JEE Main 2023 Paper 1 (B.E./B.Tech) Feb 1 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 1 February 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The value of $\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+\ldots.+\frac{1}{49!2!}+\frac{1}{51!1!}$ is : [4 marks]

(A) $\frac{2^{51}}{50!}$
(B) $\frac{2^{51}}{51!}$
(C) $\frac{2^{50}}{50!}$
(D) $\frac{2^{50}}{51!}$

Correct Answer: (D) $\frac{2^{50}}{51!}$

Solution:

Concept:

- For integers $0 \le r \le n$, $\binom{n}{r} = n!/[r!(n-r)!]$. A factorial sum can therefore be converted to a binomial-coefficient sum.
- The odd-indexed coefficients of $(1+t)^n$ sum to 2^{n-1}, obtained by subtracting its values at $t=1$ and $t=-1$.

Step 1: Convert the terms to binomial coefficients Let the required sum be T. Each denominator has factorial indices adding to 51, with the first index odd. In the last term, $1! = 0! = 1$, so it is exactly the terminal coefficient too. Thus

$$T = \frac{1}{51!}\left[\binom{51}{1}+\binom{51}{3}+\cdots+\binom{51}{49}+\binom{51}{51}\right].$$

Step 2: Evaluate the odd-indexed sum The binomial expansions give

$$2^{51} = \sum_{r=0}^{51}\binom{51}{r}, \qquad 0 = \sum_{r=0}^{51}(-1)^r\binom{51}{r}.$$

Subtracting the second equation from the first doubles only the odd-indexed terms. Hence their sum is 2^{50}, and

$$T = \frac{2^{50}}{51!}.$$

Options (A) and (C) incorrectly use $50!$; option (B) counts both even and odd coefficients, doubling the answer.

Step 3: Conclusion The value is $\boxed{2^{50}/51!}$. Hence, the correct option is (D).

Quick Tip to Solve: When the two factorial indices add to a fixed integer, multiply by that integer factorial and identify the parity of the binomial indices.

2. Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi, x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. Then $\sum_{x\in S} 2\sin^{-1}\left(x^2-1\right)$ is equal to : [4 marks]

(A) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(B) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(C) $\frac{-2\pi}{3}$

(D) None

Correct Answer: (D) None

Solution:

Concept:

- The principal range of $\cos^{-1} u$ is $[0, \pi]$, so it is nonnegative for every allowed real input.
- An empty solution set contributes zero to a sum over that set. Check attainability of the equation before applying trigonometric transformations.

Step 1: Use the principal ranges For $x \in [-1/2, 1/2]$, both inverse cosines are defined, and

$$\cos^{-1}(2x) \le \pi, \qquad 2\cos^{-1}\sqrt{1-x^2} \ge 0.$$

Their difference can equal π only if the first expression equals π and the subtracted expression equals zero. Any smaller first term or positive subtraction makes the difference strictly less than π.

Step 2: Check the simultaneous equality conditions The first equality requires $2x = -1$, hence $x = -1/2$. The second requires $\sqrt{1-x^2} = 1$, hence $x = 0$. They cannot hold simultaneously, so $S = \varnothing$. In particular, the endpoint $-1/2$ gives $\pi - 2(\pi/6) = 2\pi/3$, not π. Therefore

$$\sum_{x\in S} 2\sin^{-1}(x^2-1) = 0.$$

Options (A) and (B) are positive, and option (C) is negative; none equals zero.

Step 3: Conclusion The sum is $\boxed{0}$, which is not among the three displayed numerical expressions. Hence, the correct option is (D).

Quick Tip to Solve: For an inverse-trigonometric equation at a range endpoint, impose equality in every bound before squaring or taking cosine.

3. Let R be a relation on $\mathbb{R}$, given by $R = \{(a, b) : 3a - 3b + \sqrt{7}$ is an irrational number $\}$. Then R is [4 marks]

(A) an equivalence relation

(B) reflexive and symmetric but not transitive

(C) reflexive and transitive but not symmetric

(D) reflexive but neither symmetric nor transitive

Correct Answer: (D) reflexive but neither symmetric nor transitive

Solution:

Concept:

- A relation is reflexive when aRa for every real a, symmetric when $aRb \Rightarrow bRa$, and transitive when $aRb, bRc \Rightarrow aRc$.
- A single explicit counterexample disproves symmetry or transitivity. The sum of two irrational numbers need not be irrational.

Step 1: Test reflexivity and symmetry For every real a,

$$3a - 3a + \sqrt{7} = \sqrt{7},$$

which is irrational. Thus R is reflexive. For symmetry take $a = \sqrt{7}/3$, $b = 0$. Then

$$3a - 3b + \sqrt{7} = 2\sqrt{7}, \qquad 3b - 3a + \sqrt{7} = 0.$$

The first is irrational and the second rational. Thus aRb but not bRa, disproving symmetry.

Step 2: Disprove transitivity with three real numbers Choose $a = 0$, $b = \sqrt{7}/6$, $c = \sqrt{7}/3$. Direct substitution gives

$$3a - 3b + \sqrt{7} = \frac{\sqrt{7}}{2}, \qquad 3b - 3c + \sqrt{7} = \frac{\sqrt{7}}{2}.$$

Both are irrational, so aRb and bRc. However,

$$3a - 3c + \sqrt{7} = 0,$$

so aRc is false. Therefore transitivity fails. Options (A) and (B) require symmetry, while (C) requires transitivity; each contradicts these counterexamples.

Step 3: Conclusion R is reflexive but neither symmetric nor transitive. Hence, the correct option is (D).

Quick Tip to Solve: To disprove an irrationality-based relation property, choose differences involving the given radical so that one required expression cancels to zero.

4. The shortest distance between the lines
$\frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3}$ **and**
$\frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5}$ **is :** [4 marks]

(A) $7\sqrt{3}$

(B) $5\sqrt{3}$

(C) $4\sqrt{3}$

(D) $6\sqrt{3}$

Correct Answer: (D) $6\sqrt{3}$

Solution:

Concept:

- For lines through points $\mathbf{a}_1, \mathbf{a}_2$ with nonparallel direction vectors $\mathbf{b}_1, \mathbf{b}_2$, their shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.
- The cross product supplies a common perpendicular direction. The scalar triple product projects the vector joining the lines onto that direction.

Step 1: Read the points and direction vectors From the symmetric equations take

$$\mathbf{a}_1 = (5, 2, 4), \quad \mathbf{a}_2 = (-3, -5, 1),$$

$$\mathbf{b}_1 = (1, 2, -3), \quad \mathbf{b}_2 = (1, 4, -5).$$

The second line passes through negative x- and y-coordinates because its numerators are $x + 3$ and $y + 5$. Thus $\mathbf{a}_2 - \mathbf{a}_1 = (-8, -7, -3)$.

Step 2: Calculate the perpendicular projection The cross product is

$$\mathbf{b}_1 \times \mathbf{b}_2 = (2, 2, 2), \qquad |\mathbf{b}_1 \times \mathbf{b}_2| = 2\sqrt{3}.$$

It is nonzero, so the nonparallel-line formula applies. The numerator is

$$|(-8, -7, -3) \cdot (2, 2, 2)| = |-16 - 14 - 6| = 36.$$

Therefore

$$d = \frac{36}{2\sqrt{3}} = 6\sqrt{3}.$$

The other options $7\sqrt{3}, 5\sqrt{3}, 4\sqrt{3}$ do not equal this projection and would require a different scalar triple product.

Step 3: Conclusion The shortest distance is $\boxed{6\sqrt{3}}$ coordinate units. Hence, the correct option is (D).

Quick Tip to Solve: Read point coordinates with their signs directly from each numerator, and take an absolute value only after forming the scalar triple product.

5. Let S denote the set of all real values of λ such that the system of equations

$$\lambda x + y + z = 1$$

$$x + \lambda y + z = 1$$

$$x + y + \lambda z = 1$$

is inconsistent, then $\sum_{\lambda \in S} \left(|\lambda|^2 + |\lambda|\right)$ is equal to [4 marks]

(A) 12

(B) 2

(C) 4

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- A nonsingular coefficient matrix gives a unique solution, so inconsistency is possible only when its determinant vanishes.
- For a matrix with diagonal λ and all off-diagonal entries 1, the eigenvalues are $\lambda + 2$ and $\lambda - 1$ (twice); hence its determinant is $(\lambda + 2)(\lambda - 1)^2$.

Step 1: Locate and test the singular parameters The determinant condition gives $\lambda = 1$ or $\lambda = -2$. At $\lambda = 1$, all three equations reduce to the same equation

$$x + y + z = 1.$$

It has infinitely many real solutions, so this parameter is consistent and must not be placed in S. For every parameter other than 1 and -2, the determinant is nonzero and a unique solution exists.

Step 2: Identify the inconsistent parameter and evaluate the sum At $\lambda = -2$, adding all three equations gives

$$0x + 0y + 0z = 3,$$

which is impossible. Therefore $S = \{-2\}$, and

$$\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|) = |-2|^2 + |-2| = 4 + 2 = 6.$$

Option (B) retains only the absolute-value term, option (C) only the square, and option (A) is twice the required total. None describes the sum for the actual inconsistent set.

Step 3: Conclusion The required sum is $\boxed{6}$. Hence, the correct option is (D).

Quick Tip to Solve: A zero determinant means the unique-solution test fails; distinguish infinitely many solutions from no solution by substituting each singular parameter.

6. Let $S = \left\{x : x \in \mathbb{R} \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\right\}$. Then $n(S)$ is equal to [4 marks]

(A) 6

(B) 4

(C) 0

(D) 2

Correct Answer: (B) 4

Solution:

Concept:

- If two positive bases have product 1, their equal powers are reciprocals. Here $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) = 1$.
- For a base $a > 1$, the map $t \mapsto a^t$ is one-to-one on the real numbers. Solve first for the exponential and then for its exponent.

Step 1: Introduce one positive variable Put $a = \sqrt{3}+\sqrt{2} > 1$, $t = x^2 - 4$, and $u = a^t > 0$. The given equation becomes $u + u^{-1} = 10$. Multiplication by the positive number u gives

$$u^2 - 10u + 1 = 0, \qquad u = 5 \pm 2\sqrt{6}.$$

Both roots are positive, and neither may be discarded. Their product is 1, consistent with the original reciprocal pair.

Step 2: Recover the possible exponents Since $a^2 = (\sqrt{3}+\sqrt{2})^2 = 5 + 2\sqrt{6}$ and $a^{-2} = 5 - 2\sqrt{6}$, injectivity of the exponential gives

$$t = 2 \quad \text{or} \quad t = -2.$$

Thus $x^2 - 4 = 2$ or $x^2 - 4 = -2$, so

$$x = \pm\sqrt{6} \quad \text{or} \quad x = \pm\sqrt{2}.$$

All four values are real and distinct. Substitution gives the same two reciprocal terms in either order, so each satisfies the equation. Counting only positive roots would miss two valid solutions; there are no further exponents because the quadratic has only two roots.

Step 3: Conclusion There are four distinct real elements of S, namely $\{-\sqrt{6}, -\sqrt{2}, \sqrt{2}, \sqrt{6}\}$. The four roots directly satisfy the printed exponential equation, proving option (B) despite the different option attached to the question-sheet key. Hence, the correct option is (B).

Quick Tip to Solve: When conjugate surd bases multiply to 1, replace the two powers by u and $1/u$; remember both signs when recovering x from x^2.

7. If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ, then $3(\alpha+\beta+\gamma)$ is equal to : [4 marks]

(A) 12

(B) 10

(C) 11

(D) 9

Correct Answer: (A) 12

Solution:

Concept:

- For $z = x + iy$, $|z - a|$ is the distance from (x, y) to the real-axis point $(a, 0)$. A fixed ratio of distances defines an Apollonius circle.
- The standard circle equation $(x-h)^2 + (y-k)^2 = r^2$ gives centre (h, k) and positive radius r.

Step 1: Convert the modulus equation into Cartesian form The denominator requires $z \neq 3$. Multiplying the modulus equation gives $|z-2| = 2|z-3|$. Both sides are nonnegative, so squaring is equivalent:

$$(x-2)^2 + y^2 = 4[(x-3)^2 + y^2].$$

Expansion and collection give

$$3x^2 + 3y^2 - 20x + 32 = 0.$$

The signs of the linear terms come directly from the two real-axis points 2 and 3.

Step 2: Complete the square and evaluate the expression Dividing by 3 and completing the square gives

$$x^2 - \frac{20}{3}x + y^2 = -\frac{32}{3},$$

$$\left(x - \frac{10}{3}\right)^2 + y^2 = \frac{100}{9} - \frac{96}{9} = \frac{4}{9}.$$

Thus $\alpha = 10/3$, $\beta = 0$, and $\gamma = 2/3$. The excluded point $z = 3$ is not on this circle, so the locus needs no further deletion. Consequently

$$3(\alpha+\beta+\gamma) = 3\left(\frac{10}{3} + 0 + \frac{2}{3}\right) = 12.$$

The alternatives 10, 11 and 9 omit or miscalculate the radius contribution or centre.

Step 3: Conclusion The requested value is $\boxed{12}$. Hence, the correct option is (A).

Quick Tip to Solve: After squaring a modulus ratio, complete the square before reading the centre; the coefficient of x is minus twice the centre coordinate.

8. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5, then the sum of cubes of the remaining two observations is : [4 marks]

(A) 1792

(B) 1216

(C) 1456

(D) 1072

Correct Answer: (D) 1072

Solution:

Concept:

- The mean of N observations is $\bar{x} = \sum x_i/N$, while their variance is $\sigma^2 = \sum x_i^2/N - \bar{x}^2$. Both sums include all five observations.
- For two unknown observations u, v, use $u^2+v^2 = (u+v)^2-2uv$ and $u^3+v^3 = (u+v)^3-3uv(u+v)$.

Step 1: Recover the two basic sums The mean is 5 for five observations, so their total is $5 \times 5 = 25$. The three known observations add to $1+3+5 = 9$. Hence the two remaining observations obey

$$u + v = 25 - 9 = 16.$$

The variance is the population variance of these five observations, rather than a sample estimate with denominator four. Thus

$$8 = \frac{\sum x_i^2}{5} - 25 \quad \Longrightarrow \quad \sum x_i^2 = 165.$$

The known observations contribute $1^2 + 3^2 + 5^2 = 35$, leaving $u^2 + v^2 = 130$.

Step 2: Find the product and then the cube sum Use the square of the sum to isolate the product:

$$2uv = 16^2 - 130 = 126, \qquad uv = 63.$$

There is no need to solve separately for the observations to compute the requested symmetric expression:

$$u^3 + v^3 = 16^3 - 3(63)(16) = 4096 - 3024 = 1072.$$

As a direct check, the roots of $t^2 - 16t + 63 = 0$ are 7 and 9. Their cubes add to $343 + 729 = 1072$, and all five observations have squared deviations $16 + 4 + 0 + 4 + 16 = 40$, giving variance 8. The alternatives 1792, 1216 and 1456 therefore fail the cube-sum identity.

Step 3: Conclusion The required sum of cubes is $\boxed{1072}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the sum and product of the missing observations first; the cube-sum identity avoids unnecessary root calculations.

9. Let $f(x) = 2x + \tan^{-1}x$ and $g(x) = \log_e(\sqrt{1+x^2}+x), x \in [0,3]$. Then [4 marks]

(A) there exists $\widehat{x} \in [0,3]$ such that $f'(\widehat{x}) < g'(\widehat{x})$

(B) there exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x), \forall x \in (x_1, x_2)$

(C) $\min f'(x) = 1 + \max g'(x)$

(D) $\max f(x) > \max g(x)$

Correct Answer: (D) $\max f(x) > \max g(x)$

Solution:

Concept:

- A positive derivative makes a function increasing on an interval, so its maximum on a closed interval occurs at the right endpoint.
- If $h(0) = 0$ and $h\prime(x) > 0$ for $x > 0$, then $h(x) > 0$ there. Apply this to the difference of the two functions.

Step 1: Differentiate and compare the derivatives The logarithm is well defined since $\sqrt{1+x^2}+x > 0$. Differentiating gives

$$f\prime(x) = 2 + \frac{1}{1+x^2}, \qquad g\prime(x) = \frac{1}{\sqrt{1+x^2}}.$$

For $0 \le x \le 3$, the first derivative decreases from 3 to $21/10$, while the second decreases from 1 to $1/\sqrt{10}$. In particular, $f\prime(x) \ge 21/10 > 1 \ge g\prime(x)$ throughout. This rules out (A). It also gives $\min f\prime = 21/10$, whereas $1 + \max g\prime = 2$; hence (C) is false.

Step 2: Compare the functions and their maxima Let $h(x) = f(x) - g(x)$. At the left endpoint, $h(0) = 0$. The derivative comparison proves $h\prime(x) > 0$, so $f(x) > g(x)$ for every $x \in (0,3]$. Consequently no nonempty interior interval can satisfy $f(x) < g(x)$, and (B) is false. Both functions have positive derivatives, so their individual maxima are at $x = 3$. Evaluating at that common point gives

$$\max f = f(3) = 6 + \tan^{-1}3, \qquad \max g = g(3) = \log(\sqrt{10}+3).$$

The established strict inequality $h(3) > 0$ proves that the former maximum is larger; no decimal estimates of the logarithm or inverse tangent are required.

Step 3: Conclusion $\boxed{\max f(x) > \max g(x)}$. Hence, the correct option is (D).

Quick Tip to Solve: Compare the derivatives first, then use the equal starting values to compare the functions over the whole interval.

10. Let $f(x)=\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$, $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$. **If** α **and** β **respectively are the maximum and the minimum values of** f**, then** [4 marks]

(A) $\alpha^2-\beta^2=4\sqrt{3}$

(B) $\beta^2-2\sqrt{\alpha}=\frac{19}{4}$

(C) $\beta^2+2\sqrt{\alpha}=\frac{19}{4}$

(D) $\alpha^2+\beta^2=\frac{9}{2}$

Correct Answer: (B) $\beta^2-2\sqrt{\alpha}=\frac{19}{4}$

Solution:

Concept:

- Subtracting one row from another leaves a determinant unchanged. A common row vector plus the identity structure often simplifies a three-by-three determinant.
- On $x \in [\pi/6, \pi/3]$, the angle $2x$ lies in $[\pi/3, 2\pi/3]$, where the sine ranges from $\sqrt{3}/2$ to 1.

Step 1: Simplify the determinant Put $a=\sin^2 x$, $b=\cos^2 x$, and $c=\sin 2x$, so $a+b=1$. Apply $R_1 \leftarrow R_1-R_3$ and $R_2 \leftarrow R_2-R_3$. This gives

$$f(x)=\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ a & b & 1+c \end{vmatrix}.$$

Next add the first two columns to the third. The new third column is $(0,0,1+a+b+c)^T$, so expansion along it yields

$$f(x)=1+a+b+c=2+\sin 2x.$$

These elementary operations preserve the value; in particular the original determinant is not simply $\sin 2x$.

Step 2: Find the extrema and test the options The value $\sin 2x=1$ occurs at the interior point $x=\pi/4$. Its minimum on the given interval is $\sqrt{3}/2$, attained at either endpoint. Therefore

$$\alpha=3, \qquad \beta=2+\frac{\sqrt{3}}{2}, \qquad \beta^2=\frac{19}{4}+2\sqrt{3}.$$

Since $\sqrt{\alpha}=\sqrt{3}$, subtracting $2\sqrt{\alpha}$ gives $19/4$, as in (B). Option (C) instead gives $19/4+4\sqrt{3}$. Also $\alpha^2-\beta^2=17/4-2\sqrt{3}$, which is not $4\sqrt{3}$, ruling out (A). Finally $\alpha^2+\beta^2=55/4+2\sqrt{3}$, not $9/2$, so (D) fails too.

Step 3: Conclusion $\boxed{\beta^2-2\sqrt{\alpha}=19/4}$. Hence, the correct option is (B).

Quick Tip to Solve: When each row repeats the same trigonometric entries plus a diagonal 1, subtract rows before expanding the determinant.

11. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx} + \frac{x+a}{y-2} = 0, y(1) = 0$ is 4π.

Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is : [4 marks]

(A) $\frac{4\sqrt{3}}{3}$

(B) $2\sqrt{3}$

(C) 2

(D) $\frac{2\sqrt{3}}{3}$

Correct Answer: (A) $\frac{4\sqrt{3}}{3}$

Solution:

Concept:

- The equation $(y-2)\,dy + (x+a)\,dx = 0$ integrates to a circle. A circle of radius r encloses area πr^2.
- The normal to a circle at any point is the line through that point and its centre. This avoids differentiating again at each intersection.

Step 1: Identify the circle and determine its centre Multiply the differential equation by $y-2$ on each smooth branch and integrate:

$$\frac{(y-2)^2}{2} + \frac{(x+a)^2}{2} = C.$$

Thus the closed curve has equation $(x+a)^2 + (y-2)^2 = r^2$. Its area is 4π, so $r^2 = 4$. The condition $y(1) = 0$ means that $(1, 0)$ lies on this curve; consequently

$$(1+a)^2 + (-2)^2 = 4 \quad \Longrightarrow \quad a = -1.$$

The circle is therefore $(x-1)^2 + (y-2)^2 = 4$, with centre $(1, 2)$ and radius 2.

Step 2: Find the intersections and normal intercepts On the y-axis, $x = 0$, so $(y-2)^2 = 3$. Choose $P = (0, 2+\sqrt{3})$ and $Q = (0, 2-\sqrt{3})$; reversing these names leaves the required length unchanged. The lines from these points to the centre have slopes $-\sqrt{3}$ and $\sqrt{3}$, respectively. Written through the centre, their equations are

$$y - 2 = -\sqrt{3}(x-1), \qquad y - 2 = \sqrt{3}(x-1).$$

Putting $y = 0$ gives the x-intercepts $1 + 2/\sqrt{3}$ and $1 - 2/\sqrt{3}$. Hence

$$RS = \left|\frac{4}{\sqrt{3}}\right| = \frac{4\sqrt{3}}{3}.$$

The other proposed lengths are $2\sqrt{3}$, 2, and $2\sqrt{3}/3$; none equals this intercept difference. The vertical separation of P, Q is not the requested length.

Step 3: Conclusion The segment has length $\boxed{4\sqrt{3}/3}$ units. Hence, the correct option is (A).

Quick Tip to Solve: After integrating to identify a circle, obtain every normal by joining its point to the centre; then set the required coordinate to zero.

12. If the orthocentre of the triangle, whose vertices are (1, 2), (2, 3) and (3, 1) is (α, β), then the quadratic equation whose roots are $\alpha + 4\beta$ and $4\alpha + \beta$, is : [4 marks]

(A) $x^2 - 20x + 99 = 0$

(B) $x^2 - 22x + 120 = 0$

(C) $x^2 - 19x + 90 = 0$

(D) $x^2 - 18x + 80 = 0$

Correct Answer: (A) $x^2 - 20x + 99 = 0$

Solution:

Concept:

- The orthocentre is the intersection of the altitudes of a triangle. An altitude is perpendicular to the opposite side, so the two slopes have product -1.
- The monic quadratic with roots r, s is $x^2 - (r+s)x + rs = 0$. Compute both symmetric quantities to distinguish close options.

Step 1: Construct two altitudes Name the vertices $A = (1, 2)$, $B = (2, 3)$, $C = (3, 1)$. The slope of BC is $(1-3)/(3-2) = -2$, so the altitude from A has slope $1/2$:

$$y - 2 = \frac{1}{2}(x - 1), \qquad y = \frac{x}{2} + \frac{3}{2}.$$

The slope of AC is $(1-2)/(3-1) = -1/2$, so the altitude from B has slope 2:

$$y - 3 = 2(x - 2), \qquad y = 2x - 1.$$

Equating these expressions gives $x/2 + 3/2 = 2x - 1$, hence $x = 5/3$ and $y = 7/3$. Therefore $\alpha = 5/3$ and $\beta = 7/3$.

Step 2: Form the required quadratic and check the alternatives The requested roots are not the coordinates themselves; first form their stated linear combinations:

$$r = \alpha + 4\beta = \frac{5 + 28}{3} = 11, \qquad s = 4\alpha + \beta = \frac{20 + 7}{3} = 9.$$

Their sum is 20 and product is 99. Therefore

$$(x - 11)(x - 9) = x^2 - 20x + 99.$$

As a geometric check, AB has slope 1, so the third altitude from C is $y - 1 = -(x - 3)$, or $x + y = 4$; the calculated orthocentre satisfies this too. Among the proposed equations, only (A) has both sum 20 and product 99. The sums in (B), (C), and (D) are 22, 19, and 18 respectively, which already rules

them out.

Step 3: Conclusion The quadratic is $\boxed{x^2 - 20x + 99 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Use two perpendicular-slope altitudes to find the orthocentre, and check the third altitude before forming the requested roots.

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13. If $y = y(x)$ is the solution curve of the differential equation $\frac{dy}{dx} + y\tan x = x\sec x, 0 \le x \le \frac{\pi}{3}, y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to [4 marks]

(A) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(B) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(C) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(D) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

Correct Answer: (D) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, multiplication by the integrating factor $e^{\int P(x)\,dx}$ makes the left side a product derivative.
- Here $\cos x > 0$ throughout the stated interval, so $\int \tan x\,dx = -\log\cos x$ and the integrating factor is $\sec x$, without an absolute-value ambiguity.

Step 1: Apply the integrating factor and integrate Multiplying the differential equation by $\sec x$ gives

$$\frac{d}{dx}(y\sec x) = x\sec^2 x.$$

Integrate the right side by parts, taking $u = x$, $dv = \sec^2 x\,dx$. Since $v = \tan x$ and $\int \tan x\,dx = -\log\cos x$, the result is

$$y\sec x = x\tan x + \log\cos x + C.$$

At $x = 0$, the product $x\tan x$ and logarithm both vanish, while $y(0)\sec 0 = 1$. Thus $C = 1$, and

$$y = x\sin x + \cos x\log\cos x + \cos x.$$

The logarithm has a plus sign after the integration by parts.

Step 2: Evaluate and compare with the printed expressions At $x = \pi/6$, $\sin x = 1/2$ and $\cos x = \sqrt{3}/2$. Hence

$$y(\pi/6) = \frac{\pi}{12} + \frac{\sqrt{3}}{2}\left[1 + \log\left(\frac{\sqrt{3}}{2}\right)\right].$$

Combine 1 with the logarithm as $\log e$, then take a reciprocal:

$$1 + \log(\sqrt{3}/2) = \log(e\sqrt{3}/2) = -\log\left(\frac{2}{e\sqrt{3}}\right).$$

This is precisely (D). Option (C) reverses that logarithm sign, while (A) and (B) replace $e\sqrt{3}/2$ by a different argument involving $2\sqrt{3}/e$. These substitutions are not logarithm identities and do not satisfy the derived value.

Step 3: Conclusion $\boxed{y(\pi/6) = \frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)}$. Hence, the correct option is (D).

Quick Tip to Solve: For a linear equation with coefficient $\tan x$, use $\sec x$ as the integrating factor and track the sign of $\int \tan x\, dx$.

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14. $\lim\limits_{n\to\infty}\left[\frac{1}{1+n} + \frac{1}{2+n} + \frac{1}{3+n} + \ldots + \frac{1}{2n}\right]$ **is equal to** [4 marks]

(A) 0

(B) $\log_e 2$

(C) $\log_e\left(\frac{2}{3}\right)$

(D) $\log_e\left(\frac{3}{2}\right)$

Correct Answer: (B) $\log_e 2$

Solution:

Concept:

- A sum $\frac{1}{n}\sum_{k=1}^{n} h(k/n)$ converges to $\int_0^1 h(t)\,dt$ when h is continuous on that interval.
- The antiderivative of $1/(1+t)$ is $\log(1+t)$ for $t > -1$. The factor $1/n$ in a Riemann sum is the width of each subinterval.

Step 1: Rewrite the sum in Riemann form There are exactly n terms, with denominators $n+1, n+2, \ldots, 2n$. Factoring n out of each denominator gives

$$S_n = \sum_{k=1}^{n} \frac{1}{n+k} = \frac{1}{n}\sum_{k=1}^{n} \frac{1}{1+k/n}.$$

This is a right-endpoint Riemann sum for $h(t) = 1/(1+t)$ on $[0,1]$, using n subintervals of width $1/n$. The function is continuous there, so the limit exists and equals its integral. Although each term tends to zero, the number of terms grows with n, and the whole sum need not tend to zero.

Step 2: Evaluate the integral and exclude the distractors Take the limit by the Riemann-sum theorem:

$$\lim_{n\to\infty} S_n = \int_0^1 \frac{dt}{1+t} = [\log(1+t)]_0^1 = \log 2.$$

An equivalent interval choice is $u = 1+t$, giving $\int_1^2 du/u$; it confirms both endpoints. Since every term is at least $1/(2n)$, the full sum is at least $1/2$, excluding 0 in (A). Option (C), $\log(2/3)$, is negative,

whereas the sum is positive. Option (D), $\log(3/2)$, would use an incorrect upper or lower endpoint and also lies below $1/2$, because $\log(1+u) < u$ at $u = 1/2$. Only (B) has the integral value.

Step 3: Conclusion The limit is $\boxed{\log_e 2}$. Hence, the correct option is (B).

Quick Tip to Solve: Factor out n from the denominators, retain the width $1/n$, and read the integration interval from k/n.

15. The sum of 10 terms of the series

$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots$. **is** [4 marks]

(A) $\frac{58}{111}$

(B) $\frac{56}{111}$

(C) $\frac{55}{111}$

(D) $\frac{59}{111}$

Correct Answer: (C) $\frac{55}{111}$

Solution:

Concept:

- A quartic denominator can factor as $n^4 + n^2 + 1 = (n^2 - n + 1)(n^2 + n + 1)$.
- Consecutive quadratic factors satisfy $n^2 + n + 1 = (n+1)^2 - (n+1) + 1$, allowing cancellation in a finite sum.

Step 1: Express the general term as a difference The difference between the two factors is $2n$, so

$$\frac{n}{n^4 + n^2 + 1} = \frac{1}{2}\left(\frac{1}{n^2 - n + 1} - \frac{1}{n^2 + n + 1}\right).$$

This identity follows by putting the right-hand side over the common denominator: its numerator is $\frac{1}{2}[(n^2 + n + 1) - (n^2 - n + 1)] = n$. It holds for every positive integer in the sum.

Step 2: Cancel successive terms and use the endpoints Writing the first few terms exhibits the cancellation:

$$T = \frac{1}{2}\left[\left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{7}\right) + \cdots + \left(\frac{1}{91} - \frac{1}{111}\right)\right].$$

Every interior reciprocal occurs once positively and once negatively. Only the first positive term and the last negative term remain, giving

$$T = \frac{1}{2}\left(1 - \frac{1}{111}\right) = \frac{55}{111}.$$

The denominator 111 comes from $10^2 + 10 + 1$. Options (A), (B) and (D) have numerators 58, 56 and 59 rather than the endpoint numerator 55.

Step 3: Conclusion The sum of the ten terms is $\boxed{55/111}$. The literal ten-term sum telescopes exactly to this value; this proves option (C), rather than the different option attached to the question-sheet key. Hence, the correct option is (C).

Quick Tip to Solve: Factor a quartic with only even powers into conjugate quadratics, then check whether the second quadratic is the first evaluated at the next integer.

16. The combined equation of the two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ can be written as $(ax + by + c)(a'x + b'y + c') = 0$.

The equation of the angle bisectors of the lines represented by the equation $2x^2 + xy - 3y^2 = 0$ is : [4 marks]

(A) $3x^2 + xy - 2y^2 = 0$

(B) $x^2 - y^2 - 10xy = 0$

(C) $x^2 - y^2 + 10xy = 0$

(D) $3x^2 + 5xy + 2y^2 = 0$

Correct Answer: (B) $x^2 - y^2 - 10xy = 0$

Solution:

Concept:

- A point lies on an angle bisector when its perpendicular distances from the two lines are equal.
- The distance from (x, y) to $ax + by + c = 0$ is $|ax + by + c|/\sqrt{a^2 + b^2}$. Squaring the equality includes both bisectors.

Step 1: Factor the given pair The homogeneous quadratic factors as

$$2x^2 + xy - 3y^2 = (2x + 3y)(x - y).$$

The two lines are therefore $2x + 3y = 0$ and $x - y = 0$. The factors must be normalised separately: their normal lengths are $\sqrt{13}$ and $\sqrt{2}$. Equating the unnormalised linear expressions would generally give incorrect bisectors because the coefficients have different scales.

Step 2: Equate and simplify the distances The required combined equation is

$$\frac{(2x + 3y)^2}{13} = \frac{(x - y)^2}{2}.$$

Cross-multiplication and expansion give

$$2(4x^2 + 12xy + 9y^2) = 13(x^2 - 2xy + y^2),$$
$$-5x^2 + 50xy + 5y^2 = 0.$$

Dividing by -5 yields $x^2 - y^2 - 10xy = 0$. The opposite sign of the xy term in option C gives different slopes. Options A and D have unequal opposite quadratic coefficients and hence do not represent a perpendicular pair of bisectors; the two true angle bisectors must be perpendicular.

Step 3: Conclusion The angle bisectors are represented by $x^2 - y^2 - 10xy = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Factor a pair of lines first and equate their normalised squared distances; this avoids remembering the sign in the combined bisector formula.

17. In a binomial distribution $B(n, p)$, the sum and the product of the mean and the variance are 5 and 6 respectively, then $6(n + p - q)$ is equal to : [4 marks]

(A) 52

(B) 50

(C) 51

(D) 53

Correct Answer: (A) 52

Solution:

Concept:

- For $X \sim B(n, p)$, the mean is $\mu = np$, variance is $v = npq$, and $q = 1 - p$.
- Since $0 \leq q \leq 1$, the binomial variance cannot exceed its mean. This selects the correct order of two possible values.

Step 1: Find the mean and variance The given sum and product imply

$$\mu + v = 5, \qquad \mu v = 6.$$

Consequently, both numbers are roots of

$$z^2 - 5z + 6 = (z - 2)(z - 3) = 0.$$

Thus the unordered pair is $\{2, 3\}$. Since $v = \mu q \leq \mu$, it follows that $\mu = 3$ and $v = 2$. Interchanging them would give $q = 3/2$, which is not a probability, so that apparent alternative is impossible.

Step 2: Recover the parameters and evaluate Dividing the variance by the mean gives

$$q = \frac{v}{\mu} = \frac{2}{3}, \qquad p = 1 - q = \frac{1}{3}.$$

Now $np = 3$ gives $n = 9$, a valid non-negative integer number of trials. Therefore

$$6(n + p - q) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right) = 6\left(\frac{26}{3}\right) = 52.$$

As a consistency check, these parameters reproduce mean $9/3 = 3$ and variance $9(1/3)(2/3) = 2$, whose sum and product are 5 and 6. The other numerical choices 50, 51 and 53 do not equal the value obtained from these uniquely determined valid parameters.

Step 3: Conclusion The required value is 52. Hence, the correct option is (A).

Quick Tip to Solve: When the sum and product of binomial mean and variance are given, solve a quadratic and assign the larger root to the mean.

18. The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to [4 marks]

(A) $(\sim p) \wedge (\sim q)$

(B) $(\sim p) \vee q$

(C) $p \wedge (\sim q)$

(D) $(\sim p) \vee (\sim q)$

Correct Answer: (A) $(\sim p) \wedge (\sim q)$

Solution:

Concept:

- De Morgan's laws change the negation of a disjunction into the conjunction of the negations, and conversely.
- The distributive and complement laws give $q \vee ((\sim q) \wedge p) = (q \vee \sim q) \wedge (q \vee p) = q \vee p$.

Step 1: Simplify the original statement Write $E = q \vee ((\sim q) \wedge p)$. Distributing $\vee$ over $\wedge$ gives

$$E = (q \vee \sim q) \wedge (q \vee p).$$

The first bracket is always true. Its conjunction with another statement has the truth value of that other statement, hence $E = q \vee p$. In words, if q is already true, the whole expression is true; if q is false, the second part reduces to p.

Step 2: Negate and check the alternatives Applying De Morgan's law to the simplified form yields

$$\sim E = \sim (q \vee p) = (\sim q) \wedge (\sim p).$$

Conjunction is commutative, so this is precisely option A. To rule out the distractors explicitly, take p false and q true: the required negation is false whereas options B and D are true. Take p true and q false: the required negation is false whereas option C is true. Finally, when both propositions are false the original statement is false and option A is true, as required. Negating only the inner occurrence of q would not negate the entire expression.

Step 3: Conclusion The negation is $(\sim p) \wedge (\sim q)$. Hence, the correct option is (A).

Quick Tip to Solve: Simplify the statement before negating it; the pattern $q \vee ((\sim q) \wedge p)$ always reduces to $p \vee q$.

19. For a triangle ABC, the value of $\cos 2A + \cos 2B + \cos 2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct? [4 marks]

(A) $\overrightarrow{MA} \cdot \overrightarrow{MB} = -18$

(B) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

(C) perimeter of ΔABC is $18\sqrt{3}$

(D) area of ΔABC is $\frac{27\sqrt{3}}{2}$

Correct Answer: (D) area of ΔABC is $\frac{27\sqrt{3}}{2}$

Solution:

Concept:

- For triangle angles, $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$. The product is at most $1/8$, with equality for an equilateral triangle.
- In an equilateral triangle of side a, inradius $r = a\sqrt{3}/6$, circumradius $R = 2r$, and area $\Delta = rs$, where s is the semiperimeter.

Step 1: Identify the triangle attaining the least value If a triangle is right or obtuse, $\cos A \cos B \cos C \leq 0$, so it cannot maximise the product. For an acute triangle, $\log(\cos t)$ has second derivative $-\sec^2 t < 0$. Its concavity, with $A + B + C = \pi$, gives

$$\cos A \cos B \cos C \leq \cos^3(\pi/3) = \frac{1}{8}.$$

Equality requires $A = B = C = 60°$, and the least cosine sum is $-3/2$. Thus the triangle in the question is equilateral.

Step 2: Check every proposed statement With $r = 3$, the side is $a = 2\sqrt{3}r = 6\sqrt{3}$, so the perimeter is $18\sqrt{3}$, confirming C. The common centre M satisfies $MA = MB = R = 6$ and $\angle AMB = 120°$. Hence

$$\overrightarrow{MA} \cdot \overrightarrow{MB} = 36\cos 120° = -18,$$

confirming A. Also $\sin 120° = \sin 60° = \sqrt{3}/2$, so the two three-term sine sums in B are equal. Finally, the semiperimeter is $9\sqrt{3}$, giving

$$\Delta = rs = 3(9\sqrt{3}) = 27\sqrt{3}.$$

This is twice the area stated in D; D alone is incorrect.

Step 3: Conclusion The actual area is $27\sqrt{3}$, not $27\sqrt{3}/2$. Hence, the correct option is (D).

Quick Tip to Solve: After recognising the equilateral equality case, use $a = 2\sqrt{3}r$, $R = 2r$, and $\Delta = rs$ to check the options quickly.

20. Let the image of the point $P(2,-1,3)$ in the plane $x+2y-z=0$ be Q.
Then the distance of the plane $3x+2y+z+29=0$ from the point Q is : [4 marks]

(A) $2\sqrt{14}$
(B) $\frac{22\sqrt{2}}{7}$
(C) $\frac{24\sqrt{2}}{7}$
(D) $3\sqrt{14}$

Correct Answer: (D) $3\sqrt{14}$

Solution:

Concept:

- The reflection of P in $\mathbf{n}\cdot\mathbf{r}+d=0$ is $Q=P-2(\mathbf{n}\cdot P+d)\mathbf{n}/\|\mathbf{n}\|^2$.
- The perpendicular distance from (x_0,y_0,z_0) to $ax+by+cz+d=0$ is $|ax_0+by_0+cz_0+d|/\sqrt{a^2+b^2+c^2}$.

Step 1: Find the reflected point The reflecting plane has normal $\mathbf{n}=(1,2,-1)$ and passes through the origin. At $P=(2,-1,3)$,

$$\mathbf{n}\cdot P=2-2-3=-3, \qquad \|\mathbf{n}\|^2=1+4+1=6.$$

Therefore

$$Q=(2,-1,3)-\frac{2(-3)}{6}(1,2,-1)=(3,1,2).$$

As a geometric check, the midpoint $(5/2,0,5/2)$ lies in the reflecting plane, and $Q-P=(1,2,-1)$ is normal to it. These are the two defining properties of a reflection.

Step 2: Calculate the distance to the second plane Substitute $Q=(3,1,2)$ into the expression defining the plane $3x+2y+z+29=0$:

$$3(3)+2(1)+2+29=42.$$

Its normal has length $\sqrt{3^2+2^2+1^2}=\sqrt{14}$. Thus

$$d=\frac{|42|}{\sqrt{14}}=3\sqrt{14}.$$

The result is positive as a distance must be. Option A would correspond to numerator 28, rather than the actual 42; options B and C also differ from $42/\sqrt{14}$. Using P instead of its image, or forgetting the factor of two in the reflection formula, would give a different numerator.

Step 3: Conclusion The distance of the second plane from Q is $3\sqrt{14}$ units. Hence, the correct option is (D).

Quick Tip to Solve: Check a reflected point by verifying that its midpoint with the original point lies on the mirror plane and their joining vector is normal to it.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If $\int_0^1 \left(x^{21}+x^{14}+x^7\right)\left(2x^{14}+3x^7+6\right)^{1/7}\,dx = \frac{1}{l}(11)^{m/n}$ where $l, m, n \in \mathbb{N}, m$ and n are coprime then $l+m+n$ is equal to _____. [4 marks]

Solution:

Concept:

- Look for a substitution combining the radical base with the power of x outside it; its derivative should reproduce the remaining polynomial.
- For $u \geq 0$, the power rule gives $\int u^{1/7}\,du = (7/8)u^{8/7}$. The substitution below maps $[0,1]$ monotonically to $[0,11]$.

Step 1: Choose a substitution matching all factors Denote the required integral by I and put

$$u = x^7(2x^{14}+3x^7+6) = 2x^{21}+3x^{14}+6x^7.$$

Differentiating gives

$$du = 42x^6(x^{14}+x^7+1)\,dx.$$

Since $x \geq 0$ on the interval,

$$u^{1/7} = x(2x^{14}+3x^7+6)^{1/7}.$$

Consequently the original integrand, which is $x^7(x^{14}+x^7+1)(2x^{14}+3x^7+6)^{1/7}$, is exactly $u^{1/7}du/42$. The extra factor x in the root is essential; substituting only the radical base would not reproduce the given integrand.

Step 2: Evaluate the transformed integral and identify the integers At $x=0$, $u=0$; at $x=1$, $u=2+3+6=11$. Hence

$$I = \frac{1}{42}\int_0^{11} u^{1/7}\,du = \frac{1}{48}\left[u^{8/7}\right]_0^{11} = \frac{1}{48}11^{8/7}.$$

Comparing with the requested form gives $l=48$, $m=8$, and $n=7$. The exponent is already reduced because 8 and 7 are coprime. Differentiating $u(x)^{8/7}/48$ gives $(1/42)u(x)^{1/7}u\prime(x)$, recovering the integrand and checking the coefficient.

Step 3: Conclusion Thus $l+m+n = 48+8+7 = 63$. The requested numerical value is 63 (dimensionless).

Quick Tip to Solve: For a radical multiplied by several powers of x, try moving an outside power into the root before differentiating a proposed substitution.

22. Let $a_1 = 8, a_2, a_3, \ldots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is _____. [4 marks]

Solution:

Concept:

- In an arithmetic progression, the term at position r is $a_r = a_1 + (r-1)d$, where d is the common difference.
- The sum of equally many consecutive terms is their number times the average of the first and last of those terms. For an even number of terms, the two central terms occupy positions $n/2$ and $n/2 + 1$.

Step 1: Find the common difference The first four terms are $8, 8+d, 8+2d, 8+3d$. Their given sum therefore gives

$$32 + 6d = 50 \quad \Longrightarrow \quad d = 3.$$

Thus the sequence increases strictly by 3 at each step; its individual terms are now fixed once their positions are known.

Step 2: Use the sum at the other end The last four terms are $a_n - 3d, a_n - 2d, a_n - d, a_n$. Substituting the known difference into their sum gives

$$4a_n - 6d = 170, \qquad 4a_n - 18 = 170, \qquad a_n = 47.$$

Using $a_n = 8 + 3(n-1)$, we obtain $3(n-1) = 39$, so $n = 14$. This even value confirms that the requested two middle terms exist.

Step 3: Multiply the central pair For fourteen terms the middle positions are 7 and 8, rather than 6 and 7. Hence

$$a_7 = 8 + 6(3) = 26, \qquad a_8 = 8 + 7(3) = 29.$$

Their product is $26 \times 29 = 754$. As a check, the first and last terms have average 27.5, which also equals the average of 26 and 29.

Step 4: Conclusion The product of the two middle terms is $\boxed{754}$. This is a dimensionless numerical answer.

Quick Tip to Solve: First find the common difference from the first four terms, then the last term from the last four; determine the number of terms before selecting the middle pair.

23. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that

$$f'(x) + f(x) = \int_0^2 f(t)\,dt.$$

If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to _____. [4 marks]

Solution:

Concept:

- The definite integral $C = \int_0^2 f(t)\,dt$ is a constant with respect to x. The equation is consequently the linear equation $f' + f = C$.
- The integrating factor is e^x, so the general solution has the form $f(x) = C + Ke^{-x}$. The same constant C must also satisfy its defining integral.

Step 1: Solve using the initial value Write $a = e^{-2} = f(0)$. Multiplication by the integrating factor gives

$$\frac{d}{dx}\left(e^x f(x)\right) = Ce^x.$$

Integrating and imposing $f(0) = a$ yields $K = a - C$, and therefore

$$f(x) = C + (a - C)e^{-x}.$$

Treating the integral as zero or as $f(x)$ would change the differential equation; it is an unknown constant to be determined.

Step 2: Impose the integral condition Integrating the original equation itself from 0 to 2 gives

$$f(2) - f(0) + C = 2C,$$

so $f(2) = C + a$. On the other hand, the explicit solution evaluated at 2 gives

$$f(2) = C + (a - C)a.$$

Equating these expressions leads to $(a - C)a = a$. Since $a = e^{-2} \neq 0$, division by a gives $C = a - 1$.

Step 3: Evaluate the requested combination It follows that $f(2) = C + a = 2a - 1$. Thus

$$2f(0) - f(2) = 2a - (2a - 1) = 1.$$

For a direct consistency check, the resulting function is $f(x) = e^{-2} - 1 + e^{-x}$; its integral from 0 to 2 is $2(e^{-2} - 1) + 1 - e^{-2} = e^{-2} - 1 = C$, as required.

Step 4: Conclusion Therefore $2f(0) - f(2) = \boxed{1}$, a dimensionless value.

Quick Tip to Solve: Name a definite integral that is independent of x as a constant, but remember to impose its integral definition after solving the resulting differential equation.

24. The remainder, when $19^{200} + 23^{200}$ is divided by 49 , is _____. [4 marks]

Solution:

Concept:

- Only residues modulo 49 matter when finding the remainder. A term containing $21^2 = 441$ is divisible by 49 and therefore contributes zero.
- Expanding a pair of even powers around opposite numbers makes the terms linear in the common shift cancel. Repeated squaring can then reduce the remaining power.

Step 1: Expand around opposite residues Use $19 = 21 - 2$ and $23 = 21 + 2$. In each binomial expansion, every term containing two or more factors of 21 is a multiple of 49. The terms with exactly one factor of 21 are opposite because their other factors have exponent 199, which is odd. Consequently

$$19^{200} + 23^{200} \equiv 2 \cdot 2^{200} \pmod{49}.$$

The constant terms add because the exponent 200 is even.

Step 2: Find a useful power cycle The following small residues avoid handling the original enormous integers:

$$2^7 = 128 \equiv 30 \pmod{49}, \qquad 2^{14} \equiv 30^2 = 900 \equiv 18 \pmod{49}.$$

Multiplication gives $2^{21} \equiv 18 \cdot 30 = 540 \equiv 1 \pmod{49}$. This proves a usable cycle; it is not necessary to show that 21 is the smallest period.

Step 3: Reduce the exponent and choose the remainder Since $200 = 9 \cdot 21 + 11$, we obtain

$$2^{200} \equiv (2^{21})^9 2^{11} \equiv 2048 \equiv 39 \pmod{49}.$$

The required sum is therefore congruent to $2 \cdot 39 = 78 \equiv 29 \pmod{49}$. A remainder on division by positive 49 must lie from 0 through 48, so 29 is already the required representative.

Step 4: Conclusion The remainder on division by 49 is $\boxed{29}$.

Quick Tip to Solve: For a modulus equal to a prime square, discard binomial terms containing the square of a multiple of that prime; symmetric bases often cancel the linear terms.

25. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7, is _____. [4 marks]

Solution:

Concept:

- For finite sets, $|E \cup F| = |E| + |F| - |E \cap F|$. Numbers divisible by both positive integers are multiples of their least common multiple.
- The count of multiples of k between 100 and 999 inclusive is $N_k = \lfloor 999/k \rfloor - \lfloor 99/k \rfloor$. The subtraction removes all shorter positive integers.

Step 1: Count numbers divisible by 2 or 3 Let E and F be the three-digit multiples of 2 and 3

respectively. Their intersection consists of multiples of 6. The counts are

$$N_2 = 499 - 49 = 450, \quad N_3 = 333 - 33 = 300, \quad N_6 = 166 - 16 = 150.$$

Thus $|E \cup F| = 450 + 300 - 150 = 600$. Here "either 2 or 3" includes numbers divisible by both, so those multiples must be counted once.

Step 2: Identify the forbidden multiples of 7 A number in the counted union that is also divisible by 7 is divisible by 14 or 21. Their intersection consists of multiples of 42, not merely multiples of 7. Hence

$$N_{14} = 71 - 7 = 64, \quad N_{21} = 47 - 4 = 43, \quad N_{42} = 23 - 2 = 21.$$

Applying inclusion and exclusion again, the forbidden count is $64 + 43 - 21 = 86$.

Step 3: Remove exactly the forbidden subset The 86 forbidden numbers are already within the union of 600 numbers, so subtract them once:

$$600 - 86 = 514.$$

Subtracting every three-digit multiple of 7 would be incorrect, because multiples of 7 that are divisible by neither 2 nor 3 were never in the starting set. The two-stage count implements both restrictions without that over-subtraction.

Step 4: Conclusion The number of such three-digit integers is $\boxed{514}$.

Quick Tip to Solve: Count the desired union first, then intersect the forbidden divisibility condition with that union; use least common multiples for every intersection.

26. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is _____. [4 marks]

Solution:

Concept:

- When specified letters must be consecutive, regard them as one block during the external arrangement, and then arrange the letters inside that block.
- For a multiset of n objects with repetition counts $r_1, r_2, \ldots$, the number of distinct arrangements is $n!/(r_1!r_2!\cdots)$. Identical letters are indistinguishable.

Step 1: Count every letter carefully ASSASSINATION has thirteen letters. Its vowels are three A letters, two I letters and one O letter, for a total of six vowels. The remaining seven letters are four S letters, two N letters and one T letter. The totals $3+2+1+4+2+1 = 13$ confirm that no occurrence has been omitted.

Step 2: Arrange the vowel block with the consonants Compress the six consecutive vowels into a single object V. The external arrangement now has eight objects: V, S, S, S, S, N, N, T. The block and T are individually distinct, whereas the four S letters and two N letters repeat. Thus the number of

external arrangements is

$$\frac{8!}{4!\,2!} = 840.$$

This count permits the vowel block at either end or between consonants, as the condition allows.

Step 3: Arrange the vowels within their block The internal arrangements of A, A, A, I, I, O number

$$\frac{6!}{3!\,2!} = 60.$$

Each external arrangement combines independently with each internal arrangement, giving $840 \times 60 = 50400$. There is no extra factorial for the block: it is already counted as one external object. Conversely, using $6!$ internally would overcount exchanges of identical A or I letters.

Step 4: Conclusion The required number of words is $\boxed{50400}$. Meaning is irrelevant, so every distinct letter arrangement is counted.

Quick Tip to Solve: Keep separate inventories for vowels and consonants; repeated-letter factorials are needed both outside the block and inside it.

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27. Let A be the area bounded by the curve $y = x|x-3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then $12A$ is equal to _____. [4 marks]

Solution:

Concept:

- Geometrical area uses $\int |y|\,dx$; a portion below the x-axis contributes positively rather than cancelling a portion above it.
- On the entire interval $[-1, 2]$, we have $x < 3$, so $|x-3| = 3-x$. The resulting polynomial $y = 3x - x^2$ changes sign at $x = 0$ inside this interval.

Step 1: Resolve the absolute value and the signs Because the right endpoint is 2, there is no need to split at 3: that point lies outside the given bounds. Write

$$y = x(3-x) = 3x - x^2.$$

Since $3 - x > 0$ throughout the interval, the sign of y is the sign of x. The curve lies below the axis from -1 to 0 and above it from 0 to 2. These two regions must be added in magnitude.

Step 2: Integrate each positive area An antiderivative is $F(x) = 3x^2/2 - x^3/3$. Therefore

$$\begin{aligned} A &= -\int_{-1}^{0} (3x - x^2)\,dx + \int_{0}^{2} (3x - x^2)\,dx \\ &= -\big(F(0) - F(-1)\big) + F(2) - F(0). \end{aligned}$$

The minus sign in the first integral compensates for the negative ordinate; it is essential for geometric area.

Step 3: Simplify and multiply as requested The endpoint values are

$$F(-1) = \frac{3}{2} + \frac{1}{3} = \frac{11}{6}, \qquad F(2) = 6 - \frac{8}{3} = \frac{10}{3}.$$

Hence $A = 11/6 + 10/3 = 31/6$ square units. The question asks for twelve times this area, so

$$12A = 12 \cdot \frac{31}{6} = 62.$$

Using a single signed integral without the sign correction would instead give a smaller net area and would not answer the question.

Step 4: Conclusion The area is $31/6$ square units, and the requested value is $\boxed{12A = 62}$.

Quick Tip to Solve: Split an area integral at the zeros of the ordinate inside the integration bounds, after first resolving any absolute value in the curve formula.

28. $A(2, 6, 2), B(-4, 0, \lambda), C(2, 3, -1)$ **and** $D(4, 5, 0), |\lambda| \leq 5$ **are the vertices of a quadrilateral** $ABCD$**. If its area is 18 square units, then** $5 - 6\lambda$ **is equal to _____.** [4 marks]

Solution:

Concept:

- A diagonal divides a quadrilateral into two triangles. A triangle determined by vectors $\mathbf{u}, \mathbf{v}$ from a common vertex has area $\frac{1}{2}|\mathbf{u} \times \mathbf{v}|$.
- For quadrilateral ABCD, use diagonal AC and add the nonnegative areas of triangles ABC and ACD. The restriction $|\lambda| \leq 5$ must be applied after solving the area equation.

Step 1: Find the vectors based at A The vectors from the given coordinates are

$$\overrightarrow{AB} = (-6, -6, \lambda - 2), \quad \overrightarrow{AC} = (0, -3, -3), \quad \overrightarrow{AD} = (2, -1, -2).$$

Using the same diagonal in both triangles gives a direct sum of their areas, with no arbitrary choice of signs in scalar expressions.

Step 2: Compute the two cross products For triangle ABC, calculation of the determinant gives

$$\overrightarrow{AB} \times \overrightarrow{AC} = (3\lambda + 12, -18, 18).$$

Its magnitude is $3\sqrt{(\lambda + 4)^2 + 72}$. For triangle ACD,

$$\overrightarrow{AC} \times \overrightarrow{AD} = (3, -6, 6),$$

whose magnitude is $\sqrt{9 + 36 + 36} = 9$. Thus the total area is $\frac{3}{2}\sqrt{(\lambda + 4)^2 + 72} + \frac{9}{2}$.

Step 3: Impose the area and the parameter bound Equating the area to 18 gives

$$\frac{3}{2}\sqrt{(\lambda + 4)^2 + 72} + \frac{9}{2} = 18 \quad \Longrightarrow \quad \sqrt{(\lambda + 4)^2 + 72} = 9.$$

Squaring yields $(\lambda+4)^2 = 9$, so $\lambda = -1$ or -7. The second value violates $|\lambda| \le 5$; hence only $\lambda = -1$ remains. For this value, the cross products are $(9, -18, 18)$ and $(3, -6, 6)$, parallel with the same sense, confirming a consistent planar quadrilateral and area $(27+9)/2 = 18$.

Step 4: Conclusion We obtain $\lambda = -1$, so the required value is $5 - 6\lambda = 5 + 6 = \boxed{11}$.

Quick Tip to Solve: Use a common diagonal to add two triangle areas, and always test every algebraic root against the given bound on the parameter.

---

29. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to _____. [4 marks]

Solution:

Concept:

- Write the unknown derivative values as constants, $a = g'(1)$ and $b = g''(2)$. The given identity makes f a monic quadratic.
- An identity between polynomials holds for every real input. Differentiation and coefficient comparison therefore determine the constants consistently.

Step 1: Express both functions using two constants Put $f(x) = x^2 + ax + b$. Then $f(1) = 1 + a + b$, $f'(x) = 2x + a$ and $f''(x) = 2$. Substitution in the second identity gives

$$g(x) = (1 + a + b)x^2 + x(2x + a) + 2 = (3 + a + b)x^2 + ax + 2.$$

Let $C = 3 + a + b$, so that $g(x) = Cx^2 + ax + 2$. This introduces no additional unknown: C is simply a convenient name for the quadratic coefficient.

Step 2: Impose the derivative definitions Differentiating gives $g'(x) = 2Cx + a$. Since $a = g'(1)$, we have $a = 2C + a$, hence $C = 0$. Also $g''(x) = 2C = 0$, so $b = g''(2) = 0$. The relation $C = 3 + a + b$ now gives $a = -3$. Thus

$$f(x) = x^2 - 3x, \qquad g(x) = -3x + 2.$$

As a direct check, these functions give $g'(1) = -3$ and $g''(2) = 0$, exactly the constants used in f.

Step 3: Evaluate the required difference At the common input 4,

$$f(4) = 16 - 12 = 4, \qquad g(4) = -12 + 2 = -10.$$

The negative value of g must be subtracted with its sign retained: $f(4) - g(4) = 4 - (-10)$. No units are involved because this is a polynomial value.

Step 4: Conclusion The required numerical value is $\boxed{14}$.

Quick Tip to Solve: Name the derivative values first; the identity a = 2C + a immediately removes the quadratic term of g.

---

30. Let $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $\begin{bmatrix} \vec{u} & \vec{v} & \vec{w} \end{bmatrix}$ is $-\alpha\sqrt{3401}$, and $|\vec{u} \cdot \hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then $m + n$ is equal to _____. [4 marks]

Solution:

Concept:

- The scalar triple product is $[\vec{u}\,\vec{v}\,\vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$. For fixed $|\vec{u}| = \alpha$, its minimum is $-\alpha|\vec{v} \times \vec{w}|$.
- Equality in the negative Cauchy–Schwarz bound holds when $\vec{u}$ points opposite to $\vec{v} \times \vec{w}$. This direction also determines its component along $\hat{i}$.

Step 1: Calculate the cross product Using the stated components in their printed order,

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix} = \hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}.$$

Its squared magnitude is therefore $1 + 25\alpha^2 + 9\alpha^2 = 1 + 34\alpha^2$. Reversing the cross-product order would reverse the direction, so it is essential to retain the order in the scalar triple product.

Step 2: Use the minimum to determine alpha The least dot product occurs at an angle of $180°$, not when the vectors are perpendicular. Comparing the given minimum with the bound gives

$$-\alpha\sqrt{1 + 34\alpha^2} = -\alpha\sqrt{3401}.$$

Because $\alpha > 0$, cancellation and squaring yield $34\alpha^2 = 3400$, hence $\alpha^2 = 100$ and $\alpha = 10$.

Step 3: Find the component at the minimizing direction The equality direction is

$$\vec{u} = -\frac{\alpha}{\sqrt{3401}}(\hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}).$$

Consequently

$$|\vec{u} \cdot \hat{i}|^2 = \frac{\alpha^2}{3401} = \frac{100}{3401}.$$

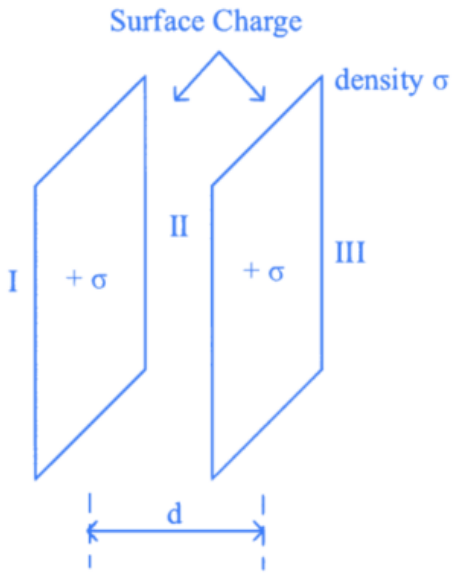
The denominator ends in 1, and has no factor 2 or 5, while all prime factors of 100 are 2 or 5. Thus the fraction is already in lowest terms, giving $m = 100$, $n = 3401$.

Step 4: Conclusion Therefore $m + n = 100 + 3401 = \boxed{3501}$, a dimensionless integer.

Quick Tip to Solve: At a prescribed triple-product minimum, use the equality direction as well as its magnitude to recover a component.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Let σ be the uniform surface charge density of two infinite thin plane sheets shown in figure. Then the electric fields in three different region E_I, E_{II} and E_{III} are: [4 marks]



(A) $\vec{E}_I = 0, \vec{E}_{II} = \frac{\sigma}{\epsilon_0}\hat{n}, E_{III} = 0$

(B) $\vec{E}_I = \frac{\sigma}{2\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{2\epsilon_0}\hat{n}$

(C) $\vec{E}_I = -\frac{\sigma}{\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{\epsilon_0}\hat{n}$

(D) $\vec{E}_I = \frac{2\sigma}{\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{2\sigma}{\epsilon_0}\hat{n}$

Correct Answer: (C) $\vec{E}_I = -\frac{\sigma}{\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{\epsilon_0}\hat{n}$

Solution:

Concept:

- Gauss's law gives the field magnitude $E_0 = \sigma/(2\epsilon_0)$ on each side of one infinite positively charged nonconducting sheet.
- The field points away from a positive sheet. Apply vector superposition separately in each region, taking $\hat{n}$ from region I toward region III.

Step 1: Add the fields outside the two sheets Both sheets in the figure carry $+\sigma$. In region I, a point lies to the left of both sheets, so both fields point along $-\hat{n}$. Their magnitudes add:

$$\vec{E}_I = -\left(\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}\right)\hat{n} = -\frac{\sigma}{\epsilon_0}\hat{n}.$$

In region III the point is to the right of both sheets, so both contributions point along $+\hat{n}$, giving $\vec{E}_{III} = \sigma\hat{n}/\epsilon_0$. The signs describe opposite directions on the two outer sides.

Step 2: Cancel the fields between the sheets In region II the left sheet produces $+\sigma\hat{n}/(2\epsilon_0)$, while the

right sheet produces $-\sigma\hat{n}/(2\epsilon_0)$. Thus

$$\vec{E}_{II} = \frac{\sigma}{2\epsilon_0}\hat{n} - \frac{\sigma}{2\epsilon_0}\hat{n} = 0.$$

The separation d does not enter because an ideal infinite sheet has a distance-independent field. Option A would describe oppositely charged sheets instead. B retains only one sheet's field outside and also misses the leftward sign. D doubles even the combined outside magnitude and has the wrong leftward direction.

Step 3: Conclusion $\boxed{\vec{E}_I = -\sigma\hat{n}/\epsilon_0, \quad \vec{E}_{II} = 0, \quad \vec{E}_{III} = \sigma\hat{n}/\epsilon_0}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal positive sheets, the fields cancel between them and add outside; check the direction on each outer side.

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32. 'n' polarizing sheets are arranged such that each makes an angle 45° with the preceeding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be $I/64$. The value of n will be: [4 marks]

(A) 4

(B) 5

(C) 3

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- An ideal polarizer transmits half the intensity of incident unpolarized light, leaving plane-polarized light.
- Malus' law applies to each later sheet: $I_{\text{next}} = I_{\text{previous}} \cos^2\theta$, with θ measured between successive transmission axes.

Step 1: Count the attenuation factors The first sheet receives the unpolarized beam and transmits $I_1 = I/2$. There are then $n-1$ further sheets, each at 45° to the sheet immediately before it. At every such sheet, $\cos^2 45^\circ = 1/2$, so the remaining transmission is multiplied by $(1/2)^{n-1}$. The polarization direction changes at each transmission; therefore the relevant angle is always the adjacent angle, not the angle from the first sheet.

Step 2: Solve the intensity equation and check the choices The final intensity is

$$I_{\text{out}} = \frac{I}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{I}{2^n}.$$

Comparison with $I/64$ gives $2^n = 64 = 2^6$, hence $n = 6$. This count includes the first polarizer. For the competing choices $n = 4, 5, 3$, the respective output intensities would be $I/16, I/32, I/8$, each exceeding the stated output. If one counted only the five applications of Malus' law after the first polarizer, one would mistakenly report 5 instead of the total number of sheets.

Step 3: Conclusion The arrangement contains $\boxed{n = 6}$ polarizing sheets. Hence, the correct option is (D).

Quick Tip to Solve: Count the first half-intensity factor separately, then count one Malus-law factor for each additional sheet.

33. Match List - I with List - II :

	List I		List II
A.	**AC generator**	**I.**	**Presence of both L and C**
B.	**Transformer**	**II.**	**Electromagnetic Induction**
C.	**Resonance phenomenon to occur**	**III.**	**Quality factor**
D.	**Sharpness of resonance**	**IV.**	**Mutual Induction**

Choose the correct answer from the options given below : [4 marks]

(A) A-II, B-I, C-III, D-IV

(B) A-IV, B-III, C-I, D-II

(C) A-II, B-IV, C-I, D-III

(D) A-IV, B-II, C-I, D-III

Correct Answer: (C) A-II, B-IV, C-I, D-III

Solution:

Concept:

- An induced emf is generated when magnetic flux linkage changes: $\mathcal{E} = -d\Phi_B/dt$. Mutual induction is induction in one coil due to changing current in another.
- Resonance in an ideal series LCR circuit occurs when inductive and capacitive reactances cancel, $\omega L = 1/(\omega C)$. Its sharpness is measured by the quality factor Q.

Step 1: Match the generator and transformer In an AC generator the coil rotates in a magnetic field, so its linked flux varies periodically. By Faraday's law this produces an alternating emf: A matches II, electromagnetic induction. A transformer has a primary coil and a secondary coil linked by a common changing magnetic flux. Alternating current in the primary induces emf in the secondary: B matches IV, mutual induction. Thus the transformer is a particular two-coil application of induction, whereas the generator is identified by the broader changing-flux principle.

Step 2: Match resonance and its sharpness Resonance requires both energy storage in an inductor and energy storage in a capacitor. Their reactances can then cancel at $\omega_0 = 1/\sqrt{LC}$, so C matches I. Sharpness describes how narrow the current-versus-frequency peak is. It is quantified by $Q = \omega_0 L/R$ for a series LCR circuit, so D matches III. The full mapping is therefore A-II, B-IV, C-I, D-III. Option A misassigns the transformer and resonance; option B misassigns the generator and transformer; option D swaps the two induction matches. Only option C has all four entries correct.

Step 3: Conclusion The matching is $\boxed{\text{A} - \text{II},\ \text{B} - \text{IV},\ \text{C} - \text{I},\ \text{D} - \text{III}}$. Hence, the correct option is (C).

Quick Tip to Solve: First lock in transformer = mutual induction; then distinguish the condition for resonance from its sharpness measure.

34. Match List I with List II:

	List I		List II
A.	Intrinsic semiconductor	I.	Fermi-level near the valence bond
B.	n-type semiconductor	II.	Fermi-level in the middle of valence and conduction band.
C.	p-type semiconductor	III.	Fermi-level near the conduction band
D.	Metals	IV.	Fermi-level inside the conduction band

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-I, C-II, D-IV

(B) A-II, B-I, C-III, D-IV

(C) A-I, B-II, C-III, D-IV

(D) A-II, B-III, C-I, D-IV

Correct Answer: (D) A-II, B-III, C-I, D-IV

Solution:

Concept:

- The Fermi level describes electron occupancy in a solid and its position relative to the valence and conduction bands distinguishes semiconductor types.
- Donor doping shifts the Fermi level toward the conduction band; acceptor doping shifts it toward the valence band. Intrinsic material is approximately mid-gap in the standard band picture.

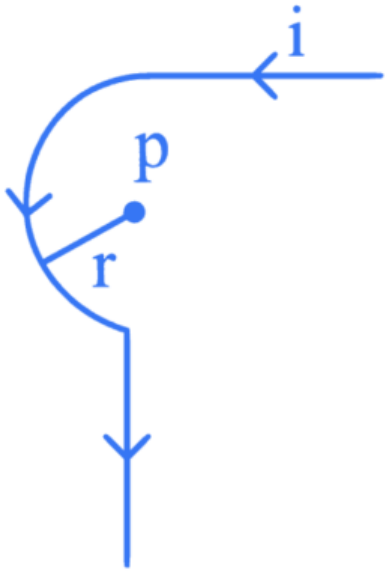
Step 1: Place the levels for the three semiconductors For an intrinsic semiconductor, electron and hole populations are equal. The conventional intrinsic band diagram places the Fermi level in the middle of the gap between valence and conduction bands. Thus A matches II. In an n-type semiconductor, donor impurities supply electrons, making electrons the majority carriers and shifting the Fermi level toward the conduction band: B matches III. In a p-type semiconductor, acceptor impurities create holes as majority carriers, and the Fermi level moves toward the valence band: C matches I. The printed word bond in entry I refers to that valence-band location.

Step 2: Identify the metallic case and compare the options A metal has available electronic states at the Fermi energy within a partially occupied band, or equivalently overlapping bands. In the band description used in the list, the Fermi level lies inside the conduction band, so D matches IV. Collecting all four entries gives A-II, B-III, C-I, D-IV. Option A places the intrinsic level near conduction rather than mid-gap. Option B interchanges the n-type and p-type locations. Option C places the intrinsic level near valence and the n-type level at mid-gap. These fail the majority-carrier and doping tests, while option D satisfies every entry.

Step 3: Conclusion The correct mapping is $\boxed{\text{A} - \text{II},\ \text{B} - \text{III},\ \text{C} - \text{I},\ \text{D} - \text{IV}}$. Hence, the correct option is (D).

Quick Tip to Solve: Remember n-type means more conduction electrons, while p-type means more valence-band holes; locate their Fermi levels accordingly.

35. Find the magnetic field at the point P in figure. The curved portion is a semicircle connected to two long straight wires. [4 marks]



(A) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{2\pi}\right)$
(B) $\frac{\mu_o i}{2r}\left(1+\frac{2}{\pi}\right)$
(C) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{\pi}\right)$
(D) $\frac{\mu_0 i}{2r}\left(1+\frac{1}{\pi}\right)$

Correct Answer: (A) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{2\pi}\right)$

Solution:

Concept:

- At the centre of a circular arc of radius r and angle θ, the magnetic field is $B_{\text{arc}} = \mu_0 i\theta/(4\pi r)$. A semicircle has $\theta = \pi$.
- A semi-infinite straight wire at perpendicular distance r contributes $\mu_0 i/(4\pi r)$. A radial straight segment contributes zero because its current element is parallel to the displacement to P.

Step 1: Evaluate the curved and radial portions The point P is the centre of the indicated semicircle, whose current travels counterclockwise around the left half of the circle. Hence

$$B_{\text{arc}} = \frac{\mu_0 i\pi}{4\pi r} = \frac{\mu_0 i}{4r},$$

directed out of the page by the right-hand rule. The downward lower straight portion lies along the radius continued through P. For every element of that portion, $d\vec{\ell}$ is parallel or antiparallel to the vector from the element to P, so $d\vec{\ell} \times \hat{r} = 0$. It makes no contribution even though it carries the same current.

Step 2: Add the upper straight-wire contribution The upper straight portion starts at the upper end of the semicircle and extends far to the right. It is tangent to the circle, with perpendicular distance r

from P, and is semi-infinite relative to its nearest point. Thus

$$B_{\text{straight}} = \frac{\mu_0 i}{4\pi r}.$$

Its leftward current also gives an outward field at P, so the contributions add:

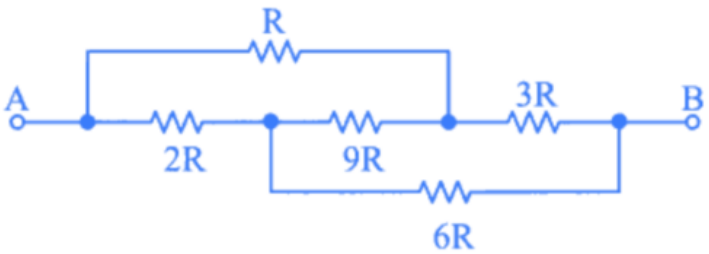
$$B = \frac{\mu_0 i}{4r} + \frac{\mu_0 i}{4\pi r} = \frac{\mu_0 i}{2r}\left(\frac{1}{2} + \frac{1}{2\pi}\right).$$

Option C incorrectly counts another tangent contribution; B and D also use an excessive curved or straight contribution. Only A contains one semicircle and one semi-infinite tangent wire.

Step 3: Conclusion $\boxed{B = \frac{\mu_0 i}{2r}\left(\frac{1}{2} + \frac{1}{2\pi}\right)}$, out of the page. Hence, the correct option is (A).

Quick Tip to Solve: Before adding straight-wire fields, test whether a segment is radial: a radial current element gives zero field at the centre.

36. The equivalent resistance between A and B of the network shown in figure; [4 marks]



(A) 21 R

(B) $\frac{8}{3}$ R

(C) $11\frac{2}{3}$ R

(D) 14 R

Correct Answer: (B) $\frac{8}{3}$ R

Solution:

Concept:

- A balanced Wheatstone bridge has equal ratios of the resistances in its two arms. The junctions then have equal potentials, so no current flows through the resistor joining them.
- Resistors in series add, whereas two resistances in parallel obey $R_{\text{eq}} = R_1 R_2/(R_1 + R_2)$. A resistor carrying zero current does not affect the terminal resistance.

Step 1: Identify the two paths from A to B Call the junction between $2R$ and $6R$ C, and the junction between R and $3R$ D. The $9R$ resistor connects C and D. Thus the bridge arms are A-C-B with

resistances $2R, 6R$, and A-D-B with resistances $R, 3R$. Their ratios are

$$\frac{2R}{6R} = \frac{R}{3R} = \frac{1}{3}.$$

Hence the bridge is balanced and C and D are at the same potential. The current in the central $9R$ resistor is zero.

Step 2: Reduce the balanced network For terminal resistance we can omit the zero-current bridge branch. The two remaining series paths have resistances

$$R_{ACB} = 2R + 6R = 8R, \qquad R_{ADB} = R + 3R = 4R.$$

These paths share only terminals A and B and are therefore in parallel:

$$R_{AB} = \frac{(8R)(4R)}{8R + 4R} = \frac{32R^2}{12R} = \frac{8}{3}R.$$

Adding every resistor would incorrectly count the bridge as a series chain. Options (A), (C) and (D) also exceed the smaller complete path resistance $4R$, whereas a parallel combination must be smaller than that path.

Step 3: Conclusion The equivalent resistance between A and B is $\boxed{8R/3}$. Hence, the correct option is (B).

Quick Tip to Solve: Label the two bridge junctions before combining resistors: equal arm ratios remove the connecting resistor regardless of its value.

37. If earth has a mass nine times and radius twice to that of a planet P. Then $\frac{v_e}{3}\sqrt{x}\ \mathrm{ms}^{-1}$ will be the minimum velocity required by a rocket to pull out of gravitational force of P, where v_e is escape velocity on earth. The value of x is [4 marks]

(A) 1

(B) 3

(C) 2

(D) 18

Correct Answer: (C) 2

Solution:

Concept:

- Escape speed is the minimum launch speed for reaching infinity with zero remaining speed, neglecting atmosphere and other gravitating bodies: $v_{\text{esc}} = \sqrt{2GM/R}$.
- For two spherical bodies the escape-speed ratio is the square root of the ratio of their M/R values; the rocket mass cancels in energy conservation.

Step 1: Express the planet quantities in Earth units Let Earth have mass M_E and radius R_E, and

planet P have mass M_P and radius R_P. The given comparisons mean

$$M_E = 9M_P, \qquad R_E = 2R_P.$$

Therefore $M_P = M_E/9$ and $R_P = R_E/2$. It is essential to invert both comparisons when expressing the planet quantities, rather than assigning the larger Earth mass or radius to P.

Step 2: Compare escape speeds and identify x The escape-speed formula follows by setting initial kinetic energy equal to the magnitude of surface gravitational potential energy: $mv^2/2 = GMm/R$. Consequently

$$\frac{v_P}{v_e} = \sqrt{\frac{M_P/R_P}{M_E/R_E}} = \sqrt{\frac{(M_E/9)/(R_E/2)}{M_E/R_E}} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}.$$

Comparing $v_P = (v_e/3)\sqrt{2}$ with the specified $(v_e/3)\sqrt{x}$ gives $x = 2$. Option (A) omits the radius factor; (B) and (D) do not give the required mass-to-radius ratio. As a check, the lighter planet has a smaller escape speed, here about $0.471v_e$.

Step 3: Conclusion The required escape speed is $v_P = \dfrac{v_e\sqrt{2}}{3}$, so $\boxed{x = 2}$. Hence, the correct option is (C).

Quick Tip to Solve: In escape-speed comparisons, form the ratio of M/R first and take its square root only at the end.

38. A steel wire with mass per unit length 7.0×10^{-3} kg m^{-1} is under tension of 70 N. The speed of transverse waves in the wire will be: [4 marks]

(A) 10 m/s

(B) 50 m/s

(C) 100 m/s

(D) 200π m/s

Correct Answer: (C) 100 m/s

Solution:

Concept:

- For a transverse disturbance on a taut wire, the wave speed is $v = \sqrt{T/\mu}$, where T is tension and μ is mass per unit length.
- Tension provides the restoring force, while linear mass density measures the inertia per unit length. The speed increases with tension and decreases with linear density.

Step 1: Substitute the tension and linear density The wire tension is $T = 70\,\text{N}$ and its linear density is $\mu = 7.0 \times 10^{-3}\,\text{kg m}^{-1}$. Both are already in SI units; the density is not $7.0\,\text{kg m}^{-1}$. Form the quotient before taking the square root:

$$\frac{T}{\mu} = \frac{70}{7.0 \times 10^{-3}} = 10 \times 10^3 = 10^4\,\text{m}^2\,\text{s}^{-2}.$$

The units follow because a newton is $\mathrm{kg\,m\,s^{-2}}$.

Step 2: Take the positive square root and compare options The speed is a positive magnitude, so

$$v = \sqrt{10^4}\,\mathrm{m\,s^{-1}} = 100\,\mathrm{m\,s^{-1}}.$$

Neither a frequency nor a wavelength is required because the tension and linear density directly determine the propagation speed. Options (A) and (B) square to 100 and $2500\,\mathrm{m^2\,s^{-2}}$, respectively, instead of the required 10^4. Option (D) would square to $40000\pi^2$, which is also incompatible. No factor of 2π belongs in this expression; that factor relates angular frequency to frequency and is irrelevant to this substitution.

Step 3: Conclusion The transverse-wave speed is $\boxed{100\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: Check the power of ten in the linear density before taking the square root; T divided by mu must have units of speed squared.

39. An object moves with speed v_1, v_2 and v_3 along a line segment AB, BC and CD respectively as shown in figure. Where AB = BC and AD = 3AB, then average speed of the object will be: [4 marks]



(A) $\frac{v_1v_2v_3}{3(v_1v_2+v_2v_3+v_3v_1)}$

(B) $\frac{(v_1+v_2+v_3)}{3}$

(C) $\frac{(v_1+v_2+v_3)}{3v_1v_2v_3}$

(D) $\frac{3v_1v_2v_3}{(v_1v_2+v_2v_3+v_3v_1)}$

Correct Answer: (D) $\frac{3v_1v_2v_3}{(v_1v_2+v_2v_3+v_3v_1)}$

Solution:

Concept:

- Average speed is total distance divided by total elapsed time, including the time spent in every segment.
- For equal distances travelled at different constant speeds, the average speed is the harmonic mean, rather than the arithmetic mean of the speeds.

Step 1: Show that all three segments have equal lengths Write $AB = BC = L$. Since the points lie in order along the straight segment and $AD = 3AB = 3L$,

$$CD = AD - AB - BC = 3L - L - L = L.$$

Thus each speed applies over the same distance, not over the same time interval. The three travel times are

$$t_1 = \frac{L}{v_1}, \qquad t_2 = \frac{L}{v_2}, \qquad t_3 = \frac{L}{v_3}.$$

The speeds are positive, so each time contributes positively to the total.

Step 2: Divide the full distance by the summed time The total distance is $3L$. Hence

$$\bar{v} = \frac{3L}{L/v_1 + L/v_2 + L/v_3} = \frac{3}{1/v_1 + 1/v_2 + 1/v_3}.$$

Multiplying numerator and denominator by $v_1 v_2 v_3$ gives

$$\bar{v} = \frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}.$$

Option (B), the arithmetic mean, would apply to equal times rather than equal distances. Option (C) has dimensions of inverse speed squared and cannot represent a speed. Option (A) is one ninth of the correct expression: setting all three speeds equal to v would give $v/9$, instead of v. The derived expression passes that equal-speed check.

Step 3: Conclusion The average speed is $\boxed{\frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}}$. Hence, the correct option is (D).

Quick Tip to Solve: For equal distances, add reciprocal speeds; for equal times, add the speeds themselves.

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40. A child stands on the edge of the cliff $10\ \text{m}$ above the ground and throws a stone horizontally with an initial speed of $5\ \text{ms}^{-1}$. Neglecting the air resistance, the speed with which the stone hits the ground will be ms^{-1} (given, $g = 10\ \text{ms}^{-2}$). [4 marks]

(A) 20

(B) 25

(C) 30

(D) 15

Correct Answer: (D) 15

Solution:

Concept:

- Horizontal and vertical motions are independent when air resistance is neglected. Gravity changes the vertical velocity while the horizontal velocity remains constant.
- The speed is the magnitude of the velocity vector: $v = \sqrt{v_x^2 + v_y^2}$. Equivalently, conservation of mechanical energy gives $v^2 = u^2 + 2gh$.

Step 1: Find the vertical velocity acquired during the fall The stone is thrown horizontally, so its initial vertical velocity is zero and its horizontal component is $v_x = 5\,\text{m}\,\text{s}^{-1}$. Take downward as positive for the vertical calculation. With vertical displacement $h = 10\,\text{m}$ and $g = 10\,\text{m}\,\text{s}^{-2}$,

$$v_y^2 = 0^2 + 2gh = 2(10)(10) = 200\,\text{m}^2\,\text{s}^{-2}.$$

Thus $v_y = 10\sqrt{2}\,\text{m}\,\text{s}^{-1}$ downward. This is only the vertical component, not the total impact speed.

Step 2: Combine the perpendicular velocity components The components are perpendicular, so we

add their squares rather than their magnitudes:

$$v^2 = v_x^2 + v_y^2 = 5^2 + 200 = 225\,\mathrm{m^2\,s^{-2}}, \qquad v = 15\,\mathrm{m\,s^{-1}}.$$

The same result follows from $\frac{1}{2}mv^2 = \frac{1}{2}m(5)^2 + mg(10)$, since the lost gravitational potential energy becomes kinetic energy. Options (A), (B) and (C) would require v^2 values of 400, 625 and 900, respectively, contradicting the available energy. The mass cancels, so no mass value is needed.

Step 3: Conclusion The stone strikes the ground with speed $\boxed{15\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: For impact speed alone, use v squared equals u squared plus 2gh; the horizontal launch speed contributes its full initial kinetic energy.

41. A proton moving with one tenth of velocity of light has a certain de Broglie wavelength of λ. An alpha particle having certain kinetic energy has the same de-Brogle wavelength λ. The ratio of kinetic energy of proton and that of alpha particle is: [4 marks]

(A) 1 : 4

(B) 2 : 1

(C) 4 : 1

(D) 1 : 2

Correct Answer: (C) 4 : 1

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where p is momentum and h is Planck's constant. Equal wavelengths imply equal momentum magnitudes.
- In the nonrelativistic treatment, kinetic energy is $K = p^2/(2m)$. At the same momentum it is inversely proportional to mass, and an alpha particle has approximately four times the proton mass.

Step 1: Use the wavelength condition to compare momenta Let the proton and alpha-particle momenta be p_p and p_α. Since both wavelengths are λ,

$$p_p = \frac{h}{\lambda} = p_\alpha = p.$$

Equal wavelengths do not mean equal velocities: the more massive alpha particle must move more slowly at the same momentum. With $m_\alpha \simeq 4m_p$, its speed is $v_\alpha = v_p/4$ in the usual nonrelativistic approximation. The given proton speed $0.1c$ is consistent with this standard approximation.

Step 2: Calculate the kinetic-energy ratio Writing each energy in terms of the common momentum avoids needing the actual wavelength or speed:

$$\frac{K_p}{K_\alpha} = \frac{p^2/(2m_p)}{p^2/(2m_\alpha)} = \frac{m_\alpha}{m_p} = 4.$$

Therefore the energy ratio in the requested order, proton to alpha particle, is $4:1$. Option (A) reverses that order. Options (B) and (D) would require a mass ratio of two rather than four. The alpha-particle charge does not enter this calculation: de Broglie wavelength is set by momentum, not by charge, and no accelerating potential is specified.

Step 3: Conclusion The kinetic-energy ratio is $\boxed{K_p : K_\alpha = 4:1}$. Hence, the correct option is (C).

Quick Tip to Solve: Equal wavelength means equal momentum; at equal momentum, the lighter particle has the greater kinetic energy.

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42. Given below are two statements:

Statement I: Acceleration due to gravity is different at different places on the surface of earth.

Statement II: Acceleration due to gravity increases as we go down below the earth's surface.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is false but Statement II is true

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- The effective acceleration due to gravity on Earth varies with latitude because of Earth's rotation and its oblate shape; it also varies with local elevation.
- In the standard uniform-density spherical-Earth model, the field inside Earth is $g(r) = GM(r)/r^2$, and enclosed mass $M(r)$ is proportional to r^3. Hence internal gravity is proportional to distance from the centre.

Step 1: Assess the surface statement Earth is not a perfect nonrotating sphere. Its equatorial radius is greater than its polar radius, making the gravitational contribution smaller near the equator. Rotation also reduces effective gravity most at the equator, while that centrifugal reduction vanishes at the poles. Consequently the surface value is generally larger at the poles and smaller at the equator. Statement I, which says that gravity differs at different surface places, is therefore true.

Step 2: Assess the depth statement and select the pair For the usual spherical uniform-density model, at depth d the distance from the centre is $r = R - d$. The enclosing outer spherical shells exert no net force on an interior point, so only enclosed mass contributes. As $M(r) = M(r/R)^3$,

$$g_d = \frac{GM(r)}{r^2} = \frac{GM}{R^3} r = g_s \left(1 - \frac{d}{R}\right).$$

Thus gravity decreases as depth increases and becomes zero at the centre in this model. Statement II asserts an increase, so it is false. Option (A) wrongly accepts Statement II, option (B) wrongly rejects Statement I, and option (C) gets both truth values reversed. Only option (D) matches the two

assessments.

Step 3: Conclusion Statement I is true and Statement II is false under the standard Earth model used here. Hence, the correct option is (D).

Quick Tip to Solve: For depth questions, use enclosed mass rather than Earth's full mass in GM/r squared; outer shells cancel inside a spherical shell.

43. A sample of gas at temperature T is adiabatically expanded to double its volume. The work done by the gas in the process is $\left(\text{given}, \gamma = \frac{3}{2}\right)$: [4 marks]

(A) $W = TR[\sqrt{2} - 2]$
(B) $W = \frac{T}{R}[\sqrt{2} - 2]$
(C) $W = \frac{R}{T}[2 - \sqrt{2}]$
(D) $W = RT[2 - \sqrt{2}]$

Correct Answer: (D) $W = RT[2 - \sqrt{2}]$

Solution:

Concept:

- For a reversible adiabatic expansion of an ideal gas, $TV^{\gamma-1}$ remains constant; $\gamma = C_P/C_V$.
- The work done by the gas is positive in expansion and equals the decrease in its internal energy: $W = nC_V(T_i - T_f)$, with $C_V = R/(\gamma - 1)$.

Step 1: Find the final temperature Let the initial volume be V and the final volume be $2V$. The adiabatic relation gives

$$TV^{1/2} = T_f(2V)^{1/2}, \qquad T_f = \frac{T}{\sqrt{2}}.$$

The temperature falls because the gas does external work without receiving heat. The temperature ratio depends only on the volume ratio and the given heat-capacity ratio, rather than on the initial pressure.

Step 2: Calculate the work and test the options Using $\gamma - 1 = 1/2$, the molar heat capacity is $C_V = 2R$. Hence the work per mole is

$$W_{\text{mol}} = 2R\left(T - \frac{T}{\sqrt{2}}\right) = RT(2 - \sqrt{2}).$$

For n moles, the corresponding total is $nRT(2 - \sqrt{2})$; the listed expressions use the molar convention. Option (A) has the opposite sign and would describe negative work by an expanding gas. Options (B) and (C) do not even have the dimensions of energy, because work requires the product RT, not either of the ratios T/R and R/T. Only (D) has both the positive sign and the correct scale.

Step 3: Conclusion The work done per mole is $\boxed{RT(2 - \sqrt{2})}$, or $nRT(2 - \sqrt{2})$ for n moles. Hence, the correct option is (D).

Quick Tip to Solve: In adiabatic expansion, calculate the fall in temperature first and check that work done by the gas is positive.

44. A block of mass $5\ \text{kg}$ is placed at rest on a table of rough surface. Now, if a force of $30\ \text{N}$ is applied in the direction parallel to surface of the table, the block slides through a distance of $50\ \text{m}$ in an interval of time $10\ \text{s}$. Coefficient of kinetic friction is (given, $g = 10\ \text{ms}^{-2}$): [4 marks]

(A) 0.25

(B) 0.75

(C) 0.60

(D) 0.50

Correct Answer: (D) 0.50

Solution:

Concept:

- With constant applied force and kinetic friction, the acceleration is constant and $s = ut + \frac{1}{2}at^2$.
- On a horizontal table with a horizontal pull, $N = mg$, $f_k = \mu_k N$, and Newton's law gives $F - f_k = ma$.

Step 1: Determine the acceleration from the displacement The block starts from rest, so $u = 0$. Substitute the measured displacement $s = 50\,\text{m}$ and time $t = 10\,\text{s}$:

$$50 = \frac{1}{2}a(10)^2, \qquad a = 1\,\text{m s}^{-2}.$$

The acceleration is smaller than $F/m = 6\,\text{m s}^{-2}$, which confirms that a substantial opposing friction force acts. Use the displacement over the entire interval; dividing distance by time would give average speed, not acceleration.

Step 2: Balance the horizontal forces and compare the choices The net force is $ma = 5 \times 1 = 5\,\text{N}$. Thus

$$f_k = 30 - 5 = 25\,\text{N}, \qquad N = mg = 5 \times 10 = 50\,\text{N}.$$

Therefore

$$\mu_k = \frac{f_k}{N} = \frac{25}{50} = 0.50.$$

The ratio is dimensionless. Option (A), 0.25, would leave a net force of $17.5\,\text{N}$, giving too large an acceleration. Option (C), 0.60, would exactly balance the applied force and give zero acceleration. Option (B), 0.75, gives friction exceeding the pull and cannot describe the stated sliding from rest in the force direction.

Step 3: Conclusion The coefficient of kinetic friction is $\boxed{0.50}$. Hence, the correct option is (D).

Quick Tip to Solve: Find acceleration from the motion data before using $F - \mu mg = ma$; do not equate the pull directly to friction.

45. Match List I with List II :

	List I		List II
A.	Microwaves	I.	Radio active decay of the nucleus
B.	Gamma rays	II.	Rapid acceleration and deceleration of electron in aerials
C.	Radio waves	III.	Inner shell electrons
D.	X-rays	IV.	Klystron valve

Choose the correct answer from the options given below : [4 marks]

(A) A-I, B-III, C-IV, D-II

(B) A-IV, B-III, C-II, D-I

(C) A-IV, B-I, C-II, D-III

(D) A-I, B-II, C-III, D-IV

Correct Answer: (C) A-IV, B-I, C-II, D-III

Solution:

Concept:

- The electromagnetic spectrum is classified by frequency, but each band also has characteristic production mechanisms.
- Nuclear transitions produce gamma rays; inner-shell electronic transitions produce characteristic X-rays. Oscillating charges in antennas produce radio waves.

Step 1: Identify the radio and microwave sources A klystron is a microwave oscillator or amplifier: it uses an electron beam interacting with resonant cavities. Thus microwaves match the klystron valve, giving A–IV. Radio waves are emitted by charges undergoing rapid alternating acceleration in transmitting aerials, so radio waves match II, giving C–II. These are electronic sources, but their particular arrangements determine the frequency range; the mere presence of electrons does not make every source an X-ray source.

Step 2: Identify the high-frequency sources and check all pairs Gamma rays arise when an excited nucleus loses energy during radioactive decay, so B–I. Characteristic X-rays result when an electron fills an inner-shell vacancy in an atom and releases the energy difference as a photon, so D–III. Combining all four matches gives

$$\mathrm{A-IV}, \quad \mathrm{B-I}, \quad \mathrm{C-II}, \quad \mathrm{D-III}.$$

Option (A) assigns microwaves to nuclear decay, which is incorrect. Option (B) swaps the nuclear and inner-shell origins of gamma rays and X-rays. Option (D) assigns radio waves to inner-shell electrons and X-rays to a klystron. Only option (C) respects all four physical sources.

Step 3: Conclusion The matching is A–IV, B–I, C–II, D–III. Hence, the correct option is (C).

Quick Tip to Solve: Remember the source hierarchy: aerials for radio, klystrons for microwaves, atomic inner shells for X-rays, and nuclei for gamma rays.

46. The mass of proton, neutron and helium nucleus are respectively $1.0073\ u, 1.0087\ u$ and $4.0015\ u$. The binding energy of helium nucleus is : [4 marks]

(A) 28.4 MeV

(B) 56.8 MeV

(C) 7.1 MeV

(D) 14.2 MeV

Correct Answer: (A) 28.4 MeV

Solution:

Concept:

- A helium nucleus contains two protons and two neutrons. Its mass defect is $\Delta m = 2m_p + 2m_n - M_{\text{He}}$.
- The total nuclear binding energy is $B = \Delta m c^2$, with $1\,\text{u}\,c^2 \simeq 931.5\,\text{MeV}$.

Step 1: Find the mass of the separated nucleons Use the nuclear mass given in the question, so no electron masses need to be added or subtracted. The two free protons and two free neutrons would have total mass

$$2(1.0073) + 2(1.0087) = 2.0146 + 2.0174 = 4.0320\,\text{u}.$$

The actual nucleus is lighter than these separated particles. The difference represents energy released when the nucleus forms, or equivalently the energy required to separate it into its constituent nucleons.

Step 2: Convert the mass defect to energy Subtract the measured helium nuclear mass:

$$\Delta m = 4.0320 - 4.0015 = 0.0305\,\text{u}.$$

Thus

$$B = 0.0305 \times 931.5\,\text{MeV} = 28.41075\,\text{MeV} \simeq 28.4\,\text{MeV}.$$

Option (C), 7.1 MeV, is approximately the binding energy per nucleon, obtained by dividing this total by four; the question asks for the whole nucleus. Option (D) is half the total and option (B) is twice the total, so neither is the mass-defect energy for this nucleus. Keeping the distinction between total and per-nucleon binding energy resolves the choices.

Step 3: Conclusion The binding energy of the helium nucleus is $\boxed{28.4\,\text{MeV}}$. Hence, the correct option is (A).

Quick Tip to Solve: Count the nucleons first, then subtract the nuclear mass; divide by the mass number only when energy per nucleon is requested.

47. $\left(P+\frac{a}{V^2}\right)(V-b)=RT$ **represents the equation of state of some gases. Where** P **is the pressure,** V **is the volume,** T **is the temperature and** a, b, R **are the constants. The physical quantity, which has dimensional formula as that of** $\frac{b^2}{a}$**, will be:** [4 marks]

(A) Energy density

(B) Bulk modulus

(C) Modulus of rigidity

(D) Compressibility

Correct Answer: (D) Compressibility

Solution:

Concept:

- Only quantities with the same dimensions can be added or subtracted. Therefore $[a/V^2]=[P]$ and $[b]=[V]$.
- Compressibility is fractional volume change per unit pressure, so its dimensions are those of inverse pressure; elastic moduli have pressure dimensions.

Step 1: Extract the dimensions of the two constants From the pressure bracket of $(P+a/V^2)(V-b)=RT$,

$$[a]=[P][V]^2.$$

From the volume bracket, $[b]=[V]$, so $[b^2]=[V]^2$. The constants are therefore constrained by dimensional homogeneity of each bracket separately. It is unnecessary to determine the dimensions of R or introduce a temperature unit, because these dimensions cancel before the right-hand side is needed.

Step 2: Simplify the ratio and identify the quantity Dividing the dimensions gives

$$\left[\frac{b^2}{a}\right]=\frac{[V]^2}{[P][V]^2}=[P]^{-1}=M^{-1}LT^2.$$

The bulk modulus is $K=-V\,dP/dV$, while compressibility is $\kappa=-(1/V)\,dV/dP=1/K$. Thus compressibility has exactly the required dimensions. Energy density is energy per volume, $ML^2T^{-2}/L^3=ML^{-1}T^{-2}$, the same as pressure. Bulk modulus and modulus of rigidity are stress divided by dimensionless strain, so they also have pressure dimensions. Consequently options (A), (B) and (C) give the reciprocal of the required dimensions, leaving (D).

Step 3: Conclusion The dimensional formula is $M^{-1}LT^2$, that of compressibility. Hence, the correct option is (D).

Quick Tip to Solve: For van der Waals type equations, obtain each constant from the bracket in which it appears before simplifying the requested ratio.

48. The average kinetic energy of a molecule of the gas is [4 marks]

(A) proportional to volume

(B) dependent on the nature of the gas

(C) proportional to absolute temperature

(D) proportional to pressure

Correct Answer: (C) proportional to absolute temperature

Solution:

Concept:

- The mean translational kinetic energy of one molecule of an ideal gas is $\langle K\rangle = \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}k_BT$.
- Here T is absolute temperature in kelvin and k_B is the universal Boltzmann constant. Each translational degree of freedom contributes $\frac{1}{2}k_BT$.

Step 1: Relate molecular motion to temperature A molecule has three independent translational velocity components. Adding their average kinetic energies gives

$$\langle K\rangle = \frac{1}{2}k_BT + \frac{1}{2}k_BT + \frac{1}{2}k_BT = \frac{3}{2}k_BT.$$

Thus doubling the absolute temperature doubles the mean translational kinetic energy. The relevant temperature is measured from absolute zero, so a numerical proportionality to Celsius temperature would be incorrect. This is a statement about the average over many molecules, not a claim that every molecule has the same instantaneous speed.

Step 2: Eliminate volume, gas identity and pressure The formula contains neither molecular mass nor molecular identity. At a fixed temperature a heavier molecule has a smaller mean-square speed, so the product $m\langle v^2\rangle$ has the same value for all ideal gases; this rejects (B). Volume and pressure can change during an isothermal expansion or compression without changing $\langle K\rangle$, rejecting (A) and (D). Pressure determines energy only when other conditions such as number density are fixed, so it is not the fundamental unconditional proportionality. The direct universal dependence in the kinetic theory formula is on absolute temperature, as stated in (C).

Step 3: Conclusion The average translational kinetic energy is proportional to absolute temperature: $\langle K\rangle = \frac{3}{2}k_BT$. Hence, the correct option is (C).

Quick Tip to Solve: At equal kelvin temperature, all ideal gases have the same mean translational kinetic energy even though their molecular speeds differ.

49. A mercury drop of radius 10^{-3} m is broken into 125 equal size droplets. Surface tension of mercury is $0.45\ \mathrm{Nm}^{-1}$. The gain in surface energy is : [4 marks]

(A) 28×10^{-5} J

(B) 17.5×10^{-5} J

(C) 5×10^{-5} J

(D) 2.26×10^{-5} J

Correct Answer: (D) 2.26×10^{-5} J

Solution:

Concept:

- Breaking a liquid drop conserves volume: $\frac{4}{3}\pi R^3 = N\frac{4}{3}\pi r^3$, where R and r are the initial and final radii.
- A liquid drop has one liquid surface, so its surface energy is $E = SA$, where S is surface tension and A is surface area.

Step 1: Find the radius of each small droplet With $N = 125 = 5^3$, volume conservation gives

$$r = \frac{R}{N^{1/3}} = \frac{10^{-3}}{5} = 2 \times 10^{-4}\,\mathrm{m}.$$

The radius scales with the cube root of the number of droplets, not with the number itself. Thus the smaller droplets retain the same combined volume as the original drop while presenting a larger total surface area.

Step 2: Calculate the area increase and the energy gain The final area is

$$A_f = 125(4\pi r^2) = 125\left(4\pi\frac{R^2}{25}\right) = 5(4\pi R^2).$$

Subtracting the original area gives

$$\Delta A = 4(4\pi R^2) = 16\pi \times 10^{-6}\,\mathrm{m}^2.$$

Therefore

$$\Delta E = S\Delta A = 0.45 \times 16\pi \times 10^{-6} \simeq 2.26 \times 10^{-5}\,\mathrm{J}.$$

The energy increases because additional interface is created. Values in (A) and (B) are much larger than this computed increase, while (C) also exceeds it by more than a factor of two. No factor of two for two surfaces is used: mercury droplets are liquid drops, not soap bubbles with inner and outer films.

Step 3: Conclusion The gain in surface energy is $\boxed{2.26 \times 10^{-5}\,\mathrm{J}}$. Hence, the correct option is (D).

Quick Tip to Solve: For N equal droplets, the final area is $N^{1/3}$ times the initial area; multiply the area increase by surface tension.

50. Which of the following frequencies does not belong to FM broadcast ? [4 marks]

(A) 106 MHz

(B) 89 MHz

(C) 64 MHz

(D) 99 MHz

Correct Answer: (C) 64 MHz

Solution:

Concept:

- Frequency modulation (FM) encodes an audio signal through changes in the instantaneous frequency of a high-frequency carrier; the carrier amplitude is ideally constant.
- The FM radio broadcast band used in this question is 88–108 MHz. A carrier belongs to this band only if its frequency lies between these limits.

Step 1: Identify the relevant broadcast interval FM is a method of modulation, whereas FM broadcast refers to the assigned radio-service frequency band. Thus the question asks for the carrier frequency outside the broadcast allocation, rather than the audio frequency being transmitted. We compare every listed carrier with the lower limit of 88 MHz and the upper limit of 108 MHz.

Step 2: Check all four frequencies For option (A), $88 < 106 < 108$, so 106 MHz lies inside the FM broadcast band. For option (B), 89 MHz is just above the lower limit and also belongs to this band. Option (D), 99 MHz, lies comfortably between the limits. In contrast, option (C) gives $64\,\text{MHz} < 88\,\text{MHz}$; it is below the band and is the only excluded frequency. The fact that all four values are expressed in MHz does not make all four FM broadcast carriers.

Step 3: Conclusion The frequency that does not belong to the FM broadcast band is 64 MHz. Hence, the correct option is (C).

Quick Tip to Solve: Recall the FM broadcast limits first, then reject any listed carrier below 88 MHz or above 108 MHz.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Two equal positive point charges are separated by a distance $2a$. The distance of a point from the centre of the line joining two charges on the equatorial line (perpendicular bisector) at which force experienced by a test charge q_0 becomes maximum is $\frac{a}{\sqrt{x}}$. The value of x is _____. [4 marks]

Solution:

Concept:

- The electric fields of equal positive charges have cancelling components parallel to the line joining them and adding components along its perpendicular bisector.
- The force magnitude on a fixed test charge is $F = |q_0|E$. Hence maximizing the force is equivalent to maximizing the resultant field magnitude.

Step 1: Write the force as a function of distance Let each charge be Q, let $k = 1/(4\pi\varepsilon_0)$, and take a point a distance $y \geq 0$ from the midpoint along the perpendicular bisector. Its distance from either charge is $r = \sqrt{a^2 + y^2}$. The component of either field along the bisector is $(kQ/r^2)(y/r)$. Adding these equal components gives

$$E(y) = \frac{2kQy}{(a^2 + y^2)^{3/2}}, \qquad F(y) = \frac{2kQ|q_0|y}{(a^2 + y^2)^{3/2}}.$$

The joining-line components cancel by symmetry; adding the full field magnitudes would therefore be incorrect.

Step 2: Differentiate and identify the maximum The multiplicative factor $2kQ|q_0|$ is independent of y. Differentiating the remaining factor gives

$$\frac{dF}{dy} = 2kQ|q_0|\frac{a^2 - 2y^2}{(a^2 + y^2)^{5/2}}.$$

Thus the stationary point satisfies $a^2 - 2y^2 = 0$, or $y = a/\sqrt{2}$. The derivative is positive before this point and negative after it; also the force vanishes at the midpoint and tends to zero at infinity. The stationary value is consequently the maximum, rather than a minimum.

Step 3: Conclusion The required distance is $a/\sqrt{2}$. Comparing it with $a/\sqrt{x}$ gives $\boxed{x = 2}$, a dimensionless number.

Quick Tip to Solve: On the perpendicular bisector, include the projection factor y/r before differentiating the resultant field.

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52. A charge particle of $2\ \mu\text{C}$ accelerated by a potential difference of $100\ \text{V}$ enters a region of uniform magnetic field of magnitude $4\ \text{mT}$ at right angle to the direction of field. The charge particle completes semicircle of radius $3\ \text{cm}$ inside magnetic field. The mass of the charge particle is _____ $\times 10^{-18}$ kg

[4 marks]

Solution:

Concept:

- Acceleration from rest through a potential difference V gives kinetic energy $qV = \frac{1}{2}mv^2$, with q the charge magnitude.
- For perpendicular entry into a uniform magnetic field, the magnetic force supplies centripetal force: $qvB = mv^2/r$. The field changes direction, not speed.

Step 1: Eliminate the unknown speed The circular-motion relation gives $mv = qBr$, so $v = qBr/m$. Substitution in the energy relation eliminates a speed that is not supplied:

$$qV = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2r^2}{2m}.$$

Therefore

$$m = \frac{qB^2r^2}{2V}.$$

The semicircular path has the same radius relation as a complete circle. Its arc length πr is not the radius and is not needed in this calculation.

Step 2: Convert all data and calculate the mass Use SI units consistently:

$$q = 2 \times 10^{-6}\,\text{C}, \quad B = 4 \times 10^{-3}\,\text{T}, \quad r = 3 \times 10^{-2}\,\text{m}, \quad V = 100\,\text{V}.$$

Then

$$\begin{aligned} m &= \frac{(2 \times 10^{-6})(4 \times 10^{-3})^2(3 \times 10^{-2})^2}{2(100)} \\ &= \frac{(2 \times 10^{-6})(16 \times 10^{-6})(9 \times 10^{-4})}{200} \\ &= 1.44 \times 10^{-16}\,\text{kg} = 144 \times 10^{-18}\,\text{kg}. \end{aligned}$$

Both the field strength and radius are squared, so their metric conversion factors must be squared as well.

Step 3: Conclusion The particle mass is $\boxed{144 \times 10^{-18}\,\text{kg}}$. The numerical value to enter is $\boxed{144}$.

Quick Tip to Solve: Combine $qV = mv^2/2$ with $mv = qBr$ before inserting numbers; this directly gives $m = qB^2r^2/(2V)$.

53. A light of energy 12.75 eV is incident on a hydrogen atom in its ground state. The atom absorbs the radiation and reaches to one of its excited states. The angular momentum of the atom in the excited state is $\frac{x}{\pi} \times 10^{-17}$ eVs. The value of x is _____ (use $h = 4.14 \times 10^{-15}$ eVs, $c = 3 \times 10^8$ ms^{-1}). [4 marks]

Solution:

Concept:

- In hydrogen, the Bohr energy levels are $E_n = -13.6/n^2\,\text{eV}$. The absorbed photon supplies the difference between final and initial atomic energies.
- The quantized orbital angular momentum in the Bohr model is $L_n = n\hbar = nh/(2\pi)$. The same final principal quantum number must be used in both formulae.

Step 1: Find the excited principal quantum number The initial ground state has energy $E_1 = -13.6\,\text{eV}$. Absorption raises the atom by $12.75\,\text{eV}$, giving

$$E_n = -13.6 + 12.75 = -0.85\,\text{eV}.$$

Equating this with the hydrogen-level energy yields

$$-\frac{13.6}{n^2} = -0.85, \qquad n^2 = \frac{13.6}{0.85} = 16, \qquad n = 4.$$

The positive integer is required because n labels an energy level. The photon energy is less than the ground-state ionization energy, so this is excitation to a bound state rather than ionization.

Step 2: Calculate angular momentum in the requested form Using the supplied Planck constant in eV seconds avoids an unnecessary conversion to joule seconds:

$$L_4 = \frac{4h}{2\pi} = \frac{2(4.14 \times 10^{-15})}{\pi} = \frac{8.28 \times 10^{-15}}{\pi}\,\text{eV s}.$$

The printed form uses 10^{-17}, so shift the coefficient by two powers of ten:

$$L_4 = \frac{828}{\pi} \times 10^{-17}\,\text{eV s}.$$

The speed of light is not needed: no photon wavelength or frequency is requested, and its energy is already provided.

Step 3: Conclusion The excited state is $n = 4$, and its orbital angular momentum is $(828/\pi) \times 10^{-17}\,\text{eV s}$. Hence $\boxed{x = 828}$, dimensionless.

Quick Tip to Solve: First subtract the photon energy from the ground-state binding energy to identify n; only then use $L = nh/(2\pi)$.

---

54. A thin cylindrical rod of length 10 cm is placed horizontally on the principle axis of a concave mirror of focal length 20 cm. The rod is placed in a such a way that mid point of the rod is at 40 cm from the pole of mirror. The length of the image formed by the mirror will be $\frac{x}{3}$ cm. The value of x is _____.

[4 marks]

Solution:

Concept:

- For a spherical mirror, $1/f = 1/u + 1/v$ with Cartesian signed distances. A concave mirror facing the object has $f < 0$, and a real object in front has $u < 0$.
- A rod along the principal axis has longitudinal image length equal to the separation of its endpoint images. Transverse magnification cannot simply multiply the rod length.

Step 1: Locate both endpoints and their images The midpoint is $40\,\text{cm}$ from the mirror and half the rod length is $5\,\text{cm}$. Thus the near and far endpoints have signed object distances $u_1 = -35\,\text{cm}$ and $u_2 = -45\,\text{cm}$, with $f = -20\,\text{cm}$. Using $v = fu/(u - f)$,

$$v_1 = \frac{(-20)(-35)}{-35 + 20} = -\frac{140}{3}\,\text{cm}, \qquad v_2 = \frac{(-20)(-45)}{-45 + 20} = -36\,\text{cm}.$$

Both endpoints lie beyond the focus, so their images are real and in front of the mirror.

Step 2: Find the separation of the image endpoints All points of the axial rod form images on the axis between these endpoints. The longitudinal mapping is continuous here because the entire rod

lies beyond the focal point. Therefore the image length is the absolute difference

$$\ell_i = |v_1 - v_2| = \left| -\frac{140}{3} + 36 \right| = \frac{32}{3}\,\text{cm}.$$

Although the midpoint lies at the centre of curvature, its transverse magnification of magnitude one does not give the image length of this finite axial rod. Different points of the rod have different object and image distances.

Step 3: Conclusion The image length is $\boxed{32/3\,\text{cm}}$. Comparing with $x/3\,\text{cm}$ gives $\boxed{x = 32}$, dimensionless.

Quick Tip to Solve: For a rod along the axis, apply the mirror equation separately at the two ends and subtract the image distances.

---

55. A small particle moves to position $5\hat{i} - 2\hat{j} + \hat{k}$ from its initial position $2\hat{i} + 3\hat{j} - 4\hat{k}$ under the action of force $5\hat{i} + 2\hat{j} + 7\hat{k}$ N. The value of work done will be _____ J. [4 marks]

Solution:

Concept:

- The work done by a constant force is the scalar product $W = \vec{F} \cdot \Delta\vec{r}$, where $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$ is displacement.
- In a Cartesian orthonormal basis, equal unit vectors have scalar product one and different unit vectors have scalar product zero. Thus work is the sum of matching component products.

Step 1: Subtract the initial position from the final position The force acts during a change of position, so first calculate the vector displacement rather than using either position vector by itself. With the position coordinates in metres,

$$\begin{aligned}\Delta\vec{r} &= (5\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + 3\hat{j} - 4\hat{k}) \\ &= (5-2)\hat{i} + (-2-3)\hat{j} + (1-(-4))\hat{k} \\ &= (3\hat{i} - 5\hat{j} + 5\hat{k})\,\text{m}.\end{aligned}$$

The displacement in the y-direction is negative, even though the corresponding force component is positive; this part of the force therefore does negative work.

Step 2: Take the scalar product with the force For $\vec{F} = (5\hat{i} + 2\hat{j} + 7\hat{k})\,\text{N}$,

$$\begin{aligned}W &= (5)(3) + (2)(-5) + (7)(5) \\ &= 15 - 10 + 35 \\ &= 40\,\text{N m} = 40\,\text{J}.\end{aligned}$$

The x- and z-components contribute positive work, while the y-component subtracts from that total. Work is a scalar, so no unit vector remains in the answer. Multiplying force magnitude by displacement magnitude without a cosine factor would incorrectly assume the two vectors are parallel.

Step 3: Conclusion The work done by the stated force is $\boxed{40\,\text{J}}$. The required numerical entry is $\boxed{40}$.

Quick Tip to Solve: Write $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$ component by component first; then multiply only corresponding components for the dot product.

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56. A series LCR circuit is connected to an ac source of $220\ \text{V}, 50\ \text{Hz}$. The circuit contain a resistance $\text{R} = 100\ \Omega$ and an inductor of inductive reactance $\text{X}_\text{L} = 79.6\ \Omega$. The capacitance of the capacitor needed to maximize the average rate at which energy is supplied will be _____ μF. [4 marks]

Solution:

Concept:

- The impedance of a series LCR circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $X_C = 1/(\omega C)$ and $\omega = 2\pi f$.
- For a fixed rms source voltage, average input power is $P = V_{\text{rms}}^2 R/Z^2$. It is maximum when the net reactance vanishes, so the circuit is in series resonance.

Step 1: Apply the condition for maximum average power The given resistance and rms voltage remain fixed while the capacitance is varied. Consequently

$$P = \frac{V_{\text{rms}}^2 R}{R^2 + (X_L - X_C)^2}$$

has its largest value when the nonnegative squared term is zero. Thus

$$X_C = X_L = 79.6\,\Omega.$$

At this point the impedance is simply $R = 100\,\Omega$, the current is in phase with the source voltage and the power factor is one. Maximizing instantaneous power at a particular time is not the same task; the question concerns the cycle-average rate of energy supply.

Step 2: Calculate the resonant capacitance With $f = 50\,\text{Hz}$, the angular frequency is $\omega = 2\pi(50) = 100\pi\ \text{rad}\,\text{s}^{-1}$. The capacitive-reactance condition gives

$$C = \frac{1}{\omega X_C} = \frac{1}{(100\pi)(79.6)}\,\text{F} \approx 3.999 \times 10^{-5}\,\text{F}.$$

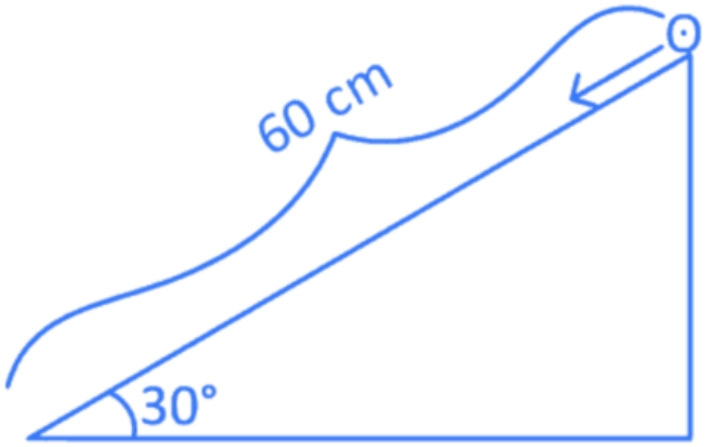
Since $1\,\mu\text{F} = 10^{-6}\,\text{F}$, this is approximately $39.99\,\mu\text{F}$, which gives $40\,\mu\text{F}$ to the required numerical precision. As a check, the maximum power itself would be $220^2/100 = 484\,\text{W}$; the voltage sets this power but does not set the resonant capacitance.

Step 3: Conclusion The required capacitance is $\boxed{40\,\mu\text{F}}$, so the numerical entry is $\boxed{40}$.

Quick Tip to Solve: For maximum average power in a series LCR circuit, set $X_C = X_L$ and use $C = 1/(2\pi f X_L)$.

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57. A solid cylinder is released from rest from the top of an inclined plane of inclination 30° and length 60 cm. If the cylinder rolls without slipping, its speed upon reaching the bottom of the inclined plane is _____ ms^{-1}. (Given $\mathrm{g} = 10\ \mathrm{ms}^{-2}$) [4 marks]



Solution:

Concept:

- For rolling without slipping, the speed of the centre of mass and angular speed satisfy $v = R\omega$.
- The moment of inertia of a uniform solid cylinder about its symmetry axis is $I = \frac{1}{2}mR^2$. Mechanical energy is conserved on a fixed rough incline when there is no slipping.

Step 1: Find the vertical drop The given 60 cm is the length along the inclined plane, rather than its vertical height. Resolving this length vertically gives

$$h = L\sin 30^\circ = 0.60 \times \frac{1}{2} = 0.30\,\mathrm{m}.$$

The cylinder starts from rest, so both its initial translational and rotational kinetic energies vanish.

Step 2: Include both forms of kinetic energy At the bottom, the lost gravitational potential energy supplies translation of the centre and rotation about it. Static friction enforces rolling, but does no work at the instantaneous point of contact on the fixed incline. Thus

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2.$$

Substitute the height and the specified gravitational acceleration:

$$v^2 = \frac{4gh}{3} = \frac{4 \times 10 \times 0.30}{3} = 4\,\mathrm{m}^2\mathrm{s}^{-2}.$$

Taking the positive root is appropriate because speed is a magnitude. The mass and radius cancel; neither is required to determine the answer.

Step 3: Conclusion The speed at the bottom is $\boxed{2\,\mathrm{m\,s}^{-1}}$. The numerical entry is 2.

Quick Tip to Solve: For a rolling solid cylinder, use $v^2 = 4gh/3$; first convert incline length into vertical height.

58. The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25% more than the potential energy is: _____ cm [4 marks]

Solution:

Concept:

- For SHM with force constant k, total energy is $E = \frac{1}{2}kA^2$, where A is amplitude.
- At displacement x from equilibrium, $U = \frac{1}{2}kx^2$ and $K = \frac{1}{2}k(A^2 - x^2)$.

Step 1: Translate the percentage condition Kinetic energy being 25 percent more than potential energy means

$$K = U + 0.25U = 1.25U = \frac{5}{4}U.$$

It does not mean that kinetic energy is 25 percent of the total energy. Adding the two energies gives $E = K + U = \frac{9}{4}U$, so the potential energy is $4/9$ of the total.

Step 2: Solve for the displacement magnitude Use the expressions for kinetic and potential energies at the same position. Their common factor cancels:

$$\frac{K}{U} = \frac{A^2 - x^2}{x^2} = \frac{5}{4}.$$

Consequently,

$$4A^2 - 4x^2 = 5x^2, \qquad x^2 = \frac{4}{9}A^2.$$

With $A = 3\,\text{cm}$,

$$|x| = \frac{2}{3}A = \frac{2}{3} \times 3 = 2\,\text{cm}.$$

As a check, $A^2 - x^2 = 9 - 4 = 5\,\text{cm}^2$, whereas $x^2 = 4\,\text{cm}^2$. Their ratio is exactly $5/4$, as required. Both sides of equilibrium give the same energies because each expression depends on the square of displacement.

Step 3: Conclusion The displacement magnitude is $\boxed{2\,\text{cm}}$, so the numerical entry is 2. The two positions are $x = +2\,\text{cm}$ and $x = -2\,\text{cm}$.

Quick Tip to Solve: If $K = rU$ in SHM, use $|x| = A/\sqrt{1+r}$; here $r = 1.25$.

59. A certain pressure 'P' is applied to 1 litre of water and 2 litre of a liquid separately. Water gets compressed to 0.01% whereas the liquid gets compressed to 0.03%. The ratio of Bulk modulus of water to that of the liquid is $\frac{3}{x}$. The value of x is _____. [4 marks]

Solution:

Concept:

- Bulk modulus is pressure increase divided by fractional volume compression: $B = P/(\Delta V/V)$, using positive compression magnitudes.
- For equal applied pressures, bulk moduli are inversely proportional to the corresponding fractional volume changes.

Step 1: Convert percentages into fractional compressions The stated changes are already percentages of the respective initial volumes. Therefore the relevant strains are

$$\epsilon_w = \frac{0.01}{100} = 10^{-4}, \qquad \epsilon_l = \frac{0.03}{100} = 3 \times 10^{-4}.$$

The water and liquid do not undergo the same fractional compression: the liquid compresses three times as much under the same pressure.

Step 2: Form the modulus ratio Using the common pressure P,

$$B_w = \frac{P}{10^{-4}}, \qquad B_l = \frac{P}{3 \times 10^{-4}}.$$

Divide these expressions so that the unspecified pressure cancels:

$$\frac{B_w}{B_l} = \frac{3 \times 10^{-4}}{10^{-4}} = 3.$$

The initial volumes of 1 litre and 2 litres are unnecessary once fractional changes have been supplied. For example, the absolute compressions would be 0.0001 litre and 0.0006 litre; dividing each by its own initial volume recovers the strains used above.

Step 3: Compare with the requested expression The question writes the ratio as $3/x$. Equating this to the calculated value gives

$$\frac{3}{x} = 3, \qquad x = 1.$$

This is also physically consistent: the more resistant water has the larger bulk modulus, and the requested ratio is greater than one.

Step 4: Conclusion The ratio is $B_w/B_l = 3$, hence $\boxed{x = 1}$. The requested parameter is dimensionless.

Quick Tip to Solve: Use percentage compression directly in an equal-pressure modulus ratio; initial volumes cancel after fractional strain is formed.

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60. In an experiment to find emf of a cell using potentiometer, the length of null point for a cell of emf 1.5 V is found to be 60 cm. If this cell is replaced by another cell of emf E, the length-of null point increases by 40 cm. The value of E is $\frac{x}{10}V$. The value of x is _____. [4 marks]

Solution:

Concept:

- For a uniform potentiometer wire carrying a fixed current, its potential gradient k is constant and balancing emf satisfies $\mathcal{E} = k\ell$.
- At null deflection, no current is drawn from the test cell, so its emf equals the potential difference across its balancing wire length.

Step 1: Find the new balancing length The first cell balances at $60\,\text{cm}$. The replacement increases

this length by $40\,\text{cm}$, so its total balancing length is

$$\ell_2 = 60 + 40 = 100\,\text{cm}.$$

The increase alone is not the new null length. Since only the test cell is replaced, the potentiometer wire and driving conditions retain the same potential gradient.

Step 2: Compare emfs using the common gradient The gradient from the first observation is

$$k = \frac{1.5}{60} = 0.025\,\text{V cm}^{-1}.$$

The second emf is therefore

$$E = k\ell_2 = 0.025 \times 100 = 2.5\,\text{V}.$$

Equivalently, the ratio method gives $E/1.5 = 100/60$. The larger balancing length correctly corresponds to a larger emf, providing a useful check on the direction of the ratio.

Step 3: Extract the parameter requested The problem expresses the new emf as $E = (x/10)\,\text{V}$. Thus

$$\frac{x}{10} = 2.5, \qquad x = 25.$$

The emf itself carries volts, while the numerical coefficient x is dimensionless. No internal resistance correction is required at the balance point because the test cell supplies no current through the detector branch.

Step 4: Conclusion The new emf is $2.5\,\text{V}$, and the requested numerical entry is $\boxed{x = 25}$.

Quick Tip to Solve: In potentiometer comparisons, add a stated increase to the original length before applying $E_2/E_1 = \ell_2/\ell_1$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Which of the following complex will show largest splitting of d-orbitals? [4 marks]

(A) $[Fe(C_2O_4)_3]^{3-}$

(B) $[Fe(CN)_6]^{3-}$

(C) $[Fe(NH_3)_6]^{3+}$

(D) $[FeF_6]^{3-}$

Correct Answer: (B) $[Fe(CN)_6]^{3-}$

Solution:

Concept:

- In an octahedral complex, ligands split the five d orbitals into lower t_{2g} and upper e_g sets, separated by Δ_o.
- For a fixed metal ion, oxidation state and coordination geometry, ligand field strength is the main comparison controlling the splitting.

Step 1: Identify the common metal oxidation state For $[Fe(C_2O_4)_3]^{3-}$, three oxalate ligands contribute charge -6, so iron is $+3$. In $[Fe(CN)_6]^{3-}$ and $[FeF_6]^{3-}$, the six singly charged ligands also leave iron in oxidation state $+3$. Ammonia is neutral, making the iron state in $[Fe(NH_3)_6]^{3+}$ again $+3$. Thus all four compare octahedral complexes of the same Fe^{3+} ion.

Step 2: Apply the spectrochemical series For these ligands, the field-strength order is

$$F^- < C_2O_4{}^{2-} < NH_3 < CN^-.$$

Cyanide is a strong-field ligand and gives the greatest separation of the d-orbital energy levels. The three bidentate oxalates provide six donor atoms, so their number of ligand molecules does not imply a different coordination number. Fluoride in option D is weak-field, oxalate in option A is weaker than ammonia and cyanide, and ammonia in option C is not as strong-field as cyanide. Therefore none of A, C or D can give the largest splitting in this comparison.

Step 3: Conclusion $[Fe(CN)_6]^{3-}$ has the largest d-orbital splitting. Hence, the correct option is (B).

Quick Tip to Solve: First equalise metal oxidation state and geometry; then rank the ligands using the spectrochemical series.

62. Which of the following are the example of double salt?

A. $FeSO_4 \cdot (NH_4)_2\,SO_4 \cdot 6H_2O$

B. $CuSO_4 \cdot 4NH_3 \cdot H_2O$

C. $K_2SO_4 \cdot Al_2\,(SO_4)_3 \cdot 24H_2O$

D. $Fe(CN)_2 \cdot 4KCN$

Choose the correct answer : [4 marks]

(A) A, B and D only

(B) B and D only

(C) A and B only

(D) A and C only

Correct Answer: (D) A and C only

Solution:

Concept:

- A double salt dissociates in water into all its constituent simple ions and loses the separate identity it had in the crystalline solid.
- A coordination compound retains a complex ion in solution; ligands inside the coordination sphere do not appear simply as all their free constituent ions.

Step 1: Classify the two double salts Compound A is Mohr's salt, $(NH_4)_2Fe(SO_4)_2 \cdot 6\,H_2O$. Its aqueous solution contains $NH_4{}^+$, Fe^{2+} and $SO_4{}^{2-}$ ions. Compound C is potash alum, conventionally written $KAl(SO_4)_2 \cdot 12\,H_2O$; the formula in the question is twice this formula unit. It dissociates into potassium, aluminium and sulfate ions. Both are standard examples of double salts.

Step 2: Distinguish the complex salts Compound B, $CuSO_4 \cdot 4\,NH_3 \cdot H_2O$, contains the tetraamminecopper(II) complex with coordinated ammonia. It does not behave as a simple mixture releasing all four ammonia ligands as independent ions. Compound D, $Fe(CN)_2 \cdot 4\,KCN$, corresponds to $K_4[Fe(CN)_6]$. On dissolution it gives potassium ions and the hexacyanidoferrate(II) complex ion rather than free iron and six free cyanide ions. Thus B and D are complex salts, not double salts.

Step 3: Match the statement combination The complete correct list is A and C only. Option A includes B and D and omits C; option B selects just the two complex salts; option C incorrectly includes B and omits C. Only the fourth answer choice selects both double salts and excludes both coordination compounds.

Step 4: Conclusion The double salts are A (Mohr's salt) and C (potash alum). Hence, the correct option is (D).

Quick Tip to Solve: Ask whether the dissolved substance retains a complex ion: Mohr's salt and alum dissociate into simple constituent ions.

63. Highest oxidation state of Mn is exhibited in Mn_2O_7. The correct statements about Mn_2O_7 are

(A) Mn is tetrahedrally surrounded by oxygen atoms.

(B) Mn is octahedrally surrounded by oxygen atoms.

(C) Contains Mn-O-Mn bridge.

(D) Contains Mn-Mn bond.

Choose the correct answer from the options given below : [4 marks]

(A) A and C only

(B) A and D only

(C) B and C only

(D) B and D only

Correct Answer: (A) A and C only

Solution:

Concept:

- In Mn_2O_7, oxygen has oxidation state -2, giving each manganese oxidation state $+7$.
- The molecular structure consists of two tetrahedral MnO_4 units sharing one bridging oxygen, with three terminal oxygens on each manganese.

Step 1: Determine the local geometry Charge balance in the neutral oxide gives

$$2z + 7(-2) = 0, \qquad z = +7.$$

The connectivity may be written as $O_3Mn{-}O{-}MnO_3$. Each Mn is bonded to three terminal oxygen atoms and one bridging oxygen atom. Four oxygen neighbours surround each metal in an approximately tetrahedral arrangement, so statement A is true. Octahedral coordination would require six neighbours and therefore statement B is false.

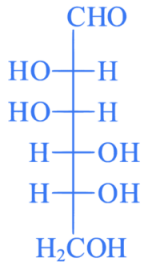
Step 2: Check the bridge and metal-metal bond The central oxygen atom is attached to both manganese atoms, creating a genuine $\mathrm{Mn-O-Mn}$ bridge. Hence statement C is true. The two manganese atoms are connected through this oxygen; there is no direct $\mathrm{Mn-Mn}$ bond in the molecular structure, so statement D is false. Counting oxygens also verifies the bridge: the six terminal oxygens plus one shared oxygen give the required total of seven.

Step 3: Eliminate the incorrect combinations The true statements are precisely A and C, corresponding to the first answer choice. The second choice introduces the nonexistent direct metal-metal bond. The third replaces the tetrahedral description by an octahedral one. The fourth includes both false statements and excludes both true statements. Oxidation state by itself is not a coordination number; the structural connectivity decides the geometry.

Step 4: Conclusion Statements A and C are correct: each Mn is tetrahedrally surrounded and a $\mathrm{Mn-O-Mn}$ bridge is present. Hence, the correct option is (A).

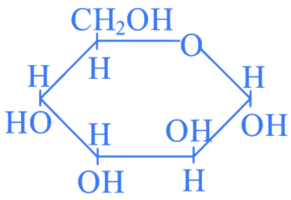
Quick Tip to Solve: For $\mathrm{Mn_2O_7}$, remember two tetrahedra sharing one oxygen: four neighbours per Mn, seven oxygens overall.

64. The correct representation in six membered pyranose form for the following sugar [X] is [4 marks]

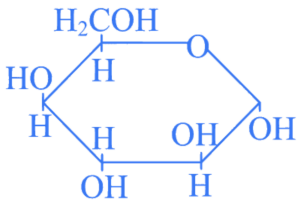


Sugar [X]

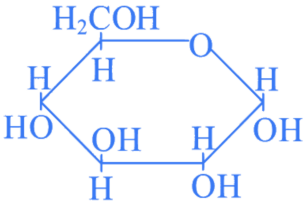
(A) Pyranose structure with C2-OH up, C3-OH down and C4-OH down.



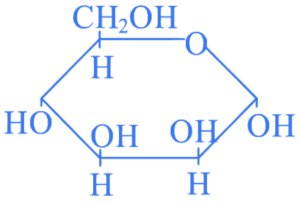
(B) Pyranose structure with C2-OH up, C3-OH down and C4-OH up.



(C) Pyranose structure with C2-OH down, C3-OH up and C4-OH down.

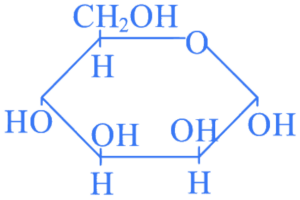


(D) Pyranose structure with C2-OH up, C3-OH up and C4-OH down.



Correct Answer: (D)

Pyranose structure with C2-OH up, C3-OH up and C4-OH down.



Solution:

Concept:

- A Fischer projection specifies the configurations of the stereogenic centres. On converting a D-aldose to its conventional Haworth projection, groups on the right point down and groups on the left point up.
- An aldohexose forms a six-membered pyranose hemiacetal when the hydroxyl oxygen at C5 bonds to the aldehyde carbon C1; the terminal CH_2OH group of a D-sugar points up.

Step 1: Read the Fischer projection The aldehyde carbon at the top is C1. Reading down the chain, the hydroxyl groups at C2 and C3 are on the left, whereas those at C4 and C5 are on the right. The C5 configuration identifies the sugar as a D-sugar. Thus the configuration pattern to preserve at C2, C3 and C4 is left, left, right; ring closure must not invert any of these three centres.

Step 2: Convert the substituent directions Place the ring oxygen at the upper right and the anomeric carbon C1 at the right-hand corner, as in the options. The C2 hydroxyl must point up, the C3 hydroxyl must also point up, and the C4 hydroxyl must point down. The C5 hydroxyl provides the ring oxygen, leaving the CH_2OH substituent above the ring. The anomeric hydroxyl is newly generated by hemiacetal formation; the drawn choices show the downward orientation.

Step 3: Compare every structure Option (A) places the C3 hydroxyl down and therefore fails to preserve C3. Option (B) has both C3 and C4 hydroxyl orientations wrong. Option (C) puts the C2 hydroxyl down. Only option (D) has the required up, up, down sequence at C2, C3 and C4, together with the upward CH_2OH group.

Step 4: Conclusion The required six-membered representation is the pyranose structure in (D). Hence, the correct option is (D).

Quick Tip to Solve: Number the Haworth ring from the anomeric carbon before applying the Fischer rule: right goes down and left goes up.

65. But-2-yne is reacted separately with one mole of Hydrogen as shown below :

A. A is more soluble than B.

B. The boiling point & melting point of A are higher and lower than B respectively.

C. A is more polar than B because dipole moment of A is zero.

D. Br_2 adds easily to B than A.

Identify the incorrect statements from the options given below : [4 marks]

$$\underline{B} \xleftarrow[\text{liq } NH_3]{Na} CH_3-C\equiv C-CH_3 \xrightarrow[+H_2]{Pd/C,\ \Delta} \underline{A}$$

(A) A and B only

(B) A, C & D only

(C) B, C & D only

(D) C and D only

Correct Answer: (D) C and D only

Solution:

Concept:

- Catalytic addition of one equivalent of hydrogen to an alkyne gives syn addition, while sodium in liquid ammonia gives anti addition.
- Geometrical isomers differ in dipole moment, intermolecular attraction, crystal packing and relative stability; these differences decide their physical properties and addition behaviour.

Step 1: Identify A and B With the specified one mole of hydrogen, addition on Pd/C places the two hydrogen atoms on the same side of the original triple bond. Product A is cis-but-2-ene. Sodium in liquid ammonia gives the anti-addition product, so B is trans-but-2-ene. The cis isomer has a nonzero resultant dipole moment; in the more symmetrical trans isomer, the bond dipoles cancel.

Step 2: Check statements A and B The greater polarity of cis-but-2-ene makes A relatively more soluble in water than B, so statement A is correct. Its stronger dipolar attractions also give A a higher boiling point. The trans isomer packs more symmetrically and efficiently in a crystal, giving B a higher melting point. Thus A has the higher boiling point and lower melting point, exactly as statement B says.

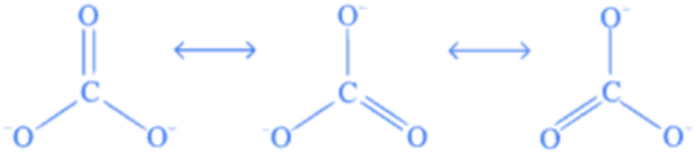
Step 3: Reject the incorrect statements Statement C is internally inconsistent: A is the more polar isomer, but its dipole moment is not zero; zero dipole moment applies to B. Statement D is also incorrect. Cis-but-2-ene is less stable because its methyl groups are on the same side, and it undergoes bromine addition more readily than the trans isomer. Thus C and D, and only these two statements, are incorrect. Options (A), (B) and (C) wrongly reject at least one correct statement A or B.

Step 4: Conclusion The incorrect statements are C and D only. Hence, the correct option is (D).

Quick Tip to Solve: For cis/trans physical properties, separate liquid polarity from crystal packing: cis often boils higher, while trans often melts higher.

66. Resonance in carbonate ion $\left(CO_3^{2-}\right)$ is

Which of the following is true? [4 marks]



(A) It is possible to identify each structure individually by some physical or chemical method.

(B) All these structures are in dynamic equilibrium with each other.

(C) Each structure exists for equal amount of time.

(D) CO_3^{2-} has a single structure i.e., resonance hybrid of the above three structures.

Correct Answer: (D) CO_3^{2-} has a single structure i.e., resonance hybrid of the above three structures.

Solution:

Concept:

- Resonance contributors are alternative Lewis representations of a single species with the same atomic framework; only electron placement changes.
- The actual species is a resonance hybrid with delocalised electrons, rather than a mixture of individually existing contributors.

Step 1: Compare the three contributors Each carbonate contributor has one carbon–oxygen double bond and two carbon–oxygen single bonds, with the two negative formal charges on singly bonded oxygens. The position of the double bond differs among the three drawings. The carbon and oxygen nuclei stay in the same arrangement throughout; the drawings differ only in the distribution of electrons. Because the three oxygen atoms are equivalent, these contributors have equal importance.

Step 2: Describe the actual carbonate ion The actual CO_3^{2-} ion is planar and has three equivalent carbon–oxygen bonds. Delocalisation spreads the pi bonding and the negative charge over all three oxygen atoms. The average carbon–oxygen bond order is $(2+1+1)/3 = 4/3$, intermediate between a single and a double bond. This is a structural property of the single ion at any instant; the ion does not alternate among three different molecular structures.

Step 3: Evaluate the options Option (A) is false because no physical or chemical method can isolate a Lewis contributor as a separate carbonate ion. Option (B) confuses resonance with a chemical equilibrium, which would require different actual species. Option (C) likewise incorrectly assigns an existence time to each contributor. Option (D) correctly states that the ion has a single actual structure, the resonance hybrid of the three representations.

Step 4: Conclusion Carbonate exists as one resonance hybrid with three equivalent C–O bonds. Hence, the correct option is (D).

Quick Tip to Solve: Resonance arrows connect drawings of one species; equilibrium arrows connect different species that can actually exist separately.

67. Match List I with List II

	List I		List II
	Test		Functional group / Class of Compound
A.	Molisch's Test	I.	Peptide
B.	Biuret Test	II.	Carbohydrate
C.	Carbylamine Test	III.	Primary amine
D.	Schiff's Test	IV.	Aldehyde

Choose the correct answer from the options given below : [4 marks]

(A) (A) - III, (B) - IV, (C) - II, (D) - I

(B) (A) - II, (B) - I, (C) - III, (D) - IV

(C) (A) - III, (B) - IV, (C) - I, (D) - II

(D) (A) - I, (B) - II, (C) - III, (D) - IV

Correct Answer: (B) (A) - II, (B) - I, (C) - III, (D) - IV

Solution:

Concept:

- Qualitative tests identify characteristic functional groups or classes by a specific colour change or chemical reaction.
- Molisch's test detects carbohydrates; Biuret test detects peptide linkages; carbylamine test detects primary amines; Schiff's reagent detects aldehydes.

Step 1: Match the carbohydrate and peptide tests In Molisch's test, concentrated sulfuric acid dehydrates carbohydrates to furfural derivatives. These condense with alpha-naphthol to give the characteristic violet ring, so A matches II. In the Biuret test, compounds with suitable peptide linkages form a violet coordination complex with copper(II) ions in alkaline solution. Hence B matches I, the peptide class, rather than the carbohydrate class.

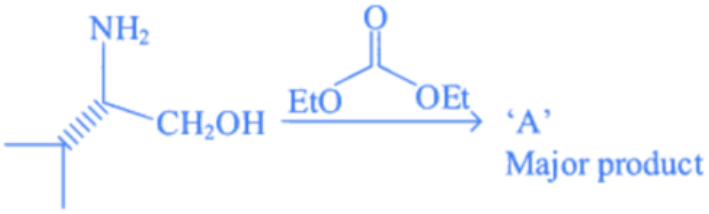
Step 2: Match the amine and aldehyde tests In the carbylamine test a primary amine reacts with chloroform and alcoholic potassium hydroxide to give a characteristically unpleasant-smelling isocyanide. Secondary and tertiary amines do not give this test, so C matches III. Aldehydes restore the pink or magenta colour of decolourised Schiff's reagent; therefore D matches IV. The relevant groups are distinguished by their reactivity, not simply by the presence of nitrogen or oxygen.

Step 3: Compare the four codes The complete sequence is A–II, B–I, C–III, D–IV, which is option (B). Option (A) incorrectly assigns Molisch's test to a primary amine and Biuret test to an aldehyde. Option (C) also misassigns these first two tests and swaps the remaining classes. Option (D) gets the carbylamine and Schiff tests right but incorrectly reverses the carbohydrate and peptide tests. Only option (B) satisfies all four matches.

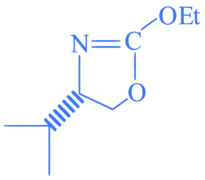
Step 4: Conclusion The correct matching is A–II, B–I, C–III, D–IV. Hence, the correct option is (B).

Quick Tip to Solve: Anchor the two very specific tests first: carbylamine means primary amine, and Biuret means peptide linkages.

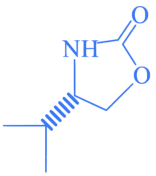
68. In the following reaction, 'A' is [4 marks]



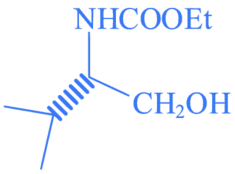
(A) Five-membered cyclic imidate with an ethoxy group.



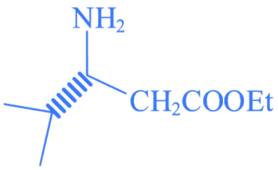
(B) Five-membered cyclic carbamate (oxazolidin-2-one).



(C) Open-chain N-ethoxycarbonyl amino alcohol.

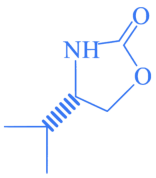


(D) Open-chain amino ester as drawn.



Correct Answer: (B)

Five-membered cyclic carbamate (oxazolidin-2-one).



Solution:

Concept:

- An amino alcohol contains two nucleophilic sites. Its amino group can attack the carbonyl carbon of a carbonate ester to form a carbamate.
- Intramolecular attack by a neighbouring hydroxyl group can close a favourable five-membered ring, with loss of ethanol from an ethoxy leaving group.

Step 1: Identify the reacting groups The substrate has an amino group and a neighbouring hydroxymethyl group: the fragment is $HOCH_2-CH(NH_2)-CH(CH_3)_2$. The reagent is diethyl carbonate, $(EtO)_2CO$. Its carbonyl carbon is electrophilic, and either ethoxy group can leave during nucleophilic acyl substitution. The amino nitrogen is the stronger nucleophile, so it attacks the carbonate carbonyl

first.

Step 2: Close the five-membered ring After the first substitution, the nitrogen bears an ethoxycarbonyl group. The neighbouring hydroxyl oxygen then attacks that same carbonyl carbon intramolecularly. Loss of the second ethoxy group as ethanol completes a five-membered cyclic carbamate, an oxazolidin-2-one. Its ring sequence is N–C(=O)–O–CH2–CH, with the isopropyl group remaining attached to the original stereogenic carbon. No bond at that stereogenic carbon is broken, so its spatial arrangement is retained.

Step 3: Distinguish the products Option (B) contains precisely this cyclic carbamate and retains an N–H bond. Option (A) is an imidate-type structure with a C=N bond and an ethoxy substituent, which is not the carbonyl substitution product. Option (C) is the open-chain carbamate intermediate and omits the favourable intramolecular ring closure. Option (D) changes the alcohol fragment into an ester side chain and does not represent carbonate cyclisation.

Step 4: Conclusion The major product is the five-membered cyclic carbamate shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: For a vicinal amino alcohol plus a carbonate ester, count the atoms in the possible ring: a five-membered cyclic carbamate is strongly favoured.

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69. A solution of $FeCl_3$ when treated with $K_4[Fe(CN)_6]$ gives a prussium blue precipitate due to the formation of : [4 marks]

(A) $Fe[Fe(CN)_6]$

(B) $Fe_4[Fe(CN)_6]_3$

(C) $Fe_3[Fe(CN)_6]_2$

(D) $K[Fe_2(CN)_6]$

Correct Answer: (B) $Fe_4[Fe(CN)_6]_3$

Solution:

Concept:

- Potassium ferrocyanide supplies the complex anion $[Fe(CN)_6]^{4-}$, while ferric chloride supplies Fe^{3+} ions.
- An ionic precipitate must be electrically neutral; combine cations and complex anions in the smallest whole-number ratio that cancels their charges.

Step 1: Determine the charges correctly Each cyanide ligand has charge -1. In ferrocyanide the central iron is in oxidation state +2, giving the overall complex charge $2 + 6(-1) = -4$. This complex remains the ferrocyanide unit when it precipitates with ferric ions. The iron supplied by ferric chloride is separately in oxidation state +3; it must not be confused with the iron inside the complex anion.

Step 2: Balance cations and complex anions The least common multiple of 3 and 4 is 12. Four ferric ions give charge +12, while three ferrocyanide ions give charge -12. Therefore the neutral product

has the formula $Fe_4[Fe(CN)_6]_3$. The precipitation can be represented as

$$4\,Fe^{3+} + 3\,[Fe(CN)_6]^{4-} \longrightarrow Fe_4[Fe(CN)_6]_3(s).$$

This ferric ferrocyanide is the characteristic Prussian blue precipitate.

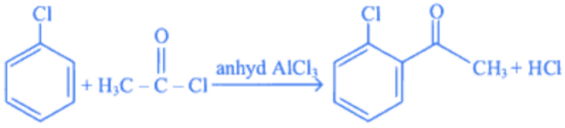
Step 3: Check the alternatives Option (B) has the required 4:3 ratio. Option (A) would pair one +3 ion with one -4 complex and would not give the required neutral ferric ferrocyanide. Option (C), with a 3:2 ratio, would likewise leave a net charge if its outer iron were ferric. Option (D) has a potassium-containing composition different from the ferric ferrocyanide formula requested here. Thus the charge balance and the standard analytical test both select (B).

Step 4: Conclusion The precipitate is ferric ferrocyanide, $Fe_4[Fe(CN)_6]_3$. Hence, the correct option is (B).

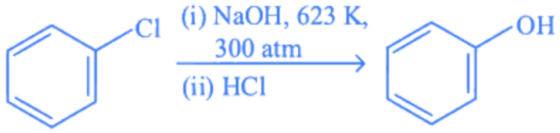
Quick Tip to Solve: Keep the iron outside the brackets separate from the iron inside: ferric ions are +3, whereas the intact ferrocyanide ion is -4.

70. Identify the incorrect option from the following [4 marks]

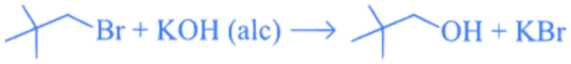
(A) Friedel–Crafts acetylation of chlorobenzene with acetyl chloride and anhydrous aluminium chloride.



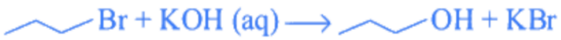
(B) Conversion of chlorobenzene to phenol using NaOH at 623 K and 300 atm, followed by HCl.



(C) Conversion of neopentyl bromide to neopentyl alcohol using alcoholic KOH.

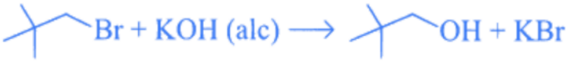


(D) Conversion of 1-bromopropane to propan-1-ol using aqueous KOH.



Correct Answer: (C)

Conversion of neopentyl bromide to neopentyl alcohol using alcoholic KOH.



Solution:

Concept:

- Aqueous potassium hydroxide generally favours nucleophilic replacement of halide by hydroxyl in primary haloalkanes; alcoholic potassium hydroxide is used for beta-elimination.

- Haloarenes require harsh conditions for hydrolysis, while a halogen already on an aromatic ring is an ortho/para director in electrophilic aromatic substitution.

Step 1: Check the aromatic transformations In option (A), chlorobenzene is treated with acetyl chloride and anhydrous aluminium chloride. Chlorine directs electrophilic substitution to the ortho and para positions, so the ortho-acetyl product drawn is a possible Friedel–Crafts acylation product. Option (B) uses hot concentrated sodium hydroxide at 623 K and 300 atm to convert chlorobenzene to sodium phenoxide; acidification with hydrochloric acid then gives phenol. These are the stated conditions for the Dow process.

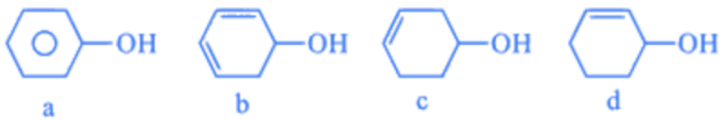
Step 2: Examine the alcoholic hydroxide option Option (C) claims conversion of neopentyl bromide, $(CH_3)_3CCH_2Br$, into neopentyl alcohol using alcoholic KOH. Alcoholic KOH is the conventional reagent for dehydrohalogenation, not the aqueous substitution conditions shown for alcohol formation. Moreover, the beta carbon of neopentyl bromide is quaternary and has no beta hydrogen, so ordinary beta-elimination cannot proceed either. Its unusually hindered primary carbon also makes direct substitution difficult. The clean alcohol conversion depicted under these conditions is therefore incorrect.

Step 3: Check aqueous hydroxide and select Option (D) uses aqueous KOH with 1-bromopropane. Hydroxide substitutes for bromide at the accessible primary carbon, giving propan-1-ol and potassium bromide, so this transformation is correct. Thus (A), (B) and (D) describe acceptable transformations, whereas (C) is the incorrect one requested. The absence of a beta hydrogen in the neopentyl substrate reinforces the need to inspect the actual structure, rather than applying a reagent mnemonic blindly.

Step 4: Conclusion The incorrect transformation is the neopentyl bromide/alcoholic KOH conversion in (C). Hence, the correct option is (C).

Quick Tip to Solve: Before predicting an alkene with alcoholic KOH, inspect the beta carbon for a hydrogen; a quaternary beta carbon blocks ordinary elimination.

71. Decreasing order of dehydration of the following alcohols is [4 marks]



(A) d > b > c > a

(B) b > d > c > a

(C) a > d > b > c

(D) b > a > d > c

Correct Answer: (B) b > d > c > a

Solution:

Concept:

- Acid-catalysed dehydration starts by protonating the hydroxyl group; water can then leave as a neutral molecule.

- For comparable cyclic alcohols, easier formation of a resonance-stabilised carbocation generally makes dehydration faster. A phenolic hydroxyl group behaves differently from an aliphatic hydroxyl group.

Step 1: Compare the carbocations from b and d In b, the hydroxyl-bearing carbon is adjacent to two carbon–carbon double bonds. After protonation and loss of water, the positive charge can be delocalised into both unsaturated portions of the ring. This doubly allylic stabilisation makes ionisation especially favourable. In d, the hydroxyl-bearing carbon is adjacent to only one double bond, so loss of water gives an allylic carbocation with less extensive delocalisation. Consequently b dehydrates more readily than d.

Step 2: Place c and the phenol a In c, the double bond is separated from the hydroxyl-bearing carbon by a saturated carbon. The carbocation initially formed on loss of water is therefore not directly allylic; its formation is less favourable than the resonance-stabilised intermediate from d. Compound a is phenol: its carbon–oxygen bond has partial double-bond character because the oxygen lone pair interacts with the aromatic ring. Ordinary dehydration would require breaking this strong aryl carbon–oxygen bond and forming an unfavourable phenyl cation, so a is last. Thus the orders putting a first or above d are excluded, and d cannot precede b.

Step 3: Conclusion The decreasing order of ease of dehydration is $b > d > c > a$. Hence, the correct option is (B).

Quick Tip to Solve: First locate the carbon carrying OH: direct adjacency to two double bonds beats one, while phenolic OH is resistant to ordinary dehydration.

72. How can photochemical smog be controlled? [4 marks]

(A) By using catalytic convertors in the automobiles/industry.

(B) By complete combustion of fuel.

(C) By using tall chimneys.

(D) By using catalyst.

Correct Answer: (A) By using catalytic convertors in the automobiles/industry.

Solution:

Concept:

- Photochemical smog develops when nitrogen oxides and hydrocarbons react in sunlight, producing secondary pollutants such as ozone and peroxyacetyl nitrate.
- Control requires reducing the release of the precursor pollutants, rather than merely dispersing the polluted air.

Step 1: Identify the function of a catalytic converter Automobile exhaust contains nitrogen oxides, carbon monoxide and unburnt hydrocarbons. A suitably designed catalytic converter promotes oxidation of carbon monoxide and hydrocarbons and reduction of nitrogen oxides to nitrogen. Its purpose is to convert harmful exhaust constituents before they enter the atmosphere. By decreasing both nitrogen oxides and hydrocarbons, it suppresses the reactants needed for the photochemical

reaction sequence. This is the specific control device named in option (A).

Step 2: Check why the other measures are insufficient Complete combustion helps decrease carbon monoxide and unburnt fuel, but high-temperature combustion can still produce nitrogen oxides from atmospheric nitrogen and oxygen. It does not by itself provide the exhaust treatment required to control all smog precursors. Tall chimneys disperse pollutants over a wider area rather than chemically removing them, so they do not prevent photochemical smog formation. The bare instruction to use a catalyst does not specify a pollutant-control reaction or an exhaust-treatment arrangement. A catalytic converter is the actual engineered application, making it the most appropriate and complete choice among the stated options.

Step 3: Conclusion Photochemical smog can be controlled by using catalytic converters in automobiles and suitable industrial exhaust systems. Hence, the correct option is (A).

Quick Tip to Solve: For photochemical smog, target the hydrocarbon and nitrogen-oxide precursors; dispersion alone does not remove them.

73. Match List I with List II :

	List I		List II
A.	**Slaked lime**	**I.**	$NaOH$
B.	**Dead burnt plaster**	**II.**	$Ca(OH)_2$
C.	**Caustic soda**	**III.**	$Na_2CO_3 \cdot 10H_2O$
D.	**Washing soda**	**IV.**	$CaSO_4$

Choose the correct answer from the options given below : [4 marks]

(A) (A) - III, (B) - IV, (C) - II, (D) - I

(B) (A) - III, (B) - II, (C) - IV, (D) - I

(C) (A) - II, (B) - IV, (C) - I, (D) - III

(D) (A) - I, (B) - IV, (C) - II, (D) - III

Correct Answer: (C) (A) - II, (B) - IV, (C) - I, (D) - III

Solution:

Concept:

- A common chemical name must be matched to the complete formula, including any water of crystallisation.
- Slaked lime is calcium hydroxide, caustic soda is sodium hydroxide, and washing soda is sodium carbonate decahydrate.

Step 1: Match the hydroxides Slaked lime is obtained by adding water to quicklime. The reaction is

$$CaO + H_2O \longrightarrow Ca(OH)_2.$$

Therefore A matches II, not I or III. Caustic soda is the strong alkali sodium hydroxide, $NaOH$, so C matches I. The two substances are both hydroxides, but their metal ions and formulae differ; their

common names cannot be interchanged. These two matches already exclude the first, second and fourth proposed combinations.

Step 2: Identify the sulfate and hydrated carbonate Dead burnt plaster is anhydrous calcium sulfate, $CaSO_4$, formed when gypsum is heated strongly enough to remove its water completely. Thus B matches IV. It must not be confused with plaster of Paris, which is the hemihydrate rather than the fully dehydrated material. Washing soda is $Na_2CO_3 \cdot 10\,H_2O$, the decahydrate of sodium carbonate, giving D matches III. The ten waters are part of the named crystalline compound; an anhydrous carbonate formula alone would not give the stated match. Combining all four identifications yields one complete code, and each entry of that code agrees with the corresponding formula in List II.

Step 3: Conclusion The matches are A–II, B–IV, C–I and D–III. Hence, the correct option is (C).

Quick Tip to Solve: Remember lime as the calcium compound and soda as the sodium compound; then check the hydrate number before matching.

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74. Match List I with List II

	List I		List II
A.	Tranquilizers	I.	Anti blood clotting
B.	Aspirin	II.	Salvarsan
C.	Antibiotic	III.	antidepressant drugs
D.	Antiseptic	IV.	soframicine

Choose the correct answer from the options given below: [4 marks]

(A) (A) - II, (B) - I, (C) - III, (D) - IV

(B) (A) - IV, (B) - II, (C) - I, (D) - III

(C) (A) - III, (B) - I, (C) - II, (D) - IV

(D) (A) - II, (B) - IV, (C) - I, (D) - III

Correct Answer: (C) (A) - III, (B) - I, (C) - II, (D) - IV

Solution:

Concept:

- Drugs are grouped by their therapeutic action: tranquilizers act on mental states, while antithrombotic drugs inhibit blood-clot formation.
- An antiseptic is used on living tissue to inhibit microbes; the classification here follows the examples and actions stated in the two lists.

Step 1: Match tranquilizers and aspirin Tranquilizers are used in the treatment of stress and certain mental disorders; the listed antidepressant drugs belong with this therapeutic category. Therefore A matches III. Aspirin inhibits platelet aggregation and is used in suitable low-dose treatment to reduce the tendency of blood to clot. Thus the listed anti-blood-clotting action corresponds to B–I. This use is distinct from the familiar analgesic and antipyretic actions of aspirin. Options assigning tranquilizers to Salvarsan or Soframicine, or assigning aspirin to the other drug names, do not match the indicated categories.

Step 2: Complete the antimicrobial matches Salvarsan is the arsenic-containing antimicrobial drug historically developed for treating syphilis; in the terminology of this matching question it is paired with the antibiotic category, so C matches II. Soframicine is used in topical antimicrobial preparations on affected skin and wounds and is paired here with antiseptic, giving D–IV. The question asks for the pairings among the supplied lists, so these two examples must be combined with the already established therapeutic actions rather than treated as interchangeable labels. The complete combination A–III, B–I, C–II, D–IV occurs only in the third option. Each of the other combinations misplaces at least the tranquilizer or aspirin entry.

Step 3: Conclusion The required matching code is A–III, B–I, C–II, D–IV. Hence, the correct option is (C).

Quick Tip to Solve: Use the most distinctive action first: aspirin prevents platelet aggregation, so its match is anti-blood clotting.

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75. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: In an Ellingham diagram, the oxidation of carbon to carbon monoxide shows a negative slope with respect to temperature.

Reason R: CO tends to get decomposed at higher temperature.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) A is correct but R is not correct

(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is not correct but R is correct

(D) Both A and R are correct and R is the correct explanation of A

Correct Answer: (A) A is correct but R is not correct

Solution:

Concept:

- An Ellingham diagram plots standard Gibbs energy of oxidation against temperature. For an approximately constant reaction entropy, its slope is $-\Delta S^\circ$.
- The temperature dependence follows $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$. An increase in the amount of gas commonly makes the reaction entropy positive.

Step 1: Test Assertion A from the oxidation reaction The oxidation referred to is

$$2\,\mathrm{C\,(s)} + \mathrm{O_2(g)} \longrightarrow 2\,\mathrm{CO\,(g)}.$$

One mole of gaseous oxygen is consumed while two moles of gaseous carbon monoxide are produced. The increase in gaseous particles gives a positive entropy change for this reaction. Consequently

$$\frac{d\Delta G^\circ}{dT} = -\Delta S^\circ < 0.$$

The carbon-to-carbon-monoxide oxidation line therefore slopes downwards as temperature increases. Assertion A is correct; its sign follows directly from reaction entropy, not from decomposition of the product.

Step 2: Test Reason R and the claimed explanation Carbon monoxide formation is increasingly favoured at high temperatures in this oxidation comparison. It is incorrect to claim that its tendency to decompose at higher temperature is responsible for the negative slope. In the related equilibrium $2\,CO\,(g) \rightleftharpoons C\,(s) + CO_2(g)$, carbon deposition is associated with lower temperatures; heating favours the carbon-monoxide side. Thus the stated reason reverses the relevant temperature trend. Reason R is false, so neither option asserting that both statements are correct can apply, and the option declaring A incorrect is also excluded.

Step 3: Conclusion Assertion A is correct, but Reason R is not correct. Hence, the correct option is (A).

Quick Tip to Solve: For an Ellingham slope, write the balanced oxidation reaction and inspect the entropy change; the slope has the opposite sign.

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76. Given below are two statements :
one is labelled as Assertion A and the other is labelled as Reason R
Assertion A : Hydrogen is an environment friendly fuel.
Reason R : Atomic number of hydrogen is 1 and it is a very light element.
In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both A and R are true but R is NOT the correct explanation of A
(B) A is false but R is true
(C) A is true but R is false
(D) Both A and R are true and R is the correct explanation of A

Correct Answer: (A) Both A and R are true but R is NOT the correct explanation of A

Solution:

Concept:

- Fuel cleanliness is judged from the products and pollutants generated during its use.
- Hydrogen has atomic number one and is the lightest element, but these physical facts do not themselves determine its combustion products.

Step 1: Evaluate the environmental statement Hydrogen burns in oxygen to form water:

$$2\,H_2 + O_2 \longrightarrow 2\,H_2O.$$

Since hydrogen contains no carbon, its combustion does not directly produce carbon dioxide, carbon monoxide or soot. In the conventional comparison with carbon-containing fuels, this makes hydrogen an environment-friendly fuel. Assertion A is therefore true. The statement concerns the fuel-use reaction; it does not establish that every production method has the same overall environmental

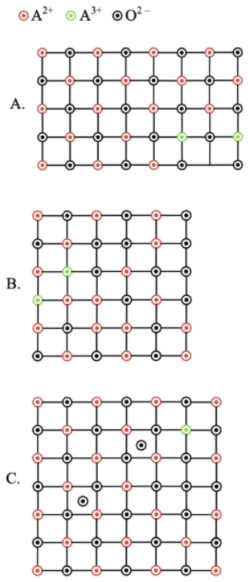
effect. The decisive chemical property in the present question is the formation of water rather than carbonaceous exhaust.

Step 2: Evaluate the reason and its logical connection Every hydrogen atom has one proton, so its atomic number is 1. Hydrogen is also a very light element, making Reason R factually true. Nevertheless, an atomic number or a low mass alone does not show that a substance burns cleanly. The explanation requires the composition of the fuel and the products formed on reacting with oxygen. Neither fact in R identifies water as the product or explains the absence of carbon-containing pollutants. Therefore R is not the correct explanation of A. The choices declaring either statement false are eliminated, while the choice claiming a correct explanatory link is rejected by this distinction.

Step 3: Conclusion Both Assertion A and Reason R are true, but R is not the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: In assertion–reason questions, verify truth first, then ask whether the reason actually explains the property named in the assertion.

77. Which of the following represents the lattice structure of $A_{0.95}O$ containing A^{2+}, A^{3+} and O^{2-} ions? [4 marks]



(A) B only

(B) A and B only

(C) A only

(D) B and C only

Correct Answer: (C) A only

Solution:

Concept:

- A lattice containing A^{2+}, A^{3+} and O^{2-} must satisfy both the composition ratio and electrical neutrality.
- A metal deficiency is compensated by converting some divalent cations into trivalent cations. The extra positive charge balances the missing positive charge at cation vacancies.

Step 1: Derive the required ion counts Take twenty oxide ions as a convenient basis. The formula $A_{0.95}O$ then requires

$$N_A = 0.95 \times 20 = 19.$$

Let x be the number of trivalent ions among these nineteen metal ions. Charge neutrality gives

$$2(19 - x) + 3x = 2(20),$$

so $x = 2$. The required lattice representation therefore has twenty oxide ions, seventeen divalent metal ions, two trivalent metal ions and one vacant position in an otherwise twenty-position metal sublattice. The count and the nature of the vacancy are both essential; merely showing a mixture of valences is insufficient.

Step 2: Compare the three displayed arrangements Arrangement A contains the required metal-site vacancy and two trivalent cations, with the oxide framework retained. Its nineteen cations per twenty oxide ions give the stated 0.95 ratio, and the charges sum to $17(2) + 2(3) = 40$, balancing twenty oxide ions. Arrangement B does not retain the required oxide and cation-site counts: its placement of ions and missing site does not give this composition. Arrangement C shows additional oxide ions at interstitial positions rather than the required simple metal-vacancy compensation. Thus B and C fail the simultaneous composition and charge-balance test. Any answer including B or C is excluded.

Step 3: Conclusion Only lattice structure A represents the specified metal-deficient composition. Hence, the correct option is (C).

Quick Tip to Solve: Use twenty oxide ions for a 0.95 metal ratio: nineteen cations must include exactly two trivalent ions to balance the charge.

78. Choose the correct statement(s) :

A. Beryllium oxide is purely acidic in nature.

B. Beryllium carbonate is kept in the atmosphere of CO_2.

C. Beryllium sulphate is readily soluble in water.

D. Beryllium shows anomalous behaviour.

Choose the correct answer from the options given below : [4 marks]

(A) A only

(B) B, C and D only

(C) A, B and C only

(D) A and B only

Correct Answer: (B) B, C and D only

Solution:

Concept:

- Beryllium differs from the other alkaline earth metals because of its small atomic and ionic sizes, relatively high ionisation enthalpy and strong polarising power.
- An amphoteric oxide reacts with both acids and bases. The stability of a carbonate against loss of carbon dioxide must be distinguished from its solubility.

Step 1: Evaluate statements A and B Beryllium oxide is amphoteric, rather than purely acidic. Its reactions with acids and strong bases show this dual behaviour:

$$BeO + 2\,H^+ \longrightarrow Be^{2+} + H_2O$$

$$BeO + 2\,OH^- + H_2O \longrightarrow [Be(OH)_4]^{2-}$$

Thus A is false. Beryllium carbonate is unstable and readily loses carbon dioxide; keeping it in a carbon-dioxide atmosphere suppresses this decomposition. Thus B is true.

Step 2: Evaluate statements C and D and select the combination Beryllium sulphate is readily soluble in water. The high hydration enthalpy of the small beryllium ion favours dissolution; the sulphates become less soluble down the group. Thus C is true. Beryllium has anomalous properties relative to the heavier group members, including substantial covalent character and amphoteric oxide and hydroxide, so D is true. The required set is therefore B, C and D. Each of the other three options contains A, which has already been disproved by the amphoteric behaviour of the oxide.

Step 3: Conclusion B, C and D are true, whereas A is false. Hence, the correct option is (B).

Quick Tip to Solve: For beryllium, remember amphoteric oxide, unstable carbonate and soluble sulphate before comparing the option combinations.

79. Given below are two statements:

Statement I : Chlorine can easily combine with oxygen to form oxides; and the product has a tendency to explode.

Statement II : Chemical reactivity of an element can be determined by its reaction with oxygen and halogens.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is true but Statement II is false

(B) Both the Statements I and II are false

(C) Statement I is false but Statement II is true

(D) Both the Statements I and II are true

Correct Answer: (C) Statement I is false but Statement II is true

Solution:

Concept:

- A compound may be known and isolable even though it is not readily obtained by direct combination of its constituent elements.
- Reactions with oxygen and halogens are useful comparisons of elemental reactivity because these electronegative elements form oxides and halides with many elements.

Step 1: Test both claims within Statement I Chlorine forms several oxides, including Cl_2O, ClO_2 and Cl_2O_7, but chlorine does not combine readily and directly with oxygen. Such oxides are obtained by suitable indirect chemical preparations. For example, chlorine can react with dry mercury(II) oxide to give chlorine monoxide:

$$2\,Cl_2 + 2\,HgO \longrightarrow HgCl_2 \cdot HgO + Cl_2O.$$

The known instability and explosive tendency of chlorine oxides does not make the preceding claim of easy combination with oxygen true. Since Statement I joins the two claims, its first false clause makes the whole statement false.

Step 2: Test Statement II and compare the alternatives The tendency to form oxides and halides, and the conditions required for those reactions, provide evidence of an element's chemical reactivity. Oxygen and halogens are therefore standard reagents for comparing reactivity, so Statement II is true. Option A reverses these truth values; B incorrectly rejects Statement II; D incorrectly accepts easy combination of chlorine and oxygen. Only option C has the truth values established from the literal wording.

Step 3: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (C). This differs from the supplied key selecting both statements as true: the printed assertion of easy combination with oxygen is false, even though chlorine oxides can be explosive.

Quick Tip to Solve: Test every clause of a compound statement: a true remark about an oxide's instability cannot rescue a false claim about its formation.

80. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Amongst $\mathrm{He}, \mathrm{Ne}, \mathrm{Ar}$ and Kr; $1\mathrm{g}$ of activated charcoal adsorbs more of Kr.

Reason R: The critical volume $\mathrm{V_c}\left(\mathrm{cm^3\ mol^{-1}}\right)$ and critical pressure $\mathrm{P_c}$ (atm) is highest for Krypton but the compressibility factor at critical point $\mathrm{Z_c}$ is lowest for Krypton.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) A is false but R is true

(C) Both A and R are true but R is NOT the correct explanation of A

(D) A is true but R is false

Correct Answer: (D) A is true but R is false

Solution:

Concept:

- Physical adsorption of a gas on activated charcoal is favoured by stronger intermolecular attractions and greater ease of liquefaction.
- The critical compressibility factor is $Z_c = P_cV_c/(RT_c)$. Its comparison needs all three critical constants; larger pressure and volume alone do not determine it.

Step 1: Evaluate the assertion Among He, Ne, Ar and Kr, krypton is the most polarisable and has the strongest dispersion attractions. It is most readily liquefied, having the highest critical temperature in this list. Its attractions to the charcoal surface are consequently strongest, so the same mass of activated charcoal adsorbs more krypton under comparable conditions. Assertion A is true. This is a physisorption comparison, rather than a chemical reaction with the charcoal.

Step 2: Evaluate the reason and select the option Although krypton has the largest critical volume and critical pressure among these gases, the claim that its critical compressibility factor is the lowest is incorrect. The tabulated critical compressibility factors of argon and krypton are very close, approximately 0.29, and argon has the smaller value (about 0.290, compared with about 0.292 for krypton). One cannot infer the stated ranking by comparing P_c and V_c without T_c. Thus the compound Reason R is false. Options A and C both require a true reason, while B requires a false assertion; all are excluded.

Step 3: Conclusion Assertion A is true, but Reason R is false. Hence, the correct option is (D).

Quick Tip to Solve: Use ease of liquefaction to compare adsorption, and compute a compressibility factor from pressure, volume and temperature together.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. 25 mL of an aqueous solution of KCl was found to require 20 mL of 1 M $AgNO_3$ solution when titrated using K_2CrO_4 as an indicator. What is the depression in freezing point of KCl solution of the given concentration? _____ (Nearest integer).

(Given : $K_f = 2.0\ K\ kg\ mol^{-1}$)

Assume 1) 100% ionization and 2) density of the aqueous solution as 1 g mL^{-1} [4 marks]

Solution:

Concept:

- Silver ions precipitate chloride ions in a one-to-one molar ratio: $Ag^+ + Cl^- \longrightarrow AgCl(s)$. The chromate indicator identifies the endpoint.
- For complete dissociation of KCl, the van't Hoff factor is $i = 2$, and the freezing-point depression is $\Delta T_f = iK_f m$, where m is the molality.

Step 1: Find the amount of dissolved potassium chloride The silver nitrate concentration and volume give

$$n(AgNO_3) = 1 \times \frac{20}{1000} = 0.020\ \text{mol}.$$

At the endpoint, this is the amount of chloride in the original 25 mL aliquot. Each formula unit of KCl supplies one chloride ion, so the aliquot contains 0.020 mol of potassium chloride. The added titrant volume must not be included when determining the concentration of the original solution.

Step 2: Calculate molality and the temperature depression The original solution has mass 25 g from its stated density. Using $M(KCl) \simeq 74.5\ g\,mol^{-1}$, the solute mass is $0.020 \times 74.5 = 1.49$ g, and the water mass is $25 - 1.49 = 23.51$ g. Thus

$$m = \frac{0.020}{0.02351} = 0.8507\ \text{mol}\,\text{kg}^{-1},$$

$$\Delta T_f = 2 \times 2.0 \times 0.8507 = 3.40\ \text{K}.$$

Rounding to the nearest integer gives 3. Even the dilute-solution approximation $m \simeq 0.8$ gives 3.2 K and the same required integer.

Step 3: Conclusion The depression in freezing point is approximately 3.40 K; the requested nearest-integer answer is $\boxed{3\ \text{K}}$.

Quick Tip to Solve: Use the titration only to find moles; for molality subtract the dissolved salt mass from the original solution mass.

82. Electrons in a cathode ray tube have been emitted with a velocity of 1000 m s^{-1}. The number of following statements which is/are true about the emitted radiation is _____.

Given : $h = 6 \times 10^{-34}$ J s, $m_e = 9 \times 10^{-31}$ kg.

(A) The de-Broglie wavelength of the electron emitted is 666.67 nm.

(B) The characteristic of electrons emitted depend upon the material of the electrodes of the cathode ray tube.

(C) The cathode rays start from cathode and move towards anode.

(D) The nature of the emitted electrons depends on the nature of the gas present in cathode ray tube.

[4 marks]

Solution:

Concept:

- An electron moving nonrelativistically with speed v has momentum $m_e v$ and de Broglie wavelength $\lambda = h/(m_e v)$.
- Cathode rays are streams of electrons. Their fundamental particle properties do not depend on the electrode material or the gas in the discharge tube.

Step 1: Calculate the wavelength and evaluate A Substitute the given constants without changing their specified precision:

$$\lambda = \frac{6 \times 10^{-34}}{(9 \times 10^{-31})(1000)} = \frac{2}{3} \times 10^{-6} \text{ m}.$$

Since $1 \text{ m} = 10^9$ nm, this is

$$\lambda = 666.67 \text{ nm}.$$

Statement A is true. The speed is much smaller than the speed of light, so the nonrelativistic momentum expression is appropriate. The wavelength describes the electron's matter-wave behaviour; it does not mean that the cathode rays have become photons.

Step 2: Assess the remaining cathode-ray statements All cathode-ray electrons have the same intrinsic mass and charge, irrespective of which electrode substance supplies them. Therefore B, which makes their characteristics material-dependent, is false. The electrons originate at the negative cathode and travel towards the positive anode under the electric field, so C is true. Changing the residual gas does not change the identity or fundamental nature of the electrons, making D false. Differences in discharge conditions must not be confused with a dependence of electron identity on the gas or electrode material.

Step 3: Conclusion Only statements A and C are true. The required number of true statements is $\boxed{2}$, a dimensionless count.

Quick Tip to Solve: Calculate the matter wavelength separately, then use the universal mass and charge of electrons to test the cathode-ray claims.

83. At what pH, given half cell $MnO_4^-(0.1\ M) \mid Mn^{2+}(0.001\ M)$ will have electrode potential of 1.282 V? _____ (Nearest Integer)

Given $E^o_{MnO_4^-|Mn^{2+}} = 1.54\ V, \frac{2.303RT}{F} = 0.059V$ [4 marks]

Solution:

Concept:

- The acidic reduction half-reaction is $MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O$, so five electrons and eight hydrogen ions enter the potential expression.
- The Nernst equation is $E = E^\circ - (0.059/n)\log Q$, with base-ten logarithms. For this reaction, $Q = [Mn^{2+}]/([MnO_4^-][H^+]^8)$.

Step 1: Write the reaction quotient in terms of pH Water has unit activity in the dilute aqueous expression. Insert the manganese-ion concentrations:

$$Q = \frac{0.001}{0.1[H^+]^8} = \frac{10^{-2}}{[H^+]^8}.$$

By definition, $pH = -\log[H^+]$, so

$$\log Q = -2 - 8\log[H^+] = -2 + 8\,pH.$$

The positive coefficient of pH is important: as acidity decreases, the reduction potential falls because hydrogen ions are reactants.

Step 2: Substitute the measured potential and solve Use $n = 5$, the given standard potential, and the given logarithmic coefficient:

$$1.282 = 1.54 - \frac{0.059}{5}(-2 + 8\,pH).$$

Rearranging gives

$$-2 + 8\,pH = \frac{(1.54 - 1.282)5}{0.059} = 21.8644,$$

$$pH = \frac{23.8644}{8} = 2.983.$$

A back-substitution at this value reproduces 1.282 V; rounding pH to the requested nearest integer gives 3. The hydrogen-ion exponent is eight, not five, because stoichiometric coefficients and electron transfer have different roles.

Step 3: Conclusion The calculated pH is approximately 2.983. The requested nearest-integer answer is $\boxed{3}$; pH is dimensionless.

Quick Tip to Solve: Balance the acidic half-reaction first: use five in the Nernst denominator and eight as the exponent of hydrogen-ion concentration.

84. A and B are two substances undergoing radioactive decay in a container. The half life of A is 15 min and that of B is 5 min. If the initial concentration of B is 4 times that of A and they both start decaying at the same time, how much time will it take for the concentration of both of them to be same? _____ min. [4 marks]

Solution:

Concept:

- Radioactive decay is first order: after time t, $N = N_0\,2^{-t/t_{1/2}}$, where $t_{1/2}$ is the half-life.
- The two substances decay simultaneously for the same elapsed time. Their initial concentration ratio must therefore be retained when equating the remaining concentrations.

Step 1: Express both concentrations at the same time Let the initial concentration of A be a, so that of B is $4a$. With time measured in minutes,

$$[A]_t = a\,2^{-t/15}, \qquad [B]_t = 4a\,2^{-t/5}.$$

Because B has the shorter half-life, it loses a larger fraction of its concentration per unit time. Its initial fourfold excess can therefore disappear, even though both concentrations are falling. The requested condition concerns equal remaining concentrations, rather than equal amounts decayed.

Step 2: Equate the concentrations and solve for time Set the two expressions equal and cancel the nonzero initial concentration a:

$$2^{-t/15} = 4\,2^{-t/5}.$$

Divide by the last power of two and use $4 = 2^2$:

$$2^{t(1/5-1/15)} = 2^2.$$

Thus

$$\frac{2t}{15} = 2 \quad \Longrightarrow \quad t = 15 \text{ min}.$$

At this time A has completed one half-life, leaving $a/2$, whereas B has completed three half-lives, leaving $4a/8 = a/2$. This direct check verifies both the elapsed time and the initial ratio.

Step 3: Conclusion The concentrations become equal after $\boxed{15 \text{ min}}$. Both then equal half the initial concentration of A.

Quick Tip to Solve: Compare the ratio of the surviving concentrations; here one half-life of A coincides with three half-lives of B.

85. At $25^{\circ}C$, the enthalpy of the following processes are given :

$H_2(g) + O_2(g)$	$\rightarrow$	$2OH(g)$	$\Delta H^{\circ} = 78\ kJ\ mol^{-1}$
$H_2(g) + \frac{1}{2}O_2(g)$	$\rightarrow$	$H_2O(g)$	$\Delta H^{\circ} = -242\ kJ\ mol^{-1}$
$H_2(g)$	$\rightarrow$	$2H(g)$	$\Delta H^{\circ} = 436\ kJ\ mol^{-1}$
$\frac{1}{2}O_2(g)$	$\rightarrow$	$O(g)$	$\Delta H^{\circ} = 249\ kJ\ mol^{-1}$

What would be the value of X for the following reaction ? _____ (Nearest integer)

$$H_2O(g) \rightarrow H(g) + OH(g)\ \Delta H^{\circ} = X\ kJ\ mol^{-1}$$

[4 marks]

Solution:

Concept:

- Hess's law makes the enthalpy change depend only on the initial and final states; equations can therefore be added and subtracted with their enthalpies.
- The standard formation enthalpy of gaseous hydrogen atoms is half the dissociation enthalpy of $H_2(g)$. The oxygen atom value is already given per mole of atoms.

Step 1: Find the formation enthalpy of the OH radical The first process forms two moles of $OH(g)$ from the elements in their standard states. Hence, for one mole,

$$\Delta_f H^{\circ}[OH(g)] = \frac{78}{2} = 39\,kJ\,mol^{-1}.$$

Similarly, half of the hydrogen dissociation equation gives

$$\Delta_f H^{\circ}[H(g)] = \frac{436}{2} = 218\,kJ\,mol^{-1}.$$

The oxygen atom enthalpy is not needed separately once the OH formation enthalpy is known.

Step 2: Apply the products-minus-reactant relation The required process breaks one O–H bond in gaseous water to produce a hydrogen atom and an OH radical. Use the formation enthalpies of precisely these species:

$$\begin{aligned} X &= \Delta_f H^{\circ}[H(g)] + \Delta_f H^{\circ}[OH(g)] - \Delta_f H^{\circ}[H_2O(g)] \\ &= 218 + 39 - (-242) \\ &= 499\,kJ\,mol^{-1}. \end{aligned}$$

The positive sign is consistent with energy being supplied for homolytic bond cleavage. Do not use the atomisation enthalpy of water: that would break both O–H bonds and produce an oxygen atom rather than the required OH radical.

Step 3: Conclusion The required enthalpy change is $499\,kJ\,mol^{-1}$, so the nearest-integer entry is $\boxed{499}$.

Quick Tip to Solve: First divide the OH-producing equation by two; then add the formation enthalpies of the two radicals and subtract that of water.

86. Sum of oxidation states of bromine in bromic acid and perbromic acid is _____. [4 marks]

Solution:

Concept:

- For a neutral molecule, the algebraic sum of the oxidation numbers of all atoms is zero.
- In these ordinary oxyacids, hydrogen has oxidation number $+1$ and oxygen has oxidation number -2. Bromic acid is $HBrO_3$; perbromic acid is $HBrO_4$.

Step 1: Calculate the bromine oxidation state in bromic acid Write the formula before assigning numbers: bromic acid contains one hydrogen, one bromine and three oxygen atoms. If the bromine oxidation number is a, electrical neutrality requires

$$1 + a + 3(-2) = 0.$$

Therefore

$$a = 6 - 1 = +5.$$

The acid being ionisable does not change this calculation: in the bromate ion, the same sum would instead equal the ion charge of -1, again giving bromine as $+5$.

Step 2: Calculate the perbromic-acid value and add Perbromic acid contains one more oxygen atom than bromic acid. If the bromine oxidation number is b,

$$1 + b + 4(-2) = 0, \qquad b = 8 - 1 = +7.$$

Each additional oxygen contributes -2, so the central bromine oxidation number rises by two while the molecule remains neutral. The requested quantity is the sum of the two bromine values, rather than their difference or the sum over every atom:

$$(+5) + (+7) = 12.$$

Oxidation numbers are formal, dimensionless bookkeeping quantities; no physical unit is attached to their sum.

Step 3: Conclusion The bromine oxidation states are $+5$ and $+7$; their sum is $\boxed{12}$.

Quick Tip to Solve: For a neutral oxyacid of formula HXO_n, the central atom has oxidation number $2n - 1$.

87. Number of isomeric compounds with molecular formula $C_9H_{10}O$ which (i) do not dissolve in $NaOH$ (ii) do not dissolve in HCl. (iii) do not give orange precipitate with 2,4-DNP (iv) on hydrogenation give identical compound with molecular formula $C_9H_{12}O$ is _____. [4 marks]

Solution:

Concept:

- The degree of unsaturation of $C_9H_{10}O$ is $(2 \times 9 + 2 - 10)/2 = 5$: a benzene ring accounts for four and an additional double bond accounts for one.

- Failure to dissolve in aqueous NaOH excludes a phenolic or carboxylic acid group. A negative 2,4-DNP test excludes aldehydes and ketones; neutral ethers are consistent with these tests.

Step 1: Identify the pair that shares a saturated ether product Keep the phenyl–oxygen bond fixed and place the remaining double bond at either possible position of a straight three-carbon chain. The two constitutional isomers are allyl phenyl ether and 1-propenyl phenyl ether:

$$\mathrm{C_6H_5{-}O{-}CH_2{-}CH{=}CH_2}, \qquad \mathrm{C_6H_5{-}O{-}CH{=}CH{-}CH_3}.$$

Each has nine carbon atoms, ten hydrogen atoms and one oxygen atom. Both are neutral ethers, so neither exhibits the acid–base dissolution tests listed, and neither contains a carbonyl group to form a 2,4-DNP precipitate.

Step 2: Check that hydrogenation gives the same connectivity Addition of one mole of hydrogen across the side-chain double bond gives, in both cases, the same ether:

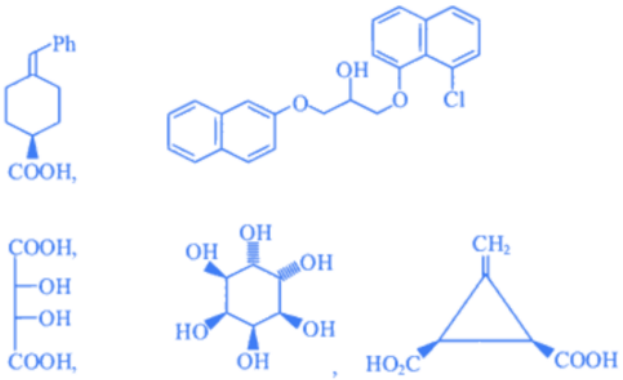
$$\mathrm{C_6H_5{-}O{-}CH_2{-}CH{=}CH_2 + H_2 \longrightarrow C_6H_5{-}O{-}CH_2{-}CH_2{-}CH_3},$$
$$\mathrm{C_6H_5{-}O{-}CH{=}CH{-}CH_3 + H_2 \longrightarrow C_6H_5{-}O{-}CH_2{-}CH_2{-}CH_3}.$$

The common product is n-propoxybenzene, with formula $\mathrm{C_9H_{12}O}$. The branched isopropenyl ether would give isopropoxybenzene, a different product, and is therefore excluded from this pair. The count concerns these constitutional compounds; the E/Z forms of the propenyl ether do not add another constitutional connectivity.

Step 3: Conclusion There are $\boxed{2}$ constitutional isomeric compounds in the required pair.

Quick Tip to Solve: Work backwards from the common saturated product and move the removable double bond along its unchanged carbon skeleton.

88. The total number of chiral compound/s from the following is _____. [4 marks]



Solution:

Concept:

- A molecule is chiral when its mirror image cannot be superposed on it. An internal mirror plane makes a structure achiral even if it contains stereogenic carbon atoms.
- A tetrahedral carbon is stereogenic only if its four attached groups are distinguishable. In rings, compare complete paths and the stereochemistry of the entire structure rather than counting wedges.

Step 1: Check the two structures in the upper row Number the drawings in reading order, left to right across the upper row and then across the lower row. Structure 1, the substituted cyclohexane with an exocyclic benzylidene group, is chiral. The unlike H and phenyl groups on the exocyclic double-bond carbon remove the symmetry that would otherwise interchange the two ring paths while retaining the shown carboxyl-group orientation. It has no internal mirror plane. Structure 2 is also chiral: its OH-bearing carbon is bonded to H, OH and two different CH_2-O-aryl groups. One terminal naphthyl group is chlorinated and the other is not, so those two substituents are unequal.

Step 2: Use internal symmetry to exclude the lower row Structure 3 is the meso form of tartaric acid shown as a Fischer projection. The two equal carboxyl ends and the indicated arrangement of the OH groups give internal symmetry; the stereogenic centres therefore do not make the whole molecule chiral. Structure 4, the depicted cyclohexanehexol, likewise possesses a symmetry plane in its shown substitution pattern. Structure 5 has the two identical carboxyl groups cis on a symmetric cyclopropane framework; reflection interchanges those groups and the corresponding ring positions. Its exocyclic methylene group does not destroy that plane. Thus all three lower-row compounds are achiral despite the stereochemical bonds shown.

Step 3: Conclusion Only structures 1 and 2 are chiral. The total number is $\boxed{2}$.

Quick Tip to Solve: Before counting stereogenic centres, look for a plane that exchanges identical substituents; meso compounds contribute zero to the chiral count.

89. **(i)** $\mathrm{X(g)} \rightleftharpoons \mathrm{Y(g)} + \mathrm{Z(g)} \quad \mathrm{K_{p1}} = 3$

(ii) $\mathrm{A(g)} \rightleftharpoons 2\,\mathrm{B(g)} \quad \mathrm{K_{p2}} = 1$

If the degree of dissociation and initial concentration of both the reactants $\mathrm{X(g)}$ and $\mathrm{A(g)}$ are equal, then the ratio of the total pressure at equilibrium $\left(\frac{p_1}{p_2}\right)$ is equal to $\mathrm{x} : 1$. The value of x is _____ (Nearest integer) [4 marks]

Solution:

Concept:

- For an initially pure gas with degree of dissociation α and net increase of one mole per mole dissociated, the total equilibrium amount is proportional to $1 + \alpha$.
- In K_p, substitute each partial pressure as its mole fraction multiplied by total pressure. Stoichiometric coefficients affect the product partial pressures, even when the net mole change is the same.

Step 1: Express the first equilibrium constant Start with one mole of X. At equilibrium the amounts of X, Y and Z are respectively $1-\alpha, \alpha, \alpha$, with total $1+\alpha$. At total pressure p_1,

$$K_{p1} = \frac{p_Y p_Z}{p_X} = \frac{\alpha^2 p_1}{(1-\alpha)(1+\alpha)} = \frac{\alpha^2 p_1}{1-\alpha^2}.$$

Therefore $3 = \alpha^2 p_1/(1-\alpha^2)$. This uses the total pressure at equilibrium, not the initial reactant pressure.

Step 2: Express the second equilibrium constant and divide For one mole initially of A, the equilibrium amounts are $1-\alpha$ of A and 2α of B, again totalling $1+\alpha$. Hence

$$K_{p2} = \frac{p_B^2}{p_A} = \frac{4\alpha^2 p_2}{1-\alpha^2} = 1.$$

The degrees of dissociation are equal, so division cancels the common factor:

$$\frac{K_{p1}}{K_{p2}} = \frac{p_1}{4p_2} = 3.$$

Consequently $p_1/p_2 = 12$. The factor of four comes from squaring the 2α product amount; omitting it would incorrectly produce a ratio of three. Equal initial concentrations do not justify setting the two final pressures equal, since equal temperatures were not stipulated.

Step 3: Conclusion The total-pressure ratio is $p_1 : p_2 = 12 : 1$, so the dimensionless nearest-integer value is $\boxed{x = 12}$.

Quick Tip to Solve: Write equilibrium mole amounts first: forming $2B$ produces a factor of four in p_B^2, unlike forming one mole each of Y and Z.

90. The density of $3M$ solution of $NaCl$ is $1.0\ g\ mL^{-1}$. Molality of the solution is _____ $\times 10^{-2}$ m. (Nearest integer).

Given: Molar mass of Na and Cl is 23 and $35.5\ g\ mol^{-1}$ respectively. [4 marks]

Solution:

Concept:

- Molarity is moles of solute per litre of solution, whereas molality is moles of solute per kilogram of solvent.
- Density converts the chosen solution volume to total solution mass. Subtract the solute mass before converting the remaining solvent mass to kilograms.

Step 1: Choose one litre and calculate its solute mass A $3\,\mathrm{mol\,L^{-1}}$ solution contains three moles of sodium chloride in $1.000\,\mathrm{L}$ of solution. The given atomic molar masses yield

$$M_{\mathrm{NaCl}} = 23 + 35.5 = 58.5\,\mathrm{g\,mol^{-1}}.$$

Thus the solute mass is

$$m_{\mathrm{solute}} = 3 \times 58.5 = 175.5\,\mathrm{g}.$$

The density is $1.0\,\mathrm{g\,mL^{-1}}$, so this litre of solution has total mass $1000\,\mathrm{g}$, not $1000\,\mathrm{g}$ of solvent.

Step 2: Obtain the solvent mass and the requested coefficient Subtract the dissolved sodium chloride from the total mass:

$$m_{\mathrm{solvent}} = 1000 - 175.5 = 824.5\,\mathrm{g} = 0.8245\,\mathrm{kg}.$$

The molality is consequently

$$b = \frac{3}{0.8245} = 3.6385688\ldots\ \mathrm{mol\,kg^{-1}}.$$

The blank multiplies $10^{-2}\,\mathrm{m}$, where m denotes molality. Accordingly, express the same value as

$$b = 363.85688\ldots \times 10^{-2}\,\mathrm{m}.$$

Only this numerical coefficient is rounded to the nearest integer. Rounding the molality itself first would lose the precision needed for the requested scaled answer.

Step 3: Conclusion The molality is approximately $3.63857\,\mathrm{mol\,kg^{-1}}$, or $\boxed{364} \times 10^{-2}\,\mathrm{m}$ to the requested precision.

Quick Tip to Solve: Use one litre for a molarity-to-molality conversion, and remember that the molality denominator is solvent mass after subtracting dissolved salt.