JEE Main 2024 Paper 1 (B.E./B.Tech) Jan 29 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 1 (Jan-Feb), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 29 January 2024, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to [4 marks]

(A) 18

(B) 16

(C) 12

(D) 15

Correct Answer: (D) 15

Solution:

Concept:

- Identical books are distinguished only by the number placed on a shelf. Identical shelves make permutations of these occupancies the same arrangement.
- The required count is therefore the number of partitions of 8 into at most four positive parts; unused shelves contribute zeros, not new choices.

Step 1: Count arrangements using one or two occupied shelves Write the positive occupancies in non-increasing order so that every physical distribution has exactly one representation. With one occupied shelf there is only (8). With two occupied shelves the possibilities are

$$(7,1),\ (6,2),\ (5,3),\ (4,4).$$

These contribute $1+4=5$ arrangements. No extra factor is attached for selecting shelves because all four shelves are identical.

Step 2: Count arrangements using three occupied shelves The largest part and the remaining sum give the exhaustive list

$$(6,1,1),\ (5,2,1),\ (4,3,1),\ (4,2,2),$$
$$(3,3,2).$$

There are five such partitions. Reordering a listed triple, for example $(1,6,1)$, does not give another arrangement.

Step 3: Count arrangements using all four shelves Every shelf must now contain at least one book. The complete list is

$$(5,1,1,1),\ (4,2,1,1),\ (3,3,1,1),$$
$$(3,2,2,1),\ (2,2,2,2).$$

This contributes another five. The counts for different numbers of occupied shelves are disjoint, so the total is $1+4+5+5=15$. The values 18, 16 and 12 in the other options disagree with this exhaustive enumeration.

Step 4: Conclusion There are 15 arrangements. Hence, the correct option is (D).

Quick Tip to Solve: For identical shelves, sort the occupancies first; counting ordered distributions would count the same arrangement repeatedly.

2. Let A be the point of intersection of the lines $3x + 2y = 14, 5x - y = 6$ and B be the point of intersection of the lines $4x + 3y = 8, 6x + y = 5$. The distance of the point $P(5, -2)$ from the line AB is [4 marks]

(A) $\frac{13}{2}$

(B) 8

(C) $\frac{5}{2}$

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- The intersection coordinates are obtained by solving each pair of linear equations simultaneously.
- The perpendicular distance of (x_0, y_0) from $ax + by + c = 0$ is $|ax_0 + by_0 + c|/\sqrt{a^2 + b^2}$.

Step 1: Find the first intersection From $5x - y = 6$, write $y = 5x - 6$. Substitution in the other equation gives

$$3x + 2(5x - 6) = 14 \quad \Rightarrow \quad 13x = 26.$$

Thus $x = 2$, $y = 4$, and $A = (2, 4)$. Both original equations check: $3(2) + 2(4) = 14$ and $5(2) - 4 = 6$.

Step 2: Find the second intersection and the joining line The second pair gives $y = 5 - 6x$, and hence

$$4x + 3(5 - 6x) = 8 \quad \Rightarrow \quad -14x = -7.$$

Therefore $B = (1/2, 2)$. The slope of the joining line is

$$m = \frac{4 - 2}{2 - 1/2} = \frac{4}{3}.$$

Using point A, its equation is $3(y - 4) = 4(x - 2)$, or

$$4x - 3y + 4 = 0.$$

As a check, the point B gives $2 - 6 + 4 = 0$.

Step 3: Calculate the required distance Insert $P = (5, -2)$ into the distance formula, preserving the negative sign of its ordinate:

$$d = \frac{|4(5) - 3(-2) + 4|}{\sqrt{4^2 + (-3)^2}} = \frac{30}{5} = 6.$$

The denominator normalizes the line coefficients; the absolute value ensures a non-negative distance. Options $13/2$, 8 and $5/2$ do not equal the calculated perpendicular distance, so they are excluded.

Step 4: Conclusion The distance is 6 units. Hence, the correct option is (D).

Quick Tip to Solve: Check both computed intersections in the joining line before applying the point-to-line distance formula.

3. If $\int \frac{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x \sin(x-\theta)}}dx = A\sqrt{\cos\theta\tan x-\sin\theta}+B\sqrt{\cos\theta-\sin\theta\cot x}+C$, where C is the integration constant, then AB is equal to [4 marks]

(A) $2\sec\theta$

(B) $8\,\text{cosec}(2\theta)$

(C) $4\,\text{cosec}(2\theta)$

(D) $4\sec\theta$

Correct Answer: (B) $8\,\text{cosec}(2\theta)$

Solution:

Concept:

- Expand $\sin(x-\theta)=\sin x\cos\theta-\cos x\sin\theta$, then separate the numerator into two terms.
- The substitutions $t=\tan x$ and $u=\cot x$ match the two square roots on the right. Work on an interval where the displayed radicals are real and denominators are nonzero.

Step 1: Separate the integrand On the real domain of the displayed fractional powers, take $\sin x>0$, $\cos x>0$, and $\sin(x-\theta)>0$. The two numerator terms then simplify without changing the square-root signs:

$$\frac{\sin^{3/2}x}{\sqrt{\sin^3 x\cos^3 x\sin(x-\theta)}}=\frac{\sec^2 x}{\sqrt{\cos\theta\tan x-\sin\theta}},$$
$$\frac{\cos^{3/2}x}{\sqrt{\sin^3 x\cos^3 x\sin(x-\theta)}}=\frac{\csc^2 x}{\sqrt{\cos\theta-\sin\theta\cot x}}.$$

The factors inside the radicals follow by dividing $\sin(x-\theta)$ by $\cos x$ and $\sin x$, respectively.

Step 2: Integrate the two terms For the first term set $v=\cos\theta\tan x-\sin\theta$. Then $dv=\cos\theta\sec^2 x\,dx$. For the second set $z=\cos\theta-\sin\theta\cot x$, giving $dz=\sin\theta\csc^2 x\,dx$. Thus, for nonzero $\cos\theta$ and $\sin\theta$,

$$I=\frac{2}{\cos\theta}\sqrt{\cos\theta\tan x-\sin\theta}+\frac{2}{\sin\theta}\sqrt{\cos\theta-\sin\theta\cot x}+C.$$

Differentiating each radical reproduces its corresponding separated term, including the positive sign in the cotangent term.

Step 3: Compare coefficients and multiply Comparison with the given form yields $A=2\sec\theta$ and $B=2\csc\theta$. Therefore

$$AB=\frac{4}{\sin\theta\cos\theta}=8\csc(2\theta).$$

The options involving only $\sec\theta$ omit the sine factor, while $4\csc(2\theta)$ is too small by a factor of two.

Step 4: Conclusion $AB=8\,\text{cosec}(2\theta)$. Hence, the correct option is (B).

Quick Tip to Solve: Split a sum over a radical before substituting; here one term becomes a tangent derivative and the other a cotangent derivative.

4. The distance of the point $(2, 3)$ from the line $2x - 3y + 28 = 0$, measured parallel to the line $\sqrt{3}x - y + 1 = 0$, is equal to [4 marks]

(A) $3 + 4\sqrt{2}$

(B) $6\sqrt{3}$

(C) $4 + 6\sqrt{3}$

(D) $4\sqrt{2}$

Correct Answer: (C) $4 + 6\sqrt{3}$

Solution:

Concept:

- A distance measured parallel to a specified line follows that line's direction, rather than the normal to the target line.
- The line $\sqrt{3}x - y + 1 = 0$ has slope $\sqrt{3}$, so a direction vector is $(1, \sqrt{3})$, whose length is 2.

Step 1: Parametrize the required direction Let $P = (2, 3)$, and let Q be the point where a line through P, parallel to the specified line, meets $2x - 3y + 28 = 0$. Every point on this parallel line has the form

$$Q = (2 + t,\ 3 + \sqrt{3}t), \qquad t \in \mathbb{R}.$$

This parametrization uses increments in the ratio $1 : \sqrt{3}$. The constant 1 in the given parallel line fixes its position but has no effect on its direction.

Step 2: Locate the intersection Substitute the parametrized coordinates into the target line:

$$2(2 + t) - 3(3 + \sqrt{3}t) + 28 = 0.$$

Collecting the constant and parameter terms gives

$$23 + (2 - 3\sqrt{3})t = 0, \qquad t = \frac{23}{3\sqrt{3} - 2}.$$

Since $3\sqrt{3} > 2$, this parameter is positive. The intersection is unique because the prescribed direction is not parallel to the target line.

Step 3: Find and simplify the segment length The displacement vector is $t(1, \sqrt{3})$, so its length is

$$\begin{aligned} PQ = |t|\sqrt{1 + 3} &= \frac{46}{3\sqrt{3} - 2} \\ &= \frac{46(3\sqrt{3} + 2)}{27 - 4} = 2(3\sqrt{3} + 2) \\ &= 4 + 6\sqrt{3}. \end{aligned}$$

The denominator in the rationalization is 23, canceling the numerator factor. Options $3 + 4\sqrt{2}$, $6\sqrt{3}$ and $4\sqrt{2}$ do not give this segment length; in particular, the middle one omits the positive constant 4.

Step 4: Conclusion The required distance is $4 + 6\sqrt{3}$ units. Hence, the correct option is (C).

Quick Tip to Solve: For a distance along a prescribed direction, parametrize a line through the point and multiply the parameter by the direction vector's length.

5. Let a unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$ make angles $\frac{\pi}{2}, \frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}, \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ respectively. If $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ then $|\hat{u} - \vec{v}|^2$ is equal to [4 marks]

(A) $\frac{11}{2}$

(B) $\frac{5}{2}$

(C) 7

(D) 9

Correct Answer: (B) $\frac{5}{2}$

Solution:

Concept:

- For unit vectors, the scalar product equals the cosine of their included angle. Each of the three given comparison vectors has magnitude one.
- The identity $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$ avoids expanding three separate coordinate squares.

Step 1: Convert the angles into equations The three scalar products give

$$\frac{x+z}{\sqrt{2}} = \cos\frac{\pi}{2} = 0, \qquad \frac{y+z}{\sqrt{2}} = \cos\frac{\pi}{3} = \frac{1}{2},$$

$$\frac{x+y}{\sqrt{2}} = \cos\frac{2\pi}{3} = -\frac{1}{2}.$$

Consequently $x + z = 0$, $y + z = 1/\sqrt{2}$, and $x + y = -1/\sqrt{2}$. The negative sign of the third cosine is essential because the corresponding angle is obtuse.

Step 2: Determine the vector and the needed product From $z = -x$, the second equation becomes $y - x = 1/\sqrt{2}$. Adding this to $x + y = -1/\sqrt{2}$ yields $2y = 0$. Therefore

$$x = -\frac{1}{\sqrt{2}}, \qquad y = 0, \qquad z = \frac{1}{\sqrt{2}}.$$

The squared norm is $1/2 + 0 + 1/2 = 1$, as required. For the specified vector $\vec{v}$,

$$|\vec{v}|^2 = \frac{3}{2}, \qquad \hat{u} \cdot \vec{v} = \frac{x+y+z}{\sqrt{2}} = 0.$$

Although $\hat{u}$ is a unit vector, $\vec{v}$ is not: its three components each have square $1/2$.

Step 3: Evaluate the squared difference Apply the norm identity:

$$|\hat{u} - \vec{v}|^2 = 1 + \frac{3}{2} - 2(0) = \frac{5}{2}.$$

A coordinate check gives $\hat{u} - \vec{v} = (-\sqrt{2}, -1/\sqrt{2}, 0)$, whose squared length is $2 + 1/2$. Thus $11/2$, 7 and 9 are all excluded; none agrees with either independent evaluation.

Step 4: Conclusion $|\hat{u} - \vec{v}|^2 = 5/2$. Hence, the correct option is (B).

Quick Tip to Solve: Use the angle data as linear dot-product equations, and check the actual norm of every vector before using a unit-vector identity.

6. The function $f(x) = \frac{x}{x^2-6x-16}, x \in \mathbb{R} - \{-2, 8\}$ [4 marks]

(A) decreases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

(B) increases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

(C) decreases in $(-2, 8)$ and increases in $(-\infty, -2) \cup (8, \infty)$

(D) decreases in $(-\infty, -2)$ and increases in $(8, \infty)$

Correct Answer: (A) decreases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Solution:

Concept:

- The derivative sign determines monotonicity on each interval on which the function is continuous and differentiable.
- The denominator factors as $(x-8)(x+2)$, so the domain has three connected intervals separated by two vertical asymptotes.

Step 1: Differentiate using the quotient rule Put $D = x^2 - 6x - 16$. For every allowed value of x, the quotient rule gives

$$\begin{aligned} f'(x) &= \frac{D - x(2x-6)}{D^2} \\ &= \frac{x^2 - 6x - 16 - 2x^2 + 6x}{(x^2-6x-16)^2} \\ &= -\frac{x^2+16}{(x^2-6x-16)^2}. \end{aligned}$$

The linear terms cancel. In particular, the numerator is not $16 - x^2$; retaining the minus sign before the entire derivative product is necessary.

Step 2: Determine the derivative sign on the domain For every real x, $x^2 + 16 > 0$. At every point of the domain the squared denominator is strictly positive. It follows that

$$f'(x) < 0 \qquad (x \neq -2, 8).$$

There are no stationary points because the numerator never vanishes. The two excluded numbers are poles, not critical points at which the derivative changes from negative to positive.

Step 3: Interpret the listed intervals The function is strictly decreasing separately on each of

$$(-\infty, -2), \qquad (-2, 8), \qquad (8, \infty).$$

This is the interval-by-interval meaning of the union notation in option (A). Option (B) incorrectly says increasing on all three components. Options (C) and (D) also claim increase on an exterior interval, contradicting the strictly negative derivative there. The derivative test must not be applied across a pole: no claim of global decrease across the disconnected domain is required.

Step 4: Conclusion The function decreases on each of the three stated domain intervals. Hence, the correct option is (A).

Quick Tip to Solve: For a rational derivative, a squared denominator is positive away from poles; inspect the numerator and split the domain at every pole.

7. If R is the smallest equivalence relation on the set $\{1,2,3,4\}$ such that $\{(1,2),(1,3)\} \subset \mathrm{R}$, then the number of elements in R is _____. [4 marks]

(A) 15

(B) 10

(C) 12

(D) 8

Correct Answer: (B) 10

Solution:

Concept:

- An equivalence relation is reflexive, symmetric and transitive. Its equivalence classes partition the underlying set.
- Each class of size k contributes every ordered pair of its members, hence exactly k^2 elements to the relation.

Step 1: Determine the forced equivalence class Since $(1,2) \in R$, symmetry forces $(2,1) \in R$. Combining $(2,1)$ with $(1,3)$ by transitivity forces $(2,3) \in R$; symmetry also forces $(3,2)$ and $(3,1)$. Reflexivity supplies $(1,1)$, $(2,2)$ and $(3,3)$. Thus 1, 2 and 3 must all belong to one class, and every pair within $\{1,2,3\}$ is unavoidable.

Step 2: Keep the remaining element separate Reflexivity also requires $(4,4)$. Nothing in the given pairs connects 4 with any of 1, 2 or 3. To obtain the smallest equivalence relation we leave 4 in its own class. The partition is therefore

$$\{1,2,3\}, \quad \{4\}.$$

The relation defined by these two classes is indeed reflexive, symmetric and transitive, so no additional pair is needed. Connecting 4 to any other element would merge both classes and produce all 16 ordered pairs, making the relation larger.

Step 3: Count and check the ordered pairs The three-element class contributes $3^2 = 9$ pairs, and the singleton contributes $1^2 = 1$. Explicitly,

$$R = \{(1,1),(1,2),(1,3),(2,1),(2,2),\\ (2,3),(3,1),(3,2),(3,3),(4,4)\}.$$

Hence $|R| = 9 + 1 = 10$. The alternatives 15, 12 and 8 do not equal this forced count; 8 would even omit some pairs required within the three-element class or its singleton complement.

Step 4: Conclusion The number of elements in R is 10. Hence, the correct option is (B).

Quick Tip to Solve: Build equivalence classes first, then add the squares of their sizes; count ordered pairs rather than unordered connections.

8. If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of the first four observations is $\frac{7}{2}$, then the variance of the first four observations in equal to [4 marks]

(A) $\frac{5}{4}$
(B) $\frac{4}{5}$
(C) $\frac{105}{4}$
(D) $\frac{77}{12}$

Correct Answer: (A) $\frac{5}{4}$

Solution:

Concept:

- For observations $x_1, \ldots, x_n$, the population variance is $\sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$, where $\bar{x}$ is the mean.
- A mean determines the sum of the observations. Subtracting the contribution of the fifth observation gives the sums needed for the first four.

Step 1: Recover the fifth observation The sum of all five observations is $5(24/5) = 24$, while the sum of the first four is $4(7/2) = 14$. Hence the last observation is

$$x_5 = 24 - 14 = 10.$$

This subtraction uses sums, rather than subtracting the two means, because the groups have different sizes.

Step 2: Recover the sum of squares Rearrange the variance formula for the five observations:

$$\frac{1}{5}\sum_{i=1}^{5} x_i^2 = \frac{194}{25} + \left(\frac{24}{5}\right)^2 = \frac{770}{25}.$$

Therefore $\sum_{i=1}^{5} x_i^2 = 154$. Removing the square of the last observation gives $\sum_{i=1}^{4} x_i^2 = 154 - 100 = 54$.

Step 3: Calculate the required variance For the first four observations, divide the remaining sum of squares by four and subtract the square of their own mean:

$$\sigma_4^2 = \frac{54}{4} - \left(\frac{7}{2}\right)^2 = \frac{54 - 49}{4} = \frac{5}{4}.$$

The divisor is four, since this is the variance of the four observations, not an unbiased sample-variance estimate. Options (B), (C) and (D) do not equal this value; (B) is its reciprocal.

Step 4: Conclusion The variance of the first four observations is $\boxed{5/4}$. Hence, the correct option is (A).

Quick Tip to Solve: First recover the missing observation from the sums; then remove its square from the total sum of squares.

9. Let $\mathrm{P}(3,2,3), \mathrm{Q}(4,6,2)$ and $\mathrm{R}(7,3,2)$ be the vertices of $\triangle \mathrm{PQR}$. Then, the angle $\angle \mathrm{QPR}$ is [4 marks]

(A) $\cos^{-1}\left(\frac{7}{18}\right)$

(B) $\frac{\pi}{6}$

(C) $\cos^{-1}\left(\frac{1}{18}\right)$

(D) $\frac{\pi}{3}$

Correct Answer: (D) $\frac{\pi}{3}$

Solution:

Concept:

- The angle $\angle QPR$ is the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$, both directed away from the vertex P.
- For nonzero vectors $\mathbf{u}, \mathbf{v}$, $\cos\theta = (\mathbf{u}\cdot\mathbf{v})/(|\mathbf{u}||\mathbf{v}|)$, with $0 \le \theta \le \pi$.

Step 1: Form the two side vectors Subtract the coordinates of the common vertex $P(3,2,3)$ from the other endpoints:

$$\overrightarrow{PQ} = (4-3, 6-2, 2-3) = (1,4,-1),$$

$$\overrightarrow{PR} = (7-3, 3-2, 2-3) = (4,1,-1).$$

Using the same starting point is essential: reversing just one vector would give the supplementary angle instead of the triangle angle.

Step 2: Calculate the dot product and lengths The dot product is

$$\overrightarrow{PQ}\cdot\overrightarrow{PR} = 1(4) + 4(1) + (-1)(-1) = 9.$$

Also,

$$|\overrightarrow{PQ}| = \sqrt{1+16+1} = \sqrt{18}, \qquad |\overrightarrow{PR}| = \sqrt{16+1+1} = \sqrt{18}.$$

The common lengths are consistent with the symmetry of the coordinate differences.

Step 3: Identify the angle Substitution gives

$$\cos\angle QPR = \frac{9}{\sqrt{18}\sqrt{18}} = \frac{1}{2}.$$

The interior angle in the principal range is therefore $\pi/3$. Option (B) has cosine $\sqrt{3}/2$; options (A) and (C) have cosines $7/18$ and $1/18$, respectively. None equals the required $1/2$.

Step 4: Conclusion $\boxed{\angle QPR = \pi/3}$ radians. Hence, the correct option is (D).

Quick Tip to Solve: For an angle at P, form both vectors by subtracting P; mixing directions changes the sign of the cosine.

10.

Let $y = \log_e\left(\frac{1-x^2}{1+x^2}\right), -1 < x < 1$**. Then at** $x = \frac{1}{2}$**, the value of** $225\left(y' - y''\right)$ **is equal to**

[4 marks]

(A) 732

(B) 736

(C) 742

(D) 746

Correct Answer: (B) 736

Solution:

Concept:

- The logarithm of a positive quotient is the difference of the logarithms: here $y = \log(1-x^2) - \log(1+x^2)$. The interval $-1 < x < 1$ makes both arguments positive.
- Apply the chain rule for the first derivative and the quotient rule for the second derivative, and substitute the evaluation point only after differentiating.

Step 1: Differentiate the logarithms Keeping both chain-rule factors and signs gives

$$y' = \frac{-2x}{1-x^2} - \frac{2x}{1+x^2} = \frac{-4x}{1-x^4}.$$

The first denominator comes from $1-x^2$; combining the two fractions produces $(1-x^2)(1+x^2) = 1-x^4$.

Step 2: Differentiate again Using the quotient rule with numerator $-4x$ and denominator $1-x^4$,

$$y'' = \frac{-4(1-x^4) - (-4x)(-4x^3)}{(1-x^4)^2} = \frac{-4(1+3x^4)}{(1-x^4)^2}.$$

In particular the second numerator contribution is negative; overlooking that sign changes the result.

Step 3: Evaluate the requested expression At $x = 1/2$, $1 - x^4 = 15/16$. Thus

$$y' = -\frac{32}{15}, \qquad y'' = -\frac{1216}{225}.$$

It follows that

$$225(y' - y'') = 225\left(-\frac{480}{225} + \frac{1216}{225}\right) = 736.$$

The other listed numbers 732, 742 and 746 fail this exact fraction calculation; no rounding is needed.

Step 4: Conclusion The requested value is $\boxed{736}$. Hence, the correct option is (B).

Quick Tip to Solve: Combine the first derivative to one fraction before differentiating again, and keep the fractions exact until the final subtraction.

11. If $\log_e a, \log_e b, \log_e c$ are in an A.P. and $\log_e a - \log_e 2b, \log_e 2b - \log_e 3c, \log_e 3c - \log_e a$ are also in an A.P, then $a : b : c$ is equal to [4 marks]

(A) $6 : 3 : 2$

(B) $9 : 6 : 4$

(C) $25 : 10 : 4$

(D) $16 : 4 : 1$

Correct Answer: (B) $9 : 6 : 4$

Solution:

Concept:

- Three quantities u, v, w form an arithmetic progression exactly when $2v = u + w$.
- For positive numbers, logarithms turn products and quotients into sums and differences. Thus $2\log b = \log a + \log c$ is equivalent to $b^2 = ac$.

Step 1: Use the first progression Because the logarithms exist, a, b, c are positive. The first A.P. condition gives

$$2\log_e b = \log_e a + \log_e c \quad \Longrightarrow \quad b^2 = ac.$$

Consequently the successive ratios a/b and b/c must be equal. This already rules out option (A), whose two ratios are 2 and $3/2$.

Step 2: Use the second progression Write its three terms as

$$u = \log_e a - \log_e(2b), \quad v = \log_e(2b) - \log_e(3c), \quad w = \log_e(3c) - \log_e a.$$

They telescope to $u + v + w = 0$. An A.P. also has $u + w = 2v$, so $3v = 0$ and $v = 0$. Therefore $\log_e(2b) = \log_e(3c)$, which implies $2b = 3c$.

Step 3: Combine the two relations We have $b/c = 3/2$, and the first relation yields $a/b = b/c = 3/2$. Taking $c = 4k$, with $k > 0$, gives $b = 6k$ and $a = 9k$. Thus the ratio is $9 : 6 : 4$. Options (C) and (D) do have equal successive ratios, but those are $5/2$ and 4; each violates $2b = 3c$.

Step 4: Conclusion $\boxed{a : b : c = 9 : 6 : 4}$. Hence, the correct option is (B).

Quick Tip to Solve: When three differences telescope to zero and are also in A.P., their middle term must be zero.

12. If $\sin\left(\frac{y}{x}\right) = \log_e|x| + \frac{\alpha}{2}$ is the solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ and $y(1) = \frac{\pi}{3}$, then α^2 is equal to [4 marks]

(A) 12

(B) 9

(C) 4

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- A given family of solutions contains an integration constant. Substituting the initial condition determines that constant directly.
- For $v = y/x$, the derivative is $dv/dx = (x\,dy/dx - y)/x^2$; hence the printed differential equation integrates to the given sine relation for $x \neq 0$.

Step 1: Confirm the form of the solution Rearrange the differential equation as

$$\cos(y/x)\left(x\frac{dy}{dx} - y\right) = x.$$

After dividing by x^2, the left side is the derivative of $\sin(y/x)$, while the right side is $1/x$. Thus

$$\frac{d}{dx}\sin(y/x) = \frac{1}{x} \quad \Longrightarrow \quad \sin(y/x) = \log_e|x| + C.$$

This agrees with the stated form with $C = \alpha/2$.

Step 2: Apply the initial condition The initial condition gives $x = 1$ and $y = \pi/3$, so $y/x = \pi/3$. Substitution in the provided relation gives

$$\sin(\pi/3) = \log_e 1 + \frac{\alpha}{2}.$$

Since $\log_e 1 = 0$ and $\sin(\pi/3) = \sqrt{3}/2$, it follows that $\alpha = \sqrt{3}$. There is no extra sign choice, because the initial condition fixes the actual sine value.

Step 3: Square and compare the options The quantity requested is the square of the parameter, not the parameter itself:

$$\alpha^2 = (\sqrt{3})^2 = 3.$$

Values 12, 9 and 4 would correspond to different parameter magnitudes and do not satisfy the initial equation $\alpha/2 = \sqrt{3}/2$.

Step 4: Conclusion $\boxed{\alpha^2 = 3}$. Hence, the correct option is (D).

Quick Tip to Solve: When a solution family is supplied, apply the initial condition to that family first; keep track of any factor multiplying its constant.

13. An integer is chosen at random from the integers $1, 2, 3, \ldots, 50$. The probability that the chosen integer is a multiple of atleast one of 4, 6 and 7 is [4 marks]

(A) $\frac{8}{25}$
(B) $\frac{9}{50}$
(C) $\frac{14}{25}$
(D) $\frac{21}{50}$

Correct Answer: (D) $\frac{21}{50}$

Solution:

Concept:

- Choosing uniformly from 1 to 50 gives 50 equally likely outcomes. The required probability is the number of favourable integers divided by 50.
- For three sets, inclusion-exclusion adds their sizes, subtracts pairwise overlaps, and adds the triple overlap. Common multiples are counted using the least common multiple.

Step 1: Count each set of multiples Let A, B, C be the sets of multiples of 4, 6 and 7, respectively, in the given range. Their sizes are

$$|A| = \lfloor 50/4 \rfloor = 12, \quad |B| = \lfloor 50/6 \rfloor = 8, \quad |C| = \lfloor 50/7 \rfloor = 7.$$

Adding these directly gives 27, but integers divisible by two of the divisors would then be counted twice.

Step 2: Count the overlaps The relevant least common multiples are 12, 28 and 42. Hence

$$|A \cap B| = 4, \qquad |A \cap C| = 1, \qquad |B \cap C| = 1.$$

The common multiple of all three divisors must be a multiple of 84, and no positive multiple of 84 lies at most 50. Therefore $|A \cap B \cap C| = 0$.

Step 3: Apply inclusion-exclusion The favourable count is

$$|A \cup B \cup C| = 12 + 8 + 7 - 4 - 1 - 1 + 0 = 21.$$

Therefore the probability is $21/50$. Option (C), $14/25 = 28/50$, even exceeds the uncorrected count 27/50. Options (A) and (B) correspond to only 16 and 9 favourable outcomes, which omit valid multiples. The union is required because the wording says at least one.

Step 4: Conclusion The probability is $\boxed{21/50}$. Hence, the correct option is (D).

Quick Tip to Solve: For at least one divisibility condition, use inclusion-exclusion and compute each intersection with an LCM.

14. Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = 12\vec{a} + 4\vec{b}$ and $\overrightarrow{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then $\frac{\text{area of the quadrilateral OA BC}}{\text{area of S}}$ is equal to _____. [4 marks]

(A) 7

(B) 6

(C) 8

(D) 10

Correct Answer: (C) 8

Solution:

Concept:

- The area of the parallelogram with adjacent vectors $\vec{a}, \vec{b}$ is $|\vec{a} \times \vec{b}|$. A triangle with those sides has half that area.
- The cross product is bilinear and antisymmetric: $\vec{a} \times \vec{a} = 0$ and $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$. These identities allow area ratios without coordinates or angles.

Step 1: Split the quadrilateral along OB The diagonal OB divides the quadrilateral $OABC$ into triangles OAB and OBC. In the basis $(\vec{a}, \vec{b})$, the vertices are $O = (0, 0), A = (1, 0), B = (12, 4), C = (0, 1)$. The points A and C lie on opposite sides of line OB, so the two triangle interiors are disjoint and their areas add.

Step 2: Calculate the two triangle areas Using $\overrightarrow{OB} = 12\vec{a} + 4\vec{b}$,

$$\text{area}(OAB) = \frac{1}{2}|\vec{a} \times (12\vec{a} + 4\vec{b})| = 2|\vec{a} \times \vec{b}|.$$

Similarly,

$$\text{area}(OBC) = \frac{1}{2}|(12\vec{a} + 4\vec{b}) \times \vec{b}| = 6|\vec{a} \times \vec{b}|.$$

The terms with a vector crossed with itself vanish.

Step 3: Form the ratio The total quadrilateral area is $(2 + 6)|\vec{a} \times \vec{b}| = 8|\vec{a} \times \vec{b}|$. Dividing by the parallelogram area gives 8. The area ratio presumes the nondegenerate parallelogram specified in the question. The numbers 7, 6 and 10 do not equal the sum of the two required triangle ratios; in particular 6 counts only triangle OBC.

Step 4: Conclusion $\boxed{\text{area}(OABC)/\text{area}(S) = 8}$. Hence, the correct option is (C).

Quick Tip to Solve: Split a quadrilateral through the origin into two triangles, then use cross-product coefficients relative to the same parallelogram.

15. The function $f(x) = 2x + 3(x)^{\frac{2}{3}}, x \in \mathbb{R}$, has [4 marks]

(A) exactly one point of local minima and no point of local maxima

(B) exactly one point of local maxima and exactly one point of local minima

(C) exactly two points of local maxima and exactly one point of local minima

(D) exactly one point of local maxima and no point of local minima

Correct Answer: (B) exactly one point of local maxima and exactly one point of local minima

Solution:

Concept:

- For a real cube root, $x^{2/3} = (\sqrt[3]{x})^2$. The function is continuous for every real x, including zero.
- A local maximum occurs when the function changes from increasing to decreasing; a local minimum occurs when it changes from decreasing to increasing. Check nondifferentiable points as well as stationary points.

Step 1: Find all possible turning points For $x \neq 0$, differentiation gives

$$f\prime(x) = 2 + 2x^{-1/3} = 2\left(1 + \frac{1}{\sqrt[3]{x}}\right).$$

Thus $f\prime(x) = 0$ gives $\sqrt[3]{x} = -1$, or $x = -1$. The derivative is undefined at $x = 0$, but the function is defined there, so zero must also be tested. These are the only candidates.

Step 2: Test the derivative signs on each interval When $x < -1$, the cube root is less than -1, hence $f\prime(x) > 0$. For $-1 < x < 0$, its reciprocal is less than -1, hence $f\prime(x) < 0$. For $x > 0$, both terms in the derivative are positive. The function therefore increases, decreases and then increases on these three intervals, respectively. At -1, the sign changes from positive to negative, giving a local maximum $f(-1) = 1$. At zero it changes from negative to positive, giving a local minimum $f(0) = 0$. No other interval contains a stationary or undefined derivative point. Options (A) and (D) omit one of these extrema, and (C) incorrectly adds a second maximum.

Step 3: Conclusion There is exactly one local maximum, at $x = -1$, and exactly one local minimum, at $x = 0$. Hence, the correct option is (B).

Quick Tip to Solve: For fractional powers, list points where the derivative fails to exist: a cusp can be an extremum even though it is not stationary.

16. If each term of a geometric progression $a_1, a_2, a_3, \ldots$ with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \ldots .. + a_n$, then $S_{20} - S_{18}$ is equal to [4 marks]

(A) -2^{15}

(B) 2^{15}

(C) -2^{18}

(D) 2^{18}

Correct Answer: (A) -2^{15}

Solution:

Concept:

- In a geometric progression with first term a and common ratio r, the term $a_k = ar^{k-1}$. Here $a = 1/8$ is nonzero.
- If a number is the arithmetic mean of two others, twice that number equals their sum. Also, subtracting partial sums cancels their common terms.

Step 1: Translate the arithmetic-mean condition Apply the condition to the first three terms:

$$2a_1 = a_2 + a_3.$$

Substituting $a_2 = a_1 r$ and $a_3 = a_1 r^2$, and dividing by the nonzero first term, gives

$$2 = r + r^2, \qquad (r-1)(r+2) = 0.$$

The possible ratios are 1 and -2. The condition $a_2 \neq a_1$ excludes $r = 1$, so the required common ratio is -2. With this ratio, the same relation holds after multiplying by any preceding power of r, so it satisfies the condition throughout the progression.

Step 2: Keep only the two terms left after cancellation The first eighteen terms cancel in the difference of sums, leaving

$$S_{20} - S_{18} = a_{19} + a_{20} = a_1 r^{18}(1 + r).$$

Consequently,

$$S_{20} - S_{18} = \frac{1}{8}(-2)^{18}(1 - 2) = -\frac{2^{18}}{2^3} = -2^{15}.$$

In particular, $a_{19} > 0$ and $a_{20} = -2a_{19}$, so their sum must be negative. This rejects the positive alternatives (B) and (D). Option (C) has the correct sign but misses the factor $1/8$ from the first term.

Step 3: Conclusion $S_{20} - S_{18} = -2^{15}$. Hence, the correct option is (A).

Quick Tip to Solve: For a difference of nearby partial sums, cancel the common terms before using a geometric-series formula; this also makes the sign easy to check.

17. Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, then (r, θ) is equal to [4 marks]

(A) $\left(2\sec\frac{11\pi}{8}, \frac{11\pi}{8}\right)$

(B) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

(C) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$

(D) $\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

Correct Answer: (B) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

Solution:

Concept:

- For $z = x + iy$, its modulus is $r = \sqrt{x^2 + y^2} > 0$, while the signs of x and y determine the quadrant of its argument.
- The identities $\tan(\pi - u) = -\tan u$ and $1 + \tan^2 u = \sec^2 u$ connect the rectangular and polar forms.

Step 1: Determine the quadrant before finding the argument Put $u = 3\pi/8$. Since $5\pi/8 = \pi - u$, we have

$$\tan\frac{5\pi}{8} = -\tan u.$$

Hence the given complex number becomes

$$z = 2 + 2i\tan u.$$

Its real part is positive, and its imaginary part is positive because $0 < u < \pi/2$. It lies in the first quadrant. Its argument must therefore be the first-quadrant angle satisfying $\tan\theta = \tan u$, namely $\theta = 3\pi/8$. Merely reading the angle inside the original tangent would give the wrong quadrant.

Step 2: Calculate the positive modulus and compare options Using the real and imaginary components,

$$r = \sqrt{2^2 + (2\tan u)^2} = 2\sqrt{1 + \tan^2 u} = 2|\sec u|.$$

Here $\cos u > 0$, so $r = 2\sec(3\pi/8)$. Equivalently, factoring this positive quantity gives

$$z = 2\sec u\,(\cos u + i\sin u).$$

Option (A) uses a third-quadrant angle and a negative proposed modulus, because $\cos(11\pi/8) < 0$. Option (C) also gives a negative modulus since $\cos(5\pi/8) < 0$. Option (D) places the argument in the second quadrant despite the positive real part.

Step 3: Conclusion $(r, \theta) = \left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$. Hence, the correct option is (B).

Quick Tip to Solve: Always check that a proposed modulus is positive and that its argument gives the correct signs of both real and imaginary parts.

18. The sum of the solutions $x \in \mathbb{R}$ of the equation $\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x} = x^3 - x^2 + 6$ is [4 marks]

(A) 3

(B) 1

(C) 0

(D) -1

Correct Answer: (D) -1

Solution:

Concept:

- Use $a^3 - b^3 = (a-b)(a^2+ab+b^2)$, together with $\cos^2 x + \sin^2 x = 1$, to simplify the denominator.
- Cancellation is valid only where the original denominator is nonzero. After solving the resulting polynomial, test its real roots in the original domain.

Step 1: Simplify the quotient while retaining its domain Write $c = \cos 2x$. Factoring the denominator gives

$$\cos^6 x - \sin^6 x = (\cos^2 x - \sin^2 x)(1 - \cos^2 x \sin^2 x).$$

Since $\cos^2 x \sin^2 x = (1-c^2)/4$, this becomes

$$\cos^6 x - \sin^6 x = \frac{c(3+c^2)}{4}.$$

The numerator is $3c + c^3 = c(3+c^2)$. As $3 + c^2 > 0$, the original expression is defined exactly when $c \neq 0$, and on this domain its value is 4. The excluded points are $x = \pi/4 + k\pi/2$, with k any integer.

Step 2: Solve the remaining polynomial equation The given equation now requires

$$4 = x^3 - x^2 + 6, \qquad x^3 - x^2 + 2 = 0.$$

Factor the polynomial:

$$x^3 - x^2 + 2 = (x+1)(x^2 - 2x + 2) = (x+1)((x-1)^2 + 1).$$

The second factor is strictly positive for every real x, so the only real root is $x = -1$. This is allowed: $\cos(-2) = \cos 2 \neq 0$, since 2 lies strictly between $\pi/2$ and π. Thus no root has been gained or lost through cancellation. There are no additional real roots to contribute to the requested sum.

Step 3: Conclusion The solution set is $\{-1\}$, so the sum is -1, rather than 3, 1 or 0. Hence, the correct option is (D).

Quick Tip to Solve: When numerator and denominator share a trigonometric factor, record its zeros before cancellation and check the final polynomial roots against them.

19. Let $x = \frac{m}{n}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos\left(2\sin^{-1}x\right) = \frac{1}{9}$ and let $\alpha, \beta(\alpha > \beta)$ be the roots of the equation $mx^2 - nx - m + n = 0$. Then the point (α, β) lies on the line [4 marks]

(A) $3x - 2y = -2$

(B) $3x + 2y = 2$

(C) $5x + 8y = 9$

(D) $5x - 8y = -9$

Correct Answer: (C) $5x + 8y = 9$

Solution:

Concept:

- The double-angle identity gives $\cos(2\sin^{-1}x) = 1 - 2x^2$ for $-1 \le x \le 1$. Positive coprime numerator and denominator select the positive rational solution.
- After determining m, n, solve the quadratic and use the specified order $\alpha > \beta$ when substituting the ordered point into a line equation.

Step 1: Determine the reduced fraction The inverse sine is defined for $x \in [-1, 1]$. Applying the cosine double-angle identity gives

$$1 - 2x^2 = \frac{1}{9}, \qquad 2x^2 = \frac{8}{9}, \qquad x^2 = \frac{4}{9}.$$

Thus the real solutions of the trigonometric equation are $x = \pm 2/3$. Because the problem writes $x = m/n$ with both m and n natural numbers, only $x = 2/3$ is eligible. The numerator and denominator are already coprime, so $m = 2$, $n = 3$. We must use this reduced pair rather than an arbitrary multiple of it.

Step 2: Find the ordered roots and test the lines The quadratic becomes

$$2x^2 - 3x - 2 + 3 = 0, \qquad 2x^2 - 3x + 1 = (2x - 1)(x - 1) = 0.$$

Its roots are 1 and $1/2$. As $\alpha > \beta$, the required point is $(1, 1/2)$. For option (C),

$$5\alpha + 8\beta = 5 + 8\left(\frac{1}{2}\right) = 9.$$

Check the other equations at the same ordered point: $3\alpha - 2\beta = 2 \neq -2$, $3\alpha + 2\beta = 4 \neq 2$, and $5\alpha - 8\beta = 1 \neq -9$. These comparisons reject (A), (B) and (D), respectively. Swapping the roots would represent a different point and is not permitted by the stated ordering.

Step 3: Conclusion The point $(\alpha, \beta) = (1, 1/2)$ lies on $5x + 8y = 9$. Hence, the correct option is (C).

Quick Tip to Solve: Use the positivity and coprimality conditions before forming the quadratic, then keep the root order fixed when testing coordinate equations.

20. Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ **and** $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$**. The sum of the prime factors of** $\left|P^{-1}AP - 2I\right|$ **is equal to** [4 marks]

(A) 66

(B) 27

(C) 23

(D) 26

Correct Answer: (D) 26

Solution:

Concept:

- Similar matrices have the same characteristic determinant: $P^{-1}AP - \lambda I = P^{-1}(A - \lambda I)P$ for invertible P.
- The determinant of a product equals the product of determinants, and $\det(P^{-1}) \det P = 1$. Prime factorisation is performed after evaluating the resulting scalar determinant.

Step 1: Remove the change-of-basis matrix First, the displayed matrix P is invertible: expansion along its first row gives

$$\det P = 1(0 - 2) - 2(25 - 14) = -2 - 22 = -24 \neq 0.$$

We may therefore insert $I = P^{-1}IP$ to obtain

$$P^{-1}AP - 2I = P^{-1}(A - 2I)P.$$

Taking determinants cancels the two factors involving P, yielding

$$\left|P^{-1}AP - 2I\right| = |A - 2I|.$$

Thus finding all entries of P^{-1} or multiplying three full matrices is unnecessary. The vertical bars here denote the determinant of the displayed square matrix.

Step 2: Evaluate and factor the determinant Subtracting two from each diagonal entry of A gives

$$A - 2I = \begin{pmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{pmatrix}.$$

Expand along the first row, retaining the alternating cofactor signs:

$$|A - 2I| = -1(6 \cdot 0 - 11 \cdot 3) + 2(6 \cdot 3 - 0 \cdot 3) = 33 + 36 = 69.$$

Its prime factorisation is $69 = 3 \cdot 23$, so the sum of its prime factors is $3 + 23 = 26$. Both factors occur once, so there is no distinction between counting distinct prime divisors and counting them with multiplicity. The other proposed sums, 66, 27 and 23, do not equal this sum.

Step 3: Conclusion The determinant is 69, whose prime factors sum to 26. Hence, the correct option is (D).

Quick Tip to Solve: Before computing an inverse, look for a similarity expression: subtracting a scalar multiple of the identity preserves the determinant reduction.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{OM} \cdot \overrightarrow{ON}$ is equal to _____. [4 marks]

Solution:

Concept:

- Write each symmetric line parametrically using its given point and direction vector. A shortest joining segment between nonparallel lines is perpendicular to both direction vectors.
- For vectors a, b, perpendicularity is equivalent to $a \cdot b = 0$. Once the endpoints are known, their position-vector dot product is the sum of products of corresponding coordinates.

Step 1: Parameterise the two endpoints Let the parameters of the first and second lines be t and s, respectively. Then

$$M = (5 + 4t, 4 + t, 5 + 3t), \qquad N = (-8 + 12s, -2 + 5s, -11 + 9s).$$

Their direction vectors are $u = (4, 1, 3)$ and $v = (12, 5, 9)$. They are not parallel, because the first and third component ratios are 3, whereas the second ratio is 5. The joining vector is

$$\overrightarrow{MN} = (-13 + 12s - 4t, -6 + 5s - t, -16 + 9s - 3t).$$

Step 2: Impose both perpendicularity conditions The conditions $\overrightarrow{MN} \cdot u = 0$ and $\overrightarrow{MN} \cdot v = 0$ give

$$-106 + 80s - 26t = 0, \qquad -330 + 250s - 80t = 0.$$

Dividing both equations by two,

$$40s - 13t = 53, \qquad 125s - 40t = 165.$$

Multiply the first by 40 and the second by 13, then subtract: $25s = 25$. Hence $s = 1$, and substitution gives $t = -1$. Therefore $M = (1, 3, 2)$, $N = (4, 3, -2)$. As a check, $\overrightarrow{MN} = (3, 0, -4)$ has dot product zero with both u and v, so these endpoints indeed define the common perpendicular.

Step 3: Calculate the requested scalar With O as the origin, the position vectors are simply the

coordinate triples of M and N. Therefore

$$\overrightarrow{OM} \cdot \overrightarrow{ON} = 1 \cdot 4 + 3 \cdot 3 + 2(-2) = 4 + 9 - 4 = 9.$$

The question asks for this scalar dot product, not the length $|MN|$, which would instead be 5. No physical unit is specified for these geometric coordinates.

Step 4: Conclusion $\overrightarrow{OM} \cdot \overrightarrow{ON} = 9$. The required numerical entry is $\boxed{9}$.

Quick Tip to Solve: For closest points on two lines, solve the two perpendicularity equations for their parameters before calculating any requested position-vector expression.

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22. Let the area of the region $\{(x, y) : 0 \le x \le 3, 0 \le y \le \min\{x^2 + 2, 2x + 2\}\}$ be A. Then 12 A is equal to _____. [4 marks]

Solution:

Concept:

- The area below the lower of two nonnegative curves is the integral of their pointwise minimum. Split the interval wherever the two curves intersect.
- The power rule gives $\int x^n \, dx = x^{n+1}/(n+1)$ for $n \neq -1$; the required quantity is twelve times the resulting area.

Step 1: Locate the intersections Set the two upper boundaries equal:

$$x^2 + 2 = 2x + 2 \quad \Longrightarrow \quad x(x - 2) = 0.$$

Thus the intersections occur at $x = 0$ and $x = 2$. The difference between the quadratic and the line is $x(x - 2)$, which is negative for $0 < x < 2$ and positive for $2 < x \le 3$. Consequently the parabola is the lower boundary on the first interval and the line is the lower boundary on the second. Both are positive, so no additional splitting at the horizontal axis is needed.

Step 2: Integrate the lower boundary on each interval The desired region extends upward from $y = 0$ to the smaller upper boundary. Hence its area is

$$\begin{aligned} A &= \int_0^2 (x^2 + 2)\, dx + \int_2^3 (2x + 2)\, dx \\ &= \left[\frac{x^3}{3} + 2x\right]_0^2 + \left[x^2 + 2x\right]_2^3 \\ &= \frac{8}{3} + 4 + (15 - 8) = \frac{41}{3}. \end{aligned}$$

This sums two adjoining regions without overlap; integrating the larger curve would instead include points that violate the minimum condition.

Step 3: Scale the area as requested The question asks for $12A$, rather than the area itself. Substitute

the exact fraction before simplifying:

$$12A = 12\left(\frac{41}{3}\right) = 4 \cdot 41 = 164.$$

Keeping the fraction exact avoids a rounding error in the final integer.

Step 4: Conclusion The area is $A = 41/3$ square units, so the required dimensionless numerical entry is $\boxed{164}$.

Quick Tip to Solve: For a minimum of two curves, check the sign of their difference on each side of an intersection before writing the integrals.

23. Let the set $C = \{(x, y) \mid x^2 - 2^y = 2023, x, y \in \mathbb{N}\}$. Then $\sum_{(x,y)\in C} (x + y)$ is equal to _____.

[4 marks]

Solution:

Concept:

- Every integer square is congruent to $0, 1$ or 4 modulo 8; in particular an odd square is congruent to 1.
- Use modular arithmetic to bound the natural exponent, then test the finitely many surviving values in the original equation.

Step 1: Exclude every exponent at least three The equation is $x^2 = 2023 + 2^y$. If $y \geq 3$, then 2^y is divisible by 8, whereas $2023 \equiv 7 \pmod 8$. It would follow that

$$x^2 \equiv 7 \pmod 8,$$

which is impossible for an integer square. To see the square residues directly, the even residues give 0 or 4, and the odd residues give 1. Thus this exclusion covers all integers x, without requiring an upper bound on x or an exhaustive search over large powers.

Step 2: Test the remaining natural exponents For positive natural numbers only $y = 1$ and $y = 2$ remain. At $y = 1$,

$$x^2 = 2023 + 2 = 2025 = 45^2,$$

so $x = 45$, since x must be natural. At $y = 2$, $x^2 = 2027$; but $45^2 = 2025 < 2027 < 2116 = 46^2$, so there is no integer solution. Even if a convention includes 0 in the natural numbers, $y = 0$ gives 2024, strictly between 44^2 and 45^2, and supplies no extra solution.

Step 3: Evaluate the sum over the set The full solution set is therefore $C = \{(45, 1)\}$. The requested summation adds $x + y$ for each pair in this set, not the two coordinates independently over different ranges. Hence

$$\sum_{(x,y)\in C} (x + y) = 45 + 1 = 46.$$

The modular exclusion proves that no untested larger exponent can contribute.

Step 4: Conclusion There is exactly one admissible pair, $(x, y) = (45, 1)$. The required numerical answer is $\boxed{46}$.

Quick Tip to Solve: When a power of two is added to a fixed integer to make a square, test modulo eight first; all exponents at least three have the same residue.

24. Let the slope of the line $45x + 5y + 3 = 0$ be $27r_1 + \frac{9r_2}{2}$ for some $r_1, r_2 \in \mathbb{R}$. Then $\lim\limits_{x\to 3}\left(\int_3^x \frac{8t^2}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x}dt\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- The slope of $Ax + By + C = 0$ is $-A/B$, provided $B \neq 0$. This supplies a relation between the two parameters.
- An integral of t^2 with respect to t treats every occurrence of x as constant. A common zero in numerator and denominator can be evaluated using their first derivatives when the denominator derivative is nonzero.

Step 1: Extract the parameter relation The line has slope $-45/5 = -9$. Therefore

$$27r_1 + \frac{9}{2}r_2 = -9, \qquad 6r_1 + r_2 = -2.$$

There is no need to determine the two parameters separately; the limit depends only on this linear combination.

Step 2: Integrate with respect to the correct variable Write

$$D(x) = \frac{3}{2}r_2x - r_2x^2 - r_1x^3 - 3x.$$

The denominator has no t, so the integral in the question equals

$$\frac{\int_3^x 8t^2\,dt}{D(x)} = \frac{\frac{8}{3}(x^3 - 27)}{D(x)}.$$

At $x = 3$, the numerator vanishes, and

$$D(3) = -27r_1 - \frac{9}{2}r_2 - 9 = 0$$

by the slope relation.

Step 3: Evaluate the common-zero limit Differentiate the numerator and denominator polynomials:

$$N'(3) = 8(3)^2 = 72,$$

and

$$D'(3) = \frac{3}{2}r_2 - 6r_2 - 27r_1 - 3$$
$$= -\frac{9}{2}r_2 - 27r_1 - 3 = 9 - 3 = 6.$$

Since this denominator derivative is nonzero, the zero is simple and the quotient tends to the ratio of these derivatives. Thus

$$\lim_{x\to 3} \frac{\frac{8}{3}(x^3 - 27)}{D(x)} = \frac{72}{6} = 12.$$

Step 4: Conclusion The requested dimensionless limit is $\boxed{12}$.

Quick Tip to Solve: In an integral with different upper-limit and integration variables, first identify which symbols are constants with respect to the integration variable.

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25. Let α, β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that $\mathrm{Im}(\alpha) > \mathrm{Im}(\beta)$. Let a, b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta} + \alpha^{98} = 3^n(a + ib), i = \sqrt{-1}$. Then $n + a + b$ is equal to _____. [4 marks]

Solution:

Concept:

- Write a complex number as $r(\cos\theta + i\sin\theta)$. De Moivre's theorem multiplies its argument by the integer exponent and raises its modulus to that exponent.
- The conjugate roots have product $\alpha\beta = 3$, so division by β can be converted into multiplication by $\alpha/3$.

Step 1: Choose the roots in the specified order The quadratic formula gives

$$x = \frac{\sqrt{6} \pm \sqrt{-6}}{2} = \frac{\sqrt{6}}{2}(1 \pm i).$$

The positive imaginary part belongs to α, so

$$\alpha = \sqrt{3}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right), \quad \beta = \overline{\alpha}.$$

Their moduli are $\sqrt{3}$, and their product is 3. The ordering condition matters because swapping them would reverse the sign of the imaginary component.

Step 2: Evaluate both high powers exactly Using the product of the roots,

$$\frac{\alpha^{99}}{\beta} = \frac{\alpha^{100}}{3}.$$

Its modulus is $3^{50}/3 = 3^{49}$, and its argument is $100\pi/4 = 25\pi$. Therefore this term is -3^{49}. Also

$$\alpha^{98} = 3^{49}\left(\cos\frac{49\pi}{2} + i\sin\frac{49\pi}{2}\right) = i3^{49},$$

since $49\pi/2 = 24\pi + \pi/2$. These reductions use the period 2π, so the signs are fixed without approximating the complex numbers.

Step 3: Match the normalized integer coefficients Adding the terms gives

$$\frac{\alpha^{99}}{\beta} + \alpha^{98} = 3^{49}(-1+i).$$

Both integers -1 and 1 are not divisible by 3, as required. Factoring out a smaller power of 3 would leave both coefficients divisible by 3, and a larger power would make them nonintegral. Hence $n = 49$, $a = -1$, $b = 1$, and $n + a + b = 49 - 1 + 1 = 49$.

Step 4: Conclusion The required numerical value is $\boxed{49}$.

Quick Tip to Solve: Use the root product to eliminate a denominator before reducing large complex powers modulo a full turn.

---

26. Let for any three distinct consecutive terms a, b, c of an A.P, the lines $ax + by + c = 0$ be concurrent at the point P and $Q(\alpha, \beta)$ be a point such that the system of equations

$$x + y + z = 6,$$
$$2x + 5y + \alpha z = \beta \text{ and}$$

$x + 2y + 3z = 4$, has infinitely many solutions. Then $(PQ)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Three consecutive terms of an arithmetic progression can be written $a = t$, $b = t + d$, $c = t + 2d$, with $d \neq 0$ when they are distinct.
- A three-equation linear system has infinitely many solutions when its coefficient rank equals its augmented rank and is less than three. Here two independent equations allow the third to be expressed as their linear combination.

Step 1: Find the common point of the family Substitute the progression into the line:

$$tx + (t+d)y + t + 2d = t(x + y + 1) + d(y + 2) = 0.$$

For the same point to lie on every such line, both coefficients must vanish:

$$x + y + 1 = 0, \qquad y + 2 = 0.$$

Thus $P = (1, -2)$. Direct substitution also checks the result: $a - 2b + c = 0$ for any arithmetic progression, so this point is indeed on every line.

Step 2: Make the second equation dependent on the others The coefficient rows of the first and third equations are $(1, 1, 1)$ and $(1, 2, 3)$; they are independent. Write the second equation as u times the

first plus v times the third. Matching the coefficients of x and y gives

$$u + v = 2, \quad u + 2v = 5,$$

hence $v = 3$ and $u = -1$. The remaining coefficient and constant must match as well:

$$\alpha = u + 3v = -1 + 9 = 8, \qquad \beta = 6u + 4v = -6 + 12 = 6.$$

These equalities ensure rank two for both matrices, so they establish infinite solutions rather than merely a vanishing determinant.

Step 3: Compute the squared separation The point determined by the system is $Q = (8, 6)$. Applying the Cartesian distance formula,

$$(PQ)^2 = (8 - 1)^2 + (6 - (-2))^2 = 7^2 + 8^2 = 49 + 64 = 113.$$

The requested quantity is already the squared distance; taking a square root would give a different answer.

Step 4: Conclusion The points are $P = (1, -2)$ and $Q = (8, 6)$, so $\boxed{(PQ)^2 = 113}$ square units.

Quick Tip to Solve: For infinite solutions, match the constants in a row dependence as well as the coefficients; determinant zero alone does not guarantee consistency.

---

27. Let $P(\alpha, \beta)$ be a point on the parabola $y^2 = 4x$. If P also lies on the chord of the parabola $x^2 = 8y$ whose mid point is $(1, \frac{5}{4})$, then $(\alpha - 28)(\beta - 8)$ is equal to _____. [4 marks]

Solution:

Concept:

- For the parabola $x^2 = 8y$, the chord whose midpoint is (h, k) has equation $hx - 4(y + k) = h^2 - 8k$, obtained by the midpoint chord relation $T = S_1$.
- Intersect that line with $y^2 = 4x$. Both possible intersections must be checked, since the question does not specify which one is the point P.

Step 1: Determine the chord equation Use the given midpoint $(h, k) = (1, 5/4)$ in the midpoint relation. This yields

$$x - 4\left(y + \frac{5}{4}\right) = 1 - 8\left(\frac{5}{4}\right) = -9,$$

or

$$x - 4y + 4 = 0, \qquad x = 4y - 4.$$

The line passes through the stated midpoint, since $1 - 4(5/4) + 4 = 0$. It is the chord of $x^2 = 8y$; this must not be confused with the other parabola on which P lies.

Step 2: Find the two possible points on the other parabola At $P = (\alpha, \beta)$, the chord equation gives

$\alpha = 4\beta - 4$, and membership in $y^2 = 4x$ gives $\beta^2 = 4\alpha$. Eliminate α:

$$\beta^2 = 16\beta - 16, \qquad (\beta - 8)^2 = 48.$$

Therefore

$$\beta = 8 \pm 4\sqrt{3}, \qquad \alpha = 28 \pm 16\sqrt{3}.$$

The signs are paired because the coordinates satisfy the same linear equation; they cannot be chosen independently.

Step 3: Evaluate the requested product without choosing a sign From $\alpha = 4\beta - 4$, subtract 28 to obtain $\alpha - 28 = 4(\beta - 8)$. Thus

$$(\alpha - 28)(\beta - 8) = 4(\beta - 8)^2 = 4 \cdot 48 = 192.$$

Both intersections produce the same value because the remaining expression is a square. Thus the unspecified choice of P does not cause any ambiguity in the requested result.

Step 4: Conclusion For either admissible point P, the required dimensionless value is $\boxed{192}$.

Quick Tip to Solve: When two intersections are possible, express the requested product as a square before calculating each point separately.

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28. Remainder when $64^{32^{32}}$ is divided by 9 is equal to _____. [4 marks]

Solution:

Concept:

- Congruences are preserved under multiplication, so if $a \equiv b \pmod{m}$, then $a^k \equiv b^k \pmod{m}$ for every positive integer k.
- The remainder on division by nine is the unique integer in $\{0, 1, \ldots, 8\}$ congruent to the given number modulo nine.

Step 1: Reduce the base before handling the exponent The base is 64, and

$$64 = 9 \cdot 7 + 1, \qquad 64 \equiv 1 \pmod{9}.$$

The entire exponent is 32^{32}, a positive integer. Once the base is congruent to one, its magnitude and the size of this exponent do not need to be calculated. In particular, reducing the exponent first would add unnecessary work and could lead to confusing the order of exponentiation.

Step 2: Raise the congruence to the full exponent Let $N = 32^{32}$. Repeated multiplication of $64 \equiv 1 \pmod{9}$ gives

$$64^N \equiv 1^N = 1 \pmod{9}.$$

Equivalently, $64 = 1 + 9 \cdot 7$, and the binomial theorem gives

$$(1 + 9 \cdot 7)^N = 1 + N(9 \cdot 7) + \binom{N}{2}(9 \cdot 7)^2 + \cdots .$$

Every term after the initial one contains a factor of nine. Thus the difference $64^N - 1$ is divisible by nine, providing an exact divisibility proof independent of the size of N.

Step 3: Identify the standard remainder The congruent value 1 is already within the allowed remainder interval from zero through eight. Consequently no further reduction is required. This argument concerns the integer as printed, $64^{32^{32}}$; it does not require approximations, logarithms, or writing out any large power. The fact that N is positive ensures the multiplication argument applies directly.

Step 4: Conclusion The remainder when $64^{32^{32}}$ is divided by 9 is $\boxed{1}$.

Quick Tip to Solve: For a huge power, reduce the base first. A base congruent to one stays one for every positive integer exponent.

---

29. Let $f(x) = \sqrt{\lim_{r\to x}\left\{\frac{2r^2[(f(r))^2 - f(x)f(r)]}{r^2-x^2} - r^3 e^{\frac{f(r)}{r}}\right\}}$ be differentiable in $(-\infty, 0)\cup(0,\infty)$ and $f(1) = 1$. Then the value of ea, such that $f(a) = 0$, is equal to _____. [4 marks]

Solution:

Concept:

- The limit of the difference quotient produces $f(x)f\prime(x)$; use differentiability before introducing a substitution.
- For a homogeneous expression involving f/x, write $v = f(x)/x$, so $f\prime = v + xv\prime$.

Step 1: Evaluate the limit Factor the numerator as $2r^2 f(r)[f(r) - f(x)]$, and factor $r^2 - x^2 = (r - x)(r + x)$. Continuity and differentiability give the limit $xf(x)f\prime(x)$. Squaring the displayed relation therefore gives

$$f^2 = xff\prime - x^3 e^{f/x}.$$

Step 2: Separate the transformed equation Substitute $f = xv$ and $f\prime = v + xv\prime$. The terms x^2v^2 cancel, leaving $vv\prime = e^v$ because $x \neq 0$. Thus

$$ve^{-v}\,dv = dx, \qquad -(v+1)e^{-v} = x + C.$$

This integration is used on the branch through the prescribed initial point.

Step 3: Apply the initial condition and locate the zero At $x = 1$, $v = 1$; hence $C = -1 - 2/e$. The implicit relation becomes

$$(v+1)e^{-v} = 1 + 2/e - x.$$

At the zero-level boundary $f(a) = 0$, one has $v = 0$, giving $a = 2/e$ and therefore $ea = 2$. Strictly, the implicit branch has an infinite derivative at this boundary: a globally differentiable function cannot actually attain that zero, since the original squared equation at a nonzero zero would read $0 = -a^3$. The stated numerical value is the zero-level boundary intended by the question.

Step 4: Conclusion The zero-level boundary of the initial-value branch is $a = 2/e$, so the intended numerical answer is $ea = 2$. The demand for differentiability at that zero is inconsistent with the

literal equation.

Quick Tip to Solve: Cancel the common difference quotient first; the substitution f/x then makes the exponential equation separable. Check singular endpoints before extending a branch.

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30. If $\int_{\pi/6}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$, where α, β and γ are rational numbers, then $3\alpha + 4\beta - \gamma$ is equal to _____. [4 marks]

Solution:

Concept:

- Use $1 - \sin 2x = (\sin x - \cos x)^2$, so the square root is an absolute value.
- The sign changes at $x = \pi/4$, which lies between the two limits. Split the integral there.

Step 1: Resolve the square root on each interval On $[\pi/6, \pi/4]$, cosine is at least sine; on $[\pi/4, \pi/3]$, sine is at least cosine. Consequently

$$I = \int_{\pi/6}^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{\pi/3} (\sin x - \cos x)\, dx.$$

Keeping the absolute value is essential: the integral of the signed difference over the whole symmetric interval would incorrectly give zero.

Step 2: Evaluate both positive contributions The antiderivatives are $\sin x + \cos x$ and $-\cos x - \sin x$, respectively. Each contribution is $\sqrt{2} - (1 + \sqrt{3})/2$. Hence

$$I = 2\sqrt{2} - 1 - \sqrt{3}.$$

Both halves have equal area by symmetry about $\pi/4$, providing an independent sign check.

Step 3: Read the coefficients Comparison with $\alpha + \beta\sqrt{2} + \gamma\sqrt{3}$ gives $\alpha = -1$, $\beta = 2$, $\gamma = -1$. Therefore

$$3\alpha + 4\beta - \gamma = -3 + 8 + 1 = 6.$$

This is a dimensionless numerical expression; no physical unit is involved.

Step 4: Conclusion The required dimensionless value is $\boxed{6}$.

Quick Tip to Solve: Whenever a square root contains a squared trigonometric expression, keep the absolute value and locate every sign change within the limits.

31. A small liquid drop of radius R is divided into 27 identical liquid drops. If the surface tension is T, then the work done in the process will be: [4 marks]

(A) $4\pi \mathrm{R}^2$ T

(B) $8\pi R^2$ T

(C) $\frac{1}{8}\pi R^2 T$

(D) $3\pi R^2$ T

Correct Answer: (B) $8\pi R^2$ T

Solution:

Concept:

- At constant surface tension T, the work required to create additional liquid surface is $W = T\Delta A$.
- The liquid volume is conserved. A liquid drop has one surface, unlike a soap film with two surfaces.

Step 1: Find the radius of each small drop Let the common final radius be r. Conservation of liquid volume gives

$$\frac{4}{3}\pi R^3 = 27\frac{4}{3}\pi r^3, \qquad r = R/3.$$

It is the volume, proportional to the cube of the radius, that must be divided equally; the radius does not become $R/27$.

Step 2: Calculate the increase in surface area The initial area is $A_i = 4\pi R^2$. The 27 final drops together have area

$$A_f = 27(4\pi r^2) = 27 \times 4\pi \frac{R^2}{9} = 12\pi R^2.$$

Thus the new area created is $\Delta A = 8\pi R^2$, a positive quantity, so positive external work is needed.

Step 3: Convert the area increase into work The required work is

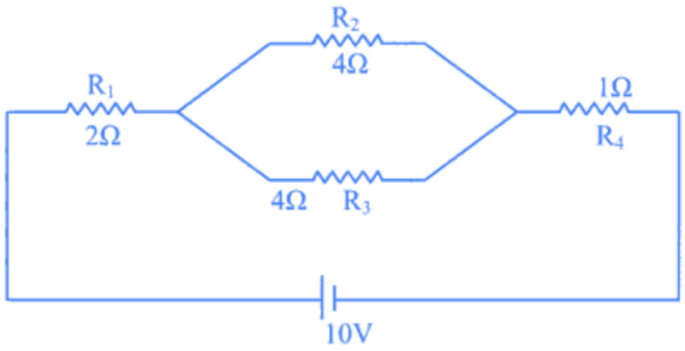
$$W = T(A_f - A_i) = 8\pi R^2 T.$$

Option (A) uses only the original area rather than the increase. Option (C) gives a decrease in the relevant area scale, and option (D) confuses the ratio of final to initial area with their difference. Surface tension has units $\mathrm{N\,m^{-1}}$, so area times surface tension has units of joules.

Step 4: Conclusion The work done is $\boxed{8\pi R^2 T}$ joules in SI units. Hence, the correct option is (B).

Quick Tip to Solve: For n equal drops formed from one drop, the final area is n to the power one-third times the initial area; subtract the initial area before multiplying by T.

32. In the given circuit, the current in resistance R_3 is : [4 marks]



(A) 2 A

(B) 1.5 A

(C) 1 A

(D) 2.5 A

Correct Answer: (C) 1 A

Solution:

Concept:

- Resistors sharing both end nodes are in parallel and have the same potential difference.
- Use $V = IR$ for the full circuit, then divide the current between the equal parallel branches.

Step 1: Reduce the parallel pair The figure shows $R_2 = 4\,\Omega$ and $R_3 = 4\,\Omega$ between the same two junctions. Their equivalent resistance is

$$R_{23} = \frac{4 \times 4}{4 + 4} = 2\,\Omega.$$

The diamond shape does not change the electrical connection: these two resistors are parallel, rather than series.

Step 2: Calculate the current supplied by the cell The $2\,\Omega$ resistor R_1, the equivalent R_{23}, and the $1\,\Omega$ resistor R_4 are in series. Therefore

$$R_{\text{total}} = 2 + 2 + 1 = 5\,\Omega, \qquad I_{\text{total}} = \frac{10}{5} = 2\,\text{A}.$$

This is the current before the junction, not the current through each of the two branches.

Step 3: Find the current through R3 The equal branch resistances share the total current equally, so $I_3 = 2/2 = 1\,\text{A}$. Equivalently the parallel pair has voltage $2 \times 2 = 4\,\text{V}$, giving $I_3 = 4/4 = 1\,\text{A}$. Option (A) is the total current; (B) and (D) violate the equal division and the circuit resistance obtained above.

Step 4: Conclusion The current through R_3 is $\boxed{1\,\text{A}}$. Hence, the correct option is (C).

Quick Tip to Solve: Redraw a diamond network by its end nodes: equal resistors connected across the same two nodes carry equal currents.

33. A physical quantity Q is found to depend on quantities a, b, c by the relation $Q = \frac{a^4b^3}{c^2}$. The percentage error in a, b and c are $3\%, 4\%$ and 5% respectively. Then, the percentage error in Q is : [4 marks]

(A) 43%

(B) 34%

(C) 66%

(D) 14%

Correct Answer: (B) 34%

Solution:

Concept:

- For a product or quotient of powers, the maximum fractional errors add with the absolute values of the exponents.
- The negative exponent of a denominator does not subtract its measurement uncertainty.

Step 1: Write the fractional-error relation Given $Q = a^4b^3c^{-2}$, logarithmic differentiation gives $dQ/Q = 4\,da/a + 3\,db/b - 2\,dc/c$. For the maximum possible uncertainty the signs are chosen to make all contributions reinforce each other, yielding

$$\frac{\Delta Q}{Q} = 4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + 2\frac{\Delta c}{c}.$$

Step 2: Insert the percentage uncertainties The individual weighted contributions are $4 \times 3\% = 12\%$, $3 \times 4\% = 12\%$, and $2 \times 5\% = 10\%$. Therefore

$$\frac{\Delta Q}{Q} \times 100\% = 12\% + 12\% + 10\% = 34\%.$$

This is the usual first-order maximum percentage error requested in such a measurement problem.

Step 3: Reject the alternative percentages The value 14% results from incorrectly retaining the minus sign of the denominator contribution: $12 + 12 - 10 = 14$. Uncertainty does not cancel that way. The values 43% and 66% do not follow from the exponent-weighted sum, and there is no instruction to use a different statistical error model.

Step 4: Conclusion The percentage error in Q is $\boxed{34\%}$. Hence, the correct option is (B).

Quick Tip to Solve: Write each variable as a power, multiply its percentage error by the absolute exponent, and add the resulting contributions.

34. A stone of mass $900\ \mathrm{g}$ is tied to a string and moved in a vertical circle of radius $1\ \mathrm{m}$ making $10\ \mathrm{rpm}$. The tension in the string, when the stone is at the lowest point is (if $\pi^2 = 9.8$ and $g = 9.8\ \mathrm{m/s^2}$) :

[4 marks]

(A) 17.8 N

(B) 97 N

(C) 9.8 N

(D) 8.82 N

Correct Answer: (C) 9.8 N

Solution:

Concept:

- At the bottom of a vertical circle, the centre is upward, so $T - mg = mv^2/r$.
- Convert revolutions per minute to angular speed using $\omega = 2\pi n/60$, then use $v = \omega r$.

Step 1: Convert the mass and angular speed The mass is $900\,\mathrm{g} = 0.9\,\mathrm{kg}$. For 10 revolutions each minute,

$$\omega = \frac{2\pi \times 10}{60} = \frac{\pi}{3}\,\mathrm{rad\,s^{-1}}.$$

With $r = 1\,\mathrm{m}$, the speed is $v = \pi/3\,\mathrm{m\,s^{-1}}$. Treating rpm as revolutions per second would give a greatly exaggerated force.

Step 2: Evaluate the radial force requirement The inward radial force needed at the bottom is

$$\frac{mv^2}{r} = m\omega^2 r = 0.9\frac{\pi^2}{9} \times 1 = 0.98\,\mathrm{N},$$

where the supplied value $\pi^2 = 9.8$ is used. Gravity contributes $mg = 0.9 \times 9.8 = 8.82\,\mathrm{N}$ downward, opposite to the required inward acceleration.

Step 3: Find the tension Tension acts upward at the lowest point. Thus

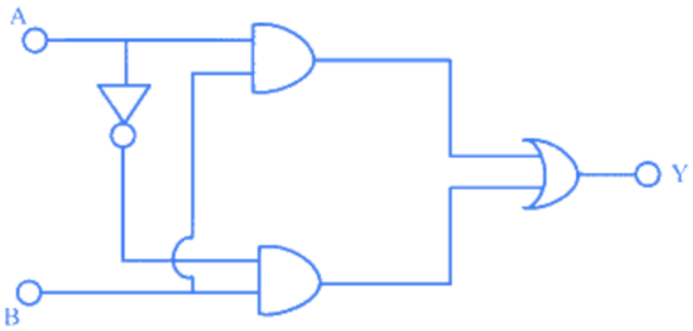
$$T = mg + \frac{mv^2}{r} = 8.82 + 0.98 = 9.8\,\mathrm{N}.$$

The value $8.82\,\mathrm{N}$ in option (D) is only the weight and omits centripetal acceleration. The values $17.8\,\mathrm{N}$ and $97\,\mathrm{N}$ do not follow from the given mass and correctly converted angular speed. The result concerns the specified lowest-point instant.

Step 4: Conclusion The tension at the lowest point is $\boxed{9.8\,\mathrm{N}}$. Hence, the correct option is (C).

Quick Tip to Solve: At the bottom, tension must supply both the weight and the centripetal force; convert rpm before squaring the angular speed.

35. The truth table for this given circuit is : [4 marks]



(A)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(B)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(C)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(D)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Correct Answer: (B)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Solution:

Concept:

- An AND gate gives the product of its inputs, an inverter gives the complement, and an OR gate gives the Boolean sum.
- Use $A + \overline{A} = 1$ and the distributive law to simplify the network before listing its truth table.

Step 1: Read the two AND branches The top AND gate receives A directly and B from the lower input, so its output is AB. The inverter changes A to $\overline{A}$; this and B feed the bottom AND gate, giving $\overline{A}B$. The curved wire bridge represents a crossing without connection, rather than a joined node.

Step 2: Combine the outputs The final gate is OR. Its output is

$$Y = AB + \overline{A}B = B(A + \overline{A}) = B.$$

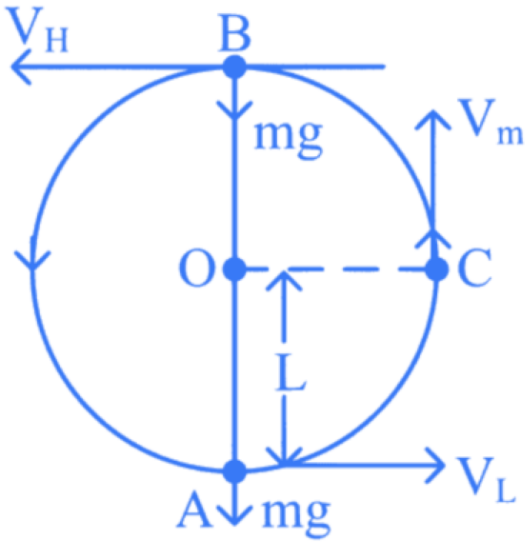
Thus A can change without changing the output when B is held fixed: with B low, both branches are low, and with B high, exactly one AND branch is high.

Step 3: Compare all four truth tables For input pairs $(A, B) = (0, 0), (0, 1), (1, 0), (1, 1)$, the outputs are respectively $0, 1, 0, 1$. This is the second displayed table. Option (A) is the AND pattern $0, 0, 0, 1$; option (C) gives the complement of B, $1, 0, 1, 0$; and option (D) is the NAND pattern $1, 1, 1, 0$. None matches the direct dependence $Y = B$.

Step 4: Conclusion The circuit has $Y = B$, with output sequence $0, 1, 0, 1$. Hence, the correct option is (B).

Quick Tip to Solve: Factor the input shared by two AND branches; complementary versions of the other input then combine to Boolean 1.

36. A bob of mass 'm' is suspended by a light string of length 'L'. It is imparted a minimum horizontal velocity at the lowest point A such that it just completes half circle reaching the top most position B. The ratio of kinetic energies $\frac{(K.E)_A}{(K.E)_B}$ is : [4 marks]



(A) 5 : 1

(B) 3 : 2

(C) 1 : 5

(D) 2 : 5

Correct Answer: (A) 5 : 1

Solution:

Concept:

- For a bob on a light string, the string can pull but cannot push. At the top of the circle the minimum speed is fixed by the condition that the tension just becomes zero.
- Tension is perpendicular to the instantaneous displacement and does no work. Thus the sum of kinetic and gravitational potential energies remains constant.

Step 1: Apply the limiting condition at the top At B both gravity and tension act towards the centre. The radial equation is $mg + T_B = mv_B^2/L$. For the smallest possible initial speed that keeps the bob on its circular path up to the top, take $T_B = 0$. Hence

$$v_B^2 = gL, \qquad K_B = \frac{1}{2}mgL.$$

The bob cannot have zero speed at B while following the string-constrained circle: gravity alone would then exceed the required centripetal force and the string would become slack earlier.

Step 2: Use the rise in gravitational potential energy From A to B the vertical rise is the diameter $2L$. Taking the potential energy at A to be zero gives

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + 2mgL.$$

Substituting the limiting top speed,

$$v_A^2 = gL + 4gL = 5gL.$$

Therefore

$$\frac{K_A}{K_B} = \frac{\frac{1}{2}m(5gL)}{\frac{1}{2}m(gL)} = 5.$$

The initial energy must exceed the final energy, excluding the ratios smaller than unity. The ratio $3 : 2$ also fails the required energy increase of $2mgL$.

Step 3: Conclusion The ratio of the kinetic energies at A and B is $5 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: For a string completing a vertical circle at the limiting speed, remember $v_{\text{bottom}}^2 = 5gL$ and $v_{\text{top}}^2 = gL$.

37. An electric field is given by $(6\hat{i}+5\hat{j}+3\hat{k})\text{N/C}$. The electric flux through a surface area $30\hat{i}\ \text{m}^2$ lying in YZ-plane (in SI unit) is : [4 marks]

(A) 60

(B) 90

(C) 180

(D) 150

Correct Answer: (C) 180

Solution:

Concept:

- Electric flux through a flat surface in a uniform field is the scalar product $\Phi = \vec{E} \cdot \vec{A}$, where the area vector is normal to the surface.
- Only the component of the field perpendicular to a surface contributes to its flux. Components parallel to that surface give zero contribution.

Step 1: Identify the normal component The surface lies in the YZ-plane, so its normal is along the X-axis. The given vector $\vec{A} = 30\hat{i}\,\text{m}^2$ specifies the positive X-normal as well as the area. For $\vec{E} = (6\hat{i}+5\hat{j}+3\hat{k})\,\text{N/C}$, the normal field is therefore $E_x = 6\,\text{N/C}$. The coefficients 5 and 3 describe tangential components; they must not be combined with the area as if they were normal components.

Step 2: Evaluate the scalar product and compare the choices Using the orthogonality relations $\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = 0$ and $\hat{i}\cdot\hat{i} = 1$,

$$\begin{aligned}\Phi &= (6\hat{i}+5\hat{j}+3\hat{k})\cdot(30\hat{i}) \\ &= 6(30)+5(0)+3(0) \\ &= 180\,\text{N m}^2/\text{C}.\end{aligned}$$

The flux is positive because the normal component of the field points in the same direction as the specified area vector. The value 150 uses the Y-component incorrectly, while 90 uses the Z-component; 60 is not the product of the stated normal component and area. The magnitude of the whole electric field is unnecessary.

Step 3: Conclusion The electric flux is $180\,\text{N m}^2/\text{C}$. Hence, the correct option is (C).

Quick Tip to Solve: For a surface in the YZ-plane, select the X-component of $\vec{E}$ before multiplying by the signed area.

38. A planet takes 200 days to complete one revolution around the Sun. If the distance of the planet from Sun is reduced to one fourth of the original distance, how many days will it take to complete one revolution : [4 marks]

(A) 20

(B) 50

(C) 100

(D) 25

Correct Answer: (D) 25

Solution:

Concept:

- For planets revolving around the same Sun, Kepler's third law is $T^2 \propto a^3$, where a is the orbital semi-major axis.
- For a circular orbit, the orbital distance is its radius. Period ratios eliminate the common factor involving the Sun's mass and the gravitational constant.

Step 1: Write the period ratio Let the original orbital distance be r_1 and the original period be $T_1 = 200$ days. The new distance is $r_2 = r_1/4$. The central gravitating mass remains the Sun, so the proportionality constant in Kepler's law is unchanged. Consequently,

$$\frac{T_2^2}{T_1^2} = \left(\frac{r_2}{r_1}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}.$$

The distance ratio is cubed before taking the square root; the period is not directly proportional to orbital distance.

Step 2: Take the positive square root A revolution time is positive, so the physical root gives

$$\frac{T_2}{T_1} = \frac{1}{8}, \qquad T_2 = \frac{200}{8} = 25 \text{ days}.$$

This reduction is also physically consistent: closer to the Sun the orbital speed is larger while the circumference of the orbit is smaller. A period of 50 days would incorrectly make the period proportional to radius; 100 days would correspond to a square-root scaling alone. A period of 20 days has no basis in the required $r^{3/2}$ dependence. In fact, substituting 25 days back gives $(25/200)^2 = 1/64$, exactly the cube of the new distance ratio.

Step 3: Conclusion The planet takes 25 days to complete one revolution. Hence, the correct option is (D).

Quick Tip to Solve: For the same central mass, use $T_2/T_1 = (r_2/r_1)^{3/2}$; a quarter-radius gives one-eighth of the period.

39. A wire of length L and radius r is clamped at one end. If its other end is pulled by a force F, its length increases by l. If the radius of the wire and the applied force both are reduced to half of their original values keeping original length constant, the increase in length will become: [4 marks]

(A) 2 times

(B) 4 times

(C) 3 times

(D) $\frac{3}{2}$ times

Correct Answer: (A) 2 times

Solution:

Concept:

- Young's modulus is $Y = (F/A)/(l/L)$, so the extension of a uniform wire is $l = FL/(AY)$. Here L is its original length.
- A circular wire has cross-sectional area $A = \pi r^2$. For the same material and original length, extension is proportional to F/r^2.

Step 1: Account for the change in cross-sectional area Initially the extension is

$$l = \frac{FL}{\pi r^2 Y}.$$

The new radius is $r' = r/2$, giving

$$A' = \pi(r/2)^2 = \frac{\pi r^2}{4} = \frac{A}{4}.$$

Thus halving the radius reduces the area by a factor of four, not two. The wire has the same original length and material, so L and Y are unchanged. This relation applies within the linear elastic regime stated implicitly by using the same Young's modulus.

Step 2: Compare the two extensions The applied force is simultaneously reduced to $F' = F/2$. Therefore

$$l' = \frac{(F/2)L}{(\pi r^2/4)Y} = 2\frac{FL}{\pi r^2 Y} = 2l.$$

Equivalently, the ratio is

$$\frac{l'}{l} = \frac{F'}{F}\frac{A}{A'} = \frac{1}{2} \times 4 = 2.$$

Although the force is smaller, the area falls more rapidly, so the tensile stress doubles and the strain doubles. Four times would be obtained if the force were kept unchanged; three times and three-halves times do not follow from the simultaneous force and radius changes. The original length must not be confused with the small extension.

Step 3: Conclusion The increase in length becomes $2l$, or twice the original extension. Hence, the correct option is (A).

Quick Tip to Solve: In wire-extension ratio problems, square the radius ratio: $l'/l = (F'/F)(r/r')^2$ when material and length stay fixed.

40. The temperature of a gas having 2.0×10^{25} molecules per cubic meter at 1.38 atm (Given, $k = 1.38 \times 10^{-23} JK^{-1}$) is : [4 marks]

(A) 500 K

(B) 300 K

(C) 200 K

(D) 100 K

Correct Answer: (A) 500 K

Solution:

Concept:

- For an ideal gas, $pV = NkT$. Dividing by volume gives $p = nkT$, where $n = N/V$ is the molecular number density.
- The Boltzmann constant is given in SI units, so convert the pressure to pascals before substituting. Temperature from this equation is an absolute temperature in kelvin.

Step 1: Convert pressure and form the denominator The molecular number density is $n = 2.0 \times 10^{25}\,\mathrm{m}^{-3}$, not a number of moles. Use Boltzmann's constant rather than the molar gas constant. Taking $1\,\mathrm{atm} \simeq 10^5\,\mathrm{Pa}$,

$$p = 1.38 \times 10^5\,\mathrm{Pa}, \qquad nk = (2.0 \times 10^{25})(1.38 \times 10^{-23}) = 276\,\mathrm{J\,m^{-3}K^{-1}}.$$

Since $\mathrm{Pa} = \mathrm{J/m^3}$, division by nk produces kelvin.

Step 2: Calculate the absolute temperature Rearrange the equation of state:

$$T = \frac{p}{nk} = \frac{1.38 \times 10^5}{276} = 500\,\mathrm{K}.$$

Using the more precise standard-atmosphere conversion $1\,\mathrm{atm} = 1.01325 \times 10^5\,\mathrm{Pa}$ gives about $507\,\mathrm{K}$, which still selects the stated 500 K option at the precision of the choices. The other temperatures, 300 K, 200 K and 100 K, would give pressures of approximately 0.828 atm, 0.552 atm and 0.276 atm under the same rounded conversion, respectively. None agrees with the given 1.38 atm.

Step 3: Conclusion The matching temperature is $500\,\mathrm{K}$. Hence, the correct option is (A).

Quick Tip to Solve: When molecules per cubic metre are supplied, use $p = nkT$; using the molar gas constant would require converting number density to molar density.

41. If the distance between object and its two times magnified virtual image produced by a curved mirror is 15 cm, the focal length of the mirror must be: [4 marks]

(A) -10 cm

(B) -12 cm

(C) 15 cm

(D) 10/3 cm

Correct Answer: (A) -10 cm

Solution:

Concept:

- The Cartesian mirror convention gives $m = -v/u$ and $1/f = 1/v + 1/u$, with a real object in front of the mirror having $u < 0$.
- A virtual image of a real object is erect. A magnified virtual image is produced by a concave mirror with the object between its pole and focus.

Step 1: Use the magnification and separation The image is erect and twice as large, so $m = +2$. Let the magnitude of the object distance be $x > 0$. Then $u = -x$, and

$$2 = -\frac{v}{-x} \quad \Rightarrow \quad v = 2x.$$

The virtual image lies behind the mirror, on the opposite side of its pole from the real object. The distance between object and image is therefore $v - u = 2x - (-x) = 3x$, rather than the difference of their unsigned distances. Since this distance is 15 cm,

$$3x = 15 \quad \Rightarrow \quad x = 5\,\text{cm}.$$

Thus $u = -5\,\text{cm}$ and $v = +10\,\text{cm}$.

Step 2: Apply the mirror equation Substitute these signed distances into the mirror equation:

$$\frac{1}{f} = \frac{1}{10} + \frac{1}{-5} = -\frac{1}{10}\,\text{cm}^{-1}, \qquad f = -10\,\text{cm}.$$

The negative focal length confirms the required concave mirror. Also, the object distance magnitude 5 cm is smaller than the focal length magnitude 10 cm, as needed for an erect magnified virtual image. The positive focal lengths in options (C) and (D) would describe convex mirrors, which make virtual images diminished for a real object. The value -12 cm does not satisfy both the given magnification and the 15 cm separation.

Step 3: Conclusion The focal length must be $-10\,\text{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: For a real object and virtual mirror image, add their unsigned distances to get object-image separation, then use signed distances in the mirror formula.

42. A plane electromagnetic wave of frequency $35\ \text{MHz}$ travels in free space along the X-direction. At a particular point (in space and time) $\vec{E} = 9.6\hat{j}\ \text{V/m}$. The value of magnetic field at this point is : [4 marks]

(A) $9.6\hat{j}T$

(B) $3.2 \times 10^{-8}\hat{i}T$

(C) $9.6 \times 10^{-8}\hat{k}T$

(D) $3.2 \times 10^{-8}\hat{k}T$

Correct Answer: (D) $3.2 \times 10^{-8}\hat{k}T$

Solution:

Concept:

- In a plane electromagnetic wave in free space, the instantaneous field magnitudes obey $E = cB$, where $c = 3.0 \times 10^8\ \text{m/s}$.
- The propagation direction is along $\vec{E} \times \vec{B}$. The electric field, magnetic field and direction of propagation are mutually perpendicular.

Step 1: Calculate the magnetic-field magnitude At the stated point, the electric field magnitude is $E = 9.6\,\text{V/m}$. The two fields oscillate in phase in a free-space travelling wave, so the relation between their instantaneous values can be applied at this same point and time. Therefore

$$B = \frac{E}{c} = \frac{9.6}{3.0 \times 10^8} = 3.2 \times 10^{-8}\,\text{T}.$$

The frequency of 35 MHz would determine the wavelength, but it is not needed when the local electric field is already specified. The factor c must divide the electric field, not multiply it.

Step 2: Determine the direction by the cross product The wave travels along positive X and $\vec{E}$ points along positive Y. We require

$$\hat{j} \times \hat{B} = \hat{i}.$$

Since $\hat{j} \times \hat{k} = \hat{i}$, the magnetic field points along positive Z. Hence

$$\vec{B} = 3.2 \times 10^{-8}\hat{k}\,\text{T}.$$

Option (A) incorrectly makes the magnetic field parallel to the electric field and has the wrong magnitude. Option (B) points along the propagation direction instead of perpendicular to it. Option (C) has the correct direction but is three times the required magnitude. Thus both the amplitude relation and the right-hand rule are essential for selecting the answer.

Step 3: Conclusion The magnetic field is $\vec{B} = 3.2 \times 10^{-8}\hat{k}\,\text{T}$. Hence, the correct option is (D).

Quick Tip to Solve: First use $B = E/c$, then choose its sign so that $\vec{E} \times \vec{B}$ points along the wave's propagation direction.

43. In an a.c. circuit, voltage and current are given by:

$V = 100\sin(100t)V$ **and** $I = 100\sin\left(100t + \frac{\pi}{3}\right)\text{mA}$ **respectively.**

The average power dissipated in one cycle is: [4 marks]

(A) 5 W

(B) 25 W

(C) 2.5 W

(D) 10 W

Correct Answer: (C) 2.5 W

Solution:

Concept:

- The average power in a sinusoidal a.c. circuit is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$, where ϕ is the phase difference between voltage and current.
- For peak values V_0 and I_0, the rms values are $V_0/\sqrt{2}$ and $I_0/\sqrt{2}$. Current must be expressed in amperes when calculating power in watts.

Step 1: Identify the amplitudes and phase difference The coefficients of the sine functions are the peak values, not the rms values. Thus $V_0 = 100\,\text{V}$, while $I_0 = 100\,\text{mA} = 0.100\,\text{A}$. The current leads the voltage by $\pi/3$, so the power factor is $\cos(\pi/3) = 1/2$. A leading phase does not make the average dissipated power negative.

Step 2: Calculate the cycle average Substitute the rms values and the power factor:

$$P_{\text{av}} = \frac{V_0 I_0}{2}\cos\phi = \frac{100(0.100)}{2}\left(\frac{1}{2}\right) = 2.5\,\text{W}.$$

Equivalently, the cycle average of $\sin\omega t\sin(\omega t + \phi)$ is $\cos\phi/2$. The value $5\,\text{W}$ omits the power factor; $10\,\text{W}$ uses the peak product without either averaging or the phase factor. The value $25\,\text{W}$ is also inconsistent with the current conversion and exceeds the apparent power $5\,\text{VA}$.

Step 3: Conclusion The average power dissipated is $2.5\,\text{W}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert milliamperes to amperes first, then use one factor of $1/2$ for peak-to-rms power and the separate factor $\cos\phi$.

44. A particle is moving in a straight line. The variation of position 'x' as a function of time 't' is given as $x = \left(t^3 - 6t^2 + 20t + 15\right) m$. The velocity of the body when its acceleration becomes zero is : [4 marks]

(A) 6 m/s

(B) 10 m/s

(C) 8 m/s

(D) 4 m/s

Correct Answer: (C) 8 m/s

Solution:

Concept:

- Instantaneous velocity is the first time derivative of position: $v = dx/dt$. Acceleration is its second derivative: $a = dv/dt = d^2x/dt^2$.
- Zero acceleration means that velocity is momentarily stationary as a function of time; it does not imply zero velocity or zero displacement.

Step 1: Differentiate the position function For the given polynomial, differentiate each term with respect to time, taking t in seconds:

$$v(t) = 3t^2 - 12t + 20 \quad (\mathrm{m\,s^{-1}}).$$

The constant displacement $15\,\mathrm{m}$ makes no contribution to velocity. Differentiate once again:

$$a(t) = 6t - 12 \quad (\mathrm{m\,s^{-2}}).$$

It is this second derivative that must vanish, rather than the original position polynomial.

Step 2: Find the time and evaluate the velocity The zero-acceleration condition gives $6t - 12 = 0$, hence $t = 2\,\mathrm{s}$. Substitute this time into the velocity expression:

$$v(2) = 3(2)^2 - 12(2) + 20 = 12 - 24 + 20 = 8\,\mathrm{m\,s^{-1}}.$$

A useful independent check is $v(t) = 3(t-2)^2 + 8$, whose minimum is indeed $8\,\mathrm{m\,s^{-1}}$ at $t = 2\,\mathrm{s}$. The values 6, 10, and $4\,\mathrm{m\,s^{-1}}$ do not satisfy the derived velocity at that time, so options A, B and D are rejected.

Step 3: Conclusion The velocity when acceleration becomes zero is $8\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a position polynomial, differentiate twice to locate zero acceleration, then substitute that time into the first derivative.

45. The bob of a pendulum was released from a horizontal position. The length of the pendulum is 10 m. If it dissipates 10% of its initial energy against air resistance, the speed with which the bob arrives at the lowest point is:

[Use, g : $10\ \text{ms}^{-2}$**]** [4 marks]

(A) $5\sqrt{6}\ \text{ms}^{-1}$

(B) $5\sqrt{5}\ \text{ms}^{-1}$

(C) $2\sqrt{5}\ \text{ms}^{-1}$

(D) $6\sqrt{5}\ \text{ms}^{-1}$

Correct Answer: (D) $6\sqrt{5}\ \text{ms}^{-1}$

Solution:

Concept:

- With the lowest point as the zero of gravitational potential energy, releasing a pendulum from the horizontal gives an initial height $h = L$ and energy $E_i = mgL$.
- Energy lost to air resistance is subtracted from the initial mechanical energy. At the lowest point the remaining mechanical energy is entirely kinetic: $K = mv^2/2$.

Step 1: Calculate the available kinetic energy The bob begins at rest. In its horizontal position it is level with the pivot, while its lowest position is a length $L = 10\,\text{m}$ below the pivot. Therefore its vertical fall is $10\,\text{m}$, not twice this length. Since 10% of its initial energy is dissipated, the fraction remaining is 0.90:

$$K_{\text{bottom}} = 0.90\,mgL.$$

The string tension does no work because it is perpendicular to the instantaneous displacement.

Step 2: Use the kinetic-energy expression Equating the remaining energy to the kinetic energy cancels the unknown bob mass:

$$\frac{1}{2}mv^2 = 0.90mgL, \qquad v^2 = 1.80gL.$$

With the specified value $g = 10\,\text{m}\,\text{s}^{-2}$,

$$v = \sqrt{1.80(10)(10)} = \sqrt{180} = 6\sqrt{5}\,\text{m}\,\text{s}^{-1}.$$

Squaring the speeds in A, B and C gives 150, 125 and $20\,\text{m}^2\,\text{s}^{-2}$, respectively, whereas the energy balance requires $180\,\text{m}^2\,\text{s}^{-2}$. Only D satisfies it.

Step 3: Conclusion The speed at the lowest point is $6\sqrt{5}\,\text{m}\,\text{s}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: An energy loss of 10% multiplies v^2, rather than v, by 0.90; use the actual vertical fall L.

46. N **moles of a polyatomic gas** $(f = 6)$ **must be mixed with two moles of a monoatomic gas so that the mixture behaves as a diatomic gas. The value of** N **is :** [4 marks]

(A) 6

(B) 2

(C) 4

(D) 3

Correct Answer: (C) 4

Solution:

Concept:

- Equipartition gives molar heat capacity $C_V = fR/2$ for an ideal gas with f active degrees of freedom. Internal energies of nonreacting ideal gases at the same temperature add.
- A monoatomic gas has $f = 3$; an ordinary diatomic gas with active translation and rotation has $f = 5$. The effective degrees of freedom of a mixture are weighted by mole numbers.

Step 1: Express the effective degrees of freedom The polyatomic contribution from N moles is $N(6R/2)$ to the heat capacity at constant volume. Two moles of monoatomic gas contribute $2(3R/2)$. The total number of moles is $N + 2$, so the mixture has molar heat capacity

$$C_{V,\text{mix}} = \frac{R}{2}\frac{6N+6}{N+2}.$$

Equal molecular weighting would be incorrect unless the two mole numbers happened to be equal.

Step 2: Match the diatomic heat capacity For the mixture to behave as a diatomic ideal gas, set the effective degree count equal to five:

$$\frac{6N+6}{N+2} = 5.$$

Thus $6N + 6 = 5N + 10$, giving $N = 4$. As a check, four moles with degree count six and two with degree count three give total degree-weighted moles $24 + 6 = 30$, divided by six moles, or five. The alternatives $N = 6, 2, 3$ give effective values $5.25, 4.5, 4.8$, respectively; none equals the required diatomic value.

Step 3: Conclusion The required amount of polyatomic gas is $N = 4$ moles. Hence, the correct option is (C).

Quick Tip to Solve: Average degrees of freedom with mole-number weights: the gas with $f = 6$ must offset the monoatomic deficit relative to $f = 5$.

47. In Young's double slit experiment, light from two identical sources are superimposing on a screen. The path difference between the two lights reaching at a point on the screen is $7\lambda/4$. The ratio of intensity of fringe at this point with respect to the maximum intensity of the fringe is : [4 marks]

(A) $\frac{1}{2}$
(B) $\frac{3}{4}$
(C) $\frac{1}{3}$
(D) $\frac{1}{4}$

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept:

- For two coherent waves of equal individual intensity I_0, the resultant intensity is $I = 2I_0(1+\cos\delta) = 4I_0\cos^2(\delta/2)$.
- The phase difference is $\delta = 2\pi\Delta/\lambda$, where Δ is the path difference. Maximum intensity is $I_{\max} = 4I_0$, obtained when the two amplitudes add in phase.

Step 1: Convert path difference into phase difference Here the two identical sources provide equal intensities and the specified path difference is $\Delta = 7\lambda/4$. Therefore

$$\delta = \frac{2\pi}{\lambda}\frac{7\lambda}{4} = \frac{7\pi}{2}.$$

Phase is periodic modulo 2π, so this is equivalent to $3\pi/2$. Its cosine is zero; the cross term in the intensity therefore vanishes at this point.

Step 2: Divide by the maximum intensity The local intensity is $I = 2I_0(1+0) = 2I_0$, while the maximum is $4I_0$. Hence

$$\frac{I}{I_{\max}} = \frac{2I_0}{4I_0} = \frac{1}{2}.$$

The same result follows from $\cos^2(7\pi/4) = 1/2$. The denominator requested is the maximum combined intensity, not the intensity of one slit. The proposed ratios $3/4$, $1/3$ and $1/4$ would require different phase differences and do not equal the calculated cosine-squared factor, rejecting B, C and D.

Step 3: Conclusion The intensity at the specified point is half the maximum fringe intensity: $I/I_{\max} = 1/2$. Hence, the correct option is (A).

Quick Tip to Solve: For identical slit intensities, directly evaluate $I/I_{\max} = \cos^2(\pi\Delta/\lambda)$; do not confuse phase with path difference.

48. Given below are two statements:

Statement I : Most of the mass of the atom and all its positive charge are concentrated in a tiny nucleus and the electrons revolve around it, is Rutherford's model.

Statement II : An atom is a spherical cloud of positive charges with electrons embedded in it, is a special case of Rutherford's model.

In the light of the above statements, choose the most appropriate from the options given below

[4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true but Statement II is false

(C) Statement I is false but Statement II is true

(D) Both statement I and statement II are false

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- Rutherford's nuclear model places the positive charge and almost all atomic mass in a nucleus much smaller than the atom, with electrons outside it.
- Thomson's model distributes positive charge throughout an extended sphere and embeds electrons within that sphere. It is a distinct atomic model, rather than a special case of a pointlike nuclear model.

Step 1: Assess Statement I The first statement describes the essential nuclear structure inferred from alpha-particle scattering. Most alpha particles pass nearly straight through the atom, indicating that it is mostly empty space. Rare large deflections require a highly concentrated positive charge and mass. The electrons occupy the region outside the tiny nucleus and revolve around it in Rutherford's description. Statement I is therefore true. The later stability limitations of this classical model do not make this description of the model false.

Step 2: Assess Statement II and select the combination An extended positive sphere with embedded electrons is the Thomson or plum-pudding model. In that picture there is no separate tiny nucleus around which electrons revolve. Spreading positive charge across the atomic volume removes the defining nuclear feature of Rutherford's model, so it cannot be called its special case. Statement II is false. Consequently A is rejected because it accepts Statement II; C reverses both truth values; D incorrectly rejects the valid description in Statement I. Only B states the required true-false combination.

Step 3: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: Concentrated nuclear positive charge identifies Rutherford; a diffuse positive sphere with embedded electrons identifies Thomson.

49. Two sources of light emit with a power of $200\ \mathrm{W}$. The ratio of number of photons of visible light emitted by each source having wavelengths $300\ \mathrm{nm}$ and $500\ \mathrm{nm}$ respectively, will be : [4 marks]

(A) $5 : 3$

(B) $3 : 5$

(C) $1 : 5$

(D) $1 : 3$

Correct Answer: (B) $3 : 5$

Solution:

Concept:

- The energy of one photon of wavelength λ is $E = hc/\lambda$, where h is Planck's constant and c is the speed of light.
- For monochromatic light of power P, the photon emission rate is $\dot{N} = P/E = P\lambda/(hc)$. Equal powers therefore imply photon counts proportional to wavelength over the same time interval.

Step 1: Relate emitted energy to photon number Each source emits $200\,\mathrm{W}$, so in an equal time t each releases the same energy Pt. A photon with the shorter wavelength carries more energy, hence fewer such photons are needed to carry that fixed emitted energy. Explicitly,

$$N_1 = \frac{Pt\lambda_1}{hc}, \qquad N_2 = \frac{Pt\lambda_2}{hc}.$$

The common power, time, Planck constant and speed of light cancel in the requested ratio.

Step 2: Substitute the two wavelengths in their stated order Using $\lambda_1 = 300\,\mathrm{nm}$ and $\lambda_2 = 500\,\mathrm{nm}$,

$$\frac{N_1}{N_2} = \frac{\lambda_1}{\lambda_2} = \frac{300}{500} = \frac{3}{5}.$$

Thus the ratio in the order stated is $3 : 5$, not $5 : 3$; the latter is the photon-energy ratio and reverses the count relation. Options C and D have numerical ratios $1/5$ and $1/3$, neither of which equals the wavelength ratio. The calculation uses the specified wavelengths directly and requires no spectral-band assumption.

Step 3: Conclusion The photon numbers emitted in equal times are in the ratio $3 : 5$. Hence, the correct option is (B).

Quick Tip to Solve: At equal optical power, longer wavelength means lower energy per photon and therefore more photons: $\dot{N} \propto \lambda$.

50. Two particles X and Y having equal charges are being accelerated through the same potential difference. Thereafter they enter normally in a region of uniform magnetic field and describes circular paths of radii R_1 and R_2 respectively. The mass ratio of X and Y is : [4 marks]

(A) $\left(\frac{R_1}{R_2}\right)$

(B) $\left(\frac{R_2}{R_1}\right)$

(C) $\left(\frac{R_2}{R_1}\right)^2$

(D) $\left(\frac{R_1}{R_2}\right)^2$

Correct Answer: (D) $\left(\frac{R_1}{R_2}\right)^2$

Solution:

Concept:

- Acceleration through a potential difference V gives a particle of charge magnitude q kinetic energy qV: $\frac{1}{2}mv^2 = qV$.
- For perpendicular entry into a uniform magnetic field, the magnetic force provides centripetal force, so $R = mv/(qB)$. The field changes the direction of velocity but does no work.

Step 1: Relate momentum to the accelerating voltage Both particles gain the same kinetic energy because their charges and accelerating potential differences are equal. Their speeds are not equal when their masses differ. Eliminating speed from the energy relation gives

$$p = mv = \sqrt{2mqV}.$$

Thus equal kinetic energies imply momentum proportional to the square root of mass, rather than momentum proportional to mass.

Step 2: Eliminate the speed from the radius Use $R = p/(qB)$, with the same field and charge for both particles:

$$R^2 = \frac{2mqV}{q^2B^2} = \frac{2mV}{qB^2}, \qquad m = \frac{qB^2R^2}{2V}.$$

Dividing the expressions for the two particles cancels every common factor:

$$\frac{m_X}{m_Y} = \frac{R_1^2}{R_2^2}.$$

Options (A) and (B) omit the square required by equal-energy acceleration; option (C) reverses the requested mass ratio.

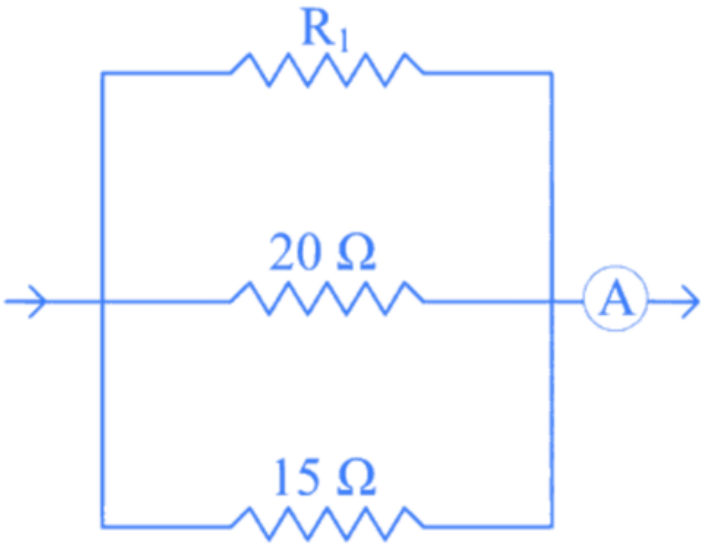
Step 3: Conclusion The required ratio is $m_X/m_Y = (R_1/R_2)^2$. Hence, the correct option is (D).

Quick Tip to Solve: For equal charges accelerated through equal voltages in the same magnetic field, remember $R \propto \sqrt{m}$, not $R \propto m$.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. In the given circuit, the current flowing through the resistance 20Ω is 0.3 A, while the ammeter reads 0.9 A. The value of R_1 is _____ Ω. [4 marks]



Solution:

Concept:

- All three resistors join the same two nodes, so their potential differences are equal. For an ohmic resistor, $V = IR$.
- Kirchhoff's junction law states that the total current equals the sum of currents in the parallel branches. The ammeter is after the branches rejoin.

Step 1: Find the common branch voltage The current through the $20\,\Omega$ resistor is $0.3\,\text{A}$. Therefore the voltage between the two common nodes is

$$V = (0.3)(20) = 6\,\text{V}.$$

The same 6 V acts across the $15\,\Omega$ resistor and across R_1; their branch currents must be evaluated with this voltage, rather than with the ammeter current.

Step 2: Use the total current to isolate the unknown branch The lower branch carries

$$I_{15} = \frac{6}{15} = 0.4\,\text{A}.$$

Since the ammeter reads the combined current, the remaining current in the upper branch is

$$I_1 = 0.9 - 0.3 - 0.4 = 0.2\,\text{A}.$$

Ohm's law now gives

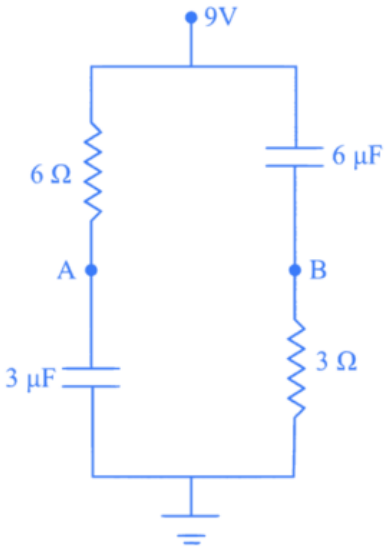
$$R_1 = \frac{6}{0.2} = 30\,\Omega.$$

As a check, the equivalent conductance is $1/30 + 1/20 + 1/15 = 0.15\,\Omega^{-1}$, giving total current $6(0.15) = 0.9\,\text{A}$, exactly the observed ammeter reading.

Step 3: Conclusion The unknown resistance is $R_1 = 30\,\Omega$, so the numerical entry is 30.

Quick Tip to Solve: First use the branch with both known resistance and current to find the voltage; subtract the known branch currents from the ammeter reading.

52. In the given figure, the charge stored in $6\mu F$ capacitor, when points A and B are joined by a connecting wire is _____ μC. [4 marks]



Solution:

Concept:

- After the connecting wire is attached, A and B become a single equipotential node. In the final steady state an ideal capacitor draws no direct current.
- A charged capacitor stores charge magnitude $Q = C\Delta V$. Its voltage must be found from the final node potentials, not by treating it as a wire.

Step 1: Identify the resistive voltage divider With A joined to B, the $6\,\Omega$ resistor connects the 9 V node to the common middle node, while the $3\,\Omega$ resistor connects that node to ground. The capacitor branches carry no current once charging is complete. The resistors therefore form a series conducting path across 9 V:

$$I = \frac{9}{6+3} = 1\,\text{A}.$$

Consequently the common node is at

$$V_A = V_B = I(3) = 3\,\text{V}$$

relative to ground.

Step 2: Evaluate the voltage and charge of the requested capacitor The upper plate of the $6\,\mu\text{F}$ capacitor is at 9 V and its lower plate is at the common 3 V node. Its voltage magnitude is therefore

$$\Delta V_6 = 9 - 3 = 6\,\text{V}.$$

Hence

$$Q_6 = (6\,\mu\text{F})(6\,\text{V}) = 36\,\mu\text{C}.$$

The lower $3\,\mu\text{F}$ capacitor instead has 3 V across it and stores $9\,\mu\text{C}$. The capacitors are not an isolated series pair after A and B are connected, so their charges need not be equal.

Step 3: Conclusion The charge stored in the $6\,\mu\text{F}$ capacitor is $36\,\mu\text{C}$; the required numerical entry is 36.

Quick Tip to Solve: In the final DC state, open the capacitor branches mentally, solve the resistor divider, then restore each capacitor and use its own node-voltage difference.

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53. In a single slit diffraction pattern, a light of wavelength 6000$\overset{o}{A}$ is used. The distance between the first and third minima in the diffraction pattern is found to be 3 mm when the screen in placed 50 cm away from slits. The width of the slit is _____ $\times 10^{-4}$ m. [4 marks]

Solution:

Concept:

- For a slit of width a, the minima satisfy $a \sin\theta_n = n\lambda$, where $n = 1, 2, 3, \ldots$.
- At small diffraction angles on a distant screen, $y_n \simeq D\theta_n = n\lambda D/a$, measured from the central maximum.

Step 1: Express the separation of the two minima The first and third minima on the same side have positions $y_1 = \lambda D/a$ and $y_3 = 3\lambda D/a$. Their separation therefore spans two intervals between adjacent minima:

$$\Delta y = y_3 - y_1 = \frac{2\lambda D}{a}.$$

It is not the distance from the centre to the third minimum, and it is not the central-maximum width measured between minima on opposite sides.

Step 2: Convert units and calculate the slit width The wavelength is 6000 angstrom, so $\lambda = 6000 \times 10^{-10} = 6 \times 10^{-7}\,\text{m}$. Also $D = 0.50\,\text{m}$ and $\Delta y = 3 \times 10^{-3}\,\text{m}$. Rearranging gives

$$a = \frac{2\lambda D}{\Delta y} = \frac{2(6 \times 10^{-7})(0.50)}{3 \times 10^{-3}} = 2 \times 10^{-4}\,\text{m}.$$

For a consistency check, the adjacent-minimum spacing is $\lambda D/a = 1.5\,\text{mm}$; two such intervals indeed give the stated 3 mm separation.

Step 3: Conclusion The slit width is $2 \times 10^{-4}\,\text{m}$. The coefficient required in the blank is 2.

Quick Tip to Solve: For minima numbered m and n on the same side, use the difference $n-m$ in the separation formula before substituting any numbers.

---

54. A body of mass $5\ \text{kg}$ moving with a uniform speed $3\sqrt{2}\ \text{ms}^{-1}$ in $X-Y$ plane along the line $y = x+4$. The angular momentum of the particle about the origin will be _____ $\text{kg m}^2\ \text{s}^{-1}$. [4 marks]

Solution:

Concept:

- For a particle moving on a straight line, its angular momentum magnitude about an origin is $L = mvd$, where d is the perpendicular distance from the origin to the line of motion.
- The perpendicular distance from $(0,0)$ to $ax + by + c = 0$ is $|c|/\sqrt{a^2+b^2}$. The magnitude stays constant in uniform straight-line motion.

Step 1: Find the perpendicular lever arm The trajectory $y = x + 4$ can be written as $x - y + 4 = 0$. Its shortest distance from the origin is

$$d = \frac{|4|}{\sqrt{1^2 + (-1)^2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\,\text{m}.$$

This is the distance to the line, not its y-intercept of 4 m. The intercept is generally not perpendicular to the velocity.

Step 2: Multiply the momentum by the lever arm The particle has mass $m = 5\,\text{kg}$ and speed $v = 3\sqrt{2}\,\text{m s}^{-1}$. Thus

$$L = mvd = 5(3\sqrt{2})(2\sqrt{2}) = 60\,\text{kg m}^2\,\text{s}^{-1}.$$

Alternatively, at $(0,4)$, motion along increasing x gives $\mathbf{v} = 3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$, so $\mathbf{L} = \mathbf{r} \times m\mathbf{v} = -60\hat{\mathbf{k}}$. Reversing travel reverses this direction but leaves the requested numerical magnitude unchanged. No specific travel direction along the line is specified.

Step 3: Conclusion The angular momentum magnitude is $60\,\text{kg m}^2\,\text{s}^{-1}$, so the numerical entry is 60.

Quick Tip to Solve: Use the perpendicular distance to the entire trajectory for mvd; a coordinate intercept works only when it is perpendicular to the motion.

---

55. Hydrogen atom is bombarded with electrons accelerated through a potential difference of V, which causes excitation of hydrogen atoms. If the experiment is being performed at $\mathrm{T} = 0\ \mathrm{K}$, the minimum potential difference needed to observe any Balmer series lines in the emission spectra will be $\frac{\alpha}{10}\ \mathrm{V}$, where $\alpha =$ _____. [4 marks]

Solution:

Concept:

- Hydrogen energy levels are $E_n = -13.6/n^2\,\mathrm{eV}$. At $T = 0\,\mathrm{K}$, the atoms initially occupy the ground state $n = 1$.
- Balmer emission consists of transitions from levels $n \geq 3$ to $n = 2$. An electron accelerated through V volts gains $V\,\mathrm{eV}$ of kinetic energy.

Step 1: Determine the lowest excited level that can emit a Balmer line Excitation to $n = 2$ alone cannot produce a Balmer line: a subsequent $2 \to 1$ transition belongs to the Lyman series. The least energetic accessible Balmer transition is $3 \to 2$, so the incident electron must be able to excite a ground-state atom at least to $n = 3$. It is the excitation energy $1 \to 3$, rather than the emitted photon energy $3 \to 2$, that sets the required accelerating voltage.

Step 2: Calculate the threshold voltage and the requested coefficient The necessary energy transfer is

$$\Delta E = E_3 - E_1 = 13.6\left(1 - \frac{1}{9}\right) = \frac{108.8}{9}\,\mathrm{eV} \simeq 12.09\,\mathrm{eV}.$$

Thus the minimum accelerating potential is approximately $12.1\ \mathrm{V}$. In the form given in the question,

$$\frac{\alpha}{10} \simeq 12.1, \qquad \alpha \simeq 121.$$

The rounding is to the integer coefficient requested; using 10.2 V would only reach $n = 2$, while 13.6 V is the larger ionisation threshold and is not necessary for Balmer emission.

Step 3: Conclusion The minimum potential is approximately $12.1\ \mathrm{V}$, giving the required dimensionless coefficient $\alpha = 121$.

Quick Tip to Solve: For the first line of a series ending at level n, first excite a ground-state atom to $n + 1$; do not use the emitted photon energy as the incident threshold.

56. A simple harmonic oscillator has an amplitude A and time period 6π second. Assuming the oscillation starts from its mean position, the time required by it to travel from $x = \mathrm{A}$ to $x = \frac{\sqrt{3}}{2}\ \mathrm{A}$ will be $\frac{\pi}{x}\ \mathrm{s}$, where $x =$ _____. [4 marks]

Solution:

Concept:

- An oscillator starting from its mean position towards the positive extreme may be written as $s = A\sin(\omega t)$, with $\omega = 2\pi/T$.
- The same position occurs twice during a cycle. A specified journey from the positive extreme must use the subsequent returning phase, rather than the earlier outward crossing.

Step 1: Find the angular frequency and the phase interval The period is $T = 6\pi\,\mathrm{s}$, hence

$$\omega = \frac{2\pi}{6\pi} = \frac{1}{3}\,\mathrm{s}^{-1}.$$

The oscillator first reaches $s = A$ at phase $\omega t_1 = \pi/2$. On its return towards the mean position, $s = \sqrt{3}A/2$ corresponds to phase $\omega t_2 = 2\pi/3$, because $\sin(2\pi/3) = \sqrt{3}/2$. The phase $\pi/3$ describes the earlier outward crossing and cannot be the arrival phase for this journey.

Step 2: Convert the phase difference into elapsed time The elapsed phase is

$$\Delta\phi = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}.$$

Therefore

$$\Delta t = \frac{\Delta\phi}{\omega} = \frac{\pi/6}{1/3} = \frac{\pi}{2}\,\mathrm{s}.$$

Equivalently, resetting time to zero at the positive extreme gives $s = A\cos(\omega\tau)$, so $\cos(\omega\tau) = \sqrt{3}/2$ immediately gives $\omega\tau = \pi/6$. This checks the interval without including the time already spent reaching the extreme.

Step 3: Conclusion The required journey takes $\pi/2\,\mathrm{s}$. Comparing with $\pi/x\,\mathrm{s}$, the dimensionless numerical answer is $x = 2$.

Quick Tip to Solve: Reset the clock at the extreme and use a cosine for the return journey; this avoids subtracting the wrong sine phases.

57. Two metallic wires P and Q have same volume and are made up of same material. If their area of cross sections are in the ratio $4:1$ and force F_1 is applied to P, an extension of Δl is produced. The force which is required to produce same extension in Q is F_2.

The value of $\frac{F_1}{F_2}$ is _____. [4 marks]

Solution:

Concept:

- Young's modulus is $Y = FL/(A\Delta L)$, where L is the original length, A the cross-sectional area and ΔL the extension, within the elastic limit.
- The volume of a uniform wire is $V = AL$. Equal volumes therefore make the original lengths inversely proportional to the cross-sectional areas.

Step 1: Use the equal-volume condition The two wires are made of the same material, so their Young's moduli are equal. Their original lengths need not be equal: the thinner wire must be longer to contain the same volume. Writing the stated area ratio explicitly gives

$$\frac{A_P}{A_Q} = 4, \qquad A_P L_P = A_Q L_Q.$$

Consequently,

$$\frac{L_Q}{L_P} = \frac{A_P}{A_Q} = 4.$$

This length ratio is essential; comparing the areas alone would miss another factor of four.

Step 2: Compare forces for the same extension From the definition of Young's modulus, the force producing extension ΔL is

$$F = \frac{YA\Delta L}{L}.$$

Both Y and ΔL cancel in the ratio, but the original lengths remain:

$$\frac{F_1}{F_2} = \frac{A_P}{A_Q}\frac{L_Q}{L_P} = 4 \times 4 = 16.$$

Equivalently, substituting $L = V/A$ gives $F = YA^2\Delta L/V$; at fixed material, volume and extension the required force is proportional to the square of the area. This confirms that the thicker, shorter wire requires the larger force.

Step 3: Conclusion The required value is $\boxed{F_1/F_2 = 16}$. This is a dimensionless force ratio.

Quick Tip to Solve: For equal-volume wires, substitute $L = V/A$ before comparing extensions: their stiffness is proportional to A^2, not merely A.

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58. A particle is moving in a circle of radius $50\ \text{cm}$ in such a way that at any instant the normal and tangential components of it's acceleration are equal. If its speed at $\text{t} = 0$ is $4\ \text{m/s}$, the time taken to complete the first revolution will be $\frac{1}{\alpha}\left[1 - e^{-2\pi}\right]$ s, where $\alpha =$ _____. [4 marks]

Solution:

Concept:

- Normal acceleration in a circle of fixed radius r is $a_n = v^2/r$, while tangential acceleration along the motion is $a_t = dv/dt$.
- With arc length s and angle θ, $ds = r\,d\theta$ and $d\theta/dt = v/r$. The first revolution corresponds to $\theta = 2\pi$.

Step 1: Relate speed to angle The equal normal and tangential accelerations give

$$\frac{dv}{dt} = \frac{v^2}{r}.$$

Use $d\theta/dt = v/r$ to replace time by angle:

$$\frac{dv}{d\theta}\frac{v}{r} = \frac{v^2}{r} \quad \Longrightarrow \quad \frac{dv}{v} = d\theta.$$

Integrating from the initial position, where $v = v_0 = 4\,\text{m}\,\text{s}^{-1}$, yields

$$\ln(v/v_0) = \theta, \qquad v = v_0 e^{\theta}.$$

Thus the speed increases continuously and cannot be treated as constant over the revolution.

Step 2: Integrate the time for one revolution Convert the radius to SI units: $r = 50\,\text{cm} = 0.50\,\text{m}$. Since $dt = r\,d\theta/v$,

$$T = \frac{r}{v_0}\int_0^{2\pi} e^{-\theta}\,d\theta = \frac{r}{v_0}(1 - e^{-2\pi}).$$

Substitute the initial speed and radius:

$$T = \frac{0.50}{4}(1 - e^{-2\pi})\,\text{s} = \frac{1}{8}(1 - e^{-2\pi})\,\text{s}.$$

The decreasing exponential appears in the time integral even though the speed itself increases exponentially with angle. Comparing this result with the supplied form identifies the requested coefficient directly.

Step 3: Conclusion Comparing $T = \frac{1}{8}(1 - e^{-2\pi})\,\text{s}$ with the stated expression gives $\boxed{\alpha = 8}$.

Quick Tip to Solve: When $a_t = a_n$, use angle as the variable: $dv/v = d\theta$. Integrating $r\,d\theta/v$ then gives the revolution time.

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59. A charge of $4.0\mu\text{C}$ is moving with a velocity of $4.0 \times 10^6\ \text{ms}^{-1}$ along the positive y axis under a magnetic field $\vec{B}$ of strength $(2\hat{k})\text{T}$. The force acting on the charge is $x\hat{i}N$. The value of x is _____.

[4 marks]

Solution:

Concept:

- The magnetic force on charge q moving with velocity $\vec{v}$ is $\vec{F} = q(\vec{v} \times \vec{B})$. Its magnitude is $qvB\sin\theta$.
- The Cartesian unit vectors obey $\hat{j} \times \hat{k} = \hat{i}$. Reversing the order of the cross product reverses its sign.

Step 1: Write the vectors in SI units The charge is positive and must be converted from microcoulombs to coulombs:

$$q = 4.0 \times 10^{-6}\,\text{C}.$$

The velocity and magnetic field are

$$\vec{v} = (4.0 \times 10^6)\hat{j}\,\text{m}\,\text{s}^{-1}, \qquad \vec{B} = 2\hat{k}\,\text{T}.$$

The velocity lies along the positive y direction and the field along the positive z direction, so their angle is 90°. The sine factor is therefore unity. The small charge and large speed have opposite powers of ten that cancel in their product.

Step 2: Evaluate the magnitude and direction together Insert the vectors in the Lorentz-force expression in the stated order:

$$\begin{aligned}\vec{F} &= (4.0 \times 10^{-6})(4.0 \times 10^6)(2)(\hat{j} \times \hat{k})\,\text{N}\\ &= 32\hat{i}\,\text{N}.\end{aligned}$$

The right-hand rule gives positive x, since the charge is positive; no additional sign reversal is required. A result along y or z would be inconsistent with magnetic force being perpendicular to both the velocity and field. Comparing with $x\hat{\imath}\,\mathrm{N}$ now fixes the numerical coefficient.

Step 3: Conclusion The magnetic force is $\boxed{\vec{F} = 32\hat{\imath}\,\mathrm{N}}$, so the requested numerical value is $\boxed{x = 32}$.

Quick Tip to Solve: Find the direction with $\vec{v} \times \vec{B}$ before multiplying magnitudes; apply a sign reversal only for a negative charge.

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60. A horizontal straight wire $5\ \mathrm{m}$ long extending from east to west falling freely at right angle to horizontal component of earths magnetic field $0.60 \times 10^{-4}\ \mathrm{Wbm^{-2}}$. The instantaneous value of emf induced in the wire when its velocity is $10\ \mathrm{ms^{-1}}$ is _____ $\times 10^{-3}$ V. [4 marks]

Solution:

Concept:

- A straight conducting rod moving perpendicular to a uniform magnetic field develops motional emf $\varepsilon = Blv$ when the rod is also perpendicular to both the field and velocity.
- The emf is the line integral of $\vec{v} \times \vec{B}$ along the rod. The magnetic force separates charges until the electric force balances it.

Step 1: Identify the effective field and geometry The rod extends horizontally from east to west and falls vertically. The horizontal component of the Earth's magnetic field is perpendicular to the falling velocity, as stated, and its north-south direction is perpendicular to the rod. Thus $\vec{v} \times \vec{B}$ is along the wire and the full length contributes to the induced emf. Use the given horizontal field component, rather than any unspecified total terrestrial field:

$$B = 0.60 \times 10^{-4}\,\mathrm{Wb\,m^{-2}} = 6.0 \times 10^{-5}\,\mathrm{T}.$$

The other quantities are $l = 5\,\mathrm{m}$ and the instantaneous speed $v = 10\,\mathrm{m\,s^{-1}}$.

Step 2: Calculate the instantaneous emf Substitute these values in the motional-emf formula:

$$\varepsilon = Blv = (6.0 \times 10^{-5})(5)(10)\,\mathrm{V}.$$

Multiplying the numerical factors gives

$$\varepsilon = 3.0 \times 10^{-3}\,\mathrm{V} = 3.0\,\mathrm{mV}.$$

Although the rod accelerates during free fall, the question specifies its speed at the instant of interest. The instantaneous emf depends on that speed alone; neither the elapsed fall time nor the acceleration is needed. The blank asks for the coefficient of $10^{-3}\,\mathrm{V}$, rather than the value expressed in volts.

Step 3: Conclusion The induced emf has magnitude $\boxed{3.0 \times 10^{-3}\,\mathrm{V}}$. Hence the required entry in the blank is $\boxed{3}$.

Quick Tip to Solve: For a falling horizontal rod, check that $\vec{v} \times \vec{B}$ lies along its length; then use the given instantaneous speed in Blv.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. On passing a gas, 'X', through Nessler's regent, a brown precipitate is obtained. The gas 'X' is [4 marks]

(A) Cl_2

(B) CO_2

(C) NH_3

(D) H_2S

Correct Answer: (C) NH_3

Solution:

Concept:

- Nessler's reagent is an alkaline solution of potassium tetraiodomercurate(II), $K_2[HgI_4]$. It is a sensitive qualitative reagent for ammonia.
- Ammonia reacts with this reagent to give a yellow-brown coloration at low concentration and a brown precipitate at higher concentration.

Step 1: Identify the characteristic positive test The decisive observation is a brown precipitate on passing the gas through Nessler's reagent. This is the characteristic positive test for ammonia, NH_3. The precipitate is commonly represented as basic mercury(II) amidoiodide, $HgO \cdot Hg(NH_2)I$. A balanced representation of its formation is

$$NH_3 + 2\,K_2[HgI_4] + 3\,KOH \longrightarrow HgO \cdot Hg(NH_2)I\downarrow + 7\,KI + 2\,H_2O.$$

The strongly alkaline medium is part of the reagent, so no additional acidification is required. This observation identifies a gas by a specific chemical reaction, rather than by colour or odour alone.

Step 2: Eliminate the other gases Option (A), Cl_2, is an oxidising halogen and does not give the characteristic ammonia precipitate with this reagent. Option (B), CO_2, is an acidic oxide; its familiar positive test is limewater turning milky, which is a different reagent and observation. Option (D), H_2S, is associated with the formation of metal sulphides, often dark precipitates, rather than the diagnostic brown ammonia product. Only option (C) supplies ammonia and matches the stated Nessler test. Therefore the gas cannot be selected merely because it reacts with an alkaline solution; the actual precipitate is the distinguishing evidence.

Step 3: Conclusion The gas X is $\boxed{NH_3}$, ammonia. Hence, the correct option is (C).

Quick Tip to Solve: Associate Nessler's reagent with ammonia: a brown precipitate is the characteristic positive result, unlike the limewater test for carbon dioxide.

62. Match List I with List II

	List - I(Bio Polymer)		List - II(Monomer)
(A)	Starch	(I)	nucleotide
(B)	Cellulose	(II)	α-glucose
(C)	Nucleic acid	(III)	β-glucose
(D)	Protein	(IV)	α-amino acid

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-II, C-I, D-III

(B) A-I, B-III, C-IV, D-II

(C) A-II, B-I, C-III, D-IV

(D) A-II, B-III, C-I, D-IV

Correct Answer: (D) A-II, B-III, C-I, D-IV

Solution:

Concept:

- Polysaccharides are formed from monosaccharide units, but the anomeric form of glucose distinguishes starch from cellulose.
- Nucleic acids are polymers of nucleotides, whereas proteins are polymers of α-amino acids linked by peptide bonds.

Step 1: Match the two glucose polymers Starch contains amylose and amylopectin, both built from α-D-glucose units. Hence entry A in List I matches entry II in List II. Cellulose is a linear polysaccharide composed of β-D-glucose units connected through $\beta(1 \rightarrow 4)$ glycosidic linkages, so B matches III. Thus the first two matches are

$$\text{A–II}, \qquad \text{B–III}.$$

The difference between α- and β-glucose is not cosmetic: it changes the geometry of the glycosidic bonds and therefore the structures and properties of the resulting polymers.

Step 2: Complete the remaining matches and compare options A nucleotide contains a nitrogenous base, a pentose sugar and a phosphate group. Nucleotides join through phosphodiester bonds to form nucleic acids, giving C–I. Proteins are formed from α-amino acids; the amino and carboxyl groups form peptide linkages, giving D–IV. Collecting all four results,

$$\text{A–II}, \quad \text{B–III}, \quad \text{C–I}, \quad \text{D–IV}.$$

Option (A) incorrectly assigns an amino acid to starch and glucose to protein. Option (B) incorrectly makes starch a nucleotide polymer and nucleic acid an amino-acid polymer. Option (C) interchanges the cellulose and nucleic-acid monomers. Option (D) alone preserves all four chemical identities.

Step 3: Conclusion The correct matching is $\boxed{\text{A–II, B–III, C–I, D–IV}}$. Hence, the correct option is (D).

Quick Tip to Solve: First distinguish starch as α-glucose and cellulose as β-glucose; then match nucleotide to nucleic acid and amino acid to protein.

63. Which of the following acts as a strong reducing agent? (Atomic number: $\mathrm{Ce} = 58, \mathrm{Eu} = 63, \mathrm{Gd} = 64, \mathrm{Lu} = 71$**)** [4 marks]

(A) Eu^{2+}

(B) Gd^{3+}

(C) Lu^{3+}

(D) Ce^{4+}

Correct Answer: (A) Eu^{2+}

Solution:

Concept:

- A reducing agent donates electrons and is itself oxidised. Among lanthanoids the +3 oxidation state is generally the most stable in aqueous solution.
- An ion in the +2 oxidation state can reduce another species if it readily loses an electron to form the corresponding +3 ion; a +4 ion can instead be an oxidising agent.

Step 1: Examine the europium ion Europium has atomic number 63. Its neutral configuration is $[\mathrm{Xe}]4f^7 6s^2$, so the divalent ion is Eu^{2+} with a half-filled $4f^7$ subshell. Despite this relative stabilisation of the divalent state, Eu^{2+} is a strong reducing agent in aqueous chemistry: it donates an electron and changes to the common trivalent state,

$$\mathrm{Eu}^{2+} \longrightarrow \mathrm{Eu}^{3+} + \mathrm{e}^{-}.$$

The electron released in this oxidation can be accepted by another species, causing its reduction. The half-filled configuration explains why the +2 state can exist, but it does not mean this ion is incapable of oxidation.

Step 2: Compare the remaining oxidation states Gd^{3+} has the stable half-filled $4f^7$ configuration, and Lu^{3+} has the filled $4f^{14}$ configuration. Removing another electron from either of these stable trivalent ions is unfavourable, so neither is the strong reducing agent in the list. Ce^{4+} has a xenon-core configuration and readily accepts an electron to form Ce^{3+}; it therefore behaves as a strong oxidising agent, the opposite role. Options (B) and (C) are thus stable trivalent ions, while option (D) is an electron acceptor. Option (A), the divalent europium ion, is the electron donor required by the question.

Step 3: Conclusion $\boxed{\mathrm{Eu}^{2+}}$ acts as the strong reducing agent. Hence, the correct option is (A).

Quick Tip to Solve: Classify the redox role by electron transfer: Eu^{2+} loses an electron to +3, whereas Ce^{4+} gains one to +3.

64. Given below are two statements:

Statement I : Fluorine has most negative electron gain enthalpy in its group.

Statement II : Oxygen has least negative electron gain enthalpy in its group.

In the light of the above statements, choose the most appropriate from the options given below

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is false but Statement II is true

(D) Statement I is true but Statement II is false

Correct Answer: (C) Statement I is false but Statement II is true

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change for adding an electron to an isolated gaseous atom: $\mathrm{X(g) + e^- \longrightarrow X^-(g)}$. A more negative value means more energy is released.
- The compact second-period atoms have unusually strong electron-electron repulsion in their small $2p$ orbitals. Consequently the electron gain enthalpies of fluorine and oxygen are less negative than those of chlorine and sulfur respectively.

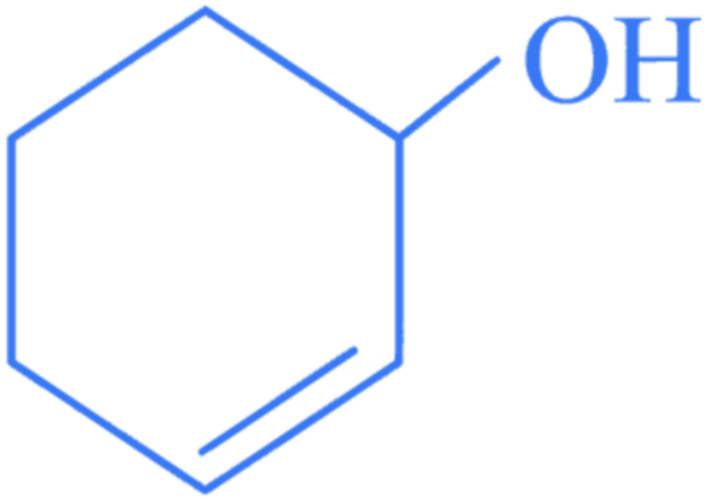
Step 1: Test Statement I Fluorine is the most electronegative element, but electronegativity is not the same quantity as electron gain enthalpy. Its incoming electron enters a compact $2p$ orbital and experiences substantial repulsion. Chlorine accepts the electron into the larger $3p$ shell with less crowding. Thus chlorine, not fluorine, has the most negative electron gain enthalpy among the halogens. Statement I is false.

Step 2: Test Statement II and compare the choices Oxygen also has a small, crowded $2p$ valence shell. In the usual Group 16 trend, oxygen has the least negative electron gain enthalpy, while sulfur has a considerably more negative value. This makes Statement II true. Option (A) incorrectly accepts Statement I; option (B) incorrectly rejects Statement II; option (D) reverses both judgements. Only option (C) combines a false first statement with a true second statement.

Step 3: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (C).

Quick Tip to Solve: Do not infer electron gain enthalpy directly from electronegativity: remember the F/Cl and O/S exceptions caused by compact 2p orbitals.

65. According to IUPAC system, the compound is named as [4 marks]



(A) Cyclohex-2-en-1-ol

(B) 1-Hydroxyhex-2-ene

(C) Cyclohex-1-en-3-ol

(D) Cyclohex-1-en-2-ol

Correct Answer: (A) Cyclohex-2-en-1-ol

Solution:

Concept:

- The principal functional group determines the suffix and receives the lowest possible locant. For an alcohol, the $-OH$ group is represented by the suffix -ol.
- A six-carbon ring uses the parent cyclohex-. After fixing the alcohol-bearing carbon as carbon 1, choose the numbering direction giving the double bond its smaller locant.

Step 1: Identify the parent and principal group The displayed skeleton is a six-membered carbon ring containing one carbon-carbon double bond and one hydroxyl group. It is therefore an unsaturated cyclic alcohol, not an open-chain hydroxyalkene. The alcohol is the principal characteristic group and must be named by the suffix -ol. Number the ring beginning at the carbon directly attached to oxygen, so the hydroxyl locant is 1.

Step 2: Locate the double bond and assemble the name From carbon 1, numbering toward the adjacent double bond places that bond between carbons 2 and 3. Numbering in the other direction would place it between carbons 5 and 6, so the smaller locant 2 is used. Combining the cyclic six-carbon parent, the double-bond locant and the alcohol suffix gives cyclohex-2-en-1-ol. Option (B) omits the cyclic parent and treats the principal alcohol as a prefix. Options (C) and (D) give the double bond priority over the hydroxyl group, incorrectly assigning the alcohol locant 3 or 2. Neither follows functional-group numbering priority.

Step 3: Conclusion The IUPAC name is cyclohex-2-en-1-ol. Hence, the correct option is (A).

Quick Tip to Solve: Fix the carbon bearing the principal -OH group as carbon 1 before minimizing the double-bond locant around the ring.

66. Match List I with List II.

	List - I Spectral Series for Hydrogen		List - II Spectral Region/Higher Energy State
(A)	Lyman	(I)	Infrared region
(B)	Balmer	(II)	UV region
(C)	Paschen	(III)	Infrared region
(D)	Pfund	(IV)	Visible region

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-IV, C-III, D-I

(B) A-I, B-III, C-II, D-IV

(C) A-II, B-III, C-I, D-IV

(D) A-I, B-II, C-III, D-IV

Correct Answer: (A) A-II, B-IV, C-III, D-I

Solution:

Concept:

- For a hydrogen spectral series, all transitions terminate at a fixed lower level n_1, while the upper level $n_2 > n_1$ varies. The wavenumber is $1/\lambda = R_H(1/n_1^2 - 1/n_2^2)$.
- Lyman, Balmer, Paschen and Pfund series terminate at $n_1 = 1, 2, 3, 5$, respectively. Their characteristic spectral regions are ultraviolet, visible, infrared and infrared.

Step 1: Match the first two series Lyman transitions end at the ground state, $n_1 = 1$, and have the greatest energy gaps among these series. They therefore lie in the ultraviolet region, giving A-II. Balmer transitions end at $n_1 = 2$ and give the familiar visible hydrogen lines, giving B-IV. This already eliminates options (B) and (D), which put Lyman in the infrared, and option (C), which puts Balmer in the infrared.

Step 2: Match the infrared series and select the complete combination Paschen transitions terminate at $n_1 = 3$, while Pfund transitions terminate at $n_1 = 5$. Their smaller energy differences correspond to longer wavelengths in the infrared. The printed List II contains two identical infrared entries, I and III; the offered matching A-II, B-IV, C-III, D-I correctly assigns both infrared series without changing their physical regions. In contrast, the other choices place either Paschen in ultraviolet or Pfund in visible, or incorrectly assign Lyman and Balmer. Thus the duplication of the infrared description does not change which offered option is correct.

Step 3: Conclusion The matching is A-II, B-IV, C-III, D-I. Hence, the correct option is (A).

Quick Tip to Solve: Remember the terminal levels 1, 2, 3, 4, 5 as Lyman, Balmer, Paschen, Brackett, Pfund; only Balmer is the usual visible series.

67. The element having the highest first ionization enthalpy is [4 marks]

(A) C

(B) Al

(C) Si

(D) N

Correct Answer: (D) N

Solution:

Concept:

- First ionization enthalpy is the energy required for $\mathrm{X(g)} \longrightarrow \mathrm{X^+(g) + e^-}$. Stronger effective nuclear attraction and smaller atomic radius generally increase this energy.
- Across a period ionization enthalpy generally increases; down a group it generally decreases. A half-filled p^3 subshell has additional stability and resists electron removal.

Step 1: Compare the valence configurations The relevant valence configurations are carbon $2s^2 2p^2$, nitrogen $2s^2 2p^3$, aluminium $3s^2 3p^1$ and silicon $3s^2 3p^2$. Nitrogen has the stable half-filled $2p$ subshell. Removing one electron destroys that half-filled arrangement and requires more energy than removing a valence electron from carbon. Nitrogen is also to the right of carbon in the same period and has the stronger effective nuclear attraction.

Step 2: Check the third-period alternatives Aluminium and silicon lose electrons from the third shell, which is farther from the nucleus and more screened than the second shell of carbon and nitrogen. Thus their first ionization enthalpies are lower than nitrogen's. Silicon exceeds aluminium across Period 3, but neither exceeds the compact half-filled nitrogen atom. The resulting order among the offered elements is Al < Si < C < N. Therefore carbon in option (A), aluminium in option (B), and silicon in option (C) are all eliminated by their lower first ionization enthalpies.

Step 3: Conclusion Nitrogen has the highest first ionization enthalpy among the given elements. Hence, the correct option is (D).

Quick Tip to Solve: Compare shell number first, then effective nuclear attraction, and check whether electron removal breaks a stable half-filled subshell.

68. Which of the following statements are correct about Zn, Cd and Hg ?

A. They exhibit high enthalpy of atomization as the d-subshell is full.

B. Zn and Cd do not show variable oxidation state while Hg shows $+I$ and $+II$.

C. Compounds of Zn, Cd and Hg are paramagnetic in nature.

D. Zn, Cd and Hg are called soft metals.

Choose the most appropriate from the options given below: [4 marks]

(A) C, D only

(B) B, C only

(C) A, D only

(D) B, D only

Correct Answer: (D) B, D only

Solution:

Concept:

- Zinc, cadmium and mercury have filled d^{10} subshells. Their usual ions also retain filled d shells, so unpaired d electrons do not provide the strong metallic bonding typical of many central transition metals.
- A species with all electrons paired is diamagnetic. Mercury can show oxidation states +1 and +2, whereas zinc and cadmium characteristically show +2.

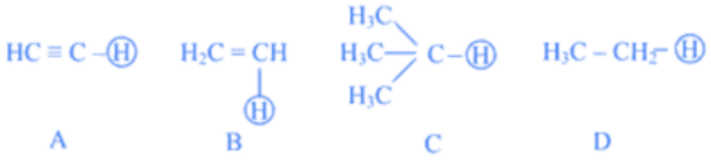
Step 1: Evaluate statements A and B Statement A is false: a full d subshell does not give these metals unusually high atomization enthalpies. Their metallic bonding is relatively weak, and they have relatively low atomization enthalpies compared with the more strongly bonded transition metals. Statement B is true in the standard treatment: zinc and cadmium show +2, while mercury also forms mercury(I), commonly present as the dimeric ion Hg_2^{2+}, as well as mercury(II), Hg^{2+}. Each Hg in the dimer has oxidation state +1.

Step 2: Evaluate statements C and D and compare combinations Statement C is false: the characteristic compounds of these metals have filled electron arrangements and no unpaired d electrons, so they are diamagnetic rather than paramagnetic. Statement D is true: these Group 12 metals are described as soft metals, consistent with their relatively weak metallic bonding; mercury is liquid at room temperature. Thus only B and D are correct. Option (A) wrongly includes C, option (B) also includes C, and option (C) wrongly includes A. The combination B, D only appears in option (D).

Step 3: Conclusion Statements B and D are correct, while A and C are false. Hence, the correct option is (D).

Quick Tip to Solve: For Group 12, connect filled d shells with weak metallic bonding and diamagnetism; remember Hg(I) as the dimeric ion Hg2(2+).

69. The ascending acidity order of the following H atoms is [4 marks]



(A) A < B < D < C

(B) A < B < C < D

(C) D < C < B < A

(D) C < D < B < A

Correct Answer: (D) C < D < B < A

Solution:

Concept:

- Acidity increases when the conjugate base formed by loss of H^+ is more stable. For a carbanion, greater s character of the carbon orbital stabilizes its negative charge.
- The s-character order is $sp\ (50\%) > sp^2\ (33\%) > sp^3\ (25\%)$. Electron-releasing alkyl groups destabilize an already negatively charged carbon through their positive inductive effect.

Step 1: Identify the four acidic sites In A the marked hydrogen is bonded to an sp carbon of ethyne. In B it is bonded to an sp2 carbon of ethene. In C it is the hydrogen on the tertiary carbon bonded to three methyl groups. In D it is a hydrogen on a primary carbon of ethane. Deprotonating these sites gives an acetylide anion, a vinyl carbanion, a tertiary alkyl carbanion and a primary alkyl carbanion, respectively.

Step 2: Rank conjugate-base stability and invert to acidity The sp carbon in the acetylide holds a lone pair closer to the nucleus and stabilizes negative charge best, making A the strongest acid. The sp2 vinyl carbon gives B greater acidity than either sp3 site. Between C and D, the three electron-releasing methyl groups attached to the negatively charged carbon in the conjugate base of C destabilize that anion more than in the primary conjugate base of D. Hence C is the weaker acid of those two. The ascending acidity order is therefore C < D < B < A. Options (A) and (B) incorrectly place terminal alkyne A first, while option (C) incorrectly makes D less acidic than the more substituted C.

Step 3: Conclusion The ascending acidity order is C < D < B < A. Hence, the correct option is (D).

Quick Tip to Solve: For C-H acidity, stabilize the resulting carbanion: more s character helps, whereas additional electron-donating alkyl groups hurt.

70. The correct IUPAC name of K_2MnO_4 is [4 marks]

(A) Potassium tetraoxidomanganate (VI)

(B) Dipotassium tetraoxidomanganate (VII)

(C) Potassium tetraoxopermanganate (VI)

(D) Potassium tetraoxidomanganese (VI)

Correct Answer: (A) Potassium tetraoxidomanganate (VI)

Solution:

Concept:

- In K_2MnO_4, potassium has oxidation number +1 and each oxygen has oxidation number -2. The sum of oxidation numbers in the neutral formula is zero.
- The anion is $MnO_4{}^{2-}$. In additive nomenclature oxygen ligands are named oxido, and an anionic manganese coordination entity uses the ending manganate followed by its oxidation number.

Step 1: Calculate the oxidation state of manganese Let the oxidation number of manganese be x. Charge balance for the given formula gives

$$2(+1) + x + 4(-2) = 0.$$

Thus

$$x = +6.$$

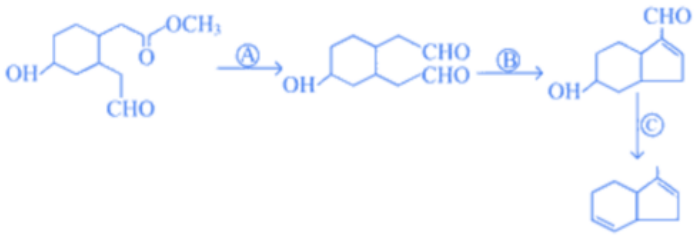
The anion is manganate(VI), not permanganate(VII). The latter would correspond to $MnO_4{}^-$, as in $KMnO_4$, and is a different chemical species.

Step 2: Name the cation and anion and rule out the other choices The four oxygen ligands give the prefix tetraoxido. Combining this with the anionic central-atom name and oxidation state produces tetraoxidomanganate(VI). The potassium counterion is named first, so the full name is potassium tetraoxidomanganate(VI). In this ionic name the cation is called potassium rather than requiring the numerical prefix dipotassium. Option (B) also incorrectly assigns +7 to manganese. Option (C) uses permanganate despite the calculated +6 oxidation state and uses the unsuitable ligand wording. Option (D) uses manganese instead of the required manganate ending for the anion. Only option (A) matches both composition and charge balance.

Step 3: Conclusion The IUPAC name is potassium tetraoxidomanganate(VI). Hence, the correct option is (A).

Quick Tip to Solve: Find the metal oxidation state from charge balance before naming an oxoanion: K2MnO4 contains Mn(VI), whereas KMnO4 contains Mn(VII).

71. Identify the reagents used for the following conversion [4 marks]



(A) $\mathrm{A} = \mathrm{LiAlH_4},\ \mathrm{B} = \mathrm{NaOH_{(aq)}}, \mathrm{C} = \mathrm{NH_2 - NH_2/KOH}$, ethylene glycol

(B) $\mathrm{A} = \mathrm{DIBAL - H}, \mathrm{B} = \mathrm{NaOH_{(aq)}}, \mathrm{C} = \mathrm{NH_2 - NH_2/KOH}$, ethylene glycol

(C) $\mathrm{A} = \mathrm{LiAlH_4},\ \mathrm{B} = \mathrm{NaOH_{(alc)}}, \mathrm{C} = \mathrm{Zn/HCl}$

(D) $\mathrm{A} = \mathrm{DIBAL - H}, \mathrm{B} = \mathrm{NaOH_{(alc)}}, \mathrm{C} = \mathrm{Zn/HCl}$

Correct Answer: (D) $\mathrm{A} = \mathrm{DIBAL - H}, \mathrm{B} = \mathrm{NaOH_{(alc)}}, \mathrm{C} = \mathrm{Zn/HCl}$

Solution:

Concept:

- DIBAL-H is a controlled reducing agent used to convert an ester group into an aldehyde, unlike the complete reduction normally achieved with lithium aluminium hydride.
- Base catalyses intramolecular aldol condensation; an acidic reducing treatment can reduce a carbonyl group and also allow dehydration of a ring alcohol.

Step 1: Identify the first reduction The first arrow converts the methyl ester side chain into an aldehyde side chain. The required reagent is DIBAL-H, which permits controlled partial ester reduction followed by work-up. Lithium aluminium hydride normally takes the ester to a primary alcohol rather than stopping at the aldehyde. This eliminates the two choices containing that stronger reducing agent and leaves the two DIBAL-H sequences to compare.

Step 2: Identify the ring-forming treatment The resulting aldehyde side chains can undergo intramolecular aldol condensation in base. Enolate formation followed by attack on the other aldehyde closes the five-membered ring, and loss of water gives the conjugated aldehyde shown. Alcoholic sodium hydroxide supplies the required base for this condensation. The choice must also explain the changes at the final arrow, rather than merely matching this ring closure.

Step 3: Identify the final reduction The final product has lost the aldehyde oxygen and also the hydroxyl group on the six-membered ring, with a new double bond appearing there. Zinc and hydrochloric acid provide the acidic reducing conditions associated with Clemmensen reduction. The acidic medium also permits protonation and dehydration of the ring alcohol. In contrast, hydrazine and potassium hydroxide in ethylene glycol give basic Wolff–Kishner conditions: they reduce carbonyl groups but do not provide the acid-catalysed dehydration required to remove the unactivated ring hydroxyl group. This final structural change distinguishes the two DIBAL-H choices.

Step 4: Conclusion $A = \mathrm{DIBAL\text{-}H}$, $B = \mathrm{NaOH\,(alc)}$, and $C = \mathrm{Zn/HCl}$. This sequence accounts for the ester reduction, intramolecular aldol condensation and final acidic reduction with dehydration. Hence, the correct option is (D).

Quick Tip to Solve: Inspect every change in the final product: loss of the ring OH group with a new double bond distinguishes acidic reduction from Wolff–Kishner conditions.

72. Chromatographic technique/s based on the principle of differential adsorption is / are

A. Column chromatography

B. Thin layer chromatography

C. Paper chromatography

Choose the most appropriate answer from the options given below: [4 marks]

(A) B only

(B) A only

(C) A & B only

(D) C only

Correct Answer: (C) A & B only

Solution:

Concept:

- Adsorption chromatography separates components through their different strengths of attachment to a solid stationary phase.
- Partition chromatography separates solutes through their different distributions between two phases; ordinary paper chromatography is classified this way.

Step 1: Classify column and thin layer methods In column chromatography the stationary phase is commonly silica gel or alumina packed in a tube. As solvent passes through, solutes repeatedly adsorb onto and desorb from this solid. A solute held more strongly moves more slowly. Thin layer chromatography uses the same principle, with a thin coating of silica gel or alumina on a support instead of a packed column. Therefore both A and B in the question are adsorption methods.

Step 2: Distinguish paper chromatography In the usual classification of paper chromatography, water retained by cellulose acts as the stationary liquid phase and the developing solvent is the mobile phase. Separation is chiefly based on differential partition of solutes between these phases. It is therefore not included in the requested pair of differential-adsorption techniques. The fact that paper is a solid support does not by itself make the method adsorption chromatography.

Step 3: Select the complete combination The valid techniques are column chromatography and thin layer chromatography together. Option (A), B only, omits the column method. Option (B), A only, omits the thin layer method. Option (D), C only, includes the partition method and excludes both required adsorption methods. Option (C) alone contains both and only the correct techniques.

Step 4: Conclusion A and B only are based on differential adsorption. Hence, the correct option is (C).

Quick Tip to Solve: For the usual textbook classification, silica or alumina means adsorption; water held on chromatography paper means partition.

73. Which of the following reaction is correct? [4 marks]

(A) $C_2H_5CONH_2 + Br_2 + NaOH \longrightarrow C_2H_5CH_2NH_2 + Na_2CO_3 + NaBr + H_2O$

(B) Cyclohexene $\xrightarrow[\text{UV light}]{Br_2, \Delta}$ 1,2-dibromocyclohexane.

(C) 1-Methylcyclohex-1-ene $\xrightarrow{HI}$ 1-iodo-1-methylcyclohexane.

(D) $CH_3CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2, 0\,^\circ C} CH_3CH_2OH + N_2 + HCl$

Correct Answer: (C) 1-Methylcyclohex-1-ene $\xrightarrow{HI}$ 1-iodo-1-methylcyclohexane.

Solution:

Concept:

- Addition of HI to an unsymmetrical alkene follows Markovnikov orientation through the more stable carbocation.
- Hofmann bromamide degradation removes the amide carbonyl carbon, while nitrous acid replaces an aliphatic amino group without the one-carbon shortening drawn in the option.

Step 1: Verify the HI addition In 1-methylcyclohex-1-ene, proton addition to the less substituted alkene carbon creates a tertiary carbocation at the carbon bearing the methyl group. Iodide attacks that centre. The product is therefore 1-iodo-1-methylcyclohexane, with iodine and methyl on the same ring carbon, exactly as option (C) shows. The ring framework is preserved.

Step 2: Reject the amide and amine conversions In option (A), $C_2H_5CONH_2$ with bromine and alkali undergoes Hofmann rearrangement to $C_2H_5NH_2$, not $C_2H_5CH_2NH_2$. The carbonyl carbon is lost. In option (D), propan-1-amine treated with nitrous acid gives an unstable aliphatic diazonium intermediate and alcohol products with the propyl carbon skeleton, not the ethanol drawn. It does not furnish the stated one-carbon-shorter alcohol.

Step 3: Check the bromination conditions Option (B) pairs the vicinal dibromide addition product with heating and UV light. Those radical conditions favour allylic substitution of cyclohexene, retaining the double bond, rather than the electrophilic addition represented. Addition of bromine across the double bond is normally considered under non-radical conditions. Thus this option does not match the specified conditions, leaving the Markovnikov HI addition as the correct reaction.

Step 4: Conclusion The addition of HI to 1-methylcyclohex-1-ene gives 1-iodo-1-methylcyclohexane. Hence, the correct option is (C).

Quick Tip to Solve: Check carbon count in Hofmann and nitrous-acid reactions, then check whether bromination conditions are ionic or radical.

74. Anomalous behavior of oxygen is due to its [4 marks]

(A) large size and low electronegativity

(B) small size and high electronegativity

(C) small size and low electronegativity

(D) large size and high electronegativity

Correct Answer: (B) small size and high electronegativity

Solution:

Concept:

- Oxygen is the first member of group 16 and has a much smaller atomic size than the heavier members.
- Its high electronegativity, high ionisation enthalpy and lack of accessible valence d orbitals help distinguish its bonding and physical behaviour.

Step 1: Identify the size effect Oxygen has only two occupied electron shells. Sulfur and the elements below it have additional shells, so their atoms are larger. The compact oxygen atom holds its bonding electron density close to the nucleus. This makes its properties differ appreciably from the heavier chalcogens. Thus both alternatives that describe oxygen as having a large size are eliminated.

Step 2: Identify the electronegativity effect Oxygen attracts shared electron pairs very strongly. Its electronegativity is higher than that of sulfur, selenium and tellurium, not lower. This affects bond polarity and is associated with strong hydrogen bonding in suitable oxygen compounds, such as water. The small size and high electronegativity together explain why extrapolating the heavier group members directly to oxygen often fails.

Step 3: Compare all combinations Option (A) has both descriptors wrong: oxygen is neither large nor weakly electronegative. Option (C) gets the small size right but assigns low electronegativity. Option (D) recognises high electronegativity but incorrectly assigns a large size. Only option (B) combines the two correct properties. Other first-member features reinforce the anomaly, but the question asks which of the listed pairs applies.

Step 4: Conclusion Oxygen shows anomalous behaviour because of its small size and high electronegativity. Hence, the correct option is (B).

Quick Tip to Solve: For a first-member anomaly, first test the pair small atomic size and high electronegativity against the listed choices.

75. Which one of the following will show geometrical isomerism? [4 marks]

(A) 3-Bromo-1-(bromomethylidene)cyclohexane: an exocyclic $=CHBr$ group at ring carbon 1 and Br at ring carbon 3.

(B) 3-Bromo-1-methylidenecyclohexane: an exocyclic $=CH_2$ group at ring carbon 1 and Br at ring carbon 3.

(C) 4-Bromo-1-methylidenecyclohexane: an exocyclic $=CH_2$ group at ring carbon 1 and Br at ring carbon 4.

(D) 4-Bromo-1-(bromomethylidene)cyclohexane: an exocyclic $=CHBr$ group at ring carbon 1 and Br at ring carbon 4.

Correct Answer: (A) 3-Bromo-1-(bromomethylidene)cyclohexane: an exocyclic $=CHBr$ group at ring carbon 1 and Br at ring carbon 3.

Solution:

Concept:

- Geometrical isomerism about a carbon–carbon double bond requires two different substituents on each double-bonded carbon.
- For a double bond attached to a ring, compare the complete paths around the ring; apparently similar neighbouring carbon atoms may lead to different substituents.

Step 1: Test the external alkene carbon Options (B) and (C) contain an exocyclic $=CH_2$ group. The external double-bonded carbon has two identical hydrogen substituents. Interchanging those hydrogens does not create a new arrangement, so these two compounds cannot show geometrical isomerism at that double bond. Options (A) and (D), with $=CHBr$, pass this first test because hydrogen and bromine are different.

Step 2: Compare the ring paths in option A In option (A), the ring bears bromine at carbon 3 relative to the carbon bearing the exocyclic double bond. Starting at the ring alkene carbon, one direction encounters the bromine-bearing carbon sooner than the other direction. The two attached ring paths are therefore constitutionally different. Consequently the ring alkene carbon also has two different substituent paths, and distinct E/Z arrangements are possible.

Step 3: Check the symmetric alternative In option (D), bromine is at the opposite ring carbon, carbon 4. The two paths from the double-bonded ring carbon to that substituted carbon are equivalent by ring symmetry. Although the external carbon bears H and Br, the ring carbon does not have two different paths. Hence option (D) fails the second requirement. Only option (A) passes both tests; restricted rotation alone is insufficient without different substituents at both ends.

Step 4: Conclusion The structure in option (A), with an exocyclic $=CHBr$ group and unsymmetrical ring paths, shows geometrical isomerism. Hence, the correct option is (A).

Quick Tip to Solve: For an exocyclic alkene, test the outer carbon first, then walk both ways around the ring to test path equivalence.

76. A reagent which gives brilliant red precipitate with Nickel ions in basic medium is [4 marks]

(A) dimethyl glyoxime

(B) sodium nitroprusside

(C) meta-dinitrobenzene

(D) neutral $FeCl_3$

Correct Answer: (A) dimethyl glyoxime

Solution:

Concept:

- Dimethylglyoxime, abbreviated DMG, is a bidentate ligand used as a sensitive qualitative reagent for nickel(II).
- In basic medium, deprotonation permits formation of a neutral, sparingly soluble chelate with a characteristic brilliant red colour.

Step 1: Identify the characteristic precipitate Nickel(II) ions react with dimethylglyoxime in alkaline or ammoniacal solution to form red nickel dimethylglyoximate. If the neutral ligand is denoted H_2dmg, its singly deprotonated form is $Hdmg^-$. Two such ligands coordinate to one nickel(II) centre, giving the neutral chelate $Ni(Hdmg)_2$. The brilliant red solid is the diagnostic observation.

Step 2: Relate the basic medium to chelation The base removes an acidic oxime proton during complex formation and favours precipitation. The two chelating ligands provide four nitrogen donor atoms around nickel in the familiar square-planar complex. A schematic ionic representation is

$$Ni^{2+} + 2\,H_2dmg + 2\,OH^- \longrightarrow Ni(Hdmg)_2(s) + 2\,H_2O.$$

The low solubility and distinctive colour make this reaction useful for identification and gravimetric determination.

Step 3: Eliminate the other reagents Sodium nitroprusside is associated with other qualitative colour tests, including the violet colour test for sulfide. Meta-dinitrobenzene is not the standard reagent for the brilliant red nickel chelate. Neutral $FeCl_3$ is widely used to test phenolic compounds through coloured complexes; it does not produce the specified nickel dimethylglyoximate precipitate. Thus none of these substitutes matches the complete observation.

Step 4: Conclusion The reagent is dimethylglyoxime. Hence, the correct option is (A).

Quick Tip to Solve: Associate nickel(II) in alkaline medium with the brilliant red DMG chelate; distinguish it from the ferric chloride test for phenols.

77. Phenol treated with chloroform in presence of sodium hydroxide, which further hydrolyzed in presence of an acid results [4 marks]

(A) Salicylic acid

(B) Benzene-1,2-diol

(C) 2-Hydroxybenzaldehyde

(D) Benzene-1,3-diol

Correct Answer: (C) 2-Hydroxybenzaldehyde

Solution:

Concept:

- Phenol with chloroform and aqueous sodium hydroxide undergoes the Reimer–Tiemann reaction, introducing a formyl group mainly ortho to the hydroxyl group.
- Base generates dichlorocarbene from chloroform; acidic work-up hydrolyses the dichloromethyl intermediate to an aldehyde.

Step 1: Generate the reactive electrophile In strong base, chloroform loses a proton and then chloride to produce dichlorocarbene, $:CCl_2$. Phenol is converted into phenoxide, whose electron donation activates the ring, especially its ortho and para positions. This reagent combination therefore signals ring formylation rather than oxidation of phenol or introduction of a second hydroxyl group.

Step 2: Follow substitution and hydrolysis The activated ring reacts chiefly at an ortho position, giving an intermediate containing a $-CHCl_2$ group next to the oxygen substituent. Hydrolysis followed by acidification converts this group into $-CHO$ and restores the phenolic hydroxyl group. The principal product is ortho-hydroxybenzaldehyde, also called salicylaldehyde or 2-hydroxybenzaldehyde. A smaller para product can occur, but it is not the named major product among these options.

Step 3: Distinguish the competing products Salicylic acid contains $-COOH$, not the aldehyde group introduced here; it is associated with carboxylation of phenoxide in the Kolbe–Schmitt process. Benzene-1,2-diol and benzene-1,3-diol contain a second hydroxyl group, whereas this reaction introduces a carbon-containing formyl substituent. These alternatives therefore represent different transformations. The option containing 2-hydroxybenzaldehyde uniquely matches Reimer–Tiemann formylation and the stated acidic hydrolysis.

Step 4: Conclusion The principal product is 2-hydroxybenzaldehyde (salicylaldehyde). Hence, the correct option is (C).

Quick Tip to Solve: Chloroform plus aqueous alkali on phenol means ortho formylation; carbon dioxide plus alkali instead points toward salicylic acid.

78. Match List I with List II

	List I (Compound)		List II (pK_a value)
(A)	Ethanol	(I)	10.0
(B)	Phenol	(II)	15.9
(C)	m-Nitrophenol	(III)	7.1
(D)	p-Nitrophenol	(IV)	8.3

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-II, C-III, D-IV

(B) A-IV, B-I, C-II, D-III

(C) A-II, B-I, C-IV, D-III

(D) A-III, B-IV, C-I, D-II

Correct Answer: (C) A-II, B-I, C-IV, D-III

Solution:

Concept:

- Acid strength increases as $pK_a = -\log_{10} K_a$ decreases. Stabilisation of the conjugate base favours loss of a proton.
- Phenoxide is resonance-stabilised; ethoxide is not. A nitro group withdraws electrons by induction and, at the para position, also by resonance.

Step 1: Match ethanol and phenol Ethanol forms ethoxide, with its negative charge mainly localised on oxygen; its ethyl group also donates electron density. Its relatively weak acidity corresponds to $pK_a = 15.9$, so A-II. Phenol forms phenoxide, which delocalises negative charge over the aromatic ring. It is appreciably more acidic than ethanol, giving $pK_a = 10.0$, so B-I.

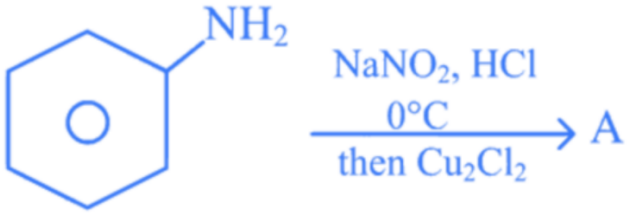
Step 2: Compare the nitrophenols The nitro group stabilises the phenoxide conjugate base through its electron-withdrawing inductive effect in both isomers. In p-nitrophenoxide it can additionally accept electron density through resonance because the para carbon participates in the phenoxide resonance system. Thus p-nitrophenol is stronger than m-nitrophenol: D-III corresponds to 7.1 and C-IV to 8.3. The sequence of increasing acidity is ethanol, phenol, m-nitrophenol, p-nitrophenol.

Step 3: Check all option mappings The complete match is A-II, B-I, C-IV, D-III. Option (A) incorrectly makes ethanol more acidic than phenol and reverses the nitrophenols. Option (B) assigns ethanol the value 8.3 and m-nitrophenol 15.9. Option (D) makes ethanol the strongest acid. Only option (C) has all four assignments correct.

Step 4: Conclusion $\boxed{\text{A-II, B-I, C-IV, D-III}}$. Hence, the correct option is (C).

Quick Tip to Solve: First assign ethanol the largest pK_a; then use the extra para resonance effect to distinguish the two nitrophenols.

79. The product A formed in the following reaction is [4 marks]



(A) 2-Chloroaniline ($2\text{-ClC}_6\text{H}_4\text{NH}_2$)

(B) Anilinium chloride ($\text{C}_6\text{H}_5\text{NH}_3^+\ \text{Cl}^-$)

(C) Chlorobenzene ($\text{C}_6\text{H}_5\text{Cl}$)

(D) 1,2-Dichlorobenzene ($1,2-\text{C}_6\text{H}_4\text{Cl}_2$)

Correct Answer: (C) Chlorobenzene ($\text{C}_6\text{H}_5\text{Cl}$)

Solution:

Concept:

- A primary aromatic amine undergoes diazotisation with nitrous acid generated from NaNO_2 and HCl at low temperature.
- The aryl diazonium group is a leaving group that can be replaced by chloride under copper-mediated conditions, with release of N_2.

Step 1: Form the diazonium intermediate The starting aromatic compound is aniline, $\text{C}_6\text{H}_5\text{NH}_2$. At 0°C, sodium nitrite and hydrochloric acid generate nitrous acid in situ. The primary amino group is converted into a diazonium group, not retained as an amino substituent. The net diazotisation is

$$\text{C}_6\text{H}_5\text{NH}_2 + \text{NaNO}_2 + 2\,\text{HCl} \longrightarrow \text{C}_6\text{H}_5\text{N}_2^+\ \text{Cl}^- + \text{NaCl} + 2\,\text{H}_2\text{O}.$$

Keeping the mixture cold permits the diazonium intermediate to form before its subsequent replacement.

Step 2: Identify the final substitution product The subsequent copper-chloride treatment replaces the diazonium group at the same ring carbon by chlorine. Nitrogen leaves as the particularly stable gas N_2. Consequently the final ring has one chlorine substituent and no amino group: chlorobenzene, $\text{C}_6\text{H}_5\text{Cl}$. The benzene skeleton is retained throughout; this is substitution at the position formerly occupied by the amino group rather than chlorination of an additional ring position.

Step 3: Eliminate the remaining structures Option (A), 2-chloroaniline, incorrectly retains NH_2 and adds chlorine next to it. Option (B), anilinium chloride, is only the acid-base salt of aniline and does not represent the diazotisation followed by replacement. Option (D), 1,2-dichlorobenzene, introduces an extra chlorine not accounted for by the single diazonium replacement. Option (C) alone is chlorobenzene.

Step 4: Conclusion The product A is chlorobenzene, $\boxed{\text{C}_6\text{H}_5\text{Cl}}$. Hence, the correct option is (C).

Quick Tip to Solve: Follow the ring carbon originally bearing NH_2: diazotisation followed by chloride replacement puts Cl at that same carbon.

80. Alkyl halide is converted into alkyl isocyanide by reaction with [4 marks]

(A) KCN

(B) NH_4CN

(C) NaCN

(D) AgCN

Correct Answer: (D) AgCN

Solution:

Concept:

- The cyanide group is ambident: bonding through carbon gives a nitrile R–CN, while bonding through nitrogen gives an isocyanide R–NC.
- Ionic alkali-metal cyanides provide free cyanide ions; predominantly covalent silver cyanide favours attachment of the alkyl group through nitrogen.

Step 1: Distinguish the two connectivity patterns An alkyl isocyanide has the connectivity R–NC; the nitrogen end of the cyanide unit must therefore become bonded to the alkyl group. This must not be confused with the isomeric alkyl cyanide, R–CN, where carbon is bonded directly to R. Although the molecular formulae may be identical, the atom through which substitution occurs decides the functional group and its chemistry.

Step 2: Select the suitable cyanide reagent In AgCN, the metal-cyanide interaction is predominantly covalent and the carbon end is involved in bonding to silver. Reaction with an alkyl halide therefore favours attack through nitrogen and formation of the isocyanide. The usual transformation is written

$$R{-}X + AgCN \longrightarrow R{-}NC + AgX.$$

Formation of silver halide accompanies substitution; the product is named an alkyl isocyanide rather than an alkyl nitrile.

Step 3: Reject the ionic cyanide choices KCN and NaCN are ionic cyanides. Their free cyanide ions preferentially react through carbon to form nitriles R–CN, so options (A) and (C) do not give the requested product as the characteristic reaction. NH_4CN likewise does not supply the covalently bound silver-cyanide arrangement responsible for the standard isocyanide preparation, so (B) is unsuitable. Among the listed reagents, silver cyanide is the required choice.

Step 4: Conclusion Alkyl halide is converted to alkyl isocyanide using $\boxed{AgCN}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the pair together: ionic KCN gives R–CN, whereas covalent AgCN gives R–NC.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The total number of molecules with zero dipole moment among $CH_4, BF_3, H_2O, HF, NH_3, CO_2$ and SO_2 is _____. [4 marks]

Solution:

Concept:

- The molecular dipole moment is the vector sum of all bond dipoles. Polar bonds do not necessarily make the whole molecule polar.
- Identical bonds arranged symmetrically can cancel, whereas bent or pyramidal geometries generally leave a nonzero resultant.

Step 1: Identify the symmetric molecules CH_4 has tetrahedral geometry with four equivalent C-H bonds. Their vector sum vanishes by symmetry. BF_3 is trigonal planar; its three equal B-F bond dipoles lie at 120° and cancel. CO_2 is linear, with two identical C=O bond dipoles pointing in opposite directions. Their magnitudes are equal, so they also cancel. Thus these three molecules have zero permanent molecular dipole moment.

Step 2: Check every remaining molecule H_2O is bent because oxygen has two lone pairs, so its O-H bond dipoles have a nonzero resultant. HF is a heteronuclear diatomic molecule; its single polar bond cannot be cancelled by another bond. NH_3 is trigonal pyramidal, so the three N-H bond dipoles do not cancel. SO_2 is bent, rather than linear, and its S-O bond dipoles also leave a nonzero resultant. None of these four belongs in the zero-dipole count.

Step 3: Count the qualifying entries The complete zero-dipole set is $\{CH_4, BF_3, CO_2\}$. Counting the molecules, rather than counting the number of polar or nonpolar bonds, gives three entries. The requested answer is a number of molecules and is therefore dimensionless; no electrical unit is attached to this count.

Step 4: Conclusion The total number of molecules with zero dipole moment is $\boxed{3}$.

Quick Tip to Solve: Check geometry before polarity: tetrahedral CH_4, planar BF_3 and linear CO_2 cancel their bond dipoles by symmetry.

82. Standard enthalpy of vapourisation for CCl_4 is $30.5\ \text{kJ mol}^{-1}$. Heat required for vapourisation of $284\ \text{g}$ of CCl_4 at constant temperature is _____ kJ.
(Given molar mass in gmol^{-1}; $C = 12, Cl = 35.5$) [4 marks]

Solution:

Concept:

- The molar enthalpy of vaporisation is the heat required to convert one mole of liquid into vapour at the stated temperature and pressure.
- For an amount n, the phase-change heat is $q = n\Delta H_{\text{vap}}$, with $n = m/M$. Use grams consistently

when the molar mass is in grams per mole.

Step 1: Calculate the molar mass One molecule of CCl_4 contains one carbon atom and four chlorine atoms. Using the given atomic masses,

$$M(CCl_4) = 12 + 4(35.5) = 154\,\text{g mol}^{-1}.$$

The four chlorine atoms must all be included. Dividing the sample mass by an atomic mass, or using the mass directly with a molar enthalpy, would give an incorrect amount of substance.

Step 2: Convert the sample mass into moles The liquid sample has mass $284\,\text{g}$, so

$$n = \frac{284\,\text{g}}{154\,\text{g mol}^{-1}} = \frac{142}{77}\,\text{mol} \approx 1.84416\,\text{mol}.$$

The sample contains more than one mole but less than two; its heat of vaporisation must therefore be between 30.5 and 61.0 kJ. Retain the fraction until the final multiplication to avoid intermediate rounding error.

Step 3: Apply the molar phase-change enthalpy At constant temperature, the energy supplied produces the phase change, so no temperature-rise term is added. Using the stated molar enthalpy,

$$q = \frac{142}{77} \times 30.5 = 56.24675\ldots\ \text{kJ}.$$

Vaporisation absorbs heat, so the required quantity is positive. The numerical-entry value to the nearest integer is 56; the unrounded calculation is approximately 56.25 kJ.

Step 4: Conclusion The heat required is $56.25\,\text{kJ}$ approximately; the nearest-integer numerical answer is $\boxed{56\,\text{kJ}}$.

Quick Tip to Solve: For a phase change at constant temperature, use mass divided by molar mass, then multiply by the molar latent enthalpy.

83. If $50\ \text{mL}$ of 0.5M oxalic acid is required to neutralise $25\ \text{mL}$ of NaOH solution, the amount of NaOH in $50\ \text{mL}$ of given NaOH solution is _____ g. [4 marks]

Solution:

Concept:

- Oxalic acid is diprotic: each mole of $H_2C_2O_4$ neutralises two moles of NaOH.
- The mole relation follows the balanced equation, and the concentration of the same homogeneous base solution is unchanged when a different aliquot is taken.

Step 1: Write the neutralisation stoichiometry The balanced reaction is

$$H_2C_2O_4 + 2\,\text{NaOH} \longrightarrow Na_2C_2O_4 + 2\,H_2O.$$

Both acidic protons of oxalic acid are neutralised by sodium hydroxide. Thus the acid-to-base mole ratio is 1:2, rather than 1:1. The given acid concentration is molarity, not normality, so this factor of

two must be included explicitly.

Step 2: Find the amount of base in the titrated aliquot The amount of oxalic acid used is

$$n_{\text{acid}} = 0.5\,\text{mol}\,\text{L}^{-1} \times 0.050\,\text{L} = 0.025\,\text{mol}.$$

Consequently the 25 mL aliquot of base contains

$$n_{\text{NaOH}} = 2(0.025) = 0.050\,\text{mol}.$$

Its concentration is therefore $0.050/0.025 = 2.0\,\text{mol}\,\text{L}^{-1}$. The conversion of millilitres to litres is essential in this calculation.

Step 3: Scale to 50 mL and convert to mass A 50 mL aliquot is twice the titrated 25 mL volume, so it contains $0.100\,\text{mol}$ of NaOH. The molar mass of sodium hydroxide is $23 + 16 + 1 = 40\,\text{g}\,\text{mol}^{-1}$. Therefore

$$m = 0.100 \times 40 = 4.0\,\text{g}.$$

This is the mass of solute in the requested 50 mL of the given solution, not the mass of the entire solution.

Step 4: Conclusion The amount of NaOH in 50 mL of the solution is $\boxed{4\,\text{g}}$.

Quick Tip to Solve: Apply both factors of two: oxalic acid supplies two acidic protons, and the requested base volume is twice the titrated aliquot.

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84. The following concentrations were observed at $500\ \text{K}$ for the formation of NH_3 from N_2 and H_2. At equilibrium ; $[N_2] = 2 \times 10^{-2}\text{M}, [H_2] = 3 \times 10^{-2}\text{M}$ and $[NH_3] = 1.5 \times 10^{-2}\text{M}$. Equilibrium constant for the reaction is _____. [4 marks]

Solution:

Concept:

- The equilibrium concentration expression follows the balanced reaction coefficients:

$$K_c = \frac{[NH_3]^2}{[N_2][H_2]^3}.$$

- Each equilibrium concentration is raised to its stoichiometric coefficient; temperature fixes the constant but is not substituted separately into this concentration expression.

Step 1: Balance the formation reaction Ammonia formation from nitrogen and hydrogen is

$$N_2(g) + 3\,H_2(g) \rightleftharpoons 2\,NH_3(g).$$

Two nitrogen atoms and six hydrogen atoms occur on each side. The coefficients 1, 3 and 2 determine the exponents in the equilibrium expression. The supplied values are already equilibrium concentrations, so there is no need to reconstruct initial amounts or calculate an additional reaction

extent.

Step 2: Substitute the equilibrium concentrations For the balanced reaction,

$$K_c = \frac{(1.5 \times 10^{-2})^2}{(2 \times 10^{-2})(3 \times 10^{-2})^3}.$$

First evaluate the numerator: $(1.5 \times 10^{-2})^2 = 2.25 \times 10^{-4}$. Next cube the hydrogen concentration: $(3 \times 10^{-2})^3 = 27 \times 10^{-6} = 2.7 \times 10^{-5}$. Multiplying this by 2×10^{-2} gives the denominator 5.4×10^{-7}.

Step 3: Evaluate the quotient and round once It follows that

$$K_c = \frac{2.25 \times 10^{-4}}{5.4 \times 10^{-7}} = \frac{2.25}{5.4} \times 10^3 = 416.666\ldots.$$

The requested nearest-integer numerical entry is therefore 417. In the usual school concentration convention, the associated units are $\mathrm{L^2\,mol^{-2}}$, because the total concentration exponent is $2 - 1 - 3 = -2$. No RT conversion is required: that would be relevant if the question requested K_p.

Step 4: Conclusion $K_c \approx 416.67\,\mathrm{L^2\,mol^{-2}}$, giving the nearest-integer answer $\boxed{417}$.

Quick Tip to Solve: Balance ammonia formation before inserting values: the hydrogen concentration is cubed and the ammonia concentration is squared.

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85. A constant current was passed through a solution of $\mathrm{AuCl_4^-}$ ion between gold electrodes. After a period of 10.0 minutes, the increase in mass of cathode was 1.314 g. The total charge passed through the solution is _____ $\times 10^{-2}$ F.

(Given atomic mass of $\mathrm{Au} = 197$) [4 marks]

Solution:

Concept:

- Faraday's law relates the deposited amount to charge: $Q/F = zn$, where n is moles of metal and z is electrons consumed per metal atom.
- In $\mathrm{AuCl_4^-}$, chloride has oxidation number -1, so gold has oxidation number $+3$. Cathodic deposition reduces gold to oxidation number zero.

Step 1: Identify the cathode reaction The cathode gains mass because gold ions are reduced to metallic gold. Charge balance gives

$$\mathrm{AuCl_4^- + 3\,e^- \longrightarrow Au + 4\,Cl^-}.$$

Thus one mole of deposited gold requires three moles of electrons, or three faradays. The negative charge of the complex does not mean that gold needs only one electron; the relevant change is the oxidation state of the metal.

Step 2: Convert the mass to charge The mass increase is the mass of gold deposited, and the given

atomic mass corresponds to a molar mass of $197\,\mathrm{g\,mol^{-1}}$. Therefore

$$n(\mathrm{Au}) = \frac{1.314}{197} = 0.006670\,\mathrm{mol}.$$

Applying the electron-to-gold ratio,

$$\frac{Q}{F} = 3\frac{1.314}{197} = 0.02001.$$

Equivalently, the charge is $2.001 \times 10^{-2}\,\mathrm{F}$. The duration would be needed to find current, but the deposited mass already determines total charge, so no extra factor of 600 seconds belongs in this calculation.

Step 3: Conclusion The total charge is approximately $2 \times 10^{-2}\,\mathrm{F}$; hence the numerical entry is $\boxed{2}$.

Quick Tip to Solve: Find the metal oxidation state inside the complex before applying Faraday's law; the complex charge is not the electron requirement.

86. The total number of anti bonding molecular orbitals, formed from $2s$ and $2p$ atomic orbitals in a diatomic molecule is _____. [4 marks]

Solution:

Concept:

- When two compatible atomic orbitals combine, they produce two molecular orbitals: one bonding combination and one antibonding combination.
- Each atom supplies one $2s$ orbital and three $2p$ orbitals. Orbital count is independent of the number of electrons occupying those orbitals.

Step 1: Count the contribution from the 2s orbitals The two atoms each contribute one $2s$ atomic orbital, giving two input orbitals. Their constructive and destructive combinations form σ_{2s} and σ^*_{2s}, respectively. Only the starred orbital is antibonding, so the $2s$ pair contributes exactly one antibonding molecular orbital. Counting both members would count a bonding orbital as well.

Step 2: Count the contribution from the 2p orbitals Choose the internuclear axis as z. The two $2p_z$ orbitals combine by end-on overlap to form σ_{2p_z} and $\sigma^*_{2p_z}$. The two $2p_x$ orbitals form π_{2p_x} and $\pi^*_{2p_x}$; similarly the two $2p_y$ orbitals form π_{2p_y} and $\pi^*_{2p_y}$. Thus the $2p$ set contributes three antibonding orbitals: one sigma-star and two separate pi-star orbitals. Although the pi-star pair is degenerate, it still consists of two orbitals. Adding the contributions gives

$$N_{\text{antibonding}} = 1 + 3 = 4.$$

The eight atomic orbitals supplied overall form eight molecular orbitals, split into four bonding and four antibonding combinations.

Step 3: Conclusion The total number of antibonding molecular orbitals is $\boxed{4}$, a dimensionless count.

Quick Tip to Solve: Count each member of a degenerate pi pair separately; one energy level may contain two molecular orbitals.

87. The half-life of radioisotope bromine - 82 is 36 hours. The fraction which remains after one day is _____ $\times 10^{-2}$.

(Given antilog $0.2006 = 1.587$) [4 marks]

Solution:

Concept:

- Radioactive decay is first order: the remaining fraction is $f = N/N_0 = 2^{-t/t_{1/2}}$, with time and half-life in the same units.
- In logarithmic form, $\log_{10}(N_0/N) = (t/t_{1/2}) \log_{10} 2$. The requested coefficient multiplies 10^{-2}, rather than being the fraction itself.

Step 1: Express the elapsed time in half-lives One day is $24\,\text{h}$, while the half-life is $36\,\text{h}$. The number of elapsed half-lives is therefore

$$\frac{t}{t_{1/2}} = \frac{24}{36} = \frac{2}{3}.$$

Hence

$$f = 2^{-2/3}.$$

Since the elapsed time is less than one half-life, more than half the initial isotope must remain. This is a useful check against mistakenly using the decayed fraction.

Step 2: Use the supplied antilogarithm Taking common logarithms of the reciprocal fraction gives

$$\log_{10}(1/f) = \frac{2}{3}\log_{10} 2 \simeq \frac{2}{3}(0.3010) = 0.2006.$$

The question supplies $\text{antilog}(0.2006) = 1.587$, so

$$f = \frac{1}{1.587} \simeq 0.6301.$$

To express this in the requested form, write

$$0.6301 = 63.01 \times 10^{-2}.$$

The nearest integer coefficient is consequently 63. In particular, 37 would describe approximately the percentage that has decayed, whereas this question asks for the amount remaining.

Step 3: Conclusion The fraction remaining is approximately 63×10^{-2}. The required dimensionless numerical entry is $\boxed{63}$.

Quick Tip to Solve: Check whether the question asks for the fraction remaining or the fraction decayed, then convert to its printed power-of-ten format.

88. The total number of 'Sigma' and 'Pi' bonds in 2-formylhex-4-enoic acid is _____. [4 marks]

Solution:

Concept:

- Every bonded pair of atoms has one sigma bond. A double bond adds one pi bond to that sigma bond, so it contains two bonds in total.
- For 2-formylhex-4-enoic acid, number the six-carbon parent chain from the carboxyl carbon. The formyl substituent at carbon 2 is $-\mathrm{CHO}$.

Step 1: Write the connectivity from the name The condensed structure is

$$\mathrm{HOOC{-}CH(CHO){-}CH_2{-}CH{=}CH{-}CH_3}.$$

The main chain has its carbon-carbon double bond between C4 and C5. There are two additional double bonds, one carbonyl in the carboxylic acid group and one in the formyl group. Therefore

$$N_\pi = 1 + 1 + 1 = 3.$$

The hydroxyl oxygen is singly bonded to the carboxyl carbon and to hydrogen; it does not create a further pi bond.

Step 2: Count sigma bonds without omitting hydrogen The molecule contains seven carbon atoms and three oxygen atoms. Its heavy-atom skeleton is an open connected chain with one branch, so ten heavy atoms have nine heavy-atom connections, each contributing one sigma bond. Explicitly these are five main-chain C–C links, the C2–formyl-carbon link, two carbonyl C–O links, and the acid C–O link. The attached hydrogens are one on C2, two on C3, one each on C4 and C5, three on C6, one on the formyl carbon, and one on the hydroxyl oxygen, for ten hydrogens. Thus

$$N_\sigma = 9 + 10 = 19, \qquad N_\sigma + N_\pi = 19 + 3 = 22.$$

Counting a double bond as two sigma bonds would give the right total for the wrong reason; separating the two types avoids that error.

Step 3: Conclusion There are 19 sigma bonds and 3 pi bonds, for a total of $\boxed{22}$ bonds.

Quick Tip to Solve: Write the condensed structure first, count one sigma per connection including every hydrogen, then add one pi for each double bond.

89. The oxidation number of iron in the compound formed during brown ring test for NO_3^- iron is _____. [4 marks]

Solution:

Concept:

- The brown ring is due to the nitrosyl complex $[\mathrm{Fe(H_2O)_5(NO)}]\mathrm{SO_4}$. The coordination entity has charge $+2$, balancing sulfate of charge -2.
- In the conventional oxidation-state assignment for this complex, coordinated nitrosyl is treated as

NO^+. Aqua ligands are neutral; charge balance then gives the formal oxidation state of iron.

Step 1: Identify the coordination entity In the nitrate brown ring test, nitric oxide formed in the reaction coordinates to iron in an aqueous solution. The brown species is commonly written $[Fe(H_2O)_5(NO)]^{2+}$, with sulfate as the counterion in its salt. All five H_2O ligands carry zero formal charge. The iron oxidation number must be assigned within this coordination ion, rather than copied directly from the initial ferrous sulfate reagent.

Step 2: Apply charge balance using the nitrosyl convention Use the conventional nitrosonium description NO^+ for the nitrosyl ligand in this oxidation-number question. If the formal oxidation number of iron is x, the sum of the metal and ligand charges is the complex charge:

$$x + 5(0) + (+1) = +2.$$

It follows that

$$x = +1.$$

The distinction between neutral free NO and the formal NO^+ ligand assignment matters here: treating the coordinated ligand as neutral would yield $+2$ and would not be the conventional assignment used for this test. Nitrosyl is a non-innocent ligand, so this formal charge assignment should not be confused with a direct measurement of localized electronic charge.

Step 3: Conclusion The formal oxidation number of iron in the brown ring complex is $+1$; the required numerical entry is $\boxed{1}$.

Quick Tip to Solve: For the brown ring oxidation-number question, write the complex charge and the conventional NO-plus ligand charge before solving for iron.

90. Molality of 0.8 M H_2SO_4 solution (density 1.06 g cm^{-3}) is _____ $\times 10^{-3}$ m. [4 marks]

Solution:

Concept:

- Molarity is moles of solute per litre of solution, whereas molality is moles of solute per kilogram of solvent. These denominators are different.
- Density converts solution volume into solution mass. Subtracting solute mass from that total gives solvent mass; H_2SO_4 has molar mass $98\,\text{g}\,\text{mol}^{-1}$.

Step 1: Choose one litre of solution Take $1.000\,\text{L} = 1000\,\text{cm}^3$ of the solution. Its mass, using the given density, is

$$m_{\text{solution}} = (1.06)(1000) = 1060\,\text{g}.$$

Since the molarity is $0.8\,\text{mol}\,\text{L}^{-1}$, this volume contains $0.8\,\text{mol}$ of sulfuric acid. The mass of acid in it is

$$m_{\text{solute}} = (0.8)(98) = 78.4\,\text{g}.$$

Dissociation does not change the number of analytical moles of acid used in this concentration conversion.

Step 2: Calculate solvent mass and molality The mass of solvent is the solution mass minus the acid mass:

$$m_{\text{solvent}} = 1060 - 78.4 = 981.6\,\text{g} = 0.9816\,\text{kg}.$$

Therefore the molality is

$$b = \frac{0.8}{0.9816} = 0.814996\,\text{mol kg}^{-1}.$$

Expressing the result with the specified multiplier gives

$$b = 814.996 \times 10^{-3}\,\text{mol kg}^{-1}.$$

Rounding the coefficient to the nearest integer gives 815. Using the whole solution mass as denominator would calculate moles per kilogram of solution, which is not molality; using a litre as the denominator would simply reproduce the given molarity.

Step 3: Conclusion The molality is $815 \times 10^{-3}\,\text{mol kg}^{-1}$ approximately, so the required entry is $\boxed{815}$.

Quick Tip to Solve: On a one-litre basis, calculate solution mass from density and subtract solute mass before converting the solvent mass to kilograms.