JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 8 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 8 April 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is : [4 marks]

(A) 16800

(B) 14800

(C) 18000

(D) 33600

Correct Answer: (A) 16800

Solution:

Concept:

- Treat all vowels as one block when they must occur together. The block is then permuted with the consonants, after which its internal arrangements are counted.
- For a collection of n letters with identical groups of sizes $r_1, r_2, \ldots$, the number of distinct permutations is $n!/(r_1!r_2!\cdots)$.

Step 1: Count the repeated vowels and consonants The word INDEPENDENCE has twelve letters. Its five vowels are one I and four E letters. The seven consonants are three N letters, two D letters, one P and one C. Equal E letters cannot be distinguished, nor can equal N or D letters. Keeping these multiplicities separate prevents counting the same written arrangement several times.

Step 2: Arrange the vowel block with the consonants Denote the single vowel block by V. The eight objects to arrange are V, N, N, N, D, D, P and C. The block is distinct from every consonant, so only the repeated N and D letters contribute divisors at this stage. Thus

$$N_{\text{outer}} = \frac{8!}{3!\,2!} = 3360.$$

Step 3: Arrange the letters inside the block The five vowel positions contain I and four identical E letters. Their number of distinct orders is

$$N_{\text{inner}} = \frac{5!}{4!} = 5.$$

Every outer arrangement can be combined with each of these five inner orders, giving

$$N = 3360 \times 5 = 16800.$$

Counting the E letters as distinct would overcount by a factor of $4!$. The other three listed numbers do not equal this product, so none gives the required count.

Step 4: Conclusion The number of arrangements is $\boxed{16800}$. Hence, the correct option is (A).

Quick Tip to Solve: Form the vowel block first, but divide separately for repeated consonants outside it and repeated vowels inside it.

2. Let $f(x)=\frac{\sin x+\cos x-\sqrt{2}}{\sin x-\cos x}, x \in [0,\pi]-\left\{\frac{\pi}{4}\right\}$. Then $f\left(\frac{7\pi}{12}\right) f''\left(\frac{7\pi}{12}\right)$ is equal to [4 marks]

(A) $\frac{2}{3\sqrt{3}}$

(B) $\frac{2}{9}$

(C) $\frac{-1}{3\sqrt{3}}$

(D) $\frac{-2}{3}$

Correct Answer: (B) $\frac{2}{9}$

Solution:

Concept:

- A phase shift turns $\sin x+\cos x$ and $\sin x-\cos x$ into single trigonometric functions. Set $t=x-\pi/4$.
- Use $(\cos t-1)/\sin t=-\tan(t/2)$, wherever the original quotient is defined. Differentiate with the chain rule before substituting the given angle.

Step 1: Simplify the function by a phase shift For $t=x-\pi/4$, we have $\sin x+\cos x=\sqrt{2}\cos t$ and $\sin x-\cos x=\sqrt{2}\sin t$. Therefore

$$f(x)=\frac{\cos t-1}{\sin t}=-\tan\left(\frac{x}{2}-\frac{\pi}{8}\right).$$

The evaluation point is not the excluded point $\pi/4$, so this identity applies in a neighbourhood of it. Write $u=x/2-\pi/8$, for which $du/dx=1/2$.

Step 2: Differentiate twice with respect to x The first derivative and its derivative are

$$f'(x)=-\frac{1}{2}\sec^2 u,$$

$$f''(x)=-\frac{1}{2}\sec^2 u\tan u.$$

In the second line the derivative of $\sec^2 u$ supplies $2\sec^2 u\tan u$, while the inner derivative supplies another factor of $1/2$. Omitting that inner factor would double the answer.

Step 3: Evaluate the required product At $x=7\pi/12$, the half-angle is $u=7\pi/24-3\pi/24=\pi/6$. Hence

$$f=-\frac{1}{\sqrt{3}},\qquad f''=-\frac{1}{2}\cdot\frac{4}{3}\cdot\frac{1}{\sqrt{3}}=-\frac{2}{3\sqrt{3}}.$$

Consequently

$$f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)=\frac{2}{9}.$$

Both factors are negative, which rules out (C) and (D). Option (A) is the magnitude of the second derivative alone, rather than the requested product.

Step 4: Conclusion The required product equals $\boxed{2/9}$. Hence, the correct option is (B).

Quick Tip to Solve: When a quotient contains both sine plus cosine and sine minus cosine, shift the angle by $\pi/4$ before differentiating.

3. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$**. If** $|\operatorname{adj}(\operatorname{adj}(\operatorname{adj} 2A))| = (16)^n$**, then** n **is equal to :** [4 marks]

(A) 9

(B) 8

(C) 10

(D) 12

Correct Answer: (C) 10

Solution:

Concept:

- For a square matrix of order m, $\det(\operatorname{adj} B) = (\det B)^{m-1}$. Here the order is three, so taking an adjugate squares its determinant.
- Scaling a matrix of order three by 2 multiplies its determinant by 2^3, not by 2. Apply this scaling before the three adjugate operations.

Step 1: Find the determinant of A Expand along the first row, keeping the minus sign of the middle cofactor:

$$\det A = 2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} - \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix}$$
$$= 2(4-1) - (2-0) = 4.$$

Thus A is nonsingular, and the determinant to start with for the matrix $2A$ is

$$\det(2A) = 2^3 \det A = 8 \cdot 4 = 32 = 2^5.$$

Step 2: Track the determinant through three adjugates Let $B_0 = 2A$, $B_1 = \operatorname{adj} B_0$, $B_2 = \operatorname{adj} B_1$ and $B_3 = \operatorname{adj} B_2$. Every one of these matrices has order three. Consequently

$$\det B_1 = 32^2,$$
$$\det B_2 = (32^2)^2 = 32^4,$$
$$\det B_3 = (32^4)^2 = 32^8 = 2^{40}.$$

Three squarings give exponent eight on 32; they do not give exponent six. Working with determinants also avoids computing any of the nine cofactors explicitly.

Step 3: Compare powers with the printed expression The given right side is $16^n = (2^4)^n = 2^{4n}$. Equating the powers of the same positive base gives $4n = 40$, so $n = 10$. In particular, the other proposed exponents 9, 8 and 12 would yield powers 2^{36}, 2^{32} and 2^{48}, respectively, and cannot equal the computed determinant.

Step 4: Conclusion The exponent is $\boxed{n = 10}$. Hence, the correct option is (C).

Quick Tip to Solve: For nested adjugates of a matrix of order three, square the determinant once per adjugate and postpone all matrix calculations.

4. $\lim\limits_{x\to 0}\left(\dfrac{1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\dfrac{\sin^3(4x)}{[\log_e(2x+1)]^5}\right)$ **is equal to _____.** [4 marks]

(A) 15

(B) 18

(C) 9

(D) 24

Correct Answer: (B) 18

Solution:

Concept:

- Use $1-\cos^2(3x)=\sin^2(3x)$, together with $\sin t/t\to 1$ and $\log(1+t)/t\to 1$ as $t\to 0$.
- Near zero the numerator vanishes to order five, exactly matching the fifth power in the logarithmic denominator. The remaining cosine factor tends to one.

Step 1: Rewrite the expression in terms of sine The identity $1-\cos^2(3x)=\sin^2(3x)$ gives the requested limit as

$$L=\lim_{x\to 0}\frac{\sin^2(3x)\sin^3(4x)}{\cos^3(4x)[\log(1+2x)]^5}.$$

All these expressions are defined for sufficiently small nonzero real x, since $1+2x>0$ and $\cos(4x)\neq 0$ there. It is unnecessary to differentiate the quotient repeatedly.

Step 2: Extract the standard limit ratios Divide each vanishing factor by its own linear argument. Then

$$L=\frac{3^2\,4^3}{2^5}\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)^2\left(\frac{\sin 4x}{4x}\right)^3 \times\left(\frac{2x}{\log(1+2x)}\right)^5\frac{1}{\cos^3(4x)}.$$

The five powers of x cancel exactly. Both sine ratios tend to one, the logarithmic ratio tends to one, and $\cos(4x)\to 1$. These two-sided limits are valid even though the odd-powered factors change sign individually.

Step 3: Calculate the remaining coefficient The value is therefore

$$L=\frac{9\cdot 64}{32}=18.$$

The factor 2^5 is essential because the logarithm has argument $1+2x$, not $1+x$. Values 15, 9 and 24 do not equal the coefficient obtained from the leading terms, ruling out the other options.

Step 4: Conclusion The limit is $\boxed{18}$. Hence, the correct option is (B).

Quick Tip to Solve: First compare the orders of vanishing, then collect the argument multipliers: here they give $3^2\cdot 4^3/2^5$.

5. In a bolt factory, machines A, B and C manufacture respectively $20\%, 30\%$ and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is : [4 marks]

(A) $\frac{2}{7}$

(B) $\frac{9}{28}$

(C) $\frac{5}{14}$

(D) $\frac{3}{7}$

Correct Answer: (C) $\frac{5}{14}$

Solution:

Concept:

- Bayes theorem reverses a conditional probability: $P(C \mid D) = P(C)P(D \mid C)/P(D)$, where D means that the selected bolt is defective.
- The total probability of a defect is a weighted sum over all machines. The weights are production shares, rather than equal probabilities for the three machines.

Step 1: Record the production shares and defect rates The machines contribute $P(A) = 0.20$, $P(B) = 0.30$, and $P(C) = 0.50$. Their conditional defect probabilities are $P(D \mid A) = 0.03$, $P(D \mid B) = 0.04$, and $P(D \mid C) = 0.02$. For example, the probability of selecting a bolt that is both from C and defective is $0.50 \times 0.02 = 0.010$. The 2 percent defect rate on its own answers a different conditional question.

Step 2: Find the probability that a random bolt is defective The three production sources partition the total output, so the law of total probability gives

$$\begin{aligned} P(D) &= P(A)P(D \mid A) + P(B)P(D \mid B) + P(C)P(D \mid C) \\ &= 0.20(0.03) + 0.30(0.04) + 0.50(0.02) \\ &= 0.006 + 0.012 + 0.010 = 0.028. \end{aligned}$$

Thus 2.8 percent of the entire output is defective. The larger production share of C compensates partly for its lower defect rate.

Step 3: Condition on the observed defect Among defective bolts, retain the contribution from C and divide it by the contribution from all three machines:

$$P(C \mid D) = \frac{0.010}{0.028} = \frac{10}{28} = \frac{5}{14}.$$

For comparison, the corresponding shares from A and B are $3/14$ and $3/7$; the three shares add to one. Option (D) belongs to B, while (A) and (B) are neither the C contribution nor its normalised share.

Step 4: Conclusion The conditional probability is $\boxed{5/14}$. Hence, the correct option is (C).

Quick Tip to Solve: Multiply each machine production share by its defect rate before normalising; do not average the defect rates without weights.

6. The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is : [4 marks]

(A) 720

(B) $7(360)^2$

(C) $7(720)^2$

(D) $126(5!)^2$

Correct Answer: (D) $126(5!)^2$

Solution:

Concept:

- For n distinct people around a round table, rotations represent the same seating, so the number of circular arrangements is $(n-1)!$. Reflections are different seatings.
- To prevent adjacent girls, first seat the boys, then place the girls in distinct gaps between successive boys. There are seven such gaps on a circle, including the closing gap.

Step 1: Seat the seven boys around the table All boys and girls are distinct people. Fix one boy to remove the rotational duplication, then arrange the remaining six boys around him. This gives

$$N_{\text{boys}} = 6! = 720.$$

Using 7! instead would count each circular arrangement seven times. We do not divide by two, because reversing the order changes each person's neighbours on the left and right.

Step 2: Choose five gaps and assign the girls The seven seated boys determine exactly seven gaps. Inserting at most one girl into a gap ensures that any pair of girls has a boy between them. Choose five gaps and then permute the five girls among them:

$$N_{\text{girls}} = \binom{7}{5} 5! = 21 \cdot 120.$$

There is no requirement that boys also be separated, so leaving two gaps empty is allowed. Every valid seating yields one unique circular order of boys and one unique selection and assignment of occupied gaps.

Step 3: Multiply and identify the listed form The total number of seatings is

$$N = 6!\binom{7}{5} 5! = 720 \cdot 21 \cdot 120.$$

Since $720 = 6 \cdot 120$, this simplifies to

$$N = 126(120)^2 = 126(5!)^2 = 1814400.$$

Option (A) counts only the boys. Option (B) equals 907200, half the result, whereas option (C) equals 3628800, twice the result. Neither extra division nor extra multiplication by two is justified for an ordinary round-table seating.

Step 4: Conclusion The required number is $\boxed{126(5!)^2}$. Hence, the correct option is (D).

Quick Tip to Solve: On a circle, seven seated boys create seven gaps; choose five of those gaps before assigning the five distinct girls.

7. The shortest distance between the lines $\frac{x-4}{4}=\frac{y+2}{5}=\frac{z+3}{3}$ and $\frac{x-1}{3}=\frac{y-3}{4}=\frac{z-4}{2}$ is : [4 marks]

(A) $3\sqrt{6}$

(B) $6\sqrt{2}$

(C) $6\sqrt{3}$

(D) $2\sqrt{6}$

Correct Answer: (A) $3\sqrt{6}$

Solution:

Concept:

- For nonparallel lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$, with directions $\mathbf{b}_1, \mathbf{b}_2$, their shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.
- The cross product is perpendicular to both lines. Projecting a joining vector onto that common normal removes the components parallel to either line.

Step 1: Read a point and direction from each line The first line passes through $\mathbf{a}_1 = (4, -2, -3)$ with direction $\mathbf{b}_1 = (4, 5, 3)$. The second passes through $\mathbf{a}_2 = (1, 3, 4)$ with direction $\mathbf{b}_2 = (3, 4, 2)$. Therefore a joining vector is

$$\mathbf{a}_2 - \mathbf{a}_1 = (-3, 5, 7).$$

In particular, $y + 2$ and $z + 3$ on the first line correspond to negative coordinates, not positive ones.

Step 2: Compute the common normal The cross product is

$$\begin{aligned}\mathbf{b}_1 \times \mathbf{b}_2 &= (5 \cdot 2 - 3 \cdot 4,\ 3 \cdot 3 - 4 \cdot 2,\ 4 \cdot 4 - 5 \cdot 3) \\ &= (-2, 1, 1).\end{aligned}$$

Its magnitude is $\sqrt{4 + 1 + 1} = \sqrt{6}$. Since it is nonzero, the lines are not parallel and the common-normal formula applies. Reversing the cross product would only change its sign and would not change the distance.

Step 3: Project the joining vector onto the unit normal The scalar triple product is

$$(-3, 5, 7) \cdot (-2, 1, 1) = 6 + 5 + 7 = 18.$$

Consequently

$$d = \frac{|18|}{\sqrt{6}} = \frac{18\sqrt{6}}{6} = 3\sqrt{6}.$$

The nonzero triple product also confirms that the lines do not intersect. Taking just the triple product would omit normalisation; the remaining option values $6\sqrt{2}$, $6\sqrt{3}$, and $2\sqrt{6}$ do not equal the computed projection.

Step 4: Conclusion The shortest distance is $\boxed{3\sqrt{6}}$ units. Hence, the correct option is (A).

Quick Tip to Solve: Check the signs of the base-point coordinates, then divide the scalar triple product by the cross-product magnitude.

8. Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q.

Let the point $G(10, 10)$ be the centroid of the triangle PQR. If $c - m = 6$, then $(PQ)^2$ is : [4 marks]

(A) 317

(B) 325

(C) 346

(D) 296

Correct Answer: (B) 325

Solution:

Concept:

- For the parabola $y^2 = 4ax$, the focus is $(a, 0)$. Here $4a = 20$, so $R = (5, 0)$.
- The centroid is the coordinatewise average of the three vertices. Two points on $y = mx + c$ also satisfy $y_1 + y_2 = m(x_1 + x_2) + 2c$.

Step 1: Use the centroid to determine the chord Write $P = (x_1, y_1)$ and $Q = (x_2, y_2)$. The centroid conditions give

$$x_1 + x_2 + 5 = 30, \qquad y_1 + y_2 = 30.$$

Thus $x_1 + x_2 = 25$. Since both endpoints lie on the given line,

$$25m + 2c = 30.$$

Combining this with $c - m = 6$, or $c = m + 6$, gives

$$27m + 12 = 30, \qquad m = \frac{2}{3}, \quad c = \frac{20}{3}.$$

The focus is needed in the horizontal centroid equation; treating it as the origin would give a different line.

Step 2: Find the endpoints and square the distance Substitute $y = (2x + 20)/3$ in the parabola:

$$\frac{(2x + 20)^2}{9} = 20x \quad \Longrightarrow \quad x^2 - 25x + 100 = 0.$$

Hence $(x - 5)(x - 20) = 0$, and the endpoints are $(5, 10)$ and $(20, 20)$. Their centroid with $(5, 0)$ is indeed $(10, 10)$. The squared distance is

$$(PQ)^2 = (20 - 5)^2 + (20 - 10)^2 = 225 + 100 = 325.$$

The question asks for the squared length, so no square root is required; the other listed numbers do not satisfy this distance calculation.

Step 3: Conclusion $(PQ)^2 = 325$. Hence, the correct option is (B).

Quick Tip to Solve: Use the centroid to obtain the sums of endpoint coordinates before solving the line-parabola intersection.

9. The area of the region $\{(x, y) : x^2 \le y \le 8 - x^2, y \le 7\}$ is : [4 marks]

(A) 18

(B) 24

(C) 20

(D) 21

Correct Answer: (C) 20

Solution:

Concept:

- A vertical strip has height equal to its upper boundary minus its lower boundary. Here the lower boundary is $y = x^2$.
- The upper boundary is $\min(8 - x^2, 7)$; symmetry about the y-axis lets us double the area for $x \ge 0$.

Step 1: Locate the intersection and the change in upper boundary The two parabolas meet where

$$x^2 = 8 - x^2, \qquad x = \pm 2, \quad y = 4.$$

Therefore the region lies between $x = -2$ and $x = 2$. The horizontal restriction becomes active when

$$8 - x^2 = 7, \qquad x = \pm 1.$$

On $0 \le x \le 1$ the upper curve is $y = 7$; on $1 \le x \le 2$ it is $y = 8 - x^2$. In particular, using the parabola as the upper boundary everywhere would count the forbidden cap above $y = 7$.

Step 2: Integrate the two strip heights The required area is consequently

$$\mathcal{A} = 2\left[\int_0^1 (7 - x^2)\,dx + \int_1^2 (8 - 2x^2)\,dx\right].$$

Evaluate each term separately:

$$\int_0^1 (7 - x^2)\,dx = 7 - \frac{1}{3} = \frac{20}{3},$$

$$\int_1^2 (8 - 2x^2)\,dx = \left[8x - \frac{2}{3}x^3\right]_1^2 = \frac{10}{3}.$$

Therefore

$$\mathcal{A} = 2\left(\frac{20}{3} + \frac{10}{3}\right) = 20.$$

Both strip heights are nonnegative on their stated intervals. The values 18, 24 and 21 do not equal this piecewise integral.

Step 3: Conclusion The area is 20 square units. Hence, the correct option is (C).

Quick Tip to Solve: When a region has two upper inequalities, find where those upper boundaries cross and split the area integral there.

10. Let $C(\alpha, \beta)$ be the circumcenter of the triangle formed by the lines

$$4x + 3y = 69$$

$4y - 3x = 17$**, and**

$x + 7y = 61$**.**

Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to : [4 marks]

(A) 15

(B) 17

(C) 16

(D) 18

Correct Answer: (B) 17

Solution:

Concept:

- The slopes of perpendicular nonvertical lines have product -1. A triangle containing their intersection therefore has a right angle.
- The circumcentre of a right triangle is the midpoint of its hypotenuse, since this point is equally distant from all three vertices.

Step 1: Identify the right angle The first two lines can be written as

$$y = -\frac{4}{3}x + 23, \qquad y = \frac{3}{4}x + \frac{17}{4}.$$

Their slopes multiply to -1, so their intersection is the right-angle vertex. The third line cuts these two perpendicular sides at the endpoints of the hypotenuse. This observation avoids solving the general perpendicular-bisector equations, while retaining the same circumcentre.

Step 2: Find the hypotenuse endpoints Intersect $4x + 3y = 69$ with $x + 7y = 61$. Substituting $x = 61 - 7y$ gives $244 - 25y = 69$, so $y = 7$ and $x = 12$. One endpoint is $(12, 7)$. Next intersect $-3x + 4y = 17$ with $x + 7y = 61$:

$$-3(61 - 7y) + 4y = 17 \quad \Longrightarrow \quad 25y = 200.$$

This gives the other endpoint $(5, 8)$. Both points satisfy the third line, as required.

Step 3: Evaluate the requested expression The midpoint of the hypotenuse is

$$C = \left(\frac{12 + 5}{2}, \frac{7 + 8}{2}\right) = \left(\frac{17}{2}, \frac{15}{2}\right).$$

Thus $\alpha - \beta = 1$ and $\alpha + \beta = 16$. Consequently

$$(\alpha - \beta)^2 + \alpha + \beta = 1^2 + 16 = 17.$$

The value 16 omits the squared difference, and 15 or 18 are inconsistent with the midpoint coordinates.

Step 4: Conclusion The required value is 17. Hence, the correct option is (B).

Quick Tip to Solve: Check whether two of the three given lines are perpendicular before using a general circumcentre formula.

11. Let $I(x)=\int \frac{(x+1)}{x(1+xe^x)^2}dx, x>0$. If $\lim_{x\to\infty} I(x)=0$, then $I(1)$ is equal to : [4 marks]

(A) $\frac{e+1}{e+2}-\log_e(e+1)$

(B) $\frac{e+1}{e+2}+\log_e(e+1)$

(C) $\frac{e+2}{e+1}-\log_e(e+1)$

(D) $\frac{e+2}{e+1}+\log_e(e+1)$

Correct Answer: (C) $\frac{e+2}{e+1}-\log_e(e+1)$

Solution:

Concept:

- Choose $t=xe^x$, because its logarithmic derivative is $dt/t=(x+1)\,dx/x$. For $x>0$, we have $t>0$, so all logarithms below are defined.
- An indefinite integral has an additive constant. The condition at infinity fixes that constant after finding an antiderivative.

Step 1: Transform and decompose the integrand With $t=xe^x$, the integral becomes

$$I(x)=\int \frac{dt}{t(1+t)^2}.$$

Resolve the rational expression into partial fractions:

$$\frac{1}{t(1+t)^2}=\frac{1}{t}-\frac{1}{1+t}-\frac{1}{(1+t)^2}.$$

Indeed, multiplying by $t(1+t)^2$ gives $(1+t)^2-t(1+t)-t=1$, confirming the signs. Integrating yields

$$I(x)=\log t-\log(1+t)+\frac{1}{1+t}+C.$$

Step 2: Apply the boundary condition and substitute one As $x\to\infty$, also $t\to\infty$. The term $\log[t/(1+t)]$ tends to zero and $1/(1+t)$ tends to zero. The given limit therefore forces $C=0$. At $x=1$, we have $t=e$, and hence

$$I(1)=1-\log(e+1)+\frac{1}{e+1}=\frac{e+2}{e+1}-\log(e+1).$$

Differentiating the antiderivative with respect to t recovers $1/[t(1+t)^2]$, so the substitution also verifies the answer. The options with a plus logarithm have the wrong sign; option (A) has the wrong rational term.

Step 3: Conclusion $I(1)=\frac{e+2}{e+1}-\log_e(e+1)$. Hence, the correct option is (C).

Quick Tip to Solve: For integrands involving both xe^x and $(x+1)/x$, use the logarithmic derivative of xe^x.

12. Let α, β, γ be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to : [4 marks]

(A) 21

(B) 19

(C) $\frac{169}{8}$

(D) $\frac{155}{8}$

Correct Answer: (B) 19

Solution:

Concept:

- For $x^3 + bx + c = 0$, Vieta gives $\alpha + \beta + \gamma = 0$, $\alpha\beta + \alpha\gamma + \beta\gamma = b$, and $\alpha\beta\gamma = -c$.
- The conditions $\beta\gamma = 1 = -\alpha$ mean both $\beta\gamma = 1$ and $\alpha = -1$. The other roots need not be real.

Step 1: Determine the coefficients and remaining root relations Since $\alpha = -1$, the zero sum of roots implies $\beta + \gamma = 1$. Thus

$$b = \alpha(\beta + \gamma) + \beta\gamma = -1 + 1 = 0.$$

Also

$$\alpha\beta\gamma = -1 = -c, \qquad c = 1.$$

The polynomial is therefore $x^3 + 1$. This directly shows that each of its roots has cube -1, including the two complex roots; there is no need to choose an ordering for them.

Step 2: Verify the individual cubes and evaluate Alternatively, β and γ satisfy $t^2 - t + 1 = 0$. Multiplication by $t + 1$ gives $t^3 + 1 = 0$, so $\beta^3 = \gamma^3 = -1$. Also $\alpha^3 = -1$. Substitute all these values in the expression, preserving the separate coefficients:

$$\begin{aligned} b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3 &= 0 + 2 - 3(-1) - 6(-1) - 8(-1) \\ &= 2 + 3 + 6 + 8 = 19. \end{aligned}$$

Since the individual cubes are equal, the unequal coefficients 6 and 8 cause no ambiguity. The fractional alternatives and 21 cannot result from the stated root relations.

Step 3: Conclusion The expression equals 19. Hence, the correct option is (B).

Quick Tip to Solve: Once the coefficients identify a polynomial such as $x^3 + 1$, read off the cube of every root without explicitly solving for complex roots.

13. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6 elements is : [4 marks]

(A) 782

(B) 772

(C) 752

(D) 792

Correct Answer: (D) 792

Solution:

Concept:

- The Cartesian product has cardinality $|A \times B| = |A|\,|B|$. Each ordered pair is one element of the product set.
- An r-element subset of an N-element set can be selected in $\binom{N}{r}$ ways. Subsets of different sizes form disjoint classes.

Step 1: Find the size of the set being sampled Each of the five elements of A can be paired with either of the two elements of B. Therefore

$$|A \times B| = 5 \cdot 2 = 10.$$

We are selecting subsets of this ten-element set, rather than separately selecting a subset of A and a subset of B. A valid subset can contain any collection of the ordered pairs; it need not itself be a Cartesian product.

Step 2: Count every permitted subset size The phrases at least 3 and at most 6 include both endpoints. Thus the required count is

$$N = \binom{10}{3} + \binom{10}{4} + \binom{10}{5} + \binom{10}{6}.$$

Evaluate the coefficients:

$$\binom{10}{3} = 120, \quad \binom{10}{4} = 210, \quad \binom{10}{5} = 252, \quad \binom{10}{6} = 210.$$

The last equality also follows from $\binom{10}{6} = \binom{10}{4}$, by taking complements. Adding gives

$$N = 120 + 210 + 252 + 210 = 792.$$

There is no factorial multiplier because a subset has no order. As a separate check, all $2^{10} = 1024$ subsets minus those of sizes 0, 1, 2, 7, 8, 9 and 10 gives $1024 - (1 + 10 + 45 + 120 + 45 + 10 + 1) = 792$. The other options do not equal this exact sum.

Step 3: Conclusion The required number of subsets is 792. Hence, the correct option is (D).

Quick Tip to Solve: First count the elements of the Cartesian product, then sum binomial coefficients over the inclusive range of allowed subset sizes.

14. Let $S_K = \frac{1+2+\ldots+K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}\left(Bn^2 + Cn + D\right)$, where $A, B, C, D \in \mathbb{N}$ and A has least value. Then [4 marks]

(A) $A + B + C + D$ is divisible by 5

(B) $A + C + D$ is not divisible by B

(C) $A + B = 5(D - C)$

(D) $A + B$ is divisible by D

Correct Answer: (D) $A + B$ is divisible by D

Solution:

Concept:

- The arithmetic-series formula gives $1+2+\cdots+j = j(j+1)/2$, so the given average is $S_j = (j+1)/2$.
- Use $\sum j = n(n+1)/2$ and $\sum j^2 = n(n+1)(2n+1)/6$. Matching polynomial coefficients determines A, B, C, D.

Step 1: Expand and sum the squared averages For each positive integer j, $S_j^2 = (j^2 + 2j + 1)/4$. Therefore

$$\begin{aligned}\sum_{j=1}^{n} S_j^2 &= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n(n+1) + n\right] \\ &= \frac{n}{24}\left[(n+1)(2n+1) + 6(n+1) + 6\right] \\ &= \frac{n}{24}(2n^2 + 9n + 13).\end{aligned}$$

For example, $n = 1$ gives 1, agreeing with S_1^2, which checks the constant term.

Step 2: Choose the least denominator and test all statements Coefficient comparison requires $B/A = 1/12$, $C/A = 3/8$, and $D/A = 13/24$. Since 13 and 24 are coprime and D is an integer, A must be a multiple of 24. The least allowed choice is consequently

$$A = 24, \quad B = 2, \quad C = 9, \quad D = 13.$$

Now $A + B + C + D = 48$, which is not divisible by 5, so (A) is false. Also $A + C + D = 46$ is divisible by $B = 2$, so (B) is false. For (C), $A + B = 26$, whereas $5(D - C) = 20$. Finally $A + B = 26 = 2D$, so statement (D) is true.

Step 3: Conclusion $A + B = 26$ is divisible by $D = 13$. Hence, the correct option is (D).

Quick Tip to Solve: After finding the polynomial, reduce its rational coefficients and use denominator divisibility to justify the least possible value of A.

15. Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ **and** $Q = PAP^T$. **If** $P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, **then** $2a + b - 3c - 4d$ **equal to :** [4 marks]

(A) 2004

(B) 2006

(C) 2007

(D) 2005

Correct Answer: (D) 2005

Solution:

Concept:

- A real matrix P is orthogonal when $P^T P = PP^T = I$; its transpose is then its inverse.
- If $Q = PAP^T$ with P orthogonal, repeated multiplication gives $Q^m = PA^m P^T$ for every positive integer m.
- For $A = I + N$ with $N^2 = 0$, the binomial expansion terminates: $A^m = I + mN$.

Step 1: Verify the orthogonality of the given matrix The two columns of P each have squared length $3/4 + 1/4 = 1$. Their scalar product is $\sqrt{3}/4 - \sqrt{3}/4 = 0$. Consequently $P^T P = I$, and since P is square, $PP^T = I$ as well. This is the condition that allows adjacent transpose factors to cancel; it is not an assumption about an arbitrary matrix.

Step 2: Reduce the large power by similarity Insert $Q = PAP^T$. For example, $Q^2 = PA(P^T P)AP^T = PA^2 P^T$, so the same cancellation repeated gives

$$P^T Q^{2007} P = (P^T P)A^{2007}(P^T P) = A^{2007}.$$

There is therefore no need to calculate the individual entries of Q or expand a matrix power directly.

Step 3: Evaluate the nilpotent contribution Write

$$A = I + N, \qquad N = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad N^2 = 0.$$

All terms containing N^2 or a higher power vanish. Hence

$$A^{2007} = I + 2007N = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}.$$

Comparing entries gives $a = d = 1$, $b = 2007$, and $c = 0$.

Step 4: Substitute the entries with their signs The required scalar is

$$2a + b - 3c - 4d = 2 + 2007 - 0 - 4 = 2005.$$

The value 2007 is only the off-diagonal entry, not the complete expression. The other offered values 2004 and 2006 also fail the final substitution.

Step 5: Conclusion The required value is $\boxed{2005}$. Hence, the correct option is (D).

Quick Tip to Solve: Before expanding a large matrix power, check whether the surrounding matrices are inverse factors and whether the reduced matrix is identity plus a nilpotent matrix.

16. If for $z = \alpha + i\beta, |z+2| = z + 4(1+i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation : [4 marks]

(A) $x^2 + 2x - 3 = 0$

(B) $x^2 + 3x - 4 = 0$

(C) $x^2 + x - 12 = 0$

(D) $x^2 + 7x + 12 = 0$

Correct Answer: (D) $x^2 + 7x + 12 = 0$

Solution:

Concept:

- The modulus of any complex number is a nonnegative real number. Equality to a complex expression forces that expression to have zero imaginary part.
- For $z = \alpha + i\beta$, $|z+2| = \sqrt{(\alpha+2)^2 + \beta^2}$.
- A monic quadratic with roots u, v is $x^2 - (u+v)x + uv = 0$.

Step 1: Equate the imaginary parts first Expand the right-hand side before taking any modulus:

$$z + 4(1+i) = (\alpha + 4) + i(\beta + 4).$$

The left-hand side $|z+2|$ is real. Therefore $\beta + 4 = 0$, giving $\beta = -4$. Also $\alpha + 4 \geq 0$, since the real side equals a modulus. Taking the modulus of the right side prematurely would discard this necessary imaginary-part condition.

Step 2: Solve and check the real equation With $\beta = -4$, the remaining equality is

$$\sqrt{(\alpha+2)^2 + 16} = \alpha + 4.$$

Squaring yields

$$\alpha^2 + 4\alpha + 20 = \alpha^2 + 8\alpha + 16,$$

and hence $4\alpha = 4$, or $\alpha = 1$. Check the unsquared equation: $\sqrt{3^2 + (-4)^2} = 5 = 1 + 4$. Thus no extraneous solution was introduced and $z = 1 - 4i$ satisfies the original equality.

Step 3: Construct the equation from the requested roots The roots asked for are not α and β themselves. They are

$$u = \alpha + \beta = 1 - 4 = -3, \qquad v = \alpha\beta = -4.$$

Their sum is -7 and their product is 12, so

$$(x+3)(x+4) = x^2 + 7x + 12.$$

The first three offered equations have root sums $-2, -3, -1$ and products $-3, -4, -12$, respectively; none has the required pair of root invariants.

Step 4: Conclusion The required quadratic equation is $\boxed{x^2 + 7x + 12 = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: When a modulus equals a complex expression, force its imaginary part to vanish before squaring, and check that the remaining real expression is nonnegative.

17. If the points with position vectors $\alpha\hat{i}+10\hat{j}+13\hat{k}, 6\hat{i}+11\hat{j}+11\hat{k}, \frac{9}{2}\hat{i}+\beta\hat{j}-8\hat{k}$ are collinear, then $(19\alpha-6\beta)^2$ is equal to : [4 marks]

(A) 16

(B) 49

(C) 36

(D) 25

Correct Answer: (C) 36

Solution:

Concept:

- Three points A, B, C are collinear exactly when their displacement vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel.
- Parallel displacement vectors satisfy $\overrightarrow{BC} = t\overrightarrow{AB}$ with the same real multiplier t in every coordinate.

Step 1: Form the displacement vectors consistently Let $A = (\alpha, 10, 13)$, $B = (6, 11, 11)$, and $C = (9/2, \beta, -8)$. Subtract the coordinates in the indicated order:

$$\overrightarrow{AB} = (6-\alpha, 1, -2), \qquad \overrightarrow{BC} = (-3/2, \beta-11, -19).$$

Each subtraction uses the end point minus the starting point. The third component of $\overrightarrow{BC}$ is $-8-11=-19$, so the negative sign in the printed position vector is essential.

Step 2: Find the common multiplier from the known coordinates Collinearity means $\overrightarrow{BC} = t\overrightarrow{AB}$. The third coordinates determine the multiplier without either unknown:

$$-19 = t(-2) \quad \Rightarrow \quad t = \frac{19}{2}.$$

The first and second coordinates must have this same multiplier, giving

$$-\frac{3}{2} = \frac{19}{2}(6-\alpha), \qquad \beta - 11 = \frac{19}{2}.$$

It follows that $6-\alpha = -3/19$, so $\alpha = 117/19$, and $\beta = 41/2$.

Step 3: Evaluate the requested combination before squaring Keeping the fractions exact makes both products simplify immediately:

$$19\alpha - 6\beta = 117 - 6\left(\frac{41}{2}\right) = 117 - 123 = -6.$$

Consequently

$$(19\alpha - 6\beta)^2 = (-6)^2 = 36.$$

For a consistency check, the three displacement ratios are all $19/2$; a different sign in one displacement would not meet the collinearity condition. The alternatives 16, 49 and 25 are the squares of 4, 7 and 5, none of which equals the magnitude of the combination obtained here.

Step 4: Conclusion The required value is $\boxed{36}$. Hence, the correct option is (C).

Quick Tip to Solve: Use a fully known coordinate pair to obtain the parallel-vector multiplier first; then solve the other coordinates with that same signed multiplier.

18. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio $1:5:20$, then the coefficient of the fourth term is [4 marks]

(A) 3654

(B) 1827

(C) 5481

(D) 2436

Correct Answer: (A) 3654

Solution:

Concept:

- The coefficient of x^r in $(1+x)^n$ is $\binom{n}{r}$, with consecutive ratio $\binom{n}{r+1}/\binom{n}{r} = (n-r)/(r+1)$.
- The fourth term of the expansion is $\binom{n}{3}x^3$, because the first term corresponds to exponent zero.

Step 1: Convert the coefficient ratios into equations Let the three consecutive coefficients be $\binom{n}{r}, \binom{n}{r+1}, \binom{n}{r+2}$, proportional to $1, 5, 20$. The ratio of the second to the first is 5, whereas the ratio of the third to the second is $20/5 = 4$. Thus

$$\frac{n-r}{r+1} = 5, \qquad \frac{n-r-1}{r+2} = 4.$$

Using 20 as the second consecutive ratio would compare the wrong pair of terms.

Step 2: Determine the exponent of the expansion Rearrange the first equation to obtain $n-r = 5r+5$, or $n = 6r+5$. The second gives $n-r-1 = 4r+8$, or $n = 5r+9$. Equating the two expressions yields

$$6r+5 = 5r+9 \quad \Rightarrow \quad r = 4, \qquad n = 29.$$

This lies in the valid range for all three coefficients. As a check, $(29-4)/5 = 5$ and $(29-5)/6 = 4$, so the required coefficient ratio is recovered.

Step 3: Calculate the coefficient of the fourth term The fourth term means the term in x^3, not the fourth member of the three-term coefficient pattern. Its coefficient is therefore

$$\binom{29}{3} = \frac{29 \cdot 28 \cdot 27}{3 \cdot 2 \cdot 1} = 29 \cdot 14 \cdot 9 = 3654.$$

The value 1827 omits a factor of two in the arithmetic, while 5481 and 2436 also differ from this exact product. The index shift is important: the coefficient of x^4 would belong to the fifth term, not the requested fourth term.

Step 4: Conclusion The coefficient of the fourth term is $\boxed{3654}$. Hence, the correct option is (A).

Quick Tip to Solve: For three binomial coefficients in a given ratio, divide neighbouring entries first; remember that the fourth term uses lower index 3.

19. If the equation of the plane containing the line
$x + 2y + 3z - 4 = 0 = 2x + y - z + 5$ **and perpendicular to the plane**

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$$

is $ax + by + cz = 4$**, then** $(a - b + c)$ **is equal to :** [4 marks]

(A) 18

(B) 22

(C) 20

(D) 24

Correct Answer: (B) 22

Solution:

Concept:

- Every plane through the intersection of $F = 0$ and $G = 0$, apart from the separately checkable member $G = 0$, can be written $F + tG = 0$.
- The normal to a plane in parametric vector form is the cross product of its two nonparallel direction vectors. Two planes are perpendicular when their normals have zero scalar product.

Step 1: Find the normal of the given vector plane The direction vectors are $\mathbf{u} = (1, 1, 1)$ and $\mathbf{v} = (1, -2, 3)$. Their cross product is

$$\mathbf{n}_0 = \mathbf{u} \times \mathbf{v} = (5, -2, -3).$$

The position vector $(1, -1, 0)$ locates the plane but does not affect its orientation, so it is not used to determine perpendicularity.

Step 2: Impose perpendicularity on the family through the line Write the desired plane as

$$(x + 2y + 3z - 4) + t(2x + y - z + 5) = 0.$$

Its normal is $\mathbf{n} = (1 + 2t, 2 + t, 3 - t)$. Therefore

$$\begin{aligned} \mathbf{n} \cdot \mathbf{n}_0 &= 5(1 + 2t) - 2(2 + t) - 3(3 - t) \\ &= -8 + 11t = 0. \end{aligned}$$

This gives $t = 8/11$. The exceptional family member $2x + y - z + 5 = 0$ has normal $(2, 1, -1)$, whose dot product with $\mathbf{n}_0$ is 11; hence it is not another solution.

Step 3: Normalize the constant and read the coefficients Multiply the resulting plane by 11 to avoid fractional coefficients:

$$11(x + 2y + 3z - 4) + 8(2x + y - z + 5) = 0.$$

Collecting terms gives

$$27x + 30y + 25z - 4 = 0, \quad \text{or} \quad 27x + 30y + 25z = 4.$$

The constant now matches the prescribed form exactly, so $a = 27, b = 30, c = 25$. Thus $a - b + c = 27 - 30 + 25 = 22$. The other offered numbers 18, 20 and 24 do not agree with these normalized coefficients; an arbitrary rescaling before fixing the constant would change their values.

Step 4: Conclusion The required value is $\boxed{22}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the family through the intersection, impose a dot product of normals equal to zero, and only then normalize the constant to the specified right-hand side.

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20. Negation of $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$ is : [4 marks]

(A) $(\sim q) \wedge p$

(B) $q \wedge (\sim p)$

(C) $p \vee (\sim q)$

(D) $(\sim p) \vee q$

Correct Answer: (B) $q \wedge (\sim p)$

Solution:

Concept:

- An implication $A \Rightarrow B$ is false precisely when A is true and B is false, so its negation is $A \wedge \neg B$.
- The equivalent disjunction form is $p \Rightarrow q \equiv \neg p \vee q$; De Morgan's law changes the negation of a disjunction into a conjunction.

Step 1: Negate the outer implication first Treat the two inner implications as single statements: $A = (p \Rightarrow q)$ and $B = (q \Rightarrow p)$. The requested expression is

$$\neg(A \Rightarrow B) = A \wedge \neg B.$$

Substituting these statements back gives

$$(p \Rightarrow q) \wedge \neg(q \Rightarrow p).$$

The outer negation does not merely negate each of the letters separately; its false condition must be applied at the level of the entire implication.

Step 2: Simplify the negated inner implication Using the false condition for $q \Rightarrow p$,

$$\neg(q \Rightarrow p) = q \wedge \neg p.$$

Also $p \Rightarrow q = \neg p \vee q$. Therefore the full negation becomes

$$(\neg p \vee q) \wedge q \wedge \neg p.$$

Whenever $q \wedge \neg p$ holds, both q and $\neg p$ are already true, so the first factor is automatically true. The expression reduces to $q \wedge \neg p$.

Step 3: Check the truth condition and the other choices The reduced expression is true only for p

false and q true. In that case $p \Rightarrow q$ is true but $q \Rightarrow p$ is false, so the original outer implication is false as required. Option (A) instead selects p true and q false, for which the antecedent of the outer implication is false and the outer implication is true. Options (C) and (D) are disjunctions and are true on additional truth assignments, so neither is the required negation.

Step 4: Conclusion The negation is $\boxed{q \wedge (\sim p)}$. Hence, the correct option is (B).

Quick Tip to Solve: For a nested implication, negate the outermost implication first and keep each inner implication grouped until the false-condition rule has been applied.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $[t]$ denote the greatest integer $\leq t$. Then $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6}(8[\operatorname{cosec} x] - 5[\cot x])\,dx$ is equal to _____.

[4 marks]

Solution:

Concept:

- The greatest integer function $[t]$ is the unique integer k such that $k \leq t < k+1$. For negative nonintegers, it rounds downward rather than toward zero.
- An integral of a step function is the sum of its constant values multiplied by the lengths of their intervals. Changing finitely many endpoint values does not affect a definite integral.

Step 1: Determine the cosecant contribution On $[\pi/6, 5\pi/6]$, sine is positive and lies between $1/2$ and 1. Consequently $1 \leq \operatorname{cosec} x \leq 2$. Except at the two outer endpoints, $[\operatorname{cosec} x] = 1$. The isolated endpoint values of 2 contribute no area. Thus

$$\int_{\pi/6}^{5\pi/6} [\operatorname{cosec} x]\,dx = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}.$$

Step 2: Split at the integer crossings of cotangent Cotangent decreases continuously from $\sqrt{3}$ to $-\sqrt{3}$ on the interval. It crosses 1, 0 and -1 at $\pi/4, \pi/2, 3\pi/4$, respectively. Apart from endpoints of the subintervals, its floor values are

x interval	$[\cot x]$	length
$(\pi/6, \pi/4)$	1	$\pi/12$
$(\pi/4, \pi/2)$	0	$\pi/4$
$(\pi/2, 3\pi/4)$	-1	$\pi/4$
$(3\pi/4, 5\pi/6)$	-2	$\pi/12$

In particular, a negative cotangent between -1 and 0 has floor -1, not 0.

Step 3: Combine the areas and include the outside factor The cotangent-floor integral is

$$\int_{\pi/6}^{5\pi/6} [\cot x]\, dx = \frac{\pi}{12} - \frac{\pi}{4} - 2\frac{\pi}{12} = -\frac{\pi}{3}.$$

By linearity, the requested value is

$$\frac{2}{\pi}\left(8\frac{2\pi}{3} - 5\left(-\frac{\pi}{3}\right)\right) = \frac{2}{\pi}\left(\frac{21\pi}{3}\right) = 14.$$

The negative floor contribution is multiplied by -5, so its net contribution to the total is positive.

Step 4: Conclusion The required numerical value is $\boxed{14}$.

Quick Tip to Solve: For floor integrals, locate the integer crossings and multiply each constant floor value by its interval length; do not cancel floors of opposite signed arguments as odd functions.

22. Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R = \{(x, y) \in A \times A : x - y$ is odd positive integer or $x - y = 2\}$. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to _____. [4 marks]

Solution:

Concept:

- A relation is symmetric when every pair (x, y) in it has its reverse (y, x) in it. Its smallest symmetric extension is $R \cup R^{-1}$.
- An integer difference is odd precisely when its endpoints have opposite parity. The condition that the difference is positive selects one orientation of each such unordered pair.

Step 1: Separate the elements by parity The even elements are $E = \{0, 4, 6, 8, 10\}$, and the odd elements are $O = \{3, 7, 9\}$. There are five even and three odd elements. Each choice of one element from each set gives one unordered pair with odd difference; these choices are all distinct.

Step 2: Count the odd positive differences For each of these unordered pairs, exactly one ordering has positive difference. Thus this part of the relation has

$$5 \times 3 = 15$$

ordered pairs. The reverse of each has negative odd difference, so none of those reverses belongs to the given relation.

Step 3: Count the pairs with difference two The pairs in $A \times A$ with $x - y = 2$ are

$$(6, 4), \quad (8, 6), \quad (10, 8), \quad (9, 7).$$

There are four. They do not overlap the odd-difference pairs because two is even. Their reverses have difference -2, so these reverses are also absent.

Step 4: Complete the symmetry minimally All original pairs have strictly positive difference. Therefore

no original pair already has its reverse in R. Add one reverse for each of the $15+4$ original pairs. This is necessary for symmetry and sufficient: each newly added pair is paired with its original reverse. No diagonal pairs need be added.

Step 5: Conclusion The minimum number of ordered pairs to add is

$$\boxed{19}.$$

Quick Tip to Solve: When every allowed difference is positive, count the original pairs: each needs exactly one new reverse, with no duplicates.

23. If the solution curve of the differential equation $(y-2\log_e x)\,dx+\left(x\log_e x^2\right)dy=0, x>1$ passes through the points $\left(e,\frac{4}{3}\right)$ and $\left(e^4,\alpha\right)$, then α is equal to _____. [4 marks]

Solution:

Concept:

- For $y\prime+P(x)y=Q(x)$, multiplication by the integrating factor $e^{\int P(x)\,dx}$ converts the left side to a product derivative.
- Here $x>1$, so $t=\ln x>0$, $dt=dx/x$, and $\ln x^2=2\ln x=2t$. The positive domain makes $\sqrt{t}$ well defined.

Step 1: Put the differential equation in linear form Dividing the equation by dx and rearranging gives

$$2x\ln x\,\frac{dy}{dx}+y=2\ln x.$$

Since $dy/dx=(1/x)dy/dt$, this becomes

$$\frac{dy}{dt}+\frac{y}{2t}=1.$$

No division by zero occurs because $t>0$.

Step 2: Apply the integrating factor The integrating factor is

$$e^{\int(1/(2t))\,dt}=\sqrt{t}.$$

Hence

$$\frac{d}{dt}(y\sqrt{t})=\sqrt{t}.$$

Integration gives

$$y\sqrt{t}=\frac{2}{3}t^{3/2}+C,\qquad y=\frac{2}{3}t+\frac{C}{\sqrt{t}}.$$

The constant must be retained until the given point is used.

Step 3: Use the point at x equals e At $x = e$, we have $t = 1$ and $y = 4/3$. Therefore

$$\frac{4}{3} = \frac{2}{3} + C \quad \Longrightarrow \quad C = \frac{2}{3}.$$

The required solution curve is consequently

$$y = \frac{2}{3}\ln x + \frac{2}{3\sqrt{\ln x}}.$$

Step 4: Evaluate the second ordinate At $x = e^4$, $t = 4$ and $\sqrt{t} = 2$. Substituting these two values separately avoids confusing $\ln x^2$ with $(\ln x)^2$:

$$\alpha = \frac{2}{3}(4) + \frac{2}{3(2)} = \frac{8}{3} + \frac{1}{3} = 3.$$

The same expression also recovers the first given ordinate.

Step 5: Conclusion The required value is

$$\boxed{\alpha = 3}.$$

Quick Tip to Solve: When both x and ln x occur in a linear equation, try t=ln x; the factor dx/x often removes x completely.

24. Let the mean and variance of 8 numbers $x, y, 10, 12, 6, 12, 4, 8$ be 9 and 9.25 respectively. If $x > y$, then $3x - 2y$ is equal to _____. [4 marks]

Solution:

Concept:

- For N observations, $\bar{x} = (\sum x_i)/N$ and the population variance is $\sigma^2 = (\sum x_i^2)/N - \bar{x}^2$.
- The identities $(x+y)^2 = x^2 + y^2 + 2xy$ and $(x-y)^2 = x^2 + y^2 - 2xy$ recover two unknown observations from their sum and sum of squares.

Step 1: Use the mean to obtain the sum The six specified observations sum to

$$10 + 12 + 6 + 12 + 4 + 8 = 52.$$

Since all eight observations have mean nine, their total is $8 \times 9 = 72$. Thus

$$x + y = 72 - 52 = 20.$$

Both occurrences of 12 are observations and must be included.

Step 2: Use the variance to obtain the square sum The total of the squares of all eight observations is

$$8(9.25 + 9^2) = 8(90.25) = 722.$$

The known square sum is

$$10^2 + 12^2 + 6^2 + 12^2 + 4^2 + 8^2 = 504.$$

Subtraction therefore gives

$$x^2 + y^2 = 722 - 504 = 218.$$

We use division by eight in the variance formula, as appropriate for the variance of the stated data.

Step 3: Recover the two observations From the sum and square sum,

$$2xy = (x + y)^2 - (x^2 + y^2) = 400 - 218 = 182,$$

so $xy = 91$. The two observations are the roots of

$$u^2 - 20u + 91 = (u - 13)(u - 7) = 0.$$

The condition $x > y$ selects $x = 13$, $y = 7$, rather than the opposite assignment.

Step 4: Calculate the requested combination Substitute the ordered values:

$$3x - 2y = 3(13) - 2(7) = 39 - 14 = 25.$$

As a check, these values have sum 20 and square sum $169 + 49 = 218$; together with the six known entries, they reproduce both the given mean and variance.

Step 5: Conclusion The requested numerical answer is

$$\boxed{25}.$$

Quick Tip to Solve: First convert mean and variance into sum and sum of squares; use the ordering condition only after finding the two roots.

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25. Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}, \vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$. If $\vec{a} \cdot \vec{c} = -12, \vec{c} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k})$ is equal to _____. [4 marks]

Solution:

Concept:

- If $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$, then $\vec{a} \times (\vec{c} - \vec{b}) = \vec{0}$. Since $\vec{a} \neq \vec{0}$, write $\vec{c} = \vec{b} + t\vec{a}$.
- Dotting this expression with a vector perpendicular to $\vec{a}$ removes the unknown multiple t. The remaining dot-product condition then determines t.

Step 1: Express c using the cross-product condition Linearity of the cross product gives

$$\vec{c} = (\alpha + 6t)\hat{i} + (11 + 9t)\hat{j} + (-2 + 12t)\hat{k}.$$

This describes all vectors with the required cross product, since adding a vector parallel to $\vec{a}$ does not affect that product.

Step 2: Determine alpha from the second dot product Let $\vec{d} = \hat{i} - 2\hat{j} + \hat{k}$. Notice that

$$\vec{a} \cdot \vec{d} = 6 - 18 + 12 = 0.$$

Thus $\vec{c} \cdot \vec{d} = \vec{b} \cdot \vec{d}$, independently of t. The given value five implies

$$\alpha - 22 - 2 = 5 \quad \Longrightarrow \quad \alpha = 29.$$

Step 3: Determine the parallel component Now $\vec{b} = (29, 11, -2)$, so

$$\vec{a} \cdot \vec{b} = 6(29) + 9(11) + 12(-2) = 249,$$

and

$$\vec{a} \cdot \vec{a} = 36 + 81 + 144 = 261.$$

The condition $\vec{a} \cdot \vec{c} = -12$ gives

$$249 + 261t = -12, \qquad t = -1.$$

Therefore $\vec{c} = \vec{b} - \vec{a} = (23, 2, -14)$.

Step 4: Evaluate and check the requested projection The dot product with $(1, 1, 1)$ is the sum of the components:

$$\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 23 + 2 - 14 = 11.$$

To check the constraints, $6(23) + 9(2) + 12(-14) = -12$, and $23 - 2(2) - 14 = 5$. Also $\vec{c} - \vec{b} = -\vec{a}$, confirming the original cross-product equality.

Step 5: Conclusion The required dot product is

$$\boxed{11}.$$

Quick Tip to Solve: Replace equal cross products by c=b+t a. A dot product perpendicular to a eliminates t before any simultaneous equations are needed.

26. If a_α is the greatest term in the sequence $\alpha_n = \frac{n^3}{n^4+147}, n = 1, 2, 3, \ldots$, then α is equal to _____. [4 marks]

Solution:

Concept:

- A sequence sampled from a differentiable function can be maximised by locating the continuous maximum and comparing the neighbouring integer indices.
- For $f(x) = x^3/(x^4 + 147)$, with $x > 0$, the denominator is positive. Thus the sign of $f\prime(x)$ is determined by its numerator.

Step 1: Differentiate the continuous extension Consider

$$f(x) = \frac{x^3}{x^4 + 147}, \qquad x > 0.$$

Applying the quotient rule gives

$$f\prime(x) = \frac{3x^2(x^4 + 147) - 4x^6}{(x^4 + 147)^2} = \frac{x^2(441 - x^4)}{(x^4 + 147)^2}.$$

This expression permits a sign test without solving a complicated rational inequality between successive terms.

Step 2: Locate where monotonicity changes The positive critical point satisfies

$$x^4 = 441, \qquad x = \sqrt{21}.$$

Since $4 < \sqrt{21} < 5$, the function increases up to a point between four and five and decreases afterwards. Every sequence term with index at most four is no larger than the fourth term, and every term with index at least five is no larger than the fifth term.

Step 3: Compare the only two candidates The neighbouring terms are

$$a_4 = \frac{64}{403}, \qquad a_5 = \frac{125}{772}.$$

Because both denominators are positive, exact cross multiplication is valid:

$$125(403) = 50375, \qquad 64(772) = 49408.$$

The first product is larger, so $a_5 > a_4$. No decimal approximation or rounding rule for the continuous maximiser is needed.

Step 4: Identify the index rather than the term value The monotonicity argument excludes all other positive integer indices, and the strict comparison excludes a tie. Thus the greatest sequence term is $a_5 = 125/772$. The question asks for α, the subscript of that term, rather than the value of the fraction itself.

Step 5: Conclusion The greatest term occurs at index

$$\boxed{\alpha = 5}.$$

Quick Tip to Solve: For a discrete maximum, compare the two integers surrounding the continuous maximiser; merely rounding that maximiser is not a proof.

27. Let $[t]$ denote the greatest integer $\leq t$. If the constant term in the expansion of $\left(3x^2 - \frac{1}{2x^5}\right)^7$ is α, then $[\alpha]$ is equal to _____. [4 marks]

Solution:

Concept:

- The term obtained by choosing the second summand r times in $(u+v)^7$ is $\binom{7}{r}u^{7-r}v^r$, where r is an integer from zero to seven.
- A constant term has total power of x equal to zero. The greatest-integer operation is applied to its coefficient after the coefficient has been evaluated exactly.

Step 1: Write the general binomial term Take $u = 3x^2$ and $v = -1/(2x^5)$. Choosing the second summand r times gives

$$T_{r+1} = \binom{7}{r}(3x^2)^{7-r}\left(-\frac{1}{2x^5}\right)^r.$$

Combining the numerical factors and the powers of x,

$$T_{r+1} = \binom{7}{r}3^{7-r}\frac{(-1)^r}{2^r}x^{14-7r}.$$

Step 2: Set the power of x equal to zero The exponent of x must vanish for the term to be independent of x:

$$14 - 7r = 0 \quad \Longrightarrow \quad r = 2.$$

This is an allowed binomial index, so the constant term exists. The exponent decreases by seven as r increases, so this is its only occurrence.

Step 3: Evaluate the constant coefficient Substituting $r = 2$ into the coefficient gives

$$\alpha = \binom{7}{2}3^5\frac{(-1)^2}{2^2} = \frac{21 \cdot 243}{4} = \frac{5103}{4}.$$

The sign is positive because the second summand has been chosen an even number of times. The power of three is five, since five of the seven factors contribute the first summand.

Step 4: Apply the greatest-integer definition The exact coefficient is

$$\frac{5103}{4} = 1275 + \frac{3}{4} = 1275.75.$$

Therefore $1275 \leq \alpha < 1276$. The greatest integer no larger than α is consequently 1275. This step is a floor operation, so rounding the decimal to the nearest integer would give the wrong result.

Step 5: Conclusion The required value is

$$\boxed{[\alpha] = 1275}.$$

Quick Tip to Solve: First solve the exponent equation for the constant term, then evaluate its coefficient and apply the floor operation at the very end.

28. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2. Then $\alpha + r$ is equal to _____.

[4 marks]

Solution:

Concept:

- Reflection in a line maps a circle to a circle with the reflected centre and the same radius. Completing squares identifies the original centre and radius.
- The reflection of (h, k) in $ax + by + c = 0$ is $(h - 2aD/(a^2 + b^2),\ k - 2bD/(a^2 + b^2))$, where $D = ah + bk + c$.

Step 1: Identify the first circle Complete squares in the equation of C_1:

$$x^2 - 4x + y^2 - 2y = \alpha - 5 \quad \Longrightarrow \quad (x - 2)^2 + (y - 1)^2 = \alpha.$$

Its centre is $(2, 1)$ and its squared radius is α. Reflection preserves the radius, so $r^2 = \alpha$.

Step 2: Reflect the centre in the given line Write the mirror as $2x - y + 1 = 0$. For $(h, k) = (2, 1)$,

$$D = 2(2) - 1 + 1 = 4, \qquad a^2 + b^2 = 2^2 + (-1)^2 = 5.$$

The reflected coordinates are therefore

$$f = 2 - \frac{2(2)(4)}{5} = -\frac{6}{5}, \qquad g = 1 - \frac{2(-1)(4)}{5} = \frac{13}{5}.$$

The midpoint of these two centres lies on the mirror, as reflection requires.

Step 3: Find the radius of the reflected circle Divide the equation of C_2 by five:

$$x^2 + y^2 - 2fx - 2gy + \frac{36}{5} = 0.$$

Its centre is (f, g), and completing squares gives

$$r^2 = f^2 + g^2 - \frac{36}{5} = \frac{36 + 169}{25} - \frac{180}{25} = 1.$$

Since a radius is nonnegative, $r = 1$.

Step 4: Use preservation of radius The radius of C_1 is $\sqrt{\alpha}$, and the reflected circle has radius one. Consequently $\alpha = r^2 = 1$, consistent with a real nondegenerate circle. The requested quantity is

$$\alpha + r = 1 + 1 = 2.$$

Notice that the constant term of the second equation must be divided by five before using the radius formula.

Step 5: Conclusion The required numerical value is

$$\boxed{\alpha + r = 2}.$$

Quick Tip to Solve: Reflect only the centre and keep the radius unchanged; normalise the circle equation before reading off its centre and squared radius.

29. The largest natural number n such that 3^n divides $66!$ is _____. [4 marks]

Solution:

Concept:

- The largest power of a prime p dividing $N!$ is its prime exponent $v_p(N!)$. Each multiple of p contributes one factor, and multiples of higher powers contribute additional factors.
- Legendre's counting formula is $v_p(N!) = \lfloor N/p \rfloor + \lfloor N/p^2 \rfloor + \cdots$, terminating when the next power exceeds N.

Step 1: Count the first factor of three The factorial contains every integer from 1 through 66. The multiples of 3 are $3, 6, 9, \ldots, 66$, so there are $\lfloor 66/3 \rfloor = 22$ of them. This counts one factor of 3 from each such integer, including the integers that contain more than one factor.

Step 2: Count the additional factors Multiples of $9 = 3^2$ supply a second factor each, giving $\lfloor 66/9 \rfloor = 7$. Multiples of $27 = 3^3$ supply a third factor each, giving $\lfloor 66/27 \rfloor = 2$. No integer in the factorial is divisible by $81 = 3^4$, because $81 > 66$. Thus no further contribution remains.

Step 3: Add the contributions and check maximality Adding the layers counts every factor of 3 exactly once:

$$v_3(66!) = 22 + 7 + 2 = 31.$$

After removing these 31 factors, every remaining factor is coprime to 3. Consequently 3^{31} divides the factorial, whereas 3^{32} cannot divide it; the requested exponent is therefore the largest one, not merely an exponent that works.

Step 4: Conclusion The largest natural number is $\boxed{n = 31}$.

Quick Tip to Solve: Count multiples of 3, then 9, then 27 separately; counting only multiples of 3 misses the extra factors in powers of 3.

30. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and $(-2, 0, 1)$ are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is _____. [4 marks]

Solution:

Concept:

- The perpendicular distance from (x_0, y_0, z_0) to $ax + by + cz + d = 0$ is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2 + b^2 + c^2}$. Equal distances allow either sign of the numerator.
- For a line through A with direction vector $\mathbf{d}$, the squared distance from P is $\|P - A\|^2 - [(P - A) \cdot \mathbf{d}]^2/\|\mathbf{d}\|^2$. This subtracts the squared parallel component.

Step 1: Find both plane-distance parameters The common denominator for the plane distances is $\sqrt{2^2 + 3^2 + (-6)^2} = 7$. The plane expression at $(-2, 0, 1)$ is $-4 - 6 + 7 = -3$. At $(5/2, 1, \lambda)$ it is

$5+3-6\lambda+7=15-6\lambda$. Hence

$$|15-6\lambda|=3, \qquad \lambda=2 \text{ or } 3.$$

The stated ordering gives $\lambda_1=3$ and $\lambda_2=2$.

Step 2: Read the target point and the line The required point is $P=(3-2,2,3)=(1,2,3)$. Parametrising the given line gives $(x,y,z)=(5,1,-7)+t(1,2,2)$. Thus choose $A=(5,1,-7)$, $\mathbf{d}=(1,2,2)$, and $\mathbf{w}=P-A=(-4,1,10)$. The minus sign in the line base point follows from its numerator $z+7$.

Step 3: Remove the component along the line The necessary scalar quantities are

$$\|\mathbf{w}\|^2=16+1+100=117, \quad \mathbf{w}\cdot\mathbf{d}=-4+2+20=18, \quad \|\mathbf{d}\|^2=9.$$

Therefore the perpendicular distance satisfies

$$D^2=117-\frac{18^2}{9}=117-36=81, \qquad D=9.$$

The positive square root is required because a geometric distance is nonnegative.

Step 4: Conclusion The distance from the specified point to the line is $\boxed{9}$ units.

Quick Tip to Solve: Keep both signs when equal plane distances introduce an absolute value; then use the direction ratios to remove the parallel projection.

Physics – Section A: Multiple Choice Questions (80 marks)

31. An air bubble of volume $1\ \text{cm}^3$ rises from the bottom of a lake $40\ \text{m}$ deep to the surface at a temperature of 12°C. The atmospheric pressure is $1\times10^5\ \text{Pa}$ the density of water is $1000\ \text{kg/m}^3$ and $\text{g}=10\ \text{m/s}^2$. There is no difference of the temperature of water at the depth of $40\ \text{m}$ and on the surface. The volume of air bubble when it reaches the surface will be: [4 marks]

(A) $4\ \text{cm}^3$

(B) $3\ \text{cm}^3$

(C) $2\ \text{cm}^3$

(D) $5\ \text{cm}^3$

Correct Answer: (D) $5\ \text{cm}^3$

Solution:

Concept:

- At depth h in water, the absolute pressure is $P=P_{\text{atm}}+\rho gh$. The atmospheric contribution must be included when applying the gas law.
- For a fixed amount of gas at constant temperature, Boyle's law gives $PV=$ constant, so $P_bV_b=P_sV_s$. Subscripts b and s denote the bottom and surface.

Step 1: Calculate the absolute bottom pressure The hydrostatic pressure due to the water column is

$$\rho g h = (1000)(10)(40) = 4 \times 10^5 \text{ Pa}.$$

Adding the atmospheric pressure gives

$$P_b = 10^5 + 4 \times 10^5 = 5 \times 10^5 \text{ Pa}.$$

At the surface the water depth is zero, so $P_s = 10^5$ Pa. The bottom pressure is five times the surface pressure.

Step 2: Apply the constant-temperature gas law The water has the same temperature at both positions, so the gas temperature cancels in $P_b V_b / T_b = P_s V_s / T_s$. The specified value 12°C need not be converted or substituted because only its equality matters. Consequently

$$V_s = \frac{P_b}{P_s} V_b = \frac{5 \times 10^5}{10^5} (1 \text{ cm}^3) = 5 \text{ cm}^3.$$

Pressure falls during the rise and the bubble expands, as this result requires.

Step 3: Compare the options The volume ratio is five, so the alternatives 2, 3, and 4 cm^3 do not satisfy the gas law with the calculated absolute pressures. In particular, 4 cm^3 would result from incorrectly using the bottom gauge pressure instead of its absolute pressure.

Step 4: Conclusion The surface volume is $\boxed{5 \text{ cm}^3}$. Hence, the correct option is (D).

Quick Tip to Solve: For bubbles rising in water, use absolute pressure at both depths: add atmospheric pressure to $\rho g h$ before taking the pressure ratio.

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32. In a reflecting telescope, a secondary mirror is used to: [4 marks]

(A) make chromatic aberration zero

(B) remove spherical aberration

(C) reduce the problem of mechanical support

(D) move the eyepiece outside the telescopic tube

Correct Answer: (D) move the eyepiece outside the telescopic tube

Solution:

Concept:

- The primary concave mirror of a reflecting telescope gathers nearly parallel rays from a distant object and directs them toward a real focus.
- A secondary mirror redirects the converging beam to an accessible observing position. The eyepiece then magnifies the image formed by the objective system.

Step 1: Identify why the primary focus needs redirection If the converging rays were used directly at the primary mirror's focus inside the telescope tube, the eyepiece and observer would need to occupy an inconvenient position in the incoming light path. The secondary mirror intercepts this beam before

it reaches the inaccessible focus and folds the optical path to a convenient location.

Step 2: State the role of the secondary mirror In a standard reflecting-telescope arrangement, the redirected beam reaches an eyepiece outside the telescopic tube, permitting observation without placing the observer in front of the primary mirror. For example, an inclined secondary can send the light to the side of the tube. The relevant function is therefore beam redirection and access to the image, rather than an increase in the primary mirror's light-collecting area.

Step 3: Eliminate the other stated purposes Chromatic aberration is absent from mirror focusing because reflection does not disperse colours as a refracting objective does; this does not require the secondary mirror. Spherical aberration is controlled by the optical shape and design of the reflecting surfaces, so its removal is not the general purpose stated here. Easier mechanical support concerns how a large mirror can be supported from behind, rather than the role of the secondary mirror.

Step 4: Conclusion The secondary mirror is used to move the eyepiece outside the telescopic tube. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish optical-path folding by the secondary mirror from the primary mirror's advantages of large aperture and freedom from chromatic aberration.

33. Certain galvanometers have a fixed core made of non magnetic metallic material. The function of this metallic material is [4 marks]

(A) to bring the coil to rest quickly

(B) to produce large deflecting torque on the coil

(C) to oscillate the coil in magnetic field for longer period of time

(D) to make the magnetic field radial

Correct Answer: (A) to bring the coil to rest quickly

Solution:

Concept:

- A conducting metallic core can support eddy currents when the magnetic flux associated with motion changes. A material need not be ferromagnetic to conduct these induced currents.
- By Lenz's law, the magnetic effect of induced currents opposes the change that produces them. The resulting damping converts mechanical energy of motion into thermal energy.

Step 1: Identify the effect of the conducting core When the galvanometer coil moves, the changing magnetic interaction with the conducting nonmagnetic metallic core induces eddy currents. These currents interact with the magnetic field and oppose the relative motion. The important property here is electrical conductivity, not strong magnetisation of the core.

Step 2: Connect the opposing torque to settling The damping torque is opposite to the angular velocity and may be represented as $\tau_d = -b\omega$, with $b > 0$. Its mechanical power is $\tau_d\omega = -b\omega^2 \leq 0$, so it removes energy from oscillation. The coil consequently reaches its steady deflection rapidly instead of swinging repeatedly about the equilibrium reading. At rest, the motion-induced damping torque vanishes and does not replace the ordinary deflecting torque.

Step 3: Rule out the other roles Producing a large deflecting torque is the role of the current-carrying coil in the applied magnetic field, rather than the nonmagnetic damping core. Prolonging oscillations is the opposite of damping. A radial magnetic field is associated with shaped magnetic pole pieces and a soft iron core; it is not the purpose of the nonmagnetic metallic material specified here.

Step 4: Conclusion The metallic material provides damping to bring the coil to rest quickly. Hence, the correct option is (A).

Quick Tip to Solve: When a galvanometer specifies a nonmagnetic metallic core, associate its conductivity with eddy-current damping, not with magnetic-field strengthening.

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34. Given below are two statements:

Statement I: If E be the total energy of a satellite moving around the earth, then its potential energy will be $\frac{E}{2}$.

Statement II: The kinetic energy of a satellite revolving in an orbit is equal to the half the magnitude of total energy E.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Statement I is incorrect but Statement II is correct

(C) Statement I is correct but Statement II is incorrect

(D) Both Statement I and Statement II are correct

Correct Answer: (A) Both Statement I and Statement II are incorrect

Solution:

Concept:

- For a satellite of mass m in a circular orbit of radius r about Earth of mass M, gravitational potential energy with zero at infinity is $U = -GMm/r$.
- Gravity supplies centripetal force, so $mv^2/r = GMm/r^2$. Consequently $K = GMm/(2r)$ and total orbital energy is $E = K + U = -GMm/(2r)$.

Step 1: Derive the relation for potential energy From the expressions for U and E,

$$U = -\frac{GMm}{r} = 2\left(-\frac{GMm}{2r}\right) = 2E.$$

Both potential and total energies are negative for the bound circular orbit. Statement I claims $U = E/2$, which reverses the actual factor; it is therefore incorrect.

Step 2: Compare kinetic energy with the magnitude of total energy The kinetic energy is positive and obeys

$$K = \frac{GMm}{2r} = -E = |E|.$$

It is equal to the full magnitude of the total energy, rather than half that magnitude. Statement II is therefore also incorrect. The familiar factor of one half relates kinetic energy to the magnitude of

potential energy: $K = |U|/2$, not to $|E|/2$.

Step 3: Select the combined verdict Since both assertions fail, any option claiming that either statement is correct is ruled out. The energy relations can also be checked by substituting $U = -2K$ into $E = K + U$, which gives $E = -K$ and hence $U = 2E$. For an elliptical orbit the corresponding standard equalities hold for orbital averages; the statements are not general instantaneous identities either.

Step 4: Conclusion Both Statement I and Statement II are incorrect. Hence, the correct option is (A).

Quick Tip to Solve: Remember the signed circular-orbit ratio $K : U : E = 1 : -2 : -1$; use $|E|$ only when a positive magnitude is requested.

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35. Dimension of $\frac{1}{\mu_0 \in_0}$ should be equal to [4 marks]

(A) T^2/L^2

(B) T/L

(C) L^2/T^2

(D) L/T

Correct Answer: (C) L^2/T^2

Solution:

Concept:

- The speed of electromagnetic waves in vacuum is $c = 1/\sqrt{\mu_0 \varepsilon_0}$, where μ_0 and ε_0 are vacuum permeability and permittivity.
- Dimensions follow multiplication and powers in the same way as algebraic quantities. Speed has dimensions $[c] = L\,T^{-1}$; squaring speed squares both dimensional powers.

Step 1: Relate the expression to wave speed Squaring the vacuum-wave-speed relation directly gives

$$\frac{1}{\mu_0 \varepsilon_0} = c^2.$$

Thus the requested product of electromagnetic constants can be recognised without calculating their separate SI units. The expression is a squared speed, rather than a speed or an inverse speed.

Step 2: Write the dimensional formula Using $[c] = LT^{-1}$,

$$\left[\frac{1}{\mu_0 \varepsilon_0}\right] = [c]^2 = L^2T^{-2} = \frac{L^2}{T^2}.$$

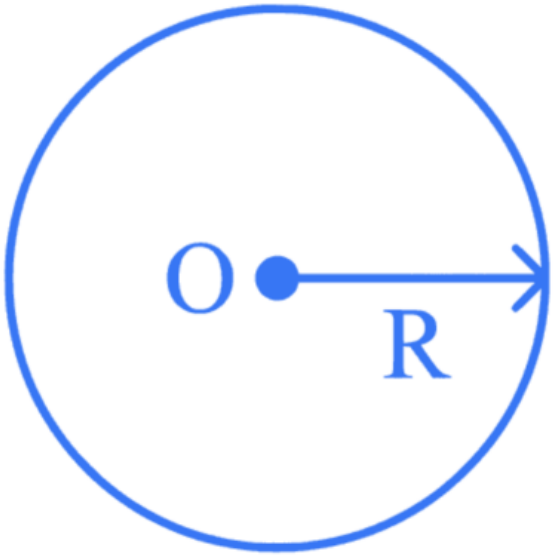
No mass or electric-current dimension remains. As an independent check, $[\mu_0] = MLT^{-2}I^{-2}$ and $[\varepsilon_0] = M^{-1}L^{-3}T^4I^2$; their product is $L^{-2}T^2$, whose reciprocal agrees.

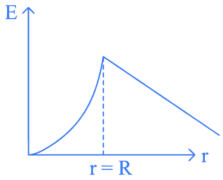
Step 3: Compare with all four expressions Option (A) is the dimension of inverse speed squared, while option (B) is that of inverse speed. Option (D) has the dimension of speed itself and misses the square. Only option (C) contains both the length power 2 and time power -2 required by the electromagnetic-wave relation.

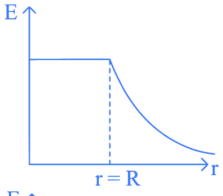
Step 4: Conclusion The dimensions are $\boxed{\mathrm{L}^2/\mathrm{T}^2}$. Hence, the correct option is (C).

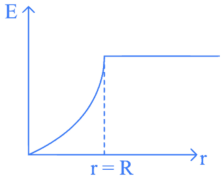
Quick Tip to Solve: Recognise $\mu_0\varepsilon_0 = 1/c^2$ before expanding SI units; then check whether the question asks for the product, reciprocal, or square root.

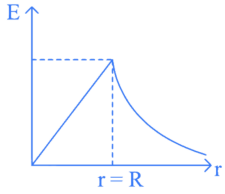
36. Graphical variation of electric field due to a uniformly charged insulating solid sphere of radius R, with distance r from the centre O is represented by: [4 marks]

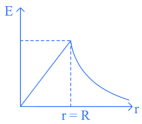


(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- Gauss's law states that the electric flux through a closed surface is $Q_{\text{enc}}/\varepsilon_0$, where Q_{enc} is its enclosed charge.
- For a uniformly charged insulating sphere, charge occupies the entire volume. Spherical symmetry makes the field radial and equal in magnitude at all points of a concentric Gaussian sphere.

Step 1: Find the field inside the sphere Let the total charge be Q, the sphere radius be R, and choose a Gaussian surface of radius $r < R$. Uniform volume density gives

$$Q_{\text{enc}} = Q\frac{r^3}{R^3}.$$

Hence,

$$E(4\pi r^2) = \frac{Qr^3}{\varepsilon_0 R^3} \quad \Rightarrow \quad E = \frac{Qr}{4\pi\varepsilon_0 R^3}.$$

The field is zero at the centre and increases linearly with distance, so the inner graph must be a straight line through the origin.

Step 2: Find the field outside and match the graph For $r \geq R$, all the charge is enclosed and

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

Both expressions give $Q/(4\pi\varepsilon_0 R^2)$ at the surface. The graph is continuous, reaches its maximum there, and then decreases as an inverse square. Option A has the wrong curved inner dependence and straight outer decrease; B gives a nonzero constant inner field; C remains constant outside. Only D has both correct portions.

Step 3: Conclusion The field increases linearly for $0 \leq r \leq R$, then decreases as $1/r^2$. Hence, the correct option is (D).

Quick Tip to Solve: For a uniform solid sphere, check three features: zero at the centre, a straight rise to the surface, and an inverse-square fall outside.

37. Given below are two statements:

Statement I: If heat is added to a system, its temperature must increase.

Statement II: If positive work is done by a system in a thermodynamic process, its volume must increase.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is false but Statement II is true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false

Correct Answer: (B) Statement I is false but Statement II is true

Solution:

Concept:

- The first law is $Q = \Delta U + W$, where Q is heat supplied and W is work done by the system. Heat transfer and temperature rise are different physical quantities.
- For the usual gas expansion or compression process, pressure-volume work is $\mathrm{d}W = P\,\mathrm{d}V$. Work done by the gas is positive during expansion and negative during compression.

Step 1: Test Statement I with an isothermal process Heat added to a system need not appear as an increase in temperature. For an ideal gas undergoing an isothermal expansion, temperature and therefore internal energy remain constant. Thus $\Delta U = 0$, and the first law gives

$$Q = W > 0, \qquad \Delta T = 0.$$

The supplied heat is converted into work rather than raising temperature. Another familiar example is heat absorbed during melting or boiling at constant pressure: it supplies latent heat while temperature remains constant. Therefore the word "must" makes Statement I false.

Step 2: Apply the sign convention to Statement II In the pressure-volume work convention used here, an expanding gas pushes its surroundings. Since pressure is positive, $\mathrm{d}W = P\,\mathrm{d}V > 0$ corresponds to $\mathrm{d}V > 0$. Thus positive expansion work is associated with increasing volume, so Statement II is true in the intended expansion/compression setting. Compression instead gives work done by the gas less than zero. The choice with I false and II true is B; A and C incorrectly accept I, while D incorrectly rejects II.

Step 3: Conclusion Statement I is false, but Statement II is true for the stated gas-work convention. Hence, the correct option is (B).

Quick Tip to Solve: Before equating added heat with a temperature rise, check whether the energy can instead become expansion work or latent heat.

38. A charge particle moving in magnetic field B, has the components of velocity along B as well as perpendicular to B. The path of the charge particle will be [4 marks]

(A) helical path with the axis along magnetic field B

(B) straight along the direction of magnetic field B

(C) circular path

(D) helical path with the axis perpendicular to the direction of magnetic field B

Correct Answer: (A) helical path with the axis along magnetic field B

Solution:

Concept:

- The magnetic Lorentz force is $\vec{F} = q\vec{v} \times \vec{B}$. Its magnitude is $|q|v_\perp B$, and it is perpendicular to the instantaneous velocity.
- Resolve the velocity into $v_\parallel$ along the field and $v_\perp$ perpendicular to it. The magnetic force acts only on the perpendicular component and does no work.

Step 1: Analyse the two velocity components The component along the magnetic field experiences no magnetic force, because its cross product with $\vec{B}$ is zero. Therefore $v_\parallel$ remains constant and produces uniform advance along the field. The perpendicular component experiences a force always

perpendicular to its direction. This supplies the centripetal force:

$$|q|v_\perp B = \frac{mv_\perp^2}{r}, \qquad r = \frac{mv_\perp}{|q|B}.$$

Thus the transverse motion is circular, with unchanged transverse speed.

Step 2: Combine the motions and identify the axis Simultaneous circular motion in a plane perpendicular to $\vec{B}$ and uniform translation along $\vec{B}$ form a helix. The centre of each successive circular turn advances along the field, so the helix axis is parallel to the field. Its pitch is $v_\parallel T$, with $T = 2\pi m/(|q|B)$. A straight line would require $v_\perp = 0$, and a circle would require $v_\parallel = 0$; both are excluded because the question specifies both components. A perpendicular helix axis is inconsistent with the direction of the force-free velocity component.

Step 3: Conclusion The charged particle follows a helical path whose axis is along the magnetic field $\vec{B}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the force-free velocity component first: it sets the helix axis, while the perpendicular component sets the circular radius.

---

39. Two forces having magnitude A and $\frac{A}{2}$ are perpendicular to each other. The magnitude of their resultant is: [4 marks]

(A) $\frac{5A}{2}$

(B) $\frac{\sqrt{5}A}{4}$

(C) $\frac{\sqrt{5}A}{2}$

(D) $\frac{\sqrt{5}A^2}{2}$

Correct Answer: (C) $\frac{\sqrt{5}A}{2}$

Solution:

Concept:

- For two force vectors $\vec{F}_1$ and $\vec{F}_2$, the resultant is their vector sum, $\vec{F} = \vec{F}_1 + \vec{F}_2$. Magnitudes may be added directly only when the forces point in the same direction.
- The magnitude obeys $F^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$, where θ is the angle between the forces. For perpendicular forces, $\cos 90^\circ = 0$.

Step 1: Use the perpendicular components Choose the direction of the first force as the horizontal axis and the second as the vertical axis. Their components can then be written as $(A, 0)$ and $(0, A/2)$. The resultant has components $(A, A/2)$. Its magnitude follows from the Pythagorean theorem:

$$F = \sqrt{A^2 + \left(\frac{A}{2}\right)^2}.$$

There is no cross term because the two vectors are perpendicular; their scalar product vanishes.

Step 2: Simplify and check the choices Since A is a force magnitude and is nonnegative,

$$F = \sqrt{A^2 + \frac{A^2}{4}} = \sqrt{\frac{5A^2}{4}} = \frac{\sqrt{5}A}{2}.$$

The result is about $1.118A$, greater than the larger component A but less than the collinear sum $1.5A$. Option A, $2.5A$, exceeds even that upper bound. Option B is smaller than the component A, which is impossible for this right-angle vector sum. Option D contains A^2, giving the wrong physical dimensions. Option C alone has the correct magnitude and dimensions.

Step 3: Conclusion The magnitude of the resultant is $\boxed{\sqrt{5}A/2}$. Hence, the correct option is (C).

Quick Tip to Solve: For perpendicular components, square and add their magnitudes. Check that the answer is larger than either component and smaller than their sum.

---

40. At any instant the velocity of a particle of mass 500 g is $\left(2t\hat{i} + 3t^2\hat{j}\right)$ ms^{-1}. If the force acting on the particle at $t = 1$ s is $(\hat{i} + x\hat{j})$N. Then the value of x will be: [4 marks]

(A) 2

(B) 4

(C) 6

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- Instantaneous acceleration is the time derivative of velocity: $\vec{a} = \mathrm{d}\vec{v}/\mathrm{d}t$. Differentiate each Cartesian component independently.
- Newton's second law gives $\vec{F} = m\vec{a}$ for constant mass. Use SI units: $500\,\mathrm{g} = 0.500\,\mathrm{kg}$, so that force is obtained in newtons.

Step 1: Differentiate before substituting the time The velocity is

$$\vec{v} = (2t\hat{i} + 3t^2\hat{j})\,\mathrm{m\,s^{-1}}.$$

The fixed unit vectors do not vary with time. Differentiating the scalar coefficients gives

$$\vec{a} = (2\hat{i} + 6t\hat{j})\,\mathrm{m\,s^{-2}}.$$

Only after differentiating should we set $t = 1\,\mathrm{s}$. At that instant, $\vec{a} = (2\hat{i} + 6\hat{j})\,\mathrm{m\,s^{-2}}$. Substituting time into the original velocity first would give an instantaneous velocity, not an acceleration.

Step 2: Multiply by mass and compare components With the mass converted to kilograms,

$$\vec{F} = 0.500(2\hat{i} + 6\hat{j}) = (\hat{i} + 3\hat{j})\,\mathrm{N}.$$

The specified force is $(\hat{i}+x\hat{j})\,\mathrm{N}$. Equality of vectors requires equality of corresponding components, so $x=3$. The horizontal component also matches the given coefficient 1, confirming the unit conversion. Option C, 6, is the vertical acceleration before multiplying by mass; option A, 2, is the horizontal acceleration. Option B, 4, matches neither force component. The required number is the vertical force coefficient, not the speed or the magnitude of acceleration.

Step 3: Conclusion The force is $(\hat{i}+3\hat{j})\,\mathrm{N}$, giving $\boxed{x=3}$. Hence, the correct option is (D).

Quick Tip to Solve: Differentiate the velocity components first, convert grams to kilograms, and compare the final force component by component.

---

41. A cylindrical wire of mass $(0.4\pm0.01)\mathrm{g}$ has length $(8\pm0.04)\mathrm{cm}$ and radius $(6\pm0.03)\mathrm{mm}$. The maximum error in its density will be: [4 marks]

(A) 1%

(B) 5%

(C) 4%

(D) 3.5%

Correct Answer: (C) 4%

Solution:

Concept:

- The density of a cylindrical wire is $\rho=m/(\pi r^2L)$, where m is mass, r is radius and L is length. The constant π has no measurement uncertainty.
- For maximum first-order error in a product or quotient, add the magnitudes of relative errors. A power multiplies its relative error, so the radius contribution is doubled.

Step 1: Write the fractional-error relation The dependence $\rho=m\pi^{-1}r^{-2}L^{-1}$ gives

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max}=\frac{\Delta m}{m}+2\frac{\Delta r}{r}+\frac{\Delta L}{L}.$$

Although radius and length occur in the denominator, their uncertainty contributions are added for the maximum possible error, rather than subtracted. The highest density occurs when the mass is overestimated and dimensions are underestimated. Each uncertainty must be divided by its own measured quantity in the same unit; converting every quantity into SI is unnecessary for these ratios.

Step 2: Evaluate each contribution and add The individual percentage contributions are

$$\frac{0.01}{0.4}\times100=2.5\%,\qquad\frac{0.04}{8}\times100=0.5\%,$$

$$2\frac{0.03}{6}\times100=1.0\%.$$

Therefore,

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max}\times100=2.5+0.5+1.0=4.0\%.$$

This is the standard first-order maximum-error estimate requested. Option D, 3.5 percent, results from forgetting to double the radius term. Option A counts only the radius contribution, and option B overstates the total. Hence option C agrees with all three measurements and the squared radius dependence.

Step 3: Conclusion The maximum percentage error in the density is $\boxed{4\%}$. Hence, the correct option is (C).

Quick Tip to Solve: Write density in powers of measured variables before adding relative errors: the squared radius makes its percentage uncertainty count twice.

---

42. Two projectiles A and B are thrown with initial velocities of $40\ \mathrm{m/s}$ and $60\ \mathrm{m/s}$ at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is $\left(g = 10\ \mathrm{m/s^2}\right)$ [4 marks]

(A) $4 : 9$

(B) $2 : \sqrt{3}$

(C) $\sqrt{3} : 2$

(D) $1 : 1$

Correct Answer: (A) $4 : 9$

Solution:

Concept:

- For a projectile launched and landing at the same level, neglecting air resistance, its range is $R = u^2 \sin(2\theta)/g$, where u is launch speed and θ is the angle above the horizontal.
- Complementary launch angles have equal double-angle sine factors: $\sin(2\theta) = \sin[2(90^\circ - \theta)]$. For unequal launch speeds the ranges then vary as the squares of the speeds.

Step 1: Substitute the launch data into the range ratio For projectile A, $u_A = 40\,\mathrm{m\,s^{-1}}$ and $\theta_A = 30^\circ$. For B, $u_B = 60\,\mathrm{m\,s^{-1}}$ and $\theta_B = 60^\circ$. Since both move under the same gravitational acceleration,

$$\frac{R_A}{R_B} = \frac{40^2 \sin 60^\circ}{60^2 \sin 120^\circ}.$$

The factor $g = 10\,\mathrm{m\,s^{-2}}$ cancels. Moreover, $\sin 120^\circ = \sin 60^\circ = \sqrt{3}/2$, so the angle factors also cancel.

Step 2: Simplify and verify the physical meaning It follows that

$$\frac{R_A}{R_B} = \frac{1600}{3600} = \frac{4}{9}.$$

As a direct check, the separate ranges are

$$R_A = 80\sqrt{3}\,\mathrm{m}, \qquad R_B = 180\sqrt{3}\,\mathrm{m},$$

which have the same ratio. Complementary angles give equal ranges only when speeds are equal, so option D cannot apply here. Options B and C leave an extra trigonometric factor even though the two sine factors are equal. The speed ratio $2 : 3$ must be squared because horizontal range depends on u^2,

not merely on the launch speed. The order requested is A to B, so the smaller range is the first term.

Step 3: Conclusion The ranges satisfy $\boxed{R_A : R_B = 4 : 9}$. Hence, the correct option is (A).

Quick Tip to Solve: When the launch angles add to a right angle, cancel their double-angle sine factors and compare the squares of the launch speeds.

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43. For a nucleus ${}^{\mathrm{A}}_{\mathrm{A}}\mathrm{X}$ having mass number A and atomic number Z

A. The surface energy per nucleon $(b_s) = -a_1 A^{2/3}$**.**

B. The Coulomb contribution to the binding energy $b_c = -a_2 \frac{Z(Z-1)}{A^{4/3}}$

C. The volume energy $b_v = a_3 A$

D. Decrease in the binding energy is proportional to surface area.

E. While estimating the surface energy, it is assumed that each nucleon interacts with 12 nucleons. (a_1, a_2 **and** a_3 **are constants)**

Choose the most appropriate answer from the options given below: [4 marks]

(A) C, D only

(B) B, C, E only

(C) B, C only

(D) A, B, C, D only

Correct Answer: (A) C, D only

Solution:

Concept:

- The liquid-drop model separates nuclear binding energy into a positive volume term and negative surface and electrostatic terms. For nuclear radius $R = r_0 A^{1/3}$, surface area scales as $A^{2/3}$.
- Total energy and energy per nucleon are different quantities: an energy per nucleon is obtained by dividing the corresponding total nuclear energy by A. The nuclear force is short ranged and saturates.

Step 1: Check statements A and B The total surface contribution is $B_s = -a_1 A^{2/3}$, but its value per nucleon is $B_s/A = -a_1 A^{-1/3}$. Statement A explicitly says per nucleon while giving the total-energy power, so it is false. The electrostatic energy of a charged sphere is proportional to $Z(Z-1)/R$. Hence the total Coulomb contribution to binding is

$$B_c = -a_2 \frac{Z(Z-1)}{A^{1/3}}.$$

The printed $A^{4/3}$ denominator would belong to the contribution per nucleon, not to the total binding energy stated in B. Thus B is false.

Step 2: Check the volume and surface statements An interior nucleon has approximately the same local nuclear environment irrespective of the size of a large nucleus. The total volume binding therefore scales with the number of nucleons: $B_v = a_3 A$, confirming C. Nucleons at the surface have fewer neighbours, so they are less tightly bound. Their number is proportional to surface area, and

the corresponding reduction in binding is proportional to $A^{2/3}$. This is precisely statement D.

Step 3: Evaluate the neighbour-count statement Surface energy is estimated by distinguishing interior nucleons from surface nucleons, which have missing neighbours. It does not assume that every nucleon has twelve interacting neighbours. A fixed count for every nucleon would remove the very surface deficit being estimated. Thus E is not correct as stated. Options B and C both include the false statement B, while option D includes both false statements A and B.

Step 4: Conclusion Only statements C and D are correct. Hence, the correct option is (A).

Quick Tip to Solve: Before comparing powers of A, decide whether the expression describes total binding energy or binding energy per nucleon; dividing by A changes every power.

44. An aluminium rod with Young's modulus $Y = 7.0 \times 10^{10}\ \mathrm{N/m^2}$ undergoes elastic strain of 0.04%. The energy per unit volume stored in the rod in SI unit is: [4 marks]

(A) 5600

(B) 2800

(C) 11200

(D) 8400

Correct Answer: (A) 5600

Solution:

Concept:

- Within the elastic limit, stress σ and longitudinal strain ε obey Hooke's law: $\sigma = Y\varepsilon$, where Y is Young's modulus.
- The elastic potential energy stored per unit original volume is $u = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}Y\varepsilon^2$. Strain is dimensionless and must be used as a fraction rather than a percentage.

Step 1: Convert the percentage strain The given strain is 0.04%, meaning a fractional extension of 0.04 per hundred units of original length. Thus

$$\varepsilon = \frac{0.04}{100} = 4.0 \times 10^{-4}.$$

The material remains in the elastic regime, so its stress rises linearly from zero to its final value during a gradual extension. Using 0.04 directly would overestimate the strain by a factor of one hundred.

Step 2: Calculate the energy density Substitute the modulus and the converted strain into the elastic-energy expression:

$$\begin{aligned} u &= \frac{1}{2}(7.0 \times 10^{10})(4.0 \times 10^{-4})^2 \\ &= \frac{1}{2}(7.0 \times 10^{10})(16.0 \times 10^{-8}) \\ &= 5.6 \times 10^{3}\ \mathrm{J\,m^{-3}}. \end{aligned}$$

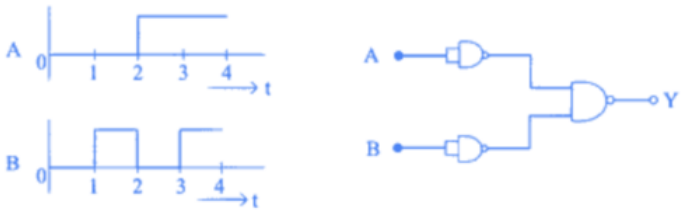
The dimensions are consistent because $\mathrm{N\,m^{-2}}$ times a squared dimensionless strain is $\mathrm{J\,m^{-3}}$. No rod dimensions are required for an energy per unit volume.

Step 3: Check the factor of one half The final stress is $Y\varepsilon = 2.8 \times 10^7$ Pa. The area under the straight stress-strain graph is a triangle, giving $u = \frac{1}{2}(2.8 \times 10^7)(4 \times 10^{-4}) = 5600\ \text{J m}^{-3}$. Option C, 11200, would result from omitting the triangular-area factor; options B and D do not equal the correctly evaluated area.

Step 4: Conclusion The energy per unit volume is $5600\ \text{J m}^{-3}$. Hence, the correct option is (A).

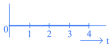
Quick Tip to Solve: Convert percentage strain first and then square it. The energy density is the triangular area under the stress-strain graph, so retain the factor $1/2$.

45. For the logic circuit shown, the output waveform at Y is: [4 marks]



(A) High for $0 < t < 1$; low for $1 < t < 4$. 

(B) Low for $0 < t < 1$; high for $1 < t < 4$. 

(C) Low throughout $0 < t < 4$. 

(D) High throughout $0 < t < 4$. 

Correct Answer: (B) Low for $0 < t < 1$; high for $1 < t < 4$. 

Solution:

Concept:

- A NAND gate gives the complement of its input product. With its inputs tied to the same signal, it acts as an inverter: $\overline{A \cdot A} = \overline{A}$.
- De Morgan's law gives $\overline{\overline{A} \cdot \overline{B}} = A + B$, where $+$ means logical OR, not arithmetic addition. OR is low only when both inputs are low.

Step 1: Simplify the gate combination The upper first-stage NAND gate has both inputs connected to A, so its output is $\overline{A}$. The lower first-stage NAND similarly produces $\overline{B}$. These two outputs feed the final NAND gate. Therefore

$$Y = \overline{\overline{A} \cdot \overline{B}} = A + B.$$

The overall circuit behaves as an OR gate despite being constructed entirely from NAND gates. It would be incorrect to treat the first two gates as ordinary AND gates and ignore their output bubbles.

Step 2: Read the successive input intervals From the input waveforms, A is low from $t = 0$ to $t = 2$ and high from $t = 2$ onward. B is low on $0 < t < 1$, high on $1 < t < 2$, low on $2 < t < 3$, and high on

$3 < t < 4$. The OR output is consequently

Interval	$(0,1)$	$(1,2)$	$(2,3)$	$(3,4)$
A	0	0	1	1
B	0	1	0	1
$Y = A + B$	0	1	1	1

Ideal switching instants only mark changes between the intervals.

Step 3: Match the resulting waveform The output begins low and rises at $t = 1$. It remains high after that because B first supplies the high input, then A remains high from $t = 2$, including the interval in which B is low. This is the waveform in option B. Option A is high in the opposite initial interval; option C misses every high interval; option D incorrectly makes the output high even when both inputs are initially low.

Step 4: Conclusion The output is low for $0 < t < 1$ and high for $1 < t < 4$, as in waveform B. Hence, the correct option is (B).

Quick Tip to Solve: Reduce a NAND network algebraically before reading its waveforms. Two tied-input NAND inverters feeding a NAND implement OR, so look only for intervals when both inputs are zero.

46. The weight of a body on the earth is 400 N. Then weight of the body when taken to a depth half of the radius of the earth will be: [4 marks]

(A) 300 N

(B) 200 N

(C) 100 N

(D) Zero

Correct Answer: (B) 200 N

Solution:

Concept:

- For the standard uniform-density spherical Earth model, the shell theorem makes the gravitational field at an interior radius depend only on mass enclosed within that radius.
- At depth d, the distance from the centre is $r = R - d$, and $g_d = g_0(r/R) = g_0(1 - d/R)$. A body's mass stays constant, so its weight has the same fractional change as g.

Step 1: Relate the enclosed mass to the radius Let the Earth have total mass M and radius R. In the uniform-density model, the mass inside radius r is $M_r = M(r/R)^3$. The attraction due to the outer spherical shells cancels inside those shells. Consequently

$$g(r) = \frac{GM_r}{r^2} = \frac{GM}{R^3}r = g_0\frac{r}{R}.$$

Although the distance to the centre decreases, the attracting enclosed mass decreases as its cube, so the interior field decreases linearly with r.

Step 2: Insert the specified depth The body is taken to depth $d = R/2$, so its distance from the centre is $r = R - R/2 = R/2$. Therefore

$$\frac{g_d}{g_0} = \frac{r}{R} = \frac{1}{2}.$$

Its mass m is unchanged. Dividing its new weight mg_d by its original weight $mg_0 = 400$ N gives

$$W_d = 400\left(\frac{1}{2}\right) = 200 \text{ N}.$$

The radius and Earth's mass cancel, so no numerical values of either are necessary.

Step 3: Check the alternatives and limiting cases The formula gives the full surface weight at zero depth and zero weight only at the centre, where $d = R$. Thus zero is not appropriate at half-depth. A weight of 300 N would correspond to $d = R/4$, and 100 N to $d = 3R/4$ in this same model. Applying the external inverse-square formula using the whole Earth's mass at $r = R/2$ would violate the shell theorem for an interior point.

Step 4: Conclusion The body weighs 200 N at the stated depth. Hence, the correct option is (B).

Quick Tip to Solve: At an interior point use $W_d/W_0 = 1 - d/R$, not the external inverse-square law. Half the radius below the surface gives half the surface weight.

47. Proton (P) and electron (e) will have same de-Broglie wavelength when the ratio of their momentum is (assume, $m_p = 1849\ m_e$): [4 marks]

(A) 1 : 1

(B) 1 : 43

(C) 1 : 1849

(D) 43 : 1

Correct Answer: (A) 1 : 1

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of momentum. The same h applies to every particle species.
- Equal wavelengths therefore require equal momentum magnitudes. Equal momentum does not mean equal velocity or equal kinetic energy when the particles have different masses.

Step 1: Equate the two matter wavelengths Write the de Broglie wavelengths of the proton and electron separately:

$$\lambda_p = \frac{h}{p_p}, \qquad \lambda_e = \frac{h}{p_e}.$$

The stated condition is $\lambda_p = \lambda_e$. Cancelling the common, nonzero Planck constant gives

$$\frac{1}{p_p} = \frac{1}{p_e} \quad \Rightarrow \quad p_p = p_e.$$

This argument uses momentum directly, so it does not require the particle speeds or their kinetic energies.

Step 2: Distinguish momentum from speed In the nonrelativistic description $p = mv$, equal momentum would imply $m_p v_p = m_e v_e$. With $m_p = 1849 m_e$, this gives $v_p/v_e = 1/1849$, but this is a speed ratio, not the momentum ratio requested. Likewise the given mass ratio has square root 43; such a factor would arise when comparing momenta at equal nonrelativistic kinetic energy through $p = \sqrt{2mK}$. Equal kinetic energy is not stated here.

Step 3: Select the correct momentum ratio The required ratio is simply

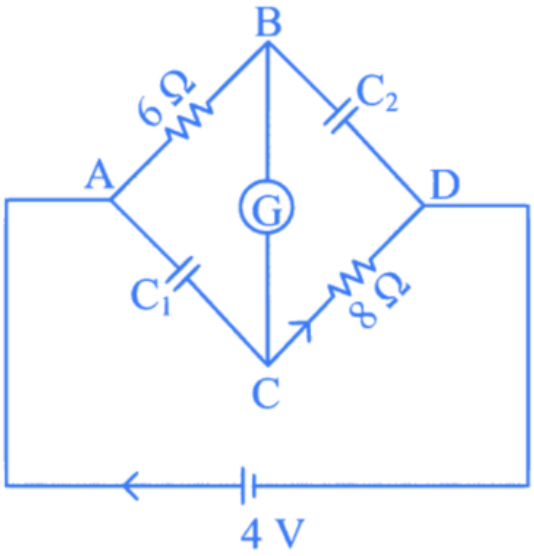
$$p_p : p_e = 1 : 1.$$

The alternatives $1 : 43$ and $43 : 1$ insert the square root of the mass ratio into a condition that involves only h and p. The alternative $1 : 1849$ confuses the equal-wavelength velocity ratio with the momentum ratio. None satisfies $h/p_p = h/p_e$, except unity. The extra mass information is therefore unnecessary for this question.

Step 4: Conclusion The momenta have equal magnitudes, so $p_p : p_e = 1 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: If two particles have equal de Broglie wavelength, equate their momenta immediately. Use the mass ratio only if the question asks for speeds or kinetic energies.

48. In this figure the resistance of the coil of galvanometer G is $2\ \Omega$. The emf of the cell is 4 V. The ratio of potential difference across C_1 and C_2 is: [4 marks]



(A) $\frac{5}{4}$

(B) $\frac{3}{4}$

(C) 1

(D) $\frac{4}{5}$

Correct Answer: (D) $\frac{4}{5}$

Solution:

Concept:

- After a DC circuit reaches steady state, an ideal capacitor carries no current. Its voltage equals the potential difference between the nodes to which it is connected.
- The conducting branch then determines the node potentials by Ohm's law. The galvanometer coil must be treated as its stated $2\ \Omega$ resistance; it is not an ideal zero-resistance wire.

Step 1: Identify the steady conducting path The capacitors occupy arms AC and BD. Once they are charged, neither of these arms carries steady current. The remaining path from A to D is $A \to B \to C \to D$, containing the $6\ \Omega$ resistor, the $2\ \Omega$ galvanometer coil and the $8\ \Omega$ resistor in series. Thus

$$R_{\text{eq}} = 6 + 2 + 8 = 16\ \Omega, \qquad I = \frac{4}{16} = \frac{1}{4}\ \text{A}.$$

The positive cell terminal is connected to A, so potential decreases along this path toward D.

Step 2: Calculate the capacitor voltages Capacitor C_1 is connected between A and C, spanning the voltage drops across the $6\ \Omega$ resistor and the galvanometer. Capacitor C_2 lies between B and D, spanning the galvanometer and the $8\ \Omega$ resistor. Therefore

$$V_{C_1} = I(6+2) = \frac{1}{4} \times 8 = 2\ \text{V},$$
$$V_{C_2} = I(2+8) = \frac{1}{4} \times 10 = 2.5\ \text{V}.$$

Each capacitor senses a different pair of node potentials; the two capacitor voltages are not merely the drops across the two outer resistors.

Step 3: Form the requested ordered ratio The requested order is the potential difference across C_1 divided by that across C_2:

$$\frac{V_{C_1}}{V_{C_2}} = \frac{I(6+2)}{I(2+8)} = \frac{8}{10} = \frac{4}{5}.$$

The current and source emf cancel from this ratio. Option A reverses the ratio. Option B would arise by incorrectly ignoring the galvanometer resistance, giving $6/8$. Option C incorrectly treats B and C as equipotential, despite the nonzero coil resistance and current.

Step 4: Conclusion The ratio is $V_{C_1}/V_{C_2} = 4/5$. Hence, the correct option is (D).

Quick Tip to Solve: For charged capacitors in a DC network, first open their branches and find node potentials through the resistors. Include the galvanometer coil resistance in every path it belongs to.

49. The engine of a train moving with speed $10\ \text{ms}^{-1}$ towards a platform sounds a whistle at frequency $400\ \text{Hz}$. The frequency heard by a passenger inside the train is: (neglect air speed. Speed of sound in air $= 330\ \text{ms}^{-1}$) [4 marks]

(A) 200 Hz

(B) 412 Hz

(C) 400 Hz

(D) 388 Hz

Correct Answer: (C) 400 Hz

Solution:

Concept:

- The observed sound frequency depends on both the source and observer velocities relative to the air. Their directions must be included consistently in the Doppler formula.
- When a source and an observer move together at the same constant velocity through still air, their fixed separation makes the source-motion and observer-motion frequency factors cancel.

Step 1: Identify the actual observer The observer is a passenger inside the train, not someone standing on the platform. The whistle source and the passenger therefore both move with the train at $u = 10\ \text{m s}^{-1}$ relative to the still air. Their separation along the train does not change. The information that the train approaches the platform would matter for a platform observer but cannot by itself determine the frequency heard by the co-moving passenger.

Step 2: Apply the two Doppler factors consistently Consider sound reaching a passenger behind the engine. Successive wavefronts travelling backward have separation $\lambda_{\text{back}} = (v + u)/f$, since the engine moves away from the region behind it between emissions. The passenger moves toward those backward-travelling wavefronts, so they pass the passenger at speed $v + u$. Consequently

$$f_{\text{heard}} = \frac{v + u}{\lambda_{\text{back}}} = f\frac{v + u}{v + u} = 400\ \text{Hz}.$$

With $v = 330\ \text{m s}^{-1}$, the common factor is $340/340 = 1$. For an observer ahead moving with the source, the corresponding factors are both $v - u$ and cancel as well.

Step 3: Compare with a stationary platform observer A stationary observer in front would instead hear $400(330/320) = 412.5$ Hz, approximately the higher-frequency alternative. A stationary observer behind would hear $400(330/340)$, approximately 388 Hz. Neither is the passenger specified. A frequency of 200 Hz would require a factor of one half, which is absent from the equal-velocity source-observer geometry.

Step 4: Conclusion The passenger hears the emitted frequency, 400 Hz. Hence, the correct option is (C).

Quick Tip to Solve: Always identify who hears the sound. A passenger moving with the whistle has equal source and observer speeds, so the two Doppler corrections cancel.

50. A TV transmitting antenna is $98\ \mathrm{m}$ high and the receiving antenna is at the ground level. If the radius of the earth is $6400\ \mathrm{km}$, the surface area covered by the transmitting antenna is approximately:

[4 marks]

(A) $4868\ \mathrm{km}^2$

(B) $1240\ \mathrm{km}^2$

(C) $3942\ \mathrm{km}^2$

(D) $1549\ \mathrm{km}^2$

Correct Answer: (C) $3942\ \mathrm{km}^2$

Solution:

Concept:

- A television signal propagates approximately along straight lines, so the horizon limits reception. For a transmitting antenna of height h above a spherical Earth of radius R, the horizon distance is $d \simeq \sqrt{2Rh}$ when $h \ll R$.
- A ground-level receiver adds no extra horizon distance. The covered region is approximately a circular disk of radius d, giving area $A = \pi d^2 \simeq 2\pi Rh$. Use a single length unit before calculating an area.

Step 1: Convert the antenna height Take distances in kilometres because the answer choices are in square kilometres. The height is $h = 98\,\mathrm{m} = 0.098\,\mathrm{km}$, while $R = 6400\,\mathrm{km}$. Since h/R is very small, the standard horizon approximation is appropriate. The receiving antenna is at ground level; therefore, there is no second height contribution to add.

Step 2: Find the radius of coverage The limiting ray from the top of the transmitting antenna is tangent to the Earth. The right triangle formed with the Earth's centre gives $d^2 = (R+h)^2 - R^2 = 2Rh + h^2$. Neglecting the tiny h^2 term,

$$d^2 \simeq 2(6400)(0.098) = 1254.4\,\mathrm{km}^2.$$

Thus the horizon radius is about $35.4\,\mathrm{km}$, not an area by itself.

Step 3: Calculate and compare the covered area The signal covers all azimuthal directions around the antenna, so multiply the squared radius by π:

$$A \simeq \pi(1254.4) \simeq 3941\,\mathrm{km}^2.$$

Using the usual rounded value of π gives approximately $3942\,\mathrm{km}^2$. Option (B) is close to d^2 and omits the circular-area factor. Options (A) and (D) do not match $2\pi Rh$ with the stated height and radius.

Step 4: Conclusion The surface area covered is approximately $3942\,\mathrm{km}^2$. Hence, the correct option is (C).

Quick Tip to Solve: For a ground-level receiver, calculate coverage directly from $A \simeq 2\pi Rh$; convert metres to kilometres before substituting.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The magnetic intensity at the center of a long current carrying solenoid is found to be $1.6\times10^3\,\mathrm{Am}^{-1}$. If the number of turns is 8 per cm, then the current flowing through the solenoid is _____ A.

[4 marks]

Solution:

Concept:

- The magnetic field intensity H inside a sufficiently long solenoid is $H = nI$, where $n = N/L$ is the number of turns per unit length and I is the current through each turn.
- Field intensity H has units $\mathrm{A\,m^{-1}}$. It is distinct from magnetic flux density $B = \mu H$, measured in tesla. When H itself is supplied, no permeability factor is needed to determine the current.

Step 1: Identify the appropriate solenoid relation The point is at the centre of a long solenoid, where end effects can be neglected and the internal magnetic intensity is nearly uniform. Ampere's circuital law applied along the solenoid gives $HL = NI$, since the enclosed current is NI. Dividing by its length gives $H = (N/L)I = nI$. The given quantity $1.6 \times 10^3\,\mathrm{A\,m^{-1}}$ is already H, rather than B.

Step 2: Express the turn density in SI units There are eight turns in each centimetre. Since one metre contains one hundred centimetres,

$$n = 8\,\mathrm{cm^{-1}} = 8 \times 100\,\mathrm{m^{-1}} = 800\,\mathrm{m^{-1}}.$$

The conversion increases the numerical value: a longer unit length contains more turns. Substituting 8 without conversion would mix centimetres and metres and overestimate the current.

Step 3: Isolate and evaluate the current Rearrange the intensity relation and substitute the measured intensity and the converted turn density:

$$I = \frac{H}{n} = \frac{1.6 \times 10^3}{800} = 2\,\mathrm{A}.$$

The units reduce as $(\mathrm{A\,m^{-1}})/(\mathrm{m^{-1}}) = \mathrm{A}$. Checking by substitution, $800 \times 2 = 1600\,\mathrm{A\,m^{-1}}$, which reproduces the stated magnetic intensity.

Step 4: Conclusion The current through the solenoid is 2 A. The numerical entry is $\boxed{2}$.

Quick Tip to Solve: Convert turns per centimetre to turns per metre by multiplying by 100, and distinguish $H = nI$ from $B = \mu nI$.

52. An organ pipe $40\ \text{cm}$ long is open at both ends. The speed of sound in air is $360\ \text{ms}^{-1}$. The frequency of the second harmonic is _____ Hz. [4 marks]

Solution:

Concept:

- An organ pipe open at both ends has displacement antinodes at both openings. Its allowed standing-wave lengths satisfy $L = m\lambda_m/2$, where the harmonic number m can be any positive integer.
- The frequency of harmonic m is $f_m = mv/(2L)$, where v is the sound speed and L is the pipe length. The second harmonic has $m = 2$ and is twice the fundamental frequency.

Step 1: Apply the open-end boundary conditions Both ends of the pipe are open, so the air there can oscillate freely and forms displacement antinodes. The fundamental contains one half-wavelength within the pipe, whereas the second harmonic contains two half-wavelengths. Hence the second-harmonic wavelength equals the full pipe length, $\lambda_2 = L$. This differs from a pipe closed at one end, in which only odd harmonics occur.

Step 2: Calculate the fundamental frequency Convert the pipe length to metres to match the sound speed:

$$L = 40\,\text{cm} = 0.40\,\text{m}.$$

The fundamental frequency is

$$f_1 = \frac{v}{2L} = \frac{360}{2(0.40)} = 450\,\text{Hz}.$$

The problem supplies the sound speed directly, so there is no need to estimate it from temperature. No end correction is specified in this ideal-pipe calculation.

Step 3: Evaluate the second harmonic The second harmonic is twice the fundamental:

$$f_2 = 2f_1 = 2(450) = 900\,\text{Hz}.$$

Equivalently, using $\lambda_2 = L = 0.40\,\text{m}$ gives $f_2 = v/\lambda_2 = 360/0.40 = 900\,\text{Hz}$. These two calculations agree. The fundamental value 450 Hz must not be entered when the question explicitly asks for the second harmonic.

Step 4: Conclusion The frequency of the second harmonic is $900\,\text{Hz}$. The numerical entry is $\boxed{900}$.

Quick Tip to Solve: For a pipe open at both ends, the second harmonic obeys $\lambda_2 = L$, so its frequency is simply v/L.

53. A current of $2\ \mathrm{A}$ flows through a wire of cross-sectional area $25.0\ \mathrm{mm}^2$. The number of free electrons in a cubic meter are 2.0×10^{28}. The drift velocity of the electrons is _____ $\times 10^{-6}\ \mathrm{ms}^{-1}$ (given, charge on electron $= 1.6 \times 10^{-19}\mathrm{C}$). [4 marks]

Solution:

Concept:

- The magnitude of current carried by free electrons is $I = neAv_d$, where n is their number per unit volume, e is the magnitude of electronic charge, A is cross-sectional area, and v_d is drift speed.
- Area conversion must be squared: $1\,\mathrm{mm}^2 = 10^{-6}\,\mathrm{m}^2$. Electron drift is opposite to conventional current, but a numerical drift-velocity magnitude is positive.

Step 1: Convert the area and collect the data The given area is

$$A = 25.0\,\mathrm{mm}^2 = 25.0 \times 10^{-6}\,\mathrm{m}^2.$$

Use $I = 2\,\mathrm{A}$, $n = 2.0 \times 10^{28}\,\mathrm{m}^{-3}$, and $e = 1.6 \times 10^{-19}\,\mathrm{C}$. The density is already the free-electron density, so no atomic mass, Avogadro constant, or additional count of conduction electrons is needed.

Step 2: Derive the drift-speed expression In a time interval Δt, electrons in a cylindrical length $v_d\Delta t$ cross the wire section. Their number is $nAv_d\Delta t$, and the magnitude of charge crossing is $neAv_d\Delta t$. Dividing by the time interval gives $I = neAv_d$. Thus

$$v_d = \frac{I}{neA}.$$

This uses the small directed drift speed, not the much larger random thermal speed.

Step 3: Substitute and extract the requested coefficient The denominator evaluates to

$$neA = (2.0)(1.6)(25.0)10^{28-19-6} = 8.0 \times 10^4.$$

Therefore,

$$v_d = \frac{2}{8.0 \times 10^4} = 2.5 \times 10^{-5}\,\mathrm{m\,s}^{-1} = 25 \times 10^{-6}\,\mathrm{m\,s}^{-1}.$$

The blank multiplies 10^{-6}, so it asks for the coefficient 25 rather than 2.5. As a check, multiplying this drift speed by neA returns the given current of 2 A.

Step 4: Conclusion The electron drift-speed magnitude is $25 \times 10^{-6}\,\mathrm{m\,s}^{-1}$. The numerical entry is $\boxed{25}$.

Quick Tip to Solve: Square the millimetre-to-metre conversion for area, then rewrite the final drift speed in the exact power of ten printed beside the blank.

54. A nucleus with mass number 242 and binding energy per nucleon as 7.6 MeV breaks into two fragment each with mass number 121. If each fragment nucleus has binding energy per nucleon as 8.1 MeV, the total gain in binding energy is _____ MeV. [4 marks]

Solution:

Concept:

- Total nuclear binding energy is the binding energy per nucleon multiplied by the mass number: $B = A\varepsilon$. Comparing per-nucleon values alone does not give the total energy change.
- The gain in binding energy in fission is $\Delta B = B_{\text{products}} - B_{\text{initial}}$. A positive gain corresponds to a decrease in rest energy and energy released, since the fragments are more tightly bound.

Step 1: Calculate the initial total binding energy The parent nucleus contains 242 nucleons, each contributing an average binding energy of $7.6\,\text{MeV}$. Its total binding energy is

$$B_i = 242(7.6) = 1839.2\,\text{MeV}.$$

Here 242 is a nucleon count rather than a mass expressed in kilograms. The supplied average binding energy already includes the energy units needed for the calculation.

Step 2: Add the binding energies of both fragments Each final nucleus has mass number 121 and binding energy per nucleon $8.1\,\text{MeV}$. Hence its binding energy is $121(8.1) = 980.1\,\text{MeV}$. Both fragments must be included:

$$B_f = 2(121)(8.1) = 1960.2\,\text{MeV}.$$

The total mass number remains $121 + 121 = 242$, so all the original nucleons are included in this energy comparison.

Step 3: Find and interpret the gain Subtract the initial binding energy from the final binding energy:

$$\Delta B = 1960.2 - 1839.2 = 121.0\,\text{MeV}.$$

The same result follows directly from the per-nucleon increase, $\Delta B = 242(8.1 - 7.6) = 242(0.5)\,\text{MeV}$. The positive sign is physically consistent with the increase in binding energy per nucleon. Using only one fragment, or entering merely 0.5 MeV, would omit part of the nucleon count.

Step 4: Conclusion The total gain in binding energy is $121\,\text{MeV}$. The numerical entry is $\boxed{121}$.

Quick Tip to Solve: When the total nucleon count is conserved and the fragments have the same binding energy per nucleon, use $\Delta B = A(\varepsilon_f - \varepsilon_i)$.

55. The momentum of a body is increased by 50%. The percentage increase in the kinetic energy of the body is _____ $\%$. [4 marks]

Solution:

Concept:

- For a nonrelativistic body of fixed mass m, momentum and kinetic energy satisfy $p = mv$ and $K = \frac{1}{2}mv^2 = p^2/(2m)$. Thus kinetic energy is proportional to the square of momentum.
- A percentage increase is measured relative to the initial value: $\%$ increase $= 100(K_f - K_i)/K_i$. The final percentage of the original value and the percentage increase are different quantities.

Step 1: Translate the momentum increase into a ratio Let the initial momentum magnitude be p_i. An increase of 50% means an addition of one half of the original momentum, so

$$p_f = p_i + 0.50p_i = 1.50p_i.$$

The same body is considered before and after the change; its mass stays fixed. This is the usual classical kinetic-energy relation, with no change in mass or relativistic correction implied.

Step 2: Use the square dependence of energy Take the ratio of the kinetic energies so that the common mass cancels:

$$\frac{K_f}{K_i} = \frac{p_f^2/(2m)}{p_i^2/(2m)} = \left(\frac{p_f}{p_i}\right)^2 = (1.50)^2 = 2.25.$$

Consequently, the final kinetic energy is 225 percent of the initial kinetic energy. A linear scaling with momentum would be incorrect because doubling speed doubles momentum but quadruples kinetic energy.

Step 3: Subtract the initial energy before taking the percentage The fractional increase is the final ratio minus one:

$$\frac{K_f - K_i}{K_i} = 2.25 - 1 = 1.25.$$

Multiplying by one hundred gives

$$\%\text{ increase} = 1.25 \times 100 = 125\%.$$

The value 225 percent describes the final energy as a percentage of the original, rather than the increase. The extra quadratic contribution also explains why simply doubling the 50 percent momentum change would underestimate the result.

Step 4: Conclusion The kinetic energy increases by 125%. The numerical entry is $\boxed{125}$.

Quick Tip to Solve: For a momentum increase of $x\%$ at fixed mass, the energy increase is $2x + x^2/100$ percent; subtract the original energy after squaring.

56. An electric dipole of dipole moment is 6.0×10^{-6} Cm placed in a uniform electric field of 1.5×10^{3} NC^{-1} in such a way that dipole moment is along electric field. The work done in rotating dipole by 180° in this field will be _____ mJ. [4 marks]

Solution:

Concept:

- The potential energy of a dipole of moment magnitude p in a uniform electric field of magnitude E is $U = -pE\cos\theta$, where θ is the angle between the dipole moment and the field.
- For controlled rotation with no change in kinetic energy, the external work is $W_{\text{ext}} = U_f - U_i$. The electric field does work of the opposite sign. The product pE has the dimensions of energy.

Step 1: Identify the initial and final orientations The dipole moment is initially along the field, so $\theta_i = 0$ and $\cos\theta_i = 1$. A rotation by 180° puts it opposite to the field, giving $\theta_f = \pi$ and $\cos\theta_f = -1$. The initial orientation has minimum potential energy; reaching the opposite orientation requires positive external work against the electric torque.

Step 2: Calculate the change in potential energy Using the two orientations,

$$U_i = -pE, \qquad U_f = +pE.$$

Therefore the work required to rotate it is

$$W_{\text{ext}} = U_f - U_i = pE - (-pE) = 2pE.$$

The factor two follows from the entire change from negative to positive potential energy. Merely using pE would measure the energy change only up to the perpendicular orientation, where the potential energy is zero.

Step 3: Substitute the values and convert to millijoules The specified values are $p = 6.0 \times 10^{-6}\,\text{C m}$ and $E = 1.5 \times 10^{3}\,\text{N C}^{-1}$. Thus

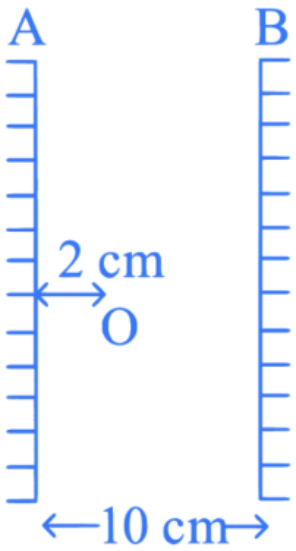
$$W_{\text{ext}} = 2(6.0 \times 10^{-6})(1.5 \times 10^{3}) = 18 \times 10^{-3}\,\text{J} = 18\,\text{mJ}.$$

The charge units cancel, leaving newton metres, equivalent to joules. The work done by the electric field itself would be $-18\,\text{mJ}$; the question asks for the work needed to perform the rotation.

Step 4: Conclusion The work required to rotate the dipole through 180° is $18\,\text{mJ}$. The numerical entry is $\boxed{18}$.

Quick Tip to Solve: For rotation from parallel to antiparallel, external work is $2pE$; convert joules to millijoules only after calculating the energy difference.

57. Two vertical parallel mirrors A and B are separated by 10 cm. A point object O is placed at a distance of 2 cm from mirror A. The distance of the second nearest image behind mirror A from the mirror A is _____ cm. [4 marks]



Solution:

Concept:

- A plane mirror forms a virtual image as far behind its reflecting plane as the object is in front. Repeated reflections occur when two reflecting surfaces face one another.
- Choose the coordinate of mirror A as $x = 0$, mirror B as $x = L = 10\,\text{cm}$, and the object as $x = a = 2\,\text{cm}$. Reflection in A maps x to $-x$, while reflection in B maps x to $2L - x$.

Step 1: Locate the nearest image behind A The direct reflection of O in A lies at $x = -a = -2\,\text{cm}$. It is therefore $2\,\text{cm}$ behind A. This is the nearest image and must be excluded when the question asks for the second nearest image. Distances behind A are measured from its plane, not from the object.

Step 2: Reflect the image formed by B in A The direct image of O in B is at

$$x_B = 2L - a = 2(10) - 2 = 18\,\text{cm}.$$

For reflection at A, this image acts as an object on the positive side of A. Its image in A is consequently at

$$x_{BA} = -x_B = -18\,\text{cm}.$$

The optical path corresponds to successive reflection at B and then A, and produces an image behind A.

Step 3: Check the ordering of repeated images The two families of image coordinates for parallel mirrors are $x = 2nL + a$ and $x = 2nL - a$, with integer n, omitting the original object. The negative coordinates nearest zero are

$$-2, \quad -18, \quad -22, \quad -38, \ldots \text{ cm}.$$

Thus the distances behind A begin $2, 18, 22, 38, \ldots$ cm. The second distance is 18; 22 belongs to the

next image, so merely adding twice the separation to the first image would skip the required one.

Step 4: Conclusion The second nearest image is $\boxed{18\,\text{cm}}$ behind mirror A. The required numerical answer is $\boxed{18}$.

Quick Tip to Solve: For parallel mirrors, list the first few negative coordinates from $2nL \pm a$; their magnitudes give the ordered distances behind the mirror at the origin.

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58. An air bubble of diameter $6\ \text{mm}$ rises steadily through a solution of density $1750\ \text{kg/m}^3$ at the rate of $0.35\ \text{cm/s}$. The co-efficient of viscosity of the solution (neglect density of air) is _____ Pas (given, $g = 10\ \text{ms}^{-2}$). [4 marks]

Solution:

Concept:

- At terminal speed the net force on a rising bubble is zero. Upthrust equals the downward viscous drag when the weight of air is negligible.
- For a sphere of radius r, Stokes drag is $6\pi\eta r v$, where η is dynamic viscosity and v is terminal speed. The buoyant force is $\frac{4}{3}\pi r^3 \rho g$, with liquid density ρ.

Step 1: Convert the radius and speed to SI units The stated $6\,\text{mm}$ is the diameter, so the radius is

$$r = \frac{6}{2}\,\text{mm} = 3 \times 10^{-3}\,\text{m}.$$

The speed is

$$v = 0.35\,\text{cm}\,\text{s}^{-1} = 3.5 \times 10^{-3}\,\text{m}\,\text{s}^{-1}.$$

Use $\rho = 1750\,\text{kg}\,\text{m}^{-3}$ and the given $g = 10\,\text{m}\,\text{s}^{-2}$. Using the diameter in the radius formula would increase the answer by a factor of four.

Step 2: Balance upthrust and viscous resistance The bubble rises steadily, so its acceleration is zero. The buoyant force acts upward and the drag opposes the upward motion. Neglecting the air density, the force balance gives

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v.$$

Cancel πr and solve for the required coefficient:

$$\eta = \frac{2r^2\rho g}{9v}.$$

This is the usual Stokes-law terminal-speed relation rearranged to find viscosity.

Step 3: Substitute and check the result Substitution gives

$$\eta = \frac{2(3 \times 10^{-3})^2(1750)(10)}{9(3.5 \times 10^{-3})}$$
$$= \frac{0.315}{0.0315} = 10\,\text{Pa}\,\text{s}.$$

Dimensionally, $r^2\rho g/v$ has units $\mathrm{kg\,m^{-1}\,s^{-1}} = \mathrm{Pa\,s}$, as required for dynamic viscosity. The slow steady rise is consistent with a highly viscous liquid.

Step 4: Conclusion The coefficient of viscosity is $\boxed{10\,\mathrm{Pa\,s}}$. The numerical entry is $\boxed{10}$.

Quick Tip to Solve: First halve the bubble diameter and convert centimetres per second to metres per second; the Stokes formula uses the square of the radius.

59. An oscillating LC circuit consists of a $75\ \mathrm{mH}$ inductor and a $1.2\ \mu\mathrm{F}$ capacitor. If the maximum charge to the capacitor is $2.7\ \mu\mathrm{C}$. The maximum current in the circuit will be _____ mA

[4 marks]

Solution:

Concept:

- In an ideal LC circuit energy oscillates between the capacitor electric field and the inductor magnetic field, with total energy constant.
- The energy is $U_C = q^2/(2C)$ in a capacitor and $U_L = Li^2/2$ in an inductor. The maximum current satisfies $I_{\max} = Q_{\max}/\sqrt{LC}$.

Step 1: Identify the two extreme states When the capacitor carries its maximum charge $Q_{\max}$, the current is zero and the whole energy is electrostatic. When the capacitor charge passes through zero, the current magnitude is maximum and the energy is magnetic. These maxima occur at different instants; they must not be added as simultaneous energies. For a lossless circuit,

$$\frac{Q_{\max}^2}{2C} = \frac{1}{2}LI_{\max}^2.$$

Step 2: Evaluate the LC time scale Convert each quantity before combining them:

$$L = 75 \times 10^{-3}\,\mathrm{H}, \qquad C = 1.2 \times 10^{-6}\,\mathrm{F},$$

$$Q_{\max} = 2.7 \times 10^{-6}\,\mathrm{C}.$$

Then

$$LC = (75 \times 10^{-3})(1.2 \times 10^{-6}) = 9.0 \times 10^{-8}\,\mathrm{s^2},$$

$$\sqrt{LC} = 3.0 \times 10^{-4}\,\mathrm{s}.$$

The square root is essential because the angular frequency is $\omega = 1/\sqrt{LC}$, rather than $1/(LC)$.

Step 3: Calculate the current amplitude Taking the positive amplitude from the energy equation,

$$\begin{aligned} I_{\max} &= \frac{Q_{\max}}{\sqrt{LC}} \\ &= \frac{2.7 \times 10^{-6}}{3.0 \times 10^{-4}} \\ &= 9.0 \times 10^{-3}\,\mathrm{A} = 9.0\,\mathrm{mA}. \end{aligned}$$

The same result follows from $q = Q_{\max}\cos\omega t$: differentiating gives a current amplitude $\omega Q_{\max}$. No factor of 2π is required because the charge-current relation uses angular frequency.

Step 4: Conclusion The maximum current is $\boxed{9\,\text{mA}}$, so the required numerical answer is $\boxed{9}$.

Quick Tip to Solve: Equate maximum electric and magnetic energies to get $I_{\max} = Q_{\max}/\sqrt{LC}$; then convert amperes into the requested milliamperes.

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60. The moment of inertia of a semicircular ring about an axis, passing through the center and perpendicular to the plane of ring, is $\frac{1}{x}\text{MR}^2$, where R is the radius and M is the mass of the semicircular ring. The value of x will be _____. [4 marks]

Solution:

Concept:

- Moment of inertia about a specified axis is $I = \int r_\perp^2\, dm$, where $r_\perp$ is the perpendicular distance of a mass element from that axis.
- For a thin circular or semicircular ring, every mass element is at the same radius R from the axis through the circle centre and perpendicular to its plane. The mass M here belongs to the semicircular ring itself.

Step 1: Interpret the specified axis The axis passes through the centre of the circle from which the semicircular arc is taken, and points perpendicular to the plane of the arc. Every point of that arc is at distance R from this axis. The semicircle does not cover all angles, but this does not change the radial distance of its mass elements. The centre of the parent circle should not be confused with the centre of mass of the semicircular wire.

Step 2: Integrate over the actual semicircular mass For any element dm,

$$dI = R^2\, dm.$$

Because R is constant along the wire,

$$I = \int_{\text{semicircle}} R^2\, dm = R^2 \int_{\text{semicircle}} dm = MR^2.$$

For example, if the linear density is uniform and equals λ, then $dm = \lambda R\, d\theta$, with $0 \le \theta \le \pi$. Hence $I = \lambda R^3\pi$ and $M = \lambda R\pi$, which again gives $I = MR^2$.

Step 3: Compare with the form in the question The given expression is

$$I = \frac{1}{x}MR^2.$$

Equating its coefficient with the integrated result gives

$$\frac{1}{x} = 1 \quad \Longrightarrow \quad x = 1.$$

A factor of one-half would apply only if the answer were being expressed using the mass of a full ring

having the same linear density. The present symbol M already denotes only the semicircular ring, so there is no additional mass reduction. This is also a ring, rather than a filled semicircular lamina, so a disk formula is inappropriate.

Step 4: Conclusion The moment of inertia is MR^2, and therefore $\boxed{x = 1}$.

Quick Tip to Solve: Whenever every mass element has the same distance from the axis, use $I = MR^2$ directly; check carefully which object the given mass describes.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Butan -1- ol has higher boiling point than ethoxyethane.

Reason R: Extensive hydrogen bonding leads to stronger association of molecules.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) A is false but R is true

(B) Both A and R are true but R is not the correct explanation of A

(C) Both A and R are true and R is the correct explanation of A

(D) A is true but R is false

Correct Answer: (C) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Boiling requires molecules to overcome intermolecular attractions and enter the vapour phase. Stronger association generally raises boiling point for comparable molecular masses.
- An alcohol has an $\mathrm{O{-}H}$ bond and oxygen lone pairs, so its molecules can donate and accept hydrogen bonds. An ether has oxygen lone pairs but no $\mathrm{O{-}H}$ bond.

Step 1: Assess the assertion using molecular structures Butan-1-ol is $\mathrm{CH_3CH_2CH_2CH_2OH}$, whereas ethoxyethane is $\mathrm{CH_3CH_2OCH_2CH_3}$. Both have the molecular formula $\mathrm{C_4H_{10}O}$, so their molar masses are equal. However, butan-1-ol molecules associate through intermolecular $\mathrm{O{-}H\cdots O}$ hydrogen bonds. Ethoxyethane molecules cannot form an analogous hydrogen-bond network among themselves because they contain no hydrogen directly bonded to oxygen. The alcohol therefore has the higher boiling point, and Assertion A is true.

Step 2: Assess the reason independently A hydrogen bond is an attractive interaction involving a hydrogen attached to an electronegative atom and a lone pair on a neighbouring electronegative atom. Extensive intermolecular hydrogen bonding links molecules into associated groups. Separating these associated molecules during vaporisation requires additional energy. Thus the reason correctly states that extensive hydrogen bonding produces stronger molecular association; Reason R is true.

Step 3: Test the explanatory link and the alternatives The association described in R occurs extensively

in butan-1-ol but not between pure ethoxyethane molecules. It explains why the alcohol needs a higher temperature to achieve the vapour pressure required for boiling. The ether can accept hydrogen bonds from water, but that does not give it self-association in its pure liquid. Option A wrongly rejects the assertion, option D wrongly rejects the reason, and option B wrongly denies the direct explanatory connection.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (C).

Quick Tip to Solve: For isomeric alcohols and ethers, look for an $O-H$ donor: an ether oxygen can accept a hydrogen bond but cannot create a self-associated network alone.

62. Which of the following complex is octahedral, diamagnetic and the most stable? [4 marks]

(A) $Na_3\,[CoCl_6]$

(B) $[Co\,(H_2O)_6]\,Cl_2$

(C) $K_3\,[Co(CN)_6]$

(D) $[Ni\,(NH_3)_6]\,Cl_2$

Correct Answer: (C) $K_3\,[Co(CN)_6]$

Solution:

Concept:

- An octahedral complex has six donor atoms surrounding the metal. Its five d orbitals split into lower t_{2g} and higher e_g sets.
- Diamagnetism requires all electrons to be paired. A strong-field ligand such as CN^- can produce low-spin d^6, with configuration $t_{2g}^6 e_g^0$, whereas weak-field ligands favour unpaired electrons.

Step 1: Find the metal oxidation states and electron counts In $[CoCl_6]^{3-}$ and $[Co(CN)_6]^{3-}$, six singly negative ligands imply cobalt in the $+3$ state. Thus each has a d^6 metal ion. In $[Co(H_2O)_6]^{2+}$, water is neutral and cobalt is $+2$, hence d^7. In $[Ni(NH_3)_6]^{2+}$, nickel is $+2$, hence d^8. All four listed coordination entities have coordination number six.

Step 2: Select the diamagnetic octahedral configuration Cyanide is a strong-field ligand, particularly with Co^{3+}. It produces a sufficiently large octahedral splitting for all six d electrons to pair in the lower orbitals:

$$[Co(CN)_6]^{3-}: \qquad t_{2g}^6 e_g^0.$$

There are no unpaired electrons, so this complex is diamagnetic. Its filled lower set gives a crystal-field stabilisation of $-2.4\Delta_o$ before including pairing energy, and no electrons occupy the upper antibonding e_g set.

Step 3: Eliminate the other complexes and assess stability Chloride is weak field, so $[CoCl_6]^{3-}$ is high-spin d^6, with $t_{2g}^4 e_g^2$ and four unpaired electrons. The aqua cobalt(II) complex is high-spin d^7, with three unpaired electrons. Octahedral nickel(II), d^8, has $t_{2g}^6 e_g^2$ and two unpaired electrons even with ammonia. These three fail the diamagnetism condition. The strong cobalt(III)-cyanide bonding and low-spin filled t_{2g} set identify the particularly stable complex among the choices.

Step 4: Conclusion $K_3[Co(CN)_6]$ contains the octahedral, diamagnetic and strongly stabilised low-spin cobalt(III) complex. Hence, the correct option is (C).

Quick Tip to Solve: Calculate oxidation state before counting d electrons; low-spin octahedral d^6 has a filled t_{2g} set and no unpaired electrons.

63. The correct order of electronegativity for given elements is: [4 marks]

(A) $C > P > At > Br$

(B) $Br > C > At > P$

(C) $Br > P > At > C$

(D) $P > Br > C > At$

Correct Answer: (B) $Br > C > At > P$

Solution:

Concept:

- Electronegativity describes an atom's tendency to attract a shared pair of electrons in a chemical bond. Comparisons must use values on the same scale.
- Across a period electronegativity generally increases, while down a group it generally decreases because greater size and shielding weaken attraction for bonding electrons. For elements in different periods and groups, numerical comparison resolves close cases.

Step 1: Compare the two halogens Bromine and astatine belong to group 17. Astatine is much lower in the group and has a larger atomic size and stronger shielding. Therefore bromine attracts a bonding pair more strongly:

$$Br > At.$$

Any proposed order that places bromine below all the other listed elements is immediately inconsistent with this comparison and with bromine's relatively high electronegativity.

Step 2: Place carbon and phosphorus using their values On the commonly used Pauling scale, useful approximate values are

$$\chi(Br) = 2.96, \qquad \chi(C) = 2.55,$$

$$\chi(At) \simeq 2.2, \qquad \chi(P) = 2.19.$$

Thus carbon lies below bromine but above astatine. Astatine and phosphorus are close; the usual tabulated values place astatine slightly above phosphorus. A simple left-to-right rule alone cannot compare all these elements because they occupy different periods.

Step 3: Arrange the sequence and reject the alternatives Putting the four values in decreasing order yields

$$Br > C > At > P.$$

This is option B. Option A incorrectly puts bromine last despite its high value. Option C incorrectly puts phosphorus above carbon and also puts carbon below astatine. Option D incorrectly puts phosphorus first, although its electronegativity is the smallest of the four on the stated scale. Electronegativity

should not be replaced by electron gain enthalpy or atomic size as if they were identical quantities.

Step 4: Conclusion The correct decreasing order is $\boxed{\mathrm{Br} > \mathrm{C} > \mathrm{At} > \mathrm{P}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use periodic trends for the obvious bromine comparison, then compare tabulated values for the close astatine-phosphorus pair.

64. The correct order of spin only magnetic moments for the following complex ions is [4 marks]

(A) $[Fe(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-} < [Mn(CN)_6]^{3-}$

(B) $[MnBr_4]^{2-} < [CoF_6]^{3-} < [Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-}$

(C) $[Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-}$

(D) $[CoF_6]^{3-} < [MnBr_4]^{2-} < [Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-}$

Correct Answer: (C) $[Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-}$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons and μ_B is the Bohr magneton. It increases with n.
- Find the metal oxidation state before counting its d electrons. Cyanide is a strong-field ligand and favours low-spin octahedral configurations; fluoride and bromide produce the high-spin configurations relevant here.

Step 1: Assign the oxidation states and d counts Each cyanide and each halide ligand has charge -1. In $[Fe(CN)_6]^{3-}$, iron is $+3$, giving d^5. In $[Mn(CN)_6]^{3-}$, manganese is also $+3$, giving d^4. Cobalt in $[CoF_6]^{3-}$ is $+3$, so it is d^6. Finally, manganese in $[MnBr_4]^{2-}$ is $+2$, hence d^5.

Step 2: Count the unpaired electrons using ligand-field strength Low-spin octahedral d^5 has configuration t_{2g}^5, with one unpaired electron. Low-spin octahedral d^4 has t_{2g}^4, with two unpaired electrons. Weak-field octahedral d^6 has $t_{2g}^4 e_g^2$, with four unpaired electrons. The tetrahedral bromide complex of Mn^{2+} is high-spin d^5, with five unpaired electrons. Thus the four moments, in increasing order, are $\sqrt{3}, \sqrt{8}, \sqrt{24}, \sqrt{35}$ Bohr magnetons.

Step 3: Compare the complete order with the options The required order is

$$[Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-}.$$

Option A incorrectly puts the low-spin manganese complex last; B and D also contradict the smallest moment of the iron cyanide complex. Only C follows the unpaired-electron counts $1 < 2 < 4 < 5$.

Step 4: Conclusion The increasing spin-only magnetic moments follow the order in option (C). Hence, the correct option is (C).

Quick Tip to Solve: Write oxidation state, d count and unpaired-electron count in three columns; compare the counts before calculating any square roots.

65.

$$2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2O$$

What is the value of x ? [4 marks]

(A) 10

(B) 2

(C) 12

(D) 6

Correct Answer: (A) 10

Solution:

Concept:

- A balanced redox equation conserves both the number of atoms of each element and total electric charge. Oxidation states show how many electrons each reactant gains or loses.
- Iodine in IO_3^- has oxidation state $+5$, whereas iodine in I^- is -1. Both form elemental iodine, whose oxidation state is zero.

Step 1: Balance iodine atoms and check the charge The products contain six iodine molecules, so there are 12 iodine atoms on the right. Two iodate ions supply two iodine atoms; the remaining ten must come from iodide ions. Therefore

$$2 + x = 2(6), \qquad x = 10.$$

The reactant charge is $2(-1) + 10(-1) + 12(+1) = 0$, equal to the charge of the neutral products. Oxygen is already balanced at six atoms and hydrogen at twelve atoms.

Step 2: Confirm the electron transfer by half-reactions Reduction of iodate in acid can be written as

$$2\,IO_3^- + 12\,H^+ + 10\,e^- \longrightarrow I_2 + 6\,H_2O.$$

Oxidation of iodide supplies the same ten electrons:

$$10\,I^- \longrightarrow 5\,I_2 + 10\,e^-.$$

Adding and cancelling electrons yields $2\,IO_3^- + 10\,I^- + 12\,H^+ \longrightarrow 6\,I_2 + 6\,H_2O$, exactly the printed equation with $x = 10$. This also shows why five iodide ions are required per iodate ion.

Step 3: Reject the other coefficients Values 2, 12 and 6 would place respectively 4, 14 and 8 iodine atoms on the reactant side instead of the required 12. They also fail the zero-charge check with the fixed acid coefficient. Thus no change to the printed oxygen or hydrogen coefficients is required.

Step 4: Conclusion The coefficient is $x = 10$. Hence, the correct option is (A).

Quick Tip to Solve: When every other coefficient is fixed, first count iodine atoms; then verify the result with total charge to catch redox-balancing errors.

66. Match List I with List II. Benzenediazonium chloride, $C_6H_5N_2^+Cl^-$, is reacted with the reagents in List I to form the products in List II.

	List I (Reagent)		List II (Product)
A	**Aniline,** $C_6H_5NH_2$	**I**	**Fluorobenzene,** C_6H_5F
B	HBF_4**, heat**	**II**	**Benzonitrile,** C_6H_5CN
C	Cu, HCl	**III**	p**-Aminoazobenzene**
D	$CuCN/KCN$	**IV**	**Chlorobenzene,** C_6H_5Cl

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-III, B-I, C-II, D-IV

(C) A-IV, B-III, C-II, D-I

(D) A-I, B-III, C-IV, D-II

Correct Answer: (A) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Aromatic diazonium salts can undergo replacement of the diazonium group with loss of N_2, or azo coupling with an activated aromatic ring.
- Aniline activates its ring towards coupling, normally at the para position. Fluorination, chlorination and cyanation require different reagent sets, so identify the new group supplied by each reagent.

Step 1: Match the coupling reaction and fluorination Reagent A is aniline. Its electron-rich ring couples with benzenediazonium chloride, primarily at the para position relative to NH_2, producing p-aminoazobenzene:

$$C_6H_5{-}N{=}N{-}C_6H_4{-}NH_2.$$

This is product III, so A-III. With reagent B, HBF_4 converts the diazonium salt to its tetrafluoroborate. Heating gives fluorobenzene with loss of nitrogen and boron trifluoride, the Balz-Schiemann reaction. Fluorobenzene is product I, giving B-I.

Step 2: Match chlorination and cyanation Copper powder with hydrochloric acid replaces the diazonium group by chlorine in the Gattermann reaction. Therefore C gives chlorobenzene, product IV. Treatment with $CuCN/KCN$ introduces the cyano group on the aromatic ring, yielding benzonitrile, C_6H_5CN, product II. Consequently D-II. The nitrogen atoms of the original diazonium group leave as molecular nitrogen in both substitution reactions.

Step 3: Assemble the mapping and compare each alternative The complete correspondence is

$$A - III, \quad B - I, \quad C - IV, \quad D - II.$$

Option B reverses the chloride and nitrile products. Option C wrongly assigns chlorobenzene to aniline coupling and an azo product to fluorination. Option D also swaps the coupling and fluorination products. Only option A assigns every reagent to its chemically appropriate product.

Step 4: Conclusion The matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (A).

Quick Tip to Solve: Memorise the reagent-to-new-group pairs: heated tetrafluoroborate gives F, Cu/HCl gives Cl, CuCN gives CN; aniline gives azo coupling.

67. Which halogen is known to cause the reaction given below :

$$2Cu^{2+} + 4X^- \rightarrow Cu_2X_2(s) + X_2$$

[4 marks]

(A) All halogens

(B) Only Iodine

(C) Only Bromine

(D) Only Chlorine

Correct Answer: (B) Only Iodine

Solution:

Concept:

- A halide can reduce Cu^{2+} to Cu^{+} only if oxidation of that halide is sufficiently favourable under the reaction conditions.
- Iodide is a stronger reducing agent than bromide or chloride. Insoluble copper(I) iodide removes Cu^{+} from solution, driving its formation along with iodine.

Step 1: Identify oxidation and reduction in the printed equation In Cu_2X_2, the two halide ions have total charge -2, so each copper is in oxidation state $+1$. The two copper(II) ions therefore gain two electrons in total. Of the four halide ions supplied, two remain as halide in the precipitate, while two lose one electron each and form the elemental halogen:

$$2\,X^- \longrightarrow X_2 + 2\,e^-.$$

Thus the halide must serve as a reducing agent, rather than merely supply the counter-ion of a copper(II) salt.

Step 2: Use the special stability of copper(I) iodide Iodide undergoes this oxidation readily in the presence of copper(II), and copper(I) is precipitated as very sparingly soluble CuI:

$$2\,Cu^{2+} + 4\,I^- \longrightarrow 2\,CuI\,(s) + I_2.$$

Writing the precipitate as Cu_2I_2 gives the same stoichiometry as the question. Removal of copper(I) by precipitation is essential to understanding why this reaction proceeds; the behaviour cannot be judged from the free aqueous copper ions alone.

Step 3: Compare the halogens in the alternatives Chloride and bromide are weaker reducing agents than iodide and do not bring about the displayed copper(II)-to-copper(I) reaction in this context. It is therefore incorrect to generalise the reaction to all halogens. The chloride-only and bromide-only choices also identify the wrong halide for this characteristic reaction, which is used in iodometric estimation of copper.

Step 4: Conclusion Here $X = I$; only iodine, through iodide ions, gives the stated reaction. Hence, the correct option is (B).

Quick Tip to Solve: For copper(II) plus iodide, remember the paired observations: a copper(I) iodide precipitate and liberated iodine; include both changes in the equation.

68. In chromyl chloride, the number of d-electrons present on chromium is same as in

(Given at no. of $\mathrm{Ti}:22,\ \mathrm{V}:23, \mathrm{Cr}:24, \mathrm{Mn}:25, \mathrm{Fe}:26$ **)** [4 marks]

(A) Mn (VII)

(B) Ti (III)

(C) Fe (III)

(D) V (IV)

Correct Answer: (A) Mn (VII)

Solution:

Concept:

- For a transition-metal ion in these oxidation states, the d-electron count is the group number minus the oxidation state. The outer s electrons are removed before the d electrons.
- Chromyl chloride is $\mathrm{CrO_2Cl_2}$, a neutral compound. Oxygen contributes -2 and chloride -1 when finding the chromium oxidation state.

Step 1: Calculate chromium oxidation state and electron count Let the chromium oxidation state in $\mathrm{CrO_2Cl_2}$ be z. Neutrality gives

$$z + 2(-2) + 2(-1) = 0, \qquad z = +6.$$

Chromium belongs to group 6. Its formal $+6$ state therefore has $6 - 6 = 0$ d electrons, conventionally written d^0. We must match this count, rather than the atomic number or the numerical oxidation state alone.

Step 2: Count the d electrons in every candidate Manganese belongs to group 7, so Mn(VII) has $7 - 7 = 0$ d electrons. Titanium is in group 4, so Ti(III) is d^1. Iron is in group 8, making Fe(III) d^5. Vanadium belongs to group 5, so V(IV) is d^1. Thus

$$\mathrm{Mn(VII)} : d^0, \quad \mathrm{Ti(III)} : d^1,$$
$$\mathrm{Fe(III)} : d^5, \quad \mathrm{V(IV)} : d^1.$$

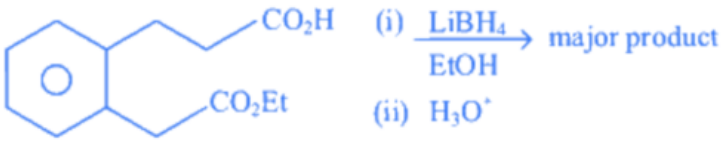
Only manganese in the listed oxidation state matches chromium in chromyl chloride.

Step 3: Distinguish electron count from oxidation state Chromium(VI) and manganese(VII) have different oxidation states, but each has lost all the valence d and s electrons counted for its neutral atom. Both therefore have a zero d-electron count. The Ti(III) and V(IV) choices are tempting because they are highly oxidised early transition ions, but each still retains one d electron. Fe(III) retains five and is also excluded.

Step 4: Conclusion Chromium in chromyl chloride and Mn(VII) are both d^0. Hence, the correct option is (A).

Quick Tip to Solve: Find the oxidation state from the actual formula first, then use group number minus oxidation state; identical oxidation states are not needed for identical d counts.

69. The major product formed in the following reaction is: [4 marks]



(A) o-$C_6H_4(CH_2CH_2CH_2OH)(CH_2CH_2OH)$

(B) o-$C_6H_4(CH_2CH_2CO_2H)(CH_2CO_2H)$

(C) o-$C_6H_4(CH_2CH_2CH_2OH)(CH_2CO_2Et)$

(D) o-$C_6H_4(CH_2CH_2CO_2H)(CH_2CH_2OH)$

Correct Answer: (D) o-$C_6H_4(CH_2CH_2CO_2H)(CH_2CH_2OH)$

Solution:

Concept:

- Lithium borohydride, $LiBH_4$, reduces esters to primary alcohols. Under the stated selective conditions, an ester is reduced in the presence of a carboxylic acid group.
- The carbon of the ester carbonyl remains in the organic skeleton as CH_2OH. Acidic work-up protonates the alkoxide product and restores any carboxylate to the carboxylic acid.

Step 1: Identify the two different carbonyl groups The starting aromatic compound bears two adjacent side chains. One is $-CH_2CH_2CO_2H$, ending in a free carboxylic acid. The other is $-CH_2CO_2Et$, ending in an ethyl ester. Although both contain carbonyl groups, they are not equivalent functional groups towards the reagent. The benzene ring and the carbon-carbon side-chain bonds remain intact throughout the transformation.

Step 2: Apply the selective ester reduction and work-up With $LiBH_4$ in ethanol followed by H_3O^+, reduce the ester side chain:

$$-CH_2CO_2Et \longrightarrow -CH_2CH_2OH.$$

The ester carbonyl carbon becomes the terminal alcohol carbon, so the original methylene next to the ring is retained. The other side chain remains $-CH_2CH_2CO_2H$ after acid work-up. The ethoxy portion of the ester leaves the principal organic skeleton; it does not become an additional carbon in the retained side chain.

Step 3: Compare the products without changing the carbon skeleton The product is consequently the ortho-disubstituted benzene with $-CH_2CH_2CO_2H$ and $-CH_2CH_2OH$ groups. Option A incorrectly reduces both groups. Option B corresponds to converting the ester to an acid rather than reducing it. Option C reverses the selectivity, reducing the acid while leaving the ester unchanged. Option D alone retains the acid and converts the ester to the primary alcohol.

Step 4: Conclusion The ester side chain becomes $-CH_2CH_2OH$, while the $-CH_2CH_2CO_2H$ side chain remains. Hence, the correct option is (D).

Quick Tip to Solve: Track the carbonyl carbon during ester reduction: it becomes the alcohol carbon. Then check separately which other functional groups the reagent leaves unchanged.

70. The reaction

$$\frac{1}{2}H_2(\ g) + AgCl(s) \rightleftharpoons H^+(aq) + Cl^-(aq) + Ag(s)$$

occurs in which of the given galvanic cell. [4 marks]

(A) $Ag|AgCl(s)|KCl\,(sol^n)\,|AgNO_3|\,Ag$

(B) $Pt\,|H_2(\ g)|\,HCl\,(sol^n)\,|AgCl(s)|Ag$

(C) $Pt\,|H_2(\ g)|\,HCl\,(sol^n)\,|AgNO_3\,(sol^n)|\,Ag$

(D) $Pt\,|H_2(\ g)|\,KCl\,(sol^n)\,|AgCl(s)|Ag$

Correct Answer: (B) $Pt\,|H_2(\ g)|\,HCl\,(sol^n)\,|AgCl(s)|Ag$

Solution:

Concept:

- In galvanic-cell notation the left electrode is written for oxidation and the right electrode for reduction. The overall reaction is obtained by adding the two half-reactions with electrons cancelled.
- A hydrogen electrode requires an inert platinum surface, hydrogen gas and an acid solution. A silver-silver chloride electrode contains silver, solid silver chloride and chloride ions in solution.

Step 1: Split the given reaction into electrode processes The hydrogen gas on the reactant side is oxidised at the left electrode:

$$\tfrac{1}{2}H_2(g) \longrightarrow H^+(aq) + e^-.$$

At the right electrode, silver chloride accepts the electron and gives silver metal and chloride:

$$AgCl(s) + e^- \longrightarrow Ag(s) + Cl^-(aq).$$

Adding the two expressions cancels one electron and reproduces all reactants and products, including the half-mole coefficient of hydrogen in the printed equation.

Step 2: Choose the solution and electrode phases Hydrochloric acid supplies both H^+ for the hydrogen electrode and Cl^- for the silver-silver chloride electrode. Platinum provides the conducting inert surface on the hydrogen side; silver metal is the conductor on the other side. The required sequence of phases is

$$Pt\ |\ H_2(g)\ |\ HCl(aq)\ |\ AgCl(s)\ |\ Ag.$$

Both electrodes can contact the common hydrochloric acid solution, so the printed notation has no need to introduce a different salt solution between separate half-cells.

Step 3: Exclude cells with the wrong participating species Option A has no hydrogen electrode and cannot oxidise hydrogen. Option C uses a silver nitrate-silver electrode, whose reduction is $Ag^+ + e^- \longrightarrow Ag$, rather than the required silver chloride reaction. Option D places the hydrogen electrode in potassium chloride, which does not provide the required acid medium. Option B contains exactly the hydrogen-acid and silver-silver chloride electrode combination.

Step 4: Conclusion The cell is $Pt|H_2(g)|HCl(aq)|AgCl(s)|Ag$. Hence, the correct option is (B).

Quick Tip to Solve: Write and add the two half-reactions before comparing cell symbols; the acid solution must supply hydrogen ions and chloride for this particular pair.

71. Sulphur (S) containing amino acids from the following are:

(a) isoleucine (b) cysteine (c) lysine (d) methionine (e) glutamic acid [4 marks]

(A) b, d

(B) a, d

(C) b, c, e

(D) a, b, c

Correct Answer: (A) b, d

Solution:

Concept:

- An amino acid contains both an amino group and a carboxyl group. Its side chain determines the additional elements present and its characteristic chemical properties.
- Sulphur occurs in the side chains of cysteine and methionine: cysteine contains a thiol group, whereas methionine contains a thioether linkage.

Step 1: Inspect the sulphur-containing side chains Cysteine, entry (b), has the side chain $-CH_2SH$. Its terminal $-SH$ group explicitly contains one sulphur atom. Two cysteine residues can also form a disulphide bond on oxidation, an important structural feature of many proteins. Methionine, entry (d), has the side chain $-CH_2CH_2SCH_3$. Here sulphur is bonded to two carbon groups, so it is a thioether rather than a thiol. Both therefore satisfy the question irrespective of their different sulphur functional groups.

Step 2: Exclude the other listed amino acids Isoleucine, entry (a), has a branched hydrocarbon side chain and contains no sulphur. Lysine, entry (c), has an additional amino group in its side chain; the extra heteroatom is nitrogen, not sulphur. Glutamic acid, entry (e), has an additional carboxyl group and hence extra oxygen, not sulphur. Thus the qualifying pair is exactly (b), (d). Option (B) incorrectly includes isoleucine, while options (C) and (D) include nonsulphur amino acids and do not give this pair.

Step 3: Conclusion The sulphur-containing amino acids are cysteine and methionine. Hence, the correct option is (A).

Quick Tip to Solve: Recall the side-chain fragments $-SH$ in cysteine and $-S-$ in methionine; these identify the two sulphur amino acids immediately.

72. The water gas on reacting with cobalt as a catalyst forms [4 marks]

(A) Methanoic acid

(B) Methanol

(C) Methanal

(D) Ethanol

Correct Answer: (B) Methanol

Solution:

Concept:

- Water gas is a mixture of carbon monoxide and hydrogen, CO and H_2. It can serve as synthesis gas for making oxygen-containing organic compounds.
- Catalytic hydrogenation adds hydrogen to carbon monoxide. For methanol formation the balanced stoichiometry is $CO + 2\,H_2 \longrightarrow CH_3OH$.

Step 1: Identify the reactants and the product class The carbon-containing component of water gas is CO. Hydrogen supplies the additional hydrogen atoms needed to reduce it to the corresponding one-carbon alcohol. In the catalytic conversion intended in the question, cobalt promotes the synthesis of methanol. The reaction does not require a second carbon-containing reagent, and the desired product retains the one-carbon skeleton of the starting carbon monoxide.

Step 2: Balance the hydrogenation reaction The net reaction is

$$CO + 2\,H_2 \xrightarrow{\text{catalyst}} CH_3OH.$$

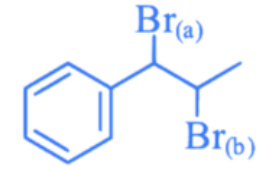
There is one carbon and one oxygen atom on each side; the four hydrogen atoms supplied by two hydrogen molecules appear in the alcohol. The CH_3OH formula corresponds to methanol. Methanal, $HCHO$, has only two hydrogen atoms and is an aldehyde, so it is not the stated alcohol-synthesis product.

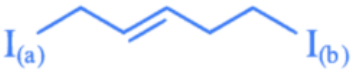
Step 3: Check the remaining choices Methanoic acid, $HCOOH$, has two oxygen atoms and is a carboxylic acid; it is not the product of the displayed hydrogenation. Ethanol, C_2H_5OH, contains two carbon atoms and requires carbon-chain growth rather than the one-carbon hydrogenation considered here. Thus neither the acid nor ethanol matches this preparation. Distinguishing methanol from methanal is essential because both names begin with the one-carbon prefix meth-.

Step 4: Conclusion Water gas gives methanol, CH_3OH, in the catalytic conversion stated. Hence, the correct option is (B).

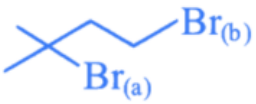
Quick Tip to Solve: For synthesis-gas questions, write $CO + 2\,H_2 \longrightarrow CH_3OH$ and check the carbon count before choosing among methanol, methanal and ethanol.

73. Choose the halogen which is most reactive towards S_N1 reaction in the given compounds (A, B, C & D) [4 marks]

A. 

B. 

C. 

D. 

(A) $A - Br_{(b)}; B - I_{(a)}; C - Br_{(a)}; D - Br_{(a)}$

(B) $A - Br_{(a)}; B - I_{(a)}; C - Br_{(a)}; D - Br_{(a)}$

(C) $A - Br_{(b)}; B - I_{(b)}; C - Br_{(b)}; D - Br_{(b)}$

(D) $A - Br_{(a)}; B - I_{(a)}; C - Br_{(b)}; D - Br_{(a)}$

Correct Answer: (D) $A - Br_{(a)}; B - I_{(a)}; C - Br_{(b)}; D - Br_{(a)}$

Solution:

Concept:

- In an S_N1 reaction, the slow ionisation step breaks the carbon–halogen bond to form a carbocation. Greater stabilisation of this carbocation usually gives faster ionisation.
- Benzylic and allylic carbocations gain resonance stabilisation; ordinary tertiary carbocations are stabilised by alkyl groups. A small rigid bridgehead cannot readily acquire the planar geometry required for a conventional carbocation.

Step 1: Compare the two bromines in A Loss of $Br_{(a)}$ places the positive charge on the carbon directly adjacent to the benzene ring. This benzylic carbocation delocalises its charge into the aromatic system. Loss of $Br_{(b)}$ instead initially gives an ordinary secondary carbocation without the same direct resonance stabilisation. Therefore A reacts most readily at $Br_{(a)}$.

Step 2: Identify the allylic iodide in B The carbon bearing $I_{(a)}$ is adjacent to the double bond, so ionisation at this end gives an allylic carbocation. Its charge is spread over the allylic system. The carbon bearing $I_{(b)}$ is farther from the double bond and does not receive this direct resonance stabilisation. Since the leaving group is iodine in both cases, carbocation stability selects $I_{(a)}$.

Step 3: Check the rigid framework C and the branched compound D In C, $Br_{(a)}$ is attached to a bridgehead of the small bridged ring. A planar carbocation there is geometrically disfavoured. The nonbridgehead, methyl-substituted carbon carrying $Br_{(b)}$ can form the more favourable carbocation, so C selects $Br_{(b)}$. In D, $Br_{(a)}$ is on a tertiary carbon, whereas $Br_{(b)}$ is on a primary terminal carbon.

Ionisation therefore favours $Br_{(a)}$. Options (A) and (C) choose the wrong site in A; option (B) incorrectly chooses the bridgehead site in C.

Step 4: Conclusion The selected combination uses the benzylic bromine, allylic iodine, nonbridgehead bromine and tertiary bromine, respectively. Hence, the correct option is (D).

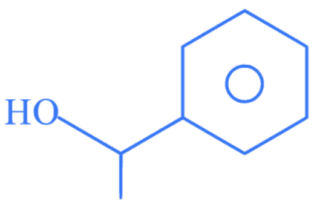
Quick Tip to Solve: Mark the carbocation left after each halogen departs; check resonance first, then ring geometry and alkyl substitution.

74. Match List I with List II:

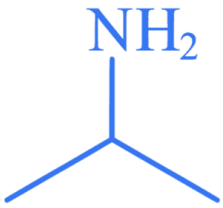
	LIST I (Reagents used)		LIST II (Compound with Functional group detected)
A.	Alkaline solution of copper sulphate and sodium citrate	I.	Compound I (shown below)
B.	Neutral $FeCl_3$	II.	Compound II (shown below)
C.	Alkaline chloroform solution	III.	Compound III (shown below)
D.	Potassium iodide and sodium hypochlorite	IV.	Compound IV (shown below)

Choose the correct answer from the options given below: [4 marks]

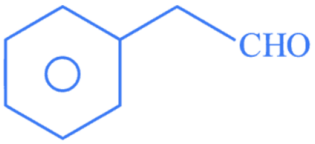
I



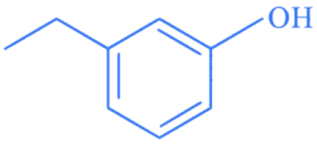
II



III



IV



(A) A-IV, B-I, C-II, D-III

(B) A-III, B-IV, C-I, D-II

(C) A-III, B-IV, C-II, D-I

(D) A-II, B-IV, C-III, D-I

Correct Answer: (C) A-III, B-IV, C-II, D-I

Solution:

Concept:

- Functional-group tests distinguish compounds by characteristic reactions. Benedict's reagent is an alkaline copper(II) sulphate–citrate solution that detects readily oxidised aliphatic aldehyde groups.
- Neutral ferric chloride detects phenolic hydroxyl groups; the carbylamine test detects primary amines. Iodoform reagents detect a methyl ketone or an alcohol oxidisable to one.

Step 1: Match the copper reagent and ferric chloride Compound III contains the side chain $-CH_2CHO$, so its aldehyde group is aliphatic even though a phenyl ring occurs elsewhere in the molecule. It reduces the alkaline copper(II) reagent to a red precipitate of Cu_2O. Hence A–III. Compound IV has an $-OH$ group directly attached to its benzene ring: it is a phenol and gives the characteristic coloured complex with neutral $FeCl_3$. Hence B–IV. The alcohol in I is not phenolic because its hydroxyl group is bonded to a side-chain carbon.

Step 2: Apply the carbylamine test Compound II is a primary amine, $(CH_3)_2CHNH_2$. Primary amines react with chloroform and alcoholic alkali to form isocyanides, the basis of the carbylamine test. In general,

$$RNH_2 + CHCl_3 + 3\,KOH \longrightarrow RNC + 3\,KCl + 3\,H_2O.$$

Thus the alkaline chloroform reagent C detects II. Neither a phenol nor an aldehyde supplies the primary amino group needed for this reaction.

Step 3: Identify the iodoform-positive alcohol Potassium iodide with sodium hypochlorite generates the active iodine reagent for the iodoform test. Compound I contains $CH_3CH(OH)^-$, which is oxidised to a methyl ketone and then gives yellow CHI_3. Therefore D–I. Combining the independently identified matches gives A–III, B–IV, C–II and D–I. Option (A) assigns the copper reagent to a phenol; (B) interchanges the amine and alcohol tests; (D) incorrectly sends the amine to the copper reagent.

Step 4: Conclusion The complete matching is A–III, B–IV, C–II and D–I. Hence, the correct option is (C).

Quick Tip to Solve: Spot the diagnostic groups first: $-CHO$ for Benedict's, phenolic $-OH$ for ferric chloride, $-NH_2$ for carbylamine and $CH_3CH(OH)^-$ for iodoform.

75. Match List I with List II:

	LIST I (Species)		**LIST II (Maximum allowed concentration in ppm in drinking water)**
A.	F^-	**I.**	**< 50 ppm**
B.	SO_4^{2-}	**II.**	**< 5 ppm**
C.	NO_3^-	**III.**	**< 2 ppm**
D.	Zn	**IV.**	**< 500 ppm**

Choose the correct answer from the options given below : [4 marks]

(A) A-IV, B-III, C-II, D-I

(B) A-I, B-II, C-III, D-IV

(C) A-II, B-I, C-III, D-IV

(D) A-III, B-IV C-I, D-II

Correct Answer: (D) A-III, B-IV C-I, D-II

Solution:

Concept:

- Drinking-water quality places upper limits on dissolved species because excess concentrations can harm health or make water unsuitable for consumption. The units here are parts per million (ppm).
- For the limits listed in this question, fluoride corresponds to below 2 ppm, sulphate to below 500 ppm, nitrate to below 50 ppm and zinc to below 5 ppm.

Step 1: Match fluoride and sulphate to their limits Fluoride is beneficial only within a controlled range; an excess can damage teeth and, at higher exposure, bones. From the supplied upper-limit categories, F^- matches III, $< 2\,\text{ppm}$. This is an upper-limit matching question, not a statement that any concentration below this bound is the optimum fluoride level. Sulphate at high concentration causes undesirable effects, including a laxative effect in drinking water. The relevant listed upper bound is $< 500\,\text{ppm}$, so $SO_4{}^{2-}$ matches IV.

Step 2: Match nitrate and zinc Excess nitrate is especially harmful to infants because conversion to nitrite can impair oxygen transport by haemoglobin. The listed maximum category for $NO_3{}^-$ is $< 50\,\text{ppm}$, which is I. Zinc is assigned the much smaller listed bound $< 5\,\text{ppm}$, which is II. Keeping the units identical is important: no conversion from ppm to a different concentration unit is needed to make these matches.

Step 3: Assemble and check the complete code The result is A–III, B–IV, C–I, D–II. Only option (D) contains all four assignments. Option (A) assigns 500 ppm to fluoride; option (B) assigns 50 ppm to fluoride; option (C) assigns 5 ppm to fluoride and 50 ppm to sulphate. These discrepancies reject all three alternatives without changing any of the limits stated in List II.

Step 4: Conclusion The matching is F^-–III, $SO_4{}^{2-}$–IV, $NO_3{}^-$–I and Zn–II. Hence, the correct option is (D).

Quick Tip to Solve: Remember the numerical sequence for fluoride, sulphate, nitrate and zinc as 2, 500, 50 and 5 ppm, then translate these values to the Roman labels.

76. Match List I with List II:

	LIST I		LIST II
A.	Saccharin	I.	High potency sweetener
B.	Aspartame	II.	First artificial sweetening agent
C.	Alitame	III.	Stable at cooking temperature
D.	Sucralose	IV.	Unstable at cooking temperature

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-III, C-IV, D-I

(B) A-II, B-IV, C-I, D-III

(C) A-II, B-IV, C-III, D-I

(D) A-IV, B-III, C-I, D-II

Correct Answer: (B) A-II, B-IV, C-I, D-III

Solution:

Concept:

- Artificial sweeteners supply sweetness in small amounts. Their usefulness depends on sweetness potency and thermal stability as well as their discovery history.
- Saccharin was the first artificial sweetening agent. Aspartame is unstable at cooking temperature, alitame is a high-potency sweetener, and sucralose remains stable at cooking temperature.

Step 1: Fix the historical and temperature-sensitive entries Saccharin is the first artificial sweetening agent among the listed substances, so A–II. Aspartame is a dipeptide-derived sweetener. It decomposes on heating and is therefore mainly suitable for cold foods and soft drinks rather than high-temperature cooking. This makes B–IV. Thermal instability does not mean that aspartame lacks sweetness: it means that the compound does not retain its useful properties during cooking.

Step 2: Distinguish alitame from sucralose Alitame has very high sweetening potency, so the matching characteristic for C is I. Its intense sweetness means that controlling the amount used is important. Sucralose is a chlorinated derivative of sucrose and is stable at cooking temperature. The characteristic assigned to D is therefore III. High potency and cooking stability are distinct descriptors in this list; one should match each named sweetener to the specific property used in the question.

Step 3: Check every proposed matching Combining the four facts yields A–II, B–IV, C–I, D–III. Option (B) gives precisely this code. Option (A) wrongly calls aspartame stable during cooking and alitame unstable. Option (C) interchanges the specified descriptions of alitame and sucralose. Option (D) incorrectly assigns cooking instability to saccharin and the first-artificial-sweetener description to sucralose. Thus the entire matching, rather than one remembered pair, confirms the selection.

Step 4: Conclusion Saccharin–II, aspartame–IV, alitame–I and sucralose–III is the correct matching. Hence, the correct option is (B).

Quick Tip to Solve: Use the anchors saccharin = first, aspartame = cold foods, alitame = high potency, and sucralose = cooking stability.

77. Given below are two statements:

Statement I : Lithium and Magnesium do not form superoxide

Statement II : The ionic radius of Li^+ is larger than ionic radius of Mg^{2+}

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is incorrect but Statement II is correct

(B) Both Statement I and Statement II are incorrect

(C) Both Statement I and Statement II are correct

(D) Statement I is correct but Statement II is incorrect

Correct Answer: (C) Both Statement I and Statement II are correct

Solution:

Concept:

- The oxide formed by a metal depends on the cation's size and charge and on the stability of the oxide, peroxide or superoxide anion. Small highly polarising cations favour the compact oxide anion.
- Lithium and magnesium show a diagonal relationship. Their ionic sizes are comparable, but the usual tabulated radius of Li^+ is slightly larger than that of Mg^{2+} for comparable coordination.

Step 1: Evaluate Statement I Lithium characteristically forms lithium oxide, Li_2O, and magnesium forms magnesium oxide, MgO, on ordinary combustion in oxygen. Neither is a characteristic superoxide former. Superoxide contains O_2^-, a relatively large molecular anion stabilised more readily by large monovalent cations such as potassium, rubidium and caesium. The small lithium ion and the doubly charged magnesium ion do not stabilise the superoxide in the same way. Statement I is therefore correct.

Step 2: Evaluate Statement II independently The ionic-radius comparison must concern the ions actually named, not neutral lithium and magnesium atoms. Common comparable ionic-radius values are about $76\,\text{pm}$ for Li^+ and $72\,\text{pm}$ for Mg^{2+}. The precise tabulated convention can change the individual values, but the stated ordering is

$$r(Li^+) > r(Mg^{2+}).$$

Thus Statement II is also correct. These ions have different electronic configurations, so an argument based on an isoelectronic pair would be inappropriate.

Step 3: Choose the statement combination Since each statement is true on its own, select the choice saying both are correct. Option (A) wrongly rejects the oxide/superoxide observation, option (B) rejects both true statements, and option (D) reverses the valid ionic-size comparison. The question asks for the truth values of two statements, so no additional claim that Statement II explains Statement I is required.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (C).

Quick Tip to Solve: Keep neutral-atom size trends separate from ionic radii; for this diagonal pair, Li^+ is slightly larger than Mg^{2+}.

78. What is the purpose of adding gypsum to cement? [4 marks]

(A) To facilitate the hydration of cement

(B) To speed up the process of setting

(C) To give a hard mass

(D) To slow down the process of setting

Correct Answer: (D) To slow down the process of setting

Solution:

Concept:

- Portland cement contains calcium silicates and calcium aluminates. Its initial setting involves hydration reactions when water is added to the powder.
- Gypsum, $CaSO_4 \cdot 2\,H_2O$, is added in a small proportion as a setting regulator. It particularly controls the very rapid hydration of tricalcium aluminate, $3\,CaO \cdot Al_2O_3$.

Step 1: Identify the process that needs control Without a regulator, the aluminate component reacts rapidly with water and can cause flash setting. The paste then loses workability before it can be mixed, transported and placed properly. The role asked about is therefore control of the initial rate of setting, rather than simply a description of the eventual strength of cement.

Step 2: Explain how gypsum retards setting In the presence of gypsum and water, the aluminate forms a calcium sulfoaluminate hydrate, commonly called ettringite. This reaction controls the rapid aluminate hydration during the early period. Consequently, the cement paste remains workable for a useful time instead of stiffening immediately after water addition.

Step 3: Compare all four choices Option (D) states the required effect: the process of setting is slowed down. Option (B) states the opposite effect. Hydration of cement does occur, but facilitating hydration in general is not the specific reason for adding gypsum, so (A) is unsuitable. Formation of a hard mass is the overall consequence of cement hydration and hardening, especially silicate hydration; it does not identify the special regulating function of gypsum, so (C) is unsuitable.

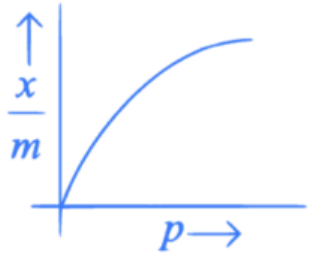
Step 4: Conclusion Gypsum is added to cement to slow down the process of setting and prevent premature stiffening. Hence, the correct option is (D).

Quick Tip to Solve: Associate gypsum with control of flash setting: it provides working time for the fresh cement paste rather than accelerating its setting.

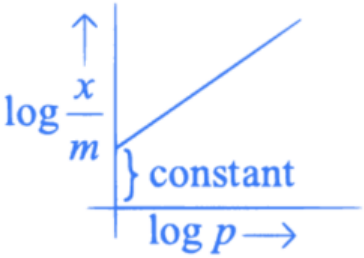
79. Which of the following represent the Freundlich adsorption isotherms?

Choose the correct answer from the options given below: [4 marks]

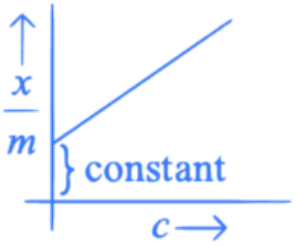
A.



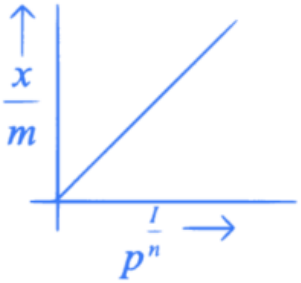
B.



C.



D.



(A) B, C, D only

(B) A, B only

(C) A, B, D only

(D) A, C, D only

Correct Answer: (C) A, B, D only

Solution:

Concept:

- For adsorption of a gas at fixed temperature, the Freundlich equation is $x/m = kp^{1/n}$, where x/m is the mass adsorbed per unit mass of adsorbent, p is pressure, and k, n are empirical constants.
- In the usual intermediate-pressure range, $0 < 1/n < 1$. Taking logarithms gives $\log(x/m) =$

$\log k + (1/n) \log p$.

Step 1: Check the ordinary pressure plot Graph A plots x/m against p. The power $1/n$ lies between zero and one, so the amount adsorbed increases with pressure but its rate of increase decreases. Thus the increasing, downward-concave curve A represents the Freundlich dependence over its range of applicability. The empirical equation should not be extrapolated to arbitrarily high pressure.

Step 2: Check the two straight-line representations Graph B plots $\log(x/m)$ against $\log p$. It must be a straight line of slope $1/n$, with intercept $\log k$, exactly as the logarithmic equation predicts. Graph D uses $p^{1/n}$ on the horizontal axis; hence $x/m = k(p^{1/n})$ is a straight line through the origin with slope k. Both B and D are valid.

Step 3: Reject the incorrect concentration plot For adsorption from solution, the corresponding expression is $x/m = kc^{1/n}$. Graph C instead shows a straight line in x/m versus c with a nonzero intercept. This is not the displayed power law. In particular, the Freundlich expression tends to zero as concentration tends to zero, unlike C. Therefore every answer combination containing C is rejected; the combination containing only A and B also omits the valid graph D.

Step 4: Conclusion The valid representations are A, B and D only. Hence, the correct option is (C).

Quick Tip to Solve: Transform the axes in the equation before comparing graphs: logarithms give intercept $\log k$, while plotting against $p^{1/n}$ gives a line through the origin.

80. Which of the following metals can be extracted through alkali leaching technique? [4 marks]

(A) Sn

(B) Cu

(C) Au

(D) Pb

Correct Answer: (C) Au

Solution:

Concept:

- Leaching selectively dissolves the useful constituent of an ore into a suitable reagent, leaving much of the unwanted material behind. A soluble complex may be formed to transfer the metal into solution.
- Gold can be extracted by cyanide leaching using a dilute solution of NaCN or KCN in an alkaline medium, with oxygen from air. The dissolved species is the dicyanoaurate(I) complex, $[Au(CN)_2]^-$.

Step 1: Identify the metal dissolved by alkaline cyanide The standard alkali-cyanide leaching process is used for gold. Oxygen enables oxidation of the gold, while cyanide stabilises the gold(I) ions as a soluble complex. The overall leaching reaction may be written as

$$4\,Au + 8\,CN^- + O_2 + 2\,H_2O \longrightarrow 4\,[Au(CN)_2]^- + 4\,OH^-.$$

Both atoms and charge balance in this equation; complex formation is essential to the dissolution.

Step 2: Explain how the dissolved metal is recovered After the ore is leached, the gold-containing solution is separated from the insoluble residue. Gold can then be displaced by zinc, which is more electropositive:

$$2\,[\mathrm{Au(CN)_2}]^- + \mathrm{Zn} \longrightarrow [\mathrm{Zn(CN)_4}]^{2-} + 2\,\mathrm{Au}.$$

Thus leaching is the selective dissolution stage; reduction or displacement subsequently gives the free metal.

Step 3: Compare the listed choices Among the listed choices, gold is the standard metal associated with the alkaline cyanide leaching technique. Copper extraction is usually discussed through concentration of sulphide ores followed by roasting, smelting and refining; copper can also be recovered by other leaching routes, but it is not the intended alkaline-cyanide example here. Tin is conventionally obtained by reduction of cassiterite, while lead is conventionally obtained from galena by roasting and reduction. These conventional extraction routes distinguish them from the gold cyanidation process asked about.

Step 4: Conclusion Gold, Au, is extracted through alkaline cyanide leaching. Hence, the correct option is (C).

Quick Tip to Solve: For leaching questions, identify both reagent and soluble species: alkaline cyanide forms $[\mathrm{Au(CN)_2}]^-$, which points directly to gold.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The number of given statement/s which is/are correct is _____.

(A) The stronger the temperature dependence of the rate constant, the higher is the activation energy.

(B) If a reaction has zero activation energy, its rate is independent of temperature.

(C) The stronger the temperature dependence of the rate constant, the smaller is the activation energy.

(D) If there is no correlation between the temperature and the rate constant then it means that the reaction has negative activation energy. [4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where k is the rate constant, A the pre-exponential factor, E_a the activation energy and T the absolute temperature.
- In the standard Arrhenius treatment with A and E_a constant over the range considered, $d\ln k/dT = E_a/(RT^2)$. This relation connects the temperature sensitivity of k to activation energy.

Step 1: Evaluate statements A and C For a fixed temperature interval and positive activation energies,

$$\ln\frac{k_2}{k_1} = \frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right).$$

With $T_2 > T_1$, the bracket is positive and fixed. A larger activation energy gives a larger relative increase in the rate constant. Therefore stronger temperature dependence corresponds to higher activation energy: statement A is correct and statement C, which claims the reverse, is incorrect.

Step 2: Evaluate the zero-energy statement If $E_a = 0$, the exponential factor is unity and $k = A$. Within the Arrhenius approximation used here, the rate constant has no temperature dependence. At unchanged reactant concentrations, the rate law therefore gives a temperature-independent rate. Statement B is correct. The result follows directly from the equation rather than from assuming that a zero barrier makes the reaction stop.

Step 3: Distinguish zero and negative activation energies No correlation between temperature and rate constant means that the temperature derivative of $\ln k$ is zero in this treatment, corresponding to zero activation energy. A negative apparent activation energy instead gives a negative derivative: the rate constant decreases as temperature increases. That is still a definite temperature dependence, so statement D is incorrect.

Step 4: Conclusion Only statements A and B are correct. The required number of correct statements is $\boxed{2}$.

Quick Tip to Solve: Use the sign and magnitude of $d\ln k/dT = E_a/(RT^2)$: positive, zero and negative activation energies give increasing, constant and decreasing rate constants, respectively.

82. The number of following factors which affect the percent covalent character of the ionic bond is _____

(A) Polarising power of cation

(B) Extent of distortion of anion

(C) Polarisability of the anion

(D) Polarising power of anion [4 marks]

Solution:

Concept:

- An ionic bond acquires covalent character when the electron cloud of the anion is distorted by the neighbouring cation. The resulting electron density is drawn into the region between the ions.
- Fajans rules relate this distortion to the polarising power of the cation and the polarisability of the anion. A small, highly charged cation and a large, readily distorted anion generally favour greater covalent character.

Step 1: Assess the cation property Statement A is relevant. The polarising power of the cation describes its ability to distort the anion electron cloud. Greater charge density generally gives greater polarising power. Thus a change in cation size or charge can change the percentage covalent character, even when the anion remains the same.

Step 2: Assess the two descriptions of the anion response Statement B is relevant because the actual extent of distortion of the anion directly determines how far the bond departs from the ideal ionic model. Greater distortion means greater electron sharing and greater covalent character. Statement C

is also relevant: polarisability is the ease with which that anion electron cloud can be distorted. A larger, more diffuse anion is usually more polarisable, so the same cation can produce more distortion in it.

Step 3: Reject the reversed role and count Statement D assigns polarising power to the anion. This is not the factor used in the Fajans-rule account of covalent character of an ionic bond: the cation is the polarising species, and the anion is the species being polarised. The phrase must not be confused with the polarisability of the anion in C. Therefore A, B and C contribute to the stated effect, while D is not one of the listed governing factors in this treatment.

Step 4: Conclusion The relevant factors are A, B and C. Hence the required number is $\boxed{3}$.

Quick Tip to Solve: Keep the two roles separate: the cation has polarising power, whereas the anion has polarisability and undergoes distortion.

83. 0.5 g **of an organic compound** (X) **with** 60% **carbon will produce** _____ $\times 10^{-1}$ g **of** CO_2 **on complete combustion.** [4 marks]

Solution:

Concept:

- On complete combustion, each carbon atom in an organic compound becomes one carbon atom in CO_2. Thus the moles of carbon in the sample equal the moles of carbon dioxide produced.
- Use the carbon mass percentage to find the mass of carbon first. With molar masses $M(\mathrm{C}) = 12\,\mathrm{g\,mol^{-1}}$ and $M(CO_2) = 44\,\mathrm{g\,mol^{-1}}$, the carbon-to-carbon-dioxide mass ratio is $12 : 44$.

Step 1: Calculate the carbon present The compound has mass $0.5\,\mathrm{g}$, of which 60% is carbon. Therefore

$$m_{\mathrm{C}} = 0.5 \times \frac{60}{100} = 0.30\,\mathrm{g}.$$

Only this carbon mass is used to determine the amount of carbon dioxide. Using the entire sample mass as carbon would overestimate the product.

Step 2: Convert carbon mass into carbon dioxide mass The combustion relation for the carbon atoms is $\mathrm{C} + \mathrm{O_2} \longrightarrow \mathrm{CO_2}$. Therefore

$$n_{\mathrm{C}} = \frac{0.30}{12} = 0.025\,\mathrm{mol}, \qquad n_{\mathrm{CO_2}} = 0.025\,\mathrm{mol}.$$

Multiplying by the molar mass gives

$$m_{\mathrm{CO_2}} = 0.025 \times 44 = 1.10\,\mathrm{g}.$$

The product can weigh more than the original carbon because it includes oxygen supplied during combustion. No molecular formula of the compound is needed.

Step 3: Express the mass in the requested scale The blank is the coefficient of 10^{-1} g, rather than the mass directly in grams. Writing

$$1.10\,\mathrm{g} = 11 \times 10^{-1}\,\mathrm{g}$$

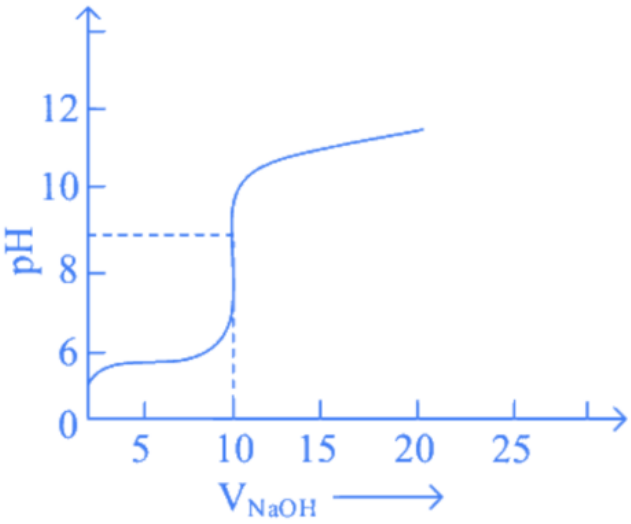
shows that the required entry is 11. As a check, multiplying the original carbon mass by $44/12$ gives the same product mass, so both the mole route and mass-ratio route agree.

Step 4: Conclusion Complete combustion produces $1.10\,\mathrm{g}$ of CO_2, so the numerical entry is $\boxed{11}$.

Quick Tip to Solve: For complete combustion, multiply the carbon mass by $44/12$, then convert the result to the power-of-ten scale printed beside the blank.

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84. The titration curve of weak acid vs. strong base with phenolphthalein as indictor) is shown below.
The $K_{\text{phenolphthalein}} = 4 \times 10^{-10}$.
Given: $\log 2 = 0.3$
The number of following statement/s which is/are correct about phenolphthalein is _____
A. It can be used as an indicator for the titration of weak acid with weak base.
B. It begins to change colour at $\mathrm{pH} = 8.4$
C. It is a weak organic base
D. It is colourless in acidic medium [4 marks]



Solution:

Concept:

- Phenolphthalein behaves as a weak organic acid indicator. Its acidic form is colourless, while its basic form is pink. Represent the indicator equilibrium as $\mathrm{HIn} \rightleftharpoons \mathrm{H^+} + \mathrm{In^-}$.
- The indicator equation is $\mathrm{pH} = \mathrm{p}K_{\mathrm{In}} + \log([\mathrm{In^-}]/[\mathrm{HIn}])$. A visible transition is conventionally taken to begin when the coloured-to-colourless concentration ratio is approximately $1:10$.

Step 1: Calculate the start of the colour transition Using $K_{\mathrm{In}} = 4 \times 10^{-10}$ and $\log 2 = 0.3$,

$$\mathrm{p}K_{\mathrm{In}} = 10 - \log 4 = 10 - 2(0.3) = 9.4.$$

At the start of the visible change, $[\mathrm{In}^-]/[\mathrm{HIn}] = 0.1$, so

$$\mathrm{pH}_{\text{start}} = 9.4 + \log(0.1) = 9.4 - 1 = 8.4.$$

Statement B is therefore correct. The transition extends over a range, so 9.4 is the midpoint rather than its beginning.

Step 2: Assess suitability and chemical nature Statement A is incorrect. A weak-acid–weak-base titration lacks a sufficiently sharp pH jump around equivalence for phenolphthalein to provide a reliable endpoint. In contrast, the displayed weak-acid–strong-base curve has its sharp rise on the alkaline side, where phenolphthalein is useful. Statement C is incorrect because phenolphthalein is treated as a weak organic acid indicator, not a weak organic base.

Step 3: Assess the colour in acid and count In acidic solution, the high hydrogen-ion concentration favours the colourless acidic indicator form. Hence statement D is correct. As base is added, the equilibrium shifts toward the pink form, which explains the observed colour transition. Thus B and D are correct, whereas A and C are incorrect; the count must include statements rather than the number of colour forms.

Step 4: Conclusion Statements B and D are correct. The required number is $\boxed{2}$.

Quick Tip to Solve: Find $\mathrm{p}K_{\mathrm{In}}$ first, then subtract one for the usual beginning of the indicator transition; do not confuse the transition midpoint with its lower limit.

85. When a $60\ \mathrm{W}$ electric heater is immersed in a gas for 100 s in a constant volume container with adiabatic walls, the temperature of the gas rises by $5^\circ\mathrm{C}$. The heat capacity of the given gas is _____ JK^{-1} (Nearest integer) [4 marks]

Solution:

Concept:

- Heat capacity is the energy needed to raise the temperature of the whole sample by one kelvin. At constant volume, the increase in internal energy is $\Delta U = C_V \Delta T$.
- An electrical heater delivers energy $E = Pt$, where P is its power and t is the operating time. Adiabatic walls prevent thermal energy from escaping to the surroundings.

Step 1: Calculate the electrical energy supplied The heater operates at a constant power of $60\,\mathrm{J\,s^{-1}}$ for $100\,\mathrm{s}$. Thus the energy transferred to the gas is

$$E = Pt = 60 \times 100 = 6000\,\mathrm{J}.$$

The electrical input is an energy transfer into the system even though its enclosing walls are adiabatic. Adiabatic does not mean that electrical work cannot enter.

Step 2: Apply the first law at fixed volume The container is rigid, so $\Delta V = 0$ and the gas performs no pressure-volume expansion work. Consequently the supplied electrical energy appears as an increase in internal energy:

$$\Delta U = 6000\,\mathrm{J}.$$

The temperature rise of $5^\circ\mathrm{C}$ is a temperature difference of $5\,\mathrm{K}$; no conversion by adding 273 is made for a temperature difference.

Step 3: Divide energy by the temperature rise For the given sample,

$$C_V = \frac{\Delta U}{\Delta T} = \frac{6000}{5} = 1200\,\mathrm{J\,K^{-1}}.$$

This is the heat capacity of the entire gas sample, not its molar heat capacity: the question supplies no number of moles, and no division by the amount of gas is required.

Step 4: Conclusion The heat capacity of the given gas is $\boxed{1200\,\mathrm{J\,K^{-1}}}$. The nearest-integer entry is $\boxed{1200}$.

Quick Tip to Solve: For a rigid insulated vessel with an electrical heater, use $C_V = Pt/\Delta T$; adiabatic insulation eliminates heat loss, not the electrical energy input.

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86. Molar mass of the hydrocarbon (X) which on ozonolysis consumes one mole of $\mathrm{O_3}$ per mole of (X) and gives one mole each of ethanal and propanone is _____ $\mathrm{g\ mol^{-1}}$

(Molar mass of $\mathrm{C}: 12\ \mathrm{g\ mol^{-1}}, \mathrm{H}: 1\ \mathrm{g\ mol^{-1}}$) [4 marks]

Solution:

Concept:

- Ozonolysis cleaves a carbon-carbon double bond and replaces each double-bond carbon by a carbonyl carbon. Each ordinary alkene double bond consumes one molecule of ozone.
- The substituents attached to each alkene carbon remain attached to the corresponding carbonyl carbon. Rejoining the two carbonyl carbons therefore reconstructs the original double bond.

Step 1: Identify the two sides of the original double bond Ethanal has the structure $\mathrm{CH_3CHO}$. Its carbonyl carbon is attached to one methyl group and one hydrogen atom. Propanone is $\mathrm{CH_3COCH_3}$; its carbonyl carbon has two methyl substituents. These substituent patterns must have been present at the two ends of the double bond in the original hydrocarbon.

Step 2: Reconstruct the hydrocarbon Remove the carbonyl oxygen atoms from the two products and join the carbonyl carbons by a double bond. This gives

$$\mathrm{CH_3CH{=}C(CH_3)_2}.$$

The compound is 2-methylbut-2-ene. It has one double bond, which agrees with consumption of one mole of $\mathrm{O_3}$ per mole of hydrocarbon. Cleavage of that bond gives one ethanal molecule and one propanone molecule, exactly as required.

Step 3: Calculate the molar mass Count five carbon atoms and ten hydrogen atoms in the reconstructed structure; the molecular formula is $\mathrm{C_5H_{10}}$. The oxygen atoms in the products were introduced by ozonolysis and are not included in the hydrocarbon formula. Using the given atomic molar masses,

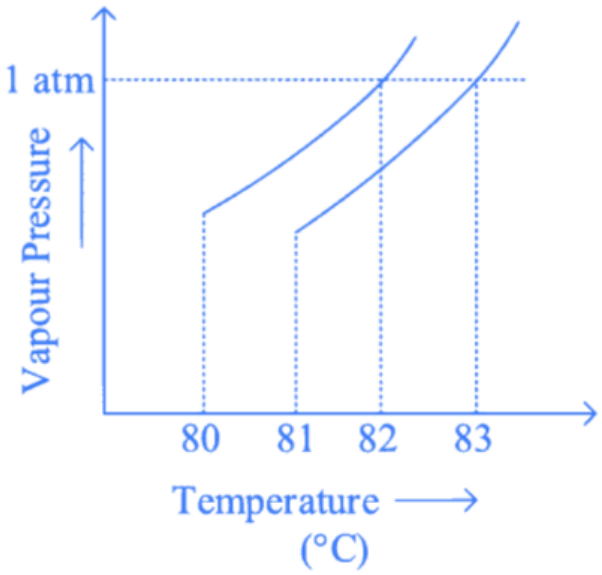
$$M = 5(12) + 10(1) = 60 + 10 = 70\,\mathrm{g\,mol^{-1}}.$$

The carbon count can also be checked directly as two carbons from ethanal plus three from propanone.

Step 4: Conclusion The hydrocarbon is $CH_3CH{=}C(CH_3)_2$, and its molar mass is $\boxed{70\,\text{g mol}^{-1}}$.

Quick Tip to Solve: To reverse ozonolysis, erase the two carbonyl oxygen atoms and connect their carbon atoms by a double bond; retain every other substituent.

87. The vapour pressure vs. temperature curve for a solution solvent system is shown below. The boiling point of the solvent is _____ °C. [4 marks]



Solution:

Concept:

- A liquid boils when its vapour pressure equals the external pressure. The normal boiling point is the temperature at which its vapour pressure becomes $1\,\text{atm}$.
- For a solution containing a nonvolatile solute, the solvent vapour pressure is lower than that of the pure solvent at the same temperature. Therefore the solution reaches atmospheric pressure at a higher temperature.

Step 1: Identify the pure-solvent curve Compare the two curves at any common temperature where both are shown. The left-hand curve is higher, so it represents the pure solvent. The right-hand curve is lower at the same temperature and represents the solution. This assignment follows vapour-pressure lowering and must be made before reading either boiling point.

Step 2: Read the intersection with atmospheric pressure The horizontal dotted line is explicitly labelled $1\,\text{atm}$. The pure-solvent curve intersects this horizontal line above the vertical dotted line marked $82°\text{C}$. Hence the normal boiling point of the solvent is read at 82 on the temperature axis. The labels 80 and 81 identify lower-temperature positions on the curves; they are not intersections with the atmospheric-pressure line.

Step 3: Check the solution boiling point for consistency The solution curve meets the same $1\,\text{atm}$

horizontal line at 83°C. Thus the figure implies

$$\Delta T_b = 83 - 82 = 1^\circ\text{C}.$$

This positive elevation is consistent with a solution having lower vapour pressure than its pure solvent. The question asks for the solvent boiling point, not the elevated boiling point of the solution.

Step 4: Conclusion The boiling point of the solvent is $\boxed{82^\circ\text{C}}$. The numerical entry is $\boxed{82}$.

Quick Tip to Solve: First identify the higher vapour-pressure curve as the pure solvent, then read its intersection with the line showing the external pressure.

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88. The number of following statement/s which is/are incorrect is _____

(A) Line emission spectra are used to study the electronic structure

(B) The emission spectra of atoms in the gas phase show a continuous spread of wavelength from red to violet

(C) An absorption spectrum is like the photographic negative of an emission spectrum

(D) The element helium was discovered in the sun by spectroscopic method [4 marks]

Solution:

Concept:

- Atomic electronic energies are quantised. Transitions between specific energy levels produce or absorb photons with $h\nu = |E_2 - E_1|$, so isolated atoms have characteristic spectral lines.
- An emission spectrum contains bright lines at emitted wavelengths; an absorption spectrum contains dark lines at wavelengths removed from a continuous background by the same atomic transitions.

Step 1: Evaluate statements A and B Statement A is correct. The positions and patterns of atomic emission lines reveal the allowed energy differences, and hence provide information about electronic structure. Statement B is incorrect. Gas-phase atoms emit a line spectrum consisting of separate characteristic wavelengths. A continuous spread from red to violet is the defining feature of a continuous visible spectrum, not an atomic line emission spectrum.

Step 2: Evaluate statements C and D Statement C is correct. For the corresponding atomic transitions, dark absorption lines occur at the positions of bright emission lines, which is why an absorption spectrum is described as the photographic negative of an emission spectrum. This comparison concerns the positions and appearance of spectral lines. Statement D is also correct: helium was first identified spectroscopically in the solar spectrum before its identification on Earth. Its characteristic spectral signature enabled that discovery.

Step 3: Count only the incorrect statements The classification is

Statement	A	B	C	D
Status	Correct	Incorrect	Correct	Correct

Only B asserts a continuous atomic emission spectrum. Thus exactly one of the four statements is

incorrect; counting the three correct statements instead would answer the opposite question.

Step 4: Conclusion Only statement B is incorrect. The required number is $\boxed{1}$.

Quick Tip to Solve: For atomic spectra, remember discrete electronic energies mean discrete spectral lines; always check whether a counting question asks for correct or incorrect statements.

89. XeF_4 **reacts with** SbF_5 **to form**
$[XeF_m]^{n+}\,[SbF_y]^{z-}$.
$m + n + y + z =$ _____ [4 marks]

Solution:

Concept:

- SbF_5 is a strong Lewis acid and fluoride-ion acceptor. Acceptance of F^- gives the hexafluoroantimonate ion $[SbF_6]^-$.
- Fluoride donation by XeF_4 forms the cation $[XeF_3]^+$. This is a fluoride-transfer reaction, and the charges and total fluorine count must balance.

Step 1: Form the fluoride-accepting anion Antimony pentafluoride accepts one fluoride ion to increase its coordination number from five to six:

$$SbF_5 + F^- \longrightarrow [SbF_6]^-.$$

Thus the anion in the question has six fluorine atoms and a single negative charge. In the notation $[SbF_y]^{z-}$, this fixes $y = 6$ and $z = 1$, where z denotes the positive magnitude of the negative charge.

Step 2: Identify the complementary xenon cation The fluoride ion comes from xenon tetrafluoride:

$$XeF_4 \longrightarrow [XeF_3]^+ + F^-.$$

Combining the donor and acceptor processes gives the salt formation

$$XeF_4 + SbF_5 \longrightarrow [XeF_3]^+[SbF_6]^-.$$

There are nine fluorine atoms on each side, and the product has equal and opposite ionic charges. Xenon remains in the +4 oxidation state: $+4 - 3 = +1$ for the cation. This check distinguishes the ionic charge from the oxidation state.

Step 3: Add the requested subscripts and charge magnitudes Comparison with the product $[XeF_m]^{n+}[SbF_y]^{z-}$ gives

$$m = 3, \quad n = 1, \quad y = 6, \quad z = 1.$$

Therefore,

$$m + n + y + z = 3 + 1 + 6 + 1 = 11.$$

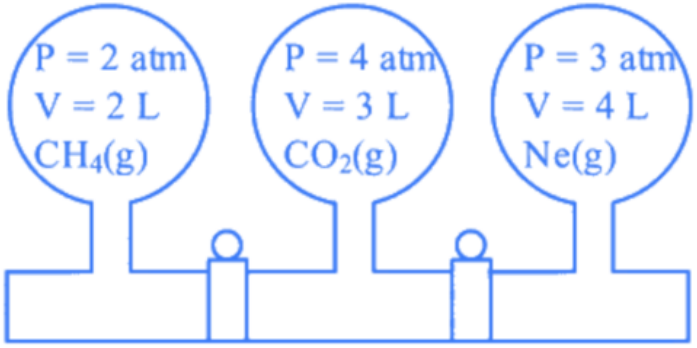
The minus sign belongs to the anion notation and does not make the quantity z itself negative in this sum.

Step 4: Conclusion The product is $[XeF_3]^+[SbF_6]^-$, so $\boxed{m+n+y+z=11}$.

Quick Tip to Solve: When SbF_5 accepts fluoride, write $[SbF_6]^-$ first; the xenon fluoride loses one fluoride and gains a unit positive charge.

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90. Three bulbs are filled with CH_4, CO_2 and Ne as shown in the picture. The bulbs are connected through pipes of zero volume. When the stopcocks are opened and the temperature is kept constant throughout, the pressure of the system is found to be _____ atm. (Nearest integer) [4 marks]



Solution:

Concept:

- For an ideal gas at constant temperature, the amount of gas is proportional to PV: $n = PV/(RT)$. Opening the stopcocks conserves the total number of moles.
- After mixing, every component occupies the full accessible volume. Dalton's law gives the total pressure as the sum of component partial pressures, so $P_f V_{\text{tot}} = \sum_i P_i V_i$.

Step 1: Read the initial pressures and volumes The methane bulb has $P_1 = 2\,\text{atm}$, $V_1 = 2\,\text{L}$; the carbon dioxide bulb has $P_2 = 4\,\text{atm}$, $V_2 = 3\,\text{L}$; and the neon bulb has $P_3 = 3\,\text{atm}$, $V_3 = 4\,\text{L}$. The connecting pipes have zero volume. Consequently the common final volume is

$$V_{\text{tot}} = 2 + 3 + 4 = 9\,\text{L}.$$

The different gas identities do not change mole conservation or the ideal-gas mixing calculation.

Step 2: Conserve total moles at the common temperature Before opening, the total amount is

$$n_{\text{tot}} = \frac{2(2) + 4(3) + 3(4)}{RT} = \frac{28\,\text{atm L}}{RT}.$$

After opening, the temperature is unchanged and the gases fill the total volume. Hence

$$P_f = \frac{n_{\text{tot}} RT}{V_{\text{tot}}} = \frac{28}{9}\,\text{atm}.$$

The factors R and T cancel because the initial and final temperatures are the same.

Step 3: Round the final pressure as requested The unrounded result is $3.111\ldots\,\text{atm}$. To the nearest

integer this becomes $3\,\mathrm{atm}$. This is a volume-weighted mean of the three initial pressures and lies between 2 and 4 atm, as expected. Adding the original pressures directly would be wrong because each gas expands into a larger volume before its partial pressure is added.

Step 4: Conclusion The equilibrium pressure is $28/9\,\mathrm{atm}$, giving the required nearest-integer answer $\boxed{3\,\mathrm{atm}}$.

Quick Tip to Solve: For connected rigid bulbs at one temperature, use $P_f = \sum PV / \sum V$, with pipe volume included only when it is nonzero.