JEE Main 2023 Paper 1 (B.E./B.Tech) Feb 1 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 1 February 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $9 = x_1 < x_2 < \ldots < x_7$ be in an A.P. with common difference d. If the standard deviation of $x_1, x_2 ..., x_7$ is 4 and the mean is $\bar{x}$, then $\bar{x} + x_6$ is equal to : [4 marks]

(A) $2\left(9 + \frac{8}{\sqrt{7}}\right)$

(B) 25

(C) $18\left(1 + \frac{1}{\sqrt{3}}\right)$

(D) 34

Correct Answer: (D) 34

Solution:

Concept:

- For seven equally weighted observations, the mean is the middle term of an arithmetic progression: $\bar{x} = 9 + 3d$. Standard deviation is the positive square root of the mean squared deviation.
- The deviations from the mean are $-3d, -2d, -d, 0, d, 2d, 3d$. Since the terms increase strictly, the common difference satisfies $d > 0$.

Step 1: Express the variance using the common difference The square of the given standard deviation is $4^2 = 16$. Adding the squares of all seven deviations and dividing by seven gives

$$\sigma^2 = \frac{(9 + 4 + 1 + 0 + 1 + 4 + 9)d^2}{7} = 4d^2.$$

The denominator is seven because these are the seven observations themselves; no sample-variance correction is used.

Step 2: Find the positive common difference Equating the two variance expressions gives $4d^2 = 16$, hence $d^2 = 4$. Although this equation has roots ± 2, only $d = 2$ is allowed by $x_1 < x_2$. Therefore the sequence is $9, 11, 13, 15, 17, 19, 21$. Its central observation, and therefore its mean, is 15.

Step 3: Evaluate the requested sum and compare options The sixth term is $x_6 = 9 + 5d = 19$, while $\bar{x} = 9 + 3d = 15$. Thus

$$\bar{x} + x_6 = 18 + 8d = 18 + 16 = 34.$$

Option (B), 25, does not equal this sum. The expressions in (A) and (C) have irrational extra terms, whereas the determined sum is the integer 34, so neither can be correct.

Step 4: Conclusion The required value is $\boxed{34}$. Hence, the correct option is (D).

Quick Tip to Solve: For an odd number of equally spaced terms, center the deviations at the middle term before computing the variance.

2. Let $P(S)$ denote the power set of $S = \{1, 2, 3, \ldots, 10\}$. Define the relations R_1 and R_2 on $P(S)$ as $\mathrm{AR_1\ B}$ if $(\mathrm{A} \cap \mathrm{B^c}) \cup (\mathrm{B} \cap \mathrm{A^c}) = \emptyset$ and $\mathrm{AR_2\ B}$ if $\mathrm{A} \cup \mathrm{B^c} = \mathrm{B} \cup \mathrm{A^c}, \forall \mathrm{A}, \mathrm{B} \in \mathrm{P(S)}$. Then : [4 marks]

(A) only R_2 is an equivalence relation

(B) both R_1 and R_2 are not equivalence relations

(C) both R_1 and R_2 are equivalence relations

(D) only R_1 is an equivalence relation

Correct Answer: (C) both R_1 and R_2 are equivalence relations

Solution:

Concept:

- An equivalence relation must be reflexive, symmetric and transitive. The equality relation on any set has all three properties.
- Complements are taken relative to S. The sets $A \cap B^c = A \setminus B$ and $B \cap A^c = B \setminus A$ describe elements belonging to exactly one of A, B.

Step 1: Reduce the first relation to equality A union is empty only when both of its constituent sets are empty. Hence

$$(A \cap B^c) \cup (B \cap A^c) = \emptyset$$

$$\iff \quad A \subseteq B \text{ and } B \subseteq A \quad \iff \quad A = B.$$

Thus R_1 is exactly equality between subsets of S, not the relation of mere nonempty intersection.

Step 2: Reduce the second relation to equality Suppose $A \cup B^c = B \cup A^c$. If an element belonged to $A \setminus B$, it would belong to the left side but to neither term on the right, a contradiction. Thus $A \subseteq B$. Interchanging the roles of the sets similarly gives $B \subseteq A$, so $A = B$. Conversely, if $A = B$, both displayed unions equal S. Consequently R_2 also holds exactly when $A = B$.

Step 3: Verify the properties and choose the statement For either relation, $A = A$ proves reflexivity; $A = B \Rightarrow B = A$ proves symmetry; and $A = B,\ B = C \Rightarrow A = C$ proves transitivity. These arguments apply to every subset, including $\emptyset$ and S. Both relations are therefore equivalence relations. Options (A) and (D) wrongly exclude one of them, and (B) wrongly excludes both.

Step 4: Conclusion Both R_1 and R_2 are equivalence relations. Hence, the correct option is (C).

Quick Tip to Solve: For set relations involving complements, test an element in the set differences; a contradiction often reduces the relation to equality.

3. The sum of the absolute maximum and minimum values of the function $f(x) = \left|x^2 - 5x + 6\right| - 3x + 2$ in the interval $[-1, 3]$ is equal to : [4 marks]

(A) 13

(B) 24

(C) 10

(D) 12

Correct Answer: (C) 10

Solution:

Concept:

- For a continuous function on a closed interval, absolute extrema occur at endpoints, interior stationary points, or points where the formula changes or the derivative fails to exist.
- The expression inside the modulus factors as $x^2 - 5x + 6 = (x - 2)(x - 3)$. Its sign determines the correct polynomial on each part of $[-1, 3]$.

Step 1: Remove the modulus on the appropriate intervals On $[-1, 2]$, both factors are nonpositive and their product is nonnegative. On $[2, 3]$, their product is nonpositive. Therefore

$$f(x) = \begin{cases} x^2 - 8x + 8, & -1 \le x \le 2, \\ -x^2 + 2x - 4, & 2 \le x \le 3. \end{cases}$$

Both formulas give -4 at $x = 2$, so the pieces join continuously.

Step 2: Check monotonicity on each piece On the first piece, $f\prime(x) = 2x - 8 < 0$ throughout $(-1, 2)$. On the second piece, $f\prime(x) = -2x + 2 < 0$ throughout $(2, 3)$. The possible stationary points $x = 4$ and $x = 1$ lie outside the respective domains of the formulas from which they arise, so they are inadmissible. The corner at $x = 2$ must still be checked.

Step 3: Compare endpoint and corner values Direct substitution gives

$$f(-1) = 17, \qquad f(2) = -4, \qquad f(3) = -7.$$

As both pieces decrease and meet continuously, the absolute maximum is 17 and the absolute minimum is -7. Their sum is $17 + (-7) = 10$. The word absolute identifies global extrema; it does not ask for the sum of their magnitudes, which would incorrectly give option (B), 24. Neither 13 nor 12 equals the requested signed sum.

Step 4: Conclusion The sum of the absolute maximum and minimum values is $\boxed{10}$. Hence, the correct option is (C).

Quick Tip to Solve: After splitting a modulus function, accept a stationary point only if it lies in the interval where that polynomial is valid.

4. Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is TRUE ? [4 marks]

(A) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-13}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

(B) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{13}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

(C) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{13}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(D) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-13}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

Correct Answer: (A) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-13}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

Solution:

Concept:

- The scalar projection of $\vec{a}$ onto a nonzero vector $\vec{b}$ is the signed number $(\vec{a} \cdot \vec{b})/|\vec{b}|$. A negative value means the component points against $\vec{b}$.
- The vector projection is $\text{proj}_{\vec{b}}\,\vec{a} = \left((\vec{a} \cdot \vec{b})/|\vec{b}|^2\right)\vec{b}$. Its direction is determined by the sign of the multiplying coefficient.

Step 1: Compute the dot product and the norm Using corresponding Cartesian components gives

$$\vec{a} \cdot \vec{b} = 5(1) + (-1)(3) + (-3)(5) = 5 - 3 - 15 = -13.$$

Also,

$$|\vec{b}|^2 = 1^2 + 3^2 + 5^2 = 35, \qquad |\vec{b}| = \sqrt{35}.$$

The negative dot product already shows that the angle between the vectors is obtuse; a positive scalar component along $\vec{b}$ would therefore be inconsistent.

Step 2: Distinguish the signed component from its magnitude The scalar projection is

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = -\frac{13}{\sqrt{35}}.$$

Its magnitude alone is $13/\sqrt{35}$, but the projection asked for carries the sign. The actual projected vector is

$$\text{proj}_{\vec{b}}\,\vec{a} = -\frac{13}{35}\vec{b}.$$

Since its coefficient is negative, this vector is opposite in direction to $\vec{b}$.

Step 3: Match both parts of the statement Option (A) states both the negative scalar projection and the opposite direction, so both clauses hold. Option (B) gives the right direction but the wrong sign for the scalar projection. Option (C) has the wrong sign and direction. Option (D) retains the signed component but incorrectly makes the projected vector point in the same direction as $\vec{b}$.

Step 4: Conclusion The scalar projection is $\boxed{-13/\sqrt{35}}$, and the projection vector points opposite to $\vec{b}$. Hence, the correct option is (A).

Quick Tip to Solve: Check projection statements twice: use the dot product for the signed scalar and the coefficient of the projection vector for its direction.

5. For the system of linear equations $\alpha x + y + z = 1, x + \alpha y + z = 1, x + y + \alpha z = \beta$, which one of the following statements is NOT correct? [4 marks]

(A) It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$

(B) It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$

(C) $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$

(D) It has no solution if $\alpha = -2$ and $\beta = 1$

Correct Answer: (B) It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$

Solution:

Concept:

- The coefficient matrix has diagonal entries α and off-diagonal entries 1. Its determinant is $(\alpha - 1)^2(\alpha + 2)$; a nonzero determinant gives a unique solution.
- Adding all three equations gives $(\alpha + 2)(x + y + z) = 2 + \beta$. A singular system must be tested for consistency before claiming infinitely many solutions.

Step 1: Check the proposed infinitely many solution cases For $\alpha = 1,\ \beta = 1$, all three equations are the same equation $x + y + z = 1$. There are two free variables, so option (A) is true. For $\alpha = 2$, however, the determinant is

$$(2 - 1)^2(2 + 2) = 4 \neq 0.$$

Thus the solution is unique for every value of β, including -1. The infinitely many solutions claim in option (B) is false.

Step 2: Verify the unique solution when beta is minus one Subtracting the first two equations at $\alpha = 2$ gives $x = y$. Put $x = y = t$. The first equation gives $z = 1 - 3t$, and the third gives $2t + 2z = -1$. Hence $2t + 2 - 6t = -1$, so

$$x = y = \frac{3}{4}, \qquad z = -\frac{5}{4}.$$

This explicit solution confirms uniqueness rather than an inconsistent or free-variable case.

Step 3: Check the remaining statements For $\alpha = 2,\ \beta = 1$, adding the equations gives $4(x+y+z) = 3$, so option (C) is true. For $\alpha = -2,\ \beta = 1$, the same addition gives $0 = 3$, which is impossible; thus option (D) is true. The question asks for the statement that is NOT correct, so it selects the sole false statement, (B).

Step 4: Conclusion The claim of infinitely many solutions for $\alpha = 2,\ \beta = -1$ is not correct. Hence, the correct option is (B).

Quick Tip to Solve: Before testing consistency in detail, evaluate the determinant: a nonzero value immediately rules out both no solution and infinitely many solutions.

6. If $y(x) = x^x, x > 0$, then $y''(2) - 2y'(2)$ is equal to [4 marks]

(A) $4(\log_e 2)^2 + 2$

(B) $8\log_e 2 - 2$

(C) $4\log_e 2 + 2$

(D) $4(\log_e 2)^2 - 2$

Correct Answer: (D) $4(\log_e 2)^2 - 2$

Solution:

Concept:

- For $x > 0$, the function $y = x^x$ is positive and satisfies $\ln y = x \ln x$. Logarithmic differentiation handles the variable base and variable exponent simultaneously.
- After the first derivative, the product rule is needed for the second derivative. In particular, $d(\ln x + 1)/dx = 1/x$.

Step 1: Obtain the first derivative Taking natural logarithms and differentiating gives

$$\ln y = x \ln x, \qquad \frac{y\prime}{y} = \ln x + 1.$$

Thus

$$y\prime = x^x(\ln x + 1).$$

It would be incorrect to treat the exponent as constant or to differentiate only $x \ln x$ without restoring the factor y. At $x = 2$, $y(2) = 2^2 = 4$.

Step 2: Differentiate once more and form the combination Use the product rule on $y\prime = y(\ln x + 1)$:

$$y\prime\prime = y\prime(\ln x + 1) + \frac{y}{x} = y\left((\ln x + 1)^2 + \frac{1}{x}\right).$$

Subtracting twice the first derivative before substituting keeps the cancellation visible:

$$y\prime\prime - 2y\prime = y\left((\ln x)^2 - 1 + \frac{1}{x}\right).$$

The terms linear in $\ln x$ cancel exactly.

Step 3: Evaluate and compare the expressions At $x = 2$, this becomes

$$y\prime\prime(2) - 2y\prime(2) = 4\left((\ln 2)^2 - 1 + \frac{1}{2}\right) = 4(\ln 2)^2 - 2.$$

Option (A) has the wrong sign on its constant. Options (B) and (C) are linear in $\ln 2$, whereas the uncancelled logarithmic contribution is quadratic. As a numerical check, the result is approximately -0.0782, also excluding those three expressions.

Step 4: Conclusion The required value is $\boxed{4(\log_e 2)^2 - 2}$. Hence, the correct option is (D).

Quick Tip to Solve: For combinations of derivatives of a variable power, factor out the original function before substituting to expose cancellations.

7. The value of the integral

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x}dx$ **is :** [4 marks]

(A) $\frac{\pi^2}{6\sqrt{3}}$

(B) $\frac{\pi^2}{6}$

(C) $\frac{\pi^2}{3\sqrt{3}}$

(D) $\frac{\pi^2}{12\sqrt{3}}$

Correct Answer: (A) $\frac{\pi^2}{6\sqrt{3}}$

Solution:

Concept:

- An odd integrand integrates to zero over a symmetric interval. Here $2-\cos 2x$ is even and positive, so $x/(2-\cos 2x)$ is odd.
- The substitution $t=\tan x$ gives $\cos 2x=(1-t^2)/(1+t^2)$ and $dx=dt/(1+t^2)$, reducing this denominator to a quadratic.

Step 1: Use symmetry to remove the odd part Let the required integral be I. Splitting its numerator gives

$$I=\int_{-\pi/4}^{\pi/4}\frac{x}{2-\cos 2x}\,dx+\frac{\pi}{4}\int_{-\pi/4}^{\pi/4}\frac{dx}{2-\cos 2x}.$$

The first integral is zero because its integrand is odd. The second integrand is even, hence

$$I=\frac{\pi}{2}\int_{0}^{\pi/4}\frac{dx}{2-\cos 2x}.$$

The denominator is at least 1 throughout, so the symmetry operation involves no singularity.

Step 2: Transform the remaining integral Put $t=\tan x$. The limits $x=0,\pi/4$ become $t=0,1$, respectively, and

$$2-\cos 2x=2-\frac{1-t^2}{1+t^2}=\frac{1+3t^2}{1+t^2}.$$

Therefore

$$\int_0^{\pi/4}\frac{dx}{2-\cos 2x}=\int_0^1\frac{dt}{1+3t^2}=\left[\frac{1}{\sqrt{3}}\tan^{-1}(\sqrt{3}t)\right]_0^1=\frac{\pi}{3\sqrt{3}}.$$

The angle is $\pi/3$, since the positive tangent argument at the upper endpoint is $\sqrt{3}$.

Step 3: Restore the symmetry factor and select the value Multiplication by the prefactor gives

$$I=\frac{\pi}{2}\cdot\frac{\pi}{3\sqrt{3}}=\frac{\pi^2}{6\sqrt{3}}.$$

Option (B) omits the factor $\sqrt{3}$ from integrating the quadratic denominator. Option (C) is twice the correct value, while option (D) is half of it; both lose a factor when using evenness or the constant numerator term.

Step 4: Conclusion The integral is $\boxed{\pi^2/(6\sqrt{3})}$. Hence, the correct option is (A).

Quick Tip to Solve: On symmetric limits, first separate the odd numerator term; only then apply a tangent substitution to the remaining even integral.

8. The area of the region given by $\{(x, y) : xy \le 8, 1 \le y \le x^2\}$ is : [4 marks]

(A) $16\log_e 2 - \frac{14}{3}$

(B) $8\log_e 2 - \frac{13}{3}$

(C) $16\log_e 2 + \frac{7}{3}$

(D) $8\log_e 2 + \frac{7}{6}$

Correct Answer: The printed region is unbounded for negative x; option A and the key describe only its bounded positive component.

Solution:

Concept:

- An area specified by inequalities includes every point satisfying them. Before integrating a bounded component, check whether other components make the complete region unbounded.
- For positive x, a vertical strip has height $\min(x^2, 8/x) - 1$. An unbounded strip contained in the region proves that its total area is infinite.

Step 1: Check the negative half-plane As printed, there is no restriction $x \ge 0$. For every $x \le -\sqrt{2}$ and $1 \le y \le 2$, we have $y \le x^2$ and $xy < 0 \le 8$. Thus the region contains the entire horizontal strip $\{(x, y) : x \le -\sqrt{2},\ 1 \le y \le 2\}$. Its height is one and its horizontal extent is infinite, so the literal region has infinite area. This is not an issue of choosing integration limits on the positive side.

Step 2: Evaluate the bounded positive component For comparison with the finite options, restrict to the positive component. The curves $y = x^2$ and $y = 8/x$ meet at $x = 2, y = 4$. Their intersections with $y = 1$ are $x = 1$ and $x = 8$, respectively. Hence its area is

$$\begin{aligned} A_+ &= \int_1^2 (x^2 - 1)\,dx + \int_2^8 (8/x - 1)\,dx \\ &= \left[\frac{x^3}{3} - x\right]_1^2 + [8\ln x - x]_2^8 \\ &= \frac{4}{3} + 8\ln 4 - 6 = 16\ln 2 - \frac{14}{3}. \end{aligned}$$

This equals option (A); the other finite expressions do not equal this bounded-component integral.

Step 3: Conclusion The region as printed has infinite area, so none of the finite options is correct literally. The official key selects (A), $16\ln 2 - 14/3$, which is the area of only the bounded positive component; it omits the unbounded negative component.

Quick Tip to Solve: For an inequality region, inspect every sign of each variable before identifying a bounded area between curves.

9. The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval (1, 2) and its other root lies in the interval (2, 3), is : [4 marks]

(A) 2

(B) 0

(C) 1

(D) 3

Correct Answer: (C) 1

Solution:

Concept:

- The roots of $2x^2 - 8x + k = 0$ are symmetric about $x = 2$, since their sum is 4. Write them as $2 - d$ and $2 + d$.
- The two open interval conditions are equivalent to $0 < d < 1$. Their product is $k/2$, which gives a strict interval for k.

Step 1: Translate the root intervals Let the smaller root be $\alpha \in (1, 2)$ and the larger be $\beta \in (2, 3)$. Vieta gives $\alpha + \beta = 8/2 = 4$, so $\alpha = 2 - d$ and $\beta = 2 + d$ with $d > 0$. The first interval gives $1 < 2 - d < 2$, or $0 < d < 1$; the second gives exactly the same condition. Thus both demands must be imposed together, and equality at an endpoint is excluded.

Step 2: Count the permitted integers Using the product,

$$\frac{k}{2} = \alpha\beta = (2 - d)(2 + d) = 4 - d^2.$$

As $0 < d^2 < 1$, we obtain

$$3 < \frac{k}{2} < 4 \quad \Longrightarrow \quad 6 < k < 8.$$

The only integer is $k = 7$. Its roots are $2 - 1/\sqrt{2}$ and $2 + 1/\sqrt{2}$, approximately 1.293 and 2.707, and therefore satisfy both open intervals. At $k = 6$, the roots are 1 and 3; at $k = 8$, both are 2. These boundary values do not qualify. There is one permissible value, so counts 0, 2 and 3 in the other options are excluded.

Step 3: Conclusion Exactly one integral value, $k = 7$, is possible. Hence, the correct option is (C).

Quick Tip to Solve: When two root intervals are symmetric about the quadratic axis, write the roots as the axis value plus and minus one positive displacement.

10. Let a, b be two real numbers such that $ab < 0$. IF the complex number $\frac{1+ai}{b+i}$ is of unit modulus and $a + ib$ lies on the circle $|z - 1| = |2z|$, then a possible value of $\frac{1+[a]}{4b}$, where $[t]$ is greatest integer function, is : [4 marks]

(A) $\left(\frac{1+\sqrt{7}}{4}\right)$

(B) $\frac{1}{2}$

(C) 0

(D) -1

Correct Answer: (C) 0

Solution:

Concept:

- For a quotient, unit modulus means numerator and denominator have equal moduli. Thus $|1 + ai|^2 = |b + i|^2$.
- The circle equation can be squared because both sides are nonnegative. The greatest integer $[a]$ is the integer at or immediately below a, including for negative values.

Step 1: Use both modulus conditions The first condition gives $1 + a^2 = b^2 + 1$, hence $a^2 = b^2$. Since $ab < 0$, neither number is zero and they have opposite signs, so $b = -a$. With $z = a + ib$, the other condition becomes

$$(a - 1)^2 + b^2 = 4(a^2 + b^2).$$

Substitution yields $2a^2 - 2a + 1 = 8a^2$, so $6a^2 + 2a - 1 = 0$. Therefore

$$a = \frac{-1 + \sqrt{7}}{6} \quad \text{or} \quad a = \frac{-1 - \sqrt{7}}{6}, \qquad b = -a.$$

Both pairs satisfy the sign restriction.

Step 2: Evaluate the greatest integer in each case For $a = (-1 + \sqrt{7})/6$, we have $0 < a < 1$, so $[a] = 0$. The expression is

$$\frac{1 + [a]}{4b} = -\frac{1}{4a} = -\frac{1 + \sqrt{7}}{4}.$$

For $a = (-1 - \sqrt{7})/6$, the bounds $-1 < a < 0$ give $[a] = -1$, while $b > 0$. The numerator is then zero and the expression equals zero. These are the only two possible values. The positive value in (A) has the wrong sign for the first case, (B) is not either value, and (D) would require $(1 + \sqrt{7})/4 = 1$, which is false.

Step 3: Conclusion A possible value is 0, obtained for $a = (-1 - \sqrt{7})/6$ and $b = (1 + \sqrt{7})/6$. Hence, the correct option is (C).

Quick Tip to Solve: For a negative number between -1 and 0, the greatest integer is -1, so a numerator containing 1 plus that integer can vanish.

11. If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$**, then :** [4 marks]

(A) $A^{30} - A^{25} = 2I$

(B) $A^{30} + A^{25} - A = I$

(C) $A^{30} = A^{25}$

(D) $A^{30} + A^{25} + A = I$

Correct Answer: (B) $A^{30} + A^{25} - A = I$

Solution:

Concept:

- A matrix $R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ satisfies $R(\alpha)R(\beta) = R(\alpha+\beta)$ by the angle-addition formulae.
- Here $A = R(\pi/3)$, so $A^n = R(n\pi/3)$. A full turn gives the identity I, and powers can be reduced modulo six.

Step 1: Establish the repeating powers The diagonal entries are $1/2 = \cos(\pi/3)$, and the upper-right entry is $\sqrt{3}/2 = \sin(\pi/3)$. The lower-left entry carries the negative sign required by this form. Consequently $A^6 = R(2\pi) = I$. Equivalently, direct multiplication gives $A^3 = -I$, whose square is I. We can therefore reduce the two large exponents without multiplying the matrix thirty times.

Step 2: Check each proposed identity The exponents satisfy $30 = 5 \cdot 6$ and $25 = 4 \cdot 6 + 1$. Hence

$$A^{30} = I, \qquad A^{25} = A.$$

It follows that

$$A^{30} + A^{25} - A = I + A - A = I.$$

For (A), the left side is $I - A$, which cannot be $2I$ because $A \neq -I$. For (C), equality would say $I = A$, contradicted by the nonzero off-diagonal entries. For (D), the left side is $I + 2A$, which is not I, since A is not the zero matrix. Thus precisely the second identity holds.

Step 3: Conclusion The valid identity is $A^{30} + A^{25} - A = I$. Hence, the correct option is (B).

Quick Tip to Solve: Identify sine-cosine matrices before expanding powers; the angle period turns large exponents into small remainders.

12. Two dice are thrown independently. Let A be the event that the number appeared on the 1^{st} die is less than the number appeared on the 2^{nd} die, B be the event that the number appeared on the 1^{st} die is even and that on the second die is odd, and C be the event that the number appeared on the 1^{st} die is odd and that on the 2^{nd} is even. Then : [4 marks]

(A) A and B are mutually exclusive

(B) the number of favourable cases of the events A, B and C are 15, 6 and 6 respectively

(C) B and C are independent

(D) the number of favourable cases of the event $(\mathrm{A} \cup \mathrm{B}) \cap \mathrm{C}$ is 6

Correct Answer: (D) the number of favourable cases of the event $(\mathrm{A} \cup \mathrm{B}) \cap \mathrm{C}$ is 6

Solution:

Concept:

- Two distinguishable fair dice produce $6 \times 6 = 36$ ordered outcomes. An outcome is written (i, j), with i on the first die.
- Mutually exclusive events have empty intersection. Independent events satisfy $P(B \cap C) = P(B)P(C)$; exclusivity alone does not imply independence.

Step 1: Count and compare the basic events For A, fixing the first number as 1 through 5 allows respectively 5, 4, 3, 2 and 1 larger second numbers. Thus $n(A) = 15$. Event B allows first die 2, 4 or 6 and second die 1, 3 or 5, giving $n(B) = 9$. Similarly $n(C) = 9$. Outcome $(2, 3)$ belongs to both A and B, so (A) is false. The counts 15, 6, 6 in (B) are false because the last two should both be 9.

Step 2: Evaluate the compound event Events B and C are disjoint, because the first number cannot be both even and odd. Yet $P(B)P(C) = (9/36)^2 = 1/16 > 0$, whereas $P(B \cap C) = 0$. Therefore they are not independent and (C) is false. By distribution of intersection,

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C) = A \cap C.$$

For first die 1, the permitted second numbers are 2, 4, 6; for first die 3, they are 4, 6; for first die 5, only 6 remains. Thus the outcomes are $(1, 2), (1, 4), (1, 6), (3, 4), (3, 6), (5, 6)$, exactly six in total.

Step 3: Conclusion The event $(A \cup B) \cap C$ has six favourable outcomes. Hence, the correct option is (D).

Quick Tip to Solve: For parity events on two dice, first test whether their intersection is empty, then simplify any union-intersection expression before counting.

13. Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}, \vec{b} = \hat{i} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to : [4 marks]

(A) $\frac{11}{7}$

(B) $\frac{11}{5}\sqrt{2}$

(C) $\frac{\sqrt{914}}{7}$

(D) $\frac{11}{7}\sqrt{2}$

Correct Answer: (D) $\frac{11}{7}\sqrt{2}$

Solution:

Concept:

- A zero cross product with a nonzero vector implies parallelism. In this problem, $(\vec{r} - \vec{c}) \times \vec{a} = \vec{0}$, so we can write $\vec{r} = \vec{c} + t\vec{a}$ for a real scalar t.
- A perpendicularity condition is a zero dot product. For Cartesian components (u, v, w), the vector magnitude is $\sqrt{u^2 + v^2 + w^2}$.

Step 1: Reduce the cross product to one parameter The given equality rearranges to

$$(\vec{r} - \vec{c}) \times \vec{a} = \vec{0}.$$

Since $\vec{a} = (2, -7, 5)$ is nonzero, write

$$\vec{r} = (1, 2, -3) + t(2, -7, 5) = (1 + 2t,\ 2 - 7t,\ -3 + 5t).$$

This representation retains every vector satisfying the cross-product equation; it does not incorrectly require $\vec{r} = \vec{c}$, because adding any multiple of $\vec{a}$ leaves that cross product unchanged.

Step 2: Impose the dot product and find the length As $\vec{b} = (1, 0, 1)$, the second condition gives

$$0 = \vec{r} \cdot \vec{b} = (1 + 2t) + (-3 + 5t) = -2 + 7t.$$

Thus $t = 2/7$ and

$$\vec{r} = \left(\frac{11}{7}, 0, -\frac{11}{7}\right).$$

Its magnitude is

$$|\vec{r}| = \sqrt{\frac{121}{49} + \frac{121}{49}} = \frac{11\sqrt{2}}{7}.$$

The equal and opposite first and third components also directly confirm $\vec{r} \cdot \vec{b} = 0$. Option (A) omits the contribution of one nonzero component, (B) uses an incorrect denominator, and (C) does not equal the computed squared length $242/49$.

Step 3: Conclusion The magnitude is $|\vec{r}| = 11\sqrt{2}/7$. Hence, the correct option is (D).

Quick Tip to Solve: An equality of cross products with the same nonzero vector determines a parallel difference; introduce one scalar parameter before using the dot product.

14. Let $S = \left\{x \in R : 0 < x < 1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$.

If $\mathrm{n(S)}$ **denotes the number of elements in** S **then :** [4 marks]

(A) $\mathrm{n(S)} = 0$

(B) $\mathrm{n(S)} = 1$ and only one element in S is less than $\frac{1}{2}$.

(C) $\mathrm{n(S)} = 1$ and the elements in S is more than $\frac{1}{2}$.

(D) $\mathrm{n(S)} = 1$ and the element in S is less than $\frac{1}{2}$.

Correct Answer: (D) $\mathrm{n(S)} = 1$ and the element in S is less than $\frac{1}{2}$.

Solution:

Concept:

- Put $\theta = \tan^{-1} x$. The restriction $0 < x < 1$ gives $0 < \theta < \pi/4$, which fixes every principal-value branch used below.
- The identities $\tan(\pi/4 - \theta) = (1-x)/(1+x)$ and $\cos 2\theta = (1-x^2)/(1+x^2)$ turn the equation into a linear equation in θ.

Step 1: Resolve the inverse functions on their branches Since $0 < \pi/4 - \theta < \pi/4$, its tangent lies on the principal branch of inverse tangent, giving

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{\pi}{4} - \theta.$$

Also $0 < 2\theta < \pi/2$, inside the principal interval $[0, \pi]$ for inverse cosine. Therefore

$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}(\cos 2\theta) = 2\theta.$$

There is no additional periodic branch or extraneous solution in this domain.

Step 2: Solve and inspect the answer choices The equation reduces to

$$2\left(\frac{\pi}{4} - \theta\right) = 2\theta, \qquad \theta = \frac{\pi}{8}.$$

Hence $S = \{\tan(\pi/8)\} = \{\sqrt{2}-1\}$. This is a single number and $0 < \sqrt{2}-1 < 1/2$, since $1 < \sqrt{2} < 3/2$. Thus (A) is false and (C) has the wrong location. Option (D) states the derived result and agrees with the official key. However, as printed, option (B) also states that there is one element and that the only element is less than one-half. Its mathematical content is identical here, so the printed choices are not mutually exclusive.

Step 3: Conclusion We obtain $n(S) = 1$ and its element $\sqrt{2} - 1 < 1/2$. The official key selects (D), which is correct; printed option (B) is also true because it expresses the same condition, so the question has duplicate correct descriptions.

Quick Tip to Solve: Before cancelling inverse trigonometric functions, place the underlying angle inside the appropriate principal-value interval.

15. Let $f : \mathbb{R} - 0, 1 \to \mathbb{R}$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. Then $f(2)$ is equal to [4 marks]

(A) $\frac{9}{4}$
(B) $\frac{7}{4}$
(C) $\frac{7}{3}$
(D) $\frac{9}{2}$

Correct Answer: (A) $\frac{9}{4}$

Solution:

Concept:

- The transformation $T(x) = 1/(1 - x)$ has period three: applying it three times returns the original argument.
- Evaluate a functional identity at arguments in the same cycle to obtain simultaneous linear equations for the unknown function values. All substituted arguments must lie in the stated domain.

Step 1: Identify the cycle containing 2 For the requested input, the successive arguments are $2, -1, 1/2$. Indeed, $T(2) = -1$, $T(-1) = 1/2$, and $T(1/2) = 2$. None equals the excluded values 0 or 1, so the identity is valid at all three arguments. Write $a = f(2)$, $b = f(-1)$, and $c = f(1/2)$ to keep track of the three unknowns.

Step 2: Substitute the three arguments The right-hand side is always one plus the argument being substituted. Thus the three equations are

$$a + b = 3, \qquad b + c = 0, \qquad c + a = \frac{3}{2}.$$

The second equation is particularly useful because it relates the other two function values without a constant term.

Step 3: Eliminate the other function values Add the first and third equations and subtract the second equation. Each of b and c cancels, leaving

$$2a = 3 + \frac{3}{2} - 0 = \frac{9}{2}, \qquad a = \frac{9}{4}.$$

As a check, $b = 3 - a = 3/4$ and $c = -b = -3/4$, and these give $a + c = 3/2$ as required. The alternatives $7/4$, $7/3$, and $9/2$ fail this eliminated equation; the last of these omits the final division by two.

Step 4: Conclusion $f(2) = \frac{9}{4}$. Hence, the correct option is (A).

Quick Tip to Solve: For an identity containing $1/(1 - x)$, generate its three-argument cycle before attempting to find a general formula for f.

16. Let $\alpha x = \exp\left(x^{\beta}y^{\gamma}\right)$ be the solution of the differential equation $2x^2y\,\mathrm{d}y - \left(1 - xy^2\right)\mathrm{d}x = 0, x > 0, y(2) = \sqrt{\log_e 2}$. Then $\alpha + \beta - \gamma$ equals : [4 marks]

(A) 1

(B) 0

(C) 3

(D) -1

Correct Answer: (A) 1

Solution:

Concept:

- Use $z = y^2$, so that $dz = 2y\,dy$. This reduces the differential equation to a first-order linear equation.
- For $z' + P(x)z = Q(x)$, multiply by the integrating factor $\exp(\int P(x)\,dx)$. Here $x > 0$, so logarithms and the integrating factor have no sign ambiguity.

Step 1: Reduce the equation to linear form The given equation is $2x^2y\,dy = (1 - xy^2)\,dx$. On setting $z = y^2$ and dividing by x^2, it becomes

$$\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2}.$$

The integrating factor is $\exp(\int dx/x) = x$, since the domain specifies positive x.

Step 2: Integrate and apply the initial value Multiplication by x gives

$$x\frac{dz}{dx} + z = \frac{1}{x}, \qquad \frac{d}{dx}(xz) = \frac{1}{x}.$$

Integration therefore yields $xz = \log_e x + C$. Since $y(2) = \sqrt{\log_e 2}$, we have $z(2) = \log_e 2$. Substituting gives $2\log_e 2 = \log_e 2 + C$, hence $C = \log_e 2$.

Step 3: Compare with the prescribed form Returning to y, the solution is

$$xy^2 = \log_e x + \log_e 2 = \log_e(2x).$$

Exponentiating gives $2x = \exp(xy^2)$. Consequently the parameters in the stated form are $\alpha = 2$, $\beta = 1$, and $\gamma = 2$, and $\alpha + \beta - \gamma = 2 + 1 - 2 = 1$. Differentiating $xy^2 = \log_e(2x)$ recovers $y^2 + 2xyy' = 1/x$, verifying the differential equation. The options 0, 3, and -1 do not follow from these parameters.

Step 4: Conclusion $\alpha + \beta - \gamma = 1$. Hence, the correct option is (A).

Quick Tip to Solve: When $y\,dy$ occurs together with y^2, replace y^2 by one variable before finding the integrating factor.

17. Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$, and be perpendicular to the plane $2x + y - z + 1 = 0$. If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to : [4 marks]

(A) $\frac{250}{83}$
(B) $\frac{250}{82}$
(C) $\frac{15}{53}$
(D) $\frac{25}{83}$

Correct Answer: (A) $\frac{250}{83}$

Solution:

Concept:

- A plane through the intersection of $F_1 = 0$ and $F_2 = 0$ can be written $F_1 + \lambda F_2 = 0$.
- Perpendicular planes have perpendicular normal vectors. The distance from (x_1, y_1, z_1) to $ax + by + cz + d = 0$ is $|ax_1 + by_1 + cz_1 + d|/\sqrt{a^2 + b^2 + c^2}$.

Step 1: Write the family of planes The required plane belongs to

$$(2x + 3y - z - 2) + \lambda(x + 2y + 3z - 6) = 0.$$

Its normal vector is $(2+\lambda, 3+2\lambda, -1+3\lambda)$. The normal of the given perpendicular plane $2x+y-z+1 = 0$ is $(2, 1, -1)$. The constant term in that plane has no effect on its normal direction.

Step 2: Enforce perpendicularity The dot product of the normals must vanish:

$$2(2 + \lambda) + (3 + 2\lambda) - (-1 + 3\lambda) = 0.$$

This simplifies to $8 + \lambda = 0$, so $\lambda = -8$. Substitution gives $-6x - 13y - 25z + 46 = 0$, or equivalently

$$6x + 13y + 25z - 46 = 0.$$

Its normal has dot product $12 + 13 - 25 = 0$ with the given normal, providing a direct check.

Step 3: Calculate the squared distance At $(-7, 1, 1)$, the plane expression is $-42+13+25-46 = -50$. Therefore

$$d^2 = \frac{(-50)^2}{6^2 + 13^2 + 25^2} = \frac{2500}{36 + 169 + 625} = \frac{2500}{830} = \frac{250}{83}.$$

Squaring removes the absolute-value sign. The denominator 82 in option (B) results from an incorrect normal-length calculation; options (C) and (D) also disagree with the squared numerator and denominator obtained from the plane.

Step 4: Conclusion $d^2 = \frac{250}{83}$. Hence, the correct option is (A).

Quick Tip to Solve: For a squared distance, square the plane expression and divide directly by the sum of squares of its normal coefficients.

18. Let $\mathrm{P}(x_0, y_0)$ be the point on the hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$. Then $\sqrt{2}(y_0 - x_0)$ is equal to : [4 marks]

(A) 3

(B) -9

(C) -3

(D) 9

Correct Answer: (B) -9

Solution:

Concept:

- The distance from (x, y) to $3x + 2y = 1$ is $|3x + 2y - 1|/\sqrt{13}$. Minimize the numerator on both branches of the hyperbola.
- At a nearest point on a smooth curve, its tangent is parallel to the given line. Both branches must still be compared because stationarity alone does not choose the global minimum.

Step 1: Find the possible extremal points Implicit differentiation of $3x^2 - 4y^2 = 36$ gives $6x - 8yy' = 0$, so $y' = 3x/(4y)$ when $y \neq 0$. The given line has slope $-3/2$. Equating slopes gives $3x/(4y) = -3/2$, or $y = -x/2$. Substitution into the hyperbola yields $2x^2 = 36$, giving the candidates $(3\sqrt{2}, -3/\sqrt{2})$ and $(-3\sqrt{2}, 3/\sqrt{2})$.

Step 2: Compare the two branches globally The hyperbola equation implies the exact identity

$$(3x + 2y)^2 = 72 + 3(x + 2y)^2 \geq 72.$$

Also $3x$ has greater magnitude than $2y$, so $3x + 2y$ is positive on the right branch and negative on the left. Thus its right-branch minimum is $6\sqrt{2}$, attained when $x + 2y = 0$; its left-branch maximum is $-6\sqrt{2}$. The least distance numerators on the two branches are therefore $6\sqrt{2} - 1$ and $6\sqrt{2} + 1$. The first is smaller, so the nearest point is the right-branch candidate.

Step 3: Evaluate the required expression We therefore have $x_0 = 3\sqrt{2}$ and $y_0 = -3/\sqrt{2}$. It follows that

$$\sqrt{2}(y_0 - x_0) = \sqrt{2}\left(-\frac{3}{\sqrt{2}} - 3\sqrt{2}\right) = -3 - 6 = -9.$$

The value 9 comes from the other branch, which is farther from the line. The values 3 and -3 omit part of the coordinate difference and fail direct substitution into the required expression.

Step 4: Conclusion $\sqrt{2}(y_0 - x_0) = -9$. Hence, the correct option is (B).

Quick Tip to Solve: For distance from a hyperbola to a line, compare both branch extrema; the constant in the line decides which branch is closer.

19. Which of the following statements is a tautology? [4 marks]

(A) $p \vee (p \wedge q)$

(B) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

(C) $p \rightarrow (p \wedge (p \rightarrow q))$

(D) $(p \wedge q) \rightarrow (\sim (p) \rightarrow q)$

Correct Answer: (D) $(p \wedge q) \rightarrow (\sim (p) \rightarrow q)$

Solution:

Concept:

- A tautology is true for every assignment of truth values to its component propositions.
- An implication $A \rightarrow B$ is false only when A is true and B is false; equivalently it is $\neg A \vee B$. A single counterexample rules out a proposed tautology.

Step 1: Simplify the proposed tautology In option (D), the inner implication is

$$\neg p \rightarrow q \equiv p \vee q.$$

Thus the complete statement is $(p \wedge q) \rightarrow (p \vee q)$. If both p and q are true, their disjunction is certainly true. If their conjunction is false, the outer implication is automatically true. This covers every assignment, proving that option (D) is always true.

Step 2: Rule out options (A) and (B) For option (A), absorption gives $p \vee (p \wedge q) \equiv p$. Taking p false makes the statement false, regardless of q, so it is not a tautology. For option (B), choose $p = q = \mathrm{T}$. Then $p \rightarrow q$ is true, so the antecedent $p \wedge (p \rightarrow q)$ is true, while the consequent $\neg q$ is false. Hence this implication is false for that assignment.

Step 3: Rule out option (C) and verify the result For option (C), take $p = \mathrm{T}$ and $q = \mathrm{F}$. The inner implication $p \rightarrow q$ is false. Therefore $p \wedge (p \rightarrow q)$ is false while the outer antecedent p is true, making the whole implication false. As a final check on (D), its algebraic form is

$$\neg(p \wedge q) \vee (p \vee q) = \neg p \vee \neg q \vee p \vee q,$$

which contains a proposition and its negation and is identically true.

Step 4: Conclusion $(p \wedge q) \rightarrow (\neg p \rightarrow q)$ is a tautology. Hence, the correct option is (D).

Quick Tip to Solve: To eliminate an implication quickly, look for an assignment making its antecedent true and its consequent false.

20. The sum $\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$ is equal to : [4 marks]

(A) $\frac{11e}{2} + \frac{7}{2e}$

(B) $\frac{13e}{4} + \frac{5}{4e} - 4$

(C) $\frac{11e}{2} + \frac{7}{2e} - 4$

(D) $\frac{13e}{4} + \frac{5}{4e}$

Correct Answer: (B) $\frac{13e}{4} + \frac{5}{4e} - 4$

Solution:

Concept:

- The even and odd parts of the exponential series give $E = \sum_{k=0}^{\infty} 1/(2k)! = (e + e^{-1})/2$ and $O = \sum_{k=0}^{\infty} 1/(2k+1)! = (e - e^{-1})/2$.
- Rewrite a polynomial numerator using falling factors $(2n)(2n-1)$ and $2n$, so that they cancel against the factorial denominator.

Step 1: Decompose the numerator The useful identity is

$$2n^2 + 3n + 4 = \frac{1}{2}(2n)(2n-1) + 2(2n) + 4.$$

This is checked by expanding the right side: it gives $2n^2 - n + 4n + 4$. Dividing by $(2n)!$ therefore expresses the required summand as

$$\frac{1}{2(2n-2)!} + \frac{2}{(2n-1)!} + \frac{4}{(2n)!}.$$

The first denominator starts at $0!$ when $n = 1$, so no exceptional term is missing.

Step 2: Sum the even and odd factorial series Add the Taylor series of e and e^{-1} to isolate the even factorial terms; subtract them to isolate the odd factorial terms. With the notation in the concept block, this gives $E = (e + e^{-1})/2$ and $O = (e - e^{-1})/2$. In the first part of the decomposed sum, setting $k = n - 1$ includes every even factorial from $0!$, so its sum is $E/2$. The second part includes every odd factorial from $1!$, and its sum is $2O$. The third part excludes the $0!$ term and consequently has sum $4(E - 1)$.

Step 3: Collect the exponential terms The total is $S = E/2 + 2O + 4(E-1) = (9/2)E + 2O - 4$. Substitute the exponential expressions and collect coefficients:

$$S = \frac{9}{4}(e + e^{-1}) + (e - e^{-1}) - 4 = \frac{13e}{4} + \frac{5}{4e} - 4.$$

The subtraction of 4 is essential: including $n = 0$ in the original series would contribute precisely 4. Thus option (D) retains an unwanted constant term. Options (A) and (C) have different coefficients of e and e^{-1}, and fail the exact even/odd decomposition above.

Step 4: Conclusion The sum is $\frac{13e}{4} + \frac{5}{4e} - 4$. Hence, the correct option is (B).

Quick Tip to Solve: With a denominator $(2n)!$, expand the numerator in falling factors of $2n$ and track the first index after each cancellation.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of this chord is equal to _____. [4 marks]

Solution:

Concept:

- Complete the square to locate the vertex and focus: $(y-k)^2 = 4a(x-h)$ has focus $(h+a, k)$.
- A focal chord lies on a straight line through the focus. Its stated intercept fixes that line, and its intersections with the parabola give the chord endpoints.

Step 1: Locate the focus and determine the chord line The parabola can be rewritten as

$$y^2 - 4y = 8x + 4, \qquad (y-2)^2 = 8(x+1).$$

Thus its vertex is $(-1, 2)$, and $4a = 8$ gives $a = 2$. The focus is therefore $(1, 2)$. The specified x-intercept is the point $(3, 0)$. Since the chord passes through this point and the focus, its slope is $(0-2)/(3-1) = -1$, so its equation is $y = 3 - x$.

Step 2: Find the intersections with the parabola On substituting $y = 3 - x$ into the completed-square equation, we obtain

$$(1-x)^2 = 8(x+1), \qquad x^2 - 10x - 7 = 0.$$

The quadratic formula gives

$$x = \frac{10 \pm \sqrt{100+28}}{2} = 5 \pm 4\sqrt{2}.$$

The corresponding ordinates are $y = -2 \mp 4\sqrt{2}$. Both roots are real and distinct, so the line meets the parabola in two endpoints of the required focal chord.

Step 3: Compute the endpoint separation The absolute difference between the two abscissae is $8\sqrt{2}$, and because the chord slope is -1, the absolute ordinate difference is also $8\sqrt{2}$. Therefore its Euclidean length is

$$L = \sqrt{(8\sqrt{2})^2 + (8\sqrt{2})^2} = \sqrt{128+128} = 16.$$

One endpoint has abscissa $5 - 4\sqrt{2} < 1$, and the other has abscissa $5 + 4\sqrt{2} > 1$; the focus lies on the intervening chord segment, consistently with the geometry.

Step 4: Conclusion The length of the focal chord is 16 coordinate units. The required numerical entry is 16.

Quick Tip to Solve: Complete the square first: the focus and the given intercept determine the chord line without needing a parabola parameter.

22. If $\int_0^{\pi} \frac{5^{\cos x}(1+\cos x \cos 3x+\cos^2 x+\cos^3 x \cos 3x)dx}{1+5^{\cos x}} = \frac{k\pi}{16}$, **then k is equal to _____.** [4 marks]

Solution:

Concept:

- For an integral on $[0, \pi]$, substitute $x \mapsto \pi - x$. If the remaining factor is unchanged and the two weights add to one, adding the two forms removes the weight.
- Even powers of cosine have standard integrals: $\int_0^{\pi} \cos^{2n} x\, dx = \pi\binom{2n}{n}/4^n$. Also, $\cos 3x = 4\cos^3 x - 3\cos x$.

Step 1: Pair the two complementary weights Put $F(x) = 1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x$. Both cosine factors change sign under reflection about $\pi/2$, so their product does not change. Thus $F(\pi - x) = F(x)$. Moreover, $5^{\cos(\pi-x)} = 5^{-\cos x}$, which gives

$$I = \int_0^{\pi} \frac{F(x)}{1 + 5^{\cos x}}\, dx.$$

Adding this expression to the original integral gives $2I = \int_0^{\pi} F(x)\, dx$. This eliminates the exponential exactly; replacing it by a numerical approximation is unnecessary.

Step 2: Expand the even polynomial and integrate Write $c = \cos x$. Factor first, and then use the triple-angle identity:

$$F = (1 + c^2)(1 + c\cos 3x) = (1 + c^2)(1 + 4c^4 - 3c^2).$$

$$F = 1 - 2c^2 + c^4 + 4c^6.$$

The required even-power integrals are $\pi/2$, $3\pi/8$, and $5\pi/16$. Consequently,

$$2I = \pi - 2\frac{\pi}{2} + \frac{3\pi}{8} + 4\frac{5\pi}{16} = \frac{13\pi}{8}.$$

It follows that $I = 13\pi/16$. In particular, remember the factor of two introduced by adding the reflected integrals; the integral itself is half of the polynomial integral.

Step 3: Conclusion Comparing $I = 13\pi/16$ with $k\pi/16$, the required dimensionless value is $\boxed{k = 13}$.

Quick Tip to Solve: Check reflection invariance of the entire polynomial before pairing the exponential weights; then retain the factor $1/2$ until the final comparison.

23. Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}}+\sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$ in the increasing powers of $2^{(x-2)\log_2 3}$, be 21 . If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is _____. [4 marks]

Solution:

Concept:

- The general term of $(a+b)^m$ is $T_{r+1}=\binom{m}{r}a^{m-r}b^r$. The sixth term therefore has $r=5$, regardless of the numerical values of a and b.
- The first, third and fifth terms of any arithmetic progression are themselves in arithmetic progression. Thus the middle of the three specified binomial coefficients is their arithmetic mean.

Step 1: Determine the exponent from the coefficients The three coefficients are m, $\binom{m}{2}$, and $\binom{m}{3}$. Hence

$$2\binom{m}{2}=m+\binom{m}{3}.$$

As the sixth term exists, $m\geq 5$, so division by m is valid. Substitution gives

$$m-1=1+\frac{(m-1)(m-2)}{6}.$$

$$m^2-9m+14=(m-7)(m-2)=0.$$

The root $m=2$ cannot supply a sixth term; therefore $m=7$. The position of each coefficient in the given A.P. is essential: using consecutive A.P. positions would still give the same mean relation, because the stated positions are equally spaced.

Step 2: Evaluate the sixth term and solve for x Set $t=3^x>0$. Then $2^{\log_2(10-3^x)}=10-t$ requires $t<10$, and $2^{(x-2)\log_2 3}=t/9$. The two bases in the binomial are therefore $a=\sqrt{10-t}$ and $b=(t/9)^{1/5}$. With $m=7$,

$$T_6=\binom{7}{5}a^2b^5=21(10-t)\frac{t}{9}.$$

Since this term is 21, $t(10-t)=9$, or $(t-1)(t-9)=0$. Both $t=1$ and $t=9$ lie in $(0,10)$, so both satisfy the logarithm and square-root domain. They yield $x=0$ and $x=2$, respectively.

Step 3: Conclusion The sum of the squares of all possible values is $0^2+2^2=\boxed{4}$, a dimensionless number.

Quick Tip to Solve: Find m before simplifying the radical bases; in the sixth term with $m=7$, the square root is squared and the fifth root is raised to its fifth power.

24. The sum of the common terms of the following three arithmetic progressions.

$3, 7, 11, 15, \ldots., 399$**,**

$2, 5, 8, 11, \ldots., 359$ **and**

$2, 7, 12, 17, \ldots., 197$**,**

is equal to _____. [4 marks]

Solution:

Concept:

- A term belongs to an arithmetic progression with first term a and difference d precisely when it is congruent to a modulo d and lies within the stated finite range.
- Common terms of several arithmetic progressions repeat with period equal to the least common multiple of their differences. Once the first common term is known, sum the resulting finite A.P.

Step 1: Combine the first two congruences Let N denote a common term. The first progression gives $N \equiv 3 \pmod 4$, while the second gives $N \equiv 2 \pmod 3$. Write $N = 3 + 4r$. Reducing modulo 3 gives $r \equiv 2 \pmod 3$, so $r = 2 + 3s$ and

$$N = 11 + 12s.$$

Thus 11 is common to the first two infinite progressions, but it is not yet necessarily in the third. The upper endpoints must also be retained: any term common to all three must be at most 197, the smallest of 399, 359 and 197.

Step 2: Impose the third progression and its endpoint The third progression requires $N \equiv 2 \pmod 5$. Substitute the previous expression:

$$11 + 12s \equiv 2 \pmod 5 \quad \Longrightarrow \quad 2s \equiv 1 \pmod 5.$$

Hence $s \equiv 3 \pmod 5$, and every common term has the form

$$N = 47 + 60u, \qquad u \in \mathbb{Z}.$$

The previous value at $u = -1$ is -13, below every given progression. The permissible values are consequently 47, 107 and 167. The next value is 227, which exceeds 197 and is excluded even though it still meets the congruences. Their common difference 60 agrees with $\operatorname{lcm}(4, 3, 5)$.

Step 3: Sum the retained terms There are exactly three terms. Applying the arithmetic-series formula to this common progression gives

$$S = \frac{3}{2}(47 + 167) = 3 \cdot 107 = 321.$$

Each of the three values also lies within the endpoints of the first two progressions, so there is no additional range restriction to apply.

Step 4: Conclusion The required sum of the common terms is the dimensionless integer $\boxed{321}$.

Quick Tip to Solve: Use the smallest final term as the upper cutoff only after solving all congruences; matching residues alone does not enforce the finite endpoints.

25. Number of integral solutions to the equation $x + y + z = 21$, where $x \geq 1, y \geq 3, z \geq 4$, is equal to _____. [4 marks]

Solution:

Concept:

- Lower bounds can be removed by subtracting them from the respective variables. This gives a bijection between the original integral solutions and nonnegative integral solutions of a new equation.
- For $u + v + w = n$ with $u, v, w \geq 0$, stars and bars gives $\binom{n+2}{2}$ solutions. The two bars select the three groups, including groups of size zero.

Step 1: Shift each variable by its minimum Define

$$u = x - 1, \qquad v = y - 3, \qquad w = z - 4.$$

The stated inequalities make u, v, w nonnegative integers, and substitution in the equation yields

$$(u + 1) + (v + 3) + (w + 4) = 21.$$

Therefore

$$u + v + w = 13.$$

This transformation neither loses nor adds a solution: from any nonnegative triple, recover the original variables by $x = u + 1$, $y = v + 3$, and $z = w + 4$. In particular, equality in a lower bound is allowed, so zero entries in the new variables must be included.

Step 2: Count by stars and bars Represent the total 13 by thirteen identical stars. Insert two separator bars to divide them into the three ordered groups of sizes u, v, and w. A bar may be at an end or next to the other bar, since empty groups are permitted. There are fifteen symbols altogether, of which any two positions may be occupied by the bars. Thus

$$\#\text{solutions} = \binom{15}{2} = \frac{15 \cdot 14}{2} = 105.$$

No division by 3! is made, because the variables are distinct: interchanging the values of u and v generally gives a different ordered solution.

Step 3: Verify the count by fixing one variable As a check, fix u between 0 and 13. Then $v + w = 13 - u$ has $14 - u$ possibilities, with v taking every integer from 0 to $13 - u$. Summing gives

$$14 + 13 + \cdots + 1 = \frac{14 \cdot 15}{2} = 105,$$

which confirms the stars-and-bars count independently.

Step 4: Conclusion There are $\boxed{105}$ integral solutions satisfying all three lower bounds. This is a count and has no physical unit.

Quick Tip to Solve: Subtract all three lower bounds first; their total is 8, leaving 13 to distribute among three nonnegative variables.

26. If the term without x in the expansion of $\left(x^{\frac{2}{3}}+\frac{\alpha}{x^3}\right)^{22}$ is 7315 , then $|\alpha|$ is equal to _____.

[4 marks]

Solution:

Concept:

- The general term T_{r+1} in $(a+b)^{22}$ is $\binom{22}{r}a^{22-r}b^r$, where r is an integer from 0 to 22.
- A term is independent of x when its total exponent of x is zero. Equate that exponent to zero before equating the coefficient to the specified constant.

Step 1: Find the only possible constant term Take $a=x^{2/3}$ and $b=\alpha x^{-3}$. The general term becomes

$$T_{r+1}=\binom{22}{r}\alpha^r x^{\frac{2}{3}(22-r)-3r}.$$

Collecting the exponents gives $(44-11r)/3$. A term without x requires

$$44-11r=0, \qquad r=4.$$

This is within the permitted integer range, so the expansion has a constant term. It is the fifth term, because the index in the term formula is $r+1$, not r. Since the exponent is a strictly decreasing linear function of r, no second term can have zero exponent.

Step 2: Use the given constant coefficient At $r=4$, the independent term is

$$T_5=\binom{22}{4}\alpha^4.$$

Evaluate its binomial coefficient explicitly:

$$\binom{22}{4}=\frac{22\cdot 21\cdot 20\cdot 19}{4\cdot 3\cdot 2\cdot 1}=7315.$$

Equating the constant term to the stated value gives

$$7315\alpha^4=7315 \quad\Longrightarrow\quad \alpha^4=1.$$

For real α, this gives $\alpha=1$ or $\alpha=-1$. Both possibilities have the same absolute value, so there is no sign ambiguity in what the question asks. The fourth power is essential: setting $\alpha=7315$ would confuse the parameter with the full binomial coefficient.

Step 3: Check the exponent directly For the fifth term, eighteen copies of $x^{2/3}$ and four copies of α/x^3 occur. Their powers of x are 12 and -12, which cancel exactly. The remaining coefficient is indeed $7315\alpha^4$, verifying the chosen term.

Step 4: Conclusion The requested dimensionless absolute value is $\boxed{|\alpha|=1}$.

Quick Tip to Solve: Solve the exponent equation before computing a coefficient; the constant term here is the fifth term, corresponding to $r=4$.

27. The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6, is _____. [4 marks]

Solution:

Concept:

- A number is divisible by 6 precisely when it is divisible by both 2 and 3. Divisibility by 2 fixes the parity of its last digit; divisibility by 3 depends only on its digit sum modulo 3.
- The allowed digits 4, 5 and 9 represent the three distinct residue classes 1, 2 and 0 modulo 3. For any desired residue, exactly one of these digits can supply it.

Step 1: Fix the last digit Among the permitted digits 4, 5 and 9, only 4 is even. Every admissible six-digit number divisible by 6 must therefore end in 4. Conversely, this last digit guarantees divisibility by 2. All three allowed digits are nonzero, so the first position has no leading-zero restriction. Repetition is allowed by the wording: the number has six positions but only three permitted digit values.

Step 2: Count completions that satisfy divisibility by three Choose the first four digits arbitrarily. There are 3^4 ordered choices. Let their digit sum be s, and let the fifth digit be d. The divisibility-by-three condition for the full six-digit number is

$$s + d + 4 \equiv 0 \pmod 3.$$

Thus $d \equiv -s - 1 \pmod 3$. Whatever the residue of s, this equation prescribes exactly one residue class for d. Since 9, 4 and 5 have residues 0, 1 and 2 respectively, exactly one allowed digit fills the fifth position. No initial four-digit choice is rejected, and none gives two different completions.

Step 3: Evaluate the total and check the fraction The resulting count is

$$3^4 \cdot 1 \cdot 1 = 81.$$

Equivalently, ending in 4 leaves $3^5 = 243$ candidate numbers. Among their first five digits, the last free digit cycles through all three residues, so exactly one third satisfy the digit-sum test. This gives $243/3 = 81$ as well. Counting all 3^6 strings without fixing the final digit would include odd numbers, while requiring each digit to appear would impose a restriction absent from the question.

Step 4: Conclusion The total number of six-digit numbers divisible by 6 is $\boxed{81}$. This is a dimensionless count.

Quick Tip to Solve: Fix the final digit as 4, choose four other positions freely, and use the remaining digit to force the required digit-sum residue modulo 3.

28. The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the points $\mathrm{A}(-3, -6, 1)$ and $\mathrm{B}(2, 4, -3)$ divides the line segment AB internally in the ratio $\mathrm{k} : 1$. If $\mathrm{a}, \mathrm{b}, \mathrm{c}(|\mathrm{a}|, |\mathrm{b}|, |\mathrm{c}|$ are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|\mathrm{a} + \mathrm{b} + \mathrm{c}|$ is equal to _____. [4 marks]

Solution:

Concept:

- A point on the line segment from A to B is $A + t(B - A)$, with $0 < t < 1$ for an internal point. Substitute these coordinates into the plane equation to find its intersection.
- If a line is $P + s\mathbf{d}$, the foot H from C satisfies $(H - C) \cdot \mathbf{d} = 0$. Thus $s = (C - P) \cdot \mathbf{d}/(\mathbf{d} \cdot \mathbf{d})$.

Step 1: Locate C on the plane Here $B - A = (5, 10, -4)$, so a general point on AB is

$$(-3 + 5t, -6 + 10t, 1 - 4t).$$

Substitute in $8x + y + 2z = 0$:

$$-28 + 42t = 0 \quad \Longrightarrow \quad t = \frac{2}{3}.$$

Thus $C = (1/3, 2/3, -5/3)$. This parameter lies between 0 and 1, confirming internal division; it also gives $AC : CB = 2 : 1$. The ratio parameter is useful as a consistency check, although the final demand concerns the perpendicular direction.

Step 2: Project C onto the given line Write the common value in the symmetric line equation as s. Because its first numerator is $1 - x$, the line is

$$(x, y, z) = (1, -4, -2) + s(-1, 2, 3).$$

Therefore $P = (1, -4, -2)$ and $\mathbf{d} = (-1, 2, 3)$. Compute

$$C - P = (-2/3, 14/3, 1/3),$$

$$(C - P) \cdot \mathbf{d} = 11, \qquad \mathbf{d} \cdot \mathbf{d} = 14.$$

The foot of the perpendicular consequently has parameter $s = 11/14$. The sign in the first direction component is essential; using $(1, 2, 3)$ would describe a different line.

Step 3: Reduce the perpendicular vector to integer ratios The vector from C to the foot H is

$$H - C = P - C + \frac{11}{14}(-1, 2, 3) = \frac{5}{42}(-1, -26, 17).$$

The integer components $-1, -26, 17$ have no common factor greater than 1. Their dot product with the line direction is $1 - 52 + 51 = 0$, verifying perpendicularity. Reversing all signs describes the same perpendicular and leaves the requested absolute value unchanged.

Step 4: Conclusion Choose $(a, b, c) = (-1, -26, 17)$. Then $|a + b + c| = |-10| = \boxed{10}$, a dimensionless value.

Quick Tip to Solve: In $(1 - x)/1 = (y + 4)/2 = (z + 2)/3$, read the direction as $(-1, 2, 3)$; the negative first component controls the projection.

29. Let $\alpha x + \beta y + \gamma z = 1$ be the equation of a plane passing through the point $(3, -2, 5)$ and perpendicular to the line joining the points $(1, 2, 3)$ and $(-2, 3, 5)$. Then the value of $\alpha\beta y$ is equal to _____. [4 marks]

Solution:

Concept:

- A plane perpendicular to a line has its normal vector parallel to the direction vector of that line. The coefficients of x, y, z in its Cartesian equation give a normal vector.
- A point (x_0, y_0, z_0) on the plane satisfies its equation. The normalization of the constant term to 1 fixes the common multiplier of the coefficients.

Step 1: Find the direction vector of the perpendicular line Subtract the coordinates of the first endpoint from those of the second endpoint. The direction vector is

$$\vec{d} = (-2 - 1, 3 - 2, 5 - 3) = (-3, 1, 2).$$

Since the plane is perpendicular to this line, its normal (α, β, γ) must be parallel to $\vec{d}$. Thus there is a nonzero real number k such that

$$\alpha = -3k, \qquad \beta = k, \qquad \gamma = 2k.$$

Reversing the chosen direction of the line changes the multiplier but leaves the resulting plane unchanged.

Step 2: Use the point to determine the normalized coefficients The plane passes through $(3, -2, 5)$, so substitution in its given equation gives

$$3\alpha - 2\beta + 5\gamma = 1.$$

Using the coefficient ratios found above,

$$-9k - 2k + 10k = 1 \quad \Rightarrow \quad -k = 1 \quad \Rightarrow \quad k = -1.$$

Therefore $\alpha = 3$, $\beta = -1$, $\gamma = -2$, and the plane is $3x - y - 2z = 1$. Direct substitution gives $9 + 2 - 10 = 1$, verifying that the stated point lies on it.

Step 3: Evaluate the requested product The product of the three plane coefficients is

$$\alpha\beta\gamma = (3)(-1)(-2) = 6.$$

The final printed symbol in the requested product is y; at the given point its coordinate is $y = -2$, so the literal product there also gives $\alpha\beta y = (3)(-1)(-2) = 6$. Thus the numerical value agrees for the coefficient product and for the stated point.

Step 4: Conclusion The required numerical value is $\boxed{6}$, a dimensionless number.

Quick Tip to Solve: Find the normal from the perpendicular line first, and normalize its coefficients using the point only at the end.

30. The line $x = 8$ is the directrix of the ellipse $\mathrm{E} : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus $(2, 0)$. If the tangent to E at the point P in the first quadrant passes through the point $(0, 4\sqrt{3})$ and intersects the x-axis at Q, then $(3\mathrm{PQ})^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For an ellipse with its major axis along the x-axis, a focus is $(ae, 0)$, the corresponding directrix is $x = a/e$, and $b^2 = a^2(1 - e^2)$.
- The tangent at (x_1, y_1) to $x^2/a^2 + y^2/b^2 = 1$ is $xx_1/a^2 + yy_1/b^2 = 1$. The distance formula then gives the segment length between two points.

Step 1: Determine the ellipse The focus and directrix give $ae = 2$ and $a/e = 8$. Multiplying, $a^2 = 16$, so $a = 4$ and $e = 1/2$. Hence

$$b^2 = 16\left(1 - \frac{1}{4}\right) = 12.$$

The ellipse is therefore

$$\frac{x^2}{16} + \frac{y^2}{12} = 1.$$

The stated focus lies on the positive major axis, so the usual horizontal-major-axis formula applies.

Step 2: Locate the point of tangency and the intercept Let $P = (x_1, y_1)$, with both coordinates positive. Its tangent is

$$\frac{xx_1}{16} + \frac{yy_1}{12} = 1.$$

Substitute the given point $(0, 4\sqrt{3})$ on this line: $(4\sqrt{3})y_1/12 = 1$, yielding $y_1 = \sqrt{3}$. Since P lies on the ellipse,

$$\frac{x_1^2}{16} + \frac{3}{12} = 1 \quad \Rightarrow \quad x_1^2 = 12.$$

The first-quadrant condition selects $x_1 = 2\sqrt{3}$. Setting $y = 0$ in the tangent gives $x_Q = 16/x_1 = 8\sqrt{3}/3$. Thus $Q = (8\sqrt{3}/3, 0)$.

Step 3: Calculate the required squared length The coordinate differences from P to Q are $2\sqrt{3}/3$ horizontally and $-\sqrt{3}$ vertically. Therefore

$$PQ^2 = \left(\frac{2\sqrt{3}}{3}\right)^2 + (\sqrt{3})^2 = \frac{4}{3} + 3 = \frac{13}{3}.$$

The factor 3 multiplying the length must also be squared:

$$(3PQ)^2 = 9PQ^2 = 9 \cdot \frac{13}{3} = 39.$$

No square root of the distance is needed because the question asks for its square.

Step 4: Conclusion The required value is $\boxed{39}$ square coordinate units.

Quick Tip to Solve: Use the given y-intercept directly in the point-form tangent; it determines the contact point y-coordinate immediately.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : For measuring the potential difference across a resistance of 600Ω, the voltmeter with resistance 1000Ω will be preferred over voltmeter with resistance 4000Ω.

Reason R : Voltmeter with higher resistance will draw smaller current than voltmeter with lower resistance.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) **A** is not correct but **R** is correct

(B) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(C) **A** is correct but **R** is not correct

(D) Both **A** and **R** are correct but **R** is not the correct explanation of **A**

Correct Answer: (A) **A** is not correct but **R** is correct

Solution:

Concept:

- A voltmeter is connected in parallel across the resistance whose potential difference is measured. Its finite resistance makes it draw current and can disturb the circuit.
- Ohm's law gives $I_V = V/R_V$ for the voltmeter current at a given terminal voltage. A larger voltmeter resistance reduces the loading of the circuit.

Step 1: Test the assertion using circuit loading The ideal voltmeter has infinite resistance and draws no current. For the given $600\,\Omega$ resistor, a $1000\,\Omega$ voltmeter is a stronger parallel load than a $4000\,\Omega$ voltmeter. For example, the resistance of the parallel combination is

$$R_{1000} = \frac{600 \times 1000}{600 + 1000} = 375\,\Omega,$$

$$R_{4000} = \frac{600 \times 4000}{600 + 4000} \approx 521.74\,\Omega.$$

The latter is closer to the original $600\,\Omega$ resistance. Thus the $4000\,\Omega$ voltmeter is preferred, and Assertion A is false.

Step 2: Test the reason independently If the voltage across either meter is V, the corresponding currents are $V/1000$ and $V/4000$. Hence

$$\frac{I_{4000}}{I_{1000}} = \frac{1}{4}.$$

The higher-resistance meter draws only one quarter of the current at the same voltage. This establishes Reason R as true. In a real source circuit, reducing this extra branch current normally makes the indicated voltage closer to the undisturbed value.

Step 3: Compare the statement combination with the choices The correct truth combination is false assertion and true reason. Option (A) states exactly this. Options (B) and (D) both require

a true assertion, so neither is valid. Option (C) requires a false reason as well as a true assertion, contradicting both tests. The reason supports preferring higher resistance, which is the opposite of the assertion.

Step 4: Conclusion Assertion A is not correct, but Reason R is correct. Hence, the correct option is (A).

Quick Tip to Solve: For voltage measurement, choose the largest meter resistance; for current measurement, choose the smallest ammeter resistance.

---

32. The escape velocities of two planets A and B are in the ratio $1:2$. If the ratio of their radii respectively is $1:3$, then the ratio of acceleration due to gravity of planet A to the acceleration of gravity of planet B will be : [4 marks]

(A) $\frac{4}{3}$

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{3}{2}$

Correct Answer: (C) $\frac{3}{4}$

Solution:

Concept:

- The escape speed from the surface of a spherical planet is $v_e = \sqrt{2GM/R}$, where M and R are the planet's mass and radius.
- The surface gravitational acceleration is $g = GM/R^2$. Eliminating the planetary mass gives $v_e^2 = 2gR$, so g is proportional to v_e^2/R.

Step 1: Eliminate the unknown planetary masses Using the escape-speed formula separately for each planet gives

$$v_A^2 = 2g_A R_A, \qquad v_B^2 = 2g_B R_B.$$

Taking the ratio and rearranging,

$$\frac{g_A}{g_B} = \left(\frac{v_A}{v_B}\right)^2 \frac{R_B}{R_A}.$$

This relation avoids having to determine the two masses. The radius ratio enters inversely because, for a fixed escape speed, a larger radius requires a smaller surface gravitational acceleration.

Step 2: Insert the given ratios The given escape-speed ratio is $v_A/v_B = 1/2$, whereas the given radius ratio is $R_A/R_B = 1/3$. Therefore

$$\frac{g_A}{g_B} = \left(\frac{1}{2}\right)^2 \times 3 = \frac{3}{4}.$$

The speeds must be squared before combining them with the radii. All ratios are dimensionless, and the factor of 2 in the escape-speed equations cancels.

Step 3: Check the options against the derived relation Option (C) gives $3/4$ and satisfies $v_A^2/v_B^2 = (g_A/g_B)(R_A/R_B) = (3/4)(1/3) = 1/4$. Option (A), $4/3$, reverses the correct gravity ratio. Options (B),

$2/3$, and (D), $3/2$, do not satisfy the squared escape-speed relation; in particular, the latter follows from using the unsquared speed ratio. Thus only the third option is consistent with both data.

Step 4: Conclusion The required ratio is $g_A : g_B = \boxed{3:4}$. Hence, the correct option is (C).

Quick Tip to Solve: Write $g = v_e^2/(2R)$ before taking ratios; square the escape-speed ratio and invert the radius ratio.

33. The ratio of average electric energy density and total average energy density of electromagnetic wave is : [4 marks]

(A) 1

(B) 3

(C) 2

(D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Concept:

- In a plane electromagnetic wave propagating in vacuum, the instantaneous electric and magnetic energy densities are $u_E = \epsilon_0 E^2/2$ and $u_B = B^2/(2\mu_0)$.
- The wave fields satisfy $E = cB$, with $c^2 = 1/(\mu_0\epsilon_0)$. Consequently, the electric and magnetic fields carry equal energy densities at each instant.

Step 1: Express both contributions in terms of the electric field Substitute $B = E/c$ into the magnetic energy density:

$$u_B = \frac{E^2}{2\mu_0 c^2} = \frac{\epsilon_0 E^2}{2} = u_E.$$

The equality holds point by point and at every instant for the electromagnetic wave. It is therefore preserved when averaging over one complete oscillation. The total energy density includes both contributions rather than only the electric field.

Step 2: Take the time averages For a sinusoidal electric field of peak amplitude E_0, $\langle E^2 \rangle = E_0^2/2$. Thus

$$\langle u_E \rangle = \frac{\epsilon_0 E_0^2}{4}, \qquad \langle u_B \rangle = \frac{\epsilon_0 E_0^2}{4}.$$

Adding them gives

$$\langle u_{\text{total}} \rangle = \frac{\epsilon_0 E_0^2}{2}.$$

The electric energy accounts for exactly half the total average energy; the common amplitude and permittivity cancel in the requested ratio.

Step 3: Form the ratio in the requested order The numerator is the average electric energy density,

and the denominator is the total average energy density. Therefore

$$\frac{\langle u_E \rangle}{\langle u_{\text{total}} \rangle} = \frac{\epsilon_0 E_0^2/4}{\epsilon_0 E_0^2/2} = \frac{1}{2}.$$

Option (A), 1, would instead describe the electric-to-magnetic ratio. Option (C), 2, reverses the requested ratio. Option (B), 3, is incompatible with two equal positive contributions. Only option (D) has the correct order and value.

Step 4: Conclusion The ratio is $\boxed{1/2}$. Hence, the correct option is (D).

Quick Tip to Solve: In a travelling electromagnetic wave, electric and magnetic energies are equal: either one is half of their total.

34. Equivalent resistance between the adjacent corners of a regular n-sided polygon of uniform wire of resistance R would be : [4 marks]

(A) $\frac{(n-1)R}{n^2}$

(B) $\frac{n^2R}{n-1}$

(C) $\frac{(n-1)R}{(2n-1)}$

(D) $\frac{(n-1)R}{n}$

Correct Answer: (A) $\frac{(n-1)R}{n^2}$

Solution:

Concept:

- For a uniform wire, resistance is proportional to length. If a wire of total resistance R forms a regular n-sided polygon, each side has resistance R/n.
- Between adjacent vertices there are two paths connected in parallel: the single intervening side and the remaining $n-1$ sides connected in series. Parallel resistances combine as $R_1R_2/(R_1+R_2)$.

Step 1: Find the resistance of each path The regular polygon has equal side lengths, and the wire has uniform material and cross section. Thus the short path between the adjacent corners consists of one side and has

$$R_1 = \frac{R}{n}.$$

The other path follows every remaining side of the polygon. Its $n-1$ equal resistances are in series, giving

$$R_2 = \frac{(n-1)R}{n}.$$

There is no junction along either path that introduces a further branch. Both paths start and end at the same chosen vertices.

Step 2: Combine the two paths in parallel The voltage between the chosen vertices is common to

both paths, so their currents add. Using the parallel formula,

$$R_{\text{eq}} = \frac{(R/n)\,((n-1)R/n)}{R/n + (n-1)R/n}.$$

The denominator is R, the resistance of the entire wire. Therefore

$$R_{\text{eq}} = \frac{(n-1)R^2/n^2}{R} = \frac{(n-1)R}{n^2}.$$

The result is smaller than the resistance R/n of the short path alone, as required for a parallel combination.

Step 3: Check the competing expressions For a triangular loop, $n = 3$, the paths are $R/3$ and $2R/3$, whose parallel value is $2R/9$. Option (A) gives exactly this value. Option (B) gives $9R/2$, which even exceeds the total wire resistance. Options (C) and (D) give $2R/5$ and $2R/3$, both greater than the short branch resistance $R/3$, so they cannot represent the parallel equivalent.

Step 4: Conclusion The equivalent resistance is $\boxed{(n-1)R/n^2}$. Hence, the correct option is (A).

Quick Tip to Solve: First split a uniform closed wire into the two paths between the terminals, then combine those two path resistances in parallel.

35. A coil is placed in magnetic field such that plane of coil is perpendicular to the direction of magnetic field. The magnetic flux through a coil can be changed :

A. By changing the magnitude of the magnetic field within the coil.

B. By changing the area of coil within the magnetic field.

C. By changing the angle between the direction of magnetic field and the plane of the coil.

D. By reversing the magnetic field direction abruptly without changing its magnitude.

Choose the most appropriate answer from the options given below : [4 marks]

(A) A, B and D only

(B) A, B and C only

(C) A and B only

(D) A and C only

Correct Answer: (B) A, B and C only

Solution:

Concept:

- The signed magnetic flux through a plane coil is $\Phi = \vec{B} \cdot \vec{A} = BA\cos\theta$, where θ is the angle between the magnetic field and the chosen area normal.
- Changing the field magnitude, the area inside the field, or the orientation changes the flux. Reversing the field changes the sign of the signed flux while preserving its magnitude for a fixed coil.

Step 1: Test the changes in magnitude, area and orientation Initially the plane is perpendicular to

the magnetic field, so its normal is parallel to the field and the flux magnitude is BA. Statement A changes B, and statement B changes the effective area A; either changes the flux magnitude. For statement C, let φ be the angle between the field and the plane. Then $|\Phi| = BA|\sin\varphi|$. Tilting the plane away from its initial perpendicular orientation changes this angular factor. Thus A, B and C are all valid ways to change the magnitude of the flux.

Step 2: Examine the meaning of field reversal Keep the coil fixed and reverse $\vec{B}$ without changing its magnitude. With the same chosen area normal, the initial and final fluxes are

$$\Phi_i = BA, \qquad \Phi_f = -BA, \qquad \Delta\Phi = -2BA.$$

Consequently, statement D does change the signed flux. The magnitude, however, is unchanged: $|\Phi_i| = |\Phi_f| = BA$. The abruptness affects the induced emf during the reversal, but does not remove the reversal of the flux sign.

Step 3: Relate the listed answers to the distinction The supplied final answer key selects option (B), A, B and C only, which corresponds to comparing the flux magnitude before and after each operation. Options (A), (C) and (D) all omit at least one of A, B and C, so none can be correct under that reading. Under the usual signed definition of magnetic flux, all four statements A, B, C and D are true, and no listed option contains all four. The wording therefore needs this magnitude-versus-signed-flux distinction to avoid a false claim about reversal.

Step 4: Conclusion The keyed answer is (B), A, B and C only, for change in flux magnitude. For signed magnetic flux, A, B, C and D all change flux, so no listed option is fully correct as worded.

Quick Tip to Solve: Always distinguish flux from its magnitude: reversing the magnetic field keeps $|\Phi|$ fixed but changes Φ to $-\Phi$.

---

36. Given below are two statements: One is labeled as Assertion A and the other is labeled as Reason R.

Assertion A : Two metallic spheres are charged to the same potential. One of them is hollow and another is solid, and both have the same radii. Solid sphere will have lower charge than the hollow one.

Reason R : Capacitance of metallic spheres depend on the radii of spheres

In light of the above statements, choose the correct answer from the options given below. [4 marks]

(A) Both **A** and **R** are true but **R** is not the correct explanation of **A**

(B) Both **A** and **R** are true and **R** is the correct explanation of **A**

(C) **A** is false but **R** is true

(D) **A** is true but **R** is false

Correct Answer: (C) **A** is false but **R** is true

Solution:

Concept:

- For an isolated metallic sphere of radius R, the potential relative to infinity is $V = Q/(4\pi\varepsilon_0 R)$, so its capacitance is $C = 4\pi\varepsilon_0 R$.
- At electrostatic equilibrium the excess charge of a conductor resides on its outer surface. For the same outer radius, filling the interior with metal does not change the capacitance.

Step 1: Evaluate Assertion A The hollow and solid metallic spheres have the same radius and are charged to the same potential. Their charge is obtained from $Q = CV$, giving

$$Q_{\text{hollow}} = 4\pi\varepsilon_0 RV = Q_{\text{solid}}.$$

Thus the solid sphere does not have a smaller charge. The assertion compares the amount of metal, but charge storage depends on the outer conducting boundary rather than the amount of metal inside. Assertion A is false.

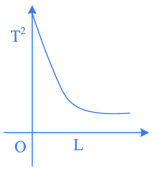
Step 2: Evaluate Reason R The expression $C = 4\pi\varepsilon_0 R$ shows directly that capacitance depends on the radius of the metallic sphere, for the same surrounding medium. Larger radius gives greater capacitance; equal radii give equal capacitances. Reason R is therefore true.

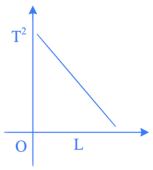
Step 3: Select the assertion–reason combination Since A is false while R is true, the alternatives asserting that both statements are true are excluded. Option D reverses the truth values and is also excluded. There is no need to decide whether a true reason explains a true assertion, because the assertion itself is already false.

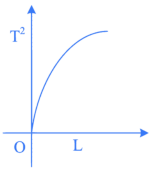
Step 4: Conclusion The two spheres carry equal charge at equal potential. A is false and R is true. Hence, the correct option is (C).

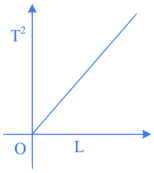
Quick Tip to Solve: For spherical conductors compare the outer radii in $C = 4\pi\varepsilon_0 R$; hollow versus solid is irrelevant when the radii and surroundings agree.

37. Choose the correct length (L) versus square of the time period (T^2) graph for a simple pendulum executing simple harmonic motion. [4 marks]

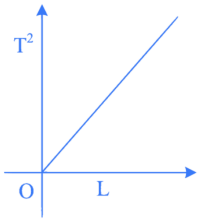
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- For a simple pendulum undergoing small-amplitude oscillations, $T = 2\pi\sqrt{L/g}$, where L is its effective length and g is the local gravitational acceleration.
- A relation $y = kx$, with constant positive k, represents a straight line through the origin with positive slope. Squaring the pendulum period exposes this linear relation.

Step 1: Square the time-period formula At a fixed location g is constant. Square the entire expression, including the coefficient 2π:

$$T^2 = \frac{4\pi^2}{g}L.$$

Thus doubling the length doubles the square of the period, even though the period itself grows only by a factor $\sqrt{2}$. The equation has no constant term and hence no nonzero vertical intercept.

Step 2: Read the axes and identify the slope The supplied option diagrams show T^2 on the vertical axis and L on the horizontal axis. Accordingly, their slope must be $4\pi^2/g > 0$. If the axes were exchanged, the slope would become $g/(4\pi^2)$, but the graph would still be a straight line through the origin. Therefore the verbal order of the variables cannot change which shape is correct.

Step 3: Rule out the other graphs Option A decreases nonlinearly with length, contradicting the increase in period. Option B is a decreasing straight line and has the wrong sign of slope. Option C rises but bends over, corresponding to a changing slope, whereas T^2/L is constant here. Only option D is a rising straight line through the origin, as required by the derived equation.

Step 4: Conclusion The required graph is a straight line through the origin with positive slope: $T^2 \propto L$. Hence, the correct option is (D).

Quick Tip to Solve: Square $T = 2\pi\sqrt{L/g}$ before choosing the graph: T versus L is curved, while T^2 versus L is linear.

---

38. If the velocity of light c, universal gravitational constant G and Planck's constant h are chosen as fundamental quantities. The dimensions of mass in the new system is : [4 marks]

(A) $\left[\mathrm{h}^1\mathrm{c}^1\mathrm{G}^{-1}\right]$
(B) $\left[\mathrm{h}^{-1/2}\mathrm{c}^{1/2}\mathrm{G}^{1/2}\right]$
(C) $\left[\mathrm{h}^{1/2}\mathrm{c}^{1/2}\mathrm{G}^{-1/2}\right]$
(D) $\left[\mathrm{h}^{1/2}\mathrm{c}^{-1/2}\mathrm{G}^{1}\right]$

Correct Answer: (C) $\left[\mathrm{h}^{1/2}\mathrm{c}^{1/2}\mathrm{G}^{-1/2}\right]$

Solution:

Concept:

- Write the required mass dimension as $[h^a c^b G^d]$, where $[h] = ML^2T^{-1}$, $[c] = LT^{-1}$, and $[G] = M^{-1}L^3T^{-2}$.
- Dimensions multiply by adding exponents. For a pure mass, the final powers of M, L, T must be $1, 0, 0$, respectively.

Step 1: Form the dimensional product Insert each constant's ordinary dimensions before equating exponents:

$$[h^a c^b G^d] = M^{a-d} L^{2a+b+3d} T^{-a-b-2d}.$$

Equating this with $M^1 L^0 T^0$ gives three independent equations:

$$a - d = 1, \qquad 2a + b + 3d = 0, \qquad -a - b - 2d = 0.$$

The negative mass exponent in G is essential; reversing it would give the wrong mass scale.

Step 2: Solve the exponent equations From the time equation, $b = -a - 2d$. Substitution into the length equation gives $a + d = 0$, so $d = -a$. The mass equation then becomes $2a = 1$. Therefore

$$a = \frac{1}{2}, \qquad d = -\frac{1}{2}, \qquad b = \frac{1}{2}.$$

The required combination is the square root of hc/G.

Step 3: Check the result and reject distractors The ratio hc/G has dimensions M^2, so its square root has precisely mass dimension. Option A is hc/G without the square root and hence has mass-squared dimension. Option B has mass exponent -1, not $+1$. In option D the mass exponent is $1/2 - 1 = -1/2$, so it also cannot represent mass. Only option C satisfies all three exponent equations.

Step 4: Conclusion The dimensions of mass in the chosen system are $\left[h^{1/2} c^{1/2} G^{-1/2}\right]$. Hence, the correct option is (C).

Quick Tip to Solve: For constants h, c, G, remember that hc/G has dimension M^2; the mass scale is its square root.

---

39. The Young's modulus of a steel wire of length $6\ \text{m}$ and cross-sectional area $3\ \text{mm}^2$, is $2 \times 10^{11}\ \text{N/m}^2$. The wire is suspended from its support on a given planet. A block of mass $4\ \text{kg}$ is attached to the free end of the wire. The acceleration due to gravity on the planet is $\frac{1}{4}$ of its value on the earth. The elongation of wire is (Take g on the earth $= 10\ \text{m/s}^2$) : [4 marks]

(A) 0.1 cm

(B) 1 cm

(C) 0.1 mm

(D) 1 mm

Correct Answer: (C) 0.1 mm

Solution:

Concept:

- Young's modulus is stress divided by longitudinal strain: $Y = (F/A)/(\Delta L/L)$. Thus the extension is $\Delta L = FL/(AY)$.
- The load is the block's weight on the stated planet, $F = mg_{\text{planet}}$. Area must be converted to square metres before substituting SI values.

Step 1: Calculate the tension on the planet The local gravitational acceleration is a quarter of the

given terrestrial value:

$$g_{\text{planet}} = \frac{10}{4} = 2.5\,\text{m}\,\text{s}^{-2}.$$

The suspended block is in equilibrium, so the wire tension is its local weight:

$$F = 4 \times 2.5 = 10\,\text{N}.$$

Using $40\,\text{N}$ would incorrectly apply Earth's gravity instead of the acceleration specified in the question.

Step 2: Convert the area and substitute Since $1\,\text{mm} = 10^{-3}\,\text{m}$, the squared conversion is

$$A = 3\,\text{mm}^2 = 3 \times 10^{-6}\,\text{m}^2.$$

With $L = 6\,\text{m}$ and $Y = 2 \times 10^{11}\,\text{N}\,\text{m}^{-2}$,

$$\Delta L = \frac{10 \times 6}{(3 \times 10^{-6})(2 \times 10^{11})} = \frac{60}{6 \times 10^{5}}\,\text{m} = 10^{-4}\,\text{m}.$$

Step 3: Convert the extension to the option units Multiply metres by 10^3 to obtain millimetres:

$$10^{-4}\,\text{m} = 0.1\,\text{mm} = 0.01\,\text{cm}.$$

Option A, $0.1\,\text{cm}$, and option D, $1\,\text{mm}$, both equal ten times this extension. Option B, $1\,\text{cm}$, is one hundred times too large. Therefore only option C agrees with the elastic-extension calculation.

Step 4: Conclusion The elongation of the wire is $0.1\,\text{mm}$. Hence, the correct option is (C).

Quick Tip to Solve: Apply the planet's gravity first, and square the millimetre-to-metre conversion when converting mm^2.

40. Choose the correct statement about Zener diode : [4 marks]

(A) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

(B) It works as a voltage regulator in both forward and reverse bias.

(C) It works as a voltage regulator only in forward bias.

(D) It works as a voltage regulator in forward bias and behaves like simple pn junction diode in reverse bias.

Correct Answer: (A) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

Solution:

Concept:

- A Zener diode is a heavily doped p–n junction designed to operate safely in its reverse-breakdown region, with current limited by the external circuit.
- In reverse breakdown, a relatively large change in current produces only a small change in voltage. This nearly constant breakdown voltage V_Z is used for regulation.

Step 1: Identify the regulating operating region When a suitable reverse voltage is applied, the

Zener diode enters breakdown at its specified Zener voltage. Connected across a load with a series current-limiting resistor, it can absorb changes in current while keeping the load voltage approximately equal to V_Z. Regulation therefore uses reverse bias in the breakdown region, not merely any small reverse voltage below breakdown.

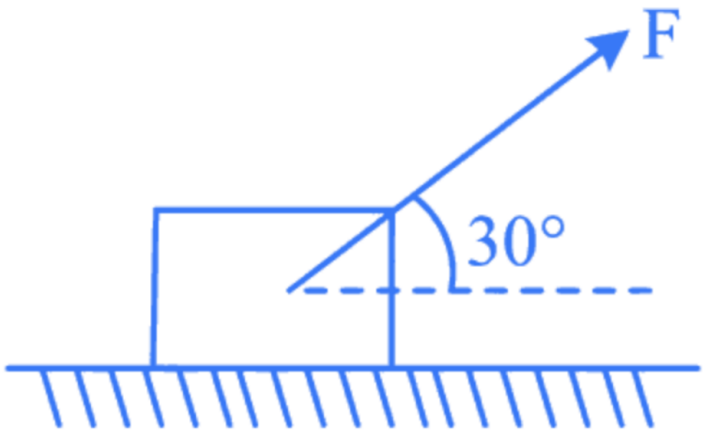
Step 2: Describe forward-bias behaviour In forward bias, the junction barrier is reduced and the diode conducts in essentially the same manner as an ordinary p–n junction diode. Its forward characteristic is not the reverse-breakdown characteristic used in a Zener regulator. The special device construction does not interchange its forward and reverse operating roles.

Step 3: Compare all four statements Option A correctly combines reverse-bias voltage regulation with ordinary forward-diode behaviour. Option B incorrectly extends the stated regulating operation to both polarities. Option C assigns it only to forward bias, the opposite of the Zener regulating region. Option D reverses the two roles: below reverse breakdown current is small, while at breakdown the diode exhibits the special regulating behaviour rather than simply the ordinary reverse characteristic.

Step 4: Conclusion A Zener diode acts as a voltage regulator in reverse breakdown and behaves like an ordinary p–n junction diode in forward bias. Hence, the correct option is (A).

Quick Tip to Solve: Associate Zener regulation with the nearly vertical reverse-breakdown characteristic; forward conduction follows the usual diode behaviour.

41. As shown in the figure a block of mass 10 kg lying on a horizontal surface is pulled by a force F acting at an angle $30°$, with horizontal. For $\mu_s = 0.25$, the block will just start to move for the value of F : [Given $g = 10\ \text{ms}^{-2}$] [4 marks]



(A) 25.2 N

(B) 35.7 N

(C) 20 N

(D) 33.3 N

Correct Answer: (A) 25.2 N

Solution:

Concept:

- At impending horizontal motion, static friction reaches its limiting magnitude $f_{\max} = \mu_s N$, where N is the normal reaction.
- An upward inclined pulling force reduces the normal reaction: vertical equilibrium gives $N + F\sin\theta = mg$. Its horizontal component is $F\cos\theta$.

Step 1: Resolve the force and find the normal reaction The figure shows the force directed upward at 30° to the horizontal. The weight is $mg = 10 \times 10 = 100\,\text{N}$. With no vertical acceleration,

$$N = mg - F\sin 30^\circ = 100 - \frac{F}{2}.$$

The normal reaction is therefore smaller than the weight; taking $N = 100\,\text{N}$ would ignore the lifting component of the pull.

Step 2: Apply the condition for impending motion At the instant the block just starts moving, the horizontal pull balances limiting friction:

$$F\cos 30^\circ = 0.25\,(100 - F\sin 30^\circ)\,.$$

Gather the terms containing F:

$$F\left(\frac{\sqrt{3}}{2} + 0.25 \times \frac{1}{2}\right) = 25.$$

Hence

$$F = \frac{25}{0.8660 + 0.125} = 25.23\,\text{N}.$$

Step 3: Check the threshold and the alternatives At this force $N \approx 87.39\,\text{N}$, so limiting friction is about $21.85\,\text{N}$; this equals $F\cos 30^\circ$. The contact force remains positive, confirming the block stays on the surface. A pull of $20\,\text{N}$ is insufficient, while $33.3\,\text{N}$ and $35.7\,\text{N}$ exceed the threshold. The required value is thus the listed $25.2\,\text{N}$.

Step 4: Conclusion The minimum pull for the block to start moving is approximately $25.2\,\text{N}$. Hence, the correct option is (A).

Quick Tip to Solve: For an upward pull use $F = \mu_s mg/(\cos\theta + \mu_s \sin\theta)$; the plus sign accounts for the reduction of the normal force.

42. Two objects A and B are placed at 15 cm and 25 cm from the pole in front of a concave mirror having radius of curvature 40 cm. The distance between images formed by the mirror is _____. [4 marks]

(A) 60 cm

(B) 40 cm

(C) 160 cm

(D) 100 cm

Correct Answer: (C) 160 cm

Solution:

Concept:

- For a spherical mirror, $1/f = 1/v + 1/u$, where the Cartesian sign convention assigns negative distances to points in front of the mirror.
- A concave mirror has $f = -R/2$. A virtual image behind the mirror has positive v, while a real image in front has negative v.

Step 1: Set the focal length and object distances The magnitude of the radius of curvature is $40\,\text{cm}$, so the signed focal length is

$$f = -\frac{40}{2} = -20\,\text{cm}.$$

Both objects are in front of the mirror, giving $u_A = -15\,\text{cm}$ and $u_B = -25\,\text{cm}$. Object A lies inside the focal length, whereas B lies outside it. Their images will therefore be on opposite sides of the mirror.

Step 2: Find each image with the mirror formula For object A,

$$\frac{1}{v_A} = \frac{1}{f} - \frac{1}{u_A} = -\frac{1}{20} + \frac{1}{15} = \frac{1}{60}, \qquad v_A = +60\,\text{cm}.$$

For object B,

$$\frac{1}{v_B} = -\frac{1}{20} + \frac{1}{25} = -\frac{1}{100}, \qquad v_B = -100\,\text{cm}.$$

The signs agree with the expected virtual and real images, respectively.

Step 3: Calculate the separation and reject the distractors The distance between two image positions is the absolute difference of their signed coordinates:

$$d = |v_A - v_B| = |60 - (-100)| = 160\,\text{cm}.$$

The values $60\,\text{cm}$ and $100\,\text{cm}$ are individual image distances, not the separation. Subtracting their unsigned magnitudes would give $40\,\text{cm}$, but that incorrectly puts both images on the same side of the mirror. Since they are on opposite sides, their distances from the pole add.

Step 4: Conclusion The distance between the two images is $160\,\text{cm}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep image distances signed until the last step. Real and virtual images on opposite sides have a separation equal to the sum of their distance magnitudes.

43. For a body projected at an angle with the horizontal from the ground, choose the correct statement.

[4 marks]

(A) Gravitational potential energy is maximum at the highest point.

(B) The vertical component of momentum is maximum at the highest point.

(C) The horizontal component of velocity is zero at the highest point.

(D) The Kinetic Energy (K.E.) is zero at the highest point of projectile motion.

Correct Answer: (A) Gravitational potential energy is maximum at the highest point.

Solution:

Concept:

- For projectile motion without air resistance, gravitational potential energy is $U = mgh$, where h is the height above the launch level. Thus its maximum occurs where the height is greatest.
- The horizontal velocity remains $v_x = u\cos\theta$, whereas the vertical velocity is $v_y = u\sin\theta - gt$. Momentum is mass times velocity, and kinetic energy depends on the square of the total speed.

Step 1: Apply the highest-point condition At the top of the trajectory, the upward motion has just stopped before the projectile starts descending. Therefore $v_y = 0$, while the horizontal component remains unchanged. The maximum height follows from the vertical equation of motion:

$$0 = u^2\sin^2\theta - 2gH \quad \Rightarrow \quad H = \frac{u^2\sin^2\theta}{2g}.$$

Consequently, $U_{\max} = mgH$, so statement (A) follows directly from the increase of potential energy with height.

Step 2: Test the remaining statements The vertical momentum at the top is $p_y = mv_y = 0$, rather than a maximum as claimed in (B). Gravity has no horizontal component, so $v_x = u\cos\theta$ is not made zero at the top; this rejects (C). The remaining horizontal motion gives

$$K_{\text{top}} = \frac{1}{2}mu^2\cos^2\theta.$$

For the usual oblique projectile this is positive, so (D) is false. Kinetic energy is minimum at the top, but a minimum need not be zero. Mechanical energy is conserved as the lost vertical kinetic energy becomes gravitational potential energy.

Step 3: Conclusion Gravitational potential energy is maximum at the highest point. Hence, the correct option is (A).

Quick Tip to Solve: At the highest point set only the vertical velocity to zero; the unchanged horizontal velocity prevents the total kinetic energy from vanishing.

44. As shown in the figure, a long straight conductor with semicircular arc of radius $\frac{\pi}{10}$m is carrying current $\mathrm{I} = 3\mathrm{A}$. The magnitude of the magnetic field, at the center O of the arc is :
(The permeability of the vacuum $= 4\pi \times 10^{-7}\ \mathrm{NA}^{-2}$) [4 marks]



(A) $4\mu\mathrm{T}$
(B) $3\mu\mathrm{T}$
(C) $6\mu\mathrm{T}$
(D) $1\mu\mathrm{T}$

Correct Answer: (B) $3\mu\mathrm{T}$

Solution:

Concept:

- By the Biot–Savart law, a current element contributes $d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{d\boldsymbol{\ell} \times \hat{\mathbf{r}}}{r^2}$. A radial straight part gives zero field at the observation point because the cross product vanishes.
- At the centre of a circular arc of radius R and angle ϕ radians, the magnetic field magnitude is $B = \mu_0 I\phi/(4\pi R)$. A semicircle has $\phi = \pi$.

Step 1: Separate the conductor into its three parts The two straight portions lie along the diameter line through O. For every element of either straight portion, the line joining that element to O is parallel or antiparallel to the current element. Therefore $\sin\alpha = 0$ in the Biot–Savart expression and both straight portions contribute zero. They must not be treated as tangent semi-infinite wires, since their geometry is radial. Only the semicircular arc contributes a nonzero magnetic field at O.

Step 2: Evaluate the semicircular contribution For the arc every current element is perpendicular to its radius to O, and every contribution has the same direction. Hence

$$B = \frac{\mu_0 I}{4R}.$$

Substitute $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$, $I = 3\,\mathrm{A}$, and $R = \pi/10\,\mathrm{m}$:

$$B = \frac{(4\pi \times 10^{-7})(3)}{4(\pi/10)} = 3 \times 10^{-6}\,\mathrm{T} = 3\,\mu\mathrm{T}.$$

The current goes clockwise around the upper arc, giving a field into the page by the right-hand rule. A full circle would instead give twice this field, explaining why $6\,\mu\mathrm{T}$ is incorrect; neither $4\,\mu\mathrm{T}$ nor $1\,\mu\mathrm{T}$ follows from the arc formula.

Step 3: Conclusion The magnetic field magnitude at O is $3\,\mu\mathrm{T}$. Hence, the correct option is (B).

Quick Tip to Solve: Check whether a straight segment is radial or tangent before applying a wire-field formula: radial pieces give zero field at the centre.

45. The threshold frequency of a metal is f_0. When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of photoelectrons is v_1. When the frequency of incident radiation is increased to $5f_0$, the maximum velocity of photoelectrons emitted is v_2. The ratio of v_1 to v_2 is : [4 marks]

(A) $\frac{v_1}{v_2} = \frac{1}{2}$
(B) $\frac{v_1}{v_2} = \frac{1}{16}$
(C) $\frac{v_1}{v_2} = \frac{1}{4}$
(D) $\frac{v_1}{v_2} = \frac{1}{8}$

Correct Answer: (A) $\frac{v_1}{v_2} = \frac{1}{2}$

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hf - \phi = h(f - f_0)$, with work function $\phi = hf_0$. The threshold frequency must be subtracted before comparing energies.
- For a nonrelativistic photoelectron, $K_{\max} = \frac{1}{2}m_e v_{\max}^2$. Thus speed varies as the square root of the frequency excess above threshold, rather than directly as the incident frequency.

Step 1: Find the two maximum kinetic energies For incident frequency $2f_0$, the photon energy is $2hf_0$. One part hf_0 supplies the work function, leaving

$$\frac{1}{2}m_e v_1^2 = h(2f_0 - f_0) = hf_0.$$

For incident frequency $5f_0$, the same metal has the same work function, leaving

$$\frac{1}{2}m_e v_2^2 = h(5f_0 - f_0) = 4hf_0.$$

The second maximum kinetic energy is therefore four times the first; the photon-frequency ratio $2/5$ is not the kinetic-energy ratio.

Step 2: Convert the energy ratio into the speed ratio Divide the two equations. The common electron mass, Planck constant, and threshold frequency cancel:

$$\frac{v_1^2}{v_2^2} = \frac{hf_0}{4hf_0} = \frac{1}{4}.$$

Since maximum speeds are positive, take the positive square root:

$$\frac{v_1}{v_2} = \sqrt{\frac{1}{4}} = \frac{1}{2}.$$

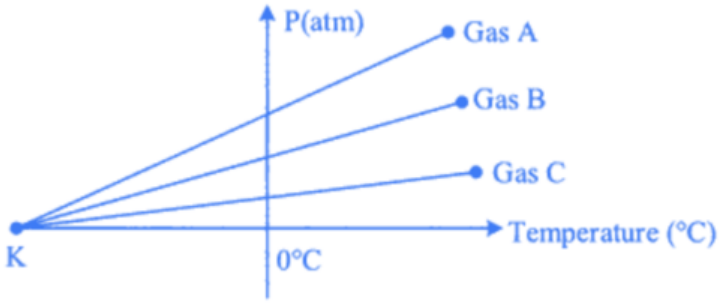
Option (C) gives the kinetic-energy ratio instead of the speed ratio. Options (B) and (D) would imply kinetic-energy ratios $1/256$ and $1/64$, respectively, contradicting the photoelectric equations. Increasing frequency increases electron energy; light intensity does not enter this ratio.

Step 3: Conclusion The ratio is $v_1 : v_2 = 1 : 2$. Hence, the correct option is (A).

Quick Tip to Solve: In a photoelectric speed comparison, first subtract the threshold frequency and then take the square root of the resulting energy ratio.

46. For three low density gases A, B, C pressure versus temperature graphs are plotted while keeping them at constant volume, as shown in the figure.

The temperature corresponding to the point 'K' is : [4 marks]



(A) $-273^{\circ}\mathrm{C}$

(B) $-373^{\circ}\mathrm{C}$

(C) $-100^{\circ}\mathrm{C}$

(D) $-40^{\circ}\mathrm{C}$

Correct Answer: (A) $-273^{\circ}\mathrm{C}$

Solution:

Concept:

- For a fixed amount of low-density gas at constant volume, the ideal-gas equation gives $P = nRT/V$, where T is absolute temperature. Pressure is proportional to Kelvin temperature.
- Celsius temperature t and Kelvin temperature satisfy $T = t + 273.15$. Therefore a straight-line pressure versus Celsius-temperature graph extrapolates to zero pressure near $-273\,^{\circ}\mathrm{C}$.

Step 1: Express pressure in terms of the horizontal-axis temperature Write the ideal-gas equation using the Celsius coordinate of the given graph:

$$P = \frac{nR}{V}(t + 273.15).$$

This has the straight-line form $P = at + b$, where $a = nR/V$ and $b = 273.15nR/V$. Different quantities of gas or different fixed volumes can give different slopes and different pressures at $0\,^{\circ}\mathrm{C}$. Those slope differences do not change the common zero-pressure intercept because the same temperature offset appears in every equation.

Step 2: Locate the shared intercept K At K all three extrapolated lines meet the temperature axis, so set $P = 0$:

$$0 = \frac{nR}{V}(t_K + 273.15) \quad \Rightarrow \quad t_K = -273.15\,^{\circ}\mathrm{C}.$$

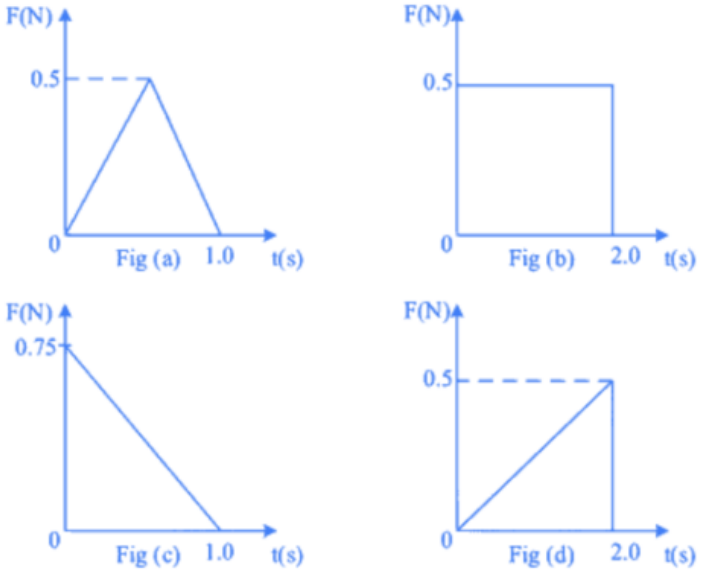
To the precision of the listed options this is $-273\,^{\circ}\mathrm{C}$, corresponding to $0\,\mathrm{K}$. Option (B), $-373\,^{\circ}\mathrm{C}$, would be below absolute zero. Options (C) and (D) correspond to positive Kelvin temperatures and hence nonzero pressures for these fixed gas samples. The graphical intercept is an ideal-gas extrapolation; an actual gas can liquefy before reaching it, which does not change the intended common intercept.

Step 3: Conclusion Point K corresponds to $-273\,^{\circ}\mathrm{C}$, approximately absolute zero. Hence, the correct

option is (A).

Quick Tip to Solve: A constant-volume pressure graph is proportional to temperature only on the Kelvin scale; its Celsius-axis intercept is approximately minus 273 degrees Celsius.

47. Figures (a), (b), (c) and (d) show variation of force with time.
The impulse is highest in figure. [4 marks]



(A) Fig (c)
(B) Fig (d)
(C) Fig (a)
(D) Fig (b)

Correct Answer: (D) Fig (b)

Solution:

Concept:

- The impulse of a force is $J = \int F\,dt = \Delta p$. On a force–time graph it is the signed area under the curve, measured in $\mathrm{N\,s}$.
- A rectangle has area base times height, while a triangle has half that area. Both force scale and duration must be read before comparing graphs; the largest peak alone does not determine the largest impulse.

Step 1: Calculate the areas for figures (a) and (b) Figure (a) is a triangular force pulse of duration $1.0\,\mathrm{s}$ and peak force $0.5\,\mathrm{N}$. Therefore

$$J_a = \frac{1}{2}(1.0)(0.5) = 0.25\,\mathrm{N\,s}.$$

Figure (b) is a rectangular pulse with constant force $0.5\,\mathrm{N}$ for $2.0\,\mathrm{s}$. Its area is

$$J_b = (2.0)(0.5) = 1.00\,\mathrm{N\,s}.$$

The sustained force in (b) acts throughout the interval, whereas the force in (a) rises and falls, reducing its average value.

Step 2: Calculate the remaining areas and compare all four Figure (c) is a triangle of base $1.0\,\mathrm{s}$ and height $0.75\,\mathrm{N}$:

$$J_c = \frac{1}{2}(1.0)(0.75) = 0.375\,\mathrm{N\,s}.$$

Figure (d) is a triangle of base $2.0\,\mathrm{s}$ and height $0.5\,\mathrm{N}$:

$$J_d = \frac{1}{2}(2.0)(0.5) = 0.50\,\mathrm{N\,s}.$$

Thus the order is $J_b > J_d > J_c > J_a$. Although (c) has the greatest peak force, its smaller duration and triangular shape make its area smaller than that of (b). The listed options associate (A) with figure (c), (B) with figure (d), (C) with figure (a), and (D) with figure (b), so only the last choice identifies the largest impulse.

Step 3: Conclusion The impulse is highest in figure b, with value $1.00\,\mathrm{N\,s}$. Hence, the correct option is (D).

Quick Tip to Solve: For each force pulse write down base, height and shape before comparing areas; a tall narrow triangle can give less impulse than a lower rectangle.

---

48. An electron of a hydrogen like atom, having $Z = 4$, jumps from 4^{th} energy state to 2^{nd} energy state. The energy released in this process, will be :

(Given Rch = 13.6 eV)

Where R = Rydberg constant

c = Speed of light in vacuum

h = Planck's constant [4 marks]

(A) 10.5 eV

(B) 40.8 eV

(C) 13.6 eV

(D) 3.4 eV

Correct Answer: (B) 40.8 eV

Solution:

Concept:

- For a one-electron hydrogen-like atom, the Bohr energy of level n is $E_n = -Rch\,Z^2/n^2$. The factor Z^2 accounts for the stronger Coulomb attraction of the nucleus.
- A transition from a higher level to a lower one emits a photon. The released positive energy is

$$E_\gamma = E_i - E_f = Rch\, Z^2(1/n_f^2 - 1/n_i^2).$$

Step 1: Determine the initial and final level energies Here $Z = 4$, $n_i = 4$, $n_f = 2$, and $Rch = 13.6\,\text{eV}$. The initial energy is

$$E_4 = -\frac{13.6(4)^2}{4^2} = -13.6\,\text{eV}.$$

The final energy is

$$E_2 = -\frac{13.6(4)^2}{2^2} = -54.4\,\text{eV}.$$

A lower principal quantum number gives a more negative bound-state energy. The electron therefore loses energy in making this transition, and that loss is carried away by the emitted photon.

Step 2: Subtract in the emission-energy order The magnitude released is the initial atomic energy minus the final atomic energy:

$$E_\gamma = (-13.6) - (-54.4) = 40.8\,\text{eV}.$$

Equivalently, insert the quantum numbers directly:

$$E_\gamma = 13.6(16)\left(\frac{1}{4} - \frac{1}{16}\right) = 13.6(3) = 40.8\,\text{eV}.$$

The atomic energy change is negative, $\Delta E_{\text{atom}} = -40.8\,\text{eV}$, whereas the question asks for the positive released energy. Option (C) gives only the magnitude of the initial level energy. Option (D) omits the nuclear-charge enhancement and is not this transition energy; option (A) likewise fails the level-energy subtraction. The result also exceeds the corresponding hydrogen transition by the required factor $Z^2 = 16$.

Step 3: Conclusion The energy released is $40.8\,\text{eV}$. Hence, the correct option is (B).

Quick Tip to Solve: For a hydrogen-like emission line, calculate the positive bracket using the lower level first and multiply the hydrogen result by the nuclear charge squared.

49. In an amplitude modulation, a modulating signal having amplitude of X Volt is superimposed with a carrier signal of amplitude Y Volt in first case. Then, in second case, the same modulating signal is superimposed with different carrier signal of amplitude 2Y Volt. The ratio of modulation index in the two cases respectively will be : [4 marks]

(A) 2 : 1

(B) 1 : 1

(C) 4 : 1

(D) 1 : 2

Correct Answer: (A) 2 : 1

Solution:

Concept:

- For ordinary amplitude modulation, the modulation index is $\mu = A_m/A_c$, where A_m is the amplitude of the modulating signal and A_c is the amplitude of the unmodulated carrier.
- When the message amplitude stays fixed, the modulation index is inversely proportional to the carrier amplitude. A ratio of modulation indices is dimensionless because both amplitudes are measured in the same unit.

Step 1: Write the modulation index for each carrier In the first case the modulating amplitude is X V and the carrier amplitude is Y V. Thus

$$\mu_1 = \frac{X}{Y}.$$

In the second case the same modulating signal is used, so its amplitude remains X V, while the new carrier amplitude is $2Y$ V. Hence

$$\mu_2 = \frac{X}{2Y}.$$

It is the denominator, rather than the numerator, that doubles. Doubling the carrier amplitude therefore halves the fractional modulation depth for this unchanged message signal.

Step 2: Form the ratio in the requested order The question asks for the first modulation index relative to the second:

$$\frac{\mu_1}{\mu_2} = \frac{X/Y}{X/(2Y)} = \frac{X}{Y}\frac{2Y}{X} = 2.$$

Consequently, $\mu_1 : \mu_2 = 2 : 1$. Option (B) would require the carrier amplitude to remain unchanged or the message amplitude also to double. Option (C) confuses an amplitude-based index with a squared-amplitude quantity such as power. Option (D) reverses the requested order. No numerical values of X and Y are necessary because the common message amplitude and original carrier amplitude cancel. This comparison uses amplitude ratios, not the ratio of total transmitted powers.

Step 3: Conclusion The modulation indices in the two cases are in the ratio $2 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the message amplitude in the numerator: doubling only the carrier amplitude halves the modulation index, so the first-to-second ratio is two.

50. A Carnot engine operating between two reservoirs has efficiency $\frac{1}{3}$. When the temperature of cold reservoir raised by x, its efficiency decreases to $\frac{1}{6}$. The value of x, if the temperature of hot reservoir is 99°C, will be : [4 marks]

(A) 66 K

(B) 62 K

(C) 16.5 K

(D) 33 K

Correct Answer: (B) 62 K

Solution:

Concept:

- A reversible Carnot engine has efficiency $\eta = 1 - T_c/T_h$, where both reservoir temperatures must be measured on the absolute Kelvin scale.
- For fixed hot-reservoir temperature T_h, increasing the cold-reservoir temperature by x changes the efficiency by $\Delta\eta = -x/T_h$. A temperature difference has the same numerical value in kelvin and degrees Celsius.

Step 1: Convert the hot temperature The value 99°C is not an absolute temperature. Convert it before using the temperature ratio:

$$T_h = 99 + 273 = 372\,\text{K}.$$

The engine initially rejects heat to a colder reservoir, whose temperature follows from the stated initial efficiency.

Step 2: Find both cold-reservoir temperatures Initially, $1/3 = 1 - T_c/372$, so

$$T_c = 372\left(1 - \frac{1}{3}\right) = 248\,\text{K}.$$

After the cold reservoir is warmed, $1/6 = 1 - T_c'/372$. Hence

$$T_c' = 372\left(1 - \frac{1}{6}\right) = 310\,\text{K}.$$

Both cold temperatures remain below the unchanged hot temperature, consistent with engine operation.

Step 3: Calculate and compare the rise The required rise is the final cold temperature minus its initial value:

$$x = T_c' - T_c = 310 - 248 = 62\,\text{K}.$$

Equivalently, $x = 372(1/3 - 1/6) = 372/6$. The values 66 K, 16.5 K and 33 K do not produce the specified efficiency drop of $1/6$; using Celsius directly in the efficiency ratio would give an incorrect result.

Step 4: Conclusion The cold reservoir is warmed by $\boxed{62\,\text{K}}$. Hence, the correct option is (B).

Quick Tip to Solve: At fixed hot temperature, multiply the drop in Carnot efficiency by the hot temperature in kelvin: $x = T_h(\eta_1 - \eta_2)$.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A square shaped coil of area $70\ \text{cm}^2$ having 600 turns rotates in a magnetic field of $0.4\ \text{wbm}^{-2}$, about an axis which is parallel to one of the side of the coil and perpendicular to the direction of field. If the coil completes 500 revolution in a minute, the instantaneous emf when the plane of the coil is inclined at 60° with the field, will be _____ V. (Take $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- The magnetic flux linkage of a coil is $N\Phi = NBA\cos\theta$, where θ is the angle between the field and the area normal.
- For uniform rotation, Faraday's law gives emf magnitude $|\mathcal{E}| = NBA\omega|\sin\theta|$. If the plane makes angle α with the field, then $\theta = 90^\circ - \alpha$.

Step 1: Convert area and rotational speed The area is $70\,\text{cm}^2 = 70 \times 10^{-4}\,\text{m}^2 = 0.007\,\text{m}^2$. The frequency and angular speed are

$$f = \frac{500}{60} = \frac{25}{3}\,\text{s}^{-1}, \qquad \omega = 2\pi f = \frac{50\pi}{3}\,\text{rad}\,\text{s}^{-1}.$$

The specified rotation axis changes the angle of the area normal to the field, so the usual rotating-coil expression applies.

Step 2: Find the peak emf Use $N = 600$ and $B = 0.4\,\text{T}$. With the supplied $\pi = 22/7$,

$$\begin{aligned}\mathcal{E}_0 &= NBA\omega \\ &= 600(0.4)(70 \times 10^{-4})\frac{50}{3}\frac{22}{7} \\ &= 88\,\text{V}.\end{aligned}$$

This is the peak value; it occurs when the coil plane is parallel to the magnetic field, rather than at the orientation in the question.

Step 3: Use the angle of the normal The plane is inclined at 60°, so the normal is inclined at 30° to the field. Therefore

$$|\mathcal{E}| = \mathcal{E}_0 \sin 30^\circ = 88\left(\frac{1}{2}\right) = 44\,\text{V}.$$

Using $\sin 60^\circ$ would incorrectly treat the plane angle as the normal angle. Since no rotation sense or terminal polarity is specified, the requested instantaneous emf is its magnitude.

Step 4: Conclusion The instantaneous induced emf has magnitude $\boxed{44\,\text{V}}$. The numerical entry is 44.

Quick Tip to Solve: For a stated angle between the coil plane and the field, use $NBA\omega\cos\alpha$; the sine formula uses the angle of the area normal.

52. A block is fastened to a horizontal spring. The block is pulled to a distance $x = 10$ cm from its equilibrium position (at $x = 0$) on a frictionless surface from rest. The energy of the block at $x = 5$ cm is 0.25 J. The spring constant of the spring is _____ Nm^{-1} [4 marks]

Solution:

Concept:

- For a spring with force constant k, the elastic potential energy at displacement x is $U = \frac{1}{2}kx^2$. Mechanical energy is conserved on the frictionless surface.
- A block released from rest at amplitude a has total energy $E = \frac{1}{2}ka^2$; its kinetic energy at displacement x is $K = \frac{1}{2}k(a^2 - x^2)$.

Step 1: Identify the energy used in the numerical entry The block starts from rest at $a = 10\,\mathrm{cm} = 0.10\,\mathrm{m}$, and subsequently moves toward equilibrium. The stated $0.25\,\mathrm{J}$ is interpreted as the kinetic energy of the block at $x = 5\,\mathrm{cm} = 0.05\,\mathrm{m}$. This is the interpretation corresponding to the numerical answer key; spring potential energy belongs to the elastic interaction.

Step 2: Apply energy conservation Initially all the mechanical energy is elastic. At the indicated position, some remains elastic and the rest is kinetic:

$$\frac{1}{2}ka^2 = \frac{1}{2}kx^2 + K.$$

Thus

$$\begin{aligned} 0.25 &= \frac{1}{2}k\left[(0.10)^2 - (0.05)^2\right] \\ &= \frac{1}{2}k(0.0100 - 0.0025) \\ &= 0.00375k. \end{aligned}$$

Using metres is essential because the specified energy is in joules.

Step 3: Evaluate and check the interpretation It follows that

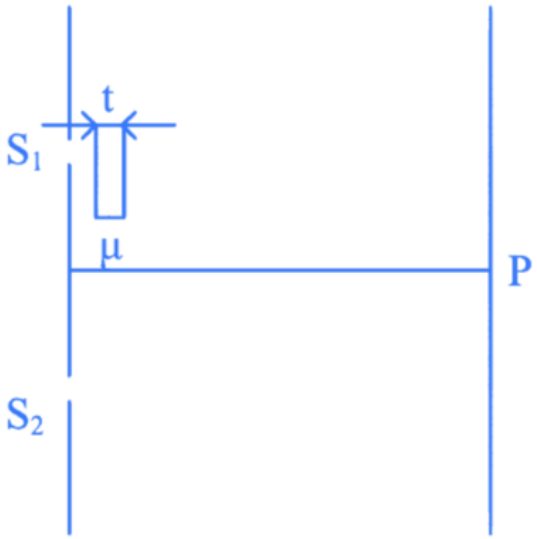
$$k = \frac{0.25}{0.00375} = 66.666\ldots\,\mathrm{N\,m}^{-1}.$$

Rounding to the integer entry gives 67. With this value, total mechanical energy is $1/3$ J, spring energy at 5 cm is $1/12$ J, and their difference is exactly $1/4$ J. The printed word "energy is not qualified: if it meant total mechanical energy, it would instead give $k = 50\,\mathrm{N\,m}^{-1}$; if it meant spring potential energy there, it would give $200\,\mathrm{N\,m}^{-1}$.

Step 4: Conclusion Interpreting the stated energy as the block's kinetic energy, $k = 66.67\,\mathrm{N\,m}^{-1}$, giving the keyed integer entry $\boxed{67}$.

Quick Tip to Solve: At a known displacement of an oscillator released from rest, distinguish $K = \frac{1}{2}k(a^2 - x^2)$ from the constant total energy $\frac{1}{2}ka^2$.

53. As shown in the figure, in Young's double slit experiment, a thin plate of thickness $t = 10\mu\text{m}$ and refractive index $\mu = 1.2$ is inserted infront of slit S_1. The experiment is conducted in air $(\mu = 1)$ and uses a monochromatic light of wavelength $\lambda = 500\ \text{nm}$. Due to the insertion of the plate, central maxima is shifted by a distance of $x\beta_0.\beta_0$ is the fringe-width befor the insertion of the plate. The value of the x is _____. [4 marks]



Solution:

Concept:

- A transparent plate replacing air over thickness t introduces extra optical path $\Delta = (\mu - 1)t$, with μ measured relative to air.
- In Young's experiment, fringe width is $\beta_0 = \lambda D/d$. An additional constant path difference shifts the central fringe by magnitude $y = \Delta D/d$, so $y/\beta_0 = \Delta/\lambda$.

Step 1: Calculate the extra optical path The plate is placed in front of S_1, and the surrounding medium is air. Thus the same geometrical thickness that previously contributed optical path t now contributes μt. The excess path is

$$\Delta = (1.2 - 1)(10\,\mu\text{m}) = 2\,\mu\text{m}.$$

The full optical path μt is not the excess: the air path must be subtracted.

Step 2: Relate the shift to the old fringe width Let the slit separation be d and screen distance be D. At the new central maximum, the geometrical path difference compensates the extra optical path through the plate. Consequently

$$|y| = \frac{D}{d}(\mu - 1)t, \qquad \beta_0 = \frac{\lambda D}{d}.$$

The shift is toward the slit in front of which the plate is inserted. The requested number is the shift magnitude in fringe-width units, so the unknown geometrical factor cancels.

Step 3: Evaluate the dimensionless multiplier Convert the wavelength consistently: $500\,\text{nm} = 0.5\,\mu\text{m}$.

Then

$$x = \frac{|y|}{\beta_0} = \frac{(\mu - 1)t}{\lambda} = \frac{2\,\mu\text{m}}{0.5\,\mu\text{m}} = 4.$$

The plate changes the phase offset between the two beams; within the thin-plate approximation used here it does not change the fringe spacing. A shift of four spacings corresponds to the added optical path of four wavelengths.

Step 4: Conclusion The central maximum shifts by $4\beta_0$ toward S_1, so the requested dimensionless value is $\boxed{x = 4}$.

Quick Tip to Solve: Count the extra wavelengths introduced by the plate: $(\mu - 1)t/\lambda$ directly gives the shift measured in fringe widths.

---

54. Nucleus A having $Z = 17$ and equal number of protons and neutrons has 1.2 MeV binding energy per nucleon.
Another nucleus B of $Z = 12$ has total 26 nucleons and 1.8 MeV binding energy per nucleons.
The difference of binding energy of B and A will be _____ MeV. [4 marks]

Solution:

Concept:

- The atomic number Z counts protons. The mass number $A = Z + N$ counts all nucleons, where N is the neutron number.
- Total binding energy equals binding energy per nucleon multiplied by the number of nucleons: $B = A(B/A)$. A difference of total binding energies must be formed after this multiplication.

Step 1: Count the nucleons in nucleus A Nucleus A has $Z_A = 17$ and equal numbers of protons and neutrons. Its neutron number is therefore also 17, giving

$$A_A = 17 + 17 = 34.$$

The quoted $1.2\,\text{MeV}$ is per nucleon, not the binding energy of the whole nucleus. Thus

$$B_A = 34(1.2) = 40.8\,\text{MeV}.$$

The letter A used to name this nucleus should not be confused with the general mass-number symbol.

Step 2: Calculate the binding energy of nucleus B The question directly states that nucleus B has 26 nucleons. Its atomic number 12 implies 12 protons and 14 neutrons, but all 26 contribute to the total binding-energy calculation. Therefore

$$B_B = 26(1.8) = 46.8\,\text{MeV}.$$

Using only the proton count would leave out the neutrons and would not calculate a total nuclear binding energy.

Step 3: Subtract the total binding energies The requested difference is the binding energy of B minus

that of A:

$$B_B - B_A = 46.8 - 40.8 = 6.0\,\text{MeV}.$$

Although B contains fewer nucleons, its greater binding energy per nucleon makes its total larger in this example. Subtracting the per-nucleon values, $1.8 - 1.2 = 0.6\,\text{MeV}$, would answer a different question. No conversion into joules is needed because the required unit is MeV.

Step 4: Conclusion The total binding energy of nucleus B exceeds that of nucleus A by $\boxed{6\,\text{MeV}}$. The numerical entry is 6.

Quick Tip to Solve: For total binding-energy differences, first multiply each per-nucleon value by its own mass number, then subtract the two totals.

---

55. Moment of inertia of a disc of mass 'M' and radius 'R' about any of its diameter is $\frac{MR^2}{4}$. The moment of inertia of this disc about an axis normal to the disc and passing through a point on its edge will be, $\frac{x}{2}$ MR^2. The value of x is _____. [4 marks]

Solution:

Concept:

- For a plane lamina, the perpendicular-axis theorem gives $I_z = I_x + I_y$, where all three axes meet at the same point and z is normal to the plane.
- The parallel-axis theorem gives $I = I_{\text{CM}} + Md^2$ for an axis parallel to the centre-of-mass axis and displaced by perpendicular distance d.

Step 1: Find the central normal moment of inertia Choose two perpendicular diameters through the centre in the plane of the uniform disc. Rotational symmetry makes their moments equal. The given moment about either diameter is $MR^2/4$, so the perpendicular-axis theorem gives

$$I_{\text{central normal}} = \frac{MR^2}{4} + \frac{MR^2}{4} = \frac{MR^2}{2}.$$

This axis is perpendicular to the disc, as is the required axis, but passes through its centre rather than its edge.

Step 2: Shift to the normal axis at the edge The required axis remains normal to the disc and passes through a point on the rim. It is therefore parallel to the central normal axis. The shortest distance between these axes is one radius, $d = R$. Apply the parallel-axis theorem:

$$I_{\text{edge normal}} = I_{\text{central normal}} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2.$$

The added term is positive: moving a parallel axis away from the centre of mass increases the moment of inertia.

Step 3: Compare the stated coefficients The question expresses the result as $I = (x/2)MR^2$.

Comparing this with the calculated expression,

$$\frac{x}{2}MR^2 = \frac{3}{2}MR^2 \quad \Longrightarrow \quad x = 3.$$

Using the diameter value directly in the parallel-axis step would be wrong because a diameter lies in the plane and is not parallel to the required normal axis. The two theorems are needed in this order.

Step 4: Conclusion The moment of inertia is $\frac{3}{2}MR^2$, so the required dimensionless coefficient is $\boxed{x = 3}$.

Quick Tip to Solve: First use perpendicular axes to obtain the central normal moment, then use parallel axes to shift that same direction to the rim.

56. The surface of water in a water tank of cross section area $750\ \text{cm}^2$ on the top of a house is h m above the tap level. The speed of water coming out through the tap of cross section area $500\ \text{mm}^2$ is $30\ \text{cm/s}$. At that instant, $\frac{dh}{dt}$ is $x \times 10^{-3}\ \text{m/s}$. The value of x will be _____. [4 marks]

Solution:

Concept:

- For incompressible water, the volume outflow rate through a tap is $Q = av$, where a is its area and v its exit speed.
- For a tank with constant cross-sectional area A, volume changes as $dV/dt = A\,dh/dt$. Outflow reduces the water height, so $A\,dh/dt = -av$.

Step 1: Convert the areas and speed Keep all quantities in SI units before using continuity:

$$\begin{aligned} A &= 750\,\text{cm}^2 = 750 \times 10^{-4} = 0.075\,\text{m}^2, \\ a &= 500\,\text{mm}^2 = 500 \times 10^{-6} = 5.00 \times 10^{-4}\,\text{m}^2, \\ v &= 30\,\text{cm}\,\text{s}^{-1} = 0.30\,\text{m}\,\text{s}^{-1}. \end{aligned}$$

Square-centimetre and square-millimetre conversion factors differ; they cannot be cancelled as if the printed area numbers used the same unit.

Step 2: Balance the water volume The instantaneous volume flow through the tap is

$$Q = av = (5.00 \times 10^{-4})(0.30) = 1.50 \times 10^{-4}\,\text{m}^3\,\text{s}^{-1}.$$

With no inflow stated, the tank loses this volume each second. Therefore

$$\frac{dh}{dt} = -\frac{Q}{A} = -\frac{1.50 \times 10^{-4}}{0.075} = -2.00 \times 10^{-3}\,\text{m}\,\text{s}^{-1}.$$

The tap level is fixed, so decreasing tank volume also decreases the height above that level.

Step 3: Distinguish derivative from speed of fall The magnitude of the rate of fall is $-dh/dt = 2 \times 10^{-3}\,\text{m}\,\text{s}^{-1}$, which corresponds to the answer-key entry 2. Literally, however, the question writes $dh/dt = x \times 10^{-3}\,\text{m}\,\text{s}^{-1}$; the signed coefficient in that expression is -2. A positive derivative would

describe the water rising, contrary to the volume balance. The given exit speed is sufficient, so Bernoulli's equation and the numerical height are unnecessary.

Step 4: Conclusion The signed result is $\boxed{dh/dt = -2 \times 10^{-3}\,\mathrm{m\,s^{-1}}}$, hence literally $x = -2$. The official answer-key value 2 is the magnitude of the rate of fall.

Quick Tip to Solve: Write $A\,dh/dt = -av$ for an emptying tank; retain the minus sign for the derivative and report a positive value only for the speed of fall.

57. A force $\mathrm{F} = (5 + 3y^2)$ acts on a particle in the y-direction, where F is in newton and y is in meter. The work done by the force during a displacement from $y = 2$ m to $y = 5$ m is _____ J. [4 marks]

Solution:

Concept:

- For a displacement along the force, the elemental work is $dW = F(y)\,dy$. The total work is the definite integral of force over the actual coordinate interval.
- A force that changes with position cannot be replaced by its value at either endpoint. Here $F(y) = 5 + 3y^2$, with force in newton and coordinate in metre.

Step 1: Set up the work integral The particle moves from the smaller coordinate $y = 2\,\mathrm{m}$ to the larger coordinate $y = 5\,\mathrm{m}$. Since the force acts in the positive y-direction and is positive throughout this interval, the work is positive. Resolve the dot product along this common direction to obtain

$$W = \int_2^5 (5 + 3y^2)\,dy.$$

The limits are the positions themselves; the change in position alone is not an upper integration limit.

Step 2: Integrate and substitute the endpoints The constant part contributes $5y$, while the position-dependent part contributes y^3:

$$W = \left[5y + y^3\right]_2^5.$$

At the upper endpoint, $5(5) + 5^3 = 25 + 125 = 150$. At the lower endpoint, $5(2) + 2^3 = 10 + 8 = 18$. Therefore,

$$W = 150 - 18 = 132\,\mathrm{J}.$$

Equivalently, the constant force contributes $5(5 - 2) = 15\,\mathrm{J}$ and the variable part contributes $125 - 8 = 117\,\mathrm{J}$. Adding these independent contributions gives the same result and checks the subtraction.

Step 3: Conclusion The work done by the force is $\boxed{132\,\mathrm{J}}$; the value to enter is 132.

Quick Tip to Solve: Integrate the force with respect to its own coordinate, then evaluate at both positions. For $3y^2$, the antiderivative is simply y^3.

58. For a train engine moving with speed of $20\ \text{ms}^{-1}$, the driver must apply brakes at a distance of 500 m before the station for the train to come to rest at the station. If the brakes were applied at half of this distance, the train engine would cross the station with speed $\sqrt{x}\ \text{ms}^{-1}$. The value of x is _____.

(Assuming same retardation is produced by brakes) [4 marks]

Solution:

Concept:

- For constant acceleration along a straight line, $v^2 = u^2 + 2as$, where u is the initial speed, v is the final speed and s is displacement.
- Choose the direction of train motion as positive. Braking then gives a negative acceleration, and the same acceleration must be used in both braking situations.

Step 1: Determine the braking acceleration In the first situation the train starts braking at $u = 20\,\text{m}\,\text{s}^{-1}$ and comes to rest after $s = 500\,\text{m}$. Substituting $v = 0$ gives

$$0 = 20^2 + 2a(500).$$

Thus

$$a = -\frac{400}{1000} = -0.4\,\text{m}\,\text{s}^{-2}.$$

The minus sign expresses retardation. The acceleration is fixed by the first stopping distance and is not doubled merely because the driver starts braking later.

Step 2: Calculate the speed after the shorter distance If braking begins only half as far from the station, the distance available is $s = 250\,\text{m}$. The train still enters this braking interval with speed $20\,\text{m}\,\text{s}^{-1}$. Hence

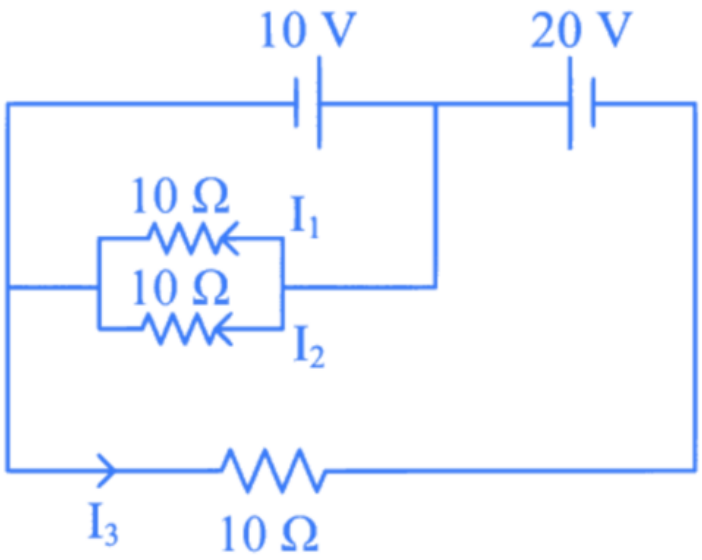
$$v^2 = 20^2 + 2(-0.4)(250) = 400 - 200 = 200.$$

Therefore $v = \sqrt{200}\,\text{m}\,\text{s}^{-1}$. Compare this expression directly with the form $\sqrt{x}\,\text{m}\,\text{s}^{-1}$ specified in the question. It is the square of the numerical speed that is requested, so the numerical entry is not $\sqrt{200}$ or 10.

Step 3: Conclusion The train crosses the station at $\sqrt{200} = 10\sqrt{2}\,\text{m}\,\text{s}^{-1}$. Thus $\boxed{x = 200}$ (dimensionless).

Quick Tip to Solve: With constant braking, the decrease in v^2 is proportional to distance. Half the stopping distance removes half the initial squared speed, leaving $v^2 = 400/2$.

59. **In the given circuit, the value of $\left|\frac{I_1+I_3}{I_2}\right|$ is _____** [4 marks]



Solution:

Concept:

- An ideal voltage source fixes the potential difference between its terminals; the longer plate of its symbol is the positive terminal.
- For a resistor, current in a chosen direction is $I = (V_{\text{start}} - V_{\text{end}})/R$. Retain the signs appropriate to the arrows printed in the circuit.

Step 1: Assign the node potentials from the batteries Call the left vertical wire node L, the junction between the batteries node M, and the right vertical wire node R. Choose $V_L = 0$. The 10 V battery has its positive terminal at M, so $V_M = 10\,\text{V}$. The 20 V battery also has its positive terminal at M, giving

$$V_M - V_R = 20\,\text{V}, \qquad V_R = -10\,\text{V}.$$

The two internal 10 ohm resistors are in parallel between M and L. The bottom 10 ohm resistor is connected between L and R, not across the 20 V source alone.

Step 2: Evaluate the currents in the marked directions The arrows for I_1 and I_2 point from M to L, so both currents are

$$I_1 = I_2 = \frac{10 - 0}{10} = 1\,\text{A}.$$

The arrow for I_3 points along the lower branch from L to R. Consequently,

$$I_3 = \frac{0 - (-10)}{10} = 1\,\text{A}.$$

All three signed values are positive relative to their printed arrows. Substitute them without changing the sign of the lower-branch current:

$$\left|\frac{I_1 + I_3}{I_2}\right| = \left|\frac{1+1}{1}\right| = 2.$$

The ampere units cancel between numerator and denominator.

Step 3: Conclusion The required current ratio is $\boxed{2}$, a dimensionless number.

Quick Tip to Solve: Read battery polarities before combining sources: here their common positive junction makes the outer-node voltage difference $20 - 10 = 10\,\mathrm{V}$.

---

60. A cubical volume is bounded by the surfaces $\mathrm{x} = 0, x = \mathrm{a}, y = 0, y = \mathrm{a}, \mathrm{z} = 0, z = \mathrm{a}$. The electric field in the region is given by $\vec{\mathrm{E}} = \mathrm{E}_0 x\hat{i}$. Where $\mathrm{E}_0 = 4 \times 10^4\ \mathrm{NC}^{-1}\,\mathrm{m}^{-1}$. If $\mathrm{a} = 2\ \mathrm{cm}$, the charge contained in the cubical volume is $\mathrm{Q} \times 10^{-14}\mathrm{C}$. The value of Q is _____.

(Take $\epsilon_0 = 9 \times 10^{-12}\ \mathrm{C}^2/\mathrm{Nm}^2$) [4 marks]

Solution:

Concept:

- Gauss law relates the outward flux of an electric field to enclosed charge: $q_{\text{enc}} = \epsilon_0 \oint \mathbf{E} \cdot d\mathbf{A}$.
- The field $\mathbf{E} = E_0 x\hat{\mathbf{i}}$ has only an x-component. Thus only the two faces perpendicular to the x-axis can contribute to the flux.

Step 1: Calculate the net flux through the cube On the face $x = 0$, the field vanishes, so its flux is zero even though its outward normal points along negative x. On the opposite face $x = a$, the field has the constant value $E_0 a$ and is parallel to its outward normal. Its area is a^2, so

$$\Phi_{x=a} = (E_0 a)a^2 = E_0 a^3.$$

On all four faces of constant y or constant z, the electric field is tangent to the surface; their dot products with the outward area vectors vanish. Therefore $\Phi_{\text{net}} = E_0 a^3$.

Step 2: Apply Gauss law and identify the requested coefficient Convert the side length before cubing it:

$$a = 2\,\mathrm{cm} = 2 \times 10^{-2}\,\mathrm{m}, \qquad a^3 = 8 \times 10^{-6}\,\mathrm{m}^3.$$

Using the given value of ϵ_0,

$$q_{\text{enc}} = (9 \times 10^{-12})(4 \times 10^4)(8 \times 10^{-6})\,\mathrm{C}.$$

The numerical factor is $9 \cdot 4 \cdot 8 = 288$, and the powers combine to $10^{-12+4-6} = 10^{-14}$. Thus $q_{\text{enc}} = 288 \times 10^{-14}\,\mathrm{C}$. It is positive because the outward flux is positive. Compare with the exact coefficient form in the question rather than entering the charge in coulomb directly.

Step 3: Conclusion The enclosed charge is $288 \times 10^{-14}\,\mathrm{C} = 2.88 \times 10^{-12}\,\mathrm{C}$. Hence $\boxed{Q = 288}$ (dimensionless coefficient).

Quick Tip to Solve: For a field linear in one coordinate, the flux of a rectangular box is its volume multiplied by the field gradient: $\Phi = E_0 a^3$ here.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. $O-O$ **bond length in** H_2O_2 **is** $\underline{X}$ **than the** $O-O$ **bond length in** F_2O_2**. The** $O-H$ **bond length in** H_2O_2 **is** $\underline{Y}$ **than that of the** $O-F$ **bond in** F_2O_2**.**

Choose the correct option for $\underline{X}$ **and** $\underline{Y}$ **from those given below :** [4 marks]

(A) X - shorter, Y - longer

(B) X - longer, Y - shorter

(C) X - shorter, Y - shorter

(D) X - longer, Y - longer

Correct Answer: (B) X - longer, Y - shorter

Solution:

Concept:

- Bond length depends on the bonded atoms and the electronic environment. A comparison between two different molecules must use the actual bonds specified, not just the presence of a single bond in a Lewis formula.
- In H_2O_2 the $O-O$ distance is approximately $148\,\text{pm}$, whereas in F_2O_2 it is approximately $122\,\text{pm}$. The $O-H$ distance is approximately $95\,\text{pm}$, much shorter than the $O-F$ distance of approximately $158\,\text{pm}$.

Step 1: Compare the oxygen-oxygen bonds The question first compares the central $O-O$ bond in the two peroxide-type molecules. The characteristic structural data give

$$d_{O-O}(H_2O_2) > d_{O-O}(F_2O_2).$$

Although both skeletal formulas display an oxygen-oxygen linkage, their terminal atoms change the electronic environment substantially. In particular, the unusually short central oxygen-oxygen bond of dioxygen difluoride cannot be treated as identical in length to the peroxide bond. Therefore the first blank is $X =$ longer.

Step 2: Compare the terminal bonds and eliminate the alternatives The second comparison is between a bond to hydrogen and a bond to fluorine. The small hydrogen atom forms the shorter oxygen-hydrogen bond, so

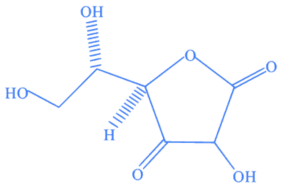
$$d_{O-H}(H_2O_2) < d_{O-F}(F_2O_2).$$

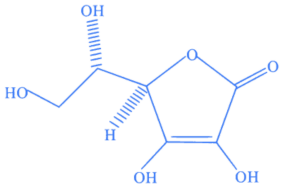
Thus $Y =$ shorter. Option (A) reverses both comparisons. Option (C) has the correct second comparison but the wrong first one. Option (D) has the correct first comparison but incorrectly calls the oxygen-hydrogen bond longer. Only option (B) satisfies both blanks simultaneously.

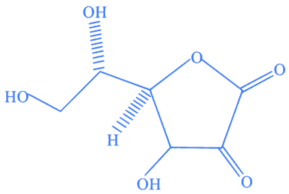
Step 3: Conclusion $\boxed{X = \text{longer}, \quad Y = \text{shorter}}$. Hence, the correct option is (B).

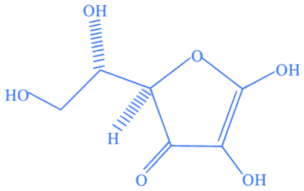
Quick Tip to Solve: Keep the two comparisons separate: the $O-O$ bond is unusually short in F_2O_2, while the $O-H$ bond is shorter than $O-F$.

62. All structures given below are of vitamin C. Most stable of them is : [4 marks]

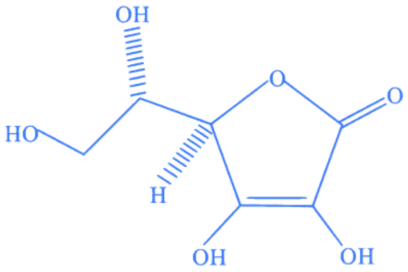
(A) 

(B) 

(C) 

(D) 

Correct Answer: (B)



Solution:

Concept:

- Tautomers have the same atomic framework but differ in the positions of hydrogen atoms and multiple bonds. Their stability depends on conjugation and the functional groups retained, rather than merely counting carbonyl bonds.
- The stable ascorbic acid structure contains a five-membered lactone ring and a conjugated enediol unit: two hydroxyl groups are attached to the two carbons of a carbon-carbon double bond.

Step 1: Identify the conjugated enediol-lactone structure Inspect the locations of the ring carbonyl and the carbon-carbon double bond in each drawing. In structure (B), the carbonyl next to the ring oxygen remains a lactone carbonyl. The adjacent ring bond is a double bond, and each of its carbons carries a hydroxyl group. The relevant sequence is

$$O{=}C{-}C(OH){=}C(OH).$$

The π electrons of the carbon-carbon double bond can interact with the neighboring carbonyl system. The enediol and lactone arrangement provides the characteristic stabilization of vitamin C. The two hydroxyl groups on the double-bonded carbons are the decisive visible feature.

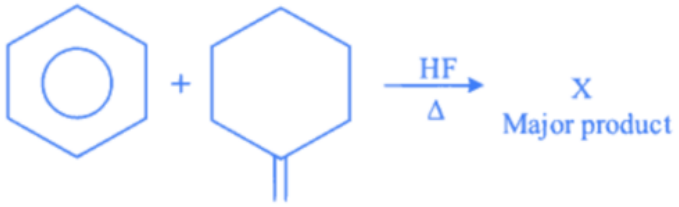
Step 2: Compare the other displayed tautomers Structure (A) replaces one enediol hydroxyl-bearing carbon by a keto group at the other side of the ring, losing the displayed conjugated enediol arrangement. Structure (C) also has a keto-hydroxy arrangement instead of the stabilized enediol pair. Structure (D) moves the carbonyl away from the lactone carbonyl position, while placing a hydroxyl on the carbon adjacent to the ring oxygen. Its conjugation does not retain the same ester-carbonyl and enediol combination as (B). Therefore selecting the structure with the greatest number of carbonyl

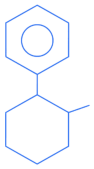
symbols would be misleading; the arrangement of those groups matters.

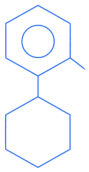
Step 3: Conclusion The conjugated enediol-lactone form shown in (B) is the most stable vitamin C structure among those given. Hence, the correct option is (B).

Quick Tip to Solve: Look for both the intact lactone carbonyl and adjacent $C(OH)=C(OH)$ unit; these two features identify the stable vitamin C tautomer.

63. 'X' **is :** [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- An alkene can be protonated by an acid to form a carbocation. Protonation that gives the more substituted, more stable carbocation determines the principal electrophile.
- Benzene reacts with a carbocation by electrophilic aromatic substitution: attack forms a sigma complex, followed by loss of a proton to restore aromaticity.

Step 1: Generate the carbocation from the exocyclic alkene The alkene in the reaction is methylenecyclohexane: the double bond joins a ring carbon to an external CH_2 group. Addition of a proton to that terminal carbon converts it into a methyl group and leaves the positive charge on the ring carbon. That carbon is then bonded to two ring carbons and one methyl carbon, so it is a tertiary carbocation. The alternative orientation would place the charge on an external primary carbon and is

less favourable. Thus the principal electrophile has the methyl substituent and the positive charge at the same ring carbon.

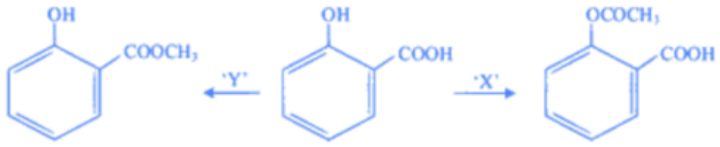
Step 2: Attach benzene and restore aromaticity The benzene π system attacks the positively charged ring carbon. Subsequent removal of a proton from the benzene sigma complex restores the aromatic ring and regenerates the acid catalyst. The final cyclohexane ring consequently bears both a phenyl group and a methyl group on the same carbon: it is 1-methyl-1-phenylcyclohexane. No hydride or alkyl shift is needed because the initially formed carbocation is already tertiary. Option (A) puts the methyl on the adjacent ring carbon, and (C) puts it farther around the ring; neither follows direct capture of this carbocation. Option (B) incorrectly places the methyl group on the benzene ring. The geminal substitution shown in (D) matches the mechanism.

Step 3: Conclusion The major product X is 1-methyl-1-phenylcyclohexane, with phenyl and methyl on the same cyclohexane carbon. Hence, the correct option is (D).

Quick Tip to Solve: For an exocyclic $=CH_2$, protonation at the external carbon makes CH_3 and leaves the more stable carbocation on the ring; attach benzene there.

64. In a reaction,

reagents 'X' and 'Y' respectively are : [4 marks]



(A) $CH_3OH/H^+, \Delta$ and $(CH_3CO)_2O/H^+$

(B) $(CH_3CO)_2O/H^+$ and $CH_3OH/H^+, \Delta$

(C) $CH_3OH/H^+, \Delta$ and $CH_3OH/H^+, \Delta$

(D) $(CH_3CO)_2O/H^+$ and $(CH_3CO)_2O/H^+$

Correct Answer: (B) $(CH_3CO)_2O/H^+$ and $CH_3OH/H^+, \Delta$

Solution:

Concept:

- Salicylic acid contains two distinct functional groups: a phenolic hydroxyl group and a carboxyl group. Each can form an ester through a different reaction.
- Acetic anhydride acetylates the phenolic oxygen, whereas methanol with an acid catalyst and heat esterifies the carboxyl group. Identify the bond that changes before selecting the reagent.

Step 1: Identify the change caused by X The central compound has adjacent $-OH$ and $-COOH$ groups. In the product to the right, the carboxyl group remains $-COOH$, but the phenolic $-OH$ becomes $-OCOCH_3$. Thus X must introduce an acetyl group on oxygen. Acid-catalysed acetic anhydride, $(CH_3CO)_2O$, does this, producing acetylsalicylic acid. Methanol would not introduce the required $-COCH_3$ group.

Step 2: Identify the change caused by Y The product to the left retains the phenolic $-OH$ but has

$-COOCH_3$ instead of $-COOH$. This is Fischer esterification of the carboxyl group with methanol:

$$C_6H_4(OH)COOH + CH_3OH \overset{H^+,\ \Delta}{\rightleftharpoons} C_6H_4(OH)COOCH_3 + H_2O.$$

Hence Y is acidified methanol under heating. Options C and D incorrectly use the same reagent for both transformations, while A reverses the two reagents.

Step 3: Conclusion $X = (CH_3CO)_2O/H^+$ and $Y = CH_3OH/H^+, \Delta$. Hence, the correct option is (B).

Quick Tip to Solve: Track the phenolic O–H and the carboxyl O–H separately: acetylation changes the former, while methanol esterifies the latter.

---

65. Given below are two statements :

Statement I : Sulphanilic acid gives esterification test for carboxyl group.

Statement II : Sulphanilic acid gives red colour in Lassigne's test for extra element detection.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct

(B) Statement I is correct but Statement II is incorrect

(C) Both Statement I and Statement II are incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (D) Statement I is incorrect but Statement II is correct

Solution:

Concept:

- Sulphanilic acid is $H_2N-C_6H_4-SO_3H$: its acidic group is sulfonic acid, not a carboxyl group. The names of acids do not by themselves establish the presence of $-COOH$.
- When nitrogen and sulfur occur together, sodium fusion can convert them into thiocyanate. Ferric ions then give a blood-red thiocyanato complex in the combined nitrogen–sulfur test.

Step 1: Test Statement I from the functional groups Sulphanilic acid contains an amino group $-NH_2$ and a sulfonic acid group $-SO_3H$ attached to benzene. It has no $-COOH$ group. The esterification test for a carboxyl group relies on producing a characteristic ester from that group with an alcohol and acid catalyst. Sulfonic acid must not be confused with carboxylic acid merely because both contain oxygen and an acidic hydrogen. Therefore Statement I is incorrect.

Step 2: Test Statement II from nitrogen and sulfur Both nitrogen and sulfur are present in sulphanilic acid. In the combined Lassaigne test, sodium fusion produces $NaSCN$. On treatment with ferric ions, thiocyanate gives the characteristic blood-red complex:

$$Fe^{3+} + SCN^- \rightleftharpoons [Fe(SCN)]^{2+}.$$

Thus Statement II is correct. A and B wrongly accept Statement I. C rejects the red-colour test even though the molecule has both required extra elements. Only D matches the two independently

established truth values.

Step 3: Conclusion Statement I is incorrect and Statement II is correct. Hence, the correct option is (D).

Quick Tip to Solve: Write sulphanilic acid as aminobenzenesulfonic acid first: it has N and S for the red combined test, but no carboxyl group.

66. Which element is not present in Nessler's reagent? [4 marks]

(A) Mercury

(B) Iodine

(C) Potassium

(D) Oxygen

Correct Answer: (D) Oxygen

Solution:

Concept:

- The characteristic active salt in Nessler's reagent is potassium tetraiodomercurate(II), $K_2[HgI_4]$, used for detecting ammonia.
- To answer an elemental-composition MCQ, inspect the chemical formula and count the elements explicitly. Distinguish the complex salt from the alkaline medium in which the reagent is used.

Step 1: Identify the salt in the reagent Nessler's reagent is conventionally identified in this question by potassium tetraiodomercurate(II). Its complex ion is $[HgI_4]^{2-}$, with mercury as the central metal and four iodide ligands. Two potassium ions balance the negative charge:

$$2\,K^+ + [HgI_4]^{2-} \longrightarrow K_2[HgI_4].$$

The formula contains potassium, mercury and iodine. It contains neither an oxide ligand nor any oxygen atom in the complex salt.

Step 2: Compare the four named elements Mercury occurs as Hg, so A is present. Iodine occurs as four iodide ligands, so B is present. Potassium is the counter-ion, so C is present. Oxygen does not occur in $K_2[HgI_4]$, making D the intended selection. Strictly, the reagent is an alkaline solution and its aqueous potassium hydroxide medium contains oxygen; the question uses the customary shorthand of referring to the Nessler complex salt rather than the complete solution. This distinction does not change the formula of the active iodometallate salt.

Step 3: Conclusion Oxygen is absent from the Nessler complex salt $K_2[HgI_4]$. Hence, the correct option is (D).

Quick Tip to Solve: Recall the active salt as potassium tetraiodomercurate(II): K, Hg and I are immediately visible in its formula.

67. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : α-halocarboxylic acid on reaction with dil. NH_3 gives good yield of α-amino carboxylic acid whereas the yield of amines is very low when prepared from alkyl halides.

Reason (R) : Amino acids exist in zwitter ion form in aqueous medium.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) (A) is not correct but (R) is correct

(B) (A) is correct but (R) is not correct

(C) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(D) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Correct Answer: (D) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Concept:

- Ammonia substitutes the halogen in an alpha-halocarboxylic acid to give an alpha-amino acid: $RCHXCOOH + 2\,NH_3 \longrightarrow RCH(NH_2)COOH + NH_4X$.
- An amino acid in aqueous medium predominantly has the zwitterionic form $RCH(NH_3{}^+)COO^-$. Protonation removes the nitrogen lone pair from availability for further alkylation.

Step 1: Evaluate the assertion An alkyl halide treated with ammonia first gives a primary amine. That amine still has a nucleophilic lone pair and can react with more alkyl halide, producing secondary and tertiary amines and finally quaternary ammonium salts. Consequently, a mixture is obtained and the isolated primary-amine yield is low. In the alpha-halo acid reaction the product is an amino acid, whose behaviour in water differs from that of an ordinary free amine. The assertion is correct.

Step 2: Evaluate the reason The carboxyl group can transfer a proton to the amino group. The resulting amino acid has a positive ammonium group and a negative carboxylate group within the same molecule:

$$RCH(NH_2)COOH \rightleftharpoons RCH(NH_3{}^+)COO^-.$$

Thus the reason is correct; a zwitterion has both charges but has zero overall charge.

Step 3: Establish the explanatory connection The protonated $-NH_3{}^+$ group is much less able to attack another halo compound than a free $-NH_2$ group. Zwitterion formation therefore suppresses successive N-alkylation and accounts for the better amino-acid yield. A and B each reject a true statement. C accepts both truths but misses their causal connection. D correctly states both the truth and the explanation.

Step 4: Conclusion Both Assertion and Reason are correct, and the Reason explains the Assertion. Hence, the correct option is (D).

Quick Tip to Solve: For ammonolysis yields, ask whether the first product retains a free nitrogen lone pair that can react again.

68. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Cu^{2+} **in water is more stable than** Cu^{+}**.**

Reason (R) : Enthalpy of hydration for Cu^{2+} **is much less than that of** Cu^{+}**.**

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) (A) is correct but (R) is not correct

(B) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(C) (A) is not correct but (R) is correct

(D) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Correct Answer: (D) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Concept:

- Aqueous ionic stability depends on the total energetic balance, including hydration, rather than on the gas-phase electronic configuration alone.
- Cu^{2+} has a greater charge density than Cu^{+}, so hydration of the divalent ion releases substantially more energy. Hydration enthalpies are negative; their magnitudes must be distinguished from signed numerical values.

Step 1: Evaluate the assertion in water Although Cu^{+} has a filled $3d^{10}$ configuration, in aqueous solution it undergoes disproportionation:

$$2\,Cu^{+} \longrightarrow Cu^{2+} + Cu\,(s).$$

The strong hydration of Cu^{2+} helps stabilise the products. Thus Cu^{2+} is more stable than Cu^{+} in water under the conditions meant by the question.

Step 2: Interpret the hydration comparison carefully Hydration of both ions is exothermic, so both hydration enthalpies are negative. Because the double charge attracts water molecules more strongly, hydration of Cu^{2+} releases substantially more energy: $|\Delta H_{hyd}(Cu^{2+})| > |\Delta H_{hyd}(Cu^{+})|$. Therefore the signed enthalpy satisfies $\Delta H_{hyd}(Cu^{2+}) < \Delta H_{hyd}(Cu^{+}) < 0$. On the literal thermodynamic reading of the printed word enthalpy, the Reason is true.

Step 3: Resolve the truth values and the key The more negative hydration enthalpy of Cu^{2+} stabilises that ion in water and compensates for the extra energy required to form the divalent ion. Thus the Reason explains the Assertion. A incorrectly rejects the signed-enthalpy comparison, B incorrectly denies the causal connection and C incorrectly rejects the Assertion. The final key selects A by comparing magnitudes, for which the claim of a smaller value is false. The printed enthalpy is signed, supporting D.

Step 4: Conclusion For signed hydration enthalpies, both statements are correct and the Reason explains the Assertion. Hence, the correct option is (D). This differs from the question-sheet key selecting the first option, which compares magnitudes; the signed inequality above proves the literal thermodynamic reading.

Quick Tip to Solve: Always state whether hydration enthalpy means a signed negative value or its magnitude before using words such as higher or lower.

69. The effect of addition of helium gas to the following reaction in equilibrium state, is :

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

[4 marks]

(A) the equilibrium will shift in the forward direction and more of Cl_2 and PCl_3 gases will be produced.

(B) addition of helium will not affect the equilibrium.

(C) helium will deactivate PCl_5 and reaction will stop.

(D) the equilibrium will go backward due to suppression of dissociation of PCl_5.

Correct Answer: (B) addition of helium will not affect the equilibrium.

Solution:

Concept:

- At fixed temperature, the equilibrium constant is $K_p = p_{PCl_3} p_{Cl_2} / p_{PCl_5}$. Helium is chemically inert, so its effect depends on whether volume or total pressure is held fixed.
- Adding inert gas at constant volume leaves each reacting partial pressure unchanged. At constant total pressure it expands the mixture, favouring the side with more gaseous moles.

Step 1: Consider addition at constant volume For each reacting species i, $p_i = n_i RT/V$. Adding helium changes neither its mole number n_i nor T or V. Hence the partial pressures of PCl_5, PCl_3 and Cl_2 remain unchanged even though the total pressure rises. The reaction quotient remains equal to K_p; no equilibrium shift occurs. Under this condition option B is correct.

Step 2: Consider addition at constant total pressure The volume increases when helium is added while total pressure is kept fixed. If each reacting partial pressure is initially multiplied by $\lambda < 1$, the new quotient is

$$Q_p = \frac{(\lambda p_{PCl_3})(\lambda p_{Cl_2})}{\lambda p_{PCl_5}} = \lambda K_p < K_p.$$

The forward reaction then occurs to restore equilibrium. It produces two gaseous moles from one and thus increases the amounts of PCl_3 and Cl_2. Option A is correct for this condition.

Step 3: Resolve the missing experimental condition The question does not state which constraint applies. Helium neither deactivates PCl_5 nor stops the reaction, so C is false. Neither standard constraint causes the backward shift claimed in D. A and B describe the two valid cases; the official final key accepts both of them.

Step 4: Conclusion At constant volume there is no shift, giving option (B); at constant total pressure the equilibrium shifts forward, giving option (A). The official answer key accepts both (A) and (B).

Quick Tip to Solve: For inert-gas additions, write the volume or pressure constraint first; total pressure alone does not determine reacting partial pressures.

70. Which one of the following sets of ions represents a collection of isoelectronic species?

(Given : Atomic Number : $\mathrm{F}:9, \mathrm{Cl}:17, \mathrm{Na}=11, \mathrm{Mg}=12, \mathrm{Al}=13, \mathrm{K}=19, \mathrm{Ca}=20, \mathrm{Sc}=21$**)** [4 marks]

(A) $\mathrm{N}^{3-}, \mathrm{O}^{2-}, \mathrm{F}^{-}, \mathrm{S}^{2-}$

(B) $\mathrm{Ba}^{2+}, \mathrm{Sr}^{2+}, \mathrm{K}^{+}, \mathrm{Ca}^{2+}$

(C) $\mathrm{Li}^{+}, \mathrm{Na}^{+}, \mathrm{Mg}^{2+}, \mathrm{Ca}^{2+}$

(D) $\mathrm{K}^{+}, \mathrm{Cl}^{-}, \mathrm{Ca}^{2+}, \mathrm{Sc}^{3+}$

Correct Answer: (D) $\mathrm{K}^{+}, \mathrm{Cl}^{-}, \mathrm{Ca}^{2+}, \mathrm{Sc}^{3+}$

Solution:

Concept:

- Isoelectronic species have the same total number of electrons, even though their nuclear charges and ionic radii can differ.
- For a cation of charge $+m$, electron count is $Z - m$; for an anion of charge $-m$, it is $Z + m$. Apply the sign to the electron count rather than to the proton count.

Step 1: Count electrons in the proposed matching set For option D the four electron counts are

$$\begin{aligned} \mathrm{K}^{+}: &\quad 19 - 1 = 18, \\ \mathrm{Cl}^{-}: &\quad 17 + 1 = 18, \\ \mathrm{Ca}^{2+}: &\quad 20 - 2 = 18, \\ \mathrm{Sc}^{3+}: &\quad 21 - 3 = 18. \end{aligned}$$

All four ions have the argon electron configuration, $1s^2 2s^2 2p^6 3s^2 3p^6$. Their different positive or negative charges do not prevent them from being isoelectronic.

Step 2: Check why the other collections fail In A, N^{3-}, O^{2-} and F^{-} each have ten electrons, but S^{2-} has eighteen, so the collection is not uniform. In B, Ba^{2+} has 54 electrons and Sr^{2+} has 36, whereas K^{+} and Ca^{2+} have eighteen. In C, Li^{+} has two electrons, Na^{+} and Mg^{2+} have ten, and Ca^{2+} has eighteen. A common ionic charge or a noble-gas configuration in each ion is insufficient: the actual electron numbers must be equal across the entire set.

Step 3: Conclusion K^{+}, Cl^{-}, Ca^{2+} and Sc^{3+} each contain eighteen electrons. Hence, the correct option is (D).

Quick Tip to Solve: Write a short row of electron counts beneath each option; one unequal count eliminates the whole collection.

71. For electron gain enthalpies of the elements denoted as $\Delta_{eg}H$, the incorrect option is : [4 marks]

(A) $\Delta_{eg}H(I) < \Delta_{eg}H(At)$

(B) $\Delta_{eg}H(Te) < \Delta_{eg}H(Po)$

(C) $\Delta_{eg}H(Cl) < \Delta_{eg}H(F)$

(D) $\Delta_{eg}H(Se) < \Delta_{eg}H(S)$

Correct Answer: (D) $\Delta_{eg}H(Se) < \Delta_{eg}H(S)$

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change for $X(g) + e^- \longrightarrow X^-(g)$. An exothermic gain has a negative value, so a more negative value is numerically smaller.
- Down a group, increasing size and shielding usually reduce the attraction for an added electron. Compact second-period atoms can be exceptions because of strong electron-electron repulsion.

Step 1: Interpret the sign before comparing The question asks for the incorrect inequality, not for the element with the greatest magnitude of electron affinity. For signed enthalpies, $-200 < -190$. Thus a less exothermic electron gain corresponds to a larger enthalpy. The ordinary downward trend makes iodine more negative than astatine and tellurium more negative than polonium; (A) and (B) are correct.

Step 2: Check the compact-atom exception Fluorine receives its electron in a compact 2p orbital, where electron-electron repulsion is unusually large. Chlorine receives it in the larger 3p shell and releases more energy. Consequently, $\Delta_{eg}H(Cl) < \Delta_{eg}H(F)$, making (C) correct. This exception should not be extended to the sulphur-selenium pair.

Step 3: Compare sulphur and selenium Sulphur has a more negative electron gain enthalpy than selenium: representative values are about -200 and $-195\,kJ\,mol^{-1}$, respectively. Hence $\Delta_{eg}H(S) < \Delta_{eg}H(Se)$. Option (D) writes this inequality in the reverse direction and is the only incorrect statement.

Step 4: Conclusion The incorrect inequality is $\Delta_{eg}H(Se) < \Delta_{eg}H(S)$. Hence, the correct option is (D).

Quick Tip to Solve: Convert every affinity comparison into signed enthalpies first: more energy released means a more negative, and therefore smaller, value.

72. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A) : An aqueous solution of KOH when used for volumetric analysis, its concentration should be checked before the use.

Reason (R) : On aging, KOH solution absorbs atmospheric CO_2.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) (A) is correct but (R) is not correct

(B) (A) is not correct but (R) is correct

(C) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(D) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Correct Answer: (D) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Concept:

- A titration calculation uses the actual concentration of the standard solution. A solution that reacts with air must be standardised again before accurate use.
- Potassium hydroxide absorbs atmospheric carbon dioxide and forms potassium carbonate: $2\,KOH + CO_2 \longrightarrow K_2CO_3 + H_2O$.

Step 1: Evaluate the assertion An aqueous potassium hydroxide solution is not a stable primary standard. Its composition can change during storage, so its nominal concentration need not equal its current hydroxide concentration. Before using its measured volume to calculate an unknown amount, its effective concentration should be determined by standardisation with a suitable standard acid. Assertion (A) is therefore correct.

Step 2: Evaluate the reason Carbon dioxide present in air dissolves into the alkaline solution. It is consumed by potassium hydroxide, producing carbonate and water according to

$$2\,KOH\,(aq) + CO_2(g) \longrightarrow K_2CO_3(aq) + H_2O\,(l).$$

Each mole of absorbed carbon dioxide converts two moles of hydroxide into carbonate. Thus aging changes the chemical composition even if no fresh solute is deliberately added. Reason (R) is correct.

Step 3: Test the explanatory connection The conversion of hydroxide into carbonate changes the concentration relevant to the titration and may alter the observed endpoint behaviour because carbonate itself is a base. This is precisely why the solution concentration is checked before use. Options (A) and (B) wrongly reject one true statement; (C) wrongly denies their direct causal connection.

Step 4: Conclusion Both the assertion and the reason are correct, and the reason correctly explains the assertion. Hence, the correct option is (D).

Quick Tip to Solve: For stored alkali solutions, check absorption of atmospheric carbon dioxide before trusting the concentration written on the bottle.

73. The complex cation which has two isomers is : [4 marks]

(A) $[Co\,(NH_3)_5\,Cl]^+$

(B) $[Co\,(H_2O)_6]^{3+}$

(C) $[Co\,(NH_3)_5\,NO_2]^{2+}$

(D) $[Co\,(NH_3)_5\,Cl]^{2+}$

Correct Answer: (C) $[Co\,(NH_3)_5\,NO_2]^{2+}$

Solution:

Concept:

- An ambidentate ligand has two different donor atoms and can attach to the same metal through either one. The resulting complexes are linkage isomers.
- For an octahedral complex of type MA_5B, all six possible positions for the single B ligand are equivalent; there is no cis-trans pair.

Step 1: Identify the ligand with two donor atoms In option (C), the nitrite ligand can coordinate through nitrogen or oxygen. Nitrogen attachment gives the nitro complex, whereas oxygen attachment gives the nitrito complex. These have the same composition and charge but different metal-ligand bonds:

$$[Co(NH_3)_5(NO_2)]^{2+} \quad \text{and} \quad [Co(NH_3)_5(ONO)]^{2+}.$$

The first notation denotes N-bonded nitrite, and the second explicitly shows the O-bonded arrangement.

Step 2: Count the alternatives for the other ligands The chloro ligand in (A) and (D) has no alternative donor atom, and ammonia coordinates through nitrogen. Changing the overall charge in (A) does not create another bonding mode. Both have five identical ammine ligands and a single chloro ligand, so relocating chlorine in an octahedron gives an equivalent arrangement rather than a distinct geometrical isomer.

Step 3: Exclude the homoleptic complex Option (B) contains six identical water ligands, each bonded through oxygen. Every position has the same ligand, so there is neither linkage isomerism nor a cis-trans distinction. Thus (A), (B) and (D) do not supply the required pair; only nitrite provides the two distinct donor linkages in (C).

Step 4: Conclusion $[Co(NH_3)_5NO_2]^{2+}$ has nitro and nitrito linkage isomers. Hence, the correct option is (C).

Quick Tip to Solve: When a complex has five identical ligands, look for an ambidentate sixth ligand before searching for geometrical isomers.

74. The structures of major products A, B and C in the following reaction are sequence. Write $R = CH_3CH_2CH_2CH(CH_3)$. The sequence is

$$RCHO \xrightarrow[NaCN,\, H_2O]{NaHSO_3,\, dil\cdot HCl} A, \qquad A \xrightarrow{LiAlH_4} B, \qquad A \xrightarrow[\Delta]{HCl/H_2O} C.$$

[4 marks]

(A) $A = RCH(OH)CN, \quad B = RCH(OH)CH_2NH_2, \quad C = RCH(OH)CO_2H$

(B) $A = RCH(OH)CN, \quad B = RCH(OH)CHO, \quad C = RCH(OH)CO_2H$

(C) $A = RCH(OH)SO_3H, \quad B = RCH_2OH, \quad C = RCH(OH)SO_2Cl$

(D) $A = RCH_2OSO_3Na, \quad B = RCH_2OH, \quad C = RCH_2Cl$

Correct Answer: (A) $A = RCH(OH)CN, \quad B = RCH(OH)CH_2NH_2, \quad C = RCH(OH)CO_2H$

Solution:

Concept:

- Cyanide adds to an aldehyde carbonyl, followed by protonation, to form a cyanohydrin $RCH(OH)CN$. The original aldehyde hydrogen stays on that carbon.
- Lithium aluminium hydride reduces a nitrile to a primary amine, whereas hot aqueous acid hydrolyses a nitrile to a carboxylic acid.

Step 1: Obtain the cyanohydrin A Let $R = CH_3CH_2CH_2CH(CH_3)$, the alkyl group already attached to the aldehyde carbonyl. The bisulphite/cyanide sequence gives cyanohydrin formation. Cyanide attacks the electrophilic carbonyl carbon and the oxygen is protonated:

$$RCHO + HCN \longrightarrow RCH(OH)CN.$$

The carbon skeleton R remains unchanged; the added carbon is the carbon of the nitrile group. Thus A contains both hydroxyl and nitrile groups on adjacent bonded centres.

Step 2: Reduce A to B Lithium aluminium hydride reduces the $-CN$ group to $-CH_2NH_2$. After work-up, the pre-existing alcohol group remains an alcohol. Therefore

$$RCH(OH)CN \xrightarrow{LiAlH_4} RCH(OH)CH_2NH_2.$$

It does not stop at an aldehyde under these conditions, so the proposed B in option (B) is incorrect.

Step 3: Hydrolyse A to C and compare Heating A with aqueous hydrochloric acid converts the nitrile carbon into a carboxylic acid carbon:

$$RCH(OH)CN \xrightarrow[\Delta]{HCl/H_2O} RCH(OH)CO_2H.$$

The hydroxyl-bearing carbon and R group are retained. Option (A) has all three products. Option (C) incorrectly retains sulphur in A and C; option (D) incorrectly proposes a sulphate-type A and misses the cyanide-derived extra carbon.

Step 4: Conclusion $A = RCH(OH)CN$, $B = RCH(OH)CH_2NH_2$, and $C = RCH(OH)CO_2H$. Hence, the correct option is (A).

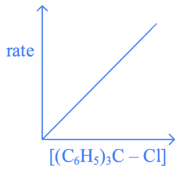
Quick Tip to Solve: Follow the nitrile carbon: reduction turns it into an aminomethyl group, while hydrolysis turns the same carbon into a carboxyl group.

75. The graph which represents the following reaction is :

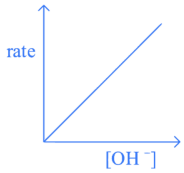
$$(C_6H_5)_3C{-}Cl \xrightarrow[\text{Pyridine}]{OH^-} (C_6H_5)_3C{-}OH.$$

[4 marks]

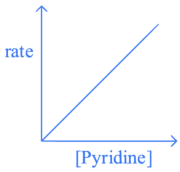
(A) Rate increases linearly through the origin with $[(C_6H_5)_3C{-}Cl]$.



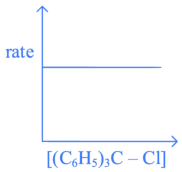
(B) Rate increases linearly through the origin with $[OH^-]$.



(C) Rate increases linearly through the origin with [pyridine].

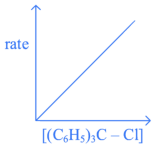


(D) Rate is constant as $[(C_6H_5)_3C{-}Cl]$ increases.



Correct Answer: (A)

Rate increases linearly through the origin with $[(C_6H_5)_3C{-}Cl]$.



Solution:

Concept:

- Triphenylmethyl chloride forms a carbocation stabilised by resonance with three phenyl rings. This favours an S_N1 substitution pathway.
- In an S_N1 reaction, slow substrate ionisation controls the rate: $v = k[(C_6H_5)_3C{-}Cl]$. The nucleophile participates after this slow step.

Step 1: Identify the favourable intermediate The carbon bonded to chlorine also bears three phenyl groups. Cleavage of the carbon-chlorine bond gives the triphenylmethyl cation, whose positive charge is delocalised into the aromatic rings. This stabilisation makes a unimolecular ionisation pathway favourable. Direct backside displacement is also hindered by the three bulky phenyl groups, so a bimolecular substitution rate law is not appropriate here.

Step 2: Write the slow step and rate law The slow step is

$$(C_6H_5)_3C{-}Cl \longrightarrow (C_6H_5)_3C^+ + Cl^-.$$

Hydroxide then captures the carbocation rapidly to give the alcohol. Because only one substrate molecule participates in the rate-determining step,

$$v = k[(C_6H_5)_3C{-}Cl].$$

At fixed temperature and solvent conditions, k is constant, so doubling the substrate concentration doubles the reaction rate.

Step 3: Match each proposed graph The rate-versus-substrate plot must be a straight line through the origin, which is graph (A). Graph (B) incorrectly assigns first-order dependence on hydroxide; its concentration does not enter the unimolecular rate law. Graph (C) assigns a linear dependence on pyridine concentration, which is likewise absent from this law. Graph (D) depicts zero-order dependence on the substrate and contradicts its participation in the slow ionisation.

Step 4: Conclusion The linear rate-versus-$[(C_6H_5)_3C-Cl]$ graph through the origin represents the reaction. Hence, the correct option is (A).

Quick Tip to Solve: For an easily stabilised tertiary or triarylmethyl carbocation, identify the slow ionisation first; its molecularity determines the rate graph.

76. The industrial activity held least responsible for global warming is : [4 marks]

(A) manufacturing of cement

(B) Electricity generation in thermal power plants

(C) steel manufacturing

(D) Industrial production of urea

Correct Answer: (D) Industrial production of urea

Solution:

Concept:

- Carbon dioxide is a greenhouse gas. Combustion of carbon-containing fuels and calcination of carbonates release it into the atmosphere.
- Industrial urea synthesis consumes carbon dioxide as a raw material: $2\,NH_3 + CO_2 \longrightarrow NH_2CONH_2 + H_2O$.

Step 1: Compare the direct process chemistry Cement manufacture involves heating limestone. The calcination step releases carbon dioxide directly:

$$CaCO_3 \longrightarrow CaO + CO_2.$$

Fuel burned to heat the kiln causes additional emissions. Thus cement manufacture is a significant contributor rather than the least responsible activity in the options.

Step 2: Check thermal power and steel production Thermal power plants commonly burn coal or other fossil fuels to produce heat and generate electricity. Carbon combustion gives $C + O_2 \longrightarrow CO_2$. Conventional steelmaking also uses coke and carbon monoxide in extracting iron; for example $Fe_2O_3 + 3\,CO \longrightarrow 2\,Fe + 3\,CO_2$. Both activities therefore have substantial carbon dioxide production associated with their main operations, excluding (B) and (C).

Step 3: Identify the carbon dioxide-consuming operation Urea is prepared by reacting ammonia with carbon dioxide, initially forming ammonium carbamate, followed by dehydration. The overall

equation is

$$2\,NH_3 + CO_2 \longrightarrow NH_2CONH_2 + H_2O.$$

Unlike the other listed process reactions, it incorporates carbon dioxide into a product. This comparison concerns the main industrial chemistry; it does not assert that the entire urea supply chain, energy use or later fertiliser use is emission-free.

Step 4: Conclusion Industrial production of urea is held least responsible among the listed activities because its synthesis consumes carbon dioxide. Hence, the correct option is (D).

Quick Tip to Solve: Compare the main process reactions: limestone calcination and fuel burning release carbon dioxide, whereas urea synthesis uses it as a feedstock.

77. The starting material for convenient preparation of deuterated hydrogen peroxide (D_2O_2) in :

[4 marks]

(A) BaO_2

(B) 2-ethylanthraquinol

(C) BaO

(D) $K_2S_2O_8$

Correct Answer: (D) $K_2S_2O_8$

Solution:

Concept:

- Deuterated hydrogen peroxide contains deuterium at both exchangeable hydrogen positions: $D-O-O-D$. A convenient route uses a hydrogen-free peroxide precursor and heavy water.
- Peroxodisulphate contains an O-O bond. Its hydrolysis can produce hydrogen peroxide; replacing ordinary water by D_2O produces D_2O_2.

Step 1: Choose the appropriate peroxide precursor Potassium peroxodisulphate, $K_2S_2O_8$, contains the peroxide linkage in the peroxodisulphate ion. It has no ordinary hydrogen atoms that would have to be retained in the desired product. Hydrolysis with heavy water provides deuterium while retaining the oxygen-oxygen linkage needed for deuterated hydrogen peroxide.

Step 2: Write and balance the heavy-water reaction The convenient preparation is represented by

$$K_2S_2O_8 + 2\,D_2O \longrightarrow 2\,KDSO_4 + D_2O_2.$$

Both potassium atoms and both sulphur atoms go into the two deuterium sulphate salt units. There are ten oxygens and four deuterium atoms on each side: eight oxygens and two deuteriums in $2\,KDSO_4$, and two oxygens and two deuteriums in D_2O_2. Thus the product is specifically the deuterated peroxide.

Step 3: Reject the less suitable starting materials Barium oxide in (C) is an oxide, not a peroxide, and lacks the required O-O linkage. The anthraquinol route in (B) is the usual industrial route for ordinary hydrogen peroxide and introduces ordinary hydrogen unless isotopic modifications are made. Barium peroxide in (A) is used for ordinary peroxide preparation with acid, but its convenient standard acid

treatment introduces hydrogen; the specified heavy-water hydrolysis route uses peroxodisulphate.

Step 4: Conclusion The suitable starting material is potassium peroxodisulphate, $K_2S_2O_8$. Hence, the correct option is (D).

Quick Tip to Solve: For deuterated peroxide, remember peroxodisulphate hydrolysis with heavy water and check that every exchangeable hydrogen comes from deuterium.

78. The correct order of bond enthalpy ($kJ\ mol^{-1}$) is : [4 marks]

(A) $C-C > Si-Si > Sn-Sn > Ge-Ge$

(B) $Si-Si > C-C > Ge-Ge > Sn-Sn$

(C) $C-C > Si-Si > Ge-Ge > Sn-Sn$

(D) $Si-Si > C-C > Sn-Sn > Ge-Ge$

Correct Answer: (C) $C-C > Si-Si > Ge-Ge > Sn-Sn$

Solution:

Concept:

- Bond enthalpy is the energy required to break one mole of a specified covalent bond in gaseous species. For similar single bonds, effective orbital overlap strongly influences the bond strength.
- Down Group 14, atomic size and the length of the element–element bond increase. Overlap of the valence orbitals becomes less effective, so the homonuclear single-bond enthalpy generally decreases.

Step 1: Compare carbon and silicon Carbon forms a short, strong single bond by effective overlap of its compact valence orbitals. The longer silicon–silicon bond has less effective overlap. Consequently the first comparison is $C{-}C > Si{-}Si$. Approximate familiar bond-enthalpy values are about 348 and $226\,kJ\,mol^{-1}$, respectively; the exact values are not needed to establish their order.

Step 2: Continue down the group Germanium and tin are larger than silicon. Their element–element bonds are longer and weaker, giving $Si{-}Si > Ge{-}Ge > Sn{-}Sn$. Combining the comparisons gives the complete sequence. This concerns homonuclear single bonds; it should not be confused with the stability of oxidation states or with the strength of bonds to oxygen.

Step 3: Eliminate the alternatives Options (B) and (D) put the silicon–silicon bond above the carbon–carbon bond, contrary to its weaker overlap. Option (A) correctly starts with carbon and silicon but reverses germanium and tin. Only option (C) preserves both the carbon–silicon comparison and the germanium–tin comparison.

Step 4: Conclusion The order is $C{-}C > Si{-}Si > Ge{-}Ge > Sn{-}Sn$. Hence, the correct option is (C).

Quick Tip to Solve: For Group 14 homonuclear single bonds, track increasing atomic size down the group: longer bonds normally mean poorer overlap and smaller bond enthalpy.

79. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Gypsum is used for making fireproof wall boards.

Reason (R) : Gypsum is unstable at high temperatures.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(B) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(C) (A) is correct but (R) is not correct

(D) (A) is not correct but (R) is correct

Correct Answer: (A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Solution:

Concept:

- Gypsum is hydrated calcium sulphate, $\mathrm{CaSO_4 \cdot 2\,H_2O}$. It is a non-combustible building material used in plasterboard and fire-resistant wall boards.
- An assertion–reason question requires three separate decisions: whether the assertion is true, whether the reason is true, and whether that reason by itself correctly explains the assertion.

Step 1: Evaluate the assertion Gypsum-based boards are indeed used as fire-resistant wall boards. They help delay the passage of heat during a fire. The hydrated material contains chemically combined water, and heating releases this water while taking up heat. Thus the assertion about its practical use is correct; it does not mean that gypsum is unchanged at every temperature.

Step 2: Evaluate the reason Gypsum loses its water of crystallisation on heating. Controlled heating first produces the hemihydrate, while stronger heating removes the remaining water. One representative dehydration is

$$2\,\mathrm{CaSO_4 \cdot 2\,H_2O} \longrightarrow 2\,\mathrm{CaSO_4 \cdot 0.5\,H_2O} + 3\,\mathrm{H_2O}.$$

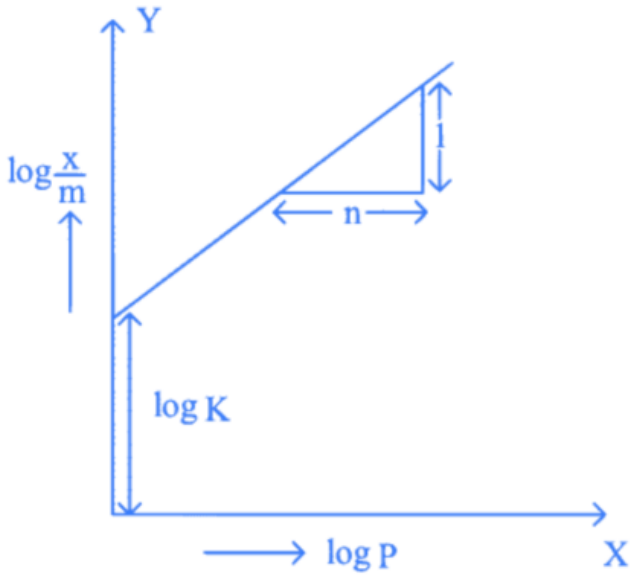
Accordingly, the statement that gypsum is unstable at high temperatures is correct as a statement about the hydrated phase.

Step 3: Test the explanatory connection Thermal instability alone does not establish that a substance is suitable for fireproof boards: many thermally unstable substances provide no such protection. The useful explanation is its non-combustible character and the absorption of heat during water release. The reason as stated gives only instability, without this protective mechanism, so it is not accepted as the correct explanation of the assertion. Options (C) and (D) incorrectly reject one of the true statements; option (B) incorrectly accepts the stated reason as sufficient.

Step 4: Conclusion Both the assertion and the reason are correct, but the reason is not the correct explanation of the assertion. Hence, the correct option is (A).

Quick Tip to Solve: For assertion–reason questions, distinguish a true associated property from a complete explanation; thermal instability alone does not establish fire resistance.

80. In figure, a straight line is given for Freundrich Adsorption ($y = 3x + 2.505$). The value of $\frac{1}{n}$ and $\log K$ are respectively. [4 marks]



(A) 0.3 and log 2.505

(B) 3 and 2.505

(C) 3 and 0.7033

(D) 0.3 and 0.7033

Correct Answer: (B) 3 and 2.505

Solution:

Concept:

- The Freundlich adsorption equation is $\frac{x}{m} = KP^{1/n}$, where x/m is the amount adsorbed per unit mass of adsorbent, P is pressure and K, n are constants for the stated conditions.
- Taking logarithms gives a straight line: $\log(x/m) = \log K + \frac{1}{n}\log P$. On a plot of $\log(x/m)$ against $\log P$, the slope is $1/n$ and the vertical intercept is $\log K$.

Step 1: Identify the plotted coordinates The vertical axis in the figure is $\log(x/m)$, while the horizontal axis is $\log P$. Thus the symbols y and x in the printed line equation refer to these plotted coordinates. In particular, the horizontal coordinate called x in $y = 3x + 2.505$ is not the adsorbed amount x in the original adsorption notation.

Step 2: Read the slope and intercept Compare the given straight line with the standard form $y = mx + c$. The coefficient of the horizontal coordinate is 3, and the value at zero horizontal coordinate is 2.505. Therefore

$$\frac{1}{n} = 3, \qquad \log K = 2.505.$$

The small slope triangle in the figure also represents rise/run as $1/n$; no reciprocal of the printed slope is required.

Step 3: Check the four pairs Options (A) and (D) use 0.3, which is not the coefficient of the horizontal coordinate. Option (C) replaces the intercept by 0.7033, although the intercept is already the logarithm

of the adsorption constant. One must not take a logarithm of that intercept again. Option (B) reads both coefficients directly and correctly.

Step 4: Conclusion The requested values are $1/n = 3$ and $\log K = 2.505$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Write the logarithmic isotherm beside the line equation before substituting: slope gives $1/n$, and the intercept already gives $\log K$.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Among following compounds, the number of those present in copper matte is _____.

A. $CuCO_3$

B. Cu_2S

C. Cu_2O

D. FeO [4 marks]

Solution:

Concept:

- Copper matte is the molten sulphide mixture obtained during extraction of copper from sulphide ores. Its principal constituents are copper(I) sulphide, Cu_2S, and iron(II) sulphide, FeS.
- An iron oxide formed during roasting is removed with silica as an iron silicate slag. Distinguish the sulphide matte from the oxide-containing slag and from later copper-refining intermediates.

Step 1: Recall the matte stage During roasting and smelting of a sulphide ore, some iron sulphide is oxidised and removed. The remaining sulphides form the molten matte, containing mainly Cu_2S together with FeS. The question asks how many of the four listed substances belong to this matte; it does not ask for the total number of all possible constituents.

Step 2: Examine each listed compound Compound A, $CuCO_3$, is a carbonate and is not a matte constituent. Compound B, Cu_2S, is the copper sulphide present in the matte. Compound C, Cu_2O, is an oxide involved later during conversion and self-reduction, rather than a constituent defining the sulphide matte. Compound D, FeO, is removed by the silica flux.

Step 3: Separate slag and count the matches The removal of iron oxide is represented by

$$FeO + SiO_2 \longrightarrow FeSiO_3.$$

The product is the slag, which separates from the sulphide melt. Thus FeO must not be counted simply because iron occurs in the original ore. Of the four listed compounds, only B matches the composition of copper matte. Although FeS is another principal matte component, it is absent from the list and cannot increase the requested count.

Step 4: Conclusion Only Cu_2S among the listed substances is present in copper matte. The required number is $\boxed{1}$ (a dimensionless count).

Quick Tip to Solve: Associate matte with metal sulphides and slag with the flux product: in copper extraction, remember $Cu_2S + FeS$ for the matte.

82. 1×10^{-5} M $AgNO_3$ **is added to** 1 L **of saturated solution of** $AgBr$**. The conductivity of this solution at** 298 K **is** _____ $\times 10^{-8}$ S m^{-1}**.**

[Given : $K_{SP}(AgBr) = 4.9 \times 10^{-13}$ **at** 298 K

$$\lambda^0_{Ag^+} = 6 \times 10^{-3} \text{ S m}^2 \text{ mol}^{-1}$$
$$\lambda^0_{Br^-} = 8 \times 10^{-3} \text{ S m}^2 \text{ mol}^{-1}$$
$$\lambda^0_{NO_3^-} = 7 \times 10^{-3} \text{ S m}^2 \text{ mol}^{-1}]$$

[4 marks]

Solution:

Concept:

- For saturated silver bromide, $K_{sp} = [Ag^+][Br^-]$. Added $AgNO_3$ supplies the common silver ion and suppresses the solubility of $AgBr$.
- At very low concentrations, conductivity is estimated by summing ionic contributions: $\kappa = \sum_i \lambda_i^0 c_i$. Here c_i must be in mol m^{-3}, so a concentration in mol L^{-1} is multiplied by 1000.

Step 1: Use the common-ion approximation The added salt gives $[Ag^+] \simeq [NO_3{}^-] = 10^{-5}$ mol L^{-1}. With silver concentration controlled mainly by the added salt,

$$[Br^-] \simeq \frac{4.9 \times 10^{-13}}{10^{-5}} = 4.9 \times 10^{-8} \text{ mol L}^{-1}.$$

This is only 0.49% of the added silver concentration, so the usual common-ion estimate neglects the small silver contribution from dissolved bromide.

Step 2: Convert concentrations and add contributions The concentrations used in the conductivity sum are 10^{-2}, 4.9×10^{-5}, and 10^{-2} mol m^{-3} for silver, bromide and nitrate, respectively. Hence

$$\begin{aligned}\kappa &\simeq (6 \times 10^{-3})(10^{-2}) + (8 \times 10^{-3})(4.9 \times 10^{-5}) \\ &\quad + (7 \times 10^{-3})(10^{-2}) \\ &= 1.30392 \times 10^{-4} \text{ S m}^{-1}.\end{aligned}$$

Both the silver and nitrate ions of the added electrolyte must be included.

Step 3: Express the requested coefficient In the requested units,

$$\kappa \simeq 13039.2 \times 10^{-8} \text{ S m}^{-1}.$$

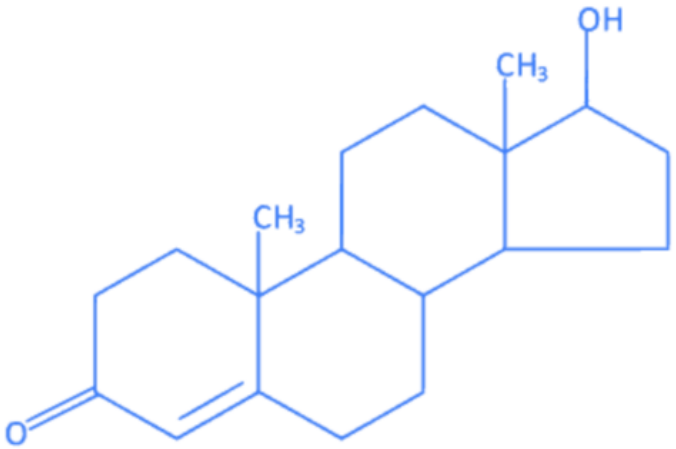
Rounding the coefficient to the nearest integer gives 13039, the common-ion-approximation result. This approximation is part of the calculation: retaining the small dissolved-salt correction to the silver concentration would require solving $s(10^{-5} + s) = K_{sp}$ rather than setting the silver concentration equal to 10^{-5} M.

Step 4: Conclusion Using the usual common-ion approximation, $\kappa \simeq 13039 \times 10^{-8}$ S m^{-1}; the required numerical coefficient is $\boxed{13039}$.

Quick Tip to Solve: First find the suppressed bromide concentration, then include all three ions in the conductivity sum and convert molarity to mol m^{-3}.

83. Testosterone, which is a steroidal hormone, has the following structure.

The total number of asymmetric carbon atom/s in testosterone is _____. [4 marks]



Testosterone

Solution:

Concept:

- An asymmetric carbon is a tetrahedral, sp^3-hybridised carbon attached to four different groups. Two paths around a ring count as different groups when they encounter different atom sequences or substituents.
- A carbonyl carbon and the two carbons of a carbon–carbon double bond are planar sp^2 centres, so they are not asymmetric carbons. A CH_2 or CH_3 carbon also fails the four-different-groups test.

Step 1: Exclude centres that cannot be asymmetric In the displayed testosterone framework, exclude the ketone carbon and both carbons of the alkene in the leftmost ring. Every methylene carbon has two identical hydrogens, and each angular methyl carbon has three identical hydrogens. These are not stereogenic even though they belong to a complicated fused-ring molecule.

Step 2: Inspect the ring junctions and alcohol carbon The saturated fused-ring junctions have distinct paths into the surrounding rings because the framework contains the carbonyl group, the double bond, two angular methyl groups and the terminal alcohol. Using conventional steroid numbering, the asymmetric junction atoms are C8, C9, C10, C13 and C14. At C10 and C13, a methyl substituent replaces the hydrogen, but each carbon still has four different groups and remains asymmetric.

Step 3: Include the carbon bearing the hydroxyl group The carbon bearing OH in the five-membered ring is C17. It is bonded to OH, hydrogen and two different paths around the substituted ring, so it is also asymmetric. The other ring carbons are either methylene or already excluded unsaturated centres. The full set is therefore

$$\mathrm{C8},\ \mathrm{C9},\ \mathrm{C10},\ \mathrm{C13},\ \mathrm{C14},\ \mathrm{C17}.$$

Counting this set gives six distinct asymmetric carbon atoms; no extra centre is obtained by counting a ring junction twice.

Step 4: Conclusion Testosterone has $\boxed{6}$ asymmetric carbon atoms (a dimensionless count).

Quick Tip to Solve: In steroid skeletons, eliminate all sp^2, CH_2 and CH_3 carbons first; then test each saturated junction and the alcohol-bearing carbon.

84. 20% **of acetic acid is dissociated when its** $5\ \mathrm{g}$ **is added to** $500\ \mathrm{mL}$ **of water. The depression in freezing point of such water is** _____ $\times 10^{-3}{}^{\circ}\mathrm{C}$.

Atomic mass of C, H **and** O **are 12,1 and 16 a.m.u. respectively.**

[Given : Molal depression constant and density of water are $1.86\ \mathrm{K\ kg\ mol^{-1}}$ **and** $1\ \mathrm{g\ cm^{-3}}$ **respectively.]**

[4 marks]

Solution:

Concept:

- For a dilute solution, freezing-point depression is $\Delta T_f = iK_f m$, where i is the van't Hoff factor, K_f is the solvent cryoscopic constant and m is solute molality.
- Acetic acid dissociates as $\mathrm{CH_3COOH} \rightleftharpoons \mathrm{H^+} + \mathrm{CH_3COO^-}$. One formula unit gives two particles when dissociated, so for degree of dissociation α, $i = 1 + \alpha$.

Step 1: Calculate moles and solvent mass The molar mass from the supplied atomic masses is

$$M(\mathrm{CH_3COOH}) = 2(12) + 4(1) + 2(16) = 60\,\mathrm{g\,mol^{-1}}.$$

Thus the amount added is $5/60 = 1/12\,\mathrm{mol}$. Water has density $1\,\mathrm{g\,cm^{-3}}$, and $500\,\mathrm{mL} = 500\,\mathrm{cm^3}$, so its mass is $500\,\mathrm{g} = 0.500\,\mathrm{kg}$. Molality uses this solvent mass, rather than the mass of the final solution.

Step 2: Account for the increased particle number The degree of dissociation is $\alpha = 20/100 = 0.20$. Starting with one mole of acid gives 0.80 mole of undissociated molecules, 0.20 mole of hydrogen ions and 0.20 mole of acetate ions, for 1.20 moles of particles. Therefore $i = 1.20$, and

$$m = \frac{1/12}{0.500} = \frac{1}{6}\,\mathrm{mol\,kg^{-1}}.$$

This is dissociation, so the factor is larger than one; using a factor below one would incorrectly model association.

Step 3: Find the depression and its coefficient Substitute into the colligative-property equation:

$$\Delta T_f = (1.20)(1.86)\left(\frac{1}{6}\right) = 0.372\,\mathrm{K}.$$

A temperature difference has the same numerical value in kelvin and degrees Celsius. Consequently $0.372\,^{\circ}\mathrm{C} = 372 \times 10^{-3}\,^{\circ}\mathrm{C}$. The question asks for this coefficient, rather than the absolute freezing temperature.

Step 4: Conclusion The freezing-point depression is $372 \times 10^{-3}\,^{\circ}\mathrm{C}$, so the required numerical coefficient is $\boxed{372}$.

Quick Tip to Solve: For a weak acid yielding two ions, use $i = 1 + \alpha$, and calculate molality from kilograms of water before applying $\Delta T_f = iK_f m$.

85. $0.3\ \mathrm{g}$ **of ethane undergoes combustion at** $27^{\circ}\mathrm{C}$ **in a bomb calorimeter. The temperature of calorimeter system (including the water) is found to rise by** $0.5^{\circ}\mathrm{C}$**. The heat evolved during combustion of ethane at constant pressure is _____** $\mathrm{kJ\ mol^{-1}}$**. (Nearest integer)**

[Given : The heat capacity of the calorimeter system is $20\ \mathrm{kJ\ K^{-1}}, \mathrm{R} = 8.3\ \mathrm{JK^{-1}\ mol^{-1}}$**.**

Assume ideal gas behaviour.

Atomic mass of C **and** H **are 12 and** $1\ \mathrm{g\ mol^{-1}}$ **respectively]** [4 marks]

Solution:

Concept:

- A bomb calorimeter operates at constant volume, so the reacting sample has $q_v = \Delta U$. Its lost heat is gained by the calorimeter: $q_{\text{cal}} = C_{\text{cal}}\Delta T$.
- For ideal gases, molar reaction quantities satisfy $\Delta H = \Delta U + \Delta n_g RT$, where Δn_g counts gaseous products minus gaseous reactants per mole of fuel.

Step 1: Find the heat released by the sample The positive temperature rise means that the calorimeter absorbs heat. Hence the sample releases the same amount with the opposite sign.

$$q_{\text{cal}} = (20\,\mathrm{kJ\,K^{-1}})(0.5\,\mathrm{K}) = 10\,\mathrm{kJ}, \qquad q_v = -10\,\mathrm{kJ}.$$

The molar mass of ethane is $2(12) + 6(1) = 30\,\mathrm{g\,mol^{-1}}$, so $0.3\,\mathrm{g}$ corresponds to $0.010\,\mathrm{mol}$. Therefore

$$\Delta U = \frac{-10}{0.010} = -1000\,\mathrm{kJ\,mol^{-1}}.$$

Step 2: Convert the constant-volume result to constant pressure At $27^{\circ}\mathrm{C} = 300\,\mathrm{K}$, the combustion reaction produces liquid water:

$$\mathrm{C_2H_6(g)} + \frac{7}{2}\,\mathrm{O_2(g)} \longrightarrow 2\,\mathrm{CO_2(g)} + 3\,\mathrm{H_2O\,(l)}.$$

Only gaseous species enter the mole correction, so $\Delta n_g = 2 - (1 + 7/2) = -5/2$. Expressing R in kilojoules gives

$$\begin{aligned}\Delta H &= -1000 + \left(-\frac{5}{2}\right)(8.3 \times 10^{-3})(300) \\ &= -1006.225\,\mathrm{kJ\,mol^{-1}}.\end{aligned}$$

The negative enthalpy sign expresses an exothermic reaction; the requested heat evolved is its positive magnitude. Using water vapour here would give the wrong gas-mole change for combustion at this temperature.

Step 3: Conclusion The heat evolved at constant pressure is $1006.225\,\mathrm{kJ\,mol^{-1}}$, or $\boxed{1006\,\mathrm{kJ\,mol^{-1}}}$ to the nearest integer. The corresponding combustion enthalpy is negative.

Quick Tip to Solve: First convert the bomb heat to molar ΔU; then apply $\Delta n_g RT$, counting gases only and keeping all energy units consistent.

86. The molality of a $10\%(\mathrm{v/v})$ solution of di-bromine solution in CCl_4 (carbon tetrachloride) is 'x'.
$x =$ _____ $\times 10^{-2}$ M. **(Nearest integer)**
[Given : molar mass of $Br_2 = 160\ \mathrm{g\ mol^{-1}}$
atomic mass of $C = 12\ \mathrm{g\ mol^{-1}}$
atomic mass of $Cl = 35.5\ \mathrm{g\ mol^{-1}}$
density of dibromine $= 3.2\ \mathrm{g\ cm^{-3}}$
density of $CCl_4 = 1.6\ \mathrm{g\ cm^{-3}}$] [4 marks]

Solution:

Concept:

- Volume percentage $10\%\,(v/v)$ means $10\,\mathrm{mL}$ of solute in $100\,\mathrm{mL}$ of solution. In the volume-additive treatment used here, the solvent occupies $90\,\mathrm{mL}$.
- Molality is $m = n_{\text{solute}}/w_{\text{solvent}}$, with solvent mass in kilograms. Density converts each component volume into mass; solution volume is not the denominator of molality.

Step 1: Calculate moles of dibromine Choose $100\,\mathrm{mL}$ of solution as the calculation basis. It contains $10\,\mathrm{mL}$ of Br_2. Since $1\,\mathrm{mL} = 1\,\mathrm{cm^3}$, its mass and amount are

$$w_{Br_2} = 10 \times 3.2 = 32\,\mathrm{g}, \qquad n_{Br_2} = \frac{32}{160} = 0.200\,\mathrm{mol}.$$

The supplied density is essential: a volume fraction cannot be treated as a mass fraction. Thus taking merely $10\,\mathrm{g}$ of dibromine would not represent the stated solution.

Step 2: Calculate solvent mass and molality The remaining $90\,\mathrm{mL}$ is carbon tetrachloride. Its mass is

$$w_{CCl_4} = 90(1.6) = 144\,\mathrm{g} = 0.144\,\mathrm{kg}.$$

Therefore

$$m = \frac{0.200}{0.144} = 1.388888\ldots\ \mathrm{mol\,kg^{-1}} = 138.8888\ldots \times 10^{-2}\,\mathrm{mol\,kg^{-1}}.$$

The total solution mass would be $176\,\mathrm{g}$, but using that mass would calculate neither the required molality nor the correct coefficient. The printed unit M denotes molarity and is inconsistent with the word molality; retain the requested molality calculation and express its physical unit correctly.

Step 3: Conclusion The molality is $1.3889\,\mathrm{mol\,kg^{-1}}$, so the requested coefficient to the nearest integer is $\boxed{x = 139}$. Molality has unit $\mathrm{mol\,kg^{-1}}$; the printed M is a unit inconsistency.

Quick Tip to Solve: Use the solvent mass, not the solution mass: on a $100\,\mathrm{mL}$ basis the two component volumes are 10 and $90\,\mathrm{mL}$, then apply their separate densities.

87. The spin only magnetic moment of $[\mathrm{Mn}\,(\mathrm{H_2O})_6]^{2+}$ complexes is _____ B.M. (Nearest integer) (Given : Atomic no. of Mn is 25) [4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mathrm{B.M.}$, where n is the number of unpaired electrons and B.M. denotes the Bohr magneton.
- Water is a neutral, weak-field ligand. In an octahedral manganese(II) aqua complex the $3d$ electrons remain high spin, obeying Hund's rule before pairing.

Step 1: Find the oxidation state and d-electron count All six water ligands are neutral, so the charge $+2$ of $[\mathrm{Mn(H_2O)_6}]^{2+}$ is the oxidation state of Mn. The neutral atom with atomic number 25 has configuration $[\mathrm{Ar}]\,3d^5 4s^2$. On ionisation, the two $4s$ electrons are removed first:

$$\mathrm{Mn}^{2+}: \quad [\mathrm{Ar}]\,3d^5.$$

Counting the neutral atom's outer electrons without first forming the ion would give an incorrect magnetic-electron count. The ligands supply electron pairs for bonding but do not change the metal's oxidation-state calculation.

Step 2: Count unpaired electrons and evaluate the moment For the weak-field water ligands, the octahedral splitting does not overcome the pairing energy. The five d electrons occupy the five d orbitals singly, corresponding to the high-spin arrangement $t_{2g}^3 e_g^2$. Thus $n = 5$, not 1 as in a low-spin d^5 ion. Substitute this unpaired-electron count in the spin-only formula:

$$\mu = \sqrt{5(5+2)} = \sqrt{35} = 5.916\ldots\ \mathrm{B.M.}$$

The square root must be evaluated before rounding. The value is close to six even though there are five unpaired electrons: the magnetic moment is not numerically equal to the unpaired-electron count.

Step 3: Conclusion The spin-only magnetic moment is $\sqrt{35}\,\mathrm{B.M.}$, which rounds to $\boxed{6\,\mathrm{B.M.}}$.

Quick Tip to Solve: For Mn^{2+} with water, remember high-spin d^5: five unpaired electrons give $\sqrt{35}$, rather than simply five, Bohr magnetons.

88. $\mathrm{A} \rightarrow \mathrm{B}$

The above reaction is of zero order. Half life of this reaction is $50\ \mathrm{min}$. The time taken for the concentration of A to reduce to one-fourth of its initial value is _____ min. (Nearest integer) [4 marks]

Solution:

Concept:

- For a zero-order reaction, the concentration decreases linearly: $[A]_t = [A]_0 - kt$. Here k is constant and is independent of the instantaneous reactant concentration.

- The zero-order half-life is $t_{1/2} = [A]_0/(2k)$. Equal concentration decreases take equal time intervals, unlike successive first-order half-lives.

Step 1: Express the rate constant using the half-life Let the initial concentration be $a = [A]_0$. After the given $50\,\text{min}$, the concentration is $a/2$. Substitution in the integrated rate law gives

$$\frac{a}{2} = a - k(50), \qquad 50k = \frac{a}{2}, \qquad k = \frac{a}{100\,\text{min}}.$$

The value of a is not supplied because it cancels from the final time ratio. There is no need to select a numerical starting concentration or to introduce logarithms, which belong to a first-order integrated law.

Step 2: Find the time for three-fourths consumption The wording asks for the concentration remaining to be one-fourth of its initial value, so the decrease is $a - a/4 = 3a/4$. Apply the same linear rate law:

$$\frac{a}{4} = a - kt, \qquad kt = \frac{3a}{4}.$$

Using the expression for k,

$$t = \frac{3a}{4}\frac{100\,\text{min}}{a} = 75\,\text{min}.$$

This can also be checked directly: decreasing from a to $a/2$ takes 50 min, while decreasing by another $a/4$ takes half that interval, 25 min. The total time is 75 min. Doubling the half-life would incorrectly treat the second half of the concentration loss as having the same duration as in first-order kinetics.

Step 3: Conclusion The concentration reaches one-fourth of its initial value after $\boxed{75\,\text{min}}$. The value is already an integer.

Quick Tip to Solve: For zero order, use the fraction consumed: three-fourths consumption takes $(3/4)/(1/2) = 3/2$ times the first half-life.

89. Among the following, the number of tranquilizer/s is/are _____.

A. Chloroliazepoxide

B. Veronal

C. Valium

D. Salvarsan [4 marks]

Solution:

Concept:

- Tranquilizers are drugs used to relieve anxiety and mental tension; the textbook classification also includes sedative and sleep-inducing agents such as barbiturates.
- A drug must be classified by its therapeutic action. An antimicrobial compound does not become a tranquilizer merely because it is also a medicinal organic compound.

Step 1: Classify the three nervous-system drugs A. The name Chloroliazepoxide refers here to chlordiazepoxide, a benzodiazepine tranquilizer used for anxiety and tension. It belongs to the group

being counted.

B. Veronal is the trade name of barbital, a barbiturate. Barbiturates are sedative-hypnotic drugs included among tranquilizers in the textbook classification, so Veronal is also counted.

C. Valium is the trade name of diazepam, another benzodiazepine tranquilizer. Like chlordiazepoxide, it acts on the central nervous system and is used to reduce anxiety. Therefore each of A, B and C satisfies the classification requested in the question.

Step 2: Exclude the antimicrobial and count D. Salvarsan is arsphenamine, an organoarsenic antimicrobial drug historically used to treat syphilis. Its therapeutic action is against infection, rather than relief of anxiety or induction of sedation, so it is excluded.

The count must include the barbiturate as well as the two benzodiazepines; counting only the benzodiazepines would omit Veronal. Conversely, counting all four names would incorrectly place Salvarsan in the nervous-system drug class. Thus the accepted entries are A, B and C, giving

$$N = 1 + 1 + 1 + 0 = 3.$$

This is a count of listed drugs, not a count of distinct chemical families: two members of the benzodiazepine family are still two separate entries.

Step 3: Conclusion The tranquilizers are A, B and C. Their number is $\boxed{3}$ (dimensionless).

Quick Tip to Solve: Group Valium and chlordiazepoxide with benzodiazepines and Veronal with barbiturates; exclude Salvarsan because it is an antimicrobial.

90. A metal M crystallizes into two lattices :- face centred cubic (fcc) and body centred cubic (bcc) with unit cell edge length of 2.0 and 2.5 $\overset{o}{A}$ respectively. The ratio of densities of lattices fcc to bcc for the metal M is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- The density of a cubic crystal is $\rho = ZM/(N_A a^3)$, where Z is the number of atoms per unit cell, M the molar mass, and a the cell edge.
- An fcc unit cell contains $Z_{\text{fcc}} = 4$ atoms, while a bcc unit cell contains $Z_{\text{bcc}} = 2$. For the same metal, M and N_A cancel in a density ratio.

Step 1: Count the atoms in each cell For an fcc lattice, the eight corner atoms each contribute one-eighth and the six face-centre atoms each contribute one-half:

$$Z_{\text{fcc}} = 8\left(\frac{1}{8}\right) + 6\left(\frac{1}{2}\right) = 4.$$

For a bcc lattice, the corner contribution is again one atom, and the single body-centre atom is entirely inside the cell:

$$Z_{\text{bcc}} = 8\left(\frac{1}{8}\right) + 1 = 2.$$

These are effective counts per cell, so shared corner and face atoms must not be counted as complete atoms. The fcc cell has twice the mass of the bcc cell because both are made of the same metal.

Step 2: Take the mass-to-volume ratio and round The edge lengths are $a_{\text{fcc}} = 2.0\,\text{Å}$ and $a_{\text{bcc}} = 2.5\,\text{Å}$, respectively. Divide the two density expressions before substituting:

$$\frac{\rho_{\text{fcc}}}{\rho_{\text{bcc}}} = \frac{4}{2}\left(\frac{2.5}{2.0}\right)^3$$
$$= 2\left(\frac{5}{4}\right)^3 = \frac{125}{32} = 3.90625.$$

Both edge lengths use the same unit, so conversion to centimetres is unnecessary in this ratio. The cell-volume ratio is the cube of the edge ratio, not the edge ratio itself. Because the fcc cell here has more atoms and a smaller volume, a density ratio greater than two provides a useful consistency check.

Step 3: Conclusion The density ratio $\rho_{\text{fcc}}/\rho_{\text{bcc}}$ is 3.90625, so its nearest integer is $\boxed{4}$ (dimensionless).

Quick Tip to Solve: For the same metal, cancel molar mass immediately: density ratio equals atom-count ratio multiplied by the inverse cell-volume ratio.