alternative donor atoms; bidentate means two donor atoms used together.

88. If 5 moles of $BaCl_2$ is mixed with 2 moles of Na_3PO_4, the maximum number of moles of $Ba_3(PO_4)_2$ formed is _____ (Nearest integer) [4 marks]

Solution:

Concept:

- The balanced precipitation equation fixes the mole ratios of reactants to product. Coefficients compare amounts in moles, rather than masses or numbers of constituent ions taken separately.
- The limiting reagent gives the smaller possible product amount: divide each available reactant amount by its stoichiometric coefficient, then multiply the smaller extent by the product coefficient.

Step 1: Balance the precipitation reaction Barium ions combine with phosphate ions to form insoluble

barium phosphate. Balancing the formula units gives

$$3\,BaCl_2 + 2\,Na_3PO_4 \longrightarrow Ba_3(PO_4)_2 + 6\,NaCl.$$

One mole of $Ba_3(PO_4)_2$ therefore requires three moles of $BaCl_2$ and two moles of Na_3PO_4. The barium and phosphate stoichiometry comes from the neutral formula $Ba_3(PO_4)_2$, with three divalent barium ions for two trivalent phosphate ions.

Step 2: Identify the limiting reagent and the product amount Calculate the product separately from each reactant:

$$n_{\text{product, from } BaCl_2} = \frac{5}{3}\,\text{mol}, \qquad n_{\text{product, from } Na_3PO_4} = \frac{2}{2} = 1\,\text{mol}.$$

Sodium phosphate allows the smaller amount and is the limiting reagent. Producing one mole uses $3\,\text{mol}$ of barium chloride and all $2\,\text{mol}$ of sodium phosphate, leaving $5 - 3 = 2\,\text{mol}$ of barium chloride unreacted. The excess barium chloride cannot produce additional precipitate after the phosphate is exhausted. Thus the theoretical maximum is exactly one mole, before any rounding.

Step 3: Conclusion The maximum amount of $Ba_3(PO_4)_2$ formed is $\boxed{1\,\text{mol}}$. Its nearest-integer numerical value is 1.

Quick Tip to Solve: Compare available moles divided by balanced coefficients: $5/3$ versus $2/2$. The smaller quotient sets the reaction extent.

89. In ammonium - phosphomolybdate, the oxidation state of Mo is + _____ [4 marks]

Solution:

Concept:

- The sum of oxidation states in a neutral compound is zero; in a polyatomic ion it equals the overall ionic charge. Oxygen has oxidation state -2 in this ordinary oxoanion.
- Ammonium phosphomolybdate has the composition $(NH_4)_3[PMo_{12}O_{40}]$. Three NH_4^+ ions balance the phosphomolybdate ion $[PMo_{12}O_{40}]^{3-}$, in which phosphorus is $+5$.

Step 1: Separate the ionic charges from oxidation states Ammonium contributes a total charge of $+3$ from its three monovalent cations. Consequently the heteropoly anion has charge -3. It contains one phosphorus atom, twelve molybdenum atoms and forty oxygen atoms. Use $+5$ for phosphorus and -2 for each oxygen in this phosphate-derived oxoanion. The charge on ammonium is not the oxidation state of its nitrogen atom, so it should enter the overall charge balance only as the charge of a whole counterion.

Step 2: Apply the oxidation-state sum to the oxoanion Let the oxidation state of each molybdenum atom be x. The anion charge gives

$$5 + 12x + 40(-2) = -3.$$

Simplifying step by step,

$$12x - 75 = -3, \qquad 12x = 72, \qquad x = +6.$$

An equivalent check uses the neutral salt directly:

$$3(+1) + 5 + 12(+6) + 40(-2) = 3 + 5 + 72 - 80 = 0.$$

This confirms that the positive counterion charge balances the anion correctly. The subscript twelve is a count of molybdenum atoms; it is not itself an oxidation state and must multiply the unknown.

Step 3: Conclusion The oxidation state of Mo in ammonium phosphomolybdate is $\boxed{+6}$. Since the question already prints the plus sign, enter 6 in the blank.

Quick Tip to Solve: Use $[PMo_{12}O_{40}]^{3-}$: one $+5$ phosphorus and forty -2 oxygens leave $+72$ for twelve Mo atoms, giving $+6$ each.

90. For the adsorption of hydrogen on platinum, the activation energy is 30 k Jmol^{-1} and for the adsorption of hydrogen on nickel, the activation energy is 41.4 k Jmol^{-1}. The logarithm of the ratio of the rates of chemisorption on equal areas of the metals at 300 K is _____ (Nearest integer)

Given: $\ln 10 = 2.3$

$$\text{R} = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$$

[4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where E_a is the activation energy, T the absolute temperature and A the pre-exponential factor.
- For the comparison of chemisorption rates on equal areas under the same conditions, take the common prefactor in the intended Arrhenius comparison. The rate ratio is then set by the difference of activation energies.

Step 1: Form the faster-to-slower adsorption-rate ratio Platinum has the smaller activation energy, $30\,\text{kJ mol}^{-1}$, so its chemisorption rate is the larger one in the Arrhenius comparison. Denote the rates by r_{Pt} and r_{Ni}. Cancelling the common pre-exponential factor gives

$$\frac{r_{\text{Pt}}}{r_{\text{Ni}}} = \exp\left(\frac{E_{a,\text{Ni}} - E_{a,\text{Pt}}}{RT}\right).$$

The activation-energy difference is

$$\Delta E_a = 41.4 - 30.0 = 11.4\,\text{kJ mol}^{-1} = 11400\,\text{J mol}^{-1}.$$

Joules are required because the given gas constant uses joules.

Step 2: Evaluate the common logarithm Taking natural logarithms first gives

$$\ln\left(\frac{r_{\mathrm{Pt}}}{r_{\mathrm{Ni}}}\right) = \frac{11400}{8.3 \times 300}.$$

Since $\ln 10 = 2.3$, divide this result by 2.3 to obtain the common logarithm:

$$\log_{10}\left(\frac{r_{\mathrm{Pt}}}{r_{\mathrm{Ni}}}\right) = \frac{11400}{2.3 \times 8.3 \times 300} = \frac{11400}{5727} \approx 1.991.$$

The positive sign is consistent with platinum adsorbing faster. The rate ratio itself is approximately $10^{1.991}$, about 98; the question requests its logarithm rather than that ratio.

Step 3: Conclusion The common logarithm of the platinum-to-nickel chemisorption-rate ratio is approximately 1.991, giving the nearest-integer answer $\boxed{2}$. A logarithm of a rate ratio is dimensionless.

Quick Tip to Solve: Put the lower-barrier metal in the numerator and use $(E_{a,\mathrm{high}} - E_{a,\mathrm{low}})/(2.3RT)$. Convert kJ to J before substitution.