JEE Main 2024 Paper 1 (B.E./B.Tech) Jan 27 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 1 (Jan-Feb), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 27 January 2024, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}\,dx = a + b\sqrt{2} + c\sqrt{3}$**, where** a, b, c **are rational numbers, then** $2a + 3b - 4c$ **is equal to :** [4 marks]

(A) 10

(B) 7

(C) 4

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- Rationalise a denominator containing the sum of two square roots by multiplying by its conjugate. Here the difference of the radicands is the constant 2.
- The power rule gives $\int \sqrt{x+k}\,dx = \frac{2}{3}(x+k)^{3/2} + C$. Evaluate both endpoints before comparing coefficients.

Step 1: Rationalise the integrand Since $(\sqrt{x+3}+\sqrt{x+1})(\sqrt{x+3}-\sqrt{x+1}) = 2$, the integral becomes

$$I = \frac{1}{2}\int_0^1 \left(\sqrt{x+3}-\sqrt{x+1}\right)dx.$$

Both square roots are positive throughout the interval, so the rationalisation introduces no domain restriction or sign change.

Step 2: Evaluate and identify the coefficients Integrating the two powers separately gives

$$I = \frac{1}{3}\left[(x+3)^{3/2} - (x+1)^{3/2}\right]_0^1 = \frac{1}{3}\left(8 - 2\sqrt{2} - 3\sqrt{3} + 1\right).$$

Thus $I = 3 - \frac{2}{3}\sqrt{2} - \sqrt{3}$, so $a = 3$, $b = -\frac{2}{3}$, and $c = -1$. In particular, the lower endpoint contributes $-3\sqrt{3} + 1$; omitting that final positive term changes the answer.

Step 3: Form the requested combination Substitute these rational coefficients, retaining the negative signs:

$$2a + 3b - 4c = 2(3) + 3\left(-\frac{2}{3}\right) - 4(-1) = 6 - 2 + 4 = 8.$$

The other proposed values, $10, 7, 4$, do not equal this combination; 10 in particular would result from losing the contribution $3b = -2$.

Step 4: Conclusion The requested value is 8. Hence, the correct option is (D).

Quick Tip to Solve: When the two radicands differ by a constant, rationalise first; the resulting difference of square roots integrates directly.

2. If $S = \{z \in C : |z - i| = |z + i| = |z - 1|\}$, then, $n(S)$ is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) 0

Correct Answer: (A) 1

Solution:

Concept:

- For $z = x + iy$, the modulus $|z - z_0|$ is the distance between their points in the Argand plane. Equality of two distances defines their perpendicular bisector.
- Squared moduli can be equated safely because moduli are nonnegative. Solve both equalities simultaneously, rather than treating their loci as separate answers.

Step 1: Use equality of distances from the imaginary points Write $z = x + iy$, with $x, y \in \mathbb{R}$. The first equality gives

$$x^2 + (y - 1)^2 = x^2 + (y + 1)^2.$$

After cancellation, $-2y = 2y$, so $y = 0$. Thus every possible member of S lies on the real axis; this condition alone describes infinitely many points, but the third distance must also agree.

Step 2: Impose the distance from the real point Now use $|z - i| = |z - 1|$ with $y = 0$:

$$x^2 + 1 = (x - 1)^2 = x^2 - 2x + 1.$$

Therefore $x = 0$, and the only candidate is $z = 0$. Conversely, this candidate works because

$$|0 - i| = |0 + i| = |0 - 1| = 1.$$

This direct check proves both existence and uniqueness, rather than merely finding a necessary condition.

Step 3: Count the resulting set The simultaneous equations leave exactly one complex number, so $S = \{0\}$. The numeral 0 is the member of this singleton, not its cardinality. Thus option (D) confuses the point with the number of points, while (B) and (C) would require additional solutions that the two linear conditions exclude.

Step 4: Conclusion We obtain $S = \{0\}$, hence $n(S) = 1$. Hence, the correct option is (A).

Quick Tip to Solve: Equate squared moduli to turn each equidistance condition into a straight line; their common points give the set to be counted.

3. Let $S = \{1, 2, 3, \ldots, 10\}$. Suppose M is the set of all the subsets of S, then the relation $\mathrm{R} = \{(\mathrm{A}, \mathrm{B}) : \mathrm{A} \cap \mathrm{B} \neq \phi; \mathrm{A}, \mathrm{B} \in \mathrm{M}\}$ is : [4 marks]

(A) symmetric only

(B) reflexive only

(C) symmetric and reflexive only

(D) symmetric and transitive only

Correct Answer: (A) symmetric only

Solution:

Concept:

- A relation on M is reflexive only if (A, A) belongs to it for every $A \in M$, including the empty subset.
- Symmetry means $A\,R\,B \Rightarrow B\,R\,A$. Transitivity requires $A\,R\,B$ and $B\,R\,C$ to imply $A\,R\,C$ for every triple.

Step 1: Test symmetry using intersection The defining condition is $A \cap B \neq \varnothing$. Since intersection is commutative,

$$A \cap B = B \cap A,$$

any pair in R has its reversed pair in R. Thus the relation is symmetric. This argument applies to every pair of subsets, without needing to enumerate the 2^{10} members of M.

Step 2: Test reflexivity at the empty subset The power set contains $\varnothing$. Taking $A = \varnothing$ yields

$$A \cap A = \varnothing,$$

so $(\varnothing, \varnothing) \notin R$. Reflexivity fails, even though every nonempty subset is related to itself. The phrase all subsets is essential: it does not exclude the empty set.

Step 3: Disprove transitivity with a counterexample Choose $A = \{1\}$, $B = \{1, 2\}$, and $C = \{2\}$, all subsets of S. Then

$$A \cap B = \{1\}, \qquad B \cap C = \{2\}, \qquad A \cap C = \varnothing.$$

Thus $A\,R\,B$ and $B\,R\,C$ hold but $A\,R\,C$ fails. Options (B) and (C) incorrectly include reflexivity; option (D) incorrectly includes transitivity.

Step 4: Conclusion The relation is symmetric only. Hence, the correct option is (A).

Quick Tip to Solve: For relations on a power set, test the empty set first for reflexivity, then use two disjoint singletons and their union to test transitivity.

4. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2+y^2-4x-16y+64 = 0$ is d, then d^2 is equal to : [4 marks]

(A) 16

(B) 24

(C) 20

(D) 36

Correct Answer: (C) 20

Solution:

Concept:

- Completing squares in a circle equation identifies its centre. The distance here is measured from that centre to the parabola, not from the circumference.
- Every real point on $y^2 = 4x$ is $(t^2, 2t)$, with $t \in \mathbb{R}$. Minimising squared distance avoids an unnecessary square root.

Step 1: Find the fixed point and parameterise the parabola Complete squares in the circle equation:

$$(x-2)^2 + (y-8)^2 = 4.$$

Its centre is therefore $C = (2, 8)$. A general point on the parabola is $P = (t^2, 2t)$, because substituting gives $(2t)^2 = 4t^2$. This parameter covers its upper and lower branches, so a minimum over real t is the required global distance.

Step 2: Minimise the squared distance Let $F(t) = CP^2$. Expanding and differentiating gives

$$F(t) = (t^2-2)^2 + (2t-8)^2 = t^4 - 32t + 68,$$

$$F'(t) = 4t^3 - 32 = 4(t^3 - 8).$$

The only real stationary point is $t = 2$. Moreover, $F'(t) < 0$ for $t < 2$ and $F'(t) > 0$ for $t > 2$, so the distance decreases then increases. Together with $F(t) \to \infty$ as $|t| \to \infty$, this proves a global minimum.

Step 3: Evaluate the minimum and compare the choices At $t = 2$, the nearest point is $P = (4, 4)$. Hence

$$d^2 = (4-2)^2 + (4-8)^2 = 4 + 16 = 20.$$

The numbers $16, 24, 36$ are not this minimum. In particular, 16 counts only the squared vertical separation and omits the horizontal contribution 4. The circle radius is irrelevant because the fixed point is its centre.

Step 4: Conclusion The minimum squared distance is $d^2 = 20$. Hence, the correct option is (C).

Quick Tip to Solve: For distance from a point to a parabola, parameterise the curve and minimise squared distance; check the derivative sign to establish the global minimum.

5. If (a, b) be the orthocentre of the triangle whose vertices are $(1, 2), (2, 3)$ and $(3, 1)$, and $\mathrm{I}_1 = \int_a^b x \sin\left(4x - x^2\right) \mathrm{d}x, \mathrm{I}_2 = \int_a^b \sin\left(4x - x^2\right) \mathrm{d}x$, then $36\frac{\mathrm{I}_1}{\mathrm{I}_2}$ is equal to : [4 marks]

(A) 80

(B) 72

(C) 66

(D) 88

Correct Answer: (B) 72

Solution:

Concept:

- The orthocentre is the intersection of two altitudes. Each altitude is perpendicular to the opposite side, so its slope is the negative reciprocal of that side slope.
- If $f(L + U - x) = f(x)$, the substitution $x = L + U - u$ gives $\int_L^U x f(x)\, dx = \frac{L+U}{2} \int_L^U f(x)\, dx$.

Step 1: Locate the orthocentre Name the vertices $A = (1, 2)$, $B = (2, 3)$, $C = (3, 1)$. Side BC has slope -2, so the altitude through A has equation $y - 2 = (x - 1)/2$. Side AC has slope $-1/2$, so the altitude through B is $y - 3 = 2(x - 2)$. Solving gives

$$a = \frac{5}{3}, \qquad b = \frac{7}{3}, \qquad a + b = 4.$$

Step 2: Reflect the integral about the midpoint Put $f(x) = \sin(4x - x^2)$. Since

$$4(4 - x) - (4 - x)^2 = 4x - x^2,$$

we have $f(4 - x) = f(x)$. Substitute $x = 4 - u$ in I_1; the endpoints interchange because $4 - a = b$:

$$I_1 = \int_a^b (4 - u) f(u)\, du = 4I_2 - I_1.$$

Thus $2I_1 = 4I_2$. No elementary antiderivative of f is needed.

Step 3: Check the ratio is defined and evaluate it On $[5/3, 7/3]$, the argument $4x - x^2 = 4 - (x - 2)^2$ lies between $35/9$ and 4, wholly inside $(\pi, 2\pi)$. Its sine is negative throughout, so $I_2 \neq 0$. We may therefore divide to obtain

$$\frac{I_1}{I_2} = 2, \qquad 36\frac{I_1}{I_2} = 72.$$

The other values $80, 66, 88$ would give a weighted mean different from the symmetry midpoint 2, and are excluded.

Step 4: Conclusion The required value is 72. Hence, the correct option is (B).

Quick Tip to Solve: Before integrating a quadratic expression in a sine, check whether reflection about the midpoint leaves it unchanged; then pair the weighted integrals.

6. Let $x = x(\mathrm{t})$ and $y = y(\mathrm{t})$ be solutions of the differential equations $\frac{\mathrm{dx}}{\mathrm{dt}} + \mathrm{ax} = 0$ and $\frac{\mathrm{dy}}{\mathrm{dt}} + \mathrm{by} = 0$ respectively, $\mathrm{a}, \mathrm{b} \in$ R. Given that $x(0) = 2; y(0) = 1$ and $3y(1) = 2x(1)$, the value of t, for which $x(\mathrm{t}) = y(\mathrm{t})$, is : [4 marks]

(A) $\log_{\frac{2}{3}} 2$

(B) $\log_{\frac{4}{3}} 2$

(C) $\log_4 3$

(D) $\log_3 4$

Correct Answer: (B) $\log_{\frac{4}{3}} 2$

Solution:

Concept:

- The solution of $u\prime + ku = 0$, with a constant k, is $u(t) = u(0)e^{-kt}$. The initial condition fixes its multiplicative constant.
- Comparing the ratio $x(t)/y(t)$ eliminates the separate unknown decay constants: only their difference is needed to find the crossing time.

Step 1: Solve the two initial-value equations Separating variables, or using the exponential form directly, yields

$$x(t) = 2e^{-at}, \qquad y(t) = e^{-bt}.$$

These formulae satisfy both the differential equations and $x(0) = 2$, $y(0) = 1$. In particular, $y(t) > 0$ for every real t, so forming their ratio is valid.

Step 2: Extract the difference of the constants The given condition at $t = 1$ says

$$3e^{-b} = 4e^{-a}, \qquad e^{b-a} = \frac{3}{4}.$$

Consequently, for arbitrary t,

$$\frac{x(t)}{y(t)} = 2e^{(b-a)t} = 2\left(\frac{3}{4}\right)^t.$$

Notice the direction of the ratio: $3y(1) = 2x(1)$ implies $x(1)/y(1) = 3/2$, consistent with this formula.

Step 3: Find when the ratio becomes one Equality $x(t) = y(t)$ requires

$$2\left(\frac{3}{4}\right)^t = 1, \qquad \left(\frac{4}{3}\right)^t = 2.$$

Thus $t = \log_{4/3} 2$. The ratio is strictly decreasing from 2, and its value at $t = 1$ is still $3/2 > 1$, so the crossing must occur after 1. Option (A) is negative, option (C) is below 1, and option (D) does not solve the displayed exponential equation.

Step 4: Conclusion The equality occurs at $t = \log_{4/3} 2$. Hence, the correct option is (B).

Quick Tip to Solve: Use the ratio of two exponential solutions; a condition at one time fixes the base of that ratio without solving for both decay constants.

7. The distance, of the point $(7, -2, 11)$ from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$, is : [4 marks]

(A) 12

(B) 18

(C) 21

(D) 14

Correct Answer: (D) 14

Solution:

Concept:

- Distance to a line along a specified direction is obtained by intersecting a line through the given point parallel to that direction with the target line.
- If a displacement is $s\boldsymbol{v}$, its length is $|s|\|\boldsymbol{v}\|$. A zero denominator in a symmetric line equation means that coordinate is constant.

Step 1: Write the line through the given point Let $P = (7, -2, 11)$. The supplied directional line has direction vector $(2, -3, 6)$; its base point does not matter when specifying the direction. Thus the line through P in that direction is

$$Q(s) = (7 + 2s, -2 - 3s, 11 + 6s).$$

The target line has parametric form

$$(x, y, z) = (6 + r, 4, 8 + 3r).$$

In particular, the printed zero denominator means $y = 4$, not division by a numerical zero.

Step 2: Find and verify the intersection Equate the y-coordinates: $-2 - 3s = 4$, giving $s = -2$. The corresponding point is

$$Q = (3, 4, -1).$$

Its target-line parameter is $r = -3$, since $6 + r = 3$ and $8 + 3r = -1$. Thus both remaining coordinates also agree, confirming an actual intersection. The negative parameter merely means motion opposite to the chosen direction vector.

Step 3: Compute the length of the displacement The displacement is $Q - P = (-4, 6, -12) = -2(2, -3, 6)$. Hence

$$PQ = 2\sqrt{2^2 + (-3)^2 + 6^2} = 2\sqrt{49} = 14.$$

The alternatives $12, 18, 21$ do not equal this vector length; 12 is only the magnitude of the change in the z-coordinate. The perpendicular-distance formula would answer a different geometric demand because the direction has already been specified.

Step 4: Conclusion The distance along the specified direction is 14 units. Hence, the correct option is (D).

Quick Tip to Solve: Use the fixed coordinate of the target line to find the directional parameter quickly, then verify the other coordinates before measuring the displacement.

8. The length of the chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose mid point is $\left(1, \frac{2}{5}\right)$, is equal to : [4 marks]

(A) $\frac{\sqrt{1691}}{5}$

(B) $\frac{\sqrt{2009}}{5}$

(C) $\frac{\sqrt{1541}}{5}$

(D) $\frac{\sqrt{1741}}{5}$

Correct Answer: (A) $\frac{\sqrt{1691}}{5}$

Solution:

Concept:

- For a chord with midpoint M, its endpoints can be written $M + t\mathbf{d}$ and $M - t\mathbf{d}$. Subtracting their ellipse equations determines the direction $\mathbf{d}$.
- The chord length is twice the distance from its midpoint to either endpoint. Coordinates must be measured using ordinary Euclidean distance, even when the ellipse equation uses unequal denominators.

Step 1: Determine the direction of the chord Write the endpoints as $(1 + u, \frac{2}{5} + v)$ and $(1 - u, \frac{2}{5} - v)$. Each satisfies the ellipse equation. Subtracting those equations cancels all quadratic and constant terms and gives

$$\frac{4u}{25} + \frac{4(2/5)v}{16} = 0 \quad \Longrightarrow \quad 8u + 5v = 0.$$

Thus choose $u = 5t$ and $v = -8t$. This direction is determined by the specified midpoint; it is not an arbitrary chord through that point.

Step 2: Find the distance to an endpoint Substitute $(x, y) = (1 + 5t, \frac{2}{5} - 8t)$ into the ellipse:

$$\frac{(1 + 5t)^2}{25} + \frac{(2/5 - 8t)^2}{16} = 1.$$

The linear terms cancel, as required for a midpoint. The remaining equation is

$$\frac{1}{20} + 5t^2 = 1, \qquad t^2 = \frac{19}{100}.$$

Hence the squared half-length is $u^2 + v^2 = 89t^2 = 1691/100$.

Step 3: Compare the complete chord length with the choices The midpoint divides the chord into two equal segments. Therefore

$$L = 2\sqrt{\frac{1691}{100}} = \frac{\sqrt{1691}}{5}.$$

The other choices would give squared lengths $2009/25$, $1541/25$, or $1741/25$, whereas substitution fixes $L^2 = 1691/25$.

Step 4: Conclusion The required chord length is $\frac{\sqrt{1691}}{5}$. Hence, the correct option is (A).

Quick Tip to Solve: Use symmetric endpoint displacements about the midpoint; subtracting the ellipse equations eliminates the quadratic terms immediately.

9. Consider the function

$$f(x)=\begin{cases}\dfrac{a(7x-12-x^2)}{b|x^2-7x+12|}, & x<3,\\ 2^{\frac{\sin(x-3)}{x-[x]}}, & x>3,\\ b, & x=3.\end{cases}$$

where $[x]$ denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a,b) such that $f(x)$ is continuous at $x=3$, then the number of elements in S is: [4 marks]

(A) Infinitely many

(B) 4

(C) 2

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- Continuity at $x=3$ requires both one-sided limits to equal $f(3)=b$. The left branch also requires $b\neq 0$.
- Near 3 from the right, $[x]=3$, and the standard limit $\sin h/h\to 1$ applies as $h\to 0$. Absolute values must be resolved separately on the left.

Step 1: Evaluate the left-hand limit Factor the quadratic:

$$x^2-7x+12=(x-3)(x-4),\qquad 7x-12-x^2=-(x-3)(x-4).$$

For $x<3$ close to 3, both factors in the product are negative, so the product is positive. Its absolute value therefore equals the product itself. The common nonzero factor cancels, giving

$$\lim_{x\to 3^-} f(x)=-\frac{a}{b}.$$

The sign is negative; dropping the absolute-value analysis would risk reversing the condition on a.

Step 2: Evaluate the right-hand limit When $3<x<4$, the greatest integer function is constant: $[x]=3$. Put $h=x-3>0$. The exponent becomes

$$\frac{\sin(x-3)}{x-[x]}=\frac{\sin h}{h}\longrightarrow 1.$$

Since the exponential function with base 2 is continuous,

$$\lim_{x\to 3^+} f(x)=2^1=2.$$

This right-hand limit determines b uniquely.

Step 3: Impose continuity and count the pairs Equate both limits to the actual value at 3:

$$b=2,\qquad -\frac{a}{b}=b.$$

Consequently $a = -b^2 = -4$. Conversely, $(a, b) = (-4, 2)$ makes both limits and the function value equal 2, and the denominator parameter is nonzero. Thus the admissible set is exactly $S = \{(-4, 2)\}$, not an infinite family or a set with two or four pairs.

Step 4: Conclusion There is exactly one ordered pair, $(-4, 2)$, so $|S| = 1$. Hence, the correct option is (D).

Quick Tip to Solve: For continuity involving a greatest integer function, first fix its constant value on a sufficiently small one-sided neighbourhood.

10. If $\mathrm{a} = \lim\limits_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ **and** $\mathrm{b} = \lim\limits_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$**, then the value of** ab^3 **is :** [4 marks]

(A) 36

(B) 25

(C) 32

(D) 30

Correct Answer: (C) 32

Solution:

Concept:

- For nested square roots, multiplying by conjugates converts radical differences into polynomial differences without approximation.
- The identity $\sin^2 x = (1 - \cos x)(1 + \cos x)$ allows an exact cancellation after rationalising the second denominator.

Step 1: Rationalise the limit defining a twice For $x \neq 0$, multiply the numerator by its conjugate, then rationalise the remaining difference:

$$\frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4} = \frac{\sqrt{1+x^4}-1}{x^4(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2})} = \frac{1}{(\sqrt{1+x^4}+1)(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2})}.$$

The transformed denominator tends to $2(2\sqrt{2})$, so $a = 1/(4\sqrt{2})$. Each cancellation is valid away from zero, which suffices for a limit.

Step 2: Rationalise the limit defining b Multiply by the conjugate of the denominator and use the trigonometric identity:

$$\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}} = \frac{\sin^2 x(\sqrt{2}+\sqrt{1+\cos x})}{1-\cos x} = (1+\cos x)(\sqrt{2}+\sqrt{1+\cos x}).$$

Thus $b = 2(2\sqrt{2}) = 4\sqrt{2}$. The apparent zero denominator in the original expression has been removed by an exact identity.

Step 3: Combine the values Now cube b before multiplying by a:

$$ab^3 = \frac{(4\sqrt{2})^3}{4\sqrt{2}} = (4\sqrt{2})^2 = 32.$$

In particular, $ab = 1$, so the requested expression can also be checked as b^2. The values 36, 25 and 30 do not equal this exact product.

Step 4: Conclusion $ab^3 = 32$. Hence, the correct option is (C).

Quick Tip to Solve: When two radical limits are related, evaluate each by conjugates and check whether their product simplifies before cubing.

---

11. Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements :

Statement I : $f(-x)$ **is the inverse of the matrix** $f(x)$**.**

Statement II : $f(x)f(y) = f(x+y)$**.**

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (C) Both Statement I and Statement II are true

Solution:

Concept:

- A matrix B is the inverse of A if $AB = BA = I$. For a rotation matrix, reversing the angle reverses the rotation.
- The addition identities for sine and cosine determine the product of two planar rotation matrices. The third coordinate here is unchanged because the last diagonal entry is 1.

Step 1: Multiply the two general matrices Matrix multiplication gives the upper-left block of $f(x)f(y)$ as

$$\begin{pmatrix} \cos x\cos y - \sin x\sin y & -\cos x\sin y - \sin x\cos y \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y \end{pmatrix}.$$

The entries outside this block remain zero, apart from the final diagonal entry 1. Using the angle-addition identities, the block becomes

$$\begin{pmatrix} \cos(x+y) & -\sin(x+y) \\ \sin(x+y) & \cos(x+y) \end{pmatrix}.$$

Thus the complete product is $f(x+y)$, establishing Statement II for every real x, y.

Step 2: Check the inverse identity Set $y = -x$ in the proved product rule. Then

$$f(x)f(-x) = f(0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I_3.$$

Reversing the order also gives $f(-x)f(x) = f(0) = I_3$. Therefore $f(-x)$ is indeed the inverse of $f(x)$, so Statement I is true. This directly verifies the defining condition instead of relying only on the geometric interpretation.

Step 3: Select the joint truth value Both statements have been proved by identities, without requiring any special angle. Option (A) incorrectly rejects the inverse property; option (B) rejects both valid identities; option (D) incorrectly rejects angle addition. Only the option declaring both statements true agrees with the computations.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (C).

Quick Tip to Solve: Prove the angle-addition product first; substituting the negative angle then proves the inverse statement in one line.

12. Four distinct points $(2k, 3k), (1, 0), (0, 1)$ and $(0, 0)$ lie on a circle for k equal to : [4 marks]

(A) $\frac{3}{13}$
(B) $\frac{2}{13}$
(C) $\frac{5}{13}$
(D) $\frac{1}{13}$

Correct Answer: (C) $\frac{5}{13}$

Solution:

Concept:

- The general equation of a circle is $x^2 + y^2 + Dx + Ey + F = 0$. Three noncollinear points determine its coefficients uniquely.
- The fourth point must satisfy that same equation. Any parameter root causing two listed points to coincide must be rejected because the points are specified to be distinct.

Step 1: Determine the circle through the fixed points Substitute $(0, 0)$ into the general circle equation to obtain $F = 0$. Substitution of $(1, 0)$ gives $1 + D = 0$, so $D = -1$. Similarly, $(0, 1)$ gives $E = -1$. These three points are noncollinear, so their circle is uniquely

$$x^2 + y^2 - x - y = 0.$$

Equivalently its centre is $(1/2, 1/2)$ and its radius is $1/\sqrt{2}$, which also confirms it is a genuine circle.

Step 2: Impose the condition on the variable point The remaining point $(2k, 3k)$ lies on this circle

exactly when

$$(2k)^2 + (3k)^2 - 2k - 3k = 0.$$

Collecting terms gives

$$13k^2 - 5k = 0 \quad \Longrightarrow \quad k(13k - 5) = 0.$$

The algebra therefore supplies two candidate values, $k = 0$ and $k = 5/13$. The distinctness requirement must still be applied; finding roots alone is not enough.

Step 3: Reject the repeated point and check the options For $k = 0$, the point $(2k, 3k)$ is $(0, 0)$, repeating a listed point, so this candidate is inadmissible. For $k = 5/13$, the point is $(10/13, 15/13)$, distinct from all three fixed points and satisfying the circle equation. The other offered nonzero fractions $3/13$, $2/13$, and $1/13$ are not roots of $k(13k - 5)$.

Step 4: Conclusion The admissible value is $k = \dfrac{5}{13}$. Hence, the correct option is (C).

Quick Tip to Solve: When a circle question specifies distinct points, check parameter roots for coincidence after substituting into the circle equation.

13. If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is : [4 marks]

(A) 10

(B) 5

(C) 7

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- For nonparallel lines through $\mathbf{a}_1, \mathbf{a}_2$ with directions $\mathbf{d}_1, \mathbf{d}_2$, the shortest distance is $D = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)|/|\mathbf{d}_1 \times \mathbf{d}_2|$.
- The scalar triple product projects the displacement between the lines onto their common normal. Its absolute value is essential because distance is nonnegative.

Step 1: Read a point and direction from each line From the symmetric equations choose the following points and directions:

$$\mathbf{a}_1 = (4, -1, 0), \quad \mathbf{d}_1 = (1, 2, -3),$$
$$\mathbf{a}_2 = (\lambda, -1, 2), \quad \mathbf{d}_2 = (2, 4, -5).$$

The second direction is not twice the first because its third entry would then be -6, not -5. Thus the lines are nonparallel, and the skew-line distance formula applies.

Step 2: Compute the common normal and distance The cross product and its length are

$$\mathbf{d}_1 \times \mathbf{d}_2 = (2, -1, 0), \qquad |\mathbf{d}_1 \times \mathbf{d}_2| = \sqrt{5}.$$

Also $\mathbf{a}_2 - \mathbf{a}_1 = (\lambda - 4, 0, 2)$, so its dot product with the normal is $2(\lambda - 4)$. Therefore

$$D = \frac{|2(\lambda - 4)|}{\sqrt{5}}.$$

Although the points differ in their third coordinates, that component does not contribute to the projection since the normal has zero third component.

Step 3: Solve the absolute-value equation Equating the given distance gives

$$\frac{2|\lambda - 4|}{\sqrt{5}} = \frac{6}{\sqrt{5}} \quad \Longrightarrow \quad |\lambda - 4| = 3.$$

Both signs must be retained: $\lambda - 4 = 3$ or $\lambda - 4 = -3$. Hence $\lambda = 7$ or 1, and their sum is 8. The proposed sums 10, 5 and 7 miss this two-root result.

Step 4: Conclusion The possible values are 1 and 7, whose sum is 8. Hence, the correct option is (D).

Quick Tip to Solve: Take the cross product before expanding the triple product; a zero component in the common normal can remove unnecessary algebra.

14. The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is : [4 marks]

(A) $\frac{30}{41}$
(B) $\frac{8}{5}$
(C) $\frac{2}{5}$
(D) $\frac{25}{41}$

Correct Answer: (A) $\frac{30}{41}$

Solution:

Concept:

- The first-quadrant portion of a line lies between its positive intercepts. Internal trisection points are found by taking one-third and two-thirds of the displacement between those endpoints.
- For slopes m_1, m_2, the tangent of the acute angle between lines is $|(m_2 - m_1)/(1 + m_1 m_2)|$. A line from the origin to (x, y) has slope y/x.

Step 1: Find the intercepts and trisection points The line $4x + 5y = 20$ meets the positive axes at $A = (5, 0)$ and $B = (0, 4)$. Its first-quadrant segment is AB. The displacement from A to B is $(-5, 4)$, so the two trisection points are

$$P = A + \frac{1}{3}(B - A) = \left(\frac{10}{3}, \frac{4}{3}\right),$$

$$Q = A + \frac{2}{3}(B - A) = \left(\frac{5}{3}, \frac{8}{3}\right).$$

Both have positive coordinates and satisfy the original line equation.

Step 2: Find the slopes of the two rays The lines through the origin and these points have slopes

$$m_1 = \frac{4/3}{10/3} = \frac{2}{5}, \qquad m_2 = \frac{8/3}{5/3} = \frac{8}{5}.$$

These are the slopes of L_1, L_2 in either order. Interchanging their names does not change the acute angle because the angle formula uses an absolute value.

Step 3: Evaluate the tangent and compare Apply the angle formula with both slopes:

$$\tan\theta = \left|\frac{\frac{8}{5} - \frac{2}{5}}{1 + \frac{2}{5}\cdot\frac{8}{5}}\right| = \frac{6/5}{41/25} = \frac{30}{41}.$$

The numbers $8/5$ and $2/5$ are the individual slopes, not the tangent of their angle. The remaining fraction $25/41$ omits the numerator factor $6/5$.

Step 4: Conclusion The tangent of the acute angle between the lines is $\frac{30}{41}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the trisection points as vector fractions of the intercept segment; then use the angle formula rather than subtracting slopes alone.

---

15. ${}^{n-1}C_r = \left(k^2 - 8\right){}^{n}C_{r+1}$ **if and only if :** [4 marks]

(A) $2\sqrt{2} < \mathrm{k} < 2\sqrt{3}$

(B) $2\sqrt{2} < \mathrm{k} \leq 3$

(C) $2\sqrt{3} < \mathrm{k} < 3\sqrt{3}$

(D) $2\sqrt{3} < \mathrm{k} \leq 3\sqrt{2}$

Correct Answer: No listed interval is literally equivalent to the equation; (B) is the intended positive-branch range.

Solution:

Concept:

- For the usual nonzero binomial coefficients, n and r are integers with $n \geq 1$ and $0 \leq r \leq n-1$. Their factorial definitions simplify a ratio directly.
- The numerator $r+1$ consequently lies between 1 and n. Apply this bound before taking a square root; squaring loses the sign of k.

Step 1: Divide the coefficients and cancel factorials Both coefficients in the printed equation are positive. Dividing by ${}^{n}C_{r+1}$ gives

$$k^2 - 8 = \frac{{}^{n-1}C_r}{{}^{n}C_{r+1}} = \frac{(n-1)!}{r!(n-r-1)!}\frac{(r+1)!(n-r-1)!}{n!} = \frac{r+1}{n}.$$

The cancellation is exact; no approximation or expansion of the binomial theorem is required.

Step 2: Bound the ratio and compare the listed intervals Since $1 \leq r+1 \leq n$, the range condition

used in the choices is

$$0 < k^2 - 8 \leq 1, \qquad 8 < k^2 \leq 9.$$

For the positive values listed in the options this becomes

$$2\sqrt{2} < k \leq 3.$$

The upper endpoint is included: $r = n - 1$ makes the ratio exactly 1. Option (A) extends above 3 and so includes values with $k^2 - 8 > 1$. Options (C) and (D) start above $2\sqrt{3} > 3$, and therefore cannot satisfy the bound. For fixed n, r, the exact value is $k = \pm\sqrt{8 + (r+1)/n}$.

Step 3: Conclusion No listed interval is literally an if-and-only-if condition for the printed equation. The exact condition is $k = \pm\sqrt{8 + (r+1)/n}$. Its magnitude lies in $2\sqrt{2} < |k| \leq 3$; the negative branch $-3 \leq k < -2\sqrt{2}$ is also permitted. Option (B), $2\sqrt{2} < k \leq 3$, is the intended positive-branch range, but requires the unstated restriction to positive k, and the exact ratio equality still applies.

Quick Tip to Solve: Cancel the factorials before bounding a binomial ratio. When recovering a variable from its square, retain both signs and check whether the choices implicitly restrict it to positive values.

---

16. The number of common terms in the progressions

$4, 9, 14, 19, \ldots\ldots$, **up to** 25^{th} **term and**

$3, 6, 9, 12, \ldots\ldots$, **up to** 37^{th} **term is :** [4 marks]

(A) 9

(B) 8

(C) 5

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- An arithmetic progression with first term a and common difference d has $t_m = a + (m-1)d$. Finite endpoints must be checked in both progressions.
- Terms common to progressions of differences 5 and 3 repeat with difference $\text{lcm}(5, 3) = 15$.

Step 1: Find the finite endpoint of each progression The first progression has first term 4 and common difference 5. Its last permitted term is

$$a_{25} = 4 + 24(5) = 124.$$

The second progression consists of multiples of 3, and its last permitted term is

$$b_{37} = 3 + 36(3) = 111.$$

A common term must therefore occur in the first progression and be divisible by 3, while being no larger than 111. Using 124 as the upper bound alone would incorrectly count a term absent from the

second progression.

Step 2: Construct and count the common progression Write a term of the first progression as $4 + 5j$, where $0 \le j \le 24$. Divisibility by 3 requires

$$4 + 5j \equiv 0 \pmod 3, \qquad 1 + 2j \equiv 0 \pmod 3,$$

so $j \equiv 1 \pmod 3$. Thus the first common term is 9 and successive common terms differ by 15. Counting those at most 111 gives

$$9 + 15m \le 111 \implies 0 \le m \le 6.$$

There are seven integer values of m. Explicitly the common terms are

$$9,\ 24,\ 39,\ 54,\ 69,\ 84,\ 99.$$

The next candidate, 114, lies in the first progression but exceeds 111. Consequently 8 or 9 terms overcount the allowed endpoint, whereas 5 terms stops too early.

Step 3: Conclusion There are $\boxed{7}$ common terms. Hence, the correct option is (D).

Quick Tip to Solve: Use the smaller of the two last terms as the upper cutoff. Find the first common term, then count at intervals equal to the least common multiple of the two differences.

---

17. Let $a_1, a_2, \ldots a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, \ldots, a_{10}$ is equal to : [4 marks]

(A) 5

(B) $\sqrt{115}$

(C) 10

(D) $\sqrt{5}$

Correct Answer: (D) $\sqrt{5}$

Solution:

Concept:

- For N observations, the population variance is $\sigma^2 = \frac{1}{N}\sum a_k^2 - \bar{a}^2$, with $\bar{a} = \frac{1}{N}\sum a_k$. Standard deviation is its nonnegative square root.
- The identity $(\sum a_k)^2 = \sum a_k^2 + 2\sum_{k<j} a_k a_j$ recovers the sum of squares from the sum of pairwise products. Each unordered pair occurs twice in the square.

Step 1: Recover the sum of squares The given pairwise sum runs over $k < j$, so it counts each distinct unordered pair once. Expanding the square of the total therefore yields

$$50^2 = \sum_{k=1}^{10} a_k^2 + 2(1100).$$

Hence

$$\sum_{k=1}^{10} a_k^2 = 2500 - 2200 = 300.$$

It would be incorrect to subtract only 1100: the two cross-products $a_k a_j$ and $a_j a_k$ both appear when the total is squared. There is no need to determine the ten observations individually.

Step 2: Calculate the mean, variance and square root The mean of the ten observations is

$$\bar{a} = \frac{50}{10} = 5.$$

Their mean square is $300/10 = 30$, so

$$\sigma^2 = 30 - 5^2 = 30 - 25 = 5, \qquad \sigma = \sqrt{5}.$$

As an equivalent check, the sum of squared deviations is

$$\sum (a_k - 5)^2 = 300 - 10(25) = 50,$$

which again gives variance $50/10 = 5$. Option (A) gives the variance rather than its square root; the values $\sqrt{115}$ and 10 do not satisfy the computed second-moment identity. The denominator is 10 because the question asks for the standard deviation of the stated observations, not an unbiased sample estimate.

Step 3: Conclusion The standard deviation is $\boxed{\sqrt{5}}$. Hence, the correct option is (D).

Quick Tip to Solve: When the pairwise product sum is supplied, first square the total to recover the sum of squares. Remember the factor 2 on the pairwise sum and the final square root.

---

18. Let $\overrightarrow{\mathrm{a}} = \hat{i} + 2\hat{j} + \hat{k}$,
$\overrightarrow{\mathrm{b}} = 3(\hat{i} - \hat{j} + \hat{k})$.
Let $\overrightarrow{c}$ **be the vector such that** $\overrightarrow{\mathrm{a}} \times \overrightarrow{c} = \overrightarrow{\mathrm{b}}$ **and** $\vec{a} \cdot \vec{c} = 3$.
Then $\vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c})$ **is equal to :** [4 marks]

(A) 32

(B) 36

(C) 24

(D) 20

Correct Answer: (C) 24

Solution:

Concept:

- The scalar triple product is invariant under cyclic permutation: $\vec{a} \cdot (\vec{c} \times \vec{b}) = \vec{b} \cdot (\vec{a} \times \vec{c})$. Reversing just one cross product would change its sign.
- The dot product distributes over a sum or difference. A cross product is perpendicular to each of

its factors, so $\vec{a} \cdot \vec{b} = 0$ when $\vec{b} = \vec{a} \times \vec{c}$.

Step 1: Evaluate the products already fixed by the data The vectors in component form are $\vec{a} = (1, 2, 1)$ and $\vec{b} = (3, -3, 3)$. Their dot product is

$$\vec{a} \cdot \vec{b} = 1(3) + 2(-3) + 1(3) = 0.$$

This also confirms the required perpendicularity implied by $\vec{a} \times \vec{c} = \vec{b}$. The squared magnitude of $\vec{b}$ is

$$\vec{b} \cdot \vec{b} = 3^2 + (-3)^2 + 3^2 = 27.$$

The factor 3 multiplies every component of $\hat{i} - \hat{j} + \hat{k}$; it is squared when computing the squared magnitude.

Step 2: Use the cyclic triple-product identity Instead of solving three component equations for $\vec{c}$, use the cross product supplied in the question:

$$\vec{a} \cdot (\vec{c} \times \vec{b}) = \vec{b} \cdot (\vec{a} \times \vec{c}) = \vec{b} \cdot \vec{b} = 27.$$

Now distribute the outer dot product in the requested expression:

$$\begin{aligned} \vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c}) &= 27 - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} \\ &= 27 - 0 - 3 = 24. \end{aligned}$$

The value 36 cannot arise after subtracting 3 from 27, while 32 and 20 likewise disagree with these exact dot products. Keeping the cyclic order avoids the erroneous negative of the triple product.

Step 3: Conclusion The required scalar is $\boxed{24}$. Hence, the correct option is (C).

Quick Tip to Solve: If a scalar triple product contains a given cross product after a cyclic permutation, use it directly. This often eliminates the need to find the unknown vector.

19. If A denotes the sum of all the coefficients in the expansion of $\left(1 - 3x + 10x^2\right)^n$ and B denotes the sum of all the coefficients in the expansion of $\left(1 + x^2\right)^n$, then : [4 marks]

(A) $\mathrm{B} = \mathrm{A}^3$

(B) $3\mathrm{A} = \mathrm{B}$

(C) $A = 3B$

(D) $\mathrm{A} = \mathrm{B}^3$

Correct Answer: (D) $\mathrm{A} = \mathrm{B}^3$

Solution:

Concept:

- If $P(x) = c_0 + c_1 x + \cdots + c_m x^m$, then the sum of all coefficients is $P(1)$. Substitution of 1 turns every power of x into 1.
- Use the same substitution separately for both polynomials, then express the resulting powers in a

common base. Substitution of -1 would instead give an alternating coefficient sum.

Step 1: Evaluate the first polynomial at one Set $x = 1$ in the full polynomial before attempting any expansion. The coefficient sum called A is

$$A = (1 - 3 \cdot 1 + 10 \cdot 1^2)^n = (1 - 3 + 10)^n = 8^n.$$

The negative sign of the linear term must be retained. It contributes -3, not $+3$, even though all powers of x become 1. Thus the sum is not obtained by adding the absolute values of the coefficients. This evaluation avoids handling the many individual terms of the trinomial expansion.

Step 2: Evaluate the second sum and compare the powers For the second polynomial the corresponding substitution gives

$$B = (1 + 1^2)^n = 2^n.$$

Since $8 = 2^3$, it follows that

$$A = 8^n = 2^{3n} = (2^n)^3 = B^3.$$

The exponent 3 therefore applies to the whole coefficient sum B. A linear relation such as $A = 3B$ or $B = 3A$ confuses multiplication by 3 with cubing. Also $B = A^3$ reverses the relation: its right-hand side would be 2^{9n}, not 2^n for positive n. For a quick check at $n = 1$, the two sums are 8 and 2 respectively, confirming $8 = 2^3$.

Step 3: Conclusion The coefficient sums satisfy $\boxed{A = B^3}$. Hence, the correct option is (D).

Quick Tip to Solve: To sum all coefficients, substitute 1 into the unexpanded polynomial. Use -1 only when the question asks for the alternating sum of coefficients.

20. The function $f : \mathbf{N} - \{1\} \to \mathbf{N}$; defined by $f(\mathrm{n}) =$ the highest prime factor of n, is : [4 marks]

(A) one-one only

(B) neither one-one nor onto

(C) onto only

(D) both one-one and onto

Correct Answer: (B) neither one-one nor onto

Solution:

Concept:

- A function is one-one (injective) only when distinct inputs always have distinct outputs. One counterexample is enough to disprove injectivity.
- A function is onto (surjective) only when every element of the stated codomain has a preimage. Its codomain here is all natural numbers, not merely the primes.

Step 1: Test whether distinct inputs can share an output The domain $\mathbf{N} \setminus \{1\}$ consists of natural numbers at least 2. Every such number has a highest prime factor, so the rule is well defined. Consider two distinct permitted inputs:

$$f(2) = 2, \qquad f(4) = 2.$$

The highest prime factor of both 2 and $4 = 2^2$ is 2. Hence $2 \neq 4$ but $f(2) = f(4)$, proving that the function is not one-one. The fact that each input has a unique highest prime factor establishes that the rule is a function; it does not establish injectivity.

Step 2: Compare the image with the whole codomain Every output of this rule is a prime number. Conversely, each prime p occurs because the input $n = p$ gives $f(p) = p$. Therefore the image is exactly the set of primes. In particular, the natural number 1 in the codomain has no preimage, and neither does any composite number such as 4:

$$f(n) \neq 1, \qquad f(n) \neq 4 \quad \text{for every } n \geq 2.$$

Thus the image is a proper subset of **N**, and the function is not onto. Options (A) and (D) wrongly require injectivity; option (C) wrongly requires surjectivity. Both properties fail independently, giving the remaining choice.

Step 3: Conclusion The function is neither one-one nor onto. Hence, the correct option is (B).

Quick Tip to Solve: For highest-prime-factor functions, use a prime and its square to disprove injectivity, then compare the prime-only image with the stated codomain to test surjectivity.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If $8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \frac{1}{4^3}(3+3p) + \cdots\cdots\infty$, then the value of p is _____. [4 marks]

Solution:

Concept:

- For $|x| < 1$, the geometric sum is $\sum_{m=0}^{\infty} x^m = 1/(1-x)$. Its differentiated form gives $\sum_{m=0}^{\infty} mx^m = x/(1-x)^2$.
- The given series is arithmetic-geometric: its numerator is $3 + mp$ and its geometric weight is x^m, with $x = 1/4$. Both component series converge absolutely.

Step 1: Separate the constant and arithmetic parts Index the first term by $m = 0$, so it is $3 + 0p = 3$, with no extra geometric factor. The entire sum can then be written as

$$8 = \sum_{m=0}^{\infty} (3 + mp) \left(\frac{1}{4}\right)^m = 3 \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m + p \sum_{m=0}^{\infty} m \left(\frac{1}{4}\right)^m.$$

The separation is valid because $|1/4| < 1$; the factor growing linearly with m does not prevent convergence against the geometric decay. Correctly starting at zero also ensures that the coefficient of p in the first term vanishes.

Step 2: Evaluate the sums and solve the linear equation Substitute $x = 1/4$ into the two standard sums:

$$\sum_{m=0}^{\infty} x^m = \frac{1}{1 - 1/4} = \frac{4}{3}, \qquad \sum_{m=0}^{\infty} mx^m = \frac{1/4}{(1 - 1/4)^2} = \frac{4}{9}.$$

Thus the printed equality becomes

$$8 = 3\left(\frac{4}{3}\right) + p\left(\frac{4}{9}\right) = 4 + \frac{4p}{9}.$$

Subtract 4 from each side and multiply by $9/4$, giving $p = 9$. Substitution back into the summed expression yields $4 + 4(9)/9 = 8$, confirming the value. The question asks for the numerical parameter itself, so no physical unit or rounding is involved.

Step 3: Conclusion The required value is $\boxed{p = 9}$.

Quick Tip to Solve: Write an arithmetic-geometric term as a constant part plus a multiple of the index. Apply the geometric sum and its differentiated sum separately, keeping the first index at zero.

22. If the solution of the differential equation

$(2x + 3y - 2)\mathrm{d}x + (4x + 6y - 7)\mathrm{d}y = 0, y(0) = 3$**, is**

$\alpha x + \beta y + 3\log_e|2x + 3y - \gamma| = 6$**, then** $\alpha + 2\beta + 3\gamma$ **is equal to _____.** [4 marks]

Solution:

Concept:

- When both coefficients depend on the same linear combination of the variables, substitute that combination as a new variable. Here let $u = 2x + 3y$, so that $\mathrm{d}u = 2\,\mathrm{d}x + 3\,\mathrm{d}y$.
- After separating the variables, integrate and use the initial condition to determine the constant. Keep the absolute value in the logarithm and compare the resulting equation with the prescribed form.

Step 1: Reduce to an equation in u and y The coefficients become $u - 2$ and $2u - 7$. Substituting $\mathrm{d}x = (\mathrm{d}u - 3\,\mathrm{d}y)/2$ gives

$$(u - 2)(\mathrm{d}u - 3\,\mathrm{d}y) + 2(2u - 7)\,\mathrm{d}y = 0.$$

$$(u - 2)\,\mathrm{d}u + (u - 8)\,\mathrm{d}y = 0.$$

At the initial point, $u = 9$, so division by $u - 8$ is valid on the solution interval containing that point.

Step 2: Integrate the separated equation Rewrite the quotient before integrating:

$$\frac{\mathrm{d}y}{\mathrm{d}u} = -\frac{u - 2}{u - 8} = -1 - \frac{6}{u - 8}.$$

$$y = -u - 6\log_e|u - 8| + C.$$

Substitute back $u = 2x + 3y$, collect the linear terms and divide by two:

$$x + 2y + 3\log_e|2x + 3y - 8| = K.$$

This normalization is necessary because the prescribed logarithmic coefficient is three.

Step 3: Apply the initial condition and compare coefficients At $x = 0, y = 3$, the logarithm is $\log_e 1 = 0$, and hence $K = 6$. Thus the equation is exactly in the requested form, with $\alpha = 1$, $\beta = 2$,

and $\gamma = 8$. The logarithmic argument and the right-hand constant both agree with the given initial condition.

Step 4: Conclusion Therefore $\alpha + 2\beta + 3\gamma = 1 + 2(2) + 3(8) = \boxed{29}$. This is a dimensionless numerical value.

Quick Tip to Solve: For coefficients involving the same expression $2x + 3y$, substitute it first; normalize the logarithmic coefficient before reading off the constants.

23. Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3), x \in$ R. Then $f'(10)$ is equal to _____. [4 marks]

Solution:

Concept:

- The quantities $f'(1)$, $f''(2)$, and $f'''(3)$ are fixed real numbers, not functions of the current variable x. Treat them as polynomial coefficients while differentiating.
- Differentiate the identity, then evaluate the derivative identities at the specified points. This produces simultaneous equations for the unknown coefficients and uniquely determines the required derivative.

Step 1: Introduce constant coefficients and differentiate Put $A = f'(1)$, $B = f''(2)$, and $C = f'''(3)$. The identity becomes $f(x) = x^3 + Ax^2 + Bx + C$. Therefore

$$f'(x) = 3x^2 + 2Ax + B, \qquad f''(x) = 6x + 2A, \qquad f'''(x) = 6.$$

The last identity also gives $C = 6$, although that constant does not affect the requested first derivative.

Step 2: Determine the coefficients consistently Evaluate the first derivative at one and the second derivative at two:

$$A = 3 + 2A + B, \qquad B = 12 + 2A.$$

The first relation is $A + B = -3$. Substitute the second relation into it:

$$A + (12 + 2A) = -3 \quad \Longrightarrow \quad A = -5, \qquad B = 2.$$

As a check, $3 + 2(-5) + 2 = -5 = A$, and $12 + 2(-5) = 2 = B$. Thus both derivative definitions are satisfied, rather than merely treating the coefficients as arbitrary.

Step 3: Evaluate the requested derivative With the determined coefficients,

$$f'(x) = 3x^2 - 10x + 2.$$

Now substitute $x = 10$:

$$f'(10) = 3(10)^2 - 10(10) + 2 = 300 - 100 + 2 = 202.$$

One can also reconstruct $f(x) = x^3 - 5x^2 + 2x + 6$; its derivatives reproduce all three constants used in the original identity.

Step 4: Conclusion The required dimensionless numerical value is $f'(10) = \boxed{202}$.

Quick Tip to Solve: Values such as $f'(1)$ are constants when differentiating with respect to x. Name them first, and only then impose their defining evaluation conditions.

24. Let the set of all $a \in \mathbf{R}$ such that the equation $\cos 2x + a \sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan 81^\circ$, then pqr is equal to _____. [4 marks]

Solution:

Concept:

- Replace $\cos 2x$ by $1 - 2\sin^2 x$ and use $-1 \le \sin x \le 1$. The range of the resulting expression determines exactly which parameters allow a real solution.
- Use complementary-angle identities $\tan(90^\circ - \theta) = \cot\theta$ and $\tan\theta + \cot\theta = 2\csc 2\theta$ to simplify the trigonometric factor without numerical approximation.

Step 1: Find the complete parameter interval Let $t = \sin x$, where $-1 \le t \le 1$. The equation becomes

$$1 - 2t^2 + at = 2a - 7 \quad \Longrightarrow \quad a(2 - t) = 8 - 2t^2.$$

Factor the right-hand side as $2(2 - t)(2 + t)$. Since $2 - t \ge 1$, cancellation introduces no excluded or extraneous values:

$$a = 2(2 + t) = 4 + 2t.$$

Its range is $[2, 6]$. Every value in this interval corresponds to $t = (a - 4)/2$ in $[-1, 1]$, so a real x exists. Hence $p = 2, q = 6$.

Step 2: Simplify the angular expression exactly The reciprocal of $\cot 63^\circ$ is $\tan 63^\circ = \cot 27^\circ$, and $\tan 81^\circ = \cot 9^\circ$. Consequently

$$r = (\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ).$$

$$r = 2\csc 18^\circ - 2\csc 54^\circ.$$

Use $\sin 18^\circ = (\sqrt{5} - 1)/4$ and $\sin 54^\circ = (\sqrt{5} + 1)/4$:

$$r = \frac{8}{\sqrt{5} - 1} - \frac{8}{\sqrt{5} + 1} = \frac{8(2)}{5 - 1} = 4.$$

Both pairs are added before the second pair is subtracted, so the complementary-angle signs are preserved.

Step 3: Form the required product The parameter endpoints are included: $t = -1$ gives $a = 2$, while $t = 1$ gives $a = 6$. Combining these endpoints with the exact value of r,

$$pqr = 2 \cdot 6 \cdot 4 = 48.$$

No rounding of trigonometric values is needed.

Step 4: Conclusion The required dimensionless value is $pqr = \boxed{48}$.

Quick Tip to Solve: Factor the parameter equation before optimizing. In the angle expression, pair each tangent with its complementary cotangent to expose the double-angle identity.

25. Let for a differentiable function $f : (0, \infty) \to \mathbf{R}$, $f(x) - f(y) \geqslant \log_e\left(\frac{x}{y}\right) + x - y, \forall x, y \in (0, \infty)$. Then $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- An inequality that holds for every ordered pair (x, y) also holds after interchanging the variables. If the resulting bounds coincide, the original inequality forces an exact identity.
- For positive x, $\mathrm{d}(\log_e x)/\mathrm{d}x = 1/x$. After finding f', use $\sum_{n=1}^{N} n^2 = N(N+1)(2N+1)/6$ to evaluate the requested sum.

Step 1: Use both orders of the variables The given inequality is

$$f(x) - f(y) \geq \log_e(x/y) + x - y.$$

Apply the same statement with x and y exchanged:

$$f(y) - f(x) \geq \log_e(y/x) + y - x.$$

Multiply this second inequality by minus one, reversing its direction, and use $\log_e(y/x) = -\log_e(x/y)$:

$$f(x) - f(y) \leq \log_e(x/y) + x - y.$$

Thus the upper and lower bounds are identical; equality must hold for every positive pair.

Step 2: Determine the derivative Fix $y = 1$ in the forced equality. It gives

$$f(x) = f(1) + \log_e x + x - 1.$$

The additive constant $f(1) - 1$ has no effect on differentiation. Therefore

$$f'(x) = \frac{1}{x} + 1, \qquad x > 0.$$

Every argument $1/n^2$ in the question is positive, so all requested derivative values lie in the given domain. In particular, $f'(1/n^2) = n^2 + 1$.

Step 3: Sum the twenty values There are twenty copies of the constant one in addition to the squares:

$$\sum_{n=1}^{20} f'(1/n^2) = \sum_{n=1}^{20} n^2 + 20.$$

$$\sum_{n=1}^{20} n^2 = \frac{20 \cdot 21 \cdot 41}{6} = 2870.$$

Adding the constant contribution gives $2870 + 20 = 2890$. The arbitrary value of $f(1)$ cancels out of the entire calculation.

Step 4: Conclusion The required dimensionless numerical sum is $\boxed{2890}$.

Quick Tip to Solve: When a functional inequality holds for all x, y, swap them immediately. Opposite matching bounds often force equality and reveal the function up to a constant.

---

26. Let the area of the region $\{(x, y) : x - 2y + 4 \geqslant 0, x + 2y^2 \geqslant 0, x + 4y^2 \leq 8, y \geqslant 0\}$ be $\frac{\mathrm{m}}{\mathrm{n}}$, where m and n are coprime numbers. Then $\mathrm{m} + \mathrm{n}$ is equal to _____. [4 marks]

Solution:

Concept:

- For a region described by inequalities in x, use horizontal strips: area equals $\int (x_{\text{right}} - x_{\text{left}})\,\mathrm{d}y$. The active left boundary is the larger of the lower bounds.
- The lower bounds can switch as y changes. Find their intersection and the final intersection with the right boundary before splitting the definite integral.

Step 1: Identify the horizontal boundaries The inequalities are equivalent to

$$x \geq 2y - 4, \qquad x \geq -2y^2, \qquad x \leq 8 - 4y^2, \qquad y \geq 0.$$

The two lower bounds meet where

$$-2y^2 = 2y - 4 \quad \Longrightarrow \quad (y + 2)(y - 1) = 0.$$

Only $y = 1$ is relevant. On $0 \leq y \leq 1$, the parabola $x = -2y^2$ is the larger lower bound. For $y \geq 1$, the line $x = 2y - 4$ is the larger one.

Step 2: Determine the upper limit of the region For the second strip interval, the line reaches the right boundary when

$$2y - 4 = 8 - 4y^2 \quad \Longrightarrow \quad 2y^2 + y - 6 = 0.$$

$$(2y - 3)(y + 2) = 0 \quad \Longrightarrow \quad y = \frac{3}{2}.$$

The first strip width $8 - 2y^2$ is positive throughout $[0, 1]$; the second width $12 - 2y - 4y^2$ is nonnegative on $[1, 3/2]$. Beyond $3/2$, the right boundary lies to the left of the required line, so no further region exists.

Step 3: Integrate the two strip widths The area is

$$\mathcal{A} = \int_0^1 (8 - 2y^2)\,\mathrm{d}y + \int_1^{3/2} (12 - 2y - 4y^2)\,\mathrm{d}y.$$

$$\int_0^1 (8-2y^2)\,dy = 8 - \frac{2}{3} = \frac{22}{3}.$$

$$\int_1^{3/2} (12-2y-4y^2)\,dy = \left[12y - y^2 - \frac{4}{3}y^3\right]_1^{3/2}.$$

$$= \frac{45}{4} - \frac{29}{3} = \frac{19}{12}.$$

Hence $\mathcal{A} = 88/12 + 19/12 = 107/12$ square units. Since 107 has no common factor with 12, this fraction is already in lowest terms.

Step 4: Conclusion Thus $m = 107$, $n = 12$, and the requested dimensionless value is $m + n = \boxed{119}$.

Quick Tip to Solve: With several lower bounds for x, integrate from their maximum to the upper bound. Split at the point where the active lower boundary changes.

27. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, B = [B_1, B_2, B_3]$**, where** B_1, B_2, B_3 **are column matrics, and**

$$AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \quad AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

If $\alpha = |B|$ **and** β **is the sum of all the diagonal elements of** B**, then** $\alpha^3 + \beta^3$ **is equal to _____.**

[4 marks]

Solution:

Concept:

- Matrix multiplication operates column by column: $AB = [AB_1, AB_2, AB_3]$. Thus the given vector equations provide the entire product matrix directly.
- For square matrices, $\det(AB) = \det A \det B$. The sum of the diagonal entries is the trace, which can be found after solving for the columns of B.

Step 1: Assemble the product and compute the determinant From the three prescribed columns,

$$AB = C = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{pmatrix}.$$

This matrix is upper triangular, so $\det C = 1 \cdot 3 \cdot 1 = 3$. Expand $\det A$ along its second column, whose only nonzero entry is the central one:

$$\det A = \det \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = 2 - 1 = 1.$$

Therefore A is invertible, and $\alpha = \det B = \det C / \det A = 3$.

Step 2: Solve a general column equation If $A(x,y,z)^{\mathsf{T}} = (a,b,c)^{\mathsf{T}}$, its rows give

$$2x + z = a, \qquad x + y = b, \qquad x + z = c.$$

Subtract the last equation from the first to obtain $x = a - c$. Then $y = b - a + c$ and $z = 2c - a$. Applying this formula successively to $(1,0,0)$, $(2,3,0)$, and $(3,2,1)$ gives

$$B = \begin{pmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{pmatrix}.$$

The order of these columns is the same as the original three vector equations.

Step 3: Extract the trace and evaluate the expression The diagonal entries of this matrix are $1, 1, -1$. Thus

$$\beta = \operatorname{tr} B = 1 + 1 - 1 = 1.$$

As a direct consistency check, multiplying the third column $(2,0,-1)^{\mathsf{T}}$ by A gives $(3,2,1)^{\mathsf{T}}$, as required. Now use the separately established determinant and trace:

$$\alpha^3 + \beta^3 = 3^3 + 1^3 = 27 + 1 = 28.$$

The trace is a sum of entries, whereas the determinant was obtained from the product rule; these two operations must not be confused.

Step 4: Conclusion The requested dimensionless numerical value is $\alpha^3 + \beta^3 = \boxed{28}$.

Quick Tip to Solve: Assemble the known product by columns first. Use its triangular determinant for $\det B$, and solve the row equations only for the entries needed for the trace.

---

28. The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is _____. [4 marks]

Solution:

Concept:

- For two nonzero vectors $\mathbf{u}, \mathbf{v}$, $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos\theta$, where $0 \le \theta \le \pi$. Their angle is acute exactly when the scalar product is strictly positive.
- Calculate the scalar product by multiplying corresponding components and adding. Solve the resulting quadratic inequality, then impose both positivity and integrality on the parameter.

Step 1: Translate the acute-angle requirement The vectors are $\mathbf{u} = (\alpha, -2, 2)$ and $\mathbf{v} = (\alpha, 2\alpha, -2)$. Both are nonzero for every real α, since their third components are nonzero. Their magnitude product is therefore positive, and the sign of $\cos\theta$ is exactly the sign of

$$\mathbf{u} \cdot \mathbf{v} = \alpha \cdot \alpha + (-2)(2\alpha) + 2(-2) = \alpha^2 - 4\alpha - 4.$$

The requirement is $\alpha^2 - 4\alpha - 4 > 0$. Equality would give a right angle and is excluded.

Step 2: Solve the strict quadratic inequality Complete the square:

$$\alpha^2 - 4\alpha - 4 = (\alpha - 2)^2 - 8.$$

Thus the scalar product is positive when

$$|\alpha - 2| > 2\sqrt{2},$$

or equivalently $\alpha < 2 - 2\sqrt{2}$ or $\alpha > 2 + 2\sqrt{2}$. The first interval consists entirely of negative numbers, since $\sqrt{2} > 1$. It cannot contain a positive integral answer. Hence only the second interval is relevant.

Step 3: Choose and check the least integer The positive threshold satisfies

$$4 < 2 + 2\sqrt{2} < 5.$$

Therefore the smallest positive integer above the threshold is five. Verify the boundary candidates directly: at $\alpha = 4$, the dot product is $16 - 16 - 4 = -4$, so the angle is obtuse; at $\alpha = 5$, it is $25 - 20 - 4 = 1 > 0$, so the angle is acute. All smaller positive integers lie between the two roots and give negative scalar products, proving minimality.

Step 4: Conclusion The least positive integral value is $\alpha = \boxed{5}$, a dimensionless parameter.

Quick Tip to Solve: For an acute-angle condition, check the sign of the dot product instead of computing the angle. Keep the inequality strict, and test the nearest integer above the positive root.

29. If α satisfies the equation $x^2+x+1=0$ and $(1+\alpha)^7 = A+B\alpha+C\alpha^2, A, B, C \geqslant 0$, then $5(3A-2B-C)$ is equal to _____. [4 marks]

Solution:

Concept:

- The non-real roots of $x^2 + x + 1 = 0$ obey $1 + \alpha + \alpha^2 = 0$ and $\alpha^3 = 1$, with $\alpha \neq 1$. These identities reduce all higher powers to a linear expression in α.
- An equality $u + v\alpha = 0$, with real coefficients u, v, requires both coefficients to vanish: otherwise it would make the non-real number α real.

Step 1: Reduce the seventh power From the quadratic relation, $1 + \alpha = -\alpha^2$. Therefore

$$(1 + \alpha)^7 = (-\alpha^2)^7 = -\alpha^{14} = -\alpha^2 = 1 + \alpha.$$

Here $\alpha^{14} = \alpha^2$ because 14 leaves remainder 2 on division by 3. The negative sign is essential because the exponent 7 is odd.

Step 2: Compare the independent coefficients Rewrite the given expression by eliminating α^2:

$$A + B\alpha + C\alpha^2 = (A - C) + (B - C)\alpha.$$

Comparison with $1+\alpha$ yields $A-C=1$ and $B-C=1$. Thus $A=C+1$ and $B=C+1$. The individual nonnegative values are not unique, but the requested combination can still be unique; there is no need to choose a particular value of C.

Step 3: Evaluate the invariant combination Substitute these two relations directly:

$$3A-2B-C=3(C+1)-2(C+1)-C=1.$$

All terms containing the free coefficient cancel. This also checks that any allowed nonnegative C gives the same answer, rather than introducing an unsupported uniqueness assumption.

Step 4: Conclusion The required dimensionless value is

$$5(3A-2B-C)=5.$$

Quick Tip to Solve: Reduce powers modulo 3, then eliminate α^2. Even if A, B, C are not individually unique, their requested linear combination may be fixed.

30. A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let

$a=P(X=3), b=P(X\geqslant 3)$ **and** $c=P(X\geqslant 6 \mid X>3)$**. Then** $\frac{b+c}{a}$ **is equal to _____.** [4 marks]

Solution:

Concept:

- For repeated independent throws, success is a six with probability $p=1/6$, and failure has probability $q=5/6$. The waiting time for the first success is geometric.
- The events $X=n$ and $X\geq n$ differ by one trial: $P(X=n)=q^{n-1}p$, whereas $P(X\geq n)=q^{n-1}$. Conditional probability uses the intersection divided by the conditioning event.

Step 1: Find the point probability and tail probability Exactly three tosses means two non-sixes followed by a six, so

$$a=P(X=3)=\left(\frac{5}{6}\right)^2\frac{1}{6}=\frac{25}{216}.$$

At least three tosses only requires the first two throws to be non-sixes. The outcome on the third throw is unrestricted:

$$b=P(X\geq 3)=\left(\frac{5}{6}\right)^2=\frac{25}{36}.$$

Step 2: Apply the conditioning event carefully The event $X>3$ means no six in the first three tosses, while $X\geq 6$ means no six in the first five tosses. Since the latter is a subset of the former,

$$c=\frac{P(X\geq 6)}{P(X>3)}=\frac{q^5}{q^3}=q^2=\frac{25}{36}.$$

Equivalently, after the first three failures, two further failures are needed. A sixth-toss success is not

required, because the event is at least six tosses.

Step 3: Form the requested ratio Both tail probabilities are equal, although they arose from different descriptions. Hence

$$\frac{b+c}{a} = \frac{25/36 + 25/36}{25/216} = \frac{50}{36}\frac{216}{25} = 12.$$

The probability factors cancel exactly, so no rounding is needed.

Step 4: Conclusion The required dimensionless ratio is $\boxed{12}$.

Quick Tip to Solve: Translate $X \geq n$ into the first $n-1$ failures and $X > n$ into the first n failures before substituting probabilities.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Position of an ant (S in metres) moving in Y-Z plane is given by $S = 2t^2\hat{j} + 5\hat{k}$ (where t is in second). The magnitude and direction of velocity of the ant at $t = 1$ s will be : [4 marks]

(A) 16 m/s in y-direction

(B) 4 m/s in x-direction

(C) 9 m/s in z-direction

(D) 4 m/s in y-direction

Correct Answer: (D) 4 m/s in y-direction

Solution:

Concept:

- Instantaneous velocity is the time derivative of position: $\vec{v} = d\vec{S}/dt$. Differentiate each Cartesian component separately; the fixed unit vectors do not change with time.
- A constant coordinate determines location, but contributes zero velocity. The direction of velocity follows the nonzero differentiated components, rather than the direction of the position vector.

Step 1: Differentiate both position components The position vector has coordinates $x = 0$, $y = 2t^2$ and $z = 5$, in metres. Therefore

$$\vec{v} = \frac{d}{dt}(2t^2)\hat{j} + \frac{d}{dt}(5)\hat{k} = 4t\hat{j} + 0\hat{k}.$$

The ant stays in the plane $z = 5$, and its changing coordinate is only y. The coefficient 5 does not represent motion along the z-axis.

Step 2: Evaluate the vector at the specified time At $t = 1\,\text{s}$, the velocity is

$$\vec{v}(1) = 4\hat{j}\,\text{m s}^{-1}, \qquad |\vec{v}| = \sqrt{0^2 + 4^2 + 0^2} = 4\,\text{m s}^{-1}.$$

Its positive $\hat{j}$ component gives the positive y-direction. This is an instantaneous result, not an average displacement divided by the elapsed time.

Step 3: Compare all listed choices Option (A) gives the right axis but an incorrect magnitude:

differentiation gives 4 rather than 16. Option (B) has the correct magnitude but assigns it to the x-axis, where the position never changes. Option (C) incorrectly interprets a constant z-coordinate as motion. Only option (D) agrees with both the calculated speed and its direction.

Step 4: Conclusion The ant has velocity of magnitude $4\,\mathrm{m\,s^{-1}}$ in the positive y-direction. Hence, the correct option is (D).

Quick Tip to Solve: Differentiate the coefficients of $\hat{i}, \hat{j}, \hat{k}$ individually; a constant component of position always contributes zero velocity.

32. Given below are two statements :

Statement (I) :Viscosity of gases is greater than that of liquids.

Statement (II) : Surface tension of a liquid decreases due to the presence of insoluble impurities.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is correct but statement II is incorrect

(B) Statement I is incorrect but Statement II is correct

(C) Both Statement I and Statement II are incorrect

(D) Both Statement I and Statement II are correct

Correct Answer: (B) Statement I is incorrect but Statement II is correct

Solution:

Concept:

- The coefficient of dynamic viscosity η relates tangential stress to velocity gradient: $\tau = \eta\, dv/dy$. Under ordinary comparable conditions, liquids have much higher dynamic viscosity than gases.
- Surface tension is a property of the interface. Insoluble surface-active impurities form a film at the liquid surface and reduce its surface tension; this differs from effects of dissolved salts, which need not lower it.

Step 1: Judge Statement I Statement I says that gases have greater viscosity than liquids. In the standard comparison intended here, this is reversed: the dynamic viscosity of water is of order $10^{-3}\,\mathrm{Pa\,s}$, whereas that of air is of order $10^{-5}\,\mathrm{Pa\,s}$ near room temperature. Liquids resist relative sliding of neighbouring layers much more strongly. Thus Statement I is incorrect. This comparison concerns dynamic viscosity, rather than kinematic viscosity, which also includes density.

Step 2: Judge Statement II An insoluble impurity spread as a film on a liquid surface reduces the effective cohesive action responsible for surface tension. The familiar example is an oily surface film, which lowers the tension of water. In the standard surface-impurity treatment used by the question, Statement II is therefore correct. The result should not be confused with soluble salts: the nature and concentration of a dissolved solute can produce a different effect.

Step 3: Match the truth values to the options The truth-value pair is I: incorrect, II: correct. Option (A) reverses both assessments; option (C) incorrectly rejects Statement II; option (D) incorrectly accepts Statement I. Option (B) alone records the pair found by considering the two physical properties

separately.

Step 4: Conclusion Statement I is incorrect, while Statement II is correct. Hence, the correct option is (B).

Quick Tip to Solve: Do not confuse dynamic viscosity with viscosity divided by density; also distinguish a surface film from a soluble salt when judging surface-tension statements.

33. If the refractive index of the material of a prism is $\cot\left(\frac{A}{2}\right)$, where A is the angle of prism then the angle of minimum deviation will be [4 marks]

(A) $\pi - 2$ A

(B) $\frac{\pi}{2} - 2$ A

(C) $\pi -$ A

(D) $\frac{\pi}{2} -$ A

Correct Answer: (A) $\pi - 2$ A

Solution:

Concept:

- For a prism in air at minimum deviation δ_m, the path is symmetric: $i = e$ and $r_1 = r_2 = A/2$. The deviation is then $\delta_m = 2i - A$.
- Snell's law gives $\mu = \sin[(A + \delta_m)/2]/\sin(A/2)$. The incidence angle lies between zero and $\pi/2$, so the physically relevant inverse-sine branch must be used.

Step 1: Substitute the given refractive index Use $\mu = \cot(A/2)$ in the minimum-deviation relation:

$$\sin\left(\frac{A+\delta_m}{2}\right) = \mu \sin\left(\frac{A}{2}\right) = \cot\left(\frac{A}{2}\right)\sin\left(\frac{A}{2}\right) = \cos\left(\frac{A}{2}\right).$$

The cancellation is exact. This replaces the refractive-index condition with a direct trigonometric condition for the angle of incidence.

Step 2: Select the physical angle and solve For the ordinary prism geometry, the incidence angle $i = (A + \delta_m)/2$ is acute or grazing. Since $\cos(A/2) = \sin(\pi/2 - A/2)$, the physical solution is

$$\frac{A+\delta_m}{2} = \frac{\pi}{2} - \frac{A}{2}.$$

Consequently $A + \delta_m = \pi - A$, giving $\delta_m = \pi - 2A$. The other sine branch would give an incidence angle greater than $\pi/2$, which does not describe the entering ray.

Step 3: Check the candidate expressions Expression (A) reproduces the derived result. Expressions (B) and (D) use $\pi/2$ where the doubled incidence contributes π. Expression (C) omits one subtraction of A. For example, at $A = \pi/3$, the formula gives $\mu = \sqrt{3}$, incidence $\pi/3$, and minimum deviation $\pi/3$, consistent with (A).

Step 4: Conclusion The angle of minimum deviation is $\boxed{\delta_m = \pi - 2A}$. Hence, the correct option is (A).

Quick Tip to Solve: At minimum deviation first find i from $\sin i = \mu \sin(A/2)$, then use $\delta_m = 2i - A$; keep the physical acute-angle branch.

34. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $\overrightarrow{\mathrm{E}}$ and $\overrightarrow{\mathrm{B}}$ represent the electric and magnetic fields respectively, then the region of space may have :

(A) $\mathrm{E} = 0,\ \mathrm{B} = 0$

(B) $\mathrm{E} = 0,\ \mathrm{B} \neq 0$

(C) $\mathrm{E} \neq 0,\ \mathrm{B} = 0$

(D) $\mathrm{E} \neq 0,\ \mathrm{B} \neq 0$

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A), (B) and (C) only

(B) (A), (C) and (D) only

(C) (A), (B) and (D) only

(D) (B), (C) and (D) only

Correct Answer: (C) (A), (B) and (D) only

Solution:

Concept:

- A proton has nonzero charge q. Constant velocity means zero acceleration, so the net electromagnetic force must vanish: $q(\vec{E} + \vec{v} \times \vec{B}) = 0$.
- The magnetic contribution vanishes when $\vec{v}$ is parallel to $\vec{B}$. When both fields are present, electric and magnetic forces may cancel for a suitable field geometry and speed.

Step 1: Test the two cases without an electric field In case (A), both fields vanish, and the Lorentz force is zero automatically. Thus uniform motion is possible. In case (B), the magnetic field is nonzero but the electric field vanishes. Choose motion parallel or antiparallel to the magnetic field; then

$$\vec{v} \times \vec{B} = 0.$$

The proton again moves without changing its velocity. A nonzero magnetic field does not always bend a trajectory: it acts only on the component of motion perpendicular to itself.

Step 2: Test the cases with an electric field In case (C), $\vec{B} = 0$ but $\vec{E} \neq 0$. The nonzero force $q\vec{E}$ accelerates the proton, so this case cannot give unchanged velocity. In case (D), both fields can be nonzero and satisfy

$$\vec{E} = -\vec{v} \times \vec{B}.$$

For example, mutually perpendicular fields can balance at speed $v = E/B$. The question asks which cases may occur, so the existence of a suitable geometry is sufficient.

Step 3: Select the combination of possible cases Cases (A), (B) and (D) are possible, while case (C) is impossible. The first, second and fourth answer combinations all include case (C), so each fails the zero-force condition. The third answer combination contains exactly the three allowed cases; its option letter is (C), distinct from the impossible field case also labelled (C).

Step 4: Conclusion The allowed situations are zero fields, a magnetic field parallel to the velocity, or nonzero fields whose electric and magnetic forces cancel. A nonzero electric field alone is excluded. Hence, the correct option is (C).

Quick Tip to Solve: Use the vector condition $\vec{E} + \vec{v} \times \vec{B} = 0$. A magnetic field parallel to velocity exerts no force, while crossed fields may cancel.

35. The acceleration due to gravity on the surface of earth is g. If the diameter of earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of earth would be : [4 marks]

(A) g/4

(B) 2g

(C) g/2

(D) 4g

Correct Answer: (D) 4g

Solution:

Concept:

- The gravitational acceleration immediately outside a spherical body of mass M and radius R is $g = GM/R^2$, where G is the universal gravitational constant.
- Diameter and radius scale in the same proportion because $D = 2R$. With fixed mass, surface gravity is inversely proportional to the square of the radius, rather than directly proportional to diameter.

Step 1: Translate the diameter change into a radius change Let the original diameter be $D = 2R$. The new diameter is $D\prime = D/2$, so its radius is

$$R\prime = \frac{D\prime}{2} = \frac{D}{4} = \frac{R}{2}.$$

Both lengths halve. The mass stays at M, exactly as specified, so there is no additional mass factor to include. In particular, we must not use a constant-density assumption, which would change the mass as the volume changes.

Step 2: Compare the new and old surface accelerations At the new surface, the distance from the centre is $R/2$. Therefore

$$g\prime = \frac{GM}{(R/2)^2} = \frac{GM}{R^2/4} = 4\frac{GM}{R^2} = 4g.$$

Equivalently, $g\prime/g = (R/R\prime)^2 = 2^2 = 4$. Moving the surface closer to the same total mass increases its gravity, as also expected from the inverse-square law.

Step 3: Eliminate the other scaling factors Options (A) and (C) predict weaker surface gravity, contrary to decreasing the distance from the same mass. Option (B) gives only a factor of two and therefore uses an inverse-linear dependence instead of the required inverse-square dependence. Option (D) contains the factor of four obtained from squaring the radius ratio.

Step 4: Conclusion The new surface gravitational acceleration is $\boxed{4g}$, directed toward the centre. Hence, the correct option is (D).

Quick Tip to Solve: For fixed mass use $g\prime/g = (R/R\prime)^2$. A halved diameter means a halved radius, so square the factor of two.

36. A train is moving with a speed of $12\ \mathrm{m/s}$ on rails which are $1.5\ \mathrm{m}$ apart. To negotiate a curve radius $400\ \mathrm{m}$, the height by which the outer rail should be raised with respect to the inner rail is (Given, $g = 10\ \mathrm{m/s^2}$) : [4 marks]

(A) 6.0 cm

(B) 5.4 cm

(C) 4.8 cm

(D) 4.2 cm

Correct Answer: (B) 5.4 cm

Solution:

Concept:

- For a banked curve negotiated without relying on lateral friction, the horizontal component of the normal reaction supplies the centripetal force: $N\sin\theta = mv^2/r$.
- The vertical component balances weight, $N\cos\theta = mg$. Hence $\tan\theta = v^2/(rg)$, where v is speed and r is the curve radius.
- For closely spaced rails with a small banking angle, the elevation is $h \simeq d\tan\theta$, where d is the separation of the rails.

Step 1: Find the required banking angle The speed is $v = 12\,\mathrm{m\,s^{-1}}$, the radius is $r = 400\,\mathrm{m}$, and $g = 10\,\mathrm{m\,s^{-2}}$. Dividing the horizontal force equation by the vertical one eliminates both mass and the normal reaction. Thus

$$\tan\theta = \frac{12^2}{400 \times 10} = 0.036.$$

The angle is small, which justifies using the standard railway-banking approximation.

Step 2: Calculate the difference in rail heights Use the rail separation $d = 1.5\,\mathrm{m}$. The outer rail must be higher so that the normal reaction has a component directed towards the centre of the curve.

$$h \simeq d\tan\theta = 1.5 \times 0.036 = 0.054\,\mathrm{m} = 5.4\,\mathrm{cm}.$$

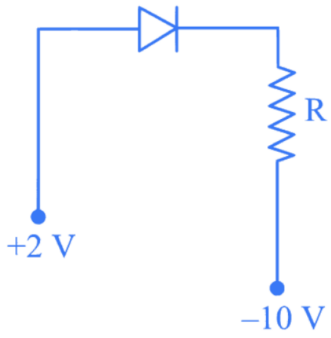
If the separation is measured along the sloping track, $h = d\sin\theta$ gives approximately $5.397\,\mathrm{cm}$, the same listed value. The alternatives 6.0, 4.8, and $4.2\,\mathrm{cm}$ do not satisfy the required banking ratio.

Step 3: Conclusion The outer rail should be raised by approximately $\boxed{5.4\,\mathrm{cm}}$. Hence, the correct option is (B).

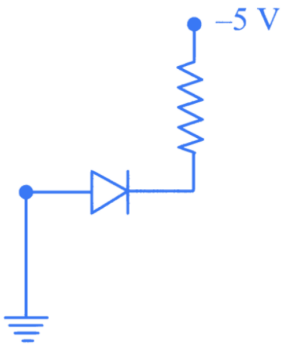
Quick Tip to Solve: First calculate the dimensionless ratio $v^2/(rg)$, multiply by the track width, and convert metres to centimetres only at the end.

37. Which of the following circuits is reverse - biased? [4 marks]

(A)



(B)



(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- A diode is forward biased when its anode (p side) is at a higher potential than its cathode (n side); it is reverse biased when the anode is at a lower potential.
- The straight bar in the diode symbol identifies the cathode. Ground represents $0\,\text{V}$; negative voltages must be compared algebraically.
- At the onset of reverse bias, the ideal diode carries negligible current, so a series resistor has negligible voltage drop. The applied end potentials determine the bias.

Step 1: Compare the potentials in the first three circuits In each diagram the diode anode is on the left and the cathode is on the right. In (A), the applied anode-side potential is $+2\,\mathrm{V}$, while the cathode-side end is at $-10\,\mathrm{V}$. The applied difference is $+12\,\mathrm{V}$, which favours forward bias. In (B), the anode is grounded and the cathode side is connected through a resistor to $-5\,\mathrm{V}$. The anode is therefore higher by $5\,\mathrm{V}$, again favouring forward bias. In (C), the anode side is supplied from $+2\,\mathrm{V}$ through the resistor and the cathode is grounded, giving forward bias.

Step 2: Identify the reverse-biased connection In (D), the anode is connected to $+2\,\mathrm{V}$, whereas the cathode is connected through a resistor to $+4\,\mathrm{V}$. With negligible reverse current, the resistor has negligible drop and the cathode remains at approximately $+4\,\mathrm{V}$. Thus

$$V_A - V_K = 2 - 4 = -2\,\mathrm{V}.$$

The anode is less positive than the cathode, so this connection increases the junction barrier and opposes ordinary diode conduction. A resistor limits current when conduction occurs; its position alone does not decide the bias.

Step 3: Conclusion Circuit (D) is reverse biased because its anode is at a lower potential than its cathode. Hence, the correct option is (D).

Quick Tip to Solve: Mark the cathode bar first, then subtract its potential from the anode potential. A negative applied anode-to-cathode voltage indicates reverse bias.

38. Identify the physical quantity that cannot be measured using spherometer : [4 marks]

(A) Radius of curvature of concave surface

(B) Specific rotation of liquids

(C) Thickness of thin plates

(D) Radius of curvature of convex surface

Correct Answer: (B) Specific rotation of liquids

Solution:

Concept:

- A spherometer is a mechanical instrument for measuring very small differences in height using a central micrometer screw and three supporting legs.
- It measures the sagitta h of a spherical surface. If the three legs form an equilateral triangle of side l, the radius of curvature is $R = l^2/(6h) + h/2$, using the magnitude of the sagitta.
- Specific rotation describes optical activity: the rotation of the plane of polarisation by a solution, divided by the optical path length and concentration in the specified convention.

Step 1: Check the geometrical measurements On a plane reference surface, note the central-screw reading. Placing the spherometer on a concave or convex spherical surface changes the screw position relative to the three feet. That difference provides the sagitta, and the measured leg separation supplies the other geometrical quantity needed to calculate the radius. Therefore both (A) and (D), the radii of curvature of concave and convex surfaces, can be determined. The concave and

convex cases differ in the sense of the height displacement, rather than in whether the instrument can measure them.

Step 2: Separate thickness from optical rotation A thin plate on a plane reference surface raises the contact point of the central screw. Comparing readings with and without the plate gives its thickness, provided the support arrangement is suitable. Thus (C) is also a valid spherometer measurement. By contrast, specific rotation of a liquid requires measuring the rotation angle of plane-polarised light through a known path length and concentration. This is the function of a polarimeter. A spherometer contains no polariser or analyser and its screw displacement cannot determine an optical rotation angle.

Step 3: Conclusion The quantity that cannot be measured with a spherometer is the specific rotation of liquids. Hence, the correct option is (B).

Quick Tip to Solve: Associate a spherometer with height, thickness and spherical curvature; associate specific rotation with a polarimeter and plane-polarised light.

39. Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of magnitude of their linear momentum is : [4 marks]

(A) $3 : 5$

(B) $5 : 4$

(C) $2 : 5$

(D) $4 : 5$

Correct Answer: (C) $2 : 5$

Solution:

Concept:

- The nonrelativistic kinetic energy of a body is $K = \frac{1}{2}mv^2$, while the magnitude of linear momentum is $p = mv$.
- Eliminating speed gives $K = p^2/(2m)$, or $p = \sqrt{2mK}$. At fixed kinetic energy, momentum is proportional to the square root of mass.

Step 1: Express the momentum ratio in terms of masses Let the momenta of the $4\,\mathrm{g}$ and $25\,\mathrm{g}$ bodies be p_1 and p_2, respectively. The question specifies equal kinetic energies, not equal speeds. Therefore

$$\frac{p_1^2}{2m_1} = \frac{p_2^2}{2m_2}.$$

Taking the positive square root is appropriate because the question asks for momentum magnitudes:

$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}.$$

There is no information about the directions of motion, and none is required for this ratio.

Step 2: Substitute and check the options Both masses are given in grams, so their common unit

cancels in the mass ratio. Hence

$$\frac{p_1}{p_2} = \sqrt{\frac{4}{25}} = \frac{2}{5}.$$

The lighter body has the smaller momentum even though it must move faster to have the same kinetic energy. One can check the result using $v_1/v_2 = \sqrt{25/4} = 5/2$, so $p_1/p_2 = (4/25)(5/2) = 2/5$. The ratio $5 : 4$ incorrectly makes the lighter body have greater momentum. The ratios $3 : 5$ and $4 : 5$ also fail the squared-ratio test $(p_1/p_2)^2 = 4/25$.

Step 3: Conclusion The ratio of momentum magnitudes in the stated order is $\boxed{2 : 5}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal kinetic energies use $p \propto \sqrt{m}$; for equal speeds use $p \propto m$. Identify the fixed quantity before forming a ratio.

---

40. 0.08 kg **air is heated at constant volume through** 5°C**. The specific heat of air at constant volume is** 0.17 kcal/kg$^\circ$C **and** J $= 4.18$ **joule/** cal**. The change in its internal energy is approximately.** [4 marks]

(A) 318 J

(B) 298 J

(C) 284 J

(D) 142 J

Correct Answer: (C) 284 J

Solution:

Concept:

- The first law of thermodynamics is $Q = \Delta U + W$, where Q is heat supplied and W is work done by the gas.
- At constant volume the expansion work $W = \int P\,dV$ is zero. The heat supplied therefore changes internal energy: $\Delta U = mc_V\Delta T$.
- A temperature rise of 5°C is a difference of $5\,$K. The conversion given is $1\,\text{cal} = 4.18\,\text{J}$, and $1\,\text{kcal} = 1000\,\text{cal}$.

Step 1: Calculate the heat supplied in kilocalories The mass of air is $m = 0.08\,\text{kg}$, its constant-volume specific heat is $c_V = 0.17\,\text{kcal}\,\text{kg}^{-1}\,^\circ\text{C}^{-1}$, and the rise in temperature is 5°C. Thus

$$Q = mc_V\Delta T = 0.08 \times 0.17 \times 5 = 0.068\,\text{kcal}.$$

The container does not expand, so none of this heat is used in pressure-volume work. In particular, using a constant-pressure specific heat would be inappropriate.

Step 2: Convert to joules and determine the sign Convert the energy before comparing with the choices:

$$\Delta U = 0.068 \times 1000 \times 4.18 = 284.24\,\text{J}.$$

Since the air is heated, its temperature and internal energy increase; the sign is positive. Rounding to the precision of the listed options gives $284\,$J. The value $142\,$J is half the computed result, while $298\,$J

and $318\,\mathrm{J}$ do not follow from the given heat capacity and conversion factor. No additional work term should be subtracted.

Step 3: Conclusion The internal energy increases by approximately $\boxed{284\,\mathrm{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: At constant volume set expansion work to zero immediately. Keep kcal and cal distinct: the joule conversion here requires both the factor 1000 and the factor 4.18.

41. The radius of third stationary orbit of electron for Bohr's atom is R. The radius of fourth stationary orbit will be: [4 marks]

(A) $\frac{4}{3}\mathrm{R}$
(B) $\frac{16}{9}R$
(C) $\frac{3}{4}R$
(D) $\frac{9}{16}\mathrm{R}$

Correct Answer: (B) $\frac{16}{9}R$

Solution:

Concept:

- For a hydrogen-like atom in the Bohr model, the radius of the nth stationary orbit is $r_n = a_0 n^2/Z$, where a_0 is the Bohr radius and Z is the nuclear charge number.
- For different orbits of the same atom, Z and a_0 are unchanged. Hence orbit radius scales as n^2, rather than as n or $1/n^2$.

Step 1: Use the third orbit to express the common factor The third orbit is specified to have radius R. Substituting $n = 3$ into the Bohr radius formula gives

$$r_3 = \frac{a_0}{Z}\,3^2 = \frac{9a_0}{Z} = R.$$

It follows that $a_0/Z = R/9$. There is no need to know the numerical value of the Bohr radius or the identity of the atom, because both orbit radii refer to the same atom. The statement concerns the orbit radius, not the electron energy.

Step 2: Calculate the fourth-orbit radius and reject other scalings For the fourth stationary orbit, $n = 4$, so

$$r_4 = \frac{16a_0}{Z} = \frac{16}{9}R.$$

Equivalently, $r_4/r_3 = (4/3)^2$. The answer $4R/3$ would incorrectly assume a linear relation between radius and orbit number. The answers $3R/4$ and $9R/16$ predict a smaller radius for the higher orbit, contrary to $r_n \propto n^2$. The inverse-square dependence belongs to the magnitude of the Bohr energy, not to the radius. The result has the correct length dimension and is larger than R, as required.

Step 3: Conclusion The fourth stationary orbit has radius $\boxed{16R/9}$. Hence, the correct option is (B).

Quick Tip to Solve: For two orbits of the same Bohr atom, write $r_m/r_n = m^2/n^2$. This cancels the nuclear charge and all physical constants.

42. A rectangular loop of length $2.5\ \text{m}$ and width $2\ \text{m}$ is placed at 60° to a magnetic field of $4\ \text{T}$. The loop is removed from the field in $10\ \text{sec}$. The average emf induced in the loop during this time is [4 marks]

(A) $-2\ \text{V}$

(B) $+2\ \text{V}$

(C) $+1\ \text{V}$

(D) $-1\ \text{V}$

Correct Answer: (C) $+1\ \text{V}$

Solution:

Concept:

- The magnetic flux through a planar loop is $\Phi = BA\cos\theta$, where θ is the angle between the magnetic field and the area normal.
- Faraday's law gives the average induced emf of a single loop as $\mathcal{E}_{\text{av}} = -\Delta\Phi/\Delta t$. Lenz's minus sign means the induced effect opposes the change of flux.

Step 1: Calculate the initial linked flux The area of the rectangle is

$$A = 2.5 \times 2 = 5\,\text{m}^2.$$

Using the stated 60° as the flux angle, measured from the loop normal, gives

$$\Phi_i = 4 \times 5 \times \cos 60^\circ = 10\,\text{Wb}.$$

The positive normal is chosen to give positive initial flux. The angle in the flux formula is always measured from that normal. If an angle were explicitly specified from the plane instead, its complementary angle would have to be used; the value $+1\,\text{V}$ in the choices corresponds to the normal-angle convention.

Step 2: Find the change of flux and average emf The loop is fully removed from the magnetic field, so its final linked flux is zero. Over the stated interval $10\,\text{s}$,

$$\Delta\Phi = \Phi_f - \Phi_i = 0 - 10 = -10\,\text{Wb}.$$

Faraday's law then gives

$$\mathcal{E}_{\text{av}} = -\frac{-10}{10} = +1\,\text{V}.$$

The positive sign is relative to the chosen positive area normal and its associated circulation direction. The negative option $-1\,\text{V}$ omits the minus sign in Faraday's law for that convention. The options of magnitude $2\,\text{V}$ omit the projection factor $\cos 60^\circ$; the total area alone is not the effective area normal to the field.

Step 3: Conclusion Using the stated angle as the angle with the area normal, the average induced emf is $\boxed{+1\,\text{V}}$, opposing the decrease in flux. Hence, the correct option is (C).

Quick Tip to Solve: Write initial and final fluxes separately before applying the minus sign. For a removed loop the final flux is zero, and the projection factor belongs in the initial flux.

43. An electric charge $10^{-6}\mu C$ is placed at origin $(0, 0)$ m of X − Y co-ordinate system. Two points P and Q are situated at $(\sqrt{3}, \sqrt{3})$m and $(\sqrt{6}, 0)$m respectively. The potential difference between the points P and Q will be : [4 marks]

(A) $\sqrt{3}$ V

(B) $\sqrt{6}$ V

(C) 0 V

(D) 3 V

Correct Answer: (C) 0 V

Solution:

Concept:

- The potential due to a point charge at distance r, taking infinity as zero potential, is $V = kq/r$, where $k = 1/(4\pi\epsilon_0)$. Potential is a scalar and depends on radial distance, not on the direction of the position vector.
- Every point on a circle centred on an isolated point charge has the same potential. Thus equal distances from the charge imply zero potential difference, even though the electric field directions may differ.

Step 1: Calculate both distances from the charge The charge is at the origin, so use the magnitude of each position vector. For the point P, both coordinates contribute to its distance, whereas Q lies on the positive x-axis.

$$r_P = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2} = \sqrt{6}\,\text{m}.$$

$$r_Q = \sqrt{(\sqrt{6})^2 + 0^2} = \sqrt{6}\,\text{m}.$$

The different coordinates therefore do not imply different distances from the charge.

Step 2: Compare the scalar potentials Substitute the equal radii into the point-charge potential formula. The given charge has the same value in both expressions, so it cancels from their difference:

$$V_P - V_Q = kq\left(\frac{1}{r_P} - \frac{1}{r_Q}\right) = kq\left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}}\right) = 0.$$

This result does not require evaluating the unusually small charge numerically. Each nonzero option would require unequal radial distances, which the coordinates rule out.

Step 3: Conclusion The two points are on the same equipotential surface, giving $V_P - V_Q = 0\,\text{V}$. Hence, the correct option is (C).

Quick Tip to Solve: Before inserting a point charge into a potential formula, compare the distances: equal radii immediately give zero potential difference.

44. A convex lens of focal length 40 cm forms an image of an extended source of light on a photoelectric cell. A current I is produced. The lens is replaced by another convex lens having the same diameter but focal length 20 cm. The photoelectric current now is : [4 marks]

(A) $\frac{I}{2}$

(B) 4 I

(C) 2 I

(D) I

Correct Answer: (D) I

Solution:

Concept:

- Photoelectric current measures the charge emitted per second. For fixed frequency above the threshold and unchanged collection conditions, it is proportional to the total rate of photons incident on the active photosensitive surface.
- A lens collects light through its aperture. For the same source illumination, equal lens diameters give the same collecting area, $A = \pi D^2/4$, and therefore the same collected optical power.

Step 1: Compare the light collected by the two lenses Let the irradiance reaching the lens be S, and let both lenses have diameter D. Their collected power, neglecting identical transmission losses, is

$$P = S\frac{\pi D^2}{4}.$$

Changing the focal length from 40 cm to 20 cm changes the image size and its concentration of light, but does not change the aperture area. The image of the source is formed on the cell in each arrangement, so the total collected light reaches the cell.

Step 2: Relate the total photon rate to the current At photon energy $h\nu$, the incident photon rate is $P/(h\nu)$. If η is the unchanged photoelectric yield and e the elementary charge,

$$I = \eta e\frac{P}{h\nu}.$$

Neither P, ν, nor the cell response changes, hence $I'/I = 1$. The choice $4I$ confuses a possible rise in image irradiance with a rise in total incident power; $2I$ similarly treats focal length as a linear multiplier of collected flux. The current cannot become $I/2$ when the same aperture still collects the same light.

Step 3: Conclusion The total photon rate on the photoelectric cell stays unchanged, so the new photoelectric current is I. Hence, the correct option is (D).

Quick Tip to Solve: For a lens forming the complete image on a photocell, compare aperture areas to find total current; a smaller image alone does not increase total photon flux.

45. A body of mass $1000\ \mathrm{kg}$ is moving horizontally with a velocity $6\ \mathrm{m/s}$. If $200\ \mathrm{kg}$ extra mass is added, the final velocity (in $\mathrm{m/s}$) is: [4 marks]

(A) 6

(B) 2

(C) 3

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- When mass is added to a moving body without an external horizontal impulse, the total horizontal momentum of the body plus added mass is conserved during the joining process.
- The added mass has zero initial horizontal velocity in this mass-loading situation. The common final speed is obtained from $mu = (m + \Delta m)v$; kinetic energy need not be conserved because the joining is inelastic.

Step 1: Write the initial and final momenta Take the original direction of motion as positive. The original body has mass $m = 1000\,\mathrm{kg}$ and speed $u = 6\,\mathrm{m\,s^{-1}}$. The added $200\,\mathrm{kg}$ contributes no initial horizontal momentum. Thus

$$p_i = (1000)(6) + (200)(0) = 6000\,\mathrm{kg\,m\,s^{-1}}.$$

After attachment the combined mass is $1200\,\mathrm{kg}$, moving together at speed v, so its momentum is $1200v$.

Step 2: Conserve momentum and check the choices Equating the momenta before and after the mass joins gives

$$1200v = 6000, \qquad v = \frac{6000}{1200} = 5\,\mathrm{m\,s^{-1}}.$$

The speed decreases because the same total momentum is now shared by a larger mass. A final speed of 6 would require extra horizontal momentum. Speeds of 2 or 3 would instead make the final momentum only 2400 or 3600 in SI units, inconsistent with the conserved value 6000. Conservation of kinetic energy is inappropriate: some mechanical energy is transformed during attachment.

Step 3: Conclusion The final velocity is $5\,\mathrm{m\,s^{-1}}$ in the original direction of motion. Hence, the correct option is (D).

Quick Tip to Solve: For mass added from rest, multiply the old speed by old mass divided by total mass; conserve momentum rather than kinetic energy.

46. A plane electromagnetic wave propagating in x-direction is described by

$$E_y = \left(200\text{Vm}^{-1}\right) \sin\left[1.5 \times 10^7 t - 0.05x\right] ;$$

The intensity of the wave is :

(Use $\epsilon_0 = 8.85 \times 10^{-12}\text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$) [4 marks]

(A) $35.4\ \text{Wm}^{-2}$

(B) $53.1\ \text{Wm}^{-2}$

(C) $26.6\ \text{Wm}^{-2}$

(D) $106.2\ \text{Wm}^{-2}$

Correct Answer: (B) $53.1\ \text{Wm}^{-2}$

Solution:

Concept:

- For a sinusoidal plane electromagnetic wave in vacuum, the time-averaged intensity is $\mathcal{I} = \frac{1}{2}\epsilon_0 c E_0^2$, where E_0 is the peak electric field. The factor one-half comes from the average of $\sin^2$.
- In the phase $\omega t - kx$, ω is angular frequency and k is wave number. Their ratio gives the propagation speed, $c = \omega/k$.

Step 1: Read the amplitude and the propagation speed Compare the given field with $E_y = E_0 \sin(\omega t - kx)$. Its peak amplitude is $200\,\text{V m}^{-1}$, not an rms value. The phase coefficients give

$$\omega = 1.5 \times 10^7\,\text{s}^{-1}, \qquad k = 0.05\,\text{m}^{-1},$$

$$c = \frac{1.5 \times 10^7}{0.05} = 3.0 \times 10^8\,\text{m s}^{-1}.$$

This agrees with the speed of electromagnetic radiation in vacuum.

Step 2: Evaluate the average energy flux Insert the specified permittivity and the peak field into the average intensity formula:

$$\mathcal{I} = \frac{1}{2}(8.85 \times 10^{-12})(3.0 \times 10^8)(200)^2.$$

$$\mathcal{I} = \frac{1}{2}(8.85)(3)(4) = 53.1\,\text{W m}^{-2}.$$

The powers of ten cancel because $200^2 = 4 \times 10^4$. The choice 106.2 omits the averaging factor and represents the maximum instantaneous energy flux. The choice 26.6 applies that factor twice, while 35.4 does not follow from the stated wave speed and amplitude.

Step 3: Conclusion The intensity of the electromagnetic wave is $53.1\,\text{W m}^{-2}$. Hence, the correct option is (B).

Quick Tip to Solve: Identify whether the quoted electric field is peak or rms: use one-half with the peak amplitude, and obtain speed directly from the phase coefficients.

47. Given below are two statements :

Statement (I) : Planck's constant and angular momentum have same dimensions.

Statement (II) : Linear momentum and moment of force have same dimensions.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are false

(C) Both Statement I and Statement II are true

(D) Statement I is false but Statement II is true

Correct Answer: (A) Statement I is true but Statement II is false

Solution:

Concept:

- Two physical quantities have the same dimensions only when the exponents of mass M, length L, and time T match separately. Equal dimensions do not require the quantities to have the same physical meaning.
- Use defining equations to determine dimensions: $E = h\nu$, angular momentum $\boldsymbol{\ell} = \mathbf{r} \times \mathbf{p}$, linear momentum $\mathbf{p} = m\mathbf{v}$, and torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$.

Step 1: Test the first statement Energy has dimensions $[E] = ML^2T^{-2}$, while frequency has dimensions $[\nu] = T^{-1}$. Therefore

$$[h] = \frac{[E]}{[\nu]} = ML^2T^{-1}.$$

Angular momentum is position crossed with momentum, giving

$$[\ell] = L(MLT^{-1}) = ML^2T^{-1}.$$

The dimensions agree exactly, so Statement I is true. Planck's constant also has the unit joule second, consistent with this result.

Step 2: Test the second statement and select the combination Linear momentum is mass times velocity:

$$[p] = M(LT^{-1}) = MLT^{-1}.$$

Moment of force means torque, which is force times perpendicular distance:

$$[\tau] = (MLT^{-2})L = ML^2T^{-2}.$$

Both the length exponent and the time exponent differ between momentum and torque. Statement II is therefore false. The alternatives asserting that both are true, both are false, or only the second is true each contradict at least one of these explicit dimensional calculations.

Step 3: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: For a moment of any vector quantity, multiply its dimensions by one factor of length: moment of momentum and moment of force are different quantities.

48. A wire of length $10\ \text{cm}$ and radius $\sqrt{7} \times 10^{-4}\ \text{m}$ connected across the right gap of a meter bridge. When a resistance of 4.5Ω is connected on the left gap by using a resistance box, the balance length is found to be at $60\ \text{cm}$ from the left end. If the resistivity of the wire is $\text{R} \times 10^{-7}\Omega\text{m}$, then value of R is : [4 marks]

(A) 63

(B) 70

(C) 66

(D) 35

Correct Answer: (C) 66

Solution:

Concept:

- At the balance point of a uniform metre bridge, the resistance ratio equals the ratio of bridge-wire lengths: $P/Q = l/(100 - l)$, where P is in the left gap and Q in the right gap.
- The resistance of a uniform wire is $Q = \rho L/A$, so its resistivity is $\rho = QA/L$. The cross-sectional area of a circular wire is $A = \pi r^2$.

Step 1: Find the resistance in the right gap The left-gap resistance is $P = 4.5\,\Omega$. Since the balance point is 60 cm from the left end, the bridge lengths on the two sides are 60 cm and 40 cm. Thus

$$\frac{4.5}{Q} = \frac{60}{40}, \qquad Q = 4.5\frac{40}{60} = 3\,\Omega.$$

The right-gap resistance is smaller, as expected because it corresponds to the shorter bridge-wire length.

Step 2: Use the specimen dimensions to find resistivity The specimen length is $L = 10\,\text{cm} = 0.10\,\text{m}$, not the 40 cm bridge segment. Its radius gives

$$A = \pi(\sqrt{7} \times 10^{-4})^2 = 7\pi \times 10^{-8}\,\text{m}^2.$$

Therefore

$$\rho = \frac{3(7\pi \times 10^{-8})}{0.10} = 21\pi \times 10^{-7}\,\Omega\,\text{m}.$$

Using $\pi = 22/7$, the coefficient is $21\pi = 66$. The choices 63, 70 and 35 do not satisfy both the measured bridge ratio and the wire dimensions. Notice that the radius must be squared before substituting the area.

Step 3: Conclusion The resistivity is approximately $66 \times 10^{-7}\,\Omega\,\text{m}$, so the dimensionless coefficient $R = 66$. Hence, the correct option is (C).

Quick Tip to Solve: Keep the bridge-wire lengths separate from the specimen length: first find its resistance using the bridge, then use its own area and length for resistivity.

49. A wire of resistance R and length L is cut into 5 equal parts. If these parts are joined parallely, then resultant resistance will be : [4 marks]

(A) $\frac{1}{25}$R

(B) $\frac{1}{5}R$

(C) 25 R

(D) 5 R

Correct Answer: (A) $\frac{1}{25}$R

Solution:

Concept:

- For a uniform wire, resistance is $R = \rho L/A$. Cutting it into equal lengths leaves its material resistivity ρ and cross-sectional area A unchanged, so each piece has resistance proportional to its new length.
- Parallel resistances share the same potential difference and their conductances add: $1/R_{\text{eq}} = \sum_i 1/R_i$. For n identical parallel resistors of resistance r, the equivalent is r/n.

Step 1: Calculate the resistance of a single piece The original wire has length L and resistance R. Cutting it into five equal parts makes each length $L/5$, without changing the radius. Hence

$$r = \frac{\rho(L/5)}{A} = \frac{1}{5}\frac{\rho L}{A} = \frac{R}{5}.$$

This is the resistance of one piece alone. It is not yet the resistance of the combination after all five pieces are connected.

Step 2: Combine the five pieces in parallel Each branch has resistance $R/5$, so add its reciprocal five times:

$$\frac{1}{R_{\text{eq}}} = 5\left(\frac{1}{R/5}\right) = \frac{25}{R}.$$

$$R_{\text{eq}} = \frac{R}{25}.$$

The answer must be smaller than the resistance $R/5$ of even one branch. Thus $R/5$ misses the parallel reduction, while $5R$ and $25R$ incorrectly increase resistance. Rejoining the pieces in series would recover R, whereas this parallel arrangement supplies five simultaneous conducting paths.

Step 3: Conclusion The resultant resistance is $R/25$. Hence, the correct option is (A).

Quick Tip to Solve: Cutting a wire into n equal pieces divides each resistance by n, and putting all n pieces in parallel divides it once more: the result is R divided by n squared.

50. The average kinetic energy of a monatomic molecule is 0.414 eV at temperature :

(Use $K_B = 1.38 \times 10^{-23}$ J/mol − K) [4 marks]

(A) 3000 K

(B) 3200 K

(C) 1600 K

(D) 1500 K

Correct Answer: (B) 3200 K

Solution:

Concept:

- The mean translational kinetic energy of one molecule in thermal equilibrium is $\overline{K} = \frac{3}{2}k_BT$. A monatomic gas has three translational degrees of freedom, each contributing $\frac{1}{2}k_BT$.
- The energy must be converted to joules before using $k_B = 1.38\times10^{-23}\,\mathrm{J\,K^{-1}}$; $1\,\mathrm{eV} = 1.60\times10^{-19}\,\mathrm{J}$. The temperature is the absolute temperature in kelvin.

Step 1: Convert the molecular energy to SI units The quoted energy belongs to one molecule, so use Boltzmann's constant rather than the molar gas constant. Convert the electronvolt to joules:

$$\overline{K} = 0.414(1.60 \times 10^{-19}) = 6.624 \times 10^{-20}\,\mathrm{J}.$$

The factor $3/2$ counts all three directions of molecular motion; using only one direction would underestimate the energy at a given temperature.

Step 2: Calculate the temperature and compare the options Rearrange the equipartition expression before substitution:

$$T = \frac{2\overline{K}}{3k_B} = \frac{2(6.624 \times 10^{-20})}{3(1.38 \times 10^{-23})} = 3200\,\mathrm{K}.$$

As a direct check, $\frac{3}{2}(1.38 \times 10^{-23})(3200) = 6.624 \times 10^{-20}\,\mathrm{J}$. At 1600 K the energy would be half the stated value, whereas 3000 K and 1500 K also give smaller energies. Thus only the second temperature reproduces the supplied energy.

Step 3: Conclusion The required absolute temperature is $\boxed{3200\,\mathrm{K}}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert eV to joules first, then use the energy per molecule with Boltzmann's constant; do not use the molar gas constant.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A particle starts from origin at $t = 0$ with a velocity $5\hat{i}$ m/s and moves in $x - y$ plane under action of a force which produces a constant acceleration of $(3\hat{i} + 2\hat{j})\text{m/s}^2$. If the x-coordinate of the particle at that instant is 84 m, then the speed of the particle at this time is $\sqrt{\alpha}$ m/s. The value of α is _____.

[4 marks]

Solution:

Concept:

- For constant acceleration, each Cartesian component obeys $x = u_x t + \frac{1}{2}a_x t^2$ and $v_x = u_x + a_x t$. The initial conditions and acceleration components must be used separately.
- The speed is the magnitude of the velocity vector: $v = \sqrt{v_x^2 + v_y^2}$. Orthogonal velocity components are combined through their squares, rather than by direct addition.

Step 1: Find the physical time from the horizontal position The particle starts at the origin with $u_x = 5\,\text{m s}^{-1}$, $u_y = 0$, $a_x = 3\,\text{m s}^{-2}$ and $a_y = 2\,\text{m s}^{-2}$. Use the specified horizontal coordinate:

$$84 = 5t + \frac{3}{2}t^2, \qquad 3t^2 + 10t - 168 = 0.$$

Factoring gives $(3t + 28)(t - 6) = 0$, so $t = 6\,\text{s}$ or $t = -28/3\,\text{s}$. Since the stated motion begins at $t = 0$, discard the negative root. Check that $5(6) + \frac{3}{2}(36) = 84\,\text{m}$.

Step 2: Combine the velocity components at that instant At the physical time, the horizontal and vertical velocities are

$$v_x = 5 + 3(6) = 23\,\text{m s}^{-1}, \qquad v_y = 0 + 2(6) = 12\,\text{m s}^{-1}.$$

Therefore the speed, which cannot be identified with just its horizontal component, is

$$v = \sqrt{23^2 + 12^2} = \sqrt{529 + 144} = \sqrt{673}\,\text{m s}^{-1}.$$

Comparing this expression with the given $\sqrt{\alpha}\,\text{m s}^{-1}$ identifies the requested coefficient without introducing a rounding error.

Step 3: Conclusion The speed is $\sqrt{673}\,\text{m s}^{-1}$, so $\boxed{\alpha = 673}$. The coefficient α is dimensionless.

Quick Tip to Solve: Use the given coordinate to find time first; then calculate both velocity components before taking their magnitude.

52. A thin metallic wire having cross sectional area of 10^{-4} m^2 is used to make a ring of radius 30 cm. A positive charge of 2π C is uniformly distributed over the ring, while another positive charge of 30 pC is kept at the centre of the ring. The tension in the ring is _____ N; provided that the ring does not get deformed (neglect the influence of gravity). (given, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ SI units) [4 marks]

Solution:

Concept:

- A point charge q at the centre produces a radial electric field $E = kq/R^2$ at every element of a ring of radius R, where $k = 1/(4\pi\epsilon_0)$.
- For a uniformly charged ring, $\lambda = Q/(2\pi R)$. A small arc subtending $d\theta$ carries $dq = \lambda R\, d\theta$. Its outward electric force is balanced by the inward radial resultant of the tensions at its ends.

Step 1: Write the force balance for a small arc Let T be the tension. The tensions at the ends of an arc of angle $d\theta$ have equal and opposite tangential components. Their net inward radial component is $2T\sin(d\theta/2)$, which approaches $T\, d\theta$ for an infinitesimal arc. The outward force due to the central positive charge is

$$dF = E\, dq = \frac{kq}{R^2}\lambda R\, d\theta.$$

Equilibrium under this central-charge loading therefore requires

$$T\, d\theta = \frac{kq\lambda}{R}\, d\theta, \qquad T = \frac{kqQ}{2\pi R^2}.$$

Using the scalar sum of forces on the entire ring would be unhelpful because the radial force vectors cancel by symmetry.

Step 2: Substitute the radius and charges Convert the radius and central charge to SI units:

$$R = 0.30\,\text{m}, \qquad q = 30 \times 10^{-12}\,\text{C}, \qquad Q = 2\pi\,\text{C}.$$

The tension required to balance the force from the central charge is then

$$T = \frac{(9 \times 10^9)(30 \times 10^{-12})(2\pi)}{2\pi(0.30)^2} = \frac{0.27}{0.09} = 3\,\text{N}.$$

The cross-sectional area would be needed for the corresponding tensile stress T/A, but it is not needed when the question asks for tension itself.

Step 3: Conclusion The required ring tension in the central-charge force balance is $\boxed{3\,\text{N}}$.

Quick Tip to Solve: For a uniformly charged circular ring under a central radial field, balance a small arc: $T = \lambda E R$. Area enters stress, not this tension calculation.

53. Two coils have mutual inductance 0.002 H. The current changes in the first coil according to the relation $\mathrm{i} = \mathrm{i}_0 \sin \omega \mathrm{t}$, where $\mathrm{i}_0 = 5$ A and $\omega = 50\pi$ rad/s. The maximum value of emf in the second coil is $\frac{\pi}{\alpha}$ V. The value of α is _____. [4 marks]

Solution:

Concept:

- The induced emf in the second coil follows Faraday's law in mutual-inductance form: $\varepsilon_2 = -M\, di_1/dt$. The minus sign specifies Lenz's-law polarity.
- If $i_1 = i_0 \sin \omega t$, its greatest rate of change is $i_0\omega$, because $|\cos \omega t| \leq 1$. Thus the induced-emf amplitude is $Mi_0\omega$, not Mi_0 alone.

Step 1: Differentiate the current in the first coil The mutual inductance is constant, so only the current is differentiated. For the supplied sinusoidal current,

$$\frac{di_1}{dt} = i_0\omega \cos \omega t.$$

Hence the emf induced in the other coil is

$$\varepsilon_2 = -Mi_0\omega \cos \omega t.$$

The emf is greatest in magnitude when $|\cos \omega t| = 1$, at a zero crossing of the current. It is zero at a current extremum, where the current is instantaneously not changing. This separates the current amplitude from the induced-emf amplitude.

Step 2: Evaluate the emf amplitude and identify the coefficient Substitute $M = 0.002\,\mathrm{H}$, $i_0 = 5\,\mathrm{A}$ and $\omega = 50\pi\,\mathrm{rad\,s^{-1}}$:

$$\varepsilon_{\max} = Mi_0\omega = (0.002)(5)(50\pi) = 0.5\pi\,\mathrm{V} = \frac{\pi}{2}\,\mathrm{V}.$$

The unit check is $\mathrm{H} \times \mathrm{A\,s^{-1}} = \mathrm{V}$. Compare the positive amplitude with $\pi/\alpha\,\mathrm{V}$:

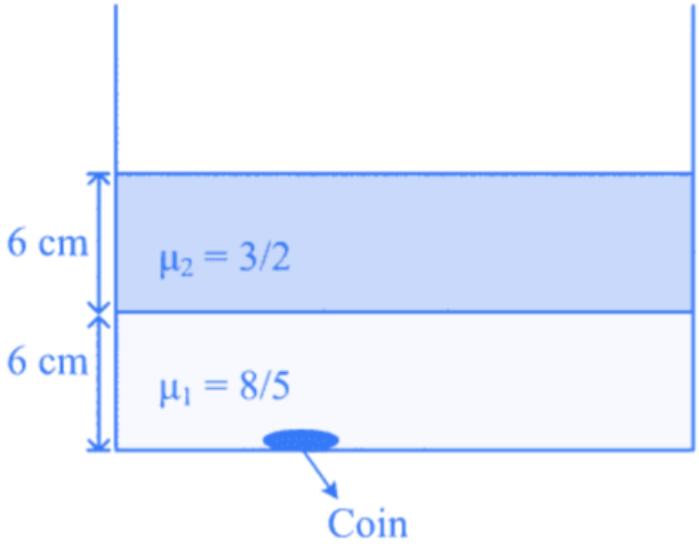
$$\frac{\pi}{\alpha} = \frac{\pi}{2} \quad \Longrightarrow \quad \alpha = 2.$$

The Lenz-law minus sign does not make the maximum magnitude negative; it controls the instantaneous direction.

Step 3: Conclusion The maximum induced emf is $\pi/2\,\mathrm{V}$, giving $\boxed{\alpha = 2}$, a dimensionless coefficient.

Quick Tip to Solve: For sinusoidal current, multiply the current amplitude by angular frequency before multiplying by mutual inductance; the emf peaks at current zero crossings.

54. Two immiscible liquids of refractive indices $\frac{8}{5}$ and $\frac{3}{2}$ respectively are put in a beaker as shown in the figure. The height of each column is 6 cm. A coin is placed at the bottom of the beaker. For near normal vision, the apparent depth of the coin is $\frac{\alpha}{4}$ cm. The value of α is _____. [4 marks]



Solution:

Concept:

- For near-normal viewing from air through a plane parallel layer of thickness t and refractive index μ, the contribution to apparent depth is t/μ. This uses the small-angle form of Snell's law.
- For several horizontal transparent layers, the apparent depth measured below the upper surface is $d_{\text{app}} = \sum_j t_j/\mu_j$. Each layer must be reduced by its own refractive index; a simple average index is generally incorrect.

Step 1: Express the apparent depth for the two layers The lower layer has index $\mu_1 = 8/5$ and the upper layer has index $\mu_2 = 3/2$. Each is $6\,\text{cm}$ thick and the coin lies at the bottom. A paraxial ray emerges into air at a small angle θ. Snell's law gives $\theta_j \simeq \theta/\mu_j$ in layer j. Its total sideways displacement is therefore $\theta(t_1/\mu_1 + t_2/\mu_2)$. Backward extension of the emerging ray locates the apparent coin at depth

$$d_{\text{app}} = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2}.$$

Thus refraction at both plane interfaces is accounted for, rather than treating the whole column as one uniform liquid.

Step 2: Calculate the depth and compare its stated form Evaluate the two contributions separately:

$$\frac{t_1}{\mu_1} = \frac{6}{8/5} = \frac{15}{4}\,\text{cm}, \qquad \frac{t_2}{\mu_2} = \frac{6}{3/2} = 4\,\text{cm}.$$

Consequently,

$$d_{\text{app}} = \frac{15}{4} + 4 = \frac{31}{4}\,\text{cm} = 7.75\,\text{cm}.$$

The apparent depth is less than the actual depth of $12\,\text{cm}$, as expected for viewing denser transparent

media from air. Comparing $31/4$ with the requested $\alpha/4$ gives the coefficient directly.

Step 3: Conclusion The coin appears $31/4\,\text{cm}$ below the upper surface, so $\boxed{\alpha = 31}$, dimensionless.

Quick Tip to Solve: For near-normal viewing through stacked plane layers, add t/μ layer by layer; do not divide the total thickness by an unweighted average index.

---

55. In a nuclear fission process, a high mass nuclide $(A \approx 236)$ with binding energy 7.6 MeV/ Nucleon dissociated into middle mass nuclides $(\text{A} \approx 118)$, having binding energy of 8.6 MeV/Nucleon. The energy released in the process would be _____ MeV. [4 marks]

Solution:

Concept:

- Total nuclear binding energy is $B = A(B/A)$, where A is the nucleon number and B/A is the binding energy per nucleon. Greater binding energy means a lower nuclear rest energy for the same constituents.
- The energy liberated in fission equals the increase in total binding energy: $Q = B_{\text{products}} - B_{\text{initial}}$. Nucleon number is conserved, so the products together still contain approximately 236 nucleons.

Step 1: Calculate the binding energy before fission The original heavy nucleus contains approximately 236 nucleons, each contributing $7.6\,\text{MeV}$ to its average binding energy. Thus its total binding energy is

$$B_{\text{initial}} = 236(7.6) = 1793.6\,\text{MeV}.$$

This is a total for one nucleus, not the energy released. The release depends on how much more tightly the daughter nuclei are bound, so the final total must also be calculated. The given mass numbers indicate two middle-mass fragments of approximately 118 nucleons apiece.

Step 2: Find the increase in binding energy Each daughter has binding energy $118(8.6)\,\text{MeV}$, so together they have

$$B_{\text{products}} = 2(118)(8.6) = 236(8.6) = 2029.6\,\text{MeV}.$$

The increase in binding energy, and hence the liberated energy, is

$$Q = 2029.6 - 1793.6 = 236\,\text{MeV}.$$

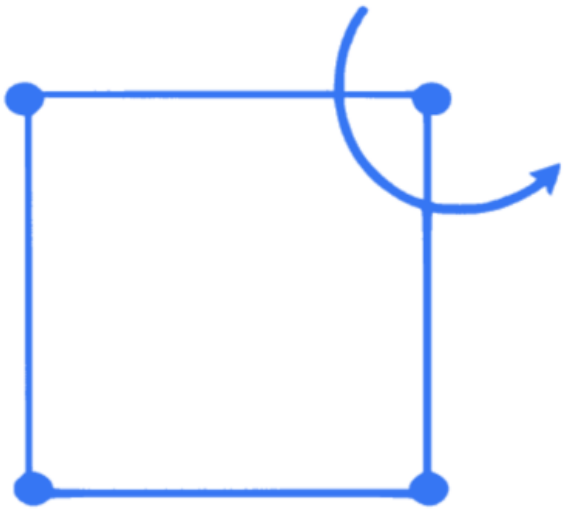
Equivalently, each conserved nucleon gains $8.6 - 7.6 = 1.0\,\text{MeV}$ of binding energy. Multiplying this gain by 236 reproduces the result. The positive sign shows release: the products have smaller total rest mass, with the difference appearing as kinetic energy and radiation rather than requiring external energy input.

Step 3: Conclusion The energy released in the fission process is $\boxed{236\,\text{MeV}}$.

Quick Tip to Solve: When all fragments have the same stated binding energy per nucleon, multiply its increase by the total conserved nucleon number; do not multiply by the number of fragments again.

---

56. Four particles each of mass $1\ \text{kg}$ are placed at four corners of a square of side $2\ \text{m}$. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is _____ kgm^2. [4 marks]



Solution:

Concept:

- For particles about a specified axis, the moment of inertia is $I = \sum_i m_i r_i^2$, where r_i is the perpendicular distance from the axis, not a path length along the square.
- The axis here is perpendicular to the square through a vertex. Therefore each distance is the straight-line distance in the plane from that vertex, and the mass located on the axis contributes zero.

Step 1: List the distances from the vertex axis Choose the vertex containing the axis as the origin. The four corners may be assigned plane coordinates $(0,0)$, $(2,0)$, $(0,2)$ and $(2,2)$, measured in metres. Because the rotation axis is perpendicular to this plane through the origin, the squared perpendicular distances are

$$r_1^2 = 0, \qquad r_2^2 = 4, \qquad r_3^2 = 4, \qquad r_4^2 = 2^2 + 2^2 = 8 \quad (\text{m}^2).$$

The opposite corner is $2\sqrt{2}\,\text{m}$ from the axis. It cannot be treated as another adjacent corner, and using the diagonal length without squaring it would give incorrect dimensions.

Step 2: Sum the four particle contributions Each mass is $1\,\text{kg}$, hence

$$I = (1)(0) + (1)(4) + (1)(4) + (1)(8) = 16\,\text{kg}\,\text{m}^2.$$

A check is available from the parallel-axis theorem. About the centre, each particle has squared distance $2\,\text{m}^2$, giving $I_{\text{centre}} = 8\,\text{kg}\,\text{m}^2$. The centre-to-vertex squared distance is $2\,\text{m}^2$, and the total mass is $4\,\text{kg}$. Thus shifting the axis gives $8 + 4(2) = 16\,\text{kg}\,\text{m}^2$, consistent with the direct sum.

Step 3: Conclusion The moment of inertia about the stated vertex axis is $\boxed{16\,\text{kg}\,\text{m}^2}$.

Quick Tip to Solve: For a square of side a with equal corner masses, a vertex axis gives squared distances $0, a^2, a^2, 2a^2$, so $I = 4ma^2$.

57. A particle executes simple harmonic motion with an amplitude of $4\ \text{cm}$. At the mean position, velocity of the particle is $10\ \text{cm/s}$. The distance of the particle from the mean position when its speed becomes $5\ \text{cm/s}$ is $\sqrt{\alpha}\ \text{cm}$, where $\alpha =$ _____. [4 marks]

Solution:

Concept:

- In simple harmonic motion, the restoring acceleration is proportional to displacement and directed towards equilibrium. Conservation of mechanical energy gives the speed relation $v^2 = \omega^2(A^2 - x^2)$, where A is amplitude and x is displacement from the mean position.
- The speed is maximum at the mean position, so $v_{\max} = \omega A$. The requested α is the square of the numerical distance expressed in centimetres and is dimensionless.

Step 1: Find the angular frequency The amplitude is $A = 4\,\text{cm}$, and the speed at equilibrium is $v_{\max} = 10\,\text{cm}\,\text{s}^{-1}$. Hence

$$\omega = \frac{v_{\max}}{A} = \frac{10}{4} = 2.5\,\text{s}^{-1}.$$

Both lengths are kept in centimetres, so no metre conversion is needed for this intermediate calculation. The mean-position speed is not the average speed over a cycle.

Step 2: Use the speed to find the displacement At the required instant $v = 5\,\text{cm}\,\text{s}^{-1}$. Dividing the speed equation by $v_{\max}^2 = \omega^2 A^2$ eliminates the angular frequency:

$$\frac{v^2}{v_{\max}^2} = 1 - \frac{x^2}{A^2}.$$

Thus

$$\frac{25}{100} = 1 - \frac{x^2}{16}, \qquad x^2 = 16\left(1 - \frac{1}{4}\right) = 12\,\text{cm}^2.$$

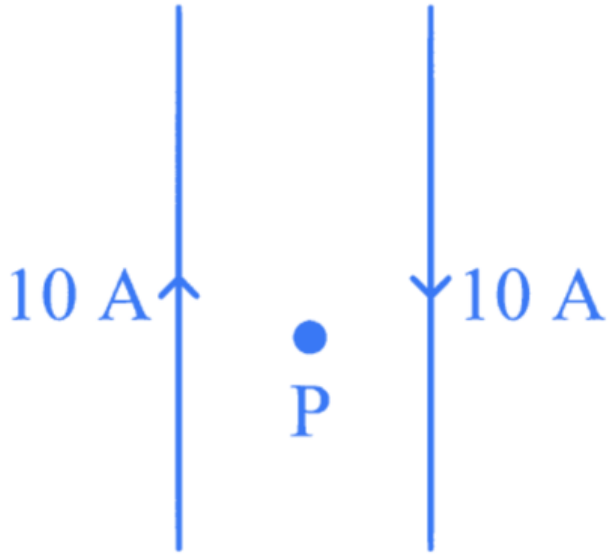
There are two possible positions, on opposite sides of equilibrium, but their distances from the mean position are equal. Taking the positive distance gives $|x| = \sqrt{12}\,\text{cm}$.

Step 3: Conclusion The distance is $2\sqrt{3}\,\text{cm} = \sqrt{12}\,\text{cm}$. Therefore the required dimensionless value is $\boxed{\alpha = 12}$.

Quick Tip to Solve: Use $x^2/A^2 = 1 - v^2/v_{\max}^2$: halving the maximum speed leaves three quarters of the squared amplitude as squared displacement.

58. Two long, straight wires carry equal currents in opposite directions as shown in figure. The separation between the wires is 5.0 cm. The magnitude of the magnetic field at a point P midway between the wires is _____ μT

(Given : $\mu_0 = 4\pi \times 10^{-7}\text{TmA}^{-1}$) [4 marks]



Solution:

Concept:

- The magnetic field at perpendicular distance r from a long straight wire is $B = \mu_0 I/(2\pi r)$. Its direction follows the right-hand thumb rule: point the thumb with the current and curl the fingers around the wire.
- Magnetic fields add as vectors. At a point between parallel wires carrying opposite currents, the two fields have the same direction, even though the currents themselves are opposite.

Step 1: Determine the distance and field directions The figure shows $I = 10\,\text{A}$ in each wire, upward in the left wire and downward in the right wire. Since P is midway across the $5.0\,\text{cm}$ separation,

$$r = \frac{5.0}{2}\,\text{cm} = 2.5 \times 10^{-2}\,\text{m}.$$

P lies to the right of the upward current and to the left of the downward current. Applying the right-hand rule to each wire gives a field into the plane of the figure in both cases. They therefore reinforce rather than cancel.

Step 2: Calculate and add the fields For either wire, substitute the given permeability:

$$B_1 = B_2 = \frac{(4\pi \times 10^{-7})(10)}{2\pi(2.5 \times 10^{-2})} = 8.0 \times 10^{-5}\,\text{T}.$$

The magnitudes may be added because the directions coincide:

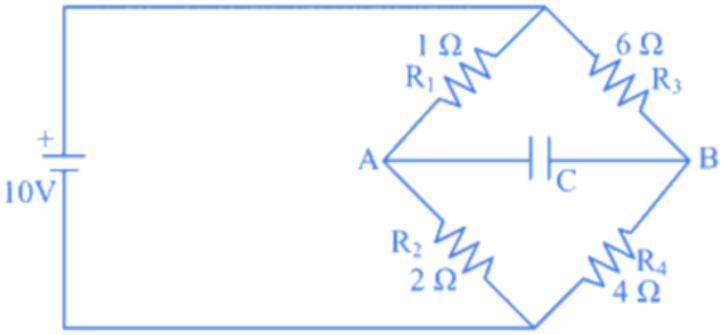
$$B_{\text{net}} = 2(8.0 \times 10^{-5}) = 1.60 \times 10^{-4}\,\text{T}.$$

Since $1\,\mu\mathrm{T} = 10^{-6}\,\mathrm{T}$, the numerical value in microtesla is $(1.60 \times 10^{-4})/10^{-6} = 160$. Using the full wire separation for r would incorrectly halve this result.

Step 3: Conclusion The magnetic field at P is $\boxed{160\,\mu\mathrm{T}}$, into the plane of the figure. The requested numerical entry is 160.

Quick Tip to Solve: At the midpoint use half the separation for each wire. Opposite currents produce reinforcing fields between the wires; same-direction currents produce cancellation there.

59. The charge accumulated on the capacitor connected in the following circuit is _____ μC (Given C = 150μF) [4 marks]



Solution:

Concept:

- After a capacitor has fully charged in a direct-current circuit, its branch carries no steady current. The resistive arms can therefore be analysed as two independent voltage dividers across the battery.
- The capacitor charge magnitude is $Q = C|V_A - V_B|$, not generally the capacitance times the battery voltage. Here its plates connect the two interior junctions A and B of an unbalanced resistor bridge.

Step 1: Find the junction potentials Take the lower battery terminal as zero potential and the upper terminal as 10 V. The left arm has $R_1 = 1\,\Omega$ above A and $R_2 = 2\,\Omega$ below A. With the capacitor branch open at steady state,

$$I_L = \frac{10}{1+2} = \frac{10}{3}\,\mathrm{A}, \qquad V_A = I_L R_2 = \frac{20}{3}\,\mathrm{V}.$$

The right arm has $R_3 = 6\,\Omega$ above B and $R_4 = 4\,\Omega$ below B, so

$$I_R = \frac{10}{6+4} = 1\,\mathrm{A}, \qquad V_B = I_R R_4 = 4\,\mathrm{V}.$$

The differing divider ratios show why the capacitor is not at zero voltage.

Step 2: Calculate the capacitor charge The capacitor voltage is the difference of the two junction potentials:

$$V_C = V_A - V_B = \frac{20}{3} - 4 = \frac{8}{3}\,\mathrm{V}.$$

Using $C = 150\,\mu\text{F}$,

$$Q = CV_C = 150 \times \frac{8}{3}\,\mu\text{C} = 400\,\mu\text{C}.$$

The plate connected to A is positive because A is at the higher potential. The opposite plate carries an equal negative charge; the accumulated charge asked for is the magnitude on either plate, rather than their algebraic sum.

Step 3: Conclusion The charge magnitude is $\boxed{400\,\mu\text{C}}$. Thus the requested numerical entry is 400.

Quick Tip to Solve: For a capacitor joining resistor-divider midpoints, first remove its steady-current branch and find each midpoint voltage relative to the same battery terminal.

---

60. If average depth of an ocean is $4000\ \text{m}$ and the bulk modulus of water is $2 \times 10^9\ \text{Nm}^{-2}$, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of ocean is $\alpha \times 10^{-2}$. The value of α is _____ (Given, $\text{g} = 10\ \text{ms}^{-2}, \rho = 1000\ \text{kg m}^{-3}$) [4 marks]

Solution:

Concept:

- The bulk modulus K measures resistance to volume compression: $K = -\Delta P/(\Delta V/V)$, where ΔV is the signed volume change. The fractional compression magnitude is therefore $|\Delta V|/V = \Delta P/K$.
- The pressure increase from the ocean surface to depth h, using the given constant density approximation, is $\Delta P = \rho g h$. This is the pressure responsible for compression relative to the surface volume.

Step 1: Calculate the pressure increase at the bottom The data are $h = 4000\,\text{m}$, $\rho = 1000\,\text{kg m}^{-3}$, and $g = 10\,\text{m s}^{-2}$. Thus

$$\Delta P = \rho g h = (1000)(10)(4000) = 4.0 \times 10^7\ \text{Pa}.$$

A pascal is N m^{-2}, the same pressure unit used for the supplied bulk modulus. No additional factor of area enters because $\rho g h$ already represents force per unit area.

Step 2: Divide by the bulk modulus With $K = 2.0 \times 10^9\ \text{Pa}$, the positive fractional compression is

$$\frac{|\Delta V|}{V} = \frac{4.0 \times 10^7}{2.0 \times 10^9} = 2.0 \times 10^{-2}.$$

Pressure and bulk modulus have identical dimensions, so this ratio is dimensionless. The signed volume change itself is negative, since pressure reduces volume, but the phrase fractional compression denotes its positive magnitude. Comparing with the stated form $\alpha \times 10^{-2}$ identifies the coefficient directly.

Step 3: Conclusion The water is compressed by a fraction 0.02, or 2%. Therefore $\boxed{\alpha = 2}$, a dimensionless number.

Quick Tip to Solve: For hydrostatic compression, calculate $\rho g h/K$ with both pressure quantities in pascals. Keep compression magnitude positive and distinguish it from signed volume change.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Two nucleotides are joined together by a linkage known as : [4 marks]

(A) Phosphodiester linkage

(B) Glycosidic linkage

(C) Disulphide linkage

(D) Peptide linkage

Correct Answer: (A) Phosphodiester linkage

Solution:

Concept:

- A nucleotide contains a nitrogenous base, a pentose sugar, and a phosphate group. A nucleic acid is formed when nucleotide units connect into a chain through their sugar-phosphate groups.
- A phosphodiester linkage contains phosphate ester bonds to two sugar residues. In ordinary DNA and RNA chains, it connects the $3'$ oxygen of one sugar to the $5'$ oxygen of the next sugar through phosphate.

Step 1: Identify the bond joining nucleotide units The repeating backbone of DNA and RNA consists of alternating sugar and phosphate units. The phosphate links a hydroxyl group of one pentose sugar to a hydroxyl group of the neighbouring sugar. Because the same phosphate forms two ester connections, the linkage is called a phosphodiester linkage. Its name describes the phosphate bridge between nucleotide residues, rather than the attachment of a base within an individual nucleotide.

Step 2: Distinguish the other biological linkages A glycosidic linkage is relevant to sugar chemistry. The N-glycosidic bond within a nucleotide joins its nitrogenous base to the sugar; it is not the sugar-phosphate bridge joining successive nucleotide units. A disulphide linkage is an $S-S$ bond between cysteine residues and helps stabilise some proteins. A peptide linkage joins the carboxyl group of one amino acid to the amino group of another, building a polypeptide. Thus options (B), (C), and (D) describe different connections and do not identify the internucleotide backbone linkage.

Step 3: Conclusion Two nucleotides are joined through a $3'$-$5'$ phosphodiester linkage. Hence, the correct option is (A).

Quick Tip to Solve: Keep the three connections separate: base to sugar is glycosidic; nucleotide to nucleotide is phosphodiester; amino acid to amino acid is peptide.

62. Highest enol content will be shown by: [4 marks]

(A) Cyclohexane-1,3-dione 

(B) Cyclohexane-1,3,5-trione 

(C) Cyclohexane-1,4-dione 

(D) Cyclohexane-1,2,3-trione 

Correct Answer: (B) Cyclohexane-1,3,5-trione 

Solution:

Concept:

- Keto-enol tautomerism shifts an alpha hydrogen to the carbonyl oxygen and changes a neighbouring single carbon-carbon bond to a double bond. The equilibrium enol content is governed by relative stability, including conjugation, hydrogen bonding, and aromaticity.
- Aromatic stabilisation is especially strong when enolisation creates a planar six-membered ring with three alternating double bonds and six pi electrons. The number of carbonyl groups alone does not determine the most stable enol.

Step 1: Recognise the structure that can form an aromatic enol Option (B) is cyclohexane-1,3,5-trione. Its three alternating carbonyl carbons each have an adjacent methylene group. Transfer of one hydrogen from each intervening methylene to a carbonyl oxygen permits all three carbonyl groups to become hydroxyl groups while introducing three alternating double bonds around the ring. The resulting fully enolised form is benzene-1,3,5-triol, also called phloroglucinol. Its aromatic ring supplies exceptional stabilisation, strongly favouring the enol form.

Step 2: Compare the remaining carbonyl arrangements Option (A), cyclohexane-1,3-dione, can form a conjugated enol and may gain intramolecular hydrogen-bond stabilisation, but it cannot generate the same fully aromatic tri-enol. Option (C), cyclohexane-1,4-dione, has separated carbonyl groups and lacks the alternating tricarbonyl arrangement needed for that aromatic product. In option (D), cyclohexane-1,2,3-trione, the three carbonyl groups are consecutive; enolisation cannot place three hydroxyl groups on alternating aromatic ring carbons as in (B). Consequently these alternatives lack the decisive aromatic stabilisation available to the enol of (B).

Step 3: Conclusion Cyclohexane-1,3,5-trione has the highest enol content because its enol form is aromatic benzene-1,3,5-triol. Hence, the correct option is (B).

Quick Tip to Solve: When comparing enol contents, first check whether enolisation creates an aromatic ring; aromatic stabilisation can outweigh the usual preference for a carbonyl group.

63. Element not showing variable oxidation state is : [4 marks]

(A) Bromine

(B) Iodine

(C) Chlorine

(D) Fluorine

Correct Answer: (D) Fluorine

Solution:

Concept:

- An oxidation state is the formal charge assigned by giving bonding electrons to the more electronegative atom. Halogens commonly show -1 in their compounds, but chlorine, bromine, and iodine can also have positive oxidation states with more electronegative partners.
- Fluorine is the most electronegative element. Therefore its oxidation state in compounds is always -1; it does not display the variable positive oxidation states characteristic of the heavier halogens.

Step 1: Apply the electronegativity rule to fluorine In a bond containing fluorine and another element, the bonding electrons are assigned to fluorine. Since fluorine has seven valence electrons and normally completes its octet with one additional assigned electron, it has oxidation state -1 in its compounds. Even in oxygen difluoride, OF_2, fluorine remains -1 and oxygen is positive. There is no more electronegative partner that would make fluorine positive in an ordinary compound.

Step 2: Check why the other halogens are variable Chlorine, bromine, and iodine are less electronegative than oxygen and fluorine, so they can have positive oxidation states as well as -1. For example chlorine is -1 in HCl and $+1$ in $HClO$; bromine is -1 in HBr and $+5$ in $HBrO_3$; iodine is -1 in HI and $+5$ in HIO_3. These examples exclude options (A), (B), and (C). All elements have oxidation state zero in their free elemental form, including F_2; the question concerns variable oxidation states in compounds.

Step 3: Conclusion Fluorine does not show variable oxidation states in its compounds: its oxidation state is -1. Hence, the correct option is (D).

Quick Tip to Solve: Assign fluorine -1 first when finding oxidation states in compounds, even when it is bonded to oxygen. Its elemental molecule is the separate zero-state case.

64. Which of the following is strongest Bronsted base? [4 marks]

(A) Aniline ($C_6H_5NH_2$)

(B) Diphenylamine ($(C_6H_5)_2NH$)

(C) Pyrrole (five-membered aromatic ring with an NH group)

(D) Pyrrolidine (saturated five-membered ring with an NH group)

Correct Answer: (D) Pyrrolidine (saturated five-membered ring with an NH group)

Solution:

Concept:

- A Bronsted base accepts a proton. For these nitrogen compounds, the availability of the nitrogen lone pair controls proton acceptance.
- Resonance delocalisation reduces the availability of a lone pair, whereas electron donation from saturated alkyl groups generally increases it.

Step 1: Identify the nitrogen environments Option (A) is aniline, in which nitrogen is directly attached to a benzene ring. Option (B) is diphenylamine, in which nitrogen is attached to two phenyl groups. Option (C) is pyrrole, an aromatic five-membered ring. Option (D) is pyrrolidine, a saturated five-membered cyclic secondary amine. These are chemically different environments even though every option contains nitrogen and an N–H bond.

Step 2: Compare lone-pair availability In aniline, the nitrogen lone pair is delocalised into the benzene ring, so it is less readily donated to a proton. In diphenylamine, conjugation with phenyl groups also reduces its availability. In pyrrole, the lone pair supplies two electrons to the aromatic sextet; protonating that lone pair sacrifices the aromatic stabilisation, making pyrrole a very weak base. Pyrrolidine has no adjacent aromatic pi system into which its nitrogen lone pair must delocalise.

Step 3: Select the strongest proton acceptor The nitrogen in pyrrolidine is an aliphatic secondary-amine nitrogen. Its lone pair remains localised and is supported by the electron-releasing inductive effect of the saturated carbon framework. It therefore accepts a proton more readily than the resonance-stabilised aromatic nitrogen compounds in (A), (B) and (C). Comparing only the presence of an N–H bond would miss the decisive difference in lone-pair availability.

Step 4: Conclusion Pyrrolidine is the strongest Bronsted base among the four compounds. Hence, the correct option is (D).

Quick Tip to Solve: First ask whether the nitrogen lone pair is localised, conjugated with a benzene ring, or required for aromaticity; a localised aliphatic-amine lone pair is usually the most available here.

65. Which of the following electronic configuration would be associated with the highest magnetic moment? [4 marks]

(A) $[\mathrm{Ar}]3\ \mathrm{d}^7$

(B) $[\mathrm{Ar}]3\ \mathrm{d}^8$

(C) $[\mathrm{Ar}]3\ \mathrm{d}^3$

(D) $[\mathrm{Ar}]3\ \mathrm{d}^6$

Correct Answer: (D) $[\mathrm{Ar}]3\ \mathrm{d}^6$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mathrm{B.M.}$, where n is the number of unpaired electrons.
- For the listed free-ion configurations, fill the five d orbitals singly with parallel spins before pairing, according to Hund's rule.

Step 1: Count the unpaired electrons The closed argon core has no unpaired electrons, so only the d subshell contributes. In $3d^3$, three orbitals are singly occupied and $n = 3$. In $3d^6$, five orbitals first receive one electron each and the sixth electron pairs in one orbital; four orbitals remain singly occupied, giving $n = 4$. Similarly, $3d^7$ has two paired orbitals and three singly occupied orbitals, while $3d^8$ has three paired orbitals and two singly occupied orbitals.

Step 2: Calculate and compare the moments The resulting spin-only values are

$$\mu(3d^7) = \sqrt{3(3+2)} = \sqrt{15} \approx 3.87\,\mathrm{B.M.},$$
$$\mu(3d^8) = \sqrt{2(2+2)} = \sqrt{8} \approx 2.83\,\mathrm{B.M.},$$
$$\mu(3d^3) = \sqrt{3(3+2)} = \sqrt{15} \approx 3.87\,\mathrm{B.M.},$$
$$\mu(3d^6) = \sqrt{4(4+2)} = \sqrt{24} \approx 4.90\,\mathrm{B.M.}.$$

Thus the moment increases with the number of unpaired electrons, not simply with the total number of d electrons.

Step 3: Reject the other configurations Options (A) and (C) each provide only three unpaired electrons and hence tie below the d^6 case. Option (B), despite having the largest d-electron count, has only two unpaired electrons because more orbitals are paired. Option (D) alone gives four unpaired electrons among the listed configurations.

Step 4: Conclusion The configuration $[\mathrm{Ar}]3d^6$ has the highest spin-only magnetic moment, approximately $4.90\,\mathrm{B.M.}$. Hence, the correct option is (D).

Quick Tip to Solve: For a free-ion d subshell, use $n = m$ up to d^5 and $n = 10 - m$ after d^5; then compare n before calculating any square roots.

66. Which of the following has highly acidic hydrogen? [4 marks]

(A) $CH_3COCOCH_2CH_2CH_3$

(B) $CH_3COCH_2CH_2COCH_3$

(C) $CH_3CH_2CH_2COCH_2CH_3$

(D) $CH_3COCH_2COCH_2CH_3$

Correct Answer: (D) $CH_3COCH_2COCH_2CH_3$

Solution:

Concept:

- Acidity increases when the conjugate base formed by loss of H^+ is stabilised. Alpha hydrogens of carbonyl compounds give resonance-stabilised enolate ions.
- A methylene group directly between two carbonyl groups is especially acidic because its conjugate base can delocalise negative charge onto either carbonyl oxygen.

Step 1: Locate the carbonyl groups Option (A), $CH_3COCOCH_2CH_2CH_3$, is a 1,2-diketone: its carbonyl carbons are adjacent, with no methylene between them. Option (B), $CH_3COCH_2CH_2COCH_3$, is a 1,4-diketone. Option (C), $CH_3CH_2CH_2COCH_2CH_3$, contains only one carbonyl. Option (D), $CH_3COCH_2COCH_2CH_3$, is a 1,3-diketone and contains a central $-CO-CH_2-CO-$ unit.

Step 2: Stabilise the conjugate base of option D Removing one hydrogen from the central methylene in (D) gives an anion whose charge is delocalised over the carbon and both adjacent carbonyl oxygens. The principal resonance contributors can be represented as

$$CH_3C(O^-)=CHCOCH_2CH_3$$

$$CH_3COCH^-COCH_2CH_3$$

$$CH_3COCH=C(O^-)CH_2CH_3.$$

Both carbonyl groups also withdraw electron density inductively. The especially stable conjugate base makes loss of this central proton favourable.

Step 3: Compare the remaining options The alpha hydrogens in (C) benefit from only one carbonyl group. Neither (A) nor (B) has a hydrogen-bearing carbon flanked directly by two carbonyl carbons. Their enolate stabilisation is therefore less effective than that of the active methylene group in (D). Merely counting two oxygen atoms is insufficient; their placement relative to the acidic C–H bond is decisive.

Step 4: Conclusion The central methylene hydrogens of $CH_3COCH_2COCH_2CH_3$ are the most acidic. Hence, the correct option is (D).

Quick Tip to Solve: Look for $-CO-CH_2-CO-$: its central carbon loses a proton to give an anion stabilised by two carbonyl groups, rather than just one.

67. A solution of two miscible liquids showing negative deviation from Raoult's law will have :

[4 marks]

(A) increased vapour pressure, increased boiling point

(B) increased vapour pressure, decreased boiling point

(C) decreased vapour pressure, decreased boiling point

(D) decreased vapour pressure, increased boiling point

Correct Answer: (D) decreased vapour pressure, increased boiling point

Solution:

Concept:

- Raoult's law predicts the vapour pressure of an ideal binary liquid mixture as $p_{\text{ideal}} = x_A p_A^\circ + x_B p_B^\circ$.
- Boiling occurs when the total equilibrium vapour pressure equals the external pressure. At a fixed external pressure, a lower vapour-pressure curve requires a higher temperature to boil.

Step 1: Interpret negative deviation For a solution showing negative deviation, the measured vapour pressure is less than the ideal Raoult-law value at the same temperature and composition:

$$p_{\text{actual}} < x_A p_A^\circ + x_B p_B^\circ.$$

The unlike A–B molecular attractions are stronger than the average attractions in the unmixed liquids. Molecules are retained more strongly in the liquid phase and have a smaller escaping tendency.

Step 2: Relate vapour pressure to boiling point At a temperature where an ideal mixture would reach atmospheric pressure, the negatively deviating solution still has a lower vapour pressure. Its temperature must be increased further until

$$p_{\text{vapour}} = p_{\text{external}}.$$

Therefore negative deviation corresponds to decreased vapour pressure and an elevated boiling point relative to ideal behaviour under the same conditions. The stronger attractions can also lead to a maximum-boiling azeotrope when the deviation is sufficiently large, though azeotrope formation is not required to select the trend.

Step 3: Check both features in each option Options (A) and (B) predict increased vapour pressure, contradicting negative deviation immediately. Option (C) correctly predicts reduced vapour pressure but incorrectly pairs it with a reduced boiling point. Option (D) has both required features: lower vapour pressure at a given temperature and a higher temperature needed for boiling.

Step 4: Conclusion The solution has decreased vapour pressure and increased boiling point. Hence, the correct option is (D).

Quick Tip to Solve: Link the two trends through the boiling condition: if the liquid produces less vapour pressure at a given temperature, heat it more to reach atmospheric pressure.

68. Consider the following complex ions

$$\mathrm{P} = [\mathrm{FeF_6}]^{3-}$$
$$\mathrm{Q} = [\mathrm{V\,(H_2O)_6}]^{2+}$$
$$\mathrm{R} = [\mathrm{Fe\,(H_2O)_6}]^{2+}$$

The correct order of the complex ions, according to their spin only magnetic moment values (in B.M.) is : [4 marks]

(A) R < Q < P

(B) R < P < Q

(C) Q < R < P

(D) Q < P < R

Correct Answer: (C) Q < R < P

Solution:

Concept:

- The oxidation state determines the metal-ion d count. For these weak-field octahedral complexes, use high-spin filling where applicable.
- The spin-only moment is $\mu = \sqrt{n(n+2)}\,\mathrm{B.M.}$, with n the number of unpaired electrons.

Step 1: Find the metal oxidation states and d counts In $[\mathrm{FeF_6}]^{3-}$, six fluoride ligands contribute -6; hence iron is $+3$, giving $\mathrm{Fe^{3+}}$, a $3d^5$ ion. Water is neutral, so $[\mathrm{V(H_2O)_6}]^{2+}$ contains $\mathrm{V^{2+}}$, a $3d^3$ ion, while $[\mathrm{Fe(H_2O)_6}]^{2+}$ contains $\mathrm{Fe^{2+}}$, a $3d^6$ ion. The 4s electrons are removed before the 3d electrons when forming these cations.

Step 2: Fill the octahedral levels Fluoride is weak-field and the ferric d^5 complex P is high spin: $t_{2g}^3 e_g^2$, with five unpaired electrons. For Q, the d^3 configuration is t_{2g}^3, with three unpaired electrons. Water gives high-spin ferrous R: $t_{2g}^4 e_g^2$, with four unpaired electrons. Thus

$$n_Q = 3, \qquad n_R = 4, \qquad n_P = 5.$$

Step 3: Calculate the order and eliminate alternatives The corresponding moments are

$$\mu_Q = \sqrt{15} \approx 3.87\,\mathrm{B.M.},$$
$$\mu_R = \sqrt{24} \approx 4.90\,\mathrm{B.M.},$$
$$\mu_P = \sqrt{35} \approx 5.92\,\mathrm{B.M.}.$$

Since the square-root expression increases with n, Q is smallest and P largest. Options (A) and (B) incorrectly put R below Q, while (D) incorrectly puts the five-unpaired-electron complex P below R.

Step 4: Conclusion The increasing spin-only magnetic-moment order is $Q < R < P$. Hence, the correct option is (C).

Quick Tip to Solve: Compute oxidation states before d counts; then use ligand strength to decide pairing. Here the unpaired counts $3, 4, 5$ determine the order without decimal calculation.

69. Choose the polar molecule from the following: [4 marks]

(A) CCl_4

(B) CO_2

(C) $CH_2 = CH_2$

(D) $CHCl_3$

Correct Answer: (D) $CHCl_3$

Solution:

Concept:

- The molecular dipole moment is the vector sum of all bond dipole moments. Polar bonds alone do not guarantee a polar molecule.
- Equivalent bonds arranged symmetrically may cancel exactly. Different substituents can break that cancellation even when the central-atom geometry remains tetrahedral.

Step 1: Test the symmetric molecules In CCl_4, four equivalent C–Cl bonds point towards the corners of a regular tetrahedron. Their dipole vectors cancel, so carbon tetrachloride is nonpolar. In CO_2, the molecule is linear. Its two equal C=O dipoles point in opposite directions and cancel:

$$\vec{\mu}_1 + \vec{\mu}_2 = \vec{0}.$$

Hence option (B) is also nonpolar despite the strong polarity of its individual carbon–oxygen bonds.

Step 2: Consider ethene Ethene, $CH_2{=}CH_2$, is planar and symmetric. Its two ends are identical; the bond-dipole contributions cancel by molecular symmetry. It therefore has no permanent net dipole moment and cannot be the required polar molecule. The existence of a double bond by itself does not imply molecular polarity.

Step 3: Find the unsymmetrical tetrahedral molecule In $CHCl_3$, three substituents are chlorine but the fourth is hydrogen. This removes the fourfold equivalence present in CCl_4. The three C–Cl bond dipoles have a nonzero vector sum along the remaining tetrahedral axis, and the C–H bond does not supply an equal opposite contribution. The resulting molecular dipole moment is nonzero. Thus (A), (B) and (C) are ruled out by cancellation, whereas (D) is polar.

Step 4: Conclusion Chloroform, $CHCl_3$, is the polar molecule. Hence, the correct option is (D).

Quick Tip to Solve: Check geometry and substituent symmetry together: replacing one chlorine of nonpolar CCl_4 by hydrogen destroys exact dipole cancellation.

70. Given below are two statements :

Statement (I) : The $4f$ and $5f$ - series of elements are placed separately in the Periodic table to preserve the principle of classification.

Statement (II) : S-block elements can be found in pure form in nature.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Elements are classified into s, p, d and f blocks according to the subshell receiving the differentiating electron. The f-block consists of the inner-transition series.
- The highly electropositive s-block metals readily form compounds. The usual statement about their natural occurrence refers to these reactive metals, rather than exceptions such as hydrogen and helium.

Step 1: Evaluate Statement I The 4f series (lanthanoids) and 5f series (actinoids) are inner-transition elements. Their differentiating electrons enter f subshells, and members within each series show broadly similar chemical behaviour. Displaying these series separately below the main body keeps the periodic table compact and preserves the grouping of elements with related electronic configurations and properties. Their separate display does not change their actual positions in periods 6 and 7. Therefore Statement I is true.

Step 2: Evaluate Statement II in its textbook context The s-block metals have low ionisation enthalpies and readily lose valence electrons to form cations. Alkali metals in particular react readily with oxygen and water; alkaline-earth metals also normally occur as minerals, salts and other compounds. They are not ordinarily found as free, pure metals in nature. Thus the intended statement about s-block metallic elements occurring in pure form is false. Hydrogen and helium have special positions and natural forms, but the occurrence generalisation tested here concerns the reactive s-block metals.

Step 3: Match the truth values to the options The required pair is

$$\text{Statement I: true}, \qquad \text{Statement II: false}.$$

Option (A) reverses both judgements. Option (B) incorrectly accepts the natural occurrence of reactive s-block metals as pure substances, and option (D) incorrectly rejects the accepted separate placement of the f-block series. Only option (C) matches the textbook classification and occurrence statements.

Step 4: Conclusion Statement I is true and Statement II is false in the standard s-block-metal context. Hence, the correct option is (C).

Quick Tip to Solve: Separate two ideas: f-block placement is a classification/display choice, whereas the combined natural occurrence of s-block metals follows from their high chemical reactivity.

71. Given below are two statements :

Statement (I) : p-nitrophenol is more acidic than m-nitrophenol and o-nitrophenol.

Statement (II) : Ethanol will give immediate turbidity with Lucas reagent.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are true

(C) Both Statement I and Statement II are false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Statement I is true but Statement II is false

Solution:

Concept:

- Acid strength increases when the conjugate base is stabilised. A nitro group withdraws electrons inductively and, at the ortho or para position, also stabilises phenoxide by resonance.
- Lucas reagent is concentrated hydrochloric acid with anhydrous zinc chloride. Turbidity results from formation of a water-insoluble alkyl chloride; its rate depends on the alcohol structure.

Step 1: Compare meta and para nitrophenol In the phenoxide ion, negative charge is delocalised to the ortho and para carbons. A para nitro group can withdraw this charge through resonance, whereas a meta nitro group mainly contributes its inductive effect. Consequently p-nitrophenol is more acidic than m-nitrophenol; merely counting the nitro groups would miss the effect of their positions.

Step 2: Compare ortho and para nitrophenol Both ortho and para nitro groups provide resonance stabilisation. In water, however, the intramolecular hydrogen bonding of o-nitrophenol stabilises its undissociated form and reduces the favourable hydration associated with ionisation. The para isomer is slightly more acidic in aqueous solution. Thus Statement I is true; the ortho isomer is not the most acidic of the three stated compounds.

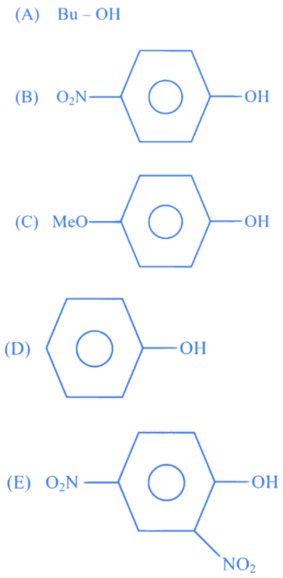
Step 3: Apply the Lucas test to ethanol Ethanol is a primary alcohol. It does not readily form a primary carbocation and does not give immediate turbidity with Lucas reagent at room temperature. Immediate turbidity is characteristic of a tertiary alcohol. Therefore Statement II is false. Options B and D wrongly make II true, while C wrongly makes I false.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: For nitrophenols compare resonance and aqueous solvation; for Lucas turbidity, classify the carbon bearing the hydroxyl group first.

72. The ascending order of acidity of $-OH$ group in the following compounds is :

Choose the correct answer from the options given below: [4 marks]



(A) (A) < (D) < (C) < (B) < (E)

(B) (C) < (A) < (D) < (B) < (E)

(C) (C) < (D) < (B) < (A) < (E)

(D) (A) < (C) < (D) < (B) < (E)

Correct Answer: (D) (A) < (C) < (D) < (B) < (E)

Solution:

Concept:

- The more stable the conjugate base formed by loss of the hydroxyl proton, the stronger the acid. Phenoxide is resonance-stabilised, unlike an ordinary alkoxide.
- An electron-withdrawing nitro group stabilises phenoxide and increases acidity. A para methoxy group donates electron density by resonance and decreases phenol acidity.

Step 1: Identify the five structures Compound A is butanol, B is p-nitrophenol, C is p-methoxyphenol, D is phenol and E is 2,4-dinitrophenol. The labels attached to the structures are compound labels, not answer choices. The ordering must therefore begin by comparing an aliphatic alcohol with the four phenolic compounds.

Step 2: Place the alcohol and electron-rich phenol Butanol gives a localised alkoxide ion, while all four phenols give resonance-stabilised phenoxide ions. Butanol A is consequently the least acidic. In C the para methoxy group has a resonance-donating effect that destabilises the negatively charged conjugate base relative to phenoxide from D. Hence the first three members are A, C and D, in that order.

Step 3: Place the nitro-substituted phenols The para nitro group of B accepts electron density from the phenoxide system, so B is more acidic than unsubstituted phenol D. Compound E has nitro groups at both ortho and para positions and stabilises its conjugate base still more strongly. Therefore the complete ascending order is $A < C < D < B < E$. Option A reverses C and D; options B and C

incorrectly put the alcohol after a phenol. Only option D preserves all comparisons.

Step 4: Conclusion The ascending acidity order is $A < C < D < B < E$. Hence, the correct option is (D).

Quick Tip to Solve: Rank alcohol below phenol, move resonance donors below phenol, and move nitro-substituted phenols above it; two nitro groups strengthen the effect.

73. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Melting point of Boron (2453 K) is unusually high in group 13 elements.

Reason (R) : Solid Boron has very strong crystalline lattice.

In the light of the above statements, choose the most appropriate answer from the options given below ; [4 marks]

(A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(B) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

Correct Answer: (B) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Concept:

- Melting requires enough thermal energy to disrupt the structural forces maintaining an ordered solid. A strongly bonded extended network generally has a high melting point.
- Boron is exceptional among group 13 elements: its small atomic size and strong covalent bonding produce a very strong crystalline lattice rather than the relatively weak metallic structures of its heavier congeners.

Step 1: Evaluate the assertion The quoted melting point of boron is $2453\,\mathrm{K}$, much higher than those of the heavier group 13 members. Boron is a hard covalent solid and its strong bonding prevents the lattice from being disrupted at modest temperatures. The assertion is therefore correct; the comparison is about the exceptional melting point, not a simple monotonic increase down the group.

Step 2: Evaluate the reason Solid boron has a complex and strongly bonded crystalline structure. Its atoms are held together through an extended covalent framework. Calling its crystalline lattice very strong is consistent with that bonding description, so the reason is also correct. A crystal need not be ionic: covalent solids can also have strong lattices.

Step 3: Test the explanatory link The strong lattice is precisely why substantial thermal energy is required to melt boron. The reason therefore gives the physical origin of the unusually high melting point and is not merely an unrelated true fact. Option A incorrectly denies this connection. Option C incorrectly rejects the strong-lattice description, and option D incorrectly rejects the high-melting-point assertion.

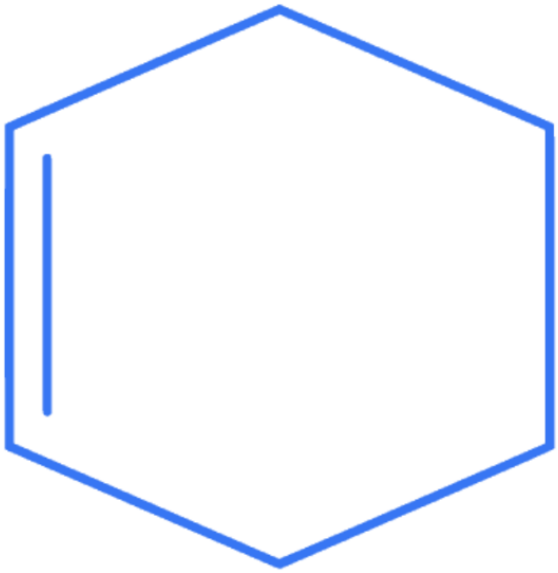
Step 4: Conclusion Both Assertion and Reason are correct, and the Reason correctly explains the

Assertion. Hence, the correct option is (B).

Quick Tip to Solve: In a melting-point assertion, connect the observed temperature to the energy needed to disrupt the actual lattice, then judge whether the stated reason supplies that link.

74. Cyclohexene

is _____ type of an organic compound. [4 marks]



(A) Benzenoid aromatic

(B) Benzenoid non-aromatic

(C) Acyclic

(D) Alicyclic

Correct Answer: (D) Alicyclic

Solution:

Concept:

- Alicyclic compounds are cyclic carbon compounds that have aliphatic, rather than aromatic, character. They may be saturated or unsaturated.
- Aromaticity requires a continuous cyclic conjugated system with the appropriate number of delocalised pi electrons. A ring or a single double bond alone does not establish aromaticity.

Step 1: Read the carbon skeleton The displayed cyclohexene structure has a six-membered ring made only of carbon atoms and contains one carbon-carbon double bond. The ring is closed, so the compound is cyclic rather than acyclic. It is also unsaturated, but unsaturation is independent of whether a compound is aromatic.

Step 2: Check whether the ring is aromatic The one double bond supplies only a localised pi bond. The remaining saturated ring carbons interrupt a continuous set of overlapping p orbitals, so there is no fully conjugated aromatic circuit around the ring. In particular, the drawing is not a benzene ring:

benzene has a conjugated six-pi-electron system, whereas cyclohexene does not.

Step 3: Choose the appropriate classification Because cyclohexene is a non-aromatic carbocyclic compound with aliphatic character, it belongs to the alicyclic class. Option A wrongly calls it benzenoid aromatic. Option B also wrongly describes it as benzenoid; having six ring carbons is not enough to make a benzene framework. Option C is excluded by the closed ring. Thus D gives the required structural class.

Step 4: Conclusion Cyclohexene is an alicyclic organic compound. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish cyclic from aromatic: first check ring closure, then check continuous conjugation. A cycloalkene with isolated unsaturation is alicyclic.

---

75. Yellow compound of lead chromate gets dissolved on treatment with hot NaOH solution. The product of lead formed is a : [4 marks]

(A) Tetraanionic complex with coordination number six

(B) Neutral complex with coordination number four

(C) Dianionic complex with coordination number six

(D) Dianionic complex with coordination number four

Correct Answer: (D) Dianionic complex with coordination number four

Solution:

Concept:

- Lead(II) is amphoteric and can dissolve in excess strong alkali through formation of a soluble hydroxido complex. Lead chromate supplies lead in oxidation state $+2$.
- The complex charge is the sum of the metal oxidation state and ligand charges. A hydroxido ligand $\mathrm{OH^-}$ is monodentate, so each such ligand contributes one to the coordination number.

Step 1: Identify the lead-containing product On treatment with hot sodium hydroxide, yellow lead chromate dissolves because lead(II) forms the tetrahydroxidoplumbate(II) ion. A balanced representation of the process is

$$\mathrm{PbCrO_4 + 4\,OH^- \longrightarrow [Pb(OH)_4]^{2-} + {CrO_4}^{2-}.}$$

The chromate part remains a chromate ion; the question asks specifically about the product containing lead.

Step 2: Calculate the charge of the complex Chromate has charge -2, so neutral $\mathrm{PbCrO_4}$ contains $\mathrm{Pb^{2+}}$. In the product the four hydroxido ligands contribute a total charge of -4. Therefore the complex charge is

$$+2 + 4(-1) = -2.$$

It is a dianion, not a tetraanion or a neutral species. The charge on the metal must not be confused with the charge on the complete coordination entity.

Step 3: Count the donor atoms Each of the four hydroxido ligands binds through one oxygen

donor atom. The coordination number is therefore four. It is not six merely because lead can show other coordination environments in different compounds. Option A has both the wrong charge and coordination number; B has the wrong charge; C has the wrong coordination number. Option D matches both independent counts.

Step 4: Conclusion The lead product is $[Pb(OH)_4]^{2-}$, a dianionic complex with coordination number four. Hence, the correct option is (D).

Quick Tip to Solve: For a hydroxido complex, calculate the overall charge using oxidation state plus ligand charges, and count coordinated oxygen donors separately.

---

76. Given below are two statements :

Statement (I) : Aqueous solution of ammonium carbonate is basic.

Statement (II) : Acidic/basic nature of salt solution of a salt of weak acid and weak base depends on K_a and K_b value of acid and the base forming it.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct

(B) Statement I is correct but Statement II is incorrect

(C) Both Statement I and Statement II are incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- Salt hydrolysis depends on the acid-base strengths of its ions. Ammonium ion is a weak acid, whereas carbonate ion is a base that can accept a proton from water.
- For a simple salt of a weak acid and a weak base, the approximate relation $\mathrm{pH} = 7 + \frac{1}{2}\log(K_b/K_a)$ at $25^\circ\mathrm{C}$ expresses how the parent base and acid strengths control the solution.

Step 1: Consider the two hydrolysing ions Ammonium carbonate dissociates to give two ammonium ions and one carbonate ion per formula unit. Their principal hydrolysis processes are

$$NH_4{}^+ + H_2O \rightleftharpoons NH_3 + H_3O^+$$

and

$$CO_3{}^{2-} + H_2O \rightleftharpoons HCO_3{}^- + OH^-.$$

Thus it is not enough to observe the presence of ammonium and immediately label the salt acidic; the carbonate contribution must also be considered.

Step 2: Determine the net nature of the solution Carbonate has an appreciable proton-accepting tendency: its base constant is K_w/K_{a2} of carbonic acid. This is far greater than the acid constant K_w/K_b of ammonium ion. Carbonate hydrolysis therefore dominates the acidic hydrolysis of ammo-

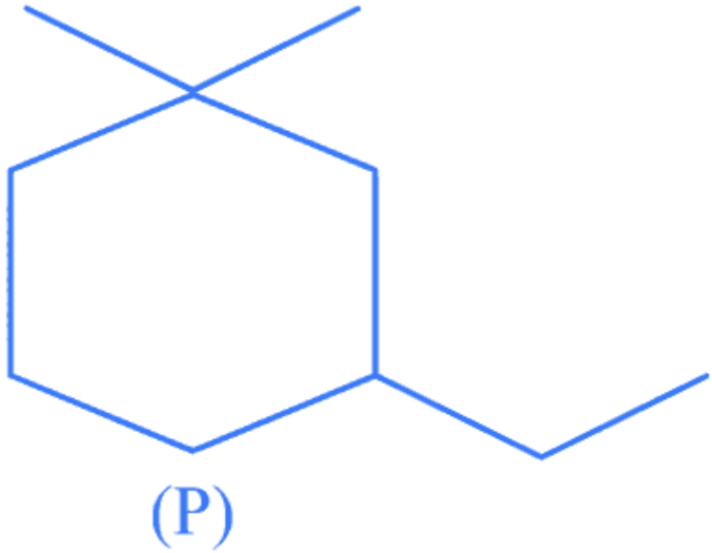
nium, making aqueous ammonium carbonate basic. The two-to-one stoichiometry does not reverse this large strength difference. Statement I is correct.

Step 3: Judge the general statement For a salt derived from a weak acid and a weak base, comparing the relevant acid and base dissociation constants determines which hydrolysis effect dominates. Statement II correctly describes that dependence. Option B wrongly rejects II; option C rejects both correct statements; option D wrongly calls the ammonium carbonate solution non-basic. Both statements must be accepted.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: Do not classify a salt from one ion alone: compare the hydrolysing acid and base strengths, especially for carbonate with ammonium.

77. IUPAC name of following compound (P) is: [4 marks]



(P)

(A) 1-Ethyl-5,5-dimethylcyclohexane

(B) 3-Ethyl-1,1-dimethylcyclohexane

(C) 1-Ethyl-3,3-dimethylcyclohexane

(D) 1,1-Dimethyl-3-ethylcyclohexane

Correct Answer: (B) 3-Ethyl-1,1-dimethylcyclohexane

Solution:

Concept:

- Choose the largest suitable carbon skeleton as parent. For this structure the six-membered carbon ring is the parent cyclohexane; the shorter side chains are substituents.
- Number a substituted ring to give the lowest set of locants at the first point of difference. After numbering, list substituent names alphabetically, ignoring multiplicative prefixes such as di-.

Step 1: Identify the substituents The top ring carbon carries two terminal single-carbon branches,

so these are two methyl substituents on the same carbon. Another ring carbon carries a two-carbon chain, which is an ethyl substituent. There are six carbon atoms in the ring and no double bond, so the parent name is cyclohexane, not hexene or an acyclic alkane.

Step 2: Compare the possible numbering sets If the carbon bearing the two methyl groups is carbon 1, number toward the ethyl group to put it at carbon 3. This produces the locant set $(1, 1, 3)$. Starting at the ethyl-bearing carbon instead gives $(1, 3, 3)$, while numbering in the less favourable direction can give $(1, 5, 5)$. Compare sorted locants at the first point of difference: $(1, 1, 3)$ is the lowest set and must be chosen.

Step 3: Assemble the final name The substituents are therefore 3-ethyl and 1,1-dimethyl. Ethyl precedes methyl alphabetically; the prefix di- is ignored in deciding that order. Hence the name is 3-ethyl-1,1-dimethylcyclohexane. Option A uses the higher locant set 1,5,5; option C uses 1,3,3; option D has the correct locants but lists dimethyl before ethyl. Alphabetical ordering is applied after the lowest numbering has been selected.

Step 4: Conclusion The IUPAC name is 3-ethyl-1,1-dimethylcyclohexane. Hence, the correct option is (B).

Quick Tip to Solve: Lowest locants decide ring numbering before alphabetical order decides the written name; do not force ethyl onto carbon 1 just because E precedes M.

---

78. $NaCl$ **reacts with conc.** H_2SO_4 **and** $K_2Cr_2O_7$ **to give reddish fumes (B), which react with** $NaOH$ **to give yellow solution (C). (B) and (C) respectively are ;** [4 marks]

(A) CrO_2Cl_2, Na_2CrO_4

(B) Na_2CrO_4, CrO_2Cl_2

(C) $CrO_2Cl_2, KHSO_4$

(D) $CrO_2Cl_2, Na_2Cr_2O_7$

Correct Answer: (A) CrO_2Cl_2, Na_2CrO_4

Solution:

Concept:

- Chloride ions heated with potassium dichromate and concentrated sulfuric acid give volatile chromyl chloride, CrO_2Cl_2, seen as reddish fumes. This is the chromyl chloride test for chloride.
- Chromyl chloride reacts with excess alkali to form the chromate ion, CrO_4^{2-}, which is yellow. Chromium remains in the +6 oxidation state in both compounds.

Step 1: Identify the reddish fumes The strongly acidic dichromate mixture converts chloride into chromyl chloride rather than into a sodium chromate vapour. A balanced overall reaction is

$$K_2Cr_2O_7 + 4\,NaCl + 6\,H_2SO_4 \longrightarrow 2\,CrO_2Cl_2 + 2\,KHSO_4 + 4\,NaHSO_4 + 3\,H_2O.$$

Thus species (B) is CrO_2Cl_2. Its volatility and reddish colour distinguish it from the nonvolatile sodium salts in the alternatives.

Step 2: Identify the yellow alkaline product On absorption in sodium hydroxide solution, chromyl

chloride is converted into sodium chromate:

$$CrO_2Cl_2 + 4\,NaOH \longrightarrow Na_2CrO_4 + 2\,NaCl + 2\,H_2O.$$

Therefore (C) is Na_2CrO_4. Option (B) reverses the two species. Option (C) gives potassium hydrogen sulfate, which is not the yellow chromate product. Option (D) gives dichromate; dichromate is orange and is favoured by acid, whereas the stated alkaline conditions favour yellow chromate.

Step 3: Conclusion The reddish fumes are chromyl chloride, CrO_2Cl_2, and the yellow solution contains sodium chromate, Na_2CrO_4. Hence, the correct option is (A).

Quick Tip to Solve: In the chromyl chloride test, remember the colour sequence: reddish volatile chromyl chloride followed by yellow chromate in alkali.

79. The correct statement regarding nucleophilic substitution reaction in a chiral alkyl halide is ; [4 marks]

(A) Retention occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

(B) Racemisation occurs in S_N1 reaction and retention occurs in S_N2 reaction.

(C) Racemisation occurs in both S_N1 and S_N2 reactions.

(D) Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

Correct Answer: (D) Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

Solution:

Concept:

- An S_N1 substitution proceeds through a carbocation intermediate. The carbon carrying the positive charge is approximately trigonal planar, so the nucleophile can approach either face.
- An S_N2 substitution is a concerted displacement. Backside attack, opposite the leaving group, gives inversion of the spatial arrangement at the reacting stereogenic carbon.

Step 1: Follow the stereochemistry of the two-step pathway In an S_N1 reaction, the carbon–halogen bond first breaks heterolytically. The chiral tetrahedral centre becomes a planar carbocation. Attack on one face and attack on the opposite face give the two configurations, so the reaction leads to racemisation rather than exclusive retention. In practice, an ion pair can obstruct one face and produce an excess of inversion, so racemisation need not be perfectly complete; it remains the characteristic description among the listed choices.

Step 2: Follow the concerted pathway and compare the choices For S_N2, bond formation to the nucleophile and bond cleavage to the leaving halide occur in one step. Backside approach turns the other three substituents over, producing Walden inversion. There is no free planar carbocation allowing random attack from both faces. Thus option (A) fails because it assigns retention to S_N1. Option (B) fails because it assigns retention to S_N2. Option (C) incorrectly assigns racemisation to the concerted reaction. Only option (D) gives both characteristic outcomes correctly.

Step 3: Conclusion S_N1 gives racemisation, while S_N2 gives inversion at the reacting chiral carbon. Hence, the correct option is (D).

Quick Tip to Solve: Connect the geometry to the result: a planar carbocation allows two faces; a single backside displacement forces inversion.

80. The electronic configuration for Neodymium is:

[Atomic Number for Neodymium 60] [4 marks]

(A) $[Xe]4f^4 6\ s^2$

(B) $[Xe]5f^4 7\ s^2$

(C) $[Xe]4f^6 6\ s^2$

(D) $[Xe]4f^1 5\ d^1 6\ s^2$

Correct Answer: (A) $[Xe]4f^4 6\ s^2$

Solution:

Concept:

- For a neutral atom, the number of electrons equals its atomic number. The noble-gas core $[Xe]$ contains 54 electrons, so neodymium with $Z = 60$ has six electrons outside this core.
- Neodymium is a lanthanoid of the 4f series. Its ground-state configuration has two electrons in 6s and four in 4f: $[Xe]\,4f^4 6s^2$.

Step 1: Count the electrons outside xenon Start with the xenon core rather than attempting to list all occupied orbitals. The electron count remaining after the core is

$$60 - 54 = 6.$$

Two of these electrons occupy the 6s subshell. For neodymium, the remaining four occupy 4f, and there is no occupied 5d subshell in the standard ground-state configuration. The resulting count is

$$54 + 4 + 2 = 60.$$

This both identifies the atom and checks the exponents in the configuration.

Step 2: Check the subshells and reject the other configurations Option (A) has the correct 4f and 6s subshells and the correct electron total. Option (B) also writes six electrons outside the core, but places them in 5f and 7s, which are not the ground-state subshells of this period-six lanthanoid. Option (C) contains $54 + 6 + 2 = 62$ electrons, so it cannot represent neutral neodymium. Option (D) contains $54 + 1 + 1 + 2 = 58$ electrons; it is associated with the cerium configuration, not neodymium. Electron counting and the identity of the f series must both be correct.

Step 3: Conclusion The ground-state electronic configuration of neodymium is $[Xe]\,4f^4 6s^2$. Hence, the correct option is (A).

Quick Tip to Solve: Subtract 54 for a xenon core, then check both the remaining electron count and the correct f-series subshell; counting alone cannot reject option (B).

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The mass of silver (Molar mass of $\mathrm{Ag}: 108\,\mathrm{g\,mol^{-1}}$) displaced by a quantity of electricity which displaces $5600\,\mathrm{mL}$ of $\mathrm{O_2}$ at S.T.P. will be _____ g. [4 marks]

Solution:

Concept:

- Faraday's law relates the amount of an electrode product to the moles of electrons passed. The same quantity of electricity means the same number of electron moles in the oxygen and silver processes.
- At STP, one mole of an ideal gas occupies $22.4\,\mathrm{L}$. Oxygen evolution requires four moles of electrons per mole of $\mathrm{O_2}$, whereas $\mathrm{Ag^+ + e^- \longrightarrow Ag}$ requires one electron per silver atom.

Step 1: Convert oxygen volume to electron moles The stated gas volume is

$$5600\,\mathrm{mL} = 5.600\,\mathrm{L}.$$

Using the STP molar volume gives

$$n(\mathrm{O_2}) = \frac{5.600}{22.4} = 0.250\,\mathrm{mol}.$$

The oxygen electrode process can be written as

$$2\,\mathrm{H_2O} \longrightarrow \mathrm{O_2} + 4\,\mathrm{H^+} + 4\,\mathrm{e^-}.$$

It follows that the charge producing this oxygen corresponds to

$$n(e^-) = 4(0.250) = 1.00\,\mathrm{mol}.$$

The four-electron factor comes from the oxidation-state change of the two oxygen atoms, not simply from the molecular subscript.

Step 2: Use the same charge to calculate silver mass Silver is deposited according to

$$\mathrm{Ag^+ + e^- \longrightarrow Ag}.$$

Thus one mole of electron charge deposits one mole of silver. For the same charge,

$$n(\mathrm{Ag}) = 1.00\,\mathrm{mol}.$$

Multiplying by the given molar mass gives

$$m(\mathrm{Ag}) = (1.00\,\mathrm{mol})(108\,\mathrm{g\,mol^{-1}}) = 108\,\mathrm{g}.$$

The Faraday constant need not be inserted numerically, because the comparison uses the same charge in both processes. Assuming a one-to-one oxygen-to-silver mole ratio would miss the four electrons

associated with each oxygen molecule.

Step 3: Conclusion The mass of silver displaced is $\boxed{108\,\text{g}}$. The numerical entry is 108.

Quick Tip to Solve: For equal charge, compare electron equivalents: one mole of oxygen corresponds to four moles of silver, not one.

82. Consider the following data for the given reaction

$$2\,\text{HI}\,(\text{g}) \longrightarrow \text{H}_2(\text{g}) + \text{I}_2(\text{g})$$

Experiment	**1**	**2**	**3**
HI $(\text{mol}\,\text{L}^{-1})$	**0.005**	**0.01**	**0.02**
Rate $(\text{mol}\,\text{L}^{-1}\,\text{s}^{-1})$	7.5×10^{-4}	3.0×10^{-3}	1.2×10^{-2}

The order of the reaction is _____. [4 marks]

Solution:

Concept:

- For this single-reactant rate comparison, write $r = k[\text{HI}]^n$, where n is the experimentally determined order and k is constant for the runs at the same temperature.
- Taking the ratio of rates eliminates k. If a concentration change by a factor f changes the rate by a factor g, then $g = f^n$. Reaction order must be inferred from data, not assumed from the balanced equation.

Step 1: Compare the first two experiments The tabulated concentrations of HI in experiments 1 and 2 are 0.005 and $0.010\,\text{mol}\,\text{L}^{-1}$. The respective rates are 7.5×10^{-4} and $3.0 \times 10^{-3}\,\text{mol}\,\text{L}^{-1}\,\text{s}^{-1}$. Therefore

$$\frac{r_2}{r_1} = \frac{3.0 \times 10^{-3}}{7.5 \times 10^{-4}} = 4, \qquad \frac{[\text{HI}]_2}{[\text{HI}]_1} = 2.$$

Applying the rate law gives

$$4 = 2^n, \qquad n = 2.$$

Doubling the concentration causes a fourfold rate increase, which identifies a second-order dependence.

Step 2: Verify the result with the third experiment Experiments 2 and 3 again double the concentration, from 0.010 to $0.020\,\text{mol}\,\text{L}^{-1}$, while the rate changes from 3.0×10^{-3} to $1.2 \times 10^{-2}\,\text{mol}\,\text{L}^{-1}\,\text{s}^{-1}$. Their ratio is also four, consistent with $n = 2$. An additional check is

$$k = \frac{r_1}{[\text{HI}]_1^2} = \frac{7.5 \times 10^{-4}}{(0.005)^2} = 30\,\text{L}\,\text{mol}^{-1}\,\text{s}^{-1}.$$

The other runs give the same value. The order is dimensionless; only the rate constant carries these units.

Step 3: Conclusion The rate law is $r = k[\text{HI}]^2$, so the order of the reaction is $\boxed{2}$.

Quick Tip to Solve: When doubling a reactant concentration quadruples the rate, its order is two; use a second pair of runs to confirm the pattern.

83. Mass of methane required to produce $22\ \mathrm{g}$ of CO_2 after complete combustion is _____ g.
(Given Molar mass in g mol-1 $\mathrm{C} = 12.0$, $\mathrm{H} = 1.0$, $\mathrm{O} = 16.0$) [4 marks]

Solution:

Concept:

- Complete combustion of methane forms carbon dioxide and water. The balanced equation fixes the mole ratio between methane consumed and carbon dioxide formed.
- Mass is converted to moles with $n = m/M$. From the given atomic masses, $M(\mathrm{CO_2}) = 44\,\mathrm{g\,mol^{-1}}$ and $M(\mathrm{CH_4}) = 16\,\mathrm{g\,mol^{-1}}$.

Step 1: Balance the reaction and find carbon dioxide moles The balanced combustion equation is

$$\mathrm{CH_4} + 2\,\mathrm{O_2} \longrightarrow \mathrm{CO_2} + 2\,\mathrm{H_2O}.$$

It conserves one carbon atom, four hydrogen atoms and four oxygen atoms on each side. Thus one mole of methane produces exactly one mole of carbon dioxide when combustion is complete. Using the supplied atomic masses,

$$M(\mathrm{CO_2}) = 12.0 + 2(16.0) = 44.0\,\mathrm{g\,mol^{-1}}.$$

Hence the required carbon dioxide amount is

$$n(\mathrm{CO_2}) = \frac{22\,\mathrm{g}}{44\,\mathrm{g\,mol^{-1}}} = 0.50\,\mathrm{mol}.$$

The coefficient 2 belongs to oxygen and water; it does not make the methane-to-carbon-dioxide ratio two.

Step 2: Convert the required methane moles into mass The one-to-one carbon balance gives

$$n(\mathrm{CH_4}) = n(\mathrm{CO_2}) = 0.50\,\mathrm{mol}.$$

Methane has molar mass

$$M(\mathrm{CH_4}) = 12.0 + 4(1.0) = 16.0\,\mathrm{g\,mol^{-1}}.$$

Consequently,

$$m(\mathrm{CH_4}) = (0.50\,\mathrm{mol})(16.0\,\mathrm{g\,mol^{-1}}) = 8.0\,\mathrm{g}.$$

A mass-ratio check gives $m(\mathrm{CH_4})/m(\mathrm{CO_2}) = 16/44$, also yielding $22(16/44) = 8\,\mathrm{g}$. The product mass is larger because oxygen from the other reactant is incorporated in carbon dioxide.

Step 3: Conclusion The mass of methane required is $\boxed{8\,\mathrm{g}}$. The numerical entry is 8.

Quick Tip to Solve: Each methane molecule contains one carbon atom and therefore forms one carbon dioxide molecule; use that mole ratio before comparing masses.

84. If three moles of an ideal gas at $300\ \mathrm{K}$ expand isothermally from $30\ \mathrm{dm}^3$ to $45\ \mathrm{dm}^3$ against a constant opposing pressure of $80\ \mathrm{kPa}$, then the amount of heat transferred is _____ J. [4 marks]

Solution:

Concept:

- For an ideal gas, internal energy depends only on temperature. An isothermal process therefore has $\Delta U = 0$, even when the expansion is irreversible.
- With the chemistry sign convention, $\Delta U = q + w$ and expansion against a constant external pressure has $w = -P_{\text{ext}}\Delta V$. Work done by the gas is negative as work on the system.

Step 1: Calculate the work against the external pressure The volume increase is

$$\Delta V = 45 - 30 = 15\,\mathrm{dm}^3.$$

Since the opposing pressure remains constant, use that pressure directly:

$$w = -(80\,\mathrm{kPa})(15\,\mathrm{dm}^3).$$

To express the result in joules, note that

$$1\,\mathrm{kPa\,dm}^3 = (10^3\,\mathrm{Pa})(10^{-3}\,\mathrm{m}^3) = 1\,\mathrm{J}.$$

Therefore

$$w = -1200\,\mathrm{J}.$$

The negative sign indicates that the expanding gas transfers energy to its surroundings through mechanical work.

Step 2: Apply the first law at constant temperature Because the gas is ideal and the temperature stays at $300\,\mathrm{K}$, its internal energy does not change:

$$0 = \Delta U = q + w.$$

Substitution of the calculated work gives

$$q = -w = +1200\,\mathrm{J}.$$

Thus heat must enter the gas to replace the energy it loses by doing work. The given number of moles and temperature would be needed for a reversible isothermal formula, but that formula cannot be substituted here: the stated process takes place against a constant opposing pressure, rather than an external pressure that continuously matches the gas pressure.

Step 3: Conclusion The gas absorbs $\boxed{1200\,\mathrm{J}}$ of heat, so $q = +1200\,\mathrm{J}$. The numerical entry is 1200.

Quick Tip to Solve: First identify the work condition: constant external pressure means $-P_{\text{ext}}\Delta V$; then use $q = -w$ for an isothermal ideal gas.

85. 3-Methylhex-2-ene on reaction with HBr in presence of peroxide forms an addition product (A). The number of possible stereoisomers for 'A' is _____. [4 marks]

Solution:

Concept:

- In the presence of peroxide, addition of HBr to an unsymmetrical alkene follows the radical, anti-Markovnikov pathway. Bromine attaches so that the intermediate carbon radical is more substituted.
- A tetrahedral carbon is stereogenic when it has four different substituents. For n independent stereogenic centres the maximum number of configurations is 2^n, reduced only if molecular symmetry produces a meso form.

Step 1: Locate the double bond and choose the radical pathway The starting alkene has the connectivity

$$CH_3-CH=C(CH_3)-CH_2-CH_2-CH_3$$

. Addition of a bromine radical to carbon 2 leaves the radical at carbon 3, where it is tertiary and therefore more stable than the alternative secondary radical. That radical then abstracts hydrogen from HBr. Thus the product is 2-bromo-3-methylhexane, not the Markovnikov bromoalkane.

Step 2: Identify the stereogenic centres in the product The product connectivity is

$$CH_3-CH(Br)-CH(CH_3)-CH_2-CH_2-CH_3$$

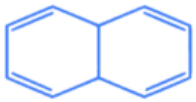
. Carbon 2 is attached to bromine, hydrogen, methyl and the substituted chain through carbon 3: all four groups differ. Carbon 3 is attached to hydrogen, methyl, propyl and the bromine-containing group through carbon 2: these too are four different groups. Hence both carbons 2 and 3 are stereogenic. The remaining carbons have repeated hydrogens and do not create further stereogenic centres.

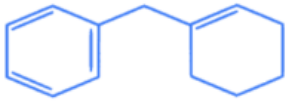
Step 3: Count distinct configurations and test symmetry The configurations are $(2R, 3R)$, $(2S, 3S)$, $(2R, 3S)$ and $(2S, 3R)$. The two stereogenic sites do not have identical substituents, and the two ends of the carbon skeleton differ. There is consequently no internal plane of symmetry that identifies either opposite-configuration structure as a meso form. The saturated product has no double bond, so no additional E/Z factor is included. Therefore $2^2 = 4$ distinct stereoisomers are possible.

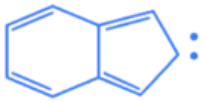
Step 4: Conclusion The addition product has two independent stereogenic centres and no meso form. The required number of possible stereoisomers is $\boxed{4}$ (a dimensionless count).

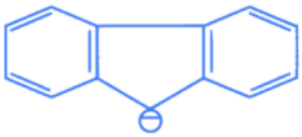
Quick Tip to Solve: Write the anti-Markovnikov product before counting centres: the former alkene carbons can both become stereogenic after addition.

86. Among the given organic compounds, the total number of aromatic compounds is [4 marks]

(A) 

(B) 

(C) 

(D) 

Solution:

Concept:

- Aromaticity requires a cyclic, planar, continuously conjugated array of p orbitals containing $(4n+2)$ delocalised π electrons, with n a nonnegative integer.
- An sp^3 carbon breaks conjugation. A properly oriented lone pair at a ring carbon can supply two electrons to a conjugated circuit. A compound containing a benzene ring can remain aromatic even when another attached ring is not aromatic.

Step 1: Check the fused structure A In structure (A), the shared bond of the two six-membered rings is single, and the two fusion carbons are joined only by single bonds to their neighbours. These junctions interrupt the p-orbital pathway. The four drawn double bonds therefore do not constitute one continuously conjugated aromatic perimeter. Simply adding their eight electrons would miss this interruption. Structure (A) is not aromatic.

Step 2: Check the benzene-containing structure B Structure (B) retains an intact benzene ring with three alternating double bonds. Its six π electrons satisfy $4n+2=6$, with $n=1$, and the ring has a continuous p-orbital system. The side chain and the partially unsaturated second ring do not destroy that benzene sextet. Thus (B) is counted as an aromatic compound even though the other ring does not itself form a separate aromatic sextet.

Step 3: Count the conjugated fused systems C and D In (C), the lone pair shown at the five-membered-ring carbon joins the conjugated fused framework. Four double bonds supply eight electrons and the participating lone pair supplies two, giving a ten-electron aromatic system: $10=4(2)+2$. In (D), the negatively charged carbon supplies its lone pair to the fused conjugated system. The six double bonds and that pair give fourteen π electrons: $14=4(3)+2$. Both frameworks allow the required continuous delocalisation, so (C) and (D) are aromatic. Combining these with (B) gives three aromatic compounds.

Step 4: Conclusion The aromatic compounds are $\boxed{\text{B, C and D}}$. Their total number is $\boxed{3}$ (dimensionless).

Quick Tip to Solve: Test continuity of p orbitals before applying the electron-count rule; also distinguish an aromatic compound from a demand that every ring be aromatic.

87. Among the following, total number of meta directing functional groups is (Integer based)

$$-OCH_3, -NO_2, -CN, -CH_3 - NHCOCH_3, -COR, -OH, -COOH, -Cl$$

[4 marks]

Solution:

Concept:

- In electrophilic aromatic substitution, a substituent is meta directing when its electron withdrawal destabilises the ortho/para sigma complexes more strongly than the meta complex.
- Groups that withdraw by resonance, such as nitro, cyano and carbonyl-containing substituents attached through carbon, are usually meta directing. Lone-pair donors and alkyl groups favour ortho/para substitution; halogens are the important deactivating ortho/para exception.

Step 1: Select the strongly electron-withdrawing groups The nitro group $-NO_2$ withdraws electron density through both inductive and resonance effects. The cyano group $-CN$, attached to the ring through its carbon atom, also withdraws by resonance. Both groups are meta directing because an ortho or para attack produces especially unfavourable resonance contributors with positive charge near the electron-withdrawing substituent.

Step 2: Classify the carbonyl groups by their attachment atom In $-COR$, the ring is bonded directly to the carbonyl carbon; the carbonyl group draws electron density away from the ring by resonance. Similarly, $-COOH$ is attached through its carbonyl carbon and is meta directing. This gives the four selected groups $-NO_2$, $-CN$, $-COR$ and $-COOH$. The attachment atom matters: $-NHCOCH_3$ is attached through nitrogen, not through a carbonyl carbon.

Step 3: Exclude the ortho/para directors and total the selection The groups $-OCH_3$ and $-OH$ donate an oxygen lone pair to the ring and direct ortho/para. The methyl group $-CH_3$ donates through hyperconjugation and is also ortho/para directing. The acetamido group $-NHCOCH_3$ can donate through nitrogen and remains ortho/para directing despite its weaker donation than an amino group. Finally, $-Cl$ deactivates inductively but directs ortho/para through lone-pair resonance donation. None of these adds to the meta count. The selected list therefore contains exactly four members.

Step 4: Conclusion The meta-directing groups are $-NO_2$, $-CN$, $-COR$ and $-COOH$. The required integer is $\boxed{4}$ (a dimensionless count).

Quick Tip to Solve: Look at the atom bonded directly to the ring: carbonyl carbon usually gives meta direction, whereas nitrogen or oxygen with a donating lone pair gives ortho/para direction.

88. The number of electrons present in all the completely filled subshells having $\mathrm{n} = 4$ and $\mathrm{s} = +\frac{1}{2}$ is _____.

(Where n = principal quantum number and s = spin quantum number) [4 marks]

Solution:

Concept:

- For a shell with principal quantum number n, the allowed orbital quantum numbers are $l = 0, 1, \ldots, n-1$. A subshell with quantum number l contains $2l+1$ orbitals.
- Each completely filled orbital contains two electrons with opposite spin projections. Hence exactly one electron per filled orbital has the specified spin projection $+\frac{1}{2}$. The specified spin condition selects half the electrons in a closed shell.

Step 1: List all subshells belonging to the fourth shell For $n = 4$, the allowed values of l are 0, 1, 2 and 3. These correspond respectively to the $4s$, $4p$, $4d$ and $4f$ subshells. The phrase all completely filled subshells requires all four of these subshells to be included; it does not restrict the count to those occupied in a particular unspecified element.

Step 2: Count the electrons of the stated spin in each subshell The $4s$ subshell has $2(0)+1 = 1$ orbital and therefore one spin-up electron when filled. The $4p$ subshell has $2(1)+1 = 3$ orbitals and three spin-up electrons. Similarly, $4d$ has five orbitals and five spin-up electrons, while $4f$ has seven orbitals and seven spin-up electrons. The counts 2, 6, 10 and 14 would instead represent both spin projections together and must not be used directly here.

Step 3: Add the subshell contributions and verify the result The required number is therefore

$$N_{+1/2} = 1 + 3 + 5 + 7 = 16.$$

As a cross-check, a shell contains n^2 orbitals, so the fourth shell contains $4^2 = 16$ orbitals. When every orbital is doubly occupied, the shell has $2n^2 = 32$ electrons in total, with sixteen of each spin projection. This independently reproduces the same selected-spin count. No extra factor of two is required after counting the orbitals, because the spin projection has already been fixed.

Step 4: Conclusion There are $\boxed{16}$ electrons with the specified spin projection in the completely filled $n = 4$ subshells. The requested number is 16 (a dimensionless count).

Quick Tip to Solve: For a completely filled shell and one specified spin projection, the electron count equals the orbital count n^2, rather than the total capacity $2n^2$.

89. Sum of bond order of CO and NO^+ is _____. [4 marks]

Solution:

Concept:

- Molecular orbital theory defines bond order as $\mathrm{BO} = (N_b - N_a)/2$, where N_b and N_a are the numbers of electrons in bonding and antibonding molecular orbitals.
- The neutral CO molecule and the NO^+ ion are isoelectronic: each has fourteen total electrons, or

ten valence electrons. Their filled core bonding and antibonding orbitals cancel in the bond-order difference.

Step 1: Count electrons in each species Carbon contributes six total electrons and oxygen contributes eight, giving $6+8=14$ electrons in CO. Nitrogen contributes seven and oxygen contributes eight; removal of one electron for the positive charge gives $7+8-1=14$ electrons in NO^+. Equivalently, their valence-electron counts are $4+6=10$ and $5+6-1=10$, respectively. It is essential to subtract, rather than add, an electron for the cation.

Step 2: Calculate the bond order of CO For the ten valence electrons, the occupied orbitals have two electrons in the bonding $2s$-derived orbital, two in the antibonding $2s$-derived orbital, and six in the bonding $2p$-derived orbitals. Thus there are eight bonding and two antibonding valence electrons. The precise relative ordering of the filled sigma and pi bonding orbitals does not change this count. Therefore

$$\mathrm{BO(CO)}=\frac{8-2}{2}=3.$$

Step 3: Calculate the ionic bond order and add The isoelectronic NO^+ ion has the same bonding-minus-antibonding electron count. Its ten valence electrons likewise give eight bonding and two antibonding electrons, so

$$\mathrm{BO}(NO^+)=\frac{8-2}{2}=3.$$

This also agrees with removing the unpaired antibonding electron from neutral NO, whose bond order is 2.5, thereby increasing the bond order by 0.5. Adding the two individual bond orders gives $3+3=6$. Bond order is a pure number and therefore has no physical unit.

Step 4: Conclusion The individual bond orders are 3 for CO and 3 for NO^+. Their sum is $\boxed{6}$ (dimensionless).

Quick Tip to Solve: Count the charge-adjusted electrons first: CO, NO^+ and N_2 are all fourteen-electron species with bond order three.

90. From the given list, the number of compounds with +4 oxidation state of Sulphur _____.

$$SO_3, H_2SO_3, SOCl_2, SF_4, BaSO_4, H_2S_2O_7$$

[4 marks]

Solution:

Concept:

- The sum of oxidation states equals the overall charge of a species. For the listed neutral compounds the sum is zero; hydrogen is $+1$, oxygen is -2, fluorine is -1, and barium is $+2$.
- Chlorine is -1 in thionyl chloride. None of the listed compounds contains a peroxide bond, so the ordinary oxygen value -2 applies, including the bridging oxygen of disulphuric acid.

Step 1: Evaluate the simple oxides and sulphurous acid Let the oxidation state of sulphur be x. In

SO_3, charge balance gives $x + 3(-2) = 0$, hence $x = +6$, so it is excluded. For H_2SO_3,

$$2(+1) + x + 3(-2) = 0 \quad \Rightarrow \quad x = +4.$$

Sulphurous acid is therefore included. The number of oxygen atoms alone is insufficient when hydrogen is also present; the full charge balance must be used.

Step 2: Check thionyl chloride and sulphur tetrafluoride In $SOCl_2$, the oxygen contributes -2 and the two chlorines together contribute -2. Thus $x - 2 - 2 = 0$, giving $x = +4$. In SF_4, each fluorine has oxidation state -1, so $x + 4(-1) = 0$, again giving $x = +4$. Both compounds meet the requested condition. The distinction between oxidation state and valency is important: this calculation uses assigned ionic charges, not a direct count of bonds alone.

Step 3: Exclude the sulphates and count the matches For $BaSO_4$,

$$+2 + x + 4(-2) = 0 \quad \Rightarrow \quad x = +6.$$

For $H_2S_2O_7$, the two sulphur atoms are equivalent, and

$$2(+1) + 2x + 7(-2) = 0 \quad \Rightarrow \quad x = +6.$$

The sum of the sulphur oxidation states in this last molecule is $+12$, which must be divided between its two sulphur atoms. Consequently neither barium sulphate nor disulphuric acid is included. Exactly H_2SO_3, $SOCl_2$ and SF_4 have sulphur in the $+4$ state.

Step 4: Conclusion The qualifying compounds are H_2SO_3, $SOCl_2$ and SF_4. Their number is $\boxed{3}$ (dimensionless).

Quick Tip to Solve: Write one charge-balance equation per compound, and divide by the number of sulphur atoms when more than one sulphur atom is present.