JEE Main 2023 Paper 1 (B.E./B.Tech) Apr 8 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 8 April 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $A = \left\{\theta \in (0, 2\pi) : \frac{1+2i\sin\theta}{1-i\sin\theta}$ is purely imaginary $\right\}$. Then the sum of the elements in A is : [4 marks]

(A) 3π

(B) π

(C) 2π

(D) 4π

Correct Answer: (D) 4π

Solution:

Concept:

- A complex number is purely imaginary when its real part is zero. Rationalising a complex denominator makes its real and imaginary parts explicit.
- For real s, the conjugate of $1 - is$ is $1 + is$, and their product is $1 + s^2 > 0$. Thus the denominator never vanishes for the angles in this question.

Step 1: Rationalise the given quotient Write $s = \sin\theta$. Multiply the numerator and denominator by the conjugate of the denominator:

$$\frac{1+2is}{1-is} = \frac{(1+2is)(1+is)}{1+s^2} = \frac{1-2s^2+3is}{1+s^2}.$$

Here $i^2 = -1$ accounts for the negative term in the real numerator. The real part is therefore $(1-2s^2)/(1+s^2)$, rather than the real numerator of the original fraction.

Step 2: Solve the real-part condition Since the denominator is strictly positive, the required condition is

$$1 - 2\sin^2\theta = 0 \quad \Longrightarrow \quad \sin\theta = \pm\frac{1}{\sqrt{2}}.$$

In the open interval $(0, 2\pi)$, this gives exactly four angles:

$$A = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}.$$

At each angle the imaginary part is nonzero because $\sin\theta \neq 0$, so each quotient is indeed purely imaginary.

Step 3: Add all the admissible angles Both positive and negative values of the sine must be included. Pairing the first with the last angle, and the two middle angles, gives

$$\sum_{\theta\in A}\theta = \frac{(1+3+5+7)\pi}{4} = 4\pi.$$

The values π, 2π, and 3π in the other options omit some of the four admissible angles or mis-add them.

Step 4: Conclusion The sum of all elements of A is $\boxed{4\pi}$. Hence, the correct option is (D).

Quick Tip to Solve: After rationalising, set only the real part to zero; then include both signs when solving a squared trigonometric equation over a full revolution.

2. $25^{190} - 19^{190} - 8^{190} + 2^{190}$ **is divisible by :** [4 marks]

(A) 14 but not by 34

(B) neither 14 nor 34

(C) both 14 and 34

(D) 34 but not by 14

Correct Answer: (D) 34 but not by 14

Solution:

Concept:

- To test divisibility by $14 = 2 \cdot 7$ and $34 = 2 \cdot 17$, test the prime factors separately. Coprime factors divide a number jointly when each factor divides it.
- Congruent bases have congruent positive integer powers. For a nonzero residue modulo the prime 7, the Fermat theorem gives $a^6 \equiv 1 \pmod 7$.

Step 1: Check the common factor 2 Let

$$N = 25^{190} - 19^{190} - 8^{190} + 2^{190}.$$

The first two terms are odd and have even difference. The last two terms are even. Hence N is even. This settles the factor 2 in both proposed divisors, but an even number need not be divisible by either 14 or 34; their odd prime factors must still be checked.

Step 2: Test divisibility by 17 Modulo 17, the pairs of bases satisfy $25 \equiv 8$ and $19 \equiv 2$. Therefore

$$N \equiv 8^{190} - 2^{190} - 8^{190} + 2^{190} \equiv 0 \pmod{17}.$$

Together with the evenness of N, and $\gcd(2, 17) = 1$, this proves $34 \mid N$. No actual expansion of the enormous powers is necessary.

Step 3: Test divisibility by 7 Modulo 7 the four bases become $4, 5, 1, 2$, respectively. Since $190 = 6 \cdot 31 + 4$, reduce the powers to fourth powers:

$$\begin{aligned} N &\equiv 4^4 - 5^4 - 1 + 2^4 \\ &\equiv 4 - 2 - 1 + 2 = 3 \pmod 7. \end{aligned}$$

The residues used here follow from $256 \equiv 4$, $625 \equiv 2$, and $16 \equiv 2 \pmod 7$. Thus 7 does not divide N, so 14 does not divide it either. This rejects the options asserting divisibility by 14, and the option asserting divisibility by neither number.

Step 4: Conclusion The expression is divisible by 34 but not by 14. Hence, the correct option is (D).

Quick Tip to Solve: For large-power divisibility questions, factor the candidate divisors and reduce the bases first; often two terms cancel immediately modulo one prime.

3. The integral $\int \left[\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right] \ln\left(\frac{ex}{2}\right) dx$ is equal to : [4 marks]

(A) $\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x + C$

(B) $\left(\frac{x}{2}\right)^x - \left(\frac{2}{x}\right)^x + C$

(C) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{2}{x}\right) + C$

(D) None

Correct Answer: (B) $\left(\frac{x}{2}\right)^x - \left(\frac{2}{x}\right)^x + C$

Solution:

Concept:

- For $x > 0$, write a variable power as an exponential: $(x/2)^x = e^{x\ln(x/2)}$. Logarithmic differentiation then handles the variable base and exponent together.
- If $u(x) = x\ln(x/2)$, then $u'(x) = \ln(x/2) + 1 = \ln(ex/2)$. The two powers in the integrand are e^u and e^{-u}.

Step 1: Identify the common inner derivative The logarithm in the integrand requires $x > 0$. Set

$$u = x\ln\left(\frac{x}{2}\right), \qquad \frac{du}{dx} = \ln\left(\frac{x}{2}\right) + 1 = \ln\left(\frac{ex}{2}\right).$$

Also $(x/2)^x = e^u$, while $(2/x)^x = e^{-u}$, because $\ln(2/x) = -\ln(x/2)$. The entire logarithmic factor is therefore absorbed by du.

Step 2: Integrate the two exponential terms The given integral becomes

$$\int (e^u + e^{-u})\, du = e^u - e^{-u} + C.$$

The minus sign is essential: differentiating e^{-u} gives $-e^{-u}$, so an antiderivative of $+e^{-u}$ is $-e^{-u}$. Substituting for u gives

$$\left(\frac{x}{2}\right)^x - \left(\frac{2}{x}\right)^x + C.$$

Step 3: Differentiate to verify and compare options Direct differentiation of the difference gives

$$\frac{d}{dx}(e^u - e^{-u}) = (e^u + e^{-u})u',$$

which is precisely the printed integrand. Option (A), with a plus sign between the powers, instead produces their difference after differentiation. Option (C) also fails: at $x = 2$, its derivative is $-1/(2\ln 2)$, whereas the given integrand is 2. Since option (B) differentiates correctly, the option stating none is also excluded.

Step 4: Conclusion The integral is $\boxed{(x/2)^x - (2/x)^x + C}$, for $x > 0$. Hence, the correct option is (B).

Quick Tip to Solve: When reciprocal variable powers occur with a logarithmic factor, set the logarithm of one power equal to a new variable; the powers become opposite exponentials.

4. If the probability that the random variable X takes values x is given by $\mathrm{P}(\mathrm{X}=x)=\mathrm{k}(x+1)3^{-x}, x=0,1,2,3,\ldots$, where k is a constant, then $\mathrm{P}(\mathrm{X}\geq 2)$ is equal to : [4 marks]

(A) $\frac{7}{18}$
(B) $\frac{20}{27}$
(C) $\frac{7}{27}$
(D) $\frac{11}{18}$

Correct Answer: (C) $\frac{7}{27}$

Solution:

Concept:

- The probabilities of all values of a discrete random variable sum to 1. This determines the unknown multiplicative constant before any event probability is computed.
- For $|r|<1$, the geometric-series identity and its derivative give $\sum_{x=0}^{\infty}(x+1)r^x=(1-r)^{-2}$. A tail probability can then be found by subtracting its short complement.

Step 1: Normalise the probability mass function Using $r=1/3$, the total probability is

$$1=k\sum_{x=0}^{\infty}(x+1)\left(\frac{1}{3}\right)^x=k\frac{1}{(1-1/3)^2}=\frac{9k}{4}.$$

Thus $k=4/9$. To obtain the series identity, differentiate $\sum_{x=0}^{\infty}r^{x+1}=r/(1-r)$ with respect to r; both sides converge for $|r|<1$. The resulting derivative on the left is $\sum(x+1)r^x$.

Step 2: Compute the two excluded probabilities The complement of $X\geq 2$ consists exactly of the disjoint outcomes $X=0$ and $X=1$, because the support is the nonnegative integers. Their probabilities are

$$P(X=0)=k=\frac{4}{9}=\frac{12}{27},\qquad P(X=1)=k\frac{2}{3}=\frac{8}{27}.$$

Consequently $P(X<2)=20/27$. This is option (B), but it is the complement of the requested event rather than the event itself.

Step 3: Subtract from the total probability The required tail is therefore

$$P(X\geq 2)=1-P(X=0)-P(X=1)=1-\frac{12}{27}-\frac{8}{27}=\frac{7}{27}.$$

It is positive and smaller than the omitted probability, as expected because the mass decreases quickly after the first values. The alternatives $7/18$ and $11/18$ do not result from the correctly normalised distribution; using an unnormalised series would give incorrect values.

Step 4: Conclusion $\boxed{P(X\geq 2)=7/27}$. Hence, the correct option is (C).

Quick Tip to Solve: Normalise first, then use the complement when the requested tail leaves only one or two support values outside it.

5. Let $\mathrm{A} = \{1, 2, 3, 4, 5, 6, 7\}$. Then the relation $\mathrm{R} = \{(x, y) \in \mathrm{A} \times \mathrm{A} : x + y = 7\}$ is : [4 marks]

(A) reflexive but neither symmetric nor transitive

(B) transitive but neither symmetric nor reflexive

(C) symmetric but neither reflexive nor transitive

(D) an equivalence relation

Correct Answer: (C) symmetric but neither reflexive nor transitive

Solution:

Concept:

- A relation on A is reflexive if (a, a) belongs to it for every $a \in A$; it is symmetric if reversing any related pair leaves a related pair.
- Transitivity requires xRy and yRz to imply xRz. One counterexample disproves reflexivity or transitivity, while symmetry must hold for every pair.

Step 1: Test reflexivity For the diagonal pair (x, x), the defining condition would become $2x = 7$. This would require $x = 7/2$, which is not an element of A. In particular $(1, 1) \notin R$, even though $1 \in A$. Hence R is not reflexive. Notice also that 7 has no partner in A, because its required partner would be 0, which is outside the given set.

Step 2: Establish symmetry If $(x, y) \in R$, then $x + y = 7$. Addition is commutative, so $y + x = 7$ and $(y, x) \in R$ as well. Explicitly,

$$R = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}.$$

Every pair has its reverse in this list, confirming the general argument. Thus symmetry holds throughout the relation, rather than merely for one example.

Step 3: Disprove transitivity and compare the choices The pairs $(1, 6)$ and $(6, 1)$ both belong to R. Transitivity would therefore require $(1, 1) \in R$. But $1 + 1 = 2 \neq 7$, so this required pair is absent. Thus R is not transitive. The first alternative incorrectly calls the relation reflexive; the second incorrectly calls it transitive and denies symmetry. An equivalence relation must have all three properties, so the fourth alternative is excluded as well. Only the stated symmetric classification matches all the tests.

Step 4: Conclusion The relation is symmetric but neither reflexive nor transitive. Hence, the correct option is (C).

Quick Tip to Solve: For a relation defined by a fixed sum, symmetry follows by interchanging the variables; a related pair and its reverse often give a quick transitivity counterexample.

6. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are coprime, then $\mathrm{m}+\mathrm{n}$ is equal to : [4 marks]

(A) 317

(B) 316

(C) 314

(D) 315

Correct Answer: (A) 317

Solution:

Concept:

- For n observations, $\bar{x} = (\sum x_i)/n$ and $\sigma^2 = (\sum x_i^2)/n - \bar{x}^2$. Both the sum and the sum of squares must be corrected when observations are replaced.
- The printed mean and variance refer to the original, incorrectly entered data. Recover its two sums first, remove the incorrect observations, and add the actual observations before recomputing the variance.

Step 1: Recover the original totals For the 12 observations with recorded mean $9/2$, the recorded sum is

$$S = 12\left(\frac{9}{2}\right) = 54.$$

The recorded variance 4 gives the original sum of squares Q:

$$4 = \frac{Q}{12} - \left(\frac{9}{2}\right)^2 \quad \Longrightarrow \quad Q = 12\left(4 + \frac{81}{4}\right) = 291.$$

These quantities are properties of the incorrectly recorded list, so the replacements must be made to them, not directly to the variance.

Step 2: Replace both incorrect entries The entered values 9 and 10 must be removed and the actual values 7 and 14 inserted. Thus

$$S_c = 54 - 9 - 10 + 7 + 14 = 56,$$

$$Q_c = 291 - 9^2 - 10^2 + 7^2 + 14^2 = 355.$$

The observation count remains 12 because two values have been replaced, not deleted. Consequently the corrected mean is $56/12 = 14/3$. In the square total each individual value is squared before addition; squaring the change in the sum would be invalid.

Step 3: Compute and reduce the corrected variance Using the corrected mean and square total,

$$\sigma_c^2 = \frac{355}{12} - \left(\frac{14}{3}\right)^2$$
$$= \frac{1065 - 784}{36} = \frac{281}{36}.$$

The numerator 281 is divisible by neither 2 nor 3, the prime factors of 36, so this fraction is already in

lowest terms. Hence $m = 281$, $n = 36$, and $m + n = 317$. The other listed sums, 316, 314 and 315, do not match the reduced fraction.

Step 4: Conclusion The corrected variance is $281/36$, so $\boxed{m + n = 317}$. Hence, the correct option is (A).

Quick Tip to Solve: For corrected-data variance, make a two-column adjustment to the sum and the sum of squares, then recompute the mean; do not keep the old mean.

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7. The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to : [4 marks]

(A) $11^3 - 11$

(B) $13^3 - 13$

(C) $12^3 - 12$

(D) $10^3 - 10$

Correct Answer: (C) $12^3 - 12$

Solution:

Concept:

- In $(a + b)^n$, the term using r factors of b is $\binom{n}{r} a^{n-r} b^r$. Separate its numerical coefficient from its power of x.
- Here each replacement of $2x^2$ by $1/(2x)$ decreases the exponent of x by 3. The general exponent is therefore $22 - 3r$, with $0 \le r \le 11$.

Step 1: Write the general term Taking r copies of the second term gives

$$T_{r+1} = \binom{11}{r} (2x^2)^{11-r} \left(\frac{1}{2x}\right)^r$$
$$= \binom{11}{r} 2^{11-2r} x^{22-3r}.$$

Both the factor 2^{-r} and the power x^{-r} from the second term must be retained. The exponent decreases strictly as r increases, so each requested power can come from only one term.

Step 2: Extract the two coefficients For the power x^{10}, solve $22 - 3r = 10$, giving $r = 4$. Its coefficient is

$$a_{10} = \binom{11}{4} 2^{11-8} = 330 \cdot 8 = 2640.$$

For x^7, solve $22 - 3r = 7$, giving $r = 5$. Its coefficient is

$$a_7 = \binom{11}{5} 2^{11-10} = 462 \cdot 2 = 924.$$

Both values of r lie in the allowed integer range, so both terms actually occur in the expansion.

Step 3: Take the absolute difference and identify it The first coefficient is larger, so the required

absolute difference is

$$|a_{10} - a_7| = 2640 - 924 = 1716.$$

Now $12^3 - 12 = 1728 - 12 = 1716$. In comparison, $11^3 - 11 = 1320$, $13^3 - 13 = 2184$, and $10^3 - 10 = 990$. Thus only the third expression equals the computed difference. This comparison also checks that neither the coefficients nor the powers themselves were mistakenly subtracted.

Step 4: Conclusion The absolute difference is $\boxed{1716 = 12^3 - 12}$. Hence, the correct option is (C).

Quick Tip to Solve: In a binomial containing positive and negative powers, solve the exponent equation first; calculate the numerical coefficient only for the required integer index.

8. The area of the quadrilateral ABCD with vertices $A(2,1,1), B(1,2,5), C(-2,-3,5)$ and $D(1,-6,-7)$ is equal to : [4 marks]

(A) 48

(B) $8\sqrt{38}$

(C) 54

(D) $9\sqrt{38}$

Correct Answer: (B) $8\sqrt{38}$

Solution:

Concept:

- The area of a triangle with adjacent vectors $\mathbf{u}, \mathbf{v}$ is $\frac{1}{2}|\mathbf{u} \times \mathbf{v}|$. The magnitude, rather than any individual component, measures its area.
- A diagonal divides a planar convex quadrilateral into two triangles. We use diagonal AC and add the areas of ABC and ACD; the cross products also check that their orientations agree.

Step 1: Form the vectors from A Subtract the coordinates of A from each of the other vertices. This gives

$$\overrightarrow{AB} = (-1, 1, 4), \quad \overrightarrow{AC} = (-4, -4, 4), \quad \overrightarrow{AD} = (-1, -7, -8).$$

All vectors must use the same starting point in each triangle; mixing position vectors with side vectors would give an incorrect cross product.

Step 2: Calculate the two triangular areas Expanding the cross products component by component gives

$$\overrightarrow{AB} \times \overrightarrow{AC} = (20, -12, 8), \qquad \overrightarrow{AC} \times \overrightarrow{AD} = (60, -36, 24).$$

The second vector is three times the first, confirming coplanarity and the consistent orientation of the two triangular pieces. Thus

$$[ABC] = \tfrac{1}{2}\sqrt{400 + 144 + 64} = 2\sqrt{38},$$

$$[ACD] = \tfrac{1}{2}\sqrt{3600 + 1296 + 576} = 6\sqrt{38}.$$

Adding, rather than subtracting, is required because these two triangles fill the quadrilateral on

opposite sides of AC.

Step 3: Compare with the options The total area is $2\sqrt{38} + 6\sqrt{38} = 8\sqrt{38}$. The value $9\sqrt{38}$ uses an incorrect triangle factor, while 48 and 54 do not equal the cross-product result. No projection onto a coordinate plane has been used, so this is the actual spatial area.

Step 4: Conclusion The area of ABCD is $\boxed{8\sqrt{38}}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Divide a quadrilateral along one diagonal and take half the magnitude of each cross product before adding.

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9. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is $(6!)\mathrm{k}$, then k is equal to : [4 marks]

(A) 5670

(B) 1890

(C) 2835

(D) 945

Correct Answer: (A) 5670

Solution:

Concept:

- For arrangements of a multiset with n letters and repeated multiplicities r_i, the number of distinct arrangements is $n!/\prod r_i!$.
- Count the unwanted arrangements with C and S together by treating CS or SC as a single block, then subtract them from all arrangements.

Step 1: Count all distinct words MATHEMATICS contains eleven letters. M, A and T each occur twice; H, E, I, C and S occur once each. The repeated copies of each of the three repeated letters are indistinguishable, so the unrestricted number is

$$N_{\text{all}} = \frac{11!}{2!\,2!\,2!} = \frac{11!}{8}.$$

The phrase with or without meaning imposes no vocabulary condition; every distinct ordering is counted.

Step 2: Subtract the adjacent C and S cases When C and S are together, replace them by one block. There are then ten objects, with M, A and T still repeated twice. The block has two internal orders, CS and SC. Hence

$$N_{\text{adjacent}} = 2\frac{10!}{8}.$$

These cases are disjoint because C and S each occur once, and every arrangement with adjacency belongs to exactly one internal order. Therefore

$$N = \frac{11! - 2(10!)}{8} = \frac{9(10!)}{8}.$$

This subtraction counts precisely the words where C and S are separated, without requiring them to be at opposite ends.

Step 3: Extract the required coefficient Since $N = (6!)k$, divide the preceding count by $6!$:

$$k = \frac{9 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{8} = 9 \cdot 10 \cdot 9 \cdot 7 = 5670.$$

The alternatives 1890, 2835 and 945 do not reproduce this complement count. In particular, 2835 is half the required coefficient; an extra division by two is not justified after the identical letters have already been accounted for.

Step 4: Conclusion $\boxed{k = 5670}$. Hence, the correct option is (A).

Quick Tip to Solve: For two specified distinct letters required to stay apart, subtract the two ordered adjacency blocks from the full multiset count.

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10. If $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $\mathrm{A}^{-1} = \alpha\mathrm{A} + \beta\mathrm{I}$ **and** $\alpha + \beta = -2$**, then** $4\alpha^2 + \beta^2 + \lambda^2$ **is equal to :** [4 marks]

(A) 12

(B) 10

(C) 19

(D) 14

Correct Answer: (D) 14

Solution:

Concept:

- For a nonsingular two-by-two matrix, Cayley–Hamilton gives $A^2 - (\operatorname{tr} A)A + (\det A)I = 0$. Multiplication by A^{-1} expresses the inverse using A and I.
- Here $\operatorname{tr} A = 11$ and $d = \det A = 10 - 5\lambda \neq 0$. Because A has a nonzero off-diagonal entry, A and I are linearly independent.

Step 1: Express the inverse as a linear combination Insert the trace and determinant into the characteristic identity:

$$A^2 - 11A + dI = 0.$$

Since the inverse is given to exist, multiply through by A^{-1} to obtain

$$A - 11I + dA^{-1} = 0, \qquad A^{-1} = -\frac{1}{d}A + \frac{11}{d}I.$$

The nonzero upper-right entry of A ensures that its coefficient and the coefficient of I can be compared uniquely with $\alpha A + \beta I$. Consequently $\alpha = -1/d$ and $\beta = 11/d$.

Step 2: Use the stated sum to determine the parameters The condition on the coefficients now becomes

$$\alpha + \beta = \frac{10}{d} = -2, \qquad d = -5.$$

Thus

$$10 - 5\lambda = -5, \qquad \lambda = 3,$$

and

$$\alpha = \frac{1}{5}, \qquad \beta = -\frac{11}{5}.$$

Notice that d is nonzero, as required, and the coefficients sum to $-10/5 = -2$, providing a direct check on the signs.

Step 3: Evaluate the requested expression Substitute the three parameter values, keeping the square on the negative coefficient:

$$4\alpha^2 + \beta^2 + \lambda^2 = \frac{4}{25} + \frac{121}{25} + 9 = 5 + 9 = 14.$$

The first two terms together are 5; omitting the factor 4 or forgetting to square beta gives a different result. The values 12, 10 and 19 therefore fail the stated coefficient identity.

Step 4: Conclusion $\boxed{4\alpha^2 + \beta^2 + \lambda^2 = 14}$. Hence, the correct option is (D).

Quick Tip to Solve: For a two-by-two inverse written as alpha A plus beta I, read both coefficients directly from the characteristic identity.

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11. If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and $\lim_{x \to \frac{1}{\alpha}} \left(\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2} \right)^{\frac{1}{2}} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$, then k is equal to : [4 marks]

(A) 2β

(B) β

(C) α

(D) 2α

Correct Answer: (D) 2α

Solution:

Concept:

- Vieta gives $\alpha\beta = 1/a$ and $\alpha + \beta = -b/a$. Thus the roots of $x^2 + bx + a$ are the reciprocals of the original roots.
- Use $1 - \cos u = 2\sin^2(u/2)$ and $\sin v/v \to 1$. The principal square root introduces an absolute value, which is essential in a two-sided limit.

Step 1: Factor the reversed quadratic Write $r = 1/\alpha$ and $s = 1/\beta$. Vieta implies $r + s = -b$ and $rs = a$, so

$$u = x^2 + bx + a = (x - r)(x - s).$$

Because $\alpha > \beta > 0$, we have $s > r > 0$. In particular, $u \to 0$ as $x \to r$, while $1 - \alpha x = -\alpha(x - r)$. This factorization makes the vanishing factors explicit before taking the limit.

Step 2: Apply the trigonometric limit carefully The expression under the limit can be written

$$\sqrt{\frac{1-\cos u}{2(1-\alpha x)^2}} = \frac{|\sin(u/2)|}{|1-\alpha x|}.$$

Factor out $|u|/2$ and use the sine limit. For $x \neq r$ sufficiently close to r, cancellation gives

$$L = \lim_{x\to r}\left|\frac{\sin(u/2)}{u/2}\right| \frac{|(x-r)(x-s)|}{2\alpha|x-r|} = \frac{|r-s|}{2\alpha}.$$

Since $s > r$, this equals $(s-r)/(2\alpha)$, a positive number. The absolute values guarantee the same value from both sides.

Step 3: Identify k The right-hand side specified in the question is $(s-r)/k$. As $s-r \neq 0$, comparison gives

$$\frac{1}{k} = \frac{1}{2\alpha}, \qquad k = 2\alpha.$$

The factor 2 comes from the half-angle identity. The alternatives involving beta do not match the denominator $1-\alpha x$, and alpha alone misses the half-angle factor.

Step 4: Conclusion $\boxed{k = 2\alpha}$. Hence, the correct option is (D).

Quick Tip to Solve: When a square root contains a squared vanishing denominator, retain absolute values and factor the reversed quadratic using reciprocal roots.

12. The value of $36\left(4\cos^2 9^\circ - 1\right)\left(4\cos^2 27^\circ - 1\right)\left(4\cos^2 81^\circ - 1\right)\left(4\cos^2 243^\circ - 1\right)$ **is :** [4 marks]

(A) 18

(B) 36

(C) 54

(D) 27

Correct Answer: (B) 36

Solution:

Concept:

- The triple-angle identity $\sin 3t = 3\sin t - 4\sin^3 t$ gives $4\cos^2 t - 1 = \sin 3t/\sin t$ whenever $\sin t \neq 0$.
- For angles multiplied successively by three, these sine ratios telescope: each numerator cancels the denominator of the next factor.

Step 1: Rewrite each cosine factor Using $\cos^2 t = 1 - \sin^2 t$, we obtain

$$4\cos^2 t - 1 = 3 - 4\sin^2 t = \frac{\sin 3t}{\sin t}.$$

All four relevant denominators are nonzero: the angles are $9^\circ, 27^\circ, 81^\circ, 243^\circ$, none of which is an integer multiple of 180°. Hence applying this ratio to each factor is valid without any limiting argument.

Step 2: Cancel the successive sine ratios Let P denote the product without its leading factor 36. Then

$$P = \frac{\sin 27^\circ}{\sin 9^\circ}\frac{\sin 81^\circ}{\sin 27^\circ}\frac{\sin 243^\circ}{\sin 81^\circ}\frac{\sin 729^\circ}{\sin 243^\circ} = \frac{\sin 729^\circ}{\sin 9^\circ}.$$

The signs are retained in the ratios: although $\sin 243^\circ$ is negative, it occurs once in a numerator and once in a denominator. There is no extra minus sign after cancellation.

Step 3: Reduce the final angle and restore the multiplier Since $729^\circ = 720^\circ + 9^\circ$, periodicity gives $\sin 729^\circ = \sin 9^\circ$. Therefore $P = 1$, and the requested value is $36P = 36$. The other choices 18, 54 and 27 would require a remaining factor of one-half, three-halves or three-quarters; the exact sine cancellation leaves none of these factors.

Step 4: Conclusion The expression equals $\boxed{36}$. Hence, the correct option is (B).

Quick Tip to Solve: Spot angles in a geometric progression with ratio three, then use the sine triple-angle ratio instead of expanding the product.

13. Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$$x + y + \sqrt{3}z = 0$$

$$-x + (\tan\theta)y + \sqrt{7}z = 0$$

$$x + y + (\tan\theta)z = 0$$

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta\in S}\theta$ is equal to : [4 marks]

(A) 40

(B) 30

(C) 10

(D) 20

Correct Answer: (D) 20

Solution:

Concept:

- A homogeneous system has a non-trivial solution precisely when its coefficient determinant is zero. Here $t = \tan\theta$ must be defined.
- Each tangent equation has period pi. Enumerate every solution in the closed interval $[-\pi, \pi]$, rather than retaining only a principal value.

Step 1: Factor the determinant The coefficient matrix is

$$M = \begin{pmatrix} 1 & 1 & \sqrt{3} \\ -1 & t & \sqrt{7} \\ 1 & 1 & t \end{pmatrix}.$$

Subtract the first row from the third, an operation that leaves the determinant unchanged. The new third row is $(0, 0, t - \sqrt{3})$, so expansion along that row gives

$$\det M = (t - \sqrt{3}) \begin{vmatrix} 1 & 1 \\ -1 & t \end{vmatrix} = (t - \sqrt{3})(t + 1).$$

Thus the required parameter values satisfy $\tan\theta = \sqrt{3}$ or $\tan\theta = -1$. The coefficient $\sqrt{7}$ does not enter the remaining minor because it lies in the deleted third column.

Step 2: List all angles in the interval For $\tan\theta = \sqrt{3}$, the general solution is $\theta = \pi/3 + n\pi$. The values in the required interval are $-2\pi/3$ and $\pi/3$. For $\tan\theta = -1$, the general solution is $\theta = -\pi/4 + n\pi$, giving $-\pi/4$ and $3\pi/4$. None is a pole of tangent, the two lists do not overlap, and neither endpoint of the interval satisfies either equation.

Step 3: Sum and scale the angles The sum is

$$\sum_{\theta \in S} \theta = -\frac{2\pi}{3} + \frac{\pi}{3} - \frac{\pi}{4} + \frac{3\pi}{4} = -\frac{\pi}{3} + \frac{\pi}{2} = \frac{\pi}{6}.$$

Consequently

$$\frac{120}{\pi} \sum_{\theta \in S} \theta = 20.$$

The other choices 40, 30 and 10 do not follow from the complete four-angle set; retaining only principal tangent values is not valid over this interval.

Step 4: Conclusion The required value is $\boxed{20}$. Hence, the correct option is (D).

Quick Tip to Solve: Subtract rows with identical first two entries before expanding the determinant, then list all tangent solutions over the full stated interval.

14. Let O be the origin and OP and OQ be the tangents to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$ at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point $(\alpha, \frac{1}{2})$, then a value of α is : [4 marks]

(A) 1

(B) $-\frac{1}{2}$

(C) $\frac{5}{2}$

(D) $\frac{3}{2}$

Correct Answer: (C) $\frac{5}{2}$

Solution:

Concept:

- The chord of contact of tangents from the origin to $x^2 + y^2 + 2gx + 2fy + c = 0$ is $gx + fy + c = 0$.
- Any circle through the intersection points of a circle $S = 0$ and a line $L = 0$ has the form $S + \mu L = 0$. Requiring it to pass through the origin fixes the parameter.

Step 1: Find the common chord PQ The given circle has $g = -3$, $f = 2$ and $c = 8$. The tangent at a point (x_1, y_1) of this circle is

$$xx_1 + yy_1 - 3(x + x_1) + 2(y + y_1) + 8 = 0.$$

Because both tangents OP and OQ pass through $(0, 0)$, their contact points satisfy $-3x_1 + 2y_1 + 8 = 0$. Thus the chord joining these points is

$$L = -3x + 2y + 8 = 0.$$

This derives the polar equation directly from the tangent condition.

Step 2: Construct the circumcircle through the origin Both P and Q satisfy the original circle S and the chord equation L. Their circumcircle with O is therefore a member of

$$S + \mu L = x^2 + y^2 - 6x + 4y + 8 + \mu(-3x + 2y + 8) = 0.$$

At O the constant term must vanish, so $8 + 8\mu = 0$, or $\mu = -1$. Substitution gives the circumcircle

$$x^2 + y^2 - 3x + 2y = 0.$$

Its equation has zero constant term, confirming that it passes through O as required.

Step 3: Insert the specified point Putting $x = \alpha$ and $y = 1/2$ gives

$$\alpha^2 + \frac{1}{4} - 3\alpha + 1 = 0, \qquad 4\alpha^2 - 12\alpha + 5 = 0.$$

Factor to obtain

$$(2\alpha - 1)(2\alpha - 5) = 0.$$

Hence the possible values are $1/2$ and $5/2$. Only $5/2$ occurs among the options; the listed negative half is not the positive half-root. The values 1 and $3/2$ also fail the quadratic.

Step 4: Conclusion A permitted listed value is $\boxed{\alpha = 5/2}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the chord of contact first, then subtract its equation from the original circle to obtain the circle through both contact points and the origin.

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15. Let a_n be the n^{th} term of the series $5 + 8 + 14 + 23 + 35 + 50 + \ldots$ and $S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to : [4 marks]

(A) 11280

(B) 11290

(C) 11310

(D) 11260

Correct Answer: (B) 11290

Solution:

Concept:

- The successive differences are $3, 6, 9, 12, 15, \ldots$. Thus the difference added to obtain the nth term is $3(n-1)$, and the first term is 5.
- Use $\sum_{k=1}^{n} k = n(n+1)/2$ and $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$ to sum the quadratic expression for the general term.

Step 1: Obtain the general term Summing the first $n-1$ differences gives

$$a_n = 5 + 3(1 + 2 + \cdots + (n-1)) = 5 + \frac{3}{2}n(n-1).$$

This reproduces 5, 8 and 14 for the first three indices, so the indexing is consistent with the given series.

Step 2: Evaluate the sum and subtract the requested term For the first 30 terms,

$$S_{30} = 150 + \frac{3}{2}\left(\sum_{k=1}^{30} k^2 - \sum_{k=1}^{30} k\right) = 150 + \frac{3}{2}(9455 - 465) = 13635.$$

Separately,

$$a_{40} = 5 + \frac{3}{2}(40)(39) = 2345.$$

Therefore $S_{30} - a_{40} = 13635 - 2345 = 11290$. The other listed numbers do not equal this difference; using 39 differences rather than 40 is essential in computing a_{40}.

Step 3: Conclusion $S_{30} - a_{40} = 11290$. Hence, the correct option is (B).

Quick Tip to Solve: When first differences form an arithmetic progression, sum those differences to obtain a quadratic formula for the term before finding the series sum.

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16. Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}, \vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}, \vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$ are also coplanar, then $6(a+b+c)$ is equal to : [4 marks]

(A) 12

(B) 6

(C) 0

(D) 4

Correct Answer: (A) 12

Solution:

Concept:

- Three vectors are coplanar precisely when their scalar triple product, the determinant of their component matrix, vanishes. Apply this condition independently to each given triple.
- Let $s = a + b + c$ and $p = abc$. Expressing the two determinant conditions in these symmetric quantities avoids solving separately for all three parameters.

Step 1: Use the first coplanarity condition The first component determinant is

$$\begin{vmatrix} 1 & 1 & a \\ 1 & b & 1 \\ c & 1 & 1 \end{vmatrix} = b - 1 - (1 - c) + a(1 - bc) = s - p - 2.$$

Hence $s - p = 2$. Coplanarity alone does not require that the three vectors be equal or that their individual components vanish.

Step 2: Use the second determinant and combine For the second triple, expansion gives

$$\begin{aligned} 0 &= \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix} \\ &= (a+b)((b+c)(c+a) - ab) \\ &\quad - c(a(c+a) - ab) + c(ab - b(b+c)) \\ &= 4abc. \end{aligned}$$

Therefore $p = 0$, and the first condition yields $s = 2$. The required expression is $6s = 12$. Values 6, 0 and 4 would give sums 1, 0 and $2/3$, contradicting $s - p = 2$ with $p = 0$.

Step 3: Conclusion $6(a + b + c) = 12$. Hence, the correct option is (A).

Quick Tip to Solve: For two coplanar triples, expand both determinants before attempting to solve for the parameters; the sum and product may be all that is needed.

17. For $\mathrm{a}, \mathrm{b} \in \mathbb{Z}$ and $|\mathrm{a}-\mathrm{b}| \leq 10$, let the angle between the plane $\mathrm{P}: \mathrm{ax}+y-\mathrm{z}=\mathrm{b}$ and the line $l: x-1=\mathrm{a}-y=z+1$ be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point $(6,-6,4)$ from the plane P is $3\sqrt{6}$, then a^4+b^2 is equal to : [4 marks]

(A) 48

(B) 85

(C) 32

(D) 25

Correct Answer: (C) 32

Solution:

Concept:

- The plane normal is $\mathbf{n}=(a,1,-1)$, while the line has direction $\mathbf{d}=(1,-1,1)$. For the acute line-plane angle θ, $\sin\theta=|\mathbf{n}\cdot\mathbf{d}|/(|\mathbf{n}||\mathbf{d}|)$.
- The distance of (x_0,y_0,z_0) from $ax+y-z-b=0$ is $|ax_0+y_0-z_0-b|/\sqrt{a^2+2}$. Use both integer and difference restrictions to remove extraneous possibilities.

Step 1: Determine the integer parameter from the angle Since $\cos\theta=1/3$, $\sin\theta=2\sqrt{2}/3$. Consequently

$$\frac{|a-2|}{\sqrt{3(a^2+2)}}=\frac{2\sqrt{2}}{3}.$$

Squaring and simplifying gives

$$9(a-2)^2=24(a^2+2), \qquad 5a^2+12a+4=0.$$

Its roots are -2 and $-2/5$. The integer condition forces $a=-2$.

Step 2: Apply the distance and difference conditions The given distance now becomes

$$\frac{|-22-b|}{\sqrt{6}}=3\sqrt{6}, \qquad |-22-b|=18.$$

Thus $b=-4$ or -40. Their differences from $a=-2$ are 2 and 38, respectively, so only $b=-4$ satisfies $|a-b|\leq 10$. Finally

$$a^4+b^2=16+16=32.$$

None of the other proposed values is the sum for the admissible pair.

Step 3: Conclusion $a=-2$, $b=-4$, and $a^4+b^2=32$. Hence, the correct option is (C).

Quick Tip to Solve: Use the sine of the angle between a line and a plane; the cosine formula instead gives the angle between the line and the normal.

18. The negation of $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to : [4 marks]

(A) $p \wedge q$

(B) $p \wedge (\sim q)$

(C) $p \wedge (q \wedge (\sim p))$

(D) $p \vee (q \vee (\sim p))$

Correct Answer: (A) $p \wedge q$

Solution:

Concept:

- De Morgan's laws state $\sim (X \vee Y) = (\sim X) \wedge (\sim Y)$ and $\sim (X \wedge Y) = (\sim X) \vee (\sim Y)$. A double negation returns the original proposition.
- Distribution and the contradiction law $p \wedge (\sim p) =$ False simplify the expression after negating it. The requested result is the negation of the entire displayed proposition.

Step 1: Negate the complete disjunction Let $E = (p \wedge (\sim q)) \vee (\sim p)$. Applying De Morgan's law to its outermost connective first gives

$$\sim E = \big((\sim p) \vee q\big) \wedge p.$$

Both the disjunction and the conjunction change when the negation passes through them, while $\sim (\sim q) = q$.

Step 2: Simplify and test the alternatives Distributing p gives

$$\sim E = (p \wedge (\sim p)) \vee (p \wedge q) = p \wedge q.$$

This is true exactly when both p and q are true. The second option instead holds when p is true and q is false. The third contains both p and $\sim p$, making it always false. The fourth contains $p \vee (\sim p)$, making it always true. These truth conditions rule out all three alternatives.

Step 3: Conclusion The negation is $p \wedge q$. Hence, the correct option is (A).

Quick Tip to Solve: Negate from the outside inward, switching each AND to OR and each OR to AND; simplify double negations only after preserving the brackets.

19. Let P be the plane passing through the line $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$ and the point $(2, 4, -3)$. If the image of the point $(-1, 3, 4)$ in the plane P is (α, β, γ) then $\alpha + \beta + \gamma$ is equal to : [4 marks]

(A) 10

(B) 12

(C) 9

(D) 11

Correct Answer: (A) 10

Solution:

Concept:

- A plane containing a given line has a normal perpendicular to its direction vector. A second vector in the plane is obtained by joining a point on the line to the other given point.
- For the plane $\mathbf{n} \cdot \mathbf{x} + d = 0$, reflection of a point $\mathbf{r}$ is $\mathbf{r} - 2(\mathbf{n} \cdot \mathbf{r} + d)\mathbf{n}/|\mathbf{n}|^2$. The factor 2 carries the point through the plane to the opposite side.

Step 1: Find the plane through the line and point The line passes through $R = (1, 2, -5)$ and has direction $\mathbf{u} = (1, -3, 7)$. For $S = (2, 4, -3)$, $\overrightarrow{RS} = (1, 2, 2)$. Their cross product is $(-20, 5, 5)$, so choose $\mathbf{n} = (-4, 1, 1)$. The plane equation is therefore

$$-4(x - 1) + (y - 2) + (z + 5) = 0, \qquad -4x + y + z + 7 = 0.$$

Both the line point and S satisfy this equation.

Step 2: Reflect the point and compute the sum For $T = (-1, 3, 4)$, $\mathbf{n} \cdot T + 7 = 18$ and $|\mathbf{n}|^2 = 18$. Thus

$$T' = (-1, 3, 4) - 2(-4, 1, 1) = (7, 1, 2).$$

The midpoint $(3, 2, 3)$ satisfies the plane equation, confirming the reflection. Hence $\alpha + \beta + \gamma = 7 + 1 + 2 = 10$. The remaining listed sums 12, 9 and 11 do not match these coordinates.

Step 3: Conclusion The image is $(7, 1, 2)$, and $\alpha + \beta + \gamma = 10$. Hence, the correct option is (A).

Quick Tip to Solve: After reflecting a point, verify that its midpoint with the image lies in the plane and their joining segment is parallel to the normal.

20. Let $\mathrm{A}(0,1), \mathrm{B}(1,1)$ and $\mathrm{C}(1,0)$ be the mid-points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4\mathrm{ax}$ passing through D is $(\alpha + \beta\sqrt{2}, 0)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to : [4 marks]

(A) $\frac{9}{2}$

(B) 12

(C) 6

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- If A, B and C are side midpoints, the original triangle has vertices $A + B - C$, $A + C - B$, and $B + C - A$. This reconstructs the sides whose angle bisectors determine the incentre.
- For a right triangle with perpendicular legs u, v and hypotenuse h, the inradius is $r = (u + v - h)/2$. The standard parabola $y^2 = 4ax$ has focus $(a, 0)$.

Step 1: Reconstruct the triangle and locate its incentre The vertices obtained from the three midpoints are

$$(0,2), \quad (0,0), \quad (2,0).$$

This is a right isosceles triangle with legs 2 and hypotenuse $2\sqrt{2}$. Its incircle is tangent to both positive coordinate axes, so its centre is (r, r), where

$$r = \frac{2 + 2 - 2\sqrt{2}}{2} = 2 - \sqrt{2}.$$

The centre lies inside the triangle since $0 < r < 1$.

Step 2: Use the point on the parabola Substituting $D = (r, r)$ into the parabola gives $r^2 = 4ar$. Since $r > 0$, division by r yields

$$a = \frac{r}{4} = \frac{1}{2} - \frac{1}{4}\sqrt{2}.$$

Thus $\alpha = 1/2$ and $\beta = -1/4$. The requested ratio is

$$\frac{\alpha}{\beta^2} = \frac{1/2}{1/16} = 8.$$

Squaring β removes its negative sign. The other choices do not equal this ratio; using the midpoint triangle itself would give an incorrect incentre.

Step 3: Conclusion $\alpha/\beta^2 = 8$. Hence, the correct option is (D).

Quick Tip to Solve: Reconstruct the original vertices before finding its incentre; the given side midpoints describe a smaller triangle with a different inradius.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $[t]$ denote the greatest integer function. If $\int_0^{2.4} [x^2]\,dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$, then $\alpha + \beta + \gamma + \delta$ is equal to _____. [4 marks]

Solution:

Concept:

- For nonnegative x, the value of $[x^2]$ changes only when x crosses $\sqrt{m}$ for an integer m. The integrand is constant between consecutive such thresholds.
- Integrate the constant value on each interval and add the resulting rectangles. The values at the finitely many jump points do not affect a definite integral.

Step 1: Partition the integration interval at every jump The upper limit is $2.4 = 12/5$, with square $144/25 = 5.76$. Hence the thresholds inside the interval are $1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}$. The function takes values 0, 1, 2, 3, 4 and 5 on the successive intervals. In particular the final interval ends at 2.4, not at $\sqrt{6}$, so its length must be treated separately.

Step 2: Add the constant pieces and collect radicals The integral is

$$\begin{aligned} I &= (\sqrt{2}-1) + 2(\sqrt{3}-\sqrt{2}) \\ &\quad + 3(2-\sqrt{3}) + 4(\sqrt{5}-2) \\ &\quad + 5\left(\frac{12}{5} - \sqrt{5}\right) \\ &= 9 - \sqrt{2} - \sqrt{3} - \sqrt{5}. \end{aligned}$$

Thus $\alpha = 9$, $\beta = -1$, $\gamma = -1$, and $\delta = -1$. The constant terms are $-1 + 6 - 8 + 12 = 9$, while each radical has coefficient -1, providing a direct check on the simplification.

Step 3: Conclusion $\alpha+\beta+\gamma+\delta = 9-1-1-1 = 6$. The required numerical value is 6 (dimensionless).

Quick Tip to Solve: For the floor of a square, split at square roots of integers rather than at integers; retain the actual upper limit on the last interval.

22. Let the area enclosed by the lines $x + y = 2, \mathrm{y} = 0, x = 0$ and the curve $f(x) = \min\left\{x^2 + \frac{3}{4}, 1 + [x]\right\}$ where $[x]$ denotes the greatest integer $\le x$, be A. Then the value of 12 A is _____. [4 marks]

Solution:

Concept:

- The lines $x = 0$, $y = 0$, and $x + y = 2$ bound the first-quadrant triangle. The required region lies below both $y = 2 - x$ and $y = f(x)$.
- The greatest-integer function is constant on each interval between successive integers. Split the integral whenever the minimum changes its defining expression.

Step 1: Resolve the minimum on the relevant intervals Only $0 \le x \le 2$ is relevant. On $[0,1)$, $[x]=0$, so compare $x^2+3/4$ with 1. They meet at $x=1/2$. Thus $f(x)=x^2+3/4$ up to $1/2$ and $f(x)=1$ from $1/2$ to 1. On $[1,2)$, both candidates defining f are at least 1, whereas the line has height $2-x \le 1$. Therefore the line, rather than f, is the upper boundary there.

Step 2: Integrate the actual upper boundary The isolated values at the jump points do not change an area integral. The three pieces give

$$\begin{aligned} A &= \int_0^{1/2} (x^2+3/4)\,dx + \int_{1/2}^{1} 1\,dx + \int_1^2 (2-x)\,dx \\ &= \left[\frac{x^3}{3}+\frac{3x}{4}\right]_0^{1/2} + \frac{1}{2} + \frac{1}{2} \\ &= \frac{1}{24}+\frac{3}{8}+1 = \frac{17}{12}. \end{aligned}$$

This is smaller than the area 2 of the whole triangle, as expected after its top-left part is removed by the curve.

Step 3: Conclusion The area is $A = 17/12$ square units, and hence the required dimensionless value is $\boxed{12A = 17}$.

Quick Tip to Solve: For a minimum involving a floor function, split at integers first, then at crossings of the candidate curves and the straight-line boundary.

23. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x+2| - 4 = 0$ respectively, where $[x]$ denotes the greatest integer $\le x$. Then $m^2 + mn + n^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Write $[x] = x - \{x\}$, where the fractional part satisfies $0 \le \{x\} < 1$. This bounds a polynomial before individual integer intervals are checked.
- An absolute-value equation must be solved separately on the two sides of the point where its argument changes sign; retain only roots satisfying their branch condition.

Step 1: Count the roots involving the greatest integer The first equation becomes

$$x^2 - 11x + 31 = \{x\} < 1.$$

Consequently $x^2 - 11x + 30 < 0$, or $(x-5)(x-6) < 0$, which forces $5 < x < 6$. Every possible root therefore has $[x] = 5$. Substitution in the original equation yields

$$x^2 - 12x + 36 = (x-6)^2 = 0.$$

Its only candidate is 6, outside the forced open interval. Directly, the original left side at 6 is 1, not zero. Hence the first equation has no real root and $m = 0$.

Step 2: Count the absolute-value roots and combine For $x \geq -2$, the second equation is

$$x^2 - 5x - 14 = (x-7)(x+2) = 0,$$

so both 7 and -2 are admitted. For $x < -2$, it becomes

$$x^2 + 5x + 6 = (x+2)(x+3) = 0.$$

Only -3 satisfies this strict branch condition. The boundary root -2 is counted once, so there are exactly three distinct real roots: $-3, -2, 7$. Thus $n = 3$, and no other roots can arise because the two branches cover the real line.

Step 3: Conclusion With $m = 0$ and $n = 3$, the required dimensionless number is

$$m^2 + mn + n^2 = 0 + 0 + 9 = \boxed{9}.$$

Quick Tip to Solve: Use the strict inequality $\{x\} < 1$ carefully: it excludes the endpoint candidate 6 here, even though that candidate solves the polynomial on the neighbouring interval.

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24. Let $\mathrm{R} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}\}$ **and** $\mathrm{S} = \{1, 2, 3, 4\}$**. Total number of onto functions** $f : \mathrm{R} \to \mathrm{S}$ **such that** $f(\mathrm{a}) \neq 1$**, is equal to _____.** [4 marks]

Solution:

Concept:

- An onto function must use every element of the codomain. For five inputs and four outputs, exactly one output has two preimages and the other three have one each.
- The four codomain labels are symmetric. Among all onto functions, each possible value of $f(a)$ occurs equally often, so excluding one label retains three quarters of them.

Step 1: Count all onto maps Partition the five distinct elements of R into four nonempty fibres. Since their sizes must be $2, 1, 1, 1$, choose the unique pair of inputs sharing an image in $\binom{5}{2} = 10$ ways. Now assign the resulting four fibres bijectively to the four labelled elements of S in $4! = 24$ ways. Thus

$$N_{\text{onto}} = \binom{5}{2} 4! = 10 \times 24 = 240.$$

There is no overcount: each onto map determines one and only one repeated-image pair, and its four fibre labels recover that map uniquely.

Step 2: Impose the restriction on the image of a Relabelling the outputs gives a bijection between onto maps with $f(a) = 1$ and those with $f(a) = j$, for each $j = 2, 3, 4$. Therefore exactly $240/4 = 60$ onto maps have $f(a) = 1$. Removing these leaves

$$N = 240 - 60 = \frac{3}{4}(240) = 180.$$

As a direct check for any fixed allowed value of $f(a)$, the remaining four inputs must hit the other

three labels. Inclusion-exclusion gives $4^4 - 3 \cdot 3^4 + 3 \cdot 2^4 - 1 = 60$, again yielding $3 \cdot 60$.

Step 3: Conclusion The total number of onto functions satisfying $f(a) \neq 1$ is $\boxed{180}$. This is a dimensionless count of functions.

Quick Tip to Solve: When there is one more input than outputs, count the unique double fibre first. A restriction on one output label can then often be handled by label symmetry.

---

25. Let k and m be positive real numbers such that the function $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2, & x \geq 1 \end{cases}$ is differentiable for all $x > 0$. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to _____. [4 marks]

Solution:

Concept:

- Both displayed branches are differentiable on their own open intervals. The only joining point requiring a check is $x = 1$. Differentiability there requires continuity and equality of the one-sided derivatives.
- Use the correct branch when evaluating a derivative: $8 \geq 1$, whereas $1/8$ lies strictly between 0 and 1.

Step 1: Match the value and derivative at the join Continuity and derivative equality at 1 give, respectively,

$$3 + k\sqrt{2} = m + k^2, \qquad 6 + \frac{k}{2\sqrt{2}} = 2m.$$

The second equation yields $m = 3 + k/(4\sqrt{2})$. Substituting in the first and using $k > 0$,

$$k\sqrt{2} - k^2 = \frac{k}{4\sqrt{2}} \quad \Longrightarrow \quad k = \sqrt{2} - \frac{1}{4\sqrt{2}} = \frac{7\sqrt{2}}{8}.$$

It follows that $m = 3 + 7/32 = 103/32$, which is positive as required. These conditions settle the only possible discontinuity in the derivative of the piecewise definition.

Step 2: Evaluate the two derivatives and their ratio On the lower branch, $f'(x) = 6x + k/(2\sqrt{x+1})$; on the upper branch, $f'(x) = 2mx$. Thus

$$f'(8) = 16m = \frac{103}{2}.$$

Since $\sqrt{1 + 1/8} = 3/(2\sqrt{2})$, the other derivative is

$$f'(1/8) = \frac{3}{4} + \frac{7\sqrt{2}/8}{3/\sqrt{2}} = \frac{3}{4} + \frac{7}{12} = \frac{4}{3}.$$

Both are nonzero and positive, so the requested quotient is well-defined. Finally,

$$\frac{8f'(8)}{f'(1/8)} = 8 \cdot \frac{103}{2} \cdot \frac{3}{4} = 309.$$

Step 3: Conclusion The required dimensionless value is $\boxed{309}$.

Quick Tip to Solve: At a piecewise join, first use derivative matching to express one parameter linearly, then substitute in continuity. Do not use the same branch at both evaluation points.

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26. Let the solution curve $x = x(y), 0 < y < \frac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2 \cos y \, dx - (1 + 3x \log_e(\cos y)) \sin y dy = 0$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$, where m and n are coprime, then mn is equal to _____. [4 marks]

Solution:

Concept:

- The substitution $t = \ln(\cos y)$ gives $dt = -\tan y \, dy$. On $0 < y < \pi/2$, $\cos y > 0$ and $t < 0$, so the logarithm and division by t are valid.
- A first-order linear equation $dx/dt + P(t)x = Q(t)$ is solved by multiplying by an integrating factor $e^{\int P(t)dt}$. A nonzero constant factor in that integrating factor does not affect the solution.

Step 1: Transform and integrate the equation Divide the given equation by $\cos y$, then use the substitution. The resulting equation is

$$t^2 \, dx + (1 + 3xt) \, dt = 0, \qquad \frac{dx}{dt} + \frac{3}{t}x = -\frac{1}{t^2}.$$

An integrating factor on the negative t-interval is t^3. Multiplying gives

$$\frac{d}{dt}(xt^3) = -t, \quad xt^3 = -\frac{t^2}{2} + C, \quad x = -\frac{1}{2t} + \frac{C}{t^3}.$$

Indeed, differentiating the product recovers both the coefficient $3/t$ and the right side, fixing the sign from $dt = -\tan y \, dy$.

Step 2: Use the initial value and identify coprime integers At $y = \pi/3$, $t = -\ln 2$. The supplied value $x = 1/(2\ln 2) = -1/(2t)$ forces $C = 0$. Therefore

$$x(y) = -\frac{1}{2\ln(\cos y)}.$$

At $y = \pi/6$, this gives

$$x(\pi/6) = -\frac{1}{2\ln(\sqrt{3}/2)} = \frac{1}{\ln 4 - \ln 3}.$$

Comparing with the required form gives $m/n = 4/3$. Because m, n are coprime positive integers, they are $m = 4, n = 3$, rather than arbitrary common multiples.

Step 3: Conclusion The required product is the dimensionless integer $\boxed{mn = 4 \times 3 = 12}$.

Quick Tip to Solve: Whenever both $\ln(\cos y)$ and $\tan y \, dy$ occur, substitute the logarithm first. Track the minus sign in its differential before finding the integrating factor.

27. If domain of the function

$$\log_e\left(\frac{6x^2+5x+1}{2x-1}\right)+\cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$$

is $(\alpha,\beta)\cup(\gamma,\delta]$, then $18\left(\alpha^2+\beta^2+\gamma^2+\delta^2\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- A real logarithm requires its argument to be strictly positive. A real inverse cosine requires its argument to lie in the closed interval $[-1,1]$. The domain of the sum is the intersection of these two conditions.
- Factor the logarithmic rational expression and use a sign chart. When multiplying the inverse-cosine inequalities by its denominator, determine the denominator sign first.

Step 1: Solve the logarithm condition The argument factors as

$$\frac{6x^2+5x+1}{2x-1}=\frac{(3x+1)(2x+1)}{2x-1}.$$

Its zeros are $-1/2, -1/3$, and its pole is $1/2$. A sign chart across these three points gives strict positivity precisely for

$$x\in(-1/2,-1/3)\cup(1/2,\infty).$$

None of the endpoints is admitted: a zero argument is invalid for the logarithm, and the denominator cannot vanish.

Step 2: Solve the inverse-cosine condition and intersect Write $N=2x^2-3x+4$. Its discriminant is $9-32<0$, so $N>0$ for all real x. If $x>5/3$, the upper inequality $N/(3x-5)\le 1$ would require $2x^2-6x+9\le 0$, impossible since its minimum is positive. If $x<5/3$, the ratio is negative, so the upper bound is automatic. Its lower bound gives

$$\frac{N}{3x-5}\ge -1 \iff N\le 5-3x \iff 2x^2\le 1.$$

Thus the inverse cosine has domain $[-1/\sqrt{2},1/\sqrt{2}]$. Intersecting with the logarithm domain yields

$$(-1/2,-1/3)\cup(1/2,1/\sqrt{2}].$$

The last endpoint is retained because inverse cosine allows its argument to equal -1, and the logarithm remains positive there.

Step 3: Conclusion Thus $\alpha=-1/2,\ \beta=-1/3,\ \gamma=1/2,\ \delta=1/\sqrt{2}$, giving

$$\boxed{18(\alpha^2+\beta^2+\gamma^2+\delta^2)=18\left(\frac{1}{4}+\frac{1}{9}+\frac{1}{4}+\frac{1}{2}\right)=20}.$$

The required value is dimensionless.

Quick Tip to Solve: Keep the logarithm endpoints open even when the inverse-cosine condition allows them. Intersect domain sets only after solving each condition separately.

28. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points $(2, -1, 0), (2, 0, -1)$, and $(5, 1, 1)$. If the foot of the perpendicular drawn from the point $(7, 4, -1)$ on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to _____. [4 marks]

Solution:

Concept:

- The cross product of two nonparallel vectors in a plane gives a normal to that plane. Solve the two plane equations simultaneously to parametrize their intersection line.
- If a line is $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{d}$, the perpendicular foot from $\boldsymbol{p}$ satisfies $(\boldsymbol{p} - \boldsymbol{a} - t\boldsymbol{d}) \cdot \boldsymbol{d} = 0$. This determines its unique projection parameter.

Step 1: Find the second plane and the intersection line Take the given points as $A = (2, -1, 0)$, $B = (2, 0, -1)$, and $C = (5, 1, 1)$. Their difference vectors are

$$\overrightarrow{AB} = (0, 1, -1), \qquad \overrightarrow{AC} = (3, 2, 1).$$

Their cross product is $(3, -3, -3)$, so the second plane has equation $x - y - z = 3$. Substituting $y = x - z - 3$ into $3x - y - 7z = 11$ gives $x = 4 + 3z$. Hence $y = 1 + 2z$, and the intersection line is

$$(x, y, z) = (4, 1, 0) + t(3, 2, 1).$$

Step 2: Project the point onto that line For the given point $P = (7, 4, -1)$, the vector from the base point $(4, 1, 0)$ to P is $(3, 3, -1)$. The projection parameter is therefore

$$t = \frac{(3, 3, -1) \cdot (3, 2, 1)}{3^2 + 2^2 + 1^2} = \frac{9 + 6 - 1}{14} = 1.$$

The foot is $H = (7, 3, 1)$. Check both requirements directly: $3(7) - 3 - 7(1) = 11$ and $7 - 3 - 1 = 3$, so H belongs to both planes. Also $P - H = (0, 1, -2)$, whose dot product with $(3, 2, 1)$ is zero. Thus this is the perpendicular foot, not merely an arbitrary point of intersection.

Step 3: Conclusion The coordinates are $(\alpha, \beta, \gamma) = (7, 3, 1)$, and the required dimensionless sum is $\boxed{\alpha + \beta + \gamma = 11}$.

Quick Tip to Solve: Project onto the common line after parametrizing it. Projecting separately onto either plane generally gives a different foot and does not solve the intersection-line problem.

29. The ordinates of the points P and Q on the parabola with focus $(3, 0)$ and directrix $x = -3$ are in the ratio $3 : 1$. If $R(\alpha, \beta)$ is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to _____. [4 marks]

Solution:

Concept:

- The focus $(a, 0)$ and directrix $x = -a$ define the parabola $y^2 = 4ax$. Here $a = 3$, so its equation is $y^2 = 12x$.

- A point with parameter t on this parabola is $(3t^2, 6t)$. The tangent there is $ty = x + 3t^2$, so ordinates and parameters have the same ratio.

Step 1: Express the points using their ordinate ratio Let the parameters of Q and P be t and $3t$, respectively. Their ordinates are then $6t$ and $18t$, as required by the ratio $3 : 1$. The stated ratio and the intersection of two tangents require distinct points, so $t \neq 0$. The tangents at the two points are therefore

$$ty = x + 3t^2, \qquad 3ty = x + 27t^2.$$

Using a signed parameter allows the points to lie below the axis as well; the calculation does not depend on the sign of t.

Step 2: Find the coordinates of the tangent intersection Subtract the first tangent equation from the second to eliminate x:

$$2ty = 24t^2 \quad \Rightarrow \quad \beta = y = 12t.$$

Substituting this result into the first tangent gives

$$\alpha = x = t(12t) - 3t^2 = 9t^2.$$

Consequently the unknown position of the two points cancels from the required ratio:

$$\frac{\beta^2}{\alpha} = \frac{(12t)^2}{9t^2} = 16.$$

The denominator is positive because $t \neq 0$, so the expression is well defined.

Step 3: Conclusion The required value is $\boxed{16}$. This is a dimensionless numerical ratio in the coordinates used.

Quick Tip to Solve: For $y^2 = 4ax$, tangents at parameters t_1, t_2 meet at $(at_1t_2, a(t_1 + t_2))$; use the ordinate ratio directly as the parameter ratio.

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30. Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to _____.

[4 marks]

Solution:

Concept:

- For three reciprocals in arithmetic progression, the middle reciprocal is their arithmetic mean: $2/y = 1/x + 1/z$.
- For $x, \sqrt{2}y, z$ in geometric progression, the square of the middle term equals the product of the outer terms: $xz = 2y^2$. Positivity permits division by xyz.

Step 1: Determine the middle number Divide the given equation by the positive product xyz:

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{\sqrt{2}}.$$

Since these three reciprocals form an arithmetic progression, their sum is three times the middle term. Thus

$$\frac{3}{y} = \frac{3}{\sqrt{2}} \quad \Rightarrow \quad y = \sqrt{2}.$$

The GP condition now yields $xz = 2(\sqrt{2})^2 = 4$. The reciprocal AP condition also becomes

$$\frac{1}{x} + \frac{1}{z} = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

Step 2: Find the sum and evaluate the expression Combine the two outer reciprocals using the product already found:

$$\frac{x+z}{xz} = \sqrt{2} \quad \Rightarrow \quad x + z = 4\sqrt{2}.$$

Therefore

$$x + y + z = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2}, \qquad 3(x+y+z)^2 = 3 \cdot 50 = 150.$$

The ordering can also be checked rather than merely assumed: the positive roots of $u^2 - 4\sqrt{2}u + 4 = 0$ are $2\sqrt{2}+2$ and $2\sqrt{2}-2$. They lie above and below $y = \sqrt{2}$, respectively, so the required $0 < z < y < x$ is consistent with the derived sum.

Step 3: Conclusion The requested numerical value is $\boxed{150}$.

Quick Tip to Solve: When reciprocals are in AP, their sum is $3/y$. Divide a symmetric equation by xyz before trying to solve separately for x and z.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A hydraulic automobile lift is designed to lift vehicles of mass 5000 kg. The area of cross section of the cylinder carrying the load is 250 cm^2. The maximum pressure the smaller piston would have to bear is [Assume $g = 10$ m/s^2] [4 marks]

(A) $20 \times 10^{+6}$ Pa

(B) $200 \times 10^{+6}$ Pa

(C) $2 \times 10^{+5}$ Pa

(D) $2 \times 10^{+6}$ Pa

Correct Answer: (D) $2 \times 10^{+6}$ Pa

Solution:

Concept:

- Pascal's law transmits an applied pressure throughout a confined fluid. The small piston must withstand the pressure required under the load-carrying piston.
- Pressure is normal force divided by area: $P = F/A$. A vehicle of mass m requires an upward support force $F = mg$; areas must be expressed in square metres.

Step 1: Convert the load and piston area to SI units The lift must support the maximum vehicle mass

$m = 5000\,\text{kg}$. With the specified gravitational acceleration, the load is

$$F = mg = 5000 \times 10 = 5.0 \times 10^4\,\text{N}.$$

Since $1\,\text{cm} = 10^{-2}\,\text{m}$, the area conversion must be squared:

$$A = 250\,\text{cm}^2 = 250 \times 10^{-4}\,\text{m}^2 = 2.5 \times 10^{-2}\,\text{m}^2.$$

This is the area of the large cylinder carrying the vehicle, which is the area needed to compute the fluid pressure.

Step 2: Apply the pressure balance and compare the options At the maximum load, the required pressure is

$$P = \frac{mg}{A} = \frac{5.0 \times 10^4}{2.5 \times 10^{-2}} = 2.0 \times 10^6\,\text{Pa}.$$

The pressure acts on the smaller piston as well; its smaller area changes the force required there, not the transmitted pressure. Option (A) is ten times this pressure, option (B) is one hundred times it, and option (C) is one tenth of it. Only option (D) has the calculated value. A direct check gives $(2.0 \times 10^6)(0.025) = 50000\,\text{N}$, exactly the vehicle weight.

Step 3: Conclusion The smaller piston must bear $\boxed{2 \times 10^6\,\text{Pa}}$. Hence, the correct option is (D).

Quick Tip to Solve: In hydraulic lifts, equate pressures rather than forces, and square the centimetre-to-metre conversion when converting the piston area.

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32. An emf of 0.08 V is induced in a metal rod of length 10 cm held normal to a uniform magnetic field of 0.4 T, when moves with a velocity of: [4 marks]

(A) $20\ \text{ms}^{-1}$

(B) $2\ \text{ms}^{-1}$

(C) $3.2\ \text{ms}^{-1}$

(D) $0.5\ \text{ms}^{-1}$

Correct Answer: (B) $2\ \text{ms}^{-1}$

Solution:

Concept:

- A conductor of length ℓ moving across a magnetic field develops motional emf because the magnetic force separates charges along the conductor.
- For the transverse motion used here, $\varepsilon = B\ell v$, where B is the magnetic flux density and v is the speed perpendicular to the field and rod.

Step 1: Identify the motional emf relation The metal rod has length $10\,\text{cm}$, so in SI units

$$\ell = 10 \times 10^{-2} = 0.10\,\text{m}.$$

Its magnetic field is $B = 0.4\,\text{T}$ and the induced emf magnitude is $\varepsilon = 0.08\,\text{V}$. The magnetic force qvB

on free charges is balanced by the electric force qE, so $E = vB$. Multiplying by the rod length gives the potential difference $E\ell = B\ell v$. We need the magnitude of the speed, so no sign convention for the rod ends is required.

Step 2: Substitute the data and test the choices Rearrange the formula before substituting:

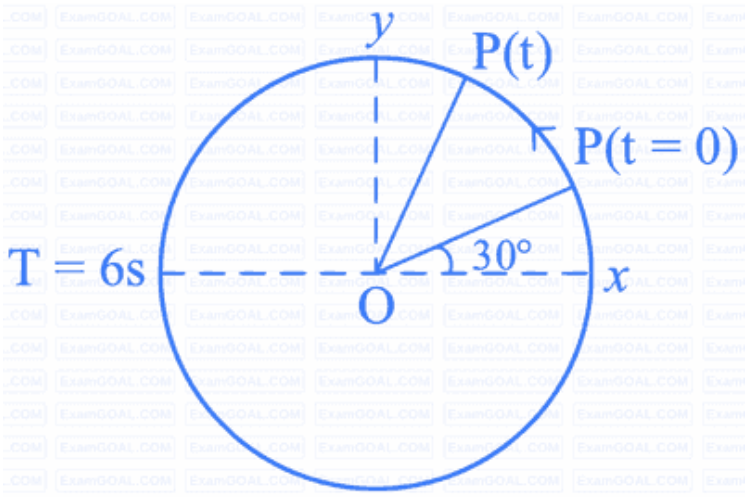
$$v = \frac{\varepsilon}{B\ell} = \frac{0.08}{0.4 \times 0.10} = \frac{0.08}{0.04} = 2\,\mathrm{m\,s^{-1}}.$$

The other listed speeds can be tested directly in $\varepsilon = 0.04v$. A speed of $20\,\mathrm{m\,s^{-1}}$ gives $0.8\,\mathrm{V}$, $3.2\,\mathrm{m\,s^{-1}}$ gives $0.128\,\mathrm{V}$, and $0.5\,\mathrm{m\,s^{-1}}$ gives $0.02\,\mathrm{V}$. None equals the stated emf. The calculated speed of $2\,\mathrm{m\,s^{-1}}$ gives $0.08\,\mathrm{V}$ exactly.

Step 3: Conclusion The rod moves with speed $\boxed{2\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the rod length to metres first. Then divide the emf by $B\ell$; a quick substitution of the chosen speed checks any decimal error.

33. For particle P revolving round the centre O with radius of circular path r and angular velocity ω, as shown in below figure, the projection of OP on the x-axis at time t is [4 marks]



(A) $x(t) = \cos\left(\omega t - \frac{\pi}{6}\omega\right)$
(B) $x(t) = \cos(\omega t)$
(C) $x(t) = r\cos\left(\omega t + \frac{\pi}{6}\right)$
(D) $x(t) = r\sin\left(\omega t + \frac{\pi}{6}\right)$
Correct Answer: (C) $x(t) = r\cos\left(\omega t + \frac{\pi}{6}\right)$

Solution:

Concept:

- The projection on the horizontal axis of a radius vector of length r making angle θ with the positive horizontal axis is $x = r\cos\theta$.
- In uniform circular motion the angle evolves as $\theta(t) = \theta_0 + \omega t$ for counterclockwise rotation, where

θ_0 is the initial phase.

Step 1: Read the initial phase and rotation direction The figure shows the initial radius $OP(t=0)$ at an angle of 30° above the positive x-axis. Therefore its initial phase is

$$\theta_0 = 30^\circ = \frac{\pi}{6}.$$

The arrow indicates counterclockwise motion from the initial position toward $P(t)$. After time t, the angle swept out is ωt, so the angle measured from the positive x-axis is

$$\theta(t) = \omega t + \frac{\pi}{6}.$$

The displayed period fixes the angular speed if needed, but the answer choices already use the symbol ω.

Step 2: Take the horizontal component and reject the alternatives The radius remains constant, so its horizontal component is

$$x(t) = r\cos\theta(t) = r\cos\left(\omega t + \frac{\pi}{6}\right).$$

At $t=0$, this gives $x(0) = r\cos(\pi/6) = \sqrt{3}r/2$, consistent with the plotted initial position. Options (A) and (B) omit the radius factor, and (B) also incorrectly starts on the positive horizontal axis. Option (A) additionally fails to represent the specified initial phase. Option (D) gives $r/2$ initially and corresponds to a sine projection rather than the horizontal component at the shown phase. Hence only (C) matches both the amplitude and the initial position.

Step 3: Conclusion The projection is $\boxed{x(t) = r\cos(\omega t + \pi/6)}$. Hence, the correct option is (C).

Quick Tip to Solve: First read the initial angle from the positive axis, then check $t=0$. This distinguishes sine from cosine and fixes the sign of the phase immediately.

34. In photo electric effect

A. The photocurrent is proportional to the intensity of the incident radiation

B. Maximum Kinetic energy with which photoelectrons are emitted depends on the intensity of incident light.

C. Max. K.E with which photoelectrons are emitted depends on the frequency of incident light.

D. The emission of photoelectrons require a minimum threshold intensity of incident radiation.

E. Max. K.E of the photoelectrons is independent of the frequency of the incident light.

Choose the correct answer from the options given below: [4 marks]

(A) A and E only

(B) A and B only

(C) B and C only

(D) A and C only

Correct Answer: (D) A and C only

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the metal work function and ν is the incident frequency.
- At a fixed frequency above threshold, increasing intensity increases the photon arrival rate and hence the photoelectron current. It does not increase the energy of an individual photon.

Step 1: Evaluate the statements about current and kinetic energy Statement A is true: for a fixed frequency above threshold and unchanged collection conditions, a greater light intensity supplies more photons each second, leading to a proportionally larger photocurrent. Statement B is false because intensity changes the photon number, whereas each photon still has energy $h\nu$. The greatest kinetic energy therefore remains unchanged when only intensity changes. Statement C is true because

$$K_{\max} = h\nu - \phi$$

explicitly increases with incident frequency for the same metal. The work function is a property of the surface, not of the beam intensity.

Step 2: Identify the threshold condition and select the pair Statement D is false: the threshold condition concerns frequency rather than intensity. Emission requires

$$h\nu \geq \phi, \qquad \nu \geq \nu_0 = \frac{\phi}{h}.$$

Even intense light below the threshold frequency cannot produce the usual single-photon photoelectric emission. Statement E is false because it claims the maximum kinetic energy is independent of frequency, contradicting Einstein's equation. Thus A and C alone are correct. Option (A) includes the false statement E, option (B) includes B, and option (C) also includes B. Option (D) is the only pair containing exactly the two true statements.

Step 3: Conclusion Only statements A and C are correct. Hence, the correct option is (D).

Quick Tip to Solve: Keep the two controls separate: intensity determines the number of emitted electrons, while frequency determines their maximum kinetic energy and whether emission can occur.

35. A bullet of mass $0.1\ \text{kg}$ moving horizontally with speed $400\ \text{ms}^{-1}$ hits a wooden block of mass $3.9\ \text{kg}$ kept on a horizontal rough surface. The bullet gets embedded into the block and moves $20\ \text{m}$ before coming to rest. The coefficient of friction between the block and the surface is _____.
(Given $g = 10\ \text{m/s}^2$) [4 marks]

(A) 0.65

(B) 0.25

(C) 0.50

(D) 0.90

Correct Answer: (B) 0.25

Solution:

Concept:

- When a bullet embeds in a block, the collision is perfectly inelastic. Horizontal momentum is conserved during the short impact, although mechanical energy is not conserved in the collision.
- After impact, kinetic friction stops the combined mass. Its deceleration has magnitude μg, and work-energy gives $\mu Mgs = \frac{1}{2}Mv^2$.

Step 1: Find the speed immediately after embedding The initial bullet momentum is $0.1 \times 400 = 40\,\text{kg}\,\text{m}\,\text{s}^{-1}$, and the block is initially at rest. The combined mass after impact is

$$M = 0.1 + 3.9 = 4.0\,\text{kg}.$$

Neglecting the external friction impulse over the very short collision interval, conservation of horizontal momentum yields

$$0.1 \times 400 = 4.0v \quad \Rightarrow \quad v = 10\,\text{m}\,\text{s}^{-1}.$$

One must not use kinetic-energy conservation to obtain this speed: embedding dissipates energy in deformation and heating.

Step 2: Use the stopping distance to find friction As the joined block and bullet slide, their initial kinetic energy is removed by the work of friction over $s = 20\,\text{m}$:

$$\frac{1}{2}Mv^2 = \mu Mgs.$$

Cancel the combined mass and substitute the specified gravitational acceleration:

$$\mu = \frac{v^2}{2gs} = \frac{10^2}{2 \times 10 \times 20} = \frac{100}{400} = 0.25.$$

For a check, $\mu g = 2.5\,\text{m}\,\text{s}^{-2}$ and $s = v^2/(2\mu g) = 20\,\text{m}$. The alternatives 0.65, 0.50 and 0.90 produce stopping distances smaller than 20 m at the same starting speed, so they are excluded.

Step 3: Conclusion The coefficient of friction is $\boxed{\mu = 0.25}$, a dimensionless quantity. Hence, the correct option is (B).

Quick Tip to Solve: Split an embedding problem into two stages: conserve momentum during impact, then use frictional work or constant deceleration during the subsequent slide.

36. Given below are two statements

Statement I : Area under velocity- time graph gives the distance travelled by the body in a given time.

Statement II : Area under acceleration- time graph is equal to the change in velocity- in the given time.

In the light of given statements, choose the correct answer from the options given below. [4 marks]

(A) Both Statement I and Statement II are False.

(B) Both Statement I and Statement II are true.

(C) Statement I is incorrect but Statement II is true.

(D) Statement I is correct but Statement II is false.

Correct Answer: (C) Statement I is incorrect but Statement II is true.

Solution:

Concept:

- Velocity is the signed rate of change of position: $v = dx/dt$. Consequently, the algebraic area under a velocity-time curve gives displacement, $\Delta x = \int v\,dt$.
- Distance travelled counts motion in either direction positively, so $s = \int |v|\,dt$. Acceleration satisfies $a = dv/dt$, giving $\Delta v = \int a\,dt$.

Step 1: Test Statement I The statement identifies area under a velocity-time graph with distance without restricting the sign of velocity. The usual signed area gives displacement. For example, travelling at $+2\,\mathrm{m\,s^{-1}}$ for one second and then at $-2\,\mathrm{m\,s^{-1}}$ for one second gives algebraic area zero, but total distance $2 + 2 = 4\,\mathrm{m}$. To obtain distance one must add the magnitudes of the areas above and below the time axis. Thus Statement I is not generally correct.

Step 2: Test Statement II and compare the choices Integrating acceleration over the specified interval gives

$$\int_{t_1}^{t_2} a(t)\,dt = v(t_2) - v(t_1).$$

This identity is valid even if acceleration changes sign; positive and negative areas give the signed change in velocity. Statement II is therefore true. Options (A) and (D) incorrectly make Statement II false, while option (B) incorrectly treats Statement I as a general distance formula.

Step 3: Conclusion Statement I is incorrect, but Statement II is true. Hence, the correct option is (C).

Quick Tip to Solve: For distance use the area under the speed-time graph, or add absolute areas under a velocity-time graph; retain signs when finding displacement.

37. The acceleration due to gravity at height h above the earth if $h << \mathrm{R}$ (Radius of earth) is given by [4 marks]

(A) $g' = g\left(1 - \frac{2h}{R}\right)$

(B) $g' = g\left(1 - \frac{2h^2}{R^2}\right)$

(C) $g' = g\left(1 - \frac{h^2}{2R^2}\right)$

(D) $g' = g\left(1 - \frac{h}{2R}\right)$

Correct Answer: (A) $g' = g\left(1 - \frac{2h}{R}\right)$

Solution:

Concept:

- Outside a spherical Earth of mass M, the gravitational acceleration at distance r from its centre is GM/r^2. At the surface, $g = GM/R^2$.
- For a small dimensionless quantity x, the first-order binomial expansion is $(1+x)^{-2} \simeq 1 - 2x$; terms containing x^2 and higher powers are neglected.

Step 1: Write the exact acceleration at altitude At height h above the surface, the distance from the centre is $R + h$, rather than h. Thus

$$g' = \frac{GM}{(R+h)^2} = g\frac{R^2}{(R+h)^2} = g\left(1 + \frac{h}{R}\right)^{-2}.$$

This expression also gives $g' = g$ at $h = 0$, as required. The decrease follows from the inverse-square dependence of gravitational attraction.

Step 2: Expand in the small height ratio Since $h \ll R$, retain only the first power of h/R:

$$\left(1 + \frac{h}{R}\right)^{-2} \simeq 1 - \frac{2h}{R}.$$

Hence the fractional decrease is $2h/R$. Options (B) and (C) omit this nonzero linear correction and instead begin with a squared height ratio. Option (D) contains a linear term, but its coefficient is $1/2$ instead of the required coefficient 2. The approximation describes a small positive altitude; it is not an exact expression at arbitrary height.

Step 3: Conclusion For $h \ll R$, $g' \simeq g\left(1 - \frac{2h}{R}\right)$. Hence, the correct option is (A).

Quick Tip to Solve: First replace the centre distance by $R + h$, then use the exponent -2 as the coefficient of the small ratio h/R.

38. The equivalent resistance between A and B as shown in figure is: [4 marks]



(A) 30 kΩ

(B) 20 kΩ

(C) 5 kΩ

(D) 10 kΩ

Correct Answer: (C) 5 kΩ

Solution:

Concept:

- Ideal connecting wires have zero resistance, so all points joined directly by a wire belong to the same electrical node and have the same potential.
- Resistors connected between the same two nodes are in parallel: $1/R_{\text{eq}} = 1/R_1 + 1/R_2 + 1/R_3$. A visual straight line does not by itself establish a series connection.

Step 1: Identify the two nodes from the wires The upper wire connects terminal A directly to D, so $V_A = V_D$. The lower wire connects C directly to terminal B, so $V_C = V_B$. The left $20\,\text{k}\Omega$ resistor is therefore between nodes A and B. The middle $20\,\text{k}\Omega$ resistor is between C and D, which are the same two nodes in reverse order. The right $10\,\text{k}\Omega$ resistor is also between these nodes. Thus all three resistors experience the same terminal voltage.

Step 2: Combine the parallel conductances Work consistently in kΩ:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{4}{20} = \frac{1}{5}.$$

Therefore $R_{\text{eq}} = 5\,\text{k}\Omega$. As a check, the two equal $20\,\text{k}\Omega$ resistors first give $10\,\text{k}\Omega$, which in parallel with the remaining $10\,\text{k}\Omega$ gives half that value. The result must be smaller than every branch resistance. Hence 10, 20 and $30\,\text{k}\Omega$, the other listed values, cannot be correct.

Step 3: Conclusion The equivalent resistance is $\boxed{5\,\text{k}\Omega}$. Hence, the correct option is (C).

Quick Tip to Solve: Colour or label points joined by uninterrupted wires before combining resistors; the two crossing bypass wires make all three branches parallel here.

39. The waves emitted when a metal target is bombarded with high energy electrons are [4 marks]

(A) Infrared rays

(B) Radio Waves

(C) Microwaves

(D) X-rays

Correct Answer: (D) X-rays

Solution:

Concept:

- An energetic electron has kinetic energy that can be converted into electromagnetic radiation when the electron is rapidly decelerated by a metal target.
- In an X-ray tube, this braking radiation produces a continuous X-ray spectrum; inner-shell ionisation followed by electronic transitions also produces characteristic X-ray lines.

Step 1: Identify the target-bombardment process High-energy electrons strike the atoms of the metal target and undergo strong acceleration or deceleration in their electric fields. Part of the incident kinetic energy emerges as photons. This is the standard physical method of producing X-rays. For an electron accelerated through potential difference V, the maximum available photon energy is

$$h\nu_{\max} = eV, \qquad \lambda_{\min} = \frac{hc}{eV}.$$

These relations connect the high electron energy to short-wavelength radiation.

Step 2: Distinguish the other spectral regions Infrared radiation is associated with thermal radiation and molecular vibrational processes; it is not the characteristic answer to energetic electron bombardment of a metal target. Radio waves are normally generated using oscillating currents in transmitting antennas. Microwaves are produced by suitable high-frequency electronic devices, such as magnetrons, rather than by the stated target mechanism. Heating of the target can occur, but the radiation-production process identified in this question is the generation of X-rays. Thus options (A), (B) and (C) do not describe the intended process.

Step 3: Conclusion The waves produced by high-energy electron bombardment of a metal target are X-rays. Hence, the correct option is (D).

Quick Tip to Solve: Associate an energetic electron beam striking a metal target with an X-ray tube; the target both decelerates electrons and supplies characteristic atomic transitions.

40. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Electromagnets are made of soft iron.

Reason R : Soft iron has high permeability and low retentivity.

In the light of above, statements, choose the most appropriate answer from the options given below.

[4 marks]

(A) Both A and R are correct and R is the correct explanation of A

(B) A is not correct but R is correct

(C) A is correct but R is not correct

(D) Both A and R are correct but R is NOT the correct explanation of A

Correct Answer: (A) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- An electromagnet should develop a strong magnetic field when its coil carries current and should lose nearly all of that magnetisation when the current is switched off.
- High magnetic permeability means a material develops a large magnetic flux density for an applied magnetising field. Low retentivity means little residual magnetisation remains when that field is removed.

Step 1: Evaluate the assertion Soft iron is commonly used as the core of an electromagnet. The field of the current-carrying coil magnetises the core and the core strengthens the resulting magnetic field. An electromagnet is intended to be controlled by the electric current, rather than to behave as a permanent magnet after the current has ceased. Therefore Assertion A is correct.

Step 2: Evaluate the reason Soft iron has high permeability, allowing it to be magnetised strongly by a comparatively small applied field. It also has low retentivity, so it does not retain substantial magnetisation after the current is switched off. These are the useful magnetic properties of a temporary magnetic core. Hence Reason R is correct; it is not describing the high-retentivity property desired for permanent magnets.

Step 3: Check the explanatory link The two properties in R directly meet the two requirements stated for an electromagnet: strong magnetisation while operating and easy demagnetisation afterwards. Thus R explains why soft iron is selected in A. Options (B) and (C) each wrongly deny one of the true statements. Option (D) accepts both facts but wrongly denies their direct causal connection.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: For an electromagnet look for high permeability and low retentivity; for a permanent magnet the ability to retain magnetisation is essential instead.

41. Electric potential at a point 'P' due to a point charge of 5×10^{-9}C is 50 V. The distance of 'P' from the point charge is:

(Assume, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{+9}$ $\mathrm{Nm^2C^{-2}}$) [4 marks]

(A) 0.9 cm

(B) 90 cm

(C) 3 cm

(D) 9 cm

Correct Answer: (B) 90 cm

Solution:

Concept:

- With zero potential at infinity, a point charge q produces electric potential $V = kq/r$ at distance r, where $k = 1/(4\pi\varepsilon_0)$.
- The distance obtained using the SI value of k, charge in coulombs and potential in volts is in metres. Convert it to centimetres only after the numerical substitution.

Step 1: Rearrange the potential expression The problem gives $q = 5 \times 10^{-9}\,\mathrm{C}$, $V = 50\,\mathrm{V}$ and $k = 9 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}}$. Solve the potential relation for the unknown distance:

$$r = \frac{kq}{V}.$$

Potential varies inversely with distance, rather than inversely with the square of distance. Using the electric-field formula in place of the potential formula would therefore be inappropriate.

Step 2: Substitute and convert the unit The powers of ten in kq cancel exactly:

$$kq = (9 \times 10^{9})(5 \times 10^{-9}) = 45\,\mathrm{V\,m}.$$

Hence

$$r = \frac{45}{50}\,\mathrm{m} = 0.9\,\mathrm{m} = 90\,\mathrm{cm}.$$

Substitution back gives $45/0.9 = 50\,\mathrm{V}$, confirming the result. The values 0.9, 3 and 9 cm would correspond respectively to 5000, 1500 and $500\,\mathrm{V}$; none reproduces the given potential. In particular, $0.9\,\mathrm{m}$ must not be misreported as $0.9\,\mathrm{cm}$.

Step 3: Conclusion The distance of P from the charge is $\boxed{90\,\mathrm{cm}}$. Hence, the correct option is (B).

Quick Tip to Solve: Cancel 10^{9} against 10^{-9} first, calculate r in metres, then multiply by 100 to compare with centimetre options.

42. The orbital angular momentum of a satellite is L, when it is revolving in a circular orbit at height h from earth surface. If the distance of satellite from the earth centre is increased by eight times to its initial value, then the new angular momentum will be - [4 marks]

(A) 9L

(B) 8L

(C) 4L

(D) 3L

Correct Answer: (D) 3L

Solution:

Concept:

- For a satellite of mass m in a circular orbit of radius r about Earth of mass M, gravitational attraction supplies centripetal force: $GMm/r^2 = mv^2/r$.
- The orbital angular momentum is $L = mvr = m\sqrt{GMr}$, so for the same satellite and Earth, L is proportional to $\sqrt{r}$, not to r.

Step 1: Obtain the radius dependence The initial centre distance is $r = R + h$, where R is Earth's radius. From the circular-orbit condition,

$$v = \sqrt{\frac{GM}{r}}, \qquad L = mr\sqrt{\frac{GM}{r}} = m\sqrt{GMr}.$$

The speed falls as the orbital radius grows, so treating speed as constant while increasing the radius would give an incorrect linear dependence of angular momentum.

Step 2: Interpret the stated increase and take the ratio The wording says the distance is increased *by* eight times its initial value. The increment is therefore $8r$, and the new centre distance is $r' = r + 8r = 9r$. This is distinct from saying it is increased *to* eight times its initial value. For the new circular orbit,

$$\frac{L'}{L} = \sqrt{\frac{r'}{r}} = \sqrt{9} = 3.$$

The value $9L$ would ignore the decrease in orbital speed, while $8L$ confuses the increment with the angular momentum ratio. A value $4L$ would require a sixteenfold centre distance, which is not the stated change.

Step 3: Conclusion The new orbital angular momentum is $\boxed{L' = 3L}$. Hence, the correct option is (D).

Quick Tip to Solve: For circular orbits use $L \propto \sqrt{r}$, with r measured from the centre; distinguish an increase by a multiple from a final radius equal to that multiple.

43. The temperature at which the kinetic energy of oxygen molecules becomes double than its value at $27^\circ\mathrm{C}$ is [4 marks]

(A) $627^\circ\mathrm{C}$

(B) $927^\circ\mathrm{C}$

(C) $327^\circ\mathrm{C}$

(D) $1227^\circ\mathrm{C}$

Correct Answer: (C) $327^\circ\mathrm{C}$

Solution:

Concept:

- The mean translational kinetic energy per gas molecule is $\overline{K} = \frac{3}{2}k_{\mathrm{B}}T$, where T is the absolute temperature and k_{B} is Boltzmann's constant.
- For the same sample and molecular species, kinetic energy is directly proportional to temperature in kelvin; Celsius temperatures cannot be used in this ratio.

Step 1: Convert the initial temperature The initial temperature is $27^\circ\mathrm{C}$. On the absolute scale this is

$$T_1 = 27 + 273 = 300\,\mathrm{K}.$$

The additive offset matters: doubling the Celsius reading would not double the energy, since zero Celsius is not zero molecular kinetic energy.

Step 2: Double the absolute temperature Using the proportionality of kinetic energy to absolute temperature,

$$\frac{\overline{K}_2}{\overline{K}_1} = \frac{T_2}{T_1} = 2, \qquad T_2 = 2(300) = 600\,\mathrm{K}.$$

Converting back gives

$$t_2 = 600 - 273 = 327^\circ\mathrm{C}.$$

The other listed temperatures correspond to 900 K, 1200 K and 1500 K, respectively, which would triple, quadruple or quintuple the original kinetic energy.

Step 3: Conclusion The required temperature is $327^\circ\mathrm{C}$. Hence, the correct option is (C).

Quick Tip to Solve: Apply energy-temperature ratios in kelvin first; subtract 273 only after finding the new absolute temperature.

44. The trajectory of projectile, projected from the ground is given by $y = x - \frac{x^2}{20}$. Where x and y are measured in meter. The maximum height attained by the projectile will be. [4 marks]

(A) 10 m

(B) 5 m

(C) 200 m

(D) $10\sqrt{2}$ m

Correct Answer: (B) 5 m

Solution:

Concept:

- A projectile trajectory is a downward-opening parabola. Its maximum height is the value of y at the vertex, where $dy/dx = 0$.
- The curvature test $d^2y/dx^2 < 0$ distinguishes a maximum from a minimum; the given coordinates are both in metres.

Step 1: Locate the vertex For the given path,

$$y = x - \frac{x^2}{20}, \qquad \frac{dy}{dx} = 1 - \frac{x}{10}.$$

Setting the slope to zero gives

$$1 - \frac{x}{10} = 0 \quad \Longrightarrow \quad x = 10\,\text{m}.$$

Also, $d^2y/dx^2 = -1/10 < 0$, so this point is indeed the highest point of the trajectory.

Step 2: Evaluate the height Substitute the horizontal coordinate of the vertex into the original equation:

$$y_{\max} = 10 - \frac{10^2}{20} = 10 - 5 = 5\,\text{m}.$$

As a check, completing the square gives

$$y = 5 - \frac{(x-10)^2}{20} \leq 5.$$

Thus 10 m is the horizontal position of the vertex, not its height. The values 200 m and $10\sqrt{2}$ m also exceed the maximum allowed by this equation.

Step 3: Conclusion The maximum height attained is $5\,\text{m}$. Hence, the correct option is (B).

Quick Tip to Solve: For a path written directly as a quadratic, find the vertex instead of first reconstructing the launch speed.

45. The width of fringe is $2\ \text{mm}$ on the screen in a double slits experiment for the light of wavelength of $400\ \text{nm}$. The width of the fringe for the light of wavelength 600 nm will be: [4 marks]

(A) 4 mm

(B) 1.33 mm

(C) 2 mm

(D) 3 mm

Correct Answer: (D) 3 mm

Solution:

Concept:

- The fringe width in Young's double-slit experiment is $\beta = \lambda D/d$, where D is the screen distance and d is the slit separation.
- For an unchanged apparatus, D/d is fixed. Fringe width is therefore directly proportional to the wavelength, not its reciprocal.

Step 1: Form a ratio for the same apparatus Let $\beta_1 = 2\,\text{mm}$ at $\lambda_1 = 400\,\text{nm}$, and let β_2 be the width at $\lambda_2 = 600\,\text{nm}$. Eliminating the common geometrical factor gives

$$\frac{\beta_2}{\beta_1} = \frac{\lambda_2 D/d}{\lambda_1 D/d} = \frac{600}{400} = \frac{3}{2}.$$

The nanometre units cancel in the ratio, so converting both wavelengths to metres is unnecessary.

Step 2: Compute and check the new width The new spacing is

$$\beta_2 = 2 \times \frac{3}{2} = 3\,\text{mm}.$$

The larger wavelength must produce wider fringes, eliminating 1.33 mm and 2 mm. A width of 4 mm would require twice the original wavelength, namely 800 nm, whereas the actual increase is only by a factor of 1.5.

Step 3: Conclusion The fringe width is $3\,\text{mm}$. Hence, the correct option is (D).

Quick Tip to Solve: When slit separation and screen distance remain fixed, multiply the old fringe width by the wavelength ratio.

46. Match List I with List II

	LIST I		LIST II
A.	**Torque**	**I.**	$ML^{-2}T^{-2}$
B.	**Stress**	**II.**	$ML^{2}T^{-2}$
C.	**Pressure gradient**	**III.**	$ML^{-1}T^{-1}$
D.	**Coefficient of viscosity**	**IV.**	$ML^{-1}T^{-2}$

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-I, C-IV, D-III

(B) A-II, B-IV, C-I, D-III

(C) A-IV, B-II, C-III, D-I

(D) A-III, B-IV, C-I, D-II

Correct Answer: (B) A-II, B-IV, C-I, D-III

Solution:

Concept:

- Dimensions follow from defining equations: force has dimensions MLT^{-2}, area has L^2, and a spatial derivative introduces an additional factor L^{-1}.
- For viscosity, Newton's law is $F = \eta A\, dv/dy$. The velocity gradient dv/dy has dimensions T^{-1}.

Step 1: Match torque and stress Torque is force times perpendicular distance, giving

$$[\tau] = (MLT^{-2})L = ML^2T^{-2}.$$

Hence A matches II. Stress is force per area, so

$$[\sigma] = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}.$$

Hence B matches IV, not I. This already rules out the first and third listed combinations.

Step 2: Match gradient and viscosity Pressure has the same dimensions as stress. Its gradient is pressure per length:

$$\left[\frac{dp}{dx}\right] = ML^{-2}T^{-2},$$

giving C-I. Finally,

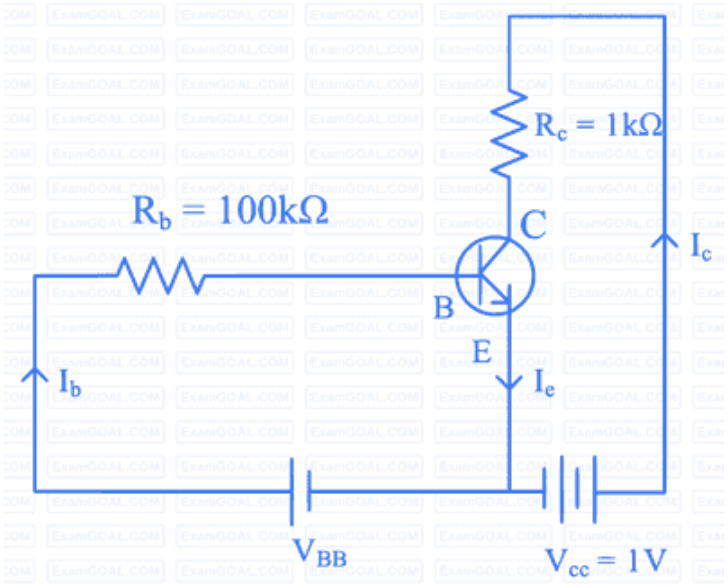
$$[\eta] = \frac{[F]}{[A][dv/dy]} = \frac{MLT^{-2}}{L^2T^{-1}} = ML^{-1}T^{-1},$$

giving D-III. The fourth combination incorrectly assigns torque the dimensions of viscosity, so it is excluded as well.

Step 3: Conclusion The complete matching is A-II, B-IV, C-I, D-III. Hence, the correct option is (B).

Quick Tip to Solve: Remember pressure as force per area; divide once more by length for a pressure gradient, and by a velocity gradient for viscosity.

47. For a given transistor amplifier circuit in CE configuration $V_{CC} = 1$ V, $R_C = 1$ kΩ, $R_b = 100$ kΩ and $\beta = 100$. Value of base current I_b is [4 marks]



(A) $I_b = 100\ \mu A$

(B) $I_b = 0.1\ \mu A$

(C) $I_b = 1.0\ \mu A$

(D) $I_b = 10\ \mu A$

Correct Answer: The independent base supply V_{BB} and the operating-point voltage V_{CE} are unspecified, so no unique base current follows.

Solution:

Concept:

- In an active-region transistor, $I_C = \beta I_B$. In the separate base-bias loop, $I_B = (V_{BB} - V_{BE})/R_B$, so the base supply or an equivalent operating-point condition is needed.
- The collector loop obeys $V_{CC} = I_C R_C + V_{CE}$. The collector supply alone limits the collector current; it does not prescribe its operating value.

Step 1: Identify the missing operating-point information The diagram shows an independent base battery V_{BB}, but no numerical value is specified for it. Combining the collector equation with the active-region current gain gives

$$I_B = \frac{V_{CC} - V_{CE}}{\beta R_C} = \frac{1 - V_{CE}}{100 \times 1000}\ \text{A},$$

with voltages expressed in volts. Thus different allowed collector-emitter voltages give different base currents for the same stated component values.

Step 2: Test the apparent intended answer If an extra ideal saturation approximation $V_{CE} \simeq 0$ is imposed, the collector-current limit is

$$I_{C,\max} \simeq \frac{1}{1000} = 1\,\text{mA}, \qquad \frac{I_{C,\max}}{\beta} = 10\,\mu\text{A}.$$

This explains the listed 10 microampere value, but saturation is not stated, and the active-region gain formula is not an exact saturation law. For example, $I_B = 1\,\mu\text{A}$ gives $I_C = 0.1\,\text{mA}$ and $V_{CE} = 0.9\,\text{V}$, also consistent with the stated collector circuit for a suitable base supply. Similarly 0.1 microampere is possible with a smaller base bias; 100 microampere would drive the transistor into saturation rather than sustain the active-region gain.

Step 3: Conclusion The base current is not uniquely determined by the printed data because V_{BB} or an operating-point condition is missing. The key's listed value $10\,\mu\text{A}$ follows only by adding the ideal saturation assumption; the equations above show why it is not forced by the question as printed.

Quick Tip to Solve: In a transistor circuit with separate base and collector batteries, check the base bias before using the collector supply to infer a base current.

48. A radio active material is reduced to $1/8$ of its original amount in 3 days. If 8×10^{-3} kg of the material is left after 5 days the initial amount of the material is [4 marks]

(A) 64 g

(B) 256 g

(C) 32 g

(D) 40 g

Correct Answer: (B) 256 g

Solution:

Concept:

- Radioactive mass follows $m(t) = m_0(1/2)^{t/T_{1/2}}$, where $T_{1/2}$ is the half-life. Equal intervals of one half-life reduce the remaining mass by the same factor of two.
- The ratio $1/8 = (1/2)^3$ corresponds to three half-lives, independent of the initial sample size.

Step 1: Determine the half-life After three days the amount is one-eighth of its initial value. Therefore

$$\left(\frac{1}{2}\right)^{3/T_{1/2}} = \left(\frac{1}{2}\right)^{3} \quad \Longrightarrow \quad T_{1/2} = 1\,\text{day}.$$

This is a statement about the fractional reduction; it does not mean that one-eighth remains after each subsequent day.

Step 2: Work back from the five-day mass Five days correspond to five half-lives, so the remaining mass is $m_0/32$. The measured mass is

$$8 \times 10^{-3}\,\text{kg} = 8\,\text{g}.$$

Hence

$$m_0 = 32(8) = 256\,\text{g}.$$

Checking forward, 256 g becomes 128, 64, 32, 16 and 8 g after successive days. Initial masses of 64 g, 32 g and 40 g would leave 2 g, 1 g and 1.25 g after five days, respectively, contradicting the

observation.

Step 3: Conclusion The initial mass was $256\,\mathrm{g}$, or $0.256\,\mathrm{kg}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the observed fraction into a power of one-half to count half-lives, then reverse the decay from the measured final mass.

49. The power radiated from a linear antenna of length l is proportional to (Given, $\lambda =$ Wavelength of wave): [4 marks]

(A) $\left(\frac{l}{\lambda}\right)^2$
(B) $\frac{l}{\lambda^2}$
(C) $\frac{l^2}{\lambda}$
(D) $\frac{l}{\lambda}$

Correct Answer: (A) $\left(\frac{l}{\lambda}\right)^2$

Solution:

Concept:

- For a short linear antenna carrying a fixed current amplitude, the radiation resistance is proportional to $(l/\lambda)^2$. The radiated power is $P = I_{\mathrm{rms}}^2 R_{\mathrm{rad}}$.
- The wavelength-to-length dependence measures electrical size: an antenna much shorter than a wavelength radiates weakly because its radiation resistance is small.

Step 1: Use the radiation-resistance dependence For a short current element one may write

$$R_{\mathrm{rad}} = C\left(\frac{l}{\lambda}\right)^2,$$

where C depends on the current distribution and the convention for antenna length. Its numerical value is irrelevant to the proportionality requested. With the current and the surrounding medium fixed, substituting into $P = I_{\mathrm{rms}}^2 R_{\mathrm{rad}}$ preserves the same dependence on length and wavelength.

Step 2: Compare the listed powers Thus

$$P \propto \left(\frac{l}{\lambda}\right)^2.$$

Doubling antenna length at fixed wavelength multiplies the power by four; doubling wavelength at fixed length reduces it by four. The expressions l/λ^2 and l^2/λ have the wrong exponent of one of the two variables. The linear ratio l/λ is dimensionless but fails the quadratic radiation law, so dimensional analysis alone would not distinguish it from the correct result.

Step 3: Conclusion The radiated power is proportional to $(l/\lambda)^2$. Hence, the correct option is (A).

Quick Tip to Solve: For a short antenna at fixed current, remember the square of its electrical length: power scales as the square of length divided by wavelength.

50. Work done by a Carnot engine operating between temperatures $127^\circ\mathrm{C}$ and $27^\circ\mathrm{C}$ is $2\ \mathrm{kJ}$. The amount of heat transferred to the engine by the reservoir is : [4 marks]

(A) 8 kJ

(B) 2 kJ

(C) 4 kJ

(D) 2.67 kJ

Correct Answer: (A) 8 kJ

Solution:

Concept:

- A Carnot engine is reversible. Its efficiency depends only on the absolute temperatures of the hot and cold reservoirs: $\eta = 1 - T_C/T_H$.
- Thermal efficiency is the fraction of the absorbed heat converted into useful work: $\eta = W/Q_H$. The remaining heat is rejected to the cold reservoir.

Step 1: Convert the reservoir temperatures The temperature ratio must use kelvin, since a ratio of Celsius readings does not describe thermodynamic efficiency. Thus the hot and cold temperatures are respectively

$$T_H = 127 + 273 = 400\,\mathrm{K}, \qquad T_C = 27 + 273 = 300\,\mathrm{K}.$$

Their temperature difference is 100 K, but this difference alone does not give the efficiency.

Step 2: Calculate the heat absorbed The Carnot efficiency is

$$\eta = 1 - \frac{300}{400} = \frac{1}{4}.$$

Therefore only one quarter of the heat supplied becomes work. With the given work output,

$$Q_H = \frac{W}{\eta} = \frac{2}{1/4} = 8\,\mathrm{kJ}.$$

The heat rejected is $Q_C = Q_H - W = 6\,\mathrm{kJ}$, which also satisfies $Q_C/Q_H = T_C/T_H = 3/4$.

Step 3: Check the alternative heat values An input of 2 kJ would imply complete conversion to work; 4 kJ would imply efficiency $1/2$; and 2.67 kJ would imply efficiency approximately $3/4$. None equals the required $1/4$. The 8 kJ input is consistent with both the Carnot relation and energy conservation.

Step 4: Conclusion The heat transferred to the engine by the hot reservoir is $\boxed{8\,\mathrm{kJ}}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert Celsius to kelvin before taking a Carnot temperature ratio; then divide the work by the efficiency to find heat input.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The number density of free electrons in copper is nearly $8 \times 10^{28}\ \mathrm{m}^{-3}$. A copper wire has its area of cross section $= 2 \times 10^{-6}\ \mathrm{m}^2$ and is carrying a current of $3.2\ \mathrm{A}$. The drift speed of the electrons is _____ $\times 10^{-6} \mathrm{ms}^{-1}$ [4 marks]

Solution:

Concept:

- The magnitude of electric current in a conductor is $I = neAv_d$, where n is the free-electron number density, e is the elementary charge, A is cross-sectional area and v_d is drift speed.
- Use $e = 1.6 \times 10^{-19}\ \mathrm{C}$. Electron drift is opposite conventional current, but the question asks for the positive speed.

Step 1: Identify the charge crossing a section In a time Δt, electrons drift through a length $v_d \Delta t$. The corresponding volume is $Av_d \Delta t$, containing $nAv_d \Delta t$ electrons. Hence the charge crossing the section per unit time has magnitude $neAv_d$. This counts the slow net drift rather than the random thermal motion of electrons.

Step 2: Substitute the given quantities Rearranging the current relation gives

$$v_d = \frac{I}{neA} = \frac{3.2}{(8 \times 10^{28})(1.6 \times 10^{-19})(2 \times 10^{-6})}.$$

The numerical denominator is $8 \times 1.6 \times 2 = 25.6$, and its power of ten is $10^{28-19-6} = 10^3$. Thus

$$v_d = \frac{3.2}{25.6 \times 10^3} = 1.25 \times 10^{-4}\,\mathrm{m\,s^{-1}}.$$

Step 3: Express the requested coefficient The blank multiplies $10^{-6}\,\mathrm{m\,s^{-1}}$, so rewrite the speed as

$$1.25 \times 10^{-4} = 125 \times 10^{-6}.$$

This is a change of scientific-notation scale, not an additional physical conversion. The small drift speed is compatible with a macroscopic current because the carrier density is extremely large.

Step 4: Conclusion The drift speed is $\boxed{125 \times 10^{-6}\,\mathrm{m\,s^{-1}}}$. The required numerical entry is $\boxed{125}$.

Quick Tip to Solve: After calculating drift speed in SI units, divide by the power of ten printed after the blank; here divide by 10^{-6}.

52. Two transparent media having refractive indices 1.0 and 1.5 are separated by a spherical refracting surface of radius of curvature 30 cm. The centre of curvature of surface is towards denser medium and a point object is placed on the principle axis in rarer medium at a distance of 15 cm from the pole of the surface. The distance of image from the pole of the surface is _____ cm. [4 marks]

Solution:

Concept:

- For paraxial refraction at one spherical surface, $n_2/v - n_1/u = (n_2 - n_1)/R$. Distances are measured from the pole with a consistent Cartesian sign convention.
- Take positive distances in the direction of incident light, from the rarer medium into the denser medium. A negative image distance then denotes a virtual image in the incident medium.

Step 1: Assign signed distances and refractive indices The object is in the incident rarer medium, so $n_1 = 1.0$ and $n_2 = 1.5$. Its position is opposite the positive direction, giving $u = -15\,\text{cm}$. The centre of curvature lies in the denser medium, in the positive direction, so $R = +30\,\text{cm}$. All distances can remain in centimetres because the surface formula uses a common length unit.

Step 2: Apply the spherical refraction formula Substituting the signed quantities,

$$\frac{1.5}{v} - \frac{1.0}{-15} = \frac{1.5 - 1.0}{30}.$$

Therefore

$$\frac{1.5}{v} + \frac{1}{15} = \frac{1}{60}, \qquad \frac{1.5}{v} = \frac{1}{60} - \frac{4}{60} = -\frac{1}{20}.$$

Solving gives

$$v = -30\,\text{cm}.$$

The negative sign follows from the equation and must not be discarded during the calculation.

Step 3: Interpret the image position The refracted rays diverge and their backward extensions meet 30 cm from the pole on the rarer-medium side. Thus the image is virtual. The requested distance is the magnitude $|v|$, rather than the signed coordinate. Substituting $v = -30$ back gives $-1/20 + 1/15 = 1/60$, confirming the result.

Step 4: Conclusion The image is virtual, at $v = -30\,\text{cm}$. Its distance from the pole is $\boxed{30\,\text{cm}}$, so the numerical entry is $\boxed{30}$.

Quick Tip to Solve: Assign the signs of the object and curvature first. Report the magnitude only when the question asks for distance rather than signed image position.

53. A guitar string of length 90 cm vibrates with a fundamental frequency of 120 Hz. The length of the string producing a fundamental frequency of 180 Hz will be _____ cm. [4 marks]

Solution:

Concept:

- For a string fixed at both ends, its fundamental mode has wavelength $\lambda_1 = 2L$. The fundamental frequency is therefore $f = v/(2L)$.
- For the same string under unchanged tension, wave speed $v = \sqrt{T/\mu}$ remains constant, where T is tension and μ is mass per unit length. Thus fL is constant.

Step 1: Relate the two vibrating lengths The question changes the effective vibrating length of the guitar string. With the same material, linear density and tension, the transverse wave speed is unchanged. Write

$$f_1 = \frac{v}{2L_1}, \qquad f_2 = \frac{v}{2L_2}.$$

Eliminating the common wave speed yields

$$f_1 L_1 = f_2 L_2.$$

A higher frequency therefore requires a shorter vibrating segment, which gives a useful direction check.

Step 2: Calculate the new length Using $L_1 = 90\,\text{cm}$, $f_1 = 120\,\text{Hz}$ and $f_2 = 180\,\text{Hz}$,

$$L_2 = L_1 \frac{f_1}{f_2} = 90\frac{120}{180} = 90\frac{2}{3} = 60\,\text{cm}.$$

There is no need to convert centimetres into metres in this ratio because the frequencies appear only as a dimensionless quotient.

Step 3: Verify that the wave speed is unchanged For the initial configuration, $v = 2(0.90)(120) = 216\,\text{m s}^{-1}$. For the new length, $v = 2(0.60)(180) = 216\,\text{m s}^{-1}$, the same value. This check also distinguishes shortening the fundamental-mode length from exciting a higher harmonic on the unchanged 90 cm segment.

Step 4: Conclusion The required vibrating length is $\boxed{60\,\text{cm}}$. The numerical entry is $\boxed{60}$.

Quick Tip to Solve: At fixed string tension and linear density, use $f_1 L_1 = f_2 L_2$; raising pitch means shortening the vibrating length.

54. A body of mass $5\ \text{kg}$ is moving with a momentum of $10\ \text{kg ms}^{-1}$. Now a force of $2\ \text{N}$ acts on the body in the direction of its motion for $5\ \text{s}$. The increase in the Kinetic energy of the body is _____ J. [4 marks]

Solution:

Concept:

- Impulse changes momentum according to $p_f - p_i = F\Delta t$ for a constant force along the motion. Here all momenta can be treated as positive scalars.
- The kinetic energy of a nonrelativistic body is $K = p^2/(2m)$. The increase is $\Delta K = (p_f^2 - p_i^2)/(2m)$, not the kinetic energy associated with the impulse alone.

Step 1: Find the final momentum The initial momentum is $p_i = 10\,\text{kg m s}^{-1}$. A force of 2 N acts along the motion for 5 s, supplying impulse

$$J = F\Delta t = 2 \times 5 = 10\,\text{N s}.$$

Since this impulse is in the same direction as the initial momentum,

$$p_f = p_i + J = 10 + 10 = 20\,\text{kg m s}^{-1}.$$

The momentum doubles; it does not remain fixed while energy is added.

Step 2: Compute and subtract the kinetic energies For the mass $m = 5\,\text{kg}$,

$$K_i = \frac{10^2}{2 \times 5} = 10\,\text{J}, \qquad K_f = \frac{20^2}{2 \times 5} = 40\,\text{J}.$$

Consequently,

$$\Delta K = K_f - K_i = 40 - 10 = 30\,\text{J}.$$

Squaring the change in momentum alone would omit the cross term involving the initial momentum and would give an incorrect increase.

Step 3: Check with the work-energy theorem The initial velocity is $u = p_i/m = 2\,\text{m s}^{-1}$, and the acceleration is $a = F/m = 0.4\,\text{m s}^{-2}$. In 5 s the displacement is $s = ut + at^2/2 = 10 + 5 = 15\,\text{m}$. The work is $Fs = 2 \times 15 = 30\,\text{J}$, agreeing with the kinetic-energy increase.

Step 4: Conclusion The increase in kinetic energy is $\boxed{30\,\text{J}}$. The required numerical entry is $\boxed{30}$.

Quick Tip to Solve: First add impulse to the initial momentum; then subtract $p^2/(2m)$ at the two endpoints. Do not square the impulse alone.

55. The ratio of wavelength of spectral lines H_α and H_β in the Balmer series is $\frac{x}{20}$. The value of x is _____.

[4 marks]

Solution:

Concept:

- The Balmer series consists of transitions to $n = 2$ in hydrogen. The H_α and H_β lines correspond to $3 \to 2$ and $4 \to 2$, respectively.
- The Rydberg formula is $1/\lambda = R_H(1/2^2 - 1/n^2)$, where R_H is the hydrogen Rydberg constant. Wavelength is inversely proportional to the wavenumber.

Step 1: Write the two wavenumbers For the first Balmer line, the upper level is 3, so

$$\frac{1}{\lambda_\alpha} = R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36}.$$

For the second Balmer line, the upper level is 4, giving

$$\frac{1}{\lambda_\beta} = R_H\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R_H}{16}.$$

Both transitions have the same final level, so the same Rydberg constant appears.

Step 2: Form the wavelength ratio in the correct order Invert the wavenumbers before comparing wavelengths:

$$\lambda_\alpha = \frac{36}{5R_H}, \qquad \lambda_\beta = \frac{16}{3R_H}.$$

Hence

$$\frac{\lambda_\alpha}{\lambda_\beta} = \frac{36}{5R_H}\frac{3R_H}{16} = \frac{108}{80} = \frac{27}{20}.$$

The Rydberg constant cancels, so its numerical value is unnecessary.

Step 3: Identify the requested numerator Comparing the calculated ratio with $x/20$ gives $x = 27$. The ratio is greater than one: the $3 \to 2$ transition releases less energy than the $4 \to 2$ transition, and therefore has the longer wavelength. This energy check prevents accidentally using the reciprocal ratio.

Step 4: Conclusion The wavelength ratio is $\lambda_\alpha/\lambda_\beta = 27/20$, so $\boxed{x = 27}$. This numerical answer is dimensionless.

Quick Tip to Solve: Remember that the first two Balmer lines start at levels 3 and 4; invert the Rydberg wavenumbers when taking a wavelength ratio.

56. A steel rod of length $1\ \text{m}$ and cross sectional area $10^{-4}\ \text{m}^2$ is heated from $0°\text{C}$ to $200°\text{C}$ without being allowed to extend or bend. The compressive tension produced in the rod is _____ $\times 10^4$ N. (Given Young's modulus of steel $= 2\times10^{11}\text{Nm}^{-2}$, coefficient of linear expansion $= 10^{-5}\ \text{K}^{-1}$) [4 marks]

Solution:

Concept:

- An unconstrained rod would acquire thermal strain $\alpha\Delta T$, where α is its coefficient of linear expansion. A fully constrained rod must have zero total longitudinal strain.
- The constraint supplies an equal and opposite elastic strain. By Young's modulus, the compressive stress magnitude is $\sigma = Y\alpha\Delta T$, and the compressive force is $F = \sigma A$.

Step 1: Find the expansion that the constraint prevents The temperature rise is $\Delta T = 200 - 0 = 200\,\text{K}$. Temperature differences have equal numerical values in kelvin and degrees Celsius. The free thermal strain would be

$$\alpha\Delta T = 10^{-5} \times 200 = 2 \times 10^{-3}.$$

For the given 1 m length, this corresponds to a free extension of $2 \times 10^{-3}\,\text{m}$. The fixed supports prevent that extension.

Step 2: Determine the compressive stress To cancel the positive thermal strain, the rod experiences elastic compressive strain of the same magnitude. Its stress magnitude is

$$\sigma = Y\alpha\Delta T = (2 \times 10^{11})(2 \times 10^{-3}) = 4 \times 10^{8}\,\text{N}\,\text{m}^{-2}.$$

With tension taken positive, the signed stress is negative, but the question requests the positive magnitude of the compressive force.

Step 3: Convert stress to force and the requested scale Multiply stress by cross-sectional area:

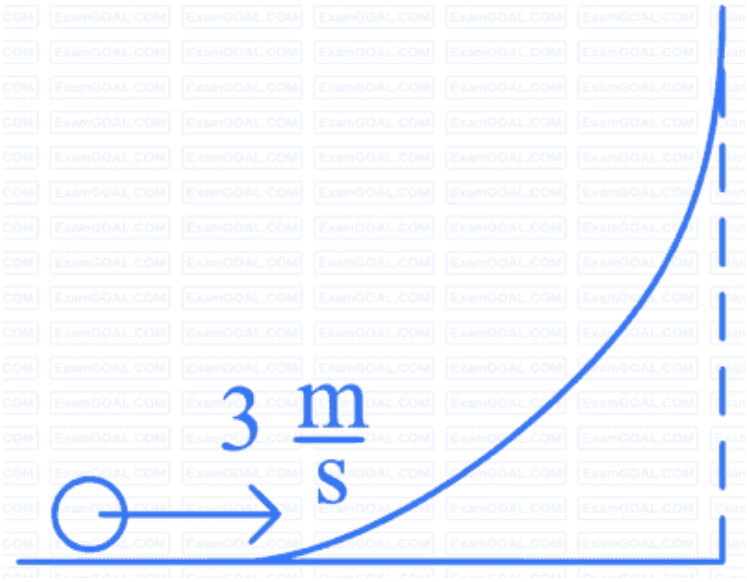
$$F = \sigma A = (4 \times 10^{8})(10^{-4}) = 4 \times 10^{4}\,\text{N}.$$

The length cancels because both the prevented thermal extension and the elastic extension are proportional to length. Thus the given 1 m length is consistent but not required in the final force formula. The rod is specified not to bend, so buckling is excluded from this model.

Step 4: Conclusion The compressive force is $\boxed{4 \times 10^4\,\text{N}}$. The coefficient to enter in the blank is $\boxed{4}$.

Quick Tip to Solve: For completely blocked thermal expansion, use $F = YA\alpha\Delta T$. The rod length cancels when thermal and elastic strains are matched.

57. A hollow spherical ball of uniform density rolls up a curved surface with an initial velocity $3\ \mathrm{m/s}$ (as shown in figure). Maximum height with respect to the initial position covered by it will be _____ cm. [4 marks]



Solution:

Concept:

- For a thin hollow spherical ball of mass m and radius R, the moment of inertia about its centre is $I = \frac{2}{3}mR^2$. Both translational and rotational kinetic energies must be counted.
- Pure rolling gives $\omega = v/R$. With no dissipative loss, the initial total kinetic energy becomes gravitational potential energy mgh at the highest point.

Step 1: Calculate the initial kinetic energy The centre moves at $v = 3\,\mathrm{m\,s^{-1}}$, while the ball also rotates. Its initial energy is therefore

$$K_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Substitute the hollow-sphere moment of inertia and the rolling condition:

$$K_i = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\frac{v^2}{R^2} = \frac{5}{6}mv^2.$$

The radius cancels; the answer does not depend on the size of the ball.

Step 2: Apply conservation of mechanical energy At the maximum height the speed and, for pure rolling, the angular speed vanish instantaneously. Static friction maintains rolling but does not dissipate energy. Taking the initial centre-of-mass level as zero potential energy,

$$mgh = \frac{5}{6}mv^2, \qquad h = \frac{5v^2}{6g}.$$

Using the standard value $g = 10\,\mathrm{m\,s^{-2}}$ for the numerical evaluation,

$$h = \frac{5(3)^2}{6(10)} = 0.75\,\mathrm{m} = 75\,\mathrm{cm}.$$

Using only translational kinetic energy would incorrectly omit the energy stored in rotation; using the solid-sphere inertia would also describe a different ball.

Step 3: Conclusion The maximum height above the initial position is $\boxed{75\,\mathrm{cm}}$. The numerical entry is 75.

Quick Tip to Solve: For a rolling body with $I = kmR^2$, use $h = (1+k)v^2/(2g)$; a hollow sphere has $k = 2/3$.

58. The ratio of magnetic field at the centre of a current carrying coil of radius r to the magnetic field at distance r from the centre of coil on its axis is $\sqrt{x} : 1$. The value of x is _____ [4 marks]

Solution:

Concept:

- The axial magnetic field of a circular coil of radius r, current I, and N turns at axial distance z is $B(z) = \mu_0 N I r^2/[2(r^2+z^2)^{3/2}]$.
- At the centre set $z = 0$. The requested ratio compares magnitudes, and the common current and number of turns cancel.

Step 1: Evaluate the two magnetic fields At the centre of the coil, the distance along the axis is zero. Hence

$$B_c = \frac{\mu_0 N I r^2}{2r^3} = \frac{\mu_0 N I}{2r}.$$

At the point whose axial distance from the centre equals the coil radius, put $z = r$, without replacing the coil radius itself:

$$B_r = \frac{\mu_0 N I r^2}{2(2r^2)^{3/2}} = \frac{\mu_0 N I}{2 \cdot 2^{3/2} r}.$$

This field is smaller because the point is away from the coil centre.

Step 2: Compare the ratio with the given form Dividing the central field by the field at axial distance r gives

$$\frac{B_c}{B_r} = 2^{3/2} = 2\sqrt{2} = \sqrt{8}.$$

The problem states this ratio as $\sqrt{x} : 1$, so its square, rather than the ratio itself, is x:

$$\sqrt{x} = \sqrt{8} \quad \Longrightarrow \quad x = 8.$$

The ratio is dimensionless because both fields have the same unit. Reversing the ratio or forgetting the power $3/2$ would not match the requested central-to-axial comparison.

Step 3: Conclusion The field ratio is $\sqrt{8} : 1$, and therefore $\boxed{x = 8}$, a dimensionless number.

Quick Tip to Solve: Write the axial ratio directly as $B(0)/B(z) = [1 + (z/r)^2]^{3/2}$; at $z = r$, square $2^{3/2}$ to find x.

59. A $600\ \text{pF}$ capacitor is charged by $200\ \text{V}$ supply. It is then disconnected from the supply and is connected to another uncharged $600\ \text{pF}$ capacitor. Electrostatic energy lost in the process is _____ μJ [4 marks]

Solution:

Concept:

- An isolated capacitor network conserves total charge. On connecting equal capacitors in parallel, both reach the same final potential.
- The energy stored in a capacitor is $U = \frac{1}{2}CV^2$. Charge is conserved during sharing, but electrostatic energy decreases through transient current and electromagnetic losses.

Step 1: Find the common final potential Initially only the first capacitor is charged. With $C = 600 \times 10^{-12}\,\text{F}$ and $V = 200\,\text{V}$, its charge is

$$Q_i = CV = (600 \times 10^{-12})(200) = 1.2 \times 10^{-7}\,\text{C}.$$

The second identical capacitor is initially uncharged. Once connected, their combined capacitance is $2C$, and the supply is no longer connected. Thus

$$V_f = \frac{Q_i}{2C} = \frac{V}{2} = 100\,\text{V}.$$

The supply does no additional work during this redistribution.

Step 2: Subtract the final stored energy from the initial energy The initial energy is

$$U_i = \frac{1}{2}(600 \times 10^{-12})(200)^2 = 12 \times 10^{-6}\,\text{J}.$$

Both capacitors finally store energy, so include both contributions:

$$U_f = 2\left[\frac{1}{2}(600 \times 10^{-12})(100)^2\right] = 6 \times 10^{-6}\,\text{J}.$$

Consequently,

$$U_{\text{lost}} = U_i - U_f = (12 - 6) \times 10^{-6}\,\text{J}.$$

The positive difference is energy dissipated during the transient; it is not a loss of electric charge. Counting only one of the two final capacitors would underestimate the retained energy.

Step 3: Conclusion The electrostatic energy lost is $\boxed{6\,\mu\text{J}}$, so the numerical entry in the stated unit is 6.

Quick Tip to Solve: When a charged capacitor shares charge with an identical uncharged one, the final voltage halves and half the initial stored energy is lost.

60. A series combination of resistor of resistance $100\ \Omega$, inductor of inductance $1\ \mathrm{H}$ and capacitor of capacitance $6.25\ \mu\mathrm{F}$ is connected to an ac source. The quality factor of the circuit will be _____

[4 marks]

Solution:

Concept:

- For a series RLC circuit, the quality factor is $Q = \omega_0 L/R = (1/R)\sqrt{L/C}$, where $\omega_0 = 1/\sqrt{LC}$ is the resonant angular frequency.
- The resistance controls energy dissipation. Quality factor is dimensionless and should be calculated with capacitance in farads and inductance in henries.

Step 1: Convert the capacitance and calculate resonance The circuit values are $R = 100\,\Omega$, $L = 1\,\mathrm{H}$, and

$$C = 6.25\,\mu\mathrm{F} = 6.25 \times 10^{-6}\,\mathrm{F}.$$

At resonance the inductive and capacitive reactances are equal. Therefore

$$\sqrt{LC} = \sqrt{6.25 \times 10^{-6}} = 2.5 \times 10^{-3}\,\mathrm{s},$$

$$\omega_0 = \frac{1}{2.5 \times 10^{-3}} = 400\,\mathrm{rad\,s^{-1}}.$$

This is an angular frequency, so no extra factor of 2π should be inserted into the formula for Q.

Step 2: Evaluate the quality factor At resonance the magnitude of the inductive reactance is $X_L = \omega_0 L = 400\,\Omega$. The quality factor is its ratio to the resistance:

$$Q = \frac{X_L}{R} = \frac{400}{100} = 4.$$

Equivalently,

$$Q = \frac{1}{100}\sqrt{\frac{1}{6.25 \times 10^{-6}}} = 4.$$

The two calculations agree. Since a ratio of reactance to resistance has no unit, the required entry is a pure number. Using capacitance as 6.25 rather than 6.25×10^{-6} would give an incorrect power of ten.

Step 3: Conclusion The quality factor of the series circuit is $\boxed{Q = 4}$, dimensionless.

Quick Tip to Solve: Use $Q = \sqrt{L/C}/R$ for a series circuit; convert microfarads to farads before taking the square root.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Match List I with List II

	LIST I Coordination Complex		LIST II Number of unpaired electrons
A.	$[Cr(CN)_6]^{3-}$	**I.**	**0**
B.	$[Fe\,(H_2O)_6]^{2+}$	**II.**	**3**
C.	$[Co\,(NH_3)_6]^{3+}$	**III.**	**2**
D.	$[Ni\,(NH_3)_6]^{2+}$	**IV.**	**4**

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-III, C-II, D-I

(B) A-II, B-I, C-IV, D-III

(C) A-II, B-IV, C-I, D-III

(D) A-III, B-IV, C-I, D-II

Correct Answer: (C) A-II, B-IV, C-I, D-III

Solution:

Concept:

- The number of unpaired electrons follows from the metal oxidation state, its d-electron count, and the ligand-field splitting relative to pairing energy.
- All four complexes are octahedral. The lower t_{2g} set holds six electrons and the upper e_g set holds four; weak fields favour high-spin filling.

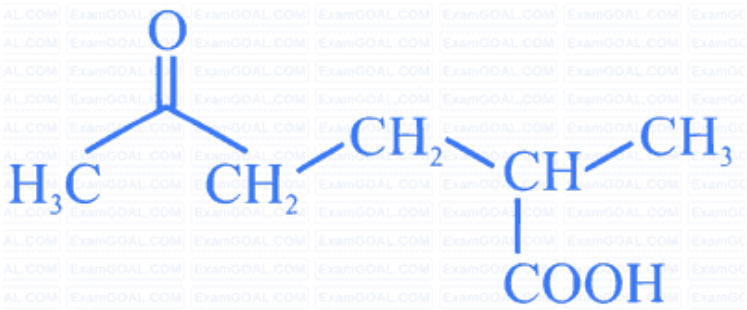
Step 1: Assign chromium and iron In $[Cr(CN)_6]^{3-}$, six cyanide ligands contribute -6, making chromium $+3$, hence d^3. Its octahedral configuration is t_{2g}^3, with three singly occupied orbitals. Thus A matches II. There is no alternative low-spin pairing for this d^3 configuration. For $[Fe(H_2O)_6]^{2+}$, water is neutral and iron is $+2$, hence d^6. Water is a weak-field ligand, so the complex is high spin, $t_{2g}^4 e_g^2$, with four unpaired electrons. Thus B matches IV.

Step 2: Assign cobalt and nickel and check the choices In $[Co(NH_3)_6]^{3+}$, cobalt is $+3$, hence d^6. The large splitting for cobalt(III) with ammonia gives the low-spin arrangement $t_{2g}^6 e_g^0$; every electron is paired, so C matches I. In $[Ni(NH_3)_6]^{2+}$, nickel is $+2$, hence d^8. Octahedral filling gives $t_{2g}^6 e_g^2$, whose two upper orbitals each have one electron, so D matches III. The complete unpaired-electron sequence is $3, 4, 0, 2$. Option A starts with four for chromium, option B assigns zero to iron, and option D starts with two for chromium; each contradicts these fillings.

Step 3: Conclusion The matching is A-II, B-IV, C-I, D-III. Hence, the correct option is (C).

Quick Tip to Solve: First obtain the metal oxidation state: d^3 and octahedral d^8 have fixed counts, while the d^6 iron and cobalt complexes require a spin-state check.

62. The correct IUPAC nomenclature for the following compound is: [4 marks]



(A) 2 - Formyl -5-methylhexan-6-oic acid

(B) 5-Methyl-2-oxohexan-6-oic acid

(C) 5-Formyl -2-methylhexanoic acid

(D) 2-Methyl-5-oxohexanoic acid

Correct Answer: (D) 2-Methyl-5-oxohexanoic acid

Solution:

Concept:

- Carboxylic acid has higher naming priority than a ketone. Its carbon must belong to the parent chain and is numbered carbon 1.
- A ketone group becomes the prefix oxo when an acid supplies the suffix. An aldehyde substituent would be formyl, which requires a terminal $-CHO$ group.

Step 1: Select and number the parent chain The displayed connectivity can be written as

$$HOOC-CH(CH_3)-CH_2-CH_2-CO-CH_3.$$

Start numbering from the carboxyl carbon, since it is the principal functional group. The longest continuous chain containing that carbon has six carbons, so its acid parent is hexanoic acid. The sequence from carbon 1 is carboxyl carbon, substituted methine carbon, two methylene carbons, carbonyl carbon, and terminal methyl carbon. Choosing the opposite direction would improperly give the acid a locant other than 1.

Step 2: Locate substituents and eliminate incorrect names The methyl branch is attached to carbon 2, and the ketone carbonyl is carbon 5. Therefore the required prefixes are 2-methyl and 5-oxo, in alphabetical order. The carbonyl at carbon 5 is bonded to a methyl carbon as well as carbon 4; it is a ketone, not an aldehyde, so neither formyl name in A or C represents the structure. Option B numbers from the wrong end and writes the acid at carbon 6, contrary to the principal-group numbering rule. The carbon count also confirms that the methyl branch is outside the six-carbon parent chain.

Step 3: Conclusion The correct IUPAC name is 2-methyl-5-oxohexanoic acid. Hence, the correct option is (D).

Quick Tip to Solve: When an acid and a ketone occur together, number from $COOH$, retain the acid suffix, and name the other carbonyl as oxo.

63. Major product 'P' formed in the following reaction is: [4 marks]



(A) $HOOC-CH_2-CH_2-CHBr-CH_2Br$

(B) $HOOC-CH_2-CH_2-CHBr-CH_3$

(C) 5-(Bromomethyl)oxolan-2-one

(D) $HOOC-CH_2-CH_2-CH_2-CH_2Br$

Correct Answer: (C) 5-(Bromomethyl)oxolan-2-one

Solution:

Concept:

- Bromine adds electrophilically to an alkene through a cyclic bromonium ion. A nearby carboxylate can open that ion intramolecularly to form a lactone.
- Sodium bicarbonate converts the acid into its carboxylate and neutralises acid formed during reaction. Formation of a five-membered ring is especially favourable for this substrate.

Step 1: Identify the reacting groups The substrate is pent-4-enoic acid, $CH_2{=}CH{-}CH_2{-}CH_2{-}COOH$. Its terminal alkene and carboxyl group are separated by the chain required for five-membered cyclisation. In the presence of $NaHCO_3$, the carboxylate oxygen is an intramolecular nucleophile. The alkene attacks Br_2, producing a bromonium ion across the terminal double bond. The more substituted alkene carbon is the favourable site for oxygen attack; the terminal carbon retains bromine.

Step 2: Close the lactone ring and compare the products The carboxylate oxygen bonds to the internal alkene carbon. The resulting ring contains the ester oxygen, that former alkene carbon, the two intervening methylene carbons, and the carboxyl carbon: five ring atoms in total. A $-CH_2Br$ substituent remains on the carbon next to the ring oxygen. This is the gamma-lactone depicted in option C, also named 5-(bromomethyl)oxolan-2-one. Option A is an open-chain vicinal dibromide and misses intramolecular capture. Option B is an open-chain monobromo acid, while D places bromine terminally without forming the cyclic ester. Neither B nor D expresses bromolactonisation under the stated conditions.

Step 3: Conclusion The major product is the five-membered bromomethyl-substituted lactone shown in option C. Hence, the correct option is (C).

Quick Tip to Solve: For an unsaturated acid with $Br_2/NaHCO_3$, count the ring formed by carboxylate attack before choosing an ordinary alkene addition product.

64. A compound 'X' when treated with phthalic anhydride in presence of concentrated H_2SO_4 yields 'Y'. 'Y' is used as an acid/base indicator. 'X' and 'Y' are respectively [4 marks]

(A) Toludine, Phenolphthalein

(B) Carbolic acid, Phenolphthalein

(C) Salicylaldehyde, Phenolphthalein

(D) Anisole, methyl orange

Correct Answer: (B) Carbolic acid, Phenolphthalein

Solution:

Concept:

- Phenol, C_6H_5OH, is also called carbolic acid. Its activated aromatic ring undergoes acid-catalysed condensation with phthalic anhydride.
- Phenolphthalein is a phthalein dye and an acid-base indicator. Its colour depends on the acid-base form present, rather than on the starting material's colour.

Step 1: Identify the starting aromatic compound The named reaction combines phthalic anhydride with phenol in the presence of concentrated H_2SO_4. The acid assists condensation, while the hydroxyl group activates the phenol ring for substitution, mainly at the para position. Two molecules of phenol participate in producing one molecule of the indicator. Thus the required starting compound X must be phenol, whose common name in the options is carbolic acid.

Step 2: Identify the indicator and eliminate alternatives The product Y is phenolphthalein. It is colourless in acidic solution and develops its characteristic pink colour in a sufficiently alkaline solution because the deprotonated form has extended conjugation. Therefore the pair must be carbolic acid and phenolphthalein. Toluidine is an aromatic amine, not phenol, so option (A) gives the wrong starting material. Salicylaldehyde contains an aldehyde group and is not the reactant for this preparation, eliminating (C). Anisole is an ether, and methyl orange is an azo indicator prepared by a different route, eliminating (D).

Step 3: Conclusion X = carbolic acid (phenol), and Y = phenolphthalein. Hence, the correct option is (B).

Quick Tip to Solve: Recall the preparation as phenol plus phthalic anhydride: the shared word fragment phthalein helps identify the indicator.

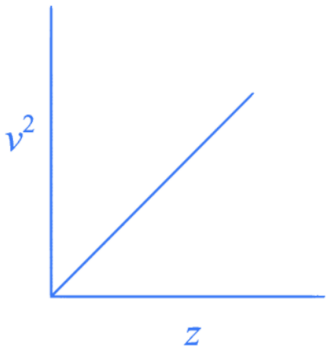
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65. Henry Moseley studied characteristic X-ray spectra of elements. The graph which represents his observation correctly is

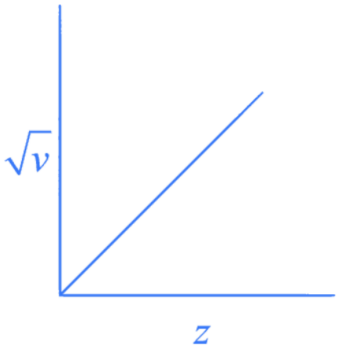
Given v = frequency of X-ray emitted

Z = atomic number [4 marks]

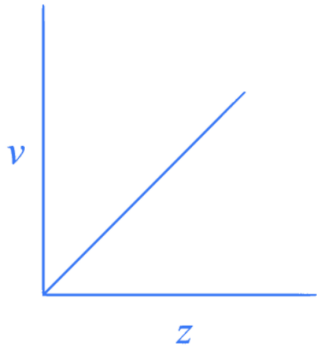
(A)



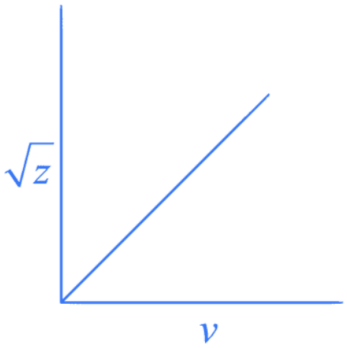
(B)



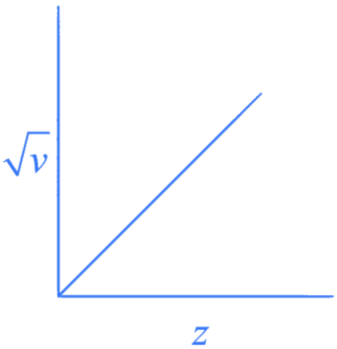
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- Moseley's law for a specified characteristic X-ray series is $\sqrt{\nu} = a(Z - b)$, where ν is the emitted frequency, Z is atomic number, and a, b are constants.

- The screening constant b accounts for the shielding of nuclear charge. A graph tests which pair of variables has a linear relationship; its essential feature here is the square root of frequency.

Step 1: Select the linear variables Characteristic X-rays arise when an electron fills a vacancy in an inner electronic shell. The relevant transition energy increases approximately as the square of the effective nuclear charge. Since photon energy is $h\nu$, this gives $\nu = a^2(Z-b)^2$, or

$$\sqrt{\nu} = aZ - ab.$$

Consequently, plotting the square root of the emitted frequency on the vertical axis against atomic number on the horizontal axis gives a straight line with positive slope. The schematic graph in option (B) has these two axes.

Step 2: Compare all four diagrams Option (A) plots ν^2 against Z; squaring the frequency does not linearise Moseley's relation. Option (C) plots ν itself against Z, which is quadratic rather than linear. Option (D) plots $\sqrt{Z}$ against ν, which also fails the required relation. The drawings are schematic: an exact Moseley plot extrapolates to $Z = b$, rather than requiring zero screening. The variable choice that represents the observation is therefore the straight-line $\sqrt{\nu}$-versus-Z plot.

Step 3: Conclusion Moseley's observation is represented by the straight-line plot of $\sqrt{\nu}$ against Z. Hence, the correct option is (B).

Quick Tip to Solve: Before choosing a Moseley graph, read the vertical axis carefully: it must contain the square root of frequency, not frequency squared.

66. Given below are two statements:

Statement I : In redox titration, the indicators used are sensitive to change in pH of the solution.

Statement II : In acid-base titration, the indicators used are sensitive to change in oxidation potential.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Statement I is correct but Statement II is incorrect

(C) Statement I is incorrect but Statement II is correct

(D) Both Statement I and Statement II are correct

Correct Answer: (A) Both Statement I and Statement II are incorrect

Solution:

Concept:

- A redox indicator changes colour when its oxidised and reduced forms interconvert. Its response is governed by the oxidation-reduction potential of the solution.
- An acid-base indicator is a weak acid or weak base whose differently protonated forms have different colours. Its colour change occurs over a characteristic pH interval.

Step 1: Evaluate Statement I In a redox titration the equivalence region is associated with a sharp

change in electrode potential. A suitable redox indicator changes between its oxidised and reduced forms near this potential. The indicator is selected to respond to this change, rather than simply to a change in pH. Although the medium and acidity can influence some redox reactions, that does not make pH the defining response of a redox indicator. Statement I therefore assigns the wrong sensitivity and is incorrect.

Step 2: Evaluate Statement II and select the combination In an acid-base titration, neutralisation produces a substantial change in pH near equivalence. An indicator such as phenolphthalein changes its acid-base form in this interval, providing the observed endpoint. The relevant change is hydrogen-ion concentration, not oxidation potential: no electron-transfer change is required for acid-base neutralisation. Statement II is therefore also incorrect. Options (B) and (C) each call one of these false statements correct, while option (D) calls both correct; all three are eliminated. Only option (A) describes the actual truth values.

Step 3: Conclusion Both Statement I and Statement II are incorrect: the two indicator sensitivities have been interchanged. Hence, the correct option is (A).

Quick Tip to Solve: Match the endpoint signal to the titration: redox uses a potential change, while acid-base uses a pH change.

67. For a good quality cement, the ratio of lime to the total of the oxides of Si, Al and Fe should be as close as to : [4 marks]

(A) 3

(B) 2

(C) 1

(D) 4

Correct Answer: (B) 2

Solution:

Concept:

- Good-quality Portland cement requires a suitable balance between basic lime, CaO, and the acidic oxide components $\mathrm{SiO_2}$, $\mathrm{Al_2O_3}$ and $\mathrm{Fe_2O_3}$.
- The approximate compositional criterion asked here is a mass ratio: lime divided by the total mass of these three oxides should be close to two. It is not a mole ratio of the elements.

Step 1: Write the specified composition ratio Use the oxide masses, since cement composition is normally stated as mass percentages. The required quantity is

$$R = \frac{m(\mathrm{CaO})}{m(\mathrm{SiO_2}) + m(\mathrm{Al_2O_3}) + m(\mathrm{Fe_2O_3})}.$$

The numerator is lime and the denominator combines the silicon, aluminium and iron oxides. For good-quality cement this ratio should be as close to 2 as possible. The statement does not refer to the separate silica-to-alumina ratio, which is a different composition criterion.

Step 2: Choose the value and reject the other ratios Cement commonly contains roughly 60–65

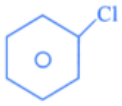
percent lime, with silica, alumina and iron oxide together contributing about 30 percent; these proportions illustrate why the ratio is near 2. The specified balance permits formation of the calcium silicate and aluminate phases responsible for setting and strength. A value of 1 supplies too little lime relative to these oxides for the stated criterion. Values of 3 or 4 supply an excessive proportion of lime and are not the desired balance. Thus only the number in option (B) satisfies the quality criterion in the question.

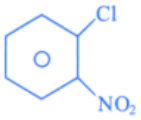
Step 3: Conclusion The ratio of lime to the total oxides of Si, Al and Fe should be close to 2. Hence, the correct option is (B).

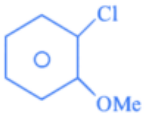
Quick Tip to Solve: Keep the two cement ratios separate: lime over all three acidic oxides is near 2; silica over alumina is a different condition.

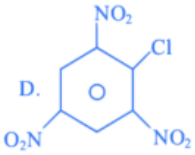
68. The correct order of reactivity of following haloarenes towards nucleophilic substitution with aqueous $NaOH$ is :

Choose the correct answer from the options given below: [4 marks]

A.


B.


C.


D.


(A) $C > A > D > B$

(B) $D > B > A > C$

(C) $A > B > D > C$

(D) $D > C > B > A$

Correct Answer: (B) $D > B > A > C$

Solution:

Concept:

- In addition-elimination nucleophilic aromatic substitution, hydroxide adds to the carbon bearing chlorine, producing a negatively charged intermediate before chloride leaves.
- Electron-withdrawing nitro groups at ortho or para positions stabilise this intermediate by reso-

nance. A methoxy group donates electron density by resonance and opposes attack by a nucleophile.

Step 1: Identify the effects of the substituents A is chlorobenzene, with no additional activating substituent. B has an ortho nitro group, which withdraws electron density and stabilises the negative charge developed during nucleophilic addition. C has an ortho methoxy group; its resonance donation makes the aromatic ring less electrophilic and destabilises the anionic addition intermediate relative to chlorobenzene. D has three nitro groups at the two ortho and one para positions relative to chlorine, so all three can strongly activate the ring towards substitution.

Step 2: Arrange the compounds and compare the options The multiply nitro-substituted compound D is most reactive because its intermediate has the greatest electron-withdrawing stabilisation. The singly nitro-substituted B comes next. Unsubstituted chlorobenzene A is less reactive than B, but is more reactive than the methoxy-substituted C. Therefore

$$D > B > A > C.$$

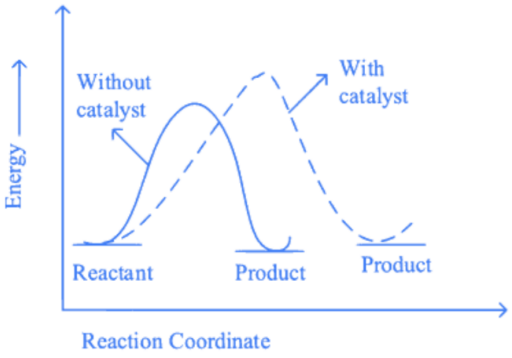
Option (A) incorrectly places the electron-rich methoxy compound first. Option (C) puts chlorobenzene ahead of the nitro-activated compounds. Option (D) places C ahead of B and A despite the methoxy group's donating effect. These contradictions eliminate the three alternatives.

Step 3: Conclusion The correct reactivity order is $D > B > A > C$. Hence, the correct option is (B).

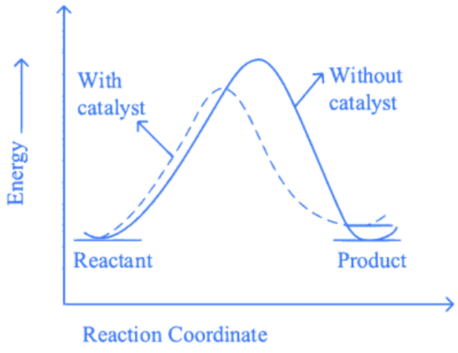
Quick Tip to Solve: For nucleophilic substitution on an aromatic chloride, count ortho and para nitro groups first: they activate, whereas methoxy donation deactivates.

69. The correct reaction profile diagram for a positive catalyst reaction. [4 marks]

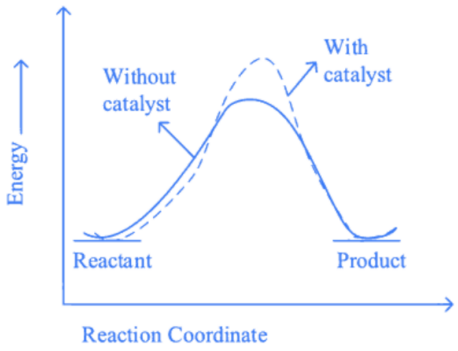
(A)



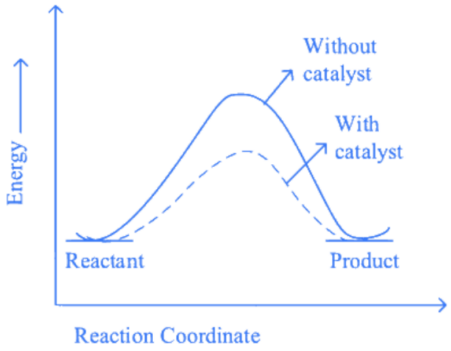
(B)



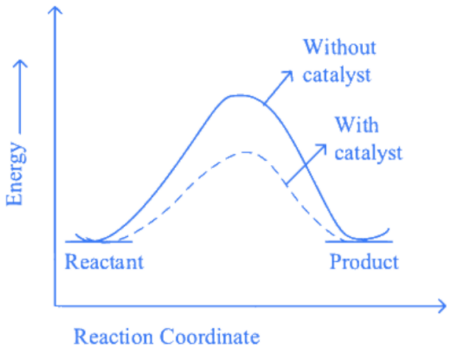
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- A positive catalyst increases reaction rate by providing an alternative pathway with a smaller activation energy, $E_a = E_{\text{TS}} - E_{\text{reactant}}$.
- A catalyst does not change the identities or energies of the initial reactants and final products. Therefore the reaction enthalpy ΔH, and the equilibrium constant at fixed temperature, remain unchanged.

Step 1: Apply both requirements to the profiles The height of the energy maximum relative to the reactant level represents the activation barrier. The curve labelled with catalyst must have a lower barrier than the curve labelled without catalyst. At the same time, both curves must begin at the same reactant energy and end at the same product energy. Lowering the maximum alone is not sufficient if the diagram also shifts the product level. The endpoints determine the overall energy change, which a catalyst cannot alter.

Step 2: Inspect the options using the labels and endpoints In option (D), the dashed catalysed path lies below the uncatalysed maximum, and both profiles share the same reactant and product energy levels. This meets both conditions. Options (A) and (C) show a higher maximum for the path labelled with catalyst, contradicting the action of a positive catalyst. Option (B) shows a lowered maximum but its catalysed and uncatalysed product levels do not coincide; it changes the overall energy difference. Hence it is not the correct complete profile. The reaction coordinate describes progress along a pathway, so its horizontal position must not be confused with activation energy.

Step 3: Conclusion Option (D) shows a lower activation barrier with unchanged reactant and product energies. Hence, the correct option is (D).

Quick Tip to Solve: Check an energy-profile option twice: first compare the labelled peak heights, then ensure the reactant and product energy levels are unchanged.

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70. Match List I with List II.

	List I: Natural amino acid		List II: One letter code
A.	Glutamic acid	I.	Q
B.	Glutamine	II.	W
C.	Tyrosine	III.	E
D.	Tryptophan	IV.	Y

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-IV, C-I, D-II

(B) A-IV, B-III, C-I, D-II

(C) A-II, B-I, C-IV, D-III

(D) A-III, B-I, C-IV, D-II

Correct Answer: (D) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Protein sequences use standard one-letter symbols for amino acids. The symbol is a convention and cannot always be inferred from the initial letter of the amino-acid name.
- The relevant symbols are glutamic acid E, glutamine Q, tyrosine Y and tryptophan W. The corresponding three-letter abbreviations are Glu, Gln, Tyr and Trp.

Step 1: Match each amino acid to its code Glutamic acid is the acidic amino acid with the one-letter code E, so entry A matches III. Glutamine is the amide derived from glutamic acid and has the code Q, so entry B matches I. These two names are similar, but their protein-sequence symbols distinguish the acid from the amide. Tyrosine, an aromatic amino acid with a phenolic group, has code Y, giving C-IV. Tryptophan, whose side chain contains an indole ring, has code W, giving D-II.

Step 2: Assemble the complete mapping and eliminate alternatives Putting the four independent matches together gives

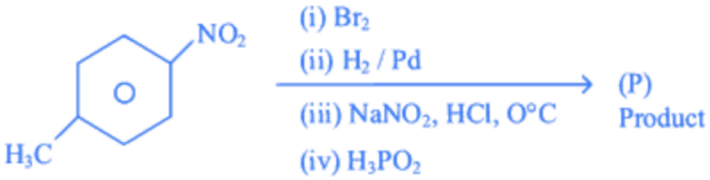
$$A\text{-III}, \quad B\text{-I}, \quad C\text{-IV}, \quad D\text{-II}.$$

This is the combination in option (D). Option (A) exchanges the assignments of glutamine and tyrosine. Option (B) gives the wrong codes for glutamic acid, glutamine and tyrosine. Option (C) incorrectly assigns W to glutamic acid and E to tryptophan. Although some individual pairings in these options are correct, a matching-list answer must contain all four correct associations. Hence checking the complete set avoids choosing an option on the basis of just one familiar symbol.

Step 3: Conclusion The matches are glutamic acid-E, glutamine-Q, tyrosine-Y and tryptophan-W: A-III, B-I, C-IV, D-II. Hence, the correct option is (D).

Quick Tip to Solve: Learn the paired acid/amide symbols together: Glu is E and Gln is Q; then use Tyr-Y and Trp-W to complete the matching.

71. The product (P) formed from the following multistep reaction is: [4 marks]



(A) 2-Bromotoluene

(B) 2-Bromo-4-methylphenol

(C) 3-Bromotoluene

(D) 3-Bromo-4-methylphenol

Correct Answer: (A) 2-Bromotoluene

Solution:

Concept:

- Electrophilic aromatic bromination follows the directing effects of groups already on the ring. A methyl group is ortho/para directing, whereas a nitro group is meta directing.
- Reduction converts an aromatic nitro group into an amino group. Diazotisation at $0^\circ C$, followed by H_3PO_2, replaces that amino-derived diazonium group by hydrogen.

Step 1: Locate the entering bromine The starting material is para-nitrotoluene. Number the methyl-bearing carbon 1, so the nitro group is at carbon 4. The methyl group favours positions 2 and 6; these same positions are meta to the nitro group and are therefore also favoured by its directing effect. Positions 2 and 6 are equivalent by symmetry. Bromination consequently gives 2-bromo-4-nitrotoluene, with bromine adjacent to methyl. It does not give the meta relationship between bromine and methyl shown in option (C).

Step 2: Track the nitro group through the remaining reagents Hydrogen over palladium reduces $-NO_2$ to $-NH_2$. Treatment with $NaNO_2/HCl$ at $0^\circ C$ converts this amino group into the corresponding arenediazonium chloride. Hypophosphorous acid then removes the diazonium group by reduction:

$$ArN_2^+ \xrightarrow{H_3PO_2} ArH + N_2$$

The methyl group and the ring bromine retain their positions. No hydrolysis with water is prescribed, so the final substituent at the original nitro position is hydrogen, not hydroxyl. Hence the phenolic structures in (B) and (D) cannot be the final product.

Step 3: Conclusion The final product is 2-bromotoluene (ortho-bromotoluene), with adjacent methyl and bromo substituents. Hence, the correct option is (A).

Quick Tip to Solve: Mark the bromination position before removing the nitro group: diazotisation followed by H_3PO_2 deletes the amino-derived substituent without moving the other groups.

72. The descending order of acidity for the following carboxylic acid is-

A. CH_3COOH

B. $F_3C - COOH$

C. $ClCH_2 - COOH$

D. $FCH_2 - COOH$

E. $BrCH_2 - COOH$

Choose the correct answer from the options given below: [4 marks]

(A) B > C > D > E > A

(B) D > B > A > E > C

(C) B > D > C > E > A

(D) E > D > B > A > C

Correct Answer: (C) B > D > C > E > A

Solution:

Concept:

- An acid is stronger when its conjugate base is more stable. Electron-withdrawing substituents stabilise a carboxylate ion through the negative inductive effect.
- For halogens at the same position, the electron-withdrawing effect decreases as $F > Cl > Br$. More withdrawing atoms produce a stronger combined effect.

Step 1: Compare the haloacetic acids All three monohalo acids have the halogen on the carbon next to the carboxyl group, so distance from the acidic group is held constant. Fluorine withdraws electron density most strongly, followed by chlorine and then bromine. Their carboxylate ions are stabilised in the same order, giving

$$FCH_2COOH > ClCH_2COOH > BrCH_2COOH.$$

Thus the relative order of the lettered compounds is $D > C > E$. This comparison concerns acidity, not atomic size or the strength of a carbon-halogen bond.

Step 2: Place the trifluoro and unsubstituted acids Compound B has three fluorine atoms on the adjacent carbon. Their combined negative inductive effect is much greater than the effect of one fluorine in D, so B is the strongest acid. Compound A is acetic acid; its methyl group donates electron

density inductively and does not stabilise the negative charge as the halo substituents do. It is therefore weaker than all three haloacetic acids. Combining the two comparisons yields

$$B > D > C > E > A.$$

Option (A) reverses the fluoro/chloro comparison, (B) incorrectly puts acetic acid above halo acids, and (D) incorrectly makes the bromo compound strongest.

Step 3: Conclusion The descending order of acidity is $B > D > C > E > A$. Hence, the correct option is (C).

Quick Tip to Solve: Compare halo acids only after fixing the halogen distance: at equal distance use electronegativity, then check whether several withdrawing atoms reinforce the effect.

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73. Given below are two statements:

Statement I : Methyl orange is a weak acid.

Statement II : The benzenoid form of methyl orange is more intense/deeply coloured than the quinonoid form.

In the light of the above statement, choose the most appropriate answer from the options given below: [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Statement I is incorrect but Statement II is correct

(C) Both statement I and Statement II are correct

(D) Statement I is correct but Statement II is incorrect

Correct Answer: (A) Both Statement I and Statement II are incorrect

Solution:

Concept:

- Methyl orange is treated as a weak basic indicator: protonation changes its molecular electronic structure and hence its visible colour.
- The yellow benzenoid form predominates in alkaline solution, whereas the red quinonoid form predominates in acidic solution and has the more intense colour.

Step 1: Judge Statement I The statement calls methyl orange a weak acid. In the usual classification of the two common indicators, phenolphthalein is a weak acid and methyl orange is a weak base. Methyl orange accepts a proton in acidic solution, producing its acid-coloured form. This reversible acid-base change is what permits it to indicate a pH range; it does not make the given classification as a weak acid correct. Therefore Statement I is incorrect.

Step 2: Judge Statement II and compare the options In an alkaline medium methyl orange displays the yellow benzenoid form. On acidification the protonated quinonoid form is produced, with a deeper red colour associated with its changed delocalised electronic structure. Thus it is the quinonoid form, not the benzenoid form, that is more intensely coloured in the comparison stated here. Statement II reverses these forms and is incorrect. The two statements must be tested separately: an incorrect

acid/base classification cannot be rescued by the presence of a colour change. Option (B) requires Statement II to be true; option (C) requires both to be true; and option (D) requires Statement I to be true. Each conflicts with at least one of the two evaluations.

Step 3: Conclusion Both Statement I and Statement II are incorrect. Hence, the correct option is (A).

Quick Tip to Solve: Keep the indicator pair distinct: phenolphthalein is weakly acidic, while methyl orange is weakly basic; its acidic quinonoid form is red and its alkaline benzenoid form is yellow.

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74. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Sodium is about 30 times as abundant as potassium in the oceans.

Reason R : Potassium is bigger in size than sodium.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both A and R are true but R is NOT the correct explanation of A

(B) A is false but R is true

(C) Both A and R are true and R is the correct explanation of A

(D) A is true but R is false

Correct Answer: (C) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- The abundance of an element in seawater depends on how readily its ions remain dissolved rather than being retained by minerals and sediments.
- Potassium lies below sodium in Group 1 and has an additional occupied shell. Both its atom and its unhydrated monovalent ion are larger than the corresponding sodium species.

Step 1: Evaluate the assertion Sodium is a principal dissolved cation of seawater, while potassium is present in a much smaller concentration. The standard comparison is that sodium is about thirty times as abundant as potassium in the oceans. The statement concerns the oceans, so a comparison of their total crustal abundances would not test this assertion. Assertion A is true.

Step 2: Evaluate the reason On moving from sodium to potassium, a new principal shell is occupied. Increased shielding and the larger shell outweigh the increased nuclear charge, giving

$$r(\mathrm{K}) > r(\mathrm{Na}).$$

The corresponding bare-ion size trend is also $r(\mathrm{K}^+) > r(\mathrm{Na}^+)$. Therefore Reason R is true; options claiming that either A or R is false are eliminated.

Step 3: Connect size to retention and oceanic abundance The larger potassium ion has lower charge density and is less strongly hydrated than sodium. It can consequently be retained more readily in suitable sites of clay and silicate minerals, whereas the more strongly hydrated sodium ion remains mobile in aqueous solution to a greater extent. Retention of potassium in these mineral structures reduces its concentration in the oceans relative to sodium. Thus the size difference supplies the

underlying reason for the contrasting retention and abundance described in the assertion; it is not merely an unrelated periodic trend. Option (A) misses this connection, while (B) and (D) incorrectly reject a true statement.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (C).

Quick Tip to Solve: For seawater abundance, connect ionic size with hydration and mineral retention; do not decide the assertion from crustal abundance alone.

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75. Which of the following have same number of significant figures?

A. 0.00253

B. 1.0003

C. 15.0

D. 163

Choose the correct answer from the options given below [4 marks]

(A) A, C and D only

(B) B and C only

(C) A, B and C only

(D) C and D only

Correct Answer: (A) A, C and D only

Solution:

Concept:

- All nonzero digits are significant. Zeros before the first nonzero digit only locate the decimal point and are not significant.
- Zeros between nonzero digits are significant, and a trailing zero after a decimal point records measurement precision and is also significant.

Step 1: Count the significant figures in A and B For A, 0.00253, the zeros before 2 are leading zeros. Only the digits 2, 5 and 3 count, so A has three significant figures. Equivalently, $0.00253 = 2.53 \times 10^{-3}$; the power of ten changes magnitude but does not add precision. For B, 1.0003, all three zeros lie between the nonzero digits 1 and 3. They are captive zeros and must be counted. B therefore has five significant figures, not three.

Step 2: Count C and D and select the complete group For C, 15.0, the zero after the decimal point is a measured final digit. It indicates precision to one decimal place, so C has three significant figures: 1, 5 and 0. For D, 163, all three digits are nonzero, giving three significant figures. The counts in the listed order are therefore

$$A : 3, \qquad B : 5, \qquad C : 3, \qquad D : 3.$$

The complete set with equal counts is A, C and D. Options (B) and (C) incorrectly include B, whose count is five. Option (D) lists only C and D and omits A, which has the same count, so it is not the complete correct selection.

Step 3: Conclusion A, C and D each have three significant figures. Hence, the correct option is (A).

Quick Tip to Solve: Write small decimals in scientific notation to discard leading zeros quickly; retain captive zeros and every explicitly written trailing decimal zero.

76. Arrange the following gases in increasing order of van der Waals constant 'a'

A. Ar

B. CH_4

C. H_2O

D. C_6H_6

Choose the correct option from the following. [4 marks]

(A) D, C, B and A

(B) A, B, C and D

(C) B, C, D and A

(D) C, D, B and A

Correct Answer: (B) A, B, C and D

Solution:

Concept:

- The van der Waals equation is $\left(P + an^2/V^2\right)(V - nb) = nRT$. The constant a measures the effect of intermolecular attractive forces.
- Greater polarizability and stronger attractive interactions generally give a larger a. The parameter a should not be confused with the excluded-volume parameter b.

Step 1: Compare argon with methane Argon is monatomic and has relatively weak dispersion attractions. Methane has a larger, more readily polarizable electron cloud and stronger overall attractions than argon. Consequently

$$a(\mathrm{Ar}) < a(\mathrm{CH_4}).$$

This already rules out the sequence starting with methane followed by water and benzene and ending with argon: argon must be at the low-attraction end, not at the high-attraction end.

Step 2: Place water and benzene Water has substantial intermolecular attraction, including hydrogen bonding, so its van der Waals attraction parameter exceeds that of methane. Benzene is much larger and has a highly polarizable extended electron cloud; its total dispersion attraction gives an even larger value of a. Indicative values in the same conventional units are about 1.36 for argon, 2.25 for methane, 5.46 for water and 18.2 for benzene. These values illustrate the increasing sequence without comparing constants in different units. Therefore

$$a(\mathrm{Ar}) < a(\mathrm{CH_4}) < a(\mathrm{H_2O}) < a(\mathrm{C_6H_6}).$$

Option (A) reverses the entire order, while (C) and (D) incorrectly place argon last. Hydrogen bonding

alone does not make water exceed the much larger benzene molecule in this parameter.

Step 3: Conclusion The increasing order is A, B, C and D, namely $Ar < CH_4 < H_2O < C_6H_6$. Hence, the correct option is (B).

Quick Tip to Solve: Use a for attractions and b for molecular exclusion. A large polarizable molecule can have a greater a than a smaller hydrogen-bonding molecule.

77. Which of these reactions is not a part of breakdown of ozone in stratosphere? [4 marks]

(A) $CF_2Cl_2(g) \xrightarrow{uv} \dot{C}l(g) + \dot{C}F_2Cl(g)$

(B) $C\dot{l}(g) + O_3(g) \longrightarrow Cl\dot{O}(g) + O_2(g)$

(C) $2Cl\dot{O} \longrightarrow ClO_2(g) + C\dot{l}(g)$

(D) $Cl\dot{O}(g) + O(g) \longrightarrow C\dot{l}(g) + O_2(g)$

Correct Answer: (C) $2Cl\dot{O} \longrightarrow ClO_2(g) + C\dot{l}(g)$

Solution:

Concept:

- Ultraviolet radiation releases chlorine radicals from chlorofluorocarbons. Chlorine then catalyses ozone destruction because it is regenerated in the reaction sequence.
- The standard propagation steps are $Cl^{\bullet} + O_3 \longrightarrow ClO^{\bullet} + O_2$ and $ClO^{\bullet} + O \longrightarrow Cl^{\bullet} + O_2$. Their net reaction is $O_3 + O \longrightarrow 2\,O_2$.

Step 1: Identify initiation and the two propagation steps In option (A), UV light cleaves a carbon-chlorine bond in CF_2Cl_2, producing a chlorine radical and a chlorodifluoromethyl radical. This is the initiation step that provides the ozone-destroying chlorine radical. In (B), the chlorine radical reacts with ozone to form chlorine monoxide radical and molecular oxygen. In (D), chlorine monoxide reacts with atomic oxygen to regenerate chlorine radical and produce another oxygen molecule. Thus (A), (B) and (D) all belong to the standard mechanism.

Step 2: Check the remaining proposed reaction Option (C) proposes that two chlorine monoxide radicals produce chlorine dioxide and chlorine radical. This is not a step of the standard chlorine-catalysed breakdown cycle given above. The essential regeneration step consumes one chlorine monoxide radical and one oxygen atom, as in (D); it does not have the form printed in (C). Adding the genuine propagation steps cancels both chlorine-containing radicals and yields

$$O_3 + O \longrightarrow 2\,O_2.$$

This demonstrates the catalytic role of chlorine and explains why a small amount can destroy many ozone molecules. Selecting (A) merely because it involves a CFC rather than ozone would ignore the required radical-initiation stage.

Step 3: Conclusion The reaction printed in option (C) is not part of the standard stratospheric ozone-breakdown sequence. Hence, the correct option is (C).

Quick Tip to Solve: To test an ozone-depletion mechanism, cancel the chlorine radicals between its propagation steps: they should regenerate chlorine and leave net conversion of ozone and atomic oxygen into oxygen.

78. The statement/s which are true about antagonists from the following is/are:

A. They bind to the receptor site.

B. Get transferred inside the cell for their action.

C. Inhibit the natural communication of the body.

D. Mimic the natural messenger.

Choose the correct answer from the options given below: [4 marks]

(A) B only

(B) A and B

(C) A and C

(D) A, C and D

Correct Answer: (C) A and C

Solution:

Concept:

- Receptors are proteins that recognise chemical messengers. Binding at a receptor can either activate its response or prevent the normal messenger from activating it.
- An antagonist has affinity for a receptor but does not produce the normal agonist response. Its occupation of the binding site blocks or inhibits the action of the natural messenger.

Step 1: Test the binding and transport statements Statement A is true: an antagonist binds to the receptor site, making that site unavailable to the natural messenger. Statement B is false in this classification. Transfer into the cell is not the defining mechanism of receptor antagonists; the relevant action is blockade at the receptor. Binding and intracellular transport must not be confused. A drug need not be carried inside a cell merely because it interferes with communication through a receptor.

Step 2: Distinguish blockade from mimicry Statement C is true because preventing the natural messenger from activating the receptor inhibits the corresponding natural communication of the body. Statement D describes an agonist: an agonist mimics the natural messenger and activates its response. An antagonist blocks that response instead. Thus B only and A and B contain the false statement B. A, C and D includes the false statement D. The combination containing exactly the true statements is A and C.

Step 3: Conclusion Only statements A and C are true. Hence, the correct option is (C).

Quick Tip to Solve: Use affinity versus activation: both agonists and antagonists bind receptors, but only an agonist mimics the messenger and activates its normal response.

79. In Hall - Heroult process, the following is used for reducing Al_2O_3 :- [4 marks]

(A) Magnesium

(B) CaF_2

(C) Graphite

(D) Na_3AlF_6

Correct Answer: (C) Graphite

Solution:

Concept:

- The Hall–Heroult process extracts aluminium by electrolysis of alumina dissolved in molten cryolite. Aluminium ions gain electrons at the cathode: $Al^{3+} + 3\,e^- \longrightarrow Al$.
- Graphite forms the consumable carbon anodes. Oxygen-containing ions are discharged there and react with carbon, so carbon is consumed in the overall reduction process.

Step 1: Identify the role of graphite The carbon anode participates in the overall conversion of alumina to aluminium, with carbon dioxide as the principal gaseous product. The net reaction is commonly represented as

$$2\,Al_2O_3 + 3\,C \longrightarrow 4\,Al + 3\,CO_2.$$

Aluminium is actually deposited through cathodic electron transfer; this is electrolytic reduction, not ordinary heating of alumina with carbon. In the choices given, graphite is the consumed material associated with reduction of the oxide.

Step 2: Separate the additives from the reducing material Cryolite, Na_3AlF_6, dissolves alumina and lowers the operating temperature while improving the properties of the conducting melt. CaF_2 is an additive that helps control the melting point and fluidity. Neither is the consumed reducing material in the net reaction. Magnesium is not used as the reducing agent in the Hall–Heroult cell. Therefore magnesium, calcium fluoride and cryolite do not answer the stated question; graphite does.

Step 3: Conclusion Graphite is used in the Hall–Heroult process and is consumed at the anode in the overall reduction of alumina. Hence, the correct option is (C).

Quick Tip to Solve: Keep the cell roles distinct: alumina supplies aluminium, cryolite and calcium fluoride condition the melt, and graphite is the consumable anode.

80. Which of the following can reduce decomposition of H_2O_2 on exposure to light : [4 marks]

(A) Urea

(B) Glass containers

(C) Alkali

(D) Dust

Correct Answer: (A) Urea

Solution:

Concept:

- Hydrogen peroxide decomposes to water and oxygen: $2\,H_2O_2 \longrightarrow 2\,H_2O + O_2$. Light, heat and catalytic impurities accelerate this decomposition.
- A stabiliser slows unwanted decomposition without being the main reactant. Small amounts of urea can be added to hydrogen peroxide as a stabiliser.

Step 1: Choose the substance that inhibits decomposition The question asks for something that reduces decomposition, not for something that reacts rapidly with the peroxide. Urea acts as a stabiliser and helps preserve a hydrogen peroxide solution against decomposition. It is therefore the appropriate chemical additive among the four alternatives. The decomposition reaction releases oxygen, so preserving the peroxide means suppressing the processes that promote this oxygen evolution.

Step 2: Reject the substances and storage description that do not stabilise it Alkali promotes decomposition rather than suppressing it; hydrogen peroxide is more stable under suitably acidic storage conditions. Dust introduces impurities and surfaces that can accelerate decomposition, so it must be excluded. Merely specifying a glass container does not provide protection from light: ordinary clear glass transmits light and can contain impurities. Hydrogen peroxide is normally protected using suitable dark storage containers and stabilising additives. Thus the unqualified glass-container option is not the chemical stabiliser sought, while urea is.

Step 3: Conclusion Urea reduces the decomposition of hydrogen peroxide by acting as a stabiliser. Hence, the correct option is (A).

Quick Tip to Solve: For peroxide-storage questions, distinguish a stabilising additive such as urea from accelerators such as alkali, dust and exposure to light.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The number of species from the following carrying a single lone pair on central atom Xenon is _____.

$$XeF_5^+, XeO_3, XeO_2F_2, XeF_5^-, XeO_3F_2, XeOF_4, XeF_4$$

[4 marks]

Solution:

Concept:

- Xenon has eight valence electrons. In these Lewis structures, each terminal fluorine uses one xenon electron in a single bond, while each terminal oxo group uses two in a formal double bond.
- For a species XeO_aF_b with signed charge q, the number of xenon lone pairs is $L = (8-2a-b-q)/2$. A positive charge removes an electron and a negative charge adds one.

Step 1: Count lone pairs species by species Apply the count to the first four species, keeping the ionic charges:

$$L(XeF_5{}^+) = (8-5-1)/2 = 1,$$
$$L(XeO_3) = (8-6)/2 = 1,$$
$$L(XeO_2F_2) = (8-4-2)/2 = 1,$$
$$L(XeF_5{}^-) = (8-5+1)/2 = 2.$$

An oxygen double bond counts as one electron domain for shape, but uses two xenon electrons in this electron count. Confusing domains with electrons would give an incorrect lone-pair total.

Step 2: Complete the list and count qualifying species For the remaining three species,

$$L(XeO_3F_2) = (8-6-2)/2 = 0,$$
$$L(XeOF_4) = (8-2-4)/2 = 1,$$
$$L(XeF_4) = (8-4)/2 = 2.$$

Exactly one lone pair is present in $XeF_5{}^+$, XeO_3, XeO_2F_2 and $XeOF_4$. The anion $XeF_5{}^-$ and neutral XeF_4 have two lone pairs, and XeO_3F_2 has none; exclude these three.

Step 3: Conclusion The number of species carrying a single lone pair on xenon is $\boxed{4}$. This is a dimensionless count.

Quick Tip to Solve: Include the ionic charge before dividing the unused xenon electrons by two; the cation and anion with five fluorines have different lone-pair counts.

82. The observed magnetic moment of the complex $[Mn(\underline{N}CS)_6]^{x-}$ is 6.06 BM. The numerical value of x is _____. [4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons. A high-spin d^5 ion has five unpaired electrons.
- Each coordinated isothiocyanato ligand, NCS^-, has charge -1. The sum of the metal oxidation state and the six ligand charges equals the charge on the complex.

Step 1: Infer the manganese electronic configuration The observed moment is $6.06\,\text{BM}$, close to the high-spin five-unpaired-electron value

$$\mu_{\text{spin}} = \sqrt{5(5+2)} = \sqrt{35} = 5.92\,\text{BM}.$$

Small differences between observed and spin-only moments are possible, so the measurement should not be forced to be exactly equal to the simple formula. The large moment identifies high-spin d^5, rather than a low-spin configuration. Manganese has atomic number 25 and neutral configuration $[Ar]3d^5 4s^2$; loss of its two $4s$ electrons gives Mn^{2+} with the required d^5 configuration.

Step 2: Balance the complex charge Let the oxidation state of manganese be z. Six singly negative ligands contribute a total charge of -6, while the complex is shown with charge $-x$. Therefore

$$z + 6(-1) = -x.$$

Substituting $z = +2$,

$$2 - 6 = -x \quad \Rightarrow \quad -4 = -x \quad \Rightarrow \quad x = 4.$$

Here x is the positive numerical magnitude of the negative charge, not the signed charge itself. The complex is consequently $[Mn(NCS)_6]^{4-}$.

Step 3: Conclusion The numerical value is $\boxed{x = 4}$, a dimensionless charge magnitude; the complex carries charge $4-$.

Quick Tip to Solve: Use the magnetic moment to find the metal electron count first, then add ligand charges; do not equate the metal oxidation state to the overall complex charge.

83. The solubility product of $BaSO_4$ is 1×10^{-10} at 298 K. The solubility of $BaSO_4$ in 0.1 M K_2SO_4(aq) solution is _____ $\times 10^{-9}$ g L^{-1} (nearest integer).

Given: Molar mass of $BaSO_4$ is 233 g mol^{-1} [4 marks]

Solution:

Concept:

- For the dissolution equilibrium $BaSO_4(s) \rightleftharpoons Ba^{2+}(aq) + {SO_4}^{2-}(aq)$, the solubility product is $K_{sp} = [Ba^{2+}][{SO_4}^{2-}]$.
- Potassium sulphate supplies the common sulphate ion. One mole of K_2SO_4 supplies one mole of

SO_4^{2-}; its two potassium ions do not double the sulphate concentration.

Step 1: Determine the molar solubility Let s be the molar solubility of barium sulphate in the solution. The background sulphate concentration is $0.1\,\text{mol}\,\text{L}^{-1}$, so

$$1 \times 10^{-10} = s(0.1 + s).$$

Because the salt is sparingly soluble and a large concentration of its common ion is already present, $s \ll 0.1$. Hence

$$s \simeq \frac{1 \times 10^{-10}}{0.1} = 1 \times 10^{-9}\,\text{mol}\,\text{L}^{-1}.$$

The inferred ratio $s/0.1 = 10^{-8}$ confirms that neglecting the added sulphate from dissolution is justified.

Step 2: Convert to the requested mass concentration Multiply the molar concentration by the given molar mass, rather than treating molarity as a mass concentration:

$$\begin{aligned}\text{mass solubility} &= sM \\ &= (1 \times 10^{-9})(233) \\ &= 233 \times 10^{-9}\,\text{g}\,\text{L}^{-1}.\end{aligned}$$

The question already supplies the factor $10^{-9}\,\text{g}\,\text{L}^{-1}$ after the blank. Therefore the number to enter is the coefficient 233. The negligible correction from retaining s in the sulphate concentration cannot change its nearest-integer value.

Step 3: Conclusion The solubility is $233 \times 10^{-9}\,\text{g}\,\text{L}^{-1}$; the requested nearest-integer coefficient is $\boxed{233}$.

Quick Tip to Solve: In a common-ion solubility calculation, use the supplied common-ion concentration in the denominator, then multiply by molar mass only at the final conversion.

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84. If the boiling points of two solvents X and Y (having same molecular weights) are in the ratio $2:1$ and their enthalpy of vaporizations are in the ratio $1:2$, then the boiling point elevation constant of X is $\underline{\text{m}}$ times the boiling point elevation constant of Y. The value of m is _____ (nearest integer) [4 marks]

Solution:

Concept:

- The boiling-point elevation constant of a solvent is $K_b = RT_b^2 M/(1000\Delta_{\text{vap}}H)$ when M is in $\text{g}\,\text{mol}^{-1}$ and the enthalpy of vaporisation is molar. The boiling temperature T_b must be absolute.
- For equal solvent molar masses, the common factors cancel. The ratio is

$$\frac{K_{b,X}}{K_{b,Y}} = \left(\frac{T_{b,X}}{T_{b,Y}}\right)^2 \frac{\Delta_{\text{vap}}H_Y}{\Delta_{\text{vap}}H_X}.$$

Step 1: Write the ratio with the correct powers The problem gives $T_{b,X} : T_{b,Y} = 2 : 1$ and $\Delta_{\text{vap}}H_X : \Delta_{\text{vap}}H_Y = 1 : 2$. The boiling temperature appears squared in K_b, while the enthalpy appears in the denominator. Thus doubling the absolute boiling temperature contributes a factor of four, and halving the enthalpy contributes a further factor of two. Equal molecular weights ensure that no additional molar-mass factor is needed.

Step 2: Substitute and identify the multiplier Using the ratios directly gives

$$\frac{K_{b,X}}{K_{b,Y}} = \left(\frac{2}{1}\right)^2 \left(\frac{2}{1}\right)$$
$$= 4 \times 2 = 8.$$

Since the question defines $K_{b,X} = mK_{b,Y}$, this ratio is precisely m. Reversing the enthalpy ratio would give the wrong result because a larger enthalpy makes a solvent less susceptible to boiling-point elevation for the same molality. Both constants have the same units, so their ratio and the requested multiplier are dimensionless.

Step 3: Conclusion The boiling-point elevation constant of X is eight times that of Y. The nearest integer is $\boxed{m = 8}$ (dimensionless).

Quick Tip to Solve: For equal solvent molar masses, remember $K_b \propto T_b^2/\Delta_{\text{vap}}H$: square the temperature ratio and invert the enthalpy ratio.

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85. For complete combustion of ethene.

$$\text{C}_2\text{H}_4(\text{g}) + 3\text{O}_2(\text{g}) \rightarrow 2\text{CO}_2(\text{g}) + 2\text{H}_2\text{O}(\text{l})$$

the amount of heat produced as measured in bomb calorimeter is $1406\ \text{kJ mol}^{-1}$ **at** $300\ \text{K}$**. The minimum value of** $\text{T}\Delta\text{S}$ **needed to reach equilibrium is (−)** _____ kJ**. (Nearest integer)**

Given : $\text{R} = 8.3\ \text{J K}^{-1}\ \text{mol}^{-1}$ [4 marks]

Solution:

Concept:

- A bomb calorimeter operates at constant volume; its heat gives the reaction internal-energy change, $q_V = \Delta U$. Heat produced by combustion corresponds to a negative change for the reacting system.
- For ideal gases, $\Delta H = \Delta U + \Delta n_g RT$, where Δn_g counts only gaseous products minus gaseous reactants. At equilibrium, $\Delta G = \Delta H - T\Delta S = 0$.

Step 1: Identify the calorimetric change The combustion releases $1406\,\text{kJ}$ per mole of ethene. Therefore the system loses energy, and $\Delta U = -1406\,\text{kJ mol}^{-1}$. This value is not yet the constant-pressure enthalpy change; the gas mole numbers change during the reaction.

Step 2: Convert internal energy to enthalpy In $\text{C}_2\text{H}_4(\text{g}) + 3\,\text{O}_2(\text{g}) \longrightarrow 2\,\text{CO}_2(\text{g}) + 2\,\text{H}_2\text{O}\,(\text{l})$, water is liquid and contributes nothing to the gas count. Thus the products contain 2 gas moles while the

reactants contain 4, giving $\Delta n_g = 2 - 4 = -2$. With the supplied gas constant, the correction is

$$\Delta n_g RT = (-2)(8.3)(300)\,\text{J mol}^{-1} = -4.98\,\text{kJ mol}^{-1}.$$

$$\Delta H = -1406 - 4.98 = -1410.98\,\text{kJ mol}^{-1}.$$

Step 3: Apply the equilibrium condition At the limiting equilibrium condition the Gibbs-energy change is zero. Hence $T\Delta S = \Delta H = -1410.98\,\text{kJ}$ for the reaction as written. The question has already supplied the negative sign, so its blank asks for the positive numerical magnitude, rounded to the nearest integer.

Step 4: Conclusion The equilibrium value is $T\Delta S \approx -1411\,\text{kJ}$. Therefore the number to enter in the blank is $\boxed{1411}$.

Quick Tip to Solve: For bomb-calorimeter data, convert ΔU to ΔH before using Gibbs energy; exclude liquid water from Δn_g.

86. The number of atomic orbitals from the following having 5 radial nodes is _____.

$$7\text{s}, 7\text{p}, 6\text{s}, 8\text{p}, 8\text{d}$$

[4 marks]

Solution:

Concept:

- For an orbital with principal quantum number n and azimuthal quantum number l, the number of radial nodes is $n - l - 1$. A radial node is a spherical surface at which the radial wavefunction vanishes.
- The subshell labels s, p, d correspond to $l = 0, 1, 2$, respectively. The question counts the listed orbital designations that satisfy the radial-node condition.

Step 1: Translate the orbital labels The principal quantum number is the leading integer, while the letter fixes l. Thus $7s$ has $(n, l) = (7, 0)$, $7p$ has $(7, 1)$, $6s$ has $(6, 0)$, $8p$ has $(8, 1)$, and $8d$ has $(8, 2)$. We must use radial nodes, rather than the total nodes $n - 1$ or angular nodes l.

Step 2: Calculate each radial-node number Apply the same expression to every entry:

$$7s : 7 - 0 - 1 = 6,$$
$$7p : 7 - 1 - 1 = 5,$$
$$6s : 6 - 0 - 1 = 5,$$
$$8p : 8 - 1 - 1 = 6,$$
$$8d : 8 - 2 - 1 = 5.$$

The two entries with six radial nodes fail the required condition. The three remaining entries each have exactly five radial nodes, despite belonging to different subshell types.

Step 3: Count the qualifying entries The qualifying orbitals are $7p$, $6s$ and $8d$. Each is one entry in

the supplied list, so the required count is $1 + 1 + 1 = 3$. No degeneracy factor is introduced: the task is to select designations from the given list, not count every magnetic quantum-number state within each subshell.

Step 4: Conclusion Exactly $\boxed{3}$ of the listed orbital designations have five radial nodes: $7p, 6s, 8d$. This is a dimensionless count.

Quick Tip to Solve: For five radial nodes, test $n - l = 6$; this immediately selects $7p$, $6s$, and $8d$.

87. The sum of oxidation state of the metals in $Fe(CO)_5$, VO^{2+} and WO_3 is _____. [4 marks]

Solution:

Concept:

- The algebraic sum of oxidation numbers equals the overall charge of the species. In these compounds, ordinary oxide oxygen has oxidation number -2.
- Carbon monoxide, CO, is a neutral ligand in a metal carbonyl. Its coordination to a metal does not give the ligand an overall charge.

Step 1: Find the iron oxidation number Let the oxidation number of iron in $Fe(CO)_5$ be x. Each of the five carbon monoxide ligands has charge zero, and the whole complex is neutral. Consequently,

$$x + 5(0) = 0 \quad \Rightarrow \quad x = 0.$$

The carbon and oxygen atoms within each CO ligand have internal oxidation-number assignments, but it is the net ligand charge that enters this metal calculation.

Step 2: Find the vanadium oxidation number In the vanadyl ion VO^{2+}, let vanadium have oxidation number y. There is one oxide oxygen and the ion has total charge $+2$, not zero. Thus

$$y - 2 = +2 \quad \Rightarrow \quad y = +4.$$

Equating the sum to zero here would incorrectly treat the ion as a neutral molecule.

Step 3: Find tungsten and add the results For neutral tungsten trioxide, WO_3, let the tungsten oxidation number be z. Its three oxygens contribute -6 in all, so

$$z + 3(-2) = 0 \quad \Rightarrow \quad z = +6.$$

The requested algebraic sum refers to the metals only. Adding iron, vanadium and tungsten gives

$$0 + (+4) + (+6) = 10.$$

Step 4: Conclusion The metal oxidation numbers are $0, +4, +6$, respectively, and their sum is $\boxed{10}$. Oxidation numbers are dimensionless.

Quick Tip to Solve: Keep CO neutral in carbonyls, and set the oxidation-number sum equal to the written ionic charge for VO^{2+}.

88. The number of incorrect statements from the following is _____.

A. The electrical work that a reaction can perform at constant pressure and temperature is equal to the reaction Gibbs energy.

B. $\mathrm{E}^{\circ}_{\mathrm{cell}}$ **cell is dependent on the pressure.**

C. $\frac{dE^{\theta}\ \textbf{cell}}{\mathrm{dT}} = \frac{\Delta_{\mathrm{r}}\mathrm{S}^{\theta}}{\mathrm{nF}}$

D. A cell is operating reversibly if the cell potential is exactly balanced by an opposing source of potential difference. [4 marks]

Solution:

Concept:

- The thermodynamic relations are $\Delta_r G = -nFE_{\text{cell}}$ and $\Delta_r G^{\circ} = -nFE^{\circ}_{\text{cell}}$. Gibbs energy determines the maximum reversible non-expansion electrical work at fixed temperature and pressure.
- Standard electrode potentials refer to standard-state activities. Actual nonstandard pressures and concentrations enter the cell potential through the reaction quotient in the Nernst equation.

Step 1: Assess statements A and B A expresses the Gibbs-energy relation for reversible electrical work at constant temperature and pressure and is accepted as correct. With the chemistry convention of work on the system, $w_{\text{elec,rev}} = \Delta_r G$; the positive work delivered by a spontaneous cell has magnitude $-\Delta_r G = nFE$. B is incorrect: E°_{cell} is the standard-state value at a specified temperature. Changing a reacting gas pressure changes Q and hence $E_{\text{cell}} = E^{\circ}_{\text{cell}} - (RT/nF)\ln Q$, rather than making the standard potential a function of that nonstandard pressure.

Step 2: Differentiate the standard Gibbs relation At constant pressure, $(\partial \Delta_r G^{\circ}/\partial T)_p = -\Delta_r S^{\circ}$. Differentiating $\Delta_r G^{\circ} = -nFE^{\circ}_{\text{cell}}$, with n and F constant, gives

$$-nF\frac{dE^{\circ}_{\text{cell}}}{dT} = -\Delta_r S^{\circ}.$$

Therefore $dE^{\circ}_{\text{cell}}/dT = \Delta_r S^{\circ}/(nF)$, so C is correct.

Step 3: Check the reversible balance D is correct. When an external opposing potential exactly balances the cell emf, there is no finite driving difference and no net current. An infinitesimal change can drive the reaction in either direction, which is the reversible limiting condition. Only statement B fails; A, C and D satisfy their thermodynamic meanings.

Step 4: Conclusion Only statement B is incorrect. The required dimensionless count is $\boxed{1}$.

Quick Tip to Solve: Distinguish E° from the Nernst-corrected E; pressure in Q changes the actual emf, not the standard-state value.

89. The ratio of sigma and π **bonds present in pyrophosphoric acid is _____.** [4 marks]

Solution:

Concept:

- Pyrophosphoric acid is $\mathrm{H_4P_2O_7}$. Its connectivity is $(\mathrm{HO})_2\mathrm{P}(=\mathrm{O})-\mathrm{O}-\mathrm{P}(=\mathrm{O})(\mathrm{OH})_2$, with one oxygen bridging the two phosphorus atoms.

- Every single bond contributes one sigma bond. Each conventional $\mathrm{P{=}O}$ double bond contributes one sigma bond and one pi bond; the four $\mathrm{O{-}H}$ bonds must also be counted.

Step 1: List the distinct bond groups The molecule contains two phosphorus centres. Each is attached to one terminal double-bonded oxygen, two hydroxyl oxygens and the common bridging oxygen. Across the whole molecule this gives two terminal $\mathrm{P{=}O}$ bonds, four $\mathrm{P{-}OH}$ single bonds, and two $\mathrm{P{-}O}$ single bonds to the bridge. There are additionally four hydroxyl $\mathrm{O{-}H}$ bonds. The bridge is one atom but has two separate bonds.

Step 2: Count sigma bonds without omitting hydrogen The two terminal double bonds contribute 2 sigma bonds. The four phosphorus-to-hydroxyl-oxygen bonds contribute 4, the two phosphorus-to-bridge bonds contribute 2, and the four oxygen-to-hydrogen bonds contribute 4. Therefore

$$N_\sigma = 2 + 4 + 2 + 4 = 12.$$

Counting only phosphorus-oxygen links would miss the four hydroxyl sigma bonds and give an incorrect ratio.

Step 3: Count pi bonds and simplify the ratio Only the two terminal $\mathrm{P{=}O}$ double bonds contribute pi bonds in the conventional bonding structure used for this question. Hence $N_\pi = 2$. The requested sigma-to-pi ratio is

$$\frac{N_\sigma}{N_\pi} = \frac{12}{2} = 6.$$

Both counts refer to bonds, so the ratio has no physical unit.

Step 4: Conclusion The ratio is $\sigma : \pi = 12 : 2 = 6 : 1$. Thus the numerical entry is $\boxed{6}$.

Quick Tip to Solve: In phosphorus oxyacids, count the hydroxyl $\mathrm{O{-}H}$ bonds as sigma bonds and count both links through a bridging oxygen.

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90. Coagulating value of the electrolytes $\mathrm{AlCl_3}$ and NaCl for $\mathrm{As_2S_3}$ are 0.09 and 50.04 respectively. The coagulating power of $\mathrm{AlCl_3}$ is x times the coagulating power of NaCl. The value of x is _____.

[4 marks]

Solution:

Concept:

- Coagulating value is the minimum electrolyte concentration required to bring about coagulation of a sol. A smaller coagulating value indicates a greater coagulating power.
- For the same sol and comparable concentration units, coagulating power is inversely proportional to coagulating value: $P \propto 1/C$. Ratios of powers therefore use the inverse ratio of the stated coagulating values.

Step 1: Identify the relevant counter-ions An arsenic trisulphide sol, $\mathrm{As_2S_3}$, is negatively charged under the usual preparation conditions. The active counter-ions are consequently cations: $\mathrm{Al^{3+}}$ from $\mathrm{AlCl_3}$ and $\mathrm{Na^+}$ from NaCl. The much smaller stated coagulating value of aluminium chloride is consistent with its higher cation charge and stronger coagulating action.

Step 2: Form the inverse concentration ratio Let P_{Al} and P_{Na} denote the coagulating powers. Since the same proportionality constant cancels,

$$\frac{P_{\mathrm{Al}}}{P_{\mathrm{Na}}} = \frac{1/0.09}{1/50.04} = \frac{50.04}{0.09}.$$

The larger value must appear in the numerator after inversion: aluminium chloride needs less concentration and therefore must have the larger power.

Step 3: Evaluate the factor Eliminate the two decimal places in numerator and denominator together:

$$x = \frac{5004}{9} = 556.$$

As an arithmetic check, $0.09 \times 556 = 50.04$. The common units of coagulating value cancel, so x is dimensionless. No additional factor of three is used for aluminium chloride: the measured coagulating values already quantify the required electrolyte concentrations.

Step 4: Conclusion The coagulating power of $\mathrm{AlCl_3}$ is $\boxed{556}$ times that of NaCl, so $x = 556$.

Quick Tip to Solve: Invert the coagulating-value ratio when comparing powers; the electrolyte with the smaller coagulating value is the stronger coagulant.