JEE Main 2024 Paper 1 (B.E./B.Tech) Jan 30 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 1 (Jan-Feb), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 30 January 2024, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $g : \mathbf{R} \to \mathbf{R}$ be a non constant twice differentiable function such that $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as $f(x) = \frac{1}{2}[g(x) + g(2-x)]$, then [4 marks]

(A) $f''(x) = 0$ for atleast two x in $(0, 2)$

(B) $f'\left(\frac{3}{2}\right) + f'\left(\frac{1}{2}\right) = 1$

(C) $f''(x) = 0$ for no x in $(0, 1)$

(D) $f''(x) = 0$ for exactly one x in $(0, 1)$

Correct Answer: (A) $f''(x) = 0$ for atleast two x in $(0, 2)$

Solution:

Concept:

- Rolle's theorem says that a continuous function on a closed interval, differentiable inside it, with equal endpoint values has a zero derivative at an interior point.
- Differentiating a reflected argument requires the chain rule: the derivative of $g(2-x)$ is $-g'(2-x)$. The symmetry of f about $x = 1$ therefore makes its first derivative antisymmetric.

Step 1: Locate three zeros of the first derivative Differentiate the definition, retaining the minus sign from the reflected argument:

$$f'(x) = \frac{g'(x) - g'(2-x)}{2}.$$

The hypothesis $g'(1/2) = g'(3/2)$ gives $f'(1/2) = 0$ and $f'(3/2) = 0$. At the centre the two terms are identical, so $f'(1) = 0$ as well. These are three distinct points, all inside $(0, 2)$.

Step 2: Apply Rolle's theorem on two disjoint intervals Since g is twice differentiable, f' is differentiable and hence continuous. Apply Rolle's theorem to f' on $[1/2, 1]$, and then on $[1, 3/2]$. There exist

$$c_1 \in (1/2, 1), \qquad c_2 \in (1, 3/2)$$

such that $f''(c_1) = f''(c_2) = 0$. The intervals do not overlap, so the zeros are distinct. The sum in option (B) is zero, not one. Option (C) contradicts the first application of Rolle's theorem. Exactly one zero, as asserted in (D), is not guaranteed: for the admissible nonconstant function $g(x) = x$, the function f is constant and f'' vanishes everywhere.

Step 3: Conclusion The second derivative vanishes at at least two distinct points in $(0, 2)$. Hence, the correct option is (A).

Quick Tip to Solve: For a function symmetrised as $g(x) + g(2-x)$, first test the derivative at the centre and at the two reflected points before using Rolle's theorem.

2. The value of $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is: [4 marks]

(A) $\frac{\pi}{8(2\sqrt{3}+3)}$

(B) $\frac{(2\sqrt{3}+3)\pi}{24}$

(C) $\frac{13\pi}{8(4\sqrt{3}+3)}$

(D) $\frac{13(2\sqrt{3}-3)\pi}{8}$

Correct Answer: (C) $\frac{13\pi}{8(4\sqrt{3}+3)}$

Solution:

Concept:

- For a continuous function h on $[0,1]$, the sum $\frac{1}{n}\sum_{k=1}^{n}h(k/n)$ tends to $\int_0^1 h(x)\,dx$. Here the spacing is $1/n$.
- Partial fractions reduce a product of quadratic denominators to standard integrals. In particular, $\int dx/(1+a^2x^2)=\arctan(ax)/a+C$, for positive a.

Step 1: Identify the Riemann sum Factor n^2 out of each denominator factor. The summand becomes

$$\frac{1}{n}\frac{1}{(1+(k/n)^2)(1+3(k/n)^2)}.$$

The remaining function is continuous throughout $[0,1]$, since both denominator factors are positive. Therefore the requested limit is

$$I=\int_0^1\frac{dx}{(1+x^2)(1+3x^2)}.$$

Step 2: Decompose and integrate Write the integrand as $A/(1+x^2)+B/(1+3x^2)$. Equating the constant and quadratic coefficients gives $A+B=1$ and $3A+B=0$, hence $A=-1/2$, $B=3/2$. Consequently,

$$I=\left[-\frac{1}{2}\arctan x+\frac{\sqrt{3}}{2}\arctan(\sqrt{3}x)\right]_0^1$$
$$=-\frac{\pi}{8}+\frac{\sqrt{3}\pi}{6}=\frac{(4\sqrt{3}-3)\pi}{24}.$$

Rationalising the coefficient in option (C) gives

$$\frac{13}{8(4\sqrt{3}+3)}=\frac{13(4\sqrt{3}-3)}{8(48-9)}=\frac{4\sqrt{3}-3}{24}.$$

Thus (C) equals the integral. The other three expressions do not equal this coefficient: in particular, the negative contribution from the first partial fraction cannot be replaced by a positive one.

Step 3: Conclusion The limit is $\frac{13\pi}{8(4\sqrt{3}+3)}$. Hence, the correct option is (C).

Quick Tip to Solve: Extract the factor $1/n$ before reading a sum as an integral, and rationalise the answer options only after evaluating the integral.

3. If the circles $(x+1)^2+(y+2)^2=r^2$ and $x^2+y^2-4x-4y+4=0$ intersect at exactly two distinct points, then [4 marks]

(A) $\frac{1}{2} < r < 7$

(B) $3 < r < 7$

(C) $5 < r < 9$

(D) $0 < r < 7$

Correct Answer: (B) $3 < r < 7$

Solution:

Concept:

- The equation $(x-h)^2+(y-k)^2=R^2$ represents a circle with centre (h,k) and radius $R>0$. Completing squares identifies these quantities in an expanded equation.
- Two circles of radii R_1, R_2, whose centres are separated by d, intersect in exactly two distinct points if and only if $|R_1-R_2|<d<R_1+R_2$. Equality gives tangency and must be excluded.

Step 1: Find the centres and radii The first circle already has standard form, with centre $C_1=(-1,-2)$ and radius r. Complete squares for the second circle:

$$x^2-4x+y^2-4y+4=0 \quad \Longrightarrow \quad (x-2)^2+(y-2)^2=4.$$

Its centre is $C_2=(2,2)$, and its radius is 2. The separation of the centres is

$$d=\sqrt{(2+1)^2+(2+2)^2}=\sqrt{9+16}=5.$$

Step 2: Impose both strict intersection inequalities Substituting the radii and the separation gives

$$|r-2|<5<r+2.$$

The right inequality requires $r>3$. The left inequality is equivalent to $-5<r-2<5$, so $-3<r<7$. Combining them, with r a positive radius, gives precisely $3<r<7$. At $r=3$ the circles touch externally; at $r=7$ they touch internally, so neither endpoint is allowed. Options (A) and (D) allow small radii whose circles are disjoint, while option (C) allows radii above 7 and omits valid radii between 3 and 5.

Step 3: Conclusion The required range is $3<r<7$. Hence, the correct option is (B).

Quick Tip to Solve: For two intersection points, apply both the sum-of-radii and difference-of-radii tests strictly; either equality describes only one point of contact.

4. Let $\overrightarrow{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\overrightarrow{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\overrightarrow{a}| = 1, \vec{a} \cdot \vec{b} = 2$ and $|\vec{b}| = 4$. If $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to: [4 marks]

(A) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(B) $\cos^{-1}\left(\frac{2}{3}\right)$

(C) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(D) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Correct Answer: (D) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Solution:

Concept:

- The cross product is perpendicular to both factors. Thus $\vec{b} \cdot (\vec{a} \times \vec{b}) = 0$, which eliminates mixed terms in scalar products involving $\vec{b}$.
- Lagrange's identity gives $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$. For nonzero vectors the angle θ is determined by $\cos\theta = (\vec{b} \cdot \vec{c})/(|\vec{b}||\vec{c}|)$.

Step 1: Evaluate the required scalar product Use the stated vector $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$. Taking its scalar product with $\vec{b}$ gives

$$\vec{b} \cdot \vec{c} = 2\vec{b} \cdot (\vec{a} \times \vec{b}) - 3|\vec{b}|^2 = 0 - 3(16) = -48.$$

This negative sign shows immediately that the required angle is obtuse.

Step 2: Find the magnitude of the second vector From the given magnitudes and scalar product,

$$|\vec{a} \times \vec{b}|^2 = (1)^2(4)^2 - (2)^2 = 12.$$

The two terms defining $\vec{c}$ are perpendicular, so their squared magnitudes add:

$$|\vec{c}|^2 = 4(12) + 9(16) = 192, \qquad |\vec{c}| = 8\sqrt{3}.$$

Therefore

$$\cos\theta = \frac{-48}{4 \cdot 8\sqrt{3}} = -\frac{\sqrt{3}}{2}.$$

Option (B) would give an acute angle, contrary to the negative scalar product. Option (C) has an inverse-cosine argument exceeding one and is not a real angle. Option (A) is obtuse but has the wrong cosine; it does not use the computed magnitude $8\sqrt{3}$.

Step 3: Conclusion The angle is $\cos^{-1}(-\sqrt{3}/2) = 150°$. Hence, the correct option is (D).

Quick Tip to Solve: When a vector combines $\vec{a} \times \vec{b}$ with $\vec{b}$, exploit their perpendicularity before expanding components.

5. The maximum area of a triangle whose one vertex is at $(0,0)$ and the other two vertices lie on the curve $y=-2x^2+54$ at points (x,y) and $(-x,y)$, where $y>0$, is : [4 marks]

(A) 108

(B) 122

(C) 88

(D) 92

Correct Answer: (A) 108

Solution:

Concept:

- The area of a triangle is half its base multiplied by the perpendicular height. Points (x,y) and $(-x,y)$ form a horizontal base of length $2|x|$, with height y above the origin when $y>0$.
- For a differentiable area function on an interval, interior maxima occur among the zeros of its derivative. The derivative sign or second derivative distinguishes a maximum from a minimum.

Step 1: Express the area using one coordinate By symmetry we may choose $x>0$; changing its sign merely exchanges the two vertices. The condition $y=54-2x^2>0$ gives $0<x<3\sqrt{3}$. The base is $2x$, and its perpendicular distance from the origin is y. Thus

$$A(x)=\frac{1}{2}(2x)y=x(54-2x^2)=54x-2x^3.$$

At either limiting degenerate configuration, $x=0$ or $x=3\sqrt{3}$, the area tends to zero. A positive maximum must therefore occur inside the interval.

Step 2: Locate and evaluate the maximum Differentiate with respect to x:

$$A'(x)=54-6x^2=0 \quad\Longrightarrow\quad x^2=9.$$

The admissible solution is $x=3$, giving $y=54-2(9)=36$. Since $A''(x)=-12x$, we have $A''(3)=-36<0$. Also A' is positive before 3 and negative afterwards, confirming the global maximum on the permitted interval. Its value is

$$A_{\max}=3\cdot 36=108 \text{ square units.}$$

The other proposed numbers do not equal the area at the unique positive critical point: 122 exceeds the proven maximum, and 88 and 92 are smaller than the area actually attained.

Step 3: Conclusion The maximum area is 108 square units. Hence, the correct option is (A).

Quick Tip to Solve: For two symmetric points on a parabola, use the horizontal segment as the triangle's base; the area then reduces directly to $x\,y$.

6. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$, **then** $\frac{1}{5}f'(0) =$ **is equal to :** [4 marks]

(A) 2

(B) 1

(C) 0

(D) 6

Correct Answer: (C) 0

Solution:

Concept:

- Subtracting one row from another leaves a determinant unchanged. Simple row differences can expose the dependence on a small number of scalar expressions before any differentiation is attempted.
- The identities $\cos^4 x + \sin^4 x = 1 - 2\sin^2 x\cos^2 x$ and $\sin^2 2x = 4\sin^2 x\cos^2 x$ allow apparently variable terms to combine into a constant.

Step 1: Simplify the determinant by row operations Put $a = 2\cos^4 x$, $b = 2\sin^4 x$, and $c = \sin^2 2x$. The determinant becomes

$$f(x) = \begin{vmatrix} a & b & 3+c \\ 3+a & b & c \\ a & 3+b & c \end{vmatrix}.$$

Apply $R_2 \leftarrow R_2 - R_1$ and $R_3 \leftarrow R_3 - R_1$, using the unchanged first row in both operations:

$$f(x) = \begin{vmatrix} a & b & 3+c \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix} = 9a + 9b + 9(3+c).$$

The signs in the expansion are positive for all three resulting terms; for example the middle cofactor contributes $-b(-9) = 9b$.

Step 2: Remove the trigonometric dependence and differentiate Combine the three variable terms:

$$\begin{aligned} a + b + c &= 2(\cos^4 x + \sin^4 x) + \sin^2 2x \\ &= 2(1 - 2\sin^2 x\cos^2 x) + 4\sin^2 x\cos^2 x = 2. \end{aligned}$$

Hence $f(x) = 9(3+2) = 45$ for every real x. Its derivative is therefore zero everywhere, including at zero. The nonzero answers $2, 1, 6$ cannot be correct because the determinant has no remaining dependence on x; the factor $1/5$ does not change a zero derivative.

Step 3: Conclusion We obtain $\frac{1}{5}f'(0) = 0$. Hence, the correct option is (C).

Quick Tip to Solve: Simplify structured determinants before differentiating: constant row differences often turn the entire determinant into a constant.

7. Let (α, β, γ) be the foot of perpendicular from the point $(1, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Then $19(\alpha + \beta + \gamma)$ is equal to : [4 marks]

(A) 99

(B) 102

(C) 101

(D) 100

Correct Answer: (C) 101

Solution:

Concept:

- A line in symmetric form can be written as $\vec{r} = \vec{a} + t\vec{d}$, where $\vec{a}$ is a point on the line and $\vec{d}$ is its direction vector.
- The foot H of the perpendicular from a point P lies on the line and satisfies $(\overrightarrow{PH}) \cdot \vec{d} = 0$. This condition is a scalar linear equation for the line parameter.

Step 1: Parameterise the foot on the given line Set the three ratios equal to t. A general point of the line, and therefore the required foot, has coordinates

$$H = (\alpha, \beta, \gamma) = (-3 + 5t,\ 1 + 2t,\ -4 + 3t).$$

The direction vector is $\vec{d} = (5, 2, 3)$. From $P = (1, 2, 3)$ to this point, the displacement is

$$\overrightarrow{PH} = (-4 + 5t,\ -1 + 2t,\ -7 + 3t).$$

Choosing the opposite displacement would give the same perpendicularity equation, since its scalar product would only change sign.

Step 2: Impose perpendicularity and calculate the sum Set the scalar product with the direction vector equal to zero:

$$5(-4 + 5t) + 2(-1 + 2t) + 3(-7 + 3t) = 0.$$

Thus $38t - 43 = 0$, giving $t = 43/38$. The individual coordinates are $\alpha = 101/38$, $\beta = 124/38$, and $\gamma = -23/38$. Alternatively, add the parameterised coordinates directly:

$$\alpha + \beta + \gamma = -6 + 10t = -6 + \frac{430}{38} = \frac{101}{19}.$$

Multiplication by 19 gives 101. This value uniquely follows from the perpendicularity equation, so $99, 102, 100$ in the other options do not satisfy the requested coordinate sum.

Step 3: Conclusion The required value is $19(\alpha + \beta + \gamma) = 101$. Hence, the correct option is (C).

Quick Tip to Solve: Once the line parameter is known, add the three coordinate expressions before substitution when the question asks only for their sum.

8. A line passing through the point $A(9,0)$ makes an angle of 30° with the positive direction of x-axis. If this line is rotated about A through an angle of 15° in the clockwise direction, then its equation in the new position is : [4 marks]

(A) $\frac{y}{\sqrt{3}+2}+x=9$
(B) $\frac{x}{\sqrt{3}+2}+y=9$
(C) $\frac{x}{\sqrt{3}-2}+y=9$
(D) $\frac{y}{\sqrt{3}-2}+x=9$

Correct Answer: (D) $\frac{y}{\sqrt{3}-2}+x=9$

Solution:

Concept:

- The slope of a nonvertical line making an inclination θ with the positive x-axis is $m=\tan\theta$. A clockwise rotation decreases its inclination.
- A line through a fixed point (x_0,y_0) has point-slope equation $y-y_0=m(x-x_0)$. Rotation about that point leaves the point on the line.

Step 1: Determine the new inclination The original inclination is 30°, measured anticlockwise from the positive horizontal direction. Rotating clockwise by 15° subtracts this angle, so the new inclination is

$$\theta=30^\circ-15^\circ=15^\circ.$$

Thus the required slope is positive and smaller than $\tan 30^\circ$; it is not the slope corresponding to 45°.

Step 2: Evaluate the slope exactly Use the tangent difference formula rather than a decimal approximation:

$$m=\tan(45^\circ-30^\circ)=\frac{1-1/\sqrt{3}}{1+1/\sqrt{3}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}.$$

Rationalisation makes the surd form convenient for comparison with the options.

Step 3: Form the equation and compare the choices The rotated line still passes through $A(9,0)$. Consequently

$$y=(2-\sqrt{3})(x-9).$$

Since $\sqrt{3}-2=-(2-\sqrt{3})$, division and rearrangement give

$$\frac{y}{\sqrt{3}-2}+x=9.$$

Option (A) has a negative slope. Options (B) and (C) fail the fixed-point test: substituting $(9,0)$ does not satisfy either equation. Option (D) satisfies both the point and slope conditions.

Step 4: Conclusion The new line is $x+\frac{y}{\sqrt{3}-2}=9$. Hence, the correct option is (D).

Quick Tip to Solve: For a rotation about a stated point, first change the inclination with the correct sign, then use point-slope form. Checking the fixed point quickly eliminates two choices.

9. Consider the system of linear equations $x+y+z=4\mu, x+2y+2\lambda z=10\mu, x+3y+4\lambda^2 z=\mu^2+15$ where $\lambda, \mu \in \mathbf{R}$. Which one of the following statements is NOT correct ? [4 marks]

(A) The system has unique solution if $\lambda \neq \frac{1}{2}$ and $\mu \neq 1, 15$

(B) The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$

(C) The system is consistent if $\lambda \neq \frac{1}{2}$

(D) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

Correct Answer: (D) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

Solution:

Concept:

- A square linear system has a unique solution whenever the determinant of its coefficient matrix is nonzero, independently of its right-hand side.
- When the determinant vanishes, use elimination to compare the reduced left-hand sides and constants. A contradiction means inconsistency; a free variable in a consistent system gives infinitely many solutions.

Step 1: Eliminate the first variable Subtract the first equation from the second, and the first from the third. This gives

$$y+(2\lambda-1)z=6\mu, \qquad 2y+(4\lambda^2-1)z=\mu^2+15-4\mu.$$

Now subtract twice the first reduced equation from the second. The resulting compatibility equation is

$$(2\lambda-1)^2 z=\mu^2-16\mu+15=(\mu-1)(\mu-15).$$

These row operations do not change the solution set.

Step 2: Classify the nonsingular case If $\lambda \neq \frac{1}{2}$, the coefficient of z is nonzero, so z is uniquely determined. Substituting back uniquely determines y, then x. Hence the system is consistent and uniquely solvable for every real μ. Statements (A) and (C) are therefore true: the additional exclusions on μ in (A) are unnecessary but do not invalidate that sufficient condition.

Step 3: Test the singular case and the incorrect statement For $\lambda = \frac{1}{2}$, the reduced last equation is

$$0=(\mu-1)(\mu-15).$$

Consistency requires $\mu = 1$ or $\mu = 15$. At either value, $y = 6\mu$, while z is free and $x+z=-2\mu$, yielding infinitely many solutions. Thus (B) is true. Statement (D) is false because its condition $\mu \neq 1$ includes $\mu = 15$, where the system is consistent, rather than inconsistent.

Step 4: Conclusion The inconsistent case is $\lambda = \frac{1}{2}$ with $\mu \notin \{1, 15\}$. Statement (D) omits the second exceptional value. Hence, the correct option is (D).

Quick Tip to Solve: For a NOT-correct question, test every exceptional parameter value. A single consistent example satisfying an asserted inconsistency condition disproves that assertion.

10. Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbf{R}$ be a differentiable function such that $f(0) = \frac{1}{2}$. If the $\lim_{x\to 0} \frac{x\int_0^x f(t)dt}{e^{x^2}-1} = \alpha$, then $8\alpha^2$ is equal to : [4 marks]

(A) 4

(B) 2

(C) 1

(D) 16

Correct Answer: (B) 2

Solution:

Concept:

- Differentiability implies continuity. Therefore, if f is differentiable at zero, then $\lim_{x\to 0} \frac{1}{x}\int_0^x f(t)\,dt = f(0)$. This holds from either side.
- The exponential standard limit is $\lim_{u\to 0} \frac{e^u - 1}{u} = 1$; taking $u = x^2$ gives $x^2/(e^{x^2} - 1) \to 1$.

Step 1: Separate the two limiting factors For $x \neq 0$, rewrite the expression exactly as

$$\frac{x\int_0^x f(t)\,dt}{e^{x^2}-1} = \left(\frac{1}{x}\int_0^x f(t)\,dt\right)\left(\frac{x^2}{e^{x^2}-1}\right).$$

This exposes the average value of f near zero and a familiar exponential limit. Both factors have finite limits; there is no need to know a complete formula for f.

Step 2: Evaluate the integral average By continuity at zero, $f(t) = f(0) + o(1)$ as $t \to 0$, hence

$$\int_0^x f(t)\,dt = xf(0) + o(x).$$

Dividing by x gives the limit $f(0) = \frac{1}{2}$. The signed integral for negative x has the same limiting average, so this argument proves the two-sided limit.

Step 3: Calculate the requested expression The second factor tends to one because $e^{x^2} - 1 = x^2 + o(x^2)$. It follows that

$$\alpha = \frac{1}{2}\cdot 1 = \frac{1}{2}, \qquad 8\alpha^2 = 8\left(\frac{1}{2}\right)^2 = 2.$$

The quantity being asked for is $8\alpha^2$, not α. Thus (A), (C) and (D), giving 4, 1 and 16 respectively, do not follow from the computed limit.

Step 4: Conclusion $\alpha = \frac{1}{2}$ and $8\alpha^2 = 2$. Hence, the correct option is (B).

Quick Tip to Solve: When a short integral appears in a limit, factor out its interval length. Its average tends to the integrand at the endpoint, reducing the problem to standard limits.

11. If the length of the minor axis of an ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is : [4 marks]

(A) $\frac{1}{\sqrt{3}}$

(B) $\frac{2}{\sqrt{5}}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{\sqrt{5}}{3}$

Correct Answer: (B) $\frac{2}{\sqrt{5}}$

Solution:

Concept:

- For an ellipse with semimajor axis a, semiminor axis b, and centre-to-focus distance c, the relation is $a^2 = b^2 + c^2$.
- The full minor-axis length is $2b$, the distance between the foci is $2c$, and the eccentricity is $e = c/a$, with $0 < e < 1$.

Step 1: Translate the given lengths correctly The question compares the full minor axis with half of the full focal separation. Therefore

$$2b = \frac{1}{2}(2c) = c.$$

In particular $c = 2b$, rather than $b = c$ or $b = 2c$. The lengths refer to a nondegenerate ellipse, so $b > 0$, permitting cancellation of b below.

Step 2: Use the ellipse identity Substitute $c = 2b$ into the relation between the semiaxes and focal distance:

$$a^2 = b^2 + (2b)^2 = 5b^2.$$

Since semiaxis lengths are positive, $a = \sqrt{5}\,b$. The result also satisfies $a > c$, because $\sqrt{5} > 2$, as required for an ellipse.

Step 3: Compute and check the eccentricity The eccentricity is consequently

$$e = \frac{c}{a} = \frac{2b}{\sqrt{5}\,b} = \frac{2}{\sqrt{5}}.$$

Another useful check is $b/a = 1/\sqrt{5}$, so $e^2 = 1 - b^2/a^2 = 1 - 1/5 = 4/5$. Option (A) would give $e^2 = 1/3$, option (C) would give $3/4$, and option (D) would give $5/9$; none satisfies the required value $4/5$. The condition fixes the shape independently of the overall size of the ellipse.

Step 4: Conclusion The eccentricity is $e = \frac{2}{\sqrt{5}}$. Hence, the correct option is (B).

Quick Tip to Solve: Write full axis lengths and full focal separation before simplifying. Confusing a semiaxis with an axis introduces a factor-of-two error in eccentricity questions.

12. Let M denote the median of the following frequency distribution

Class	0 - 4	4 - 8	8 - 12	12 - 16	16 - 20
Frequency	3	9	10	8	6

Then 20M is equal to : [4 marks]

(A) 104

(B) 52

(C) 208

(D) 416

Correct Answer: (C) 208

Solution:

Concept:

- For grouped continuous data, the median is $M = L + \frac{N/2-C}{f}h$, where L is the median-class lower boundary, C the preceding cumulative frequency, f its frequency and h its width.
- The median class is the class in which the cumulative frequency first reaches or exceeds $N/2$. The printed adjacent intervals are already continuous; no half-unit boundary correction is needed.

Step 1: Find the total and cumulative frequencies Adding the frequencies gives

$$N = 3 + 9 + 10 + 8 + 6 = 36, \qquad N/2 = 18.$$

The cumulative frequencies corresponding to the successive classes are

$$3, \quad 12, \quad 22, \quad 30, \quad 36.$$

The eighteenth observation lies after the cumulative total 12 but before 22, so the median class is 8–12.

Step 2: Interpolate within the median class For this class, the lower boundary is $L = 8$, the preceding cumulative frequency is $C = 12$, its frequency is $f = 10$, and its width is $h = 4$. Substitution gives

$$M = 8 + \frac{18 - 12}{10} \times 4 = 8 + \frac{24}{10} = 10.4.$$

The answer lies within the selected class, which provides a direct check on both the class selection and the interpolation.

Step 3: Multiply by the requested factor and test the choices The question asks for $20M$, so

$$20M = 20(10.4) = 208.$$

The listed values 104, 52 and 416 are not equal to this product, ruling out (A), (B) and (D). The class midpoint 10 is only an approximation to the median; the grouped-data formula gives the required interpolation.

Step 4: Conclusion The median is $M = 10.4$, and therefore $20M = 208$. Hence, the correct option is (C).

Quick Tip to Solve: Prepare the cumulative-frequency row first. In the median formula, use the cumulative frequency before the median class, rather than the cumulative total including that class.

13. Two integers x and y are chosen with replacement from the set $\{0, 1, 2, 3, \ldots, 10\}$. Then the probability that $|x - y| > 5$, is : [4 marks]

(A) $\frac{31}{121}$
(B) $\frac{60}{121}$
(C) $\frac{62}{121}$
(D) $\frac{30}{121}$

Correct Answer: (D) $\frac{30}{121}$

Solution:

Concept:

- With replacement, each of the two independent choices has 11 possibilities. The sample space consists of $11^2 = 121$ equally likely ordered pairs (x, y).
- The event $|x - y| > 5$ splits into the disjoint cases $x - y > 5$ and $y - x > 5$. They have equal counts by interchanging the two coordinates.

Step 1: Count pairs with a positive difference exceeding five For a fixed difference $d = x - y$, where $d \in \{6, 7, 8, 9, 10\}$, the smaller integer y can take the values $0, 1, \ldots, 10 - d$. There are therefore $11 - d$ pairs. Summing over these five allowed positive differences gives

$$5 + 4 + 3 + 2 + 1 = 15.$$

The strict inequality excludes difference 5; including it would add unwanted outcomes.

Step 2: Include the reversed pairs For each counted pair (x, y) with $x - y > 5$, the reversed pair (y, x) has $y - x > 5$ in the original coordinates. These two events cannot overlap, because a difference cannot be simultaneously positive and negative. Thus the number of favourable ordered pairs is

$$15 + 15 = 30.$$

No pair with equal coordinates is favourable, and the fact that replacement permits equal choices changes only the sample space.

Step 3: Divide by the complete sample space All 121 ordered pairs have probability $1/121$, hence

$$P(|x - y| > 5) = \frac{30}{121}.$$

Option (A) counts 31 favourable pairs, one too many; options (B) and (C) would require 60 and 62 favourable pairs. The exact enumeration gives 30, ruling out all three. As a check, the complementary event contains $121 - 30 = 91$ outcomes, including all 11 diagonal pairs.

Step 4: Conclusion The required probability is $\frac{30}{121}$. Hence, the correct option is (D).

Quick Tip to Solve: For choices from consecutive integers, count pairs by their positive difference, then double for the absolute value. Keep ordered outcomes in both numerator and denominator.

14. Let $A(2,3,5)$ and $C(-3,4,-2)$ be opposite vertices of a parallelogram $ABCD$. If the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$, then the area of the parallelogram is equal to : [4 marks]

(A) $\frac{1}{2}\sqrt{410}$

(B) $\frac{1}{2}\sqrt{306}$

(C) $\frac{1}{2}\sqrt{586}$

(D) $\frac{1}{2}\sqrt{474}$

Correct Answer: (D) $\frac{1}{2}\sqrt{474}$

Solution:

Concept:

- If adjacent side vectors of a parallelogram are $\mathbf{u}$ and $\mathbf{v}$, its area is $|\mathbf{u} \times \mathbf{v}|$.
- Its diagonal vectors can be written $\mathbf{u} + \mathbf{v}$ and $\mathbf{v} - \mathbf{u}$. Their cross product has magnitude twice the area, so Area $= \frac{1}{2}|\mathbf{d}_1 \times \mathbf{d}_2|$.

Step 1: Determine the two diagonal vectors The opposite vertices $A(2,3,5)$ and $C(-3,4,-2)$ give

$$\overrightarrow{AC} = C - A = -5\hat{i} + \hat{j} - 7\hat{k}.$$

The other diagonal is already supplied as

$$\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}.$$

These are diagonals, rather than adjacent sides, so their cross-product magnitude must be halved.

Step 2: Evaluate their cross product Using the determinant expansion,

$$\begin{aligned}\overrightarrow{AC} \times \overrightarrow{BD} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 1 & -7 \\ 1 & 2 & 3 \end{vmatrix} \\ &= (3+14)\hat{i} - (-15+7)\hat{j} + (-10-1)\hat{k} \\ &= 17\hat{i} + 8\hat{j} - 11\hat{k}.\end{aligned}$$

The middle component uses the alternating signs in the determinant. Reversing a diagonal changes the cross-product direction but not its magnitude.

Step 3: Find the area and compare the options The cross-product magnitude is

$$\sqrt{17^2 + 8^2 + (-11)^2} = \sqrt{289 + 64 + 121} = \sqrt{474}.$$

Hence the area is $\sqrt{474}/2$. The radicands 410, 306 and 586 in (A), (B) and (C) do not equal this sum of squares. As a check,

$$|AC|^2|BD|^2 - (AC \cdot BD)^2 = 75 \cdot 14 - (-24)^2 = 474.$$

Step 4: Conclusion The area of the parallelogram is $\dfrac{\sqrt{474}}{2}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: With two diagonal vectors, take half their cross-product magnitude. With two adjacent side vectors, use the whole magnitude; identifying which vectors are given prevents a factor-of-two error.

15. Let $y = y(x)$ be the solution of the differential equation $\sec x\, dy + \{2(1-x)\tan x + x(2-x)\}dx = 0$ such that $y(0) = 2$. Then $y(2)$ is equal to: [4 marks]

(A) $2\{\sin(2) + 1\}$

(B) 2

(C) 1

(D) $2\{1 - \sin(2)\}$

Correct Answer: (B) 2

Solution:

Concept:

- Multiply the differential equation by $\cos x$ to isolate dy/dx; use $\tan x \cos x = \sin x$.
- The product rule gives $\frac{d}{dx}[u(x)\sin x] = u\prime(x)\sin x + u(x)\cos x$. Here $u(x) = x(2-x)$, so $u\prime(x) = 2(1-x)$.

Step 1: Rewrite the differential equation The equation is first order and the coefficient of dy is $\sec x$. Multiplying by $\cos x$, at points where the equation is defined, gives

$$\frac{dy}{dx} = -2(1-x)\sin x - x(2-x)\cos x.$$

There is no factor of y on the right; the task is to identify an antiderivative.

Step 2: Recognise a product derivative and integrate Since $\frac{d}{dx}[x(2-x)] = 2(1-x)$, both terms are precisely the negative derivative of $x(2-x)\sin x$:

$$\frac{dy}{dx} = -\frac{d}{dx}\big[x(2-x)\sin x\big].$$

Integration therefore yields

$$y(x) = C - x(2-x)\sin x.$$

Differentiating this expression recovers the rewritten equation, confirming the sign of the product term.

Step 3: Apply the initial value and evaluate At $x = 0$, the product vanishes, so $2 = y(0) = C$. Thus

$$y(x) = 2 - x(2-x)\sin x.$$

At $x = 2$, it again vanishes because $2 - x = 0$, giving $y(2) = 2$. The sine terms in options (A) and (D) cannot survive this zero factor; option (C) also fails the evaluated expression.

Step 4: Conclusion $\boxed{y(2) = 2}$. Hence, the correct option is (B).

Quick Tip to Solve: When a derivative contains $u\prime\sin x + u\cos x$, recognise $(u\sin x)\prime$ before attempting integration by parts.

16. If $z = x + iy, xy \neq 0$, satisfies the equation $z^2 + i\bar{z} = 0$, then $\left|z^2\right|$ is equal to : [4 marks]

(A) 9

(B) $\frac{1}{4}$

(C) 4

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- Complex conjugation preserves modulus: $|\bar{z}| = |z|$, while $|z^2| = |z|^2$ and $|i| = 1$.
- For $z = x + iy$, equality of complex numbers also requires equality of their real and imaginary parts. The condition $xy \neq 0$ excludes $z = 0$.

Step 1: Take moduli of the given equality Rearrange the equation as $z^2 = -i\bar{z}$. Taking absolute values removes the phase factors but preserves the necessary equality:

$$|z|^2 = |-i|\,|\bar{z}| = |z|.$$

Put $r = |z|$, which is a nonnegative real number. Then

$$r^2 = r, \qquad r(r-1) = 0.$$

Because $xy \neq 0$, neither coordinate is zero, and in particular $z \neq 0$. Hence $r = 0$ is inadmissible and $r = 1$.

Step 2: Check consistency with the coordinate condition Expanding the original equation provides an independent check:

$$(x + iy)^2 + i(x - iy) = (x^2 - y^2 + y) + i(2xy + x).$$

The imaginary part gives $x(2y + 1) = 0$. As $x \neq 0$, we get $y = -1/2$. The real part then gives

$$x^2 - \frac{1}{4} - \frac{1}{2} = 0, \qquad x^2 = \frac{3}{4}.$$

Both $x = \sqrt{3}/2$ and $x = -\sqrt{3}/2$ are nonzero and satisfy the condition. In each case $x^2 + y^2 = 1$, exactly as the modulus argument predicts.

Step 3: Evaluate the requested modulus The quantity asked for is $|z^2|$, not the real part of z^2 or just y^2. Since $|z^2| = |z|^2 = 1$, values 9, $1/4$, and 4 in the other options violate the modulus equality together with the nonzero condition.

Step 4: Conclusion $\boxed{|z^2| = 1}$. Hence, the correct option is (D).

Quick Tip to Solve: When an equation relates z^m to a multiple of $\bar{z}$, taking moduli can determine the required magnitude without finding the argument.

17. If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, $n \in N$, then the roots of the equation $x^2 + nx + (n-3) = 0$ belong to : [4 marks]

(A) $(0, \infty)$

(B) Z

(C) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$

(D) $(-\infty, 0)$

Correct Answer: (D) $(-\infty, 0)$

Solution:

Concept:

- Use $\sin 2x = 2\sin x \cos x$ and $\sin^2 x + \cos^2 x = 1$ to reduce the equation to a linear equation in $\sin x$.
- For $0 < c < 1$, the solutions of $\sin x = c$ in each positive period are $a + 2k\pi$ and $\pi - a + 2k\pi$, with $a = \sin^{-1} c \in (0, \pi/2)$.

Step 1: Simplify the trigonometric equation The two cubic-looking terms combine rather than requiring a cubic equation:

$$2\sin^3 x + \sin 2x \cos x = 2\sin x(\sin^2 x + \cos^2 x) = 2\sin x.$$

The full equation becomes $6\sin x - 4 = 0$, so $\sin x = 2/3$. Write $a = \sin^{-1}(2/3)$, where $0 < a < \pi/2$.

Step 2: Count the roots in the allowed interval The first four nonnegative solutions, in increasing order, are

$$a, \quad \pi - a, \quad 2\pi + a, \quad 3\pi - a.$$

The third lies strictly between 2π and $5\pi/2$, while the fourth lies strictly between $5\pi/2$ and 3π. Therefore an endpoint of the form $n\pi/2$, for a natural number n, includes exactly three solutions only when $n = 5$. The endpoint is not itself a solution, so there is no boundary-count ambiguity.

Step 3: Locate both quadratic roots Substituting $n = 5$ into the quadratic gives

$$x^2 + 5x + 2 = 0, \qquad x = \frac{-5 \pm \sqrt{17}}{2}.$$

Since $\sqrt{17} < 5$, both numerators are negative. Thus both roots belong to $(-\infty, 0)$. They are irrational, ruling out $\mathbb{Z}$; neither is positive, and the smaller root is below $-\sqrt{17}/2$, ruling out option (C).

Step 4: Conclusion Both roots are negative real numbers. Hence, the correct option is (D).

Quick Tip to Solve: Count the sine roots at successive half-period endpoints first; then use the sum and product of the quadratic roots to check their signs.

18. If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3-x)\}^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then $\alpha+\beta+\gamma$ is equal to : [4 marks]

(A) 11

(B) 12

(C) 9

(D) 8

Correct Answer: (A) 11

Solution:

Concept:

- The real inverse cosine $\cos^{-1} u$ requires $-1 \le u \le 1$.
- The logarithm $\log_e(3-x)$ requires $3-x>0$; its reciprocal also requires $\log_e(3-x) \neq 0$. All conditions must hold together.

Step 1: Impose the inverse cosine restriction We require

$$-1 \le \frac{2-|x|}{4} \le 1.$$

Multiplication by the positive number 4 gives $-4 \le 2-|x| \le 4$. The left inequality yields $|x| \le 6$. The right gives $|x| \ge -2$, which is automatic because an absolute value is nonnegative. Thus this term alone has domain $[-6, 6]$, including both endpoints.

Step 2: Impose both logarithmic restrictions The argument of a real logarithm must be strictly positive:

$$3-x>0 \quad \Longrightarrow \quad x<3.$$

The superscript -1 outside braces means the reciprocal of the logarithm. Its denominator cannot be zero:

$$\log_e(3-x) \neq 0 \quad \Longrightarrow \quad 3-x \neq 1 \quad \Longrightarrow \quad x \neq 2.$$

Here $x=2$ makes the logarithm itself defined but makes its reciprocal undefined; $x=3$ makes the logarithm undefined already.

Step 3: Intersect the domains and identify the parameters The intersection is

$$[-6,6] \cap (-\infty, 3) \setminus \{2\} = [-6, 3) \setminus \{2\}.$$

Comparing this with $[-\alpha, \beta) \setminus \{\gamma\}$, we obtain $\alpha=6$, $\beta=3$, $\gamma=2$, and therefore $\alpha+\beta+\gamma=11$. The values 12, 9, and 8 do not equal this sum; in particular, omitting the forbidden point would give an incorrect parameter identification.

Step 4: Conclusion $\boxed{\alpha+\beta+\gamma=11}$. Hence, the correct option is (A).

Quick Tip to Solve: For a reciprocal logarithm, check three things separately: positive argument, nonzero logarithm, and every other function's domain.

19. Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $S_{15} - S_5$ is : [4 marks]

(A) 405

(B) 390

(C) 410

(D) 395

Correct Answer: (D) 395

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, $S_n = \frac{n}{2}[2a + (n-1)d]$.
- The difference $S_{15} - S_5$ is the sum of terms numbered 6 through 15, which are ten consecutive terms of the same progression.

Step 1: Determine the common difference Apply the sum formula to the two supplied totals:

$$10(2a + 19d) = 790 \quad \Longrightarrow \quad 2a + 19d = 79,$$

$$5(2a + 9d) = 145 \quad \Longrightarrow \quad 2a + 9d = 29.$$

Subtracting the second linear equation from the first eliminates a, giving $10d = 50$. Therefore $d = 5$. Substitution into $2a + 9d = 29$ gives $2a = -16$, hence $a = -8$. A negative first term is allowed and must not be replaced by its absolute value.

Step 2: Calculate the requested block sum The first term remaining after subtracting S_5 is $t_6 = a + 5d = -8 + 25 = 17$; the last is $t_{15} = a + 14d = -8 + 70 = 62$. There are $15 - 5 = 10$ terms. Their sum is

$$S_{15} - S_5 = \frac{10}{2}(17 + 62) = 5 \times 79 = 395.$$

The endpoint average method avoids computing two larger sums separately.

Step 3: Verify the result and select the option As a direct check,

$$S_{15} = \frac{15}{2}(-16 + 70) = 405, \qquad S_5 = \frac{5}{2}(-16 + 20) = 10.$$

Thus the difference is $405 - 10 = 395$. Option (A) is only S_{15}, before the required subtraction. The remaining alternatives 390 and 410 do not equal the sum of the ten terms found above.

Step 4: Conclusion $\boxed{S_{15} - S_5 = 395}$. Hence, the correct option is (D).

Quick Tip to Solve: Interpret a difference of partial sums as one consecutive block: determine its first term, last term, and number of terms.

20. The area (in square units) of the region bounded by the parabola $y^2 = 4(x-2)$ and the line $y = 2x - 8$, is : [4 marks]

(A) 7

(B) 8

(C) 9

(D) 6

Correct Answer: (C) 9

Solution:

Concept:

- For curves expressed as $x = x(y)$, the enclosed area is $\int_{y_1}^{y_2} (x_{\text{right}} - x_{\text{left}})\, dy$.
- Horizontal strips are convenient here because the parabola $y^2 = 4(x-2)$ gives one expression $x = 2 + y^2/4$, avoiding separate square-root branches.

Step 1: Find the intersection ordinates Write the line as $x = 4 + y/2$. At an intersection,

$$2 + \frac{y^2}{4} = 4 + \frac{y}{2} \quad \Longrightarrow \quad y^2 - 2y - 8 = 0.$$

Factoring gives $(y-4)(y+2) = 0$, so the limits are $y = -2$ and $y = 4$. The corresponding points are $(3, -2)$ and $(6, 4)$.

Step 2: Identify the right and left boundaries For an ordinate between the intersections, the horizontal width is

$$x_{\text{line}} - x_{\text{parabola}} = 4 + \frac{y}{2} - \left(2 + \frac{y^2}{4}\right) = 2 + \frac{y}{2} - \frac{y^2}{4}.$$

It equals $(4-y)(y+2)/4$, which is nonnegative throughout $[-2, 4]$, so the line is the right-hand boundary. This sign check is essential: reversing the subtraction would produce a negative signed integral instead of geometric area.

Step 3: Integrate the width The required area is

$$\begin{aligned} A &= \int_{-2}^{4} \left(2 + \frac{y}{2} - \frac{y^2}{4}\right) dy \\ &= \left[2y + \frac{y^2}{4} - \frac{y^3}{12}\right]_{-2}^{4} \\ &= \frac{20}{3} - \left(-\frac{7}{3}\right) = 9. \end{aligned}$$

The integrand vanishes at both endpoints, as expected where the curves meet, and remains positive inside. Thus alternatives 7, 8, and 6 are ruled out by the exact definite integral.

Step 4: Conclusion The enclosed area is $\boxed{9 \text{ square units}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a sideways parabola and an oblique line, integrate with respect to y; one horizontal-strip integral often replaces two vertical-strip integrals.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. A group of 40 students appeared in an examination of 3 subjects - Mathematics, Physics and Chemistry. It was found that all students passed in atleast one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, atmost 11 students passed in both Mathematics and Physics, atmost 15 students passed in both Physics and Chemistry, atmost 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is _____. [4 marks]

Solution:

Concept:

- Let M, P, C be the sets passing Mathematics, Physics and Chemistry. Inclusion-exclusion gives $|M \cup P \cup C| = |M| + |P| + |C| - |M \cap P| - |P \cap C| - |M \cap C| + |M \cap P \cap C|$.
- Every pairwise intersection contains the triple intersection. All region counts must be nonnegative integers, so an upper bound must also be checked for attainability.

Step 1: Relate the pair intersections to the triple intersection Put $t = |M \cap P \cap C|$, and denote the three pairwise intersection sizes by a, b, c. These are inclusive intersections: students who pass all three are counted in each. Since every student passes at least one subject, the union has size 40. Therefore

$$40 = 20 + 25 + 16 - (a + b + c) + t, \qquad a + b + c = 21 + t.$$

Step 2: Obtain an integer upper bound Because $a \geq t$, $b \geq t$, and $c \geq t$, their sum is at least $3t$. Combining with the previous equation gives

$$3t \leq 21 + t \quad \Longrightarrow \quad 2t \leq 21.$$

As t counts students, t is an integer, hence $t \leq 10$. This proves that 11 or more students cannot pass all three, independently of how the remaining students are distributed.

Step 3: Construct a distribution attaining the bound Take $t = 10$, $|M \cap P| = 10$, $|P \cap C| = 11$, and $|M \cap C| = 10$. All three satisfy the stated upper bounds $11, 15, 15$. Then exactly one student passes Physics and Chemistry only, and none passes either of the other pairs only. Assign 10 students to Mathematics only, 14 to Physics only, and 5 to Chemistry only. The totals are

$$|M| = 10 + 10 = 20, \quad |P| = 14 + 1 + 10 = 25, \quad |C| = 5 + 1 + 10 = 16.$$

Their union contains $10 + 14 + 5 + 1 + 10 = 40$ students. Thus every condition is satisfied and the upper bound is achieved.

Step 4: Conclusion The maximum possible number passing all three subjects is $\boxed{10 \text{ students}}$.

Quick Tip to Solve: For a maximum overlap, derive an integer upper bound from inclusion-exclusion and then give one feasible allocation to prove it is attainable.

22. **Let $y = y(x)$ be the solution of the differential equation**

$$(1 - x^2)\,dy = \left[xy + (x^3 + 2)\sqrt{3(1 - x^2)}\right]\,dx,$$

where $-1 < x < 1$ and $y(0) = 0$. If $y(1/2) = m/n$, where m and n are co-prime numbers, then $m + n$ is equal to _____. [4 marks]

Solution:

Concept:

- For a linear equation $y' + P(x)y = Q(x)$, multiplication by the integrating factor $I(x) = e^{\int P(x)\,dx}$ makes the left side the derivative of $I(x)y(x)$.
- The interval $-1 < x < 1$ ensures that $1 - x^2 > 0$, so its positive square root and the divisions used below are well defined.

Step 1: Put the equation in linear form Divide the given equation by $1 - x^2$ and move its term containing y to the left:

$$y' - \frac{x}{1 - x^2}y = \frac{\sqrt{3}(x^3 + 2)}{\sqrt{1 - x^2}}.$$

Here $P(x) = -x/(1 - x^2)$. Since $\int -x/(1 - x^2)\,dx = \frac{1}{2}\ln(1 - x^2)$, the integrating factor is $I(x) = \sqrt{1 - x^2}$.

Step 2: Integrate and impose the initial condition Multiplying both sides by the integrating factor gives

$$\frac{d}{dx}\left(y\sqrt{1 - x^2}\right) = \sqrt{3}(x^3 + 2).$$

Consequently,

$$y\sqrt{1 - x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right) + C.$$

At $x = 0$, the initial value is $y = 0$; hence $C = 0$. The initial condition therefore selects a unique solution on the given interval.

Step 3: Evaluate the required fraction At $x = 1/2$, the factor on the left is $\sqrt{3}/2$, while the polynomial in parentheses is $1/64 + 1 = 65/64$. Thus

$$y\left(\frac{1}{2}\right)\frac{\sqrt{3}}{2} = \frac{65\sqrt{3}}{64}, \qquad y\left(\frac{1}{2}\right) = \frac{65}{32}.$$

The numerator is odd and the denominator is a power of two, so the fraction is already in lowest terms. Therefore $m = 65$ and $n = 32$.

Step 4: Conclusion The required numerical value is $\boxed{m + n = 65 + 32 = 97}$. This is a dimensionless integer.

Quick Tip to Solve: After finding the integrating factor, simplify its product with the right side before integrating: the square roots cancel here and leave only a polynomial.

23. The value of $9\int_0^9 \left[\sqrt{\frac{10x}{x+1}}\right] dx$, where $[t]$ denotes the greatest integer less than or equal to t, is

[4 marks]

Solution:

Concept:

- The greatest integer function is constant between consecutive integer crossings. Split the integration interval at those crossings and integrate each constant value separately.
- For $f(x) = \sqrt{10x/(x+1)}$, solve $f(x) = k$ by squaring: the crossing point is $x = k^2/(10-k^2)$ whenever it lies in the integration interval.

Step 1: Locate all the integer crossings For $0 \le x \le 9$, the function inside the square root is nonnegative. Moreover,

$$\frac{d}{dx}\left(\frac{10x}{x+1}\right) = \frac{10}{(x+1)^2} > 0.$$

Hence $f(x)$ increases continuously from $f(0) = 0$ to $f(9) = 3$. Its crossings of 1 and 2 occur at

$$\frac{10x}{x+1} = 1 \Rightarrow x = \frac{1}{9}, \qquad \frac{10x}{x+1} = 4 \Rightarrow x = \frac{2}{3}.$$

The crossing of 3 is the upper endpoint $x = 9$; there are no further crossings inside the interval.

Step 2: Integrate the resulting constant pieces The floor is zero on $[0, 1/9)$, one on $[1/9, 2/3)$, and two on $[2/3, 9)$. Although its value at $x = 9$ is three, a change at one isolated point contributes no area to a definite integral. Therefore

$$\begin{aligned}\int_0^9 [f(x)]\,dx &= 0\left(\frac{1}{9}\right) + 1\left(\frac{2}{3} - \frac{1}{9}\right) + 2\left(9 - \frac{2}{3}\right) \\ &= \frac{5}{9} + \frac{50}{3} = \frac{155}{9}.\end{aligned}$$

Each term is the height of one step multiplied by its interval width.

Step 3: Include the factor outside the integral The question asks for nine times the integral, rather than the integral alone. Multiplying the computed area gives

$$9\int_0^9 \left[\sqrt{\frac{10x}{x+1}}\right] dx = 9\left(\frac{155}{9}\right) = 155.$$

This also explains why the requested result is an integer even though individual crossing points are fractional.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{155}$.

Quick Tip to Solve: Find integer crossings of the expression inside the floor first; do not attempt to integrate the square root and then apply the floor.

24. Let $\alpha, \beta \in \mathbf{N}$ be roots of the equation $x^2 - 70x + \lambda = 0$, where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin \mathbf{N}$. If λ assumes the minimum possible value, then $\frac{(\sqrt{\alpha-1}+\sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|}$ is equal to : [4 marks]

Solution:

Concept:

- By the relations between roots and coefficients, $\alpha + \beta = 70$ and $\alpha\beta = \lambda$. Thus the problem is a constrained minimization of a product of two positive integers with fixed sum.
- The conditions $\lambda/2, \lambda/3 \notin \mathbf{N}$ mean that the positive integer λ is divisible by neither 2 nor 3.

Step 1: Find the smallest admissible pair of roots Interchange the root names if necessary and put $\alpha = a \leq 35$, so $\beta = 70 - a$. Then $\lambda = a(70 - a)$. For $1 \leq a < 35$, this product increases as a increases, since its derivative as a real polynomial is $70 - 2a > 0$. To make the product odd, both roots must be odd. The first possible smaller roots are therefore $a = 1, 3, 5, \ldots$. At $a = 1$, the product is 69, which is divisible by 3; at $a = 3$, it is 201, again divisible by 3. Both are excluded.

Step 2: Check the first allowable product The next odd value is $a = 5$, giving $\beta = 65$ and

$$\lambda = 5 \cdot 65 = 325.$$

This product is odd and its digit sum is 10, so it is divisible by neither 2 nor 3. Thus it satisfies both exclusions. By the increasing-product argument, no later value of the smaller root can give a lower admissible product. The minimum is therefore $\lambda = 325$, with roots 5 and 65, in either order.

Step 3: Substitute into the requested expression For these roots,

$$\sqrt{\alpha - 1} + \sqrt{\beta - 1} = \sqrt{4} + \sqrt{64} = 2 + 8 = 10,$$

$$\lambda + 35 = 325 + 35 = 360, \qquad |\alpha - \beta| = |5 - 65| = 60.$$

Hence the expression reduces exactly to

$$\frac{10 \cdot 360}{60} = 60.$$

The absolute value in the denominator makes the result independent of which root was named α.

Step 4: Conclusion The minimum admissible product is 325, and the required dimensionless value is $\boxed{60}$.

Quick Tip to Solve: For positive integer roots with a fixed sum, test the smallest root first, applying divisibility exclusions before evaluating the requested expression.

25. Number of integral terms in the expansion of $\left\{7^{1/2}+11^{1/6}\right\}^{824}$ is equal to _____. [4 marks]

Solution:

Concept:

- The general term of $(a+b)^N$ is $T_{r+1}=\binom{N}{r}a^{N-r}b^r$, where $r=0,1,\ldots,N$. Here the coefficient is always a nonzero integer.
- Because 7 and 11 are distinct primes, a product of their rational powers can be rational only when each prime has an integer exponent. An integer binomial coefficient cannot cancel a fractional prime exponent.

Step 1: Write the indexed general term Select the factor $11^{1/6}$ exactly r times from the 824 factors. This gives

$$T_{r+1}=\binom{824}{r}7^{(824-r)/2}\,11^{r/6}, \qquad 0\le r\le 824.$$

A term can be an integer only if the exponent of 7 and that of 11 are both integers. This follows from unique prime factorization: the integer coefficient contributes integer prime exponents, and adding those cannot remove a nonintegral fractional part.

Step 2: Apply the two divisibility conditions The exponent of 11 is an integer precisely when $r\equiv 0 \pmod 6$. The exponent of 7 is an integer precisely when $824-r$ is even, equivalently when r is even. Every multiple of six is already even, so the simultaneous requirement is simply

$$r=6j \quad \text{for a nonnegative integer } j.$$

Conversely, for each such index within the allowed range, both exponents are nonnegative integers. The binomial coefficient and both prime powers are then integers, so the whole term is indeed integral.

Step 3: Count the allowed indices including the first one The range on the term index gives

$$0\le 6j\le 824 \quad\Rightarrow\quad 0\le j\le \left\lfloor\frac{824}{6}\right\rfloor=137.$$

Thus the allowed values are $j=0,1,\ldots,137$, giving 138 indices. In particular, $r=0$ contributes the first term 7^{412}, while the last permitted multiple of six is 822. The endpoint $r=824$ is excluded because its exponent of 11 is not integral.

Step 4: Conclusion The expansion contains $\boxed{138}$ integral terms.

Quick Tip to Solve: Count multiples of six from zero, not from six: the first binomial term is integral here and contributes the extra one in the count.

26. Let the latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$ subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{l}{\mathrm{m}}(1 + \sqrt{\mathrm{n}})$, where l and m are co-prime numbers, then $\mathrm{l}^2 + \mathrm{m}^2 + \mathrm{n}^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, the eccentricity satisfies $e^2 = 1 + b^2/a^2$, and a focus is $(ae, 0)$.
- The latus rectum through that focus has endpoints $(ae, b^2/a)$ and $(ae, -b^2/a)$. Symmetry about the transverse axis bisects the angle it subtends at the centre.

Step 1: Relate the subtended angle to the latus rectum The given hyperbola has $a = 3$, since $a^2 = 9$. For the right-hand latus rectum, the two endpoint vectors from the origin have equal positive horizontal component $3e$ and opposite vertical components $\pm b^2/3$. The total angle is $\pi/3$, so each vector makes an angle $\pi/6$ with the positive horizontal axis. Consequently,

$$\tan\frac{\pi}{6} = \frac{b^2/3}{3e} = \frac{b^2}{9e}.$$

As $\tan(\pi/6) = 1/\sqrt{3}$, this becomes $b^2 = 3\sqrt{3}\,e$.

Step 2: Solve for the positive parameter Put $t = b^2 > 0$. Squaring the preceding relation and using $e^2 = 1 + t/9$, we obtain

$$t^2 = 27e^2 = 27\left(1 + \frac{t}{9}\right) = 27 + 3t.$$

Therefore

$$t^2 - 3t - 27 = 0, \qquad t = \frac{3 \pm \sqrt{117}}{2} = \frac{3 \pm 3\sqrt{13}}{2}.$$

The negative-sign root is negative and cannot represent b^2. The positive-sign root satisfies the unsquared relation because both its sides are positive.

Step 3: Read off the requested integers The accepted value can be written in the requested form as

$$b^2 = \frac{3}{2}(1 + \sqrt{13}).$$

Thus $l = 3$, $m = 2$, and $n = 13$, with l and m coprime. We now square each of these integers, including the radicand n itself, to obtain

$$l^2 + m^2 + n^2 = 3^2 + 2^2 + 13^2 = 9 + 4 + 169 = 182.$$

Step 4: Conclusion The hyperbola has $b^2 = \frac{3}{2}(1 + \sqrt{13})$, and the required dimensionless integer is $\boxed{182}$.

Quick Tip to Solve: The angle subtended by a full latus rectum is twice the angle made by either endpoint vector with the transverse axis; use the half-angle in the tangent ratio.

27. If d_1 is the shortest distance between the lines $x+1=2y=-12z, x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}, \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}\,d_1}{d_2}$ is : [4 marks]

Solution:

Concept:

- For lines $\mathbf{r}=\mathbf{a}+s\mathbf{u}$ and $\mathbf{r}=\mathbf{b}+t\mathbf{v}$ with nonparallel directions, their shortest distance is $d=|(\mathbf{b}-\mathbf{a})\cdot(\mathbf{u}\times\mathbf{v})|/|\mathbf{u}\times\mathbf{v}|$.
- A direction vector may be multiplied by any nonzero scalar without changing its line. Integer direction vectors simplify the cross products, provided each coordinate is scaled equally.

Step 1: Parametrize the first pair of lines In $x+1=2y=-12z$, take a point $\mathbf{a}=(-1,0,0)$ and direction $\mathbf{u}=(12,6,-1)$. In $x=y+2=6z-6$, a point is $\mathbf{b}=(0,-2,1)$ and a direction is $\mathbf{v}=(6,6,1)$. Their cross product and connecting vector are

$$\mathbf{u}\times\mathbf{v}=(12,-18,36)=6(2,-3,6), \qquad \mathbf{b}-\mathbf{a}=(1,-2,1).$$

Using the common-normal direction $(2,-3,6)$, whose norm is 7, gives

$$d_1=\frac{|(1,-2,1)\cdot(2,-3,6)|}{7}=\frac{|2+6+6|}{7}=2.$$

Step 2: Compute the distance for the second pair The given symmetric equations provide points $\mathbf{A}=(1,-8,4)$, $\mathbf{B}=(1,2,6)$ and direction vectors $\mathbf{U}=(2,-7,5)$, $\mathbf{V}=(2,1,-3)$. We have

$$\mathbf{U}\times\mathbf{V}=(16,16,16), \qquad \mathbf{B}-\mathbf{A}=(0,10,2).$$

Dividing the cross product by 16 leaves the equivalent common normal $(1,1,1)$, of length $\sqrt{3}$. Thus

$$d_2=\frac{|0+10+2|}{\sqrt{3}}=\frac{12}{\sqrt{3}}=4\sqrt{3}.$$

Both distances are positive; each pair is genuinely skew rather than intersecting.

Step 3: Form the requested dimensionless ratio Substitute the distances only after both common-normal calculations are complete:

$$\frac{32\sqrt{3}\,d_1}{d_2}=\frac{32\sqrt{3}\cdot 2}{4\sqrt{3}}=16.$$

The distance units cancel between numerator and denominator. The factors $\sqrt{3}$ cancel as well, leaving an integer as required.

Step 4: Conclusion The distances are $d_1=2$ and $d_2=4\sqrt{3}$ in coordinate-length units; the required dimensionless value is $\boxed{16}$.

Quick Tip to Solve: In chained equalities such as $x=y+2=6z-6$, first set the common value to zero to obtain a point, then read and scale the direction vector.

28. Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to _____. [4 marks]

Solution:

Concept:

- The unsquared terms $1, 4, 8, 13, 19, 26, \ldots$ have successive differences $3, 4, 5, 6, 7, \ldots$. Summing these differences gives a quadratic expression for their general term.
- Compute $4\alpha - \beta$ term by term before summing. Its fourth-power terms cancel, leaving only standard sums of cubes, squares, first powers and constants.

Step 1: Find the general term of the unsquared sequence Let a_n be its nth term. The increment from a_{n-1} to a_n is $n + 1$, for $n \geq 2$. Starting from $a_1 = 1$,

$$a_n = 1 + \sum_{j=2}^{n}(j+1) = \frac{n(n+1)}{2} + n - 1 = \frac{n^2 + 3n - 2}{2}.$$

Substitution gives $a_1 = 1$ and $a_2 = 4$, confirming the indexing. The quantity α is the sum of the squares of these a_n, not the sum of a_n themselves.

Step 2: Cancel the fourth powers before summing Using the formula for a_n,

$$\begin{aligned} 4\alpha - \beta &= \sum_{n=1}^{10}\left[(n^2 + 3n - 2)^2 - n^4\right] \\ &= \sum_{n=1}^{10}(6n^3 + 5n^2 - 12n + 4). \end{aligned}$$

To verify the expansion, the cross product $2n^2(3n-2)$ contributes $6n^3 - 4n^2$, and $(3n-2)^2$ contributes $9n^2 - 12n + 4$; their square terms therefore combine to $5n^2$.

Step 3: Evaluate the standard sums and isolate k For the integers from 1 to 10,

$$\sum n = 55, \qquad \sum n^2 = 385, \qquad \sum n^3 = 55^2 = 3025.$$

There are ten constant terms. Hence

$$\begin{aligned} 4\alpha - \beta &= 6(3025) + 5(385) - 12(55) + 4(10) \\ &= 18150 + 1925 - 660 + 40 = 19455. \end{aligned}$$

Comparing with $55k + 40$ gives

$$55k = 19455 - 40 = 19415, \qquad k = \frac{19415}{55} = 353.$$

The explicit subtraction of 40 keeps the constant term from being counted twice.

Step 4: Conclusion The required dimensionless integer is $\boxed{k = 353}$.

Quick Tip to Solve: When two sums have the same highest-degree term, subtract their summands first; cancellation can remove the need for a fourth-power summation formula.

29. Let $\mathrm{A} = \{1, 2, 3, \ldots, 7\}$ and let $\mathrm{P(A)}$ denote the power set of A. If the number of functions $f : \mathrm{A} \to \mathrm{P(A)}$ such that $\mathrm{a} \in f(\mathrm{a}), \forall \mathrm{a} \in \mathrm{A}$ is $\mathrm{m^n}, \mathrm{m}$ and $\mathrm{n} \in \mathrm{N}$ and m is least, then $\mathrm{m} + \mathrm{n}$ is equal to _____. [4 marks]

Solution:

Concept:

- A function assigns exactly one subset of A to each argument. The condition $a \in f(a)$ restricts the permitted subsets independently at each argument.
- A subset containing one fixed element of a seven-element set has 2^6 possibilities: each of the other six elements may independently be included or omitted.

Step 1: Count the choices for one image Fix an argument $a \in A$. Its image must be a subset containing a, so the membership of a is already decided. For every element of $A \setminus \{a\}$, we can choose inclusion or exclusion. Thus

$$\#\{S \subseteq A : a \in S\} = 2^{7-1} = 64.$$

This count includes the singleton $\{a\}$ and the full set A; it does not include the empty set. There is no restriction on the size of the image subset.

Step 2: Multiply the independent choices There are seven distinct arguments, and choosing the image at one argument imposes no constraint on any other image. In particular, two arguments are allowed to have the same image; injectivity has not been required. The multiplication principle therefore gives

$$N = \underbrace{64 \cdot 64 \cdots 64}_{7 \text{ factors}} = (2^6)^7 = 2^{42}.$$

The count is of functions, not merely the number of allowed subsets for a single argument.

Step 3: Minimise the natural-number base The count is greater than one, so $m = 1$ is impossible for every natural exponent. The smallest possible natural base is consequently $m = 2$, which works with $n = 42$. Although expressions such as 4^{21} represent the same count, their bases are larger and do not satisfy the least-base requirement.

$$m + n = 2 + 42 = 44.$$

Step 4: Conclusion The least base representation is 2^{42}, and the requested dimensionless numerical value is $\boxed{44}$.

Quick Tip to Solve: For a map into a power set with $a \in f(a)$, fix the required element first, then count the remaining independent membership decisions.

30. If the function

$$f(x)=\begin{cases}\frac{1}{|x|}, & |x|\geqslant 2\\ \mathrm{a}x^2+2\ \mathrm{b}, & |x|<2\end{cases}$$

is differentiable on R, then $48(a+b)$ is equal to _____. [4 marks]

Solution:

Concept:

- At a joining point, differentiability requires both continuity and equal one-sided derivatives. Each formula is differentiable away from the joins $x=\pm 2$.
- For $x>0$, $1/|x|=1/x$, whereas for $x<0$, $1/|x|=-1/x$. The interior polynomial has derivative $2ax$.

Step 1: Impose continuity at the positive join The outer definition includes $x=2$, so $f(2)=1/2$. The limiting value from inside the interval must equal it:

$$4a+2b=\frac{1}{2}.$$

Because both pieces are even functions of x, the same equality also ensures continuity at $x=-2$. There is no singularity at the origin: the formula used there is the polynomial ax^2+2b, not the reciprocal expression.

Step 2: Match the derivatives at the joins At $x=2$, the right derivative of $1/x$ equals $-1/4$, and the left derivative of the polynomial equals $4a$. Hence

$$4a=-\frac{1}{4},\qquad a=-\frac{1}{16}.$$

Check the negative join explicitly. The derivative of $-1/x$ at -2 is $1/4$; the interior derivative is $-4a=1/4$, so the derivative condition there is satisfied too. Checking the sign of the absolute-value branch prevents an erroneous contradiction.

Step 3: Find the remaining parameter and required value Substitute $a=-1/16$ into the continuity equation:

$$-\frac{1}{4}+2b=\frac{1}{2},\qquad 2b=\frac{3}{4},\qquad b=\frac{3}{8}.$$

It follows that

$$a+b=-\frac{1}{16}+\frac{6}{16}=\frac{5}{16},\qquad 48(a+b)=48\frac{5}{16}=15.$$

Both pieces are differentiable in their own open domains, and the two joins have now been checked, establishing differentiability throughout the real line.

Step 4: Conclusion The parameters are $a=-1/16$ and $b=3/8$. Therefore the requested dimensionless value is $\boxed{15}$.

Quick Tip to Solve: For an even piecewise function, solve the positive join first and verify the reflected derivative has the opposite sign at the negative join.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The ratio of the magnitude of the kinetic energy to the potential energy of an electron in the 5th excited state of a hydrogen atom is : [4 marks]

(A) 4

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

Correct Answer: (C) $\frac{1}{2}$

Solution:

Concept:

- For an electron moving in a Coulomb orbit, the attractive electric force supplies centripetal force: $mv^2/r = ke^2/r^2$, where $k = 1/(4\pi\varepsilon_0)$.
- With zero potential energy at infinite separation, $U = -ke^2/r$, while the kinetic energy is $K = mv^2/2 > 0$. The requested magnitude ratio is $K/|U|$.

Step 1: Relate kinetic and potential energies Multiply the centripetal-force equation by r, and then divide by two:

$$mv^2 = \frac{ke^2}{r}, \qquad K = \frac{1}{2}mv^2 = \frac{ke^2}{2r}.$$

The electron-proton interaction is attractive, so its potential energy is negative. Its magnitude is $|U| = ke^2/r$, exactly twice the kinetic energy. This relation follows from the inverse-square Coulomb interaction and applies to every allowed Bohr orbit.

Step 2: Apply the relation to the excited state The fifth excited state corresponds to $n = 6$, since the ground state has $n = 1$. If desired, the familiar hydrogen energies give

$$K = \frac{13.6}{6^2}\,\text{eV}, \qquad U = -\frac{27.2}{6^2}\,\text{eV}.$$

Their common factor $1/6^2$ cancels, just as the orbit radius cancels in the force-based derivation:

$$\frac{K}{|U|} = \frac{ke^2/(2r)}{ke^2/r} = \frac{1}{2}.$$

Step 3: Distinguish the energy quantities The value 1 would arise from comparing kinetic energy with the magnitude of the total energy, because $E = K + U = -K$. That is not the ratio requested here. The choices 4 and $1/4$ do not obey $|U| = 2K$. Although the signed ratio K/U is negative, the question and positive options call for the ratio of magnitudes, which is positive.

Step 4: Conclusion The magnitude of kinetic energy is half the magnitude of potential energy: $\boxed{K/|U| = 1/2}$. Hence, the correct option is (C).

Quick Tip to Solve: In a Coulomb orbit remember $U = -2K$ and $E = -K$; decide whether the denominator is potential energy or total energy before forming the ratio.

32. The work function of a substance is $3.0\ \text{eV}$. The longest wavelength of light that can cause the emission of photoelectrons from this substance is approximately; [4 marks]

(A) 215 nm

(B) 400 nm

(C) 414 nm

(D) 200 nm

Correct Answer: (C) 414 nm

Solution:

Concept:

- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, where ϕ is the work function and $K_{\max}$ is the maximum kinetic energy of an emitted electron.
- At threshold, $K_{\max} = 0$, so the longest wavelength permitting emission is $\lambda_0 = hc/\phi$. Photon energy decreases as wavelength increases.

Step 1: Identify the threshold condition At the longest effective wavelength, a photon supplies just enough energy to liberate an electron, with no surplus kinetic energy. Consequently

$$\frac{hc}{\lambda_0} = \phi = 3.0\,\text{eV}.$$

A wavelength larger than λ_0 has less photon energy and cannot produce photoemission, regardless of intensity. Shorter wavelengths can emit electrons, but they are not the longest wavelength asked for.

Step 2: Substitute the constants consistently Use $hc \simeq 1240\,\text{eV nm}$, so that the electron-volt units cancel directly:

$$\lambda_0 = \frac{1240\,\text{eV nm}}{3.0\,\text{eV}} \simeq 413.3\,\text{nm}.$$

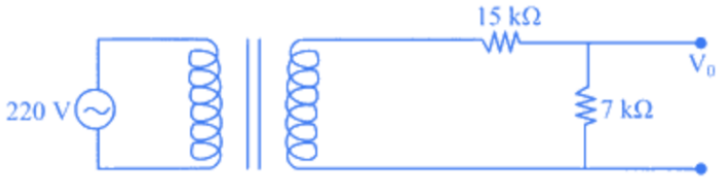
The equivalent SI calculation, with $\phi = 3(1.602 \times 10^{-19})\,\text{J}$, yields approximately $4.14 \times 10^{-7}\,\text{m}$. This is about $414\,\text{nm}$ to the precision of the listed answers.

Step 3: Compare all the listed wavelengths The option $414\,\text{nm}$ is the approximation to the calculated threshold. Light of wavelength $400\,\text{nm}$ has photon energy about $3.10\,\text{eV}$, so emission occurs but with excess kinetic energy; it is not the threshold wavelength. The still shorter $215\,\text{nm}$ and $200\,\text{nm}$ photons also exceed the work function substantially. Thus emission alone does not select those shorter wavelengths.

Step 4: Conclusion The longest effective wavelength is approximately $\boxed{414\,\text{nm}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $hc \approx 1240\,\text{eV nm}$ when the work function is in eV; the threshold wavelength then follows directly in nm.

33. Primary coil of a transformer is connected to $220\ \text{V}$ ac. Primary and secondary turns of the transforms are 100 and 10 respectively. Secondary coil of transformer is connected to two series resistances shown in figure. The output voltage (V_0) is : [4 marks]



(A) 7 V

(B) 44 V

(C) 22 V

(D) 15 V

Correct Answer: (A) 7 V

Solution:

Concept:

- An ideal transformer satisfies $V_s/V_p = N_s/N_p$, with voltages expressed using the same convention, such as rms values.
- The secondary feeds series resistors $15\,\text{k}\Omega$ and $7\,\text{k}\Omega$. The output terminals in the figure are across the $7\,\text{k}\Omega$ resistor, so use voltage division.

Step 1: Calculate the secondary voltage The primary has $N_p = 100$ turns and the secondary has $N_s = 10$ turns. Since the secondary has fewer turns, the transformer steps down the voltage:

$$V_s = V_p \frac{N_s}{N_p} = 220 \frac{10}{100} = 22\,\text{V}.$$

The two external resistances share this secondary voltage. The stated $220\,\text{V}$ is applied to the primary, not directly to the resistor network.

Step 2: Find the common current and output drop The resistors form a series load of total resistance

$$R_{\text{tot}} = 15\,\text{k}\Omega + 7\,\text{k}\Omega = 22\,\text{k}\Omega.$$

Thus the secondary current and output voltage are

$$I_s = \frac{22}{22000}\,\text{A} = 1.0\,\text{mA}, \qquad V_0 = I_s(7000\,\Omega) = 7\,\text{V}.$$

Equivalently, $V_0 = 22 \times 7/(15+7)$. The output is measured between the right-hand terminals, which span the lower-valued resistor only.

Step 3: Check the options against the circuit The drop across the other resistor is $15\,\text{V}$, and $15 + 7 = 22\,\text{V}$ checks the loop sum. Therefore $15\,\text{V}$ refers to the wrong resistor, and $22\,\text{V}$ refers to the whole secondary. The option $44\,\text{V}$ exceeds the available secondary voltage and cannot be a passive divider output. The physically consistent choice is the $7\,\text{V}$ drop.

Step 4: Conclusion The secondary voltage is $22\,\mathrm{V}$, and the output across $7\,\mathrm{k\Omega}$ is $\boxed{7\,\mathrm{V}}$. Hence, the correct option is (A).

Quick Tip to Solve: Apply the turns ratio first, then inspect which resistor the output terminals span before using the voltage-divider ratio.

34. A particle of mass m is projected with a velocity 'u' making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is : [4 marks]

(A) $\frac{\mathrm{mu}^3}{\sqrt{2}\,\mathrm{g}}$

(B) zero

(C) $\frac{\sqrt{3}}{2}\frac{\mathrm{mu}^2}{\mathrm{g}}$

(D) $\frac{\sqrt{3}}{16}\frac{\mathrm{mu}^3}{\mathrm{g}}$

Correct Answer: (D) $\frac{\sqrt{3}}{16}\frac{\mathrm{mu}^3}{\mathrm{g}}$

Solution:

Concept:

- The angular momentum about the launch point is $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$. Its magnitude equals momentum times the perpendicular distance from that point to the instantaneous line of motion.
- For projectile motion without air resistance, the horizontal speed remains $u\cos\theta$, and the vertical speed is zero at the maximum height $h = u^2\sin^2\theta/(2g)$.

Step 1: Evaluate the velocity and height at the apex With $\theta = 30^\circ$, the horizontal velocity at the highest point is

$$v_x = u\cos 30^\circ = \frac{\sqrt{3}}{2}u, \qquad v_y = 0.$$

The maximum height follows from the vertical-motion equation:

$$h = \frac{(u\sin 30^\circ)^2}{2g} = \frac{u^2}{8g}.$$

Zero vertical velocity does not mean zero velocity: the horizontal component remains nonzero throughout the flight.

Step 2: Take the cross product about the launch point Write the apex position as $\mathbf{r} = x\hat{\mathbf{i}} + h\hat{\mathbf{j}}$. Since its momentum is horizontal,

$$\mathbf{L} = (x\hat{\mathbf{i}} + h\hat{\mathbf{j}}) \times (mv_x\hat{\mathbf{i}}) = -mhv_x\hat{\mathbf{k}}.$$

The horizontal position contributes zero because it is parallel to momentum. Therefore the magnitude is

$$L = mhv_x = m\frac{u^2}{8g}\frac{\sqrt{3}u}{2} = \frac{\sqrt{3}}{16}\frac{mu^3}{g}.$$

Step 3: Check units and eliminate the alternatives The quantity mu^3/g has units $\mathrm{kg\,m^2\,s^{-1}}$, as angular momentum must. The choice involving mu^2/g instead has units $\mathrm{kg\,m}$, so it is dimensionally incorrect. Zero is excluded by the nonzero height and horizontal momentum. The remaining coefficient $1/\sqrt{2}$ does not match the product of $\sin^2 30^\circ/2$ and $\cos 30^\circ$.

Step 4: Conclusion The angular-momentum magnitude at maximum height is $\boxed{\dfrac{\sqrt{3}\,mu^3}{16g}}$, in angular-momentum units. Hence, the correct option is (D).

Quick Tip to Solve: At a projectile's highest point the momentum is horizontal, so its perpendicular lever arm about the launch point is simply the height.

35. The diffraction pattern of a light of wavelength 400 nm diffracting from a slit of width 0.2 mm is focused on the focal plane of a convex lens of focal length 100 cm. The width of the 1^{st} secondary maxima will be : [4 marks]

(A) 2 mm

(B) 0.2 mm

(C) 0.02 mm

(D) 2 cm

Correct Answer: (A) 2 mm

Solution:

Concept:

- Single-slit minima satisfy $a\sin\theta_m = m\lambda$, with $m = 1, 2, \ldots$, slit width a, and wavelength λ.
- In the paraxial focal plane of a lens, $y_m \simeq f\theta_m \simeq mf\lambda/a$. A secondary bright band lies between adjacent minima and has width $f\lambda/a$.

Step 1: Identify the boundaries of the first secondary maximum On one side of the central maximum, the first secondary bright band lies between the first and second minima. Their positions in the focal plane are

$$y_1 = \frac{f\lambda}{a}, \qquad y_2 = \frac{2f\lambda}{a}.$$

The width of that bright band is consequently

$$\Delta y = y_2 - y_1 = \frac{f\lambda}{a}.$$

This differs from the central maximum, which extends from $-y_1$ to $+y_1$ and has twice that width. The focal length replaces the usual screen distance in the far-field formula.

Step 2: Convert all distances to SI and substitute The given quantities are

$$\lambda = 400 \times 10^{-9}\,\mathrm{m}, \qquad a = 0.2 \times 10^{-3}\,\mathrm{m}, \qquad f = 100\,\mathrm{cm} = 1\,\mathrm{m}.$$

Therefore

$$\Delta y = \frac{1(400 \times 10^{-9})}{0.2 \times 10^{-3}}\,\mathrm{m} = 2.0 \times 10^{-3}\,\mathrm{m} = 2.0\,\mathrm{mm}.$$

The angle scale $\lambda/a = 0.002$ is small, confirming that the paraxial position approximation is appropriate.

Step 3: Compare the numerical choices The result is $2\,\mathrm{mm}$. The alternatives $0.2\,\mathrm{mm}$ and $0.02\,\mathrm{mm}$ are too small by factors of ten and one hundred, respectively, while $2\,\mathrm{cm} = 20\,\mathrm{mm}$ is too large by a factor of ten. These discrepancies typically arise from mixing millimetres, centimetres and metres before substitution.

Step 4: Conclusion The width of the first secondary bright band is $\boxed{2\,\mathrm{mm}}$. Hence, the correct option is (A).

Quick Tip to Solve: For single-slit diffraction, adjacent minima are separated by $f\lambda/a$; only the central bright maximum spans twice this distance.

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36. A series L.R circuit connected with an ac source $E = (25 \sin 1000t)V$ has a power factor of $\frac{1}{\sqrt{2}}$. If the source of emf is changed to $\mathrm{E} = (20 \sin 2000\mathrm{t})\mathrm{V}$, the new power factor of the circuit will be : [4 marks]

(A) $\frac{1}{\sqrt{3}}$
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{\sqrt{5}}$
(D) $\frac{1}{\sqrt{7}}$

Correct Answer: (C) $\frac{1}{\sqrt{5}}$

Solution:

Concept:

- In a series resistor-inductor circuit, the impedance is $Z = \sqrt{R^2 + (\omega L)^2}$, where R is resistance, L is inductance and ω is angular frequency.
- The power factor is $\cos\phi = R/Z$. It depends on the ratio of reactance to resistance, and not on the applied voltage amplitude for fixed circuit components.

Step 1: Use the initial power factor to relate the circuit elements The initial source has angular frequency $\omega_1 = 1000\,\mathrm{rad\,s^{-1}}$. Its power factor gives

$$\frac{R}{\sqrt{R^2 + (1000L)^2}} = \frac{1}{\sqrt{2}}.$$

Squaring and rearranging yields $2R^2 = R^2 + (1000L)^2$, hence $1000L = R$, since both quantities are positive. Initially the inductive reactance equals the resistance.

Step 2: Calculate the reactance at the new frequency The new source has $\omega_2 = 2000\,\mathrm{rad\,s^{-1}} = 2\omega_1$. With the same inductor and resistor,

$$X_{L2} = \omega_2 L = 2\omega_1 L = 2R.$$

The voltage amplitude changes from 25 V to 20 V, but this changes current magnitude rather than the impedance angle. Thus it does not supply an extra factor in the power factor.

Step 3: Evaluate the new power factor and compare the options The new impedance is $Z_2 = \sqrt{R^2 + 4R^2} = \sqrt{5}R$, so

$$\cos\phi_2 = \frac{R}{\sqrt{5}R} = \frac{1}{\sqrt{5}}.$$

Option (B) would apply only if the reactance-to-resistance ratio stayed unchanged. Options (A) and (D) correspond to different ratios and do not satisfy the doubled-frequency condition. The smaller value also agrees with the increased inductive phase lag.

Step 4: Conclusion The new power factor is $\boxed{1/\sqrt{5}}$. Hence, the correct option is (C).

Quick Tip to Solve: First infer X_L/R from the original power factor, then multiply that ratio by the frequency ratio; ignore a voltage-amplitude change.

37. The electrostatic potential due to an electric dipole at a distance 'r' varies as : [4 marks]

(A) $\frac{1}{r^3}$

(B) $\frac{1}{\mathrm{r}}$

(C) $\frac{1}{r^2}$

(D) r

Correct Answer: (C) $\frac{1}{r^2}$

Solution:

Concept:

- An electric dipole consists of equal and opposite charges separated by a small distance. Its dipole moment $\vec{p}$ points from the negative charge to the positive charge.
- For an ideal dipole, or at distances much larger than its charge separation, the potential is $V(r,\theta) = p\cos\theta/(4\pi\varepsilon_0 r^2)$, where θ is measured from the dipole axis.

Step 1: Add the potentials of the two charges If the charges are $+q$ and $-q$, the scalar potential is

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right).$$

The leading monopole term, proportional to $1/r$, cancels because the total charge is zero. The first nonzero term therefore decreases faster than the potential of a single isolated charge.

Step 2: Identify the radial dependence of the dipole term Let the separation of the charges be d, so $p = qd$. At a fixed direction and for $r \gg d$, expanding the two reciprocal distances gives

$$V = \frac{qd\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}.$$

The dipole moment and angular factor do not change when only the observation distance is varied. Consequently $V \propto r^{-2}$. On the equatorial plane the angular factor is zero, so the potential vanishes;

this does not change the characteristic radial dependence of the dipole potential.

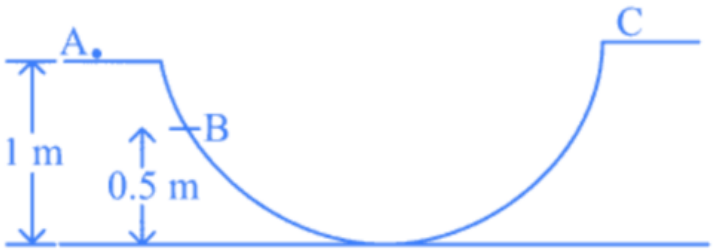
Step 3: Distinguish potential from field The inverse-cube dependence in option (A) describes the characteristic dipole electric field, obtained by spatially differentiating the potential. Option (B) describes monopole potential, whose contribution cancels for a neutral dipole. Option (D) would increase with distance and is inconsistent with the vanishing potential at infinity. Only option (C) has the required inverse-square dependence.

Step 4: Conclusion The electrostatic dipole potential varies as $\boxed{1/r^2}$ at fixed angle. Hence, the correct option is (C).

Quick Tip to Solve: For a dipole remember the paired powers: potential varies as r^{-2}, while electric field varies as r^{-3}.

---

38. A particle is placed at the point A of a frictionless track ABC as shown in figure. It is gently pushed towards right. The speed of the particle when it reaches the point B is :
(Take $g = 10\ \text{m/s}^2$). [4 marks]



(A) $2\sqrt{10}\ \text{m/s}$
(B) $10\ \text{m/s}$
(C) $\sqrt{10}\ \text{m/s}$
(D) $20\ \text{m/s}$
Correct Answer: (C) $\sqrt{10}\ \text{m/s}$

Solution:

Concept:

- On a fixed frictionless track, the normal reaction is perpendicular to the instantaneous displacement and does no work. Gravity is conservative, so the particle's mechanical energy is conserved.
- Taking a common horizontal reference level, the energy at height h is $mgh + \frac{1}{2}mv^2$. The speed depends on the vertical height lost, rather than on the length or curvature of the track.

Step 1: Read the heights from the figure The particle starts at A, which is $1\,\text{m}$ above the indicated horizontal reference. Point B is $0.5\,\text{m}$ above the same reference. Thus the vertical fall from A to B is

$$\Delta h = h_A - h_B = 1 - 0.5 = 0.5\,\text{m}.$$

The gentle push initiates motion to the right; its initial speed is negligible in comparison with the speed acquired under gravity.

Step 2: Apply energy conservation between A and B Write the initial potential energy equal to the potential plus kinetic energy at B:

$$mgh_A = mgh_B + \frac{1}{2}mv_B^2.$$

The mass cancels. Rearranging and using the given $g = 10\,\mathrm{m\,s^{-2}}$,

$$v_B^2 = 2g(h_A - h_B) = 2(10)(0.5) = 10\,\mathrm{m^2\,s^{-2}}.$$

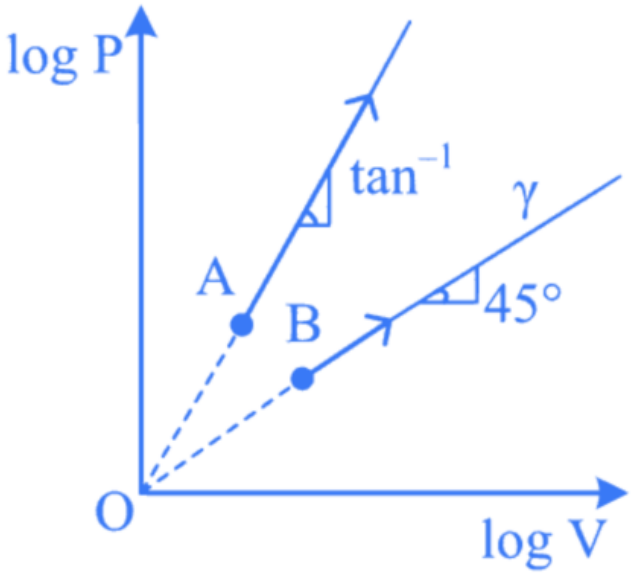
Taking the positive square root is appropriate because the question asks for speed, a nonnegative scalar.

Step 3: Check the result against the alternatives The result is $v_B = \sqrt{10}\,\mathrm{m\,s^{-1}}$, approximately $3.16\,\mathrm{m\,s^{-1}}$. The other listed speeds would require height drops of $2\,\mathrm{m}$, $5\,\mathrm{m}$, and $20\,\mathrm{m}$, respectively, from $\Delta h = v^2/(2g)$. None equals the actual $0.5\,\mathrm{m}$ drop. The height of C is irrelevant to the energy balance up to B.

Step 4: Conclusion The speed at B is $\boxed{\sqrt{10}\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the difference between the two marked heights, not the full height of A above the bottom, in $v = \sqrt{2g\Delta h}$.

39. Two thermodynamical processes are shown in the figure. The molar heat capacity for process A and B are C_A and C_B. The molar heat capacity at constant pressure and constant volume are represented by C_P and C_V, respectively. Choose the correct statement. [4 marks]



(A) $C_P > C_B > C_A > C_V$

(B) $C_P > C_V > C_A = C_B$

(C) $C_A = 0$ and $C_B = \infty$

(D) $C_A = \infty, C_B = 0$

Correct Answer: (A) $C_P > C_B > C_A > C_V$

Solution:

Concept:

- For one mole of an ideal gas, $\delta Q = C_V\,dT + P\,dV$ and $PV = RT$. The molar heat capacity along a specified path is $C = \delta Q/dT$.
- A straight line with slope s in a $\log P$ versus $\log V$ graph means $d\log P/d\log V = s$, hence $P \propto V^s$. Also $C_P = C_V + R$ and $\gamma = C_P/C_V > 1$.

Step 1: Read the slopes of the two processes The graph has $\log P$ on the vertical axis and $\log V$ on the horizontal axis. Process A makes the angle $\tan^{-1}\gamma$ with the horizontal, so its slope is $s_A = \gamma$. Process B makes 45°, giving $s_B = 1$. Both slopes are positive. In particular A is not an adiabatic line: an ideal-gas adiabat would have the negative slope $-\gamma$.

Step 2: Derive the heat capacity for an arbitrary positive slope From $P \propto V^s$ and $T = PV/R$, we get $T \propto V^{s+1}$. Therefore

$$\frac{dT}{T} = (s+1)\frac{dV}{V}, \qquad \frac{P\,dV}{dT} = \frac{R}{s+1}.$$

Substituting this work term into the first law yields

$$C = C_V + \frac{R}{s+1}.$$

This formula includes the heat used to increase internal energy as well as the work done during the expansion.

Step 3: Compare the capacities and eliminate the other choices For the two shown paths,

$$C_A = C_V + \frac{R}{\gamma+1}, \qquad C_B = C_V + \frac{R}{2}.$$

Because $\gamma > 1$, we have $0 < R/(\gamma+1) < R/2 < R$. Adding C_V throughout gives $C_V < C_A < C_B < C_P$. Option (B) incorrectly makes both capacities below C_V and equal. Options (C) and (D) would require adiabatic or isothermal paths; neither positive slope shown has those properties.

Step 4: Conclusion The required ordering is $\boxed{C_P > C_B > C_A > C_V}$. Hence, the correct option is (A).

Quick Tip to Solve: On a $\log P$-$\log V$ graph use the signed slope s: $C = C_V + R/(1+s)$. A positive slope must not be mistaken for an adiabat.

40. The electric field of an electromagnetic wave in free space is represented as $\overrightarrow{\mathrm{E}} = \mathrm{E}_0 \cos(\omega \mathrm{t} - \mathrm{kz})\hat{i}$. The corresponding magnetic induction vector will be : [4 marks]

(A) $\overrightarrow{\mathrm{B}} = \mathrm{E}_0 \mathrm{C} \cos(\omega \mathrm{t} + \mathrm{k}z)\hat{j}$

(B) $\overrightarrow{\mathrm{B}} = \frac{\mathrm{E}_0}{\mathrm{C}} \cos(\omega \mathrm{t} - \mathrm{kz})\hat{j}$

(C) $\overrightarrow{\mathrm{B}} = \mathrm{E}_0 \mathrm{C} \cos(\omega \mathrm{t} - \mathrm{k}z)\hat{j}$

(D) $\overrightarrow{\mathrm{B}} = \frac{\mathrm{E}_0}{\mathrm{C}} \cos(\omega \mathrm{t} + \mathrm{kz})\hat{j}$

Correct Answer: (B) $\overrightarrow{\mathrm{B}} = \frac{\mathrm{E}_0}{\mathrm{C}} \cos(\omega \mathrm{t} - \mathrm{kz})\hat{j}$

Solution:

Concept:

- In a plane electromagnetic wave in free space, $\vec{E}$, $\vec{B}$, and the propagation direction are mutually perpendicular, with $\vec{E} \times \vec{B}$ along propagation.
- The fields oscillate in phase and their magnitudes obey $E = cB$, where c is the speed of light (denoted C in the options).

Step 1: Determine the propagation direction from the phase The electric-field phase is $\omega t - kz$. Holding that phase constant gives

$$\omega t - kz = \text{constant}, \qquad \frac{dz}{dt} = \frac{\omega}{k} > 0.$$

Thus the wave travels along the positive z-axis. Its electric field is along $\hat{i}$, the x-axis. A magnetic field along $+\hat{j}$ makes $\hat{i} \times \hat{j} = \hat{k}$, which is the required propagation direction.

Step 2: Fix the magnetic amplitude and phase The magnetic field amplitude must be $B_0 = E_0/c$, rather than $E_0 c$. Since electric and magnetic fields are in phase in vacuum, they must share the same cosine argument. Combining amplitude, direction and phase gives

$$\vec{B} = \frac{E_0}{c} \cos(\omega t - kz)\, \hat{j}.$$

When the cosine changes sign, both fields reverse together, so their cross product continues to point along $+z$. This is consistent with a single travelling wave carrying energy in one direction.

Step 3: Check each competing expression Options (A) and (C) have the amplitude $E_0 c$, violating $E/B = c$ and the required units. Option (D) has the correct amplitude but uses $\omega t + kz$, representing a phase travelling along negative z and not matching the given electric field. Option (B) alone combines the correct amplitude with the same phase and a magnetic direction that gives positive-z energy flow.

Step 4: Conclusion The magnetic induction is $\boxed{\vec{B} = (E_0/c)\cos(\omega t - kz)\hat{j}}$. Hence, the correct option is (B).

Quick Tip to Solve: Check an electromagnetic-wave option in three steps: $B_0 = E_0/c$, equal phase, then $\vec{E} \times \vec{B}$ along the direction of travel.

41. At which temperature the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at $47^\circ\mathrm{C}$? [4 marks]

(A) 20 K

(B) 80 K

(C) 4 K

(D) -73 K

Correct Answer: (A) 20 K

Solution:

Concept:

- The root-mean-square speed of a gas is $v_{\text{rms}} = \sqrt{3RT/M}$, where T is absolute temperature and M is molar mass in consistent units.
- Equal rms speeds of two gases require equal values of T/M. Hydrogen and oxygen here are diatomic molecules, with molar masses $M_{H_2} = 2$ and $M_{O_2} = 32\,\mathrm{g\,mol^{-1}}$.

Step 1: Convert the oxygen temperature to kelvin The oxygen temperature is $47^\circ\mathrm{C}$. The kinetic-theory relation uses absolute temperature, so

$$T_O = 47 + 273 = 320\,\mathrm{K}.$$

Using the school approximation 273 for the Celsius-to-kelvin offset is consistent with the integer options. Celsius temperatures cannot be substituted directly into the square-root speed law because their zero is not absolute zero.

Step 2: Equate the squared rms speeds For equal speeds, the common factor $3R$ cancels after squaring:

$$\frac{3RT_H}{M_{H_2}} = \frac{3RT_O}{M_{O_2}}, \qquad \frac{T_H}{2} = \frac{320}{32}.$$

Hence

$$T_H = 320\left(\frac{2}{32}\right) = 20\,\mathrm{K}.$$

Both masses may be written in grams per mole here because only their ratio occurs; converting both to kilograms per mole gives exactly the same factor $1/16$.

Step 3: Check the magnitude and the alternative temperatures Hydrogen is sixteen times lighter than oxygen, so it needs one-sixteenth the absolute temperature to have the same rms speed. At 80 K the hydrogen rms speed would be twice the oxygen value, since the squared-speed ratio would be four. At 4 K it would be too slow. A negative kelvin temperature such as the option $-73\,\mathrm{K}$ is not a physical temperature for this ordinary gas calculation.

Step 4: Conclusion The hydrogen temperature is $\boxed{20\,\mathrm{K}}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal rms speeds use $T_1/T_2 = M_1/M_2$, and convert the stated Celsius temperature to kelvin before forming the ratio.

42. Match List I with List II.

	List I		List II
(A)	**Coefficient of viscosity**	**(I)**	$[\mathrm{ML}^2\ \mathrm{T}^{-2}]$
(B)	**Surface tension**	**(II)**	$[\mathrm{ML}^2\ \mathrm{T}^{-1}]$
(C)	**Angular momentum**	**(III)**	$[\mathrm{ML}^{-1}\ \mathrm{T}^{-1}]$
(D)	**Rotational kinetic energy**	**(IV)**	$[\mathrm{ML}^0\ \mathrm{T}^{-2}]$

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(B) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(C) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer: (D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Solution:

Concept:

- Dimensional formulae express quantities using mass M, length L and time T. They follow from the defining physical relation rather than from numerical SI conversion factors.
- Force has dimensions $[F] = \mathrm{MLT}^{-2}$, velocity has $[v] = \mathrm{LT}^{-1}$, and angular velocity has $[\omega] = \mathrm{T}^{-1}$, since an angle in radians is dimensionless.

Step 1: Find viscosity and surface-tension dimensions For viscous shear, $F = \eta A(dv/dy)$, where A is area and the velocity gradient has dimensions T^{-1}. Thus

$$[\eta] = \frac{\mathrm{MLT}^{-2}}{\mathrm{L}^2\mathrm{T}^{-1}} = \mathrm{ML}^{-1}\mathrm{T}^{-1}.$$

Coefficient of viscosity therefore matches (III). Surface tension is force per unit length, so

$$[S] = \frac{\mathrm{MLT}^{-2}}{\mathrm{L}} = \mathrm{ML}^{0}\mathrm{T}^{-2},$$

matching (IV). The zero exponent means the length dimension cancels entirely.

Step 2: Find angular momentum and rotational-energy dimensions Angular momentum is position crossed with linear momentum, giving

$$[L_{\mathrm{ang}}] = \mathrm{L}\,(\mathrm{MLT}^{-1}) = \mathrm{ML}^2\mathrm{T}^{-1},$$

which is (II). Rotational kinetic energy is $K = \frac{1}{2}I\omega^2$, with $[I] = \mathrm{ML}^2$. Hence

$$[K] = \mathrm{ML}^2\,\mathrm{T}^{-2} = \mathrm{ML}^2\mathrm{T}^{-2},$$

which is (I), as expected for any form of energy.

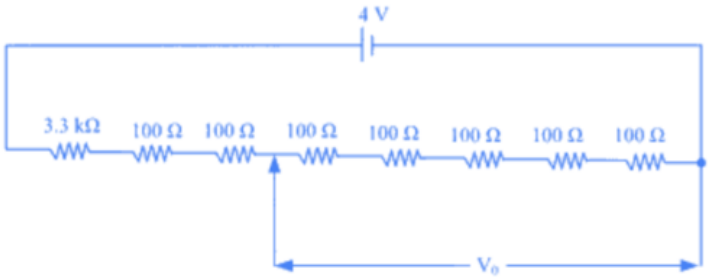
Step 3: Assemble the complete matching code The four matches in the order printed in List I are (A)-(III), (B)-(IV), (C)-(II), and (D)-(I). Option (D) gives all four. Options (A) and (B) already assign an incorrect formula to viscosity. Option (C) correctly matches the last two quantities but interchanges

viscosity and surface tension; their time and length exponents distinguish them.

Step 4: Conclusion The dimensions of viscosity, surface tension, angular momentum and rotational kinetic energy match (III), (IV), (II) and (I), respectively. Hence, the correct option is (D).

Quick Tip to Solve: Derive viscosity from shear force and surface tension from force per length; then use momentum times distance and energy for the last two matches.

43. A potential divider circuit is shown in figure. The output voltage V_0 is : [4 marks]



(A) 2 mV

(B) 4 V

(C) 0.5 V

(D) 12 mV

Correct Answer: (C) 0.5 V

Solution:

Concept:

- In a series circuit, the same current flows through every resistor. The total resistance is the sum of all the resistances connected along the conducting path.
- For an unloaded potential divider, the voltage across a selected series resistance is $V_0 = V R_{\text{out}}/R_{\text{total}}$. The output terminals specify which resistors contribute to R_{out}.

Step 1: Read the resistance chain The circuit contains one $3.3\,\text{k}\Omega$ resistor and seven $100\,\Omega$ resistors in series. Convert the kilohm value before adding. The output tap is after the first two small resistors, so the output is taken across the remaining five, not across the entire chain.

$$R_{\text{total}} = 3300 + 7(100) = 4000\,\Omega.$$

$$R_{\text{out}} = 5(100) = 500\,\Omega.$$

Step 2: Calculate the common current and output The supply provides $4\,\text{V}$ across the complete chain, so Ohm's law gives

$$I = \frac{4}{4000} = 1.0 \times 10^{-3}\,\text{A}.$$

Each $100\,\Omega$ resistor therefore drops $0.1\,\mathrm{V}$. The output spans five such drops:

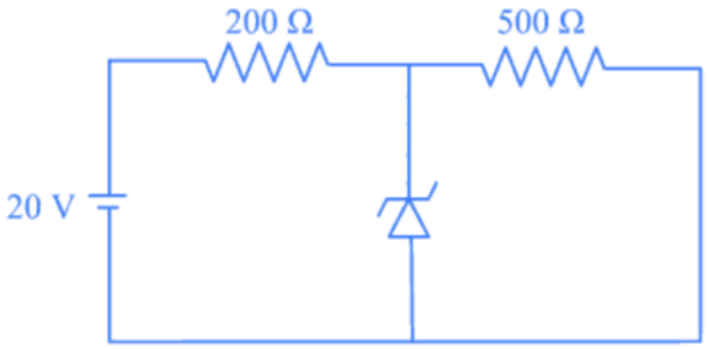
$$V_0 = IR_{\text{out}} = (1.0 \times 10^{-3})(500) = 0.5\,\mathrm{V}.$$

Step 3: Check the alternatives The output is one eighth of the supply because $500/4000 = 1/8$; it cannot equal the full $4\,\mathrm{V}$ in option B. Options A and D are millivolt values, inconsistent with a current of $1\,\mathrm{mA}$ through $500\,\Omega$. Thus only the half-volt value has the required voltage-divider ratio.

Step 4: Conclusion The output voltage is $\boxed{V_0 = 0.5\,\mathrm{V}}$. Hence, the correct option is (C).

Quick Tip to Solve: Count only the resistors between the two output terminals, but include every series resistor when calculating the supply current.

44. A Zener diode of breakdown voltage $10\ \mathrm{V}$ is used as a voltage regulator as shown in the figure. The current through the Zener diode is : [4 marks]



(A) 0

(B) 30 mA

(C) 20 mA

(D) 50 mA

Correct Answer: (B) 30 mA

Solution:

Concept:

- A reverse-biased Zener diode in breakdown maintains approximately its breakdown voltage across itself and a parallel load. Here the regulated output is $V_Z = 10\,\mathrm{V}$.
- The series current splits at the junction: $I_S = I_L + I_Z$, where I_L is the load current and I_Z is the Zener current. The series resistor does not carry only the diode current.

Step 1: Check that breakdown can occur The supply is $20\,\mathrm{V}$, the series resistance is $200\,\Omega$, and the load resistance is $500\,\Omega$. If the diode did not conduct, the divider would produce

$$V_L = 20\frac{500}{200 + 500} = \frac{100}{7}\,\mathrm{V} \approx 14.29\,\mathrm{V}.$$

This exceeds $10\,\mathrm{V}$, so the reverse-biased Zener enters breakdown and clamps the output to $10\,\mathrm{V}$.

Step 2: Find the series and load currents The series resistor has the remaining supply voltage across it:

$$I_S = \frac{20 - 10}{200} = 0.050\,\mathrm{A} = 50\,\mathrm{mA}.$$

The load is in parallel with the diode, so its voltage is also $10\,\mathrm{V}$:

$$I_L = \frac{10}{500} = 0.020\,\mathrm{A} = 20\,\mathrm{mA}.$$

Step 3: Apply the junction rule Conservation of current at the output junction gives

$$I_Z = I_S - I_L = 50 - 20 = 30\,\mathrm{mA}.$$

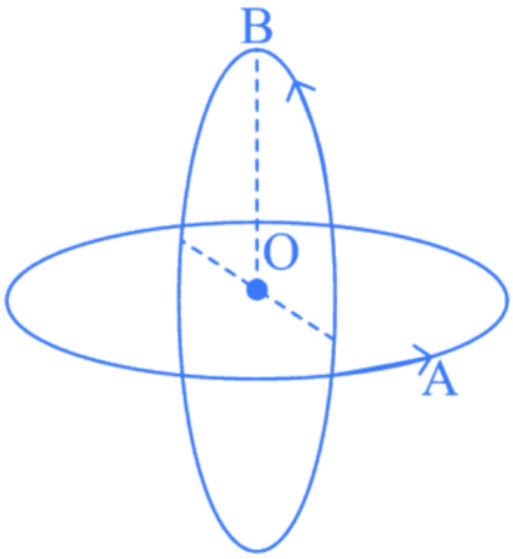
The positive result is consistent with the breakdown assumption. Option D is the total series current, while option C is the load current, so neither is the Zener current. Option A would require the load alone to take the entire series current, which is not the case.

Step 4: Conclusion The current through the Zener diode is $\boxed{30\,\mathrm{mA}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a Zener regulator, calculate both the series current and the load current at the clamped voltage; their difference flows through the diode.

45. Two insulated circular loop A and B of radius 'a' carrying a current of 'I' in the anti clockwise direction as shown in the figure. The magnitude of the magnetic induction at the centre will be :

[4 marks]



(A) $\frac{\sqrt{2}\mu_0 I}{a}$

(B) $\frac{\mu_0 I}{\sqrt{2}a}$

(C) $\frac{\mu_0 \mathrm{I}}{2\mathrm{a}}$

(D) $\frac{2\mu_0 I}{a}$

Correct Answer: (B) $\frac{\mu_0 I}{\sqrt{2}a}$

Solution:

Concept:

- A circular loop of radius a, carrying current I, produces a magnetic field of magnitude $B_0 = \mu_0 I/(2a)$ at its centre. Its direction is normal to the plane of the loop.
- Magnetic fields superpose as vectors. For perpendicular components, the resultant magnitude is found by Pythagoras rather than by adding their magnitudes.

Step 1: Identify the two field directions The figure shows the circular loops in mutually perpendicular planes with a common centre O. Apply the right-hand rule separately: curling the fingers along each loop's current makes the thumb point along that loop's magnetic field. Since the two loop normals are perpendicular, the two fields at O are perpendicular as well. The clockwise or anticlockwise sense sets the signs along these normals; it does not make the fields parallel.

Step 2: Calculate each loop contribution Both loops have the same radius and carry the same current, so their central field magnitudes are equal:

$$B_A = B_B = B_0 = \frac{\mu_0 I}{2a}.$$

There is one turn in each loop, and the insulation prevents electrical contact from changing the stated current paths. Thus the single-loop expression applies independently to A and B.

Step 3: Add the fields vectorially For the 90° angle between the two fields,

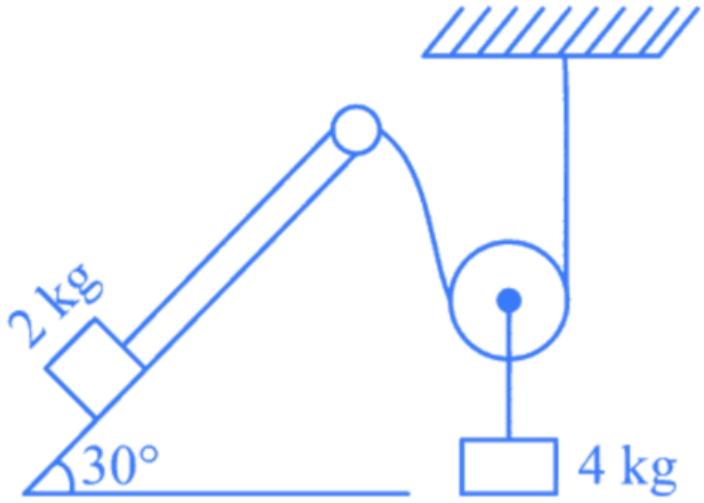
$$B = \sqrt{B_A^2 + B_B^2} = \sqrt{2}\,\frac{\mu_0 I}{2a} = \frac{\mu_0 I}{\sqrt{2}\,a}.$$

Option C accounts for just one loop. Option A is twice the resultant, and option D is even larger than the sum $2B_0$, so neither can represent two equal perpendicular fields. The resultant lies along the angle bisector of their field directions.

Step 4: Conclusion The magnetic induction has magnitude $\boxed{B = \frac{\mu_0 I}{\sqrt{2}\,a}}$. Hence, the correct option is (B).

Quick Tip to Solve: Draw the normal to each loop mentally: equal perpendicular fields give a resultant $\sqrt{2}$ times one field, not twice one field.

46. All surfaces shown in figure are assumed to be frictionless and the pulleys and the string are light. The acceleration of the block of mass 2 kg is : [4 marks]



(A) $\frac{g}{2}$

(B) $\frac{g}{4}$

(C) g

(D) $\frac{g}{3}$

Correct Answer: (D) $\frac{g}{3}$

Solution:

Concept:

- An ideal light string has the same tension T throughout. The movable pulley supporting the $4\,\text{kg}$ mass is pulled upward by two string segments, so its upward force is $2T$.
- The inextensible-string constraint relates the accelerations: a downward displacement y of the movable pulley lengthens two segments by a total $2y$, requiring the $2\,\text{kg}$ block to move $2y$ up the incline.

Step 1: Choose consistent accelerations Let a be the acceleration of the $2\,\text{kg}$ block up the frictionless 30° incline. The $4\,\text{kg}$ block then accelerates downward at $a/2$. The incline-normal force balances the perpendicular weight component, leaving only tension and $2g\sin 30^\circ$ along the slope.

Step 2: Write Newton's law for each mass For the block on the slope, with upward along the incline positive,

$$T - 2g\sin 30^\circ = 2a \quad \Rightarrow \quad T - g = 2a.$$

For the hanging mass and its light movable pulley, take downward as positive:

$$4g - 2T = 4\left(\frac{a}{2}\right) = 2a.$$

The factor of two in the pulley support force and the factor of one half in its acceleration have different origins and must both be retained.

Step 3: Eliminate the tension and test the choices Substitute $T = g + 2a$ from the first equation into

the second:

$$4g - 2(g + 2a) = 2a.$$

$$2g = 6a \quad \Rightarrow \quad a = \frac{g}{3}.$$

The tension is then $5g/3$ in newtons when g is expressed in SI units, giving a positive acceleration in the chosen directions. Values $g/2$, $g/4$, and g fail the two force equations together with the same string constraint.

Step 4: Conclusion The $2\,\text{kg}$ block accelerates up the incline with magnitude $\boxed{g/3}$. Hence, the correct option is (D).

Quick Tip to Solve: For a single movable pulley, write the two-to-one displacement constraint before applying Newton's law; the two masses do not have equal accelerations.

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47. An electric toaster has resistance of 60Ω at room temperature (27°C). The toaster is connected to a $220\ \text{V}$ supply. If the current flowing through it reaches $2.75\ \text{A}$, the temperature attained by toaster is around : (if $\alpha = 2 \times 10^{-4}/^\circ\text{C}$) [4 marks]

(A) 1235 °C

(B) 1667 °C

(C) 694 °C

(D) 1694 °C

Correct Answer: (D) 1694 °C

Solution:

Concept:

- For a metallic resistance with temperature coefficient α, the linear model is $R_T = R_{27}[1+\alpha(T-27)]$, using the stated room-temperature resistance as the reference.
- Ohm's law gives the hot operating resistance as $R_T = V/I$. The temperature rise is then $(R_T/R_{27} - 1)/\alpha$, and the starting temperature must be added to obtain the final temperature.

Step 1: Find the resistance during operation The specified steady current is $2.75\ \text{A}$ at a supply voltage of $220\ \text{V}$. Therefore,

$$R_T = \frac{220}{2.75} = 80\,\Omega.$$

This exceeds the $60\,\Omega$ resistance at 27°C, as expected for a positive temperature coefficient. The resistance rise is $20\,\Omega$, not the complete hot resistance.

Step 2: Calculate the temperature increase Substitute the two resistances and the given coefficient:

$$80 = 60\left[1 + 2 \times 10^{-4}(T - 27)\right].$$

Dividing by 60 and subtracting unity gives

$$\frac{1}{3} = 2 \times 10^{-4}(T - 27).$$

Hence,

$$T - 27 = \frac{1/3}{2 \times 10^{-4}} = 1666.67^\circ\text{C}.$$

Step 3: Recover the final temperature and compare Add the initial 27°C to the rise:

$$T = 1666.67 + 27 = 1693.67^\circ\text{C} \approx 1694^\circ\text{C}.$$

Option B is close to the temperature increase alone and omits the initial temperature. Options A and C do not give the required fractional resistance increase $(80 - 60)/60 = 1/3$ when substituted in the linear relation. The calculated value selects option D within the rounding requested by the word around.

Step 4: Conclusion The toaster reaches approximately $\boxed{1694^\circ\text{C}}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the reference temperature with its resistance: the formula first gives a temperature rise, so add 27°C at the end.

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48. A spherical body of mass $100\ \text{g}$ is dropped from a height of $10\ \text{m}$ from the ground. After hitting the ground, the body rebounds to a height of $5\ \text{m}$. The impulse of force imparted by the ground to the body is given by : (given, $\text{g} = 9.8\ \text{m/s}^2$) [4 marks]

(A) $43.2\ \text{kg ms}^{-1}$

(B) $2.39\ \text{kg ms}^{-1}$

(C) $4.32\ \text{kg ms}^{-1}$

(D) $23.9\ \text{kg ms}^{-1}$

Correct Answer: (B) $2.39\ \text{kg ms}^{-1}$

Solution:

Concept:

- Free fall from rest through height h produces speed $v = \sqrt{2gh}$. The rebound height similarly gives the speed just after collision with the ground.
- Impulse equals the change in momentum: $J = m(v_f - v_i)$. When a body reverses direction, its incoming and outgoing momenta have opposite signs, so their speed magnitudes add in the impulse magnitude.

Step 1: Calculate the speeds immediately before and after impact Convert the mass to SI units: $m = 100\,\text{g} = 0.100\,\text{kg}$. Before impact the body has fallen $10\,\text{m}$, so

$$u = \sqrt{2(9.8)(10)} = \sqrt{196} = 14\,\text{m s}^{-1}.$$

To rebound to $5\,\text{m}$, it must leave the ground with speed

$$v = \sqrt{2(9.8)(5)} = \sqrt{98} \approx 9.899\,\text{m s}^{-1}.$$

Step 2: Account for the reversal of direction Take upward as positive. The velocity just before impact

is $-14\,\mathrm{m\,s^{-1}}$, while the velocity immediately after impact is $+9.899\,\mathrm{m\,s^{-1}}$. Thus, for the short ground collision,

$$J = 0.100[9.899 - (-14)] = 2.3899\,\mathrm{kg\,m\,s^{-1}}.$$

The impulse is upward. As usual for a brief collision, the impulse of weight during contact is negligible compared with the contact impulse; no finite contact time is specified.

Step 3: Check the magnitude and options The incoming momentum magnitude is $1.4\,\mathrm{kg\,m\,s^{-1}}$, and the outgoing magnitude is about $0.990\,\mathrm{kg\,m\,s^{-1}}$. Their sum is 2.39, not their difference. Option D is ten times too large, consistent with a mass-conversion error. Options A and C also fail the momentum change calculated from the two given heights.

Step 4: Conclusion The impulse imparted by the ground is approximately $\boxed{2.39\,\mathrm{kg\,m\,s^{-1}}}$ upward. Hence, the correct option is (B).

Quick Tip to Solve: For a rebound, give incoming and outgoing velocities opposite signs before subtracting them; the two speed magnitudes then add.

49. Young's modules of material of a wire of length 'L' and cross-sectional area A is Y. If the length of the wire is doubled and cross-sectional area is halved then Young's modules will be : [4 marks]

(A) 4 Y

(B) 2 Y

(C) $\frac{Y}{4}$

(D) Y

Correct Answer: (D) Y

Solution:

Concept:

- Young's modulus is the ratio of longitudinal stress to longitudinal strain in the elastic regime: $Y = (F/A)/(\Delta L/L) = FL/(A\Delta L)$.
- At a fixed temperature, Young's modulus depends on the material, not on the specimen's dimensions. Changing length or area changes its extension and spring constant, while its elastic modulus remains the same.

Step 1: Separate the modulus from the wire stiffness For the original wire, an applied axial force F produces extension

$$\Delta L = \frac{FL}{AY}.$$

The force per unit extension, $k = AY/L$, is the wire's stiffness. This quantity depends on geometry, but it is distinct from Young's modulus. The question asks for the modulus of the same material rather than for k or for its extension.

Step 2: Track what happens to the extension For the new dimensions $L' = 2L$ and $A' = A/2$, apply

the same sufficiently small elastic force. Using the unchanged material modulus,

$$\Delta L' = \frac{F(2L)}{(A/2)Y} = 4\frac{FL}{AY} = 4\Delta L.$$

Both effects increase the extension: doubling the length doubles it, and halving the area doubles it once again. Holding the old extension fixed while changing both dimensions would be inconsistent with the same applied force.

Step 3: Verify the stress-to-strain ratio The new stress is $F/(A/2) = 2F/A$, while the new strain is

$$\frac{\Delta L'}{L'} = \frac{4\Delta L}{2L} = 2\frac{\Delta L}{L}.$$

Both stress and strain double, so their ratio is unchanged:

$$Y' = \frac{2F/A}{2\Delta L/L} = Y.$$

Option C instead describes the factor by which the stiffness changes: $k' = k/4$. Options A and B incorrectly attribute geometric changes directly to a material property.

Step 4: Conclusion The Young's modulus remains $\boxed{Y}$. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish the material modulus Y from the wire stiffness AY/L: dimensions change the latter and the extension, while Y stays fixed.

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50. The gravitational potential at a point above the surface of earth is -5.12×10^7 J/kg and the acceleration due to gravity at that point is 6.4 m/s^2. Assume that the mean radius of earth to be 6400 km. The height of this point above the earth's surface is : [4 marks]

(A) 1600 km

(B) 1200 km

(C) 540 km

(D) 1000 km

Correct Answer: (A) 1600 km

Solution:

Concept:

- Outside a spherically symmetric Earth, the gravitational potential is $\Phi = -GM/r$, with zero potential at infinity. Here r is measured from the centre of Earth.
- The magnitude of gravitational acceleration at the same point is $g = GM/r^2$. Dividing the magnitude of potential by acceleration eliminates the unknown product GM.

Step 1: Find the distance from the centre The negative sign of potential indicates an attractive

gravitational field; use its magnitude to obtain a positive distance. Thus

$$r = \frac{|\Phi|}{g} = \frac{5.12 \times 10^7}{6.4} = 8.0 \times 10^6\,\text{m} = 8000\,\text{km}.$$

The units are consistent because $(\text{J/kg})/(\text{m/s}^2) = \text{m}$.

Step 2: Subtract the radius of Earth The requested height is measured from the surface, so subtract $R = 6400\,\text{km}$, rather than using the centre distance directly:

$$h = r - R = 8000 - 6400 = 1600\,\text{km}.$$

For the other proposed heights, the centre distances would be 7600, 6940, and $7400\,\text{km}$; none equals the required ratio $|\Phi|/g = 8000\,\text{km}$.

Step 3: Conclusion The height above the surface of Earth is $\boxed{1600\,\text{km}}$. Hence, the correct option is (A).

Quick Tip to Solve: At one exterior point, use $r = |\Phi|/g$, then subtract Earth's radius to convert centre distance into height.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The horizontal component of earth's magnetic field at a place is 3.5×10^{-5} T. A very long straight conductor carrying current of $\sqrt{2}$ A in the direction from South east to North West is placed. The force per unit length experienced by the conductor is _____ $\times 10^{-6}$ N/m. [4 marks]

Solution:

Concept:

- The magnetic force on a straight conductor of length L carrying current I in a magnetic field is $F = ILB\sin\theta$, where θ is the angle between the current and field.
- The horizontal component of Earth's magnetic field points towards magnetic north. A horizontal current directed from southeast to northwest makes an angle of 45° with north.

Step 1: Identify the angle with the horizontal field The wire points northwest, halfway between north and west. Its angle with the given northward horizontal field is therefore $\theta = 45^\circ$. Equivalently, only the westward component of current contributes to the force from this field; the northward component is parallel to it and experiences no such force.

Step 2: Calculate the force per unit length Use the horizontal field value given in the question:

$$\frac{F}{L} = IB_H \sin 45^\circ.$$

Substituting $I = \sqrt{2}\,\text{A}$, $B_H = 3.5 \times 10^{-5}\,\text{T}$, and $\sin 45^\circ = 1/\sqrt{2}$,

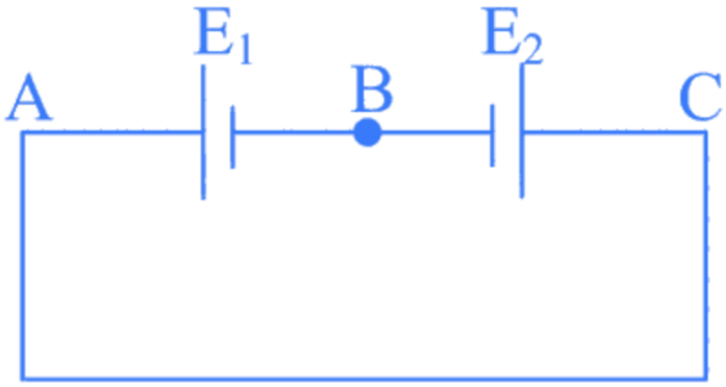
$$\frac{F}{L} = \sqrt{2}(3.5 \times 10^{-5})\frac{1}{\sqrt{2}} = 3.5 \times 10^{-5}\,\text{N/m}.$$

Rewrite the power of ten in the requested form: $3.5 \times 10^{-5} = 35 \times 10^{-6}$.

Step 3: Conclusion The force per unit length due to the given horizontal field is $35 \times 10^{-6}\,\text{N/m}$, so the required numerical entry is $\boxed{35}$.

Quick Tip to Solve: Resolve the current perpendicular to magnetic north: the northwest current contributes $I \sin 45^\circ$ to the force from the horizontal field.

52. Two cells are connected in opposition as shown. Cell E_1 is of $8\ \text{V}$ emf and 2Ω internal resistance; the cell E_2 is of $2\ \text{V}$ emf and 4Ω internal resistance. The terminal potential difference of cell E_2 is _____ V. [4 marks]



Solution:

Concept:

- Kirchhoff's loop rule gives the current from the algebraic sum of emfs divided by the total resistance. Opposing cells contribute the difference of their emfs.
- A cell being charged has current entering its positive terminal. Its terminal voltage is then $V = \mathcal{E} + Ir$, whereas a discharging cell has $V = \mathcal{E} - Ir$.

Step 1: Calculate the current driven by the stronger cell The diagram places the two cells in opposition and shows no external resistance. The $8\,\text{V}$ cell drives current against the $2\,\text{V}$ emf. The two internal resistances are in series, so

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2} = \frac{8-2}{2+4} = 1\,\text{A}.$$

The weaker cell is consequently being charged rather than supplying energy to the circuit.

Step 2: Find the terminal voltage of the weaker cell Current enters the positive terminal of E_2, so its emf and internal resistive drop add when calculating the terminal voltage:

$$V_2 = \mathcal{E}_2 + Ir_2 = 2 + (1)(4) = 6\,\text{V}.$$

As a check, the stronger cell is discharging and its terminal voltage is $V_1 = 8 - (1)(2) = 6\,\text{V}$. Their terminal voltages agree because the connecting wires have negligible resistance.

Step 3: Conclusion The terminal potential difference of cell E_2 is $\boxed{6\,\text{V}}$, so the numerical entry is 6.

Quick Tip to Solve: First decide whether the cell is charging: current entering its positive terminal requires $V = \mathcal{E} + Ir$, not the discharging formula.

53. A electron of hydrogen atom on an excited state is having energy $\text{E}_\text{n} = -0.85\ \text{eV}$. The maximum number of allowed transitions to lower energy level is _____. [4 marks]

Solution:

Concept:

- The Bohr energy of the nth state of hydrogen is $E_n = -13.6/n^2\,\text{eV}$, where the ionisation limit is taken as zero energy.
- If all possible downward transitions among the levels from n to 1 are considered, the maximum number of distinct transitions is the number of pairs of levels, $n(n-1)/2$.

Step 1: Determine the excited principal quantum number Insert the given energy into the hydrogen energy formula:

$$-0.85 = -\frac{13.6}{n^2}.$$

The negative signs cancel, giving

$$n^2 = \frac{13.6}{0.85} = 16, \qquad n = 4.$$

Only the positive integer root represents a principal quantum number, so the electron is initially in the fourth energy level.

Step 2: Count all possible downward transitions The levels available during de-excitation are $4, 3, 2, 1$. Their possible downward transitions are $4 \to 3$, $4 \to 2$, $4 \to 1$, $3 \to 2$, $3 \to 1$, and $2 \to 1$. Thus

$$N = 3 + 2 + 1 = \frac{4(4-1)}{2} = 6.$$

This counts the maximum set of spectral transitions possible in de-excitation, including cascades. Counting only the three direct transitions out of level 4 would omit the transitions from intermediate levels; a single cascade need not emit all six lines.

Step 3: Conclusion The maximum number of distinct downward transitions is $\boxed{6}$, a dimensionless count.

Quick Tip to Solve: Find n from the energy first, then count pairs of accessible levels; include transitions between intermediate levels as well as direct ones.

54. A capacitor of capacitance C and potential V has energy E. It is connected to another capacitor of capacitance $2\mathrm{C}$ and potential $2\ \mathrm{V}$. Then the loss of energy is $\frac{x}{3}\mathrm{E}$, where x is _____. [4 marks]

Solution:

Concept:

- The electrostatic energy stored in a capacitor is $U = \frac{1}{2}CV^2$. When charged capacitors are connected with like terminals together, their total charge is conserved.
- After charge sharing, connected capacitors have a common potential $V_f = Q_{\text{total}}/(C_1 + C_2)$. The decrease of stored field energy is dissipated during redistribution.

Step 1: Find the initial charge and energy The first capacitor has $Q_1 = CV$ and energy $E = \frac{1}{2}CV^2$. For the second capacitor, $C_2 = 2C$ and $V_2 = 2V$, hence $Q_2 = (2C)(2V) = 4CV$. Its energy is

$$U_2 = \frac{1}{2}(2C)(2V)^2 = 4CV^2 = 8E.$$

The total initial energy is therefore $U_i = E + 8E = 9E$.

Step 2: Apply charge conservation and compare energies The total charge on the like-connected plates is $Q_1 + Q_2 = 5CV$, and the total parallel capacitance is $3C$. Therefore

$$V_f = \frac{5CV}{3C} = \frac{5V}{3}.$$

The final energy becomes

$$U_f = \frac{1}{2}(3C)\left(\frac{5V}{3}\right)^2 = \frac{25}{6}CV^2 = \frac{25}{3}E.$$

Consequently,

$$\Delta U = U_i - U_f = 9E - \frac{25}{3}E = \frac{2}{3}E.$$

Only charge, not stored electrostatic energy, is conserved during the connection.

Step 3: Conclusion Comparing $\Delta U = \frac{2}{3}E$ with $\frac{x}{3}E$ gives $\boxed{x = 2}$, which is dimensionless.

Quick Tip to Solve: Use charge conservation to find the common voltage before calculating the final energy; adding the initial energies alone cannot give the loss.

55. The distance between object and its two times magnified real image as produced by a convex lens is $45\ \mathrm{cm}$. The focal length of the lens used is _____ cm. [4 marks]

Solution:

Concept:

- For a thin lens, the Cartesian sign convention gives $1/f = 1/v - 1/u$, with a real object on the incident side having $u < 0$ and its real image having $v > 0$.
- The transverse magnification is $m = v/u$. A real image formed by a convex lens is inverted, so two times magnification means $m = -2$, even though its size ratio is positive.

Step 1: Use magnification to locate the object and image Let the positive object distance magnitude be a, so $u = -a$. The magnification condition gives $v/u = -2$, hence $v = 2a$. Since a real object and real image lie on opposite sides of the lens, their separation is the sum of distance magnitudes:

$$a + 2a = 45\,\text{cm}.$$

Thus $a = 15\,\text{cm}$, and the Cartesian distances are $u = -15\,\text{cm}$, $v = 30\,\text{cm}$.

Step 2: Apply the thin-lens equation Substitute the signed distances consistently:

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{-15} = \frac{1}{30} + \frac{1}{15} = \frac{1}{10}\,\text{cm}^{-1}.$$

Therefore $f = 10\,\text{cm}$. The result is positive, as required for a convex lens. Also, the object lies between f and $2f$, consistent with a real, enlarged image beyond $2f$.

Step 3: Conclusion The focal length of the convex lens is $\boxed{10\,\text{cm}}$, so the numerical entry is 10.

Quick Tip to Solve: For a real magnified image, split the object-image separation in the ratio $1 : |m|$, then substitute the signed distances into the lens formula.

---

56. In a closed organ pipe, the frequency of fundamental note is 30 Hz. A certain amount of water is now poured in the organ pipe so that the fundamental frequency is increased to 110 Hz. If the organ pipe has a cross-sectional area of 2 cm^2, the amount of water poured in the organ tube is _____ g. (Take speed of sound in air is 330 m/s) [4 marks]

Solution:

Concept:

- A pipe closed at one end has fundamental frequency $f = v/(4L)$, where L is the length of its air column and v is the speed of sound.
- Pouring water into the closed pipe raises its lower boundary and shortens the air column. The added water volume is $A(L_i - L_f)$; water has density $1\,\text{g/cm}^3$.

Step 1: Calculate the initial and final air-column lengths Use the fundamental mode before adding water, with $f_i = 30\,\text{Hz}$:

$$L_i = \frac{330}{4(30)} = 2.75\,\text{m}.$$

After adding water, the fundamental frequency is $f_f = 110\,\text{Hz}$, giving

$$L_f = \frac{330}{4(110)} = 0.75\,\text{m}.$$

The higher frequency corresponds to the shorter air column, as expected for a fixed sound speed.

Step 2: Convert the rise in water level into mass The water occupies the decrease in air-column length:

$$\Delta L = L_i - L_f = 2.75 - 0.75 = 2.00\,\text{m} = 200\,\text{cm}.$$

Using the uniform cross-sectional area $A = 2\,\text{cm}^2$,

$$V_w = A\Delta L = (2)(200) = 400\,\text{cm}^3.$$

With density $\rho_w = 1\,\text{g/cm}^3$, the added mass is

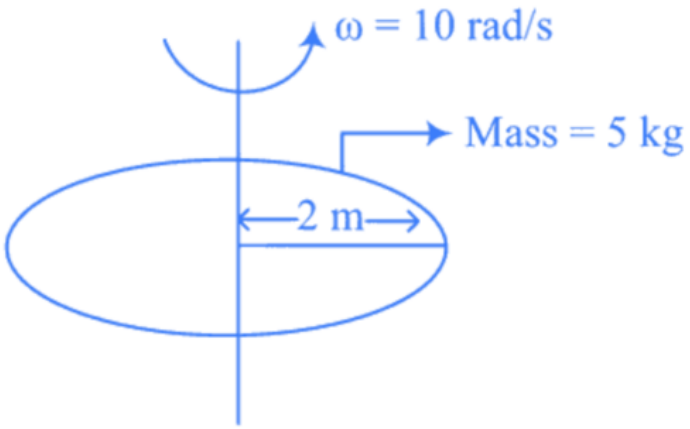
$$m_w = \rho_w V_w = (1)(400) = 400\,\text{g}.$$

The same closed-pipe fundamental formula is used in both states; the frequency increase is not a change of harmonic.

Step 3: Conclusion The mass of water poured into the organ pipe is $\boxed{400\,\text{g}}$, so the required entry is 400.

Quick Tip to Solve: Calculate both air-column lengths using the fundamental frequency, subtract them, and multiply by area and water density using consistent centimetre units.

57. Consider a Disc of mass 5 kg, radius 2 m, rotating with angular velocity of 10 rad/s about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both the discs continue to rotate together without slipping is _____ J. [4 marks]



Solution:

Concept:

- The moment of inertia of a uniform disc about its central perpendicular axis is $I = \frac{1}{2}MR^2$, where M is its mass and R its radius.
- In the absence of an external torque about the rotation axis, total angular momentum is conserved. Friction between the discs is internal and dissipates mechanical energy while bringing them to a common angular speed.

Step 1: Calculate the moment of inertia Each disc has mass $M = 5\,\text{kg}$ and radius $R = 2\,\text{m}$. Therefore,

for each disc,

$$I = \frac{1}{2} \times 5 \times 2^2 = 10\,\text{kg m}^2.$$

Initially only the lower disc rotates; the identical upper disc is at rest. The initial total moment of inertia carrying angular momentum is thus that of one rotating disc.

Step 2: Conserve angular momentum to find the final speed Let ω_f be their common angular speed after slipping ceases. The initial angular momentum is $I\omega_i$, while the final angular momentum is $2I\omega_f$. Hence

$$10 \times 10 = (10 + 10)\omega_f, \qquad \omega_f = 5\,\text{rad s}^{-1}.$$

The angular speed halves because the total rotating inertia doubles; it does not remain at its initial value.

Step 3: Subtract the final rotational energy from the initial energy The initial and final kinetic energies are

$$K_i = \frac{1}{2}(10)(10)^2 = 500\,\text{J}, \qquad K_f = \frac{1}{2}(20)(5)^2 = 250\,\text{J}.$$

The mechanical energy lost to friction is $K_i - K_f$. Angular momentum conservation does not imply kinetic-energy conservation during this slipping process.

Step 4: Conclusion The energy dissipated is $500 - 250 = \boxed{250\,\text{J}}$. The numerical entry is 250.

Quick Tip to Solve: For an identical stationary disc placed coaxially on a rotating disc, the common speed and total rotational kinetic energy each become half their initial values.

58. The displacement and the increase in the velocity of a moving particle in the time interval of t to $(t+1)$s are 125 m and 50 m/s, respectively. The distance travelled by the particle in $(t+2)^{\text{th}}$s is _____ m. [4 marks]

Solution:

Concept:

- For uniformly accelerated motion, displacement over a one-second interval beginning with velocity v is $s = v(1) + \frac{1}{2}a(1)^2$.
- The n^{th} second is the interval from time $n-1$ to n. Consecutive one-second displacements differ by $a(1\,\text{s})^2$.

Step 1: Find the acceleration and velocity at time t Using the uniform acceleration described by this kinematics question, the increase of velocity by $50\,\text{m s}^{-1}$ in one second gives

$$a = \frac{50}{1} = 50\,\text{m s}^{-2}.$$

Let v_t be the velocity at the beginning of the stated interval. Its displacement is

$$125 = v_t(1) + \frac{1}{2}(50)(1)^2, \qquad v_t = 100\,\text{m s}^{-1}.$$

Thus the velocity at time $t+1$ is $150\,\mathrm{m\,s^{-1}}$.

Step 2: Identify the requested second precisely The printed $(t+2)^{\text{th}}$ second runs from time $t+1$ to time $t+2$. It is the immediately following one-second interval, rather than the interval starting at time $t+2$. Its displacement is

$$s = 150(1) + \frac{1}{2}(50)(1)^2 = 175\,\mathrm{m}.$$

Velocity is positive throughout this interval, so the distance travelled equals the displacement.

Step 3: Check the alternative numerical value The next interval after that, from $t+2$ to $t+3$, starts at $200\,\mathrm{m\,s^{-1}}$ and has displacement

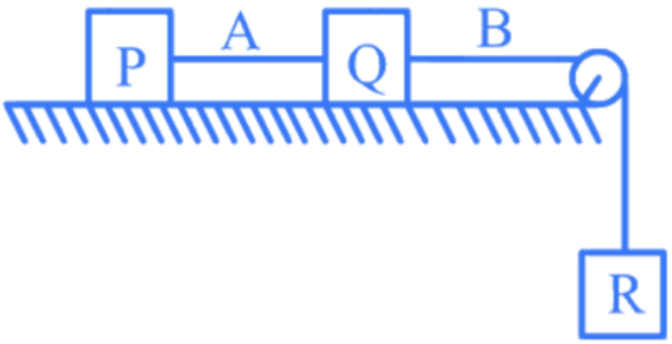
$$200(1) + \frac{1}{2}(50)(1)^2 = 225\,\mathrm{m}.$$

This is the $(t+3)^{\text{th}}$ second under the usual ordinal-second convention. It cannot be substituted for the explicitly printed $(t+2)^{\text{th}}$ second.

Step 4: Conclusion For the interval asked for as printed, the distance is $\boxed{175\,\mathrm{m}}$. The supplied key value 225 describes the interval $[t+2, t+3]$, whereas the printed $(t+2)^{\text{th}}$ second is $[t+1, t+2]$; thus it differs from the literal result.

Quick Tip to Solve: Write the endpoints of an ordinal second before applying a formula: the nth second is [n-1,n], not [n,n+1].

59. Each of three blocks P, Q and R shown in figure has a mass of $3\ \mathrm{kg}$. Each of the wires A and B has cross-sectional area $0.005\ \mathrm{cm}^2$ and Young's modulus $2 \times 10^{11}\ \mathrm{N\ m^{-2}}$. Neglecting friction, the longitudinal strain on wire B is _____ $\times 10^{-4}$. (Take $\mathrm{g} = 10\ \mathrm{m/s^2}$) [4 marks]



Solution:

Concept:

- For a wire under tension T, longitudinal strain is $\epsilon = \Delta L/L = T/(AY)$, where A is cross-sectional area and Y is Young's modulus.
- Apply Newton's second law to all three blocks to find their common acceleration, then isolate the two blocks on the table to find the tension in wire B.

Step 1: Find the acceleration of the connected blocks The smooth table and negligible friction allow the blocks to share a common acceleration magnitude a. For the three-block system, the tensions are internal. The driving force is the weight of the hanging block, $3g = 30\,\mathrm{N}$, while the total mass is $3+3+3 = 9\,\mathrm{kg}$. Hence

$$a = \frac{30}{9} = \frac{10}{3}\,\mathrm{m\,s^{-2}}.$$

The two blocks on the table move towards the pulley while the hanging block moves downward.

Step 2: Calculate the tension in wire B Wire B pulls both blocks P and Q, whose combined mass is $6\,\mathrm{kg}$. The tension in wire A cancels when these two blocks are taken together. Therefore

$$T_B = (3+3)a = 6 \times \frac{10}{3} = 20\,\mathrm{N}.$$

Checking with the hanging block gives $30 - T_B = 3a = 10\,\mathrm{N}$, confirming the same tension. The tension is less than its weight because the block accelerates downward.

Step 3: Convert the area and apply Young's modulus The area must be converted from square centimetres, not centimetres:

$$A = 0.005 \times 10^{-4} = 5 \times 10^{-7}\,\mathrm{m^2}.$$

With $Y = 2 \times 10^{11}\,\mathrm{N\,m^{-2}}$,

$$\epsilon_B = \frac{20}{(5\times10^{-7})(2\times10^{11})} = \frac{20}{10^5} = 2\times10^{-4}.$$

Strain is a ratio of lengths and is dimensionless.

Step 4: Conclusion The longitudinal strain in wire B is $\boxed{2\times10^{-4}}$, so the required numerical coefficient is $\boxed{2}$ (dimensionless).

Quick Tip to Solve: To find tension B quickly, treat the two tabletop blocks as a single mass; remember that converting square centimetres to square metres introduces a factor of 10 to the power -4.

60. A ceiling fan having 3 blades of length $80\ \mathrm{cm}$ each is rotating with an angular velocity of 1200 rpm. The magnetic field of earth in that region is $0.5\mathrm{G}$ and angle of dip is $30°$. The emf induced across the blades is $\mathrm{N}\pi \times 10^{-5}\ \mathrm{V}$. The value of N is _____. [4 marks]

Solution:

Concept:

- A radial conducting blade of length l rotating at angular speed ω in a perpendicular magnetic field $B_\perp$ has hub-to-tip emf $\mathcal{E} = \frac{1}{2}B_\perp\omega l^2$.
- For a horizontal ceiling fan, only the vertical component of Earth's field contributes: $B_\perp = B\sin\delta$, where the dip angle δ is measured from the horizontal.

Step 1: Convert the speed and identify the field component The blade length is $l = 80\,\mathrm{cm} = 0.80\,\mathrm{m}$,

and its angular speed is

$$\omega = 1200\frac{2\pi}{60} = 40\pi \,\text{rad}\,\text{s}^{-1}.$$

Since $1\,\text{G} = 10^{-4}\,\text{T}$, the total terrestrial field is $0.5 \times 10^{-4}\,\text{T}$. Its vertical component is

$$B_{\perp} = (0.5 \times 10^{-4})\sin 30^{\circ} = 2.5 \times 10^{-5}\,\text{T}.$$

The horizontal component does not give a radial motional emf in the horizontal blades.

Step 2: Apply the rotating-conductor formula A small blade element at radius r has speed ωr, so its contribution is $d\mathcal{E} = B_{\perp}\omega r\,dr$. Integrating from the hub to the tip yields

$$\mathcal{E} = \int_0^l B_{\perp}\omega r\,dr = \frac{1}{2}B_{\perp}\omega l^2.$$

Substitution gives

$$\mathcal{E} = \frac{1}{2}(2.5 \times 10^{-5})(40\pi)(0.80)^2 = 32\pi \times 10^{-5}\,\text{V}.$$

Step 3: Compare with the requested form Each of the three blades has the same hub-to-tip emf. The blades share a common hub and are not connected as three independent voltage sources in series, so their number does not multiply this potential difference. Comparing $32\pi \times 10^{-5}\,\text{V}$ with $N\pi \times 10^{-5}\,\text{V}$ determines the dimensionless coefficient directly.

Step 4: Conclusion The induced emf across a blade is $32\pi \times 10^{-5}\,\text{V}$, giving $\boxed{N = 32}$ (dimensionless).

Quick Tip to Solve: For a horizontal rotating blade, use the vertical field B sin(dip), convert rpm to radians per second, and do not multiply the hub-to-tip emf by the number of blades.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Aluminium chloride in acidified aqueous solution forms an ion having geometry [4 marks]

(A) Square planar

(B) Octahedral

(C) Trigonal bipyramidal

(D) Tetrahedral

Correct Answer: (B) Octahedral

Solution:

Concept:

- In aqueous solution, the small highly charged Al^{3+} ion is hydrated by water molecules, which donate oxygen lone pairs to the metal.
- The coordination number counts donor atoms directly attached to the central ion. Six equivalent water ligands give an octahedral arrangement around aluminium.

Step 1: Identify the species formed in acidified water Aluminium chloride supplies aluminium(III)

ions on dissolution. In acidified aqueous solution, hydration gives the hexaaquaaluminium(III) ion,

$$AlCl_3 + 6\,H_2O \longrightarrow [Al(H_2O)_6]^{3+} + 3\,Cl^-.$$

The acidic medium suppresses the extensive hydrolysis that would otherwise produce hydroxo species or aluminium hydroxide. The question concerns the aqueous hydrated ion, rather than the covalent aluminium chloride molecule in a different medium.

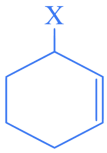
Step 2: Count donor atoms and assign the geometry Each water molecule is monodentate and coordinates through one oxygen atom. The ion $[Al(H_2O)_6]^{3+}$ therefore has six metal–oxygen coordinate bonds. The six donors occupy the vertices of an octahedron: four in an equatorial square and one on either side of that square. The typical adjacent ligand–metal–ligand angle is $90°$, with opposite positions separated by $180°$.

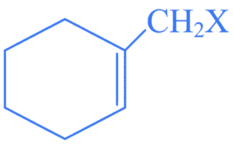
Step 3: Eliminate the other coordination geometries Square planar and tetrahedral geometries both correspond to four directly coordinated ligands, so neither describes this six-coordinate hydrate. Trigonal bipyramidal geometry corresponds to five coordinated ligands. Confusing the three chlorides in the formula of the dissolved salt with the coordination number of its hydrated cation would therefore give an incorrect geometry.

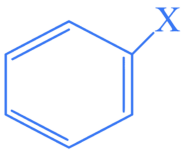
Step 4: Conclusion The ion formed is $[Al(H_2O)_6]^{3+}$, which has octahedral geometry. Hence, the correct option is (B).

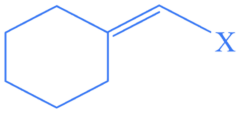
Quick Tip to Solve: For a metal salt in water, first identify the hydrated ion; the number of counterions in the salt formula is not its coordination number.

62. Example of vinylic halide is : [4 marks]

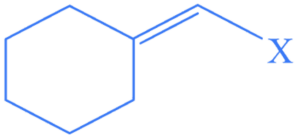
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- A vinylic halide contains a halogen bonded directly to an sp^2-hybridised carbon of a non-aromatic carbon–carbon double bond.

- An allylic halide instead has the halogen on an sp^3 carbon next to a double bond; an aryl halide has it attached directly to an aromatic ring carbon.

Step 1: Locate the carbon directly bonded to X The classification depends on the carbon carrying the halogen, denoted by X, and not simply on whether a double bond occurs somewhere in the molecule. A carbon on an alkene double bond is sp^2-hybridised. A neighbouring saturated carbon is sp^3-hybridised. Follow the C–X bond in each structure and inspect that particular carbon before deciding the class.

Step 2: Examine options A, B and C In (A), X is attached to a saturated ring carbon adjacent to the cyclohexene double bond. It is an allylic halide. In (B), X is carried by the external CH_2X group; that carbon is also saturated and is attached to an alkene carbon, so this is again an allylic halide. In (C), X is bonded directly to a benzene ring. Although that carbon is sp^2-hybridised, aromatic halogen compounds form the distinct class of aryl halides, rather than vinylic halides.

Step 3: Check the exocyclic double bond in option D In (D), the double bond extends from the ring to an external carbon, and X is attached directly to that external alkene carbon. Thus the relevant fragment is $C = CH - X$: the C–X bond originates on a carbon that participates in the non-aromatic double bond itself. The fact that the other end of the double bond belongs to a ring does not change this classification.

Step 4: Conclusion Option (D) has X directly bonded to a carbon of the exocyclic carbon–carbon double bond and is a vinylic halide. Hence, the correct option is (D).

Quick Tip to Solve: Trace the C–X bond: X on an alkene carbon means vinylic; X on the saturated carbon next to the alkene means allylic; X on benzene means aryl.

63. Given below are two statements :

Statement (I) : The gas liberated on warming a salt with dil H_2SO_4, turns a piece of paper dipped in lead acetate into black, it is a confirmatory test for sulphide ion.

Statement (II) : In statement-I the colour of paper turns black because of formation of lead sulphite. In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are true

(D) Statement I is false but Statement II is true

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- Sulphide ions react with dilute acids to release hydrogen sulphide gas: $S^{2-} + 2\,H^+ \longrightarrow H_2S\uparrow$.
- Hydrogen sulphide reacts with lead(II) ions to form black lead(II) sulphide, PbS. This black precipitate identifies the sulphide ion in the stated test.

Step 1: Evaluate Statement I On warming a sulphide-containing salt with dilute sulphuric acid, the sulphide ion accepts protons and produces H_2S. Lead acetate paper supplies Pb^{2+} ions, which react with the gas to deposit black lead sulphide. The visible blackening of the paper is the stated confirmatory observation. Thus Statement I correctly describes both the evolution of the gas and the qualitative test.

Step 2: Identify the black compound and evaluate Statement II The reaction on the moist lead acetate paper can be written as

$$Pb(CH_3COO)_2 + H_2S \longrightarrow PbS\downarrow + 2\,CH_3COOH.$$

The black deposit is PbS, lead(II) sulphide. It is not lead sulphite, $PbSO_3$. Sulphide contains the ion S^{2-}, whereas sulphite contains the different polyatomic ion $SO_3{}^{2-}$. Replacing the suffix therefore changes the chemical species and makes Statement II false.

Step 3: Choose the matching truth combination The results are: Statement I true, Statement II false. Option (A) wrongly rejects the valid test, while (C) wrongly accepts lead sulphite as the black product. Option (D) reverses both truth values. Only option (B) agrees with the acid reaction and the chemical identity of the precipitate observed on the paper.

Step 4: Conclusion Statement I is true and Statement II is false: the black compound is PbS, not lead sulphite. Hence, the correct option is (B).

Quick Tip to Solve: In the lead acetate paper test, remember the product by its formula PbS: black lead sulphide confirms sulphide, while sulphite contains oxygen.

64. The final product A, formed in the following multistep reaction sequence is :

$$C_6H_5Br \xrightarrow[\text{(ii) } NH_3,\,\Delta;\ \text{(iii) } Br_2,\,NaOH]{\text{(i) Mg, ether; then } CO_2,\,H^+} A$$

[4 marks]

(A) C_6H_5COBr

(B) C_6H_5COOH

(C) $C_6H_5NH_2$

(D) $C_6H_5CONH_2$

Correct Answer: (C) $C_6H_5NH_2$

Solution:

Concept:

- An aryl bromide reacts with magnesium in dry ether to give a Grignard reagent, C_6H_5MgBr. The carbon bonded to magnesium behaves as a nucleophile.
- Carbonation of a Grignard reagent adds one carbon and acidification gives a carboxylic acid. Heating its ammonium salt produces the corresponding primary amide.
- Hofmann bromamide degradation with Br_2/NaOH converts $RCONH_2$ into RNH_2, losing the car-

bonyl carbon.

Step 1: Form and carbonate the Grignard reagent Magnesium inserts into the aryl carbon–bromine bond under anhydrous conditions. The resulting phenylmagnesium bromide attacks the electrophilic carbon of carbon dioxide. Acid work-up protonates the carboxylate:

$$C_6H_5Br \xrightarrow{Mg,\ ether} C_6H_5MgBr \xrightarrow[H^+]{CO_2} C_6H_5COOH.$$

The isolated product of the first stage is benzoic acid; its carboxyl carbon comes from carbon dioxide.

Step 2: Convert the acid into the amide Ammonia first forms ammonium benzoate with benzoic acid. Heating the salt eliminates water to form benzamide:

$$C_6H_5COOH + NH_3 \longrightarrow C_6H_5COONH_4 \xrightarrow{\Delta} C_6H_5CONH_2 + H_2O.$$

Thus the intermediate just before the last reagent is the primary amide, not an acyl bromide.

Step 3: Apply the carbon-loss rearrangement Bromine in aqueous sodium hydroxide causes Hofmann rearrangement of benzamide. The phenyl group migrates from the carbonyl carbon to nitrogen; hydrolysis of the isocyanate intermediate produces aniline. The balanced overall change is

$$C_6H_5CONH_2 + Br_2 + 4\,NaOH \longrightarrow C_6H_5NH_2 + 2\,NaBr + Na_2CO_3 + 2\,H_2O.$$

Option (A), benzoyl bromide, is not produced by these reagents. Benzoic acid in (B) and benzamide in (D) are intermediates consumed by the subsequent stages.

Step 4: Conclusion The final product A is aniline, $C_6H_5NH_2$. Hence, the correct option is (C).

Quick Tip to Solve: Track the carbon count: carbonation adds one carbon, while Hofmann rearrangement removes that same carbonyl carbon and attaches NH_2 to phenyl.

65.

$$C_6H_5COCl + H_2 \xrightarrow{Pd-BaSO_4} C_6H_5CHO$$

This reduction reaction is known as: [4 marks]

(A) Wolff-Kishner reduction

(B) Etard reduction

(C) Stephen reduction

(D) Rosenmund reduction

Correct Answer: (D) Rosenmund reduction

Solution:

Concept:

- Rosenmund reduction selectively hydrogenates an acid chloride, RCOCl, to an aldehyde, RCHO, using hydrogen over palladium supported on barium sulfate.
- The moderated activity of the supported catalyst limits further hydrogenation of the newly formed

aldehyde. The carbonyl group remains in the product.

Step 1: Identify the functional-group conversion The reactant is benzoyl chloride, C_6H_5COCl, and the product drawn is benzaldehyde, C_6H_5CHO. Thus chlorine attached to the carbonyl carbon is replaced by hydrogen; the benzene ring and the carbonyl carbon remain unchanged. The equation is

$$C_6H_5COCl + H_2 \xrightarrow{Pd-BaSO_4} C_6H_5CHO + HCl.$$

This is partial reduction of a carboxylic acid derivative, not complete removal of the carbonyl oxygen.

Step 2: Match the reagent to its named reaction Hydrogen with $Pd/BaSO_4$ is the characteristic Rosenmund reagent. Its reduced catalytic activity permits isolation of the aldehyde rather than continued reduction to benzyl alcohol. Therefore the shown starting functional group, product and reagent all identify the same named reduction.

Step 3: Eliminate the other named reactions Wolff–Kishner reduction uses hydrazine and strong base with heat to convert an aldehyde or ketone carbonyl group into a methylene group; it would not leave the shown aldehyde. The Etard reaction uses chromyl chloride to oxidise a benzylic methyl group to an aldehyde, so it is not this hydrogenation of an acid chloride. Stephen reduction starts from a nitrile, uses $SnCl_2/HCl$, and gives an aldehyde after hydrolysis. None matches the acid chloride and supported palladium shown here.

Step 4: Conclusion The conversion of benzoyl chloride into benzaldehyde with $H_2/Pd-BaSO_4$ is Rosenmund reduction. Hence, the correct option is (D).

Quick Tip to Solve: Look first at the starting group: acid chloride plus $H_2/Pd-BaSO_4$ means Rosenmund; nitrile plus $SnCl_2/HCl$ means Stephen.

66. Compound A formed in the following reaction reacts with B gives the product C. Find out A and B.

$$CH_3-C\equiv CH + Na \longrightarrow A \xrightarrow{B} CH_3-C\equiv C-CH_2-CH_2-CH_3 + NaBr$$

[4 marks]

(A) $A = CH_3-C\equiv C^- Na^+, \quad B = CH_3-CH_2-CH_2-Br$

(B) $A = CH_3-CH=CH_2, \quad B = CH_3-CH_2-CH_2-Br$

(C) $A = CH_3-C\equiv C^- Na^+, \quad B = CH_3-CH_2-CH_3$

(D) $A = CH_3-CH_2-CH_3, \quad B = CH_3-C\equiv CH$

Correct Answer: (A) $A = CH_3-C\equiv C^- Na^+, \quad B = CH_3-CH_2-CH_2-Br$

Solution:

Concept:

- A terminal alkyne contains an acidic hydrogen on an sp-hybridised carbon. Sodium metal replaces that hydrogen to form a sodium alkynide, liberating hydrogen gas.
- The alkynide carbon is a nucleophile. It reacts with a primary alkyl bromide by S_N2 substitution, creating a carbon–carbon bond and eliminating bromide.

Step 1: Identify A from deprotonation The starting alkyne is propyne, $CH_3-C\equiv CH$. Its terminal hydrogen is replaced by sodium without reducing the triple bond. Thus

$$2\,CH_3-C\equiv CH + 2\,Na \longrightarrow 2\,CH_3-C\equiv C^-\,Na^+ + H_2.$$

Therefore A is sodium propynide, $CH_3-C\equiv C^-\,Na^+$. Neither propene nor propane can be the product of this metal–hydrogen replacement reaction.

Step 2: Determine the alkyl group introduced by B The product is $CH_3-C\equiv C-CH_2-CH_2-CH_3$, hex-2-yne. Compare it with the alkynide: the new group attached to the negatively charged terminal carbon is an n-propyl group, $-CH_2-CH_2-CH_3$. To obtain the displayed sodium bromide by-product, B must supply that group attached to bromine:

$$B = CH_3-CH_2-CH_2-Br.$$

The primary bromide is suitable for substitution by the carbon nucleophile.

Step 3: Verify the substitution and reject the other pairs Attack at the carbon bearing bromine gives

$$CH_3-C\equiv C^-\,Na^+ + Br-CH_2-CH_2-CH_3 \longrightarrow CH_3-C\equiv C-CH_2-CH_2-CH_3 + NaBr.$$

Option (A) supplies both the required alkynide and the required primary bromide. Option (B) gives neutral propene as A, which cannot perform this substitution. In (C), propane has no leaving group, and (D) gives two neutral hydrocarbons rather than the nucleophile–electrophile pair. The product also preserves the original three-carbon alkyne unit, confirming the carbon accounting.

Step 4: Conclusion $A = CH_3-C\equiv C^-\,Na^+$ and $B = CH_3-CH_2-CH_2-Br$. Hence, the correct option is (A).

Quick Tip to Solve: For alkynide alkylation, disconnect the new bond at the former terminal alkyne carbon; the attached alkyl fragment must come from an alkyl halide.

67. Which of the following molecule/species is most stable? [4 marks]

(A) Cyclohexa-1,3-diene 

(B) Cyclopropenyl cation 

(C) Cyclopentadienyl cation 

(D) Cyclopropenyl anion 

Correct Answer: (B) Cyclopropenyl cation 

Solution:

Concept:

- An aromatic ring must be cyclic, planar and continuously conjugated, with a p orbital at every ring atom. Its delocalised pi-electron count is $4n+2$, where $n = 0, 1, 2, \ldots$.
- A fully conjugated planar ring containing $4n$ pi electrons is antiaromatic and destabilised. A break in the p-orbital circuit makes a ring nonaromatic rather than aromatic.

Step 1: Count the pi electrons in the three-membered cation The cyclopropenyl cation in option (B) has one carbon–carbon double bond and one positively charged carbon with an empty p orbital. All three ring carbons can therefore take part in continuous cyclic conjugation. The double bond contributes two pi electrons; the empty p orbital contributes none. Hence

$$N_\pi = 2 = 4(0) + 2.$$

It obeys Huckel's rule and receives aromatic stabilisation through delocalisation of the positive charge around the triangle.

Step 2: Classify the other three structures Option (A) is cyclohexa-1,3-diene: its saturated carbons interrupt cyclic conjugation. It is a conjugated diene, but it does not possess an aromatic circuit. The cyclopentadienyl cation in (C) has two double bonds, supplying four pi electrons, and an empty p orbital on the cationic carbon. A planar conjugated form has 4π electrons and is antiaromatic. Option (D), the cyclopropenyl anion, has two electrons from the double bond and two from the carbanionic lone pair, again giving four pi electrons if cyclic conjugation is maintained. It is not the aromatic two-electron species.

Step 3: Use aromatic stabilisation to choose the species The comparison tests the special stabilisation of a closed cyclic delocalised electron shell, rather than the mere presence of a double bond or a formal charge. Of the structures shown, only the cyclopropenyl cation has the uninterrupted ring of p orbitals and the $4n + 2$ count. The nonaromatic diene lacks this stabilisation, while the four-electron ions are destabilised in the conjugated planar arrangement and may distort to avoid antiaromaticity.

Step 4: Conclusion The aromatic cyclopropenyl cation is the most stable species in this aromaticity comparison. Hence, the correct option is (B).

Quick Tip to Solve: Count a carbocation p orbital as zero pi electrons and a conjugating carbanion lone pair as two; this distinguishes the two triangle ions immediately.

68. What happens to freezing point of benzene when small quantity of napthalene is added to benzene? [4 marks]

(A) Increases

(B) Decreases

(C) Remains unchanged

(D) First decreases and then increases

Correct Answer: (B) Decreases

Solution:

Concept:

- For a dilute solution of a nonvolatile solute, freezing-point depression is $\Delta T_f = T_f^\circ - T_f = iK_f m$, where K_f is the solvent cryoscopic constant and m is solute molality.
- Naphthalene dissolves in benzene as neutral molecules. It is not an electrolyte, so the ideal dilute-solution van't Hoff factor is $i = 1$.

Step 1: Identify solvent and solute Benzene is the solvent because a small quantity of naphthalene is being added to it. Naphthalene acts as the dissolved molecular solute. The small amount specified corresponds to the dilute-solution situation in which freezing-point depression can be applied. No ionic dissociation is involved; the solute nevertheless changes the equilibrium properties of the solvent.

Step 2: Apply the sign of freezing-point depression Both K_f and the molality of an added positive amount of solute are positive. Therefore

$$\Delta T_f = K_f m > 0, \qquad T_f = T_f^\circ - K_f m < T_f^\circ.$$

The solution freezes below the freezing point of pure benzene. Microscopically, mixing lowers the chemical potential of liquid benzene, so a lower temperature is needed before the liquid solvent equilibrates with solid benzene.

Step 3: Compare all four proposed changes A rise, option (A), has the wrong sign for the freezing-point effect of a dilute nonvolatile solute. Option (C) ignores the added solute particles and would correspond to zero molality in this model. Option (D) proposes a reversal as solute is added, but the dilute relation predicts an increasing positive depression with increasing molality; it provides no such reversal. Hence a small addition lowers the freezing point. This concerns the freezing temperature of the benzene solution, not the melting point of pure naphthalene; confusing the two substances can suggest the incorrect direction.

Step 4: Conclusion The freezing point of benzene decreases when a small quantity of naphthalene is dissolved in it. Hence, the correct option is (B).

Quick Tip to Solve: Write $T_f = T_f^\circ - iK_f m$ before choosing the direction: dissolved nonvolatile solute subtracts a positive quantity from the solvent freezing point.

69. Match List I with List II.

	List I: Molecule		List II: Shape
(A)	BrF_5	(I)	T-Shape
(B)	H_2O	(II)	See saw
(C)	ClF_3	(III)	Bent
(D)	SF_4	(IV)	Square pyramidal

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)

(B) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(C) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(D) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

Correct Answer: (B) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Solution:

Concept:

- VSEPR theory determines molecular shape by first counting bonding domains and lone pairs around the central atom. A multiple bond, if present, counts as one electron domain.
- Lone pairs affect the domain arrangement but are omitted when naming the molecular shape. Thus the electron-pair geometry and the molecular shape need not be identical.

Step 1: Assign the shapes of bromine pentafluoride and water In BrF_5, bromine has seven valence electrons. Five bonding pairs leave one lone pair, giving six electron domains: AX_5E. The domain arrangement is octahedral, with one position occupied by the lone pair, so the molecular shape is square pyramidal. Therefore (A) matches (IV). In H_2O, oxygen has two bonds and two lone pairs, AX_2E_2. Its four domains are approximately tetrahedral, but the positions of the atoms give a bent molecular shape. Thus (B) matches (III).

Step 2: Assign the shapes of chlorine trifluoride and sulfur tetrafluoride Chlorine in ClF_3 has three bonding pairs and two lone pairs, AX_3E_2. Its five-domain arrangement is trigonal bipyramidal. Both lone pairs prefer equatorial sites to reduce 90-degree repulsions, leaving a T-shaped arrangement of the three fluorines. Hence (C) matches (I). Sulfur in SF_4 has four bonds and one lone pair, AX_4E. The lone pair occupies an equatorial site of the trigonal bipyramid; the four remaining atomic positions form a see-saw. Thus (D) matches (II).

Step 3: Compare the complete mapping with the options Collecting the assignments gives

$$(\mathrm{A}) - (\mathrm{IV}), \quad (\mathrm{B}) - (\mathrm{III}), \quad (\mathrm{C}) - (\mathrm{I}), \quad (\mathrm{D}) - (\mathrm{II}).$$

Only option (B) contains all four matches. Option (A) incorrectly calls BrF_5 T-shaped and water see-saw. Option (C) interchanges the first two shapes. Option (D) assigns the see-saw shape to BrF_5 and the T-shape to water, contradicting their respective domain counts.

Step 4: Conclusion The complete matching derived above selects option (B). Hence, the correct option is (B).

Quick Tip to Solve: Count lone pairs before naming the shape: AX_5E is square pyramidal, AX_3E_2 T-shaped and AX_4E see-saw.

70. Match List I with List II.

	List I: Species		List II: Electronic distribution
(A)	Cr^{2+}	**(I)**	$3d^8$
(B)	Mn^{+}	**(II)**	$3d^3 4s^1$
(C)	Ni^{2+}	**(III)**	$3d^4$
(D)	V^{+}	**(IV)**	$3d^5 4s^1$

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(B) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(C) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(D) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Correct Answer: (A) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Solution:

Concept:

- For transition-metal cations, electrons are removed from the outer $4s$ subshell before the $3d$ subshell. The inner $[Ar]$ core remains unchanged here.
- Use the actual neutral ground-state configuration: chromium is $[Ar]3d^5 4s^1$, whereas manganese, nickel and vanadium are $[Ar]3d^5 4s^2$, $[Ar]3d^8 4s^2$ and $[Ar]3d^3 4s^2$, respectively.

Step 1: Find the chromium and manganese ion configurations For Cr^{2+}, remove the single $4s$ electron of neutral chromium first and then one $3d$ electron:

$$[Ar]3d^5 4s^1 \longrightarrow [Ar]3d^5 \longrightarrow [Ar]3d^4.$$

Thus (A) matches (III). For Mn^{+}, remove just one of the two $4s$ electrons from the neutral atom. The result is $[Ar]3d^5 4s^1$, so (B) matches (IV). The singly charged manganese ion retains one $4s$ electron; removing both would describe a different ion.

Step 2: Find the nickel and vanadium ion configurations Nickel loses both of its $4s$ electrons on forming Ni^{2+}, leaving $[Ar]3d^8$, so (C) matches (I). Vanadium loses one $4s$ electron on forming V^{+}, leaving $[Ar]3d^3 4s^1$, so (D) matches (II). The listed distributions omit the common argon core, as is customary when comparing valence configurations.

Step 3: Match the entire sequence The ordered mapping is

$$(A) - (III), \quad (B) - (IV), \quad (C) - (I), \quad (D) - (II).$$

It is precisely option (A). Option (B) assigns nickel's $3d^8$ distribution to chromium, and option (C) swaps the chromium and manganese entries. Option (D) puts vanadium's configuration against chromium and gives V^{+} chromium's distribution. These fail both the electron counts and the rule for removing outer $4s$ electrons first.

Step 4: Conclusion The complete matching derived above selects option (A). Hence, the correct option is (A).

Quick Tip to Solve: Write the neutral configuration first, including chromium's exception, then remove $4s$ electrons before $3d$ electrons.

71. The Lassiagne's extract is boiled with dil HNO_3 before testing for halogens because, [4 marks]

(A) Silver halides are soluble in HNO_3.

(B) $AgCN$ is soluble in HNO_3.

(C) Na_2S and $NaCN$ are decomposed by HNO_3.

(D) Ag_2S is soluble in HNO_3.

Correct Answer: (C) Na_2S and $NaCN$ are decomposed by HNO_3.

Solution:

Concept:

- Lassaigne's sodium-fusion extract may contain NaX, $NaCN$, and Na_2S when halogen, nitrogen, and sulphur occur in the compound.
- The silver nitrate test detects halide ions through precipitation of AgX. Other anions which precipitate silver must be removed first.

Step 1: Identify the interfering ions Adding $AgNO_3$ directly to an untreated extract can produce white $AgCN$ from cyanide and black Ag_2S from sulphide. These precipitates interfere with the observation of a silver halide and can give a misleading positive result. Thus acidification is a pretreatment of the extract, rather than a reaction meant to dissolve the desired halide precipitate.

Step 2: Remove the interference before testing Boiling with dilute nitric acid decomposes the sodium cyanide and sodium sulphide, releasing volatile products which are expelled on heating. The acid decomposition can be represented by

$$NaCN + HNO_3 \longrightarrow NaNO_3 + HCN\uparrow$$

$$Na_2S + 2\,HNO_3 \longrightarrow 2\,NaNO_3 + H_2S\uparrow$$

After removal of the interfering species, addition of silver nitrate gives the halide test. Silver halides are not soluble in dilute nitric acid, so option (A) reverses the required property. Options (B) and (D) describe silver salts instead of the removal of cyanide and sulphide from the extract before silver nitrate is added.

Step 3: Conclusion Na_2S and $NaCN$ are decomposed by nitric acid, removing interference in the halogen test. Hence, the correct option is (C).

Quick Tip to Solve: For Lassaigne's halogen test, remember the order: boil the extract with dilute nitric acid first, then add silver nitrate; the acid removes cyanide and sulphide interference.

72. Given below are two statements :

Statement (I) : The orbitals having same energy are called as degenerate orbitals.

Statement (II) : In hydrogen atom, 3p and 3d orbitals are not degenerate orbitals.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is true but Statement II is false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are false

(D) Both Statement I and Statement II are true

Correct Answer: (A) Statement I is true but Statement II is false

Solution:

Concept:

- Degenerate orbitals are distinct orbitals with the same energy; they need not have the same shape or orientation.
- For the hydrogen atom in the usual nonrelativistic atomic model, the orbital energy depends only on the principal quantum number n: $E_n = -13.6/n^2\,\text{eV}$.

Step 1: Judge Statement I from the definition Statement I says that orbitals having the same energy are degenerate orbitals. This is precisely the definition of degeneracy, so Statement I is true. Equal energy is the deciding condition; equality of the azimuthal quantum number or of orbital shape is not required. For example, different orientations within one subshell provide a familiar instance of degeneracy.

Step 2: Compare the hydrogen 3p and 3d energies Both $3p$ and $3d$ have $n = 3$. Their azimuthal quantum numbers differ, but that difference does not change their energy in the one-electron hydrogen atom. Substitution gives

$$E_{3p} = E_{3d} = -\frac{13.6}{9}\,\text{eV}.$$

Therefore these orbitals are degenerate, and the assertion that they are not degenerate is false. The familiar energy splitting between different subshells of the same shell belongs to many-electron atoms, where electron shielding and penetration affect the energies. Applying that many-electron ordering to hydrogen would lead to the wrong choice here. Option (B) reverses both truth values; (C) wrongly rejects the definition; and (D) wrongly accepts the claimed splitting.

Step 3: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: When degeneracy is asked, identify the atom first: in hydrogen compare only n; do not apply the many-electron subshell ordering to its $3p$ and $3d$ orbitals.

73. Sugar which does not give reddish brown precipitate with Fehling's reagent, is : [4 marks]

(A) Glucose

(B) Sucrose

(C) Lactose

(D) Maltose

Correct Answer: (B) Sucrose

Solution:

Concept:

- Fehling's reagent is an alkaline copper(II) complex. A reducing sugar converts it to reddish brown Cu_2O on heating.
- A sugar needs a free anomeric hemiacetal or hemiketal centre that can open to a reactive carbonyl form to behave as a reducing sugar under the test conditions.

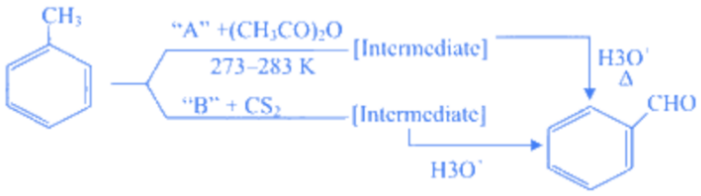
Step 1: Inspect the anomeric centres of sucrose Sucrose joins the anomeric carbon of glucose to the anomeric carbon of fructose through its glycosidic linkage. Consequently neither sugar unit has a free anomeric hydroxyl group in the intact disaccharide. The linkage locks both centres, preventing the normal ring-chain opening needed to provide a reducing carbonyl group. Sucrose is therefore non-reducing and does not precipitate reddish brown copper(I) oxide in this test.

Step 2: Check the other three sugars Glucose has a free hemiacetal group and can form its open-chain aldehyde; it reduces Fehling's reagent. Lactose and maltose each have one anomeric centre involved in the glycosidic bond but retain another free anomeric centre. Each therefore has a reducing end and gives the precipitate. Thus options (A), (C), and (D) all describe reducing sugars, whereas option (B) alone describes a non-reducing sugar. Hydrolysis of sucrose would produce glucose and fructose and change its behaviour, but the question asks about sucrose itself, with no prior hydrolysis step.

Step 3: Conclusion Sucrose does not give the reddish brown precipitate with Fehling's reagent. Hence, the correct option is (B).

Quick Tip to Solve: Count free anomeric centres rather than the number of sugar units: sucrose links both anomeric centres, while maltose and lactose retain one reducing end.

74. In the given reactions, identify the reagent A and reagent B. [4 marks]



(A) $A - CrO_3$ $B - CrO_3$

(B) $A - CrO_3$ $B - CrO_2Cl_2$

(C) A- CrO_2Cl_2 $B - CrO_3$

(D) $A - CrO_2Cl_2$ $B - CrO_2Cl_2$

Correct Answer: (B) $A - CrO_3$ $B - CrO_2Cl_2$

Solution:

Concept:

- Toluene can be oxidised selectively at its benzylic methyl group to benzaldehyde if the reaction conditions prevent further oxidation to benzoic acid.
- Chromium trioxide in acetic anhydride traps the oxidation product as benzylidene diacetate; chromyl chloride in carbon disulphide performs the Etard oxidation.

Step 1: Identify reagent A from acetic anhydride The upper route uses reagent A with acetic anhydride at 273–283 K, followed by acidic hydrolysis with heating. These conditions identify chromium trioxide, CrO_3. The benzylic group is converted into benzylidene diacetate, $C_6H_5CH(OCOCH_3)_2$, which protects the aldehyde during oxidation. Acidic hydrolysis then releases benzaldehyde, C_6H_5CHO. Thus the presence of acetic anhydride and the separate hydrolysis step specifically identify the first reagent.

Step 2: Identify reagent B from carbon disulphide The lower route uses reagent B in CS_2, followed by hydrolysis. This is the Etard reaction: chromyl chloride, CrO_2Cl_2, forms a chromium complex at the benzylic carbon, and hydrolysis gives benzaldehyde. Both routes reach the same aldehyde through different intermediates; they do not require the same chromium reagent. Option (A) assigns chromium trioxide to the Etard route and is incorrect. Option (C) interchanges the two reagents, while (D) incorrectly puts chromyl chloride into the acetic-anhydride route.

Step 3: Conclusion Reagent A is CrO_3, and reagent B is CrO_2Cl_2. Hence, the correct option is (B).

Quick Tip to Solve: Use the solvent as the clue: CS_2 with chromyl chloride signals Etard oxidation; acetic anhydride with CrO_3 signals protection as the diacetate before hydrolysis.

75. Diamagnetic Lanthanoid ions are : [4 marks]

(A) La^{3+} & Ce^{4+}

(B) Nd^{3+} & Ce^{4+}

(C) Lu^{3+} & Eu^{3+}

(D) Nd^{3+} & Eu^{3+}

Correct Answer: (A) La^{3+} & Ce^{4+}

Solution:

Concept:

- An ion is diamagnetic when all its electrons are paired. In lanthanoid ions, check the occupancy of the $4f$ subshell after removing the outer electrons.
- An empty $4f^0$ shell and a completely filled $4f^{14}$ shell have no unpaired electrons; partially occupied $4f$ shells generally contain unpaired electrons.

Step 1: Determine the configurations in option A Lanthanum has the neutral configuration $[Xe]5d^16s^2$. Removing three electrons gives La^{3+} : $[Xe]$, with $4f^0$. Cerium loses its two $6s$ electrons, its $5d$ electron and its $4f$ electron on forming Ce^{4+}, also giving $[Xe]$, or $4f^0$. Both ions therefore have closed-shell configurations and are diamagnetic. Their different oxidation states do not change this conclusion because the electron counts, rather than the charge labels alone, determine magnetism.

Step 2: Eliminate pairs containing open f shells Nd^{3+} has configuration $[Xe]4f^3$, with three unpaired electrons, so it is paramagnetic. Eu^{3+} has configuration $[Xe]4f^6$, with unpaired electrons in the standard orbital description, and is also classified as paramagnetic. Lu^{3+} is $[Xe]4f^{14}$ and is diamagnetic, but the pair in option (C) also contains Eu^{3+}, so that pair fails. Options (B) and (D) contain Nd^{3+} and fail immediately. Only option (A) contains two ions without unpaired electrons.

Step 3: Conclusion The required pair is La^{3+} and Ce^{4+}. Hence, the correct option is (A).

Quick Tip to Solve: Write the ionic $4f^n$ configuration before judging magnetism: La^{3+} and Ce^{4+} are $4f^0$, while Lu^{3+} is $4f^{14}$.

76. Choose the correct statements from the following :

(A) Ethane-1, 2-diamine is a chelating ligand.

(B) Metallic aluminium is produced by electrolysis of aluminium oxide in presence of cryolite.

(C) Cyanide ion is used as ligand for leaching of silver.

(D) Phosphine act as a ligand in Wilkinson catalyst.

(E) The stability constants of Ca^{2+} and Mg^{2+} are similar with EDTA complexes.

Choose the correct answer from the options given below : [4 marks]

(A) (B), (C), (E) only

(B) (A), (D), (E) only

(C) (C), (D), (E) only

(D) (A), (B), (C) only

Correct Answer: (D) (A), (B), (C) only

Solution:

Concept:

- A chelating ligand binds through two or more donor atoms to the same metal centre, forming a ring. Coordination also explains leaching and the behaviour of homogeneous catalysts.
- The stability of a metal–ligand complex is measured by its formation constant; sharing the same ligand does not make different metal ions have equal stability constants.

Step 1: Check statements A, B and C (A) Ethane-1,2-diamine has two nitrogen donor atoms and forms a chelate ring when both coordinate to a metal ion, so (A) is correct. (B) Aluminium is obtained by electrolysis of alumina dissolved in molten cryolite; cryolite lowers the working melting temperature and improves conductivity, so (B) is correct. (C) Cyanide forms the soluble silver complex $[Ag(CN)_2]^-$, enabling silver leaching; subsequent displacement recovers the metal. Thus (C) is also correct. These statements describe three different applications but each follows the appropriate chemical process.

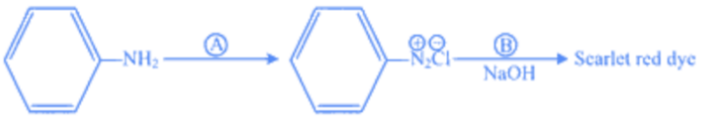
Step 2: Check D and E and match the combination (D) Wilkinson's catalyst is $[RhCl(PPh_3)_3]$; its phosphorus donor ligand is triphenylphosphine, not phosphine PH_3. The wording in (D) therefore makes it false. (E) The EDTA formation constants of calcium and magnesium are appreciably different: the calcium complex is substantially more stable, which permits selective complexometric applications. Their logarithmic formation constants are approximately 10.7 and 8.7 respectively, a difference of about two powers of ten rather than similar constants. Thus (E) is false. Every option except (D) includes the false statement (E); the only listed combination containing exactly the true statements is (A), (B), (C).

Step 3: Conclusion The first three statements are correct; the last two are false. Hence, the correct option is (D).

Quick Tip to Solve: For this combination question, identify Wilkinson's ligand as triphenylphosphine first, then distinguish the calcium and magnesium EDTA constants; both checks eliminate every choice containing (E).

77. Following is a confirmatory test for aromatic primary amines. Identify reagent (A) and (B).

[4 marks]



(A) A: $NaNO_2 + HCl$, 0–5 °C; B: naphthalen-2-ol (β-naphthol).

(B) A: $NaNO_2 + HCl$, 0–5 °C; B: aniline.

(C) A: $NaNO_2 + HCl$, 0–5 °C; B: naphthalen-1-ol (α-naphthol).

(D) A: HNO_3/H_2SO_4; B: phenol.

Correct Answer: (A) A: $NaNO_2 + HCl$, 0–5 °C; B: naphthalen-2-ol (β-naphthol).

Solution:

Concept:

- A primary aromatic amine reacts with nitrous acid at 0–5 °C to form an aromatic diazonium salt. Nitrous acid is generated in situ from sodium nitrite and hydrochloric acid.
- A diazonium salt couples with activated aromatic rings, especially phenoxide or naphthoxide ions, to form a coloured azo compound containing the $-N=N-$ linkage.

Step 1: Identify reagent A in the first conversion The reaction scheme changes aniline, $C_6H_5NH_2$, into benzenediazonium chloride, $C_6H_5N_2{}^+Cl^-$. Hence A must provide nitrous acid in hydrochloric acid at low temperature:

$$NaNO_2 + HCl \longrightarrow HNO_2 + NaCl.$$

The mixture is kept at 0–5 °C to preserve the diazonium salt. The nitrating mixture in option (D) does not carry out this diazotisation, so that option fails at the first step.

Step 2: Identify reagent B from the scarlet dye test The standard confirmatory azo-dye test couples the cold diazonium solution with alkaline β-naphthol, also called naphthalen-2-ol. Sodium hydroxide generates the more nucleophilic naphthoxide ion; electrophilic azo coupling produces the strongly coloured scarlet-red dye. In option (A), the hydroxyl group is at the beta position of the naphthalene system, matching this reagent. Option (B) instead shows aniline, while (C) shows the alpha isomer, naphthalen-1-ol. They are not the beta-naphthol reagent used for the specified confirmatory scarlet-red dye test. The positions of the hydroxyl groups in the drawings therefore distinguish (A) from the otherwise similar option (C).

Step 3: Conclusion A is $NaNO_2 + HCl$ at 0–5 °C, and B is β-naphthol (naphthalen-2-ol). Hence, the correct option is (A).

Quick Tip to Solve: For the aromatic primary-amine dye test, pair cold diazotisation with alkaline beta-naphthol; inspect the hydroxyl position to distinguish beta-naphthol from its alpha isomer.

78. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : $CH_2 = CH - CH_2 - Cl$ **is an example of allyl halide.**

Reason (R) : Allyl halides are the compounds in which the halogen atom is attached to sp^2 **hybridised carbon atom.**

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) (A) is true but (R) is false

(B) (A) is false but (R) is true

(C) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(D) Both (A) and (R) are true and (R) is the correct explanation of (A)

Correct Answer: (A) (A) is true but (R) is false

Solution:

Concept:

- An allylic halide has halogen bonded to a saturated, sp^3-hybridised carbon immediately adjacent to a carbon-carbon double bond.
- In a vinylic halide the halogen is bonded directly to an sp^2-hybridised carbon of the double bond. Classification depends on the carbon bearing halogen, not merely on the presence of a double bond.

Step 1: Test the assertion Write the connectivity as $CH_2{=}CH{-}CH_2{-}Cl$. The chlorine-bearing carbon is the terminal CH_2 carbon. It has single bonds to two hydrogen atoms, chlorine and the adjacent carbon: four sigma bonds and approximately tetrahedral geometry. It is therefore sp^3-hybridised. This saturated carbon is directly next to the $C{=}C$ bond, so it occupies the allylic position. The compound is allyl chloride, and Assertion (A) is true.

Step 2: Test the reason independently Reason (R) says that the halogen in an allyl halide is attached to an sp^2-hybridised carbon. That is the definition of a vinylic attachment, as in $CH_2{=}CH{-}Cl$, rather than an allylic attachment. The two alkene carbons of allyl chloride are sp^2, but neither of those carbons carries chlorine. Hence Reason (R) is false.

Step 3: Match the truth values to the options The required combination is true assertion and false reason. Option (B) reverses both truth values. Options (C) and (D) both require a true reason, so neither can apply; the question of whether R explains A does not arise once R has been shown false.

Step 4: Conclusion Assertion (A) is true, whereas Reason (R) is false. Hence, the correct option is (A).

Quick Tip to Solve: Circle the carbon bonded to the halogen first: next to a double bond means allylic; on the double bond means vinylic.

79. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : There is a considerable increase in covalent radius from N to P. However from As to Bi only a small increase in covalent radius is observed.

Reason (R) : Covalent and ionic radii in a particular oxidation state increases down the group. In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) (A) is false but (R) is true

(B) Both (A) and (R) are true and (R) is the correct explanation of (A)

(C) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(D) (A) is true but (R) is false

Correct Answer: (C) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Solution:

Concept:

- Down Group 15, extra electron shells generally increase covalent radii and ionic radii when ions of the same oxidation state are compared.
- The size increase is not uniform. Intervening filled d and f subshells shield nuclear charge poorly, partly contracting the outer electron cloud and moderating the increase.

Step 1: Evaluate the assertion Nitrogen uses the second shell for its valence orbitals, whereas phosphorus uses the third shell. The added shell produces a substantial increase in covalent radius from N to P. For the heavier Group 15 elements, the increase is less pronounced because the effect of added shells is partly offset by increased effective nuclear attraction associated with poor shielding by intervening d and f electrons. Thus the stated considerable N-to-P increase and smaller As-to-Bi increase are both correct; Assertion (A) is true.

Step 2: Evaluate the reason For atoms down the group, and for ions compared in a particular oxidation state, size generally increases as occupied principal shells are added. Reason (R) states this general trend correctly. It does not claim that the increments are equal, so the smaller increases among the heavier members do not make R false.

Step 3: Test the explanatory link The assertion distinguishes the magnitudes of the increases in two portions of the group. The reason only states the overall direction of increase. It provides no account of why the N-to-P increase is large while that from As to Bi is comparatively small. That distinction requires the shielding and contraction effects discussed above. Thus R is true but is not the correct explanation of the complete assertion. Option (A) wrongly makes A false, option (D) wrongly makes R false, and option (B) incorrectly supplies an explanatory link.

Step 4: Conclusion Both the assertion and reason are true, but the reason does not correctly explain the assertion. Hence, the correct option is (C).

Quick Tip to Solve: In assertion-reason questions, an overall trend does not by itself explain why the size of the change differs between particular elements.

80. Structure of 4-Methylpent-2-enal is : [4 marks]

(A) $CH_2{=}CH{-}CH(CH_3){-}CH_2{-}CHO$

(B) $CH_3{-}CH_2{-}C(CH_3){=}CH{-}CHO$

(C) $CH_3{-}CH(CH_3){-}CH{=}CH{-}CHO$

(D) $CH_3{-}CH_2{-}CH{=}C(CH_3){-}CHO$

Correct Answer: (C) $CH_3{-}CH(CH_3){-}CH{=}CH{-}CHO$

Solution:

Concept:

- The suffix -al denotes an aldehyde. The carbon of the $-CHO$ group belongs to the parent chain and must receive position 1.
- Pent-2-enal has five carbons in the aldehyde-containing parent chain and a double bond between C2 and C3. The prefix 4-methyl places a $-CH_3$ substituent on C4.

Step 1: Build and number the parent chain Begin from the aldehyde end rather than the end nearest the alkene. Numbering the five-carbon chain from $-CHO$ gives C1 at the aldehyde carbon, then C2, C3, C4 and C5. Place the double bond between C2 and C3. Before the substituent is added, the connectivity is $CHO{-}CH{=}CH{-}CH_2{-}CH_3$. The aldehyde group has priority for numbering over the carbon-carbon double bond.

Step 2: Attach the methyl substituent at C4 Replacing one hydrogen at C4 by a methyl group gives

$$CHO{-}CH{=}CH{-}CH(CH_3){-}CH_3.$$

Written in the left-to-right orientation used in the options, the same structure is

$$CH_3{-}CH(CH_3){-}CH{=}CH{-}CHO.$$

The longest chain containing both the aldehyde carbon and double bond has five carbons. Counting from the right gives the double bond at 2 and the methyl group at 4, as required.

Step 3: Eliminate the other structures Option (A), $CH_2{=}CH{-}CH(CH_3){-}CH_2{-}CHO$, has the double bond at position 4 and methyl at 3. Option (B), $CH_3{-}CH_2{-}C(CH_3){=}CH{-}CHO$, has methyl at position 3. Option (D), $CH_3{-}CH_2{-}CH{=}C(CH_3){-}CHO$, has methyl at position 2. Only option (C) combines the aldehyde at C1, the double bond at C2 and the methyl group at C4.

Step 4: Conclusion The required structure is $CH_3{-}CH(CH_3){-}CH{=}CH{-}CHO$, which is 4-methylpent-2-enal. Hence, the correct option is (C).

Quick Tip to Solve: Always count the aldehyde carbon as C1 before locating either a double bond or an alkyl substituent.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The total number of molecular orbitals formed from 2s and 2p atomic orbitals of a diatomic molecule is _____. [4 marks]

Solution:

Concept:

- In the linear combination of atomic orbitals method, the number of independent molecular orbitals formed equals the number of atomic orbitals combined.
- Each atom supplies one $2s$ orbital and three $2p$ orbitals: $2p_x$, $2p_y$ and $2p_z$. Orbital count is distinct from electron count or the number of occupied orbitals.

Step 1: Count the atomic orbitals on both atoms The label $2p$ denotes a subshell, not a single orbital. Its three spatial orientations must all be included. Thus one atom contributes

$$N_{\text{AO,one atom}} = 1 + 3 = 4.$$

A diatomic molecule has two atomic centres, and both supply their $2s$ and $2p$ orbitals. The input basis therefore contains

$$N_{\text{AO,total}} = 2(4) = 8.$$

No $1s$ orbitals are included because the question restricts the count to $2s$ and $2p$.

Step 2: Count the corresponding molecular orbitals The pair of $2s$ orbitals generates a bonding σ_{2s} orbital and an antibonding σ^*_{2s} orbital. The six $2p$ atomic orbitals generate six molecular orbitals: one bonding sigma, two bonding pi, one antibonding sigma and two antibonding pi orbitals. Taking the molecular axis as z, these are σ_{2p_z}, π_{2p_x}, π_{2p_y}, and their three antibonding counterparts. Their energy ordering does not affect their total number.

Step 3: Include bonding and antibonding orbitals The total is two orbitals from the s basis plus six from the p basis:

$$N_{\text{MO}} = 2 + 6 = 8.$$

Counting only the four bonding orbitals would omit half of the independent combinations. Whether these orbitals are occupied depends on the particular molecule, which is unnecessary for the requested count.

Step 4: Conclusion The total number of molecular orbitals is $\boxed{8}$. This is a dimensionless count of orbitals.

Quick Tip to Solve: Count spatial atomic orbitals on both atoms, including all three p orientations; the same number of molecular orbitals must result.

82. The mass of sodium acetate (CH_3COONa) required to prepare $250\ mL$ of $0.35\ M$ aqueous solution is _____ g. (Molar mass of CH_3COONa is $82.02\ g\ mol^{-1}$) [4 marks]

Solution:

Concept:

- Molarity is the amount of solute per litre of final solution: $c = n/V$, where n is in moles and V is in litres.
- The mass of solute is $m = nM_{\mathrm{m}}$, where M_{m} is molar mass. Hence $m = cVM_{\mathrm{m}}$, with consistent units.

Step 1: Convert the solution volume into litres The required volume is the final solution volume, not a volume of solvent to be added. Convert it using $1000\,\mathrm{mL} = 1\,\mathrm{L}$:

$$V = 250\,\mathrm{mL} = 0.250\,\mathrm{L}.$$

The concentration $0.35\,\mathrm{M}$ means $0.35\,\mathrm{mol\,L^{-1}}$. Using 250 directly with this concentration would overestimate the amount by a factor of 1000.

Step 2: Calculate the required amount of sodium acetate Apply the definition of molarity to the formula units of CH_3COONa:

$$n = cV = (0.35\,\mathrm{mol\,L^{-1}})(0.250\,\mathrm{L}) = 0.0875\,\mathrm{mol}.$$

Dissociation into sodium and acetate ions does not double the required amount of sodium acetate: the stated molarity is that of the dissolved solute formula units.

Step 3: Convert the amount to mass and round the entry Use the supplied molar mass of $82.02\,\mathrm{g\,mol^{-1}}$, without adding water of crystallisation:

$$m = (0.0875\,\mathrm{mol})(82.02\,\mathrm{g\,mol^{-1}}) = 7.17675\,\mathrm{g}.$$

Thus approximately $7.18\,\mathrm{g}$ would be weighed to prepare the solution, dissolving it and making up the final volume to 250 mL. For the numerical integer entry, 7.17675 rounds to 7 because its fractional part is below one half.

Step 4: Conclusion The calculated mass is $7.17675\,\mathrm{g}$ (approximately $7.18\,\mathrm{g}$); the required integer entry is $\boxed{7}$ in grams.

Quick Tip to Solve: Use $m = cVM_{\mathrm{m}}$ with volume in litres, and round only after the final multiplication.

83. $0.05\ \text{cm}$ **thick coating of silver is deposited on a plate of** $0.05\ \text{m}^2$ **area. The number of silver atoms deposited on plate are _____** $\times 10^{23}$**. (At mass** $\text{Ag} = 108,\ \text{d} = 7.9\ \text{g cm}^{-3}$**)** [4 marks]

Solution:

Concept:

- For a uniform coating, volume is area times thickness: $V = At$. Its mass is $m = \rho V$, where ρ is the mass density.
- The number of silver atoms is $N = (m/M_{\text{Ag}})N_{\text{A}}$, with $M_{\text{Ag}} = 108\,\text{g mol}^{-1}$ and $N_{\text{A}} \approx 6.022 \times 10^{23}\,\text{mol}^{-1}$.

Step 1: Convert the plate area to square centimetres The density is given in g cm^{-3}, and the thickness is already in centimetres, so express the area in cm^2. Squaring the length conversion gives

$$1\,\text{m}^2 = (100\,\text{cm})^2 = 10^4\,\text{cm}^2.$$

$$A = 0.05 \times 10^4 = 500\,\text{cm}^2.$$

The stated area is the coated area; no additional factor of two for a second face is specified.

Step 2: Calculate the coating volume and mass Multiply the area by the given thickness $t = 0.05\,\text{cm}$:

$$V = (500\,\text{cm}^2)(0.05\,\text{cm}) = 25\,\text{cm}^3.$$

Now use the density supplied in the question, $7.9\,\text{g cm}^{-3}$:

$$m = (7.9)(25) = 197.5\,\text{g}.$$

The density and volume units cancel correctly to give mass in grams.

Step 3: Convert silver mass to atoms Silver is an element, so each mole of Ag contributes one Avogadro number of silver atoms:

$$n = \frac{197.5}{108} = 1.8287\,\text{mol}.$$

$$N = \frac{197.5}{108}(6.022 \times 10^{23}) \approx 11.012 \times 10^{23}\ \text{atoms}.$$

The blank asks for the coefficient of 10^{23}, not the full atom count. Rounding that coefficient to the nearest integer gives 11; this corresponds to approximately 1.10×10^{24} atoms.

Step 4: Conclusion The number deposited is approximately 11.012×10^{23} silver atoms, so the required coefficient is $\boxed{11}$.

Quick Tip to Solve: When density is in g cm^{-3}, convert square metres using 10^4, then multiply by thickness before converting mass to atoms.

84. The compound formed by the reaction of ethanal with semicarbazide contains _____ number of nitrogen atoms. [4 marks]

Solution:

Concept:

- Aldehydes react with semicarbazide, $H_2N{-}NH{-}C(=O){-}NH_2$, to form semicarbazones by condensation at the aldehyde carbonyl group.
- The terminal hydrazino $-NH_2$ group forms the new $C{=}N$ bond. Water is eliminated, while all three nitrogen atoms of semicarbazide remain in the organic product.

Step 1: Identify the reacting groups Ethanal has the structure $CH_3{-}CHO$. Its carbonyl carbon is electrophilic and can react with the terminal hydrazino nitrogen of semicarbazide. Semicarbazide contains three nitrogen atoms in the sequence $H_2N{-}NH{-}C(=O){-}NH_2$: the terminal hydrazino nitrogen, the adjacent nitrogen and the amide nitrogen. The amide group remains part of the reagent-derived fragment during semicarbazone formation.

Step 2: Write the condensation product Addition at the carbonyl group followed by loss of water replaces the aldehyde oxygen by the semicarbazide-derived nitrogen group. The balanced connectivity is

$$CH_3CHO + H_2NNHCONH_2 \longrightarrow CH_3CH{=}NNHCONH_2 + H_2O.$$

The product is ethanal semicarbazone. Its expanded connectivity is

$$CH_3{-}CH{=}N{-}NH{-}C(=O){-}NH_2.$$

The eliminated water contains no nitrogen, so this reaction does not remove any of the three nitrogen atoms present in semicarbazide.

Step 3: Count nitrogen atoms in one product molecule The $CH{=}N$ unit contributes one nitrogen. The following $-NH-$ contributes a second, and the terminal $-CONH_2$ contributes a third. Therefore

$$N_{\text{nitrogen per molecule}} = 1 + 1 + 1 = 3.$$

The answer is a count in one molecule of the compound, rather than a number of moles or the number of nitrogen-containing functional groups. Confusing semicarbazide with hydrazine would incorrectly give only two nitrogen atoms.

Step 4: Conclusion Ethanal semicarbazone contains $\boxed{3}$ nitrogen atoms per molecule; the required numerical entry is 3.

Quick Tip to Solve: Write the retained fragment $=N{-}NH{-}C(=O){-}NH_2$ and count all three nitrogen symbols; condensation loses water, not nitrogen.

85. The pH at which $Mg(OH)_2$ $\left[K_{sp} = 1 \times 10^{-11}\right]$ begins to precipitate from a solution containing 0.10 M Mg^{2+} ions is _____. [4 marks]

Solution:

Concept:

- Precipitation begins when the ionic product first reaches the solubility product. For $Mg(OH)_2$, $K_{sp} = [Mg^{2+}][OH^-]^2$; the exponent two follows from the two hydroxide ions in its formula.
- For an aqueous solution at the usual $25^\circ C$ reference temperature, $pH + pOH = 14$, with $pOH = -\log_{10}[OH^-]$.

Step 1: Write the precipitation threshold The solution initially contains $0.10\,mol\,L^{-1}$ of magnesium ions. At the instant precipitation just starts, no appreciable amount has yet been removed, so this is the magnesium concentration used in the ionic product. Thus

$$1 \times 10^{-11} = 0.10[OH^-]^2.$$

The hydroxide concentration must be squared; using a first power would correspond to a different salt.

Step 2: Calculate the critical hydroxide concentration Dividing by $0.10 = 10^{-1}$ gives

$$[OH^-]^2 = 10^{-10}, \qquad [OH^-] = 10^{-5}\,mol\,L^{-1}.$$

Concentration is positive, so only the positive square root is physical. Below this concentration the ionic product is below K_{sp}, and the solution is unsaturated with respect to the hydroxide.

Step 3: Convert the threshold to pH The corresponding hydroxide exponent is

$$pOH = -\log_{10}(10^{-5}) = 5.$$

Therefore

$$pH = 14 - 5 = 9.$$

Increasing the pH beyond this threshold increases hydroxide concentration and causes precipitation.

Step 4: Conclusion The hydroxide begins to precipitate at $\boxed{pH = 9}$. The numerical entry is $\boxed{9}$; pH is dimensionless.

Quick Tip to Solve: At the onset of precipitation, retain the initial metal-ion concentration and solve for hydroxide first; convert pOH to pH only afterward.

86.

$$2MnO_4^- + bI^- + cH_2O \rightarrow xI_2 + yMnO_2 + z\overline{O}H$$

If the above equation is balanced with integer coefficients, the value of z is _____. [4 marks]

Solution:

Concept:

- In basic medium, balance redox reactions using H_2O, OH^- and electrons. Both atoms and total electric charge must balance.
- Manganese changes from $+7$ in $MnO_4{}^-$ to $+4$ in MnO_2, accepting three electrons per manganese. Iodide changes from -1 to zero in I_2, releasing one electron per iodide ion.

Step 1: Balance the reduction half-reaction Balance manganese first, then oxygen with water and hydrogen with hydroxide in the basic-medium form. The reduction half-reaction is

$$MnO_4{}^- + 2\,H_2O + 3\,e^- \longrightarrow MnO_2 + 4\,OH^-.$$

Both sides contain one Mn, six O and four H atoms. The total charge is -4 on each side, confirming the placement of the three electrons.

Step 2: Balance oxidation and equalise electrons Two iodide ions form one iodine molecule:

$$2\,I^- \longrightarrow I_2 + 2\,e^-.$$

The least common multiple of three and two is six. Multiply the manganese half-reaction by two and the iodine half-reaction by three. This also produces the required fixed coefficient two before permanganate in the question.

Step 3: Add and verify the complete equation Cancel the six electrons to obtain

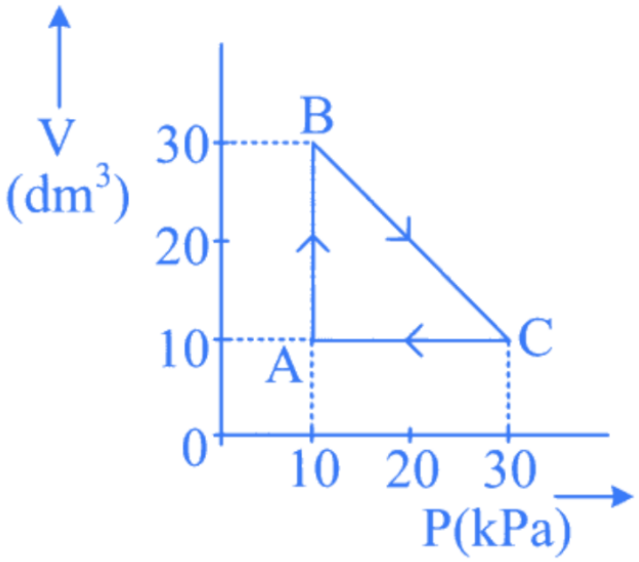
$$2\,MnO_4{}^- + 6\,I^- + 4\,H_2O \longrightarrow 3\,I_2 + 2\,MnO_2 + 8\,OH^-.$$

Hence $b = 6$, $c = 4$, $x = 3$, $y = 2$, and $z = 8$. Oxygen is $8 + 4 = 12$ on the left and $4 + 8 = 12$ on the right; hydrogen is eight on each side. Total charge is $-2 - 6 = -8$ on the left and -8 on the right.

Step 4: Conclusion The coefficient of hydroxide is $\boxed{z = 8}$, a dimensionless integer coefficient.

Quick Tip to Solve: With two permanganate ions reduced to manganese dioxide in basic medium, the balanced reduction half-reaction immediately supplies eight hydroxide ions.

87. An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by tracing the path $\mathrm{A} \to \mathrm{B} \to \mathrm{C} \to \mathrm{A}$ as shown in the diagram above. The total work done in the process is _____ J. [4 marks]



Solution:

Concept:

- For pressure-volume work, the work done on the gas in the chemistry sign convention is $w = -\int P\,dV$. Expansion gives negative work and compression gives positive work.
- The plotted axes have volume vertically and pressure horizontally. Read coordinates in that order carefully: $A = (10\,\mathrm{kPa}, 10\,\mathrm{dm}^3)$, $B = (10, 30)$, and $C = (30, 10)$. Also $1\,\mathrm{kPa\,dm}^3 = 1\,\mathrm{J}$.

Step 1: Calculate work along the isobaric expansion Along $A \to B$, the pressure remains $10\,\mathrm{kPa}$, while volume increases from 10 to $30\,\mathrm{dm}^3$. Therefore

$$w_{AB} = -P\Delta V = -10(30 - 10) = -200\,\mathrm{J}.$$

The gas expands and does work on its surroundings, so the work on the gas is negative.

Step 2: Calculate work along the straight compression path Along $B \to C$, pressure increases linearly from 10 to $30\,\mathrm{kPa}$ while volume decreases from 30 to $10\,\mathrm{dm}^3$. A straight line permits the arithmetic mean pressure:

$$w_{BC} = -\frac{10 + 30}{2}(10 - 30) = 400\,\mathrm{J}.$$

The volume decrease makes the work on the gas positive. Using the end pressure alone would overestimate this contribution.

Step 3: Close the cycle and combine contributions Along $C \to A$, the volume stays at $10\,\mathrm{dm}^3$, so $dV = 0$ and $w_{CA} = 0$. The complete cycle therefore gives

$$w = w_{AB} + w_{BC} + w_{CA} = -200 + 400 + 0 = 200\,\mathrm{J}.$$

The magnitude also equals the triangular enclosed area, $\frac{1}{2}(30-10)(30-10) = 200\,\mathrm{kPa\,dm}^3$. Returning

to the initial state makes internal-energy change zero, but does not make the cyclic work zero.

Step 4: Conclusion The total work on the gas is $\boxed{+200\,\text{J}}$, so the numerical entry is $\boxed{200}$. With the opposite convention, work done by the gas would be $-200\,\text{J}$.

Quick Tip to Solve: When the volume axis is vertical, determine the sign from expansion and compression using $-\int P\,dV$; do not apply a memorised clockwise-cycle sign without checking the axes.

88. The rate of First order reaction is $0.04\ \text{mol L}^{-1}\ \text{s}^{-1}$ at 10 minutes and $0.03\ \text{mol L}^{-1}\ \text{s}^{-1}$ at 20 minutes after initiation. Half life of the reaction is _____ minutes.

(Given $\log 2 = 0.3010, \log 3 = 0.4771$) [4 marks]

Solution:

Concept:

- For a first-order reaction, the instantaneous rate is $r = k[A]$. Since $[A]_t = [A]_0 e^{-kt}$, the rate itself also decays exponentially with the same rate constant.
- At two times, $k = \dfrac{2.303}{t_2 - t_1}\log_{10}(r_1/r_2)$, while $t_{1/2} = 2.303\log_{10} 2/k$. Use a common time unit throughout.

Step 1: Relate the two measured rates The measurements refer to 10 and 20 minutes after initiation, so their time separation is $10\,\text{min}$. Their ratio is

$$\frac{r_{10}}{r_{20}} = \frac{0.04}{0.03} = \frac{4}{3}.$$

The common rate units cancel in the ratio. Although rates are expressed per second, using minutes for the time separation simply produces k in min^{-1}; no rate conversion is needed for a dimensionless ratio.

Step 2: Evaluate the logarithm and rate constant Using the supplied logarithms,

$$\log_{10}\frac{4}{3} = 2\log_{10} 2 - \log_{10} 3 = 2(0.3010) - 0.4771 = 0.1249.$$

Consequently

$$k = \frac{2.303}{10}(0.1249) \simeq 0.02876\,\text{min}^{-1}.$$

The positive value is consistent with a rate that decreases as the reaction proceeds.

Step 3: Calculate and round the half-life Substitution into the first-order half-life expression gives

$$t_{1/2} = \frac{2.303(0.3010)}{2.303(0.1249)/10} = \frac{3.010}{0.1249} \simeq 24.10\,\text{min}.$$

Thus the requested numerical value, expressed to the nearest integer minute, is 24. The initial concentration is unnecessary because it cancels when the two rate equations are divided.

Step 4: Conclusion The half-life is approximately $\boxed{24.1\,\text{min}}$, giving the integer numerical entry $\boxed{24}$ in minutes.

Quick Tip to Solve: For first-order kinetics, a ratio of rates is a ratio of concentrations. Use the time difference, and cancel the factor 2.303 before evaluating the half-life.

89. On a thin layer chromatographic plate, an organic compound moved by 3.5 cm, while the solvent moved by 5 cm. The retardation factor of the organic compound is _____ $\times 10^{-1}$. [4 marks]

Solution:

Concept:

- The retardation factor in thin-layer chromatography is $R_f = d_{\text{solute}}/d_{\text{front}}$, where the two distances are measured from the common origin to the solute spot and the solvent front, respectively.
- Both distances must be measured from the same baseline and in the same units. Their ratio is dimensionless, and a solute spot behind the solvent front normally has $0 \le R_f \le 1$.

Step 1: Identify the two distances The compound has moved $3.5\,\text{cm}$, whereas the solvent front has moved $5\,\text{cm}$. These are the numerator and denominator, respectively. The denominator is the distance of the solvent front from the origin, rather than the separation $5 - 3.5 = 1.5\,\text{cm}$ between the front and the compound spot.

Step 2: Calculate the dimensionless ratio Insert the distances directly because both are already in centimetres:

$$R_f = \frac{3.5\,\text{cm}}{5\,\text{cm}} = 0.70.$$

The centimetres cancel, so the result carries no length unit. It is less than one, as required because the compound travels a shorter distance than the solvent front. Reversing the ratio would produce a value exceeding one and would not be the definition of R_f.

Step 3: Match the scale used in the answer blank The question writes the result as a blank multiplied by 10^{-1}. If the blank is x, then

$$x \times 10^{-1} = 0.70.$$

Thus

$$x = \frac{0.70}{0.10} = 7.$$

The physical retardation factor remains 0.70; seven is its coefficient in the particular scaled form requested, rather than the unscaled value of R_f.

Step 4: Conclusion The retardation factor is $\boxed{R_f = 0.70 = 7 \times 10^{-1}}$, so the blank is $\boxed{7}$. The factor is dimensionless.

Quick Tip to Solve: Calculate the actual ratio first, check it lies between zero and one, and then convert it to the power-of-ten form printed beside the blank.

90. If IUPAC name of an element is "Unununnium" then the element belongs to n^{th} group of Periodic table. The value of n is _____. [4 marks]

Solution:

Concept:

- Temporary systematic element names encode each digit of the atomic number: the root un denotes one, and the suffix -ium completes the element name.
- For transition elements, the group number corresponds to the combined $(n-1)d$ and ns electron count. The column containing copper, silver and gold is group 11; roentgenium occupies its period-7 position.

Step 1: Decode the systematic name The name Unununnium contains the roots un-un-un, corresponding to the digits 1, 1 and 1. Thus its atomic number is

$$Z = 111.$$

The repeated root gives three successive digits; it does not mean adding three ones to obtain an atomic number of three. The ending -ium identifies an element name and supplies no additional digit.

Step 2: Locate the element in the periodic table Atomic number 111 identifies roentgenium, Rg, in period 7 of the d block. It lies in the same column as copper, silver and gold. Its position follows the group-10 element darmstadtium, atomic number 110, and precedes the group-12 element copernicium, atomic number 112. This identifies the intervening group as 11 without needing a detailed ground-state configuration for this heavy element. Its atomic number locates it in the transition series, rather than among the group-1 alkali metals.

Step 3: State the requested group number The relevant column is group 11 in the modern 1–18 group-numbering scheme. The question defines n as the group number, so

$$n = 11.$$

Do not enter 111, which is the atomic number decoded from the name, or 7, which is the period number. These identify different properties of the same position in the periodic table.

Step 4: Conclusion Unununnium belongs to group $\boxed{11}$, hence $\boxed{n = 11}$. The group number is dimensionless.

Quick Tip to Solve: Decode all three numerical roots first; then locate that atomic number in the periodic table. Atomic number, period and group are different entries.