JEE Main 2023 Paper 1 (B.E./B.Tech) Jan 30 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 30 January 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $a_1 = 1, a_2, a_3, a_4, \ldots.$ be consecutive natural numbers.
Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \ldots.. + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to : [4 marks]

(A) $\frac{\pi}{4} - \cot^{-1}(2022)$

(B) $\frac{\pi}{4} - \tan^{-1}(2022)$

(C) $\cot^{-1}(2022) - \frac{\pi}{4}$

(D) $\tan^{-1}(2022) - \frac{\pi}{4}$

Correct Answer: (D) $\tan^{-1}(2022) - \frac{\pi}{4}$

Solution:

Concept:

- Consecutive natural numbers beginning with 1 satisfy $a_k = k$. The summand therefore contains $1 + k(k+1)$ in its denominator.
- For positive $u > v$, the difference $\tan^{-1} u - \tan^{-1} v$ lies in $(0, \pi/2)$, so the tangent subtraction formula identifies its principal inverse tangent without adding any multiple of π.

Step 1: Rewrite each summand as a difference Use the difference formula with $u = k+1$ and $v = k$. Since their difference is 1, we obtain

$$\tan\big(\tan^{-1}(k+1) - \tan^{-1} k\big) = \frac{1}{1 + k(k+1)}.$$

Both the angle difference and the inverse tangent of the positive fraction are between 0 and $\pi/2$. Consequently,

$$\tan^{-1}\left(\frac{1}{1 + k(k+1)}\right) = \tan^{-1}(k+1) - \tan^{-1} k.$$

Step 2: Telescope the sum and compare the options Adding for $k = 1, 2, \ldots, 2021$ cancels every intermediate term. Thus

$$S = \tan^{-1}(2022) - \tan^{-1}(1) = \tan^{-1}(2022) - \frac{\pi}{4}.$$

For a positive argument, $\cot^{-1}(2022) = \pi/2 - \tan^{-1}(2022)$. Hence option (A) is another expression for the same positive number. Options (B) and (C) are its negative and cannot equal a sum of positive summands. The principal-value ranges are essential when using this complementary-angle identity.

Step 3: Conclusion The sum is $\boxed{\tan^{-1}(2022) - \pi/4}$. Hence, the correct option is (D). The official answer key accepts both letters A and D, because these two expressions are equal.

Quick Tip to Solve: When the numerator is the difference of consecutive terms and the denominator is 1 plus their product, try a difference of inverse tangents before adding.

2. For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$$x - y + z = 5$$
$$2x + 2y + \alpha z = 8$$
$$3x - y + 4z = \beta$$

has infinitely many solutions. Then α and β are the roots of : [4 marks]

(A) $x^2 + 18x + 56 = 0$

(B) $x^2 - 10x + 16 = 0$

(C) $x^2 + 14x + 24 = 0$

(D) $x^2 - 18x + 56 = 0$

Correct Answer: (D) $x^2 - 18x + 56 = 0$

Solution:

Concept:

- A system has infinitely many solutions when its coefficient and augmented matrices have the same rank, smaller than the number of unknowns.
- Row dependence must hold for the constants as well as for the coefficients. A zero determinant alone would also allow an inconsistent system.

Step 1: Eliminate the first two coefficients of the third equation The first two coefficient rows are independent: their x, y coefficient determinant is $1 \cdot 2 - (-1) \cdot 2 = 4 \neq 0$. If the full system is to have rank below 3, the third row must be a linear combination of these two rows. Write it as $uR_1 + vR_2$. Comparing the x and y coefficients gives

$$u + 2v = 3, \qquad -u + 2v = -1.$$

Adding yields $4v = 2$, so $v = 1/2$ and $u = 2$. Comparing the z coefficients now gives

$$2 + \frac{\alpha}{2} = 4 \quad \Longrightarrow \quad \alpha = 4.$$

Step 2: Impose consistency and construct the quadratic The right-hand sides must obey the same relation. Therefore

$$\beta = 2(5) + \frac{1}{2}(8) = 14.$$

At these values, the third equation is exactly twice the first plus half the second. The rank is 2, leaving one free unknown and hence infinitely many solutions. The monic quadratic having roots $\alpha = 4$ and $\beta = 14$ is

$$(X - 4)(X - 14) = X^2 - 18X + 56 = 0.$$

Option (A) reverses the required sign of the sum of roots. Option (B) has the wrong sum and product, and option (C) has a negative root sum and the wrong product.

Step 3: Conclusion The required equation is $\boxed{x^2 - 18x + 56 = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: For infinitely many solutions, first find the row combination from two coefficient columns, then apply that same combination to every remaining coefficient and the constants.

3. The number of ways of selecting two numbers a and $b, a \in \{2, 4, 6, \ldots., 100\}$ and $b \in \{1, 3, 5, \ldots .., 99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is : [4 marks]

(A) 186

(B) 54

(C) 108

(D) 268

Correct Answer: (C) 108

Solution:

Concept:

- An even integer plus an odd integer is odd. The permissible sums must therefore be odd numbers congruent to 2 modulo 23.
- For a fixed sum s, choosing the even number a determines $b = s - a$ uniquely; the restrictions on both numbers give an interval for a.

Step 1: List the possible sums The least possible sum is $2 + 1 = 3$, and the greatest is $100 + 99 = 199$. Write a permissible sum as $s = 23k + 2$. Since s is odd, k must be odd. The range $3 \le s \le 199$ leaves exactly

$$s = 25,\ 71,\ 117,\ 163.$$

The intermediate values with even k are even sums and cannot arise from one even and one odd integer. This parity restriction is applied before counting, so no invalid sums enter the total.

Step 2: Count the even choices for each sum For each s, the condition $1 \le b = s - a \le 99$, together with $2 \le a \le 100$, requires

$$\max(2, s - 99) \le a \le \min(100, s - 1), \qquad a \text{ even.}$$

For $s = 25$, the choices $2, 4, \ldots, 24$ give 12 pairs. For $s = 71$, the choices $2, 4, \ldots, 70$ give 35 pairs. For $s = 117$, the choices $18, 20, \ldots, 100$ give 42 pairs. Finally, for $s = 163$, the choices $64, 66, \ldots, 100$ give 19 pairs. The sum classes are disjoint, and each choice determines exactly one admissible odd b. Thus

$$N = 12 + 35 + 42 + 19 = 108.$$

The other listed numbers, 186, 54 and 268, do not equal this complete count; no division by 2 is appropriate because the two selections come from different sets.

Step 3: Conclusion The number of selections is $\boxed{108}$. Hence, the correct option is (C).

Quick Tip to Solve: Use parity to prune possible remainder classes, then count one variable in the intersection of the intervals imposed by both sets.

4. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots, a_{100}$ is 25 . Then S is : [4 marks]

(A) $\{9\}$

(B) ϕ

(C) $\{99\}$

(D) N

Correct Answer: (D) N

Solution:

Concept:

- The mean deviation about the arithmetic mean is $\text{MD} = \frac{1}{n}\sum_{i=1}^{n}|a_i - \bar{a}|$. Absolute distances, rather than signed deviations, must be added.
- Adding the same constant to every observation adds that constant to the mean and leaves every deviation unchanged. Thus mean deviation is invariant under translation.

Step 1: Express the mean and eliminate the starting integer Let the first positive integer be $a_1 = m$, where $m \in \mathbb{N}$. The observations are $m, m+1, \ldots, m+99$. Their mean is the average of the endpoints:

$$\bar{a} = \frac{m + (m+99)}{2} = m + \frac{99}{2}.$$

The observation with offset j, for $j = 0, 1, \ldots, 99$, has absolute deviation

$$\left|m + j - \left(m + \frac{99}{2}\right)\right| = \left|j - \frac{99}{2}\right|.$$

The starting value has cancelled exactly, so any computed mean deviation will hold for every positive integer m.

Step 2: Sum the symmetric distances There are 50 observations on each side of the mean. Their distances are $1/2, 3/2, \ldots, 99/2$, each appearing twice. Therefore

$$\sum_{j=0}^{99}\left|j - \frac{99}{2}\right| = 2\left(\frac{1}{2} + \frac{3}{2} + \cdots + \frac{99}{2}\right)$$
$$= 2\left[\frac{50}{2}\left(\frac{1}{2} + \frac{99}{2}\right)\right] = 2500.$$

Dividing by the number of observations gives $\text{MD} = 2500/100 = 25$. Both 9 and 99 work, but they are merely two members of the full solution set; neither singleton is the complete set. The empty set is ruled out by this calculation, which proves that every permitted starting integer works.

Step 3: Conclusion Every positive integer can be a_1, so $\boxed{S = \mathbb{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: For consecutive equally spaced data, subtract the starting value immediately; pairing equal distances on opposite sides of the mean halves the work.

5. Let $a, b, c > 1, a^3, b^3$ and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then abc is equal to : [4 marks]

(A) 343

(B) 216

(C) $\frac{343}{8}$

(D) $\frac{125}{8}$

Correct Answer: (B) 216

Solution:

Concept:

- Three positive quantities U, V, W are in geometric progression if $V^2 = UW$. Change logarithm bases to express all three in a common set of variables.
- For an arithmetic progression with first term A, common difference d, and n terms, the sum is $S_n = \frac{n}{2}[2A + (n-1)d]$.

Step 1: Use the logarithmic progression to compare a and c Put $u = \ln a$, $v = \ln b$, and $w = \ln c$. All three are strictly positive because $a, b, c > 1$; division by them is legitimate. The three stated logarithms are respectively v/u, u/w, and w/v. The geometric-progression condition gives

$$\left(\frac{u}{w}\right)^2 = \frac{v}{u} \cdot \frac{w}{v} = \frac{w}{u}.$$

Multiplying by uw^2 yields $u^3 = w^3$, hence $u = w$ and $a = c$. Since a^3, b^3, c^3 are in arithmetic progression,

$$2b^3 = a^3 + c^3 = 2a^3,$$

so $b = a$ as well. Write the common positive value as t.

Step 2: Evaluate the arithmetic-progression sum The first term and common difference of the other progression now reduce to

$$A = \frac{t + 4t + t}{3} = 2t, \qquad d = \frac{t - 8t + t}{10} = -\frac{3t}{5}.$$

Applying the 20-term sum formula,

$$S_{20} = 10\left[4t - 19\left(\frac{3t}{5}\right)\right] = -74t.$$

The supplied sum is -444, so $-74t = -444$ gives $t = 6$, consistent with $t > 1$. Consequently $abc = t^3 = 216$. The remaining choices 343, $343/8$, and $125/8$ correspond to different common values and fail the supplied sum condition.

Step 3: Conclusion The product is $\boxed{abc = 216}$. Hence, the correct option is (B).

Quick Tip to Solve: When cyclic logarithms form a progression, change to natural logarithms first. Their products often cancel, turning the condition into a simple equality of powers.

6. Let f, g and h be the real valued functions defined on $\mathbb{R}$ as

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

$$g(x) = \begin{cases} \frac{\sin(x+1)}{(x+1)}, & x \neq -1 \\ 1, & x = -1 \end{cases}$$

and $h(x) = 2[x] - f(x)$, where $[x]$ is the greatest integer $\leq x$. Then the value of $\lim_{x \to 1} g(h(x-1))$ is : [4 marks]

(A) 1

(B) -1

(C) $\sin(1)$

(D) 0

Correct Answer: (A) 1

Solution:

Concept:

- The greatest-integer function must be evaluated separately immediately to the left and right of an integer. The sign function also changes at zero.
- For a composite expression, find the actual inner-function values on a punctured neighbourhood before applying the outer function, especially when the outer function has a separately defined value.

Step 1: Compute the inner function on both sides of zero Set $t = x - 1$, so $x \to 1$ corresponds to $t \to 0$. For $-1 < t < 0$, the greatest integer is $[t] = -1$ and $f(t) = t/|t| = -1$. Therefore

$$h(t) = 2(-1) - (-1) = -1.$$

For $0 < t < 1$, we instead have $[t] = 0$ and $f(t) = 1$. Thus

$$h(t) = 2(0) - 1 = -1.$$

The changes cancel: the inner function is exactly -1 on both punctured intervals.

Step 2: Apply the defined value of the outer function Because $h(x-1) = -1$ for every x close enough to 1 with $x \neq 1$, the composite expression is

$$g(h(x-1)) = g(-1) = 1.$$

The value $g(-1)$ comes from the second line of its definition. Substitution into the first line would give the undefined form $0/0$, so that line must not be used at its excluded argument. Both one-sided limits are 1. The alternatives -1, $\sin 1$, and 0 do not equal the value of this locally constant composite.

Step 3: Conclusion The limit is $\boxed{1}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the floor and sign changes together: they can cancel exactly, making the inner function constant and turning the composite limit into one defined function value.

7. Let q be the maximum integral value of p in $[0, 10]$ for which the roots of the equation $x^2 - px + \frac{5}{4}p = 0$ are rational. Then the area of the region $\{(x, y) : 0 \le y \le (x - q)^2, 0 \le x \le q\}$ is : [4 marks]

(A) $\frac{125}{3}$

(B) 243

(C) 164

(D) 25

Correct Answer: (B) 243

Solution:

Concept:

- For $x^2 - px + 5p/4 = 0$, the quadratic formula gives roots $(p \pm \sqrt{p^2 - 5p})/2$. With integral p, rational roots require a nonnegative discriminant with rational square root.
- The area between a nonnegative curve $y = F(x)$ and the x-axis over $[a, b]$ is $\int_a^b F(x)\,dx$. Here the upper boundary is a square, so no sign splitting is needed.

Step 1: Find the largest eligible integer parameter The permitted integral values of p range from 0 through 10. To find the maximum, start at the upper endpoint. At $p = 10$, the discriminant is

$$\Delta = 10^2 - 5(10) = 50,$$

whose square root is $5\sqrt{2}$, an irrational number. Thus $p = 10$ does not qualify. At the next integer, $p = 9$,

$$\Delta = 9^2 - 5(9) = 36, \qquad x = \frac{9 \pm 6}{2} = \frac{15}{2}, \frac{3}{2}.$$

Both roots are rational. There is no integer strictly between 9 and 10, so no smaller values need to be tested to establish the maximum: $q = 9$.

Step 2: Integrate over the specified region For $0 \le x \le q$, the ordinate runs from 0 to $(x - q)^2$. Therefore the area is

$$A = \int_0^q (x - q)^2\,dx = \left[\frac{(x - q)^3}{3}\right]_0^q$$
$$= 0 - \frac{(-q)^3}{3} = \frac{q^3}{3}.$$

Substituting $q = 9$ gives $A = 729/3 = 243$ square units. The choice $125/3$ would arise from using $q = 5$, which qualifies but is not the maximum. The choices 164 and 25 do not equal the integral with the established endpoint.

Step 3: Conclusion The area is $\boxed{243}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: When only the maximum integer parameter is needed, test downward from the upper endpoint. Stop at the first valid value, then use the integral formula with that value.

8. If the functions $f(x) = \frac{x^3}{3} + 2bx + \frac{ax^2}{2}$
and $g(x) = \frac{x^3}{3} + ax + bx^2, a \neq 2b$
have a common extreme point, then $a + 2b + 7$ **is equal to :** [4 marks]

(A) 6
(B) $\frac{3}{2}$
(C) 3
(D) 4

Correct Answer: (A) 6

Solution:

Concept:

- For a differentiable function, a local maximum or minimum at an interior point requires the first derivative to vanish. Use the common extremum abscissa in both derivative equations.
- If the second derivative is nonzero at a stationary point, its sign determines whether that point is a local maximum or a local minimum.

Step 1: Differentiate both functions Let the common extremum occur at $x = t$. Differentiating the given cubic functions gives

$$f'(x) = x^2 + ax + 2b, \qquad g'(x) = x^2 + 2bx + a.$$

Consequently the same number t must satisfy both equations

$$t^2 + at + 2b = 0, \qquad t^2 + 2bt + a = 0.$$

These equations compare stationary points; equating the two functions alone would not impose the necessary extremum condition.

Step 2: Eliminate the square term Subtract the second stationary-point equation from the first:

$$(a - 2b)t + (2b - a) = (a - 2b)(t - 1) = 0.$$

The question explicitly excludes $a = 2b$. We may therefore divide by $a - 2b$, obtaining $t = 1$. Substitution into either original derivative equation gives

$$1 + a + 2b = 0 \quad \Rightarrow \quad a + 2b = -1.$$

It follows immediately that $a + 2b + 7 = 6$. The other proposed values would give a different sum and contradict this derivative equation.

Step 3: Check the extremum restriction At the common stationary abscissa, the second derivatives are $f''(1) = 2 + a$ and $g''(1) = 2 + 2b$. A zero second derivative here would make that cubic have a stationary inflection instead of an extremum, so such parameters are excluded by the stated hypothesis. This restriction does not change the necessary relation $a + 2b = -1$; the hypothesis is consistent, for example with $a = 0$, $b = -1/2$, giving nonzero second derivatives.

Step 4: Conclusion The required value is $a + 2b + 7 = 6$. Hence, the correct option is (A).

Quick Tip to Solve: For two polynomial functions with a common stationary abscissa, subtract their derivative equations before trying to solve the quadratics.

9. The parabolas : $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then : [4 marks]

(A) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(B) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(C) d, e, f are in A.P.

(D) d, e, f are in G.P.

Correct Answer: (A) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Solution:

Concept:

- Three positive numbers a, b, c are in geometric progression exactly when $b^2 = ac$.
- A point on the intersection must satisfy both parabola equations. Three quantities p, q, r are in arithmetic progression when $p + r = 2q$.

Step 1: Locate the intersection on the first parabola The intersection is on $y = 1$. Substituting this into the first parabola gives

$$ax^2 + 2bx + c = 0.$$

Because $b^2 = ac$, its discriminant is $4b^2 - 4ac = 0$. Thus the only possible abscissa at this height is

$$x = -\frac{b}{a}.$$

The common point is therefore $(-b/a, 1)$. Positivity ensures the divisions used here and below are valid.

Step 2: Apply the second parabola equation Put this same point into $dx^2 + 2ex + fy = 0$:

$$\frac{db^2}{a^2} - \frac{2eb}{a} + f = 0.$$

Using $b^2 = ac$, divide by c to obtain

$$\frac{d}{a} - \frac{2eb}{ac} + \frac{f}{c} = 0 \quad \Rightarrow \quad \frac{d}{a} + \frac{f}{c} = 2\frac{e}{b}.$$

The middle ratio is the arithmetic mean of the other two, exactly the condition in option (A). No relation involving their geometric mean was used or implied.

Step 3: Rule out the other claimed necessities For example, take $(a, b, c) = (1, 2, 4)$ and $(d, e, f) = (1, 3, 8)$. These positive values satisfy both equations at $(-2, 1)$, while the ratios are $1, 3/2, 2$. They are in A.P. but not in G.P., since $(3/2)^2 \neq 1 \cdot 2$. Also $1, 3, 8$ are in neither A.P. nor G.P. Thus options (B), (C) and (D) do not follow from the hypotheses.

Step 4: Conclusion $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. Hence, the correct option is (A).

Quick Tip to Solve: When a G.P. condition makes a quadratic discriminant zero, use its repeated root in the other equation.

10. Let $\vec{a}$ and $\vec{b}$ be two vectors, Let $|\vec{a}| = 1, |\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$, then the value of $\vec{b} \cdot \vec{c}$ is : [4 marks]

(A) -48

(B) -60

(C) -84

(D) -24

Correct Answer: (A) -48

Solution:

Concept:

- The cross product $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$, so $\vec{b} \cdot (\vec{a} \times \vec{b}) = 0$.
- The dot product distributes over addition and satisfies $\vec{b} \cdot \vec{b} = |\vec{b}|^2$. A magnitude must be squared when computing the self-dot product.

Step 1: Take the dot product with the given expression Use $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$ directly, rather than resolving the vectors into coordinates. Distributivity gives

$$\vec{b} \cdot \vec{c} = 2\vec{b} \cdot (\vec{a} \times \vec{b}) - 3\vec{b} \cdot \vec{b}.$$

The scalar 2 multiplies the cross-product contribution only; the coefficient of the component along $\vec{b}$ remains -3.

Step 2: Remove the perpendicular contribution Since $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$, the first dot product is zero. Equivalently it is a scalar triple product with the vector $\vec{b}$ repeated, and such a triple product vanishes. Hence

$$\vec{b} \cdot \vec{c} = -3|\vec{b}|^2 = -3(4)^2 = -48.$$

The negative sign records the opposite direction of the $-3\vec{b}$ component. The perpendicular component of $\vec{c}$ has no effect on this projection.

Step 3: Check the supplied information and choices The data $|\vec{a}| = 1$ and $\vec{a} \cdot \vec{b} = 2$ are consistent: they imply an angle with cosine $2/(1 \cdot 4) = 1/2$. However, this angle is unnecessary for the requested dot product, because perpendicularity holds for any $\vec{a}$. Options -60, -84 and -24 cannot result from the surviving term $-3(16)$; including an extra cross-product contribution would violate perpendicularity.

Step 4: Conclusion $\vec{b} \cdot \vec{c} = -48$. Hence, the correct option is (A).

Quick Tip to Solve: Before calculating a cross product, check whether the later dot product uses one of its own factors; that contribution is then zero.

11. The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is : [4 marks]

(A) $[2\sqrt{2}, \sqrt{11}]$

(B) $[\sqrt{5}, \sqrt{13}]$

(C) $[\sqrt{2}, \sqrt{7}]$

(D) $[\sqrt{5}, \sqrt{10}]$

Correct Answer: (D) $[\sqrt{5}, \sqrt{10}]$

Solution:

Concept:

- Each real square root requires a nonnegative radicand, so the domain is determined before finding the range.
- For nonnegative u, v with fixed sum, their product is largest when $u = v$. The square function preserves inequalities between nonnegative quantities.

Step 1: Find the domain and square the expression The inequalities $3 - x \geq 0$ and $2 + x \geq 0$ give $-2 \leq x \leq 3$. On this interval $f(x) \geq 0$, and

$$f(x)^2 = 3 - x + 2 + x + 2\sqrt{(3-x)(2+x)} = 5 + 2\sqrt{(3-x)(2+x)}.$$

The two radicands have the fixed sum 5. This simplifies the range question to bounding their product, without solving for x in terms of $f(x)$.

Step 2: Obtain and attain both bounds Their product is nonnegative and satisfies

$$(3-x)(2+x) = \frac{25}{4} - \left(x - \frac{1}{2}\right)^2 \leq \frac{25}{4}.$$

Thus $5 \leq f(x)^2 \leq 10$, and therefore $\sqrt{5} \leq f(x) \leq \sqrt{10}$. The minimum occurs at $x = -2$ or $x = 3$, where one radicand is zero. The maximum occurs at $x = 1/2$, where both radicands equal $5/2$, giving $2\sqrt{5/2} = \sqrt{10}$.

Step 3: Confirm that the full interval is the range The sum of the square-root functions is continuous throughout the closed domain $[-2, 3]$. It attains the two bounds above, so the intermediate value theorem gives every value between them. The endpoints are included. Option (A) has the wrong minimum, option (B) has the wrong maximum, and option (C) has both bounds wrong; only option (D) matches both attained extremes.

Step 4: Conclusion The range is $[\sqrt{5}, \sqrt{10}]$. Hence, the correct option is (D).

Quick Tip to Solve: For a sum of two square roots whose radicands add to a constant, square once and maximize their product.

12. The solution of the differential equation $\frac{dy}{dx} = -\left(\frac{x^2+3y^2}{3x^2+y^2}\right), y(1) = 0$ **is :** [4 marks]

(A) $\log_e|x+y| + \frac{xy}{(x+y)^2} = 0$

(B) $\log_e|x+y| - \frac{xy}{(x+y)^2} = 0$

(C) $\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$

(D) $\log_e|x+y| - \frac{2xy}{(x+y)^2} = 0$

Correct Answer: (C) $\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$

Solution:

Concept:

- The differential equation is homogeneous because its numerator and denominator are quadratic forms of the same degree.
- The substitutions $u = x + y$, $v = x - y$ turn symmetric quadratic expressions into simpler forms. The initial condition determines the integration constant.

Step 1: Transform the differential equation Write the equation in differential form:

$$(x^2 + 3y^2)\,dx + (3x^2 + y^2)\,dy = 0.$$

Let $u = x + y$ and $v = x - y$. Then $x = (u + v)/2$, $y = (u - v)/2$, so substitution and collection of terms give

$$(u^2 + v^2)\,du - uv\,dv = 0.$$

At the initial point $(1, 0)$, both u and v equal 1. We work on the local solution branch with $u > 0$, so division by u is legitimate.

Step 2: Integrate after a second substitution Put $w = v^2$, so $dw = 2v\,dv$. The transformed equation becomes

$$\frac{dw}{du} - \frac{2w}{u} = 2u.$$

The integrating factor is u^{-2}. Therefore

$$\frac{d}{du}\left(\frac{w}{u^2}\right) = \frac{2}{u}, \qquad \frac{v^2}{u^2} = 2\log_e|u| + C.$$

Since $v^2 = u^2 - 4xy$, this relation can be rearranged as

$$\log_e|x+y| + \frac{2xy}{(x+y)^2} = C_1.$$

At $(1, 0)$, the left side is zero, hence $C_1 = 0$.

Step 3: Verify the coefficient and sign For $F = \log_e|x+y| + 2xy/(x+y)^2$, direct differentiation gives

$$F_x = \frac{x^2 + 3y^2}{(x+y)^3}, \qquad F_y = \frac{3x^2 + y^2}{(x+y)^3}.$$

Thus $F = 0$ implies $y' = -F_x/F_y$, exactly the required differential equation. Options (A) and (B)

have the wrong coefficient of xy; option (D) has the wrong sign. The verified local solution passes through the prescribed initial point.

Step 4: Conclusion The solution is $\log_e|x+y| + \dfrac{2xy}{(x+y)^2} = 0$. Hence, the correct option is (C).

Quick Tip to Solve: For quadratic forms symmetric in x, y, try the sum and difference variables; differentiating the final implicit answer checks the sign quickly.

13. Let $x = (8\sqrt{3}+13)^{13}$ and $y = (7\sqrt{2}+9)^9$. If $[t]$ denotes the greatest integer $\le t$, then : [4 marks]

(A) $[x]$ is odd but $[y]$ is even

(B) $[x]$ and $[y]$ are both odd

(C) $[x]+[y]$ is even

(D) $[x]$ is even but $[y]$ is odd

Correct Answer: (C) $[x]+[y]$ is even

Solution:

Concept:

- Adding conjugate binomial powers cancels the terms with odd powers of the radical. The remaining sum is twice an integer, hence is even.
- If N is an integer and $0 < \delta < 1$, then $[N+\delta] = N$. The sign of the small conjugate power is crucial for a greatest-integer calculation.

Step 1: Use the conjugate of the first base Set $\alpha = 13+8\sqrt{3}$, $\beta = 13-8\sqrt{3}$. Since $13 < 8\sqrt{3} < 14$, we have $-1 < \beta < 0$, and the odd power satisfies $-1 < \beta^{13} < 0$. By the binomial theorem,

$$\alpha^{13} + \beta^{13} = 2\sum_{j=0}^{6} \binom{13}{2j} 13^{13-2j}(8\sqrt{3})^{2j} = 2M$$

for an integer M. Therefore $x = 2M - \beta^{13}$ lies strictly between $2M$ and $2M+1$, so $[x] = 2M$ is even.

Step 2: Repeat for the second base Let $\gamma = 9+7\sqrt{2}$ and $\delta = 9-7\sqrt{2}$. The inequalities $9 < 7\sqrt{2} < 10$ imply $-1 < \delta < 0$, hence $-1 < \delta^9 < 0$. Again conjugate terms cancel the odd radical powers:

$$\gamma^9 + \delta^9 = 2\sum_{j=0}^{4} \binom{9}{2j} 9^{9-2j}(7\sqrt{2})^{2j} = 2N,$$

where N is an integer. Thus $2N < y < 2N+1$, giving $[y] = 2N$, also even.

Step 3: Compare the parity statements Both greatest integers are even, so their sum $[x]+[y] = 2(M+N)$ is even. Option (A) is false because $[x]$ is not odd; option (B) is false because neither is odd; option (D) is false because $[y]$ is not odd. No approximation of the enormous positive powers is needed: the exact cancellation and the bounds on their small conjugates determine both parities.

Step 4: Conclusion $[x]$ and $[y]$ are both even, and therefore $[x]+[y]$ is even. Hence, the correct option is (C).

Quick Tip to Solve: Find the sign of the small conjugate power before taking the floor; a negative conjugate puts the large power just above an even integer.

14. A vector $\vec{v}$ in the first octant is inclined to the x-axis at $60°$, to the y-axis at 45 and to the z-axis at an acute angle. If a plane passing through the points $(\sqrt{2}, -1, 1)$ and (a, b, c), is normal to $\vec{v}$, then : [4 marks]

(A) $a + b + \sqrt{2}c = 1$

(B) $\sqrt{2}a + b + c = 1$

(C) $\sqrt{2}a - b + c = 1$

(D) $a + \sqrt{2}b + c = 1$

Correct Answer: (D) $a + \sqrt{2}b + c = 1$

Solution:

Concept:

- The direction cosines l, m, n of a vector satisfy $l^2 + m^2 + n^2 = 1$. In the first octant all three are positive.
- A plane normal to (l, m, n) through (x_0, y_0, z_0) has equation $l(x - x_0) + m(y - y_0) + n(z - z_0) = 0$.

Step 1: Determine the normal direction The angles with the x- and y-axes give

$$l = \cos 60° = \frac{1}{2}, \qquad m = \cos 45° = \frac{1}{\sqrt{2}}.$$

Consequently

$$n^2 = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}.$$

The angle with the z-axis is acute, so $n = 1/2$, not $-1/2$. Multiplying the direction-cosine triple by 2, a convenient normal vector for the plane is $(1, \sqrt{2}, 1)$.

Step 2: Write the plane through the known point Use the given point $(\sqrt{2}, -1, 1)$ in the point-normal equation:

$$(x - \sqrt{2}) + \sqrt{2}(y + 1) + (z - 1) = 0.$$

The two constant terms involving $\sqrt{2}$ cancel, leaving

$$x + \sqrt{2}y + z = 1.$$

Because (a, b, c) is another point on this same plane, substitution gives $a + \sqrt{2}b + c = 1$. The first-octant information has fixed the signs of every normal component.

Step 3: Check the alternative equations A plane perpendicular to the vector must have a normal proportional to $(1, \sqrt{2}, 1)$. Option (A) places the factor $\sqrt{2}$ on the wrong coordinate. Options (B) and (C) similarly give normals $(\sqrt{2}, 1, 1)$ and $(\sqrt{2}, -1, 1)$, neither proportional to the required direction. Only option (D) has both the correct normal and the constant determined by the known point. This also checks the sign arising from its negative y-coordinate.

Step 4: Conclusion The required relation is $a + \sqrt{2}b + c = 1$. Hence, the correct option is (D).

Quick Tip to Solve: Use the sum of squared direction cosines to find the missing angle, then scale the normal vector to remove fractions before writing the plane.

15. If a plane passes through the points $(-1, k, 0), (2, k, -1), (1, 1, 2)$ and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is : [4 marks]

(A) $\frac{17}{5}$

(B) $\frac{6}{13}$

(C) $\frac{13}{6}$

(D) $\frac{5}{17}$

Correct Answer: (C) $\frac{13}{6}$

Solution:

Concept:

- A plane through three non-collinear points has a normal vector given by the cross product of two independent displacement vectors lying in it.
- A line parallel to a plane has direction vector perpendicular to the normal: if the normal is $\mathbf{n}$ and the line direction is $\mathbf{d}$, then $\mathbf{n} \cdot \mathbf{d} = 0$.

Step 1: Extract the actual line direction Write the common ratio in the given line as t. Its coordinates are $x = 1 + t$, $y = t - \frac{1}{2}$, and $z = -1 - t$. Therefore a direction vector is $\mathbf{d} = (1, 1, -1)$. The coefficient 2 in the numerator $2y + 1$ cancels the denominator 2; using $(1, 2, -1)$ would describe a different line.

Step 2: Calculate a normal to the plane Name the points $A = (-1, k, 0)$, $B = (2, k, -1)$, and $C = (1, 1, 2)$. Then

$$\overrightarrow{AB} = (3, 0, -1), \qquad \overrightarrow{AC} = (2, 1 - k, 2).$$

Their cross product is

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = (1 - k, -8, 3(1 - k)).$$

The middle component is always nonzero, so these vectors do define a plane, including at the value of k found below.

Step 3: Apply parallelism and evaluate The perpendicularity condition gives

$$\mathbf{n} \cdot \mathbf{d} = (1 - k) - 8 - 3(1 - k) = 2k - 10 = 0.$$

Thus $k = 5$, and neither factor in the requested denominator vanishes. Substitute only after determining this value:

$$\frac{k^2 + 1}{(k - 1)(k - 2)} = \frac{25 + 1}{(5 - 1)(5 - 2)} = \frac{26}{12} = \frac{13}{6}.$$

The other three offered fractions are not equal to this reduced value; in particular, taking its reciprocal would give option (B), not the expression asked.

Step 4: Conclusion The required value is $\frac{13}{6}$. Hence, the correct option is (C).

Quick Tip to Solve: Parameterise a symmetric line first when a numerator contains a multiple of a coordinate; its denominator alone need not give the direction ratio.

16. $\lim\limits_{n\to\infty} \frac{3}{n}\left\{4+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots+\left(3-\frac{1}{n}\right)^2\right\}$ **is equal to :** [4 marks]

(A) 0

(B) $\frac{19}{3}$

(C) 19

(D) 12

Correct Answer: (C) 19

Solution:

Concept:

- For a continuous function f on $[a,b]$, equally spaced Riemann sums with width Δx converge to $\int_a^b f(x)\,dx$.
- A fixed number of endpoint terms can be added or omitted without changing the limit when their total contribution tends to zero. An outer constant multiplying the sum must still be retained.

Step 1: Identify the sampled interval The initial 4 is 2^2, so the expression inside braces is exactly

$$\sum_{j=0}^{n-1}\left(2+\frac{j}{n}\right)^2.$$

There are n terms, starting at $x=2$ and ending at $x=3-1/n$. These are the left endpoints of the n equal subdivisions of $[2,3]$, each of width $1/n$. The values remain bounded between 4 and 9, so there is no divergent endpoint contribution.

Step 2: Convert the sum to an integral Keep the factor 3 outside the Riemann sum:

$$L=3\lim_{n\to\infty}\frac{1}{n}\sum_{j=0}^{n-1}\left(2+\frac{j}{n}\right)^2=3\int_2^3 x^2\,dx.$$

Continuity of x^2 on this closed interval guarantees that this conversion is valid. Equivalently, with $t=j/n$, the integral is $3\int_0^1(2+t)^2\,dt$, which has the same endpoints and value.

Step 3: Evaluate and compare the options Using the antiderivative $x^3/3$,

$$L=3\left[\frac{x^3}{3}\right]_2^3=27-8=19.$$

The value $19/3$ in option (B) is the integral before multiplication by 3. Option (A) incorrectly ignores the increasing number of summands. Also every summand is at least 4 and the sum is multiplied by $3/n$, so a limiting value below 12 is impossible. Option (D) corresponds to treating all sample values as the leftmost value 4, whereas the function increases throughout the interval.

Step 4: Conclusion The limit is 19. Hence, the correct option is (C).

Quick Tip to Solve: Count the terms and identify the mesh width before integrating; here the mesh is $1/n$, while the additional factor 3 remains outside.

17. Consider the following statements:

P : I have fever

Q: I will not take medicine

R : I will take rest.

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to : [4 marks]

(A) $((\sim P)\vee \sim Q)\wedge((\sim P)\vee \sim R)$

(B) $(P\vee \sim Q)\wedge(P\vee \sim R)$

(C) $((\sim P)\vee \sim Q)\wedge((\sim P)\vee R)$

(D) $(P\vee Q)\wedge((\sim P)\vee R)$

Correct Answer: (C) $((\sim P)\vee \sim Q)\wedge((\sim P)\vee R)$

Solution:

Concept:

- The implication $P \Rightarrow S$ is equivalent to $(\sim P)\vee S$; it fails only when its antecedent is true and its consequent is false.
- The distributive law gives $X\vee(Y\wedge Z)=(X\vee Y)\wedge(X\vee Z)$. The negation must be applied to the proposition exactly as it is defined.

Step 1: Translate the consequent carefully The proposition Q already means "I will not take medicine". Therefore "I will take medicine" is $\sim Q$, while "I will take rest" is R. The word "and" joins these two requirements, so the complete consequent is $(\sim Q)\wedge R$. The statement to be translated is consequently

$$P\Rightarrow((\sim Q)\wedge R).$$

Replacing medicine by Q would reverse the intended meaning.

Step 2: Eliminate the implication and distribute Apply the implication identity first, without changing the conjunction in the consequent:

$$P\Rightarrow((\sim Q)\wedge R)\equiv(\sim P)\vee((\sim Q)\wedge R).$$

Now distribute the disjunction across the conjunction:

$$(\sim P)\vee((\sim Q)\wedge R)\equiv((\sim P)\vee\sim Q)\wedge((\sim P)\vee R).$$

This is precisely the expression in option (C). If P is false, both brackets become true, as any implication with a false antecedent must.

Step 3: Check why the alternatives fail Option (A) uses $\sim R$ and would require not taking rest when fever is present. Option (B) has P, rather than $\sim P$, in its disjunctions; it becomes automatically true when fever is present, even if neither action is taken. Option (D) also fails the false-antecedent test: for P false and Q false, its first bracket is false, whereas the original implication is true. These checks rule out the other forms independently of superficial symbol matching.

Step 4: Conclusion The equivalent statement is $((\sim P)\vee\sim Q)\wedge((\sim P)\vee R)$. Hence, the correct option is (C).

Quick Tip to Solve: Write the meaning of each proposition beside its letter first. A negatively worded definition such as Q needs one more negation to express the positive action.

18. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to : [4 marks]

(A) 76

(B) 81

(C) 64

(D) 72

Correct Answer: (D) 72

Solution:

Concept:

- For the parabola $y^2 = 4ax$, a tangent with nonzero slope m is $y = mx + a/m$, touching at $(a/m^2, 2a/m)$.
- A line is tangent to a circle when its perpendicular distance from the centre equals the radius. The foot of the perpendicular gives the circle's contact point.

Step 1: Impose the two tangency conditions Here $a = 4$, so write a nonvertical common tangent as $y = mx + 4/m$. The vertical tangent $x = 0$ of the parabola passes through the circle's centre and is not tangent to that circle, so no common tangent is lost by using finite nonzero m. The circle has centre $(0, 0)$ and radius $\sqrt{8}$. Hence

$$\frac{|4/m|}{\sqrt{m^2+1}} = \sqrt{8} \quad \Longrightarrow \quad m^4 + m^2 - 2 = 0.$$

Putting $u = m^2$ gives $(u - 1)(u + 2) = 0$; only $u = 1$ is nonnegative.

Step 2: Find the two contact points Take $m = 1$, for which the tangent is $y = x + 4$. It intersects the x-axis at $A = (-4, 0)$, as required. Its contact point on the parabola is $R = (4, 8)$. The perpendicular from the circle's centre has equation $y = -x$. Intersecting it with $y = x + 4$ yields $Q = (-2, 2)$. Direct substitution confirms $(-2)^2 + 2^2 = 8$ and $8^2 = 16 \cdot 4$.

Step 3: Calculate the squared separation The coordinate differences along the tangent are $4-(-2) = 6$ and $8 - 2 = 6$. Therefore

$$(QR)^2 = 6^2 + 6^2 = 72.$$

The other tangent, $m = -1$, is the reflection of this one in the x-axis; its contact points are $(-2, -2)$ and $(4, -8)$, giving the same squared distance. Thus either of the tangents specified in the question gives 72, while 76, 81 and 64 fail the coordinate distance calculation.

Step 4: Conclusion The required squared distance is $(QR)^2 = 72$. Hence, the correct option is (D).

Quick Tip to Solve: Use the slope form of a parabola tangent, then the centre-to-line distance for the circle; this avoids solving two separate double-root equations.

19. Let $\lambda \in \mathbb{R}, \vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$.
If $((\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})) \times (\vec{a} - \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$,
then $|\lambda(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|^2$ is equal to : [4 marks]

(A) 136

(B) 140

(C) 144

(D) 132

Correct Answer: (B) 140

Solution:

Concept:

- The vector triple-product identity is $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \mathbf{v}(\mathbf{u} \cdot \mathbf{w}) - \mathbf{w}(\mathbf{u} \cdot \mathbf{v})$.
- Cross products are distributive and antisymmetric. In particular, $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b}) = -2\mathbf{a} \times \mathbf{b}$, so the required squared magnitude can be reduced to that of one cross product.

Step 1: Simplify the nested cross product Put $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ and apply the triple-product identity:

$$(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a}(\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2) - \mathbf{b}(|\mathbf{a}|^2 + \mathbf{a} \cdot \mathbf{b}).$$

Cross this result with $\mathbf{a} - \mathbf{b}$. Terms involving a vector crossed with itself vanish, leaving

$$\big((\mathbf{a} + \mathbf{b}) \times \mathbf{c}\big) \times (\mathbf{a} - \mathbf{b}) = (|\mathbf{a}|^2 - |\mathbf{b}|^2)\mathbf{c}.$$

The order of these cross products matters; they are not associative.

Step 2: Determine the parameter From the given coordinates,

$$|\mathbf{a}|^2 = \lambda^2 + 13, \qquad |\mathbf{b}|^2 = \lambda^2 + 5.$$

Their difference is 8, so the displayed vector equation gives $\mathbf{c} = (1, -5, -3)$. Directly calculating the same cross product gives

$$\mathbf{a} \times \mathbf{b} = (4 - 3\lambda, -3 - 2\lambda, -\lambda^2 - 2).$$

Its second component requires $-3 - 2\lambda = -5$, hence $\lambda = 1$. This also satisfies both other components, confirming consistency of the full vector equation.

Step 3: Evaluate the requested magnitude Use the reduced cross product rather than expanding another determinant:

$$|\lambda(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b})|^2 = 4\lambda^2|\mathbf{c}|^2.$$

Since $\lambda = 1$ and $|\mathbf{c}|^2 = 1 + 25 + 9 = 35$, the answer is $4 \cdot 35 = 140$. The values 136, 144 and 132 do not equal this squared norm. In particular, the factor 2 in the cross product becomes a factor 4 after squaring.

Step 4: Conclusion The required squared magnitude is 140. Hence, the correct option is (B).

Quick Tip to Solve: For a nested vector expression, apply the triple-product identity before substituting coordinates; here the difference of squared norms becomes the constant 8.

20. If P is a 3×3 real matrix such that $P^T = aP + (a-1)I$, where $a > 1$, then : [4 marks]

(A) $|AdjP| = 1$

(B) $|AdjP| > 1$

(C) $|AdjP| = \frac{1}{2}$

(D) P is a singular matrix

Correct Answer: (A) $|AdjP| = 1$

Solution:

Concept:

- Transposition is linear, $(P^T)^T = P$, and $I^T = I$. A relation involving P and its transpose can therefore be combined with its own transpose.
- For an $n \times n$ matrix, $\det(\text{adj}\, P) = (\det P)^{n-1}$. For an invertible matrix, $\text{adj}\, P = (\det P)P^{-1}$.

Step 1: Transpose the given relation The equation is $P^T = aP + (a-1)I$, where $a > 1$. Transposing both sides gives

$$P = aP^T + (a-1)I.$$

The scalar a is unchanged by transposition. Substitute the original expression for P^T into this new relation to eliminate the transpose completely:

$$P = a\{aP + (a-1)I\} + (a-1)I.$$

This uses the same scalar in both equations and imposes a much stronger restriction than merely taking determinants of the original equation.

Step 2: Solve for the matrix Collect the identity terms to obtain

$$P = a^2P + (a^2-1)I.$$

Thus

$$(1-a^2)P = (a^2-1)I.$$

Because $a > 1$, the factor $a^2 - 1$ is nonzero and division is permitted. Consequently $P = -I$. A direct check gives $a(-I) + (a-1)I = -I = P^T$, so this matrix satisfies the original equation for every allowed a.

Step 3: Compute the adjugate determinant For this matrix of order three, $\det P = (-1)^3 = -1$. Hence

$$\det(\text{adj}\, P) = (\det P)^2 = (-1)^2 = 1.$$

Alternatively, $P^{-1} = -I$, so $\text{adj}\, P = (-1)(-I) = I$, whose determinant is 1. Options (B) and (C) contradict this exact value. Option (D) is false because $\det P = -1 \neq 0$, so P is invertible rather than singular.

Step 4: Conclusion $\det(\text{adj}\, P) = 1$. Hence, the correct option is (A).

Quick Tip to Solve: When a transpose equation has scalar coefficients, transpose it once more and substitute; verify that the scalar you divide by is nonzero.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f : A \to A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to _____. [4 marks]

Solution:

Concept:

- The condition is imposed only for pairs $m, n \in A$ whose product is also in A. It is not a multiplicativity condition on all positive integers.
- After identifying all admissible products, count the compatible choices for linked function values and multiply by the independent choices for the remaining inputs. All images must belong to the given codomain A.

Step 1: Determine the value at 1 Take $m = n = 1$. Then $f(1) = f(1)^2$, so $f(1)$ is either 0 or 1. Since $0 \notin A$, necessarily $f(1) = 1$. Every pair involving 1 then gives $f(1 \cdot n) = 1 \cdot f(n)$, which is automatically satisfied for any chosen image of n. Such pairs impose no further restriction.

Step 2: List every nontrivial product in the domain Consider pairs with both factors greater than 1. The only product lying in $A = \{1, 2, 3, 5, 8, 9\}$ is $3 \cdot 3 = 9$. Indeed, $2 \cdot 2 = 4 \notin A$, $2 \cdot 3 = 6 \notin A$, and a factor of 5, 8 or 9 paired with any member greater than 1 produces at least 10. Thus the only remaining equation is

$$f(9) = f(3)^2.$$

Although 8 is a power of 2, the intermediate input 4 is absent from A; hence there is no equation forcing $f(8) = f(2)^3$.

Step 3: Count the linked and free choices We need both $f(3)$ and its square to be in A. Among the six possible images, only 1 and 3 have squares in A: they give $(f(3), f(9)) = (1, 1)$ and $(3, 9)$, respectively. Therefore there are two compatible choices for this pair of function values. The values $f(2), f(5), f(8)$ are unrestricted, each with six independent choices. The multiplication principle now gives

$$N = 1 \cdot 2 \cdot 6^3 = 432.$$

Every counted assignment satisfies the complete list of admissible products above, so there is no overcount or missing constraint.

Step 4: Conclusion The number of functions is $\boxed{432}$ (a dimensionless count).

Quick Tip to Solve: List the products that stay inside the domain before applying multiplicativity. A missing intermediate factor can make a seemingly linked function value completely free.

22. The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is _____. [4 marks]

Solution:

Concept:

- A number is odd exactly when its units digit is odd. Here the possible final digits are $1, 3, 5$, and these cases are disjoint.
- For a multiset of n digits with repetition counts $r_1, r_2, \ldots$, the number of distinct arrangements is $n!/(r_1!r_2!\cdots)$. Identical copies of a digit are not separately labelled.

Step 1: Fix the final digit as 1 The six remaining digits are $2, 2, 2, 3, 3, 5$. Arrange them in the first six positions, dividing by the permutations within the three identical 2s and the two identical 3s. Every resulting arrangement is a valid seven-digit number because none of the available digits is zero. Thus the contribution is

$$N_1 = \frac{6!}{3!2!} = \frac{720}{12} = 60.$$

Step 2: Fix the final digit as 3 or 5 If the final digit is 3, the remaining digits are $1, 2, 2, 2, 3, 5$, with only the 2 repeated. Choosing either physical copy of 3 for the end does not produce a different number. If the final digit is 5, the remaining multiset is $1, 2, 2, 2, 3, 3$. Hence

$$N_3 = \frac{6!}{3!} = 120, \qquad N_5 = \frac{6!}{3!2!} = 60.$$

Step 3: Add the disjoint cases Each odd number ends in exactly one of the three specified digits, so there is no overlap and no missing admissible last digit. Therefore

$$N = N_1 + N_3 + N_5 = 60 + 120 + 60 = 240.$$

As a check, all distinct arrangements number $7!/(3!2!) = 420$. The even arrangements end in 2 and number $6!/(2!2!) = 180$, leaving $420 - 180 = 240$ odd ones.

Step 4: Conclusion The required number of seven-digit odd numbers is $\boxed{240}$. This is a count, so it has no physical unit.

Quick Tip to Solve: Fix the units digit first; then reduce its repetition count before applying the multiset permutation formula.

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23. Let A be the area of the region

$\{(x, y) : y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\}$.

Then 540 A is equal to : [4 marks]

Solution:

Concept:

- The required area is the integral of the upper boundary minus the larger of the two lower boundaries, over values of x where the inequalities are simultaneously possible.

- The boundaries x^2 and $(1-x)^2$ exchange under $x \mapsto 1-x$, whereas $2x(1-x)$ is unchanged. Thus the region is symmetric about $x = 1/2$.

Step 1: Find the feasible interval The upper curve must lie above both lower curves. Their comparisons give

$$2x(1-x) \geq x^2 \implies x \in [0, 2/3],$$

$$2x(1-x) \geq (1-x)^2 \implies x \in [1/3, 1].$$

The common interval is therefore $[1/3, 2/3]$. The lower parabolas meet where $x^2 = (1-x)^2$, namely $x = 1/2$. For $x \leq 1/2$, the greater lower boundary is $(1-x)^2$; after that it is x^2.

Step 2: Integrate one symmetric half On the left half the vertical height is

$$2x(1-x) - (1-x)^2 = 4x - 3x^2 - 1.$$

This height is nonnegative between $1/3$ and $1/2$. Doubling its integral accounts for the right half without counting either lower parabola twice:

$$\begin{aligned} A &= 2\int_{1/3}^{1/2} (4x - 3x^2 - 1)\, dx \\ &= 2\left[2x^2 - x^3 - x\right]_{1/3}^{1/2} \\ &= 2\left[-\frac{1}{8} + \frac{4}{27}\right] = 2 \cdot \frac{5}{216} = \frac{5}{108}. \end{aligned}$$

Step 3: Evaluate the requested multiple The question asks for $540A$, rather than the area alone. Hence

$$540A = 540 \cdot \frac{5}{108} = 5 \cdot 5 = 25.$$

The area is positive, as it must be for a nonempty bounded region. Integrating from 0 to 1 would include intervals where no point satisfies all three inequalities, so the intersection bounds are essential.

Step 4: Conclusion The area is $5/108$ square units, and the requested numerical value is $\boxed{540A = 25}$.

Quick Tip to Solve: When two lower curves are specified, subtract their maximum. Their crossing point determines where to split the area integral.

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24. If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to _____. [4 marks]

Solution:

Concept:

- A common root must satisfy both equations. Subtracting monic quadratics eliminates the squared term and produces a simpler necessary relation between the root and the parameter.
- After obtaining a candidate parameter, substitute back into the original equations. The condition $a > 0$ selects the positive parameter and must be retained when comparing square roots.

Step 1: Eliminate the squared term Let the common real root be r. Then

$$r^2 - 5ar + 1 = 0, \qquad r^2 - ar - 5 = 0.$$

Subtract the second equation from the first:

$$-4ar + 6 = 0 \quad \Longrightarrow \quad ar = \frac{3}{2}.$$

Since $a > 0$, division by a is valid and $r = 3/(2a) > 0$. A common root cannot be zero because the constant terms in both quadratics are nonzero.

Step 2: Determine the positive parameter Use the second equation and the product $ar = 3/2$:

$$r^2 = ar + 5 = \frac{3}{2} + 5 = \frac{13}{2}.$$

Together with $r = 3/(2a)$, this yields

$$\frac{9}{4a^2} = \frac{13}{2} \quad \Longrightarrow \quad a^2 = \frac{9}{26}.$$

Thus $a = 3/\sqrt{26}$. The negative square root is excluded by the printed condition on a.

Step 3: Verify the common root and identify beta For the positive candidate, take $r = \sqrt{26}/2$. Then $ar = 3/2$ and $r^2 = 13/2$, so the two original left sides become

$$\frac{13}{2} - 5 \cdot \frac{3}{2} + 1 = 0, \qquad \frac{13}{2} - \frac{3}{2} - 5 = 0.$$

The candidate is therefore sufficient as well as necessary. Comparing the given representation with this value gives

$$\frac{3}{\sqrt{2\beta}} = \frac{3}{\sqrt{26}} \quad \Longrightarrow \quad 2\beta = 26.$$

Both radicals are taken positive, consistently with the positive value of a.

Step 4: Conclusion The parameter is $a = 3/\sqrt{26}$, so the required dimensionless value is $\boxed{\beta = 13}$.

Quick Tip to Solve: Subtract the two monic quadratics first; the resulting value of the product of the parameter and root often avoids a resultant calculation.

25. Let $P\,(a_1, b_1)$ and $Q\,(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to : [4 marks]

Solution:

Concept:

- For perpendicular vectors u, v, $|u + v|^2 = |u|^2 + |v|^2$. In particular, a right triangle has squared hypotenuse equal to the sum of the squared legs.

- The area of a right triangle with perpendicular legs of lengths d and r is $dr/2$. All radii of the same circle have the same length.

Step 1: Find the distance from the origin to the centre The centre is $C(\sqrt{2}, \sqrt{3})$, so

$$OC^2 = (\sqrt{2})^2 + (\sqrt{3})^2 = 2 + 3 = 5, \qquad OC = \sqrt{5}.$$

The condition $OC \perp CP$ means the triangle OCP is right-angled at C. Here $CP = r$ is the radius. It is not the distance from the origin to P, so these lengths must not be interchanged.

Step 2: Use the triangle area to determine the radius Apply the right-triangle area formula to the supplied area:

$$\frac{1}{2}\, OC \cdot CP = \frac{\sqrt{35}}{2}.$$

Substituting $OC = \sqrt{5}$ gives

$$\sqrt{5}\, r = \sqrt{35} \quad \Longrightarrow \quad r = \sqrt{7}, \qquad r^2 = 7.$$

The positive radius is used because it is a length. The same radius applies to both distinct points P and Q, since they lie on the given circle.

Step 3: Add the squared distances of the two points Both CP and CQ are perpendicular to OC. Consequently both triangles OCP and OCQ have a right angle at C, irrespective of which side of OC the points occupy. Pythagoras gives

$$OP^2 = OC^2 + CP^2 = 5 + 7 = 12,$$

$$OQ^2 = OC^2 + CQ^2 = 5 + 7 = 12.$$

By the coordinate distance formula, $OP^2 = a_1^2 + b_1^2$ and $OQ^2 = a_2^2 + b_2^2$. Thus the requested sum is

$$a_1^2 + a_2^2 + b_1^2 + b_2^2 = OP^2 + OQ^2 = 24.$$

There is no need to determine the individual coordinates of either point.

Step 4: Conclusion The requested sum of squared coordinates is $\boxed{24}$, in squared coordinate units.

Quick Tip to Solve: Use the two right triangles at the centre: the area determines the radius, and Pythagoras gives both squared distances directly.

26. Let a line L pass through the point $P(2, 3, 1)$ and be parallel to the line $x+3y-2z-2 = 0 = x-y+2z$. If the distance of L from the point $(5, 3, 8)$ is α, then $3\alpha^2$ is equal to : [4 marks]

Solution:

Concept:

- The direction of the intersection of two planes is perpendicular to both plane normals. A cross product of the normals therefore provides a direction vector for their intersection line.

- For a line through P with direction d, and a point R, the squared perpendicular distance is $|R-P|^2 - ((R-P)\cdot d)^2/|d|^2$, obtained by subtracting the squared parallel projection.

Step 1: Determine a direction vector of the given line The two plane equations are $x+3y-2z-2=0$ and $x-y+2z=0$. Their normal vectors are $n_1=(1,3,-2)$ and $n_2=(1,-1,2)$. Hence

$$n_1 \times n_2 = (4,-4,-4) = 4(1,-1,-1).$$

We may take $d=(1,-1,-1)$; multiplying a direction vector by a nonzero scalar does not change a line's direction. Since L is parallel to the intersection line, the same d is a direction vector for L.

Step 2: Form the displacement from a point on L The desired line passes through $P=(2,3,1)$, while the external point is $R=(5,3,8)$. Thus

$$v = R-P = (3,0,7), \qquad |v|^2 = 9+49 = 58.$$

The quantities for its projection along the line are

$$v\cdot d = 3+0-7 = -4, \qquad |d|^2 = 1+1+1 = 3.$$

The negative scalar product only indicates that the projection points opposite to d; its square is what enters the distance formula.

Step 3: Subtract the parallel component Decompose v into perpendicular and parallel components relative to L. Pythagoras for those components gives

$$\alpha^2 = 58 - \frac{(-4)^2}{3} = \frac{174-16}{3} = \frac{158}{3}.$$

For a separate check, $v\times d=(7,10,-3)$, whose squared length is $49+100+9=158$. The cross-product distance formula gives the same result $\alpha^2=158/3$. Finally multiply by the factor asked in the question:

$$3\alpha^2 = 158.$$

Step 4: Conclusion The distance is $\alpha=\sqrt{158/3}$ coordinate units, and the requested value is $\boxed{3\alpha^2=158}$.

Quick Tip to Solve: Use only the plane normals to find the parallel direction; the plane constants do not affect that direction or the line through the specified point.

27. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colour is q. If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to : [4 marks]

Solution:

Concept:

- Replacement restores the original six-ball bag after every draw, so draws are independent and each of the six colours has probability $1/6$ at every position.
- Exactly three equal colours in four draws means a frequency pattern $(3, 1)$. Count the repeated colour, the exceptional colour, and the position of that exceptional draw.

Step 1: Calculate the probability for two matching draws The first ball can have any colour. Once it is replaced, exactly one of the six equally likely colours matches it on the second draw. Therefore

$$p = 1 \cdot \frac{1}{6} = \frac{1}{6}.$$

Equivalently, there are $6^2 = 36$ ordered two-draw outcomes and six matching-colour outcomes, giving $p = 6/36$. Replacement is essential: there is only one ball of each colour, but it can be selected repeatedly.

Step 2: Count the four-draw outcomes with exactly three alike Choose the colour appearing three times in six ways. Choose the position of the single different colour in $\binom{4}{1} = 4$ ways, then choose its colour in five ways. Hence the number of favourable ordered outcomes is

$$6 \cdot 4 \cdot 5 = 120.$$

There are $6^4 = 1296$ equally likely ordered outcomes, so

$$q = \frac{120}{1296} = \frac{5}{54}.$$

The exceptional colour must differ from the repeated one; choosing from all six colours would incorrectly include four identical draws. No valid pattern is counted twice, because its repeated colour and its exceptional position are uniquely determined.

Step 3: Reduce the ratio to coprime integers Use a common denominator to simplify the ratio:

$$p : q = \frac{1}{6} : \frac{5}{54} = \frac{9}{54} : \frac{5}{54} = 9 : 5.$$

Since 9 and 5 have greatest common divisor 1, these are the required coprime values $m = 9$ and $n = 5$. Their sum is

$$m + n = 9 + 5 = 14.$$

The ratio is a ratio of probabilities, so it is dimensionless and no unit conversion is involved.

Step 4: Conclusion The probabilities are $p = 1/6$ and $q = 5/54$. Thus $m : n = 9 : 5$, and the requested dimensionless answer is $\boxed{14}$.

Quick Tip to Solve: For exactly three alike in four draws, count a repeated colour and one different colour; the fourth draw must not be allowed to match the triple.

28. If $\int \sqrt{\sec 2x - 1}dx = \alpha \log_e \left|\cos 2x + \beta + \sqrt{\cos 2x \left(1 + \cos \frac{1}{\beta}x\right)}\right| +$ constant, then $\beta - \alpha$ is equal to _____.

[4 marks]

Solution:

Concept:

- Use $\sec 2x - 1 = 2\sin^2 x/\cos 2x$. A real radical requires $\cos 2x > 0$; for evaluating the displayed logarithmic form take the usual local branch $0 < x < \pi/4$, where $\sin x$ and $\cos x$ are positive.
- The standard primitive $\int du/\sqrt{u^2 - 1} = \log|u + \sqrt{u^2 - 1}| + C$ follows by differentiating the logarithm. Multiplying its argument by a positive constant changes only the integration constant.

Step 1: Reduce the radical using a cosine substitution On the stated branch,

$$\sqrt{\sec 2x - 1} = \frac{\sqrt{2}\sin x}{\sqrt{2\cos^2 x - 1}}.$$

Put $u = \sqrt{2}\cos x$, so that $du = -\sqrt{2}\sin x\, dx$. The integral becomes

$$I = -\int \frac{du}{\sqrt{u^2 - 1}} = -\log|\sqrt{2}\cos x + \sqrt{\cos 2x}| + C.$$

Here $u > 1$, so all radicals in this calculation are real and the logarithm's argument is positive.

Step 2: Rewrite the logarithm in the printed form Let $c = \cos 2x$. Since $2\cos^2 x = 1 + c$, squaring the positive argument gives

$$\begin{aligned}(\sqrt{2}\cos x + \sqrt{c})^2 &= 1 + 2c + 2\sqrt{c(1+c)} \\ &= 2\left(c + \frac{1}{2} + \sqrt{c(1+c)}\right).\end{aligned}$$

Consequently,

$$I = -\frac{1}{2}\log\left|\cos 2x + \frac{1}{2} + \sqrt{\cos 2x(1 + \cos 2x)}\right| + C.$$

The factor 2 inside the logarithm contributes only an additive constant. Comparing $\cos((1/\beta)x)$ with $\cos 2x$ gives $\beta = 1/2$, and the logarithmic coefficient is $\alpha = -1/2$.

Step 3: Check the sign and compute the difference Differentiating the first logarithmic form yields

$$I' = \frac{\sqrt{2}\sin x}{\sqrt{2\cos^2 x - 1}} = \sqrt{\sec 2x - 1}$$

on $0 < x < \pi/4$, verifying the negative coefficient. Thus

$$\beta - \alpha = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1.$$

As usual for a radical involving a square of a trigonometric function, the primitive is interpreted locally: $\sqrt{\sin^2 x} = |\sin x|$, and a corresponding sign adjustment is needed on branches with negative sine.

Step 4: Conclusion In the displayed local antiderivative, $\beta = 1/2$ and $\alpha = -1/2$. The required dimensionless value is $\boxed{\beta - \alpha = 1}$.

Quick Tip to Solve: After integrating with a cosine substitution, square the logarithm's argument to create the printed double-angle expression; remember the resulting factor of one half.

29. 50^{th} root of a number x is 12 and 50^{th} root of another number y is 18 . Then the remainder obtained on dividing $(x + y)$ by 25 is _____. [4 marks]

Solution:

Concept:

- The given roots imply $x = 12^{50}$ and $y = 18^{50}$. The required answer is the least nonnegative residue of their sum modulo 25.
- If $a \equiv b \pmod{25}$, replacing a factor by its residue preserves products and positive integer powers. Reducing at intermediate stages avoids constructing the enormous original numbers.

Step 1: Find a short power of 12 First reduce successive small powers, keeping the modulus fixed throughout:

$$12^2 = 144 \equiv 19, \qquad 12^4 \equiv 19^2 = 361 \equiv 11 \pmod{25}.$$

Consequently,

$$12^5 \equiv 11 \times 12 = 132 \equiv 7, \qquad 12^{10} \equiv 7^2 = 49 \equiv -1 \pmod{25}.$$

The exponent 50 is five times 10, so $x = (12^{10})^5 \equiv (-1)^5 = -1 \pmod{25}$.

Step 2: Reduce the second number and add For the other base there is an even shorter reduction:

$$18^2 = 324 \equiv 24 \equiv -1 \pmod{25}.$$

Since $50 = 2 \times 25$, we obtain

$$y = (18^2)^{25} \equiv (-1)^{25} = -1 \pmod{25}.$$

Thus $x + y \equiv -2 \pmod{25}$. A negative residue is convenient during the calculation, but the remainder on division must lie from 0 to 24. Add one modulus: $-2 + 25 = 23$. The two odd outer powers are important: neither negative residue becomes positive.

Step 3: Conclusion The remainder obtained on dividing $x + y$ by 25 is $\boxed{23}$.

Quick Tip to Solve: Look for a small power congruent to 1 or -1 before handling a large exponent; convert a negative final residue to the range 0 to 24.

30. The 8^{th} common term of the series

$$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots$$
$$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots$$

is : [4 marks]

Solution:

Concept:

- The first progression has first term 3 and common difference 4; the second has first term 1 and common difference 5. A common term must satisfy both arithmetic-progression membership conditions.
- For two integer progressions, successive common terms differ by the least common multiple of their differences. Here $\text{lcm}(4, 5) = 20$. The first positive common term must be located before using the nth-term formula.

Step 1: Find the first common term Write a general term of each progression using positive integer indices n, m:

$$a_n = 3 + 4(n-1) = 4n - 1, \qquad b_m = 1 + 5(m-1) = 5m - 4.$$

A common term therefore obeys $4n - 1 = 5m - 4$, or $4n + 3 = 5m$. Reducing modulo 5 gives $n \equiv 3 \pmod 5$. The smallest positive such index is $n = 3$, giving $a_3 = 11$; indeed $b_3 = 11$ too. The earlier terms 3 and 7 do not appear in the second progression, so 11 is the first common term, not merely an arbitrary intersection.

Step 2: Construct the common progression and select its eighth term For the next common term the first progression advances by five terms and the second by four terms, increasing the value by 20 in either case. Equivalently, the index solutions are $n = 3 + 5k$, $m = 3 + 4k$, for integers $k \geq 0$. Thus the common terms are

$$11,\ 31,\ 51,\ 71, \ldots, \qquad c_r = 11 + 20(r-1).$$

For $r = 8$, this gives $c_8 = 11 + 7 \times 20 = 151$. As a direct membership check, $(151 + 1)/4 = 38$ and $(151 + 4)/5 = 31$ are both positive integers. Using eight increments would instead select the ninth common term.

Step 3: Conclusion The eighth common term is $\boxed{151}$.

Quick Tip to Solve: Find the earliest intersection first, then use the LCM of the two common differences as the common progression's step.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A vehicle travels 4 km with speed of 3 km/h and another 4 km with speed of 5 km/h, then its average speed is [4 marks]

(A) 3.75 km/h

(B) 4.25 km/h

(C) 3.50 km/h

(D) 4.00 km/h

Correct Answer: (A) 3.75 km/h

Solution:

Concept:

- Average speed is total distance travelled divided by total elapsed time: $v_{\text{av}} = D/T$. Equal distances do not mean equal travel times.
- For two equal distances covered at speeds u and v, this definition gives the harmonic mean $2uv/(u+v)$, rather than the arithmetic mean.

Step 1: Calculate the time for each part Use the same distance and speed units throughout, so each travel time is in hours. The time taken over the first 4 km is

$$t_1 = \frac{4}{3}\,\text{h}.$$

The next 4 km is covered faster, taking

$$t_2 = \frac{4}{5}\,\text{h}.$$

Hence the total time is $T = 4/3 + 4/5 = 32/15\,\text{h}$. The slower leg takes longer, so it must contribute a larger share of the elapsed time; simply averaging the two speeds would ignore that fact.

Step 2: Divide the total distance by the total time The total distance is $D = 4 + 4 = 8\,\text{km}$. Therefore

$$v_{\text{av}} = \frac{8}{32/15} = \frac{15}{4} = 3.75\,\text{km}\,\text{h}^{-1}.$$

The equal-distance formula gives the same check:

$$\frac{2 \times 3 \times 5}{3+5} = \frac{30}{8} = 3.75\,\text{km}\,\text{h}^{-1}.$$

Option D, 4.00, is the arithmetic mean and would apply to equal times, which are not given. Option B, 4.25, even exceeds that arithmetic mean, inconsistent with the longer time spent at 3 km/h. Option C, 3.50, does not reproduce the actual total time: $8/3.50 \neq 32/15$.

Step 3: Conclusion The average speed is $\boxed{3.75\,\text{km}\,\text{h}^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal-distance legs, add the times or use the harmonic mean; reserve the arithmetic mean for equal-time legs.

32. Match List I with List II:

List I		List II	
A.	**Torque**	**I.**	$\mathrm{kg\,m^{-1}\,s^{-2}}$
B.	**Energy density**	**II.**	$\mathrm{kg\,m\,s^{-1}}$
C.	**Pressure gradient**	**III.**	$\mathrm{kg\,m^{-2}\,s^{-2}}$
D.	**Impulse**	**IV.**	$\mathrm{kg\,m^{2}\,s^{-2}}$

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-I, C-III, D-II

(B) A-IV, B-I, C-II, D-III

(C) A-I, B-IV, C-III, D-II

(D) A-IV, B-III, C-I, D-II

Correct Answer: (A) A-IV, B-I, C-III, D-II

Solution:

Concept:

- Build each derived SI unit from force, length and time. Newton's second law gives $[F] = \mathrm{kg\,m\,s^{-2}}$.
- Torque is force times perpendicular distance, energy density is energy per volume, pressure gradient is pressure change per distance, and impulse is force times time.

Step 1: Match torque and energy density For torque, multiply the force unit by a length:

$$[\tau] = \mathrm{N\,m} = \mathrm{kg\,m^{2}\,s^{-2}}.$$

This is entry IV, so A-IV. Energy also has unit joule, $\mathrm{kg\,m^{2}\,s^{-2}}$, but energy density divides energy by volume, not area:

$$[u] = \frac{\mathrm{J}}{\mathrm{m^{3}}} = \mathrm{kg\,m^{-1}\,s^{-2}}.$$

This is entry I, giving B-I. Equal dimensions of torque and energy do not make torque an energy density; the division by volume changes the power of length.

Step 2: Match pressure gradient and impulse Pressure is force per area, so its unit is $\mathrm{kg\,m^{-1}\,s^{-2}}$. A gradient is a change per unit distance, giving

$$[\nabla P] = \frac{\mathrm{kg\,m^{-1}\,s^{-2}}}{\mathrm{m}} = \mathrm{kg\,m^{-2}\,s^{-2}}.$$

Thus C-III. Finally, impulse equals the change in momentum, or force multiplied by time:

$$[I] = \mathrm{N\,s} = \mathrm{kg\,m\,s^{-1}},$$

so D-II. Option B interchanges the gradient and impulse units; option C interchanges torque and energy-density units; option D interchanges energy density and pressure gradient. Only option A retains all four independently derived matches.

Step 3: Conclusion The complete matching is $\boxed{\mathrm{A-IV,\ B-I,\ C-III,\ D-II}}$. Hence, the correct option is (A).

Quick Tip to Solve: A spatial gradient adds one inverse-length factor: derive pressure first, then divide its unit by metre.

33. An object is allowed to fall from a height R above the earth, where R is the radius of earth. Its velocity when it strikes the earth's surface, ignoring air resistance, will be [4 marks]

(A) $\sqrt{\frac{gR}{2}}$

(B) $\sqrt{gR}$

(C) $\sqrt{2gR}$

(D) $2\sqrt{gR}$

Correct Answer: (B) $\sqrt{gR}$

Solution:

Concept:

- Outside a spherical earth, gravitational potential energy is $U(r) = -GMm/r$, where r is measured from the earth's centre.
- With air resistance ignored, mechanical energy is conserved. The surface acceleration g satisfies $g = GM/R^2$; it cannot be treated as constant over a fall of height R.

Step 1: Set the initial and final distances from the centre A height R above the surface corresponds to initial radial distance $r_i = R + R = 2R$, while impact occurs at $r_f = R$. The object is allowed to fall, so its initial speed is zero. Its initial mechanical energy is therefore

$$E_i = -\frac{GMm}{2R}.$$

At the surface the energy is

$$E_f = \frac{1}{2}mv^2 - \frac{GMm}{R}.$$

The negative potential energy becomes more negative during the fall, and the lost potential energy supplies the positive kinetic energy.

Step 2: Use conservation and express the result with surface gravity Equating the two energies and rearranging gives

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R} = \frac{GMm}{2R}, \qquad v^2 = \frac{GM}{R} = gR.$$

Thus the impact speed is $\sqrt{gR}$, directed radially towards the earth. Option C results from the constant-g formula $v^2 = 2gR$, unsuitable over this large altitude. Option A corresponds to half the actual kinetic energy, and option D would require four times the actual v^2. As a physical check, the speed must be below escape speed $\sqrt{2gR}$, since the object starts at a finite distance rather than infinity.

Step 3: Conclusion The object strikes the surface with speed $\boxed{\sqrt{gR}}$, directed downwards. Hence, the correct option is (B).

Quick Tip to Solve: For heights comparable to the earth's radius, use $-GMm/r$ and measure both radii from the centre; constant surface gravity overestimates the fall speed.

34. A force is applied to a steel wire 'A', rigidly clamped at one end. As a result elongation in the wire is 0.2 mm. If same force is applied to another steel wire ' B ' of double the length and a diameter 2.4 times that of the wire ' A ', the elongation in the wire ' B ' will be (wires having uniform circular cross sections) [4 marks]

(A) 6.9×10^{-2} mm

(B) 6.06×10^{-2} mm

(C) 2.77×10^{-2} mm

(D) 3.0×10^{-2} mm

Correct Answer: (A) 6.9×10^{-2} mm

Solution:

Concept:

- Within the elastic limit, Young's modulus is $Y = (F/A)/(\Delta L/L)$, so the extension of a uniform wire is $\Delta L = FL/(AY)$.
- For circular cross sections, $A = \pi d^2/4$. For wires of the same material under the same force, extension is proportional to L/d^2, not L/d.

Step 1: Form the extension ratio Both wires are steel, so they have the same Young modulus. The applied force is also unchanged. Taking the ratio cancels these two common quantities:

$$\frac{\Delta L_B}{\Delta L_A} = \frac{L_B}{L_A}\frac{A_A}{A_B} = \frac{L_B}{L_A}\left(\frac{d_A}{d_B}\right)^2.$$

Insert $L_B/L_A = 2$ and $d_B/d_A = 2.4$. The area ratio is $A_B/A_A = (2.4)^2 = 5.76$, hence $\Delta L_B/\Delta L_A = 2/5.76$. Although the doubled length increases compliance, the greater cross-sectional area has a stronger opposing effect.

Step 2: Calculate the extension and compare the options Keep the supplied extension in millimetres because the ratio is dimensionless:

$$\Delta L_B = 0.2 \times \frac{2}{5.76}\,\text{mm} = 0.069444\ldots\ \text{mm} \simeq 6.9 \times 10^{-2}\,\text{mm}.$$

This matches option A to the precision printed. Option B gives $0.0606\,\text{mm}$, while options C and D give $0.0277\,\text{mm}$ and $0.030\,\text{mm}$; none equals the ratio result. The answer is smaller than 0.2 mm, as expected because the area rises by a factor 5.76 whereas the length rises only by a factor 2. No numerical value of Young's modulus or applied force is needed.

Step 3: Conclusion The elongation in wire B is approximately $\boxed{6.9 \times 10^{-2}\,\text{mm}}$. Hence, the correct option is (A).

Quick Tip to Solve: In an extension ratio, cancel the common material and load first; remember that multiplying diameter by 2.4 multiplies area by 2.4^2.

35. An electron accelerated through a potential difference V_1 has a de-Broglie wavelength of λ. When the potential is changed to V_2, its de-Broglie wavelength increases by 50%. The value of $\left(\frac{V_1}{V_2}\right)$ is equal to [4 marks]

(A) $\frac{3}{2}$

(B) 4

(C) 3

(D) $\frac{9}{4}$

Correct Answer: (D) $\frac{9}{4}$

Solution:

Concept:

- An electron accelerated from rest through potential difference V gains kinetic energy eV. In the nonrelativistic regime, $p^2/(2m_e) = eV$.
- The de Broglie relation $\lambda = h/p$ therefore gives $\lambda = h/\sqrt{2m_e eV}$. For the same electron, wavelength is inversely proportional to the square root of the accelerating potential.

Step 1: Express the stated percentage increase as a ratio Initially the wavelength is $\lambda_1 = \lambda$. An increase by 50 percent means adding half of the original value, so

$$\lambda_2 = \lambda + 0.50\lambda = \frac{3}{2}\lambda, \qquad \frac{\lambda_2}{\lambda_1} = \frac{3}{2}.$$

It does not mean that the final wavelength is only $\lambda/2$. A larger wavelength corresponds to a smaller momentum, so the new accelerating potential must be lower. Thus V_1/V_2 should be greater than one, consistent with the requested order of the ratio.

Step 2: Square the wavelength ratio to obtain the potential ratio Using the same constants in the two acceleration experiments,

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{V_1}{V_2}}, \qquad \frac{V_1}{V_2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}.$$

Equivalently $V_2 = 4V_1/9$, which is indeed a reduction of voltage. Option A, $3/2$, uses the wavelength ratio without squaring it. Options B and C would imply wavelength ratios 2 and $\sqrt{3}$, respectively, rather than 1.5, so they do not satisfy the stated increase. The potential ratio is dimensionless; constants such as the electron mass, charge and Planck's constant cancel completely.

Step 3: Conclusion The required ratio is $\boxed{V_1/V_2 = 9/4}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert a percentage wavelength change into the final-to-initial wavelength ratio first, then square it to obtain the inverse voltage ratio.

36. A machine gun of mass $10\ \text{kg}$ fires $20\ \text{g}$ bullets at the rate of 180 bullets per minute with a speed of $100\ \text{m s}^{-1}$ each. The recoil velocity of the gun is [4 marks]

(A) $0.02\ \text{m/s}$

(B) $1.5\ \text{m/s}$

(C) $2.5\ \text{m/s}$

(D) $0.6\ \text{m/s}$

Correct Answer: (D) $0.6\ \text{m/s}$

Solution:

Concept:

- In the absence of an external horizontal impulse, the momentum acquired by the bullets is balanced by opposite momentum of the gun. Initially the gun and its ammunition are at rest.
- The firing rate is a number per unit time. For a firing interval t, the number discharged is $N = nt$, and the usual constant-gun-mass recoil estimate is $Mv = Nmu$.

Step 1: Convert the mass and firing rate The mass of each bullet must be converted to kilograms before using momentum conservation. The given values become

$$m = 20 \times 10^{-3} = 0.020\,\text{kg}, \qquad n = \frac{180}{60} = 3\,\text{s}^{-1}.$$

Thus three bullets are discharged in one second. Their forward momentum, using the given bullet speed, is

$$p = 3(0.020)(100) = 6\,\text{kg m s}^{-1}.$$

Step 2: Balance the opposite momenta Take the direction of firing as positive. Using the one-second firing interval implicit in the listed choices and treating the gun mass as the stated $10\,\text{kg}$, conservation of momentum gives

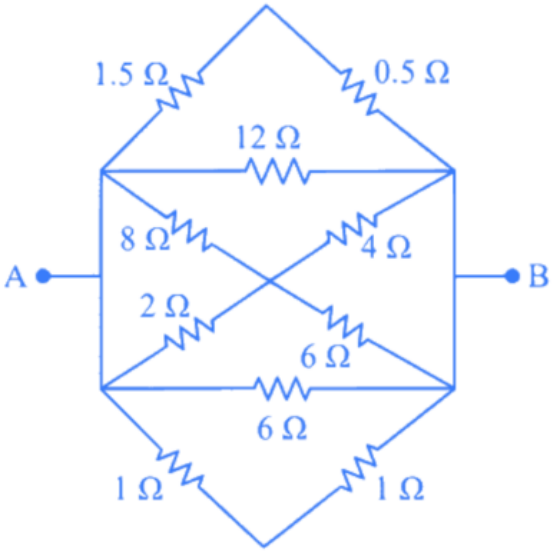
$$0 = Mv_{\text{gun}} + p, \qquad v_{\text{gun}} = -\frac{6}{10} = -0.6\,\text{m s}^{-1}.$$

The minus sign means recoil opposite to the bullets. The other listed speeds do not balance the momentum of the three bullets fired in one second. Strictly, a firing rate alone gives recoil force, $nmu = 6\,\text{N}$; a recoil velocity requires a firing duration. For a general interval t, this constant-mass model gives $|v| = 0.6t\,\text{m s}^{-1}$ when t is expressed in seconds.

Step 3: Conclusion The intended recoil speed for one second of firing is $0.6\,\text{m s}^{-1}$, opposite to the direction of firing. Hence, the correct option is (D). The stated rate alone does not specify a duration-independent recoil velocity.

Quick Tip to Solve: Convert bullets per minute to bullets per second first. Distinguish momentum per second (force) from momentum accumulated during a specified firing interval.

37. The equivalent resistance between A and B is _____. [4 marks]



(A) $\frac{1}{2}\Omega$
(B) $\frac{2}{3}\Omega$
(C) $\frac{3}{2}\Omega$
(D) $\frac{1}{3}\Omega$

Correct Answer: (B) $\frac{2}{3}\Omega$

Solution:

Concept:

- Ideal connecting wires form equipotential nodes. Both left vertices belong to terminal A, and both right vertices belong to terminal B; their positions on the drawing do not make them distinct nodes.
- Resistances in parallel add by reciprocals, whereas resistances in series add directly. Identify the central junction before reducing the crossed interior branches.

Step 1: Reduce the central branch Let the crossing junction inside the network be C. The $8\,\Omega$ and $2\,\Omega$ resistors both connect A to C, so they are parallel. Similarly, $4\,\Omega$ and the diagonal $6\,\Omega$ both connect C to B. Hence

$$R_{AC} = \frac{8 \times 2}{8+2} = \frac{8}{5}\,\Omega, \qquad R_{CB} = \frac{4 \times 6}{4+6} = \frac{12}{5}\,\Omega.$$

These two equivalent resistors are in series along the central path:

$$R_{\text{central}} = \frac{8}{5} + \frac{12}{5} = 4\,\Omega.$$

Step 2: Combine all paths between the terminals The top outer path has resistance $1.5 + 0.5 = 2\,\Omega$, and the bottom outer path has resistance $1 + 1 = 2\,\Omega$. The horizontal $12\,\Omega$ and $6\,\Omega$ resistors are also

directly across the terminals. Thus there are five parallel paths, of resistances $2, 12, 4, 6, 2\,\Omega$:

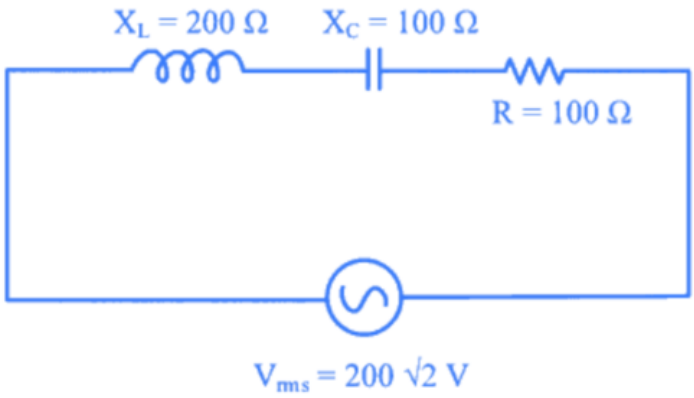
$$\frac{1}{R_{AB}} = \frac{1}{2} + \frac{1}{12} + \frac{1}{4} + \frac{1}{6} + \frac{1}{2} = \frac{6+1+3+2+6}{12} = \frac{3}{2}\,\Omega^{-1}.$$

Therefore $R_{AB} = 2/3\,\Omega$. The proposed $1/2$ and $1/3\,\Omega$ would require larger total conductance, while $3/2\,\Omega$ is the reciprocal result rather than the resistance.

Step 3: Conclusion The equivalent resistance is $\boxed{R_{AB} = \frac{2}{3}\,\Omega}$. Hence, the correct option is (B).

Quick Tip to Solve: Merge the vertices joined by bare wire before using series or parallel rules. At the central junction, reduce the left and right parallel pairs separately.

38. In the given circuit, rms value of current (I_{rms}) through the resistor R is: [4 marks]



(A) $2\sqrt{2}$ A

(B) 2 A

(C) $\frac{1}{2}$ A

(D) 20 A

Correct Answer: (B) 2 A

Solution:

Concept:

- In a series LCR circuit, the same alternating current passes through the inductor, capacitor and resistor. The rms current through R is therefore the rms current of the entire series circuit.
- The impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. Inductive and capacitive voltage contributions oppose in phase, so their reactances subtract rather than add.

Step 1: Find the net reactance and impedance Read the circuit values as $X_L = 200\,\Omega$, $X_C = 100\,\Omega$, $R = 100\,\Omega$, and $V_{\text{rms}} = 200\sqrt{2}\,\text{V}$. The net reactance is inductive:

$$X = X_L - X_C = 200 - 100 = 100\,\Omega.$$

The resistive and reactive voltage drops are perpendicular in a phasor representation. Their corresponding impedance components therefore combine quadratically:

$$Z = \sqrt{100^2 + 100^2} = 100\sqrt{2}\,\Omega.$$

Adding R, X_L and X_C arithmetically would ignore their phase relationships.

Step 2: Apply the rms form of Ohm's law Both the stated voltage and requested current are rms quantities, so no additional factor of $\sqrt{2}$ is needed:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200\sqrt{2}}{100\sqrt{2}} = 2\,\text{A}.$$

This is also the current in the resistor, because there is no branch in the circuit. Option (A), $2\sqrt{2}\,\text{A}$, is the peak current rather than the rms current. Option (C) does not give the required applied voltage when multiplied by the impedance, and option (D) is ten times the actual current. The net reactance is nonzero, so the circuit cannot be treated as a pure $100\,\Omega$ resistor.

Step 3: Conclusion The rms current through R is $\boxed{2\,\text{A}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the difference of reactances inside the impedance formula. When the source already gives rms voltage, divide by impedance directly to obtain rms current.

39. A thin prism P_1 with an angle 6° and made of glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism P_2 is [4 marks]

(A) 4.5°

(B) 7.8°

(C) 1.3°

(D) 6°

Correct Answer: (A) 4.5°

Solution:

Concept:

- For a thin prism of small refracting angle A, its average deviation is approximately $\delta = (\mu - 1)A$, where μ is the average refractive index of its glass.
- Two opposed prisms produce no average deviation when their average deviations have equal magnitudes and opposite directions. This condition differs from cancelling dispersion.

Step 1: Write the condition for cancelling average deviation The two prisms must be oppositely oriented, so their mean deviations subtract. With $A_1 = 6^\circ$, $\mu_1 = 1.54$ and $\mu_2 = 1.72$, the zero-average-deviation condition is

$$(\mu_1 - 1)A_1 = (\mu_2 - 1)A_2.$$

The prism angles can be used in degrees on both sides because the same angle-conversion factor

would multiply both sides. Consequently the ratio of the prism angles is unaffected by the choice of angular unit.

Step 2: Calculate the required second angle Solving for the unknown refracting angle gives

$$A_2 = \frac{\mu_1 - 1}{\mu_2 - 1} A_1 = \frac{0.54}{0.72} \times 6^\circ = \frac{3}{4} \times 6^\circ = 4.5^\circ.$$

As a check, the first deviation is $0.54 \times 6^\circ = 3.24^\circ$, and the magnitude of the second is $0.72 \times 4.5^\circ = 3.24^\circ$. Their opposite directions cancel the average bending, while the glasses can still have different dispersive effects. An angle of 6° would overcompensate because the second refractive index is greater. The 7.8° and 1.3° choices also fail the equality of the two mean deviations. No dispersive-power values are required for the stated condition.

Step 3: Conclusion The angle of the second prism is $\boxed{4.5^\circ}$, with its orientation opposing that of the first prism. Hence, the correct option is (A).

Quick Tip to Solve: For dispersion without average deviation, equate $(\mu - 1)A$ for the opposed prisms. A glass with larger $\mu - 1$ needs the smaller prism angle.

40. A point source of 100 W emits light with 5% efficiency. At a distance of 5 m from the source, the intensity produced by the electric field component is: [4 marks]

(A) $\frac{1}{40\pi}\frac{W}{m^2}$
(B) $\frac{1}{10\pi}\frac{W}{m^2}$
(C) $\frac{1}{20\pi}\frac{W}{m^2}$
(D) $\frac{1}{2\pi}\frac{W}{m^2}$

Correct Answer: (A) $\frac{1}{40\pi}\frac{W}{m^2}$

Solution:

Concept:

- For isotropic radiation from a point source, the total light intensity at distance r is $I = P_{\text{light}}/(4\pi r^2)$. Only the useful radiated light power enters this expression.
- The electric and magnetic fields of a travelling electromagnetic wave carry equal average energy densities. Each field component therefore accounts for half of the total wave intensity.

Step 1: Calculate the total light intensity The electrical input power is $100\,\text{W}$, but the light-emission efficiency is only 5%. Thus

$$P_{\text{light}} = \frac{5}{100}(100) = 5\,\text{W}.$$

At $r = 5\,\text{m}$, this light is distributed over a sphere of area

$$4\pi r^2 = 4\pi(5)^2 = 100\pi\,\text{m}^2.$$

The total intensity of the electromagnetic radiation is consequently

$$I = \frac{5}{100\pi} = \frac{1}{20\pi}\,\mathrm{W\,m^{-2}}.$$

Step 2: Take the electric-field contribution The question asks specifically for the contribution associated with the electric field, rather than the sum of the electric and magnetic contributions. Using the equal energy sharing gives

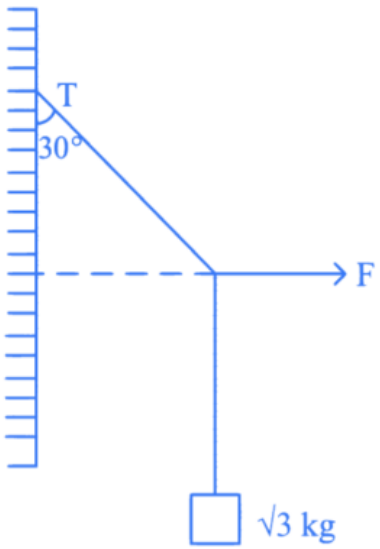
$$I_E = \frac{I}{2} = \frac{1}{2}\left(\frac{1}{20\pi}\right) = \frac{1}{40\pi}\,\mathrm{W\,m^{-2}}.$$

Option (C) is the total intensity and misses this final factor of one half. Option (B) is twice the total intensity, and option (D) is much greater still; neither follows from the useful light power and spherical spreading. The inverse-square dependence applies to the full intensity and to each component contribution alike.

Step 3: Conclusion The intensity associated with the electric-field component is $\boxed{\frac{1}{40\pi}\,\mathrm{W\,m^{-2}}}$. Hence, the correct option is (A).

Quick Tip to Solve: First apply the source efficiency and inverse-square spreading. Then take half only when the question explicitly asks for one field component's energy contribution.

41. A block of $\sqrt{3}$ kg is attached to a string whose other end is attached to the wall. An unknown force F is applied so that the string makes an angle of 30° with the wall. The tension T is: (Given $g = 10\ ms^{-2}$) [4 marks]



(A) 15 N

(B) 10 N

(C) 25 N

(D) 20 N

Correct Answer: (D) 20 N

Solution:

Concept:

- A stationary junction has zero resultant force. Resolve the sloping tension into vertical and horizontal components before applying the equilibrium conditions.
- The given 30° is measured from the vertical wall. Therefore the upward component of the tension is $T\cos 30^\circ$, and the horizontal component is $T\sin 30^\circ$.

Step 1: Balance the vertical forces at the junction The lower vertical string supports a block of mass $\sqrt{3}\,\text{kg}$. Since the block is at rest, the downward pull on the junction is its weight:

$$W = mg = (\sqrt{3})(10) = 10\sqrt{3}\,\text{N}.$$

The applied force F shown in the figure is horizontal, so it has no vertical component. Hence the entire weight must be balanced by the upward component of the sloping string tension:

$$T\cos 30^\circ = 10\sqrt{3}.$$

Step 2: Solve for the tension and check horizontal balance Substitute $\cos 30^\circ = \sqrt{3}/2$ into the vertical equation:

$$T\frac{\sqrt{3}}{2} = 10\sqrt{3} \quad \Rightarrow \quad T = 20\,\text{N}.$$

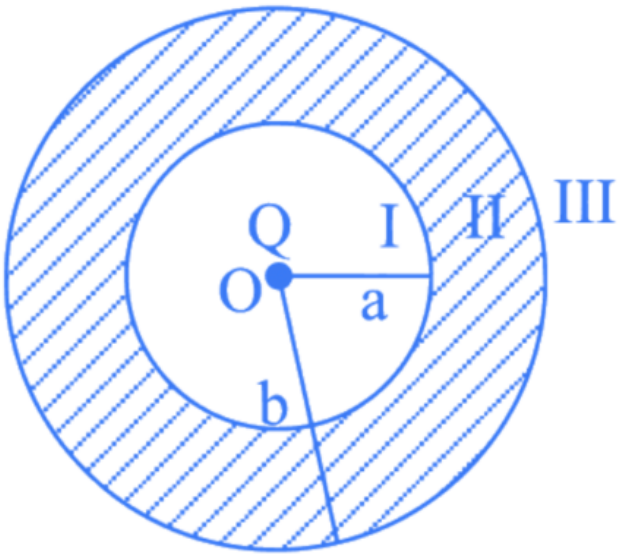
The horizontal balance then gives $F = T\sin 30^\circ = 20(1/2) = 10\,\text{N}$, which is the applied force, not the requested tension. This explains why option (B) is tempting but incorrect. Options (A) and (C) would have vertical tension components different from the block's weight, so neither can give equilibrium. The tension must exceed mg because only its vertical component supports the load, which is also consistent with $20 > 10\sqrt{3}$.

Step 3: Conclusion The tension in the inclined string is $\boxed{T = 20\,\text{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: Check whether the stated angle is measured from the wall or the horizontal. For an angle from the vertical, use the cosine component to support the weight.

42. As shown in the figure, a point charge Q is placed at the centre of conducting spherical shell of inner radius a and outer radius b. The electric field due to charge Q in three different regions I, II and III is given by:

(I : $r < a$, II : $a < r < b$, III: $r > b$) [4 marks]



(A) $E_I = 0, E_{II} = 0, E_{III} \neq 0$

(B) $E_I \neq 0, E_{II} = 0, E_{III} = 0$

(C) $E_I \neq 0, E_{II} = 0, E_{III} \neq 0$

(D) $E_I = 0, E_{II} = 0, E_{III} = 0$

Correct Answer: (C) $E_I \neq 0, E_{II} = 0, E_{III} \neq 0$

Solution:

Concept:

- In electrostatic equilibrium, the electric field inside the material of a conductor is zero. This restriction applies to the metal, not automatically to an empty cavity containing a charge.
- For a spherically symmetric charge distribution, Gauss's law gives $E(4\pi r^2) = Q_{\text{enclosed}}/\varepsilon_0$. A central point charge preserves the spherical symmetry.

Step 1: Examine the cavity and the conducting material For region I, $0 < r < a$, a spherical Gaussian surface encloses the central point charge Q. The induced spherical surface charges lie outside it and do not produce a field inside their surfaces. Thus

$$E_{\text{I}} = \frac{Q}{4\pi\varepsilon_0 r^2} \neq 0.$$

For region II, $a < r < b$, the Gaussian surface lies in the conductor. Electrostatic equilibrium requires

$$E_{\text{II}} = 0.$$

The inner surface therefore carries induced charge $-Q$, cancelling the net enclosed charge for a Gaussian surface in the metal.

Step 2: Examine the region outside the shell For the initially neutral conducting shell shown, the compensating charge on its outer surface is $+Q$. A Gaussian sphere with $r > b$ encloses the central charge and both induced surface charges:

$$Q_{\text{enclosed}} = Q - Q + Q = Q.$$

Consequently

$$E_{\text{III}} = \frac{Q}{4\pi\varepsilon_0 r^2} \neq 0.$$

Options (A) and (D) incorrectly set the cavity field to zero despite the enclosed central charge. Option (B) incorrectly eliminates the exterior field; shielding makes the field zero within the conducting material but does not remove the net charge seen from outside a neutral shell. At the charge itself the point-charge field is singular, so the region-I formula is understood for $r > 0$.

Step 3: Conclusion The fields satisfy $\boxed{E_{\text{I}} \neq 0, \quad E_{\text{II}} = 0, \quad E_{\text{III}} \neq 0}$. Hence, the correct option is (C).

Quick Tip to Solve: Apply the zero-field rule only inside the metal. Then use the enclosed charge separately for a Gaussian sphere in the cavity and for one outside the entire shell.

43. A flask contains hydrogen and oxygen in the ratio of $2:1$ by mass at temperature 27°C. The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is: [4 marks]

(A) 1 : 1

(B) 4 : 1

(C) 1 : 4

(D) 2 : 1

Correct Answer: (A) 1 : 1

Solution:

Concept:

- The mean translational kinetic energy of one molecule of an ideal gas is $\langle K \rangle = \frac{3}{2} k_{\text{B}} T$, where T is absolute temperature and k_{B} is Boltzmann's constant.
- Thermal equilibrium fixes the same temperature for both species. Molecular mass affects mean-square speed, but not the mean translational kinetic energy at a given temperature.

Step 1: Identify the common temperature The hydrogen and oxygen occupy the same flask at 27°C. Their common absolute temperature is $T = 27 + 273 = 300\,\text{K}$. The supplied mass ratio describes how much of each gas is present; it does not assign different temperatures to the molecules. We require energy per molecule rather than the total energy of either sample.

Step 2: Compare energies per molecule Applying equipartition separately to the two gases gives

$$\langle K_{\text{H}_2} \rangle = \frac{3}{2} k_{\text{B}} T, \qquad \langle K_{\text{O}_2} \rangle = \frac{3}{2} k_{\text{B}} T.$$

Hence their ratio is unity. Equivalently, the lighter hydrogen molecules move faster, so their smaller

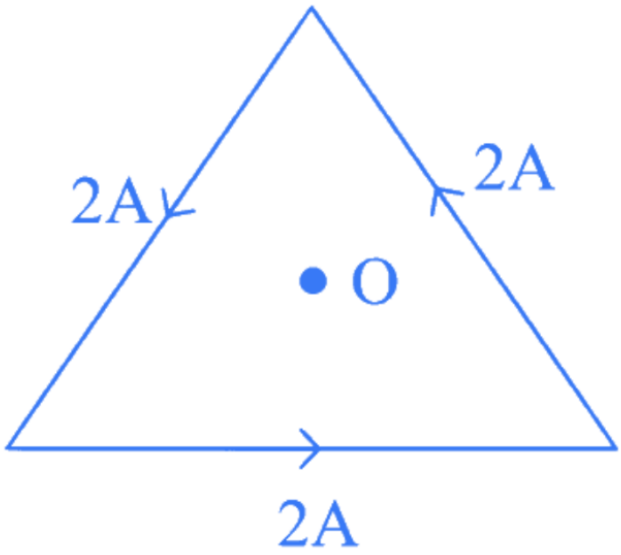
mass is exactly offset in $\frac{1}{2}m\langle v^2\rangle$. A ratio based directly on the sample masses would incorrectly compare extensive quantities. The ratios $4:1$, $1:4$ and $2:1$ therefore do not describe the requested mean energy.

Step 3: Conclusion The average kinetic energies per molecule are equal: $\langle K_{H_2}\rangle : \langle K_{O_2}\rangle = 1:1$. Hence, the correct option is (A).

Quick Tip to Solve: For a gas mixture at one temperature, use energy per molecule before considering the mass or number of molecules in each sample.

44. As shown in the figure, a current of 2 A flowing in an equilateral triangle of side $4\sqrt{3}$ cm. The magnetic field at the centroid O of the triangle is

(Neglect the effect of earth's magnetic field) [4 marks]



(A) $4\sqrt{3}\times 10^{-4}$ T

(B) $4\sqrt{3}\times 10^{-5}$ T

(C) $3\sqrt{3}\times 10^{-5}$ T

(D) $\sqrt{3}\times 10^{-4}$ T

Correct Answer: (C) $3\sqrt{3}\times 10^{-5}$ T

Solution:

Concept:

- For a straight wire at perpendicular distance d, $B=\frac{\mu_0 I}{4\pi d}(\sin\theta_1+\sin\theta_2)$. The angles are measured from the perpendicular to the wire.
- At the centroid of an equilateral current loop, all three sides produce equal fields normal to the plane. Their directions agree by the right-hand rule.

Step 1: Find the perpendicular distance Let the side be $a=4\sqrt{3}\,\text{cm}$. The altitude is $h=\sqrt{3}a/2$, and

the centroid is one third of this altitude from each side. Thus

$$d = \frac{h}{3} = \frac{a}{2\sqrt{3}} = 2\,\text{cm} = 2 \times 10^{-2}\,\text{m}.$$

The perpendicular bisects a side, giving $\tan\theta = (a/2)/d = \sqrt{3}$, so both endpoint angles are $60°$.

Step 2: Add the three side contributions For one side, substitute $I = 2\,\text{A}$ and $\sin 60° = \sqrt{3}/2$:

$$B_1 = \frac{10^{-7} \times 2}{2 \times 10^{-2}} \left(2\frac{\sqrt{3}}{2}\right) = \sqrt{3} \times 10^{-5}\,\text{T}.$$

All three contributions point out of the paper for the shown anticlockwise current. Therefore

$$B = 3B_1 = 3\sqrt{3} \times 10^{-5}\,\text{T}.$$

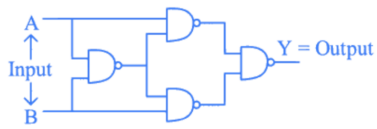
The other listed magnitudes either use an incorrect centroid distance or omit/miscount the equal finite-side contributions; the infinite-wire formula would not apply to each finite segment.

Step 3: Conclusion The magnetic field at O is $3\sqrt{3} \times 10^{-5}\,\text{T}$, out of the plane. Hence, the correct option is (C).

Quick Tip to Solve: For an equilateral loop, use its inradius $a/(2\sqrt{3})$ as the wire distance, then multiply one finite-side field by three.

45. The output Y for the inputs A and B of circuit is given by

Truth table of the shown circuit is: [4 marks]



(A)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(B)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(C)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(D)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Correct Answer: (A)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Solution:

Concept:

- A NAND gate produces the complement of the AND of its inputs: $\text{NAND}(X, Z) = \overline{XZ}$. A small circle at the output denotes inversion.
- Trace intermediate outputs before evaluating the final gate. In Boolean algebra $\overline{XZ} = \overline{X} + \overline{Z}$, and juxtaposition denotes AND.

Step 1: Label the three intermediate outputs The first NAND receives A, B, so set $N = \overline{AB}$. The upper middle NAND has inputs A, N, giving $P = \overline{AN}$. The lower middle NAND receives B, N, giving $Q = \overline{BN}$. The last NAND therefore gives

$$Y = \overline{PQ} = AN + BN = N(A + B).$$

Substituting $N = \overline{AB} = \overline{A} + \overline{B}$ and using $A\overline{A} = B\overline{B} = 0$ yields $Y = A\overline{B} + \overline{A}B$.

Step 2: Evaluate all four input combinations The result is exclusive OR: the output is one exactly when the inputs differ. Direct evaluation gives

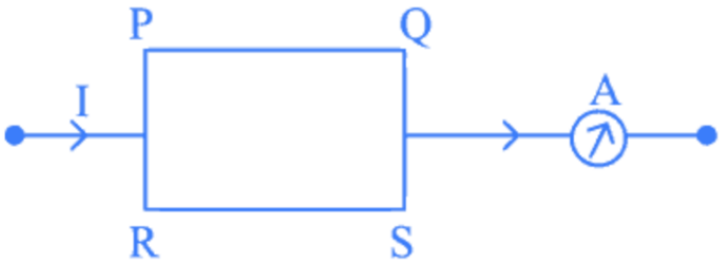
A	B	N	P	Q	Y
0	0	1	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	1	1	0

Thus the output sequence in the printed order is $0, 1, 1, 0$. Option (B) reverses that sequence and is XNOR. Option (C) is the NAND truth table, whereas option (D) is OR; neither accounts for the complete four-gate circuit.

Step 3: Conclusion The circuit implements $Y = A \oplus B$, with outputs $0, 1, 1, 0$ for inputs $00, 01, 10, 11$. Hence, the correct option is (A).

Quick Tip to Solve: For a NAND network, name each wire output and propagate all four input pairs; the four-NAND pattern here is the standard XOR construction.

46. A current carrying rectangular loop PQRS is made of uniform wire. The length $PR = QS = 5$ cm and $PQ = RS = 100$ cm. If ammeter current reading changes from I to $2I$, the ratio of magnetic forces per unit length on the wire PQ due to wire RS in the two cases respectively $\left(f^{I}_{PQ} : f^{2t}_{PQ}\right)$ is: [4 marks]



(A) 1 : 4

(B) 1 : 3

(C) 1 : 2

(D) 1 : 5

Correct Answer: (A) 1 : 4

Solution:

Concept:

- The magnetic force per unit length between long parallel wires separated by d is $f = \mu_0 i_1 i_2/(2\pi d)$. For fixed geometry it is proportional to the product of the two currents.
- A uniform resistive network has current ratios fixed by its branch resistances. Doubling the total applied current therefore doubles every branch current.

Step 1: Track the branch-current scaling The incoming current splits between the upper and lower paths of the uniform rectangular wire and recombines before the ammeter. Their resistance ratio depends only on their lengths and cross section, which remain unchanged. Thus, whatever the fixed fractions carried by PQ and RS, write $i_{PQ} = \alpha I$ and $i_{RS} = \beta I$. When the ammeter reads $2I$, these become $2\alpha I$ and $2\beta I$, respectively. In the symmetric drawing each is $I/2$, but their exact fractions are unnecessary for this ratio.

Step 2: Compare the magnetic forces The separation of the two long horizontal wires stays $5\,\text{cm}$, so the common geometric factor cancels. Hence

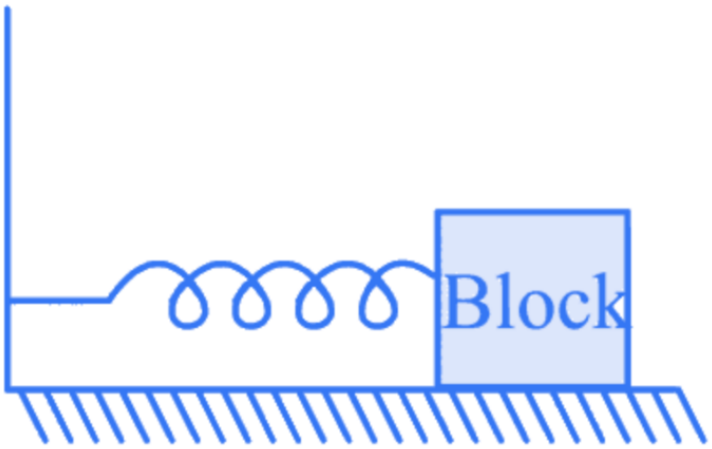
$$f^{I}_{PQ} = \frac{\mu_0 \alpha \beta I^2}{2\pi d}, \qquad f^{2I}_{PQ} = \frac{\mu_0 (2\alpha I)(2\beta I)}{2\pi d} = 4 f^{I}_{PQ}.$$

Both branch currents change, which is why a factor of two alone is insufficient. The ratios $1 : 3$ and $1 : 5$ also have no basis in the unchanged geometry. Even retaining finite-length corrections would leave the same quadratic current scaling.

Step 3: Conclusion The required ratio is $f^{I}_{PQ} : f^{2I}_{PQ} = 1 : 4$. Hence, the correct option is (A).

Quick Tip to Solve: When every current in a fixed circuit is multiplied by a factor s, a mutual magnetic force between two branches is multiplied by s^2.

47. For a simple harmonic motion in a mass spring system shown, the surface is frictionless. When the mass of the block is 1 kg, the angular frequency is ω_1. When the mass block is 2 kg the angular frequency is ω_2. The ratio ω_2/ω_1 is [4 marks]



(A) $1/\sqrt{2}$

(B) $1/2$

(C) 2

(D) $\sqrt{2}$

Correct Answer: (A) $1/\sqrt{2}$

Solution:

Concept:

- For an ideal horizontal spring, Hooke's law is $F = -kx$, where k is the spring constant and x is displacement from equilibrium.
- Comparing $m\ddot{x} = -kx$ with the SHM equation $\ddot{x} = -\omega^2 x$ gives $\omega = \sqrt{k/m}$. The same spring retains the same k.

Step 1: Write the two angular frequencies The frictionless surface eliminates a dissipative force, and the horizontal spring provides the restoring force. Gravity and the normal force balance vertically, so neither changes the horizontal spring constant. With the first block $m_1 = 1\,\text{kg}$,

$$\omega_1 = \sqrt{\frac{k}{m_1}}.$$

Replacing it by the second block $m_2 = 2\,\text{kg}$, without changing the spring, gives $\omega_2 = \sqrt{k/m_2}$.

Step 2: Divide and interpret the result Taking the ratio cancels the unknown spring constant:

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{k/m_2}{k/m_1}} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}.$$

The heavier block accelerates less for the same displacement and restoring force, so its oscillation is slower. The value $1/2$ incorrectly assumes inverse proportionality to mass rather than its square root. Values 2 and $\sqrt{2}$ predict a higher frequency for the heavier block and have the ratio reversed. The

amplitude does not enter this ideal linear-spring relation.

Step 3: Conclusion The angular-frequency ratio is $\omega_2/\omega_1 = 1/\sqrt{2}$. Hence, the correct option is (A).

Quick Tip to Solve: With the same ideal spring, frequency scales as $m^{-1/2}$, while the period scales as $m^{1/2}$; check which ratio is requested.

48. Match List I with List II:

List I	List II
A. Attenuation	I. Combination of a receiver and transmitter.
B. Transducer	II. process of retrieval of information from the carrier wave at receiver
C. Demodulation	III. converts one form of energy into another
D. quad Repeater	IV. Loss of strength of a signal while propagating through a medium.

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-II, C-III, D-IV

(B) A-IV, B-III, C-I, D-II

(C) A-IV, B-III, C-II, D-I

(D) A-II, B-III, C-IV, D-I

Correct Answer: (C) A-IV, B-III, C-II, D-I

Solution:

Concept:

- A communication system transmits information through a physical channel. Its components are identified by the operation performed on the signal, rather than by a similar-sounding name.
- Attenuation describes signal-strength loss; a transducer changes the physical form of energy; demodulation recovers information; a repeater receives and retransmits.

Step 1: Match attenuation and transducer As a signal propagates, absorption and spreading can reduce its strength. This loss is attenuation, so A matches IV. A transducer changes energy from one form to another: for example, a microphone converts sound into an electrical signal. Consequently B matches III. A transducer is not specifically the receiver-transmitter combination, and attenuation does not retrieve the information carried by a modulated wave.

Step 2: Match demodulation and repeater The receiver must extract the original information from the modulated carrier wave. This recovery process is demodulation, so C matches II. A repeater receives a weakened signal and retransmits it onward, allowing the communication range to increase; it combines a receiver and transmitter, so D matches I. Collecting these results gives

$$\text{A} - \text{IV}, \quad \text{B} - \text{III}, \quad \text{C} - \text{II}, \quad \text{D} - \text{I}.$$

Option (A) mismatches all four definitions. Option (B) gets attenuation and transducer right but

interchanges the demodulator and repeater descriptions. Option (D) assigns information recovery to attenuation and signal loss to demodulation, so it also fails.

Step 3: Conclusion The correct matching is A-IV, B-III, C-II, D-I. Hence, the correct option is (C).

Quick Tip to Solve: Anchor the pair attenuation–loss, then distinguish recovering information (demodulation) from receiving and sending onward (repeater).

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49. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: The nuclear density of nuclides ${}^{10}_{5}\mathrm{B}, {}^{6}_{3}\mathrm{Li}, {}^{56}_{26}\mathrm{Fe}, {}^{20}_{10}\mathrm{Ne}$ **and** ${}^{209}_{83}\mathrm{Bi}$ **can be arranged as** $\rho^{\mathrm{N}}_{\mathrm{Bi}} > \rho^{\mathrm{N}}_{\mathrm{Fe}} > \rho^{\mathrm{N}}_{\mathrm{Ne}} > \rho^{\mathrm{N}}_{\mathrm{B}} > \rho^{\mathrm{N}}_{\mathrm{Li}}$

Reason R: The radius R **of nucleus is related to its mass number** A **as** $R = R_0 A^{1/3}$**, where** R_0 **is a constant.**

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) *Both* **A** and **R** are true and **R** is the correct explanation of **A**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) **A** is false but **R** is true

(D) **A** is true but **R** is false

Correct Answer: (C) **A** is false but **R** is true

Solution:

Concept:

- The empirical nuclear-radius law is $R = R_0 A^{1/3}$, with mass number A and an approximately universal constant R_0.
- The nuclear mass is approximately Am_n, with m_n the mean nucleon mass. Thus mean nuclear density is mass divided by the spherical nuclear volume.

Step 1: Test the reason using the radius law The stated relation $R = R_0 A^{1/3}$ is the standard empirical dependence of nuclear size on mass number. Cubing gives $R^3 = R_0^3 A$, so the spherical volume is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A.$$

The volume therefore grows linearly with the number of nucleons, rather than staying fixed as heavier nuclides are considered. Reason R is true.

Step 2: Evaluate the assertion from density Using the same nuclear model for each listed nuclide,

$$\rho^N = \frac{Am_n}{(4/3)\pi R_0^3 A} = \frac{3m_n}{4\pi R_0^3}.$$

The mass number cancels, so the densities are approximately equal rather than increasing in the strict order claimed. Small nuclear-structure and binding-energy corrections are not the mass-number ordering proposed in Assertion A. Thus A is false. Both options (A) and (B) require a true assertion

and are excluded. Option (D) reverses the truth values by calling the standard radius law false. The reason actually supplies the scaling that contradicts the assertion.

Step 3: Conclusion Assertion A is false and Reason R is true: mean nuclear density is approximately independent of mass number. Hence, the correct option is (C).

Quick Tip to Solve: For nuclear density, cube the radius law first: both mass and volume are proportional to A, which cancels from their ratio.

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50. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Efficiency of a reversible heat engine will be highest at $-273^{\circ}\mathrm{C}$ temperature of cold reservoir.

Reason R: The efficiency of Carnot's engine depends not only on the temperature of the cold reservoir but it depends on the temperature of the hot reservoir too and is given as $\eta = \left(1 - \frac{T_2}{T_1}\right)$

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) A is false but **R** is true

(D) A is true but **R** is false

Correct Answer: (A) Both **A** and **R** are true and **R** is the correct explanation of **A**

Solution:

Concept:

- A reversible engine operating between hot and cold reservoirs at absolute temperatures T_1 and T_2 has efficiency $\eta = 1 - T_2/T_1$, with $T_1 > T_2$.
- Temperatures in the Carnot relation must be in kelvin. Absolute zero is conventionally written as $-273^{\circ}\mathrm{C}$ at the precision used here; the ideal limiting efficiency is unity.

Step 1: Evaluate the assertion Hold the hot-reservoir temperature fixed and positive. Lowering the cold-reservoir temperature reduces the fraction of heat rejected and increases the work obtained per unit heat absorbed. In the absolute-zero limit,

$$T_2 \longrightarrow 0 \quad \Rightarrow \quad \eta \longrightarrow 1.$$

Thus the stated cold-reservoir temperature gives the highest theoretical efficiency, so Assertion A is true. This is a limiting idealisation, not a claim that absolute zero can be attained by an actual engine.

Step 2: Evaluate the reason The reason gives the correct Carnot expression,

$$\eta = 1 - \frac{T_2}{T_1}.$$

Both reservoir temperatures enter this expression: reducing T_2 at fixed T_1, or increasing T_1 at fixed T_2, increases efficiency. Hence Reason R is true.

Step 3: Check the explanatory link The formula in R directly establishes the limiting statement in A: $T_2 = 0$ makes the subtracted ratio vanish. Therefore R explains A, ruling out (B). Options (C) and (D) each incorrectly label one true statement as false.

Step 4: Conclusion Both A and R are true, and R correctly explains A in the ideal absolute-zero limit. Hence, the correct option is (A).

Quick Tip to Solve: Convert reservoir temperatures to kelvin before using the Carnot formula; Celsius values cannot be inserted into a temperature ratio.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. In an ac generator, a rectangular coil of 100 turns each having area 14×10^{-2} m^2 is rotated at 360 rev/min about an axis perpendicular to a uniform magnetic field of magnitude 3.0 T. The maximum value of the emf produced will be _____ V.

(**Take** $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- For N identical turns of area A rotating in a uniform field B, the flux linkage is $N\Phi = NBA\cos\omega t$ when the rotation axis is perpendicular to the field.
- Faraday's law gives $e = NBA\omega\sin\omega t$, so the maximum emf is $e_0 = NBA\omega$. The angular speed $\omega = 2\pi f$ must be in radians per second.

Step 1: Convert the rotational speed The quoted rate counts complete revolutions per minute. Divide by 60 to obtain the number of cycles each second, then multiply by 2π radians per revolution:

$$f = \frac{360}{60} = 6 \text{ Hz}, \qquad \omega = 2\pi(6) = 12\pi \text{ rad s}^{-1}.$$

With the prescribed value $\pi = 22/7$, this is $264/7$ rad s^{-1}.

Step 2: Substitute the coil data Use the area of each turn rather than multiplying the area by the turn count twice. Here $N = 100$, $A = 14 \times 10^{-2} = 0.14$ m^2, and $B = 3.0$ T. Consequently,

$$\begin{aligned} e_0 &= NBA\omega \\ &= 100 \times 3.0 \times 0.14 \times 12 \times \frac{22}{7} \\ &= 42 \times \frac{264}{7} = 1584 \text{ V}. \end{aligned}$$

The maximum is reached when the plane of the coil is parallel to the magnetic field, because the flux then changes most rapidly. The requested maximum is the amplitude, so no factor $1/\sqrt{2}$ for the rms emf is needed.

Step 3: Conclusion The maximum induced emf is $\boxed{1584 \text{ V}}$; enter 1584.

Quick Tip to Solve: Convert rpm to hertz first and distinguish the peak value $NBA\omega$ from the rms value $NBA\omega/\sqrt{2}$.

52. A faulty thermometer reads 5°C in melting ice and 95°C in stream. The correct temperature on absolute scale will be _____ K when the faulty thermometer reads 41°C. [4 marks]

Solution:

Concept:

- A uniformly calibrated thermometer has a linear relation between its reading F and the correct Celsius temperature t. Two fixed points determine both its zero offset and its scale interval.
- At ordinary pressure, melting ice and steam correspond to 0°C and 100°C. The absolute temperature is $T = t + 273$ in the integer convention used for this numerical answer.

Step 1: Set up the fixed-point ratio The faulty instrument has lower fixed point 5 and upper fixed point 95, whereas the correct Celsius interval runs from 0 to 100. Equal fractional positions between these fixed points represent the same temperature. Therefore

$$\frac{F-5}{95-5} = \frac{t-0}{100-0}.$$

Subtracting the faulty zero is essential; simply scaling the observed reading would leave the offset uncorrected.

Step 2: Find the true Celsius temperature For the observed faulty reading $F = 41$, the rise above its ice-point indication is $41 - 5 = 36$, over a full faulty interval of 90. Thus

$$\frac{t}{100} = \frac{36}{90} = \frac{2}{5}, \qquad t = 40^\circ\text{C}.$$

This is a reasonable interpolation because 41 lies between 5 and 95.

Step 3: Convert to the absolute scale Add the Kelvin offset after correcting the Celsius reading:

$$T = 40 + 273 = 313 \text{ K}.$$

Using the exact offset gives 313.15 K, which also rounds to 313 for the requested numerical response. Applying the Kelvin conversion directly to the faulty reading would incorrectly retain both calibration errors.

Step 4: Conclusion The correct absolute temperature is $\boxed{313 \text{ K}}$ at integer precision; enter 313.

Quick Tip to Solve: First remove the faulty ice-point offset and rescale its fixed-point interval; only then add the Kelvin offset.

53. In a Young's double slit experiment, the intensities at two points, for the path differences $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ (λ being the wavelength of light used) are I_1 and I_2 respectively. If I_0 denotes the intensity produced by each one of the individual slits, then $\frac{I_1+I_2}{I_0}$ = _____. [4 marks]

Solution:

Concept:

- Two coherent slits of equal individual intensity I_0 produce resultant intensity $I = 2I_0(1+\cos\phi) = 4I_0\cos^2(\phi/2)$.
- The phase difference corresponding to path difference Δ is $\phi = 2\pi\Delta/\lambda$. The given I_0 belongs to one slit, not to the combined central maximum.

Step 1: Calculate the first intensity For the first point the path difference is $\lambda/4$. Convert it into phase before taking the interference term:

$$\phi_1 = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}.$$

Since $\cos(\pi/2) = 0$,

$$I_1 = 2I_0\left(1+\cos\frac{\pi}{2}\right) = 2I_0.$$

The two individual intensities add here because the interference term vanishes.

Step 2: Calculate the second intensity The second path difference is $\lambda/3$, giving

$$\phi_2 = \frac{2\pi}{\lambda}\frac{\lambda}{3} = \frac{2\pi}{3}.$$

Now $\cos(2\pi/3) = -1/2$, so destructive interference partly reduces the total:

$$I_2 = 2I_0\left(1-\frac{1}{2}\right) = I_0.$$

This is positive and below the incoherent sum $2I_0$, as expected.

Step 3: Form the required dimensionless ratio Add the intensities rather than their amplitudes, because the two points are separate observation locations:

$$\frac{I_1+I_2}{I_0} = \frac{2I_0+I_0}{I_0} = 3.$$

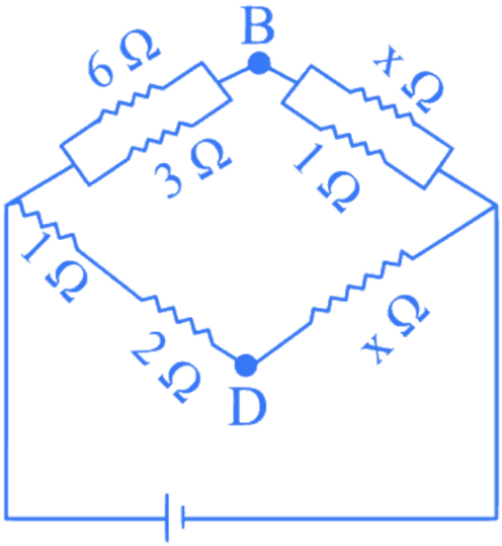
The wavelength cancels when the specified path differences are expressed as fractions of it; no screen distance or slit separation is required.

Step 4: Conclusion The required ratio is $\boxed{3}$, a dimensionless number; enter 3.

Quick Tip to Solve: For equal slits, use $I/I_0 = 2[1+\cos(2\pi\Delta/\lambda)]$; do not confuse the single-slit intensity with $4I_0$.

54. If the potential difference between B and D is zero, the value of x is $\frac{1}{n}\Omega$. The value of n is _____.

[4 marks]



Solution:

Concept:

- Zero potential difference between the two bridge junctions means their potential-divider fractions are equal. Thus the ratio of the left and right arm resistances is the same on both paths.
- Combine each parallel pair before applying bridge balance: $R_a \parallel R_b = R_a R_b/(R_a + R_b)$. Series resistances add directly.

Step 1: Reduce the four bridge arms Call the left supply junction L and the right supply junction R. The two resistors between L and B are in parallel, so

$$R_{LB} = \frac{6 \times 3}{6+3} = 2\ \Omega.$$

Between B and R, the $x\ \Omega$ and $1\ \Omega$ resistors are parallel, giving

$$R_{BR} = \frac{x}{x+1}\ \Omega.$$

On the lower path the $1\ \Omega$ and $2\ \Omega$ resistors are in series: $R_{LD} = 3\ \Omega$, while $R_{DR} = x\ \Omega$. Here x denotes the numerical resistance in ohms.

Step 2: Impose equal junction potentials Because $V_B = V_D$, the same fractional supply drop occurs from L to either junction. Equivalently, the bridge-balance condition is

$$\frac{R_{LB}}{R_{BR}} = \frac{R_{LD}}{R_{DR}}.$$

Substitution yields

$$\frac{2}{x/(x+1)} = \frac{3}{x}.$$

For positive resistance x, multiply by x to obtain $2(x+1)=3$, hence $x=1/2$.

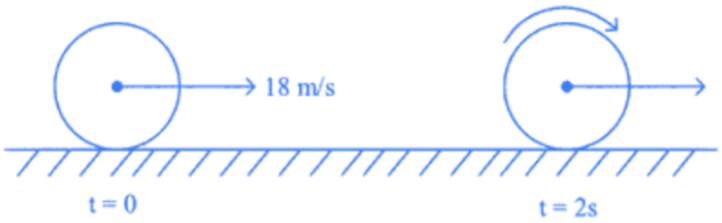
Step 3: Identify the requested integer The resulting physical resistance is $x\ \Omega=(1/2)\ \Omega$. Comparing this with the question's form $(1/n)\ \Omega$ gives $1/n=1/2$, so $n=2$. As a check, the top arm ratio is $2/(1/3)=6$, equal to the bottom ratio $3/(1/2)=6$.

Step 4: Conclusion The resistance is $0.5\ \Omega$, so the dimensionless requested value is $\boxed{n=2}$; enter 2.

Quick Tip to Solve: Reduce parallel pairs within bridge arms first, then compare the two left-to-right arm ratios using a consistent orientation.

---

55. A uniform disc of mass 0.5 kg and radius r is projected with velocity 18 m/s at $t=0$ s on a rough horizontal surface. It starts off with a purely sliding motion at $t=0$ s. After 2 s it acquires a purely rolling motion (see figure). The total kinetic energy of the disc after 2 s will be _____ J (given, coefficient of friction is 0.3 and $g=10\ \text{m/s}^2$). [4 marks]



Solution:

Concept:

- While a disc slides on a rough horizontal plane, kinetic friction $f=\mu mg$ decreases its centre-of-mass speed and supplies a torque that increases its angular speed.
- A uniform disc has moment of inertia $I=mr^2/2$. At the onset of rolling without slipping, $v=\omega r$, and total kinetic energy includes both translation and rotation.

Step 1: Find the speed after sliding The normal force is mg, so the magnitude of the frictional deceleration is

$$a=\frac{f}{m}=\mu g=0.3\times 10=3\ \text{m s}^{-2}.$$

Friction acts opposite the translational motion. Over the stated 2 s of sliding,

$$v=18-3(2)=12\ \text{m s}^{-1}.$$

The initial motion has no rotation, so friction must create all the final angular motion.

Step 2: Check the rolling condition The torque about the centre has magnitude fr. Thus

$$\alpha=\frac{fr}{I}=\frac{\mu mgr}{mr^2/2}=\frac{2\mu g}{r}=\frac{6}{r}.$$

Starting from rest in rotation gives $\omega=\alpha t=12/r$ after two seconds, hence $\omega r=12\ \text{m s}^{-1}=v$. This confirms the given transition to pure rolling.

Step 3: Add both kinetic-energy contributions At this instant,

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{4}mr^2\frac{v^2}{r^2} = \frac{3}{4}mv^2.$$

Substitute the mass and speed:

$$K = \frac{3}{4}(0.5)(12)^2 = 54 \text{ J}.$$

The translational part is 36 J, and the rotational part is 18 J. Their sum is less than the initial 81 J, because sliding friction dissipates mechanical energy.

Step 4: Conclusion The total kinetic energy after 2 s is $\boxed{54 \text{ J}}$; enter 54.

Quick Tip to Solve: For a rolling uniform disc use $K = 3mv^2/4$; retaining only $mv^2/2$ misses its rotational energy.

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56. The velocity of a particle executing SHM varies with displacement (x) as $4v^2 = 50 - x^2$. The time period of oscillations is $\frac{x}{7}s$. The value of x is _____. (Take $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- For simple harmonic motion with displacement u, amplitude A, and angular frequency ω, the speed obeys $v^2 = \omega^2(A^2 - u^2)$.
- The coefficient of the squared displacement is $-\omega^2$. Once ω is known, the period is $T = 2\pi/\omega$; the amplitude does not enter the period.

Step 1: Rewrite and compare the speed relation The printed relation is $4v^2 = 50 - x^2$. Divide every term by four before comparing with the standard SHM form:

$$v^2 = \frac{50}{4} - \frac{x^2}{4}.$$

Matching coefficients with $v^2 = \omega^2 A^2 - \omega^2 x^2$ gives

$$\omega^2 = \frac{1}{4}\ \text{s}^{-2}, \qquad \omega = \frac{1}{2}\ \text{rad s}^{-1}.$$

The positive root is used because angular frequency is a positive magnitude.

Step 2: Calculate the period The time for one complete oscillation is therefore

$$T = \frac{2\pi}{1/2} = 4\pi \text{ s}.$$

With the value specified in the question,

$$T = 4 \times \frac{22}{7} = \frac{88}{7} \text{ s}.$$

The constant term $50/4$ fixes the amplitude through $\omega^2 A^2 = 50/4$, but is not needed to determine this

period.

Step 3: Compare with the requested form The symbol x is reused in the question for the numerical numerator in $T = (x/7)$ s; it is separate from the instantaneous displacement appearing in the speed relation. Equating the two period expressions gives

$$\frac{x}{7} = \frac{88}{7} \quad \Rightarrow \quad x = 88.$$

Taking $\omega = 1/4$ instead of its square root would incorrectly double the period.

Step 4: Conclusion The period is $88/7$ s, so the requested dimensionless numerator is $\boxed{x = 88}$; enter 88.

Quick Tip to Solve: After dividing the velocity equation into standard form, take the square root of the displacement coefficient to obtain ω.

57. A body of mass 2 kg is initially at rest. It starts moving unidirectionally under the influence of a source of constant power P. Its displacement in 4 s is $\frac{1}{3}\alpha^2\sqrt{P}m$. The value of α will be _____. [4 marks]

Solution:

Concept:

- Constant power means a constant rate of energy transfer: $P = dK/dt$, where $K = mv^2/2$ is the kinetic energy.
- For a body starting from rest with all supplied work becoming kinetic energy, $Pt = mv^2/2$. Displacement is the time integral of speed, not final speed multiplied by time.

Step 1: Obtain the time-dependent speed The initial kinetic energy is zero. Integrating the constant power from the start gives $K(t) = Pt$. Thus

$$\frac{1}{2}mv^2 = Pt, \qquad v(t) = \sqrt{\frac{2P}{m}}\, t^{1/2}.$$

The positive root describes the stated unidirectional motion. For the given mass $m = 2\,\text{kg}$, the numerical SI relation is $v(t) = \sqrt{P}\, t^{1/2}$. The speed increases, but the acceleration is not constant, so the usual constant-acceleration displacement formula cannot be used.

Step 2: Integrate the speed over four seconds Using the displacement measured from the initial position,

$$s = \int_0^4 v(t)\, dt = \sqrt{\frac{2P}{m}} \int_0^4 t^{1/2}\, dt.$$

Therefore

$$s = \frac{2}{3}\sqrt{\frac{2P}{m}}\, 4^{3/2} = \frac{16}{3}\sqrt{P}\ \text{m}$$

when the numerical value of P is expressed in watts. The integral is finite at the initial instant even though the ideal constant-power force becomes large near zero speed.

Step 3: Compare the prescribed coefficients The given displacement is $s = \frac{1}{3}\alpha^2\sqrt{P}\,\text{m}$. Comparing

the coefficient with $16\sqrt{P}/3$ gives

$$\alpha^2 = 16.$$

The requested positive numerical parameter is consequently $\alpha = 4$.

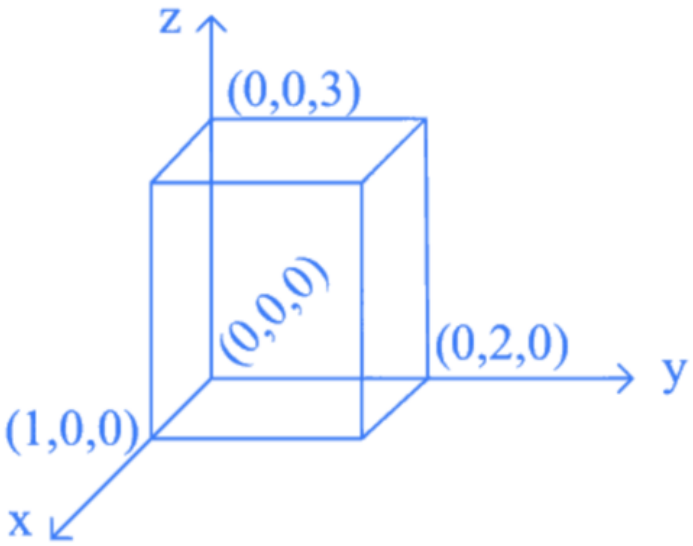
Step 4: Conclusion The displacement is $16\sqrt{P}/3\,\mathrm{m}$, so the required numerical value is $\boxed{\alpha = 4}$ (dimensionless).

Quick Tip to Solve: For constant power from rest, first use energy to get $v \propto \sqrt{t}$; integrating gives $s \propto t^{3/2}$.

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58. As shown in figure, a cuboid lies in a region with electric field $E = 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}$ N/C. The magnitude of charge within the cuboid is $n\varepsilon_0 C$.

The value of n is _____ (if dimension of cuboid is $1 \times 2 \times 3$ m^3) [4 marks]



Solution:

Concept:

- Gauss's law gives $Q_{\text{enclosed}} = \varepsilon_0 \Phi_E$, where $\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A}$ is the outward electric flux.
- Only the field component normal to a face contributes. Opposite faces have opposite outward normals, so each pair contributes the field difference times its area.

Step 1: Read the bounds and compute the x-face flux The figure places the cuboid in $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$, with all lengths in metres. Each face normal to the x-axis has area $2 \times 3 = 6\,\mathrm{m}^2$. Since $E_x = 2x^2$, its values at the two faces are zero and $2\,\mathrm{N/C}$. The outward flux of this face pair is

$$\Phi_x = [E_x(1) - E_x(0)](6) = (2 - 0)6 = 12\,\mathrm{N\,m^2/C}.$$

Step 2: Add the y-face and z-face contributions The faces normal to y have area $1 \times 3 = 3\,\mathrm{m}^2$. With

$E_y = -4y$, their net contribution is

$$\Phi_y = [E_y(2) - E_y(0)](3) = (-8 - 0)3 = -24\,\mathrm{N\,m^2/C}.$$

The negative value means more flux enters than leaves through this pair. The z-component is the same constant 6 at both z-faces. Their equal and opposite fluxes cancel:

$$\Phi_z = [6 - 6](1 \times 2) = 0.$$

Step 3: Apply Gauss law and take the magnitude The total signed flux is

$$\Phi_E = 12 - 24 + 0 = -12\,\mathrm{N\,m^2/C}.$$

Hence $Q_{\text{enclosed}} = -12\varepsilon_0\,\mathrm{C}$ in the notation of the question. The question asks for its magnitude, not its sign, so $|Q_{\text{enclosed}}| = 12\varepsilon_0\,\mathrm{C}$. Comparing with $n\varepsilon_0\,\mathrm{C}$ gives the required coefficient.

Step 4: Conclusion The charge is negative, but its magnitude is $12\varepsilon_0\,\mathrm{C}$. Thus $\boxed{n = 12}$ (dimensionless).

Quick Tip to Solve: For an axis-aligned cuboid, calculate each opposite-face pair as the normal-field difference multiplied by the face area; take the absolute value only at the end.

59. A stone tied to $180\ \mathrm{cm}$ long string at its end is making 28 revolutions in horizontal circle in every minute. The magnitude of acceleration of stone is $\frac{1936}{x} ms^{-2}$. The value of x _____. (Take $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- The magnitude of the centripetal acceleration in uniform circular motion is $a = \omega^2 r = 4\pi^2 f^2 r$, directed towards the centre.
- Frequency f must be in revolutions per second and radius r in metres before substituting into an SI acceleration formula.

Step 1: Convert the radius and revolution rate The string gives the radius of the horizontal circular path:

$$r = 180\,\mathrm{cm} = \frac{180}{100}\,\mathrm{m} = \frac{9}{5}\,\mathrm{m}.$$

The stone completes 28 revolutions in 60 seconds, so

$$f = \frac{28}{60}\,\mathrm{s^{-1}} = \frac{7}{15}\,\mathrm{s^{-1}}.$$

The rate 28 cannot be substituted directly for f, because it is a rate per minute, whereas the acceleration is required in $\mathrm{m\,s^{-2}}$.

Step 2: Calculate the angular speed and acceleration Using the specified value $\pi = 22/7$,

$$\omega = 2\pi f = 2\left(\frac{22}{7}\right)\left(\frac{7}{15}\right) = \frac{44}{15}\,\mathrm{rad\,s^{-1}}.$$

The acceleration magnitude is therefore

$$a = \left(\frac{44}{15}\right)^2\left(\frac{9}{5}\right) = \frac{1936\times 9}{225\times 5} = \frac{1936}{125}\,\mathrm{m\,s^{-2}}.$$

The factor 9 cancels between numerator and denominator; keeping fractions throughout avoids rounding the coefficient.

Step 3: Match the denominator in the question The required form is $a = 1936/x\,\mathrm{m\,s^{-2}}$. Its numerator is already exactly the same as the calculated numerator, so comparing denominators yields $x = 125$. As a scale check, ω is about $2.93\,\mathrm{rad\,s^{-1}}$, giving an acceleration about $15.5\,\mathrm{m\,s^{-2}}$, consistent with this fraction.

Step 4: Conclusion The acceleration magnitude is $\frac{1936}{125}\,\mathrm{m\,s^{-2}}$. Hence $\boxed{x = 125}$ (dimensionless).

Quick Tip to Solve: Convert rpm to revolutions per second before squaring the frequency; here $\pi = 22/7$ cancels the factor 7 in $f = 7/15$.

---

60. A radioactive nucleus decays by two different process. The half life of the first process is 5 minutes and that of the second process is 30 s. The effective half-life of the nucleus is calculated to be $\frac{\alpha}{11}$ s. The value of α is _____. [4 marks]

Solution:

Concept:

- For independent decay channels available to the same parent nucleus, the decay constants add: $\lambda = \lambda_1 + \lambda_2$.
- Half-life and decay constant obey $T_{1/2} = \ln 2/\lambda$. Consequently reciprocal partial half-lives add when competing channels act simultaneously.

Step 1: Put the partial half-lives in the same unit The first process has partial half-life

$$T_1 = 5\,\mathrm{min} = 300\,\mathrm{s},$$

while the second has $T_2 = 30\,\mathrm{s}$. Their respective decay constants are

$$\lambda_1 = \frac{\ln 2}{300}\,\mathrm{s^{-1}}, \qquad \lambda_2 = \frac{\ln 2}{30}\,\mathrm{s^{-1}}.$$

These are alternative decay routes of one parent nucleus, rather than two successive stages of a decay chain.

Step 2: Add the decay rates to find the effective half-life If N parent nuclei are present, their total

loss rate is $-dN/dt = (\lambda_1 + \lambda_2)N$. Thus

$$\lambda = \ln 2 \left(\frac{1}{300} + \frac{1}{30} \right) = \frac{11 \ln 2}{300} \, \mathrm{s}^{-1}.$$

Substitution into the half-life definition gives

$$T_{\text{eff}} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{11 \ln 2/300} = \frac{300}{11} \, \mathrm{s}.$$

The common factor $\ln 2$ cancels; there is no need to approximate it.

Step 3: Identify the requested numerator The effective half-life is written as $\alpha/11\,\mathrm{s}$. Comparison with $300/11\,\mathrm{s}$ yields $\alpha = 300$. The result is about $27.27\,\mathrm{s}$, smaller than either partial half-life, as expected because both decay processes remove parent nuclei at the same time. Adding half-lives or averaging them would incorrectly make the decay slower.

Step 4: Conclusion The effective half-life is $\frac{300}{11}\,\mathrm{s}$; therefore $\boxed{\alpha = 300}$ (dimensionless).

Quick Tip to Solve: For two competing decay modes use $T_{\text{eff}} = T_1 T_2/(T_1 + T_2)$, after converting both half-lives to the same unit.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Formulae for Nessler's reagent is : [4 marks]

(A) $KHgI_3$

(B) HgI_2

(C) KHg_2I_2

(D) K_2HgI_4

Correct Answer: (D) K_2HgI_4

Solution:

Concept:

- Nessler's reagent is an alkaline solution of potassium tetraiodomercurate(II), used to detect ammonia or ammonium ions.
- The tetraiodomercurate(II) complex ion is $[HgI_4]^{2-}$: mercury has oxidation state +2 and each iodide ligand has charge -1.

Step 1: Identify the mercury complex in the reagent Mercuric iodide can react with excess iodide to form the soluble tetraiodomercurate(II) complex:

$$HgI_2 + 2\,I^- \longrightarrow [HgI_4]^{2-}.$$

There are four iodide ligands around the mercury(II) centre. The total charge of the complex is $+2 + 4(-1) = -2$. Nessler's reagent contains this complex in an alkaline medium; merely identifying a mercury iodide compound is not sufficient.

Step 2: Balance the charge with potassium ions Two potassium ions, each of charge $+1$, are required per complex dianion. The corresponding salt is

$$K_2[HgI_4],$$

commonly written as K_2HgI_4. Its name is potassium tetraiodomercurate(II). The alkali provides the conditions needed in the ammonia test, but it does not change the formula of the mercury complex salt asked for here.

Step 3: Check the alternative formulae Option (A), $KHgI_3$, contains only three iodides and is not the tetraiodomercurate salt. Option (B), HgI_2, is mercuric iodide, the starting mercury compound rather than the reagent complex. Option (C), KHg_2I_2, has the wrong mercury-to-iodide ratio and potassium count. Only option (D), K_2HgI_4, has two potassium counterions and the required four iodide ligands.

Step 4: Conclusion Nessler's reagent contains $\boxed{K_2HgI_4}$ in alkaline solution. Hence, the correct option is (D).

Quick Tip to Solve: Remember Nessler's reagent as alkaline potassium tetraiodomercurate(II): four iodides give a dianion, requiring two potassium ions.

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62. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A:

$$CH_3COCH_2C(OH)(CH_3)_2$$

can be easily reduced using $Zn-Hg/HCl$ **to**

$$CH_3CH_2CH_2C(OH)(CH_3)_2.$$

Reason R: $Zn-Hg/HCl$ **is used to reduce carbonyl group to** $-CH_2-$ **group.**

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) A is false but R is true

(C) A is true but R is false

(D) Both A and R are true but R is not the correct explanation of A

Correct Answer: (B) A is false but R is true

Solution:

Concept:

- Clemmensen reduction uses $Zn-Hg$ and concentrated HCl to convert an aldehyde or ketone carbonyl group into a methylene group.
- The strongly acidic reaction conditions must be compatible with the other functional groups. A tertiary alcohol is acid-sensitive and can undergo dehydration or substitution.

Step 1: Evaluate the assertion for the complete molecule The reactant shown has the condensed formula

$$CH_3COCH_2C(OH)(CH_3)_2.$$

It contains both a ketone group and a tertiary alcohol group. The proposed product is

$$CH_3CH_2CH_2C(OH)(CH_3)_2.$$

It retains the tertiary alcohol unchanged while removing the carbonyl oxygen. In strongly acidic HCl, the tertiary alcohol can be protonated; loss of water gives a relatively stable tertiary carbocation, followed by competing dehydration or substitution. Therefore the depicted selective conversion cannot be described as an easy clean reduction under the given conditions. Assertion A is false.

Step 2: Evaluate the stated role of zinc amalgam Reason R describes the usual Clemmensen transformation of aldehydes and ketones:

$$CH_3COCH_3 \xrightarrow{Zn-Hg/HCl} CH_3CH_2CH_3.$$

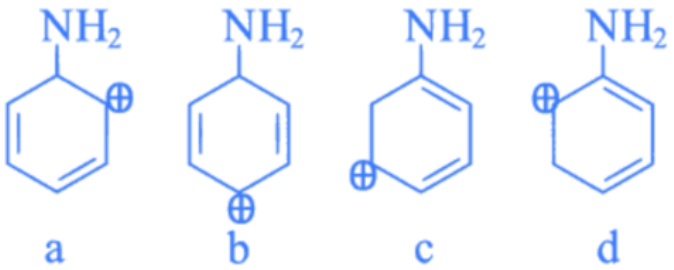
This is the standard carbonyl-to-methylene reduction, so Reason R is true. It describes the carbonyl functional group transformation correctly, but it does not establish that every other group in a polyfunctional substrate survives the acidic medium.

Step 3: Select the correct truth-value combination Combining the evaluations gives false Assertion A and true Reason R. Options (A) and (D) both require the assertion to be true, so neither is possible. Option (C) also declares A true and additionally declares R false, contradicting both evaluations. Option (B) is the only alternative with the correct truth values. The distinction is reagent compatibility, rather than a denial of the normal Clemmensen reduction.

Step 4: Conclusion Assertion A is false because the tertiary alcohol is not compatible with a clean selective reduction in strongly acidic conditions; Reason R is true. Hence, the correct option is (B).

Quick Tip to Solve: Before choosing a carbonyl reduction, inspect every other functional group: acid-sensitive alcohols can prevent the clean product expected from Clemmensen conditions.

63. The most stable carbocation for the following is: [4 marks]



(A) b

(B) d

(C) c

(D) a

Correct Answer: (C) c

Solution:

Concept:

- A carbocation is stabilised when its vacant p orbital is conjugated with adjacent carbon-carbon double bonds, allowing delocalisation of the positive charge.
- An amino group can donate its nitrogen lone pair by resonance when it lies in a continuous conjugated path to the electron-deficient centre. This can produce an iminium contributor with complete octets.

Step 1: Inspect the conjugated path in structure c In structure c, start at the positively charged lower-left carbon and follow the ring through the bottom carbon, the lower-right carbon and the upper-right carbon to the top carbon bearing NH_2. Along that path the vacant orbital and the two double bonds form a conjugated pentadienyl cation. Shifting the adjacent double bond moves the positive charge to the lower-right carbon; shifting the next double bond moves it onto the amino-bearing top carbon.

Step 2: Use nitrogen donation to obtain an octet contributor When the positive charge is on the carbon bonded to NH_2, the nitrogen lone pair can form a carbon-nitrogen double bond. The resulting iminium contributor has a positive nitrogen with four bonds, while every carbon now has an octet. Moving positive charge from an electron-deficient carbon to an octet-bearing nitrogen provides substantial resonance stabilisation. The essential condition is the uninterrupted orbital connection to that carbon.

Step 3: Compare the other arrangements and option letters In a and b the top amino-bearing carbon is saturated, so it interrupts conjugation of the nitrogen lone pair with the cationic system. In d the amino group is attached to an internal carbon of the conjugated path: the ordinary cation resonance positions do not place the positive charge on the amino-bearing carbon, so it lacks the equivalent fully octet iminium contributor. Thus a, b and d do not gain the decisive stabilisation available to c. In the printed option order, c is option (C), while (A), (B) and (D) denote b, d and a respectively.

Step 4: Conclusion The most stable carbocation is $\boxed{c}$, whose charge can delocalise to nitrogen in an octet-bearing iminium contributor. Hence, the correct option is (C).

Quick Tip to Solve: Trace the actual alternating-bond path before applying amino-group donation; the strongest case allows a resonance contributor with no electron-deficient carbon.

64. Match List I with List II:

List I (Complexes)		List II (Hybridisation)	
A.	$[Ni(CO)_4]$	**I.**	sp^3
B.	$[Cu(NH_3)_4]^{2+}$	**II.**	dsp^2
C.	$[Fe(NH_3)_6]^{2+}$	**III.**	sp^3d^2
D.	$[Fe(H_2O)_6]^{2+}$	**IV.**	d^2sp^3

[4 marks]

(A) A-I, B-II, C-III, D-IV

(B) A-II, B-I, C-III, D-IV

(C) A-I, B-II, C-IV, D-III

(D) A-II, B-I, C-IV, D-III

Correct Answer: (C) A-I, B-II, C-IV, D-III

Solution:

Concept:

- Valence bond theory relates coordination geometry to hybrid orbitals: tetrahedral complexes use sp^3, square-planar complexes use dsp^2, and octahedral complexes use d^2sp^3 or sp^3d^2.
- For an octahedral complex, ligand-induced pairing distinguishes inner-orbital d^2sp^3 bonding from outer-orbital sp^3d^2 bonding. Oxidation state must be found before assigning the metal configuration.

Step 1: Assign the four-coordinate complexes Carbon monoxide is neutral, so nickel in $[Ni(CO)_4]$ is in oxidation state zero. Its electrons pair to give a filled $3d$ set, and four sp^3 orbitals form the tetrahedral complex: A-I. In $[Cu(NH_3)_4]^{2+}$, copper is $+2$, with a $3d^9$ configuration. The standard square-planar valence-bond description uses dsp^2 hybridisation, so B-II. Do not interchange these geometries merely because both coordination numbers are four.

Step 2: Distinguish the two octahedral iron complexes Both iron complexes contain Fe^{2+}, whose configuration is $3d^6$. In the standard valence-bond assignment used here, ammonia causes pairing of the $3d$ electrons, leaving two inner d orbitals available. Thus $[Fe(NH_3)_6]^{2+}$ is assigned d^2sp^3, giving C-IV. Water is a weak-field ligand and does not produce that pairing; $[Fe(H_2O)_6]^{2+}$ uses outer d orbitals and is sp^3d^2, giving D-III. Option A reverses the iron assignments, while B and D incorrectly interchange the nickel and copper assignments.

Step 3: Conclusion The matching is A-I, B-II, C-IV, D-III. Hence, the correct option is (C).

Quick Tip to Solve: First fix tetrahedral nickel carbonyl and square-planar tetraamminecopper(II); then distinguish the inner- and outer-orbital iron complexes.

65. The $Cl - Co - Cl$ bond angle values in a fac- $[Co\,(NH_3)_3\,Cl_3]$ complex is/are : [4 marks]

(A) 90° & 120°

(B) 90°

(C) 180°

(D) 90° & 180°

Correct Answer: (B) 90°

Solution:

Concept:

- An octahedral MA_3B_3 complex has facial and meridional geometrical isomers. In the facial isomer, three identical ligands occupy the vertices of one triangular face.
- Adjacent coordination directions in an ideal octahedron make 90°, whereas opposite directions make 180°. The ligand arrangement, not coordination number alone, determines which angles occur.

Step 1: Locate the three chloride ligands The coordination number in $[Co(NH_3)_3Cl_3]$ is six, so the reference geometry is octahedral. Choose its six bond directions as $+x, -x, +y, -y, +z, -z$. A facial arrangement can be represented by placing the chloride ligands along $+x, +y, +z$. These are three vertices on a single face; the three ammonia ligands occupy the opposite face. Any equivalent rotated placement gives the same angular relations. In particular, a facial set never includes a pair of opposite vertices.

Step 2: Evaluate every chloride–cobalt–chloride angle There are $\binom{3}{2} = 3$ pairs of chloride ligands. The pairs $(+x, +y)$, $(+y, +z)$ and $(+z, +x)$ are all perpendicular, so each gives

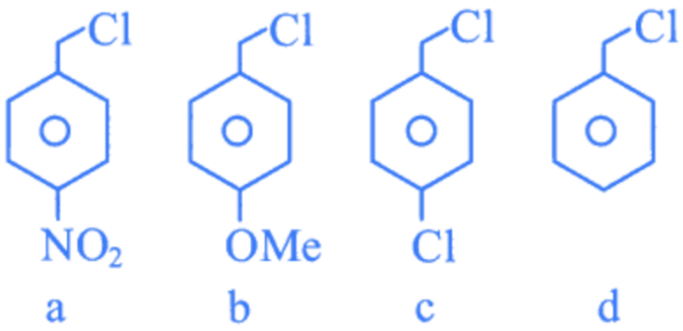
$$\angle \mathrm{Cl}{-}\mathrm{Co}{-}\mathrm{Cl} = 90^\circ.$$

No pair lies along opposite axes, so 180° does not occur between chloride ligands in the facial isomer. A value of 120° would be characteristic of a planar trigonal arrangement and is not an octahedral bond angle. Thus A fails because of its 120°, C fails because it gives only trans angles, and D describes the presence of a trans pair characteristic of the meridional arrangement.

Step 3: Conclusion All three $Cl{-}Co{-}Cl$ angles in the facial complex are 90°. Hence, the correct option is (B).

Quick Tip to Solve: For fac-MA_3B_3, put the three identical ligands on $+x, +y, +z$: every pair is cis, so every angle between them is 90°.

66. Decreasing order towards SN 1 reaction for the following compounds is: [4 marks]



(A) $a > c > d > b$

(B) $\mathrm{d} > \mathrm{b} > \mathrm{c} > \mathrm{a}$

(C) $a > b > c > d$

(D) $\mathrm{b} > \mathrm{d} > \mathrm{c} > \mathrm{a}$

Correct Answer: (D) $\mathrm{b} > \mathrm{d} > \mathrm{c} > \mathrm{a}$

Solution:

Concept:

- An $S_{\mathrm{N}}1$ reaction begins with slow ionisation of the carbon–halogen bond. With the same leaving group, a more stable carbocation generally gives a faster reaction.
- A benzyl carbocation delocalises positive charge into the aromatic ring. A para electron-donating group stabilises it by resonance; a strong para electron-withdrawing group destabilises it.

Step 1: Compare the benzylic intermediates Each structure contains a benzylic $-\mathrm{CH_2Cl}$ group, so ionisation gives a substituted benzyl carbocation and Cl^-. The leaving group and the benzylic reaction centre are common to all four; the ring substituent is the decisive difference. Compound b has a para methoxy group. Donation of an oxygen lone pair through the aromatic π system stabilises the delocalised positive charge strongly, making b the fastest. Compound d is unsubstituted benzyl chloride and provides the reference benzyl-cation stability.

Step 2: Rank the withdrawing substituents and test the options Compound c has para chlorine. Although chlorine can donate a lone pair by resonance, its net electron-withdrawing inductive effect makes it less activating toward benzylic ionisation than hydrogen in this comparison. Compound a has para nitro, which withdraws strongly by both induction and resonance; it destabilises the positive-charge-containing resonance system most strongly. Consequently,

$$b > d > c > a.$$

Option A incorrectly puts the nitro compound first and methoxy last. Option B puts the unsubstituted compound above the strongly resonance-stabilised methoxy compound. Option C again gives nitro the highest reactivity. Only D follows the required carbocation-stability order. This is a comparison of ionisation rates, not of nucleophile attack in an $S_{\mathrm{N}}2$ process.

Step 3: Conclusion The decreasing $S_{\mathrm{N}}1$ reactivity is $b > d > c > a$. Hence, the correct option is (D).

Quick Tip to Solve: For para-substituted benzyl halides, rank how well the ring substituent stabilises a positive charge: methoxy helps strongly, while nitro opposes ionisation.

67. $KMnO_4$ **oxidises** I^- **in acidic and neutral/faintly alkaline solutions, respectively, to :** [4 marks]

(A) IO_3^- & IO_3^-

(B) I_2 & I_2

(C) I_2 & IO_3^-

(D) IO_3^- & I_2

Correct Answer: (C) I_2 & IO_3^-

Solution:

Concept:

- The products of permanganate oxidation depend on the reaction medium. Iodide can be oxidised to iodine or further to iodate.
- Iodine has oxidation state -1 in I^-, zero in I_2, and $+5$ in $IO_3{}^-$. Balanced redox equations must conserve both atoms and charge.

Step 1: Identify the product in acidic solution In acidic solution, permanganate is reduced from manganese(VII) to Mn^{2+}. The standard reaction with iodide liberates iodine:

$$2\,MnO_4{}^- + 10\,I^- + 16\,H^+ \longrightarrow 2\,Mn^{2+} + 5\,I_2 + 8\,H_2O.$$

Each iodide ion loses one electron on becoming iodine in I_2, and each permanganate ion gains five electrons. The ten transferred electrons balance in the equation above. Thus the first member of the requested pair is I_2, which rules out options A and D immediately.

Step 2: Identify the product in neutral or faintly alkaline solution In this medium, permanganate is reduced to brown MnO_2, and iodide is oxidised further to iodate:

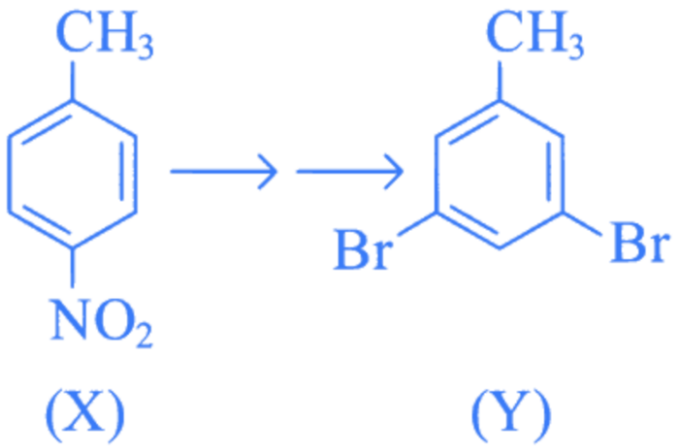
$$2\,MnO_4{}^- + I^- + H_2O \longrightarrow 2\,MnO_2 + IO_3{}^- + 2\,OH^-.$$

Iodine changes from -1 to $+5$, losing six electrons. Two manganese atoms each gain three electrons in changing from $+7$ to $+4$, so the electron balance is six on either side. The equation also balances charge: both sides carry a total of -3. The second product is therefore $IO_3{}^-$, not I_2. Option B gets the acidic product right but fails for the neutral/faintly alkaline medium. Read the products in the order of media stated in the question.

Step 3: Conclusion The respective products are I_2 and $IO_3{}^-$. Hence, the correct option is (C).

Quick Tip to Solve: Write the medium above each permanganate reaction before choosing products: acidic iodide gives iodine; neutral/faintly alkaline iodide gives iodate.

68. In the above conversion of compound (X) to product (Y), the sequence of reagents to be used will be: [4 marks]



(A) (i) Br_2, Fe (ii) Fe, H^+ (iii) $LiAlH_4$

(B) (i) $Br_2(aq)$ (ii) $LiAlH_4$ (iii) H_3O^+

(C) (i) Fe, H^+ (ii) Br_2 (aq) (iii) HNO_2 (iv) H_3PO_2

(D) (i) Fe, H^+ (ii) $Br_2(aq)$ (iii) HNO_2 (iv) $CuBr$

Correct Answer: (C) (i) Fe, H^+ (ii) Br_2 (aq) (iii) HNO_2 (iv) H_3PO_2

Solution:

Concept:

- Reduction of an aromatic nitro group gives an amino group. The amino group strongly activates the ring and directs electrophilic substitution to its ortho and para positions.
- An aromatic primary amine forms a diazonium salt with nitrous acid in cold acidic conditions. Hypophosphorous acid replaces the diazonium group by hydrogen; copper(I) bromide replaces it by bromine.

Step 1: Use the amino group to place the bromine atoms The starting compound X is para-nitrotoluene, and product Y is 3,5-dibromotoluene: the nitro-derived group must disappear while bromines appear at the two positions ortho to that group. First use Fe/H^+ to reduce $-NO_2$ to $-NH_2$, giving para-toluidine. Aqueous bromine then brominates the two available ortho positions relative to $-NH_2$. Its para position is already occupied by methyl, so this step introduces the two required bromines at positions 3 and 5 relative to methyl.

Step 2: Remove the directing group without adding another bromine Treating the resulting aromatic primary amine with HNO_2 under diazotisation conditions changes $-NH_2$ into an aryl diazonium group. Finally, H_3PO_2 reduces that group to hydrogen with loss of nitrogen, preserving both bromines and the methyl group. The sequence is therefore

$$Fe/H^+ ; \quad Br_2(aq) ; \quad HNO_2 ; \quad H_3PO_2.$$

Option D would use $CuBr$ and introduce a third bromine at the former amino position, producing the wrong substitution pattern. Options A and B end in reductions that do not provide the needed diazoti-

sation followed by deamination; they do not remove the nitrogen substituent in the required way. The amino group acts as a temporary directing group and must be removed only after bromination.

Step 3: Conclusion The required sequence is (i) $\mathrm{Fe}, \mathrm{H}^{+}$, (ii) $\mathrm{Br}_2(\mathrm{aq})$, (iii) HNO_2, (iv) $\mathrm{H}_3\mathrm{PO}_2$. Hence, the correct option is (C).

Quick Tip to Solve: When bromines must remain but an aromatic amino group must disappear, diazotise and use $\mathrm{H}_3\mathrm{PO}_2$; CuBr would replace that group with an extra bromine.

69. Match List I with List II:

	List I (Mixture)		List II (Separation Technique)
A.	$\mathrm{CHCl}_3 + \mathrm{C}_6\mathrm{H}_5\mathrm{NH}_2$	**I.**	**Steam distillation**
B.	$\mathrm{C}_6\mathrm{H}_{14} + \mathrm{C}_5\mathrm{H}_{12}$	**II.**	**Differential extraction**
C.	$\mathrm{C}_6\mathrm{H}_5\mathrm{NH}_2 + \mathrm{H}_2\mathrm{O}$	**III.**	**Distillation**
D.	**Organic compound in** $\mathrm{H}_2\mathrm{O}$	**IV.**	**Fractional distillation**

[4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-II, B-I, C-III, D-IV

(C) A-IV, B-I, C-III, D-II

(D) A-III, B-IV, C-I, D-II

Correct Answer: (D) A-III, B-IV, C-I, D-II

Solution:

Concept:

- Distillation separates volatile liquids by their boiling behaviour. A fractionating column provides repeated vaporisation and condensation, improving separation of volatile components.
- Steam distillation is suited to a steam-volatile organic liquid that is poorly miscible with water. Differential extraction uses different distribution of a solute between two immiscible solvents.

Step 1: Match the two mixtures of organic liquids Chloroform and aniline have widely separated boiling points, approximately $61^{\circ}\mathrm{C}$ and $184^{\circ}\mathrm{C}$, respectively. Ordinary distillation separates the much more volatile chloroform from the higher-boiling aniline, so A-III. The pentane–hexane mixture consists of two miscible, volatile, chemically similar hydrocarbons. A fractionating column improves separation by repeated enrichment of the vapour in the more volatile pentane; this is the fractional-distillation match B-IV in the list. Steam distillation is not the appropriate entry for a pair of miscible hydrocarbons without water.

Step 2: Match the water-containing mixtures and select the code Aniline is only sparingly soluble in water and is steam volatile, so aniline with water is separated by steam distillation: C-I. The combined vapour pressures allow the mixture to distil below the normal boiling point of the organic component. For an organic compound dissolved in water, differential extraction transfers it into a suitable immiscible organic solvent in which it is more soluble: D-II. Separation in a funnel then gives

the organic phase, from which the solvent can be removed. The complete mapping is

$$\text{A-III, B-IV, C-I, D-II.}$$

Options A, B and C all assign steam distillation to the hexane–pentane mixture, so they fail the hydrocarbon pairing. Options B and C also mismatch the chloroform–aniline separation. Only D gives all four appropriate techniques together.

Step 3: Conclusion The matches are A-III, B-IV, C-I, D-II. Hence, the correct option is (D).

Quick Tip to Solve: Identify steam distillation by the poorly water-miscible, steam-volatile liquid; then distinguish ordinary distillation, fractionation and solvent extraction for the other mixtures.

70. Maximum number of electrons that can be accommodated in shell with $n = 4$ are: [4 marks]

(A) 50

(B) 32

(C) 72

(D) 16

Correct Answer: (B) 32

Solution:

Concept:

- For principal quantum number n, the allowed azimuthal quantum numbers are $l = 0, 1, \ldots, n-1$. A subshell with quantum number l contains $2l + 1$ orbitals.
- By the Pauli exclusion principle, one orbital accommodates at most two electrons with opposite spin. Therefore a complete shell has a maximum capacity of $2n^2$ electrons.

Step 1: Count the orbitals in the fourth shell For $n = 4$, the four allowed values of l are 0, 1, 2 and 3, corresponding to the $4s$, $4p$, $4d$ and $4f$ subshells. Their orbital counts are respectively 1, 3, 5 and 7. Hence the number of spatial orbitals in the entire shell is

$$1 + 3 + 5 + 7 = 16 = 4^2.$$

All four subshells belong to the same principal shell, even though their energies and filling order in a many-electron atom differ. The question asks for capacity, so no ground-state electron-filling restriction removes any of them.

Step 2: Apply the two-electron limit and compare choices Each of the 16 orbitals can contain two electrons of opposite spin. Consequently,

$$N_{\max} = 2(16) = 2(4)^2 = 32.$$

The same result follows by adding subshell capacities: $4s$ holds 2, $4p$ holds 6, $4d$ holds 10 and $4f$ holds 14, giving $2 + 6 + 10 + 14 = 32$. Option D, 16, counts orbitals rather than electrons. Option A, 50, is the capacity $2(5)^2$ of the fifth shell; option C, 72, is $2(6)^2$, the sixth-shell capacity. Neither uses

the given principal quantum number. A shell capacity is a maximum and does not assert that every atom actually fills that shell completely.

Step 3: Conclusion The shell with $n = 4$ accommodates at most 32 electrons. Hence, the correct option is (B).

Quick Tip to Solve: Use n^2 for the number of orbitals and $2n^2$ for the number of electrons; this prevents confusing 16 orbitals with 32 electrons.

71. Bond dissociation energy of "E-H" bond of the "H_2E " hydrides of group 16 elements (given below), follows order.

A. O

B. S

C. Se

D. Te

Choose the correct from the options given below: [4 marks]

(A) A > B > C > D

(B) D > C > B > A

(C) B > A > C > D

(D) A > B > D > C

Correct Answer: (A) A > B > C > D

Solution:

Concept:

- Bond dissociation enthalpy measures the energy needed to break one mole of a specified gaseous covalent bond homolytically. A stronger E–H bond has a higher bond dissociation energy.
- Down group 16, the central atom becomes larger, the E–H bond becomes longer and overlap with the compact hydrogen $1s$ orbital becomes less effective.

Step 1: Arrange the central atoms by size The elements in the question occur down group 16 in the order oxygen, sulphur, selenium and tellurium. Their atomic sizes increase in that order. Hydrogen has the same small $1s$ orbital in all four hydrides; it therefore overlaps most effectively with the smaller oxygen orbital and progressively less effectively with the larger orbitals down the group.

Step 2: Translate bond length into bond strength The E–H bond length increases from H_2O to H_2Te. Consequently the bond becomes easier to break, and its dissociation energy decreases:

$$D(\mathrm{O{-}H}) > D(\mathrm{S{-}H}) > D(\mathrm{Se{-}H}) > D(\mathrm{Te{-}H}).$$

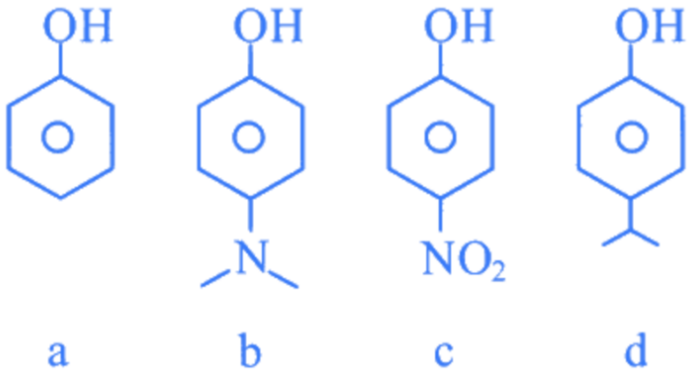
This trend also explains the decreasing thermal stability of these hydrides down the group.

Step 3: Compare the listed orders With the question's element labels, oxygen is A, sulphur B, selenium C and tellurium D. Hence the required order is A > B > C > D. Option (B) reverses the entire trend, option (C) wrongly places the S–H bond above O–H, and option (D) wrongly places Te–H above Se–H.

Step 4: Conclusion The bond dissociation energies decrease as $\mathrm{O{-}H} > \mathrm{S{-}H} > \mathrm{Se{-}H} > \mathrm{Te{-}H}$, or $\mathrm{A} > \mathrm{B} > \mathrm{C} > \mathrm{D}$. Hence, the correct option is (A).

Quick Tip to Solve: For group 16 hydrides, follow increasing central-atom size down the group: longer E–H bonds are weaker, so bond strength and thermal stability both decrease.

72. The correct order of $\mathrm{pK_a}$ values for the following compounds is: [4 marks]



(A) $\mathrm{a} > \mathrm{b} > \mathrm{c} > \mathrm{d}$

(B) $c > a > d > b$

(C) $\mathrm{b} > \mathrm{d} > \mathrm{a} > \mathrm{c}$

(D) $b > a > d > c$

Correct Answer: (C) $\mathrm{b} > \mathrm{d} > \mathrm{a} > \mathrm{c}$

Solution:

Concept:

- Phenol acidity is controlled by the stability of its conjugate base, the phenoxide ion. An electron-withdrawing substituent stabilises the negative charge, whereas an electron-donating substituent destabilises it.
- The relationship $\mathrm{p}K_a = -\log K_a$ means stronger acids have smaller $\mathrm{p}K_a$ values. The requested order must therefore be the reverse of the acid-strength order.

Step 1: Identify the four substituents The structures are (a) phenol, (b) para-dimethylaminophenol, (c) para-nitrophenol and (d) para-isopropylphenol. In each molecule the acidic proton belongs to the phenolic hydroxyl group. Compare the effect of each para substituent on the phenoxide ion formed after that proton is lost.

Step 2: Compare their conjugate-base stabilisation The nitro group has strong electron-withdrawing inductive and resonance effects. It stabilises the phenoxide ion by allowing delocalisation of negative charge and makes (c) the strongest acid. The isopropyl group in (d) donates electron density through its positive inductive effect and weakens acidity relative to unsubstituted phenol (a). The dimethylamino group in (b) is a powerful resonance donor; its lone pair supplies electron density into the aromatic system and destabilises the phenoxide conjugate base more strongly than the alkyl

group does.

Step 3: Reverse the acid-strength order The resulting acid-strength order is $c > a > d > b$. Reversing it to express decreasing $\mathrm{p}K_a$ gives

$$\mathrm{p}K_a(b) > \mathrm{p}K_a(d) > \mathrm{p}K_a(a) > \mathrm{p}K_a(c).$$

Option (B) states the acidity order instead. Option (A) incorrectly makes (a) exceed the donor-substituted compounds, while option (D) incorrectly places unsubstituted phenol above the alkyl-substituted phenol in $\mathrm{p}K_a$.

Step 4: Conclusion The decreasing $\mathrm{p}K_a$ order is $b > d > a > c$. Hence, the correct option is (C).

Quick Tip to Solve: First rank phenoxide stability, then reverse that order for $\mathrm{p}K_a$: nitro lowers the value, while para electron donors raise it.

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73. $1\ \mathrm{L}, 0.02\mathrm{M}$ **solution of** $[\mathrm{Co}\,(\mathrm{NH_3})_5\,\mathrm{SO_4}]$ **Br is mixed with** $1\ \mathrm{L}, 0.02\mathrm{M}$ **solution of** $[\mathrm{Co}\,(\mathrm{NH_3})_5\,\mathrm{Br}]\,\mathrm{SO_4}$**. The resulting solution is divided into two equal parts** (X) **and treated with excess of** $\mathrm{AgNO_3}$ **solution and** $\mathrm{BaCl_2}$ **solution respectively as shown below:**

The number of moles of Y **and** Z **respectively are** [4 marks]

$$1\,\mathrm{L\ solution}\ (X) \xrightarrow{+\,\mathrm{AgNO_3\ solution\ (excess)}} Y$$

$$1\,\mathrm{L\ solution}\ (X) \xrightarrow{+\,\mathrm{BaCl_2\ solution\ (excess)}} Z$$

(A) $0.01, 0.01$

(B) $0.01, 0.02$

(C) $0.02, 0.01$

(D) $0.02, 0.02$

Correct Answer: (A) $0.01, 0.01$

Solution:

Concept:

- Only counterions outside the coordination sphere are released as free ions in the usual precipitation tests. Coordinated ligands inside square brackets do not give their normal counterion precipitation immediately.
- Use $n = MV$, with volume in litres. Each precipitate forms in a one-to-one mole ratio with its free counterion: $\mathrm{Ag^+ + Br^- \longrightarrow AgBr\,(s)}$ and $\mathrm{Ba^{2+} + {SO_4}^{2-} \longrightarrow BaSO_4(s)}$.

Step 1: Count the free ions in the mixture The first complex, $[\mathrm{Co(NH_3)_5SO_4}]\mathrm{Br}$, supplies free bromide, while its sulphate group is coordinated. Its initial amount is $0.02 \times 1 = 0.02\,\mathrm{mol}$, giving $0.02\,\mathrm{mol}$ of free $\mathrm{Br^-}$. The second complex, $[\mathrm{Co(NH_3)_5Br}]\mathrm{SO_4}$, supplies free sulphate while its bromide is coordinated. It also contributes $0.02\,\mathrm{mol}$ of free counterion.

Step 2: Account for division into equal portions The combined solution has volume $2\,\mathrm{L}$. Dividing it into two equal $1\,\mathrm{L}$ portions divides the amounts of both freely dissolved ions equally. Each portion therefore contains

$$n(\mathrm{Br}^-) = 0.010\,\mathrm{mol}, \qquad n(\mathrm{SO_4}^{2-}) = 0.010\,\mathrm{mol}.$$

Mixing the solutions does not double either individual counterion amount; each species comes from just one of the salts.

Step 3: Apply the precipitation stoichiometry Excess $\mathrm{AgNO_3}$ precipitates the $0.010\,\mathrm{mol}$ of free bromide in the first portion as $Y = \mathrm{AgBr}$. Excess $\mathrm{BaCl_2}$ precipitates the $0.010\,\mathrm{mol}$ of free sulphate in the second portion as $Z = \mathrm{BaSO_4}$. Options (B), (C) and (D) incorrectly retain an undivided ion amount or count coordinated ions as freely precipitable counterions.

Step 4: Conclusion $Y = \mathrm{AgBr}$ and $Z = \mathrm{BaSO_4}$, with amounts $0.010\,\mathrm{mol}$ and $0.010\,\mathrm{mol}$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Mark the ions outside square brackets first, calculate their moles, and halve those moles when the mixed solution is split into two equal portions.

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74. The wave function (Ψ) of $2\ \mathrm{s}$ is given by

$$\Psi_{2\ \mathrm{s}} = \frac{1}{2\sqrt{2\pi}} \left(\frac{1}{a_0}\right)^{1/2} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$

At $r = r_0$, radial node is formed. Thus, r_0 in terms of a_0 [4 marks]

(A) $r_0 = a_0$

(B) $r_0 = 4a_0$

(C) $r_0 = 2a_0$

(D) $r_0 = \frac{a_0}{2}$

Correct Answer: (C) $r_0 = 2a_0$

Solution:

Concept:

- A radial node is a finite positive radius at which the radial wave function vanishes, making the probability density zero over a spherical surface.
- For the given $2s$ function, the prefactor is a nonzero constant and $e^{-r/(2a_0)}$ is nonzero at every finite radius. Thus only the factor $2 - r/a_0$ can produce the radial node.

Step 1: Set the wave function equal to zero At $r = r_0$, impose $\Psi_{2s}(r_0) = 0$. Since a_0 is a positive length, the constant multiplying the radial expression cannot be zero. Likewise, an exponential of a finite real number is strictly positive. Dividing by these nonzero factors reduces the nodal condition to

$$2 - \frac{r_0}{a_0} = 0.$$

The overall normalisation does not affect a zero of the wave function.

Step 2: Solve for the radial position Rearranging the remaining factor gives

$$\frac{r_0}{a_0} = 2 \quad \Longrightarrow \quad r_0 = 2a_0.$$

This is a finite positive radius and therefore defines a spherical radial node. It is consistent with the node-count rule $n - l - 1 = 2 - 0 - 1 = 1$: a $2s$ orbital has exactly one radial node.

Step 3: Check the other proposed radii At $r = a_0$, the factor $2 - r/a_0$ equals 1; at $r = 4a_0$, it equals -2; and at $r = a_0/2$, it equals $3/2$. None of these values gives a vanishing wave function. Thus options (A), (B) and (D) cannot represent the radial node. The exponential tending to zero at infinity is not an additional finite radial node.

Step 4: Conclusion The radial node occurs at $\boxed{r_0 = 2a_0}$. Hence, the correct option is (C).

Quick Tip to Solve: To locate radial nodes, set the polynomial radial factor to zero; nonzero normalisation constants and finite exponential factors cannot supply the node.

75. Given below are two statements:

Statement I: During Electrolytic refining, the pure metal is made to act as anode and its impure metallic form is used as cathode.

Statement II: During the Hall-Heroult electrolysis process, purified Al_2O_3 is mixed with Na_3AlF_6 to lower the melting point of the mixture.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Statement I is correct but Statement II is incorrect

(B) Statement I is incorrect but Statement II is correct

(C) Both Statement I and Statement II are incorrect

(D) Both Statement I and Statement II are correct

Correct Answer: (B) Statement I is incorrect but Statement II is correct

Solution:

Concept:

- Electrolytic refining transfers metal from an impure anode into solution and then deposits purified metal at the cathode. Oxidation occurs at the anode and reduction at the cathode.
- In the Hall–Heroult process, alumina is dissolved in molten cryolite, Na_3AlF_6. Cryolite lowers the operating melting temperature and helps provide a suitable conducting electrolyte.

Step 1: Test the electrode assignment in Statement I The impure metal is made the anode, where metal atoms undergo oxidation:

$$\mathrm{M} \longrightarrow \mathrm{M}^{n+} + n\,\mathrm{e}^-.$$

A thin sheet of pure metal is the cathode, where dissolved metal ions gain electrons:

$$\mathrm{M}^{n+} + n\,\mathrm{e}^- \longrightarrow \mathrm{M}.$$

The metal collected at the cathode is purified; insoluble impurities can accumulate as anode mud. Statement I interchanges the pure and impure electrode materials and is therefore incorrect.

Step 2: Test the role of cryolite in Statement II Pure alumina has a very high melting point, so electrolysing it alone would require an impractically high temperature. In the Hall–Heroult cell, purified Al_2O_3 is mixed with Na_3AlF_6, producing a molten mixture that can operate at a lower temperature than pure alumina. The statement describes this standard role of cryolite correctly; it is not claiming that cryolite is the source of aluminium metal.

Step 3: Choose the matching truth combination The required combination is Statement I false and Statement II true. Option (A) gives the reverse combination. Option (C) incorrectly rejects the lowering of melting temperature by cryolite, and option (D) incorrectly accepts the reversed electrode arrangement. Only option (B) describes both evaluations correctly.

Step 4: Conclusion Statement I is incorrect, while Statement II is correct. Hence, the correct option is (B).

Quick Tip to Solve: Remember refining as impure anode dissolves, pure cathode grows; remember cryolite as the medium that lowers the temperature needed for alumina electrolysis.

76. The water quality of a pond was analyzed and its BOD was found to be 4. The pond has : [4 marks]

(A) Very clean water

(B) Highly polluted water

(C) Water has high amount of fluoride compounds

(D) Slightly polluted water

Correct Answer: (A) Very clean water

Solution:

Concept:

- Biochemical oxygen demand (BOD) is the amount of dissolved oxygen consumed by microorganisms while decomposing biodegradable organic matter under prescribed test conditions.
- Higher BOD indicates a greater biodegradable organic load. In the standard textbook comparison, very clean water has BOD below about $5\,\text{mg}\,\text{L}^{-1}$, whereas highly polluted water can have BOD of $17\,\text{mg}\,\text{L}^{-1}$ or more.

Step 1: Interpret the measured BOD A BOD value of 4 means a relatively small oxygen demand, conventionally expressed as $4\,\text{mg}\,\text{L}^{-1}$. It is well below the values characteristic of strongly polluted water. The measurement concerns biodegradable material; it does not directly identify a particular dissolved inorganic pollutant such as fluoride.

Step 2: Distinguish the qualitative classifications For the standard textbook comparison, clean water has BOD below about $5\,\text{mg}\,\text{L}^{-1}$, while a highly polluted sample may have BOD of $17\,\text{mg}\,\text{L}^{-1}$ or more. Here $4 < 5$, so the value falls in the clean-water range. Among the qualitative options supplied, very clean water is therefore the matching description. The small value does not support moving the sample into a pollution category.

Step 3: Eliminate unsupported interpretations Option (B), highly polluted water, would require a much larger oxygen demand. Option (C) is unsupported because BOD cannot establish a high fluoride concentration: it measures oxygen consumed in biological degradation of organic matter. Option (D), slightly polluted water, is not the best match to a value already below the clean-water threshold. This classification addresses organic pollution only; it does not establish that the water is free from every possible contaminant or safe to drink.

Step 4: Conclusion With BOD $4\,\mathrm{mg\,L^{-1}}$, the pond is classified as having very clean water. Hence, the correct option is (A).

Quick Tip to Solve: Compare BOD with the clean-water benchmark of about $5\,\mathrm{mg\,L^{-1}}$. A small BOD indicates little biodegradable organic pollution, not a fluoride concentration.

77. Chlorides of which metal are soluble in organic solvents : [4 marks]

(A) K

(B) Be

(C) Ca

(D) Mg

Correct Answer: (B) Be

Solution:

Concept:

- Fajans' rules relate covalent character to polarisation of an anion by a cation. A smaller cation of higher charge density distorts a large, polarisable anion more strongly.
- Compounds with appreciable covalent character dissolve more readily in suitable organic solvents than strongly ionic salts do. Among the listed metals, beryllium produces the smallest doubly charged cation.

Step 1: Compare the polarising powers Potassium forms the relatively large singly charged K^+ ion and its chloride is predominantly ionic. Beryllium, magnesium and calcium form doubly charged cations. In group 2, cation size increases from Be^{2+} to Mg^{2+} to Ca^{2+}. For the same positive charge, the smallest ion has the greatest charge density and polarising power.

Step 2: Identify the chloride with covalent bonding The exceptionally small Be^{2+} ion strongly distorts the electron cloud of Cl^-. Consequently $BeCl_2$ has pronounced covalent character rather than behaving as a simple strongly ionic lattice. It can therefore dissolve in appropriate organic solvents. This is the anomalous behaviour of beryllium that distinguishes it from the other alkaline earth metals listed.

Step 3: Rule out the alternatives KCl is strongly ionic because potassium has low polarising power. $CaCl_2$ is also predominantly ionic because the calcium ion is relatively large. $MgCl_2$ has more polarisation than calcium chloride but lacks the exceptionally strong covalent character of beryllium chloride. Hence neither potassium, calcium nor magnesium gives the characteristic organic-solvent-soluble chloride asked for here; beryllium does.

Step 4: Conclusion The metal is beryllium: its chloride, $BeCl_2$, has strong covalent character and is soluble in organic solvents. Hence, the correct option is (B).

Quick Tip to Solve: For unusual group 2 properties, check beryllium first: its small, highly polarising ion gives covalent compounds such as organic-solvent-soluble $BeCl_2$.

78. Which of the following reaction is correct? [4 marks]

(A) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

(B) $2LiNO_3 \longrightarrow 2Li + 2NO_2 + O_2$

(C) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 2\,N_2O_4 + O_2$

(D) $2LiNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2$

Correct Answer: (A) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

Solution:

Concept:

- Lithium differs from the other alkali metals because its small cation strongly polarises the nitrate ion. On heating, lithium nitrate gives lithium oxide, nitrogen dioxide and oxygen.
- The decomposition equation must conserve each element. At the high temperature of decomposition, the nitrogen oxide is represented as NO_2, not its low-temperature dimer N_2O_4.

Step 1: Identify the characteristic products Unlike sodium and potassium nitrates, lithium nitrate does not simply give a nitrite. Its behaviour resembles that of alkaline-earth nitrates: the solid residue is Li_2O, accompanied by brown NO_2 gas and O_2. Thus an equation giving lithium metal cannot describe ordinary thermal decomposition of the nitrate.

Step 2: Balance and compare all four equations Balancing lithium first and then nitrogen gives the equation

$$4\,LiNO_3 \xrightarrow{\Delta} 2\,Li_2O + 4\,NO_2 + O_2.$$

There are four lithium atoms, four nitrogen atoms and twelve oxygen atoms on each side. Option (B) wrongly produces lithium metal. Option (C) writes the nitrogen oxide as its dimer; $2\,NO_2 \rightleftharpoons N_2O_4$ favours dissociation on heating, so it is not the standard thermal-decomposition equation. Option (D) even creates sodium atoms from a lithium salt and violates conservation of elements.

Step 3: Conclusion Lithium nitrate decomposes on heating to Li_2O, NO_2 and O_2 in the ratio $2 : 4 : 1$ for four moles of nitrate. Hence, the correct option is (A).

Quick Tip to Solve: For alkali-metal nitrates, remember lithium as the exception: it yields the oxide; the heavier alkali-metal nitrates normally yield nitrites.

79. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Antihistamines do not affect the secretion of acid in stomach.

Reason R : Antiallergic and antacid drugs work on different receptors.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) A is false but R is true

(B) Both A and R are true but R is not the correct explanation of A

(C) Both A and R are true and R is the correct explanation of A

(D) A is true but R is false

Correct Answer: (C) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Histamine can act at different receptor subtypes. Antiallergic antihistamines act mainly at H1 receptors, whereas histamine-stimulated acid secretion in the stomach involves H2 receptors.
- Drug action depends on receptor selectivity. Blocking the receptor responsible for allergy does not automatically block every physiological action of the same chemical messenger.

Step 1: Evaluate Assertion A In this question, antihistamines means the usual antiallergic drugs. They interfere with histamine action at H1 receptors and relieve responses such as sneezing and other allergic symptoms. The stomach acid-secretion response is mediated by a different receptor subtype. Consequently these antiallergic antihistamines do not inhibit the secretion of gastric acid in the manner of H2 blockers. Assertion A is therefore true in the intended textbook context.

Step 2: Evaluate Reason R Antiallergic drugs and the histamine-receptor antagonists used to control gastric acidity target different receptor subtypes. H1 antagonism is associated with antiallergic action, while H2 antagonism reduces acid secretion. Thus Reason R is true. Here the reference to antacid drugs concerns acid-controlling receptor blockers; simple acid-neutralising substances act by neutralisation rather than receptor blockade.

Step 3: Connect the statements and eliminate the alternatives The difference in receptor targets explains why an antiallergic antihistamine need not suppress gastric acid secretion. Therefore the reason directly explains the assertion. Option (A) incorrectly rejects A; option (D) incorrectly rejects R. Option (B) accepts both but misses their explanatory connection.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (C).

Quick Tip to Solve: Separate H1-mediated allergic responses from H2-mediated gastric secretion before judging an assertion about antihistamines.

80. Boric acid is solid, whereas BF_3 is gas at room temperature because of : [4 marks]

(A) Strong van der Waal's interaction in Boric acid

(B) Strong covalent bond in BF_3

(C) Strong hydrogen bond in Boric acid

(D) Strong ionic bond in Boric acid

Correct Answer: (C) Strong hydrogen bond in Boric acid

Solution:

Concept:

- Physical state depends mainly on attractions between particles. A strong covalent bond inside a molecule does not by itself imply a high boiling point or a solid molecular lattice.
- Boric acid, $B(OH)_3$, contains hydroxyl groups that form intermolecular hydrogen bonds. BF_3 has no hydrogen atom available for such self-association.

Step 1: Identify the attractive forces in boric acid Each boric acid molecule has three hydroxyl groups. Hydrogen attached to oxygen can interact with an oxygen atom of a neighbouring molecule, producing extensive intermolecular hydrogen bonding. The molecules are thereby associated in layers, and this cohesive arrangement stabilises the solid at room temperature. Energy must be supplied to disrupt these intermolecular contacts.

Step 2: Compare with boron trifluoride Boron trifluoride consists of discrete trigonal-planar BF_3 molecules. Its B–F bonds are strong covalent bonds within each molecule, but they do not join the molecules into the hydrogen-bonded network present in boric acid. The relatively weak attractions between its molecules allow it to remain gaseous at room temperature.

Step 3: Test the proposed explanations Option (A) misses the characteristic strong interaction in boric acid: hydrogen bonding. Option (B) describes an intramolecular bond in BF_3, not the intermolecular origin of the difference in physical state. Option (D) is incorrect because boric acid is a molecular covalent substance rather than an ionic crystal. Option (C) identifies the relevant attraction.

Step 4: Conclusion Strong intermolecular hydrogen bonding makes boric acid solid at room temperature. Hence, the correct option is (C).

Quick Tip to Solve: When comparing molecular physical states, inspect intermolecular attractions first; do not confuse bond strength inside a molecule with cohesion between molecules.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The strength of 50 volume solution of hydrogen peroxide is _____ $\mathrm{g/L}$ (Nearest integer).

Given:

Molar mass of H_2O_2 is $34\ \mathrm{g\ mol^{-1}}$

Molar volume of gas at $\mathrm{STP} = 22.7\ \mathrm{L}$ [4 marks]

Solution:

Concept:

- Volume strength is the volume of oxygen at STP released by complete decomposition of one volume of hydrogen peroxide solution. Thus one litre of a 50-volume solution releases 50 litres of oxygen.
- The decomposition is $2\,H_2O_2 \longrightarrow 2\,H_2O + O_2$; two moles of peroxide produce one mole of oxygen. Use the supplied molar gas volume of $22.7\,\mathrm{L\,mol^{-1}}$.

Step 1: Convert the evolved oxygen volume to moles Take exactly $1.00\,\mathrm{L}$ of solution. The volume-strength definition gives $V_{O_2} = 50.0\,\mathrm{L}$ at STP. Hence

$$n_{O_2} = \frac{50.0}{22.7} = 2.20264\,\mathrm{mol}.$$

This oxygen amount refers to one litre of the original solution, not to one litre of pure peroxide.

Step 2: Apply the stoichiometric ratio and obtain the mass The balanced reaction requires twice as many moles of peroxide as oxygen produced:

$$n_{H_2O_2} = 2\left(\frac{50}{22.7}\right) = 4.40529\,\mathrm{mol}.$$

Using the given molar mass, the mass in one litre of solution is

$$m = \frac{2 \times 50 \times 34}{22.7} = 149.7797\,\mathrm{g}.$$

Therefore the strength is $149.7797\,\mathrm{g\,L^{-1}}$. The factor of two comes from the reaction coefficients; omitting it would underestimate the strength by half.

Step 3: Conclusion The strength, rounded to the nearest integer, is $\boxed{150\,\mathrm{g\,L^{-1}}}$. The numerical entry is 150.

Quick Tip to Solve: For volume strength V and molar gas volume Vm, peroxide strength in g/L is 68V/Vm; use the molar gas volume stated in the question.

82. A short peptide on complete hydrolysis produces 3 moles of glycine (G), two moles of leucine (L) and two moles of valine (V) per mole of peptide. The number of peptide linkages in it are _____. [4 marks]

Solution:

Concept:

- A peptide linkage is the amide bond $-\mathrm{CO}-\mathrm{NH}-$ joining two amino-acid residues. Complete hydrolysis cleaves these bonds and releases the constituent amino acids.
- An open-chain peptide containing n amino-acid residues has n-1 peptide linkages. The identities and order of the amino acids do not affect this count.

Step 1: Count the amino-acid residues in one peptide molecule The hydrolysis amounts are given per mole of peptide. Three moles of glycine, two moles of leucine and two moles of valine therefore mean that every peptide molecule contains three glycine residues, two leucine residues and two valine residues. Their total is

$$n = 3 + 2 + 2 = 7.$$

The molecule is consequently a heptapeptide; there is no need to determine the sequence of G, L and V to count its links.

Step 2: Relate the number of joins to the number of residues To connect seven residues into a single linear chain, connect the first to the second, the second to the third, and continue until the sixth is connected to the seventh. There are six adjacent pairs. Equivalently, formation of a linear peptide from n amino acids eliminates n-1 water molecules, one for each peptide linkage. Therefore

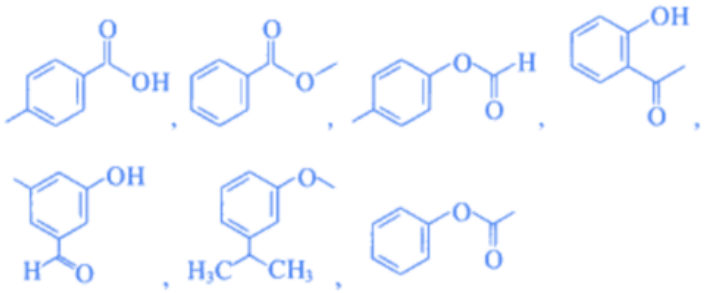
$$N_{\text{peptide linkages}} = 7 - 1 = 6.$$

The free amino and carboxyl end groups do not count as peptide linkages. A cyclic structure would require an explicit indication; none is specified here.

Step 3: Conclusion The peptide contains $\boxed{6}$ peptide linkages per molecule. This is a dimensionless count, so the numerical entry is 6.

Quick Tip to Solve: Add all amino-acid counts from complete hydrolysis, then subtract one for a linear peptide; repeated amino acids still count as separate residues.

83. Number of compounds from the following which will not dissolve in cold $NaHCO_3$ and $NaOH$ solutions but will dissolve in hot $NaOH$ solution is _____. [4 marks]



Solution:

Concept:

- Carboxylic acids form soluble carboxylate salts with cold $NaHCO_3$ or NaOH. Phenols generally form phenoxide salts with NaOH, but not with bicarbonate.
- Neutral esters are not dissolved by these cold acid-base tests, but undergo alkaline hydrolysis on heating. Formation of ionic carboxylate salts causes their dissolution in hot aqueous alkali.

Step 1: Identify the compounds that already respond to cold alkali Read the structures from left to right in each row. The first compound in the top row is a methyl-substituted benzoic acid, so its carboxylic acid group reacts even with bicarbonate. The fourth compound in that row contains a phenolic hydroxyl group and dissolves in cold sodium hydroxide. The first compound in the bottom row is also a phenol, with methyl and aldehyde substituents, and likewise forms a phenoxide with cold alkali. These three fail the condition of being insoluble in both cold reagents.

Step 2: Select the neutral esters that react on heating The remaining top-row compounds are methyl benzoate and a methylphenyl methanoate (an aryl formate). The final compound in the bottom row is phenyl ethanoate. All three contain an ester group. On heating with aqueous sodium hydroxide they undergo saponification, represented generally by

$$R_1COOR_2 + NaOH \longrightarrow R_1COONa + R_2OH.$$

The carboxylate product is ionic and water-soluble. The other bottom-row compound is an aromatic ether with an isopropyl substituent; it has neither an acidic hydroxyl group nor an ester linkage and is not hydrolysed by hot aqueous alkali in this test.

Step 3: Conclusion The qualifying compounds are methyl benzoate, the methylphenyl methanoate and phenyl ethanoate: $\boxed{3}$ compounds. The numerical entry is the dimensionless count 3.

Quick Tip to Solve: Use the tests in order: remove carboxylic acids and phenols first, then count neutral esters that become soluble through hot alkaline hydrolysis.

84. An organic compound undergoes first-order decomposition. If the time taken for the 60% decomposition is 540 s, then the time required for 90% decomposition will be _____ s. (Nearest integer).

Given: $\ln 10 = 2.3; \log 2 = 0.3$ [4 marks]

Solution:

Concept:

- For a first-order reaction, $kt = \ln([A]_0/[A]_t) = 2.303 \log_{10}([A]_0/[A]_t)$. The logarithm uses the fraction remaining, not the fraction decomposed.
- For two conversions in the same reaction, k is unchanged, so the time ratio equals the ratio of the corresponding logarithms. Use the supplied approximate logarithms consistently.

Step 1: Write the first-order expression at 60 percent decomposition When 60 percent has decomposed, 40 percent remains. The concentration ratio is therefore $[A]_0/[A]_t = 1/0.4 = 2.5$, giving

$$k(540) = 2.303 \log_{10}(2.5).$$

Using the given value $\log_{10} 2 = 0.3$,

$$\log_{10}(2.5) = \log_{10}(10/4) = 1 - 2\log_{10} 2 = 1 - 0.6 = 0.4.$$

This step is why the supplied logarithm matters; substituting an independently more precise value would not follow the numerical approximation specified.

Step 2: Form the ratio for 90 percent decomposition At 90 percent decomposition, 10 percent remains, so the concentration ratio is 10 and its common logarithm is 1. Thus

$$kt_{90} = 2.303 \log_{10} 10 = 2.303.$$

Dividing by the expression for 540 seconds eliminates both k and the logarithm-conversion factor:

$$\frac{t_{90}}{540} = \frac{1}{0.4} = 2.5, \qquad t_{90} = 540 \times 2.5 = 1350\,\text{s}.$$

The answer is the total elapsed time from the initial state, rather than the additional time after reaching 60 percent decomposition.

Step 3: Conclusion The required total time for 90 percent decomposition is $\boxed{1350\,\text{s}}$, already an integer. The numerical entry is 1350.

Quick Tip to Solve: For first-order conversion questions, form a ratio of logarithms of the reciprocal fractions remaining; this avoids separately calculating the rate constant.

85. 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of $27^{\circ}C$. The work done is $3\ kJ\ mol^{-1}$. The final temperature of the gas is _____ K (Nearest integer).

Given $C_V = 20\ J\ mol^{-1}\ K^{-1}$ [4 marks]

Solution:

Concept:

- For an adiabatic process, heat exchange is zero: $q = 0$. The first law in the chemistry sign convention is $\Delta U = q + w$, where work done on the gas is positive.
- For an ideal gas, internal energy depends only on temperature, and $\Delta U = nC_V(T_f - T_i)$. Here $n = 1\,\text{mol}$ and $C_V = 20\,\text{J}\,\text{mol}^{-1}\text{K}^{-1}$.

Step 1: Set the initial temperature and work sign The initial temperature is $T_i = 27 + 273 = 300\,\text{K}$. During expansion the gas does work on its surroundings, so its energy decreases. The stated work magnitude is $3\,\text{kJ}\,\text{mol}^{-1}$; for one mole the work on the gas is

$$w = -3000\,\text{J}.$$

Reversibility does not change this sign or the first-law balance.

Step 2: Apply the adiabatic energy balance Because no heat enters to replace the energy used in expansion, the entire work output comes from internal energy. Therefore

$$\Delta U = q + w = 0 - 3000 = -3000\,\text{J}.$$

Substituting into the ideal-gas relation gives

$$1 \times 20(T_f - 300) = -3000.$$

Hence

$$T_f - 300 = -150, \qquad T_f = 150\,\text{K}.$$

The decrease is physically consistent with adiabatic expansion; adding the work magnitude instead would incorrectly predict heating.

Step 3: Conclusion The final temperature is $\boxed{150\,\text{K}}$. The nearest-integer entry is $\boxed{150}$.

Quick Tip to Solve: For an adiabatically expanding ideal gas, subtract $W_{\text{by}}/(nC_V)$ from the initial Kelvin temperature; do not use C_P.

86. The electrode potential of the following half cell at $298\ \mathrm{K}$

$\mathrm{X}\left|\mathrm{X}^{2+}(0.001\mathrm{M})\right\|\mathrm{Y}^{2+}(0.01\mathrm{M})\left|\mathrm{Y}\right.$ **is** _____ $\times 10^{-2}\ \mathrm{V}$ **(Nearest integer)**

Given: $\mathrm{E}^{0}_{\mathrm{X}^{2+}|\mathrm{X}} = -2.36\ \mathrm{V}$

$$\mathrm{E}^{0}_{\mathrm{Y}^{2+}|\mathrm{Y}} = +0.36\ \mathrm{V}$$

$$\frac{2.303\mathrm{RT}}{\mathrm{F}} = 0.06\ \mathrm{V}$$

[4 marks]

Solution:

Concept:

- The written cell has oxidation at the left electrode and reduction at the right electrode. Its potential is $E_{\text{cell}} = E_{\text{right}} - E_{\text{left}}$, using reduction potentials.
- For a reaction transferring n electrons, the Nernst equation is $E = E^{\circ} - (2.303RT/nF)\log Q$; pure solids have unit activity.

Step 1: Write the cell reaction and standard potential The left electrode supplies electrons: $\mathrm{X} \longrightarrow \mathrm{X}^{2+} + 2\,\mathrm{e}^{-}$. At the right electrode, $\mathrm{Y}^{2+} + 2\,\mathrm{e}^{-} \longrightarrow \mathrm{Y}$. Thus the net reaction is

$$\mathrm{X}\,(\mathrm{s}) + \mathrm{Y}^{2+}(\mathrm{aq}) \longrightarrow \mathrm{X}^{2+}(\mathrm{aq}) + \mathrm{Y}\,(\mathrm{s}).$$

It transfers two electrons, so $n = 2$, not four. The standard cell potential is

$$E^{\circ}_{\text{cell}} = 0.36 - (-2.36) = 2.72\,\mathrm{V}.$$

Step 2: Calculate the concentration correction For this reaction the dissolved product is X^{2+} and the dissolved reactant is Y^{2+}. Consequently

$$Q = \frac{[\mathrm{X}^{2+}]}{[\mathrm{Y}^{2+}]} = \frac{0.001}{0.01} = 0.1.$$

With the supplied logarithmic coefficient,

$$E_{\text{cell}} = 2.72 - \frac{0.06}{2}\log(0.1) = 2.72 - 0.03(-1) = 2.75\,\mathrm{V}.$$

Since $Q < 1$, the correction increases the potential, which also checks the sign.

Step 3: Conclusion The potential is $\boxed{2.75\,\mathrm{V}} = \boxed{275 \times 10^{-2}\,\mathrm{V}}$. The required nearest-integer coefficient is $\boxed{275}$.

Quick Tip to Solve: Write the net cell reaction before constructing Q. A tenfold concentration ratio with two electrons changes the potential by $0.03\,\mathrm{V}$ here.

87. Consider the following equation:

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g), \Delta H = -190 \text{ kJ}$$

The number of factors which will increase the yield of SO_3 at equilibrium from the following is _____.

A. Increasing temperature

B. Increasing pressure

C. Adding more SO_2

D. Adding more O_2

E. Addition of catalyst [4 marks]

Solution:

Concept:

- Le Chatelier's principle predicts that an equilibrium shifts to oppose a change in temperature, pressure or concentration. A higher equilibrium yield requires a shift toward products.
- The forward reaction $2\,SO_2(g) + O_2(g) \rightleftharpoons 2\,SO_3(g)$ is exothermic, with $\Delta H = -190\,kJ$. There are three stoichiometric moles of gas on the left and two on the right.

Step 1: Assess temperature and pressure changes Factor A raises the temperature. Heat behaves as a product for this exothermic forward reaction, so adding heat favours the reverse reaction. The equilibrium amount of SO_3 decreases; A is excluded. Factor B increases pressure, as by compressing the reaction mixture at fixed temperature. The equilibrium then favours the side containing fewer gas molecules. Since the right side has two rather than three gas moles, B increases the yield.

Step 2: Assess reactant additions and the catalyst Factor C adds SO_2, and factor D adds O_2. In either case the system consumes some of the added reactant by shifting forward, producing more SO_3. Thus C and D both qualify. Factor E adds a catalyst, which lowers the activation barriers for both directions and speeds the approach to equilibrium. It changes neither the equilibrium constant nor the final equilibrium composition at a given temperature, so E does not qualify. The successful factors are therefore B, C and D.

Step 3: Conclusion Increasing pressure and adding either reactant increase the equilibrium yield. The number of qualifying factors is $\boxed{3}$ (a dimensionless count).

Quick Tip to Solve: For an exothermic gas equilibrium, compare gas coefficients for pressure and treat heat as a product for temperature. A catalyst changes the rate, not the equilibrium yield.

88. Lead storage battery contains 38% by weight solution of H_2SO_4. The van't Hoff factor is 2.67 at this concentration. The temperature in Kelvin at which the solution in the battery will freeze is _____. (Nearest integer).

Given $K_f = 1.8\ \text{K kg mol}^{-1}$ [4 marks]

Solution:

Concept:

- The depression in freezing point is $\Delta T_f = iK_f m$, where i is the given van't Hoff factor and m is moles of solute per kilogram of solvent.
- A mass percentage refers to the mass of the entire solution. In a 38% by-mass solution, $100\,\text{g}$ of solution contains $38\,\text{g}$ of H_2SO_4 and $62\,\text{g}$ of water.

Step 1: Convert mass percentage to molality Choose $100\,\text{g}$ of solution as the basis. The molar mass of H_2SO_4 is $2(1) + 32 + 4(16) = 98\,\text{g mol}^{-1}$. Thus

$$n(H_2SO_4) = \frac{38}{98}\,\text{mol}, \qquad m_{\text{water}} = 0.062\,\text{kg}.$$

The molality is

$$m = \frac{38/98}{0.062} = 6.254\,\text{mol kg}^{-1}.$$

Using $0.100\,\text{kg}$ as the denominator would wrongly count the solute as solvent.

Step 2: Calculate the freezing point Use the supplied factor $i = 2.67$, which already describes the effective number of dissolved particles at this concentration; do not multiply by an additional dissociation factor. Then

$$\Delta T_f = 2.67 \times 1.8 \times 6.254 \approx 30.06\,\text{K}.$$

Water freezes at about $273\,\text{K}$, so the solution freezes at

$$T_f = 273 - 30.06 = 242.94\,\text{K}.$$

Using $273.15\,\text{K}$ instead gives $243.09\,\text{K}$; either convention rounds to the same integer.

Step 3: Conclusion The freezing temperature, rounded to the nearest integer, is $\boxed{243\,\text{K}}$. The numerical entry is $\boxed{243}$.

Quick Tip to Solve: For a mass-percent solution, use $m = 1000w/[M(100 - w)]$. The solvent mass is $100 - w$, and the given i already accounts for dissociation.

89. The graph of $\log \frac{x}{m}$ vs $\log p$ for an adsorption process is a straight line inclined at an angle of 45° with intercept equal to 0.6020. The mass of gas adsorbed per unit mass of adsorbent at the pressure of 0.4 atm is _____ $\times 10^{-1}$ (Nearest integer).

Given: $\log 2 = 0.3010$ [4 marks]

Solution:

Concept:

- The Freundlich adsorption isotherm is $x/m = kp^{1/n}$, where x is adsorbed gas mass and m is adsorbent mass. In logarithmic form, $\log(x/m) = \log k + (1/n)\log p$.
- On a plot of $\log(x/m)$ against $\log p$, the slope equals $1/n$ and the vertical intercept equals $\log k$. Here the logarithms are base ten.

Step 1: Determine the exponent and adsorption constant The inclination is 45°, so the slope is

$$\frac{1}{n} = \tan 45^\circ = 1.$$

The intercept is 0.6020. Using the supplied value $\log 2 = 0.3010$,

$$\log k = 0.6020 = 2\log 2 = \log 4.$$

Therefore $k = 4$ for the stated pressure and mass conventions. The intercept is the logarithm of k, so substituting 0.6020 directly as the constant would be incorrect.

Step 2: Substitute the pressure and convert the requested scale Because the exponent $1/n$ is one, adsorption per unit mass varies directly with pressure in the given relation. At $p = 0.4\,\text{atm}$,

$$\frac{x}{m} = kp^{1/n} = 4(0.4)^1 = 1.6.$$

The requested blank multiplies 10^{-1}, rather than representing x/m itself. If its value is N, then

$$N \times 10^{-1} = 1.6 \quad \Rightarrow \quad N = 16.$$

The mass ratio is dimensionless when both masses use the same unit; the required coefficient is already an integer.

Step 3: Conclusion The adsorbed mass per unit adsorbent mass is $1.6 = 16 \times 10^{-1}$. Hence the nearest-integer entry is $\boxed{16}$.

Quick Tip to Solve: Use slope $= \tan\theta$ and intercept $= \log k$, then check the power of ten beside the blank before entering the answer.

90. Iron oxide FeO, crystallises in a cubic lattice with a unit cell edge length of 5.0 Å. If density of the FeO in the crystal is $4.0\ \mathrm{g\ cm^{-3}}$, then the number of FeO units present per unit cell is _____. (Nearest integer)

Given: Molar mass of Fe and O is 56 and $16\ \mathrm{g\ mol^{-1}}$ respectively. $\mathrm{N_A} = 6.0 \times 10^{23}\ \mathrm{mol^{-1}}$ [4 marks]

Solution:

Concept:

- For a cubic unit cell of edge length a, the volume is a^3. Its mass is ZM/N_A, where Z is the number of formula units per cell and M is the formula molar mass.
- The density relation is $\rho = ZM/(N_A a^3)$, or $Z = \rho N_A a^3/M$. Use compatible units: with density in $\mathrm{g\,cm^{-3}}$, convert the edge length to centimetres.

Step 1: Find the cell volume and formula molar mass The molar mass of one mole of FeO formula units is

$$M = 56 + 16 = 72\,\mathrm{g\,mol^{-1}}.$$

Since $1\,\text{Å} = 10^{-8}\,\mathrm{cm}$, the edge is $a = 5.0 \times 10^{-8}\,\mathrm{cm}$. Cubing both the numerical factor and the power of ten gives

$$a^3 = (5.0 \times 10^{-8})^3 = 125 \times 10^{-24} = 1.25 \times 10^{-22}\,\mathrm{cm^3}.$$

Step 2: Use the density to count formula units The mass of a unit cell is

$$m_{\text{cell}} = \rho a^3 = 4.0 \times 1.25 \times 10^{-22} = 5.0 \times 10^{-22}\,\mathrm{g}.$$

One FeO formula unit has mass

$$m_{\text{formula}} = \frac{72}{6.0 \times 10^{23}} = 1.2 \times 10^{-22}\,\mathrm{g}.$$

Their ratio is

$$Z = \frac{5.0 \times 10^{-22}}{1.2 \times 10^{-22}} = 4.1667.$$

The supplied data are rounded and the question explicitly asks for the nearest integer, so this corresponds to four formula units. Count FeO units, not the total number of constituent ions.

Step 3: Conclusion The number of FeO formula units per unit cell, to the nearest integer, is $\boxed{4}$ (a dimensionless count).

Quick Tip to Solve: Convert angstroms to centimetres before cubing. In the density formula, M is the molar mass of the whole formula unit, here 72, not that of iron alone.