JEE Main 2023 Paper 2A (B.Arch) Apr 12 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 400**

General Instructions

(i) This paper is Paper 2A (B.Arch.) of JEE (Main) 2023, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 12 April 2023, Shift 2. The total duration is 3 hours.

(ii) The paper has 82 questions: Mathematics (30), Aptitude Test (50), Drawing Test (2), in that order. Maximum marks: 400.

(iii) Mathematics has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) The Drawing Test questions are answered on the drawing sheet; marks for each are indicated against it.

(vii) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If $f(x) = \frac{x^3}{x-1} + \frac{x^3}{x+1}$ and $g(x) = \sqrt{x}$, then $fog : [0,2] - \{1\} \to \mathbb{R}$ is : [4 marks]

(A) one-one but not onto

(B) one-one and onto

(C) onto but not one-one

(D) neither one-one nor onto

Correct Answer: (A) one-one but not onto

Solution:

Concept:

- For a composite function, first substitute the inner function and retain every restriction inherited from the original denominators. Here the domain is $[0,2] \setminus \{1\}$.
- A function is one-one when distinct domain elements have distinct images. Strict monotonicity proves this on an interval, but disconnected intervals must also have disjoint image sets.
- A function into $\mathbb{R}$ is onto only if its range contains every real number; a single omitted real value disproves surjectivity.

Step 1: Simplify the composite Write $F = f \circ g$. Since $g(x) = \sqrt{x}$, combining the fractions gives

$$F(x) = \frac{x^{3/2}}{\sqrt{x}-1} + \frac{x^{3/2}}{\sqrt{x}+1} = \frac{2x^2}{x-1}.$$

This identity is valid on the stated domain, which excludes the pole at $x = 1$.

Step 2: Check injectivity on both domain pieces Differentiate the simplified expression:

$$F'(x) = \frac{4x(x-1) - 2x^2}{(x-1)^2} = \frac{2x(x-2)}{(x-1)^2}.$$

The derivative is negative on $(0,1)$ and $(1,2)$. Thus each restriction is strictly decreasing. On $[0,1)$, the values decrease from $F(0) = 0$ to $-\infty$, giving $(-\infty, 0]$. On $(1,2]$, they decrease from $+\infty$ to $F(2) = 8$, giving $[8, \infty)$. These image sets do not overlap, so two inputs from different pieces cannot have the same image either.

Step 3: Compare the range with the codomain The complete range is $(-\infty, 0] \cup [8, \infty)$. Every real number strictly between 0 and 8 is omitted; for example, no input gives 1. Therefore the function is not onto $\mathbb{R}$. Options (B) and (C) incorrectly assert surjectivity, and option (D) incorrectly denies the injectivity established on and between the two pieces.

Step 4: Conclusion The composite is one-one but not onto $\mathbb{R}$. Hence, the correct option is (A).

Quick Tip to Solve: For a domain broken by a pole, check monotonicity on each piece and then compare their ranges; monotonicity on separate pieces alone is not enough.

2. The sum of squares of the values of $\alpha \in \mathbb{R}$ such that the argument of $\frac{\alpha+i}{\alpha-i}$ is 60°, $\mathrm{i} = \sqrt{-1}$, is :

[4 marks]

(A) 3

(B) $\frac{10}{3}$

(C) $\frac{11}{3}$

(D) 4

Correct Answer: (A) 3

Solution:

Concept:

- The principal argument 60° places a nonzero complex number in the first quadrant, so both its real and imaginary parts must be positive.
- Multiplication by the conjugate of the denominator expresses a complex quotient in Cartesian form. For real α, the denominator $\alpha^2 + 1$ is always positive.
- The identity $\tan\theta = \operatorname{Im} z / \operatorname{Re} z$ determines an angle only modulo 180°; candidate roots must be checked against the required quadrant.

Step 1: Write the quotient in Cartesian form Multiply numerator and denominator by $\alpha + i$:

$$z = \frac{\alpha+i}{\alpha-i} = \frac{(\alpha+i)^2}{\alpha^2+1} = \frac{\alpha^2-1}{\alpha^2+1} + i\frac{2\alpha}{\alpha^2+1}.$$

Because the argument is 60°, the real part is positive and the imaginary part is positive. Consequently, $\alpha^2 > 1$ and $\alpha > 0$, together giving $\alpha > 1$.

Step 2: Solve the tangent equation Using the real and imaginary parts,

$$\frac{2\alpha}{\alpha^2-1} = \sqrt{3}, \qquad \sqrt{3}\alpha^2 - 2\alpha - \sqrt{3} = 0.$$

The quadratic formula gives

$$\alpha = \frac{2 \pm \sqrt{4+12}}{2\sqrt{3}} = \sqrt{3} \quad \text{or} \quad -\frac{1}{\sqrt{3}}.$$

The second root violates $\alpha > 1$. In fact, its quotient has negative real and imaginary parts, so its principal argument is -120°, although its tangent is the same.

Step 3: Square only the admissible value For $\alpha = \sqrt{3}$, the quotient is $\frac{1}{2} + i\frac{\sqrt{3}}{2}$, which has exactly the required argument. There is therefore only one admissible real value, and its square is 3. Adding the square of the rejected root would give $10/3$, option (B), but that counts a wrong-quadrant solution. Options (C) and (D) likewise do not equal the sum over admissible roots.

Step 4: Conclusion The required sum is $(\sqrt{3})^2 = 3$. Hence, the correct option is (A).

Quick Tip to Solve: After solving an equation involving the tangent of an argument, always test the quadrant; tangent equality alone can introduce an angle differing by 180°.

3. For the system of equations

$$x + \lambda y - z = 1$$

$$x + 2y + \lambda z = 2$$

$x + 2y + z = 2$,

which one of the following is NOT correct? [4 marks]

(A) it has unique solution if λ is not a root of the equation $t^2 - 3t + 2 = 0$

(B) it has unique solution if λ is not a root of the equation $t^2 - t - 2 = 0$

(C) it has infinitely many solutions if $\lambda = 1$

(D) it has no solution if $\lambda = 2$

Correct Answer: (B) it has unique solution if λ is not a root of the equation $t^2 - t - 2 = 0$

Solution:

Concept:

- A square linear system has a unique solution when its coefficient determinant is nonzero. At a zero determinant, consistency must be checked separately.
- Subtracting two equations with common coefficients isolates a variable without dividing by a parameter expression that might vanish.
- The question asks for the statement that is NOT correct. Each option must therefore be tested as a claim about all parameter values satisfying its stated condition.

Step 1: Find the singular parameter values Subtract the third equation from the second, and subtract the third from the first:

$$(\lambda - 1)z = 0, \qquad (\lambda - 2)y - 2z = -1.$$

Equivalently, the coefficient determinant is

$$\begin{vmatrix} 1 & \lambda & -1 \\ 1 & 2 & \lambda \\ 1 & 2 & 1 \end{vmatrix} = (\lambda - 1)(\lambda - 2).$$

Thus the unique-solution condition is $\lambda \neq 1, 2$. These are precisely the roots excluded by $t^2 - 3t + 2 = (t - 1)(t - 2)$, so statement (A) is correct.

Step 2: Check the two singular cases For $\lambda = 1$, the second and third equations coincide, and subtraction gives $y + 2z = 1$. Taking $z = s$ yields $y = 1 - 2s$ and $x = 3s$, for any real s. Hence there are infinitely many solutions, as (C) states. For $\lambda = 2$, the first subtraction forces $z = 0$, but the other subtraction then reads $0 = -1$. This contradiction means no solution exists, confirming (D).

Step 3: Disprove the remaining statement The polynomial in (B) factors as $t^2 - t - 2 = (t - 2)(t + 1)$. Its condition excludes 2 and -1, but permits $\lambda = 1$. At this permitted value the system has infinitely many solutions, not a unique solution. This single counterexample disproves (B). Conversely, $\lambda = -1$ actually gives a unique solution, illustrating why its exclusion is unrelated to the true singular pair.

Step 4: Conclusion Statement (B) is NOT correct: it permits $\lambda = 1$, for which the system is not uniquely solvable. Hence, the correct option is (B).

Quick Tip to Solve: When asked which parameter condition is false, test a singular value that the proposed condition still permits; it often supplies an immediate counterexample.

4. If $a_n = \left(2n^2 - n + 2\right)(n!)$ then $\sum_{n=1}^{20} a_n$ is equal to : [4 marks]

(A) $37(20!) - 1$

(B) $37(20!) + 1$

(C) $39(21!) + 1$

(D) $39(21!) - 1$

Correct Answer: (C) $39(21!) + 1$

Solution:

Concept:

- A factorial sum often telescopes after writing its summand as the difference of two consecutive expressions involving factorials.
- The recurrence $(n+1)! = (n+1)n!$ lets us compare polynomial coefficients after factoring out $n!$.
- For $a_n = b_{n+1} - b_n$, the finite sum from 1 to N equals $b_{N+1} - b_1$. Both boundary terms, including their signs, must be retained.

Step 1: Look for a consecutive factorial difference Seek $b_n = (an + b)n!$, so that

$$b_{n+1} - b_n = \big[(an + a + b)(n + 1) - (an + b)\big]n!.$$

Expanding the bracket gives $an^2 + (a + b)n + a$. Matching with $2n^2 - n + 2$ gives $a = 2$ and $b = -3$. Thus choose $b_n = (2n - 3)n!$. This coefficient comparison constructs the telescoping form without computing any large factorial.

Step 2: Verify the identity before summing With that choice,

$$\begin{aligned} b_{n+1} - b_n &= (2n - 1)(n + 1)! - (2n - 3)n! \\ &= \big[(2n - 1)(n + 1) - (2n - 3)\big]n! \\ &= (2n^2 - n + 2)n! = a_n. \end{aligned}$$

For example, at $n = 1$, the difference is $1 \cdot 2! - (-1) \cdot 1! = 3$, which equals the original first summand. This also checks the potentially easy-to-miss negative lower boundary.

Step 3: Cancel the intermediate terms Therefore

$$\begin{aligned} \sum_{n=1}^{20} a_n &= (b_2 - b_1) + (b_3 - b_2) + \cdots + (b_{21} - b_{20}) \\ &= b_{21} - b_1 = 39(21!) - (-1) = 39(21!) + 1. \end{aligned}$$

Options (A) and (B) use the wrong upper factorial and coefficient, stopping at 20 rather than the shifted index 21. Option (D) has the right upper boundary but subtracts 1 instead of subtracting the lower boundary -1.

Step 4: Conclusion The sum is $39(21!) + 1$. Hence, the correct option is (C).

Quick Tip to Solve: For a quadratic polynomial times $n!$, try a linear polynomial times $n!$ as the telescoping term, and check the lower boundary sign explicitly.

5.

$$\lim_{x \to 0^+} \frac{1}{\sqrt{x}} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$$

[4 marks]

(A) is equal to 0

(B) is equal to $\frac{1}{5}$

(C) is equal to 1

(D) does not exist

Correct Answer: (A) is equal to 0

Solution:

Concept:

- Combine differences of reciprocal functions before taking a limit, since their individual divergent parts may cancel.
- Near zero, $\sin x = x - x^3/6 + O(x^5)$, so $x - \sin x$ is of order x^3. Comparing this order with the denominator determines the limit.
- The approach is $x \to 0^+$, ensuring that the real square root $\sqrt{x}$ is defined. The standard limit $\sin x/x \to 1$ applies on this side as well.

Step 1: Combine the two reciprocal terms For sufficiently small positive x, both x and $\sin x$ are nonzero. The expression becomes

$$\frac{1}{\sqrt{x}} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = \frac{x - \sin x}{x \sin x \sqrt{x}}.$$

Do not treat the reciprocal terms separately: each tends to positive infinity, and subtraction of their limits is not a valid operation.

Step 2: Separate standard factors and evaluate them Rewrite the quotient in a form where each factor has a known behaviour:

$$\frac{x - \sin x}{x \sin x \sqrt{x}} = \frac{x - \sin x}{x^3} \cdot \frac{x}{\sin x} \cdot \sqrt{x}.$$

Taylor's expansion gives

$$\frac{x - \sin x}{x^3} = \frac{1}{6} + O(x^2), \qquad \frac{x}{\sin x} \longrightarrow 1.$$

The remaining factor $\sqrt{x}$ tends to zero, so the product tends to $(1/6) \cdot 1 \cdot 0 = 0$. In particular, the expression behaves as $\sqrt{x}/6$, rather than as a nonzero constant.

Step 3: Check existence and eliminate the distractors All three factors are defined on a sufficiently small interval to the right of zero. The first two tend to finite values, while the third tends to zero; hence the one-sided limit exists. This excludes (D). Neither $1/5$ nor 1 agrees with the vanishing leading term, excluding (B) and (C). The apparent divergence of the original reciprocal terms disappears after their common leading part cancels.

Step 4: Conclusion The stated right-hand limit is 0. Hence, the correct option is (A).

Quick Tip to Solve: For reciprocal trigonometric differences, combine first and compare powers: here a cubic numerator divided by an order-$x^{5/2}$ denominator leaves $\sqrt{x}$.

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6. Let $x = x(t)$ be the solution curve of the differential equation $\frac{dx}{dt} = -kx$, and $x(0) = 100$, $x\left(\frac{1}{2}\right) = 80$. If $x(t_\alpha) = 5$, then t_α is equal to : [4 marks]

(A) $\frac{\log_e 5 + \log_e 4}{2(\log_e 5 - \log_e 4)}$

(B) $\frac{\log_e 5 + \log_e 4}{\log_e 5 - \log_e 4}$

(C) $\frac{\log_e 5 - \log_e 4}{2(\log_e 5 + \log_e 4)}$

(D) $\frac{\log_e 5 - \log_e 4}{\log_e 5 + \log_e 4}$

Correct Answer: (A) $\frac{\log_e 5 + \log_e 4}{2(\log_e 5 - \log_e 4)}$

Solution:

Concept:

- The separable equation $dx/dt = -kx$, with constant k, has the positive solution $x(t) = Ce^{-kt}$ when the initial value is positive.
- The initial condition determines the multiplicative constant. A second measured value fixes the decay constant through a logarithm of a dimensionless ratio.
- Natural logarithm rules $\ln(a/b) = \ln a - \ln b$ and $\ln(ab) = \ln a + \ln b$ put the resulting time into the form of the given options.

Step 1: Use the initial value to determine the solution Separate variables and integrate:

$$\frac{dx}{x} = -k\,dt, \qquad \ln x = -kt + C_1.$$

Since $x(0) = 100 > 0$, the solution is

$$x(t) = 100e^{-kt}.$$

This positive exponential branch never crosses zero and is valid at the requested value $x = 5$. The integration constant is therefore fixed by the initial condition before the later observations are used.

Step 2: Determine the rate from the half-time observation Substitute $x(1/2) = 80$:

$$80 = 100e^{-k/2}, \qquad e^{-k/2} = \frac{4}{5}.$$

Taking natural logarithms yields

$$-\frac{k}{2} = \ln 4 - \ln 5, \qquad k = 2(\ln 5 - \ln 4) > 0.$$

The positive sign of k is consistent with a fall from 100 to 80. The factor 2 is essential because the observed interval is one half of the time unit, not one full unit.

Step 3: Solve for the time at which the value is five Now impose the target condition:

$$5 = 100e^{-kt_\alpha}, \qquad e^{-kt_\alpha} = \frac{1}{20}, \qquad t_\alpha = \frac{\ln 20}{k}.$$

Since $\ln 20 = \ln 5 + \ln 4$, substitution gives

$$t_\alpha = \frac{\ln 5 + \ln 4}{2(\ln 5 - \ln 4)}.$$

Option (B) loses the half-time factor. Options (C) and (D) reverse the essential logarithmic ratio; they would not satisfy $e^{-kt_\alpha} = 1/20$. As a check, the result is about 6.71 time units, greater than the time to reach 80.

Step 4: Conclusion The required time is $t_\alpha = \dfrac{\ln 5 + \ln 4}{2(\ln 5 - \ln 4)}$, in the time unit used for t. Hence, the correct option is (A).

Quick Tip to Solve: First compute the decay constant from the observed fractional time; then use $t = \ln(\text{initial/target})/k$, keeping that time factor.

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7. The slope of the tangent to the curve $y = y(x) = \int_{\sin^{-1}(x)}^{\cos^{-1}(x)} \sqrt{1 + 4\sin^2 t}\, dt; 0 \le x \le 1$ at the point $\left(\frac{1}{\sqrt{2}}, 0\right)$ on the curve is : [4 marks]

(A) $-2\sqrt{6}$

(B) $2\sqrt{6}$

(C) $-4\sqrt{3}$

(D) $4\sqrt{3}$

Correct Answer: (A) $-2\sqrt{6}$

Solution:

Concept:

- For a continuous integrand $g(t)$, the variable-limit rule is $\frac{d}{dx}\int_{u(x)}^{v(x)} g(t)\, dt = g(v(x))v\prime(x) - g(u(x))u\prime(x)$. The upper and lower limits contribute separately.
- The slope of the tangent is $y\prime(x)$. For $-1 < x < 1$, the derivatives of $\sin^{-1} x$ and $\cos^{-1} x$ are respectively $1/\sqrt{1-x^2}$ and $-1/\sqrt{1-x^2}$.

Step 1: Check the point on the curve At $x = 1/\sqrt{2}$, both inverse trigonometric limits equal $\pi/4$. An integral with equal limits is zero, so the stated point has $y = 0$. The slope can therefore be evaluated there without finding an antiderivative of the integrand.

Step 2: Differentiate both moving limits Put $g(t) = \sqrt{1 + 4\sin^2 t}$. The identities $\sin(\sin^{-1} x) = x$ and $\sin(\cos^{-1} x) = \sqrt{1 - x^2}$ apply on the stated interval. Thus

$$y'(x) = -\frac{\sqrt{5 - 4x^2}}{\sqrt{1 - x^2}} - \frac{\sqrt{1 + 4x^2}}{\sqrt{1 - x^2}}.$$

The second term is negative because the lower-limit contribution is subtracted; the first is negative because the upper limit decreases.

Step 3: Evaluate and compare the options At $x^2 = 1/2$, both numerators are $\sqrt{3}$, while the common denominator is $1/\sqrt{2}$. Therefore

$$y'(1/\sqrt{2}) = -\frac{2\sqrt{3}}{1/\sqrt{2}} = -2\sqrt{6}.$$

The positive values in (B) and (D) have the wrong sign. Option (C) has the wrong magnitude: each of the two contributions is $-\sqrt{6}$, not $-2\sqrt{3}$.

Step 4: Conclusion The tangent has slope $\boxed{-2\sqrt{6}}$. Hence, the correct option is (A).

Quick Tip to Solve: Before differentiating an integral with two variable limits, write the upper contribution minus the lower contribution; then substitute the point.

8. Let $\alpha, \beta, \gamma \left(0 < \alpha, \beta, \gamma < \frac{\pi}{2}\right)$ be the angles between non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{b}$ and $\vec{c}$, $\vec{c}$ and $\vec{a}$ respectively. If θ is the angle that the vector $\vec{a}$ makes with the plane containing $\vec{b}$ and $\vec{c}$, then

[4 marks]

(A) $\cos^2\theta = \operatorname{cosec}^2\beta\left(\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma\right)$

(B) $\cos^2\theta = \sec^2\beta\left(\cos^2\alpha + \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma\right)$

(C) $\sin^2\theta = \operatorname{cosec}^2\beta\left(\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma\right)$

(D) $\sin^2\theta = \sec^2\beta\left(\cos^2\alpha + \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma\right)$

Correct Answer: (A) $\cos^2\theta = \operatorname{cosec}^2\beta\left(\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma\right)$

Solution:

Concept:

- For a unit vector, the squared length of its orthogonal projection onto a plane is $\cos^2\theta$, where θ is the angle between the vector and the plane.
- Dot products of unit vectors equal the cosines of their included angles. Choose an orthonormal basis in the plane so that the mutual angle β is represented directly.

Step 1: Choose coordinates in the plane Normalize the three nonzero vectors without changing their angles. Let the unit vector along $\vec{b}$ be $\hat{u} = (1, 0, 0)$, and that along $\vec{c}$ be $\hat{v} = (\cos\beta, \sin\beta, 0)$. Since $0 < \beta < \pi/2$, their directions are independent and $\sin\beta \neq 0$. Write the unit vector along $\vec{a}$ as (p, q, r).

Step 2: Use the two dot products The angle with $\vec{b}$ gives $p = \cos\alpha$. The angle with $\vec{c}$ gives

$$p\cos\beta + q\sin\beta = \cos\gamma, \qquad q = \frac{\cos\gamma - \cos\alpha\cos\beta}{\sin\beta}.$$

The normal coordinate r is unnecessary because only the projection length is wanted.

Step 3: Find the projection length The projection of the unit vector onto the chosen plane has coordinates $(p, q, 0)$. Hence

$$\begin{aligned}\cos^2\theta &= p^2 + q^2 \\ &= \cos^2\alpha + \frac{(\cos\gamma - \cos\alpha\cos\beta)^2}{\sin^2\beta} \\ &= \frac{\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma}{\sin^2\beta}.\end{aligned}$$

We used $\sin^2\beta + \cos^2\beta = 1$ in collecting the $\cos^2\alpha$ terms.

Step 4: Reject the other expressions The denominator is $\sin^2\beta$, and expansion gives a negative cross term. Therefore (B) and (D), which use $\sec^2\beta$ and a positive cross term, do not match. Option (C) has the right algebraic expression but incorrectly identifies it with the squared normal component $\sin^2\theta$.

Step 5: Conclusion $\cos^2\theta = \operatorname{cosec}^2\beta(\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma)$. Hence, the correct option is (A).

Quick Tip to Solve: For the angle with a plane, use the in-plane projection for cosine and the normal component for sine; fixing this first prevents complementary-angle errors.

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9. The domain of the function $f(x) = \sin^{-1}\left(\log_2((x-1)(x-2))\right)$ is : [4 marks]

(A) $[0, 3]$

(B) $\left[0, \frac{3-\sqrt{3}}{2}\right] \cup \left[\frac{3+\sqrt{3}}{2}, 3\right]$

(C) $\left(\frac{3-\sqrt{3}}{2}, 1\right) \cup \left(2, \frac{3+\sqrt{3}}{2}\right)$

(D) $\left[0, \frac{3-\sqrt{6}}{2}\right] \cup \left[\frac{3+\sqrt{6}}{2}, 3\right]$

Correct Answer: (B) $\left[0, \frac{3-\sqrt{3}}{2}\right] \cup \left[\frac{3+\sqrt{3}}{2}, 3\right]$

Solution:

Concept:

- The real inverse sine $\sin^{-1}u$ is defined exactly when $-1 \le u \le 1$. Its endpoints are allowed.
- For base $2 > 1$, the logarithm is increasing. Thus $-1 \le \log_2 v \le 1$ is equivalent to $2^{-1} \le v \le 2$, which also guarantees the logarithm has a positive argument.

Step 1: Combine both domain restrictions Here the logarithm argument is $v = (x-1)(x-2) =$

$x^2 - 3x + 2$. The outer inverse sine requires

$$-1 \le \log_2((x-1)(x-2)) \le 1.$$

Exponentiating the entire double inequality gives

$$\frac{1}{2} \le x^2 - 3x + 2 \le 2.$$

We must intersect the solutions of the lower and upper bounds, rather than taking their union.

Step 2: Solve the upper bound The inequality $x^2 - 3x + 2 \le 2$ reduces to $x(x-3) \le 0$. The upward-opening quadratic is nonpositive between its roots, including both roots, so this restriction gives $x \in [0, 3]$. This alone is insufficient: values near 1 or 2 violate positivity of the logarithm argument.

Step 3: Solve the lower bound and intersect The other inequality is $x^2 - 3x + 3/2 \ge 0$. Its roots are

$$x = \frac{3 \pm \sqrt{9-6}}{2} = \frac{3 \pm \sqrt{3}}{2}.$$

An upward-opening quadratic is nonnegative outside its root interval. Intersecting those two exterior intervals with $[0, 3]$ yields

$$\left[0, \frac{3-\sqrt{3}}{2}\right] \cup \left[\frac{3+\sqrt{3}}{2}, 3\right].$$

Step 4: Check endpoints and the proposed alternatives At $x = 0, 3$, the logarithm argument is 2 and the inverse-sine input is 1. At the other two endpoints the argument is $1/2$ and the input is -1. All are valid. Option (A) includes invalid middle values; (C) uses the wrong intervals and excludes valid endpoints; (D) has $\sqrt{6}$ instead of the discriminant $\sqrt{3}$.

Step 5: Conclusion The domain is $\boxed{[0, (3-\sqrt{3})/2] \cup [(3+\sqrt{3})/2, 3]}$. Hence, the correct option is (B).

Quick Tip to Solve: For an inverse sine of a logarithm, impose the outer bound first and exponentiate it; the resulting positive lower bound automatically handles logarithm positivity.

10. Let α and β be the roots of the equation $2x^2 - 5x - 1 = 0$. For $n \in \mathbb{N}$, let $P_n = \alpha^n + \beta^n$. Then $\frac{2P_{11}(2P_{10}-5P_9)}{P_8(5P_{10}+P_9)}$ is equal to : [4 marks]

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) -1

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- If a number r is a root of $2x^2 - 5x - 1 = 0$, then $2r^2 = 5r + 1$. Multiplying this identity by powers of r gives recurrences for higher powers.
- Adding the identities for both roots gives the recurrence $2P_n = 5P_{n-1} + P_{n-2}$, where $P_n = \alpha^n + \beta^n$. This avoids calculating the roots or their large powers.

Step 1: Derive the common recurrence For $r = \alpha$ or $r = \beta$, multiply $2r^2 - 5r - 1 = 0$ by r^{n-2}, with $n \geq 2$. Adding both equations produces

$$2(\alpha^n + \beta^n) - 5(\alpha^{n-1} + \beta^{n-1}) - (\alpha^{n-2} + \beta^{n-2}) = 0.$$

Therefore $2P_n - 5P_{n-1} = P_{n-2}$, with the plus sign on the right fixed by the negative constant term in the polynomial.

Step 2: Identify the two factors in the expression Using $n = 10$ gives $2P_{10} - 5P_9 = P_8$. Using $n = 11$ gives $5P_{10} + P_9 = 2P_{11}$. Thus the two apparently complicated parenthesized factors are already present in the denominator and numerator, respectively.

Step 3: Verify the cancellation is legitimate The roots are $\alpha = (5 + \sqrt{33})/4$ and $\beta = (5 - \sqrt{33})/4$. The first is positive, the second negative, and $\alpha > |\beta|$. Consequently $P_8 = \alpha^8 + \beta^8 > 0$ and $P_{11} = \alpha^{11} - |\beta|^{11} > 0$. The denominator $P_8(2P_{11})$ is therefore nonzero.

Step 4: Simplify and compare Substitute the recurrence identities before performing any numerical calculation:

$$\frac{2P_{11}(2P_{10} - 5P_9)}{P_8(5P_{10} + P_9)} = \frac{2P_{11}P_8}{P_8(2P_{11})} = 1.$$

The factors cancel exactly. Neither negative alternative can result, and the factor 2 cancels as well, ruling out the positive one-half in (B).

Step 5: Conclusion The given expression equals $\boxed{1}$. Hence, the correct option is (D).

Quick Tip to Solve: Whenever a problem contains power sums of quadratic roots, multiply the root equation by a suitable power and match the displayed combinations before computing any roots.

11. If the image of the point $(1, 1, 2)$ in the plane $2x - y + z + 3 = 0$ is the point P, then the distance of P from origin is [4 marks]

(A) $2\sqrt{3}$

(B) $3\sqrt{2}$

(C) 4

(D) 6

Correct Answer: (B) $3\sqrt{2}$

Solution:

Concept:

- For a plane $ax + by + cz + d = 0$, a normal vector is $\vec{n} = (a, b, c)$. Reflection moves a point along this normal through twice its signed normal displacement from the plane.
- The image of $X = (x_0, y_0, z_0)$ is $X - 2\lambda\vec{n}$, where $\lambda = (ax_0 + by_0 + cz_0 + d)/(a^2 + b^2 + c^2)$. Distance from the origin is the Euclidean norm of the image coordinates.

Step 1: Find the normal displacement Here $X = (1, 1, 2)$, the plane is $2x - y + z + 3 = 0$, and $\vec{n} = (2, -1, 1)$. Its squared length is $4 + 1 + 1 = 6$. Substituting the point into the plane equation gives $2 - 1 + 2 + 3 = 6$, so $\lambda = 6/6 = 1$. Keep the constant term $+3$: omitting it would change the reflecting plane.

Step 2: Compute the reflected point The foot of the perpendicular from X is

$$H = X - \lambda\vec{n} = (1, 1, 2) - (2, -1, 1) = (-1, 2, 1).$$

Indeed $2(-1) - 2 + 1 + 3 = 0$, confirming that H lies in the plane. Since H is the midpoint between the point and its image,

$$P = 2H - X = (-2, 4, 2) - (1, 1, 2) = (-3, 3, 0).$$

Step 3: Check the geometry of the image The displacement $P - X = (-4, 2, -2) = -2\vec{n}$ is normal to the plane, as reflection requires. The plane expression at P is $-6 - 3 + 0 + 3 = -6$, opposite to its value $+6$ at X. Thus the points are at equal perpendicular distances on opposite sides, providing a sign check.

Step 4: Find the distance from the origin Use the coordinates of P, not the original point or the perpendicular foot:

$$OP = \sqrt{(-3)^2 + 3^2 + 0^2} = \sqrt{18} = 3\sqrt{2}.$$

The squared distances in (A), (C), and (D) are 12, 16, and 36, respectively, whereas the actual squared distance is 18; only (B) agrees.

Step 5: Conclusion The image is $P = (-3, 3, 0)$, whose distance from the origin is $\boxed{3\sqrt{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For reflection in a plane, find the perpendicular foot first and use image = twice the foot minus the original point; verify the midpoint lies in the plane.

12. For three non-coplanar vectors $\vec{a}, \vec{b}, \vec{c}$, if $(\vec{b}+\vec{c}) \cdot \{(\vec{c}+\vec{a}) \times (\vec{a}+\vec{b})\} = \alpha[\vec{a}\ \vec{b}\ \vec{c}]$ and $(\vec{a}+\vec{b}) \cdot \{(\vec{b}+\vec{c}) \times (\vec{a}+\vec{b}+\vec{c})\} = \beta[\vec{a}\ \vec{b}\ \vec{c}]$, then $\alpha+\beta$ is equal to : [4 marks]

(A) -3

(B) -1

(C) 1

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- The scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$ is linear in each vector, changes sign on swapping two vectors, and vanishes if two vectors coincide.
- If three vectors are expressed as linear combinations of a non-coplanar basis $\vec{a}, \vec{b}, \vec{c}$, their scalar triple product equals the determinant of their coefficient rows times $[\vec{a}\ \vec{b}\ \vec{c}]$.

Step 1: Write the coefficient matrix for the first product Let $T = [\vec{a}\ \vec{b}\ \vec{c}]$. Non-coplanarity ensures $T \neq 0$, so its coefficient can be identified uniquely. In the ordered basis $(\vec{a}, \vec{b}, \vec{c})$, the coefficient rows for $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$, and $\vec{a}+\vec{b}$ are $(0,1,1)$, $(1,0,1)$, and $(1,1,0)$. Preserve that order because the cross product is not commutative.

Step 2: Evaluate alpha Therefore

$$\alpha = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -1(0-1) + 1(1-0) = 2.$$

Equivalently, the only nonzero terms in the multilinear expansion are $[\vec{b}\ \vec{c}\ \vec{a}]$ and $[\vec{c}\ \vec{a}\ \vec{b}]$. Both are cyclic permutations of T, so each has a positive sign.

Step 3: Evaluate beta independently The coefficient rows for the second product are $(1,1,0)$, $(0,1,1)$, and $(1,1,1)$. Hence

$$\beta = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1(1-1) - 1(0-1) = 1.$$

As another check, subtracting the first row from the third gives $(0,0,1)$; expansion along that row leaves the triangular two-by-two determinant of value 1.

Step 4: Combine the two coefficients Both coefficients multiply the same nonzero triple product T, so $\alpha+\beta = 2+1 = 3$. The negative alternatives (A) and (B) would require an unjustified reversal of vector order. Option (C) incorrectly loses one of the two positive cyclic terms in the first product or subtracts the coefficients instead of adding them.

Step 5: Conclusion $\alpha = 2$, $\beta = 1$, and $\boxed{\alpha+\beta = 3}$. Hence, the correct option is (D).

Quick Tip to Solve: For triple products of sums, write coefficient rows in exactly the displayed order and evaluate a small determinant; cyclic order preserves sign, while a swap reverses it.

13. If $I(x) = \int \frac{dx}{1-2\sin^2 x \cos^2 x}$, then $\tan\left(\sqrt{2}\left(I\left(\frac{\pi}{8}\right) - I(0)\right)\right)$ is equal to : [4 marks]

(A) $\frac{1}{\sqrt{2}}$

(B) 1

(C) $\sqrt{2}$

(D) 2

Correct Answer: (A) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- The difference of two values of an antiderivative is the definite integral between the corresponding limits; the arbitrary constant cancels.
- Use $\sin^2 x \cos^2 x = \frac{1}{4}\sin^2(2x)$, followed by $t = \tan(2x)$. On $0 \le x \le \pi/8$, this substitution is continuous and increasing.

Step 1: Convert the required difference into a definite integral Write $J = I(\pi/8) - I(0)$. The denominator is at least $1/2$, so the integrand has no singularity on this interval. The double-angle identity gives

$$J = \int_0^{\pi/8} \frac{dx}{1 - \frac{1}{2}\sin^2(2x)}.$$

This formulation avoids choosing an integration constant and keeps the required endpoint difference explicit.

Step 2: Substitute the tangent of the double angle Set $t = \tan(2x)$. Then $dx = dt/[2(1+t^2)]$ and $\sin^2(2x) = t^2/(1+t^2)$. The limits become 0 and 1, respectively. Hence

$$\begin{aligned} J &= \int_0^1 \frac{dt}{2+t^2} \\ &= \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)\right]_0^1 = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right). \end{aligned}$$

Both factors of two from the double-angle substitution have been retained.

Step 3: Apply the outer tangent and compare the choices Multiplying by $\sqrt{2}$ leaves precisely $\tan^{-1}(1/\sqrt{2})$. This angle lies between 0 and $\pi/4$, so taking its tangent gives $1/\sqrt{2}$. In particular, the value is smaller than one; this rules out options (B), (C) and (D), whose values are $1, \sqrt{2}, 2$.

Step 4: Conclusion $\tan\{\sqrt{2}[I(\pi/8) - I(0)]\} = \boxed{1/\sqrt{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a rational expression in $\sin^2(2x)$, try $t = \tan(2x)$; transform the limits before integrating to avoid branch or constant errors.

14. Let X have a binomial distribution B $(6, p)$. If the sum of the mean and the variance of X is $\frac{21}{8}$, then $\frac{P(2\le X<4)}{P(4<X<6)}$ is equal to : [4 marks]

(A) 65

(B) 195

(C) $\frac{195}{2}$

(D) $\frac{225}{2}$

Correct Answer: (C) $\frac{195}{2}$

Solution:

Concept:

- For $X \sim B(n,p)$, the mean is np, the variance is $np(1-p)$, and $P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$.
- A binomial random variable takes integer values only. Thus $2 \le X < 4$ means $X = 2, 3$, whereas $4 < X < 6$ means only $X = 5$.

Step 1: Determine the valid success probability Put $q = 1-p$. The given sum of the mean and variance gives

$$6p + 6p(1-p) = \frac{21}{8} \quad \Longrightarrow \quad 16p^2 - 32p + 7 = 0.$$

Factor the quadratic as $(4p-1)(4p-7) = 0$. Its roots are $1/4$ and $7/4$. Since a probability must lie in $[0,1]$, only $p = 1/4$ is admissible, and consequently $q = 3/4$. Substitution checks the sum: $6/4 + 18/16 = 21/8$.

Step 2: Expand precisely the events in the ratio The strict inequalities in the denominator exclude both 4 and 6. Therefore

$$P(2 \le X < 4) = 15p^2q^4 + 20p^3q^3,$$
$$P(4 < X < 6) = 6p^5q.$$

The denominator is nonzero because $p, q > 0$. There is no additional $X = 4$ or $X = 6$ term in either of these expressions.

Step 3: Cancel common factors and evaluate Dividing first, instead of calculating separate small probabilities, gives

$$\frac{15p^2q^4 + 20p^3q^3}{6p^5q} = \frac{15}{6}\left(\frac{q}{p}\right)^3 + \frac{20}{6}\left(\frac{q}{p}\right)^2$$
$$= \frac{15}{6}(27) + \frac{20}{6}(9) = \frac{135}{2} + 30 = \frac{195}{2}.$$

Here $q/p = 3$. The values 65 and 195 in options (A) and (B) do not equal this sum, while $225/2$ in (D) would overcount the probability ratio.

Step 4: Conclusion The required probability ratio is $\boxed{195/2}$. Hence, the correct option is (C).

Quick Tip to Solve: List the allowed integer values before applying a binomial formula; then divide the terms symbolically and use q/p to simplify.

15. If the statement

$$((\mathrm{p} * (\sim \mathrm{q})) \wedge (\mathrm{p} \vee \mathrm{q})) \Leftrightarrow \mathrm{p}$$

is a tautology, then $*$ is : [4 marks]

(A) $\wedge$

(B) $\vee$

(C) $\Rightarrow$

(D) $\Leftrightarrow$

Correct Answer: (B) $\vee$

Solution:

Concept:

- A biconditional $A \Leftrightarrow p$ is a tautology exactly when A and p have identical truth values for every assignment of p, q.
- The distributive identity $(p \vee r) \wedge (p \vee s) = p \vee (r \wedge s)$, together with $q \wedge \neg q = F$, can simplify the proposed connective.

Step 1: Test disjunction by a logical identity Choose $* = \vee$. The expression on the left of the biconditional becomes

$$\begin{aligned}(p \vee \neg q) \wedge (p \vee q) &= p \vee (\neg q \wedge q) \\ &= p \vee F = p.\end{aligned}$$

Thus the whole printed statement reduces to $p \Leftrightarrow p$, which is true for both values of p, independently of q. This establishes that disjunction works for every possible assignment, rather than merely for one test case.

Step 2: Reject conjunction and implication For $* = \wedge$, take $p = T, q = T$. Then $p \wedge \neg q = F$, so the outer conjunction is false, although the right side of the biconditional is $p = T$. Option (A) therefore fails.

For $* = \Rightarrow$, use the same assignment. The implication $p \Rightarrow \neg q$ is $T \Rightarrow F$, which is false. Multiplying logically by $p \vee q = T$ still leaves false. Hence option (C) also makes the biconditional false.

Step 3: Reject the remaining biconditional connective For $* = \Leftrightarrow$, again put $p = T, q = T$. Since $\neg q = F$, the inner biconditional $p \Leftrightarrow \neg q$ is false. Its conjunction with $p \vee q$ is false, and the outer biconditional compares this false value with true. Thus option (D) fails as well.

A single counterexample is sufficient to reject each of these three connectives. The distributive proof for disjunction supplies the positive all-cases verification required for a tautology.

Step 4: Conclusion The required connective is $\boxed{\vee}$, logical disjunction. Hence, the correct option is (B).

Quick Tip to Solve: To prove a tautology, simplify the expression identically. To reject another connective, one truth assignment making the biconditional false is enough.

16. If the circles $x^2+y^2-2x-4y+4=0$ and $x^2+y^2-6x-10y+20+2\sqrt{13}=0$ touch each other at the point (a,b), then $(3a-2\mathrm{b})^2$ is equal to : [4 marks]

(A) 1

(B) 4

(C) 9

(D) 13

Correct Answer: (A) 1

Solution:

Concept:

- Completing squares identifies a circle's centre and radius. At tangency, the point of contact and the two centres are collinear.
- For externally tangent circles, the contact point lies between the centres, at a distance equal to the first radius from the first centre.

Step 1: Find both centres and radii The first circle can be written as

$$(x-1)^2+(y-2)^2=1,$$

so its centre is $C_1=(1,2)$ and radius is $r_1=1$. The second becomes

$$(x-3)^2+(y-5)^2=14-2\sqrt{13}.$$

Its centre is $C_2=(3,5)$. Since $14-2\sqrt{13}=(\sqrt{13}-1)^2$ and $\sqrt{13}>1$, its positive radius is $r_2=\sqrt{13}-1$.

Step 2: Locate the contact point along the line of centres The centre separation is

$$C_1C_2=\sqrt{(3-1)^2+(5-2)^2}=\sqrt{13}=r_1+r_2.$$

This confirms external tangency. The vector from the first centre to the second is $(2,3)$, and its unit vector is $(2,3)/\sqrt{13}$. Moving one radius from C_1 toward C_2 gives

$$(a,b)=\left(1+\frac{2}{\sqrt{13}},\ 2+\frac{3}{\sqrt{13}}\right).$$

The distance from this point to the second centre is $\sqrt{13}-1$, as required, so the direction of the displacement is correct.

Step 3: Evaluate the requested combination Instead of approximating the coordinates, retain their exact radical form:

$$3a-2b=3+\frac{6}{\sqrt{13}}-4-\frac{6}{\sqrt{13}}=-1,$$

$$(3a-2b)^2=1.$$

The radical contributions cancel because $(3,-2)$ is perpendicular to the line-of-centres direction $(2,3)$. The other options 4, 9 and 13 are distinct from the exact squared value; using the centre separation itself would incorrectly lead toward option (D).

Step 4: Conclusion $(3a-2b)^2 = \boxed{1}$. Hence, the correct option is (A).

Quick Tip to Solve: At tangency, move one radius from a centre toward the other centre. Keep radicals exact when the requested linear combination may cancel them.

17. If the angle between the line $l : \frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{2}$ and the plane $\mathrm{P} : \lambda x + 4\mathrm{y} - 7 = 0, \lambda \neq 0$, is $\mathrm{cosec}^{-1}\left(\frac{3}{2}\right)$, then the sum of co-ordinates of the point where line l crosses the plane P is : [4 marks]

(A) -33

(B) -2

(C) 3

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- If a line has direction vector $\mathbf{d}$ and a plane has normal $\mathbf{n}$, the acute line-plane angle θ satisfies $\sin\theta = |\mathbf{d}\cdot\mathbf{n}|/(|\mathbf{d}|\,|\mathbf{n}|)$.
- The symmetric equation of the line provides a parameter: $x = 1+2t,\ y = -1+t,\ z = 2+2t$. Substituting this into the plane determines the crossing point.

Step 1: Use the given angle to determine the plane Here $\mathbf{d} = (2,1,2)$, so $|\mathbf{d}| = 3$, and $\mathbf{n} = (\lambda, 4, 0)$. Since $\theta = \mathrm{cosec}^{-1}(3/2)$, its sine is $2/3$. Hence

$$\frac{|2\lambda+4|}{3\sqrt{\lambda^2+16}} = \frac{2}{3} \quad\Longrightarrow\quad |2\lambda+4| = 2\sqrt{\lambda^2+16}.$$

Both sides are nonnegative, so squaring preserves the equation.

Step 2: Solve and verify the parameter of the plane Squaring and expanding gives

$$4\lambda^2 + 16\lambda + 16 = 4\lambda^2 + 64,$$

so $16\lambda = 48$ and $\lambda = 3$. This satisfies the given restriction $\lambda \neq 0$. In the original angle equation, the numerator is 10 and the normal length is 5, giving $10/(3\cdot 5) = 2/3$; thus the root is not an extraneous consequence of squaring.

Step 3: Find the crossing point and its coordinate sum Substitute the line coordinates into the plane $3x + 4y - 7 = 0$:

$$3(1+2t) + 4(-1+t) - 7 = 0 \quad\Longrightarrow\quad 10t - 8 = 0.$$

Thus $t = 4/5$, and the point is $(13/5, -1/5, 18/5)$. Its sum is

$$x + y + z = \frac{13 - 1 + 18}{5} = 6.$$

Equivalently, the sum on the line is $2 + 5t$, which also gives 6. Neither -33, -2, nor 3 matches this

value; these would fail either the plane substitution or the line-plane angle condition.

Step 4: Conclusion The sum of the coordinates of the intersection point is $\boxed{6}$. Hence, the correct option is (D).

Quick Tip to Solve: Use sine for the angle with the plane and cosine for the angle with its normal. After finding the plane, substitute the parametric line directly.

18. Let three distinct normal be drawn to the parabola $y^2 + 4y - 6x - 8 = 0$ from a point (a, b) on the axis of the parabola. Then : [4 marks]

(A) $a \in (1, \infty)$ and $b = -2$

(B) $a \in (0, \infty)$ and $b = -2$

(C) $a \in (1, \infty)$ and $b = 2$

(D) $a \in (2, \infty)$ and $b = 2$

Correct Answer: (A) $a \in (1, \infty)$ and $b = -2$

Solution:

Concept:

- Completing the square translates the parabola into $Y^2 = 4cX$. Its axis is $Y = 0$, and a normal at the parameter point $(ct^2, 2ct)$ has equation $Y = -tX + 2ct + ct^3$.
- Three distinct normals from a point correspond to three distinct real roots of the normal-parameter equation. A repeated root does not count as a new normal.

Step 1: Translate the parabola and identify its axis Completing the square gives

$$(y + 2)^2 = 6(x + 2).$$

Set $X = x + 2$ and $Y = y + 2$. The standard parameter is $c = 3/2$, and the axis in the original coordinates is $y = -2$. Since (a, b) lies on the axis, necessarily $b = -2$. Its translated coordinates are $(A, 0)$, where $A = a + 2$. Options (C) and (D), which require $b = 2$, are therefore impossible.

Step 2: Obtain the equation for the normal parameters For completeness, the tangent slope at $(ct^2, 2ct)$ is $1/t$, so the normal slope is $-t$. The corresponding normal equation is

$$Y - 2ct = -t(X - ct^2),$$

which rearranges to $Y = -tX + 2ct + ct^3$. At $t = 0$, the normal is the axis itself, so this equation includes the vertex normal as well. Requiring the normal to pass through $(A, 0)$ yields

$$0 = t(ct^2 + 2c - A).$$

Step 3: Require three distinct real normals One root is always $t = 0$. The other two satisfy

$$t^2 = \frac{A - 2c}{c} = \frac{2(a - 1)}{3}.$$

They are distinct and real, and different from zero, precisely when $a > 1$. At $a = 1$, all three parameter roots coincide at zero; for $a < 1$, the two additional roots are nonreal. Thus the open condition is essential. Option (B) is too broad: it includes points with $0 < a \leq 1$, from which three distinct real normals cannot be drawn.

Step 4: Conclusion The required condition is $\boxed{a \in (1, \infty),\ b = -2}$. Hence, the correct option is (A).

Quick Tip to Solve: For a point on the parabola's axis, factor the normal cubic by t. Three distinct normals require a positive value of the remaining t^2.

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19. Given ${}^{n}c_r = \frac{n!}{r!(n-r)!}$, $0 \leq \mathrm{r} \leq \mathrm{n}$ and n is a non-negative integer. Then a possible value of k for which the equality

${}^{50}\mathrm{C}_{\mathrm{k}-1} + \sum_{r=1}^{50} {}^{100-r}\mathrm{C}_{\mathrm{k}-2} = {}^{100}\mathrm{C}_{49}$ **holds, is** [4 marks]

(A) 40

(B) 49

(C) 50

(D) 25

Correct Answer: (C) 50

Solution:

Concept:

- Pascal's identity is $\binom{j+1}{t+1} = \binom{j}{t+1} + \binom{j}{t}$. Its rearranged form makes a sum of consecutive binomial coefficients telescope.
- For a fixed upper index n, symmetry gives $\binom{n}{r} = \binom{n}{n-r}$. The required value of k must also keep every lower index in its allowed range.

Step 1: Rewrite the summation index Put $j = 100 - r$. As r increases from 1 to 50, j runs from 99 down to 50. Reversing this finite sum does not change its value, so the expression on the left is

$$\binom{50}{k-1} + \sum_{j=50}^{99} \binom{j}{k-2}.$$

The upper index varies but the lower index remains fixed, which is the situation in which Pascal's identity is useful.

Step 2: Telescope with Pascal's identity Use $\binom{j}{k-2} = \binom{j+1}{k-1} - \binom{j}{k-1}$. Consecutive terms cancel, giving

$$\sum_{j=50}^{99} \binom{j}{k-2} = \binom{100}{k-1} - \binom{50}{k-1},$$

$$\binom{50}{k-1} + \sum_{j=50}^{99} \binom{j}{k-2} = \binom{100}{k-1}.$$

Thus it suffices to have $\binom{100}{k-1} = \binom{100}{49}$.

Step 3: Select the admissible listed value Taking $k-1=49$ gives $k=50$, which satisfies every coefficient's domain. Symmetry would suggest the other lower index 51, but that would require $k=52$, for which the displayed $\binom{50}{51}$ is outside the specified domain. Options 40, 49 and 25 instead give lower indices 39, 48 and 24; none equals 49 or its symmetric index 51. The coefficients increase strictly up to the central index, so these other indices cannot give the required value.

Step 4: Conclusion A possible admissible value is $\boxed{k=50}$. Hence, the correct option is (C).

Quick Tip to Solve: For a sum whose upper binomial index changes by one, subtract adjacent terms using Pascal's identity before trying numerical values.

20. If an unbiased die, marked with $-3,-2,-1,0,1,2$ on its faces, is thrown four times, then the probability of getting -1 as the sum of outputs is : [4 marks]

(A) $\frac{7}{81}$
(B) $\frac{35}{324}$
(C) $\frac{8}{81}$
(D) $\frac{26}{243}$

Correct Answer: (B) $\frac{35}{324}$

Solution:

Concept:

- Four independent throws of an unbiased six-faced die have 6^4 equally likely ordered outcomes. A probability is the number of favourable ordered outcomes divided by this total.
- Shifting each face value by 3 converts the allowed values to the integers 0 through 5. Stars and bars counts non-negative solutions, and inclusion-exclusion removes any exceeding 5.

Step 1: Convert the required sum to a non-negative sum Let the four original outputs be x_1, x_2, x_3, x_4, where each belongs to $\{-3,-2,-1,0,1,2\}$. Define $y_i = x_i + 3$, so $0 \le y_i \le 5$. The required condition becomes

$$x_1 + x_2 + x_3 + x_4 = -1 \quad \Longleftrightarrow \quad y_1 + y_2 + y_3 + y_4 = 11.$$

This shift is one-to-one and preserves the ordering of the throws; no additional permutation factor is needed.

Step 2: Count solutions and enforce the upper bounds Without the restrictions $y_i \le 5$, the number of non-negative solutions is $\binom{14}{3} = 364$. If a specified variable is at least 6, subtract 6 from it. The remaining non-negative variables then sum to 5, giving $\binom{8}{3} = 56$ solutions. There are four choices of this variable. Two variables cannot both be at least 6, since their sum would already be at least 12, exceeding 11. Hence the inclusion-exclusion count is exactly

$$N = \binom{14}{3} - 4\binom{8}{3} = 364 - 224 = 140.$$

Step 3: Divide by the sample-space size There are $6^4 = 1296$ possible ordered outcomes. Consequently,

$$P = \frac{140}{1296} = \frac{35}{324}.$$

For comparison, the other proposed fractions correspond to favourable counts 112, 128 and $416/3$, respectively, when multiplied by 1296. They therefore do not equal the derived count 140; the last is not even an integer count of equally likely outcomes.

Step 4: Conclusion The required probability is $\boxed{\frac{35}{324}}$. Hence, the correct option is (B).

Quick Tip to Solve: Shift consecutive negative face values to start at zero, then check whether one or more upper-bound violations are possible before applying inclusion-exclusion.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. $(\sin 5^\circ \sin 55^\circ \sin 65^\circ \sin 75^\circ)^{-1}$ **is equal to _____.** [4 marks]

Solution:

Concept:

- The product identity $4\sin\theta\sin(60^\circ - \theta)\sin(60^\circ + \theta) = \sin 3\theta$ combines three angles symmetric about 60°.
- Complementary angles satisfy $\sin(90^\circ - \alpha) = \cos\alpha$, and $2\sin\alpha\cos\alpha = \sin 2\alpha$. All angles here are measured in degrees.

Step 1: Establish the three-factor identity The product of the two factors around 60° is

$$\sin(60^\circ - \theta)\sin(60^\circ + \theta) = \sin^2 60^\circ - \sin^2\theta = \frac{3}{4} - \sin^2\theta.$$

Multiplying by $4\sin\theta$ gives $3\sin\theta - 4\sin^3\theta = \sin 3\theta$. This derivation shows why the identity applies to the particular three sine factors instead of requiring decimal approximations.

Step 2: Apply it at five degrees Take $\theta = 5^\circ$. The other two angles are then exactly 55° and 65°, so

$$\sin 5^\circ \sin 55^\circ \sin 65^\circ = \frac{1}{4}\sin 15^\circ.$$

The fourth factor is $\sin 75^\circ = \cos 15^\circ$. Therefore the entire product is

$$\begin{aligned} P &= \frac{1}{4}\sin 15^\circ \cos 15^\circ \\ &= \frac{1}{8}\sin 30^\circ = \frac{1}{8}\cdot\frac{1}{2} = \frac{1}{16}. \end{aligned}$$

Step 3: Take the reciprocal of the complete product The exponent -1 outside the brackets asks for the reciprocal of all four factors together, not the negative of the product. Each angle lies in the first

quadrant, so every sine is positive and the product is nonzero. Thus taking the reciprocal is valid and gives

$$P^{-1} = \left(\frac{1}{16}\right)^{-1} = 16.$$

This is a dimensionless trigonometric value; no physical unit or rounding is required.

Step 4: Conclusion The numerical value is $\boxed{16}$.

Quick Tip to Solve: Look for a triple of angles of the form $\theta, 60^\circ - \theta, 60^\circ + \theta$; combine them before handling the remaining factor.

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22. If the shortest distance between the lines $3x+2y-4z-5=0=5x-7y-17z+2$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+1}{2}$ is $\frac{10}{\sqrt{k}}$, then k is equal to _____. [4 marks]

Solution:

Concept:

- A line given as the intersection of two planes has a direction vector perpendicular to both plane normals, hence parallel to their cross product.
- For lines through A, B with nonparallel direction vectors $\mathbf{u}, \mathbf{v}$, the shortest distance is $d = |(\overrightarrow{AB}) \cdot (\mathbf{u} \times \mathbf{v})|/|\mathbf{u} \times \mathbf{v}|$.

Step 1: Find a point and direction on the first line The plane normals are $\mathbf{n}_1 = (3, 2, -4)$ and $\mathbf{n}_2 = (5, -7, -17)$. Their cross product is

$$\mathbf{n}_1 \times \mathbf{n}_2 = (-62, 31, -31) = -31(2, -1, 1).$$

We may therefore choose $\mathbf{u} = (2, -1, 1)$. Setting $z = 0$, the plane equations reduce to $3x + 2y = 5$ and $5x - 7y = -2$, which give $x = y = 1$. Thus $A = (1, 1, 0)$ belongs to the first line.

Step 2: Read the second line and form the common normal The symmetric equation gives a point $B = (2, 1, -1)$ and direction $\mathbf{v} = (3, -5, 2)$. Notice that $z + 1$ means the point has $z = -1$. Compute

$$\mathbf{u} \times \mathbf{v} = (3, -1, -7),$$
$$|\mathbf{u} \times \mathbf{v}| = \sqrt{9 + 1 + 49} = \sqrt{59},$$
$$\overrightarrow{AB} = (1, 0, -1).$$

The cross product is nonzero, so the lines are not parallel and the stated shortest-distance formula applies.

Step 3: Evaluate the scalar triple product Project the displacement from one line to the other onto their common normal:

$$\left|\overrightarrow{AB} \cdot (\mathbf{u} \times \mathbf{v})\right| = |1 \cdot 3 + 0 \cdot (-1) + (-1)(-7)| = 10,$$
$$d = \frac{10}{\sqrt{59}}.$$

Comparing this positive distance with the specified form $10/\sqrt{k}$ gives $\sqrt{k} = \sqrt{59}$. The integer k is dimensionless; the distance is in coordinate-length units.

Step 4: Conclusion The required value is $\boxed{k = 59}$.

Quick Tip to Solve: For a line defined by two planes, first cross their normals; then use the scalar triple product, keeping the signs of the chosen points separate from direction ratios.

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23. For $p \in \mathbb{N}$, if the angle between pair of tangents drawn to the ellipse $3x^2 + 2y^2 = 5$ from the point $(1, p)$ is $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$, then the distance of the vertex of the parabola $y = x^2 - px + p + 1$ from the point $(-7, 8)$ is equal to _____. [4 marks]

Solution:

Concept:

- For the ellipse $x^2/a^2 + y^2/b^2 = 1$, a line $y = mx + c$ is tangent precisely when $c^2 = a^2m^2 + b^2$. Here $a^2 = 5/3$ and $b^2 = 5/2$.
- If two nonvertical lines have slopes m_1, m_2, the tangent of their acute angle is $|(m_1 - m_2)/(1 + m_1m_2)|$. Completing the square locates the parabola's vertex.

Step 1: Obtain the equation for the tangent slopes A line of slope m through $(1, p)$ is $y = mx + p - m$. The tangency condition therefore gives

$$(p - m)^2 = \frac{5}{3}m^2 + \frac{5}{2} \quad \Longrightarrow \quad 4m^2 + 12pm + 15 - 6p^2 = 0.$$

The vertical line through this point is $x = 1$, which is not an ellipse tangent because the vertical tangents are $x = \pm\sqrt{5/3}$. Thus the slope method includes both tangents. Their slopes satisfy

$$m_1 + m_2 = -3p, \qquad m_1m_2 = \frac{15 - 6p^2}{4}.$$

Step 2: Use the prescribed angle to determine p From the sum and product,

$$|m_1 - m_2| = \sqrt{(m_1 + m_2)^2 - 4m_1m_2} = \sqrt{15(p^2 - 1)}.$$

The given positive angle then implies

$$\frac{4\sqrt{15(p^2 - 1)}}{|19 - 6p^2|} = \frac{12}{\sqrt{5}}.$$

Squaring and simplifying, with $t = p^2$, gives

$$25(t - 1) = 3(19 - 6t)^2, \qquad (t - 4)(108t - 277) = 0.$$

Since p is natural, $p^2 = 277/108$ is impossible, while $p^2 = 4$ yields $p = 2$. Substitution into the unsquared relation gives $12\sqrt{5}/5 = 12/\sqrt{5}$, so no extraneous value has been retained.

Step 3: Find the vertex and its distance With $p = 2$, complete the square:

$$y = x^2 - 2x + 3 = (x-1)^2 + 2.$$

The vertex is $V = (1, 2)$, not the point of contact on the ellipse. Its distance from $(-7, 8)$ is

$$\sqrt{(1+7)^2 + (2-8)^2} = \sqrt{64 + 36} = 10.$$

The requested value is a distance in coordinate-length units.

Step 4: Conclusion The vertex is $(1, 2)$, and the required distance is $\boxed{10}$ coordinate-length units.

Quick Tip to Solve: Use the sum and product of the two tangent slopes in the angle formula; solving for each slope individually adds unnecessary algebra.

24. Let P be a polygon with n vertices such that the line segment joining any two points of P remains entirely in P. If the number of diagonals of P is $n + 25$, then n is equal to _____. [4 marks]

Solution:

Concept:

- The condition that every segment joining two points of the polygon remains inside it says that the polygon is convex. Thus each nonadjacent pair of vertices determines one diagonal inside the polygon.
- Among n vertices there are $\binom{n}{2} = n(n-1)/2$ unordered pairs. The n adjacent pairs form sides, so the number of diagonals is $D = n(n-3)/2$.

Step 1: Count all vertex pairs and remove the sides Choose two distinct vertices without regard to order. This gives $n(n-1)/2$ line segments. Exactly n of them are the boundary sides of the polygon, including the side joining the last vertex to the first. The remaining segments are diagonals:

$$D = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}.$$

Equivalently, each vertex connects diagonally to $n - 3$ other vertices, and division by 2 removes the double counting from the two endpoints.

Step 2: Translate the given count into an equation The problem states that the number of diagonals is $n + 25$, so equate the two expressions for the same count:

$$\frac{n(n-3)}{2} = n + 25.$$

Multiplying by 2 and bringing all terms to one side gives

$$n^2 - 3n = 2n + 50 \quad \Longrightarrow \quad n^2 - 5n - 50 = 0.$$

The right-hand side is not the total number of vertex pairs, since polygon sides have already been excluded in the diagonal formula.

Step 3: Factor and check the allowable number of vertices Factor the quadratic:

$$(n-10)(n+5)=0.$$

The algebraic roots are 10 and -5. A polygon has a positive integral number of vertices with $n \geq 3$, so the negative root is inadmissible. For $n=10$, the diagonal count is $10(10-3)/2=35$, and the stated count is $10+25=35$, confirming the result. The number of vertices is a dimensionless count rather than a physical measurement.

Step 4: Conclusion The polygon has $\boxed{n=10}$ vertices.

Quick Tip to Solve: To count polygon diagonals, choose all pairs of vertices and subtract the sides; if counting from each vertex, remember to divide by two.

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25. Let $f(x)$ be a polynomial of degree 5 such that $\lim_{x\to 0}\frac{f(x)}{x^2}=1$, $f(-1)=-1$, $f(x)-14x$ has an extrema at $x=1$ and $f(x)-10x$ has an extrema at $x=-1$. Then $f(2)$ is equal to _____. [4 marks]

Solution:

Concept:

- The limit $\lim_{x\to 0} f(x)/x^2=1$ determines the coefficients of the constant, linear and quadratic terms of the polynomial.
- At an interior local extremum of a differentiable function, its derivative is zero. Thus the two stated extrema prescribe values of $f\prime(1)$ and $f\prime(-1)$.

Step 1: Use the limiting condition Write the general polynomial of degree five and divide it by x^2. A finite limit forces the constant and linear coefficients to vanish, and the limit 1 fixes the coefficient of x^2. Therefore, for real constants a, b, c with $c \neq 0$,

$$f(x)=x^2+ax^3+bx^4+cx^5.$$

The given value at -1 now yields $1-a+b-c=-1$, or

$$a-b+c=2.$$

Step 2: Convert both extrema into equations Differentiate the polynomial:

$$f\prime(x)=2x+3ax^2+4bx^3+5cx^4.$$

For $f(x)-14x$, stationarity at 1 gives $f\prime(1)=14$. For $f(x)-10x$, stationarity at -1 gives $f\prime(-1)=10$. Substituting gives

$$3a+4b+5c=12, \qquad 3a-4b+5c=12.$$

Subtracting these equations produces $8b=0$, so $b=0$.

Step 3: Find the polynomial and evaluate it The remaining equations are $a+c=2$ and $3a+5c=12$.

Subtracting three times the first from the second gives $2c = 6$, hence $c = 3$ and $a = -1$. Thus

$$f(x) = x^2 - x^3 + 3x^5, \qquad f(2) = 4 - 8 + 3(32) = 92.$$

As a check on the extrema, $f''(1) = 56$ and $f''(-1) = -52$, both nonzero. The stationary points are genuine extrema, and the leading coefficient is nonzero as required for degree five.

Step 4: Conclusion The required numerical value is $\boxed{f(2) = 92}$. This is a dimensionless polynomial value.

Quick Tip to Solve: Apply the limit before introducing unknown coefficients. Subtract the derivative equations at opposite arguments to isolate the even-power coefficient b.

26. The number of 7 digits integers formed by using the digits 2, 3, 4, 5 only and having the sum of digits equal to 18 is _____. [4 marks]

Solution:

Concept:

- Each of the seven digits belongs to $\{2, 3, 4, 5\}$. Subtracting 2 from each digit converts the sum condition into a bounded nonnegative integer equation.
- With repetition permitted, distinct arrangements of the digits give distinct integers. No leading-zero restriction arises because every allowed digit is nonzero.

Step 1: Measure the excess over the smallest integer Let the seven ordered digits be $d_1, \ldots, d_7$, and put $e_i = d_i - 2$. Then every e_i is one of $0, 1, 2, 3$, while the required sum becomes

$$e_1 + \cdots + e_7 = 18 - 7(2) = 4.$$

Thus we must distribute four excess units among seven labelled positions, subject to at most three units per position. The positions are distinguished because changing a digit position usually changes the integer.

Step 2: Count and remove the forbidden distributions Without the upper bounds, the number of nonnegative ordered solutions is given by stars and bars:

$$\binom{4 + 7 - 1}{7 - 1} = \binom{10}{6} = 210.$$

A solution violates a bound only if one e_i equals 4. Since the total excess is exactly 4, all other variables must then be zero. There are exactly seven such forbidden solutions, one for each position, and they cannot overlap. Therefore the required count is

$$210 - 7 = 203.$$

Step 3: Check by the possible digit multiplicities The allowed partitions of the excess are $3 + 1$, $2 + 2$,

$2+1+1$, and $1+1+1+1$. Their counts are, respectively,

$$7 \cdot 6 = 42, \quad \binom{7}{2} = 21, \quad 7\binom{6}{2} = 105, \quad \binom{7}{4} = 35.$$

They correspond to replacing selected baseline digits 2 by 5 and 3, two 4s, one 4 and two 3s, or four 3s. Their sum $42 + 21 + 105 + 35 = 203$ confirms the count without any forbidden digit.

Step 4: Conclusion The number of seven-digit integers is $\boxed{203}$. This is a count, with no physical unit.

Quick Tip to Solve: Subtract the minimum allowed digit from every position. Here only seven stars-and-bars solutions exceed the upper bound, since the total excess is just four.

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27. The remainder when $(556)^{40}$ is divided by 7 is _____. [4 marks]

Solution:

Concept:

- If two integers are congruent modulo 7, their equal positive integer powers are also congruent modulo 7. Reduce the base before doing any exponentiation.
- A short repeating cycle of powers determines a large power. The remainder must be the unique integer among $0, 1, \ldots, 6$ in the same congruence class.

Step 1: Reduce the base modulo seven Division of 556 by 7 gives

$$556 = 7 \cdot 79 + 3.$$

Consequently $556 \equiv 3 \pmod 7$, so

$$556^{40} \equiv 3^{40} \pmod 7.$$

This reduction does not replace the exponent by a remainder arbitrarily; we must first establish a cycle for powers of the reduced base.

Step 2: Find a power that returns to one Compute small powers modulo 7:

$$3^2 = 9 \equiv 2 \pmod 7, \qquad 3^3 \equiv 3 \cdot 2 = 6 \equiv -1 \pmod 7.$$

Squaring the last congruence gives $3^6 \equiv 1 \pmod 7$. This follows directly from integer arithmetic and lets us remove groups of six factors of 3. Equivalently, the binomial theorem applied to $(7 \cdot 79 + 3)^{40}$ leaves only the constant term 3^{40} modulo 7, since all other terms contain a factor of 7.

Step 3: Reduce the exponent using the established cycle Since $40 = 6 \cdot 6 + 4$, we obtain

$$3^{40} = (3^6)^6\, 3^4 \equiv 1^6\, 3^4 \pmod 7.$$

Now $3^4 = 81 = 7 \cdot 11 + 4$, so the residue is 4. One may also square $3^2 \equiv 2$ to get $3^4 \equiv 4$. The number 4 already lies between 0 and 6, so it is the actual nonnegative remainder, rather than only another representative of its congruence class.

Step 4: Conclusion The remainder when 556^{40} is divided by 7 is $\boxed{4}$, a dimensionless integer.

Quick Tip to Solve: Reduce 556 to 3 first, then use $3^3 \equiv -1 \pmod 7$. Squaring gives a six-power cycle and leaves only the fourth power to evaluate.

28. Let $[t]$ denote the greatest integer less than or equal to t. Then the value of $10 - 10\int_{-2}^{2}\left[x + x^3\right] dx$ is _____. [4 marks]

Solution:

Concept:

- For any noninteger real u, $\lfloor u \rfloor + \lfloor -u \rfloor = -1$. At integer u, the sum instead equals zero.
- The polynomial $g(x) = x + x^3$ is odd and strictly increasing. Symmetric substitution pairs the two floor values; finitely many exceptional points do not alter a definite integral.

Step 1: Establish the floor-function identity If $n < u < n + 1$ for an integer n, then $\lfloor u \rfloor = n$ while $-n - 1 < -u < -n$, giving $\lfloor -u \rfloor = -n - 1$. Their sum is therefore -1. When u is itself an integer the sum is zero, so this exception must be acknowledged rather than treating the identity as pointwise universal.

Step 2: Pair the integrand at opposite arguments Put

$$I = \int_{-2}^{2} \lfloor x + x^3 \rfloor \, dx.$$

Replacing x by $-x$ and using oddness gives

$$I = \int_{-2}^{2} \lfloor -(x + x^3) \rfloor \, dx.$$

Adding these equal integrals yields

$$2I = \int_{-2}^{2} \left(\lfloor g(x) \rfloor + \lfloor -g(x) \rfloor \right) dx.$$

Here $g\prime(x) = 1 + 3x^2 > 0$, and g ranges from -10 to 10 on the interval. Each integer value has at most one preimage, so there are only finitely many exceptional points where the integrand sum is zero.

Step 3: Evaluate the integral and the requested expression A bounded function with only finitely many jumps is integrable. Changing its values at the finitely many integer preimages has no effect on its integral. Therefore the paired integrand integrates exactly like the constant -1 across an interval of length 4:

$$2I = \int_{-2}^{2} (-1) \, dx = -4, \qquad I = -2.$$

Substitute this value with the correct outer minus sign:

$$10 - 10I = 10 - 10(-2) = 30.$$

The floor function itself is not odd; using ordinary odd-function cancellation on it would incorrectly give zero for the integral.

Step 4: Conclusion The required value is $\boxed{30}$, with no physical unit.

Quick Tip to Solve: For an odd inner function inside a greatest-integer integral, pair x with $-x$. The floor values sum to -1 away from isolated integer inputs.

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29. Let $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. If $k, l \in \mathbb{N}$ be such that $A^k B^l = I$, then the minimum value of $k + l$ is _____. [4 marks]

Solution:

Concept:

- The determinant of a matrix product is the product of its determinants. Since $\det A = 1$ and $\det B = -1$, the equation imposes a parity condition on l.
- The matrix B squares to the identity, while A is the plane rotation through $-\pi/4$. Matrix powers compose these rotation angles.

Step 1: Use the determinant to restrict the exponent of B Taking determinants in $A^k B^l = I$ gives

$$1^k(-1)^l = 1.$$

Hence l must be even. Direct multiplication shows $B^2 = I$, so for every allowed even l, we have $B^l = I$. The original equation then reduces to $A^k = I$. This argument does not assume that A and B commute; it uses only the product as printed.

Step 2: Determine the least positive power of A In rotation notation,

$$A = \begin{bmatrix} \cos(\pi/4) & \sin(\pi/4) \\ -\sin(\pi/4) & \cos(\pi/4) \end{bmatrix}.$$

Thus A^k is rotation through $-k\pi/4$. It equals the identity exactly when its cosine is 1 and sine is zero, meaning $k\pi/4$ is an integer multiple of 2π. Consequently k must be a positive multiple of 8. A multiplication check gives $A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A^4 = -I$, and $A^8 = I$. In particular, the tempting smaller exponent 4 yields minus the identity and cannot work with even l.

Step 3: Minimize the sum over natural-number exponents Both exponents are positive natural numbers. The smallest possible even exponent is $l = 2$, and the smallest possible positive exponent giving $A^k = I$ is $k = 8$. These choices are simultaneously attainable because

$$A^8 B^2 = I\, I = I.$$

Every other admissible pair has $k \geq 8$ and $l \geq 2$, so none has a smaller sum. Therefore

$$\min(k+l) = 8+2 = 10.$$

The determinant condition also excludes every odd l, regardless of the value of k.

Step 4: Conclusion The minimum value of $k+l$ is $\boxed{10}$, a dimensionless integer.

Quick Tip to Solve: Take determinants before multiplying powers. The reflection matrix forces an even exponent, after which only the rotation matrix order needs to be found.

30. Let $A_1 = [1]$, $A_2 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $A_3 = \begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$, ... **Then the sum of the diagonal elements of** A_{20} **is _____.** [4 marks]

Solution:

Concept:

- The matrices use consecutive positive integers in row order. The matrix A_n contains n^2 entries, so the first entry is one more than the total entries in all preceding matrices.
- Moving one position along the main diagonal of an n-column array advances by $n+1$ consecutive integers. Its n diagonal entries form an arithmetic progression.

Step 1: Locate the first entry of the twentieth matrix The matrices before A_{20} contain

$$1^2 + 2^2 + \cdots + 19^2 = \frac{19 \cdot 20 \cdot 39}{6} = 2470$$

entries. Since the entries begin at 1 and continue without gaps, A_{20} starts with 2471. This agrees with the examples: after the first entry 1, the second matrix begins at 2; after $1+4=5$ entries, the third begins at 6. The same cumulative-square rule therefore gives the required first diagonal element.

Step 2: Find the progression on the main diagonal Write the first entry as $a = 2471$. In a matrix with 20 columns filled row by row, the entry in row r, column c is

$$a + 20(r-1) + (c-1).$$

For the main diagonal $c = r$, it becomes $a + 21(r-1)$. Thus the twenty diagonal entries are

$$2471,\ 2492,\ 2513,\ \ldots,\ 2471 + 19 \cdot 21.$$

The common difference is 21, not 20: each diagonal move goes down one full row and right one additional column. The final diagonal element is $2471 + 399 = 2870$.

Step 3: Add the twenty diagonal entries Use the arithmetic-progression sum with first term 2471,

last term 2870, and 20 terms:

$$S = \frac{20}{2}(2471 + 2870) = 10 \cdot 5341 = 53410.$$

Equivalently, summing the entry formula gives

$$S = 20(2471) + 21\frac{19 \cdot 20}{2} = 49420 + 3990 = 53410.$$

Both computations count exactly one element from each row and each column of the main diagonal. There is no addition of the other diagonal, since the diagonal elements of a square matrix mean its principal diagonal entries.

Step 4: Conclusion The sum of the diagonal elements of A_{20} is $\boxed{53410}$, a dimensionless sum.

Quick Tip to Solve: Find the starting entry from the sum of the preceding squares, then use diagonal spacing $n + 1$ in an n-column row-filled matrix.

Aptitude Test (200 marks)

31. Which of the following is not a sustainable development goal targeted to be achieved by 2030?

[4 marks]

(A) gender equality

(B) zero hunger

(C) good health & well being

(D) space Research

Correct Answer: (D) space Research

Solution:

Concept:

- The United Nations 2030 Agenda contains 17 Sustainable Development Goals (SDGs), addressing social well-being, economic development and environmental protection together.
- An activity can help society without being the name of one of the 17 goals. The question asks which listed expression is not an SDG, so compare the options with the actual goal titles.

Step 1: Check the three development objectives Gender Equality is Goal 5. It concerns equality and the empowerment of women and girls, so option (A) belongs to the SDG framework. Zero Hunger is Goal 2, addressing hunger, food security, nutrition and sustainable agriculture; option (B) is therefore also an SDG. Good Health and Well-being is Goal 3, which concerns healthy lives and well-being for people at all ages. Thus option (C) is another recognised goal, rather than the required exception.

Step 2: Identify the exception Space research is a field of scientific investigation, not the title of a Sustainable Development Goal. Space technology may support development through weather observations, communications and environmental monitoring, but those applications do not create a separate SDG named Space Research. The word "not" in the question requires this distinction: the first

three options directly name goals in the 2030 Agenda, whereas the fourth names an area of research.

Step 3: Conclusion Space Research is not one of the Sustainable Development Goals targeted for achievement by 2030. Hence, the correct option is (D).

Quick Tip to Solve: For a negative SDG question, first recognise the official goal titles: Zero Hunger is 2, Good Health and Well-being is 3, and Gender Equality is 5.

32. KANDARIYA MAHADEV TEMPLE is situated in which state of India? [4 marks]

(A) Madhya Pradesh

(B) Maharashtra

(C) Rajasthan

(D) Karnataka

Correct Answer: (A) Madhya Pradesh

Solution:

Concept:

- Architectural awareness questions often connect a named monument with its town and state. Identify the particular monument before choosing among states that may also have famous temples.
- The Kandariya Mahadeva Temple belongs to the Khajuraho group of temples in Madhya Pradesh. It is a major example of the northern Indian Nagara tradition associated with the Chandella rulers.

Step 1: Locate the named temple The name Kandariya Mahadev, also written Kandariya Mahadeva, refers to the prominent Shiva temple at Khajuraho. Its ascending cluster of spires and tall central shikhara are characteristic features of this temple. The location chain needed for the question is therefore Kandariya Mahadeva Temple, then Khajuraho, then Madhya Pradesh. The question asks for the state, so the answer must be the last member of that chain rather than the town or the architectural style.

Step 2: Compare the four states Option (A), Madhya Pradesh, contains Khajuraho and therefore matches the temple's location. Maharashtra, Rajasthan and Karnataka each possess important monuments and temple traditions, but none is the state containing this particular Khajuraho temple. A state cannot be selected simply because it is associated with historic temples in general. Here the proper name fixes the monument, and the known location of that monument rules out options (B), (C) and (D).

Step 3: Conclusion Kandariya Mahadev Temple is situated at Khajuraho in Madhya Pradesh. Hence, the correct option is (A).

Quick Tip to Solve: Memorise monument-location pairs as a three-link chain: Kandariya Mahadeva Temple–Khajuraho–Madhya Pradesh.

33. Vertical elements in a building are:- [4 marks]

(A) Column, Lintel, Wall

(B) Lintel, Beam, Arch

(C) Column, Pier, Post

(D) Floor, Column, Arch

Correct Answer: (C) Column, Pier, Post

Solution:

Concept:

- Building elements are classified by their usual orientation and structural role. Vertical members stand upright and commonly transmit loads down towards the foundation.
- A column, a pier and a post are upright supporting members. A lintel or beam spans across an opening or between supports, while a floor extends primarily in a horizontal plane.

Step 1: Classify the members in the complete vertical group A column is a vertical structural member supporting loads from beams, slabs or other parts of a building. A pier is an upright supporting mass or member, commonly of masonry, and a post is likewise an upright support. Their sizes, materials and particular uses can differ, but their normal orientation is vertical. Consequently all three entries in option (C), Column, Pier, Post, satisfy the condition together.

Step 2: Eliminate groups containing spanning members Option (A) includes a lintel. Although a wall and column can be vertical elements, a lintel normally runs horizontally over a door or window, so this group is unsuitable. Option (B) contains a lintel, a beam and an arch, which are spanning elements rather than a group of upright supports. Option (D) includes a floor, a horizontal surface, and an arch, a curved spanning element; the presence of a column does not make the whole group vertical. Each word in a proposed group must meet the condition, not just one member of that group.

Step 3: Conclusion Column, Pier and Post are the group consisting entirely of vertical building elements. Hence, the correct option is (C).

Quick Tip to Solve: For a group-classification question, reject the entire option as soon as one member fails: lintels, beams and floors are not upright supports.

34. Cavity Walls are used for [4 marks]

(A) Waterproofing

(B) Heat Insulation

(C) Aesthetics

(D) Cost saving

Correct Answer: (B) Heat Insulation

Solution:

Concept:

- A cavity wall has two separate wall leaves with a continuous intervening air space, tied together where needed. The cavity interrupts the direct solid-material path for heat transfer through the wall.
- Air is a relatively poor conductor of heat. An appropriately designed cavity, often supplemented by insulation, improves the thermal resistance of the building envelope.

Step 1: Identify the function of the air gap In a solid wall, heat can pass by conduction through the continuous masonry from the warmer side to the cooler side. Separating the wall into two leaves introduces an air layer between them and reduces the direct conduction path. This helps limit unwanted heat gain in hot conditions and heat loss in cold conditions. Heat insulation, option (B), therefore describes a principal purpose of cavity-wall construction. The cavity does not make all modes of heat transfer vanish; it improves resistance rather than acting as a perfect thermal barrier.

Step 2: Distinguish the other proposed purposes A properly detailed cavity wall can also help control rain penetration, because moisture reaching the outer leaf can drain away instead of bridging directly to the inner leaf. However, waterproofing is not achieved merely by having a cavity: drainage, flashings and damp-proof detailing remain necessary. Among the listed general purposes, thermal insulation is the intended answer. Aesthetics describes appearance, which does not require a hidden air gap, and cost saving is not an inherent result of constructing two wall leaves with ties and careful detailing. Thus neither (C) nor (D) identifies the characteristic thermal benefit.

Step 3: Conclusion Cavity walls are used for heat insulation through the thermal resistance provided by the separated leaves and intervening air space. Hence, the correct option is (B).

Quick Tip to Solve: Associate a wall cavity with an air gap and reduced heat transfer; distinguish that main thermal function from moisture-control detailing.

35. Lintel is related to: [4 marks]

(A) Window

(B) Roof

(C) Foundation

(D) Pipe

Correct Answer: (A) Window

Solution:

Concept:

- A lintel is a horizontal member spanning over a wall opening, such as a window or door. It supports the masonry or other loading immediately above that opening.
- The lintel transfers its load to the wall or supports on either side, allowing the opening below it to remain clear. Its location and spanning function distinguish it from the sill below a window.

Step 1: Relate the member to a window opening A window interrupts the continuity of a wall. Masonry over the opening cannot simply continue unsupported across the gap, so a spanning member is provided above it. That member is the lintel. It bears on the sides of the opening and carries the load over the window to the surrounding structure. Therefore the building component directly associated with a lintel among the options is the window. A door opening can also have a lintel, but a door is not one of the listed alternatives.

Step 2: Separate it from the other building systems A roof covers the top of the building and is supported through its own system of slabs, beams, trusses or other members; "roof" is not the specific opening-related association tested here. A foundation transfers the building's loads to the ground and lies at its base, rather than spanning above a wall opening. A pipe conveys services such as water or drainage and does not perform the lintel's structural role. Thus options (B), (C) and (D) describe different building elements or systems, while option (A) identifies the opening whose upper edge commonly requires a lintel.

Step 3: Conclusion A lintel is related to a window, spanning above the opening and supporting the construction over it. Hence, the correct option is (A).

Quick Tip to Solve: Remember the vertical order around a window: sill below the opening, jambs at the sides, and lintel above the opening.

36. Choose the correct option among the following by looking at the image given above: [4 marks]



(A) Lodhi Garden

(B) Jantar Mantar

(C) Rajghat

(D) Bahai Temple

Correct Answer: (B) Jantar Mantar

Solution:

Concept:

- Identify a monument from its characteristic built forms rather than from incidental trees or background buildings. Astronomical observatories contain geometric masonry instruments designed for observation and measurement.
- Jantar Mantar is associated with large constructed astronomical instruments. Curved, circular and inclined masonry surfaces distinguish these instruments from ordinary garden pavilions or temple buildings.

Step 1: Examine the structure in the photograph The photograph shows a low masonry construction with a conspicuous curved circular form, sloping edges and upright masonry divisions. It reads as a purpose-built geometric instrument rather than a conventional enclosed building with rooms and a roof. These open geometric forms are characteristic of the astronomical instruments at Jantar Mantar. The recognisable evidence lies in the instrument itself; the palms, planting and multistorey building behind it are background features and are not the basis for choosing the answer.

Step 2: Compare the alternative landmarks Lodhi Garden is known for its landscaped setting and historic tomb structures; the photographed instrument is not a tomb or domed pavilion. Rajghat is a memorial associated with a simple black platform, not this circular astronomical construction. The Bahai Temple, commonly called the Lotus Temple, has a distinctive large lotus-like arrangement of white petal forms, which is absent here. Of the four alternatives, Jantar Mantar alone directly matches the visible masonry instrument and its geometrical organisation. No particular city needs to

be inferred, since the option asks only for the landmark name.

Step 3: Conclusion The image shows an astronomical masonry structure belonging to Jantar Mantar. Hence, the correct option is (B).

Quick Tip to Solve: Recognise Jantar Mantar by its large geometric masonry instruments; do not identify a monument from the landscaping or background buildings alone.

37. ARCHITECT ACT 1972 was enforced in which month [4 marks]

(A) 31st Dec

(B) 31st May

(C) 31st July

(D) 31st March

Correct Answer: No option gives the commencement date, 1 September 1972. The keyed B gives the enactment/publication date, 31 May 1972.

Solution:

Concept:

- The date of enactment or publication of an Act must be distinguished from the date on which it comes into force.
- The Architects Act, 1972 bears the date 31 May 1972; the Council of Architecture records its commencement as 1 September 1972.

Step 1: Separate the two dates The date 31 May 1972 appearing with the Act is its enactment/publication date. It explains why May appears among the alternatives. However, the question uses the word enforced, which ordinarily means commencement, rather than enactment. The two events need not occur on the same day.

Step 2: Check the offered alternatives The Council of Architecture states that the Act came into force on 1 September 1972. None of the alternatives gives September. December, July and March do not give the stated commencement, and 31 May is the enactment date. Therefore the literal wording has no correct listed option. The keyed choice (B) identifies the enactment date, and is understandable only if enforced is being used loosely for enacted.

Step 3: Conclusion Literally, commencement was 1 September 1972, so none of the listed options is correct for enforcement. The keyed option (B), 31 May, is the enactment/publication date; this distinction makes the printed question defective.

Quick Tip to Solve: Keep separate entries for enactment and commencement when learning dates of legislation; a date printed beside an Act title need not be its enforcement date.

38. The dome of Rashtrapati Bhawan is influenced by which of the following: [4 marks]

(A) Gurudwara

(B) Mosque

(C) Stupa

(D) Temple

Correct Answer: (C) Stupa

Solution:

Concept:

- Architectural influence is identified by characteristic form and ornament, rather than by the present function of a building.
- The central dome of Rashtrapati Bhavan incorporates an Indian Buddhist stupa reference, especially the railing associated with the Sanchi Stupa.

Step 1: Recognise the stupa reference The rounded central dome and the railing around its base recall the stupa vocabulary. In particular, the encircling railing connects the design to Sanchi. This is a question about the influence on a feature, not a claim that the whole presidential residence is itself a Buddhist religious building.

Step 2: Compare the alternatives A gurudwara is a Sikh religious building and a mosque belongs to Islamic architecture. Although domes also occur in those traditions, the specific reference asked here is the stupa. Temple is a broad category and does not identify the characteristic Sanchi reference. Among the four listed choices, stupa is therefore the precise match.

Step 3: Conclusion The dome is influenced by the stupa, particularly the Sanchi Stupa tradition. Hence, the correct option is (C).

Quick Tip to Solve: For Rashtrapati Bhavan, connect the central dome and its surrounding railing with Sanchi rather than identifying every dome as a mosque influence.

39. Dominion tower is designed by [4 marks]

(A) Bjarke Ingels

(B) Zaha Hadid

(C) Frei otto

(D) F.L. Wright

Correct Answer: (B) Zaha Hadid

Solution:

Concept:

- Architectural general awareness pairs the name of a completed building with the architect or practice responsible for its design.
- Dominion Tower, also called the Dominion Office Building in Moscow, is a project of Zaha Hadid Architects.

Step 1: Identify the named project Dominion Tower refers to the Moscow office building by Zaha Hadid Architects. Its striking arrangement of horizontally offset floors and its flowing internal circulation are useful recognition cues. The question asks for the designer, so the building-to-architect association is the decisive fact.

Step 2: Eliminate the other architects Bjarke Ingels is associated with the BIG practice; Frei Otto is especially known for lightweight tensile structures; Frank Lloyd Wright is associated with organic architecture and buildings such as Fallingwater. None is the designer of Dominion Tower. Zaha Hadid is the architect among the given names who matches this particular project.

Step 3: Conclusion Dominion Tower was designed by Zaha Hadid and her architectural practice. Hence, the correct option is (B).

Quick Tip to Solve: Learn architect-building pairs with one visual cue: Dominion Tower combines offset floor plates with the name Zaha Hadid.

40. The largest river island of the world is present in which state of India? [4 marks]

(A) Maharastra

(B) Assam

(C) Uttar Pradesh

(D) West Bengal

Correct Answer: (B) Assam

Solution:

Concept:

- The river island intended in this question is Majuli, located in the Brahmaputra river system.
- Majuli is in Assam in northeastern India and is widely described as the largest inhabited river island.

Step 1: Identify the island The familiar Indian geography reference in the question is Majuli. It is a river island of the Brahmaputra system in Assam. Recognising the island first avoids confusing a river island with a coastal or marine island, and also fixes the state without having to compare river lengths.

Step 2: Match its location to the choices Assam is the state containing Majuli. Maharashtra and Uttar Pradesh do not contain this Brahmaputra island. West Bengal includes the lower Ganga delta region, but Majuli is upstream in Assam rather than in that delta. The qualifier largest is often given as largest inhabited river island; that qualification does not change the state being asked for.

Step 3: Conclusion The intended island is Majuli in Assam. Hence, the correct option is (B).

Quick Tip to Solve: Remember the chain Majuli–Brahmaputra–Assam; it identifies both the river system and the state in this common geography question.

41. Who prepared the first layout of the town of Jamshedpur? [4 marks]

(A) Charles Correa

(B) Otto Konigsberger

(C) Le Corbuseir

(D) Messrs Julin Kennedy Sahhin

Correct Answer: (D) Messrs Julin Kennedy Sahhin

Solution:

Concept:

- An initial town layout must be distinguished from later extension plans or master plans for the same settlement.
- The first layout of Jamshedpur is associated with the Pittsburgh firm named in the alternatives as Messrs Julin Kennedy Sahhin, commonly rendered Julian Kennedy and Sahlin.

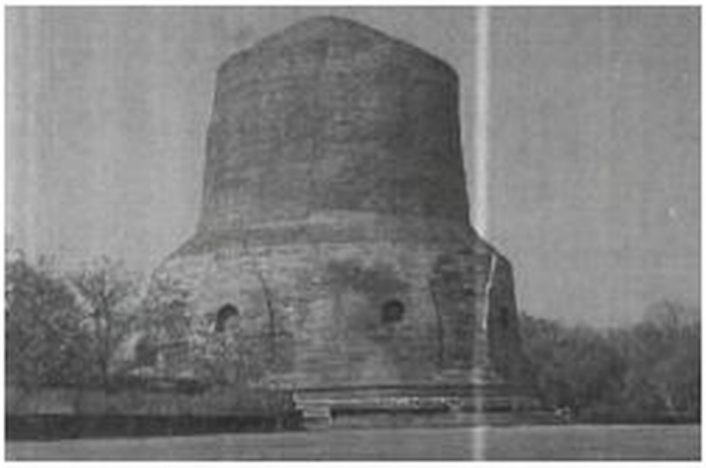
Step 1: Focus on the word first Jamshedpur developed as a planned industrial township around the Tata steel works. Its original layout was prepared by the American engineering firm associated with Julian Kennedy and Sahlin. The question asks for this early layout, rather than for every planner subsequently involved in the growing town.

Step 2: Distinguish the listed planners Otto Koenigsberger is associated with later planning work for Jamshedpur, so his name is a tempting distraction when first is overlooked. Charles Correa and Le Corbusier are major names in modern Indian architecture and planning, but they do not identify the initial layout in this question. The firm in the final alternative is the required historical association despite the spelling used there.

Step 3: Conclusion The first layout was prepared by the firm given as Messrs Julin Kennedy Sahhin in the options. Hence, the correct option is (D).

Quick Tip to Solve: In town-planning questions, underline first or later: the original Jamshedpur layout and its subsequent master plans have different attributions.

42. Identify the historical monument which is given in the image. [4 marks]



(A) Sanchi Stupa

(B) Victory Tower

(C) Sarnath Stupa

(D) Nalanda

Correct Answer: (C) Sarnath Stupa

Solution:

Concept:

- A monument can be recognised by its overall mass, profile and surviving architectural details.
- The Dhamek Stupa at Sarnath has a prominent cylindrical upper mass above a broader lower base, unlike the low hemispherical form of the Great Stupa at Sanchi.

Step 1: Read the visible form The photograph shows a large, solid-looking cylindrical monument. Its upper portion has a tall drum-like profile, while the lower portion is wider and displays niches and masonry detail. These features identify the Dhamek Stupa at Sarnath. The recognition comes from the monument itself rather than from any surrounding buildings.

Step 2: Compare each alternative Sanchi Stupa is characteristically a broad hemispherical mound with railings and gateways, which does not match the tall cylindrical body shown. A victory tower is a much more slender, storeyed vertical monument. Nalanda is known for its monastic and university ruins, rather than this distinctive surviving cylindrical stupa. Thus Sarnath Stupa is the matching option.

Step 3: Conclusion The image depicts the Dhamek Stupa at Sarnath. Hence, the correct option is (C).

Quick Tip to Solve: Compare silhouettes first: Sanchi is hemispherical, while the Dhamek Stupa at Sarnath is conspicuously cylindrical.

43. Who was known as "Father of modern concrete"? [4 marks]

(A) John Paul

(B) John Nash

(C) John Smeaton

(D) Joseph Stein

Correct Answer: (C) John Smeaton

Solution:

Concept:

- The title "Father of modern concrete identifies the engineer associated with the systematic development and practical use of hydraulic lime concrete.
- Hydraulic binders harden through reaction with water and can set in damp or underwater conditions. They differ from ordinary air lime, which principally hardens by carbonation.

Step 1: Identify the contribution behind the title John Smeaton investigated the properties of limes and aggregates while developing materials for the Eddystone Lighthouse. A lighthouse exposed to seawater required a durable binder suitable for wet conditions. His work established the importance of hydraulic lime and helped make concrete a reliable engineering material. This experimental approach to selecting and proportioning materials is the contribution referred to by the title in the question.

Step 2: Compare all four names John Smeaton is therefore the match in option (C). John Nash is known as an architect, especially for buildings and urban design in Britain, rather than for this development of hydraulic concrete. Joseph Stein is associated with architecture, including major work in India, and does not carry the stated title. John Paul is also not the engineer associated with this historical contribution. The question asks for the named pioneer, so the hydraulic-lime connection decides the choice rather than the fact that a person worked in building design.

Step 3: Conclusion John Smeaton was known as the "Father of modern concrete. Hence, the correct option is (C).

Quick Tip to Solve: Link Smeaton with the Eddystone Lighthouse and hydraulic lime; this distinguishes his concrete contribution from the work of building architects.

44. Who is the architect of TWA Airport, New York? [4 marks]

(A) Eero Saarinen

(B) Mies Vander Rohe

(C) Joseph Allen Stein

(D) Alvar Aalto

Correct Answer: (A) Eero Saarinen

Solution:

Concept:

- Architectural-awareness questions are answered by matching the particular landmark with its designer, rather than choosing an architect merely because they belong to the same modern period.
- The TWA Flight Center at New York's John F. Kennedy Airport is a landmark of expressive modern architecture designed by Eero Saarinen.

Step 1: Identify the airport building intended The name TWA Airport in this question refers to the celebrated TWA terminal or Flight Center in New York. Its sweeping curved concrete roof and sculptural spaces were designed to suggest the excitement and movement of flight. These features belong to Eero Saarinen's design for the terminal. The building is the decisive reference; an airport complex can contain many terminals and buildings by different designers.

Step 2: Eliminate the other architects Eero Saarinen in option (A) is the designer of the TWA Flight Center. Mies van der Rohe is associated with a restrained vocabulary of steel-and-glass buildings and is not its architect. Joseph Allen Stein is known for his architectural work in India, including the India International Centre, rather than this New York terminal. Alvar Aalto is associated with Finnish modernism and is likewise not the designer of TWA. Thus neither the international fame of the other architects nor their association with modern design changes the building-designer pairing.

Step 3: Conclusion The TWA Flight Center in New York was designed by Eero Saarinen. Hence, the correct option is (A).

Quick Tip to Solve: Associate Saarinen with TWA's wing-like, curving terminal roof; identifying a landmark's distinctive form helps recall its architect.

45. Starting from a point, Maria walked 12 meters towards North, she turned right and walked 10 meters. She again turned right and walked 12 meters, then she turned left and walked 5 meters. How far is she now and in which direction from the starting point? Consider the starting point is in south direction. [4 marks]

(A) 27 meters towards East

(B) 5 meters towards East

(C) 10 meters towards West

(D) 15 meters towards East

Correct Answer: (D) 15 meters towards East

Solution:

Concept:

- Distance from a starting point is the magnitude of the final displacement, not the sum of all the distances walked along the route.
- Take east as the positive horizontal direction and north as the positive vertical direction. A right or left turn is measured relative to the direction the walker currently faces.

Step 1: Track every turn and position Let the starting point be $O = (0, 0)$, with coordinates measured in metres. After walking 12 metres north, Maria is at $(0, 12)$. Facing north, a right turn points east, so walking 10 metres brings her to $(10, 12)$. The next right turn, from east, points south. Walking 12 metres south brings her to $(10, 0)$. She is now facing south; a left turn from south points east, not west. The final 5 metres therefore take her to $(15, 0)$.

Step 2: Calculate the direct separation and check the options The northward and southward movements cancel exactly. Both horizontal movements are eastward, so the net displacement is

$$\Delta x = 10 + 5 = 15 \text{ m}, \qquad \Delta y = 12 - 12 = 0.$$

Hence the straight-line distance is $\sqrt{15^2 + 0^2} = 15$ m, due east. Option (A), 27 metres east, does not cancel the vertical movements correctly. Option (B) keeps only the final leg and loses the earlier eastward movement. Option (C) has both the wrong direction and the wrong magnitude. The total walking distance would be 39 metres, but that is not the separation requested. The reference to the starting point does not change the north, east and south directions explicitly given for the route.

Step 3: Conclusion Maria is 15 metres east of her starting point. Hence, the correct option is (D).

Quick Tip to Solve: Cancel equal opposite movements first, then add movements in the same direction. For the last turn, imagine facing south before turning left.

46. Who among the following is not Pritzker prize winner? [4 marks]

(A) Wang Shu

(B) B.V. Doshi

(C) Arata Isozaki

(D) Denise Scott Brown

Correct Answer: (D) Denise Scott Brown

Solution:

Concept:

- The Pritzker Architecture Prize is awarded to named architects. Being an influential architect, a collaborator, or the partner of a laureate does not itself make a person a laureate.
- For this paper, compare the named recipients known by the 2023 examination date. The word "not reverses the usual identification task.

Step 1: Check the three listed laureates Wang Shu received the Pritzker Architecture Prize in 2012. B. V. Doshi received it in 2018, becoming the first Indian architect to receive the prize. Arata Isozaki received it in 2019. These three names therefore satisfy the description Pritzker prize winner and must be excluded from the answer to a question asking who is not a winner. Checking each candidate avoids confusing different generations or countries of modern architecture.

Step 2: Distinguish Denise Scott Brown from a laureate collaborator Denise Scott Brown is a major architect and theorist, but she was not a named recipient of the Pritzker Prize at the time of this paper. Robert Venturi, with whom she worked closely, was the named recipient in 1991. Their professional collaboration does not make the award automatically a joint award. Consequently, option (D) is the name that fails the recipient test. Options (A), (B) and (C) cannot be selected because each has an identifiable award year. This distinction concerns formal prize attribution, rather than any judgement about Scott Brown's architectural importance or contribution.

Step 3: Conclusion Denise Scott Brown is the person among these options who was not a Pritzker Prize winner at the time of the paper. Hence, the correct option is (D).

Quick Tip to Solve: For negative award questions, attach an award year to each confirmed laureate; do not transfer an individual award to a collaborator.

47. Which of the following is not Antonio Gaudi's project? [4 marks]

(A) Casa Mila

(B) Sagrada Familia

(C) Casa Batllo

(D) Casa Milano

Correct Answer: (D) Casa Milano

Solution:

Concept:

- A building-attribution question is solved by checking whether each named project belongs to the architect's body of work. A similar-sounding name alone is insufficient evidence.
- Antoni Gaudi is closely associated with Barcelona and Catalan modernisme. Casa Mila, the Sagrada Familia and Casa Batllo are established examples of his work.

Step 1: Recognise the three confirmed Gaudi projects Casa Mila, also called La Pedrera, is Gaudi's celebrated Barcelona building with an undulating stone facade. The Sagrada Familia is the basilica in Barcelona with which he is especially associated. Casa Batllo is another of his major Barcelona projects, recognisable for its organic forms, richly treated surfaces and distinctive roof. These identifications confirm that options (A), (B) and (C) are all associated with Gaudi. The question asks for the exception, so recognising a famous Gaudi project is a reason to eliminate that option.

Step 2: Select the name outside that group Casa Milano is not one of the Gaudi projects represented by this group. In particular, it must not be confused with Casa Mila: the similarity of their names is the distractor. The word Casa simply means house and occurs in many building names; it does not establish a common architect. Since the first three names are confirmed Gaudi works and the fourth is not, the exception is option (D). No inference from stylistic similarity is necessary here; the known building-designer attributions settle all four alternatives.

Step 3: Conclusion Casa Milano is not Antoni Gaudi's project among the listed choices. Hence, the correct option is (D).

Quick Tip to Solve: Memorise the trio Casa Mila, Casa Batllo and Sagrada Familia with Gaudi; check carefully for the misleading change from Mila to Milano.

48. Which one of the following is most earthquake resistant structure? [4 marks]

(A) RCC framed structure

(B) Load bearing structure

(C) Mud walls structure

(D) Random stone masonary structure

Correct Answer: (A) RCC framed structure

Solution:

Concept:

- Earthquake resistance depends on a continuous load path, lateral strength and ductility: a structure must transmit shaking forces to its foundations while tolerating deformation.
- An RCC frame uses reinforced-concrete beams and columns. Concrete resists compression, while steel reinforcement provides tensile resistance and, with suitable detailing, ductile behaviour.

Step 1: Compare the structural systems under lateral shaking Among the listed conventional systems, a properly designed and detailed RCC framed structure provides the best earthquake resistance. Its connected beams and columns form a skeleton that carries loads to the foundation. Reinforcement crossing joints and appropriate confinement enable the frame to resist lateral actions and deform without the sudden loss of capacity characteristic of brittle construction. The answer therefore concerns an engineered structural system, not merely the presence of concrete in a building.

Step 2: Rule out the less resistant alternatives A conventional load-bearing structure relies heavily on walls to carry loads; unreinforced masonry walls are vulnerable to tensile and shear cracking during shaking. Ordinary mud walls have low tensile strength and may lack effective ties between walls, roof and foundation. Random stone masonry may have irregular bonding and weak connections, making separation and local collapse more likely under lateral motion. These descriptions do not mean that every RCC building is automatically safe or that masonry cannot be strengthened. Proper seismic design, construction quality and detailing are essential. However, when comparing the four systems as listed, the ductile engineered frame is the appropriate selection.

Step 3: Conclusion A properly designed and detailed RCC framed structure is the most earthquake resistant of the listed alternatives. Hence, the correct option is (A).

Quick Tip to Solve: For earthquake-resistance comparisons, look for a tied, ductile load-carrying frame; compressive strength alone does not prevent brittle seismic failure.

49. "CHALK" belongs to which category? [4 marks]

(A) Rock

(B) Mineral

(C) Wood

(D) Polymer

Correct Answer: (A) Rock

Solution:

Concept:

- Building materials may be classified by their natural origin and structure. Rock is a naturally occurring geological material made of one or more minerals; a mineral is a constituent with a characteristic composition and crystal structure.
- Natural chalk is a soft, fine-grained sedimentary limestone. Its principal mineral constituent is calcite, or calcium carbonate, but the geological material called chalk is classified as a rock.

Step 1: Identify what the question names The question names CHALK as a material, rather than asking for its main mineral. Chalk develops through the accumulation and consolidation of microscopic calcareous remains in marine environments. Its fine texture and softness are characteristics of this sedimentary rock. Therefore, classify the material itself before considering its constituents.

Step 2: Distinguish the material from its mineral constituent Calcite is the mineral that chiefly makes up chalk; calcium carbonate describes its main chemical composition. Calling the whole material a mineral would confuse a constituent with the rock formed from it. A rock need not contain several different minerals: a rock dominated by a single mineral can still be a rock.

Step 3: Eliminate the unrelated material classes Option (A), Rock, fits the geological classification. Option (B), Mineral, names the constituent category rather than chalk itself. Option (C), Wood, denotes a material obtained from woody plants and cannot describe this marine sedimentary material. Option (D), Polymer, denotes a material based on large molecular chains and is unrelated to natural chalk. Thus the word CHALK belongs to the rock category among the four choices.

Step 4: Conclusion Chalk is a soft sedimentary rock, specifically a fine-grained limestone. Hence, the correct option is (A).

Quick Tip to Solve: When a material is mostly one mineral, distinguish the rock name from its constituent: chalk is the rock; calcite is its main mineral.

50. What are the three parts of colour theory? [4 marks]

(A) hyper, value and chrome

(B) honey, violet and chroma

(C) hue, value and chroma

(D) hue, chrome and violet

Correct Answer: (C) hue, value and chroma

Solution:

Concept:

- In colour description, hue identifies the colour family, value describes relative lightness or darkness, and chroma describes the strength or purity of the colour.
- These three attributes describe different aspects of a colour. A colour name alone does not specify how light it is or how vivid it appears.

Step 1: Identify the three independent attributes Hue answers which colour family is seen, such as red, yellow or blue. Value answers how light or dark the colour appears, from a dark value towards a light value. Chroma answers how vivid or dull the colour is relative to a neutral appearance. Together, hue, value and chroma provide a systematic description rather than merely a list of colour names.

Step 2: Use a familiar colour to separate the ideas For a red surface, the word red identifies its hue. A light red and a dark red can retain that hue while differing in value. A vivid red and a greyed red can differ in chroma. This shows why the three terms are complementary: each provides information that the other two alone do not supply. Violet, in contrast, is one particular hue, rather than one of the three general attributes.

Step 3: Compare the offered word groups Option (C) gives all three standard attributes: hue, value and chroma. Option (A) substitutes hyper and chrome, so it does not give the required terminology. Option (B) includes honey and violet and omits both hue and value as attributes. Option (D) contains hue but replaces the other two attributes with chrome and violet. Chrome is not the attribute chroma, and violet is a colour family, so those groups cannot describe the required three-part classification.

Step 4: Conclusion The three attributes are hue, value and chroma. Hence, the correct option is (C).

Quick Tip to Solve: Ask three separate questions about a colour: which family (hue), how light (value), and how vivid (chroma).

51. Identify the three primary colours? [4 marks]

(A) Red, Blue and Green

(B) Red, Yellow and Blue

(C) Black, White and Yellow

(D) Blue, White and Yellow

Correct Answer: (B) Red, Yellow and Blue

Solution:

Concept:

- In the traditional artist's colour wheel used for painting and design aptitude, the primary colours are red, yellow and blue, commonly abbreviated RYB.
- The colour model matters: red, green and blue are the additive primaries for light, whereas this conventional art question uses the traditional painting colour wheel.

Step 1: Identify the colour system used in the art context The question appears in design and colour aptitude and asks for the familiar three primary colours. In the traditional painting colour wheel these are red, yellow and blue. They are treated as the basic starting colours for constructing secondary colours. The intended grouping is therefore RYB, rather than a list that includes neutral white or black.

Step 2: Check the primary set through secondary colours The traditional wheel associates red and yellow with orange, yellow and blue with green, and blue and red with violet. These relations provide a memory check on the primary triad. Green belongs among the secondary colours in this traditional scheme, so replacing yellow with green would change the colour model rather than give the conventional painting answer.

Step 3: Eliminate the other combinations Option (B) lists red, yellow and blue and matches the traditional primary triad. Option (A) lists red, blue and green, the additive-light RGB primaries used in illuminated displays; that is a different colour system. Options (C) and (D) both include white, and option (C) also includes black. White and black are neutral achromatic references in the traditional wheel, rather than members of its primary triad. Thus neither combination supplies the three traditional artistic primaries.

Step 4: Conclusion For the traditional artistic colour wheel used here, the primaries are red, yellow and blue. Hence, the correct option is (B).

Quick Tip to Solve: Remember RYB for the traditional painter's wheel and RGB for emitted light; always identify the colour model before choosing.

52. Match List I with List II:

List I	List II
A. Kamakhya Temple	I. Madhya Pradesh
B. Padmanabha Temple	II. Rajasthan
C. Kandariya Mahadev	III. Assam
D. Karni Mata Temple	IV. Kerala

Choose the *correct* answer from the options given below: [4 marks]

(A) A-II, B-I, C-III, D-IV

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-IV, C-I, D-II

(D) A-I, B-II, C-IV, D-III

Correct Answer: (C) A-III, B-IV, C-I, D-II

Solution:

Concept:

- Temple-location matching requires the state in which each named monument stands. Matching the city first often makes the corresponding state easier to recall.
- The four pairs are Kamakhya–Assam, Padmanabha–Kerala, Kandariya Mahadev–Madhya Pradesh and Karni Mata–Rajasthan.

Step 1: Match Kamakhya and Padmanabha Kamakhya Temple is on Nilachal Hill in Guwahati, which is in Assam. Therefore A matches III. The Padmanabha Temple, commonly called the Sree Padmanabhaswamy Temple, is in Thiruvananthapuram in Kerala. Therefore B matches IV. These two location pairs already distinguish the correct option from the other three listed sequences.

Step 2: Match Kandariya Mahadev and Karni Mata Kandariya Mahadev Temple is part of the Khajuraho temple group in Madhya Pradesh. Hence C matches I. Karni Mata Temple is at Deshnoke near Bikaner in Rajasthan. Hence D matches II. It is important to use the states rather than mix the monument names with their cities: Guwahati, Thiruvananthapuram, Khajuraho and Deshnoke locate the respective monuments.

Step 3: Check the complete code against every option Collecting the four matches gives A-III, B-IV, C-I and D-II. Option (C) contains exactly this sequence. Option (A) incorrectly starts by placing Kamakhya in Rajasthan and Padmanabha in Madhya Pradesh. Option (B) places Kamakhya in Kerala and Padmanabha in Assam. Option (D) places Kamakhya in Madhya Pradesh and Padmanabha in Rajasthan. Each of those alternatives contradicts the identified locations; checking C and D also confirms that the complete four-way match is consistent.

Step 4: Conclusion The correct matching is A-III, B-IV, C-I, D-II. Hence, the correct option is (C).

Quick Tip to Solve: Link each monument to its city and then its state: Guwahati–Assam, Thiruvananthapuram–Kerala, Khajuraho–Madhya Pradesh, Deshnoke–Rajasthan.

53. A square has an area of 100 sq.mt. How much will be its area on a scale of 1 : 20? [4 marks]

(A) 5 sq.mt.

(B) 50 sq.mt.

(C) 0.25 sq.mt.

(D) 25 sq.mt.

Correct Answer: (C) 0.25 sq.mt.

Solution:

Concept:

- A drawing scale of $1 : 20$ is a ratio of corresponding lengths: each drawing length is one-twentieth of the real length when both use the same unit.
- Area scales as the square of the linear scale factor. If $k = L_{\text{drawing}}/L_{\text{actual}}$, then $A_{\text{drawing}} = k^2 A_{\text{actual}}$.

Step 1: Convert the given area into the square's side The actual square has area $100\,\text{m}^2$. If its side is s, then $s^2 = 100$. Taking the positive root because a side length is positive gives

$$s = \sqrt{100} = 10\,\text{m}.$$

This separates the linear quantity to which the scale applies from the area given in the question.

Step 2: Apply the linear scale and recompute the area At scale $1 : 20$, the side of the drawn square is

$$s_d = \frac{10}{20} = 0.5\,\text{m}.$$

Its area is therefore

$$A_d = (0.5)^2 = 0.25\,\text{m}^2.$$

Equivalently, square the scale factor directly:

$$A_d = 100\left(\frac{1}{20}\right)^2 = \frac{100}{400} = 0.25\,\text{m}^2.$$

Step 3: Check units and eliminate linear-scaling errors The answer is an area, so its unit remains square metres when the lengths are expressed in metres. Option (A), 5 square metres, comes from dividing 100 by 20 only once; that incorrectly applies a length factor directly to an area. Options (B), 50 square metres, and (D), 25 square metres, do not correspond to the required area reduction by a factor of 400. Option (C) gives the result obtained both by scaling each side and by squaring the linear scale factor.

Step 4: Conclusion The area on the scale drawing is $0.25\,\text{m}^2$. Hence, the correct option is (C).

Quick Tip to Solve: For scale $1 : n$, divide lengths by n, areas by n^2, and volumes by n^3; do not apply the length divisor directly to area.

54. Given below are two statements:

Statement I: Shakti Sthal is a memorial in Raj Ghat marking the cremation site of Indira Gandhi.

Statement II: A.P. Kanvinde in the landscape designer known for designing Shakti Sthal.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- A two-statement question requires an independent truth assessment of each statement before choosing the option describing the pair.
- Shakti Sthal is Indira Gandhi's memorial at her cremation site in the Raj Ghat memorial precinct in Delhi. Its landscape design is associated with Ravindra Bhan, not A. P. Kanvinde.

Step 1: Assess the identification in Statement I Statement I identifies Shakti Sthal as the memorial marking Indira Gandhi's cremation site in the Raj Ghat area. This identification is correct. The Raj Ghat memorial precinct includes memorials to several national leaders, so the reference to the area should be distinguished from Raj Ghat, the particular memorial to Mahatma Gandhi. The memorial name Shakti Sthal here refers to Indira Gandhi.

Step 2: Assess the designer attribution in Statement II Statement II attributes the landscape design of Shakti Sthal to A. P. Kanvinde. That attribution is incorrect: the landscape architect associated with the design is Ravindra Bhan. Kanvinde was a prominent Indian architect, but being a well-known architect does not make this particular project attribution true. The question tests the connection between a named memorial and its designer, rather than general recognition of an architect's name.

Step 3: Select the option for the ordered truth pair The assessments give Statement I true and Statement II false. Option (C) states precisely that combination. Option (A) would require the Kanvinde attribution to be correct, so it fails. Option (B) would also reject the correct identification of Shakti Sthal and Indira Gandhi. Option (D) reverses both truth values, treating the correct memorial identification as false and the incorrect designer attribution as true. Therefore only option (C) fits the two statements together.

Step 4: Conclusion Statement I is true, but Statement II is false; Shakti Sthal's landscape designer was Ravindra Bhan. Hence, the correct option is (C).

Quick Tip to Solve: Check the memorial's dedicatee and the designer separately: Shakti Sthal–Indira Gandhi; landscape design–Ravindra Bhan.

55. What is the full form of 'UDAY'? [4 marks]

(A) Ujjwala Discom Assitance Yojana

(B) Unnati Development Ayojan Yojana

(C) Ujwal Distribution Assurance Yojana

(D) Ujwal Discom Assurance Yojana

Correct Answer: (D) Ujwal Discom Assurance Yojana

Solution:

Concept:

- UDAY is the name of the electricity-distribution reform programme. Its expansion must retain the official wording, rather than substitute a similar word that fits an initial.
- In the name Ujwal Discom Assurance Yojana, Discom means a distribution company. The four principal words give the letters U, D, A and Y respectively.

Step 1: Identify the sector and the key word The programme concerns electricity distribution companies, commonly called discoms. This identifies the D as Discom. Although distribution describes the sector, it is not the word used at this position in the programme's official name. Consequently, an expansion can sound plausible while still failing the exact-name test.

Step 2: Match all four words against the choices The complete name is Ujwal Discom Assurance Yojana. Option (D) reproduces these words. Option (A) changes Ujwal to Ujjwala and Assurance to Assistance; these are different words, not alternative expansions. Option (B), Unnati Development Ayojan Yojana, does not give the programme name. Option (C) retains Ujwal, Assurance and Yojana but replaces Discom with Distribution, so it too is not the stated full form.

Step 3: Conclusion UDAY stands for Ujwal Discom Assurance Yojana. Hence, the correct option is (D).

Quick Tip to Solve: Remember UDAY as a programme for discoms: the D is Discom and the A is Assurance. Check these two words before choosing a similar-sounding expansion.

56. What is the acronym for 'AYUSH'? [4 marks]

(A) Ayurveda, Yoga and Naturopathy, Unani, Siddha and Homeopathy

(B) Ayurveda, Yogic and Unani Siddha and Homeopathy

(C) Acroveda, Yoga, Unani, Siddha and Homeopathy

(D) Ayurveda, Yoga, Naturopathy, Unani Surgery and Homeopathy

Correct Answer: (A) Ayurveda, Yoga and Naturopathy, Unani, Siddha and Homeopathy

Solution:

Concept:

- AYUSH groups Ayurveda, Yoga and Naturopathy, Unani, Siddha, and Homeopathy. It is an umbrella abbreviation for these systems, not a sentence formed by assigning a separate initial to every word.
- The letters correspond to Ayurveda (A), Yoga and Naturopathy (Y), Unani (U), Siddha (S), and

Homeopathy (H). Yoga and Naturopathy form the combined entry under Y.

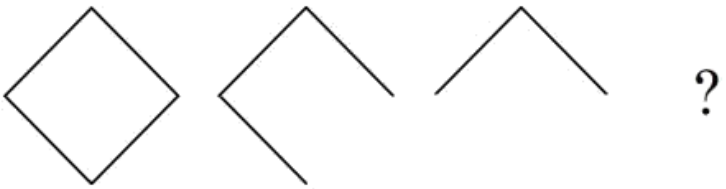
Step 1: Expand the grouped letters Begin with A for Ayurveda. Next comes Yoga and Naturopathy, followed by Unani. The last two entries are Siddha and Homeopathy. Reading the groups in order gives Ayurveda, Yoga and Naturopathy, Unani, Siddha and Homeopathy. The conjunctions join the systems; they do not introduce additional letters in the abbreviation.

Step 2: Compare the alternatives precisely Option (A) includes every system in the correct form. Option (B) uses Yogic instead of Yoga and omits Naturopathy. Option (C) begins with Acroveda, which is not Ayurveda, and also omits Naturopathy. Option (D) changes the Unani and Siddha portion into Unani Surgery and leaves out Siddha as the named system. It therefore fails even though several of its other words occur in the expansion.

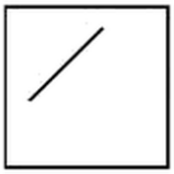
Step 3: Conclusion AYUSH expands to Ayurveda, Yoga and Naturopathy, Unani, Siddha and Homeopathy. Hence, the correct option is (A).

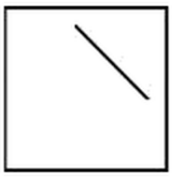
Quick Tip to Solve: Learn the expansion in five groups: Ayurveda; Yoga and Naturopathy; Unani; Siddha; Homeopathy. Keeping Yoga and Naturopathy together prevents the common omission.

57. Select the correct option from answer figure which will be best suitable to continue the same pattern as established by the problem figure. [4 marks]

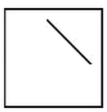


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- In a visual series, first separate the changing feature from the fixed geometry. Here the original figure is a diamond made from four sloping sides, and the series progressively removes one side at a time.
- A valid continuation must preserve the size, slope and position of every surviving side. The square around an answer choice is only its enclosing frame and does not belong to the changing diamond.

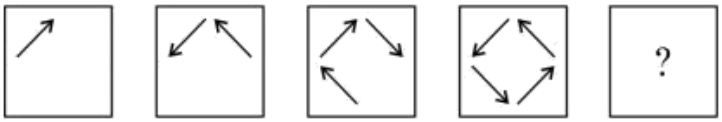
Step 1: Track the sides already removed The first problem figure is a complete diamond. In the second, the lower-right side has disappeared, leaving three sides. In the third, the lower-left side has also disappeared, leaving the two upper sides joined at the top vertex. Thus the side count progresses as four, three, two, and the deletion moves around the diamond in order.

Step 2: Delete the next side and compare the options The next deletion removes the upper-left side. Only the upper-right side remains: a segment descending to the right from the former top vertex. Option (C) shows this segment. Option (B) has the opposite, rising-to-the-right slope, which would leave the wrong upper side. Options (A) and (D) each retain two sides joined as a V or an inverted V; neither continues the decrease to one side.

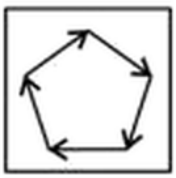
Step 3: Conclusion The continuation is the single upper-right side of the diamond, sloping downward to the right. Hence, the correct option is (C).

Quick Tip to Solve: Number the diamond sides around its perimeter, then cross out the deleted sides in order. Compare the slope of the one surviving segment, ignoring the answer frame.

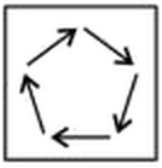
58. Choose the correct option from the answer figures which completes the question series. [4 marks]

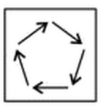


(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- A figure series may change several features together. Count the arrows, note their circulation around the centre, and check whether neighbouring arrows are separate or joined.
- The arrow count grows from one to two to three to four. The direction reverses at each step: the first and third circulate clockwise, while the second and fourth circulate anticlockwise.

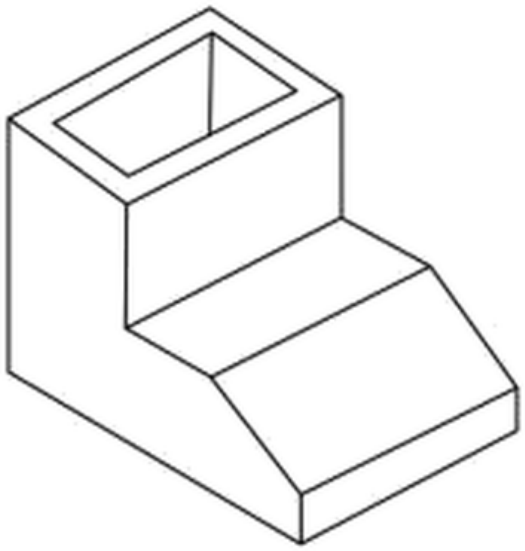
Step 1: Extend the number and direction rules The first square contains one arrow pointing towards the upper-right. The second has two arrows with the reverse circulation. The third arranges three arrows clockwise around a triangular route, and the fourth has four arrows anticlockwise around a diamond route. Continuing both changes requires five arrows arranged around a pentagonal route and directed clockwise.

Step 2: Keep the arrow style and eliminate the distractors Option (D) has five distinct arrows progressing clockwise around the pentagonal route: up the left, towards the top, down the right and back along the bottom. The small gaps between consecutive arrows preserve the separate-arrow style of the problem figures. Option (A) reverses the required circulation. Option (C) is an unarrowed pentagon and loses the directional information. Option (B) closes the arrows into an adjoining pentagonal outline instead of retaining the separated arrow arrangement. Option (D) matches count, direction and construction together.

Step 3: Conclusion The next figure contains five separate arrows circulating clockwise. Hence, the correct option is (D).

Quick Tip to Solve: Use three checks for an arrow series: how many arrows, which circulation, and whether the strokes touch. Matching only the pentagonal outline leaves a tempting distractor.

59. Identify the number of surfaces in the given 3D object. [4 marks]



(A) 12
(B) 13
(C) 10
(D) 9

Correct Answer: (B) 13

Solution:

Concept:

- A surface is one continuous planar face of the solid, including faces hidden from the observer and the walls of an opening. Count faces rather than visible line segments or corners.
- The body has a stepped-and-sloping outer profile and a rectangular hollow at the raised end. The outer body and the inner walls can be counted separately, then added without counting the open space itself as a face.

Step 1: Count the exterior faces systematically The two large side faces contribute two surfaces. Across the width, the outer profile gives seven more: the bottom, rear vertical face, upper horizontal rim, vertical drop to the intermediate level, intermediate horizontal tread, inclined ramp, and low front vertical face. Thus the exterior contributes $2 + 7 = 9$ surfaces. The horizontal rim is one connected planar surface, even though it surrounds the opening; its separate strips must not be counted again as different planes.

Step 2: Include the walls inside the opening The rectangular opening contributes four internal wall surfaces, one on each side. Its empty mouth is not an additional solid face. Add these walls to the exterior count:

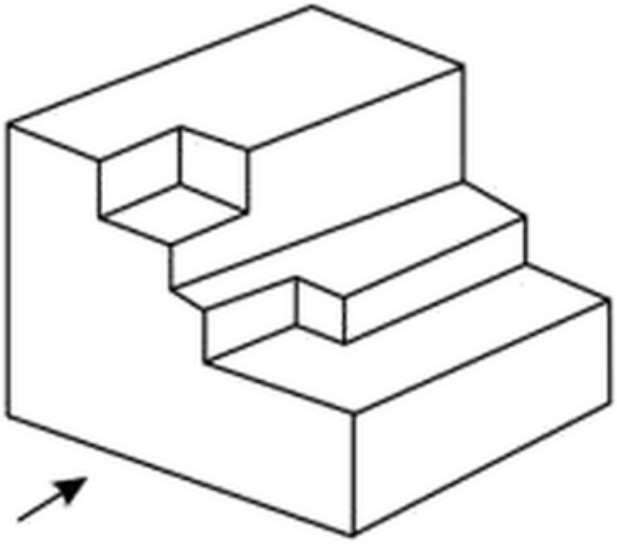
$$N = 9 + 4 = 13.$$

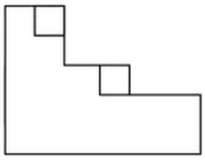
Option (D), 9, stops at the exterior and omits the inner walls. Option (C), 10, does not account for all four walls. Option (A), 12, still misses one surface. A systematic exterior-plus-interior count gives the complete total without relying only on the faces visible in the perspective view.

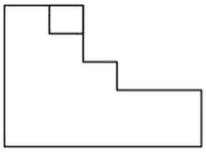
Step 3: Conclusion The object has 13 surfaces in total. Hence, the correct option is (B).

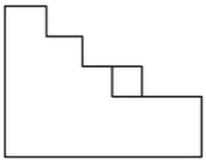
Quick Tip to Solve: For a hollow stepped solid, list the outer profile faces first, include both side faces, then add the inner walls. Do not count the empty opening as a face.

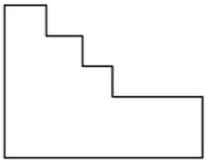
60. The question figure shows the 3D view of an object. Identify the view of the object looking in the direction of the arrow. [4 marks]

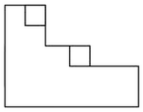


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- An orthographic view projects the object along the viewing arrow. Depth along that direction disappears, while height and horizontal position in the viewing plane remain.
- First obtain the outer silhouette of the stepped body, then project the edges of recesses. A notch that ends inside the body can appear as an internal corner rather than changing the entire outer silhouette.

Step 1: Project the main body along the arrow The arrow approaches the broad side of the staircase-shaped object from the lower left. Looking along it gives a side elevation with a high left portion, successive drops towards the right, and a common horizontal bottom. The receding depth edges of the perspective drawing collapse in this projection; they must not be copied as diagonal edges into the answer.

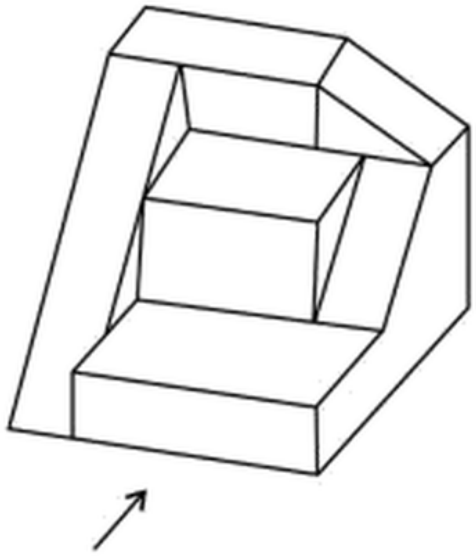
Step 2: Place both recesses in the elevation The upper recess projects as a square-cornered inset at

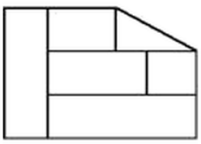
the upper step. The lower recess similarly produces an inset corner at the intermediate step. Both features must be retained, together with the stepped outer boundary. Option (A) shows these two internal corners in the correct positions. Option (B) keeps only the upper one and misses the lower recess. Option (C) keeps only the lower one and loses the upper recess. Option (D) shows the plain staircase boundary without either internal feature. Thus silhouette alone does not decide the answer; the two recessed regions distinguish it.

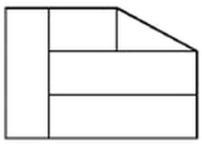
Step 3: Conclusion The required view is the stepped elevation with both internal recess outlines. Hence, the correct option is (A).

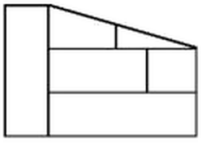
Quick Tip to Solve: In a directional-view question, match the silhouette first and then count the projected recesses. Here the two internal corners distinguish the correct view from the plain stair profile.

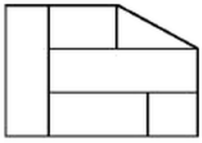
61. The 3D figure shows the view of an object. Identify the correct view looking in the direction of arrow, amongst the answer figures. [4 marks]

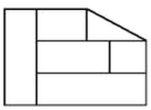


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- An orthographic view is obtained by looking parallel to the arrow: depth in that direction collapses, while horizontal position and height remain.
- Visible changes of surface produce edges in the projection. A line belonging to an intermediate raised block must not be transferred to the lower front step.

Step 1: Establish the outline from the viewing direction Look from the lower-left arrow toward the object. The long member at the left projects as a tall, narrow rectangular strip. Across the upper part, the rear member remains horizontal for most of its width and then descends toward the right-hand outer corner. Therefore the upper silhouette consists of a horizontal segment followed by a sloping segment. It is not one continuous slope across the whole width.

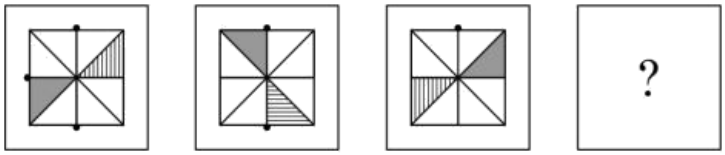
Step 2: Project the steps and the intermediate block The broad front step forms the lowest horizontal band. Its visible front face has no internal vertical division. The intermediate raised block lies above this band, and its right boundary is visible as a vertical edge within the middle band. The rear upper member also has an internal vertical edge where its horizontal upper portion meets the inclined right portion. Retain these two edges at their respective heights; do not extend either edge into the lower band.

Step 3: Compare all four answer figures Figure (A) contains the tall left strip, the horizontal-then-sloping top, the upper internal division and the vertical division in the middle band, while leaving the lowest band uninterrupted. Figure (B) omits the visible middle-band division. Figure (C) replaces the upper horizontal portion by a continuous slope. Figure (D) puts the extra division in the lowest band instead of the middle band. These are structural differences, so a similar outside outline alone is insufficient.

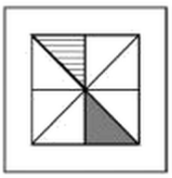
Step 4: Conclusion The view retaining the correct upper outline and the internal edge of the intermediate block is figure (A). Hence, the correct option is (A).

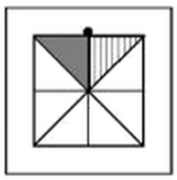
Quick Tip to Solve: Check the outline first, then locate each internal vertical edge by its height: the extra edge belongs to the middle step, not the bottom band.

62. **Choose the correct option from the answer figures which completes the sequence.** [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- In a figure sequence, track the filled sector, hatched sector and boundary dots separately. A valid continuation must satisfy every changing feature.
- The triangular sectors here turn through a quarter-turn clockwise at each stage. This is a rotation about the square centre, not a reflection.

Step 1: Follow the solid grey sector In the first figure the solid grey triangle is in the lower-left part of the square, adjoining the left midpoint. In the second it has moved to the upper-left part, adjoining the top midpoint. In the third it is in the upper-right part, adjoining the right midpoint. Each move is a clockwise rotation through a right angle about the centre. Applying the same move once more places it in the lower-right part, adjoining the bottom midpoint.

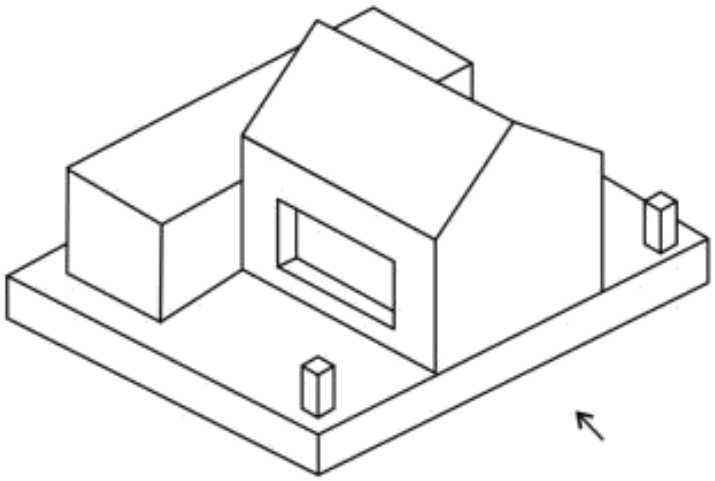
Step 2: Follow the hatched triangle and the dots The hatched triangular region also advances through a clockwise quarter-turn: from the upper-right sector to the lower-right sector and then to the lower-left sector. Its next position is therefore the upper-left sector. The parallel hatch lines rotate with the region, so the fourth pattern has horizontal hatching there. The midpoint dots decrease from three in the first figure to two in the second and one in the third. The fourth figure consequently has no remaining midpoint dot.

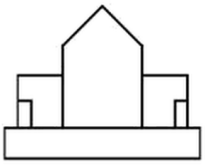
Step 3: Use all features to select the continuation Figure (B) has the grey lower-right triangle, the horizontally hatched upper-left triangle, and no dot. Figure (A) places the grey triangle in the upper-right region and retains a bottom dot. Figure (C) puts the grey triangle in the upper-left region, hatches the upper-right region and retains a top dot. Figure (D) repeats the earlier grey lower-left and hatched upper-right arrangement instead of advancing the sequence. Only (B) meets the rotation and dot rules together.

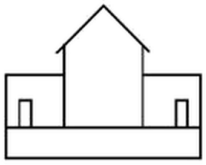
Step 4: Conclusion The next pattern has grey shading at the lower right and horizontal hatching at the upper left, with no midpoint dot. Hence, the correct option is (B).

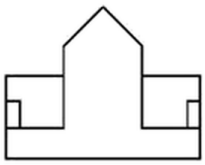
Quick Tip to Solve: Track one shaded triangle through the first three frames, then verify the other shading and the disappearing dots before choosing the answer.

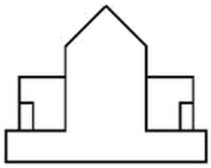
63. The question figure shows the 3D view of an object, identify the correct view looking in the direction of the arrow? [4 marks]

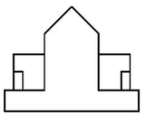


(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- The arrow fixes the viewing direction. In an end elevation, the pitched roof appears as a triangular gable above the central wall.
- A projection preserves visible boundaries of the particular object. Do not introduce a dividing line across a face that is continuous in that view.

Step 1: Determine which face is seen The arrow points toward the end of the building rather than toward the long wall containing the rectangular opening. Consequently the required central silhouette

is a tall gable: two vertical sides joined by the two slopes of the roof. The window on the long side does not appear on this end face. The lower adjoining parts project on either side of the central gable.

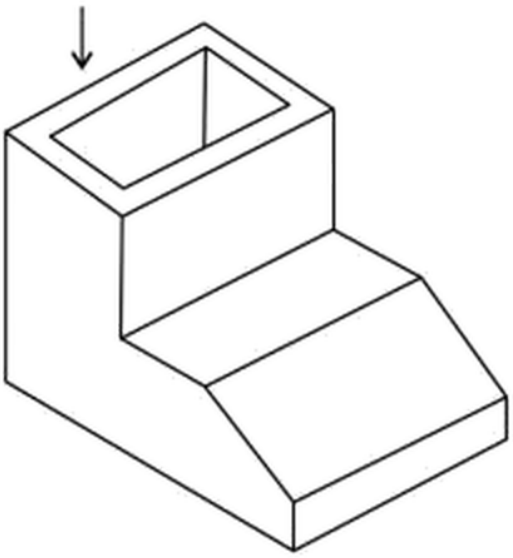
Step 2: Place the side blocks, posts and base The low blocks are set in from the outer ends of the platform, leaving a small projection of the base at both ends. The two small upright posts project near the outer edges of the lower side regions. The central end face continues down to the bottom outline of the platform without a separate horizontal line drawn across its width. Thus the correct figure must preserve both the small exposed base ends and the uninterrupted central lower face.

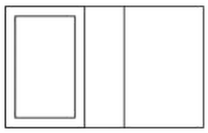
Step 3: Eliminate the competing outlines Figure (A) introduces a full horizontal line separating the entire base strip from the central face, which is absent in the required projection. Figure (B) adds projecting roof eaves and changes the placement of the low blocks and posts. Figure (C) makes the low side portions terminate flush with the outer base ends, losing the small projecting shoulders. Figure (D) retains the gabled central outline, the inset low side portions, the two short posts and the small platform extensions at the outer ends. Its central lower face is not crossed by an extra line.

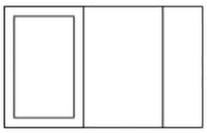
Step 4: Conclusion The end elevation with the correct gable, inset side portions and projecting platform ends is figure (D). Hence, the correct option is (D).

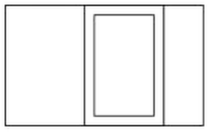
Quick Tip to Solve: After finding the gable outline, compare the tiny platform projections at both ends and check whether an extra line crosses the central lower face.

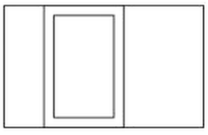
64. The question figure shows the 3D view of an object. Identify the top view of the object in the direction of the arrow. [4 marks]

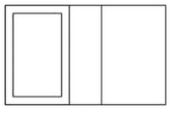


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- A top view looks vertically down on the object. It records the horizontal footprint and boundaries between visible upper surfaces; vertical height itself is suppressed.
- A rectangular recess is shown by its inner rectangular rim in plan. Vertical walls are not drawn as extra broad strips merely because they are tall.

Step 1: Locate the hollow upper part in plan The rear, taller portion has a rectangular opening surrounded by a rim. In the top view this gives an outer rectangular region containing a smaller rectangle. Follow the orientation used in the answer figures: this hollow portion occupies the left-hand end of the plan. It cannot move to the middle or to the right simply because the side faces look large in the pictorial view.

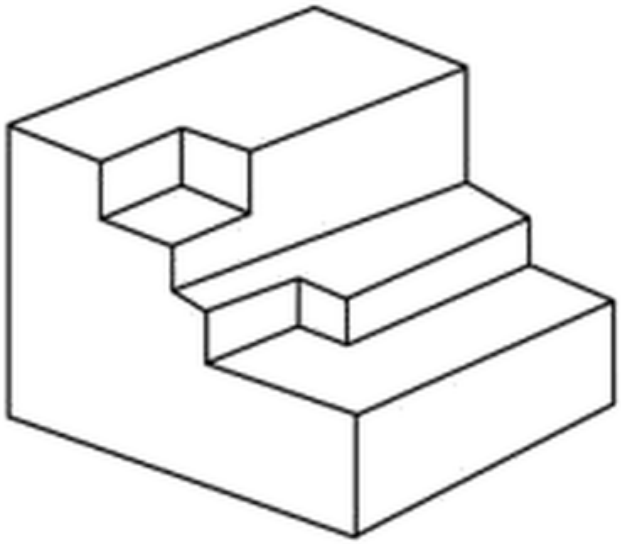
Step 2: Project the ledge and the inclined face In front of the hollow portion there is a narrow level ledge, followed by the broader inclined upper face descending toward the front edge. Both have the same overall transverse span as the body, so the outer plan stays rectangular. Their junctions appear as transverse straight lines. The sloping surface still occupies horizontal area in plan; its drop in height does not create an additional narrow vertical-face band at the far end.

Step 3: Check the positions and widths in the options Figure (A) places the rectangular hollow region at the left, a narrow ledge next to it and the broad sloping-face projection toward the right. Figure (B) changes that order of widths by making the middle region broad and the far-right region narrow, so it does not represent the given ledge and slope. Figures (C) and (D) put the recessed rectangular region in the middle rather than at the end. Matching the outer rectangular outline alone is therefore inadequate; the location of the opening and the two surface boundaries decide the answer.

Step 4: Conclusion The plan with the hollow region at the left, followed by the narrow ledge and broad inclined-face projection, is figure (A). Hence, the correct option is (A).

Quick Tip to Solve: Ignore the vertical wall heights. Mark the recess first, then follow the upper surfaces from the hollow end toward the sloping front face.

65. Identify the total number of surfaces in the given 3D object. [4 marks]



(A) 12

(B) 13

(C) 15

(D) 16

Correct Answer: (C) 15

Solution:

Concept:

- Count each distinct planar face of the solid, including the hidden underside, rear and opposite side. A nonrectangular but continuous flat face is still one surface.
- A notch may add walls and a floor, but a floor coplanar and continuous with an existing tread is not a separate surface. Count faces rather than lines or corners.

Step 1: Count the upward-facing surfaces There are three principal horizontal levels: the top tread, the middle tread and the lowest tread. The recess cut into the upper level has its own horizontal floor, giving a fourth upward-facing surface. At the lower notch, the exposed horizontal region joins the lowest tread in the same plane, so it is counted with that tread rather than as an additional floor. Thus the number of distinct upward-facing surfaces is four.

Step 2: Count the vertical faces by their function The three step levels provide three principal riser faces. The two recesses contribute two internal vertical walls each, giving four recess walls. Finally count the outside left side, the opposite outside side and the rear outside face, adding three. These groups are separate exposed planes, even when a notch makes the boundary of one face irregular. Therefore the vertical-face count is three plus four plus three, which equals ten. Include the two outer faces hidden by the pictorial viewing direction; visibility in the sketch is not the same as existence on the solid.

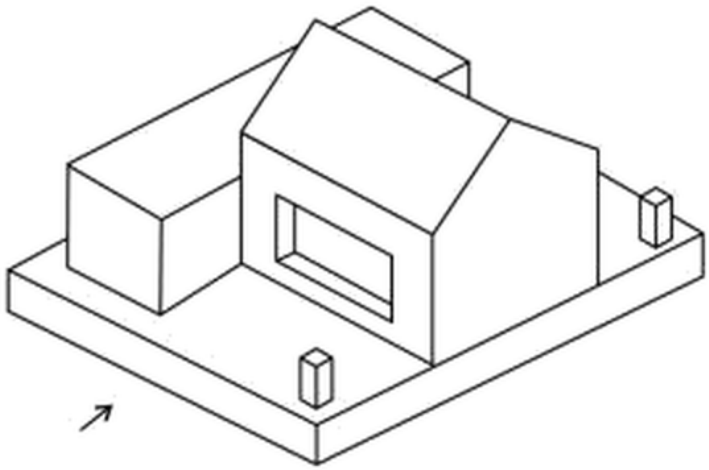
Step 3: Include the underside and check the total The object also has one bottom surface. The complete count is therefore four upward-facing surfaces, ten vertical surfaces and one underside, a

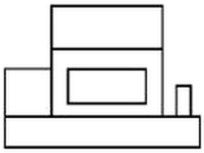
total of fifteen. A count of twelve omits hidden outside faces; thirteen still omits part of the complete boundary. Sixteen overcounts the lower recess floor as a new surface even though it is continuous with the lowest tread. The count is of planar faces, so neither every drawn polygon nor every straight edge should be counted independently.

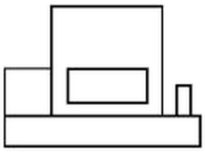
Step 4: Conclusion The solid has $4 + 10 + 1 = 15$ distinct planar surfaces. Hence, the correct option is (C).

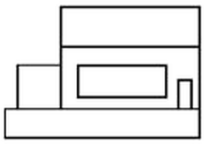
Quick Tip to Solve: Group the faces into horizontal tops, vertical walls and the bottom; treat the lower notch floor as part of the continuous lowest tread.

66. The question figure shows the 3D view of an object. Identify the correct view looking in the direction of the arrow. [4 marks]

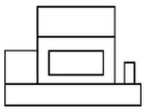


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- The viewing arrow determines which elevation is required. Looking toward the long wall makes that wall and its rectangular opening visible.
- In an orthographic elevation, the pitched roof above a long side appears as a rectangular band,

bounded below by the eaves line; its depth is suppressed.

Step 1: Identify the visible wall and roof The lower-left arrow points toward the long face containing the rectangular opening. The required elevation must therefore contain that opening. The roof is seen from the side: its ridge and eaves project as horizontal lines, with a rectangular roof band above the main wall. A gabled triangular outline would correspond to looking from the other direction and is not appropriate here.

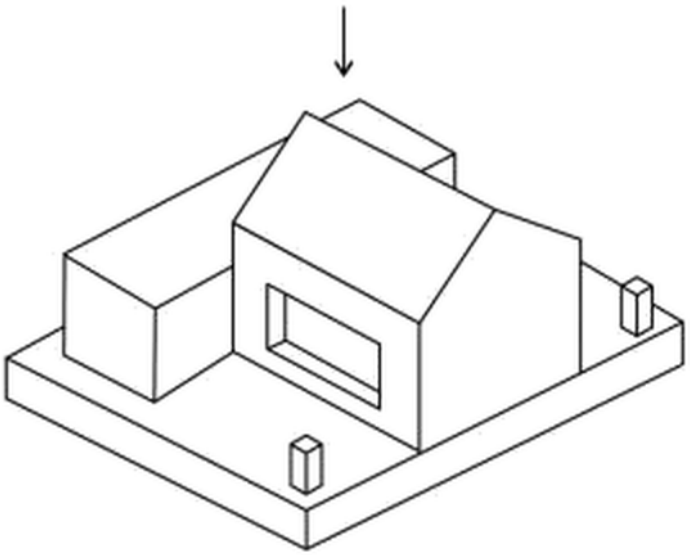
Step 2: Project the surrounding lower elements The low rectangular block occupies the left-hand side and rises partway up the central wall. At the right, the small upright post appears above the platform; it is lower than the main wall and separated from it. The platform forms a continuous horizontal base strip. Retain the relative heights of the block and post rather than enlarging either to match the wall. The opening stays within the main wall below the eaves line.

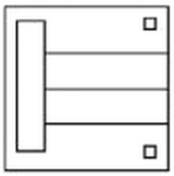
Step 3: Compare the distinguishing details Figure (A) shows the left low block, the roof band with its horizontal eaves line, the rectangular opening below it and the isolated short right-hand post on the platform. Figure (B) omits the horizontal division between the roof band and wall, making the whole central height one undivided rectangle. Figure (C) changes the roof-band and wall proportions and places the right post against the higher wall region. Figure (D) leaves out the rectangular opening entirely. Consequently only figure (A) reproduces the required visible details and their vertical positions together.

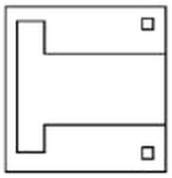
Step 4: Conclusion The long-side elevation containing the opening and the separate roof band is figure (A). Hence, the correct option is (A).

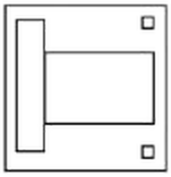
Quick Tip to Solve: Use the rectangular opening to identify the viewing side, then check the eaves line and the height of the small post against the answer figures.

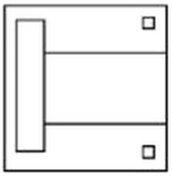
67. In the question figure it is a 3D view of an object. Identify the correct top view among the given options. [4 marks]

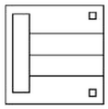


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- A top view is the perpendicular projection of the object onto a horizontal plane. It preserves the footprint and every visible change of roof plane, but suppresses vertical heights.
- A pitched roof has two sloping faces meeting at a ridge. The ridge remains a visible line in plan, whereas a window on a vertical wall does not appear as a window in the top view.

Step 1: Trace the footprint and the separate objects Look vertically down along the indicated arrow. The outer base gives a rectangular boundary. The long flat block at the left is represented by a narrow rectangle parallel to the left boundary. The two small upright posts project to small squares near the upper-right and lower-right corners of the base. Their height is irrelevant to the plan; both must nevertheless remain visible as separate footprints.

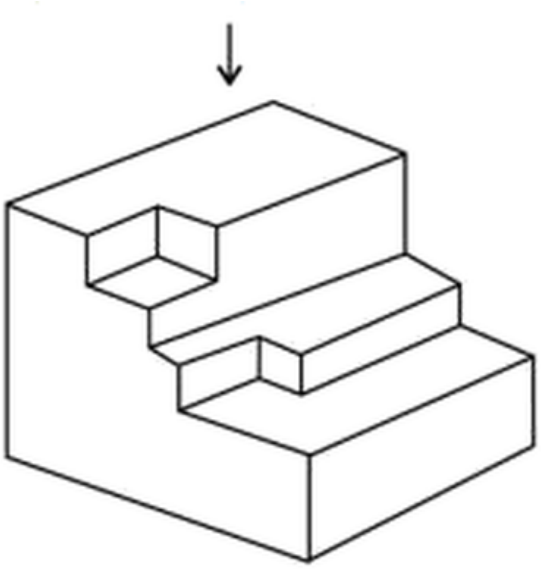
Step 2: Project the pitched roof The central house projects as a broad rectangular roof adjoining the long block. Both eaves and the ridge of the pitched roof must be retained. Therefore three parallel transverse roof lines occur across the central portion: the two outer limits and the ridge between them. The rectangular window is on the vertical front wall, so it does not create an additional rectangular opening in the plan.

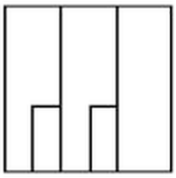
Step 3: Eliminate the other projections Option (A) contains the narrow left block, both post squares and the three required roof lines. Option (B) loses the roof-plane separation and merges the footprint into an unsuitable outline. Option (C) represents the central part as a closed inner rectangle rather than the roof extending to its indicated edge. Option (D) has the roof limits but omits the intervening ridge line. Thus only (A) preserves all the visible roof edges.

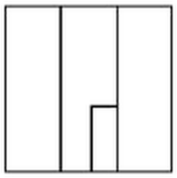
Step 4: Conclusion The correct top view is the figure with the roof ridge, both roof limits, the long left block and the two post squares. Hence, the correct option is (A).

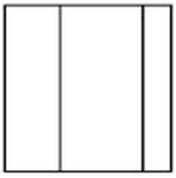
Quick Tip to Solve: For a pitched-roof top view, check the ridge before comparing the outer outline; vertical wall windows disappear in plan.

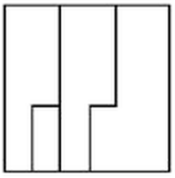
68. The question figure shows the 3D view of an object. Identify the most appropriate top view of the object from the given direction of the arrow. [4 marks]

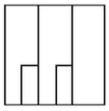


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- An orthographic top view shows horizontal surfaces and the boundaries separating their different levels. Heights are eliminated, but the plan positions of the level changes are retained.
- A notch cut into the edge of a step produces a short return in the corresponding boundary. Project each notch separately instead of treating the whole object as an ordinary staircase.

Step 1: Separate the three horizontal levels The object has a high rear platform, an intermediate step and a low front platform. Looking vertically from the arrow, their combined outer footprint is rectangular. The two transitions between levels give two principal lines inside this rectangle. These must be placed in the order of the levels, without adding perspective slants or drawing the vertical riser heights as widths.

Step 2: Retain the two recessed portions The high level has a rectangular recess near its left/front

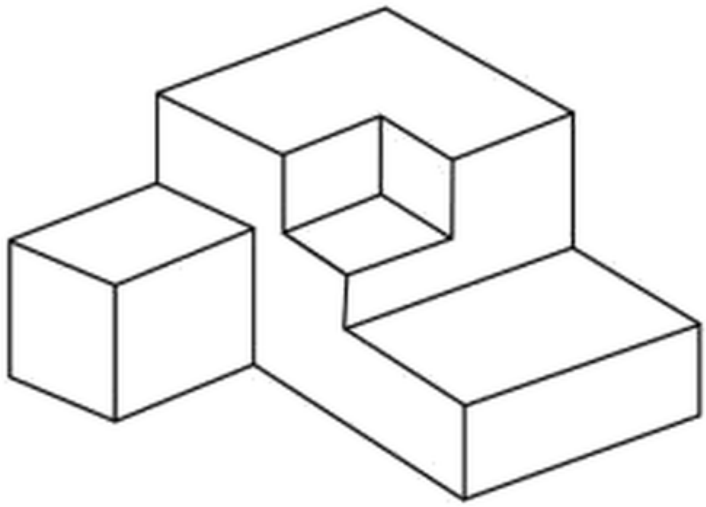
edge. Its projection produces a short return in the first internal division. The intermediate level has another rectangular recess near the same end of the next division. It contributes a second, separate short return. A top view therefore needs both notched divisions; neither recess disappears simply because the view is taken from above.

Step 3: Compare the internal lines in the choices Option (A) retains the rectangular outer contour, both main level divisions and both short rectangular returns. Option (B) retains only one of the two returns, so it misses one recess. Option (C) shows straight divisions with neither return, which would fit an unnotched staircase. Option (D) makes the second division turn into the wrong continuing boundary rather than preserving the straight division with its local return. It does not match the stepped ledge arrangement shown.

Step 4: Conclusion The complete top projection has two principal divisions with a local rectangular return at each of the two recesses. Hence, the correct option is (A).

Quick Tip to Solve: Project the highest step first, then add each lower step boundary and its own notch; a plain rectangular outline alone cannot distinguish these options.

69. Identify the total number of surfaces in the given object. [4 marks]



(A) 13

(B) 14

(C) 11

(D) 10

Correct Answer: (B) 14

Solution:

Concept:

- A surface count includes the hidden exterior faces as well as the faces visible in the perspective view. Internal contact faces between joined portions are not exterior surfaces.

- Count each continuous planar face once, even if its boundary is concave. In particular, a bend in the outline of a face does not necessarily make an additional face.

Step 1: Count the horizontal surfaces There are four visible upward-facing surfaces: the highest notched top, the floor of the small recess, the top of the left projecting block and the broad lower platform at the front. Each is a distinct planar patch at its own location or level. The underside supplies one further horizontal exterior surface. The number of horizontal surfaces is therefore $4 + 1 = 5$.

Step 2: Count the visible upright surfaces There are seven visible vertical surfaces. The left projection contributes its two exposed upright sides. The long irregular upright side adjoining that projection and continuing towards the lower platform is one continuous planar surface, not one face for every bend in its outline. The small recess contributes two inner upright walls. The broad upright face above the front platform contributes another surface, and the outer upright front edge of the low platform contributes the last. This gives $2 + 1 + 2 + 1 + 1 = 7$ visible vertical surfaces.

Step 3: Include the rear sides and total the count Two further exterior upright surfaces face away from the observer and are hidden in this view. They close the rear and far side of the object; do not count the buried junctions between its joined portions. Thus there are $7 + 2 = 9$ vertical surfaces altogether. Combining the horizontal and vertical groups gives

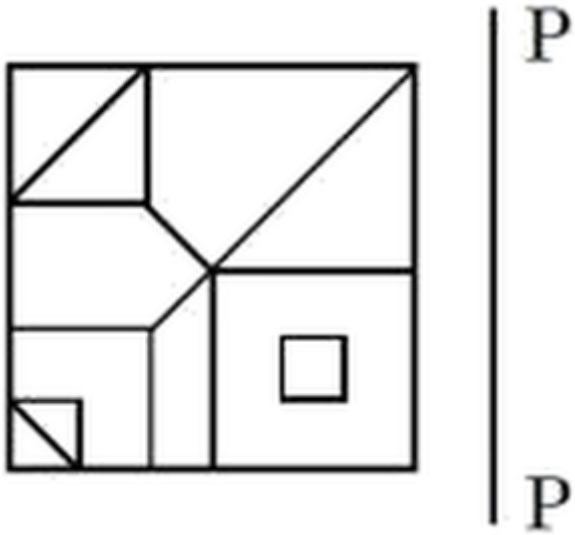
$$N = 5 + 9 = 14.$$

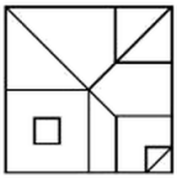
The counts 13, 11 and 10 omit exterior surfaces; in particular, 11 counts only the visible patches rather than the complete solid.

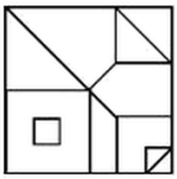
Step 4: Conclusion The object has 14 exterior surfaces, including the underside and the two hidden upright surfaces. Hence, the correct option is (B).

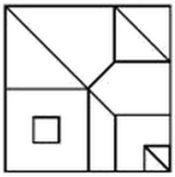
Quick Tip to Solve: Separate upward, downward and upright faces, then inspect the rear: counting only regions visible in the sketch misses hidden exterior surfaces.

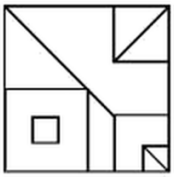
70. Identify the mirror image of the question figure, amongst the answer figures with respect to P-P. [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Reflection in the vertical line P–P reverses left and right while preserving the vertical level of every point. It is not a rotation or a reflection in a horizontal line.
- The slope of each oblique segment reverses under a left–right reflection. Internal junctions, small corner triangles and isolated shapes must all transform together.

Step 1: Reflect the easily located square and corner triangle The small isolated square is in the lower-right portion of the question figure. Its reflection must therefore be in the lower-left portion at the same height. The tiny triangle in the lower-left corner must move to the lower-right corner. Its diagonal descends from left to right in the original, so after reflection it ascends from left to right. This last orientation is a useful check because several choices place the small corner region correctly but give its diagonal the wrong slope.

Step 2: Reflect the upper triangular subdivision The upper-left corner subdivision moves to the upper-right. Its original diagonal rises from left to right; the reflected diagonal must fall from left to right, from the upper-left corner of that small subdivision to its lower-right corner. The long diagonal running from the centre towards the upper-right outer corner also reverses and runs towards the upper-left outer corner. All the connecting horizontal and vertical segments must still meet the central junction.

Step 3: Check all the answer figures Option (B) has the small square at lower-left, the correctly sloped tiny triangle at lower-right and the correctly reflected upper-right subdivision. Option (A) leaves the upper-right small diagonal in the wrong orientation. Option (C) reverses the lower-right small diagonal incorrectly. Option (D) fails the corner-diagonal tests. Matching just the isolated square would not distinguish these errors; matching both corner triangles selects (B).

Step 4: Conclusion The complete left–right reflection in P–P is answer figure (B). Hence, the correct option is (B).

Quick Tip to Solve: After moving the isolated square to the other side, check the slopes of both tiny corner triangles; their heights stay fixed but their slopes reverse.

71. Identify the correct mirror image of the question image with respect to P-P. [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- A vertical mirror reverses the order of a horizontal row and also reverses the shape of every individual character. Merely writing the characters in reverse order is insufficient.
- Top and bottom are preserved by this reflection. A tall stroke that lies on the left side of a character must lie on its right side in the mirror image.

Step 1: Reverse the order of the five characters The original sequence is J, P, E, G, followed by the

drawn numeral 4. Its mirror sequence begins with the reflected 4, then the reflected G, reflected E, reflected P and reflected J. The long top bar and hook of J must be reversed, the bowl of P must lie on the opposite side of its stem, and E must point to the left. These checks eliminate figures that reverse the sequence but leave an individual letter unchanged.

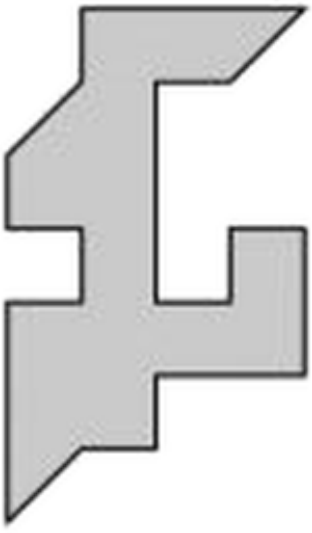
Step 2: Use the asymmetric strokes of 4 as a decisive check In the given figure, the numeral has a tall upright on its left, a shorter upright to its right, and a horizontal joining stroke. Reflection must put the tall upright on the right and the shorter upright on the left. Option (B) does exactly this. Option (D), although its letters are reversed correctly, keeps the tall upright of 4 on the left. It therefore repeats the original orientation of this asymmetric character instead of reflecting it.

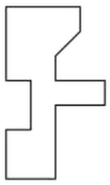
Step 3: Check the remaining letters against the alternatives In option (B), G opens towards the left, E has its vertical spine on the right, P has its bowl on the left, and the final J has its hook reversed. Option (A) does not reverse P correctly, and option (C) does not reverse E correctly. Thus the character-by-character geometric test selects (B), independently of any answer-key label. A vertical mirror cannot preserve the handedness of the asymmetric 4 while reversing the other characters.

Step 4: Conclusion The literal mirror image is option (B). The supplied key selects the fourth alternative, but that figure leaves the tall upright of the numeral 4 on its original left side; a vertical reflection moves that upright to the right, as in (B). Hence, the correct option is (B).

Quick Tip to Solve: For a mirror-image word, check one asymmetric stroke inside each character as well as the reversed character order; here the tall upright of 4 is decisive.

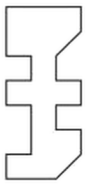
72. Which of the answer figures fit best to the left side of the given question figure? [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Two planar pieces fit along an edge only when their adjoining boundaries coincide without an overlap or gap. A projection on one piece must correspond to a recess of the other.
- Compare the matching boundary in order from top to bottom. The lengths and positions of the vertical runs, the diagonal shoulder and the rectangular returns must all agree.

Step 1: Trace the left boundary of the given piece Starting at the top of the shaded figure, the left boundary first descends vertically, then slopes down towards the left, then continues vertically. It next turns to the right to form the upper side of a rectangular indentation, descends through the indentation height and turns left again. Finally it continues straight down along the lower-left side. These features define the edge that the answer figure must meet; the other outside edges of the pieces are irrelevant to the actual join.

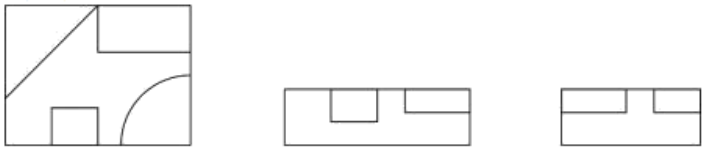
Step 2: Match the complementary right boundary Place the right edge of an answer figure against this left edge without changing the indicated upright orientation. Option (B) provides the same upper vertical run and diagonal shoulder. Its central rightward tab fills the given rectangular indentation, and the straight lower part continues along the remaining lower-left boundary. The tab has the correct depth: it fills the recess rather than projecting through the shaded piece. Every turn on one boundary consequently has a corresponding turn on the other.

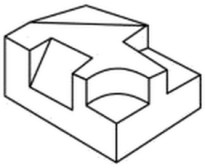
Step 3: Reject the other outlines Option (A) has an excessively long rightward central tab; aligning the upper shoulder would cause its tab to overlap the shaded material. Option (C) has the wrong upper and middle return arrangement, so its boundary cannot follow the complete left edge. Option (D) includes an additional sloping change near its lower end where the given piece has a straight boundary. It would leave a gap at the bottom after the upper features were aligned. Only (B) passes the full top-to-bottom boundary comparison.

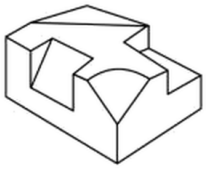
Step 4: Conclusion Answer figure (B) fits the left side of the given shaded figure without an overlap or gap. Hence, the correct option is (B).

Quick Tip to Solve: Trace the contact edge from top to bottom and compare each turn in sequence; a nearly matching silhouette can still fail at one tab or lower corner.

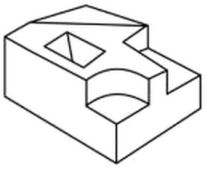
73. In the question figure, the top, front and right side view is given. Identify the most appropriate 3D view of the object from the answer figures. [4 marks]

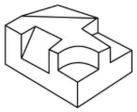


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- Orthographic views are perpendicular projections of the same solid. The top view fixes the plan positions of recesses; front and right-side views fix their heights and visible outlines.
- A candidate is valid only if it satisfies all three views simultaneously. An inclined planar cut has a straight plan edge, whereas a cylindrical recess has a curved plan edge.

Step 1: Read the distinguishing features in plan The upper-left part of the top view contains a diagonal straight boundary, so the corresponding corner must be cut by an inclined plane. Near the lower-left side there is an open rectangular recess, rather than a closed hole. The lower-right corner has a circular arc, indicating a curved recess. Retain these three features together when examining the solids.

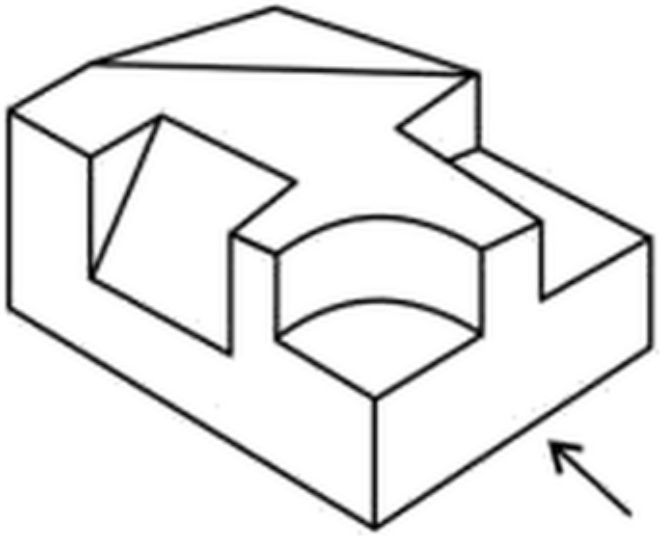
Step 2: Check the front and side projections The front elevation has a rectangular outer boundary, with an inset rectangular feature on the left and a higher horizontal ledge on the right. The right-side elevation has the corresponding stepped horizontal boundaries. Option (A) has the inclined rear cut, the open rectangular recess and the cylindrical corner recess in the required positions, and these project to the two given elevations.

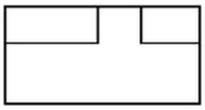
Step 3: Eliminate the other three solids Option (B) replaces the curved corner recess by an angular V-shaped cut, which would give straight plan edges instead of the arc. Option (C) rounds the rear boundary, contradicting the straight diagonal in the top view. Option (D) closes the rectangular recess into an internal opening, whereas the plan shows it meeting the outside edge.

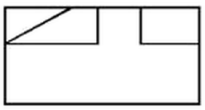
Step 4: Conclusion Only the solid in (A) reproduces all three supplied orthographic views. Hence, the correct option is (A).

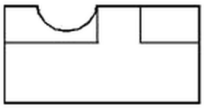
Quick Tip to Solve: Use the arc, the diagonal and the open rectangular notch as three separate checks; reject a solid as soon as one fails.

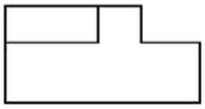
74. The question figure shows the 3D view of an object. Identify the most appropriate elevation (front view) looking in the direction of arrow. [4 marks]

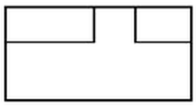


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- An elevation is an orthographic projection along the indicated viewing direction. Distances parallel to the viewing direction disappear; vertical heights remain unchanged.
- A curved horizontal edge in perspective need not remain curved in elevation. Project its endpoints and check whether the curve is being viewed edge-on.

Step 1: Fix the viewing direction and outside boundary The arrow points towards the object from the lower-right side. Look straight towards that face without rotating the object. The low base occupies the full width, while the higher material behind its recesses still reaches the common top height. The elevation therefore has a full rectangular outer boundary, rather than a downward step at its right end.

Step 2: Project the recess edges The curved corner cut is a vertical cylindrical recess. Its horizontal curved edges become horizontal segments in this elevation; it does not make a semicircular hole in the upper silhouette. The rear inclined cut recedes in the line of sight and does not create a

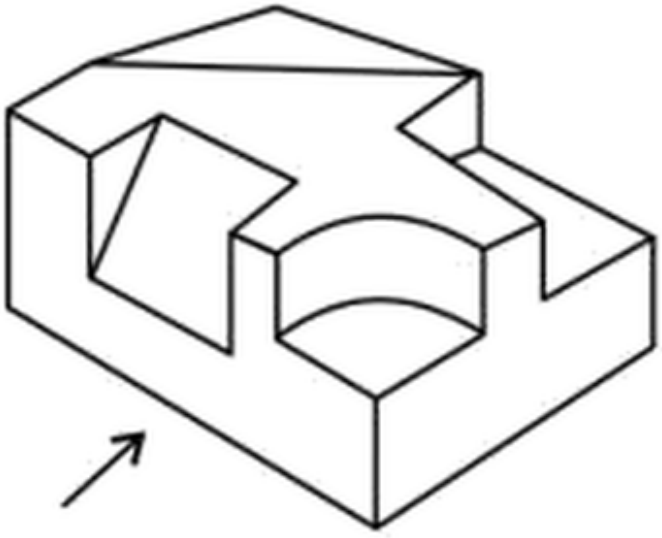
visible diagonal in this view. The resulting internal lines are two horizontal ledges joined to vertical boundaries, with the taller central portion retained.

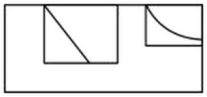
Step 3: Compare each answer figure Option (A) shows exactly the rectangular outline and the two appropriate horizontal ledges. Option (B) adds a diagonal in the left region, incorrectly retaining a depth feature. Option (C) turns the cylindrical recess into a semicircle in the top outline, confusing the plan curve with the elevation. Option (D) lowers the right-hand silhouette and discards the higher material behind that ledge.

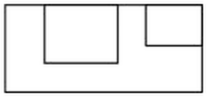
Step 4: Conclusion The required elevation is the rectangular profile with stepped internal ledges shown in (A). Hence, the correct option is (A).

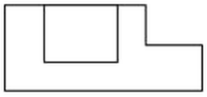
Quick Tip to Solve: Before copying a curve or slope into an elevation, check whether its direction is parallel to the viewing arrow.

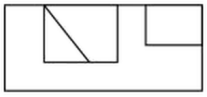
75. The question figure shows the 3D view of an object. Identify the most appropriate view of the object looking in the direction of the arrow. [4 marks]

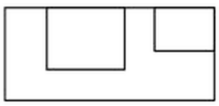


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- For an orthographic view, project every point along parallel sight lines in the arrow direction. Perspective sloping edges that run into the object can disappear in projection.
- The outer silhouette and the edges of visible recesses must both match. A horizontal circular arc on a vertical cylindrical cut projects to a straight horizontal edge in a side elevation.

Step 1: Look from the indicated side Here the arrow points from the lower-left towards the object, so the view is along the other horizontal axis from the preceding type of projection. Keep heights vertical and suppress distances along this axis. The higher rear material supplies the full rectangular outline, even where a lower ledge is visible in front of it.

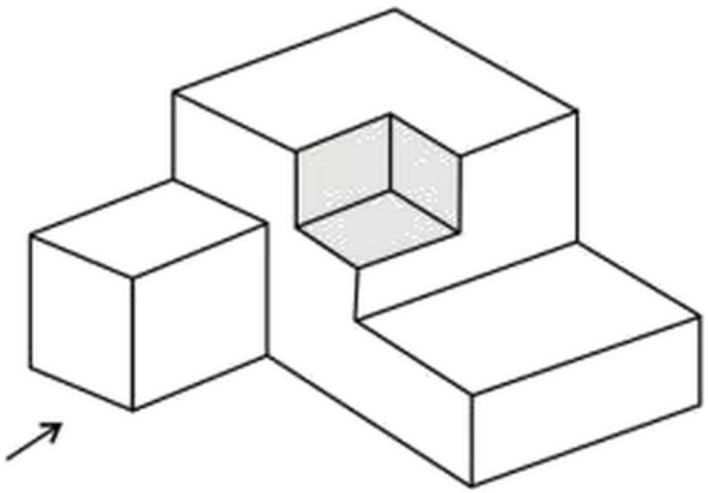
Step 2: Trace the two recesses The rectangular opening on the left gives a deep rectangular U-shaped internal boundary. Its sloping edge in the perspective drawing recedes along the viewing direction, so it does not remain a diagonal across the projected opening. At the right, the cylindrical recess gives a higher rectangular ledge in elevation. The curved top and bottom edges are seen edge-on and therefore are not drawn as curved arcs.

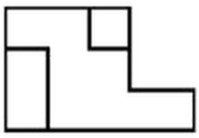
Step 3: Reject projections containing depth information Option (B) retains the correct full rectangle and the two rectangular inset boundaries. Option (A) preserves both an oblique line in the left opening and an arc on the right, neither of which belongs in this projection. Option (C) lowers the right outer edge and loses the higher rear portion. Option (D) has the correct right ledge but incorrectly retains the left diagonal.

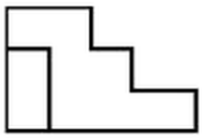
Step 4: Conclusion The projected view has straight rectangular recess boundaries as in (B). Hence, the correct option is (B).

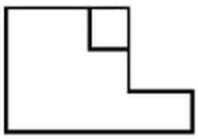
Quick Tip to Solve: Project the endpoints of a suspicious oblique edge first: if they fall on the same projected boundary, do not retain a diagonal.

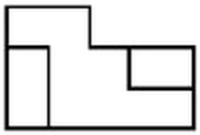
76. The question figure is the 3D view of an object. Identify the most appropriate front view of the object looking in the direction of the arrow. [4 marks]

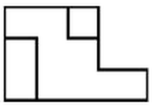


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- An orthographic front view preserves width and height while eliminating depth. A nearer block can hide the lower portion of material behind it without reducing the higher rear silhouette.
- To reconstruct a stepped elevation, first find the outside profile and then add visible internal edges, including the boundaries of recesses.

Step 1: Determine the outer stepped profile Follow the arrow towards the assembly from the lower-left. The rear upper block produces a high horizontal region on the left and middle of the elevation. On the right, only the low platform continues outward, so the outline drops to a low horizontal ledge before returning to the base. Preserve the common bottom line of the complete object.

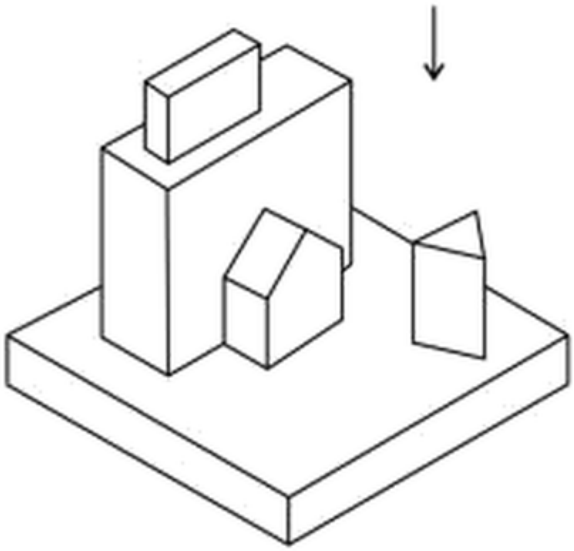
Step 2: Add the near block and upper recess The small near cube appears at the lower-left in this view. Its top is below the top of the rear block, and its right-hand vertical edge is visible, so a rectangular subdivision must remain at the left. The square recess cut from the upper block is visible towards the upper-right of the high region; its horizontal bottom edge and vertical left boundary make a small square subdivision there.

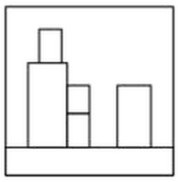
Step 3: Test the options against both features Option (A) contains the left lower rectangular subdivision, the upper-right square recess and the low rightward extension simultaneously. Option (B) turns the recess into an additional outside step. Option (C) omits the visible lower-left cube boundary. Option (D) places the rectangular subdivision on the low right-hand extension rather than preserving the upper recess in the high block.

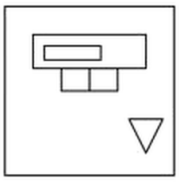
Step 4: Conclusion The front view with both the near-cube edge and the upper square recess is (A). Hence, the correct option is (A).

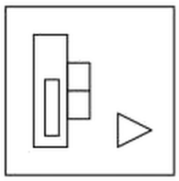
Quick Tip to Solve: After matching the outer staircase profile, check the small near cube and the recessed square separately; the silhouette alone is insufficient.

77. Question figure shows 3D view of an object. Identify the most appropriate top view of the given 3D object looking in the direction of an arrow, from the given answers figures. [4 marks]

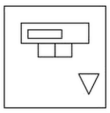


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- A top view is the projection vertically downward. Heights disappear, but the plan footprints of separate parts and the visible roof boundaries remain.
- A rectangular prism has a rectangular footprint; a gabled roof has a ridge in plan; a triangular prism has a triangular top outline when viewed vertically.

Step 1: Project the large block and its cap The broad upright main block becomes a long rectangle in plan, with the thinner raised cap appearing as a smaller narrow rectangle inside it. The cap is not seen standing above the main rectangle in a top view: that would be an elevation. Use the large block as the reference for positioning all the remaining parts.

Step 2: Locate the small roofed block and triangular part The small house-shaped attachment projects outside the broad side of the main rectangle. Its two sloping roof faces meet at a ridge, so its plan is a

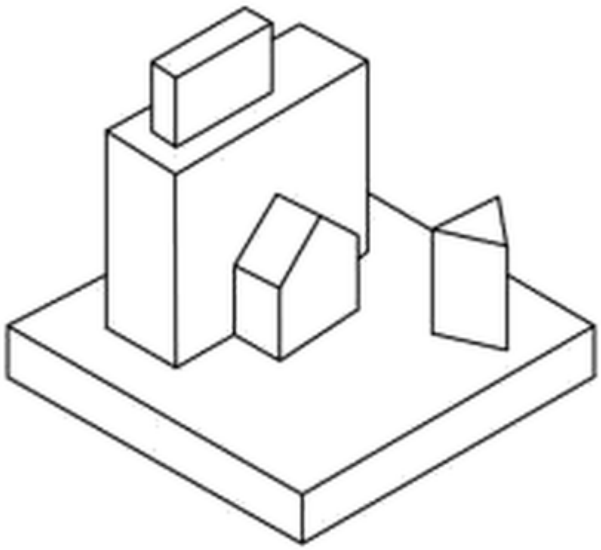
small rectangle divided by a central line. The separate triangular prism lies away from this attachment on the other part of the base. Its triangular plan must retain the orientation and relative placement shown by the three-dimensional arrangement.

Step 3: Compare the complete plans Option (B) shows the long main rectangle, its inset narrow cap, the divided rectangular roof attachment on the adjacent side and the separate triangular outline in the matching relative arrangement. Options (A) and (D) are elevations: they retain vertical heights, and (D) additionally substitutes a circular feature. Option (C) changes the orientation and placement of the parts relative to the base and the triangular component; merely rotating one feature does not reproduce the complete plan.

Step 4: Conclusion The overhead arrangement of all the components is shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: In a top view remove height first, then match the cap, roof ridge and triangular footprint as a single arrangement.

78. Identify the number of surfaces in the problem figure. [4 marks]



(A) 24

(B) 25

(C) 23

(D) 20

Correct Answer: (B) 25

Solution:

Concept:

- Count surfaces of the assembled solid, including its unseen outside faces. Contact faces between attached pieces are internal and are not counted as exposed surfaces.

- A single connected coplanar region counts as one surface even if its outline has a notch. Divide the object into simple components and count only their exposed faces.

Step 1: Count the base and large upright block The rectangular base contributes its bottom face, four side faces and its exposed upper face: six surfaces. The upper region is continuous around the attached objects, so it is one surface, not several separate rectangles. The large upright rectangular block contributes its top and four vertical faces: five more surfaces. Its bottom is attached to the base and is not exposed. The front face remains one connected planar region around the roofed attachment.

Step 2: Count the cap and the house-shaped attachment The narrow rectangular cap on top of the large block has one top and four vertical exposed faces, giving five surfaces; its underside is hidden at the joint. The small house-shaped prism has two sloping roof surfaces, two vertical side surfaces and one exposed pentagonal end surface, again five. Its rear end meets the large upright block and its bottom meets the base, so neither is an outside surface.

Step 3: Add the triangular prism and compare totals The separate triangular prism contributes one triangular top and three vertical side faces, giving four surfaces. Its bottom is attached to the base. Therefore the total is

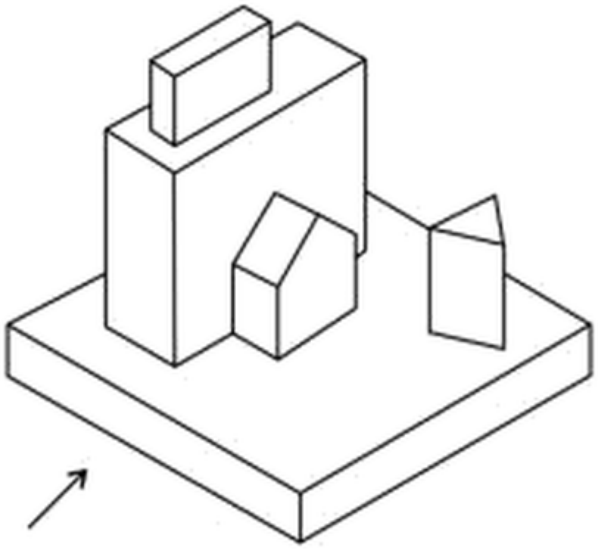
$$6 + 5 + 5 + 5 + 4 = 25.$$

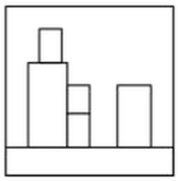
The totals 24, 23 and 20 omit one or more exposed surfaces, often an unseen rear face or the underside of the base; none equals the systematic component count.

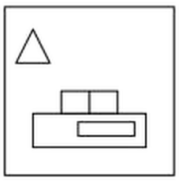
Step 4: Conclusion The assembled object has 25 exposed surfaces, including the unseen outer faces. Hence, the correct option is (B).

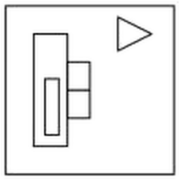
Quick Tip to Solve: Exclude joined contact faces, but include the base underside and the hidden rear outside faces; keep a component-by-component subtotal.

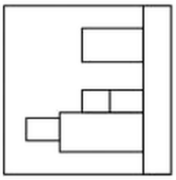
79. The question figure shows 3D view of an object. Identify the most appropriate elevation of the given 3D object, looking in the direction of an arrow, from the given answer figures. [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- An elevation is an orthographic projection onto a vertical plane perpendicular to the viewing direction. Depth in that direction disappears, while height remains vertical.
- The arrow fixes the observer's position. Compare the projected outline, relative heights and overlap of the individual solids, rather than their apparent sizes in the perspective drawing.

Step 1: Establish the viewing direction Place the observer at the lower-left end of the arrow and look towards the assembly. The rectangular base then appears as a low horizontal strip. The tall rectangular slab stands on the left of the view, with its narrow upright plate projecting above its top. These two features already require a view with a horizontal base and a tall left-hand rectangular component.

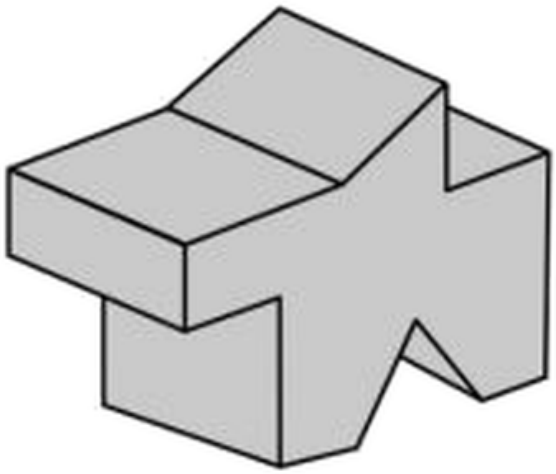
Step 2: Project the smaller components Looking along the arrow removes the depth of the house-shaped component next to the slab. Its end elevation appears beside the tall slab as a shorter rectangle, with the junction of the roof and wall represented by an internal horizontal division. The separate triangular-topped upright on the right also projects as a rectangular upright in this direction: its triangular top is not seen face-on. The relative positions and the height ordering agree with answer figure (A).

Step 3: Eliminate the other orientations Figure (B) shows the triangular component as a separate triangle and the tall slab as a long low rectangle, which corresponds to looking from another direction. Figure (C) similarly presents a plan-like arrangement, with the triangle to the side and the thin plate represented within the slab's rectangular outline. Figure (D) turns the base into a vertical strip and does not preserve the specified upright elevation. None has the horizontal base and left-hand height profile required by the arrow.

Step 4: Conclusion The elevation has a horizontal base, a tall slab with a plate above it on the left, a shorter divided component beside it and a separate rectangular upright on the right. Hence, the correct option is (A).

Quick Tip to Solve: First project the base and the tallest element along the arrow. These two silhouettes often eliminate the wrong views before the smaller roof details are considered.

80. Find out the total number of surfaces of the 3D object. [4 marks]



(A) 12

(B) 13

(C) 14

(D) 15

Correct Answer: (C) 14

Solution:

Concept:

- Count distinct planar boundary faces of the whole solid, including faces hidden from the observer. An edge is the boundary between faces, not an additional surface.
- For a solid formed by extending a polygonal profile through a uniform depth, each edge of the profile produces one connecting face. There are also two parallel end faces, so a profile with n edges has $n + 2$ surfaces.

Step 1: Identify the repeated end profile The object can be read as a stepped and notched profile extended through a constant depth. The large visible end face contains an upper change of slope, the right-hand step, a triangular notch at the bottom and the lower-left recess. The parallel end face at the opposite end has the same outline even though much of it is hidden. These two irregular end faces count as two surfaces, rather than being split into smaller faces by imaginary construction lines.

Step 2: Trace all the profile edges Start at the lower-left corner of the large end face and follow its boundary once. Pass along the short bottom ledge, up one side of the triangular notch, down its other side, along the right bottom ledge and up the outer right wall. Continue over the right upper ledge, up the inner upright of the top step, across the sloping roof and across the adjoining top ledge. Finally go down the left lip, along the underside of the projecting ledge and down the long recessed wall to the starting point. This gives twelve straight boundary segments. The notch and the recess both contribute real boundary segments and must not be omitted.

Step 3: Add the connecting and end surfaces Each of the twelve segments extends through the depth

to form one surface. Including the two end faces gives

$$N = 12 + 2 = 14.$$

The alternatives 12 and 13 undercount the complete boundary, for example by omitting one or both end faces. The alternative 15 adds a face that is not present, such as an imaginary division of a single planar end region. Count the actual surfaces of the solid, not only the polygons visible in the drawing.

Step 4: Conclusion The solid has twelve connecting surfaces and two end surfaces, making $\boxed{14}$ surfaces in total. Hence, the correct option is (C).

Quick Tip to Solve: For a constant-depth notched solid, trace one end profile without lifting your pencil, count its edges, and add two for the end faces.

Drawing Test (100 marks)

81. Draw the Sketch of the given image in detail. [50 marks]



Solution:

Concept:

- A convincing architectural sketch preserves the reference's silhouette, proportions, perspective and tonal hierarchy before adding ornament. The low viewpoint makes the tower appear tall and causes its vertical divisions to converge upwards.
- The reference is a monochrome close view of a tapering, richly carved masonry tower. Repeated projecting shafts, horizontal ornamental bands and deep grooves supply both its structure and its texture.

Step 1: Place the tower and establish the overall silhouette Use a portrait-oriented composition matching the reference. Lightly mark the top, the lower cropped boundary and the tower's central upward direction. Give the broad foreground base most of the lower width, then taper the outline

strongly towards the small upper portion. Preserve the close, upward-looking view instead of drawing an entire distant monument. Leave only a narrow surrounding background, as in the photograph, so that the mass of the architecture remains dominant.

Step 2: Construct the perspective and major projections Block in the principal cylindrical and angular vertical projections as long tapering masses. Follow each visible shaft upwards to keep it continuous through successive horizontal bands. Let the apparent widths of the shafts diminish towards the top. Establish the deep central and side recesses separately from the forward-facing rounded surfaces. Draw the horizontal divisions with the perspective and curvature visible in the image; avoid making every band an identical flat ruler line across the whole tower.

Step 3: Develop the carved bands and masonry detail Locate the broad ornamental belts near the lower foreground first, using light guide lines around the curved shafts. Within them, suggest the repeated relief lettering and decorative motifs through their actual visible rhythm and dark-light pattern. Do not replace them with invented readable inscriptions. Add the finer stone joints and smaller repeating details higher up with increasingly abbreviated marks. The foreground carving should be larger and clearer than the distant carving, so that detail reinforces the recession rather than flattening the surface.

Step 4: Build light, shadow and surface texture Reserve light paper for the illuminated ridges and place the darkest tones in the deep vertical grooves and under projecting carved bands. Use graduated graphite shading around each rounded shaft, moving from the lighter face through a middle tone to the shaded edge. Keep cast shadows beneath the raised ornament sharper than the gentle tonal changes on cylindrical masonry. Indicate weathered stone with restrained broken marks following the forms. Render the background as a simple, quieter midtone so that it does not compete with the architectural detail.

Step 5: Refine the focal area and check the drawing Strengthen selected lower contours and the principal carved bands while keeping the small upper details lighter and less distinct. Check the tower's taper, the continuity of every major shaft and the placement of the deepest recesses against the reference. Remove construction lines and accidental smudges. The final sketch should read clearly at a distance as the same upward-looking tower, and at close range should show controlled stone texture and carefully observed relief detail. Keep the tonal treatment monochrome to match the given image.

Step 6: Conclusion A suitable finished response is a detailed monochrome architectural sketch with a broad cropped foreground, strong upward taper, correctly aligned projecting shafts, repeated carved bands and convincing light and shadow. Accurate perspective and an orderly progression from large forms to fine ornament give the sketch both height and depth.

Quick Tip to Solve: Secure the silhouette and the continuous vertical shafts before drawing any carving. Ornamental detail cannot correct a tower whose overall taper or perspective is wrong.

82. Create an interesting sculpture, using basic 3D shapes of sports elements eg. different Balls, Bats etc. You can scale each object to any size of your choice, and use them as per your choice. Draw the effect of light and shadow on your composition. Use colours of your choice. [50 marks]

Solution:

Concept:

- A sculptural composition combines three-dimensional masses into a balanced whole. Spheres, elongated prisms and cylinders can remain recognisable as sports equipment while being arranged at deliberately varied scales.
- A single light source, consistent perspective, overlap and contact shadows make the assembly look solid. Colour should unify the forms while allowing a clear focal element.

Step 1: Choose a theme and organise the masses Develop the theme "Energy of Sport" as an upward-moving arrangement on a low circular plinth. Place one oversized basketball near the lower centre as the dominant spherical mass. Add a smaller football partly behind it on the left and a tennis ball in the front-right foreground. Let two enlarged cricket bats rise diagonally behind the central ball, their broad blades helping create a triangular overall silhouette. Keep the objects recognisable by preserving their characteristic shapes and a few essential surface markings, even though their sizes are deliberately changed.

Step 2: Make the sculpture balanced and connected Anchor the principal ball on the plinth and arrange the bats so that their lower ends meet the base behind it. Use the smaller balls to counterbalance the diagonals rather than distributing all elements equally. Overlap the football with the central sphere, and let one bat partially conceal the other, creating front, middle and rear layers. Keep small areas of open space between the raised handles so that the silhouette remains readable. Every heavy object should either rest on the base or connect convincingly to another supporting form; avoid an unintentionally floating assembly.

Step 3: Establish the perspective and volumes View the composition slightly from above at a three-quarter angle so that the plinth's top is an ellipse and its thickness is visible. Build the balls as spheres and the bat blades as tapered prisms with visible side thickness. Align each bat's handle with its own blade rather than using unrelated axes. Draw ball seams and panel boundaries as curves wrapping around the spheres. Reduce or hide the rear contours where objects overlap, and maintain a consistent viewpoint throughout the arrangement.

Step 4: Apply a controlled colour scheme Use warm orange for the dominant basketball, cream and charcoal panels for the football, and yellow-green for the tennis ball. Give the bats pale wood tones with a small matching orange accent near the handles to repeat the focal colour. Keep the plinth a subdued grey-blue so that it supports the sports forms without competing with them. Preserve the local colour in the lit areas, then darken the same colours in shadow instead of outlining every object heavily in black. Use the brightest contrast around the dominant central sphere.

Step 5: Render light and cast shadows consistently Set the light source above and to the left. Leave highlights on the upper-left parts of the balls, blend through midtones and darken their lower-right turning surfaces. Shade the right-facing planes of the bats more strongly than their illuminated broad faces. Put compact dark contact shadows where the balls and bats meet the base. Cast the assembly's shadows towards the lower right across the plinth, with overlapping forms interrupting them correctly.

A highlight, halftone, core shadow and small reflected-light edge will make each sphere read as a rounded mass.

Step 6: Refine the final composition Strengthen the foreground edges and focal surface markings, then soften less important rear details. Check that the triangular arrangement leads the eye upwards and that the small front ball balances the taller bats. Remove construction lines, keep the background simple and retain clean negative spaces around the upper forms. The completed work should communicate an intentional sculpture made from sports elements, rather than a flat collection of unrelated equipment symbols.

Step 7: Conclusion A successful response is a balanced, coloured three-dimensional sculpture composition with an oversized central ball, smaller supporting sports balls and rising bat forms. Consistent perspective, visible thickness, a restrained palette and upper-left lighting with lower-right cast shadows unify the assembly and make its volumes convincing.

Quick Tip to Solve: Decide the main silhouette and one light direction before adding seams or colour. Check contact shadows last: they are what make the sports forms appear supported and connected.