JEE Main 2023 Paper 1 (B.E./B.Tech) Jan 25 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 25 January 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The vector $\overrightarrow{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\overrightarrow{b}$. Then the projection of $3\overrightarrow{a} + \sqrt{2}\overrightarrow{b}$ on $\overrightarrow{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is : [4 marks]

(A) $\sqrt{6}$

(B) $2\sqrt{3}$

(C) 1

(D) $3\sqrt{2}$

Correct Answer: (D) $3\sqrt{2}$

Solution:

Concept:

- A rotation preserves vector length. A right-angle rotation gives $\vec{a} \cdot \vec{b} = 0$, and passing through the positive y-axis selects the orientation in the plane containing $\vec{a}$ and $\hat{j}$.
- The scalar projection of $\vec{v}$ on $\vec{c}$ is $\vec{v} \cdot \vec{c}/|\vec{c}|$; it is a scalar rather than the vector projection.

Step 1: Construct the perpendicular direction Here $|\vec{a}|^2 = 1 + 4 + 1 = 6$ and $\vec{a} \cdot \hat{j} = 2$. Remove from $\hat{j}$ its component parallel to $\vec{a}$:

$$\vec{u} = \hat{j} - \frac{2}{6}\vec{a} = \frac{1}{3}(\hat{i} + \hat{j} - \hat{k}).$$

This vector points toward the y-axis from the original direction and satisfies $\vec{u} \cdot \vec{a} = 0$. Its magnitude is $1/\sqrt{3}$, so multiplying its unit vector by $|\vec{a}| = \sqrt{6}$ gives the rotated vector

$$\vec{b} = \sqrt{2}(\hat{i} + \hat{j} - \hat{k}).$$

The opposite sign would rotate away from the specified y-axis. As checks, $|\vec{b}| = \sqrt{6}$ and $\vec{a} \cdot \vec{b} = 0$.

Step 2: Calculate the scalar projection Combine the vectors before taking the dot product:

$$3\vec{a} + \sqrt{2}\vec{b} = (-3, 6, 3) + (2, 2, -2) = (-1, 8, 1).$$

For $\vec{c} = (5, 4, 3)$,

$$|\vec{c}| = \sqrt{25 + 16 + 9} = 5\sqrt{2}, \qquad (-1, 8, 1) \cdot (5, 4, 3) = -5 + 32 + 3 = 30.$$

Hence the required projection is

$$\frac{30}{5\sqrt{2}} = 3\sqrt{2}.$$

The dot product 30 itself is not the projection: division by the magnitude of the target vector is essential. The alternatives $\sqrt{6}$, $2\sqrt{3}$, and 1 do not equal this quotient.

Step 3: Conclusion The required scalar projection is $3\sqrt{2}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the perpendicular part of the axis direction first; normalize it to the original length, then use a single dot product for the projection.

2. The minimum value of the function $f(x) = \int_0^2 e^{|x-t|}dt$ is : [4 marks]

(A) 2

(B) $2(e-1)$

(C) $e(e-1)$

(D) $2e-1$

Correct Answer: (B) $2(e-1)$

Solution:

Concept:

- Split an integral containing $|x-t|$ at $t=x$ when x lies between its limits. Outside that interval the sign is constant.
- For positive numbers with fixed product, AM-GM gives the least possible sum. Equality requires the two numbers to be equal.

Step 1: Evaluate the integral on the central interval For $0 \le x \le 2$, the exponent is $x-t$ for $0 \le t \le x$, and $t-x$ for $x \le t \le 2$. Thus

$$f(x) = \int_0^x e^{x-t}\,dt + \int_x^2 e^{t-x}\,dt.$$

Evaluating both pieces gives

$$f(x) = (e^x - 1) + (e^{2-x} - 1) = e^x + e^{2-x} - 2.$$

The product of the two exponential terms is e^2. Therefore AM-GM yields

$$e^x + e^{2-x} \ge 2e, \qquad f(x) \ge 2e - 2.$$

Equality holds when $e^x = e^{2-x}$, which gives $x = 1$, a point inside the required interval.

Step 2: Check points outside the integration interval For $x \le 0$, every t in the integral satisfies $t \ge x$, so

$$f(x) = e^{-x}\int_0^2 e^t\,dt = e^{-x}(e^2-1) \ge e^2 - 1.$$

For $x \ge 2$, similarly

$$f(x) = e^{x}\int_0^2 e^{-t}\,dt = e^{x-2}(e^2-1) \ge e^2 - 1.$$

Since $(e^2-1) - 2(e-1) = (e-1)^2 > 0$, neither exterior interval gives a smaller value. Option A overlooks the strict increase of the exponential away from zero, option C is the exterior boundary scale, and option D exceeds the attained minimum by 1. The value in option B is attained exactly at $x = 1$.

Step 3: Conclusion The global minimum is $f(1) = 2(e-1)$. Hence, the correct option is (B).

Quick Tip to Solve: For an absolute-value integral with symmetric limits around a midpoint, split at the variable and test the midpoint; still check the exterior intervals for a global minimum.

3. Let $x = 2$ be a local minima of the function $f(x) = 2x^4 - 18x^2 + 8x + 12, x \in (-4, 4)$. If M is local maximum value of the function f in $(-4, 4)$, then M = [4 marks]

(A) $18\sqrt{6} - \frac{33}{2}$
(B) $18\sqrt{6} - \frac{31}{2}$
(C) $12\sqrt{6} - \frac{33}{2}$
(D) $12\sqrt{6} - \frac{31}{2}$

Correct Answer: (C) $12\sqrt{6} - \frac{33}{2}$

Solution:

Concept:

- A differentiable function has stationary interior extrema where $f\prime(x) = 0$. The sign of $f\prime\prime$ distinguishes a strict local maximum from a minimum.
- At a stationary point, its polynomial equation can reduce higher powers and simplify evaluation of the function exactly.

Step 1: Find and classify all stationary points Differentiate the printed polynomial:

$$f\prime(x) = 8x^3 - 36x + 8 = 4(x - 2)(2x^2 + 4x - 1).$$

Hence its stationary points are

$$x = 2, \qquad x = -1 \pm \frac{\sqrt{6}}{2}.$$

All three lie in $(-4, 4)$. The second derivative is $f\prime\prime(x) = 24x^2 - 36$. At $x = 2$, it equals 60, agreeing with the stated local minimum. Put $r = -1 + \sqrt{6}/2$. This number lies between 0 and $1/2$, so $24r^2 - 36 < 0$; it is the local maximum point. The other root is less than -2, so its second derivative is positive and it is another local minimum. Endpoints are excluded and cannot supply the requested local maximum.

Step 2: Evaluate the maximum exactly Because $2r^2 + 4r - 1 = 0$, we have

$$r^2 = \frac{1}{2} - 2r, \qquad r^4 = \left(\frac{1}{2} - 2r\right)^2 = \frac{9}{4} - 10r.$$

Substitute these identities rather than expanding a fourth power of a radical:

$$\begin{aligned} f(r) &= 2\left(\frac{9}{4} - 10r\right) - 18\left(\frac{1}{2} - 2r\right) + 8r + 12 \\ &= \frac{15}{2} + 24r \\ &= \frac{15}{2} + 24\left(-1 + \frac{\sqrt{6}}{2}\right) \\ &= 12\sqrt{6} - \frac{33}{2}. \end{aligned}$$

This calculation gives option C: the coefficient of the radical is 12, excluding A and B, and the constant is $-33/2$, excluding D. The classification ensures this is a maximum value, rather than merely a

stationary value.

Step 3: Conclusion The local maximum value is $M = 12\sqrt{6} - \frac{33}{2}$. Hence, the correct option is (C).

Quick Tip to Solve: After locating a radical stationary point, use its quadratic relation to reduce powers before substitution; this avoids lengthy fourth-power expansions.

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4. The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2, then their new variance is equal to : [4 marks]

(A) 3.92

(B) 4.08

(C) 3.96

(D) 4.04

Correct Answer: (C) 3.96

Solution:

Concept:

- For n observations, the population variance is $\sigma^2 = \sum x_i^2/n - \bar{x}^2$. Both the mean and the mean square change when one observation changes.
- Replacing an observation a by b changes the sum by $b - a$ and the sum of squares by $b^2 - a^2$, while the number of observations remains fixed.

Step 1: Determine the number of students Let the number of students be n. The original sum of their scores is $10n$. Replacing 8 by 12 increases the sum by 4, and the mean rises from 10 to 10.2. Therefore

$$10n + 4 = 10.2n, \qquad 0.2n = 4, \qquad n = 20.$$

The count does not change: one score is corrected, rather than an additional student being added. As a check, the original sum is 200 and the corrected sum is 204, whose quotient by 20 is indeed 10.2.

Step 2: Update the second moment and subtract the new mean square The original variance is 4, hence the original mean square is

$$\frac{\sum x_i^2}{20} = 4 + 10^2 = 104, \qquad \sum x_i^2 = 2080.$$

The correction changes the sum of squares by

$$12^2 - 8^2 = 144 - 64 = 80.$$

Thus the new mean square is

$$\frac{2080 + 80}{20} = 108.$$

Subtract the square of the corrected mean, not the original mean:

$$\sigma^2_{\text{new}} = 108 - (10.2)^2 = 108 - 104.04 = 3.96.$$

The variance decreases slightly because a value below the old mean moves to a comparable distance above it while the mean itself shifts upward. Neither 3.92 nor 4.08 nor 4.04 equals the exact difference of the two corrected moments; simply modifying the old variance by a linear score change would omit the mean shift.

Step 3: Conclusion The new variance is 3.96. Hence, the correct option is (C).

Quick Tip to Solve: For a corrected observation, update the sum and sum of squares separately; compute variance only after using the corrected mean.

5. The value of $\lim\limits_{n\to\infty} \frac{1+2-3+4+5-6+\ldots..+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ **is :** [4 marks]

(A) $\frac{3}{2\sqrt{2}}$

(B) $\frac{3}{2}(\sqrt{2}+1)$

(C) $3(\sqrt{2}+1)$

(D) $\frac{\sqrt{2}+1}{2}$

Correct Answer: (B) $\frac{3}{2}(\sqrt{2}+1)$

Solution:

Concept:

- Group a repeating sign pattern before summing. Here each block has three consecutive integers and the signs $+, +, -$.
- When radicals contain fourth-degree polynomials, each radical grows as n^2. Divide numerator and denominator by n^2 to expose the leading coefficients.

Step 1: Sum the numerator in complete blocks For the k-th block,

$$(3k-2)+(3k-1)-3k = 3k-3.$$

There are exactly n such blocks, ending at $-3n$. Therefore the numerator is

$$\sum_{k=1}^{n}(3k-3) = 3\frac{n(n+1)}{2} - 3n = \frac{3}{2}n(n-1).$$

The first block is zero and the second is 3, consistent with this formula. Treating the expression as an alternating series with one positive and one negative term would give the wrong leading coefficient because its actual period is three.

Step 2: Divide by the common growth rate Since $n > 0$, factoring n^4 from each radical gives

$$\sqrt{2n^4+4n+3} = n^2\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}},$$

$$\sqrt{n^4+5n+4} = n^2\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}.$$

Divide the original ratio by n^2 in numerator and denominator and take the limit:

$$L = \frac{\frac{3}{2}(1-1/n)}{\sqrt{2+4/n^3+3/n^4}-\sqrt{1+5/n^3+4/n^4}} \longrightarrow \frac{3}{2(\sqrt{2}-1)}.$$

The limiting denominator is nonzero, so direct substitution at this stage is valid. Rationalizing yields

$$L = \frac{3}{2}(\sqrt{2}+1).$$

Option A replaces the difference of leading radicals by a single radical, option C doubles the numerator coefficient, and option D loses a factor of 3.

Step 3: Conclusion The limit is $\frac{3}{2}(\sqrt{2}+1)$. Hence, the correct option is (B).

Quick Tip to Solve: First sum the signed blocks exactly; then compare the degree of that sum with the growth of each radical before choosing a normalization.

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6. Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S = \{x \in \mathbb{Z} : x(66-x) \geq \frac{5}{9}M\}$ and the event $A = \{x \in S : x \text{ is a multiple of } 3\}$. Then P(A) is equal to : [4 marks]

(A) $\frac{1}{3}$
(B) $\frac{1}{5}$
(C) $\frac{7}{22}$
(D) $\frac{15}{44}$

Correct Answer: (A) $\frac{1}{3}$

Solution:

Concept:

- For a fixed sum, the product of two positive integers is greatest when they are as equal as possible. Completing the square gives this maximum directly.
- In a finite equally likely sample space, $P(A) = |A|/|S|$. Integer endpoints of a closed interval must both be counted.

Step 1: Find the maximum and the complete sample space The two integers may be written as x and $66-x$. Their product is

$$x(66-x) = 1089-(x-33)^2 \leq 1089.$$

Equality is possible for the positive integers 33 and 33, so $M = 1089$. Now the defining inequality becomes

$$1089-(x-33)^2 \geq \frac{5}{9}(1089) = 605.$$

Thus

$$(x-33)^2 \le 484 = 22^2, \qquad -22 \le x-33 \le 22.$$

Since x must be an integer, $S = \{11, 12, \ldots, 55\}$. The inequality is inclusive, so both endpoints are present and $|S| = 55 - 11 + 1 = 45$.

Step 2: Count the multiples of three in the sample space The first allowed multiple of 3 is 12 and the last is 54. The favorable outcomes form the arithmetic progression

$$12, 15, 18, \ldots, 54.$$

Their number is

$$|A| = \frac{54-12}{3} + 1 = 15.$$

Equivalently, it is $\lfloor 55/3 \rfloor - \lfloor 10/3 \rfloor = 18 - 3 = 15$. Hence

$$P(A) = \frac{15}{45} = \frac{1}{3}.$$

The count includes 33 only once, because the outcomes are integers x, not unordered pairs of integers. In particular, 44 is not the sample-space size: omitting the extra 1 in an inclusive interval would incorrectly lead toward the alternative $15/44$. Neither $1/5$ nor $7/22$ is the quotient of the actual favorable and total counts.

Step 3: Conclusion The required probability is $\frac{1}{3}$. Hence, the correct option is (A).

Quick Tip to Solve: Complete the square to turn a product condition into an integer interval, and use last minus first plus one when both boundary integers are allowed.

7. Let $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}dx$. If $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$, then $f(4)$ is equal to [4 marks]

(A) $\log_e 19 - \log_e 20$

(B) $\log_e 17 - \log_e 18$

(C) $\frac{1}{2}(\log_e 19 - \log_e 17)$

(D) $\frac{1}{2}(\log_e 17 - \log_e 19)$

Correct Answer: (D) $\frac{1}{2}(\log_e 17 - \log_e 19)$

Solution:

Concept:

- For a rational expression in x^2 containing $2x\,dx$, substitute $u = x^2$. The resulting integral can be resolved into simple fractions.
- An indefinite integral represents a family of antiderivatives. A prescribed function value determines the additive constant, which must be retained until that condition is applied.

Step 1: Integrate the rational expression Put $u = x^2$, so that $du = 2x\,dx$. Then

$$f(x) = \int \frac{du}{(u+1)(u+3)}.$$

The partial-fraction identity is

$$\frac{1}{(u+1)(u+3)} = \frac{1}{2}\left(\frac{1}{u+1} - \frac{1}{u+3}\right).$$

Integrating both terms gives

$$f(x) = \frac{1}{2}\log_e\left(\frac{x^2+1}{x^2+3}\right) + C.$$

Both quadratic expressions are strictly positive for every real x, so these real logarithms require no additional domain restriction. Differentiation verifies the sign and factor: half of $2x/(x^2+1) - 2x/(x^2+3)$ equals the printed integrand.

Step 2: Use the given value and evaluate at four At $x = 3$, the expression becomes

$$f(3) = \frac{1}{2}\log_e\frac{10}{12} + C = \frac{1}{2}(\log_e 5 - \log_e 6) + C.$$

This is equal to the supplied value only when $C = 0$. Therefore at $x = 4$,

$$f(4) = \frac{1}{2}\log_e\frac{17}{19} = \frac{1}{2}(\log_e 17 - \log_e 19).$$

This value is negative since $17/19 < 1$, immediately excluding the reversed positive logarithmic difference in option C. Options A and B use different ratios and omit the necessary factor of one half; they do not arise from substituting $x^2 = 16$ into the verified antiderivative. The initial-value condition has been applied explicitly rather than assuming the integration constant vanishes.

Step 3: Conclusion The value is $f(4) = \frac{1}{2}(\log_e 17 - \log_e 19)$. Hence, the correct option is (D).

Quick Tip to Solve: Use the initial condition before substituting the target input; logarithmic antiderivatives often look correct while hiding a nonzero integration constant.

8. The distance of the point P(4, 6, −2) from the line passing through the point (−3, 2, 3) and parallel to a line with direction ratios 3, 3, −1 is equal to : [4 marks]

(A) 3

(B) $\sqrt{14}$

(C) $\sqrt{6}$

(D) $2\sqrt{3}$

Correct Answer: (B) $\sqrt{14}$

Solution:

Concept:

- The perpendicular distance from a point P to a line through A with direction vector $\mathbf{b}$ is the length of the component of $\overrightarrow{AP}$ perpendicular to $\mathbf{b}$.
- The projection formula gives $d^2 = |\overrightarrow{AP}|^2 - (\overrightarrow{AP} \cdot \mathbf{b})^2/|\mathbf{b}|^2$. Direction ratios need not first be converted to a unit vector.

Step 1: Form the displacement and direction vectors Take $A = (-3, 2, 3)$, $P = (4, 6, -2)$ and $\mathbf{b} = (3, 3, -1)$. The line is $A + t\mathbf{b}$, so the displacement from its known point to P is

$$\overrightarrow{AP} = (4 + 3, 6 - 2, -2 - 3) = (7, 4, -5).$$

Its squared length is $49 + 16 + 25 = 90$, while $|\mathbf{b}|^2 = 9 + 9 + 1 = 19$. These quantities include all three coordinate differences.

Step 2: Remove the parallel component The scalar product is

$$\overrightarrow{AP} \cdot \mathbf{b} = 7(3) + 4(3) + (-5)(-1) = 38.$$

Thus the square of the parallel component is $38^2/19 = 76$, and Pythagoras gives

$$d^2 = 90 - 76 = 14, \qquad d = \sqrt{14}.$$

As a check, the foot parameter is $t = 38/19 = 2$, giving the foot $(3, 8, 1)$. Its displacement from P is $(1, -2, -3)$, orthogonal to $(3, 3, -1)$, and its squared length is again 14. The other proposed distances have squares 9, 6 and 12, so none equals the required perpendicular distance.

Step 3: Conclusion The distance is $\boxed{\sqrt{14}}$ coordinate units. Hence, the correct option is (B).

Quick Tip to Solve: Compute the squared distance by subtracting the squared projection; taking a square root only at the end avoids unnecessary cross-product expansion.

9. Let $y(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$. Then $y' - y''$ at $x = -1$ is equal to [4 marks]

(A) 496

(B) 976

(C) 464

(D) 944

Correct Answer: (A) 496

Solution:

Concept:

- When a product contains a factor vanishing at the evaluation point, separate that factor before differentiating. Here write $y = (1+x)g(x)$.
- For a nonzero product g, logarithmic differentiation gives g'/g as the sum of each factor derivative divided by that factor.

Step 1: Isolate the vanishing factor Let

$$g(x) = (1+x^2)(1+x^4)(1+x^8)(1+x^{16}).$$

At $x = -1$, every factor in g equals 2, so $g(-1) = 2^4 = 16$. Differentiate $y = (1+x)g$ twice using the product rule:

$$y' = g + (1+x)g', \qquad y'' = 2g' + (1+x)g''.$$

The factors $1+x$ vanish at the specified point. Consequently, only $g(-1)$ and $g'(-1)$ are needed; there is no need to expand a degree-31 polynomial.

Step 2: Evaluate the logarithmic derivative For $n = 2, 4, 8, 16$, the contribution at -1 is

$$\left.\frac{nx^{n-1}}{1+x^n}\right|_{x=-1} = -\frac{n}{2}.$$

Adding gives

$$\frac{g'(-1)}{g(-1)} = -\frac{2+4+8+16}{2} = -15, \qquad g'(-1) = -240.$$

It follows that $y'(-1) = 16$ and $y''(-1) = -480$. Therefore

$$y'(-1) - y''(-1) = 16 - (-480) = 496.$$

The negative sign of the second derivative is essential. The values 976, 464 and 944 do not equal this difference; in particular 464 comes from subtracting 16 instead of adding it to 480.

Step 3: Conclusion $\boxed{y'(-1) - y''(-1) = 496}$. Hence, the correct option is (A).

Quick Tip to Solve: At a simple zero of a product, differentiate the nonvanishing product logarithmically and use the vanishing factor to eliminate higher derivatives.

10. Consider the lines L_1 and L_2 given by

$$L_1 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2}$$

$L_2 : \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$.

A line L_3 having direction ratios 1, -1, -2, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is [4 marks]

(A) $4\sqrt{3}$

(B) $2\sqrt{6}$

(C) 4

(D) $3\sqrt{2}$

Correct Answer: (B) $2\sqrt{6}$

Solution:

Concept:

- A point on a line in symmetric form is obtained by equating the three ratios to one parameter.
- If the segment PQ has direction ratios $(1, -1, -2)$, then $\overrightarrow{PQ} = k(1, -1, -2)$ and its length is $|k|\sqrt{6}$.

Step 1: Parametrise the two intersection points Since P lies on L_1 and Q lies on L_2, write

$$P = (1 + 2t, 3 + t, 2 + 2t), \qquad Q = (2 + u, 2 + 2u, 3 + 3u).$$

Subtracting these coordinates in the same order gives

$$\overrightarrow{PQ} = (1 + u - 2t, -1 + 2u - t, 1 + 3u - 2t).$$

This difference must be parallel to the prescribed direction of L_3; the actual values of t and u are not necessarily equal merely because both points lie on that transversal.

Step 2: Impose parallelism and solve the parameters Equate the components to $(k, -k, -2k)$:

$$\begin{aligned} 1 + u - 2t &= k, \\ -1 + 2u - t &= -k, \\ 1 + 3u - 2t &= -2k. \end{aligned}$$

Adding the first two equations gives $3u - 3t = 0$, hence $u = t$. The first equation now gives $k = 1 - t$. Substitute these into the third:

$$1 + t = -2(1 - t), \qquad t = u = 3, \qquad k = -2.$$

Thus $P = (7, 6, 8)$, $Q = (5, 8, 12)$, and $\overrightarrow{PQ} = (-2, 2, 4)$.

Step 3: Calculate the segment length The distance formula gives

$$PQ = \sqrt{(-2)^2 + 2^2 + 4^2} = \sqrt{24} = 2\sqrt{6}.$$

This also equals $|k|\sqrt{1+1+4} = 2\sqrt{6}$. The other options have squared lengths 48, 16 and 18, rather than 24, so they are ruled out by the coordinate calculation.

Step 4: Conclusion The segment has length $\boxed{2\sqrt{6}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a transversal with known direction ratios, introduce one common multiplier for Q minus P; solving three linear equations is quicker than finding its full line equation.

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11. The points of intersection of the line $ax + by = 0, (a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is : [4 marks]

(A) $x^2 + y^2 + 5x + 5y + 12 = 0$

(B) $x^2 + y^2 + 3x + 5y + 8 = 0$

(C) $x^2 + y^2 - 5x - 5y + 12 = 0$

(D) $x^2 + y^2 + 3x + 3y + 4 = 0$

Correct Answer: (A) $x^2 + y^2 + 5x + 5y + 12 = 0$

Solution:

Concept:

- A circle with diameter endpoints A and B has centre $(A + B)/2$ and radius $|AB|/2$. A reflection preserves its radius.
- The reflection of (h, k) in $lx+my+n = 0$ is $(h-2lT, k-2mT)$, where $T = (lh+mk+n)/(l^2+m^2)$.

Step 1: Determine the diameter endpoints The given circle passes through the origin, and every line $ax + by = 0$ also passes through the origin. Its intersection A, which has y-coordinate zero, is therefore $A = (0, 0)$. For B, substituting x=1 into the circle gives

$$1 + \beta^2 - 2 = 0, \qquad \beta = \pm 1.$$

If $\beta = 1$, the line through the origin and B is $x - y = 0$, which has $a \neq b$ and is permitted. If $\beta = -1$, the line is $x + y = 0$, which would have $a = b$, contrary to the condition. Hence $B = (1, 1)$.

Step 2: Reflect the centre and preserve the radius The diameter circle has centre $C = (1/2, 1/2)$ and squared radius

$$r^2 = \frac{|AB|^2}{4} = \frac{1^2 + 1^2}{4} = \frac{1}{2}.$$

For reflection in $x + y + 2 = 0$,

$$T = \frac{1/2 + 1/2 + 2}{2} = \frac{3}{2}.$$

Thus its reflected centre is

$$C\prime = (1/2 - 3, 1/2 - 3) = (-5/2, -5/2).$$

The image is therefore

$$(x + 5/2)^2 + (y + 5/2)^2 = 1/2,$$

or $x^2 + y^2 + 5x + 5y + 12 = 0$. The remaining options have different centres: $(-3/2, -5/2)$, $(5/2, 5/2)$ and $(-3/2, -3/2)$. None is the reflection of C, so they cannot describe the image circle.

Step 3: Conclusion The image is $\boxed{x^2 + y^2 + 5x + 5y + 12 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the condition on a and b to choose the sign of the second intersection, then reflect only the centre; a reflection leaves the radius unchanged.

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12. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set $S = \left\{z \in \mathbb{C} : |z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2\right\}$ represents a

[4 marks]

(A) hyperbola with the length of the transverse axis 7

(B) hyperbola with eccentricity 2

(C) straight line with the sum of its intercepts on the coordinate axes equals -18

(D) straight line with the sum of its intercepts on the coordinate axes equals 14

Correct Answer: (D) straight line with the sum of its intercepts on the coordinate axes equals 14

Solution:

Concept:

- For $z = x + iy$, $|z - (a + ib)|^2 = (x - a)^2 + (y - b)^2$, the squared Euclidean distance in the Argand plane.
- The difference of two squared distances cancels the common $x^2 + y^2$ terms and gives a straight line, rather than a hyperbola.

Step 1: Expand the two squared moduli Put $z = x + iy$. Using $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$, the defining relation becomes

$$(x - 2)^2 + (y - 3)^2 - \left((x - 3)^2 + (y - 4)^2\right) = |(-1) - i|^2.$$

The right-hand side is $1 + 1 = 2$. On the left, expanding and collecting the x terms gives $(x - 2)^2 - (x - 3)^2 = 2x - 5$; similarly the y terms give $2y - 7$. Hence the locus satisfies

$$2x - 5 + 2y - 7 = 2.$$

The variables are real coordinates, and these algebraic steps are reversible, so every point on the resulting line belongs to S.

Step 2: Find the intercepts and compare the options Simplifying gives

$$2x + 2y = 14, \qquad x + y = 7.$$

At y=0, x=7; at x=0, y=7. Therefore the sum of the coordinate intercepts is

$$7 + 7 = 14.$$

Options (A) and (B) are excluded because no quadratic terms remain, so the locus is not a hyperbola.

Option (C) has the wrong intercept sum and sign. As an additional check, the point $(7, 0)$ has squared distances $25 + 9 = 34$ and $16 + 16 = 32$, whose difference is indeed 2.

Step 3: Conclusion The locus is the straight line $\boxed{x + y = 7}$, whose intercepts sum to $\boxed{14}$. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish a difference of distances from a difference of squared distances: the latter becomes linear after the common quadratic terms cancel.

13. Let S_1 and S_2 be respectively the sets of all $a \in \mathbb{R} - \{0\}$ for which the system of linear equations

$$ax + 2ay - 3az = 1$$

$$(2a + 1)x + (2a + 3)y + (a + 1)z = 2$$

$$(3a + 5)x + (a + 5)y + (a + 2)z = 3$$

has unique solution and infinitely many solutions. Then [4 marks]

(A) $n(S_1) = 2$ and S_2 is an infinite set

(B) $S_1 = \Phi$ and $S_2 = \mathbb{R} - \{0\}$

(C) $S_1 = \mathbb{R} - \{0\}$ and $S_2 = \Phi$

(D) S_1 is an infinite set and $n(S_2) = 2$

Correct Answer: (C) $S_1 = \mathbb{R} - \{0\}$ and $S_2 = \Phi$

Solution:

Concept:

- A square linear system has a unique solution whenever its coefficient determinant is nonzero. Infinite solutions require a singular coefficient matrix and consistency.
- Here $a \neq 0$, so the first equation may be divided by a before eliminating x from the other two equations.

Step 1: Eliminate x using the first equation Divide the first equation by a to obtain

$$x + 2y - 3z = \frac{1}{a}.$$

Subtract $(2a + 1)$ times this equation from the second, and $(3a + 5)$ times it from the third. The resulting two equations are

$$(1 - 2a)y + (7a + 4)z = -\frac{1}{a},$$
$$(-5a - 5)y + (10a + 17)z = -\frac{5}{a}.$$

These row operations preserve the solutions for every allowed a. Once y and z are determined, x is uniquely recovered from the divided first equation.

Step 2: Show that the reduced determinant never vanishes The determinant of the two remaining coefficients is

$$\begin{aligned}\Delta &= (1-2a)(10a+17)+(5a+5)(7a+4)\\ &= (-20a^2-24a+17)+(35a^2+55a+20)\\ &= 15a^2+31a+37.\end{aligned}$$

Its discriminant is $31^2-4(15)(37)=961-2220=-1259<0$. Since the leading coefficient is positive, $\Delta>0$ for every real a. The determinant of the original coefficient matrix is $a\Delta$, which is nonzero because a=0 is excluded.

Step 3: Identify the parameter sets Thus every nonzero real parameter gives a unique solution, and no allowed parameter gives infinitely many solutions:

$$S_1=\mathbb{R}\setminus\{0\},\qquad S_2=\varnothing.$$

Options (A) and (B) incorrectly make the unique-solution set finite or empty. Option (D) requires two singular consistent values, but the determinant has no allowed real zero at all.

Step 4: Conclusion $\boxed{S_1=\mathbb{R}\setminus\{0\},\ S_2=\varnothing}$. Hence, the correct option is (C).

Quick Tip to Solve: Before checking ranks at singular parameters, factor or analyse the determinant; if it never vanishes on the allowed domain, the consistency classification is immediate.

14. Let $y=y(x)$ be the solution curve of the differential equation $\frac{dy}{dx}=\frac{y}{x}\left(1+xy^2(1+\log_e x)\right), x>0, y(1)=3$. Then $\frac{y^2(x)}{9}$ is equal to : [4 marks]

(A) $\frac{x^2}{5-2x^3(2+\log_e x^3)}$

(B) $\frac{x^2}{3x^3(1+\log_e x^2)-2}$

(C) $\frac{x^2}{7-3x^3(2+\log_e x^2)}$

(D) $\frac{x^2}{2x^3(2+\log_e x^3)-3}$

Correct Answer: (A) $\frac{x^2}{5-2x^3(2+\log_e x^3)}$

Solution:

Concept:

- The equation is a Bernoulli equation $y'-y/x=(1+\ln x)y^3$. The substitution $v=y^{-2}$ converts it into a first-order linear equation.
- For $v'+2v/x=-2(1+\ln x)$, the integrating factor on x>0 is x^2. The initial value determines the integration constant.

Step 1: Transform to a linear equation Since $y(1)=3\neq 0$, use $v=1/y^2$ on the solution interval containing 1. Then $v'=-2y^{-3}y'$. Multiplying the given equation by $-2y^{-3}$ yields

$$v'+\frac{2}{x}v=-2(1+\ln x).$$

After multiplication by x^2, this becomes

$$\frac{d}{dx}(x^2 v) = -2x^2(1 + \ln x).$$

The integrating factor is valid because x>0, as specified.

Step 2: Integrate and impose the initial condition Integration by parts gives

$$\int x^2 \ln x \, dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9}.$$

Therefore

$$x^2 v = C - \frac{2x^3 \ln x}{3} - \frac{4x^3}{9}.$$

At x=1, $v(1) = 1/9$, so $1/9 = C - 4/9$, giving $C = 5/9$. Consequently,

$$\frac{x^2}{y^2} = \frac{5 - 4x^3 - 6x^3 \ln x}{9}.$$

Inverting and dividing by 9 gives the requested expression.

Step 3: Rewrite in the printed logarithmic form For positive x, $\ln x^3 = 3 \ln x$. Thus

$$\frac{y^2}{9} = \frac{x^2}{5 - 2x^3(2 + \ln x^3)}.$$

This is valid on the interval containing 1 where its denominator remains positive. Options (B) and (D) have the opposite derivative at x=1; option (C) gives derivative 26 for $y^2/9$, whereas the differential equation gives $2y(1)y\prime(1)/9 = 20$. The expression in (A) has that required derivative and initial value.

Step 4: Conclusion $\boxed{\frac{y^2(x)}{9} = \frac{x^2}{5 - 2x^3(2 + \ln x^3)}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a cubic Bernoulli term, use the reciprocal square of y; after solving, rewrite logarithmic powers only for x>0 and check the initial value.

15. Let $f : (0, 1) \to \mathbb{R}$ be a function defined $f(x) = \frac{1}{1-e^{-x}}$, and $g(x) = (f(-x) - f(x))$. Consider two statements

(I) g is an increasing function in (0, 1)

(II) g is one-one in (0, 1)

Then, [4 marks]

(A) Both (I) and (II) are true

(B) Neither (I) nor (II) is true

(C) Only (II) is true

(D) Only (I) is true

Correct Answer: With the stated domain of f, $f(-x)$ and hence g are undefined. Extending the displayed formula gives the intended option (A).

Solution:

Concept:

- For a differentiable function, a positive derivative throughout an interval establishes strict increase there. Strict increase also implies that distinct inputs have distinct outputs.
- The displayed exponential expression can be simplified by multiplying numerator and denominator by e^x. The exponential satisfies $e^x > 1$ for $x \in (0, 1)$.

Step 1: Simplify the displayed expression for g Using the displayed formula to evaluate the expression $f(-x) - f(x)$, we obtain

$$f(-x) = \frac{1}{1-e^x} = -\frac{1}{e^x - 1}, \qquad f(x) = \frac{e^x}{e^x - 1}.$$

Consequently,

$$g(x) = -\frac{e^x + 1}{e^x - 1}.$$

Its denominator is nonzero throughout the interval under consideration. Strictly, the stated domain of f does not include $-x$; the evaluation above uses the natural extension given by its displayed formula, which the definition of g calls for.

Step 2: Differentiate and test both statements Apply the quotient rule, preserving the minus sign in front of the quotient:

$$g\prime(x) = -\frac{e^x(e^x - 1) - (e^x + 1)e^x}{(e^x - 1)^2} = \frac{2e^x}{(e^x - 1)^2} > 0.$$

Thus g is strictly increasing on $(0, 1)$, proving statement (I). If $x_1 < x_2$, strict increase gives $g(x_1) < g(x_2)$, so equal outputs cannot arise at distinct inputs. Statement (II) is therefore true as well. Option (B) rejects both valid conclusions, while (C) and (D) each omit one valid conclusion.

Step 3: Conclusion The literal domain makes g undefined, so no listed option is strictly justified as written. Under the natural extension of the displayed formula used in g, both (I) and (II) are true. Hence, the correct option is (A).

Quick Tip to Solve: Simplify the difference before differentiating; once its derivative is strictly positive, one-one behaviour follows immediately.

16. The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is [4 marks]

(A) a tautology

(B) equivalent to $(\sim p) \vee (\sim q)$

(C) a contradiction

(D) $p \vee q$

Correct Answer: (A) a tautology

Solution:

Concept:

- An implication $P \Rightarrow Q$ is false only when P is true and Q is false. Equivalently, $P \Rightarrow Q \equiv \neg P \vee Q$.
- A tautology is true for every assignment of truth values to its component statements. A contradiction is false for every such assignment.

Step 1: Reduce the implication algebraically Let the complete statement be $S = (p \wedge \neg q) \Rightarrow (p \Rightarrow \neg q)$. First rewrite the inner implication, then the outer one:

$$p \Rightarrow \neg q \equiv \neg p \vee \neg q.$$

$$\begin{aligned} S &\equiv \neg(p \wedge \neg q) \vee (\neg p \vee \neg q) \\ &\equiv (\neg p \vee q) \vee (\neg p \vee \neg q) \\ &\equiv \neg p \vee (q \vee \neg q) \equiv \text{True}. \end{aligned}$$

The complementary pair $q \vee \neg q$ ensures truth independently of p.

Step 2: Check the classification and reject the alternatives If the antecedent $p \wedge \neg q$ is true, then p is true and q false; the consequent $p \Rightarrow \neg q$ is then true. If the antecedent is false, the outer implication is true automatically. Hence every possible case gives true. Option (B) is false when $p = q = \text{True}$, whereas S remains true. Option (C) asserts the opposite classification. Option (D), $p \vee q$, is false when both variables are false, again unlike S.

Step 3: Conclusion The statement is true for every truth assignment, so it is a tautology. Hence, the correct option is (A).

Quick Tip to Solve: For nested implications, replace each arrow by negation of its antecedent OR its consequent, starting with the inner arrow.

17. If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1+x)^{10}$, then $\sum_{r=1}^{10} r^3\left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to [4 marks]

(A) 3025

(B) 4895

(C) 5445

(D) 1210

Correct Answer: (D) 1210

Solution:

Concept:

- The coefficient of x^k in $(1+x)^{10}$ is $\binom{10}{k}$. Symmetry gives $\binom{10}{10-r} = \binom{10}{r}$.
- The adjacent coefficient ratio simplifies through factorial cancellation. The standard sums are $\sum r = n(n+1)/2$, $\sum r^2 = n(n+1)(2n+1)/6$, and $\sum r^3 = [n(n+1)/2]^2$.

Step 1: Find the adjacent coefficient ratio The coefficient specified in the question is

$$a_r = \binom{10}{10-r} = \binom{10}{r}, \qquad a_{r-1} = \binom{10}{r-1}.$$

For every integer r from 1 to 10, both coefficients are defined and the denominator is nonzero. Cancelling factorials yields

$$\frac{a_r}{a_{r-1}} = \frac{(r-1)!(11-r)!}{r!(10-r)!} = \frac{11-r}{r}.$$

Therefore the summand reduces to $r^3(a_r/a_{r-1})^2 = r(11-r)^2$; it is not necessary to expand the whole binomial.

Step 2: Evaluate the resulting power sums Expand the quadratic factor and sum term by term:

$$\begin{aligned} S &= \sum_{r=1}^{10}(121r - 22r^2 + r^3) \\ &= 121(55) - 22(385) + 3025 \\ &= 6655 - 8470 + 3025 = 1210. \end{aligned}$$

Here 55, 385, and 3025 are respectively the sums of the first ten positive integers, their squares, and their cubes. The value 3025 in option (A) accounts only for the cubic contribution; the linear and quadratic contributions must also be retained. Options (B) and (C) likewise do not equal the complete sum.

Step 3: Conclusion The required sum is 1210. Hence, the correct option is (D).

Quick Tip to Solve: For sums involving adjacent binomial coefficients, simplify their ratio first; the factorials often reduce the problem to elementary power sums.

18. Let $x, y, z > 1$ and $A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$. Then $|\text{adj } (\text{adj } A^2)|$ is equal to [4 marks]

(A) 6^4

(B) 2^8

(C) 4^8

(D) 2^4

Correct Answer: (B) 2^8

Solution:

Concept:

- Change of logarithmic base gives $\log_x y = (\ln y)/(\ln x)$. Since $x, y, z > 1$, all three natural logarithms are nonzero.
- For a square matrix of order three, $\det(\text{adj } B) = (\det B)^2$. Also $\det(A^2) = (\det A)^2$.

Step 1: Remove the logarithmic ratios by a similarity transformation Put $u = \ln x$, $v = \ln y$, $w = \ln z$, and $D = \text{diag}(u, v, w)$. Then

$$DAD^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix} = M.$$

For example, the first-row second-column entry becomes $u(v/u)/v = 1$; the other off-diagonal entries simplify in the same way. Similarity preserves the determinant, because $\det D$ and $\det D^{-1}$ cancel. Subtracting the first row of M from each remaining row gives

$$\det A = \det M = \det \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = 2.$$

Step 2: Apply the adjugate determinant identity twice Let $B = A^2$. The required vertical bars denote a determinant. Since each matrix involved has order three,

$$\begin{aligned} \det(\text{adj}(\text{adj } B)) &= [\det(\text{adj } B)]^2 \\ &= [(\det B)^2]^2 = (\det B)^4. \end{aligned}$$

Now $\det B = (\det A)^2 = 4$, so the answer is $4^4 = 2^8$. Option (A) uses an incorrect base determinant, option (C) introduces too large an exponent, and option (D) stops before both adjugate operations have been accounted for.

Step 3: Conclusion $\left|\text{adj}(\text{adj } A^2)\right| = 2^8$. Hence, the correct option is (B).

Quick Tip to Solve: For an order-three matrix, each adjugate squares its determinant; first find the determinant of the innermost matrix.

19. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b} \cdot \vec{c} = 0$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}-\vec{c}}{2}$. If $\vec{d}$ be a vector such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal to [4 marks]

(A) $\frac{1}{2}$

(B) $-\frac{1}{4}$

(C) $\frac{1}{4}$

(D) $\frac{3}{4}$

Correct Answer: (C) $\frac{1}{4}$

Solution:

Concept:

- The vector triple-product identity is $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$.
- The scalar product of cross products satisfies $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$.

Step 1: Extract the required scalar products The nonzero vectors $\mathbf{b}$ and $\mathbf{c}$ are perpendicular, so they are linearly independent. Using the triple-product identity in the given equation gives

$$\mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b}) = \tfrac{1}{2}\mathbf{b} - \tfrac{1}{2}\mathbf{c}.$$

Coefficients of the two independent vectors must therefore agree:

$$\mathbf{a} \cdot \mathbf{c} = \frac{1}{2}, \qquad \mathbf{a} \cdot \mathbf{b} = \frac{1}{2}.$$

The separate condition on $\mathbf{d}$ now gives $\mathbf{b} \cdot \mathbf{d} = \mathbf{a} \cdot \mathbf{b} = 1/2$. No information about the component of $\mathbf{d}$ perpendicular to $\mathbf{b}$ is required.

Step 2: Evaluate the product of the cross products Apply the second identity, keeping its order and minus sign unchanged:

$$\begin{aligned}(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) &= (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) \\ &\quad - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}) \\ &= \frac{1}{2} \cdot \frac{1}{2} - (\mathbf{a} \cdot \mathbf{d})\, 0 = \frac{1}{4}.\end{aligned}$$

Option (A) retains only one of the two factors. Option (B) has the wrong sign, typically caused by reversing a cross product. Option (D) is incompatible with the exact contraction above.

Step 3: Conclusion The required scalar product is $1/4$. Hence, the correct option is (C).

Quick Tip to Solve: Use perpendicularity to eliminate the unknown scalar product before trying to determine the vector $\mathbf{d}$ itself.

20. The distance of the point $(6, -2\sqrt{2})$ from the common tangent $y = mx + c, m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is : [4 marks]

(A) $\frac{1}{3}$

(B) 5

(C) $\frac{14}{3}$

(D) $5\sqrt{3}$

Correct Answer: (B) 5

Solution:

Concept:

- A line is tangent to a parabola when substitution produces a quadratic with a repeated root, so its discriminant vanishes.
- The perpendicular distance of (x_0, y_0) from $Ax + By + C = 0$ is $|Ax_0 + By_0 + C|/\sqrt{A^2 + B^2}$.

Step 1: Impose tangency to both curves Substitute $y = mx + c$, or equivalently $x = (y - c)/m$, into $x = 2y^2$. Because $m > 0$, division by m is valid:

$$2my^2 - y + c = 0.$$

Its discriminant is $1 - 8mc$, giving $c = 1/(8m)$. For $x = 1 + y^2$, substitution gives

$$my^2 - y + (m + c) = 0,$$

whose zero discriminant implies $c = 1/(4m) - m$. Equating the two expressions for c,

$$\frac{1}{8m} = \frac{1}{4m} - m \quad \Longrightarrow \quad m^2 = \frac{1}{8}.$$

The positive slope condition selects $m = 1/(2\sqrt{2})$, and hence $c = \sqrt{2}/4$.

Step 2: Write the line in standard form and calculate distance Multiply $y = x/(2\sqrt{2}) + \sqrt{2}/4$ by $2\sqrt{2}$ to obtain the tangent

$$x - 2\sqrt{2}\,y + 1 = 0.$$

For the point $(6, -2\sqrt{2})$, the numerator of the distance formula is $|6+8+1| = 15$, while its denominator is $\sqrt{1+8} = 3$. Thus

$$d = \frac{15}{3} = 5.$$

Options (A) and (C) do not use the complete numerator, and option (D) does not match the normal length 3. Direct substitution of the given point into the standard line equation confirms the numerator and its sign.

Step 3: Conclusion The distance from the point to the common tangent of positive slope is 5 units. Hence, the correct option is (B).

Quick Tip to Solve: For sideways parabolas, substitute $x = (y - c)/m$ and apply the repeated-root condition in y to find tangent intercepts.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The constant term in the expansion of $\left(2x + \frac{1}{x^7} + 3x^2\right)^5$ is _____. [4 marks]

Solution:

Concept:

- In the multinomial expansion, selecting the three terms a, b, c times respectively gives the coefficient $5!/(a!b!c!)$, where $a + b + c = 5$.
- A constant term has zero total exponent of x. Here a selection of $2x$, x^{-7}, and $3x^2$ contributes exponent $a - 7b + 2c$.

Step 1: Find all selections producing zero exponent Let a, b, c count the choices of $2x$, x^{-7}, and $3x^2$. Each is a nonnegative integer and

$$a + b + c = 5, \qquad a - 7b + 2c = 0.$$

Eliminating $a = 5 - b - c$ yields $5 - 8b + c = 0$, or $c = 8b - 5$. Nonnegativity of c requires $b \geq 1$. Also $a = 10 - 9b \geq 0$, so $b \leq 10/9$. The only integer possibility is therefore

$$b = 1, \qquad c = 3, \qquad a = 1.$$

This exhausts the possible constant contributions; no additional term needs to be added.

Step 2: Compute the coefficient including multiplicity The corresponding product is $(2x)^1(x^{-7})^1(3x^2)^3$. Its exponent is $1 - 7 + 6 = 0$, as required. There are $5!/(1!1!3!) = 20$ ways to choose these factors from the five identical brackets. Hence its constant coefficient is

$$\frac{5!}{1!1!3!}\, 2^1\, 1^1\, 3^3 = 20 \cdot 2 \cdot 27 = 1080.$$

The factor 20 is essential: multiplying the selected terms once would count just one ordering, although every ordering appears in the expansion. Since the selection found in the first step is unique, this coefficient is the entire constant term.

Step 3: Conclusion The constant term, and thus the required numerical answer, is $\boxed{1080}$. It is dimensionless.

Quick Tip to Solve: Combine the number-of-factors equation with the zero-exponent equation first, then check nonnegative integer solutions before applying the multinomial coefficient.

22. Let S = {1, 2, 3, 5, 7, 10, 11}. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is _____. [4 marks]

Solution:

Concept:

- The sum of a subset is divisible by 3 exactly when its residue modulo 3 is zero. Elements with residue zero can be chosen independently without changing this condition.
- If a subset contains k elements of residue 1 and l elements of residue 2, its sum is congruent to $k + 2l \equiv k - l \pmod 3$. Choosing such elements gives $\binom{3}{k}\binom{3}{l}$ subsets.

Step 1: Group the elements by their residues The residue classes in the given set are $\{3\}$, $\{1, 7, 10\}$ and $\{2, 5, 11\}$, with respective residues 0, 1 and 2. Thus there are three elements of each nonzero residue. For any admissible selection from these six elements, the element 3 may either be omitted or included. This produces two distinct subsets having the same residue.

Step 2: Count all admissible pairs of selection sizes Since $0 \le k, l \le 3$, the condition $k - l \equiv 0 \pmod 3$ allows precisely $(k, l) = (0, 0), (0, 3), (1, 1), (2, 2), (3, 0), (3, 3)$. Their contributions are respectively $1, 1, 9, 9, 1, 1$. Therefore the number of selections from the two nonzero classes is

$$1 + 1 + 9 + 9 + 1 + 1 = 22.$$

The terms 9 arise because each of the two classes offers three choices when selecting either one or two elements.

Step 3: Include the zero-residue element and remove the empty set Allowing the independent choice of 3 gives $2 \times 22 = 44$ subsets whose sums are divisible by 3. This count includes the empty subset exactly once: it occurs when neither nonzero class contributes anything and 3 is also omitted. The subset $\{3\}$ is nonempty and must remain in the count. Hence the required count is $44 - 1 = 43$.

Step 4: Conclusion The number of nonempty subsets is $\boxed{43}$. This is a dimensionless count.

Quick Tip to Solve: Count subsets modulo 3, include every zero-residue element independently, and subtract the empty subset only at the end.

23. Let x and y be distinct integers where $1 \le x \le 25$ and $1 \le y \le 25$. Then, the number of ways of choosing x and y, such that $x + y$ is divisible by 5, is _____. [4 marks]

Solution:

Concept:

- Divisibility by 5 depends only on residues modulo 5. The integers from 1 to 25 contain exactly five representatives of each residue $0, 1, 2, 3, 4$.
- The variables x and y specify an ordered choice. For different residue classes there are 5×5 choices in a fixed order; in the same class the distinctness restriction leaves 5×4 choices.

Step 1: List the compatible residue classes The condition $x + y \equiv 0 \pmod 5$ gives the ordered residue pairs $(0, 0), (1, 4), (4, 1), (2, 3), (3, 2)$. This is exhaustive because for each residue of x, the residue of y

is uniquely its additive inverse modulo 5. No other pair can have a sum divisible by 5.

Step 2: Count the choices from different classes For $(1, 4)$, five choices for x and five choices for y give 25 ordered pairs. The same count applies separately to $(4, 1), (2, 3), (3, 2)$. Distinctness is automatic here because an integer cannot belong to two different residue classes. These four cases therefore contribute

$$4 \times 5 \times 5 = 100.$$

Reversing a pair interchanges the values assigned to the two named variables and is counted separately.

Step 3: Enforce distinctness for two multiples of five The zero class is $\{5, 10, 15, 20, 25\}$. Choose x in five ways, then choose y from the four remaining integers in this class, giving $5 \times 4 = 20$. The five diagonal pairs with $x = y$ are forbidden. Adding the disjoint residue cases gives $100 + 20 = 120$. Equivalently, each of the 25 choices for x initially permits five choices for y, and the five equal multiples-of-five pairs must be removed: $125 - 5 = 120$.

Step 4: Conclusion The required number of choices is $\boxed{120}$, a dimensionless count of ordered pairs.

Quick Tip to Solve: For named variables count ordered pairs first. Remove equal pairs only from a residue class that is its own additive inverse.

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24. For some a, b, c $\in \mathbb{N}$, let $f(x) = ax - 3$ and $g(x) = x^b + c, x \in \mathbb{R}$. If $(fog)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$, then $(fog)(ac) + (gof)(b)$ is equal to _____. [4 marks]

Solution:

Concept:

- An inverse function reverses the relation between input and output. If $h^{-1}(x) = \sqrt[3]{(x-7)/2}$, then $h(t) = 2t^3 + 7$.
- Composition means substitution: $(f \circ g)(t) = f(g(t))$ and $(g \circ f)(t) = g(f(t))$. The order matters because the inner function is evaluated first.

Step 1: Recover the composite function from its inverse Write $t = h^{-1}(x)$. Cubing the stated inverse relation gives $t^3 = (x - 7)/2$, hence $x = 2t^3 + 7$. Therefore $h(t) = 2t^3 + 7$. The real cube root is defined for every real argument and is one-to-one, so this inversion introduces no extra choices or sign branches.

Step 2: Compare coefficients to identify the constants Using the definitions of the two functions gives

$$(f \circ g)(t) = a(t^b + c) - 3 = at^b + ac - 3.$$

This must equal $2t^3 + 7$ for every real t. Comparing the nonconstant power, its coefficient and the constant term yields $b = 3$, $a = 2$, and $ac - 3 = 7$. Thus $ac = 10$, so $c = 5$. These values satisfy the given requirement that all three parameters are natural numbers.

Step 3: Evaluate the two composites in the required order The first required input is $ac = 10$, so

$$(f \circ g)(ac) = 2(10)^3 + 7 = 2007.$$

For the other term first evaluate $f(b) = f(3) = 2(3) - 3 = 3$, and then evaluate $g(3) = 3^3 + 5 = 32$. Consequently

$$(f \circ g)(ac) + (g \circ f)(b) = 2007 + 32 = 2039.$$

As a direct check, $f(g(10)) = 2(1005) - 3 = 2007$, agreeing with the recovered composite.

Step 4: Conclusion The requested value is $\boxed{2039}$. The functions and their values here are dimensionless.

Quick Tip to Solve: Invert the stated inverse first, then compare the resulting composite with its algebraic definition before substituting the requested inputs.

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25. Let $S = \left\{\alpha : \log_2(9^{2\alpha-4} + 13) - \log_2\left(\frac{5}{2}.3^{2\alpha-4} + 1\right) = 2\right\}$. Then the maximum value of β for which the equation $x^2 - 2\left(\sum\limits_{\alpha\in s}\alpha\right)^2 x + \sum\limits_{\alpha\in s}(\alpha+1)^2\beta = 0$ has real roots, is _____. [4 marks]

Solution:

Concept:

- For positive arguments, $\log_2 U - \log_2 V = 2$ is equivalent to $U = 4V$. An exponential substitution turns the given logarithmic equation into a quadratic.
- A real quadratic $x^2+Bx+C = 0$ has real roots exactly when its discriminant B^2-4C is nonnegative. Equality includes a repeated real root and determines the limiting parameter value.

Step 1: Find every element of the set Put $t = 3^{2\alpha-4} > 0$. Then $9^{2\alpha-4} = t^2$, and both logarithmic arguments are positive. Combining the logarithms gives

$$t^2 + 13 = 4\left(\frac{5}{2}t + 1\right).$$

Thus $t^2 - 10t + 9 = 0$, or $(t-1)(t-9) = 0$. Both roots are positive, so both survive the exponential substitution. From $3^{2\alpha-4} = 1$ and $3^{2\alpha-4} = 9$, obtain $\alpha = 2$ and $\alpha = 3$. Hence $S = \{2, 3\}$.

Step 2: Compute the two coefficients of the required quadratic The sum occurring in the coefficient of x is $\sum_{\alpha\in S}\alpha = 2 + 3 = 5$; its square is 25. The other sum is

$$\sum_{\alpha\in S}(\alpha+1)^2 = (2+1)^2 + (3+1)^2 = 9 + 16 = 25.$$

Therefore the equation in the question reduces exactly to $x^2 - 50x + 25\beta = 0$. Note that the square of the first sum differs from the sum of the squares; keeping these operations in their printed order is essential.

Step 3: Apply the condition for real roots The discriminant is

$$D = (-50)^2 - 4(1)(25\beta) = 2500 - 100\beta.$$

Real roots require $2500 - 100\beta \geq 0$, which is equivalent to $\beta \leq 25$. The endpoint is allowed: when $\beta = 25$, the quadratic becomes $(x-25)^2 = 0$, which has a repeated real root. Thus 25 is attained and

is the maximum, rather than merely an upper bound.

Step 4: Conclusion The maximum value is $\boxed{\beta = 25}$, a dimensionless parameter.

Quick Tip to Solve: Use a positive exponential substitution to find the set first, then keep each printed summation and squaring operation separate when forming the discriminant.

---

26. Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as $A, A+1, A+2$, respectively. Let a, b, c be the $7^{th}, 9^{th}, 17^{th}$ terms of A_1, A_2, A_3, respective such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$.

If $a = 29$, then the sum of first 20 terms of an AP whose first term is $c - a - b$ and common difference is $\frac{d}{12}$, is equal to _____. [4 marks]

Solution:

Concept:

- For an arithmetic progression with first term A and difference d, its n-th term is $A + (n-1)d$. The three progressions share the same difference but have different first terms.
- Subtracting one row from another does not change a determinant. The sum of the first n terms of an AP with first term u and difference r is $S_n = \frac{n}{2}[2u + (n-1)r]$.

Step 1: Express the three selected terms From the seventh, ninth and seventeenth positions in the respective progressions,

$$a = A + 6d, \qquad b = A + 1 + 8d, \qquad c = A + 2 + 16d.$$

The offsets 1 and 2 arise from the first terms of the second and third progressions; they must not be absorbed into the common difference.

Step 2: Simplify the determinant and recover the original parameters Apply $R_2 \leftarrow R_2 - R_3$. The second row becomes $(2b - c, 0, 0)$. Expanding in that row yields

$$D = -(2b - c)\begin{vmatrix} 7 & 1 \\ 17 & 1 \end{vmatrix} = 10(2b - c).$$

The given relation $D + 70 = 0$ implies $c - 2b = 7$. Substituting the AP expressions gives $c - 2b = -A$, because the d-terms and constant offsets cancel. Thus $A = -7$. Now $a = 29 = A + 6d$ gives $d = 6$. Therefore $b = -7 + 1 + 48 = 42$ and $c = -7 + 2 + 96 = 91$.

Step 3: Form the new progression and sum its first twenty terms The required first term is $u = c - a - b = 91 - 29 - 42 = 20$, while its common difference is $r = d/12 = 6/12 = 1/2$. Hence

$$S_{20} = \frac{20}{2}\left[2(20) + 19\left(\frac{1}{2}\right)\right] = 10\left(40 + \frac{19}{2}\right) = 495.$$

The twentieth term is $20 + 19/2 = 59/2$; averaging this with the first term and multiplying by 20 gives

the same result.

Step 4: Conclusion The required sum of the first twenty terms is $\boxed{495}$, with no physical unit.

Quick Tip to Solve: Subtract the determinant rows that share their last two entries; this removes two columns at once and often cancels the AP difference as well.

27. If the area enclosed by the parabolas $\mathrm{P_1 : 2y = 5x^2}$ and $\mathrm{P_2 : x^2 - y + 6 = 0}$ is equal to the area enclosed by $\mathrm{P_1}$ and $\mathrm{y = \alpha x, \alpha > 0}$, then α^3 is equal to _____. [4 marks]

Solution:

Concept:

- The area enclosed between two graphs is the integral of the upper ordinate minus the lower ordinate over the interval between their intersections.
- For the upward parabola $y = \frac{5}{2}x^2$ and the line $y = \alpha x$, with $\alpha > 0$, the intersections are $x = 0$ and $x = 2\alpha/5$. The line is above the parabola between these points.

Step 1: Find the region between the two parabolas The second parabola is $y = x^2 + 6$. Equating it to $y = \frac{5}{2}x^2$ gives $\frac{3}{2}x^2 = 6$, so the intersections have abscissae $x = -2, 2$. On this interval, $x^2 + 6$ lies above $\frac{5}{2}x^2$, as is also evident at $x = 0$. Their vertical separation is $6 - \frac{3}{2}x^2$, which is nonnegative throughout the interval.

Step 2: Evaluate the first enclosed area Using the symmetry of the integrand,

$$A_1 = \int_{-2}^{2} \left(6 - \frac{3}{2}x^2\right) dx = 2\left[6x - \frac{x^3}{2}\right]_0^2.$$

Consequently $A_1 = 2(12 - 4) = 16$. Both intersections must be used; integrating from 0 to 2 alone would count only half of this region.

Step 3: Compute the area under the line and equate the areas The other enclosed region has area

$$A_2 = \int_0^{2\alpha/5} \left(\alpha x - \frac{5}{2}x^2\right) dx = \left[\frac{\alpha x^2}{2} - \frac{5x^3}{6}\right]_0^{2\alpha/5}.$$

Substitution gives $A_2 = 2\alpha^3/25 - 4\alpha^3/75 = 2\alpha^3/75$. Equating this to 16 yields $2\alpha^3/75 = 16$, hence $\alpha^3 = 600$. The positive value is consistent with the stipulated $\alpha > 0$.

Step 4: Conclusion The requested value is $\boxed{\alpha^3 = 600}$. The two common areas are 16 square coordinate units.

Quick Tip to Solve: Locate all intersections before integrating, and use symmetry only for the two-parabola region; the line-parabola region lies on the positive side alone.

28. If the sum of all the solutions of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}, -1<x<1, x\neq 0$, is $\alpha-\frac{4}{\sqrt{3}}$, then α is equal to _____. [4 marks]

Solution:

Concept:

- The principal range of $\tan^{-1}$ is $(-\pi/2, \pi/2)$, while that of $\cot^{-1}$ is $(0, \pi)$. Branch restrictions must be applied before combining the two inverse functions.
- With $t=\tan^{-1}x$ and $-1<x<1$, we have $-\pi/4<t<\pi/4$ and $\tan 2t = 2x/(1-x^2)$. Since $x\neq 0$, both signs of t must be considered separately.

Step 1: Convert the arguments into double-angle expressions For the stated interval, $2t\in(-\pi/2,\pi/2)$, so

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)=2t.$$

The second argument is $\cot 2t$. When $t>0$, the angle $2t$ already belongs to the principal cotangent-inverse range. When $t<0$, add π to place it in that range, since cotangent has period π.

Step 2: Solve on the positive and negative branches For $t>0$, the equation is $2t+2t=\pi/3$, giving $t=\pi/12$ and $x=\tan(\pi/12)=2-\sqrt{3}$. For $t<0$, it is $2t+(2t+\pi)=\pi/3$, giving $t=-\pi/6$ and $x=-1/\sqrt{3}$. Both values lie strictly between -1 and 1 and neither is zero. These two cases exhaust the permitted domain, and each resulting linear equation has exactly one solution.

Step 3: Add the solutions and identify the parameter The sum is

$$(2-\sqrt{3})-\frac{1}{\sqrt{3}}=2-\frac{3+1}{\sqrt{3}}=2-\frac{4}{\sqrt{3}}.$$

Comparing it with the form $\alpha-4/\sqrt{3}$ specified in the question gives $\alpha=2$. Directly, the positive root makes both inverse values $\pi/6$; the negative root gives $-\pi/3$ and $2\pi/3$. Each pair indeed adds to $\pi/3$, confirming that the negative branch was not an extraneous solution.

Step 4: Conclusion The required dimensionless value is $\boxed{\alpha=2}$.

Quick Tip to Solve: For reciprocal tangent and cotangent arguments, split by sign before using principal inverse values; a negative angle needs an added π for cotangent inverse.

29. The vertices of a hyperbola H are $(\pm 6, 0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x+y=2\sqrt{2}$. If d is the length of the line segment of N between H and the y-axis then d^2 is equal to _____. [4 marks]

Solution:

Concept:

- The standard equation is $x^2/a^2-y^2/b^2=1$, where the vertices are $(\pm a, 0)$ and $e^2=1+b^2/a^2$.
- The tangent and normal are perpendicular. Implicit differentiation determines the tangent slope, and the negative reciprocal gives the normal slope at the same point.

Step 1: Find the hyperbola The given vertices give $a = 6$, so $a^2 = 36$. Using the eccentricity, obtain

$$b^2 = a^2(e^2 - 1) = 36\left(\frac{5}{4} - 1\right) = 9.$$

Thus the required equation is $x^2/36 - y^2/9 = 1$. We must find the point on its first-quadrant branch whose normal has the prescribed direction.

Step 2: Use the normal direction Differentiating the hyperbola gives

$$\frac{2x}{36} - \frac{2y}{9}\frac{dy}{dx} = 0, \qquad m_{\text{tangent}} = \frac{x}{4y}.$$

Consequently $m_{\text{normal}} = -4y/x$. The given parallel line has slope $-\sqrt{2}$, so $4y/x = \sqrt{2}$, or $y = x/(2\sqrt{2})$. Substitution into the hyperbola gives

$$\frac{x^2}{36} - \frac{x^2}{72} = 1, \qquad x^2 = 72.$$

The first-quadrant point is therefore $P = (6\sqrt{2}, 3)$; its positive coordinates are essential for selecting the stated branch.

Step 3: Measure the normal segment The normal through P has slope $-\sqrt{2}$. Its intersection with the y-axis has horizontal displacement of magnitude $x_P = 6\sqrt{2}$ from P, and vertical displacement of magnitude $\sqrt{2}x_P = 12$. Hence the squared length is

$$d^2 = (6\sqrt{2})^2 + 12^2 = 72 + 144 = 216.$$

Equivalently, for a line of slope m, the segment from a point of abscissa x_P to the y-axis has squared length $x_P^2(1 + m^2)$.

Step 4: Conclusion The required value is $\boxed{d^2 = 216}$ square coordinate units. Thus the numerical entry is 216.

Quick Tip to Solve: Find the normal slope by implicit differentiation first; then use $d^2 = x_P^2(1 + m^2)$ to avoid calculating the y-intercept.

30. Let the equation of the plane passing through the line $x - 2y - z - 5 = 0 = x + y + 3z - 5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$. Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is _____. [4 marks]

Solution:

Concept:

- Every plane through the intersection of $P_1 = 0$ and $P_2 = 0$ is represented by $P_1 + \lambda P_2 = 0$, including the limiting member when needed.
- The direction of the line formed by two intersecting planes is the cross product of their normal vectors. A plane parallel to that direction has its normal perpendicular to the direction.

Step 1: Form the plane family Using the first pair of planes, write

$$(x - 2y - z - 5) + \lambda(x + y + 3z - 5) = 0.$$

Its normal is $(1+\lambda, -2+\lambda, -1+3\lambda)$, and its right-hand constant is $5(1+\lambda)$. This expression guarantees that every point of the given intersection line lies on the sought plane.

Step 2: Impose the parallel direction The normals of the planes defining the second line are $(1, 1, 2)$ and $(2, 3, 1)$. Their cross product is

$$(1, 1, 2) \times (2, 3, 1) = (-5, 3, 1).$$

The required dot product vanishes:

$$-5(1 + \lambda) + 3(-2 + \lambda) + (-1 + 3\lambda) = 0.$$

This simplifies to $\lambda - 12 = 0$. Substitution yields

$$13x + 10y + 35z = 65.$$

The constant already equals the specified 65, so the coefficients are $a = 13, b = 10, c = 35$, with no further rescaling.

Step 3: Find the perpendicular distance The point asked for is therefore $(13, 10, 35)$. For a point (x_0, y_0, z_0), the distance from $Ax+By+Cz+D = 0$ is the absolute residual divided by $\sqrt{A^2 + B^2 + C^2}$. Apply this to the final plane:

$$d = \frac{|2(13) + 2(10) - 35 + 16|}{\sqrt{2^2 + 2^2 + (-1)^2}} = \frac{27}{3} = 9.$$

The absolute value makes the distance nonnegative; its denominator normalises the plane coefficients.

Step 4: Conclusion The required distance is $\boxed{9}$ coordinate units, so the numerical entry is 9.

Quick Tip to Solve: After finding the plane family parameter, match its constant to the stated constant before treating its coefficients as point coordinates.

31. A parallel plate capacitor has plate area 40 cm^2 and plates separation 2 mm. The space between the plates is filled with a dielectric medium of a thickness 1 mm and dielectric constant 5. The capacitance of the system is : [4 marks]

(A) $10\varepsilon_0$ F

(B) $24\varepsilon_0$ F

(C) $\frac{3}{10}\varepsilon_0$ F

(D) $\frac{10}{3}\varepsilon_0$ F

Correct Answer: (D) $\frac{10}{3}\varepsilon_0$ F

Solution:

Concept:

- A dielectric slab perpendicular to the electric field and an air gap act as two capacitors in series, since both regions carry the same free charge on equal facing areas.
- For plate area A, separation d, slab thickness t and relative permittivity K, the effective separation is $d - t + t/K$, so $C = \varepsilon_0 A/(d - t + t/K)$.

Step 1: Identify and convert the two layers The dielectric occupies $1\,\text{mm}$ of the $2\,\text{mm}$ plate spacing. The remaining air thickness is therefore also $1\,\text{mm}$. The given area is

$$A = 40\,\text{cm}^2 = 40 \times 10^{-4}\,\text{m}^2 = 4 \times 10^{-3}\,\text{m}^2.$$

Both layers have this same cross-sectional area. They lie consecutively along the electric field, so a parallel-capacitor combination would not describe this arrangement.

Step 2: Combine the voltage drops The air layer has capacitance $C_a = \varepsilon_0 A/(d-t)$, while the dielectric layer has $C_d = K\varepsilon_0 A/t$. Adding their reciprocal capacitances gives

$$\frac{1}{C} = \frac{d-t}{\varepsilon_0 A} + \frac{t}{K\varepsilon_0 A}.$$

Thus the effective thickness is

$$(d-t) + \frac{t}{K} = 10^{-3} + \frac{10^{-3}}{5} = 1.2 \times 10^{-3}\,\text{m}.$$

Only the slab thickness is divided by K; the air part retains its full contribution.

Step 3: Calculate and compare the options Substituting area and effective separation yields

$$C = \frac{\varepsilon_0(4 \times 10^{-3})}{1.2 \times 10^{-3}} = \frac{10}{3}\varepsilon_0\,\text{F}.$$

As a useful bound, the all-air capacitance is $2\varepsilon_0$ F, and the fully dielectric-filled capacitance is $10\varepsilon_0$ F. A partial filling must lie strictly between these. Option A represents the full filling, option B exceeds even that bound, and option C is below the all-air value. Only option D matches both the calculation and these bounds.

Step 4: Conclusion The capacitance is $\boxed{\frac{10}{3}\varepsilon_0\,\mathrm{F}}$. Hence, the correct option is (D).

Quick Tip to Solve: For layers stacked along the field, replace each thickness by thickness divided by its dielectric constant, then add the effective thicknesses.

32. Match List I with List II

	List I		List II
A.	**Surface tension**	**I.**	$\mathrm{kg\ m^{-1}\ s^{-1}}$
B.	**Pressure**	**II.**	$\mathrm{kg\ ms^{-1}}$
C.	**Viscosity**	**III.**	$\mathrm{kg\ m^{-1}\ s^{-2}}$
D.	**Impulse**	**IV.**	$\mathrm{kg\ s^{-2}}$

Choose the correct answer from the options given below : [4 marks]

(A) A-IV, B-III, C-I, D-II

(B) A-IV, B-III, C-II, D-I

(C) A-I, B-I, C-III, D-IV

(D) A-III, B-IV, C-I, D-II

Correct Answer: (A) A-IV, B-III, C-I, D-II

Solution:

Concept:

- The SI unit of force is $\mathrm{N} = \mathrm{kg\,m\,s^{-2}}$. Units of derived quantities follow by substituting this expression into their defining relations.
- Surface tension is force per length, pressure is force per area, dynamic viscosity is shear stress divided by velocity gradient, and impulse is force integrated over time.

Step 1: Match surface tension and pressure Surface tension S is the tangential force divided by the length on which it acts. Therefore

$$[S] = \frac{\mathrm{N}}{\mathrm{m}} = \mathrm{kg\,s^{-2}},$$

which is entry IV. Pressure p is normal force per unit area, giving

$$[p] = \frac{\mathrm{N}}{\mathrm{m^2}} = \mathrm{kg\,m^{-1}\,s^{-2}},$$

which is entry III. Thus A must match IV and B must match III; interchanging the force-per-length and force-per-area definitions changes a power of length and is incorrect.

Step 2: Match viscosity and impulse Newton's law of viscosity is $\tau = \eta\, dv/dy$. The velocity gradient has unit $\mathrm{s^{-1}}$, while shear stress τ has the unit of pressure. Consequently

$$[\eta] = \frac{\mathrm{kg\,m^{-1}\,s^{-2}}}{\mathrm{s^{-1}}} = \mathrm{kg\,m^{-1}\,s^{-1}},$$

which is entry I. Impulse equals change in momentum, or force multiplied by time:

$$[J] = \mathrm{N\,s} = \mathrm{kg\,m\,s^{-1}},$$

which is entry II. Here viscosity means dynamic viscosity, not kinematic viscosity, which would have a different unit.

Step 3: Select the complete mapping The four independently derived matches are A-IV, B-III, C-I and D-II. Option A contains precisely this sequence. Option B incorrectly swaps viscosity and impulse; option C assigns the wrong units already to surface tension and pressure; option D interchanges the required matches for surface tension and pressure. The powers of time and length decide each match without needing numerical values.

Step 4: Conclusion The correct matching is $\boxed{\text{A-IV, B-III, C-I, D-II}}$. Hence, the correct option is (A).

Quick Tip to Solve: Write every unit using kilograms, metres and seconds, and derive viscosity from stress divided by velocity gradient rather than memorising it.

33. The root mean square velocity of molecules of gas is [4 marks]

(A) Proportional to temperature (T)

(B) Inversely proportional to square root of temperature $\left(\sqrt{\frac{1}{T}}\right)$

(C) Proportional to square of temperature (T^2)

(D) Proportional to square root of temperature ($\sqrt{T}$)

Correct Answer: (D) Proportional to square root of temperature ($\sqrt{T}$)

Solution:

Concept:

- For an ideal gas in thermal equilibrium, the mean translational kinetic energy per molecule is $\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}k_{\mathrm{B}}T$, where T is absolute temperature.
- The RMS speed is $v_{\mathrm{rms}} = \sqrt{\langle v^2\rangle} = \sqrt{3k_{\mathrm{B}}T/m} = \sqrt{3RT/M}$. For a particular gas, molecular mass m and molar mass M are fixed.

Step 1: Relate kinetic energy to RMS speed RMS speed is defined by taking the square root of the mean squared molecular speed. It is not the mean velocity vector, which is zero for a gas at rest as a whole. Substituting $\langle v^2\rangle = v_{\mathrm{rms}}^2$ into the kinetic-energy relation gives

$$mv_{\mathrm{rms}}^2 = 3k_{\mathrm{B}}T.$$

Thus the square of the RMS speed varies linearly with absolute temperature. This distinction between a speed and its square is the main point of the question.

Step 2: Take the square root at fixed molecular mass Rearranging and taking the nonnegative square root yields

$$v_{\mathrm{rms}} = \sqrt{\frac{3k_{\mathrm{B}}}{m}}\,\sqrt{T}.$$

For the same gas, the factor $\sqrt{3k_B/m}$ is constant, so $v_{rms} \propto \sqrt{T}$. In a ratio of two equilibrium states,

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}.$$

For example, quadrupling the Kelvin temperature doubles the RMS speed; it does not quadruple or reduce it. Celsius temperature cannot be substituted into this proportionality.

Step 3: Rule out the other dependences Option A would make speed itself proportional to temperature, although kinetic energy is proportional to speed squared. Option B predicts decreasing speed on heating and conflicts with the energy increase. Option C would imply mean kinetic energy proportional to T^4, rather than T. Option D alone has the square-root dependence required by the molecular energy relation. The result concerns a fixed gas; comparing different gases also requires their molecular masses.

Step 4: Conclusion The RMS molecular speed is proportional to $\boxed{\sqrt{T}}$, with temperature measured in kelvin. Hence, the correct option is (D).

Quick Tip to Solve: First decide whether the energy law fixes speed or speed squared. Since thermal kinetic energy is proportional to T, its associated speed varies as $\sqrt{T}$.

34. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Photodiodes are used in forward bias usually for measuring the light intensity.

Reason R : For a p-n junction diode, at applied voltage V the current in the forward bias is more than the current in the reverse bias for $|V_z| > \pm v \geq |v_0|$ where v_0 is the threshold voltage and V_z is the breakdown voltage.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both A and R are true but R is NOT the correct explanation of A

(B) A is true but R is false

(C) Both A and R are true and R is the correct explanation of A

(D) A is false but R is true

Correct Answer: (D) A is false but R is true

Solution:

Concept:

- A photodiode is ordinarily operated under reverse bias. Light generates electron-hole pairs in or near its depletion region, and the reverse electric field collects the carriers to produce photocurrent.
- In an ordinary p-n junction, forward bias above the threshold permits substantial majority-carrier current, while reverse bias below breakdown gives only a small reverse current in the dark.

Step 1: Evaluate Assertion A The assertion says that photodiodes are usually forward biased for measuring light intensity. In normal intensity-detection operation they are reverse biased. The reverse photocurrent changes with incident optical intensity and can therefore serve as the measured signal.

Reverse bias widens the depletion region and assists carrier collection. Forward-bias current would contain a large injection contribution, making the small illumination-induced change less useful for this standard operating mode. Hence Assertion A is false.

Step 2: Evaluate Reason R The voltage range stated is the ordinary operating comparison: the forward-bias magnitude is at least the threshold, and the reverse-bias magnitude is less than the breakdown magnitude. In this range forward bias reduces the potential barrier and allows a large majority-carrier current. Reverse bias increases the barrier and restricts the dark current primarily to minority carriers. Thus, for corresponding applied voltage magnitudes in the indicated range, the forward current exceeds the reverse current. Breakdown is expressly excluded; in breakdown the reverse current could instead become large. Reason R is therefore true.

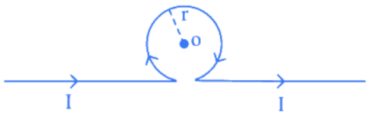
Step 3: Combine the truth values The two truth values must be assessed separately before looking for an explanation link. We have A false and R true, which is option D. Options A and C both require the assertion to be true, so neither can apply. Option B also makes the assertion true and, additionally, rejects the true current comparison. Since the assertion is false, there is no question of accepting R as a correct explanation of it. The correct physical statement about the instrument is reverse-bias operation.

Step 4: Conclusion Assertion A is false, whereas Reason R is true. Hence, the correct option is (D).

Quick Tip to Solve: For a photodiode, remember reverse bias and an illumination-dependent reverse photocurrent; assess the ordinary diode current comparison separately.

35. Match List I with List II

[4 marks]

	List I (Current configuration)		List II (Magnitude of Magnetic Field at point O)
A.		I.	$B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 2]$
B.		II.	$B_0 = \frac{\mu_0 I}{4\, r}$
C.		III.	$B_0 = \frac{\mu_0 I}{2\pi r}[\pi - 1]$
D.		IV.	$B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 1]$

Choose the correct answer from the options given below :

(A) A-III, B-IV, C-I, D-II

(B) A-II, B-I, C-IV, D-III

(C) A-III, B-I, C-IV, D-II

(D) A-I, B-III, C-IV, D-II

Correct Answer: (C) A-III, B-I, C-IV, D-II

Solution:

Concept:

- At the centre of a circular arc of radius r and angle θ in radians, $B_{\text{arc}} = \mu_0 I\theta/(4\pi r)$. A full circle has angle 2π, and a semicircle has angle π.
- A semi-infinite straight tangent at distance r gives $\mu_0 I/(4\pi r)$. A radial straight segment gives zero at the centre because each current element is parallel to its displacement vector. Field directions follow the right-hand rule.

Step 1: Use a common field factor Let $k = \mu_0 I/(4\pi r)$. Then a full circular turn gives $2\pi k$, a semicircle gives πk, and each semi-infinite tangent gives k. In configuration A, the almost closed full turn is clockwise, so its field points into the page. The two straight tangent portions carry current to the right below O and produce an opposing field out of the page. Their sum is $2k$, giving magnitude

$$B_A = (2\pi - 2)k = \frac{\mu_0 I}{2\pi r}(\pi - 1).$$

Therefore A matches III.

Step 2: Add the fields in B and C In configuration B, the semicircle and both straight tangent portions produce fields in the same direction at O. Its magnitude is therefore

$$B_B = (\pi + 2)k,$$

so B matches I. In configuration C, the semicircle and the horizontal semi-infinite tangent again have the same field direction. The upper vertical straight segment is radial relative to O, so its field at O vanishes. Thus

$$B_C = (\pi + 1)k,$$

and C matches IV. The distinction between a radial segment and a tangent is essential; a radial segment cannot be assigned the tangent-wire field.

Step 3: Evaluate D and select the mapping In configuration D, the two horizontal straight portions lie along the diameter line through O. They give zero field there because their current elements and displacement vectors are collinear. Only the lower semicircular arc contributes:

$$B_D = \pi k = \frac{\mu_0 I}{4r},$$

which is II. The complete mapping is therefore A-III, B-I, C-IV, D-II. Option A swaps the required B and C matches; option B misidentifies A and D; option D swaps the required A and B matches. Only option C agrees with every configuration.

Step 4: Conclusion The matching is $\boxed{\text{A-III, B-I, C-IV, D-II}}$. Hence, the correct option is (C).

Quick Tip to Solve: Express every contribution in units of $\mu_0 I/(4\pi r)$, put its direction sign in first, and discard radial segments before adding.

36. In Young's double slits experiment, the position of 5$^{\text{th}}$ bright fringe from the central maximum is 5 cm. The distance between slits and screen is 1 m and wavelength of used monochromatic light is 600 nm. The separation between the slits is : [4 marks]

(A) 60 μm

(B) 48 μm

(C) 36 μm

(D) 12 μm

Correct Answer: (A) 60 μm

Solution:

Concept:

- The bright fringes of a Young double-slit arrangement satisfy $d\sin\theta = n\lambda$, where d is slit separation and n is the order counted from the central maximum.
- For a screen distance D much greater than the slit separation, the fringe position is $y_n = n\lambda D/d$. The fringe width is therefore $\beta = \lambda D/d$.

Step 1: Identify the fringe order and convert the data The central bright fringe has order zero, so the fifth bright fringe corresponds to $n = 5$. The stated distance is its displacement from the central maximum, rather than the distance between adjacent bright fringes. Use SI units throughout:

$$y_5 = 5\,\text{cm} = 0.05\,\text{m}, \qquad D = 1\,\text{m}, \qquad \lambda = 600\times10^{-9}\,\text{m}.$$

Thus the adjacent-fringe spacing is $y_5/5 = 0.01\,\text{m}$.

Step 2: Calculate the slit separation Rearrange the bright-fringe position formula to isolate the separation.

$$d = \frac{n\lambda D}{y_n} = \frac{5(600\times10^{-9})(1)}{0.05}\,\text{m} = 6.0\times10^{-5}\,\text{m} = 60\,\mu\text{m}.$$

The units reduce to a length, as required. The angle is small since $y_5/D = 0.05$, consistent with the usual fringe-position approximation.

Step 3: Check the alternatives The result is option (A). Values $48\,\mu$m and $36\,\mu$m would put the fifth bright fringe at $6.25\,$cm and about $8.33\,$cm, respectively. The value $12\,\mu$m results from incorrectly treating the given fifth-fringe displacement as one fringe width; it would actually give $y_5 = 25\,$cm.

Step 4: Conclusion The slit separation is $\boxed{60\,\mu\text{m}}$. Hence, the correct option is (A).

Quick Tip to Solve: First divide the displacement of the nth bright fringe by n to get the fringe width; do not substitute that displacement directly as one fringe width.

37. T is the time period of simple pendulum on the earth's surface. Its time period becomes x T when taken to a height R (equal to earth's radius) above the earth's surface. Then, the value of x will be : [4 marks]

(A) 4

(B) $\frac{1}{2}$

(C) 2

(D) $\frac{1}{4}$

Correct Answer: (C) 2

Solution:

Concept:

- For a simple pendulum of unchanged length ℓ, the small-oscillation period is $T = 2\pi\sqrt{\ell/g}$, so its period varies inversely as the square root of local gravity.
- Outside a spherical Earth, gravity at distance r from the centre is GM/r^2. A height h above the surface corresponds to $r = R + h$, not $r = h$.

Step 1: Determine gravity at the new position At the surface the distance from the centre is R, and $g = GM/R^2$. At the stated height $h = R$, the new distance is $R + R = 2R$. Hence

$$g_h = \frac{GM}{(2R)^2} = \frac{GM}{4R^2} = \frac{g}{4}.$$

Gravity becomes one quarter of its original value. The inverse-square law must be applied to the distance from the centre; the small-height linear approximation is unsuitable when the height equals the radius.

Step 2: Take the ratio of the periods The length of the same pendulum remains unchanged, so it cancels when the two periods are divided.

$$\frac{T_h}{T} = \sqrt{\frac{g}{g_h}} = \sqrt{\frac{g}{g/4}} = \sqrt{4} = 2.$$

Since the question writes $T_h = xT$, the dimensionless multiplier is $x = 2$. The weaker restoring acceleration makes oscillations slower, which correctly predicts an increased period.

Step 3: Eliminate the other numerical multipliers Option (A), 4, confuses the change in gravity with the change in period and omits the square root. Options (B) and (D) predict a period smaller than at the surface, although gravity has decreased. They use either the reversed ratio or the gravity ratio itself. Only option (C) has both the required increase and the correct square-root dependence.

Step 4: Conclusion The period becomes $2T$, so $\boxed{x = 2}$. Hence, the correct option is (C).

Quick Tip to Solve: At height h, use the centre-to-pendulum distance R+h; for unchanged length, the period ratio outside Earth is simply (R+h)/R.

38. Assume that the earth is a solid sphere of uniform density and a tunnel is dug along its diameter throughout the earth. It is found that when a particle is released in this tunnel, it executes a simple harmonic motion. The mass of the particle is 100 g. The time period of the motion of the particle will be (approximately)

(Take g = 10 m s^{-2} , radius of earth = 6400 km) [4 marks]

(A) 12 hours

(B) 1 hour 24 minutes

(C) 24 hours

(D) 1 hour 40 minutes

Correct Answer: (B) 1 hour 24 minutes

Solution:

Concept:

- Inside a sphere of uniform density, only the mass within radius r contributes to the net gravitational field there. This enclosed mass is $M(r) = Mr^3/R^3$.
- The gravitational acceleration inside Earth is directed towards the centre and has magnitude gr/R. Along a diameter this gives the SHM equation $\ddot{x} = -(g/R)x$.

Step 1: Find the restoring acceleration Let x be the signed displacement from the centre along the tunnel. For a uniform Earth, the enclosed mass varies as the cube of distance, while the field has an inverse-square factor. Combining them leaves an acceleration proportional to distance.

$$a_x = -\frac{GM}{R^3}x = -\frac{g}{R}x.$$

Comparing this with $a_x = -\omega^2 x$ gives $\omega^2 = g/R$. The particle mass cancels from Newton's second law, so the given $100\,\mathrm{g}$ does not affect the period.

Step 2: Evaluate the period in seconds The Earth radius is $R = 6400\,\mathrm{km} = 6.4 \times 10^6\,\mathrm{m}$. Therefore

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4 \times 10^6}{10}}\,\mathrm{s} = 2\pi(800)\,\mathrm{s} \approx 5027\,\mathrm{s}.$$

This is the full oscillation period, including a return to the same starting point with the same direction of motion.

Step 3: Convert the time and compare the choices Dividing by 60 gives approximately $83.8\,\mathrm{min}$, or 1 hour 23.8 minutes. Rounded to the precision of the choices, this is 1 hour 24 minutes. The twelve-hour and twenty-four-hour choices are far too long for $2\pi\sqrt{R/g}$, and one hour forty minutes is about sixteen minutes larger than the calculated value. A transit from one surface end to the other is only half this full period.

Step 4: Conclusion The approximate period is $\boxed{1 \text{ hour } 24 \text{ minutes}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a uniform-density Earth tunnel, remember omega squared equals g/R; distinguish the full oscillation period from the half-period surface-to-surface transit.

39. A car travels a distance of 'x' with speed v_1 and then same distance 'x' with speed v_2 in the same direction. The average speed of the car is : [4 marks]

(A) $\frac{v_1 v_2}{2(v_1+v_2)}$
(B) $\frac{2v_1 v_2}{v_1+v_2}$
(C) $\frac{2x}{v_1+v_2}$
(D) $\frac{v_1+v_2}{2}$

Correct Answer: (B) $\frac{2v_1 v_2}{v_1+v_2}$

Solution:

Concept:

- Average speed is total path length divided by total elapsed time. It is not generally the arithmetic mean of two speeds.
- When equal distances are covered at positive speeds v_1 and v_2, their travel times are x/v_1 and x/v_2. The slower leg takes more time and has greater influence on the average.

Step 1: Add the distances and travel times The first leg covers distance x at speed v_1, so its duration is $t_1 = x/v_1$. The second leg covers the same distance at speed v_2, with duration $t_2 = x/v_2$. Since the motion continues in the same direction without an extra stop, the total path length is $2x$ and the elapsed time is

$$t = t_1 + t_2 = \frac{x}{v_1} + \frac{x}{v_2} = \frac{x(v_1+v_2)}{v_1 v_2}.$$

Step 2: Divide total distance by total time Using the definition gives

$$v_{\text{av}} = \frac{2x}{x/v_1 + x/v_2} = \frac{2xv_1v_2}{x(v_1+v_2)} = \frac{2v_1v_2}{v_1+v_2}.$$

The common distance cancels, so the answer depends only on the two speeds. This expression is their harmonic mean. It lies between the smaller and larger speeds, as an average speed must.

Step 3: Check dimensions and limiting cases If $v_1 = v_2 = v$, the result becomes v, as required for uniform-speed travel. Option (A) instead becomes $v/4$, which immediately rules it out. Option (C) has dimensions of time because it divides distance by speed. Option (D), the arithmetic mean, applies to equal time intervals; these legs have equal distances and generally unequal times. Thus option (B) alone satisfies the definition for the stated journey.

Step 4: Conclusion The average speed is $\boxed{\frac{2v_1v_2}{v_1+v_2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal-distance legs use the harmonic mean; for equal-time legs use the arithmetic mean. Always start from total distance divided by total time.

40. Electron beam used in an electron microscope, when accelerated by a voltage of 20 kV, has a de-Broglie wavelength of λ_0. IF the voltage is increased to 40 kV, then the de-Broglie wavelength associated with the electron beam would be : [4 marks]

(A) 3 λ_0

(B) 9 λ_0

(C) $\frac{\lambda_0}{\sqrt{2}}$

(D) $\frac{\lambda_0}{2}$

Correct Answer: (C) $\frac{\lambda_0}{\sqrt{2}}$

Solution:

Concept:

- The de Broglie wavelength of a particle is $\lambda = h/p$, where h is Planck's constant and p is its momentum.
- For an electron accelerated from rest through potential difference V, the standard nonrelativistic relation is $eV = p^2/(2m_e)$, giving $\lambda = h/\sqrt{2m_e eV}$.

Step 1: Relate the wavelength to accelerating voltage An electron gains kinetic energy eV on crossing the accelerating potential. Equating this to $p^2/(2m_e)$ yields $p = \sqrt{2m_e eV}$. Consequently,

$$\lambda = \frac{h}{\sqrt{2m_e eV}} \quad \Rightarrow \quad \lambda \propto V^{-1/2}.$$

The mass, charge and Planck constant are unchanged between the two settings. Their values therefore cancel from the wavelength ratio, so no numerical evaluation of the initial wavelength is needed.

Step 2: Take the ratio for the two voltages The voltage is increased from $V_1 = 20\,\text{kV}$ to $V_2 = 40\,\text{kV}$, exactly a factor of two. Since both are expressed in the same unit, their ratio can be used directly.

$$\frac{\lambda_2}{\lambda_0} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{20}{40}} = \frac{1}{\sqrt{2}}.$$

Thus doubling the kinetic energy increases momentum by $\sqrt{2}$, rather than by two, and the wavelength decreases by the reciprocal factor.

Step 3: Compare all four wavelength choices Options (A) and (B) increase the wavelength and therefore contradict the increased electron momentum. Option (D) would require the momentum to double, which in this relation requires the voltage to become four times its initial value. Option (C) is precisely the inverse-square-root scaling of the accelerating potential used for this question. Relativistic corrections are beyond the approximation encoded by these choices.

Step 4: Conclusion The new wavelength is $\boxed{\lambda_0/\sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: For the same particle accelerated through a potential, compare wavelengths using the square root of the inverse voltage ratio; doubling voltage does not halve wavelength.

41. A uniform metallic wire carries a current 2 A, when 3.4 V battery is connected across it. The mass of uniform metallic wire is 8.92 $\times$ 10^{-3} kg, density is 8.92 $\times$ 10^{3} kg/m^3 and resistivity is 1.7 $\times$ 10^{-8} Ω-m. The length of wire is : [4 marks]

(A) $l = 100$ m

(B) $l = 6.8$ m

(C) $l = 5$ m

(D) $l = 10$ m

Correct Answer: (D) $l = 10$ m

Solution:

Concept:

- For a uniform wire of length L, cross-sectional area A and resistivity ρ_e, its resistance is $R_w = \rho_e L/A$. Ohm's law gives $R_w = V/I$.
- If the wire mass is m and its material density is ρ_m, its volume is $AL = m/\rho_m$. Combining the mass and resistance equations eliminates the unknown area.

Step 1: Calculate resistance and material volume The stated potential difference and current give

$$R_w = \frac{3.4}{2}\,\Omega = 1.7\,\Omega.$$

Distinguish the material density from electrical resistivity: they have different units and roles. The wire volume is

$$\mathcal{V} = \frac{m}{\rho_m} = \frac{8.92 \times 10^{-3}}{8.92 \times 10^{3}}\,\mathrm{m}^3 = 10^{-6}\,\mathrm{m}^3.$$

The identical numerical factors cancel, leaving the ratio of the powers of ten.

Step 2: Eliminate the cross-sectional area Since $\mathcal{V} = AL$, the area is $A = \mathcal{V}/L$. Substitution into the resistance formula gives

$$R_w = \frac{\rho_e L}{\mathcal{V}/L} = \frac{\rho_e L^2}{\mathcal{V}}.$$

Hence

$$L^2 = \frac{R_w \mathcal{V}}{\rho_e} = \frac{1.7 \times 10^{-6}}{1.7 \times 10^{-8}}\,\mathrm{m}^2 = 100\,\mathrm{m}^2, \qquad L = 10\,\mathrm{m}.$$

Choose the positive root because a physical length is positive.

Step 3: Verify and reject the other lengths The resulting area is $A = 10^{-7}\,\mathrm{m}^2$. Substitution gives $\rho_e L/A = 1.7\,\Omega$, matching the measured resistance. For the fixed volume, resistance is proportional to L^2; lengths 100, 6.8 and 5 m would give resistances 170, 0.78608 and $0.425\,\Omega$, respectively. None matches $1.7\,\Omega$.

Step 4: Conclusion The length of the uniform wire is $\boxed{10\,\mathrm{m}}$. Hence, the correct option is (D).

Quick Tip to Solve: When both mass and density are given, find volume first and use R = resistivity times length squared divided by volume; remember to take the square root at the end.

42. A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2 m long and 0.2 m in diameter. The magnetic intensity at the center of the solenoid when a current of 2 A flows through it is : [4 marks]

(A) $1\ \text{A m}^{-1}$

(B) $2.4 \times 10^{-3}\ \text{A m}^{-1}$

(C) $1.2 \times 10^{3}\ \text{A m}^{-1}$

(D) $2.4 \times 10^{3}\ \text{A m}^{-1}$

Correct Answer: (C) $1.2 \times 10^{3}\ \text{A m}^{-1}$

Solution:

Concept:

- For a long uniformly wound solenoid carrying current I, the magnetic field intensity is $H = nI = NI/L$, where n is the number of turns per unit axial length.
- Magnetic intensity H is measured in A m^{-1}. It differs from flux density B, measured in tesla; in air or glass treated as nonmagnetic, $B \approx \mu_0 H$.

Step 1: Find the number of turns per unit length The solenoid has $N = 1200$ turns distributed uniformly over axial length $L = 2\,\text{m}$. Thus

$$n = \frac{N}{L} = \frac{1200}{2}\,\text{m}^{-1} = 600\,\text{m}^{-1}.$$

The stated diameter is $0.2\,\text{m}$, so the length is ten times the diameter. The standard long-solenoid approximation is appropriate at its centre. Use the axial length for turn density, not the circumference of the supporting tube.

Step 2: Calculate the magnetic intensity Multiply turns per unit length by the current:

$$H = nI = (600\,\text{m}^{-1})(2\,\text{A}) = 1200\,\text{A m}^{-1} = 1.2 \times 10^{3}\,\text{A m}^{-1}.$$

This is the requested magnetic intensity. No factor of μ_0 is included: that factor would convert H to the magnetic flux density B. The single-layer construction ensures that all the stated turns are distributed along the given length.

Step 3: Check the options and the approximation Option (C) has both the calculated value and the correct unit. Option (D) doubles the answer by using the total turns without dividing by the two-metre length. Options (A) and (B) are inconsistent with the large turn density and the current. A finite-solenoid centre correction would multiply the long-solenoid result by $L/\sqrt{L^2 + 4a^2}$, with radius $a = 0.1\,\text{m}$; this is about 0.995, so the stated choice remains the same at the intended precision.

Step 4: Conclusion The magnetic intensity is $\boxed{1.2 \times 10^{3}\,\text{A m}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Check whether a solenoid question asks for H or B: use NI/L for H in ampere per metre, and multiply by permeability only when B is required.

43. An electromagnetic wave is transporting energy in the negative z direction. At a certain point and certain time the direction of electric field of the wave is along positive y direction. What will be the direction of the magnetic field of the wave at that point and instant? [4 marks]

(A) Negative direction of y

(B) Positive direction of z

(C) Positive direction of x

(D) Negative direction x

Correct Answer: (C) Positive direction of x

Solution:

Concept:

- The energy flow in an electromagnetic wave is along its Poynting vector, $\vec{S} = \mu_0^{-1}(\vec{E} \times \vec{B})$. Thus the order of the two fields in the cross product matters.
- The electric field, magnetic field and direction of propagation are mutually perpendicular. Unit vectors obey $\hat{i} \times \hat{j} = \hat{k}$, and reversing the order changes the sign.

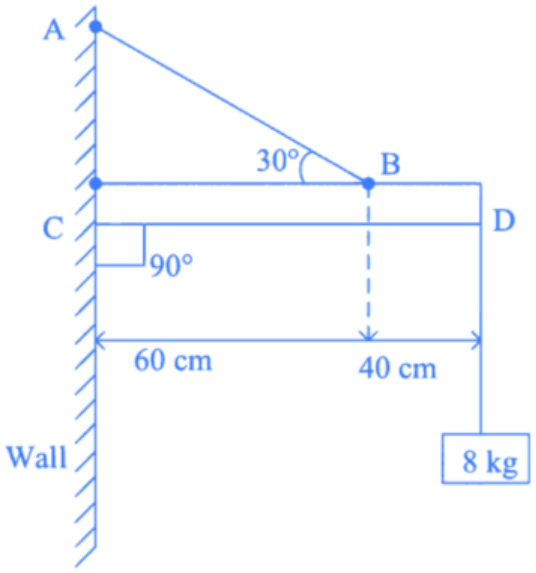
Step 1: Write the required cross product The wave transports energy along negative z, so its Poynting vector is proportional to $-\hat{k}$. The electric field at the specified instant is along $+\hat{j}$. We therefore need a magnetic-field direction for which $\hat{j} \times \hat{B} = -\hat{k}$. It is the instantaneous direction that is relevant; the magnitudes and frequency are unnecessary.

Step 2: Apply the right-hand rule and test the alternatives Since $\hat{j} \times \hat{i} = -\hat{k}$, a magnetic field along positive x gives exactly the stated energy flow. A field along negative x instead gives positive z, so option (D) reverses the propagation. A field along negative y is parallel to the electric field and gives zero cross product; this rules out (A). A field along positive z gives energy flow along positive x, ruling out (B).

Step 3: Conclusion The magnetic field is along the positive x direction. Hence, the correct option is (C).

Quick Tip to Solve: Always use $\vec{E} \times \vec{B}$ for the wave direction. Swapping the fields reverses the answer even when the axes are perpendicular.

44. An object of mass 8 kg is hanging from one end of a uniform rod CD of mass 2 kg and length 1 m pivoted at its end C on a vertical wall as shown in figure. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is (Take g = 10 m/s^2) [4 marks]



(A) 90 N

(B) 240 N

(C) 30 N

(D) 300 N

Correct Answer: (D) 300 N

Solution:

Concept:

- For a rigid body in equilibrium the total torque about any point vanishes. Taking moments about the hinge removes the unknown hinge reaction because its line of action passes through that point.
- A uniform rod has its weight acting at its midpoint. Only the component of cable tension perpendicular to the horizontal rod contributes a moment about the hinge.

Step 1: Identify the forces and their lever arms The hanging object exerts $8g = 80\,\mathrm{N}$ downward at D, $1.0\,\mathrm{m}$ from C. The rod weighs $2g = 20\,\mathrm{N}$, acting $0.50\,\mathrm{m}$ from C. The cable is attached at B, $0.60\,\mathrm{m}$ from C, and makes 30° with the rod. Its upward component is $T\sin 30^\circ$. Its horizontal component has no torque about C.

Step 2: Balance the clockwise and anticlockwise moments The weights turn the rod clockwise, while the upward component of tension turns it anticlockwise. Thus

$$T\sin 30^\circ(0.60) = 80(1.0) + 20(0.50).$$

Substituting $\sin 30^\circ = 1/2$,

$$0.30T = 80 + 10 = 90, \qquad T = 300\,\mathrm{N}.$$

The values 90, 240 and 30 N fail this torque equation: their counterclockwise moments would be 27, 72 and 9 N m, respectively, instead of the required 90 N m.

Step 3: Conclusion The tension in cable AB is $300\,\mathrm{N}$. Hence, the correct option is (D).

Quick Tip to Solve: In a hinged-rod problem, take moments about the hinge first and use the cable attachment distance, which need not equal the full rod length.

45. In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of oscillator becomes x times its initial resonant frequency ω_0. The value of x is : [4 marks]

(A) 1/4

(B) 1/16

(C) 4

(D) 16

Correct Answer: (A) 1/4

Solution:

Concept:

- An ideal LC oscillator has angular resonant frequency $\omega_0 = 1/\sqrt{LC}$, where L is its inductance and C its capacitance.
- The ordinary frequency is $f_0 = 1/(2\pi\sqrt{LC})$. Both frequencies have the same scaling with LC, so the requested ratio is independent of which frequency convention is used.

Step 1: Express the new oscillator parameters Let the original inductance and capacitance be L and C. The changes specified in the question give $L\prime = 2L$ and $C\prime = 8C$. It is their product, rather than their sum or their individual factors alone, that enters the frequency formula. Hence

$$L\prime C\prime = (2L)(8C) = 16LC.$$

The product increases, so the new frequency must be smaller than the original frequency.

Step 2: Take the frequency ratio The new angular frequency is

$$\omega\prime = \frac{1}{\sqrt{16LC}} = \frac{1}{4\sqrt{LC}} = \frac{\omega_0}{4}.$$

Consequently $x = \omega\prime/\omega_0 = 1/4$. Option (B), $1/16$, incorrectly omits the square root. Options (C) and (D), 4 and 16, predict an increase despite both energy-storage parameters increasing, and therefore have the wrong direction of change.

Step 3: Conclusion The new resonant frequency is one quarter of its initial value, so $x = 1/4$. Hence, the correct option is (A).

Quick Tip to Solve: Multiply the factors by which L and C change, take their square root, then invert it to obtain the frequency factor.

46. A car is moving with a constant speed of 20 m/s in a circular horizontal track of radius 40 m. A bob is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be : (Take g = 10 m/s^2) [4 marks]

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

Correct Answer: (C) $\frac{\pi}{4}$

Solution:

Concept:

- Uniform circular motion has centripetal acceleration $a_c = v^2/r$, directed horizontally towards the centre even when the speed is constant.
- For a bob at rest relative to the turning car, the vertical component of tension balances its weight while the horizontal component supplies the required centripetal force.

Step 1: Resolve the tension along vertical and radial directions Let the string make an angle θ with the vertical and let the bob have mass m. Its vertical acceleration is zero, so $T\cos\theta = mg$. Its horizontal acceleration is towards the centre of the track, so $T\sin\theta = mv^2/r$. The bob lies outward from its roof attachment, allowing the tension to pull it inward. No component of weight supplies the horizontal force.

Step 2: Eliminate the tension and calculate the angle Dividing the horizontal equation by the vertical equation cancels both T and m:

$$\tan\theta = \frac{v^2}{rg} = \frac{20^2}{40 \times 10} = 1.$$

For the physical inclination between zero and $\pi/2$, this gives $\theta = \pi/4$. Option (A) would make the vertical component of tension zero, so it cannot balance the weight. Options (B) and (D) have tangents $1/\sqrt{3}$ and $\sqrt{3}$, respectively, neither equal to the required acceleration ratio 1.

Step 3: Conclusion The string makes an angle $\pi/4$, or 45°, with the vertical. Hence, the correct option is (C).

Quick Tip to Solve: For a bob in a horizontally accelerating vehicle, use $\tan\theta = a/g$; in circular motion substitute $a = v^2/r$.

47. A Carnot engine with efficiency 50% takes heat from a source at 600 K. In order to increase the efficiency to 70%, keeping the temperature of sink same, the new temperature of the source will be : [4 marks]

(A) 300 K

(B) 1000 K

(C) 900 K

(D) 360 K

Correct Answer: (B) 1000 K

Solution:

Concept:

- The efficiency of a Carnot engine is $\eta = 1 - T_c/T_h$, where T_c and T_h are the sink and source temperatures on the absolute Kelvin scale.
- With the sink temperature fixed, increasing the source temperature lowers the ratio T_c/T_h and increases the efficiency. The heat quantities do not need to be known separately.

Step 1: Find the unchanged sink temperature Initially the source temperature is $T_{h1} = 600\,\text{K}$ and the efficiency is $\eta_1 = 50\% = 0.50$. Therefore

$$0.50 = 1 - \frac{T_c}{600}, \qquad T_c = 600(1 - 0.50) = 300\,\text{K}.$$

This is the sink temperature, which remains unchanged in the second operating condition. It must not be confused with the new source temperature being asked for.

Step 2: Use the target efficiency to find the source temperature For the required efficiency $\eta_2 = 70\% = 0.70$,

$$0.70 = 1 - \frac{300}{T_{h2}}, \qquad T_{h2} = \frac{300}{0.30} = 1000\,\text{K}.$$

At 300 K the source would equal the sink and the efficiency would be zero, eliminating (A). At 900 K it would be $1 - 300/900 = 2/3$, rather than 0.70, eliminating (C). At 360 K it would be $1 - 300/360 = 1/6$, eliminating (D).

Step 3: Conclusion The source must be raised to $1000\,\text{K}$. Hence, the correct option is (B).

Quick Tip to Solve: First determine the sink from the initial efficiency. Then use the same sink value with the target efficiency; never scale efficiency directly with temperature.

48. The ratio of the density of oxygen nucleus ($^{16}_{8}O$) and helium nucleus ($^{4}_{2}\mathrm{He}$) is [4 marks]

(A) 4 : 1

(B) 1 : 1

(C) 2 : 1

(D) 8 : 1

Correct Answer: (B) 1 : 1

Solution:

Concept:

- The empirical nuclear radius is $R = R_0 A^{1/3}$, where A is the mass number and R_0 is approximately the same constant for different nuclei.
- The nuclear mass is approximately Am_N, where m_N is the mean nucleon mass. Nuclear density is mass divided by the spherical nuclear volume, not the volume of the entire atom.

Step 1: Obtain the dependence of nuclear density on mass number For a spherical nucleus, the volume is

$$V = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}R_0^3 A.$$

The factor A therefore appears in both nuclear mass and nuclear volume. Substituting into the density gives

$$\rho = \frac{Am_N}{(4\pi/3)R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

Thus nuclear density is approximately independent of mass number within the nuclear-radius model used here.

Step 2: Compare oxygen and helium explicitly For oxygen $A_O = 16$, whereas for helium $A_{He} = 4$. Their masses are in the ratio $16 : 4 = 4 : 1$, but their volumes are also in the ratio $16 : 4 = 4 : 1$. Hence

$$\frac{\rho_O}{\rho_{He}} = \frac{16/4}{16/4} = 1.$$

Option (A) uses the mass ratio alone and ignores the different volumes. Options (C) and (D) likewise retain an unjustified dependence on nuclear size rather than the cancellation of mass and volume.

Step 3: Conclusion The oxygen and helium nuclei have the same approximate nuclear density, giving the ratio $1 : 1$. Hence, the correct option is (B).

Quick Tip to Solve: Remember that nuclear radius scales as $A^{1/3}$, so nuclear volume scales as A and cancels the A in nuclear mass.

49. A message signal of frequency 5 kHz is used to modulate a carrier signal of frequency 2 MHz. The bandwidth for amplitude modulation is : [4 marks]

(A) 10 kHz

(B) 2.5 kHz

(C) 20 kHz

(D) 5 kHz

Correct Answer: (A) 10 kHz

Solution:

Concept:

- For ordinary amplitude modulation by a single sinusoidal message of frequency f_m, the spectrum contains a carrier at f_c and sidebands at $f_c - f_m$ and $f_c + f_m$.
- The transmission bandwidth is the highest spectral frequency minus the lowest spectral frequency. Therefore double-sideband AM requires $B = 2f_m$, independently of the carrier frequency.

Step 1: Locate the two sideband frequencies The carrier frequency is $f_c = 2\,\text{MHz} = 2000\,\text{kHz}$, and the message frequency is $f_m = 5\,\text{kHz}$. The lower and upper sideband frequencies are therefore

$$f_{LSB} = 2000 - 5 = 1995\,\text{kHz},$$

$$f_{USB} = 2000 + 5 = 2005\,\text{kHz}.$$

The carrier lies midway between them. Its large absolute value determines the position of the band, rather than the width occupied around that position.

Step 2: Subtract the lower edge from the upper edge The total AM bandwidth is

$$B = f_{USB} - f_{LSB} = 2005 - 1995 = 10\,\text{kHz}.$$

Equivalently, $B = 2f_m = 2(5) = 10\,\text{kHz}$. Option (D), 5 kHz, measures only one sideband separation from the carrier. Option (B), 2.5 kHz, incorrectly halves the message frequency; option (C), 20 kHz, doubles the correct two-sideband width again. Neither the carrier alone nor a single separation is the full bandwidth.

Step 3: Conclusion The bandwidth required for amplitude modulation is $10\,\text{kHz}$. Hence, the correct option is (A).

Quick Tip to Solve: For standard AM, subtract $f_c - f_m$ from $f_c + f_m$: the carrier cancels and the bandwidth is twice the message frequency.

50. A bowl filled with very hot soup cools from 98°C to 86°C in 2 minutes when the room temperature is 22°C. How long it will take to cool from 75°C to 69°C? [4 marks]

(A) 2 minutes

(B) 0.5 minute

(C) 1.4 minutes

(D) 1 minute

Correct Answer: (C) 1.4 minutes

Solution:

Concept:

- Newton's law of cooling states that the rate of fall of temperature is proportional to the excess temperature above the surroundings: $-dT/dt = k(T - T_s)$, where T_s is constant and k is the cooling constant.
- For cooling from T_1 to T_2, integration gives $t = k^{-1}\ln[(T_1 - T_s)/(T_2 - T_s)]$. Ratios of cooling times eliminate the unknown constant of the same bowl of soup.

Step 1: Determine the cooling constant The room temperature is $22°\text{C}$, so the initial interval has excess temperatures $98 - 22 = 76°\text{C}$ and $86 - 22 = 64°\text{C}$. Thus

$$2 = \frac{1}{k}\ln\frac{76}{64}, \qquad k = \frac{1}{2}\ln\frac{76}{64}\ \text{min}^{-1}.$$

The excess temperature, rather than the absolute Celsius temperature, controls the rate. The soup cools more slowly as it approaches room temperature.

Step 2: Calculate the second cooling time For the required interval the excess temperatures are $75 - 22 = 53°\text{C}$ and $69 - 22 = 47°\text{C}$. Hence

$$t = 2\frac{\ln(53/47)}{\ln(76/64)} \approx 1.398\ \text{min} \approx 1.4\ \text{min}.$$

As a check, the average-excess approximation gives $t/2 = (6/12)(70/50) = 0.7$, also giving 1.4 minutes. Options (A), (B) and (D), namely 2, 0.5 and 1 minute, do not satisfy this ratio; simply halving the time ignores the lower cooling rate during the later interval.

Step 3: Conclusion The required cooling time is approximately 1.4 minutes. Hence, the correct option is (C).

Quick Tip to Solve: Always subtract the room temperature before comparing cooling intervals. The ratio of logarithms gives the exact Newton-cooling result.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ then, the unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right)$. The value of x is _____. [4 marks]

Solution:

Concept:

- The cross product is perpendicular to both input vectors; its components follow the determinant rule with the ordered basis $\hat{i}, \hat{j}, \hat{k}$. Reversing the vector order reverses the direction.
- A unit vector along a nonzero vector $\vec{A}$ is $\vec{A}/|\vec{A}|$, where $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$. The coefficient of the stated direction must therefore normalise its magnitude to one.

Step 1: Evaluate the cross product Using the supplied components in the order $\vec{P} \times \vec{Q}$,

$$(\vec{P} \times \vec{Q})_x = \sqrt{3}(2.5) - 2\sqrt{3} = \frac{\sqrt{3}}{2},$$
$$\left(\vec{P} \times \vec{Q}\right)_y = 2(4) - 3(2.5) = \frac{1}{2},$$
$$\left(\vec{P} \times \vec{Q}\right)_z = 3\sqrt{3} - 4\sqrt{3} = -\sqrt{3}.$$

Thus $\vec{P} \times \vec{Q} = \frac{1}{2}(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k})$. Its positive scalar factor confirms that the direction in the question has the correct orientation.

Step 2: Normalise the direction vector The magnitude of the vector in parentheses is

$$\sqrt{(\sqrt{3})^2 + 1^2 + (-2\sqrt{3})^2} = \sqrt{3 + 1 + 12} = 4.$$

Equivalently the actual cross product has magnitude $4/2 = 2$. Dividing that cross product by 2 therefore gives

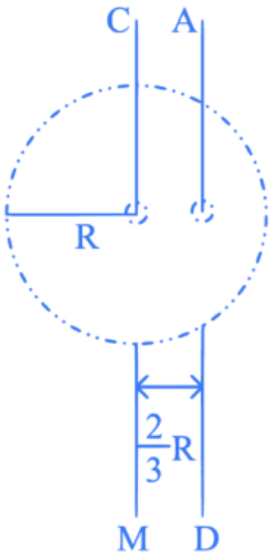
$$\hat{n} = \frac{1}{4}(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}).$$

The squared length of this result is $(3 + 1 + 12)/16 = 1$, checking the normalisation. Since it must point along, rather than opposite to, the cross product, the positive value is selected.

Step 3: Conclusion Comparing with the given form gives $x = 4$, a dimensionless normalising factor.

Quick Tip to Solve: Check the magnitude of the vector inside the parentheses first, then verify its sign against the actual cross product.

52. I_{CM} **is the moment of inertia of a circular disc about an axis (CM) passing through its center and perpendicular to the plane of disc.** I_{AB} **is it's moment of inertia about an axis AB perpendicular to plane and parallel to axis CM at a distance** $\frac{2}{3}$**R from center. Where R is the radius of the disc. The ratio of** I_{AB} **and** I_{CM} **is** $x : 9$**. The value of** x **is _____.** [4 marks]



Solution:

Concept:

- For a uniform circular disc of mass M and radius R, the central moment of inertia about an axis perpendicular to its plane is $I_{CM} = \frac{1}{2}MR^2$. This is different from the moment about an in-plane diameter.
- The parallel-axis theorem gives $I_{AB} = I_{CM} + Md^2$, where d is the perpendicular separation of the two parallel axes and the first axis passes through the centre of mass.

Step 1: Substitute the axis separation The diagram and question place AB parallel to the central perpendicular axis, at separation $d = 2R/3$. The displacement increases the inertia, because the extra term is positive:

$$\begin{aligned} I_{AB} &= \frac{1}{2}MR^2 + M\left(\frac{2R}{3}\right)^2 \\ &= \left(\frac{1}{2} + \frac{4}{9}\right)MR^2 \\ &= \frac{17}{18}MR^2. \end{aligned}$$

The distance must be squared; adding a term linear in R would also fail the dimensional check for moment of inertia.

Step 2: Form the required ratio Both moments refer to the same disc, so M and R^2 cancel:

$$\frac{I_{AB}}{I_{CM}} = \frac{(17/18)MR^2}{(1/2)MR^2} = \frac{17}{9}.$$

Therefore $I_{AB} : I_{CM} = 17 : 9$. The ratio exceeds unity, as expected for a parallel axis displaced from

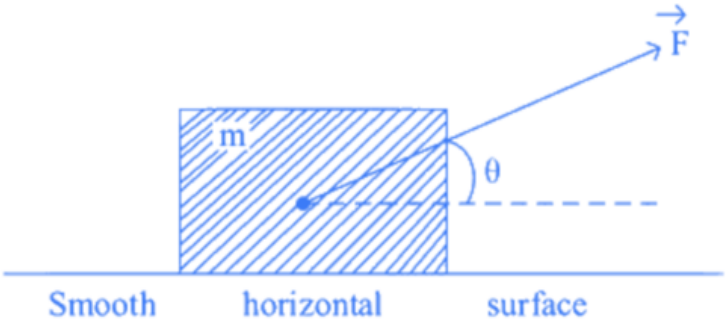
the centre. It is also less than the tangent-axis ratio $3:1$, since the separation $2R/3$ is less than the radius. These checks support both the axis selection and arithmetic.

Step 3: Conclusion The ratio is $17:9$, so the required dimensionless value is $x = 17$.

Quick Tip to Solve: Write the ratio as $1 + Md^2/I_{\mathrm{CM}}$; the mass and radius factors cancel immediately for a specified fraction of the radius.

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53. An object of mass 'm' initially at rest on a smooth horizontal plane starts moving under the action of force F = 2N. In the process of its linear motion, the angle θ (as shown in figure) between the direction of force and horizontal varies as $\theta = \mathrm{k}x$, where k is a constant and x is the distance covered by the object from its initial position. The expression of kinetic energy of the object will be $E = \frac{n}{k}\sin\theta$. The value of n is _____. [4 marks]



Solution:

Concept:

- The work-energy theorem gives $K - K_0 = W$. Since the block starts at rest, $K_0 = 0$. On the smooth horizontal plane the normal force and gravity do no work because they are perpendicular to the displacement.
- The component of the applied force along the displacement is $F\cos\theta$. Here its magnitude is constant but its direction changes with position, so the work is the integral $W = \int F\cos\theta\,dx$.

Step 1: Set up the work integral At a displacement s from the initial position, the angle is ks; using a separate integration variable avoids confusing it with the final displacement x. Therefore

$$dW = 2\cos(ks)\,ds, \qquad K(x) = \int_0^x 2\cos(ks)\,ds.$$

Only the horizontal component contributes. Multiplying the final horizontal force by the entire displacement would be incorrect, because that component changes throughout the motion.

Step 2: Integrate and compare coefficients Since k is constant, the antiderivative of $\cos(ks)$ is $\sin(ks)/k$. Consequently

$$K(x) = \frac{2}{k}\big[\sin(ks)\big]_0^x = \frac{2}{k}\sin(kx) = \frac{2}{k}\sin\theta.$$

This vanishes at the initial position, as required by the given initial rest condition. Differentiating the result with respect to x recovers $2\cos(kx)$, the applied horizontal force. Since k has units of inverse length, the coefficient 2 N divided by k has units of joules.

Step 3: Conclusion Comparing $E = (2/k)\sin\theta$ with the specified form gives the requested numerical coefficient $n = 2$ (the force coefficient is 2 N).

Quick Tip to Solve: For a force of fixed magnitude but changing angle, integrate its component along the displacement; do not use its final component times the full distance.

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54. A ray of light is incident from air on a glass plate having thickness $\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence of a ray is equal to the critical angle for glass-air interface. The lateral displacement of the ray when it passes through the plate is _____ $\times 10^{-2}$ cm. (given $\sin 15^\circ = 0.26$) [4 marks]

Solution:

Concept:

- The critical angle c at a glass-air interface obeys $\sin c = 1/\mu$, where μ is the refractive index of glass relative to air. The given incident angle equals this angle, even though the ray enters from air.
- Snell's law at entry is $\sin i = \mu \sin r$. A slab of perpendicular thickness t produces lateral displacement $d = t\sin(i - r)/\cos r$; the emerging ray is parallel to the incident ray.

Step 1: Determine the incident and refracted angles With $\mu = \sqrt{2}$, the critical angle satisfies

$$\sin c = \frac{1}{\sqrt{2}}, \qquad c = 45^\circ.$$

Thus the incident angle is $i = 45^\circ$. Applying Snell's law from air into glass,

$$\sin r = \frac{\sin 45^\circ}{\sqrt{2}} = \frac{1}{2}, \qquad r = 30^\circ.$$

The ray bends towards the normal on entry, so the refracted angle is smaller than the incident angle. There is no total internal reflection at entry from the rarer medium.

Step 2: Calculate the displacement in the requested units Substitute $t = \sqrt{3}\,\text{cm}$, $i - r = 15^\circ$, $\cos 30^\circ = \sqrt{3}/2$, and the supplied $\sin 15^\circ = 0.26$:

$$d = \frac{\sqrt{3}\,(0.26)}{\sqrt{3}/2}\,\text{cm} = 0.52\,\text{cm}.$$

Expressing this as a multiple of the unit printed next to the blank gives

$$0.52\,\text{cm} = 52 \times 10^{-2}\,\text{cm}.$$

The thickness is already in centimetres, so there is no need to convert to metres. The requested number is 52, not 0.52; the latter is the displacement measured directly in centimetres.

Step 3: Conclusion The lateral displacement is $0.52\,\text{cm} = 52 \times 10^{-2}\,\text{cm}$. The number to enter is 52.

Quick Tip to Solve: Find both angles before using the slab formula, and retain the unit printed after the numerical blank when deciding what number to enter.

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55. The wavelength of the radiation emitted is λ_0 when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will $\frac{20}{x}\lambda_0$. The value of x is _____. [4 marks]

Solution:

Concept:

- The ground state of hydrogen has $n = 1$, so the first, second and third excited states have $n = 2, 3, 4$, respectively. An emission transition goes from the larger quantum number to the smaller one.
- The Rydberg formula is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$. Wavelength is inversely proportional to this difference; the same Rydberg constant cancels when taking a ratio for two hydrogen transitions.

Step 1: Identify the reference transition The second excited to first excited transition is $n_i = 3$ to $n_f = 2$. Hence

$$\frac{1}{\lambda_0} = R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36}, \qquad \lambda_0 = \frac{36}{5R_H}.$$

The wording refers to excited states rather than orbit numbers in this first transition. Treating the second excited state as the second orbit would identify the wrong initial energy level.

Step 2: Evaluate the second wavelength and take the ratio The third excited state has $n_i = 4$, while the second orbit has $n_f = 2$. Therefore

$$\frac{1}{\lambda} = R_H\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R_H}{16}, \qquad \lambda = \frac{16}{3R_H}.$$

Dividing the two wavelengths gives

$$\frac{\lambda}{\lambda_0} = \frac{16}{3R_H}\frac{5R_H}{36} = \frac{20}{27}.$$

The second transition releases more energy than the $3 \to 2$ transition, so its wavelength should be shorter. The factor $20/27 < 1$ verifies this physical expectation. Comparing it with $20/x$ determines the dimensionless integer.

Step 3: Conclusion The emitted wavelength is $\lambda = (20/27)\lambda_0$, so $x = 27$, dimensionless.

Quick Tip to Solve: Translate excited-state names into orbit numbers first. For wavelength ratios, invert the corresponding Rydberg differences or photon-energy ratios.

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56. The distance between two consecutive points with phase difference of 60° in a wave of frequency 500 Hz is 6.0 m. The velocity with which wave is travelling is _____ km/s [4 marks]

Solution:

Concept:

- For a progressive sinusoidal wave at a fixed instant, the phase change over a distance Δx is $\Delta\phi = (2\pi/\lambda)\Delta x$, where λ is the wavelength. In degrees, one full wavelength corresponds to $360°$.
- The propagation speed is $v = f\lambda$, where frequency f counts oscillations per second. Using wavelength in metres and frequency in hertz gives speed in metres per second, which is then converted to kilometres per second.

Step 1: Find the wavelength from the spatial phase change The two consecutive points in the question differ in phase by $60°$ and are $6.0\,\mathrm{m}$ apart. A $60°$ change is one-sixth of a full cycle, so

$$\frac{\Delta x}{\lambda} = \frac{60°}{360°} = \frac{1}{6}.$$

Consequently

$$\lambda = 6\Delta x = 6(6.0) = 36\,\mathrm{m}.$$

The distance given is not a full wavelength. It is the distance for only the specified phase increment; confusing the two would underestimate the wavelength and speed by a factor of six.

Step 2: Calculate and convert the propagation speed Substitute the given frequency and the derived wavelength:

$$v = f\lambda = (500\,\mathrm{s^{-1}})(36\,\mathrm{m}) = 18000\,\mathrm{m\,s^{-1}}.$$

Since $1000\,\mathrm{m} = 1\,\mathrm{km}$,

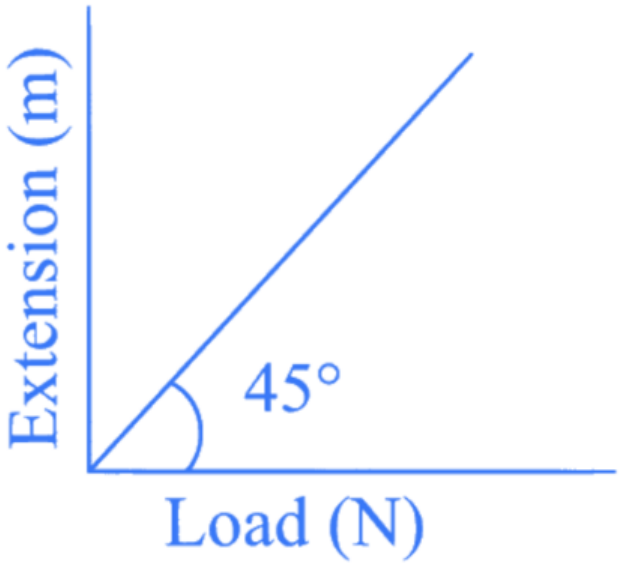
$$v = \frac{18000}{1000}\,\mathrm{km\,s^{-1}} = 18\,\mathrm{km\,s^{-1}}.$$

A time-domain check gives a period $1/500 = 0.002\,\mathrm{s}$. The wave advances one wavelength, 36 m, in that time, again giving $18000\,\mathrm{m\,s^{-1}}$. The requested unit is kilometres per second, so the entered number is 18.

Step 3: Conclusion The wave travels at $18\,\mathrm{km\,s^{-1}}$. The required numerical answer is 18.

Quick Tip to Solve: A spatial phase difference of $60°$ means a separation of $\lambda/6$. Recover the full wavelength before multiplying by frequency.

57. As shown in the figure, in an experiment to determine Young's modulus of a wire, the extension-load curve is plotted. The curve is a straight line passing through the origin and makes an angle of 45° with the load axis. The length of wire is 62.8 cm and its diameter is 4 mm. The Young's modulus is found to be $x \times 10^4$ Nm^{-2}. The value of x is _____. [4 marks]



Solution:

Concept:

- Young's modulus is longitudinal stress divided by longitudinal strain: $Y = (F/A)/(\Delta l/l) = Fl/(A\Delta l)$. Here l is original length and A is the cross-sectional area.
- On an extension-versus-load graph, the slope is $\Delta l/F$, not $F/\Delta l$. Therefore $Y = l/(A\,\text{slope})$, with both axes expressed in their labelled SI units.

Step 1: Read the slope from the graph The vertical axis is extension in metres and the horizontal axis is load in newtons. The straight line passes through the origin, so the ratio of extension to load is constant. Its angle with the load axis is $45°$; hence the numerical slope in the indicated units is

$$s = \frac{\Delta l}{F} = \tan 45° = 1\,\text{m N}^{-1}.$$

This is the compliance of the wire. Taking the reciprocal gives its force per unit extension, $1\,\text{N m}^{-1}$.

Step 2: Convert the wire dimensions to SI units The length is $l = 62.8\,\text{cm} = 0.628\,\text{m}$. The given diameter is $d = 4\,\text{mm}$, so the radius is $r = 2 \times 10^{-3}\,\text{m}$, not $4 \times 10^{-3}\,\text{m}$. Thus

$$A = \pi r^2 = 4\pi \times 10^{-6}\,\text{m}^2.$$

Using the radius correctly is essential because the area depends on its square.

Step 3: Calculate the modulus and identify the coefficient Substitute the area and the graph slope into the stress-strain relation:

$$Y = \frac{0.628}{(4\pi \times 10^{-6})(1)}\,\text{N m}^{-2}.$$

With $\pi \simeq 3.14$, the denominator is 1.256×10^{-5}, giving

$$Y = 5.0 \times 10^4 \,\mathrm{N\,m^{-2}}.$$

Comparing this with the requested form determines the dimensionless coefficient directly.

Step 4: Conclusion The Young's modulus is $5.0 \times 10^4 \,\mathrm{N\,m^{-2}}$; therefore $\boxed{x = 5}$.

Quick Tip to Solve: For extension on the vertical axis, use the reciprocal slope in Young modulus; also halve the diameter before calculating the area.

58. An LCR series circuit of capacitance 62.5 nF and resistance of 50 Ω, is connected to an A.C. source of frequency 2.0 kHz. For maximum value of amplitude of current in circuit, the value of inductance is _____ mH.

(Take $\pi^2 = 10$) [4 marks]

Solution:

Concept:

- The impedance of a series LCR circuit is $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$. The current amplitude is $I_0 = V_0/Z$ for a fixed source amplitude.
- Maximum current occurs at series resonance, where inductive and capacitive reactances cancel: $\omega L = 1/(\omega C)$, with $\omega = 2\pi f$.

Step 1: Set the net reactance equal to zero The resistance is fixed at $50\,\Omega$, so it cannot be reduced by varying the inductance. The squared reactance term in the impedance is always nonnegative. Its minimum value is zero, reached when

$$\omega L - \frac{1}{\omega C} = 0.$$

At this setting $Z = R$, the voltage and current are in phase, and the current amplitude is the largest possible. This explains why the resistance does not enter the equation for the required inductance.

Step 2: Convert the capacitance and frequency The source frequency is $f = 2.0 \times 10^3\,\mathrm{Hz}$, and the capacitance is $C = 62.5 \times 10^{-9}\,\mathrm{F}$. The resonance condition gives

$$L = \frac{1}{\omega^2 C} = \frac{1}{4\pi^2 f^2 C}.$$

The frequency must be used in hertz and the capacitance in farads to obtain inductance in henries; inserting kilohertz or nanofarads as bare numbers would give the wrong scale.

Step 3: Evaluate and convert to millihenries Using the prescribed value $\pi^2 = 10$,

$$\begin{aligned} 4\pi^2 f^2 C &= 4(10)(2.0 \times 10^3)^2(62.5 \times 10^{-9}) \\ &= 40(4.0 \times 10^6)(62.5 \times 10^{-9}) = 10. \end{aligned}$$

Consequently

$$L = \frac{1}{10}\,\mathrm{H} = 0.100\,\mathrm{H} = 100\,\mathrm{mH}.$$

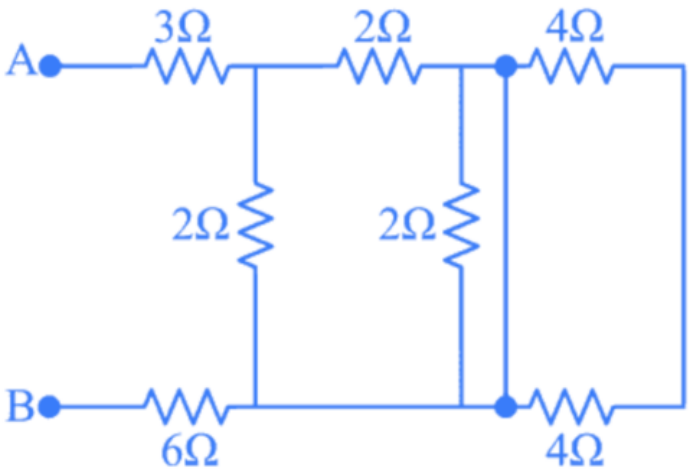
This value makes the two reactances equal, as required.

Step 4: Conclusion The required inductance is $\boxed{100\,\mathrm{mH}}$.

Quick Tip to Solve: For a series circuit at maximum current, first cancel the reactances; the resistance sets the peak current but does not change the resonance inductance.

59. In the given circuit, the equivalent resistance between the terminal A and B is _____ Ω.

[4 marks]



Solution:

Concept:

- All points joined by an ideal wire have the same electric potential. A resistor connected between the same node at both ends has zero voltage across it and carries no current.
- Two resistors sharing both endpoint nodes are in parallel: $1/R_p = 1/R_1 + 1/R_2$. Resistors traversed consecutively by the same total current are in series.

Step 1: Identify the wire that shorts the middle branch Label the junction after the upper $3\,\Omega$ resistor as C and the junction after the lower $6\,\Omega$ resistor as D. The two solid dots beside the right-hand $2\,\Omega$ resistor are joined by a direct vertical wire. Since the bottom horizontal wire is also connected to D, the upper dot and lower dot are both at the potential of D. The right-hand vertical $2\,\Omega$ resistor is therefore bypassed, rather than being in series with the upper resistor.

Step 2: Remove branches with zero potential difference The two right-hand $4\,\Omega$ resistors form a loop whose two ends both connect to D. With no source within that loop there is no current through it. They do not contribute to resistance between A and B. Between C and D there remain two $2\,\Omega$

resistors: the left vertical one and the upper horizontal one. Both share the same pair of nodes, so

$$R_{CD} = \frac{2 \times 2}{2 + 2} = 1\,\Omega.$$

Step 3: Add the surviving series resistances Current entering A passes through $3\,\Omega$, divides between the two equal branches from C to D, recombines at D, and leaves through $6\,\Omega$ to B. The three equivalent sections are in series:

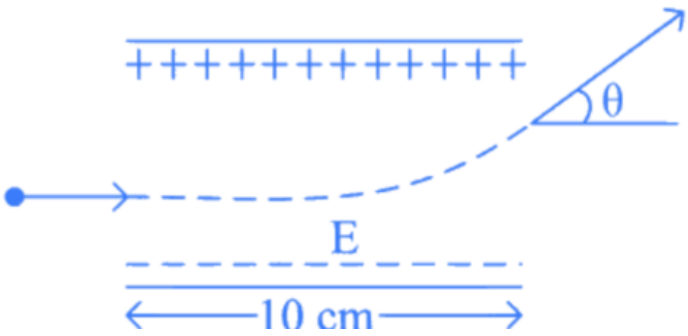
$$R_{AB} = 3 + R_{CD} + 6 = 3 + 1 + 6 = 10\,\Omega.$$

A check is that the middle network must be smaller than either of its two parallel $2\,\Omega$ branches, which the value $1\,\Omega$ satisfies.

Step 4: Conclusion The equivalent resistance is $\boxed{R_{AB} = 10\,\Omega}$.

Quick Tip to Solve: Mark every continuous ideal wire as one node before combining resistors; the vertical short removes the middle vertical resistor and the entire right-hand loop.

60. A uniform electric field of 10 N/C is created between two parallel charged plates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy 0.5 eV. The length of each plate is 10 cm. The angle (θ) of deviation of the path of electron as it comes out of the field is _____ (in degree). [4 marks]



Solution:

Concept:

- The electric force on an electron has magnitude eE and points opposite to the field. Its transverse acceleration is $a_y = eE/m$, where e is the magnitude of the electron charge.
- The horizontal velocity stays constant because the electric field is vertical. The exit direction satisfies $\tan\theta = v_y/v_x$, while the initial kinetic energy is $K = mv_x^2/2$.

Step 1: Separate the horizontal and vertical motion The electron enters horizontally between the plates, so its initial vertical velocity is zero. The upper plate is positive and the field points downward; because the electron is negatively charged, it accelerates upward toward that plate. There is no horizontal force, so v_x remains equal to its initial speed throughout the passage. For a plate length l,

the time spent in the field is therefore

$$t = \frac{l}{v_x}.$$

Step 2: Find the velocity ratio at the exit During this time the electron acquires transverse speed

$$v_y = \frac{eE}{m}\,t = \frac{eEl}{mv_x}.$$

The exit angle is determined by the tangent to the trajectory, hence by the velocity components, not by displacement components. Thus

$$\tan\theta = \frac{v_y}{v_x} = \frac{eEl}{mv_x^2} = \frac{eEl}{2K}.$$

The electron mass cancels on using the given kinetic energy, so no numerical mass or speed calculation is needed.

Step 3: Substitute consistent energy units Here $E = 10\,\mathrm{N\,C^{-1}}$ and $l = 0.10\,\mathrm{m}$, giving $El = 1.0\,\mathrm{V}$. Thus $eEl = 1.0\,\mathrm{eV}$, while the initial energy is $K = 0.5\,\mathrm{eV}$. Substitution gives

$$\tan\theta = \frac{1.0\,\mathrm{eV}}{2(0.5\,\mathrm{eV})} = 1.$$

Therefore $\theta = 45^\circ$. The sign of the electron charge determines the upward direction, while the requested magnitude is positive.

Step 4: Conclusion The electron leaves at an upward angle $\boxed{\theta = 45^\circ}$ to its original direction; the numerical entry in degrees is 45.

Quick Tip to Solve: Use the exit velocity ratio, not the slope of the line joining entry and exit: $\tan\theta = eEl/(2K)$. Expressing both energies in eV cancels the charge.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Inert gases have positive electron gain enthalpy. Its correct order is : [4 marks]

(A) He $<$ Ne $<$ Kr $<$ Xe

(B) He $<$ Xe $<$ Kr $<$ Ne

(C) Xe $<$ Kr $<$ Ne $<$ He

(D) He $<$ Kr $<$ Xe $<$ Ne

Correct Answer: (B) He $<$ Xe $<$ Kr $<$ Ne

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change for adding an electron to an isolated gaseous atom: $\mathrm{X(g) + e^- \longrightarrow X^-(g)}$. A positive value means this addition requires energy.
- Noble gases have closed valence shells. An incoming electron has to occupy a higher principal

shell, so the trend cannot be inferred merely by applying the usual electron-affinity trend of the halogens.

Step 1: Recognise why all the values are positive Helium has the closed-shell configuration $1s^2$, and the other noble gases have filled ns^2np^6 valence shells. Adding an electron cannot complete the shell further; it must start occupation beyond the filled valence shell. The resulting gaseous anion is energetically unfavourable relative to the atom and a free electron. The question therefore compares positive endothermic changes, not the magnitudes of negative electron affinities.

Step 2: Use the established noble-gas values The approximate electron gain enthalpies, in $\mathrm{kJ\,mol^{-1}}$, are

$$\mathrm{He}: +48, \quad \mathrm{Ne}: +116,$$
$$\mathrm{Kr}: +96, \quad \mathrm{Xe}: +77.$$

Helium has the smallest positive value among these four. Neon has the largest: the incoming electron must enter its next shell in the presence of the particularly compact, stable closed shell. The values for xenon and krypton lie between those of helium and neon.

Step 3: Arrange the values and eliminate the alternatives Increasing positive enthalpy means increasing the numerical energy required for electron attachment. The values above give

$$48 < 77 < 96 < 116,$$

hence

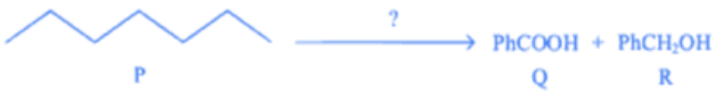
$$\mathrm{He} < \mathrm{Xe} < \mathrm{Kr} < \mathrm{Ne}.$$

Option (A) incorrectly places neon below krypton and xenon. Option (C) incorrectly makes helium the greatest. Option (D) exchanges xenon and krypton, even though $+77 < +96$. Only option (B) preserves all four comparisons.

Step 4: Conclusion The order is $\boxed{\mathrm{He} < \mathrm{Xe} < \mathrm{Kr} < \mathrm{Ne}}$. Hence, the correct option is (B).

Quick Tip to Solve: For noble-gas electron gain enthalpies, remember the positive values: helium is lowest and neon highest; xenon lies below krypton.

62. The correct sequence of reagents for the preparation of Q and R is : [4 marks]



(A) $(i)Cr_2O_3, 770K, 20\ atm; (ii)CrO_2Cl_2, H_3O^+; (iii)NaOH; (iv)H_3O^+$

(B) $(i)CrO_2Cl_2, H_3O^+; (ii)Cr_2O_3, 770K, 20\ atm; (iii)NaOH; (iv)H_3O^+$

(C) $(i)KMnO_4, OH^-; (ii)Mo_2O_3, \Delta; (iii)NaOH; (iv)H_3O^+$

(D) $(i)Mo_2O_3, \Delta; (ii)CrO_2Cl_2, H_3O^+; (iii)NaOH; (iv)H_3O^+$

Correct Answer: (A) $(i)Cr_2O_3, 770K, 20\ atm; (ii)CrO_2Cl_2, H_3O^+; (iii)NaOH; (iv)H_3O^+$

Solution:

Concept:

- An unbranched seven-carbon alkane undergoes dehydrocyclisation over Cr_2O_3 at high temperature and pressure to give toluene.
- Etard oxidation converts the benzylic methyl group of toluene into an aldehyde. Benzaldehyde has no alpha hydrogen and undergoes the Cannizzaro disproportionation in concentrated alkali.

Step 1: Convert the alkane into the aromatic precursor The skeletal structure P contains seven carbon atoms, so it represents n-heptane. To obtain the aromatic products Q and R, first form the benzene ring while retaining the seventh carbon as a methyl substituent. The specified aromatisation reagent is Cr_2O_3 at $770\,K$ and $20\,atm$:

$$C_7H_{16} \longrightarrow C_6H_5CH_3 + 4\,H_2.$$

The product is toluene. Oxidising a side chain before producing this aromatic intermediate is not the required sequence.

Step 2: Stop the oxidation at benzaldehyde Toluene is treated with chromyl chloride, CrO_2Cl_2, followed by acidic hydrolysis. This is the Etard reaction, which converts its methyl group into $-CHO$, giving benzaldehyde, C_6H_5CHO. The aldehyde stage is essential because the desired final mixture contains both benzoic acid and benzyl alcohol. Strong oxidation directly to the carboxylic acid would not provide a precursor that disproportionates to those two products.

Step 3: Disproportionate the aldehyde and identify the sequence Benzaldehyde lacks an alpha hydrogen, so concentrated $NaOH$ gives the Cannizzaro reaction:

$$2\,C_6H_5CHO + NaOH \longrightarrow C_6H_5COONa + C_6H_5CH_2OH.$$

Acidification with H_3O^+ converts benzoate into benzoic acid Q, while benzyl alcohol is R. Hence the steps are aromatisation, Etard oxidation/hydrolysis, alkali, then acidification. Option (B) reverses the first two stages. Option (C) starts with permanganate oxidation, and option (D) uses Mo_2O_3 instead of the specified aromatisation catalyst; neither gives the required standard route.

Step 4: Conclusion The sequence is (i) Cr_2O_3, $770\,K$, $20\,atm$; (ii) CrO_2Cl_2, H_3O^+; (iii) $NaOH$; (iv) H_3O^+. Hence, the correct option is (A).

Quick Tip to Solve: Benzoic acid and benzyl alcohol together suggest Cannizzaro of benzaldehyde; work backward to Etard oxidation of toluene and then aromatisation of n-heptane.

63. In the cumene to phenol preparation in presence of air, the intermediate is [4 marks]

(A) $C_6H_5CH(CH_3)_2$ (cumene)

(B) $C_6H_5OCH(CH_3)_2$ (isopropyl phenyl ether)

(C) $C_6H_5OCOCH_3$ (phenyl ethanoate)

(D) $C_6H_5C(CH_3)_2OOH$ (cumene hydroperoxide)

Correct Answer: (D) $C_6H_5C(CH_3)_2OOH$ (cumene hydroperoxide)

Solution:

Concept:

- Cumene is isopropylbenzene, $C_6H_5CH(CH_3)_2$. Oxidation by air introduces a hydroperoxide group at its benzylic carbon.
- A hydroperoxide contains the $-O-O-H$ linkage. Acid-catalysed cleavage of cumene hydroperoxide produces phenol and acetone in the industrial cumene process.

Step 1: Locate the site of oxidation in cumene The benzylic carbon in cumene is bonded to a phenyl group, two methyl groups, and one hydrogen. Removal of this benzylic hydrogen in the radical oxidation sequence gives a resonance-stabilised benzylic radical. Oxygen from air is trapped at that carbon; subsequent hydrogen transfer gives a hydroperoxide. The two methyl groups and the carbon-phenyl bond remain present at this stage. Thus the intermediate is not obtained by inserting an oxygen directly between the phenyl ring and an isopropyl group.

Step 2: Write the hydroperoxide and its cleavage products The oxidation stage can be represented by

$$C_6H_5CH(CH_3)_2 + O_2 \longrightarrow C_6H_5C(CH_3)_2OOH.$$

The product has an $-OOH$ substituent on the original benzylic carbon. On acid-catalysed rearrangement and cleavage it yields

$$C_6H_5C(CH_3)_2OOH \longrightarrow C_6H_5OH + CH_3COCH_3.$$

This explains both the phenol product and the acetone coproduct, and distinguishes the oxidation intermediate from the final products.

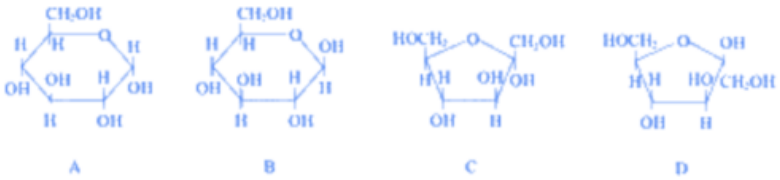
Step 3: Compare the four printed structures Option (A) is unchanged cumene and has no oxygen, so it cannot be the oxidation intermediate. Option (B) is isopropyl phenyl ether and has only one oxygen atom; it lacks the characteristic peroxide bond. Option (C) is phenyl ethanoate, an ester containing a carbonyl group, which is not formed in this initial air-oxidation stage. Option (D) retains the phenyl-bearing carbon and both methyl groups while attaching $-O-O-H$ to that carbon, exactly as required for cumene hydroperoxide.

Step 4: Conclusion The intermediate is cumene hydroperoxide, $\boxed{C_6H_5C(CH_3)_2OOH}$. Hence, the correct option is (D).

Quick Tip to Solve: In the cumene process, look for an O–O bond at the benzylic carbon: air first gives the hydroperoxide, and acid cleavage then gives phenol plus acetone.

64. Match items of Row I with those of Row II. [4 marks]

Row I :



Row II :

(i) α-D-(−)-Fructofuranose

(ii) β-D-(−)-Fructofuranose

(iii) α-D-(−) Glucopyranose

(iv) β-D-(−)-Glucopyranose

Correct match is

(A) A → iii, B → iv, C → i, D → ii

(B) A → iv, B → iii, C → i, D → ii

(C) A → i, B → ii, C → iii, D → iv

(D) A → iii, B → iv, C → ii, D → i

Correct Answer: (A) A → iii, B → iv, C → i, D → ii

Solution:

Concept:

- A pyranose has a six-membered ring, whereas a furanose has a five-membered ring. The ring oxygen counts as one member.
- For a D-sugar in the usual Haworth orientation, the anomeric hydroxyl is alpha when it is trans to the reference terminal CH_2OH group and beta when it is cis.

Step 1: Identify the ring sizes Structures A and B each contain a six-membered ring with one oxygen, so they are glucopyranoses. Structures C and D have five-membered rings and two hydroxymethyl substituents, so they are fructofuranoses. This already excludes option (C), which interchanges the glucose and fructose ring types. The anomeric carbon is C1 in glucose but C2 in fructose; use that carbon when comparing the hydroxyl orientation.

Step 2: Assign alpha and beta at the anomeric carbon In A the anomeric hydroxyl is down while the reference hydroxymethyl group is up: A is alpha-D-glucopyranose and matches (iii). In B the anomeric hydroxyl is up, so B is beta-D-glucopyranose and matches (iv). In C the anomeric hydroxyl is down relative to the reference group, giving alpha-D-fructofuranose, (i). In D that hydroxyl is up, giving beta-D-fructofuranose, (ii). These assignments concern relative configuration; the optical-rotation

sign does not decide alpha versus beta.

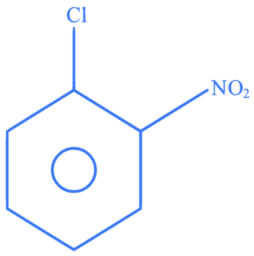
Step 3: Check the complete matching The mapping is $A \to (iii),\ B \to (iv),\ C \to (i),\ D \to (ii)$. Option (B) reverses the two glucopyranose anomers, and option (D) reverses the two fructofuranose anomers. Neither respects the hydroxyl orientations in the structures.

Step 4: Conclusion The correct matching is $A \to iii,\ B \to iv,\ C \to i,\ D \to ii$. Hence, the correct option is (A).

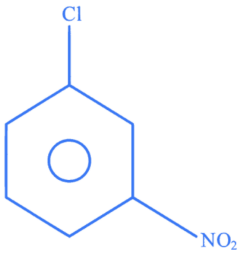
Quick Tip to Solve: First count ring members, then inspect only the anomeric hydroxyl relative to the reference hydroxymethyl group; do not use the optical-rotation sign to identify anomers.

65. The compound which will have the lowest rate towards nucleophilic aromatic substitution on treatment with OH^- is [4 marks]

(A) 1-Chloro-2-nitrobenzene



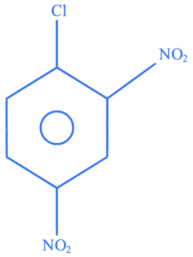
(B) 1-Chloro-3-nitrobenzene



(C) 1-Chloro-4-nitrobenzene

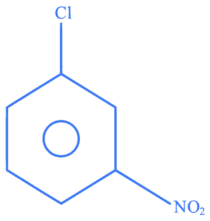


(D) 1-Chloro-2,4-dinitrobenzene



Correct Answer: (B)

1-Chloro-3-nitrobenzene



Solution:

Concept:

- In addition-elimination nucleophilic aromatic substitution, OH^- adds at the carbon bearing chlorine and forms an anionic sigma complex.
- An electron-withdrawing nitro group stabilises this intermediate strongly by resonance only when it is ortho or para to the leaving group.

Step 1: Locate the nitro groups Option (A) is ortho-nitrochlorobenzene, (B) is meta-nitrochlorobenzene, (C) is para-nitrochlorobenzene and (D) has nitro groups at both the ortho and para positions. All four have the same leaving group, chlorine, so the decisive comparison is the position and number of the electron-withdrawing groups rather than a change of leaving-group identity.

Step 2: Compare stabilisation of the intermediate Attack of OH^- produces resonance contributors placing negative charge at positions ortho and para to the attacked carbon. In (A) and (C), this charge can be delocalised further into a nitro group. In (D), two suitably placed nitro groups stabilise the intermediate, strongly activating substitution. In (B), the nitro group is meta and cannot receive the negative charge through these resonance contributors. Its inductive withdrawal alone gives much less activation.

Step 3: Select the slowest compound The meta compound therefore has the least stabilised addition intermediate and the highest barrier among these choices. Options (A), (C) and (D) are rejected because each has at least one resonance-activating nitro group ortho or para to chlorine. This is the nucleophilic substitution trend; the familiar directing rules for electrophilic substitution must not be used in its place.

Step 4: Conclusion 1-Chloro-3-nitrobenzene has the lowest substitution rate towards OH^-. Hence, the correct option is (B).

Quick Tip to Solve: For nucleophilic aromatic substitution, circle nitro groups ortho or para to the leaving group: a meta nitro group lacks the crucial resonance activation.

66. Match List I with List II

	List IElements		List IIColour imparted to the flame
A.	K	I.	Brick Red
B.	Ca	II.	Violet
C.	Sr	III.	Apple Green
D.	Ba	IV.	Crimson Red

Choose the correct answer from the options given below : [4 marks]

(A) A-II, B-IV, C-I, D-III

(B) A-II, B-I, C-IV, D-III

(C) A-II, B-I, C-III, D-IV

(D) A-IV, B-III, C-II, D-I

Correct Answer: (B) A-II, B-I, C-IV, D-III

Solution:

Concept:

- Heating a suitable salt in a flame excites electrons in the metal species. Their return to lower energy levels produces characteristic visible emission.
- Use the standard flame-test colours: potassium violet, calcium brick red, strontium crimson red and barium apple green.

Step 1: Match the first two elements Potassium, listed as A, gives a violet or lilac flame, so it matches entry II. Calcium, listed as B, gives a brick-red flame, so it matches entry I. The violet flame of potassium may be masked by traces of sodium; cobalt glass can help observe it, but this does not alter the characteristic colour being matched here. These are emission colours, not the colours of aqueous ions or precipitates.

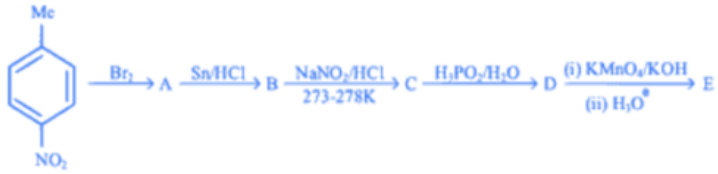
Step 2: Match the remaining elements Strontium, listed as C, gives a crimson-red flame and therefore matches entry IV. Barium, listed as D, gives an apple-green flame and matches entry III. Calcium and strontium both give reddish flames, but their standard descriptions are distinct: calcium is brick red and strontium is crimson red. Barium is the green member in this list.

Step 3: Eliminate inconsistent codes Combining all four gives $A - II,\ B - I,\ C - IV,\ D - III$. Option (A) interchanges the calcium and strontium reds. Option (C) assigns apple green to strontium and crimson red to barium. Option (D) fails even the potassium and calcium matches. Thus only the second code satisfies every entry of both lists.

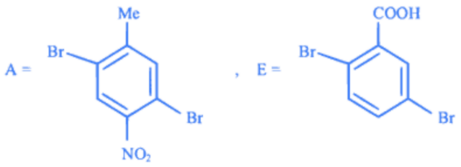
Step 4: Conclusion The matching is $A - II,\ B - I,\ C - IV,\ D - III$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the two red flame tests separate: calcium is brick red, strontium is crimson red; barium is apple green and potassium is violet.

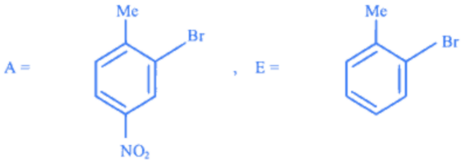
67. Identify the product formed (A and E) [4 marks]



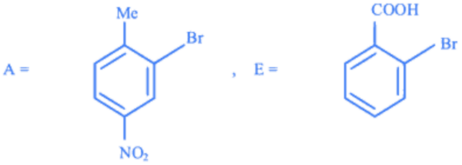
(A) A: 2,5-dibromo-4-nitrotoluene; E: 2,5-dibromobenzoic acid



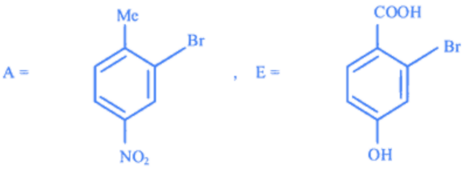
(B) A: 2-bromo-4-nitrotoluene; E: 2-bromotoluene



(C) A: 2-bromo-4-nitrotoluene; E: 2-bromobenzoic acid

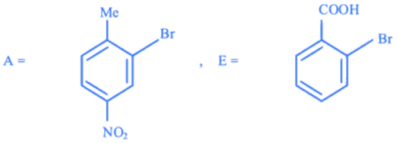


(D) A: 2-bromo-4-nitrotoluene; E: 2-bromo-4-hydroxybenzoic acid



Correct Answer: (C)

A: 2-bromo-4-nitrotoluene; E: 2-bromobenzoic acid



Solution:

Concept:

- A methyl group directs electrophilic substitution ortho/para, while a nitro group directs it meta. Their directing effects can reinforce one another.
- Sn/HCl reduces nitro to amino; diazotisation followed by H_3PO_2 replaces the amino-derived diazonium group with hydrogen. Alkaline permanganate oxidises a benzylic methyl group to carboxyl after acidification.

Step 1: Identify the bromination product A The starting compound is para-nitrotoluene. Number the methyl-bearing carbon as C1 and the nitro-bearing carbon as C4. Both substituents favour bromination at C2 or the symmetry-equivalent C6: these positions are ortho to methyl and meta to nitro. Thus A is 2-bromo-4-nitrotoluene, the monobrominated product shown in options (B), (C) and (D), rather than the dibrominated structure in (A).

Step 2: Follow reduction and removal of nitrogen Tin and hydrochloric acid convert the nitro group into an amino group, giving 2-bromo-4-aminotoluene B. With $\mathrm{NaNO_2/HCl}$ at 273–278 K, the amino group becomes the diazonium group C. Hypophosphorous acid then replaces that diazonium group by hydrogen. Hence D is 2-bromotoluene. This step is reductive deamination, not hydrolysis to a phenol.

Step 3: Oxidise the side chain and select E Alkaline $\mathrm{KMnO_4}$, followed by $\mathrm{H_3O^+}$, oxidises the methyl side chain to COOH without moving the bromine. E is consequently 2-bromobenzoic acid. Option (B) incorrectly stops at bromotoluene, and (D) incorrectly retains an oxygen substituent where the diazonium group was removed. Only (C) has both the correct A and final E.

Step 4: Conclusion A is 2-bromo-4-nitrotoluene and E is 2-bromobenzoic acid. Hence, the correct option is (C).

Quick Tip to Solve: Mark the nitro carbon once: after reduction, diazotisation and hypophosphorous acid it carries hydrogen; only the methyl side chain becomes carboxyl in the final oxidation.

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68. '25 volume' hydrogen peroxide means [4 marks]

(A) 1 L marketed solution contains 75 g of H_2O_2.

(B) 1 L marketed solution contains 250 g of H_2O_2.

(C) 1 L marketed solution contains 25 g of H_2O_2.

(D) 100 mL marketed solution contains 25 g of H_2O_2.

Correct Answer: (A) 1 L marketed solution contains 75 g of H_2O_2.

Solution:

Concept:

- Volume strength is the volume of oxygen at STP liberated by one volume of peroxide solution on complete decomposition.
- The stoichiometry is $2\,H_2O_2 \longrightarrow 2\,H_2O + O_2$. At STP, one mole of oxygen occupies approximately $22.4\,L$.

Step 1: Translate the volume strength into oxygen A 25-volume solution gives 25 volumes of oxygen from one volume of the solution. Choose $1\,L$ of marketed solution as the basis. It must release $25\,L$ of oxygen at STP, not contain 25 grams of peroxide. The corresponding oxygen amount is

$$n(O_2) = \frac{25}{22.4}\,\text{mol}.$$

This definition relates a gas volume to a solution volume.

Step 2: Convert oxygen into peroxide mass Two moles of peroxide produce one mole of oxygen, and

the molar mass of peroxide is $34\,\mathrm{g\,mol^{-1}}$. Therefore

$$m(\mathrm{H_2O_2}) = 2 \times \frac{25}{22.4} \times 34 = 75.9\,\mathrm{g}$$

in one litre of solution. Equivalently, strength in grams per litre is $25 \times 68/22.4$, approximately $76\,\mathrm{g\,L^{-1}}$. The option using 75 g is the intended approximate representation.

Step 3: Compare the listed concentrations Options (B) and (D) each imply $250\,\mathrm{g\,L^{-1}}$, which would give about 82 volumes, not 25. Option (C) gives $25\,\mathrm{g\,L^{-1}}$, corresponding to only about 8.2 volumes. The first concentration alone is close to the required value; the small difference between 75 and 75.9 reflects the approximation in the choices.

Step 4: Conclusion A litre of 25-volume hydrogen peroxide contains approximately $76\,\mathrm{g}$ of $\mathrm{H_2O_2}$, represented by 75 g in the choices. Hence, the correct option is (A).

Quick Tip to Solve: Multiply volume strength by $68/22.4$ to obtain peroxide concentration in grams per litre; volume strength itself is not a mass concentration.

69. The radius of the 2^{nd} orbit of $\mathrm{Li^{2+}}$ is x. The expected radius of the 3^{rd} orbit of $\mathrm{Be^{3+}}$ is [4 marks]

(A) $\frac{16}{27}x$
(B) $\frac{4}{9}x$
(C) $\frac{9}{4}x$
(D) $\frac{27}{16}x$

Correct Answer: (D) $\frac{27}{16}x$

Solution:

Concept:

- For a one-electron ion, the Bohr radius of orbit n is $r_n = a_0 n^2/Z$, where Z is nuclear charge and a_0 is the hydrogen ground-state radius.
- Both $\mathrm{Li^{2+}}$ and $\mathrm{Be^{3+}}$ contain one electron, so the same hydrogen-like formula applies to each.

Step 1: Express the known radius Lithium has nuclear charge $Z = 3$. Its doubly charged ion has one remaining electron, and the specified second orbit has $n = 2$. Thus

$$x = r_2(\mathrm{Li^{2+}}) = a_0 \frac{2^2}{3} = \frac{4a_0}{3}.$$

The ionic charge is not the value of Z: the nucleus still contains three protons. Rearranging gives $a_0 = 3x/4$.

Step 2: Calculate the requested radius Beryllium has nuclear charge $Z = 4$. Its triply charged ion is also hydrogen-like. For its third orbit,

$$r_3(\mathrm{Be^{3+}}) = a_0 \frac{3^2}{4} = \frac{9a_0}{4}.$$

Substituting the expression obtained from the lithium ion gives

$$r_3 = \frac{9}{4} \times \frac{3x}{4} = \frac{27x}{16}.$$

The result is a length because the multiplying factor is dimensionless.

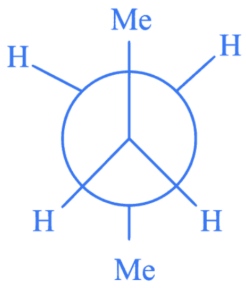
Step 3: Check the ratio and reject alternatives The radius rises with n^2, but the larger beryllium nuclear charge partly offsets this increase. The net ratio is $(9/4)(3/4) = 27/16$. Option (A) is the reciprocal of the required ratio. Option (C) includes only the orbit-number factor and ignores the nuclear-charge change. Option (B) also fails the combined n^2/Z scaling; neither ion can be treated as neutral hydrogen.

Step 4: Conclusion The radius of the third orbit of Be^{3+} is $\frac{27}{16}x$. Hence, the correct option is (D).

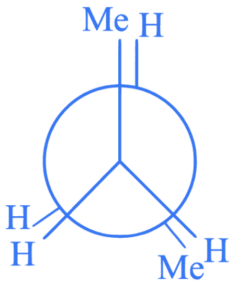
Quick Tip to Solve: For two hydrogen-like ions, take the ratio $r_2/r_1 = (n_2/n_1)^2(Z_1/Z_2)$; the common Bohr constant cancels immediately. Use nuclear charges, not the net ionic charges.

70. Which of the following conformations will be the most stable? [4 marks]

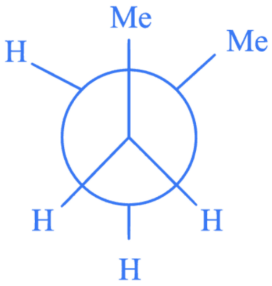
(A) Anti staggered conformation of butane



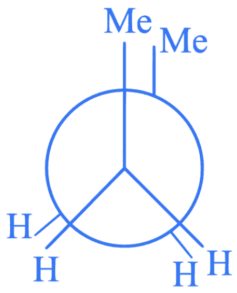
(B) Eclipsed conformation with methyl-hydrogen eclipsing pairs



(C) Gauche staggered conformation of butane

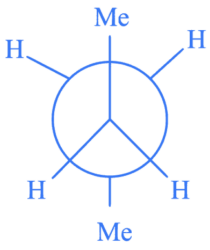


(D) Fully eclipsed conformation with the methyl groups eclipsing



Correct Answer: (A)

Anti staggered conformation of butane



Solution:

Concept:

- Rotation about the central carbon-carbon single bond of butane produces conformations with different torsional and steric energies.
- Staggered conformations minimise eclipsing interactions. Among staggered forms, separating the two methyl groups by 180° gives the anti form and minimises their steric repulsion.

Step 1: Read the Newman projections The central junction represents the front carbon and the circle represents the rear carbon. In (A), one methyl group is upward on the front carbon while the other is downward on the rear carbon. Their dihedral angle is 180°, and all front bonds lie between rear bonds. Thus (A) is the anti staggered conformation. In (C), the methyl groups are separated by only 60°, giving the gauche staggered form.

Step 2: Compare torsional and steric interactions Both staggered drawings avoid direct bond eclipsing, but the methyl groups are bulkier than hydrogen. Their closer approach in the gauche form causes a steric interaction absent in the anti arrangement. Consequently anti has lower energy than gauche. Option (B) is an eclipsed arrangement with methyl-hydrogen eclipsing interactions, so it has additional torsional strain. Option (D) brings the two methyl groups into an eclipsed arrangement, combining large steric repulsion with eclipsing strain.

Step 3: Select the lowest-energy form The most stable conformation must minimise both sources of strain, rather than merely maximise the apparent separation of hydrogen labels on the drawing. The anti staggered projection in (A) does this: the bulky methyl substituents are farthest apart while the carbon-hydrogen bonds remain staggered. The gauche and both eclipsed alternatives therefore lie higher in energy.

Step 4: Conclusion The anti staggered conformation shown in (A) is the most stable. Hence, the correct option is (A).

Quick Tip to Solve: In a butane Newman projection, find the two methyl groups first: staggered with 180° separation is anti and is lower in energy than gauche or eclipsed forms.

71. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Acetal / Ketal is stable in basic medium.

Reason R : The high leaving tendency of alkoxide ion gives the stability to acetal/ketal in basic medium.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) A is false but R is true.

(B) Both A and R are true but R is NOT the correct explanation of A

(C) Both A and R are true and R is the correct explanation of A

(D) A is true but R is false.

Correct Answer: (D) A is true but R is false.

Solution:

Concept:

- An acetal or ketal has two alkoxy groups on the same carbon. Hydrolysis converts it back into a carbonyl compound and alcohol.
- Acid catalysis activates an alkoxy group by protonation, allowing neutral alcohol to leave. An unprotonated alkoxide, RO^-, is a poor leaving group.

Step 1: Evaluate the assertion Acetals and ketals are stable towards aqueous base under the usual hydrolysis conditions. In basic medium neither alkoxy oxygen is activated by protonation, so cleavage would require expulsion of an alkoxide ion. This unfavourable departure prevents the hydrolysis pathway that operates readily in aqueous acid. Thus Assertion A is true.

Step 2: Evaluate the reason The reason claims that the alkoxide ion has a high leaving tendency. Alkoxide is a strong base and is therefore a poor leaving group. If it left readily, carbon–oxygen cleavage would be easier and the acetal would be less stable, contrary to the assertion. The actual stability is associated with the low leaving tendency of the unprotonated alkoxy group. Thus Reason R is false.

Step 3: Select the truth combination Option (A) reverses both truth values, while options (B) and (C) both require a true reason. Neither can hold because the stated high leaving tendency is incorrect. The combination true assertion and false reason occurs only in option (D).

Step 4: Conclusion A is true but R is false: acetal/ketal is stable in basic medium because alkoxide is a poor leaving group. Hence, the correct option is (D).

Quick Tip to Solve: For acetal hydrolysis, first ask whether an alkoxy oxygen can be protonated; a neutral alcohol leaves much more readily than an alkoxide ion.

72. The correct order in aqueous medium of basic strength in case of methyl substituted amines is : [4 marks]

(A) $NH_3 > Me_3N > MeNH_2 > Me_2NH$

(B) $Me_3N > Me_2NH > MeNH_2 > NH_3$

(C) $Me_2NH > MeNH_2 > Me_3N > NH_3$

(D) $Me_2NH > Me_3N > MeNH_2 > NH_3$

Correct Answer: (C) $Me_2NH > MeNH_2 > Me_3N > NH_3$

Solution:

Concept:

- The basicity of an amine in water depends on electron donation to nitrogen and on hydration of both the base and its conjugate acid.
- Methyl groups exert a positive inductive effect, whereas crowding around nitrogen reduces solvation of the protonated ion. These effects must be considered together in aqueous solution.

Step 1: Compare methyl substitution with ammonia Methyl groups push electron density towards nitrogen and favour acceptance of a proton. Consequently, all three methyl-substituted amines are stronger aqueous bases than NH_3. This already rejects option (A), which places ammonia first. The gas-phase order based chiefly on inductive donation cannot simply be carried over to water.

Step 2: Balance inductive donation and hydration Dimethylamine combines the electron donation of two methyl groups with effective hydration of its conjugate acid, $(CH_3)_2NH_2^+$. It is therefore strongest in this series. Methylammonium ions are hydrated more effectively than the more crowded trimethylammonium ion, so methylamine is stronger than trimethylamine in water despite having fewer electron-donating methyl groups.

Step 3: Arrange the four bases The net aqueous order is

$$(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N > NH_3.$$

Option (B) incorrectly makes trimethylamine strongest. Option (D) places trimethylamine above methylamine and neglects the decisive hydration difference. Only option (C) includes both comparisons correctly.

Step 4: Conclusion The aqueous basic-strength order is $Me_2NH > MeNH_2 > Me_3N > NH_3$. Hence, the correct option is (C).

Quick Tip to Solve: For methylamines in water remember secondary, primary, tertiary, ammonia; do not apply the gas-phase inductive-effect order.

73. Match the List-I with List-II :

	List-I Cations		List-II Group reagents
A.	Pb^{2+}, Cu^{2+}	**i)**	H_2S **gas in presence of dilute HCl**
B.	Al^{3+}, Fe^{3+}	**ii)**	$(NH_4)_2CO_3$ **in presence of** NH_4OH
C.	Co^{2+}, Ni^{2+}	**iii)**	NH_4OH **in presence of** NH_4Cl
D.	Ba^{2+}, Ca^{2+}	**iv)**	**H_2S in presence of NH_4OH**

Correct match is - [4 marks]

(A) A → i, B → iii, C → iv, D → ii

(B) A → i, B → iii, C → ii, D → iv

(C) A → iii, B → i, C → iv, D → ii

(D) A → iv, B → ii, C → iii, D → i

Correct Answer: (A) A → i, B → iii, C → iv, D → ii

Solution:

Concept:

- Cation group separation uses controlled concentrations of sulfide, hydroxide and carbonate ions to precipitate salts selectively.
- Acid suppresses ionisation of H_2S, whereas ammoniacal medium raises the sulfide concentration. NH_4Cl controls the hydroxide concentration through the common-ion effect.

Step 1: Match the acid-sulfide and hydroxide groups Pb^{2+} and Cu^{2+} form very insoluble sulfides with H_2S even in dilute hydrochloric acid, so A matches (i). Lead can occur in this group if it has not been completely removed as chloride. Al^{3+} and Fe^{3+} are precipitated as hydroxides by NH_4OH in the presence of NH_4Cl, so B matches (iii).

Step 2: Match the alkaline-sulfide and carbonate groups Co^{2+} and Ni^{2+} need the higher sulfide concentration available from H_2S in ammoniacal medium; hence C matches (iv). Ba^{2+} and Ca^{2+} are precipitated as carbonates by ammonium carbonate in ammoniacal medium, so D matches (ii).

Step 3: Check the option mappings Together the matches are $A \to i,\ B \to iii,\ C \to iv,\ D \to ii$. Option (B) interchanges the nickel/cobalt sulfide group and the barium/calcium carbonate group. Option (C) interchanges the first two reagents. Option (D) assigns all four groups incorrectly. Option (A) preserves every selective precipitation condition.

Step 4: Conclusion The correct matching is $A \to i,\ B \to iii,\ C \to iv,\ D \to ii$. Hence, the correct option is (A).

Quick Tip to Solve: Use the sequence acidic sulfide, buffered hydroxide, alkaline sulfide, carbonate to identify these four cation groups quickly.

74. Reaction of thionyl chloride with white phosphorus forms a compound [A], which on hydrolysis gives [B], a dibasic acid. [A] and [B] are respectively : [4 marks]

(A) P_4O_6 and H_3PO_3

(B) $POCl_3$ and H_3PO_4

(C) PCl_5 and H_3PO_4

(D) PCl_3 and H_3PO_3

Correct Answer: (D) PCl_3 and H_3PO_3

Solution:

Concept:

- White phosphorus reacts with thionyl chloride to give phosphorus trichloride. Hydrolysis of phosphorus trichloride produces phosphorous acid.
- The basicity of an oxyacid counts ionisable oxygen-bound hydrogens. In phosphorous acid, H_3PO_3, one hydrogen is bonded directly to phosphorus.

Step 1: Identify the chloride produced The reaction gives phosphorus trichloride, PCl_3, rather than an oxide or phosphorus pentachloride. One balanced representation is

$$P_4 + 8\,SOCl_2 \longrightarrow 4\,PCl_3 + 4\,SO_2 + 2\,S_2Cl_2.$$

This identifies compound [A] independently of the subsequent acid-basicity clue. The phosphorus changes from its elemental state to the +3 oxidation state.

Step 2: Hydrolyse the chloride Water replaces the three chlorine atoms and forms phosphorous acid:

$$PCl_3 + 3\,H_2O \longrightarrow H_3PO_3 + 3\,HCl.$$

Thus compound [B] is H_3PO_3. Its structural description is $H - P(=O)(OH)_2$: only the two hydroxyl hydrogens are ionisable, while the phosphorus-bound hydrogen does not contribute to acid basicity.

Step 3: Use both clues to reject alternatives Option (A) has the required acid but gives P_4O_6 as [A], contrary to the thionyl-chloride reaction. Options (B) and (C) give phosphoric acid, H_3PO_4, which has three hydroxyl groups and is tribasic. Only option (D) gives both the correct chloride and the dibasic hydrolysis product.

Step 4: Conclusion [A] is PCl_3 and [B] is the dibasic acid H_3PO_3. Hence, the correct option is (D).

Quick Tip to Solve: For phosphorus oxyacids count P–OH groups, not total hydrogens: the P–H hydrogen of phosphorous acid is not ionisable.

75. Which one of the following reactions does not occur during extraction of copper? [4 marks]

(A) $2FeS + 3O_2 \rightarrow 2FeO + 2SO_2$

(B) $FeO + SiO_2 \rightarrow FeSiO_3$

(C) $CaO + SiO_2 \rightarrow CaSiO_3$

(D) $2Cu_2s + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

Correct Answer: (C) $CaO + SiO_2 \rightarrow CaSiO_3$

Solution:

Concept:

- During extraction of copper from sulfide ores, roasting and conversion oxidise iron sulfide and part of the copper sulfide.
- Silica is added as an acidic flux to combine with basic iron(II) oxide and remove it as iron silicate slag. The identity of the impurity determines the slag reaction.

Step 1: Identify oxidation reactions in the process Iron sulfide is oxidised to iron(II) oxide, releasing sulfur dioxide:

$$2\,FeS + 3\,O_2 \longrightarrow 2\,FeO + 2\,SO_2.$$

Part of copper(I) sulfide is also oxidised to copper(I) oxide:

$$2\,Cu_2S + 3\,O_2 \longrightarrow 2\,Cu_2O + 2\,SO_2.$$

Thus the reactions represented by options (A) and (D) occur. The lowercase letter in the displayed sulfide formula does not describe a different chemical species.

Step 2: Identify the slag-forming reaction The iron(II) oxide formed is removed by the silica flux:

$$FeO + SiO_2 \longrightarrow FeSiO_3.$$

This is the slag-forming reaction of option (B). The operation removes iron while retaining copper-containing material for subsequent self-reduction, in which copper(I) oxide reacts with remaining copper(I) sulfide to yield copper.

Step 3: Locate the reaction from another flux system $CaO + SiO_2 \longrightarrow CaSiO_3$ forms calcium silicate slag when lime is used to remove a silica impurity, as in iron extraction. In the stated copper process the basic impurity is iron oxide and the supplied flux is silica; calcium oxide is not the flux involved. Hence option (C) is the reaction absent from this extraction sequence.

Step 4: Conclusion The reaction $CaO + SiO_2 \longrightarrow CaSiO_3$ does not occur in the described extraction of copper. Hence, the correct option is (C).

Quick Tip to Solve: For copper sulfide metallurgy remember silica removes FeO as iron silicate; calcium silicate slag belongs to a different impurity–flux combination.

76. Which of the following statements is incorrect for antibiotics? [4 marks]

(A) An antibiotic should promote the growth or survival of microorganisms.

(B) An antibiotic must be a product of metabolism.

(C) An antibiotic should be effective in low concentrations.

(D) An antibiotic is a synthetic substance produced as a structural analogue of naturally occurring antibiotic.

Correct Answer: (A) An antibiotic should promote the growth or survival of microorganisms.

Solution:

Concept:

- Antibiotics act at low concentration against microorganisms by inhibiting their growth or destroying them; they do not nourish the infectious organism.
- The term originally described metabolic products of microorganisms. Its broader medicinal usage also includes suitable synthetic structural analogues of naturally occurring antibiotics.

Step 1: Apply the biological-action criterion A substance used as an antibiotic must interfere with microbial growth or survival. Depending on its action, it may be bacteriostatic, suppressing multiplication, or bactericidal, killing susceptible organisms. Statement (A) says it should promote growth or survival, which is the opposite of this therapeutic purpose. It is therefore the incorrect statement sought.

Step 2: Check the concentration and origin statements Effectiveness at low concentration is a defining feature, so statement (C) is correct. Statement (B) expresses the classical origin-based description: antibiotics were identified as products of the metabolism of organisms, particularly certain fungi and bacteria. The word product distinguishes such biologically generated agents from unrelated substances that simply happen to kill microbes at very high concentration.

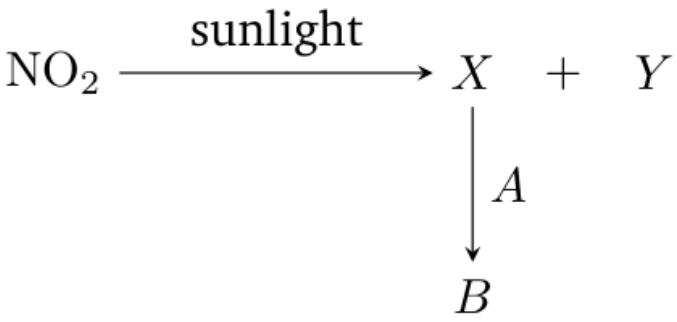
Step 3: Interpret the synthetic-analogue statement The expanded use of the term includes synthetic substances that are structural analogues of natural antibiotics and have the required antimicrobial action. Therefore statement (D) expresses this broader category. The origin descriptions in (B) and (D) refer to the classical class and its accepted extension; they do not make microbial growth promotion a valid property. The clear reversal of biological action singles out (A).

Step 4: Conclusion An antibiotic should inhibit growth or destroy susceptible microorganisms, so the growth-promoting statement is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: When asked for an incorrect antibiotic property, check the direction of its action first: inhibition or killing, never promotion of microbial survival.

77. Some reactions of NO_2 relevant to photochemical smog formation are

Identify A, B, X and Y : [4 marks]



(A) $X = [O], Y = NO, A = O_2, B = O_3$

(B) $X = N_2O, Y = [O], A = O_3, B = NO$

(C) $X = \frac{1}{2}O_2, Y = NO_2, A = O_3, B = O_2$

(D) $X = NO, Y = [O], A = O_2, B = N_2O_3$

Correct Answer: (A) $X = [O], Y = NO, A = O_2, B = O_3$

Solution:

Concept:

- Sunlight photodissociates nitrogen dioxide to nitrogen monoxide and reactive atomic oxygen. The atom then combines with molecular oxygen to form ozone.
- The reaction diagram makes X the species that reacts with A to form B. This connectivity fixes the assignments, rather than merely listing photolysis products in any order.

Step 1: Identify the photolysis products The first reaction is

$$NO_2 \xrightarrow{\text{sunlight}} NO + O.$$

The products are nitrogen monoxide and an oxygen atom, denoted $[O]$. In the given diagram X is the product from which the downward reaction starts. Therefore X must be the reactive oxygen atom that participates in ozone production, leaving Y as nitrogen monoxide.

Step 2: Identify the downward reaction Atomic oxygen combines with the oxygen already present in air:

$$O + O_2 \longrightarrow O_3.$$

A third body may carry away excess energy in the atmosphere, but it is not one of the labels in the stated scheme. Since A is the reactant added to X and B is the product below the arrow, A is O_2 and B is O_3. Ozone is a major oxidising component of photochemical smog.

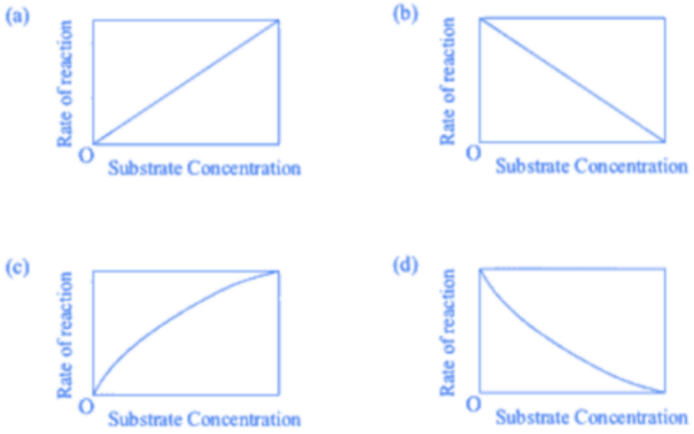
Step 3: Test all proposed assignments Option (A) gives exactly X as atomic oxygen, Y as NO, A as molecular oxygen and B as ozone. Option (B) incorrectly produces nitrous oxide in the first step. Option (C) does not represent the stated atomic-oxygen photolysis and ozone-formation sequence. Option (D) assigns NO to the downward path and gives N_2O_3, rather than the ozone produced from atomic and molecular oxygen. Thus no other assignment fits both arrows.

Step 4: Conclusion $X = [O],\ Y = NO,\ A = O_2,\ B = O_3$. Hence, the correct option is (A).

Quick Tip to Solve: Follow the branch in the scheme: the photolysis product that reacts with atmospheric oxygen is the oxygen atom, and its product is ozone.

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78. The variation of the rate of an enzyme catalyzed reaction with substrate concentration is correctly represented by graph : [4 marks]



(A) (a)

(B) (b)

(C) (d)

(D) (c)

Correct Answer: (D) (c)

Solution:

Concept:

- An enzyme provides a finite number of active sites. The reaction proceeds through an enzyme–substrate complex, and its rate depends on how many of these sites are occupied.
- For simple enzyme kinetics, $v = V_{max}[S]/(K_M + [S])$, where $[S]$ is substrate concentration, K_M is the Michaelis constant and V_{max} is the saturation rate.

Step 1: Examine the low-concentration behaviour When $[S] \ll K_M$, the denominator is approximately K_M. Thus

$$v \simeq \frac{V_{max}}{K_M}[S].$$

The rate initially increases with substrate concentration because additional substrate molecules find free active sites. A graph that falls as substrate concentration increases cannot represent this ordinary enzyme-catalysed reaction.

Step 2: Apply saturation at high concentration As $[S]$ becomes large, almost every active site is

occupied and the available enzyme becomes the limiting factor. Therefore

$$\lim_{[S]\to\infty} v = V_{\max}.$$

The curve rises with a progressively smaller slope and tends towards a maximum. Graph (c) has this rising, flattening form. Graph (a) remains linear and has no saturation; graphs (b) and (d) decrease, contrary to the required trend. In the listed options, graph (c) is option D, not option C.

Step 3: Conclusion The correct curve shows increasing rate followed by enzyme saturation. Hence, the correct option is (D).

Quick Tip to Solve: Identify the saturation curve first, then map its small-letter graph label to the separately ordered option letter.

79. Compound A reacts with NH_4Cl and forms a compound B. Compound B reacts with H_2O and excess of CO_2 to form compound C which on passing through or reaction with saturated NaCl solution forms sodium hydrogen carbonate. Compound A, B and C, are respectively : [4 marks]

(A) $Ca(OH)_2, NH_4^{\oplus}, (NH_4)_2CO_3$

(B) $CaCl_2, NH_4^{\oplus}, (NH_4)_2CO_3$

(C) $CaCl_2, NH_3, NH_4HCO_3$

(D) $Ca(OH)_2, NH_3, NH_4HCO_3$

Correct Answer: (D) $Ca(OH)_2, NH_3, NH_4HCO_3$

Solution:

Concept:

- The Solvay process uses ammonia, carbon dioxide and concentrated sodium chloride solution to obtain sodium hydrogen carbonate. Ammonia is regenerated using slaked lime.
- A strong base liberates NH_3 from an ammonium salt; excess CO_2 in aqueous ammonia produces NH_4HCO_3.

Step 1: Identify the base and the liberated gas The compound that reacts with ammonium chloride to release ammonia is calcium hydroxide, or slaked lime:

$$Ca(OH)_2 + 2\,NH_4Cl \longrightarrow CaCl_2 + 2\,NH_3 + 2\,H_2O.$$

Hence $A = Ca(OH)_2$ and $B = NH_3$. Calcium chloride is a product of this reaction, not the base that liberates ammonia. This eliminates options B and C. The ammonium ion in option A is not the gaseous ammonia required in the next step.

Step 2: Follow the carbonation and precipitation steps On adding water and excess carbon dioxide, ammonia yields ammonium hydrogen carbonate:

$$NH_3 + CO_2 + H_2O \longrightarrow NH_4HCO_3.$$

Thus $C = NH_4HCO_3$, rather than ammonium carbonate under the specified excess-carbon-dioxide

conditions. Reaction with saturated brine then gives

$$NH_4HCO_3 + NaCl \longrightarrow NaHCO_3 \downarrow + NH_4Cl.$$

Sodium hydrogen carbonate separates because it is sparingly soluble in the ammoniated brine. These successive reactions establish all three compounds in the stated order, and only option D contains the complete sequence.

Step 3: Conclusion $A = Ca(OH)_2$, $B = NH_3$, and $C = NH_4HCO_3$. Hence, the correct option is (D).

Quick Tip to Solve: In the Solvay sequence, first find the ammonia-liberating base; excess carbon dioxide then points to hydrogen carbonate.

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80. A cubic solid is made up of two elements X and Y. Atoms of X are present on every alternate corner and one at the center of cube. Y is at $\frac{1}{3}^{rd}$ of the total faces. The empirical formula of the compound is : [4 marks]

(A) $X_2Y_{1.5}$

(B) X_3Y_2

(C) $XY_{2.5}$

(D) $X_{2.5}Y$

Correct Answer: (B) X_3Y_2

Solution:

Concept:

- An atom at a cube corner is shared by eight unit cells and contributes $1/8$ atom to one cell; a body-centred atom belongs wholly to that cell.
- An atom at a face centre is shared between two neighbouring cells and contributes $1/2$. The empirical formula follows from the smallest whole-number ratio of effective atom counts.

Step 1: Count the effective X atoms A cube has eight corners. Occupation of every alternate corner means that four corners contain X atoms. Their total contribution is

$$N_{X,\text{corners}} = 4\left(\frac{1}{8}\right) = \frac{1}{2}.$$

There is also one X atom at the centre of the cube, contributing one complete atom. Therefore

$$N_X = \frac{1}{2} + 1 = \frac{3}{2}.$$

Counting the four corner atoms as four complete atoms would ignore the sharing of each corner among neighbouring unit cells.

Step 2: Count Y and reduce the ratio A cube has six faces, and one-third of its faces are occupied by

Y. Hence two face centres contain Y, giving

$$N_Y = \frac{1}{3}(6)\left(\frac{1}{2}\right) = 1.$$

Consequently

$$N_X : N_Y = \frac{3}{2} : 1 = 3 : 2.$$

The empirical formula is therefore X_3Y_2. Option A has the different ratio $4 : 3$, while options C and D introduce a factor 2.5 that does not follow from the stated occupancy. None of them gives the calculated ratio 3:2.

Step 3: Conclusion The empirical formula is X_3Y_2. Hence, the correct option is (B).

Quick Tip to Solve: Apply the sharing fractions before reducing a unit-cell composition to an empirical formula: corner 1/8, face 1/2, body centre 1.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The density of a monobasic strong acid (Molar mass 24.2 g/mol) is 1.21 kg/L. The volume of its solution required for the complete neutralization of 25 mL of 0.24 M NaOH is _____ $\times 10^{-2}$ mL (Nearest integer) [4 marks]

Solution:

Concept:

- A monobasic strong acid supplies one mole of H^+ per mole of acid. Its neutralization with $NaOH$ therefore has a 1:1 acid-to-base mole ratio.
- Molarity is moles per litre. Using the given density and molar mass, the acid concentration is obtained from the mass contained in one litre.

Step 1: Convert density into acid concentration The given density is $1.21\,\text{kg}\,\text{L}^{-1} = 1210\,\text{g}\,\text{L}^{-1}$. A litre of the acid therefore contains

$$n = \frac{1210\,\text{g}}{24.2\,\text{g}\,\text{mol}^{-1}} = 50\,\text{mol}.$$

Its molarity is $50\,\text{mol}\,\text{L}^{-1}$. The factor of 1000 in converting kilograms to grams is essential: the molar mass is expressed in grams per mole. No extra acid dissociation factor is needed because the acid is monobasic and strong.

Step 2: Use the neutralization mole balance The sodium hydroxide volume is $25\,\text{mL} = 0.025\,\text{L}$, so

$$n(\text{NaOH}) = 0.24 \times 0.025 = 0.006\,\text{mol}.$$

From $H^+ + OH^- \longrightarrow H_2O$, the required acid amount is also $0.006\,\text{mol}$. Its volume is

$$V = \frac{0.006}{50} = 1.2 \times 10^{-4}\,\text{L} = 0.12\,\text{mL}.$$

To express this in the form requested, write

$$0.12\,\text{mL} = 12 \times 10^{-2}\,\text{mL}.$$

The numerical entry is the coefficient 12, rather than 0.12 or 120.

Step 3: Conclusion The required acid volume is $0.12\,\text{mL} = 12 \times 10^{-2}\,\text{mL}$. The nearest-integer entry is $\boxed{12}$.

Quick Tip to Solve: Convert density to grams per litre before dividing by molar mass, then convert the final volume to the exact power-of-ten unit in the blank.

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82. A litre of buffer solution contains 0.1 mole of each of NH_3 and NH_4Cl. On the addition of 0.02 mole of HCl by dissolving gaseous HCl, the pH of the solution is found to be _____ $\times 10^{-3}$ (Nearest integer)

[Given : $pK_b(NH_3) = 4.745$

$$\log 2 = 0.301$$

$$\log 3 = 0.477$$

$$T = 298\ K]$$

[4 marks]

Solution:

Concept:

- A mixture of NH_3 and NH_4Cl is a basic buffer. Added strong acid consumes ammonia and forms additional ammonium ions.
- At 298 K, $pOH = pK_b + \log([NH_4^+]/[NH_3])$, and $pH = 14 - pOH$. The ratio can be calculated directly from moles in the same final solution.

Step 1: Update the buffer amounts after neutralization The added hydrogen chloride reacts essentially completely according to

$$NH_3 + HCl \longrightarrow NH_4Cl.$$

Initially each buffer component is present in an amount of $0.1\,\text{mol}$. Adding $0.02\,\text{mol}$ of gaseous HCl therefore leaves

$$n(NH_3) = 0.10 - 0.02 = 0.08\,\text{mol}$$

and produces a total of

$$n(NH_4^+) = 0.10 + 0.02 = 0.12\,\text{mol}.$$

Both components remain, so the buffer equation is applicable; the added HCl must not be treated as unreacted free strong acid.

Step 2: Calculate pOH and convert to the required pH entry The common volume cancels from the concentration ratio. Using the given logarithms,

$$\log \frac{0.12}{0.08} = \log \frac{3}{2} = 0.477 - 0.301 = 0.176.$$

Hence

$$\text{pOH} = 4.745 + 0.176 = 4.921,$$

and

$$\text{pH} = 14.000 - 4.921 = 9.079.$$

The decrease from the initial pH of 9.255 is consistent with addition of acid. The blank is scaled by 10^{-3}, so

$$9.079 = 9079 \times 10^{-3}.$$

pH is dimensionless; the entry is 9079.

Step 3: Conclusion The final pH is $9.079 = 9079 \times 10^{-3}$. The required nearest-integer entry is $\boxed{9079}$.

Quick Tip to Solve: For an added acid, subtract its moles from the weak base and add the same moles to the conjugate acid before applying the buffer equation.

83. Consider the cell

$$\text{Pt(s)}|\text{H}_2\text{(g)}\,(1\,\text{atm})|\text{H}^+\,(\text{aq}, [\text{H}^+] = 1)||\text{Fe}^{3+}\text{(aq)}, \text{Fe}^{2+}\text{(aq)}|\text{Pt(s)}$$

Given $\text{E}^\circ_{\text{Fe}^{3+}/\text{Fe}^{2+}} = 0.771\,\text{V}$ **and** $\text{E}^\circ_{\text{H}^+/1/2\,\text{H}_2} = 0\,\text{V},\ \text{T} = 298\,\text{K}$

If the potential of the cell is 0.712 V, the ratio of concentration of Fe^{2+} to Fe^{3+} is _____ (Nearest integer) [4 marks]

Solution:

Concept:

- The standard hydrogen electrode has zero potential when $p_{\text{H}_2} = 1\,\text{atm}$ and $[\text{H}^+] = 1$. Thus the left electrode is under standard conditions here.
- For the reduction $\text{Fe}^{3+} + \text{e}^- \longrightarrow \text{Fe}^{2+}$, the electron number is $n = 1$, and at 298 K $E = E^\circ - 0.059 \log([\text{Fe}^{2+}]/[\text{Fe}^{3+}])$.

Step 1: Write the reaction quotient in the correct order The iron couple forms the right-hand electrode, so its reduction potential is measured relative to the hydrogen electrode on the left. The cell reaction per electron is

$$\text{Fe}^{3+} + \frac{1}{2}\,\text{H}_2 \longrightarrow \text{Fe}^{2+} + \text{H}^+.$$

With hydrogen pressure and hydrogen-ion activity equal to unity, the reaction quotient reduces to

$$Q = \frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]}.$$

The standard cell potential is $0.771 - 0 = 0.771\,\text{V}$. Platinum is an inert conductor and does not enter the reaction quotient.

Step 2: Substitute the measured cell potential Let the required ratio be r. Applying the Nernst equation gives

$$0.712 = 0.771 - 0.059 \log r.$$

Rearranging while retaining the negative sign in the equation,

$$\log r = \frac{0.771 - 0.712}{0.059} = 1.$$

Therefore

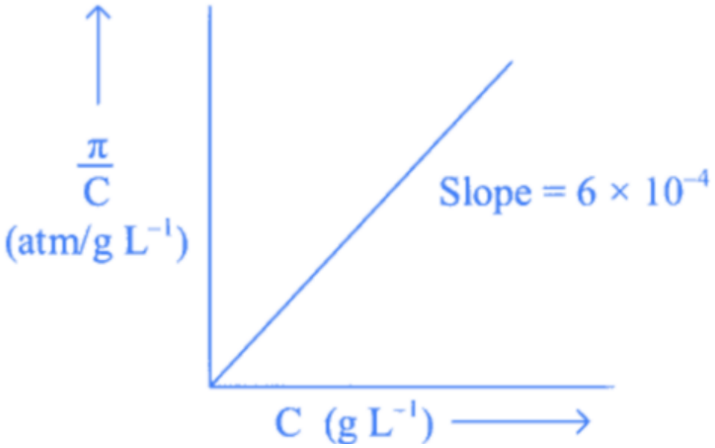
$$r = 10^1 = 10.$$

Using the more precise coefficient 0.0591 gives approximately 9.96, which has the same nearest integer. Because the observed potential is below its standard value, the quotient must exceed one; taking the reciprocal would incorrectly give 0.1.

Step 3: Conclusion The concentration ratio is $[\text{Fe}^{2+}]/[\text{Fe}^{3+}] \approx 10$, a dimensionless ratio. The nearest-integer entry is $\boxed{10}$.

Quick Tip to Solve: For a reduction half-cell, put the reduced species above the oxidized species in the Nernst quotient; a lower potential then means a larger quotient.

84. The osmotic pressure of solutions of PVC in cyclohexanone at 300 K are plotted on the graph. The molar mass of PVC is _____ g mol^{-1} (Nearest integer)
(Given : R = 0.083 L atm K^{-1} mol^{-1}) [4 marks]



Solution:

Concept:

- For a dilute polymer solution, $\pi = (C/M)RT$, where C is mass concentration and M is molar mass. Thus a plot of π against C has slope RT/M.
- For a plot of reduced osmotic pressure π/C against C, the dilute-solution expansion is $\pi/C =$

$RT/M + BC + \cdots$. Its intercept, rather than its slope, determines the molar mass.

Step 1: Inspect what the printed graph actually supplies The vertical axis is labelled π/C, the horizontal axis is labelled C, and the rising line is labelled with slope 6×10^{-4}. For these axes, the slope describes the concentration-dependent correction to reduced osmotic pressure. The molar mass would require the numerical vertical intercept

$$\left(\frac{\pi}{C}\right)_{C\to 0} = \frac{RT}{M}.$$

No finite numerical intercept is supplied. The drawn line meets the origin, which would give a zero intercept and cannot determine a finite molar mass for PVC. Consequently the literal graph is inconsistent with extracting a finite molar mass from the stated slope.

Step 2: State the conventional numerical result and its condition If the supplied number 6×10^{-4} is intended to represent RT/M, as for the slope of a π-versus-C plot, the calculation is

$$M = \frac{RT}{6 \times 10^{-4}} = \frac{0.083 \times 300}{6 \times 10^{-4}} = 41500\,\mathrm{g\,mol^{-1}}.$$

The units follow because RT has units $\mathrm{L\,atm\,mol^{-1}}$, and π/C has units $\mathrm{atm\,L\,g^{-1}}$. This gives the conventional numerical entry, but interpreting the slope of the printed reduced-pressure plot as its intercept would not be mathematically justified.

Step 3: Conclusion The conventional intended entry is $\boxed{41500}$, corresponding to $M = 41500\,\mathrm{g\,mol^{-1}}$ when $6 \times 10^{-4} = RT/M$. As printed, however, the π/C-versus-C graph gives a slope instead of the required intercept, so it does not uniquely establish this molar mass.

Quick Tip to Solve: Check the plotted axes before using a slope: molar mass comes from the slope of a pressure plot, but from the intercept of a reduced-pressure plot.

85. In sulphur estimation, 0.471 g of an organic compound gave 1.4439 g of barium sulphate. The percentage of sulphur in the compound is _____ (Nearest Integer)
(Given : Atomic mass Ba: 137 u, S: 32 u, O: 16 u) [4 marks]

Solution:

Concept:

- In the gravimetric estimation of sulphur, sulphur in the organic sample is converted quantitatively into sulphate and precipitated as barium sulphate, $\mathrm{BaSO_4}$. Each mole of the precipitate contains exactly one mole of sulphur atoms.
- The mass fraction of sulphur in a pure precipitate is its atomic mass divided by the formula mass of the precipitate. The sulphur percentage in the original sample is $100 m_{\mathrm{S}}/m_{\text{sample}}$, using masses in the same units.

Step 1: Find the molar mass of the precipitate The formula has one barium atom, one sulphur atom and four oxygen atoms. Use the given atomic masses, rather than treating the mass of the entire

precipitate as sulphur mass.

$$M(\mathrm{BaSO_4}) = 137 + 32 + 4(16) = 233\,\mathrm{g\,mol^{-1}}.$$

Thus only the fraction $32/233$ of the collected barium sulphate mass is due to sulphur.

Step 2: Convert precipitate mass into sulphur mass The amount of sulphur is the same as the amount of barium sulphate because their mole ratio is one to one.

$$m_{\mathrm{S}} = 1.4439\left(\frac{32}{233}\right) = 0.198304\,\mathrm{g}.$$

The barium and oxygen in the precipitate come from the analytical reagents and cannot be counted as part of the sulphur originally present.

Step 3: Calculate and round the sample percentage Divide the recovered sulphur mass by the mass of the organic sample, not by the precipitate mass.

$$\%\mathrm{S} = \frac{0.198304}{0.471} \times 100 = 42.1028\%.$$

The question asks for the nearest integer, so round only this final percentage.

Step 4: Conclusion The sulphur content is approximately 42.10%; the required nearest-integer percentage is $\boxed{42\%}$, so the numerical entry is 42.

Quick Tip to Solve: For sulphur estimation, directly use $\%\mathrm{S} = (32/233)(m_{\mathrm{BaSO_4}}/m_{\mathrm{sample}})100$; do not use the whole precipitate mass as sulphur.

86. An athlete is given 100 g of glucose ($C_6H_{12}O_6$) for energy. This is equivalent to 1800kJ of energy. The 50% of this energy gained is utilized by the athlete for sports activities at the event. In order to avoid storage of energy, the weight of extra water he would need to perspire is _____ g (Nearest integer)

Assume that there is no other way of consuming stored energy.

Given : The enthalpy of evaporation of water is 45 kJ mol^{-1}

Molar mass of C, H & O are 12, 1 and 16 g mol^{-1}. [4 marks]

Solution:

Concept:

- Energy conservation requires the energy not used for sports to leave the body through the only remaining route specified: evaporation of perspired water. The latent heat absorbed is $q = n\Delta H_{\mathrm{vap}}$.
- The given $1800\,\mathrm{kJ}$ is already the energy supplied by the stated glucose serving. The fraction left after activity is $1 - 0.50 = 0.50$. Water has molar mass $2(1) + 16 = 18\,\mathrm{g\,mol^{-1}}$.

Step 1: Find the energy that must be dissipated The athlete uses half of the supplied energy during the event. The other half would be stored unless it is removed by evaporation, under the assumption

in the question.

$$E_{\text{sports}} = 0.50(1800) = 900\,\text{kJ}.$$

$$q_{\text{evaporation}} = 1800 - 900 = 900\,\text{kJ}.$$

It would be incorrect to remove all $1800\,\text{kJ}$, since sports activities already consume part of it.

Step 2: Calculate the amount of water evaporated Each mole of water needs $45\,\text{kJ}$ to evaporate. Divide the excess energy by this molar enthalpy, with both energies in kilojoules.

$$n(\text{H}_2\text{O}) = \frac{q_{\text{evaporation}}}{\Delta H_{\text{vap}}} = \frac{900\,\text{kJ}}{45\,\text{kJ mol}^{-1}} = 20\,\text{mol}.$$

Evaporation absorbs energy from the body, providing the required cooling.

Step 3: Convert moles into the required mass Multiply by the molar mass of water. The glucose molar mass is not needed because its total supplied energy was explicitly given.

$$m(\text{H}_2\text{O}) = nM = 20 \times 18 = 360\,\text{g}.$$

The result is already an integer; the instruction to give the nearest integer does not change it.

Step 4: Conclusion The extra mass of water that must evaporate is $\boxed{360\,\text{g}}$. The required numerical entry is 360.

Quick Tip to Solve: First subtract the useful energy expenditure; divide the remaining energy by the molar latent heat and then multiply by $18\,\text{g mol}^{-1}$.

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87. How many of the following metal ions have similar value of spin only magnetic moment in gaseous state? _____

(Given : Atomic number V, 23; Cr, 24; Fe, 26; Ni, 28)

V^{3+}, Cr^{3+}, Fe^{2+}, Ni^{3+} [4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu_{\text{so}} = \sqrt{n(n+2)}\,\mu_{\text{B}}$, where n is the number of unpaired electrons and μ_{B} is the Bohr magneton. Ions have equal spin-only moments when their unpaired-electron counts agree.
- For transition-metal cations, remove the $4s$ electrons before the $3d$ electrons. In the gaseous state there is no ligand field to impose a low-spin configuration; distribute the d electrons according to Hund's rule.

Step 1: Determine the ionic electron configurations The neutral configurations are $\text{V} : [Ar]3d^3 4s^2$, $\text{Cr} : [Ar]3d^5 4s^1$, $\text{Fe} : [Ar]3d^6 4s^2$ and $\text{Ni} : [Ar]3d^8 4s^2$. Removing the indicated number of electrons gives

$$\text{V}^{3+} : [Ar]3d^2, \quad \text{Cr}^{3+} : [Ar]3d^3,$$
$$\text{Fe}^{2+} : [Ar]3d^6, \quad \text{Ni}^{3+} : [Ar]3d^7.$$

The exceptional neutral chromium configuration does not alter its final d^3 cation configuration.

Step 2: Count unpaired electrons using Hund's rule The five degenerate d orbitals fill singly before pairing. Thus d^2 and d^3 have two and three unpaired electrons respectively. For d^6, one pair and four single electrons remain, while d^7 has two pairs and three single electrons. Consequently the unpaired counts for the listed ions are $2, 3, 4, 3$, respectively.

Step 3: Compare the spin-only values The corresponding moments are

$$\sqrt{8}, \quad \sqrt{15}, \quad \sqrt{24}, \quad \sqrt{15} \quad (\mu_B).$$

Only Cr^{3+} and Ni^{3+} share the same value. The question asks how many ions share it, rather than the number of matching pairs.

Step 4: Conclusion The two ions Cr^{3+} and Ni^{3+} both have three unpaired electrons and $\mu_{so} = \sqrt{15}\,\mu_B$. The answer is $\boxed{2}$ ions.

Quick Tip to Solve: For gaseous d^n ions, the unpaired count is n up to d^5, then $10 - n$; therefore d^3 and d^7 give equal spin-only moments.

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88. The number of paramagnetic species from the following is _____.

$$[Ni(CN)_4]^{2-}, [Ni(CO)_4], [NiCl_4]^{2-}$$

$$[Fe(CN)_6]^{4-}, [Cu(NH_3)_4]^{2+}$$

$$[Fe(CN)_6]^{3-} \text{ and } [Fe(H_2O)_6]^{2+}$$

[4 marks]

Solution:

Concept:

- A complex is paramagnetic if it retains at least one unpaired electron. First determine the metal oxidation state and d-electron count, then account for ligand strength and coordination geometry.
- CN^- and CO are strong-field ligands, whereas Cl^- and H_2O normally give high-spin arrangements here. A d^9 copper(II) complex retains one unpaired electron.

Step 1: Check the three nickel species In $[Ni(CN)_4]^{2-}$, nickel is $+2$, hence d^8. The strong-field cyanide ligands give the usual square-planar arrangement with all electrons paired: diamagnetic. In $Ni(CO)_4$, CO is neutral, so nickel is zero-valent and d^{10}; all electrons are paired, again diamagnetic. In $[NiCl_4]^{2-}$, nickel is $+2$, d^8, but the weak-field tetrahedral chloride complex has two unpaired electrons and is paramagnetic.

Step 2: Check iron and copper with strong-field ligands For $[Fe(CN)_6]^{4-}$, iron is $+2$, d^6. Strong-field cyanide gives low-spin $t_{2g}^6 e_g^0$, so the complex is diamagnetic. For $[Fe(CN)_6]^{3-}$, iron is $+3$, d^5, and

low-spin t_{2g}^5 still has one unpaired electron. In $[Cu(NH_3)_4]^{2+}$, neutral ammonia gives copper(II), d^9, with one unpaired electron. Both of the latter species are paramagnetic.

Step 3: Include the aqua complex and total the species In $[Fe(H_2O)_6]^{2+}$, iron is $+2$, d^6. Water gives a high-spin octahedral arrangement $t_{2g}^4 e_g^2$, with four unpaired electrons. Count the paramagnetic complexes rather than summing all their unpaired electrons: the nickel chloride, copper ammonia, iron(III) cyanide and iron(II) aqua complexes qualify.

Step 4: Conclusion There are $\boxed{4}$ paramagnetic species: $[NiCl_4]^{2-}$, $[Cu(NH_3)_4]^{2+}$, $[Fe(CN)_6]^{3-}$ and $[Fe(H_2O)_6]^{2+}$.

Quick Tip to Solve: A strong ligand does not always mean diamagnetic: low-spin d^5 still has one unpaired electron, while low-spin d^6 has none.

89. For the first order reaction A $\rightarrow$ B, the half life is 30 min. The time taken for 75% completion of the reaction is _____ min. (Nearest integer)

Given : log 2 = 0.3010

log 3 = 0.4771

log 5 = 0.6989

[4 marks]

Solution:

Concept:

- For a first-order reaction, the fraction remaining after time t is $[A]_t/[A]_0 = e^{-kt}$, where k is the rate constant. Its half-life is independent of the starting concentration.
- The integrated law in common logarithms is $t = (2.303/k)\log([A]_0/[A]_t)$, and $t_{1/2} = (2.303/k)\log 2$. Completion percentage refers to the amount reacted, so it must be subtracted from 100% to obtain the remaining fraction.

Step 1: Translate completion into concentration remaining A completion of 75% means three quarters of the original reactant has changed into product. Therefore only one quarter is still present:

$$\frac{[A]_t}{[A]_0} = 1 - \frac{75}{100} = \frac{1}{4}.$$

Do not substitute $0.75[A]_0$ as the concentration left; that would describe only 25% completion.

Step 2: Use the sequence of first-order half-lives Each successive interval of $30\,\text{min}$ halves the reactant still present. The first interval leaves 50% of the initial amount and the second leaves 25%.

$$\frac{[A]_t}{[A]_0} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

$$t = 2t_{1/2} = 2(30) = 60\,\text{min}.$$

The second halving is of the remaining material, not another half of the original amount.

Step 3: Verify using the logarithmic rate law Dividing the integrated rate law by the half-life formula

cancels the unknown rate constant:

$$\frac{t}{30} = \frac{\log 4}{\log 2} = \frac{2(0.3010)}{0.3010} = 2.$$

This agrees with the successive-halving calculation. The supplied values of $\log 3$ and $\log 5$ are not needed for this remaining fraction.

Step 4: Conclusion The time for 75% completion is $\boxed{60\ \text{min}}$. The required nearest-integer numerical entry is 60.

Quick Tip to Solve: For first-order kinetics, 75%, 87.5% and 93.75% completion correspond to two, three and four half-lives, respectively.

90. The total number of lone pairs of electrons on oxygen atoms of ozone is _____. [4 marks]

Solution:

Concept:

- An ozone molecule, O_3, contains three oxygen atoms with six valence electrons each, giving a total of 18 valence electrons. A lone pair consists of two valence electrons not shared in a bond.
- The octet-satisfying Lewis contributors of ozone contain one single O–O bond and one double O=O bond. They are resonance contributors of the same bent molecule, so their lone-pair counts must not be added together.

Step 1: Allocate the bonding electrons Connect the central oxygen to both terminal oxygen atoms. To give every oxygen an octet with the available 18 electrons, one connection is single and the other is double in a Lewis contributor. The single bond uses two electrons and the double bond uses four:

$$N_{\text{bonding electrons}} = 2 + 4 = 6.$$

Thus there are three shared electron pairs in all, counting the two pairs of the double bond separately.

Step 2: Count nonbonding electrons and pairs Subtract the bonding electrons from the molecule's total valence electron count. Every remaining electron belongs to a lone pair in the closed-shell Lewis structure:

$$N_{\text{nonbonding electrons}} = 18 - 6 = 12.$$

$$N_{\text{lone pairs}} = \frac{12}{2} = 6.$$

The question asks for lone pairs on all three oxygen atoms, rather than only on the central atom.

Step 3: Verify the distribution among oxygen atoms The singly bonded terminal oxygen has three lone pairs and formal charge -1. The central oxygen has one lone pair and formal charge $+1$. The doubly bonded terminal oxygen has two lone pairs and formal charge zero. Their sum is

$$3 + 1 + 2 = 6.$$

The other resonance contributor interchanges the two terminal atoms' roles. It leaves the total of six

lone pairs unchanged and does not represent an additional molecule.

Step 4: Conclusion The total number of lone pairs on the oxygen atoms in one ozone molecule is $\boxed{6}$ (a dimensionless count).

Quick Tip to Solve: Count lone pairs by subtracting bonding electrons from total valence electrons and dividing by two; never add electron counts from separate resonance contributors.