JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 6 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 6 April 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the coefficient of x^7 in $\left(ax^2 + \frac{1}{2bx}\right)^{11}$ and x^{-7} in $\left(ax - \frac{1}{3bx^2}\right)^{11}$ are equal, then : [4 marks]

(A) $243ab = 64$

(B) $32ab = 729$

(C) $64ab = 243$

(D) $729ab = 32$

Correct Answer: The equality permits a=0 as well as 729ab=32; option (D) is the intended answer for nonzero a.

Solution:

Concept:

- In $(u+v)^{11}$, the term selecting v exactly r times is $\binom{11}{r}u^{11-r}v^r$. The power of x determines which term contributes to a requested coefficient.
- A coefficient contains the numerical factors and powers of the parameters, but not the requested power of x. Since the displayed denominators contain b, it is nonzero.

Step 1: Locate the first coefficient Choose r copies of $1/(2bx)$. The resulting term is

$$\binom{11}{r}\frac{a^{11-r}}{2^r b^r}x^{22-3r}.$$

For the power x^7, solve $22-3r=7$, giving $r=5$. Hence its coefficient is $\binom{11}{5}a^6/(2^5b^5)$. The factor 2^5 is essential; it comes from each of the five selected reciprocal terms.

Step 2: Locate the second coefficient In the second expansion, choosing the negative reciprocal term s times gives

$$\binom{11}{s}\frac{(-1)^s a^{11-s}}{3^s b^s}x^{11-3s}.$$

The exponent condition $11-3s=-7$ gives $s=6$. This is even, so the sign is positive and the coefficient is $\binom{11}{6}a^5/(3^6b^6)$.

Step 3: Equate and simplify The binomial coefficients are equal because $\binom{11}{5}=\binom{11}{6}$. For nonzero a, equality therefore gives

$$\frac{a^6}{32b^5}=\frac{a^5}{729b^6} \quad\Longrightarrow\quad 729ab=32.$$

The other proposed relations give different values of ab, so they cannot match these two nonzero coefficients. If $a=0$, both coefficients vanish; that degenerate value supplies no one of the listed relations. The intended nonzero-coefficient interpretation is the one represented by the options.

Step 4: Conclusion For $a\neq 0$, the intended answer is option (D), $729ab=32$. As printed, however, $a=0$ also makes both requested coefficients zero and satisfies none of the options. Thus the key selects (D) under an unstated nonzero restriction, while the literal hypotheses do not force any listed relation.

Quick Tip to Solve: Write the exponent of x before simplifying coefficients. Here the two exponent equations select complementary indices 5 and 6, cancelling the binomial factors.

2. The area bounded by the curves $y = |x-1| + |x-2|$ and $y = 3$ is equal to : [4 marks]

(A) 5

(B) 4

(C) 6

(D) 3

Correct Answer: (B) 4

Solution:

Concept:

- An absolute value changes its formula at the zero of its argument. The breakpoints here are $x = 1$ and $x = 2$, so the lower curve must be handled on three intervals.
- The enclosed area is the integral of upper ordinate minus lower ordinate between the two intersections. The resulting quantity is measured in square units.

Step 1: Find the intersections and pieces For $x \le 1$, both arguments are nonpositive and $y = 3 - 2x$. For $1 \le x \le 2$, their sum is $y = 1$. For $x \ge 2$, it is $y = 2x - 3$. Equating the outer pieces to 3 gives the left intersection $x = 0$ and right intersection $x = 3$. The horizontal middle part never meets $y = 3$, so the entire enclosed interval is $[0, 3]$.

Step 2: Integrate each vertical gap The line $y = 3$ is above the absolute-value curve throughout the interior. Thus

$$\begin{aligned} A &= \int_0^1 2x\,dx + \int_1^2 2\,dx + \int_2^3 (6-2x)\,dx \\ &= [x^2]_0^1 + 2[x]_1^2 + [6x - x^2]_2^3 \\ &= 1 + 2 + 1 = 4. \end{aligned}$$

The left and right pieces are equal triangular areas of one square unit each; the middle piece is a rectangle of width one and height two. This geometric check verifies the integration limits and signs. Values 5, 6 and 3 do not equal the sum of these three nonoverlapping areas.

Step 3: Conclusion The bounded area is 4 square units. Hence, the correct option is (B).

Quick Tip to Solve: For a sum of two absolute values, split at both zeros first; here the middle interval is flat, and the two outer triangles give a quick check.

3. All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is : [4 marks]

(A) 578

(B) 576

(C) 580

(D) 582

Correct Answer: (D) 582

Solution:

Concept:

- Dictionary order compares the first unequal letters. Count all permutations beginning with a smaller available letter at each successive position of PUBLIC.
- All six letters are distinct, so a fixed prefix with k positions remaining has $k!$ completions. A serial number starts at one, whereas the preceding-word count starts at zero.

Step 1: Count the earlier first and second letters The alphabetical list is B, C, I, L, P, U. Before the initial P, four choices B, C, I and L are possible. Each has $5!$ completions, contributing $4(120) = 480$. After fixing P, the remaining letters are B, C, I, L, U. Four are smaller than U, so words with the same first letter but an earlier second letter contribute $4(4!) = 96$. The subtotal is 576; this is not yet the rank because subsequent positions must also be compared.

Step 2: Complete the prefix count and add one With PU fixed, the next letter B is the smallest remaining, contributing zero. With PUB fixed, C and I are smaller than L, contributing $2(2!) = 4$. With PUBL fixed, C is smaller than I, contributing $1(1!) = 1$. After PUBLI the final C is forced. Therefore

$$N_{\text{before}} = 480 + 96 + 0 + 4 + 1 = 581.$$

Adding PUBLIC itself gives

$$\text{rank}(\text{PUBLIC}) = 581 + 1 = 582.$$

The distractor 576 stops after the first two positions; 578 and 580 also omit necessary later-prefix contributions or the final one-based adjustment.

Step 3: Conclusion The serial number of PUBLIC is 582. Hence, the correct option is (D).

Quick Tip to Solve: Keep two separate quantities: the number of earlier words and the one-based rank. Add one only after all prefixes have been counted.

4. Let $a \neq b$ be two non-zero real numbers. Then the number of elements in the set $X = \{z \in \mathbb{C} : \mathrm{Re}\,(az^2 + bz) = a$ and $\mathrm{Re}\,(bz^2 + az) = b\}$ is equal to : [4 marks]

(A) 0

(B) 2

(C) 1

(D) Infinite

Correct Answer: The count is 0 for $a + b \neq 0$ and infinite for $a = -b$; both cases satisfy the printed hypotheses.

Solution:

Concept:

- Write $z = x + iy$, with real x, y. Then $\mathrm{Re}\, z = x$ and $\mathrm{Re}\, z^2 = x^2 - y^2$. Real coefficients can be taken outside the real-part operator.
- The determinant of the two linear equations in $x^2 - y^2$ and x is $a^2 - b^2$. The stated condition $a \neq b$ does not exclude $a = -b$, so that case must be retained.

Step 1: Reduce the two conditions Put $u = x^2 - y^2$. The defining equations become

$$au + bx = a, \qquad bu + ax = b.$$

Subtracting these equations and using $a - b \neq 0$ yields

$$u - x = 1.$$

Adding them yields $(a + b)(u + x) = (a + b)$. Division by $a + b$ is allowed only when that sum is nonzero; otherwise it would discard a permitted case.

Step 2: Treat the nonsingular case If $a + b \neq 0$, addition gives $u + x = 1$. Combined with $u - x = 1$, this implies $u = 1$ and $x = 0$. But then $u = x^2 - y^2 = -y^2 \leq 0$, contradicting $u = 1$. Hence X is empty in this case and has zero elements. This accounts for option (A) under the additional condition $a \neq -b$.

Step 3: Retain the permitted singular case If $a = -b \neq 0$, the two original equations are equivalent to

$$x^2 - y^2 - x = 1.$$

For every real y, choose

$$x = \frac{1 + \sqrt{5 + 4y^2}}{2}.$$

These values satisfy the equation, giving infinitely many distinct complex numbers. For example $a = 1, b = -1$ meets every stated parameter condition and yields this infinite set. Thus option (D) applies to this permitted case, whereas neither 1 nor 2 is the general count.

Step 4: Conclusion As printed, the answer is not unique: $|X| = 0$ if $a + b \neq 0$, but $|X|$ is infinite if $a = -b$. The supplied key selecting zero requires the additional restriction $a \neq -b$; the explicit admissible counterexample $a = 1, b = -1$ proves why it does not hold for all stated parameters.

Quick Tip to Solve: Before dividing by a parameter expression, check whether the hypotheses exclude its zero. Here a difference being nonzero does not ensure the sum is nonzero.

5. Let P be a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to : [4 marks]

(A) 18

(B) 22

(C) 24

(D) 40

Correct Answer: (C) 24

Solution:

Concept:

- The identity $P^2 = I - P$ reduces every positive power to a linear combination of I and P. Multiplication by P then produces a two-coefficient recurrence.
- If $P^n = A_n I + B_n P$, then $P^{n+1} = B_n I + (A_n - B_n)P$. This follows by replacing P^2 before comparing expressions.

Step 1: Reduce the relevant powers Starting with $P^2 = I - P$, repeated multiplication gives

$$\begin{aligned} P^3 &= -I + 2P, & P^4 &= 2I - 3P, \\ P^5 &= -3I + 5P, & P^6 &= 5I - 8P, \\ P^7 &= -8I + 13P, & P^8 &= 13I - 21P. \end{aligned}$$

The coefficients alternate in sign and follow the Fibonacci recurrence. In particular, the negative coefficients of P occur at even exponents, and their magnitudes increase. This makes the pair involving 21 and 8 the natural candidate for the prescribed sum and difference.

Step 2: Match both given identities Adding and subtracting these two powers gives

$$\begin{aligned} P^8 + P^6 &= 18I - 29P, \\ P^8 - P^6 &= 8I - 13P. \end{aligned}$$

They match the question simultaneously, with $\alpha = 8$, $\beta = 6$, $\gamma = 18$ and $\delta = 8$. Equivalently, adding the supplied equations gives $2P^\alpha = (\gamma+\delta)I - 42P$; subtracting gives $2P^\beta = (\gamma-\delta)I - 16P$, consistent with the same reduced powers.

Step 3: Evaluate the requested combination Substitute the matched values without interchanging the order in the difference:

$$\alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24.$$

The alternatives 18, 22 and 40 are not the value of this combination; 18 alone is merely the coefficient γ, while retaining the positive δ instead of subtracting it would give the distractor 40.

Step 4: Conclusion The required value is 24. Hence, the correct option is (C).

Quick Tip to Solve: For a matrix obeying a quadratic, reduce powers recursively to I and P; verify both the sum and the ordered difference before reading off exponents.

6. Among the statements :

(S1) : $2023^{2022} - 1999^{2022}$ **is divisible by 8**

(S2) : $13(13)^n - 12n - 13$ **is divisible by 144 for infinitely many** $n \in \mathbb{N}$ [4 marks]

(A) both (S1) and (S2) are incorrect

(B) only (S1) is correct

(C) only (S2) is correct

(D) both (S1) and (S2) are correct

Correct Answer: (D) both (S1) and (S2) are correct

Solution:

Concept:

- Two integers congruent modulo a positive integer have congruent positive integral powers. An expression is divisible by the modulus precisely when its residue is zero.
- In a binomial expansion of $(1+12)^m$, every term of degree at least two in 12 contains $12^2 = 144$. Only the constant and linear terms affect the remainder modulo 144.

Step 1: Test statement S1 modulo eight Both bases have the same residue:

$$2023 \equiv 7 \pmod 8, \qquad 1999 \equiv 7 \pmod 8.$$

Their 2022nd powers therefore have the same residue and their difference is divisible by 8. Alternatively, their difference is 24, already divisible by 8, and $A^m - B^m$ contains the factor $A - B$. Either argument proves S1 without evaluating large powers. The even exponent also makes both residues equal to one, though that additional observation is not required.

Step 2: Expand the expression in S2 Write $13(13)^n = 13^{n+1} = (1+12)^{n+1}$. The binomial theorem gives

$$\begin{aligned}(1+12)^{n+1} &= 1 + 12(n+1) + 144K \\ &= 13 + 12n + 144K,\end{aligned}$$

where K is an integer formed from the remaining binomial terms. Thus

$$13(13)^n - 12n - 13 = 144K.$$

This is divisible by 144 for every positive integer n, which is stronger than holding for infinitely many of them. Therefore S2 is also true. Options declaring one or both statements incorrect fail these explicit residue computations.

Step 3: Conclusion Both (S1) and (S2) are correct. Hence, the correct option is (D).

Quick Tip to Solve: For divisibility by a square such as 144, expand about 1 using 13=1+12; discard terms containing 12 squared only after recording why they are multiples of 144.

7. If the solution curve $f(x, y) = 0$ of the differential equation

$(1 + \log_e x)\frac{dx}{dy} - x\log_e x = e^y, x > 0$,

passes through the points $(1, 0)$ and $(\alpha, 2)$, then α^α is equal to : [4 marks]

(A) $e^{\sqrt{2}e^2}$

(B) $e^{2e^{\sqrt{2}}}$

(C) e^{e^2}

(D) e^{2e^2}

Correct Answer: (D) e^{2e^2}

Solution:

Concept:

- The product-rule derivative of $x\log_e x$, with x regarded as a function of y, is $(1 + \log_e x)\,dx/dy$. This identifies a substitution for the entire left side.
- A linear equation $dv/dy - v = e^y$ has integrating factor e^{-y}. Multiplication by it converts the equation to an exact derivative of ve^{-y}.

Step 1: Introduce the product substitution Let $v = x\log_e x$, valid for $x > 0$. By the chain rule,

$$\frac{dv}{dy} = (1 + \log_e x)\frac{dx}{dy}.$$

Consequently the supplied equation is

$$\frac{dv}{dy} - v = e^y.$$

This avoids trying to separate x and y directly, since the logarithmic product is already the dependent variable whose derivative appears.

Step 2: Integrate and impose the first point Multiply through by the integrating factor e^{-y}:

$$\frac{d}{dy}(ve^{-y}) = 1.$$

Integration gives $ve^{-y} = y + C$, or

$$x\log_e x = e^y(y + C).$$

At $(x, y) = (1, 0)$, the left side is $1\log_e 1 = 0$, so $C = 0$. The particular curve is therefore $x\log_e x = ye^y$. This substitution back into the differential equation also checks the integration sign.

Step 3: Use the second point and exponentiate At $(\alpha, 2)$, the curve gives

$$\alpha\log_e \alpha = 2e^2.$$

Since $\alpha > 0$, the power identity $\alpha^\alpha = e^{\alpha\log_e \alpha}$ applies, giving

$$\alpha^\alpha = e^{2e^2}.$$

There is no need to solve for α itself. Options (A), (B) and (C) have exponents different from the exactly determined $2e^2$; a square root does not arise in the integration.

Step 4: Conclusion The required value is e^{2e^2}. Hence, the correct option is (D).

Quick Tip to Solve: When both x log x and its derivative occur, substitute the whole product. If the target is alpha to the power alpha, determine alpha log alpha rather than alpha itself.

8. Let $f(x)$ be a function satisfying $f(x)+f(\pi-x)=\pi^2, \forall x \in \mathbb{R}$. Then $\int_0^{\pi} f(x)\sin x dx$ is equal to :

[4 marks]

(A) π^2
(B) $\frac{\pi^2}{2}$
(C) $2\pi^2$
(D) $\frac{\pi^2}{4}$

Correct Answer: (A) π^2

Solution:

Concept:

- For an integrable function on $[0, a]$, substitution $x = a - t$ gives $\int_0^a g(x)\,dx = \int_0^a g(a-t)\,dt$. The same integral can therefore be averaged with its reflected form.
- Here the reflected function has the fixed sum $f(x) + f(\pi - x) = \pi^2$, and the weight is symmetric because $\sin(\pi - x) = \sin x$.

Step 1: Reflect the variable Write $I = \int_0^{\pi} f(x)\sin x\,dx$. Put $x = \pi - t$, so $dx = -dt$, and reverse the transformed limits. Then

$$I = \int_0^{\pi} f(\pi - t)\sin(\pi - t)\,dt = \int_0^{\pi} f(\pi - x)\sin x\,dx.$$

The dummy variable can be renamed after the substitution. This step uses only symmetry; we do not need an explicit formula for f.

Step 2: Add the two representations Adding the original and reflected expressions combines the unknown functions exactly as in the given identity:

$$2I = \int_0^{\pi} [f(x) + f(\pi - x)]\sin x\,dx = \pi^2 \int_0^{\pi} \sin x\,dx.$$

The elementary remaining integral is

$$\int_0^{\pi} \sin x\,dx = [-\cos x]_0^{\pi} = 1 - (-1) = 2.$$

Thus $2I = 2\pi^2$, and division by two gives $I = \pi^2$.

Step 3: Check the alternatives Option (A) is exactly π^2. Option (B) incorrectly halves the result again; option (C) is the sum $2I$, rather than I; and option (D) includes two unnecessary factors of one half. The symmetric weight integrates to two, which is essential when applying the averaging argument.

Step 4: Conclusion The integral equals $\boxed{\pi^2}$. Hence, the correct option is (A).

Quick Tip to Solve: When a question gives $f(x) + f(a - x)$, reflect the definite integral first and check whether its weight is unchanged; adding the two forms removes the unknown function.

9. $\lim_{n\to\infty}\left\{\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right).....\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)\right\}$ **is equal to :** [4 marks]

(A) $\sqrt{2}$

(B) 1

(C) $\frac{1}{\sqrt{2}}$

(D) 0

Correct Answer: (D) 0

Solution:

Concept:

- A finite product of nonnegative factors can be bounded by replacing each factor by a common upper bound. If that upper bound is a fixed number smaller than one, the resulting geometric sequence tends to zero.
- The squeeze theorem states that if $0 \le P_n \le r^n$ and $0 < r < 1$, then $P_n \to 0$. A product limit is not found simply by replacing every factor by its limiting value.

Step 1: Bound each factor The product has the n factors

$$P_n = \prod_{k=1}^{n}\left(\sqrt{2}-2^{1/(2k+1)}\right).$$

For every $k \ge 1$, the exponent satisfies $0 < 1/(2k+1) < 1/2$. Since the exponential function with base two is increasing,

$$1 < 2^{1/(2k+1)} < \sqrt{2}.$$

Subtracting from $\sqrt{2}$ gives

$$0 < \sqrt{2}-2^{1/(2k+1)} < \sqrt{2}-1.$$

In particular the factors are positive; there is no change of sign as more factors are added.

Step 2: Squeeze the complete product Multiplying the bounds for all n factors yields

$$0 < P_n < (\sqrt{2}-1)^n.$$

Now $0 < \sqrt{2}-1 < 1$, so the geometric upper bound tends to zero as $n \to \infty$. The lower bound is zero, and the squeeze theorem therefore proves $\lim_{n\to\infty} P_n = 0$. This argument treats the increasing number of factors explicitly and does not interchange an infinite product with a single-factor limit.

Step 3: Reject the nonzero values Options (A), (B), and (C) are the positive constants $\sqrt{2}$, 1, and $1/\sqrt{2}$. Each is incompatible with the geometric upper bound, which eventually becomes smaller than any prescribed positive constant. Only option (D) has the required value zero.

Step 4: Conclusion The limit of the product is $\boxed{0}$. Hence, the correct option is (D).

Quick Tip to Solve: For products with an increasing number of factors, seek one uniform bound below one; a geometric squeeze often avoids logarithms or product expansions.

10. In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is α and the number of persons who speak only Hindi is β, then the eccentricity of the ellipse $25\left(\beta^2x^2+\alpha^2y^2\right)=\alpha^2\beta^2$ is : [4 marks]

(A) $\frac{\sqrt{129}}{12}$

(B) $\frac{3\sqrt{15}}{12}$

(C) $\frac{\sqrt{119}}{12}$

(D) $\frac{\sqrt{117}}{12}$

Correct Answer: (C) $\frac{\sqrt{119}}{12}$

Solution:

Concept:

- Inclusion-exclusion gives $|E\cap H|=|E|+|H|-|E\cup H|$, where E and H are the sets of English and Hindi speakers.
- For an ellipse $x^2/a^2+y^2/b^2=1$ with $a>b>0$, eccentricity is $e=\sqrt{1-b^2/a^2}$. The larger denominator determines the major semiaxis.

Step 1: Count the exclusive speakers Every one of the 100 persons speaks at least one language, so $|E\cup H|=100$. The number speaking both is

$$|E\cap H|=75+40-100=15.$$

Subtract these bilingual persons from the individual language totals:

$$\alpha=75-15=60,\qquad \beta=40-15=25.$$

As a check, the three disjoint groups have sizes 60, 25, and 15, which sum to 100. The symbols denote exclusive groups, not the totals 75 and 40.

Step 2: Put the ellipse in standard form Divide the given equation by the positive quantity $\alpha^2\beta^2$:

$$\frac{25x^2}{\alpha^2}+\frac{25y^2}{\beta^2}=1.$$

Using the exclusive counts gives

$$\frac{x^2}{144}+\frac{y^2}{25}=1.$$

Hence the major semiaxis is $a=12$ and the minor semiaxis is $b=5$. Their squares, rather than the semiaxes themselves, are the denominators.

Step 3: Calculate and compare eccentricities The required eccentricity is

$$e=\sqrt{1-\frac{25}{144}}=\sqrt{\frac{119}{144}}=\frac{\sqrt{119}}{12}.$$

This is option (C). The squared values in options (A), (B), and (D) are respectively $129/144$, $135/144$, and $117/144$; none equals the required $119/144$. The result lies between zero and one, as an ellipse requires.

Step 4: Conclusion The eccentricity is $\boxed{\sqrt{119}/12}$. Hence, the correct option is (C).

Quick Tip to Solve: First subtract the overlap to find the exclusive language counts; then divide the ellipse equation by its right-hand side before identifying its semiaxes.

11. Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to : [4 marks]

(A) 3

(B) 4

(C) 1

(D) 2

Correct Answer: (B) 4

Solution:

Concept:

- Three independent fair six-faced dice give 6^3 equally likely ordered outcomes. Treating the dice as distinguishable makes counting direct and avoids unequal weighting of unordered triples.
- For pairwise different faces, the first die has six choices, the second has five remaining choices, and the third has four remaining choices. Reduce the resulting probability before identifying p and q.

Step 1: Count the favourable ordered triples The first die can show any of its six numbers. After its result is fixed, the second die must avoid that number, leaving five choices. The third die must avoid both numbers already seen, leaving four choices. Therefore

$$N_{\text{favourable}} = 6 \cdot 5 \cdot 4 = 120.$$

This counts every possible ordering once. For example, $(1, 2, 3)$ and $(2, 1, 3)$ are distinct elementary outcomes because their first and second dice differ.

Step 2: Reduce the probability There are

$$N_{\text{total}} = 6 \cdot 6 \cdot 6 = 216$$

equally likely outcomes in total. Thus

$$P(\text{all different}) = \frac{120}{216} = \frac{5}{9}.$$

The fraction is reduced because 5 and 9 have no common factor greater than one. Equivalently, the successive probabilities are 1, $5/6$, and $4/6$, whose product is also $5/9$.

Step 3: Use the coprime numerator and denominator The required representation is consequently $p = 5$, $q = 9$, and

$$q - p = 9 - 5 = 4.$$

Option (B) equals four. Options (A), (C), and (D), namely three, one, and two, do not equal the

difference of the reduced numerator and denominator. Using 120 and 216 directly would violate the coprimality condition.

Step 4: Conclusion The required difference is $\boxed{4}$. Hence, the correct option is (B).

Quick Tip to Solve: For all-distinct dice outcomes, use descending choices $6, 5, 4$, but reduce the probability before using a numerator or denominator named in the question.

12. For the system of equations

$$x + y + z = 6$$

$$x + 2y + \alpha z = 10$$

$x + 3y + 5z = \beta$**, which one of the following is NOT true?** [4 marks]

(A) System has a unique solution for $\alpha = 3, \beta \neq 14$.

(B) System has infinitely many solutions for $\alpha = 3, \beta = 14$.

(C) System has no solution for $\alpha = 3, \beta = 24$.

(D) System has a unique solution for $\alpha = -3, \beta = 14$.

Correct Answer: (A) System has a unique solution for $\alpha = 3, \beta \neq 14$.

Solution:

Concept:

- Subtracting equations produces an equivalent system with the same solution set. A nonzero pivot for each variable gives a unique solution; a contradictory row gives no solution.
- If a parameter makes the final pivot zero, inspect its right-hand side separately: a zero right-hand side can leave a free variable, while a nonzero value is inconsistent.

Step 1: Eliminate the first variable Subtract the first equation from the second and from the third, respectively:

$$y + (\alpha - 1)z = 4, \qquad 2y + 4z = \beta - 6.$$

Now subtract twice the first reduced equation from the second. The coefficient and constant become

$$(6 - 2\alpha)z = \beta - 14.$$

This calculation retains every original coefficient: the third equation contains $5z$, so subtracting the first leaves $4z$. The last pivot vanishes precisely at $\alpha = 3$.

Step 2: Inspect the exceptional value When $\alpha = 3$, the final equation reduces to $0 = \beta - 14$. If $\beta = 14$, it is an identity. The remaining equations are $y + 2z = 4$ and $x + y + z = 6$, so putting $z = t$ gives

$$y = 4 - 2t, \qquad x = 2 + t, \qquad t \in \mathbb{R}.$$

Thus there are infinitely many solutions, proving (B). If $\beta \neq 14$, the final row is contradictory and

there is no solution. In particular $\beta = 24$ gives $0 = 10$, proving (C). Therefore the unique-solution claim in (A) is false.

Step 3: Check the remaining alternative For $\alpha = -3$, the last coefficient is $6 - 2(-3) = 12 \neq 0$. At $\beta = 14$, it gives $z = 0$, then $y = 4$ and $x = 2$. These values satisfy all three original equations and are uniquely determined, so (D) is true. Since (B), (C), and (D) hold, only (A) answers the question asking for the statement that is NOT true.

Step 4: Conclusion The unique-solution claim for $\alpha = 3, \beta \neq 14$ is false. Hence, the correct option is (A).

Quick Tip to Solve: After elimination, check both sides of a zero-pivot equation; zero equals zero gives dependence, while zero equals a nonzero constant gives inconsistency.

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13. Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{\lceil x \rceil - x}}$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. Then among the statements

(S1) : $A \cap B = (1, \infty) - \mathbb{N}$ **and**

(S2) : $A \cup B = (1, \infty)$ [4 marks]

(A) only (S2) is true

(B) only (S1) is true

(C) neither (S1) nor (S2) is true

(D) both (S1) and (S2) are true

Correct Answer: (B) only (S1) is true

Solution:

Concept:

- A reciprocal square root is real and defined only when its radicand is strictly positive. For nonintegral x, the ceiling satisfies $0 < \lceil x \rceil - x < 1$; at an integer the difference is zero.
- The domain contains all real inputs where the expression is defined. The range must be found from all attainable radicands, including whether their limiting endpoints are attained.

Step 1: Determine all admissible inputs If $x = m$ is an integer, then $\lceil x \rceil = m$, so the denominator is zero and $f(x)$ is undefined. If $m-1 < x < m$, where m is an integer, then $\lceil x \rceil = m$ and $0 < m - x < 1$. Hence all nonintegers, positive and negative, are allowed:

$$A = \mathbb{R} \setminus \mathbb{Z}.$$

No additional restriction such as $x > 1$ belongs to the domain.

Step 2: Determine every possible output As x runs over $(m-1, m)$, the quantity $t = m - x$ runs over the whole interval $(0, 1)$. Consequently $1/\sqrt{t} > 1$, with no finite upper bound. Conversely, for any proposed output $y > 1$, take $t = 1/y^2 \in (0, 1)$ and $x = m - t$. Then the function equals y, proving

$$B = (1, \infty).$$

The endpoint one is excluded because it would need $t = 1$, which does not occur at a noninteger input.

Step 3: Check the two set statements Intersecting B with the nonintegers removes precisely the positive integers above one:

$$A \cap B = (1, \infty) \setminus \mathbb{N}.$$

Thus (S1) is true. However, A includes $x = 1/2$, so $1/2 \in A \cup B$ although $1/2 \notin (1, \infty)$. This counterexample proves (S2) false. In fact the union includes every noninteger real and also every integer greater than one. Options (A) and (D) wrongly accept (S2), while (C) wrongly rejects (S1).

Step 4: Conclusion Only statement (S1) is true. Hence, the correct option is (B).

Quick Tip to Solve: For ceiling-function denominators, separate integers from open unit intervals first; a negative noninteger input can quickly disprove an overly restricted proposed union.

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14. A plane P contains the line of intersection of the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$. If P passes through the point $(0, 2, -2)$, then the square of distance of the point $(12, 12, 18)$ from the plane P is : [4 marks]

(A) 310

(B) 620

(C) 1240

(D) 155

Correct Answer: (B) 620

Solution:

Concept:

- Planes through the intersection of two nonparallel planes $F_1 = 0$ and $F_2 = 0$ can be written as $F_1 + \lambda F_2 = 0$, together with the second plane itself. A given extra point fixes the needed member.
- The squared distance from (x_0, y_0, z_0) to $ax + by + cz + d = 0$ is $(ax_0 + by_0 + cz_0 + d)^2/(a^2 + b^2 + c^2)$. The coefficients of the plane provide its normal vector.

Step 1: Use the family of planes Write the two given planes as

$$F_1 = x + y + z - 6 = 0, \qquad F_2 = 2x + 3y + 4z + 5 = 0.$$

Any simultaneous solution of these equations lies on $F_1 + \lambda F_2 = 0$. At the required point $(0, 2, -2)$, the two left-hand sides are -6 and 3, respectively. Thus the point condition gives

$$-6 + 3\lambda = 0, \qquad \lambda = 2.$$

The point is not on $F_2 = 0$, so the omitted family member cannot be the required plane.

Step 2: Obtain the plane equation Substitute $\lambda = 2$ and collect terms:

$$(x + y + z - 6) + 2(2x + 3y + 4z + 5) = 0,$$

$$5x + 7y + 9z + 4 = 0.$$

The normal is $(5, 7, 9)$, with squared length $25 + 49 + 81 = 155$. Substitution of $(0, 2, -2)$ gives $14 - 18 + 4 = 0$, confirming the extra point lies on the plane. Keeping the constant $+5$ in F_2 is essential because the original right-hand side is -5.

Step 3: Evaluate the squared distance At $(12, 12, 18)$, the plane expression is

$$5(12) + 7(12) + 9(18) + 4 = 60 + 84 + 162 + 4 = 310.$$

Therefore

$$d^2 = \frac{310^2}{155} = \frac{96100}{155} = 620.$$

Option (B) matches this value. Option (A) is just the unsquared plane expression, (D) is the normal squared length, and (C) is twice the required squared distance. None of those quantities substitutes for the complete point-to-plane formula.

Step 4: Conclusion The required squared distance is $\boxed{620}$ square coordinate units. Hence, the correct option is (B).

Quick Tip to Solve: For a plane containing an intersection line, combine the two zero-form plane equations and use the extra point before applying the distance formula.

15. If $\gcd(m, n) = 1$ **and** $1^2 - 2^2 + 3^2 - 4^2 + \ldots.. + (2021)^2 - (2022)^2 + (2023)^2 = 1012\ m^2 n$ **then** $m^2 - n^2$ **is equal to :** [4 marks]

(A) 220

(B) 200

(C) 240

(D) 180

Correct Answer: The gcd condition permits both $(m, n) = (17, 7)$, yielding 240, and $(1, 2023)$, yielding -4092528; a unique value is not determined.

Solution:

Concept:

- Pairing consecutive terms converts an alternating sum of squares into a sum of odd integers, because $a^2 - b^2 = (a - b)(a + b)$.
- The sum of the first k positive integers is $k(k + 1)/2$. Prime factorisation then identifies a square factor and its remaining coprime factor.

Step 1: Pair the first 2022 terms There are 1011 pairs before the last positive square. Each pair has the form $(2j - 1)^2 - (2j)^2 = 1 - 4j$. Therefore, denoting the given sum by S, we obtain

$$S = \sum_{j=1}^{1011} (1 - 4j) + 2023^2.$$

This grouping preserves every sign; the final term must not be paired with a nonexistent negative term.

Step 2: Evaluate and factor the sum Use the arithmetic-series formula and $2023 = 2(1011) + 1$:

$$\begin{aligned} S &= 1011 - 4\frac{1011(1012)}{2} + 2023^2 \\ &= 1011 - 2(1011)(1012) + 2023^2 \\ &= 1012(2023). \end{aligned}$$

Comparing with $1012m^2n$ gives $m^2n = 2023$. Its prime factorisation is

$$2023 = 7 \times 289 = 7 \times 17^2.$$

The nontrivial square factor is thus $m^2 = 289$, and $n = 7$. The numbers 17 and 7 are coprime, as required.

Step 3: Calculate the requested difference With $m^2 = 289$ and $n^2 = 49$, the requested difference is

$$m^2 - n^2 = 289 - 49 = 240.$$

In particular, the square-free factor 7 is n, not n^2. The alternatives 220, 200 and 180 do not equal this difference. There is also the trivial square divisor $m^2 = 1$, giving $n = 2023$ and $m^2 - n^2 = -4092528$. It is not among the listed answers. Thus 240 is the only listed value consistent with the equation; the gcd condition alone does not exclude that additional unlisted case.

Step 4: Conclusion The printed conditions do not determine a unique value: $m^2 - n^2$ can be 240 or -4092528. Option (C) gives the nontrivial-square case, but it requires the additional restriction $m > 1$; the gcd condition alone does not supply it.

Quick Tip to Solve: After extracting a square factor, test the trivial square divisor 1 too: coprimality does not exclude it. State an ambiguity when both cases meet every condition.

16. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ represent three coterminous edges of a parallelopiped of volume V. Then the volume of the parallelopiped, whose coterminous edges are represented by $\vec{a}, \vec{b}+\vec{c}$ and $\vec{a}+2\vec{b}+3\vec{c}$ is equal to : [4 marks]

(A) 3 V

(B) 2 V

(C) 6 V

(D) V

Correct Answer: (D) V

Solution:

Concept:

- The volume generated by three coterminous edge vectors is the absolute scalar triple product: $V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$.
- The scalar triple product is linear in each position. It vanishes if two entries coincide, and interchanging two entries reverses its sign.

Step 1: Write the new scalar triple product Let $T = [\vec{a}, \vec{b}, \vec{c}]$, where brackets denote the signed scalar triple product. The new signed volume is

$$T_1 = [\vec{a}, \vec{b}+\vec{c}, \vec{a}+2\vec{b}+3\vec{c}].$$

The original geometric volume is $|T| = V$. Keeping the sign until the final absolute value avoids counting contributions that cancel.

Step 2: Expand by linearity Expand the last entry first. The term containing $\vec{a}$ is zero, since the first entry is also $\vec{a}$:

$$\begin{aligned} T_1 &= 2[\vec{a}, \vec{b}+\vec{c}, \vec{b}] + 3[\vec{a}, \vec{b}+\vec{c}, \vec{c}] \\ &= 2[\vec{a}, \vec{c}, \vec{b}] + 3[\vec{a}, \vec{b}, \vec{c}] \\ &= -2T + 3T = T. \end{aligned}$$

The terms $[\vec{a}, \vec{b}, \vec{b}]$ and $[\vec{a}, \vec{c}, \vec{c}]$ vanish. The negative sign in the surviving term with coefficient 2 is essential.

Step 3: Take the geometric volume Taking the absolute value gives $V_1 = |T_1| = |T| = V$. Equivalently, the coefficient matrix with rows $(1,0,0), (0,1,1), (1,2,3)$ has determinant $3-2=1$, so the volume scale factor is one. The options $3V$, $2V$, and $6V$ ignore the subtraction created by the reversed vector order and are therefore incorrect.

Step 4: Conclusion The new parallelepiped has volume $\boxed{V}$. Hence, the correct option is (D).

Quick Tip to Solve: Express the new edges in terms of the old ones and take the absolute determinant of their coefficient matrix; adding edge vectors need not increase volume.

17. If the tangents at the points P and Q on the circle $x^2 + y^2 - 2x + y = 5$ meet at the point $R\left(\frac{9}{4}, 2\right)$, then the area of the triangle PQR is : [4 marks]

(A) $\frac{13}{8}$

(B) $\frac{5}{8}$

(C) $\frac{5}{4}$

(D) $\frac{13}{4}$

Correct Answer: (B) $\frac{5}{8}$

Solution:

Concept:

- Tangents from an external point have a chord of contact perpendicular to the line joining that point to the circle centre.
- For radius r, external-point distance d, and chord midpoint M, one has $CM = r^2/d$ and $MP = \sqrt{r^2 - CM^2}$.

Step 1: Find the centre, radius and external distance Complete squares in the circle equation:

$$(x-1)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{4}.$$

Thus the centre is $C = (1, -1/2)$ and $r = 5/2$. For $R = (9/4, 2)$,

$$CR = \sqrt{\left(\frac{5}{4}\right)^2 + \left(\frac{5}{2}\right)^2} = \frac{5\sqrt{5}}{4}.$$

Since $CR > r$, two real tangents exist.

Step 2: Determine the chord length and triangle height Let M be the midpoint of the chord of contact PQ. In the right triangle CPR, the altitude from P to CR meets it at M; projection gives $CM \cdot CR = CP^2$. Hence

$$CM = \frac{25/4}{5\sqrt{5}/4} = \sqrt{5}.$$

The half-chord follows from the right triangle CMP:

$$MP = \sqrt{\frac{25}{4} - 5} = \frac{\sqrt{5}}{2}, \qquad PQ = \sqrt{5}.$$

The distance from R to the chord is $RM = CR - CM = \sqrt{5}/4$. Use this distance, rather than the full distance from the centre to R, as the triangle height.

Step 3: Evaluate the area Using PQ as base gives

$$[PQR] = \frac{1}{2}(PQ)(RM) = \frac{1}{2}(\sqrt{5})\left(\frac{\sqrt{5}}{4}\right) = \frac{5}{8}.$$

Neither the centre-to-chord distance nor a tangent length is the altitude for this base. The resulting area excludes $13/8$, $5/4$, and $13/4$.

Step 4: Conclusion The required area is $\boxed{5/8}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For tangents from an external point, use $CM = r^2/CR$; then half the chord times its distance from the external point gives the area.

18. Among the statements

(S1) : $(p \Rightarrow q) \vee ((\sim p) \wedge q)$ **is a tautology**

(S2) : $(q \Rightarrow p) \Rightarrow ((\sim p) \wedge q)$ **is a contradiction** [4 marks]

(A) neither (S1) and (S2) is True

(B) only (S2) is True

(C) both (S1) and (S2) are True

(D) only (S1) is True

Correct Answer: (A) neither (S1) and (S2) is True

Solution:

Concept:

- An implication $p \Rightarrow q$ is equivalent to $(\sim p) \vee q$; it is false only when p is true and q is false.
- A tautology is true for every truth assignment, whereas a contradiction is false for every assignment. One counterexample disproves either universal claim.

Step 1: Simplify the first expression Use the implication identity followed by absorption:

$$\begin{aligned}(p \Rightarrow q) \vee ((\sim p) \wedge q) &= ((\sim p) \vee q) \vee ((\sim p) \wedge q) \\ &= (\sim p) \vee q.\end{aligned}$$

The conjunction already implies $(\sim p) \vee q$, so it adds no new true cases. At $p = \mathrm{T}, q = \mathrm{F}$, this expression is false. Therefore it is not a tautology, and statement (S1) is false.

Step 2: Simplify the second expression Apply the same implication identity to the outer implication:

$$\begin{aligned}(q \Rightarrow p) \Rightarrow ((\sim p) \wedge q) &= \sim ((\sim q) \vee p) \vee ((\sim p) \wedge q) \\ &= (q \wedge (\sim p)) \vee ((\sim p) \wedge q) \\ &= (\sim p) \wedge q.\end{aligned}$$

De Morgan's law supplies the second line. For $p = \mathrm{F}, q = \mathrm{T}$, the final conjunction is true; consequently the expression is not a contradiction. Statement (S2) is also false.

Step 3: Select the combination Both assertions about the logical expressions fail: the first expression has a false case and the second has a true case. Thus neither statement is true. The alternatives claiming only (S2), both statements, or only (S1) are each excluded by the corresponding explicit truth assignment.

Step 4: Conclusion Neither (S1) nor (S2) is true. Hence, the correct option is (A).

Quick Tip to Solve: Test a claimed tautology at an implication's false case, and test a claimed contradiction at a case making its final conjunction true.

19. Let the line L pass through the point $(0,1,2)$, intersect the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and be parallel to the plane $2x+y-3z=4$. Then the distance of the point $\mathrm{P}(1,-9,2)$ from the line L is : [4 marks]

(A) 9

(B) $\sqrt{74}$

(C) $\sqrt{69}$

(D) $\sqrt{54}$

Correct Answer: (B) $\sqrt{74}$

Solution:

Concept:

- A line is parallel to a plane if its direction vector is perpendicular to the plane normal; here the normal is $(2,1,-3)$.
- The squared distance from a point to a line through A, of direction $\vec{d}$, is $|\overrightarrow{AP}|^2-(\overrightarrow{AP}\cdot\vec{d})^2/|\vec{d}|^2$.

Step 1: Find the intersection point and line direction Write a general point of the given line as

$$B=(1+2t,\ 2+3t,\ 3+4t).$$

The required line passes through $A=(0,1,2)$, so a direction vector is

$$\overrightarrow{AB}=(1+2t,\ 1+3t,\ 1+4t).$$

Impose its perpendicularity to the plane normal:

$$2(1+2t)+(1+3t)-3(1+4t)=-5t=0.$$

Thus $t=0$, $B=(1,2,3)$, and we may use $\vec{d}=(1,1,1)$.

Step 2: Resolve the point displacement For $P=(1,-9,2)$,

$$\overrightarrow{AP}=(1,-10,0),\quad |\overrightarrow{AP}|^2=101,\quad \overrightarrow{AP}\cdot\vec{d}=-9,\quad |\vec{d}|^2=3.$$

The negative dot product only specifies the direction of the projection; its squared length is positive. The projection of $\overrightarrow{AP}$ along the line is $(-9/3)(1,1,1)=(-3,-3,-3)$, of squared length 27.

Step 3: Subtract the squared projection The component perpendicular to the line has squared length

$$D^2=101-\frac{(-9)^2}{3}=101-27=74.$$

As a direct check, the closest point on the required line is $A+(-3)\vec{d}=(-3,-2,-1)$. Its displacement from P is $(4,-7,3)$, whose squared length is $16+49+9=74$. Therefore 9, $\sqrt{69}$, and $\sqrt{54}$ are excluded.

Step 4: Conclusion The distance is $\boxed{\sqrt{74}}$ units. Hence, the correct option is (B).

Quick Tip to Solve: First use the plane normal to determine the line direction; only then apply the projection formula for point-to-line distance.

20. The sum of all values of α, for which the points whose position vectors are $\hat{i}-2\hat{j}+3\hat{k}, 2\hat{i}-3\hat{j}+4\hat{k}, (\alpha+1)\hat{i}+2\hat{k}$ and $9\hat{i}+(\alpha-8)\hat{j}+6\hat{k}$ are coplanar, is equal to : [4 marks]

(A) 6

(B) 4

(C) -2

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- Four points A, B, C, D are coplanar precisely when their displacement vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ have zero scalar triple product.
- For a quadratic $a\alpha^2+b\alpha+c=0$, the sum of its roots is $-b/a$. Displacement vectors must all use the same origin.

Step 1: Form three displacement vectors Read the points from the given position vectors:

$$A=(1,-2,3),\ B=(2,-3,4),\ C=(\alpha+1,0,2),\ D=(9,\alpha-8,6).$$

Subtract the coordinates of A from each of the other points:

$$\overrightarrow{AB}=(1,-1,1),\quad \overrightarrow{AC}=(\alpha,2,-1),\quad \overrightarrow{AD}=(8,\alpha-6,3).$$

In particular the middle component of $\overrightarrow{AD}$ is $\alpha-6$, not $\alpha-8$.

Step 2: Apply the zero-determinant condition The required determinant is

$$\begin{vmatrix}1 & -1 & 1\\ \alpha & 2 & -1\\ 8 & \alpha-6 & 3\end{vmatrix}=0.$$

Expanding along the first row gives

$$\begin{aligned}0&=\{6+(\alpha-6)\}+(3\alpha+8)+\{\alpha(\alpha-6)-16\}\\&=\alpha^2-2\alpha-8\\&=(\alpha-4)(\alpha+2).\end{aligned}$$

All terms arise from coordinate differences, so the determinant correctly tests the affine plane through the four points.

Step 3: Add both admissible values The two values are $\alpha=4$ and $\alpha=-2$, so their sum is $4+(-2)=2$. These values are distinct and both make the determinant vanish; neither is lost by division. The option 4 is just one root, and -2 is the other root, rather than their sum; 6 incorrectly adds their absolute values.

Step 4: Conclusion The sum of all admissible values is $\boxed{2}$. Hence, the correct option is (D).

Quick Tip to Solve: For four-point coplanarity, subtract one point from the other three before forming the determinant; use the quadratic coefficients if only the root sum is needed.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If the mean and variance of the frequency distribution

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is _____. [4 marks]

Solution:

Concept:

- For frequencies f_i, the total frequency is $N = \sum f_i$ and the mean is $\bar{x} = (\sum f_i x_i)/N$.
- The population variance is $\sigma^2 = (\sum f_i x_i^2)/N - \bar{x}^2$. Thus the mean and variance yield two equations for the unknown frequencies.

Step 1: Use the given mean The fixed frequencies sum to $4 + 4 + 15 + 8 + 4 + 5 = 40$, giving $N = 40 + \alpha + \beta$. Their contribution to the first moment is

$$4(2) + 4(4) + 15(8) + 8(10) + 4(14) + 5(16) = 360.$$

Therefore the mean condition is

$$\frac{360 + 6\alpha + 12\beta}{40 + \alpha + \beta} = 9.$$

Multiplication and simplification give $360 + 6\alpha + 12\beta = 360 + 9\alpha + 9\beta$, so $\alpha = \beta$.

Step 2: Use the second moment The fixed contribution to $\sum f_i x_i^2$ is

$$4(2^2) + 4(4^2) + 15(8^2) + 8(10^2) + 4(14^2) + 5(16^2) = 3904.$$

Since $\bar{x}^2 + \sigma^2 = 81 + 15.08 = 96.08$,

$$\frac{3904 + 36\alpha + 144\beta}{40 + \alpha + \beta} = 96.08.$$

Set $\beta = \alpha$ and simplify:

$$3904 + 180\alpha = 3843.2 + 192.16\alpha, \qquad 12.16\alpha = 60.8.$$

It follows that $\alpha = 5$, and hence $\beta = 5$.

Step 3: Check and evaluate the expression The frequencies now total 50. Their first moment is $360+30+60 = 450$, so the mean is 9. The second moment is $(3904+180+720)/50 = 4804/50 = 96.08$; subtracting 81 recovers the stated variance 15.08. Finally,

$$\alpha^2 + \beta^2 - \alpha\beta = 25 + 25 - 25 = 25.$$

The nonnegative integral frequencies satisfy both supplied statistical conditions.

Step 4: Conclusion The required numerical value is $\boxed{25}$ (dimensionless).

Quick Tip to Solve: Use the mean first to relate the missing frequencies, then use $E(X^2) = \sigma^2 + \bar{x}^2$ to avoid expanding squared deviations.

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22. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is _____. [4 marks]

Solution:

Concept:

- The hyperbola $x^2/A^2 - y^2/B^2 = 1$ has eccentricity $e_h = \sqrt{1 + B^2/A^2}$. For an ellipse with horizontal major axis, $e_e^2 = 1 - b^2/a^2$.
- Curves intersect orthogonally when their tangent slopes have product -1. An ellipse with semiaxes $a > b$ has latus-rectum length $L = 2b^2/a$.

Step 1: Find the ratio of the squared semiaxes The given hyperbola is $x^2/(1/2) - y^2/(1/2) = 1$, so $e_h = \sqrt{2}$. Its reciprocal gives $e_e = 1/\sqrt{2}$. Thus, for a horizontal major axis,

$$1 - \frac{b^2}{a^2} = \frac{1}{2}, \qquad b^2 = \frac{a^2}{2}.$$

The right-angle condition below also forces $a > b$, confirming this orientation.

Step 2: Apply the orthogonal-intersection condition Differentiate the two curve equations implicitly. At their intersection, the ellipse has slope $-b^2x/(a^2y)$, whereas the hyperbola has slope x/y. Their product is -1, giving

$$\frac{b^2x^2}{a^2y^2} = 1, \qquad y^2 = \frac{b^2}{a^2}x^2 = \frac{x^2}{2}.$$

Substitution into $x^2 - y^2 = 1/2$ gives $x^2 = 1$ and $y^2 = 1/2$. A vertical-major-axis ellipse would give $a^2/b^2 < 1$, incompatible with the hyperbola's $x^2/y^2 > 1$.

Step 3: Determine the latus rectum Insert the intersection coordinates into the ellipse:

$$\frac{1}{a^2} + \frac{1/2}{a^2/2} = 1 \quad \Longrightarrow \quad a^2 = 2, \quad b^2 = 1.$$

Therefore the square of its latus-rectum length is

$$L^2 = \frac{4b^4}{a^2} = \frac{4}{2} = 2.$$

The intersections have nonzero coordinates, so both tangent slopes used above are well defined.

Step 4: Conclusion The required square of the latus-rectum length is $\boxed{2}$ square coordinate units; the numerical entry is 2.

Quick Tip to Solve: Use the eccentricity to fix the axis ratio first; orthogonal tangent slopes then determine the actual semiaxis lengths.

23. The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is _____.

[4 marks]

Solution:

Concept:

- The distinct vowels of UNIVERSE are U, I and E; its consonants are N, V, R and S. Although E occurs twice in the original word, repetition of a letter is forbidden here.
- Choose the required letters using combinations, then arrange the four distinct selected letters using $4!$ permutations.

Step 1: Choose two vowels and two consonants Only three distinct vowel types are available: U, I and E. We must select two different vowels, so there are

$$\binom{3}{2} = 3$$

choices: UI, UE and IE. Selecting two of the four different consonants gives

$$\binom{4}{2} = 6$$

choices. The two printed E copies must not be treated as distinct selections: they produce identical words, and EE would repeat a letter.

Step 2: Arrange every chosen set of four letters For each selection, the two vowels and two consonants are four different letters. They can occupy the four positions in $4! = 24$ ways, since the question imposes no adjacency or alternating-position restriction. The total is therefore

$$\binom{3}{2}\binom{4}{2} 4! = 3 \times 6 \times 24 = 432.$$

Every admissible word determines its unique vowel pair and consonant pair, so no word is counted in two different selections.

Step 3: Check by assigning vowel positions As an independent count, select the two vowel positions in $\binom{4}{2} = 6$ ways. Fill them with two distinct vowels in $3 \times 2 = 6$ orders, then fill the remaining positions with consonants in $4 \times 3 = 12$ orders. This gives $6 \times 6 \times 12 = 432$, confirming that words without dictionary meaning are included equally.

Step 4: Conclusion The number of admissible four-letter words is $\boxed{432}$. This is a count, so it has no physical unit.

Quick Tip to Solve: When repetition is forbidden, list distinct letter types before choosing them; duplicate printed copies of E create no new words.

24. The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ is _____. [4 marks]

Solution:

Concept:

- Complementary angles satisfy $\tan(90^\circ - \theta) = \cot\theta$. Also $\tan\theta + \cot\theta = 2\csc 2\theta$.
- The exact values $\sin 18^\circ = (\sqrt{5}-1)/4$ and $\sin 54^\circ = (\sqrt{5}+1)/4$ allow the remaining difference to be evaluated without approximation.

Step 1: Group complementary tangent terms Let the required expression be T. Because $81^\circ = 90^\circ - 9^\circ$ and $63^\circ = 90^\circ - 27^\circ$,

$$T = (\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ).$$

For any of these angles,

$$\tan\theta + \cot\theta = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{2}{\sin 2\theta}.$$

All denominators are nonzero for the acute angles used.

Step 2: Insert the exact double-angle values The grouped expression becomes

$$T = 2\left(\frac{1}{\sin 18^\circ} - \frac{1}{\sin 54^\circ}\right).$$

Using the exact sine values and rationalising separately,

$$\frac{1}{\sin 18^\circ} = \frac{4}{\sqrt{5}-1} = \sqrt{5}+1,$$

$$\frac{1}{\sin 54^\circ} = \frac{4}{\sqrt{5}+1} = \sqrt{5}-1.$$

Consequently,

$$T = 2\big[(\sqrt{5}+1) - (\sqrt{5}-1)\big] = 2(2) = 4.$$

The positive sign is consistent with $\sin 18^\circ < \sin 54^\circ$, so the first reciprocal is larger.

Step 3: Conclusion The required value is $\boxed{4}$. The expression is dimensionless.

Quick Tip to Solve: Pair the complementary angles before using tangent addition; each pair collapses immediately to twice a cosecant.

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25. For $\alpha, \beta, z \in \mathbb{C}$ and $\lambda > 1$, if $\sqrt{\lambda - 1}$ is the radius of the circle $|z-\alpha|^2 + |z-\beta|^2 = 2\lambda$, then $|\alpha - \beta|$ is equal to _____. [4 marks]

Solution:

Concept:

- For two fixed complex points α, β, their midpoint is $m = (\alpha+\beta)/2$. Squared distances to the endpoints simplify when measured from this midpoint.
- The equation $|z-m|^2 = R^2$ describes a circle centred at m with radius $R > 0$.

Step 1: Rewrite the locus about the midpoint Put $d = (\alpha - \beta)/2$, so $\alpha = m + d$ and $\beta = m - d$. Writing $w = z - m$, expansion with complex conjugates gives

$$|w - d|^2 + |w + d|^2 = 2|w|^2 + 2|d|^2.$$

The mixed terms cancel because they have opposite signs. Since $|d|^2 = |\alpha - \beta|^2/4$, the given equation is

$$2|z - m|^2 + \frac{|\alpha - \beta|^2}{2} = 2\lambda.$$

Step 2: Compare the radius with the prescribed radius Dividing by two and isolating the squared distance gives

$$|z - m|^2 = \lambda - \frac{|\alpha - \beta|^2}{4}.$$

Thus the squared radius is $R^2 = \lambda - |\alpha - \beta|^2/4$. The question specifies $R = \sqrt{\lambda - 1}$, so

$$\lambda - \frac{|\alpha - \beta|^2}{4} = \lambda - 1.$$

Cancelling λ gives $|\alpha - \beta|^2 = 4$. Taking the nonnegative square root is mandatory because a modulus is a distance.

Step 3: Check that the locus is a genuine circle The hypothesis $\lambda > 1$ makes the specified squared radius $\lambda - 1$ positive. The derived separation therefore gives precisely that nondegenerate circle, rather than an empty locus or a single point. No particular location of the midpoint is required.

Step 4: Conclusion The distance between the two complex points is $\boxed{|\alpha - \beta| = 2}$ coordinate units; enter 2.

Quick Tip to Solve: For a sum of two squared distances, shift to the midpoint; the cross terms cancel and the endpoint separation becomes explicit.

26. If the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is _____. [4 marks]

Solution:

Concept:

- Write each symmetric line using its own parameter and equate the three coordinates at an intersection. Read the sign of a direction component carefully when the numerator is $2 - y$.
- After intersection gives a relation between α and β, complete the square to find the unrestricted quadratic's minimum.

Step 1: Find the intersection parameters Let the common ratios of the first and second lines be t and s, respectively. Their coordinates are

$$(x, y, z) = (1 + 2t,\ 2 + 3t,\ 3 + \alpha t),$$

$$(x, y, z) = (4 + 5s,\ 1 + 2s,\ \beta s).$$

In particular, $(2 - y)/(-3) = t$ gives $y = 2 + 3t$, not $2 - 3t$. Equality of the first two coordinates yields

$$2t - 5s = 3, \qquad 3t - 2s = -1.$$

Elimination gives $s = -1$ and $t = -1$.

Step 2: Use the third coordinate At these parameters, the first line has $z = 3 - \alpha$, while the second has $z = -\beta$. Hence

$$3 - \alpha = -\beta, \qquad \beta = \alpha - 3.$$

Conversely, every pair satisfying this relation makes the lines meet at $(-1, -1, 3 - \alpha)$, so there is no further intersection restriction on the optimisation.

Step 3: Minimise the product and take its magnitude Substitution gives

$$8\alpha\beta = 8\alpha(\alpha - 3) = 8\left(\alpha - \frac{3}{2}\right)^2 - 18.$$

The squared term is nonnegative; equality is attained at $\alpha = 3/2$, giving $\beta = -3/2$. Both values are nonzero, so the symmetric forms are valid at the minimiser. The minimum value is -18, and its magnitude is 18. This is the magnitude of the minimum, rather than the minimum of the absolute product.

Step 4: Conclusion The minimum of $8\alpha\beta$ is -18; the requested magnitude is $\boxed{18}$, a dimensionless numerical value.

Quick Tip to Solve: Keep separate parameters for the two lines; solve the first two coordinates before using the third to relate the unknown directions.

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27. If

$(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots + 20(21)^{19} = k(20)^{19}$,

then k is equal to _____. [4 marks]

Solution:

Concept:

- Dividing every term by 20^{19} turns the sum into $\sum_{j=0}^{19}(j+1)r^j$, where $r = 21/20$.
- Subtracting r times a weighted geometric series from itself removes the increasing coefficients; the remaining unweighted series has the usual geometric-sum formula.

Step 1: Express the required coefficient as a finite series The general term on the left is $(j+1)21^j 20^{19-j}$, for $j = 0, 1, \ldots, 19$. Since $20^{19} \neq 0$, dividing the given equality by it gives

$$k = S = 1 + 2r + 3r^2 + \cdots + 20r^{19}, \qquad r = \frac{21}{20}.$$

The upper limit is 19, with coefficient 20; retaining this endpoint is essential.

Step 2: Subtract the shifted series Multiplying by r and subtracting term by term gives

$$(1-r)S = 1 + r + r^2 + \cdots + r^{19} - 20r^{20}.$$

Since $r \neq 1$, the unweighted sum is $(1-r^{20})/(1-r)$. Multiplying the last equality by $1-r$ therefore yields

$$(1-r)^2 S = 1 - r^{20} - 20r^{20}(1-r)$$

$$= 1 - 21r^{20} + 20r^{21} = 1 + r^{20}(20r - 21).$$

This keeps the large powers symbolic instead of evaluating them separately.

Step 3: Exploit the special ratio For $r = 21/20$, the factor $20r - 21$ is exactly zero. Therefore

$$(1-r)^2 S = 1, \qquad S = \frac{1}{(-1/20)^2} = 400.$$

Although $1-r$ is negative, its square is positive. The result is also larger than $1 + 2 + \cdots + 20 = 210$, as expected because $r > 1$ increases every term beyond the first.

Step 4: Conclusion The coefficient is $\boxed{k = 400}$. It is dimensionless.

Quick Tip to Solve: For increasing coefficients times geometric powers, subtract the shifted sum and keep the endpoint symbolic; here $20r - 21 = 0$ cancels it.

28. The number of points, where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the x-axis, is _____.

[4 marks]

Solution:

Concept:

- A polynomial is continuous on the real line. Opposite signs at two endpoints guarantee at least one root strictly between them by the intermediate value theorem.
- A nonzero polynomial of degree five has at most five distinct real roots. Five disjoint sign-changing intervals therefore establish the exact number of crossings.

Step 1: Find convenient sign-testing points Write $f(x) = x^5 - 20x^3 + 50x + 2$. Direct substitution at six increasing integer points gives

$$f(-5) = -3125 + 2500 - 250 + 2 = -873,$$
$$f(-2) = -32 + 160 - 100 + 2 = 30,$$
$$f(-1) = -1 + 20 - 50 + 2 = -29,$$
$$f(1) = 1 - 20 + 50 + 2 = 33,$$
$$f(2) = 32 - 160 + 100 + 2 = -26,$$
$$f(5) = 3125 - 2500 + 250 + 2 = 877.$$

The signs alternate as negative, positive, negative, positive, negative, positive; none of these endpoint values is zero.

Step 2: Locate five distinct real roots By continuity, there is at least one root in each of the disjoint open intervals

$$(-5,-2), \quad (-2,-1), \quad (-1,1), \quad (1,2), \quad (2,5).$$

Each interval has endpoints of opposite signs. Because the intervals are disjoint, the roots furnished by them are distinct, so the polynomial has at least five real zeros.

Step 3: Use the degree to conclude that each root is a crossing The polynomial has degree five and leading coefficient one, so it cannot have more than five distinct zeros. The five already established must therefore be all its roots. Their multiplicities are each at least one and sum to at most five, forcing every multiplicity to be exactly one. A real polynomial changes sign across a simple root, so each of these five zeros represents a crossing of the axis, not a tangential contact. No numerical root approximation or sketch is needed.

Step 4: Conclusion The curve crosses the x-axis at exactly $\boxed{5}$ points. This is a dimensionless count.

Quick Tip to Solve: To count real roots of an odd-degree polynomial, try small integer test points first; enough disjoint sign changes can settle the count without differentiation.

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29. Let a curve $y = f(x), x \in (0, \infty)$ pass through the points $P\left(1, \frac{3}{2}\right)$ and $Q\left(a, \frac{1}{2}\right)$. If the tangent at any point $R(b, f(b))$ to the given curve cuts the y-axis at the point $S(0, c)$ such that $bc = 3$, then $(PQ)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The tangent to $y = f(x)$ at abscissa x has slope $y\prime$ and equation $Y - y = y\prime(X - x)$. Its intercept on the vertical axis is therefore $c = y - xy\prime$.
- A first-order linear differential equation can be solved by recognizing the derivative of y/x. Once the curve is known, the squared distance between two points is the sum of the squared coordinate differences.

Step 1: Translate the tangent condition into an equation The condition holds at every positive abscissa, so replace b by the variable x. Setting $X = 0$ in the tangent equation gives its intercept. Hence

$$x(y - xy\prime) = 3.$$

Dividing by x^2, which is permitted because $x > 0$, gives

$$y\prime - \frac{y}{x} = -\frac{3}{x^2}.$$

Step 2: Integrate and use the given point Divide the equation by x. The left side is exactly the quotient-rule derivative:

$$\frac{d}{dx}\left(\frac{y}{x}\right) = -\frac{3}{x^3}.$$

Integrating on the positive half-line,

$$\frac{y}{x} = \frac{3}{2x^2} + C, \qquad y = \frac{3}{2x} + Cx.$$

Since $P = (1, 3/2)$ lies on the curve, $3/2 = 3/2 + C$, so $C = 0$. Thus the required curve is $y = 3/(2x)$.

Step 3: Locate the second point and calculate the distance At $Q = (a, 1/2)$, substitute into the curve equation:

$$\frac{1}{2} = \frac{3}{2a} \quad \Longrightarrow \quad a = 3.$$

Consequently $Q = (3, 1/2)$. Apply the distance formula without taking a square root, because the question asks for its square:

$$(PQ)^2 = (3-1)^2 + \left(\frac{1}{2} - \frac{3}{2}\right)^2 = 4 + 1 = 5.$$

The tangent intercept for this curve is $3/x$, confirming the original condition $xc = 3$.

Step 4: Conclusion The requested numerical value is $\boxed{5}$ (squared coordinate units).

Quick Tip to Solve: Write the tangent intercept as $y - xy\prime$ first; after division by x, the linear equation exposes $(y/x)\prime$.

30. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$, $x \in \mathbb{R} - \{-1\}$, $n \in \mathbb{N}$, $n > 2$.

If $f^n(x) = (f \circ f \circ \cdots \circ f)(x)$, with n compositions, then $\lim\limits_{n\to\infty} \int_0^1 x^{n-2} f^n(x)\, dx$ is equal to _____.

[4 marks]

Solution:

Concept:

- The notation $f^n(x)$ here means n-fold composition, not the ordinary power $(f(x))^n$. For each fixed n, the exponent parameter stays unchanged throughout all compositions.
- On $[0, 1]$, every denominator $(1 + x^n)^{1/n}$ is at least one. Nonnegative integral bounds can therefore determine the limit by the squeeze theorem without exchanging a limit and an integral.

Step 1: Find the repeated-composition pattern For a fixed integer $n > 2$, write $g_k = f \circ \cdots \circ f$, with k copies. The first two iterates are

$$g_1(x) = \frac{x}{(1 + x^n)^{1/n}}, \qquad g_2(x) = \frac{x}{(1 + 2x^n)^{1/n}}.$$

Indeed, $g_1(x)^n = x^n/(1 + x^n)$, which simplifies the denominator in $f(g_1(x))$. The same substitution proves inductively that

$$g_k(x) = \frac{x}{(1 + kx^n)^{1/n}}.$$

All quantities used are real and nonnegative on the interval of integration.

Step 2: Bound the required integral Put $k = n$ in the iterate formula. If I_n denotes the integral in the

question,

$$I_n = \int_0^1 \frac{x^{n-1}}{(1+nx^n)^{1/n}}\,dx.$$

The denominator is at least one, so pointwise on $[0,1]$ we have

$$0 \le \frac{x^{n-1}}{(1+nx^n)^{1/n}} \le x^{n-1}.$$

Integration preserves both inequalities and yields

$$0 \le I_n \le \int_0^1 x^{n-1}\,dx = \frac{1}{n}.$$

Step 3: Apply the squeeze theorem The lower bound is zero for every n, while $1/n$ tends to zero. Thus the sequence of nonnegative integrals also tends to zero. This argument is valid for the literal composition notation and avoids the incorrect substitution of the ordinary nth power. As a check, putting $u = 1 + nx^n$ gives

$$I_n = \frac{(n+1)^{(n-1)/n} - 1}{n(n-1)},$$

which has the same limit.

Step 4: Conclusion The requested limit is $\boxed{0}$, a dimensionless number.

Quick Tip to Solve: For limits of positive integrals, seek a simple upper bound first: here the entire integral is at most $\int_0^1 x^{n-1}\,dx = 1/n$.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A particle starts with an initial velocity of $10.0\ \text{ms}^{-1}$ along x-direction and accelerates uniformly at the rate of $2.0\ \text{ms}^{-2}$. The time taken by the particle to reach the velocity of $60.0\ \text{ms}^{-1}$ is _____.

[4 marks]

(A) 30s

(B) 6s

(C) 3s

(D) 25s

Correct Answer: (D) 25s

Solution:

Concept:

- For constant acceleration in one dimension, the change in velocity is acceleration multiplied by elapsed time: $v - u = at$. The velocities and acceleration must be expressed along the same chosen axis.
- The initial speed is already nonzero. Therefore the required change is the final velocity minus the initial velocity, rather than the final velocity alone. SI values of velocity and acceleration give time

in seconds.

Step 1: Identify the initial and final velocities Take the positive axis along the stated initial direction. The given motion has

$$u = 10.0\,\mathrm{m\,s^{-1}}, \quad v = 60.0\,\mathrm{m\,s^{-1}}, \quad a = 2.0\,\mathrm{m\,s^{-2}}.$$

The positive acceleration increases the positive velocity. The required increment is

$$\Delta v = 60.0 - 10.0 = 50.0\,\mathrm{m\,s^{-1}}.$$

No displacement is supplied or needed because the velocity-time relation directly connects these three data.

Step 2: Solve the velocity-time equation Rearranging $v = u + at$ gives

$$t = \frac{v - u}{a} = \frac{50.0}{2.0} = 25\,\mathrm{s}.$$

The unit check is $(\mathrm{m\,s^{-1}})/(\mathrm{m\,s^{-2}}) = \mathrm{s}$. Substitution verifies the result: $10.0 + 2.0(25) = 60.0\,\mathrm{m\,s^{-1}}$.

Step 3: Check the alternatives Option (A), $30\,\mathrm{s}$, would give a final velocity of $70\,\mathrm{m\,s^{-1}}$; it results from wrongly using v/a. Option (B), $6\,\mathrm{s}$, gives $22\,\mathrm{m\,s^{-1}}$, while option (C), $3\,\mathrm{s}$, gives $16\,\mathrm{m\,s^{-1}}$. Only option (D) produces the required final velocity under the stated constant acceleration.

Step 4: Conclusion The elapsed time is $\boxed{25\,\mathrm{s}}$. Hence, the correct option is (D).

Quick Tip to Solve: Compute the velocity increment before dividing by acceleration; using the final velocity alone silently assumes that the particle started from rest.

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32. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: The phase difference of two light waves change if they travel through different media having same thickness, but different indices of refraction.

Reason R: The wavelengths of waves are different in different media.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) Both A and R are correct but R is NOT the correct explanation of A

(B) A is correct but R is not correct

(C) A is not correct but R is correct

(D) Both A and R are correct and R is the correct explanation of A

Correct Answer: (D) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- Light retains its frequency ν on entering a stationary medium. Its speed is c/n, so its wavelength becomes $\lambda = \lambda_0/n$, where $\lambda_0 = c/\nu$ is the vacuum wavelength.
- A wave accumulates phase $2\pi L/\lambda$ while traversing thickness L. Consequently the refractive index controls the optical path nL, even if two media have the same geometrical thickness.

Step 1: Evaluate Assertion A Let the two media have equal thickness L and refractive indices n_1 and n_2. For light of the same frequency the respective phase advances are

$$\phi_1 = \frac{2\pi n_1 L}{\lambda_0}, \qquad \phi_2 = \frac{2\pi n_2 L}{\lambda_0}.$$

Their relative phase therefore acquires the difference

$$\phi_1 - \phi_2 = \frac{2\pi L}{\lambda_0}(n_1 - n_2).$$

Equal geometrical distances do not ensure equal phase advances. Assertion A expresses this standard dependence and is correct.

Step 2: Evaluate Reason R The frequency is fixed by the source, while the propagation speed changes in a medium. Since $v = \nu\lambda$, the wavelength must change with the speed. In particular,

$$\lambda_1 = \frac{\lambda_0}{n_1}, \qquad \lambda_2 = \frac{\lambda_0}{n_2}.$$

Different refractive indices give different wavelengths for the same incident light. Thus Reason R is also correct.

Step 3: Test the explanatory connection The phase accumulated across a fixed thickness equals 2π times the number of wavelengths within that thickness. Different wavelengths cause different numbers of oscillation cycles and therefore different phase advances. Reason R directly explains Assertion A. Option (A) incorrectly denies this connection; (B) incorrectly rejects the wavelength change; (C) incorrectly rejects the assertion. The appropriate combined statement is option (D).

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (D).

Quick Tip to Solve: Use $\phi = 2\pi nL/\lambda_0$: equal physical thickness gives equal phase only when the optical paths are equal.

33. The temperature of an ideal gas is increased from $200\ \mathrm{K}$ to $800\ \mathrm{K}$. If r.m.s. speed of gas at $200\ \mathrm{K}$ is v_0. Then, r.m.s. speed of the gas at $800\ \mathrm{K}$ will be: [4 marks]

(A) v_0

(B) $2v_0$

(C) $4v_0$

(D) $\frac{v_0}{4}$

Correct Answer: (B) $2v_0$

Solution:

Concept:

- For an ideal gas, $v_{\mathrm{rms}} = \sqrt{3k_{\mathrm{B}}T/m} = \sqrt{3RT/M}$, where T is absolute temperature and m or M is the molecular or molar mass.
- For the same gas, mass is unchanged, so the root-mean-square speed is proportional to $\sqrt{T}$. Temperature ratios must use kelvin, not Celsius, because molecular kinetic energy is proportional to absolute temperature.

Step 1: Form the ratio for the same gas Initially $T_1 = 200\,\mathrm{K}$ and $v_{\mathrm{rms},1} = v_0$. Finally $T_2 = 800\,\mathrm{K}$. Dividing the two kinetic-theory expressions cancels both the gas constant and the gas mass:

$$\frac{v_{\mathrm{rms},2}}{v_0} = \sqrt{\frac{T_2}{T_1}}.$$

There is no need to know the identity of the gas, its amount, its pressure or its volume for this ratio.

Step 2: Substitute the temperature ratio The absolute temperature increases by a factor of four:

$$\frac{T_2}{T_1} = \frac{800}{200} = 4.$$

Taking the square root gives

$$\frac{v_{\mathrm{rms},2}}{v_0} = \sqrt{4} = 2, \qquad v_{\mathrm{rms},2} = 2v_0.$$

This is consistent with $\frac{1}{2}mv_{\mathrm{rms}}^2 = \frac{3}{2}k_{\mathrm{B}}T$: quadrupling temperature quadruples mean-square speed, rather than speed itself.

Step 3: Rule out the remaining choices Option (A) would mean no change in mean kinetic energy despite heating. Option (C), $4v_0$, would multiply mean-square speed and temperature by sixteen, which is too large. Option (D), $v_0/4$, predicts cooling and is opposite to the stated rise in temperature. Only option (B) obeys the square-root temperature dependence and increases the speed by the correct factor.

Step 4: Conclusion The final root-mean-square speed is $\boxed{2v_0}$. Hence, the correct option is (B).

Quick Tip to Solve: For one gas, compare absolute temperatures and take a square root: a fourfold temperature increase doubles the r.m.s. speed.

34. The energy density associated with electric field $\vec{E}$ and magnetic field $\vec{B}$ of an electromagnetic wave in free space is given by (ϵ_0- permittivity of free space, μ_0- permeability of free space) [4 marks]

(A) $U_E = \frac{\epsilon_0 E^2}{2}, U_B = \frac{B^2}{2\mu_0}$

(B) $U_E = \frac{E^2}{2\epsilon_0}, U_B = \frac{\mu_0 B^2}{2}$

(C) $U_E = \frac{\epsilon_0 E^2}{2}, U_B = \frac{\mu_0 B^2}{2}$

(D) $U_E = \frac{E^2}{2\epsilon_0}, U_B = \frac{B^2}{2\mu_0}$

Correct Answer: (A) $U_E = \frac{\epsilon_0 E^2}{2}, U_B = \frac{B^2}{2\mu_0}$

Solution:

Concept:

- The instantaneous electric-field energy per unit volume in vacuum is $U_E = \frac{1}{2}\epsilon_0 E^2$. The magnetic-field contribution is $U_B = B^2/(2\mu_0)$. Each has SI unit $\mathrm{J\,m^{-3}}$.
- For a travelling electromagnetic wave in vacuum, $E = cB$ and $c^2 = 1/(\epsilon_0\mu_0)$. These relations show that the electric and magnetic contributions are equal at each point and instant.

Step 1: Identify the electric contribution The energy stored in a vacuum electric field depends quadratically on field magnitude. The proportionality coefficient is the permittivity, giving

$$U_E = \frac{1}{2}\epsilon_0 E^2.$$

The expression $E^2/(2\epsilon_0)$ is incorrect for energy density written in terms of E; inverse permittivity would appear if the expression were instead written in terms of electric displacement D. Therefore options (B) and (D) already have an incorrect electric term.

Step 2: Identify the magnetic contribution The vacuum magnetic energy density is

$$U_B = \frac{B^2}{2\mu_0}.$$

Here permeability is in the denominator because the field variable supplied is B. Equivalently, the general form $BH/2$ with $H = B/\mu_0$ gives the same result. The expressions $\mu_0 B^2/2$ in options (B) and (C) do not have the required dependence or dimensions.

Step 3: Check equality for a vacuum wave Using $B = E/c$,

$$U_B = \frac{E^2}{2\mu_0 c^2} = \frac{1}{2}\epsilon_0 E^2 = U_E.$$

Thus option (A) gives both correct contributions and their equality for the wave. The total instantaneous density is $U_E + U_B = \epsilon_0 E^2$. These are instantaneous formulas; an additional time-average factor applies only if one replaces instantaneous fields by sinusoidal peak amplitudes.

Step 4: Conclusion $\boxed{U_E = \epsilon_0 E^2/2, \quad U_B = B^2/(2\mu_0)}$. Hence, the correct option is (A).

Quick Tip to Solve: Remember the paired forms $\epsilon_0 E^2/2$ and $B^2/(2\mu_0)$; verify them quickly with $E = cB$ and equal wave energy contributions.

35. A 2 meter long scale with least count of 0.2 cm is used to measure the locations of objects on an optical bench. While measuring the focal length of a convex lens, the object pin and the convex lens are placed at 80 cm mark and 1 m mark, respectively. The image of the object pin on the other side of lens coincides with image pin that is kept at 180 cm mark. The $\%$ error in the estimation of focal length is: [4 marks]

(A) 1.70

(B) 0.51

(C) 1.02

(D) 0.85

Correct Answer: (A) 1.70

Solution:

Concept:

- Using positive object and real-image distances u and v, the convex-lens focal length is $f = uv/(u+v)$. The distances are differences of two independently measured bench positions.
- For maximum absolute error, errors in a difference add. If each scale position has uncertainty equal to its least count, both u and v have uncertainty twice the least count. First-order propagation uses the absolute magnitudes of the differential coefficients.

Step 1: Find the distances and their errors Convert the lens position to centimetres: $1\,\text{m} = 100\,\text{cm}$. Hence

$$u = 100 - 80 = 20\,\text{cm}, \qquad v = 180 - 100 = 80\,\text{cm}.$$

Each distance uses two scale readings of least count $0.2\,\text{cm}$, so the maximum uncertainty is

$$\Delta u = \Delta v = 0.2 + 0.2 = 0.4\,\text{cm}.$$

The nominal focal length is $f = 20(80)/(20+80) = 16\,\text{cm}$.

Step 2: Differentiate the focal-length expression For $f = uv/(u+v)$, differentiation and division by f give

$$\frac{df}{f} = \frac{v}{u+v}\frac{du}{u} + \frac{u}{u+v}\frac{dv}{v}.$$

For the maximum-error estimate take positive magnitudes. The factors $v/(u+v)$ and $u/(u+v)$ account for the denominator in the lens formula; simply adding the fractional errors in u and v would overestimate the result.

Step 3: Calculate the percentage uncertainty Substituting the distances and uncertainties,

$$\begin{aligned}\frac{\Delta f}{f} &= \frac{80}{100}\frac{0.4}{20} + \frac{20}{100}\frac{0.4}{80} \\ &= 0.016 + 0.001 = 0.017.\end{aligned}$$

Thus the percentage error is $100(0.017) = 1.70\%$. Option (D), 0.85%, uses only one least count for each distance and misses that each is a difference of two readings. Options (B) and (C) also disagree with the propagated uncertainty. The matching choice is (A).

Step 4: Conclusion The percentage error in focal length is $\boxed{1.70\%}$. Hence, the correct option is (A).

Quick Tip to Solve: On an optical bench, a distance is a difference of two position readings: add their absolute uncertainties before propagating error through the lens formula.

36. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: When you squeeze one end of a tube to get toothpaste out from the other end, Pascal's principle is observed.

Reason R: A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) Both A and R are correct but R is NOT the correct explanation of A

(B) A is not correct but R is correct

(C) A is correct but R is not correct

(D) Both A and B are correct and R is the correct explanation of A

Correct Answer: (D) Both A and B are correct and R is the correct explanation of A

Solution:

Concept:

- Pascal's principle states that an increase of pressure applied to an enclosed, incompressible fluid is communicated undiminished throughout the fluid and to its container walls.
- Pressure is force per unit area, $p = F/A$. A pressure increase can produce an outward force $\Delta F = \Delta p\, A$ at another part of the fluid boundary.

Step 1: Examine the assertion Squeezing the flexible tube pushes its wall inward and raises the pressure in the toothpaste. The paste transmits this increase through its bulk, including towards the open nozzle at the other end. There the paste is driven outward into the lower-pressure surroundings. Thus the described observation is the familiar application of Pascal's principle, so assertion A is correct.

Step 2: Examine the reason The reason correctly states the pressure-transmission law for an enclosed incompressible fluid. It is the additional pressure that is transmitted undiminished; the statement does not require the pressure at different heights to have the same absolute value. Therefore reason R is also correct.

Step 3: Connect the two statements The transmission described in R explains why force applied at the closed end produces a pressure increase and an outward push at the far nozzle. Hence R explains A. Option (A) incorrectly denies this explanatory link, while (B) and (C) incorrectly reject one of the two correct statements. The last option is the intended both-correct, correct-explanation choice.

Step 4: Conclusion Both A and R are correct, and R correctly explains A. Hence, the correct option is (D), the explanatory choice (whose printed wording has B in place of R).

Quick Tip to Solve: For a Pascal-principle assertion, identify where pressure is applied and where the transmitted pressure produces an output force.

37. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Diffusion current in a p-n junction is greater than the drift current in magnitude if the junction is forward biased.

Reason R: Diffusion current in a p-n junction is from the n-side to the p-side if the junction is forward biased.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) Both A and R are correct but R is NOT the correct explanation of A

(B) A is correct but R is not correct

(C) A is not correct but R is correct

(D) Both A and R are correct and R is the correct explanation of A

Correct Answer: (B) A is correct but R is not correct

Solution:

Concept:

- Diffusion of majority carriers across a p-n junction gives conventional diffusion current from the p-side to the n-side.
- The depletion-region electric field causes minority-carrier drift current in the opposite direction. At equilibrium the two currents have equal magnitudes; forward bias produces a net forward current.

Step 1: Judge the magnitude statement Forward bias reduces the junction barrier and narrows the depletion layer. Majority carriers cross the junction much more readily, increasing the diffusion current substantially. The minority-carrier drift component is approximately the reverse saturation current and changes little with applied forward voltage. Thus the diffusion-current magnitude exceeds the drift-current magnitude, making assertion A correct.

Step 2: Determine the conventional current direction Electrons diffuse from the n-region into the p-region, but their negative charge makes their conventional current point from p to n. Holes diffuse from p to n, so their conventional current also points from p to n. These contributions add rather than oppose one another. Therefore the total diffusion current is from the p-side to the n-side.

Step 3: Evaluate the reason and options Reason R reverses the conventional-current direction by assigning the direction of electron motion to current. It is consequently false even though A is true. Options (A) and (D) require R to be true, so both fail. Option (C) requires A to be false and also gives the wrong truth value for R.

Step 4: Conclusion A is correct but R is not correct. Hence, the correct option is (B).

Quick Tip to Solve: Always reverse the electron-flow arrow when finding conventional current; hole-flow and conventional-current arrows agree.

38. A capacitor of capacitance $150.0\ \mu\text{F}$ is connected to an alternating source of emf given by $\text{E} = 36\sin(120\pi\text{t})\text{V}$. The maximum value of current in the circuit is approximately equal to : [4 marks]

(A) $\frac{1}{\sqrt{2}}A$

(B) $2\sqrt{2}A$

(C) $\sqrt{2}A$

(D) $2A$

Correct Answer: (D) $2A$

Solution:

Concept:

- For a capacitor, $q = CV$ and $i = dq/dt = C\,dV/dt$. A sinusoidal voltage therefore produces a current that leads it by a quarter cycle.
- If $V = V_0 \sin\omega t$, then the maximum current is $I_0 = \omega C V_0$. Here V_0 is the voltage amplitude, not its rms value.

Step 1: Read the voltage amplitude and angular frequency Compare the given expression $E = 36\sin(120\pi t)\,\text{V}$ with $V_0 \sin\omega t$. It gives $V_0 = 36\,\text{V}$ and $\omega = 120\pi\,\text{rad}\,\text{s}^{-1}$. Convert the capacitance before substitution: $C = 150.0 \times 10^{-6}\,\text{F}$. The coefficient of time already is angular frequency; an additional factor of 2π must not be inserted.

Step 2: Differentiate and calculate the amplitude Using the capacitor equation gives

$$i(t) = C\frac{d}{dt}\{36\sin(120\pi t)\} = 36(120\pi)C\cos(120\pi t).$$

Hence

$$I_0 = (120\pi)(150.0 \times 10^{-6})(36) = 0.648\pi\ \text{A} \simeq 2.04\,\text{A}.$$

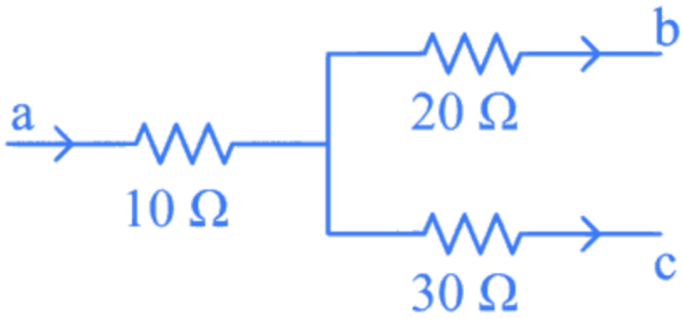
The cosine attains magnitude one, which is why this coefficient is the maximum current.

Step 3: Compare with all the choices The closest listed value is $2\,\text{A}$. The value $\sqrt{2}\,\text{A}$ is approximately the rms current, obtained by dividing the amplitude by $\sqrt{2}$, and does not answer the question. The values $1/\sqrt{2}\,\text{A}$ and $2\sqrt{2}\,\text{A}$ disagree with the directly calculated amplitude.

Step 4: Conclusion The maximum current is approximately $2\,\text{A}$. Hence, the correct option is (D).

Quick Tip to Solve: For a sine voltage across a capacitor, read off the amplitude and angular frequency and multiply them by C; divide by square root of two only for rms current.

39. Figure shows a part of an electric circuit. The potentials at points a, b and c are $30\ \text{V}, 12\ \text{V}$ and $2\ \text{V}$ respectively. The current through the $20\ \Omega$ resistor will be, [4 marks]



(A) 0.2 A

(B) 0.6 A

(C) 0.4 A

(D) 1.0 A

Correct Answer: (C) 0.4 A

Solution:

Concept:

- Kirchhoff's junction law states that the total current entering a junction equals the total current leaving it, because charge does not accumulate there.
- For a resistor joining potentials V_1 and V_2, the current from the first end to the second is $(V_1 - V_2)/R$. A negative answer reverses the chosen direction.

Step 1: Assign the junction potential Let the potential of the common junction of the three resistors be V. The $10\,\Omega$ resistor connects it to point a at $30\,\text{V}$, the $20\,\Omega$ resistor connects it to point b at $12\,\text{V}$, and the $30\,\Omega$ resistor connects it to point c at $2\,\text{V}$. The two right-hand resistors do not have the same end potentials, so treating them as ordinary parallel resistors would be incorrect.

Step 2: Apply current conservation and solve Take current from a towards the junction as entering, and currents towards b and c as leaving. Then

$$\frac{30 - V}{10} = \frac{V - 12}{20} + \frac{V - 2}{30}.$$

Multiplying throughout by 60 gives

$$6(30 - V) = 3(V - 12) + 2(V - 2).$$

Therefore $180 - 6V = 5V - 40$, so $11V = 220$ and $V = 20\,\text{V}$.

Step 3: Find and check the requested current The current in the $20\,\Omega$ resistor is

$$I_{20} = \frac{20 - 12}{20} = 0.4\,\text{A},$$

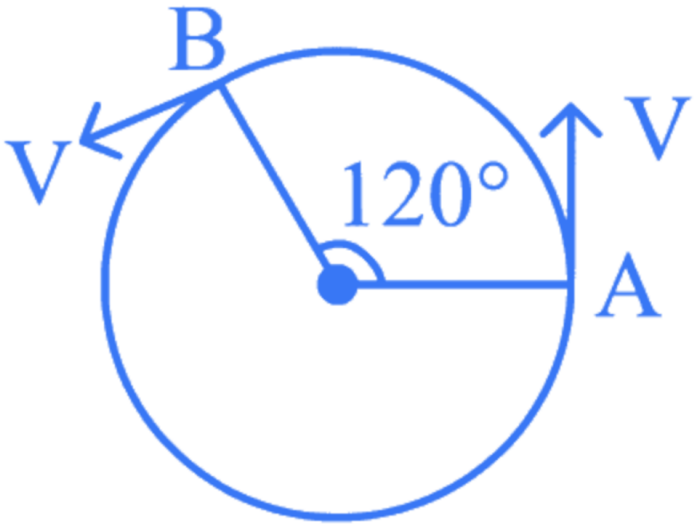
directed from the junction towards b. For a check, the entering current is $(30 - 20)/10 = 1.0\,\text{A}$, and

the other outgoing current is $(20-2)/30 = 0.6\,\text{A}$. Indeed $1.0 = 0.4 + 0.6$. Options (B) and (D) give currents in the other branches; (A) satisfies neither the resistor voltage difference nor the junction equation.

Step 4: Conclusion The current through the $20\,\Omega$ resistor is $0.4\,\text{A}$, towards b. Hence, the correct option is (C).

Quick Tip to Solve: When branch ends have different specified potentials, first solve for the common junction potential; do not combine those branches as parallel resistors.

40. As shown in the figure, a particle is moving with constant speed π m/s. Considering its motion from A to B, the magnitude of the average velocity is : [4 marks]



(A) π m/s
(B) $1.5\sqrt{3}$ m/s
(C) $\sqrt{3}$ m/s
(D) $2\sqrt{3}$ m/s

Correct Answer: (B) $1.5\sqrt{3}$ m/s

Solution:

Concept:

- The magnitude of average velocity is displacement divided by elapsed time. Average speed instead uses the length of the actual path.
- For radius r and swept angle θ in radians, arc length is $r\theta$, while the chord joining the end positions has length $2r\sin(\theta/2)$.

Step 1: Calculate the travel time from the arc The figure shows motion from A to B counterclockwise through $120°$, or $2\pi/3$ radians. Let the unspecified circle radius be r metres. Since the speed is

constant at $\pi\,\mathrm{m\,s^{-1}}$, the distance travelled and time taken are

$$s = r\frac{2\pi}{3}, \qquad \Delta t = \frac{s}{\pi} = \frac{2r}{3}\,\mathrm{s}.$$

Use the indicated smaller arc, as fixed by the velocity arrows and the marked angle.

Step 2: Find the displacement and average velocity The displacement is the straight chord AB, irrespective of the curved path between the endpoints. Its magnitude is

$$|\Delta\mathbf{r}| = 2r\sin 60^\circ = \sqrt{3}\,r.$$

Consequently

$$|\mathbf{v}_{\mathrm{av}}| = \frac{\sqrt{3}\,r}{2r/3} = \frac{3\sqrt{3}}{2}\,\mathrm{m\,s^{-1}}.$$

The radius cancels, so no numerical radius is needed.

Step 3: Check the result against the options This equals $1.5\sqrt{3}\,\mathrm{m\,s^{-1}}$, as in (B). Its approximate value, $2.60\,\mathrm{m\,s^{-1}}$, is smaller than the speed $\pi\,\mathrm{m\,s^{-1}}$, as required because a chord is shorter than its arc. Option (A) substitutes speed for average velocity, while (C) and (D) use incorrect chord-to-time ratios; (D) even exceeds the constant speed.

Step 4: Conclusion The magnitude of average velocity is $1.5\sqrt{3}\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a circular arc at constant speed v, use average-velocity magnitude $v\,2\sin(\theta/2)/\theta$, taking theta in radians.

41. The work functions of Aluminium and Gold are 4.1 eV and and 5.1 eV respectively. The ratio of the slope of the stopping potential versus frequency plot for Gold to that of Aluminium is [4 marks]

(A) 1.5

(B) 1.24

(C) 1

(D) 2

Correct Answer: (C) 1

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where h is Planck's constant, ν is incident frequency, and ϕ is the work function.
- The stopping-potential magnitude satisfies $eV_s = K_{\max}$, where e is the magnitude of electronic charge. Thus $V_s = (h/e)\nu - \phi/e$.

Step 1: Identify what sets the graph slope Compare the stopping-potential equation with the straight-line form $y = mx + c$. With frequency on the horizontal axis and stopping potential on the vertical

axis, the coefficient of frequency is the slope:

$$m = \frac{dV_s}{d\nu} = \frac{h}{e}.$$

Both constants are universal and do not depend on the emitting metal. The relationship is used for frequencies above the respective threshold, where photoemission occurs.

Step 2: Distinguish the work-function effect Aluminium and gold have different work functions, $4.1\,\text{eV}$ and $5.1\,\text{eV}$. These values change the vertical intercept $-\phi/e$, and the threshold frequency $\nu_0 = \phi/h$, but do not change the coefficient h/e. Thus the two straight-line graphs have different intercepts but are parallel; the larger work function does not imply a steeper graph.

Step 3: Calculate the ratio and reject alternatives The required gold-to-aluminium ratio is

$$\frac{m_{\text{Au}}}{m_{\text{Al}}} = \frac{h/e}{h/e} = 1.$$

The approximate value 1.24 comes from dividing the work functions $5.1/4.1$; that compares threshold frequencies, not slopes. Values 1.5 and 2 likewise contradict the same universal slope for both metals.

Step 4: Conclusion The ratio of slopes is 1, a dimensionless quantity. Hence, the correct option is (C).

Quick Tip to Solve: In a stopping-potential versus frequency graph, work function shifts the intercept, while the slope always equals h/e for any metal.

42. A body cools in 7 minutes from 60°C to 40°C. The temperature of the surrounding is 10°C. The temperature of the body after the next 7 minutes will be: [4 marks]

(A) 34°C

(B) 28°C

(C) 32°C

(D) 30°C

Correct Answer: (B) 28°C

Solution:

Concept:

- Newton's law of cooling gives $d(T - T_s)/dt = -k(T - T_s)$ for constant surrounding temperature T_s and cooling constant k.
- The excess temperature therefore decays exponentially: $T(t) - T_s = (T(0) - T_s)e^{-kt}$. Equal time intervals multiply the excess temperature by the same factor.

Step 1: Find the temperature-excess ratio for seven minutes The surrounding temperature is $T_s = 10^\circ\text{C}$. At the beginning the excess temperature is $60 - 10 = 50^\circ\text{C}$, and after seven minutes it is $40 - 10 = 30^\circ\text{C}$. Temperature differences have the same numerical value in kelvin and degrees Celsius,

so these differences may be used directly in the cooling equation. Hence

$$e^{-7k} = \frac{30}{50} = \frac{3}{5}.$$

Step 2: Apply the same ratio over the next interval The next interval also lasts seven minutes, and the body and its surroundings have the same cooling conditions. Therefore the new excess temperature is

$$T(14) - 10 = 30\left(\frac{3}{5}\right) = 18^\circ\text{C}.$$

Restoring the surrounding temperature gives

$$T(14) = 10 + 18 = 28^\circ\text{C}.$$

We multiply the excess above the surroundings, rather than the absolute Celsius reading.

Step 3: Check the physical behaviour and options The temperature falls by 20°C in the first interval but by only 12°C in the second. This slower cooling is expected as the thermal difference decreases. A constant temperature-drop assumption would be incorrect. The listed values 34, 32, and 30°C fail the equal-interval ratio $(T-10)/30 = 3/5$; only 28°C satisfies it.

Step 4: Conclusion After the next seven minutes, the body is at 28°C. Hence, the correct option is (B).

Quick Tip to Solve: For equal cooling intervals, form a geometric progression using temperature above the surroundings, not the Celsius temperature itself.

43. A dipole comprises of two charged particles of identical magnitude q and opposite in nature. The mass 'm' of the positive charged particle is half of the mass of the negative charged particle. The two charges are separated by a distance 'l'. If the dipole is placed in a uniform electric field '$\bar{E}$'; in such a way that dipole axis makes a very small angle with the electric field, '$\bar{E}$'. The angular frequency of the oscillations of the dipole when released is given by: [4 marks]

(A) $\sqrt{\frac{3qE}{2ml}}$

(B) $\sqrt{\frac{4qE}{ml}}$

(C) $\sqrt{\frac{8qE}{3ml}}$

(D) $\sqrt{\frac{8qE}{ml}}$

Correct Answer: (A) $\sqrt{\frac{3qE}{2ml}}$

Solution:

Concept:

- The torque on a rigid electric dipole is restoring near stable alignment: $\tau = -pE\sin\theta$, where $p = ql$. For a small angular displacement, $\sin\theta \simeq \theta$.

- Rotational simple harmonic motion obeys $I\ddot{\theta} = -pE\theta$, so its angular frequency is $\Omega = \sqrt{pE/I}$. The relevant moment of inertia is about the centre of mass.

Step 1: Locate the centre of mass The positive charge has mass m, whereas the negative charge has mass $2m$. Let their distances from the centre of mass be r_+ and r_-. Then $mr_+ = 2mr_-$, and $r_+ + r_- = l$. Hence

$$r_+ = \frac{2l}{3}, \qquad r_- = \frac{l}{3}.$$

The unequal masses mean that the geometrical midpoint is not the centre of rotation. The uniform electric field exerts equal and opposite forces, so their resultant force vanishes and the centre of mass has no electric acceleration.

Step 2: Calculate the inertia and restoring frequency Treating the charges as particles,

$$I = m\left(\frac{2l}{3}\right)^2 + 2m\left(\frac{l}{3}\right)^2 = \frac{2ml^2}{3}.$$

Therefore

$$\Omega^2 = \frac{qlE}{2ml^2/3} = \frac{3qE}{2ml}.$$

The coefficient is $3/2$, ruling out the coefficients 4, $8/3$, and 8 in the other alternatives. The result also has dimensions of inverse time squared before taking its square root.

Step 3: Conclusion The angular frequency is $\boxed{\Omega = \sqrt{3qE/(2ml)}}$. Hence, the correct option is (A).

Quick Tip to Solve: For two unequal point masses a fixed distance apart, use $I = \mu l^2$, with reduced mass $\mu = m_1 m_2/(m_1 + m_2)$, before applying the dipole torque.

44. The ratio of speed of sound in hydrogen gas to the speed of sound in oxygen gas at the same temperature is: [4 marks]

(A) $1 : 1$

(B) $1 : 2$

(C) $1 : 4$

(D) $4 : 1$

Correct Answer: (D) $4 : 1$

Solution:

Concept:

- For sound travelling through an ideal gas, $v = \sqrt{\gamma RT/M}$, where $\gamma = C_P/C_V$, R is the molar gas constant, T is absolute temperature and M is molar mass.
- At the same temperature, the common factor RT cancels in a speed ratio. Hydrogen and oxygen are both diatomic gases, so their heat-capacity ratios are taken equal in this comparison.

Step 1: Write the ratio before substituting masses Use the gas formula separately for the two gases:

$$\frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{\gamma_{H_2}}{\gamma_{O_2}} \frac{M_{O_2}}{M_{H_2}}}.$$

With the diatomic ideal-gas value $\gamma = 7/5$ for each, the heat-capacity ratio cancels. Thus sound is faster in the gas with smaller molar mass. Equal temperature alone does not make the speeds equal, because the inertial response of the gases differs.

Step 2: Substitute the molecular masses and compare the options The molar masses are $M_{H_2} = 2\,\mathrm{g\,mol^{-1}}$ and $M_{O_2} = 32\,\mathrm{g\,mol^{-1}}$. The same mass unit may be used because only their ratio occurs:

$$\frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4.$$

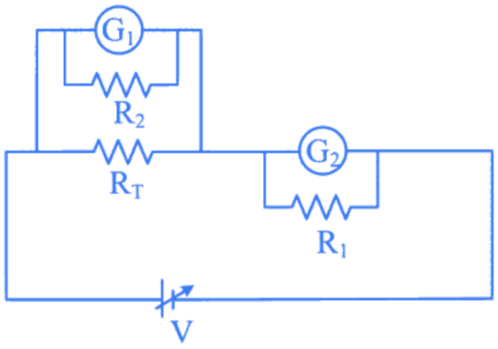
Therefore the requested order, hydrogen to oxygen, gives $4 : 1$. The alternatives $1 : 2$ and $1 : 4$ incorrectly make the lighter gas slower; $1 : 1$ incorrectly ignores the molar masses.

Step 3: Conclusion The ratio is $\boxed{v_{H_2} : v_{O_2} = 4 : 1}$. Hence, the correct option is (D).

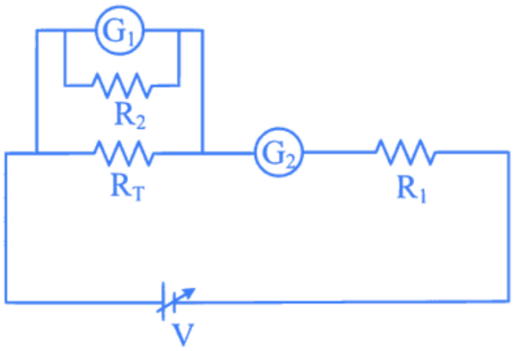
Quick Tip to Solve: For gases with the same γ at the same temperature, reverse the molar-mass ratio and then take its square root to obtain the speed ratio.

45. A student is provided with a variable voltage source V, a test resistor $R_T = 10\ \Omega$, two identical galvanometers G_1 and G_2 and two additional resistors, $R_1 = 10\ M\Omega$ and $R_2 = 0.001\ \Omega$. For conducting an experiment to verify ohm's law, the most suitable circuit is: [4 marks]

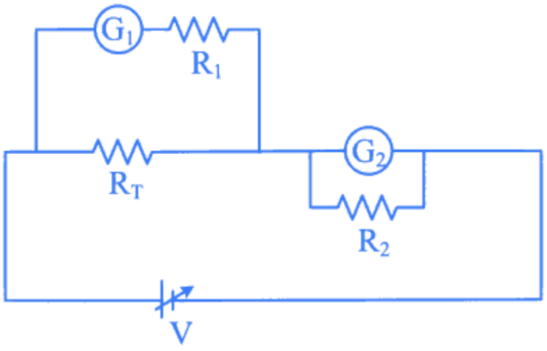
(A)



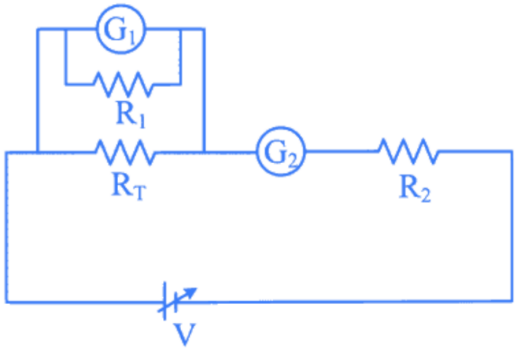
(B)



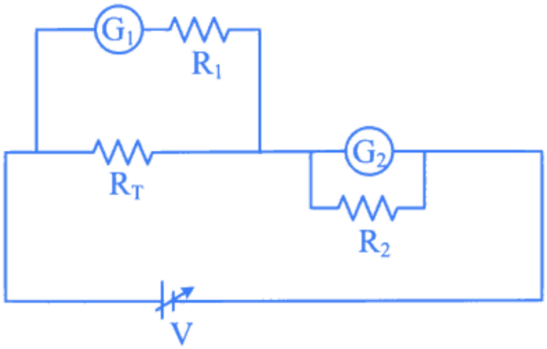
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- To verify Ohm's law for R_T, measure its current with a low-resistance ammeter in series and its potential difference with a high-resistance voltmeter in parallel.
- A galvanometer becomes a voltmeter when a large resistance is placed in series with it. A small resistance connected in parallel acts as a shunt and converts a galvanometer into an ammeter.

Step 1: Build the two measuring instruments The available resistances are widely separated: $R_1 = 10\,\text{M}\Omega = 10^7\,\Omega$ and $R_2 = 0.001\,\Omega$. Connect G_1 in series with R_1, making the branch resistance large so it draws very little current when connected across the $10\,\Omega$ test resistor. Connect G_2 in parallel with R_2. Most of the measured current then passes through the small shunt, giving a low-resistance ammeter combination.

Step 2: Check the placement in each circuit In circuit (C), the G_1-R_1 series branch is connected across R_T, while the parallel combination of G_2 and R_2 lies in the main current path. Changing the

source voltage allows corresponding readings of voltage and current, so the ratio V_T/I_T can be tested for constancy. In (A), R_2 shunts the meter across the test resistor and R_1 is placed in parallel with the series meter: the required instrument roles are reversed. Circuit (B) retains the wrong low-resistance branch across the test resistor and puts the large resistance in the main path. In (D), the voltmeter resistance is parallel rather than series with G_1, and the current meter has no shunt.

Step 3: Conclusion Circuit (C) correctly places the high-resistance voltmeter across R_T and the shunted galvanometer in series with the current path. Hence, the correct option is (C).

Quick Tip to Solve: Identify each converted meter before checking its position: large series resistance means voltmeter; tiny parallel shunt means ammeter.

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46. A small particle of mass m moves in such a way that its potential energy $U = \frac{1}{2}m\,\omega^2 r^2$ where ω is constant and r is the distance of the particle from origin. Assuming Bohr's quantization of momentum and circular orbit, the radius of n^{th} orbit will be proportional to, [4 marks]

(A) $\sqrt{n}$

(B) n^2

(C) $\frac{1}{n}$

(D) n

Correct Answer: (A) $\sqrt{n}$

Solution:

Concept:

- For a central potential $U(r)$, the radial force is $F_r = -dU/dr$. In a circular orbit its inward magnitude supplies the centripetal force mv^2/r.
- Bohr's orbital quantization condition is $mvr = n\hbar$, where n is a positive integer and $\hbar = h/(2\pi)$. It quantizes angular momentum, rather than the linear momentum alone.

Step 1: Find the speed in a circular orbit The given potential has the form of an isotropic harmonic restoring potential:

$$U = \frac{1}{2}m\omega^2 r^2, \qquad F_r = -m\omega^2 r.$$

Taking the inward magnitude for the circular force balance gives

$$\frac{mv^2}{r} = m\omega^2 r.$$

Since $r > 0$, this reduces to $v^2 = \omega^2 r^2$, or $v = \omega r$ when ω denotes the positive oscillator frequency. Thus orbital speed increases linearly with radius in this potential.

Step 2: Apply quantization and isolate the radius Substitute this speed into the angular-momentum condition:

$$mvr = m\omega r^2 = n\hbar.$$

Solving for the radius gives

$$r_n^2 = \frac{n\hbar}{m\omega}, \qquad r_n = \sqrt{\frac{\hbar}{m\omega}}\sqrt{n}.$$

The quantities m, ω and $\hbar$ are constant, so the radius varies as $\sqrt{n}$. A dependence n^2 belongs to the Coulomb potential and cannot be transferred here. The alternatives $1/n$ and n also contradict the directly obtained relation $r_n^2 \propto n$.

Step 3: Conclusion The radius is proportional to $\boxed{\sqrt{n}}$. Hence, the correct option is (A).

Quick Tip to Solve: Derive the force from the stated potential first; the familiar hydrogen relation $r_n \propto n^2$ applies only to a Coulomb force.

47. A child of mass 5 kg is going round a merry-go-round that makes 1 rotation in 3.14 s. The radius of the merry-go-round is 2 m. The centrifugal force on the child will be [4 marks]

(A) 50 N

(B) 80 N

(C) 100 N

(D) 40 N

Correct Answer: (D) 40 N

Solution:

Concept:

- For uniform circular motion of radius r and period T, the angular speed is $\Omega = 2\pi/T$ and the acceleration magnitude is $\Omega^2 r$.
- In the rotating frame, the centrifugal force is directed radially outward with magnitude $m\Omega^2 r$. In an inertial frame the corresponding net real force is centripetal and has the same magnitude.

Step 1: Convert the period into angular speed The child completes one revolution in $T = 3.14\,\text{s}$; one revolution corresponds to 2π radians. Hence

$$\Omega = \frac{2\pi}{T} \simeq \frac{2 \times 3.14}{3.14} = 2\,\text{rad}\,\text{s}^{-1}.$$

The given time is the period, not the frequency, so it must appear in the denominator. The tangential speed could equivalently be found as $v = \Omega r = 4\,\text{m}\,\text{s}^{-1}$.

Step 2: Calculate the outward inertial force Use the mass $m = 5\,\text{kg}$ and radius $r = 2\,\text{m}$:

$$F_{\text{cf}} = m\Omega^2 r = 5 \times 2^2 \times 2 = 40\,\text{N}.$$

As an independent arithmetic check,

$$\frac{mv^2}{r} = \frac{5 \times 4^2}{2} = 40\,\text{N}.$$

The alternatives 50, 80 and $100\,\text{N}$ do not follow from the stated mass, period and radius. The child's

weight is not the requested force: gravity is vertical, whereas this centrifugal force lies along the radius in the horizontal rotating frame.

Step 3: Conclusion The centrifugal force has magnitude $\boxed{40\,\text{N}}$, directed radially outward in the merry-go-round frame. Hence, the correct option is (D).

Quick Tip to Solve: When one revolution takes approximately π seconds, Ω is approximately $2\,\text{rad}\,\text{s}^{-1}$; square this before multiplying by mr.

48. The weight of a body on the surface of the earth is 100 N. The gravitational force on it when taken at a height, from the surface of earth, equal to one-fourth the radius of the earth is: [4 marks]

(A) 50 N

(B) 64 N

(C) 25 N

(D) 100 N

Correct Answer: (B) 64 N

Solution:

Concept:

- Outside a spherical Earth, the gravitational force is $F = GMm/r^2$, where r is distance from Earth's centre, M is Earth's mass and m is the body's mass.
- At the surface $r = R$, so the weight is $W = GMm/R^2$. Dividing the force at altitude by this surface weight cancels the unknown masses and gravitational constant.

Step 1: Convert altitude into centre-to-centre distance The stated height above the surface is $h = R/4$. The inverse-square law must use the distance from Earth's centre:

$$r = R + h = R + \frac{R}{4} = \frac{5R}{4}.$$

Using $R/4$ as the radius would incorrectly place the body inside the Earth and predict an increased force. Here the body is outside the surface, so the force must be smaller than the surface value $100\,\text{N}$.

Step 2: Apply the inverse-square ratio Form the ratio of the two gravitational forces:

$$\frac{F}{W} = \frac{GMm/(5R/4)^2}{GMm/R^2} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}.$$

Substituting $W = 100\,\text{N}$,

$$F = 100 \times \frac{16}{25} = 64\,\text{N}.$$

This eliminates 50 and $25\,\text{N}$, which correspond to different distances. The choice $100\,\text{N}$ would require the body to remain at the surface. A linear altitude approximation is unnecessary because the exact ratio is simple and the height is a substantial fraction of the radius.

Step 3: Conclusion The gravitational force is $\boxed{64\,\text{N}}$, directed towards Earth's centre. Hence, the correct option is (B).

Quick Tip to Solve: For a height R/n, write $r = R(n+1)/n$ first; then multiply the surface weight by $[n/(n+1)]^2$.

49. Choose the incorrect statement from the following: [4 marks]

(A) The linear speed of a planet revolving around the sun remains constant.

(B) When a body falls towards earth, the displacement of earth towards the body is negligible.

(C) The speed of satellite in a given circular orbit remains constant.

(D) For a planet revolving around the sun in an elliptical orbit, the total energy of the planet remains constant.

Correct Answer: (A) The linear speed of a planet revolving around the sun remains constant.

Solution:

Concept:

- An isolated gravitational two-body system conserves mechanical energy and angular momentum. In an elliptical orbit the distance from the attracting centre changes, so speed generally changes too.
- The vis-viva equation is $v^2 = GM(2/r - 1/a)$, where a is the semi-major axis and r is the instantaneous distance from the Sun. Circular motion is its constant-radius special case.

Step 1: Test the constant-speed claim A planet in an elliptical orbit approaches and recedes from the Sun. Its semi-major axis a remains fixed, but r varies. From

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right),$$

the planet moves faster near perihelion and slower near aphelion. Thus statement (A), which asserts a constant linear speed without restricting the orbit to a circle, is incorrect. Kepler's equal-area law expresses the same result: a smaller radius requires a larger transverse velocity.

Step 2: Verify the remaining three statements Statement (B) is correct because the falling body and Earth attract each other with equal forces, but Earth's much greater mass gives it a negligible acceleration and displacement. Their displacements relative to the centre of mass are inversely proportional to their masses. Statement (C) is correct: in a given circular orbit the radius is constant and $v = \sqrt{GM/r}$ is constant. Statement (D) is also correct because gravity is conservative. In an ellipse,

$$E = K + U = -\frac{GMm}{2a}$$

remains constant, although kinetic and potential energies change separately. Conservation of total energy therefore does not imply a constant speed.

Step 3: Conclusion The incorrect statement is the claim that a planet's linear speed around the Sun remains constant. Hence, the correct option is (A).

Quick Tip to Solve: For elliptical motion, distinguish conserved total energy and areal velocity from ordinary speed, which increases as the planet approaches the Sun.

50. For an amplitude modulated wave the minimum amplitude is 3 V, while the modulation index is 60%. The maximum amplitude of the modulated wave is: [4 marks]

(A) 5 V

(B) 12 V

(C) 10 V

(D) 15 V

Correct Answer: (B) 12 V

Solution:

Concept:

- For ordinary amplitude modulation, the envelope has extrema $A_{\max} = A_c(1+m)$ and $A_{\min} = A_c(1-m)$, where A_c is the carrier amplitude and m is the modulation index.
- The modulation index also equals $m = (A_{\max} - A_{\min})/(A_{\max} + A_{\min})$. A percentage must first be converted to a fraction.

Step 1: Find the carrier amplitude The stated modulation index is $60\% = 0.60$, and the minimum envelope amplitude is $3\,\text{V}$. The minimum is the carrier amplitude reduced by the modulation amplitude, so use the minus sign:

$$3 = A_c(1 - 0.60) = 0.40A_c.$$

Therefore

$$A_c = \frac{3}{0.40} = 7.5\,\text{V}.$$

The carrier amplitude must lie between the minimum and maximum envelope values; it is not itself the required maximum.

Step 2: Calculate and check the maximum At the positive crest of the modulating signal, the carrier envelope is increased by the factor $1 + m$:

$$A_{\max} = 7.5(1 + 0.60) = 7.5 \times 1.60 = 12\,\text{V}.$$

As a check, substituting the two extrema into the modulation-index definition gives

$$(12 - 3)/(12 + 3) = 9/15 = 0.60,$$

exactly the given value.

Step 3: Rule out the remaining options The extrema must have the ratio

$$\frac{A_{\max}}{A_{\min}} = \frac{1 + 0.60}{1 - 0.60} = 4.$$

Thus the given minimum of $3\,\text{V}$ requires a maximum of $12\,\text{V}$. The values 5, 10 and $15\,\text{V}$ in (A), (C) and (D) fail this ratio, so none produces the specified modulation depth.

Step 4: Conclusion The maximum amplitude is $\boxed{12\,\text{V}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $A_{\max}/A_{\min} = (1+m)/(1-m)$ when one envelope extreme and the modulation index are given.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Experimentally it is found that 12.8 eV energy is required to separate a hydrogen atom into a proton and an electron. So the orbital radius of the electron in a hydrogen atom is $\frac{9}{x} \times 10^{-10}$ m. The value of the x is _____.

$\left(1\text{eV} = 1.6 \times 10^{-19}\text{ J}, \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{Nm}^2/\text{C}^2 \text{ and electronic charge } = 1.6 \times 10^{-19}\text{C}\right)$ [4 marks]

Solution:

Concept:

- In a circular Coulomb orbit, $mv^2/r = ke^2/r^2$, so the kinetic energy is $K = ke^2/(2r)$, where $k = 1/(4\pi\epsilon_0)$.
- The electrostatic potential energy is $U = -ke^2/r$; hence the total orbital energy is $E = -ke^2/(2r)$. The positive separation energy is $E_b = -E$.

Step 1: Convert the measured separation energy Separating the electron and proton means taking their total energy from its negative bound value to zero at infinite separation. Consequently the stated energy is the binding energy, not the full magnitude of the potential energy. In SI units it is

$$E_b = 12.8 \times 1.6 \times 10^{-19} = 2.048 \times 10^{-18}\,\text{J}.$$

We use the measured value and constants supplied in the problem.

Step 2: Calculate the radius from the binding energy Using $E_b = ke^2/(2r)$, rearrange before substitution:

$$r = \frac{ke^2}{2E_b}.$$

The numerator is

$$ke^2 = (9 \times 10^9)(1.6 \times 10^{-19})^2 = 2.304 \times 10^{-28}\,\text{J m}.$$

Thus

$$r = \frac{2.304 \times 10^{-28}}{4.096 \times 10^{-18}} = 0.5625 \times 10^{-10}\,\text{m}.$$

The units reduce to metres, as required.

Step 3: Compare with the requested parametrisation The required radius is written as $(9/x) \times 10^{-10}$ m. Comparing coefficients of the same power of ten gives

$$\frac{9}{x} = 0.5625 = \frac{9}{16}, \qquad x = 16.$$

Retaining the given energy throughout avoids introducing the usual hydrogen ground-state energy in place of the measurement stated here.

Step 4: Conclusion The orbital radius is 5.625×10^{-11} m, so the required dimensionless value is $\boxed{x = 16}$.

Quick Tip to Solve: The separation energy is half the magnitude of the Coulomb potential energy; use $r = ke^2/(2E_b)$, not ke^2/E_b.

52. Two concentric circular coils with radii $1\ \text{cm}$ and $1000\ \text{cm}$, and number of turns 10 and 200 respectively are placed coaxially with centers coinciding. The mutual inductance of this arrangement will be _____ $\times 10^{-8}$H. (Take, $\pi^2 = 10$) [4 marks]

Solution:

Concept:

- The magnetic field at the centre of a circular coil of radius R, N_L turns and current I is $B = \mu_0 N_L I/(2R)$.
- The smaller coil lies in an almost uniform field because $r \ll R$. Its flux linkage is $N_s B\pi r^2$, and mutual inductance is this linkage divided by I.

Step 1: Convert the dimensions and select the source coil Take current I through the large coil, whose radius and turns are

$$R = 1000\,\text{cm} = 10\,\text{m}, \qquad N_L = 200.$$

The small coil has

$$r = 1\,\text{cm} = 10^{-2}\,\text{m}, \qquad N_s = 10.$$

Since $r/R = 10^{-3}$, the central-field approximation across the small coil is appropriate. Their coincident axes make the field normal to the small coil.

Step 2: Form the mutual-inductance expression The flux through each small turn is the field from the large coil multiplied by the small turn area:

$$\Phi = B\pi r^2 = \frac{\mu_0 N_L I \pi r^2}{2R}.$$

There are N_s receiving turns, so

$$M = \frac{N_s \Phi}{I} = \frac{\mu_0 N_s N_L \pi r^2}{2R}.$$

Both turn counts occur once; the area belongs to the small coil, while the field denominator contains the large radius.

Step 3: Substitute the values Using $\mu_0 = 4\pi \times 10^{-7}\,\text{H}\,\text{m}^{-1}$ gives

$$M = \frac{(4\pi \times 10^{-7})(10)(200)\pi(10^{-2})^2}{2(10)} = 4\pi^2 \times 10^{-9}\,\text{H}.$$

With the instructed value $\pi^2 = 10$,

$$M = 4 \times 10^{-8}\,\text{H}.$$

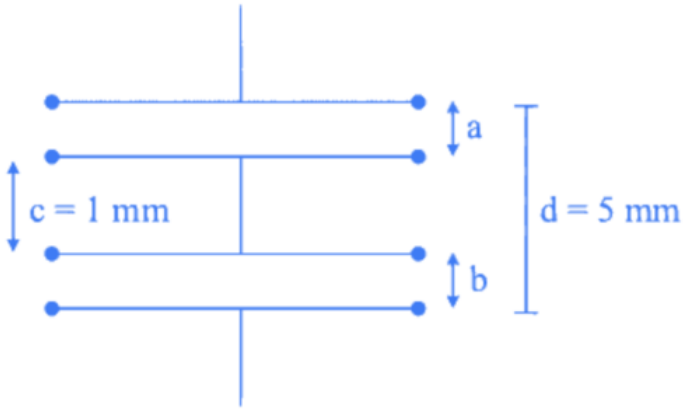
This is a small coupling, consistent with a tiny coil centred inside a much larger one.

Step 4: Conclusion The mutual inductance is $\boxed{4 \times 10^{-8}\,\text{H}}$; the requested coefficient is $\boxed{4}$.

Quick Tip to Solve: For very unequal concentric coils, calculate the large coil field and multiply by the small coil area and its turns.

53. As shown in the figure, two parallel plate capacitors having equal plate area of $200\ \mathrm{cm}^2$ are joined in such a way that $a \neq b$. The equivalent capacitance of the combination is $x\epsilon_0\ \mathrm{F}$. The value of x is _____.

[4 marks]



Solution:

Concept:

- An air-filled parallel-plate capacitor has capacitance $C = \epsilon_0 A/s$, where A is the facing plate area and s is the separation.
- Capacitors in series have equal charge magnitude, and their potential differences add: $1/C_{\mathrm{eq}} = 1/C_1 + 1/C_2$. A wire-connected pair of plates is an equipotential conductor.

Step 1: Identify the two active plate gaps In the figure, the two middle plates are joined by a conducting wire. The upper terminal plate is separated from the upper middle plate by a, and the lower terminal plate is separated from the lower middle plate by b. These two gaps form capacitors in series through the common middle conductor. The middle gap c has zero potential difference because its bounding plates are directly connected; it contributes no additional capacitor between the terminals.

Step 2: Find the sum of the active separations The total span between the outer plates is $d = a + c + b$. The figure gives $d = 5\,\mathrm{mm}$ and $c = 1\,\mathrm{mm}$. Therefore

$$a + b = d - c = 4\,\mathrm{mm} = 4 \times 10^{-3}\,\mathrm{m}.$$

The separate values of a and b are unnecessary, even though they are unequal. Convert the area as

$$A = 200\,\mathrm{cm}^2 = 200 \times 10^{-4} = 0.020\,\mathrm{m}^2.$$

Step 3: Combine the two capacitances Write $C_1 = \epsilon_0 A/a$ and $C_2 = \epsilon_0 A/b$. Their series combination obeys

$$\frac{1}{C_{\mathrm{eq}}} = \frac{a}{\epsilon_0 A} + \frac{b}{\epsilon_0 A} = \frac{a+b}{\epsilon_0 A}.$$

Hence

$$C_{\mathrm{eq}} = \frac{\epsilon_0 (0.020)}{0.004} = 5\epsilon_0\,\mathrm{F},$$

in the numerical notation of the question.

Step 4: Conclusion The equivalent capacitance is $\boxed{5\epsilon_0\,\mathrm{F}}$, so the requested value is $\boxed{x = 5}$.

Quick Tip to Solve: First merge plates joined by a wire into one equipotential node; then add only the separations of the two active series gaps.

54. A body is dropped on ground from a height 'h_1' and after hitting the ground, it rebounds to a height 'h_2'. If the ratio of velocities of the body just before and after hitting ground is 4 , then percentage loss in kinetic energy of the body is $\frac{x}{4}$. The value of x is _____. [4 marks]

Solution:

Concept:

- For an unchanged mass, kinetic energy is $K = \frac{1}{2}mv^2$; therefore the ratio of kinetic energies equals the square of the ratio of speeds.
- The percentage kinetic energy loss is $100(K_{\text{before}} - K_{\text{after}})/K_{\text{before}}$. Speed magnitudes are used because kinetic energy is a scalar.

Step 1: Express the final speed using the given ratio Let u denote the speed immediately before the body hits the ground, and v the speed immediately after rebound. The given before-to-after ratio is

$$\frac{u}{v} = 4, \qquad v = \frac{u}{4}.$$

Although the velocity direction reverses in the collision, this does not introduce a negative kinetic energy. The ratio must be squared when comparing energies rather than being used directly as an energy ratio.

Step 2: Calculate the fraction of energy retained The mass of the body is unchanged. Thus

$$K_{\text{before}} = \frac{1}{2}mu^2, \qquad K_{\text{after}} = \frac{1}{2}m\left(\frac{u}{4}\right)^2 = \frac{1}{16}K_{\text{before}}.$$

Only one-sixteenth of the incident kinetic energy remains as kinetic energy just after the impact. Consequently the fraction lost in the collision is

$$1 - \frac{1}{16} = \frac{15}{16}.$$

Step 3: Convert to a percentage and determine x Multiplying the fractional loss by 100 gives

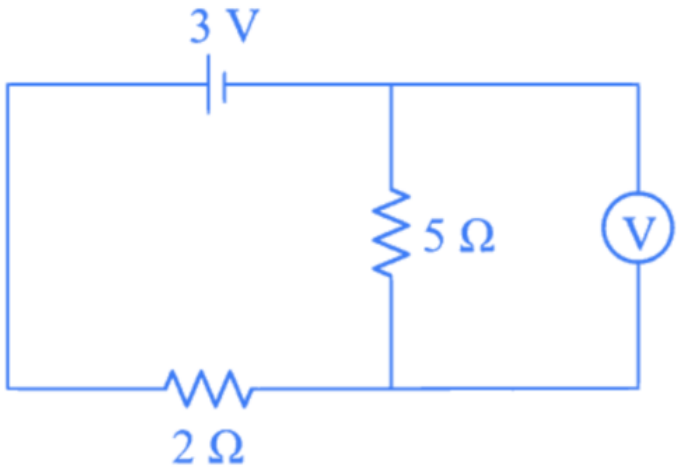
$$\text{percentage loss} = \frac{15}{16} \times 100 = 93.75 = \frac{375}{4}.$$

The question writes this percentage as $x/4$, so comparison yields $x = 375$. The heights h_1 and h_2 are consistent with this result: neglecting air resistance, their ratio is $h_2/h_1 = v^2/u^2 = 1/16$, but their individual values are not needed.

Step 4: Conclusion The kinetic energy loss is 93.75%, and hence $\boxed{x = 375}$.

Quick Tip to Solve: Square the rebound-to-incident speed ratio first; subtract that retained energy fraction from one before multiplying by 100.

55. As shown in the figure, the voltmeter reads $2\ \text{V}$ across $5\ \Omega$ resistor. The resistance of the voltmeter is _____ Ω. [4 marks]



Solution:

Concept:

- A real voltmeter of resistance R_V draws current $I_V = V/R_V$. When connected across a resistor, both branches have the same potential difference.
- Kirchhoff's current law gives the total current in the series resistor as the sum of the currents through the parallel resistor and voltmeter. Kirchhoff's voltage law gives the remaining series voltage.

Step 1: Find the total current supplied by the battery The diagram has a $3\,\text{V}$ battery and a $2\,\Omega$ resistor in series with the parallel combination of the $5\,\Omega$ resistor and the voltmeter. Since the voltmeter reads $2\,\text{V}$, this entire parallel combination has a $2\,\text{V}$ potential difference. Thus the voltage across the series resistor is

$$V_2 = 3 - 2 = 1\,\text{V}.$$

The current through it, which is the total circuit current, is

$$I = \frac{1}{2} = 0.50\,\text{A}.$$

Step 2: Find the current through the voltmeter The current in the $5\,\Omega$ branch is fixed by its own voltage:

$$I_5 = \frac{2}{5} = 0.40\,\text{A}.$$

The total current splits between this resistor and the voltmeter, so

$$I_V = I - I_5 = 0.50 - 0.40 = 0.10\,\text{A}.$$

This nonzero meter current is why the voltmeter cannot be treated as ideal or assigned infinite resistance.

Step 3: Apply Ohm's law to the meter branch The voltmeter itself has the same $2\,\text{V}$ across it as the

resistor to which it is connected. Consequently

$$R_V = \frac{2}{0.10} = 20\,\Omega.$$

As a check, the parallel equivalent is

$$(5 \times 20)/(5 + 20) = 4\,\Omega,$$

which, in series with $2\,\Omega$, draws $3/6 = 0.50\,\text{A}$ and drops $0.50 \times 4 = 2\,\text{V}$ across the meter.

Step 4: Conclusion The voltmeter resistance is $\boxed{20\,\Omega}$, so the requested numerical value is $\boxed{20}$.

Quick Tip to Solve: Find the total current from the series voltage drop, then subtract the current in the measured resistor to obtain the meter current.

56. A proton with a kinetic energy of 2.0 eV moves into a region of uniform magnetic field of magnitude $\frac{\pi}{2} \times 10^{-3}$ T. The angle between the direction of magnetic field and velocity of proton is $60°$. The pitch of the helical path taken by the proton is _____ cm. (Take, mass of proton $= 1.6 \times 10^{-27}$ kg and Charge on proton $= 1.6 \times 10^{-19}$C). [4 marks]

Solution:

Concept:

- The kinetic energy determines the speed through $K = \frac{1}{2}mv^2$. Resolve this speed into $v_{\parallel} = v\cos\theta$ and $v_{\perp} = v\sin\theta$ relative to the magnetic field.
- A uniform magnetic field gives circular motion of the transverse component with period $T = 2\pi m/(qB)$. The parallel component stays constant, so the helical pitch is $p = v_{\parallel}T$.

Step 1: Calculate the proton speed Convert the given kinetic energy into joules using the proton charge magnitude:

$$K = 2.0 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}\,\text{J}.$$

Therefore

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{6.4 \times 10^{-19}}{1.6 \times 10^{-27}}} = \sqrt{4.0 \times 10^{8}} = 2.0 \times 10^{4}\,\text{m s}^{-1}.$$

This speed is much smaller than the speed of light, so the nonrelativistic kinetic energy formula is appropriate.

Step 2: Find the parallel component and the period The angle with the field is $60°$, so the component producing advance along the field is

$$v_{\parallel} = v\cos 60° = 1.0 \times 10^{4}\,\text{m s}^{-1}.$$

Using the given mass, charge and field gives

$$T = \frac{2\pi(1.6 \times 10^{-27})}{(1.6 \times 10^{-19})(\pi/2)10^{-3}} = 4.0 \times 10^{-5}\,\text{s}.$$

The perpendicular component controls the circular radius; it must not be used for the axial advance.

Step 3: Compute the advance in one revolution One pitch is the distance travelled parallel to the field in one complete transverse revolution. Thus

$$p = v_{\parallel} T = (1.0 \times 10^{4})(4.0 \times 10^{-5}) = 0.40\,\mathrm{m} = 40\,\mathrm{cm}.$$

The field changes the direction of the perpendicular velocity but does no work, which is consistent with using the same speed throughout the motion.

Step 4: Conclusion The pitch of the helical path is $\boxed{40\,\mathrm{cm}}$; enter $\boxed{40}$.

Quick Tip to Solve: Pitch uses $v\cos\theta$, while the helical radius uses $v\sin\theta$; both share the cyclotron period $2\pi m/(qB)$.

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57. A simple pendulum with length 100 cm and bob of mass 250 g is executing S.H.M. of amplitude 10 cm. The maximum tension in the string is found to be $\frac{x}{40}$ N. The value of x is _____. [4 marks]

Solution:

Concept:

- In small-amplitude simple harmonic motion, the angular frequency of a pendulum is $\omega = \sqrt{g/L}$. Its maximum speed is $v_{\max} = \omega A$, where A is the small arc displacement amplitude.
- At the lowest point the radial force balance is $T - mg = mv^2/L$. Both the speed and the tension are maximum there; the centripetal term must be added to the weight.

Step 1: Convert the data and find the speed Use $L = 1.00\,\mathrm{m}$, $m = 0.250\,\mathrm{kg}$, and $A = 0.100\,\mathrm{m}$. Taking $g = 10\,\mathrm{m\,s^{-2}}$, the usual value for this calculation, gives

$$v_{\max}^2 = \frac{g}{L}A^2 = \frac{10}{1}(0.100)^2 = 0.100\,\mathrm{m^2\,s^{-2}}.$$

The amplitude is a displacement, so it must be squared in this expression.

Step 2: Add the radial force to the weight At the bottom the acceleration points upward towards the pivot. Therefore

$$T_{\max} = mg + \frac{mv_{\max}^2}{L} = 0.250(10) + \frac{0.250(0.100)}{1} = 2.525\,\mathrm{N}.$$

Equating this with the given form yields $x/40 = 2.525$, hence $x = 40(2.525) = 101$.

Step 3: Conclusion The maximum tension is $2.525\,\mathrm{N} = 101/40\,\mathrm{N}$; hence the required dimensionless coefficient is $\boxed{x = 101}$.

Quick Tip to Solve: For a pendulum in small oscillations, write $T_{\max} = mg[1 + (A/L)^2]$ at the bottom; do not set the maximum tension equal to the weight.

---

58. A metal block of mass m is suspended from a rigid support through a metal wire of diameter $14\ \mathrm{mm}$. The tensile stress developed in the wire under equilibrium state is $7 \times 10^5 \mathrm{Nm}^{-2}$. The value of mass m is _____ kg. (Take, $\mathrm{g} = 9.8\ \mathrm{ms}^{-2}$ and $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- Tensile stress is force divided by cross-sectional area: $\sigma = F/S$. For a circular wire of diameter d, $S = \pi d^2/4$.
- In static equilibrium the upward tensile force equals the weight of the suspended block, $F = mg$. Consequently $m = \sigma\pi d^2/(4g)$.

Step 1: Convert the diameter and calculate the area The diameter is $14\,\mathrm{mm} = 14 \times 10^{-3}\,\mathrm{m}$, so the radius is $7 \times 10^{-3}\,\mathrm{m}$. Using the supplied value of π,

$$S = \frac{22}{7}(7 \times 10^{-3})^2 = 154 \times 10^{-6}\,\mathrm{m}^2.$$

The stress is given in SI units, so using millimetres directly would give an incorrect force.

Step 2: Use equilibrium to recover the mass The force supported by the wire is

$$F = \sigma S = (7 \times 10^5)(154 \times 10^{-6}) = 107.8\,\mathrm{N}.$$

Since the block is at rest, $mg = 107.8\,\mathrm{N}$. Hence

$$m = \frac{107.8}{9.8} = 11\,\mathrm{kg}.$$

No extension or Young modulus is needed: the specified stress already gives the force per unit area.

Step 3: Conclusion The required mass of the metal block is $\boxed{m = 11\,\mathrm{kg}}$.

Quick Tip to Solve: Use the radius, not the diameter, in πr^2, and convert the diameter to metres before squaring.

59. A ring and a solid sphere rotating about an axis passing through their centers have same radii of gyration. The axis of rotation is perpendicular to plane of ring. The ratio of radius of ring to that of sphere is $\sqrt{\frac{2}{x}}$. The value of x is _____. [4 marks]

Solution:

Concept:

- The radius of gyration k about a chosen axis is defined by $I = Mk^2$, where I is the moment of inertia and M is the mass.
- A thin ring about its central perpendicular axis has $I_r = M_r R_r^2$. A uniform solid sphere about any diameter has $I_s = \frac{2}{5} M_s R_s^2$.

Step 1: Find each radius of gyration For the ring, divide its moment of inertia by its own mass:

$$k_r^2 = \frac{I_r}{M_r} = R_r^2.$$

For the sphere, similarly,

$$k_s^2 = \frac{I_s}{M_s} = \frac{2}{5}R_s^2.$$

The two bodies need not have the same mass: the mass cancels separately in each definition.

Step 2: Equate the radii of gyration and compare The given equality is $k_r = k_s$, so

$$R_r^2 = \frac{2}{5}R_s^2, \qquad \frac{R_r}{R_s} = \sqrt{\frac{2}{5}}.$$

Radii are positive, so the positive square root is appropriate. Comparing this with the printed ratio $\sqrt{2/x}$ gives $2/x = 2/5$, hence $x = 5$. Using the hollow-sphere formula instead would describe a different body.

Step 3: Conclusion The radius ratio is $R_r/R_s = \sqrt{2/5}$, and the required dimensionless value is $\boxed{x = 5}$.

Quick Tip to Solve: Equal radii of gyration mean equal I/M, not necessarily equal moments of inertia; cancel each body's own mass first.

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60. A beam of light consisting of two wavelengths $7000\ \overset{o}{A}$ and $5500\ \overset{o}{A}$ is used to obtain interference pattern in Young's double slit experiment. The distance between the slits is 2.5 mm and the distance between the plane of slits and the screen is 150 cm. The least distance from the central fringe, where the bright fringes due to both the wavelengths coincide, is $n \times 10^{-5}$ m. The value of n is _____.

[4 marks]

Solution:

Concept:

- A bright fringe of order m occurs at $y = m\lambda D/d$, where D is the screen distance and d the slit separation.
- For two bright fringes to coincide away from the centre, their positive integer orders must satisfy $m_1\lambda_1 = m_2\lambda_2$. The smallest such orders locate the first coincidence.

Step 1: Find the least pair of coincident orders The wavelengths are $\lambda_1 = 7000 \times 10^{-10}$ m and $\lambda_2 = 5500 \times 10^{-10}$ m. The coincidence equation becomes

$$7000m_1 = 5500m_2, \qquad 14m_1 = 11m_2.$$

Since 14 and 11 are coprime, the least positive solution is $m_1 = 11$, $m_2 = 14$. The zero-order coincidence is the central fringe and is excluded.

Step 2: Calculate the common fringe position Convert $D = 150\,\text{cm} = 1.50\,\text{m}$ and $d = 2.5\,\text{mm} =$

$2.5 \times 10^{-3}\,\mathrm{m}$. Then

$$y = \frac{11(7000 \times 10^{-10})(1.50)}{2.5 \times 10^{-3}} = 4.62 \times 10^{-3}\,\mathrm{m}.$$

In the required form this is $462 \times 10^{-5}\,\mathrm{m}$. The shorter wavelength gives the same result using order 14.

Step 3: Conclusion The least noncentral coincidence is at $4.62\,\mathrm{mm} = 462 \times 10^{-5}\,\mathrm{m}$, so $\boxed{n = 462}$.

Quick Tip to Solve: Reduce the wavelength ratio to coprime integers; the fringe orders are the integers in the reverse ratio.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Formation of which complex, among the following, is not a confirmatory test of Pb^{2+} ions :

[4 marks]

(A) lead chromate

(B) lead iodide

(C) lead sulphate

(D) lead nitrate

Correct Answer: (D) lead nitrate

Solution:

Concept:

- Confirmatory precipitation tests identify Pb^{2+} through characteristic poorly soluble salts. A soluble nitrate does not provide such a diagnostic precipitate.
- Lead chromate is yellow, lead iodide is yellow, and lead sulphate is a white sparingly soluble salt. Lead nitrate is soluble in water.

Step 1: Check the three precipitation tests Chromate ions give yellow lead chromate:

$$Pb^{2+} + {CrO_4}^{2-} \longrightarrow PbCrO_4(s).$$

Iodide ions give yellow lead iodide, PbI_2, which can yield golden-yellow crystals on cooling a hot solution. Sulphate ions give white $PbSO_4$. Thus the products in choices A, B and C can serve as precipitation evidence for lead ions.

Step 2: Identify the product that stays dissolved All nitrates are soluble in water under ordinary qualitative-analysis conditions, including $Pb(NO_3)_2$. Adding nitrate ions to an aqueous solution of lead ions therefore produces no characteristic insoluble product. The question uses the term complex broadly; the deciding distinction here is the characteristic precipitate, rather than coordination-complex formation.

Step 3: Conclusion Formation of soluble lead nitrate is not a confirmatory test for Pb^{2+}. Hence, the correct option is (D).

Quick Tip to Solve: For lead-salt tests, remember the yellow chromate and iodide precipitates; a nitrate normally remains soluble.

62. Element not present in Nessler's reagent is : [4 marks]

(A) I

(B) N

(C) K

(D) Hg

Correct Answer: (B) N

Solution:

Concept:

- Nessler's reagent is an alkaline solution of potassium tetraiodomercurate(II), $K_2[HgI_4]$, usually in potassium hydroxide.
- The complex contains potassium, mercury and iodine; the alkaline medium additionally contains oxygen and hydrogen. Nitrogen belongs to the ammonia being detected, not to the original reagent.

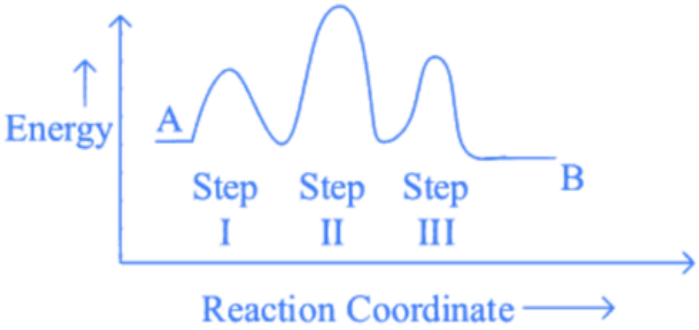
Step 1: Read the elements in the reagent formula In $K_2[HgI_4]$, the two potassium ions balance the charge on $[HgI_4]^{2-}$. The central metal is mercury and the four ligands are iodide ions. Thus iodine (choice A), potassium (choice C), and mercury (choice D) are all explicitly present. The KOH medium introduces no nitrogen either.

Step 2: Distinguish the reagent from the analyte Nessler's reagent is used to test for ammonia or ammonium ions in an alkaline medium, giving a yellow-brown coloration or precipitate. Although ammonia contains nitrogen, that nitrogen is supplied by the sample being tested. It is not part of the reagent's initial chemical composition. Therefore the presence of nitrogen in the test product must not be confused with its presence in the reagent bottle.

Step 3: Conclusion Nitrogen, N, is absent from Nessler's reagent. Hence, the correct option is (B).

Quick Tip to Solve: Write $K_2[HgI_4]$ first; distinguish the nitrogen-containing analyte from the reagent used to detect it.

63. Consider the following reaction that goes from A to B in three steps as shown below:
Choose the correct option [4 marks]



(A) 2 intermediates; 3 activated complexes; rate-determining step III.
(B) 2 intermediates; 3 activated complexes; rate-determining step II.
(C) 2 intermediates; 3 activated complexes; rate-determining step I.
(D) 3 intermediates; 2 activated complexes; rate-determining step II.

Correct Answer: (B) 2 intermediates; 3 activated complexes; rate-determining step II.

Solution:

Concept:

- A local minimum between adjacent energy maxima represents an intermediate, while each maximum represents an activated complex or transition state.
- The rate-determining step has the largest forward activation-energy barrier, measured from the preceding minimum to that step's maximum, rather than merely from the baseline.

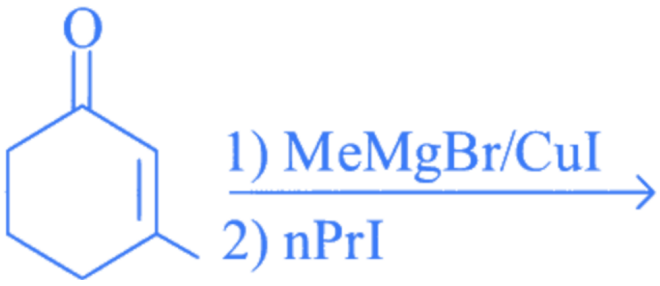
Step 1: Count the stationary features on the curve The energy profile has three peaks, one for each of steps I, II and III. These are three activated complexes. Between these peaks lie two valleys, corresponding to two intermediates. The initial species A and the final species B are reactant and product, so neither is counted as an intermediate.

Step 2: Compare the forward activation barriers The rise from the first valley to the peak of step II is greater than either the rise from A to peak I or the rise from the second valley to peak III. Step II therefore has the largest activation barrier and is the slowest, rate-determining step. Choices A and C name the wrong rate-determining step. Choice D exchanges the numbers of intermediates and activated complexes.

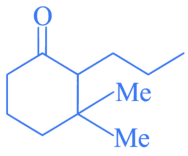
Step 3: Conclusion There are 2 intermediates and 3 activated complexes, and step II is rate determining. Hence, the correct option is (B).

Quick Tip to Solve: Count valleys for intermediates and peaks for transition states; measure each barrier upward from its own preceding minimum.

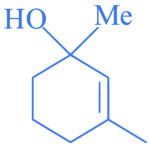
64. Find out the major product from the following reaction. [4 marks]



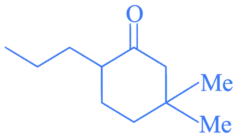
(A) 3,3-Dimethyl-2-propylcyclohexan-1-one.



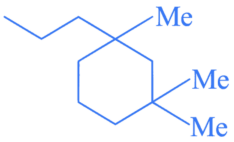
(B) 1,3-Dimethylcyclohex-2-en-1-ol.



(C) 5,5-Dimethyl-2-propylcyclohexan-1-one.

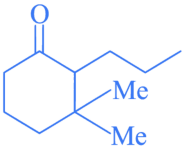


(D) 1,3,3-Trimethyl-1-propylcyclohexane.



Correct Answer: (A)

3,3-Dimethyl-2-propylcyclohexan-1-one.



Solution:

Concept:

- An α,β-unsaturated ketone can undergo either carbonyl addition or conjugate addition. Copper(I) converts the organomagnesium reagent into a softer methyl-transfer reagent that favours conjugate addition.
- Conjugate addition puts the incoming methyl group at the β-carbon and leaves an enolate. Trapping that enolate with a primary alkyl iodide forms a carbon–carbon bond at its α-carbon.

Step 1: Locate the two reactive carbon atoms Number the carbonyl carbon as C1. The original double bond is C2=C3, and C3 already carries a methyl group. Therefore C2 is the alpha carbon and C3 is the beta carbon. The carbonyl oxygen must be retained in the final ketone; the double bond is consumed during conjugate addition.

Step 2: Add methyl and trap the resulting enolate With $\mathrm{MeMgBr/CuI}$, methyl adds to C3 rather than

C1. C3 consequently has two methyl groups. The resulting enolate has its nucleophilic carbon at C2. The second reagent, $CH_3CH_2CH_2I$, is a primary iodide, so the enolate displaces iodide by an S_N2 reaction. A normal propyl group is introduced at C2 and the carbonyl is regenerated.

Step 3: Compare the product connectivities The product has a ketone at C1, propyl at C2 and geminal methyl groups at C3. Option (A) has precisely this arrangement. Option (B) represents carbonyl addition and leaves the original alkene. Option (C) puts propyl at the other alpha position, rather than the enolate carbon produced by conjugate addition. Option (D) removes the carbonyl altogether, although neither reagent is a deoxygenating reagent.

Step 4: Conclusion The major product is 3,3-dimethyl-2-propylcyclohexan-1-one. Hence, the correct option is (A).

Quick Tip to Solve: For an enone followed by a copper-assisted organometallic reagent and an alkyl halide, mark beta addition first and alpha alkylation second.

65. If the radius of the first orbit of hydrogen atom is α_0, then de Broglie's wavelength of electron in 3^{rd} orbit is : [4 marks]

(A) $\frac{\pi\alpha^3}{6}$

(B) $3\pi\alpha_0$

(C) $6\pi\alpha_0$

(D) $\frac{\pi\alpha^3}{3}$

Correct Answer: (C) $6\pi\alpha_0$

Solution:

Concept:

- For a hydrogen atom in the Bohr model, the orbit radius is $r_n = n^2\alpha_0$, where α_0 is the radius of the first orbit.
- The standing-wave condition is $2\pi r_n = n\lambda_n$, where n is the principal quantum number and $\lambda_n = h/(mv_n)$ is the electron de Broglie wavelength.

Step 1: Find the radius of the third orbit The orbit requested is the third orbit, so $n = 3$. The radius increases as the square of the orbit number, not linearly. Hence

$$r_3 = 3^2\alpha_0 = 9\alpha_0.$$

This is an orbital radius, whereas the circumference is 2π times this value. Keeping those two quantities separate avoids a missing factor of 2π.

Step 2: Apply the standing-wave condition Three complete electron wavelengths fit around the third Bohr orbit. Therefore

$$3\lambda_3 = 2\pi r_3 = 18\pi\alpha_0, \qquad \lambda_3 = 6\pi\alpha_0.$$

Equivalently, combining the two general relations gives $\lambda_n = 2\pi n\alpha_0$; substituting $n = 3$ gives the same result. The wavelength grows linearly with n, even though the radius grows quadratically.

Step 3: Reject the other expressions Option (B), $3\pi\alpha_0$, misses a factor of two in the orbit circumference.

The expressions in options (A) and (D) contain a cubic power of the radius symbol and do not follow the standing-wave condition; a wavelength must have the dimensions of length. Only option (C) has both the correct dependence and the correct numerical coefficient.

Step 4: Conclusion The electron wavelength in the third orbit is $\boxed{6\pi\alpha_0}$. Hence, the correct option is (C).

Quick Tip to Solve: Combine $r_n = n^2\alpha_0$ with $n\lambda_n = 2\pi r_n$ before substituting the orbit number.

66. Which one of the following elements will remain as liquid inside pure boiling water? [4 marks]

(A) Li

(B) Br

(C) Cs

(D) Ga

Correct Answer: (D) Ga

Solution:

Concept:

- An element can remain liquid in boiling water only when its melting point is below the water temperature and its boiling point is above that temperature.
- Physical melting is not enough: the element must also persist without being rapidly consumed by reaction with water. At ordinary pressure boiling water is approximately $100\,^\circ\mathrm{C}$.

Step 1: Apply the melting and boiling point tests Gallium melts near $30\,^\circ\mathrm{C}$ and has a very high boiling point, far above $100\,^\circ\mathrm{C}$. Thus gallium is liquid at the temperature of boiling water and does not boil away. Lithium melts near $181\,^\circ\mathrm{C}$, so boiling water is below its melting point; in addition lithium reacts with water. Lithium cannot be the required persistent liquid.

Step 2: Check bromine and caesium Bromine boils near $59\,^\circ\mathrm{C}$, below the temperature of boiling water. It will not remain as a bulk liquid at $100\,^\circ\mathrm{C}$ under ordinary pressure. Caesium melts near room temperature, so its melting point alone might make it seem suitable. However, caesium reacts extremely vigorously with water to form caesium hydroxide and hydrogen; it does not remain as elemental liquid caesium inside the water.

Step 3: Select the element satisfying both conditions Gallium meets the phase-temperature condition and does not undergo the violent water reaction characteristic of the alkali metals in the list. Consequently it can remain as liquid droplets in boiling water. Option (A) fails the melting and reaction tests, option (B) fails the boiling point test, and option (C) fails the chemical stability test. This leaves gallium as the unique appropriate choice.

Step 4: Conclusion Gallium, Ga, remains liquid inside boiling water. Hence, the correct option is (D).

Quick Tip to Solve: When a question places an element in water, check reaction with water as well as melting and boiling points.

67. Given below are two statements: one is labelled as "Assertion A" and the other is labelled as "Reason R"

Assertion A : In the complex $Ni(CO)_4$ and $Fe(CO)_5$, the metals have zero oxidation state.

Reason R : Low oxidation states are found when a complex has ligands capable of π-donor character in addition to the σ-bonding.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) A is not correct but R is correct

(B) A is correct but R is not correct

(C) Both A and R are correct but R is NOT the correct explanation of A

(D) Both A and R are correct and R is the correct explanation of A

Correct Answer: (B) A is correct but R is not correct

Solution:

Concept:

- The oxidation state of a metal is obtained by assigning the usual formal charge to each ligand and matching the total complex charge. Carbon monoxide is a neutral ligand.
- Carbon monoxide donates a lone pair through a sigma bond and accepts electron density from filled metal orbitals into its vacant antibonding pi orbitals. This pi acceptance stabilises low metal oxidation states.

Step 1: Evaluate the assertion Both $Ni(CO)_4$ and $Fe(CO)_5$ are neutral molecules. Each CO ligand is neutral. If the metal oxidation state is x, charge balance gives

$$x + 4(0) = 0 \quad \text{and} \quad x + 5(0) = 0,$$

respectively. Thus nickel and iron each have oxidation state zero. Assertion A is correct; coordination number and oxidation state are different quantities.

Step 2: Evaluate the reason The reason attributes low oxidation states to pi-donor ligands. The relevant property of carbon monoxide is instead pi-acceptor character. The carbon lone pair forms the ligand-to-metal sigma donation, while filled metal d orbitals donate back into the ligand π^* orbitals. This back-donation removes some electron density from an electron-rich metal centre, helping stabilise its low oxidation state. A pi-donor ligand has the opposite direction of pi electron donation.

Step 3: Match the truth values to the options The assertion is true but the reason, as worded with pi-donor character, is false. Option (A) reverses these truth values. Options (C) and (D) both call the reason true, so neither can apply. Only option (B) correctly states that A is correct and R is not correct. Replacing pi-donor by pi-acceptor would repair the reason, but the printed statement must be judged as given.

Step 4: Conclusion Assertion A is correct, whereas Reason R is not correct. Hence, the correct option is (B).

Quick Tip to Solve: For metal carbonyls remember the two directions: CO-to-metal sigma donation and metal-to-CO pi back-donation.

68. Group-13 elements react with O_2 in amorphous form to form oxides of type M_2O_3 (M = element). Which among the following is the most basic oxide? [4 marks]

(A) Al_2O_3

(B) Tl_2O_3

(C) B_2O_3

(D) Ga_2O_3

Correct Answer: (B) Tl_2O_3

Solution:

Concept:

- The metallic character of Group 13 elements generally increases down the group. Greater metallic character tends to make the corresponding oxide more ionic and more basic.
- Boron oxide is acidic, aluminium and gallium oxides are amphoteric, and the heavier Group 13 oxides show increasing basic character. Compare the oxides at the same stated oxidation state.

Step 1: Identify the common oxidation state Every candidate is of the type M_2O_3. Taking oxygen as -2, charge balance gives $2x + 3(-2) = 0$, hence $x = +3$ for the Group 13 element. The comparison is therefore among trivalent oxides. It is not a comparison of Tl_2O with Tl_2O_3, and the inert-pair effect does not change which formula has been asked about.

Step 2: Compare the elements down the group The relevant order of elements is B, Al, Ga, then Tl farther down Group 13. Boron is a non-metal and B_2O_3 is an acidic oxide. Al_2O_3 is amphoteric, reacting with both acids and strong bases. Ga_2O_3 is also amphoteric and more metallic in character than the boron oxide. Thallium is the most metallic element among those listed, so its stated trivalent oxide has the greatest basic character.

Step 3: Eliminate the less basic candidates Option (C) is ruled out because an acidic oxide cannot be the most basic member of this list. Options (A) and (D) are amphoteric oxides of elements higher in the group. The heavier thallium oxide is therefore the most basic of the four. Read the element symbol in option (B) as Tl, with a lower-case letter l, rather than titanium, whose symbol is Ti.

Step 4: Conclusion The most basic oxide is Tl_2O_3. Hence, the correct option is (B).

Quick Tip to Solve: For a fixed oxide formula within a main-group family, use increasing metallic character down the group to compare basicity.

69. During the reaction of permanganate with thiosulphate, the change in oxidation of manganese occurs by value of 3. Identify which of the below medium will favour the reaction. [4 marks]

(A) aqueous neutral

(B) both aqueous acidic and faintly alkaline

(C) both aqueous acidic and neutral

(D) aqueous acidic

Correct Answer: (A) aqueous neutral

Solution:

Concept:

- In permanganate, MnO_4^-, manganese has oxidation state $+7$. The reduction product depends strongly on whether the solution is acidic, neutral or strongly alkaline.
- Acidic permanganate usually gives Mn^{2+}; neutral or weakly alkaline permanganate gives MnO_2. The oxidation-state decrease is obtained by subtracting the product value from $+7$.

Step 1: Determine the required manganese product The stated change in manganese oxidation state is three units. Since permanganate is being reduced while thiosulphate is oxidised, manganese must move from $+7$ to $+4$:

$$+7 - 3 = +4.$$

In MnO_2, the two oxygen atoms contribute -4, so manganese is indeed $+4$. Thus the required product is manganese dioxide rather than a manganese(II) salt.

Step 2: Connect that product with the reaction medium The standard neutral-medium reduction half-equation is

$$MnO_4^- + 2\,H_2O + 3\,e^- \longrightarrow MnO_2 + 4\,OH^-.$$

It explicitly requires three electrons per manganese atom. Thiosulphate supplies the reducing equivalents, while the medium determines which manganese reduction product is favoured. Aqueous neutral medium therefore gives the oxidation-state change specified in the question.

Step 3: Check all the alternatives In acidic solution the familiar half-equation is

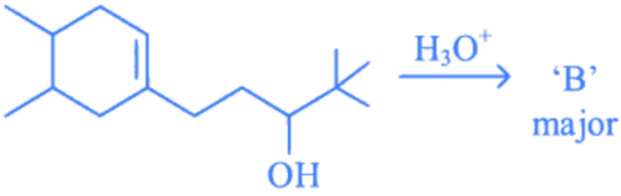
$$MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O.$$

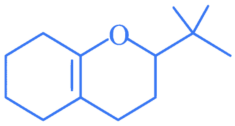
That changes manganese by five units, not three. Consequently every option that includes acidic medium as a valid case is incorrect: this eliminates (B), (C) and (D). Weakly alkaline conditions can also produce manganese dioxide, but option (B) joins that with acidic conditions, so the whole statement in that option is false. Option (A) is the valid listed choice.

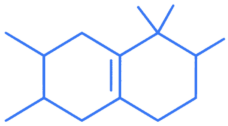
Step 4: Conclusion Aqueous neutral medium favours the stated three-unit change, $Mn(VII) \longrightarrow Mn(IV)$. Hence, the correct option is (A).

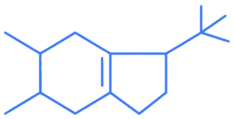
Quick Tip to Solve: First infer the final oxidation state from the stated change; then match the manganese product to the medium.

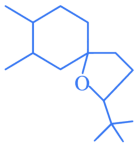
70. In the following reaction, 'B' is [4 marks]



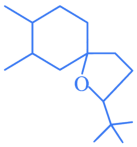
(A) The fused six-membered cyclic ether shown, with an oxygen atom in the second ring. 

(B) The fused six-membered carbocycle shown, with a gem-dimethyl carbon and an adjacent methyl-substituted carbon. 

(C) The fused five-membered carbocycle shown, bearing a tert-butyl substituent. 

(D) The spiro five-membered cyclic ether shown, bearing a tert-butyl substituent. 

Correct Answer: (D)

The spiro five-membered cyclic ether shown, bearing a tert-butyl substituent. 

Solution:

Concept:

- In aqueous acid an alkene can be protonated to form the more stable carbocation. The adjacent alcohol oxygen can then act as an intramolecular nucleophile.
- Intramolecular alcohol capture of a carbocation forms a cyclic oxonium ion. Loss of a proton gives a neutral cyclic ether and regenerates the acid catalyst. Count atoms along the actual tether to determine the ring size.

Step 1: Protonate the alkene at the correct end The alkene lies within the original six-membered ring. One alkene carbon carries the side chain containing the hydroxyl group; the other does not. Add a proton to the less substituted alkene carbon. The positive charge then appears on the tether-bearing carbon, which is bonded to three carbon atoms and is a tertiary carbocation. Protonation in the

opposite direction would produce the less stable secondary carbocation.

Step 2: Close the five-membered oxygen ring The side chain from the carbocation centre is $-CH_2-CH_2-CH(OH)-C(CH_3)_3$. The alcohol oxygen attacks the tertiary carbocation within the same molecule. The newly formed ring contains the original carbocation carbon, the two methylene carbons, the alcohol-bearing carbon and the oxygen: five atoms in all. It shares only the original carbocation carbon with the cyclohexane ring, so the product is spirocyclic rather than fused. The tert-butyl group remains attached to the carbon next to oxygen.

Step 3: Deprotonate and compare the products The attacking oxygen initially has three bonds and a positive charge. Water removes its extra proton, leaving the neutral spiro cyclic ether and restoring the acid catalyst. The original ring double bond has been consumed, so the cyclohexane ring is saturated; both original ring methyl substituents are retained. Option (D) has exactly these features. Option (A) shows a fused six-membered ether and also loses the original methyl substituents. Option (B) would require alcohol dehydration, methyl migration and carbon cyclisation rather than the direct intramolecular capture in aqueous acid. Option (C) is a carbon-only five-membered ring and does not retain oxygen as the cyclic ether.

Step 4: Conclusion The major product is the five-membered spiro cyclic ether bearing the intact tert-butyl group, as shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: For an alkenol in aqueous acid, protonate the alkene to its more stable carbocation, then count the tether atoms before drawing intramolecular oxygen attack.

71. The IUPAC name of $K_3\,[Co\,(C_2O_4)_3]$ is:- [4 marks]

(A) Potassium tris(oxalato)cobalt(III)

(B) Potassium tris(oxalato)cobaltate(III)

(C) Potassium trioxalatocobaltate(III)

(D) Potassium trioxalatocobalt(III)

Correct Answer: (B) Potassium tris(oxalato)cobaltate(III)

Solution:

Concept:

- A coordination salt is named with its cation first, followed by its complex anion. An anionic cobalt complex uses the metal name cobaltate.
- Oxalate, $C_2O_4{}^{2-}$, is a bidentate ligand. Three oxalato ligands are designated tris(oxalato), with the oxidation state of the metal in Roman numerals.

Step 1: Determine the complex charge Each potassium ion is K^+. The three potassium ions provide total charge $+3$, so the coordination entity must have charge -3 for the compound to be neutral:

$$K_3[Co(C_2O_4)_3] \longrightarrow 3\,K^+ + [Co(C_2O_4)_3]^{3-}.$$

Potassium is therefore the cation named first; the cobalt-containing entity is an anion.

Step 2: Find the oxidation state and assemble the name Let the oxidation state of cobalt be x. Each

of the three oxalato ligands carries charge -2, hence

$$x + 3(-2) = -3 \quad \Longrightarrow \quad x = +3.$$

The coordination entity is consequently tris(oxalato)cobaltate(III). Combining it with the counter-ion gives potassium tris(oxalato)cobaltate(III). Option (A) incorrectly uses cobalt for an anionic complex. Option (C) has cobaltate but uses the simple multiplier tri instead of tris for the oxalato ligand here. Option (D) has both of these naming errors. The coordination number is six because each oxalato ligand binds through two donor atoms; this does not change the count of three ligands in the name.

Step 3: Conclusion The IUPAC name is **potassium tris(oxalato)cobaltate(III)**. The supplied key selects the third choice, but its trioxalato prefix does not follow the tris(oxalato) form for these three chelating ligands; the charge balance and naming rules above give (B). Hence, the correct option is (B).

Quick Tip to Solve: First calculate the charge inside the brackets: an anionic complex changes cobalt to cobaltate, while three chelating oxalato ligands use tris(oxalato).

72. The strongest acid from the following is [4 marks]

(A) 3-methylphenol (m-cresol) 

(B) 3-nitrophenol 

(C) Phenol 

(D) 3-chlorophenol 

Correct Answer: (B) 3-nitrophenol 

Solution:

Concept:

- Phenol dissociates as $\mathrm{ArOH} \rightleftharpoons \mathrm{ArO^-} + \mathrm{H^+}$. A substituent that stabilises the phenoxide ion increases acidity by favouring proton loss.
- At the meta position, the electron-withdrawing inductive effect is decisive for the substituents shown; electron-donating groups destabilise the negative charge.

Step 1: Identify the groups and their electronic effects All four structures have a phenolic OH group. Option (A) is 3-methylphenol, (B) is 3-nitrophenol, (C) is phenol, and (D) is 3-chlorophenol. The methyl group is electron releasing through its positive inductive effect, so it does not assist stabilisation of the conjugate base. The nitro group is strongly electron withdrawing, whereas chlorine withdraws inductively less strongly than the nitro group. Phenol has no additional substituent.

Step 2: Compare the conjugate bases After removal of the hydroxyl proton, the oxygen bears the negative charge of a phenoxide ion, which is also delocalised in the aromatic ring. The meta nitro substituent withdraws electron density through sigma bonds and stabilises this anion most effectively among the given choices. Direct resonance interaction of the nitro group with the usual ortho/para phenoxide charge positions is not needed for this comparison. Meta chlorine also raises acidity relative

to phenol through its negative inductive effect, but less than nitro. The resulting acidity order is

$$\text{3-nitrophenol} > \text{3-chlorophenol} > \text{phenol} > \text{3-methylphenol.}$$

Thus (A) is weakened by methyl donation, (C) lacks the stabilising withdrawing group, and (D) is less acidic than the nitro-substituted compound.

Step 3: Conclusion **3-Nitrophenol** is the strongest acid among the four structures. Hence, the correct option is (B).

Quick Tip to Solve: Compare the stability of the phenoxide ions, not the O–H bond alone: a meta nitro group withdraws strongly by induction and increases acidity.

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73. Match List I with List II.

	LIST I Natural Amino Acid		LIST II One Letter Code
A.	**Arginine**	**I.**	**D**
B.	**Aspartic acid**	**II.**	**N**
C.	**Asparagine**	**III.**	**A**
D.	**Alanine**	**IV.**	**R**

Choose the correct answer from the options given below: [4 marks]

(A) (A) - IV, (B) - I, (C) - III, (D) - II

(B) (A) - III, (B) - I, (C) - II, (D) - IV

(C) (A) - IV, (B) - I, (C) - II, (D) - III

(D) (A) - I, (B) - III, (C) - IV, (D) - II

Correct Answer: (C) (A) - IV, (B) - I, (C) - II, (D) - III

Solution:

Concept:

- The standard one-letter codes identify amino acids unambiguously in protein sequences; they must not be inferred simply from the first letter of each name.
- For the amino acids in this question, arginine is R, aspartic acid is D, asparagine is N and alanine is A.

Step 1: Assign the standard code to each amino acid Arginine has the one-letter code R, which occurs at entry IV of List II. Therefore the first match is A–IV. Aspartic acid has code D, corresponding to entry I, so B–I. Although aspartic acid and asparagine have similar names, the former has a carboxylic acid side chain and the latter its amide; they have different codes. Asparagine is N, giving C–II. Alanine, whose side chain is a methyl group, uses A, giving D–III.

Step 2: Compare the complete mapping with each option The required sequence is

$$\begin{aligned} \text{A (Arginine)} &\longrightarrow \text{IV (R)}, \\ \text{B (Aspartic acid)} &\longrightarrow \text{I (D)}, \\ \text{C (Asparagine)} &\longrightarrow \text{II (N)}, \\ \text{D (Alanine)} &\longrightarrow \text{III (A)}. \end{aligned}$$

Option (C) alone reproduces all four correspondences. Option (A) assigns A to asparagine and N to alanine, so its last two matches are interchanged. Option (B) incorrectly gives A to arginine and R to alanine. Option (D) gives D to arginine and A to aspartic acid, and its remaining assignments are also wrong. Checking every match prevents accepting an option just because its first or second entry happens to be correct.

Step 3: Conclusion The correct matching is $\text{A} - \text{IV},\ \text{B} - \text{I},\ \text{C} - \text{II},\ \text{D} - \text{III}$. Hence, the correct option is (C).

Quick Tip to Solve: Remember the easily confused pair: aspartic acid is D and asparagine is N; then use arginine R and alanine A to check the full sequence.

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74. From the figure of column chromatography given below, identify incorrect statements.

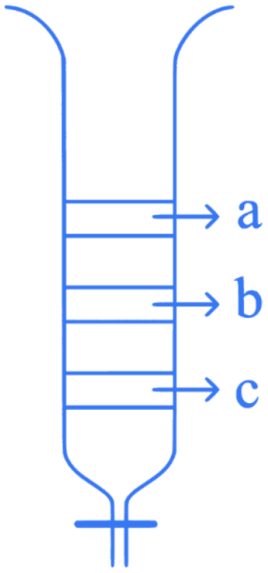
A. Compound 'c' is more polar than 'a' and 'b'

B. Compound 'a' is least polar

C. Compound 'b' comes out of the column before 'c' and after 'a'

D. Compound 'a' spends more time in the column

Choose the correct answer from the options given below: [4 marks]



(A) B and D only

(B) B, C and D only

(C) A, B and D only

(D) A, B and C only

Correct Answer: (D) A, B and C only

Solution:

Concept:

- In ordinary adsorption column chromatography, the polar stationary phase, such as silica or alumina, retains more polar solutes more strongly.
- The mobile phase moves down the column. A band that has travelled farther down is less strongly adsorbed and elutes earlier.

Step 1: Read the band order in the column The figure places band a highest, b in the middle and c lowest, nearest the outlet. Thus c has travelled farthest and is the first to leave the column. Band b follows c, and a is retained longest. For the usual polar adsorbent, the polarity and retention-time orders are

$$a > b > c,$$

whereas the elution sequence is c, then b, then a. The topmost band is not the least polar one: it is the band held most strongly by the stationary phase.

Step 2: Evaluate all four statements Statement A says that c is more polar than both a and b. It is incorrect: c is least polar and has moved farthest. Statement B calls a least polar; this is also incorrect because a is most strongly retained. Statement C says that b emerges before c and after a. Both directions are reversed: b emerges after c and before a, so C is incorrect. Statement D says that a spends more time in the column. This agrees with a being highest and most strongly adsorbed, so D is correct. Therefore the incorrect statements are exactly A, B and C. Options (A), (B) and (C) all include D among the incorrect statements, contradicting its correct retention-time interpretation; only (D) leaves D out and includes all three false statements.

Step 3: Conclusion The incorrect statements are **A, B and C only**. Hence, the correct option is (D).

Quick Tip to Solve: For a downward-moving column with a polar adsorbent, read bottom to top for elution order: c, b, a. Reverse that order for polarity and retention time.

75. The volume of 0.02 M aqueous HBr required to neutralize 10.0 mL of 0.01 M aqueous $Ba(OH)_2$ is (Assume complete neutralization) [4 marks]

(A) 7.5 mL

(B) 5.0 mL

(C) 10.0 mL

(D) 2.5 mL

Correct Answer: (C) 10.0 mL

Solution:

Concept:

- Complete neutralisation equates moles of hydrogen ions supplied by the acid with moles of hydroxide ions supplied by the base.
- HBr is monoprotic, while one mole of $Ba(OH)_2$ supplies two moles of hydroxide ions. The reaction ratio is therefore two moles of acid per mole of base.

Step 1: Write the balanced neutralisation and find the base amount The balanced chemical equation is

$$\mathrm{Ba(OH)_2 + 2\,HBr \longrightarrow BaBr_2 + 2\,H_2O.}$$

The base volume is $10.0\,\mathrm{mL} = 0.0100\,\mathrm{L}$, so its amount is

$$n_{\mathrm{Ba(OH)_2}} = (0.01\,\mathrm{mol\,L^{-1}})(0.0100\,\mathrm{L}) = 1.0 \times 10^{-4}\,\mathrm{mol}.$$

Because each formula unit contains two hydroxide groups, the amount of hydroxide to be neutralised is $2.0 \times 10^{-4}\,\mathrm{mol}$.

Step 2: Calculate the required acid volume and check the options The reaction requires $n_{\mathrm{HBr}} = 2n_{\mathrm{Ba(OH)_2}} = 2.0 \times 10^{-4}\,\mathrm{mol}$. Hence

$$V_{\mathrm{HBr}} = \frac{n}{M} = \frac{2.0 \times 10^{-4}}{0.02}\,\mathrm{L} = 0.010\,\mathrm{L} = 10.0\,\mathrm{mL}.$$

The acid molarity is twice the base molarity, exactly compensating for the two hydroxide ions produced by each base unit; equal volumes therefore neutralise one another. Option (B), 5.0 mL, would supply only half the required hydrogen ions because it ignores the factor of two in the base. Options (A) and (D), 7.5 and 2.5 mL, likewise supply insufficient acid and leave hydroxide unneutralised.

Step 3: Conclusion The required volume of $0.02\,\mathrm{M}$ aqueous HBr is $\boxed{10.0\,\mathrm{mL}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use acid equivalents equals base equivalents: $M_{\mathrm{HBr}}V_{\mathrm{HBr}} = 2M_{\mathrm{Ba(OH)_2}}V_{\mathrm{Ba(OH)_2}}$; do not drop the two hydroxide groups.

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76. The product, which is not obtained during the electrolysis of brine solution is : [4 marks]

(A) NaOH

(B) Cl_2

(C) H_2

(D) HCl

Correct Answer: (D) HCl

Solution:

Concept:

- Brine is an aqueous concentrated solution of sodium chloride. During electrolysis, chloride is oxidised at the anode and water is reduced at the cathode.
- The industrial chlor-alkali process produces chlorine, hydrogen and sodium hydroxide; separating the electrode products prevents their subsequent reactions.

Step 1: Identify the electrode products At the anode, chloride ions lose electrons to form chlorine gas:

$$\mathrm{2\,Cl^- \longrightarrow Cl_2 + 2\,e^-.}$$

At the cathode, water is reduced preferentially to sodium ions in the aqueous medium:

$$2\,H_2O + 2\,e^- \longrightarrow H_2 + 2\,OH^-.$$

Thus chlorine is an anode product and hydrogen is a cathode product. Sodium metal is not deposited from the aqueous brine; sodium ions remain in solution and balance the newly formed hydroxide ions.

Step 2: Combine the changes and eliminate the alternatives Adding the half-reactions and including the spectator sodium ions gives

$$2\,NaCl + 2\,H_2O \longrightarrow 2\,NaOH + H_2 + Cl_2.$$

The solution in the cathode compartment becomes alkaline because hydroxide ions accumulate along with sodium ions, giving sodium hydroxide. Hence option (A), $NaOH$, is obtained, option (B), Cl_2, is obtained, and option (C), H_2, is obtained. Hydrochloric acid is not a product of this electrolysis, so (D) is the required exception. Although collected hydrogen and chlorine can react in a separate chemical process to make hydrogen chloride, that subsequent reaction is not the direct electrode or solution product asked for here. The word not is therefore crucial when choosing the option.

Step 3: Conclusion HCl is **not obtained** as a product during electrolysis of brine. Hence, the correct option is (D).

Quick Tip to Solve: Recall the chlor-alkali trio: chlorine at the anode, hydrogen at the cathode and sodium hydroxide in solution. Choose the species outside this list.

77. Ion having highest hydration enthalpy among the given alkaline earth metal ions is : [4 marks]

(A) Sr^{2+}

(B) Ca^{2+}

(C) Be^{2+}

(D) Ba^{2+}

Correct Answer: (C) Be^{2+}

Solution:

Concept:

- Hydration enthalpy is the enthalpy change when gaseous ions become surrounded by water molecules. Stronger ion–dipole attraction releases more energy.
- For ions of the same charge, smaller ionic radius gives higher charge density and a larger magnitude of the negative hydration enthalpy.

Step 1: Compare the ionic radii at fixed charge All four choices are Group 2 cations with charge $+2$: Sr^{2+}, Ca^{2+}, Be^{2+} and Ba^{2+}. Since their charges are equal, the key variable is ionic size. Ionic radius increases down the group as additional electron shells are added. Among the given ions, the radius order is

$$r(Be^{2+}) < r(Ca^{2+}) < r(Sr^{2+}) < r(Ba^{2+}).$$

Beryllium therefore concentrates the same positive charge into the smallest volume.

Step 2: Infer the strength of hydration Water molecules orient their oxygen ends towards the cation. The intense electric field of the small Be^{2+} ion attracts these dipoles most strongly, producing the greatest energy release on hydration. Consequently the magnitudes obey

$$|\Delta H_{hyd}(Be^{2+})| > |\Delta H_{hyd}(Ca^{2+})| > |\Delta H_{hyd}(Sr^{2+})| > |\Delta H_{hyd}(Ba^{2+})|.$$

Thus calcium, strontium and barium, in options (B), (A) and (D), are all less strongly hydrated than beryllium. In this standard question, highest hydration enthalpy refers to the largest magnitude of the exothermic hydration enthalpy; its algebraic value for beryllium is the most negative, not the most positive.

Step 3: Conclusion $\boxed{Be^{2+}}$ has the highest hydration enthalpy in magnitude among the given ions. Hence, the correct option is (C).

Quick Tip to Solve: For equal ionic charge, choose the smallest ion for strongest hydration; remember that stronger hydration means a more negative enthalpy change.

78. Structures of $BeCl_2$ in solid state, vapour phase and at very high temperature respectively are :

[4 marks]

(A) Polymeric, Dimeric, Monomeric

(B) Dimeric, Polymeric, Monomeric

(C) Polymeric, Monomeric, Dimeric

(D) Monomeric, Dimeric, Polymeric

Correct Answer: (A) Polymeric, Dimeric, Monomeric

Solution:

Concept:

- Beryllium chloride, $BeCl_2$, is electron deficient: an isolated beryllium atom has only four electrons around it in its two covalent bonds.
- Chloride bridges donate lone-pair electron density to electron-deficient beryllium centres. Aggregation therefore changes with physical state and temperature.

Step 1: Identify the solid-state arrangement In the solid, chloride atoms bridge neighbouring beryllium centres, giving an extended polymeric chain. Each beryllium is surrounded by four chlorine atoms in a tetrahedral arrangement. The formula of the compound remains $BeCl_2$; the coordination number in the chain does not change that empirical formula.

Step 2: Follow the changes on heating In the vapour phase the chain breaks into smaller units, represented here by the bridged dimer Be_2Cl_4. Each dimer has two bridging and two terminal chlorides. At very high temperature the bridges break further and the monomer $BeCl_2$ is obtained. The isolated monomer is linear, with two bonds arranged at $180°$. Increasing temperature thus favours progressively smaller aggregates.

Step 3: Compare the order with the options The required order is solid, vapour, then very high

temperature, so it is polymeric, dimeric, monomeric. Option (B) reverses the solid and ordinary-vapour assignments. Option (C) puts the monomer before the dimer as temperature rises, and option (D) makes the solid monomeric and the hottest state polymeric; both contradict thermal dissociation.

Step 4: Conclusion The sequence is $\boxed{\text{Polymeric, Dimeric, Monomeric}}$. Hence, the correct option is (A).

Quick Tip to Solve: Read the temperature sequence first: chloride bridges are progressively broken on heating, so polymer becomes dimer and then monomer.

79. The group of chemicals used as pesticide is : [4 marks]

(A) Dieldrin, Sodium arsinite, Tetrachloroethene

(B) Aldrin, Sodium chlorate, Sodium arsinite

(C) DDT, Aldrin

(D) Sodium chlorate, DDT, PAN

Correct Answer: (C) DDT, Aldrin

Solution:

Concept:

- Pesticides are chemicals used to control pests; the broad category includes insecticides and herbicides.
- DDT and aldrin are organochlorine insecticides. PAN is a secondary air pollutant, whereas tetrachloroethene is commonly used as a dry-cleaning solvent.

Step 1: Identify the intended organochlorine group DDT and aldrin are both familiar synthetic organochlorine pesticides used against insect pests. Thus every substance listed in option (C) belongs to the insecticide group. This is the intended classification behind the keyed answer. Dieldrin, appearing in option (A), is also an insecticide, but its presence alone does not make the entire group correct.

Step 2: Eliminate the groups containing non-pesticides Option (A) includes tetrachloroethene, used as a dry-cleaning solvent rather than classified here as a pesticide. Option (D) includes PAN, peroxyacetyl nitrate, a component of photochemical smog that injures plants and irritates the eyes. It is an atmospheric pollutant rather than a pesticide. These two groups therefore fail even though they contain some pesticidal substances.

Step 3: Check the breadth of the printed term Option (B) contains aldrin together with sodium chlorate and sodium arsenite (spelt arsinite in the question). The last two have been used as herbicides. Consequently, under the broad meaning of pesticide, which includes herbicides, option (B) also qualifies. If the intended term were organochlorine insecticides, option (C) would be uniquely correct. The printed word pesticide does not itself impose that restriction.

Step 4: Conclusion The keyed and intended answer is $\boxed{\text{DDT, Aldrin}}$, option (C). However, the literal broad term pesticide also admits the herbicides in (B), so the printed wording does not give a unique scientifically defensible choice.

Quick Tip to Solve: Check every member of a chemical group. Distinguish insecticides from the broader pesticide category, which also contains herbicides.

80. Given below are two statements:

Statement I: Morphine is a narcotic analgesic. It helps in relieving pain without producing sleep.

Statement II: Morphine and its derivatives are obtained from opium poppy.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Analgesics relieve pain. Narcotic analgesics such as morphine act on the central nervous system and can produce sleep or stupor.
- Morphine is an opium alkaloid. Opium is obtained from the latex of the opium poppy, *Papaver somniferum*.

Step 1: Evaluate every clause of Statement I The first clause, that morphine is a narcotic analgesic, is correct. The second clause says that it relieves pain without producing sleep. This is not the characteristic description of morphine: narcotic analgesics depress the central nervous system and can induce sleep or stupor. A statement containing a correct classification and an incorrect characteristic is false overall. Relief of pain without the narcotic sleep-producing effect is associated with the non-narcotic class in this comparison.

Step 2: Evaluate Statement II Morphine is obtained from opium, the dried latex collected from the unripe capsules of the opium poppy. Morphine-related narcotic drugs are obtained from this natural starting material, including derivatives prepared from the alkaloid. Thus the statement identifying opium poppy as their source is true. It does not assert that every chemically prepared derivative is already present unchanged in the plant.

Step 3: Match the truth values to the answer The pair of truth values is false for Statement I and true for Statement II. Options (A) and (B) wrongly treat Statement I as true; option (C) wrongly rejects the correct source stated in Statement II. Only option (D) retains the correct judgement for both statements.

Step 4: Conclusion $\boxed{\text{Statement I is false; Statement II is true.}}$ Hence, the correct option is (D).

Quick Tip to Solve: Judge both clauses of a statement: correctly calling morphine a narcotic does not rescue the incorrect claim that it cannot produce sleep.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The number of species having a square planar shape from the following is _____.

$$XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, [Cu(NH_3)_4]^{2+}, [FeCl_4]^{2-}, [PtCl_4]^{2-}$$

[4 marks]

Solution:

Concept:

- For main-group species, VSEPR theory counts bonding pairs and lone pairs around the central atom. An AX_4E_2 arrangement has square planar molecular shape.
- For coordination complexes, ligand strength and the metal electronic configuration determine geometry; four-coordinate complexes are not automatically tetrahedral.

Step 1: Classify the main-group species In XeF_4, xenon has four bonding pairs and two lone pairs. The six electron domains form an octahedral arrangement, with the two lone pairs opposite each other; the four fluorine atoms are square planar. $BrF_4{}^-$ has the same AX_4E_2 electron-domain pattern and is also square planar. In contrast, SF_4 is AX_4E, so its shape is seesaw. Both SiF_4 and $BF_4{}^-$ have four bonding domains and no central lone pair, giving tetrahedral shapes.

Step 2: Classify the transition-metal complexes The four-coordinate ammine complex $[Cu(NH_3)_4]^{2+}$ is treated as square planar in this question. $[PtCl_4]^{2-}$ contains platinum(II), a d^8 ion whose four-coordinate complex is square planar. In $[FeCl_4]^{2-}$, iron is in oxidation state $+2$, with a d^6 configuration; chloride is weak field and this complex is tetrahedral. Thus the three complexes contribute two square planar species, not three.

Step 3: Add the two groups The qualifying species are XeF_4, $BrF_4{}^-$, $[Cu(NH_3)_4]^{2+}$ and $[PtCl_4]^{2-}$. The main-group set contributes two and the complex-ion set contributes two, so the total is $2 + 2 = 4$.

Step 4: Conclusion The number of square planar species is $\boxed{4}$ (a dimensionless count).

Quick Tip to Solve: Separate VSEPR molecules from metal complexes before counting; four attached ligands alone do not establish square planar geometry.

82. The standard reduction potentials at $298\ \mathrm{K}$ for the following half cells are given below:

$$NO_3^- + 4H^+ + 3e^- \rightarrow NO(g) + 2H_2O \quad E^\theta = 0.97\ V$$

$$V^{2+}(aq) + 2e^- \rightarrow V \qquad E^\theta = -1.19\ V$$

$$Fe^{3+}(aq) + 3e^- \rightarrow Fe \qquad E^\theta = -0.04\ V$$

$$Ag^+(aq) + e^- \rightarrow Ag(s) \qquad E^\theta = 0.80\ V$$

$$Au^{3+}(aq) + 3e^- \rightarrow Au(s) \qquad E^\theta = 1.40\ V$$

The number of metal(s) which will be oxidized by NO_3^- in aqueous solution is _____. [4 marks]

Solution:

Concept:

- For spontaneous oxidation of a metal by nitrate, nitrate is reduced at the cathode and the metal is oxidized at the anode.
- Using reduction potentials for both couples, $E^\circ_{\text{cell}} = E^\circ_{\text{nitrate/NO}} - E^\circ_{\text{metal ion/metal}}$. A positive value indicates favourable reaction under the stated standard conditions.

Step 1: Establish the comparison criterion The nitrate reduction potential is $0.97\,\mathrm{V}$. Therefore a metal can be oxidized when its listed reduction potential is smaller than $0.97\,\mathrm{V}$. We use the values exactly as given; the nitrate half-reaction includes acid, so this is the supplied acidic nitrate reduction couple. The number of electrons determines how the reactions are balanced, but electrode potentials are intensive quantities and must not be multiplied by stoichiometric coefficients.

Step 2: Calculate the cell potential for each metal For vanadium, $E^\circ_{\text{cell}} = 0.97 - (-1.19) = 2.16\,\mathrm{V}$. For iron it is $0.97 - (-0.04) = 1.01\,\mathrm{V}$. For silver it is $0.97 - 0.80 = 0.17\,\mathrm{V}$. All three are positive, so oxidation of V, Fe and Ag by nitrate is thermodynamically favourable according to the supplied couples. For gold the value is $0.97 - 1.40 = -0.43\,\mathrm{V}$, which is negative.

Step 3: Count the favourable metal oxidations Vanadium, iron and silver satisfy the positive-cell-potential criterion, whereas gold does not. Counting each metal once gives three. Different electron counts in their half-reactions do not create additional metals or change the sign test.

Step 4: Conclusion The metals are V, Fe and Ag, so the required number is $\boxed{3}$ (dimensionless).

Quick Tip to Solve: Compare every metal reduction potential directly with 0.97 V; never multiply an electrode potential when balancing electrons.

83. Consider the following data

Heat of combustion of $H_2(g)$ $= -241.8\ \text{kJ mol}^{-1}$

Heat of combustion of $C(s)$ $= -393.5\ \text{kJ mol}^{-1}$

Heat of combustion of $C_2H_5OH(l)$ $= -1234.7\ \text{kJ mol}^{-1}$

The heat of formation of $C_2H_5OH(l)$ **is** $(-)$ **_____** kJ mol^{-1} **(Nearest integer).** [4 marks]

Solution:

Concept:

- Hess's law makes the enthalpy change independent of the reaction path. For a reaction, $\Delta H = \sum \nu \Delta H_f(\text{products}) - \sum \nu \Delta H_f(\text{reactants})$.
- The supplied combustion of hydrogen forms water vapour, and combustion of carbon forms carbon dioxide. Elements in their reference states have zero formation enthalpy.

Step 1: Write the balanced ethanol combustion One mole of liquid ethanol undergoes the reaction

$$\mathrm{C_2H_5OH\,(l) + 3\,O_2(g) \longrightarrow 2\,CO_2(g) + 3\,H_2O\,(g).}$$

The coefficients are fixed by two carbon atoms and six hydrogen atoms in ethanol. The water state must be consistent with the supplied hydrogen combustion value, $-241.8\,\text{kJ mol}^{-1}$; substituting the enthalpy for liquid water instead would use a different thermochemical dataset.

Step 2: Apply the enthalpy balance Let $x = \Delta H_f(\mathrm{C_2H_5OH\,(l)})$. The supplied carbon and hydrogen combustion values give the formation enthalpies of the combustion products. Since oxygen is an element in its reference state,

$$-1234.7 = 2(-393.5) + 3(-241.8) - x.$$

The product sum is

$$-787.0 - 725.4 = -1512.4\,\text{kJ mol}^{-1}.$$

Therefore $-1234.7 = -1512.4 - x$, and hence $x = -277.7\,\text{kJ mol}^{-1}$.

Step 3: Enter the requested magnitude The question already prints a negative sign before the blank. The blank therefore asks for the positive magnitude of the negative formation enthalpy, rounded to the nearest integer. Rounding 277.7 gives 278; inserting an additional negative sign in the blank would reverse the intended sign.

Step 4: Conclusion The formation enthalpy is approximately $\boxed{-278\,\text{kJ mol}^{-1}}$. The number to enter in the blank is $\boxed{278}$.

Quick Tip to Solve: Balance the combustion first and preserve the water state. When a minus sign is printed before the blank, enter the positive magnitude.

84. Number of isomeric aromatic amines with molecular formula $C_8H_{11}N$, which can be synthesized by Gabriel Phthalimide synthesis is _____. [4 marks]

Solution:

Concept:

- Gabriel phthalimide synthesis prepares primary amines through nucleophilic substitution on suitable alkyl halides. Aryl halides do not undergo the required ordinary substitution at the ring carbon.
- For $C_8H_{11}N$, the degree of unsaturation is $(2 \times 8 + 2 + 1 - 11)/2 = 4$, consistent with a benzene ring. Both constitutional and optical isomers must be considered.

Step 1: Restrict the position of the amino group The amines obtainable by Gabriel synthesis must have $-NH_2$ attached to an aliphatic carbon, even though the molecule contains an aromatic ring. Ring-bound amino compounds such as ethylanilines and xylidines are excluded because their preparation would require replacement at an aryl carbon. Secondary amines are also excluded because Gabriel synthesis gives primary amines.

Step 2: Enumerate the constitutional arrangements There are three positional isomers of methylbenzylamine: o-, m- and p-$CH_3C_6H_4CH_2NH_2$. Each has formula $C_8H_{11}N$ and is obtainable from the corresponding methylbenzyl halide. A fourth arrangement is $C_6H_5CH_2CH_2NH_2$, 2-phenylethan-1-amine, obtained from a primary phenethyl halide. A fifth arrangement is $C_6H_5CH(NH_2)CH_3$, 1-phenylethan-1-amine, obtainable from the corresponding benzylic halide.

Step 3: Include the optical pair In 1-phenylethan-1-amine, the carbon bearing NH_2 is attached to four different groups: hydrogen, methyl, phenyl and amino. It therefore exists as two enantiomers. None of the other four constitutional arrangements has a stereogenic centre. The three methylbenzylamines contribute three isomers, phenethylamine contributes one, and the chiral benzylic amine contributes two, giving $3 + 1 + 2 = 6$.

Step 4: Conclusion Including the two enantiomers of 1-phenylethan-1-amine, the required number is $\boxed{6}$ (dimensionless).

Quick Tip to Solve: After listing primary side-chain amines, check for a carbon bonded to four different groups; counting only structural isomers would give five.

85. The equilibrium composition for the reaction $PCl_3 + Cl_2 \rightleftharpoons PCl_5$ at 298 K is given below:

$$[PCl_3]_{eq} = 0.2 \text{ mol L}^{-1}, [Cl_2]_{eq} = 0.1 \text{ mol L}^{-1}, [PCl_5]_{eq} = 0.40 \text{ mol L}^{-1}$$

If 0.2 mol of Cl_2 is added at the same temperature, the equilibrium concentrations of PCl_5 is _____ $\times 10^{-2}$ mol L^{-1}

Given : K_c for the reaction at 298 K is 20 [4 marks]

Solution:

Concept:

- For $PCl_3 + Cl_2 \rightleftharpoons PCl_5$, the concentration equilibrium constant is $K_c = [PCl_5]/([PCl_3][Cl_2])$. It stays unchanged when the temperature remains at 298 K.
- Adding chlorine makes the reaction quotient smaller than K_c, so the reaction moves forward. A change x in product concentration consumes the same concentration of each reactant. An amount in moles must first be divided by the vessel volume to obtain this concentration change.

Step 1: Express the addition as a concentration Let the fixed vessel volume be V litres and let $d = 0.2/V$, expressed in mol L^{-1}. Immediately after adding chlorine, its concentration is $0.10 + d$. At the new equilibrium the three concentrations, in mol L^{-1}, are

$$[PCl_3] = 0.20 - x, \quad [Cl_2] = 0.10 + d - x, \quad [PCl_5] = 0.40 + x.$$

The product increase must satisfy $0 < x < 0.20$.

Step 2: Apply the unchanged equilibrium constant Substitution gives

$$20 = \frac{0.40 + x}{(0.20 - x)(0.10 + d - x)}.$$

Expanding and collecting terms yields

$$20x^2 - (7 + 20d)x + 4d = 0.$$

The physically permitted root is

$$x = \frac{7 + 20d - \sqrt{(7 + 20d)^2 - 320d}}{40}.$$

The larger root must be rejected because it would consume more reactant than is present.

Step 3: Evaluate the numerical interpretation For $V = 1.00$ L, the addition is $d = 0.20$ mol L^{-1}. Thus

$$20x^2 - 11x + 0.80 = 0, \qquad x = \frac{11 - \sqrt{57}}{40} = 0.08625.$$

Consequently

$$[PCl_5] = 0.48625 \text{ mol L}^{-1} = 48.625 \times 10^{-2} \text{ mol L}^{-1},$$

whose numerical coefficient rounds to 49. The printed question specifies an added amount, but no vessel volume; therefore this numerical result is conditional on a one-litre vessel. Other volumes give

different values through $d = 0.2/V$.

Step 4: Conclusion The numerical entry is $\boxed{49}$ for a 1 L vessel, corresponding to $[PCl_5] \simeq 0.49\,\text{mol}\,L^{-1}$. As printed without the vessel volume, the question does not determine a unique final concentration; its general value is $0.40 + x$ with x as derived above.

Quick Tip to Solve: Before constructing an equilibrium table, distinguish an added number of moles from an added molarity: divide by the vessel volume, then reject any root giving a negative equilibrium concentration.

86. In an ice crystal, each water molecule is hydrogen bonded to _____ neighbouring molecules.

[4 marks]

Solution:

Concept:

- A hydrogen bond in ice is an intermolecular attraction between a hydrogen attached to oxygen in one H_2O molecule and a lone pair on oxygen in another molecule.
- An oxygen atom in water has two covalent $O-H$ bonds and two lone pairs. In ordinary ice, these four directions produce a tetrahedral hydrogen-bond network.

Step 1: Count the donated hydrogen bonds Each water molecule contains two hydrogen atoms, each already bonded covalently to its own oxygen. Each of these hydrogens can form one hydrogen bond with an oxygen atom of a neighbouring water molecule. Therefore one molecule donates two hydrogen bonds to two different neighbouring molecules. The hydrogen bonds do not replace or break the two covalent bonds within the water molecule.

Step 2: Count the accepted hydrogen bonds The same oxygen has two lone pairs. Each lone-pair direction can accept a hydrogen bond from a hydrogen of another neighbouring molecule. This gives two more hydrogen-bonded neighbours. Thus the local coordination is

$$N_{\text{neighbours}} = 2 \text{ donor neighbours} + 2 \text{ acceptor neighbours} = 4.$$

The arrangement is approximately tetrahedral, with the oxygen atom at the centre and four neighbouring oxygen atoms around it.

Step 3: Distinguish local coordination from bond counting The question asks for the neighbours of one molecule, so all four neighbours are counted. Dividing by two would instead count the number of distinct hydrogen bonds per molecule when the bonds in the entire crystal are totalled: every intermolecular bond belongs to two molecules. That sharing convention does not reduce the coordination number. The open tetrahedral network also explains the relatively low density of ice compared with liquid water.

Step 4: Conclusion Each water molecule in an ice crystal is hydrogen bonded to $\boxed{4}$ neighbouring water molecules. This is a dimensionless count.

Quick Tip to Solve: For ice, remember two hydrogen-bond donations from the two hydrogens and two acceptances at the oxygen lone pairs; together they give four neighbours.

87. Among the following, the number of compounds which will give positive iodoform reaction is _____

(a) 1-Phenylbutan-2-one

(b) 2-Methylbutan-2-ol

(c) 3-Methylbutan-2-ol

(d) 1-Phenylethanol

(e) 3,3-dimethylbutan-2-one

(f) 1-Phenylpropan-2-ol [4 marks]

Solution:

Concept:

- A methyl ketone, $\mathrm{RCOCH_3}$, gives the iodoform test with iodine and alkali, forming yellow $\mathrm{CHI_3}$. The carbonyl group must be directly bonded to a methyl group.
- A secondary alcohol of the form $\mathrm{RCH(OH)CH_3}$ is first oxidised to a methyl ketone and also gives the test. A tertiary alcohol lacks the necessary hydrogen on the carbon bearing OH.

Step 1: Check the two ketones Compound (a), 1-phenylbutan-2-one, is $\mathrm{C_6H_5CH_2COCH_2CH_3}$. Neither carbon next to the carbonyl is a methyl carbon, so it does not contain $\mathrm{COCH_3}$ and is negative. Compound (e), 3,3-dimethylbutan-2-one, is $\mathrm{CH_3COC(CH_3)_3}$. It has the required methyl-ketone group and is positive; the bulky group on the other side of the carbonyl does not remove that group.

Step 2: Check the alcohols individually Compound (b), 2-methylbutan-2-ol, is $\mathrm{(CH_3)_2C(OH)CH_2CH_3}$, a tertiary alcohol. The carbon bearing OH has no hydrogen and cannot undergo the required oxidation to a methyl ketone, so it is negative. Compound (c), 3-methylbutan-2-ol, is $\mathrm{CH_3CH(OH)CH(CH_3)_2}$, which oxidises to $\mathrm{CH_3COCH(CH_3)_2}$, so it is positive. Compound (d), 1-phenylethanol, is $\mathrm{C_6H_5CH(OH)CH_3}$, which oxidises to acetophenone, $\mathrm{C_6H_5COCH_3}$, and is positive. Compound (f), 1-phenylpropan-2-ol, is $\mathrm{C_6H_5CH_2CH(OH)CH_3}$, which oxidises to $\mathrm{C_6H_5CH_2COCH_3}$, and is also positive.

Step 3: Count the positive compounds The positive members are exactly (c), (d), (e) and (f). Thus

$$N = 1 + 1 + 1 + 1 = 4.$$

For a methyl ketone, the overall haloform reaction can be represented as

$$\mathrm{RCOCH_3 + 3\,I_2 + 4\,OH^- \longrightarrow RCOO^- + CHI_3 + 3\,I^- + 3\,H_2O.}$$

The alcohols qualify because oxidation creates this same reactive methyl-ketone unit, not simply because they contain an alcohol group.

Step 4: Conclusion The number of compounds giving a positive iodoform reaction is $\boxed{4}$: (c), (d), (e) and (f). The entry is dimensionless.

Quick Tip to Solve: Expand each name into a condensed structure and circle $\mathrm{COCH_3}$ or $\mathrm{CH(OH)CH_3}$; a methyl group elsewhere in the molecule is insufficient.

88. Consider the following pairs of solution which will be isotonic at the same temperature. The number of pairs of solutions is / are _____.

A. 1 M **aq.** NaCl **and** 2 M **aq. urea**

B. 1 M **aq.** $CaCl_2$ **and** 1.5 M **aq.** KCl

C. 1.5 M **aq.** $AlCl_3$ **and** 2 M **aq.** Na_2SO_4

D. 2.5 M **aq.** KCl **and** 1 M **aq.** $Al_2(SO_4)_3$ [4 marks]

Solution:

Concept:

- Osmotic pressure is $\pi = iCRT$, where C is molarity, T is absolute temperature and i is the van't Hoff factor. Two solutions at the same temperature are isotonic when their values of iC are equal.
- Use complete dissociation for these ionic solutes: $i = 2$ for NaCl and KCl, $i = 3$ for $CaCl_2$ and Na_2SO_4, $i = 4$ for $AlCl_3$, and $i = 5$ for $Al_2(SO_4)_3$. Urea is a nonelectrolyte with $i = 1$.

Step 1: Compare pairs A and B For pair A, dissociation $NaCl \longrightarrow Na^+ + Cl^-$ gives two ions per formula unit, whereas urea remains molecular. Hence

$$(iC)_{NaCl} = 2(1) = 2, \qquad (iC)_{urea} = 1(2) = 2.$$

Pair A is isotonic. For pair B, $CaCl_2 \longrightarrow Ca^{2+} + 2\,Cl^-$ gives three ions and KCl gives two. Therefore

$$(iC)_{CaCl_2} = 3(1) = 3, \qquad (iC)_{KCl} = 2(1.5) = 3.$$

Pair B is isotonic too.

Step 2: Compare pairs C and D For pair C, one $AlCl_3$ formula unit produces four ions and one Na_2SO_4 unit produces three. Thus

$$(iC)_{AlCl_3} = 4(1.5) = 6, \qquad (iC)_{Na_2SO_4} = 3(2) = 6.$$

Pair C is isotonic. In pair D, $Al_2(SO_4)_3 \longrightarrow 2\,Al^{3+} + 3\,{SO_4}^{2-}$ produces five ions per formula unit. Consequently

$$(iC)_{KCl} = 2(2.5) = 5, \qquad (iC)_{Al_2(SO_4)_3} = 5(1) = 5.$$

Pair D is also isotonic.

Step 3: Count the qualifying pairs All four pairs have equal particle concentrations within their respective pair. The common factor RT cancels in every comparison, so its numerical value is unnecessary. The pairs need not have the same particle concentration as other pairs: only the two members of each individual pair must match. Thus A, B, C and D all qualify in the complete-dissociation model used for this question.

Step 4: Conclusion The number of isotonic pairs is $\boxed{4}$, a dimensionless count.

Quick Tip to Solve: For each member calculate molarity times the total number of ions per formula unit; compare these products only within the pair, rather than comparing molarities alone.

89. The number of colloidal systems from the following, which will have 'liquid' as the dispersion medium, is _____.

Gem stones, paints, smoke, cheese, milk, hair cream, insecticide sprays, froth, soap lather [4 marks]

Solution:

Concept:

- A colloid is classified by the physical states of its dispersed phase and dispersion medium. The medium is the continuous phase in which the smaller dispersed particles or droplets are distributed.
- A sol has solid dispersed in liquid, an emulsion has liquid dispersed in liquid, and a foam has gas dispersed in liquid. Each of these has a liquid dispersion medium; gels, solid sols and aerosols have different media.

Step 1: Identify systems with a liquid medium Paints are sols: fine solid pigment particles are dispersed in a liquid vehicle. Milk is an emulsion with liquid fat droplets dispersed in water. Hair cream is also treated as an emulsion, with liquid droplets in a continuous liquid phase. Froth is a foam consisting of gas bubbles dispersed in liquid. Soap lather is another foam, with air dispersed in a soap solution. Thus the five qualifying systems are paints, milk, hair cream, froth and soap lather.

Step 2: Exclude systems with solid or gaseous media Gem stones are solid sols: the dispersed material is embedded in a solid medium, so they do not qualify. Cheese is classified as a gel, with liquid dispersed in a solid framework; its medium is solid. Smoke is a solid aerosol, with solid particles dispersed in gas. Insecticide sprays are liquid aerosols, with liquid droplets dispersed in gas. In the last case, the liquid is the dispersed phase, not the medium; therefore a spray must not be counted merely because it contains liquid droplets.

Step 3: Total the qualifying classifications The liquid-medium categories represented in the list are one sol (paints), two emulsions (milk and hair cream), and two foams (froth and soap lather). Hence

$$N = 1 + 2 + 2 = 5.$$

The full list has nine systems: the remaining four have either a solid medium (gem stones and cheese) or a gaseous medium (smoke and insecticide sprays). This checks that every named system has been considered exactly once.

Step 4: Conclusion The number of colloidal systems with a liquid dispersion medium is $\boxed{5}$. The answer is a dimensionless count.

Quick Tip to Solve: Ask which phase is continuous: a liquid spray has gas as its medium, whereas a foam has liquid as its medium even though its dispersed bubbles are gas.

90. Number of crystal systems from the following where body centred unit cell can be found, is _____.

Cubic, tetragonal, orthorhombic, hexagonal, rhombohedral, monoclinic, triclinic [4 marks]

Solution:

Concept:

- A body-centred conventional unit cell has lattice points at all eight corners and one at the body centre. Its net lattice-point count is $8(1/8) + 1 = 2$. The question asks for crystal systems admitting this centring as a distinct Bravais lattice.
- The seven crystal systems together give fourteen Bravais lattices. Use their allowed conventional centring types rather than inferring a new lattice merely by putting an extra point in a cell.

Step 1: List the systems that admit body centring The cubic system admits primitive, body-centred and face-centred lattices. The tetragonal system admits primitive and body-centred lattices. The orthorhombic system admits primitive, base-centred, body-centred and face-centred lattices. Thus the body-centred type, conventionally denoted I, appears in exactly cubic, tetragonal and orthorhombic systems. Their metric restrictions differ, but all three permit a distinct conventional I-centred cell.

Step 2: Check the remaining four systems The hexagonal system has the primitive hexagonal Bravais lattice. The rhombohedral system has the rhombohedral lattice. The monoclinic system has primitive and base-centred conventional lattices. The triclinic system has only a primitive lattice. None of these four systems adds a distinct body-centred conventional Bravais type to the fourteen-lattice classification. A nonstandard centred description that can be transformed to an existing conventional lattice is not an additional allowed type.

Step 3: Count systems rather than lattice points The required systems are

$$\text{cubic}, \quad \text{tetragonal}, \quad \text{orthorhombic}.$$

Therefore

$$N = 3.$$

As a consistency check, the conventional counts over all seven systems are $3+2+4+1+1+2+1 = 14$. The two lattice points per body-centred cell are a separate property; that number does not answer how many crystal systems contain the body-centred type.

Step 4: Conclusion Body-centred unit cells occur as distinct conventional Bravais lattices in $\boxed{3}$ crystal systems: cubic, tetragonal and orthorhombic. The requested count is dimensionless.

Quick Tip to Solve: Remember the three systems allowing I-centring: cubic, tetragonal and orthorhombic; do not confuse their number with the two lattice points in one body-centred cell.