JEE Main 2023 Paper 1 (B.E./B.Tech) Jan 31 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 31 January 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ be three vectors. If $\vec{r}$ is a vector such that, $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then $25|\vec{r}|^2$ is equal to : [4 marks]

(A) 336

(B) 449

(C) 339

(D) 560

Correct Answer: (C) 339

Solution:

Concept:

- Two vectors have zero cross product exactly when they are parallel, provided the fixed vector is nonzero. Thus the cross-product condition determines a line of possible vectors.
- The dot product tests perpendicularity: $\vec{r} \cdot \vec{a} = 0$. In an orthonormal basis, the squared magnitude is the sum of the squares of the three components.

Step 1: Express the unknown vector using the cross product Subtract the two sides and use distributivity:

$$(\vec{r} - \vec{c}) \times \vec{b} = \vec{0}.$$

Since $\vec{b} = (1, -1, 2) \neq \vec{0}$, there is a real number λ such that $\vec{r} = \vec{c} + \lambda\vec{b}$. Consequently its components are

$$\vec{r} = (5 + \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (3 + 2\lambda)\hat{k}.$$

This form automatically satisfies the cross-product condition for every value of the parameter.

Step 2: Use perpendicularity to determine the parameter Substitute these components into the remaining condition:

$$\vec{r} \cdot \vec{a} = (5 + \lambda) + 2(-3 - \lambda) + 3(3 + 2\lambda) = 8 + 5\lambda = 0.$$

Hence $\lambda = -8/5$, and the unique vector is

$$\vec{r} = \frac{17}{5}\hat{i} - \frac{7}{5}\hat{j} - \frac{1}{5}\hat{k}.$$

As a check, its dot product with $\vec{a}$ is $(17 - 14 - 3)/5 = 0$.

Step 3: Calculate the requested squared magnitude Square all three components before adding; the negative components contribute positive squares:

$$25|\vec{r}|^2 = 17^2 + (-7)^2 + (-1)^2 = 289 + 49 + 1 = 339.$$

The values 336, 449 and 560 do not equal this sum, so they are excluded.

Step 4: Conclusion $25|\vec{r}|^2 = 339$. Hence, the correct option is (C).

Quick Tip to Solve: Use the zero cross product to write a one-parameter vector first; the dot product then gives one linear equation.

2. Let $f : \mathbb{R} - \{2, 6\} \to \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$.

Then range of f is [4 marks]

(A) $\left(-\infty, -\frac{21}{4}\right] \cup [1, \infty)$

(B) $\left(-\infty, -\frac{21}{4}\right) \cup (0, \infty)$

(C) $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$

(D) $\left(-\infty, -\frac{21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

Correct Answer: (C) $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$

Solution:

Concept:

- A real number y belongs to the range if $f(x) = y$ has at least one real solution in the stated domain. For a quadratic equation in x, a nonnegative discriminant gives real roots.
- When the leading coefficient of that quadratic vanishes, solve the remaining linear equation separately. Also check that a real root is not one of the excluded poles.

Step 1: Convert the range problem into an equation Let $y = (x + 1)^2/(x^2 - 8x + 12)$, where $x \neq 2, 6$. Cross multiplication gives

$$(y - 1)x^2 + (-8y - 2)x + (12y - 1) = 0.$$

For $y \neq 1$, this is a quadratic equation. Its roots are real precisely when its discriminant is nonnegative. We must subsequently check the domain because multiplying by a denominator does not itself preserve the original restrictions.

Step 2: Find all admissible output values The discriminant simplifies to

$$\begin{aligned} D &= (-8y - 2)^2 - 4(y - 1)(12y - 1) \\ &= 64y^2 + 32y + 4 - (48y^2 - 52y + 4) \\ &= 16y^2 + 84y = 4y(4y + 21). \end{aligned}$$

Thus $y \leq -21/4$ or $y \geq 0$. For the exceptional value $y = 1$, the equation becomes $-10x + 11 = 0$, giving the valid input $x = 11/10$. Therefore 1 is included rather than being lost through the quadratic test.

Step 3: Check poles and boundary values At $x = 2$, the cross-multiplied left side is -9, and at $x = 6$ it is -49; neither is zero for any y. Thus none of the real roots supplied by the discriminant is an excluded pole. The value 0 is attained at $x = -1$, and the discriminant-zero endpoint $-21/4$ is attained at $x = 16/5$. Options (A) and (D) omit attainable nonnegative values, while (B) wrongly omits the endpoints.

Step 4: Conclusion The range is $(-\infty, -21/4] \cup [0, \infty)$. Hence, the correct option is (C).

Quick Tip to Solve: After a discriminant range test, check the output where the quadratic coefficient vanishes and verify that its roots avoid the poles.

3. The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0, x \in \mathbb{R}$ has : [4 marks]

(A) two solutions and both are negative

(B) two solutions and only one of them is negative

(C) four solutions two of which are negative

(D) no solution

Correct Answer: (A) two solutions and both are negative

Solution:

Concept:

- The substitution $t = e^{x}$ gives $t > 0$, and every positive t corresponds to exactly one real $x = \log t$. Negative algebraic roots must therefore be discarded.
- The sign of $\log t$ is negative for $0 < t < 1$, zero for $t = 1$, and positive for $t > 1$. A factorisation can determine both the count and the signs without approximating logarithms.

Step 1: Factor the polynomial in the exponential Put $t = e^{x}$. The given equation becomes

$$t^4 + 8t^3 + 13t^2 - 8t + 1 = 0.$$

Observe that this polynomial is $(t^2 + 4t - 1)^2 - t^2$, a difference of squares. Hence

$$(t^2 + 3t - 1)(t^2 + 5t - 1) = 0.$$

Expanding the product reproduces the middle coefficient $15 - 2 = 13$, so the factorisation retains the printed equation exactly.

Step 2: Retain only positive roots The first factor gives $t = (-3 \pm \sqrt{13})/2$, and the second gives $t = (-5 \pm \sqrt{29})/2$. Each minus-sign root is negative and cannot equal an exponential. The two surviving roots are

$$t_1 = \frac{\sqrt{13} - 3}{2}, \qquad t_2 = \frac{\sqrt{29} - 5}{2}.$$

They are positive because $\sqrt{13} > 3$ and $\sqrt{29} > 5$. They are distinct: if both factors vanished at the same positive t, subtracting their equations would give $2t = 0$, impossible.

Step 3: Determine the signs of the two real solutions Since $\sqrt{13} < 5$ and $\sqrt{29} < 7$, both positive roots are less than 1. Therefore $x_1 = \log t_1 < 0$ and $x_2 = \log t_2 < 0$. There are exactly two real solutions. This excludes (B), which requires a nonnegative solution, (C), which counts inadmissible roots, and (D), which denies the positive roots.

Step 4: Conclusion The equation has two real solutions, and both are negative. Hence, the correct option is (A).

Quick Tip to Solve: For a polynomial in e^{x}, count positive roots only; compare each with 1 to determine the signs of the logarithms.

4. Let (a, b) $\subset (0, 2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0, \theta \in (0, 2\pi)$, holds. If $\alpha x^2 + \beta x + \sin^{-1}\left(x^2 - 6x + 10\right) + \cos^{-1}\left(x^2 - 6x + 10\right) = 0$ and $\alpha - \beta = b - a$, then α is equal to :

[4 marks]

(A) $\frac{\pi}{16}$
(B) $\frac{\pi}{48}$
(C) $\frac{\pi}{8}$
(D) $\frac{\pi}{12}$

Correct Answer: (D) $\frac{\pi}{12}$

Solution:

Concept:

- For $-1 \le u \le 1$, principal values satisfy $\sin^{-1} u + \cos^{-1} u = \pi/2$. The inverse sine is increasing on this domain.
- A real inverse trigonometric expression requires its argument to lie in $[-1, 1]$. Apply this restriction before treating an equation containing it as an ordinary quadratic.

Step 1: Find the largest interval of angles Use the complementary inverse-function identity with $u = \sin\theta$:

$$\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) = 2\sin^{-1}(\sin\theta) - \frac{\pi}{2}.$$

It is positive exactly when $\sin^{-1}(\sin\theta) > \pi/4$, equivalently $\sin\theta > 1/\sqrt{2}$. In $(0, 2\pi)$, this gives the single largest interval $(a, b) = (\pi/4, 3\pi/4)$. Thus $\alpha - \beta = b - a = \pi/2$. The endpoints are excluded because the original inequality is strict.

Step 2: Use the domain of the second inverse expression Its common argument is

$$x^2 - 6x + 10 = (x - 3)^2 + 1.$$

This is at least 1 for every real x. To be an allowable argument for either inverse function it must also be at most 1. Therefore $(x - 3)^2 = 0$, so $x = 3$ is the only permissible input. At this input, the inverse sine is $\pi/2$ and the inverse cosine is zero.

Step 3: Solve for the coefficient Substitute $x = 3$ into the given equation:

$$9\alpha + 3\beta + \frac{\pi}{2} = 0.$$

Since $\beta = \alpha - \pi/2$, this reduces to

$$12\alpha - \pi = 0, \qquad \alpha = \frac{\pi}{12}.$$

The other three proposed values fail this equation, so the domain restriction and interval width together select a unique coefficient.

Step 4: Conclusion $\alpha = \pi/12$. Hence, the correct option is (D).

Quick Tip to Solve: Complete the square inside inverse functions first: an argument bounded below by 1 can meet their real domain only at its minimum.

5. Let $\alpha > 0$. If $\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}\,dx = \frac{16+20\sqrt{2}}{15}$, then α is equal to : [4 marks]

(A) 4

(B) 2

(C) $2\sqrt{2}$

(D) $\sqrt{2}$

Correct Answer: (B) 2

Solution:

Concept:

- Rationalisation uses $(\sqrt{x+\alpha}-\sqrt{x})(\sqrt{x+\alpha}+\sqrt{x}) = \alpha$, which is positive here. It removes the difference of radicals from the denominator.
- The substitution $x = \alpha t$ changes the interval to $[0, 1]$ and separates the parameter as a positive power. Standard power integration then evaluates the remaining constant.

Step 1: Rationalise and scale the variable Write the integral as $I(\alpha)$. Multiplying numerator and denominator by the conjugate gives

$$I(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} x\left(\sqrt{x+\alpha}+\sqrt{x}\right)dx.$$

Now put $x = \alpha t$, so $dx = \alpha\,dt$. Because $\alpha > 0$, the limits become 0 and 1 and the radicals scale by $\sqrt{\alpha}$. Hence

$$I(\alpha) = \alpha^{3/2}\int_0^1 \left(t\sqrt{t+1}+t^{3/2}\right)dt.$$

Step 2: Evaluate the constant integral For the first term set $u = t+1$, so $t = u-1$:

$$\int_0^1 t\sqrt{t+1}\,dt = \left[\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right]_1^2 = \frac{4\sqrt{2}+4}{15}.$$

The second term contributes $\int_0^1 t^{3/2}\,dt = 2/5$. Consequently

$$I(\alpha) = \alpha^{3/2}\frac{10+4\sqrt{2}}{15}.$$

Both the coefficient and the parameter power are positive.

Step 3: Compare with the prescribed value The required numerator can be written as

$$16+20\sqrt{2} = 2\sqrt{2}\,(10+4\sqrt{2}).$$

Cancelling the common positive factor yields $\alpha^{3/2} = 2\sqrt{2} = 2^{3/2}$, so $\alpha = 2$. The map $\alpha \mapsto \alpha^{3/2}$ is strictly increasing for positive α, excluding 4, $2\sqrt{2}$ and $\sqrt{2}$ without further integration.

Step 4: Conclusion $\alpha = 2$. Hence, the correct option is (B).

Quick Tip to Solve: When a parameter appears in both a radical and the upper limit, scale the variable by that parameter before integrating.

6. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha(>0)$, and the mean and standard deviation of marks of class B of n students be respectively 55 and 30 $-\alpha$. If the mean and variance of the marks of the combined class of $100+\mathrm{n}$ studants are respectively 50 and 350 , then the sum of variances of classes A and B is : [4 marks]

(A) 450

(B) 900

(C) 650

(D) 500

Correct Answer: Both 500 (D) and 900 (B) satisfy the printed conditions.

Solution:

Concept:

- The mean of combined groups is the population-weighted mean. Their combined variance includes both within-group variances and squared displacements of group means from the overall mean.
- A standard deviation is nonnegative, and zero is valid when every observation in that group is identical. A condition of strict positivity on one group does not impose strict positivity on the other.

Step 1: Determine the size of class B The combined mean gives

$$\frac{100(40)+55n}{100+n}=50.$$

Thus $4000+55n=5000+50n$, implying $n=200$. The group proportions are therefore $1/3$ for class A and $2/3$ for class B. Their mean displacements from the overall mean 50 are respectively -10 and 5.

Step 2: Apply the combined-variance identity Using the stated standard deviations, the variance equation is

$$350=\frac{100(\alpha^2+100)+200((30-\alpha)^2+25)}{300}.$$

After multiplication and simplification this becomes

$$\alpha^2+2(30-\alpha)^2=900,$$

$$3\alpha^2-120\alpha+900=0, \qquad (\alpha-10)(\alpha-30)=0.$$

Both roots satisfy $\alpha>0$; for either root the second standard deviation is nonnegative.

Step 3: Check both admissible cases If $\alpha=10$, the two variances sum to $10^2+20^2=500$, giving option (D). If $\alpha=30$, they sum to $30^2+0^2=900$, giving option (B). The latter is a valid case: all 200 students in class B may have marks 55, so their standard deviation is zero. Nothing printed rules this out. The values 450 and 650 do not arise from the variance equation. A unique answer of 500 would require the additional restriction $30-\alpha>0$.

Step 4: Conclusion As printed, both 500 (D) and 900 (B) are possible. The usual answer 500 follows only if class B is also required to have positive standard deviation; the stated conditions do not impose that restriction.

Quick Tip to Solve: Keep a zero standard deviation unless the question explicitly excludes it; test every algebraic root against the actual conditions.

7. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to : [4 marks]

(A) 24

(B) $\frac{381}{4}$

(C) 9

(D) $\frac{33}{4}$

Correct Answer: (A) 24

Solution:

Concept:

- An arithmetic progression has $a_k = a_1 + (k-1)d$. Fixing one term lets us express every other term using the common difference alone.
- A quadratic with positive leading coefficient has its minimum at its vertex. For a positive number n of terms, $S_n = n[2a_1 + (n-1)d]/2$, so a zero sum forces the bracket to vanish.

Step 1: Express and minimise the specified product Since $a_7 = 3$, we have $a_1 = 3 - 6d$ and $a_4 = 3 - 3d$. Their product is

$$a_1 a_4 = (3 - 6d)(3 - 3d) = 18d^2 - 27d + 9.$$

Complete the square to determine the minimum:

$$18d^2 - 27d + 9 = 18\left(d - \frac{3}{4}\right)^2 - \frac{9}{8}.$$

The square cannot be negative, so the minimum occurs uniquely at $d = 3/4$. Therefore $a_1 = 3 - 6(3/4) = -3/2$. This does not require either factor to be positive.

Step 2: Find the positive number of terms Apply the zero-sum condition with these values:

$$0 = S_n = \frac{n}{2}\left[-3 + \frac{3}{4}(n-1)\right].$$

A sum of the first n terms has positive integer n, so the factor n is nonzero. Consequently

$$-3 + \frac{3}{4}(n-1) = 0, \qquad n - 1 = 4, \qquad n = 5.$$

Indeed the five terms are $-3/2, -3/4, 0, 3/4, 3/2$, which cancel in opposite pairs.

Step 3: Evaluate the factorial and the distant term The subscript in the question is $n(n+2) = 5 \cdot 7 = 35$; it is an index, not a product of terms. Hence

$$a_{35} = -\frac{3}{2} + 34 \cdot \frac{3}{4} = 24.$$

Also $n! = 5! = 120$. Thus

$$n! - 4a_{n(n+2)} = 120 - 4(24) = 24.$$

The fractions $381/4$, $33/4$, and the value 9 do not equal this final difference.

Step 4: Conclusion $n! - 4a_{n(n+2)} = 24$. Hence, the correct option is (A).

Quick Tip to Solve: Express the progression about its known seventh term, minimise the product in the common difference, and only then use the sum condition.

8. The complex number $z = \frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ is equal to : [4 marks]

(A) $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$
(B) $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$
(C) $\sqrt{2}i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$
(D) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$
Correct Answer: (D) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

Solution:

Concept:

- The modulus and principal argument of $a + ib$ determine its polar representation $r(\cos\theta + i\sin\theta)$. Here $r = \sqrt{a^2+b^2}$, with the quadrant fixing θ.
- For division of two nonzero complex numbers, divide their moduli and subtract their arguments. Arguments that differ by 2π describe the same number.

Step 1: Express the numerator in polar form The numerator $-1 + i$ lies in the second quadrant. Its modulus is $\sqrt{(-1)^2+1^2} = \sqrt{2}$, and its argument is $3\pi/4$, rather than $\pi/4$. Thus

$$i - 1 = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right).$$

The denominator already has modulus one and argument $\pi/3$.

Step 2: Divide and compare the printed options Subtracting the denominator argument gives

$$\arg z = \frac{3\pi}{4} - \frac{\pi}{3} = \frac{5\pi}{12}, \qquad |z| = \sqrt{2}.$$

Therefore

$$z = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right).$$

Option (A) has modulus one, so it cannot be the quotient. Option (B) has argument $\pi/12$, which is different. In option (C), multiplication by i adds $\pi/2$, giving argument $\pi/2 - 5\pi/12 = \pi/12$; hence it too differs from the required quotient.

Step 3: Conclusion The quotient has modulus $\sqrt{2}$ and argument $5\pi/12$. Hence, the correct option is (D).

Quick Tip to Solve: Fix the numerator quadrant before subtracting arguments; $-1 + i$ has argument $3\pi/4$, not $\pi/4$.

9. Among the relations

$$S = \left\{(a, b) : a, b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0\right\}$$

and $T = \left\{(a, b) : a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\right\}$, [4 marks]

(A) S is transitive but T is not

(B) both S and T are symmetric

(C) neither S nor T is transitive

(D) T is symmetric but S is not

Correct Answer: (D) T is symmetric but S is not

Solution:

Concept:

- A relation is symmetric if (a, b) implies (b, a), and transitive if (a, b) and (b, c) imply (a, c). A single counterexample disproves either property.
- The integers are closed under negation and addition. These two facts apply directly to differences of squares in relation T.

Step 1: Test symmetry and transitivity of T If $(a, b) \in T$, then $a^2 - b^2 = n$ for some integer n. Reversing the pair gives

$$b^2 - a^2 = -n \in \mathbb{Z},$$

so T is symmetric. If also $(b, c) \in T$, write $b^2 - c^2 = m \in \mathbb{Z}$. Then

$$a^2 - c^2 = (a^2 - b^2) + (b^2 - c^2) = n + m \in \mathbb{Z}.$$

Thus T is transitive as well.

Step 2: Test S using nonzero counterexamples Take $a = 1$ and $b = -3$. Then $2 + a/b = 5/3 > 0$, so $(1, -3) \in S$. However $2 + b/a = -1 < 0$, so $(-3, 1) \notin S$. Hence S is not symmetric. It is also not transitive: $(-3, -1) \in S$ because $2 + 3 = 5 > 0$, and $(-1, 1) \in S$ because $2 - 1 = 1 > 0$, but $(-3, 1) \notin S$ because $2 - 3 = -1 < 0$. All these values satisfy the nonzero domain restriction. Options (A) and (C) wrongly deny transitivity of T, and (B) wrongly asserts symmetry of S.

Step 3: Conclusion Relation T is symmetric, whereas S is not symmetric. Hence, the correct option is (D).

Quick Tip to Solve: For relations defined by an integer-valued difference, reverse and add the differences; for ratio inequalities, try numbers with opposite signs.

10. The absolute minimum value, of the function

$f(x) = |x^2 - x + 1| + [x^2 - x + 1]$,

where $[t]$ denotes the greatest integer function, in the interval $[-1, 2]$, is : [4 marks]

(A) $\frac{3}{4}$

(B) $\frac{3}{2}$

(C) $\frac{1}{4}$

(D) $\frac{5}{4}$

Correct Answer: (A) $\frac{3}{4}$

Solution:

Concept:

- Completing the square gives the range of a quadratic without differentiation: $x^2 - x + 1 = (x - 1/2)^2 + 3/4$.
- For $u \geq 0$, $|u| = u$, and $\lfloor u \rfloor \geq 0$. If the smallest attainable u lies below one, its greatest integer is zero.

Step 1: Find the range of the quadratic Set $u = x^2 - x + 1$. Completing the square gives

$$u = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}.$$

Equality holds at $x = 1/2$, which lies in the given interval $[-1, 2]$. Both endpoints give $u = 3$, and the quadratic is continuous, so its range over this interval is $[3/4, 3]$. In particular, it is always positive, allowing the absolute value to be removed.

Step 2: Establish and attain a global lower bound The required function becomes $f(x) = u + \lfloor u \rfloor$. Since $u \geq 3/4 > 0$, its floor is nonnegative. Consequently

$$f(x) \geq u \geq \frac{3}{4}.$$

At $x = 1/2$, $u = 3/4$ and $\lfloor 3/4 \rfloor = 0$, yielding

$$f(1/2) = \frac{3}{4} + 0 = \frac{3}{4}.$$

This attains the lower bound, so it is the absolute minimum, despite the jumps caused by the floor function. Option (C) is below the proven bound. Options (B) and (D) exceed the value already attained and therefore cannot be the minimum.

Step 3: Conclusion The absolute minimum value is $\boxed{3/4}$, attained at $x = 1/2$. Hence, the correct option is (A).

Quick Tip to Solve: Bound the positive quadratic first, then check the floor at its vertex; a floor function does not require differentiation.

11. If $\phi(x) = \frac{1}{\sqrt{x}} \int_{\frac{\pi}{4}}^{x} \left(4\sqrt{2}\sin t - 3\phi'(t)\right) dt, x > 0$,

then $\phi'\left(\frac{\pi}{4}\right)$ **is equal to :** [4 marks]

(A) $\frac{4}{6+\sqrt{\pi}}$

(B) $\frac{4}{6-\sqrt{\pi}}$

(C) $\frac{8}{\sqrt{\pi}}$

(D) $\frac{8}{6+\sqrt{\pi}}$

Correct Answer: (D) $\frac{8}{6+\sqrt{\pi}}$

Solution:

Concept:

- For a continuous integrand g, the fundamental theorem gives $\frac{d}{dx}\int_a^x g(t)\,dt = g(x)$. An integral with identical upper and lower limits is zero.
- Multiply by $\sqrt{x}$ before differentiating. The product rule then yields a relation involving only $\phi(x)$ and $\phi\prime(x)$, which can be evaluated at the lower limit.

Step 1: Use the integral at its lower limit Write $a = \pi/4$. At $x = a$, the integral in the question vanishes, and $\sqrt{a} \neq 0$. Hence $\phi(a) = 0$. Multiplying the given equation by $\sqrt{x}$, we have

$$\sqrt{x}\,\phi(x) = \int_a^x \left(4\sqrt{2}\sin t - 3\phi\prime(t)\right) dt.$$

This avoids differentiating an inverse-square-root factor times an integral.

Step 2: Differentiate and substitute the known value Applying the product rule on the left and the fundamental theorem on the right gives

$$\frac{\phi(x)}{2\sqrt{x}} + \sqrt{x}\,\phi\prime(x) = 4\sqrt{2}\sin x - 3\phi\prime(x).$$

At $x = a$, use $\phi(a) = 0$, $\sqrt{a} = \sqrt{\pi}/2$ and $\sin a = 1/\sqrt{2}$. Thus

$$\left(3 + \frac{\sqrt{\pi}}{2}\right)\phi\prime(a) = 4, \qquad \phi\prime(a) = \frac{8}{6+\sqrt{\pi}}.$$

Option (A) is smaller by a factor of two. Option (B) has the wrong sign in the denominator as well as the wrong numerator. Option (C) neglects the $-3\phi\prime(t)$ contribution inside the integral.

Step 3: Conclusion $\phi\prime(\pi/4) = \boxed{\frac{8}{6+\sqrt{\pi}}}$. Hence, the correct option is (D).

Quick Tip to Solve: At the lower integration limit, first find the function value from the zero integral; only then differentiate the multiplied equation.

12. Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is : [4 marks]

(A) $\frac{5}{2}$

(B) 3

(C) 2

(D) $\frac{3}{2}$

Correct Answer: (C) 2

Solution:

Concept:

- For a horizontal hyperbola centred at (h, k), the foci are $(h \pm c, k)$, with $c^2 = a^2 + b^2$ and eccentricity $e = c/a$.
- The length of the complete latus rectum of a hyperbola is $2b^2/a$. Translating the centre changes positions but does not change this length.

Step 1: Identify the centre and semi-axes The midpoint of the two foci $(1 - \sqrt{2}, 0)$ and $(1 + \sqrt{2}, 0)$ is $(1, 0)$. The distance of either focus from this centre is $c = \sqrt{2}$, not $1 + \sqrt{2}$. Given $e = \sqrt{2}$,

$$a = \frac{c}{e} = \frac{\sqrt{2}}{\sqrt{2}} = 1.$$

The hyperbola identity now gives

$$b^2 = c^2 - a^2 = 2 - 1 = 1.$$

Thus both squared semi-axis parameters equal one, while the hyperbola remains translated one unit to the right.

Step 2: Calculate the full focal chord length The standard form is $(x - 1)^2 - y^2 = 1$. At a focus, $x - 1 = \pm\sqrt{2}$, so the equation gives $y^2 = 2 - 1 = 1$. The latus rectum therefore runs from $y = -1$ to $y = 1$, and its full length is two units. Equivalently,

$$L = \frac{2b^2}{a} = \frac{2 \cdot 1}{1} = 2.$$

This direct substitution also checks that the full chord, rather than its half-length, is being calculated. Options (A), (B) and (D) give lengths different from the computed focal chord and do not satisfy this standard formula.

Step 3: Conclusion The length of the latus rectum is $\boxed{2}$ units. Hence, the correct option is (C).

Quick Tip to Solve: Measure c from the midpoint of the foci; the translation of the hyperbola has no effect on its latus rectum length.

13.

$$\lim_{x\to\infty} \frac{(\sqrt{3x+1}+\sqrt{3x-1})^6+(\sqrt{3x+1}-\sqrt{3x-1})^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$$

[4 marks]

(A) is equal to 9

(B) is equal to $\frac{27}{2}$

(C) does not exist

(D) is equal to 27

Correct Answer: (D) is equal to 27

Solution:

Concept:

- As $x \to \infty$, divide the numerator by its leading power x^3 and the denominator by x^6. This also accounts for the extra factor x^3 outside the fraction.
- The functions $\sqrt{3+1/x}$ and $\sqrt{3-1/x}$ both approach $\sqrt{3}$. Also $\sqrt{1-1/x^2} \to 1$. Their sums remain finite and can be raised to the sixth power.

Step 1: Normalize each radical expression Denote the numerator by $N(x)$ and the denominator by $D(x)$. For sufficiently large positive x, factor $\sqrt{x}$ out of each numerator radical. This gives

$$\frac{N(x)}{x^3} = \left(\sqrt{3+1/x}+\sqrt{3-1/x}\right)^6+\left(\sqrt{3+1/x}-\sqrt{3-1/x}\right)^6.$$

Similarly, factor x out of each denominator bracket:

$$\frac{D(x)}{x^6} = \left(1+\sqrt{1-1/x^2}\right)^6+\left(1-\sqrt{1-1/x^2}\right)^6.$$

The expression to be evaluated is exactly $(N(x)/x^3)/(D(x)/x^6)$.

Step 2: Take the two finite limits and divide The normalized numerator tends to

$$(2\sqrt{3})^6+0^6 = 64\cdot 27 = 1728,$$

and the normalized denominator tends to

$$2^6+0^6 = 64.$$

Since the latter is nonzero, the quotient limit exists and is

$$\frac{1728}{64} = 27.$$

The printed numerator is $(\sqrt{3x+1}+\sqrt{3x-1})^6$, so this leading-power calculation unambiguously yields 27; 9 and $27/2$ are different constants, and nonexistence is ruled out by the finite nonzero denominator limit.

Step 3: Conclusion The limit is $\boxed{27}$. Hence, the correct option is (D).

Quick Tip to Solve: Normalize by the leading powers before expanding sixth powers; the small difference terms then tend to zero immediately.

14. Let $y = y(x)$ be the solution of the differential equation

$$\left(3y^2 - 5x^2\right) y \, \mathrm{d}x + 2x \left(x^2 - y^2\right) \mathrm{d}y = 0$$

such that $y(1) = 1$. Then $\left|(y(2))^3 - 12y(2)\right|$ is equal to : [4 marks]

(A) 64

(B) $16\sqrt{2}$

(C) 32

(D) $32\sqrt{2}$

Correct Answer: The stated initial-value problem has no differentiable solution; formal integration suggests option (D).

Solution:

Concept:

- The differential equation is homogeneous in x, y. Put $y = vx$, so $dy = v\,dx + x\,dv$, to reduce it to a separable equation.
- An implicit first integral is sufficient because the requested expression combines y^3 and x^2y. There is no need to solve a cubic explicitly for y.

Step 1: Reduce and separate the equation Substituting $y = vx$ into the given equation and dividing by x^3 yields

$$v(3v^2 - 5)\,dx + 2(1 - v^2)(v\,dx + x\,dv) = 0.$$

Combining the terms in dx gives

$$v(v^2 - 3)\,dx + 2x(1 - v^2)\,dv = 0,$$

hence

$$\frac{2(1 - v^2)}{v(3 - v^2)}\,dv = \frac{dx}{x}.$$

We use the branch fixed by the supplied initial value $x = y = 1$.

Step 2: Integrate and determine the constant The partial fraction decomposition is

$$\frac{2(1 - v^2)}{v(3 - v^2)} = \frac{2}{3v} - \frac{4v}{3(3 - v^2)}.$$

Integration therefore gives

$$\frac{2}{3}\ln|v| + \frac{2}{3}\ln|3 - v^2| = \ln x + C.$$

On the branch through $v = 1, x = 1$, this becomes

$$v(3 - v^2) = 2x^{3/2}.$$

Substitute $v = y/x$ and multiply by x^3:

$$3x^2y - y^3 = 2x^{9/2}.$$

The constant is two because $3(1)^2(1) - (1)^3 = 2$.

Step 3: Evaluate the requested combination At $x = 2$, the implicit relation directly gives

$$12y(2) - (y(2))^3 = 2 \cdot 2^{9/2} = 32\sqrt{2}.$$

Taking the absolute value removes the reversed sign in the question:

$$\left|(y(2))^3 - 12y(2)\right| = 32\sqrt{2}.$$

The other printed values 64, $16\sqrt{2}$ and 32 differ from this result; keeping the exponent $9/2$ during resubstitution avoids those factors-of-two errors.

Step 4: Check existence at the initial point There is a defect in the stated initial-value problem. Writing the equation in terms of a derivative gives $(3y^2 - 5x^2)y + 2x(x^2 - y^2)y\prime = 0$. At $x = 1, y = 1$, this requires $-2 + 0 \cdot y\prime(1) = 0$, which is impossible for a finite derivative. Moreover $v(3 - v^2)$ has local maximum two at $v = 1$, while $2x^{3/2} > 2$ for $x > 1$. Thus the formal curve through the initial point cannot supply the assumed real function up to $x = 2$.

Step 5: Conclusion Formal integration gives $\boxed{32\sqrt{2}}$, corresponding to option (D). Strictly, the printed initial-value problem has no differentiable solution, so $y(2)$ is undefined; (D) is its formal intended answer rather than a value attained by the stated solution.

Quick Tip to Solve: Check the differential equation at the initial point before dividing: a zero derivative coefficient with a nonzero remaining term makes the initial-value problem inconsistent.

15. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $\mathrm{P}\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1 + a)x - (1 - a)y = 0$, is equal to : [4 marks]

(A) $(0, 4]$

(B) $(4, \infty)$

(C) $(2, 12]$

(D) $(8, \infty)$

Correct Answer: (D) $(8, \infty)$

Solution:

Concept:

- A chord midpoint M satisfies $(P - M) \cdot (M - C) = 0$, since the radius from the centre C to the midpoint is perpendicular to the chord.
- Two distinct real chord midpoints on the specified line require a quadratic equation with positive discriminant.

Step 1: Find the centre and radius Dividing the circle equation by 2 gives its centre as $C = ((1 + a)/4, (1 - a)/4)$. Write $P = (u, v)$, where $u = (1 + a)/2$ and $v = (1 - a)/2$. Thus $C = P/2$, $u - v = a$, and $u^2 + v^2 = (1 + a^2)/2$. Substitution confirms that P lies on the circle; hence every genuine chord from this fixed point has a well-defined midpoint.

Step 2: Put the midpoint on the bisecting line A point on $x + y = 0$ has the form $M = (t, -t)$. The chord-midpoint condition therefore becomes

$$(u - t)(t - u/2) + (v + t)(-t - v/2) = 0.$$

Expanding and collecting terms gives

$$2t^2 - \frac{3a}{2}t + \frac{1 + a^2}{4} = 0.$$

Each real root determines a chord through P, with its other endpoint equal to $2M - P$.

Step 3: Require two distinct chords The discriminant is

$$\Delta = \frac{9a^2}{4} - 8\frac{1 + a^2}{4} = \frac{a^2 - 8}{4}.$$

Thus two distinct midpoints, and therefore two distinct chords, occur exactly when $a^2 > 8$. A repeated root at $a^2 = 8$ supplies only one midpoint and is excluded. No midpoint equals P, since the coordinates of P sum to 1 whereas those of M sum to zero. Consequently neither counted chord degenerates to the tangent at P. Options (A), (B) and (C) include values that fail the strict discriminant condition.

Step 4: Conclusion The required set is $\boxed{(8, \infty)}$. Hence, the correct option is (D).

Quick Tip to Solve: Parameterise a chord midpoint on the given line and impose its perpendicularity to the chord; count distinct chords by the discriminant.

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16. Let the plane $\mathrm{P} : 8x + \alpha_1 y + \alpha_2 z + 12 = 0$ be parallel to the line $\mathrm{L} : \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1 , then the distance between P and L is : [4 marks]

(A) $\frac{6}{\sqrt{14}}$

(B) $\sqrt{14}$

(C) $\sqrt{\frac{2}{7}}$

(D) $\sqrt{\frac{7}{2}}$

Correct Answer: (B) $\sqrt{14}$

Solution:

Concept:

- A line is parallel to a plane when its direction vector is perpendicular to the plane normal.
- For $Ax + By + Cz + D = 0$, the distance from (x_0, y_0, z_0) is $|Ax_0 + By_0 + Cz_0 + D|/\sqrt{A^2 + B^2 + C^2}$.

Step 1: Use the given intercept The intercept on the y-axis is 1, so $(0, 1, 0)$ lies on the plane. Substituting it gives $\alpha_1 + 12 = 0$, hence $\alpha_1 = -12$. This is the signed intercept, so its substitution directly fixes the coefficient rather than only its magnitude.

Step 2: Apply the parallelism condition The direction vector of the given line is $(2, 3, 5)$, and the

plane normal is $(8, -12, \alpha_2)$. Their dot product must vanish:

$$8(2) - 12(3) + 5\alpha_2 = 0.$$

Thus $5\alpha_2 = 20$, giving $\alpha_2 = 4$. The plane is therefore $8x - 12y + 4z + 12 = 0$, or, after division by 4,

$$2x - 3y + z + 3 = 0.$$

Step 3: Evaluate the perpendicular distance Taking the common parameter of the line to be zero gives the point $Q = (-2, 3, -4)$. All points of a parallel line have the same perpendicular distance from the plane, so this one point is sufficient. Substitution into the reduced equation yields

$$d = \frac{|2(-2) - 3(3) - 4 + 3|}{\sqrt{2^2 + (-3)^2 + 1^2}} = \frac{14}{\sqrt{14}} = \sqrt{14}.$$

This is option (B), not (A); the numerator is 14, not 6. The other square-root values likewise do not equal this exact point-to-plane distance.

Step 4: Conclusion The distance is $\boxed{\sqrt{14}}$ units. Hence, the correct option is (B).

Quick Tip to Solve: Reduce the plane equation before using the distance formula, and use the zero-parameter point of a parallel line.

17. The number of values of $\mathrm{r} \in \{\mathrm{p}, \mathrm{q}, \sim \mathrm{p}, \sim \mathrm{q}\}$ for which $((\mathrm{p} \wedge \mathrm{q}) \Rightarrow (\mathrm{r} \vee \mathrm{q})) \wedge ((\mathrm{p} \wedge \mathrm{r}) \Rightarrow \mathrm{q})$ is a tautology, is : [4 marks]

(A) 2

(B) 1

(C) 4

(D) 3

Correct Answer: (A) 2

Solution:

Concept:

- An implication $A \Rightarrow B$ fails only when A is true and B is false.
- A conjunction is a tautology precisely when both its components are true for every assignment of the underlying propositions.

Step 1: Simplify the first implication Consider $(p \wedge q) \Rightarrow (r \vee q)$. Whenever its antecedent is true, q is true, so $r \vee q$ must also be true, regardless of r. Whenever the antecedent is false, the implication is true by definition. Therefore the first implication is a tautology for each of the four proposed choices of r. It imposes no additional restriction on that choice.

Step 2: Test the second implication The full expression is therefore a tautology exactly when $(p \wedge r) \Rightarrow q$ is a tautology. For $r = p$, it reduces to $p \Rightarrow q$, which fails at $p = \mathrm{T}, q = \mathrm{F}$. For $r = q$, the antecedent already contains q, so $(p \wedge q) \Rightarrow q$ is always true. For $r = \sim p$, the antecedent $p \wedge \sim p$

is always false, making the implication always true. For $r = \sim q$, choosing $p = \mathrm{T}, q = \mathrm{F}$ makes the antecedent true and the consequent false, so this choice fails.

Step 3: Count the successful choices The successful choices are precisely $r = q$ and $r = \sim p$. There are two, with each accounting for a different mechanism: a consequent contained in the antecedent, or an impossible antecedent. The choices $r = p$ and $r = \sim q$ have explicit counterexamples, so neither can be counted. Thus counts 1, 3 and 4 in the remaining options are excluded.

Step 4: Conclusion The number of values is $\boxed{2}$. Hence, the correct option is (A).

Quick Tip to Solve: For an implication, search first for the sole failure pattern: true antecedent and false consequent.

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18. The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ is $(2, \mathrm{a}, 4), \mathrm{a} \in \mathrm{N}$. If the volume of the tetrahedron OABC is 144 unit^3, then which of the following points is NOT on P ? [4 marks]

(A) $(3, 0, 4)$

(B) $(0, 6, 3)$

(C) $(0, 4, 4)$

(D) $(2, 2, 4)$

Correct Answer: (A) $(3, 0, 4)$

Solution:

Concept:

- If F is the foot from the origin to a plane, then $\overrightarrow{OF}$ is normal to that plane and the plane passes through F.
- A plane with positive axis intercepts u, v, w forms a tetrahedron of volume $uvw/6$ with the origin.

Step 1: Write the plane using its normal Let $F = (2, a, 4)$. A normal vector is $(2, a, 4)$, so the equation through F is

$$2(x - 2) + a(y - a) + 4(z - 4) = 0.$$

Writing $S = a^2 + 20$, this becomes $2x + ay + 4z = S$. Since a is a natural number, it is positive here, and all three axis intercepts are positive.

Step 2: Use the tetrahedron volume The intercepts are $S/2, S/a, S/4$. Therefore

$$144 = \frac{1}{6}\frac{S}{2}\frac{S}{a}\frac{S}{4} = \frac{S^3}{48a}, \qquad (a^2 + 20)^3 = 6912a.$$

The unique positive solution is $a = 2$. To justify uniqueness, minimise $f(a) = (a^2 + 20)^3/a$ for positive real a. Its logarithmic derivative is

$$\frac{f\prime(a)}{f(a)} = \frac{6a}{a^2 + 20} - \frac{1}{a} = \frac{5(a^2 - 4)}{a(a^2 + 20)}.$$

It is negative below 2 and positive above 2. The minimum is $f(2) = 6912$, so equality holds only there.

Step 3: Test each proposed point The plane reduces to $2x + 2y + 4z = 24$, or $x + y + 2z = 12$. At (A), the left side is $3 + 0 + 8 = 11$, so (A) is not on the plane. At (B) it is $0 + 6 + 6 = 12$; at (C) it is $0 + 4 + 8 = 12$; and at (D) it is $2 + 2 + 8 = 12$. Thus the sole point not on the plane is actually (A); the explicit substitutions distinguish a failure of membership from a point that satisfies the equation.

Step 4: Conclusion The point not on the plane is $\boxed{(3, 0, 4)}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the foot coordinates as the plane normal, then substitute every option into the resulting simplest plane equation.

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19. Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the line joining the points $(4, 1, -3)$ and $(2, 4, 3)$. Then the distance of P from the point $(3, -2, 2)$ is : [4 marks]

(A) 5

(B) 4

(C) 6

(D) 7

Correct Answer: (A) 5

Solution:

Concept:

- A plane perpendicular to a line has a normal vector parallel to the direction vector of that line.
- The perpendicular distance to a plane is the absolute value of its equation at the point, divided by the length of its normal.

Step 1: Find a normal vector The vector from $(4, 1, -3)$ to $(2, 4, 3)$ is $(-2, 3, 6)$. Reversing its sign, choose the equally valid normal $\mathbf{n} = (2, -3, -6)$. The direction itself is what matters, so either choice produces the same plane and the same nonnegative distance. Its magnitude is $\sqrt{4 + 9 + 36} = 7$.

Step 2: Build the plane through the given point The plane passes through $(1, -1, -5)$. Its point-normal equation is

$$2(x - 1) - 3(y + 1) - 6(z + 5) = 0.$$

Expanding the constant terms gives

$$2x - 3y - 6z - 35 = 0.$$

As a sign check, substitution of the known point gives $2 + 3 + 30 - 35 = 0$, confirming that the plane contains it. The coefficient vector remains parallel to the joining line as required.

Step 3: Calculate and compare the distance At the point $(3, -2, 2)$, the plane expression equals

$$2(3) - 3(-2) - 6(2) - 35 = 6 + 6 - 12 - 35 = -35.$$

Hence

$$d = \frac{|-35|}{7} = 5.$$

The absolute value is necessary because a distance cannot be negative; the negative numerator only identifies the side of the plane. The options 4, 6 and 7 do not equal this ratio. In particular, 7 is the normal length, not the required distance, and must not be selected in place of the completed calculation.

Step 4: Conclusion The required distance is $\boxed{5}$ units. Hence, the correct option is (A).

Quick Tip to Solve: After expanding a point-normal equation, substitute its defining point back in to catch sign errors in the constant term.

20. If a point $\mathrm{P}(\alpha, \beta, \gamma)$ satisfying

$$\begin{pmatrix} \alpha & \beta & \gamma \end{pmatrix} \begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

lies on the plane $2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to : [4 marks]

(A) $\frac{11}{5}$

(B) 11

(C) -1

(D) $\frac{5}{4}$

Correct Answer: (B) 11

Solution:

Concept:

- A row vector multiplied on the right by a matrix gives one scalar equation from each column of that matrix.
- The homogeneous equations determine the coordinate ratios; the nonhomogeneous plane condition then fixes their common scale.

Step 1: Expand the matrix multiplication The three columns yield

$$\begin{aligned} 2\alpha + 9\beta + 8\gamma &= 0, \\ 10\alpha + 3\beta + 4\gamma &= 0, \\ 8\alpha + 8\beta + 8\gamma &= 0. \end{aligned}$$

The third equation simplifies to $\gamma = -\alpha - \beta$. Using columns rather than rows is essential because the coordinate vector appears on the left of the matrix, as a row vector.

Step 2: Determine the coordinate ratios Substitute $\gamma = -\alpha - \beta$ into the first equation. It gives

$$2\alpha + 9\beta - 8\alpha - 8\beta = 0,$$

hence $\beta = 6\alpha$ and $\gamma = -7\alpha$. The second equation then becomes $10\alpha + 18\alpha - 28\alpha = 0$, confirming consistency instead of adding a new restriction. Thus all vectors solving the homogeneous system

have the form $(\alpha, \beta, \gamma) = t(1, 6, -7)$.

Step 3: Use the plane and evaluate the expression The point also satisfies $2\alpha + 4\beta + 3\gamma = 5$. Substituting the ratios gives

$$2t + 24t - 21t = 5, \qquad 5t = 5, \qquad t = 1.$$

Therefore the point is $(1, 6, -7)$, and

$$6\alpha + 9\beta + 7\gamma = 6 + 54 - 49 = 11.$$

The value $11/5$ would incorrectly leave the scale unfixed or divide the answer again after already solving for t. The values -1 and $5/4$ likewise do not result from the coordinates satisfying both original conditions.

Step 4: Conclusion The required value is $\boxed{11}$. Hence, the correct option is (B).

Quick Tip to Solve: For a left null vector, use the matrix columns, find coordinate ratios first, and use the extra plane only to fix the scale.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let the area of the region
$\{(x, y) : |2x - 1| \le y \le |x^2 - x|, 0 \le x \le 1\}$ **be A.**
Then $(6\ \mathrm{A} + 11)^2$ **is equal to** [4 marks]

Solution:

Concept:

- On $0 \le x \le 1$, $|x^2 - x| = x - x^2$; the other boundary $|2x - 1|$ changes form at $x = 1/2$.
- An enclosed vertical strip exists only where its upper boundary is at least its lower boundary. Symmetry about $x = 1/2$ halves the integration work.

Step 1: Find where the region exists On the left half, $|2x - 1| = 1 - 2x$. The boundary intersections satisfy

$$x - x^2 = 1 - 2x, \qquad x^2 - 3x + 1 = 0.$$

The root in $[0, 1/2]$ is $t = (3 - \sqrt{5})/2$. On this half the upper curve exceeds the lower one only for $t \le x \le 1/2$. Both boundary functions are unchanged by $x \mapsto 1 - x$, so the full region extends symmetrically to $x = 1 - t$. Integrating over the entire interval from 0 to 1 without restricting the strip would incorrectly include negative vertical differences.

Step 2: Integrate the positive strip height The vertical height on the left half is $(x - x^2) - (1 - 2x) = 3x - x^2 - 1$. Thus

$$A = 2\int_t^{1/2} (3x - x^2 - 1)\, dx = 2\left[\frac{3x^2}{2} - \frac{x^3}{3} - x\right]_t^{1/2}.$$

At $x = 1/2$, the bracket is $-1/6$. Since $t^2 = 3t - 1$, we also have $t^3 = 8t - 3$. These relations give the bracket at t as $5t/6 - 1/2$. Therefore

$$A = \frac{2 - 5t}{3} = \frac{5\sqrt{5} - 11}{6}.$$

This is positive, as required for an area.

Step 3: Compute the requested numerical value Substitute this exact area before squaring:

$$6A + 11 = 5\sqrt{5} - 11 + 11 = 5\sqrt{5}.$$

Consequently $(6A + 11)^2 = 25 \cdot 5 = 125$. Keeping the radical exact avoids a rounding error in the final integer. The expression is dimensionless, so no unit of area is attached to this final numerical answer, although A itself denotes the area of the region.

Step 4: Conclusion The requested numerical value is $\boxed{125}$.

Quick Tip to Solve: Use symmetry only after locating the boundary intersections; replace powers of the intersection root using its quadratic equation.

22. The coefficient of x^{-6}, in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$, is [4 marks]

Solution:

Concept:

- The binomial term with index r is $T_{r+1} = \binom{9}{r}(4x/5)^{9-r}(5/(2x^2))^r$, with $0 \le r \le 9$.
- The power of x in a product is the sum of its powers. First match this exponent, and only then evaluate the numerical coefficient.

Step 1: Match the requested power Taking the second summand r times and the first summand $9 - r$ times gives the exponent

$$(9 - r) - 2r = 9 - 3r.$$

For the required term this must equal -6, so

$$9 - 3r = -6 \quad \Longrightarrow \quad r = 5.$$

This is an allowed integer index. The sixth term, rather than the term with index six, is therefore the required one.

Step 2: Evaluate its coefficient Substitute $r = 5$ into the numerical part of the general term:

$$\begin{aligned}[x^{-6}] &= \binom{9}{5}\left(\frac{4}{5}\right)^4\left(\frac{5}{2}\right)^5 \\ &= 126 \cdot \frac{4^4\,5^5}{5^4\,2^5} \\ &= 126 \cdot 2^3 \cdot 5 = 5040.\end{aligned}$$

All factors are positive, so the coefficient is positive. The remaining variable factor is exactly $x^{4-10} = x^{-6}$, which checks the selected index.

Step 3: Conclusion The coefficient of x^{-6} is $\boxed{5040}$. This is a dimensionless coefficient.

Quick Tip to Solve: For a Laurent binomial, solve the linear exponent equation before evaluating factorials; the index and the term number differ by one.

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23. Let $\mathrm{A} = [\mathrm{a}_{ij}]\,, \mathrm{a}_{ij} \in \mathbb{Z} \cap [0, 4], 1 \le i, j \le 2$**.**

The number of matrices A such that the sum of all entries is a prime number $\mathrm{p} \in (2, 13)$ **is _____.**

[4 marks]

Solution:

Concept:

- A 2×2 matrix is specified by four ordered entries. Thus matrices are counted as ordered quadruples (a, b, c, d), each in $\{0, 1, 2, 3, 4\}$.
- Without upper bounds, the number of nonnegative solutions to $a+b+c+d = s$ is $\binom{s+3}{3}$. Inclusion-exclusion removes entries at least five.

Step 1: List the permitted prime sums The strict interval $(2, 13)$ contains exactly the primes $3, 5, 7, 11$. Each required matrix belongs to exactly one of these four sum classes. For a fixed sum s, start from all nonnegative ordered quadruples and exclude those in which an entry is at least five. Shifting that entry down by five preserves nonnegativity and changes the total to $s - 5$.

Step 2: Apply inclusion-exclusion to each sum Let $N(s)$ be the number with sum s. Then

$$N(s) = \binom{s+3}{3} - 4\binom{s-2}{3} + 6\binom{s-7}{3},$$

where a binomial coefficient is zero when its upper argument is less than three. Triple violations are impossible since $s \le 11 < 15$. Hence

$$\begin{aligned}N(3) &= 20, \\ N(5) &= 56 - 4 = 52, \\ N(7) &= 120 - 40 = 80, \\ N(11) &= 364 - 336 + 24 = 52.\end{aligned}$$

The factor four chooses the violating entry and the factor six chooses a pair of violating entries.

Step 3: Add the disjoint cases Because the entry sum cannot take two different values at once, add the four counts:

$$20 + 52 + 80 + 52 = 204.$$

As a check, replacing every entry u by $4 - u$ changes the sum s to $16 - s$. Thus sums five and eleven must have equal counts, as obtained.

Step 4: Conclusion The required number of matrices is $\boxed{204}$, a dimensionless count.

Quick Tip to Solve: Treat matrix entries as ordered slots; use stars and bars with inclusion-exclusion rather than combinations of an unordered multiset.

24. If ${}^{2n+1}\mathrm{P}_{n-1} : {}^{2n-1}\mathrm{P}_{n} = 11 : 21$**,**
then $n^2 + n + 15$ **is equal to :** [4 marks]

Solution:

Concept:

- For integers $m \geq r \geq 0$, ${}^{m}P_{r} = m!/(m-r)!$. Both the upper and lower arguments must satisfy the permutation domain.
- A ratio of factorials should be simplified before solving its resulting equation; domain restrictions remove nonintegral or negative roots.

Step 1: Expand and simplify the ratio The two permutations are

$${}^{2n+1}P_{n-1} = \frac{(2n+1)!}{(n+2)!}, \qquad {}^{2n-1}P_{n} = \frac{(2n-1)!}{(n-1)!}.$$

The ratio condition therefore becomes

$$\frac{(2n+1)(2n)(n-1)!}{(n+2)!} = \frac{11}{21}.$$

Since the permutation arguments require an integer $n \geq 1$, cancel the nonzero factor n to obtain

$$\frac{2(2n+1)}{(n+1)(n+2)} = \frac{11}{21}.$$

Step 2: Find the admissible integer Cross-multiplying positive denominators gives

$$42(2n+1) = 11(n+1)(n+2).$$

Expanding and collecting terms,

$$11n^2 - 51n - 20 = 0 \quad \Longrightarrow \quad (n-5)(11n+4) = 0.$$

Thus $n = 5$ or $n = -4/11$. The latter is not a positive integer and cannot be a permutation parameter. For $n = 5$, the simplified ratio is $22/42 = 11/21$, confirming the root.

Step 3: Evaluate the requested expression Using the admissible parameter, rather than either formal algebraic root, gives

$$n^2 + n + 15 = 5^2 + 5 + 15 = 25 + 5 + 15 = 45.$$

No rounding is needed because the result is an exact integer.

Step 4: Conclusion The required value is $\boxed{45}$, dimensionless.

Quick Tip to Solve: Cancel factorials as products of consecutive factors; check that every root is an allowed integer permutation parameter.

---

25. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that
$|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$.
If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}}\right)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The vector identity $\vec{c} \times \vec{a} = -\vec{a} \times \vec{c}$ converts the given relation into a parallel-vector condition.
- For vectors $\vec{u}, \vec{v}$, $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$ and $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin\theta$. The square of a vector ratio here denotes the squared magnitude of its numerator divided by the scalar denominator squared.

Step 1: Evaluate the scalar data The given lengths imply $|\vec{b}| = 1/2$, $|\vec{c}| = 2$. As their angle is $2\pi/3$,

$$\vec{b} \cdot \vec{c} = \frac{1}{2} \cdot 2 \cos\frac{2\pi}{3} = -\frac{1}{2}.$$

Set $\vec{d} = 2\vec{b} + 3\vec{c}$. Its squared length is

$$|\vec{d}|^2 = 4\left(\frac{1}{4}\right) + 9(4) + 12\left(-\frac{1}{2}\right) = 31.$$

Thus $|\vec{d}| = |\vec{a}| = \sqrt{31}$, and $\vec{d} \neq 0$.

Step 2: Use the parallelism condition The given cross-product equation is equivalent to

$$\vec{a} \times (2\vec{b} + 3\vec{c}) = \vec{0}.$$

Hence $\vec{a} = t\vec{d}$. Equality of their nonzero lengths gives $t = \pm 1$. Consequently

$$\vec{a} \cdot \vec{b} = \pm\left(2|\vec{b}|^2 + 3\vec{c} \cdot \vec{b}\right) = \pm(-1),$$

and $\vec{a} \times \vec{c} = \pm 2(\vec{b} \times \vec{c})$. The denominator is nonzero, so the requested ratio is defined.

Step 3: Square the magnitude ratio The sign choice disappears on squaring. Therefore

$$\begin{aligned}\frac{|\vec{a}\times\vec{c}|^2}{(\vec{a}\cdot\vec{b})^2} &= 4|\vec{b}\times\vec{c}|^2\\ &= 4\left(\frac{1}{2}\cdot 2\sin\frac{2\pi}{3}\right)^2\\ &= 4\cdot\frac{3}{4}=3.\end{aligned}$$

Both parallel and antiparallel choices for $\vec{a}$ produce the same value.

Step 4: Conclusion The required squared ratio is $\boxed{3}$, a dimensionless value.

Quick Tip to Solve: Move both cross products to the same left factor. This reveals a parallel vector and avoids introducing coordinate components.

26. Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space $[0, 60]$ is less than or equal to a . If $\mathrm{P(A)} = \frac{11}{36}$, then a is equal to _____. [4 marks]

Solution:

Concept:

- Two independently and uniformly selected numbers $X, Y \in [0, 60]$ correspond to a uniform point in a square of area 60^2.
- For a uniform point, probability equals favourable area divided by total area. The complement of $|X - Y| \le a$ consists of two equal right triangles when $0 \le a \le 60$.

Step 1: Express the complementary area The event is neither impossible nor certain because its probability is $11/36$. Thus its threshold must satisfy $0 < a < 60$. In the square $0 \le X, Y \le 60$, the lines $Y = X + a$ and $Y = X - a$ bound the desired diagonal band. Outside that band lie two right triangles, each with perpendicular side lengths $60 - a$. The combined excluded area is therefore

$$2\cdot\frac{1}{2}(60-a)^2 = (60-a)^2.$$

The boundary lines have area zero, so the non-strict inequality does not change the probability.

Step 2: Solve the probability equation Dividing the complement area by the square area gives

$$1 - \frac{(60-a)^2}{3600} = \frac{11}{36}.$$

Hence

$$\frac{(60-a)^2}{3600} = \frac{25}{36} \quad\Longrightarrow\quad (60-a)^2 = 2500.$$

Since $0 < a < 60$, the length $60 - a$ is positive, so it equals 50, not -50. Thus $a = 10$. The other formal root, 110, lies outside the domain and would make the event certain.

Step 3: Verify the threshold For $a = 10$, the two excluded triangles together have area $50^2 = 2500$;

the band area is $3600 - 2500 = 1100$. Its probability is

$$\frac{1100}{3600} = \frac{11}{36},$$

exactly the given value.

Step 4: Conclusion The required threshold is $\boxed{a = 10}$, in the same units as the selected numbers.

Quick Tip to Solve: For the difference of two uniform numbers on an interval, calculate the two outside triangles instead of integrating the diagonal band.

---

27. If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^l}\right)^9$ is -84 and the coefficient of x^{-3l} is $2^\alpha\beta$, where $\beta < 0$ is an odd number, then $|\alpha l - \beta|$ is equal to _____. [4 marks]

Solution:

Concept:

- Choosing the second summand r times gives a term with exponent $\frac{5}{2}(9-r) - lr$ and coefficient $(-1)^r\binom{9}{r}2^{3r-9}$.
- The constant term has exponent zero. To express a negative integer uniquely as $2^\alpha\beta$ with β odd, extract all factors of two.

Step 1: Identify the constant term index The general term is

$$T_{r+1} = (-1)^r\binom{9}{r}2^{3r-9}x^{\frac{5}{2}(9-r)-lr}.$$

Its coefficient is negative only for odd r. For the possible odd indices $r = 1, 3, 5, 7, 9$, the coefficient magnitudes are respectively

$$\frac{9}{64}, \quad 84, \quad 8064, \quad 147456, \quad 262144.$$

Therefore the constant coefficient -84 occurs only at $r = 3$. Requiring its exponent to vanish yields

$$\frac{5}{2}(9-3) - 3l = 0 \quad \Longrightarrow \quad l = 5.$$

Step 2: Find the term with the second requested exponent With $l = 5$, the exponent becomes $\frac{45}{2} - \frac{15r}{2}$. Set it equal to $-3l = -15$:

$$\frac{45 - 15r}{2} = -15 \quad \Longrightarrow \quad r = 5.$$

The corresponding coefficient is

$$(-1)^5\binom{9}{5}2^{15-9} = -126 \cdot 64 = -8064.$$

It is negative, consistent with the condition $\beta < 0$.

Step 3: Extract the full power of two Since $126 = 2 \cdot 63$,

$$-8064 = 2^7(-63).$$

The factor -63 is odd, so this is the required factorisation: $\alpha = 7$, $\beta = -63$. Finally,

$$|\alpha l - \beta| = |7 \cdot 5 - (-63)| = |35 + 63| = 98.$$

The minus sign before β is essential in the final substitution.

Step 4: Conclusion The requested value is $\boxed{98}$, dimensionless.

Quick Tip to Solve: Keep the coefficient and the power of the variable separate; after locating the term, extract every factor of two so the remaining factor is odd.

28. Let A be a $n \times n$ matrix such that $|\mathrm{A}| = 2$. If the determinant of the matrix $\mathrm{Adj}\left(2 \cdot \mathrm{Adj}\left(2\ \mathrm{A}^{-1}\right)\right) \cdot$ is 2^{84}, then n is equal to : [4 marks]

Solution:

Concept:

- For an invertible $n \times n$ matrix M, $\det(\mathrm{Adj}\, M) = (\det M)^{n-1}$.
- Scaling all entries of an $n \times n$ matrix by two multiplies its determinant by 2^n. Also $\det(A^{-1}) = 1/\det A$.

Step 1: Evaluate the inner determinant Because $\det A = 2 \neq 0$, the inverse exists and

$$\det(A^{-1}) = \frac{1}{2}.$$

Put $B = 2A^{-1}$. Multiplication by two scales each of the n rows, giving

$$\det B = 2^n \cdot \frac{1}{2} = 2^{n-1}.$$

Applying the adjugate identity once gives

$$\det(\mathrm{Adj}\, B) = \left(2^{n-1}\right)^{n-1} = 2^{(n-1)^2}.$$

The exponent is squared because both the inner determinant and the adjugate identity contribute a factor $n - 1$.

Step 2: Apply the outer scaling and adjugate Now put $C = 2\,\mathrm{Adj}\, B$. Its determinant is

$$\det C = 2^n\, 2^{(n-1)^2} = 2^{n+(n-1)^2}.$$

The matrix in the question is $\mathrm{Adj}\, C$, so its determinant is

$$2^{(n-1)\{n+(n-1)^2\}}.$$

Equating it to 2^{84}, using injectivity of real powers of base two, yields

$$(n-1)(n^2-n+1)=84.$$

Step 3: Solve for the matrix order Expanding and factoring after moving the right side gives

$$n^3-2n^2+2n-85=0 \quad \Longrightarrow \quad (n-5)(n^2+3n+17)=0.$$

The quadratic has discriminant $9-68=-59<0$, so the only real root, and hence the only positive integer order, is $n=5$. Checking the unexpanded exponent gives $4(5+16)=84$, as required.

Step 4: Conclusion The matrix order is $\boxed{n=5}$, a dimensionless positive integer.

Quick Tip to Solve: Work from the innermost matrix outward, applying determinant scaling before each adjugate power; do not scale a determinant by just two.

29. Let S be the set of all $\mathrm{a}\in\mathrm{N}$ such that the area of the triangle formed by the tangent at the point $\mathrm{P(b, c)}$, $\mathrm{b, c}\in\mathbb{N}$, on the parabola $y^2=2\mathrm{a}x$ and the lines $x=\mathrm{b}, y=0$ is 16 unit^2, then $\sum\limits_{\mathrm{a}\in\mathrm{S}}\mathrm{a}$ is equal to : [4 marks]

Solution:

Concept:

- The tangent to $y^2=2ax$ at (b,c) is $cy=a(x+b)$, with $c^2=2ab$.
- The area of a right triangle is half the product of its perpendicular sides. The conditions $a,b,c\in\mathbb{N}$ must all hold; they restrict the possible tangent points.

Step 1: Find the tangent intercept and triangle area Because $c>0$, the tangent meets the horizontal axis at $(-b,0)$. The vertical line $x=b$ meets that axis at $(b,0)$ and meets the tangent at (b,c). These three points form a right triangle with horizontal base $2b$ and vertical height c. Therefore its prescribed area gives

$$\frac{1}{2}(2b)c=16 \quad \Longrightarrow \quad bc=16.$$

The horizontal base is $2b$, rather than b, because the tangent intercept is on the negative side of the origin.

Step 2: Apply the integer restrictions Substitute $b=16/c$ into the parabola equation:

$$c^2=2a\frac{16}{c} \quad \Longrightarrow \quad a=\frac{c^3}{32}.$$

Since b is a positive integer, c must be a positive divisor of 16. Thus $c\in\{1,2,4,8,16\}$. For $c=1,2$, the corresponding values $1/32, 1/4$ of a are not integers. For the other divisors the triples (c,b,a) are

$$(4,4,2), \qquad (8,2,16), \qquad (16,1,128).$$

Each triple satisfies both $bc=16$ and $c^2=2ab$, so none is extraneous. This divisor list is exhaustive.

Step 3: Conclusion The admissible set is $S = \{2, 16, 128\}$. Hence the required dimensionless sum is

$$\sum_{a \in S} a = 2 + 16 + 128 = \boxed{146}.$$

Quick Tip to Solve: Find the tangent intercept before calculating the area: here it doubles the apparent horizontal base. Then use the integer-coordinate condition to enumerate divisors.

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30. The sum $1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + 5 \cdot 9^2 - \ldots + 15 \cdot 29^2$ is _____. [4 marks]

Solution:

Concept:

- The term with index n is $(-1)^{n+1} n(2n-1)^2$. Pairing consecutive terms converts the alternating cubic polynomial into an ordinary quadratic sum.
- For positive integer m, $\sum_{j=1}^{m} j = m(m+1)/2$ and $\sum_{j=1}^{m} j^2 = m(m+1)(2m+1)/6$.

Step 1: Pair the first fourteen terms Put $f(n) = n(2n-1)^2 = 4n^3 - 4n^2 + n$. There are fifteen terms, beginning positive and ending positive. Pair indices $1, 2$, then $3, 4$, up to $13, 14$, leaving index 15 unpaired. Thus

$$S = \sum_{j=1}^{7} \big[f(2j-1) - f(2j) \big] + f(15).$$

Expanding the two members of a pair gives

$$f(2j-1) = 32j^3 - 64j^2 + 42j - 9,$$
$$f(2j) = 32j^3 - 16j^2 + 2j.$$

Subtracting cancels the cubic terms, so each pair contributes $-48j^2 + 40j - 9$.

Step 2: Evaluate the quadratic sum and last term For seven pairs, the required elementary sums are

$$\sum_{j=1}^{7} j = 28, \qquad \sum_{j=1}^{7} j^2 = 140.$$

Consequently the sum of the paired terms is

$$-48(140) + 40(28) - 9(7) = -6720 + 1120 - 63 = -5663.$$

The final positive term has index 15 and odd-number factor 29:

$$f(15) = 15 \cdot 29^2 = 15 \cdot 841 = 12615.$$

Adding this term to the paired contribution yields $12615 - 5663 = 6952$. As a sign check, every pair is negative because the even-indexed magnitude is larger; the large final positive term then leaves a positive total.

Step 3: Conclusion The given sum equals

$$\boxed{6952}.$$

It is a dimensionless numerical value.

Quick Tip to Solve: With an odd number of alternating terms, pair from the beginning and keep the final positive term separate; this prevents losing its sign.

Physics – Section A: Multiple Choice Questions (80 marks)

31. For a solid rod, the Young's modulus of elasticity is $3.2 \times 10^{11}\text{Nm}^{-2}$ and density is 8×10^3 kg m^{-3}. The velocity of longitudinal wave in the rod will be. [4 marks]

(A) 3.65×10^3 ms^{-1}

(B) 6.32×10^3 ms^{-1}

(C) 18.96×10^3 ms^{-1}

(D) 145.75×10^3 ms^{-1}

Correct Answer: (B) 6.32×10^3 ms^{-1}

Solution:

Concept:

- The longitudinal wave speed in a thin solid rod is $v = \sqrt{Y/\rho}$, where Y is Young's modulus and ρ is mass density.
- Elastic stiffness provides the restoring force and density provides inertia. Their ratio has units of speed squared, so the square root must be taken after dividing the numerical values.

Step 1: Form the modulus-to-density ratio Use the supplied values $Y = 3.2 \times 10^{11}\,\text{N m}^{-2}$ and $\rho = 8 \times 10^3\,\text{kg m}^{-3}$. For a longitudinal disturbance along the rod, the relevant elastic modulus is Young's modulus; the bulk-modulus expression for a fluid is not needed. We obtain

$$\frac{Y}{\rho} = \frac{3.2 \times 10^{11}}{8 \times 10^3} = 0.4 \times 10^8 = 4.0 \times 10^7\,\text{m}^2\,\text{s}^{-2}.$$

Here the exponent changes by subtraction, $11 - 3 = 8$, before normalising the coefficient.

Step 2: Take the square root and compare the choices The speed is positive, so take the positive square root:

$$v = \sqrt{4.0 \times 10^7} = 2\sqrt{10} \times 10^3 \simeq 6.32 \times 10^3\,\text{m s}^{-1}.$$

Squaring the value in option (B) gives approximately $3.994 \times 10^7\,\text{m}^2\,\text{s}^{-2}$, consistent with the calculated ratio to the precision of the options. The values in (A), (C) and (D) do not square to 4.0×10^7; therefore they cannot satisfy the wave-speed relation for these data. The units also confirm that the square-root calculation has produced speed.

Step 3: Conclusion The longitudinal wave velocity has magnitude $\boxed{6.32 \times 10^3\,\text{m s}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Normalise the power of ten before taking a square root: $4 \times 10^7 = 40 \times 10^6$, making the factor 10^3 immediate.

32. Heat energy of $735\ \mathrm{J}$ is given to a diatomic gas allowing the gas to expand at constant pressure. Each gas molecule rotates around an internal axis but do not oscillate. The increase in the internal energy of the gas will be : [4 marks]

(A) $572\ \mathrm{J}$

(B) $441\ \mathrm{J}$

(C) $525\ \mathrm{J}$

(D) $735\ \mathrm{J}$

Correct Answer: (C) $525\ \mathrm{J}$

Solution:

Concept:

- A non-vibrating diatomic molecule has five active degrees of freedom: three translational and two rotational. Equipartition gives $C_V = 5R/2$.
- For an ideal gas, $C_P = C_V + R = 7R/2$. At constant pressure, supplied heat is $Q = nC_P\Delta T$, whereas the internal-energy increase is $\Delta U = nC_V\Delta T$.

Step 1: Identify how the heat is divided The molecule is allowed to rotate but not vibrate, so no vibrational energy contribution is included. For a linear diatomic molecule the two perpendicular rotational modes are active, giving $f = 5$. Thus

$$C_V = \frac{fR}{2} = \frac{5R}{2}, \qquad C_P = \frac{7R}{2}.$$

At constant pressure the gas expands while being heated. The supplied 735 J must therefore provide both an internal-energy increase and work against the surrounding pressure; it cannot all remain as internal energy.

Step 2: Use the heat-capacity ratio Eliminate the unknown amount of gas and temperature rise by dividing the internal-energy equation by the heat equation:

$$\frac{\Delta U}{Q} = \frac{nC_V\Delta T}{nC_P\Delta T} = \frac{5}{7}.$$

Hence

$$\Delta U = \frac{5}{7}(735) = 5(105) = 525\,\mathrm{J}.$$

The remaining energy is $W = Q - \Delta U = 210\,\mathrm{J}$, positive for expansion and equal to $2Q/7$, which checks the first law. Option (D) would imply no expansion work. Option (B) uses the monatomic ratio $3/5$, which is inappropriate here, and option (A) does not follow the five-degree-of-freedom ratio.

Step 3: Conclusion The internal energy increases by $\boxed{525\,\mathrm{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: For constant-pressure heating of a non-vibrating diatomic ideal gas, internal energy receives $5/7$ of the heat and expansion work receives $2/7$.

33. The radius of electron's second stationary orbit in Bohr's atom is R. The radius of 3rd orbit will be [4 marks]

(A) 2.25R

(B) 3R

(C) $\frac{R}{3}$

(D) 9R

Correct Answer: (A) 2.25R

Solution:

Concept:

- For the same hydrogen-like atom, the Bohr orbit radius varies as the square of its principal quantum number: $r_n = a_0 n^2/Z$.
- When comparing two orbits in one atom, the common factor a_0/Z cancels. The stated radius R belongs to the second orbit, not the first.

Step 1: Relate the given radius to the Bohr scale Let $k = a_0/Z$, where a_0 is the Bohr radius and Z is the nuclear charge number. The radius formula is then $r_n = kn^2$. For the second stationary orbit,

$$R = r_2 = k(2)^2 = 4k \quad \Longrightarrow \quad k = \frac{R}{4}.$$

This step identifies the correct reference length. Using R as the first-orbit radius would replace k by R and overestimate every subsequent radius by a factor of four.

Step 2: Calculate the third-orbit radius For the third orbit, $n = 3$, so

$$r_3 = k(3)^2 = 9k = \frac{9}{4}R = 2.25R.$$

Equivalently, directly taking the ratio gives $r_3/r_2 = 3^2/2^2 = 9/4$. The result is larger than R, as expected for an orbit with higher principal quantum number. Option (B), $3R$, does not use the square-law ratio. Option (C), $R/3$, incorrectly predicts a smaller orbit. Option (D), $9R$, would apply only if R denoted the first-orbit radius. Only option (A) uses both quantum numbers correctly; the ratio is dimensionless and multiplies the given length.

Step 3: Conclusion The third stationary orbit has radius $\boxed{r_3 = 2.25R}$. Hence, the correct option is (A).

Quick Tip to Solve: In Bohr-radius comparisons, write $r_m/r_n = m^2/n^2$ before substituting; do not treat a stated higher-orbit radius as the Bohr radius.

34. If the two metals A and B are exposed to radiation of wavelength 350 nm. The work functions of metals A and B are 4.8eV and 2.2eV. Then choose the correct option. [4 marks]

(A) Metal B will not emit photo-electrons

(B) Both metals A and B will not emit photo-electrons

(C) Metal A will not emit photo-electrons

(D) Both metals A and B will emit photo-electrons

Correct Answer: (C) Metal A will not emit photo-electrons

Solution:

Concept:

- A photon of wavelength λ has energy $E = hc/\lambda$. A metal emits photoelectrons only when this energy is at least its work function ϕ.
- The useful conversion is $hc \simeq 1240\,\text{eV nm}$. Above threshold, Einstein's equation gives $K_{\max} = E - \phi$; a negative difference means no emission.

Step 1: Calculate the incident photon energy Both metals receive radiation of the same wavelength, $350\,\text{nm}$, so each incident photon has the same energy regardless of the metal:

$$E = \frac{1240}{350}\,\text{eV} \simeq 3.54\,\text{eV}.$$

Compare this single-photon energy with each work function separately. Increasing the number of incident photons would not enable emission from a metal whose work function exceeds the photon energy; the threshold condition concerns energy per photon.

Step 2: Compare the energy with both thresholds For metal A, $\phi_A = 4.8\,\text{eV}$, which is greater than $3.54\,\text{eV}$. The energy deficit is about $1.26\,\text{eV}$, so no photoelectrons are emitted from A. For metal B, $\phi_B = 2.2\,\text{eV}$, and

$$K_{\max,B} = 3.54 - 2.2 = 1.34\,\text{eV} > 0.$$

Thus B does emit. Option (A) denies this allowed emission and is false. Option (B) incorrectly denies emission from both metals. Option (D) incorrectly allows emission from A below its threshold. Option (C) correctly states the non-emission from A and is consistent with B emitting.

Step 3: Conclusion Metal A will not emit photoelectrons, whereas metal B will emit them. Hence, the correct option is (C).

Quick Tip to Solve: Convert wavelength to photon energy once, then compare that energy with each work function independently; do not compare the work functions with each other alone.

35. The number of turns of the coil of a moving coil galvanometer is increased in order to increase current sensitivity by 50%. The percentage change in voltage sensitivity of the galvanometer will be : [4 marks]

(A) 0%

(B) 75%

(C) 100%

(D) 50%

Correct Answer: (A) 0%

Solution:

Concept:

- For a moving-coil galvanometer, current sensitivity is $S_I = \theta/I = NBA/\kappa$, where κ is the torsional constant.
- Voltage sensitivity is $S_V = \theta/V = S_I/G$, with G the coil resistance. For the same wire and turn size, both the number of turns and the coil resistance increase in the same ratio.

Step 1: Find the required change in turns Keep the magnetic field, area per turn, suspension and wire cross-section unchanged while adding turns. Then current sensitivity is proportional to N. A 50 percent increase means

$$S_I' = 1.5S_I \quad \Longrightarrow \quad N' = 1.5N.$$

Each added turn contributes the same wire length as an original turn, so the total length of wire also rises by a factor of 1.5. Since $G = \rho_{\mathrm{w}} L/a_{\mathrm{w}}$, with wire resistivity and cross-sectional area fixed, the new coil resistance is $G' = 1.5G$.

Step 2: Compare voltage sensitivities Voltage sensitivity must account for the reduced current produced by a given applied voltage when resistance increases. Using $S_V = S_I/G$,

$$\frac{S_V'}{S_V} = \frac{S_I'}{S_I}\frac{G}{G'} = 1.5 \times \frac{1}{1.5} = 1.$$

Therefore

$$\frac{S_V' - S_V}{S_V} \times 100\% = 0\%.$$

The increased magnetic torque per unit current is cancelled by the increased coil resistance at fixed voltage. Option (D) would result from ignoring that resistance change. Options (B) and (C) likewise imply an increase despite this exact cancellation, and are excluded. The conclusion uses the usual change of turns with the same coil wire and geometry per turn.

Step 3: Conclusion The voltage sensitivity remains unchanged, giving a percentage change of $\boxed{0\%}$. Hence, the correct option is (A).

Quick Tip to Solve: When turns are added to a galvanometer coil, track its resistance as well as NBA: current sensitivity rises, but voltage sensitivity need not rise.

36. Match List I with List II

LIST I		LIST II	
A.	Microwaves	I.	Physiotherapy
B.	UV rays	II.	Treatment of cancer
C.	Infra-red light	III.	Lasik eye surgery
D.	X-ray	IV.	Aircraft navigation

Choose the correct answer from the options given below: [4 marks]

(A) A - IV, B - III, C - I, D - II

(B) A - II, B - IV, C - III, D - I

(C) A - III, B - II, C - I, D - IV

(D) A - IV, B - I, C - II, D - III

Correct Answer: (A) A - IV, B - III, C - I, D - II

Solution:

Concept:

- Electromagnetic waves are selected for an application according to their wavelength, penetration and interaction with matter.
- Microwaves are used in radar, infrared radiation produces heating, ultraviolet light is used in precision corneal treatment, and energetic X-rays can destroy malignant tissue.

Step 1: Match the navigation and heating applications Microwaves can be generated as narrow beams and their reflected signals detected. Radar therefore uses them for locating aircraft and assisting navigation: A matches IV. Infrared radiation is strongly associated with radiant heating. Controlled infrared heating of tissues is used in physiotherapy: C matches I. These two matches already eliminate options (B), (C) and (D), since each breaks at least one of them.

Step 2: Complete the medical matches Ultraviolet radiation from an excimer laser is used to remove very thin layers of corneal tissue in LASIK eye surgery. Thus B matches III. High-energy X-rays are ionising and can damage rapidly dividing cells; they are used in radiotherapy for cancer, so D matches II. This is distinct from diagnostic X-ray imaging, which uses smaller doses to form images. The complete matching is therefore A–IV, B–III, C–I and D–II; it agrees with all four entries in option (A).

Step 3: Conclusion The matching is A–IV, B–III, C–I and D–II. Hence, the correct option is (A).

Quick Tip to Solve: Anchor the matching with microwave radar and infrared heating, then use the two remaining medical applications to finish.

37. The H amount of thermal energy is developed by a resistor in $10\ \mathrm{s}$ when a current of $4\ \mathrm{A}$ is passed through it. If the current is increased to $16\ \mathrm{A}$, the thermal energy developed by the resistor in $10\ \mathrm{s}$ will be : [4 marks]

(A) $\frac{\mathrm{H}}{4}$

(B) $16\mathrm{H}$

(C) H

(D) $4\mathrm{H}$

Correct Answer: (B) $16\mathrm{H}$

Solution:

Concept:

- Joule's law gives the heat generated in a resistor as $Q = I^2Rt$, where I is current, R resistance and t time.
- For the same resistor and the same duration, thermal energy is proportional to the square of the current.

Step 1: Express the original thermal energy Let the resistance be R, assumed unchanged in the comparison. The original current is $I_1 = 4\,\mathrm{A}$, and the duration is $t_1 = 10\,\mathrm{s}$. Hence the given energy satisfies

$$H = (4)^2R(10) = 160R.$$

Here H denotes the original amount of energy, not a current or a resistance. Keeping the same resistance and interval is essential when using the squared-current relation.

Step 2: Compare the new and original energies The new current is $I_2 = 16\,\mathrm{A}$, four times its original value, while $t_2 = 10\,\mathrm{s}$. Dividing the two expressions removes the unknown resistance:

$$\frac{Q_2}{H} = \frac{I_2^2Rt_2}{I_1^2Rt_1} = \left(\frac{16}{4}\right)^2 = 16.$$

Thus $Q_2 = 16H$. Option (A) predicts a reduction despite a larger current, option (C) ignores the changed current, and option (D) uses direct proportionality to current instead of its square. None of those obeys Joule heating for a fixed resistor.

Step 3: Conclusion The thermal energy produced in the next comparison interval is $16H$. Hence, the correct option is (B).

Quick Tip to Solve: At fixed resistance and time, multiply the heat by the square of the factor by which the current changes.

38. A body weight W, is projected vertically upwards from earth's surface to reach a height above the earth which is equal to nine times the radius of earth. The weight of the body at that height will be : [4 marks]

(A) $\frac{W}{91}$
(B) $\frac{W}{3}$
(C) $\frac{W}{100}$
(D) $\frac{W}{9}$

Correct Answer: (C) $\frac{W}{100}$

Solution:

Concept:

- Outside a spherical Earth, the gravitational force on a body of mass m is $F = GMm/r^2$, where r is measured from Earth's centre.
- The weight at the surface is $W = GMm/R^2$; at altitude h, replace r by $R + h$.

Step 1: Identify the distance from the centre Write Earth's radius as R. The specified height is $h = 9R$ above the surface, so the distance from the centre is

$$r = R + h = R + 9R = 10R.$$

It would be incorrect to put $9R$ directly into the inverse-square formula, because altitude and distance from the centre have different origins. The details of the initial projection do not affect the gravitational force at this location.

Step 2: Take the ratio of gravitational weights Use the same mass of Earth and body in both expressions, so that the common factors cancel:

$$\frac{W_h}{W} = \frac{GMm/(10R)^2}{GMm/R^2} = \frac{1}{100}.$$

Therefore $W_h = W/100$. This is the gravitational weight even if the speed becomes momentarily zero at the highest point. Option (A) does not follow the inverse-square law; options (B) and (D) also give much too large a weight. In particular, neither the height ratio nor its square alone is the required denominator; the factor is $(1 + h/R)^2$.

Step 3: Conclusion The body's weight at height $9R$ is $W/100$. Hence, the correct option is (C).

Quick Tip to Solve: In altitude problems first write the distance from the centre as radius plus height, then square that distance ratio.

39. Considering a group of positive charges, which of the following statements is correct ? [4 marks]

(A) Net potential of the system cannot be zero at a point but net electric field can be zero at that point

(B) Net potential of the system at a point can be zero but net electric field can't be zero at that point.

(C) Both the net potential and the net electric field cannot be zero at a point.

(D) Both the net potential and the net field can be zero at a point.

Correct Answer: (A) Net potential of the system cannot be zero at a point but net electric field can be zero at that point

Solution:

Concept:

- With potential zero at infinity, the potential of positive point charges is $V = k\sum_i q_i/r_i$, a scalar sum of positive terms at finite distances.
- The electric field is a vector sum, $\vec{E} = k\sum_i (q_i/r_i^2)\hat{r}_i$, so contributions pointing in opposite directions may cancel.

Step 1: Examine whether the potential can vanish For every charge in the stated group, $q_i > 0$. At a finite point other than a charge position, every distance r_i is positive and finite. Consequently, each term kq_i/r_i is positive, and their sum cannot equal zero:

$$V = k\sum_i \frac{q_i}{r_i} > 0.$$

This uses the conventional reference of zero potential at infinity. Potential tends to zero infinitely far away, but that limiting reference does not supply a finite cancellation point.

Step 2: Check whether vector fields can cancel Consider two equal positive charges separated by a distance $2a$. At their midpoint each field has magnitude kq/a^2, but one points to the right and the other to the left. Their resultant is zero. Meanwhile the two potentials add to $2kq/a > 0$. This example proves that zero net field is possible without zero net potential. Option (B) reverses the scalar and vector behaviour. Option (C) incorrectly rules out field cancellation, and option (D) incorrectly allows the positive potentials to cancel at a finite point.

Step 3: Conclusion Net potential cannot be zero at a finite point, but net electric field can be zero there. Hence, the correct option is (A).

Quick Tip to Solve: For like positive charges add potentials as positive scalars, but draw the field directions before deciding whether fields cancel.

40. A body of mass $10\ \text{kg}$ is moving with an initial speed of $20\ \text{m/s}$. The body stops after $5\ \text{s}$ due to friction between body and the floor. The value of the coefficient of friction is:

(Take acceleration due to gravity $g = 10\ \text{ms}^{-2}$) [4 marks]

(A) 0.3

(B) 0.2

(C) 0.5

(D) 0.4

Correct Answer: (D) 0.4

Solution:

Concept:

- For constant acceleration, $v = u + at$; friction acts opposite the sliding velocity and therefore produces a negative acceleration.
- On a horizontal floor the normal force is $N = mg$. Kinetic friction $f = \mu N$ gives a deceleration of magnitude μg.

Step 1: Find the acceleration from the stopping time Take the original direction of motion as positive. The initial velocity is $u = 20\,\text{m s}^{-1}$, the final velocity is zero, and the stopping time is $5\,\text{s}$. Thus

$$a = \frac{v - u}{t} = \frac{0 - 20}{5} = -4\,\text{m s}^{-2}.$$

The negative sign shows that the acceleration opposes motion; the coefficient of friction itself must be positive. The magnitude of the retarding force is $m|a| = 10(4) = 40\,\text{N}$.

Step 2: Relate the retarding force to the normal force With no vertical acceleration, the normal reaction equals the weight:

$$N = mg = 10(10) = 100\,\text{N}.$$

Friction alone stops the body, so

$$\mu = \frac{f}{N} = \frac{40}{100} = 0.4.$$

Equivalently $\mu = |a|/g$, showing why the mass cancels. The other options can be checked by predicting stopping times $t = u/(\mu g)$: 0.3 gives about $6.67\,\text{s}$, 0.2 gives $10\,\text{s}$, and 0.5 gives $4\,\text{s}$, all different from the stated $5\,\text{s}$.

Step 3: Conclusion The coefficient of friction is the dimensionless value $\mu = 0.4$. Hence, the correct option is (D).

Quick Tip to Solve: For stopping under horizontal kinetic friction, use $\mu = u/(gt)$; the mass cancels from Newton's law.

41. A hypothetical gas expands adiabatically such that its volume changes from 08 litres to 27 litres. If the ratio of final pressure of the gas to initial pressure of the gas is $\frac{16}{81}$. Then the ratio of $\frac{Cp}{Cv}$ will be. [4 marks]

(A) $\frac{3}{1}$
(B) $\frac{4}{3}$
(C) $\frac{1}{2}$
(D) $\frac{3}{2}$

Correct Answer: (B) $\frac{4}{3}$

Solution:

Concept:

- For the stated adiabatic expansion, use $PV^{\gamma} = \text{constant}$, with $\gamma = C_p/C_v$.
- Between the initial and final states, $P_f/P_i = (V_i/V_f)^{\gamma}$; writing ratios as powers of the same base makes the exponent easy to compare.

Step 1: Insert the state ratios The initial volume written as 08 litres is $8\,\text{L}$, and the final volume is $27\,\text{L}$. Both volumes use the same unit, so their ratio is dimensionless. The pressure ratio is specified directly. Therefore

$$\frac{16}{81} = \left(\frac{8}{27}\right)^{\gamma}.$$

The final pressure is smaller than the initial pressure, as expected for expansion. Inverting only one of these two ratios would produce an incorrect negative value of the exponent.

Step 2: Compare powers of a common base Write each ratio using the base $2/3$:

$$\frac{8}{27} = \left(\frac{2}{3}\right)^{3}, \qquad \frac{16}{81} = \left(\frac{2}{3}\right)^{4}.$$

The adiabatic equation becomes $(2/3)^{3\gamma} = (2/3)^4$, and hence $3\gamma = 4$. It follows that $\gamma = 4/3$. The alternative exponents 3, 1/2 and 3/2 would give powers 9, 3/2 and 9/2 of the same base rather than the required power 4. Thus they do not reproduce the specified pressure ratio.

Step 3: Conclusion The ratio of heat capacities is $C_p/C_v = 4/3$. Hence, the correct option is (B).

Quick Tip to Solve: When adiabatic volume and pressure ratios contain perfect powers, rewrite both with one base and compare exponents.

42. A long conducting wire having a current I flowing through it, is bent into a circular coil of N turns. Then it is bent into a circular coil of n turns. The magnetic field is calculated at the centre of coils in both the cases. The ratio of the magnetic field in first case to that of second case is : [4 marks]

(A) $N^2 : n^2$

(B) N : n

(C) n : N

(D) $n^2 : N^2$

Correct Answer: (A) $N^2 : n^2$

Solution:

Concept:

- The magnetic field at the centre of a circular coil of T turns and radius r is $B = \mu_0 TI/(2r)$.
- For the same wire of length L, close circular turns satisfy $L = 2\pi rT$. More turns therefore mean a smaller coil radius.

Step 1: Express each radius in terms of wire length Let the unchanged wire length be L, neglecting any connecting leads as in the ideal coil model. In the first arrangement,

$$L = 2\pi r_N N, \qquad r_N = \frac{L}{2\pi N}.$$

In the second arrangement the same wire makes n turns, giving

$$r_n = \frac{L}{2\pi n}.$$

Thus the radii are not equal: increasing the number of turns reduces the radius inversely. Keeping the radius fixed would incorrectly treat the two coils as using different lengths of wire.

Step 2: Substitute the radii into the field formula The current remains I in both configurations. Substitute the length constraint into the centre-field expression:

$$B_N = \frac{\mu_0 NI}{2[L/(2\pi N)]} = \frac{\mu_0 \pi I N^2}{L}, \qquad B_n = \frac{\mu_0 \pi I n^2}{L}.$$

Dividing cancels current, wire length and permeability, giving $B_N/B_n = N^2/n^2$. Option (B) accounts only for the explicit turn factor and misses the radius change. Options (C) and (D) reverse the required first-to-second order; the latter is the reciprocal of the correct ratio.

Step 3: Conclusion The first-to-second magnetic-field ratio is $N^2 : n^2$. Hence, the correct option is (A).

Quick Tip to Solve: For fixed wire length and current, the field scales as turns squared: one factor comes from turns and another from the smaller radius.

43. A microscope is focused on an object at the bottom of a bucket. If liquid with refractive index $\frac{5}{3}$ is poured inside the bucket, then the microscope has to be raised by 30 cm to focus the object again. The height of the liquid in the bucket is : [4 marks]

(A) 50 cm

(B) 18 cm

(C) 75 cm

(D) 12 cm

Correct Answer: (C) 75 cm

Solution:

Concept:

- For near-normal viewing from air through a plane liquid surface, apparent depth is $d_{\text{app}} = h/\mu$, where h is real depth and μ the liquid refractive index.
- The object appears raised by $h - d_{\text{app}}$. A microscope must be raised by this same amount to focus on the apparent image.

Step 1: Relate the microscope shift to the liquid depth The bucket bottom and the actual object remain fixed when liquid is poured in. Refraction makes the bottom appear nearer the liquid surface. For the given index,

$$d_{\text{app}} = \frac{h}{5/3} = \frac{3h}{5}.$$

Hence the upward image shift is $h - 3h/5 = 2h/5$. The shift is not the apparent depth itself; it is the difference between real and apparent positions.

Step 2: Substitute the measured rise and compare the options The microscope rises by $30\,\text{cm}$, so

$$\frac{2h}{5} = 30\,\text{cm}, \qquad h = 30 \times \frac{5}{2} = 75\,\text{cm}.$$

As a check, the apparent depth is $45\,\text{cm}$, and the image is $75 - 45 = 30\,\text{cm}$ above the actual bottom. Depths $50, 18, 12\,\text{cm}$ would instead give shifts $20, 7.2, 4.8\,\text{cm}$, respectively, so none satisfies the observation.

Step 3: Conclusion The liquid height is $\boxed{75\,\text{cm}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a microscope raised after adding liquid, use shift $= h(1 - 1/\mu)$; do not equate the shift with apparent depth.

44. Match List I with List II.

LIST I		LIST II	
A.	**Angular momentum**	**I.**	$[ML^2T^{-2}]$
B.	**Torque**	**II.**	$[ML^{-2}T^{-2}]$
C.	**Stress**	**III.**	$[ML^2T^{-1}]$
D.	**Pressure gradient**	**IV.**	$[ML^{-1}T^{-2}]$

Choose the correct answer from the options given below: [4 marks]

(A) A - I, B - IV, C - III, D - II

(B) A - III, B - I, C - IV, D - II

(C) A - IV, B - II, C - I, D - III

(D) A - II, B - III, C - IV, D - I

Correct Answer: (B) A - III, B - I, C - IV, D - II

Solution:

Concept:

- Dimensions follow from the defining equations. Force has dimensions $[MLT^{-2}]$, velocity $[LT^{-1}]$, and area $[L^2]$.
- Angular momentum is position crossed with momentum; torque is position crossed with force. Stress is force per area, while a pressure gradient is pressure change per length.

Step 1: Determine angular momentum and torque Angular momentum has magnitude proportional to mvr, giving

$$[L_{\text{ang}}] = [m][v][r] = ML^2T^{-1}.$$

It therefore matches III. Torque has magnitude $rF\sin\theta$; the sine has no dimensions, so

$$[\tau] = [r][F] = ML^2T^{-2}.$$

Torque therefore matches I. The extra inverse time in torque compared with angular momentum also agrees with $\tau = dL_{\text{ang}}/dt$.

Step 2: Determine stress and pressure gradient Stress and pressure are force divided by area:

$$[\sigma] = [P] = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}.$$

Thus stress matches IV. A spatial pressure gradient divides pressure by another length:

$$\left[\frac{dP}{dx}\right] = \frac{ML^{-1}T^{-2}}{L} = ML^{-2}T^{-2}.$$

It matches II. Combining the independent results gives A–III, B–I, C–IV, D–II. Options (A), (C) and (D) already mismatch angular momentum, and their remaining assignments cannot repair that failure.

Step 3: Conclusion The matching is $\boxed{\text{A–III, B–I, C–IV, D–II}}$. Hence, the correct option is (B).

Quick Tip to Solve: Differentiate a quantity with respect to distance by dividing its dimensions by L; a pressure gradient has one more inverse length than pressure.

45. Under the same load, wire A having length $5.0\ \text{m}$ and cross section $2.5\times10^{-5}\ \text{m}^2$ stretches uniformly by the same amount as another wire B of length $6.0\ \text{m}$ and a cross section of $3.0\times10^{-5}\ \text{m}^2$ stretches. The ratio of the Young's modulus of wire A to that of wire B will be : [4 marks]

(A) $1:2$

(B) $1:4$

(C) $1:1$

(D) $1:10$

Correct Answer: (C) $1:1$

Solution:

Concept:

- Young modulus is longitudinal stress divided by longitudinal strain: $Y = (F/A)/(\Delta L/L) = FL/(A\Delta L)$.
- When load F and extension ΔL are equal for two wires, their Young moduli are proportional to L/A, not to length alone.

Step 1: Write the ratio before substituting Let the original lengths be L_A, L_B, and cross-sectional areas be A_A, A_B. Both wires carry the same force and stretch by the same amount. Dividing the two elastic-extension relations gives

$$\frac{Y_A}{Y_B} = \frac{FL_A/(A_A\Delta L)}{FL_B/(A_B\Delta L)} = \frac{L_A A_B}{A_A L_B}.$$

The common force and extension cancel. This is why neither the numerical load nor the numerical extension is required.

Step 2: Use the given lengths and areas Substituting the data in consistent SI units,

$$\frac{Y_A}{Y_B} = \frac{5.0\,(3.0\times10^{-5})}{(2.5\times10^{-5})\,6.0} = \frac{15.0}{15.0} = 1.$$

Equivalently, $L_A/A_A = 2.0\times10^{5}\,\text{m}^{-1}$ and $L_B/A_B = 2.0\times10^{5}\,\text{m}^{-1}$. Both geometrical factors are identical. Ratios $1:2$, $1:4$, and $1:10$ would predict unequal extensions for the same force because the values of L/A are equal.

Step 3: Conclusion The required ratio is $\boxed{Y_A : Y_B = 1:1}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal loads and equal extensions, compare L/A. Here length and area increase by the same factor, so the modulus stays the same.

46. An alternating voltage source $\mathrm{V} = 260\sin(628\mathrm{t})$) is connected across a pure inductor of $5\mathrm{mH}$ Inductive reactance in the circuit is : [4 marks]

(A) 6.28Ω

(B) 0.318Ω

(C) 0.5Ω

(D) 3.14Ω

Correct Answer: (D) 3.14Ω

Solution:

Concept:

- For an ideal inductor driven sinusoidally at angular frequency ω, inductive reactance is $X_L = \omega L$.
- Compare the voltage with $V = V_0\sin(\omega t)$. The coefficient of time is angular frequency in radians per second, not ordinary frequency in hertz.

Step 1: Identify the angular frequency and convert inductance The source is $V = 260\sin(628t)$. Therefore its peak voltage is $260\,\mathrm{V}$ and its angular frequency is $\omega = 628\,\mathrm{rad\,s^{-1}}$. The peak voltage does not enter the reactance formula. Convert the inductance before multiplying:

$$L = 5\,\mathrm{mH} = 5\times10^{-3}\,\mathrm{H}.$$

There is no additional factor 2π, because the angular frequency has already been supplied in the sine argument.

Step 2: Calculate and check the reactance Using the ideal-inductor relation,

$$X_L = 628\,(5\times10^{-3}) = 3.14\,\Omega.$$

The product has units of ohms because $\mathrm{H} = \Omega\,\mathrm{s}$. The reciprocal $1/3.14 \simeq 0.318$ would be an admittance magnitude in siemens, not the requested reactance. The values $6.28\,\Omega$ and $0.5\,\Omega$ likewise do not equal ωL for the given inductance and angular frequency.

Step 3: Conclusion The inductive reactance is $\boxed{3.14\,\Omega}$. Hence, the correct option is (D).

Quick Tip to Solve: Read ω directly from $\sin(\omega t)$, then convert millihenry to henry before evaluating ωL.

47. A stone of mass $1\ \text{kg}$ is tied to end of a massless string of length $1\ \text{m}$. If the breaking tension of the string is $400\ \text{N}$, then maximum linear velocity, the stone can have without breaking the string, while rotating in horizontal plane, is : [4 marks]

(A) $20\ \text{ms}^{-1}$

(B) $40\ \text{ms}^{-1}$

(C) $400\ \text{ms}^{-1}$

(D) $10\ \text{ms}^{-1}$

Correct Answer: (A) $20\ \text{ms}^{-1}$

Solution:

Concept:

- For uniform circular motion, the required inward force is mv^2/r. The speed limit is reached when the string tension reaches its breaking value.
- For the usual horizontal-circle model, the string supplies the centripetal force, giving $v_{\max} = \sqrt{T_{\max} r/m}$. Tension grows as the square of speed.

Step 1: Apply the breaking-tension condition Use $m = 1\,\text{kg}$, the circle radius $r = 1\,\text{m}$, and limiting tension $T_{\max} = 400\,\text{N}$. At the largest allowed speed,

$$\frac{mv_{\max}^2}{r} = T_{\max}, \qquad v_{\max}^2 = \frac{400 \times 1}{1} = 400\,\text{m}^2\text{s}^{-2}.$$

Taking the positive square root gives a speed of $20\,\text{m s}^{-1}$. Speeds larger than this require more inward force than the string can sustain.

Step 2: Check the offered speeds and the horizontal-plane interpretation At $40\,\text{m s}^{-1}$, the needed tension is $1600\,\text{N}$, and at $400\,\text{m s}^{-1}$ it is $160000\,\text{N}$; both exceed the limit. At $10\,\text{m s}^{-1}$, tension is only $100\,\text{N}$, so this is possible but not maximal. If the horizontal circle is produced as a freely suspended conical pendulum, $T\cos\theta = mg$, $r = l\sin\theta$, and $T\sin\theta = mv^2/r$. These give $v^2 = Tl/m - mg^2l/T$, which at the given tension is approximately $399.76\,\text{m}^2\text{s}^{-2}$, again $v \simeq 20\,\text{m s}^{-1}$ at the precision of the options.

Step 3: Conclusion The maximum speed corresponding to the given choices is $\boxed{20\,\text{m s}^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Find a tension-limited speed from $v^2 \propto T$; doubling speed would require four times the tension.

48. A body is moving with constant speed, in a circle of radius $10\ \mathrm{m}$. The body completes one revolution in $4\ \mathrm{s}$. At the end of 3rd second, the displacement of body (in m) from its starting point is : [4 marks]

(A) 15π

(B) 30

(C) $10\sqrt{2}$

(D) 5π

Correct Answer: (C) $10\sqrt{2}$

Solution:

Concept:

- Displacement joins the initial and final positions directly. Its magnitude is the chord length, whereas distance travelled follows the circular arc.
- For radius R and angular separation θ, the chord magnitude is $2R|\sin(\theta/2)|$. Uniform circular motion has angular speed $2\pi/T$.

Step 1: Find the angular position after three seconds The period is $T = 4\,\mathrm{s}$, so the angular speed and angle swept in $3\,\mathrm{s}$ are

$$\omega = \frac{2\pi}{4} = \frac{\pi}{2}\,\mathrm{rad\,s^{-1}}, \qquad \theta = \omega t = \frac{3\pi}{2}.$$

The body has completed three quarters of a revolution. The smaller angle between its initial and final radius vectors is therefore $\pi/2$. It has not returned to the starting point.

Step 2: Calculate the chord rather than the arc The initial and final radius vectors are perpendicular and each has length $10\,\mathrm{m}$. Their endpoints form a right triangle, so

$$|\Delta\mathbf{r}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}\,\mathrm{m}.$$

The chord formula gives the same result, $2(10)\sin(3\pi/4) = 10\sqrt{2}\,\mathrm{m}$. In contrast, $15\pi\ \mathrm{m}$ is the distance travelled along the three-quarter circular arc. The value 5π is a quarter-circle arc length, not this displacement, and $30\,\mathrm{m}$ exceeds the diameter $20\,\mathrm{m}$, so it cannot be any chord of the circle.

Step 3: Conclusion The displacement magnitude is $\boxed{10\sqrt{2}\,\mathrm{m}}$. Hence, the correct option is (C).

Quick Tip to Solve: For displacement on a circle, locate the two radius vectors and find their chord. Any answer greater than the diameter is impossible.

49. Given below are two statements :

Statement I : For transmitting a signal, size of antenna (l) should be comparable to wavelength of signal (at least $l = \frac{\lambda}{4}$ in dimension)

Statement II: In amplitude modulation, amplitude of carrier wave remains constant (unchanged).

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) Statement I is incorrect but Statement II is correct

(B) Both Statement I and Statement II are correct

(C) Both Statement I and Statement II are incorrect

(D) Statement I is correct but Statement II is incorrect

Correct Answer: (D) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- Efficient radiation requires an antenna with dimensions comparable to the wavelength; a quarter-wave antenna has characteristic length $l = \lambda/4$.
- In amplitude modulation, the carrier amplitude varies with the instantaneous message signal. The carrier frequency remains constant; amplitude does not.

Step 1: Evaluate Statement I A very short antenna compared with the signal wavelength is an inefficient radiator. The usual practical wavelength scale is a quarter wavelength, so an antenna dimension l of order $\lambda/4$ is suitable for transmission. This is precisely the criterion described in Statement I, which is therefore correct. It also explains why low-frequency message signals are commonly placed on a high-frequency carrier: the shorter carrier wavelength allows practical antenna dimensions.

Step 2: Evaluate Statement II and choose the combination An amplitude-modulated voltage can be written as

$$v(t) = A_c\,[1 + \mu_m \cos(\omega_m t)] \cos(\omega_c t),$$

where μ_m is modulation index and ω_m, ω_c are message and carrier angular frequencies. The amplitude envelope $A_c[1 + \mu_m \cos(\omega_m t)]$ changes with time whenever modulation is present. Thus Statement II is incorrect: it confuses a constant carrier frequency with a constant modulated amplitude. Options (A) and (C) wrongly reject Statement I, while option (B) wrongly accepts Statement II. Only the combination in (D) agrees with both independent evaluations.

Step 3: Conclusion Statement I is correct and Statement II is incorrect. Hence, the correct option is (D).

Quick Tip to Solve: Remember what the modulation name says changes: AM changes amplitude, while the carrier frequency stays fixed.

50. Given below are two statements :

Statement I: In a typical transistor, all three regions emitter, base and collector have same doping level.

Statement II: In a transistor, collector is the thickest and base is the thinnest segment.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) **Statement I** is correct but **Statement II** is incorrect

(B) Both **Statement I** and **Statement II** are incorrect

(C) **Statement I** is incorrect but **Statement II** is correct

(D) Both **Statement I** and **Statement II** are correct

Correct Answer: (C) **Statement I** is incorrect but **Statement II** is correct

Solution:

Concept:

- A bipolar junction transistor has an emitter that injects majority charge carriers, a thin base that controls their flow, and a collector that collects most injected carriers.
- Doping and dimensions serve different purposes. The emitter is heavily doped, the base is lightly doped and very thin, and the collector is moderately doped with a large physical size to dissipate heat.

Step 1: Evaluate Statement I The three regions cannot have equal doping in a typical transistor. Heavy emitter doping provides a large carrier supply. The lightly doped base limits recombination, allowing most injected carriers to reach the collector. The collector doping is lower than that of the emitter. Thus the claim that emitter, base and collector all have the same doping level is false.

Step 2: Evaluate Statement II The base is the thinnest region so that carriers cross it with little recombination. The collector has the largest physical extent and is the thickest of the three regions in the usual transistor structure. Its larger size also helps dissipate the heat produced during operation. Therefore Statement II is true; thickness must not be confused with doping concentration.

Step 3: Match the truth values to the options The required combination is false for Statement I and true for Statement II. Option A reverses both truth values. Option B incorrectly calls Statement II false. Option D incorrectly calls Statement I true. Only option C preserves both evaluations.

Step 4: Conclusion Statement I is incorrect but Statement II is correct. Hence, the correct option is (C).

Quick Tip to Solve: Check doping and size separately: the emitter is heavily doped, while the collector is physically largest and the base is thinnest.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A water heater of power $2000\ \text{W}$ is used to heat water. The specific heat capacity of water is $4200\ \text{J kg}^{-1}\ \text{K}^{-1}$. The efficiency of heater is 70%. Time required to heat $2\ \text{kg}$ of water from 10°C to 60°C is _____ s.

(Assume that the specific heat capacity of water remains constant over the temperature range of the water). [4 marks]

Solution:

Concept:

- The heat needed for a temperature rise without a change of phase is $Q = mc\Delta T$, where m is mass and c is specific heat capacity.
- Heater efficiency is the fraction of electrical input energy transferred to the water: $\eta = Q/(Pt)$. Thus the useful heating power is ηP, not the full rated power.

Step 1: Calculate the heat absorbed by water The water stays liquid over the stated temperature interval, so no latent-heat term is required. The temperature difference has the same numerical value in kelvin and degrees Celsius:

$$\Delta T = 60 - 10 = 50\,\text{K}.$$

Using the given constant specific heat capacity,

$$Q = 2 \times 4200 \times 50 = 420000\,\text{J}.$$

This is the energy that must actually reach the water, rather than the total electrical energy consumed.

Step 2: Find the useful heating power The efficiency is $70\% = 0.70$. The rate of energy transfer into the water is therefore

$$P_{\text{useful}} = 0.70 \times 2000 = 1400\,\text{J s}^{-1}.$$

The remaining input power does not contribute to the specified temperature increase. Using 2000 W directly would underestimate the heating time.

Step 3: Calculate the heating time Divide the required heat by the useful power:

$$t = \frac{Q}{\eta P} = \frac{420000}{1400} = 300\,\text{s}.$$

Equivalently, the electrical input energy is $420000/0.70 = 600000\,\text{J}$, which at 2000 W takes the same 300 seconds. The dimensions are joules divided by joules per second, giving seconds.

Step 4: Conclusion The required heating time is $\boxed{300\,\text{s}}$.

Quick Tip to Solve: Apply efficiency to the power delivered to the water: use $t = mc\Delta T/(\eta P)$, with efficiency written as a fraction.

52. A series LCR circuit consists of $\mathrm{R} = 80\Omega, \mathrm{X_L} = 100\Omega$, and $\mathrm{X_C} = 40\Omega$. The input voltage is 2500 $\cos(100\pi \mathrm{t})\mathrm{V}$. The amplitude of current, in the circuit, is _____ A. [4 marks]

Solution:

Concept:

- The impedance magnitude of a series LCR circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. Inductive and capacitive reactances oppose each other in phase.
- For a sinusoidal input $v = V_0 \cos \omega t$, the current amplitude is $I_0 = V_0/Z$. Peak voltage must be paired with peak current; RMS quantities are a separate consistent pair.

Step 1: Combine the reactances The supplied reactances already correspond to the applied frequency, so there is no need to infer an inductance or capacitance from the angular frequency. Their net value is

$$X = X_L - X_C = 100 - 40 = 60\,\Omega.$$

The positive sign means the circuit is net inductive. Adding the two reactance magnitudes would be incorrect because their voltage contributions are opposite in phase.

Step 2: Calculate the impedance magnitude The resistance and net reactance contribute perpendicular phasor components. Hence

$$Z = \sqrt{80^2 + 60^2} = \sqrt{6400 + 3600} = 100\,\Omega.$$

The 80-60-100 triangle provides a simple arithmetic check. Although the current lags the voltage in this circuit, only the impedance magnitude is needed to calculate its amplitude.

Step 3: Extract the peak voltage and obtain the current In $v = 2500\cos(100\pi t)\,\mathrm{V}$, the coefficient of the cosine is the peak voltage $V_0 = 2500\,\mathrm{V}$. Therefore

$$I_0 = \frac{2500}{100} = 25\,\mathrm{A}.$$

There is no factor of $\sqrt{2}$ here: introducing it would convert a peak quantity to RMS even though the question explicitly asks for amplitude.

Step 4: Conclusion The amplitude of the current is $\boxed{25\,\mathrm{A}}$.

Quick Tip to Solve: Subtract the reactances before combining them with resistance, and match peak voltage to peak current when amplitude is requested.

53. Two light waves of wavelengths 800 and $600\ \text{nm}$ are used in Young's double slit experiment to obtain interference fringes on a screen placed $7\ \text{m}$ away from plane of slits. If the two slits are separated by $0.35\ \text{mm}$, then shortest distance from the central bright maximum to the point where the bright fringes of the two wavelength coincide will be _____ mm. [4 marks]

Solution:

Concept:

- In Young's double-slit experiment, the bright fringe of order m occurs at $y = m\lambda D/d$, using the usual small-angle fringe relation.
- Two bright fringes coincide when their path differences match: $m_1\lambda_1 = m_2\lambda_2$. The shortest nonzero distance corresponds to the smallest positive integer orders satisfying this condition.

Step 1: Find the first common path difference Let m_1 refer to the 800 nm wave and m_2 to the 600 nm wave. Coincidence requires

$$800m_1 = 600m_2, \qquad 4m_1 = 3m_2.$$

The smallest positive solution is $m_1 = 3$, $m_2 = 4$. Thus the first common path difference away from the central maximum is

$$\delta = 3(800) = 4(600) = 2400\,\text{nm}.$$

The central maximum itself is common at zero distance, but the question seeks the first subsequent common bright point.

Step 2: Convert all lengths to SI units The screen distance is $D = 7\,\text{m}$, and slit separation is

$$d = 0.35\,\text{mm} = 3.5 \times 10^{-4}\,\text{m}.$$

The common path difference is

$$\delta = 2400 \times 10^{-9} = 2.4 \times 10^{-6}\,\text{m}.$$

Converting before substitution avoids mixing nanometres with millimetres.

Step 3: Locate the coincidence on the screen Using $y = D\delta/d$,

$$y = \frac{7 \times 2.4 \times 10^{-6}}{3.5 \times 10^{-4}} = 4.8 \times 10^{-2}\,\text{m} = 48\,\text{mm}.$$

As a check, the separate fringe widths are 16 mm and 12 mm; their first positive common multiple is 48 mm, corresponding to orders 3 and 4 respectively.

Step 4: Conclusion The shortest nonzero distance from the central bright maximum is $\boxed{48\,\text{mm}}$.

Quick Tip to Solve: Reduce the wavelength ratio first: 800:600 = 4:3, so the first common bright fringe has orders 3 and 4, respectively.

54. Two bodies are projected from ground with same speeds $40\ \mathrm{ms}^{-1}$ at two different angles with respect to horizontal. The bodies were found to have same range. If one of the body was projected at an angle of 60°, with horizontal then sum of the maximum heights, attained by the two projectiles, is m. (Given $\mathrm{g} = 10\ \mathrm{ms}^{-2}$) [4 marks]

Solution:

Concept:

- For a projectile launched and landing at the same level, $R = u^2 \sin 2\theta/g$ and $H = u^2 \sin^2\theta/(2g)$, neglecting air resistance.
- Two distinct acute launch angles with the same speed and horizontal range are complementary: $\theta_1 + \theta_2 = 90^\circ$. Their squared sines therefore add to one.

Step 1: Determine the second projection angle Both projectiles have the same speed and experience the same gravitational acceleration. Equal ranges give

$$\sin 2\theta_1 = \sin 2\theta_2.$$

For two different upward projection angles, the relevant solution is $2\theta_2 = 180^\circ - 2\theta_1$. Since $\theta_1 = 60^\circ$, the other angle is $\theta_2 = 30^\circ$. Taking 60 degrees again would contradict the stated different angles.

Step 2: Calculate the individual maximum heights The common factor in the height formula is

$$\frac{u^2}{2g} = \frac{40^2}{2 \times 10} = 80\,\mathrm{m}.$$

The vertical components, rather than the full speeds, determine the heights. Hence

$$H_1 = 80 \sin^2 60^\circ = 80 \times \frac{3}{4} = 60\,\mathrm{m},$$

$$H_2 = 80 \sin^2 30^\circ = 80 \times \frac{1}{4} = 20\,\mathrm{m}.$$

The steeper projectile therefore reaches the larger height, as expected.

Step 3: Add and check the result The requested sum is

$$H_1 + H_2 = 60 + 20 = 80\,\mathrm{m}.$$

An independent shortcut uses complementary angles:

$$H_1 + H_2 = \frac{u^2}{2g}(\sin^2\theta + \cos^2\theta) = \frac{u^2}{2g}.$$

This reproduces the calculated value and shows that the sum is independent of which complementary pair is chosen.

Step 4: Conclusion The sum of the maximum heights is $\boxed{80\,\mathrm{m}}$.

Quick Tip to Solve: For two distinct equal-range projections at the same speed, the angles are complementary and the sum of maximum heights is $u^2/(2g)$.

55. The displacement equations of two interfering waves are given by

$y_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$ cm, $y_2 = 5[\sin\omega t + \sqrt{3}\cos\omega t]$cm **respectively.**

The amplitude of the resultant wave is _____ cm. [4 marks]

Solution:

Concept:

- Two sinusoidal displacements of the same angular frequency add algebraically. Their resultant amplitude depends on their relative phase, not only on their individual amplitudes.
- The identity $\sin(x+\phi) = \sin x\cos\phi + \cos x\sin\phi$ converts a sum of sine and cosine into a single phase-shifted sine wave.

Step 1: Express the second wave in standard form Factor the coefficients so that they correspond to the cosine and sine of one phase angle:

$$y_2 = 10\left(\frac{1}{2}\sin\omega t + \frac{\sqrt{3}}{2}\cos\omega t\right) \text{ cm}.$$

Since $\cos(\pi/3) = 1/2$ and $\sin(\pi/3) = \sqrt{3}/2$, this becomes

$$y_2 = 10\sin(\omega t + \pi/3)\,\text{cm}.$$

Its amplitude is 10 cm, not 5 cm; the multiplier outside the original bracket alone is not the amplitude.

Step 2: Compare the two phases and add the displacements The first wave is already $y_1 = 10\sin(\omega t + \pi/3)\,\text{cm}$. Thus the two waves have equal amplitudes and exactly the same phase. Their phase difference is zero, so the interference is fully constructive:

$$y = y_1 + y_2 = 20\sin(\omega t + \pi/3)\,\text{cm}.$$

The coefficient of this single sine is the resultant amplitude.

Step 3: Verify using the general amplitude relation For two amplitudes A_1, A_2 with phase difference $\Delta\phi$,

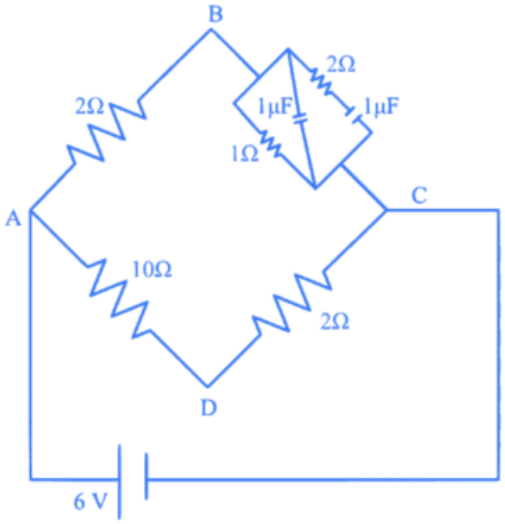
$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\phi.$$

Putting $A_1 = A_2 = 10\,\text{cm}$ and $\Delta\phi = 0$ gives $A^2 = 100 + 100 + 200 = 400\,\text{cm}^2$, confirming the direct addition. Amplitudes add here only because the phases agree.

Step 4: Conclusion The amplitude of the resultant wave is $\boxed{20\,\text{cm}}$.

Quick Tip to Solve: Convert each wave to one sine with a phase before adding amplitudes; a sine-plus-cosine bracket can hide a larger amplitude.

56. For the given circuit, in the steady state, $|V_B - V_D|$ = _____ V. [4 marks]



Solution:

Concept:

- After a DC circuit reaches steady state, an ideal capacitor carries no current. A branch containing a series capacitor is open for the steady conduction-current calculation.
- Resistor voltage drops satisfy Ohm's law $V = IR$. Parallel branches share the same endpoint potential difference, so each conducting branch can be analysed with that common supply voltage.

Step 1: Reduce the upper branch in the steady state In the small network between B and C, the middle branch contains a capacitor and the outer right branch contains a resistor in series with a capacitor. Neither branch conducts in steady state. The remaining 1 ohm resistor directly connects B to C. Therefore the conducting upper path from A to C consists of the 2 ohm resistor from A to B in series with this 1 ohm resistor. Its total resistance is

$$R_{ABC} = 2 + 1 = 3\,\Omega.$$

The capacitor values affect the charging transient but do not change this final DC reduction.

Step 2: Find the potential at B Take $V_C = 0$ as the reference. The battery has its positive terminal at A, so $V_A = 6\,\text{V}$. The current along the upper path is

$$I_{ABC} = \frac{6}{3} = 2\,\text{A}.$$

The potential drop from B to C across the 1 ohm resistor is therefore

$$V_B - V_C = 2 \times 1 = 2\,\text{V}.$$

Thus $V_B = 2\,\text{V}$ in the chosen reference.

Step 3: Find the potential at D and compare The lower path A-D-C contains 10 ohm and 2 ohm

resistors in series. It independently has the same 6 V across its ends, giving

$$I_{ADC} = \frac{6}{10+2} = 0.5\,\text{A}.$$

The drop from D to C is

$$V_D - V_C = 0.5 \times 2 = 1\,\text{V}.$$

Hence $V_D = 1\,\text{V}$, and the required absolute difference is $|2-1|$. Choosing another common reference would shift both potentials equally and leave this difference unchanged.

Step 4: Conclusion The steady-state potential difference is $\boxed{|V_B - V_D| = 1\,\text{V}}$.

Quick Tip to Solve: Open the capacitor branches first, then use the two resistive voltage dividers across A and C to find B and D relative to the same node.

57. Two discs of same mass and different radii are made of different materials such that their thicknesses are 1 cm and 0.5 cm respectively. The densities of materials are in the ratio $3:5$. The moment of inertia of these discs respectively about their diameters will be in the ratio of $\frac{x}{6}$. The value of x is _____. [4 marks]

Solution:

Concept:

- For a uniform disc of radius R, density ρ, thickness t and mass M, $M = \rho\pi R^2 t$. Equal masses therefore require squared radii inversely proportional to density times thickness.
- The moment of inertia of a thin uniform disc about a diameter is $I = MR^2/4$. Because the masses are equal, the inertia ratio is just the ratio of the squared radii.

Step 1: Relate the radii using equal masses Label the discs in the order given, so $t_1 = 1\,\text{cm}$, $t_2 = 0.5\,\text{cm}$ and $\rho_1/\rho_2 = 3/5$. Conservation of the specified equal mass gives

$$\rho_1 \pi R_1^2 t_1 = \rho_2 \pi R_2^2 t_2.$$

Hence

$$\frac{R_1^2}{R_2^2} = \frac{\rho_2 t_2}{\rho_1 t_1} = \frac{5(0.5)}{3(1)} = \frac{5}{6}.$$

The first disc has the smaller radius despite its lower density, because it is twice as thick.

Step 2: Compare the diametral inertias Use the thin-disc formula for each disc, keeping the order unchanged:

$$\frac{I_1}{I_2} = \frac{MR_1^2/4}{MR_2^2/4} = \frac{R_1^2}{R_2^2} = \frac{5}{6}.$$

The common mass and numerical factor cancel. Comparing this with the question's $x/6$ gives $x = 5$. The thicknesses enter through the masses; the standard disc model treats their direct contribution to diametral inertia as negligible.

Step 3: Conclusion The required ratio is $I_1 : I_2 = 5 : 6$, so $\boxed{x = 5}$ (dimensionless).

Quick Tip to Solve: For equal-mass discs, first compare density times thickness; invert that ratio to obtain the squared-radius and inertia ratios.

58. A ball is dropped from a height of 20 m. If the coefficient of restitution for the collision between ball and floor is 0.5, after hitting the floor, the ball rebounds to a height of _____ m. [4 marks]

Solution:

Concept:

- The coefficient of restitution e is the ratio of separation speed to approach speed along the collision normal. For a stationary floor, it equals the rebound speed divided by the incident speed.
- Under gravity, a fall from rest through height h gives $u^2 = 2gh$. A rebound with speed v reaches height $H = v^2/(2g)$; thus height depends on the square of the speed.

Step 1: Find the speed immediately after collision Take speed magnitudes to avoid confusing the reversal of direction with the coefficient. Just before the first impact, the ball has fallen $h = 20\,\text{m}$ from rest, so

$$u = \sqrt{2g(20)}.$$

Since the floor remains at rest and $e = 0.5$,

$$v = eu = 0.5\sqrt{2g(20)}.$$

The ball is moving upward after impact. Its speed is half the incident speed, rather than half its incident kinetic energy.

Step 2: Convert rebound speed into height From just after collision until the top of the rebound, gravity brings the upward speed to zero. Therefore

$$0 = v^2 - 2gH, \qquad H = \frac{v^2}{2g}.$$

Substituting the previous result,

$$H = \frac{(0.5)^2\,2g(20)}{2g} = 0.25(20) = 5\,\text{m}.$$

Gravity cancels because the downward and upward flights occur in the same gravitational field. The retained kinetic energy fraction is $e^2 = 1/4$, consistent with a rebound to one quarter of the original height.

Step 3: Conclusion The ball rebounds to $\boxed{5\,\text{m}}$; the numerical entry is 5.

Quick Tip to Solve: For a bounce on a fixed horizontal floor, use rebound height = initial height times the square of the coefficient of restitution.

59. If the binding energy of ground state electron in a hydrogen atom is $13.6\,\text{eV}$, then, the energy required to remove the electron from the second excited state of Li^{2+} will be : $x \times 10^{-1}\text{eV}$. The value of x is _____. [4 marks]

Solution:

Concept:

- A one-electron ion of atomic number Z has Bohr energy $E_n = -13.6Z^2/n^2\,\text{eV}$, with the zero of energy at a free electron infinitely far from the nucleus.
- Ionising an electron from level n requires the positive binding-energy magnitude $-E_n$. The ground state is $n = 1$, the first excited state is $n = 2$, and the second excited state is $n = 3$.

Step 1: Identify the electronic state and nuclear charge Lithium has atomic number $Z = 3$. The ion Li^{2+} retains just one electron, so the hydrogen-like formula applies directly. The phrase second excited state specifies $n = 3$, not $n = 2$. With the given hydrogen ground-state binding energy as the reference,

$$E_3 = -13.6\frac{3^2}{3^2}\,\text{eV} = -13.6\,\text{eV}.$$

Nuclear attraction increases the binding by $Z^2 = 9$, while occupying $n = 3$ reduces it by the same factor.

Step 2: Calculate the removal energy and requested coefficient To remove the electron completely, the final state is the continuum limit $n \to \infty$, where $E_\infty = 0$. Thus

$$\Delta E = E_\infty - E_3 = 0 - (-13.6) = 13.6\,\text{eV}.$$

This is an ionisation process, not a transition to another finite bound level. Match the specified numerical form:

$$13.6\,\text{eV} = 136 \times 10^{-1}\,\text{eV}.$$

The exponent belongs to the requested energy unit expression; the coefficient x itself is dimensionless.

Step 3: Conclusion The required removal energy is $13.6\,\text{eV}$, giving $\boxed{x = 136}$.

Quick Tip to Solve: Translate the excited-state name into n before substituting; for a hydrogen-like ion, compare Z squared with n squared.

60. Two parallel plate capacitors C_1 and C_2 each having capacitance of $10\mu F$ are individually charged by a 100 V D.C. source. Capacitor C_1 is kept connected to the source and a dielectric slab is inserted between it plates. Capacitor C_2 is disconnected from the source and then a dielectric slab is inserted in it. Afterwards the capacitor C_1 is also disconnected from the source and the two capacitors are finally connected in parallel combination. The common potential of the combination will be _____ V.

(Assuming Dielectric constant $= 10$) [4 marks]

Solution:

Concept:

- Inserting a dielectric of relative permittivity K completely between the plates changes capacitance from C_0 to KC_0. With a battery connected, voltage is fixed; with an isolated capacitor, charge is fixed.
- When charged capacitors are connected in parallel with corresponding polarities together, total charge is conserved and $V_f = (Q_1 + Q_2)/(C_1 + C_2)$. Both dielectric slabs remain in place.

Step 1: Find each charge after dielectric insertion Initially each capacitor has $C_0 = 10\,\mu\text{F}$ and $V_0 = 100\,\text{V}$, so

$$Q_0 = C_0 V_0 = 1000\,\mu\text{C}.$$

With $K = 10$, each final capacitance is $100\,\mu\text{F}$. Capacitor 1 remains connected while its slab is inserted, so its voltage stays 100 V and the source supplies extra charge:

$$Q_1 = (100\,\mu\text{F})(100\,\text{V}) = 10000\,\mu\text{C}.$$

Capacitor 2 was disconnected first, so $Q_2 = 1000\,\mu\text{C}$, and its new voltage is $Q_2/(100\,\mu\text{F}) = 10\,\text{V}$.

Step 2: Conserve charge in the final parallel connection After capacitor 1 is also disconnected, the source supplies no further charge. Connecting the charged capacitors with the same polarity gives

$$Q_{\text{total}} = 10000 + 1000 = 11000\,\mu\text{C}.$$

Their parallel capacitance is

$$C_{\text{total}} = 100 + 100 = 200\,\mu\text{F}.$$

Therefore

$$V_f = \frac{11000\,\mu\text{C}}{200\,\mu\text{F}} = 55\,\text{V}.$$

This lies between the preconnection voltages of 10 V and 100 V, as expected for charge redistribution between equal capacitances.

Step 3: Conclusion The common potential is $\boxed{55\,\text{V}}$, so the numerical entry is 55.

Quick Tip to Solve: Track whether the battery is attached during each insertion: connected fixes voltage, isolated fixes charge. Add charges only after both are disconnected.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. When a hydrocarbon A undergoes complete combustion it requires 11 equivalents of oxygen and produces 4 equivalents of water. What is the molecular formula of A ? [4 marks]

(A) C_9H_8

(B) C_5H_8

(C) $C_{11}H_4$

(D) $C_{11}H_8$

Correct Answer: (A) C_9H_8

Solution:

Concept:

- Complete combustion of a hydrocarbon C_aH_b is

$$C_aH_b + (a^{+}b/_4)O_2 \longrightarrow aCO_2 + (b/_2)H_2O.$$

  The oxygen coefficient accounts for the oxygen needed by both carbon and hydrogen.
- The relative quantities in the question are combustion stoichiometric amounts per amount of hydrocarbon. The water amount first determines hydrogen content; the oxygen amount then determines carbon content.

Step 1: Determine the hydrogen content from water Each water molecule contains two hydrogen atoms, so production of four stoichiometric amounts of water requires eight hydrogens in each molecule of the hydrocarbon:

$$\frac{b}{2} = 4 \quad \Rightarrow \quad b = 8.$$

This immediately excludes $C_{11}H_4$, option (C), which would produce only two water molecules per molecule of fuel. The other three proposed formulae all contain eight hydrogen atoms, so the oxygen requirement must distinguish them.

Step 2: Use the oxygen requirement and test the options The oxygen coefficient is 11, giving

$$a + \frac{8}{4} = 11, \qquad a = 9.$$

The balanced combustion equation is therefore

$$C_9H_8 + 11\,O_2 \longrightarrow 9\,CO_2 + 4\,H_2O.$$

Carbon, hydrogen and oxygen are all balanced: the products contain 18 oxygen atoms in carbon dioxide and four in water, requiring 11 oxygen molecules. Option (B), C_5H_8, needs only $5 + 2 = 7$ oxygen molecules. Option (D), $C_{11}H_8$, needs $11+2 = 13$. Neither meets the stated oxygen requirement, whereas option (A) meets both data simultaneously.

Step 3: Conclusion The molecular formula is $\boxed{C_9H_8}$. Hence, the correct option is (A).

Quick Tip to Solve: Use water production to fix the hydrogen subscript first, then subtract one quarter of that subscript from the oxygen coefficient to find carbon.

62. Cyclohexylamine when treated with nitrous acid yields (P). On treating (P) with PCC results in (Q). When (Q) is heated with dil. NaOH we get (R) The final product (R) is : [4 marks]

(A) 2-(Cyclohex-1-en-1-yl)cyclohexanone

(B) 2-Cyclohexylidenecyclohexanone

(C) 2-(2-Oxocyclohexyl)cyclohexanone

(D) 2-Cyclohexyl-2-hydroxycyclohexanone

Correct Answer: (B) 2-Cyclohexylidenecyclohexanone

Solution:

Concept:

- An aliphatic primary amine reacts with nitrous acid to replace its amino group by hydroxyl, releasing nitrogen. Cyclohexylamine therefore gives cyclohexanol as the relevant alcohol product.
- PCC oxidises a secondary alcohol to a ketone. A ketone with alpha hydrogen undergoes base-catalysed aldol condensation; heating favours dehydration to a conjugated alpha,beta-unsaturated ketone.

Step 1: Identify the alcohol and ketone intermediates The amino-bearing ring carbon of cyclohexylamine becomes the hydroxyl-bearing carbon when treated with nitrous acid, so P is cyclohexanol. It is a secondary alcohol because this carbon is attached to two other ring carbons. PCC oxidises its carbon-hydrogen and oxygen-hydrogen bonds to form a carbonyl at the same ring carbon; thus Q is cyclohexanone. No carbon skeleton expansion or new carbon-carbon bond is needed in these two steps.

Step 2: Follow the aldol condensation and locate the double bond Dilute NaOH removes an alpha hydrogen from one cyclohexanone molecule. Its enolate attacks the carbonyl carbon of a second molecule, forming a new bond between the donor alpha carbon and the acceptor carbonyl carbon. Protonation gives a beta-hydroxy ketone. On heating, loss of water places the double bond across this newly formed link, producing 2-cyclohexylidenecyclohexanone. This is the structure in option (B), with one retained carbonyl and an exocyclic double bond conjugated to it. Option (A) has its alkene within the second ring rather than conjugated through the inter-ring link. Option (C) retains two carbonyl groups and is not the aldol-condensation product. Option (D) has hydroxyl on the donor alpha carbon and is also not the heated dehydration product.

Step 3: Conclusion The final product is 2-cyclohexylidenecyclohexanone, the conjugated enone shown in option (B). Hence, the correct option is (B).

Quick Tip to Solve: In a self-aldol condensation, join the enolate alpha carbon to the other molecule's carbonyl carbon; heating makes that new link a double bond.

63. Compound $A, C_5H_{10}O_5$, given a tetraacetate with Ac_2O and oxidation of A with $Br_2 - H_2O$ gives an acid, $C_5H_{10}O_6$. Reduction of A with HI gives isopentane. The possible structure of A is [4 marks]

(A) $HOCH_2-CO-CH(CH_2OH)-CH(OH)_2$

(B) $OHC-CH(OH)-C(OH)(CH_2OH)_2$

(C) $OHC-(CHOH)_3-CH_2OH$

(D) $HOCH_2-CO-C(OH)(CH_2OH)_2$

Correct Answer: (B) $OHC-CH(OH)-C(OH)(CH_2OH)_2$

Solution:

Concept:

- Formation of a tetraacetate with acetic anhydride establishes four acetylatable hydroxyl groups. Mild oxidation by bromine water converts an aldehyde group to a carboxyl group without oxidising an ordinary ketone.
- Exhaustive reduction with HI replaces oxygen-containing groups by hydrogen while retaining the carbon skeleton. Isopentane is 2-methylbutane, so a branched five-carbon skeleton is required.

Step 1: Apply the hydroxyl and aldehyde tests The starting formula $C_5H_{10}O_5$ contains five oxygen atoms. Four must occur in hydroxyl groups to form the stated tetraacetate, leaving one oxygen in the carbonyl group. Bromine-water oxidation adds one oxygen to give $C_5H_{10}O_6$, which is the conversion $-CHO \longrightarrow -COOH$. Thus the remaining group must be an aldehyde. Options (A) and (D) show ketone carbonyl groups, so they fail this mild-oxidation test even though they show four hydroxyl groups.

Step 2: Distinguish the aldehydes by the carbon skeleton Option (B) has the condensed structure

$$OHC-CH(OH)-C(OH)(CH_2OH)_2.$$

It has four hydroxyl groups: one on the second carbon, one on the central carbon and one on each terminal hydroxymethyl group. It also has an aldehyde and the correct five-carbon formula. On exhaustive reduction with HI, this skeleton becomes $CH_3-CH_2-CH(CH_3)_2$, which is isopentane. Option (C), $OHC-(CHOH)_3-CH_2OH$, also has an aldehyde and four hydroxyl groups but possesses a straight five-carbon chain; it would give n-pentane instead. Only option (B) satisfies all three observations together.

Step 3: Conclusion The branched polyhydroxy aldehyde in option (B) satisfies all the tests. Hence, the correct option is (B).

Quick Tip to Solve: Combine functional-group tests with the hydrocarbon skeleton test: bromine water selects the aldehyde, and HI reduction distinguishes branching.

64. An organic compound $[A]$ $(C_4H_{11}N)$, shows optical activity and gives N_2 gas on treatment with HNO_2. The compound $[A]$ reacts with $PhSO_2Cl$ producing a compound which is soluble in KOH. The structure of A is : [4 marks]

(A) $CH_3CH_2CH_2NHCH_3$

(B) $CH_3CH_2CH(NH_2)CH_3$

(C) $CH_3CH_2N(CH_3)_2$

(D) $CH_3CH_2CH_2CH_2NH_2$

Correct Answer: (B) $CH_3CH_2CH(NH_2)CH_3$

Solution:

Concept:

- Aliphatic primary amines react with nitrous acid to form alcohols with liberation of nitrogen: $RNH_2 + HNO_2 \longrightarrow ROH + N_2 + H_2O$.
- In the Hinsberg test a primary amine forms a sulphonamide containing an acidic N–H bond; its potassium salt dissolves in aqueous KOH. Optical activity requires a chiral structure.

Step 1: Identify the class of amine The evolution of N_2 with nitrous acid identifies an aliphatic primary amine. This agrees independently with the Hinsberg observation: reaction with $PhSO_2Cl$ leaves one hydrogen on nitrogen, giving $PhSO_2NHR$. The sulphonyl group makes that hydrogen acidic, so alkali removes it and forms a soluble salt. A secondary amine instead gives an N,N-disubstituted sulphonamide without this acidic hydrogen; a tertiary amine cannot form the corresponding sulphonamide.

Step 2: Apply the optical-activity condition Among the primary amines shown, option B is butan-2-amine, $CH_3CH_2CH(NH_2)CH_3$. Its second carbon is bonded to four different groups: hydrogen, amino, methyl and ethyl. It is therefore a stereogenic centre and an enantiomerically enriched sample can rotate plane-polarized light. Option D, butan-1-amine, is primary but has no stereogenic carbon. Option A is a secondary amine and option C is tertiary; neither satisfies both chemical tests. Nitrogen inversion also prevents treating the ordinary acyclic nitrogen centre in A as a stable source of optical activity.

Step 3: Conclusion The compound is butan-2-amine, $CH_3CH_2CH(NH_2)CH_3$. Hence, the correct option is (B).

Quick Tip to Solve: First identify a primary amine from nitrogen evolution and alkali-soluble Hinsberg product; then check its carbon atoms for four different substituents.

65. Which of the following elements have half-filled f-orbitals in their ground state?

(Given : atomic number $\mathrm{Sm} = 62; \mathrm{Eu} = 63; \mathrm{Tb} = 65; \mathrm{Gd} = 64, \mathrm{Pm} = 61$ **)**

A. Sm

B. Eu

C. Tb

D. Gd

E. Pm

Choose the correct answer from the options given below : [4 marks]

(A) B and D only

(B) A and B only

(C) C and D only

(D) A and E only

Correct Answer: (A) B and D only

Solution:

Concept:

- An f subshell has seven orbitals and can hold fourteen electrons. Half-filled means the configuration f^7, with one electron in every f orbital.
- Use ground-state configurations of neutral atoms, not the configurations of their common trivalent ions. The stability of $4f^7$ affects gadolinium.

Step 1: Write the relevant ground states All five atoms have a xenon core and two outer 6s electrons. Their configurations are $\mathrm{Pm} : [\mathrm{Xe}]4f^5 6s^2$, $\mathrm{Sm} : [\mathrm{Xe}]4f^6 6s^2$, and $\mathrm{Eu} : [\mathrm{Xe}]4f^7 6s^2$. Gadolinium retains a half-filled f subshell with one electron in 5d: $\mathrm{Gd} : [\mathrm{Xe}]4f^7 5d^1 6s^2$. Terbium has $\mathrm{Tb} : [\mathrm{Xe}]4f^9 6s^2$. Each configuration accounts for the stated atomic number.

Step 2: Select the half-filled f subshells Europium and gadolinium each have exactly seven 4f electrons, so both meet the condition. Samarium has only six and promethium has five, so neither has one electron in all seven orbitals. Terbium has nine: two electrons must pair beyond the half-filled arrangement. Consequently the element labels are B and D. The choice A and B incorrectly includes samarium; C and D includes terbium; A and E contains neither of the required atoms. Do not count the extra gadolinium 5d electron as an eighth 4f electron.

Step 3: Conclusion Only Eu and Gd have half-filled f subshells in their ground states: B and D only. Hence, the correct option is (A).

Quick Tip to Solve: Remember the pair Eu: $4f^7 6s^2$ and Gd: $4f^7 5d^1 6s^2$; the extra electron in Gd enters 5d.

66. Evaluate the following statements for their correctness.

A. The elevation in boiling point temperature of water will be same for $0.1\text{M}\,\text{NaCl}$ and 0.1M urea.

B. Azeotropic mixtures boil without change in their composition.

C. Osmosis always takes place from hypertonic to hypotonic solution.

D. The density of $32\%\,\text{H}_2\text{SO}_4$ solution having molarity $4.09\ \text{M}$ is approximately $1.26\ \text{g mL}^{-1}$

E. A negatively charged sol is obtained when KI solution is added to silver nitrate solution.

Choose the correct answer from the options given below : [4 marks]

(A) A, B and D only

(B) A and C only

(C) B and D only

(D) B, D and E only

Correct Answer: (C) B and D only

Solution:

Concept:

- Boiling-point elevation is $\Delta T_b = iK_b m$; osmosis is solvent flow through a semipermeable membrane from lower to higher solute concentration.
- Azeotropes have identical liquid and vapour compositions at their boiling point. Sol charge depends on preferential adsorption of the ion present in excess.

Step 1: Evaluate statements A, B and C A is false: sodium chloride dissociates into ions, whereas urea remains molecular. For comparable dilute concentrations their particle concentrations and van't Hoff factors differ, so their boiling-point elevations are not the same. B is true: an azeotropic mixture boils at constant composition because its equilibrium vapour has the same composition as the liquid. C reverses the normal direction and is false: solvent moves from the hypotonic solution toward the hypertonic solution, unless sufficient external pressure reverses the process.

Step 2: Calculate the density in D Take one litre of solution. Its acid mass is $4.09 \times 98 = 400.82\,\text{g}$. Since this is 32 percent of the solution mass,

$$m_{\text{solution}} = \frac{400.82}{0.32} = 1252.56\,\text{g}.$$

$$\rho = \frac{1252.56}{1000} = 1.253\,\text{g mL}^{-1} \simeq 1.26\,\text{g mL}^{-1}.$$

Thus D is correct to the stated approximate precision.

Step 3: Check the sol preparation in E Adding potassium iodide solution to silver nitrate solution leaves the receiving silver nitrate medium in excess during the usual preparation. The silver iodide particles adsorb excess Ag^+ ions and form a positively charged sol. A negatively charged sol is obtained by the reverse addition, with iodide in excess. Hence E is false. Options containing A, C or E therefore fail; only the combination B and D remains.

Step 4: Conclusion The correct statements are B and D only. Hence, the correct option is (C).

Quick Tip to Solve: For mass percent, compute solute mass in one litre from molarity, divide by the mass fraction, then divide the solution mass by 1000 mL.

67. Arrange the following orbitals in decreasing order of energy.

A. $\mathrm{n}=3, \mathrm{l}=0,\ \mathrm{m}=0$

B. $\mathrm{n}=4, \mathrm{l}=0,\ \mathrm{m}=0$

C. $\mathrm{n}=3, \mathrm{l}=1,\ \mathrm{m}=0$

D. $\mathrm{n}=3, \mathrm{l}=2,\ \mathrm{m}=1$

The correct option for the order is : [4 marks]

(A) $\mathrm{B}>\mathrm{D}>\mathrm{C}>\mathrm{A}$

(B) $\mathrm{A}>\mathrm{C}>\mathrm{B}>\mathrm{D}$

(C) $\mathrm{D}>\mathrm{B}>\mathrm{A}>\mathrm{C}$

(D) $\mathrm{D}>\mathrm{B}>\mathrm{C}>\mathrm{A}$

Correct Answer: (D) $\mathrm{D}>\mathrm{B}>\mathrm{C}>\mathrm{A}$

Solution:

Concept:

- For the usual multi-electron atom filling order, orbital energy increases with $n+l$. If two orbitals share this sum, the one with smaller n has lower energy.
- The quantum numbers $l=0,1,2$ identify s, p and d subshells. In the absence of an external field, the magnetic quantum number does not change the subshell energy.

Step 1: Identify the subshells and their sums A has $n=3, l=0$, so it is 3s with $n+l=3$. B is 4s and has $n+l=4$. C is 3p and also has $n+l=4$. D is 3d with $n+l=5$. The given values of m merely select an orientation within each subshell; they do not reverse the energy order. Thus 3d is highest among these orbitals and 3s is lowest.

Step 2: Resolve the tie and compare the choices The only tie occurs between 3p and 4s. Their common sum is four, so compare their principal quantum numbers. Because 3 is smaller than 4, 3p has lower energy than 4s. The increasing order is therefore

$$3s<3p<4s<3d.$$

Reversing it gives

$$D>B>C>A.$$

Option A wrongly places 4s above 3d; option B places the lowest-energy 3s first in decreasing order; option C swaps 3s and 3p. Only option D respects both the sum rule and the tie-breaker. This is the standard Aufbau ordering intended by the question, rather than the hydrogenic energy rule depending solely on n.

Step 3: Conclusion The decreasing energy order is $D>B>C>A$. Hence, the correct option is (D).

Quick Tip to Solve: Write $n+l$ beside every orbital first; use n only to break a tie, and reverse the final list if decreasing energy is asked.

68. In Dumas method for the estimation of N_2, the sample is heated with copper oxide and the gas evolved is passed over : [4 marks]

(A) Copper oxide

(B) Copper gauze

(C) Pd

(D) Ni

Correct Answer: (B) Copper gauze

Solution:

Concept:

- In Dumas analysis an organic compound is oxidised by heating with excess copper(II) oxide. Carbon and hydrogen form carbon dioxide and water, while nitrogen is ultimately measured as N_2.
- Hot metallic copper reduces any nitrogen oxides produced during combustion to nitrogen. Absorbers remove water and carbon dioxide before the nitrogen volume is measured.

Step 1: Distinguish combustion from gas purification The sample is already heated with CuO, which supplies oxygen for complete oxidation. The emerging gas may contain oxides of nitrogen as well as molecular nitrogen. Measuring that mixture directly would not give a valid nitrogen estimate because nitrogen oxides have a different composition and may be absorbed in later stages. The gas must therefore pass over a reducing agent after the combustion zone.

Step 2: Identify the reducing material Red-hot copper gauze is used to convert nitrogen oxides into nitrogen. For example,

$$2\,NO + 2\,Cu \longrightarrow N_2 + 2\,CuO.$$

Copper also removes excess oxygen by forming copper oxide. After water and carbon dioxide are removed, the remaining nitrogen can be collected and its volume corrected to the required conditions. Copper oxide in option A is the combustion oxidant, not the required reducing gauze. Palladium and nickel in options C and D are not the material used in this analytical stage; their familiar catalytic applications do not replace the hot-copper reduction step. Thus distinguish the copper oxide used with the sample from the metallic copper used in the downstream tube.

Step 3: Conclusion The evolved gases are passed over red-hot copper gauze. Hence, the correct option is (B).

Quick Tip to Solve: In Dumas analysis remember the sequence: CuO oxidises the sample, hot Cu removes nitrogen oxides and oxygen, and absorbers remove water and carbon dioxide.

69. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A): The first ionization enthalpy of $3\,\mathrm{d}$ series elements is more than that of group 2 metals

Reason (R): In 3d series of elements successive filling of d-orbitals takes place.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) (A) is true but (R) is false

(C) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(D) (A) is false but (R) is true

Correct Answer: (D) (A) is false but (R) is true

Solution:

Concept:

- First ionization enthalpy is the energy required to remove one electron from an isolated gaseous atom. It depends on nuclear attraction, shielding, radius and electronic configuration.
- The 3d transition series is characterised by progressive occupation of the 3d subshell, with familiar configuration exceptions such as chromium and copper.

Step 1: Test the assertion as stated The assertion makes an unrestricted comparison between the 3d series and group 2 metals. There is no universal inequality putting every 3d first ionization enthalpy above those of group 2. For example, magnesium has a first ionization enthalpy of approximately $738\,\mathrm{kJ\,mol^{-1}}$, whereas scandium is approximately $633\,\mathrm{kJ\,mol^{-1}}$. Thus a 3d element can have a lower value than a group 2 metal. Beryllium likewise has a high value because of its small size. The first ionization enthalpies also vary irregularly across the 3d series. The broad assertion is therefore false; it cannot be inferred merely from the term transition metal.

Step 2: Evaluate the reason independently The reason correctly describes the characteristic electronic change across the first transition series: electrons progressively occupy 3d orbitals. Differences in the distribution between 3d and 4s in individual ground states do not invalidate this overall description. Hence R is true. Since A is false, there is no valid explanatory link to test between two true statements. Options A and C both incorrectly declare A true, and option B incorrectly declares R false. Only the combination of a false assertion and a true reason matches the printed statements.

Step 3: Conclusion The assertion is false and the reason is true. Hence, the correct option is (D).

Quick Tip to Solve: Test an unrestricted periodic-trend assertion with a specific counterexample before accepting it; then judge the reason separately.

70. A hydrocarbon ' X ' with formula C_6H_8 uses two moles of H_2 on catalytic hydrogenation of its one mole. On ozonolysis, ' X ' yields two moles of methane dicarbaldehyde. The hydrocarbon ' X ' is: [4 marks]

(A) hexa-1, 3, 5-triene

(B) cyclohexa-1, 4-diene

(C) cyclohexa - 1,3 - diene

(D) 1-methylcyclopenta-1, 4-diene

Correct Answer: (B) cyclohexa-1, 4-diene

Solution:

Concept:

- Catalytic hydrogenation consumes one mole of hydrogen per ordinary carbon–carbon double bond. Ozonolysis cleaves each double bond and turns its two carbon atoms into carbonyl carbons.
- Methane dicarbaldehyde is $OHC-CH_2-CHO$, also called propane-1,3-dial. Two identical molecules of it require two equivalent three-carbon fragments.

Step 1: Count the unsaturations For C_6H_8, the degree of unsaturation is

$$\frac{2(6)+2-8}{2}=3.$$

Consumption of two moles of H_2 accounts for two double bonds. One degree of unsaturation remains, which can be a ring. Thus a cyclic diene is consistent, whereas the acyclic hexa-1,3,5-triene would have three double bonds and consume three moles of hydrogen; it also has formula C_6H_8 but fails the hydrogen-uptake test.

Step 2: Reconnect the ozonolysis fragments In cyclohexa-1,4-diene the two double bonds lie opposite each other in a six-membered ring. Cutting both leaves two equal chains, each with the sequence carbonyl carbon–methylene carbon–carbonyl carbon. Each product is therefore $OHC-CH_2-CHO$, exactly methane dicarbaldehyde. In cyclohexa-1,3-diene, the unequal spacings between the double bonds give a two-carbon dialdehyde and a four-carbon dialdehyde, not two equal three-carbon products. The methyl-substituted five-membered diene is unsymmetrical and includes a substituted alkene carbon, producing a ketonic carbonyl rather than the required pair of identical dialdehydes. Only the 1,4-diene satisfies both tests.

Step 3: Conclusion The hydrocarbon is cyclohexa-1,4-diene. Hence, the correct option is (B).

Quick Tip to Solve: For cyclic-diene ozonolysis, cut both double bonds and count the carbon atoms in each remaining arc; equal three-carbon arcs identify the 1,4-diene.

71. Given below are two statements :

Statement I: Upon heating a borax bead dipped in cupric sulphate in a luminous flame, the colour of the bead becomes green

Statement II: The green colour observed is due to the formation of copper(I) metaborate

In light of the above statements, choose the most appropriate answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are false

(B) Both Statement I and Statement II are true

(C) Statement I is false but Statement II is true

(D) Statement I is true but Statement II is false

Correct Answer: (A) Both Statement I and Statement II are false

Solution:

Concept:

- On strong heating, borax loses its water of crystallisation and forms a transparent mixture of sodium metaborate and boric anhydride: $\mathrm{Na_2B_4O_7} \longrightarrow 2\,\mathrm{NaBO_2} + \mathrm{B_2O_3}$.
- The boric anhydride dissolves a metallic oxide to form a coloured metaborate. The oxidation state of the metal, and hence the bead colour, depends on whether the flame is oxidising or reducing.

Step 1: Identify the effect of the luminous flame A luminous flame provides reducing conditions. With copper salts, the familiar green colour in the hot oxidising flame is not retained under the reducing conditions specified here. The reducing flame produces an opaque red bead containing finely divided metallic copper. Therefore Statement I, which calls the bead green in a luminous flame, is false.

Step 2: Check the identity of the green compound The green hot bead in an oxidising flame is associated with copper(II) metaborate, not copper(I) metaborate. The oxide first reacts with boric anhydride as follows:

$$\mathrm{CuO} + \mathrm{B_2O_3} \longrightarrow \mathrm{Cu(BO_2)_2}.$$

Each metaborate group has charge -1, so two such groups require $\mathrm{Cu^{2+}}$. On cooling the oxidised bead is blue. Statement II incorrectly assigns the green colour to a copper(I) compound, so it is also false.

Step 3: Compare the statement combinations Both statements have now been tested separately. Option (B) incorrectly accepts both; option (C) incorrectly accepts Statement II; and option (D) incorrectly accepts Statement I. Only the combination in (A) rejects both errors.

Step 4: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (A).

Quick Tip to Solve: In the copper borax-bead test, distinguish the hot oxidising green bead from the reducing red opaque bead before identifying the copper oxidation state.

72. In the following halogenated organic compounds, the one with the maximum number of chlorine atoms in its structure is : [4 marks]

(A) Gammaxene

(B) Chloropicrin

(C) Chloral

(D) Freon-12

Correct Answer: (A) Gammaxene

Solution:

Concept:

- Common names must first be converted to molecular formulae. The number of chlorine atoms is the subscript attached to Cl, not the total number of halogen or hydrogen atoms.
- Gammaxene is the gamma isomer of hexachlorocyclohexane, $\mathrm{C_6H_6Cl_6}$. Chloropicrin, chloral and Freon-12 have different carbon frameworks and fewer chlorine atoms.

Step 1: Write the formulae of all four compounds Gammaxene has formula $\mathrm{C_6H_6Cl_6}$, so one molecule contains six chlorine atoms. Chloropicrin is trichloronitromethane, $\mathrm{CCl_3NO_2}$, with three chlorine atoms. Chloral is trichloroacetaldehyde, $\mathrm{CCl_3CHO}$, also with three chlorine atoms. Freon-12 is dichlorodifluoromethane, $\mathrm{CCl_2F_2}$, containing two chlorine atoms and two fluorine atoms.

Step 2: Compare the chlorine counts The requested comparison is therefore

$$N_{\mathrm{Cl}}(\text{Gammaxene}) = 6 > N_{\mathrm{Cl}}(\text{Chloropicrin}) = 3 = N_{\mathrm{Cl}}(\text{Chloral}) > N_{\mathrm{Cl}}(\text{Freon-12}) = 2.$$

The fluorine atoms of Freon-12 do not contribute to its chlorine count. Similarly, the three chlorine atoms in the trichloromethyl group of chloral must not be mistaken for six because its molecule contains two carbon atoms.

Step 3: Eliminate the competing choices Options (B) and (C) tie with each other at three chlorine atoms, but each is below Gammaxene. Option (D) has only two chlorine atoms. Thus all three alternatives fail the requirement of a maximum count, while the six chlorines of Gammaxene give an unambiguous maximum.

Step 4: Conclusion Gammaxene has the maximum number of chlorine atoms: six per molecule. Hence, the correct option is (A).

Quick Tip to Solve: Associate the names with formulae: Gammaxene has six Cl atoms, chloropicrin and chloral have three each, and Freon-12 has two.

73. The incorrect statement for the use of indicators in acid-base titration is : [4 marks]

(A) Phenolphthalein is a suitable indicator for a weak acid vs strong base titration.

(B) Methyl orange may be used for a weak acid vs weak base titration.

(C) Methyl orange is a suitable indicator for a strong acid vs weak base titration.

(D) Phenolphthalein may be used for a strong acid vs strong base titration.

Correct Answer: (B) Methyl orange may be used for a weak acid vs weak base titration.

Solution:

Concept:

- An acid-base indicator is suitable when its colour-transition range lies within the steep pH change near the equivalence point. A visible colour change alone does not establish a reliable endpoint.
- Phenolphthalein changes colour in the alkaline region, approximately pH 8.2–10.0; methyl orange changes in the acidic region, approximately pH 3.1–4.4.

Step 1: Check the titrations with a sharp endpoint For a weak acid titrated with a strong base, the salt hydrolyses to give an alkaline equivalence solution. The steep portion of the curve includes the phenolphthalein range, making (A) correct. For a strong acid titrated with a weak base, the conjugate acid of the weak base makes the equivalence solution acidic; methyl orange is suitable, so (C) is correct.

Step 2: Examine the weak-acid and weak-base case When both the acid and base are weak, the pH change near equivalence is gradual rather than sharply vertical. The methyl orange transition cannot then identify the equivalence point accurately. Even if the equivalence pH happens to lie near an indicator range, there is no sufficiently sharp jump to justify the general statement in (B). A different endpoint-detection method is needed.

Step 3: Check the remaining option and select the incorrect one A strong acid versus strong base titration has a large sharp pH jump spanning the transition ranges of both methyl orange and phenolphthalein. Thus the use of phenolphthalein mentioned in (D) is acceptable. Options (A), (C) and (D) are valid uses; the weak acid versus weak base claim in (B) is the sole incorrect statement.

Step 4: Conclusion Methyl orange is unsuitable for accurately locating the endpoint of a weak acid versus weak base titration. Hence, the correct option is (B).

Quick Tip to Solve: Choose an indicator from the steep part of the titration curve. For a weak acid and weak base, the missing sharp pH jump is the decisive objection.

74. The normal rain water is slightly acidic and its pH value is 5.6 because of which one of the following? [4 marks]

(A) $N_2O_5 + H_2O \rightarrow 2HNO_3$

(B) $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$

(C) $4NO_2 + O_2 + 2H_2O \rightarrow 4HNO_3$

(D) $CO_2 + H_2O \rightarrow H_2CO_3$

Correct Answer: (D) $CO_2 + H_2O \rightarrow H_2CO_3$

Solution:

Concept:

- Pure water is neutral, but ordinary rainwater equilibrates with atmospheric carbon dioxide. Dissolved CO_2 produces a small amount of weak carbonic acid.
- The relation $\mathrm{pH} = -\log_{10}[H^+]$ shows that even a small hydrogen-ion concentration can lower the pH below 7. Normal rainwater has pH close to 5.6.

Step 1: Identify the naturally present atmospheric acid source Carbon dioxide is a normal constituent of the atmosphere. Its dissolution and hydration in rainwater are represented by

$$CO_2 + H_2O \rightleftharpoons H_2CO_3.$$

Carbonic acid is weak and partially ionises:

$$H_2CO_3 \rightleftharpoons H^+ + HCO_3{}^-.$$

The released hydrogen ions explain why normal rain is slightly acidic without requiring industrial pollution.

Step 2: Distinguish normal rain from acid rain At pH 5.6, the hydrogen-ion concentration is about $10^{-5.6} = 2.5 \times 10^{-6}\ \mathrm{mol\,L^{-1}}$, consistent with weak acidity. Options (A) and (C) instead form nitric acid from nitrogen oxides; option (B) forms sulfuric acid from sulfur dioxide. These stronger acids contribute to polluted acid rain and can depress the pH appreciably below the normal value.

Step 3: Compare the four reactions All four reactions can describe acid formation, but the question asks specifically for the natural cause of the normal pH value. The nitric- and sulfuric-acid reactions in (A), (B) and (C) do not provide that explanation. The carbonic-acid reaction in (D) is the one associated with ordinary rainwater exposed to atmospheric carbon dioxide.

Step 4: Conclusion Normal rainwater is slightly acidic because dissolved CO_2 forms H_2CO_3. Hence, the correct option is (D).

Quick Tip to Solve: Link pH 5.6 with atmospheric carbon dioxide; distinguish this natural carbonic acidity from stronger sulfuric and nitric acids in polluted rain.

75. The element playing significant role in neuromuscular function and interneuronal transmission is : [4 marks]

(A) Be

(B) Ca

(C) Mg

(D) Li

Correct Answer: (B) Ca

Solution:

Concept:

- Calcium occurs biologically as Ca^{2+}, which acts as an intracellular signal. Changes in its concentration help couple electrical excitation to cellular responses.
- Neuromuscular function includes muscle activation, while interneuronal transmission involves the release of neurotransmitters at synapses. Calcium ions are central to both processes.

Step 1: Connect calcium with muscle contraction Excitation of a muscle cell releases calcium ions from intracellular stores. In skeletal muscle, Ca^{2+} binds to troponin and allows the contractile proteins actin and myosin to interact. The availability of calcium therefore helps regulate contraction and relaxation, explaining its significant role in neuromuscular function.

Step 2: Connect calcium with transmission between neurons When a nerve impulse reaches a presynaptic terminal, voltage-gated calcium channels open. Calcium enters the terminal and triggers synaptic vesicles to fuse with the membrane, releasing neurotransmitter. That transmitter carries the signal to the next cell. Thus the same ion provides the link between an electrical impulse and chemical communication at a synapse.

Step 3: Exclude the other listed elements Beryllium has no corresponding normal physiological role and is toxic. Magnesium is essential in many enzyme systems and energy metabolism, but it is not the element identified by this particular pair of functions. Lithium has pharmacological uses, but it is not the calcium signal governing muscle activation and synaptic transmitter release. Hence the listed element matching both functions is calcium.

Step 4: Conclusion Calcium plays the significant role described in neuromuscular function and interneuronal transmission. Hence, the correct option is (B).

Quick Tip to Solve: Use the common calcium link: it activates muscle contraction and triggers neurotransmitter release from presynaptic vesicles.

76. Given below are two statements :

Statement I : H_2O_2 **is used in the synthesis of Cephalosporin**

Statement II : H_2O_2 **is used for the restoration of aerobic conditions to sewage wastes.**

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement **I** is incorrect but Statement **II** is correct

(B) Statement **I** is correct but Statement **II** is incorrect

(C) Both Statement **I** and Statement **II** are incorrect

(D) Both Statement **I** and Statement **II** are correct

Correct Answer: (D) Both Statement **I** and Statement **II** are correct

Solution:

Concept:

- Hydrogen peroxide, H_2O_2, is an oxidising agent used in chemical manufacture and environmental treatment. Its usefulness is not limited to bleaching or disinfection.
- Its decomposition releases oxygen: $2\,H_2O_2 \longrightarrow 2\,H_2O + O_2$. The products make it useful where oxygen must be supplied without adding a persistent chemical residue.

Step 1: Evaluate the pharmaceutical statement Hydrogen peroxide is used in the synthesis of pharmaceutical chemicals, including cephalosporin. In this application its controlled oxidising action is useful in the chemical transformations needed during manufacture. The statement concerns a reagent in synthesis; it does not mean that hydrogen peroxide itself is the antibiotic or the final drug. Statement I is therefore correct.

Step 2: Evaluate the sewage-treatment statement Aerobic conditions require oxygen to be available for oxygen-dependent biological processes. Adding hydrogen peroxide can provide oxygen through its decomposition:

$$2\,H_2O_2 \longrightarrow 2\,H_2O + O_2.$$

This supports restoration of aerobic conditions in sewage wastes, especially when oxygen has been depleted. It can also act as an oxidant in waste treatment. Thus Statement II correctly identifies another established application.

Step 3: Select the matching truth combination The two statements describe separate uses and both are valid. Option (A) wrongly rejects the pharmaceutical use; option (B) wrongly rejects the sewage-treatment use; and option (C) wrongly rejects both. Option (D) alone accepts both correct statements. No claim that one statement explains the other is needed because the question asks only for their correctness.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (D).

Quick Tip to Solve: Remember hydrogen peroxide as both an industrial oxidant and an oxygen source: these explain its pharmaceutical and sewage-treatment applications.

77. Match List I with List II

LIST I		LIST II	
A.	**Physisorption**	**I.**	**Single Layer Adsorption**
B.	**Chemisorption**	**II.**	$20-40\ \text{kJ mol}^{-1}$
C.	$N_2(g)+3H_2(g) \xrightarrow{Fe(s)} 2NH_3(g)$	**III.**	**Chromatography**
D.	**Analytical Application or Adsorption**	**IV.**	**Heterogeneous catalysis**

Choose the correct answer from the options given below: [4 marks]

(A) A - III, B - IV, C - I, D - II

(B) A - II, B - III, C - I, D - IV

(C) A - II, B - I, C - IV, D - III

(D) A - IV, B - II, C - III, D - I

Correct Answer: (C) A - II, B - I, C - IV, D - III

Solution:

Concept:

- Physisorption is caused mainly by weak van der Waals forces and usually has a low enthalpy of adsorption, about $20–40\ \text{kJ mol}^{-1}$. Chemisorption involves formation of chemical bonds at the surface.
- Surface adsorption also underlies heterogeneous catalysis and chromatographic separation. Match each example with its defining property rather than treating every surface process as the same phenomenon.

Step 1: Match the two types of adsorption Physisorption has relatively weak intermolecular attraction and is associated with the low adsorption enthalpy range listed as II. Thus A–II. In chemisorption, bonding occurs at specific surface sites; after a monolayer forms, further molecules cannot form the same bonds to the occupied surface. Therefore chemisorption corresponds to single-layer adsorption, giving B–I.

Step 2: Classify the ammonia synthesis example In $N_2(g)+3\,H_2(g) \xrightarrow{Fe\,(s)} 2\,NH_3(g)$, the reactants are gases while the catalyst is solid iron. The catalytic process occurs at the iron surface and the catalyst is in a different phase from the reacting gases. This is heterogeneous catalysis, so C–IV. It is not an analytical separation or merely a statement about a monolayer.

Step 3: Identify the analytical application and compare options Chromatography separates components because they interact with the stationary phase to different extents; adsorption is central to adsorption chromatography. Hence D–III. Combining the matches gives A–II, B–I, C–IV, D–III. Option (A) already mismatches physisorption; option (B) assigns chromatography to chemisorption and single-layer adsorption to ammonia synthesis; option (D) likewise mismatches the listed properties. Only (C) contains all four correct assignments.

Step 4: Conclusion The correct matching is A–II, B–I, C–IV, D–III. Hence, the correct option is (C).

Quick Tip to Solve: Anchor this matching with physisorption as low adsorption enthalpy and chemisorption as a monolayer; then identify iron-catalysed ammonia synthesis by its different phases.

78. **Which of the following compounds are not used as disinfectants ?**

A. Chloroxylenol

B. Bithional

C. Veronal

D. Prontosil

E. Terpineol

Choose the correct answer from the options given below : [4 marks]

(A) A, B

(B) C, D

(C) B, D, E

(D) A, B, E

Correct Answer: (B) C, D

Solution:

Concept:

- Disinfectants destroy or inhibit microorganisms on non-living objects. Antiseptic formulations are applied to living tissues; the use and concentration distinguish these closely related categories.
- Drug classification depends on therapeutic action. A tranquilliser affects the central nervous system, whereas an antibacterial drug acts against bacterial infection in the body.

Step 1: Classify the disinfectant ingredients Chloroxylenol (A) is a phenolic antimicrobial ingredient used in disinfectant and antiseptic preparations. Bithional (B), also written bithionol, has antimicrobial action and is used in medicated soaps and related antiseptic preparations. Terpineol (E) is also an antimicrobial ingredient used in disinfectant formulations. Thus A, B and E belong to the antimicrobial disinfectant/antiseptic group rather than the group the question asks us to select.

Step 2: Identify the two other therapeutic classes Veronal (C) is a barbiturate sedative or tranquilliser. It is used for its action on the nervous system, not as a surface disinfectant. Prontosil (D) is an antibacterial sulphonamide prodrug; its antibacterial therapeutic action does not make it a disinfectant. The compounds that are not used as disinfectants are therefore C and D.

Step 3: Eliminate the remaining combinations Option (A) selects A and B, both antimicrobial ingredients, so it fails. Option (C) includes B and E together with D, so it contains two compounds from the wrong category. Option (D) lists A, B and E, the disinfectant-related set itself. Only option (B) lists precisely the two requested non-disinfectants.

Step 4: Conclusion Veronal and Prontosil, labelled C and D, are not used as disinfectants. Hence, the correct option is (B).

Quick Tip to Solve: Separate surface disinfection from treatment inside the body: an antibacterial medicine such as Prontosil is not automatically a disinfectant.

79. The Lewis acid character of boron tri halides follows the order : [4 marks]

(A) $BI_3 > BBr_3 > BCl_3 > BF_3$

(B) $BCl_3 > BF_3 > BBr_3 > BI_3$

(C) $BF_3 > BCl_3 > BBr_3 > BI_3$

(D) $BBr_3 > BI_3 > BCl_3 > BF_3$

Correct Answer: (A) $BI_3 > BBr_3 > BCl_3 > BF_3$

Solution:

Concept:

- A Lewis acid accepts an electron pair. Boron trihalides are electron-deficient because the boron atom has only six electrons in its valence shell and an available vacant orbital.
- Halogen lone-pair donation into the vacant boron orbital reduces the electron deficiency of boron. In the standard boron trihalide comparison, this back donation is strongest for fluorine and weakens down the halogen group.

Step 1: Compare the orbital overlap In BF_3, both the fluorine donor orbital and the boron acceptor orbital are compact 2p orbitals. Their good overlap favours $p\pi$-$p\pi$ back bonding. Donation of electron density towards boron partly compensates its deficiency, making it less eager to accept an external electron pair. Thus the very high electronegativity of fluorine alone does not predict the observed order.

Step 2: Follow the change down the group For chlorine, bromine and iodine, the relevant halogen orbitals become successively larger relative to the boron 2p orbital. Effective back donation diminishes. The electron-pair accepting tendency therefore increases from the fluoride to the iodide:

$$BF_3 < BCl_3 < BBr_3 < BI_3.$$

Written from strongest to weakest Lewis acid, this is the reverse sequence.

Step 3: Match all four alternatives Option (A) gives the iodide first, followed by bromide, chloride and fluoride, exactly as required. Option (B) incorrectly puts chloride above iodide and bromide. Option (C) reverses the trend and makes fluoride strongest. Option (D) incorrectly makes bromide stronger than iodide. These errors cannot be repaired by using electronegativity alone.

Step 4: Conclusion The Lewis acid character follows $BI_3 > BBr_3 > BCl_3 > BF_3$. Hence, the correct option is (A).

Quick Tip to Solve: For the standard boron trihalide order, remember that stronger halogen-to-boron back bonding means weaker acceptance of an external lone pair.

80. Which one of the following statements is incorrect? [4 marks]

(A) Cast iron is obtained by melting pig iron with scrap iron and coke using hot air blast.

(B) The malleable iron is prepared from cast iron by oxidising impurities in a reverberatory furnace.

(C) van Arkel method is used to purify tungsten.

(D) Boron and Indium can be purified by zone refining method.

Correct Answer: (C) van Arkel method is used to purify tungsten.

Solution:

Concept:

- A refining method must suit the physical or chemical properties of the metal. Zone refining uses the greater solubility of impurities in molten metal than in solid metal.
- The van Arkel method relies on forming a volatile metal iodide and decomposing it at a hot filament. Titanium and zirconium are characteristic examples; tungsten is instead purified by the hydrogen-reduction route.

Step 1: Check the statements about iron Pig iron is remelted with scrap iron and coke using a hot air blast to obtain cast iron, so statement (A) describes the accepted preparation. Wrought or malleable iron is obtained by oxidising impurities in cast iron in a reverberatory furnace lined with an oxidising material. Carbon and other impurities are removed, giving a softer, workable iron. Thus statement (B) is also accepted in this metallurgy context.

Step 2: Test the named refining method The van Arkel process is associated with conversion of impure titanium or zirconium into volatile iodides followed by thermal decomposition on a hot filament. It is not the standard purification method for tungsten. Tungsten can be obtained in high purity by reduction of tungsten oxide with hydrogen, for example:

$$\mathrm{WO_3 + 3\,H_2 \longrightarrow W + 3\,H_2O.}$$

Consequently statement (C), which assigns tungsten to the van Arkel method, is incorrect.

Step 3: Confirm the remaining statement Boron and indium are among substances for which zone refining can be used when very high purity is required. A narrow molten zone is moved along the solid and carries impurities towards one end, which is removed. Statement (D) is therefore correct. Since the question asks for the incorrect statement, the three valid statements (A), (B) and (D) must be rejected as answers.

Step 4: Conclusion The incorrect statement is that the van Arkel method is used to purify tungsten. Hence, the correct option is (C).

Quick Tip to Solve: Match van Arkel refining with titanium and zirconium; match zone refining with substances requiring very high purity and favourable impurity solubility.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Assume carbon burns according to the following equation :

$$2C_{(s)} + O_{2(\ g)} \rightarrow 2CO(g)$$

when $12\ g$ **carbon is burnt in** $48\ g$ **of oxygen, the volume of carbon monoxide produced is _____** $\times 10^{-1}\ L$ **at STP [nearest integer]**

[Given: Assume CO **as ideal gas, Mass of** C **is** $12\ g\ mol^{-1}$**, Mass of** O **is** $16\ g\ mol^{-1}$ **and molar volume of an ideal gas at STP is** $22.7\ L\ mol^{-1}$ **]** [4 marks]

Solution:

Concept:

- For a balanced reaction, stoichiometric coefficients relate amounts in moles, not directly masses. The limiting reactant is exhausted first and fixes the amount of product.
- At the stated STP, the question supplies the molar volume $22.7\,L\,mol^{-1}$. The requested blank is a numerical coefficient multiplying $10^{-1}\,L$.

Step 1: Convert each starting mass to moles The atomic molar mass of carbon is $12\,g\,mol^{-1}$. Oxygen is present as O_2, so its molecular molar mass is twice the stated atomic value:

$$M(O_2) = 2(16) = 32\,g\,mol^{-1}.$$

$$n(C) = \frac{12}{12} = 1.00\,mol, \qquad n(O_2) = \frac{48}{32} = 1.50\,mol.$$

Using 16 as the molar mass of oxygen gas would confuse atoms with molecules.

Step 2: Determine the limiting reactant and product amount The stipulated equation is $2\,C + O_2 \longrightarrow 2\,CO$. One mole of carbon therefore needs only half a mole of oxygen. The available $1.50\,mol$ of oxygen exceeds that requirement, so carbon is limiting and oxygen remains in excess. Carbon and carbon monoxide have equal coefficients, giving

$$n(CO) = 1.00\,mol.$$

We use the given formation of carbon monoxide rather than replacing it with a combustion equation producing carbon dioxide.

Step 3: Calculate and express the volume in the requested form For ideal carbon monoxide at the stated STP,

$$V = nV_m = 1.00(22.7) = 22.7\,L.$$

If the blank is x, then

$$x \times 10^{-1} = 22.7, \qquad x = 227.$$

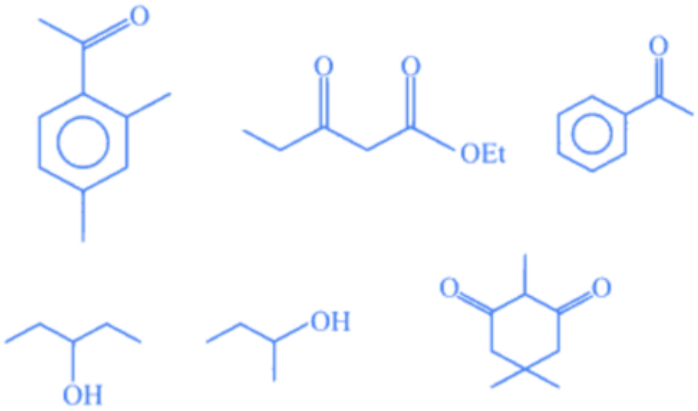
The number is already an integer, so nearest-integer rounding does not change it.

Step 4: Conclusion The carbon monoxide volume is $22.7\,L = 227 \times 10^{-1}\,L$. The required numerical

answer is $\boxed{227}$.

Quick Tip to Solve: First identify the limiting reagent, then convert the final physical volume into the power-of-ten unit attached to the blank.

82. The number of molecules which gives the haloform test among the following molecules is _____. [4 marks]



Solution:

Concept:

- The haloform test is positive for methyl ketones containing CH_3CO^-. In alkaline halogen solution, their methyl group is successively halogenated and cleaved to form haloform.
- A secondary alcohol of the form $CH_3CH(OH)R$ is also positive because it is oxidised to a methyl ketone under the test conditions. A methyl group elsewhere in a molecule is insufficient.

Step 1: Inspect the three molecules in the upper row Number the displayed structures left to right, starting in the upper row. The first is an aromatic methyl ketone: the acetyl group attached to the substituted benzene ring contains $ArCOCH_3$, so it gives the test. The second is $CH_3CH_2COCH_2COOEt$; its ketone carbonyl has an ethyl group on one side and a methylene group on the other. It has no $COCH_3$ group and is negative. The third is acetophenone, $C_6H_5COCH_3$, a methyl ketone and therefore positive.

Step 2: Inspect the alcohols in the lower row The lower-left structure is pentan-3-ol, with condensed formula

$$CH_3CH_2CH(OH)CH_2CH_3.$$

Oxidation gives pentan-3-one, not a methyl ketone, so it is negative. The lower-middle structure is butan-2-ol:

$$CH_3CH(OH)CH_2CH_3.$$

It is oxidised to butan-2-one, which contains CH_3CO^-, and is positive.

Step 3: Check the cyclic diketone and total the positives The lower-right cyclic diketone has carbonyl groups within the ring. Neither carbonyl carbon is bonded directly to a methyl carbon. Its ring

methyl substituents do not create the required methyl-ketone arrangement, so it is negative. The positive compounds are therefore the upper-left aromatic methyl ketone, upper-right acetophenone and lower-middle butan-2-ol: three molecules in all.

Step 4: Conclusion Three displayed molecules give the haloform test. The required count is $\boxed{3}$.

Quick Tip to Solve: Circle the actual CH_3CO^- unit, or a $CH_3CH(OH)^-$ unit that oxidises to it; do not count isolated methyl substituents.

83. Amongst the following, the number of species having the linear shape is _____.

$XeF_2, I_3^+, C_3O_2, I_3^-, CO_2, SO_2, BeCl_2$ **and** $BCl_2^{\ominus}$ [4 marks]

Solution:

Concept:

- VSEPR theory predicts the arrangement of electron domains around a central atom. Molecular shape describes atom positions, with lone pairs omitted from the named shape.
- A two-coordinate centre is linear when it has two bonding domains and no lone pair, or when its lone pairs occupy positions that leave the two bonds opposite. Multiple bonds count as one domain for this purpose.

Step 1: Classify xenon difluoride and the iodine ions XeF_2 is AX_2E_3: its three lone pairs occupy equatorial positions in a trigonal-bipyramidal electron arrangement, leaving the fluorine atoms opposite, hence linear. I_3^- has the same central-iodine AX_2E_3 arrangement and is linear. In I_3^+, the central iodine has two bonding pairs and two lone pairs, an AX_2E_2 arrangement. It is bent rather than linear.

Step 2: Classify the oxides and beryllium chloride Carbon suboxide has the cumulene skeleton $O{=}C{=}C{=}C{=}O$, whose carbon centres each have two bonding directions; its atomic skeleton is linear. Carbon dioxide, $O{=}C{=}O$, likewise has two bonding domains and is linear. Sulphur dioxide has two bonding domains and a lone pair on sulphur; its molecular shape is bent. Monomeric $BeCl_2$ has two bond domains and no central lone pair, so its shape is linear.

Step 3: Check the remaining boron species and count For BCl_2^-, boron supplies three valence electrons and the negative charge adds one. After forming two B–Cl bonds, one lone pair remains at boron. Its AX_2E electron arrangement is trigonal planar and its molecular shape is bent. The linear species are therefore XeF_2, C_3O_2, I_3^-, CO_2 and $BeCl_2$. The other three species are bent.

Step 4: Conclusion The number of species with linear shape is $\boxed{5}$.

Quick Tip to Solve: Count central-atom lone pairs carefully in the two iodine ions: the change from I_3^- to I_3^+ changes the molecular shape.

84. Enthalpies of formation of $CCl_4(g), H_2O(g), CO_2(g)$ and $HCl(g)$ are $-105, -242, -394$ and -92 kJ mol^{-1} respectively. The magnitude of enthalpy of the reaction given below is _____ kJ mol^{-1}. (nearest integer)

$$CCl_4(g) + 2H_2O(g) \rightarrow CO_2(g) + 4HCl(g)$$

[4 marks]

Solution:

Concept:

- Hess's law makes reaction enthalpy independent of the path. Using standard enthalpies of formation, $\Delta H_r = \sum \nu \Delta H_f(\text{products}) - \sum \nu \Delta H_f(\text{reactants})$.
- Each formation enthalpy must be multiplied by its coefficient in the balanced equation. The sign of the reaction enthalpy identifies heat release or absorption, whereas its magnitude is nonnegative.

Step 1: Sum the product formation enthalpies The products are one mole of gaseous carbon dioxide and four moles of gaseous hydrogen chloride per reaction as written. Keeping the supplied negative signs,

$$\begin{aligned}\sum \nu \Delta H_f(\text{products}) &= (-394) + 4(-92) \\ &= -394 - 368 \\ &= -762\,\text{kJ mol}^{-1}.\end{aligned}$$

The factor four is essential because the given HCl formation enthalpy applies to one mole of HCl.

Step 2: Sum the reactant formation enthalpies The reactants are one mole of gaseous carbon tetrachloride and two moles of water vapour. We must use the supplied vapour value, not a liquid-water value:

$$\begin{aligned}\sum \nu \Delta H_f(\text{reactants}) &= (-105) + 2(-242) \\ &= -105 - 484 \\ &= -589\,\text{kJ mol}^{-1}.\end{aligned}$$

These sums use the stoichiometric equation exactly as given, with no reversal or rescaling.

Step 3: Subtract and take the requested magnitude Subtracting the reactant sum from the product sum gives

$$\Delta H_r = -762 - (-589) = -173\,\text{kJ mol}^{-1}.$$

The reaction is exothermic because its enthalpy change is negative. The blank asks for the magnitude, so

$$|\Delta H_r| = 173\,\text{kJ mol}^{-1}.$$

This value is already an integer; rounding therefore leaves it unchanged.

Step 4: Conclusion The reaction enthalpy is $-173\,\text{kJ mol}^{-1}$, whose magnitude is $173\,\text{kJ mol}^{-1}$. The required numerical answer is $\boxed{173}$.

Quick Tip to Solve: Write products minus reactants with both negative sums in brackets; take the absolute value only after completing the subtraction.

85. At 298 K, the solubility of silver chloride in water is 1.434×10^{-3} g L^{-1}. The value of $-\log K_{sp}$ for silver chloride is _____.

(Given mass of Ag is 107.9 g mol^{-1} and mass of Cl is 35.5 g mol^{-1}) [4 marks]

Solution:

Concept:

- Solubility product is the product of the equilibrium ionic concentrations, with each raised to its stoichiometric coefficient. For silver chloride, $AgCl(s) \rightleftharpoons Ag^+(aq) + Cl^-(aq)$.
- For a sparingly soluble 1:1 salt in pure water, each ionic concentration equals its molar solubility s, so $K_{sp} = s^2$. Convert mass solubility to moles before using this expression.

Step 1: Find the formula mass The molar mass is the sum of the given atomic masses. Thus

$$M(AgCl) = 107.9 + 35.5 = 143.4\,g\,mol^{-1}.$$

The given solubility is grams per litre, whereas the ionic concentrations required in the equilibrium expression are moles per litre.

Step 2: Convert the solubility and calculate the product Divide the mass dissolved in one litre by its molar mass:

$$s = \frac{1.434 \times 10^{-3}}{143.4} = 1.000 \times 10^{-5}\,mol\,L^{-1}.$$

Each mole of dissolved silver chloride supplies one mole of each ion. Therefore

$$K_{sp} = [Ag^+][Cl^-] = (10^{-5})(10^{-5}) = 10^{-10}.$$

No common ion is present to change this 1:1 concentration relation.

Step 3: Take the negative logarithm The logarithm is to base ten, as indicated by the conventional notation log. Therefore

$$-\log K_{sp} = -\log(10^{-10}) = 10.$$

The negative sign converts the negative logarithm of the small solubility product into a positive number.

Step 4: Conclusion The required value of $-\log K_{sp}$ is $\boxed{10}$, a dimensionless logarithmic value.

Quick Tip to Solve: For a 1:1 salt in pure water, first find molar solubility and square it; do not square a solubility still expressed in grams per litre.

86. The resistivity of a 0.8M solution of an electrolyte is $5 \times 10^{-3}\Omega$ cm.

Its molar conductivity is _____ $\times 10^4\ \Omega^{-1}\ cm^2\ mol^{-1}$. (Nearest integer) [4 marks]

Solution:

Concept:

- Conductivity κ is the reciprocal of resistivity ρ: $\kappa = 1/\rho$. With resistivity in $\Omega\,cm$, conductivity is in $\Omega^{-1}\,cm^{-1}$.

- Molar conductivity is conductivity divided by the amount concentration in moles per cubic centimetre. If concentration c is in moles per litre, $\Lambda_m = 1000\kappa/c$.

Step 1: Calculate conductivity from resistivity The electrolyte has resistivity $5 \times 10^{-3}\,\Omega\,\mathrm{cm}$. Taking the reciprocal gives

$$\kappa = \frac{1}{5 \times 10^{-3}} = 200\,\Omega^{-1}\,\mathrm{cm}^{-1}.$$

A reciprocal is necessary: resistivity describes opposition to current, whereas conductivity describes the ability to conduct it.

Step 2: Use a consistent concentration unit The concentration is $0.8\,\mathrm{mol\,L^{-1}}$. Since one litre contains $1000\,\mathrm{cm}^3$, this equals

$$c = \frac{0.8}{1000} = 8 \times 10^{-4}\,\mathrm{mol\,cm^{-3}}.$$

Dividing conductivity by this concentration produces the required area-per-mole units rather than a result smaller by a factor of one thousand.

Step 3: Find the requested coefficient Now substitute in the molar conductivity formula:

$$\Lambda_m = \frac{200}{8 \times 10^{-4}} = 250000 = 25 \times 10^4\,\Omega^{-1}\,\mathrm{cm}^2\,\mathrm{mol}^{-1}.$$

The blank multiplies 10^4, so it asks for the coefficient 25, not for the full value 250000. This coefficient is already an integer and needs no rounding change.

Step 4: Conclusion The molar conductivity is $25 \times 10^4\,\Omega^{-1}\,\mathrm{cm}^2\,\mathrm{mol}^{-1}$; the nearest integer to enter is $\boxed{25}$.

Quick Tip to Solve: When resistivity uses centimetres and concentration uses molarity, use $\Lambda_m = 1000/(\rho c)$, then extract the power of ten printed beside the blank.

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87. A sample of a metal oxide has formula $\mathrm{M_{0.83}O_{1.00}}$. The metal M can exist in two oxidation states $+2$ and $+3$.

In the sample of $\mathrm{M_{0.83}O_{1.00}}$, the percentage of metal ions existing in $+2$ oxidation state is _____ %. (nearest integer) [4 marks]

Solution:

Concept:

- An electrically neutral oxide has total positive charge equal to total negative charge. Oxygen in this metal oxide is present as O^{2-}.
- If the metal occupies only the $+2$ and $+3$ states, the numbers in those states must add to the total metal content. The percentage sought uses all metal ions as the denominator, not the oxygen content.

Step 1: Choose the formula-unit basis Take the composition exactly as printed, with 1.00 oxygen atom and 0.83 metal atoms on a formula-unit basis. Let x be the number of metal ions in the $+2$ state

and y the number in the $+3$ state on this basis. The metal balance is

$$x + y = 0.83.$$

Fractional values here describe the average composition of a crystal and need not be whole numbers.

Step 2: Apply electrical neutrality The single oxide ion contributes negative charge of magnitude 2. The positive charges must therefore satisfy

$$2x + 3y = 2.$$

Multiply the metal balance by two and subtract it from the charge balance:

$$y = 2 - 2(0.83) = 0.34.$$

Hence

$$x = 0.83 - 0.34 = 0.49.$$

As a check, $2(0.49) + 3(0.34) = 0.98 + 1.02 = 2.00$, which exactly neutralizes the oxide charge.

Step 3: Calculate the fraction of divalent metal ions There are 0.49 divalent ions among 0.83 total metal ions on the chosen basis. Thus

$$\%\mathrm{M}^{2+} = \frac{0.49}{0.83} \times 100 = 59.036\ldots\%.$$

The requested nearest integer is obtained only after this final division; rounding the composition prematurely would change the result.

Step 4: Conclusion The percentage of metal ions in the $+2$ oxidation state is approximately 59.04%, so the required nearest integer is $\boxed{59}$.

Quick Tip to Solve: For a mixed-valence oxide, write one equation for total metal and another for total charge; divide the desired ion count by the total metal count.

---

88. If the CFSE of $[\mathrm{Ti}\,(\mathrm{H_2O})_6]^{3+}$ is -96.0 kJ/mol, this complex will absorb maximum at wavelength _____ nm. (nearest integer)

Assume Planck's constant (h) $= 6.4 \times 10^{-34}$Js, Speed of light (c) $= 3.0 \times 10^8$ m/s and Avogadro's Constant $(\mathrm{N_A}) = 6 \times 10^{23}$/mol [4 marks]

Solution:

Concept:

- Titanium has atomic number 22; Ti^{3+} is a d^1 ion. In an octahedral field its one electron occupies the lower t_{2g} level.
- One t_{2g} electron contributes $-0.4\Delta_o$ to CFSE. The absorption promotes that electron across the full splitting Δ_o, with photon energy $E = hc/\lambda$.

Step 1: Obtain the full octahedral splitting Water is neutral, so titanium is in the $+3$ oxidation state

in $[Ti(H_2O)_6]^{3+}$. For this d^1 octahedral complex,

$$\text{CFSE} = -0.4\Delta_o = -96.0\,\text{kJ mol}^{-1}.$$

Consequently

$$\Delta_o = \frac{96.0}{0.4} = 240\,\text{kJ mol}^{-1}.$$

The absorption energy is the full separation between the two levels; it is not simply the magnitude of the CFSE.

Step 2: Convert the energy to one photon The splitting just calculated is per mole of complexes. Divide by the supplied Avogadro constant to obtain the energy absorbed by a single complex:

$$E = \frac{240 \times 10^3}{6 \times 10^{23}} = 4.0 \times 10^{-19}\,\text{J}.$$

The factor 10^3 converts kilojoules to joules so that the result is consistent with Planck's constant in joule seconds.

Step 3: Calculate the absorption wavelength Using the constants given in the question,

$$\lambda = \frac{hc}{E} = \frac{(6.4 \times 10^{-34})(3.0 \times 10^8)}{4.0 \times 10^{-19}} = 4.8 \times 10^{-7}\,\text{m}.$$

Since $1\,\text{nm} = 10^{-9}\,\text{m}$, this is $480\,\text{nm}$. The wavelength is positive, as required for an absorbed photon.

Step 4: Conclusion The complex absorbs maximally at $\boxed{480\,\text{nm}}$; the nearest integer to enter is 480.

Quick Tip to Solve: For an octahedral d^1 complex, divide the magnitude of CFSE by 0.4 before using the photon-energy formula; CFSE itself is not the absorption energy.

---

89. The rate constant for a first order reaction is $20\ \text{min}^{-1}$. The time required for the initial concentration of the reactant to reduce to its $\frac{1}{32}$ level is _____ $\times 10^{-2}$ min. (Nearest integer)

(Given : $\ln 10 = 2.303$ and $\log 2 = 0.3010$) [4 marks]

Solution:

Concept:

- The integrated first-order rate equation is $kt = \ln([A]_0/[A]_t)$, where $[A]_0$ is the initial concentration and $[A]_t$ the concentration remaining at time t.
- A first-order rate constant has reciprocal-time units. With k in min^{-1}, the time calculated from the integrated equation is in minutes.

Step 1: Translate the remaining fraction into a logarithm The reactant concentration is reduced to one thirty-second of its initial value, so

$$[A]_t = \frac{[A]_0}{32}, \qquad \frac{[A]_0}{[A]_t} = 32 = 2^5.$$

This means five half-lives have elapsed. The logarithm involves the ratio of concentrations, so the actual numerical value of the initial concentration cancels.

Step 2: Evaluate the time with the supplied constants Using the given common logarithm of 2,

$$\ln 2 = (\ln 10)\log 2 = (2.303)(0.3010) = 0.693203.$$

Hence

$$\ln 32 = 5\ln 2 = 3.466015.$$

Substitute in the first-order rate law:

$$t = \frac{\ln 32}{k} = \frac{3.466015}{20} = 0.17330075\,\text{min}.$$

The positive time follows because the final concentration is smaller than the initial concentration.

Step 3: Express the answer in the printed scale The blank is followed by $10^{-2}\,\text{min}$. Write the calculated time in exactly this form:

$$0.17330075\,\text{min} = 17.330075 \times 10^{-2}\,\text{min}.$$

Therefore the coefficient rounds to 17. As a check, one half-life is $0.693203/20 = 0.03466015\,\text{min}$; five such intervals reproduce the calculated time.

Step 4: Conclusion The time is approximately $0.1733\,\text{min} = 17.33 \times 10^{-2}\,\text{min}$, so the required nearest integer is $\boxed{17}$.

Quick Tip to Solve: Recognize $1/32 = (1/2)^5$: the required time is five half-lives, then convert to the power of ten specified beside the blank.

---

90. The number of alkali metal(s), from $\text{Li}, \text{K}, \text{Cs}$, Rb having ionization enthalpy greater than $400\ \text{kJ mol}^{-1}$ and forming stable super oxide is _____. [4 marks]

Solution:

Concept:

- First ionization enthalpy generally decreases down Group 1 because the outer electron is farther from the nucleus and increasingly shielded by inner shells.
- The relatively large superoxide ion O_2^- is stabilized in ionic solids by larger alkali-metal cations. Potassium, rubidium and caesium form stable superoxides, whereas lithium preferentially forms the normal oxide.

Step 1: Apply the ionization-enthalpy condition Use the approximate first ionization enthalpies of the four listed metals: lithium 520, potassium 419, rubidium 403, and caesium $376\,\text{kJ mol}^{-1}$. Thus lithium, potassium and rubidium exceed the threshold of $400\,\text{kJ mol}^{-1}$. Caesium is below it and must be excluded even though it forms a stable superoxide. Rubidium is just above the specified boundary.

Step 2: Apply the stable-superoxide condition Among the listed metals, the stable superoxides are

KO_2, RbO_2, and CsO_2. Their larger cations accommodate the large superoxide anion effectively. Lithium is not included in this group: its small cation favours the normal oxide Li_2O. The question requires the standard stable oxide chemistry of these metals, not merely their ability to react with oxygen.

Step 3: Take the intersection of the two groups The ionization-enthalpy test gives Li, K and Rb, whereas the stable-superoxide test gives K, Rb and Cs. Only potassium and rubidium occur in both groups. The word "and" therefore requires an intersection: counting every metal that meets either condition would incorrectly retain lithium or caesium. There are exactly two qualifying metals.

Step 4: Conclusion The qualifying metals are potassium and rubidium, so the required number is $\boxed{2}$ (dimensionless).

Quick Tip to Solve: Apply both filters separately: Li fails stable-superoxide formation, Cs fails the ionization threshold, and K and Rb satisfy both conditions.