JEE Main 2023 Paper 1 (B.E./B.Tech) Jan 31 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 31 January 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is : [4 marks]

(A) 0

(B) 1

(C) 3

(D) 2

Correct Answer: (B) 1

Solution:

Concept:

- The square roots are principal nonnegative roots. Their radicands must be nonnegative before any squaring is performed.
- For nonnegative quantities, squaring preserves equality. Any candidates obtained after squaring must still satisfy the original domain and equation.

Step 1: Establish the real domain The first radicand factors as $(x-1)(x-3)$, while the second is $(x-3)(x+3)$. Requiring both to be nonnegative gives

$$x \le -3 \quad \text{or} \quad x \ge 3.$$

On this set the third radicand $2(2x-1)(x-3)$ is also nonnegative, so these are the relevant possible values.

Step 2: Square and isolate the product of radicals Squaring once and collecting the two radicands gives

$$2\sqrt{(x^2-4x+3)(x^2-9)} = 2x^2-10x+12.$$

Therefore

$$\sqrt{(x-1)(x-3)^2(x+3)} = (x-2)(x-3).$$

The right side is nonnegative throughout the domain already found. Squaring this equality produces

$$(x-3)^2\big((x-1)(x+3)-(x-2)^2\big) = 0,$$
$$(x-3)^2(6x-7) = 0.$$

The algebraic candidates are $x=3$ and $x=7/6$.

Step 3: Check the candidates in the printed equation At $x=3$, all three radicals vanish, so the equality holds. At $x=7/6$, $x^2-9<0$, so this value is not in the real domain and must be rejected. There are no other candidates because every real solution necessarily satisfies the polynomial equation derived above. Thus counts zero, two and three all contradict the domain check.

Step 4: Conclusion The only real root is $x=3$, so the number of real roots is 1. Hence, the correct option is (B).

Quick Tip to Solve: Find the common domain first; after squaring, reject $7/6$ immediately because the second square root is not real.

2. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is : [4 marks]

(A) $\frac{3}{7}$

(B) $\frac{5}{6}$

(C) $\frac{5}{7}$

(D) $\frac{2}{7}$

Correct Answer: (C) $\frac{5}{7}$

Solution:

Concept:

- Bayes theorem weights each possible bag composition by the likelihood of observing the two black balls.
- Use the conventional equally likely composition hypotheses: the number K of black balls is initially equally likely to be any integer from 0 through 6. Drawing is without replacement.

Step 1: Assign the likelihood of the observation Let E be the event that both drawn balls are black. If the bag has exactly k black balls, the unordered pairs of black balls number $\binom{k}{2}$, out of $\binom{6}{2} = 15$ pairs in total. Thus

$$P(E \mid K = k) = \frac{\binom{k}{2}}{15}.$$

For $k = 0, 1$, the likelihood is zero. Those compositions cannot explain the observed draw.

Step 2: Apply Bayes theorem to the required compositions At least five black balls means $K = 5$ or $K = 6$. With the equal prior probabilities, the common prior factor and the denominator 15 cancel from Bayes theorem:

$$\begin{aligned} P(K \geq 5 \mid E) &= \frac{\binom{5}{2} + \binom{6}{2}}{\binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \binom{5}{2} + \binom{6}{2}} \\ &= \frac{10 + 15}{1 + 3 + 6 + 10 + 15} \\ &= \frac{25}{35} = \frac{5}{7}. \end{aligned}$$

The observation does not make the five surviving compositions equally likely: a bag with six black balls is much more likely to produce two black balls than a bag with only two.

Step 3: Check the interpretation and options The result lies between zero and one and reflects the greater likelihood of a black draw from bags rich in black balls. Counting only how many compositions survive would ignore the observation's different likelihoods. The values $3/7$, $5/6$ and $2/7$ do not equal the weighted ratio. As in the usual formulation of this problem, the result uses equal prior weights for the possible compositions; an arbitrary different prior would require its own Bayes calculation.

Step 4: Conclusion With the conventional equally likely composition hypotheses, the required probability is $5/7$. Hence, the correct option is (C).

Quick Tip to Solve: After observing two black balls, weight composition k by $\binom{k}{2}$, rather than counting the remaining compositions equally.

3. If $\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0, 0<\alpha<13$**, then** $\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$ **is equal to :** [4 marks]

(A) 16

(B) π

(C) $16-5\pi$

(D) 0

Correct Answer: (B) π

Solution:

Concept:

- The principal values of $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ lie in $[-\pi/2,\pi/2]$, $[0,\pi]$ and $(-\pi/2,\pi/2)$, respectively.
- The tangent subtraction identity is $\tan(A-B)=(\tan A-\tan B)/(1+\tan A\tan B)$. The angle interval must also be checked.

Step 1: Determine the value of alpha Put $A=\tan^{-1}(77/36)$ and $B=\cos^{-1}(4/5)$. Both are acute, and $\tan B=3/4$. The given equation says $\sin^{-1}(\alpha/17)=A-B$. Since $77/36>3/4$, this difference is acute. Hence

$$\tan(A-B)=\frac{77/36-3/4}{1+(77/36)(3/4)}=\frac{200}{375}=\frac{8}{15}.$$

The associated right triangle has hypotenuse 17, so $\sin(A-B)=8/17$. Therefore $\alpha=8$, which satisfies $0<\alpha<13$.

Step 2: Reduce the angle to the correct branches The argument in the requested expression is now 8 radians, not $8/17$. We have

$$\frac{5\pi}{2}<8<3\pi.$$

Subtracting 2π puts 8 in the second quadrant: $\pi/2<8-2\pi<\pi$. The principal arcsine is therefore

$$\sin^{-1}(\sin 8)=\pi-(8-2\pi)=3\pi-8.$$

The principal arccosine already lies in its allowed interval after this reduction, giving

$$\cos^{-1}(\cos 8)=8-2\pi.$$

Step 3: Add the principal values Adding the two reduced angles yields

$$(3\pi-8)+(8-2\pi)=\pi.$$

Using inverse functions as though they simply cancel would produce 16, but that ignores their principal intervals. The alternatives zero and $16-5\pi$ also use incompatible branch reductions for this second-quadrant angle.

Step 4: Conclusion The required sum is π. Hence, the correct option is (B).

Quick Tip to Solve: After obtaining $\alpha=8$, reduce 8 modulo 2π and choose the principal arcsine and arccosine branches separately.

4. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296 , respectively, then the sum of common ratios of all such GPs is [4 marks]

(A) 7

(B) 14

(C) 3

(D) $\frac{9}{2}$

Correct Answer: (A) 7

Solution:

Concept:

- Four positive consecutive terms of a geometric progression may be written symmetrically as $a/t^3, a/t, at, at^3$, with $a, t > 0$. Their common ratio is t^2.
- The identities $t^3 + t^{-3} = (t + t^{-1})^3 - 3(t + t^{-1})$ and $t^2 + t^{-2} = (t + t^{-1})^2 - 2$ simplify the sum constraint.

Step 1: Use the product to determine the middle scale Multiplying the four symmetric terms cancels every power of t, leaving $a^4 = 1296 = 6^4$. Positivity requires $a = 6$. The sum condition consequently becomes

$$6(t^{-3} + t^{-1} + t + t^3) = 126.$$

Let $u = t + t^{-1}$. Since $t > 0$, the arithmetic-geometric mean inequality gives $u \geq 2$. The sum equation reduces to

$$u^3 - 2u = 21.$$

Step 2: Find every admissible parameter Factor the cubic:

$$u^3 - 2u - 21 = (u - 3)(u^2 + 3u + 7) = 0.$$

The quadratic factor has discriminant $9 - 28 = -19$, so it has no real roots. Therefore $u = 3$ is the only possible real value. Solving $t + t^{-1} = 3$ gives

$$t = \frac{3 + \sqrt{5}}{2} \quad \text{or} \quad t = \frac{3 - \sqrt{5}}{2}.$$

Both values are positive and reciprocal, so both give valid positive progressions. Reversing the progression interchanges these two cases.

Step 3: Sum the distinct common ratios The two ratios are t^2 and t^{-2}, rather than t and t^{-1}, because the exponents of consecutive terms differ by two. Thus

$$r_1 + r_2 = t^2 + t^{-2} = u^2 - 2 = 9 - 2 = 7.$$

There are no further ratios, since the cubic supplied only the one admissible value of u. The option 3 is the sum of the auxiliary parameters, not the requested ratios. The values 14 and $9/2$ do not satisfy this reciprocal-ratio calculation.

Step 4: Conclusion The sum of the common ratios of all such progressions is 7. Hence, the correct option is (A).

Quick Tip to Solve: Use a symmetric parametrisation so that the product fixes the scale instantly; remember that the common ratio is t^2.

5. For the system of linear equations

$$x + y + z = 6$$

$$\alpha x + \beta y + 7z = 3$$

$$x + 2y + 3z = 14$$

which of the following is NOT true ? [4 marks]

(A) If $\alpha = \beta = 7$, then the system has no solution

(B) For every point $(\alpha, \beta) \neq (7, 7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions

(C) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

(D) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution

Correct Answer: (B) For every point $(\alpha, \beta) \neq (7, 7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions

Solution:

Concept:

- The first and third equations are independent and determine a line of possible triples. The remaining equation either selects one point, contains the whole line, or contradicts it.
- An equation $Az = B$ has one solution when $A \neq 0$, infinitely many when $A = B = 0$, and no solution when $A = 0$, $B \neq 0$.

Step 1: Reduce the fixed equations to one parameter Subtract the first equation from the third to obtain $y + 2z = 8$. Thus $y = 8 - 2z$, and substitution in $x + y + z = 6$ gives $x = z - 2$. Putting these into the second equation yields

$$\alpha(z - 2) + \beta(8 - 2z) + 7z = 3,$$

or

$$(\alpha - 2\beta + 7)z = 3 + 2\alpha - 8\beta.$$

This single equation now gives every consistency case without separately solving three unknowns.

Step 2: Locate the infinite-solution case The coefficient vanishes on $\alpha - 2\beta + 7 = 0$, so $\alpha = 2\beta - 7$. On this line, the right side is

$$3 + 2(2\beta - 7) - 8\beta = -11 - 4\beta.$$

It vanishes only at

$$\beta = -\frac{11}{4}, \qquad \alpha = -\frac{25}{2}.$$

Therefore only this one point on the coefficient-zero line gives infinitely many solutions. Every other point on that line gives no solution. Statement (B), which claims infinitely many solutions at every

point except $(7, 7)$, is false.

Step 3: Verify the other three statements At $\alpha = \beta = 7$, the reduced equation is $0z = -39$, a contradiction, so (A) is true. The unique infinite-solution point satisfies

$$-\frac{25}{2} + 2\left(-\frac{11}{4}\right) + 18 = 0,$$

which verifies (C). Finally, for $\alpha = \beta$, the coefficient is $7 - \alpha$. If $\alpha \neq 7$, it is nonzero, so z, and hence x, y, are uniquely determined. Thus (D) is true, leaving exactly one false statement.

Step 4: Conclusion Statement (B) is NOT true: on the stated line only $(-25/2, -11/4)$ gives infinitely many solutions. Hence, the correct option is (B).

Quick Tip to Solve: Eliminate the two fixed equations first; the coefficient and constant in one parameter reveal all three consistency cases.

---

6. Let R be a relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)\ \mathrm{R}\ (c, d)$ if and only if $ad(b - c) = bc(a - d)$. Then R is [4 marks]

(A) symmetric and transitive but not reflexive

(B) reflexive and symmetric but not transitive

(C) transitive but neither reflexive nor symmetric

(D) symmetric but neither reflexive nor transitive

Correct Answer: (D) symmetric but neither reflexive nor transitive

Solution:

Concept:

- A relation is reflexive when every element is related to itself, symmetric when each related pair can be reversed, and transitive when two successive links imply a direct link.
- Natural numbers here are positive. Hence division by $abcd$ is valid, and a reciprocal invariant simplifies the printed condition.

Step 1: Rewrite the defining condition Divide $ad(b - c) = bc(a - d)$ by $abcd$:

$$\frac{1}{c} - \frac{1}{b} = \frac{1}{d} - \frac{1}{a}.$$

Rearranging gives

$$\left(\frac{1}{a} - \frac{1}{b}\right) = -\left(\frac{1}{c} - \frac{1}{d}\right).$$

Define $F(a, b) = 1/a - 1/b$. The relation is precisely $F(a, b) = -F(c, d)$. This expresses the relation between ordered pairs, whose components are themselves natural numbers.

Step 2: Test symmetry and reflexivity If $F(a, b) = -F(c, d)$, then reversing the equality gives $F(c, d) = -F(a, b)$. The reversed pair is therefore also related, proving symmetry for all elements. Reflexivity would require $F(a, b) = -F(a, b)$ for every ordered pair, which means $F(a, b) = 0$ for every

pair. This fails, for example, at $(1,2)$, where $F = 1/2$. Indeed, the printed condition for $(1,2)$ related to itself reads $2 = -2$, which is false. Thus the relation is not reflexive.

Step 3: Give a counterexample to transitivity Take $P = (1,2)$ and $Q = (2,1)$. Their invariant values are $F(P) = 1/2$ and $F(Q) = -1/2$. Consequently $P\,R\,Q$ and $Q\,R\,P$, but $P\,R\,P$ is false. These two valid successive links fail to imply the required direct link, disproving transitivity. Option (A) wrongly asserts transitivity, (B) wrongly asserts reflexivity, and (C) both denies symmetry and asserts transitivity. Only (D) matches all three tests.

Step 4: Conclusion The relation is symmetric but neither reflexive nor transitive. Hence, the correct option is (D).

Quick Tip to Solve: After dividing by $abcd$, look for an invariant related to its negative; a nonzero invariant immediately supplies counterexamples.

---

7. If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\le x$, is $[2,6)$, then its range is [4 marks]

(A) $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$
(B) $\left(\frac{5}{37}, \frac{2}{5}\right]$
(C) $\left(\frac{5}{26}, \frac{2}{5}\right]$
(D) $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$
Correct Answer: (B) $\left(\frac{5}{37}, \frac{2}{5}\right]$

Solution:

Concept:

- On each interval $[n, n+1)$, the greatest-integer function has the constant value $[x] = n$. Treat these intervals separately before taking their union.
- For positive x, the function $n/(1+x^2)$ with fixed positive n is strictly decreasing. The left endpoint is included, while the limit at the excluded right endpoint is not.

Step 1: Find the four interval images The domain splits into $[2,3)$, $[3,4)$, $[4,5)$ and $[5,6)$. On these intervals the numerator is respectively 2, 3, 4 and 5. Thus their images are

$$I_2 = \left(\frac{1}{5}, \frac{2}{5}\right],$$
$$I_3 = \left(\frac{3}{17}, \frac{3}{10}\right],$$
$$I_4 = \left(\frac{2}{13}, \frac{4}{17}\right],$$
$$I_5 = \left(\frac{5}{37}, \frac{5}{26}\right].$$

For example, the first lower bound is the limit $2/(1+3^2) = 1/5$, but it is not attained within $[2,3)$. At $x = 3$, the numerator changes to 3, so endpoint handling must be done within each interval.

Step 2: Check that no gaps remain in the union The ranges overlap consecutively: $3/10 > 1/5$, $4/17 > 3/17$, and $5/26 > 2/13$. Hence I_3 overlaps I_2, I_4 overlaps I_3, and I_5 overlaps I_4. Their union is therefore one interval, rather than several pieces. Its smallest limiting value is $5/37$, approached only as $x \to 6^-$, and its maximum is $2/5$, attained at $x = 2$.

Step 3: Compare the offered ranges The lower endpoint $5/37$ is excluded because 6 is excluded and no earlier interval reaches that low. The upper endpoint $2/5$ is included. There is no need to delete isolated interior values: the overlapping images already attain every interior value in the union. Thus options (A) and (D), which delete four points, are incorrect; (C) uses $5/26$, the initial value of the last piece, as though it were the infimum.

Step 4: Conclusion The range is $\left(\frac{5}{37}, \frac{2}{5}\right]$. Hence, the correct option is (B).

Quick Tip to Solve: Compute the image on each floor-function interval, then check overlaps; jumps in the function do not necessarily create gaps in its range.

---

8. A wire of length 20 m is to be cut into two pieces. A piece of length l_1 is bent to make a square of area A_1 and the other piece of length l_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum then $(\pi l_1) : l_2$ is equal to : [4 marks]

(A) 6 : 1

(B) 1 : 6

(C) 4 : 1

(D) 3 : 1

Correct Answer: (A) 6 : 1

Solution:

Concept:

- The square has side $l_1/4$, so its area is $A_1 = l_1^2/16$. The circle has circumference l_2, giving $A_2 = l_2^2/(4\pi)$.
- The total length is fixed: $l_1 + l_2 = 20$, with both lengths nonnegative. A stationary point of a strictly convex quadratic is its unique global minimum.

Step 1: Express the quantity using one length Put $l_1 = x$, so $l_2 = 20 - x$ and $0 \le x \le 20$. The weighted area is therefore

$$F(x) = 2A_1 + 3A_2 = \frac{x^2}{8} + \frac{3(20-x)^2}{4\pi}.$$

The weights multiply areas, not perimeters; retaining these factors is essential when comparing the optimal lengths.

Step 2: Differentiate and locate the minimum Differentiating the expression gives

$$F\prime(x) = \frac{x}{4} - \frac{3(20-x)}{2\pi}.$$

At a stationary point, $\pi x = 6(20 - x)$, hence $x = 120/(\pi + 6)$. This lies strictly between 0 and 20, so both pieces have positive length. Moreover,

$$F''(x) = \frac{1}{4} + \frac{3}{2\pi} > 0.$$

Thus this stationary point is the required minimum over the allowed interval, rather than a maximum or an inadmissible endpoint.

Step 3: Evaluate the requested ratio The stationary equation directly states $\pi l_1 = 6l_2$. Therefore $(\pi l_1) : l_2 = 6 : 1$. The reversed ratio $1 : 6$ confuses the order of the lengths, while $4 : 1$ and $3 : 1$ fail the derivative equation.

Step 4: Conclusion The minimum occurs when $\pi l_1 = 6l_2$, so the required ratio is $6 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: For weighted areas with a fixed total perimeter, write both areas in terms of the perimeter before differentiating; use the stationary equation directly for the ratio.

---

9. Let $\alpha \in (0, 1)$ and $\beta = \log_e(1 - \alpha)$. Let $P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots + \frac{x^n}{n}, x \in (0, 1)$. Then the integral $\int_0^\alpha \frac{t^{50}}{1-t} dt$ is equal to [4 marks]

(A) $-(\beta + P_{50}(\alpha))$

(B) $\beta - P_{50}(\alpha)$

(C) $P_{50}(\alpha) - \beta$

(D) $\beta + P_{50} - (\alpha)$

Correct Answer: (A) $-(\beta + P_{50}(\alpha))$

Solution:

Concept:

- The finite geometric identity gives $(1 - t)(1 + t + \cdots + t^{49}) = 1 - t^{50}$. It converts the rational integrand into a logarithmic term and a finite polynomial.
- Here $0 < \alpha < 1$, so $1 - t > 0$ throughout integration. Thus the antiderivative of $1/(1 - t)$ is $-\ln(1 - t)$ with no domain ambiguity.

Step 1: Separate the remainder of the geometric sum Rearranging the finite identity, for $0 \le t \le \alpha < 1$, gives

$$\frac{t^{50}}{1-t} = \frac{1}{1-t} - \sum_{k=0}^{49} t^k.$$

This is an exact finite decomposition; no infinite series expansion or exchange of a limit and integral is required. The highest term in the polynomial is t^{49}, so integration produces the term $\alpha^{50}/50$.

Step 2: Integrate and apply the limits Integrating term by term in this finite sum,

$$\int_0^{\alpha} \frac{t^{50}}{1-t}\,dt = \left[-\ln(1-t) - \sum_{k=0}^{49} \frac{t^{k+1}}{k+1}\right]_0^{\alpha}$$
$$= -\ln(1-\alpha) - \sum_{j=1}^{50} \frac{\alpha^j}{j}.$$

Both the logarithm and every polynomial term vanish at the lower limit. Using the question's definitions, this is $-\beta - P_{50}(\alpha)$.

Step 3: Check the signs against the options The integrand is positive, so the answer must be positive. Since $-\ln(1-\alpha)$ exceeds its first fifty positive series terms, $-\beta - P_{50}(\alpha) > 0$, as required. Option (B) has a negative logarithm minus a positive sum; option (C) adds the polynomial instead of subtracting it, and (D) has the opposite overall sign.

Step 4: Conclusion The integral equals $-\big(\beta + P_{50}(\alpha)\big)$. Hence, the correct option is (A).

Quick Tip to Solve: To integrate $t^n/(1-t)$, subtract the first n terms of the finite geometric sum from $1/(1-t)$.

10. Let the shortest distance between the lines
$L : \frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}, \lambda \geq 0$ **and**
$L_1 : x+1 = y-1 = 4-z$ **be** $2\sqrt{6}$**. If** (α, β, γ) **lies on** L**,**
then which of the following is NOT possible? [4 marks]

(A) $\alpha + 2\gamma = 24$
(B) $2\alpha + \gamma = 7$
(C) $\alpha - 2\gamma = 19$
(D) $2\alpha - \gamma = 9$

Correct Answer: (A) $\alpha + 2\gamma = 24$

Solution:

Concept:

- For lines through $\vec{a}_1, \vec{a}_2$ with nonparallel directions $\vec{d}_1, \vec{d}_2$, their shortest distance is $d = |(\vec{a}_2 - \vec{a}_1) \cdot (\vec{d}_1 \times \vec{d}_2)|/|\vec{d}_1 \times \vec{d}_2|$.
- A zero denominator in a symmetric line equation means that coordinate is fixed. Parametrising the line also reveals linear combinations of coordinates that remain constant.

Step 1: Obtain points and directions Write the first line as $(x, y, z) = (5, \lambda, -\lambda) + t(-2, 0, 1)$. Write the second as $(x, y, z) = (-1, 1, 4) + s(1, 1, -1)$. Thus

$$\vec{d}_1 \times \vec{d}_2 = (-1, -1, -2), \qquad |\vec{d}_1 \times \vec{d}_2| = \sqrt{6}.$$

The displacement between the chosen points is $(-6, 1-\lambda, 4+\lambda)$.

Step 2: Determine the nonnegative parameter The scalar triple product is

$$(-6, 1-\lambda, 4+\lambda)\cdot(-1,-1,-2) = -3-\lambda.$$

The given distance therefore implies $|\lambda+3|/\sqrt{6} = 2\sqrt{6}$, or $|\lambda+3| = 12$. Since $\lambda \geq 0$, we obtain $\lambda = 9$; the other algebraic value -15 violates the stated condition.

Step 3: Test each proposed coordinate relation A point on L now has $\alpha = 5-2t$, $\beta = 9$, $\gamma = -9+t$. Hence

$$\alpha + 2\gamma = -13$$

for every real t, so it cannot be 24. For the remaining relations, $2\alpha+\gamma = 1-3t = 7$ gives $t = -2$; $\alpha - 2\gamma = 23-4t = 19$ gives $t = 1$; and $2\alpha-\gamma = 19-5t = 9$ gives $t = 2$. Each is an allowed point on the infinite line.

Step 4: Conclusion The relation $\alpha + 2\gamma = 24$ is impossible, while all other listed relations occur. Hence, the correct option is (A).

Quick Tip to Solve: After finding the line parameter, search for a linear combination that cancels the running parameter; a false constant relation is immediately impossible.

---

11. Let $\vec{a} = 2\hat{i}+\hat{j}+\hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a}+\vec{b}+\vec{c}| = |\vec{a}+\vec{b}-\vec{c}|$ and $\vec{b}\cdot\vec{c} = 0$. Consider the following two statements:

(A) $|\vec{a}+\lambda\vec{c}| \geq |\vec{a}|$ for all $\lambda \in \mathbb{R}$.

(B) $\vec{a}$ and $\vec{c}$ are always parallel.

Then, [4 marks]

(A) only (B) is correct

(B) both (A) and (B) are correct

(C) only (A) is correct

(D) neither (A) nor (B) is correct

Correct Answer: (C) only (A) is correct

Solution:

Concept:

- For any vectors $\vec{u}, \vec{v}$, the identity $|\vec{u}+\vec{v}|^2 - |\vec{u}-\vec{v}|^2 = 4\vec{u}\cdot\vec{v}$ turns equality of lengths into orthogonality.
- If two nonzero vectors are perpendicular, their squared sum obeys the Pythagorean identity. Nonzero parallel vectors instead have a nonzero scalar product.

Step 1: Extract the orthogonality condition Square the given equality; this preserves it because vector lengths are nonnegative. Taking $\vec{u} = \vec{a}+\vec{b}$ yields

$$4(\vec{a}+\vec{b})\cdot\vec{c} = 0.$$

Since $\vec{b}\cdot\vec{c} = 0$ is already given, subtracting it leaves $\vec{a}\cdot\vec{c} = 0$. Thus $\vec{a}$ and $\vec{c}$ are perpendicular, not

parallel. Notice that $|\vec{a}|^2 = 2^2 + 1^2 + 1^2 = 6$, so $\vec{a}$ is nonzero.

Step 2: Verify statement (A) for every real scalar Expand the squared length for an arbitrary $\lambda \in \mathbb{R}$:

$$\begin{aligned} |\vec{a} + \lambda\vec{c}|^2 &= |\vec{a}|^2 + 2\lambda\vec{a} \cdot \vec{c} + \lambda^2|\vec{c}|^2 \\ &= 6 + \lambda^2|\vec{c}|^2 \geq 6. \end{aligned}$$

Taking nonnegative square roots gives $|\vec{a} + \lambda\vec{c}| \geq |\vec{a}|$. This includes negative λ and $\lambda = 0$, where equality occurs, so statement (A) is true on the entire specified domain.

Step 3: Reject statement (B) and choose the combination If $\vec{c} = k\vec{a}$ for a nonzero scalar k, then $\vec{a} \cdot \vec{c} = 6k \neq 0$, contradicting the condition just proved. Both vectors are nonzero, so a zero-vector exception is unavailable. Statement (B) is false. Consequently the options asserting only (B), both statements, or neither statement all fail.

Step 4: Conclusion The first statement is true and the second is false. Hence, the correct option is (C).

Quick Tip to Solve: Equal lengths of a vector sum and difference imply a zero dot product; square first, then cancel the common squared lengths.

---

12. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to : [4 marks]

(A) 4094

(B) 2050

(C) 6144

(D) 4097

Correct Answer: (D) 4097

Solution:

Concept:

- A matrix satisfying $A^2 = A$ is idempotent, so every positive integer power A^k equals A.
- The sum of diagonal entries is the trace. Since the identity commutes with every matrix, the binomial theorem applies to $(I + A)^n$.

Step 1: Check the special matrix relation The first diagonal block of A is the scalar 1. For the lower block $B = \begin{pmatrix} 4 & -1 \\ 12 & -3 \end{pmatrix}$, direct multiplication gives

$$B^2 = \begin{pmatrix} 16 - 12 & -4 + 3 \\ 48 - 36 & -12 + 9 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 12 & -3 \end{pmatrix} = B.$$

The off-block entries are zero, so this also proves $A^2 = A$ for the full matrix. Repeated multiplication

then gives $A^k = A$ for every integer $k \geq 1$.

Step 2: Reduce the eleventh power As $AI = IA$, expand by the binomial theorem and use $A^k = A$ for every positive exponent:

$$(I + A)^{11} = I + \sum_{k=1}^{11} \binom{11}{k} A = I + (2^{11} - 1)A.$$

The coefficient is 2047: the sum of all binomial coefficients is 2^{11}, and the constant term is already retained as I.

Step 3: Take the trace Here $\mathrm{tr}(A) = 1 + 4 - 3 = 2$ and $\mathrm{tr}(I) = 3$. Linearity of trace therefore gives

$$\mathrm{tr}((I + A)^{11}) = 3 + 2047(2) = 4097.$$

The number 4094 omits the identity contribution. Neither 2050 nor 6144 equals the trace obtained from the exact power formula; treating every diagonal entry as an independent eigenvalue would also be incorrect.

Step 4: Conclusion The sum of diagonal elements of $(A + I)^{11}$ is 4097. Hence, the correct option is (D).

Quick Tip to Solve: Before multiplying a high matrix power, test $A^2 = A$ or another low-degree relation; idempotence collapses the whole binomial expansion.

---

13. For all $z \in C$ on the curve $C_1 : |z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2. Then :

[4 marks]

(A) the curves C_1 and C_2 intersect at 4 points

(B) the curve C_2 lies inside C_1

(C) the curve C_1 lies inside C_2

(D) the curves C_1 and C_2 intersect at 2 points

Correct Answer: (A) the curves C_1 and C_2 intersect at 4 points

Solution:

Concept:

- A point on $|z| = 4$ can be written $z = 4(\cos\theta + i\sin\theta)$. Its reciprocal is $(\cos\theta - i\sin\theta)/4$.
- The parametrisation $x = a\cos\theta$, $y = b\sin\theta$ describes an ellipse with semiaxes a, b. Comparing its distance from the origin with 4 counts intersections with the circle.

Step 1: Find the image curve explicitly Let $w = z + 1/z = x + iy$. Substituting the polar form of z gives

$$w = \frac{17}{4}\cos\theta + i\frac{15}{4}\sin\theta.$$

Thus the locus C_2 is the ellipse

$$\frac{x^2}{(17/4)^2} + \frac{y^2}{(15/4)^2} = 1.$$

The original curve C_1 is the circle $x^2 + y^2 = 16$, both centred at the origin.

Step 2: Solve for the intersections On the ellipse, the squared distance from the origin is

$$x^2 + y^2 = \frac{289\cos^2\theta + 225\sin^2\theta}{16} = \frac{225 + 64\cos^2\theta}{16}.$$

Equating this to 16 yields $64\cos^2\theta = 31$, so $\cos^2\theta = 31/64$. This number lies strictly between 0 and 1, giving exactly four parameter values in one full revolution, one in each quadrant. They yield four distinct points; neither coordinate vanishes. In fact, $x^2 = 8959/1024$ and $y^2 = 7425/1024$, with independent signs.

Step 3: Rule out containment and tangency The ellipse reaches $17/4 > 4$ on the horizontal axis and only $15/4 < 4$ on the vertical axis. It therefore has points both outside and inside the circle. Neither whole curve lies inside the other. The four interior-quadrant intersections also exclude the alternative claiming only two points.

Step 4: Conclusion The curves C_1 and C_2 intersect at four distinct points. Hence, the correct option is (A).

Quick Tip to Solve: For $z + 1/z$ with $|z| = r$, the image ellipse has semiaxes $r + 1/r$ and $|r - 1/r|$; compare both with the circle radius.

---

14. The value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{2+3\sin x}{\sin x(1+\cos x)}\,dx$ is equal to : [4 marks]

(A) $\frac{10}{3} - \sqrt{3} + \log_e \sqrt{3}$

(B) $\frac{7}{2} - \sqrt{3} - \log_e \sqrt{3}$

(C) $\frac{10}{3} - \sqrt{3} - \log_e \sqrt{3}$

(D) $-2 + 3\sqrt{3} + \log_e \sqrt{3}$

Correct Answer: (A) $\frac{10}{3} - \sqrt{3} + \log_e \sqrt{3}$

Solution:

Concept:

- The tangent half-angle substitution $t = \tan(x/2)$ gives $\sin x = 2t/(1+t^2)$, $\cos x = (1-t^2)/(1+t^2)$ and $dx = 2\,dt/(1+t^2)$.
- The integration interval $[\pi/3, \pi/2]$ maps to $[1/\sqrt{3}, 1]$, where $t > 0$. Thus the logarithmic antiderivative is simply $\ln t$, and there are no singularities on the interval.

Step 1: Transform the full integrand Under the substitution, $1 + \cos x = 2/(1+t^2)$, so

$$\sin x(1 + \cos x) = \frac{4t}{(1+t^2)^2}.$$

Keeping the differential factor, the complete expression becomes

$$\frac{2+3\sin x}{\sin x(1+\cos x)}\,dx = \left(2+\frac{6t}{1+t^2}\right)\frac{1+t^2}{2t}\,dt$$
$$= \left(t+\frac{1}{t}+3\right)dt.$$

The constant 3 is especially easy to lose if the numerator is simplified before the differential is included.

Step 2: Integrate with the transformed limits Therefore the required value is

$$I = \int_{1/\sqrt{3}}^{1}\left(t+\frac{1}{t}+3\right)dt$$
$$= \left[\frac{t^2}{2}+\ln t+3t\right]_{1/\sqrt{3}}^{1}$$
$$= \frac{7}{2}-\left(\frac{1}{6}-\ln\sqrt{3}+\sqrt{3}\right)$$
$$= \frac{10}{3}-\sqrt{3}+\ln\sqrt{3}.$$

The lower-limit logarithm is negative because $1/\sqrt{3}<1$; subtracting it supplies the positive logarithm in the answer.

Step 3: Compare the exact expressions Option (A) matches all three terms. Option (C) reverses the logarithm sign, (B) changes both the constant and that sign, and (D) has different algebraic terms. The integrand is positive throughout the interval, consistent with the positive value obtained.

Step 4: Conclusion The integral equals $\frac{10}{3}-\sqrt{3}+\log_e\sqrt{3}$. Hence, the correct option is (A).

Quick Tip to Solve: When both $\sin x$ and $1+\cos x$ occur in a denominator, try $t=\tan(x/2)$; transform the limits before evaluating the antiderivative.

---

15. Let $y=f(x)=\sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right)\right)\right)$. Then, at x = 1, [4 marks]

(A) $2y'+\sqrt{3}\pi^2 y=0$

(B) $y'+3\pi^2 y=0$

(C) $\sqrt{2}y'-3\pi^2 y=0$

(D) $2y'+3\pi^2 y=0$

Correct Answer: (D) $2y'+3\pi^2 y=0$

Solution:

Concept:

- For a composite function, differentiate each layer and multiply the derivatives. Here the layers are a polynomial, its power, a cosine, and the cube of a sine.
- If $y=\sin^3 v$, then $y'=3\sin^2 v\cos v\,v'$. Evaluate the inner arguments exactly before simplifying the

trigonometric values.

Step 1: Evaluate the nested arguments Write the three inner functions separately:

$$h = -4x^3 + 5x^2 + 1.$$

$$u = \frac{\pi h^{3/2}}{3\sqrt{2}}.$$

$$v = \frac{\pi}{3}\cos u.$$

Then $y = \sin^3 v$. At the required point,

$$h(1) = 2, \qquad h'(1) = -12 + 10 = -2.$$

$$u(1) = \frac{\pi}{3\sqrt{2}}(2\sqrt{2}) = \frac{2\pi}{3}, \qquad v(1) = -\frac{\pi}{6}.$$

Thus $y(1) = (-1/2)^3 = -1/8$; the negative sign must be retained.

Step 2: Differentiate from the inside out The power rule and chain rule give

$$u' = \frac{\pi}{2\sqrt{2}}\sqrt{h}\,h', \qquad u'(1) = -\pi.$$

Consequently,

$$v'(1) = -\frac{\pi}{3}\sin\frac{2\pi}{3}(-\pi) = \frac{\sqrt{3}\pi^2}{6}.$$

Apply the outer derivative:

$$y'(1) = 3\left(\frac{1}{4}\right)\left(\frac{\sqrt{3}}{2}\right)\frac{\sqrt{3}\pi^2}{6} = \frac{3\pi^2}{16}.$$

Both negative signs in the derivative of the cosine cancel.

Step 3: Test the proposed relations Since $y(1) \neq 0$, divide by it to compare the options efficiently:

$$\frac{y'(1)}{y(1)} = \frac{3\pi^2/16}{-1/8} = -\frac{3\pi^2}{2}.$$

Options A and B would require ratios $-\sqrt{3}\pi^2/2$ and $-3\pi^2$, respectively. Option C requires a positive ratio $3\pi^2/\sqrt{2}$. None equals the calculated negative ratio.

Step 4: Conclusion At $x = 1$, $2y' + 3\pi^2 y = 0$. Hence, the correct option is (D).

Quick Tip to Solve: Name each inner function and evaluate its argument at the specified point; comparing y'/y then avoids testing four full expressions.

16. Let a differentiable function f satisfy $f(x) + \int_3^x \frac{f(t)}{t}\,dt = \sqrt{x+1},\ x \geq 3$. Then $12f(8)$ is equal to: [4 marks]

(A) 19

(B) 34

(C) 17

(D) 1

Correct Answer: (C) 17

Solution:

Concept:

- The fundamental theorem of calculus gives $\frac{d}{dx}\int_3^x f(t)/t\,dt = f(x)/x$. The stated differentiability makes this operation valid for $x \geq 3$.
- A linear equation $f' + f/x = g(x)$ has integrating factor x on this positive domain, so its left-hand side becomes $(xf)'$.

Step 1: Extract the initial value and differentiate At $x = 3$, the integral has equal upper and lower limits and vanishes. Therefore $f(3) = \sqrt{4} = 2$. Differentiating the given identity yields

$$f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}.$$

Multiply by x, which is nonzero throughout the domain:

$$\frac{d}{dx}\{xf(x)\} = \frac{x}{2\sqrt{x+1}}.$$

The initial value is essential; differentiation alone would leave an undetermined constant.

Step 2: Integrate using a square-root substitution Set $u = \sqrt{x+1}$; then $x = u^2 - 1$ and $dx = 2u\,du$. Hence

$$\int \frac{x}{2\sqrt{x+1}}\,dx = \int (u^2 - 1)\,du = \frac{u^3}{3} - u.$$

It follows that

$$xf(x) = \frac{(x+1)^{3/2}}{3} - \sqrt{x+1} + C.$$

At $x = 3$, this gives $6 = 8/3 - 2 + C$, so $C = 16/3$.

Step 3: Evaluate the required multiple For $x = 8$, $\sqrt{x+1} = 3$, giving

$$8f(8) = 9 - 3 + \frac{16}{3} = \frac{34}{3}.$$

$$f(8) = \frac{17}{12}, \qquad 12f(8) = 17.$$

The value 34 is the numerator in $8f(8) = 34/3$, not the requested quantity. Neither 19 nor 1 results from applying the initial condition and the final factor 12 correctly.

Step 4: Conclusion The requested value is $12f(8) = 17$. Hence, the correct option is (C).

Quick Tip to Solve: Before differentiating an integral equation, substitute its lower limit to obtain the initial condition that determines the integration constant.

17. Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point $(3, 2)$. Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is : [4 marks]

(A) $2\left(2+\sqrt{2}\right)$

(B) $4\left(1+\sqrt{2}\right)$

(C) $3+2\sqrt{2}$

(D) $2\left(1+\sqrt{2}\right)$

Correct Answer: (B) $4\left(1+\sqrt{2}\right)$

Solution:

Concept:

- Completing squares in a circle equation gives its centre and radius. Rolling along a straight tangent translates the centre by the same distance parallel to that tangent.
- Reflection in $ax + by + c = 0$ sends (h, k) to $(h, k) - 2(ah + bk + c)(a, b)/(a^2 + b^2)$. The area of a trapezium is half the sum of its parallel sides times their perpendicular separation.

Step 1: Find the original centre and upward direction The circle can be written

$$(x-2)^2 + (y-3)^2 = 2.$$

Its centre is $O = (2, 3)$. The radius to $P = (3, 2)$ has direction $(1, -1)$, so its tangent is $T : x-y-1 = 0$. An upward unit vector along this line is $(1, 1)/\sqrt{2}$. Rolling 4 units therefore moves the centre by $(2\sqrt{2}, 2\sqrt{2})$, giving

$$A = (2 + 2\sqrt{2},\ 3 + 2\sqrt{2}).$$

The radius stays unchanged because rolling is a rigid motion.

Step 2: Reflect the translated centre For the line $x - y - 1 = 0$, the signed expression at A is

$$\begin{aligned} x_A - y_A - 1 &= (2-3) - 1 \\ &= -2. \end{aligned}$$

The identical radical terms cancel. Using the reflection formula with normal vector $(1, -1)$,

$$B = A - \frac{2(-2)}{2}(1, -1) = A + (2, -2).$$

$$B = (4 + 2\sqrt{2},\ 1 + 2\sqrt{2}).$$

As a check, the midpoint of AB lies on T, and AB is perpendicular to T.

Step 3: Compute the trapezium area Both centres are above the x-axis. Thus $AM = 3 + 2\sqrt{2}$ and $BN = 1 + 2\sqrt{2}$ are the parallel vertical sides. The perpendicular separation is the horizontal distance

$$MN = |(4 + 2\sqrt{2}) - (2 + 2\sqrt{2})| = 2.$$

Therefore

$$[AMNB] = \frac{1}{2}(4 + 4\sqrt{2}) \cdot 2 = 4(1 + \sqrt{2}).$$

The other three options are unequal to this exact area; using AB as the separation would be incorrect since it is oblique to the parallel sides.

Step 4: Conclusion The area is $4(1 + \sqrt{2})$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For a trapezium whose parallel sides are vertical, use the difference of the centres' x-coordinates as its perpendicular width.

---

18. If the maximum distance of normal to the ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1, b < 2$, from the origin is 1, then the eccentricity of the ellipse is : [4 marks]

(A) $\frac{\sqrt{3}}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\sqrt{3}}{2}$

Correct Answer: (D) $\frac{\sqrt{3}}{2}$

Solution:

Concept:

- Use the ellipse parametrisation $(a\cos t, b\sin t)$, with $a = 2 > b > 0$. The normal there has equation $ax\sin t - by\cos t = (a^2 - b^2)\sin t\cos t$.
- The distance from the origin to $Ax + By + C = 0$ is $|C|/\sqrt{A^2 + B^2}$. The eccentricity is $e = \sqrt{1 - b^2/a^2}$.

Step 1: Express the distance of a general normal For $a = 2$, the normal at $(2\cos t, b\sin t)$ is

$$2x\sin t - by\cos t = (4 - b^2)\sin t\cos t.$$

Its distance from the origin is

$$d(t) = \frac{(4 - b^2)|\sin t\cos t|}{\sqrt{4\sin^2 t + b^2\cos^2 t}}.$$

Normals at the axis vertices pass through the origin and have zero distance, so a positive maximum occurs away from those endpoints.

Step 2: Find the maximum exactly For positive a, b, expand the following difference:

$$\begin{aligned} a^2\sin^2 t + b^2\cos^2 t - (a + b)^2\sin^2 t\cos^2 t \\ = (a\sin^2 t - b\cos^2 t)^2 \geq 0. \end{aligned}$$

Hence the denominator in the distance formula is at least $(a+b)|\sin t\cos t|$, and $d \leq (a^2 - b^2)/(a+b) = a - b$. Equality is attained when $a\sin^2 t = b\cos^2 t$, which is possible since $a, b > 0$. Therefore the maximum is exactly $a - b = 2 - b$, not merely an upper bound.

Step 3: Use the given maximum and compare options The condition gives $2 - b = 1$, so $b = 1$. Substitution into the eccentricity formula yields

$$e^2 = 1 - \frac{1}{4} = \frac{3}{4}, \qquad e = \frac{\sqrt{3}}{2}.$$

Eccentricity is nonnegative. The squares of options A, B and C are $3/16$, $1/4$ and $1/2$, respectively; none equals the required $3/4$.

Step 4: Conclusion The ellipse has eccentricity $\sqrt{3}/2$. Hence, the correct option is (D).

Quick Tip to Solve: The largest distance of a normal from the centre of an ellipse is the difference of its semiaxes; establish equality as well as the bound.

---

19. (S1) $(p \Rightarrow q) \vee (p \wedge (\sim q))$ **is a tautology**

(S2) $((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$ **is a contradiction.**

Then [4 marks]

(A) only (S2) is correct

(B) both (S1) and (S2) are correct

(C) only (S1) is correct

(D) both (S1) and (S2) are wrong

Correct Answer: (C) only (S1) is correct

Solution:

Concept:

- An implication $p \Rightarrow q$ is equivalent to $(\sim p) \vee q$; it fails only when p is true and q is false.
- A tautology is true for every truth assignment. A contradiction is false for every truth assignment; one true assignment suffices to disprove a claim of contradiction.

Step 1: Simplify the first statement Put $R = (p \Rightarrow q)$. Then $R = (\sim p) \vee q$. Its negation, by De Morgan's law, is $\sim R = p \wedge (\sim q)$. Thus the expression in S1 is

$$R \vee (\sim R),$$

which is always true. More directly, whenever the implication fails, the second term is true; whenever the implication holds, the disjunction is already true. Therefore S1 correctly describes a tautology.

Step 2: Test the second statement independently The implication in S2 becomes

$$(\sim p) \Rightarrow (\sim q) \equiv p \vee (\sim q).$$

Consequently its entire expression is

$$(p \vee \sim q) \wedge (\sim p \vee q).$$

This is true when p, q are both true or both false, and false when their truth values differ. In particular,

with $p = q = \mathrm{T}$, both clauses are true. It therefore cannot be a contradiction. With $p = \mathrm{T}$, $q = \mathrm{F}$, the second clause is false, so it is not a tautology either.

Step 3: Select the matching assessment S1 is correct, while S2 is incorrect. The fact that S2 is false for some inputs does not make it a contradiction; it would need to be false for all four inputs. Option A wrongly accepts S2 and rejects S1, option B wrongly accepts S2, and option D wrongly rejects S1. Only the option asserting that S1 alone is correct survives these independent checks.

Step 4: Conclusion Only (S1) is correct. Hence, the correct option is (C).

Quick Tip to Solve: Replace implications first. To disprove a contradiction, exhibit a truth assignment that makes the expression true.

---

20. Let $\mathrm{y} = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$. Then $S = \left\{x \in \mathbb{R} : \tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x)+1}) = \frac{\pi}{2}\right\}$: [4 marks]

(A) is an empty set

(B) contains exactly one element

(C) contains exactly two elements

(D) is an infinite set

Correct Answer: (C) contains exactly two elements

Solution:

Concept:

- A point of a parabola is equidistant from its focus and directrix. Squaring the two nonnegative distances gives an equation for $f(x)$.
- Real square roots require nonnegative radicands, and the argument of the principal real $\sin^{-1}$ must lie in $[-1, 1]$. These domain conditions must hold before manipulating the equation.

Step 1: Determine the parabola For (x, y), the squared distance to the focus $(-1/2, 0)$ is $(x+1/2)^2+y^2$. The squared distance to the directrix $y = -1/2$ is $(y + 1/2)^2$. Equating them gives

$$(x + 1/2)^2 + y^2 = y^2 + y + 1/4.$$

Therefore

$$y = f(x) = x^2 + x.$$

In particular, the parabola opens upwards; its vertex is $(-1/2, -1/4)$, consistent with the supplied focus and directrix.

Step 2: Apply the simultaneous domain restrictions The expression $\sqrt{f(x)}$ is real only if $f(x) \geq 0$. Meanwhile $\sqrt{f(x)+1}$, as the argument of $\sin^{-1}$, must be real and at most 1. Thus

$$0 \leq f(x) + 1 \leq 1, \qquad -1 \leq f(x) \leq 0.$$

Intersecting this with $f(x) \geq 0$ forces $f(x) = 0$. These restrictions are simultaneous, not alternative

conditions. No positive value of $f(x)$ can satisfy the inverse-sine domain.

Step 3: Verify all admissible roots Solve $x^2 + x = x(x+1) = 0$, obtaining $x = 0$ and $x = -1$. At either root the original left-hand side is

$$\tan^{-1} 0 + \sin^{-1} 1 = 0 + \frac{\pi}{2} = \frac{\pi}{2}.$$

Both roots therefore satisfy the equation exactly, and the domain argument excludes every other real number. The set is neither empty nor a singleton, and it cannot be infinite because all its elements are among these two polynomial roots.

Step 4: Conclusion $S = \{-1, 0\}$ contains exactly two elements. Hence, the correct option is (C).

Quick Tip to Solve: Intersect the square-root and inverse-sine domains before using trigonometric identities; here they immediately force the parabola's value to zero.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The remainder on dividing 5^{99} by 11 is _____. [4 marks]

Solution:

Concept:

- Congruence modulo 11 permits replacing an integer by its remainder at every multiplication. If $a \equiv b \pmod{11}$, their nonnegative integer powers have the same remainder.
- A short cycle of powers reduces a large exponent: find a positive k with $5^k \equiv 1 \pmod{11}$, and divide the exponent by k.

Step 1: Find a usable cycle Compute the small powers while reducing modulo 11:

$$5^2 = 25 \equiv 3, \qquad 5^3 \equiv 15 \equiv 4 \pmod{11}.$$

$$5^4 \equiv 20 \equiv 9, \qquad 5^5 \equiv 45 \equiv 1 \pmod{11}.$$

Thus every complete group of five factors of 5 contributes a factor congruent to 1. We do not need to evaluate the 70-digit integer 5^{99} or perform long division on it. The cycle has returned to 1, which is enough for reducing the exponent.

Step 2: Reduce the exponent and choose the standard remainder The division algorithm gives $99 = 5 \cdot 19 + 4$. Hence

$$5^{99} = (5^5)^{19} 5^4 \equiv 1^{19} \cdot 9 \equiv 9 \pmod{11}.$$

The required remainder must be an integer from 0 through 10. Since 9 is already in this range, no further adjustment is needed. As an independent consistency check, Fermat's theorem gives $5^{10} \equiv 1$; reducing 99 modulo 10 leaves 9, and $5^9 \equiv 5^5 \cdot 5^4 \equiv 9$, giving the same result. Reducing the exponent alone without first establishing a power cycle would not be justified.

Step 3: Conclusion The remainder on division by 11 is $\boxed{9}$, a dimensionless integer.

Quick Tip to Solve: Reduce successive small powers modulo the divisor until reaching 1; then retain only the exponent's remainder modulo that cycle length.

22. Let 5 digit numbers be constructed using the digits $0, 2, 3, 4, 7, 9$ with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is _____. [4 marks]

Solution:

Concept:

- For numbers of equal length, numerical order agrees with lexicographic order: compare digits from the left until the first unequal position.
- If a digit is fixed and there are k unfilled places, repetition gives 6^k possible suffixes. A five-digit number cannot start with zero.

Step 1: Count numbers with a smaller first digit The available digits in increasing order are $0, 2, 3, 4, 7, 9$. Before the target 42923, the first digit may be either 2 or 3. Zero is excluded here because the number must have five digits. The remaining four positions are unrestricted, so this gives

$$2 \cdot 6^4 = 2592.$$

Step 2: Keep the first digit and compare the second and third With first digit 4, a second digit smaller than 2 can only be 0. These numbers contribute $6^3 = 216$. With prefix 42, a third digit smaller than 9 may be 0, 2, 3, 4 or 7. Each leaves two free positions, contributing

$$5 \cdot 6^2 = 180.$$

These groups are disjoint because their first difference from the target occurs in different positions.

Step 3: Count the remaining prefixes With prefix 429, a fourth digit smaller than 2 is only 0; there are 6 choices for its last digit. With prefix 4292, a last digit smaller than 3 is either 0 or 2, adding 2 numbers. Thus the number strictly smaller than the target is

$$2592 + 216 + 180 + 6 + 2 = 2996.$$

Step 4: Convert the count into the serial number Ascending serial numbers start at 1. The count above excludes the target itself, so its position is one more:

$$2996 + 1 = 2997.$$

Repetition creates no additional restriction on any suffix.

Step 5: Conclusion The serial number of 42923 is $\boxed{2997}$. This is a dimensionless position in the ordered list.

Quick Tip to Solve: At each digit, count smaller permitted choices times the number of unrestricted suffixes; add one only after counting all smaller numbers.

23. Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term $\beta x^{-\alpha}, \beta \in \mathbb{N}$. Then α is equal to _____. [4 marks]

Solution:

Concept:

- The general term of $(u+v)^{30}$ is $T_{r+1} = \binom{30}{r} u^{30-r} v^r$, where $r = 0, 1, \ldots, 30$.
- A term $\beta x^{-\alpha}$ with $\alpha > 0$ has a strictly negative exponent. The smallest positive α corresponds to the negative exponent closest to zero.

Step 1: Write the general term Take $u = x^{2/3}$ and $v = 2x^{-3}$, with $x \neq 0$. Choosing the second summand in exactly r factors gives

$$T_{r+1} = \binom{30}{r} 2^r x^{(2/3)(30-r)-3r}.$$

The coefficient is a positive integer for every allowed integer r, so the condition on β imposes no further restriction.

Step 2: Simplify the exponent Combine both contributions to the power of x:

$$\frac{2}{3}(30-r) - 3r = 20 - \frac{11r}{3}.$$

Hence $\alpha = 11r/3 - 20$, whenever that expression is positive. Notice that the exponent decreases strictly as r increases.

Step 3: Locate the first negative exponent The required inequality is

$$20 - \frac{11r}{3} < 0 \quad \Longleftrightarrow \quad r > \frac{60}{11}.$$

The smallest allowed integer is therefore $r = 6$. At $r = 5$, the exponent is $5/3 > 0$, so that term is not eligible. At $r = 6$, the exponent is $20 - 22 = -2$.

Step 4: Check the coefficient and minimality The corresponding term is

$$T_7 = \binom{30}{6} 2^6 x^{-2}.$$

Its coefficient is a natural number. Every larger integer r makes the exponent still more negative and therefore makes α larger. Thus this is the desired smallest positive value.

Step 5: Conclusion The smallest positive exponent parameter is $\boxed{\alpha = 2}$, a dimensionless number.

Quick Tip to Solve: Simplify the general-term exponent first, then find the first integer index making it negative; do not require every exponent to be an integer.

24. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11 , is equal to _____. [4 marks]

Solution:

Concept:

- Four-digit numbers less than or equal to 2800 form the inclusive interval $[1000, 2800]$.
- The number of multiples of d in $[L, U]$ is $\lfloor U/d \rfloor - \lfloor (L-1)/d \rfloor$. Inclusion-exclusion counts an inclusive either-or condition without double-counting.

Step 1: Count the multiples of 3 The smallest eligible multiple is $1002 = 3 \cdot 334$, and the largest is $2799 = 3 \cdot 933$. Counting both endpoints gives

$$N_3 = 933 - 334 + 1 = 600.$$

Equivalently, subtract the multiples no greater than 999 from those no greater than 2800. Numbers with fewer than four digits must not be included.

Step 2: Count the multiples of 11 The first multiple of 11 in the interval is $1001 = 11 \cdot 91$. The final one is $2794 = 11 \cdot 254$, since $11 \cdot 255 = 2805 > 2800$. Consequently,

$$N_{11} = 254 - 91 + 1 = 164.$$

These numbers include some multiples of 3 already counted in the first group.

Step 3: Count and subtract the intersection Since 3 and 11 are coprime, an integer is divisible by both exactly when it is divisible by $\text{lcm}(3, 11) = 33$. The first eligible such multiple is $1023 = 33 \cdot 31$, and the last is $2772 = 33 \cdot 84$. Thus

$$N_{33} = 84 - 31 + 1 = 54.$$

Each of these was counted once in each of the previous groups and must be subtracted once.

Step 4: Apply inclusion-exclusion The word either is used inclusively: divisibility by at least one of the two numbers is enough. The union therefore has size

$$N = N_3 + N_{11} - N_{33} = 600 + 164 - 54 = 710.$$

The endpoint 2800 itself contributes nothing because it is divisible by neither divisor.

Step 5: Conclusion There are $\boxed{710}$ such four-digit numbers. The requested count is dimensionless.

Quick Tip to Solve: For an inclusive interval, subtract the multiple count through 999, not through 1000; then remove the common multiples once.

25. Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then $12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \ldots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- In an arithmetic progression, $a_n = a + (n-1)d$, where a is the first term and d is the common difference.
- For consecutive positive terms, rationalisation gives $1/(\sqrt{a_n} + \sqrt{a_{n+1}}) = (\sqrt{a_{n+1}} - \sqrt{a_n})/d$. This turns the sum into a telescoping sum.

Step 1: Determine the arithmetic progression Using the given relation gives

$$a + 4d = 2(a + 6d) \quad \Longrightarrow \quad a = -8d.$$

The other condition is $a_{11} = a + 10d = 18$. Substitution yields $2d = 18$, so $d = 9$ and $a = -72$. Therefore

$$a_n = 9n - 81 = 9(n - 9).$$

Although the earlier terms need not be positive, every term under the displayed square roots is positive.

Step 2: Rationalise a typical denominator The sum contains the pairs (a_{10}, a_{11}) through (a_{17}, a_{18}). Since $a_{n+1} - a_n = 9$, multiplication by the conjugate gives

$$\frac{1}{\sqrt{a_n} + \sqrt{a_{n+1}}} = \frac{\sqrt{a_{n+1}} - \sqrt{a_n}}{9}.$$

The numerator order is important: it must give the positive difference of the increasing square roots.

Step 3: Cancel the intermediate square roots If S denotes the sum inside the parentheses, then

$$S = \frac{1}{9}\sum_{n=10}^{17}(\sqrt{a_{n+1}} - \sqrt{a_n}) = \frac{\sqrt{a_{18}} - \sqrt{a_{10}}}{9}.$$

Each intermediate radical appears once with each sign. Only the final positive and initial negative terms survive.

Step 4: Evaluate the surviving endpoints The formula for the progression gives $a_{10} = 9$ and $a_{18} = 81$. Thus

$$12S = \frac{12}{9}(\sqrt{81} - \sqrt{9}) = \frac{12}{9}(9 - 3) = 8.$$

The factor 12 multiplies the entire telescoped sum, rather than an individual term.

Step 5: Conclusion The value of the given expression is $\boxed{8}$, a dimensionless number.

Quick Tip to Solve: When consecutive A.P. terms occur inside a sum of square roots in the denominator, rationalise before substituting each term separately.

26. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$. Then $(\vec{a} \cdot \vec{b})^2$ is equal to _____.

[4 marks]

Solution:

Concept:

- For vectors making angle θ, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$ and $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$.
- The identity $(\vec{a} \cdot \vec{b})^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2$ follows from $\sin^2\theta + \cos^2\theta = 1$.

Step 1: Square the given magnitudes The data supply the lengths, rather than their squares. Accordingly,

$$|\vec{a}|^2 = 14, \qquad |\vec{b}|^2 = 6, \qquad |\vec{a} \times \vec{b}|^2 = 48.$$

Squaring removes the radicals exactly; no approximation is required. In particular, the cross-product contribution is 48, not $\sqrt{48}$.

Step 2: Relate the scalar and vector products Both products use the same angle between the two vectors. Squaring their definitions and adding gives

$$\begin{aligned}(\vec{a} \cdot \vec{b})^2 + |\vec{a} \times \vec{b}|^2 &= |\vec{a}|^2|\vec{b}|^2(\cos^2\theta + \sin^2\theta) \\ &= |\vec{a}|^2|\vec{b}|^2.\end{aligned}$$

Thus the required scalar-product square is determined even without knowing the vectors individually.

Step 3: Substitute and simplify Rearrange the identity to isolate the requested quantity:

$$(\vec{a} \cdot \vec{b})^2 = 14 \cdot 6 - 48 = 84 - 48 = 36.$$

The result is nonnegative, as any real square must be. The given cross-product magnitude is also admissible because its square is less than 84.

Step 4: Check why no sign choice is needed One can independently obtain $\sin^2\theta = 48/84 = 4/7$, hence $\cos^2\theta = 3/7$. Multiplying by 84 again gives 36. The dot product itself could be either $+6$ or -6, but the question asks for its square, which has the same value in both cases.

Step 5: Conclusion The required value is $\boxed{(\vec{a} \cdot \vec{b})^2 = 36}$. No physical units are specified for these vectors.

Quick Tip to Solve: When the squared dot product is requested, subtract the squared cross-product magnitude from the product of squared lengths; finding the angle is unnecessary.

27. Let for $x \in \mathbb{R}$,

$$f(x) = \frac{x + |x|}{2} \text{ and } g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}.$$

Then area bounded by the curve $y = (f \circ g)(x)$ and the lines $y = 0, 2y - x = 15$ is equal to _____.

[4 marks]

Solution:

Concept:

- The function $f(t) = (t + |t|)/2$ equals zero for $t < 0$ and equals t for $t \geq 0$. Composition requires applying this rule to $t = g(x)$.
- The area of a bounded region is found by integrating upper boundary minus lower boundary, splitting the interval wherever the lower or upper boundary changes.

Step 1: Find the composed curve When $x < 0$, $g(x) = x < 0$, so $f(g(x)) = 0$. When $x \geq 0$, $g(x) = x^2 \geq 0$, so $f(g(x)) = x^2$. Thus the curve coincides with the negative part of the x-axis and follows the right half of the parabola for nonnegative x.

Step 2: Locate the boundary intersections The line is $y = (x + 15)/2$, meeting $y = 0$ at $x = -15$. Its intersection with the nonnegative parabola satisfies

$$2x^2 - x - 15 = (2x + 5)(x - 3) = 0.$$

Only $x = 3$ belongs to the parabolic branch. The root $-5/2$ is not on that branch of the composed curve. The other junction is the origin.

Step 3: Split the enclosed area For $-15 \leq x \leq 0$, the lower boundary is the axis. For $0 \leq x \leq 3$, it is the parabola. In both intervals the line is above it. Therefore

$$A = \int_{-15}^{0} \frac{x + 15}{2}\, dx + \int_{0}^{3} \left(\frac{x + 15}{2} - x^2 \right) dx.$$

The piecewise definition prevents using the parabola on the negative interval.

Step 4: Evaluate the two integrals The first integral is $225/4$. The second is

$$\left[\frac{x^2}{4} + \frac{15x}{2} - \frac{x^3}{3} \right]_0^3 = \frac{9}{4} + \frac{45}{2} - 9 = \frac{63}{4}.$$

Adding gives $A = (225 + 63)/4 = 72$. Equivalently, subtract the area $\int_0^3 x^2 dx = 9$ from the triangle of base 18 and height 9.

Step 5: Conclusion The bounded area is $\boxed{72 \text{ square units}}$.

Quick Tip to Solve: Form the piecewise composition before solving intersections; a root of the full parabola may lie outside the branch actually present.

28. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3, then α is equal to _____. [4 marks]

Solution:

Concept:

- For total frequency N, the mean is $\bar{x} = \sum f_i x_i / N$, and the variance is $\sigma^2 = \sum f_i x_i^2 / N - \bar{x}^2$.
- A frequency is nonnegative. Centering the observations at a convenient value can simplify the weighted sums without changing their variance.

Step 1: Calculate the total frequency and first moment Keeping the unknown frequency separate,

$$N = 3 + 6 + 16 + \alpha + 9 + 5 + 6 = 45 + \alpha.$$

The known weighted sum is $2(3) + 3(6) + 4(16) + 6(9) + 7(5) + 8(6) = 225$. Including the observation 5 gives

$$\sum f_i x_i = 225 + 5\alpha = 5(45 + \alpha).$$

Since $N > 0$, the mean is exactly $\bar{x} = 5$, independently of α.

Step 2: Use deviations from the mean The deviations of $2, 3, 4, 5, 6, 7, 8$ from 5 are $-3, -2, -1, 0, 1, 2, 3$. Their weighted squared deviations are

$$3(9) + 6(4) + 16(1) + \alpha(0) + 9(1) + 5(4) + 6(9) = 150.$$

The unknown frequency contributes nothing to this numerator because it occurs at the mean, but it still contributes to the denominator.

Step 3: Impose the stated variance The given variance 3 is the population variance of this frequency distribution, so divide by the total frequency, not by one less than the total:

$$3 = \frac{150}{45 + \alpha}.$$

Hence $45 + \alpha = 50$, giving $\alpha = 5$. This satisfies the requirement that a frequency be a nonnegative integer.

Step 4: Check using the raw second moment The known weighted square sum is 1275. With $\alpha = 5$,

$$\frac{\sum f_i x_i^2}{N} = \frac{1275 + 25(5)}{50} = 28.$$

Subtracting $\bar{x}^2 = 25$ gives variance $28 - 25 = 3$, confirming the required value with the original data.

Step 5: Conclusion The unknown frequency is $\boxed{\alpha = 5}$. A frequency is a dimensionless count.

Quick Tip to Solve: If the unknown frequency is attached to the mean, it changes only the denominator of the centered variance formula, making the equation especially simple.

29. Let the line $L : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1, -1, -3)$ on the line L. If α is the area of triangle PQR, then α^2 is equal to _____. [4 marks]

Solution:

Concept:

- A point on the line is $X(t) = (1+2t, -1-t, 3+t)$, with direction vector $\mathbf{d} = (2, -1, 1)$. Substitution into the plane gives its intersection with the line.
- At the perpendicular foot Q, the vector $R - Q$ is perpendicular to $\mathbf{d}$. A right triangle with perpendicular sides PQ and RQ has area $\alpha = \frac{1}{2} PQ\, RQ$.

Step 1: Locate the intersection point Substitute the coordinates of a general point on the line into the given plane. This uses all three coordinates with the same parameter, so the resulting point lies on both objects:

$$2(1 + 2t) + (-1 - t) + 3(3 + t) = 16.$$

Thus $10 + 6t = 16$, giving $t = 1$ and $P = (3, -2, 4)$.

Step 2: Find the perpendicular foot Write $Q = (1 + 2s, -1 - s, 3 + s)$. Since $R = (1, -1, -3)$,

$$R - Q = (-2s, s, -6 - s).$$

Its scalar product with the line direction must be zero:

$$-4s - s - 6 - s = 0 \quad \Rightarrow \quad s = -1.$$

Hence $Q = (-1, 0, 2)$. The parameter is not the parameter of P; these are different points on the same line.

Step 3: Calculate the squared area The required squared lengths can be found without introducing square roots:

$$PQ^2 = 4^2 + (-2)^2 + 2^2 = 24, \qquad RQ^2 = 2^2 + (-1)^2 + (-5)^2 = 30.$$

As a check, $(P - Q) \cdot (R - Q) = 8 + 2 - 10 = 0$. Thus the right angle really is at Q, and

$$\alpha^2 = \frac{1}{4} PQ^2 RQ^2 = \frac{1}{4}(24)(30) = 180.$$

Step 4: Conclusion The required numerical value is $\boxed{\alpha^2 = 180}$, in fourth powers of the coordinate length unit.

Quick Tip to Solve: When an area squared is requested for a right triangle, multiply the squared perpendicular side lengths and divide by four; no square roots are needed.

30. Let θ be the angle between the planes $P_1 : \vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 9$ and $P_2 : \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 15$. Let L be the line that meets P_2 at the point $(4, -2, 5)$ and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $\left(\tan^2\theta\right)\left(\cot^2\alpha\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- The angle between two planes is the acute angle between their normal vectors: $\cos\theta = |\mathbf{n}_1 \cdot \mathbf{n}_2|/(|\mathbf{n}_1||\mathbf{n}_2|)$.
- The acute angle α between a line and a plane is complementary to the angle between that line and the normal to the plane. Therefore $\alpha + \theta = 90^\circ$ here.

Step 1: Extract and compare the normals The vector coefficients in the plane equations give $\mathbf{n}_1 = (1, 1, 2)$ and $\mathbf{n}_2 = (2, -1, 1)$. Their scalar product is

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = 2 - 1 + 2 = 3,$$

while $|\mathbf{n}_1| = |\mathbf{n}_2| = \sqrt{6}$. The constants 9 and 15 locate the planes but do not change the angle between their normals. Therefore

$$\cos\theta = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{1}{2}.$$

Step 2: Relate the line angle to the plane angle The angle between the planes is $\theta = 60^\circ$. The given line makes this angle with the normal of P_2. A normal is perpendicular to the plane, so the angle of the line with the plane itself is

$$\alpha = 90^\circ - 60^\circ = 30^\circ.$$

The point $(4, -2, 5)$ fixes a point of incidence; indeed $2(4) - (-2) + 5 = 15$. Its coordinates are not needed to determine this angular relationship.

Step 3: Evaluate the requested product Use the angles in the order specified. We have $\tan^2 60^\circ = 3$ and $\cot^2 30^\circ = 3$, so

$$(\tan^2\theta)(\cot^2\alpha) = 3 \times 3 = 9.$$

Equivalently, $\cot\alpha = \tan\theta$, making the requested product $\tan^4\theta$. This also checks that a reciprocal has not been introduced inadvertently.

Step 4: Conclusion The dimensionless numerical answer is $\boxed{9}$.

Quick Tip to Solve: First complement the angle with the normal to obtain the angle with the plane. Here this turns the product directly into the fourth power of the tangent.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The initial speed of a projectile fired from ground is u. At the highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}u$. The time of flight of the projectile is : [4 marks]

(A) $\frac{u}{g}$

(B) $\frac{2u}{g}$

(C) $\frac{u}{2g}$

(D) $\frac{\sqrt{3}u}{g}$

Correct Answer: (A) $\frac{u}{g}$

Solution:

Concept:

- In ideal projectile motion the horizontal velocity $u\cos\theta$ stays constant, while the vertical velocity becomes zero at the highest point.
- For launch and landing at the same height, the time of flight is $T = 2u\sin\theta/g$, where g is the magnitude of gravitational acceleration.

Step 1: Use the speed at the highest point At the top, only the horizontal component of velocity remains. Therefore the stated speed gives

$$u\cos\theta = \frac{\sqrt{3}}{2}u \quad \Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{2}.$$

For an upward launch, $0 < \theta < 90^\circ$, so $\theta = 30^\circ$ and $u\sin\theta = u/2$. The speed at the top is not zero: only the vertical component vanishes there.

Step 2: Find the complete time in the air Use vertical displacement measured from the launch point. At landing,

$$0 = (u\sin\theta)T - \frac{1}{2}gT^2.$$

Apart from the initial instant $T = 0$, this yields

$$T = \frac{2u\sin\theta}{g} = \frac{2u(1/2)}{g} = \frac{u}{g}.$$

The ascent time alone is $u/(2g)$, so it must be doubled to include the descent.

Step 3: Compare all four expressions Option (A) gives exactly the calculated total time. Option (B), $2u/g$, would correspond to vertical launch with initial vertical speed u, inconsistent with the nonzero speed at the top. Option (C) gives only the ascent time. Option (D) incorrectly uses a quantity proportional to the horizontal component instead of the vertical component in the flight-time formula. The dimensions u/g are those of time, as required.

Step 4: Conclusion The time of flight is $\boxed{T = u/g}$, in seconds when SI units are used. Hence, the correct option is (A).

Quick Tip to Solve: At maximum height, read the surviving horizontal component from the given speed, then obtain the vertical component before calculating flight time.

32. A bar magnet with a magnetic moment 5.0Am^2 is placed in parallel position relative to a magnetic field of 0.4 T. The amount of required work done in turning the magnet from parallel to antiparallel position relative to the field direction is _____. [4 marks]

(A) zero

(B) 1 J

(C) 2 J

(D) 4 J

Correct Answer: (D) 4 J

Solution:

Concept:

- A magnetic dipole of magnetic moment m in a uniform field B has potential energy $U = -mB\cos\theta$, where θ is the angle between its moment and the field.
- The external work required for a slow rotation equals the change in potential energy: $W_{\text{ext}} = U_f - U_i$. The work of the magnetic field has the opposite sign.

Step 1: Evaluate the initial and final energies In the parallel orientation, $\theta_i = 0$, and hence

$$U_i = -mB = -5.0(0.4) = -2.0\,\text{J}.$$

In the antiparallel orientation, $\theta_f = 180^\circ$, so

$$U_f = -mB\cos 180^\circ = +mB = +2.0\,\text{J}.$$

Parallel alignment is the minimum-energy state, while antiparallel alignment is the maximum-energy state.

Step 2: Calculate the external work The magnet must be turned against the tendency of the magnetic torque to restore parallel alignment. Thus the required external work is positive:

$$W_{\text{ext}} = U_f - U_i = 2.0 - (-2.0) = 4.0\,\text{J}.$$

Equivalently, integrate the resisting torque $mB\sin\theta$ from zero to π; its integral is $2mB$, again giving 4.0 J.

Step 3: Eliminate the other options Zero work would imply no change in potential energy, whereas the two end orientations have different energies even though the torque vanishes at both ends. The value 1 J has no basis in this energy difference. The value $2\,\text{J} = mB$ is the work from parallel to perpendicular orientation only. A complete half-turn requires twice that value, which is option (D).

Step 4: Conclusion The required work is $\boxed{4\,\text{J}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a full reversal of a dipole, remember that its energy changes from minus mB to plus mB, so the external work is twice mB.

33. Spherical insulating ball and a spherical metallic ball of same size and mass are dropped from the same height. Choose the correct statement out of the following

{Assume negligible air friction} [4 marks]

(A) Metal ball will reach the earth's surface earlier than the insulating ball

(B) Both will reach the earth's surface simultaneously.

(C) Insulating ball will reach the earth's surface earlier than the metal ball

(D) Time taken by them to reach the earth's surface will be independent of the properties of their materials

Correct Answer: (C) Insulating ball will reach the earth's surface earlier than the metal ball

Solution:

Concept:

- A changing magnetic flux through a conducting body can induce closed eddy currents. By Lenz's law, their magnetic effect opposes the change that produces them.
- During a fall through the Earth's nonuniform magnetic field, magnetic braking can act on a conducting metallic ball. An insulating ball cannot support comparable circulating conduction currents.

Step 1: Identify the difference between the materials Both balls have the same mass and size and experience the same downward gravitational force. Negligible air friction removes aerodynamic resistance from consideration. The intended distinction is their electrical conductivity: a metal permits circulating induced currents, whereas an insulator does not. As the metal falls through the spatially varying terrestrial magnetic field, the magnetic flux linked with possible current loops changes.

Step 2: Apply Lenz's law to the falling metal The induced eddy currents oppose the flux change associated with the motion. Their effect is magnetic braking: part of the gravitational potential energy becomes Joule heat in the metal rather than kinetic energy. In symbols, the downward equation can be expressed as

$$ma = mg - F_{\text{mag}},$$

where F_{mag} is the retarding magnetic force. For the insulating ball this eddy-current contribution is absent, so its descent is faster in this comparison.

Step 3: Select the statement consistent with braking Option (C) states that the insulating ball reaches the surface earlier, matching the eddy-current argument. Option (A) reverses the direction of magnetic braking. Options (B) and (D) describe the ideal gravity-only model, in which both balls have acceleration g; they omit the conducting ball's interaction with the terrestrial field. The predicted difference is very small, but negligible air friction alone does not remove magnetic induction.

Step 4: Conclusion The insulating ball reaches the Earth's surface earlier because the metallic ball is retarded by eddy-current effects. Hence, the correct option is (C).

Quick Tip to Solve: When a falling-conductor comparison appears under electromagnetic induction, check for eddy-current energy loss as well as gravitational acceleration.

34. The effect of increase in temperature on the number of electrons in conduction band (n_e) and resistance of a semiconductor will be as: [4 marks]

(A) n_e decreases, resistance increases

(B) Both n_e and resistance increase

(C) n_e increases, resistance decreases

(D) Both n_e and resistance decrease

Correct Answer: (C) n_e increases, resistance decreases

Solution:

Concept:

- Thermal excitation promotes valence-band electrons across the forbidden energy gap into the conduction band of a semiconductor, creating electron-hole pairs.
- Conductivity is $\sigma = e(n_e\mu_e + n_h\mu_h)$, where the carrier densities are n_e, n_h and the corresponding mobilities are μ_e, μ_h. Resistance is inversely related to conductivity for fixed geometry.

Step 1: Determine the carrier-density change Increasing temperature supplies more thermal energy to the covalent-bond electrons. More electrons can cross the relatively small band gap, leaving holes behind in the valence band. Thus the number of conduction-band electrons increases. For an intrinsic semiconductor, the trend is reflected in

$$n_i \propto T^{3/2} \exp\left(-\frac{E_g}{2k_B T}\right),$$

with E_g the energy gap. Raising T strongly increases the thermally generated carrier density.

Step 2: Connect carrier density to resistance Both the promoted electrons and the holes contribute to current. In the usual semiconductor temperature range tested here, the increase in carrier concentration dominates the mobility reduction caused by extra lattice scattering. Therefore conductivity increases and resistivity decreases. Since

$$R = \rho\frac{\ell}{A} = \frac{\ell}{\sigma A},$$

a sample of fixed length ℓ and cross-sectional area A has decreasing resistance.

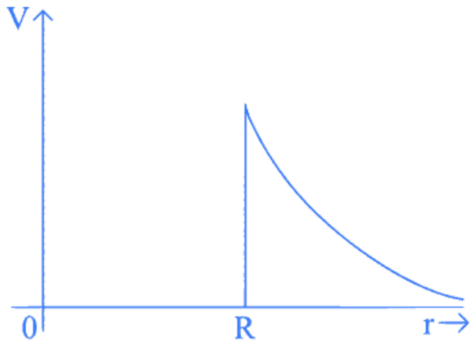
Step 3: Check the signs in every option Option (C) combines increased conduction-band electron number with decreased resistance and is therefore correct. Options (A) and (D) incorrectly predict fewer conduction electrons on heating. Option (B) has the correct carrier-density change but the wrong resistance change. Applying the usual positive temperature coefficient of a metal would miss the strong thermally activated carrier generation in a semiconductor.

Step 4: Conclusion On increasing temperature, $\boxed{n_e \text{ increases and resistance decreases}}$. Hence, the correct option is (C).

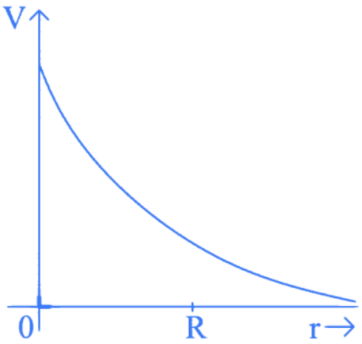
Quick Tip to Solve: For a semiconductor, heating creates many additional mobile carriers; this is why its resistance trend differs from the familiar metallic-wire trend.

35. Which of the following correctly represents the variation of electric potential (V) of a charged spherical conductor of radius (R) with radial distance (r) from the center? [4 marks]

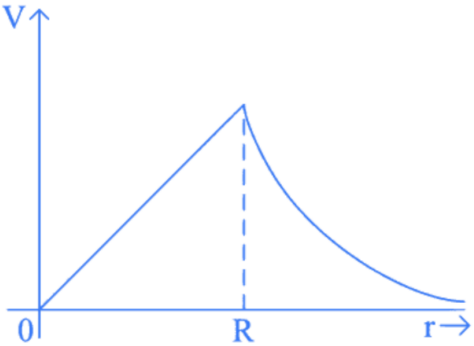
(A)



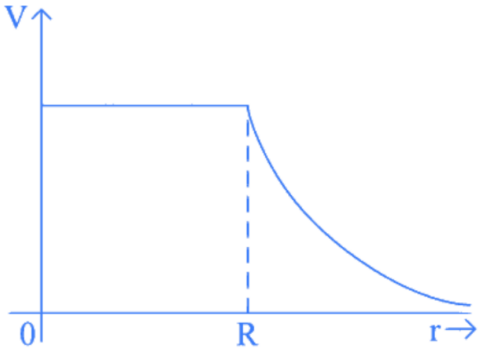
(B)



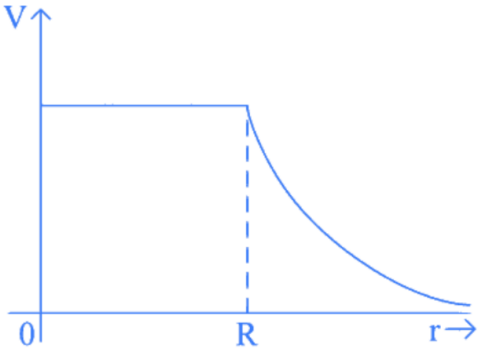
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- In electrostatic equilibrium the electric field inside a conductor is zero, so its entire interior and surface have the same potential.
- With zero potential at infinity, a spherical conductor carrying charge Q has $V = kQ/R$ for $0 \le r \le R$, and $V = kQ/r$ for $r \ge R$, where $k = 1/(4\pi\varepsilon_0)$.

Step 1: Find the interior potential All excess charge resides on the surface of the conducting sphere. Inside the material and its spherical interior, $E = 0$. Since $E_r = -dV/dr$, the potential has zero radial derivative there. It is therefore constant, rather than zero or increasing with radius. Continuity at the surface fixes this constant to the surface value

$$V(R) = \frac{kQ}{R}.$$

Thus the graph begins with a horizontal segment from the centre to $r = R$.

Step 2: Find the exterior variation Outside a spherically symmetric charged conductor, the field and potential are those of a point charge at the centre. Hence

$$V(r) = \frac{kQ}{r} \qquad (r \ge R).$$

For the positive potential shown in the options this falls smoothly towards zero at infinity. At $r = R$, both pieces give kQ/R, so the potential has no jump. Its slope changes at the charged surface because the electric field changes there.

Step 3: Match the complete graph Option (D) shows exactly a constant positive potential up to the surface, followed by an inverse-distance decrease. Option (A) incorrectly has zero potential inside and a discontinuity at the surface. Option (B) incorrectly decreases throughout the interior. Option (C) incorrectly rises linearly from the centre to the surface. These interior behaviours all conflict with the zero electric field of a conductor in electrostatic equilibrium.

Step 4: Conclusion The potential is constant inside and decreases as $1/r$ outside, continuously at $r = R$. Hence, the correct option is (D).

Quick Tip to Solve: Use the interior field first: zero electric field means a flat potential graph, while the exterior of a spherical conductor gives the inverse-radius curve.

36. The drift velocity of electrons for a conductor connected in an electrical circuit is $\mathrm{V_d}$. The conductor in now replaced by another conductor with same material and same length but double the area of cross section. The applied voltage remains same. The new drift velocity of electrons will be [4 marks]

(A) $\frac{V_d}{4}$

(B) $\mathrm{V_d}$

(C) $2\ \mathrm{V_d}$

(D) $\frac{V_d}{2}$

Correct Answer: (B) $\mathrm{V_d}$

Solution:

Concept:

- Drift velocity is the average velocity acquired by conduction electrons in an electric field. Its magnitude is $v_d = eE\tau/m_e$, where τ is the relaxation time.
- For a uniform conductor of length L, $E = V/L$. Also, $I = neAv_d$, with n the electron number density and A the cross-sectional area.

Step 1: Identify the quantities that remain fixed The replacement has the same material, so its electron number density and relaxation time are unchanged under the stated conditions. The applied voltage and the length are also unchanged. Thus the electric field driving the electrons remains the same:

$$E' = \frac{V}{L} = E.$$

Increasing the area provides more conducting electrons in parallel; it does not increase the field along the wire.

Step 2: Compare the drift velocities Substituting the unchanged field into the microscopic drift relation gives

$$v_d' = \frac{e\tau}{m_e}\frac{V}{L} = V_d.$$

As a check, the resistance is $R = \rho L/A$, so doubling A halves the resistance and doubles the current. However,

$$v_d' = \frac{2I}{ne(2A)} = \frac{I}{neA} = V_d.$$

Hence doubling the current does not double the drift velocity here. The values $V_d/4$ and $V_d/2$ would require a reduction in field or a different constraint, while $2V_d$ incorrectly treats the area as fixed when using the doubled current.

Step 3: Conclusion The new drift velocity has the same magnitude, $\boxed{v_d' = V_d}$. Hence, the correct option is (B).

Quick Tip to Solve: With voltage and length fixed, compare V/L first. A change in area changes the current but leaves the drift speed unchanged.

37. A rod with circular cross-section area $2\ \text{cm}^2$ and length $40\ \text{cm}$ is wound uniformly with 400 turns of an insulated wire. If a current of $0.4\ \text{A}$ flows in the wire windings, the total magnetic flux produced inside windings is $4\pi \times 10^{-6}\ \text{Wb}$. The relative permeability of the rod is

(Given : Permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}\text{NA}^{-2}$) [4 marks]

(A) $\frac{5}{16}$

(B) 125

(C) $\frac{32}{5}$

(D) 12.5

Correct Answer: (A) $\frac{5}{16}$

Solution:

Concept:

- The magnetic field inside a uniformly wound rod is $B = \mu_0\mu_r NI/\ell$, where N is the number of turns and ℓ is its length.
- The flux through one turn is BA. The total flux linked with all windings is $\Phi_{\text{total}} = NBA = \mu_0\mu_r N^2 IA/\ell$. Relative permeability μ_r is dimensionless.

Step 1: Convert the data and identify the total flux The area and length in SI units are

$$A = 2 \times 10^{-4}\,\text{m}^2, \qquad \ell = 0.40\,\text{m}.$$

There are $N = 400$ turns carrying $I = 0.4\,\text{A}$. The stated total flux inside the windings is the sum of the fluxes linked with all turns, so it is NBA, rather than just BA for a single turn. Its given value is $4\pi \times 10^{-6}\,\text{Wb}$.

Step 2: Calculate the relative permeability Rearranging the total-linkage relation and substituting all quantities gives

$$\mu_r = \frac{\Phi_{\text{total}}\ell}{\mu_0 N^2 IA} = \frac{(4\pi \times 10^{-6})(0.40)}{(4\pi \times 10^{-7})(400)^2(0.4)(2 \times 10^{-4})}.$$

Cancelling the common factors carefully,

$$\mu_r = \frac{4}{12.8} = 0.3125 = \frac{5}{16}.$$

Using the given flux as a single-turn flux would instead give 125, which is option (B); that overlooks the word total and the 400 linked turns. Neither $32/5$ nor 12.5 satisfies the total-flux equation with the supplied data.

Step 3: Conclusion The relative permeability of the rod is $\boxed{\mu_r = 5/16}$, with no unit. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish flux per turn BA from total flux linkage NBA. Including the number of turns twice is essential in the total-linkage formula.

38. At a certain depth "d " below surface of earth, value of acceleration due to gravity becomes four times that of its value at a height $3\mathrm{R}$ above earth surface. Where R is Radius of earth (Take $\mathrm{R} = 6400\ \mathrm{km}$). The depth d is equal to [4 marks]

(A) 5260 km

(B) 2560 km

(C) 640 km

(D) 4800 km

Correct Answer: (D) 4800 km

Solution:

Concept:

- At height h above the Earth, $g_h = g_0[R/(R+h)]^2$, where g_0 is the surface gravitational acceleration.
- Using the standard uniform-density Earth model, at depth d, $g_d = g_0(1 - d/R)$. The height is measured from the surface, whereas $R + h$ is measured from the centre.

Step 1: Calculate gravity at the specified height The stated height is $h = 3R$, so the distance from the centre is $R + 3R = 4R$. Therefore

$$g_h = g_0\left(\frac{R}{4R}\right)^2 = \frac{g_0}{16}.$$

The factor 16 comes from squaring the centre-distance ratio. Using $3R$ itself as the centre distance would not represent a height of $3R$ above the surface.

Step 2: Apply the condition at depth The question requires gravity at depth to be four times this value:

$$g_d = 4g_h = \frac{g_0}{4}.$$

Combining this with the depth relation,

$$g_0\left(1 - \frac{d}{R}\right) = \frac{g_0}{4} \quad \Rightarrow \quad \frac{d}{R} = \frac{3}{4}.$$

Substituting the supplied Earth radius,

$$d = \frac{3}{4}(6400\,\mathrm{km}) = 4800\,\mathrm{km}.$$

This leaves the position at radius $R - d = 1600\,\mathrm{km}$, one quarter of the Earth radius, which gives the required $g_0/4$. The other proposed depths do not satisfy $1 - d/R = 1/4$.

Step 3: Conclusion The depth below the surface is $\boxed{4800\,\mathrm{km}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a height quoted above the surface, first add R. At depth, gravity varies with the remaining radius $R - d$, not inversely with its square.

39. Two polaroide A and B are placed in such a way that the pass-axis of polaroids are perpendicular to each other. Now, another polaroid C is placed between A and B bisecting angle between them. If intensity of unpolarized light is I_0 then intensity of transmitted light after passing through polaroid B will be: [4 marks]

(A) $\frac{I_0}{4}$

(B) $\frac{I_0}{8}$

(C) Zero

(D) $\frac{I_0}{2}$

Correct Answer: (B) $\frac{I_0}{8}$

Solution:

Concept:

- An ideal first polaroid transmits half the intensity of unpolarized incident light, producing linearly polarized light.
- Malus' law gives $I_{\text{out}} = I_{\text{in}} \cos^2\theta$ for polarized light passing through a polaroid whose transmission axis makes angle θ with its polarization.

Step 1: Find the intensity after the first and middle polaroids The order of transmission is A, C, B. Initially the light is unpolarized with intensity I_0, so after A its intensity is

$$I_A = \frac{I_0}{2}.$$

The axes of A and B differ by 90°, and C bisects that angle. Thus C is at 45° to A. Applying Malus' law,

$$I_C = I_A \cos^2 45^\circ = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}.$$

The transmitted light is now polarized along C, rather than along A.

Step 2: Pass the light through the last polaroid The angle between C and B is also 45°. Thus a second application of Malus' law gives

$$I_B = I_C \cos^2 45^\circ = \frac{I_0}{4} \times \frac{1}{2} = \frac{I_0}{8}.$$

Zero intensity would occur for the crossed pair A and B without the middle polaroid, but C changes the polarization direction before the light reaches B. The value $I_0/4$ is only the intensity after C, and $I_0/2$ is only the intensity after A; neither includes all three transmissions.

Step 3: Conclusion The final transmitted intensity is $\boxed{I_B = I_0/8}$. Hence, the correct option is (B).

Quick Tip to Solve: Follow adjacent axes in their physical order. After each polaroid, the polarization direction becomes that polaroid's axis.

40. 100 balls each of mass m moving with speed v simultaneously strike a wall normally and reflected back with same speed, in time t s. The total force exerted by the balls on the wall is [4 marks]

(A) $\frac{200mv}{t}$

(B) $\frac{100mv}{t}$

(C) $\frac{mv}{100t}$

(D) $200mvt$

Correct Answer: (A) $\frac{200mv}{t}$

Solution:

Concept:

- Average force equals the total change of momentum divided by the interaction time: $F_{\text{avg}} = \Delta p/\Delta t$.
- Momentum is a vector. Reflection at unchanged speed reverses the normal momentum, so the magnitude of its change is twice the incident momentum. The wall receives an equal and opposite impulse.

Step 1: Find the momentum change for one ball Take the direction towards the wall as positive. A ball has initial momentum $p_i = mv$ and, after reflection, final momentum $p_f = -mv$. Its momentum change is therefore

$$\Delta p_{\text{ball}} = p_f - p_i = -mv - mv = -2mv.$$

Although its speed and kinetic energy are unchanged, its velocity and momentum are not. The negative sign shows that the impulse on the ball is away from the wall. By Newton's third law, the impulse delivered to the wall is $+2mv$.

Step 2: Add the impulses and divide by time All 100 balls strike during the same stated interval t, so their impulses add:

$$J_{\text{wall}} = 100(2mv) = 200mv.$$

Consequently the magnitude of the total average force on the wall is

$$F_{\text{wall}} = \frac{J_{\text{wall}}}{t} = \frac{200mv}{t}.$$

Option (B) omits the factor two from reversal. Option (C) incorrectly divides by the number of balls, although their forces add. Option (D) multiplies impulse by time and has units $\mathrm{kg\,m}$, rather than the force unit $\mathrm{kg\,m\,s^{-2}}$.

Step 3: Conclusion The total average force exerted on the wall has magnitude $\boxed{200mv/t}$, directed towards the wall. Hence, the correct option is (A).

Quick Tip to Solve: In an elastic normal rebound, use $2mv$ per ball. Add all impulses before dividing by the common collision interval.

41. If $\mathrm{R}, \mathrm{X_L}$, and $\mathrm{X_C}$ represent resistance, inductive reactance and capacitive reactance. Then which of the following is dimensionless : [4 marks]

(A) $\frac{R}{X_L X_C}$

(B) $RX_L X_C$

(C) $\frac{R}{\sqrt{X_L X_C}}$

(D) $R\frac{X_L}{X_C}$

Correct Answer: (C) $\frac{R}{\sqrt{X_L X_C}}$

Solution:

Concept:

- Resistance and reactance both relate an applied voltage to a current, so $[R] = [X_L] = [X_C] = [\Omega]$.
- A quantity is dimensionless only when every base-dimension exponent cancels. Products add exponents, ratios subtract them, and square roots halve them.

Step 1: Establish the common dimensions Electrical resistance has dimensions

$$D = [R] = \frac{[V]}{[I]} = \mathrm{ML^2T^{-3}I^{-2}}.$$

Here I denotes the electric-current dimension. For inductive reactance $X_L = \omega L$ and capacitive reactance $X_C = 1/(\omega C)$, their respective voltage-current relations also give this same dimension D. Their values can differ, but dimensions do not depend on those numerical values or on the circuit frequency.

Step 2: Test every expression Replace each resistance or reactance by its dimension D:

$$\begin{aligned}
[R/(X_L X_C)] &= D/D^2 = D^{-1}, \\
[RX_L X_C] &= D^3, \\
\left[R/\sqrt{X_L X_C}\right] &= D/\sqrt{D^2} = 1, \\
[RX_L/X_C] &= D(D/D) = D.
\end{aligned}$$

Thus only the third expression cancels all powers of mass, length, time and current. In particular, X_L/X_C alone is dimensionless, but multiplying it by R restores the resistance dimension, so option (D) is not dimensionless. Options (A) and (B) retain inverse-resistance and cubic-resistance dimensions respectively.

Step 3: Conclusion The dimensionless expression is $\boxed{R/\sqrt{X_L X_C}}$. Hence, the correct option is (C).

Quick Tip to Solve: When all quantities have the same dimensions, replace each by a single symbol and count its total exponent; dimensionless means exponent zero.

42. The maximum potential energy of a block executing simple harmonic motion is 25 J. A is amplitude of oscillation. At $A/2$, the kinetic energy of the block is [4 marks]

(A) 9.75 J

(B) 37.5 J

(C) 18.75 J

(D) 12.5 J

Correct Answer: (C) 18.75 J

Solution:

Concept:

- For simple harmonic motion, $U(x) = \frac{1}{2}kx^2$, where x is displacement from equilibrium and k is the effective force constant.
- The total mechanical energy is $E = \frac{1}{2}kA^2 = U_{\max}$. At any displacement, conservation of energy gives $K = E - U(x)$.

Step 1: Relate the given maximum energy to total energy At either extreme position, $x = \pm A$, the block momentarily stops. Its kinetic energy is zero there, so its potential energy is the entire mechanical energy. Hence

$$E = U_{\max} = 25\,\text{J}.$$

The given amplitude need not be assigned a numerical value: ratios of displacement to amplitude determine how the fixed total energy is divided between potential and kinetic energy.

Step 2: Evaluate the energy at half the amplitude At the stated displacement $x = A/2$, potential energy depends on the square of displacement:

$$\frac{U(A/2)}{E} = \frac{(A/2)^2}{A^2} = \frac{1}{4}.$$

Therefore

$$U(A/2) = \frac{25}{4} = 6.25\,\text{J},$$

and the remaining energy is kinetic:

$$K = 25 - 6.25 = 18.75\,\text{J}.$$

The value 12.5 J incorrectly treats potential energy as linear in displacement. The value 37.5 J exceeds the entire available mechanical energy and is impossible. Finally, 9.75 J does not equal $E[1 - (x/A)^2]$ at $x/A = 1/2$, so it also fails the energy relation.

Step 3: Conclusion The kinetic energy at displacement $A/2$ is $\boxed{18.75\,\text{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $K/E = 1 - (x/A)^2$. At half the amplitude, one quarter of the energy is potential and three quarters is kinetic.

43. If 1000 droplets of water of surface tension $0.07\ \mathrm{N/m}$, having same radius $1\ \mathrm{mm}$ each, combine to from a single drop. In the process the released surface energy is :-

$$\left(\text{Take } \pi = \frac{22}{7}\right)$$

[4 marks]

(A) 7.92×10^{-4} J

(B) 7.92×10^{-6} J

(C) 8.8×10^{-5} J

(D) 9.68×10^{-4} J

Correct Answer: (A) 7.92×10^{-4} J

Solution:

Concept:

- Surface energy of a liquid drop is $U = \sigma A$, where σ is surface tension and A is the single free surface area. A water drop has one surface, unlike a soap bubble.
- Coalescence conserves the total water volume. The released energy equals initial surface energy minus final surface energy, since the combined drop has less surface area.

Step 1: Find the radius after coalescence Let $r = 1\,\mathrm{mm} = 10^{-3}\,\mathrm{m}$ be each initial radius and R the final radius. Conservation of volume gives

$$\frac{4}{3}\pi R^3 = 1000\frac{4}{3}\pi r^3 \quad \Rightarrow \quad R = 10r = 10^{-2}\,\mathrm{m}.$$

The radius increases tenfold; it does not increase by the number of droplets. The final area is therefore the area of one sphere of radius $10r$.

Step 2: Calculate the decrease in area and energy The initial and final areas are $A_i = 1000(4\pi r^2)$ and $A_f = 4\pi(10r)^2 = 100(4\pi r^2)$. Hence

$$\begin{aligned} E_{\text{released}} &= \sigma(A_i - A_f) \\ &= 0.07 \times 4 \times \frac{22}{7} \times 900 \times 10^{-6} \\ &= 7.92 \times 10^{-4}\,\mathrm{J}. \end{aligned}$$

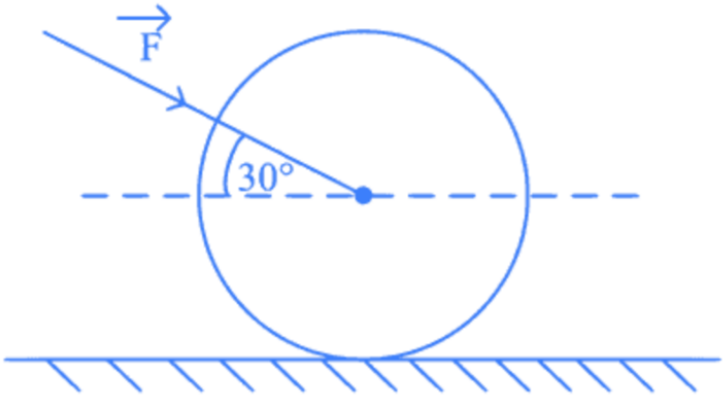
The result is positive because energy is released as the surface shrinks. Option B is smaller by a factor of one hundred, C does not use the full area decrease, and D is larger than even the initial surface energy 8.8×10^{-4} J.

Step 3: Conclusion The released surface energy is 7.92×10^{-4} J. Hence, the correct option is (A).

Quick Tip to Solve: For n equal droplets, first use $R = n^{1/3}r$, then compute $4\pi\sigma r^2(n - n^{2/3})$; do not conserve surface area.

44. As shown in figure, a $70\ \text{kg}$ garden roller is pushed with a force of $\vec{F} = 200\ \text{N}$ at an angle of 30° with horizontal. The normal reaction on the roller is

(Given $g = 10\ \text{m s}^{-2}$) [4 marks]



(A) 800 N

(B) 600 N

(C) $200\sqrt{3}$ N

(D) $800\sqrt{2}$ N

Correct Answer: (A) 800 N

Solution:

Concept:

- The normal reaction is the upward contact force exerted by the ground. A roller remaining on a horizontal surface has zero vertical acceleration, so its upward and downward forces balance.
- A force F directed at an angle θ below the horizontal has downward component $F\sin\theta$ and horizontal component $F\cos\theta$.

Step 1: Resolve the pushing force The arrow in the figure points downward toward the roller centre. Therefore the push adds to the weight pressing the roller against the ground. Its vertical component is

$$F_y = F\sin 30^\circ = 200 \times \frac{1}{2} = 100\,\text{N}$$

downward. Its horizontal component is $200\cos 30^\circ = 100\sqrt{3}\,\text{N}$; that component can move the roller along the ground but does not enter the vertical balance.

Step 2: Balance the vertical forces The weight acts downward and is

$$mg = 70 \times 10 = 700\,\text{N}.$$

Taking upward as positive and denoting the normal reaction by N,

$$N - mg - F\sin 30^\circ = 0 \quad \Rightarrow \quad N = 700 + 100 = 800\,\text{N}.$$

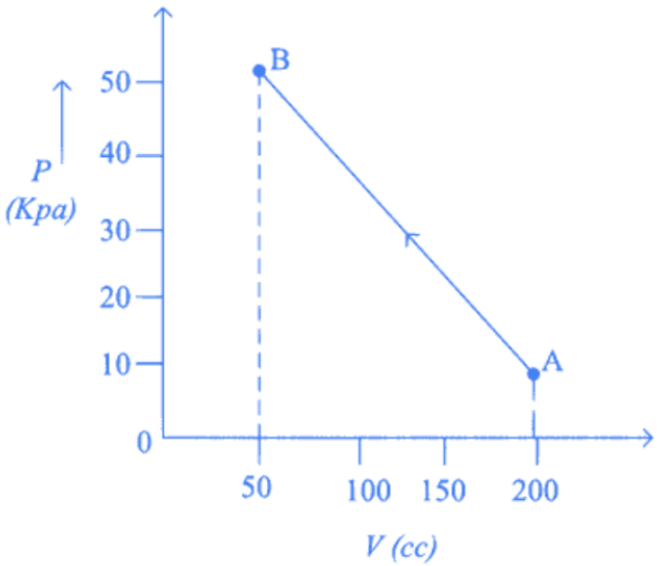
Option B, $600\,\text{N}$, would correspond to an upward pulling component and contradicts the shown push. Option C involves a horizontal component rather than the required vertical support, while option

D cannot follow from the sum of the two downward forces. No knowledge of friction or rolling acceleration is needed for this vertical equilibrium condition.

Step 3: Conclusion The normal reaction on the roller is $800\,\mathrm{N}$. Hence, the correct option is (A).

Quick Tip to Solve: Look at the force arrow before choosing the sign: pushing downward increases the normal reaction, while pulling upward decreases it.

45. The pressure of a gas changes linearly with volume from A to B as shown in figure. If no heat is supplied to or extracted from the gas then change in the internal energy of the gas will be [4 marks]



(A) 6 J

(B) 4.5 J

(C) zero

(D) -4.5 J

Correct Answer: (B) 4.5 J

Solution:

Concept:

- Using work done by the gas as positive, the first law is $\Delta U = Q - W$, where Q is heat supplied and $W = \int P\,dV$. No supplied or extracted heat means $Q = 0$.
- For a straight line on a pressure-volume graph, work equals average pressure times the signed volume change: $W = (P_A + P_B)(V_B - V_A)/2$.

Step 1: Read the endpoints and the direction The process proceeds from A to B, so the volume decreases. The graph gives

$$P_A = 10\,\mathrm{kPa}, \quad V_A = 200\,\mathrm{cm}^3,$$
$$P_B = 50\,\mathrm{kPa}, \quad V_B = 50\,\mathrm{cm}^3.$$

This is compression, and the work done by the gas is negative. The shaded-area magnitude would be

a trapezium, not just the triangular region above the lower endpoint pressure.

Step 2: Evaluate work and apply the energy balance Since $1\,\text{kPa} = 10^3\,\text{Pa}$ and $1\,\text{cm}^3 = 10^{-6}\,\text{m}^3$,

$$\begin{aligned} W &= \frac{10+50}{2} \times 10^3 \times (50-200) \times 10^{-6} \\ &= -4.5\,\text{J}, \\ \Delta U &= 0 - (-4.5) = +4.5\,\text{J}. \end{aligned}$$

The surroundings do positive work on the gas, increasing its internal energy despite zero heat transfer. Option A does not equal the trapezium area; option C confuses no heat transfer with no energy change. Option D is the work done by the gas, rather than its change in internal energy.

Step 3: Conclusion The internal energy increases by $4.5\,\text{J}$. Hence, the correct option is (B).

Quick Tip to Solve: For an adiabatic compression, check that ΔU is positive. Convert both graph axes to SI units before multiplying pressure by volume.

---

46. The correct relation between $\gamma = \frac{c_p}{c_v}$ and temperature T is : [4 marks]

(A) $\gamma \propto T$

(B) $\gamma \propto \frac{1}{\sqrt{T}}$

(C) $\gamma \propto \frac{1}{T}$

(D) $\gamma \propto T^{\circ}$

Correct Answer: (D) $\gamma \propto T^{\circ}$

Solution:

Concept:

- In the ideal-gas kinetic theory model with a fixed number f of active degrees of freedom, the molar heat capacities are $C_V = fR/2$ and $C_P = C_V + R$.
- The ratio $\gamma = C_P/C_V$ is dimensionless. Mass-specific and molar heat capacities give the same ratio because the conversion factor cancels.

Step 1: Express the ratio in terms of degrees of freedom Equipartition assigns energy $RT/2$ per mole to each quadratic degree of freedom. Thus the internal energy is $U = fRT/2$, and differentiating with respect to temperature at constant volume gives $C_V = fR/2$. Using Mayer's relation,

$$C_P = \frac{fR}{2} + R = \frac{(f+2)R}{2}.$$

Both heat capacities are constants in this fixed-degree-of-freedom model; their ratio does not acquire a temperature factor.

Step 2: Compare the proposed temperature dependences Taking the ratio yields

$$\gamma = \frac{(f+2)R/2}{fR/2} = 1 + \frac{2}{f}.$$

For example, a monatomic ideal gas has $f = 3$ and $\gamma = 5/3$; a rigid diatomic ideal gas has $f = 5$ and $\gamma = 7/5$. In either case the ratio is independent of temperature while the active degrees of freedom remain fixed. Consequently $\gamma \propto T^0$, since $T^0 = 1$. Options A, B and C would make the ratio respectively increase or decrease with temperature and do not follow from these heat capacities. At temperatures that activate additional molecular modes, heat capacities can vary; no such change is specified here.

Step 3: Conclusion Within the ideal-gas model used here, $\gamma \propto T^0$. Hence, the correct option is (D).

Quick Tip to Solve: Use $\gamma = 1 + 2/f$: for a fixed molecular model, the ratio depends on the active degrees of freedom rather than on temperature directly.

---

47. If a source of electromagnetic radiation having power $15\ \text{kW}$ produces 10^{16} photons per second, the radiation belongs to a part of spectrum is.

(Take Planck constant $h = 6 \times 10^{-34}\text{Js}$) [4 marks]

(A) Gamma rays

(B) Radio waves

(C) Micro waves

(D) Ultraviolet rays

Correct Answer: (A) Gamma rays

Solution:

Concept:

- Radiation power is energy emitted per second. If $\dot{N}$ identical photons are emitted each second, then $P = \dot{N}E$, where E is the energy of one photon.
- Planck's relation $E = h\nu$ determines photon frequency. The wavelength is $\lambda = c/\nu$; high photon energy corresponds to high frequency and short wavelength.

Step 1: Find the energy carried by one photon Convert the source power into SI units before dividing by the photon emission rate:

$$P = 15\,\text{kW} = 1.5 \times 10^4\,\text{J s}^{-1},$$

$$E = \frac{P}{\dot{N}} = \frac{1.5 \times 10^4}{10^{16}} = 1.5 \times 10^{-12}\,\text{J}.$$

The factors of seconds cancel. Power alone does not determine the spectral region; the photon count is essential because it specifies how that power is distributed among photons.

Step 2: Calculate the frequency and identify the region Using the supplied value of Planck's constant,

$$\nu = \frac{E}{h} = \frac{1.5 \times 10^{-12}}{6 \times 10^{-34}} = 2.5 \times 10^{21}\,\text{Hz}.$$

For a scale check, its wavelength is

$$\lambda = \frac{3 \times 10^8}{2.5 \times 10^{21}} = 1.2 \times 10^{-13}\,\text{m}.$$

Such extremely high frequency and short wavelength place this radiation in the gamma-ray region. Radio waves and microwaves have vastly lower frequencies and much longer wavelengths. Ultraviolet radiation also has lower frequencies and wavelengths many orders of magnitude longer than the calculated value, so it cannot describe this source.

Step 3: Conclusion The radiation has frequency 2.5×10^{21} Hz and belongs to the gamma-ray region. Hence, the correct option is (A).

Quick Tip to Solve: Combine the two equations directly as $\nu = P/(h\dot{N})$, taking care to convert kilowatts to watts before evaluating the exponent.

---

48. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R

Assertion A : The beam of electrons show wave nature and exhibit interference and diffraction.

Reason R : Davisson Germer Experimentally verified the wave nature of electrons.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) A is not correct but R is correct.

(B) Both A and R are correct and R is the correct explanation of A

(C) Both A and R are correct but R is Not the correct explanation of A

(D) A is correct but R is not correct

Correct Answer: (B) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- De Broglie associated a wavelength $\lambda = h/p$ with a moving particle of momentum p. Electron beams can therefore display interference and diffraction under suitable experimental conditions.
- Diffraction from a crystal provides experimental evidence of wave behaviour. Constructive interference from crystal planes obeys the Bragg condition $2d\sin\theta = n\lambda$.

Step 1: Evaluate the assertion An electron beam can form an interference pattern when coherent alternatives are available, and it can diffract from the regularly spaced planes of a crystal. These are characteristic wave phenomena, even though individual electrons are detected as localized events. The de Broglie wavelength becomes appreciable for electron momenta used in diffraction experiments. Thus the assertion that electrons show wave nature and exhibit interference and diffraction is correct; the particle character of their detection does not invalidate it.

Step 2: Evaluate the reason Davisson and Germer scattered electrons from a nickel crystal and observed pronounced diffraction maxima. The wavelength inferred from the crystal diffraction agreed

with the de Broglie wavelength calculated using electron momentum. Their experiment therefore verified the wave nature of electrons. The reason is correct; it names a real experimental observation rather than a hypothetical prediction.

Step 3: Connect the experimental evidence to the assertion The observed diffraction maxima provide direct experimental support for the wave behaviour claimed in the assertion. In the sense of experimental justification used by this question, the reason explains why the wave nature of an electron beam is accepted. Both statements are consequently correct and the reason supports the assertion. Options A and D incorrectly reject one true statement; option C overlooks this direct connection between the experimental verification and the asserted wave behaviour.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: For an assertion about wave behaviour, identify a wave-specific observation such as diffraction; Davisson–Germer supplies that experimental evidence for electrons.

49. The amplitude of $15\sin(1000\pi t)$ is modulated by $10\sin(4\pi t)$ signal. The amplitude modulated signal contains frequency (ies) of

A. 500 Hz

B. 2 Hz

C. 250 Hz

D. 498 Hz

E. 502 Hz

Choose the correct answer from the options given below: [4 marks]

(A) B only

(B) A only

(C) A, D and E only

(D) A and B only

Correct Answer: (C) A, D and E only

Solution:

Concept:

- A sinusoid written as $A\sin(\omega t)$ has frequency $f = \omega/(2\pi)$. In amplitude modulation, the high-frequency carrier amplitude varies with the low-frequency message.
- Ordinary single-tone amplitude modulation contains the carrier frequency f_c and the two sideband frequencies $f_c - f_m$ and $f_c + f_m$, where f_m is the modulating frequency.

Step 1: Extract the carrier and message frequencies The carrier is $15\sin(1000\pi t)$, while the message is $10\sin(4\pi t)$. Their angular frequencies are the coefficients of time, so

$$f_c = \frac{1000\pi}{2\pi} = 500\,\text{Hz}, \qquad f_m = \frac{4\pi}{2\pi} = 2\,\text{Hz}.$$

The amplitudes 15 and 10 affect the carrier and sideband strengths, not their frequencies. In particular, 1000π is an angular frequency, not a frequency in hertz.

Step 2: Generate the sidebands and select the listed components The modulated waveform has the form

$$s(t) = [15 + 10\sin(4\pi t)]\sin(1000\pi t).$$

Expanding its product term with $\sin a \sin b = [\cos(a-b) - \cos(a+b)]/2$ gives oscillations at the sum and difference angular frequencies. Hence the sidebands are

$$f_L = 500 - 2 = 498\,\text{Hz}, \qquad f_U = 500 + 2 = 502\,\text{Hz}.$$

Together with the retained carrier, these are entries A, D and E. The $2\,\text{Hz}$ message is not a separate component of this transmitted AM waveform, and $250\,\text{Hz}$ is neither carrier nor sideband. Thus options containing only A or B, or A and B, omit the required sidebands or include the wrong component.

Step 3: Conclusion The AM signal contains $500\,\text{Hz}$, $498\,\text{Hz}$ and $502\,\text{Hz}$: A, D and E only. Hence, the correct option is (C).

Quick Tip to Solve: First divide every angular frequency by 2π; then write the three AM spectral lines as f_c and $f_c \pm f_m$.

50. A free neutron decays into a proton but a free proton does not decay into neutron. This is because [4 marks]

(A) neutron is an uncharged particle

(B) neutron has larger rest mass than proton

(C) neutron is a composite particle made of a proton and an electron

(D) proton is a charged particle

Correct Answer: (B) neutron has larger rest mass than proton

Solution:

Concept:

- An isolated particle can decay spontaneously only when the total rest energy of its final products is smaller than its initial rest energy. The released energy is the decay Q-value.
- In beta-minus decay, a neutron changes into a proton, an electron and an electron antineutrino: $n \to p + e^- + \bar{\nu}_e$. Electric charge and lepton number are conserved.

Step 1: Compare the rest energies The neutron rest energy is approximately $939.565\,\text{MeV}$, whereas the proton rest energy is $938.272\,\text{MeV}$. Their difference is about $1.293\,\text{MeV}$. Creating the electron needs $0.511\,\text{MeV}$, leaving a positive energy release:

$$Q = (m_n - m_p - m_e)c^2 \simeq 0.782\,\text{MeV}.$$

Thus free neutron decay is energetically allowed; this remaining energy appears as kinetic energy of the products.

Step 2: Test the reverse process and the options A free proton changing into a neutron would need an increase in rest energy. In the charge-conserving beta-plus channel $p \to n + e^+ + \nu_e$, even the neutron alone is heavier than the starting proton, and creating the positron costs additional energy. No external energy is supplied to a free proton, so this process cannot occur spontaneously. Options (A) and (D) identify electric charge, but charge conservation is already satisfied by the accompanying leptons and does not explain the energy difference. Option (C) is false: a neutron is not a pre-existing proton-electron composite; it is a baryon with quark content udd.

Step 3: Conclusion The neutron has a larger rest mass than the proton, making neutron beta decay energetically possible while the corresponding free-proton decay is forbidden. Hence, the correct option is (B).

Quick Tip to Solve: For a proposed free-particle decay, compare total initial and final rest energies, including any electron or positron that must be created.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A lift of mass $\mathrm{M} = 500\ \mathrm{kg}$ is descending with speed of $2\ \mathrm{ms}^{-1}$. Its supporting cable begins to slip thus allowing it to fall with a constant acceleration of $2\ \mathrm{ms}^{-2}$. The kinetic energy of the lift at the end of fall through to a distance of $6\ \mathrm{m}$ will be _____ kJ. [4 marks]

Solution:

Concept:

- For constant acceleration along a straight line, $v^2 = u^2 + 2as$, where u is initial speed, v is final speed and s is displacement along the chosen positive direction.
- The translational kinetic energy is $K = \frac{1}{2}Mv^2$. Acceleration changes the speed; the stated downward acceleration is the net acceleration, not the gravitational acceleration alone.

Step 1: Choose the direction and substitute the data Take vertically downward as positive because the lift is initially descending and continues to fall. The initial velocity is $u = +2\,\mathrm{m\,s^{-1}}$, the acceleration is $a = +2\,\mathrm{m\,s^{-2}}$, and the displacement is $s = +6\,\mathrm{m}$. The mass is $M = 500\,\mathrm{kg}$. The cable still exerts a force while slipping, so replacing the given acceleration by g would incorrectly describe free fall.

Step 2: Calculate the final kinetic energy Use the kinematic relation without first calculating the time:

$$v^2 = (2)^2 + 2(2)(6) = 4 + 24 = 28\,\mathrm{m^2\,s^{-2}}.$$

Therefore

$$K_f = \frac{1}{2}(500)(28) = 7000\,\mathrm{J} = 7\,\mathrm{kJ}.$$

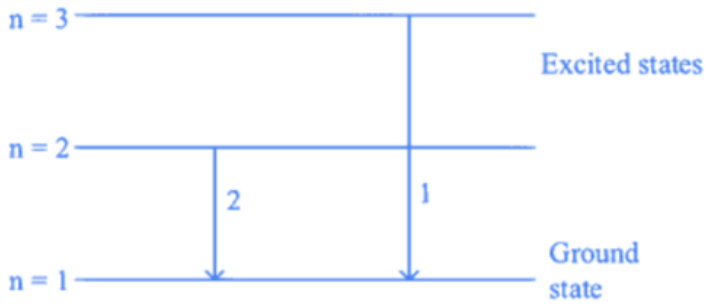
As an independent energy check, the initial kinetic energy is $\frac{1}{2}(500)(2)^2 = 1000\,\mathrm{J}$. The net force does work $Mas = (500)(2)(6) = 6000\,\mathrm{J}$, giving the same final energy $1000 + 6000 = 7000\,\mathrm{J}$. This also shows why the initial motion cannot be omitted.

Step 3: Conclusion The lift has kinetic energy $\boxed{7\,\mathrm{kJ}}$ after descending through $6\,\mathrm{m}$. The numerical

entry is 7.

Quick Tip to Solve: When initial speed, acceleration and distance are given, compute the final squared speed directly; do not assume the object starts from rest.

52. For hydrogen atom, λ_1 and λ_2 are the wavelengths corresponding to the transitions 1 and 2 respectively as shown in figure. The ratio of λ_1 and λ_2 is $\frac{x}{32}$. The value of x is _____. [4 marks]



Solution:

Concept:

- The energy of the hydrogen level with principal quantum number n is $E_n = -13.6/n^2\,\text{eV}$. An emitted photon carries the positive energy difference between the initial and final levels.
- The photon relation $E = hc/\lambda$ means that wavelength is inversely proportional to the transition energy. Equivalently, $1/\lambda = R(1/n_f^2 - 1/n_i^2)$ for emission.

Step 1: Read the two transitions Transition 1 in the figure is from $n = 3$ to $n = 1$, while transition 2 is from $n = 2$ to $n = 1$. Both terminate in the ground state, so both belong to the Lyman series. The arrow lengths are schematic; use the level numbers to calculate the gaps rather than measuring their drawn lengths. Their reciprocal wavelengths are

$$\frac{1}{\lambda_1} = R\left(1 - \frac{1}{9}\right) = \frac{8R}{9}, \qquad \frac{1}{\lambda_2} = R\left(1 - \frac{1}{4}\right) = \frac{3R}{4}.$$

Step 2: Form the wavelength ratio and identify x Divide the second reciprocal wavelength by the first, because the required ratio is λ_1/λ_2:

$$\frac{\lambda_1}{\lambda_2} = \frac{3R/4}{8R/9} = \frac{3}{4} \times \frac{9}{8} = \frac{27}{32}.$$

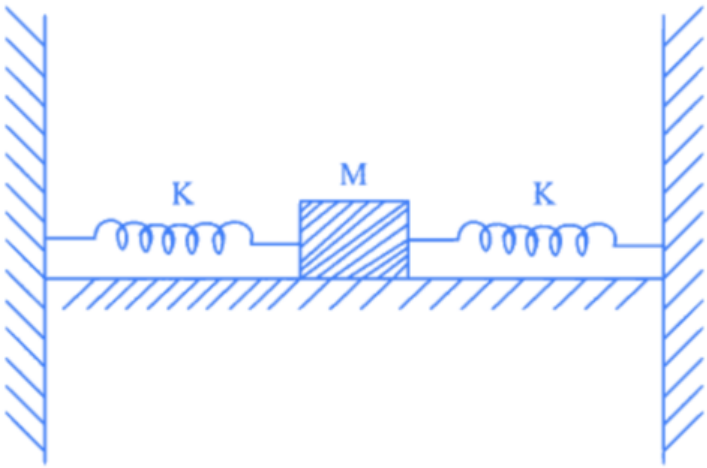
Comparison with the stated ratio $x/32$ gives $x = 27$. The Rydberg constant cancels, so neither its numerical value nor a wavelength unit is needed. The result is less than one, as it should be: the $3 \to 1$ transition releases more energy and hence emits the shorter wavelength.

Step 3: Conclusion The ratio is $\lambda_1/\lambda_2 = 27/32$, so $\boxed{x = 27}$. The requested coefficient is dimensionless.

Quick Tip to Solve: For two spectral lines, calculate their energy gaps first and reverse the ratio when converting an energy ratio into a wavelength ratio.

53. In the figure given below, a block of mass $M = 490\ \mathrm{g}$ placed on a frictionless table is connected with two springs having same spring constant $(\mathrm{K} = 2\ \mathrm{N\ m^{-1}})$. If the block is horizontally displaced through 'X' m then the number of complete oscillations it will make in 14π seconds will be _____.

[4 marks]



Solution:

Concept:

- Hooke's law gives a restoring force proportional to displacement. If both springs oppose the same displacement of a block, their restoring forces add.
- For $F = -k_{\text{eff}}x$, the angular frequency is $\omega = \sqrt{k_{\text{eff}}/M}$, the period is $T = 2\pi\sqrt{M/k_{\text{eff}}}$, and the number of oscillations in time t is t/T.

Step 1: Find the effective spring constant When the block moves a distance x to the right, the left spring extends by x and pulls it left. At the same time, the right spring compresses by x and also pushes it left. Each force has magnitude Kx, so

$$F = -Kx - Kx = -2Kx.$$

Hence $k_{\text{eff}} = 2K = 4\,\mathrm{N\,m^{-1}}$. The springs do not act in series here: the block itself moves between two fixed ends, and both forces act directly on it.

Step 2: Calculate the period and the number of cycles Convert the block mass to SI units: $M = 490\,\mathrm{g} = 0.490\,\mathrm{kg}$. The period is

$$T = 2\pi\sqrt{\frac{0.490}{4}} = 2\pi\sqrt{0.1225} = 0.7\pi\ \mathrm{s}.$$

Over the given time interval $t = 14\pi\,\mathrm{s}$, the number of complete oscillations is

$$N = \frac{14\pi}{0.7\pi} = 20.$$

The displacement specified as X determines the amplitude, but it does not enter the period of ideal small-amplitude spring oscillations. Since the ratio is exactly an integer, there is no incomplete final cycle to discard.

Step 3: Conclusion The block completes $\boxed{20}$ oscillations in $14\pi\,\mathrm{s}$. The numerical entry is the dimensionless count 20.

Quick Tip to Solve: For a block attached between two fixed springs, displace it mentally and add the restoring forces; extension of one and compression of the other both restore it.

54. The speed of a swimmer is $4\ \mathrm{km\ h^{-1}}$ in still water. If the swimmer makes his strokes normal to the flow of river of width $1\ \mathrm{km}$, he reaches a point $750\ \mathrm{m}$ down the stream on the opposite bank. The speed of the river water is _____ $\mathrm{km\ h^{-1}}$ [4 marks]

Solution:

Concept:

- The swimmer's velocity relative to the bank equals the vector sum of his velocity relative to water and the river velocity relative to the bank.
- Velocity components perpendicular and parallel to the flow act independently. Crossing time is the river width divided by the component of velocity across the river.

Step 1: Find the time to cross the river The swimmer makes his strokes normal to the river flow. Thus his full still-water speed, $4\,\mathrm{km\,h^{-1}}$, is directed across the river relative to the water. The river contributes only a downstream component and does not change the cross-river speed. With width $w = 1\,\mathrm{km}$,

$$t = \frac{w}{v_\perp} = \frac{1}{4}\,\mathrm{h} = 0.25\,\mathrm{h}.$$

No diagonal distance is needed to find this time; the opposite bank is reached when the perpendicular displacement becomes one kilometre.

Step 2: Use the downstream drift to find river speed The downstream displacement is $750\,\mathrm{m} = 0.750\,\mathrm{km}$. Because the swimmer has no downstream component relative to the water, this entire drift is caused by the current. Therefore

$$v_r = \frac{d}{t} = \frac{0.750}{0.25} = 3\,\mathrm{km\,h^{-1}}.$$

A direct check is $d/w = v_r/v_\perp = 3/4 = 0.75$, agreeing with the specified drift-to-width ratio. The speed relative to the bank would be $\sqrt{4^2 + 3^2} = 5\,\mathrm{km\,h^{-1}}$, but that resultant is different from the river speed asked for.

Step 3: Conclusion The speed of the river water is $\boxed{3\,\mathrm{km\,h^{-1}}}$. The numerical entry is 3.

Quick Tip to Solve: Use the river width for crossing time and the downstream drift for current speed. Keep the distances in the same unit before taking their ratio.

55. A solid sphere of mass $1\ \text{kg}$ rolls without slipping on a plane surface. Its kinetic energy is 7×10^{-3} J. The speed of the centre of mass of the sphere is _____ cm s^{-1} [4 marks]

Solution:

Concept:

- Rolling without slipping combines centre-of-mass translation and rotation about the centre. The total energy is $K=\frac{1}{2}Mv^2+\frac{1}{2}I\omega^2$.
- A solid sphere has $I=\frac{2}{5}MR^2$ about a diameter. The no-slip constraint is $v=\omega R$, which eliminates the unknown radius from the energy equation.

Step 1: Express total kinetic energy in terms of speed Substitute the solid-sphere moment of inertia and $\omega=v/R$:

$$K=\frac{1}{2}Mv^2+\frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{v^2}{R^2}$$
$$=\frac{1}{2}Mv^2+\frac{1}{5}Mv^2=\frac{7}{10}Mv^2.$$

Both contributions must be included because the sphere rolls. Using only $\frac{1}{2}Mv^2$ would assign all the given energy to translation and overestimate its centre-of-mass speed.

Step 2: Substitute and convert the speed unit The mass is $M=1\,\text{kg}$, and the total energy is $K=7\times10^{-3}\,\text{J}$. Hence

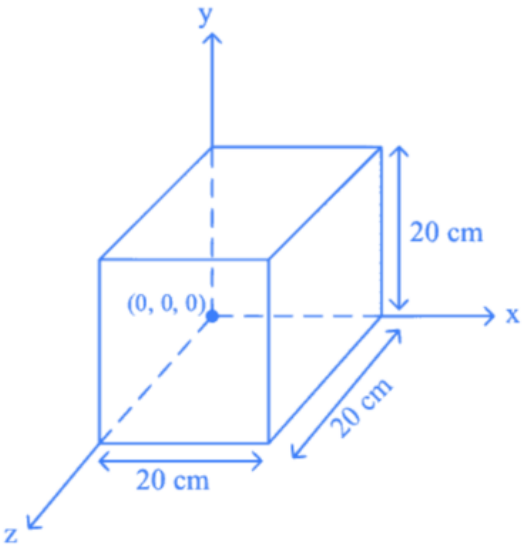
$$v^2=\frac{10K}{7M}=\frac{10(7\times10^{-3})}{7(1)}=10^{-2}\,\text{m}^2\,\text{s}^{-2}.$$

Taking the positive square root gives $v=0.10\,\text{m s}^{-1}$. Since one metre is one hundred centimetres, $v=10\,\text{cm s}^{-1}$. For verification, the translational energy is $0.005\,\text{J}$ and the rotational energy is $0.002\,\text{J}$; their sum is the stated $0.007\,\text{J}$.

Step 3: Conclusion The speed of the centre of mass is $\boxed{10\,\text{cm s}^{-1}}$. The numerical entry is 10.

Quick Tip to Solve: For a rolling solid sphere, use total energy $7Mv^2/10$; calculate in SI units and convert the final speed to the requested unit only at the end.

56. Expression for an electric field is given by $\overrightarrow{\mathrm{E}} = 4000x^2\hat{i}\,\frac{\mathrm{V}}{\mathrm{m}}$. The electric flux through the cube of side $20\ \mathrm{cm}$ when placed in electric field (as shown in the figure) is _____ $\mathrm{V\ cm}$. [4 marks]



Solution:

Concept:

- Electric flux through a surface is $\Phi = \int \vec{E} \cdot d\vec{A}$. For a closed surface, each area vector points outward; incoming flux has a negative sign.
- When the field is purely along x and depends only on x, the four faces parallel to the field have zero flux. Only the two faces normal to x contribute, giving $\Phi = [E_x(x_2) - E_x(x_1)]A$.

Step 1: Locate the contributing faces The figure puts one vertex of the cube at the origin and extends the cube from $x = 0$ to $x = a$, where $a = 20\,\mathrm{cm} = 0.20\,\mathrm{m}$. The area of either face normal to x is $A = a^2 = 0.040\,\mathrm{m}^2$. The outward normals of the other four faces are along y or z, perpendicular to $\vec{E} = 4000x^2\hat{i}$, so their scalar products with the field vanish. The field is constant over each x-face because every point of that face has the same x coordinate.

Step 2: Evaluate the net outward flux and convert units At the left x-face, $E_x(0) = 0$. At the right x-face,

$$E_x(a) = 4000(0.20)^2 = 160\,\mathrm{V\,m^{-1}}.$$

Therefore the closed-surface flux is

$$\Phi = [160 - 0](0.040) = 6.4\,\mathrm{V\,m}.$$

The requested unit is volt-centimetre, not volt-metre. Since $1\,\mathrm{V\,m} = 100\,\mathrm{V\,cm}$,

$$\Phi = 6.4 \times 100 = 640\,\mathrm{V\,cm}.$$

The positive sign means net outward flux, consistent with the increase in field magnitude between the two x-faces. Multiplying the field by all six face areas would be incorrect because flux depends on surface orientation.

Step 3: Conclusion The net flux through the cube is $\boxed{640\,\mathrm{V\,cm}}$. The numerical entry is 640.

Quick Tip to Solve: For a field along one coordinate axis, subtract the field strengths on the two perpendicular faces, multiply by one face area, then check the requested flux unit.

57. A thin rod having a length of $1\ \mathrm{m}$ and area of cross-section $3 \times 10^{-6}\ \mathrm{m}^2$ is suspended vertically from one end. The rod is cooled from $210^\circ\mathrm{C}$ to $160^\circ\mathrm{C}$. After cooling, a mass M is attached at the lower end of the rod such that the length of rod again becomes $1\ \mathrm{m}$. Young's modulus and coefficient of linear expansion of the rod are $2 \times 10^{11}\ \mathrm{N\ m^{-2}}$ and $2 \times 10^{-5}\ \mathrm{K^{-1}}$, respectively. The value of M is _____ kg.

(Take $\mathrm{g} = 10\ \mathrm{m\ s^{-2}}$) [4 marks]

Solution:

Concept:

- Cooling produces a free contraction of magnitude $\Delta L_{\text{th}} = L\alpha\Delta T$, where ΔT is the temperature fall. The coefficient α is the fractional length change per kelvin.
- Within the elastic limit, Young's modulus is stress divided by longitudinal strain: $Y = (F/A)/(\Delta L/L)$. The applied mass supplies the tensile force $F = Mg$.

Step 1: Find the thermal contraction The temperature decreases through $210 - 160 = 50\,\mathrm{K}$. A temperature interval has the same numerical value in kelvin and degrees Celsius. Before attaching the mass, the rod therefore shortens by

$$\Delta L_{\text{th}} = 1(2 \times 10^{-5})(50) = 10^{-3}\,\mathrm{m}.$$

Its unstrained cooled length is approximately $0.999\,\mathrm{m}$. The added load must compensate this contraction, rather than compress the rod further.

Step 2: Balance the thermal and elastic strains Using the usual linear, small-strain approximation, both length changes are referred to the original length. Restoration of the original length requires

$$\frac{Mg}{AY} = \alpha\Delta T.$$

Thus the required tensile stress is $Y\alpha\Delta T = 2 \times 10^8\,\mathrm{Pa}$. Multiplying by the area converts this stress into a force:

$$F = (2 \times 10^8)(3 \times 10^{-6}) = 600\,\mathrm{N}.$$

Consequently,

$$M = \frac{600}{10} = 60\,\mathrm{kg}.$$

The length cancels because the thermal and elastic changes are both proportional to it. Products of the already small strains are neglected, as in the linear expansion law.

Step 3: Conclusion The required attached mass is $M = 60\,\mathrm{kg}$. The numerical entry is $\boxed{60}$.

Quick Tip to Solve: When a load exactly restores thermal contraction, equate strains directly: $Mg/A = Y\alpha\Delta T$.

58. In a medium the speed of light wave decreases to 0.2 times to its speed in free space The ratio of relative permittivity to the refractive index of the medium is $x : 1$. The value of x is _____.
(Given speed of light in free space $= 3 \times 10^8$ m s^{-1} and for the given medium $\mu_r = 1$) [4 marks]

Solution:

Concept:

- The absolute refractive index is $n = c/v$, where c is the vacuum speed and v is the speed in the medium. Both speeds must be expressed in the same units.
- For the stated electromagnetic medium, $n = \sqrt{\varepsilon_r \mu_r}$. Relative permittivity and relative permeability, as well as refractive index, are dimensionless.

Step 1: Calculate the refractive index The phrase that the speed decreases to 0.2 times its free-space value means $v = 0.2c$, not a reduction of only twenty percent. In particular,

$$v = 0.2(3 \times 10^8) = 6 \times 10^7 \,\mathrm{m\,s^{-1}}.$$

The refractive index is therefore

$$n = \frac{c}{v} = \frac{3 \times 10^8}{6 \times 10^7} = 5.$$

The value exceeds one, consistently with the reduced propagation speed. The speed ratio must be inverted when finding the index.

Step 2: Obtain the permittivity and the requested ratio Squaring the material relation gives $n^2 = \varepsilon_r \mu_r$. The question specifies $\mu_r = 1$, so the relative permittivity is

$$\varepsilon_r = \frac{n^2}{\mu_r} = \frac{25}{1} = 25.$$

The question asks for its ratio to the index, rather than for the permittivity alone. Hence

$$\frac{\varepsilon_r}{n} = \frac{25}{5} = 5, \qquad \varepsilon_r : n = 5 : 1.$$

Thus neither the speed fraction 0.2 nor the permittivity 25 is the requested value of x. Equivalently, for a nonmagnetic medium this ratio equals the refractive index itself.

Step 3: Conclusion The ratio is $5 : 1$, so $\boxed{x = 5}$. The answer is dimensionless.

Quick Tip to Solve: For $\mu_r = 1$, use $\varepsilon_r = n^2$; therefore $\varepsilon_r / n = n = c/v$.

59. Two identical cells, when connected either in parallel or in series gives same current in an external resistance $5\ \Omega$. The internal resistance of each cell will be _____ Ω. [4 marks]

Solution:

Concept:

- A real cell is modelled by an emf E in series with its internal resistance r. The current through an external resistance R follows Ohm's law for the complete circuit.

- Two identical cells connected in aiding series have emf $2E$ and internal resistance $2r$. In parallel with like terminals joined, their emf remains E and their equivalent internal resistance is $r/2$.

Step 1: Write the two external currents Let $R = 5\,\Omega$. In the series arrangement the same current passes through both internal resistances and the external resistor, so

$$I_s = \frac{2E}{R + 2r}.$$

For identical cells in parallel there is no circulating current arising from unequal emfs. Their internal resistances act in parallel and the total external current is

$$I_p = \frac{E}{R + r/2}.$$

This expression is the current in the external resistor; the current supplied by each parallel cell is half of it.

Step 2: Use the equality of currents The stated condition is $I_s = I_p$. Cancelling the common, nonzero emf and cross-multiplying gives

$$\frac{2}{R + 2r} = \frac{1}{R + r/2},$$
$$2R + r = R + 2r.$$

It follows that $r = R$, and hence $r = 5\,\Omega$. As a direct check, series connection gives $2E/15$, while parallel connection gives $E/7.5$; these are the same current. The result does not require the value of the emf, because both arrangements use identical cells.

Step 3: Conclusion The internal resistance of each cell is $\boxed{r = 5\,\Omega}$. The numerical entry is 5.

Quick Tip to Solve: For two identical cells producing equal load currents in series and parallel, the crossover occurs at $R = r$.

---

60. An inductor of 0.5 mH, a capacitor of 20 μF and resistance of 20 Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the amplitude of current of the circuit is $\sqrt{x}$ A. The value of x is _____ [4 marks]

Solution:

Concept:

- In a series LCR circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and $\tan\phi = (X_L - X_C)/R$. Voltage and current are in phase when the net reactance vanishes.
- An AC voltage rating is its rms value unless a peak value is explicitly stated. For a sinusoidal current, $I_0 = \sqrt{2} I_{\text{rms}}$, where I_0 is its amplitude.

Step 1: Apply the phase condition Since the current is in phase with the source emf, $\phi = 0$ and therefore $X_L = X_C$. The circuit is at series resonance; its impedance is purely resistive:

$$Z = R = 20\,\Omega.$$

The given inductance and capacitance specify which frequency produces this condition, but they do not increase the resonant impedance. For example,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.5\times10^{-3})(20\times10^{-6})}} = 10^4\,\text{rad}\,\text{s}^{-1}.$$

At this angular frequency both reactances equal $5\,\Omega$ and cancel.

Step 2: Convert the rms current to its amplitude Using the source rating $V_{\text{rms}} = 220\,\text{V}$, the rms current is

$$I_{\text{rms}} = \frac{220}{20} = 11\,\text{A}.$$

The question asks for the amplitude, not this rms current. Accordingly,

$$I_0 = 11\sqrt{2}\,\text{A} = \sqrt{242}\,\text{A}.$$

Comparing with the stated form $\sqrt{x}\,\text{A}$ gives $x = 242$. Squaring 11 alone would give the square of the rms current and would miss the factor of two needed for the peak current.

Step 3: Conclusion The amplitude is $11\sqrt{2}\,\text{A}$; consequently $\boxed{x = 242}$.

Quick Tip to Solve: At series resonance use $I_{\text{rms}} = V_{\text{rms}}/R$, then multiply by $\sqrt{2}$ if the question asks for amplitude.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. When Cu^{2+} ion is treated with KI, a white precipitate, X appears in solution. The solution is titrated with sodium thiosulphate, the compound Y is formed. X and Y respectively are : [4 marks]

(A) $X = CuI_2, \quad Y = Na_2S_4O_6$

(B) $X = Cu_2I_2, \quad Y = Na_2S_4O_6$

(C) $X = CuI_2, \quad Y = Na_2S_2O_3$

(D) $X = Cu_2I_2, \quad Y = Na_2S_4O_5$

Correct Answer: (B) $X = Cu_2I_2, \quad Y = Na_2S_4O_6$

Solution:

Concept:

- Iodide reduces copper(II) to copper(I), while it is itself oxidised to iodine. The insoluble copper(I) iodide separates as a white precipitate, driving the redox reaction forward.
- In iodometric titration, iodine oxidises thiosulphate ions to tetrathionate ions. Sodium ions remain counterions; the product salt is sodium tetrathionate.

Step 1: Identify the white precipitate Copper(II) does not simply give a stable precipitate of copper(II) iodide. Reduction to copper(I) accompanies precipitation:

$$2\,Cu^{2+} + 4\,I^- \longrightarrow 2\,CuI\,(s) + I_2.$$

The copper oxidation state changes from $+2$ to $+1$. Thus X is copper(I) iodide, which is written as Cu_2I_2 in the options. This has the same elemental ratio as CuI. The alternatives that identify X as CuI_2, namely (A) and (C), assign copper the wrong oxidation state.

Step 2: Identify the titration product The liberated iodine is reduced back to iodide by thiosulphate:

$$I_2 + 2\,S_2O_3^{2-} \longrightarrow 2\,I^- + S_4O_6^{2-}.$$

In terms of the sodium salts, this is

$$I_2 + 2\,Na_2S_2O_3 \longrightarrow 2\,NaI + Na_2S_4O_6.$$

Therefore $Y = Na_2S_4O_6$. The formula $Na_2S_2O_3$ denotes the added titrant, not its oxidation product, and $Na_2S_4O_5$ in (D) is not tetrathionate. Only option (B) combines the correct white copper(I) iodide with the correct sulphur oxyanion product.

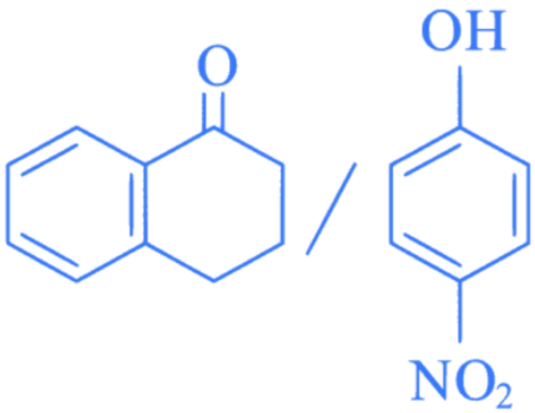
Step 3: Conclusion $X = Cu_2I_2$ and $Y = Na_2S_4O_6$. Hence, the correct option is (B).

Quick Tip to Solve: In iodometry remember the pair: iodide liberates iodine from copper(II), then thiosulphate becomes tetrathionate.

62. Match items of column I and II.

	Column I (Mixture of compounds)		Column II (Separation Technique)
A.	H_2O/CH_2Cl_2	i.	Crystallization
B.	The pair of structures shown below	ii.	Differential solvent extraction
C.	Kerosene / Naphthalene	iii.	Column chromatography
D.	$C_6H_{12}O_6$/NaCl	iv.	Fractional Distillation

Correct match is [4 marks]



(A) A-(ii), B-(iv), C-(i), D-(iii)

(B) A-(iii), B-(iv), C-(ii), D-(i)

(C) A-(i), B-(iii), C-(ii), D-(iv)

(D) A-(ii), B-(iii), C-(iv), D-(i)

Correct Answer: (D) A-(ii), B-(iii), C-(iv), D-(i)

Solution:

Concept:

- Differential solvent extraction relies on partition between immiscible liquid phases. Column chromatography separates compounds through different adsorption or distribution behaviour on a stationary phase.
- Fractional distillation uses differences in volatility, whereas crystallization exploits differences in solubility and the ability of a solute to form crystals under suitable conditions.

Step 1: Match the first two mixtures Water and dichloromethane form immiscible liquid layers. Their phase behaviour is the basis of differential solvent extraction, giving A-(ii). In practice the layers are separated in a separating funnel. Row B contains a fused cyclic ketone and a nitrophenol. These organic compounds have different polarities and interactions with an adsorbent such as silica or alumina. They can therefore travel through a chromatographic column at different rates, giving B-(iii). The illustrated compounds are not assigned to fractional distillation simply because both are organic; their adsorption differences are the separation principle tested here.

Step 2: Complete the matching and reject the alternatives Kerosene and naphthalene are matched with fractional distillation in the listed techniques, so C-(iv). Their differing boiling behaviour allows separation on heating and fractionation. Glucose and sodium chloride are matched with crystallization, D-(i), using their different solubility behaviour to crystallize the components selectively.

$$\mathrm{A}-(\mathrm{ii}), \quad \mathrm{B}-(\mathrm{iii}), \quad \mathrm{C}-(\mathrm{iv}), \quad \mathrm{D}-(\mathrm{i}).$$

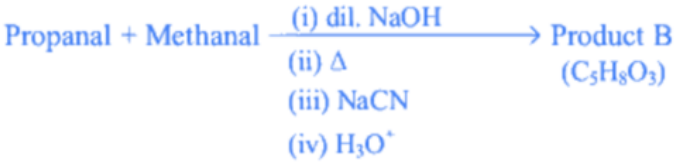
Option (A) assigns the ketone/nitrophenol pair to distillation and glucose/salt to chromatography, so it fails. Option (B) wrongly assigns water/dichloromethane to chromatography. Option (C) wrongly assigns that immiscible liquid pair to crystallization. Only (D) preserves all four matches.

Step 3: Conclusion The matching is A-(ii), B-(iii), C-(iv), D-(i). Hence, the correct option is (D).

Quick Tip to Solve: Start a separation matching question with the clearest physical property: immiscible water and dichloromethane immediately fix A-(ii).

63. Consider the following reaction

The correct statement for product B is. It is [4 marks]



(A) racemic mixture and is neutral

(B) optically active and adds one mole of bromine

(C) racemic mixture and gives a gas with saturated $NaHCO_3$ solution

(D) optically active alcohol and is neutral

Correct Answer: (C) racemic mixture and gives a gas with saturated $NaHCO_3$ solution

Solution:

Concept:

- Propanal has alpha hydrogen and undergoes crossed aldol addition with methanal. Heating dehydrates the beta-hydroxy aldehyde to an alpha,beta-unsaturated aldehyde.
- Cyanide undergoes conjugate addition to an alpha,beta-unsaturated carbonyl compound. Acidic hydrolysis converts the resulting nitrile group into a carboxylic acid; formation of a stereocentre without a chiral influence gives a racemic mixture.

Step 1: Follow aldol addition and dehydration The enolate of propanal attacks methanal, giving the beta-hydroxy aldehyde $HOCH_2CH(CH_3)CHO$. On heating this loses water to give methacrolein:

$$HOCH_2CH(CH_3)CHO \xrightarrow{\Delta} CH_2{=}C(CH_3)CHO + H_2O.$$

The carbon count is four after combining the three-carbon propanal with the one-carbon methanal. The aldehyde group survives the dehydration step and remains available for the subsequent cyanide addition.

Step 2: Identify the final functional groups and stereochemistry Cyanide adds at the beta carbon of the conjugated aldehyde. Protonation gives the saturated cyano aldehyde $NCCH_2CH(CH_3)CHO$. Acidic hydrolysis of the nitrile then gives

$$HOOCCH_2CH(CH_3)CHO.$$

This product has formula $C_5H_8O_3$, agreeing with the formula in the reaction. Its carbon bearing the methyl group is attached to four different groups: hydrogen, methyl, formyl and carboxymethyl. It is therefore a stereocentre. With no chiral reagent or catalyst, its two configurations form equally; the product is racemic and has no net optical activity. Its carboxylic acid group releases carbon dioxide with bicarbonate:

$$RCOOH + NaHCO_3 \longrightarrow RCOONa + CO_2\uparrow + H_2O.$$

Thus (A) fails because the product is acidic, while (B) and (D) fail because the mixture is not optically active. The carbon-carbon double bond is consumed in conjugate addition, and the final product is an

aldehyde acid rather than a neutral alcohol.

Step 3: Conclusion Product B is a racemic aldehyde acid and evolves CO_2 with saturated $NaHCO_3$. Hence, the correct option is (C).

Quick Tip to Solve: For cyanide addition to a conjugated aldehyde, track the beta carbon; then convert CN to COOH during acid hydrolysis.

64. Nd^{2+} = _____ [4 marks]

(A) $4f^4 6s^2$

(B) $4f^3$

(C) $4f^4$

(D) $4f^2 6s^2$

Correct Answer: (C) $4f^4$

Solution:

Concept:

- Neodymium has atomic number 60. Its neutral ground-state configuration is $[Xe]\,4f^4 6s^2$, where the xenon core contains 54 electrons.
- In forming a lanthanoid cation, the outermost $6s$ electrons are removed before the inner $4f$ electrons. The ionic charge counts the total electrons lost.

Step 1: Write the neutral configuration Begin with the xenon core and distribute the six remaining electrons between the occupied subshells. The configuration of neutral neodymium is $[Xe]\,4f^4 6s^2$. This accounts for $54 + 4 + 2 = 60$ electrons. The configuration in option (A) is therefore that of the neutral atom, rather than the required doubly charged cation.

Step 2: Remove the two outer electrons The highest principal shell occupied is $n = 6$, so formation of Nd^{2+} removes both $6s$ electrons. Thus

$$Nd \longrightarrow Nd^{2+} + 2\,e^-, \qquad [Xe]\,4f^4 6s^2 \longrightarrow [Xe]\,4f^4.$$

The resulting ion has 58 electrons, as required by $60 - 2 = 58$.

Step 3: Check the alternative configurations Option (B), $4f^3$, represents loss of three electrons from the neutral atom after removal of the $6s$ pair, and corresponds to the familiar trivalent ion. Option (D) leaves both outer $6s$ electrons while removing inner $4f$ electrons; that is not the ground-state ionisation sequence. Only option (C) has both the correct electron count and the correct occupied subshell.

Step 4: Conclusion The electronic configuration of Nd^{2+} is $[Xe]\,4f^4$, written as $4f^4$ beyond the noble-gas core. Hence, the correct option is (C).

Quick Tip to Solve: For a lanthanoid ion, write the neutral configuration first and remove the outer $6s$ electrons before changing the $4f$ count.

65. The correct order of basicity of oxides of vanadium is : [4 marks]

(A) $V_2O_3 > V_2O_5 > V_2O_4$

(B) $V_2O_3 > V_2O_4 > V_2O_5$

(C) $V_2O_5 > V_2O_4 > V_2O_3$

(D) $V_2O_4 > V_2O_3 > V_2O_5$

Correct Answer: (B) $V_2O_3 > V_2O_4 > V_2O_5$

Solution:

Concept:

- For oxides of the same transition metal, increasing the oxidation state generally increases acidic character and decreases basic character.
- Oxygen is assigned oxidation number -2 in these oxides. Charge balance therefore determines the oxidation state of vanadium in each formula.

Step 1: Find the oxidation states Let the oxidation number of vanadium be x. Neutrality gives

$$\begin{aligned} V_2O_3: &\quad 2x - 6 = 0 \Rightarrow x = +3, \\ V_2O_4: &\quad 2x - 8 = 0 \Rightarrow x = +4, \\ V_2O_5: &\quad 2x - 10 = 0 \Rightarrow x = +5. \end{aligned}$$

The formulas must be compared through these oxidation states rather than by their oxygen atom counts alone.

Step 2: Apply the trend in oxide character At the lower oxidation state, vanadium retains greater metallic character, and its oxide is more basic. At higher oxidation states, stronger polarisation of the oxide ion increases covalent and acidic character. Thus the oxide with vanadium in the $+3$ state is the most basic of the three, while the $+5$ oxide is the least basic. The $+4$ oxide lies between them.

Step 3: Compare all proposed orders The decreasing basicity order is therefore

$$V_2O_3 > V_2O_4 > V_2O_5.$$

Option (A) incorrectly places the $+5$ oxide above the $+4$ oxide. Option (C) reverses the entire trend. Option (D) incorrectly makes the $+4$ oxide more basic than the $+3$ oxide. Option (B) is the only order consistent with the oxidation-state trend throughout.

Step 4: Conclusion The correct order of basicity is $V_2O_3 > V_2O_4 > V_2O_5$. Hence, the correct option is (B).

Quick Tip to Solve: When comparing oxides of one metal, calculate its oxidation states first: a higher oxidation state usually means a more acidic, less basic oxide.

66. The correct increasing order of the ionic radii is [4 marks]

(A) $Cl^- < Ca^{2+} < K^+ < S^{2-}$

(B) $K^+ < S^{2-} < Ca^{2+} < Cl^-$

(C) $Ca^{2+} < K^+ < Cl^- < S^{2-}$

(D) $S^{2-} < Cl^- < Ca^{2+} < K^+$

Correct Answer: (C) $Ca^{2+} < K^+ < Cl^- < S^{2-}$

Solution:

Concept:

- Isoelectronic ions contain the same number of electrons. Within such a series, ionic radius decreases as nuclear charge increases.
- The four ions S^{2-}, Cl^-, K^+ and Ca^{2+} each contain 18 electrons and have the argon electronic configuration.

Step 1: Confirm the common electron count Sulfur has atomic number 16 and gains two electrons; chlorine has atomic number 17 and gains one. Potassium has atomic number 19 and loses one electron; calcium has atomic number 20 and loses two. Consequently,

$$16 + 2 = 17 + 1 = 19 - 1 = 20 - 2 = 18.$$

All four ions therefore have the same filled shells, so neutral-atom sizes cannot be used directly to rank them.

Step 2: Compare nuclear attraction For the same electron arrangement, the number of protons increases in the order $16 < 17 < 19 < 20$. The more positively charged nucleus attracts the common electron cloud more strongly and contracts it. Thus sulfur forms the largest ion in this set and calcium forms the smallest. Potassium must lie between calcium and chlorine because its nuclear charge lies between theirs.

Step 3: Write the increasing-radius order Reversing the nuclear-charge order gives

$$Ca^{2+} < K^+ < Cl^- < S^{2-}.$$

Option (A) incorrectly places chloride below both cations. Option (B) puts potassium below calcium and sulfide below chloride, contrary to the isoelectronic trend. Option (D) starts with sulfide as the smallest ion, although it has the least nuclear attraction in this series.

Step 4: Conclusion The correct increasing order is $Ca^{2+} < K^+ < Cl^- < S^{2-}$. Hence, the correct option is (C).

Quick Tip to Solve: Count electrons before using a periodic trend. For equal electron counts, greater atomic number means smaller ionic radius.

67. The correct order of melting points of dichlorobenzenes is [4 marks]

(A) p-dichlorobenzene $>$ o-dichlorobenzene $>$ m-dichlorobenzene

(B) p-dichlorobenzene $>$ m-dichlorobenzene $>$ o-dichlorobenzene

(C) m-dichlorobenzene $>$ p-dichlorobenzene $>$ o-dichlorobenzene

(D) o-dichlorobenzene $>$ m-dichlorobenzene $>$ p-dichlorobenzene

Correct Answer: (A) p-dichlorobenzene $>$ o-dichlorobenzene $>$ m-dichlorobenzene

Solution:

Concept:

- Melting depends on the stability and packing of molecules in the solid crystal. Molecular symmetry can improve packing and raise the melting point.
- The para isomer of dichlorobenzene is more symmetrical than its ortho and meta isomers. The established melting-point order is para greater than ortho greater than meta.

Step 1: Identify the three substitution patterns In the displayed structures, the para isomer has chlorine atoms at positions 1 and 4, the ortho isomer at positions 1 and 2, and the meta isomer at positions 1 and 3. These are positional isomers with the same molecular formula, $C_6H_4Cl_2$. Their molar masses are therefore identical; the difference in melting behaviour comes from molecular arrangement and crystal packing.

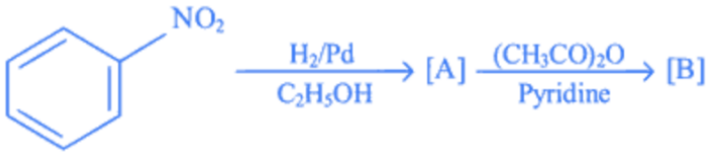
Step 2: Compare their solid-state packing The para molecule has the most regular and symmetrical arrangement of substituents. It packs efficiently in a crystal lattice, so melting requires more thermal energy. Its melting point is much higher than those of the other two isomers. For the remaining pair, ortho-dichlorobenzene melts at a higher temperature than meta-dichlorobenzene. The known approximate values, about $53^\circ C$, $-17^\circ C$ and $-25^\circ C$, respectively, give the required comparison.

Step 3: Match the structures to the option The decreasing order is $p > o > m$, which is the sequence of structures in option (A). Option (B) reverses the ortho-meta pair. Option (C) places the meta isomer above the para isomer, and option (D) places the para isomer last. Both conflict with the especially efficient crystal packing of the symmetrical para isomer.

Step 4: Conclusion The melting-point order is p-dichlorobenzene $>$ o-dichlorobenzene $>$ m-dichlorobenzene. Hence, the correct option is (A).

Quick Tip to Solve: For melting points of positional isomers, check symmetry and crystal packing; do not transfer a boiling-point order directly to melting points.

68. Consider the above reaction and identify the product B. [4 marks]



(A) $C_6H_5-NH-C(=O)-CH_3$

(B) $C_6H_5-NH-CH_2-COOH$

(C) $C_6H_5-CH_2-C(=O)-NH_2$

(D) $C_6H_5-C(=O)-CH_2-NH_2$

Correct Answer: (A) $C_6H_5-NH-C(=O)-CH_3$

Solution:

Concept:

- Catalytic hydrogenation with H_2/Pd reduces a nitro group on an aromatic ring to an amino group under the stated reaction conditions.
- Aniline reacts with acetic anhydride by N-acetylation. Pyridine acts as a base, and the product is the amide acetanilide, $C_6H_5NHCOCH_3$.

Step 1: Identify the intermediate A The starting aromatic compound is nitrobenzene. Hydrogen in the presence of palladium converts its $-NO_2$ group into $-NH_2$, while the benzene skeleton is retained in this sequence. The balanced reduction is

$$C_6H_5NO_2 + 3\,H_2 \xrightarrow{Pd} C_6H_5NH_2 + 2\,H_2O.$$

Thus intermediate A is aniline, rather than a compound containing a new carbon side chain.

Step 2: Acetylate the amino group Aniline has a nitrogen lone pair that attacks an acetyl carbonyl of acetic anhydride. Acetate leaves, and transfer of a proton gives the neutral amide. The overall change is

$$C_6H_5NH_2 + (CH_3CO)_2O \longrightarrow C_6H_5NHCOCH_3 + CH_3COOH.$$

Pyridine assists by accepting acid. One hydrogen on nitrogen is replaced by the acetyl group CH_3CO^-; nitrogen remains directly bonded to the aromatic ring.

Step 3: Check the product connectivity Option (A) has the required $Ph-NH-C(=O)-CH_3$ linkage. Option (B) instead introduces a carboxymethyl group on nitrogen. Option (C) places a methylene group between the ring and an amide carbonyl. Option (D) has the carbonyl directly attached to the ring and an amino group on the adjacent carbon. None of those skeletal changes follows from reduction followed by N-acetylation.

Step 4: Conclusion Product B is acetanilide, $C_6H_5NHCOCH_3$. Hence, the correct option is (A).

Quick Tip to Solve: Track which atom is acylated: acetic anhydride attaches $COCH_3$ to aniline nitrogen without inserting carbon between nitrogen and the ring.

69. A protein 'X' with molecular weight of $70,000$ u, on hydrolysis gives amino acids. One of these amino acid is [4 marks]

(A) $NH_2-CH_2-CH(CH_3)-CH_2-CH_2-COOH$

(B) $CH_3-C(CH_3)(NH_2)-CH_2-CH_2-COOH$

(C) $CH_3-CH(CH_3)-CH_2-CH(NH_2)-COOH$

(D) $CH_3-CH(CH_3)-CH(NH_2)-CH_2-COOH$

Correct Answer: (C) $CH_3-CH(CH_3)-CH_2-CH(NH_2)-COOH$

Solution:

Concept:

- Proteins are polymers of alpha-amino acids connected through peptide bonds. Hydrolysis cleaves the peptide links and regenerates the constituent alpha-amino acids.
- An alpha-amino acid has its amino group on the carbon immediately next to the carboxyl carbon: the characteristic connectivity is $R-CH(NH_2)-COOH$.

Step 1: Locate the alpha carbon in each option Start at the carboxyl group $-COOH$, whose carbon is numbered 1. The adjacent carbon is carbon 2, or the alpha carbon. For an amino acid released by ordinary protein hydrolysis, the amino group must be attached at that position. The molecular weight $70,000$ u identifies a macromolecular protein but does not determine which side chain occurs in this structural test.

Step 2: Identify the protein amino acid Option (C) is $CH_3-CH(CH_3)-CH_2-CH(NH_2)-COOH$. Its amino group is on the carbon directly adjacent to $-COOH$, so it has the required alpha-amino-acid structure. Reading from the carboxyl end, its side chain is $-CH_2-CH(CH_3)_2$. This is leucine, a standard amino acid found in proteins.

Step 3: Eliminate the other connectivities In option (A), the amino group is at the remote end of the chain, not on the alpha carbon. In option (B), two methylene groups separate the amino-bearing carbon from the carboxyl carbon, so it is not an alpha-amino acid. In option (D), one methylene group lies between the amino-bearing carbon and the carboxyl group; this is a beta-amino arrangement. Only option (C) can be selected as the normal protein-hydrolysis product among the four.

Step 4: Conclusion The amino acid is leucine, $CH_3CH(CH_3)CH_2CH(NH_2)COOH$, shown in option (C). Hence, the correct option is (C).

Quick Tip to Solve: For a protein-hydrolysis MCQ, find $COOH$ first and check whether NH_2 is on its immediately neighbouring carbon.

70. Choose the correct set of reagents for the following conversion.

trans $(\mathrm{Ph}-\mathrm{CH}=\mathrm{CH}-\mathrm{CH_3}) \rightarrow$ **cis** $(\mathrm{Ph}-\mathrm{CH}=\mathrm{CH}-\mathrm{CH_3})$ [4 marks]

(A) $\mathrm{Br_2}, \mathrm{alc}\cdot\mathrm{KOH}, \mathrm{NaNH_2}, \mathrm{H_2}$ Lindlar Catalyst

(B) $\mathrm{Br_2}, \mathrm{aq}\cdot\mathrm{KOH}, \mathrm{NaNH_2}, \mathrm{Na}$ (Liq $\mathrm{NH_3}$)

(C) $\mathrm{Br_2}, \mathrm{alc}\cdot\mathrm{KOH}, \mathrm{NaNH_2}, \mathrm{Na}$ (Liq $\mathrm{NH_3}$)

(D) $\mathrm{Br_2}, \mathrm{aq}\cdot\mathrm{KOH}, \mathrm{NaNH_2}, \mathrm{H_2}$ Lindlar Catalyst

Correct Answer: (A) $\mathrm{Br_2}, \mathrm{alc}\cdot\mathrm{KOH}, \mathrm{NaNH_2}, \mathrm{H_2}$ Lindlar Catalyst

Solution:

Concept:

- An alkene can be converted to an alkyne by bromine addition followed by two dehydrohalogenations. The second elimination from a vinyl halide needs a strong base such as $\mathrm{NaNH_2}$.
- Partial hydrogenation of an alkyne with hydrogen and Lindlar catalyst gives the cis alkene. Sodium in liquid ammonia instead gives the trans alkene.

Step 1: Convert the alkene into a vicinal dibromide Add $\mathrm{Br_2}$ across the carbon-carbon double bond of the starting trans alkene. The carbon skeleton remains unchanged, and the product has bromine on each of the two carbons that originally formed the double bond:

$$\mathrm{Ph{-}CH{=}CH{-}CH_3} \xrightarrow{\mathrm{Br_2}} \mathrm{Ph{-}CHBr{-}CHBr{-}CH_3}.$$

This addition removes the original alkene geometry, allowing it to be rebuilt through an alkyne intermediate.

Step 2: Carry out two eliminations Alcoholic KOH removes one equivalent of HBr from the vicinal dibromide to give a bromoalkene. The stronger base $\mathrm{NaNH_2}$ then removes the remaining equivalent from the vinyl halide, giving $\mathrm{Ph{-}C{\equiv}C{-}CH_3}$. Aqueous KOH favours substitution rather than the required elimination sequence, so the options using it are unsuitable for this conversion.

Step 3: Regenerate the cis double bond Treat the alkyne with $\mathrm{H_2}$ and Lindlar catalyst. Syn addition of hydrogen produces the cis alkene and stops before complete hydrogenation:

$$\mathrm{Ph{-}C{\equiv}C{-}CH_3} \xrightarrow{\mathrm{H_2,\ Lindlar}} \mathrm{cis{-}Ph{-}CH{=}CH{-}CH_3}.$$

This selects option (A). Option (C) has the correct elimination reagents but uses sodium in liquid ammonia, which produces the trans alkene. Options (B) and (D) additionally use aqueous rather than alcoholic potassium hydroxide.

Step 4: Conclusion The required reagent sequence is $\mathrm{Br_2}$, alcoholic KOH, $\mathrm{NaNH_2}$, then $\mathrm{H_2}$/Lindlar catalyst. Hence, the correct option is (A).

Quick Tip to Solve: To change trans to cis, pass through the alkyne and choose the final reduction carefully: Lindlar gives cis; sodium in liquid ammonia gives trans.

71. Which transition in the hydrogen spectrum would have the same wavelength as the Balmer type transition from $\mathrm{n} = 4$ to $\mathrm{n} = 2$ of He^+ spectrum [4 marks]

(A) $\mathrm{n} = 3$ to $\mathrm{n} = 4$

(B) $\mathrm{n} = 2$ to $\mathrm{n} = 1$

(C) $\mathrm{n} = 1$ to $\mathrm{n} = 2$

(D) $\mathrm{n} = 1$ to $\mathrm{n} = 3$

Correct Answer: (B) $\mathrm{n} = 2$ to $\mathrm{n} = 1$

Solution:

Concept:

- The wavenumber of an emitted line of a one-electron species is $1/\lambda = RZ^2(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$.
- Equal wavelengths require equal positive energy differences. Helium ion has nuclear charge $Z = 2$, whereas hydrogen has $Z = 1$.

Step 1: Find the helium-ion wavenumber For the stated Balmer transition, the electron falls from level 4 to level 2. Therefore the energy released is positive and the corresponding wavenumber is

$$\frac{1}{\lambda} = 4R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 4R\left(\frac{3}{16}\right) = \frac{3R}{4}.$$

The factor of four is essential: using only the principal quantum numbers would compare the two species incorrectly.

Step 2: Compare the hydrogen transitions For hydrogen, the downward transition from level 2 to level 1 gives

$$\frac{1}{\lambda} = R\left(1 - \frac{1}{4}\right) = \frac{3R}{4}.$$

It therefore emits a photon of exactly the required wavelength. Option (A), from 3 to 4, is upward and has energy magnitude $7Rhc/144$, which is different. Option (D), from 1 to 3, is also upward and has magnitude $8Rhc/9$. Option (C) is the reverse, absorptive transition from 1 to 2; it has the same energy magnitude but is not an emission transition. In the usual emission-spectrum interpretation of the stated Balmer transition, the required hydrogen transition is downward.

Step 3: Conclusion The matching hydrogen emission is $n = 2 \longrightarrow n = 1$, in the Lyman series. Hence, the correct option is (B). The reverse upward transition would absorb light of the same wavelength.

Quick Tip to Solve: Compare $Z^2(1/n_f^2 - 1/n_i^2)$, and check the direction separately: downward transitions emit, while their reverse transitions absorb.

72. Which one of the following statements is correct for electrolysis of brine solution? [4 marks]

(A) O_2 is formed at cathode

(B) H_2 is formed at anode

(C) Cl_2 is formed at cathode

(D) OH^- is formed at cathode

Correct Answer: (D) OH^- is formed at cathode

Solution:

Concept:

- Brine is concentrated aqueous NaCl. Reduction occurs at the cathode and oxidation at the anode.
- In aqueous brine, water is reduced preferentially to Na^+; chloride is oxidised at the anode under the chlor-alkali conditions.

Step 1: Write the cathode reaction The cathode supplies electrons to species in solution. Depositing sodium from aqueous brine is not the favoured process; water is reduced instead:

$$2\,H_2O + 2\,e^- \longrightarrow H_2 + 2\,OH^-.$$

Thus hydrogen gas is liberated at the cathode, and hydroxide ions are formed there at the same time. Both atoms and charge balance: the left side has charge -2, as does the right side. The production of hydroxide makes the cathode compartment alkaline.

Step 2: Check the anode and reject the distractors At the anode of a brine electrolytic cell, chloride loses electrons:

$$2\,Cl^- \longrightarrow Cl_2 + 2\,e^-.$$

Adding the half-reactions gives the ionic overall reaction

$$2\,Cl^- + 2\,H_2O \longrightarrow Cl_2 + H_2 + 2\,OH^-.$$

The sodium ions remain as counterions, so sodium hydroxide accumulates in the cathode solution. Option (A) is incorrect because oxygen is not formed at the cathode; a cathodic reaction is reduction. Option (B) places hydrogen at the wrong electrode. Option (C) places chlorine at the wrong electrode. Only option (D) correctly identifies a species formed at the cathode.

Step 3: Conclusion Hydroxide ions, OH^-, are formed at the cathode along with hydrogen gas. Hence, the correct option is (D).

Quick Tip to Solve: For aqueous brine, memorise the balanced cathode equation: water gains electrons to produce hydrogen and hydroxide together.

73. Cobalt chloride when dissolved in water forms pink colored complex $\underline{X}$ which has octahedral geometry. This solution on treating with conc HCl forms deep blue complex, $\underline{Y}$ which has a $\underline{Z}$ geometry. X, Y and Z, respectively, are [4 marks]

(A) $X = [Co(H_2O)_6]^{2+}, Y = [CoCl_4]^{2-}, Z =$ Tetrahedral

(B) $X = [Co(H_2O)_6]^{2+}, Y = [CoCl_6]^{3-}, Z =$ Octahedral

(C) $X = [Co(H_2O)_4Cl_2]^{+}, Y = [CoCl_4]^{2-}, Z =$ Tetrahedral

(D) $X = [Co(H_2O)_6]^{3+}, Y = [CoCl_6]^{3-}, Z =$ Octahedral

Correct Answer: (A) $X = [Co(H_2O)_6]^{2+}, Y = [CoCl_4]^{2-}, Z =$ Tetrahedral

Solution:

Concept:

- Aqueous cobalt(II) forms the pink hexaaquacobalt(II) ion, $[Co(H_2O)_6]^{2+}$, with coordination number six and octahedral geometry.
- A high chloride concentration favours replacement of water ligands by chloride, producing blue tetrahedral $[CoCl_4]^{2-}$. Ligand substitution does not change cobalt oxidation state.

Step 1: Identify the pink aqua complex Water is a neutral ligand, so the charge of the aqua complex is the oxidation state of cobalt. Dissolving cobalt(II) chloride supplies Co^{2+}; six water molecules coordinate to it. Thus

$$X = [Co(H_2O)_6]^{2+}.$$

Its six donor oxygen atoms occupy an octahedral arrangement. A $3+$ aqua ion would contain cobalt(III), which is not produced merely by dissolving cobalt(II) chloride in water.

Step 2: Identify the chloride-rich complex and its shape Concentrated hydrochloric acid provides a high concentration of chloride ions. The ligand-substitution equilibrium is

$$[Co(H_2O)_6]^{2+} + 4\,Cl^- \rightleftharpoons [CoCl_4]^{2-} + 6\,H_2O.$$

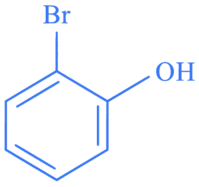
The product has four chloride ligands and tetrahedral geometry. Its charge is $+2 + 4(-1) = -2$, consistent with cobalt remaining in oxidation state $+2$. Options (B) and (D) instead propose a six-coordinate cobalt(III) chloride complex and an octahedral product. Option (C) assigns the wrong pink starting complex: the standard pink aqueous ion contains six waters, not four waters and two chlorides. Therefore only the first combination matches both colours and coordination geometries.

Step 3: Conclusion $X = [Co(H_2O)_6]^{2+}$, $Y = [CoCl_4]^{2-}$, and Z is tetrahedral. Hence, the correct option is (A).

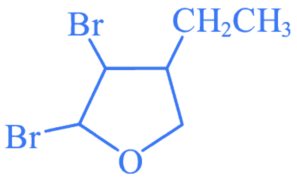
Quick Tip to Solve: Use ligand count and charge together: six neutral waters give the pink octahedral $2+$ ion; four chlorides give the blue tetrahedral $2-$ ion.

74. An organic compound 'A' with emperical formula C_6H_6O gives sooty flame on burning. Its reaction with bromine solution in low polarity solvent results in high yield of B. B is [4 marks]

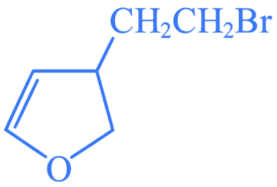
(A) 2-Bromophenol



(B) 2,3-Dibromo-4-ethyloxolane



(C) 4-(2-Bromoethyl)-4,5-dihydrofuran



(D) 4-Bromophenol



Correct Answer: (D)

4-Bromophenol



Solution:

Concept:

- Phenol, C_6H_5OH, has empirical formula C_6H_6O; its aromatic ring explains its sooty flame.
- The hydroxyl group activates the ring toward electrophilic substitution and directs incoming bromine to ortho and para positions.
- Bromination in a low-polarity solvent favours monobromination; the para product is usually obtained in higher yield because it is less sterically crowded.

Step 1: Identify the aromatic starting compound The formula contains six carbon atoms and one oxygen atom, and the sooty flame indicates a high carbon content characteristic of an aromatic compound. The structure relevant to the bromination options is phenol, C_6H_5OH. Its oxygen lone pair donates electron density into the benzene ring by resonance, increasing electron density especially at the two ortho positions and the para position. Consequently bromine substitutes a ring hydrogen at these positions rather than adding across the aromatic ring.

Step 2: Apply the solvent condition and select the major product The question specifies a low-polarity solvent rather than bromine water. Under these conditions the main process is controlled monobromination, giving ortho- and para-bromophenol, with the para isomer in higher yield.

$$C_6H_5OH + Br_2 \longrightarrow \text{p-}BrC_6H_4OH + HBr.$$

The para site is farther from the hydroxyl group and is less sterically hindered than the ortho site. Option (A) is ortho-bromophenol, a possible minor product, but not the higher-yield product

requested. Options (B) and (C) depict oxygen-containing nonbenzene rings and cannot result from simple electrophilic substitution of phenol. Option (D) preserves the benzene ring and places bromine opposite the hydroxyl group, as required.

Step 3: Conclusion The major product is para-bromophenol (4-bromophenol), the structure in option (D). Hence, the correct option is (D).

Quick Tip to Solve: Distinguish bromine water from bromine in a low-polarity solvent: the former gives 2,4,6-tribromophenol; controlled nonpolar conditions favour monobromination.

75. Match List I with List II

	List I		List II
A.	XeF_4	**I.**	**See-saw**
B.	SF_4	**II.**	**Square-planar**
C.	NH_4^+	**III.**	**Bent T-shaped**
D.	BrF_3	**IV.**	**Tetrahedral**

Choose the correct answer from the options given below : [4 marks]

(A) A - II, B - I, C - III, D - IV

(B) A - II, B - I, C - IV, D - III

(C) A - IV, B - I, C - II, D - III

(D) A - IV, B - III, C - II, D - I

Correct Answer: (B) A - II, B - I, C - IV, D - III

Solution:

Concept:

- VSEPR theory determines molecular shape from bonding pairs and lone pairs around the central atom; molecular shape describes atom positions, not lone-pair positions.
- In trigonal-bipyramidal electron arrangements, lone pairs preferentially occupy equatorial sites. In an octahedral arrangement, two lone pairs occupy opposite sites.

Step 1: Determine the shapes of xenon tetrafluoride and sulfur tetrafluoride In XeF_4, xenon has four bonding pairs and two lone pairs, giving six electron domains. The electron-domain geometry is octahedral; opposite lone pairs leave the four fluorine atoms in one plane. Hence XeF_4 is square-planar: A–II. In SF_4, sulfur has four bonding pairs and one lone pair, giving five electron domains. With the lone pair equatorial, the four S–F bonds form a see-saw shape. Thus B–I. These results already exclude options (C) and (D), which assign xenon tetrafluoride a tetrahedral shape.

Step 2: Determine the shapes of ammonium and bromine trifluoride In $NH_4{}^+$, nitrogen forms four N–H bonds and has no lone pair. Four equivalent bonding domains give a tetrahedral shape, so C–IV. In BrF_3, bromine has three bonding pairs and two lone pairs, again giving five electron domains. Both lone pairs occupy equatorial sites of the trigonal bipyramid, leaving two axial bonds and one equatorial bond. The resulting molecular shape is T-shaped, described as bent T-shaped in List II, so D–III. Option (A) interchanges the ammonium and bromine-trifluoride shapes and is therefore

incorrect. Only option (B) satisfies all four assignments simultaneously.

Step 3: Conclusion The matching is A–II, B–I, C–IV, D–III. Hence, the correct option is (B).

Quick Tip to Solve: Write the central-atom pattern first: AX_4E_2 square-planar, AX_4E see-saw, AX_4 tetrahedral, and AX_3E_2 T-shaped.

---

76. The methods NOT involved in concentration of ore are

A. Liquation

B. Leaching

C. Electrolysis

D. Hydraulic washing

E. Froth floatation

Choose the correct answer from the options given below : [4 marks]

(A) A and C only

(B) C, D and E only

(C) B, D and C only

(D) B, D and E only

Correct Answer: (A) A and C only

Solution:

Concept:

- Concentration of ore separates valuable mineral from gangue before extraction of the metal.
- Hydraulic washing uses density differences, froth flotation uses differences in surface wettability, and leaching uses selective dissolution.
- Refining purifies an already extracted crude metal. Liquation and electrolytic refining belong to this later stage rather than ore concentration.

Step 1: Classify the genuine concentration methods Leaching, method B, dissolves the desired ore component selectively in a suitable reagent, leaving insoluble gangue behind; the valuable component is then recovered from solution. Hydraulic washing, method D, uses a stream of water to remove lighter gangue from heavier ore particles. Froth flotation, method E, separates suitably wetted sulphide ore particles by attaching them to air bubbles while gangue remains in water. Thus B, D and E are all methods involved in concentration of ores and must not be included in the requested set.

Step 2: Identify the methods belonging to other stages Liquation, method A, is a refining process useful when a low-melting metal can be melted and drained away from higher-melting impurities. It treats an impure metal rather than a raw ore-gangue mixture. Electrolysis, method C, can be used for extraction of reactive metals or for electrolytic refining, but it is not one of the listed ore-concentration methods. Therefore the methods not involved in concentration are A and C only. Option (B) incorrectly includes hydraulic washing and froth flotation. Option (C) incorrectly includes leaching and hydraulic washing. Option (D) consists entirely of actual concentration methods, so it reverses the demand of the question.

Step 3: Conclusion Liquation and electrolysis, A and C only, are not ore-concentration methods. Hence, the correct option is (A).

Quick Tip to Solve: Classify the stage before choosing: ore concentration removes gangue, extraction produces metal, and refining purifies crude metal.

77. Identify X, Y and Z in the following reaction. (Equation not balanced) [4 marks]

$$ClO^{\bullet} + NO_2 \rightarrow \underline{X} \xrightarrow{H_2O} \underline{Y} + \underline{Z}$$

(A) $X = ClONO_2, Y = HOCl, Z = NO_2$

(B) $X = ClNO_3, Y = Cl_2, Z = NO_2$

(C) $X = ClONO_2, Y = HOCl, Z = HNO_3$

(D) $X = ClNO_2, Y = HCl, Z = HNO_3$

Correct Answer: (C) $X = ClONO_2, Y = HOCl, Z = HNO_3$

Solution:

Concept:

- The chlorine monoxide radical combines with nitrogen dioxide to form chlorine nitrate, $ClONO_2$, an atmospheric reservoir species.
- Hydrolysis of chlorine nitrate produces hypochlorous acid, HOCl, and nitric acid, HNO_3. Atom balance distinguishes the products unambiguously.

Step 1: Identify the intermediate X The first step combines the chlorine monoxide unit with nitrogen dioxide:

$$ClO^{\bullet} + NO_2 \longrightarrow ClONO_2.$$

The product is chlorine nitrate, whose connectivity is represented by Cl–O–NO_2. Its formula contains one chlorine, one nitrogen and three oxygen atoms. Therefore $X = ClONO_2$. The notation $ClNO_3$ in one distractor gives the same atom count but does not rescue that option, since its proposed hydrolysis products must also be checked. The formula $ClNO_2$ in option (D) has too few oxygen atoms for the indicated combination.

Step 2: Hydrolyse chlorine nitrate and verify every product The water-mediated reaction is

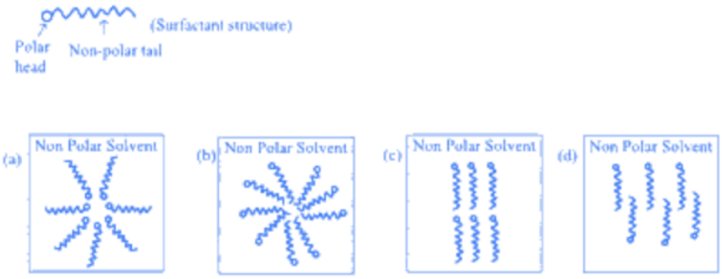
$$ClONO_2 + H_2O \longrightarrow HOCl + HNO_3.$$

The left side contains one chlorine, one nitrogen, two hydrogen and four oxygen atoms; the right side contains the same numbers. Consequently $Y = HOCl$ and $Z = HNO_3$. Option (A) incorrectly makes nitrogen dioxide instead of nitric acid and does not account for the hydrogen from water. Option (B) proposes chlorine gas and nitrogen dioxide, which do not give the stated hydrolysis products. Option (D) incorrectly uses hydrogen chloride rather than hypochlorous acid, in addition to having the wrong intermediate. Option (C) alone identifies the intermediate and both hydrolysis products consistently.

Step 3: Conclusion $X = ClONO_2$, $Y = HOCl$, and $Z = HNO_3$. Hence, the correct option is (C).

Quick Tip to Solve: Check hydrolysis by atom balance: adding water to $ClONO_2$ must distribute its two hydrogens between HOCl and HNO_3.

78. Adding surfactants in non polar solvent, the micelles structure will look like [4 marks]



(A) d

(B) a

(C) c

(D) b

Correct Answer: (B) a

Solution:

Concept:

- A surfactant has a polar, hydrophilic head and a nonpolar hydrocarbon tail. The part in favourable contact with the solvent determines the orientation of its aggregate.
- In a nonpolar solvent, reverse micelles have polar heads directed towards the interior and nonpolar tails directed outwards into the solvent.

Step 1: Identify the solvent-facing part The surrounding liquid is nonpolar. Its molecules interact favourably with the hydrocarbon tails of the surfactant, rather than with its polar heads. Aggregation therefore places the tails on the outside, where they contact the nonpolar solvent. The polar heads collect in the core, away from the bulk liquid. This reverses the orientation of an ordinary micelle in water.

Step 2: Compare the four structures In structure (a), the small circles representing polar heads point into the central region, while the zigzag hydrocarbon chains extend outwards. It therefore shows a reverse micelle. Structure (b) has heads on the exterior and tails towards the centre, the orientation suitable for a polar solvent. Structures (c) and (d) show extended arrangements rather than the required inward-head radial aggregate. Thus neither represents the reverse micellar structure asked for.

Step 3: Match the structure to its printed option The diagrams are labelled with lower-case letters, whereas the answer choices are labelled with capital letters. The required diagram is (a), and the printed choice containing (a) is option (B). Selecting option (A) merely because it is first would instead choose diagram (d).

Step 4: Conclusion The reverse micelle is diagram a, with polar heads inside and nonpolar tails outside. Hence, the correct option is (B).

Quick Tip to Solve: First identify the solvent: nonpolar solvent means tails outside. Then map the diagram letter to the printed answer choice.

79. H_2O_2 **acts as a reducing agent in :** [4 marks]

(A) $Mn^{2+} + 2H_2O_2 \rightarrow MnO_2 + 2H_2O$

(B) $2Fe^{2+} + 2H^+ + H_2O_2 \rightarrow 2Fe^{3+} + 2H_2O$

(C) $Na_2S + 4H_2O_2 \rightarrow Na_2SO_4 + 4H_2O$

(D) $2NaOCl + H_2O_2 \rightarrow 2NaCl + H_2O + O_2$

Correct Answer: (D) $2NaOCl + H_2O_2 \rightarrow 2NaCl + H_2O + O_2$

Solution:

Concept:

- In H_2O_2, oxygen has oxidation number -1. A reducing agent loses electrons and is itself oxidised while reducing another species.
- Oxidation of peroxide oxygen to O_2 changes its oxidation number from -1 to zero. Reduction to water or oxide changes it from -1 to -2.

Step 1: Inspect the reaction with hypochlorite In option (D), hydrogen peroxide produces molecular oxygen. Its oxidation half-reaction is

$$H_2O_2 \longrightarrow O_2 + 2\,H^+ + 2\,e^-.$$

Peroxide thus supplies electrons. At the same time chlorine in OCl^- changes from $+1$ to -1 in chloride, so hypochlorite is reduced. This identifies hydrogen peroxide as the reducing agent in this option. The redox role is determined by electron transfer, independently of the coefficients printed in the reaction.

Step 2: Rule out the other reactions In (A), manganese changes from $+2$ in Mn^{2+} to $+4$ in MnO_2, so manganese is oxidised and peroxide acts as the oxidant. In (B), Fe^{2+} is oxidised to Fe^{3+}; peroxide oxygen is reduced to water. In (C), sulfur changes from -2 in sulfide to $+6$ in sulfate, again indicating that peroxide oxidises the other reactant. These three options describe the oxidising behaviour of peroxide, not its reducing behaviour.

Step 3: Conclusion In the reaction with $NaOCl$, peroxide is oxidised to O_2 and acts as a reducing agent. Hence, the correct option is (D).

Quick Tip to Solve: Look for oxygen gas among the peroxide products: conversion of peroxide oxygen from -1 to zero signals reducing action.

80. Which of the following artificial sweeteners has the highest sweetness value in comparison to cane sugar ? [4 marks]

(A) Alitame

(B) Sucralose

(C) Saccharin

(D) Aspartame

Correct Answer: (A) Alitame

Solution:

Concept:

- Relative sweetness compares the sweetness produced by a substance with that of cane sugar, or sucrose, taken as one under comparable conditions.
- The standard approximate relative sweetness values are alitame 2000, sucralose 600, saccharin 550 and aspartame 100.

Step 1: List the relative sweetness values To compare artificial sweeteners, use their sweetness relative to the same reference rather than their chemical class or stability. Alitame is approximately 2000 times as sweet as cane sugar, whereas sucralose is approximately 600 times as sweet. Saccharin is approximately 550 times as sweet and aspartame approximately 100 times as sweet. These are the usual textbook comparison values; the largest value identifies the required compound.

Step 2: Arrange the compounds in decreasing order The comparison gives

$$\text{Alitame} > \text{Sucralose} > \text{Saccharin} > \text{Aspartame}.$$

Alitame therefore provides the greatest sweetness relative to cane sugar among the four choices. Option (B), sucralose, is less sweet than alitame despite being much sweeter than sucrose. Option (C), saccharin, also has a lower relative sweetness. Option (D), aspartame, has the smallest listed comparison value. The question asks only about sweetness, so considerations such as heat stability or suitability in particular foods do not change this ranking.

Step 3: Conclusion Alitame has the highest sweetness value among the given sweeteners, approximately 2000 times that of cane sugar. Hence, the correct option is (A).

Quick Tip to Solve: Remember the comparison sequence: alitame 2000, sucralose 600, saccharin 550, aspartame 100 times sucrose.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Zinc reacts with hydrochloric acid to give hydrogen and zinc chloride. The volume of hydrogen gas produced at STP from the reaction of 11.5 g of zinc with excess HCl is _____ L (Nearest integer)

(Given : Molar mass of Zn is $65.4\ g\ mol^{-1}$ and Molar volume of H_2 at STP $= 22.7$ L) [4 marks]

Solution:

Concept:

- The balanced reaction is $Zn + 2\,HCl \longrightarrow ZnCl_2 + H_2$. One mole of zinc gives one mole of hydrogen when the acid is in excess.
- Convert mass to amount using $n = m/M$, then use $V = nV_m$. Here the supplied STP molar volume is $22.7\,L\,mol^{-1}$.

Step 1: Find the amount of zinc Hydrochloric acid is present in excess, so zinc is the limiting reactant. All of the available zinc can react; the acid does not impose an additional limit. With the molar mass supplied in the question,

$$n(Zn) = \frac{11.5\,g}{65.4\,g\,mol^{-1}} = 0.17584\,mol.$$

The unit conversion is from grams to moles, and no factor of two is needed at this stage.

Step 2: Use the reaction ratio and molar volume The coefficients of zinc and hydrogen in the balanced equation are both one. Therefore

$$n(H_2) = n(Zn) = 0.17584\,mol.$$

The coefficient two belongs to hydrochloric acid and must not be used to double the hydrogen amount. Applying the given molar volume,

$$V(H_2) = \frac{11.5}{65.4} \times 22.7\,L = 3.9916\,L.$$

Use 22.7 rather than substituting another commonly memorised STP molar volume.

Step 3: Round only the final volume The unrounded volume is just below four litres. Rounding it to the nearest integer gives four, since its fractional part is greater than one half. Keeping intermediate digits avoids changing the requested rounding.

Step 4: Conclusion The hydrogen volume is approximately $3.992\,L$, so the nearest-integer answer is $\boxed{4\,L}$.

Quick Tip to Solve: For zinc with excess acid, the Zn:H2 mole ratio is 1:1. Multiply mass divided by molar mass by the given molar volume.

82. The logarithm of equilibrium constant for the reaction $Pd^{2+} + 4Cl^- \rightleftharpoons PdCl_4^{2-}$ is _____ (Nearest integer)

Given : $\frac{2.303R\ T}{F} = 0.06\ V$

$$Pd^{2+}_{(aq)} + 2e^- \rightleftharpoons Pd(s) \quad E^{\ominus} = 0.83\ V$$

$$PdCl_4^{2-}(aq) + 2e^- \rightleftharpoons Pd(s) + 4Cl^-(aq) E^{\ominus} = 0.65\ V$$

[4 marks]

Solution:

Concept:

- Standard electrode potentials are combined through Gibbs energies: $\Delta G^\circ = -nFE^\circ$. Potentials themselves are not extensive quantities.
- For an equilibrium, $\Delta G^\circ = -RT \ln K$; thus $\log_{10} K = nE^\circ/(2.303RT/F)$ for the corresponding formal cell reaction.

Step 1: Combine the two half-reactions Keep the reduction $Pd^{2+} + 2\,e^- \longrightarrow Pd$, whose potential is $0.83\,V$. Reverse the second given reduction to obtain $Pd + 4\,Cl^- \longrightarrow PdCl_4{}^{2-} + 2\,e^-$. Adding cancels both the electrons and solid palladium, giving exactly $Pd^{2+} + 4\,Cl^- \longrightarrow PdCl_4{}^{2-}$. The formal cell potential for this combination is

$$E^\circ = 0.83 - 0.65 = 0.18\,V.$$

Its positive sign is consistent with favourable complex formation.

Step 2: Calculate the logarithm of the equilibrium constant Two electrons occur in each of the half-reactions, so the Gibbs-energy combination uses $n = 2$, even though the net complexation reaction contains no free electrons. Equating its Gibbs energy to the equilibrium expression gives

$$-2FE^\circ = -2.303RT \log_{10} K.$$

Substitution yields

$$\log_{10} K = \frac{2(0.18)}{0.06} = 6.$$

The coordination number four is the number of chloride ligands; it is not the electron-transfer number. The equilibrium constant itself is 10^6, whereas the quantity requested is its logarithm.

Step 3: Conclusion The logarithm is $\boxed{\log_{10} K = 6}$, a dimensionless quantity. Its nearest-integer value is $\boxed{6}$.

Quick Tip to Solve: Add half-reactions through Gibbs energies. For these two palladium reductions, subtract the potentials and use n=2, not four ligands.

83. The enthalpy change for the conversion of $\frac{1}{2}Cl_2(\,g)$ to Cl^-(aq) is (−) _____ $kJmol^{-1}$ (Nearest integer)

Given : $\Delta_{dis}H^{\ominus}_{Cl^{\theta}_{2(\,g)}} = 240\ kJ\ mol^{-1}, \Delta_{eg}H^{\ominus}_{Cl_{(g)}} = -350\ kJ\ mol^{-1}$,

$\Delta_{hyd}H^{\Theta}_{Cl^-_{(g)}} = -380\ kJ\ mol^{-1}$ [4 marks]

Solution:

Concept:

- Hess law states that the enthalpy change for a net conversion is the sum of the enthalpy changes of steps that produce exactly that conversion.
- Conversion of chlorine gas to aqueous chloride combines atomisation, electron gain by a gaseous atom, and hydration of the resulting gaseous ion.

Step 1: Atomise the required amount of chlorine The supplied dissociation enthalpy refers to breaking one mole of chlorine molecules into two moles of atoms. The question starts with only half a mole of chlorine molecules, so the first step is

$$\tfrac{1}{2}Cl_2(g) \longrightarrow Cl(g), \qquad \Delta H_1 = \frac{240}{2} = +120\,kJ\,mol^{-1}.$$

This supplies one mole of chlorine atoms, matching the one mole of chloride ions in the requested conversion. Using the full dissociation value would count twice the required atomisation.

Step 2: Add electron gain and hydration The gaseous atom accepts an electron:

$$Cl(g) + e^- \longrightarrow Cl^-(g), \qquad \Delta H_2 = -350\,kJ\,mol^{-1}.$$

The gaseous chloride ion then hydrates:

$$Cl^-(g) \longrightarrow Cl^-(aq), \qquad \Delta H_3 = -380\,kJ\,mol^{-1}.$$

Each value applies to one mole of atoms or ions, so neither of these terms is halved. Both processes are exothermic and retain their negative signs.

Step 3: Sum the steps and interpret the blank The intermediate gaseous atom and gaseous ion cancel in the thermochemical cycle. Adding the enthalpies gives

$$\Delta H = 120 - 350 - 380 = -610\,kJ\,mol^{-1}.$$

The printed answer format already places a minus sign before the blank. Consequently the number to enter in the blank is the positive magnitude 610, rather than a second negative sign.

Step 4: Conclusion The enthalpy change is $\boxed{-610\,kJ\,mol^{-1}}$. With the minus sign supplied, the nearest-integer entry is $\boxed{610}$.

Quick Tip to Solve: Halve only the Cl2 bond dissociation term. Electron gain and hydration already refer to one mole of chlorine atoms or ions.

84. On complete combustion, 0.492 g of an organic compound gave 0.792 g of CO_2. The % of carbon in the organic compound is _____ (Nearest integer) [4 marks]

Solution:

Concept:

- On complete combustion, the carbon in an organic sample is converted to carbon dioxide. Each mole of CO_2 contains one mole of carbon atoms.
- The carbon mass fraction in CO_2 is $12/44$. Percentage carbon in the original sample equals $100 m_C / m_{\text{sample}}$.

Step 1: Find the mass of carbon in carbon dioxide The collected carbon dioxide contains oxygen supplied during combustion as well as carbon from the sample. Its entire mass must therefore not be treated as carbon. Using the molar masses $M(CO_2) = 44\,\text{g mol}^{-1}$ and $M(C) = 12\,\text{g mol}^{-1}$,

$$m_C = 0.792 \times \frac{12}{44}\,\text{g} = 0.216\,\text{g}.$$

Equivalently, the carbon dioxide amount is $0.792/44 = 0.018\,\text{mol}$, so the carbon amount is also $0.018\,\text{mol}$.

Step 2: Calculate the percentage in the original compound The original compound weighed $0.492\,\text{g}$. Carbon percentage is based on this original sample mass, not on the product mass:

$$\%\text{C} = \frac{0.216}{0.492} \times 100 = 43.9024\%.$$

No molecular formula is needed, because complete combustion directly determines the total amount of carbon. The remaining sample mass belongs to other elements, whose identities are not needed for this calculation.

Step 3: Apply the requested rounding The percentage lies between 43 and 44 and is much closer to 44. Rounding only after the mass conversion and percentage calculation therefore gives 44. Rounding the carbon mass too early could unnecessarily reduce the accuracy.

Step 4: Conclusion The carbon content is approximately 43.90%; to the nearest integer, it is $\boxed{44\%}$. The numerical entry is $\boxed{44}$.

Quick Tip to Solve: First multiply CO2 mass by 12/44 to get carbon mass; then divide by the original sample mass and multiply by 100.

85. At 27°C, a solution containing 2.5 g of solute in 250.0 mL of solution exerts an osmotic pressure of 400 Pa. The molar mass of the solute is _____ g mol^{-1} (Nearest integer)
(Given : R $= 0.083$ L bar K^{-1} mol^{-1}) [4 marks]

Solution:

Concept:

- For a dilute solution, osmotic pressure obeys $\Pi V = nRT$, where n is the amount of solute, T is absolute temperature and V is solution volume.

- With solute mass w and molar mass M, use $n = w/M$, giving $M = wRT/(\Pi V)$. The pressure and volume units must agree with those of the supplied gas constant.

Step 1: Convert the temperature, volume and pressure The temperature must be in kelvin, so $T = 27 + 273 = 300\,\text{K}$. The solution volume is $250.0\,\text{mL} = 0.2500\,\text{L}$. Since the given gas constant uses bar, convert the pressure using $1\,\text{bar} = 10^5\,\text{Pa}$:

$$\Pi = \frac{400}{10^5} = 0.004\,\text{bar}.$$

Using 400 directly with this gas constant would mix pascals and bar and give an incorrect molar mass.

Step 2: Substitute in the osmotic-pressure equation The dissolved mass is $w = 2.5\,\text{g}$, and the volume is that of the complete solution. Rearrange the equation before inserting the values:

$$M = \frac{wRT}{\Pi V} = \frac{2.5 \times 0.083 \times 300}{0.004 \times 0.2500} = \frac{62.25}{0.001000} = 62250\,\text{g mol}^{-1}.$$

The litre and bar units cancel those in R, leaving grams per mole. The very small osmotic pressure at this mass concentration is consistent with a large molar mass.

Step 3: Conclusion The molar mass of the solute, to the nearest integer, is $\boxed{62250\,\text{g mol}^{-1}}$. Enter 62250.

Quick Tip to Solve: Before using $M = wRT/(\Pi V)$, align pressure and volume with the units of R: here use bar and litres.

86. The oxidation state of phosphorus in hypophosphoric acid is + _____. [4 marks]

Solution:

Concept:

- For a neutral molecule, the algebraic sum of the oxidation states of all atoms is zero. Hydrogen bonded to a nonmetal normally has oxidation state $+1$, while ordinary oxygen has -2.
- Hypophosphoric acid has the molecular formula $H_4P_2O_6$. Its two phosphorus atoms are equivalent and connected by a phosphorus–phosphorus bond; a bond between identical atoms contributes zero to oxidation-state assignment.

Step 1: Identify the correct acid formula Use hypophosphoric acid, $H_4P_2O_6$, rather than hypophosphorous acid, H_3PO_2, or phosphoric acid, H_3PO_4. These similar names refer to different compounds and give different phosphorus oxidation states. Hypophosphoric acid contains four hydrogen atoms, two phosphorus atoms and six oxygen atoms. There is no peroxide oxygen here, so each oxygen is assigned -2. Its hydrogen atoms contribute $+1$ each.

Step 2: Balance the oxidation-state sum Let the oxidation state of each phosphorus atom be x. The hydrogen contribution is $+4$, the oxygen contribution is -12, and the phosphorus contribution is $2x$. Electrical neutrality therefore requires

$$4(+1) + 2x + 6(-2) = 0.$$

Thus

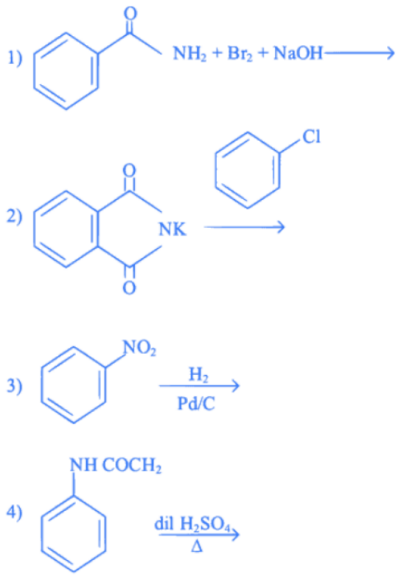
$$4 + 2x - 12 = 0 \quad \Longrightarrow \quad 2x = 8 \quad \Longrightarrow \quad x = +4.$$

As a check, substituting $+4$ gives $4 + 8 - 12 = 0$. The two equivalent phosphorus centres each have this value; it is not a total to be shared again between them.

Step 3: Conclusion The oxidation state of phosphorus in hypophosphoric acid is $\boxed{+4}$. Since the plus sign is already printed, enter 4.

Quick Tip to Solve: Write $H_4P_2O_6$ first: confusing hypophosphoric acid with hypophosphorous acid changes the answer.

87. How many of the transformations given below would result in aromatic amines? [4 marks]



Solution:

Concept:

- An aromatic amine has nitrogen directly bonded to an aromatic-ring carbon, as in aniline, $C_6H_5NH_2$. The presence of a benzene ring elsewhere in a molecule does not by itself make the amine aromatic.
- Hofmann bromamide degradation converts $RCONH_2$ to RNH_2, losing the carbonyl carbon. Nitro-group reduction and hydrolysis of an N-acyl aromatic amine can also generate an aromatic amine.

Step 1: Examine transformations 1 and 2 In transformation 1, benzamide reacts with bromine and sodium hydroxide. Hofmann rearrangement removes the carbonyl carbon and gives aniline:

$$C_6H_5CONH_2 \xrightarrow{Br_2/NaOH} C_6H_5NH_2.$$

Nitrogen is now directly attached to the phenyl ring, so transformation 1 qualifies. Transformation 2 uses potassium phthalimide with chlorobenzene. Gabriel synthesis requires nucleophilic substitution on a suitable alkyl halide; the aryl carbon–chlorine bond of chlorobenzene does not undergo that ordinary substitution. Thus this proposed Gabriel route does not furnish an aromatic amine and is

excluded.

Step 2: Examine transformations 3 and 4 In transformation 3, catalytic hydrogenation with H_2 and Pd/C reduces the nitro group of nitrobenzene to an amino group while retaining the nitrogen–aryl bond:

$$C_6H_5NO_2 + 3\,H_2 \xrightarrow{Pd/C} C_6H_5NH_2 + 2\,H_2O.$$

Transformation 4 hydrolyses the N-acyl group of the depicted anilide using dilute sulfuric acid and heat. Cleavage of the amide linkage regenerates the aromatic amino group. In the acidic reaction mixture it is protonated as the anilinium salt; the amine is obtained on the usual isolation/neutralisation. Hence both 3 and 4 are counted.

Step 3: Conclusion Transformations 1, 3 and 4 produce aromatic amines. The required number is $\boxed{3}$.

Quick Tip to Solve: For Gabriel synthesis, inspect the carbon bearing the halogen: chlorobenzene is an aryl halide and does not behave like an alkyl halide.

88. A $\rightarrow$ B

The rate constants of the above reaction at 200 K and 300 K are 0.03 min^{-1} and 0.05 min^{-1} respectively. The activation energy for the reaction is _____ J (Nearest integer)

(Given : $\ln 10 = 2.3$

$$R = 8.3\ J\ K^{-1}\ mol^{-1}$$

$$\log 5 = 0.70$$

$$\log 3 = 0.48$$

$\log 2 = 0.30$**)** [4 marks]

Solution:

Concept:

- The two-temperature Arrhenius equation is $\ln(k_2/k_1) = (E_a/R)(1/T_1 - 1/T_2)$, with temperatures in kelvin and E_a the molar activation energy.
- Convert the rate-constant ratio to a natural logarithm using the supplied $\ln 10 = 2.3$: $\ln x = 2.3 \log x$. Both rate constants have the same units, so their ratio is dimensionless.

Step 1: Evaluate the logarithm with the given approximations Take $T_1 = 200\,K$, $k_1 = 0.03\,min^{-1}$, $T_2 = 300\,K$ and $k_2 = 0.05\,min^{-1}$. Since the rate constant increases with temperature, the activation energy should be positive. Using the numerical logarithms supplied in the question,

$$\log \frac{k_2}{k_1} = \log \frac{5}{3} = 0.70 - 0.48 = 0.22,$$

$$\ln \frac{k_2}{k_1} = 2.3(0.22) = 0.506.$$

No conversion from minutes to seconds is required, because the identical rate-constant units cancel.

Step 2: Calculate and round the activation energy The reciprocal-temperature difference is

$$\frac{1}{200} - \frac{1}{300} = \frac{3-2}{600} = \frac{1}{600}\,\mathrm{K}^{-1}.$$

Rearrange the Arrhenius equation and substitute $R = 8.3\,\mathrm{J\,K^{-1}\,mol^{-1}}$:

$$E_a = \frac{R\ln(k_2/k_1)}{1/T_1 - 1/T_2} = 8.3 \times 0.506 \times 600 = 2519.88\,\mathrm{J\,mol^{-1}}.$$

Use the provided approximate logarithms throughout and round only this final result. The question expresses its requested numerical blank in joules; the activation energy obtained with the molar gas constant is conventionally a molar energy.

Step 3: Conclusion The activation energy is $\boxed{2520\,\mathrm{J\,mol^{-1}}}$, to the nearest integer. Enter 2520 in the stated blank.

Quick Tip to Solve: Use the logarithms supplied in the question before rounding: replacing them with calculator logarithms would change this numerical result.

89. For reaction : $\mathrm{SO_2(\,g) + \frac{1}{2}O_2(\,g) \rightleftharpoons SO_3(\,g)}$

$\mathrm{K_p = 2\times10^{12}}$ **at** $27^\circ\mathrm{C}$ **and** $1\ \mathrm{atm}$ **pressure. The** $\mathrm{K_c}$ **for the same reaction is** _____ $\times10^{13}$**. (Nearest integer)**

(Given $\mathrm{R = 0.082\ L\ atm\ K^{-1}\ mol^{-1}}$**)** [4 marks]

Solution:

Concept:

- For a gaseous equilibrium, $K_p = K_c(RT)^{\Delta n_g}$, where Δn_g is the sum of gaseous product coefficients minus the sum of gaseous reactant coefficients.
- Fractional stoichiometric coefficients must be retained exactly as printed. For $\mathrm{SO_2(g) + \frac{1}{2}\,O_2(g) \rightleftharpoons SO_3(g)}$, the change in gaseous amount is $1 - (1 + 1/2) = -1/2$.

Step 1: Determine the correct exponent and rearrange The reaction produces one mole of gaseous sulfur trioxide from one mole of sulfur dioxide and half a mole of oxygen, in the stoichiometric sense. Thus

$$\Delta n_g = 1 - 1 - \frac{1}{2} = -\frac{1}{2}.$$

Substitution into the equilibrium-constant relation gives

$$K_p = K_c(RT)^{-1/2} = \frac{K_c}{\sqrt{RT}}, \qquad K_c = K_p\sqrt{RT}.$$

The negative exponent is essential: dividing K_p by $\sqrt{RT}$ would reverse the rearrangement. Do not

double the equation, since doing so would also square its equilibrium constant.

Step 2: Calculate the coefficient of the requested power of ten At $27^\circ\mathrm{C}$, use $T = 300\,\mathrm{K}$ and the supplied $R = 0.082\,\mathrm{L\,atm\,K^{-1}\,mol^{-1}}$. Then

$$RT = 0.082(300) = 24.6,$$

$$K_c = 2 \times 10^{12}\sqrt{24.6} \simeq 9.9197 \times 10^{12} = 0.99197 \times 10^{13}.$$

The blank asks for the coefficient multiplying 10^{13}, rather than the full constant. Rounding 0.99197 to the nearest integer gives 1. The quoted total pressure does not enter this conversion; temperature and the reaction coefficients determine it.

Step 3: Conclusion $K_c \simeq 0.99197 \times 10^{13}$, so the coefficient to the nearest integer is $\boxed{1}$.

Quick Tip to Solve: Keep the printed half-mole coefficient when finding Δn_g, then round the coefficient of 10^{13}, not the full value of K_c.

---

90. The total pressure of a mixture of non-reacting gases $\mathrm{X}(0.6\ \mathrm{g})$ and $\mathrm{Y}(0.45\ \mathrm{g})$ in a vessel is $740\ \mathrm{mm}$ of Hg.

The partial pressure of the gas X is _____ mm of Hg. (Nearest Integer)

(Given : molar mass $\mathrm{X} = 20$ and $\mathrm{Y} = 45\ \mathrm{g\ mol^{-1}}$) [4 marks]

Solution:

Concept:

- Dalton's law states that a component's partial pressure is its mole fraction times the total pressure: $p_X = x_X P_{\text{total}}$. This applies to a mixture of non-reacting ideal gases.
- Calculate mole fractions from amounts of substance, using $n = m/M$ and $x_X = n_X/(n_X + n_Y)$. Mass fractions cannot replace mole fractions when the gases have different molar masses.

Step 1: Convert both gas masses into moles For gas X, divide its mass by its molar mass:

$$n_X = \frac{0.60\,\mathrm{g}}{20\,\mathrm{g\,mol^{-1}}} = 0.030\,\mathrm{mol}.$$

For gas Y, the corresponding calculation is

$$n_Y = \frac{0.45\,\mathrm{g}}{45\,\mathrm{g\,mol^{-1}}} = 0.010\,\mathrm{mol}.$$

Although the gas masses are fairly similar, the number of moles of X is three times that of Y, because X has the smaller molar mass. The total amount is therefore $0.030 + 0.010 = 0.040\,\mathrm{mol}$.

Step 2: Find the mole fraction and apply Dalton law The mole fraction of X is

$$x_X = \frac{0.030}{0.040} = \frac{3}{4} = 0.75.$$

Both gases occupy the same vessel at the same temperature. Their pressures are therefore proportional

to their mole numbers, and the total pressure of $740\,\text{mm Hg}$ can be used directly:

$$p_X = 0.75 \times 740 = 555\,\text{mm Hg}.$$

For a consistency check, $x_Y = 0.25$, so $p_Y = 185\,\text{mm Hg}$. The two partial pressures sum to $555 + 185 = 740\,\text{mm Hg}$, reproducing the given total pressure without needing a temperature or vessel volume.

Step 3: Conclusion The partial pressure of gas X is $\boxed{555\,\text{mm Hg}}$. The nearest-integer entry is 555.

Quick Tip to Solve: Convert masses to moles before taking the fraction. Here the mole ratio $X : Y = 3 : 1$, so X supplies three quarters of the pressure.