JEE Main 2024 Paper 1 (B.E./B.Tech) Jan 29 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 1 (Jan-Feb), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 29 January 2024, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Suppose $f(x) = \frac{\left(2^x+2^{-x}\right)\tan x\sqrt{\tan^{-1}(x^2-x+1)}}{(7x^2+3x+1)^3}$. Then the value of $f'(0)$ is equal to [4 marks]

(A) π

(B) $\sqrt{\pi}$

(C) 0

(D) $\frac{\pi}{2}$

Correct Answer: (B) $\sqrt{\pi}$

Solution:

Concept:

- For a differentiable product $f(x) = u(x)v(x)$, the product rule is $f'(x) = u'(x)v(x) + u(x)v'(x)$. When one factor is zero at the evaluation point, one term vanishes and the calculation becomes much shorter.
- The derivative of $\tan x$ is $\sec^2 x$, with angles in radians. Here $\tan 0 = 0$, $\sec^2 0 = 1$, and the principal inverse tangent satisfies $\tan^{-1} 1 = \pi/4$. The square root is the positive square root.

Step 1: Separate the zero factor Write the function as $f(x) = h(x)\tan x$, where

$$h(x) = \frac{(2^x + 2^{-x})\sqrt{\tan^{-1}(x^2 - x + 1)}}{(7x^2 + 3x + 1)^3}.$$

Near zero, the denominator is nonzero and the argument of the square root is positive, so h is differentiable there. The factor $\tan x$ vanishes at zero; we can therefore avoid differentiating every factor of h.

Step 2: Apply the product rule at zero The product rule gives

$$f'(x) = h'(x)\tan x + h(x)\sec^2 x.$$

Substituting zero removes the first term:

$$f'(0) = h'(0)\cdot 0 + h(0)\cdot 1 = h(0).$$

Notice that $f(0) = 0$ does not imply $f'(0) = 0$: the slope depends on the nonzero coefficient multiplying $\tan x$ near the origin.

Step 3: Evaluate the remaining factor Now substitute $x = 0$ into the undifferentiated factor:

$$h(0) = \frac{(1+1)\sqrt{\tan^{-1} 1}}{1^3} = 2\sqrt{\frac{\pi}{4}} = \sqrt{\pi}.$$

Thus option (C) incorrectly confuses the value of the function with its derivative. Options (A) and (D) do not equal the coefficient obtained after taking the square root; retaining the radical is essential.

Step 4: Conclusion For the printed function the derivative is $f'(0) = \sqrt{\pi}$, not zero: the product rule leaves $h(0)\sec^2 0$. Hence, the correct option is (B).

Quick Tip to Solve: At a point where one factor vanishes, apply the product rule before expanding or differentiating the other factors; most terms may disappear immediately.

2. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear. If $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}, \vec{b} + 6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$, then $\alpha + \beta$ is equal to [4 marks]

(A) 30

(B) -30

(C) -25

(D) 35

Correct Answer: (D) 35

Solution:

Concept:

- Collinear vectors are scalar multiples of one another. Two non-collinear non-zero vectors are linearly independent, so coefficients in their linear combination can be compared.
- Express the third vector in the basis formed by the two independent vectors before using the final zero-vector equation.

Step 1: Translate collinearity into equations From the first condition write $\vec{a} + 5\vec{b} = \lambda\vec{c}$, giving $\vec{a} = -5\vec{b} + \lambda\vec{c}$. From the second write $\vec{b} + 6\vec{c} = \mu\vec{a}$. The vector on the left cannot be zero, since b and c are non-collinear. Thus $\mu \neq 0$, and substitution into the second equation is legitimate.

Step 2: Compare the independent coefficients Substitution gives

$$\vec{b} + 6\vec{c} = -5\mu\vec{b} + \mu\lambda\vec{c}.$$

Comparing the coefficients of b and c separately yields $1 = -5\mu$ and $6 = \mu\lambda$. Hence $\mu = -1/5$ and $\lambda = -30$. It follows that $\vec{a} = -5\vec{b} - 30\vec{c}$. The negative signs are essential: the collinear vectors need not point in the same direction.

Step 3: Use the final relation Now

$$\vec{a} + \alpha\vec{b} + \beta\vec{c} = (\alpha - 5)\vec{b} + (\beta - 30)\vec{c} = \vec{0}.$$

Independence forces $\alpha = 5$ and $\beta = 30$, so their sum is 35. The alternatives 30 and -30 confuse a coefficient with the sum, while -25 uses an incorrect sign for one of the independent coefficients.

Step 4: Conclusion $\alpha + \beta = 5 + 30 = 35$. Hence, the correct option is (D).

Quick Tip to Solve: Choose the given non-collinear pair as a vector basis, then compare their coefficients instead of taking magnitudes.

3. Let $\left(5, \frac{a}{4}\right)$ be the circumcenter of a triangle with vertices $\mathrm{A}(a,-2), \mathrm{B}(a,6)$ and $C\left(\frac{a}{4},-2\right)$. Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then $\alpha+\beta+\gamma$ is [4 marks]

(A) 60

(B) 62

(C) 53

(D) 30

Correct Answer: (C) 53

Solution:

Concept:

- A horizontal side and a vertical side meeting at a vertex form a right angle. The circumcentre of a right triangle is the midpoint of its hypotenuse.
- The circumradius is half the hypotenuse; area is half the product of the perpendicular sides, and perimeter is the sum of all three side lengths.

Step 1: Identify the right angle and centre AB is vertical because both endpoints have x-coordinate a, while AC is horizontal because both endpoints have y-coordinate -2. Thus the angle at A is a right angle, and BC is the hypotenuse. Its midpoint is

$$\left(\frac{a+a/4}{2}, \frac{6-2}{2}\right) = \left(\frac{5a}{8}, 2\right).$$

Equating this with the given circumcentre $(5, a/4)$ gives $a = 8$ from either coordinate, consistently.

Step 2: Calculate the three side lengths The vertices are now $A = (8,-2)$, $B = (8,6)$, and $C = (2,-2)$. Their perpendicular side lengths are $AB = 8$ and $AC = 6$. By Pythagoras,

$$BC = \sqrt{8^2+6^2} = \sqrt{100} = 10.$$

The triangle is non-degenerate and the circumcentre is indeed $(5,2)$, midway between B and C.

Step 3: Combine radius, area and perimeter Therefore

$$\alpha = 10/2 = 5, \qquad \beta = \tfrac{1}{2}(8)(6) = 24, \qquad \gamma = 8+6+10 = 24.$$

Their requested numerical sum is $5+24+24 = 53$. In particular, 30 is the sum of the side lengths only if a side is misread; the alternatives 60 and 62 do not equal the radius-plus-area-plus-perimeter computed from these vertices.

Step 4: Conclusion $\alpha+\beta+\gamma = 53$. Hence, the correct option is (C).

Quick Tip to Solve: Before using equal-distance equations for a circumcentre, check whether two sides are horizontal and vertical: a right triangle makes the centre immediate.

4. If $\alpha, -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is [4 marks]

(A) $\frac{10-\sqrt{10}}{12}$

(B) $\frac{\sqrt{10}-10}{6}$

(C) $\frac{\sqrt{10}-10}{12}$

(D) $\frac{10-\sqrt{10}}{6}$

Correct Answer: (C) $\frac{\sqrt{10}-10}{12}$

Solution:

Concept:

- On $-\pi/2 < \alpha < \pi/2$, cosine is positive. With $t = \tan\alpha$, this fixes $\cos\alpha = 1/\sqrt{1+t^2}$ and $\sin\alpha = t/\sqrt{1+t^2}$.
- Squaring an equation can introduce extraneous roots. Preserve the sign condition from the unsquared equation and test every algebraic candidate.

Step 1: Express the equation through tangent Using the positive value of cosine on the stated interval, the equation becomes

$$\frac{4+5t}{\sqrt{1+t^2}} = 1, \qquad 4+5t > 0.$$

Thus $t > -4/5$. This inequality must be retained while solving: replacing cosine by either square-root sign would lose the interval information.

Step 2: Solve the quadratic equation Squaring gives

$$(4+5t)^2 = 1+t^2, \qquad 24t^2+40t+15 = 0.$$

The quadratic formula then yields

$$t = \frac{-40 \pm \sqrt{1600-1440}}{48} = \frac{-10 \pm \sqrt{10}}{12}.$$

Both numbers are negative, but only one meets the sign condition in the original equation.

Step 3: Reject the extraneous root and compare options The root $(-10-\sqrt{10})/12$ is approximately -1.097 and violates $t > -0.8$, so it gives a negative left numerator. The other root $(\sqrt{10}-10)/12$ is approximately -0.570 and is admissible. Options A and D are positive and cannot satisfy $4\cos\alpha + 5\sin\alpha = 1$ in this interval with positive tangent. Option B has the wrong factor of two and fails the derived quadratic.

Step 4: Conclusion $\tan\alpha = (\sqrt{10}-10)/12$. Hence, the correct option is (C).

Quick Tip to Solve: When converting a sine-cosine equation to tangent, first fix the sign of cosine using the given interval, and retain that sign after squaring.

5. If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to [4 marks]

(A) 7

(B) 6

(C) 5

(D) 4

Correct Answer: (B) 6

Solution:

Concept:

- In a geometric progression, every even-positioned term equals the immediately preceding odd-positioned term multiplied by the common ratio r.
- For an even number of terms, pair the terms as first with second, third with fourth, and so on; the total sum is $(1 + r)$ times the odd-position sum.

Step 1: Pair all sixty-four terms Let the first term be a and let $S_o = a + ar^2 + \cdots + ar^{62}$ be the sum of the 32 odd-positioned terms. The even-position sum is

$$S_e = ar + ar^3 + \cdots + ar^{63} = rS_o.$$

This identity follows term by term, so no long formula involving r^{64} is needed. There are exactly as many odd positions as even positions.

Step 2: Use the specified ratio of sums The condition is $S_o + S_e = 7S_o$. In the non-zero odd-sum case represented by the offered positive common ratios, substitute $S_e = rS_o$ to obtain

$$(1 + r)S_o = 7S_o, \qquad 1 + r = 7, \qquad r = 6.$$

For $r = 6$, the even-position sum is six times the odd-position sum, and the total is consequently seven times that sum, exactly as required.

Step 3: Check the alternatives For the offered ratios 7, 5 and 4, the total-to-odd-sum ratios would be 8, 6 and 5 respectively. Only the offered ratio 6 produces 7. The distinction between the common ratio and the ratio of the two sums is why simply choosing 7 would be incorrect. The pairing method also avoids introducing a denominator that would need separate treatment at $r = 1$.

Step 4: Conclusion The common ratio among the offered values is $r = 6$. Hence, the correct option is (B).

Quick Tip to Solve: For an even-length GP, write even-position sum = r times odd-position sum before using any finite-series formula.

6. A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws, is [4 marks]

(A) $\frac{5}{11}$
(B) $\frac{5}{6}$
(C) $\frac{1}{6}$
(D) $\frac{6}{11}$

Correct Answer: (A) $\frac{5}{11}$

Solution:

Concept:

- For independent fair-die throws, success (a 2) has probability $p = 1/6$ and failure has probability $q = 5/6$. The first success on throw n has probability $q^{n-1}p$.
- Different stopping throws are mutually exclusive. Add their probabilities, using the infinite geometric-series formula when the common ratio has absolute value less than one.

Step 1: List the even stopping times An even number of throws means that the first 2 appears on throw 2, 4, 6, and so forth. For throw 2 there must first be one failure, giving probability qp. For throw 4 there must first be three failures, giving q^3p. A 2 on an earlier throw is excluded because the experiment stops at its first appearance.

Step 2: Sum the geometric series The desired probability is therefore

$$P = qp + q^3p + q^5p + \cdots = \frac{pq}{1 - q^2}.$$

Here $q^2 = 25/36 < 1$, so this infinite series converges. Substituting gives

$$P = \frac{(1/6)(5/6)}{1 - 25/36} = \frac{5/36}{11/36} = \frac{5}{11}.$$

The stopping time is finite with probability one, since the probability of n consecutive failures tends to zero.

Step 3: Check the parity and other options The analogous odd-stopping probability is $p/(1 - q^2) = 6/11$, and the two probabilities sum to one. Odd stopping is slightly more likely because the first possible throw is odd. Thus 6/11 corresponds to the other parity. The values 1/6 and 5/6 refer to success and failure on one throw, not to the parity of a random stopping time.

Step 4: Conclusion The probability that the first 2 occurs on an even-numbered throw is $5/11$. Hence, the correct option is (A).

Quick Tip to Solve: For first-success parity, sum p q, p q cubed, and so on; successive even stopping times differ by two failures.

7. In an A.P., the sixth term $a_6 = 2$. If the product $a_1a_4a_5$ is the greatest, then the common difference of the A.P. is equal to [4 marks]

(A) $\frac{2}{3}$

(B) $\frac{5}{8}$

(C) $\frac{3}{2}$

(D) $\frac{8}{5}$

Correct Answer: As printed with unrestricted real common difference, the product has no greatest value, so no option is a global maximizer. Option (D), $8/5$, gives only the local maximum and is the intended stationary-maximum choice.

Solution:

Concept:

- The nth term of an AP is $a_n = a_1 + (n-1)d$. The fixed sixth term lets every factor in the product be written as a linear expression in d.
- A stationary point with negative second derivative is a local maximum. A global greatest value additionally requires control of the function over its entire allowed domain.

Step 1: Write the product as a polynomial Since $a_6 = a_1 + 5d = 2$, we have $a_1 = 2 - 5d$, $a_4 = 2 - 2d$, and $a_5 = 2 - d$. Therefore the product is

$$F(d) = (2-5d)(2-2d)(2-d) = 8 - 32d + 34d^2 - 10d^3.$$

The printed statement places no positivity condition on the terms and gives no interval for the common difference.

Step 2: Find and classify the stationary points Differentiation gives

$$F\prime(d) = -32 + 68d - 30d^2 = -2(3d-2)(5d-8).$$

Thus the stationary points are $d = 2/3$ and $d = 8/5$. Since $F\prime\prime(d) = 68 - 60d$, its values at these points are 28 and -28 respectively. Hence $2/3$ is a local minimum and $8/5$ is a local maximum. The alternatives 5/8 and 3/2 are not stationary points.

Step 3: Check whether a greatest value actually exists The leading term of F is $-10d^3$, so

$$\lim_{d \to -\infty} F(d) = +\infty.$$

Consequently F has no greatest value on the unrestricted real domain of the common difference. The local maximum at $8/5$ does not resolve this: sufficiently large negative common differences give still larger products. No offered value is a valid global maximizer of the literal question.

Step 4: Conclusion As printed with unrestricted real common difference, the product has no greatest value, so no option is a global maximizer. Option (D), $8/5$, gives only the local maximum and is the intended stationary-maximum choice.

Quick Tip to Solve: For optimization of an odd-degree polynomial, inspect its end behaviour before calling a stationary local maximum the greatest value.

8.

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ **and** $|2\ \mathrm{A}|^3 = 2^{21}$ **where** $\alpha, \beta \in Z$**, Then a value of** α **is**

[4 marks]

(A) 9

(B) 17

(C) 3

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- For a square matrix of order three, multiplying every entry by 2 multiplies its determinant by 2^3: $|2A| = 8|A|$. The outside power is applied only after evaluating this determinant.
- The first row gives $|A| = \alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)$. Since both parameters are integers, each candidate must make $\alpha^2 - 16$ an integer square.

Step 1: Evaluate the determinant before cubing Expanding along the first row leaves the lower-right two-by-two block. Thus the stated equality becomes

$$[8(\alpha^2 - \beta^2)]^3 = 2^{21}.$$

Taking the real cube root gives

$$8(\alpha^2 - \beta^2) = 2^7 = 128, \qquad \alpha^2 - \beta^2 = 16.$$

The cube root is positive; there is no independent negative sign to introduce.

Step 2: Test the integer condition for each listed value For $\alpha = 9, 17, 3, 5$, respectively, the required values of β^2 are

$$81 - 16 = 65, \quad 289 - 16 = 273, \quad 9 - 16 = -7, \quad 25 - 16 = 9.$$

Neither 65 nor 273 is a perfect square: they lie between consecutive squares $8^2, 9^2$ and $16^2, 17^2$. The value -7 is impossible for a real integer square. However, 9 is the square of either 3 or -3, so the last candidate satisfies every condition.

Step 3: Check a valid parameter pair Choose $\alpha = 5$ and $\beta = 3$. Then $|A| = 25 - 9 = 16$, hence $|2A| = 128 = 2^7$. Its cube is exactly 2^{21}, confirming the original expression rather than only a rearranged equation.

Step 4: Conclusion A permitted value is $\boxed{\alpha = 5}$, with $\beta = \pm 3$. Hence, the correct option is (D).

Quick Tip to Solve: Apply the determinant scaling rule before the outer power, then test whether each candidate leaves an integer square for β^2.

9. Let R be a relation on $Z \times Z$ defined by $(a, b)R(c, d)$ if and only if $ad - bc$ is divisible by 5. Then R is [4 marks]

(A) Reflexive and transitive but not symmetric

(B) Reflexive and symmetric but not transitive

(C) Reflexive but neither symmetric nor transitive

(D) Reflexive, symmetric and transitive

Correct Answer: (B) Reflexive and symmetric but not transitive

Solution:

Concept:

- A relation is reflexive when every element is related to itself, and symmetric when reversing a related pair preserves the relation.
- Transitivity requires the implication uRv and $vRw \Rightarrow uRw$ for every triple. One counterexample is sufficient to disprove it; divisibility by 5 includes zero.

Step 1: Check reflexivity and symmetry directly For any $(a, b) \in \mathbb{Z} \times \mathbb{Z}$, comparison with itself gives

$$ab - ba = 0.$$

Since 5 divides zero, every ordered pair is related to itself. Thus R is reflexive. If $(a, b)R(c, d)$, then $ad - bc = 5k$ for an integer k. In the reverse direction the determinant is

$$cb - da = -(ad - bc) = -5k,$$

which is also divisible by 5. Therefore $(c, d)R(a, b)$, establishing symmetry.

Step 2: Construct a counterexample to transitivity Take the three elements $u = (1, 0)$, $v = (0, 0)$, and $w = (0, 1)$. Then

$$1 \cdot 0 - 0 \cdot 0 = 0, \qquad 0 \cdot 1 - 0 \cdot 0 = 0.$$

These show uRv and vRw. But the determinant for the first and last elements is

$$1 \cdot 1 - 0 \cdot 0 = 1,$$

and 5 does not divide 1. Hence u is not related to w, so transitivity fails. All three elements belong to the stated set; in particular the zero pair is not excluded.

Step 3: Identify the matching classification Options claiming a lack of symmetry contradict the sign-change argument. The option claiming all three properties contradicts the explicit triple above. The only remaining classification includes reflexivity and symmetry while excluding transitivity.

Step 4: Conclusion The relation is reflexive and symmetric, but not transitive. Hence, the correct option is (B).

Quick Tip to Solve: For determinant-based relations, test the zero vector as the middle element before trying to prove transitivity by division.

10. If $f(x) = \begin{cases} 2+2x, & -1 \le x < 0 \\ 1-\frac{x}{3}, & 0 \le x \le 3 \end{cases}; g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ x, & 0 < x \le 1 \end{cases}$ **, then range of** $(fog)(x)$ **is** [4 marks]

(A) $[0, 1)$

(B) $[0, 3)$

(C) $(0, 1]$

(D) $[0, 1]$

Correct Answer: (D) $[0, 1]$

Solution:

Concept:

- In $(f \circ g)(x) = f(g(x))$, the output of the inner function determines which branch of the outer function is used.
- The range must track endpoint inclusion. For a decreasing affine function on a closed interval, both extreme values are attained.

Step 1: Find all possible inputs supplied to the outer function On $-3 \le x \le 0$, we have $g(x) = -x$, whose image is $[0, 3]$. On $0 < x \le 1$, we have $g(x) = x$, whose image is $(0, 1]$. Combining these gives

$$\text{Range}(g) = [0, 3] \cup (0, 1] = [0, 3].$$

Thus every value entering f is nonnegative. The branch $2 + 2x$ of f, defined for negative inputs, is never used in this composition.

Step 2: Apply the correct branch and retain both endpoints Write $t = g(x)$. Because $0 \le t \le 3$, the applicable rule is

$$f(t) = 1 - \frac{t}{3}.$$

This decreases from 1 at $t = 0$ to 0 at $t = 3$, and it takes every intermediate value. Both inputs are available: $g(0) = 0$ and $g(-3) = 3$. Consequently

$$(f \circ g)(0) = 1, \qquad (f \circ g)(-3) = 0.$$

Neither endpoint may be removed.

Step 3: Compare the proposed intervals The intervals $[0, 1)$ and $(0, 1]$ each incorrectly omit an attained endpoint. The interval $[0, 3)$ includes values greater than 1, whereas $1 - t/3 \le 1$ for every allowed t. The closed interval from zero to one is therefore exact.

Step 4: Conclusion $\boxed{\text{Range}(f \circ g) = [0, 1]}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the range of the inner function first; it can eliminate an entire branch of the outer piecewise function.

11. Let O be the origin and the position vectors of A and B be $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$ respectively. If the internal bisector of $\angle$AOB meets the line AB at C, then the length of OC is [4 marks]

(A) $\frac{3}{2}\sqrt{34}$
(B) $\frac{2}{3}\sqrt{31}$
(C) $\frac{2}{3}\sqrt{34}$
(D) $\frac{3}{2}\sqrt{31}$

Correct Answer: (C) $\frac{2}{3}\sqrt{34}$

Solution:

Concept:

- The internal angle-bisector theorem gives $AC/CB = OA/OB$ in triangle OAB. The lengths of position vectors are ordinary Euclidean norms.
- If $AC : CB = m : n$, the internal section formula is $\vec{OC} = (n\vec{OA} + m\vec{OB})/(m+n)$. The weight on each endpoint is the opposite segment length.

Step 1: Compute the two sides adjoining the bisected angle The given points are $A = (2, 2, 1)$ and $B = (2, 4, 4)$. Therefore

$$OA = \sqrt{2^2 + 2^2 + 1^2} = 3, \qquad OB = \sqrt{2^2 + 4^2 + 4^2} = 6.$$

The bisector at the origin meets the opposite side internally. Applying the angle-bisector theorem yields

$$AC : CB = OA : OB = 3 : 6 = 1 : 2.$$

Thus C lies closer to A, as the ratio also suggests.

Step 2: Locate the intersection on the opposite side Using the internal division ratio gives

$$\vec{OC} = \frac{2\vec{OA} + \vec{OB}}{3}.$$

Substituting all components separately,

$$C = \left(\frac{2(2)+2}{3}, \frac{2(2)+4}{3}, \frac{2(1)+4}{3}\right) = \left(2, \frac{8}{3}, 2\right).$$

This point is indeed on AB, one-third of the way from A to B, which provides a direct check of the order of the section weights.

Step 3: Calculate the distance from the origin The required distance is the norm of this vector:

$$OC = \sqrt{2^2 + \left(\frac{8}{3}\right)^2 + 2^2} = \sqrt{\frac{36+64+36}{9}} = \frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}.$$

The alternatives containing $\sqrt{31}$ disagree with the squared-coordinate sum, and the factor $3/2$ does not arise from the internal section ratio.

Step 4: Conclusion $\boxed{OC = \frac{2}{3}\sqrt{34}}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the two vector lengths first, then use the internal section formula with opposite weights; do not simply average the endpoints.

12. In a $\triangle ABC$, suppose $y = x$ is the equation of the bisector of the angle B and the equation of the side AC is $2x - y = 2$. If $2AB = BC$ and the points A and B are respectively $(4, 6)$ and (α, β), then $\alpha + 2\beta$ is equal to [4 marks]

(A) 42

(B) 39

(C) 48

(D) 45

Correct Answer: (A) 42

Solution:

Concept:

- The vertex of an angle lies on its bisector. Hence B must have equal coordinates when its angle bisector is $y = x$.
- The internal angle-bisector theorem gives $AD/DC = AB/BC$, where D is the intersection of the bisector with side AC. A triangle must be nondegenerate.

Step 1: Locate the intersection and determine the third vertex Solving $y = x$ and $2x - y = 2$ gives $D = (2, 2)$. Since $2AB = BC$, the side ratio is $AB : BC = 1 : 2$, so

$$AD : DC = 1 : 2, \qquad D = \frac{2A + C}{3}.$$

With $A = (4, 6)$, this yields

$$C = 3D - 2A = (6, 6) - (8, 12) = (-2, -6).$$

The point D lies between A and C, consistently with the internal bisector.

Step 2: Use the given side-length ratio to find the vertex Let $B = (t, t)$. Squaring $BC = 2AB$ is legitimate because these lengths are nonnegative:

$$(t + 2)^2 + (t + 6)^2 = 4\left[(t - 4)^2 + (t - 6)^2\right].$$

Expand and collect terms:

$$2t^2 + 16t + 40 = 8t^2 - 80t + 208,$$

$$t^2 - 16t + 28 = 0, \qquad (t - 2)(t - 14) = 0.$$

Thus the algebraic candidates are $t = 2$ and $t = 14$.

Step 3: Exclude the degenerate configuration and evaluate the expression When $t = 2$, the vertex B equals D and lies on side AC, so the three points are collinear and do not form the stated triangle. Therefore $B = (14, 14)$, giving $\alpha = \beta = 14$. The requested quantity is

$$\alpha + 2\beta = 14 + 28 = 42.$$

The other offered values do not satisfy this nondegenerate solution.

Step 4: Conclusion $\boxed{\alpha + 2\beta = 42}$. Hence, the correct option is (A).

Quick Tip to Solve: After solving squared side-length equations, reject any root placing the vertex on the opposite side; it represents a degenerate triangle.

13. For $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, if $y(x) = \int \frac{\operatorname{cosec} x + \sin x}{\operatorname{cosec} x \sec x + \tan x \sin^2 x} dx$, and $\lim_{x \to (\pi/2)^-} y(x) = 0$ then $y\left(\frac{\pi}{4}\right)$ is equal to [4 marks]

(A) $-\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(B) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(C) $\frac{1}{2} \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(D) $\frac{1}{\sqrt{2}} \tan^{-1}\left(-\frac{1}{2}\right)$

Correct Answer: (D) $\frac{1}{\sqrt{2}} \tan^{-1}\left(-\frac{1}{2}\right)$

Solution:

Concept:

- Multiply numerator and denominator by $\sin x \cos x$ to remove cosecant, secant and tangent together. The substitution $t = \sin x$ then uses $dt = \cos x\, dx$.
- The derivative of $\tan^{-1} v$ is $v'/(1 + v^2)$. The endpoint condition determines the constant on $0 < x < \pi/2$, the component containing the requested argument.

Step 1: Reduce the integrand to a rational function For positive $x < \pi/2$, multiplying both numerator and denominator gives

$$y(x) = \int \frac{\cos x(1 + \sin^2 x)}{1 + \sin^4 x}\, dx = \int \frac{1 + t^2}{1 + t^4}\, dt.$$

The differential has been included in this substitution. In particular, the denominator contains t^4, not t^2, so the basic $\tan^{-1} t$ formula alone is insufficient.

Step 2: Recognise and verify an inverse-tangent primitive Put $v = (t^2 - 1)/(\sqrt{2}\, t)$. Then

$$v' = \frac{1 + t^{-2}}{\sqrt{2}}, \qquad 1 + v^2 = \frac{1 + t^4}{2t^2}.$$

Consequently

$$\frac{d}{dt}\left[\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t^2 - 1}{\sqrt{2}\, t}\right)\right] = \frac{1 + t^2}{1 + t^4}.$$

This exact differentiation confirms the scale factor and the sign. Therefore the displayed primitive plus a constant is $y(x)$ after replacing t with $\sin x$.

Step 3: Apply the limiting condition and evaluate As $x \to (\pi/2)^-$, we have $t \to 1$, so the inverse-tangent argument tends to zero. The stated limit therefore makes the integration constant zero. At $x = \pi/4$, $t = 1/\sqrt{2}$, and

$$\frac{t^2 - 1}{\sqrt{2}\, t} = -\frac{1}{2}.$$

Thus $y(\pi/4) = \frac{1}{\sqrt{2}}\tan^{-1}(-1/2)$. Options with positive signs cannot agree because the primitive increases to zero at the upper endpoint. The remaining negative alternative has a different inverse-tangent argument.

Step 4: Conclusion $\boxed{y(\pi/4) = \frac{1}{\sqrt{2}}\tan^{-1}(-1/2)}$. Hence, the correct option is (D).

Quick Tip to Solve: For $(1+t^2)/(1+t^4)$, try an inverse tangent involving $t - 1/t$, and differentiate it to check the factor exactly.

14. Let A be a square matrix such that $AA^T = I$. Then $\frac{1}{2}A\left[(A+A^T)^2 + (A-A^T)^2\right]$ is equal to

[4 marks]

(A) $A^2 + A^T$

(B) $A^3 + I$

(C) $A^3 + A^T$

(D) $A^2 + I$

Correct Answer: (C) $A^3 + A^T$

Solution:

Concept:

- For a square matrix satisfying $AA^T = I$, the transpose equals the inverse, so $A^TA = I$ as well. In particular, $A(A^T)^2 = A^T$.
- Matrix multiplication need not commute. Expand both squares in their original order; the mixed products cancel when their sum is taken.

Step 1: Expand the two squares without assuming commutativity The two expansions are

$$(A+A^T)^2 = A^2 + AA^T + A^TA + (A^T)^2,$$

$$(A-A^T)^2 = A^2 - AA^T - A^TA + (A^T)^2.$$

Adding them cancels each mixed product with its own negative:

$$(A+A^T)^2 + (A-A^T)^2 = 2A^2 + 2(A^T)^2.$$

This cancellation is valid for any square matrix and does not require replacing AA^T by A^TA during the expansion.

Step 2: Apply the outer factor and use the given identity The expression in the question therefore reduces to

$$\frac{1}{2}A\left[2A^2 + 2(A^T)^2\right] = A^3 + A(A^T)^2.$$

By associativity and the given condition,

$$A(A^T)^2 = (AA^T)A^T = IA^T = A^T.$$

Thus the complete expression is $A^3 + A^T$. The outside matrix multiplies on the left, exactly as written in the question.

Step 3: Distinguish the resulting expression from the other options The leading power is A^3, since A multiplies A^2, ruling out the alternatives starting with A^2. The remaining term is A^T, rather than I, because only one transpose factor is consumed by $AA^T = I$. Orthogonality does not generally imply $A^T = I$; hence replacing this term by the identity is invalid.

Step 4: Conclusion The matrix expression equals $\boxed{A^3 + A^T}$. Hence, the correct option is (C).

Quick Tip to Solve: Add the expanded plus and minus squares first; cancellation removes both mixed matrix products before orthogonality is needed.

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15. If $z = \frac{1}{2} - 2i$ is such that $|z+1| = \alpha z + \beta(1+i), i = \sqrt{-1}$ and $\alpha, \beta \in \mathbb{R}$, then $\alpha + \beta$ is equal to [4 marks]

(A) 2

(B) -4

(C) 3

(D) -1

Correct Answer: (C) 3

Solution:

Concept:

- For a complex number $u + iv$, its modulus is the nonnegative real number $\sqrt{u^2 + v^2}$. A modulus is not a complex expression with a free imaginary component.
- Equality of two complex numbers requires equality of their real parts and equality of their imaginary parts separately. Here the unknown coefficients α, β are real.

Step 1: Calculate the modulus first Substituting the given number gives $z + 1 = \frac{3}{2} - 2i$. Therefore

$$|z+1| = \sqrt{\left(\frac{3}{2}\right)^2 + (-2)^2} = \sqrt{\frac{9}{4} + 4} = \frac{5}{2}.$$

This fixes the left side as a positive real number. In particular, replacing the modulus by $z + 1$ itself would give the wrong equation.

Step 2: Equate imaginary and real parts Expand the right side without taking its modulus:

$$\alpha z + \beta(1+i) = \left(\frac{\alpha}{2} + \beta\right) + i(-2\alpha + \beta).$$

Its imaginary part must vanish, so $\beta = 2\alpha$. Its real part must be $5/2$, giving

$$\frac{\alpha}{2} + 2\alpha = \frac{5}{2}, \qquad \alpha = 1, \quad \beta = 2.$$

Both equations are needed; merely cancelling the imaginary part does not determine the scale.

Step 3: Check the coefficients and compare choices With these values,

$$z + 2(1+i) = \frac{1}{2} - 2i + 2 + 2i = \frac{5}{2} = |z+1|.$$

Thus the coefficients satisfy the full original equation, not just one component. Their sum is $1+2=3$. The proposed values $2, -4, -1$ cannot equal this uniquely determined sum, so the other three options are excluded.

Step 4: Conclusion $\alpha = 1$, $\beta = 2$, and hence $\boxed{\alpha + \beta = 3}$. Hence, the correct option is (C).

Quick Tip to Solve: Compute the real modulus first, then set the imaginary part of the other side to zero before solving its real part.

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16. Consider the function $f : \left[\frac{1}{2}, 1\right] \to \mathbb{R}$ defined by $f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$. Consider the statements

(I) The curve $y = f(x)$ intersects the x-axis exactly at one point.

(II) The curve $y = f(x)$ intersects the x-axis at $x = \cos\frac{\pi}{12}$.

Then [4 marks]

(A) Both (I) and (II) are correct.

(B) Only (I) is correct.

(C) Both (I) and (II) are incorrect.

(D) Only (II) is correct.

Correct Answer: (A) Both (I) and (II) are correct.

Solution:

Concept:

- A continuous function with opposite signs at the endpoints has a zero between them. Strict increase throughout the interior ensures that this zero is unique.
- The identity $4\cos^3\theta - 3\cos\theta = \cos 3\theta$ identifies roots of a cubic expressed in terms of cosine. The proposed root must also lie in the specified domain.

Step 1: Prove existence and uniqueness on the interval The function is a polynomial and therefore continuous on $[1/2, 1]$. At the endpoints,

$$f(1/2) = -\sqrt{2} - 1 < 0, \qquad f(1) = \sqrt{2} - 1 > 0.$$

Hence at least one root lies in the interior. Its derivative is

$$f\prime(x) = 3\sqrt{2}(4x^2 - 1).$$

For every $x \in (1/2, 1)$, this derivative is strictly positive. Thus the function is strictly increasing there and cannot have two distinct zeros. Statement (I) is true.

Step 2: Test the stated intersection exactly Set $x = \cos\theta$. Then

$$f(\cos\theta) = \sqrt{2}\cos 3\theta - 1.$$

For $\theta = \pi/12$, $3\theta = \pi/4$, so

$$f(\cos(\pi/12)) = \sqrt{2} \cdot \frac{1}{\sqrt{2}} - 1 = 0.$$

Also $\cos(\pi/12)$ is between $1/2$ and 1, so this is an admissible root and is the unique root already proved to exist. Statement (II) is true.

Step 3: Choose the combination of statements The uniqueness claim and the proposed location have been checked independently. Both hold. The option saying only (I) fails because the exact substitution establishes (II); the option saying only (II) fails because strict monotonicity establishes (I). The option rejecting both contradicts both calculations.

Step 4: Conclusion Both (I) and (II) are correct, and the unique intersection is at $x = \cos(\pi/12)$. Hence, the correct option is (A).

Quick Tip to Solve: Use derivative signs for the number of roots, then the triple-angle identity for the exact root; the two statements need separate checks.

17. Let PQR be a triangle with $R(-1, 4, 2)$. Suppose $M(2, 1, 2)$ is the mid point of PQ. The distance of the centroid of $\triangle$PQR from the point of intersection of the lines $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is [4 marks]

(A) 69

(B) $\sqrt{99}$

(C) $\sqrt{69}$

(D) 9

Correct Answer: (C) $\sqrt{69}$

Solution:

Concept:

- The centroid of three points has position vector equal to one third of the sum of their position vectors. Since M is the midpoint of PQ, the sum of the position vectors of P and Q is twice that of M.
- Two intersecting lines have a point satisfying both parametric equations. The Euclidean distance between points is the square root of the sum of the squared coordinate differences.

Step 1: Find the centroid Let G denote the centroid. We do not need P and Q separately, because their sum is fixed by the midpoint. Thus

$$G = \frac{2M + R}{3} = \left(\frac{4-1}{3}, \frac{2+4}{3}, \frac{4+2}{3}\right) = (1, 2, 2).$$

This also places G two thirds of the way from R towards M, as required by the median property.

Step 2: Determine the intersection The first line has equations $x = 2,\ y = 2t,\ z = -3 - t$. The zero denominator means a constant x-coordinate, rather than an undefined line. The second line is $x = 1 + s,\ y = -3 - 3s,\ z = -1 + s$. Equating x gives $s = 1$, so y is -6 and z is 0. Then $t = -3$ gives the same y and z on the first line. Their intersection is therefore $H = (2, -6, 0)$.

Step 3: Calculate the distance The displacement from G to H is $(1, -8, -2)$. Hence

$$GH = \sqrt{(2-1)^2 + (-6-2)^2 + (0-2)^2} = \sqrt{1 + 64 + 4} = \sqrt{69}.$$

The value 69 is the squared distance, not the distance; 9 and $\sqrt{99}$ do not have the required squared value.

Step 4: Conclusion The required distance is $\sqrt{69}$, since the squared coordinate differences sum to 69. Thus the printed question gives $\sqrt{69}$, not $\sqrt{99}$. Hence, the correct option is (C).

Quick Tip to Solve: Use twice the midpoint in the centroid formula; there is no need to recover the two individual vertices.

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18. $\lim\limits_{x \to \frac{\pi}{2}} \left(\frac{1}{\left(x - \frac{\pi}{2}\right)^2} \int\limits_{x^3}^{\left(\frac{\pi}{2}\right)^3} \cos\left(t^{\frac{1}{3}}\right) dt \right)$ **is equal to** [4 marks]

(A) $\frac{3\pi^2}{4}$

(B) $\frac{3\pi^2}{8}$

(C) $\frac{3\pi}{4}$

(D) $\frac{3\pi}{8}$

Correct Answer: (B) $\frac{3\pi^2}{8}$

Solution:

Concept:

- The fundamental theorem of calculus differentiates an integral with a variable endpoint; a variable lower endpoint contributes a minus sign.
- If numerator and denominator both vanish, L'Hopital's rule can be applied when its differentiability conditions hold. Here two applications resolve the quadratic vanishing.

Step 1: Identify the vanishing numerator Put $a = \pi/2$ and

$$F(x) = \int_{x^3}^{a^3} \cos(t^{1/3})\, dt.$$

The required limit is $F(x)/(x-a)^2$. Since $F(a) = 0$, it has the form $0/0$. Around $a > 0$, the integrand is smooth and the fundamental theorem gives

$$F\prime(x) = -3x^2 \cos x.$$

In particular $F\prime(a) = 0$, which explains why division by a squared difference can have a finite nonzero

limit.

Step 2: Apply the limit rule twice For the first application,

$$L = \lim_{x \to a} \frac{-3x^2 \cos x}{2(x-a)}.$$

This is again $0/0$. Differentiating once more gives

$$L = \lim_{x \to a} \frac{-6x \cos x + 3x^2 \sin x}{2} = \frac{3a^2}{2} = \frac{3\pi^2}{8}.$$

Both derivatives exist in a neighbourhood of the limiting point, and the final denominator is nonzero. The positive sign is also consistent with the integral: near the upper endpoint its value vanishes to second order.

Step 3: Compare the value with the options Option (B) is exactly $3\pi^2/8$. Option (A) is twice the answer and would result from overlooking the factor 2 in the second derivative of the denominator. Options (C) and (D) contain only one power of π; the factor x^2 from differentiating the endpoint supplies a^2, so neither can agree.

Step 4: Conclusion The limit equals $\boxed{3\pi^2/8}$. Hence, the correct option is (B).

Quick Tip to Solve: When an endpoint integral is divided by $(x-a)^2$, check both its value and first derivative at a; if both vanish, evaluate half its second derivative.

19. A function $y = f(x)$ satisfies $f(x)\sin 2x + \sin x - \left(1 + \cos^2 x\right) f'(x) = 0$ with condition $f(0) = 0$. Then, $f\left(\frac{\pi}{2}\right)$ is equal to [4 marks]

(A) 2

(B) 1

(C) -1

(D) 0

Correct Answer: (B) 1

Solution:

Concept:

- The product rule is $(uy)\prime = uy\prime + u\prime y$. Matching the coefficient of y with the derivative of the coefficient of $y\prime$ often avoids a separate integrating-factor calculation.
- An initial condition fixes the integration constant. Because $1+\cos^2 x \geq 1$, division by this coefficient is valid for every real x.

Step 1: Rearrange the equation into a product derivative Write $y = f(x)$. The supplied equation becomes

$$(1 + \cos^2 x)y\prime - y \sin 2x = \sin x.$$

Since

$$\frac{d}{dx}(1+\cos^2 x) = -2\sin x\cos x = -\sin 2x,$$

its left side is exactly

$$\frac{d}{dx}\left[(1+\cos^2 x)y\right].$$

The sign of the second term matters: using a plus sign for the derivative of $\cos^2 x$ would not reproduce the differential equation.

Step 2: Integrate and impose the initial condition Integrating gives

$$(1+\cos^2 x)y = -\cos x + C.$$

At $x = 0$, the given condition is $y = 0$, so

$$2 \cdot 0 = -1 + C, \qquad C = 1.$$

Consequently the complete solution satisfying the condition is

$$f(x) = \frac{1-\cos x}{1+\cos^2 x}.$$

Substitution at zero verifies the condition, while differentiating the product recovers the original equation.

Step 3: Evaluate at the requested argument At $x = \pi/2$, the cosine is zero. Thus

$$f(\pi/2) = \frac{1-0}{1+0} = 1.$$

The result is positive, excluding -1; it is nonzero, excluding 0. The denominator here is 1, so the initial denominator value 2 must not be used at this argument; option 2 also fails. Only the displayed option 1 agrees.

Step 4: Conclusion $\boxed{f(\pi/2) = 1}$. Hence, the correct option is (B).

Quick Tip to Solve: Before finding an integrating factor, differentiate the coefficient of y'; it may already supply the coefficient of y in a product derivative.

20. If the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{\sin x^{2123}}} \right) dx = \frac{\pi}{4}(\pi + a) - 2$, then the value of a is [4 marks]

(A) $-\frac{3}{2}$

(B) 3

(C) $\frac{3}{2}$

(D) 2

Correct Answer: (B) 3

Solution:

Concept:

- On a symmetric interval, if $g(x)$ is even and $h(-x) = -h(x)$, then $\int_{-b}^{b} g(x)/(1+e^{h(x)})\,dx = \frac{1}{2}\int_{-b}^{b} g(x)\,dx$. This follows by pairing x and $-x$.
- The same pairing works for $1+\pi^x$, because $\pi^{-x} = 1/\pi^x$. Use integration by parts for the remaining polynomial times cosine.

Step 1: Apply reflection separately to both terms Let $b = \pi/2$. The numerator $x^2 \cos x$ is even, and

$$\frac{1}{1+\pi^x} + \frac{1}{1+\pi^{-x}} = 1.$$

Therefore its integral is $\int_0^b x^2 \cos x\,dx$. The other numerator $1+\sin^2 x$ is even. The exponent $\sin(x^{2123})$ is odd since 2123 is odd; its exponential at $-x$ is the reciprocal of its value at x. Hence the second integral is $\int_0^b (1+\sin^2 x)\,dx$. No explicit integration of the exponential expressions is needed.

Step 2: Evaluate the elementary integrals Twice integrating by parts gives

$$\int x^2 \cos x\,dx = x^2 \sin x + 2x \cos x - 2\sin x.$$

Thus

$$\int_0^{\pi/2} x^2 \cos x\,dx = \frac{\pi^2}{4} - 2.$$

Also $\sin^2 x = (1-\cos 2x)/2$, so

$$\int_0^{\pi/2} (1+\sin^2 x)\,dx = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}.$$

Both reduced integrals were taken over the half interval only after the reflection identity supplied the factor one-half.

Step 3: Match the stated form Adding gives

$$I = \frac{\pi^2}{4} + \frac{3\pi}{4} - 2 = \frac{\pi}{4}(\pi+3) - 2.$$

Comparison with $\pi(\pi+a)/4 - 2$ gives $a = 3$. The choices $-3/2, 3/2, 2$ would give a different coefficient of π, so they are ruled out by the exact sum.

Step 4: Conclusion $\boxed{a = 3}$. Hence, the correct option is (B).

Quick Tip to Solve: For a logistic-looking denominator on symmetric limits, add the integrand at x and $-x$ before attempting any substitution.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The area (in sq. units) of the part of the circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is $\frac{\pi\alpha}{2\beta} - \frac{65}{2} + \frac{\alpha}{\beta}\sin^{-1}\left(\frac{12}{13}\right)$, where α, β are coprime numbers. Then $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- A line at perpendicular distance d from the centre of a circle of radius r cuts off a minor segment of area $r^2 \arccos(d/r) - d\sqrt{r^2 - d^2}$. This is sector area minus the isosceles-triangle area.
- The half-plane that does not contain the centre gives the minor segment. Use inverse trigonometric identities with explicit angle ranges to compare its area with the expression given.

Step 1: Identify the segment and its dimensions The circle has centre $O = (0, 0)$ and radius 13. Below the line means $y < 5x - 13$. The origin does not satisfy this inequality, so the required region is the smaller segment. The centre-to-line distance is

$$d = \frac{|13|}{\sqrt{5^2 + (-1)^2}} = \frac{13}{\sqrt{26}}.$$

Half the chord length is

$$\sqrt{169 - d^2} = \frac{65}{\sqrt{26}}.$$

Their product is $845/26 = 65/2$, the triangle term appearing in the question.

Step 2: Write the area and simplify its angle Let $\theta = \arccos(1/\sqrt{26})$, which lies in $(\pi/4, \pi/2)$. The minor-segment area is

$$A = 169\theta - \frac{65}{2}.$$

Since $\cos 2\theta = 2/26 - 1 = -12/13$, and $2\theta - \pi/2 \in (0, \pi/2)$,

$$\sin(2\theta - \pi/2) = \frac{12}{13}.$$

Therefore

$$\theta = \frac{\pi}{4} + \frac{1}{2}\sin^{-1}(12/13).$$

The stated ranges justify this inverse-sine branch rather than an angle giving the major segment.

Step 3: Compare coefficients and use coprimality Substituting the angle gives

$$A = \frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2}\sin^{-1}(12/13).$$

Comparing with $\pi\alpha/(2\beta) - 65/2 + (\alpha/\beta)\sin^{-1}(12/13)$ yields $\alpha/\beta = 169/2$. As 169 and 2 have no common factor, the coprime numbers are $\alpha = 169$, $\beta = 2$. Their sum, rather than their ratio or the area, is the requested numerical response.

Step 4: Conclusion The area is $\frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2}\sin^{-1}(12/13)$ square units. Thus $\boxed{\alpha + \beta = 171}$, a dimensionless number.

Quick Tip to Solve: Test whether the centre lies in the requested half-plane before choosing minor or major segment; then compare the inverse-sine coefficient.

22. If the mean and variance of the data $65, 68, 58, 44, 48, 45, 60, \alpha, \beta, 60$ where $\alpha > \beta$, are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For N observations with arithmetic mean $\bar{x}$, the population variance is $\sigma^2 = \frac{1}{N}\sum x_i^2 - \bar{x}^2$. Here the divisor is the number of observations, not one less than that number.
- The mean determines the sum of the observations. The variance and mean together determine their sum of squares, so the requested sum can be found without first finding the two unknown observations.

Step 1: Use the mean to find the missing sum There are ten observations, including both occurrences of 60. The sum of the eight specified observations is

$$65 + 68 + 58 + 44 + 48 + 45 + 60 + 60 = 448.$$

Since the mean is 56, the total sum is $10 \times 56 = 560$. Consequently,

$$\alpha + \beta = 560 - 448 = 112.$$

Counting the repeated observation separately is necessary because each occurrence contributes to the mean.

Step 2: Convert the variance into a sum of squares Insert the given population variance and mean into the variance identity:

$$66.2 = \frac{\sum x_i^2}{10} - 56^2.$$

Hence

$$\sum x_i^2 = 10(66.2 + 3136) = 32022.$$

The contribution from the eight specified entries is

$$65^2 + 68^2 + 58^2 + 44^2 + 48^2 + 45^2 + 60^2 + 60^2 = 25678.$$

Subtract this contribution to obtain

$$\alpha^2 + \beta^2 = 32022 - 25678 = 6344.$$

As a check, the sum 112 and this sum of squares give $\alpha\beta = (112^2 - 6344)/2 = 3100$, so the observations are 62 and 50. Taking $\alpha = 62 > 50 = \beta$ satisfies the stated inequality.

Step 3: Conclusion The required numerical value is $\boxed{6344}$.

Quick Tip to Solve: When a question asks for the sum of squares of missing observations, use variance directly; solving separately for the observations is only an optional check.

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23. Equations of two diameters of a circle are $2x - 3y = 5$ and $3x - 4y = 7$. The line joining the points $\left(-\frac{22}{7}, -4\right)$ and $\left(-\frac{1}{7}, 3\right)$ intersects the circle at only one point $P(\alpha, \beta)$. Then, $17\beta - \alpha$ is equal to _____. [4 marks]

Solution:

Concept:

- Every diameter passes through the centre of its circle. Thus the intersection of two distinct diameter lines determines the centre even when the radius is not supplied.
- A line meeting a circle at exactly one point is a tangent. Its point of contact is the perpendicular projection of the centre onto the tangent line.

Step 1: Locate the centre and write the tangent line Let the centre be $C(h, k)$. Solving the two diameter equations gives

$$2h - 3k = 5, \qquad 3h - 4k = 7.$$

Subtract twice the second equation from three times the first to obtain $-k = 1$. Therefore $k = -1$, and the first equation gives $h = 1$. The line through the two given points has slope

$$(3 + 4)/\left(-\frac{1}{7} + \frac{22}{7}\right) = \frac{7}{3}.$$

Its equation, using either point, is

$$7x - 3y + 10 = 0.$$

Substitution of both supplied points verifies this line.

Step 2: Project the centre onto the tangent The normal vector of the line is $(7, -3)$, with squared length $49 + 9 = 58$. The signed line expression at $C(1, -1)$ is $7 + 3 + 10 = 20$. The foot of the perpendicular is therefore

$$P = C - \frac{20}{58}(7, -3) = \left(-\frac{41}{29}, \frac{1}{29}\right).$$

This is the point of contact because the line has exactly one intersection with the circle. It lies on the line: $7(-41/29) - 3(1/29) + 10 = 0$. Also, CP is parallel to $(7, -3)$, so it is perpendicular to the

tangent, as required. Thus

$$17\beta - \alpha = \frac{17}{29} + \frac{41}{29} = 2.$$

No separate calculation of the radius is needed.

Step 3: Conclusion The required numerical value is $\boxed{2}$.

Quick Tip to Solve: When two diameter equations and a tangent are given, first find their intersection and then use the perpendicular-foot formula for the contact point.

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24. A line with direction ratios $2, 1, 2$ meets the lines $x = y + 2 = z$ and $x + 2 = 2y = 2z$ respectively at the points P and Q. If the length of the perpendicular from the point $(1, 2, 12)$ to the line PQ is l, then l^2 is _____. [4 marks]

Solution:

Concept:

- A line with direction ratios $(2, 1, 2)$ has direction vector $\mathbf{d} = (2, 1, 2)$. The displacement between any two of its points is a scalar multiple of this vector.
- For a point A and a point P on the line, the squared perpendicular distance is $AP^2 - (\overrightarrow{PA} \cdot \mathbf{d})^2/|\mathbf{d}|^2$. This subtracts the squared component parallel to the line.

Step 1: Find the intersection points of the required line Parameterise the first line as $P = (t, t - 2, t)$, and the second as $Q = (u - 2, u/2, u/2)$. Because PQ has the stated direction ratios, write

$$Q - P = k(2, 1, 2).$$

Comparing coordinates gives

$$u - 2 - t = 2k, \quad u/2 - t + 2 = k, \quad u/2 - t = 2k.$$

Subtracting the third equation from the second yields $2 = -k$, hence $k = -2$. The first and third equations now give $u = t - 2$ and $u = 2t - 8$, respectively. Therefore $t = 6$, $u = 4$, and

$$P = (6, 4, 6), \qquad Q = (2, 2, 2).$$

Indeed $Q - P = (-4, -2, -4) = -2(2, 1, 2)$.

Step 2: Check whether the connecting segment is perpendicular For $A = (1, 2, 12)$, the displacement from P is

$$\overrightarrow{PA} = (-5, -2, 6).$$

Its dot product with the line direction is

$$(-5)(2) + (-2)(1) + (6)(2) = -10 - 2 + 12 = 0.$$

Thus PA itself is perpendicular to PQ; the foot is exactly P, rather than another point on the line.

Therefore

$$l^2 = |\overrightarrow{PA}|^2 = (-5)^2 + (-2)^2 + 6^2 = 65.$$

This also follows from the distance formula because its parallel-component term vanishes.

Step 3: Conclusion The required squared distance is $\boxed{l^2 = 65}$ square coordinate units.

Quick Tip to Solve: Before expanding a cross product for distance from a line, check the dot product: if it vanishes, your chosen point is already the perpendicular foot.

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25. If $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \ldots + \frac{{}^{11}C_9}{10} = \frac{n}{m}$ with $\gcd(n, m) = 1$, then $n + m$ is equal to _____. [4 marks]

Solution:

Concept:

- The identity $\binom{n}{r}/(r+1) = \binom{n+1}{r+1}/(n+1)$ follows directly by cancelling factorials. It converts a denominator depending on the index into a constant denominator.
- The sum of all binomial coefficients of order N is 2^N. For a truncated sum, explicitly remove every coefficient outside the requested index range before reducing the fraction.

Step 1: Shift the binomial index Write the given sum as

$$S = \sum_{r=1}^{9} \frac{\binom{11}{r}}{r+1}.$$

For each term, factorial cancellation gives

$$\frac{\binom{11}{r}}{r+1} = \frac{11!}{(r+1)!(11-r)!} = \frac{1}{12}\binom{12}{r+1}.$$

Therefore the new index runs from 2 to 10, inclusive, and

$$S = \frac{1}{12}\sum_{j=2}^{10}\binom{12}{j}.$$

The lower and upper limits matter: the last displayed original term is $\binom{11}{9}/10$, so neither of the two remaining higher-index terms belongs to the sum.

Step 2: Remove the missing coefficients and reduce By the binomial theorem, $\sum_{j=0}^{12}\binom{12}{j} = 2^{12} = 4096$. The indices omitted from the required sum are $0, 1, 11, 12$, whose coefficients are $1, 12, 12, 1$. Hence

$$S = \frac{4096 - (1 + 12 + 12 + 1)}{12} = \frac{4070}{12} = \frac{2035}{6}.$$

The numerator 2035 is odd and has digit sum 10, so it is divisible by neither 2 nor 3. Therefore it is coprime to 6, establishing that this is already the required lowest-terms representation. Thus $n = 2035$ and $m = 6$, and

$$n + m = 2035 + 6 = 2041.$$

Step 3: Conclusion The required numerical value is $\boxed{2041}$.

Quick Tip to Solve: In sums of $\binom{n}{r}/(r+1)$, raise the upper index to $n+1$; then carefully list the missing end coefficients before using 2^{n+1}.

26. If the points of intersection of two distinct conics $x^2+y^2=4b$ and $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ lie on the curve $y^2=3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is _____.

[4 marks]

Solution:

Concept:

- At every common point, both conic equations and the stated relation $y^2=3x^2$ hold. Substituting squared coordinates avoids unnecessary choices of signs.
- Four points $(\pm a, \pm c)$ form an axis-parallel rectangle of side lengths $2a$ and $2c$, and therefore of area $4ac$. The condition that the conics are distinct must also be enforced.

Step 1: Determine the parameter from the common points At an intersection, use $y^2=3x^2$ in the circle equation to get

$$x^2+3x^2=4b, \qquad x^2=b, \quad y^2=3b.$$

A nondegenerate rectangle requires real, nonzero coordinates, so $b>0$. Substitution into the second conic gives

$$\frac{b}{16}+\frac{3b}{b^2}=1, \qquad \frac{b}{16}+\frac{3}{b}=1.$$

Multiplying by $16b$, which is nonzero, yields

$$b^2-16b+48=0=(b-4)(b-12).$$

Thus the algebraic candidates are $b=4$ and $b=12$.

Step 2: Exclude coincident conics and calculate the rectangle When $b=4$, the first equation becomes $x^2+y^2=16$. The second becomes $x^2/16+y^2/16=1$, exactly the same circle. This contradicts the explicit requirement of two distinct conics, so this value must be rejected. Hence $b=12$. The four common points have

$$x=\pm 2\sqrt{3}, \qquad y=\pm 6.$$

They lie on the circle and ellipse and form a rectangle of width $4\sqrt{3}$ and height 12. Its area is

$$A=(4\sqrt{3})(12)=48\sqrt{3}.$$

Consequently the quantity requested is

$$3\sqrt{3}\,A=3\sqrt{3}\times 48\sqrt{3}=432.$$

The distinctness condition is what makes this answer unique.

Step 3: Conclusion The required numerical value is $\boxed{432}$.

Quick Tip to Solve: After solving a conic parameter equation, test each root against geometric qualifiers such as distinctness; an algebraic root may make the two curves coincide.

27. All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" is _____. [4 marks]

Solution:

Concept:

- Dictionary order is alphabetical order, compared at the first differing position. To find the rank of a word, count words preceding it and then add one for the word itself.
- For a multiset of letters, the number of distinct arrangements is the factorial of the total number divided by the factorials of each repeated multiplicity. Here T occurs twice; E, G, N, W and Y occur once each.

Step 1: Count words before the prefixes G and GT The alphabetically sorted letters are E, G, N, T, T, W, Y. Before the initial G, only E is possible. Fixing E first leaves six letters, including two identical Ts, giving

$$\frac{6!}{2!} = 360$$

words. With the first letter fixed as G, the letters smaller than the second letter T are E and N. Fixing either leaves five letters with two identical Ts, giving

$$2 \times \frac{5!}{2!} = 120$$

further words. The running total preceding the target is therefore $360 + 120 = 480$. These groups are disjoint because their first differing position is different.

Step 2: Continue the prefix count and include the target After fixing GT, the remaining letters are E, N, T, W and Y, each now occurring once. At the third position of GTWENTY the letter is W. Each of E, N and T is smaller, and each choice allows $4!$ arrangements of the remaining letters. This contributes

$$3 \times 4! = 72.$$

The preceding-word count is now $480 + 72 = 552$. After GTW, the next letter E is the smallest available; after GTWE, N is the smallest available; after GTWEN, T is the smallest available. Hence these positions contribute no additional preceding words, and the final Y is forced. Adding one for GTWENTY itself gives

$$\text{rank} = 552 + 1 = 553.$$

The repeated-letter divisor is used only while both Ts remain unplaced.

Step 3: Conclusion The serial number of GTWENTY is $\boxed{553}$.

Quick Tip to Solve: Update letter multiplicities after fixing each prefix. Once the first T has been used, the remaining letters are distinct, so no $2!$ divisor remains.

28. Let α, β be the roots of the equation $x^2 - x + 2 = 0$ with $\text{Im}(\alpha) > \text{Im}(\beta)$. Then

$$\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$$

is equal to _____. [4 marks]

Solution:

Concept:

- If z is a root of $x^2 - x + 2 = 0$, then $z^2 = z - 2$. Every higher power can therefore be reduced to an expression linear in z, avoiding expansion of complex radicals.
- Vieta's formula gives $\alpha + \beta = 1$. The two roots are $(1 \pm i\sqrt{7})/2$; the imaginary-part condition selects the plus sign for α, although the expression below becomes independent of that selection.

Step 1: Reduce the fourth and sixth powers For either root z, start with $z^2 = z - 2$. Squaring and applying the same identity again gives

$$z^4 = (z-2)^2 = z^2 - 4z + 4 = (z-2) - 4z + 4 = 2 - 3z.$$

Multiply by z^2 to obtain

$$z^6 = (2 - 3z)(z - 2) = -3z^2 + 8z - 4.$$

Replacing z^2 by $z - 2$ once more gives

$$z^6 = -3(z-2) + 8z - 4 = 5z + 2.$$

In particular, $\alpha^6 = 5\alpha + 2$, while $\alpha^4 = 2 - 3\alpha$ and $\beta^4 = 2 - 3\beta$. All these reductions use the original quadratic, so they hold exactly for both nonreal roots.

Step 2: Combine the terms and cancel the remaining root First combine the symmetric fourth-power terms using the sum of the roots:

$$\alpha^4 + \beta^4 = 4 - 3(\alpha + \beta) = 4 - 3 = 1.$$

Also $\alpha^2 = \alpha - 2$. Consequently the expression in the question is

$$\begin{aligned} \alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2 &= (5\alpha + 2) + 1 - 5(\alpha - 2) \\ &= 5\alpha + 3 - 5\alpha + 10 \\ &= 13. \end{aligned}$$

The coefficient of α cancels exactly. Thus the result is real, despite the roots themselves being nonreal. The imaginary-part ordering is consistent with the roots and does not require any separate approximation.

Step 3: Conclusion The required numerical value is $\boxed{13}$.

Quick Tip to Solve: Reduce powers modulo the quadratic relation before inserting complex roots. Linear terms often cancel, leaving a simple exact real value.

29. **Let $f(x) = 2^x - x^2, x \in \mathbb{R}$. If m and n are respectively the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersect the x-axis, then the value of $\mathrm{m} + \mathrm{n}$ is _____.** [4 marks]

Solution:

Concept:

- The intersections of a graph with the x-axis are its real zeros. Thus count the zeros of $f(x) = 2^x - x^2$ and of $f'(x) = (\ln 2)2^x - 2x$ separately.
- A strictly monotone continuous function has at most one zero on an interval. A strictly convex function has at most two zeros, since its graph lies strictly below any chord joining two distinct points.

Step 1: Count the zeros of the function For $x < 0$, $f'(x) = (\ln 2)2^x - 2x > 0$, so f is strictly increasing there. Also $f(x) \to -\infty$ as $x \to -\infty$, while $f(0) = 1$. Hence exactly one negative zero exists. For $x > 0$, the equation $2^x = x^2$ is equivalent to $h(x) = x \ln 2 - 2 \ln x = 0$. Now $h\prime(x) = \ln 2 - 2/x$ changes sign once, from negative to positive. Thus h decreases then increases and has at most two positive zeros. Since $2^2 = 2^2$ and $2^4 = 4^2$, those zeros are exactly 2 and 4. Zero itself is not a root. Therefore $m = 3$.

Step 2: Count the zeros of the derivative Put $g(x) = f'(x) = (\ln 2)2^x - 2x$. For $x \le 0$, both terms are nonnegative and the exponential term is positive, so there are no roots. On the whole real line,

$$g''(x) = (\ln 2)^3 2^x > 0.$$

Thus g is strictly convex and can have at most two roots. Furthermore, $g(0) = \ln 2 > 0$, $g(2) = 4 \ln 2 - 4 < 0$, and $g(x) \to +\infty$ as $x \to +\infty$. Continuity gives one root between 0 and 2, and another beyond 2; strict convexity excludes any further roots. Hence $n = 2$.

Step 3: Conclusion Therefore $m = 3$, $n = 2$, and the required dimensionless value is

$$\boxed{m + n = 5}.$$

Quick Tip to Solve: For exponential-versus-polynomial root counts, combine obvious exact roots with monotonicity or convexity to prove that no hidden roots remain.

30. **If the solution curve $y = y(x)$ of the differential equation $\left(1 + y^2\right)\left(1 + \log_e x\right) dx + x dy = 0, x > 0$ passes through the point $(1, 1)$ and $y(e) = \frac{\alpha - \tan\left(\frac{3}{2}\right)}{\beta + \tan\left(\frac{3}{2}\right)}$, then $\alpha + 2\beta$ is _____.** [4 marks]

Solution:

Concept:

- The differential equation is separable because the factors involving y can be collected with dy, while those involving x can be collected with dx. The restriction $x > 0$ makes $\ln x$ well defined.
- Use $\int dy/(1 + y^2) = \tan^{-1} y$ and the substitution $t = \ln x$, so $dt = dx/x$. The initial condition fixes the integration constant before evaluating at $x = e$.

Step 1: Separate and integrate the equation Dividing by $x(1+y^2)$, which is nonzero for real y and positive x, gives

$$\frac{dy}{1+y^2} = -\frac{1+\ln x}{x}\,dx.$$

Integrating both sides yields

$$\tan^{-1} y = -\ln x - \frac{(\ln x)^2}{2} + C.$$

The squared logarithm comes from integrating $(\ln x)/x$, rather than from integrating $\ln x$ alone. At the point $(1,1)$, the logarithms vanish and $\tan^{-1} 1 = \pi/4$. Consequently $C = \pi/4$, and the solution on the branch through the given point is

$$y(x) = \tan\left(\frac{\pi}{4} - \ln x - \frac{(\ln x)^2}{2}\right).$$

Step 2: Evaluate and match the required expression At $x = e$, $\ln e = 1$, so the angle is $\pi/4 - 3/2$. It lies between $-\pi/2$ and $\pi/2$, as does the continuous angle along $1 \le x \le e$; thus no tangent pole intervenes. Applying the tangent subtraction formula,

$$y(e) = \tan\left(\frac{\pi}{4} - \frac{3}{2}\right) = \frac{1-\tan(3/2)}{1+\tan(3/2)}.$$

Comparing the numerator and denominator with the stated form identifies $\alpha = 1$ and $\beta = 1$. The argument $3/2$ is in radians, and no degree conversion is appropriate.

Step 3: Conclusion The constants are $\alpha = 1$ and $\beta = 1$. Therefore the required dimensionless value is

$$\boxed{\alpha + 2\beta = 3}.$$

Quick Tip to Solve: When $(1+\ln x)/x$ appears, substitute $t = \ln x$; apply the initial condition before expanding a tangent of a difference.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The resistance $R = \frac{V}{I}$ where $\mathrm{V} = (200 \pm 5)\mathrm{V}$ and $I = (20 \pm 0.2)\mathrm{A}$, the percentage error in the measurement of R is : [4 marks]

(A) 5.5%

(B) 3%

(C) 7%

(D) 3.5%

Correct Answer: (D) 3.5%

Solution:

Concept:

- For a quotient $R = V/I$, the maximum fractional uncertainty is the sum of the fractional uncertainties of the numerator and denominator: $\Delta R/R = \Delta V/V + \Delta I/I$.
- Percentage uncertainty means fractional uncertainty multiplied by 100. The voltage error must be divided by the measured voltage and the current error by the measured current before adding them.

Step 1: Find the two relative uncertainties The central measured voltage is $200\,\mathrm{V}$, with absolute uncertainty $5\,\mathrm{V}$. Therefore

$$\frac{\Delta V}{V} \times 100 = \frac{5}{200} \times 100 = 2.5\%.$$

Likewise, the measured current is $20\,\mathrm{A}$, with uncertainty $0.2\,\mathrm{A}$, giving

$$\frac{\Delta I}{I} \times 100 = \frac{0.2}{20} \times 100 = 1.0\%.$$

Each ratio is dimensionless because the units in its numerator and denominator cancel. The absolute error numbers 5 and 0.2 cannot be added directly, since they refer to different physical quantities.

Step 2: Add the percentage errors for the quotient The worst-case uncertainty increases when the voltage increases while the current decreases, or conversely. Hence error magnitudes add even though current is in the denominator:

$$\frac{\Delta R}{R} \times 100 = 2.5\% + 1.0\% = 3.5\%.$$

As a consistency check, the nominal resistance is $R = 200/20 = 10\,\Omega$; the corresponding first-order absolute uncertainty is $0.035 \times 10 = 0.35\,\Omega$. The other listed percentages, 5.5%, 3%, and 7%, do not equal the sum of the two correctly normalized uncertainties. Subtraction or an unrequested statistical quadrature rule would not give the maximum-error convention used here.

Step 3: Conclusion The percentage error in the measured resistance is $\boxed{3.5\%}$. Hence, the correct option is (D).

Quick Tip to Solve: For products and quotients, add fractional error magnitudes first; convert to a percentage only after each absolute error is normalized.

32. The de-Broglie wavelength of an electron is the same as that of a photon. If velocity of electron is 25% of the velocity of light, then the ratio of K.E. of electron and K.E. of photon will be: [4 marks]

(A) $\frac{1}{4}$
(B) $\frac{8}{1}$
(C) $\frac{1}{8}$
(D) $\frac{1}{1}$

Correct Answer: (C) $\frac{1}{8}$

Solution:

Concept:

- An electron has de Broglie wavelength $\lambda = h/p_e$, while a photon satisfies $\lambda = h/p_\gamma$. Equal wavelengths therefore imply equal momenta, not equal kinetic energies.
- In the nonrelativistic electron treatment used by the given options, $K_e = p_e^2/(2m_e) = p_e v/2$. A photon has zero rest mass and energy $E_\gamma = p_\gamma c$; its entire energy is kinetic.

Step 1: Use the common momentum Let the common wavelength be λ. Then

$$p_e = \frac{h}{\lambda} = p_\gamma = p.$$

For the electron, $p = m_e v$, so

$$K_e = \frac{1}{2} m_e v^2 = \frac{1}{2} pv.$$

For the photon, the energy corresponding to the same wavelength is

$$K_\gamma = E_\gamma = \frac{hc}{\lambda} = pc.$$

Keeping the momentum common avoids introducing an unnecessary electron mass or a numerical value of Planck's constant. The equality of wavelengths fixes p, whereas energy depends on the particle's energy-momentum relation.

Step 2: Form the ratio and substitute the speed Dividing electron energy by photon energy cancels their common momentum:

$$\frac{K_e}{K_\gamma} = \frac{pv/2}{pc} = \frac{v}{2c}.$$

The stated electron speed is 25% of light speed, so $v = c/4$. Therefore

$$\frac{K_e}{K_\gamma} = \frac{c/4}{2c} = \frac{1}{8}.$$

Option (A), $1/4$, omits the factor $1/2$ in the electron kinetic energy. Option (B), $8/1$, reverses the requested order of the ratio. Option (D), $1/1$, incorrectly equates energies merely because the wavelengths and momenta are equal. The requested ratio follows the usual school-level kinetic-energy formula reflected in the options.

Step 3: Conclusion The ratio of the kinetic energy of the electron to that of the photon is $\boxed{1:8}$. Hence, the correct option is (C).

Quick Tip to Solve: At equal wavelengths, cancel the common momentum: a nonrelativistic electron contributes $pv/2$, while a photon contributes pc.

33. A convex mirror of radius of curvature $30\ \text{cm}$ forms an image that is half the size of the object. The object distance is : [4 marks]

(A) -45 cm

(B) -15 cm

(C) 45 cm

(D) 15 cm

Correct Answer: (B) -15 cm

Solution:

Concept:

- For a spherical mirror, $f = R/2$ and $1/f = 1/v + 1/u$, where distances follow the Cartesian sign convention. A convex mirror has positive focal length and forms a virtual upright image of a real object.
- The transverse magnification of a mirror is $m = h_i/h_o = -v/u$. An upright image half the object size has $m = +1/2$, not $-1/2$.

Step 1: Use the image size to relate the distances The given radius of curvature is $30\,\text{cm}$, so

$$f = +\frac{30}{2} = +15\,\text{cm}.$$

The positive sign places the focus behind the convex mirror. Because the image is virtual and erect, its magnification is positive:

$$m = \frac{1}{2} = -\frac{v}{u} \quad \Rightarrow \quad v = -\frac{u}{2}.$$

For a real object, $u < 0$, so this relation correctly gives $v > 0$. Thus it also checks the physical side of the mirror on which the image must form. Using a negative magnification would describe an inverted image, which is inconsistent with this convex-mirror situation.

Step 2: Substitute into the mirror equation Insert $v = -u/2$ into the mirror formula:

$$\frac{1}{15} = \frac{1}{-u/2} + \frac{1}{u} = -\frac{2}{u} + \frac{1}{u} = -\frac{1}{u}.$$

Hence $u = -15\,\text{cm}$, and the image distance is $v = +7.5\,\text{cm}$. The image is between the pole and the focus, as expected for a convex mirror. Substitution gives $1/7.5 - 1/15 = 1/15$, confirming the focal length. Option (A), $-45\,\text{cm}$, would produce magnification $1/4$, rather than $1/2$. Options (C) and (D) give positive object distances and therefore do not represent the real object in front of the mirror under the Cartesian convention.

Step 3: Conclusion The signed object distance is $\boxed{-15\,\text{cm}}$, corresponding to an object 15 cm in front of the mirror. Hence, the correct option is (B).

Quick Tip to Solve: For a convex mirror with an upright half-size image, first write $v = -u/2$; then the mirror equation determines the signed object distance directly.

34. The deflection in moving coil galvanometer falls from 25 divisions to 5 division when a shunt of 24Ω is applied. The resistance of galvanometer coil will be : [4 marks]

(A) 48Ω

(B) 100Ω

(C) 96Ω

(D) 12Ω

Correct Answer: (C) 96Ω

Solution:

Concept:

- For the same moving-coil galvanometer, deflection is proportional to the current through its coil. Thus the ratio of coil currents equals the ratio of deflections when its sensitivity is unchanged.
- A shunt is connected in parallel with the galvanometer. The coil and shunt therefore have equal potential differences, so $I_g G = I_s S$, where G and S are their resistances.

Step 1: Find the fraction of current diverted through the shunt Let I be the current that originally passed entirely through the galvanometer and produced 25 divisions. When the shunt is applied to carry the same measured current, the coil deflects through only 5 divisions. Therefore

$$\frac{I_g}{I} = \frac{5}{25} = \frac{1}{5}.$$

The current through the coil is $I_g = I/5$, while the remainder flows through the parallel shunt:

$$I_s = I - I_g = I - \frac{I}{5} = \frac{4I}{5} = 4I_g.$$

The reduction factor is five, but the shunt current is four times the coil current, because one of those five equal current portions still passes through the coil.

Step 2: Equate the parallel-branch voltage drops The given shunt resistance is $S = 24\,\Omega$. Equal voltages across the parallel branches give

$$I_g G = I_s S = (4I_g)(24).$$

Cancelling the nonzero coil current yields

$$G = 4 \times 24 = 96\,\Omega.$$

Since the shunt has one-quarter of the coil resistance, it carries four times as much current, confirming the current split and the observed deflection ratio. Option (A), $48\,\Omega$, would give a threefold total-to-coil current ratio; option (B), $100\,\Omega$, is not the exact result of the stated numbers. Option (D), $12\,\Omega$, would divert too little current to reduce the deflection to one-fifth.

Step 3: Conclusion The resistance of the galvanometer coil is $\boxed{96\,\Omega}$. Hence, the correct option is (C).

Quick Tip to Solve: If a shunt reduces deflection by a factor n, the shunt carries $(n-1)$ times the coil current, giving $G = (n-1)S$.

35. A capacitor of capacitance $100\mu\text{F}$ is charged to a potential of $12\ \text{V}$ and connected to a $6.4\ \text{mH}$ inductor to produce oscillations. The maximum current in the circuit would be : [4 marks]

(A) 2.0 A

(B) 3.2 A

(C) 1.5 A

(D) 1.2 A

Correct Answer: (C) 1.5 A

Solution:

Concept:

- In an ideal LC circuit, energy oscillates between the capacitor electric field and the inductor magnetic field. The total energy is conserved when resistance and other losses are neglected.
- The capacitor energy is $U_C = CV^2/2$, and the inductor energy is $U_L = LI^2/2$. The current is maximum when the capacitor is momentarily uncharged, so all the initial energy is magnetic.

Step 1: Convert the circuit values to SI units The initially charged capacitor has

$$C = 100\,\mu\text{F} = 100 \times 10^{-6}\,\text{F} = 10^{-4}\,\text{F}.$$

The inductance is

$$L = 6.4\,\text{mH} = 6.4 \times 10^{-3}\,\text{H}.$$

The initial capacitor voltage is $V_0 = 12\,\text{V}$, so its stored energy is

$$U_0 = \frac{1}{2}CV_0^2 = \frac{1}{2}(10^{-4})(12)^2 = 7.2 \times 10^{-3}\,\text{J}.$$

Initially the current is zero. During discharge the capacitor energy decreases while the inductor magnetic energy increases; at the maximum current, the capacitor energy has fallen to zero.

Step 2: Equate the initial electric energy to maximum magnetic energy Conservation of energy gives

$$\frac{1}{2}CV_0^2 = \frac{1}{2}LI_{\text{max}}^2 \quad \Rightarrow \quad I_{\text{max}} = V_0\sqrt{\frac{C}{L}}.$$

Substituting in SI units,

$$I_{\text{max}} = 12\sqrt{\frac{10^{-4}}{6.4 \times 10^{-3}}} = 12\sqrt{\frac{1}{64}} = 12 \times \frac{1}{8} = 1.5\,\text{A}.$$

Checking directly, $\frac{1}{2}(6.4 \times 10^{-3})(1.5)^2 = 7.2 \times 10^{-3}\,\text{J}$, exactly the initial energy. The other currents, $2.0\,\text{A}$, $3.2\,\text{A}$, and $1.2\,\text{A}$, give different magnetic energies, so none conserves the stated initial energy at the instant when the capacitor is uncharged.

Step 3: Conclusion The maximum current in the oscillating LC circuit is $\boxed{1.5\,\text{A}}$. Hence, the correct option is (C).

Quick Tip to Solve: For the peak LC current, use $I_{\text{max}} = V_0\sqrt{C/L}$ after converting microfarads and millihenrys to farads and henrys.

36. At what distance above and below the surface of the earth a body will have same weight. (take radius of earth as R.) [4 marks]

(A) $\frac{\sqrt{3}R-R}{2}$

(B) $\frac{R}{2}$

(C) $\frac{\sqrt{5}R-R}{2}$

(D) $\sqrt{5}R-R$

Correct Answer: (C) $\frac{\sqrt{5}R-R}{2}$

Solution:

Concept:

- Outside a spherical Earth, the acceleration is $g_h = g_0R^2/(R+h)^2$, where g_0 is its surface value.
- For the standard uniform-density Earth model, at depth d, $g_d = g_0(1-d/R)$. Equal weights of the same body mean equal gravitational accelerations.

Step 1: Express the common distance Let the distance above and below the surface be x, as requested. The body is at radius $R+x$ above and at radius $R-x$ below the surface. The inverse-square expression applies outside Earth; the linear depth expression applies inside. Therefore the equality to solve is

$$\frac{g_0R^2}{(R+x)^2} = g_0\left(1-\frac{x}{R}\right).$$

The mass of the body cancels because its weight is mass times local gravitational acceleration.

Step 2: Solve for the nonzero distance Put $y = x/R$ and cancel g_0. Multiplication by $(1+y)^2$ gives

$$1 = (1-y)(1+y)^2 = 1+y-y^2-y^3.$$

Thus $y(1-y-y^2) = 0$. The root $y = 0$ describes the surface itself, rather than distinct positions above and below it. The remaining quadratic gives

$$y^2+y-1 = 0, \qquad y = \frac{-1\pm\sqrt{5}}{2}.$$

Only the positive root lies in the physical interval $0 < y < 1$.

Step 3: Check the alternatives The common distance is consequently $x = R(\sqrt{5}-1)/2$, approximately $0.618R$. Option A has $y = (\sqrt{3}-1)/2$, and option B has $y = 1/2$; neither satisfies $y^2+y = 1$. Option D has $y = \sqrt{5}-1 > 1$, so it is not a physical depth within Earth. The positive quadratic root selects option C uniquely.

Step 4: Conclusion The required equal distance above and below the surface is $\boxed{x = (\sqrt{5}R-R)/2}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the exact altitude formula when the distance is comparable with Earth's radius; the small-height approximation would lose the nonzero solution.

37. A galvanometer having coil resistance 10Ω shows a full scale deflection for a current of $3\ \mathrm{mA}$. For it to measure a current of $8\ \mathrm{A}$, the value of the shunt should be: [4 marks]

(A) $3.75 \times 10^{-3}\Omega$

(B) $3 \times 10^{-3}\Omega$

(C) $4.85 \times 10^{-3}\Omega$

(D) $2.75 \times 10^{-3}\Omega$

Correct Answer: (A) $3.75 \times 10^{-3}\Omega$

Solution:

Concept:

- A shunt is a small resistance connected in parallel with a galvanometer so most of the total current bypasses its coil.
- Parallel branches have equal potential differences: $I_g G = (I - I_g)S$, with coil resistance G, full-scale coil current I_g, and shunt resistance S.

Step 1: Find the coil voltage The full-scale coil current is $I_g = 3\,\mathrm{mA} = 0.003\,\mathrm{A}$, while $G = 10\,\Omega$. The voltage across the coil at full scale must therefore remain

$$V_g = I_g G = 0.003 \times 10 = 0.030\,\mathrm{V}.$$

The coil cannot safely carry the full external current of $8\,\mathrm{A}$; the shunt supplies the parallel bypass needed for the ammeter conversion.

Step 2: Calculate the bypass resistance At an external current $I = 8\,\mathrm{A}$, current conservation at the junction gives

$$I_s = I - I_g = 8 - 0.003 = 7.997\,\mathrm{A}.$$

The shunt experiences the same $0.030\,\mathrm{V}$ as the coil, so

$$S = \frac{0.030}{7.997} = 0.0037514\,\Omega \simeq 3.75 \times 10^{-3}\,\Omega.$$

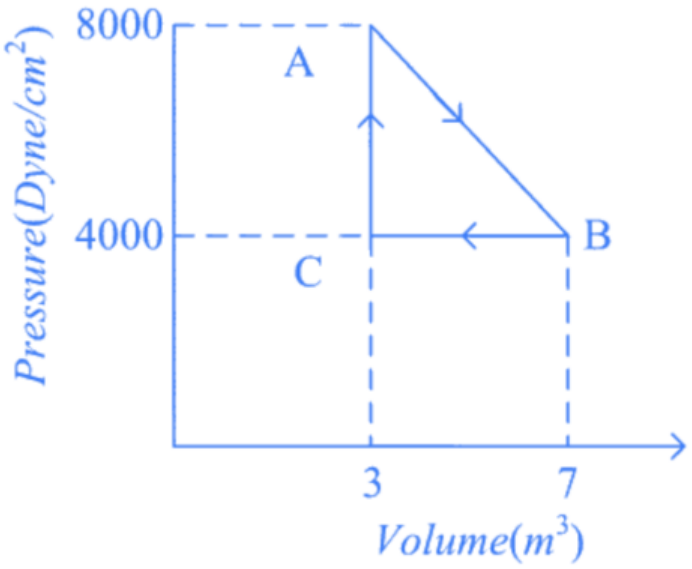
Retaining the subtraction before rounding shows why the approximate answer is accurate.

Step 3: Compare with the choices Because $I_g \ll I$, the quick estimate $S \simeq I_g G/I = 0.03/8$ gives the same listed value. The values in options B, C and D would not provide the necessary $7.997\,\mathrm{A}$ bypass at the coil voltage. For example, $3 \times 10^{-3}\,\Omega$ would carry $10\,\mathrm{A}$, exceeding the required bypass current. A shunt resistance much smaller than the coil resistance is physically consistent.

Step 4: Conclusion The shunt required is $\boxed{S \simeq 3.75 \times 10^{-3}\,\Omega}$. Hence, the correct option is (A).

Quick Tip to Solve: For ammeter conversion, subtract the full-scale coil current from the desired range before using $S = I_g G/(I - I_g)$.

38. A thermodynamic system is taken from an original state A to an intermediate state B by a linear process as shown in the figure. It's volume is then reduced to the original value from B to C by an isobaric process. The total work done by the gas from A to B and B to C would be : [4 marks]



(A) 800 J

(B) 2200 J

(C) 33800 J

(D) 1200 J

Correct Answer: (A) 800 J

Solution:

Concept:

- The work done by a gas on a pressure-volume path is $W = \int P\,dV$. Expansion gives positive work and compression gives negative work.
- For a straight-line path, the integral is average pressure times volume change. Here $1\,\text{dyne}\,\text{cm}^{-2} = 0.1\,\text{Pa}$.

Step 1: Read and convert the graph The diagram gives $A : (V, P) = (3\,\text{m}^3, 8000\,\text{dyne}\,\text{cm}^{-2})$, $B : (7\,\text{m}^3, 4000\,\text{dyne}\,\text{cm}^{-2})$, and $C : (3\,\text{m}^3, 4000\,\text{dyne}\,\text{cm}^{-2})$. Since $1\,\text{dyne} = 10^{-5}\,\text{N}$ and $1\,\text{cm}^2 = 10^{-4}\,\text{m}^2$, the pressures are $800\,\text{Pa}$ and $400\,\text{Pa}$. Converting the pressure unit is essential before reporting joules.

Step 2: Evaluate the two signed works On the linear expansion from A to B the average pressure is $(800 + 400)/2 = 600\,\text{Pa}$. Hence

$$W_{AB} = 600(7 - 3) = 2400\,\text{J}.$$

On the isobaric compression from B to C, the pressure stays at $400\,\text{Pa}$ and the volume change is negative:

$$W_{BC} = 400(3 - 7) = -1600\,\text{J}.$$

Adding the two contributions gives

$$W_{ABC} = 2400 - 1600 = 800\,\text{J}.$$

Step 3: Check the sign and alternatives The result is also the triangular area between AB and BC: $\frac{1}{2}(800 - 400)(7 - 3) = 800\,\text{J}$. The return vertical segment CA would have zero work, so this geometric check applies to the specified path even though only AB and BC are requested. A value of 2200, 33800, or $1200\,\text{J}$ does not equal the signed integral of these two segments; treating compression as positive or missing the dyne conversion gives a different, incorrect quantity.

Step 4: Conclusion The total work done by the gas is $\boxed{800\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: On a pressure-volume graph, keep compression work negative and convert the pressure axis to pascals before finding an area.

39. Two charges of $5Q$ and $-2Q$ are situated at the points $(3a, 0)$ and $(-5a, 0)$ respectively. The electric flux through a sphere of radius '$4a$' having center at origin is : [4 marks]

(A) $\frac{2Q}{\varepsilon_0}$
(B) $\frac{7Q}{\varepsilon_0}$
(C) $\frac{3Q}{\varepsilon_0}$
(D) $\frac{5Q}{\varepsilon_0}$

Correct Answer: (D) $\frac{5Q}{\varepsilon_0}$

Solution:

Concept:

- Gauss's law for a closed surface is $\Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}}/\varepsilon_0$.
- An external charge may alter the field at points on the surface, but its net contribution to the flux through the entire closed surface is zero.

Step 1: Identify the enclosed charge The sphere has centre at the origin and radius $4a$. A point on the x-axis at $(x, 0)$ is inside when its distance $|x|$ from the origin is less than $4a$. Therefore the charge $+5Q$ at $(3a, 0)$ is inside, because $3a < 4a$. The charge $-2Q$ at $(-5a, 0)$ is outside, because its distance is $5a > 4a$. Neither charge lies on the spherical surface.

Step 2: Apply the closed-surface flux law Only the algebraic charge enclosed by the sphere enters Gauss's law. In this case

$$Q_{\text{enclosed}} = 5Q, \qquad \Phi_E = \frac{5Q}{\varepsilon_0}.$$

The external negative charge produces field lines that enter and leave the sphere, contributing equal and opposite signed fluxes. It must not be subtracted from the enclosed charge. The positive sign of the final result means the net flux is outward.

Step 3: Reject the other combinations Option C, $3Q/\varepsilon_0$, would incorrectly count both charges as enclosed by summing $5Q - 2Q$. Option B, $7Q/\varepsilon_0$, would add their magnitudes and also incorrectly

include the external charge. Option A does not represent the enclosed charge either. No explicit field integration is needed: the surface may have a nonuniform field, but the total flux is fixed solely by the internal charge. The location of the charge within the sphere does not change this result.

Step 4: Conclusion The electric flux through the sphere is $\boxed{5Q/\varepsilon_0}$. Hence, the correct option is (D).

Quick Tip to Solve: For net flux, first compare each charge's distance from the centre with the radius; count only enclosed charges, with their signs.

40. A biconvex lens of refractive index 1.5 has a focal length of 20 cm in air. Its focal length when immersed in a liquid of refractive index 1.6 will be: [4 marks]

(A) -16 cm

(B) $+16$ cm

(C) $+160$ cm

(D) -160 cm

Correct Answer: (D) -160 cm

Solution:

Concept:

- For a thin lens in a surrounding medium, $1/f = (n_l/n_m - 1)(1/R_1 - 1/R_2)$. Here n_l is the lens index and n_m is the medium index.
- The geometrical curvature factor stays fixed when a lens is moved between media. A lens less refractive than its surroundings reverses the sign of its power.

Step 1: Determine the curvature factor In air, take the surrounding index as unity. With $n_l = 1.5$ and $f_{\text{air}} = 20\,\text{cm}$, define $K = 1/R_1 - 1/R_2$. The lens maker formula gives

$$\frac{1}{20} = (1.5 - 1)K = \frac{1}{2}K, \qquad K = \frac{1}{10}\,\text{cm}^{-1}.$$

There is no need to find the two radii separately because the same combination appears in both environments.

Step 2: Use the relative refractive index In the liquid, $n_m = 1.6$, so the relative index is

$$\frac{n_l}{n_m} = \frac{1.5}{1.6} = \frac{15}{16}.$$

Consequently

$$\frac{1}{f_{\text{liquid}}} = \left(\frac{15}{16} - 1\right)\frac{1}{10} = -\frac{1}{160}\,\text{cm}^{-1}, \qquad f_{\text{liquid}} = -160\,\text{cm}.$$

The negative focal length denotes a diverging lens in this liquid, despite its biconvex shape.

Step 3: Check the magnitude and sign The air-to-liquid focal-length ratio is $(1.5 - 1)/(1.5/1.6 - 1) = -8$, which confirms $-8 \times 20 = -160\,\text{cm}$. Options B and C have a positive sign and cannot describe this lens in a medium of higher refractive index. Option A has the correct sign but is smaller in magnitude

by a factor of ten. The relative index is close to unity, so the optical power is weak and the focal length magnitude should increase.

Step 4: Conclusion The focal length in the liquid is $\boxed{-160\,\text{cm}}$. Hence, the correct option is (D).

Quick Tip to Solve: In a surrounding liquid, replace the lens index by its relative index n_l/n_m; a value below one reverses the power.

41. A block of mass $100\ \text{kg}$ slides over a distance of $10\ \text{m}$ on a horizontal surface. If the co-efficient of friction between the surfaces is 0.4, then the work done against friction $(\text{in } J)$ is : [4 marks]

(A) 3900

(B) 4500

(C) 4200

(D) 4000

Correct Answer: (D) 4000

Solution:

Concept:

- On a horizontal surface with no vertical applied force, the normal reaction is $N = mg$. The magnitude of sliding friction is $f_k = \mu N$.
- The work done against friction over distance s is the positive energy expenditure $W = \mu mgs$; work done by friction itself is negative.

Step 1: Calculate the friction force The mass is $100\,\text{kg}$, the coefficient of friction is 0.4, and the displacement is $10\,\text{m}$. Using the conventional approximation $g = 10\,\text{m}\,\text{s}^{-2}$ for the numerical options, the normal reaction and friction magnitude are

$$N = mg = 100 \times 10 = 1000\,\text{N}, \qquad f_k = \mu N = 0.4 \times 1000 = 400\,\text{N}.$$

Friction acts opposite to the direction of sliding throughout the motion.

Step 2: Evaluate the energy spent The question asks for work done against friction, so report the positive magnitude of the energy transferred to thermal energy. The friction magnitude is constant on the horizontal surface; hence

$$W_{\text{against}} = f_k s = 400 \times 10 = 4000\,\text{J}.$$

Equivalently, the frictional work on the block is $W_{\text{friction}} = -4000\,\text{J}$, because the angle between friction and displacement is 180°. These signs refer to different wordings, not different force magnitudes.

Step 3: Match the listed answer Option D is $4000\,\text{J}$, the result under the stated conventional approximation. Options B and C exceed the product μmgs without any additional force or distance in the question. Option A is $3900\,\text{J}$, whereas choosing $g = 9.8\,\text{m}\,\text{s}^{-2}$ would give $3920\,\text{J}$, not exactly that listed number. The intended standard numerical approximation therefore selects D. The frictional energy loss does not depend on the speed of sliding in this model.

Step 4: Conclusion The work done against friction is $\boxed{4000\,\mathrm{J}}$, using $g = 10\,\mathrm{m\,s^{-2}}$. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish positive work against friction from negative work by friction; on a horizontal plane their magnitude is μmgs.

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42. Given below are two statements:

Statement I : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in hot water.

Statement II : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in cold water.

In the light of the above statements, choose the most appropriate from the options given below

[4 marks]

(A) Both Statement I and Statement II are false

(B) Both Statement I and Statement II are true

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- For a wetting liquid in a narrow capillary, the equilibrium rise is $h = 2T\cos\theta/(\rho g r)$, where T is surface tension and r is tube radius.
- The surface tension of water decreases as temperature rises. In the usual capillary comparison this decrease dominates the smaller accompanying density change.

Step 1: Compare the liquid properties The same capillary is used for cold and hot water, so its radius r and the gravitational acceleration g are unchanged. Water wets clean glass, and its contact angle is approximately zero in the standard treatment, making $\cos\theta$ approximately unity. Thus the rise is governed principally by T/ρ:

$$h \simeq \frac{2T}{\rho g r}.$$

Heating reduces the cohesive surface-tension force that supports the column of water against gravity.

Step 2: Evaluate Statement I Hot water has a smaller surface tension than cold water. Although its density also decreases slightly, the surface-tension decrease is the deciding effect in this familiar comparison. Therefore $h_{\mathrm{hot}} < h_{\mathrm{cold}}$. Statement I says that the height is smaller in hot water, precisely this result, so Statement I is true. The order of immersion does not reverse the equilibrium temperature dependence assumed in the question.

Step 3: Evaluate Statement II and the alternatives Statement II asserts the opposite ordering, namely that the height is smaller in cold water. That contradicts the capillary-rise comparison, so Statement II is false. The statements cannot both be true for the same pair of temperatures and the same tube.

Option A wrongly rejects the correct first statement; option B accepts the incorrect second statement; and option D reverses both truth values. Only option C gives the required true-false combination.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: For water in the same clean capillary, recall that heating lowers surface tension and hence lowers the equilibrium capillary rise.

43. If the radius of curvature of the path of two particles of same mass are in the ratio $3:4$, then in order to have constant centripetal force, their velocities will be in the ratio of : [4 marks]

(A) $1:\sqrt{3}$

(B) $2:\sqrt{3}$

(C) $\sqrt{3}:2$

(D) $\sqrt{3}:1$

Correct Answer: (C) $\sqrt{3}:2$

Solution:

Concept:

- For a particle of mass m, speed v, and radius of curvature r, the inward force needed is $F_c = mv^2/r$. The radius describes the local curvature of the path.
- With the same mass and the same centripetal force, v^2/r is fixed. Thus speed is proportional to the square root of the radius, not to the radius itself.

Step 1: Equate the centripetal forces Let the speeds be v_1, v_2 and the radii be r_1, r_2. Since the particles have equal masses and are required to experience the same centripetal force,

$$\frac{mv_1^2}{r_1} = \frac{mv_2^2}{r_2}.$$

Cancelling the common mass and rearranging gives

$$\frac{v_1^2}{v_2^2} = \frac{r_1}{r_2} = \frac{3}{4}.$$

Step 2: Take the positive square root and compare options Speeds are nonnegative, so the physical ratio is the positive square root:

$$\frac{v_1}{v_2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}.$$

The first particle has the smaller radius, so it must also have the smaller speed. Option (B) reverses the required ratio. Options (A) and (D) give squared speed ratios of $1/3$ and 3, respectively, rather than $3/4$. Only option (C) satisfies the force equation.

Step 3: Conclusion The velocities, interpreted as their speeds along the paths, are in the ratio $\boxed{\sqrt{3}:2}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal masses and equal centripetal forces, write $v \propto \sqrt{r}$ before taking any ratio. Check that the smaller radius has the smaller speed.

44. Match List I with List II

	List - I		List - II
(A)	$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_o i_c + \mu_o \varepsilon_o \frac{d\phi_E}{dt}$	**(I)**	**Gauss' law for electricity**
(B)	$\oint \vec{E} \cdot \overrightarrow{dl} = \frac{d\phi_B}{dt}$	**(II)**	**Gauss' law for magnetism**
(C)	$\oint \vec{E} \cdot \overrightarrow{dA} = \frac{Q}{\varepsilon_o}$	**(III)**	**Faraday law**
(D)	$\oint \vec{B} \cdot \overrightarrow{dA} = 0$	**(IV)**	**Ampere - Maxwell law**

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-III, C-I, D-II

(B) A-I, B-II, C-III, D-IV

(C) A-IV, B-I, C-III, D-II

(D) A-II, B-III, C-I, D-IV

Correct Answer: (A) A-IV, B-III, C-I, D-II

Solution:

Concept:

- The integral Maxwell equations connect electric and magnetic fields with charges, currents and changing fluxes. A line integral describes circulation, whereas a closed-surface integral describes net outward flux.
- Gauss laws concern flux through closed surfaces. Ampere–Maxwell and Faraday laws concern circulation around closed curves.

Step 1: Identify the two circulation equations Equation (A) relates the circulation of the magnetic field to conduction current i_c and displacement current $\varepsilon_0 \, d\Phi_E/dt$. It is the Ampere–Maxwell law, so A matches IV. Equation (B) relates the circulation of the electric field to the rate of change of magnetic flux, identifying Faraday law, so B matches III. In the conventional common orientation of loop and surface, Faraday law carries a minus sign; the printed expression omits it. This sign omission does not change which named law the expression is intended to represent.

Step 2: Identify the two flux equations and choose the complete mapping Equation (C) gives electric flux as enclosed charge divided by permittivity:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}.$$

It is Gauss law for electricity, so C matches I. Equation (D) gives zero net magnetic flux through a closed surface, expressing the absence of isolated magnetic poles; it is Gauss law for magnetism, so D matches II. The complete mapping is therefore A-IV, B-III, C-I, D-II. Option (B) misidentifies the circulation laws, option (C) interchanges the electric-flux and induction identifications, and option (D) misidentifies the magnetic circulation and flux equations.

Step 3: Conclusion The correct matching is $\boxed{\text{A-IV, B-III, C-I, D-II}}$. Hence, the correct option is (A).

Quick Tip to Solve: First separate circulation integrals from flux integrals. A charge term identifies electric Gauss law, while zero closed-surface magnetic flux identifies magnetic Gauss law.

45. Two vessels A and B are of the same size and are at same temperature. A contains 1 g of hydrogen and B contains 1 g of oxygen. $\mathrm{P_A}$ and $\mathrm{P_B}$ are the pressures of the gases in A and B respectively, then $\frac{P_A}{P_B}$ is: [4 marks]

(A) 4

(B) 32

(C) 8

(D) 16

Correct Answer: (D) 16

Solution:

Concept:

- For an ideal gas, $PV = nRT$, where n is the number of moles. At the same temperature and volume, pressure is directly proportional to the number of moles.
- The mole number is $n = m/M$, where m is the mass and M the molar mass. Hydrogen and oxygen gases are diatomic, with molar masses 2 and $32\,\mathrm{g\,mol^{-1}}$, respectively.

Step 1: Calculate the amount of each gas Each vessel contains $1\,\mathrm{g}$, but equal masses of different gases do not generally contain equal numbers of molecules. For molecular hydrogen in vessel A,

$$n_A = \frac{1}{2} = 0.5\,\mathrm{mol}.$$

For molecular oxygen in vessel B,

$$n_B = \frac{1}{32}\,\mathrm{mol}.$$

Thus the lighter gas contains more molecules in the same mass.

Step 2: Form the pressure ratio at equal volume and temperature Apply the gas equation separately to the two vessels. The volume V, temperature T, and universal gas constant R cancel:

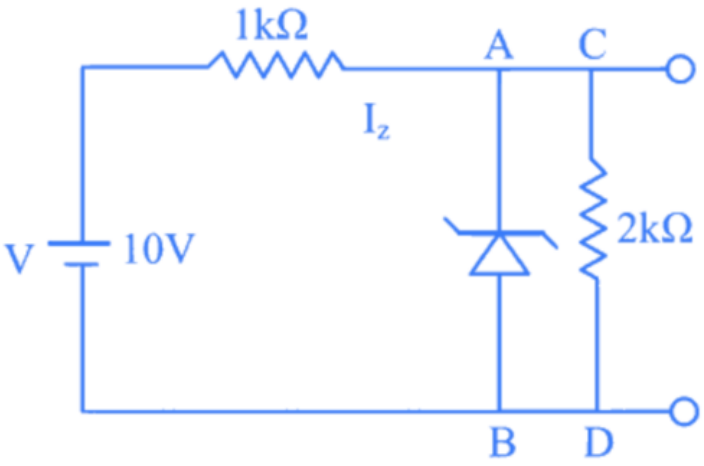
$$\frac{P_A}{P_B} = \frac{n_A RT/V}{n_B RT/V} = \frac{n_A}{n_B} = \frac{1/2}{1/32} = 16.$$

The pressure ratio is dimensionless. The value 32 would incorrectly use an atomic molar mass for hydrogen while retaining molecular oxygen. The values 4 and 8 also fail the mole-number ratio. The smaller molecular mass produces the larger pressure here, which checks the direction of the result.

Step 3: Conclusion The required ratio is $\boxed{P_A/P_B = 16}$. Hence, the correct option is (D).

Quick Tip to Solve: At equal temperature and volume, equal gas masses give $P \propto 1/M$. Use molecular molar masses for hydrogen and oxygen, then invert their ratio.

46. In the given circuit, the breakdown voltage of the Zener diode is 3.0 V. What is the value of I_z ? [4 marks]



(A) 3.3 mA

(B) 10 mA

(C) 5.5 mA

(D) 7 mA

Correct Answer: (C) 5.5 mA

Solution:

Concept:

- A reverse-biased Zener diode in breakdown holds the voltage across itself approximately equal to its breakdown voltage V_Z. The parallel load has the same voltage.
- Kirchhoff current law at the regulated node gives $I_s = I_L + I_Z$, where I_s is the current through the series resistor and I_L is the load current.

Step 1: Check breakdown and calculate the series current The circuit has a $10\,\text{V}$ source, a $1\,\text{k}\Omega$ series resistor and a $2\,\text{k}\Omega$ load in parallel with the Zener. Without Zener conduction, the divider output would be $10(2/3) = 6.67\,\text{V}$, exceeding $3.0\,\text{V}$. The Zener therefore enters reverse breakdown and clamps the output to $3.0\,\text{V}$. The voltage across the series resistor is $10 - 3 = 7\,\text{V}$, giving

$$I_s = \frac{7}{1000} = 7.0\,\text{mA}.$$

Step 2: Subtract the load current from the available current The load resistor also has $3.0\,\text{V}$ across it. Hence

$$I_L = \frac{3}{2000} = 1.5\,\text{mA}.$$

At the top junction, the incoming current splits between the load and Zener:

$$I_Z = I_s - I_L = 7.0 - 1.5 = 5.5\,\text{mA}.$$

This is positive, confirming that the assumed breakdown operating condition is consistent. Option

(D) is the series current rather than the Zener current. Option (B) ignores the regulated voltage drop when computing current, while option (A) does not satisfy the node-current balance.

Step 3: Conclusion The Zener current is $\boxed{I_Z = 5.5\,\text{mA}}$. Hence, the correct option is (C).

Quick Tip to Solve: In a Zener regulator, calculate both resistor currents using the clamped output voltage. The Zener receives the series current left over after the load current is supplied.

47. A body starts moving from rest with constant acceleration covers displacement S_1 in first $(p-1)$ seconds and S_2 in first p seconds. The displacement $\mathrm{S}_1+\mathrm{S}_2$ will be made in time : [4 marks]

(A) $(2p+1)s$

(B) $(2p-1)s$

(C) $\left(2p^2-2p+1\right)s$

(D) $\sqrt{(2p^2-2p+1)}s$

Correct Answer: (D) $\sqrt{(2p^2-2p+1)}s$

Solution:

Concept:

- A body starting from rest with constant acceleration a has displacement $s=\frac{1}{2}at^2$ after elapsed time t. The same relation applies to each stated displacement measured from the initial instant.
- For fixed acceleration, elapsed time is proportional to the square root of displacement. Displacements can be added, but their corresponding elapsed times cannot simply be added.

Step 1: Express the two displacements from the start For the first $(p-1)$ seconds, the displacement is

$$S_1=\frac{1}{2}a(p-1)^2.$$

For the first p seconds, it is

$$S_2=\frac{1}{2}ap^2.$$

Both are measured from rest at the same initial instant. In particular, S_2 is not the displacement only during the next one-second interval. Adding the two specified displacements gives

$$S_1+S_2=\frac{1}{2}a\left((p-1)^2+p^2\right).$$

Step 2: Find the time for the summed displacement Let t be the elapsed time from rest needed to cover S_1+S_2. Then

$$\frac{1}{2}at^2=\frac{1}{2}a(2p^2-2p+1).$$

Cancel the nonzero common acceleration factor and take the positive root:

$$t=\sqrt{2p^2-2p+1}\,\text{s}.$$

Option (C) gives the squared numerical time and misses the square root. Options (A) and (B) treat

the time dependence as linear; neither follows the quadratic displacement law. As a check, setting $p = 1$ gives $S_1 = 0$, and the result correctly reduces to one second.

Step 3: Conclusion The required time is $\boxed{\sqrt{2p^2 - 2p + 1}\,\text{s}}$. Hence, the correct option is (D).

Quick Tip to Solve: When displacements from rest are added, add the squares of the corresponding times: $t^2 = t_1^2 + t_2^2$. Do not add the elapsed times themselves.

48. The electric current through a wire varies with time as $I = I_0 + \beta t$, where $I_0 = 20$ A and $\beta = 3$ A/s. The amount of electric charge crossed through a section of the wire in 20 s is : [4 marks]

(A) 80 C

(B) 800 C

(C) 1000 C

(D) 1600 C

Correct Answer: (C) 1000 C

Solution:

Concept:

- Current is the rate of flow of charge: $I = dQ/dt$. Therefore charge passing a section during an interval is the time integral of current over that interval.
- For current varying linearly with time, the average current equals the arithmetic mean of its initial and final values. This gives an independent check on the integral.

Step 1: Integrate the given current over the interval The current is $I(t) = I_0 + \beta t$, with $I_0 = 20\,\text{A}$ and $\beta = 3\,\text{A}\,\text{s}^{-1}$. Over the first $20\,\text{s}$, the charge is

$$Q = \int_0^{20} (I_0 + \beta t)\, dt.$$

Integrating the constant term and the term proportional to time separately gives

$$Q = I_0 t + \frac{1}{2}\beta t^2 \quad \text{at } t = 20\,\text{s}.$$

Step 2: Substitute the values and check the average current The constant-current contribution is $20 \times 20 = 400\,\text{C}$, and the increasing-current contribution is $\frac{1}{2} \times 3 \times 20^2 = 600\,\text{C}$. Thus

$$Q = 400 + 600 = 1000\,\text{C}.$$

The final current is $20 + 3(20) = 80\,\text{A}$, so the average current is $(20 + 80)/2 = 50\,\text{A}$. Multiplying by $20\,\text{s}$ again gives $1000\,\text{C}$. The value 80 in option (A) is the final current, not the transferred charge. Option (D) uses final current throughout the interval and overestimates the charge; option (B) also disagrees with the integral.

Step 3: Conclusion The charge crossing the wire section is $\boxed{1000\,\text{C}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a straight-line current-versus-time graph, use trapezium area: $Q = \frac{1}{2}(I_{\text{initial}} + I_{\text{final}})\Delta t$. The result has units of coulomb.

49. The potential energy function (in J) of a particle in a region of space is given as $U = \left(2x^2 + 3y^3 + 2z\right)$. Here x, y and z are in meter. The magnitude of x-component of force (in N) acting on the particle at point $P(1, 2, 3)\mathrm{m}$ is : [4 marks]

(A) 4

(B) 2

(C) 8

(D) 6

Correct Answer: (A) 4

Solution:

Concept:

- For a conservative force, $\vec{F} = -\nabla U$. Its Cartesian component is $F_x = -\partial U/\partial x$, with the other coordinates held fixed during differentiation.
- The signed component gives direction along the chosen axis. Its magnitude is $|F_x|$, which is nonnegative. Energy gradient has units $\mathrm{J\,m^{-1}} = \mathrm{N}$.

Step 1: Differentiate the potential with respect to the required coordinate The given energy function is

$$U(x, y, z) = 2x^2 + 3y^3 + 2z.$$

To find the x-component of force, differentiate only with respect to x. While taking this partial derivative, y and z are constant, so their terms contribute zero:

$$\frac{\partial U}{\partial x} = 4x, \qquad F_x = -4x.$$

There is no need to evaluate the full potential or calculate the other force components.

Step 2: Evaluate the component and take its magnitude At the specified point $P(1, 2, 3)\,\mathrm{m}$, the x-coordinate is $1\,\mathrm{m}$. Therefore

$$F_x(P) = -4(1) = -4\,\mathrm{N}.$$

The force component points in the negative x direction because potential increases toward positive x here. The question asks for its magnitude, so

$$|F_x(P)| = 4\,\mathrm{N}.$$

Option (B) confuses the coefficient of x^2 with its derivative. Options (C) and (D) do not result from differentiating the x-dependent term and evaluating it at $x = 1$. The other coordinates affect other components, not this partial derivative.

Step 3: Conclusion The magnitude of the x-component is $\boxed{4\,\mathrm{N}}$. Hence, the correct option is (A).

Quick Tip to Solve: For one force component, differentiate the potential only with respect to that coordinate, include the minus sign, and take an absolute value only if magnitude is requested.

50. The explosive in a Hydrogen bomb is a mixture of $_1H^2, _1H^3$ and $_3Li^6$ in some condensed form. The chain reaction is given by

$$_3Li^6 + _0n^1 \rightarrow _2He^4 + _1H^3$$
$$_1H^2 + _1H^3 \rightarrow _2He^4 + _0n^1$$

During the explosion the energy released is approximately

[Given ; $M(Li) = 6.01690$ amu, $M\left(_1H^2\right) = 2.01471$ amu, $M\left(_2He^4\right) = 4.00388$ amu**, and** 1 amu $= 931.5$ MeV] [4 marks]

(A) 22.22 MeV

(B) 28.12 MeV

(C) 16.48 MeV

(D) 12.64 MeV

Correct Answer: (A) 22.22 MeV

Solution:

Concept:

- The energy released by a nuclear reaction is its mass defect multiplied by the mass-energy conversion factor: $Q = \Delta mc^2$. Here each atomic mass unit corresponds to $931.5\,\text{MeV}$.
- Add consecutive reactions and cancel intermediate particles on opposite sides before calculating the mass defect. The tritium nucleus and neutron are regenerated or consumed inside this cycle, so their masses need not be supplied.

Step 1: Add the two nuclear reactions The first reaction produces one tritium nucleus and consumes a neutron. The second consumes that tritium nucleus and produces a neutron. Cancelling both common particles leaves the net transformation

$$^6_3\text{Li} + ^2_1\text{H} \longrightarrow 2^4_2\text{He}.$$

The mass numbers balance as $6+2 = 2\times 4$, while the charges balance as $3+1 = 2\times 2$. Atomic masses may be used because there are four electrons on each side, so the electron masses cancel as well.

Step 2: Calculate the net mass defect and released energy Use the supplied masses without substituting masses from a different table:

$$m_{\text{initial}} = 6.01690 + 2.01471 = 8.03161\,\text{u},$$
$$m_{\text{final}} = 2(4.00388) = 8.00776\,\text{u},$$
$$\Delta m = 8.03161 - 8.00776 = 0.02385\,\text{u}.$$

Thus

$$Q = 0.02385 \times 931.5 = 22.216275\,\text{MeV} \simeq 22.22\,\text{MeV}.$$

The positive mass defect confirms that energy is released. The values 28.12, 16.48 and $12.64\,\text{MeV}$ do not follow from the net mass balance; including an uncancelled neutron or tritium mass would count an intermediate particle incorrectly.

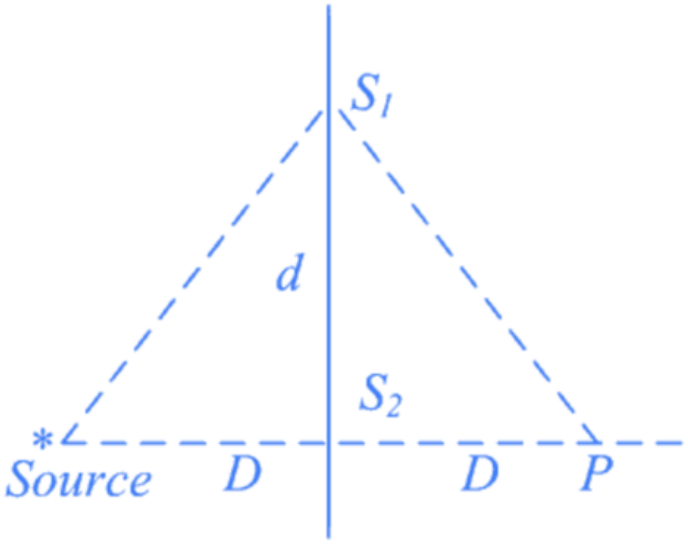
Step 3: Conclusion The energy released per net reaction is approximately $\boxed{22.22\,\text{MeV}}$. Hence, the correct option is (A).

Quick Tip to Solve: First cancel the neutron and tritium between the two reactions; calculate the mass defect only for lithium plus deuterium forming two helium nuclei.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. In a double slit experiment shown in figure, when light of wavelength $400\ \text{nm}$ is used, dark fringe is observed at P. If $\text{D} = 0.2\ \text{m}$, the minimum distance between the slits S_1 and S_2 is _____ mm. [4 marks]



Solution:

Concept:

- For coherent waves initially in phase, a dark fringe occurs when the total path difference is $(2n+1)\lambda/2$, with $n = 0, 1, 2, \ldots$. The smallest slit separation corresponds to the first such minimum.
- The source and observation point are both a horizontal distance D from the lower slit. Therefore both the incident and outgoing portions contribute to the path difference; the source cannot be treated as illuminating both slits with equal phase.

Step 1: Find the complete optical path difference In the figure the source, lower slit S_2 and point P lie on a straight horizontal line. The slit separation is d, so the path through S_2 is $2D$. Each oblique segment through S_1 has length $\sqrt{D^2+d^2}$. Hence

$$\Delta = 2\sqrt{D^2+d^2} - 2D.$$

This is twice the excess of one oblique segment. Keeping only the outgoing path difference would

miss the additional phase difference acquired before the slits.

Step 2: Apply the first dark-fringe condition For minimum positive d, set $\Delta = \lambda/2$. Then

$$\sqrt{D^2 + d^2} = D + \frac{\lambda}{4}, \qquad d^2 = \frac{D\lambda}{2} + \frac{\lambda^2}{16}.$$

With $D = 0.2\,\mathrm{m}$ and $\lambda = 400 \times 10^{-9}\,\mathrm{m}$,

$$d^2 = 4.0 \times 10^{-8} + 1.0 \times 10^{-14}\,\mathrm{m}^2.$$

Thus $d \simeq 2.0 \times 10^{-4}\,\mathrm{m} = 0.20\,\mathrm{mm}$. Since $d/D \simeq 10^{-3}$, the equivalent small-angle formula $\Delta \simeq d^2/D$ gives the same precision. Higher odd-half-wavelength differences require larger separations.

Step 3: Conclusion The minimum slit separation is $\boxed{0.20\,\mathrm{mm}}$; the requested numerical value is 0.20.

Quick Tip to Solve: When the source is off the perpendicular bisector, add the path differences before and after the slits before applying the dark-fringe condition.

52. A cylinder is rolling down on an inclined plane of inclination $60°$. It's acceleration during rolling down will be $\frac{x}{\sqrt{3}} m/s^2$, where $x =$ _____ (use $\mathrm{g} = 10\ \mathrm{m/s^2}$). [4 marks]

Solution:

Concept:

- For pure rolling of a uniform solid cylinder, $I = \frac{1}{2}MR^2$, where M is its mass and R its radius. Translational and rotational accelerations are related by $a = \alpha R$.
- Static friction supplies the torque about the centre. It acts up the incline during downward rolling and does not dissipate mechanical energy at a stationary point of contact.

Step 1: Combine translation and rotation Take the direction down the slope as positive. The component of gravity along the slope is $Mg\sin\theta$, while friction f points uphill. Newton's second law and the torque equation give

$$Mg\sin\theta - f = Ma, \qquad fR = I\alpha.$$

With $\alpha = a/R$, the friction is $f = Ia/R^2$. Substitution yields

$$Mg\sin\theta = a\left(M + \frac{I}{R^2}\right), \qquad a = \frac{g\sin\theta}{1 + I/(MR^2)}.$$

This is smaller than the acceleration of a frictionless sliding particle because part of the gravitational energy produces rotation.

Step 2: Substitute the cylinder inertia and compare coefficients For the cylinder, $I/(MR^2) = 1/2$. Therefore

$$a = \frac{10\sin 60°}{1 + 1/2} = \frac{10(\sqrt{3}/2)}{3/2} = \frac{10\sqrt{3}}{3} = \frac{10}{\sqrt{3}}\,\mathrm{m\,s^{-2}}.$$

The problem writes the acceleration as $x/\sqrt{3}\,\mathrm{m\,s^{-2}}$. Comparing the coefficients of $1/\sqrt{3}$ gives $x = 10$. The mass and radius cancel, as expected for bodies of the same shape rolling on a fixed incline. Using $I = MR^2$ would describe a hollow cylindrical hoop rather than the solid cylinder used here.

Step 3: Conclusion The acceleration is $\boxed{10/\sqrt{3}\,\mathrm{m\,s^{-2}}}$ down the plane, so $\boxed{x = 10}$ (dimensionless).

Quick Tip to Solve: For a solid cylinder, remember the factor $1+I/(MR^2) = 3/2$; divide the sliding acceleration by this factor before comparing with the requested form.

---

53. An electron is moving under the influence of the electric field of a uniformly charged infinite plane sheet S having surface charge density $+\sigma$. The electron at $t = 0$ is at a distance of 1 m from S and has a speed of 1 m/s. The maximum value of σ if the electron strikes S at $t = 1$ s is $\alpha\left[\frac{m\epsilon_0}{e}\right]\frac{C}{m^2}$, the value of α is _____. [4 marks]

Solution:

Concept:

- A uniformly charged infinite nonconducting plane sheet produces an electric field of magnitude $E = \sigma/(2\epsilon_0)$ on either side. The factor is two, unlike the field immediately outside a conducting surface.
- An electron of charge $-e$ accelerates toward a positively charged sheet. Its acceleration magnitude is constant: $a = e\sigma/(2m\epsilon_0)$. Only the initial velocity component perpendicular to the sheet affects the time of impact.

Step 1: Express the acceleration in terms of initial normal velocity Choose the direction away from the sheet as positive, and let u be the initial velocity component in that direction. Initially the normal distance is $y_0 = 1\,\mathrm{m}$; at time t,

$$y(t) = 1 + ut - \frac{1}{2}at^2.$$

At impact $t = 1\,\mathrm{s}$, the condition $y(1) = 0$ becomes

$$0 = 1 + u - \frac{1}{2}a, \qquad a = 2(1 + u),$$

where the numerical substitution uses SI units. Since the full initial speed is $1\,\mathrm{m\,s^{-1}}$, the normal component satisfies $-1 \le u \le 1\,\mathrm{m\,s^{-1}}$. Any tangential component decreases the largest possible outward normal component.

Step 2: Maximise the permitted charge density The surface charge density is proportional to a, so it is largest when the electron initially moves directly away from the sheet: $u = +1\,\mathrm{m\,s^{-1}}$. This gives $a_{\max} = 4\,\mathrm{m\,s^{-2}}$. The trajectory is then $y(t) = 1 + t - 2t^2 = (1 - t)(1 + 2t)$, which is positive for $0 \le t < 1$; thus the first impact really occurs at one second. Using the electric-force equation,

$$\frac{e\sigma_{\max}}{2m\epsilon_0} = 4, \qquad \sigma_{\max} = 8\frac{m\epsilon_0}{e}\,\mathrm{C\,m^{-2}}$$

in the numerical SI form requested.

Step 3: Conclusion The coefficient in the maximum surface charge density is $\boxed{\alpha = 8}$ (dimensionless). The maximum occurs for an initially outward normal velocity.

Quick Tip to Solve: To maximise the attractive field for a fixed arrival time, give the electron its largest allowed initial velocity away from the sheet, then use the normal displacement equation.

54. When a hydrogen atom going from $n = 2$ to $n = 1$ emits a photon, its recoil speed is $\frac{x}{5}$ m/s. Where $x =$ _____. (Use, mass of hydrogen atom $= 1.6 \times 10^{-27}$ kg) [4 marks]

Solution:

Concept:

- The hydrogen energy levels are $E_n = -13.6/n^2\,\text{eV}$. The energy released in a downward transition is the positive difference between the initial and final atomic energies.
- A photon has momentum $p_\gamma = E_\gamma/c$. For an atom initially at rest, momentum conservation gives the atom equal and opposite recoil momentum: $Mv = p_\gamma$.

Step 1: Calculate the transition energy For the transition from $n = 2$ to $n = 1$,

$$E_\gamma \simeq 13.6\left(1 - \frac{1}{2^2}\right) = 13.6 \times \frac{3}{4} = 10.2\,\text{eV}.$$

In SI units, using $1\,\text{eV} = 1.6 \times 10^{-19}$ J, this is

$$E_\gamma = 10.2 \times 1.6 \times 10^{-19} = 1.632 \times 10^{-18}\,\text{J}.$$

The small part of the transition energy given to atomic recoil is negligible at the precision of the supplied atomic mass; treating all the transition energy as photon energy is the usual nonrelativistic approximation for hydrogen spectral lines.

Step 2: Use momentum conservation to find the recoil speed The emitted photon carries momentum

$$p_\gamma = \frac{1.632 \times 10^{-18}}{3.0 \times 10^8} = 5.44 \times 10^{-27}\,\text{kg m s}^{-1}.$$

For $M = 1.6 \times 10^{-27}$ kg,

$$v = \frac{p_\gamma}{M} = \frac{5.44 \times 10^{-27}}{1.6 \times 10^{-27}} = 3.4\,\text{m s}^{-1}.$$

This is the speed, with recoil directed opposite to the photon. Comparing with $v = x/5\,\text{m s}^{-1}$ gives $x = 5(3.4) = 17$. As a check, the recoil kinetic energy $Mv^2/2$ is only about 9.2×10^{-27} J, far below the transition energy, justifying the approximation.

Step 3: Conclusion The recoil speed is $\boxed{3.4\,\text{m s}^{-1}}$, opposite to the photon, and the requested coefficient is $\boxed{x = 17}$ (dimensionless).

Quick Tip to Solve: Use photon momentum E/c, not $\sqrt{2mE}$; the latter formula applies to a nonrelativistic particle with rest mass, not a photon.

55. A square loop of side $10\ \text{cm}$ and resistance 0.7Ω is placed vertically in east-west plane. A uniform magnetic field of $0.20T$ is set up across the plane in north east direction. The magnetic field is decreased to zero in $1\ \text{s}$ at a steady rate. Then, magnitude of induced emf is $\sqrt{x} \times 10^{-3}\ \text{V}$. The value of x is _____. [4 marks]

Solution:

Concept:

- The magnetic flux through a plane loop is $\Phi = BA\cos\theta$, where θ is the angle between the magnetic field and the normal to the loop, rather than the loop plane.
- Faraday's law gives the magnitude of the induced emf in a single-turn loop: $|\mathcal{E}| = |\Delta\Phi|/\Delta t$ for a uniform rate of change. Resistance is needed for induced current, but not for the emf.

Step 1: Identify the normal component of the field The square loop is vertical in the east-west plane, so its horizontal normal is along the north-south direction. A field pointing north-east makes an angle of 45° with this normal. Thus only the northward component $B\cos 45^\circ = B/\sqrt{2}$ threads the loop. The eastward component lies within its plane and contributes no flux. The side length is $10\,\text{cm} = 0.10\,\text{m}$, giving

$$A = (0.10)^2 = 1.0 \times 10^{-2}\,\text{m}^2.$$

The loop is stationary, so the flux changes only because the field decreases.

Step 2: Calculate the emf and compare with the given expression The field falls steadily from $0.20\,\text{T}$ to zero in $1\,\text{s}$. Hence

$$|\mathcal{E}| = \frac{A\cos 45^\circ\,|\Delta B|}{\Delta t} = \frac{(1.0 \times 10^{-2})(0.20)}{\sqrt{2}(1)}\,\text{V}.$$

Simplifying,

$$|\mathcal{E}| = \frac{2 \times 10^{-3}}{\sqrt{2}} = \sqrt{2} \times 10^{-3}\,\text{V}.$$

The problem writes this as $\sqrt{x} \times 10^{-3}\,\text{V}$, so $\sqrt{x} = \sqrt{2}$ and $x = 2$. Dividing by the resistance $0.7\,\Omega$ would instead give current in amperes, which is not the quantity requested. Lenz's law fixes the polarity; only the magnitude is needed here.

Step 3: Conclusion The induced emf has magnitude $\boxed{\sqrt{2} \times 10^{-3}\,\text{V}}$, so $\boxed{x = 2}$ (dimensionless).

Quick Tip to Solve: Draw the loop normal mentally before taking the cosine. A north-east field has equal north and east components, but only the component normal to the east-west plane contributes.

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56. The magnetic potential due to a magnetic dipole at a point on its axis situated at a distance of $20\ \text{cm}$ from its center is $1.5 \times 10^{-5}\ \text{T m}$. The magnetic moment of the dipole is _____ $A\ \text{m}^2$. (Given : $\frac{\mu_o}{4\pi} = 10^{-7}\text{Tm}A^{-1}$) [4 marks]

Solution:

Concept:

- The scalar magnetic potential of a dipole is $V_m = \mu_0\mathcal{M}\cos\theta/(4\pi r^2)$, where $\mathcal{M}$ is the dipole magnetic moment and r is the distance from its centre.

- On the dipole axis, the magnitude has $|\cos\theta| = 1$, so $|V_m| = (\mu_0/4\pi)\mathcal{M}/r^2$. The potential is measured in tesla metres, while magnetic field is measured in tesla.

Step 1: Identify the axial potential relation The observation point is on the axis at $20\,\mathrm{cm}$, so convert the distance to SI units:

$$r = 20 \times 10^{-2} = 0.20\,\mathrm{m}, \qquad r^2 = 0.040\,\mathrm{m}^2.$$

The supplied quantity $1.5 \times 10^{-5}\,\mathrm{T\,m}$ is magnetic potential, not magnetic field. Therefore use the inverse-square potential relation rather than the inverse-cube axial field relation. On the axis the magnitude is

$$V_m = \frac{\mu_0}{4\pi}\frac{\mathcal{M}}{r^2}.$$

The sign of the potential depends on which axial side is chosen; its given positive magnitude determines the magnitude of the dipole moment.

Step 2: Solve for the magnetic moment Rearranging before substitution gives

$$\mathcal{M} = \frac{V_m r^2}{\mu_0/(4\pi)}.$$

Insert the stated values:

$$\mathcal{M} = \frac{(1.5 \times 10^{-5})(0.20)^2}{10^{-7}} = \frac{6.0 \times 10^{-7}}{10^{-7}} = 6.0\,\mathrm{A\,m}^2.$$

The units also provide a check: $(\mathrm{T\,m})(\mathrm{m}^2)/(\mathrm{T\,m\,A}^{-1}) = \mathrm{A\,m}^2$. Substituting $6.0\,\mathrm{A\,m}^2$ back into the potential formula gives $10^{-7} \times 6.0/0.040 = 1.5 \times 10^{-5}\,\mathrm{T\,m}$, exactly the specified potential.

Step 3: Conclusion The magnetic moment has magnitude $\boxed{6\,\mathrm{A\,m}^2}$; the requested numerical value is 6.

Quick Tip to Solve: Check the units before choosing a dipole formula: magnetic potential has units $\mathrm{T\,m}$ and varies as r^{-2}, whereas axial magnetic field varies as r^{-3}.

57. A 16Ω wire is bend to form a square loop. A $9\ \mathrm{V}$ battery with internal resistance 1Ω is connected across one of its sides. If a $4\mu F$ capacitor is connected across one of its diagonals, the energy stored by the capacitor will be $\frac{x}{2}\mu J$, where $x =$ _____ [4 marks]

Solution:

Concept:

- A capacitor in a steady direct-current circuit draws no current. Its final voltage equals the potential difference between its two connection points.
- For resistors in parallel, $R_{\mathrm{eq}}^{-1} = R_1^{-1} + R_2^{-1}$. The battery internal resistance is in series with this external equivalent resistance.
- The electrostatic energy stored is $U = \frac{1}{2}CV^2$, where C is capacitance and V is the diagonal voltage.

Step 1: Find the resistance seen by the battery Label the square vertices successively A, B, C and D, with the battery connected across AB and the capacitor across AC. The uniform wire has total

resistance $16\,\Omega$, so each side has resistance $4\,\Omega$. Between A and B there are two paths: the single side AB and the three remaining sides in series. Therefore

$$R_{\text{ext}} = \frac{4 \times 12}{4 + 12} = 3\,\Omega.$$

Including the internal resistance gives $R_{\text{total}} = 3 + 1 = 4\,\Omega$.

Step 2: Find the voltage across the diagonal The current delivered by the battery is $I = 9/4\,\text{A}$, and its terminal voltage is $V_{AB} = IR_{\text{ext}} = 27/4\,\text{V}$. The current through the three-side path A-D-C-B is

$$I_{\text{long}} = \frac{27/4}{12} = \frac{9}{16}\,\text{A}.$$

The diagonal AC spans two of those equal sides. Consequently $V_{AC} = I_{\text{long}}(4 + 4) = 9/2\,\text{V}$. The other diagonal has the same voltage magnitude by symmetry.

Step 3: Calculate the energy and identify the coefficient Use the diagonal voltage, rather than the battery emf, in the capacitor energy formula:

$$U = \frac{1}{2}(4 \times 10^{-6})\left(\frac{9}{2}\right)^2 = 40.5 \times 10^{-6}\,\text{J} = \frac{81}{2}\,\mu\text{J}.$$

Comparing with the specified form $x/2\,\mu\text{J}$ determines the requested number.

Step 4: Conclusion The stored energy is $40.5\,\mu\text{J}$, so the required dimensionless coefficient is $\boxed{x = 81}$.

Quick Tip to Solve: First remove the capacitor for the steady-state resistor calculation; then find the voltage between its actual terminals, not the battery voltage.

58. When the displacement of a simple harmonic oscillator is one third of its amplitude, the ratio of total energy to the kinetic energy is $\frac{x}{8}$, where $x =$ _____. [4 marks]

Solution:

Concept:

- For a simple harmonic oscillator of mass m, angular frequency ω and amplitude A, total mechanical energy is $E = \frac{1}{2}m\omega^2A^2$.
- At displacement y from equilibrium, potential energy is $U = \frac{1}{2}m\omega^2y^2$, and kinetic energy is $K = E - U$.
- The energy ratios depend on the squared fractional displacement $(y/A)^2$, so the sign of the displacement does not affect this calculation.

Step 1: Write the displacement as a fraction of amplitude To distinguish the displacement from the coefficient called x in the question, denote displacement by y. We are given $|y| = A/3$. Thus $y^2/A^2 = 1/9$. At this position the oscillator has not reached a turning point, so both potential and kinetic energies are nonzero. The total energy remains the same throughout the undamped motion.

Step 2: Find the fraction of energy that is kinetic Dividing the potential energy by total energy cancels

the common factor $\frac{1}{2}m\omega^2$:

$$\frac{U}{E} = \frac{y^2}{A^2} = \frac{1}{9}.$$

The remaining energy is kinetic, giving

$$\frac{K}{E} = 1 - \frac{1}{9} = \frac{8}{9}.$$

This is also obtained from $v^2 = \omega^2(A^2 - y^2)$, followed by $K = \frac{1}{2}mv^2$.

Step 3: Invert the fraction in the requested order The question asks for total energy divided by kinetic energy, rather than kinetic energy divided by total energy. Hence

$$\frac{E}{K} = \frac{1}{8/9} = \frac{9}{8}.$$

Comparison with $x/8$ yields $x = 9$. The ratio exceeds unity, as it must because total energy includes the positive potential-energy contribution at this position.

Step 4: Conclusion The ratio of total to kinetic energy is $\boxed{9/8}$. Therefore the requested dimensionless number is $\boxed{x = 9}$.

Quick Tip to Solve: Compute $K/E = 1 - (y/A)^2$ first, then invert only if the requested ratio has total energy in the numerator.

59. A ball rolls off the top of a stairway with horizontal velocity u. The steps are 0.1 m high and 0.1 m wide. The minimum velocity u with which that ball just hits the step 5 of the stairway will be $\sqrt{x}$ ms^{-1} where $x =$ _____ [use $g = 10\ \mathrm{m/s}^2$]. [4 marks]

Solution:

Concept:

- Take the launch point as origin, horizontal distance as s, and downward displacement as y. A horizontal projectile obeys $s = ut$ and $y = \frac{1}{2}gt^2$.
- Eliminating time gives the trajectory $y = gs^2/(2u^2)$. The limiting speed for reaching a later step is found by just clearing the last obstructing corner.
- Each step has height $h = 0.1\,\mathrm{m}$ and width $b = 0.1\,\mathrm{m}$; the fourth outer corner is at $(4b, 4h)$.

Step 1: Locate the limiting corner before the fifth step The fifth tread extends horizontally from $s = 4b$ to $s = 5b$ at depth $5h$. To hit it, the projectile must first pass the fourth step. At the minimum-speed boundary it just grazes the fourth outer corner, whose coordinates are $s = 0.4\,\mathrm{m}$, $y = 0.4\,\mathrm{m}$. A slightly smaller speed makes the downward displacement at that horizontal location larger, so an earlier step intercepts the ball.

Step 2: Apply the projectile equation at that corner At the limiting corner,

$$4h = \frac{g(4b)^2}{2u^2}.$$

Rearranging before substitution avoids confusing the step number with the requested coefficient:

$$u^2 = \frac{2gb^2}{h} = \frac{2(10)(0.1)^2}{0.1} = 2\,\mathrm{m^2 s^{-2}}.$$

For the earlier corners (nb, nh), $n < 4$, this trajectory has $y = 0.025n^2\,\mathrm{m} < 0.1n\,\mathrm{m}$, so it clears them.

Step 3: Check that the next tread is reached At the limiting speed, reaching depth $5h = 0.5\,\mathrm{m}$ requires

$$s = \sqrt{\frac{2u^2(5h)}{g}} = \sqrt{0.2}\,\mathrm{m} \approx 0.447\,\mathrm{m}.$$

This lies between 0.4 m and 0.5 m, exactly within the fifth tread. Thus the limiting horizontal speed has the required fifth-step landing geometry.

Step 4: Conclusion The minimum limiting speed is $\boxed{u = \sqrt{2}\,\mathrm{m\,s^{-1}}}$. In the requested form, $\boxed{x = 2}$ (dimensionless).

Quick Tip to Solve: To find the minimum speed for hitting step n, test the obstructing corner of step $n - 1$, then check that the landing lies on tread n.

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60. In a test experiment on a model aeroplane in wind tunnel, the flow speeds on the upper and lower surfaces of the wings are $70\ \mathrm{ms^{-1}}$ and $65\ \mathrm{ms^{-1}}$ respectively. If the wing area is $2\ \mathrm{m^2}$, the lift of the wing is _____ N.

(Given density of air $= 1.2\ \mathrm{kg\ m^{-3}}$) [4 marks]

Solution:

Concept:

- For steady incompressible flow at effectively equal heights, Bernoulli's equation is $p + \frac{1}{2}\rho v^2 =$ constant.
- The faster upper-surface flow has lower static pressure. The difference $p_{\text{lower}} - p_{\text{upper}}$ produces an upward lift on the wing.
- For a uniform pressure difference over wing area A, the lift is $F = (p_{\text{lower}} - p_{\text{upper}})A$.

Step 1: Relate the two surface pressures Apply Bernoulli's equation to the upper and lower airstreams, neglecting their small height difference in this wing model:

$$p_{\text{upper}} + \frac{1}{2}\rho v_{\text{upper}}^2 = p_{\text{lower}} + \frac{1}{2}\rho v_{\text{lower}}^2.$$

Rearranging gives

$$p_{\text{lower}} - p_{\text{upper}} = \frac{1}{2}\rho(v_{\text{upper}}^2 - v_{\text{lower}}^2).$$

The order of the two squared speeds is important: it gives a positive upward pressure difference.

Step 2: Evaluate the pressure difference Insert $\rho = 1.2\,\mathrm{kg\,m^{-3}}$, $v_{\text{upper}} = 70\,\mathrm{m\,s^{-1}}$, and $v_{\text{lower}} =$

$65\,\mathrm{m\,s^{-1}}$:

$$\Delta p = \frac{1}{2}(1.2)(70^2 - 65^2).$$

Using the difference of squares, $70^2 - 65^2 = (70-65)(70+65) = 675\,\mathrm{m^2 s^{-2}}$. Therefore $\Delta p = 0.6(675) = 405\,\mathrm{Pa}$.

Step 3: Multiply by the stated wing area The pressure difference acts over the given area $A = 2\,\mathrm{m^2}$, so

$$F = \Delta p A = 405 \times 2 = 810\,\mathrm{N}.$$

No additional factor of two is needed for the two surfaces: their opposing pressure forces have already been combined in Δp. The result is positive upward because the lower pressure acts on the upper surface.

Step 4: Conclusion The lift of the wing is $\boxed{810\,\mathrm{N}}$, directed upward.

Quick Tip to Solve: Use the difference of the squared speeds, not the square of their difference; multiply the resulting pressure difference by area once.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Identify the incorrect pair from the following : [4 marks]

(A) Fluoroapatite $-3Ca_3(PO_4)_2 \cdot CaF_2$

(B) Carnallite $-KCl \cdot MgCl_2 \cdot 6H_2O$

(C) Cryolite $-Na_3AlF_6$

(D) Fluorspar $-BF_3$

Correct Answer: (D) Fluorspar $-BF_3$

Solution:

Concept:

- A mineral name must be matched with its characteristic composition; a fluorine-containing compound is not automatically the mineral fluorspar.
- Fluorspar is calcium fluoride, CaF_2, whereas BF_3 is the molecular compound boron trifluoride.

Step 1: Check the first three mineral formulae Fluoroapatite contains calcium phosphate and calcium fluoride, conventionally written as $3\,Ca_3(PO_4)_2 \cdot CaF_2$, equivalently $Ca_{10}(PO_4)_6F_2$. Thus pair (A) is correct. Carnallite is the hydrated double salt $KCl \cdot MgCl_2 \cdot 6\,H_2O$, so (B) is also correct. Cryolite is sodium hexafluoroaluminate, Na_3AlF_6, which confirms pair (C). These are standard mineral compositions rather than interchangeable names for fluoride salts.

Step 2: Identify the incorrect name-formula pairing Fluorspar is composed of calcium and fluoride ions, in the charge-balanced formula CaF_2: one Ca^{2+} ion is balanced by two F^- ions. The formula printed in (D), BF_3, instead contains boron and fluorine and represents boron trifluoride. It contains no calcium, so it cannot describe fluorspar. The task asks for the incorrect pair, which is why the three correctly matched mineral formulae must be rejected as answers.

Step 3: Keep the selection tied to the question wording There is exactly one mismatch among the four entries. Correcting only that entry gives fluorspar–CaF_2; no change is needed to fluoroapatite, carnallite, or cryolite. The fluoride content common to several entries does not make them equivalent minerals.

Step 4: Conclusion The incorrect pair is fluorspar–BF_3; fluorspar is actually CaF_2. Hence, the correct option is (D).

Quick Tip to Solve: Remember fluorspar as the simple calcium fluoride mineral; distinguish it from cryolite, which contains both sodium and aluminium.

62. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : The first ionisation enthalpy decreases across a period.

Reason R : The increasing nuclear charge outweighs the shielding across the period.

In the light of the above statements, choose the most appropriate from the options given below :

[4 marks]

(A) **A** is false but **R** is true

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) Both **A** and **R** are true and **R** is the correct explanation of **A**

(D) **A** is true but **R** is false

Correct Answer: (A) **A** is false but **R** is true

Solution:

Concept:

- First ionisation enthalpy is the energy required to remove the outermost electron from an isolated gaseous atom: $M(g) \longrightarrow M^+(g) + e^-$.
- Across a period, nuclear charge rises while electrons are added mainly to the same shell. Their incomplete shielding increases effective nuclear attraction.
- A stronger attraction and smaller atomic size generally increase the energy needed for electron removal, with familiar subshell and pairing exceptions.

Step 1: Evaluate Assertion A The assertion claims that first ionisation enthalpy decreases across a period. The general left-to-right trend is an increase, because outer electrons are bound more strongly as effective nuclear charge rises and atomic radius decreases. The assertion therefore reverses the general periodic trend and is false. Local exceptions, such as the effect of beginning a p subshell or electron pairing, do not convert this overall trend into a decrease.

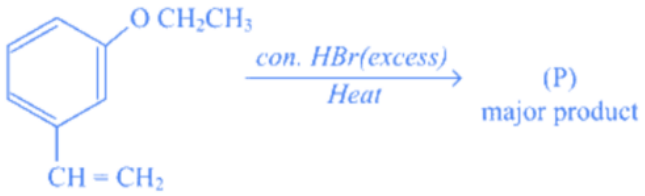
Step 2: Evaluate Reason R The reason states that increasing nuclear charge outweighs shielding across the period. This is true: added electrons enter the same principal shell, so their shielding of one another is relatively weak compared with the increased nuclear charge. Consequently the effective attraction experienced by the valence electron rises. This reasoning supports increasing ionisation enthalpy and thus directly exposes the mistake in the assertion.

Step 3: Match the truth values to the options We have A false and R true. Option (A) states exactly this combination. Options (B) and (C) both require the assertion to be true, so both are excluded. Option (D) requires a true assertion and false reason, reversing both results. Since the assertion is false, the reason cannot be its correct explanation even though the reason itself is valid.

Step 4: Conclusion Assertion A is false, while Reason R is true. Hence, the correct option is (A).

Quick Tip to Solve: For an assertion-reason question, decide the truth of each statement separately before testing the explanatory link.

63. The major product(P) in the following reaction is [4 marks]



(A) 3-(2-bromoethyl)phenol

(B) 1-bromo−3-(1-bromoethyl)benzene

(C) 3-(1-bromoethyl)phenol

(D) 1-bromo−3-(2-bromoethyl)benzene

Correct Answer: (C) 3-(1-bromoethyl)phenol

Solution:

Concept:

- Hot concentrated HBr cleaves an aryl alkyl ether at the alkyl–oxygen bond: the aryl–oxygen bond is resistant to substitution.
- Without a peroxide reagent, HBr adds to a styrene side chain in the Markovnikov direction through the resonance-stabilised benzylic carbocation.
- With excess reagent, both the ether group and the alkene side chain must be considered; the relative ring substitution positions remain unchanged.

Step 1: Cleavage gives a phenolic group The starting material has an ethoxy group and a vinyl group at meta positions on the benzene ring. Protonation of the ether oxygen facilitates attack by Br^- at the primary ethyl carbon. Cleavage therefore gives the phenol and C_2H_5Br. It does not replace the aryl oxygen by bromine: the aryl carbon cannot undergo the ordinary substitution pathway, and the aryl–oxygen bond has partial double-bond character.

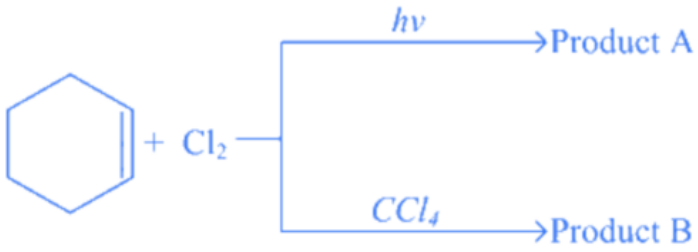
Step 2: Add hydrogen bromide to the vinyl side chain For the substituent $Ar-CH=CH_2$, addition of a proton to the terminal carbon forms the more stable benzylic carbocation $Ar-CH^+-CH_3$. Bromide then attacks that carbon, giving $Ar-CH(Br)-CH_3$. Thus the final ring substituents are OH and $CH(Br)CH_3$, still meta to each other. No peroxide is specified, so the anti-Markovnikov terminal bromide is not the major product.

Step 3: Compare the four printed structures Option (C) has both required changes. Option (A) has the correct phenolic group but puts bromine at the terminal carbon, contrary to the ionic addition. Options (B) and (D) replace the ring ether oxygen with bromine, which is not the outcome of aryl alkyl ether cleavage; (D) also gives the wrong addition orientation.

Step 4: Conclusion The major product is 3-(1-bromoethyl)phenol, with meta OH and $CH(Br)CH_3$ groups. Hence, the correct option is (C).

Quick Tip to Solve: For a substrate bearing ether and alkene groups, apply ether cleavage and alkene addition separately, then combine the two unchanged ring positions.

64. Identify product A and product B : [4 marks]



(A) A: 3,6-dichlorocyclohex-1-ene; B: 1,2-dichlorocyclohexane.

(B) A: 1,2-dichlorocyclohex-1-ene; B: 1,4-dichlorocyclohexane.

(C) A: 1,2-dichlorocyclohexane; B: 1,2-dichlorocyclohexane.

(D) A: 3,6-dichlorocyclohex-1-ene; B: 3,6-dichlorocyclohex-1-ene.

Correct Answer: (A) A: 3,6-dichlorocyclohex-1-ene; B: 1,2-dichlorocyclohexane.

Solution:

Concept:

- Chlorination depends on conditions: irradiation initiates radical substitution at allylic carbon atoms, whereas chlorine in CCl_4 adds electrophilically across a carbon–carbon double bond.
- An allylic carbon is adjacent to the double bond. Radical substitution replaces its hydrogen with chlorine while retaining the alkene; addition consumes the double bond and installs chlorine on both of its carbon atoms.

Step 1: Identify product A under irradiation In cyclohexene the two positions next to the double bond are allylic. The product A drawn among the alternatives has substitution at both these positions, giving 3,6-dichlorocyclohex-1-ene. Its carbon–carbon double bond remains in the ring. This distinguishes it from a saturated vicinal dichloride and from chlorine attached directly to the two alkene carbons.

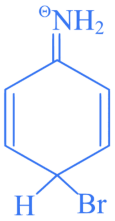
Step 2: Identify product B and compare the alternatives In CCl_4, the pi bond reacts with chlorine by addition, so the original two alkene carbons each acquire one chlorine. Product B is 1,2-dichlorocyclohexane. Only option (A) combines the allylic substitution product shown for A with this vicinal addition product B. Option (B) has vinylic chlorine and a 1,4-dichloride; option (C) assigns addition under both conditions; option (D) retains the double bond in B, contradicting addition.

Step 3: Conclusion A is 3,6-dichlorocyclohex-1-ene and B is 1,2-dichlorocyclohexane, as paired in the printed alternatives. Hence, the correct option is (A).

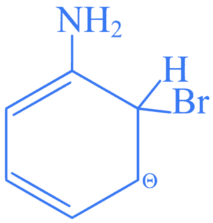
Quick Tip to Solve: First inspect whether the double bond survives: light favours allylic substitution, while chlorine in an inert solvent gives a vicinal dichloride.

65. The arenium ion which is not involved in the bromination of aniline is _____. [4 marks]

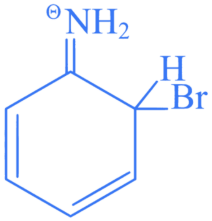
(A) Para attack: NH_2^+ double-bonded to C1; double bonds C2=C3 and C5=C6; H and Br bonded to C4.



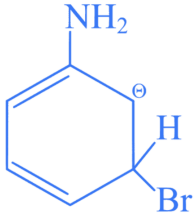
(B) Ortho attack: NH_2 bonded to C1; double bonds C1=C6 and C4=C5; positive charge at C3; H and Br bonded to C2.



(C) Ortho attack: NH_2^+ double-bonded to C1; double bonds C3=C4 and C5=C6; H and Br bonded to C2.

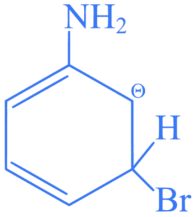


(D) Meta attack: NH_2 bonded to C1; double bonds C1=C6 and C4=C5; positive charge at C2; H and Br bonded to C3.



Correct Answer: (D)

Meta attack: NH_2 bonded to C1; double bonds C1=C6 and C4=C5; positive charge at C2; H and Br bonded to C3.



Solution:

Concept:

- The amino group donates its nitrogen lone pair into the aromatic ring by resonance. Aniline is therefore strongly activated towards electrophilic substitution at its ortho and para positions.
- An arenium ion is the nonaromatic sigma complex formed after electrophilic attack. For ortho or para attack, resonance can place the positive charge on the carbon bonded to nitrogen, allowing extra stabilization by nitrogen donation.

Step 1: Locate the carbon carrying bromine Number the carbon bearing NH_2 as C1. Bromine bonded to C2 corresponds to ortho attack, bromine at C4 to para attack, and bromine at C3 to meta attack. The incoming bromine and the retained hydrogen lie on the same tetrahedral carbon in each sigma complex; this carbon is the reliable way to identify the substitution position.

Step 2: Separate the stabilized and unstabilized pathways Option (A) has H and Br on C4 and a nitrogen-positive resonance contributor, so it belongs to para bromination. Options (B) and (C) both have H and Br on C2; they are ring-positive and nitrogen-positive contributors of the ortho sigma complex. Option (D) instead puts H and Br on C3, so it represents meta attack. This pathway lacks the extra nitrogen-donation stabilization of the ortho and para intermediates and is not the arenium ion involved in the usual bromination of aniline.

Step 3: Conclusion The meta-attack sigma complex in option (D) is not involved in the ortho/para bromination pathway of aniline. Hence, the correct option is (D).

Quick Tip to Solve: Identify the carbon bearing both H and Br before following the resonance bonds; it fixes whether the sigma complex arose from ortho, meta or para attack.

66. Given below are two statements :

Statement I : The electronegativity of group 14 elements from Si to Pb, gradually decreases.

Statement II : Group 14 contains non-metallic, metallic, as well as metalloid elements.

In the light of the above statements, choose the most appropriate from the options given below :

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Both Statement I and Statement II are false

(D) Statement I is true but Statement II is false

Correct Answer: (A) Statement I is false but Statement II is true

Solution:

Concept:

- Electronegativity measures the attraction of a bonded atom for the shared electron pair. Down-group trends must account for shielding and effective nuclear attraction, and need not be monotonic in the heavier p-block elements.
- Group 14 contains carbon, silicon, germanium, tin and lead. Its members span nonmetallic,

metalloid and metallic behaviour, with metallic character generally increasing down the group.

Step 1: Evaluate Statement I using the actual trend From silicon to lead, electronegativity does not gradually decrease. Representative Pauling values are approximately $\mathrm{Si} = 1.90$, $\mathrm{Ge} = 2.01$, $\mathrm{Sn} = 1.96$, and $\mathrm{Pb} = 2.33$. The rise from Si to Ge already contradicts a steady fall, and Pb is more electronegative than Sn. Thus, the simple size-based decrease is not a valid description of these four elements, and Statement I is false.

Step 2: Evaluate Statement II and select the truth combination Carbon is a nonmetal; silicon and germanium are metalloids; tin and lead are metals. These examples establish every category named in Statement II, so it is true. Option (A) states precisely this false/true combination. Option (B) incorrectly accepts the monotonic electronegativity claim; option (C) incorrectly rejects the diversity of group 14; option (D) reverses both truth values.

Step 3: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (A).

Quick Tip to Solve: For heavier p-block groups, check the actual electronegativity sequence before applying a general down-group rule.

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67. Chlorine undergoes disproportionation in alkaline medium as shown below:

$$\mathrm{aCl_2(g) + bOH^-(aq) \longrightarrow cClO^-(aq) + dCl^-(aq) + eH_2O\,(l)}$$

The values of a, b, c and d in a balanced redox reaction are respectively: [4 marks]

(A) 3, 4, 4 and 2

(B) 1, 2, 1 and 1

(C) 2, 4, 1 and 3

(D) 2, 2, 1 and 3

Correct Answer: (B) 1, 2, 1 and 1

Solution:

Concept:

- Disproportionation converts one element in a single oxidation state into both a higher and a lower oxidation state. In $\mathrm{Cl_2}$, chlorine is initially in oxidation state zero.
- In $\mathrm{ClO^-}$, oxygen is -2, so chlorine is $+1$; in $\mathrm{Cl^-}$, chlorine is -1. Equal one-electron oxidation and reduction changes therefore require one chlorine atom in each product.

Step 1: Balance chlorine and electron transfer One chlorine molecule supplies two chlorine atoms. One atom loses an electron on becoming hypochlorite and the other gains that electron on becoming chloride. Taking the smallest integral coefficients therefore gives $a = 1$, $c = 1$, and $d = 1$. This equal distribution is specific to the hypochlorite product printed here; it should not be confused with chlorate formation in hot concentrated alkali.

Step 2: Balance oxygen, hydrogen and charge Place two hydroxide ions on the left and one water

molecule on the right:

$$Cl_2(g) + 2\,OH^-(aq) \longrightarrow ClO^-(aq) + Cl^-(aq) + H_2O\,(l).$$

There are two chlorine atoms, two oxygen atoms and two hydrogen atoms on each side; each side has net charge -2. Thus $b = 2$ and $e = 1$. Option (B) gives these coefficients. Option (A) fails chlorine balance, while options (C) and (D) do not give the equal oxidized and reduced chlorine counts required by electron conservation.

Step 3: Conclusion The coefficients are $a, b, c, d = 1, 2, 1, 1$. Hence, the correct option is (B).

Quick Tip to Solve: Balance the oxidation-state changes first: chlorine going from 0 to +1 and from 0 to -1 must occur in equal atom counts.

68. In chromyl chloride test for confirmation of Cl^- ion, a yellow solution is obtained. Acidification of the solution and addition of amyl alcohol and 10% H_2O_2 turns organic layer blue indicating formation of chromium pentoxide. The oxidation state of chromium in that is [4 marks]

(A) +6

(B) +5

(C) +3

(D) +10

Correct Answer: (A) +6

Solution:

Concept:

- The oxidation-state sum of all atoms equals the overall molecular charge. Oxygen is usually -2, but each oxygen in a peroxide linkage $O-O$ has oxidation state -1.
- Blue chromium pentoxide, CrO_5, contains one oxo oxygen and two peroxide groups. Its five oxygen atoms therefore cannot all be assigned oxidation state -2.

Step 1: Assign the oxygen contributions correctly The single oxo oxygen contributes -2. Each of the two peroxide groups contains two oxygen atoms at -1, so the four peroxidic oxygens contribute -4 in total. Thus the combined oxygen contribution is -6. The peroxide groups arise from hydrogen peroxide; their presence is the central feature of the blue chromium pentoxide test.

Step 2: Use charge balance and eliminate alternatives Let the oxidation state of chromium be x. The molecule is neutral, hence

$$x + (-2) + 4(-1) = 0, \qquad x = +6.$$

Option (D), $+10$, incorrectly treats every oxygen as oxide. Options (B) and (C), $+5$ and $+3$, do not balance the combined -6 oxygen contribution. Extraction of the blue species into the organic layer changes its location, but does not change this oxidation-state calculation.

Step 3: Conclusion Chromium has oxidation state $+6$ in CrO_5. Hence, the correct option is (A).

Quick Tip to Solve: Whenever hydrogen peroxide makes a metal peroxide complex, identify the O–O groups before assigning oxygen oxidation states.

69. Type of amino acids obtained by hydrolysis of proteins is : [4 marks]

(A) γ

(B) β

(C) α

(D) δ

Correct Answer: (C) α

Solution:

Concept:

- Proteins are polypeptides: their amino-acid units are linked by peptide or amide bonds, $-\mathrm{CO}-\mathrm{NH}-$. Hydrolysis breaks these bonds by addition of water.
- An alpha-amino acid has its amino group on the carbon immediately adjacent to the carboxyl carbon. Its general neutral formula is $\mathrm{H_2N}-\mathrm{CH(R)}-\mathrm{COOH}$, where R is the side chain.

Step 1: Identify the building blocks released by hydrolysis In a protein, the amino group of one alpha-amino acid condenses with the carboxyl group of another, eliminating water and forming a peptide bond. Reversing this condensation by hydrolysis restores the amino and carboxyl groups. It releases the original amino-acid residues without relocating the amino group along the carbon skeleton.

Step 2: Distinguish alpha from the other positional names In $\mathrm{H_2N}-\mathrm{CH(R)}-\mathrm{COOH}$, the amino-bearing carbon is adjacent to the carboxyl carbon, so it is the alpha carbon. A beta-amino acid would have an extra carbon between that carbon and the carboxyl group; gamma and delta amino acids place the amino group still farther away. These are not the standard monomeric building blocks of proteins. Therefore option (C) identifies the correct type, while options (A), (B) and (D) specify the wrong position of the amino group.

Step 3: Conclusion Protein hydrolysis yields α-amino acids. Hence, the correct option is (C).

Quick Tip to Solve: For protein monomers, look for the amino and carboxyl groups attached to the same alpha carbon, with the side chain represented by R.

70. The correct set of four quantum numbers for the valence electron of rubidium atom $(Z = 37)$ is : [4 marks]

(A) $5, 1, 1, +\frac{1}{2}$

(B) $5, 0, 0, +\frac{1}{2}$

(C) $5, 0, 1, +\frac{1}{2}$

(D) $5, 1, 0, +\frac{1}{2}$

Correct Answer: (B) $5, 0, 0, +\frac{1}{2}$

Solution:

Concept:

- The four quantum numbers are n (shell), l (subshell), m_l (orbital orientation) and m_s (spin). Their restrictions include $l = 0, 1, \ldots, n-1$, $m_l = -l, \ldots, +l$, and $m_s = \pm\frac{1}{2}$.
- An s orbital has $l = 0$, which permits only $m_l = 0$. Rubidium has atomic number 37 and one electron beyond the closed-shell krypton configuration.

Step 1: Write the ground-state electron configuration Krypton contains 36 electrons. The next electron enters the 5s subshell, giving rubidium the configuration $[\mathrm{Kr}]5s^1$. This is the outermost, or valence, electron. The leading 5 fixes $n = 5$, and the letter s fixes $l = 0$; the electron is not in a 5p orbital despite sharing the same principal shell with it.

Step 2: Apply the allowed magnetic and spin values For $l = 0$, the only permitted magnetic quantum number is $m_l = 0$. A single electron may be represented with $m_s = +\frac{1}{2}$, as in the options. The resulting set is

$$(n, l, m_l, m_s) = \left(5, 0, 0, +\frac{1}{2}\right).$$

Options (A) and (D) use $l = 1$, describing p orbitals rather than the occupied 5s orbital. Option (C) sets $m_l = 1$ while $l = 0$, which is forbidden even independently of rubidium configuration.

Step 3: Conclusion The correct set is $5, 0, 0, +\frac{1}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Obtain n and l from the valence configuration first; once the orbital is s, its magnetic quantum number must be zero.

71. The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is [4 marks]

(A) electromeric energy

(B) resonance energy

(C) ionization energy

(D) hyperconjugation energy

Correct Answer: (B) resonance energy

Solution:

Concept:

- A resonance hybrid is the actual electronic structure of a molecule represented by several contributing Lewis structures. The nuclei keep the same arrangement while electrons are delocalised.
- Delocalisation stabilises the actual molecule below the most stable individual contributor. The energy lowering relative to that contributor is called resonance energy.

Step 1: Identify the reference energy The reference specified in the question is the lowest-energy resonance contributor, not the least stable contributor and not an average of their energies. A single Lewis structure localises electrons and cannot describe the full stabilisation due to delocalisation. The actual molecule is one hybrid; it does not rapidly switch between the drawn contributors.

Step 2: Name the stabilisation and reject the alternatives If the actual hybrid has energy E_{hybrid} and the best individual contributor has energy E_{best}, the positive stabilisation magnitude is

$$E_{\text{res}} = E_{\text{best}} - E_{\text{hybrid}}.$$

This is resonance energy. Electromeric effect describes temporary complete displacement of a multiple-bond electron pair under an attacking reagent. Ionisation energy is the energy needed to remove an electron from a gaseous species. Hyperconjugation refers specifically to delocalisation involving a neighbouring sigma bond; it is not the general energy difference defined here.

Step 3: Conclusion The stated difference is the resonance energy, measuring the extra stability of the actual resonance hybrid. Hence, the correct option is (B).

Quick Tip to Solve: For resonance energy, compare the hybrid with its most stable contributor; the hybrid has the lower energy.

72. Match List I with List II.

	List I (Substances)		List II (Element Present)
(A)	Ziegler catalyst	(I)	Rhodium
(B)	Blood Pigment	(II)	Cobalt
(C)	Wilkinson catalyst	(III)	Iron
(D)	Vitamin B_{12}	(IV)	Titanium

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-III, C-I, D-II

(B) A-II, B-IV, C-I, D-III

(C) A-III, B-II, C-IV, D-I

(D) A-II, B-III, C-IV, D-I

Correct Answer: (A) A-IV, B-III, C-I, D-II

Solution:

Concept:

- Identify the central transition-metal element in each named catalyst or biological coordination compound. The ligand names and the functions of the compound are separate from the identity of its metal.
- Ziegler catalysts use titanium compounds, haemoglobin contains iron, Wilkinson catalyst contains rhodium, and vitamin B_{12} contains cobalt.

Step 1: Match the catalysts Ziegler or Ziegler–Natta catalysts conventionally contain a titanium halide, such as $TiCl_4$, with an organoaluminium cocatalyst. Titanium is the element offered in List II, so A matches IV. Wilkinson catalyst is chlorotris(triphenylphosphine)rhodium(I), conventionally written $RhCl(PPh_3)_3$, where Ph denotes phenyl. Its central metal is rhodium, so C matches I. The presence of an aluminium cocatalyst in a Ziegler system does not make any other listed transition metal the correct match.

Step 2: Match the biological compounds and compare every code The blood pigment haemoglobin has an iron-containing haem group; therefore B matches III. Vitamin B_{12}, also called cobalamin, contains cobalt, so D matches II. Combining the four independent matches gives

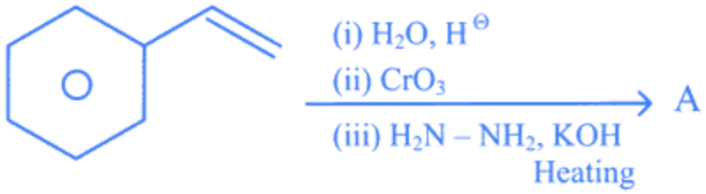
$$A-IV, \quad B-III, \quad C-I, \quad D-II.$$

Only the first code agrees with all four. The second puts cobalt in the Ziegler catalyst and titanium in blood pigment. The third puts iron in the Ziegler catalyst and titanium in Wilkinson catalyst. The fourth likewise assigns the catalysts incorrectly, despite its correct blood-pigment match.

Step 3: Conclusion The complete matching is A-IV, B-III, C-I, D-II. Hence, the correct option is (A).

Quick Tip to Solve: Remember the metal associations Ti–Ziegler, Rh–Wilkinson, Fe–haemoglobin and Co–cobalamin before checking the answer codes.

73. The final product A formed in the following multistep reaction sequence is [4 marks]



(A) $C_6H_5CH_2CH_3$

(B) $C_6H_5CH_2COOH$

(C) $C_6H_5CH_2CH{=}NNH_2$

(D) $C_6H_5C(=NNH_2)CH_3$

Correct Answer: (A) $C_6H_5CH_2CH_3$

Solution:

Concept:

- Acid-catalysed hydration adds water across an alkene by Markovnikov addition. For styrene, the favoured benzylic carbocation places the hydroxyl group on the carbon attached to phenyl.
- Chromium trioxide oxidises a secondary alcohol to a ketone. Hydrazine with potassium hydroxide on heating then reduces a carbonyl group to methylene in the Wolff–Kishner reaction.

Step 1: Hydrate styrene and oxidise the alcohol The starting alkene is styrene, $C_6H_5CH{=}CH_2$. Protonation that produces the benzylic carbocation is favoured because this carbocation is resonance-stabilised by the phenyl group. Water attacks this carbon and loses a proton, giving 1-phenylethanol. Oxidation of this secondary alcohol gives acetophenone:

$$C_6H_5CH{=}CH_2 \xrightarrow{H_2O,\,H^+} C_6H_5CH(OH)CH_3$$

$$C_6H_5CH(OH)CH_3 \xrightarrow{CrO_3} C_6H_5COCH_3.$$

The two-carbon side chain is retained throughout these transformations.

Step 2: Complete the reduction and test the offered products The ketone first reacts with hydrazine to form its hydrazone. Under hot strongly basic conditions, the Wolff–Kishner process removes the carbonyl oxygen and ultimately releases nitrogen, giving the hydrocarbon:

$$C_6H_5COCH_3 \xrightarrow[\text{heat}]{H_2N-NH_2,\,KOH} C_6H_5CH_2CH_3.$$

Thus the final product is ethylbenzene. Option B is phenylacetic acid and would require a different connectivity and oxidation sequence. Option C is an aldehyde-derived hydrazone with the wrong carbon skeleton around the carbonyl-derived carbon. Option D is the acetophenone hydrazone intermediate; the specified base and heating carry the reaction beyond that intermediate to the hydrocarbon.

Step 3: Conclusion Product A is ethylbenzene, $C_6H_5CH_2CH_3$. Hence, the correct option is (A).

Quick Tip to Solve: Track the same two side-chain carbons through alkene, secondary alcohol, ketone and finally methylene; hot hydrazine/KOH does not stop at the hydrazone.

74. Which of the following is not correct? [4 marks]

(A) ΔG is positive for a spontaneous reaction

(B) ΔG is positive for a non-spontaneous reaction

(C) ΔG is zero for a reversible reaction

(D) ΔG is negative for a spontaneous reaction

Correct Answer: (A) ΔG is positive for a spontaneous reaction

Solution:

Concept:

- At constant temperature and pressure, the Gibbs energy change determines whether a reaction is thermodynamically spontaneous in the stated direction.
- The criterion is $\Delta G < 0$ for a spontaneous change, $\Delta G > 0$ for a non-spontaneous change, and $\Delta G = 0$ at equilibrium or for an infinitesimal reversible change.

Step 1: State the sign criterion with its conditions For a process at fixed temperature and pressure, Gibbs energy is the appropriate thermodynamic potential. A spontaneous forward process lowers this potential, so its change is negative:

$$\Delta G = G_{\text{final}} - G_{\text{initial}} < 0.$$

A positive change means that the forward direction requires an external driving influence; the reverse direction is favoured. This thermodynamic criterion does not say how rapidly a process occurs, since a spontaneous reaction can still face an activation-energy barrier.

Step 2: Check each statement, remembering the negative wording Option A says that a spontaneous reaction has positive ΔG. This directly contradicts the criterion and is therefore the incorrect statement requested. Option B correctly assigns positive ΔG to a non-spontaneous reaction in the forward direction. Option C uses reversible reaction in the equilibrium sense: the forward and reverse tendencies balance and $\Delta G = 0$. Option D correctly states the negative sign for a spontaneous reaction. Thus B, C and D are accepted thermodynamic statements under the constant-temperature, constant-pressure conditions of the Gibbs criterion.

Step 3: Conclusion The incorrect statement is that ΔG is positive for a spontaneous reaction; it must be negative under the Gibbs criterion. Hence, the correct option is (A).

Quick Tip to Solve: Underline the word “not first. At fixed temperature and pressure, spontaneity means decreasing Gibbs energy, so the sign is negative.

75. The interaction between π bond and lone pair of electrons present on an adjacent atom is responsible for [4 marks]

(A) Resonance effect

(B) Electromeric effect

(C) Hyperconjugation

(D) Inductive effect

Correct Answer: (A) Resonance effect

Solution:

Concept:

- Resonance arises when adjacent orbitals overlap to delocalise electrons over a conjugated set of atoms. A lone pair adjacent to a pi bond can participate when its orbital has a suitable orientation.
- The contributing Lewis structures differ in electron distribution while the atomic positions stay fixed. The actual species is a resonance hybrid rather than a sequence of alternating Lewis structures.

Step 1: Identify the interacting orbitals The question specifically describes a pi bond and a lone pair on an adjacent atom. The lone-pair orbital can overlap with the neighbouring pi system, permitting electron density to spread over more than two atoms. A familiar example is donation of the nitrogen lone pair towards a neighbouring carbonyl group in an amide. The resulting delocalisation gives partial multiple-bond character and stabilises the conjugated structure; it is represented by resonance contributors.

Step 2: Distinguish the other electronic effects This interaction is called the resonance effect, so option A fits the specified pair of electron sources. Electromeric effect is a temporary complete transfer of a pi-electron pair under the influence of an attacking reagent, rather than the persistent conjugation described here. Hyperconjugation involves a sigma bond, commonly a carbon–hydrogen bond, interacting with an adjacent pi bond or suitable vacant orbital; the question instead names a lone pair. Inductive effect is polarisation transmitted through sigma bonds because of electronegativity differences, and does not require this adjacent lone-pair/pi-orbital overlap.

Step 3: Conclusion The adjacent lone pair and pi bond interact through conjugation, producing the resonance effect. Hence, the correct option is (A).

Quick Tip to Solve: Identify the electron source: lone pair plus adjacent pi bond suggests resonance; an adjacent sigma bond suggests hyperconjugation.

76. In which one of the following metal carbonyls, CO forms a bridge between metal atoms? [4 marks]

(A) $[Os_3(CO)_{12}]$

(B) $[Ru_3(CO)_{12}]$

(C) $[Mn_2(CO)_{10}]$

(D) $[Co_2(CO)_8]$

Correct Answer: (D) $[Co_2(CO)_8]$

Solution:

Concept:

- A terminal carbonyl ligand binds through carbon to a single metal centre. A bridging carbonyl, denoted μ-CO, links two metal atoms.
- The standard bridged form of dicobalt octacarbonyl contains six terminal carbonyls and two bridging carbonyls, together with a cobalt–cobalt bond.

Step 1: Identify the carbonyl with bridges Among the listed compounds, $Co_2(CO)_8$ has the familiar bridged structure containing two μ-CO ligands. Each of these carbonyl groups connects the two cobalt atoms rather than terminating at just one metal. The remaining six carbonyl ligands are terminal. Consequently the carbonyl count is

$$6 \text{ terminal CO} + 2 \text{ bridging CO} = 8 \text{ CO ligands.}$$

This is the structural feature sought; the presence of more than one metal atom alone does not prove that any ligand bridges them.

Step 2: Rule out the other clusters In the usual structures of $Os_3(CO)_{12}$ and $Ru_3(CO)_{12}$, the three metals form a triangular framework and the carbonyl ligands are terminal. Thus the metal–metal links of the cluster do not imply carbonyl bridges. $Mn_2(CO)_{10}$ consists of two manganese atoms joined by a metal–metal bond, with five terminal carbonyls on each metal; its CO ligands are not bridging. Dicobalt octacarbonyl also has an unbridged form under suitable conditions, but its established bridged form distinguishes it from the other choices in this standard structural question.

Step 3: Conclusion The metal carbonyl showing bridging CO ligands is $Co_2(CO)_8$, in its bridged form. Hence, the correct option is (D).

Quick Tip to Solve: Do not infer a CO bridge from a metal–metal bond: remember the two bridging CO groups in the standard bridged structure of dicobalt octacarbonyl.

77. $KMnO_4$ **decomposes on heating at** $513\ K$ **to form** O_2 **along with** [4 marks]

(A) K_2MnO_4 & Mn

(B) MnO_2 & K_2O_2

(C) K_2MnO_4 & MnO_2

(D) Mn & KO_2

Correct Answer: (C) K_2MnO_4 & MnO_2

Solution:

Concept:

- On moderate heating, potassium permanganate decomposes to potassium manganate, manganese dioxide and oxygen. Potassium remains in the manganate salt.
- The products are checked by conserving every element and by tracking the decrease in manganese oxidation state from $+7$ to $+6$ and $+4$, coupled to oxygen evolution.

Step 1: Write and balance the thermal decomposition At the given temperature, the standard decomposition equation is

$$2\,KMnO_4 \xrightarrow{\text{heat}} K_2MnO_4 + MnO_2 + O_2.$$

Both sides contain two potassium atoms and two manganese atoms. The left side contains eight oxygen atoms, while the right side contains four in potassium manganate, two in manganese dioxide and two in oxygen gas, also giving eight. Therefore the two solid products accompanying the oxygen are K_2MnO_4 and MnO_2.

Step 2: Check oxidation states and eliminate the other pairs In permanganate manganese has oxidation state $+7$. It becomes $+6$ in manganate and $+4$ in manganese dioxide. The two manganese atoms therefore gain a total of four electrons, while two oxide oxygen atoms form O_2, losing four electrons altogether. This confirms the internal redox balance. Option A incorrectly substitutes elemental manganese for manganese dioxide. Option B proposes potassium peroxide instead of the manganate salt. Option D likewise predicts manganese metal and potassium superoxide, neither of which is a product of this stated thermal decomposition. Only option C matches the balanced reaction and the known solid products.

Step 3: Conclusion Along with O_2, the products are K_2MnO_4 and MnO_2. Hence, the correct option is (C).

Quick Tip to Solve: Remember the balanced pattern $2\,KMnO_4 \longrightarrow K_2MnO_4 + MnO_2 + O_2$; an oxygen-atom count quickly checks all three products.

78. Appearance of blood red colour, on treatment of the sodium fusion extract of an organic compound with $FeSO_4$ in presence of concentrated H_2SO_4 indicates the presence of element/s [4 marks]

(A) Br

(B) S

(C) N and S

(D) N

Correct Answer: (C) N and S

Solution:

Concept:

- Sodium fusion converts covalently bonded elements in an organic compound into water-soluble ionic salts. Nitrogen and sulphur present together can form sodium thiocyanate, NaSCN.
- Thiocyanate ions produce a characteristic blood-red complex with ferric ions. Nitrogen alone instead gives the Prussian-blue test, whereas sulphur alone forms sulphide.

Step 1: Identify the species formed during fusion When nitrogen and sulphur occur together, fusion with sodium incorporates both into thiocyanate. The diagnostic conversion is represented by

$$\mathrm{Na + C + N + S \longrightarrow NaSCN.}$$

The aqueous sodium fusion extract therefore contains SCN^-. This ion, rather than bromide or sulphide alone, is responsible for the blood-red colour in the iron test.

Step 2: Relate the red colour to the iron complex In the acidified iron test, ferric ions available after oxidation of ferrous ions react with thiocyanate:

$$\mathrm{Fe^{3+} + SCN^- \rightleftharpoons [Fe(SCN)]^{2+}.}$$

The ferric thiocyanate complex is blood red. Thus both nitrogen and sulphur must have supplied the atoms of thiocyanate. Bromine does not produce this complex. Sulphur alone gives sodium sulphide, detected by violet sodium nitroprusside colour or black lead sulphide; nitrogen alone gives cyanide and the Prussian-blue test, not this blood-red response.

Step 3: Conclusion The blood-red ferric thiocyanate colour indicates the simultaneous presence of nitrogen and sulphur. Hence, the correct option is (C).

Quick Tip to Solve: Keep the iron-test colours distinct: nitrogen alone gives Prussian blue; nitrogen together with sulphur gives blood-red thiocyanate.

79. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Aryl halides cannot be prepared by replacement of hydroxyl group of phenol by halogen atom.

Reason R : Phenols react with halogen acids violently.

In the light of the above statements, choose the most appropriate from the options given below :

[4 marks]

(A) A is true but R is false

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is false but R is true

(D) Both A and R are true and R is the correct explanation of A

Correct Answer: (A) A is true but R is false

Solution:

Concept:

- The oxygen lone pair of phenol is delocalised into the aromatic ring, giving its aryl carbon–oxygen bond partial double-bond character.
- Replacement of the hydroxyl group by halide requires cleavage of this strong aryl carbon–oxygen bond. The usual substitution behaviour of alcohols with hydrogen halides does not apply to phenol.

Step 1: Evaluate Assertion A In an alcohol, protonation of the hydroxyl group can enable displacement of water and formation of an alkyl halide. In phenol the hydroxyl-bearing carbon is an aromatic sp^2 carbon and the carbon–oxygen bond is strengthened by resonance. Ordinary hydrogen-halide substitution therefore does not convert phenol into an aryl halide. Aryl substitution by the normal S_N1 route would require an unstable phenyl cation, while normal backside S_N2 attack at that aromatic carbon is not feasible. Assertion A is true in this standard preparation context.

Step 2: Evaluate Reason R Phenols do not react violently with halogen acids to replace their hydroxyl group. Their resistance to that reaction is precisely why the alcohol-to-halide preparation cannot be transferred to phenol. Thus Reason R is false; it neither describes the observed reaction nor explains the assertion.

Step 3: Select the matching truth combination The required combination is a true assertion and a false reason. Options (B) and (D) both incorrectly treat the reason as true, and option (C) incorrectly reverses the truth of the assertion. Only option (A) agrees with the separate chemical assessments.

Step 4: Conclusion A is true but R is false. Hence, the correct option is (A).

Quick Tip to Solve: For phenol substitution questions, check whether the bond being broken is aryl C–O: resonance makes it behave differently from an alcohol C–O bond.

80. In alkaline medium, MnO_4^- oxidises I^- to [4 marks]

(A) I_2

(B) IO_3^-

(C) IO^-

(D) IO_4^-

Correct Answer: (B) IO_3^-

Solution:

Concept:

- Permanganate is an oxidising agent whose products depend on the medium. In neutral or moderately alkaline solution its manganese is commonly reduced to manganese dioxide.
- Iodide is oxidised by alkaline permanganate to iodate, $IO_3{}^-$; iodine changes oxidation state from -1 to $+5$, losing six electrons per iodine atom.

Step 1: Write the oxidation half-reaction The product expected for iodide under these alkaline oxidation conditions is iodate. Balancing oxygen with water, hydrogen with hydroxide and charge with electrons gives

$$I^- + 6\,OH^- \longrightarrow IO_3{}^- + 3\,H_2O + 6\,e^-.$$

The six released electrons agree with the oxidation-number rise from -1 in iodide to $+5$ in iodate.

Step 2: Combine with permanganate reduction The reduction half-reaction producing manganese dioxide in alkaline medium is

$$MnO_4{}^- + 2\,H_2O + 3\,e^- \longrightarrow MnO_2 + 4\,OH^-.$$

Multiply it by two and add it to the iodide half-reaction. Cancelling electrons, water and hydroxide gives

$$2\,MnO_4{}^- + I^- + H_2O \longrightarrow 2\,MnO_2 + IO_3{}^- + 2\,OH^-.$$

Both atom counts and the net charge are balanced.

Step 3: Reject the other oxidation products Iodine is the familiar product when iodide is oxidised in acidic conditions, so option (A) ignores the stated medium. Hypoiodite contains iodine in oxidation state $+1$, and periodate contains iodine in $+7$; neither is the standard iodide product of the stated alkaline permanganate reaction. The balanced reaction identifies iodate directly.

Step 4: Conclusion Alkaline permanganate oxidises I^- to $IO_3{}^-$. Hence, the correct option is (B).

Quick Tip to Solve: In permanganate questions, identify the medium before selecting the iodine product; the alkaline iodide reaction gives iodate.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The mass of zinc produced by the electrolysis of zine sulphate solution with a steady current of 0.015 A for 15 minutes is _____ $\times 10^{-4}$ g.

(Atomic mass of zinc $= 65.4$ amu) [4 marks]

Solution:

Concept:

- Faraday's law relates deposited mass to charge: $m = MQ/(nF)$, where M is molar mass, Q is charge, n is electrons transferred per metal ion and $F \approx 96500\,\mathrm{C\,mol^{-1}}$.
- Zinc sulphate supplies Zn^{2+} ions. Cathodic deposition is $Zn^{2+} + 2\,e^- \longrightarrow Zn$, so two moles of electrons deposit one mole of zinc.

Step 1: Calculate the charge passed Convert the time to seconds before multiplying by current, because an ampere is one coulomb per second:

$$t = 15 \times 60 = 900\,\mathrm{s}.$$

The steady current therefore supplies

$$Q = It = 0.015 \times 900 = 13.5\,\mathrm{C}.$$

The small charge is consistent with a deposit of only a few milligrams; using minutes directly would underestimate it by a factor of sixty.

Step 2: Use the two-electron reduction of zinc Each zinc ion needs two electrons, not one. Taking the molar mass as $65.4\,\mathrm{g\,mol^{-1}}$, Faraday's law gives

$$m = \frac{65.4 \times 13.5}{2 \times 96500}\,\mathrm{g} = 0.0045746\,\mathrm{g}.$$

Equivalently, the moles of electrons are $13.5/96500$, and half this amount is the moles of deposited zinc. Multiplication by the molar mass gives the same mass.

Step 3: Express the mass in the requested scale The question requests the coefficient of 10^{-4} g, rather than the mass in grams itself. Hence

$$0.0045746\,\mathrm{g} = 45.746 \times 10^{-4}\,\mathrm{g}.$$

The coefficient rounded to the integer numerical response is 46. Keep the unrounded mass until this final conversion so that intermediate rounding does not alter the answer.

Step 4: Conclusion The zinc deposit is approximately 46×10^{-4} g; the required numerical entry is 46.

Quick Tip to Solve: For a metal deposit, first identify the ionic charge to obtain n, then convert the final mass to the power-of-ten scale printed in the blank.

82. A solution of H_2SO_4 is $31.4\%H_2SO_4$ by mass and has a density of $1.25\ g/mL$. The molarity of the H_2SO_4 solution is _____ M (nearest integer)

[Given molar mass of $H_2SO_4 = 98\ g\ mol^{-1}$] [4 marks]

Solution:

Concept:

- Mass percentage means grams of solute per 100 grams of solution. It is not grams per 100 millilitres.
- Molarity is $c = n/V$, where n is the moles of solute and V is the volume of the complete solution in litres. Density converts solution mass to solution volume.

Step 1: Choose a one-litre sample of solution Take $1.000\,L = 1000\,mL$ of the given solution. Its mass follows from the solution density:

$$m_{solution} = 1.25 \times 1000 = 1250\,g.$$

This choice makes the eventual number of moles numerically equal to the molarity. The density applies to the complete solution, so no separate solvent-volume estimate is needed.

Step 2: Calculate the acid mass and moles The acid contributes 31.4% of the solution mass. Therefore the mass of H_2SO_4 in this litre is

$$m_{acid} = \frac{31.4}{100} \times 1250 = 392.5\,g.$$

Use the given molar mass, rather than the equivalent mass of this diprotic acid:

$$n = \frac{392.5}{98} = 4.00510\,mol.$$

Molarity counts acid molecules; multiplying by two would calculate an acid-equivalent concentration instead.

Step 3: Divide by solution volume and round The selected sample volume is exactly one litre, so

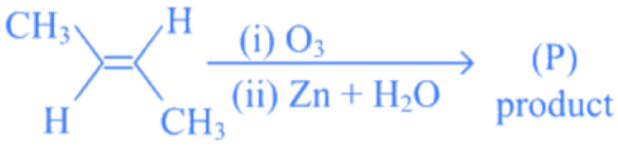
$$c = \frac{4.00510}{1.000} = 4.00510\,mol\,L^{-1}.$$

Rounding this value to the nearest integer as requested gives 4. The calculation consistently uses the mass percentage and the density together; treating the percentage as mass per unit volume would omit the density factor.

Step 4: Conclusion The molarity is approximately $4.005\,M$, or $4\,M$ to the nearest integer. The required entry is 4.

Quick Tip to Solve: For mass percentage w and density rho in grams per millilitre, use molarity = 10 rho w divided by molar mass; do not confuse molarity with normality.

83. Consider the given reaction. The total number of oxygen atom/s present per molecule of the product (P) is _____. [4 marks]



Solution:

Concept:

- Ozonolysis cleaves an alkene at its carbon–carbon double bond. Under reductive work-up with zinc and water, each double-bond carbon becomes a carbonyl carbon.
- A double-bond carbon bonded to a hydrogen yields an aldehyde on cleavage; one bonded to two carbon groups yields a ketone. Count oxygen atoms per product molecule, not over all product molecules.

Step 1: Identify the substituents on the alkene The displayed reactant is trans-but-2-ene, $CH_3CH{=}CHCH_3$. Each carbon of its double bond carries one methyl group and one hydrogen. The two double-bond carbons therefore have identical substitution patterns, even though the methyl groups are drawn on opposite sides. This symmetry means cleavage will produce two molecules of the same carbonyl compound.

Step 2: Apply ozonolysis with reductive work-up Ozone attacks the double bond and forms an ozonide intermediate. The subsequent Zn + H_2O work-up converts that intermediate into carbonyl compounds without oxidising the resulting aldehyde further. Cleavage can be summarised as

$$CH_3CH{=}CHCH_3 \xrightarrow[2.\,Zn,\,H_2O]{1.\,O_3} 2\,CH_3CHO.$$

Each original alkene carbon retains its methyl substituent and hydrogen and acquires a double bond to oxygen. The product P is therefore ethanal, not ethanoic acid.

Step 3: Count atoms in one product molecule One molecule of ethanal has the formula C_2H_4O, with its single oxygen in the aldehyde carbonyl group. Although one molecule of the alkene gives two molecules of ethanal, the question explicitly asks for oxygen atoms per molecule of product. Thus summing the oxygen atoms over both molecules would incorrectly give two. Neither the three oxygen atoms in ozone nor the temporary ozonide determines the oxygen count in the isolated carbonyl product.

Step 4: Conclusion The product P is ethanal, CH_3CHO, which contains 1 oxygen atom per molecule. The required numerical entry is 1.

Quick Tip to Solve: For reductive ozonolysis, cut the C=C bond and put one oxygen on each alkene carbon; then count atoms in one resulting molecule.

84. For the reaction $N_2O_{4(g)} \rightleftarrows 2NO_{2(g)}, K_p = 0.492$ atm at 300 K.K_c for the reaction at same temperature is _____ $\times 10^{-2}$.

(Given : $R = 0.082$ L atm mol^{-1} K^{-1}) [4 marks]

Solution:

Concept:

- For a gaseous equilibrium expressed using partial pressures and molar concentrations, $K_p = K_c(RT)^{\Delta n}$, with Δn equal to gaseous product coefficients minus gaseous reactant coefficients.
- Use the gas constant in units consistent with the pressure convention of K_p. Here the supplied $R = 0.082\,\mathrm{L\,atm\,mol^{-1}\,K^{-1}}$ matches the pressure in atmospheres.

Step 1: Determine the change in gaseous moles For the equilibrium

$$\mathrm{N_2O_4(g) \rightleftharpoons 2\,NO_2(g)},$$

there are two moles of gaseous products for one mole of gaseous reactant. Hence

$$\Delta n = 2 - 1 = 1.$$

Only the stoichiometric coefficients in the balanced equation are used. The equilibrium concentrations or extent of dissociation are unnecessary for converting the two equilibrium constants.

Step 2: Substitute into the equilibrium-constant relation With $\Delta n = 1$, the conversion reduces to

$$K_p = K_c RT, \qquad K_c = \frac{K_p}{RT}.$$

At the specified temperature,

$$RT = 0.082 \times 300 = 24.6\,\mathrm{L\,atm\,mol^{-1}}.$$

Therefore

$$K_c = \frac{0.492}{24.6} = 0.020.$$

Dividing, rather than multiplying, is essential because K_c is the unknown and RT multiplies it in the original equation.

Step 3: Read the coefficient requested in the blank Express the concentration-based constant in scientific notation:

$$K_c = 2.0 \times 10^{-2}.$$

The requested blank precedes $\times 10^{-2}$, so the numerical response is the coefficient 2, not 0.02. Under the conventional units used here, the concentration quotient has units of $\mathrm{mol\,L^{-1}}$, consistent with the net increase of one gaseous mole.

Step 4: Conclusion $K_c = 2.0 \times 10^{-2}$ in the stated concentration convention; the required coefficient is 2.

Quick Tip to Solve: Compute the gaseous coefficient difference first. For this dissociation it is +1, so divide Kp by RT and only then extract the requested power-of-ten coefficient.

85. The number of species from the following which are paramagnetic and with bond order equal to one is _____.

$$H_2, He_2^+, O_2^+, N_2^{2-}, O_2^{2-}, F_2, Ne_2^+, B_2$$

[4 marks]

Solution:

Concept:

- Bond order is $BO = (N_b - N_a)/2$, where N_b and N_a are bonding and antibonding electron counts. Core bonding and antibonding pairs cancel in this difference.
- A species is paramagnetic only when at least one molecular orbital contains an unpaired electron. Bond order and magnetic behaviour must both meet the stated conditions.

Step 1: Classify the first four species For H_2, two electrons occupy the bonding σ_{1s} orbital: bond order is 1, but both electrons are paired. In $He_2{}^+$, three electrons give bond order $(2-1)/2 = 1/2$, although one electron is unpaired. Removing an antibonding electron from O_2 gives $O_2{}^+$ with bond order $5/2$, so it also fails. Adding two electrons to N_2 gives $N_2{}^{2-}$ with two singly occupied degenerate antibonding orbitals, but its bond order falls from 3 to 2, not to 1.

Step 2: Classify the remaining four species In $O_2{}^{2-}$, both antibonding π^* orbitals are filled: bond order is 1 but all electrons are paired. Likewise F_2 has bond order 1 and is diamagnetic. $Ne_2{}^+$ has one unpaired electron but bond order $1/2$. Finally, the valence configuration of B_2 is $(\sigma_{2s})^2(\sigma^*_{2s})^2(\pi_{2p_x})^1(\pi_{2p_y})^1$. Hund's rule gives two unpaired electrons and $(4-2)/2 = 1$. Thus only this final species satisfies both tests.

Step 3: Conclusion Only B_2 is paramagnetic with bond order equal to one. The required number of species is $\boxed{1}$.

Quick Tip to Solve: Use two separate filters: first retain bond order 1, then remove every species whose electrons are all paired.

86. For a reaction taking place in three steps at same temperature, overall rate constant $K = \frac{K_1 K_2}{K_3}$. If Ea_1, Ea_2 and Ea_3 are 40, 50 and 60 kJ/mol respectively, the overall Ea is _____ kJ/mol. [4 marks]

Solution:

Concept:

- For an elementary rate constant, the Arrhenius form is $K_i = A_i \exp[-E_{a,i}/(RT)]$. Here A_i is its pre-exponential factor and T is absolute temperature.
- Products of rate constants add their activation energies in the exponent; division subtracts the activation energy of the denominator. Use the given relation for the overall constant.

Step 1: Substitute each Arrhenius expression The three constants are evaluated at the same temperature, so their exponential factors have a common denominator RT. Substitute them directly into the

specified expression, retaining the denominator as division:

$$K = \frac{A_1 e^{-E_{a,1}/RT} A_2 e^{-E_{a,2}/RT}}{A_3 e^{-E_{a,3}/RT}}.$$

The numerator contributes two negative exponents, whereas dividing by the third exponential reverses its sign. This is why simply adding all three activation energies would be incorrect.

Step 2: Identify the overall activation energy Combining the factors gives

$$K = \frac{A_1 A_2}{A_3} \exp\left[-\frac{E_{a,1} + E_{a,2} - E_{a,3}}{RT}\right].$$

Compare this with $K = A\exp[-E_a/(RT)]$: the overall pre-exponential factor is $A_1 A_2/A_3$, and the overall activation energy is $E_a = E_{a,1} + E_{a,2} - E_{a,3}$. Therefore,

$$E_a = 40 + 50 - 60 = 30\,\text{kJ mol}^{-1}.$$

All three data use the same energy unit, so no conversion is needed for the requested answer.

Step 3: Conclusion The overall activation energy is $\boxed{30\,\text{kJ mol}^{-1}}$. Hence the number to enter is $\boxed{30}$.

Quick Tip to Solve: Take the logarithm of the given product or quotient: the signs multiplying the logarithms are also the signs multiplying the activation energies.

87. From the compounds given below, number of compounds which give positive Fehling's test is _____.

Benzaldehyde, Acetaldehyde, Acetone, Acetophenone, Methanal, 4nitrobenzaldehyde, cyclohexane carbaldehyde. [4 marks]

Solution:

Concept:

- Fehling's reagent contains complexed copper(II) ions in alkaline solution. A suitable aldehyde reduces these ions to brick-red Cu_2O while its aldehyde group is oxidised to carboxylate.
- Ordinary aliphatic aldehydes give a positive test, whereas aromatic aldehydes and simple ketones do not. Classify the carbonyl group and its directly attached group before counting.

Step 1: Identify the positive compounds Acetaldehyde, CH_3CHO, is an aliphatic aldehyde and gives the red precipitate. Methanal, $HCHO$, also reduces the reagent. Cyclohexanecarbaldehyde has an aldehyde group attached to a saturated cyclohexyl group; its carbonyl carbon is not directly attached to an aromatic ring. It is therefore an aliphatic aldehyde and also gives a positive test. The presence of a ring alone does not make a compound aromatic.

Step 2: Exclude the remaining compounds and count Benzaldehyde and 4-nitrobenzaldehyde have their $-CHO$ group directly bonded to a benzene ring. They are aromatic aldehydes and do not give the ordinary Fehling test. Acetone and acetophenone are ketones, with two carbon groups attached to the carbonyl carbon; neither is an alpha-hydroxy ketone or another reducing exception. They

therefore fail the test as well. The accepted oxidation can be represented generally by

$$\mathrm{RCHO} + 2\,\mathrm{Cu^{2+}} + 5\,\mathrm{OH^-} \longrightarrow \mathrm{RCOO^-} + \mathrm{Cu_2O}\downarrow + 3\,\mathrm{H_2O}.$$

Thus three named compounds reduce the copper(II) reagent.

Step 3: Conclusion Acetaldehyde, methanal and cyclohexanecarbaldehyde give positive Fehling tests. The required count is $\boxed{3}$.

Quick Tip to Solve: Check whether the aldehyde carbon is bonded directly to an aromatic ring; a saturated cyclohexyl ring does not exclude a positive Fehling test.

88. Number of compounds with one lone pair of electrons on central atom amongst following is _____.

$$\mathrm{O_3}, \mathrm{H_2O}, \mathrm{SF_4}, \mathrm{ClF_3}, \mathrm{NH_3}, \mathrm{BrF_5}, \mathrm{XeF_4}$$

[4 marks]

Solution:

Concept:

- The required quantity is the number of lone pairs on the central atom, not the total number of lone pairs in the molecule. Terminal atoms must therefore be excluded from the count.
- Construct the central-atom Lewis arrangement and count nonbonding electron pairs. Multiple bonds form one electron domain in VSEPR, but their electrons still belong to bonding rather than lone pairs.

Step 1: Identify central atoms with exactly one lone pair In ozone, a resonance form has one single and one double bond at the central oxygen. The central oxygen carries one lone pair, producing a bent arrangement. Sulphur in $\mathrm{SF_4}$ uses four of its six valence electrons for four S–F bonds, leaving two electrons as one lone pair; the arrangement is $\mathrm{AX_4E}$. Nitrogen in $\mathrm{NH_3}$ forms three N–H bonds and retains one lone pair, giving $\mathrm{AX_3E}$. Bromine in $\mathrm{BrF_5}$ forms five bonds and retains one lone pair, giving $\mathrm{AX_5E}$.

Step 2: Check the other molecules separately Oxygen in $\mathrm{H_2O}$ forms two bonds and retains four nonbonding electrons, or two lone pairs. Chlorine in $\mathrm{ClF_3}$ forms three bonds from seven valence electrons and retains four nonbonding electrons, again two lone pairs. Xenon in $\mathrm{XeF_4}$ forms four bonds from eight valence electrons and retains four nonbonding electrons, giving two lone pairs. These three must not be included. The complete central-atom lone-pair sequence in the order printed is therefore $1, 2, 1, 2, 1, 1, 2$. Exactly four entries equal one.

Step 3: Conclusion The qualifying species are $\mathrm{O_3}$, $\mathrm{SF_4}$, $\mathrm{NH_3}$ and $\mathrm{BrF_5}$. The required number is $\boxed{4}$.

Quick Tip to Solve: Count only central-atom nonbonding electrons and divide by two. For ozone, first draw a valid resonance form rather than applying a single-bond formula blindly.

89. Number of compounds among the following which contain sulphur as heteroatom is _____.

Furan, Thiophene, Pyridine, Pyrrole, Cysteine, Tyrosine [4 marks]

Solution:

Concept:

- A heteroatom in an organic compound is an atom other than carbon or hydrogen. The question specifically asks for sulphur, so the presence of oxygen or nitrogen alone is insufficient.
- Identify the characteristic ring atom or side-chain functional group of each named compound. Sulphur need not be in a ring to count as a heteroatom.

Step 1: Classify the heterocyclic compounds Furan is a five-membered aromatic ring containing one oxygen atom, so it contains no sulphur. Thiophene is the corresponding five-membered aromatic heterocycle containing one sulphur atom and qualifies. Pyridine has a six-membered aromatic ring with one nitrogen atom; pyrrole has a five-membered aromatic ring with one nitrogen atom. Although all four are heterocycles, only thiophene has sulphur. Counting all heterocycles would answer a different question.

Step 2: Inspect the amino-acid side chains Cysteine has the structure $\mathrm{HS{-}CH_2{-}CH(NH_2){-}COOH}$. Its side-chain thiol group, $-\mathrm{SH}$, contains sulphur, which counts even though it is not part of a ring. Tyrosine has a hydroxyphenyl side chain and can be represented as $\mathrm{HO{-}C_6H_4{-}CH_2{-}CH(NH_2){-}COOH}$; its heteroatoms are oxygen and nitrogen, with no sulphur. Therefore the six names divide into two sulphur-containing compounds, thiophene and cysteine, and four compounds without sulphur. The amino and carboxyl groups shared by the amino acids do not themselves introduce sulphur.

Step 3: Conclusion Only thiophene and cysteine contain sulphur as a heteroatom. The required number of compounds is $\boxed{2}$.

Quick Tip to Solve: Remember the sulphur pair in this list: thiophene has ring sulphur, whereas cysteine has a side-chain thiol group.

90. The osmotic pressure of a dilute solution is 7×10^5 Pa at 273 K. Osmotic pressure of the same solution at 283 K is _____ $\times 10^4 \mathrm{Nm^{-2}}$. [4 marks]

Solution:

Concept:

- For a dilute solution, van't Hoff's relation is $\pi = CRT$, where C is molar concentration, R is the gas constant and T is absolute temperature.
- For the same dilute solution at unchanged concentration, π/T is constant. Temperature must be in kelvin. Also, $1\,\mathrm{Pa} = 1\,\mathrm{N\,m^{-2}}$.

Step 1: Relate the pressures at the two temperatures The solute and solution concentration are unchanged, so the factors C and R cancel between the two states. Hence

$$\frac{\pi_2}{\pi_1} = \frac{T_2}{T_1}, \qquad \pi_2 = \pi_1 \frac{T_2}{T_1}.$$

Substituting the printed values gives

$$\pi_2 = 7 \times 10^5 \frac{283}{273}\,\text{Pa}.$$

Since $283 > 273$, the final pressure must exceed the initial pressure; reversing the temperature ratio would fail this physical check.

Step 2: Express the result in the requested scale The numerical pressure is

$$\pi_2 = 7.25641 \times 10^5\,\text{Pa} = 72.5641 \times 10^4\,\text{N}\,\text{m}^{-2}.$$

The blank is the coefficient multiplying 10^4, so the value before rounding is 72.5641, rather than 7.25641 or the full pressure in pascals. Rounding this coefficient to the nearest integer for the numerical entry gives 73. The roughly 3.66 percent temperature rise produces the same fractional rise in osmotic pressure, consistent with the calculated result.

Step 3: Conclusion The osmotic pressure is approximately $7.2564 \times 10^5\,\text{N}\,\text{m}^{-2}$. The coefficient in the specified $10^4\,\text{N}\,\text{m}^{-2}$ scale rounds to $\boxed{73}$.

Quick Tip to Solve: First calculate the pressure in pascals, then divide by the power of ten printed beside the blank before rounding the entry.