JEE Main 2023 Paper 1 (B.E./B.Tech) Jan 30 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2023, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 30 January 2023, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let the solution curve $y = y(x)$ of the differential equation

$$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{3/2}} y = 2x \exp\left\{\frac{x^3 - \tan^{-1} x^3}{\sqrt{(1+x^6)}}\right\}$$ **pass through the origin. Then $y(1)$ is equal to :**

[4 marks]

(A) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$

(B) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$

(C) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$

(D) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$

Correct Answer: (B) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$

Solution:

Concept:

- A first-order linear equation $y' + P(x)y = Q(x)$ is solved by multiplying by the integrating factor $\exp(\int P(x)\,dx)$.
- Identify an exponent whose derivative gives the coefficient of the unknown function. The product rule then converts the left side into one exact derivative.

Step 1: Recognise the derivative Put $F(x) = (x^3 - \tan^{-1}(x^3))/\sqrt{1+x^6}$. Differentiate the quotient, keeping the derivative of the inverse tangent. On collecting terms,

$$F'(x) = \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{3/2}}.$$

Thus the printed equation is $y' - F'y = 2xe^F$. Its integrating factor is e^{-F}, with the minus sign essential.

Step 2: Integrate and apply the initial condition Multiplying the equation by this factor gives

$$\frac{d}{dx}\left(ye^{-F(x)}\right) = 2x.$$

Hence $ye^{-F(x)} = x^2 + C$. The curve passes through the origin, so $y(0) = 0$, while $F(0) = 0$; therefore $C = 0$. Consequently $y(x) = x^2 e^{F(x)}$.

Step 3: Evaluate the required value At $x = 1$, $\tan^{-1} 1 = \pi/4$, so

$$F(1) = \frac{1 - \pi/4}{\sqrt{2}} = \frac{4-\pi}{4\sqrt{2}}.$$

The multiplier x^2 is one here. The other options use an incorrect constant, an incorrect sign for the inverse tangent, or the negative of the required exponent.

Step 4: Conclusion

$$y(1) = \exp\left(\frac{4-\pi}{4\sqrt{2}}\right).$$

Hence, the correct option is (B).

Quick Tip to Solve: When the forcing term contains an exponential, differentiate its exponent first; the same derivative may provide the integrating factor immediately.

2. The minimum number of elements that must be added to the relation $\mathrm{R} = \{(\mathrm{a}, \mathrm{b}), (\mathrm{b}, \mathrm{c})\}$ on the set $\{a, b, c\}$ so that it becomes symmetric and transitive is : [4 marks]

(A) 7

(B) 3

(C) 4

(D) 5

Correct Answer: (A) 7

Solution:

Concept:

- Symmetry requires (v, u) whenever (u, v) belongs to the relation.
- Transitivity requires (u, w) whenever both (u, v) and (v, w) belong. These rules must hold for the added pairs as well as the original pairs.

Step 1: Add the reversed pairs The given pairs are (a, b) and (b, c). Symmetry forces (b, a) and (c, b). These are two distinct new ordered pairs. Merely inserting them is insufficient: the resulting relation would still fail transitivity.

Step 2: Close every two-step chain The chain $(a, b), (b, c)$ forces (a, c), and symmetry then forces (c, a). In addition, $(a, b), (b, a)$ forces (a, a), while the reversed chain forces (b, b). The chain $(c, b), (b, c)$ forces (c, c). Thus all three diagonal pairs are compulsory even though reflexivity was not explicitly requested.

Step 3: Prove the count is minimal The new pairs are

$$\{(b, a), (c, b), (a, c), (c, a), (a, a), (b, b), (c, c)\}.$$

Every one has been forced by symmetry or transitivity, so none can be omitted. Together with the original two, they form the full Cartesian product $\{a, b, c\} \times \{a, b, c\}$, which is certainly symmetric and transitive. Hence the lower bound is attained. The smaller counts 3, 4 and 5 cannot include all seven forced pairs.

Step 4: Conclusion Exactly $9 - 2 = 7$ ordered pairs must be added. Hence, the correct option is (A).

Quick Tip to Solve: For a symmetric relation, a two-way connection forces loops by transitivity; a connected three-element chain then forces every ordered pair.

3. Let the system of linear equations

$$x + y + kz = 2$$

$$2x + 3y - z = 1$$

$$3x + 4y + 2z = k$$

have infinitely many solutions. Then the system

$$(k+1)x + (2k-1)y = 7$$

$$(2k+1)x + (k+5)y = 10$$

has : [4 marks]

(A) unique solution satisfying $x - y = 1$

(B) infinitely many solutions

(C) no solution

(D) unique solution satisfying $x + y = 1$

Correct Answer: (D) unique solution satisfying $x + y = 1$

Solution:

Concept:

- Infinitely many solutions require the coefficient rows to be linearly dependent and the same dependence to hold for the constants.
- For two linear equations in two unknowns, a nonzero coefficient determinant establishes uniqueness; solve to test the additional condition stated in an option.

Step 1: Determine the parameter The first two coefficient rows are independent, since their first two entries have determinant $1 \cdot 3 - 1 \cdot 2 = 1$. To reproduce the first two entries $(3, 4)$ of the third row, their only possible combination is their sum. Its third entry is $k - 1$. Therefore dependence requires $k - 1 = 2$, giving $k = 3$. The constants also obey $2 + 1 = 3 = k$, so the system is consistent and indeed has one free variable.

Step 2: Substitute into the second system With $k = 3$, the new equations become

$$4x + 5y = 7, \qquad 7x + 8y = 10.$$

Their coefficient determinant is $4 \cdot 8 - 5 \cdot 7 = -3$, which is nonzero. Thus neither infinitely many solutions nor no solution is possible.

Step 3: Solve and identify the condition Multiply the first equation by 7 and the second by 4, then subtract:

$$3y = 49 - 40 = 9.$$

Thus $y = 3$; substitution in $4x + 5y = 7$ gives $4x = -8$, so $x = -2$. These values satisfy both original equations of the new system. Their sum is $x + y = 1$, whereas their difference is $x - y = -5$, excluding the other unique-solution option.

Step 4: Conclusion The system has the unique solution $(x, y) = (-2, 3)$, satisfying $x + y = 1$. Hence, the correct option is (D).

Quick Tip to Solve: Before expanding a three-by-three determinant, check whether one coefficient row is visibly the sum of two others; also check the constants.

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4. If [t] denotes the greatest integer $\leq$ t, then the value of $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx$ is : [4 marks]

(A) $e^8 - e$

(B) $e^7 - 1$

(C) $e^9 - e$

(D) $e^8 - 1$

Correct Answer: (A) $e^8 - e$

Solution:

Concept:

- The floor function is constant between consecutive integers. Values at finitely many endpoints do not affect a definite integral.
- The substitution $u = x^3$ converts $3x^2\,dx$ into du, and a sum of powers of e is a finite geometric progression.

Step 1: Remove the constant floor term For $1 \leq x < 2$, $[x] = 1$. The different endpoint value at $x = 2$ has no effect on the integral. If the requested expression is V, then

$$V = 3(e-1)\int_1^2 x^2 e^{[x^3]}\,dx.$$

The factor e from $e^{[x]}$ has cancelled the denominator of the given prefactor.

Step 2: Change the variable and split the interval Use $u = x^3$, so $du = 3x^2 dx$, with bounds 1 and 8. Hence

$$V = (e-1)\int_1^8 e^{[u]}\,du.$$

On each interval $[j, j+1)$, where $j = 1, \ldots, 7$, the integrand equals e^j. Every interval has length one, giving

$$V = (e-1)(e + e^2 + \cdots + e^7).$$

Step 3: Sum the geometric progression The first term is e, common ratio e, and there are seven terms. Therefore

$$e + e^2 + \cdots + e^7 = \frac{e(e^7 - 1)}{e - 1}.$$

Multiplication by $e-1$ yields e^8-e. The terms must start at e, rather than 1, and end at e^7, rather than e^8; the other three options correspond to changing one of these endpoints or losing the leading factor.

Step 4: Conclusion The required value is e^8-e. Hence, the correct option is (A).

Quick Tip to Solve: First substitute the expression inside the floor function; the resulting unit intervals often turn the integral into a geometric sum.

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5. If the coefficient of x^{15} in the expansion of $\left(ax^3+\frac{1}{bx^{1/3}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{1/3}-\frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b) : [4 marks]

(A) a = 3b

(B) ab = 1

(C) ab = 3

(D) a = b

Correct Answer: (B) ab = 1

Solution:

Concept:

- The term with index r in $(A+B)^{15}$ is $\binom{15}{r}A^{15-r}B^r$, where $0\le r\le 15$.
- Locate a coefficient by equating the total exponent of x to the requested exponent. Retain the sign of the second summand and use positivity before taking roots.

Step 1: Locate the first coefficient In the first expansion, the general term is

$$\binom{15}{r}a^{15-r}b^{-r}x^{45-10r/3}.$$

To obtain x^{15}, solve $45-10r/3=15$, giving $r=9$. Its coefficient is therefore $\binom{15}{9}a^6b^{-9}$. The exponent decreases by $10/3$ at every successive term, so there is exactly one contributing term.

Step 2: Locate the second coefficient In the second expansion, the general term is

$$\binom{15}{r}(-1)^r a^{15-r}b^{-r}x^{5-10r/3}.$$

The condition $5-10r/3=-15$ gives $r=6$. Because this index is even, the coefficient is positive, namely $\binom{15}{6}a^9b^{-6}$.

Step 3: Equate and simplify The binomial coefficients agree because $\binom{15}{9}=\binom{15}{6}$. Equality of the requested coefficients thus gives

$$\frac{a^6}{b^9}=\frac{a^9}{b^6}\quad\Longrightarrow\quad a^3b^3=1.$$

Since $a, b > 0$, this is exactly $ab = 1$. Neither $ab = 3$ nor a fixed ratio a/b follows; for example $(a, b) = (2, 1/2)$ satisfies the condition but not $a = b$ or $a = 3b$.

Step 4: Conclusion Every permissible ordered pair satisfies $ab = 1$. Hence, the correct option is (B).

Quick Tip to Solve: Find the term indices before comparing coefficients; complementary indices make their binomial coefficients cancel.

6. Suppose $f : \mathbb{R} \to (0, \infty)$ be a differentiable function such that $5f(x + y) = f(x) \cdot f(y), \forall x, y \in \mathbb{R}$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to : [4 marks]

(A) 6875

(B) 6525

(C) 6575

(D) 6825

Correct Answer: (D) 6825

Solution:

Concept:

- A positive multiplicative functional equation determines a geometric progression on the nonnegative integers.
- For first term a and ratio $r \neq 1$, the sum of N terms is $a(r^N - 1)/(r - 1)$. Count the term at index zero.

Step 1: Find the value at zero Set $x = y = 0$. The equation becomes $5f(0) = f(0)^2$. The range is $(0, \infty)$, so $f(0) \neq 0$ and division gives $f(0) = 5$. This positivity also excludes any negative choice when extracting a cube root later.

Step 2: Determine the integer ratio Set $y = 1$ in the functional equation to obtain

$$f(x + 1) = \frac{f(1)}{5} f(x).$$

Write $r = f(1)/5 > 0$. Iteration from zero gives $f(n) = 5r^n$ for every nonnegative integer n. In particular,

$$320 = f(3) = 5r^3 \quad \Longrightarrow \quad r^3 = 64, \qquad r = 4.$$

Only integer values are required, so a full solution for real arguments is unnecessary.

Step 3: Sum all six values The indices $0, 1, 2, 3, 4, 5$ supply six terms. Their sum is

$$\sum_{n=0}^{5} f(n) = 5 \sum_{n=0}^{5} 4^n = 5 \frac{4^6 - 1}{4 - 1} = 5 \frac{4096 - 1}{3} = 6825.$$

As a direct check, the values are $5, 20, 80, 320, 1280, 5120$. Adding them gives the same total. The other proposed totals do not equal this six-term progression; omitting the zero-index term would give 6820, also incorrect.

Step 4: Conclusion The required sum is 6825. Hence, the correct option is (D).

Quick Tip to Solve: For a multiplicative equation with a constant factor, determine the value at zero first and then divide by it to identify the progression ratio.

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7. If the solution of the equation $\log_{\cos x} \cot x + 4 \log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha+\sqrt{\beta}}{2}\right)$, where α, β are integers, then $\alpha + \beta$ is equal to : [4 marks]

(A) 3

(B) 6

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Concept:

- The change-of-base identity is $\log_a b = \ln b / \ln a$. On $(0, \pi/2)$, both sine and cosine lie strictly between zero and one, so these logarithms are defined.
- Use $\ln(\tan x) = \ln(\sin x) - \ln(\cos x)$ and $\sin^2 x + \cos^2 x = 1$ to reduce the equation to algebra.

Step 1: Express both logarithms using one ratio Put $t = \ln(\sin x)/\ln(\cos x)$. Both logarithms are negative, so $t > 0$. Change of base gives

$$\log_{\cos x} \cot x = 1 - t, \qquad \log_{\sin x} \tan x = 1 - \frac{1}{t}.$$

Substitute these expressions into the equation: $1 - t + 4(1 - 1/t) = 1$.

Step 2: Solve the ratio equation Rearrangement yields $t + 4/t = 4$. Multiplying by the positive nonzero t, we get

$$t^2 - 4t + 4 = 0, \qquad (t-2)^2 = 0.$$

Thus $\ln(\sin x) = 2\ln(\cos x)$, which is equivalent to $\sin x = \cos^2 x$. No roots were introduced by this multiplication, since t cannot vanish.

Step 3: Recover the angle and the requested integers Let $s = \sin x$, with $0 < s < 1$. The equation becomes $s = 1 - s^2$, or $s^2 + s - 1 = 0$. Of its roots, only

$$s = \frac{-1 + \sqrt{5}}{2}$$

belongs to $(0, 1)$. Sine is strictly increasing on the specified interval, so the angle is uniquely $\sin^{-1}((-1+\sqrt{5})/2)$. Comparing with the stated form gives $\alpha = -1$, $\beta = 5$. The other options do not equal their sum.

Step 4: Conclusion $\alpha + \beta = -1 + 5 = 4$. Hence, the correct option is (C).

Quick Tip to Solve: When logs have sine and cosine as reciprocal-style bases, take the ratio of their natural logarithms; the two terms then become expressions in t and $1/t$.

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8. If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is : [4 marks]

(A) $5 - \frac{3}{2}\sqrt{3}$

(B) $4 - 2\sqrt{3}$

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- Use complementary-angle identities: $\cot(90^\circ - \theta) = \tan\theta$, and the period 180° of tangent.
- The subtraction formula gives $\tan 15^\circ = \tan(45^\circ - 30^\circ) = 2 - \sqrt{3}$. Reciprocal surds can be simplified by multiplying numerator and denominator by the conjugate.

Step 1: Reduce every term to an angle of fifteen degrees The first term is already $\tan 15^\circ$. For the next two terms, use cotangent and the signs in the second quadrant:

$$\frac{1}{\tan 75^\circ} = \tan 15^\circ, \qquad \frac{1}{\tan 105^\circ} = -\tan 15^\circ.$$

Also, $\tan 195^\circ = \tan(180^\circ + 15^\circ) = \tan 15^\circ$. Thus the four terms do not all have the same sign; the negative reciprocal term cancels one positive term. Their sum is $2\tan 15^\circ = 2a$, giving $a = \tan 15^\circ$.

Step 2: Evaluate the reciprocal sum Apply the tangent subtraction identity explicitly:

$$a = \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} = 2 - \sqrt{3}.$$

Since $(2 - \sqrt{3})(2 + \sqrt{3}) = 1$, we have $1/a = 2 + \sqrt{3}$. Therefore

$$a + \frac{1}{a} = (2 - \sqrt{3}) + (2 + \sqrt{3}) = 4.$$

Option (B) is twice a, rather than the requested reciprocal sum. Option (C) omits the reciprocal contribution, while the radical expression in (A) also differs from the exact cancellation obtained above.

Step 3: Conclusion The requested value is $\boxed{4}$. Hence, the correct option is (D).

Quick Tip to Solve: First use tangent periodicity and complementary angles to combine the four terms; rationalize only after finding a.

9. Let a unit vector $\widehat{OP}$ make angles α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$. If $\widehat{\mathrm{OP}}$ is perpendicular to the plane through points $(1,2,3), (2,3,4)$ and $(1,5,7)$, then which one of the following is true? [4 marks]

(A) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$
(B) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$
(C) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$
(D) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

Correct Answer: (A) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

Solution:

Concept:

- A normal to a plane through three non-collinear points is the cross product of two independent vectors lying in that plane.
- The components of a unit vector are its direction cosines $(\cos\alpha, \cos\beta, \cos\gamma)$. A positive component gives an acute direction angle and a negative component gives an obtuse angle.

Step 1: Find the two possible unit normals Take the first point as a common starting point. The two vectors in the plane are

$$\mathbf{u} = (2,3,4) - (1,2,3) = (1,1,1), \qquad \mathbf{v} = (1,5,7) - (1,2,3) = (0,3,4).$$

Their cross product is

$$\mathbf{u} \times \mathbf{v} = (4-3,\ 0-4,\ 3-0) = (1,-4,3).$$

It is nonzero, so it gives the required normal direction. Its magnitude is $\sqrt{1+16+9} = \sqrt{26}$. The unit normal can point in either direction, giving the alternatives $\pm(1,-4,3)/\sqrt{26}$.

Step 2: Use the given acute angle to select the orientation The condition $0 < \beta < \pi/2$ requires $\cos\beta > 0$. Thus the unit vector must be

$$\widehat{OP} = \frac{(-1,4,-3)}{\sqrt{26}}.$$

Consequently $\cos\alpha = -1/\sqrt{26} < 0$ and $\cos\gamma = -3/\sqrt{26} < 0$. Direction angles with positive axes lie between zero and π, so both α and γ are strictly between $\pi/2$ and π. Options (B) and (C) incorrectly make one of these angles acute, and (D) incorrectly makes both acute.

Step 3: Conclusion Both $\alpha \in (\pi/2, \pi)$ and $\gamma \in (\pi/2, \pi)$. Hence, the correct option is (A).

Quick Tip to Solve: A cross product has a sign ambiguity. Use the prescribed sign of one direction cosine before classifying the other angles.

10. If an unbiased die, marked with $-2, -1, 0, 1, 2, 3$ on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is : [4 marks]

(A) $\frac{27}{288}$

(B) $\frac{521}{2592}$

(C) $\frac{440}{2592}$

(D) $\frac{881}{2592}$

Correct Answer: (B) $\frac{521}{2592}$

Solution:

Concept:

- A product of nonzero real numbers is positive precisely when the number of negative factors is even. Any zero outcome makes the product zero and must be excluded.
- For independent throws, multiply the probabilities of the outcomes. There are two negative faces, three positive faces and one zero face on this die.

Step 1: List the admissible numbers of negative outcomes In five throws a positive product allows zero, two or four negative results, with every remaining result positive. The probability of a negative result in a throw is $2/6$, and that of a positive result is $3/6$. If exactly j results are negative and the other $5 - j$ are positive, the probability is

$$\binom{5}{j}\left(\frac{2}{6}\right)^{j}\left(\frac{3}{6}\right)^{5-j}.$$

The binomial coefficient chooses the positions of the negative results. No zero face is allowed in these events; omitting this restriction would count products that are not positive.

Step 2: Add the three disjoint probabilities Using $j = 0, 2, 4$, we obtain

$$\begin{aligned} P &= \frac{3^5 + \binom{5}{2}2^2 3^3 + \binom{5}{4}2^4 3}{6^5} \\ &= \frac{243 + 1080 + 240}{7776} = \frac{1563}{7776} = \frac{521}{2592}. \end{aligned}$$

As a check, the even-parity counting identity gives the same numerator: $((3+2)^5 + (3-2)^5)/2 = 1563$. Options (A), (C) and (D) do not equal this sum of all admissible cases; in particular a single all-positive case is not the complete event.

Step 3: Conclusion The probability is $\boxed{521/2592}$, a dimensionless number. Hence, the correct option is (B).

Quick Tip to Solve: Count zero, two and four negative outcomes, and exclude the zero face before applying the even-parity binomial identity.

11. Among the statements :

$$(\mathrm{S1})\ ((\mathrm{p} \vee \mathrm{q}) \Rightarrow \mathrm{r}) \Leftrightarrow (\mathrm{p} \Rightarrow \mathrm{r})$$

$$(\mathrm{S2})\ ((\mathrm{p} \vee \mathrm{q}) \Rightarrow \mathrm{r}) \Leftrightarrow ((\mathrm{p} \Rightarrow \mathrm{r}) \vee (\mathrm{q} \Rightarrow \mathrm{r}))$$

[4 marks]

(A) only (S1) is a tautology

(B) neither (S1) nor (S2) is a tautology

(C) both (S1) and (S2) are tautologies

(D) only (S2) is a tautology

Correct Answer: (B) neither (S1) nor (S2) is a tautology

Solution:

Concept:

- A tautology is true for every assignment of truth values to its variables. One assignment making a proposition false is sufficient to show that it is not a tautology.
- The implication $A \Rightarrow B$ is false only when A is true and B is false. A biconditional is false when its two sides have different truth values.

Step 1: Choose a truth assignment that distinguishes the implications Set $p = \mathrm{F}$, $q = \mathrm{T}$ and $r = \mathrm{F}$. Then $p \vee q = \mathrm{T}$, and therefore

$$(p \vee q) \Rightarrow r = \mathrm{T} \Rightarrow \mathrm{F} = \mathrm{F}.$$

On the other hand, $p \Rightarrow r = \mathrm{F} \Rightarrow \mathrm{F} = \mathrm{T}$, because an implication with a false antecedent is true. The two sides of (S1) have opposite truth values under this assignment. Hence (S1) is false for this case and cannot be a tautology.

Step 2: Test the second biconditional with the same assignment For the chosen values we also have $q \Rightarrow r = \mathrm{T} \Rightarrow \mathrm{F} = \mathrm{F}$. Thus

$$(p \Rightarrow r) \vee (q \Rightarrow r) = \mathrm{T} \vee \mathrm{F} = \mathrm{T}.$$

The left side of (S2) was already found to be false, so (S2) is likewise false. The usual valid distributive identity uses conjunction here:

$$((p \vee q) \Rightarrow r) \equiv ((p \Rightarrow r) \wedge (q \Rightarrow r)).$$

Replacing that conjunction by disjunction is the defect in (S2). Options (A), (C) and (D) each assert that at least one of the two statements is a tautology, contrary to the explicit counterexample.

Step 3: Conclusion Neither (S1) nor (S2) is a tautology. Hence, the correct option is (B).

Quick Tip to Solve: To disprove a tautology, seek one counterexample; choosing a false consequent often makes implications easy to compare.

12. If $a_n = \frac{-2}{4n^2-16n+15}$, then $a_1 + a_2 + ... + a_{25}$ is equal to : [4 marks]

(A) $\frac{51}{144}$

(B) $\frac{49}{138}$

(C) $\frac{50}{141}$

(D) $\frac{52}{147}$

Correct Answer: (C) $\frac{50}{141}$

Solution:

Concept:

- Factor a quadratic denominator before summing a rational sequence. Partial fractions can convert a sum into differences of successive terms.
- In a telescoping series, intermediate contributions cancel, but the first and last uncancelled terms must retain their signs.

Step 1: Factor and decompose the general term The denominator factors as

$$4n^2 - 16n + 15 = (2n - 5)(2n - 3).$$

Since these two linear factors differ by two, the negative numerator gives the decomposition

$$a_n = \frac{-2}{(2n-5)(2n-3)} = \frac{1}{2n-3} - \frac{1}{2n-5}.$$

Combining the fractions on the right produces the numerator $(2n - 5) - (2n - 3) = -2$, verifying the sign. None of the denominator factors vanishes at an integer index in the required range, so this expression applies to all twenty-five terms.

Step 2: Cancel the intermediate fractions and evaluate the endpoints Write the beginning and end of the sum to see its structure:

$$\begin{aligned} S_{25} &= \left(\frac{1}{-1} - \frac{1}{-3}\right) + \left(\frac{1}{1} - \frac{1}{-1}\right) + \cdots + \left(\frac{1}{47} - \frac{1}{45}\right) \\ &= \frac{1}{47} - \frac{1}{-3} = \frac{1}{47} + \frac{1}{3} = \frac{50}{141}. \end{aligned}$$

Every intermediate odd reciprocal occurs once with each sign. The negative starting denominator is why the surviving initial contribution is positive. The fractions in (A), (B) and (D) correspond to different endpoints and are not the exact sum for the index range 1 through 25.

Step 3: Conclusion The sum is $\boxed{50/141}$. Hence, the correct option is (C).

Quick Tip to Solve: After partial fractions, check the first and last surviving denominators directly: here they are -3 and 47, not 1 and 49.

13. A straight line cuts off the intercepts $\mathrm{OA} = \mathrm{a}$ and $\mathrm{OB} = \mathrm{b}$ on the positive directions of x-axis and y axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of $\triangle\mathrm{OAB}$ is $\frac{98}{3}\sqrt{3}$, then $\mathrm{a}^2 - \mathrm{b}^2$ is equal to : [4 marks]

(A) $\frac{392}{3}$

(B) 98

(C) 196

(D) $\frac{196}{3}$

Correct Answer: (A) $\frac{392}{3}$

Solution:

Concept:

- A line with positive intercepts a, b has equation $x/a + y/b = 1$, and the triangle it forms with the coordinate axes has area $ab/2$.
- A normal vector to this line is $(1/a, 1/b)$. Its component ratio gives its inclination, allowing the angle condition to determine the ratio of intercepts.

Step 1: Translate the normal angle into an intercept ratio The positive normal from the origin into the first quadrant makes angle $\pi/6$ with the positive y-axis, so it makes angle $\pi/3$ with the positive x-axis. Both normal components are positive because both intercepts are positive. Therefore

$$\tan\frac{\pi}{3} = \frac{1/b}{1/a} = \frac{a}{b} = \sqrt{3}.$$

Thus $a = \sqrt{3}\,b$. Notice that the slope of the line itself is negative; the stated angle belongs to its perpendicular and must not be used as the line's inclination.

Step 2: Use the area to determine the squared intercepts From the given area,

$$\frac{ab}{2} = \frac{98\sqrt{3}}{3} \quad \Longrightarrow \quad ab = \frac{196\sqrt{3}}{3}.$$

Substitute $a = \sqrt{3}b$, obtaining $\sqrt{3}b^2 = 196\sqrt{3}/3$, hence $b^2 = 196/3$. It follows that $a^2 = 3b^2 = 196$, and

$$a^2 - b^2 = 196 - \frac{196}{3} = \frac{392}{3}.$$

Option (C) is a^2 alone and (D) is b^2 alone, so neither is the requested difference. Option (B) also fails the area and ratio calculation above.

Step 3: Conclusion The value is $\boxed{392/3}$ square coordinate units. Hence, the correct option is (A).

Quick Tip to Solve: The angle is measured from the y-axis: complement it before using tangent, and distinguish a line from its normal.

14. If $\mathrm{P(h,k)}$ be a point on the parabola $x = 4y^2$, which is nearest to the point $\mathrm{Q}(0, 33)$, then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to : [4 marks]

(A) 8

(B) 2

(C) 6

(D) 4

Correct Answer: (C) 6

Solution:

Concept:

- Minimizing distance is equivalent to minimizing its square, since the square-root function is increasing on nonnegative numbers.
- Complete the square to convert a translated parabola into $(y - k)^2 = 4a(x - h)$. Its directrix is the vertical line $x = h - a$.

Step 1: Minimize the squared distance along the first parabola Write a general point on $x = 4y^2$ as $P = (4k^2, k)$. Its squared distance from $Q = (0, 33)$ is

$$f(k) = 16k^4 + (k - 33)^2.$$

At a stationary point,

$$f'(k) = 64k^3 + 2k - 66 = 0 \quad \Longleftrightarrow \quad 32k^3 + k - 33 = 0.$$

The value $k = 1$ satisfies this equation. Moreover $f''(k) = 192k^2 + 2 > 0$ for every real k, so f is strictly convex. This stationary point is its unique global minimum; also $f(k)$ tends to infinity as $|k|$ grows. Hence the nearest point is $P = (4, 1)$.

Step 2: Find the second parabola's directrix and the required distance The directrix belongs to the second parabola in the question, not the curve used in the minimization. Complete its square:

$$y^2 = 4x + 4y \quad \Longleftrightarrow \quad (y - 2)^2 = 4(x + 1).$$

Its vertex is $(-1, 2)$ and its parameter is $a = 1$, so its directrix is $x = -1 - 1 = -2$. The perpendicular distance of $(4, 1)$ from this vertical line is

$$|4 - (-2)| = 6.$$

The values 8, 2 and 4 in the other options are not this horizontal distance; no distance from Q or from the first parabola's directrix is requested.

Step 3: Conclusion The required distance is $\boxed{6}$ coordinate units. Hence, the correct option is (C).

Quick Tip to Solve: Use the first parabola only to find P; then complete the square of the second parabola before locating its directrix.

15. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a} = \alpha\vec{b} - \hat{n}, (\alpha \neq 0)$ and $\vec{b} \cdot \vec{c} = 12$, then $\left|\vec{c} \times (\vec{a} \times \vec{b})\right|$ is equal to : [4 marks]

(A) 15

(B) 9

(C) 6

(D) 12

Correct Answer: (D) 12

Solution:

Concept:

- The vector triple-product identity is $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \mathbf{v}(\mathbf{u} \cdot \mathbf{w}) - \mathbf{w}(\mathbf{u} \cdot \mathbf{v})$. The order of the two terms is essential.
- A unit vector has magnitude one. Perpendicular vectors have zero scalar product, so $\mathbf{c} \cdot \hat{\mathbf{n}} = 0$.

Step 1: Find the second scalar product Use the given linear relation before expanding the cross products. Taking its scalar product with $\mathbf{c}$ gives

$$\mathbf{a} \cdot \mathbf{c} = (\alpha\mathbf{b} - \hat{\mathbf{n}}) \cdot \mathbf{c} = 12\alpha.$$

The perpendicularity condition removes the unit-vector contribution; no individual components of the three non-zero vectors are required.

Step 2: Apply the triple-product identity Expand with the outer vector $\mathbf{c}$, keeping the order $\mathbf{a} \times \mathbf{b}$ unchanged:

$$\begin{aligned} \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) &= \mathbf{a}(\mathbf{c} \cdot \mathbf{b}) - \mathbf{b}(\mathbf{c} \cdot \mathbf{a}) \\ &= 12\mathbf{a} - 12\alpha\mathbf{b} \\ &= 12(\mathbf{a} - \alpha\mathbf{b}) = -12\hat{\mathbf{n}}. \end{aligned}$$

Thus the unknown multiplier cancels exactly. Taking magnitudes yields $12|\hat{\mathbf{n}}| = 12$. The values 15, 9 and 6 would require a different scalar product or a non-unit normal; none is compatible with the stated data.

Step 3: Conclusion The required magnitude is $\boxed{12}$. Hence, the correct option is (D).

Quick Tip to Solve: For a nested cross product, expand it into scalar products first; the given linear relation can then eliminate the unknown vector coefficient.

16. The coefficient of x^{301} in $(1+x)^{500}+x(1+x)^{499}+x^2(1+x)^{498}+\ldots+x^{500}$ is : [4 marks]

(A) ${}^{500}C_{300}$

(B) ${}^{501}C_{200}$

(C) ${}^{500}C_{301}$

(D) ${}^{501}C_{302}$

Correct Answer: (B) ${}^{501}C_{200}$

Solution:

Concept:

- The finite geometric sum satisfies $\sum_{k=0}^{n} a^{n-k}b^k = (a^{n+1}-b^{n+1})/(a-b)$ when $a \neq b$.
- The coefficient of x^r in $(1+x)^n$ is $\binom{n}{r}$, and binomial symmetry gives $\binom{n}{r} = \binom{n}{n-r}$.

Step 1: Collapse the polynomial sum Let the displayed polynomial be $S(x)$. Its general term is $x^k(1+x)^{500-k}$, with $k = 0, 1, \ldots, 500$. Choose $a = 1+x$, $b = x$; then $a - b = 1$. Hence

$$S(x) = \sum_{k=0}^{500} x^k(1+x)^{500-k} = (1+x)^{501} - x^{501}.$$

This identity can also be checked by multiplying the original sum by $(1+x)-x$: all intermediate terms cancel. The extra x^{501} affects only that highest power.

Step 2: Extract the requested power The coefficient of x^{301} in the subtractive term x^{501} is zero. Therefore

$$[x^{301}]S(x) = \binom{501}{301} = \binom{501}{200}.$$

Options (A) and (C) incorrectly retain upper index 500 after summing all terms. Option (D) selects power 302 instead of 301; symmetry would pair 302 with 199, rather than 200. Thus only the coefficient with upper index 501 and lower index 200 matches the required term.

Step 3: Conclusion The coefficient is $\boxed{\binom{501}{200}}$. Hence, the correct option is (B).

Quick Tip to Solve: When adjacent terms are powers of x and 1+x, exploit their difference of one before expanding hundreds of binomial terms.

17. Let $A = \begin{pmatrix} \mathrm{m} & \mathrm{n} \\ \mathrm{p} & \mathrm{q} \end{pmatrix}, \mathrm{d} = |\mathrm{A}| \neq 0$ **and** $|\mathrm{A} - \mathrm{d}(\mathrm{AdjA})| = 0$**. Then** [4 marks]

(A) $1 + \mathrm{d}^2 = \mathrm{m}^2 + \mathrm{q}^2$

(B) $1 + d^2 = (m + q)^2$

(C) $(1 + d)^2 = m^2 + q^2$

(D) $(1 + d)^2 = (m + q)^2$

Correct Answer: (D) $(1 + d)^2 = (m + q)^2$

Solution:

Concept:

- For $A = \begin{pmatrix} m & n \\ p & q \end{pmatrix}$, its determinant is $d = mq - np$, and $\operatorname{adj} A = \begin{pmatrix} q & -n \\ -p & m \end{pmatrix}$.
- A two-by-two determinant is the product of its diagonal entries minus the product of its off-diagonal entries. The condition $d \neq 0$ permits division by d.

Step 1: Write the matrix inside the determinant Subtract the scalar multiple of the adjugate entry by entry:

$$A - d \operatorname{adj} A = \begin{pmatrix} m - dq & (1 + d)n \\ (1 + d)p & q - dm \end{pmatrix}.$$

The off-diagonal entries contain $1 + d$, because the corresponding adjugate entries already have negative signs. This is the step that distinguishes a square of $1 + d$ from the expression $1 + d^2$.

Step 2: Expand and simplify the zero determinant The stated condition gives

$$\begin{aligned} 0 &= (m - dq)(q - dm) - (1 + d)^2 np \\ &= (1 + d^2)mq - d(m^2 + q^2) - (1 + 2d + d^2)np \\ &= (1 + d^2)(mq - np) - d(m^2 + q^2 + 2np) \\ &= d\{1 + d^2 - m^2 - q^2 - 2np\}. \end{aligned}$$

After division by d, substitute $np = mq - d$. This yields

$$0 = 1 + d^2 - m^2 - q^2 - 2mq + 2d = (1 + d)^2 - (m + q)^2.$$

Options (A) and (B) omit the linear term $2d$, while (C) omits the cross term $2mq$. Neither omission is justified by the hypotheses.

Step 3: Conclusion Consequently $\boxed{(1 + d)^2 = (m + q)^2}$. Hence, the correct option is (D).

Quick Tip to Solve: For a two-by-two adjugate equation, write all four entries explicitly and use mq-np=d only after expanding the determinant.

18. The line l_1 passes through the point (2, 6, 2) and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is : [4 marks]

(A) 9

(B) 7

(C) $\frac{19}{3}$

(D) $\frac{13}{3}$

Correct Answer: (A) 9

Solution:

Concept:

- A line perpendicular to $ax + by + cz = k$ has direction vector (a, b, c), the normal to that plane.
- For non-parallel lines through P and Q with direction vectors u and v, their shortest distance is $D = |(Q - P) \cdot (u \times v)|/|u \times v|$. This projects the joining vector onto the common perpendicular.

Step 1: Identify the points and directions The first line passes through $P = (2, 6, 2)$. Since it is perpendicular to the stated plane, take $u = (2, 1, -2)$. From the symmetric equation of the second line, take $Q = (-1, -4, 0)$ and $v = (2, -3, 2)$. Hence $Q - P = (-3, -10, -2)$. The constant 10 fixes the position of the plane but does not change its normal vector or the specified first line.

Step 2: Project onto the common perpendicular Calculate

$$u \times v = (-4, -8, -8), \qquad |u \times v| = \sqrt{16 + 64 + 64} = 12.$$

The nonzero cross product confirms that the lines are not parallel. Their scalar triple product is

$$(Q - P) \cdot (u \times v) = (-3)(-4) + (-10)(-8) + (-2)(-8) = 108.$$

Therefore $D = 108/12 = 9$. One may instead use the proportional common-normal vector $(1, 2, 2)$, obtaining $27/3 = 9$. The alternatives 7, $19/3$ and $13/3$ do not equal this perpendicular projection; they cannot follow from the specified points and direction ratios.

Step 3: Conclusion The shortest distance is $\boxed{9}$ coordinate units. Hence, the correct option is (A).

Quick Tip to Solve: Reduce the cross-product direction by a common factor before taking its dot product and magnitude; the shortest-distance quotient stays unchanged.

19. The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to $x + 90y + 2 = 0$ is : [4 marks]

(A) 2

(B) 3

(C) 4

(D) 0

Correct Answer: (C) 4

Solution:

Concept:

- At a regular point of $y = f(x)$, the tangent slope is $f\prime(x)$, and a non-horizontal tangent has normal slope $-1/f\prime(x)$.
- Parallel non-vertical lines have the same slope. Distinct real x-coordinates on a graph y=f(x) correspond to distinct points, even if a polynomial root is repeated.

Step 1: Translate the normal-slope requirement The given line is $y = -x/90 - 1/45$, so its slope is $-1/90$. Equating the normal slope to this value gives

$$-\frac{1}{f\prime(x)} = -\frac{1}{90}, \qquad f\prime(x) = 90.$$

The finite nonzero target slope excludes vertical or horizontal normals. Differentiate the polynomial carefully:

$$f\prime(x) = 270x^4 - 540x^3 - 210x^2 + 360x + 210.$$

Step 2: Factor and count the real solutions Subtract 90 and divide by 30:

$$9x^4 - 18x^3 - 7x^2 + 12x + 4 = 0.$$

Its factorisation is

$$(x - 2)(x - 1)(3x + 1)(3x + 2) = 0.$$

For example, multiplying the two quadratics $(x^2 - 3x + 2)(9x^2 + 9x + 2)$ reproduces the displayed quartic. The four distinct roots are $2, 1, -1/3, -2/3$. At every one the tangent slope is 90 and the normal slope is $-1/90$. Each supplies one point on the given graph. Thus counts 2 and 3 omit valid roots, and count 0 incorrectly concludes that no matching slope exists.

Step 3: Conclusion There are $\boxed{4}$ such points. Hence, the correct option is (C).

Quick Tip to Solve: Convert a prescribed normal slope to the tangent slope before differentiating; count distinct real roots of the resulting derivative equation.

20. Let

$$y = x + 2, \qquad 4y = 3x + 6, \qquad 3y = 4x + 1$$

be three tangent lines to the circle

$$(x - h)^2 + (y - k)^2 = r^2.$$

Then $h + k$ is equal to: [4 marks]

(A) 6

(B) 5 (1 + $\sqrt{2}$)

(C) 5

(D) $5\sqrt{2}$

Correct Answer: Both (B) and (C) occur for positive-radius circles tangent to the three stated lines; a centre-location condition is needed for a unique answer.

Solution:

Concept:

- A circle of centre (h, k) is tangent to $ax + by + c = 0$ exactly when its radius equals $|ah + bk + c|/\sqrt{a^2 + b^2}$. Absolute values allow centres on either side of each line.
- Three non-concurrent lines generally have an incircle and three excircles. Without a condition locating the centre, all positive-radius circles tangent to the three full lines must be considered.

Step 1: Equate the distances to the last two lines The three radius equations are

$$r = \frac{|h - k + 2|}{\sqrt{2}} = \frac{|3h - 4k + 6|}{5} = \frac{|4h - 3k + 1|}{5}.$$

The last equality splits into two angle-bisector equations. Equal signs give $3h - 4k + 6 = 4h - 3k + 1$, or $h + k = 5$. Opposite signs give $3h - 4k + 6 = -4h + 3k - 1$, or $k = h + 1$. Both must be retained because the question imposes no location restriction.

Step 2: Exhibit both listed possibilities In the second branch, $k = h + 1$, the first distance is $1/\sqrt{2}$. The second distance equation becomes $|2 - h|/5 = 1/\sqrt{2}$, giving

$$h = 2 \pm \frac{5}{\sqrt{2}}, \quad k = 3 \pm \frac{5}{\sqrt{2}}, \quad h + k = 5 \pm 5\sqrt{2}.$$

Both radii are positive. In particular the plus sign realizes option (B). The other branch realizes option (C): put $k = 5 - h$, then $|7h - 14|/5 = |2h - 3|/\sqrt{2}$; its real solutions give positive radii, so centres with $h + k = 5$ exist. Options (A) and (D) are not these sums. The supplied key chooses (C), but the explicit positive-radius centre $(2 + 5/\sqrt{2}, 3 + 5/\sqrt{2})$ proves that (B) also satisfies all printed tangencies.

Step 3: Conclusion The stated conditions do not determine a unique sum: $h + k$ can be 5 or $5 \pm 5\sqrt{2}$. The supplied key selects (C), but both listed options (B) and (C) are possible for the question as printed.

Quick Tip to Solve: Keep both signs when equating distances to lines. Choosing just one angle bisector can silently discard excircles allowed by the wording.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. $\lim\limits_{x\to 0}\dfrac{48}{x^4}\displaystyle\int_0^x \dfrac{t^3}{1+t^6}\,\mathrm{d}t$ **is equal to _____.** [4 marks]

Solution:

Concept:

- The fundamental theorem of calculus gives $\frac{d}{dx}\int_0^x g(t)\,dt = g(x)$ when g is continuous near zero.
- For a quotient tending to the indeterminate form $0/0$, L'Hopital's rule replaces it by the quotient of derivatives when the hypotheses hold. Here the denominator derivative is nonzero away from zero.

Step 1: Check the indeterminate form Let $F(x) = \int_0^x t^3/(1+t^6)\,dt$. The integrand is continuous for every real t because $1+t^6 > 0$. Thus $F(0) = 0$, and the required expression is $48F(x)/x^4$, a $0/0$ form as x tends to zero. Continuity and differentiability hold on both sides of zero, so the limit can be checked without a one-sided restriction.

Step 2: Differentiate the numerator and denominator Apply the fundamental theorem followed by L'Hopital's rule once:

$$\lim_{x\to 0}\frac{48F(x)}{x^4} = \lim_{x\to 0}\frac{48x^3/(1+x^6)}{4x^3}$$
$$= \lim_{x\to 0}\frac{12}{1+x^6} = 12.$$

The cancellation is performed only for $x \neq 0$, as is appropriate inside a limit. As an independent check, substitute $t = xu$; the expression becomes $48\int_0^1 u^3/(1+x^6u^6)\,du$, whose limit is $48\int_0^1 u^3du = 12$. The orientation of the substitution remains correct for negative x as well, since the four powers of x combine to x^4.

Step 3: Conclusion The required limit is $\boxed{12}$, a dimensionless value, agreeing with the stated numerical answer key.

Quick Tip to Solve: For an integral divided by x to a power, use the fundamental theorem in L'Hopital's rule, or scale t=xu to expose the leading power.

22. Number of 4-digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3 and 5, and are divisible by 15, is equal to _____. [4 marks]

Solution:

Concept:

- A number is divisible by 15 exactly when it is divisible by both 3 and 5. Divisibility by 3 depends only on the sum of its digits.
- Repetition is allowed and all available digits are nonzero, so every ordered choice for the first three places gives a valid four-digit number once its last digit is fixed.

Step 1: Fix the units digit The allowed digits are 1, 2, 3 and 5. Divisibility by 5 forces the units digit to be 5. Let the other digits be a, b, c. Their sum must satisfy

$$a+b+c+5 \equiv 0 \pmod 3, \qquad a+b+c \equiv 1 \pmod 3.$$

The residues 0, 1 and 2 are represented by 3, by 1, and by the two digits 2 and 5, respectively. Thus their multiplicities are $(1, 1, 2)$.

Step 2: Count ordered pairs by residue For the first two positions, a residue sum of 0 arises from $(0,0), (1,2), (2,1)$, giving $1+2+2=5$ pairs. A sum of 1 arises from $(0,1), (1,0), (2,2)$, giving $1+1+4=6$ pairs. A sum of 2 arises from $(0,2), (2,0), (1,1)$, giving $2+2+1=5$ pairs. The total is $5+6+5=16$, as required for all 4^2 ordered pairs.

Step 3: Complete the third position To make the first three digits have residue 1, pair residues 0, 1 and 2 require third-digit residues 1, 0 and 2, respectively. The numbers of choices for those digits are 1, 1 and 2. Hence the count is

$$5 \cdot 1 + 6 \cdot 1 + 5 \cdot 2 = 21.$$

We count ordered placements, so changing the order of two unequal digits correctly produces a different number.

Step 4: Conclusion The required number of four-digit numbers divisible by 15 is $\boxed{21}$. This is a dimensionless count.

Quick Tip to Solve: Fix the last digit first, then count the other positions by residues modulo 3; do not list every four-digit number.

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23. Let $z = 1+i$ and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}\arg(z_1)$ is equal to _____. [4 marks]

Solution:

Concept:

- For $z = 1+i$, its conjugate is $\bar{z} = 1-i$, and the reciprocal is obtained by multiplying numerator and denominator by the conjugate.
- The principal argument is determined by the quadrant as well as the ratio of imaginary to real parts. A negative real part with a positive imaginary part places a number in the second quadrant.

Step 1: Evaluate the numerator and denominator First use $i^2 = -1$ to obtain

$$1 + i\bar{z} = 1 + i(1-i) = 2 + i.$$

Also $1 - z = -i$, so

$$\bar{z}(1-z) = (1-i)(-i) = -1 - i.$$

The reciprocal of z is

$$\frac{1}{z} = \frac{1-i}{(1+i)(1-i)} = \frac{1-i}{2}.$$

Therefore the complete denominator, retaining both its real and imaginary parts, becomes

$$-1-i+\frac{1-i}{2}=\frac{-1-3i}{2}.$$

It is nonzero, so the given quotient is well-defined.

Step 2: Simplify the quotient Substitution gives

$$z_1=\frac{2+i}{(-1-3i)/2}=\frac{4+2i}{-1-3i}.$$

Multiply numerator and denominator by $-1+3i$:

$$z_1=\frac{(4+2i)(-1+3i)}{(-1)^2+3^2}=\frac{-10+10i}{10}=-1+i.$$

In the numerator, the real part is $-4-6=-10$, while the imaginary part is $12i-2i=10i$.

Step 3: Determine the principal argument The point representing $-1+i$ has coordinates $(-1,1)$, so it lies in quadrant II. Its reference angle is $\pi/4$, because the absolute real and imaginary parts are equal. Hence

$$\arg(z_1)=\pi-\frac{\pi}{4}=\frac{3\pi}{4},\qquad \frac{12}{\pi}\arg(z_1)=\frac{12}{\pi}\frac{3\pi}{4}=9.$$

Taking only the inverse tangent of the ratio would miss the quadrant.

Step 4: Conclusion The requested dimensionless value is $\boxed{9}$, using the principal argument $3\pi/4$.

Quick Tip to Solve: After simplifying a complex quotient, mark the signs of its real and imaginary parts before choosing its argument.

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24. Let α be the area of the larger region bounded by the curve $y^2=8x$ and the lines $y=x$ and $x=2$, which lies in the first quadrant. Then the value of 3α is equal to _____. [4 marks]

Solution:

Concept:

- In the first quadrant the parabola $y^2=8x$ has upper branch $y=\sqrt{8x}$. The area between two graphs is the integral of the upper ordinate minus the lower ordinate.
- A vertical line inside a bounded region can divide it into two regions. Compute their areas separately before deciding which one is larger.

Step 1: Locate the intersections The line $y=x$ meets the parabola where $x^2=8x$, so $x=0$ or $x=8$. The two intersection points are $(0,0)$ and $(8,8)$. At $x=2$, the line gives $y=2$, while the positive branch of the parabola gives $y=4$. For $0<x<8$, $\sqrt{8x}>x$; thus the parabola is above the line throughout the bounded first-quadrant region.

Step 2: Find the area to the left of the vertical line The region on the left is bounded by the two

curves from $x = 0$ to $x = 2$ and by the vertical segment at $x = 2$. Its area is

$$A_L = \int_0^2 \left(\sqrt{8x} - x\right) dx = \left[\frac{4\sqrt{2}}{3}x^{3/2} - \frac{x^2}{2}\right]_0^2.$$

Since $2^{3/2} = 2\sqrt{2}$, this gives

$$A_L = \frac{16}{3} - 2 = \frac{10}{3}.$$

Step 3: Find and compare the other area The full area between the line and parabola is

$$A = \left[\frac{4\sqrt{2}}{3}x^{3/2} - \frac{x^2}{2}\right]_0^8 = \frac{128}{3} - 32 = \frac{32}{3}.$$

Therefore the region to the right of $x = 2$ has area

$$A_R = A - A_L = \frac{32}{3} - \frac{10}{3} = \frac{22}{3}.$$

This exceeds $10/3$, so the larger region is the right-hand one, and $\alpha = 22/3$.

Step 4: Conclusion The larger region has area $\alpha = \frac{22}{3}$ square units. Consequently, $\boxed{3\alpha = 22}$.

Quick Tip to Solve: When a line divides an enclosed region, calculate the whole area and one piece; subtraction identifies the other piece and prevents selecting the smaller one.

25. Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the number of one-one functions $f : \mathrm{S} \to \mathrm{P(S)}$, where $\mathrm{P(S)}$ denote the power set of S, such that $f(n) \subset f(\mathrm{m})$ where $n < m$ is _____. [4 marks]

Solution:

Concept:

- The condition $f(n) \subset f(m)$ for $n < m$, together with injectivity, makes $f(1), \ldots, f(6)$ a strictly increasing chain of six subsets of a six-element set.
- Encode a nested chain by the stage at which each element first enters it. Every strict transition must introduce at least one new element.

Step 1: Encode the chain by disjoint groups Write $A_j = f(j)$. Partition the six labelled elements of S into $B_0 = A_1$, the five difference groups $B_j = A_{j+1} \setminus A_j$ for $1 \le j \le 5$, and $B_6 = S \setminus A_6$. These seven groups are disjoint and their union is S. Each of $B_1, \ldots, B_5$ is nonempty because each inclusion is strict. The initial and unused groups B_0, B_6 may be empty. Conversely, such a partition uniquely reconstructs every set in the chain, so counting these assignments counts functions without duplication.

Step 2: Count assignments with one optional element First suppose each of the five mandatory groups contains exactly one element. There is then one element left, which must belong to either B_0 or B_6 in this case. Choose that element in 6 ways, assign the other five bijectively to the ordered

mandatory groups in $5!$ ways, and choose its optional group in 2 ways. This gives

$$6 \cdot 5! \cdot 2 = 1440.$$

The groups are distinguished by their positions in the chain.

Step 3: Count assignments with a double introduction The remaining possibility is that both optional groups are empty and one mandatory group contains two elements. Choose that group in 5 ways, choose its pair of elements in $\binom{6}{2}$ ways, and assign the remaining four elements to the other four mandatory groups in $4!$ ways. Thus this case contributes

$$5\binom{6}{2}4! = 5 \cdot 15 \cdot 24 = 1800.$$

No group can have three elements, and no two mandatory groups can both have two: five nonempty groups would then require at least seven elements.

Step 4: Conclusion The two disjoint cases give $1440 + 1800 = \boxed{3240}$ one-one functions. This is a dimensionless count.

Quick Tip to Solve: For a strictly nested chain, count element-entry stages: each strict inclusion needs a nonempty difference group, while the first and never-used groups may be empty.

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26. Let $f^1(x) = \frac{3x+2}{2x+3}, x \in \mathbf{R} - \left\{\frac{-3}{2}\right\}$ For $\mathrm{n} \geq 2$, define $f^{\mathrm{n}}(x) = f^1 \mathrm{o} f^{\mathrm{n}-1}(x)$. If $f^5(x) = \frac{\mathrm{a}x+\mathrm{b}}{\mathrm{b}x+\mathrm{a}}, \gcd(\mathrm{a}, \mathrm{b}) = 1$, then $\mathrm{a} + \mathrm{b}$ is equal to _____. [4 marks]

Solution:

Concept:

- A transformation $x \mapsto (ax+b)/(bx+a)$ is represented by the coefficient matrix $\begin{pmatrix} a & b \\ b & a \end{pmatrix}$; composing transformations multiplies these matrices.
- For symmetric coefficients, the sum and difference of the two coefficients evolve separately. Cancel common coefficient factors only after forming the required iterate.

Step 1: Set up the coefficient recurrence Write $f^n(x) = (A_n x + B_n)/(B_n x + A_n)$ wherever the successive compositions are defined. For the first transformation, $A_1 = 3$ and $B_1 = 2$. Substituting the expression for f^n into f^1 and clearing the inner denominator yields

$$f^{n+1}(x) = \frac{(3A_n + 2B_n)x + (3B_n + 2A_n)}{(2A_n + 3B_n)x + (2B_n + 3A_n)}.$$

Therefore $A_{n+1} = 3A_n + 2B_n$ and $B_{n+1} = 2A_n + 3B_n$.

Step 2: Track the sum and difference Adding the recurrences gives

$$A_{n+1} + B_{n+1} = 5(A_n + B_n).$$

Since $A_1 + B_1 = 5$, it follows that $A_n + B_n = 5^n$. Subtracting gives

$$A_{n+1} - B_{n+1} = A_n - B_n.$$

The initial difference is 1, so $A_n - B_n = 1$ for every positive integer n. These relations determine both coefficients directly and avoid five long substitutions.

Step 3: Extract the fifth iterate in lowest terms At $n = 5$,

$$A_5 + B_5 = 5^5 = 3125, \qquad A_5 - B_5 = 1.$$

Solving these two equations gives

$$A_5 = \frac{3125 + 1}{2} = 1563, \qquad B_5 = \frac{3125 - 1}{2} = 1562.$$

These consecutive positive integers have greatest common divisor 1, so there is no common coefficient factor to cancel. Thus the required coprime coefficient pair is $(a, b) = (1563, 1562)$. The rational identity is used on the domain where the original successive compositions exist.

Step 4: Conclusion The reduced fifth iterate is $f^5(x) = \dfrac{1563x + 1562}{1562x + 1563}$, and the requested dimensionless value is $\boxed{a + b = 3125}$.

Quick Tip to Solve: For a symmetric fractional linear transformation, track coefficient sum and difference; they multiply by the two matrix eigenvalues at every composition.

---

27. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then $\mathrm{a} + 3\mathrm{b} - 5$ **is equal to _____.** [4 marks]

Solution:

Concept:

- For N observations with mean μ, the sum is $N\mu$. Their population variance is $\sigma^2 = \frac{1}{N}\sum x_i^2 - \mu^2$.
- When deleting an observation, subtract its value from the sum and its square from the sum of squares, then use the reduced number of observations in both averages.

Step 1: Recover the original totals There are seven observations with mean 8, so their sum is

$$\sum_{i=1}^{7} x_i = 7 \cdot 8 = 56.$$

Rearranging the variance identity before making any deletion gives

$$\frac{1}{7}\sum_{i=1}^{7} x_i^2 = 16 + 8^2 = 80.$$

Hence the original sum of squares is

$$\sum_{i=1}^{7} x_i^2 = 7 \cdot 80 = 560.$$

This is the raw second moment total; it must not be confused with the sum of squared deviations about the original mean.

Step 2: Calculate the new mean and variance Deleting the observation 14 leaves six observations. Their sum is $56 - 14 = 42$, so

$$a = \frac{42}{6} = 7.$$

Their sum of squares is

$$560 - 14^2 = 560 - 196 = 364.$$

Using the new mean, the variance is therefore

$$b = \frac{364}{6} - 7^2 = \frac{182}{3} - 49 = \frac{35}{3}.$$

The denominator is 6 because variance here is the mean squared deviation of the remaining observations; there is no sample-variance correction.

Step 3: Substitute into the required expression Now combine the two updated quantities, rather than the original mean and variance:

$$a + 3b - 5 = 7 + 3\left(\frac{35}{3}\right) - 5 = 7 + 35 - 5 = 37.$$

As an additional check, the new variance is positive and smaller than 16, consistent with removing the relatively high observation 14. The exact fraction is retained until multiplication by 3, so no rounding enters the result.

Step 4: Conclusion The remaining observations have mean $a = 7$ and variance $b = 35/3$. The requested numerical value is $\boxed{37}$. No measurement unit is specified.

Quick Tip to Solve: Reconstruct the sum and sum of squares first; after removing a value, recompute variance using the new mean and the new observation count.

28. Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$**, where** $a, b, c \in \mathbb{Z}$ **and** $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ **Then** $a^2 - b + c$ **is equal to _____.** [4 marks]

Solution:

Concept:

- The exponential series is $e = \sum_{n=0}^{\infty} 1/n!$. Falling-factorial decompositions reduce sums involving powers of n divided by $n!$ to shifted copies of this series.
- The even and odd reciprocal-factorial sums are $\cosh 1 = (e + e^{-1})/2$ and $\sinh 1 = (e - e^{-1})/2$,

respectively. Separate any exceptional initial term before shifting indices.

Step 1: Separate the two convergent sums Cancel the factorials term by term in the stated summand:

$$T = \sum_{n=0}^{\infty} \frac{n^3}{n!} + \sum_{n=0}^{\infty} \frac{2n-1}{(2n)!} = T_1 + T_2.$$

Both series converge absolutely by the ratio test, so the separation and index shifts are valid. In particular, the $n = 0$ contribution to T_2 is -1; it must be kept, since a negative factorial cannot be used to rewrite that term.

Step 2: Evaluate the cubic factorial sum Use

$$n^3 = n(n-1)(n-2) + 3n(n-1) + n.$$

Each nonzero falling-factorial term cancels the corresponding factors of $n!$. Thus

$$\begin{aligned} T_1 &= \sum_{n=3}^{\infty} \frac{1}{(n-3)!} + 3\sum_{n=2}^{\infty} \frac{1}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \\ &= e + 3e + e = 5e. \end{aligned}$$

The starting indices ensure no undefined factorials appear.

Step 3: Evaluate the even-factorial sum For $n \geq 1$, $(2n-1)/(2n)! = 1/(2n-1)! - 1/(2n)!$. Consequently

$$\begin{aligned} T_2 &= -1 + \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=1}^{\infty} \frac{1}{(2n)!} \\ &= -1 + \sinh 1 - (\cosh 1 - 1) \\ &= \sinh 1 - \cosh 1 = -e^{-1}. \end{aligned}$$

It follows that $T = 5e - 1/e$, giving the integer coefficients $a = 5$, $b = -1$, and $c = 0$.

Step 4: Conclusion The series equals $5e - \frac{1}{e}$. Hence the requested dimensionless value is $a^2 - b + c = 25 - (-1) + 0 = \boxed{26}$.

Quick Tip to Solve: Expand powers into falling factorials, and isolate the zero-index term before converting even-factorial sums into their odd and even exponential parts.

29. If the equation of the plane passing through the point $(1, 1, 2)$ and perpendicular to the line $x - 3y + 2z - 1 = 0 = 4x - y + z$ is $\mathrm{A}x + \mathrm{B}y + \mathrm{C}z = 1$, then $140(\mathrm{C} - \mathrm{B} + \mathrm{A})$ is equal to _____.

[4 marks]

Solution:

Concept:

- The direction of the intersection of two planes is perpendicular to both their normals, so it is given by their cross product.

- A plane perpendicular to a line has its normal parallel to the direction of that line. A plane with normal (a, b, c) through (x_0, y_0, z_0) has equation $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

Step 1: Find the direction of the given line The line is the intersection of $x - 3y + 2z - 1 = 0$ and $4x - y + z = 0$. Their normals are $\mathbf{n}_1 = (1, -3, 2)$ and $\mathbf{n}_2 = (4, -1, 1)$. Thus a direction vector of the line is

$$\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 = (-1, 7, 11).$$

Indeed, its dot products with the two normals are $-1 - 21 + 22 = 0$ and $-4 - 7 + 11 = 0$, confirming the direction.

Step 2: Construct and normalize the required plane Use $(-1, 7, 11)$ as the normal to the required plane. Passing through $(1, 1, 2)$ gives

$$-(x - 1) + 7(y - 1) + 11(z - 2) = 0.$$

Collecting the constants yields

$$-x + 7y + 11z = 28.$$

The requested right side is 1, so divide every coefficient by 28. Consequently $A = -1/28$, $B = 7/28$, and $C = 11/28$. The normalization is essential because a plane equation can otherwise be multiplied by any nonzero constant.

Step 3: Evaluate the specified combination Substitute the normalized coefficients, preserving the minus sign before B:

$$140(C - B + A) = 140\left(\frac{11 - 7 - 1}{28}\right) = 5 \times 3 = 15.$$

As a point check, the normalized left side at $(1, 1, 2)$ is $(-1 + 7 + 22)/28 = 1$, as required.

Step 4: Conclusion The plane is $-x + 7y + 11z = 28$, and the requested dimensionless numerical value is $\boxed{15}$.

Quick Tip to Solve: For a line expressed as two intersecting planes, cross their normals first; normalize the final plane before reading off its coefficients.

30. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1 : \vec{r}(3\hat{i} - 5\hat{j} + \hat{k}) = 7$ and $P_2 : \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is _____. [4 marks]

Solution:

Concept:

- The acute angle between two planes equals the acute angle between their normal vectors: $\cos\theta = |\mathbf{n}_1 \cdot \mathbf{n}_2|/(|\mathbf{n}_1||\mathbf{n}_2|)$.
- The squared perpendicular distance from (x_0, y_0, z_0) to $ax + by + cz = d$ is $(ax_0 + by_0 + cz_0 - d)^2/(a^2 + b^2 + c^2)$.

Step 1: Obtain the equation for the parameter The normals are $(3, -5, 1)$ and $(\lambda, 1, -3)$. Their dot product is $3\lambda - 8$, and their squared lengths are 35 and $\lambda^2 + 10$. Since $\sin\theta = 2\sqrt{6}/5$, we have $\cos^2\theta = 1 - 24/25 = 1/25$. Therefore

$$\frac{(3\lambda - 8)^2}{35(\lambda^2 + 10)} = \frac{1}{25}.$$

Squaring introduces no sign restriction because the plane angle uses an absolute dot product.

Step 2: Solve and order the two roots Cross multiplication and expansion give

$$25(9\lambda^2 - 48\lambda + 64) = 35\lambda^2 + 350.$$

Hence

$$19\lambda^2 - 120\lambda + 125 = 0 = (19\lambda - 25)(\lambda - 5).$$

Since $25/19 < 5$, the ordered values are $\lambda_1 = 25/19$ and $\lambda_2 = 5$. The point in the question is consequently

$$(38\lambda_1, 10\lambda_2, 2) = (50, 50, 2).$$

Step 3: Apply the distance formula The first plane is $3x - 5y + z = 7$. At the point found above, the signed plane expression is $150 - 250 + 2 - 7 = -105$. Its sign affects which side of the plane contains the point, but not its distance. Thus

$$d^2 = \frac{(-105)^2}{3^2 + (-5)^2 + 1^2} = \frac{11025}{35} = 315.$$

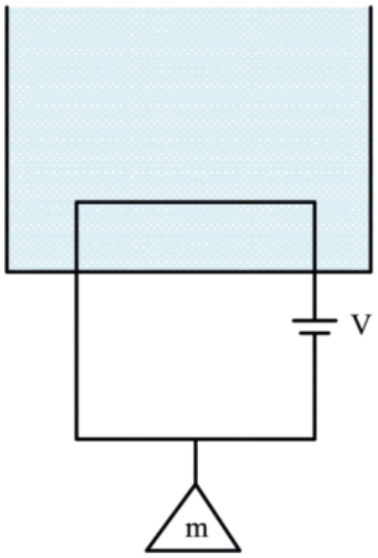
Step 4: Conclusion The square of the perpendicular distance is $\boxed{315}$ square coordinate units.

Quick Tip to Solve: Convert the given sine to cosine squared before using normals, and order the parameter roots before constructing the point.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A massless square loop, of wire of resistance 10Ω, supporting a mass of 1 g, hangs vertically with one of its sides in a uniform magnetic field of 10^3G, directed outwards in the shaded region. A dc voltage V is applied to the loop. For what value of V, the magnetic force will exactly balance the weight of the supporting mass of 1 g ?

(If sides of the loop $= 10$ cm, g $= 10$ ms^{-2}) [4 marks]



(A) 1 V

(B) $\frac{1}{10}$V

(C) 10 V

(D) 100 V

Correct Answer: (C) 10 V

Solution:

Concept:

- A straight conductor of active length L, perpendicular to a uniform magnetic field B, experiences force $F = BIL$. Only the portion within the field contributes.
- For a massless loop supporting a mass m, equilibrium requires an upward magnetic force mg. Ohm's law relates current to applied voltage by $I = V/R$.

Step 1: Identify the force and convert the data The upper horizontal side lies in the shaded field; the lower side is outside it. The forces on the vertical portions are horizontal and cancel. With the shown battery polarity, the current on the upper side is directed so that its magnetic force is upward. Convert the supplied values to SI units:

$$B = 10^3\,\text{G} = 0.1\,\text{T}, \quad L = 0.10\,\text{m}, \quad m = 10^{-3}\,\text{kg}.$$

The downward weight is $mg = 10^{-3} \times 10 = 0.01\,\text{N}$.

Step 2: Find the balancing current and voltage For exact balance, set the upward force equal to the weight:

$$BIL = mg \quad \Rightarrow \quad I = \frac{0.01}{0.1 \times 0.10} = 1\,\text{A}.$$

The stated resistance is the resistance of the entire loop, so the required applied voltage is

$$V = IR = 1 \times 10 = 10\,\text{V}.$$

No extra factor of four is introduced: only one horizontal side produces the required vertical magnetic force.

Step 3: Check the other voltage choices At $1\,\text{V}$, the current is $0.1\,\text{A}$ and the magnetic force is only $0.001\,\text{N}$, one tenth of the weight. At $0.1\,\text{V}$, the force is smaller still, $0.0001\,\text{N}$. At $100\,\text{V}$, the force is $0.1\,\text{N}$, ten times the weight. Thus none of those voltages gives equilibrium.

Step 4: Conclusion The balancing voltage is $\boxed{10\,\text{V}}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert gauss to tesla before substituting into $BIL = mg$, and use the length of the side actually inside the field.

---

32. The height of liquid column raised in a capillary tube of certain radius when dipped in liquid A vertically is, 5 cm. If the tube is dipped in a similar manner in another liquid B of surface tension and density double the values of liquid A, the height of liquid column raised in liquid B would be _____ m. [4 marks]

(A) 0.05

(B) 0.20

(C) 0.5

(D) 0.10

Correct Answer: (A) 0.05

Solution:

Concept:

- The capillary rise is $h = 2S\cos\theta/(\rho g r)$, where S is surface tension, ρ is density, r is tube radius and θ is the contact angle.
- For the same tube and the same wetting conditions, the rise depends on the ratio S/ρ, rather than on surface tension alone.

Step 1: Form the ratio of the two heights The same capillary is dipped vertically into the two liquids, so its radius and gravitational acceleration are unchanged. Under the common wetting conditions implicit in the comparison, the contact-angle factor is the same. Divide the capillary-rise expressions to eliminate these common quantities:

$$\frac{h_B}{h_A} = \frac{S_B}{S_A}\frac{\rho_A}{\rho_B}.$$

This ratio method avoids needing the actual radius or either absolute density.

Step 2: Insert both changes and convert the unit The question doubles both surface tension and density: $S_B = 2S_A$ and $\rho_B = 2\rho_A$. Thus

$$\frac{h_B}{h_A} = 2 \times \frac{1}{2} = 1.$$

The extra upward surface-tension force is exactly compensated by the extra weight per unit column height. Therefore

$$h_B = h_A = 5\,\text{cm} = \frac{5}{100}\,\text{m} = 0.05\,\text{m}.$$

Step 3: Compare the alternatives Option (D), $0.10\,\text{m}$, would follow if the surface tension doubled while the density stayed fixed, which is not the stated change. Option (B), $0.20\,\text{m}$, incorrectly combines the two doublings as a factor of four. Option (C), $0.5\,\text{m}$, also fails the centimetre-to-metre conversion and is ten times the actual rise. Only $0.05\,\text{m}$ preserves the unchanged ratio.

Step 4: Conclusion The height is $\boxed{0.05\,\text{m}}$. Hence, the correct option is (A).

Quick Tip to Solve: In capillary comparisons, track surface tension in the numerator and density in the denominator; equal scale factors cancel.

33. A small object at rest, absorbs a light pulse of power $20\ \text{mW}$ and duration $300\ \text{ns}$. Assuming speed of light as $3 \times 10^8\ \text{m/s}$, the momentum of the object becomes equal to : [4 marks]

(A) $1 \times 10^{-17}\ \text{kg m/s}$

(B) $0.5 \times 10^{-17}\ \text{kg m/s}$

(C) $3 \times 10^{-17}\ \text{kg m/s}$

(D) $2 \times 10^{-17}\ \text{kg m/s}$

Correct Answer: (D) $2 \times 10^{-17}\ \text{kg m/s}$

Solution:

Concept:

- A light pulse of constant power P and duration Δt carries energy $E = P\Delta t$. Convert power and time into SI units before multiplying.
- Electromagnetic radiation of energy E carries momentum E/c. Complete absorption transfers this momentum to the object, along the direction of the incident light.

Step 1: Calculate the energy of the pulse The pulse power is $20\,\text{mW} = 20 \times 10^{-3}\,\text{W}$, and its duration is $300\,\text{ns} = 300 \times 10^{-9}\,\text{s}$. Therefore

$$E = P\Delta t = (20 \times 10^{-3})(300 \times 10^{-9}) = 6.0 \times 10^{-9}\,\text{J}.$$

This is the total energy incident on and absorbed by the object during the stated pulse, rather than an energy per second.

Step 2: Use conservation of momentum The object is initially at rest, so its initial momentum is zero.

On absorbing the pulse it receives the entire incident electromagnetic momentum:

$$p = \frac{E}{c} = \frac{6.0 \times 10^{-9}}{3.0 \times 10^{8}} = 2.0 \times 10^{-17}\,\mathrm{kg\,m\,s^{-1}}.$$

The direction is that of propagation of the light. The factor of two used for perfect reflection does not apply because the question specifies absorption.

Step 3: Test the numerical alternatives The pulse energy fixes the momentum uniquely. Options (A) and (B) are smaller by factors of two and four, respectively, while option (C) would require 9×10^{-9} J of absorbed energy. None equals the calculated 6×10^{-9} J divided by c. The dimensions also agree: $\mathrm{J}/(\mathrm{m\,s^{-1}}) = \mathrm{kg\,m\,s^{-1}}$.

Step 4: Conclusion The object acquires momentum $\boxed{2 \times 10^{-17}\,\mathrm{kg\,m\,s^{-1}}}$, along the light pulse. Hence, the correct option is (D).

Quick Tip to Solve: For an absorbed pulse, combine the two relations immediately as $p = P\Delta t/c$; reserve $2E/c$ for perfect reflection.

---

34. Choose the correct relationship between Poisson ratio (σ), bulk modulus (K) and modulus of rigidity (η) of a given solid object : [4 marks]

(A) $\sigma = \frac{3K+2\eta}{6K+2\eta}$

(B) $\sigma = \frac{3K-2\eta}{6K+2\eta}$

(C) $\sigma = \frac{6K+2\eta}{3K-2\eta}$

(D) $\sigma = \frac{6K-2\eta}{3K-2\eta}$

Correct Answer: (B) $\sigma = \frac{3K-2\eta}{6K+2\eta}$

Solution:

Concept:

- For a homogeneous isotropic elastic solid, Young's modulus Y, bulk modulus K, rigidity modulus η and Poisson ratio σ satisfy $Y = 3K(1-2\sigma)$.
- The second independent relation is $Y = 2\eta(1+\sigma)$. Eliminating Y expresses the dimensionless Poisson ratio in terms of the two moduli.

Step 1: Equate the expressions for Young modulus Both standard elastic relations describe the same solid within its linear elastic range. Set them equal:

$$3K(1-2\sigma) = 2\eta(1+\sigma).$$

Expand carefully, keeping the sign of the volumetric-strain term:

$$3K - 6K\sigma = 2\eta + 2\eta\sigma.$$

Bulk modulus measures resistance to volume change, while rigidity modulus measures resistance to shear; their units are the same.

Step 2: Collect the terms containing Poisson ratio Move the constant rigidity term to the left and the terms proportional to σ to the right:

$$3K - 2\eta = 6K\sigma + 2\eta\sigma = \sigma(6K + 2\eta).$$

Dividing by the positive modulus combination gives

$$\sigma = \frac{3K - 2\eta}{6K + 2\eta}.$$

The numerator and denominator both have pressure units, which cancel as required for a strain ratio.

Step 3: Check the alternatives using a familiar limiting value For a solid with $\sigma = 0$, the two modulus relations imply $3K = 2\eta$. The correct expression then vanishes. Option (A) instead has a nonzero positive numerator. Options (C) and (D) place $3K - 2\eta$ in the denominator and become undefined at this perfectly admissible value, so they cannot be the general relationship. Option (B) matches the algebra and remains finite.

Step 4: Conclusion The required relationship is $\boxed{\sigma = (3K - 2\eta)/(6K + 2\eta)}$. Hence, the correct option is (B).

Quick Tip to Solve: Remember the two formulas for Young modulus and eliminate it; this is safer than memorizing the final fraction with several similar signs.

35. The pressure (P) and temperature (T) relationship of an ideal gas obeys the equation $PT^2 =$ constant. The volume expansion coefficient of the gas will be : [4 marks]

(A) $3T^2$

(B) $\frac{3}{T^2}$

(C) $\frac{3}{T^3}$

(D) $\frac{3}{T}$

Correct Answer: (D) $\frac{3}{T}$

Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nRT$, with temperature expressed on the absolute Kelvin scale.
- The volume expansion coefficient along the stated process is $\beta = (1/V)(dV/dT)$. The relation between pressure and temperature must be used before differentiating.

Step 1: Eliminate the changing pressure Let the positive constant in the prescribed relation be C. Then

$$PT^2 = C \quad \Rightarrow \quad P = \frac{C}{T^2}.$$

Pressure therefore decreases as the gas is heated along this process. Substitute into the ideal-gas

equation for a fixed amount of gas:

$$\frac{C}{T^2}V = nRT \quad \Rightarrow \quad V = \frac{nR}{C}T^3.$$

Thus the volume is proportional to the cube of absolute temperature, not simply to temperature.

Step 2: Differentiate the volume relation Because n, R, and C remain constant,

$$\frac{dV}{dT} = 3\frac{nR}{C}T^2.$$

Divide by the current volume rather than the initial volume:

$$\beta = \frac{1}{V}\frac{dV}{dT} = \frac{3(nR/C)T^2}{(nR/C)T^3} = \frac{3}{T}.$$

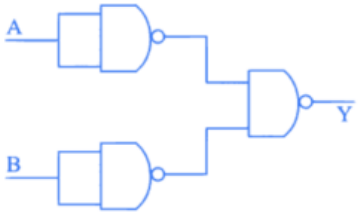
This coefficient includes both the direct temperature increase and the additional expansion caused by the falling pressure.

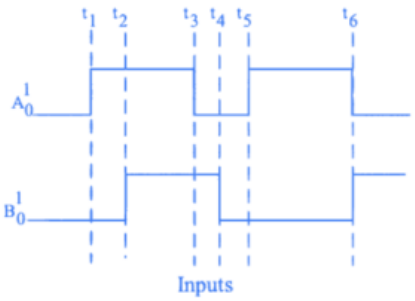
Step 3: Check dimensions and exclude the other options An expansion coefficient has dimensions of inverse temperature, since it is fractional volume change per unit rise in temperature. Option (A), $3T^2$, has dimensions of temperature squared. Options (B) and (C) have dimensions of inverse temperature squared and cubed. All three are inconsistent. The standard constant-pressure ideal-gas result $1/T$ cannot be used here because pressure explicitly varies as T^{-2}.

Step 4: Conclusion Along the specified process, the volume expansion coefficient is $\boxed{3/T}$, in K^{-1} when T is in kelvin. Hence, the correct option is (D).

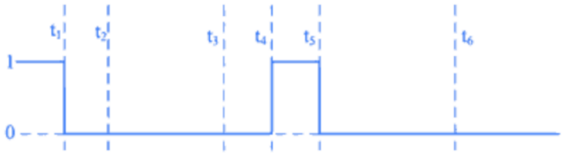
Quick Tip to Solve: If an ideal-gas process gives $P \propto T^a$, then $V \propto T^{1-a}$, so its fractional volume coefficient is $(1-a)/T$.

36. The output waveform of the given logical circuit for the following inputs A and B as shown below, is : [4 marks]

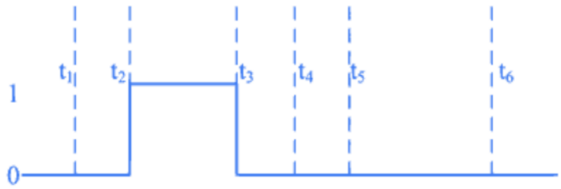




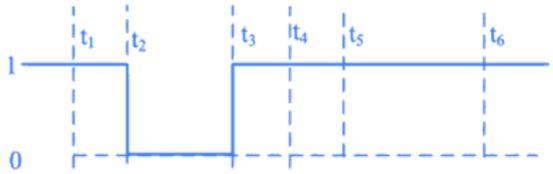
(A)



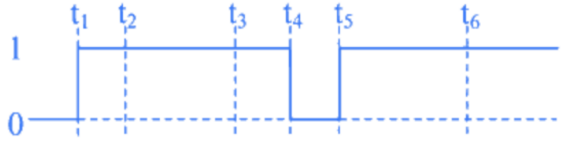
(B)



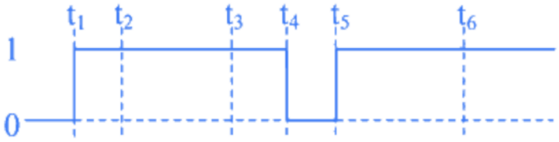
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- A NAND gate gives the complement of the product of its inputs: $Y = \overline{XY}$. When both inputs are joined, it acts as a NOT gate.
- De Morgan's law gives $\overline{\overline{A}\,\overline{B}} = A + B$, where the plus sign denotes logical OR, not ordinary addition.

Step 1: Reduce the three-gate circuit The two input terminals of the upper NAND gate are both connected to A. Its output is therefore $\overline{AA} = \overline{A}$. Similarly the lower NAND gate gives $\overline{B}$. These two inverted signals feed the last NAND gate, so its output is

$$Y = \overline{\overline{A}\,\overline{B}} = A + B.$$

Thus the complete circuit is an OR gate: its output is zero only when both original inputs are zero.

Step 2: Follow the input changes in time Before t_1, both inputs are zero, so Y is zero. From t_1 to t_2, A alone is high; from t_2 to t_3, both are high; and from t_3 to t_4, B alone is high. Hence Y remains high throughout $t_1 < t < t_4$. Both inputs are low between t_4 and t_5, so Y is low there. A becomes high at t_5; at t_6, A falls while B rises, leaving the OR output high.

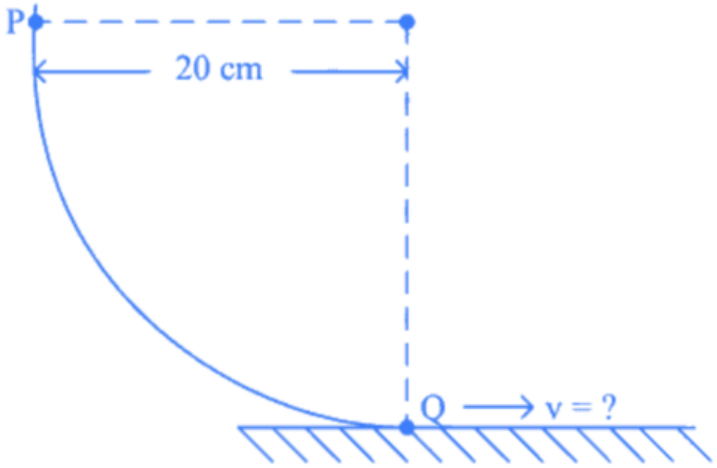
Step 3: Compare the four waveforms Option (D) has exactly these transitions: low initially, high from t_1 to t_4, low until t_5, then high. Option (A) has the opposite initial value. Option (B) is high only during the simultaneous-high interval, which corresponds to AND. Option (C) is low during that same interval, which corresponds to NAND of the original inputs.

Step 4: Conclusion The output represents $A + B$, with the waveform shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: First replace each NAND with joined inputs by an inverter; then apply De Morgan before tracing any waveform.

37. As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be :

(g = 10 m/s^2) [4 marks]



(A) 0.25 m/s

(B) 4 m/s

(C) 0

(D) 2 m/s

Correct Answer: (D) 2 m/s

Solution:

Concept:

- With no friction, the loss of gravitational potential energy becomes translational kinetic energy: $mgh = \frac{1}{2}mu^2$. The normal force does no work on the sliding ball.
- In a direct elastic collision, both momentum and kinetic energy are conserved. Equal masses exchange velocities when one of them is initially at rest.

Step 1: Find the speed at the bottom The marked horizontal radius is $20\,\text{cm}$. The starting point is level with the centre of the quadrant, while its lowest point lies one radius below the centre. Thus the vertical fall is $h = 0.20\,\text{m}$, not the length of the curved track. Starting from rest, the ball P reaches Q with speed

$$u = \sqrt{2gh} = \sqrt{2(10)(0.20)} = \sqrt{4} = 2\,\text{m}\,\text{s}^{-1}.$$

This is the incident speed immediately before impact.

Step 2: Apply the equal-mass elastic collision conditions Let the final velocities of P and Q along the horizontal tangent be v_P and v_Q. Conservation of momentum gives $u = v_P + v_Q$. For an elastic collision the relative speed of separation equals the relative speed of approach, giving $v_Q - v_P = u$. Adding these equations yields $2v_Q = 2u$, so

$$v_Q = u = 2\,\text{m}\,\text{s}^{-1}, \qquad v_P = 0.$$

The kinetic energy is transferred completely from P to Q.

Step 3: Check the alternatives The speed $4\,\mathrm{m\,s^{-1}}$ would give Q four times the available kinetic energy, and is impossible. Zero would describe the final speed of P, not Q. The value $0.25\,\mathrm{m\,s^{-1}}$ satisfies neither the energy gained in the descent nor the equal-mass exchange of velocities.

Step 4: Conclusion Ball Q moves horizontally to the right with speed $2\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a smooth descent use the vertical drop; for an elastic impact of equal masses with one at rest, transfer the incident velocity directly.

38. The charge flowing in a conductor changes with time as $\mathrm{Q(t)} = \alpha\mathrm{t} - \beta\mathrm{t}^2 + \gamma\mathrm{t}^3$. Where α, β and γ are constants. Minimum value of current is : [4 marks]

(A) $\beta - \frac{\alpha^2}{3\gamma}$

(B) $\alpha - \frac{3\beta^2}{\gamma}$

(C) $\alpha - \frac{\beta^2}{3\gamma}$

(D) $\alpha - \frac{\gamma^2}{3\beta}$

Correct Answer: (C) $\alpha - \frac{\beta^2}{3\gamma}$

Solution:

Concept:

- Instantaneous current is the rate of flow of charge: $I = dQ/dt$. Differentiate the charge polynomial before seeking its minimum.
- An upward-opening quadratic has its minimum at its vertex. Here a finite unrestricted minimum requires $\gamma > 0$; the stationary time must also lie in the physical time interval.

Step 1: Differentiate the charge function For $Q(t) = \alpha t - \beta t^2 + \gamma t^3$, the constants remain unchanged during differentiation. Applying the power rule to each term gives

$$I(t) = \alpha - 2\beta t + 3\gamma t^2.$$

It is the current, rather than Q itself, whose lowest value is requested. Setting $dQ/dt = 0$ would locate a stationary charge and would not minimize the current.

Step 2: Locate and evaluate the minimum Differentiate I once more:

$$\frac{dI}{dt} = -2\beta + 6\gamma t.$$

The stationary time is $t_* = \beta/(3\gamma)$, and $d^2I/dt^2 = 6\gamma$, positive for the minimum intended here. Substitution gives

$$I_{\min} = \alpha - \frac{2\beta^2}{3\gamma} + \frac{\beta^2}{3\gamma} = \alpha - \frac{\beta^2}{3\gamma}.$$

Equivalently, complete the square:

$$I = 3\gamma\left(t - \frac{\beta}{3\gamma}\right)^2 + \alpha - \frac{\beta^2}{3\gamma}.$$

The squared term makes the minimum explicit.

Step 3: Identify the matching expression Only option (C) has the correct subtraction term $\beta^2/(3\gamma)$. Option (A) interchanges the roles of the constant and linear coefficients, option (B) has an incorrect factor of nine in the correction, and option (D) exchanges the powers of the coefficients. For a restricted interval the vertex must belong to that interval; otherwise its endpoint must be checked.

Step 4: Conclusion The intended minimum is $I_{\min} = \alpha - \beta^2/(3\gamma)$, for an upward-opening current parabola with accessible vertex. Hence, the correct option is (C).

Quick Tip to Solve: For $at^2 + bt + c$ with $a > 0$, use $I_{\min} = c - b^2/(4a)$, after identifying coefficients from the current, not the charge.

39. Heat is given to an ideal gas in an isothermal process.

A. Internal energy of the gas will decrease.

B. Internal energy of the gas will increase.

C. Internal energy of the gas will not change.

D. The gas will do positive work.

E. The gas will do negative work.

Choose the correct answer from the options given below : [4 marks]

(A) B and D only

(B) C and E only

(C) A and E only

(D) C and D only

Correct Answer: (D) C and D only

Solution:

Concept:

- The internal energy of a fixed amount of ideal gas depends only on its temperature: $\Delta U = nC_V\Delta T$.
- Using work done by the gas as positive, the first law is $Q = \Delta U + W$. Heat supplied to the gas corresponds to $Q > 0$.

Step 1: Use the isothermal condition An isothermal process proceeds at constant temperature, so $\Delta T = 0$. For an ideal gas this immediately gives

$$\Delta U = nC_V(0) = 0.$$

Internal energy therefore neither decreases nor increases. Statement C is true, while statements A and B are false. This conclusion depends on the ideal-gas assumption: internal energy is then fixed by

temperature alone, even though heat can enter during the process.

Step 2: Determine the sign of the work The problem says heat is given to the gas, so $Q > 0$. With $\Delta U = 0$, the first law becomes

$$W = Q > 0.$$

The supplied heat is converted into work done by the gas, rather than stored as an increase in its internal energy. Thus statement D is true and statement E is false. For a reversible isothermal expansion this also agrees with $W = nRT\ln(V_f/V_i)$, positive when the gas expands.

Step 3: Select the true pair The simultaneously correct statements are C and D only. Option (A) includes B, which wrongly asserts increasing internal energy. Option (B) includes E and therefore assigns the wrong sign to the work. Option (C) includes both a decrease of internal energy and negative work; both contradict the constant-temperature energy balance. Option (D) contains exactly the two correct statements.

Step 4: Conclusion Internal energy is unchanged and the gas does positive work: C and D only. Hence, the correct option is (D).

Quick Tip to Solve: For an ideal gas, isothermal means $\Delta U = 0$; then read heat and work signs directly from $Q = W$.

40. Electric field in a certain region is given by $\vec{\mathrm{E}} = \left(\frac{\mathrm{A}}{x^2}\hat{i} + \frac{\mathrm{B}}{y^3}\hat{j}\right)$. The SI unit of A and B are :

[4 marks]

(A) $\mathrm{Nm^2C}; \mathrm{Nm^3C}$

(B) $\mathrm{Nm^3C^{-1}}; \mathrm{Nm^2C^{-1}}$

(C) $\mathrm{Nm^3C}; \mathrm{Nm^2C}$

(D) $\mathrm{Nm^2C^{-1}}; \mathrm{Nm^3C^{-1}}$

Correct Answer: (D) $\mathrm{Nm^2C^{-1}}; \mathrm{Nm^3C^{-1}}$

Solution:

Concept:

- Electric field is force per unit charge, so its SI unit is $\mathrm{N\,C^{-1}}$. Each Cartesian component must have this same unit.
- Coordinates x and y are lengths, measured in metres, while unit vectors $\hat{i}$ and $\hat{j}$ are dimensionless.

Step 1: Extract the unit of A from the x-component The x-component is $E_x = A/x^2$. The denominator contributes two powers of length. Multiplying by x^2 gives $A = E_x x^2$, hence

$$[A] = [E]\,\mathrm{m^2} = \mathrm{N\,m^2\,C^{-1}}.$$

Dividing this unit by $\mathrm{m^2}$ returns $\mathrm{N\,C^{-1}}$, as required. A is a coefficient with units; it is not itself an electric field.

Step 2: Extract the unit of B from the y-component The y-component is $E_y = B/y^3$. The cube in the

denominator requires three powers of length in the coefficient:

$$[B] = [E]\,\mathrm{m}^3 = \mathrm{N\,m^3\,C^{-1}}.$$

Its length power differs from that of A because the coordinate powers in the two components differ. Vector addition allows the two field components to combine only after these denominators have been included.

Step 3: Check both entries of each option Option (D) gives the required pair in the correct order. Option (B) reverses the powers of metre and would fail the unit check for each component. Options (A) and (C) put coulomb in the numerator, whereas electric field requires inverse coulomb. The force unit N is present in both coefficients; no additional change to its mass or time dimensions is needed.

Step 4: Conclusion The units of A and B are respectively $\mathrm{N\,m^2\,C^{-1}}$ and $\mathrm{N\,m^3\,C^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Multiply the field unit by the coordinate power in the denominator; check the order of the two coefficients before choosing.

41. Two isolated metallic solid spheres of radii R and $2\mathrm{R}$ are charged such that both have same charge density σ. The spheres are then connected by a thin conducting wire. If the new charge density of the bigger sphere is σ'. The ratio $\frac{\sigma'}{\sigma}$ is : [4 marks]

(A) $\frac{5}{3}$
(B) $\frac{5}{6}$
(C) $\frac{9}{4}$
(D) $\frac{4}{3}$

Correct Answer: (B) $\frac{5}{6}$

Solution:

Concept:

- A sphere of radius r and surface charge density σ carries charge $q = 4\pi r^2\sigma$. Metallic excess charge resides on its surface.
- When separated conducting spheres are connected, their potentials become equal. Using $V = kq/r$, the final charges are proportional to radii, while total charge is conserved.

Step 1: Calculate the initial total charge The small sphere initially carries $q_1 = 4\pi R^2\sigma$. The bigger sphere has four times its surface area, so $q_2 = 4\pi(2R)^2\sigma = 16\pi R^2\sigma$. Therefore the conserved total is

$$Q = q_1 + q_2 = 20\pi R^2\sigma.$$

Equal initial surface charge densities do not imply equal initial charges or equal initial potentials.

Step 2: Share the charge at equal potential Let the final charges on the small and bigger spheres be q_s and q_b. Equal potentials give

$$\frac{kq_s}{R} = \frac{kq_b}{2R}, \qquad q_b = 2q_s.$$

Along with $q_s + q_b = Q$, this gives

$$q_b = \frac{2}{3}Q = \frac{40}{3}\pi R^2\sigma.$$

The bigger sphere takes two-thirds of the total, rather than four-fifths as it initially held. Charge flows until the potential difference vanishes.

Step 3: Convert the final charge to density The bigger sphere retains its area $16\pi R^2$, so

$$\sigma' = \frac{q_b}{16\pi R^2} = \frac{5}{6}\sigma.$$

Its density decreases, consistent with charge leaving the initially higher-potential bigger sphere. Option (A) misses the factor two in the area conversion; options (C) and (D) also predict an increase and do not satisfy charge conservation together with equal potentials.

Step 4: Conclusion The requested ratio is $\sigma'/\sigma = 5/6$. Hence, the correct option is (B).

Quick Tip to Solve: At equal potential charge scales as radius, but surface density scales inversely with radius; conserve total charge before finding density.

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42. Speed of an electron in Bohr's 7^{th} orbit for Hydrogen atom is 3.6×10^6 m/s. The corresponding speed of the electron in 3^{rd} orbit, in m/s is : [4 marks]

(A) $\left(1.8 \times 10^6\right)$

(B) $\left(7.5 \times 10^6\right)$

(C) $\left(8.4 \times 10^6\right)$

(D) $\left(3.6 \times 10^6\right)$

Correct Answer: (C) $\left(8.4 \times 10^6\right)$

Solution:

Concept:

- For a fixed hydrogen-like atom in the Bohr model, the orbital speed varies inversely with the principal quantum number: $v_n \propto Z/n$.
- For the same atom Z is unchanged, so nv_n is constant and a ratio eliminates the common physical constants.

Step 1: Relate the two orbital speeds The Bohr speed relation can be written $v_n = CZ/n$, where C contains universal constants. Taking the ratio for the third and seventh orbits of the same hydrogen atom gives

$$\frac{v_3}{v_7} = \frac{Z/3}{Z/7} = \frac{7}{3}.$$

The smaller orbit has the larger speed. The ratio is inverse in n, rather than inverse in n^2; the latter dependence belongs to the magnitude of the orbital energy, not the speed.

Step 2: Substitute the stated seventh-orbit value Use the speed provided in the question directly:

$$v_3 = \frac{7}{3}\left(3.6 \times 10^6\right) = 7(1.2 \times 10^6) = 8.4 \times 10^6\,\mathrm{m\,s^{-1}}.$$

The numerical data determine the requested proportional answer, so no separate memorised numerical value of the first-orbit speed is needed. Both sides of the ratio have the same speed unit.

Step 3: Compare the offered speeds Option (C) is the computed value and is larger than the given seventh-orbit speed, as expected. Option (A) would reduce the speed on moving to a lower orbit, reversing the physical trend. Option (D) would leave it unchanged despite a change in n. Although option (B) increases the speed, its multiplier is not the exact ratio $7/3$, so it does not follow from the stated input.

Step 4: Conclusion The corresponding speed in the third orbit, using the stated data, is $8.4 \times 10^6\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For two Bohr orbits of the same atom, write $n_1 v_1 = n_2 v_2$; do not use the energy scaling $1/n^2$ for speed.

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43. The magnetic moments associated with two closely wound circular coils A and B of radius $\mathrm{r_A} = 10$ cm and $\mathrm{r_B} = 20$ cm respectively are equal if : (Where $\mathrm{N_A}, \mathrm{I_A}$ and $\mathrm{N_B}, \mathrm{I_B}$ are number of turn and current of A and B respectively) [4 marks]

(A) $4\,\mathrm{N_A I_A} = \mathrm{N_B I_B}$

(B) $2\,\mathrm{N_A I_A} = \mathrm{N_B I_B}$

(C) $\mathrm{N_A} = 2\,\mathrm{N_B}$

(D) $\mathrm{N_A I_A} = 4\,\mathrm{N_B I_B}$

Correct Answer: (D) $\mathrm{N_A I_A} = 4\,\mathrm{N_B I_B}$

Solution:

Concept:

- The magnetic dipole moment of a closely wound circular coil is $m = NIA = NI\pi r^2$, where N is its number of turns, I its current and A the area of each turn.
- Equal magnetic moments therefore require equal products NIr^2. The radius enters through area and must be squared; circumference does not determine the dipole moment.

Step 1: Write the equality of moments For the two coils, write $m_A = N_A I_A \pi r_A^2$ and $m_B = N_B I_B \pi r_B^2$. Their stated equality gives

$$N_A I_A \pi (0.10)^2 = N_B I_B \pi (0.20)^2.$$

The same unit must be used for both radii. Using centimetres on both sides would give the same ratio, since the conversion factor cancels.

Step 2: Compare the turn-current products Cancel π, divide by $(0.10)^2$, and use the radius ratio:

$$N_A I_A = N_B I_B \left(\frac{0.20}{0.10}\right)^2 = 4N_B I_B.$$

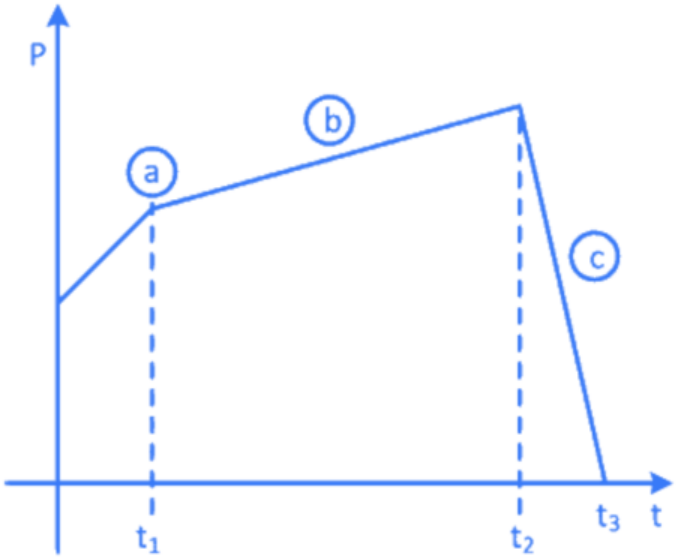
Coil B has four times the area of coil A, so A needs four times the turn-current product to compensate. Option (A) reverses this compensation; option (B) uses the radius ratio instead of its square. Option (C) gives a turns-only condition and does not control the currents, so it cannot ensure equal moments in general.

Step 3: Conclusion The required condition is $N_A I_A = 4N_B I_B$. Hence, the correct option is (D).

Quick Tip to Solve: For equal coil moments, first compare areas: doubling the radius quadruples the area and divides the required turn-current product by four.

44. The figure represents the momentum time ($\mathrm{p} - \mathrm{t}$) curve for a particle moving along an axis under the influence of the force. Identify the regions on the graph where the magnitude of the force is maximum and minimum respectively?

If $(t_3 - t_2) < t_1$ [4 marks]



(A) b and c

(B) c and a

(C) a and b

(D) c and b

Correct Answer: (D) c and b

Solution:

Concept:

- Newton's second law in momentum form is $F = dp/dt$. Thus the force is the slope of a momentum-time graph, and its magnitude is the absolute value of that slope.
- A downward-sloping segment represents negative force, not necessarily a small force. Compare the steepness of the segments rather than their heights above the time axis.

Step 1: Compare the rising segments In region a, momentum rises appreciably during the interval from zero to t_1, so its slope is positive. Region b rises much more gently between t_1 and t_2. Therefore

$$|F_b| < |F_a|.$$

The larger momentum reached in b does not make its force larger: force measures the rate of momentum change, not momentum itself.

Step 2: Compare the falling segment In region c, the momentum falls from its largest plotted value to zero during $t_3 - t_2$. This change is larger than the rise in a, while the given condition $(t_3 - t_2) < t_1$ makes its time interval shorter. Hence

$$|F_c| = \frac{p(t_2)}{t_3 - t_2} > |F_a| > |F_b|.$$

Its negative slope only specifies the direction of force. Option (A) interchanges the extremes; option (B) incorrectly makes a the minimum; option (C) misses the steeper fall in c.

Step 3: Conclusion The force magnitude is maximum in region c and minimum in region b. Hence, the correct option is (D).

Quick Tip to Solve: On a momentum-time plot, compare absolute slopes; a steep downward line can represent the largest force magnitude.

45. If the gravitational field in the space is given as $\left(-\frac{K}{r^2}\right)$. Taking the reference point to be at $\mathrm{r} = 2$ cm with gravitational potential $\mathrm{V} = 10$ J/kg. Find the gravitational potential at $\mathrm{r} = 3$ cm in SI unit (Given, that $\mathrm{K} = 6$ Jcm/kg) [4 marks]

(A) 9

(B) 11

(C) 10

(D) 12

Correct Answer: (B) 11

Solution:

Concept:

- The radial gravitational field and potential obey $g_r = -dV/dr$. For $g_r = -K/r^2$, the potential difference is found by integrating $dV/dr = K/r^2$.

- The zero of potential is arbitrary. The stated value at $r = 2\,\text{cm}$ fixes the additive constant; it is incorrect to impose zero potential at infinity as an extra condition.

Step 1: Convert the constant and write the difference The supplied constant is $K = 6\,\text{J cm kg}^{-1} = 0.06\,\text{J m kg}^{-1}$. Use $r_1 = 0.02\,\text{m}$ and $r_2 = 0.03\,\text{m}$. Integrating between these positions gives

$$V(r_2) - V(r_1) = \int_{r_1}^{r_2} \frac{K}{r^2}\,dr = K\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

The positive difference is consistent with potential increasing as one moves outward against an attractive field.

Step 2: Evaluate and apply the reference value Substitute the radii and the SI value of K:

$$\Delta V = 0.06\left(\frac{1}{0.02} - \frac{1}{0.03}\right) = 0.06\left(50 - \frac{100}{3}\right) = 1\,\text{J kg}^{-1}.$$

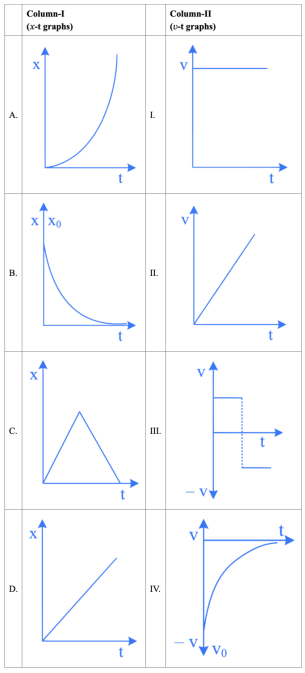
Hence $V(r_2) = 10 + 1 = 11\,\text{J kg}^{-1}$. Option (A) has the wrong sign for the outward change, option (C) ignores the nonzero field, and option (D) gives twice the actual potential increase.

Step 3: Conclusion The potential is $11\,\text{J kg}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Calculate a potential difference from the field before adding the given reference potential; convert the length in the units of K too.

46. Match Column-I with Column-II. Choose the correct answer from the options given below.

[4 marks]



(A) A- II, B-III, C-IV, D-I

(B) A- II, B-IV, C-III, D-I

(C) A- I, B-III, C-IV, D-II

(D) A- I, B-II, C-III, D-IV

Correct Answer: (B) A- II, B-IV, C-III, D-I

Solution:

Concept:

- Instantaneous velocity is the slope of the position-time graph: $v = dx/dt$. An upward slope means positive velocity; a downward slope means negative velocity.
- A straight position-time segment has constant velocity. For a curved graph, compare the changing steepness of the tangent, rather than the height of the position coordinate.

Step 1: Match the increasing position curve A Curve A rises and becomes progressively steeper. Thus its slope is positive and increases with time. The corresponding velocity must rise from zero into the positive region, as in graph II. Hence $A \to$ II. Graph I cannot represent A because its velocity is constant, while graph IV has negative velocity.

Step 2: Match the decaying position curve B Curve B falls from x_0, so its slope and velocity are negative. The curve gradually flattens, meaning that the magnitude of the negative velocity decreases towards zero. Graph IV lies below the time axis and approaches it from below, exactly as required. Thus $B \to$ IV. Graph III would instead require two straight position-time segments with an abrupt change of slope.

Step 3: Match the triangular position graph C The first straight segment of C has a constant positive slope. After the peak, the second straight segment has a constant negative slope. Therefore the velocity changes from a positive constant to a negative constant. This is the step-shaped velocity graph III, so $C \to$ III. The position is continuous at the peak even though the idealised velocity changes suddenly there.

Step 4: Match the straight position graph D and compare the options Graph D is a rising straight line with an unchanging positive slope. Its velocity is therefore a positive constant, represented by graph I. Hence $D \to$ I. Option A incorrectly interchanges the matches of B and C; option C incorrectly assigns constant velocity to A and increasing velocity to D; option D also gives the wrong matches for A, B and D.

Step 5: Conclusion The complete matching is $A \to$ II, $B \to$ IV, $C \to$ III, $D \to$ I. Hence, the correct option is (B).

Quick Tip to Solve: Read the sign and changing size of the slope on each position-time graph. A falling curve can have negative velocity that increases towards zero as the curve flattens.

47. A ball of mass $200\ \mathrm{g}$ rests on a vertical post of height $20\ \mathrm{m}$. A bullet of mass $10\ \mathrm{g}$, travelling in horizontal direction, hits the centre of the ball. After collision both travels independently. The ball hits the ground at a distance $30\ \mathrm{m}$ and the bullet at a distance of $120\ \mathrm{m}$ from the foot of the post. The value of initial velocity of the bullet will be (if $g = 10\ \mathrm{m/s^2}$) : [4 marks]

(A) 120 m/s

(B) 360 m/s

(C) 400 m/s

(D) 60 m/s

Correct Answer: (B) 360 m/s

Solution:

Concept:

- After the horizontal collision, both objects start their vertical fall from height h with zero vertical velocity. Their common flight time is $t = \sqrt{2h/g}$.
- Horizontal momentum is conserved during the brief impact because the external horizontal impulse is negligible. Kinetic energy need not be conserved, and the bullet and ball do not stick together.

Step 1: Find the outgoing horizontal speeds With $h = 20\,\mathrm{m}$ and $g = 10\,\mathrm{m\,s^{-2}}$,

$$t = \sqrt{\frac{2(20)}{10}} = 2\,\mathrm{s}.$$

Horizontal speeds remain constant during the subsequent falls. The ball therefore leaves with $v_B = 30/2 = 15\,\mathrm{m\,s^{-1}}$, while the bullet leaves with $v_b = 120/2 = 60\,\mathrm{m\,s^{-1}}$. The larger bullet range describes its speed after impact, not its initial speed.

Step 2: Conserve momentum through impact Let the incident bullet speed be u. The ball is initially at rest. Using masses $0.010\,\mathrm{kg}$ and $0.200\,\mathrm{kg}$,

$$0.010u = (0.010)(60) + (0.200)(15) = 0.60 + 3.00.$$

Thus

$$u = \frac{3.60}{0.010} = 360\,\mathrm{m\,s^{-1}}.$$

Option (D) is just the outgoing bullet speed; options (A) and (C) fail the momentum balance. No restitution coefficient is needed, since both final horizontal speeds have already been determined from their ranges.

Step 3: Conclusion The initial speed of the bullet is $360\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the common fall time to turn both ranges into final speeds, then conserve horizontal momentum across the collision.

48. A person has been using spectacles of power -1.0 dioptre for distant vision and a separate reading glass of power 2.0 dioptres. What is the least distance of distinct vision for this person : [4 marks]

(A) 50 cm

(B) 40 cm

(C) 30 cm

(D) 10 cm

Correct Answer: (A) 50 cm

Solution:

Concept:

- A reading glass forms a virtual image of an object at the normal reading distance $25\,\text{cm}$, placing that image at the unaided near point of the person.
- The lens formula is $P = 1/f = 1/v - 1/u$ with distances in metres. For an object on the incident side, $u < 0$; a virtual image on that same side has $v < 0$.

Step 1: Set up the reading-glass equation The separate reading glass has power $+2.0\,\text{D}$, so its focal length is $f = +0.50\,\text{m}$. Let the person's unaided near point be at a positive distance D from the eye. With spectacles close to the eye, the lens forms the virtual image there: $v = -D$. For comfortable reading at the normal distance, $u = -0.25\,\text{m}$. Consequently

$$2 = \frac{1}{-D} - \frac{1}{-0.25} = 4 - \frac{1}{D}.$$

Step 2: Solve and distinguish the two corrections Rearranging gives $1/D = 2\,\text{m}^{-1}$, hence

$$D = 0.50\,\text{m} = 50\,\text{cm}.$$

The negative-power lens specified for distant vision corrects the far point; it is a separate pair and is not combined with the reading glass here. A near point of $40\,\text{cm}$ would require $1.5\,\text{D}$, and $30\,\text{cm}$ would require about $0.67\,\text{D}$. A near point of $10\,\text{cm}$ would need negative, rather than the given positive, reading correction under this setup.

Step 3: Conclusion The least distance of distinct vision for the unaided person is $50\,\text{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: For a separate reading glass, use $P = 4 - 1/D$ with D in metres; the distance-vision prescription fixes a different limit of vision.

49. In a series LR circuit with $X_L = R$, power factor P_1. If a capacitor of capacitance C with $X_C = X_L$ is added to the circuit the power factor becomes P_2. The ratio of P_1 to P_2 will be : [4 marks]

(A) $1 : \sqrt{2}$

(B) $1 : 3$

(C) $1 : 2$

(D) $1 : 1$

Correct Answer: (A) $1 : \sqrt{2}$

Solution:

Concept:

- For a series AC circuit the power factor is $\cos\phi = R/Z$, where R is resistance and Z is impedance. An ideal inductor and capacitor contribute reactance, not average power dissipation.
- The impedances are $Z_{LR} = \sqrt{R^2 + X_L^2}$ and $Z_{LCR} = \sqrt{R^2 + (X_L - X_C)^2}$. Opposite inductive and capacitive reactances cancel when their magnitudes are equal.

Step 1: Calculate the original power factor Initially only R and L are in series and the question gives $X_L = R$. Therefore

$$Z_1 = \sqrt{R^2 + R^2} = \sqrt{2}R, \qquad P_1 = \frac{R}{Z_1} = \frac{1}{\sqrt{2}}.$$

The voltage leads the current by $45°$; consequently the power factor is less than unity even though the same current flows through both components.

Step 2: Add the capacitor and form the ratio The new capacitor has $X_C = X_L$, so the net reactance becomes zero:

$$X_L - X_C = 0, \quad Z_2 = R, \quad P_2 = \frac{R}{R} = 1.$$

This is series resonance at the operating frequency. Thus

$$P_1 : P_2 = \frac{1}{\sqrt{2}} : 1 = 1 : \sqrt{2}.$$

Ratios $1 : 3$ and $1 : 2$ would require a different original impedance; $1 : 1$ incorrectly treats the original LR circuit as already having unity power factor. The cancellation changes the phase relation without removing the resistor.

Step 3: Conclusion The power-factor ratio is $P_1 : P_2 = 1 : \sqrt{2}$. Hence, the correct option is (A).

Quick Tip to Solve: When equal inductive and capacitive reactances cancel in series, the new power factor is unity; calculate only the original R/Z.

50. A sinusoidal carrier voltage is amplitude modulated. The resultant amplitude modulated wave has maximum and minimum amplitude of 120 V and 80 V respectively. The amplitude of each sideband is : [4 marks]

(A) 15 V

(B) 20 V

(C) 5 V

(D) 10 V

Correct Answer: (D) 10 V

Solution:

Concept:

- For a sinusoidal amplitude-modulated wave, the envelope limits are $V_{\max} = V_c(1+m)$ and $V_{\min} = V_c(1-m)$, where V_c is the carrier amplitude and m is the modulation index.
- The product of the carrier and modulating sinusoids generates two frequency components. Each sideband has voltage amplitude $mV_c/2$, not mV_c.

Step 1: Find the carrier amplitude and modulation index Adding the two envelope amplitudes removes the modulating contribution, while subtracting them isolates it. Thus

$$V_c = \frac{120+80}{2} = 100\,\text{V}, \qquad m = \frac{120-80}{120+80} = 0.20.$$

The modulation index is below unity, so the envelope does not undergo overmodulation.

Step 2: Separate the two sidebands The modulated voltage contains the term $mV_c \cos\omega_m t \cos\omega_c t$. Using the product-to-sum identity gives two cosines, at $\omega_c + \omega_m$ and $\omega_c - \omega_m$, each with half this amplitude. Therefore

$$V_{\text{SB}} = \frac{mV_c}{2} = \frac{0.20 \times 100}{2} = 10\,\text{V}.$$

The value 20 V is the envelope variation amplitude and equals the sum of the two sideband amplitudes; it is not the amplitude of either sideband separately. Neither 15 V nor 5 V satisfies the product-to-sum coefficient.

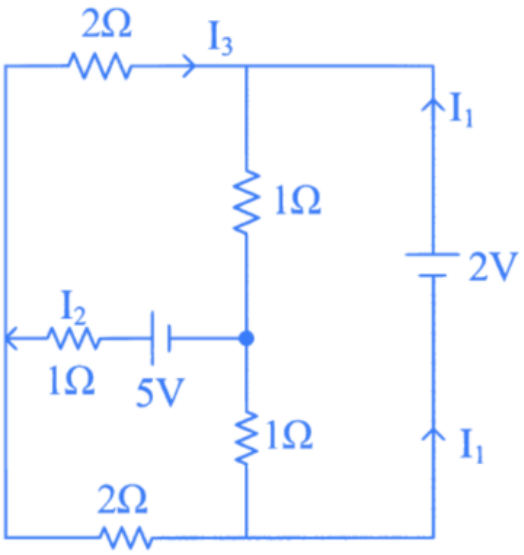
Step 3: Conclusion Each sideband has amplitude $10\,\text{V}$. Hence, the correct option is (D).

Quick Tip to Solve: For sinusoidal AM, each sideband amplitude is one quarter of the difference between the maximum and minimum envelope amplitudes.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. In the following circuit, the magnitude of current I_1, is _____ A. [4 marks]



Solution:

Concept:

- Kirchhoff's junction law states that the algebraic sum of currents leaving a node is zero. A resistor current from node a to node b is $(V_a - V_b)/R$.
- An ideal cell fixes the potential difference between its terminals. Its long plate is positive; the 2 V cell makes the upper rail 2 V above the lower rail.

Step 1: Assign the node potentials Take the lower right node as zero potential. The upper right node is then $2\,\text{V}$. Let the common left node have potential V_L, and the central junction between the two vertical resistors have potential V_M. The 5 V cell has its positive terminal on the left, so the potential immediately left of that cell is $V_M + 5$. The current from the left node towards this branch is consequently $V_L - V_M - 5$, in amperes.

Step 2: Apply the junction equations At the left node and central junction respectively, Kirchhoff's law gives

$$\frac{V_L - 2}{2} + \frac{V_L}{2} + V_L - V_M - 5 = 0,$$

$$(V_M - 2) + V_M + (V_M + 5 - V_L) = 0.$$

These simplify to $2V_L - V_M = 6$ and $3V_M - V_L = -3$. Solving gives $V_M = 0\,\text{V}$ and $V_L = 3\,\text{V}$.

Step 3: Obtain the current through the 2 V cell At the top junction, the upper left resistor supplies $(3-2)/2 = 0.5\,\text{A}$, while the upper central resistor carries $(2-0)/1 = 2\,\text{A}$ downwards. The 2 V branch must supply the remaining current upwards:

$$I_1 + 0.5 = 2 \quad \Rightarrow \quad I_1 = 1.5\,\text{A}.$$

The positive result agrees with the upward arrow in the diagram.

Step 4: Conclusion The required magnitude is $\boxed{1.5\,\mathrm{A}}$.

Quick Tip to Solve: When ideal cells connect nodes, fix their potential differences first; this reduces the number of unknown currents and avoids unnecessary loop equations.

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52. In an experiment for estimating the value of focal length of converging mirror, image of an object placed at 40 cm from the pole of the mirror is formed at distance 120 cm from the pole of the mirror. These distances are measured with a modified scale in which there are 20 small divisions in 1 cm. The value of error in measurement of focal length of the mirror is $\frac{1}{\mathrm{K}}$ cm. The value of K is _____.

[4 marks]

Solution:

Concept:

- For a concave mirror forming a real image, the magnitude of the focal length is $f = uv/(u+v)$, where u, v denote the positive object and image distances.
- The maximum first-order uncertainty of a function is the sum of the absolute partial-derivative contributions. The scale reading uncertainty is its least count, $\Delta u = \Delta v = 1/20\,\mathrm{cm}$.

Step 1: Find the focal length and reading uncertainty The given distances are $u = 40\,\mathrm{cm}$ and $v = 120\,\mathrm{cm}$. Using magnitudes avoids confusing the mirror sign convention with the magnitude of the uncertainty. The nominal focal length is

$$f = \frac{40 \times 120}{40 + 120} = 30\,\mathrm{cm}.$$

There are 20 subdivisions per centimetre, hence the smallest measurable interval is $0.05\,\mathrm{cm}$. Use this for each measured distance.

Step 2: Differentiate the combined expression Because the same measurements occur in both numerator and denominator, propagate errors in the complete expression rather than treating those combinations as independent measurements. The derivatives are

$$\frac{\partial f}{\partial u} = \frac{v^2}{(u+v)^2}, \qquad \frac{\partial f}{\partial v} = \frac{u^2}{(u+v)^2}.$$

Both coefficients are positive. Therefore the maximum uncertainty is

$$\Delta f = \frac{v^2 \Delta u + u^2 \Delta v}{(u+v)^2}.$$

Step 3: Evaluate the uncertainty and compare Substituting the distances and their common uncertainty gives

$$\Delta f = \frac{120^2 + 40^2}{160^2} \times \frac{1}{20} = \frac{16000}{25600} \times \frac{1}{20} = \frac{1}{32}\,\mathrm{cm}.$$

Comparing with the specified expression $\Delta f = (1/K)\,\mathrm{cm}$ determines the numerical coefficient directly.

Step 4: Conclusion The error is $1/32\,\text{cm}$, so $\boxed{K = 32}$ (dimensionless).

Quick Tip to Solve: For $f = uv/(u+v)$, differentiate first: errors from u and v are weighted by $v^2/(u+v)^2$ and $u^2/(u+v)^2$.

53. The general displacement of a simple harmonic oscillator is $x = A\sin\omega t$. Let T be its time period. The slope of its potential energy (U) - time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is _____.

[4 marks]

Solution:

Concept:

- An oscillator with spring constant k has potential energy $U = \frac{1}{2}kx^2$, measured from its equilibrium position.
- The slope of a potential-energy versus time graph is dU/dt, not U itself. The angular frequency and period satisfy $\omega = 2\pi/T$.

Step 1: Express potential energy in terms of time The displacement is $x = A\sin\omega t$. Substituting it into the elastic potential energy gives

$$U(t) = \frac{1}{2}kA^2\sin^2\omega t.$$

The energy varies with twice the displacement frequency, because squaring the sine makes both positive and negative displacements contribute positively. The potential energy is zero at equilibrium and maximum at the extreme positions.

Step 2: Differentiate to find the graph slope Apply the chain rule before looking for the time of maximum slope:

$$\frac{dU}{dt} = \frac{1}{2}kA^2\,2\sin\omega t\cos\omega t\,\omega = \frac{1}{2}kA^2\omega\sin(2\omega t).$$

The coefficient is positive and constant. Thus the greatest positive slope occurs when $\sin(2\omega t) = 1$, rather than when $\sin^2\omega t = 1$. At an energy maximum the slope would instead vanish.

Step 3: Identify the first time and the requested coefficient The first positive time satisfying the maximum-slope condition has

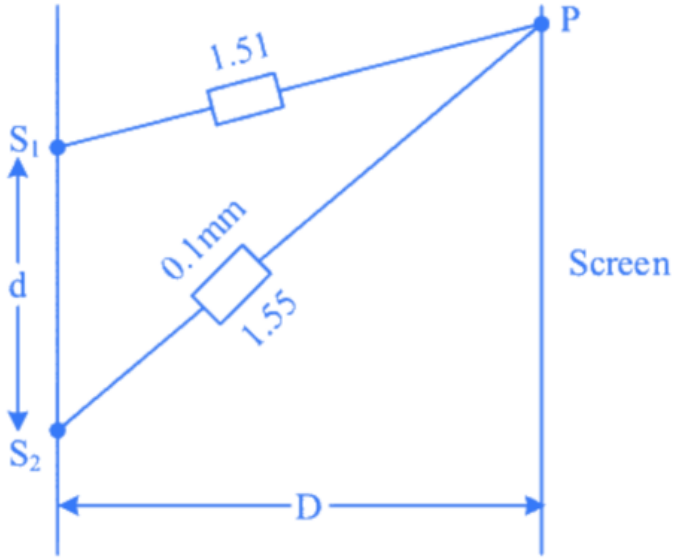
$$2\omega t = \frac{\pi}{2} \quad \Rightarrow \quad t = \frac{\pi}{4\omega}.$$

Since $T = 2\pi/\omega$, this becomes $t = T/8$. Further maxima of the positive slope recur after every $T/2$; the expression in the question selects the first such instant in the cycle. Matching $t = T/\beta$ now fixes the coefficient.

Step 4: Conclusion The maximum positive slope first occurs at $t = T/8$; hence $\boxed{\beta = 8}$ (dimensionless).

Quick Tip to Solve: Maximum energy and maximum energy-growth rate occur at different phases: differentiate the time expression before choosing the phase.

54. In Young's double slit experiment, two slits S_1 and S_2 are 'd' distance apart and the separation from slits to screen is D (as shown in figure). Now if two transparent slabs of equal thickness 0.1 mm but refractive index 1.51 and 1.55 are introduced in the path of beam ($\lambda = 4000\ \mathring{A}$) from S_1 and S_2 respectively. The central bright fringe spot will shift by _____ number of fringes. [4 marks]



Solution:

Concept:

- A slab of thickness t and refractive index μ adds optical path $(\mu - 1)t$ relative to the air it replaces.
- In Young's experiment a change in optical path difference of one wavelength shifts the pattern by one fringe width, $b = \lambda D/d$.

Step 1: Find the unequal optical paths added by the slabs The two slabs have the same physical thickness, so their common air-path replacement cancels when the optical paths are subtracted. The remaining difference is

$$\delta = |(\mu_2 - 1)t - (\mu_1 - 1)t| = |\mu_2 - \mu_1|t.$$

Here $\mu_1 = 1.51$, $\mu_2 = 1.55$ and $t = 0.1\,\mathrm{mm} = 10^{-4}\,\mathrm{m}$. Thus

$$\delta = (1.55 - 1.51)10^{-4} = 4 \times 10^{-6}\,\mathrm{m}.$$

Step 2: Convert the optical-path change to fringe widths The stated wavelength is 4000 angstroms, so

$$\lambda = 4000 \times 10^{-10} = 4 \times 10^{-7}\,\mathrm{m}.$$

The central maximum moves until the geometric path difference cancels the additional optical difference. Its displacement magnitude is $s = \delta D/d$, while the fringe width is $b = \lambda D/d$. Their ratio therefore does not require either slit separation or screen distance:

$$N = \frac{s}{b} = \frac{\delta}{\lambda} = \frac{4 \times 10^{-6}}{4 \times 10^{-7}} = 10.$$

The pattern moves towards the slit with the larger refractive index, since that geometrical path must become shorter to restore equal optical paths.

Step 3: Conclusion The central bright fringe shifts by $\boxed{10}$ fringes (a dimensionless count).

Quick Tip to Solve: For equal-thickness slabs in the two paths, use the refractive-index difference: the fringe shift is $|\mu_2 - \mu_1|t/\lambda$.

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55. A thin uniform rod of length 2 m, cross sectional area 'A' and density 'd' is rotated about an axis passing through the centre and perpendicular to its length with angular velocity ω. If value of ω in terms of its rotational kinetic energy E is $\sqrt{\frac{\alpha E}{Ad}}$ then value of α is _____. [4 marks]

Solution:

Concept:

- A uniform rod has mass $M = \rho AL$. Here the question uses d for the mass density, so $M = dAL$.
- The moment of inertia of a thin rod about a central axis perpendicular to its length is $I = ML^2/12$, and its rotational kinetic energy is $E = I\omega^2/2$.

Step 1: Express the mass and moment of inertia With a constant cross-sectional area and density, each equal length of the rod contains equal mass, so the uniform-rod moment-of-inertia formula applies. Substituting the mass into that formula gives

$$I = \frac{(dAL)L^2}{12} = \frac{AdL^3}{12}.$$

This is the central perpendicular axis specified in the question. The end-axis formula $ML^2/3$ would describe a different rotation and would give a wrong coefficient.

Step 2: Relate the angular speed to kinetic energy Insert the moment of inertia into the kinetic-energy expression and solve for angular speed:

$$E = \frac{1}{2}\left(\frac{AdL^3}{12}\right)\omega^2 = \frac{AdL^3\omega^2}{24},$$

$$\omega = \sqrt{\frac{24E}{AdL^3}}.$$

Keep the length factor until this step: mass is proportional to L and the moment of inertia contains two additional powers of L. This explains why the denominator contains the cube of the rod length.

Step 3: Insert the specified length For $L = 2\,\text{m}$, the numerical cube of the SI length is 8. Thus the required SI form becomes

$$\omega = \sqrt{\frac{24E}{8Ad}} = \sqrt{\frac{3E}{Ad}}.$$

Comparing the coefficient under the square root with the given expression $\sqrt{\alpha E/(Ad)}$ gives its requested numerical value. The numerical coefficient incorporates the specified length in metres.

Step 4: Conclusion Using SI quantities and the given 2 m rod, the required numerical value is $\boxed{\alpha = 3}$.

Quick Tip to Solve: For a uniform rod with area and density given, write $I = AdL^3/12$ directly and insert the length only after solving for angular speed.

56. A horse rider covers half the distance with $5\ \mathrm{m/s}$ speed. The remaining part of the distance was travelled with speed $10\ \mathrm{m/s}$ for half the time and with speed $15\ \mathrm{m/s}$ for other half of the time. The mean speed of the rider averaged over the whole time of motion is $\frac{x}{7}\ \mathrm{m/s}$. The value of x is _____. [4 marks]

Solution:

Concept:

- Average speed over the full journey is total distance divided by total elapsed time. Speeds may be averaged arithmetically only for equal time intervals.
- For two segments of equal distance, the combined average speed is the harmonic mean $2v_1v_2/(v_1+v_2)$. The second half here must first be reduced to its own average speed.

Step 1: Average the second half of the journey Let the full distance be D, and let the time spent over its second half be t_2. The rider travels at 10 m/s for $t_2/2$ and at 15 m/s for $t_2/2$. The distance covered in this time is

$$\frac{D}{2} = 10\frac{t_2}{2} + 15\frac{t_2}{2} = \frac{25t_2}{2}.$$

Hence the mean speed for this half is $v_2 = (10+15)/2 = 12.5\,\mathrm{m\,s^{-1}}$, and $t_2 = D/25$.

Step 2: Add the travel times of both halves The first half-distance is covered at $5\,\mathrm{m\,s^{-1}}$, so its duration is

$$t_1 = \frac{D/2}{5} = \frac{D}{10}.$$

The total time is therefore

$$t = t_1 + t_2 = \frac{D}{10} + \frac{D}{25} = \frac{7D}{50}.$$

Here D is expressed in metres and the resulting times are in seconds. The arbitrary total distance cancels when the complete average speed is calculated.

Step 3: Compute the complete average speed Divide total distance by the summed travel time:

$$\bar{v} = \frac{D}{7D/50} = \frac{50}{7}\,\mathrm{m\,s^{-1}}.$$

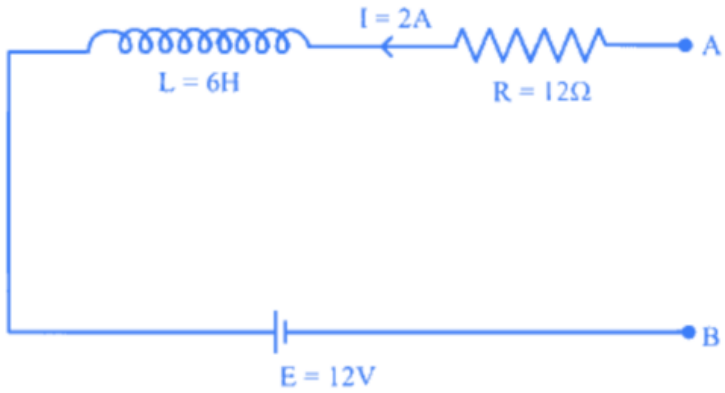
It lies between 5 and 12.5 m/s and closer to 5 m/s because more time is spent covering the slow half. An arithmetic mean of all three listed speeds would give each speed equal time weight, contrary to the stated equal-distance condition for the two halves.

Step 4: Conclusion The mean speed is $\frac{50}{7}\,\mathrm{m\,s^{-1}}$, so $\boxed{x = 50}$ (dimensionless).

Quick Tip to Solve: Reduce equal-time stages with an arithmetic mean first; then combine equal-distance stages by adding their times or using a harmonic mean.

57. As per the given figure, if $\frac{dI}{dt} = -1\ A/s$ then the value of V_{AB} at this instant will be _____ V.

[4 marks]



Solution:

Concept:

- Across a resistor, the potential drop in the direction of conventional current is IR. The given current flows from A towards the left through the resistor and coil.
- For an ideal inductor with the passive sign convention, the drop in the current direction is $L\,dI/dt$. A decreasing current makes this drop negative: the coil supplies energy to maintain the current.
- The voltage V_{AB} means $V_A - V_B$. Add signed potential drops along a continuous path from A to B; do not add the magnitudes of all circuit elements.

Step 1: Evaluate the resistor and inductor drops The resistor carries $I = 2\,\text{A}$, so its drop from right to left is

$$IR = 2(12) = 24\,\text{V}.$$

For the $6\,\text{H}$ inductor, the signed drop in the same direction is

$$L\frac{dI}{dt} = 6(-1) = -6\,\text{V}.$$

Thus the potential actually rises by $6\,\text{V}$ while crossing the inductor from right to left. This sign is essential because the current is falling.

Step 2: Include the battery polarity The longer battery plate is on the left, so its left terminal is $12\,\text{V}$ above the right terminal connected to B. Starting at A and following the upper branch to the left, then through the battery to B, gives

$$V_A - V_B = IR + L\frac{dI}{dt} + E = 24 - 6 + 12 = 30\,\text{V}.$$

No external closing wire is needed to define the terminal potential difference; the instantaneous element voltages determine it.

Step 3: Conclusion

$$\boxed{V_{AB} = 30\,\text{V}}.$$

The requested numerical entry is 30; A is at the higher potential.

Quick Tip to Solve: Write the inductor voltage as a signed $L\,dI/dt$, using the marked current direction, before adding the battery voltage.

58. In a screw gauge, there are 100 divisions on the circular scale and the main scale moves by 0.5 mm on a complete rotation of the circular scale. The zero of circular scale lies 6 divisions below the line of graduation when two studs are brought in contact with each other. When a wire is placed between the studs, 4 linear scale divisions are clearly visible while 46^{th} division the circular scale coincide with the reference line. The diameter of the wire is _____ $\times 10^{-2}$ mm. [4 marks]

Solution:

Concept:

- The pitch is the distance advanced in one complete turn. The least count is pitch divided by the number of circular-scale divisions.
- The observed reading equals the main-scale reading plus the circular-scale reading times least count. A positive zero error must be subtracted from this observed reading.
- A zero lying below the reference line gives a positive reading when the studs touch. Here the corresponding positive zero error is six least counts.

Step 1: Find the least count and zero error The pitch is $0.5\,\text{mm}$ and there are 100 circular divisions. Hence

$$\text{LC} = \frac{0.5}{100} = 0.005\,\text{mm}.$$

The positive zero error is

$$e = 6(0.005) = 0.030\,\text{mm}.$$

The zero correction is therefore $-0.030\,\text{mm}$. It is the correction, rather than the error itself, that is added to an observed reading.

Step 2: Correct the observed wire reading Four visible linear divisions correspond to $4(0.5) = 2.0\,\text{mm}$. The circular contribution is $46(0.005) = 0.230\,\text{mm}$. Therefore

$$d_{\text{obs}} = 2.0 + 0.230 = 2.230\,\text{mm},$$

and the corrected diameter is

$$d = 2.230 - 0.030 = 2.200\,\text{mm}.$$

Expressing this in the units requested in the blank,

$$2.200\,\text{mm} = 220 \times 10^{-2}\,\text{mm}.$$

The correction decreases the reading because the instrument already shows a positive value for zero separation.

Step 3: Conclusion The wire diameter is $\boxed{2.20\,\text{mm}}$, so the required entry is $\boxed{220}$.

Quick Tip to Solve: Calculate the signed zero correction separately, then apply it only once after adding the main and circular readings.

59. A point source of light is placed at the centre of curvature of a hemispherical surface. The source emits a power of $24\ \mathrm{W}$. The radius of curvature of hemisphere is $10\ \mathrm{cm}$ and the inner surface is completely reflecting. The force on the hemisphere due to the light falling on it is _____ $\times 10^{-8}$ N. [4 marks]

Solution:

Concept:

- For a point source emitting isotropically, the intensity at radius r is $I = P/(4\pi r^2)$. Only half the emitted power reaches a hemisphere.
- Normal incidence on a perfectly reflecting surface reverses photon momentum, giving radiation pressure $p = 2I/c$. Rays from the centre of curvature are normal to every surface element.
- The resultant force is along the symmetry axis. Transverse components cancel in opposite pairs; the axial component of each elemental force contains a factor $\cos\theta$.

Step 1: Write the force on a surface element Let θ be the angle between the radius to a surface element and the hemisphere axis. Since intensity is constant over this surface, the outward radial force is

$$dF = \frac{2I}{c}\,dA.$$

Its axial component is $dF\cos\theta$. Simply using the total incident power and treating all rays as parallel would overestimate the resultant, since the radial forces point in different directions.

Step 2: Integrate the axial components Using $dA = r^2\sin\theta\,d\theta\,d\phi$, the projected area integral is

$$\int_{\text{hemisphere}} \cos\theta\,dA = 2\pi r^2 \int_0^{\pi/2} \sin\theta\cos\theta\,d\theta = \pi r^2.$$

Thus

$$F = \frac{2I}{c}\pi r^2 = \frac{2}{c}\frac{P}{4\pi r^2}\pi r^2 = \frac{P}{2c}.$$

With $P = 24\,\mathrm{W}$ and $c = 3\times 10^8\,\mathrm{m\,s^{-1}}$,

$$F = \frac{24}{6\times 10^8} = 4\times 10^{-8}\,\mathrm{N}.$$

The radius cancels because the lower intensity at larger radius is offset by the larger surface area.

Step 3: Conclusion The force magnitude is $\boxed{4\times 10^{-8}\,\mathrm{N}}$; the requested entry is $\boxed{4}$.

Quick Tip to Solve: For radiation forces on curved surfaces, integrate the vector component or use the projected area; do not add force magnitudes.

60. A capacitor of capacitance $900\mu\text{F}$ is charged by a $100\ \text{V}$ battery. The capacitor is disconnected from the battery and connected to another uncharged identical capacitor such that one plate of uncharged capacitor connected to positive plate and another plate of uncharged capacitor connected to negative plate of the charged capacitor. The loss of energy in this process is measured as $x \times 10^{-2}$ J. The value of x is _____. [4 marks]

Solution:

Concept:

- Capacitor charge and stored energy are $Q = CV$ and $U = \frac{1}{2}CV^2$. Disconnecting the battery leaves the total charge of the two-capacitor system conserved.
- Connecting corresponding plates puts the identical capacitors in parallel. Their final potential difference is common, and their total capacitance is $2C$.
- The decrease of electrostatic energy appears during transient charge flow as heating and electromagnetic radiation. Charge conservation does not imply conservation of stored electrostatic energy.

Step 1: Find the common final potential The initial capacitance is $C = 900 \times 10^{-6}\,\text{F}$ and the voltage is $V = 100\,\text{V}$. The total initial positive charge is

$$Q = CV = 900 \times 10^{-6}(100) = 0.090\,\text{C}.$$

The second capacitor initially has zero charge. Once the like plates are connected,

$$2CV_f = CV, \qquad V_f = \frac{V}{2} = 50\,\text{V}.$$

Each capacitor therefore carries half the original charge.

Step 2: Compare the initial and final energies Initially only one capacitor stores energy:

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}(900 \times 10^{-6})(100)^2 = 4.50\,\text{J}.$$

Finally both capacitors store energy at $50\,\text{V}$:

$$U_f = 2\left[\frac{1}{2}C(50)^2\right] = 2.25\,\text{J}.$$

Hence

$$\Delta U = U_i - U_f = 2.25\,\text{J} = 225 \times 10^{-2}\,\text{J}.$$

The final energy is half the initial energy, as expected when a charged capacitor shares charge with an identical uncharged one.

Step 3: Conclusion The energy loss is $\boxed{2.25\,\text{J}}$, and therefore $\boxed{x = 225}$.

Quick Tip to Solve: For identical capacitors sharing charge, use $V_f = V/2$ and remember that both capacitors contribute to the final energy.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. To inhibit the growth of tumours, identify the compounds used from the following :

A. EDTA

B. Coordination Compounds of Pt

C. D - Penicillamine

D. Cis - Platin

Choose the correct answer from the option given below : [4 marks]

(A) A and B Only

(B) C and D Only

(C) B and D Only

(D) A and C Only

Correct Answer: (C) B and D Only

Solution:

Concept:

- Certain platinum coordination compounds are antitumour agents. Cisplatin is the cis isomer of $[Pt(NH_3)_2Cl_2]$, a square-planar platinum(II) complex.
- Chelating agents can also be medicines, but their medical uses differ. Binding a toxic metal ion for its removal is not the same function as inhibiting tumour growth.

Step 1: Identify the antitumour compounds Statement B refers to the platinum coordination compounds used in cancer chemotherapy. Statement D names cisplatin, a specific member of that class. Cisplatin can form coordination bonds to DNA after activation, producing lesions that hinder DNA replication and cell division. Thus B and D both satisfy the stated application; the broad class and its named example are both included in the list.

Step 2: Distinguish the chelating agents and eliminate options EDTA in statement A is used in chelation, for example in treatment of lead poisoning. D-penicillamine in statement C is a chelator used to remove excess copper, as in Wilson disease. These are not the tumour-inhibiting examples asked for in this question. Option (A) includes EDTA and misses cisplatin; option (B) includes D-penicillamine; option (D) selects the two chelators instead of the platinum anticancer agents. Only option (C) includes both B and D and excludes A and C. The classification is based on the specified therapeutic use, rather than merely on whether a compound can form a coordination complex.

Step 3: Conclusion The compounds are B and D only: platinum coordination compounds and cisplatin. Hence, the correct option is (C).

Quick Tip to Solve: Associate cisplatin with cancer treatment, EDTA with lead chelation, and D-penicillamine with copper removal.

62. The major products 'A' and 'B', respectively, are

$$CH_2{=}C(CH_3)_2 \xrightarrow{\text{cold } H_2SO_4} A, \qquad CH_2{=}C(CH_3)_2 \xrightarrow[80\ ^\circ C]{H_2SO_4} B.$$

[4 marks]

(A) $(CH_3)_3C{-}OSO_3H$ and $(CH_3)_2C{=}CH{-}C(CH_3)_3$

(B) $(CH_3)_3C{-}OSO_3H$ and $CH_3{-}CH(CH_3){-}CH_2{-}CH_2{-}CH(CH_3){-}CH_3$

(C) $CH_3{-}CH(CH_3){-}CH_2{-}CH_2{-}CH(CH_3){-}CH_3$ and $(CH_3)_3C{-}OSO_3H$

(D) $(CH_3)_2C{=}CH{-}C(CH_3)_3$ and $(CH_3)_3C{-}OSO_3H$

Correct Answer: (A) $(CH_3)_3C{-}OSO_3H$ and $(CH_3)_2C{=}CH{-}C(CH_3)_3$

Solution:

Concept:

- Protonation of 2-methylpropene, $CH_2{=}C(CH_3)_2$, produces the relatively stable tertiary tert-butyl carbocation. Markovnikov addition places the proton on the terminal carbon.
- Cold concentrated sulfuric acid traps the carbocation with hydrogen sulfate to form an alkyl hydrogen sulfate. At the stated higher temperature, acid-catalysed alkene dimerisation becomes the relevant pathway.
- Dimerisation forms a carbon-carbon bond between a carbocation and another alkene molecule. Loss of a proton then regenerates the acid catalyst and leaves an alkene rather than a saturated alkane.

Step 1: Determine product A in cold acid Protonation followed by attack of HSO_4^- gives tert-butyl hydrogen sulfate:

$$CH_2{=}C(CH_3)_2 + H_2SO_4 \longrightarrow (CH_3)_3C{-}OSO_3H.$$

The oxygen of the sulfate group bonds to the tertiary carbon. This fixes A as the alkyl hydrogen sulfate shown first in options (A) and (B), eliminating options (C) and (D), which reverse the role of the products.

Step 2: Determine product B and compare the remaining options At $80^\circ C$, the tert-butyl carbocation attacks another 2-methylpropene molecule to give $(CH_3)_3C{-}CH_2{-}C^+(CH_3)_2$. Removal of a proton gives the more substituted major alkene

$$(CH_3)_3C{-}CH{=}C(CH_3)_2.$$

This is 2,4,4-trimethylpent-2-ene, the unsaturated dimer shown as B in option (A). The saturated product drawn in option (B) cannot result from simple acid-catalysed dimerisation and proton elimination; it would require net hydrogen addition. Thus only the ordered pair in option (A) has both the cold-addition product and the hot dimerisation product correctly assigned.

Step 3: Conclusion A is tert-butyl hydrogen sulfate and B is 2,4,4-trimethylpent-2-ene. Hence, the correct option is (A).

Quick Tip to Solve: Cold sulfuric acid adds across the double bond; hot acid can couple two alkene molecules, with proton loss preserving a double bond.

63. In the wet tests for identification of various cations by precipitation, which transition element cation doesn't belong to group IV in qualitative inorganic analysis? [4 marks]

(A) Co^{2+}

(B) Ni^{2+}

(C) Zn^{2+}

(D) Fe^{3+}

Correct Answer: (D) Fe^{3+}

Solution:

Concept:

- Classical qualitative cation analysis separates groups using selective precipitation. The group reagent and the medium control the available sulfide or hydroxide concentration.
- Group IV contains ions such as Co^{2+}, Ni^{2+}, Mn^{2+} and Zn^{2+}, precipitated as sulfides in an ammoniacal medium.
- Group III contains Fe^{3+}, Al^{3+} and Cr^{3+}, precipitated as hydroxides using ammonium hydroxide in the presence of ammonium chloride.

Step 1: Classify the listed divalent cations Cobalt(II), nickel(II) and zinc(II) belong to analytical group IV. In the prescribed basic medium their sulfides precipitate:

$$Co^{2+} + S^{2-} \longrightarrow CoS\,(s), \qquad Ni^{2+} + S^{2-} \longrightarrow NiS\,(s).$$

Likewise $Zn^{2+} + S^{2-} \longrightarrow ZnS\,(s)$. Therefore options (A), (B) and (C) all name members of group IV and cannot answer the negative wording of the question.

Step 2: Identify the ion separated in an earlier group Iron(III) is precipitated in group III as ferric hydroxide:

$$Fe^{3+} + 3\,OH^- \longrightarrow Fe(OH)_3(s).$$

Its removal occurs before the group-IV sulfide tests. The analytical group number should not be confused with an element's periodic-table group: this question concerns the laboratory separation scheme. Even though the wording mentions transition elements, the decisive distinction among the offered ions is that ferric ion belongs to analytical group III, while the other three listed cations are assigned to analytical group IV.

Step 3: Conclusion Fe^{3+} does not belong to group IV in qualitative cation analysis. Hence, the correct option is (D).

Quick Tip to Solve: Remember group III as Fe, Al and Cr hydroxides, then group IV as Co, Ni, Mn and Zn sulfides.

64. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Ketoses give Seliwanoff's test faster than Aldoses.

Reason (R) : Ketoses undergo β-elimination followed by formation of furfural.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) (A) is false but (R) is true

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) Both (A) and (R) are true and (R) is the correct explanation of (A)

Correct Answer: (C) (A) is true but (R) is false

Solution:

Concept:

- Seliwanoff's reagent is resorcinol in concentrated hydrochloric acid; rapid acid dehydration of a ketose produces the characteristic red condensation product.
- Aldoses respond more slowly because their conversion to a ketose precedes rapid dehydration. For a hexose the dehydrated product is 5-hydroxymethylfurfural.

Step 1: Evaluate the assertion On heating with Seliwanoff's reagent, a ketohexose such as fructose is dehydrated rapidly. The resulting 5-hydroxymethylfurfural condenses with resorcinol to give a cherry-red colour. An aldohexose such as glucose takes longer to develop this response. Therefore the assertion correctly compares the rates and is true.

Step 2: Evaluate the stated mechanism The reason attributes the result to a specific β-elimination followed by formation of furfural. This is not the correct description of the Seliwanoff reaction of ketohexoses: acid-catalysed dehydration gives 5-hydroxymethylfurfural, which then reacts with resorcinol. Furfural itself is the corresponding dehydration product of pentoses. Thus the stated reason is false; it should not be accepted merely because dehydration involves removal of water.

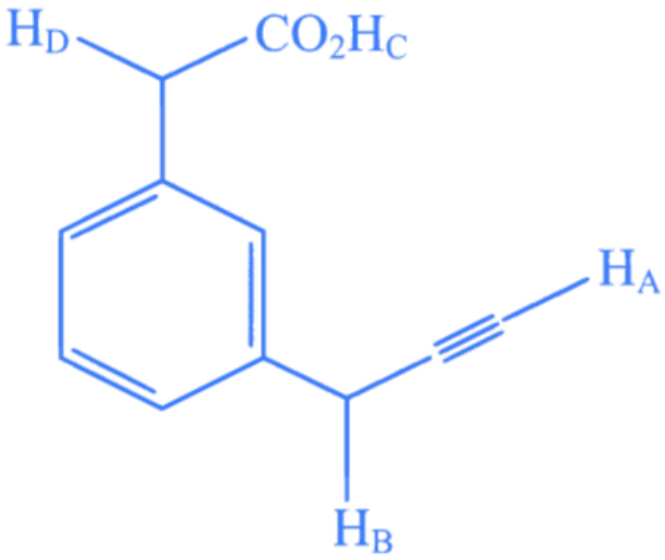
Step 3: Select the truth combination We have a true assertion and a false reason. The option claiming that the assertion is false is excluded immediately. Both options that call the reason true are also excluded, regardless of whether they describe it as an explanation. The remaining truth combination is assertion true, reason false.

Step 4: Conclusion The assertion is true but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: In Seliwanoff's test distinguish rapid ketose dehydration from the subsequent colour reaction, and distinguish furfural from hydroxymethylfurfural.

65. What is the correct order of acidity of the protons marked $\mathrm{A}-\mathrm{D}$ in the given compounds ?

[4 marks]



(A) $\mathrm{H_D} > \mathrm{H_C} > \mathrm{H_B} > \mathrm{H_A}$

(B) $\mathrm{H_C} > \mathrm{H_A} > \mathrm{H_D} > \mathrm{H_B}$

(C) $\mathrm{H_C} > \mathrm{H_D} > \mathrm{H_B} > \mathrm{H_A}$

(D) $\mathrm{H_C} > \mathrm{H_D} > \mathrm{H_A} > \mathrm{H_B}$

Correct Answer: (D) $\mathrm{H_C} > \mathrm{H_D} > \mathrm{H_A} > \mathrm{H_B}$

Solution:

Concept:

- The more stable the conjugate base formed by removing a proton, the greater the acidity. Resonance delocalisation and electron-withdrawing groups stabilise negative charge.
- A carboxylic acid proton is far more acidic than carbon-bound protons. An sp-hybridised carbon stabilises a negative charge more effectively than an otherwise comparable sp^3 carbon.

Step 1: Identify the four proton environments H_C belongs to the carboxylic acid group. Its removal gives a carboxylate ion in which the negative charge is shared between two oxygen atoms. H_D is at the benzylic carbon adjacent to the carboxyl group. H_A is the terminal acetylene proton, while H_B is on the benzylic/propargylic saturated carbon. These environments must be distinguished before comparing acidity.

Step 2: Rank the stabilisation of the anions The oxygen-stabilised carboxylate makes H_C the most acidic. Removing H_D produces a carbon anion stabilised by the adjacent carbonyl group as well as its benzylic environment; this is more favourable than removing the terminal alkyne proton. The acetylide ion from H_A benefits from the high s-character of its sp carbon. The carbon anion from H_B, although benzylic/propargylic, lacks the extra carbonyl stabilisation available at H_D.

Step 3: Compare the proposed sequences Consequently the decreasing acidity is

$$H_C > H_D > H_A > H_B.$$

The first option incorrectly places a carbon-bound proton above the carboxyl proton. The second incorrectly places the terminal alkyne proton above the carbonyl-stabilised proton. The third reverses the last two entries, incorrectly making H_B more acidic than H_A.

Step 4: Conclusion The required order is $H_C > H_D > H_A > H_B$. Hence, the correct option is (D).

Quick Tip to Solve: First isolate the carboxyl O–H proton; then compare the carbon anions using adjacent carbonyl stabilisation and hybridisation.

66. Which of the following is correct order of ligand field strength? [4 marks]

(A) $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

(B) $CO < en < NH_3 < C_2O_4^{2} < S^{2-}$

(C) $S^{2-} < NH_3 < en < CO < C_2O_4^{2}$

(D) $NH_3 < en < CO < S^{2-} < C_2O_4^{2}$

Correct Answer: (A) $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

Solution:

Concept:

- Ligand field strength measures the splitting of metal d-orbitals produced by a ligand; for the same metal environment a stronger-field ligand gives a larger splitting.
- The relevant segment of the spectrochemical series is $S^{2-} < {C_2O_4}^{2-} < NH_3 < en < CO$, where en denotes ethane-1,2-diamine.

Step 1: Place the sulphur and oxygen donor ligands The sulphide ion is a weak-field ligand in the spectrochemical series. Oxalate, an oxygen donor ligand, produces a larger splitting than sulphide but a smaller splitting than ammonia. Thus the first comparisons must be

$$S^{2-} < {C_2O_4}^{2-} < NH_3.$$

Ligand charge alone cannot determine field strength: a doubly charged ligand need not be stronger than a neutral ligand.

Step 2: Place the nitrogen donors and carbon monoxide Ethane-1,2-diamine is placed above ammonia in the spectrochemical series. Carbon monoxide is a very strong-field ligand: its effective bonding, including metal-to-ligand back donation, produces a large d-orbital splitting. Therefore the last portion is

$$NH_3 < en < CO.$$

Joining both portions gives the complete increasing order asked for.

Step 3: Reject the other arrangements The reverse sequence starting with carbon monoxide makes the strongest ligand the weakest and is wrong. The sequence ending with oxalate incorrectly places an oxygen donor above carbon monoxide. The sequence putting sulphide after carbon monoxide likewise contradicts the weak-field character of sulphide. Only the first arrangement preserves all the required pairwise comparisons.

Step 4: Conclusion The increasing ligand field strength is $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$. Hence, the correct option is (A).

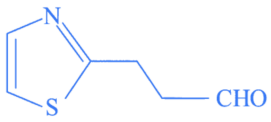
Quick Tip to Solve: Remember this short series as sulphide, oxalate, ammonia, en, CO; do not rank these ligands by charge alone.

67. Which of the following compounds would give the following set of qualitative analysis ?

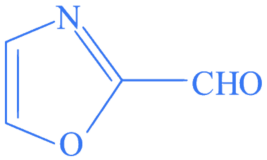
(i) Fehling's Test : Positive

(ii) Na fusion extract upon treatment with sodium nitroprusside gives a blood red colour but not prussian blue. [4 marks]

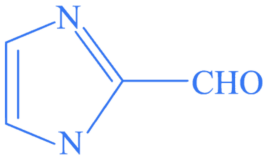
(A) 3-(Thiazol-2-yl)propanal



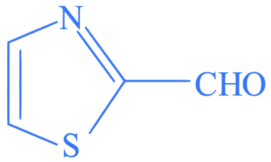
(B) Oxazole-2-carbaldehyde



(C) Imidazole-2-carbaldehyde

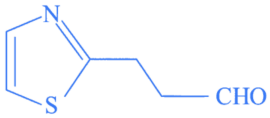


(D) Thiazole-2-carbaldehyde



Correct Answer: (A)

3-(Thiazol-2-yl)propanal



Solution:

Concept:

- Fehling's solution is reduced by an aliphatic aldehyde, giving a brick-red precipitate of Cu_2O; an aldehyde attached directly to an aromatic ring normally does not give this test.
- Sodium fusion of a compound containing both nitrogen and sulphur can give $NaSCN$. Thiocyanate gives a blood-red complex with ferric ions, whereas sulphide with sodium nitroprusside gives a violet colour.

Step 1: Apply the aldehyde test Option (A) contains a terminal $-CHO$ group at the end of a saturated two-carbon chain attached to a thiazole ring. The aldehyde is therefore aliphatic and gives Fehling's test. In the other three structures the $-CHO$ group is directly attached to the heteroaromatic ring.

They do not supply the aliphatic aldehyde arrangement selected by the positive Fehling test.

Step 2: Check nitrogen and sulphur together The thiazole ring in (A) contains both nitrogen and sulphur. Their sodium-fusion product can be sodium thiocyanate rather than sodium cyanide:

$$\mathrm{Na + C + N + S \longrightarrow NaSCN.}$$

Consequently the usual cyanide-based Prussian-blue test need not appear. Options (B) and (C) contain no sulphur and cannot meet this combined nitrogen-and-sulphur interpretation. Option (D) has both elements but fails the aliphatic-aldehyde criterion.

Step 3: Distinguish the stated reagent from the intended test There is a chemical inconsistency in the printed colour test. Blood red is the thiocyanate test with ferric ions, represented by $\mathrm{Fe^{3+} + SCN^- \rightleftharpoons [FeSCN]^{2+}}$, not the sodium-nitroprusside test; sodium nitroprusside with sulphide gives violet. Thus no option fulfils the second test exactly as worded. If that reagent is read as the usual ferric-ion thiocyanate test, the unique structural choice is (A). This identifies the intended compound without claiming the printed reagent produces a blood-red colour.

Step 4: Conclusion The intended compound is 3-(thiazol-2-yl)propanal, option (A). Strictly, the sodium-nitroprusside/blood-red statement is chemically inconsistent, so the literal question has no fully correct option. Hence, the correct option is (A) under the intended thiocyanate-test interpretation.

Quick Tip to Solve: Use Fehling's test to identify the aliphatic aldehyde, then distinguish ferric thiocyanate's blood red from nitroprusside's violet sulphide test.

68. During the qualitative analysis of $\mathrm{SO_3^{2-}}$ using dilute $\mathrm{H_2SO_4}$, $\mathrm{SO_2}$ gas is evolved which turns $\mathrm{K_2Cr_2O_7}$ solution (acidified with dilute $\mathrm{H_2SO_4}$) : [4 marks]

(A) blue

(B) black

(C) red

(D) green

Correct Answer: (D) green

Solution:

Concept:

- Acidification of sulphite liberates sulphur dioxide: $\mathrm{{SO_3}^{2-} + 2\,H^+ \longrightarrow SO_2 + H_2O}$. Sulphur dioxide acts as a reducing agent.
- Acidified dichromate contains chromium(VI), which is reduced to chromium(III). The characteristic colour changes from orange to green.

Step 1: Identify the gas and its reducing action Dilute sulphuric acid supplies hydrogen ions to the sulphite ion. The unstable sulphurous acid formed decomposes into water and sulphur dioxide. Sulphur in $\mathrm{SO_2}$ has oxidation state $+4$, so it can be oxidised to sulphate, in which sulphur has oxidation state $+6$. Each sulphur atom therefore loses two electrons. The evolved gas reduces acidified potassium dichromate rather than merely dissolving in it.

Step 2: Balance the electron transfer The relevant half-reactions are

$$SO_2 + 2\,H_2O \longrightarrow {SO_4}^{2-} + 4\,H^+ + 2\,e^-$$

and

$${Cr_2O_7}^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

Multiplying the first by three and adding gives

$${Cr_2O_7}^{2-} + 3\,SO_2 + 2\,H^+ \longrightarrow 2\,Cr^{3+} + 3\,{SO_4}^{2-} + H_2O.$$

Both charge and all atoms are balanced.

Step 3: Connect the product to its colour The orange chromium(VI) reagent is converted into green chromium(III) ions in the acidic solution. A blue, black or red solution is not the characteristic product of this dichromate reduction. Those alternatives do not correspond to the chromium oxidation-state change established by the half-reactions.

Step 4: Conclusion The acidified potassium dichromate solution turns green because Cr^{3+} is formed. Hence, the correct option is (D).

Quick Tip to Solve: For acidified dichromate tests, track chromium: orange Cr(VI) becomes green Cr(III) when the test gas is a reducing agent.

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69. For OF_2 molecule consider the following :

A. Number of lone pairs on oxygen is 2 .

B. FOF angle is less than $104.5°$.

C. Oxidation state of O is -2.

D. Molecule is bent 'V' shaped.

E. Molecular geometry is linear.

correct options are: [4 marks]

(A) A, C, D only

(B) A, B, D only

(C) C, D, E only

(D) B, E, A only

Correct Answer: (B) A, B, D only

Solution:

Concept:

- The central oxygen atom has six valence electrons. Two single bonds to fluorine leave four nonbonding electrons, giving two lone pairs and two bond pairs.
- VSEPR predicts a bent AX_2E_2 molecular shape. Fluorine has oxidation state -1 in its compounds, so oxygen is positive in oxygen fluorides.

Step 1: Count the electron pairs and establish shape The Lewis structure of OF_2 has two $O-F$ single bonds and two lone pairs on oxygen. Thus statement A is true. Four electron domains have an approximately tetrahedral electron-pair arrangement, but the molecular shape considers only the positions of the atoms. With two bonded atoms and two lone pairs, the molecule is bent or V-shaped. Hence D is true and E, which calls the molecular geometry linear, is false.

Step 2: Compare the bond angle with water The very electronegative fluorine atoms draw the bonding electron density away from oxygen. This decreases bond-pair repulsion near the central oxygen relative to water, allowing the lone pairs to compress the F–O–F angle further. The angle is about 103°, smaller than the 104.5° H–O–H angle of water. Statement B is therefore true.

Step 3: Calculate oxygen oxidation state and select Let oxygen's oxidation state be x. Since the molecule is neutral and each fluorine is -1,

$$x + 2(-1) = 0 \quad \Rightarrow \quad x = +2.$$

Statement C, assigning oxygen -2, is false. The true statements are exactly A, B and D. The first and third combinations include false statement C; the third and fourth also include false statement E. Only the second combination contains all and only the true statements.

Step 4: Conclusion Statements A, B and D alone are correct. Hence, the correct option is (B).

Quick Tip to Solve: Separate electron-pair geometry from molecular shape, and assign fluorine its oxidation state before calculating oxygen's.

70. Match List I with List II.

	List I		List II
A.	$C_6H_5Cl + CH_3Cl \xrightarrow{Na} C_6H_5CH_3$	**I.**	**Fittig reaction**
B.	$C_6H_5Cl \xrightarrow{2\,Na} C_6H_5-C_6H_5$	**II.**	**Wurtz Fittig reaction**
C.	$C_6H_5N_2{}^+Cl^- \xrightarrow{Cu_2Cl_2} C_6H_5Cl + N_2$	**III.**	**Finkelstein reaction**
D.	$C_5H_5Cl + NaI \longrightarrow C_2H_5I + NaCl$	**IV.**	**Sandemyer reaction**

Choose the correct answer from the options given below: [4 marks]

(A) A - IV, B - II, C - III, D - I

(B) A - II, B - I, C - IV, D - III

(C) A - II, B - I, C - III, D - IV

(D) A - III, B - II, C - IV, D - I

Correct Answer: (B) A - II, B - I, C - IV, D - III

Solution:

Concept:

- Wurtz–Fittig coupling combines an aryl halide and an alkyl halide with sodium; Fittig coupling joins aryl groups using sodium.
- Sandmeyer substitution replaces a diazonium group by a halogen using a copper(I) salt. Finkelstein

reaction exchanges an alkyl chloride or bromide for iodide using sodium iodide.

Step 1: Identify the two sodium couplings In reaction A, chlorobenzene and methyl chloride react with sodium to form methylbenzene. The coupling partners are one aryl halide and one alkyl halide, so this is Wurtz–Fittig reaction: A matches II. Reaction B produces biphenyl from an aryl halide under sodium coupling conditions. Aryl–aryl coupling is the Fittig reaction: B matches I. Distinguish these by inspecting both coupling partners rather than only the sodium reagent.

Step 2: Identify diazonium substitution Reaction C starts with benzenediazonium chloride and gives chlorobenzene with liberation of nitrogen in the presence of cuprous chloride, printed as Cu_2Cl_2. This is the characteristic Sandmeyer reaction, so C matches IV. It cannot be Finkelstein reaction because the starting group being displaced is a diazonium group, not a chlorine or bromine atom on an alkyl carbon.

Step 3: Identify the iodide exchange and compare choices Reaction D displays sodium iodide and formation of an organic iodide together with sodium chloride. This is the characteristic Finkelstein exchange pattern, giving D–III. The printed reactant formula C_5H_5Cl does not conserve carbon with the displayed C_2H_5I product; the line is not a balanced chemical equation as printed. The exchange identity is nevertheless unambiguous from the reagents and products. The combined matching A–II, B–I, C–IV, D–III is only in option (B). The other choices misidentify at least one of these diagnostic transformations.

Step 4: Conclusion A–II, B–I, C–IV, D–III. Hence, the correct option is (B); the printed formula mismatch in row D does not change the named exchange reaction.

Quick Tip to Solve: Sodium plus aryl and alkyl halides means Wurtz–Fittig; sodium plus aryl halides means Fittig; diazonium plus Cu(I) means Sandmeyer.

71. Benzyl isocyanide can be obtained by:

A. $C_6H_5CH_2Br \xrightarrow{AgCN}$

B. $C_6H_5CH_2NH_2 \xrightarrow[aq.\ KOH]{CHCl_3}$

C. $C_6H_5CH_2NHCH_3 \xrightarrow[aq.\ KOH]{CHCl_3}$

D. $C_6H_5CH_2OTs \xrightarrow{KCN}$

Choose the correct answer from the options given below : [4 marks]

(A) B and C

(B) A and D

(C) A and B

(D) Only B

Correct Answer: (C) A and B

Solution:

Concept:

- An isocyanide has the connectivity $R-NC$, whereas a nitrile has $R-CN$. The atom through which cyanide attacks therefore decides the product.
- The carbylamine reaction converts a primary amine into an isocyanide using chloroform and strong alkali. Secondary amines do not give this reaction.

Step 1: Examine reactions A and B In A, benzyl bromide reacts with silver cyanide. Because silver cyanide is predominantly covalent, attack occurs through nitrogen, giving benzyl isocyanide, $C_6H_5CH_2NC$. In B, benzylamine is a primary amine; its nitrogen is attached to one benzyl group and has two hydrogens. The chloroform and alkaline conditions give the carbylamine reaction. Its balanced overall equation is

$$C_6H_5CH_2NH_2 + CHCl_3 + 3\,KOH \longrightarrow C_6H_5CH_2NC + 3\,KCl + 3\,H_2O.$$

Thus both A and B provide the required connectivity.

Step 2: Reject reactions C and D The nitrogen in C is bonded to benzyl and methyl groups, so this compound is a secondary amine. It cannot undergo the characteristic primary-amine carbylamine reaction to give benzyl isocyanide. In D, the benzyl tosylate reacts with ionic potassium cyanide. Its free cyanide ion attacks mainly through carbon, replacing the tosylate leaving group and giving $C_6H_5CH_2CN$, a nitrile. This product contains a carbon-carbon bond to the cyanide carbon and is different from benzyl isocyanide. Therefore combinations containing C or D are rejected; Only B also omits the valid route A.

Step 3: Conclusion Benzyl isocyanide is obtained by reactions A and B. Hence, the correct option is (C).

Quick Tip to Solve: Check both the class of amine and the cyanide reagent: primary amine gives carbylamine, AgCN gives isocyanide, and KCN gives nitrile.

72. Match List I with List II

	List I(Atomic number)		List II(Block of periodic table)
A.	37	I.	p-block
B.	78	II.	d-block
C.	52	III.	f-block
D.	65	IV.	s-block

Choose the correct answer from the options given below : [4 marks]

(A) A - II, B - IV, C - I, D - III

(B) A - IV, B - III, C - II, D - I

(C) A - IV, B - II, C - I, D - III

(D) A - I, B - III, C - IV, D - II

Correct Answer: (C) A - IV, B - II, C - I, D - III

Solution:

Concept:

- The block of an element is identified by the subshell receiving its differentiating electron: s, p, d or f. It is not decided simply by the highest principal quantum number.
- Transition metals belong to the d-block, while the lanthanoids are f-block elements even though their outermost occupied shell also contains s electrons.

Step 1: Identify the elements and their configurations Atomic number 37 is rubidium, with configuration $[\mathrm{Kr}]5s^1$. Its valence electron enters an s orbital, so A belongs to the s-block, IV. Atomic number 78 is platinum, a transition metal with ground-state configuration $[\mathrm{Xe}]4f^{14}5d^9 6s^1$. Despite its exceptional distribution between d and s orbitals, it belongs to the d-block; hence B matches II.

Step 2: Complete the matches and compare the options Atomic number 52 is tellurium, with configuration $[\mathrm{Kr}]4d^{10}5s^2 5p^4$. Filling of the p subshell places it in the p-block, so C matches I. Atomic number 65 is terbium, a lanthanoid with configuration $[\mathrm{Xe}]4f^9 6s^2$, placing it in the f-block, so D matches III. Collecting these results gives

$$\mathrm{A}-\mathrm{IV}, \quad \mathrm{B}-\mathrm{II}, \quad \mathrm{C}-\mathrm{I}, \quad \mathrm{D}-\mathrm{III}.$$

Option A incorrectly interchanges the blocks for rubidium and platinum. Option B incorrectly treats platinum as f-block and tellurium as d-block, while option D misplaces several elements, including rubidium. Only option C contains all four correct matches.

Step 3: Conclusion The matching is A-IV, B-II, C-I, D-III. Hence, the correct option is (C).

Quick Tip to Solve: First identify familiar anchors: atomic number 37 is alkali metal Rb and 65 is lanthanoid Tb; their s- and f-block positions quickly narrow the choices.

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73. Match List I with List II

	List I(molecules/ions)		List II(No. of lone pairs of e^- on central atom)
A.	IF_7	**I.**	**Three**
B.	ICl_4^-	**II.**	**One**
C.	XeF_6	**III.**	**Two**
D.	XeF_2	**IV.**	**Zero**

Choose the correct answer from the options given below : [4 marks]

(A) A - II, B - I, C - IV, D - III

(B) A - IV, B - I, C - II, D - III

(C) A - IV, B - III, C - II, D - I

(D) A - II, B - III, C - IV, D - I

Correct Answer: (C) A - IV, B - III, C - II, D - I

Solution:

Concept:

- For a central atom bonded to monovalent halogens, subtract one central-atom electron for each single bond from its available valence electrons. The remaining electrons form lone pairs.
- Add one electron for a negative ionic charge. Count lone pairs on the central atom only; the terminal halogens also have lone pairs, but these are not requested.

Step 1: Count lone pairs on iodine Iodine contributes seven valence electrons. In IF_7, it forms seven iodine-fluorine single bonds, using one of its own electrons in each bond. No nonbonding electrons remain on iodine, so A has zero lone pairs and matches IV. For ICl_4^-, the negative charge raises the available central-atom electron count to eight. Four are used for four iodine-chlorine bonds; four remain as two lone pairs. Thus B matches III.

Step 2: Count lone pairs on xenon and select the mapping Xenon has eight valence electrons. Six single bonds in XeF_6 leave two electrons on xenon, giving one lone pair; C therefore matches II. Two single bonds in XeF_2 leave six electrons on xenon, giving three lone pairs; D therefore matches I. The counts can be summarized by

$$IF_7 : (7-7)/2 = 0, \quad ICl_4^- : (7+1-4)/2 = 2,$$
$$XeF_6 : (8-6)/2 = 1, \quad XeF_2 : (8-2)/2 = 3.$$

Options A and D incorrectly give iodine in IF_7 a lone pair. Option B assigns three rather than two lone pairs to ICl_4^-, and two rather than three to XeF_2. Only C matches all counts.

Step 3: Conclusion The correct matching is A-IV, B-III, C-II, D-I. Hence, the correct option is (C).

Quick Tip to Solve: For halogen-bonded central atoms, use (central valence electrons + negative charge - number of bonds)/2 to count central lone pairs.

74. Caprolactam when heated at high temperature in presence of water, gives [4 marks]

(A) Dacron

(B) Nylon 6

(C) Teflon

(D) Nylon 6,6

Correct Answer: (B) Nylon 6

Solution:

Concept:

- Caprolactam is a cyclic amide containing six carbon atoms. Heating it with water opens the lactam ring, providing an amino-acid unit that can form a polyamide.
- Nylon-6 is formed from a single six-carbon monomer. Nylon-6,6 instead uses two different six-carbon monomers, a diamine and a dicarboxylic acid.

Step 1: Open the caprolactam ring Water hydrolyses the amide bond in the caprolactam ring to give

6-aminohexanoic acid, also called 6-aminocaproic acid:

$$H_2N-(CH_2)_5-COOH.$$

This molecule has an amino group at one end and a carboxylic acid group at the other. Each molecule can therefore join another by forming an amide linkage, so repeated reaction produces a long chain rather than a small isolated product.

Step 2: Identify the repeating unit and reject the alternatives The polyamide chain contains the repeating unit

$$[-NH-(CH_2)_5-CO-]_n .$$

There are six carbon atoms in the monomer-derived unit: five methylene carbon atoms and one carbonyl carbon. This is nylon-6. Dacron is a polyester made from ethylene glycol and terephthalic acid, so its ester linkages do not match the amide linkage obtained here. Teflon is the addition polymer of tetrafluoroethene and contains a carbon-fluorine backbone pattern. Nylon-6,6 requires hexamethylenediamine and adipic acid, rather than caprolactam alone. The presence of water initiates ring opening; it does not change the identity to nylon-6,6.

Step 3: Conclusion Caprolactam gives nylon-6 on heating at high temperature in the presence of water. Hence, the correct option is (B).

Quick Tip to Solve: Remember the monomer pairings: caprolactam gives nylon-6, while hexamethylenediamine plus adipic acid gives nylon-6,6.

75. Formation of photochemical smog involves the following reaction in which A, B and C are respectively.

i. $NO_2 \xrightarrow{h\upsilon} A + B$

ii. $B + O_2 \rightarrow C$

iii. $A + C \rightarrow NO_2 + O_2$

Choose the correct answer from the options given below : [4 marks]

(A) NO, O & O_3

(B) N, O_2 & O_3

(C) O, N_2O & NO

(D) O, NO & NO_3^-

Correct Answer: (A) NO, O & O_3

Solution:

Concept:

- Sunlight initiates photochemical smog chemistry by photodissociating nitrogen dioxide into nitric oxide and an oxygen atom.
- Atomic oxygen combines with molecular oxygen to form ozone. Ozone can then oxidise nitric

oxide back to nitrogen dioxide.

Step 1: Identify the products of photolysis The first reaction is

$$NO_2 \xrightarrow{h\nu} NO + O.$$

Thus A is nitric oxide, NO, and B is atomic oxygen, O. Both atoms and the overall neutral charge are conserved: the nitrogen atom remains in nitric oxide while one of the two oxygen atoms is liberated. Molecular nitrogen and molecular oxygen are not the products of this elementary photolysis step.

Step 2: Use the next two reactions as checks The liberated oxygen atom reacts with oxygen gas:

$$O + O_2 \longrightarrow O_3.$$

Hence C is ozone, O_3. Substitution in the third equation gives

$$NO + O_3 \longrightarrow NO_2 + O_2,$$

which is the familiar oxidation of nitric oxide by ozone and reproduces the printed products. Option B incorrectly gives nitrogen atoms and oxygen molecules in the first step. Option C would require nitrogen to appear in B even though B must combine with oxygen to make C. Option D assigns nitric oxide to B and a nitrate ion to C; those choices do not reproduce either the neutral products or the oxygen balance of the stated cycle. Only the first option agrees with all three equations, not merely with the presence of ozone in smog.

Step 3: Conclusion A, B and C are respectively NO, O and O_3. Hence, the correct option is (A).

Quick Tip to Solve: Start the smog cycle with nitrogen dioxide photolysis, then check the third equation; NO consumes ozone to regenerate nitrogen dioxide.

76. In the extraction of copper, its sulphide ore is heated in a reverberatory furnace after mixing with silica to: [4 marks]

(A) remove FeO as $FeSiO_3$

(B) remove calcium as $CaSiO_3$

(C) decrease the temperature needed for roasting of Cu_2S

(D) separate CuO as $CuSiO_3$

Correct Answer: (A) remove FeO as $FeSiO_3$

Solution:

Concept:

- A flux reacts with gangue or an impurity oxide to produce a fusible slag, enabling separation from the useful metal-containing phase.
- Silica, SiO_2, is an acidic flux. In copper sulphide metallurgy it removes basic iron(II) oxide as iron silicate.

Step 1: Identify the impurity removed by silica The copper sulphide concentrate commonly contains

iron sulphide. During oxidation, iron sulphide is converted to iron(II) oxide. A representative oxidation equation is

$$2\,FeS + 3\,O_2 \longrightarrow 2\,FeO + 2\,SO_2.$$

The iron oxide must be removed from the copper-bearing material. Adding silica supplies an acidic oxide that can combine with this basic impurity oxide in the heated furnace.

Step 2: Form the slag and compare the proposed purposes The relevant slag-forming reaction is

$$FeO + SiO_2 \longrightarrow FeSiO_3.$$

The product is iron silicate slag, which separates from the copper matte. The role of silica is therefore removal of FeO, not a decrease in the temperature needed for roasting copper sulphide. Calcium removal as calcium silicate is not the stated impurity-removal step in this process. Removing copper oxide as copper silicate would send valuable copper into the slag and is not the intended purpose of silica addition. This distinction also separates the flux from a reducing agent: silica is used to capture the oxide impurity chemically, rather than to reduce copper directly. The balanced equation preserves one iron atom and combines the oxide oxygen with the two oxygen atoms of silica to form the silicate.

Step 3: Conclusion Silica removes FeO as $FeSiO_3$ slag. Hence, the correct option is (A).

Quick Tip to Solve: Match the acidic flux silica with a basic impurity oxide: FeO plus SiO2 forms iron silicate slag in copper extraction.

77. Amongst the following compounds, which one is an antacid? [4 marks]

(A) Ranitidine

(B) Brompheniramine

(C) Terfenadine

(D) Meprobamate

Correct Answer: (A) Ranitidine

Solution:

Concept:

- Antacids relieve excessive gastric acidity. In the therapeutic classification used here, drugs that suppress acid secretion also belong to the treatment of hyperacidity.
- Histamine acts at different receptors in different tissues. Blocking gastric H2 receptors suppresses acid secretion, whereas H1 receptor antagonists principally treat allergic responses.

Step 1: Identify the gastric-acid drug Ranitidine is a histamine H2-receptor antagonist. It acts on the histamine-mediated stimulation of acid-secreting cells in the stomach and reduces gastric acid production. Therefore it is the compound in the list associated with treatment of excessive stomach acidity. Its action differs from a simple inorganic neutralising agent such as a carbonate: it controls secretion instead of merely consuming acid already present.

Step 2: Classify the remaining compounds Brompheniramine is an antihistamine used for allergic

symptoms. Its relevant action is associated with H1 receptors, so it is not the required gastric-acid drug. Terfenadine is also an H1 antihistamine used against allergic responses; describing both drugs simply as antihistamines would overlook the distinction between the receptor types. Meprobamate belongs to the class of tranquillisers and acts on the nervous system rather than serving as an antacid. Thus options B and C are excluded by their antiallergic role, and option D is excluded by its tranquillising role. Among the given choices, only ranitidine has the acid-suppressing action asked for in the question. The comparison depends on therapeutic function, rather than on whether a drug contains nitrogen or has a similar-sounding name.

Step 3: Conclusion Ranitidine is the antacid among the listed compounds. Hence, the correct option is (A).

Quick Tip to Solve: Keep H1 and H2 blockers separate: H1 antihistamines treat allergies; ranitidine blocks gastric H2 receptors and reduces acid secretion.

78. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In expensive scientific instruments, silica gel is kept in watch-glasses or in semipermeable membrane bags.

Reason (R) : Silica gel adsorbs moisture from air via adsorption, thus protects the instrument from water corrosion (rusting) and / or prevents malfunctioning.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) (A) is true but (R) is false

(C) (A) is false but (R) is true

(D) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Correct Answer: (A) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- Adsorption is accumulation of molecules at a solid surface; silica gel is a porous solid with a large surface area.
- A desiccant removes water vapour from the surrounding air. Lower humidity reduces moisture-related corrosion and damage to sensitive instruments.

Step 1: Assess the assertion Silica gel is commonly placed beside sensitive scientific instruments in an open watch-glass or a moisture-permeable bag. Such containment keeps the granules separate from the instrument while allowing water vapour in the surrounding air to reach them. Thus the described storage practice is appropriate, and Assertion (A) is true.

Step 2: Assess the reason The extensive surface of silica gel adsorbs water vapour. It therefore decreases the moisture available near metal parts, optical surfaces and electrical components. This

helps prevent rusting where relevant and reduces malfunctioning caused by humidity. Reason (R) correctly describes the useful property of silica gel and is true.

Step 3: Connect the two statements The reason explains why silica gel is deliberately kept near the instrument: its removal of moisture provides protection during storage. The watch-glass or bag makes the desiccant accessible to the air. Option (B) incorrectly calls the reason false, option (C) incorrectly calls the assertion false, and option (D) denies the direct explanatory connection.

Step 4: Conclusion Both (A) and (R) are true, and (R) is the correct explanation of (A). Hence, the correct option is (A).

Quick Tip to Solve: For a desiccant assertion-reason question, connect its water-vapour adsorption directly to the purpose of keeping the protected object dry.

79. Lithium aluminium hydride can be prepared from the reaction of : [4 marks]

(A) $LiCl$ and Al_2H_6

(B) LiH and $Al(OH)_3$

(C) LiH and Al_2Cl_6

(D) $LiCl, Al$ and H_2

Correct Answer: (C) LiH and Al_2Cl_6

Solution:

Concept:

- Lithium aluminium hydride, $LiAlH_4$, is prepared by reacting lithium hydride with anhydrous aluminium chloride in dry ether.
- Aluminium chloride may be written in its dimeric form, Al_2Cl_6. Four hydride ions are needed for each aluminium atom in the product.

Step 1: Identify the required reactants The desired product contains the tetrahydridoaluminate ion, AlH_4^-, accompanied by Li^+. Lithium hydride supplies hydride ions, and aluminium chloride supplies aluminium in a form whose chloride ions can be replaced. Dry conditions are essential because water or an alcohol would consume hydride rather than allow the reducing reagent to be isolated.

Step 2: Balance the preparation reaction For one aluminium atom the balanced reaction is

$$4\,LiH + AlCl_3 \longrightarrow LiAlH_4 + 3\,LiCl.$$

Using the dimeric formula printed in the options gives

$$8\,LiH + Al_2Cl_6 \longrightarrow 2\,LiAlH_4 + 6\,LiCl.$$

Both atom balance and the required hydride-to-aluminium ratio are satisfied.

Step 3: Eliminate the other combinations Option (A) lacks lithium hydride, the reagent used in this preparation. In option (B), the hydroxyl groups of $Al(OH)_3$ react with hydride to evolve hydrogen, so it is not the anhydrous chloride route. Option (D) lists lithium chloride, aluminium and hydrogen

rather than the required hydride donor and aluminium halide pair. Only option (C) contains both necessary reactants.

Step 4: Conclusion Lithium hydride and Al_2Cl_6 prepare $LiAlH_4$. Hence, the correct option is (C).

Quick Tip to Solve: Balance the monomeric $AlCl_3$ equation first; double it when the option gives the dimer Al_2Cl_6.

80. The alkaline earth metal sulphate(s) which are readily soluble in water is/are :

A. $BeSO_4$

B. $MgSO_4$

C. $CaSO_4$

D. $SrSO_4$

E. $BaSO_4$

Choose the correct answer from the options given below : [4 marks]

(A) A and B

(B) B and C

(C) B only

(D) A only

Correct Answer: (A) A and B

Solution:

Concept:

- Dissolution of an ionic solid depends on the balance between lattice enthalpy, hydration enthalpy and entropy.
- Down Group 2, sulphate solubility decreases markedly because the hydration enthalpy becomes less favourable as the cation grows, while lattice enthalpy changes relatively little for the large sulphate ion.

Step 1: Apply the sulphate solubility trend The sulphate ion is large, so changes in the Group 2 cation size do not reduce the lattice enthalpy enough to compensate for the loss of hydration energy down the group. The smaller cations at the top of the group are hydrated strongly. Consequently their sulphates dissolve much more readily than the sulphates of the larger cations.

Step 2: Classify the five listed salts $BeSO_4$ and $MgSO_4$ are readily soluble in water, so statements A and B qualify. $CaSO_4$ is only sparingly soluble and does not meet the readily-soluble criterion. $SrSO_4$ and $BaSO_4$ have still lower solubilities. Thus C, D and E are excluded from the requested set.

Step 3: Compare the offered combinations The correct combination must include both A and B and omit C. Option (B) incorrectly includes calcium sulphate and omits beryllium sulphate. Option (C), B only, omits the readily soluble beryllium salt. Option (D), A only, omits magnesium sulphate. Option (A) alone contains the complete and correct pair.

Step 4: Conclusion The readily soluble sulphates are $BeSO_4$ and $MgSO_4$, namely A and B. Hence, the correct option is (A).

Quick Tip to Solve: For Group 2 sulphates, remember the downward decrease in solubility; do not confuse it with the increasing solubility of the hydroxides.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Some amount of dichloromethane (CH_2Cl_2) is added to $671.141\ mL$ of chloroform $(CHCl_3)$ to prepare $2.6 \times 10^{-3}M$ solution of $CH_2Cl_2(DCM)$. The concentration of DCM is _____ ppm (by mass).

Given :

atomic mass : C = 12

H = 1

Cl = 35.5

density of $CHCl_3 = 1.49\ g\ cm^{-3}$ [4 marks]

Solution:

Concept:

- Molarity is moles of solute per litre of solution: $M = n/V$. For a very dilute solution the solution volume is taken approximately equal to the solvent volume.
- Mass-based parts per million is $\text{ppm} = (m_{\text{solute}}/m_{\text{solution}})10^6$, with both masses in the same units.

Step 1: Find the solvent mass and solute molar mass The given density converts the chloroform volume into mass:

$$m_{\text{solvent}} = 671.141 \times 1.49 = 1000.00009\,\text{g}.$$

Using the given atomic masses, the molar mass of dichloromethane is

$$M_{CH_2Cl_2} = 12 + 2(1) + 2(35.5) = 85\,\text{g mol}^{-1}.$$

The approximately one-kilogram solvent mass makes the conversion to mass ppm particularly simple.

Step 2: Calculate the dissolved solute mass This is a very dilute solution, so use $V_{\text{solution}} \simeq 0.671141\,\text{L}$. The solute amount and mass are

$$n = 2.6 \times 10^{-3} \times 0.671141 = 1.7449666 \times 10^{-3}\,\text{mol},$$

$$m_{\text{DCM}} = 1.7449666 \times 10^{-3} \times 85 = 0.148322161\,\text{g}.$$

This small solute mass is consistent with the dilute-volume approximation.

Step 3: Convert the mass fraction to ppm Include the solute in the denominator for a mass fraction of solution:

$$m_{\text{solution}} = 1000.00009 + 0.148322161 = 1000.148412161\,\text{g},$$

$$\text{ppm} = \frac{0.148322161}{1000.148412161} \times 10^6 \simeq 148.30.$$

Neglecting the tiny solute correction in the denominator also gives about 148.32 ppm; both yield the same integer value.

Step 4: Conclusion The concentration of dichloromethane is approximately $\boxed{148\,\text{ppm}}$ by mass.

Quick Tip to Solve: Convert molarity to grams of solute first, then use density to obtain the solvent mass; mass ppm uses total solution mass, not its volume.

82. If compound A reacts with B following first order kinetics with rate constant $2.011 \times 10^{-3}\ s^{-1}$. The time taken by A (in seconds) to reduce from 7 g to 2 g will be _____. (Nearest Integer)

$$[\log 5 = 0.698, \log 7 = 0.845, \log 2 = 0.301]$$

[4 marks]

Solution:

Concept:

- For a first-order reaction, $kt = \ln([A]_0/[A]_t) = 2.303 \log_{10}([A]_0/[A]_t)$.
- At unchanged reaction volume, the mass of the same reactant is proportional to its concentration. Its molar mass therefore cancels in the initial-to-final ratio.

Step 1: Set up the initial-to-final ratio The amount of A decreases from $7\,g$ to $2\,g$. Because both masses refer to the same compound, the ratio in the first-order law is $7/2$, not the mass lost divided by the initial mass. Thus

$$t = \frac{2.303}{k} \log_{10}\left(\frac{7}{2}\right).$$

The logarithm is positive, as it must be for a positive elapsed time.

Step 2: Use the supplied logarithms The question supplies decimal logarithms. Therefore

$$\log_{10}\left(\frac{7}{2}\right) = \log 7 - \log 2 = 0.845 - 0.301 = 0.544.$$

Substitute the rate constant in inverse seconds:

$$t = \frac{2.303 \times 0.544}{2.011 \times 10^{-3}}\,s.$$

The factor 2.303 converts the decimal logarithm to the natural logarithm required by the integrated law.

Step 3: Evaluate and round the elapsed time The numerator is $2.303 \times 0.544 = 1.252832$. Hence

$$t = \frac{1.252832}{0.002011}\,s \simeq 622.99\,s.$$

The logarithmic factor is dimensionless, so division by a rate constant in s^{-1} correctly gives seconds. Rounding only this final value avoids any premature rounding error.

Step 4: Conclusion The required time, to the nearest integer, is $\boxed{623\,s}$.

Quick Tip to Solve: For first-order decay of a mass, use $\log(m_0/m_t)$; the molar mass cancels, and the mass lost is not the denominator.

83. $600\ \text{mL}$ **of** $0.01\ \text{M}\ \text{HCl}$ **is mixed with** $400\ \text{mL}$ **of** $0.01\ \text{M}\ \text{H}_2\text{SO}_4$**. The** pH **of the mixture is** _____ $\times 10^{-2}$**. (Nearest integer)**

[Given $\log 2 = 0.30$

$$\log 3 = 0.48$$

$$\log 5 = 0.69$$

$$\log 7 = 0.84$$

$$\log 11 = 1.04]$$

[4 marks]

Solution:

Concept:

- In the strong-acid treatment of this mixture, HCl supplies one mole of H^+ per mole and H_2SO_4 supplies two.
- Add hydrogen-ion amounts before dividing by the total volume. The pH is $-\log_{10}[\text{H}^+]$, and it is dimensionless.

Step 1: Calculate the hydrogen-ion amounts Convert the two solution volumes to litres. Hydrochloric acid contributes

$$n_{\text{H}^+,1} = 0.600 \times 0.01 = 0.006\,\text{mol}.$$

With the strong-acid stoichiometric treatment intended here, sulphuric acid contributes

$$n_{\text{H}^+,2} = 2 \times 0.400 \times 0.01 = 0.008\,\text{mol}.$$

The factor of two is essential; simply averaging the two given molarities would miss the diprotic acid contribution.

Step 2: Obtain the final concentration The total hydrogen-ion amount is $0.006 + 0.008 = 0.014\,\text{mol}$. Taking the mixed volumes as additive,

$$V_{\text{total}} = 0.600 + 0.400 = 1.000\,\text{L},$$

$$[\text{H}^+] = \frac{0.014}{1.000} = 1.4 \times 10^{-2}\,\text{mol}\,\text{L}^{-1}.$$

The acids reinforce each other; there is no neutralisation because neither solution supplies a base.

Step 3: Calculate pH and the requested coefficient Using the supplied logarithms,

$$\log 1.4 = \log 14 - 1 = \log 2 + \log 7 - 1 = 0.30 + 0.84 - 1 = 0.14.$$

Therefore

$$\text{pH} = 2 - 0.14 = 1.86 = 186 \times 10^{-2}.$$

The blank asks for the integer coefficient of 10^{-2}, so enter 186 rather than 1.86. This uses the complete-acid-dissociation convention of the supplied numerical data.

Step 4: Conclusion The pH is 1.86, so the requested dimensionless integer is $\boxed{186}$.

Quick Tip to Solve: In mixed-acid questions add moles of hydrogen ions first, and check whether the answer blank asks for pH itself or a scaled coefficient.

84. When 2 litre of ideal gas expands isothermally into vacuum to a total volume of 6 litre, the change in internal energy is _____ J. (Nearest integer) [4 marks]

Solution:

Concept:

- The internal energy of an ideal gas depends only on temperature. For a fixed amount, $\Delta U = nC_V\Delta T$.
- For expansion against constant external pressure, the chemistry sign convention gives $w = -P_{\text{ext}}\Delta V$. Expansion into a vacuum has $P_{\text{ext}} = 0$.

Step 1: Use the isothermal condition Isothermal means that the temperature remains constant during the change of state. Thus $T_f = T_i$, so $\Delta T = 0$. For an ideal gas the intermolecular potential energy does not introduce any additional volume-dependent internal-energy term. Consequently the change from 2 litre to 6 litre does not, by itself, imply a change in internal energy.

Step 2: Evaluate the internal-energy change Apply the ideal-gas relation directly:

$$\Delta U = nC_V(T_f - T_i) = nC_V(0) = 0\,\text{J}.$$

Neither the amount of gas nor the heat capacity is required because the temperature difference vanishes. This conclusion follows from the stated ideal-gas and isothermal conditions, independently of the magnitude of the volume increase.

Step 3: Check the free-expansion energy balance The external pressure is zero because the gas expands into a vacuum. Although $\Delta V = 6 - 2 = 4\,\text{L}$, the work is

$$w = -P_{\text{ext}}\Delta V = 0.$$

The first law, $\Delta U = q + w$, then gives zero net heat transfer for these stated initial and final conditions. The reversible isothermal work formula $-nRT\ln(V_f/V_i)$ must not be used for free expansion into a vacuum.

Step 4: Conclusion The change in internal energy is $\boxed{0\,\text{J}}$; the nearest-integer entry is 0.

Quick Tip to Solve: For an ideal gas, check the temperature first: an isothermal change always gives zero internal-energy change, even when the volume changes substantially.

85. Consider the cell

$$\mathrm{Pt_{(s)}\,|H_2(\,g, 1\ atm)|\,H^+(aq, 1M)||Fe^{3+}(aq), Fe^{2+}(aq)\,|\,Pt(s)}$$

When the potential of the cell is 0.712 V at 298 K, the ratio $\left[Fe^{2+}\right] / \left[Fe^{3+}\right]$ is _____. (Nearest integer)

Given : $Fe^{3+} + e^- = Fe^{2+}, E^{\theta}Fe^{3+}, Fe^{2+} \mid Pt = 0.771$

$$\frac{2.303RT}{F} = 0.06\ V$$

[4 marks]

Solution:

Concept:

- The standard hydrogen electrode has zero potential. With hydrogen at 1 atm and hydrogen-ion concentration 1 M, its electrode potential remains zero.
- For the one-electron reduction $Fe^{3+} + e^- \longrightarrow Fe^{2+}$, the Nernst equation is $E = E^\circ - (0.06/1)\log_{10}([Fe^{2+}]/[Fe^{3+}])$, using the coefficient supplied.

Step 1: Identify the cell reaction The hydrogen electrode is written on the left and acts as the anode; the iron couple is the cathode. Oxidation of hydrogen supplies electrons and iron(III) accepts them. The overall reaction can be written per electron as $Fe^{3+} + \frac{1}{2}H_2 \longrightarrow Fe^{2+} + H^+$. Since the hydrogen conditions are standard, neither its pressure nor hydrogen-ion concentration introduces an additional factor into the reaction quotient.

Step 2: Substitute the measured potential Let $r = [Fe^{2+}]/[Fe^{3+}]$. The standard cell potential is $0.771 - 0 = 0.771$ V. Hence

$$0.712 = 0.771 - 0.06\log_{10} r.$$

$$\log_{10} r = \frac{0.059}{0.06} = 0.98333\ldots.$$

The decrease below the standard potential implies $r > 1$, consistent with an increased concentration of the reduction product.

Step 3: Evaluate the concentration ratio Taking the base-ten antilogarithm gives

$$r = 10^{0.98333\ldots} = 9.624\ldots.$$

The ratio compares two concentrations expressed in the same units, so it is dimensionless. Round this value only at the end, as requested; replacing the supplied 0.06 V by a more precise thermal coefficient would disregard the question data.

Step 4: Conclusion The nearest-integer value of $[Fe^{2+}]/[Fe^{3+}]$ is $\boxed{10}$ (dimensionless).

Quick Tip to Solve: For an iron(III)/iron(II) electrode, put the reduced species in the numerator of the Nernst quotient; a lower potential then means more iron(II).

86. A trisubstituted compound 'A', $C_{10}H_{12}O_2$ gives neutral $FeCl_3$ test positive. Treatment of compound 'A' with $NaOH$ and CH_3Br gives $C_{11}H_{14}O_2$, with hydroiodic acid gives methyl iodide and with hot conc. $NaOH$ gives a compound $B, C_{10}H_{12}O_2$. Compound 'A' also decolorises alkaline $KMnO_4$. The number of π bond/s present in the compound 'A' is _____. [4 marks]

Solution:

Concept:

- The degree of unsaturation of a compound containing only carbon, hydrogen and oxygen is $\text{DBE} = (2C + 2 - H)/2$; oxygen does not enter this count.
- A benzene ring accounts for four degrees of unsaturation: one ring and three π bonds. An additional alkene contributes one more π bond.

Step 1: Interpret the oxygen-containing groups A positive neutral $FeCl_3$ test identifies a phenolic hydroxyl group. With $NaOH$ and CH_3Br, its hydrogen is replaced by methyl, producing a net addition of CH_2, exactly the change from $C_{10}H_{12}O_2$ to $C_{11}H_{14}O_2$. Liberation of methyl iodide on treatment with hydroiodic acid indicates a methoxy group. Thus the two oxygens can be assigned to phenolic $-OH$ and aryl $-OCH_3$ groups rather than to a carbonyl.

Step 2: Use the substitution and unsaturation clues The trisubstituted aromatic compound has a benzene ring bearing these two oxygen-containing groups and a three-carbon side chain. Decolorisation of alkaline $KMnO_4$ is consistent with a carbon-carbon double bond in that side chain. A phenol bearing methoxy and allyl substituents has formula $C_{10}H_{12}O_2$; base-induced allyl-to-propenyl isomerisation retains this formula, consistent with compound B. The exact positions of the three substituents are unnecessary for counting bonds.

Step 3: Count the pi bonds From the molecular formula,

$$\text{DBE} = \frac{2(10) + 2 - 12}{2} = 5.$$

One of these five units is the aromatic ring itself. The other four units are double bonds: three within benzene and one in the side chain. The methoxy and hydroxyl groups contribute no additional π bond. Counting all five degrees of unsaturation as double bonds would incorrectly include the ring closure.

Step 4: Conclusion Compound A contains $3 + 1 = \boxed{4}$ π bonds; this is a dimensionless bond count.

Quick Tip to Solve: Calculate DBE first, then subtract ring closures before converting it into a count of pi bonds; oxygen atoms do not affect DBE.

87. The number of electrons involved in the reduction of permanganate to manganese dioxide in acidic medium is _____. [4 marks]

Solution:

Concept:

- The number of electrons accepted by a metal centre equals the decrease in its oxidation number for the specified product.
- Oxygen has oxidation number -2 in both $\mathrm{MnO_4^-}$ and $\mathrm{MnO_2}$. Acidic half-reactions are balanced with water for oxygen and hydrogen ions for hydrogen.

Step 1: Find the oxidation state in permanganate Let the oxidation state of manganese in $\mathrm{MnO_4^-}$ be x. Its four oxygen atoms contribute -8, and the total ionic charge is -1:

$$x + 4(-2) = -1 \quad \Longrightarrow \quad x = +7.$$

The negative charge of the ion is included in this equation; the oxidation state of manganese is not the same as the charge on the whole ion.

Step 2: Find the oxidation state in the stated product Manganese dioxide is neutral. If its manganese oxidation state is y, then

$$y + 2(-2) = 0 \quad \Longrightarrow \quad y = +4.$$

Thus each manganese atom changes from $+7$ to $+4$, a decrease of three. Reduction requires acceptance of electrons, so three electrons enter on the reactant side for each permanganate ion.

Step 3: Verify with the balanced acidic half-reaction The complete half-reaction is

$$\mathrm{MnO_4^-} + 4\,\mathrm{H^+} + 3\,\mathrm{e^-} \longrightarrow \mathrm{MnO_2} + 2\,\mathrm{H_2O}.$$

There is one manganese on each side, four oxygen atoms on each side, and four hydrogen atoms on each side. The charge on the left is $-1 + 4 - 3 = 0$, matching the neutral right side. Acidic permanganate often forms $\mathrm{Mn^{2+}}$, requiring five electrons, but that is a different product. Here the question expressly specifies manganese dioxide.

Step 4: Conclusion The reduction to $\mathrm{MnO_2}$ involves $\boxed{3}$ electrons per permanganate ion (a dimensionless count).

Quick Tip to Solve: Choose the oxidation-state change from the product actually named: manganese dioxide means $+4$, even when the medium is acidic.

88. A solution containing 2 g of a non-volatile solute in 20 g of water boils at 373.52 K. The molecular mass of the solute is _____ g mol^{-1}. (Nearest integer)

Given, water boils at 373 K, K_b for water $= 0.52$ K kg mol^{-1} [4 marks]

Solution:

Concept:

- For a nonvolatile molecular solute, boiling-point elevation is $\Delta T_b = K_b m$, taking the van't Hoff factor as one when no association or dissociation is specified.
- Molality m is moles of solute per kilogram of solvent, not per kilogram of solution. Molar mass is the solute mass divided by its amount in moles.

Step 1: Calculate the temperature elevation The pure solvent boils at $373\,\mathrm{K}$, whereas the solution boils at $373.52\,\mathrm{K}$. Therefore

$$\Delta T_b = 373.52 - 373 = 0.52\,\mathrm{K}.$$

A temperature difference has the same numerical value in kelvin and degrees Celsius, but the absolute boiling temperature itself must not be substituted for the elevation. The increase has the expected positive sign for a nonvolatile solute.

Step 2: Determine the amount of solute Using the given ebullioscopic constant,

$$m = \frac{\Delta T_b}{K_b} = \frac{0.52\,\mathrm{K}}{0.52\,\mathrm{K\,kg\,mol^{-1}}} = 1.00\,\mathrm{mol\,kg^{-1}}.$$

The water mass is $20\,\mathrm{g} = 0.020\,\mathrm{kg}$, so

$$n = m\,w_{\text{water}} = 1.00 \times 0.020 = 0.020\,\mathrm{mol}.$$

The 2 g solute mass is not added to the solvent mass when computing molality.

Step 3: Find and check the molar mass The solute molar mass is

$$M = \frac{w_{\text{solute}}}{n} = \frac{2\,\mathrm{g}}{0.020\,\mathrm{mol}} = 100\,\mathrm{g\,mol^{-1}}.$$

As a check, 2 g of a 100 g molar-mass substance gives 0.020 mol; in 0.020 kg water this is exactly 1 molal. Its boiling-point elevation is then $0.52 \times 1 = 0.52\,\mathrm{K}$, reproducing the stated boiling point.

Step 4: Conclusion The nearest-integer molar mass of the solute is $\boxed{100\,\mathrm{g\,mol^{-1}}}$.

Quick Tip to Solve: Subtract the pure-solvent boiling point first and convert only the solvent mass to kilograms when using the molality formula.

89. The energy of one mole of photons of radiation of frequency 2×10^{12} Hz in J mol^{-1} is _____. (Nearest integer)

[Given : $h = 6.626 \times 10^{-34}$ Js

$N_A = 6.022 \times 10^{23}$ mol^{-1}**]** [4 marks]

Solution:

Concept:

- Planck's relation gives the energy of one photon: $\varepsilon = h\nu$, where h is Planck's constant and ν is frequency.
- One mole of photons contains N_A photons, so the molar photon energy is $E_m = N_A h\nu$. Frequency must be in inverse seconds for the result to be in joules.

Step 1: Find the energy of an individual photon Use the given frequency $\nu = 2 \times 10^{12}\,\mathrm{s}^{-1}$ and Planck constant $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$:

$$\varepsilon = (6.626 \times 10^{-34})(2 \times 10^{12}) = 1.3252 \times 10^{-21}\,\mathrm{J}.$$

The seconds in the Planck constant cancel the inverse seconds in frequency. No wavelength conversion or factor of the speed of light is needed because frequency is already supplied.

Step 2: Convert from one photon to one mole Multiply the single-photon energy by the Avogadro constant:

$$E_m = (1.3252 \times 10^{-21})(6.022 \times 10^{23})\,\mathrm{J\,mol^{-1}}.$$

Collecting the coefficients and powers separately gives

$$E_m = (1.3252 \times 6.022) \times 10^{2} = 798.03544\,\mathrm{J\,mol^{-1}}.$$

The power of ten is $10^{-21+23} = 10^2$; losing this factor would incorrectly turn a molar energy into a single-photon-scale quantity.

Step 3: Round in the requested units The question asks for joules per mole and the nearest integer. Thus 798.03544 rounds to 798, since its fractional part is less than 0.5. In kilojoules per mole the same energy would be about 0.798, but that number is not the entry requested here. Retaining the supplied constants until this last step avoids premature rounding.

Step 4: Conclusion The energy of one mole of these photons is $\boxed{798\,\mathrm{J\,mol^{-1}}}$ to the nearest integer.

Quick Tip to Solve: Write the single expression $N_A h\nu$, combine exponents separately, and check whether the answer is required in joules or kilojoules.

90. A $300\ \text{mL}$ bottle of soft drink has $0.2\ \text{M}\ CO_2$ dissolved in it. Assuming CO_2 behaves as an ideal gas, the volume of the dissolved CO_2 at STP is _____ mL. (Nearest integer)

Given : At STP, molar volume of an ideal gas is $22.7\ \text{L mol}^{-1}$ [4 marks]

Solution:

Concept:

- Molarity is amount of dissolved solute per litre of solution: $n = cV_{\text{solution}}$, with solution volume in litres.
- At the specified STP, ideal-gas volume is $V_{\text{gas}} = nV_m$. Use the given molar volume $V_m = 22.7\,\text{L mol}^{-1}$ rather than replacing it with another STP convention.

Step 1: Calculate the dissolved amount of carbon dioxide The bottle contains $300\,\text{mL} = 0.300\,\text{L}$ of solution, with dissolved CO_2 concentration $0.2\,\text{mol L}^{-1}$. Therefore

$$n_{CO_2} = cV_{\text{solution}} = 0.2 \times 0.300 = 0.060\,\text{mol}.$$

The 300 mL is the liquid volume used with molarity; it is not the gas volume at STP. Carbon dioxide is counted as molecular gas, so no factor for the number of atoms in a molecule is introduced.

Step 2: Apply the stated molar gas volume Assuming ideal-gas behaviour as instructed, the volume corresponding to 0.060 mol at STP is

$$V_{CO_2} = n_{CO_2}V_m = (0.060\,\text{mol})(22.7\,\text{L mol}^{-1}) = 1.362\,\text{L}.$$

The mole units cancel. This calculation gives the volume the dissolved gas would occupy at the specified STP; it does not require knowing the pressure of the gas inside the bottle or applying Henry's law.

Step 3: Convert to the answer unit The requested unit is millilitres. Since one litre equals one thousand millilitres,

$$V_{CO_2} = 1.362 \times 1000 = 1362\,\text{mL}.$$

This is already an integer. Using 22.4 instead of the supplied 22.7 would give a different result, so the numerical constant explicitly stated in the question must control the calculation.

Step 4: Conclusion The dissolved carbon dioxide occupies $\boxed{1362\,\text{mL}}$ at the stated STP.

Quick Tip to Solve: First obtain moles from liquid volume and molarity; then use the given gas molar volume and convert litres to millilitres only at the end.