# JEE Main 2024 Paper 1 (B.E./B.Tech) Jan 27 Shift 2 Question Paper with Solutions PDF

## Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

**Time Allowed: 3 Hours** **Maximum Marks: 300**

### General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 1 (Jan-Feb), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 27 January 2024, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

## Mathematics – Section A: Multiple Choice Questions (80 marks)

**1. Considering only the principal values of inverse trigonometric functions, the number of positive real values of $x$ satisfying $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$ is :** [4 marks]

(A) more than 2

(B) 2

(C) 0

(D) 1

**Correct Answer:** (D) 1

**Solution:**

**Concept:**

- The principal value of $\tan^{-1} u$ lies in $(-\pi/2, \pi/2)$. For positive $u$, it lies in $(0, \pi/2)$.
- For angles $A, B$, $\tan(A+B) = (\tan A + \tan B)/(1 - \tan A \tan B)$, provided the denominator is nonzero. A candidate obtained using tangent must be checked in the original principal-value equation.

**Step 1:** Convert the angle equation into a quadratic Put $A = \tan^{-1} x$ and $B = \tan^{-1}(2x)$. The required sum is $\pi/4$, so its tangent equals 1. Thus

$$\frac{3x}{1-2x^2} = 1 \quad \Longrightarrow \quad 2x^2 + 3x - 1 = 0.$$

The value $1 - 2x^2 = 0$ cannot satisfy the given angle equation, since it would make the tangent of the sum undefined.

**Step 2:** Select and verify the positive root The two algebraic roots are

$$x = \frac{-3 \pm \sqrt{17}}{4}.$$

Since $\sqrt{17} > 3$, exactly one root is positive: $x_0 = (\sqrt{17} - 3)/4$. Moreover, $0 < x_0 < 1/2$, so $2x_0^2 < 1$. Both principal angles are positive, and their sum belongs to $(0, \pi)$. The positive denominator in the tangent formula places this sum in $(0, \pi/2)$; there, tangent is one-to-one. Consequently its tangent being 1 implies that the sum is exactly $\pi/4$, with no added multiple of $\pi$.

**Step 3:** Count the solutions and exclude the other choices The negative quadratic root is inadmissible because the question requires positive $x$. The positive root passes the original angle restriction. Thus zero solutions, two solutions and more than two solutions are all excluded. This also agrees with the strict increase of both inverse-tangent terms for $x > 0$.

**Step 4:** Conclusion There is exactly 1 positive real solution, namely $x = (\sqrt{17} - 3)/4$. Hence, the correct option is (D).

**Quick Tip to Solve:** After taking tangent of an inverse-trigonometric equation, check the angle interval; the quadratic alone does not enforce principal values.

**2. Let the position vectors of the vertices $\mathrm{A}, \mathrm{B}$ and $\mathrm{C}$ of a triangle be $2\hat{i}+2\hat{j}+\hat{k}, \hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}+2\hat{k}$ respectively. Let $l_1, l_2$ and $l_3$ be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides $\mathrm{AB}, \mathrm{BC}$ and $\mathrm{CA}$ respectively, then $l_1^2+l_2^2+l_3^2$ equals:** [4 marks]

(A) $\frac{1}{4}$
(B) $\frac{1}{5}$
(C) $\frac{1}{3}$
(D) $\frac{1}{2}$

**Correct Answer:** (D) $\frac{1}{2}$

**Solution:**

**Concept:**

- The distance between two points is the magnitude of the difference of their position vectors. Equal side lengths identify an equilateral triangle even when its vertices lie in three-dimensional space.
- In an equilateral triangle, the orthocentre, centroid and incentre coincide. The perpendicular distance from this common centre to each side is one-third of the altitude.

**Step 1:** Calculate all three side lengths The vertices are $A=(2,2,1)$, $B=(1,2,2)$ and $C=(2,1,2)$. Their side vectors are

$$\overrightarrow{AB}=(-1,0,1), \qquad \overrightarrow{BC}=(1,-1,0), \qquad \overrightarrow{CA}=(0,1,-1).$$

Each vector has squared magnitude $1+1=2$. Therefore $AB=BC=CA=\sqrt{2}$, so the triangle is equilateral in the plane through its three vertices.

**Step 2:** Find the altitude and the centre-to-side distance For side length $a=\sqrt{2}$, the perpendicular altitude bisects the opposite side. By Pythagoras its length is

$$h=\sqrt{a^2-(a/2)^2}=\frac{\sqrt{3}}{2}a=\frac{\sqrt{6}}{2}.$$

The centroid divides a median in the ratio $2:1$, measured from the vertex. Because the median is also an altitude here, the common centre is $h/3$ from the side, rather than $2h/3$ from it. Thus

$$l_1=l_2=l_3=\frac{h}{3}=\frac{\sqrt{6}}{6}.$$

**Step 3:** Add the squared perpendicular lengths The requested quantities are distances to the sides, not distances to the vertices. Squaring and adding gives

$$l_1^2+l_2^2+l_3^2=3\left(\frac{\sqrt{6}}{6}\right)^2=3\cdot\frac{1}{6}=\frac{1}{2}.$$

The alternatives $1/4$, $1/5$ and $1/3$ are inconsistent with these equal perpendicular distances; they cannot equal the computed sum.

**Step 4:** Conclusion $l_1^2+l_2^2+l_3^2=\boxed{\frac{1}{2}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** When three-dimensional vertices give equal side lengths, use the equilateral triangle in its own plane; the centre-to-side distance is one-third of its altitude.

**3. Consider the function $f : (0, 2) \to \mathbf{R}$ defined by $f(x) = \frac{x}{2} + \frac{2}{x}$, and the function $g$ defined by**

$$g(x) = \begin{cases} \min\{f(t) : 0 < t \leq x\}, & 0 < x \leq 1, \\ \frac{3}{2} + x, & 1 < x < 2. \end{cases}$$

**Then,** [4 marks]

(A) $g$ is continuous but not differentiable at $x = 1$

(B) $g$ is continuous and differentiable for all $x \in (0, 2)$

(C) $g$ is not continuous for all $x \in (0, 2)$

(D) $g$ is neither continuous nor differentiable at $x = 1$

**Correct Answer:** (A) $g$ is continuous but not differentiable at $x = 1$

**Solution:**

**Concept:**

- To evaluate a minimum over $0 < t \leq x$, first determine whether the function increases or decreases on that interval. A decreasing function attains this minimum at its right endpoint.
- At a joining point, continuity requires equal left limit, right limit and function value. Differentiability additionally requires equal left and right derivatives.

**Step 1:** Evaluate the minimum in the first branch For $f(t) = t/2 + 2/t$, differentiation gives

$$f\prime(t) = \frac{1}{2} - \frac{2}{t^2}.$$

On $0 < t \leq 1$, we have $2/t^2 \geq 2$, so $f\prime(t) \leq -3/2 < 0$. Thus $f$ is strictly decreasing throughout every interval $(0, x]$ with $0 < x \leq 1$. Its minimum is therefore $f(x)$, attained at $t = x$.

**Step 2:** Check continuity at the junction The first branch includes $x = 1$. Therefore

$$g(1) = \frac{1}{2} + 2 = \frac{5}{2}, \qquad \lim_{x \to 1^-} g(x) = \frac{5}{2}.$$

From the second branch,

$$\lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} \left(\frac{3}{2} + x\right) = \frac{5}{2}.$$

The limits equal the actual value. The branches are also continuous elsewhere on $(0, 2)$.

**Step 3:** Compare the one-sided derivatives Evaluate the left and right derivatives at 1:

$$g\prime_-(1) = \frac{1}{2} - 2 = -\frac{3}{2}, \qquad g\prime_+(1) = 1.$$

They differ, so $g$ is not differentiable at 1. Option (B) wrongly requires matching slopes, while (C) and (D) wrongly deny the continuity just established.

**Step 4:** Conclusion The function $g$ is continuous but not differentiable at $x = 1$. Hence, the correct option is (A).

**Quick Tip to Solve:** For an endpoint minimum in a piecewise definition, simplify the minimum first; then check both values and slopes at the joining point separately.

**4. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :** [4 marks]

(A) $\frac{3}{256}$
(B) $\frac{5}{256}$
(C) $\frac{3}{715}$
(D) $\frac{5}{715}$

**Correct Answer:** (C) $\frac{3}{715}$

**Solution:**

**Concept:**

- An unordered draw of $r$ balls from $n$ distinct balls has $\binom{n}{r}$ equally likely outcomes. Balls of the same colour may be counted as distinct for this calculation.
- For successive events $E$ and $F$, $P(E \cap F) = P(E)P(F \mid E)$. Without replacement, the second probability must use the remaining contents of the urn.

**Step 1:** Find the probability of four white balls first Initially there are $6 + 9 = 15$ balls. Let $E$ be the event that all four balls in the first draw are white. We can select these balls in $\binom{6}{4}$ ways from $\binom{15}{4}$ possible first draws. Hence

$$P(E) = \frac{\binom{6}{4}}{\binom{15}{4}} = \frac{15}{1365} = \frac{1}{91}.$$

This is a group draw of four balls; their internal order does not change the colour event.

**Step 2:** Use the changed urn for the second draw Conditional on $E$, four white balls have been removed. The urn now contains 2 white and 9 black balls, a total of 11. If $F$ denotes an all-black second draw, then

$$P(F \mid E) = \frac{\binom{9}{4}}{\binom{11}{4}} = \frac{126}{330} = \frac{21}{55}.$$

The numerator still uses all nine black balls, because the first successful draw removed only white balls. The denominator must use 11 rather than 15.

**Step 3:** Multiply and compare the options The order of the two successful draws is specified, so no extra factor of two is introduced. Thus

$$P(E \cap F) = \frac{1}{91} \cdot \frac{21}{55} = \frac{21}{5005} = \frac{3}{715}.$$

This value is about $0.00420$. Neither fraction with denominator 256 has this value, and $5/715$ has an incorrect numerator. Using an unchanged urn for the second draw would incorrectly treat the events as independent.

**Step 4:** Conclusion The required probability is $\boxed{3/715}$. Hence, the correct option is (C).

**Quick Tip to Solve:** After a successful first draw, write the remaining numbers of each colour before calculating the conditional probability of the second draw.

**5. Let the image of the point $(1, 0, 7)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the point $(\alpha, \beta, \gamma)$. Then which one of the following points lies on the line passing through $(\alpha, \beta, \gamma)$ and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with $y$-axis and $z$-axis respectively and an acute angle with $x$-axis ?** [4 marks]

(A) $(1, -2, 1 + \sqrt{2})$

(B) $(3, -4, 3 + 2\sqrt{2})$

(C) $(3, 4, 3 - 2\sqrt{2})$

(D) $(1, 2, 1 - \sqrt{2})$

**Correct Answer:** (C) $(3, 4, 3 - 2\sqrt{2})$

**Solution:**

**Concept:**

- The image of a point $P$ in a line is $P\prime = 2H - P$, where $H$ is the perpendicular projection of $P$ onto that line.
- If $l, m, n$ are direction cosines of a line, then $l^2 + m^2 + n^2 = 1$. The signs must match the given acute or obtuse angles with the positive coordinate axes.

**Step 1:** Project the given point onto the reflecting line Write a general point of the line as $H = (t, 1 + 2t, 2 + 3t)$, whose direction vector is $d = (1, 2, 3)$. For $P = (1, 0, 7)$, perpendicularity gives

$$(P - H) \cdot d = (1 - t) + 2(-1 - 2t) + 3(5 - 3t) = 14 - 14t = 0.$$

Thus $t = 1$ and $H = (1, 3, 5)$. Since $H$ is the midpoint of the original point and its reflected image,

$$(\alpha, \beta, \gamma) = 2(1, 3, 5) - (1, 0, 7) = (1, 6, 3).$$

**Step 2:** Find the required direction ratios The direction cosines with the positive $y$- and $z$-axes are

$$m = \cos(2\pi/3) = -\frac{1}{2}, \qquad n = \cos(3\pi/4) = -\frac{1}{\sqrt{2}}.$$

Consequently $l^2 = 1 - 1/4 - 1/2 = 1/4$. The angle with the positive $x$-axis is acute, so $l = 1/2$. The direction ratios can therefore be taken as $(1, -1, -\sqrt{2})$.

**Step 3:** Test the listed points on the line The line through the image is

$$(x, y, z) = (1, 6, 3) + s(1, -1, -\sqrt{2}).$$

For option (C), $x = 3$ gives $s = 2$, and the other coordinates become $y = 4$, $z = 3 - 2\sqrt{2}$, exactly as listed. Both (A) and (D) have $x = 1$, which would require $s = 0$ and $y = 6$, contrary to their listed values. Option (B) has $x = 3$, but its $y = -4$ differs from the necessary value 4. Thus only (C) satisfies all coordinates.

**Step 4:** Conclusion The point is $\boxed{(3, 4, 3 - 2\sqrt{2})}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Find the projection first and reflect using twice the foot minus the original point; then use the acute-angle condition to fix the remaining direction-cosine sign.

**6. Let $A$ and $B$ be two finite sets with $m$ and $n$ elements respectively. The total number of subsets of the set $A$ is 56 more than the total number of subsets of $B$. Then the distance of the point $P(m, n)$ from the point $Q(-2, -3)$ is :** [4 marks]

(A) 8

(B) 10

(C) 4

(D) 6

**Correct Answer:** (B) 10

**Solution:**

**Concept:**

- A finite set with $k$ elements has $2^k$ subsets, including the empty set and the whole set. Each element is independently included or excluded.
- Factor a difference of powers of two to separate its power-of-two factor from an odd factor. The distance between $(x_1, y_1)$ and $(x_2, y_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

**Step 1:** Translate the subset count into an equation The total numbers of subsets of $A$ and $B$ are $2^m$ and $2^n$, respectively. Since $A$ has 56 more subsets,

$$2^m - 2^n = 56.$$

The right-hand side is positive, so $m > n$. Hence $m - n$ is a positive integer and we may factor the equation as

$$2^n \left(2^{m-n} - 1\right) = 56 = 2^3 \cdot 7.$$

**Step 2:** Determine the two set sizes uniquely For positive integer $m - n$, $2^{m-n}$ is even. Therefore $2^{m-n} - 1$ is odd, so all factors of two on the left come from $2^n$. Comparing the exact powers of two on both sides gives $n = 3$. The odd factors then give

$$2^{m-3} - 1 = 7 \quad \Longrightarrow \quad 2^{m-3} = 8 = 2^3,$$

so $m = 6$. A check gives $2^6 - 2^3 = 64 - 8 = 56$, confirming the original subset condition.

**Step 3:** Calculate the distance between the specified points Thus $P = (m, n) = (6, 3)$, while $Q = (-2, -3)$. Their horizontal and vertical differences are 8 and 6. The straight-line distance is

$$PQ = \sqrt{(6 + 2)^2 + (3 + 3)^2} = \sqrt{64 + 36} = 10.$$

Option (A), 8, gives only the horizontal difference; option (D), 6, gives only the vertical difference. Option (C), 4, is neither the distance nor either coordinate difference. Both coordinates must be included in the Euclidean distance.

**Step 4:** Conclusion The distance is $\boxed{10}$ coordinate units. Hence, the correct option is (B).

**Quick Tip to Solve:** In $2^m - 2^n = c$ with $m > n$, the exact power of two dividing $c$ determines $n$, because the remaining factor is odd.

**7. If $\alpha, \beta$ are the roots of the equation, $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then :** [4 marks]

(A) $2S_{12} = S_{11} + S_{10}$

(B) $S_{12} = S_{11} + S_{10}$

(C) $S_{11} = S_{10} + S_{12}$

(D) $2S_{11} = S_{12} + S_{10}$

**Correct Answer:** (B) $S_{12} = S_{11} + S_{10}$

**Solution:**

**Concept:**

- Each root $r$ of $x^2 - x - 1 = 0$ satisfies $r^2 = r + 1$. Multiplication by a nonnegative integer power of $r$ gives a relation between three successive powers.
- A fixed linear combination of two sequences obeying the same linear recurrence obeys that recurrence as well. The coefficients 2023 and 2024 do not change the recurrence.

**Step 1:** Obtain the recurrence for each root Since $\alpha$ and $\beta$ solve the given quadratic,

$$\alpha^2 = \alpha + 1, \qquad \beta^2 = \beta + 1.$$

Multiply the first equation by $\alpha^{10}$ and the second by $\beta^{10}$. This yields

$$\alpha^{12} = \alpha^{11} + \alpha^{10}, \qquad \beta^{12} = \beta^{11} + \beta^{10}.$$

There is no need to substitute explicit radical expressions for the roots.

**Step 2:** Combine using the fixed coefficients Multiply these identities by 2023 and 2024, respectively, and add. Grouping equal powers gives

$$\begin{aligned} 2023\alpha^{12} + 2024\beta^{12} = &(2023\alpha^{11} + 2024\beta^{11}) \\ &+ (2023\alpha^{10} + 2024\beta^{10}). \end{aligned}$$

Using the definition of $S_n$, this is exactly

$$S_{12} = S_{11} + S_{10}.$$

**Step 3:** Exclude the remaining equations To see that the other alternatives do not hold accidentally, note that the roots are $(1 \pm \sqrt{5})/2$. One root is greater than 1 and the other has absolute value less than 1. Since the coefficients are positive and $\alpha^{10}, \beta^{10} > 0$, we have $S_{10} > 0$; also $S_{12} > 0$. Therefore (A) would force $S_{12} = 0$, and (C) would force $S_{10} = 0$, both impossible. Option (D), combined with the recurrence, would require $S_{11} = 2S_{10}$. For either root $r < 2$, and with even tenth powers, each term in $S_{11} - 2S_{10} = 2023\alpha^{10}(\alpha - 2) + 2024\beta^{10}(\beta - 2)$ is negative. Thus (D) also fails.

**Step 4:** Conclusion The required relation is $\boxed{S_{12} = S_{11} + S_{10}}$. Hence, the correct option is (B).

**Quick Tip to Solve:** For a weighted sum of powers of quadratic roots, multiply the root equation by a suitable power; this produces the recurrence without evaluating the roots.

**8. Let $e_1$ be the eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and $e_2$ be the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \mathrm{a} > \mathrm{b}$, which passes through the foci of the hyperbola. If $\mathrm{e_1 e_2} = 1$, then the length of the chord of the ellipse parallel to the $x$-axis and passing through $(0, 2)$ is :** [4 marks]

(A) $\frac{8\sqrt{5}}{3}$

(B) $3\sqrt{5}$

(C) $4\sqrt{5}$

(D) $\frac{10\sqrt{5}}{3}$

**Correct Answer:** (D) $\frac{10\sqrt{5}}{3}$

**Solution:**

**Concept:**

- For the hyperbola $x^2/A^2 - y^2/B^2 = 1$, the focal distance is $c = \sqrt{A^2 + B^2}$ and eccentricity is $e = c/A$.
- For an ellipse with major axis along the $x$-axis, $e^2 = 1 - b^2/a^2$. A horizontal chord at height $y_0$ has length $2a\sqrt{1 - y_0^2/b^2}$.

**Step 1:** Find the hyperbola's foci Here the transverse semi-axis is $A = 4$, and $B = 3$. Hence

$$c^2 = 16 + 9 = 25, \qquad c = 5, \qquad e_1 = \frac{5}{4}.$$

The foci are $(5, 0)$ and $(-5, 0)$. They lie on the ellipse, so substitution into its equation gives $25/a^2 = 1$. Since semi-axes are positive, $a = 5$. The foci of the hyperbola are points on the ellipse, not necessarily its foci.

**Step 2:** Determine the ellipse The eccentricity condition gives

$$e_2 = \frac{1}{e_1} = \frac{4}{5}.$$

Therefore

$$\frac{16}{25} = 1 - \frac{b^2}{25}, \qquad b^2 = 9.$$

Thus the ellipse is $x^2/25 + y^2/9 = 1$. Its smaller semi-axis is 3, so the specified line $y = 2$ genuinely meets it in two distinct points.

**Step 3:** Find both endpoints of the chord A chord parallel to the $x$-axis through $(0, 2)$ lies on $y = 2$. Substitute this ordinate:

$$\frac{x^2}{25} + \frac{4}{9} = 1, \qquad x^2 = \frac{125}{9}, \qquad x = \pm\frac{5\sqrt{5}}{3}.$$

The endpoints have the same ordinate, so their distance is the difference of their abscissae, namely $10\sqrt{5}/3$. Options (A), (B) and (C) are different from this endpoint separation; using only the positive abscissa would give merely half the chord.

**Step 4:** Conclusion The required chord length is $\boxed{10\sqrt{5}/3}$ units. Hence, the correct option is (D).

**Quick Tip to Solve:** First use the hyperbola's foci to fix the ellipse's major semi-axis; then set the chord's ordinate and double the positive abscissa.

**9.**

**The $20^{\text{th}}$ term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, -129\frac{1}{4}$ is :**

[4 marks]

(A) $-115$

(B) $-100$

(C) $-110$

(D) $-118$

**Correct Answer:** (A) $-115$

**Solution:**

**Concept:**

- In an arithmetic progression, adjacent terms differ by a constant $d$. Counting from the last term reverses the sign of this difference.
- The $r$th term from the end is $l-(r-1)d$, where $l$ is the last term and $d$ is the forward common difference.

**Step 1:** Identify the forward difference Write the mixed fractions as improper fractions so that their signs remain clear. The first two terms are $20 = 80/4$ and $19\frac{1}{4} = 77/4$. Thus

$$d = \frac{77}{4} - \frac{80}{4} = -\frac{3}{4}.$$

The next two terms are $74/4$ and $71/4$, confirming the same difference. The last term is

$$l = -129\frac{1}{4} = -\frac{517}{4}.$$

Here the minus sign applies to the entire mixed number.

**Step 2:** Count nineteen backward steps The last term itself is the first term when counting from the end. Reaching the twentieth therefore takes nineteen steps, each adding $3/4$:

$$T_{20,\text{end}} = l - 19d = -\frac{517}{4} - 19\left(-\frac{3}{4}\right).$$

Hence

$$T_{20,\text{end}} = \frac{-517+57}{4} = -\frac{460}{4} = -115.$$

Subtracting $3/4$ while moving backward would incorrectly make the terms still more negative.

**Step 3:** Check its position and the alternatives For a second check, the total number $N$ of terms satisfies

$$-\frac{517}{4} = 20 + (N-1)\left(-\frac{3}{4}\right), \qquad N-1 = 199, \qquad N = 200.$$

The twentieth from the end is therefore the 181st from the start, whose value is $20 - 180(3/4) = -115$. Thus (A) is confirmed. The values $-100$, $-110$ and $-118$ do not result from nineteen backward differences and cannot be the requested term.

**Step 4:** Conclusion The twentieth term from the end is $\boxed{-115}$. Hence, the correct option is (A).

**Quick Tip to Solve:** When counting backward in an AP, start at the last term and reverse the common difference; the twentieth term needs nineteen steps.

**10. Let $f : \mathbf{R} - \left\{\frac{-1}{2}\right\} \to \mathbf{R}$ and $g : \mathbf{R} - \left\{\frac{-5}{2}\right\} \to \mathbf{R}$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then, the domain of the function fog is :** [4 marks]

(A) $\mathbf{R} - \left\{-\frac{7}{4}\right\}$

(B) $\mathbf{R}$

(C) $\mathbf{R} - \left\{-\frac{5}{2}, -\frac{7}{4}\right\}$

(D) $\mathbf{R} - \left\{-\frac{5}{2}\right\}$

**Correct Answer:** (D) $\mathbf{R} - \left\{-\frac{5}{2}\right\}$

**Solution:**

**Concept:**

- The composite $(f \circ g)(x) = f(g(x))$ requires both $x \in \mathrm{Dom}(g)$ and $g(x) \in \mathrm{Dom}(f)$.
- Handle the absolute value separately for $x \geq 0$ and $x < 0$; a root obtained in one branch must satisfy that branch's sign condition.

**Step 1:** Apply the inner function's restriction The denominator of $g(x) = (|x| + 1)/(2x + 5)$ must be nonzero, so

$$x \neq -\frac{5}{2}.$$

The outer function $f(u) = (2u + 3)/(2u + 1)$ is undefined only at $u = -1/2$. Consequently we must check whether any permitted $x$ makes $g(x) = -1/2$; merely excluding the original pole is not sufficient without this check.

**Step 2:** Test whether the forbidden outer input occurs For $x \neq -5/2$, cross-multiplication is valid and gives

$$\frac{|x| + 1}{2x + 5} = -\frac{1}{2} \quad \Longleftrightarrow \quad 2|x| + 2x = -7.$$

If $x < 0$, the left side is $2(-x) + 2x = 0$, never $-7$. If $x \geq 0$, it is $4x \geq 0$, also never $-7$. Thus $-1/2$ is not attained by $g$, and the outer function removes no further input.

**Step 3:** Reject the apparent extra exclusion Ignoring the absolute value would give $4x = -7$, hence $x = -7/4$. But that calculation used $|x| = x$, which requires $x \geq 0$, so the negative candidate is invalid. Indeed, at $x = -7/4$,

$$g(x) = \frac{7/4 + 1}{-7/2 + 5} = \frac{11}{6},$$

an allowed input to $f$. Option (A) both excludes this legitimate point and admits the pole; (B) admits the pole; (C) unnecessarily excludes $-7/4$. Only (D) retains every allowed input and removes the actual pole.

**Step 4:** Conclusion The domain is $\boxed{\mathbb{R} \setminus \{-5/2\}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For a composition, check the inner pole first, then solve for the outer forbidden input; reject any absolute-value branch root with the wrong sign.

**11.**

**If** $\lim\limits_{x\to 0}\dfrac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\dfrac{1}{3}$**, then** $2\alpha-\beta$ **is equal to :**

[4 marks]

(A) 2

(B) 1

(C) 5

(D) 7

**Correct Answer:** (C) 5

**Solution:**

**Concept:**

- A finite limit with a denominator of order $x^2$ requires the numerator's constant and linear terms to vanish.
- Near zero, $\sin x = x + O(x^3)$, $\cos x = 1 - x^2/2 + O(x^4)$, $\log(1-x) = -x - x^2/2 + O(x^3)$, and $\tan^2 x = x^2 + O(x^4)$.

**Step 1:** Cancel the constant and linear terms The logarithm is defined in a neighbourhood of zero because $1 - x > 0$ there. Expand the numerator only as far as the denominator requires:

$$3+\alpha\sin x+\beta\cos x+\log(1-x) = (3+\beta)+(\alpha-1)x - \frac{\beta+1}{2}x^2+O(x^3).$$

The denominator is $3x^2 + O(x^4)$. A nonzero constant or linear coefficient would prevent the stated finite limit. Therefore

$$3+\beta=0, \qquad \alpha-1=0,$$

giving $\beta=-3$ and $\alpha=1$.

**Step 2:** Verify the prescribed quadratic ratio With these values, the numerator's quadratic coefficient is

$$-\frac{\beta+1}{2}=-\frac{-3+1}{2}=1.$$

Thus the fraction becomes

$$\frac{x^2+O(x^3)}{3x^2+O(x^4)}\longrightarrow\frac{1}{3},$$

exactly as required. This check matters: cancelling the two lower-order terms alone would not establish agreement with an arbitrary limit value. It also confirms that the minus sign in $\log(1-x)$ has been handled correctly.

**Step 3:** Evaluate the requested expression Finally,

$$2\alpha-\beta=2(1)-(-3)=5.$$

This is option (C). The numbers 2, 1 and 7 in the other options do not equal the expression for the uniquely forced pair $(\alpha,\beta)=(1,-3)$; retaining either would contradict the coefficient conditions needed for a finite limit.

**Step 4:** Conclusion We obtain $\alpha = 1$, $\beta = -3$, and $\boxed{2\alpha - \beta = 5}$. Hence, the correct option is (C).

**Quick Tip to Solve:** When the denominator starts with $x^2$, cancel the numerator's constant and linear terms before comparing quadratic coefficients.

**12. If $y = y(x)$ is the solution curve of the differential equation $\left(x^2 - 4\right) \mathrm{d}y - \left(y^2 - 3y\right) \mathrm{d}x = 0, x > 2, y(4) = \frac{3}{2}$ and the slope of the curve is never zero, then the value of $y(10)$ equals :** [4 marks]

(A) $\frac{3}{1+(8)^{1/4}}$

(B) $\frac{3}{1-(8)^{1/4}}$

(C) $\frac{3}{1-2\sqrt{2}}$

(D) $\frac{3}{1+2\sqrt{2}}$

**Correct Answer:** (A) $\frac{3}{1+(8)^{1/4}}$

**Solution:**

**Concept:**

- The differential equation is separable: $dy/[y(y-3)] = dx/(x^2-4)$. Nonzero slope excludes the equilibrium values $y = 0, 3$.
- Use partial fractions and retain the sign fixed by the initial condition when removing logarithmic absolute values.

**Step 1:** Separate and integrate Since $x > 2$, $x^2 - 4$ is positive. The initial value $y(4) = 3/2$ lies between the two equilibrium solutions, so the nonconstant solution stays in $0 < y < 3$ on its interval. Partial fractions give

$$\frac{1}{y(y-3)} = \frac{1}{3}\left(\frac{1}{y-3} - \frac{1}{y}\right),$$

and

$$\frac{1}{x^2-4} = \frac{1}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right).$$

Integration therefore yields

$$\frac{1}{3}\log\left|\frac{y-3}{y}\right| = \frac{1}{4}\log\frac{x-2}{x+2} + C.$$

**Step 2:** Use the initial condition and choose the sign Multiplying by three and exponentiating gives

$$\left|\frac{y-3}{y}\right| = K\left(\frac{x-2}{x+2}\right)^{3/4}.$$

At $x = 4$, the left side is 1 and the fraction on the right is $1/3$, so $K = 3^{3/4}$. As $0 < y < 3$, the absolute value equals $(3-y)/y$. Hence

$$\frac{3-y}{y} = \left(\frac{3(x-2)}{x+2}\right)^{3/4}, \qquad y = \frac{3}{1 + (3(x-2)/(x+2))^{3/4}}.$$

This positive branch satisfies the initial value.

**Step 3:** Substitute the requested abscissa At $x = 10$, the quantity in parentheses is $3 \cdot 8/12 = 2$. Thus

$$y(10) = \frac{3}{1 + 2^{3/4}} = \frac{3}{1 + 8^{1/4}}.$$

The denominator has a plus sign; the minus-sign alternatives (B) and (C) would give negative values and do not satisfy the chosen branch. Option (D) uses $2^{3/2}$ instead of $2^{3/4}$, contradicting the integrated exponent.

**Step 4:** Conclusion The requested value is $\boxed{y(10) = 3/(1 + 8^{1/4})}$. Hence, the correct option is (A).

**Quick Tip to Solve:** In a separable equation with logarithms, use the initial value both to determine the constant and to choose the correct sign after exponentiating.

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**13. If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, for the least value of $n \in \mathbf{N}$, then $\sum_{k=1}^{n} \frac{k}{2^k}$ is equal to:** [4 marks]

(A) $\frac{1}{2^{14}}\left(2^{15} - 15\right)$

(B) $1 - \frac{15}{2^{13}}$

(C) $\frac{1}{2^{15}}\left(2^{14} - 14\right)$

(D) $\frac{1}{2^{13}}\left(2^{14} - 15\right)$

**Correct Answer:** (D) $\frac{1}{2^{13}}\left(2^{14} - 15\right)$

**Solution:**

**Concept:**

- Use $\tan^2\theta = \sec^2\theta - 1$ where the original equation is defined, and reject secant values whose magnitude is below one.
- For $0 < r < 1$, differentiating the finite geometric sum gives $\sum_{k=1}^{n} kr^k = r[1 - (n+1)r^n + nr^{n+1}]/(1-r)^2$.

**Step 1:** Reduce the trigonometric equation Put $s = \sec\theta$. The equation becomes

$$2(s^2 - 1) - 5s = 1, \qquad 2s^2 - 5s - 3 = (2s + 1)(s - 3) = 0.$$

The root $s = -1/2$ is impossible for real $\theta$, since it would mean $\cos\theta = -2$. Therefore only $\cos\theta = 1/3$ remains. If $\phi = \arccos(1/3)$, then $0 < \phi < \pi/2$, and the solutions are $2m\pi \pm \phi$ wherever they lie in the interval.

**Step 2:** Count until the seventh solution appears In increasing nonnegative order the first seven solutions are

$$\phi,\ 2\pi - \phi,\ 2\pi + \phi,\ 4\pi - \phi,\ 4\pi + \phi,\ 6\pi - \phi,\ 6\pi + \phi.$$

At $n = 12$, the upper endpoint is $6\pi$, so exactly six of them are included. At $n = 13$, it is $13\pi/2 = 6\pi + \pi/2$, which includes the seventh because $\phi < \pi/2$. The eighth, $8\pi - \phi > 15\pi/2$, is still excluded. Thus the least natural number giving exactly seven is $n = 13$.

**Step 3:** Evaluate the finite weighted geometric sum Using the finite-sum formula with $r = 1/2$,

$$\sum_{k=1}^{n} \frac{k}{2^k} = 2 - \frac{n+2}{2^n}.$$

Hence for $n = 13$,

$$S = 2 - \frac{15}{2^{13}} = \frac{2^{14} - 15}{2^{13}}.$$

This is (D). Option (A) instead gives $2 - 15/2^{14}$, with the wrong denominator; (B) has leading term 1; (C) is close to $1/2$. None equals the computed finite sum.

**Step 4:** Conclusion The least value is $n = 13$, and the sum is $\boxed{(2^{14} - 15)/2^{13}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** List the roots around successive multiples of $2\pi$; then identify the first allowed half-$\pi$ endpoint beyond the seventh root.

**14. Let $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$ and $f''(x) > 0$ for all $x \in (0,3)$. If $g$ is decreasing in $(0,\alpha)$ and increasing in $(\alpha,3)$, then $8\alpha$ is :** [4 marks]

(A) 0

(B) 24

(C) 18

(D) 20

**Correct Answer:** (C) 18

**Solution:**

**Concept:**

- The condition $f''(x) > 0$ means that $f'$ is strictly increasing on $(0,3)$.
- Apply the chain rule to each composite term. For a strictly increasing function, the sign of $f'(u) - f'(v)$ equals the sign of $u - v$.

**Step 1:** Differentiate with the correct chain factors For $0 < x < 3$, the arguments $x/3$ and $3 - x$ both lie in $(0,3)$, so the given condition applies to both. Differentiate:

$$\begin{aligned} g'(x) &= 3f'(x/3) \cdot \frac{1}{3} + f'(3-x) \cdot (-1) \\ &= f'(x/3) - f'(3-x). \end{aligned}$$

The prefactor 3 in the first term cancels its inner derivative, while the second term has a minus sign.

**Step 2:** Locate the unique sign change Because $f'$ is strictly increasing, its values are equal exactly when their arguments are equal:

$$\frac{x}{3} = 3 - x \quad \Longleftrightarrow \quad 4x = 9 \quad \Longleftrightarrow \quad x = \frac{9}{4}.$$

For $0 < x < 9/4$, we have $x/3 < 3 - x$, hence $g'(x) < 0$. For $9/4 < x < 3$, the inequality reverses, hence $g'(x) > 0$. This establishes exactly the decreasing-then-increasing pattern, without needing an explicit formula for $f$.

**Step 3:** Evaluate and check the choices The transition point is $\alpha = 9/4$, which lies inside the stated interval $(0, 3)$. Therefore

$$8\alpha = 8 \cdot \frac{9}{4} = 18.$$

Options (A) and (B) would place the transition at the boundary values 0 and 3, although the derivative actually changes sign at an interior point. Option (D) would give $\alpha = 5/2$, where the two arguments are $5/6$ and $1/2$, so strict increase of $f'$ makes $g' > 0$, not zero.

**Step 4:** Conclusion The minimum occurs at $\alpha = 9/4$, giving $\boxed{8\alpha = 18}$. Hence, the correct option is (C).

**Quick Tip to Solve:** When $f'$ is strictly increasing, compare its input arguments directly instead of trying to determine the unknown function.

**15. Let R be the interior region between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of $a$, for which the points $(a^2, a + 1)$ lie in $R$, is :** [4 marks]

(A) $(-3, 0) \cup \left(\frac{2}{3}, 1\right)$
(B) $(-3, 0) \cup \left(\frac{1}{3}, 1\right)$
(C) $(-3, -1) \cup \left(\frac{1}{3}, 1\right)$
(D) $(-3, -1) \cup \left(-\frac{1}{3}, 1\right)$
**Correct Answer:** (B) $(-3, 0) \cup \left(\frac{1}{3}, 1\right)$

**Solution:**

**Concept:**

- A straight line divides the plane into two open half-planes. Substitute the origin into each line expression to identify the sides containing it.
- The word interior excludes the bounding lines. After substituting the moving point, solve the resulting strict quadratic inequalities and take their intersection.

**Step 1:** Identify the required sides For the origin, $3x - y + 1 = 1 > 0$, whereas $x + 2y - 5 = -5 < 0$. Thus the region containing it is described by $3x - y + 1 > 0$ and $x + 2y - 5 < 0$. The inequalities are strict because points on either line are boundary points, not interior points.

**Step 2:** Substitute the coordinates of the point Put $x = a^2$ and $y = a + 1$. The first condition gives

$$3a^2 - (a + 1) + 1 = a(3a - 1) > 0,$$

so $a < 0$ or $a > \frac{1}{3}$. The second condition gives

$$a^2 + 2(a + 1) - 5 = (a + 3)(a - 1) < 0,$$

so $-3 < a < 1$. A quadratic with positive leading coefficient is negative between its distinct roots and

positive outside them.

**Step 3:** Intersect the intervals and check the choices Both conditions must hold simultaneously. Intersecting $(-\infty, 0) \cup (\frac{1}{3}, \infty)$ with $(-3, 1)$ yields

$$(-3, 0) \cup \left(\frac{1}{3}, 1\right).$$

Each endpoint is excluded: at $-3$ or $1$ the point lies on the second line, and at $0$ or $\frac{1}{3}$ it lies on the first. Option (A) wrongly replaces the lower positive endpoint by $\frac{2}{3}$; (C) wrongly removes admissible negative values between $-1$ and $0$; (D) includes values that violate the first inequality.

**Step 4:** Conclusion The required set is $\boxed{(-3, 0) \cup (\frac{1}{3}, 1)}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Use a test point to fix each half-plane sign before substituting the parameter. Keep the inequalities strict when the region is described as interior.

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**16. Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$. Then :** [4 marks]

(A) $\alpha \in \mathbf{N}$ and $\beta \in \mathbf{N}$

(B) $\alpha \in \mathbf{N}$ and $\beta \notin \mathbf{N}$

(C) $\alpha \notin \mathbf{N}$ and $\beta \in \mathbf{N}$

(D) $\alpha \notin \mathbf{N}$ and $\beta \notin \mathbf{N}$

**Correct Answer:** (A) $\alpha \in \mathbf{N}$ and $\beta \in \mathbf{N}$

**Solution:**

**Concept:**

- For positive integers $m, n$, the multinomial coefficient $\dfrac{(mn)!}{(m!)^n}$ counts assignments of $mn$ distinct objects to $n$ labelled groups of size $m$, so it is a positive integer.
- A factorial in an exponent is evaluated first: $3! = 6$ and $4! = 24$. These exponents are numbers of groups, not powers to be approximated.

**Step 1:** Express alpha as a counting number Since $4! = 24$ and $3! = 6$,

$$\alpha = \frac{24!}{(4!)^6}.$$

Take 24 distinct objects and divide them among six labelled boxes, each containing four objects. Choosing the boxes successively gives

$$\binom{24}{4}\binom{20}{4}\binom{16}{4}\binom{12}{4}\binom{8}{4}\binom{4}{4} = \frac{24!}{(4!)^6}.$$

Every binomial coefficient is a positive integer, hence so is their product.

**Step 2:** Apply the same interpretation to beta Here $5! = 120$ and $4! = 24$, so

$$\beta = \frac{120!}{(5!)^{24}}.$$

This counts ways of distributing 120 distinct objects into 24 labelled boxes of five objects each. Equivalently, it is

$$\binom{120}{5}\binom{115}{5}\cdots\binom{10}{5}\binom{5}{5},$$

which is again a product of positive integers. The group sizes sum to 120, so there is no leftover object and the multinomial interpretation applies exactly.

**Step 3:** Select the joint statement Both ratios count nonempty finite collections and therefore belong to $\mathbf{N}$. Option (B) incorrectly denies the integrality of $\beta$, option (C) incorrectly denies that of $\alpha$, and option (D) incorrectly denies both. No evaluation of the enormous factorials is needed; the counting interpretation proves divisibility exactly.

**Step 4:** Conclusion $\boxed{\alpha \in \mathbf{N} \text{ and } \beta \in \mathbf{N}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** When a factorial ratio resembles $(mn)!/(m!)^n$, look for equally sized labelled groups. A multinomial interpretation proves integrality without large-number calculation.

**17.**

**The integral** $\displaystyle\int \frac{\left(x^8 - x^2\right)\mathrm{d}x}{\left(x^{12} + 3x^6 + 1\right)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$ **is equal to :**

[4 marks]

(A) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/3} + C$

(B) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right) + C$

(C) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/2} + C$

(D) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{3} + C$

**Correct Answer:** (A) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/3} + C$

**Solution:**

**Concept:**

- If the integrand has the form $u\prime(x)/u(x)$, its antiderivative is $\log_e|u(x)| + C$ on each interval where $u \neq 0$.
- The derivative of $\tan^{-1} v$ is $v\prime/(1+v^2)$. Here choose $v = x^3 + x^{-3}$, with $x \neq 0$.

**Step 1:** Differentiate the inverse tangent Let $u = \tan^{-1}(x^3 + x^{-3})$. Since

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^3 + x^{-3}) = 3x^2 - 3x^{-4},$$

the chain rule gives

$$\frac{du}{dx} = \frac{3x^2 - 3x^{-4}}{1 + (x^3 + x^{-3})^2}.$$

Expanding the denominator and multiplying its numerator and denominator by $x^6$ puts this into the useful form

$$\frac{du}{dx} = \frac{3(x^8 - x^2)}{x^{12} + 3x^6 + 1}.$$

**Step 2:** Integrate the logarithmic derivative The original integral is therefore

$$I = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\log_e |u| + C.$$

Returning to $x$,

$$I = \log_e \left(\left|\tan^{-1}(x^3 + x^{-3})\right|^{1/3}\right) + C.$$

For real nonzero $x$, $x^3 + x^{-3}$ cannot vanish, so the logarithm is defined. The integration constant can be chosen separately on the intervals $x > 0$ and $x < 0$.

**Step 3:** Verify the coefficient against the options Differentiating $\frac{1}{3}\log_e |u|$ gives $u\prime/(3u)$, exactly the printed integrand. The logarithm of $|u|^k$ has derivative $ku\prime/u$. Options (B), (C), and (D) have coefficients $1$, $\frac{1}{2}$, and $3$, respectively, whereas the required coefficient is $\frac{1}{3}$. This comparison also checks that the factor 3 introduced by differentiation was not lost.

**Step 4:** Conclusion The integral is $\boxed{\log_e \left(\left|\tan^{-1}(x^3 + x^{-3})\right|^{1/3}\right) + C}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Differentiate the nested inverse tangent before trying substitutions. Clear its negative powers with $x^6$ to reveal the numerator already present in the integral.

**18. The values of $\alpha$, for which** $\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$**, lie in the interval** [4 marks]

(A) $(-2, 1)$

(B) $\left(-\frac{3}{2}, \frac{3}{2}\right)$

(C) $(-3, 0)$

(D) $(0, 3)$

**Correct Answer:** (C) $(-3, 0)$

**Solution:**

**Concept:**

- Adding a multiple of one column to another leaves a determinant unchanged. A column operation that creates two zero entries makes expansion straightforward.
- All real roots must belong to the proposed interval. An interval containing only one root does not satisfy the statement about the values of $\alpha$.

**Step 1:** Create zeros in the third column Denote the determinant by $D$ and perform $C_3 \leftarrow C_3 - C_2 - \alpha C_1$. The first two entries of the new third column are zero, because $(\alpha + \frac{3}{2}) - \frac{3}{2} - \alpha = 0$ and $(\alpha + \frac{1}{3}) - \frac{1}{3} - \alpha = 0$. Its last entry is

$$0 - (3\alpha + 1) - \alpha(2\alpha + 3) = -(2\alpha^2 + 6\alpha + 1).$$

The other two columns remain unchanged.

**Step 2:** Expand and solve the resulting quadratic Expanding along the third column, whose bottom entry has positive cofactor sign, gives

$$D = -(2\alpha^2 + 6\alpha + 1)\begin{vmatrix} 1 & 3/2 \\ 1 & 1/3 \end{vmatrix}.$$

The minor is $\frac{1}{3} - \frac{3}{2} = -\frac{7}{6}$, so

$$D = \frac{7}{6}(2\alpha^2 + 6\alpha + 1).$$

Consequently $D = 0$ exactly when

$$\alpha = \frac{-6 \pm \sqrt{36 - 8}}{4} = \frac{-3 \pm \sqrt{7}}{2}.$$

**Step 3:** Locate both roots Since $2 < \sqrt{7} < 3$, the smaller root lies strictly between $-3$ and $-\frac{5}{2}$, and the larger lies strictly between $-\frac{1}{2}$ and $0$. Thus both lie in $(-3, 0)$. The smaller root is below $-2$, excluding option (A), and below $-\frac{3}{2}$, excluding (B). Both roots are negative, excluding (D). Neither root equals an interval endpoint.

**Step 4:** Conclusion The two values are $\boxed{\alpha = (-3 \pm \sqrt{7})/2}$, both in $(-3, 0)$. Hence, the correct option is (C).

**Quick Tip to Solve:** Look for a column that equals a simple linear combination of the others in two rows. Making those entries zero reduces a three-by-three determinant to one minor.

**19. For $0 < a < 1$, the value of the integral $\int_0^{\pi} \frac{\mathrm{d}x}{1 - 2a\cos x + a^2}$ is:** [4 marks]

(A) $\frac{\pi^2}{\pi + a^2}$
(B) $\frac{\pi^2}{\pi - a^2}$
(C) $\frac{\pi}{1 - a^2}$
(D) $\frac{\pi}{1 + a^2}$

**Correct Answer:** (C) $\frac{\pi}{1 - a^2}$

**Solution:**

**Concept:**

- The substitution $t = \tan(x/2)$ gives $\cos x = (1 - t^2)/(1 + t^2)$ and $\mathrm{d}x = 2\,\mathrm{d}t/(1 + t^2)$.

- For positive $A, B$, $\int_0^\infty \mathrm{d}t/(A^2 + B^2t^2) = \pi/(2AB)$. Here $1 - a$ and $1 + a$ are both positive.

**Step 1:** Transform the limits and denominator As $x$ runs from $0$ to $\pi$, $t = \tan(x/2)$ runs from $0$ to $+\infty$. The denominator becomes

$$1 - 2a\frac{1 - t^2}{1 + t^2} + a^2 = \frac{(1 - a)^2 + (1 + a)^2t^2}{1 + t^2}.$$

Cancelling the common factor with that from $\mathrm{d}x$ yields

$$I = 2\int_0^\infty \frac{\mathrm{d}t}{(1 - a)^2 + (1 + a)^2t^2}.$$

The original denominator is positive because it equals $(1 - a)^2 + 2a(1 - \cos x)$.

**Step 2:** Evaluate the elementary integral Set $v = \frac{1+a}{1-a}t$, which preserves the limits since $0 < a < 1$. Then

$$I = \frac{2}{(1 - a)(1 + a)}\int_0^\infty \frac{\mathrm{d}v}{1 + v^2}.$$

Using $\tan^{-1}(0) = 0$ and $\tan^{-1}(+\infty) = \pi/2$, we obtain

$$I = \frac{2}{1 - a^2}\left[\tan^{-1} v\right]_0^\infty = \frac{\pi}{1 - a^2}.$$

**Step 3:** Check the alternatives The exact expression has denominator $1 - a^2$, selecting (C). Option (D) is less than $\pi$ for positive $a$, whereas the result is greater than $\pi$. Option (A) likewise gives less than $\pi$. Option (B), although greater than $\pi$, has the wrong dependence on $a$: as $a$ approaches 1 from below the actual integral diverges, while (B) stays finite. These checks agree with the evaluated integral.

**Step 4:** Conclusion $\boxed{\int_0^\pi \frac{\mathrm{d}x}{1 - 2a\cos x + a^2} = \frac{\pi}{1 - a^2}}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For a denominator linear in $\cos x$, the tangent half-angle substitution often turns the integral into $1/(A^2 + B^2t^2)$. Check signs before evaluating the inverse tangent.

**20. The position vectors of the vertices $\mathrm{A}, \mathrm{B}$ and $\mathrm{C}$ of a triangle are $2\hat{i}-3\hat{j}+3\hat{k}, 2\hat{i}+2\hat{j}+3\hat{k}$ and $-\hat{i}+\hat{j}+3\hat{k}$ respectively. Let $l$ denotes the length of the angle bisector $\mathrm{AD}$ of $\angle\mathrm{BAC}$ where $\mathrm{D}$ is on the line segment $\mathrm{BC}$, then $2l^2$ equals :** [4 marks]

(A) 45

(B) 50

(C) 42

(D) 49

**Correct Answer:** (A) 45

**Solution:**

**Concept:**

- The internal angle bisector satisfies $BD/DC = AB/AC$. Equal adjacent sides therefore make the angle bisector meet the opposite side at its midpoint.
- The squared length of a vector $(u, v, w)$ is $u^2+v^2+w^2$. Computing the squared length directly avoids unnecessary square roots.

**Step 1:** Compute the two sides adjacent to A The coordinates are $A=(2,-3,3)$, $B=(2,2,3)$, and $C=(-1,1,3)$. Subtracting the position vector of $A$ gives

$$\overrightarrow{AB}=(0,5,0), \qquad \overrightarrow{AC}=(-3,4,0).$$

Hence

$$AB=\sqrt{25}=5, \qquad AC=\sqrt{9+16}=5.$$

The third coordinates cancel, so all the calculations can be carried out in their common plane $z=3$.

**Step 2:** Locate the angle-bisector foot Because $AB=AC$, the angle bisector theorem gives $BD : DC = 1 : 1$. The point $D$ is therefore the midpoint of $BC$, not an arbitrary point on its line. Its position vector is

$$\vec{D}=\frac{\vec{B}+\vec{C}}{2}=\left(\frac{1}{2},\frac{3}{2},3\right).$$

Thus

$$\overrightarrow{AD}=\vec{D}-\vec{A}=\left(-\frac{3}{2},\frac{9}{2},0\right).$$

**Step 3:** Find the requested squared length Now

$$l^2=\left(-\frac{3}{2}\right)^2+\left(\frac{9}{2}\right)^2=\frac{9}{4}+\frac{81}{4}=\frac{45}{2}.$$

It follows that $2l^2=45$. As a check, $BC^2=(-3)^2+(-1)^2=10$; the median-length identity gives $l^2=25-10/4=45/2$ again. Therefore 50, 42, and 49 in options (B), (C), and (D) do not equal the required quantity. In particular, 50 would incorrectly ignore the subtraction arising from the nonzero base length.

**Step 4:** Conclusion $\boxed{2l^2=45}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Check the two sides adjacent to the bisected angle first. If they are equal, use the midpoint of the opposite side and calculate the squared vector length directly.

## Mathematics – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**21. The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12 . If $\mu$ and $\sigma^2$ denote the mean and variance of the correct observations respectively, then $15\left(\mu+\mu^2+\sigma^2\right)$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- For $n$ observations, $\bar{x}=\sum x_i/n$ and $\sigma^2=\sum x_i^2/n-\bar{x}^2$. Hence $n(\bar{x}^2+\sigma^2)=\sum x_i^2$.
- When a misread observation is corrected, subtract its old contribution and add its true contribution in both the sum and the sum of squares. The number of observations stays fixed.

**Step 1:** Recover the incorrect totals There are $n=15$ observations. The originally calculated mean 12 gives the incorrect sum

$$S_{\text{old}}=15\times 12=180.$$

Since the originally calculated standard deviation is 3, the incorrect variance is $3^2=9$. Thus the corresponding sum of squares is

$$T_{\text{old}}=15(12^2+9)=15\times 153=2295.$$

Squaring the standard deviation is essential before using the variance identity.

**Step 2:** Replace the misread observation The true observation is 12, but it was recorded as 10. Consequently the corrected totals are

$$S=180-10+12=182,$$

and

$$T=2295-10^2+12^2=2295-100+144=2339.$$

Only one observation is being replaced. There is no change to $n$, and the change in the sum of squares is 44 rather than the square of the change in the sum.

**Step 3:** Combine the moments efficiently The corrected mean is $\mu=182/15$, while the corrected variance satisfies

$$\sigma^2=\frac{2339}{15}-\left(\frac{182}{15}\right)^2.$$

In the requested combination, the squared mean cancels through the moment identity:

$$15(\mu+\mu^2+\sigma^2)=15\mu+15(\mu^2+\sigma^2)=S+T.$$

Substituting the corrected totals gives

$$S+T=182+2339=2521.$$

This uses the population variance convention implicit in the original standard-deviation formula.

**Step 4:** Conclusion The required numerical value is $\boxed{2521}$, a dimensionless number for the observations as given.

**Quick Tip to Solve:** Before computing a corrected variance explicitly, inspect the requested expression. Here $n(\mu^2+\sigma^2)$ is simply the corrected sum of squares, so only two totals are needed.

---

**22. The coefficient of $x^{2012}$ in the expansion of $(1-x)^{2008}\left(1+x+x^2\right)^{2007}$ is equal to _____.**

[4 marks]

**Solution:**

**Concept:**

- The identity $(1-x)(1+x+x^2)=1-x^3$ combines the two factors before a coefficient is extracted.
- In $(1-x^3)^n$, every power of $x$ is a multiple of three; multiplication by $1-x$ shifts these powers by either zero or one.

**Step 1:** Combine the factors The exponent of the first factor exceeds that of the second by one. Separate this one factor and use the difference-of-cubes identity:

$$\begin{aligned}(1-x)^{2008}(1+x+x^2)^{2007} &= (1-x)\big((1-x)(1+x+x^2)\big)^{2007}\\ &= (1-x)(1-x^3)^{2007}.\end{aligned}$$

This is an exact polynomial identity, so coefficient comparison is valid for every power.

**Step 2:** Identify the possible exponents The binomial expansion is

$$\sum_{k=0}^{2007}(-1)^k\binom{2007}{k}x^{3k}-\sum_{k=0}^{2007}(-1)^k\binom{2007}{k}x^{3k+1}.$$

Thus an exponent is congruent to either 0 or 1 modulo 3. But $2012=3(670)+2$, which has remainder 2. Neither $3k=2012$ nor $3k+1=2012$ admits an integer $k$; there is no term of the requested degree.

**Step 3:** Conclusion The coefficient of $x^{2012}$ is $\boxed{0}$. The requested numerical entry is 0, a dimensionless coefficient.

**Quick Tip to Solve:** For a high-power product containing $1+x+x^2$, first pair it with $1-x$; the resulting powers modulo 3 can eliminate the requested coefficient immediately.

---

**23. The lines $\frac{x-2}{2}=\frac{y}{-2}=\frac{z-7}{16}$ and $\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}$ intersect at the point $P$. If the distance of P from the line $\frac{x+1}{2}=\frac{y-1}{3}=\frac{z-1}{1}$ is $l$, then $14l^2$ is equal to _____.**

[4 marks]

**Solution:**

**Concept:**

- A symmetric line can be parametrised using one common parameter. Equating coordinates of two lines determines their point of intersection.
- The squared perpendicular distance from $P$ to a line through $Q$ with direction vector $\mathbf{d}$ is $l^2 = |\overrightarrow{QP} \times \mathbf{d}|^2/|\mathbf{d}|^2$.

**Step 1:** Find the intersection point Use parameters $s$ and $t$ for the first two lines:

$$P = (2+2s, -2s, 7+16s) = (-3+4t, -2+3t, -2+t).$$

The first two coordinate equations give $2s-4t=-5$ and $2s+3t=2$. Subtracting yields $7t=7$, hence $t=1$ and $s=-1/2$. The third coordinates agree: $7+16(-1/2)=-1=-2+1$. Therefore $P=(1,1,-1)$.

**Step 2:** Calculate the perpendicular distance The third line passes through $Q=(-1,1,1)$ and has direction $\mathbf{d}=(2,3,1)$. Thus $\overrightarrow{QP}=(2,0,-2)$ and

$$\overrightarrow{QP} \times \mathbf{d} = (6,-6,6).$$

Its squared magnitude is $36+36+36=108$, while $|\mathbf{d}|^2=4+9+1=14$. Consequently $l^2=108/14$. This cross-product construction measures distance to the entire line, not merely to its chosen point $Q$.

**Step 3:** Conclusion

$$14l^2 = 14\left(\frac{108}{14}\right) = \boxed{108}.$$

The requested numerical entry is 108 (in squared coordinate units).

**Quick Tip to Solve:** After finding an intersection using two coordinates, check the third before calculating a distance; use the squared cross-product formula to avoid unnecessary square roots.

---

**24. Let $f(x)=\int_0^x g(t)\log_e\left(\frac{1-t}{1+t}\right)\,dt$, where $g$ is a continuous odd function. If $\int_{-\pi/2}^{\pi/2}\left(f(x)+\frac{x^2\cos x}{1+e^x}\right)dx=\left(\frac{\pi}{\alpha}\right)^2-\alpha$, then $\alpha$ is equal to _____.**

[4 marks]

**Solution:**

**Concept:**

- For a real logarithm, its argument must be positive. Here $(1-t)/(1+t)>0$ holds exactly on $-1<t<1$.
- On this domain, $h(t)=\log((1-t)/(1+t))$ is odd. Since $g$ is odd, $g(t)h(t)$ is even and its integral from zero is odd.
- For the even numerator $x^2\cos x$, pairing the values at $x$ and $-x$ uses $1/(1+e^x)+1/(1+e^{-x})=1$.

**Step 1:** Check the domain of the printed expression The printed logarithm has no absolute-value signs. At $t = 1$ its argument is zero; for $t > 1$ or $t < -1$ its argument is negative. Hence the displayed definition does not give a real-valued $f(x)$ when $|x| > 1$. The outer bounds are $\pm\pi/2$, outside this domain. A continuous odd $g$ alone cannot cure this: $g(t) = t$ already leaves a non-real logarithm on part of the integration path. Thus the real definite integral, literally stated, is undefined.

**Step 2:** Calculate the formal parity value The numerical key corresponds to extending the odd-logarithm parity through the outer interval. Under that additional interpretation the $f$ contribution is zero, and pairing $x$ with $-x$ gives

$$\begin{aligned} J &= \int_0^{\pi/2} x^2 \cos x \, dx \\ &= \left[x^2 \sin x + 2x \cos x - 2 \sin x\right]_0^{\pi/2} \\ &= \frac{\pi^2}{4} - 2. \end{aligned}$$

Matching this with $(\pi/\alpha)^2 - \alpha$ gives $\alpha = 2$. Indeed for positive $\alpha$, the right-hand side decreases strictly, so this positive value is unique. This calculation explains the intended key; it does not remove the real-domain defect in the printed definition.

**Step 3:** Conclusion The keyed numerical entry is $\boxed{2}$, obtained by the formal parity calculation above. As printed with a real logarithm, however, the integral is not defined on the full interval, so 2 is an intended value rather than a rigorously determined real answer to the literal statement. The parameter $\alpha$ is dimensionless.

**Quick Tip to Solve:** Before applying odd/even cancellation to a logarithmic integral, check the logarithm is real on the entire interval; symmetry alone cannot define a missing domain.

**25. If the area of the region $\{(x, y) : 0 \le y \le \min\{2x, 6x - x^2\}\}$ is A, then 12 A is equal to _____.**

[4 marks]

**Solution:**

**Concept:**

- The condition $0 \le y \le \min\{u(x), v(x)\}$ uses the lower nonnegative curve as the upper boundary.
- For each interval with a single upper boundary, the area is $\int (\text{upper} - \text{lower})\, dx$; the lower boundary here is the x-axis.

**Step 1:** Find the valid intervals and switching point Both upper-bound candidates must be nonnegative. The line requires $x \ge 0$, and $6x - x^2 = x(6 - x) \ge 0$ requires $0 \le x \le 6$. Equating the curves gives $2x = 6x - x^2$, or $x(x - 4) = 0$. On $0 < x < 4$, the difference $(6x - x^2) - 2x = x(4 - x)$ is positive, so the line is lower. On $4 < x < 6$, the parabola is lower. No points with positive vertical width occur outside $[0, 6]$.

**Step 2:** Integrate the lower envelope The area therefore splits at $x = 4$:

$$\begin{aligned} A &= \int_0^4 2x\,dx + \int_4^6 (6x - x^2)\,dx \\ &= \left[x^2\right]_0^4 + \left[3x^2 - \frac{x^3}{3}\right]_4^6 \\ &= 16 + \left(36 - \frac{80}{3}\right) \\ &= \frac{76}{3}. \end{aligned}$$

The line and parabola agree at the splitting point, so the two integrals cover the region without gaps or double counting. The area is positive, as required by the geometric description.

**Step 3:** Conclusion

$$12A = 12\left(\frac{76}{3}\right) = \boxed{304}.$$

The requested numerical entry is 304, with $A = 76/3$ square coordinate units.

**Quick Tip to Solve:** When an area uses a minimum, compare the curves algebraically on each side of every intersection; integrating the larger curve would include points outside the region.

**26. If the sum of squares of all real values of $\alpha$, for which the lines $2x - y + 3 = 0, 6x + 3y + 1 = 0$ and $\alpha x + 2y - 2 = 0$ do not form a triangle is $p$, then the greatest integer less than or equal to $p$ is _____.**

[4 marks]

**Solution:**

**Concept:**

- Three distinct lines fail to form a triangle when two are parallel or when all three are concurrent.
- For $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, parallelism requires $a_1b_2 - a_2b_1 = 0$; concurrency is checked by substitution of an intersection point.

**Step 1:** List the parallel cases The first two lines have slopes 2 and $-2$, so they are not parallel. The third line has slope $-\alpha/2$. It is parallel to the first when $-\alpha/2 = 2$, giving $\alpha = -4$, and parallel to the second when $-\alpha/2 = -2$, giving $\alpha = 4$. Neither case makes coincident lines: for $\alpha = -4$, division by $-2$ gives $2x - y + 1 = 0$, differing from the first; for $\alpha = 4$, the intercept also differs from the second. Both cases fail to enclose a triangle.

**Step 2:** Find the concurrent case and sum the squares Solve the first two lines. From $y = 2x + 3$, substitution in $6x + 3y + 1 = 0$ gives $12x + 10 = 0$, so their intersection is $(-5/6, 4/3)$. The third line passes through it when

$$-\frac{5\alpha}{6} + \frac{8}{3} - 2 = 0, \qquad \alpha = \frac{4}{5}.$$

These three values exhaust the ways three noncoincident lines can fail to form a triangle. Thus

$$p = (-4)^2 + 4^2 + \left(\frac{4}{5}\right)^2 = 32 + \frac{16}{25} = 32.64.$$

**Step 3:** Conclusion The greatest integer less than or equal to $p$ is $\lfloor 32.64 \rfloor = \boxed{32}$. The requested entry is dimensionless.

**Quick Tip to Solve:** For three lines that do not form a triangle, check both parallelism and concurrency; checking the determinant for concurrency alone misses the parallel cases.

---

**27. Let $A$ be a $2 \times 2$ real matrix and $I$ be the identity matrix of order 2. If the roots of the equation $|A - xI| = 0$ be $-1$ and 3, then the sum of the diagonal elements of the matrix $A^2$ is** [4 marks]

**Solution:**

**Concept:**

- For a $2 \times 2$ matrix, the characteristic polynomial is $x^2 - (\operatorname{tr} A)x + \det A$.
- The Cayley–Hamilton theorem states that a matrix satisfies its own characteristic equation. The trace, the sum of diagonal entries, is linear.

**Step 1:** Recover the characteristic polynomial The roots of $|A - xI| = 0$ are $-1$ and 3. For order two this determinant has leading coefficient 1, so

$$|A - xI| = (x + 1)(x - 3) = x^2 - 2x - 3.$$

Comparing coefficients gives $\operatorname{tr} A = 2$ and $\det A = -3$. This uses the characteristic roots rather than presuming that the diagonal entries themselves are $-1$ and 3; a general real matrix need not be diagonal.

**Step 2:** Apply Cayley–Hamilton and take traces Substitution of $A$ in its characteristic polynomial yields

$$A^2 - 2A - 3I = 0, \qquad A^2 = 2A + 3I.$$

Now take the trace on both sides. Since $I$ has two unit diagonal entries, $\operatorname{tr} I = 2$, and hence

$$\operatorname{tr}(A^2) = 2 \operatorname{tr} A + 3 \operatorname{tr} I = 2(2) + 3(2) = 10.$$

As an independent check, the eigenvalues of $A^2$ are the squares 1 and 9, whose sum is also 10. No knowledge of the off-diagonal entries is required.

**Step 3:** Conclusion The sum of the diagonal elements of $A^2$ is $\operatorname{tr}(A^2) = \boxed{10}$. This is a dimensionless matrix quantity.

**Quick Tip to Solve:** For a two-by-two matrix, use $\operatorname{tr}(A^2) = (\operatorname{tr} A)^2 - 2 \det A$; the characteristic roots determine both quantities without finding the matrix.

---

**28. Consider a circle $(x-\alpha)^2+(y-\beta)^2=50$, where $\alpha,\beta>0$. If the circle touches the line $y+x=0$ at the point $P$, whose distance from the origin is $4\sqrt{2}$, then $(\alpha+\beta)^2$ is equal to _____.**

[4 marks]

**Solution:**

**Concept:**

- The circle $(x-\alpha)^2+(y-\beta)^2=50$ has centre $C=(\alpha,\beta)$ and radius $r=\sqrt{50}$.
- A tangent line is at perpendicular distance equal to the radius from the centre. The distance from $(a,b)$ to $x+y=0$ is $|a+b|/\sqrt{2}$.

**Step 1:** Use the tangency condition The radius to the contact point is perpendicular to the tangent. Therefore the shortest distance from $C$ to $x+y=0$ is $\sqrt{50}$:

$$\frac{|\alpha+\beta|}{\sqrt{2}}=\sqrt{50}.$$

Both centre coordinates are positive, so $\alpha+\beta>0$, and the absolute value may be removed. Squaring gives $(\alpha+\beta)^2/2=50$. This already determines the required quantity, without needing to locate the contact point.

**Step 2:** Check compatibility with the given contact distance The perpendicular foot from $(\alpha,\beta)$ to $x+y=0$ is

$$P=\left(\frac{\alpha-\beta}{2},\frac{\beta-\alpha}{2}\right).$$

Its distance from the origin is $|\alpha-\beta|/\sqrt{2}$. The supplied distance $4\sqrt{2}$ therefore gives $|\alpha-\beta|=8$. Together with $\alpha+\beta=10$, this yields the two possible centres $(9,1)$ and $(1,9)$. Both satisfy the positivity condition and give contact points $(4,-4)$ or $(-4,4)$, each at the required distance. Thus the extra condition is consistent and causes no change in the requested sum.

**Step 3:** Conclusion

$$(\alpha+\beta)^2=2(50)=\boxed{100}.$$

The numerical entry is 100 (in squared coordinate units).

**Quick Tip to Solve:** For a circle tangent to a line, write the centre-to-line distance first; it may give the requested expression directly, while the contact-point data serves as a consistency check.

**29. Let the complex numbers $\alpha$ and $\frac{1}{\bar{\alpha}}$ lie on the circles $|z-z_0|^2=4$ and $|z-z_0|^2=16$ respectively, where $z_0=1+i$. Then, the value of $100|\alpha|^2$ is _____.**

[4 marks]

**Solution:**

**Concept:**

- Write $\alpha=x+iy$, with $x,y$ real, and set $r=|\alpha|^2=x^2+y^2$. Since the reciprocal is defined, $r>0$.
- The conjugate reciprocal has the same direction as $\alpha$: $1/\bar{\alpha}=\alpha/r$. Expand the two circle equations and eliminate $x+y$.

**Step 1:** Expand the first circle The centre is $z_0 = 1 + i$, so the first squared distance gives

$$(x-1)^2 + (y-1)^2 = 4.$$

Thus

$$r - 2(x+y) + 2 = 4, \qquad 2(x+y) = r - 2.$$

This relation uses the squared radius 4 directly; the radius itself is 2. Keep $r$ as the squared modulus, since that is the quantity the question asks for.

**Step 2:** Expand the circle for the reciprocal As $1/\bar{\alpha} = (x+iy)/r$, the second circle equation is

$$\left(\frac{x}{r} - 1\right)^2 + \left(\frac{y}{r} - 1\right)^2 = 16.$$

Use $x^2 + y^2 = r$ to obtain

$$\frac{1}{r} - \frac{2(x+y)}{r} + 2 = 16.$$

Multiplication by the positive, nonzero $r$ yields

$$1 - 2(x+y) = 14r.$$

**Step 3:** Eliminate the coordinates and check consistency Substitute $2(x+y) = r - 2$ into the last equation:

$$1 - (r-2) = 14r \quad \Longrightarrow \quad 15r = 3 \quad \Longrightarrow \quad r = \frac{1}{5}.$$

Thus the necessary value requested by elimination is $100r = 20$, agreeing with the answer key. There is a defect in the existence premise as printed: these equations also require $x + y = -9/10$, whereas real $x, y$ must satisfy

$$(x+y)^2 \le 2(x^2+y^2) = \frac{2}{5}.$$

Here $81/100 > 2/5$, so no actual complex number satisfies both stated circles. This does not change the formal numerical value obtained from the two given equations.

**Step 4:** Conclusion The algebraic value expected by the question and its key is $\boxed{20}$. Strictly, the printed conditions have no simultaneous complex-number solution, as the coordinate inequality above shows.

**Quick Tip to Solve:** Use the conjugate reciprocal identity first; it turns both circle conditions into equations in the squared modulus and the coordinate sum.

**30. If the solution curve, of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passing through the point $(2, 1)$ is $\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{\beta}\log_e\left(\alpha + \left(\frac{y-1}{x-1}\right)^2\right) = \log_e|x-1|$, then $5\beta + \alpha$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- Translate the origin to remove constants: $X = x - 1$, $Y = y - 1$. Then $dY/dX = dy/dx$, and the new equation is homogeneous in $X, Y$.
- For a homogeneous differential equation, put $v = Y/X$, so $Y = vX$ and $dY/dX = v + X\,dv/dX$. Work locally near the given point, where $X = 1 \neq 0$.

**Step 1:** Shift the origin and introduce the ratio The numerator becomes $x + y - 2 = X + Y$, and the denominator becomes $x - y = X - Y$. Therefore

$$v + X\frac{dv}{dX} = \frac{1+v}{1-v}.$$

Isolating the derivative and combining fractions gives

$$X\frac{dv}{dX} = \frac{1+v-v(1-v)}{1-v} = \frac{1+v^2}{1-v}.$$

The original equation excludes $X = Y$; our local solution passes through $(X, Y) = (1, 0)$, away from that singular line.

**Step 2:** Separate and integrate Separate the ratio variable from $X$:

$$\frac{1-v}{1+v^2}\,dv = \frac{dX}{X}.$$

Integrating each term separately,

$$\int \frac{dv}{1+v^2} - \int \frac{v\,dv}{1+v^2} = \log_e|X| + C.$$

The first integral is $\tan^{-1} v$; in the second, the derivative of $1 + v^2$ is $2v$. Hence

$$\tan^{-1} v - \frac{1}{2}\log_e(1+v^2) = \log_e|X| + C.$$

**Step 3:** Use the initial point and compare coefficients At $(x, y) = (2, 1)$, we have $X = 1$, $Y = 0$, and $v = 0$. Both sides without $C$ vanish, so $C = 0$. Restoring $v = (y-1)/(x-1)$, comparison with the displayed solution gives

$$\alpha = 1, \qquad \frac{1}{\beta} = \frac{1}{2}, \qquad \beta = 2.$$

The logarithm argument is positive because $1 + v^2 > 0$, consistent with this real solution near the initial point.

**Step 4:** Conclusion Therefore $5\beta + \alpha = 5(2) + 1 = \boxed{11}$, a dimensionless numerical value.

**Quick Tip to Solve:** Shift the intersection of the two linear expressions to the origin before using the homogeneous substitution; then determine the integration constant at the given point.

## Physics – Section A: Multiple Choice Questions (80 marks)

**31. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).**

**Assertion (A) : In Vernier calliper if positive zero error exists, then while taking measurements, the reading taken will be more than the actual reading.**

**Reason (R) : The zero error in Vernier Calliper might have happened due to manufacturing defect or due to rough handling.**

**In the light of the above statements, choose the correct answer from the options given below :**

[4 marks]

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(B) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

**Correct Answer:** (B) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

**Solution:**

**Concept:**

- A positive zero error means that the instrument shows a positive reading even when its jaws are closed on zero length. Corrected length equals observed length minus the signed zero error.
- In an assertion-reason question, truth of both statements and a causal explanation of the assertion are separate tests. A cause of a defect need not explain the sign of its effect.

**Step 1:** Evaluate the assertion using the correction equation Let the true length be $L$ and the positive zero error be $e > 0$. The scale offset remains added to the measured length, so

$$L_{\text{observed}} = L + e, \qquad L = L_{\text{observed}} - e.$$

Consequently the uncorrected reading is greater than the actual length. This is exactly what Assertion (A) says, so the assertion is true. The sign of the offset is essential: a negative zero error would instead require adding a positive correction.

**Step 2:** Evaluate the reason independently Manufacturing imperfections can prevent the main-scale and Vernier-scale zeros from coinciding when the jaws are closed. Rough handling can also disturb the alignment or damage the jaws. Thus either can produce a zero error, and Reason (R) is true. However, these causes can produce an error of either sign. Merely naming them does not establish that the observed reading is too large.

**Step 3:** Test whether the reason explains the assertion The excess reading follows from the positive offset in the measurement equation, not from the general origin of zero error. Therefore R does not provide the required explanation of A. Option (A) incorrectly treats the reason as explanatory; option (C) incorrectly declares R false; and option (D) incorrectly declares A false. Only option (B) retains the truth of both statements while rejecting the claimed explanatory connection.

**Step 4:** Conclusion The assertion and reason are both true, but the reason does not explain why

positive zero error increases the reading. Hence, the correct option is (B).

**Quick Tip to Solve:** Write actual reading = observed reading minus signed zero error, then test whether the proposed reason explains the sign rather than merely the source of the error.

**32. A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle $(\theta)$ of thread deflection in the extreme position will be :** [4 marks]

(A) $\tan^{-1}\left(\frac{1}{2}\right)$

(B) $2\tan^{-1}\left(\frac{1}{2}\right)$

(C) $2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

(D) $\tan^{-1}(\sqrt{2})$

**Correct Answer:** (B) $2\tan^{-1}\left(\frac{1}{2}\right)$

**Solution:**

**Concept:**

- At a turning point the speed is zero, so radial acceleration $v^2/l$ is zero; the remaining tangential acceleration has magnitude $g\sin\theta$.
- At the lowest point the tangential component of gravity vanishes. The acceleration is then entirely radial, with magnitude $v_0^2/l$, where $l$ is the thread length.
- Conservation of mechanical energy relates the lowest-point speed to the height lost: $v_0^2 = 2gl(1-\cos\theta)$.

**Step 1:** Express the two acceleration magnitudes At the extreme position the ball momentarily rests, but it still has a tangential acceleration directed toward equilibrium:

$$a_{\text{extreme}} = g\sin\theta.$$

At the bottom the ball moves fastest and gravity has no tangential component, so

$$a_{\text{bottom}} = \frac{v_0^2}{l} = 2g(1-\cos\theta).$$

These are magnitudes, so compare the full accelerations at those particular positions rather than comparing tensions.

**Step 2:** Use the equality and solve with a half-angle identity The required equality is

$$\sin\theta = 2(1-\cos\theta).$$

For a nonzero swing, $\sin(\theta/2) \neq 0$. Using the half-angle identities gives

$$2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 4\sin^2\frac{\theta}{2}, \qquad \tan\frac{\theta}{2} = \frac{1}{2}.$$

Thus $\theta = 2\tan^{-1}(1/2)$. The discarded zero-angle solution represents a stationary ball, not the stated swing.

**Step 3:** Check the available angles For the selected angle, $\cos\theta = 3/5$ and $\sin\theta = 4/5$, so both acceleration magnitudes are $4g/5$. Option (A) has $\tan(\theta/2) = \sqrt{5}-2$, option (C) has $\tan(\theta/2) = 1/\sqrt{5}$, and option (D) has $\tan(\theta/2) = (\sqrt{3}-1)/\sqrt{2}$. None equals $1/2$, so none satisfies the acceleration equality for a nonzero swing.

**Step 4:** Conclusion The extreme deflection is $\boxed{\theta = 2\tan^{-1}(1/2)}$. Hence, the correct option is (B).

**Quick Tip to Solve:** At the extreme use tangential acceleration; at the bottom use centripetal acceleration. Energy conservation eliminates the speed and thread length.

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**33. Given below are two statements :**

**Statement (I) : The limiting force of static friction depends on the area of contact and independent of materials.**

**Statement (II) : The limiting force of kinetic friction is independent of the area of contact and depends on materials.**

**In the light of the above statements, choose the most appropriate answer from the options given below :** [4 marks]

(A) Statement I is incorrect but Statement II is correct

(B) Both Statement I and Statement II are correct

(C) Both Statement I and Statement II are incorrect

(D) Statement I is correct but Statement II is incorrect

**Correct Answer:** (A) Statement I is incorrect but Statement II is correct

**Solution:**

**Concept:**

- For ordinary dry sliding contact, limiting static friction is $f_{s,\max} = \mu_s N$, where $N$ is the normal reaction and $\mu_s$ depends on the contacting surfaces.
- Kinetic friction is approximately $f_k = \mu_k N$. Under the usual school-level dry-friction model, neither coefficient depends on the apparent contact area.

**Step 1:** Evaluate Statement I against the static-friction law Statement I claims both dependence on contact area and independence from the materials. Both claims contradict the usual dry-friction law. At a fixed normal force, the limiting static force is

$$f_{s,\max} = \mu_s N.$$

Changing the pair of materials or their surface condition generally changes $\mu_s$. Changing the apparent area alone does not enter this model. Also distinguish limiting static friction from the actual static force: before sliding, the actual force adjusts up to this limiting value. Statement I is therefore incorrect.

**Step 2:** Evaluate Statement II for sliding contact Once surfaces slide relative to one another, the friction magnitude is approximately

$$f_k = \mu_k N.$$

For a fixed $N$, it depends on the surface pair through $\mu_k$, but is independent of their apparent area of contact in this standard model. These are precisely the dependencies asserted by Statement II, so Statement II is correct. The printed use of the word limiting does not alter the intended comparison of static and kinetic dry friction.

**Step 3:** Select the combination and eliminate alternatives The truth combination is I false and II true. Option (B) fails because I is false. Option (C) fails because II is true. Option (D) reverses both truth values. Option (A) alone preserves the material dependence and area independence of friction at a fixed normal reaction. These conclusions use the stated conventional laws, rather than effects such as adhesion or deformation outside that model.

**Step 4:** Conclusion Statement I is incorrect, but Statement II is correct. Hence, the correct option is (A).

**Quick Tip to Solve:** Check both clauses of a statement against the coefficient law: the surface materials enter through the coefficient, while apparent contact area does not.

**34. Primary side of a transformer is connected to $230\ \text{V}, 50\ \text{Hz}$ supply. Turns ratio of primary to secondary winding is $10:1$. Load resistance connected to secondary side is $46\Omega$. The power consumed in it is :** [4 marks]

(A) 11.5 W

(B) 12.5 W

(C) 10.0 W

(D) 12.0 W

**Correct Answer:** (A) 11.5 W

**Solution:**

**Concept:**

- An ideal transformer has the same rms-voltage ratio as its turns ratio: $V_s/V_p = N_s/N_p$. Here primary and secondary quantities are denoted by subscripts $p, s$.
- A resistor on an AC supply consumes average power $P = V_{\text{rms}}^2/R = I_{\text{rms}}^2 R$. Use the secondary rms voltage across the load.

**Step 1:** Find the secondary voltage from the turns ratio The given ratio is primary to secondary, $N_p : N_s = 10:1$, so this is a step-down transformer. With primary rms voltage $230\,\text{V}$,

$$V_s = V_p \frac{N_s}{N_p} = 230 \times \frac{1}{10} = 23\,\text{V}.$$

The stated $50\,\text{Hz}$ frequency permits transformer action but does not multiply the voltage ratio or enter the resistive-power calculation. No transformer losses are specified, so use the ideal ratio.

**Step 2:** Calculate the load current and average power The load resistance is $46\,\Omega$, giving the rms secondary current

$$I_s = \frac{V_s}{R} = \frac{23}{46} = 0.5\,\text{A}.$$

Therefore the average power consumed by the resistor is

$$P = V_s I_s = 23(0.5) = 11.5\,\text{W}.$$

Equivalently, $V_s^2/R = 23^2/46 = 529/46 = 11.5\,\text{W}$. Both calculations use rms quantities, so no additional factor of one-half is required.

**Step 3:** Check the listed powers The calculated power is exactly $11.5\,\text{W}$, matching option (A). Values $12.5\,\text{W}$, $10.0\,\text{W}$, and $12.0\,\text{W}$ do not equal $529/46$ and therefore fail the load relation. As a consistency check, ideal input power is also $11.5\,\text{W}$, corresponding to primary current $11.5/230 = 0.05\,\text{A}$, one-tenth the secondary current.

**Step 4:** Conclusion The load consumes $\boxed{11.5\,\text{W}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Reduce voltage by the primary-to-secondary turns ratio first; square that secondary rms voltage and divide by the load resistance.

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**35. A heavy iron bar of weight $12\ \text{kg}$ is having its one end on the ground and the other on the shoulder of a man. The rod makes an angle $60^\circ$ with the horizontal, the weight experienced by the man is :** [4 marks]

(A) $3\ \text{kg}$

(B) $6\ \text{kg}$

(C) $6\sqrt{3}\ \text{kg}$

(D) $12\ \text{kg}$

**Correct Answer:** (B) $6\ \text{kg}$

**Solution:**

**Concept:**

- A uniform heavy bar has its weight acting vertically downward through its midpoint. The ground and the shoulder support its ends in static equilibrium.
- Take moments about the end on the ground to remove that support reaction. For a vertical force, its moment arm is the horizontal separation from the pivot.

**Step 1:** Identify the forces and moment arms Let the bar length be $L$, its weight be $W = 12g\,\text{N}$, and the upward support force supplied by the shoulder be $F$. The vertical load on the man has the same magnitude $F$, by Newton's third law. At an inclination of $60^\circ$, the shoulder is horizontally $L\cos 60^\circ$ from the grounded end, while the centre of mass is $(L/2)\cos 60^\circ$ from that end. Both moments therefore contain the same cosine factor.

**Step 2:** Apply the torque balance Taking anticlockwise and clockwise moments about the end on the

ground,

$$FL\cos 60^\circ = W\frac{L}{2}\cos 60^\circ.$$

Since the bar has nonzero length and $\cos 60^\circ = 1/2 \neq 0$, cancellation yields

$$F = \frac{W}{2} = 6g\,\mathrm{N}.$$

The inclination cancels because the weight acts at the midpoint and both load and shoulder reaction are vertical. Using $W\sin 60^\circ$ would instead resolve force along the bar and would not express this torque balance.

**Step 3:** Express the load in the units used by the options The options use kilograms for the supported weight, meaning the mass-equivalent load. Thus

$$\frac{F}{g} = 6\,\mathrm{kg}.$$

The other end supports the remaining $6g\,\mathrm{N}$, so vertical force balance is also satisfied. Option (A), $3\,\mathrm{kg}$, incorrectly adds an extra factor of one-half; option (C) retains an angle factor that cancels; option (D) assigns the entire bar weight to one support and fails moment balance.

**Step 4:** Conclusion The force experienced by the man is $6g\,\mathrm{N}$, equivalent to a $\boxed{6\,\mathrm{kg}}$ load. Hence, the correct option is (B). The supplied key selects the 3 kg alternative, but that would give only half the moment needed to balance the midpoint weight; the printed bar therefore supports the 6 kg result.

**Quick Tip to Solve:** Take moments about the ground contact. For a uniform bar supported at its ends, the midpoint weight and end reaction share the same inclination factor.

**36. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).**

**Assertion (A) : The angular speed of the moon in its orbit about the earth is more than the angular speed of the earth in its orbit about the sun.**

**Reason (R) : The moon takes less time to move around the earth than the time taken by the earth to move around the sun.**

**In the light of the above statements, choose the most appropriate answer from the options given below :** [4 marks]

(A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(B) (A) is correct but (R) is not correct

(C) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(D) (A) is not correct but (R) is correct

**Correct Answer:** (C) Both (A) and (R) are correct and (R) is the correct explanation of (A)

**Solution:**

**Concept:**

- For one complete revolution the angular displacement is $2\pi$ radians. The mean angular speed of an orbit is therefore $\omega = 2\pi/T$, where $T$ is its period.
- Angular speed measures the angle swept per unit time, whereas linear orbital speed also depends on the orbital radius through $v = r\omega$. The two must not be confused.

**Step 1:** Evaluate the assertion The Moon completes one revolution about the Earth in approximately $27.3$ days, while the Earth completes one revolution about the Sun in approximately $365.25$ days. Comparing their mean angular speeds gives

$$\frac{\omega_{\text{Moon}}}{\omega_{\text{Earth}}} = \frac{T_{\text{Earth}}}{T_{\text{Moon}}} = \frac{365.25}{27.3} \approx 13.4 > 1.$$

Thus the Moon sweeps a larger orbital angle in the same time. Assertion (A) is correct.

**Step 2:** Evaluate the reason Reason (R) states that the lunar orbital period is smaller than the terrestrial orbital period. The periods above confirm this statement. Both periods must be expressed in the same unit, as they are here; comparison of orbital distances is not required.

**Step 3:** Test the explanatory connection The relation $\omega = 2\pi/T$ shows directly that a smaller period gives a larger angular speed. Consequently the fact in (R) is precisely the explanation for (A). Option (A) incorrectly denies this connection, while options (B) and (D) each declare one true statement incorrect. The comparison concerns angular speeds, so the much larger Earth–Sun orbital radius cannot reverse it.

**Step 4:** Conclusion The assertion and the reason are both true, and the reason correctly explains the assertion. Hence, the correct option is (C).

**Quick Tip to Solve:** For two complete orbital revolutions, compare periods in the same unit: the shorter period always gives the larger mean angular speed.

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**37. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).**

**Assertion (A) : The property of body, by virtue of which it tends to regain its original shape when the external force is removed, is Elasticity.**

**Reason (R) : The restoring force depends upon the bonded inter atomic and inter molecular force of solid.**

**In the light of the above statements, choose the correct answer from the options given below :**

[4 marks]

(A) (A) is false but (R) is true

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) Both (A) and (R) are true and (R) is the correct explanation of (A)

(D) (A) is true but (R) is false

**Correct Answer:** (C) Both (A) and (R) are true and (R) is the correct explanation of (A)

**Solution:**

**Concept:**

- Elasticity is the tendency of a body to recover its original dimensions after the deforming external force is removed, provided the elastic limit has not been exceeded.
- Atoms and molecules in a solid occupy equilibrium separations. Changing those separations produces internal restoring forces that oppose the deformation.

**Step 1:** Evaluate the assertion If an external force stretches or compresses a solid slightly, its dimensions change. When that force is withdrawn, an elastic body returns towards its original shape and size. This recovery tendency is the definition of elasticity, so assertion (A) is true. It should not be confused with plasticity, in which a permanent deformation remains after removal of the load.

**Step 2:** Evaluate the reason A solid is held together by interatomic and intermolecular interactions. At equilibrium their net force vanishes. Deformation displaces the particles from their equilibrium positions, and the resulting unbalanced internal forces tend to restore their original arrangement. For a small displacement $x$, the effective restoring force may be written $F_{\text{restoring}} = -kx$, with $k > 0$; the minus sign indicates opposition to the displacement. Thus (R) is also true.

**Step 3:** Connect the microscopic cause to elastic recovery The restoring forces supplied by the bonds are the physical reason that the body tends to regain its original configuration after the external force disappears. Hence (R) explains (A), rather than merely accompanying it. Option (B) misses this causal connection. Options (A) and (D) are excluded because neither of the two statements is false. The usual definition applies to elastic deformation, not to loading beyond the elastic limit.

**Step 4:** Conclusion The assertion and the reason are both true, and the reason correctly explains the assertion. Hence, the correct option is (C).

**Quick Tip to Solve:** In elasticity questions, link macroscopic recovery to microscopic restoring forces; bonds oppose displacement from the equilibrium arrangement.

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**38. The atomic mass of $_6C^{12}$ is $12.000000\ \text{u}$ and that of $_6C^{13}$ is $13.003354\ \text{u}$. The required energy to remove a neutron from $_6C^{13}$, if mass of neutron is $1.008665\ \text{u}$, will be :** [4 marks]

(A) 62.5 MeV

(B) 6.25 MeV

(C) 4.95 MeV

(D) 49.5 MeV

**Correct Answer:** (C) 4.95 MeV

**Solution:**

**Concept:**

- The neutron separation energy is the energy required for $^{13}_{6}\text{C} \rightarrow {}^{12}_{6}\text{C} + {}^{1}_{0}\text{n}$. It is the increase in rest mass multiplied by $c^2$.
- Use $1\,\text{u}\,c^2 \approx 931.5\,\text{MeV}$. Atomic masses can be used here because both carbon atoms contain six electrons, whose masses cancel in the difference.

**Step 1:** Write the mass balance for neutron removal The initial atom is carbon-13. After removal of one neutron, the carbon atom has the same atomic number but mass number 12. The final products have total mass

$$m_f = m(^{12}\mathrm{C}) + m_n = 12.000000 + 1.008665 = 13.008665\,\mathrm{u}.$$

The final mass exceeds the initial mass $13.003354\,\mathrm{u}$, so external energy must be supplied. Taking the opposite sign would describe released energy rather than the required separation energy.

**Step 2:** Convert the mass increase into energy The required mass increase and its energy equivalent are

$$\Delta m = 13.008665 - 13.003354 = 0.005311\,\mathrm{u},$$
$$E_{\text{sep}} = \Delta m c^2 = 0.005311 \times 931.5\,\mathrm{MeV}$$
$$\approx 4.947\,\mathrm{MeV} \approx 4.95\,\mathrm{MeV}.$$

This is the separation energy of the last neutron, not the total binding energy of the entire nucleus. The values $49.5$ and $62.5\,\mathrm{MeV}$ have the wrong scale, and $6.25\,\mathrm{MeV}$ does not follow from the stated mass difference. Keeping all supplied decimal places until the last multiplication avoids an appreciable rounding error.

**Step 3:** Conclusion The energy required to remove the neutron is approximately $4.95\,\mathrm{MeV}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For neutron removal, subtract the parent mass from daughter-plus-neutron mass. Equal atomic numbers let the electron masses cancel automatically.

**39. The threshold frequency of a metal with work function $6.63$ eV is :** [4 marks]

(A) $16 \times 10^{15}$ Hz

(B) $16 \times 10^{12}$ Hz

(C) $1.6 \times 10^{15}$ Hz

(D) $1.6 \times 10^{12}$ Hz

**Correct Answer:** (C) $1.6 \times 10^{15}$ Hz

**Solution:**

**Concept:**

- The work function $\phi$ is the minimum photon energy needed to liberate an electron. At threshold, its maximum kinetic energy is zero, so $h\nu_0 = \phi$.
- Convert electron volts to joules before using $h$ in SI units: $1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$, and $h \approx 6.626 \times 10^{-34}\,\mathrm{J\,s}$.

**Step 1:** Express the work function in SI units The stated energy $6.63\,\mathrm{eV}$ is an energy per electron, not a frequency. Its SI value is

$$\phi = 6.63 \times 1.602 \times 10^{-19}\,\mathrm{J} = 1.062126 \times 10^{-18}\,\mathrm{J}.$$

Using the numerical value 6.63 directly with the SI value of Planck's constant would omit the

electron-volt conversion and give a physically incorrect scale.

**Step 2:** Apply the threshold condition Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$. At the minimum frequency for emission, set $K_{\max} = 0$:

$$\nu_0 = \frac{\phi}{h} = \frac{1.062126 \times 10^{-18}}{6.626 \times 10^{-34}}\,\mathrm{s}^{-1} \approx 1.60 \times 10^{15}\,\mathrm{Hz}.$$

The unit check gives $\mathrm{J}/(\mathrm{J\,s}) = \mathrm{s}^{-1}$, as required. Below this frequency no electron can be emitted by a single photon, regardless of light intensity.

**Step 3:** Check the powers of ten in the choices Option (A), $16 \times 10^{15}$ Hz, is ten times too large. Option (B) equals $1.6 \times 10^{13}$ Hz, one hundred times too small, and option (D) is one thousand times too small. Only $1.6 \times 10^{15}$ Hz has both the correct coefficient and exponent.

**Step 4:** Conclusion The threshold frequency is $\nu_0 \approx 1.6 \times 10^{15}$ Hz. Hence, the correct option is (C).

**Quick Tip to Solve:** Use $h\nu_0 = \phi$ with consistent energy units, and normalize each option before comparing its power of ten.

**40. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio of $\frac{\mathrm{Cp}}{\mathrm{Cv}}$ for the gas is :** [4 marks]

(A) $\frac{7}{5}$
(B) $\frac{3}{2}$
(C) $\frac{9}{7}$
(D) $\frac{5}{3}$

**Correct Answer:** (B) $\frac{3}{2}$

**Solution:**

**Concept:**

- For a fixed amount of ideal gas in a reversible adiabatic process, $PV^{\gamma} =$ constant, where $\gamma = C_p/C_v$.
- The ideal-gas law $PV = nRT$ eliminates the volume and gives $P \propto T^{\gamma/(\gamma-1)}$. The temperature must be absolute temperature.

**Step 1:** Eliminate volume from the adiabatic law For the same gas sample, $nR$ is constant and $V = nRT/P$. Substitute this expression into $PV^{\gamma} = K$:

$$P\left(\frac{nRT}{P}\right)^{\gamma} = K.$$

Collecting the temperature and pressure factors gives

$$T^{\gamma}P^{1-\gamma} = \text{constant}, \qquad P^{\gamma-1} \propto T^{\gamma}.$$

Therefore $P \propto T^{\gamma/(\gamma-1)}$. The exponent is not simply $\gamma$, because volume changes during the process.

**Step 2:** Match the given power of temperature The question states $P \propto T^3$, so equality of exponents requires

$$\frac{\gamma}{\gamma - 1} = 3 \quad \Longrightarrow \quad \gamma = 3\gamma - 3 \quad \Longrightarrow \quad \gamma = \frac{3}{2}.$$

As a check, substituting $\gamma = 3/2$ gives $(3/2)/(1/2) = 3$, exactly the observed exponent. The result is dimensionless, as a ratio of two heat capacities must be.

**Step 3:** Reject the other heat-capacity ratios The other choices predict different pressure–temperature powers: $\gamma = 7/5$ gives $7/2$, $\gamma = 9/7$ gives $9/2$, and $\gamma = 5/3$ gives $5/2$. None equals 3. Identifying a familiar molecular-gas ratio without using the stated dependence would therefore choose the wrong answer.

**Step 4:** Conclusion The heat-capacity ratio is $C_p/C_v = 3/2$. Hence, the correct option is (B).

**Quick Tip to Solve:** If an ideal gas obeys $P \propto T^m$ adiabatically, use $\gamma = m/(m-1)$; do not identify the temperature exponent directly with $\gamma$.

**41. A bullet is fired into a fixed target looses one third of its velocity after travelling $4$ cm. It penetrates further $\mathrm{D} \times 10^{-3}$ m before coming to rest. The value of $\mathrm{D}$ is :** [4 marks]

(A) 23

(B) 32

(C) 42

(D) 52

**Correct Answer:** (B) 32

**Solution:**

**Concept:**

- For motion through a target with constant retarding force, the deceleration has constant magnitude $a$. Then $v^2 = u^2 - 2as$, where $u$ is the entry speed and $s$ is penetration.
- Equivalently, the work done against a constant resisting force is the loss of kinetic energy: $Fs = \frac{1}{2}m(u^2 - v^2)$. Distance is proportional to the kinetic energy lost, not the speed lost.

**Step 1:** Determine the speed after the first segment The bullet loses one third of its initial velocity, so its remaining speed after $4\,\mathrm{cm} = 0.040\,\mathrm{m}$ is $v = 2u/3$. Under the uniform-resistance model for penetration,

$$u^2 - \left(\frac{2u}{3}\right)^2 = 2a(0.040).$$

Thus the first segment removes a fraction $1 - 4/9 = 5/9$ of the initial kinetic energy, while $4/9$ remains. Losing one third of the speed is not the same as losing one third of the kinetic energy.

**Step 2:** Find the remaining stopping distance Let $s$ be the further distance. The same deceleration brings the remaining speed to zero:

$$\left(\frac{2u}{3}\right)^2 = 2as.$$

Dividing this equation by that for the first segment eliminates both the unknown entry speed and the deceleration:

$$\frac{s}{0.040} = \frac{4u^2/9}{5u^2/9} = \frac{4}{5}, \qquad s = 0.032\,\text{m} = 32 \times 10^{-3}\,\text{m}.$$

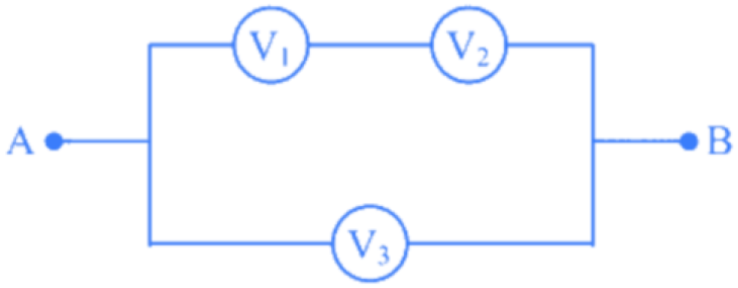
This is the additional distance after the first $4\,\text{cm}$, rather than the total penetration of $7.2\,\text{cm}$.

**Step 3:** Compare the requested coefficient The question writes the further penetration as $D \times 10^{-3}\,\text{m}$. Matching this form gives $D = 32$. The alternatives 23, 42 and 52 do not have the required energy-loss ratio $4/5$. The physical assumption of uniform target resistance is the standard constant-deceleration model implicit in this penetration calculation.

**Step 4:** Conclusion The further penetration is $0.032\,\text{m} = 32 \times 10^{-3}\,\text{m}$, so $D = 32$. Hence, the correct option is (B).

**Quick Tip to Solve:** For constant penetration resistance, compare losses of speed squared: after losing one third of the speed, the remaining-to-lost energy ratio is $4 : 5$.

**42. Three voltmeters, all having different internal resistances are joined as shown in figure. When some potential difference is applied across $A$ and $B$, their readings are $V_1, V_2$ and $V_3$. Choose the correct option.** [4 marks]




(A) $V_1 = V_2$

(B) $V_1 \neq V_3 - V_2$

(C) $V_1 + V_2 = V_3$

(D) $V_1 + V_2 > V_3$

**Correct Answer:** (C) $V_1 + V_2 = V_3$

**Solution:**

**Concept:**

- All branches connected between the same pair of nodes have the same total potential difference. Ideal connecting wires have no potential drop.
- Kirchhoff's voltage law states that the algebraic sum of potential changes around a closed loop is zero. A real voltmeter measures the potential drop across its own internal resistance.

**Step 1:** Identify the two branches between the terminals In the diagram, the upper branch from $A$ to $B$ contains voltmeters 1 and 2 in series. The lower branch contains voltmeter 3 alone. Both

branches start and end at the same nodes, so each spans the applied terminal potential difference $V_{AB}$. Consequently, the lower-branch reading is $V_3 = V_{AB}$. The branch currents need not be equal because the branch resistances need not be equal.

**Step 2:** Add the potential drops in the upper branch If the current in the upper branch is $I$, its meter readings are $V_1 = IR_1$ and $V_2 = IR_2$. The sum is

$$V_1 + V_2 = I(R_1 + R_2) = V_{AB}.$$

Combining this with the lower branch gives $V_1 + V_2 = V_3$. Finite internal resistances change the currents and division of voltage, but cannot change the node-to-node voltage equality required by the circuit connections.

**Step 3:** Check each alternative against the circuit Option (A) would require equal internal resistances of the two upper voltmeters; the question specifies different resistances, so their readings are unequal for a nonzero applied voltage. Rearranging the correct relation gives $V_1 = V_3 - V_2$, which directly contradicts option (B). Option (D) would violate Kirchhoff's voltage law, since the sum of upper-branch drops cannot exceed the same applied voltage measured across the lower branch.

**Step 4:** Conclusion The readings satisfy $V_1 + V_2 = V_3$, independently of their different internal resistances. Hence, the correct option is (C).

**Quick Tip to Solve:** First mark the common end nodes of a circuit: voltage drops add along a series path, while complete parallel paths have equal total voltage.

**43. The equation of state of a real gas is given by $\left(\mathrm{P} + \frac{\mathrm{a}}{\mathrm{V}^2}\right)(\mathrm{V} - \mathrm{b}) = \mathrm{RT}$, where $\mathrm{P}, \mathrm{V}$ and $\mathrm{T}$ are pressure, volume and temperature respectively and $\mathrm{R}$ is the universal gas constant. The dimensions of $\frac{\mathrm{a}}{\mathrm{b}^2}$ is similar to that of :** [4 marks]

(A) P

(B) RT

(C) PV

(D) R

**Correct Answer:** (A) P

**Solution:**

**Concept:**

- Only quantities having the same dimensions can be added or subtracted in a physical equation. Thus the two terms in each bracket must be dimensionally homogeneous.
- Pressure has dimensions $[P] = ML^{-1}T^{-2}$, while volume has dimensions $[V] = L^3$. Constants in an equation need not be dimensionless.

**Step 1:** Compare the terms in the pressure bracket In the real-gas equation, $a/V^2$ is added to the pressure $P$. Consequently it must itself have the dimensions of pressure, irrespective of the numerical

values of the gas constants. Hence

$$[a/V^2] = [P], \qquad [a] = [P][V]^2.$$

In fundamental dimensions this gives $[a] = ML^5T^{-2}$.

**Step 2:** Compare the terms in the volume bracket The factor $V - b$ subtracts $b$ from a volume. Therefore $[b] = [V] = L^3$, so $[b^2] = L^6$. Combining this with the first comparison gives

$$\left[\frac{a}{b^2}\right] = \frac{[P][V]^2}{[V]^2} = [P] = ML^{-1}T^{-2}.$$

This argument uses both brackets; treating either $a$ or $b$ as a pure number would give inconsistent dimensions.

**Step 3:** Eliminate the other proposed quantities The equation also shows that $[RT] = [PV] = ML^2T^{-2}$, an energy dimension. These are different from pressure by a volume factor, excluding (B) and (C). Finally, $[R] = [PV]/[T_{\text{temperature}}]$ includes inverse temperature, so (D) cannot match either.

**Step 4:** Conclusion The dimensions of $a/b^2$ are the dimensions of pressure $P$, namely $ML^{-1}T^{-2}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Compare added terms first: $a/V^2$ behaves as pressure and $b$ as volume. Cancel the volume factors only after identifying both.

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**44. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).**

**Assertion (A) : Work done by electric field on moving a positive charge on an equipotential surface is always zero.**

**Reason (R) : Electric lines of forces are always perpendicular to equipotential surfaces.**

**In the light of the above statements, choose the most appropriate answer from the options given below :** [4 marks]

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(B) (A) is correct but (R) is not correct

(C) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(D) (A) is not correct but (R) is correct

**Correct Answer:** (A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

**Solution:**

**Concept:**

- The work done by an electrostatic field in moving charge $q$ from point 1 to point 2 is $W = q(V_1 - V_2)$. An equipotential surface has the same potential at all its points.
- The electric field satisfies $\vec{E} = -\nabla V$. A nonzero electrostatic field is normal to the local equipotential surface, so its component along a tangent displacement vanishes.

**Step 1:** Evaluate the assertion using potential difference Both the initial and final positions lie on the same equipotential surface. Thus $V_1 = V_2$, and

$$W = q(V_1 - V_2) = 0.$$

The positive sign of the charge does not change this result: multiplication by a zero potential difference gives zero work. The assertion is therefore correct, including for a curved path restricted to the surface.

**Step 2:** Evaluate the reason geometrically If the electric field had a tangential component on the surface, moving through a short tangent displacement would change the potential. This contradicts the definition of an equipotential surface. Therefore electric field lines meet such surfaces perpendicularly wherever the field is nonzero. The reason is correct.

**Step 3:** Check whether the reason explains the assertion For each infinitesimal displacement $d\vec{l}$ along the surface, the force $q\vec{E}$ is perpendicular to it. Its elementary work is

$$dW = q\vec{E} \cdot d\vec{l} = 0.$$

Integrating these zero contributions over the path gives zero total work. Thus the reason supplies precisely the physical explanation for the assertion. Options (B) and (D) falsely reject one true statement; (C) falsely denies their explanatory connection.

**Step 4:** Conclusion Both the assertion and the reason are correct, and the reason correctly explains the assertion. Hence, the correct option is (A).

**Quick Tip to Solve:** For an equipotential-surface question, check both routes: zero potential difference gives zero work, and a normal electric field gives zero tangential force.

**45. When a polaroid sheet is rotated between two crossed polaroids then the transmitted light intensity will be maximum for a rotation of :** [4 marks]

(A) $90^\circ$

(B) $30^\circ$

(C) $45^\circ$

(D) $60^\circ$

**Correct Answer:** (C) $45^\circ$

**Solution:**

**Concept:**

- Malus law gives the intensity after an analyser as $I_{\text{out}} = I_{\text{in}} \cos^2 \phi$, where $\phi$ is the angle between the incident polarisation direction and the analyser axis.
- When a third polaroid lies between crossed outer polaroids, the two successive angles are $\theta$ and $90^\circ - \theta$. Both transmissions must be included.

**Step 1:** Apply Malus law to the inserted sheet Let $I_1$ be the intensity emerging from the first outer

polaroid, whose axis defines zero angle. If the inserted sheet is at angle $\theta$ to that axis, the intensity after this sheet is

$$I_2 = I_1 \cos^2\theta.$$

It also changes the polarisation direction of the transmitted light to its own transmission axis. Maximising only this first transmission would miss the effect of the final crossed polaroid.

**Step 2:** Include the final crossed polaroid The final outer polaroid has its axis at $90^\circ$ to the first. Its angle with the inserted sheet is therefore $90^\circ - \theta$, giving

$$I = I_1 \cos^2\theta \cos^2(90^\circ - \theta) = I_1 \cos^2\theta \sin^2\theta = \frac{I_1}{4}\sin^2(2\theta).$$

For a fixed incident intensity, only the squared sine varies.

**Step 3:** Locate the maximum and compare the choices The largest possible value of $\sin^2(2\theta)$ is one. In the interval between the crossed axes, this occurs at $2\theta = 90^\circ$, or $\theta = 45^\circ$. At $90^\circ$ the product is zero. At either $30^\circ$ or $60^\circ$, the product is $3I_1/16$, which is below the maximum $I_1/4$. Hence neither unequal-angle arrangement is optimal.

**Step 4:** Conclusion Maximum light is transmitted when the inserted polaroid bisects the angle between the crossed axes, at $45^\circ$. Hence, the correct option is (C).

**Quick Tip to Solve:** For three polaroids, multiply both Malus-law factors. With crossed outer axes, rewrite the product as $\frac{1}{4}\sin^2(2\theta)$.

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**46. An object is placed in a medium of refractive index 3 . An electromagnetic wave of intensity $6 \times 10^8$ W/m$^2$ falls normally on the object and it is absorbed completely. The radiation pressure on the object would be (speed of light in free space $= 3 \times 10^8$ m/s ) :** [4 marks]

(A) $6\ \text{Nm}^{-2}$

(B) $36\ \text{Nm}^{-2}$

(C) $18\ \text{Nm}^{-2}$

(D) $2\ \text{Nm}^{-2}$

**Correct Answer:** (A) $6\ \text{Nm}^{-2}$

**Solution:**

**Concept:**

- For radiation completely absorbed at normal incidence, the radiation pressure is the incident momentum transferred per unit area per unit time.
- In the medium, the wave speed is $v = c/n$. Using the radiation-pressure relation for this question, $p_{\text{rad}} = I/v = nI/c$, where $I$ is the intensity in the medium.

**Step 1:** Find the wave speed in the medium The refractive index is $n = 3$, while the free-space speed is $c = 3 \times 10^8\,\text{m s}^{-1}$. The corresponding wave speed in the medium is

$$v = \frac{c}{n} = \frac{3 \times 10^8}{3} = 1 \times 10^8\,\text{m s}^{-1}.$$

The given intensity is already the incident energy arriving per second per unit area; it is not multiplied by an additional transmission factor.

**Step 2:** Calculate the momentum transfer on absorption The object completely absorbs the incident wave. The pressure is therefore $I/v$, without the extra factor of two that would apply to complete normal reflection in this model. Substitution gives

$$p_{\text{rad}} = \frac{6 \times 10^8\,\text{W m}^{-2}}{1 \times 10^8\,\text{m s}^{-1}} = 6\,\text{N m}^{-2}.$$

The power-of-ten factors cancel, and the units reduce as $(\text{J s}^{-1}\text{m}^{-2})/(\text{m s}^{-1}) = \text{J m}^{-3} = \text{N m}^{-2}$.

**Step 3:** Check the remaining numerical choices Using $I/c$ would yield $2\,\text{N m}^{-2}$, option (D), but it ignores the specified refractive index. The values 18 and 36 require extra factors not present in $nI/c$; in particular, neither another multiplication by $n$ nor a reflection multiplier belongs in the stated absorption calculation. This rules out (B) and (C).

**Step 4:** Conclusion The radiation pressure on the fully absorbing object is $6\,\text{N m}^{-2}$. Hence, the correct option is (A).

**Quick Tip to Solve:** First replace $c$ by the wave speed $c/n$. For complete absorption use $I/v$; reserve the extra factor two for reflection.

**47. The total kinetic energy of 1 mole of oxygen at $27^\circ\text{C}$ is : [Use universal gas constant $(R) = 8.31$ J/ mole K]** [4 marks]

(A) 6232.5 J

(B) 5670.5 J

(C) 6845.5 J

(D) 5942.0 J

**Correct Answer:** (A) 6232.5 J

**Solution:**

**Concept:**

- By equipartition, each active quadratic degree of freedom contributes $\frac{1}{2}RT$ to the energy of one mole of an ideal gas.
- Oxygen is diatomic. Near room temperature its three translational and two rotational degrees of freedom are active, while vibrations are effectively frozen; hence $f = 5$.

**Step 1:** Choose the active molecular degrees of freedom The requested total kinetic energy includes rotational as well as translational motion. At $27^\circ\text{C}$, a diatomic oxygen molecule has three independent translational components and two thermally active rotational components. Its vibration is not appreciably excited at this temperature. Thus use

$$K = \frac{f}{2}\nu RT = \frac{5}{2}\nu RT,$$

where $\nu = 1$ is the number of moles. The formula $\frac{3}{2}\nu RT$ would give only the translational contribution.

**Step 2:** Convert the temperature and substitute The absolute temperature is $T = 27 + 273 = 300\,\text{K}$. Using the specified value $R = 8.31\,\text{J}\,\text{mol}^{-1}\text{K}^{-1}$,

$$K = \frac{5}{2} \times 1 \times 8.31 \times 300\,\text{J}.$$

First $8.31 \times 300 = 2493$; multiplying by $2.5$ then gives

$$K = 6232.5\,\text{J}.$$

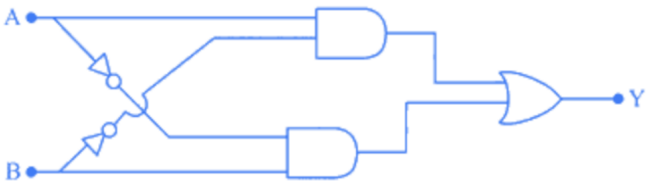
The mole and kelvin units cancel against the gas-constant units, leaving energy.

**Step 3:** Compare against the offered energies Only 6232.5 J equals the equipartition value for five active degrees of freedom at 300 K. Each of the other three values would imply a different temperature or a nonphysical effective degree count for this calculation. In particular, replacing kelvin temperature with 27 would underestimate the result drastically; the Celsius value cannot be inserted directly in an equipartition formula.

**Step 4:** Conclusion The total kinetic energy of one mole of oxygen is $6232.5\,\text{J}$. Hence, the correct option is (A).

**Quick Tip to Solve:** In a room-temperature diatomic-gas question, distinguish total kinetic energy $5RT/2$ from translational kinetic energy $3RT/2$.

**48. The truth table of the given circuit diagram is :** [4 marks]




(A)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(B)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(C)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(D)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**Correct Answer:** (A)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**Solution:**

**Concept:**

- A NOT gate complements its input, an AND gate outputs one only when both inputs are one, and an OR gate outputs one when either input is one.
- The two AND branches in the circuit form $A\overline{B}$ and $\overline{A}B$. Their OR combination is exclusive OR: $Y = A\overline{B} + \overline{A}B$.

**Step 1:** Trace the two branches before combining them The upper AND gate receives the direct input $A$ and the inverted input $\overline{B}$. The lower AND gate receives $B$ and $\overline{A}$. The crossed wires do not join: the bridge indicates that each complemented signal continues to the opposite branch. Thus the branch outputs are

$$X_1 = A\overline{B}, \qquad X_2 = \overline{A}B.$$

The final OR gate produces $Y = X_1 + X_2$, with the plus sign denoting logical OR.

**Step 2:** Evaluate every pair of binary inputs For $(A, B) = (0, 0)$, both AND terms contain a zero, so $Y = 0$. For $(0, 1)$, the lower term is $1 \cdot 1 = 1$, giving $Y = 1$. For $(1, 0)$, the upper term is $1 \cdot 1 = 1$, again giving $Y = 1$. Finally, for $(1, 1)$, both complemented inputs are zero and both AND outputs vanish. The resulting table is

| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

.

**Step 3:** Identify the matching option The output is one precisely when the two input bits differ, which identifies the XOR function and matches table (A). Table (B) gives one for the 00 input and is therefore incorrect. Table (C) gives zero for both unequal pairs and describes AND instead. Table (D) gives one for equal pairs and describes XNOR, the complement of this circuit output.

**Step 4:** Conclusion The circuit is an XOR circuit with outputs $0, 1, 1, 0$ for inputs $00, 01, 10, 11$, respectively. Hence, the correct option is (A).

**Quick Tip to Solve:** Trace each AND gate separately before reading the final OR. The expression $A\overline{B} + \overline{A}B$ immediately identifies unequal-input detection.

**49. A current of $200\mu A$ deflects the coil of a moving coil galvanometer through $60°$. The current to cause deflection through $\frac{\pi}{10}$ radian is :** [4 marks]

(A) 120 $\mu$A

(B) 180 $\mu$A

(C) 30 $\mu$A

(D) 60 $\mu$A

**Correct Answer:** (D) 60 $\mu$A

**Solution:**

**Concept:**

- In a moving-coil galvanometer with a radial magnetic field, the magnetic torque is $NABI$, while the restoring torque of the suspension is $k\theta$.
- At equilibrium $NABI = k\theta$, so for an unchanged instrument $I/\theta = k/(NAB)$ is constant. All angles used in a ratio must be in the same unit.

**Step 1:** Relate the current to the angular deflection For the first equilibrium, $NABI_1 = k\theta_1$, and for the second, $NABI_2 = k\theta_2$. The coil, magnetic field and restoring system remain the same, so division eliminates all instrument constants:

$$\frac{I_2}{I_1} = \frac{\theta_2}{\theta_1}.$$

This is a linear current-deflection relation, not a tangent relation; the radial magnetic field keeps the magnetic torque proportional to current throughout the operating range.

**Step 2:** Convert the initial angle to radians The initial angle is $60° = \pi/3$ radians. The desired angle is $\pi/10$ radians. Therefore

$$\frac{\theta_2}{\theta_1} = \frac{\pi/10}{\pi/3} = \frac{3}{10}.$$

Equivalently, the desired deflection is $18°$, whose ratio to $60°$ is also $3/10$. This verifies the unit conversion before substitution.

**Step 3:** Calculate and compare the current Using $I_1 = 200\,\mu$A,

$$I_2 = 200 \times \frac{3}{10} = 60\,\mu\text{A}.$$

It must be below 200 microamperes because the required deflection is smaller. The alternative currents 120, 180 and 30 microamperes would produce deflections of $36°$, $54°$ and $9°$, respectively, by the same calibration; none equals $18°$. Thus all three alternatives are excluded quantitatively.

**Step 4:** Conclusion The required current is $60\,\mu$A. Hence, the correct option is (D).

**Quick Tip to Solve:** Use $I_2/I_1 = \theta_2/\theta_1$ for a moving-coil galvanometer, and convert degrees and radians to the same angular unit before taking the ratio.

**50. Wheatstone bridge principle is used to measure the specific resistance $(S_1)$ of given wire, having length $L$, radius $r$. If $X$ is the resistance of wire, then specific resistance is ; $S_1 = X\left(\frac{\pi r^2}{L}\right)$. If the length of the wire gets doubled then the value of specific resistance will be :** [4 marks]

(A) $\frac{S_1}{4}$

(B) 2 $S_1$

(C) $\frac{S_1}{2}$

(D) $S_1$

**Correct Answer:** (D) $S_1$

**Solution:**

**Concept:**

- Resistivity, also called specific resistance, is a property of the material at a specified temperature. For a uniform wire, $X = \rho L/A$, where $X$ is resistance and $A = \pi r^2$ is cross-sectional area.
- Changing the length or area changes the measured resistance, whereas $\rho = XA/L$ removes this geometrical dependence. The material and temperature have not been changed in the question.

**Step 1:** Identify the measured material property The given expression $S_1 = X\pi r^2/L$ is precisely the definition of resistivity, so $S_1 = \rho$. It must not be treated as the resistance itself. The Wheatstone bridge determines $X$; the subsequent multiplication by area and division by length converts that result into a material property.

**Step 2:** Apply the length change consistently For a wire of the same area with length $L' = 2L$, the resistance is $X' = 2X$. Thus

$$S_1' = \frac{X'\pi r^2}{L'} = \frac{(2X)\pi r^2}{2L} = S_1.$$

Even if the wire is stretched so its radius also changes, the relation $X' = \rho L'/A'$ still gives $X'A'/L' = \rho$, at unchanged material and temperature.

**Step 3:** Rule out geometrical scaling of resistivity Option (B) confuses the doubled resistance of a fixed-area wire with its resistivity. Options (A) and (C) also assign a geometrical scaling to a material property. Neither a factor of one-fourth nor one-half survives when the changed resistance and dimensions are substituted together.

**Step 4:** Conclusion The specific resistance remains $\boxed{S_1}$. Hence, the correct option is (D).

**Quick Tip to Solve:** When dimensions change, first decide whether the quantity is resistance or resistivity: only resistance directly depends on the wire geometry.

## Physics – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**51. A closed organ pipe $150\ \text{cm}$ long gives 7 beats per second with an open organ pipe of length $350\ \text{cm}$, both vibrating in fundamental mode. The velocity of sound is _____ $\text{m/s}$.** [4 marks]

**Solution:**

**Concept:**

- The fundamental frequency of a closed pipe is $f_c = v/(4L_c)$, because its length is one-quarter wavelength. The open pipe has fundamental frequency $f_o = v/(2L_o)$.
- The beat frequency is the positive difference $b = |f_c - f_o|$. Both pipes contain the same medium, so the same sound speed $v$ must be used in the two expressions.

**Step 1:** Convert the pipe lengths to metres The closed pipe has length $L_c = 150\,\text{cm} = 1.50\,\text{m}$, and the open pipe has length $L_o = 350\,\text{cm} = 3.50\,\text{m}$. Since both vibrate in their fundamental modes, no higher harmonic multiplier is needed. Their wavelengths are respectively $4L_c = 6.00\,\text{m}$ and $2L_o = 7.00\,\text{m}$.

**Step 2:** Write the two fundamental frequencies For sound speed $v$ in metres per second,

$$f_c = \frac{v}{6}, \qquad f_o = \frac{v}{7}.$$

The closed pipe has the higher frequency here: its fundamental wavelength is shorter, even though its physical length is less. This establishes the sign of the frequency difference before applying the beat condition.

**Step 3:** Use the observed beat rate The two frequencies differ by $7\,\text{Hz}$, hence

$$7 = \frac{v}{6} - \frac{v}{7} = \frac{v(7-6)}{42} = \frac{v}{42}.$$

Therefore $v = 7 \times 42 = 294\,\text{m s}^{-1}$. As a direct check, the resulting frequencies are $49\,\text{Hz}$ and $42\,\text{Hz}$, whose difference is exactly the stated seven beats per second.

**Step 4:** Conclusion The velocity of sound is $\boxed{294\,\text{m s}^{-1}}$, so the numerical entry is 294.

**Quick Tip to Solve:** For mixed open and closed pipes, write the fundamental wavelengths first; the beat rate is a difference of frequencies, not wavelengths.

**52. A series LCR circuit with $\text{L} = \frac{100}{\pi}\text{mH}, \text{C} = \frac{10^{-3}}{\pi}\text{F}$ and $\text{R} = 10\Omega$, is connected across an ac source of $220\ \text{V}, 50\ \text{Hz}$ supply. The power factor of the circuit would be _____.** [4 marks]

**Solution:**

**Concept:**

- For a series LCR circuit, $X_L = \omega L$, $X_C = 1/(\omega C)$, and $\omega = 2\pi f$. Inductive and capacitive reactances oppose each other in the net impedance.

- The impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the power factor is $\cos\phi = R/Z$. Power factor is dimensionless.

**Step 1:** Convert inductance and calculate angular frequency The inductance in henries and capacitance in farads are

$$L = \frac{100}{\pi} \times 10^{-3} = \frac{0.1}{\pi}\,\text{H}, \qquad C = \frac{10^{-3}}{\pi}\,\text{F}.$$

With $f = 50\,\text{Hz}$, the angular frequency is $\omega = 2\pi(50) = 100\pi\,\text{rad}\,\text{s}^{-1}$. Converting millihenries is essential; using the numerical value in millihenries directly would change the reactance by a factor of a thousand.

**Step 2:** Compare inductive and capacitive reactances Substituting the supplied values gives

$$X_L = (100\pi)\frac{0.1}{\pi} = 10\,\Omega,$$

$$X_C = \frac{1}{(100\pi)(10^{-3}/\pi)} = 10\,\Omega.$$

Thus the circuit is at resonance. Although the inductor and capacitor each have a nonzero reactance, their opposite contributions cancel in the series impedance.

**Step 3:** Evaluate impedance and power factor The net reactance is zero, so

$$Z = \sqrt{10^2 + (10-10)^2} = 10\,\Omega, \qquad \cos\phi = \frac{R}{Z} = \frac{10}{10} = 1.$$

The source voltage sets the current magnitude, but it is not needed to calculate the phase angle here. At resonance the source current and voltage are in phase, consistent with unity power factor.

**Step 4:** Conclusion The circuit power factor is $\boxed{1}$, a dimensionless quantity.

**Quick Tip to Solve:** Check whether $\omega L = 1/(\omega C)$ before calculating a full impedance; equality immediately gives unity power factor in a series LCR circuit.

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**53. Two charges of $-4\mu$C and $+4\mu$C are placed at the points $\text{A}(1,0,4)\text{m}$ and $\text{B}(2,-1,5)\text{m}$ located in an electric field $\vec{\text{E}} = 0.20\hat{i}\ \text{V/cm}$. The magnitude of the torque acting on the dipole is $8\sqrt{\alpha} \times 10^{-5}\text{Nm}$, where $\alpha =$ _____.** [4 marks]

**Solution:**

**Concept:**

- The electric dipole moment points from the negative charge to the positive charge: $\boldsymbol{p} = q(\boldsymbol{r}_+ - \boldsymbol{r}_-)$. The displacement vector must use the given coordinates in metres.
- In a uniform electric field, the dipole torque is $\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E}$. Only the component of dipole separation perpendicular to the field contributes to its magnitude.

**Step 1:** Form the dipole moment vector The displacement from the negative charge at A to the

positive charge at B is

$$\overrightarrow{AB} = (2-1)\hat{\imath} + (-1-0)\hat{\jmath} + (5-4)\hat{k} = \hat{\imath} - \hat{\jmath} + \hat{k} \quad \text{m}.$$

The charge magnitude is $q = 4 \times 10^{-6}\,\text{C}$, hence $\boldsymbol{p} = 4 \times 10^{-6}(\hat{\imath} - \hat{\jmath} + \hat{k})\,\text{C m}$. The sign convention is fixed by pointing from minus to plus.

**Step 2:** Convert the field and take the cross product The field magnitude in SI units is $0.20\,\text{V/cm} = 20\,\text{V/m}$, directed along positive x. Therefore

$$\boldsymbol{\tau} = 4 \times 10^{-6}(\hat{\imath} - \hat{\jmath} + \hat{k}) \times (20\hat{\imath}) = 8 \times 10^{-5}(\hat{\jmath} + \hat{k})\,\text{N m}.$$

Here $\hat{\imath} \times \hat{\imath} = 0$, $-\hat{\jmath} \times \hat{\imath} = \hat{k}$, and $\hat{k} \times \hat{\imath} = \hat{\jmath}$.

**Step 3:** Match the required torque form The two nonzero components are perpendicular and equal, so

$$|\boldsymbol{\tau}| = 8 \times 10^{-5}\sqrt{1^2 + 1^2} = 8\sqrt{2} \times 10^{-5}\,\text{N m}.$$

Comparing this with $8\sqrt{\alpha} \times 10^{-5}\,\text{N m}$ identifies $\alpha = 2$. The x displacement does not enter the magnitude because it is parallel to the applied field.

**Step 4:** Conclusion The required value is $\boxed{\alpha = 2}$, dimensionless; the torque magnitude is $8\sqrt{2} \times 10^{-5}\,\text{N m}$.

**Quick Tip to Solve:** For a field along x, calculate the dipole separation in the yz plane and convert V/cm to V/m before multiplying by charge.

**54. A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is $\frac{7}{x}$, where $x$ is _____.** [4 marks]

**Solution:**

**Concept:**

- For rolling without slipping, total kinetic energy is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, with $v = R\omega$. A ring has $I = mR^2$; a solid sphere has $I = \frac{2}{5}mR^2$.
- When a body rolls from rest down a vertical height $h$ without dissipative losses, static friction does no work and conservation of energy gives total kinetic energy $K = mgh$.

**Step 1:** Apply energy conservation to each rolling body For the ring,

$$m_r gh = \frac{1}{2}m_r v_r^2 + \frac{1}{2}m_r R^2 \frac{v_r^2}{R^2} = m_r v_r^2.$$

For the solid sphere,

$$m_s gh = \frac{1}{2}m_s v_s^2 + \frac{1}{2}\left(\frac{2}{5}m_s R^2\right)\frac{v_s^2}{R^2} = \frac{7}{10}m_s v_s^2.$$

The two speeds differ because different fractions of the released potential energy go into rotation. This does not change the total energy gained per unit mass over a given vertical descent.

**Step 2:** Compare total rather than translational energy At the same vertical descent, the total kinetic energies are $K_r = m_r gh$ and $K_s = m_s gh$. In the equal-mass comparison used for the stated numerical result, both equal $mgh$, so

$$\frac{K_r}{K_s} = 1 = \frac{7}{x}.$$

It follows that $x = 7$. The ratio $7/10$ sometimes obtained in this problem concerns only translational kinetic energies of equal masses at the same descent; it excludes rotational energy and is not the total kinetic-energy ratio.

**Step 3:** Identify the scope of the numerical comparison The printed statement specifies equal radii but does not explicitly specify masses or the instant of comparison. In general, at equal descent the total-energy ratio is $m_r/m_s$; therefore identical radii alone do not establish a unique total-energy ratio. The numerical value 7 corresponds to the usual equal-mass, equal-height interpretation of this comparison.

**Step 4:** Conclusion For the conventional equal-mass comparison over the same descent, $\boxed{x = 7}$ (dimensionless). More generally, the literal data give $x = 7m_s/m_r$ at equal descent; the masses are not stated explicitly.

**Quick Tip to Solve:** Include rotational energy when comparing rolling bodies: at a common height the total energy is $mgh$, even though their translational speeds differ.

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**55. A body falling under gravity covers two points $A$ and $B$ separated by $80\ \text{m}$ in $2\ \text{s}$. The distance of upper point A from the starting point is _____ $\text{m}$ (use $\text{g} = 10\ \text{ms}^{-2}$).** [4 marks]

**Solution:**

**Concept:**

- For constant gravitational acceleration, the displacement during an interval $t$ starting with downward speed $u$ is $s = ut + \frac{1}{2}gt^2$. Take downward as positive throughout.
- A body falling from rest obeys $v^2 = 2gh$, where $h$ is its distance below the release point. First find the speed at the upper point A, then use this relation for the preceding fall.

**Step 1:** Find the speed at the upper point Let the downward speed as the body passes A be $v_A$. Between A and B it travels $80\,\text{m}$ in $2\,\text{s}$, while gravity accelerates it at $10\,\text{m}\,\text{s}^{-2}$. Applying the interval displacement formula,

$$80 = v_A(2) + \frac{1}{2}(10)(2)^2 = 2v_A + 20.$$

Thus $2v_A = 60$ and $v_A = 30\,\text{m}\,\text{s}^{-1}$. This is not the average speed of $40\,\text{m}\,\text{s}^{-1}$ over AB, because the speed increases during that interval.

**Step 2:** Calculate the fall before reaching A The falling body begins from rest at its starting point. If that point is a distance $h$ above A, then

$$v_A^2 = 0^2 + 2gh, \qquad h = \frac{30^2}{2(10)} = \frac{900}{20} = 45\,\text{m}.$$

Equivalently, the time to reach A is $v_A/g = 3\,\text{s}$, and the distance in those three seconds is $\frac{1}{2}(10)(3)^2 = 45\,\text{m}$.

**Step 3:** Check the complete fall The body reaches B after a total of $5\,\text{s}$. Its total distance then is $\frac{1}{2}(10)(5)^2 = 125\,\text{m}$. Subtracting the first $45\,\text{m}$ gives $125 - 45 = 80\,\text{m}$, exactly the specified separation, and the time difference is $5 - 3 = 2\,\text{s}$.

**Step 4:** Conclusion The upper point A is $\boxed{45\,\text{m}}$ below the starting point, so the numerical entry is $45$.

**Quick Tip to Solve:** Use the known interval to determine the speed at its first point; then trace backward to the release point with $v^2 = 2gh$.

**56. The electric potential at the surface of an atomic nucleus $(z = 50)$ of radius $9 \times 10^{-13}$ cm is _____ $\times 10^6$ V.** [4 marks]

**Solution:**

**Concept:**

- Outside a spherically symmetric charge distribution, the potential with zero at infinity is $V = kQ/r$. At the nuclear surface use $r = R$ and nuclear charge $Q = Ze$.
- The atomic number $Z$ counts protons. Use $e = 1.6 \times 10^{-19}\,\text{C}$ and $k = 9 \times 10^9\,\text{N m}^2\,\text{C}^{-2}$, with the given radius converted from centimetres to metres.

**Step 1:** Calculate the nuclear charge in coulombs The nucleus has atomic number $Z = 50$, so its positive charge is

$$Q = Ze = 50(1.6 \times 10^{-19}) = 8.0 \times 10^{-18}\,\text{C}.$$

Only the nuclear charge is being considered: the requested potential is at the surface of the nucleus. The expression is therefore not the potential of a neutral atom as a whole; electron shielding is not included in this nuclear-surface calculation.

**Step 2:** Convert the surface radius to SI units The given radius is $9 \times 10^{-13}\,\text{cm}$. Since one centimetre is $10^{-2}\,\text{m}$,

$$R = 9 \times 10^{-13} \times 10^{-2} = 9 \times 10^{-15}\,\text{m}.$$

The exponent decreases by two when converting this length into metres. Failing to convert the radius would reduce the computed potential by a factor of one hundred.

**Step 3:** Evaluate the surface potential and its coefficient Substitute the charge and radius into $V = kQ/R$:

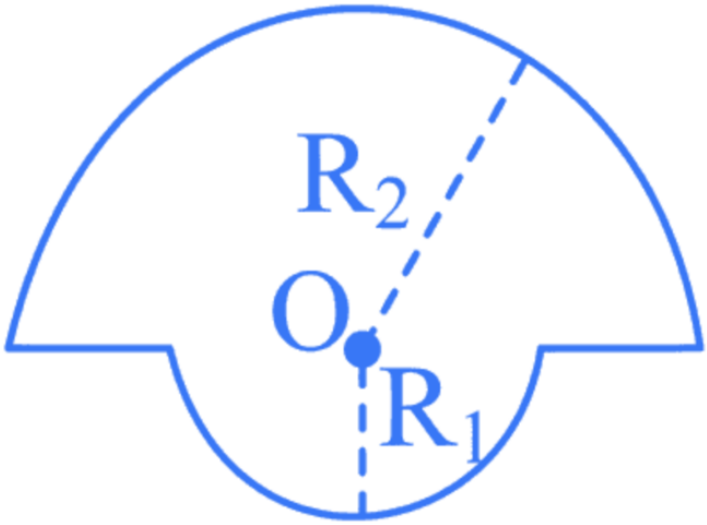
$$V = \frac{(9 \times 10^9)(8.0 \times 10^{-18})}{9 \times 10^{-15}} = 8.0 \times 10^{9-18+15} = 8.0 \times 10^6\,\text{V}.$$

The potential is positive because the nucleus is positively charged. The question has already factored out $10^6$ V, so the blank requires only its numerical coefficient, rather than the complete value in volts.

**Step 4:** Conclusion The surface potential is $\boxed{8 \times 10^6\,\text{V}}$; the required numerical coefficient is $\boxed{8}$.

**Quick Tip to Solve:** Convert the nuclear radius into metres before using $kZe/R$, then report just the coefficient when the question already supplies a power of ten.

**57. The magnetic field at the centre of a wire loop formed by two semicircular wires of radii $R_1 = 2\pi\text{m}$ and $R_2 = 4\pi\text{m}$, carrying current $\text{I} = 4\ \text{A}$ as per figure given below is $\alpha \times 10^{-7}\ \text{T}$. The value of $\alpha$ is _____. (Centre O is common for all segments)** [4 marks]




**Solution:**

**Concept:**

- For a circular arc of radius $R$, angle $\theta$ and current $I$, the field magnitude at its centre is $B = \mu_0 I\theta/(4\pi R)$. A semicircle has $\theta = \pi$.
- The straight connecting segments lie along radii through $O$. Their contribution at $O$ vanishes because $d\ell$ is parallel to the displacement vector in the Biot–Savart law.

**Step 1:** Identify the contributions from the loop The wire is a single closed loop, with an upper semicircle of radius $R_2$, a lower semicircle of radius $R_1$, and two radial connectors. Following current continuously around the loop, both semicircles circulate about $O$ in the same sense. The right-hand rule therefore gives fields in the same perpendicular direction. The result asks for a magnitude, so reversing the entire current would reverse the field but would not change the required coefficient.

**Step 2:** Add the fields of the two semicircles Using $\mu_0 = 4\pi \times 10^{-7}\,\text{T m A}^{-1}$, each semicircle contributes $\mu_0 I/(4R)$. Thus

$$B = \frac{\mu_0 I}{4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

Insert the two different radii before simplifying:

$$B = \frac{(4\pi \times 10^{-7})(4)}{4}\left(\frac{1}{2\pi} + \frac{1}{4\pi}\right) = 4\pi \times 10^{-7}\frac{3}{4\pi} = 3 \times 10^{-7}\,\text{T}.$$

The radial segments add zero; neither a full-circle factor nor a subtraction of the two arc fields is appropriate.

**Step 3:** Conclusion The magnetic field magnitude is $3 \times 10^{-7}\,\text{T}$. Comparing with $\alpha \times 10^{-7}\,\text{T}$, the required dimensionless value is $\boxed{\alpha = 3}$.

**Quick Tip to Solve:** Follow the current around the entire loop before adding arc fields: these two semicircles have the same circulation sense, while radial connectors give zero field at the centre.

**58. A parallel beam of monochromatic light of wavelength 5000 $\overset{o}{A}$ is incident normally on a single narrow slit of width $0.001$ mm. The light is focused by convex lens on screen, placed on its focal plane. The first minima will be formed for the angle of diffraction of _____ (degree).** [4 marks]

**Solution:**

**Concept:**

- For a slit of width $a$, destructive interference occurs at $a\sin\theta = m\lambda$, where $m = 1, 2, \ldots$. The first minimum corresponds to $m = 1$.
- The diffraction angle must be found from the sine relation when it is not small. A lens places each diffracted direction at a corresponding point of its focal plane.

**Step 1:** Convert the wavelength and slit width to metres The wavelength is stated in angstroms, with $1$ angstrom $= 10^{-10}\,\mathrm{m}$. Therefore

$$\lambda = 5000 \times 10^{-10} = 5.0 \times 10^{-7}\,\mathrm{m}.$$

The slit width is

$$a = 0.001\,\mathrm{mm} = 0.001 \times 10^{-3} = 1.0 \times 10^{-6}\,\mathrm{m}.$$

Both lengths are now in the same unit. Their ratio is dimensionless and equals one half. The slit is only twice the wavelength, so there is no justification for treating the diffraction angle as very small.

**Step 2:** Apply the condition for the first minimum For the first dark direction on either side of the central maximum, set $m = 1$:

$$\sin\theta = \frac{\lambda}{a} = \frac{5.0 \times 10^{-7}}{1.0 \times 10^{-6}} = \frac{1}{2}.$$

The first positive angle satisfying this relation is

$$\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ.$$

There is a corresponding minimum at $-30^\circ$. The question requests the positive magnitude in degrees. The focal length is unnecessary, because it would be needed only to convert this angular position into a distance on the screen. Using $\theta \simeq \lambda/a$ here would incorrectly give $0.5$ radians rather than the exact angle.

**Step 3:** Conclusion The first diffraction minima lie at angular magnitudes $\boxed{30^\circ}$, so the numerical entry in degrees is $\boxed{30}$.

**Quick Tip to Solve:** Calculate $\lambda/a$ first. If it is not much smaller than one, use the inverse sine instead of the small-angle approximation.

**59. The reading of pressure metre attached with a closed pipe is $4.5 \times 10^4$ N/m$^2$. On opening the valve, water starts flowing and the reading of pressure metre falls to $2.0 \times 10^4$ N/m$^2$. The velocity of water is found to be $\sqrt{V}$ m/s. The value of $V$ is _____.** [4 marks]

**Solution:**

**Concept:**

- For steady incompressible flow at a fixed height, Bernoulli's equation relates pressure and speed through $P + \frac{1}{2}\rho v^2 =$ constant.
- The water is initially stationary, so opening the valve converts a fall in pressure into kinetic energy per unit volume. Use $\rho = 1000\,\mathrm{kg\,m^{-3}}$ for water.

**Step 1:** Find the pressure available to accelerate the water When the valve is closed, water has zero flow speed and the pressure reading is $P_1 = 4.5 \times 10^4\,\mathrm{Pa}$. With the valve open, the pressure becomes $P_2 = 2.0 \times 10^4\,\mathrm{Pa}$. The pressure reduction is

$$\Delta P = P_1 - P_2 = (4.5 - 2.0) \times 10^4 = 2.5 \times 10^4\,\mathrm{Pa}.$$

Only this difference is needed. Whether the meter reports gauge pressure or absolute pressure does not affect the difference, because any common atmospheric reference cancels.

**Step 2:** Equate the pressure decrease to the kinetic term At the same elevation, the gravitational terms cancel in Bernoulli's equation. Taking the initial velocity as zero gives

$$P_1 = P_2 + \frac{1}{2}\rho v^2.$$

Consequently,

$$v^2 = \frac{2(P_1 - P_2)}{\rho} = \frac{2(2.5 \times 10^4)}{1000} = 50\,\mathrm{m^2\,s^{-2}}.$$

The units check because pressure divided by mass density has units of speed squared. Hence $v = \sqrt{50}\,\mathrm{m\,s^{-1}}$, approximately $7.07\,\mathrm{m\,s^{-1}}$. The question defines this speed as $\sqrt{V}\,\mathrm{m\,s^{-1}}$, so it is the value under the square root that must be entered, rather than the speed itself.

**Step 3:** Conclusion The speed is $\sqrt{50}\,\mathrm{m\,s^{-1}}$. Therefore the required numerical parameter is $\boxed{V = 50}$.

**Quick Tip to Solve:** Use the pressure drop, not either pressure alone, and compare your final speed with the exact form $\sqrt{V}$ requested in the question.

**60. If Rydberg's constant is $R$, the longest wavelength of radiation in Paschen series will be $\frac{\alpha}{7R}$, where $\alpha =$ _____.** [4 marks]

**Solution:**

**Concept:**

- In the hydrogen spectrum, the Rydberg formula is $1/\lambda = R(1/n_f^2 - 1/n_i^2)$, with $n_i > n_f$.
- Paschen transitions end at $n_f = 3$. The longest wavelength corresponds to the smallest emitted photon energy, hence the nearest higher initial level.

**Step 1:** Select the transition of smallest energy All Paschen emissions terminate at the third Bohr level. The possible initial levels are $n_i = 4, 5, 6, \ldots$. As the initial level increases, $1/n_i^2$ decreases, so the difference $1/9 - 1/n_i^2$ increases. Thus the wavenumber increases and the wavelength decreases as we go to higher initial levels. The longest wavelength is therefore the first line of the series, corresponding to the transition $4 \to 3$. It is not the series limit $\infty \to 3$, which has the largest energy and shortest wavelength in this series.

**Step 2:** Evaluate the wavelength and compare coefficients Put $n_f = 3$ and $n_i = 4$ into the Rydberg relation:

$$\frac{1}{\lambda_{\max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = R\left(\frac{1}{9} - \frac{1}{16}\right).$$

Taking the common denominator gives

$$\frac{1}{\lambda_{\max}} = R\frac{16-9}{144} = \frac{7R}{144}.$$

Invert only after completing the subtraction:

$$\lambda_{\max} = \frac{144}{7R}.$$

Since $R$ has units of inverse length, this expression correctly has dimensions of length. The denominator already matches the question's $7R$, so the numerator is exactly the required dimensionless coefficient.

**Step 3:** Conclusion The longest Paschen wavelength is $144/(7R)$. Thus $\boxed{\alpha = 144}$, a dimensionless numerical value.

**Quick Tip to Solve:** For the longest wavelength of a series ending at level $n$, use the adjacent transition $(n+1) \to n$; the series limit gives the shortest wavelength.

## Chemistry – Section A: Multiple Choice Questions (80 marks)

**61. Identity the incorrect pair from the following :** [4 marks]

(A) Haber process - Iron

(B) Polythene preparation - $TiCl_4, Al(CH_3)_3$

(C) Photography - AgBr

(D) Wacker process - $PtCl_2$

**Correct Answer:** (D) Wacker process - $PtCl_2$

**Solution:**

**Concept:**

- Named industrial processes are distinguished by the specific metal or metal compound used as catalyst; metals in the same periodic group are not interchangeable in these standard associations.
- The Wacker oxidation converts ethene to ethanal using a palladium(II) chloride catalyst, with copper(II) chloride involved in catalyst regeneration.

**Step 1:** Check the first three associations The Haber process synthesises ammonia from nitrogen and hydrogen using finely divided iron as the principal catalyst, so pair (A) is correct. In polythene preparation, titanium tetrachloride combined with an organoaluminium compound forms a Ziegler–Natta type catalyst. The stated $TiCl_4$, $Al(CH_3)_3$ pair in (B) belongs to that catalyst class and is treated as the correct association here. Silver bromide, $AgBr$, is light-sensitive and is used in photographic emulsions, so (C) is also a correct pair. These three options therefore do not satisfy the demand for the incorrect association.

**Step 2:** Identify the metal compound in the Wacker process The characteristic catalyst for the Wacker process is palladium(II) chloride, $PdCl_2$, rather than platinum(II) chloride, $PtCl_2$. The oxidation can be represented overall as

$$2\,CH_2{=}CH_2 + O_2 \longrightarrow 2\,CH_3CHO.$$

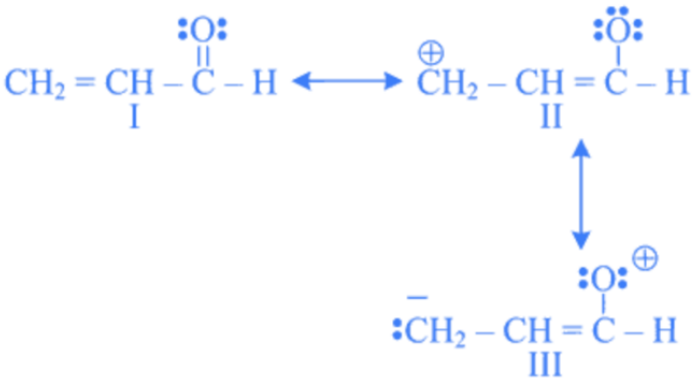
Palladium mediates the alkene oxidation, while copper chloride and oxygen participate in regeneration of the active catalyst. Option (D) names platinum in place of palladium. Even though platinum and palladium are related transition metals, that replacement makes the stated standard process–catalyst pair incorrect. The question asks for a mismatched pair, so the incorrect pair itself is the answer.

**Step 3:** Conclusion The incorrect pair is Wacker process–$PtCl_2$; the correct catalyst association is $PdCl_2$. Hence, the correct option is (D).

**Quick Tip to Solve:** Remember Wacker oxidation by its palladium catalyst: distinguish $PdCl_2$ from the similarly written platinum compound $PtCl_2$.

**62. The order of relative stability of the contributing structure is :**

**Choose the correct answer from the options given below :** [4 marks]




(A) I = II = III

(B) I > II > III

(C) III > II > I

(D) II > I > III

**Correct Answer:** (B) I > II > III

**Solution:**

**Concept:**

- Resonance contributors are ranked by favourable electron distribution: complete octets, more covalent bonding, and less charge separation generally increase their importance.
- When opposite charges occur, a negative charge is more favourable on electronegative oxygen than on carbon; a positive charge on oxygen is correspondingly less favourable.

**Step 1:** Compare the neutral and charge-separated contributors Contributor I is the neutral conjugated aldehyde structure, $CH_2{=}CH{-}CHO$. Its carbon atoms and oxygen have their normal valences and complete octets, and no formal charges are separated. It therefore makes the largest contribution to the resonance hybrid. In contributor II, electron delocalisation produces a positive charge at the terminal carbon and a negative charge on oxygen. Although the negative charge is placed on a suitable electronegative atom, this form has charge separation and an electron-deficient positively charged carbon. It is less stable than neutral I, so $\mathrm{I} > \mathrm{II}$.

**Step 2:** Rank the two ionic contributors and eliminate options Contributor III reverses the favourable charge distribution: the terminal carbon bears a negative charge while oxygen bears a positive charge. This places electron excess on the less electronegative atom and electron deficiency on the more electronegative atom. It is therefore less favourable than II, which puts negative charge on oxygen. Combining the two comparisons gives

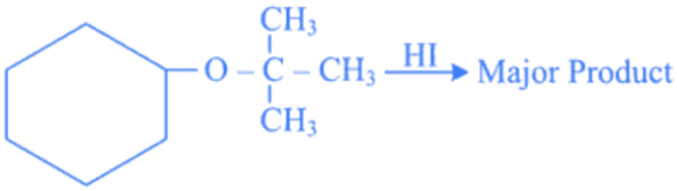
$$\mathrm{I} > \mathrm{II} > \mathrm{III}.$$

Option (A) incorrectly treats the unequal contributors as equally stable. Option (C) makes the least favourable charge arrangement the most stable. Option (D) incorrectly places the charge-separated structure II above the neutral octet-complete form I. Only option (B) agrees with both comparisons. The drawings are contributors of one resonance hybrid, not distinct molecules in equilibrium.

**Step 3:** Conclusion The relative stability is $\boxed{\mathrm{I} > \mathrm{II} > \mathrm{III}}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Rank the neutral octet-complete contributor first, then compare ionic forms by asking which one places the negative charge on oxygen rather than carbon.

**63. Major product formed in the following reaction is a mixture of :** [4 marks]




(A) Cyclohexyl iodide and $(CH_3)_3CI$

(B) Cyclohexanol and $(CH_3)_3COH$

(C) Cyclohexyl iodide and $(CH_3)_3COH$

(D) Cyclohexanol and $(CH_3)_3CI$

**Correct Answer:** (D) Cyclohexanol and $(CH_3)_3CI$

**Solution:**

**Concept:**

- Hydrogen iodide protonates the ether oxygen, converting the oxygen-containing group into a good leaving group and allowing cleavage of a carbon–oxygen bond.
- When one ether carbon is tertiary, cleavage preferentially forms the stable tertiary carbocation. Iodide then traps that carbocation, and the other fragment becomes an alcohol.

**Step 1:** Identify the two groups attached to oxygen The ether in the reaction joins a cyclohexyl group and a tert-butyl group through oxygen: $C_6H_{11}-O-C(CH_3)_3$. The carbon attached to oxygen on the tert-butyl side is tertiary, whereas the corresponding carbon on the cyclohexyl side is secondary. After oxygen is protonated by HI, cleavage on the tertiary side can produce the relatively stable tert-butyl carbocation and neutral cyclohexanol. This tertiary pathway is favoured over an iodide displacement at a crowded tertiary centre. The oxygen stays attached to the cyclohexyl fragment during this cleavage.

**Step 2:** Form the products and compare all options Iodide combines with the tert-butyl carbocation, giving tert-butyl iodide. The net cleavage is

$$C_6H_{11}-O-C(CH_3)_3 + HI \longrightarrow C_6H_{11}OH + (CH_3)_3CI.$$

Thus the product pair is cyclohexanol plus tert-butyl iodide. Option (A) makes both fragments iodides and does not represent the direct cleavage pair. Option (B) gives two alcohols and omits the incorporation of iodide. Option (C) assigns iodide to the secondary cyclohexyl carbon while retaining the tertiary fragment as an alcohol, the opposite of the favoured cleavage. Option (D) correctly retains oxygen in cyclohexanol and puts iodine on the tertiary fragment. This identifies the major products of the stated ether-cleavage reaction.

**Step 3:** Conclusion The major product mixture is cyclohexanol and tert-butyl iodide, $(CH_3)_3CI$. Hence, the correct option is (D).

**Quick Tip to Solve:** For ether cleavage by HI, check for a tertiary alkyl group before applying the usual less-hindered-carbon substitution rule.

**64. Given below are two statements :**

**Statement (I) : Oxygen being the first member of group 16 exhibits only -2 oxidation state.**

**Statement (II) : Down the group 16 stability of +4 oxidation state decreases and +6 oxidation state increases.**

**In the light of the above statements, choose the most appropriate answer from the options given below :** [4 marks]

(A) Statement I is correct but Statement II is incorrect

(B) Both Statement I and Statement II are incorrect

(C) Statement I is incorrect but Statement II is correct

(D) Both Statement I and Statement II are correct

**Correct Answer:** (B) Both Statement I and Statement II are incorrect

**Solution:**

**Concept:**

- Oxidation state is assigned by giving the bonding electrons to the more electronegative atom. Oxygen usually has oxidation state $-2$, but exceptions must be checked before accepting the word "only.
- In the heavier group 16 elements, the inert-pair effect makes the lower positive oxidation state, $+4$, progressively more stable relative to $+6$.

**Step 1:** Test the oxidation states of oxygen In an ordinary oxide such as $MgO$, oxygen is $-2$. However, the oxygen atoms in $H_2O_2$ each have oxidation state $-1$, and those in superoxide $O_2^-$ have average oxidation state $-\frac{1}{2}$. In elemental $O_2$ the value is zero. Fluorine is more electronegative than oxygen, so in $OF_2$ oxygen has oxidation state $+2$. Thus being the first member of the group does not restrict oxygen to $-2$. Statement I is incorrect.

**Step 2:** Apply the inert-pair trend Down the group, the outer $ns^2$ pair becomes less available for bonding. The state involving all six valence electrons therefore becomes less stable relative to the state using four. Consequently the stability of $+4$ increases and that of $+6$ decreases down group 16, opposite to Statement II. Statement II is also incorrect. Options A and C each accept one false statement, while D accepts both, so all three are ruled out.

**Step 3:** Conclusion Both Statement I and Statement II are incorrect. Hence, the correct option is (B).

**Quick Tip to Solve:** For a statement containing "only, look for one clear exception. For heavy p-block elements, use the inert-pair effect to identify the more stable lower oxidation state.

**65. The technique used for purification of steam volatile water immiscible substances is :** [4 marks]

(A) Fractional distillation

(B) Distillation

(C) Fractional distillation under reduced pressure

(D) Steam distillation

**Correct Answer:** (D) Steam distillation

**Solution:**

**Concept:**

- Steam distillation separates a steam-volatile organic substance that is immiscible with water. Each liquid contributes its own vapour pressure to the mixture.
- The mixture boils when $p_{\text{water}} + p_{\text{organic}} = p_{\text{atm}}$, so the organic substance can distil below its normal boiling temperature.

**Step 1:** Use the two conditions in the question The substance is volatile in steam, so it can enter the vapour along with water. It is also immiscible with water, so separate liquid phases exist and the vapour pressures of the two liquids add. The total can reach atmospheric pressure at a temperature below the normal boiling point of the organic substance. Passing steam therefore carries the volatile substance into the condenser without requiring it to be heated to its own boiling point.

**Step 2:** Identify the specific purification method The condensed distillate contains water and the organic compound; because they are immiscible, they form layers that can be separated. These are precisely the operating conditions of steam distillation. Fractional distillation is mainly used for miscible liquids with close boiling points, so A is not the specific technique here. Ordinary distillation in B does not exploit the independent water vapour pressure. Reduced-pressure fractionation in C lowers the external pressure, whereas the given conditions point to distillation with steam.

**Step 3:** Conclusion The required technique is steam distillation. Hence, the correct option is (D).

**Quick Tip to Solve:** The combination "steam volatile and "water immiscible directly identifies steam distillation; close boiling points instead suggest fractional distillation.

**66. Which structure of protein remains intact after coagulation of egg white on boiling?** [4 marks]

(A) Quaternary

(B) Primary

(C) Secondary

(D) Tertiary

**Correct Answer:** (B) Primary

**Solution:**

**Concept:**

- The primary structure of a protein is its amino-acid sequence, joined through covalent peptide bonds. It is different from the folding of that chain.

- Denaturation disrupts the interactions maintaining higher levels of protein organisation. Coagulation of egg white on heating is an example of denaturation followed by aggregation.

**Step 1:** Distinguish sequence from folding The order of amino-acid residues along a polypeptide chain constitutes its primary structure. Secondary structure describes local arrangements such as an $\alpha$-helix or $\beta$-sheet, supported chiefly by hydrogen bonding. Tertiary structure is the overall three-dimensional folding of a chain, and quaternary structure describes the association of multiple polypeptide subunits. These levels must not be confused with the covalent sequence.

**Step 2:** Apply the effect of boiling On boiling egg white, heat disrupts interactions responsible for the folded protein structure. The unfolded chains aggregate to give the familiar coagulated solid. Under this treatment peptide bonds are not normally hydrolysed, so the amino-acid sequence remains intact even though the native conformation and biological activity are lost. Therefore secondary and tertiary organisation are not retained, and quaternary organisation, where present, is also disrupted. This excludes C, D and A respectively; only the primary structure in B is preserved.

**Step 3:** Conclusion The primary structure remains intact after coagulation of egg white on boiling. Hence, the correct option is (B).

**Quick Tip to Solve:** In denaturation, ask whether peptide bonds are broken: loss of folding preserves the amino-acid sequence, whereas hydrolysis of peptide bonds changes the primary structure.

**67. Identify B formed in the reaction.**

$$\mathrm{Cl-(CH_2)_4-Cl} \xrightarrow{\textbf{excess } \mathrm{NH_3}} \mathrm{A} \xrightarrow{\mathrm{NaOH}} \mathrm{B + H_2O + NaCl}$$

[4 marks]

(A) 


(B) $(CH_2)_4N{-}N(CH_2)_4$, with each $(CH_2)_4N$ unit a five-membered ring

(C) $Cl^-\ H_3N^+{-}(CH_2)_4{-}NH_3{}^+\ Cl^-$

(D) $H_2N{-}(CH_2)_4{-}NH_2$

**Correct Answer:** (A) 


**Solution:**

**Concept:**

- Ammonia is a nucleophile and replaces halide in a primary alkyl halide by nucleophilic substitution. A molecule with two appropriately spaced leaving groups can undergo ring closure.
- A four-carbon chain joining the two leaving groups can form a five-membered ring containing four carbon atoms and one nitrogen atom. Alkali converts an ammonium salt into the free amine.

**Step 1:** Follow substitution and ring closure In $Cl(CH_2)_4Cl$, ammonia first attacks a terminal primary carbon and replaces chloride. The nitrogen-containing end can then attack the other terminal carbon intramolecularly. Joining nitrogen to both ends of the four-carbon chain produces a five-membered saturated heterocycle. The initially formed cyclic ammonium species is the hydrochloride salt of pyrrolidine, represented by A. Count the ring atoms as four carbon atoms plus nitrogen; the nitrogen

retains hydrogen rather than acquiring another nitrogen atom.

**Step 2:** Liberate the free cyclic amine Treatment with $\mathrm{NaOH}$ removes the extra proton from the cyclic ammonium ion. The proton and hydroxide give water, and the chloride counterion gives $\mathrm{NaCl}$, exactly as displayed in the second reaction. B is therefore pyrrolidine, the five-membered ring containing $\mathrm{NH}$, shown in A. The N–N-linked pair of rings in B would require formation of an N–N bond, which these reagents do not provide. C is an ionic diammonium salt rather than a neutral product of alkali treatment, and D is the open-chain diamine rather than the ring-closure product.

**Step 3:** Conclusion B is pyrrolidine, the saturated five-membered ring containing one $\mathrm{NH}$ group. Hence, the correct option is (A).

**Quick Tip to Solve:** For an $\alpha,\omega$-dihaloalkane reacting with ammonia, count the ring formed by linking nitrogen to both chain ends; four carbons give a favourable five-membered ring.

**68. Identify from the following species in which $\mathrm{d^2sp^3}$ hybridization is shown by central atom :**

[4 marks]

(A) $\left[\mathrm{Co}\left(\mathrm{NH_3}\right)_6\right]^{3+}$

(B) $\mathrm{SF_6}$

(C) $\left[\mathrm{Pt}\left(\mathrm{Cl_4}\right)\right]^{2-}$

(D) $\mathrm{BrF_5}$

**Correct Answer:** (A) $\left[\mathrm{Co}\left(\mathrm{NH_3}\right)_6\right]^{3+}$

**Solution:**

**Concept:**

- In valence-bond theory, $d^2sp^3$ hybridisation uses two inner $(n-1)d$ orbitals, one $ns$ orbital and three $np$ orbitals to form six octahedral bonding orbitals.
- The oxidation state and electron configuration of the central metal, together with ligand-induced pairing, distinguish an inner-orbital octahedral complex from other species.

**Step 1:** Analyse the cobalt complex Ammonia is neutral, so cobalt has oxidation state $+3$ in $[\mathrm{Co(NH_3)_6}]^{3+}$. Cobalt has atomic number 27 and its ion is $\mathrm{Co^{3+}}$ : $3d^6$. With $\mathrm{Co^{3+}}$, the six ammonia ligands produce a low-spin arrangement: the six $3d$ electrons pair in three orbitals, leaving two inner $3d$ orbitals vacant. These two orbitals, together with $4s$ and three $4p$ orbitals, provide the six $d^2sp^3$ hybrids for the octahedral complex.

**Step 2:** Exclude the other central atoms In the conventional school-level hybridisation description, $\mathrm{SF_6}$ is assigned $sp^3d^2$, which uses outer orbitals rather than inner $d^2sp^3$ orbitals. The platinum(II) complex $[\mathrm{PtCl_4}]^{2-}$ is a four-coordinate, square-planar $d^8$ species with $dsp^2$ hybridisation. $\mathrm{BrF_5}$ has five bonds and one lone pair around bromine; its electron-pair geometry is octahedral and its shape is square pyramidal, conventionally assigned $sp^3d^2$. None of B, C or D has the specified inner-orbital hybridisation.

**Step 3:** Conclusion The central cobalt atom in $[\mathrm{Co(NH_3)_6}]^{3+}$ shows $d^2sp^3$ hybridisation. Hence, the correct option is (A).

**Quick Tip to Solve:** Do not equate every octahedral arrangement with $d^2sp^3$: identify whether the two d orbitals belong to the inner or outer shell.

**69. Phenolic group can be identified by a positive:** [4 marks]

(A) Tollen's test

(B) Phthalein dye test

(C) Carbylamine test

(D) Lucas test

**Correct Answer:** (B) Phthalein dye test

**Solution:**

**Concept:**

- Phenols undergo condensation with phthalic anhydride on heating in concentrated sulfuric acid to form a phthalein dye, such as phenolphthalein.
- The dye gives a characteristic colour in alkaline medium. Functional-group tests must be matched with the group they actually detect.

**Step 1:** Identify the phenolic response When phenol is heated with phthalic anhydride in the presence of concentrated $H_2SO_4$, condensation gives phenolphthalein. Addition of alkali converts this indicator into its coloured form, giving the characteristic pink response. Thus the phthalein dye test is a positive identification test for the phenolic group among the choices. The diagnostic point is formation of a coloured phthalein derivative, not reduction of a metal ion or substitution of an alcoholic hydroxyl group.

**Step 2:** Compare the remaining tests Tollens' test is primarily a test for aldehydes and other suitable reducing compounds, which reduce the silver complex to metallic silver; it is not the given phenol test. The carbylamine test uses chloroform and alcoholic alkali to identify primary amines through an isocyanide product, so C tests a nitrogen functional group. Lucas reagent, concentrated hydrochloric acid with anhydrous zinc chloride, distinguishes suitable alcohols through alkyl-chloride formation and turbidity. Phenol does not undergo the corresponding ordinary substitution at an aryl carbon, so D is also unsuitable. These observations exclude A, C and D.

**Step 3:** Conclusion The phenolic group can be identified by a positive phthalein dye test. Hence, the correct option is (B).

**Quick Tip to Solve:** Associate phthalein dye formation with phenols, carbylamine with primary amines, Tollens' reagent with aldehydes, and Lucas reagent with alcohol classification.

**70. Match List - I with List - II.**

| | List - I (Reaction) | | List - II (Reagent(s)) |
|---|---|---|---|
| **(A)** | $C_6H_5OH \longrightarrow$ **ortho-**$HOC_6H_4COOH$ | **(I)** | $Na_2Cr_2O_7, H_2SO_4$ |
| **(B)** | $C_6H_5OH \longrightarrow$ **ortho-**$HOC_6H_4CHO$ | **(II)** | **(i)** $NaOH$, **(ii)** $CH_3Cl$ |
| **(C)** | $C_6H_5OH \longrightarrow$ **para-benzoquinone** | **(III)** | **(i)** $NaOH, CHCl_3$, **(ii)** $NaOH$, **(iii)** $HCl$ |
| **(D)** | $C_6H_5OH \longrightarrow C_6H_5OCH_3$ | **(IV)** | **(i)** $NaOH$, **(ii)** $CO_2$, **(iii)** $HCl$ |

**Choose the correct answer from the options given below:** [4 marks]

(A) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(B) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(C) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(D) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

**Correct Answer:** (C) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

**Solution:**

**Concept:**

- The reaction of sodium phenoxide with carbon dioxide followed by acidification is the Kolbe–Schmitt reaction, producing mainly salicylic acid.
- Reimer–Tiemann formylation, oxidation and Williamson ether synthesis are distinguished by the new functional group formed on phenol.

**Step 1:** Match carboxylation and formylation In reaction A, phenol becomes ortho-hydroxybenzoic acid: the new substituent is $COOH$. First $NaOH$ forms sodium phenoxide, then $CO_2$ introduces the carboxylate group and $HCl$ releases salicylic acid. Hence A matches IV. In reaction B, the new ortho group is $CHO$, identifying Reimer–Tiemann formylation. Chloroform in alkali supplies the formyl carbon; subsequent alkaline treatment and acid work-up give salicylaldehyde. Therefore B matches III.

**Step 2:** Match oxidation and ether formation In reaction C, phenol is oxidised to para-benzoquinone, with carbonyl groups at opposite ring positions. Acidified sodium dichromate, $Na_2Cr_2O_7/H_2SO_4$, is the oxidising reagent, so C matches I. Reaction D converts the hydroxyl group into $OCH_3$, giving anisole. Sodium hydroxide forms phenoxide, which attacks methyl chloride in a Williamson ether synthesis; D therefore matches II. This gives A–IV, B–III, C–I and D–II. Option A assigns the wrong reagent to A and C, B assigns the oxidation reagent to formylation, and D exchanges the carboxylation and methylation reagents.

**Step 3:** Conclusion The matching is A–IV, B–III, C–I, D–II. Hence, the correct option is (C).

**Quick Tip to Solve:** Identify the group introduced before recalling the reaction: $COOH$ means $CO_2$, $CHO$ means chloroform/alkali, and $OCH_3$ means methyl halide plus phenoxide.

**71. Which among the following halide/s will not show $S_N1$ reaction?**

**(A)** $H_2C{=}CH{-}CH_2Cl$

**(B)** $CH_3{-}CH{=}CH{-}Cl$

**(C)** $C_6H_5{-}CH_2{-}Cl$

**(D)** $(CH_3)_2CH{-}Cl$

**Choose the most appropriate answer from the options given below:** [4 marks]

(A) (B) and (C) only

(B) (A), (B) and (D) only

(C) (B) only

(D) (A) and (B) only

**Correct Answer:** (C) (B) only

**Solution:**

**Concept:**

- An $S_N1$ substitution starts with heterolytic cleavage of the carbon–halogen bond. The rate-determining ionisation produces a carbocation, so its stability is decisive.
- Allylic and benzylic carbocations are resonance-stabilised. A secondary alkyl carbocation is also accessible under suitable ionising conditions, whereas a vinylic carbocation is very unstable.

**Step 1:** Identify the carbon carrying chlorine In (A), chlorine is on the saturated carbon adjacent to a double bond: this is allyl chloride, not a vinyl chloride. In (B), chlorine is bonded directly to a double-bond carbon. Compound (C) is benzyl chloride, with chlorine on the carbon next to the aromatic ring. Compound (D) is isopropyl chloride, a secondary alkyl halide. This distinction is essential because the presence of a nearby double bond is not enough to classify a halide as vinylic.

**Step 2:** Compare the cations formed on ionisation For (A), loss of chloride produces an allyl cation with charge delocalised over two carbons:

$$CH_2{=}CH{-}CH_2{}^{+} \longleftrightarrow {}^{+}CH_2{-}CH{=}CH_2.$$

For (C), the benzyl cation is stabilised by delocalisation into the benzene ring. For (D), the secondary carbocation is stabilised by the electron-releasing alkyl groups and hyperconjugation. These three can undergo the unimolecular substitution pathway under suitable conditions. In (B), loss of chloride would give a highly unstable vinyl cation; the vinylic carbon–chlorine bond also resists ionisation. Thus (B) does not show the ordinary $S_N1$ reaction.

**Step 3:** Match the requested combination The question asks which halide will *not* react by this pathway. The required list therefore contains (B) alone. Option (A) incorrectly excludes benzyl chloride; option (B) incorrectly excludes allyl and secondary alkyl chlorides; option (D) incorrectly excludes allyl chloride. Do not confuse the compound labels (A)–(D) with the final answer-option labels.

**Step 4:** Conclusion Only the vinylic halide, $CH_3{-}CH{=}CH{-}Cl$, fails to show the ordinary $S_N1$ reaction. Hence, the correct option is (C).

**Quick Tip to Solve:** Locate the carbon bonded to the halogen first: allylic and benzylic positions stabilise a cation, while a vinylic position does not.

**72. Which of the following statements is not correct about rusting of iron?** [4 marks]

(A) Rusting of iron is envisaged as setting up of electrochemical cell on the surface of iron object.

(B) Dissolved acidic oxides $SO_2, NO_2$ in water act as catalyst in the process of rusting.

(C) Coating of iron surface by tin prevents rusting, even if the tin coating is peeling off.

(D) When $pH$ lies above 9 or 10, rusting of iron does not take place.

**Correct Answer:** (C) Coating of iron surface by tin prevents rusting, even if the tin coating is peeling off.

**Solution:**

**Concept:**

- Rusting is an electrochemical process in which iron is oxidised at anodic sites while dissolved oxygen is reduced at cathodic sites in the presence of moisture.
- A protective metal coating works as a physical barrier while intact. If it is damaged, the relative electrode potentials decide whether exposed iron becomes the anode and corrodes.

**Step 1:** Describe the electrochemical process At an anodic patch, iron dissolves according to

$$\mathrm{Fe} \longrightarrow \mathrm{Fe}^{2+} + 2\,\mathrm{e}^{-}.$$

Electrons move through the metal to cathodic patches, where oxygen is reduced. Subsequent reactions produce hydrated iron(III) oxide, rust. Thus statement (A) correctly describes tiny electrochemical cells on the surface. Acidic oxides such as $SO_2$ and $NO_2$ dissolve in moisture and create acidic conditions that accelerate corrosion, as stated in (B).

**Step 2:** Examine what happens when the tin coating peels Tin is more noble than iron: its standard reduction potential is more positive. Where a scratch or peeling exposes iron to water and oxygen, the exposed iron acts as the anode and tin supplies a relatively large cathodic area. The iron can therefore corrode faster at the damaged place. Tin protects iron only as an intact barrier; it does not provide sacrificial protection after the coating is breached. Consequently, statement (C) is incorrect.

**Step 3:** Check the remaining statement Under strongly alkaline conditions, around pH above 9 or 10 in the context of the question, a protective oxide or hydroxide film suppresses the ordinary rusting process. Statement (D) is therefore the accepted description here. Statements (A), (B) and (D) do not supply the requested incorrect claim. A zinc coating behaves differently from tin because zinc is more readily oxidised than iron and can protect a damaged surface sacrificially.

**Step 4:** Conclusion The claim that tin prevents rusting even when its coating is peeling off is incorrect. Hence, the correct option is (C).

**Quick Tip to Solve:** For a damaged metal coating, compare the metals: zinc sacrifices itself, whereas tin makes exposed iron the anode.

**73. Which of the following cannot function as an oxidising agent?** [4 marks]

(A) $SO_4^{2-}$

(B) $MnO_4^-$

(C) $N^{3-}$

(D) $BrO_3^-$

**Correct Answer:** (C) $N^{3-}$

**Solution:**

**Concept:**

- An oxidising agent accepts electrons and is itself reduced. Its relevant atom must be capable of decreasing its oxidation number.
- The oxidation numbers of oxygen and the total charge of an ion allow the oxidation state of its central atom to be calculated. Nitrogen has its lowest ordinary oxidation state, $-3$, in nitride.

**Step 1:** Calculate the oxidation states Let the central oxidation states be $s$, $m$ and $b$. For sulfate, permanganate and bromate respectively,

$$\begin{aligned} s + 4(-2) &= -2 \quad \Rightarrow s = +6, \\ m + 4(-2) &= -1 \quad \Rightarrow m = +7, \\ b + 3(-2) &= -1 \quad \Rightarrow b = +5. \end{aligned}$$

In the monatomic nitride ion $N^{3-}$, nitrogen has oxidation state $-3$ directly. These values describe electron bookkeeping, not the ionic charge on the central atom in an oxoanion.

**Step 2:** Identify the species that cannot be reduced further Nitrogen in $N^{3-}$ has already reached its lowest ordinary oxidation state. It cannot act as an oxidising agent through a further decrease in nitrogen oxidation state. It can instead lose electron density and be oxidised, so its possible redox role is reducing rather than oxidising. This eliminates nitride as an electron-accepting oxidant in the usual chemical sense.

**Step 3:** Rule out the other options Manganese in permanganate is in the high $+7$ state and readily accepts electrons, forming lower states such as $+2$ or $+4$. Bromine in bromate at $+5$ can similarly be reduced, for example to bromide at $-1$. Sulfur in sulfate is at $+6$, and sulfate can act as an oxidising species under appropriate conditions, as illustrated by reduction of sulfur(VI) in hot concentrated sulfuric acid. The question asks whether oxidation is possible, not which species is a strong oxidant in dilute water; these three therefore do not meet the negative condition.

**Step 4:** Conclusion $N^{3-}$ cannot function as the oxidising agent among the listed species. Hence, the correct option is (C).

**Quick Tip to Solve:** An oxidising agent must be reduced. A central atom already at its lowest common oxidation state is the first species to reject.

**74. Given below are two statements :**

**Statement (I) : In the Lanthanoids, the formation $Ce^{+4}$ is favoured by its noble gas configuration.**

**Statement (II) : $Ce^{+4}$ is a strong oxidant reverting to the common +3 state.**

**In the light of the above statements, choose the most appropriate answer from the options given below :** [4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are true

(D) Statement I is false but Statement II is true

**Correct Answer:** (C) Both Statement I and Statement II are true

**Solution:**

**Concept:**

- Lanthanoids most commonly form the $+3$ oxidation state. Additional stability of empty, half-filled or fully filled subshells can also permit other oxidation states.
- An oxidising agent accepts an electron. Formation of a particularly simple electronic configuration does not by itself mean that the higher oxidation state cannot be reduced in solution.

**Step 1:** Check the electronic configuration in Statement I Cerium has atomic number 58. The configurations relevant to its ions are

$$Ce^{3+} : [Xe]\,4f^1, \qquad Ce^{4+} : [Xe].$$

Removal of the remaining $4f$ electron from $Ce^{3+}$ yields the xenon noble-gas configuration. This special configuration favours the existence of the $+4$ state compared with the usual behaviour of many other lanthanoids. Statement I is therefore true.

**Step 2:** Check the oxidising behaviour in Statement II The tetravalent ion accepts one electron to return to the common trivalent state:

$$Ce^{4+} + e^- \longrightarrow Ce^{3+}.$$

This reduction is favourable enough that cerium(IV) is used as a strong oxidant. Statement II therefore correctly identifies both its oxidising character and the oxidation state to which it returns. Noble-gas configuration is one contribution to stability; solution behaviour also involves ionisation and solvation energies.

**Step 3:** Compare the truth values with the choices Both statements are true. There is no contradiction between favourable formation of a xenon configuration and the ability of cerium(IV) to oxidise another substance in a suitable medium. Option (A) wrongly rejects both statements, option (B) wrongly rejects the observed oxidising behaviour, and option (D) wrongly rejects the electronic-configuration argument. The wording asks only for their truth values, rather than an assertion–reason explanation relation.

**Step 4:** Conclusion Both Statement I and Statement II are true. Hence, the correct option is (C).

**Quick Tip to Solve:** Write $Ce^{3+} = [Xe]4f^1$ and $Ce^{4+} = [Xe]$; then check redox behaviour separately from configuration.

**75. Choose the correct option having all the elements with $d^{10}$ electronic configuration from the following :** [4 marks]

(A) $^{46}Pd, ^{28}Ni, ^{26}Fe, ^{24}Cr$

(B) $^{29}Cu, ^{30}Zn, ^{48}Cd, ^{47}Ag$

(C) $^{27}Co, ^{28}Ni, ^{26}Fe, ^{24}Cr$

(D) $^{28}Ni, ^{24}Cr, ^{26}Fe, ^{29}Cu$

**Correct Answer:** (B) $^{29}Cu, ^{30}Zn, ^{48}Cd, ^{47}Ag$

**Solution:**

**Concept:**

- A $d^{10}$ configuration means that all five orbitals of the relevant d subshell are doubly occupied, containing ten electrons in total.
- Ground-state configurations must be used rather than a blindly applied filling sequence. Copper and silver have filled d subshells with one electron in the outer s subshell.

**Step 1:** Test every element in the candidate set For the elements in option (B), the ground-state configurations are

$$\begin{aligned}
\text{Cu } (Z = 29) &: [\text{Ar}]\, 3d^{10}4s^{1}, \\
\text{Zn } (Z = 30) &: [\text{Ar}]\, 3d^{10}4s^{2}, \\
\text{Cd } (Z = 48) &: [\text{Kr}]\, 4d^{10}5s^{2}, \\
\text{Ag } (Z = 47) &: [\text{Kr}]\, 4d^{10}5s^{1}.
\end{aligned}$$

Each atom has ten d electrons. The numbers written beside the symbols in the question identify their atomic numbers; they do not ask for ionic configurations or for mass-number calculations.

**Step 2:** Eliminate the groups containing incompletely filled d subshells The other groups contain elements whose neutral atoms do not have a filled d subshell:

$$\begin{aligned}
\text{Cr} &: [\text{Ar}]\, 3d^{5}4s^{1}, \\
\text{Fe} &: [\text{Ar}]\, 3d^{6}4s^{2}, \\
\text{Co} &: [\text{Ar}]\, 3d^{7}4s^{2}, \\
\text{Ni} &: [\text{Ar}]\, 3d^{8}4s^{2}.
\end{aligned}$$

Thus options (A), (C) and (D) each contain several counterexamples. Palladium in (A) does have $[\text{Kr}]4d^{10}$, but one qualifying element cannot make the entire group correct.

**Step 3:** Apply the word all The requirement is that *every* listed element have the $d^{10}$ configuration. Copper alone in (D), or palladium alone in (A), does not satisfy that condition for the other members. Only the copper, zinc, cadmium and silver set survives the full check. This also illustrates why filled d subshells occur with different numbers of outer s electrons.

**Step 4:** Conclusion The complete qualifying set is Cu, Zn, Cd and Ag. Hence, the correct option is (B).

**Quick Tip to Solve:** Check each element in the group; remember the filled-d exceptions Cu, Ag and Pd rather than relying only on the usual filling order.

**76. The incorrect statement regarding conformations of ethane is :** [4 marks]

(A) The dihedral angle in staggered conformation is $60°$.

(B) Ethane has infinite number of conformations.

(C) Eclipsed conformation is the most stable conformation.

(D) The conformations of ethane are inter-convertible to one-another.

**Correct Answer:** (C) Eclipsed conformation is the most stable conformation.

**Solution:**

**Concept:**

- Ethane conformations arise by rotation around its carbon–carbon sigma bond. The dihedral angle measures the relative rotation of a front carbon–hydrogen bond and a rear one in a Newman view.
- Eclipsed ethane has maximum torsional strain, whereas staggered ethane has minimum torsional strain and the lower potential energy.

**Step 1:** Compare eclipsed and staggered geometries In an eclipsed conformation, the front and rear carbon–hydrogen bonds align in projection, giving a dihedral angle of $0°$. In a staggered conformation, the rear bonds lie midway between the front bonds and the dihedral angle is $60°$. The eclipsed arrangement has greater torsional strain because of the unfavourable interactions between aligned bonding electron pairs. The staggered arrangement reduces these interactions and is the more stable form. Hence statement (A) is correct.

**Step 2:** Check the number and interconversion of conformations Rotation about a single bond is continuous, so there are infinitely many intermediate conformations between the special eclipsed and staggered positions. This makes statement (B) correct; the two named extreme forms are not the only possible orientations. Rotation carries one conformation into another without breaking any bond, so statement (D) is also correct. Conformational interconversion must not be confused with changing the connectivity of the molecule.

**Step 3:** Select the incorrect stability claim Statement (C) says that the eclipsed conformation is the most stable. It reverses the energy order. The eclipsed conformation is a maximum on the rotational energy profile, while staggered conformations are minima. A finite torsional barrier separates them, but rotation at ordinary temperatures is rapid. The incorrect statement is therefore (C); none of the geometric, multiplicity or interconversion statements supplies the requested error.

**Step 4:** Conclusion The eclipsed conformation is less stable than the staggered conformation, so statement (C) is incorrect. Hence, the correct option is (C).

**Quick Tip to Solve:** For ethane, associate staggered with $60°$ and an energy minimum; associate eclipsed with $0°$ and an energy maximum.

**77. The final product A, formed in the following reaction sequence is:**

$C_6H_5-CH=CH_2 \xrightarrow{\text{(i) } BH_3} \xrightarrow{\text{(ii) } H_2O_2, OH^-} \xrightarrow{\text{(iii) HBr}} \xrightarrow{\text{(iv) Mg, ether, then } HCHO/H_3O^+} \textbf{A.}$ [4 marks]

(A) $C_6H_5-CH(CH_3)-CH_2OH$

(B) $C_6H_5-CH(CH_3)-CH_3$

(C) $C_6H_5-CH_2-CH_2-CH_2-OH$

(D) $C_6H_5-CH_2-CH_2-CH_3$

**Correct Answer:** (C) $C_6H_5-CH_2-CH_2-CH_2-OH$

**Solution:**

**Concept:**

- Hydroboration–oxidation adds water across an alkene in the anti-Markovnikov orientation: the hydroxyl group goes to the less substituted alkene carbon.
- A primary alcohol can be converted to a bromoalkane by HBr. Magnesium in dry ether converts the bromoalkane to a Grignard reagent, whose reaction with formaldehyde adds one carbon and yields a primary alcohol after hydrolysis.

**Step 1:** Apply hydroboration–oxidation to styrene The starting material is $C_6H_5-CH=CH_2$. With $BH_3$, followed by alkaline hydrogen peroxide, the hydroxyl group is introduced at the terminal carbon:

$$C_6H_5-CH=CH_2 \xrightarrow[2.\ H_2O_2/OH^-]{1.\ BH_3} C_6H_5-CH_2-CH_2OH.$$

The product is 2-phenylethanol. A Markovnikov hydration would instead place the hydroxyl group on the carbon next to the phenyl group, but that is not the reaction specified here.

**Step 2:** Form the organomagnesium reagent Hydrogen bromide replaces the primary alcohol group with bromine. Magnesium then inserts into the carbon–bromine bond in dry ether:

$$C_6H_5-CH_2-CH_2OH \xrightarrow{HBr} C_6H_5-CH_2-CH_2Br,$$
$$C_6H_5-CH_2-CH_2Br \xrightarrow[\text{dry ether}]{Mg} C_6H_5-CH_2-CH_2MgBr.$$

The carbon framework remains unbranched. Dry conditions are required before the final hydrolysis because a Grignard reagent is destroyed by proton donors.

**Step 3:** Add formaldehyde and hydrolyse The nucleophilic carbon bonded to magnesium attacks formaldehyde. Its carbonyl carbon becomes a new terminal $CH_2OH$ carbon after acidic work-up:

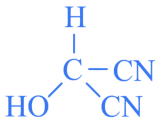
$$C_6H_5-CH_2-CH_2MgBr \xrightarrow[H_3O^+]{HCHO} C_6H_5-CH_2-CH_2-CH_2OH.$$

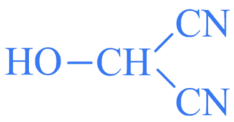
Option (A) has a branched skeleton and is not obtained from this anti-Markovnikov sequence. Options (B) and (D) are hydrocarbons, whereas addition to formaldehyde followed by hydrolysis yields an alcohol. Option (C) alone has the correct unbranched skeleton, oxygen group and one-carbon extension.

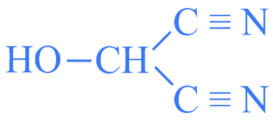
**Step 4:** Conclusion The final product is 3-phenylpropan-1-ol, $C_6H_5-CH_2-CH_2-CH_2OH$. Hence, the correct option is (C).

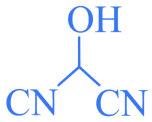
**Quick Tip to Solve:** After hydroboration, track the terminal carbon carefully: a Grignard reagent plus formaldehyde lengthens the chain by one carbon and gives a primary alcohol.

**78. Bond line formula of $HOCH(CN)_2$ is :** [4 marks]

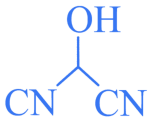
(A) 

(B) 

(C) 

(D) 

**Correct Answer:** (D)



**Solution:**

**Concept:**

- In a bond-line formula, an unlabelled line end or junction represents a carbon atom. Hydrogens attached to such carbon atoms are supplied implicitly to satisfy carbon valency four.
- Heteroatoms and functional-group labels remain explicit. In $HOCH(CN)_2$, the central carbon bears one hydroxyl group, two cyano groups and one hydrogen.

**Step 1:** Identify the central carbon and its substituents The condensed formula is $HO{-}CH(CN)_2$. The central carbon is tetrahedral: it has a single bond to oxygen and single bonds to the carbon atoms of two cyano groups, $-C{\equiv}N$. Its fourth bond is to hydrogen. Thus a skeletal drawing needs a central junction with three visible substituent bonds, while its carbon and attached hydrogen may both be omitted.

**Step 2:** Apply the conventions to the four representations In (D), the unlabelled junction carries OH and two CN labels. The three drawn single bonds leave one implicit hydrogen on that junction, so its connectivity is exactly the required formula. Option (A) explicitly writes the central C and H; it is a displayed formula. Option (B) explicitly writes the central CH and is a condensed structural representation. Option (C) likewise writes CH and expands the cyano triple bonds. These drawings can convey the same connectivity, but they do not omit the central carbon and its attached hydrogen in the bond-line convention asked for here.

**Step 3:** Conclusion The bond-line formula is the drawing with an unlabelled central carbon junction bonded to OH and two CN groups, with one implicit H. Hence, the correct option is (D).

**Quick Tip to Solve:** When a question asks for bond-line notation, check that carbon junctions and hydrogens bonded to carbon are implicit; functional-group labels such as OH and CN stay visible.

**79. The quantity which changes with temperature is :** [4 marks]

(A) Molality

(B) Molarity

(C) Mole fraction

(D) Mass percentage

**Correct Answer:** (B) Molarity

**Solution:**

**Concept:**

- Molarity is the amount of solute per litre of solution: $M = n_{\text{solute}}/V_{\text{solution}}$. A change in solution volume therefore changes molarity.
- For a fixed nonreacting sample with no evaporation or loss of material, temperature changes its volume but not the masses or numbers of moles of its components.

**Step 1:** Identify the concentration with a volume denominator On heating an ordinary solution, thermal expansion generally increases its volume. Since the solute amount remains constant,

$$M(T) = \frac{n_{\text{solute}}}{V(T)}.$$

Thus molarity changes when temperature changes. It normally decreases as the solution expands. The essential distinction is the presence of solution volume in its definition, rather than any change in the solute itself.

**Step 2:** Check each of the mass-based or mole-based alternatives Molality is $m = n_{\text{solute}}/m_{\text{solvent}}$, with solvent mass in kilograms, so it does not depend on thermal expansion. Mole fraction is $x_i = n_i/\sum_j n_j$; neither numerator nor denominator changes for the fixed composition. Mass percentage is $100 m_{\text{solute}}/m_{\text{solution}}$, so it too remains unchanged. Therefore (A), (C) and (D) lack the volume dependence that makes (B) temperature-sensitive. These conclusions refer to concentration definitions for a sample whose chemical composition is unchanged.

**Step 3:** Conclusion Molarity changes with temperature because the volume of the solution changes. Hence, the correct option is (B).

**Quick Tip to Solve:** Look at the denominator of the concentration expression: a litre of solution can expand with temperature, while a kilogram of solvent and a mole ratio remain fixed.

**80. The molecular formula of second homologue in the homologous series of mono carboxylic acids is** [4 marks]

(A) $C_2H_4O_2$

(B) $C_2H_2O_2$

(C) $CH_2O$

(D) $C_3H_6O_2$

**Correct Answer:** (A) $C_2H_4O_2$

**Solution:**

**Concept:**

- Members of a homologous series have the same functional group, with successive members differing by one $CH_2$ unit.
- The saturated acyclic monocarboxylic acid series starts with methanoic acid, $HCOOH$. Its members have molecular formula $C_nH_{2n}O_2$, where $n \geq 1$.

**Step 1:** Start with the smallest acid in the series A monocarboxylic acid contains one $-COOH$ group. The smallest member is $HCOOH$, which has one carbon, two hydrogen atoms and two oxygen atoms. Its molecular formula is therefore $CH_2O_2$. The carbon atom in the carboxyl group is part of the total carbon count; it must not be omitted when numbering the members of this homologous series.

**Step 2:** Add one homologous increment and compare the options Adding one $CH_2$ unit gives ethanoic acid:

$$HCOOH \xrightarrow{+CH_2} CH_3COOH.$$

Counting its atoms gives

$$CH_3COOH = C_2H_4O_2.$$

This agrees with (A). Formula $C_2H_2O_2$ in (B) has too few hydrogens for the second saturated member; $CH_2O$ in (C) has only one oxygen and is not a carboxylic acid formula. Formula $C_3H_6O_2$ in (D) is propanoic acid, the third member. Thus adding one carbon beyond the first member selects ethanoic acid, not propanoic acid.

**Step 3:** Conclusion The second homologue is ethanoic acid, $CH_3COOH$, with molecular formula $C_2H_4O_2$. Hence, the correct option is (A).

**Quick Tip to Solve:** Begin the monocarboxylic acid series at HCOOH, then add CH2 once: the carbon of the COOH group already counts as the first carbon.

## Chemistry – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**81. 1 mole of $PbS$ is oxidised by "X" moles of $O_3$ to get "Y" moles of $O_2$. $X + Y =$ _____.** [4 marks]

**Solution:**

**Concept:**

- Ozone is an oxidising agent and oxidises black lead sulphide, $PbS$, to lead sulphate, $PbSO_4$. Ozone itself gives molecular oxygen.
- In this oxidation, each $O_3$ supplies one oxygen atom to the substrate and leaves one $O_2$ molecule. Stoichiometric coefficients determine the required mole ratios.

**Step 1:** Identify the oxidation product and oxygen requirement The sulphur in $PbS$ has oxidation state $-2$, whereas sulphur in sulphate has oxidation state $+6$. Lead remains in the $+2$ state. Oxidation therefore converts the sulphide unit into a sulphate unit, introducing four oxygen atoms per formula unit of $PbS$. The familiar ozone test is the change from black lead sulphide to white lead sulphate; the product here is not lead oxide plus a separate sulphur compound.

**Step 2:** Balance the reaction and read both mole numbers The four required oxygen atoms are furnished by four ozone molecules, each leaving a molecule of oxygen:

$$PbS + 4\,O_3 \longrightarrow PbSO_4 + 4\,O_2.$$

Check oxygen: the left side has $4 \times 3 = 12$ oxygen atoms and the right side has $4 + 4 \times 2 = 12$. Pb and S are each one on either side. Hence for one mole of $PbS$, ozone consumed is $X = 4$ moles and oxygen produced is $Y = 4$ moles. These are coefficients for the same balanced equation, so they can be added directly:

$$X + Y = 4 + 4 = 8.$$

**Step 3:** Conclusion $X = 4$ and $Y = 4$; therefore the required numerical value is $\boxed{8}$ (dimensionless sum of the mole-number coefficients).

**Quick Tip to Solve:** For ozone oxidation, count one transferred oxygen atom per O3 and one O2 left behind; forming sulphate from sulphide requires four such transfers.

**82. The Spin only magnetic moment value of square planar complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is _____ B.M. (Nearest integer)**

**(Given atomic number for $Pt = 78$)** [4 marks]

**Solution:**

**Concept:**

- The spin-only magnetic moment is $\mu_{so} = \sqrt{n(n+2)}$ B.M., where $n$ is the number of unpaired electrons.
- Neutral ammine and methylamine ligands contribute no formal charge, while each chloride ligand

has charge $-1$. A square-planar $d^8$ platinum(II) complex has all electrons paired.

**Step 1:** Find the oxidation state and d-electron count The chloride outside the coordination brackets is $Cl^-$, so the complex ion must have charge $+1$. Inside the brackets, two $NH_3$ ligands and one $NH_2CH_3$ ligand are neutral; only the coordinated chloride has charge $-1$. If the platinum oxidation state is $x$, then

$$x - 1 = +1 \quad \Rightarrow \quad x = +2.$$

Platinum is a group 10 element. Removing the two valence electrons to form $Pt^{2+}$ gives the $5d^8$ configuration relevant to the coordination complex.

**Step 2:** Pair the square-planar electrons and evaluate the moment In the stated square-planar geometry, the $d_{x^2-y^2}$ orbital is highest in energy. The eight d electrons fill the other four d orbitals as pairs, leaving that highest orbital empty. Platinum(II), a 5d ion, thus has no unpaired electrons in this square-planar complex. It is diamagnetic. Substituting $n = 0$ into the requested spin-only expression gives

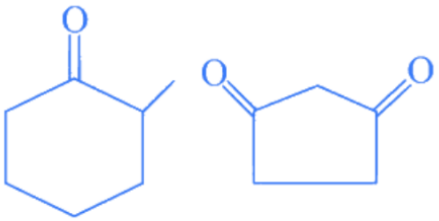
$$\mu_{so} = \sqrt{0(0+2)} = 0\,\text{B.M.}$$

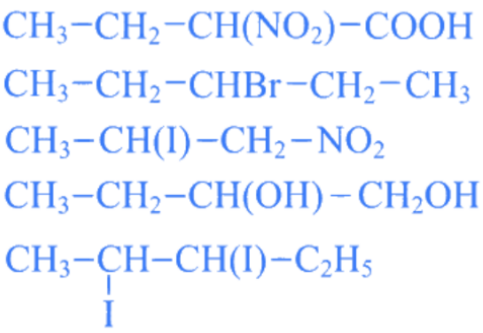
There is consequently no rounding ambiguity when the answer is expressed to the nearest integer.

**Step 3:** Conclusion The complex has zero unpaired electrons and a spin-only magnetic moment of $\boxed{0\,\text{B.M.}}$. The nearest-integer numerical entry is $0$.

**Quick Tip to Solve:** First use the counter-ion to find Pt(II); then recognise that square-planar d8 platinum has four paired d orbitals and an empty highest orbital.

**83. Total number of compounds with Chiral carbon atoms from following is _____.** [4 marks]





**Solution:**

**Concept:**

- A chiral carbon atom is tetrahedral and bonded to four different substituents. Carbonyl carbons are trigonal planar and cannot be chiral carbon centres.
- In a ring, the two paths leaving a candidate carbon must be compared atom by atom. Two ring bonds need not be equivalent: a nearby carbonyl group can distinguish the paths.

**Step 1:** Check the two cyclic structures and the first two chains The first drawing is 2-methylcyclohexanone. Its methyl-bearing ring carbon is attached to H, methyl, a ring path immediately reaching the carbonyl carbon, and a different ring path first reaching a methylene carbon. Its four groups differ, so this compound has a chiral carbon. The second drawing, cyclopentane-1,3-dione, has only carbonyl and methylene carbons and has no chiral carbon. In $CH_3CH_2CH(NO_2)COOH$, the methine carbon is attached to H, nitro, ethyl and carboxyl groups, so it is chiral. By contrast, $CH_3CH_2CHBrCH_2CH_3$ has two identical ethyl groups at the bromine-bearing carbon and is not chiral there.

**Step 2:** Check the remaining chains and count compounds In $CH_3CH(I)CH_2NO_2$, the iodine-bearing carbon has H, I, methyl and nitromethyl groups, giving one chiral carbon. In $CH_3CH_2CH(OH)CH_2OH$, the hydroxyl-bearing methine has H, OH, ethyl and hydroxymethyl groups, also giving a chiral carbon. Finally, $CH_3CH(I)CH(I)C_2H_5$ contains chiral methine carbons: at each such carbon the H, I and two carbon-chain substituents differ. It contributes one compound to the count, even though it has more than one chiral centre. The qualifying compounds are therefore the first cyclic structure and the first, third, fourth and fifth displayed open-chain structures:

$$N = 1 + 1 + 1 + 1 + 1 = 5.$$

**Step 3:** Conclusion Five of the seven displayed compounds contain at least one chiral carbon atom. The required dimensionless count is $\boxed{5}$.

**Quick Tip to Solve:** Count compounds with a qualifying centre, not the total number of centres; compare both ring paths and watch for two identical ethyl groups.

**84. Time required for completion of $99.9\%$ of a First order reaction is _____ times of half life $(t_{1/2})$ of the reaction.** [4 marks]

**Solution:**

**Concept:**

- For a first-order reaction, $kt = \ln([A]_0/[A]_t)$, where $k$ is the rate constant and $[A]_t$ is the concentration remaining at time $t$.
- The half-life is $t_{1/2} = \ln 2/k$. Taking a ratio cancels the rate constant, so the required multiple depends only on the fraction remaining.

**Step 1:** Convert percentage completion into the remaining fraction Completion of $99.9\%$ means that

$100 - 99.9 = 0.1\%$ of the reactant remains. Relative to the initial amount, the residual fraction is

$$\frac{[A]_t}{[A]_0} = \frac{0.1}{100} = 0.001 = 10^{-3}.$$

It is the fraction remaining, rather than the fraction reacted, that enters the integrated first-order law. Substitution gives

$$kt = \ln\left(\frac{1}{0.001}\right) = \ln 1000 = 3\ln 10.$$

**Step 2:** Divide by the half-life and evaluate the multiple Using the half-life expression,

$$\frac{t}{t_{1/2}} = \frac{(\ln 1000)/k}{(\ln 2)/k} = \frac{3\ln 10}{\ln 2}.$$

With $\ln 10 \simeq 2.302585$ and $\ln 2 \simeq 0.693147$,

$$\frac{t}{t_{1/2}} \simeq 9.9658 \simeq 10.$$

As a check, after ten half-lives the fraction remaining is $2^{-10} = 1/1024 \simeq 0.0009766$, corresponding to about $99.902\%$ completion, essentially the specified $99.9\%$. The ratio has no unit because both numerator and denominator are times.

**Step 3:** Conclusion The time is approximately $9.966\,t_{1/2}$, or $\boxed{10}$ times the half-life to the nearest integer. The requested multiplier is dimensionless.

**Quick Tip to Solve:** For percentage completion, first subtract from 100 to obtain percentage remaining; then use log(initial/remaining) divided by log 2 to count half-lives.

**85. The hydrogen electrode is dipped in a solution of $\mathrm{pH} = 3$ at $25^\circ\mathrm{C}$. The potential of the electrode will be _____ $\times 10^{-2}$ V.**

$$\left(\frac{2.303\mathrm{RT}}{\mathrm{F}} = 0.059\ \mathrm{V}\right)$$

[4 marks]

**Solution:**

**Concept:**

- The conventional hydrogen electrode potential is the reduction potential for $2\,\mathrm{H}^+ + 2\,\mathrm{e}^- \longrightarrow \mathrm{H}_2$. Its standard reduction potential is zero relative to the standard hydrogen electrode.
- For unit hydrogen pressure, the Nernst equation at $25^\circ\mathrm{C}$ gives $E = -0.059\,\mathrm{pH}$ in volts. The electron number is two, but the squared hydrogen-ion activity cancels the factor of two.

**Step 1:** Write the reaction quotient with its sign For the reduction reaction, the quotient is $Q = p_{\mathrm{H_2}}/a^2_{\mathrm{H^+}}$. Under the usual hydrogen-electrode convention $p_{\mathrm{H_2}} = 1$, and taking the stated pH as the

hydrogen-ion activity measure gives $\log a_{H^+} = -3$. Thus the reduction potential is

$$E = 0 - \frac{0.059}{2} \log\left(\frac{1}{a_{H^+}^2}\right) = 0.059 \log a_{H^+}.$$

The minus sign before the reaction-quotient term must be retained; acidic solution does not automatically make this potential positive.

**Step 2:** Substitute the pH and express the requested coefficient Substituting $\mathrm{pH} = 3$,

$$E = -0.059(3) = -0.177\,\mathrm{V} = -17.7 \times 10^{-2}\,\mathrm{V}.$$

On rounding the numerical coefficient to the nearest integer, it is $-18$. Reversing the electrode reaction would instead give an oxidation potential of $+0.177\,\mathrm{V}$, and the magnitude also rounds to 18. Those are distinct conventions and should not be confused with the conventional reduction potential. The pH is dimensionless; volts belong to the electrode potential, not to the coefficient in the answer blank.

**Step 3:** Conclusion The hydrogen reduction potential is $\boxed{-0.177\,\mathrm{V}}$, so the rounded coefficient is $\boxed{-18}$. The listed key value 18 corresponds to its magnitude (or to the oxidation potential); the printed question does not specify either, and the conventional reduction-potential sign gives -18.

**Quick Tip to Solve:** Write the half-reaction first: reduction uses $E = -0.059\,\mathrm{pH}$ at unit hydrogen pressure; the oxidation potential has the opposite sign.

**86. The number of non-polar molecules from the following is _____.**

$HF$, $H_2O$, $SO_2$, $H_2$, $CO_2$, $CH_4$, $NH_3$, $HCl$, $CHCl_3$, $BF_3$. [4 marks]

**Solution:**

**Concept:**

- Molecular polarity depends on the vector sum of all bond dipoles, not merely on whether an individual bond is polar. Equal dipoles cancel in a sufficiently symmetric geometry.
- A homonuclear diatomic molecule is non-polar because its identical atoms have equal electronegativity. Lone-pair-induced bent or pyramidal geometries commonly prevent bond-dipole cancellation.

**Step 1:** Identify the molecules with zero resultant dipole $H_2$ is a homonuclear diatomic molecule, so its bond has no permanent dipole. $CO_2$ is linear, and its two equal carbon–oxygen bond dipoles point in opposite directions. $CH_4$ is a regular tetrahedron with four equivalent bonds, whose vector sum is zero. $BF_3$ is trigonal planar with three equal boron–fluorine dipoles separated by $120^\circ$; their vector sum also vanishes. These four molecules are therefore non-polar even though the bonds in the last three can be polar.

**Step 2:** Exclude every remaining molecule and count $HF$ and $HCl$ are heteronuclear diatomic molecules, each with one nonzero bond dipole that cannot cancel. $H_2O$ and $SO_2$ are bent, so their bond dipoles do not point exactly opposite one another. $NH_3$ is trigonal pyramidal, with a nonzero resultant along its symmetry axis. In tetrahedral $CHCl_3$, three chlorine atoms and one hydrogen are not four identical substituents; their bond dipoles do not cancel. Hence these six are polar. Counting

only the zero-resultant cases gives

$$N = 1 + 1 + 1 + 1 = 4.$$

**Step 3:** Conclusion The non-polar molecules are $H_2$, $CO_2$, $CH_4$ and $BF_3$. Their total number is $\boxed{4}$, a dimensionless count.

**Quick Tip to Solve:** Check molecular geometry after bond polarity: linear $CO_2$, regular tetrahedral $CH_4$, and trigonal planar $BF_3$ cancel their equal dipoles.

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**87.** $9.3$ g **of aniline is subjected to reaction with excess of acetic anhydride to prepare acetanilide. The mass of acetanilide produced if the reaction is** $100\%$ **completed is** _____ $\times 10^{-1}$ g**.**
**(Given molar mass in** g mol$^{-1}$

$$\mathrm{N}: 14, \mathrm{O}: 16,$$
$$\mathrm{C}: 12, \mathrm{H}: 1 \textbf{ )}$$

[4 marks]

**Solution:**

**Concept:**

- Aniline undergoes acetylation with acetic anhydride to form acetanilide. One mole of aniline forms one mole of acetanilide when acetylation is complete.
- The limiting-reagent calculation uses $n = m/M$, where $n$ is amount in moles, $m$ is mass in grams and $M$ is molar mass. Excess acetic anhydride leaves aniline as the limiting reactant.

**Step 1:** Write the reaction and calculate the molar masses The amino group of aniline is acetylated without loss of the benzene ring:

$$\mathrm{C_6H_5NH_2} + (\mathrm{CH_3CO})_2\mathrm{O} \longrightarrow \mathrm{C_6H_5NHCOCH_3} + \mathrm{CH_3COOH}.$$

The stoichiometric coefficients of aniline and acetanilide are both one. With the supplied atomic masses,

$$\begin{aligned} M(\mathrm{C_6H_7N}) &= 6(12) + 7(1) + 14 \\ &= 93\,\mathrm{g\,mol^{-1}}, \\ M(\mathrm{C_8H_9NO}) &= 8(12) + 9(1) + 14 + 16 \\ &= 135\,\mathrm{g\,mol^{-1}}. \end{aligned}$$

The product has gained an acetyl group at nitrogen; its mass is therefore larger than the original aniline mass.

**Step 2:** Use the limiting reactant and convert the answer scale The amount of aniline initially taken is

$$n(\text{aniline}) = \frac{9.3\,\mathrm{g}}{93\,\mathrm{g\,mol^{-1}}} = 0.100\,\mathrm{mol}.$$

Because acetic anhydride is in excess and conversion is complete, $n(\text{acetanilide}) = 0.100\,\text{mol}$. Therefore,

$$m(\text{acetanilide}) = 0.100(135) = 13.5\,\text{g}.$$

The blank multiplies $10^{-1}\,\text{g}$, so write $13.5\,\text{g} = 135 \times 10^{-1}\,\text{g}$. The coefficient in the blank is 135 rather than 13.5.

**Step 3:** Conclusion The mass of acetanilide is $\boxed{13.5\,\text{g} = 135 \times 10^{-1}\,\text{g}}$. The required numerical entry is $\boxed{135}$.

**Quick Tip to Solve:** Use the one-to-one mole ratio first, then multiply by the product molar mass; only at the end convert grams to the power of ten printed beside the blank.

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**88. Volume of** $3\text{M}$ $\text{NaOH}$ **(formula weight** $40\ \text{g mol}^{-1}$ **) which can be prepared from** $84\ \text{g}$ **of** $\text{NaOH}$ **is** _____ $\times 10^{-1}\text{dm}^3$**.** [4 marks]

**Solution:**

**Concept:**

- Molarity is the amount of solute divided by the final solution volume in litres: $c = n/V$. Thus $V = n/c$, with $n$ in moles and $c$ in $\text{mol L}^{-1}$.
- The amount of solute is obtained from its mass and formula weight, $n = m/M$. One litre is exactly one cubic decimetre, so $1\,\text{L} = 1\,\text{dm}^3$.

**Step 1:** Convert the given sodium hydroxide mass to moles The sample contains $84\,\text{g}$ of $\text{NaOH}$, and its given formula weight is $40\,\text{g mol}^{-1}$. The amount available is therefore

$$n(\text{NaOH}) = \frac{84\,\text{g}}{40\,\text{g mol}^{-1}} = 2.10\,\text{mol}.$$

The formula weight must divide the mass; multiplying them would not give moles. The concentration refers to dissolved formula units of $\text{NaOH}$, so its dissociation into sodium and hydroxide ions does not introduce an extra factor of two into the molarity calculation.

**Step 2:** Find the final solution volume and the coefficient The required concentration is $3\,\text{M} = 3\,\text{mol L}^{-1}$. From the definition of molarity,

$$V = \frac{n}{c} = \frac{2.10\,\text{mol}}{3\,\text{mol L}^{-1}} = 0.700\,\text{L}.$$

Converting to cubic decimetres and the scale in the blank,

$$V = 0.700\,\text{dm}^3 = 7 \times 10^{-1}\,\text{dm}^3.$$

This is the final volume of the prepared solution, not a separately measured volume of water to add. As a check, $3\,\text{mol L}^{-1} \times 0.700\,\text{L} = 2.10\,\text{mol}$, which corresponds to the original 84 g.

**Step 3:** Conclusion The solution volume is $\boxed{0.700\,\text{dm}^3 = 7 \times 10^{-1}\,\text{dm}^3}$, so the required numerical entry is $\boxed{7}$.

**Quick Tip to Solve:** Combine mass and molarity in $V = m/(Mc)$, keeping volume in litres; then use $1\,\text{L} = 1\,\text{dm}^3$ before filling the scaled blank.

**89. Total number of ions from the following with noble gas configuration is _____.**

$\mathrm{Sr}^{2+}(z=38), \mathrm{Cs}^{+}(z=55), \mathrm{La}^{2+}(z=57), \mathrm{Pb}^{2+}(z=82), \mathrm{Yb}^{2+}(z=70)$ **and** $\mathrm{Fe}^{2+}(z=26)$ [4 marks]

**Solution:**

**Concept:**

- A positive ion has fewer electrons than its atomic number: an ion with charge $+r$ has $Z-r$ electrons. A noble-gas configuration must match a complete noble-gas electron configuration.
- A filled $d$ or $f$ subshell alone is not a noble-gas configuration. For transition-metal cations, the outer $ns$ electrons are removed before electrons from the $(n-1)d$ subshell.

**Step 1:** Find the ions that exactly match noble gases For strontium and caesium, the electron counts are

$$N_e(\mathrm{Sr}^{2+}) = 38-2 = 36, \qquad N_e(\mathrm{Cs}^{+}) = 55-1 = 54.$$

These are exactly the electron counts of krypton and xenon, respectively. Their configurations are $\mathrm{Sr}^{2+} : [\mathrm{Kr}]$ and $\mathrm{Cs}^{+} : [\mathrm{Xe}]$. Both therefore satisfy the condition. It is the configuration of each ion that matters, not whether the neutral atom itself is a noble gas.

**Step 2:** Check the other four ions explicitly Lanthanum has configuration $[\mathrm{Xe}]5d^1 6s^2$; removing its two $6s$ electrons gives $\mathrm{La}^{2+} : [\mathrm{Xe}]5d^1$, not xenon. Lead loses its two $6p$ electrons to give $\mathrm{Pb}^{2+} : [\mathrm{Xe}]4f^{14}5d^{10}6s^2$, with 80 electrons. Ytterbium loses its two $6s$ electrons to give $\mathrm{Yb}^{2+} : [\mathrm{Xe}]4f^{14}$, with 68 electrons. Although its $4f$ subshell is filled, it is not a noble gas. Finally $\mathrm{Fe}^{2+} : [\mathrm{Ar}]3d^6$, with 24 electrons, also differs from a noble gas. Thus only the first two ions qualify; the other four are excluded by their remaining electrons outside the noble-gas core.

**Step 3:** Conclusion Only $\mathrm{Sr}^{2+}$ and $\mathrm{Cs}^{+}$ have noble-gas configurations. The required total is $\boxed{2}$, a dimensionless count.

**Quick Tip to Solve:** First subtract the ionic charge from $Z$; then check whether the configuration is exactly a noble gas, rather than merely containing a filled subshell.

**90. For a certain thermochemical reaction** $\mathrm{M} \rightarrow \mathrm{N}$ **at** $\mathrm{T} = 400\ \mathrm{K}, \Delta\mathrm{H}^{\ominus} = 77.2\ \mathrm{kJ\ mol^{-1}}, \Delta\mathrm{S} = 122\ \mathrm{JK^{-1}}, \log$ **equilibrium constant** $(\log K)$ **is** _____ $\times 10^{-1}$**.** [4 marks]

**Solution:**

**Concept:**

- The standard reaction Gibbs energy is $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$, with the enthalpy and entropy terms expressed in compatible units.
- The equilibrium constant satisfies $\Delta G^\circ = -RT\ln K = -2.303RT\log_{10} K$. A positive standard Gibbs energy therefore requires $K<1$, so its logarithm is negative.

**Step 1:** Calculate the standard Gibbs energy at 400 K Take the supplied entropy change in its molar thermochemical sense, $\Delta S^\circ = 122\ \mathrm{J\,mol^{-1}\,K^{-1}}$. Convert the enthalpy before subtraction:

$$\Delta H^\circ = 77.2 \times 10^3 = 77200\ \mathrm{J\,mol^{-1}}.$$

The entropy contribution at the specified temperature is

$$T\Delta S^\circ = 400(122) = 48800\,\mathrm{J\,mol^{-1}}.$$

Hence

$$\Delta G^\circ = 77200 - 48800 = 28400\,\mathrm{J\,mol^{-1}}.$$

It is positive: the favorable positive entropy term is too small at 400 K to offset the endothermic enthalpy change.

**Step 2:** Calculate the logarithm and retain the sign Using $R = 8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$,

$$\log_{10} K = -\frac{28400}{2.303(8.314)(400)} \simeq -3.708.$$

The sign is also checked without arithmetic: positive $\Delta G^\circ$ implies $\ln K < 0$. Thus

$$\log_{10} K \simeq -37.08 \times 10^{-1}.$$

Rounding the coefficient to the nearest integer gives $-37$. A positive coefficient would imply $K > 1$ and a negative standard Gibbs energy, contradicting the values printed in the question. The equilibrium constant and its logarithm are dimensionless.

**Step 3:** Conclusion For the printed positive enthalpy and entropy changes, $\boxed{\log_{10} K \simeq -3.71}$, so the rounded coefficient is $\boxed{-37}$. The listed key gives 37, but its positive sign contradicts $\Delta G^\circ = +28.4\,\mathrm{kJ\,mol^{-1}}$ calculated from the printed data.

**Quick Tip to Solve:** Before calculating $\log K$, inspect the sign of $\Delta H^\circ - T\Delta S^\circ$; a positive value forces $K < 1$ and a negative logarithm.