# JEE Main 2024 Paper 1 (B.E./B.Tech) Jan 31 Shift 1 Question Paper with Solutions PDF

## Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

**Time Allowed: 3 Hours** **Maximum Marks: 300**

### General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 1 (Jan-Feb), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 31 January 2024, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

## Mathematics – Section A: Multiple Choice Questions (80 marks)

**1.**

$$\lim_{x\to 0} \frac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2}$$

[4 marks]

(A) is equal to 1

(B) does not exist

(C) is equal to $-1$

(D) is equal to 2

**Correct Answer:** (D) is equal to 2

**Solution:**

**Concept:**

- The standard exponential limit is $\lim_{u\to 0}(e^u - 1 - u)/u^2 = 1/2$, obtained from the quadratic term of the exponential expansion.
- For real $x$, $|\sin x|^2 = \sin^2 x$, and $\lim_{x\to 0} \sin x/x = 1$. Squaring removes the sign difference on the two sides of zero.

**Step 1:** Introduce the small exponential argument Put $u = 2|\sin x|$. Then $u \to 0$ as $x \to 0$, and the numerator is exactly $e^u - 1 - u$. For sufficiently small nonzero $x$, $u \neq 0$, so multiplication and division by $u^2$ are legitimate. Thus

$$\frac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2} = \frac{e^u - 1 - u}{u^2}\,\frac{4\sin^2 x}{x^2}.$$

The absolute value must be retained until its square is taken; replacing it by $\sin x$ separately on only one side would not establish a two-sided limit.

**Step 2:** Evaluate the two factors The expansion $e^u = 1 + u + u^2/2 + O(u^3)$ gives the first factor tending to $1/2$. The second factor tends to $4$, since $\sin x/x \to 1$. Consequently

$$L = \frac{1}{2} \cdot 4 = 2.$$

Both one-sided limits agree because the remaining trigonometric factor is squared. The value 1 omits the coefficient arising from $u = 2|\sin x|$; the value $-1$ also contradicts the nonnegative numerator $e^u - 1 - u$. The absolute value therefore does not cause nonexistence of the limit.

**Step 3:** Conclusion The two-sided limit equals 2. Hence, the correct option is (D).

**Quick Tip to Solve:** When the constant and linear terms of an exponential cancel, retain its quadratic term and square the entire small argument, including its numerical coefficient.

**2. For $\alpha, \beta, \gamma \neq 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha+\beta+\gamma)(\alpha-\gamma+\beta) = 3\alpha\beta$, then $\gamma$ equals**

[4 marks]

(A) $\sqrt{3}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

**Correct Answer:** (B) $\frac{\sqrt{3}}{2}$

**Solution:**

**Concept:**

- Principal inverse sine values belong to $[-\pi/2, \pi/2]$. Three nonzero such angles with sum $\pi$ must all be positive.
- For a triangle with angles $A, B, C$, the sine rule and cosine rule imply $\cos C = (\sin^2 A + \sin^2 B - \sin^2 C)/(2\sin A \sin B)$.

**Step 1:** Interpret the inverse sines as triangle angles Let $A = \sin^{-1}\alpha$, $B = \sin^{-1}\beta$, and $C = \sin^{-1}\gamma$. If one of these were negative, the other two, each at most $\pi/2$, could not make the total $\pi$. None is zero by the hypothesis. Hence all three are positive and their sum is $\pi$, so they are the angles of a triangle. Their sines are $\alpha, \beta, \gamma$, all positive; in particular division by $\alpha\beta$ is permitted.

**Step 2:** Apply the algebraic relation to the cosine rule Expand the given product as a difference of squares:

$$(\alpha+\beta)^2 - \gamma^2 = 3\alpha\beta \quad \Longrightarrow \quad \alpha^2 + \beta^2 - \gamma^2 = \alpha\beta.$$

By the sine rule, the side lengths are proportional to $\alpha, \beta, \gamma$. The cosine rule therefore gives

$$\cos C = \frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta} = \frac{1}{2}.$$

As $0 < C \leq \pi/2$, this fixes $C = \pi/3$, not another periodic angle. Thus $\gamma = \sin C = \sqrt{3}/2$. Option (A) is outside the sine range, while the smaller values in (C) and (D) do not give $\cos C = 1/2$.

**Step 3:** Conclusion $\gamma = \sqrt{3}/2$. Hence, the correct option is (B).

**Quick Tip to Solve:** When three principal inverse sines sum to pi, view their arguments as sides proportional to the sines of triangle angles, then use the cosine rule.

**3. Let $\mathrm{S}$ be the set of positive integral values of $a$ for which $\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32} < 0, \forall x \in \mathbb{R}$. Then, the number of elements in $\mathrm{S}$ is :** [4 marks]

(A) 0

(B) $\infty$

(C) 3

(D) 1

**Correct Answer:** (A) 0

**Solution:**

**Concept:**

- A statement required for every real $x$ must hold at each convenient test value, including $x = 0$. One counterexample disproves it.
- The denominator satisfies $x^2 - 8x + 32 = (x-4)^2 + 16 > 0$ for every real $x$, so the fraction has the sign of its numerator.

**Step 1:** Check the denominator and use a necessary condition Completing the square gives

$$x^2 - 8x + 32 = (x-4)^2 + 16.$$

There are no real poles and the denominator is always strictly positive. For the fraction to be negative for all real $x$, its numerator would therefore have to be negative at every real input. We can test that necessary requirement at zero before doing any discriminant calculation. This is valid because zero is included in the universal quantifier $\forall x \in \mathbb{R}$.

**Step 2:** Exclude every positive integer parameter At $x = 0$, the left side becomes

$$\frac{9a+4}{32}.$$

For every positive integral $a$, we have $a \geq 1$, so $9a + 4 \geq 13 > 0$. The fraction is positive, contradicting the required strict negative sign. Thus no positive integral value can belong to $S$. Independently, the positive leading coefficient $a$ would make the numerator positive for sufficiently large $|x|$, giving the same obstruction. The set is empty, rather than infinite or a set of one or three elements. The number being requested is its cardinality, not a permissible value of $a$.

**Step 3:** Conclusion $S = \varnothing$, and consequently $|S| = 0$. Hence, the correct option is (A).

**Quick Tip to Solve:** For an inequality claimed for all real x, first test zero or a point that kills the variable terms; it can eliminate the parameter without solving the full quadratic inequality.

**4. Let $\alpha, \beta, \gamma, \delta \in \mathbb{Z}$ and let $A(\alpha, \beta), B(1, 0), C(\gamma, \delta)$ and $D(1, 2)$ be the vertices of a parallelogram ABCD. If $AB = \sqrt{10}$ and the points A and C lie on the line $3y = 2x + 1$, then $2(\alpha + \beta + \gamma + \delta)$ is equal to** [4 marks]

(A) 8

(B) 5

(C) 12

(D) 10

**Correct Answer:** (A) 8

**Solution:**

**Concept:**

- In a parallelogram, the diagonals bisect each other. Thus the vector identity $A + C = B + D$ holds when the vertices are listed in cyclic order.
- The requested sum of coordinates can be found from this midpoint identity without locating the individual vertices or choosing a square-root sign.

**Step 1:** Equate the diagonal midpoints The opposite vertices are $A, C$ and $B, D$. The midpoint of the known diagonal is

$$M_{BD} = \left(\frac{1+1}{2}, \frac{0+2}{2}\right) = (1, 1).$$

Since diagonals bisect one another, the midpoint of $AC$ must be the same point:

$$\left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right) = (1, 1).$$

It follows that $\alpha + \gamma = 2$ and $\beta + \delta = 2$. This conclusion is independent of the individual positions along the specified diagonal line.

**Step 2:** Evaluate the expression and check consistency Adding the coordinate identities gives $\alpha + \beta + \gamma + \delta = 4$, so

$$2(\alpha + \beta + \gamma + \delta) = 2 \cdot 4 = 8.$$

The additional conditions are consistent: choose $A = (-2, -1)$ and $C = (4, 3)$. Both have integer coordinates and satisfy $3y = 2x + 1$, and $AB = \sqrt{(1+2)^2 + (0+1)^2} = \sqrt{10}$. Their midpoint is indeed $(1, 1)$. This example verifies existence; the midpoint argument establishes the same requested result for every allowed configuration. The other numerical options cannot satisfy that fixed coordinate sum.

**Step 3:** Conclusion The required value is 8. Hence, the correct option is (A).

**Quick Tip to Solve:** When a parallelogram question asks for sums of opposite-vertex coordinates, use A+C=B+D before solving for the separate vertices.

**5. For $0 < c < b < a$, let $(a+b-2c)x^2 + (b+c-2a)x + (c+a-2b) = 0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements**

**(I) If $\alpha \in (-1, 0)$, then $b$ cannot be the geometric mean of $a$ and $c$**

**(II) If $\alpha \in (0, 1)$, then $b$ may be the geometric mean of $a$ and $c$** [4 marks]

(A) only (II) is true

(B) Both (I) and (II) are true

(C) only (I) is true

(D) Neither (I) nor (II) is true

**Correct Answer:** (B) Both (I) and (II) are true

**Solution:**

**Concept:**

- The coefficients of the quadratic add to zero, so $x = 1$ is a root. The product of roots gives the other root directly.
- For positive numbers, $b$ is the geometric mean of $a, c$ exactly when $b^2 = ac$. Under $a > b > c$, write $a = t^2c$, $b = tc$ with $t > 1$.

**Step 1:** Find the root different from one Substitution of $x = 1$ gives zero because

$$(a+b-2c) + (b+c-2a) + (c+a-2b) = 0.$$

The leading coefficient $a + b - 2c$ is positive under $a > b > c$, so this is genuinely quadratic. By the product-of-roots formula, the other root is

$$\alpha = \frac{a+c-2b}{a+b-2c}.$$

The signs of this expression depend on the relative position of $b$; we now test specifically the geometric-mean possibility in the two statements.

**Step 2:** Test the geometric-mean condition If $b = \sqrt{ac}$, put $a = t^2c$, $b = tc$, where $t > 1$ follows from the strict ordering. Then

$$\alpha = \frac{t^2+1-2t}{t^2+t-2} = \frac{(t-1)^2}{(t-1)(t+2)} = \frac{t-1}{t+2}.$$

Both numerator and denominator are positive, and the numerator is smaller than the denominator. Hence $0 < \alpha < 1$. Therefore a negative $\alpha \in (-1, 0)$ cannot occur when $b$ is the geometric mean, proving (I). The geometric-mean case does produce roots in $(0, 1)$; for example $(a, b, c) = (4, 2, 1)$ gives $\alpha = 1/4$, proving the possibility in (II). In fact $t = (1 + 2\alpha)/(1 - \alpha) > 1$ realizes every $\alpha$ in that interval. Options asserting only one statement or neither are ruled out.

**Step 3:** Conclusion Both statements (I) and (II) are true. Hence, the correct option is (B).

**Quick Tip to Solve:** If a quadratic has coefficients summing to zero, remove the root one using the product of roots; then parameterize a geometric progression by c, tc, t squared c.

**6. If $f(x)=\frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ and $(f \circ f)(x)=g(x)$, where $g: \mathbb{R}-\left\{\frac{2}{3}\right\} \rightarrow \mathbb{R}-\left\{\frac{2}{3}\right\}$, then $(gogog)(4)$ is equal to** [4 marks]

(A) $-4$

(B) $\frac{19}{20}$

(C) $-\frac{19}{20}$

(D) 4

**Correct Answer:** (D) 4

**Solution:**

**Concept:**

- To find $f \circ f$, substitute the complete rational expression for $x$ in both numerator and denominator of $f$.
- If $f(f(x))=x$ throughout its domain, then $f$ is an involution and $g=f \circ f$ is the identity function. Repeated compositions of the identity do not change an input.

**Step 1:** Compute the double composition For $x \neq 2/3$, substitute $f(x)=(4x+3)/(6x-4)$:

$$f(f(x))=\frac{4\frac{4x+3}{6x-4}+3}{6\frac{4x+3}{6x-4}-4}.$$

Combining the terms separately gives numerator $(16x+12+18x-12)/(6x-4)=34x/(6x-4)$, and denominator $(24x+18-24x+16)/(6x-4)=34/(6x-4)$. Therefore

$$g(x)=\frac{34x/(6x-4)}{34/(6x-4)}=x.$$

No zero denominator was cancelled: $6x-4 \neq 0$ by the domain restriction and 34 is nonzero.

**Step 2:** Check the domain and compose the identity The inner value never equals the forbidden input: $(4x+3)/(6x-4)=2/3$ would give $12x+9=12x-8$, an impossibility. Thus the double composition is valid for every specified input. Since $4 \neq 2/3$, each application of $g$ at 4 is allowed, and

$$(g \circ g \circ g)(4)=g(g(g(4)))=g(g(4))=4.$$

The value $19/20$ in option (B) is merely $f(4)$, not the requested repeated composition of $g$. The negative alternatives are also inconsistent with the identity result. Distinguishing $f$ from $g$ prevents stopping after just one fractional-linear evaluation.

**Step 3:** Conclusion $(g \circ g \circ g)(4)=4$. Hence, the correct option is (D).

**Quick Tip to Solve:** For a fractional linear map with opposite diagonal coefficients, check whether the double composition is the identity before evaluating many nested functions.

**7. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = \frac{(\tan x)+y}{\sin x(\sec x - \sin x \tan x)}, x \in \left(0, \frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right) = 2$. Then, $y\left(\frac{\pi}{3}\right)$ is** [4 marks]

(A) $\sqrt{3}\,(2 + \log_e 3)$

(B) $\sqrt{3}\,(1 + 2\log_e 3)$

(C) $\sqrt{3}\left(2 + \log_e \sqrt{3}\right)$

(D) $\frac{\sqrt{3}}{2}\,(2 + \log_e 3)$

**Correct Answer:** (C) $\sqrt{3}\left(2 + \log_e \sqrt{3}\right)$

**Solution:**

**Concept:**

- A linear equation $y' + P(x)y = Q(x)$ is solved with integrating factor $e^{\int P(x)\,dx}$. Multiplication turns its left side into the derivative of a product.
- On $0 < x < \pi/2$, sine, cosine and tangent are positive, so $\log(\tan x)$ is real and no absolute-value sign is needed.

**Step 1:** Simplify the equation and choose an integrating factor First simplify the bracket:

$$\sec x - \sin x \tan x = \frac{1 - \sin^2 x}{\cos x} = \cos x.$$

The equation becomes

$$y' - \frac{y}{\sin x \cos x} = \sec^2 x.$$

Since $\frac{d}{dx}\log(\tan x) = 1/(\sin x \cos x)$, an integrating factor is

$$e^{-\int dx/(\sin x \cos x)} = \frac{1}{\tan x} = \cot x.$$

The domain ensures the divisions are valid and the factor is nonzero.

**Step 2:** Integrate and impose the initial condition Multiplication by $\cot x$ gives

$$\frac{d}{dx}(y \cot x) = \cot x \sec^2 x = \frac{1}{\sin x \cos x}.$$

Integrating yields $y \cot x = \log(\tan x) + C$, hence $y = \tan x[\log(\tan x) + C]$. At $x = \pi/4$, tangent is 1 and the logarithm is zero, so the stated value $y(\pi/4) = 2$ fixes $C = 2$. Now substitute $\tan(\pi/3) = \sqrt{3}$:

$$y(\pi/3) = \sqrt{3}\,[2 + \log \sqrt{3}].$$

The identity $\log \sqrt{3} = \frac{1}{2}\log 3$ shows why replacing it by $\log 3$, as in option (A), doubles the logarithmic contribution. Options (B) and (D) have different constants or prefactors and fail this derived expression.

**Step 3:** Conclusion $y(\pi/3) = \sqrt{3}(2 + \log_e \sqrt{3})$. Hence, the correct option is (C).

**Quick Tip to Solve:** Simplify trigonometric coefficients before integrating; here the denominator becomes sin x cos x and the integrating factor is immediately cot x.

**8. Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable $x$ to be the number of rotten apples in a draw of two apples, the variance of $x$ is** [4 marks]

(A) $\frac{57}{153}$
(B) $\frac{40}{153}$
(C) $\frac{37}{153}$
(D) $\frac{47}{153}$

**Correct Answer:** (B) $\frac{40}{153}$

**Solution:**

**Concept:**

- The draw is without replacement: two different apples are selected from eighteen apples, of which three are rotten. Thus $X$, the number of rotten apples drawn, has possible values $0, 1, 2$.
- For equally likely unordered pairs, $P(X = r) = \binom{3}{r}\binom{15}{2-r}/\binom{18}{2}$. The variance is $\mathrm{Var}(X) = E(X^2) - [E(X)]^2$.

**Step 1:** Construct the probability distribution There are $\binom{18}{2} = 153$ possible pairs. Choosing zero, one or two rotten apples gives respectively

$$P(X = 0) = \frac{105}{153}, \qquad P(X = 1) = \frac{45}{153}, \qquad P(X = 2) = \frac{3}{153}.$$

The numerators add to 153, so all possible pairs have been counted once. Order need not be included because the question concerns only the number of rotten apples.

**Step 2:** Calculate both moments Weight each possible value by its probability. For the second moment, square the value of the random variable before weighting:

$$E(X) = \frac{0 \cdot 105 + 1 \cdot 45 + 2 \cdot 3}{153} = \frac{1}{3},$$
$$E(X^2) = \frac{0^2 \cdot 105 + 1^2 \cdot 45 + 2^2 \cdot 3}{153} = \frac{57}{153} = \frac{19}{51}.$$

**Step 3:** Subtract the square of the mean Consequently,

$$\mathrm{Var}(X) = \frac{19}{51} - \frac{1}{9} = \frac{57 - 17}{153} = \frac{40}{153}.$$

Option (A) gives the second moment rather than the variance. Options (C) and (D) equal neither the moment calculation nor the variance; they do not follow from the three probabilities above.

**Step 4:** Conclusion The variance of the number of rotten apples is $\boxed{40/153}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Build the distribution for 0, 1 and 2 rotten apples; remember that the second moment is not itself the variance.

**9. Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then $\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$ is equal to** [4 marks]

(A) 24

(B) 32

(C) 36

(D) 28

**Correct Answer:** (B) 32

**Solution:**

**Concept:**

- If the cross product of two vectors is zero and one is nonzero, the vectors are parallel. Therefore $(\vec{p} - \vec{c}) \times \vec{b} = \vec{0}$ implies $\vec{p} = \vec{c} + t\vec{b}$ for a real scalar $t$.
- The condition $\vec{p} \cdot \vec{a} = 0$ means that $\vec{p}$ is perpendicular to $\vec{a}$. It determines the scalar left free by the cross-product equation.

**Step 1:** Reduce the vector equation to one parameter Subtract the two cross products:

$$(\vec{p} - \vec{c}) \times \vec{b} = \vec{0}.$$

Since $\vec{b} = (4, 1, 7) \neq \vec{0}$, write

$$\vec{p} = (1, -3, 4) + t(4, 1, 7).$$

This step uses parallelism, rather than cancelling a vector from a cross product, which would not be a valid algebraic operation.

**Step 2:** Use the dot-product constraint Calculate the two scalar products carefully, including the negative third component of $\vec{a}$:

$$\vec{c} \cdot \vec{a} = 3 - 3 - 8 = -8,$$
$$\vec{b} \cdot \vec{a} = 12 + 1 - 14 = -1.$$

Thus $0 = \vec{p} \cdot \vec{a} = -8 - t$, giving $t = -8$. Therefore

$$\vec{p} = (1, -3, 4) - 8(4, 1, 7) = (-31, -11, -52).$$

**Step 3:** Evaluate and check the required scalar The requested vector is $(1, -1, -1)$, so

$$\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k}) = -31 + 11 + 52 = 32.$$

As a check, $(-31, -11, -52) \cdot (3, 1, -2) = -93 - 11 + 104 = 0$, and $\vec{p} - \vec{c} = -8\vec{b}$, satisfying both original conditions. The values 24, 36 and 28 in the other options disagree with this direct dot product.

**Step 4:** Conclusion $\boxed{\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k}) = 32}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Equal cross products with the same nonzero vector mean the two vectors differ by a multiple of that vector.

**10. The sum of the series $\frac{1}{1-3\cdot1^2+1^4}+\frac{2}{1-3\cdot2^2+2^4}+\frac{3}{1-3\cdot3^2+3^4}+\ldots$ up to 10 -terms is** [4 marks]

(A) $\frac{45}{109}$

(B) $-\frac{55}{109}$

(C) $\frac{55}{109}$

(D) $-\frac{45}{109}$

**Correct Answer:** (B) $-\frac{55}{109}$

**Solution:**

**Concept:**

- A quartic denominator can sometimes be split into two quadratics whose difference is proportional to the numerator. Here $n^4-3n^2+1=(n^2-n-1)(n^2+n-1)$.
- For a telescoping sum, express the general term as $F(n)-F(n+1)$. All interior terms then cancel, leaving only boundary contributions.

**Step 1:** Factor the general denominator Let $a_n=n/(n^4-3n^2+1)$. The denominator is a difference of squares:

$$(n^2-1)^2-n^2=(n^2-n-1)(n^2+n-1).$$

The difference between the two factors is $2n$, which matches twice the numerator. Neither factor vanishes at an integer $1\le n\le 10$, so the following decomposition is valid for every term.

**Step 2:** Write a difference of successive fractions Using the factorization,

$$a_n=\frac{1}{2}\left(\frac{1}{n^2-n-1}-\frac{1}{n^2+n-1}\right).$$

Define $F(n)=1/(n^2-n-1)$. Since $(n+1)^2-(n+1)-1=n^2+n-1$, we have $a_n=\frac{1}{2}[F(n)-F(n+1)]$.

**Step 3:** Cancel the interior terms The sum of the first ten terms is therefore

$$\begin{aligned}S_{10}&=\frac{1}{2}(F(1)-F(11))\\&=\frac{1}{2}\left(-1-\frac{1}{121-11-1}\right)\\&=\frac{1}{2}\left(-1-\frac{1}{109}\right)=-\frac{55}{109}.\end{aligned}$$

The first term is negative, and the positive remaining terms do not reverse the sign. This excludes the positive options (A) and (C). Option (D) has the wrong boundary numerator; the terminal contribution must be subtracted from $-1$, not added.

**Step 4:** Conclusion The sum is $\boxed{-55/109}$. Hence, the correct option is (B).

**Quick Tip to Solve:** In a quartic series with numerator n, try a difference of squares and compare the two quadratic factors at n and n+1.

**11. Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is** [4 marks]

(A) $\frac{4}{25}$
(B) $\frac{2}{3}$
(C) $\frac{2}{25}$
(D) $\frac{4}{75}$

**Correct Answer:** (D) $\frac{4}{75}$

**Solution:**

**Concept:**

- Replacement restores the original composition of the box after the first draw. The colour probabilities on the second draw are therefore unchanged by the first outcome.
- For independent events $R_1$ (red first) and $W_2$ (white second), $P(R_1 \cap W_2) = P(R_1)P(W_2)$. The order named in the question is fixed.

**Step 1:** Count the marbles and identify the events The box contains

$$N = 10 + 30 + 20 + 15 = 75$$

marbles. A red marble is required specifically on the first draw, so $P(R_1) = 10/75 = 2/15$. The blue and orange marbles must remain in the total, even though neither colour is a desired outcome: either can still be selected on a draw.

**Step 2:** Apply replacement before the second draw After the first marble is put back, the box again contains 75 marbles, including 30 white marbles. Consequently

$$P(W_2 \mid R_1) = \frac{30}{75} = \frac{2}{5}.$$

The denominator is not 74, since no marble has been permanently removed. The independent multiplication rule is thus applicable.

**Step 3:** Multiply for the specified order The required probability is

$$P(R_1 \cap W_2) = \frac{10}{75} \cdot \frac{30}{75} = \frac{300}{5625} = \frac{4}{75}.$$

There is no factor of two: white followed by red is a different ordered event and is not requested. As a check, this probability is smaller than $P(R_1) = 2/15$, as any intersection must be. Option (A) is twice the value in (C), but neither equals this product; option (B) is even greater than the probability of drawing red first, so it cannot describe the specified intersection.

**Step 4:** Conclusion The required probability is $\boxed{4/75}$. Hence, the correct option is (D).

**Quick Tip to Solve:** With replacement, keep the original total on both draws; multiply only the probabilities for the order actually requested.

**12. The distance of the point $Q(0, 2, -2)$ form the line passing through the point $P(5, -4, 3)$ and perpendicular to the lines $\vec{r} = (-3\hat{i}+2\hat{k})+\lambda(2\hat{i}+3\hat{j}+5\hat{k}), \lambda \in \mathbb{R}$ and $\vec{r} = (\hat{i}-2\hat{j}+\hat{k})+\mu(-\hat{i}+3\hat{j}+2\hat{k}), \mu \in \mathbb{R}$ is :** [4 marks]

(A) $\sqrt{74}$

(B) $\sqrt{86}$

(C) $\sqrt{54}$

(D) $\sqrt{20}$

**Correct Answer:** (A) $\sqrt{74}$

**Solution:**

**Concept:**

- A line perpendicular to two nonparallel directions has direction given by their cross product. The position vectors of the two given lines do not affect this perpendicular direction.
- If $\vec{v} = \overrightarrow{PQ}$ and $\vec{d}$ is a direction of the required line through P, then the squared perpendicular distance is $D^2 = |\vec{v}|^2 - (\vec{v} \cdot \vec{d})^2/|\vec{d}|^2$.

**Step 1:** Determine the required direction The given direction vectors are $\vec{u} = (2, 3, 5)$ and $\vec{w} = (-1, 3, 2)$. Their cross product is

$$\vec{u} \times \vec{w} = (6 - 15, -(4 + 5), 6 + 3) = (-9, -9, 9).$$

We can therefore use the simpler parallel direction $\vec{d} = (1, 1, -1)$. Check: $\vec{d} \cdot \vec{u} = 2 + 3 - 5 = 0$, and $\vec{d} \cdot \vec{w} = -1 + 3 - 2 = 0$.

**Step 2:** Separate the displacement into parallel and perpendicular parts For $P = (5, -4, 3)$ and $Q = (0, 2, -2)$,

$$\vec{v} = \overrightarrow{PQ} = (-5, 6, -5).$$

Hence

$$|\vec{v}|^2 = 25 + 36 + 25 = 86, \qquad \vec{v} \cdot \vec{d} = -5 + 6 + 5 = 6, \qquad |\vec{d}|^2 = 3.$$

The component along the line must be removed; the full distance PQ is not the distance from Q to the line.

**Step 3:** Calculate the perpendicular distance Using orthogonal projection,

$$D^2 = 86 - \frac{6^2}{3} = 86 - 12 = 74.$$

Thus $D = \sqrt{74}$. Option (B), $\sqrt{86}$, is the distance to P itself and ignores the possible motion along the line. The other proposed squares, 54 and 20, do not result from removing the squared parallel component 12 from 86.

**Step 4:** Conclusion The perpendicular distance is $\boxed{\sqrt{74}}$ units. Hence, the correct option is (A).

**Quick Tip to Solve:** First find a direction perpendicular to both given lines; then subtract the squared projection from the squared displacement.

**13. If the system of linear equations**

$$x - 2y + z = -4$$
$$2x + \alpha y + 3z = 5$$
$$3x - y + \beta z = 3$$

**has infinitely many solutions, then $12\alpha + 13\beta$ is equal to** [4 marks]

(A) 60

(B) 54

(C) 64

(D) 58

**Correct Answer:** (D) 58

**Solution:**

**Concept:**

- Three linear equations in three unknowns have infinitely many solutions when the coefficient and augmented matrices have equal rank less than three.
- Eliminating x using the first equation produces two equations in y and z. For infinitely many solutions these equations must be proportional, including their constant terms; a zero determinant alone does not establish consistency.

**Step 1:** Eliminate the first unknown Subtract twice the first equation from the second, and three times the first from the third:

$$(\alpha + 4)y + z = 13,$$
$$5y + (\beta - 3)z = 15.$$

The right sides are 13 and 15 because $5 - 2(-4) = 13$ and $3 - 3(-4) = 15$. Neither reduced equation is identically zero.

**Step 2:** Match the proportional coefficients The second reduced equation must be $15/13$ times the first. Matching its y and z coefficients gives

$$\frac{5}{\alpha + 4} = \frac{15}{13}, \qquad \beta - 3 = \frac{15}{13}.$$

Thus

$$\alpha + 4 = \frac{13}{3}, \qquad \alpha = \frac{1}{3}, \qquad \beta = \frac{54}{13}.$$

These values make both reduced equations the same straight line in the y-z plane, rather than parallel inconsistent lines.

**Step 3:** Verify freedom and evaluate the expression With these parameters the first reduced equation determines z in terms of a freely chosen y, while the original first equation then determines x. Thus there really are infinitely many solutions. The requested combination is

$$12\alpha + 13\beta = 12\left(\frac{1}{3}\right) + 13\left(\frac{54}{13}\right) = 4 + 54 = 58.$$

The values 60, 54 and 64 in the other options fail this final substitution; 54 alone would omit the contribution $12\alpha = 4$.

**Step 4:** Conclusion $\boxed{12\alpha + 13\beta = 58}$. Hence, the correct option is (D).

**Quick Tip to Solve:** When testing infinite solutions, compare the constants as well as the variable coefficients after elimination.

**14. Let $g(x)$ be a linear function and $f(x) = \begin{cases} g(x) & , x \le 0 \\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} & , x > 0 \end{cases}$, is continuous at $x = 0$. If $f'(1) = f(-1)$, then the value $g(3)$ is** [4 marks]

(A) $\log_e\left(\frac{4}{9}\right) - 1$

(B) $\frac{1}{3}\log_e\left(\frac{4}{9e^{1/3}}\right)$

(C) $\log_e\left(\frac{4}{9e^{1/3}}\right)$

(D) $\frac{1}{3}\log_e\left(\frac{4}{9}\right) + 1$

**Correct Answer:** (C) $\log_e\left(\frac{4}{9e^{1/3}}\right)$

**Solution:**

**Concept:**

- Continuity at zero requires the value of the linear branch to equal the right-hand limit of the exponential branch. A base tending to a fixed number below one, raised to a power tending to positive infinity, tends to zero.
- For a positive function $h(x) = u(x)^{1/x}$, logarithmic differentiation gives $h\prime(x) = h(x)\,\frac{d}{dx}[\log u(x)/x]$. Here the base is positive for x greater than zero.

**Step 1:** Use continuity to determine the intercept As $x \to 0^+$, the base $(1+x)/(2+x) \to 1/2$. More precisely,

$$\log f(x) = \frac{\log(1+x) - \log(2+x)}{x} \longrightarrow -\infty,$$

so $f(x) \to 0$. Write $g(x) = mx + c$. Continuity at zero gives $c = g(0) = 0$, hence $g(x) = mx$.

**Step 2:** Differentiate the positive branch at one For $x > 0$, let $L(x) = \log(1+x) - \log(2+x)$. Then

$$f\prime(x) = f(x)\left[-\frac{L(x)}{x^2} + \frac{1}{x}\left(\frac{1}{1+x} - \frac{1}{2+x}\right)\right].$$

At $x = 1$, $f(1) = 2/3$, $L(1) = \log(2/3)$, and the bracketed rational difference is $1/2 - 1/3 = 1/6$. Consequently

$$f\prime(1) = \frac{2}{3}\left[\log\left(\frac{3}{2}\right) + \frac{1}{6}\right].$$

**Step 3:** Determine the slope and evaluate g The given equality is $f\prime(1) = f(-1) = g(-1) = -m$.

Therefore

$$g(3) = 3m = -3f\prime(1)$$
$$= -2\log\left(\frac{3}{2}\right) - \frac{1}{3}$$
$$= \log\left(\frac{4}{9e^{1/3}}\right).$$

Option (A) subtracts 1 instead of 1/3. Option (B) divides the entire correct logarithm by three, and option (D) has both a wrong coefficient of the logarithm and a wrong constant. Thus only (C) agrees with the derivative condition.

**Step 4:** Conclusion $\boxed{g(3) = \log_e\left(4/(9e^{1/3})\right)}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Do not use the standard e-limit when the base tends to 1/2; take logarithms to establish the zero limit first.

**15. The area of the region** $\left\{(x, y) : y^2 \le 4x,\ x < 4,\ \dfrac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0,\ x \ne 3\right\}$ **is** [4 marks]

(A) $\frac{32}{3}$
(B) $\frac{16}{3}$
(C) $\frac{8}{3}$
(D) $\frac{64}{3}$

**Correct Answer:** (A) $\frac{32}{3}$

**Solution:**

**Concept:**

- The condition $y^2 \le 4x$ confines the region to $x \ge 0$, between $y = -2\sqrt{x}$ and $y = 2\sqrt{x}$.
- A strict product inequality is analysed by a sign chart. Boundaries consisting of finitely many curves or vertical lines have zero area.

**Step 1:** Separate the sign of the rational factor For $0 < x < 4$, let $R(x) = x(x-1)(x-2)/((x-3)(x-4))$. Its critical points are 0, 1, 2, 3 and 4. Counting negative factors shows that its signs on $(0,1), (1,2), (2,3), (3,4)$ are respectively $+, -, +, -$. The inequality in the question is $yR(x) > 0$, so $y$ must have the same sign as $R(x)$.

**Step 2:** Find the vertical thickness of the selected region On $(0,1)$ and $(2,3)$, only the upper half $0 < y \le 2\sqrt{x}$ survives. On $(1,2)$ and $(3,4)$, only the lower half $-2\sqrt{x} \le y < 0$ survives. Thus every allowed vertical slice has thickness $2\sqrt{x}$, even though the selected half alternates. The excluded lines $x = 0, 1, 2, 3, 4$ do not affect the integral.

**Step 3:** Integrate the common slice thickness Adding the four intervals is equivalent to integrating over the full interval from 0 to 4:

$$A = \int_0^4 2\sqrt{x}\,dx = \left[\frac{4}{3}x^{3/2}\right]_0^4 = \frac{32}{3}.$$

The full parabolic region would have thickness $4\sqrt{x}$ and twice this area. Therefore option (D) counts both halves, while (B) and (C) omit additional allowed slices.

**Step 4:** Conclusion The required area is $\boxed{32/3}$ square units. Hence, the correct option is (A).

**Quick Tip to Solve:** When a sign inequality selects one of two symmetric halves at every x, integrate one half; the alternating sign need not change its thickness.

**16. The solution curve of the differential equation $y\frac{dx}{dy} = x\left(\log_e x - \log_e y + 1\right), x > 0, y > 0$ passing through the point $(e, 1)$ is** [4 marks]

(A) $\left|\log_e \frac{y}{x}\right| = y^2$

(B) $\left|\log_e \frac{y}{x}\right| = x$

(C) $\left|\log_e \frac{x}{y}\right| = y$

(D) $2\left|\log_e \frac{x}{y}\right| = y + 1$

**Correct Answer:** (C) $\left|\log_e \frac{x}{y}\right| = y$

**Solution:**

**Concept:**

- When logarithms occur as $\log x - \log y$, use the positive ratio $v = x/y$, so $\log x - \log y = \log v$.
- Treat $x$ as a function of $y$: if $x = vy$, then $dx/dy = v + y\,dv/dy$. The prescribed point fixes the integration constant.

**Step 1:** Substitute the ratio and simplify Put $x = vy$, where $v > 0$. Substitution in the printed differential equation gives

$$y\left(v + y\frac{dv}{dy}\right) = vy(\log v + 1).$$

Cancelling $yv$ terms and dividing by $y$, permissible because $y > 0$, yields

$$y\frac{dv}{dy} = v\log v.$$

The initial value is $v = e$ at $y = 1$, so the relevant branch has $\log v > 0$.

**Step 2:** Integrate and apply the initial point Separating variables on that branch gives

$$\frac{dv}{v\log v} = \frac{dy}{y}, \qquad \log|\log v| = \log y + C.$$

Equivalently $\log v = Ky$. At $(x, y) = (e, 1)$, we have $1 = K$, hence

$$\log\frac{x}{y} = y, \qquad x = ye^y.$$

This explicit form stays positive for $y > 0$, so the sign chosen during separation is consistent everywhere on the required curve.

**Step 3:** Check the equation and compare the options From $x = ye^y$, $dx/dy = e^y(1+y)$. Hence $y\,dx/dy = x(1+y) = x(1+\log(x/y))$, verifying the original equation. Since $\log(x/y) = y > 0$, its absolute value is also $y$. Option (A) would require $y = y^2$ for every point, option (B) would require $y = x$, and option (D) would require $2y = y + 1$; these are not identities along the solution curve.

**Step 4:** Conclusion The curve is $\log(x/y) = y$, represented by $\boxed{|\log(x/y)| = y}$ in the given choices. Hence, the correct option is (C).

**Quick Tip to Solve:** Use the initial point to choose the sign after integrating a logarithm; then differentiate the resulting implicit curve to verify it.

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**17. Let $a$ be the sum of all coefficients in the expansion of $\left(1-2x+2x^2\right)^{2023}\left(3-4x^2+2x^3\right)^{2024}$ and $b = \lim_{x\to 0}\left(\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equation $cx^2+dx+e=0$ and $2bx^2+ax+4=0$ have a common root, where $c, d, e \in \mathbb{R}$, then $\mathrm{d} : \mathrm{c} : \mathrm{e}$ equals** [4 marks]

(A) $2:1:4$

(B) $1:1:4$

(C) $1:2:4$

(D) $4:1:4$

**Correct Answer:** (B) $1:1:4$

**Solution:**

**Concept:**

- The sum of the coefficients of a polynomial $P(x)$ is $P(1)$. The fundamental theorem of calculus differentiates an integral with variable upper limit.
- A real quadratic with a nonreal root also has its conjugate root. Thus two real quadratics sharing a nonreal root are proportional.

**Step 1:** Evaluate the coefficient sum and the limit For the polynomial in the question,

$$a = (1-2+2)^{2023}(3-4+2)^{2024} = 1.$$

The integrand tends to zero and the integral is zero at $x = 0$, so the quotient defining $b$ is of type $0/0$. Applying differentiation to numerator and denominator gives

$$b = \lim_{x\to 0}\frac{\log(1+x)}{2x(1+x^{2024})} = \frac{1}{2},$$

because $\log(1+x)/x \to 1$ and $1 + x^{2024} \to 1$.

**Step 2:** Determine the nature of the shared root The second quadratic becomes

$$2bx^2 + ax + 4 = x^2 + x + 4 = 0.$$

Its discriminant is $1 - 16 = -15 < 0$, so its roots are the nonreal conjugates $(-1 \pm i\sqrt{15})/2$. Since

$c, d, e$ are real, sharing either of these roots forces $cx^2+dx+e$ to have the other root as well. Therefore it must be a nonzero scalar multiple of $x^2+x+4$.

**Step 3:** Read the coefficients in the requested order Writing $cx^2+dx+e=k(x^2+x+4)$ gives $c=k, d=k, e=4k$, so

$$d:c:e=1:1:4.$$

The order is $d, c, e$, not $c, d, e$, although the first two happen to be equal here. The other proposed ratios violate either $d=c$ or $e=4c$. This reasoning uses real coefficients essentially: a polynomial with arbitrary complex coefficients need not include the conjugate of a shared root.

**Step 4:** Conclusion The requested ratio is $\boxed{1:1:4}$. Hence, the correct option is (B).

**Quick Tip to Solve:** After simplifying a common-root problem, check the discriminant: one shared nonreal root forces two real quadratics to be proportional.

**18.**

**If** $f(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ **for all** $x\in\mathbb{R}$**, then** $2f(0)+f'(0)$ **is equal to**

[4 marks]

(A) 24

(B) 18

(C) 42

(D) 48

**Correct Answer:** (C) 42

**Solution:**

**Concept:**

- A determinant is a polynomial in its entries. To find $f(0)$ and $f'(0)$, only its constant and linear terms are needed.
- Determinants are multilinear in rows: their derivative is the sum of determinants obtained by differentiating one row at a time.

**Step 1:** Evaluate the constant term At $x=0$, the determinant is

$$f(0)=\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}.$$

Expanding along the first row gives

$$f(0)=-1(2(-2)-6\cdot 0)+1(2\cdot 4-0\cdot 0)=4+8=12.$$

The alternating cofactor signs are $+, -, +$, which is why the middle contribution is positive after its

negative minor is included.

**Step 2:** Differentiate one row at a time The row derivatives at zero are $(0,0,3)$, $(0,2,0)$, and $(-1,0,0)$. Consequently,

$$f'(0) = \begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}$$
$$= 3(2 \cdot 4) + 0 - 6 = 18.$$

The middle determinant vanishes because its first column is zero. In the last determinant the cofactor of the bottom-left entry is positive, giving $(-1)(1 \cdot 6 - 1 \cdot 0) = -6$.

**Step 3:** Combine the two requested quantities Therefore

$$2f(0) + f'(0) = 2(12) + 18 = 42.$$

As a quick independent check, retaining only linear terms in the original determinant yields rows $(0, 1, 1+3x)$, $(2, 2x, 6)$, and $(-x, 4, -2)$. Their determinant begins $12 + 18x$, confirming both values. Option (A) is only $2f(0)$, option (B) is only the derivative, and option (D) does not equal their sum.

**Step 4:** Conclusion The required value is $\boxed{42}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For a determinant evaluated at zero, discard terms of degree two or higher when only its value and first derivative are requested.

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**19. If one of the diameters of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle C, whose center is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle C is :** [4 marks]

(A) 4

(B) $3\sqrt{2}$

(C) 6

(D) $\sqrt{20}$

**Correct Answer:** (C) 6

**Solution:**

**Concept:**

- Completing squares gives the centre and radius of a circle. A diameter has its midpoint at that centre and half-length equal to the radius.
- The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord, so radius squared equals half-chord squared plus centre-to-chord distance squared.

**Step 1:** Find the diameter midpoint and half-length Complete the squares in the first circle:

$$x^2 - 10x + y^2 + 4y + 13 = 0 \quad \Longrightarrow \quad (x-5)^2 + (y+2)^2 = 16.$$

Thus its centre is $O = (5, -2)$, its radius is 4, and every diameter has midpoint $O$ and total length 8. The diameter specified in the question is a chord of circle $C$, so its midpoint remains $O$, and its half-length remains 4.

**Step 2:** Locate the centre of the second circle Let the centre of circle $C$ be $H = (h, k)$. Solving the given lines,

$$2h + 3k = 12, \qquad 3h - 2k = 5,$$

gives $13h = 39$, hence $h = 3$ and $k = 2$. The distance between $H = (3, 2)$ and the chord midpoint $O = (5, -2)$ is

$$HO = \sqrt{(5-3)^2 + (-2-2)^2} = \sqrt{20}.$$

This is the perpendicular distance from $H$ to the common chord, not the radius of circle $C$.

**Step 3:** Use the right triangle formed by the chord If $A$ is either endpoint of that diameter, then $OA = 4$ and $HO \perp OA$. Pythagoras therefore gives

$$HA^2 = HO^2 + OA^2 = 20 + 16 = 36, \qquad HA = 6.$$

Both endpoints have the same distance from $H$, so they lie on the same second circle as required. The choices 4 and $\sqrt{20}$ are respectively the first radius and the distance between centres; $3\sqrt{2}$ has squared value 18 instead of 36.

**Step 4:** Conclusion The radius of circle $C$ is $\boxed{6}$ units. Hence, the correct option is (C).

**Quick Tip to Solve:** For a diameter reused as another circle's chord, retain its midpoint and half-length; combine these with the new centre using Pythagoras.

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**20. If the foci of a hyperbola are same as that of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ and the eccentricity of the hyperbola is $\frac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2}, \frac{14}{3}\sqrt{\frac{2}{5}}\right)$ on the hyperbola, is equal to** [4 marks]

(A) $14\sqrt{\frac{2}{5}} - \frac{4}{3}$

(B) $7\sqrt{\frac{2}{5}} + \frac{8}{3}$

(C) $7\sqrt{\frac{2}{5}} - \frac{8}{3}$

(D) $14\sqrt{\frac{2}{5}} - \frac{16}{3}$

**Correct Answer:** (C) $7\sqrt{\frac{2}{5}} - \frac{8}{3}$

**Solution:**

**Concept:**

- For an ellipse with semimajor axis $a_e$ and semiminor axis $b_e$, $c^2 = a_e^2 - b_e^2$ and eccentricity is $c/a_e$.
- For a hyperbola, eccentricity is $e_h = c/a_h$. For its upper branch with foci on the y-axis, the smaller focal distance is $e_h y - a_h$.

**Step 1:** Determine the common foci and hyperbola parameters The ellipse has semimajor axis 5 along the y-axis and semiminor axis 3. Thus

$$c = \sqrt{25 - 9} = 4, \qquad e_e = \frac{4}{5}.$$

The hyperbola shares the foci $(0, \pm 4)$, and

$$e_h = \frac{15}{8} \cdot \frac{4}{5} = \frac{3}{2}, \qquad a_h = \frac{c}{e_h} = \frac{8}{3}.$$

Its other squared parameter is $b_h^2 = c^2 - a_h^2 = 16 - 64/9 = 80/9$, so its equation is $y^2/(64/9) - x^2/(80/9) = 1$.

**Step 2:** Check the given point and identify the nearer focus For the stated point, $x^2 = 2$ and $y^2 = (196/9)(2/5) = 392/45$. Substitution gives

$$\frac{392/45}{64/9} - \frac{2}{80/9} = \frac{49}{40} - \frac{9}{40} = 1,$$

so it is on the upper branch. Since $y > 0$, the nearer focus is $(0, 4)$. The focal-distance identity follows from $x^2 = b_h^2(y^2/a_h^2 - 1)$: expanding $x^2 + (y - c)^2$ gives $(e_h y - a_h)^2$.

**Step 3:** Evaluate the smaller focal distance Here $e_h y - a_h > 0$, so the distance is

$$\frac{3}{2}\left(\frac{14}{3}\sqrt{\frac{2}{5}}\right) - \frac{8}{3} = 7\sqrt{\frac{2}{5}} - \frac{8}{3}.$$

The plus sign in option (B) gives the farther focal distance. Options (A) and (D) have an incorrect coefficient of the square-root term and therefore cannot be either distance for this point. The difference between the two actual distances is $2a_h = 16/3$, as a hyperbola requires.

**Step 4:** Conclusion The smaller focal distance is $\boxed{7\sqrt{2/5} - 8/3}$ units. Hence, the correct option is (C).

**Quick Tip to Solve:** For confocal conics, keep c fixed. For a vertical hyperbola use the point's y-coordinate in the focal-distance formula.

## Mathematics – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**21. If $\alpha$ denotes the number of solutions of $|1-i|^x = 2^x$ and $\beta = \left(\frac{|z|}{\arg(z)}\right)$, where $z = \frac{\pi}{4}(1+i)^4\left[\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right], i = \sqrt{-1}$, then the distance of the point $(\alpha, \beta)$ from the line $4x - 3y = 7$ is _____.** [4 marks]

**Solution:**

**Concept:**

- The modulus of $1-i$ is $\sqrt{2}$. Exponential equality with the same positive base different from 1 reduces to equality of exponents.
- The principal argument of a positive imaginary number is $\pi/2$. The perpendicular distance from $(x_0, y_0)$ to $Ax + By + C = 0$ is $|Ax_0 + By_0 + C|/\sqrt{A^2 + B^2}$.

**Step 1:** Find the number of exponential solutions The equation defining $\alpha$ is

$$(\sqrt{2})^x = 2^x \quad \Longleftrightarrow \quad 2^{x/2} = 2^x.$$

Since $2^t$ is strictly increasing for real $t$, we obtain $x/2 = x$, hence $x = 0$. This is exactly one real solution, so $\alpha = 1$. The number $\alpha$ is the number of solutions, not the value of the solution itself.

**Step 2:** Simplify the complex expression before taking its argument Writing $s = \sqrt{\pi}$, note that $1 - si = -i(s+i)$ and $s - i = -i(1+si)$. Each denominator is nonzero, and consequently

$$\frac{1-si}{s+i} = -i, \qquad \frac{s-i}{1+si} = -i.$$

Also $(1+i)^2 = 2i$, so $(1+i)^4 = -4$. Therefore

$$z = \frac{\pi}{4}(-4)(-2i) = 2\pi i.$$

This lies on the positive imaginary axis, giving $|z| = 2\pi$ and principal argument $\arg z = \pi/2$. Thus $\beta = (2\pi)/(\pi/2) = 4$.

**Step 3:** Calculate the perpendicular distance The point is now $(\alpha, \beta) = (1, 4)$. Rewrite the line as $4x - 3y - 7 = 0$ and apply the distance formula:

$$d = \frac{|4(1) - 3(4) - 7|}{\sqrt{4^2 + (-3)^2}} = \frac{|-15|}{5} = 3.$$

The absolute value makes the distance nonnegative, and the denominator normalises the line coefficients. Since the coordinates and line are dimensionless, this is a distance of 3 coordinate units.

**Step 4:** Conclusion The required numerical value is $\boxed{3}$, corresponding to a perpendicular distance of 3 units.

**Quick Tip to Solve:** Before rationalising complicated complex fractions, test whether the numerator is a simple multiple of the denominator; both fractions here equal -i.

**22. In the expansion of $(1+x)\left(1-x^2\right)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5, x \neq 0$, the sum of the coefficients of $x^3$ and $x^{-13}$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- The binomial expansion gives the coefficient of $x^r$ in $(1+x)^n$ as $\binom{n}{r}$, with the coefficient zero outside $0 \leq r \leq n$.
- A Laurent polynomial can be converted into an ordinary polynomial by extracting its negative power of $x$. Multiplication by $x^{-15}$ reduces every exponent by 15.

**Step 1:** Rewrite the cubed factor Recognise the four terms as a perfect cube. Since $x \neq 0$,

$$1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}=\left(1+\frac{1}{x}\right)^3.$$

Raising to the fifth power gives $x^{-15}(1+x)^{15}$. Therefore the entire expression becomes

$$x^{-15}(1-x^2)(1+x)^{16}.$$

This form keeps the exponent shift separate from the coefficient calculation and avoids expanding fifteen factors individually.

**Step 2:** Extract the two required coefficients Write $P(x)=(1-x^2)(1+x)^{16}$. The coefficient of $x^3$ in the original expression is the coefficient of $x^{18}$ in $P$. As $(1+x)^{16}$ has degree 16, only the product of $-x^2$ with its leading term contributes, giving $-1$. The coefficient of $x^{-13}$ corresponds to $x^2$ in $P$, because $-13+15=2$. Hence it is

$$\binom{16}{2}-\binom{16}{0}=120-1=119.$$

The minus sign in the second contribution comes from $1-x^2$, and is essential in both calculations.

**Step 3:** Conclusion The sum of the two coefficients is $-1+119=\boxed{118}$, a dimensionless integer.

**Quick Tip to Solve:** For a Laurent polynomial, first factor out the overall negative power; then shift each requested exponent before using binomial coefficients.

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**23. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=1, |\vec{b}|=4$, and $\vec{a}\cdot\vec{b}=2$. If $\vec{c}=(2\vec{a}\times\vec{b})-3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\alpha$, then $192\sin^2\alpha$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- The cross-product identity is $|\vec{a}\times\vec{b}|^2=|\vec{a}|^2|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2$.
- A cross product is perpendicular to both original vectors. Consequently the two components of $\vec{c}=2(\vec{a}\times\vec{b})-3\vec{b}$ are perpendicular, so their squared magnitudes add.

**Step 1:** Find the magnitude of the perpendicular component The given lengths and dot product yield

$$|\vec{a} \times \vec{b}|^2 = (1)^2(4)^2 - 2^2 = 16 - 4 = 12.$$

Thus $|2\vec{a} \times \vec{b}|^2 = 4(12) = 48$. The other component, $-3\vec{b}$, has squared magnitude $9(16) = 144$. The mixed dot product is zero, since $(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$. Hence

$$|\vec{c}|^2 = 48 + 144 = 192.$$

Both vectors whose angle is required are nonzero, so the angle formula is well-defined.

**Step 2:** Calculate the squared sine of the angle Dotting $\vec{c}$ with $\vec{b}$ eliminates its cross-product component:

$$\vec{b} \cdot \vec{c} = 0 - 3|\vec{b}|^2 = -48.$$

Therefore

$$\cos^2 \alpha = \frac{(\vec{b} \cdot \vec{c})^2}{|\vec{b}|^2|\vec{c}|^2} = \frac{48^2}{16 \cdot 192} = \frac{3}{4}.$$

The dot product is negative, so $\alpha$ is obtuse; squaring does not change the required sine square. Using $\sin^2 \alpha = 1 - \cos^2 \alpha$, we obtain $\sin^2 \alpha = 1/4$. Equivalently, the perpendicular component contributes $48/192$ of the squared magnitude of $\vec{c}$.

**Step 3:** Conclusion The requested value is $192 \sin^2 \alpha = 192/4 = \boxed{48}$, which is dimensionless.

**Quick Tip to Solve:** Decompose the constructed vector into its parts parallel and perpendicular to the reference vector; the squared perpendicular fraction directly gives the sine square.

---

**24. If the integral $525 \int_0^{\frac{\pi}{2}} \sin 2x \cos^{\frac{11}{2}} x \left(1 + \mathrm{Cos}^{\frac{5}{2}} x\right)^{\frac{1}{2}} dx$ is equal to $(n\sqrt{2} - 64)$, then $n$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- Use $\sin 2x = 2 \sin x \cos x$ and choose a substitution whose derivative absorbs the sine factor and a suitable power of cosine.
- On $[0, \pi/2]$, cosine is nonnegative. Thus the fractional powers and the substitutions $u = \cos^{5/2} x$, $v = \sqrt{1 + u}$ are real throughout the integration interval.

**Step 1:** Reduce the trigonometric integral to an algebraic integral Let $I$ denote the complete integral including its factor 525. Put $u = \cos^{5/2} x$, so

$$du = -\frac{5}{2} \sin x \cos^{3/2} x \, dx.$$

Combining $\sin 2x$ with $\cos^{11/2} x$ gives $2 \sin x \cos^{13/2} x$. The remaining cosine power after the substitution is $\cos^5 x = u^2$. The limits change from $x = 0, \pi/2$ to $u = 1, 0$, respectively. Reversing them

absorbs the minus sign:

$$I = 420 \int_0^1 u^2\sqrt{1+u}\,du.$$

**Step 2:** Evaluate the polynomial integral exactly Now put $v = \sqrt{1+u}$. Then $u = v^2 - 1$, $du = 2v\,dv$, and the limits are $1$ and $\sqrt{2}$. Hence

$$I = 840 \int_1^{\sqrt{2}} (v^6 - 2v^4 + v^2)\,dv.$$

An antiderivative gives

$$I = 840 \left[\frac{v^7}{7} - \frac{2v^5}{5} + \frac{v^3}{3}\right]_1^{\sqrt{2}}.$$

At the upper limit the bracket is $22\sqrt{2}/105$; at the lower limit it is $8/105$. Thus $I = 176\sqrt{2} - 64$. In particular, both the radical coefficient and the constant term match the requested form.

**Step 3:** Conclusion Comparing $I = 176\sqrt{2} - 64$ with $n\sqrt{2} - 64$ gives the dimensionless integer $\boxed{n = 176}$.

**Quick Tip to Solve:** When a fractional cosine power occurs inside a square root, first substitute that inner power, then substitute the square root to obtain a polynomial.

---

**25. Let $S = (-1, \infty)$ and $f : S \to \mathbb{R}$ be defined as**

$$f(x) = \int_{-1}^{x} \left(e^t - 1\right)^{11} (2t-1)^5 (t-2)^7 (t-3)^{12} (2t-10)^{61} dt,$$

**Let $\mathrm{p}$ = Sum of squares of the values of $x$, where $f(x)$ attains local maxima on $S$, and $\mathrm{q}$ = Sum of the values of $\mathrm{x}$, where $f(x)$ attains local minima on $S$. Then, the value of $p^2 + 2q$ is _____.** [4 marks]

**Solution:**

**Concept:**

- The fundamental theorem of calculus gives $f\prime(x)$ as the integrand evaluated at $t = x$. Local maxima require a positive-to-negative derivative sign change; minima require the reverse.
- An odd power has the sign of its base and changes sign at a simple zero of that base. An even power stays nonnegative and cannot by itself change the derivative sign.

**Step 1:** Locate the critical points and simplify the signs Differentiating the given integral yields

$$f\prime(x) = (e^x - 1)^{11}(2x-1)^5(x-2)^7(x-3)^{12}(2x-10)^{61}.$$

Its zeros in $S = (-1, \infty)$ are $0, 1/2, 2, 3, 5$. Since the exponential is increasing, $e^x - 1$ has the same sign as $x$. Away from its zeros, the derivative therefore has the sign of

$$x(x - \tfrac{1}{2})(x-2)(x-5).$$

The factor $(x-3)^{12}$ is positive on either side of 3, so this critical point must be checked without assuming an extremum.

**Step 2:** Apply the sign test and calculate the required sums The derivative signs on the successive intervals are

| $x$ | $(-1,0)$ | $(0,1/2)$ | $(1/2,2)$ | $(2,3)$ | $(3,5)$ | $(5,\infty)$ |
|---|---|---|---|---|---|---|
| $f\prime(x)$ | $+$ | $-$ | $+$ | $-$ | $-$ | $+$ |

Thus local maxima occur at $x=0,2$, while local minima occur at $x=1/2,5$. At $x=3$ the derivative vanishes but is negative on both sides, so there is no extremum. The excluded endpoint $-1$ contributes nothing. Therefore

$$p=0^2+2^2=4, \qquad q=\frac{1}{2}+5=\frac{11}{2}.$$

Remember that $p$ is a sum of squares, whereas $q$ is a sum of the locations themselves.

**Step 3:** Conclusion The required dimensionless value is $p^2+2q=4^2+11=\boxed{27}$.

**Quick Tip to Solve:** For a product with large powers, discard positive even factors for sign testing and retain only the signs of the odd-power bases.

**26. Let $\mathrm{Q}$ and $\mathrm{R}$ be the feet of perpendiculars from the point $\mathrm{P}(a,a,a)$ on the lines $x=y, z=1$ and $x=-y, z=-1$ respectively. If $\angle\mathrm{QPR}$ is a right angle, then $12a^2$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- The foot of a perpendicular lies on its line and its displacement from the original point has zero dot product with the line direction.
- An angle between two nonzero vectors is a right angle exactly when their dot product vanishes. Use vectors starting at $P$ to enforce the angle $QPR$.

**Step 1:** Find the two feet of perpendiculars A general point on $x=y, z=1$ is $(s,s,1)$, and the line direction is $(1,1,0)$. The perpendicular condition from $P(a,a,a)$ is

$$(s-a,s-a,1-a)\cdot(1,1,0)=2(s-a)=0.$$

Thus $Q=(a,a,1)$. For the second line use $R=(t,-t,-1)$ and direction $(1,-1,0)$. Its perpendicular condition gives

$$(t-a,-t-a,-1-a)\cdot(1,-1,0)=2t=0.$$

Consequently $R=(0,0,-1)$, independently of $a$. This is a projection condition, not a requirement that the two lines intersect.

**Step 2:** Impose the right angle at the point P The displacement vectors are

$$\overrightarrow{PQ}=(0,0,1-a), \qquad \overrightarrow{PR}=(-a,-a,-1-a).$$

Their dot product is

$$\overrightarrow{PQ} \cdot \overrightarrow{PR} = (1-a)(-1-a) = a^2 - 1.$$

The prescribed right angle therefore requires $a^2 = 1$. When $a = 1$, $P = Q$, so the angle itself is undefined and that candidate is inadmissible. The value $a = -1$ gives $\overrightarrow{PQ} = (0, 0, 2)$ and $\overrightarrow{PR} = (1, 1, 0)$, both nonzero and perpendicular. It satisfies the actual geometric condition and still gives $a^2 = 1$.

**Step 3:** Conclusion The admissible value $a = -1$ yields the dimensionless number $12a^2 = 12(1) = \boxed{12}$.

**Quick Tip to Solve:** After solving a perpendicularity equation, check that neither angle-defining vector is zero; a vanishing dot product alone also admits degenerate cases.

**27. Let the foci and length of the latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > bbe(\pm 5, 0)$ and $\sqrt{50}$, respectively. Then, the square of the eccentricity of the hyperbola $\frac{x^2}{b^2} - \frac{y^2}{a^2 b^2} = 1$ equals** [4 marks]

**Solution:**

**Concept:**

- For the ellipse with semi-major axis $a$ and semi-minor axis $b$, the focal distance satisfies $c^2 = a^2 - b^2$ and the latus rectum has length $2b^2/a$.
- For a hyperbola $x^2/A^2 - y^2/B^2 = 1$, the eccentricity satisfies $e^2 = 1 + B^2/A^2$. The semi-axis squares are the complete denominators.

**Step 1:** Determine the two ellipse parameters The foci at $(\pm 5, 0)$ give $c = 5$. Hence

$$a^2 - b^2 = 25.$$

The specified latus rectum is $\sqrt{50} = 5\sqrt{2}$, so

$$\frac{2b^2}{a} = 5\sqrt{2}, \qquad b^2 = \frac{5a}{\sqrt{2}}.$$

Eliminating $b^2$ gives

$$a^2 - \frac{5}{\sqrt{2}}a - 25 = 0.$$

The discriminant is $25/2 + 100 = 225/2$, whose square root is $15/\sqrt{2}$. The roots are $5\sqrt{2}$ and $-5/\sqrt{2}$. As an axis length must be positive, choose $a = 5\sqrt{2}$. Therefore $a^2 = 50$ and $b^2 = 25$, which also verify the original focal equation.

**Step 2:** Read the hyperbola denominators and evaluate its eccentricity The required hyperbola is

$$\frac{x^2}{b^2} - \frac{y^2}{a^2 b^2} = 1.$$

Thus its transverse semi-axis square is $A^2 = b^2 = 25$, while its conjugate semi-axis square is $B^2 =$

$a^2b^2 = 50 \cdot 25$. Apply the hyperbola identity:

$$e^2 = 1 + \frac{B^2}{A^2} = 1 + \frac{a^2b^2}{b^2} = 1 + a^2 = 51.$$

This uses the hyperbola eccentricity, not the ellipse eccentricity $c/a$. The latter belongs only to the preliminary ellipse data.

**Step 3:** Conclusion The square of the eccentricity of the given hyperbola is the dimensionless value $\boxed{51}$.

**Quick Tip to Solve:** First solve the ellipse data for its axis parameters, then rename the hyperbola denominators as new semi-axis squares to avoid mixing the two eccentricities.

**28. The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- The word DISTRIBUTION contains nine distinct letter types: I occurs three times, T occurs twice, and D, S, R, B, U, O, N occur once each.
- For a chosen multiset of four letters with multiplicities $m_1, m_2, \ldots$, the number of distinct arrangements is $4!/(m_1!m_2!\cdots)$. Distinguish words by letter sequence, not by copies of the same letter.

**Step 1:** Count words with all distinct letters or one pair If all four letters are distinct, choose their types from the nine available and arrange them:

$$N_1 = \binom{9}{4} 4! = 126 \cdot 24 = 3024.$$

For multiplicity pattern $2 + 1 + 1$, the repeated type can only be I or T, giving two choices. After choosing it, choose two distinct singleton types from the remaining eight types. These may include the other repeat-capable type, but only one copy of it is used in this case. Therefore

$$N_2 = 2\binom{8}{2}\frac{4!}{2!} = 2 \cdot 28 \cdot 12 = 672.$$

There is no overcount: a word of this pattern has exactly one doubled letter type.

**Step 2:** Count the remaining feasible multiplicity patterns For the pattern $2 + 2$, the two doubled types must be I and T. Its contribution is

$$N_3 = \frac{4!}{2!2!} = 6.$$

For the pattern $3 + 1$, only I can occur three times. The singleton may be any of the other eight letter

types, so

$$N_4 = 8\frac{4!}{3!} = 8 \cdot 4 = 32.$$

A pattern $4$ is impossible because no type occurs four times in the original word. The four cases are mutually exclusive and exhaust all partitions of four compatible with the available copies, so they can be added directly.

**Step 3:** Conclusion The total number of distinct four-letter words is $3024 + 672 + 6 + 32 = \boxed{3734}$. This is a dimensionless count.

**Quick Tip to Solve:** Classify by multiplicity patterns before arranging: for four letters check 1+1+1+1, 2+1+1, 2+2 and 3+1, discarding patterns unsupported by the inventory.

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**29. Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3), (1, 4)\}$ be a relation on A. Let S be the equivalence relation on A such that $R \subset S$ and the number of elements in S is n. Then, the minimum value of $n$ is _____.**

[4 marks]

**Solution:**

**Concept:**

- An equivalence relation is reflexive, symmetric and transitive. Its equivalence classes form a partition of the underlying set.
- If two elements occur together in a related pair, they must lie in the same equivalence class. A class of size $k$ contributes $k^2$ ordered pairs.

**Step 1:** Identify the class forced by the given pairs The inclusion $R \subset S$ requires $1 \sim 2$, $2 \sim 3$, and $1 \sim 4$. Transitivity of the first two gives $1 \sim 3$. Thus 2, 3 and 4 all belong to the class containing 1. In particular, they cannot be placed in separate classes to reduce the number of pairs. Symmetry ensures that the reversed pairs must also occur.

**Step 2:** Count all pairs and verify attainability The only possible partition is therefore the single class $A = \{1, 2, 3, 4\}$. Every element is related to every other element, including itself. Consequently,

$$S = A \times A, \qquad |S| = |A|^2 = 4^2 = 16.$$

This relation is reflexive, symmetric and transitive, and contains all three given pairs. Hence the lower bound is actually attained. There are no additional elements outside this class, and no smaller equivalence relation can satisfy the required inclusions.

**Step 3:** Conclusion The minimum value is $\boxed{n = 16}$. This is a count of ordered pairs and has no physical unit.

**Quick Tip to Solve:** Join elements linked by the prescribed pairs; each connected component must become one equivalence class, contributing its size squared.

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**30. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{4^x}{4^x+2}$, and let**

$$M = \int_{f(a)}^{f(1-a)} x \sin^4(x(1-x))\,dx, \qquad N = \int_{f(a)}^{f(1-a)} \sin^4(x(1-x))\,dx; \quad a \neq \frac{1}{2}.$$

**If $\alpha M = \beta N$, where $\alpha, \beta \in \mathbb{N}$, then the least value of $\alpha^2 + \beta^2$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- For limits summing to 1, the substitution $x = 1 - u$ preserves the interval with reversed endpoints.
- The function $g(x) = \sin^4(x(1-x))$ is symmetric: $g(1-x) = g(x)$. This pairs the weights $x$ and $1-x$.

**Step 1:** Show that the endpoints are complementary Put $t = 4^a > 0$. Then $4^{1-a} = 4/t$, so

$$f(a) = \frac{t}{t+2}, \qquad f(1-a) = \frac{4/t}{4/t+2} = \frac{2}{t+2}.$$

Their sum is exactly 1. Write $p = f(a)$, so that the upper endpoint is $1 - p$. The condition $a \neq 1/2$ implies $t \neq 2$, and hence the endpoints are distinct.

**Step 2:** Apply the reflection and minimise the integers Under $x = 1-u$, with the limits and differential both transformed,

$$M = \int_p^{1-p} (1-u)g(u)\,du = N - M.$$

Therefore $2M = N$. On the open interval $(0, 1)$, we have $0 < x(1-x) \leq 1/4$, so $g(x) > 0$. Thus the integral between the distinct endpoints is nonzero, even when its orientation is negative. The equation $\alpha M = \beta N$ now gives $\alpha = 2\beta$. Positive integers therefore have the form $(\alpha, \beta) = (2k, k)$, $k \geq 1$, and

$$\alpha^2 + \beta^2 = 5k^2 \geq 5.$$

Equality occurs for $\alpha = 2$, $\beta = 1$.

**Step 3:** Conclusion The least value is $\boxed{5}$, a dimensionless number.

**Quick Tip to Solve:** Before integrating a symmetric function times x, add the integral to its reflected form; complementary limits often give twice the weighted integral.

## Physics – Section A: Multiple Choice Questions (80 marks)

**31. Four identical particles of mass $m$ are kept at the four corners of a square. If the gravitational force exerted on one of the masses by the other masses is $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$, the length of the sides of the square is** [4 marks]

(A) 4L

(B) 3L

(C) 2L

(D) $\frac{L}{2}$

**Correct Answer:** (A) 4L

**Solution:**

**Concept:**

- Newton's gravitational force between two particles is $Gm^2/d^2$, directed along the line joining them.
- The two adjacent-corner forces are perpendicular and equal. Their resultant lies along the diagonal, in the same direction as the force due to the opposite corner.

**Step 1:** Combine the forces along the diagonal Let the side length be $s$. Each adjacent mass pulls with magnitude $F_0 = Gm^2/s^2$. Their vector sum has magnitude $\sqrt{2}F_0$, directed toward the diagonally opposite corner. That corner is a distance $\sqrt{2}s$ away, so its force has magnitude $Gm^2/(2s^2)$. Since these two contributions are parallel, they add directly:

$$F = \frac{Gm^2}{s^2}\left(\sqrt{2}+\frac{1}{2}\right) = \frac{(2\sqrt{2}+1)Gm^2}{2s^2}.$$

**Step 2:** Compare coefficients and check the choices Equating this to the stated force and cancelling the common positive factors gives

$$\frac{1}{2s^2} = \frac{1}{32L^2}, \qquad s^2 = 16L^2.$$

A length is positive, so $s = 4L$. The choices $3L$, $2L$ and $L/2$ would respectively produce denominators $18L^2$, $8L^2$ and $L^2/2$ in this coefficient, rather than $32L^2$. One must not add all three magnitudes before resolving the adjacent forces: those two forces are mutually perpendicular.

**Step 3:** Conclusion The side length is $\boxed{4L}$. Hence, the correct option is (A).

**Quick Tip to Solve:** At a square corner, first add the two equal perpendicular forces; their resultant aligns with the diagonal force.

**32. Two charges $q$ and $3q$ are separated by a distance '$r$' in air. At a distance $x$ from charge $q$, the resultant electric field is zero. The value of $x$ is :** [4 marks]

(A) $\frac{r}{3(1+\sqrt{3})}$

(B) $\frac{(1+\sqrt{3})}{r}$

(C) $\frac{r}{(1+\sqrt{3})}$

(D) $r(1+\sqrt{3})$

**Correct Answer:** (C) $\frac{r}{(1+\sqrt{3})}$

**Solution:**

**Concept:**

- The magnitude of the field of a point charge is $E = k|q|/d^2$, with direction away from a positive charge and toward a negative charge.
- The charges $q$ and $3q$ have the same sign. Their fields oppose between the charges and point in the same direction outside the segment.

**Step 1:** Locate the only possible cancellation point Place charge $q$ at coordinate 0 and charge $3q$ at coordinate $r$. A zero field must lie between them; there the distances from the charges are $x$ and $r - x$. Equal magnitudes are required because the fields have opposite directions. Thus

$$\frac{k|q|}{x^2} = \frac{3k|q|}{(r-x)^2}.$$

Both distances are positive. Taking the positive square root therefore gives $r - x = \sqrt{3}\,x$, rather than its negative counterpart.

**Step 2:** Solve and test the alternatives Rearranging the distance relation,

$$r = (1+\sqrt{3})x, \qquad x = \frac{r}{1+\sqrt{3}}.$$

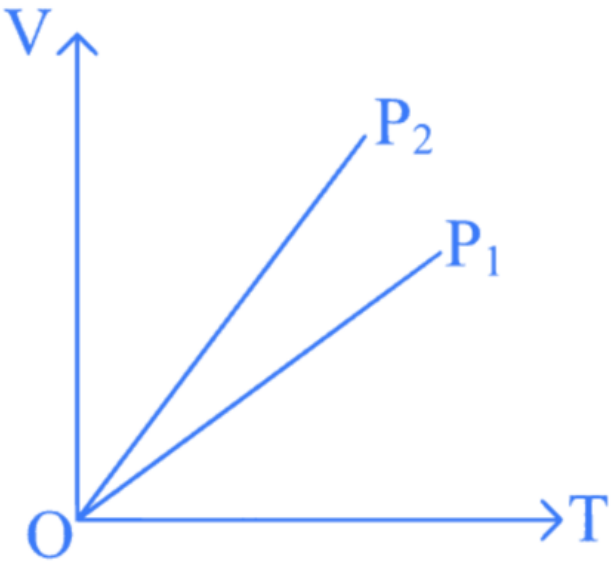
This lies below $r/2$, correctly placing the cancellation point nearer the smaller charge. Choice (A) inserts an extra factor of 3 and fails the squared-distance balance. Choice (B) has dimensions of inverse length and cannot represent a distance. Choice (D) is greater than $r$, putting the point outside the charges, where their fields reinforce. Thus only the stated fraction is consistent both algebraically and physically.

**Step 3:** Conclusion $\boxed{x = r/(1+\sqrt{3})}$, measured from charge $q$ toward charge $3q$. Hence, the correct option is (C).

**Quick Tip to Solve:** For two like charges, locate the zero field between them and use the square root of their charge ratio to compare distances.

**33. The given figure represents two isobaric processes for the same mass of an ideal gas, then**

[4 marks]




(A) $P_2 > P_1$

(B) $P_1 > P_2$

(C) $P_1 = P_2$

(D) $P_2 \geq P_1$

**Correct Answer:** (B) $P_1 > P_2$

**Solution:**

**Concept:**

- For a fixed mass of an ideal gas, $PV = nRT$, where $n$ is constant and temperature is measured on the absolute scale.
- At constant pressure, the volume-temperature graph satisfies $V = (nR/P)T$. Its slope is inversely proportional to pressure.

**Step 1:** Read the axes and compare the slopes The vertical axis of the given graph is volume $V$, and the horizontal axis is temperature $T$. Both straight lines represent isobaric processes for the same gas sample. The line labelled $P_2$ is visibly steeper than the one labelled $P_1$. If their slopes are $s_2$ and $s_1$, respectively, then $s_2 > s_1 > 0$. The labels identify pressures, not the slope values themselves.

**Step 2:** Use the ideal-gas equation to infer pressure From the ideal-gas law,

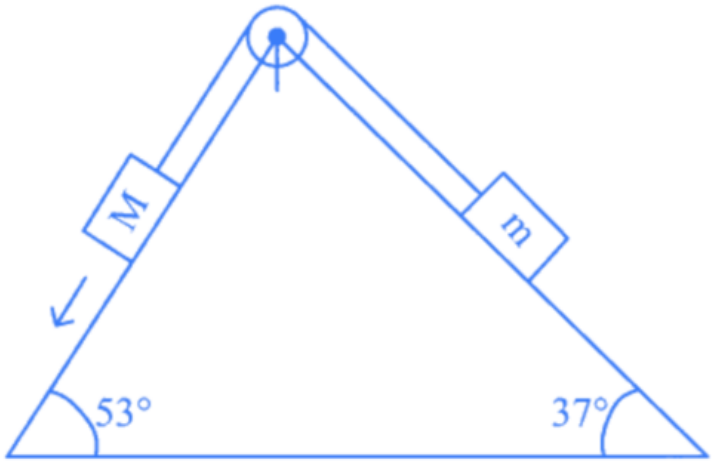
$$s_1 = \frac{nR}{P_1}, \qquad s_2 = \frac{nR}{P_2}, \qquad \frac{P_1}{P_2} = \frac{s_2}{s_1} > 1.$$

Therefore $P_1 > P_2$. Physically, at any common positive temperature the steeper line gives the larger volume; a fixed amount of gas at that temperature has lower pressure when its volume is larger. Choice (A) reverses this relation. Choice (C) would require coincident lines of equal slope, and choice (D) wrongly includes pressures at least as large as $P_1$.

**Step 3:** Conclusion The pressures satisfy $\boxed{P_1 > P_2}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Always read the axes first: for V versus T an isobar has slope nR/P, whereas a P versus T isochore has slope nR/V.

**34. In the given arrangement of a doubly inclined plane two blocks of masses $M$ and $m$ are placed. The blocks are connected by a light string passing over an ideal pulley as shown. The coefficient of friction between the surface of the plane and the blocks is 0.25. The value of $m$, for which $M = 10$ kg will move down with an acceleration of $2\ \mathrm{m/s^2}$, is: (take $\mathrm{g} = 10\ \mathrm{m/s^2}$ and $\tan 37^\circ = 3/4$)** [4 marks]




(A) 4.5 kg

(B) 6.5 kg

(C) 9 kg

(D) 2.25 kg

**Correct Answer:** (A) 4.5 kg

**Solution:**

**Concept:**

- Apply Newton's second law along each incline, taking the stated direction of each block's acceleration as positive.
- Kinetic friction opposes motion, with magnitude $\mu mg\cos\theta$. The light string and ideal pulley give equal tension and equal acceleration magnitudes.

**Step 1:** Find the tension using the descending block The figure gives the incline of $M = 10\,\mathrm{kg}$ as $53^\circ$. From $\tan 37^\circ = 3/4$, use $\sin 53^\circ = 4/5$ and $\cos 53^\circ = 3/5$. Block $M$ moves down, so tension and friction act up its plane. Its equation is

$$Mg\sin 53^\circ - T - \mu Mg\cos 53^\circ = Ma.$$

Substituting the data gives

$$80 - T - 15 = 20, \qquad T = 45\,\mathrm{N}.$$

Friction on this block is $0.25 \times 10 \times 10 \times 3/5 = 15\,\mathrm{N}$.

**Step 2:** Use the ascending block to determine its mass The block of mass $m$ accelerates up the $37^\circ$ incline. Gravity and friction both act down that plane, so

$$T - mg\sin 37^\circ - \mu mg\cos 37^\circ = ma.$$

Since $\sin 37^\circ = 3/5$, $\cos 37^\circ = 4/5$, this becomes

$$45 - 6m - 2m = 2m, \qquad 45 = 10m.$$

Thus $m = 4.5\,\text{kg}$. The other listed masses would require tensions $65$, $90$ and $22.5\,\text{N}$, respectively, inconsistent with the $45\,\text{N}$ fixed by the descending block. Both friction signs follow the actual directions of motion.

**Step 3:** Conclusion The required mass is $\boxed{m = 4.5\,\text{kg}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Write one equation per block in its direction of motion; on the ascending block, gravity and friction act together against the tension.

**35. The relation between time '$t$' and distance '$x$' is $t = \alpha x^2 + \beta x$, where $\alpha$ and $\beta$ are constants. The relation between acceleration $(a)$ and velocity $(v)$ is :** [4 marks]

(A) $a = -5\alpha v^5$

(B) $a = -3\alpha v^2$

(C) $a = -2\alpha v^3$

(D) $a = -4\alpha v^4$

**Correct Answer:** (C) $a = -2\alpha v^3$

**Solution:**

**Concept:**

- The reciprocal derivative gives velocity: $v = dx/dt = (dt/dx)^{-1}$, wherever the time-distance relation describes differentiable motion.
- When velocity is expressed as a function of position, the chain rule gives $a = dv/dt = v\,dv/dx$.

**Step 1:** Differentiate time with respect to distance From the given relation $t = \alpha x^2 + \beta x$, differentiate with respect to $x$:

$$\frac{dt}{dx} = 2\alpha x + \beta.$$

Hence

$$v = \frac{1}{2\alpha x + \beta}.$$

This reciprocal is essential: $dt/dx$ has dimensions of time per length, whereas velocity has dimensions of length per time. We work on a branch where $2\alpha x + \beta \neq 0$, so the velocity is finite.

**Step 2:** Apply the chain rule and eliminate position Differentiating the velocity with respect to position gives

$$\frac{dv}{dx} = -\frac{2\alpha}{(2\alpha x + \beta)^2} = -2\alpha v^2.$$

Multiplication by $dx/dt = v$ yields

$$a = v\frac{dv}{dx} = -2\alpha v^3.$$

The parameter $\beta$ is absorbed into the instantaneous velocity. Since $[\alpha] = \text{s}\,\text{m}^{-2}$, the dimensions of $\alpha v^3$

are $\mathrm{m\,s^{-2}}$, as required. The powers $v^5$, $v^2$ and $v^4$ in the other choices do not give the derivative or the correct dimensions with this same $\alpha$.

**Step 3:** Conclusion The required relation is $\boxed{a = -2\alpha v^3}$. Hence, the correct option is (C).

**Quick Tip to Solve:** When time is given as a function of position, invert dt/dx to obtain velocity, then use a = v dv/dx.

**36. An artillery piece of mass $M_1$ fires a shell of mass $M_2$ horizontally. Instantaneously after the firing, the ratio of kinetic energy of the artillery and that of the shell is:** [4 marks]

(A) $M_1/(M_1 + M_2)$
(B) $\frac{M_2}{M_1}$
(C) $\frac{M_1}{M_2}$
(D) $M_2/(M_1 + M_2)$

**Correct Answer:** (B) $\frac{M_2}{M_1}$

**Solution:**

**Concept:**

- For the artillery and shell together, horizontal momentum is conserved during firing because the impulsive internal forces cancel and the external horizontal impulse is negligible.
- For a body of mass $m$, kinetic energy can be written as $K = p^2/(2m)$. Equal magnitudes of momentum therefore mean energies inversely proportional to mass.

**Step 1:** Apply momentum conservation Let the shell leave with horizontal speed $v$, and let the artillery recoil with speed $u$ in the opposite direction. Before firing both are at rest, so the initial horizontal momentum is zero. Taking the shell direction as positive gives

$$M_2 v - M_1 u = 0, \qquad M_1 u = M_2 v.$$

The two momenta have equal magnitude, even though their velocities and kinetic energies are generally unequal. The recoil direction affects momentum, but it does not give a negative kinetic energy.

**Step 2:** Compare the two kinetic energies Using the separate masses given in the question,

$$\frac{K_{\text{artillery}}}{K_{\text{shell}}} = \frac{\frac{1}{2}M_1 u^2}{\frac{1}{2}M_2 v^2} = \frac{M_1}{M_2}\left(\frac{M_2}{M_1}\right)^2 = \frac{M_2}{M_1}.$$

Option (C) reverses the required ratio. Options (A) and (D) contain the combined mass and could describe fractions of a total energy, rather than the ratio of these two energies. As a check, a very heavy artillery piece recoils slowly and has much less kinetic energy than the light shell, exactly as this ratio predicts.

**Step 3:** Conclusion The requested ratio is $\boxed{K_{\text{artillery}}/K_{\text{shell}} = M_2/M_1}$. Hence, the correct option is (B).

**Quick Tip to Solve:** For two bodies recoiling from rest, first set their momentum magnitudes equal; their kinetic-energy ratio is then the inverse mass ratio.

**37. The parameter that remains the same for molecules of all gases at a given temperature is :**

[4 marks]

(A) kinetic energy

(B) mass

(C) momentum

(D) speed

**Correct Answer:** (A) kinetic energy

**Solution:**

**Concept:**

- In kinetic theory, the mean translational kinetic energy of one gas molecule is $\langle K \rangle = \frac{3}{2} k_{\mathrm{B}} T$, where $T$ is absolute temperature and $k_{\mathrm{B}}$ is Boltzmann's constant.
- Temperature fixes the average translational energy per molecule. Molecular mass, speed and momentum can still depend on which gas is considered.

**Step 1:** Identify the quantity fixed by temperature For any gas obeying the kinetic-theory model,

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_{\mathrm{B}} T.$$

The right-hand side contains no molecular mass or chemical identity. Thus molecules of different gases at the same absolute temperature have the same average translational kinetic energy. The statement concerns this thermal average; individual molecules in a gas have a distribution of speeds and energies, so they need not all have an identical instantaneous energy.

**Step 2:** Check the remaining parameters Molecular mass is a property of the species, so option (B) cannot be common to all gases. From the same energy relation,

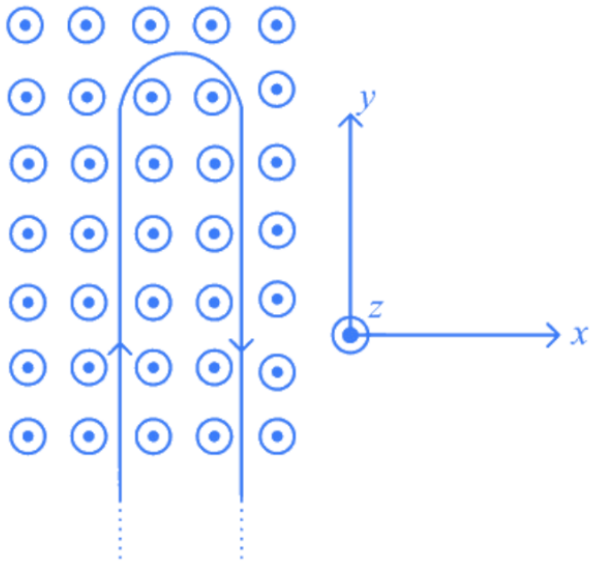
$$v_{\mathrm{rms}} = \sqrt{\frac{3 k_{\mathrm{B}} T}{m}}, \qquad p_{\mathrm{rms}} = m v_{\mathrm{rms}} = \sqrt{3 m k_{\mathrm{B}} T}.$$

The characteristic speed decreases with increasing molecular mass, whereas the characteristic momentum increases with mass. Therefore neither speed nor momentum remains the same across different gases at fixed temperature. Equating temperatures does not equate these mass-dependent quantities; it equates their average translational kinetic energies.

**Step 3:** Conclusion The common parameter is the mean translational kinetic energy per molecule, $\frac{3}{2} k_{\mathrm{B}} T$. Hence, the correct option is (A).

**Quick Tip to Solve:** At a common temperature, compare energy per molecule using $3 k_{\mathrm{B}} T/2$; do not equate the speeds of heavy and light gas molecules.

**38. A rigid wire consists of a semicircular portion of radius $R$ and two straight sections. The wire is partially immerged in a perpendicular magnetic field $B = B_0\hat{k}$ as shown in figure. The magnetic force on the wire if it has a current $i$ is:** [4 marks]




(A) $iBR\hat{j}$

(B) $-2iBR\hat{j}$

(C) $2iBR\hat{j}$

(D) $-iBR\hat{j}$

**Correct Answer:** (B) $-2iBR\hat{j}$

**Solution:**

**Concept:**

- The magnetic force on an element of wire carrying current $i$ is $d\vec{F} = i\,d\vec{\ell} \times \vec{B}$, with $d\vec{\ell}$ directed along the current.
- In a uniform field the total force depends on the displacement from the entry to the exit of the wire segment in the field: $\vec{F} = i(\vec{r}_{\text{exit}} - \vec{r}_{\text{entry}}) \times \vec{B}$.

**Step 1:** Find the effective displacement in the field The figure shows current travelling upward in the left straight part, over the semicircle, and downward in the right straight part. The straight lengths immersed in the field are equal, so their vertical displacements cancel. Equivalently, their forces are equal and opposite along the horizontal direction. The entry and exit points have the same height and are separated horizontally by the diameter of the semicircle. Hence the net displacement along the current through the field is

$$\Delta\vec{r} = 2R\hat{\imath}.$$

The geometrical length $\pi R$ of the curved portion is not the effective length in this vector integral.

**Step 2:** Evaluate the cross product and compare options The dots in the figure represent a field out of the plane, in the positive $z$-direction: $\vec{B} = B_0\hat{k}$. Therefore

$$\vec{F} = i(2R\hat{\imath}) \times (B_0\hat{k}) = -2iB_0R\hat{\jmath}.$$

Here $\hat{\imath} \times \hat{k} = -\hat{\jmath}$, so the force is downward. Options (A) and (C) have the wrong direction. Option (D) has the right direction but uses the radius instead of the diameter and is smaller by a factor of two. The field-free portions contribute no magnetic force.

**Step 3:** Conclusion The net magnetic force is $\boxed{\vec{F} = -2iB_0R\hat{\jmath}}$, or $-2iBR\hat{\jmath}$ in the option notation. Hence, the correct option is (B).

**Quick Tip to Solve:** For a bent wire in a uniform field, replace its shape by the vector from its entry point to its exit point, directed with the current.

**39. When a metal surface is illuminated by light of wavelength $\lambda$, the stopping potential is $8$ V. When the same surface is illuminated by light of wavelength $3\lambda$, stopping potential is $2$ V. The threshold wavelength for this surface is:** [4 marks]

(A) $3\lambda$

(B) $9\lambda$

(C) $5\lambda$

(D) $4.5\lambda$

**Correct Answer:** (B) $9\lambda$

**Solution:**

**Concept:**

- Einstein's photoelectric equation is $eV_s = hc/\lambda - \phi$, where $V_s$ is stopping potential, $e$ the elementary charge and $\phi$ the metal's work function.
- At the threshold wavelength $\lambda_0$, the maximum kinetic energy vanishes, so $\phi = hc/\lambda_0$. The work function is unchanged when the incident wavelength changes.

**Step 1:** Write and subtract the two energy equations For the first illumination, the photon energy is $hc/\lambda$ and the stopping potential is $8\,\mathrm{V}$. For the second, the photon energy is $hc/(3\lambda)$ and the stopping potential is $2\,\mathrm{V}$. Thus

$$\frac{hc}{\lambda} - \phi = 8e\,\mathrm{V}, \qquad \frac{hc}{3\lambda} - \phi = 2e\,\mathrm{V}.$$

Subtract the second relation from the first to eliminate the unknown work function:

$$\frac{2hc}{3\lambda} = 6e\,\mathrm{V}, \qquad \frac{hc}{\lambda} = 9e\,\mathrm{V}.$$

**Step 2:** Determine the work function and threshold wavelength Substituting the photon energy into the first equation gives $\phi = (9-8)\,\mathrm{eV} = 1\,\mathrm{eV}$. Since $hc/\lambda_0 = \phi$, the ratio of wavelengths is

$$\frac{\lambda_0}{\lambda} = \frac{hc/\lambda}{\phi} = \frac{9\,\mathrm{eV}}{1\,\mathrm{eV}} = 9.$$

The result reproduces the second stopping potential: a photon at $3\lambda$ has energy $3\,\mathrm{eV}$, leaving $2\,\mathrm{eV}$ after overcoming the work function. The alternatives $3\lambda$, $5\lambda$ and $4.5\lambda$ do not satisfy both measured potentials.

**Step 3:** Conclusion The threshold wavelength is $\boxed{\lambda_0 = 9\lambda}$. Hence, the correct option is (B).

**Quick Tip to Solve:** With two stopping potentials for the same metal, subtract the photoelectric equations first; this removes the work function immediately.

**40. The refractive index of a prism with apex angle $A$ is $\cot A/2$. The angle of minimum deviation is :** [4 marks]

(A) $\delta_m = 180^\circ - 3$ A

(B) $\delta_m = 180^\circ - 4A$

(C) $\delta_m = 180^\circ - 2A$

(D) $\delta_m = 180^\circ - A$

**Correct Answer:** (C) $\delta_m = 180^\circ - 2A$

**Solution:**

**Concept:**

- At minimum deviation a prism in air has a symmetric ray path: $r_1 = r_2 = A/2$ and $i = e$. Its refractive index obeys $\mu = \sin[(A + \delta_m)/2]/\sin(A/2)$.
- The stated refractive index $\cot(A/2)$ is the ratio $\cos(A/2)/\sin(A/2)$. Use the physical incident angle between zero and $90^\circ$ when solving the sine equation.

**Step 1:** Substitute the specified refractive index For the symmetric path, Snell's law at the first face gives $\sin i = \mu \sin(A/2)$. Inserting the given cotangent of the half-apex angle,

$$\sin i = \cot\left(\frac{A}{2}\right)\sin\left(\frac{A}{2}\right) = \cos\left(\frac{A}{2}\right).$$

Use the complementary-angle identity $\cos(A/2) = \sin(90^\circ - A/2)$. Since the physical angle of incidence is in the principal acute range, this gives $i = 90^\circ - A/2$. Taking the other sine branch would give an incidence angle exceeding $90^\circ$, which is not the required refracting path.

**Step 2:** Obtain the deviation and eliminate the alternatives The total deviation through a prism is $\delta = i + e - A$. At minimum deviation $e = i$, so

$$\delta_m = 2i - A = 2\left(90^\circ - \frac{A}{2}\right) - A = 180^\circ - 2A.$$

Both appearances of the apex angle must be included: one arises through the incidence angle and the other is subtracted in the prism-deviation formula. Option (D) omits that final subtraction. Options (A) and (B) subtract too many multiples of $A$ and do not give $\sin i = \cos(A/2)$ when substituted back into the minimum-deviation condition.

**Step 3:** Conclusion The angle of minimum deviation is $\boxed{\delta_m = 180^\circ - 2A}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Use $\mu \sin(A/2) = \sin i$ first. A cotangent of the half-apex angle turns this directly into the complementary angle for $i$.

**41. The fundamental frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. If length of the open pipe is $60$ cm, the length of the closed pipe will be:** [4 marks]

(A) 15 cm

(B) 60 cm

(C) 45 cm

(D) 30 cm

**Correct Answer:** (A) 15 cm

**Solution:**

**Concept:**

- For an organ pipe open at both ends, the fundamental frequency is $v/(2L_o)$, and all integer harmonics occur. Its first overtone is its second harmonic, $v/L_o$.
- For a pipe closed at one end, the fundamental frequency is $v/(4L_c)$. The closed end is a displacement node and the open end a displacement antinode, giving a quarter-wavelength fundamental.

**Step 1:** Identify the two frequencies that are equated Write $L_o$ for the open-pipe length and $L_c$ for the closed-pipe length. For the open pipe, the first overtone lies one allowed mode above the fundamental, so it is the second harmonic:

$$f_{o,\text{first overtone}} = 2\left(\frac{v}{2L_o}\right) = \frac{v}{L_o}.$$

For the closed pipe, the question specifies the fundamental itself, not its first overtone. Consequently,

$$f_{c,\text{fundamental}} = \frac{v}{4L_c}.$$

The same sound speed is used for both pipes; no separate gas or temperature is specified.

**Step 2:** Equate the frequencies and calculate the length The stated equality gives

$$\frac{v}{4L_c} = \frac{v}{L_o}, \qquad 4L_c = L_o.$$

With $L_o = 60\,\text{cm}$, we obtain

$$L_c = \frac{60}{4}\,\text{cm} = 15\,\text{cm}.$$

Option (D), $30\,\text{cm}$, would follow from equating the fundamentals of the two pipes, but that is not the comparison asked here. Option (C), $45\,\text{cm}$, would involve a higher odd harmonic of the closed pipe, and option (B) ignores the different boundary conditions. Substitution of $15\,\text{cm}$ gives $v/(60\,\text{cm})$, exactly the open pipe's first overtone.

**Step 3:** Conclusion The closed organ pipe has length $\boxed{15\,\text{cm}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Name the harmonic before using a pipe formula: the first overtone of an open pipe is its second harmonic, while its fundamental is the first harmonic.

**42. If the wavelength of the first member of Lyman series of hydrogen is $\lambda$. The wavelength of the second member will be** [4 marks]

(A) $\frac{27}{5}\lambda$

(B) $\frac{5}{27}\lambda$

(C) $\frac{27}{32}\lambda$

(D) $\frac{32}{27}\lambda$

**Correct Answer:** (C) $\frac{27}{32}\lambda$

**Solution:**

**Concept:**

- The Lyman series consists of transitions from $n = 2, 3, 4, \ldots$ to the ground state $n = 1$. Its first two members correspond to $2 \to 1$ and $3 \to 1$, respectively.
- The Rydberg formula is $1/\lambda_n = R_H(1 - 1/n^2)$, where $R_H$ is the hydrogen Rydberg constant. Wavelength is the reciprocal of the wavenumber.

**Step 1:** Write the wavenumbers of the first two members For the first Lyman line the upper state is $n = 2$, and its wavelength is the specified $\lambda$. Therefore

$$\frac{1}{\lambda} = R_H\left(1 - \frac{1}{4}\right) = \frac{3}{4}R_H.$$

For the second member, the upper state increases to $n = 3$, while the lower state stays at $n = 1$. Calling its wavelength $\lambda_2$,

$$\frac{1}{\lambda_2} = R_H\left(1 - \frac{1}{9}\right) = \frac{8}{9}R_H.$$

The second member does not mean a transition ending at $n = 2$; that would belong to a different spectral series.

**Step 2:** Form the wavelength ratio and check its size Taking the ratio of the reciprocal wavenumbers cancels $R_H$:

$$\frac{\lambda_2}{\lambda} = \frac{(3/4)R_H}{(8/9)R_H} = \frac{3}{4} \times \frac{9}{8} = \frac{27}{32}.$$

The transition $3 \to 1$ releases more energy than $2 \to 1$, so the second wavelength must be smaller than the first. Option (D) has the inverted ratio and is greater than one; option (A) is also too large. Although option (B) is less than one, it does not follow from the energy-level differences. As a further scale check, both lines end in the ground state, so their wavelengths are reasonably close, consistent with a ratio of about $0.844$.

**Step 3:** Conclusion The wavelength of the second Lyman member is $\boxed{\lambda_2 = \frac{27}{32}\lambda}$. Hence, the correct option is (C).

**Quick Tip to Solve:** List the actual transitions before taking ratios, and invert the wavenumber ratio when converting it into a wavelength ratio.

**43. A coil is places perpendicular to a magnetic field of $5000\ \mathrm{T}$. When the field is changed to $3000\ \mathrm{T}$ in $2\ \mathrm{s}$, an induced emf of $22\ \mathrm{V}$ is produced in the coil. If the diameter of the coil is $0.02\ \mathrm{m}$, then the number of turns in the coil is:** [4 marks]

(A) 35

(B) 70

(C) 7

(D) 140

**Correct Answer:** (B) 70

**Solution:**

**Concept:**

- Faraday's law relates induced emf to the rate of change of flux linkage: $|\mathcal{E}| = N|\Delta\Phi|/\Delta t$, where $N$ is the number of turns.
- For a coil whose plane is perpendicular to the field, the normal is parallel to the field, so the magnetic flux through each turn is $\Phi = BA$. The area is $A = \pi(d/2)^2$.

**Step 1:** Find the area of one turn The given size is a diameter, so first divide it by two. The radius is $r = 0.02/2 = 0.01\,\mathrm{m}$, and hence

$$A = \pi(0.01)^2 = \pi \times 10^{-4}\,\mathrm{m}^2.$$

Using the diameter directly as the radius would overestimate the area by a factor of four and give an incorrect turn count.

**Step 2:** Calculate the flux change per second The magnetic field decreases from $5000$ to $3000\,\mathrm{T}$. Its magnitude of change is $2000\,\mathrm{T}$ over $2\,\mathrm{s}$. Thus

$$\frac{|\Delta\Phi|}{\Delta t} = A\frac{|\Delta B|}{\Delta t} = \pi \times 10^{-4}\frac{2000}{2} = 0.1\pi\,\mathrm{Wb\,s^{-1}}.$$

The decrease fixes the polarity through Lenz's law; only the magnitude is needed to find the number of turns.

**Step 3:** Determine and check the number of turns All turns contribute to the flux linkage, giving

$$22 = N(0.1\pi), \qquad N = \frac{220}{\pi} \simeq 70.$$

Using $\pi = 22/7$ gives exactly 70. The proposed values 35, 7 and 140 would instead give approximately 11, 2.2 and 44 V, respectively, so they cannot reproduce the stated emf.

**Step 4:** Conclusion The coil has $\boxed{70}$ turns. Hence, the correct option is (B).

**Quick Tip to Solve:** Convert diameter to radius before evaluating the area, and use the field change per second rather than the final field.

**44. A coin is placed on a disc. The coefficient of friction between the coin and the disc is $\mu$. If the distance of the coin from the center of the disc is $r$, the maximum angular velocity which can be given to the disc, so that the coin does not slip away, is :** [4 marks]

(A) $\sqrt{\frac{r}{\mu g}}$

(B) $\sqrt{\frac{\mu g}{r}}$

(C) $\frac{\mu g}{r}$

(D) $\frac{\mu}{\sqrt{rg}}$

**Correct Answer:** (B) $\sqrt{\frac{\mu g}{r}}$

**Solution:**

**Concept:**

- For a coin rotating with the disc at angular speed $\omega$, the required inward centripetal force is $m\omega^2 r$.
- On a horizontal disc the normal force is $N = mg$, so the maximum available static friction is $f_{s,\max} = \mu mg$. Static friction adjusts up to this limit.

**Step 1:** Identify the force that makes the coin rotate The weight acts downward and the normal reaction upward. These balance because there is no vertical acceleration. In the horizontal plane, static friction is the force that supplies the inward acceleration needed for the coin to follow a circle of radius $r$. No additional inward contact force is present. Therefore the required friction is

$$f_s = m\omega^2 r.$$

Its direction is toward the centre of the disc; it is not directed tangentially for uniform rotation.

**Step 2:** Apply the limiting condition for no slipping The required friction cannot exceed the available static friction. Consequently,

$$m\omega^2 r \le \mu mg.$$

Cancelling the positive mass gives $\omega^2 \le \mu g/r$. At the largest allowed speed the inequality becomes equality:

$$\omega_{\max} = \sqrt{\frac{\mu g}{r}}.$$

The mass cancels because both the friction limit and the required centripetal force are proportional to it.

**Step 3:** Check the physical and dimensional behaviour The result increases with friction and decreases with distance from the centre, as expected. Also $g/r$ has dimensions $\mathrm{s}^{-2}$, so its square root has dimensions $\mathrm{s}^{-1}$. Option (A) has the dimensions of time. Option (C) has dimensions $\mathrm{s}^{-2}$, and option (D) has dimensions $\mathrm{s\,m}^{-1}$, so none is an angular speed.

**Step 4:** Conclusion The maximum angular velocity is $\boxed{\sqrt{\mu g/r}}$. Hence, the correct option is (B).

**Quick Tip to Solve:** At the slipping threshold set the required centripetal force equal to the maximum static friction; cancel the mass before solving for speed.

**45. Two conductors have the same resistances at $0^\circ C$ but their temperature coefficients of resistance are $\alpha_1$ and $\alpha_2$. The respective temperature coefficients for their series and parallel combinations are :** [4 marks]

(A) $\alpha_1 + \alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$
(B) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$
(C) $\alpha_1 + \alpha_2, \frac{\alpha_1+\alpha_2}{2}$
(D) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1 + \alpha_2$

**Correct Answer:** (B) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

**Solution:**

**Concept:**

- For a small temperature rise $T$ from $0^\circ C$, write $R_i(T) = R_0(1 + \alpha_i T)$. The equal initial resistance of each conductor is $R_0$.
- The temperature coefficient of an equivalent resistance at the reference temperature is $\alpha_{eq} = R_{eq}(0)^{-1}(dR_{eq}/dT)_{T=0}$. Keep terms only to first order in temperature.

**Step 1:** Combine the resistances in series Adding the two temperature-dependent resistances gives

$$R_s(T) = R_0[2 + (\alpha_1 + \alpha_2)T].$$

The initial series resistance is $2R_0$. Express the result in the standard relative-change form:

$$R_s(T) = 2R_0\left[1 + \frac{\alpha_1 + \alpha_2}{2}T\right].$$

Thus $\alpha_s = (\alpha_1 + \alpha_2)/2$. Adding absolute resistance changes does not mean adding the relative coefficients without dividing by the initial equivalent resistance.

**Step 2:** Combine the conductors in parallel For parallel connection,

$$R_p(T) = R_0\frac{(1 + \alpha_1 T)(1 + \alpha_2 T)}{2 + (\alpha_1 + \alpha_2)T}.$$

Put $S = \alpha_1 + \alpha_2$. To first order, the numerator is $1 + ST$, while the reciprocal denominator is $\frac{1}{2}(1 - ST/2)$. Their product gives

$$R_p(T) \simeq \frac{R_0}{2}\left[1 + \frac{S}{2}T\right].$$

Since the initial parallel resistance is $R_0/2$, its temperature coefficient is also $S/2$.

**Step 3:** Compare the proposed pairs Both combinations therefore have the arithmetic mean of the individual coefficients at the reference temperature. Options (A) and (C) incorrectly give the sum for the series coefficient. Option (D) gives the sum for the parallel coefficient. A useful check is $\alpha_1 = \alpha_2 = \alpha$: each combination must then retain coefficient $\alpha$, not $2\alpha$.

**Step 4:** Conclusion The respective coefficients are $\boxed{(\alpha_1 + \alpha_2)/2,\ (\alpha_1 + \alpha_2)/2}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Always divide the derivative of equivalent resistance by its own initial resistance; for equal initial resistances both coefficients become the mean.

**46. If the percentage errors in measuring the length and the diameter of a wire are $0.1\%$ each. The percentage error in measuring its resistance will be:** [4 marks]

(A) 0.144%

(B) 0.2%

(C) 0.1%

(D) 0.3%

**Correct Answer:** (D) 0.3%

**Solution:**

**Concept:**

- A uniform wire of fixed resistivity $\rho$, length $l$ and diameter $d$ has resistance $R = \rho l/A = 4\rho l/(\pi d^2)$.
- For the maximum fractional error in a product of powers, add the magnitudes of fractional errors multiplied by their exponents. The negative power of diameter contributes a positive error bound.

**Step 1:** Express resistance in terms of the measured quantities The cross-sectional area is $A = \pi d^2/4$. Substituting into $R = \rho l/A$ shows that resistance is proportional to $l$ and inversely proportional to $d^2$:

$$R = \frac{4\rho l}{\pi d^2}.$$

The question gives measurement errors only for length and diameter, so resistivity is treated as fixed. The factor $4/\pi$ is exact and contributes no measurement error.

**Step 2:** Propagate the limiting errors Logarithmic differentiation gives the signed relative variation

$$\frac{dR}{R} = \frac{dl}{l} - 2\frac{dd}{d}.$$

Measurement-error limits must allow the two changes to reinforce one another. Taking the sum of magnitudes therefore gives

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} + 2\frac{\Delta d}{d}.$$

The minus sign in the differential is not a reason to subtract uncertainties; a decrease in measured diameter and an increase in measured length both increase the calculated resistance.

**Step 3:** Insert the percentage errors and compare Each given percentage error is $0.1\%$. Thus

$$\text{Percentage error in } R = 0.1\% + 2(0.1\%) = 0.3\%.$$

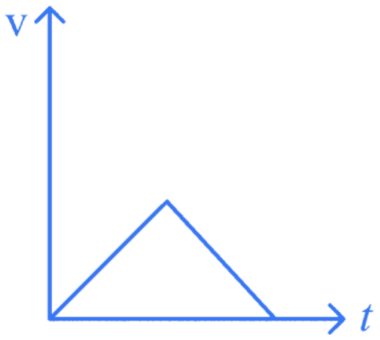
Option (B) counts diameter only once and ignores the square in area. Option (C) omits a contribution, and option (A) is not the maximum-error sum for the powers in this formula. A statistical root-sum-square estimate is not the limiting error requested in this standard propagation question.

**Step 4:** Conclusion The percentage error in resistance is $\boxed{0.3\%}$. Hence, the correct option is (D).

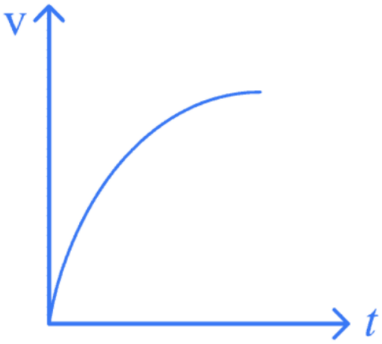
**Quick Tip to Solve:** Write resistance using diameter explicitly: its power is minus two, so its percentage uncertainty counts twice in the maximum-error sum.

47. **A small steel ball is dropped into a long cylinder containing glycerine. Which one of the following is the correct representation of the velocity time graph for the transit of the ball?** [4 marks]

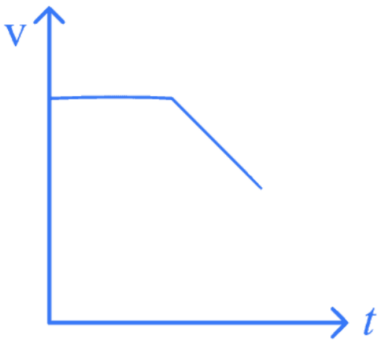
(A)



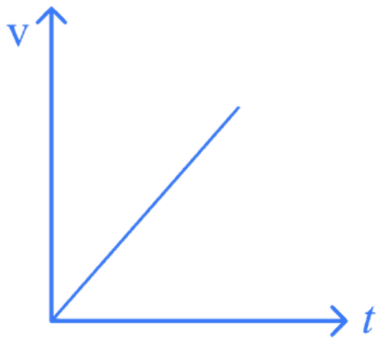

(B)



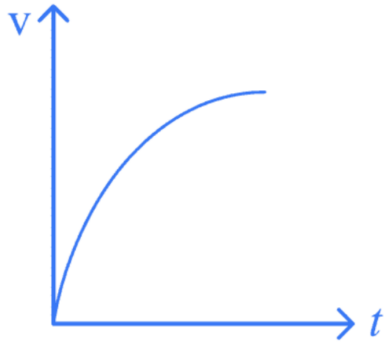

(C)




(D)



**Correct Answer:** (B)




**Solution:**

**Concept:**

- A falling steel sphere experiences downward weight and upward buoyancy and viscous drag. In slow viscous flow, Stokes' drag is $F_d = 6\pi\eta a v$, where $a$ is the sphere radius.
- As speed increases, drag increases, reducing the net downward force. Terminal speed is reached when weight equals buoyancy plus drag, so the acceleration tends to zero.

**Step 1:** Identify the initial motion Take downward as positive. At release the speed is zero, so the viscous drag is initially zero. Steel is denser than glycerine, making the weight greater than the buoyant force. The ball therefore begins from rest with positive downward acceleration. A valid velocity-time graph starts at the origin and initially rises; it cannot begin at a nonzero speed as in option (C).

**Step 2:** Find how the acceleration changes with speed If $V$ is the sphere volume and $\rho_f$ the liquid density, Newton's law gives

$$m\frac{dv}{dt} = mg - \rho_f V g - 6\pi\eta a v.$$

Define $k = 6\pi\eta a$ and $v_T = (mg - \rho_f V g)/k$. The equation becomes $dv/dt = (k/m)(v_T - v)$. With $v(0) = 0$, its solution is

$$v(t) = v_T\left(1 - e^{-kt/m}\right).$$

Hence the speed grows, but the slope of the graph continuously decreases.

**Step 3:** Match the limiting behaviour to the options At long times the speed approaches a positive constant $v_T$, and the graph becomes horizontal. Option (B) shows precisely this rising, concave-down approach. Option (A) incorrectly makes the speed fall back to zero without a change in the forces. Option (C) starts with nonzero velocity and later decelerates. Option (D) shows constant acceleration indefinitely and ignores the increasing viscous resistance. The long cylinder permits the approach to terminal motion before the bottom interferes.

**Step 4:** Conclusion The ball accelerates from rest and approaches a constant terminal speed, as shown in graph (B). Hence, the correct option is (B).

**Quick Tip to Solve:** For an object released in a viscous liquid, look for zero initial speed followed by a curve whose slope decreases toward zero at terminal speed.

**48. A force is represented by $F = ax^2 + bt^{\frac{1}{2}}$**
**where $x$ = distance and $t$ = time. The dimensions of $b^2/a$ are:** [4 marks]

(A) $\left[\mathrm{ML}^2\ \mathrm{T}^{-3}\right]$
(B) $\left[\mathrm{ML}^3\ \mathrm{T}^{-3}\right]$
(C) $\left[MLT^{-2}\right]$
(D) $\left[ML^{-1}T^{-1}\right]$

**Correct Answer:** (B) $\left[\mathrm{ML}^3\ \mathrm{T}^{-3}\right]$

**Solution:**

**Concept:**

- Every term added in a physical equation must have the same dimensions. In $F = ax^2 + bt^{1/2}$, both terms separately have force dimensions $[F] = MLT^{-2}$.
- Use $[x] = L$ and $[t] = T$, keeping the fractional time exponent when finding the dimensions of $b$. The dimensions of a quotient are obtained by subtracting exponents.

**Step 1:** Determine the dimensions of the coefficient a Equating the dimensions of the first term with those of force gives

$$[a][x]^2 = MLT^{-2}.$$

Since $[x]^2 = L^2$, the coefficient must have dimensions

$$[a] = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}.$$

The coefficient is not itself a force: multiplying it by the square of a distance must supply the force unit.

**Step 2:** Determine the dimensions of b and form the ratio The second term gives $[b]T^{1/2} = MLT^{-2}$. Dividing by the square root of time yields

$$[b] = MLT^{-5/2}, \qquad [b^2] = M^2L^2T^{-5}.$$

Therefore

$$\left[\frac{b^2}{a}\right] = \frac{M^2L^2T^{-5}}{ML^{-1}T^{-2}} = ML^3T^{-3}.$$

In the length exponent, subtracting $-1$ increases the power from 2 to 3; in the time exponent, $-5 - (-2) = -3$.

**Step 3:** Check each dimension against the options Option (B) has precisely the mass, length and time powers just derived. Option (A) loses one power of length, option (C) has force dimensions rather than those of the specified coefficient ratio, and option (D) has both the length and time powers incorrect. A direct shortcut confirms the result: $[b^2/a] = [F]^2T^{-1}/([F]L^{-2}) = [F]L^2/T = ML^3T^{-3}$.

**Step 4:** Conclusion The required dimensions are $\boxed{[ML^3T^{-3}]}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Find each coefficient by dividing force dimensions by the variable factor beside it; retain the half-power of time until b is squared.

**49. In a plane EM wave, the electric field oscillates sinusoidally at a frequency of $5 \times 10^{10}$ Hz and an amplitude of $50\ \mathrm{Vm}^{-1}$. The total average energy density of the electromagnetic field of the wave is : [Use $\varepsilon_0 = 8.85 \times 10^{-12}\mathrm{C}^2/\mathrm{Nm}^2$ ]** [4 marks]

(A) $4.425 \times 10^{-8}\mathrm{Jm}^{-3}$

(B) $2.212 \times 10^{-10}\mathrm{Jm}^{-3}$

(C) $2.212 \times 10^{-8}\mathrm{Jm}^{-3}$

(D) $1.106 \times 10^{-8}\mathrm{Jm}^{-3}$

**Correct Answer:** (D) $1.106 \times 10^{-8}\mathrm{Jm}^{-3}$

**Solution:**

**Concept:**

- The electric energy density is $u_E = \frac{1}{2}\varepsilon_0 E^2$, and the magnetic energy density is $u_B = B^2/(2\mu_0)$. For a plane electromagnetic wave, these two contributions are equal.
- For a sinusoidal electric field with amplitude $E_0$, the time average satisfies $\langle E^2 \rangle = E_0^2/2$. The stated amplitude is the peak field, not the rms field.

**Step 1:** Combine the electric and magnetic contributions In a plane wave $B = E/c$, and $c^2 = 1/(\mu_0\varepsilon_0)$. Consequently $u_B = \varepsilon_0 E^2/2 = u_E$. Adding the two fields gives the instantaneous total energy density

$$u = u_E + u_B = \varepsilon_0 E^2.$$

This equality prevents us from accidentally calculating only the electric contribution when the question asks for the electromagnetic total.

**Step 2:** Average the squared sinusoidal field Write the field at a fixed location as $E = E_0 \sin(\omega t + \phi)$. Over a complete cycle the mean of $\sin^2$ is $1/2$, independently of frequency and phase. Therefore

$$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2.$$

The frequency $5 \times 10^{10}\,\mathrm{Hz}$ need not enter the numerical calculation. The amplitude alone fixes the average density for the given permittivity.

**Step 3:** Substitute the amplitude and compare values Using $E_0 = 50\,\mathrm{V\,m}^{-1}$,

$$\langle u \rangle = \frac{1}{2}(8.85 \times 10^{-12})(50)^2 = 1.10625 \times 10^{-8}\,\mathrm{J\,m}^{-3}.$$

Option (D) is this value rounded to four significant figures. Option (C) is approximately the peak total density, omitting the sinusoidal average. Option (A) is four times the average, while option (B) has an incorrect power of ten. No factor of the wave frequency is needed in the energy-density formula.

**Step 4:** Conclusion The total average energy density is $\boxed{1.106 \times 10^{-8}\,\mathrm{J\,m}^{-3}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For a sinusoidal plane wave use half of epsilon-zero times the peak electric field squared; the electric and magnetic parts contribute equally.

**50. Identify the logic operation performed by the given circuit.** [4 marks]




(A) AND

(B) NOR

(C) OR

(D) NAND

**Correct Answer:** (C) OR

**Solution:**

**Concept:**

- An inverter changes a Boolean input to its complement: input $A$ becomes $\overline{A}$. A small circle on the output of a gate represents inversion.
- A NAND gate outputs the complement of the product of its inputs. The De Morgan identity gives $\overline{\overline{A} \cdot \overline{B}} = A + B$, where $+$ denotes the Boolean inclusive-OR operation.

**Step 1:** Identify the input inversions Trace each input independently before identifying the final gate. The triangular symbol and small output circle on the upper wire make the upper input $\overline{A}$. The identical symbol on the lower wire makes the lower input $\overline{B}$. These are the two signals entering the last gate; they are not the original inputs $A$ and $B$.

**Step 2:** Write and simplify the output The last symbol has an AND-shaped body and an output circle, so it is NAND. Its output is therefore

$$Y = \overline{\overline{A} \cdot \overline{B}} = A + B.$$

The outer complement converts the product into a sum and complements both terms, cancelling the two input complements. Thus the whole network is an OR gate even though its final component is a NAND gate.

**Step 3:** Check the truth table and alternatives For input pairs $(A, B) = (0,0), (0,1), (1,0), (1,1)$, the output values are respectively $0, 1, 1, 1$. This is the inclusive-OR truth table. AND fails at the unequal input pairs; NOR is the complement of these outputs; NAND would give output 1 for $(0,0)$ and output 0 for $(1,1)$. Hence none of those other operations represents this complete circuit.

**Step 4:** Conclusion The circuit performs the OR operation, $Y = A + B$. Hence, the correct option is (C).

**Quick Tip to Solve:** Include every inversion bubble before simplifying a gate network: NAND with both inputs inverted becomes OR.

## Physics – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**51. A solid circular disc of mass $50\ \text{kg}$ rolls along a horizontal floor so that its center of mass has a speed of $0.4\ \text{m/s}$. The absolute value of work done on the disc to stop it is _____ J.** [4 marks]

**Solution:**

**Concept:**

- A rolling rigid body possesses translational kinetic energy of its centre of mass and rotational kinetic energy about that centre: $K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$.
- For a uniform solid disc, $I = \frac{1}{2}MR^2$. Rolling without slipping gives $v = \omega R$, where $R$ is the disc radius.

**Step 1:** Include both forms of kinetic energy The disc has mass $M = 50\,\text{kg}$ and centre-of-mass speed $v = 0.4\,\text{m s}^{-1}$. Its motion is rolling, so stopping it requires removing the energy of translation as well as the energy of rotation. Using only the centre-of-mass kinetic energy would omit the rotational contribution. The radius is not supplied because it cancels when the rolling constraint is used.

**Step 2:** Evaluate the initial energy Substitute the solid-disc moment of inertia and $\omega = v/R$:

$$\begin{aligned} K_i &= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2} \\ &= \frac{3}{4}Mv^2 = \frac{3}{4}(50)(0.4)^2 \\ &= \frac{3}{4}(50)(0.16) = 6\,\text{J}. \end{aligned}$$

Equivalently, the translational part is $4\,\text{J}$ and the rotational part is $2\,\text{J}$. Their sum is the total energy to be removed.

**Step 3:** Apply the work-energy theorem Once the disc is stopped, both its linear and angular speeds vanish, so $K_f = 0$. The net stopping work is

$$W = K_f - K_i = 0 - 6 = -6\,\text{J}.$$

The negative sign means that the stopping force removes mechanical energy. The question asks for the absolute value, so report the positive magnitude rather than the signed stopping work.

**Step 4:** Conclusion The absolute value of the work required to stop the disc is $\boxed{6\,\text{J}}$. The numerical entry is $6$.

**Quick Tip to Solve:** For a solid disc rolling without slipping, use total energy $3Mv^2/4$; stopping work is negative, but its absolute value is positive.

**52. An electron moves through a uniform magnetic field $\vec{B} = B_0\hat{i} + 2B_0\hat{j}T$. At a particular instant of time, the velocity of electron is $\vec{u} = 3\hat{i} + 5\hat{j}$ m/s. If the magnetic force acting on electron is $\vec{F} = 5e\hat{k}N$, where $e$ is the charge of electron, then the value of $B_0$ is _____ $T$.** [4 marks]

**Solution:**

**Concept:**

- The magnetic part of the Lorentz force is $\vec{F} = q(\vec{u} \times \vec{B})$. Here the question denotes the electron charge by $e$; retain the same signed charge symbol on both sides.
- For two vectors in the $xy$-plane, their cross product is perpendicular to that plane: $(u_x\hat{i} + u_y\hat{j}) \times (B_x\hat{i} + B_y\hat{j}) = (u_xB_y - u_yB_x)\hat{k}$.

**Step 1:** Identify the components and charge convention The stated velocity has components $u_x = 3\,\mathrm{m\,s^{-1}}$, $u_y = 5\,\mathrm{m\,s^{-1}}$, while the field has $B_x = B_0$ and $B_y = 2B_0$. Both have zero $z$-component, so the magnetic force must lie along the $z$-axis. In the printed statement, $e$ is the charge of the electron and the force is also written using this same $e$. Thus it cancels algebraically; do not introduce an additional minus sign into only one side.

**Step 2:** Compute the cross product explicitly Use $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{j} \times \hat{i} = -\hat{k}$. Parallel-component products vanish:

$$\begin{aligned}\vec{u} \times \vec{B} &= (3\hat{i} + 5\hat{j}) \times (B_0\hat{i} + 2B_0\hat{j}) \\ &= 6B_0\hat{k} - 5B_0\hat{k} = B_0\hat{k}.\end{aligned}$$

The order is velocity crossed with field. Reversing the order would incorrectly reverse the force direction.

**Step 3:** Match the given force The Lorentz force becomes $\vec{F} = eB_0\hat{k}$. Equate it to the specified force:

$$eB_0\hat{k} = 5e\hat{k} \quad \Longrightarrow \quad B_0 = 5\,\mathrm{T}.$$

The charge is nonzero, so division by the common $e$ is valid. A dimensional check gives force divided by charge and speed, which is the tesla, as required for a magnetic field.

**Step 4:** Conclusion Using the charge convention stated in the question, $\boxed{B_0 = 5\,\mathrm{T}}$. The numerical entry is 5.

**Quick Tip to Solve:** For planar velocity and field, calculate $u_xB_y - u_yB_x$ first, and use the same signed charge convention on both sides.

**53. A parallel plate capacitor with plate separation $5\ \text{mm}$ is charged up by a battery. It is found that on introducing a dielectric sheet of thickness $2\ \text{mm}$, while keeping the battery connections intact, the capacitor draws $25\%$ more charge from the battery than before. The dielectric constant of the sheet is _____.**

[4 marks]

**Solution:**

**Concept:**

- With a battery still connected, the potential difference $V$ stays constant, so $Q = CV$ implies that the charge ratio equals the capacitance ratio.
- A dielectric slab of thickness $t$ and relative permittivity $K$, filling the plate area, leaves an air gap $d - t$. The two layers act in series: $C = \varepsilon_0 A/(d - t + t/K)$.

**Step 1:** Translate the charge increase into a capacitance ratio Initially the plates are separated by $d = 5\,\text{mm}$, so $C_0 = \varepsilon_0 A/d$ and $Q_0 = C_0 V$. Introducing the slab draws an extra $25\%$ of the original charge from the battery. Consequently the final total charge is $Q = 1.25Q_0$, rather than $0.25Q_0$. Since the battery holds the voltage fixed, $C/C_0 = Q/Q_0 = 1.25$.

**Step 2:** Use the air and dielectric layers in series The slab has thickness $t = 2\,\text{mm}$, leaving $3\,\text{mm}$ of air. The reciprocal capacitances add:

$$\frac{1}{C} = \frac{d-t}{\varepsilon_0 A} + \frac{t}{K\varepsilon_0 A}.$$

Dividing the resulting capacitance by $C_0$ gives

$$\frac{C}{C_0} = \frac{d}{d - t + t/K} = \frac{5}{3 + 2/K}.$$

All the lengths in this ratio may be expressed in millimetres because the common unit cancels.

**Step 3:** Solve for the dielectric constant Equate the measured ratio to the expression above:

$$\frac{5}{3+2/K} = \frac{5}{4} \quad \Longrightarrow \quad 3 + \frac{2}{K} = 4 \quad \Longrightarrow \quad K = 2.$$

The result exceeds unity, as expected for a dielectric. Substitution gives an effective separation of $4\,\text{mm}$, hence $C/C_0 = 5/4$, confirming the required increase. Using $C = KC_0$ would be incorrect because the dielectric does not occupy the whole plate separation.

**Step 4:** Conclusion The dielectric constant of the sheet is $\boxed{K = 2}$, a dimensionless quantity. The numerical entry is 2.

**Quick Tip to Solve:** For a partial dielectric slab, replace the air-gap thickness $t$ by $t/K$, and keep voltage fixed when the battery remains connected.

**54. The depth below the surface of sea to which a rubber ball be taken so as to decrease its volume by $0.02\%$ is _____ $m$.**

**(Take density of sea water $= 10^3 \text{kgm}^{-3}$, Bulk modulus of rubber $= 9 \times 10^8\ \text{Nm}^{-2}$, and $g = 10\ \text{ms}^{-2}$)**

[4 marks]

**Solution:**

**Concept:**

- The bulk modulus is $B = -\Delta P/(\Delta V/V)$. For compression, the positive fractional decrease $f = |\Delta V|/V$ therefore gives $\Delta P = Bf$.
- At depth $h$ in water of density $\rho$, the increase of pressure relative to the surface is $\Delta P = \rho g h$. The atmospheric contribution is already present at the surface and cancels.

**Step 1:** Convert the percentage decrease to a fraction The required volume decrease is $0.02\%$, so the fractional compression is

$$f = \frac{0.02}{100} = 0.0002 = 2 \times 10^{-4}.$$

This is a decrease, hence the signed volume change is negative, but its magnitude $f$ is positive. The percentage must be divided by 100 before it is used in the bulk-modulus equation; inserting $0.02$ itself would overestimate the required depth by a factor of 100.

**Step 2:** Find the necessary increase in pressure Use the given rubber bulk modulus $B = 9 \times 10^8\ \text{N m}^{-2}$:

$$\Delta P = Bf = (9 \times 10^8)(2 \times 10^{-4}) = 18 \times 10^4 = 1.8 \times 10^5\ \text{Pa}.$$

This is the additional pressure needed to compress the ball from its surface volume. We compare the deeper position with the surface, so atmospheric pressure is not added again.

**Step 3:** Relate the pressure increase to depth With $\rho = 10^3\ \text{kg m}^{-3}$ and $g = 10\ \text{m s}^{-2}$, the hydrostatic pressure gradient is $\rho g = 10^4\ \text{Pa m}^{-1}$. Thus

$$h = \frac{\Delta P}{\rho g} = \frac{1.8 \times 10^5}{10^3 \times 10} = 18\ \text{m}.$$

The units reduce to metres. Substituting this depth back gives a pressure increment of $1.8 \times 10^5\ \text{Pa}$, and division by the bulk modulus reproduces $2 \times 10^{-4}$.

**Step 4:** Conclusion The required depth below the sea surface is $\boxed{18\ \text{m}}$. The numerical entry is 18.

**Quick Tip to Solve:** Convert percentage compression to a fraction first, then use $h = Bf/(\rho g)$ with the pressure increase from the surface.

**55. A body starts falling freely from height $H$ hits an inclined plane in its path at height $h$. As a result of this perfectly elastic impact, the direction of the velocity of the body becomes horizontal. The value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is _____.** [4 marks]

**Solution:**

**Concept:**

- A body released from rest falls a vertical distance $s$ in time $t = \sqrt{2s/g}$, provided its initial vertical velocity is zero.
- The elastic impact redirects the velocity horizontally. Immediately after the impact the vertical component is zero, so the remaining vertical fall from height $h$ begins with zero vertical speed.

**Step 1:** Write the two travel times The body begins from rest at height $H$, and first falls through $H - h$ before reaching the inclined plane. The time before impact is

$$t_1 = \sqrt{\frac{2(H-h)}{g}}.$$

After impact the velocity is horizontal, so it contributes no initial vertical speed to the remaining fall. The time from height $h$ to the ground is therefore

$$t_2 = \sqrt{\frac{2h}{g}}.$$

The horizontal speed affects horizontal distance, but not this vertical time. The collision itself is treated as instantaneous.

**Step 2:** Maximise the total time at fixed release height For a fixed initial height $H$, the total is

$$T = \sqrt{\frac{2}{g}}\left(\sqrt{H-h} + \sqrt{h}\right), \qquad 0 \le h \le H.$$

The prefactor is constant. Squaring the positive bracket gives

$$\left(\sqrt{H-h} + \sqrt{h}\right)^2 = H + 2\sqrt{h(H-h)}.$$

Thus maximising the time is equivalent to maximising $h(H-h)$.

**Step 3:** Locate and check the maximum Complete the square:

$$h(H-h) = \frac{H^2}{4} - \left(h - \frac{H}{2}\right)^2 \le \frac{H^2}{4}.$$

Equality occurs at $h = H/2$, which lies in the allowed interval. The endpoints give only the uninterrupted fall time $\sqrt{2H/g}$, whereas the midpoint gives $2\sqrt{H/g}$, so this interior point is the maximum. At that point the two vertical falling distances, and hence their two time contributions, are equal.

**Step 4:** Conclusion The maximum fall time occurs for $h = H/2$; consequently $\boxed{H/h = 2}$, a dimensionless ratio. The numerical entry is 2.

**Quick Tip to Solve:** When an impact resets the vertical velocity to zero, split the fall into two zero-initial-vertical-speed stages; equal distances maximise their total time.

**56. A particle performs simple harmonic motion with amplitude $A$. Its speed is increased to three times at an instant when its displacement is $\frac{2A}{3}$. The new amplitude of motion is $\frac{nA}{3}$. The value of $n$ is _____.** [4 marks]

**Solution:**

**Concept:**

- For SHM with angular frequency $\omega$ and amplitude $A$, conservation of oscillator energy gives $v^2 = \omega^2(A^2 - x^2)$.
- An instantaneous speed change leaves the instantaneous displacement unchanged. If the restoring system is unchanged, its $\omega$ is also unchanged; the new amplitude $A_{\text{new}}$ follows from the new energy.

**Step 1:** Find the speed just before the change At the given instant the displacement is $x = 2A/3$. Apply the SHM speed relation using the original amplitude:

$$v^2 = \omega^2\left(A^2 - \frac{4A^2}{9}\right) = \frac{5}{9}\omega^2 A^2.$$

The particle is away from its equilibrium position, so some of its energy is potential energy. Its speed at this position is smaller than the original maximum speed $\omega A$.

**Step 2:** Apply the threefold speed increase The new speed is $v_{\text{new}} = 3v$, so its square becomes nine times as large:

$$v_{\text{new}}^2 = 9v^2 = 5\omega^2 A^2.$$

During the instantaneous change, the particle remains at $x = 2A/3$. The potential energy therefore stays the same at that instant, while the kinetic energy increases by a factor of nine. The total energy does not become nine times its original value because its potential-energy part has not been multiplied.

**Step 3:** Determine the new turning point Use the speed relation for the new oscillation at the same position:

$$\begin{aligned} A_{\text{new}}^2 &= x^2 + \frac{v_{\text{new}}^2}{\omega^2} \\ &= \frac{4A^2}{9} + 5A^2 = \frac{49A^2}{9}. \end{aligned}$$

An amplitude is nonnegative, so $A_{\text{new}} = 7A/3$. Comparing this with the specified form $nA/3$ yields $n = 7$. This new amplitude exceeds the old one, consistently with the added kinetic energy.

**Step 4:** Conclusion The new amplitude is $\boxed{7A/3}$, and hence $\boxed{n = 7}$. The parameter $n$ is dimensionless.

**Quick Tip to Solve:** At an instantaneous kick, keep $x$ fixed and compute $A_{\text{new}}^2 = x^2 + v_{\text{new}}^2/\omega^2$; do not multiply the original amplitude by the speed factor.

**57. The mass defect in a particular reaction is $0.4$ g. The amount of energy liberated is $n \times 10^7$ kWh, where $n =$ _____. (speed of light $= 3 \times 10^8$ m/s)** [4 marks]

**Solution:**

**Concept:**

- Einstein's mass-energy relation is $E = \Delta m c^2$, where $\Delta m$ is the decrease in total rest mass and $c$ is the speed of light. A mass defect corresponds to energy released, so the released energy is positive.
- The SI calculation gives joules when mass is in kilograms and speed is in metres per second. The electrical energy unit is converted using $1\,\text{kWh} = 1000 \times 3600\,\text{J} = 3.6 \times 10^6\,\text{J}$.

**Step 1:** Convert the mass defect to kilograms The stated defect is a mass, not a percentage of the initial mass. Use the full given value and first change grams to kilograms:

$$\Delta m = 0.4\,\text{g} = 0.4 \times 10^{-3}\,\text{kg} = 4 \times 10^{-4}\,\text{kg}.$$

This conversion must be made before multiplying by the square of the speed of light; using grams directly would overestimate the energy by a factor of one thousand.

**Step 2:** Calculate and convert the released energy With the supplied speed of light, the energy is

$$E = (4 \times 10^{-4})(3 \times 10^8)^2 = 3.6 \times 10^{13}\,\text{J}.$$

The speed must be squared, including its power of ten. Now express this result in kilowatt-hours:

$$E = \frac{3.6 \times 10^{13}}{3.6 \times 10^6} = 10^7\,\text{kWh}.$$

Dividing by the joules in one kilowatt-hour changes the unit without changing the physical amount of energy. Comparing with the requested form $n \times 10^7$ kWh gives the coefficient directly.

**Step 3:** Conclusion The energy liberated is $1 \times 10^7$ kWh; hence the required dimensionless coefficient is $\boxed{n = 1}$.

**Quick Tip to Solve:** Convert grams to kilograms before using $mc^2$, then divide joules by $3.6 \times 10^6$ to obtain kWh.

**58. A small square loop of wire of side $l$ is placed inside a large square loop of wire of side $L$ $(L = l^2)$. The loops are coplanar and their centers coincide. The value of the mutual inductance of the system is $\sqrt{x} \times 10^{-7}$H, where $x =$ _____.** [4 marks]

**Solution:**

**Concept:**

- Mutual inductance is linked flux per unit current: $M = \Phi/I$. For a small inner loop, the field of the outer loop is taken as uniform over its area and equal to its value at the common centre.
- The magnetic field from one straight wire segment is $B = \mu_0 I(\sin\theta_1 + \sin\theta_2)/(4\pi r)$. At the centre of a square, each side has $r = L/2$ and $\theta_1 = \theta_2 = 45°$.

**Step 1:** Add the fields from the four sides Pass a current $I$ through the large square loop. Each straight side produces a magnetic field perpendicular to the plane at the centre. The directions of these four contributions are the same, so their magnitudes add. For one side,

$$B_1 = \frac{\mu_0 I}{4\pi(L/2)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}\mu_0 I}{2\pi L}.$$

Thus the field used for the small concentric square is

$$B = 4B_1 = \frac{2\sqrt{2}\mu_0 I}{\pi L}.$$

The four-side factor is essential; the field of only one side would not represent the closed loop.

**Step 2:** Find the flux and identify the coefficient The area of the inner square is $l^2$, and its area normal is parallel to the field. In the stated small-loop approximation,

$$M = \frac{Bl^2}{I} = \frac{2\sqrt{2}\mu_0 l^2}{\pi L}.$$

Using the given relation $L = l^2$ in the specified length values and $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$,

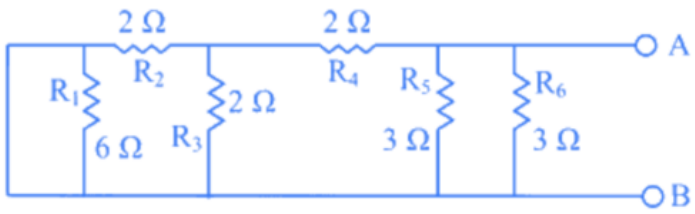
$$M = 8\sqrt{2} \times 10^{-7}\,\mathrm{H} = \sqrt{128} \times 10^{-7}\,\mathrm{H}.$$

The current cancels as it must: mutual inductance is a geometrical property of this fixed pair of loops, independent of the chosen test current.

**Step 3:** Conclusion The mutual inductance has the required form $\sqrt{128} \times 10^{-7}\,\mathrm{H}$, giving $\boxed{x = 128}$ (dimensionless).

**Quick Tip to Solve:** For a small concentric loop, find the outer square's central field first, multiply by $l^2$, and only then divide the flux by current.

**59. Equivalent resistance of the following network is _____ $\Omega$.** [4 marks]




**Solution:**

**Concept:**

- An ideal wire makes its connected points equipotential. A resistor bypassed by such a wire has zero voltage across it and contributes nothing to the equivalent resistance.
- Resistors sharing the same two nodes are in parallel: $R_{\mathrm{p}}^{-1} = \sum_i R_i^{-1}$. Series resistors share a branch with no current diversion, so their resistances add.

**Step 1:** Identify the short circuit and reduce the left branch The wire running down the left edge connects the top of $R_1$ directly to the lower rail, which is terminal B. Therefore the $6\,\Omega$ resistor $R_1$ is short-circuited; no terminal current needs to pass through it. The left end of $R_2$ is consequently at B potential. Both $R_2$ and $R_3$, each $2\,\Omega$, then connect the central top junction to B. They are parallel, not series:

$$R_{23} = \frac{2 \times 2}{2 + 2} = 1\,\Omega.$$

This node identification uses the actual uninterrupted wire and does not depend on the apparent horizontal or vertical positions of the resistors.

**Step 2:** Combine the three branches between A and B The branch through $R_4$ contains its $2\,\Omega$ resistance in series with $R_{23}$. Its total is

$$R_{423} = 2 + 1 = 3\,\Omega.$$

The other two branches are $R_5 = 3\,\Omega$ and $R_6 = 3\,\Omega$, each directly joining A to the bottom rail B. Thus three equal $3\,\Omega$ branches are in parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1\,\Omega^{-1}.$$

Equivalently, a test voltage of one volt sends one-third ampere through each branch, for a total of one ampere.

**Step 3:** Conclusion The equivalent resistance between the labelled terminals A and B is $\boxed{1\,\Omega}$.

**Quick Tip to Solve:** Trace ideal wires before reducing resistors: the left edge shorts $R_1$ and makes $R_2$ parallel to $R_3$.

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**60. Two waves of intensity ratio $1 : 9$ cross each other at a point. The resultant intensities at that point, when (a) Waves are incoherent is $I_1$ (b) Waves are coherent is $I_2$ and differ in phase by $60°$. If $\frac{I_1}{I_2} = \frac{10}{x}$ then $x =$ _____.** [4 marks]

**Solution:**

**Concept:**

- For incoherent waves, the time-averaged interference term vanishes and intensities add. For coherent waves, $I = I_a + I_b + 2\sqrt{I_a I_b}\cos\phi$, where $\phi$ is their fixed phase difference.
- Intensity is proportional to the square of amplitude. Therefore the intensity ratio $1 : 9$ corresponds to an amplitude ratio $1 : 3$, and the interference term depends on the geometric mean of the intensities.

**Step 1:** Evaluate the incoherent resultant Let the individual wave intensities be $I_0$ and $9I_0$, with $I_0 > 0$. Choosing a common scale avoids assigning an unnecessary absolute value. With incoherent waves the relative phase fluctuates, so the average of its cosine is zero. The measured resultant is therefore

$$I_1 = I_0 + 9I_0 = 10I_0.$$

It would be incorrect to add amplitudes and square them for this case, since no fixed phase relationship is maintained during the intensity measurement.

**Step 2:** Include the coherent interference term For the coherent case the phase difference is fixed at $60^\circ$. Consequently,

$$I_2 = I_0 + 9I_0 + 2\sqrt{I_0(9I_0)}\cos 60^\circ$$
$$= 10I_0 + 6I_0\left(\frac{1}{2}\right) = 13I_0.$$

The interference is constructive but not fully constructive: the result lies between the incoherent sum $10I_0$ and the in-phase maximum $(\sqrt{I_0} + \sqrt{9I_0})^2 = 16I_0$. Form the requested ratio:

$$\frac{I_1}{I_2} = \frac{10I_0}{13I_0} = \frac{10}{13}.$$

The common intensity unit and scale cancel, leaving a dimensionless ratio.

**Step 3:** Conclusion Comparing $I_1/I_2 = 10/13$ with $10/x$, the required value is $\boxed{x = 13}$ (dimensionless).

**Quick Tip to Solve:** Use the square root of the intensity product in the interference term; an intensity ratio $1:9$ gives amplitudes $1:3$.

## Chemistry – Section A: Multiple Choice Questions (80 marks)

**61. Match List I with List II.**

| | List I | | List II |
|---|---|---|---|
| **(A)** | **Glucose/$NaHCO_3/\Delta$** | **(I)** | **Gluconic acid** |
| **(B)** | **Glucose/$HNO_3$** | **(II)** | **No reaction** |
| **(C)** | **Glucose/$HI/\Delta$** | **(III)** | **n-hexane** |
| **(D)** | **Glucose/Bromine water** | **(IV)** | **Saccharic acid** |

**Choose the correct answer from the options given below:** [4 marks]

(A) A-II, B-IV, C-III, D-I

(B) A-III, B-II, C-I, D-IV

(C) A-IV, B-I, C-III, D-II

(D) A-I, B-IV, C-III, D-II

**Correct Answer:** (A) A-II, B-IV, C-III, D-I

**Solution:**

**Concept:**

- Glucose contains an aldehyde group in its open-chain form and a terminal primary alcohol group. Oxidants of different strengths therefore produce different oxidation products.
- Mild bromine water oxidises the aldehyde group to a carboxylic acid; strong nitric acid oxidises both terminal groups. Prolonged heating with HI reduces the carbon chain to an alkane.

**Step 1:** Match the first two reagents For A, sodium hydrogencarbonate is the reagent used to detect sufficiently acidic carboxylic acids by carbon dioxide evolution. Glucose has no carboxylic acid group and its alcoholic groups do not give that test. It therefore corresponds to no reaction, A-II. For B, nitric acid is a strong oxidising agent: it converts both the aldehyde end and the terminal primary alcohol end into carboxylic acid groups. The resulting six-carbon dicarboxylic acid is saccharic acid, so B-IV. It is not merely gluconic acid, which would retain the terminal alcohol group.

**Step 2:** Complete the reduction and mild oxidation matches For C, prolonged heating of glucose with HI removes its oxygen functionality while preserving the straight six-carbon skeleton. The hydrocarbon obtained is n-hexane; hence C-III. For D, bromine water is sufficiently mild to oxidise the aldehyde group without oxidising the terminal primary alcohol. It gives gluconic acid, so D-I. Together these are

$$\boxed{\text{A} - \text{II},\ \text{B} - \text{IV},\ \text{C} - \text{III},\ \text{D} - \text{I}}.$$

Option B incorrectly makes bicarbonate produce an alkane and nitric acid cause no reaction. Option C interchanges the oxidising strengths and incorrectly leaves bromine water unreactive. Option D incorrectly assigns gluconic acid to bicarbonate and no reaction to bromine water. These exclusions also check the option letter against the complete four-part match.

**Step 3:** Conclusion The correct matching is A-II, B-IV, C-III, D-I. Hence, the correct option is (A).

**Quick Tip to Solve:** Distinguish the oxidants by the ends they oxidise: bromine water changes only the aldehyde end, while nitric acid changes both ends.

**62. Consider the oxides of group 14 elements $SiO_2, GeO_2, SnO_2, PbO_2, CO$ and $GeO$. The amphoteric oxides are** [4 marks]

(A) $SnO_2, CO$

(B) $SiO_2, GeO_2$

(C) $SnO_2, PbO_2$

(D) $GeO, GeO_2$

**Correct Answer:** (C) $SnO_2, PbO_2$

**Solution:**

**Concept:**

- An amphoteric oxide reacts with both acids and bases. Across a group, increasing metallic character generally changes oxide behaviour from acidic towards amphoteric and basic.
- For the standard Group 14 classification, $CO$ is neutral, $SiO_2$ and $GeO_2$ are acidic, while $SnO_2$ and $PbO_2$ are amphoteric.

**Step 1:** Classify the non-amphoteric choices Carbon monoxide is a neutral oxide: unlike a typical acidic oxide, it does not produce a corresponding acid simply by reacting with water, and it is not classified as amphoteric in this list. Silicon dioxide is acidic and reacts with strong bases to give silicates. Germanium dioxide is classified as acidic in the Group 14 trend used here. Thus the pair containing $CO$, option A, cannot be the requested pair. Option B gives acidic oxides rather than the

two amphoteric dioxides.

**Step 2:** Identify the two amphoteric dioxides Tin dioxide and lead dioxide show amphoteric behaviour and form compounds with both acidic and alkaline reagents. Their reaction with alkali can be represented by hydroxo complexes, for example

$$SnO_2 + 2\,OH^- + 2\,H_2O \longrightarrow [Sn(OH)_6]^{2-}.$$

An acid-side reaction illustrating the tin oxide behaviour is

$$SnO_2 + 4\,HCl \longrightarrow SnCl_4 + 2\,H_2O.$$

These equations show why identifying an oxide merely as a metal oxide is insufficient: its ability to react on both sides of the acid-base classification is what matters. Option D includes $GeO$, which is not one of the required amphoteric dioxides, together with $GeO_2$. The pair that satisfies the stated classification is therefore $SnO_2$ and $PbO_2$.

**Step 3:** Conclusion The amphoteric oxides in the given list are $SnO_2$ and $PbO_2$. Hence, the correct option is (C).

**Quick Tip to Solve:** For the Group 14 dioxide trend, remember acidic $SiO_2$, $GeO_2$, followed by amphoteric $SnO_2$, $PbO_2$.

**63. The compound that is white in color is** [4 marks]

(A) ammonium sulphide

(B) ammonium arsinomolybdate

(C) lead iodide

(D) lead sulphate

**Correct Answer:** (D) lead sulphate

**Solution:**

**Concept:**

- Qualitative inorganic analysis identifies ions using characteristic precipitates, their colours and their solubilities. Lead(II) sulphate is the white, sparingly soluble sulphate precipitate.
- Lead(II) iodide and ammonium arsinomolybdate are characteristic yellow products. The ammonium sulphide reagent used in salt analysis is commonly encountered as a yellow solution.

**Step 1:** Identify the characteristic white precipitate Lead(II) ions combine with sulphate ions to form lead sulphate:

$$Pb^{2+}(aq) + {SO_4}^{2-}(aq) \longrightarrow PbSO_4(s).$$

This precipitate is white. Its low solubility makes precipitation a useful analytical observation: the colour belongs to the solid product rather than to an arbitrary solution concentration. Thus lead sulphate is the intended white compound among the analytical substances listed. The lead ion alone does not determine the colour of every lead salt; the identity of its counter-ion matters.

**Step 2:** Compare the alternatives in their analytical context Lead iodide forms a bright yellow

precipitate:

$$Pb^{2+}(aq) + 2\,I^{-}(aq) \longrightarrow PbI_2(s).$$

It therefore cannot replace white lead sulphate in this question. Ammonium arsinomolybdate is also yellow, being the characteristic yellow product associated with arsenate in the molybdate test. Ammonium sulphide is used as a sulphide reagent and is commonly supplied as a yellow solution in qualitative analysis, so it is not the intended white analytical precipitate. The distinction between the white sulphate and yellow iodide is particularly useful because both contain the same lead(II) cation. Comparing all the listed analytical colours selects the sulphate rather than relying on the presence of lead in its name.

**Step 3:** Conclusion Lead sulphate, $PbSO_4$, is the characteristic white precipitate in this list. Hence, the correct option is (D).

**Quick Tip to Solve:** Keep the two lead precipitates separate: $PbSO_4$ is white, whereas $PbI_2$ is yellow.

**64. Give below are two statements:**

**Statement - I: Noble gases have very high boiling points.**

**Statement - II: Noble gases are monoatomic gases. They are held together by strong dispersion forces. Because of this they are liquefied at very low temperature. Hence, they have very high boiling points.**

**In the light of the above statements, choose the correct answer from the options given below:**

[4 marks]

(A) Both Statement I and Statement II are false.

(B) Statement I is true but Statement II is false.

(C) Statement I is false but Statement II is true.

(D) Both Statement I and Statement II are true.

**Correct Answer:** (A) Both Statement I and Statement II are false.

**Solution:**

**Concept:**

- Noble gases have closed valence shells and normally exist as separate atoms. Monoatomicity describes the particles present, not the strength of attraction between them.
- The attractions between noble-gas atoms are London dispersion forces. Their generally small magnitude gives low melting and boiling points; boiling requires overcoming these interatomic attractions, not breaking covalent bonds.

**Step 1:** Evaluate Statement I Noble gases have very low boiling points compared with substances held together by strong intermolecular attractions. Only a small thermal energy is needed to separate their atoms into the gaseous state. Therefore the claim that noble gases have very high boiling points is false. Down the group the boiling point increases as polarizability and dispersion attraction increase, but this relative trend does not make the general statement true.

**Step 2:** Evaluate Statement II as a whole The opening claim that noble gases are monoatomic is correct. However, the statement then calls the dispersion forces strong and concludes that very

low liquefaction temperatures imply very high boiling points. Both inferences are incorrect. Weak dispersion forces account for liquefaction only at low temperatures. At a specified pressure the boiling temperature and condensation temperature are the same equilibrium temperature, so low-temperature liquefaction is consistent with a low boiling point.

**Step 3:** Match the two truth values A statement containing a correct opening clause followed by false claims is false as a whole. Here Statement I is false and Statement II is also false. Options (B), (C) and (D) each classify at least one statement as true, so none matches the physical reasoning.

**Step 4:** Conclusion Both Statement I and Statement II are false. Hence, the correct option is (A).

**Quick Tip to Solve:** Judge every clause in a compound statement. For noble gases, weak dispersion attraction means low boiling and condensation temperatures.

**65. A species having carbon with sextet of electrons and can act as electrophile is called** [4 marks]

(A) pentavalent carbon

(B) carbon free radical

(C) carbanion

(D) carbocation

**Correct Answer:** (D) carbocation

**Solution:**

**Concept:**

- An electrophile accepts an electron pair. Electron deficiency at carbon is therefore an important test for identifying an electrophilic intermediate.
- A conventional trivalent carbocation has three covalent bonds, six electrons around carbon and an empty orbital. A carbanion instead has a lone pair, while a carbon free radical has one unpaired electron.

**Step 1:** Count the electrons at carbon A sextet means six valence-shell electrons around the carbon atom, rather than the eight required for an octet. Three ordinary covalent bonds account for these six electrons. In a typical carbocation such as $CH_3{}^+$, carbon has three bonds and an empty p orbital. Its positive charge and incomplete octet make the carbon centre electron deficient.

**Step 2:** Connect the sextet with electrophilic behaviour The empty orbital can accept an electron pair from a nucleophile to form an additional bond. This supplies the electron-deficient carbon with an octet. For example, the bond-forming step $CH_3{}^+ + OH^- \longrightarrow CH_3OH$ illustrates electron-pair acceptance at carbon; the oxygen lone pair provides the new bond. The species satisfying both the electron count and the chemical role is thus a carbocation.

**Step 3:** Eliminate the other species A carbon free radical normally has three bonds and one unpaired electron, giving seven electrons around carbon rather than six. A carbanion normally has three bonds and a lone pair, giving an octet and usually nucleophilic behaviour. Pentavalent carbon is not the ordinary trivalent, sextet intermediate described here. Therefore options (A), (B) and (C) fail the stated combination of properties.

**Step 4:** Conclusion A carbocation has a carbon sextet and acts as an electrophile. Hence, the correct option is (D).

**Quick Tip to Solve:** For ordinary carbon intermediates count bond electrons and nonbonding electrons: carbocation 6, radical 7, carbanion 8.

**66. Identify the mixture that shows positive deviations from Raoult's Law** [4 marks]

(A) $(CH_3)_2\,CO + CS_2$

(B) $(CH_3)_2\,CO + C_6H_5NH_2$

(C) $CHCl_3 + C_6H_6$

(D) $CHCl_3 + (CH_3)_2\,CO$

**Correct Answer:** (A) $(CH_3)_2\,CO + CS_2$

**Solution:**

**Concept:**

- For an ideal binary solution, $p_A = x_A p_A^\circ$ and $p_B = x_B p_B^\circ$. Positive deviation means the observed vapour pressure exceeds this ideal prediction.
- Positive deviation occurs when unlike molecular attractions are weaker than the average like attractions. Molecules then escape more readily; stronger unlike attractions instead produce negative deviation.

**Step 1:** Examine acetone and carbon disulphide Acetone, $(CH_3)_2CO$, is polar and its pure liquid has appreciable dipole–dipole attractions. Carbon disulphide, $CS_2$, is nonpolar. On mixing, some acetone–acetone contacts are replaced by weaker acetone–carbon disulphide contacts. The unlike attractions do not compensate for the original like attractions, so the molecules have a greater tendency to enter the vapour phase. This mixture therefore shows positive deviation.

**Step 2:** Check the mixtures containing stronger unlike attractions Acetone and aniline, $C_6H_5NH_2$, can form hydrogen-bonded contacts involving the aniline N–H group and acetone oxygen; this favours negative deviation. Chloroform and acetone also form specific attractive contacts, with the acidic hydrogen of $CHCl_3$ interacting with acetone oxygen. Thus option (D) is the familiar negative-deviation example, not a positive one.

**Step 3:** Distinguish the remaining pair Chloroform and benzene have favourable specific unlike interactions involving the chloroform hydrogen and the benzene electron cloud. They are not the weaker-contact positive-deviation mixture sought here. Of the four listed pairs, acetone with carbon disulphide is the positive-deviation example; the other three involve strengthened unlike attractions rather than the required weakening.

**Step 4:** Conclusion The mixture $(CH_3)_2CO + CS_2$ shows positive deviation from Raoult's law. Hence, the correct option is (A).

**Quick Tip to Solve:** Compare unlike contacts with the original like contacts: weaker unlike attraction raises vapour pressure above the Raoult-law prediction.

**67. The correct statements from following are:**

**A. The strength of anionic ligands can be explained by crystal field theory.**

**B. Valence bond theory does not give a quantitative interpretation of kinetic stability of coordination compounds.**

**C. The hybridization involved in formation of $[Ni(CN)_4]^{2-}$ complex is $dsp^2$.**

**D. The number of possible isomer(s) of cis- $[PtCl_2(en)_2]^{2+}$ is one**

**Choose the correct answer from the options given below:** [4 marks]

(A) B, C only

(B) B, D only

(C) A, C only

(D) A, D only

**Correct Answer:** (A) B, C only

**Solution:**

**Concept:**

- Crystal field theory models ligands through an electrostatic field; it explains d-orbital splitting but does not explain the full experimental ligand-strength order through electrostatics alone.
- Valence bond theory uses orbital hybridization and electron pairing to describe geometry. Kinetic stability concerns substitution rates, while optical isomerism depends on whether mirror images are superposable.

**Step 1:** Assess Statements A and B Statement A is false. A purely electrostatic crystal-field model cannot explain the observed relative strengths of anionic ligands: charge alone does not reproduce their spectrochemical order. Covalent and orbital-interaction ideas are needed to account for that limitation. Statement B is true: valence bond theory does not quantitatively predict the kinetic stability or substitution rates of coordination compounds. A hybridization assignment alone is not a rate calculation.

**Step 2:** Find the hybridization in Statement C In $[Ni(CN)_4]^{2-}$, four cyanide ligands contribute total charge $-4$, so nickel is in oxidation state $+2$. Thus $Ni^{2+}$ has configuration $3d^8$. Cyanide is a strong-field ligand and promotes pairing, allowing an inner d orbital to participate with one s and two p orbitals. The resulting $dsp^2$ hybridization gives a square-planar complex. Statement C is true.

**Step 3:** Count the cis-complex stereoisomers In cis-$[PtCl_2(en)_2]^{2+}$, the two bidentate ethylenediamine ligands give a chiral octahedral arrangement. Its mirror-image configurations form two optical isomers, conventionally labelled $\Delta$ and $\Lambda$. Thus the assertion that the cis form has only one possible isomer is false. Statements A and D are false, leaving B and C alone true; all other options include a false statement or omit a true one.

**Step 4:** Conclusion Only statements B and C are correct. Hence, the correct option is (A).

**Quick Tip to Solve:** For cis complexes with two bidentate en ligands, test for an optical pair before counting the cis form as a single stereoisomer.

**68. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R:**

**Assertion A: Alcohols react both as nucleophiles and electrophiles.**

**Reason R: Alcohols react with active metals such as sodium, potassium and aluminum to yield corresponding alkoxides and liberate hydrogen.**

**In the light of the above statements, choose the correct answer from the options given below:**

[4 marks]

(A) $A$ is false but $R$ is true.

(B) Both $A$ and $R$ are true but $R$ is NOT the correct explanation of $A$.

(C) Both $A$ and $R$ are true and $R$ is the correct explanation of $A$.

(D) $A$ is true but $R$ is false.

**Correct Answer:** (B) Both $A$ and $R$ are true but $R$ is NOT the correct explanation of $A$.

**Solution:**

**Concept:**

- Alcohol oxygen has lone pairs and can donate an electron pair, giving nucleophilic behaviour. Under suitable acidic conditions an alcohol can also provide an electrophilic carbon after activation of the hydroxyl group.
- Alcohols react with active metals by replacing the hydroxyl hydrogen with metal to form alkoxides and hydrogen gas. This reaction demonstrates the acidic character of the O–H bond.

**Step 1:** Evaluate Assertion A Alcohols can act as nucleophiles because an oxygen lone pair attacks an electron-deficient centre; for example, alcohol oxygen attacks an acyl carbon in an appropriate ester-forming reaction. They can also undergo nucleophilic substitution after protonation of oxygen makes water a leaving group. The carbon attached to this activated group is attacked by a nucleophile. Hence the statement that alcohols show both nucleophilic and electrophilic behaviour is true.

**Step 2:** Evaluate Reason R Active metals react with alcohols to give the corresponding alkoxides and liberate hydrogen. For sodium, the balanced equation is

$$2\,\mathrm{ROH} + 2\,\mathrm{Na} \longrightarrow 2\,\mathrm{RONa} + \mathrm{H_2}.$$

Potassium similarly gives potassium alkoxide, while aluminium can form aluminium alkoxide under suitable conditions. The stated reaction is therefore correct, so Reason R is true. In this process the alcohol loses its hydroxyl hydrogen.

**Step 3:** Test whether the reason explains the assertion The metal reaction establishes the acidic behaviour of the O–H group; it does not explain both donation of an oxygen lone pair and nucleophilic attack at an activated carbon. Therefore it is not a complete explanation of the dual nucleophilic/electrophilic behaviour in Assertion A. Options (A) and (D) incorrectly make one statement false, and option (C) incorrectly claims the explanatory connection.

**Step 4:** Conclusion Both A and R are true, but R is not the correct explanation of A. Hence, the correct option is (B).

**Quick Tip to Solve:** In assertion–reason items, separate truth from explanation: alkoxide formation with a metal tests O–H acidity, not both reaction roles of an alcohol.

**69. The product (C) in the below mentioned reaction is :**

$$CH_3 - CH_2 - CH_2 - Br \xrightarrow[\Delta]{KOH_{(alc)}} A \xrightarrow{HBr} B \xrightarrow[KOH_{(aq)}]{\Delta} C$$

[4 marks]

(A) Propan-1-ol

(B) Propyne

(C) Propan-2-ol

(D) Propene

**Correct Answer:** (C) Propan-2-ol

**Solution:**

**Concept:**

- Hot alcoholic KOH favours beta-elimination from a haloalkane, whereas aqueous KOH favours replacement of halogen by hydroxyl.
- Addition of HBr to an unsymmetrical alkene without peroxide follows Markovnikov orientation: bromine attaches to the carbon that gives the more stable carbocation intermediate.

**Step 1:** Identify intermediate A by elimination The starting compound is 1-bromopropane, $CH_3CH_2CH_2Br$. Alcoholic KOH and heat remove hydrogen from the adjacent carbon and bromine from the terminal carbon, forming a double bond.

$$CH_3CH_2CH_2Br + KOH \longrightarrow CH_3CH{=}CH_2 + KBr + H_2O.$$

Thus A is propene. There is only one adjacent carbon available for this beta-elimination, so there is no competing positional alkene here.

**Step 2:** Apply Markovnikov addition to find B No peroxide is specified in the HBr step. Hydrogen therefore adds to the terminal carbon of propene, generating the more stable secondary carbocation rather than a primary one. Bromide then attaches to the middle carbon:

$$CH_3CH{=}CH_2 + HBr \longrightarrow CH_3CHBrCH_3.$$

Consequently B is 2-bromopropane, not 1-bromopropane. Keeping the carbon position of bromine correct is essential for the next step.

**Step 3:** Hydrolyse B with aqueous alkali Aqueous KOH supplies hydroxide for substitution at the carbon bearing bromine.

$$CH_3CHBrCH_3 + KOH \longrightarrow CH_3CH(OH)CH_3 + KBr.$$

The final product C is therefore propan-2-ol. Propan-1-ol would require the wrong orientation in the preceding addition. Propene is intermediate A rather than C, and no reagent in the sequence generates the triple bond required for propyne.

**Step 4:** Conclusion Product C is propan-2-ol, $CH_3CH(OH)CH_3$. Hence, the correct option is (C).

**Quick Tip to Solve:** Track the functional group at every arrow: alcoholic alkali gives an alkene, ordinary HBr puts Br on the middle carbon, aqueous alkali replaces Br by OH.

**70. Identify the factor from the following that does not affect electrolytic conductance of a solution.**

[4 marks]

(A) The nature of solvent used.

(B) The nature of the electrolyte added.

(C) The nature of the electrode used.

(D) Concentration of the electrolyte.

**Correct Answer:** (C) The nature of the electrode used.

**Solution:**

**Concept:**

- An electrolyte conducts through the motion of ions in solution. At fixed cell geometry, conductance is $G = \kappa A/l$, where $\kappa$ is conductivity, $A$ electrode area and $l$ separation.
- Conductivity depends on the number and mobility of ions. Solvent properties, electrolyte identity, concentration and temperature affect ion formation or motion.

**Step 1:** Identify the quantities that govern ion transport Current within the liquid is carried by dissolved positive and negative ions moving in opposite directions. The identity and degree of dissociation of an electrolyte determine which ions are present and how many charge carriers it supplies. Concentration changes their number per unit volume and their mutual interactions. Hence the electrolyte identity and concentration in options (B) and (D) can affect conductance; they cannot be the requested factor.

**Step 2:** Examine the solvent effect The solvent controls dissociation through solvation and dielectric properties, and it affects ionic mobility through viscosity. The same electrolyte can therefore conduct differently in different solvents even when comparable amounts are dissolved. Thus option (A) also names a relevant factor. This reasoning concerns transport in the liquid and does not require a chemical change of the dissolved electrolyte.

**Step 3:** Distinguish electrode material from cell geometry Changing only the nature or material of suitable electrodes does not determine the intrinsic electrolytic conductance of a given solution. Their area and separation do matter through the cell constant, so these must be kept fixed when comparing measurements. Electrode polarization or chemical reactions are practical measurement complications, not the solution property asked here. Among the listed choices, electrode nature is therefore the factor that does not affect the solution's electrolytic conductance under the standard measurement conditions.

**Step 4:** Conclusion The nature of the electrode used is the listed factor that does not affect electrolytic conductance of the solution. Hence, the correct option is (C).

**Quick Tip to Solve:** Distinguish the material of an electrode from its geometry: ion properties set conductivity, while area and spacing set the cell constant.

**71. Integrated rate law equation for a first order gas phase reaction is given by (where $\mathrm{P_i}$ is initial pressure and $\mathrm{P_t}$ is total pressure at time $t$)** [4 marks]

(A) $k = \frac{2.303}{t} \times \log \frac{P_i}{(2P_i - P_t)}$

(B) $\mathrm{k} = \frac{2.303}{\mathrm{t}} \times \log \frac{(2\mathrm{P_i} - \mathrm{P_t})}{\mathrm{P_i}}$

(C) $k = \frac{2.303}{t} \times \frac{P_i}{(2P_i - P_t)}$

(D) $\mathrm{k} = \frac{2.303}{\mathrm{t}} \times \log \frac{2\mathrm{P_i}}{(2\mathrm{P_i} - \mathrm{P_t})}$

**Correct Answer:** (A) $k = \frac{2.303}{t} \times \log \frac{P_i}{(2P_i - P_t)}$

**Solution:**

**Concept:**

- The integrated first-order law is $kt = \ln(p_0/p_A)$, where $p_A$ is the reactant partial pressure at time $t$, at constant temperature and volume.
- For the pressure-increase expression in these options, one molecule of gaseous reactant gives two molecules of gaseous products. Total pressure is not the remaining reactant pressure.

**Step 1:** Express the remaining reactant pressure Let the pressure corresponding to reactant consumed be $x$. Initially only reactant is present at pressure $P_i$. Its remaining partial pressure is $P_i - x$, while the two gaseous products together contribute $2x$. Hence

$$P_t = (P_i - x) + 2x = P_i + x.$$

Thus $x = P_t - P_i$, and

$$p_A = P_i - x = 2P_i - P_t.$$

**Step 2:** Apply the logarithmic rate law Substitute this partial pressure, rather than total pressure, into the integrated law:

$$k = \frac{1}{t} \ln \frac{P_i}{2P_i - P_t} = \frac{2.303}{t} \log \frac{P_i}{2P_i - P_t}.$$

During reaction, $P_i < P_t < 2P_i$, so the logarithm and $k$ are positive. Option B reverses the ratio and gives a negative constant; C omits the required logarithm. D incorrectly doubles the initial reactant pressure and does not recover a zero logarithm at $t = 0$.

**Step 3:** Conclusion The pressure form of the integrated first-order law is $k = (2.303/t) \log[P_i/(2P_i - P_t)]$. Hence, the correct option is (A).

**Quick Tip to Solve:** Build a pressure balance first: the first-order logarithm uses remaining reactant partial pressure, not total pressure.

**72. For the given reaction, choose the correct expression of $K_C$ from the following :-**

$$Fe^{3+}_{(aq)} + SCN^{-}_{(aq)} \rightleftharpoons (FeSCN)^{2+}_{(aq)}$$

[4 marks]

(A) $K_C = \frac{[Fe^{3+}][SCN^-]}{[FeSCN^{2+}]}$

(B) $K_C = \frac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^-]}$

(C) $K_C = \frac{[FeSCN^{2+}]^2}{[Fe^{3+}][SCN^-]}$

(D) $K_C = \frac{[FeSCN^{2+}]}{[Fe^{3+}]^2[SCN^-]^2}$

**Correct Answer:** (B) $K_C = \frac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^-]}$

**Solution:**

**Concept:**

- For $aA + bB \rightleftharpoons cC$, the concentration equilibrium constant is $K_c = [C]^c/([A]^a[B]^b)$, with equilibrium concentrations.
- Stoichiometric coefficients determine powers; ionic charges do not become concentration exponents.

**Step 1:** Read the balanced reaction The reaction is $Fe^{3+}(aq) + SCN^-(aq) \rightleftharpoons FeSCN^{2+}(aq)$. One iron(III) ion and one thiocyanate ion form one complex ion. Every stoichiometric coefficient is therefore one. The charges $+3, -1, +2$ ensure charge conservation, since $+3 - 1 = +2$, but they do not change those coefficients.

**Step 2:** Write products over reactants Place the complex concentration in the numerator and the concentrations of the two reacting ions in the denominator:

$$K_c = \frac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^-]}.$$

All three species are aqueous solutes, so all appear. Option A is the reciprocal and belongs to the reverse reaction. Option C wrongly squares the complex concentration, and option D wrongly squares the reactant concentrations. Neither squared expression follows the printed stoichiometry.

**Step 3:** Conclusion The equilibrium expression is $K_c = [FeSCN^{2+}]/([Fe^{3+}][SCN^-])$. Hence, the correct option is (B).

**Quick Tip to Solve:** Use coefficients in the balanced chemical equation as powers; never use the charges written on the ions.

**73. The linear combination of atomic orbitals to form molecular orbitals takes place only when the combining atomic orbitals**

**A. have the same energy**

**B. have the minimum overlap**

**C. have same symmetry about the molecular axis**

**D. have different symmetry about the molecular axis**

**Choose the most appropriate from the options given below:** [4 marks]

(A) B, C, D only

(B) A, B, C only

(C) B and D only

(D) A and C only

**Correct Answer:** (D) A and C only

**Solution:**

**Concept:**

- Effective linear combination requires atomic orbitals of the same or comparable energies and compatible symmetry about the molecular axis.
- An appreciable overlap integral is required for significant interaction and bonding-antibonding splitting.

**Step 1:** Check energy and overlap Statement A represents the energy-matching requirement: orbitals with the same, or sufficiently close, energy combine effectively. It is accepted here. Statement B demands minimum overlap, which is the opposite of the effective-overlap condition. Significant overlap makes the interaction stronger; an almost vanishing overlap does not favour useful molecular-orbital formation.

**Step 2:** Check symmetry and select the combination Statement C is correct because orbitals must have compatible symmetry with respect to the internuclear axis. Then their overlap does not cancel by symmetry. For example, an s orbital and a p orbital directed along that axis can combine, whereas a perpendicular p orbital has cancelling positive and negative contributions. Statement D proposes different symmetry and is therefore false. Thus the accepted statements are A and C only. Option A includes false B and D; B includes false B; and C consists of the two false statements B and D.

**Step 3:** Conclusion The required conditions among the listed statements are A and C only. Hence, the correct option is (D).

**Quick Tip to Solve:** Remember the three LCAO checks together: comparable energy, compatible symmetry and appreciable overlap.

**74. Identify correct statements from below:**

**A. The chromate ion is square planar.**

**B. Dichromates are generally prepared from chromates.**

**C. The green manganate ion is diamagnetic.**

**D. Dark green coloured $K_2MnO_4$ disproportionates in a neutral or acidic medium to give permanganate.**

**E. With increasing oxidation number of transition metal, ionic character of the oxides decreases.**

**Choose the correct answer from the options given below:** [4 marks]

(A) B, D, E only

(B) A, B, C only

(C) A, D, E only

(D) B, C, D only

**Correct Answer:** (A) B, D, E only

**Solution:**

**Concept:**

- Chromate $CrO_4^{2-}$ is tetrahedral. Acidification connects chromate and dichromate through a reversible condensation equilibrium.
- Manganese(VI) has a $d^1$ configuration and is paramagnetic. Higher metal oxidation states generally increase covalent character in oxides.

**Step 1:** Evaluate chromate and manganate statements A is false: the four oxygen atoms of $CrO_4^{2-}$ surround chromium tetrahedrally, not in a square plane. B is true: dichromates are prepared from chromates by acidification, represented by

$$2\,CrO_4^{2-} + 2\,H^+ \rightleftharpoons Cr_2O_7^{2-} + H_2O.$$

C is false: in green $MnO_4^{2-}$, manganese is in oxidation state $+6$, leaving one d electron. That unpaired electron makes the ion paramagnetic, rather than diamagnetic.

**Step 2:** Check disproportionation and oxide character D is true: green manganate disproportionates in neutral or acidic medium, forming permanganate and manganese dioxide:

$$3\,MnO_4^{2-} + 4\,H^+ \longrightarrow 2\,MnO_4^- + MnO_2 + 2\,H_2O.$$

E is also true. As metal oxidation number increases, the metal centre polarises oxygen more strongly and metal-oxygen bonding becomes more covalent, so ionic character decreases. The true set is B, D and E. Every other listed combination contains A or C, already shown false.

**Step 3:** Conclusion Only statements B, D and E are correct. Hence, the correct option is (A).

**Quick Tip to Solve:** For magnetic character, find the metal oxidation state and d-electron count before associating a colour with an ion.

**75. The metals that are employed in the battery industries are**

**A.** Fe

**B.** Mn

**C.** Ni

**D.** Cr

**E.** Cd

**Choose the correct answer from the options given below:** [4 marks]

(A) A, B, C, D and E

(B) A, B, C and D only

(C) B, D and E only

(D) B, C and E only

**Correct Answer:** (D) B, C and E only

**Solution:**

**Concept:**

- The standard battery applications tested here include manganese dioxide in dry cells and nickel-cadmium rechargeable cells.
- Use the characteristic textbook applications of the listed metals to choose the intended set; electrode compounds can contain the metal without using its elemental form.

**Step 1:** Identify the characteristic battery materials Manganese, statement B, is used as $MnO_2$ in common dry-cell cathode mixtures. Nickel, statement C, is used in rechargeable battery systems; in nickel-cadmium cells the positive electrode involves nickel oxyhydroxide. Cadmium, statement E, forms the negative electrode of the nickel-cadmium rechargeable cell. Thus B, C and E all belong to the standard battery-industry group.

**Step 2:** Compare the proposed groups The textbook group selected by this question is manganese, nickel and cadmium. Iron and chromium, A and D, are not included in that conventional group. This identifies option D. Option A adds both A and D, option B includes them and omits cadmium, while option C includes chromium and omits nickel. The phrase is being used in this standard textbook sense; it is not a claim that no specialised electrochemical battery can ever contain iron or a chromium compound. In particular, nickel-iron systems exist, but they do not change the listed conventional answer group.

**Step 3:** Conclusion The standard battery-industry metals selected here are manganese, nickel and cadmium: B, C and E only. Hence, the correct option is (D).

**Quick Tip to Solve:** Link the textbook examples directly: dry cell to manganese dioxide, and Ni-Cd rechargeable cell to nickel plus cadmium.

**76. Match List I with List II**

| | List - I (Technique) | | List - II (Application |
|---|---|---|---|
| (A) | Distillation | (I) | Separation of glycerol from spent-lye |
| (B) | Fractional distillation | (II) | Aniline - Water mixture |
| (C) | Steam distillation | (III) | Separation of crude oil fractions |
| (D) | Distillation under reduced pressure | (IV) | Chloroform - Aniline |

**Choose the correct answer from the options given below:** [4 marks]

(A) A-I, B-II, C-IV, D-III

(B) A-II, B-III, C-I, D-IV

(C) A-IV, B-I, C-II, D-III

(D) A-IV, B-III, C-II, D-I

**Correct Answer:** (D) A-IV, B-III, C-II, D-I

**Solution:**

**Concept:**

- Simple distillation separates liquids with sufficiently separated boiling points; fractional distillation uses repeated vaporisation-condensation for closer boiling fractions.
- Steam distillation is suitable for steam-volatile, water-immiscible substances, whereas reduced pressure lowers boiling temperature for high-boiling or heat-sensitive liquids.

**Step 1:** Match the ordinary and fractional methods Chloroform and aniline have widely separated boiling points, so ordinary distillation separates them: A-IV. Crude oil contains many liquid components and is separated into boiling-range fractions using a fractionating column: B-III. Its repeated condensation and vaporisation enrich different fractions at different levels; a single simple distillation would not provide the required fractionation.

**Step 2:** Match the steam and vacuum methods Aniline is steam-volatile and only sparingly miscible with water, so steam distillation is used for aniline-water mixtures: C-II. The combined vapour pressures allow boiling below the normal boiling point of aniline. Glycerol is a high-boiling substance that can decompose on strong heating. Distillation under reduced pressure lowers its boiling temperature and separates it from spent-lye: D-I. Together these matches are A-IV, B-III, C-II, D-I. Options A and B fail already on the ordinary-distillation match; C has the first match right but incorrectly assigns the remaining three.

**Step 3:** Conclusion The matching is A-IV, B-III, C-II and D-I. Hence, the correct option is (D).

**Quick Tip to Solve:** Use the signature pairs first: crude oil means fractional distillation; glycerol means reduced-pressure distillation.

**77. 'Adsorption' principle is used for which of the following purification method?** [4 marks]

(A) Chromatography

(B) Sublimation

(C) Distillation

(D) Extraction

**Correct Answer:** (A) Chromatography

**Solution:**

**Concept:**

- Adsorption is accumulation of molecules at a surface. In adsorption chromatography, components bind to a solid stationary phase with different strengths.
- The mobile phase carries components along; weaker adsorption produces faster movement, while stronger adsorption produces slower movement.

**Step 1:** Apply the adsorption principle In adsorption chromatography, a mixture passes over an adsorbent such as silica gel or alumina. Its components repeatedly adsorb on and desorb from the stationary surface. Differences in adsorption strength cause different travel rates and hence separation into distinct bands or spots. This is the purification method that directly uses the surface-adsorption principle in the question.

**Step 2:** Distinguish the remaining methods Sublimation relies on a solid changing directly into vapour and then depositing as a solid; the key property is sublimability. Distillation relies on differences in volatility or boiling behaviour and a vapour-liquid separation. Differential extraction relies on different solubilities, or partitioning, between two immiscible liquid solvents. None of those is the adsorption-based separation asked here. Although chromatography has several forms, the adsorption form specifically supplies the intended principle and clearly distinguishes option A from the other three methods.

**Step 3:** Conclusion The purification method based on adsorption is chromatography. Hence, the correct option is (A).

**Quick Tip to Solve:** In adsorption chromatography, remember that stronger attraction to the stationary surface means slower movement with the mobile phase.

**78. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R:**

**Assertion A: $\mathrm{pK_a}$ value of phenol is 10.0 while that of ethanol is 15.9 .**

**Reason R: Ethanol is stronger acid than phenol.**

**In the light of the above statements, choose the correct answer from the options given below:**

[4 marks]

(A) A is false but R is true.

(B) Both $A$ and $R$ are true but $R$ is NOT the correct explanation of $A$.

(C) Both A and R are true and R is the correct explanation of A.

(D) $A$ is true but $R$ is false.

**Correct Answer:** (D) $A$ is true but $R$ is false.

**Solution:**

**Concept:**

- The acid dissociation constant describes the equilibrium $\mathrm{HA} \rightleftharpoons \mathrm{H^+ + A^-}$; its logarithmic measure is $pK_a = -\log_{10} K_a$. A smaller $pK_a$ therefore means a stronger acid.
- Conjugate-base stability controls acidity. The negative charge of phenoxide is delocalised into the aromatic ring, whereas the negative charge of ethoxide remains mainly on oxygen.

**Step 1:** Evaluate Assertion A The stated values, $pK_a(\text{phenol}) = 10.0$ and $pK_a(\text{ethanol}) = 15.9$, are the usual approximate aqueous values. They show that phenol has the larger acid dissociation constant. Thus Assertion A is true; the presence of an aromatic ring does not make the phenolic proton less acidic than the alcohol proton.

**Step 2:** Evaluate Reason R Compare the dissociation constants directly:

$$\frac{K_a(\text{phenol})}{K_a(\text{ethanol})} = 10^{15.9-10.0} = 10^{5.9}.$$

Phenol is consequently about $8 \times 10^5$ times stronger on this comparison. Resonance stabilises phenoxide, while the electron-releasing ethyl group does not provide such stabilisation to ethoxide. The claim that ethanol is the stronger acid reverses this order, so R is false.

**Step 3:** Compare the answer choices Option (A) incorrectly makes A false and R true. Options (B) and (C) both require R to be true, so neither can apply. Since R itself is false, it cannot be an explanation of A. Only the true-assertion, false-reason combination survives.

**Step 4:** Conclusion Assertion A is true, but Reason R is false: phenol is more acidic than ethanol. Hence, the correct option is (D).

**Quick Tip to Solve:** Convert a comparison of $pK_a$ values into acidity before judging the reason: lower $pK_a$ means stronger acid.

**79. Given below are two statements:**

**Statement I: IUPAC name of $HO - CH_2 - (CH_2)_3 - CH_2 - COCH_3$ is 7-hydroxyheptan-2-one.**

**Statement II: 2-oxoheptan-7-ol is the correct IUPAC name for above compound. In the light of the above statements, choose the most appropriate answer from the options given below:** [4 marks]

(A) Statement I is incorrect but Statement II is correct.

(B) Statement I is correct but Statement II is incorrect.

(C) Both Statement I and Statement II are correct.

(D) Both Statement I and Statement II are incorrect.

**Correct Answer:** (B) Statement I is correct but Statement II is incorrect.

**Solution:**

**Concept:**

- For a compound containing both a ketone and an alcohol group, the ketone is the senior characteristic group. It receives the suffix -one; the alcohol is expressed by the prefix hydroxy-.
- The parent chain must contain the carbonyl carbon and is numbered to give the suffix functional group the lower locant. Only after this priority is satisfied do other substituent locants enter the name.

**Step 1:** Select and number the parent chain Write the chain from the methyl end:

$$CH_3COCH_2CH_2CH_2CH_2CH_2OH.$$

There are seven carbon atoms, including the carbonyl carbon, so the parent is heptane. Numbering from the methyl end places the ketone carbonyl at C2 and the hydroxyl substituent at C7. Numbering from the alcohol end would instead put the senior ketone group at C6, so that numbering is rejected.

**Step 2:** Assess Statement I Use heptan-2-one as the parent suffix name and attach the hydroxy substituent at C7. The resulting name is 7-hydroxyheptan-2-one, exactly the name given in Statement I. The structure and all locants agree with that statement; it is correct.

**Step 3:** Assess Statement II and eliminate options The proposed name 2-oxoheptan-7-ol treats the alcohol as the principal suffix group and relegates the ketone to an oxo prefix. This violates the ketone-over-alcohol functional-group priority. Statement II is therefore incorrect. Option (A) reverses the truth values; (C) accepts the invalid second name; (D) rejects the valid first name. Only option (B) has the required combination.

**Step 4:** Conclusion Statement I is correct, but Statement II is incorrect. The correct IUPAC name is 7-hydroxyheptan-2-one. Hence, the correct option is (B).

**Quick Tip to Solve:** Choose the senior functional group before numbering: a ketone uses -one and the accompanying alcohol becomes hydroxy-.

**80. The correct sequence of electron gain enthalpy of the elements listed below is**

**A. Ar**

**B. Br**

**C. F**

**D. S**

**Choose the most appropriate from the options given below:** [4 marks]

(A) $A > D > C > B$

(B) $A > D > B > C$

(C) $D > C > B > A$

(D) $C > B > D > A$

**Correct Answer:** (B) $A > D > B > C$

**Solution:**

**Concept:**

- Electron gain enthalpy is the signed enthalpy change for $X(g) + e^- \longrightarrow X^-(g)$. An exothermic electron addition has a negative value, whereas an endothermic addition has a positive value.
- Noble gases resist electron addition because the incoming electron must enter a higher shell. Halogens gain an electron favourably, while sulfur has a less negative electron gain enthalpy than fluorine or bromine.

**Step 1:** Compare argon with sulfur and the halogens Argon has a closed-shell configuration. Adding an electron is energetically unfavourable, so its electron gain enthalpy is positive. Sulfur, bromine and fluorine have negative values. Consequently Ar must come first in a descending sequence of the signed enthalpies. This immediately excludes options (C) and (D), which put Ar last.

**Step 2:** Order the three negative values The approximate electron gain enthalpies, in $\mathrm{kJ\,mol^{-1}}$, are

$$\Delta H_{eg}(S) \approx -200, \qquad \Delta H_{eg}(Br) \approx -325, \qquad \Delta H_{eg}(F) \approx -328.$$

Hence the descending signed order is $S > Br > F$. Although fluorine has a compact shell with appreciable electron repulsion, its value remains slightly more negative than that of bromine; the familiar fluorine anomaly is especially relevant when comparing it with chlorine.

**Step 3:** Translate back to the listed letters The labels are A = Ar, B = Br, C = F and D = S. Combining the positive argon value with the three negative values gives A > D > B > C. Option (A) incorrectly exchanges F and Br. The negative sign must be retained: a larger energy release corresponds to a smaller signed enthalpy, not a larger one.

**Step 4:** Conclusion The correct sequence is $Ar > S > Br > F$, or A > D > B > C. Hence, the correct option is (B).

**Quick Tip to Solve:** Write representative signed values before ordering electron gain enthalpies; do not confuse a more negative value with a greater value.

## Chemistry – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**81. Consider the following reaction at** $298\ \mathrm{K} \cdot \frac{3}{2}\mathrm{O}_{2(g)} \rightleftharpoons \mathrm{O}_{3(g)} \cdot \mathrm{K_P} = 2.47 \times 10^{-29}$**.** $\Delta_r G^{\ominus}$ **for the reaction is _____** $\mathrm{kJ}$**. (Given** $\mathrm{R} = 8.314\ \mathrm{JK^{-1}\ mol^{-1}}$**)** [4 marks]

**Solution:**

**Concept:**

- The standard reaction Gibbs energy is related to the thermodynamic equilibrium constant by $\Delta_r G^\circ = -RT \ln K_p$. The logarithm is natural and the equilibrium constant is dimensionless in this expression.
- With $R$ in $\mathrm{J\,K^{-1}\,mol^{-1}}$, the calculation first gives joules per mole of reaction. Divide by 1000 to express the requested energy in kilojoules.

**Step 1:** Use the reaction exactly as written The stated equilibrium is $\frac{3}{2}\mathrm{O_2(g)} \rightleftharpoons \mathrm{O_3(g)}$, with $K_p = 2.47 \times 10^{-29}$ at $T = 298\,\mathrm{K}$. The quoted constant already belongs to this stoichiometric reaction. There is no reason to square it or multiply the Gibbs energy by two; doing so would correspond to a different reaction.

**Step 2:** Evaluate the logarithm Separate the coefficient and the power of ten:

$$\begin{aligned} \ln K_p &= \ln(2.47) - 29 \ln 10 \\ &\approx 0.9042 - 29(2.302585) \\ &\approx -65.8708. \end{aligned}$$

The negative logarithm is expected because the equilibrium constant is far below unity. This provides an immediate sign check for the next step.

**Step 3:** Calculate and convert the energy Substitute the given gas constant without changing its units:

$$\begin{aligned} \Delta_r G^\circ &= -(8.314)(298)(-65.8708) \\ &\approx 1.632 \times 10^5\ \mathrm{J\,mol^{-1}} \\ &\approx 163.2\,\mathrm{kJ\,mol^{-1}}. \end{aligned}$$

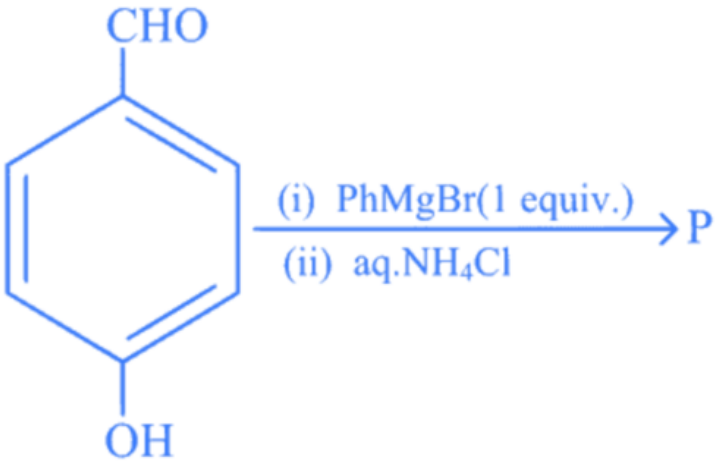
The positive standard Gibbs energy is consistent with the extremely small equilibrium constant: ozone formation is not favoured under standard-state conditions at this temperature.

**Step 4:** Conclusion The standard Gibbs energy change for the reaction as written is approximately $+163\,\mathrm{kJ}$ per mole of reaction. The required numerical entry is 163.

**Quick Tip to Solve:** Check the sign first: $K_p < 1$ gives $\ln K_p < 0$, so $\Delta_r G^\circ$ must be positive.

**82. The product of the following reaction is P.**

**The number of hydroxyl groups present in the product P is _____.** [4 marks]




**Solution:**

**Concept:**

- A Grignard reagent is strongly basic and reacts rapidly with an acidic hydroxyl proton. One mole of $\mathrm{PhMgBr}$ is consumed for each mole of phenolic OH, producing benzene and a magnesium phenoxide.
- An aqueous ammonium chloride work-up protonates oxygen-magnesium species. The final organic product must be assessed after both printed operations, rather than at the intermediate salt stage.

**Step 1:** Identify the competing functional groups The starting compound is para-hydroxybenzaldehyde, $\mathrm{HO-C_6H_4-CHO}$. It contains one phenolic OH and one aldehyde carbonyl. Although a Grignard reagent can add to an aldehyde, the acidic phenolic proton first quenches the reagent by proton transfer. The question supplies only one equivalent of phenylmagnesium bromide, so that equivalent is used up in the acid-base reaction.

**Step 2:** Track the single equivalent of reagent The first operation can be represented as

$$\mathrm{HO-C_6H_4-CHO + PhMgBr \longrightarrow BrMgO-C_6H_4-CHO + PhH.}$$

The aromatic aldehyde group remains unchanged because there is no further equivalent of Grignard reagent available for carbonyl addition. At this intermediate stage the oxygen is present as a magnesium phenoxide, so it is not bonded to hydrogen.

**Step 3:** Apply the printed aqueous work-up The second operation is aqueous $\mathrm{NH_4Cl}$, which supplies a proton to the phenoxide during work-up:

$$\mathrm{BrMgO-C_6H_4-CHO \xrightarrow{aq\cdot NH_4Cl} HO-C_6H_4-CHO.}$$

Thus the final organic product regains its original phenolic OH. No additional alcoholic OH is created, since no carbonyl addition occurred. Counting the intermediate magnesium salt instead of the worked-up product would give zero, but would omit the explicitly printed second operation.

**Step 4:** Conclusion The final product after aqueous $NH_4Cl$ work-up has $\boxed{1}$ hydroxyl group. The supplied key value 0 counts the phenoxide intermediate; the printed aqueous work-up restores the phenolic OH, so the literal final-product answer is 1.

**Quick Tip to Solve:** For a Grignard reagent with an acidic OH, consume reagent equivalents by proton transfer first, then separately apply the aqueous work-up.

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**83. The ionization energy of sodium in $\text{kJ mol}^{-1}$, if electromagnetic radiation of wavelength $242\ \text{nm}$ is just sufficient to ionize sodium atom is _____.** [4 marks]

**Solution:**

**Concept:**

- The energy of a photon of wavelength $\lambda$ is $E = h\nu = hc/\lambda$. At the threshold for ionization the photon energy equals the energy required to remove the electron from one gaseous atom.
- To obtain a molar ionization energy, multiply the energy per atom by Avogadro's constant $N_A$. Convert nanometres to metres before substitution and joules to kilojoules at the end.

**Step 1:** Interpret the threshold condition The words "just sufficient to ionize" specify the threshold radiation. There is no excess photon energy to assign to the kinetic energy of the ejected electron. Therefore the first ionization energy per atom of sodium equals the energy of one photon of wavelength 242 nm. This is an energy-per-particle calculation first, followed by conversion to a mole of particles.

**Step 2:** Calculate the photon energy Use $h = 6.626 \times 10^{-34}\,\text{J s}$ and $c = 2.99792458 \times 10^{8}\,\text{m s}^{-1}$, with $\lambda = 242 \times 10^{-9}\,\text{m}$:

$$E = \frac{(6.626 \times 10^{-34})(2.99792458 \times 10^{8})}{242 \times 10^{-9}} \approx 8.2084 \times 10^{-19}\,\text{J}.$$

The seconds and metres cancel, leaving joules for one photon and hence for one sodium atom.

**Step 3:** Convert to molar ionization energy Multiply by $N_A = 6.022 \times 10^{23}\,\text{mol}^{-1}$:

$$\begin{aligned} E_{mol} &= (8.2084 \times 10^{-19})(6.022 \times 10^{23}) \\ &\approx 4.943 \times 10^{5}\,\text{J mol}^{-1} \\ &\approx 494.3\,\text{kJ mol}^{-1}. \end{aligned}$$

Keeping these constants through the conversion gives approximately $494.3\,\text{kJ mol}^{-1}$. The requested nearest-integer entry is accordingly 494.

**Step 4:** Conclusion The ionization energy is approximately $494\,\text{kJ mol}^{-1}$; the required numerical entry is 494.

**Quick Tip to Solve:** Use $E_{mol} = N_A hc/\lambda$ directly, and retain precise constants until the final integer rounding.

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**84. Number of alkanes obtained on electrolysis of a mixture of $CH_3COONa$ and $C_2H_5COONa$ is _____.**

[4 marks]

**Solution:**

**Concept:**

- In Kolbe electrolysis a carboxylate ion loses an electron and carbon dioxide at the anode, producing an alkyl radical: $RCOO^- \longrightarrow R\cdot + CO_2 + e^-$.
- Two alkyl radicals couple to form an alkane. A mixture of two different carboxylates can therefore give two self-coupling products and one cross-coupling product, provided those structures are distinct.

**Step 1:** Identify the two radicals Sodium acetate, $CH_3COONa$, furnishes $CH_3COO^-$. Anodic decarboxylation removes the carboxyl carbon as $CO_2$, leaving a methyl radical, $CH_3\cdot$. Sodium propionate, $C_2H_5COONa$, similarly gives the ethyl radical, $C_2H_5\cdot$. The carbon of the carboxylate group is not retained in either alkyl radical.

**Step 2:** List all distinct radical couplings There are three unordered pairings:

$$CH_3\cdot + .CH_3 \longrightarrow CH_3CH_3 \quad (\text{ethane}),$$
$$CH_3\cdot + .C_2H_5 \longrightarrow CH_3CH_2CH_3 \quad (\text{propane}),$$
$$C_2H_5\cdot + .C_2H_5 \longrightarrow CH_3CH_2CH_2CH_3 \quad (\text{butane}).$$

Each radical joins at its radical-bearing carbon, so the ethyl-ethyl coupling gives normal butane. It does not produce a branched four-carbon skeleton.

**Step 3:** Count products without duplication The methyl-ethyl and ethyl-methyl descriptions refer to the same propane molecule, so they are counted only once. The two self-couplings have different carbon counts and are distinct from the cross-coupling. Thus ethane, propane and butane exhaust the alkane products from these two radicals. Carbon dioxide and cathodic hydrogen are not alkanes and are outside the requested count.

**Step 4:** Conclusion The distinct alkanes are ethane, propane and butane. Their number is $\boxed{3}$, a dimensionless product count.

**Quick Tip to Solve:** Write the radicals after decarboxylation, then list unordered radical pairs; reversing a cross-pair does not create another alkane.

**85. The 'Spin only' Magnetic moment for $[Ni(NH_3)_6]^{2+}$ is _____ $\times 10^{-1}$ BM. (given = Atomic number of Ni : 28)**

[4 marks]

**Solution:**

**Concept:**

- The spin-only magnetic moment depends on the number $n$ of unpaired electrons: $\mu = \sqrt{n(n+2)}\,\text{BM}$, where BM is the Bohr magneton.
- Find the metal ion electron configuration before filling the octahedral $t_{2g}$ and $e_g$ orbitals. Ligand

lone pairs form coordinate bonds and are not themselves unpaired metal electrons.

**Step 1:** Find the oxidation state and electron count Each ammonia ligand, $NH_3$, is neutral. Therefore the charge $+2$ on $[Ni(NH_3)_6]^{2+}$ is the oxidation state of nickel. The neutral atom, atomic number 28, has configuration $[Ar]3d^8 4s^2$. Remove the two $4s$ electrons first to obtain $Ni^{2+}$ with configuration $[Ar]3d^8$. Thus eight metal $d$ electrons must be distributed; using neutral nickel would give the wrong starting configuration.

**Step 2:** Fill the octahedral orbitals and count unpaired electrons Six ligands give an octahedral complex. The lower three $t_{2g}$ orbitals contain six electrons as three pairs, and the higher two $e_g$ orbitals contain the remaining two electrons separately according to Hund's rule. Hence the occupancy is $t_{2g}^6 e_g^2$, with $n = 2$. An octahedral $d^8$ configuration retains these two unpaired electrons; it is not the square-planar arrangement that can make some nickel complexes diamagnetic.

**Step 3:** Calculate the moment and the requested coefficient Insert $n = 2$ into the spin-only expression:

$$\mu = \sqrt{2(2+2)} = \sqrt{8} = 2.828\,\text{BM}.$$

If the blank is $x$, then $x \times 10^{-1} = 2.828$, so $x = 28.28$. The numerical entry is the nearest integer, 28; the moment itself is approximately $2.83\,\text{BM}$, not 28 BM.

**Step 4:** Conclusion The spin-only moment is $2.828\,\text{BM} = 28.28 \times 10^{-1}\,\text{BM}$. The requested integer entry is $\boxed{28}$.

**Quick Tip to Solve:** For octahedral nickel(II), count two unpaired electrons in $d^8$, then convert the magnetic moment to the power of ten used in the blank.

**86. One Faraday of electricity liberates $x \times 10^{-1}$ gram atom of copper from copper sulphate. $x$ is _____.** [4 marks]

**Solution:**

**Concept:**

- One Faraday is the charge carried by one mole of electrons: $Q = F$ corresponds to $n(e^-) = 1\,\text{mol}$.
- A gram atom is one mole of atoms. The number of moles of metal deposited follows the electron-to-metal ratio in the cathode half-reaction, not simply the number of Faradays.

**Step 1:** Write the cathode reaction Copper sulphate supplies $Cu^{2+}$ ions in solution. Their reduction at the cathode is

$$Cu^{2+} + 2\,e^- \longrightarrow Cu\,(s).$$

The charge on copper is $+2$, so each copper ion requires two electrons to become a neutral copper atom. Sulphate is the counter-ion and does not change the number of electrons needed per deposited copper atom. The relevant stoichiometric ratio is two moles of electrons for one mole of copper.

**Step 2:** Convert one Faraday into copper gram atoms For a charge $Q$, the electron amount is $Q/F$ moles. Therefore

$$n(\text{Cu}) = \frac{Q}{2F} = \frac{F}{2F} = \frac{1}{2}\,\text{mol}.$$

Since a gram atom is a mole of atoms, the deposited copper is $0.5$ gram atom. This is equivalent to half the molar mass in grams, but the question asks for gram atoms, so no atomic-mass value is required. One Faraday deposits one chemical equivalent of copper, which here is half a gram atom.

**Step 3:** Express the amount in the required form The question gives the deposited amount as $x \times 10^{-1}$ gram atom. Equating it to the calculated amount gives

$$x \times 10^{-1} = 0.5, \qquad x = \frac{0.5}{0.1} = 5.$$

The factor of two belongs to the reduction stoichiometry; the factor of ten belongs only to the numerical format. Keeping these operations separate avoids reporting 0.5 in a blank that asks for its scaled coefficient.

**Step 4:** Conclusion One Faraday deposits $0.5 = 5 \times 10^{-1}$ gram atom of copper. Hence $\boxed{x = 5}$, a dimensionless coefficient.

**Quick Tip to Solve:** Write the ion charge first: one Faraday deposits $1/z$ mole of a metal ion with charge $+z$. Then apply the scale requested in the blank.

**87. Number of moles of methane required to produce $22$ g $CO_{2(g)}$ after combustion is $x \times 10^{-2}$ moles. The value of $x$ is _____.** [4 marks]

**Solution:**

**Concept:**

- Balance the combustion equation before converting masses into amounts: reaction coefficients give mole ratios, not mass ratios.
- The mole amount is $n = m/M$, where $m$ is the mass and $M$ is the molar mass. Carbon dioxide has molar mass $44\,\text{g mol}^{-1}$.

**Step 1:** Balance the complete combustion reaction Methane burns in sufficient oxygen to form carbon dioxide and water:

$$CH_4 + 2\,O_2 \longrightarrow CO_2 + 2\,H_2O.$$

There is one carbon atom in every methane molecule and one in every carbon dioxide molecule, so the coefficients of methane and carbon dioxide are both one. The four hydrogen atoms require two water molecules. Counting oxygen then requires two oxygen molecules. Consequently one mole of methane produces one mole of carbon dioxide in complete combustion.

**Step 2:** Calculate the amount of carbon dioxide and methane The mass of carbon dioxide is $22\,\text{g}$, and its molar mass is $12 + 2(16) = 44\,\text{g mol}^{-1}$. Thus

$$n(CO_2) = \frac{22}{44} = 0.50\,\text{mol}.$$

From the balanced 1:1 mole ratio,

$$n(CH_4) = n(CO_2) = 0.50\,\text{mol}.$$

The oxygen coefficient does not double the methane requirement: it describes the oxygen consumed. Nor should 22 g be divided by the molar mass of methane, because 22 g is the measured product mass.

**Step 3:** Find the coefficient of the stated power of ten The methane requirement is written in the question as $x \times 10^{-2}$ moles. Therefore

$$x \times 10^{-2} = 0.50, \qquad x = \frac{0.50}{0.01} = 50.$$

As a useful mass check, half a mole of methane is 8 g and consumes 32 g of oxygen, producing 22 g of carbon dioxide and 18 g of water. Both sides then have total mass 40 g, consistent with the balanced equation.

**Step 4:** Conclusion The required methane amount is $0.50\,\text{mol} = 50 \times 10^{-2}\,\text{mol}$. Therefore $\boxed{x = 50}$.

**Quick Tip to Solve:** For complete methane combustion, the methane-to-carbon-dioxide mole ratio is 1:1. Convert the carbon-dioxide mass to moles before applying the power of ten.

**88. Molar mass of the salt from $\text{NaBr}, \text{NaNO}_3, \text{KI}$ and $\text{CaF}_2$ which does not evolve coloured vapours on heating with concentrated $\text{H}_2\text{SO}_4$ is _____ $\text{g mol}^{-1}$.**

**(Molar mass in $\text{g mol}^{-1}$ : $\text{Na} : 23,\ \text{N} : 14,\ \text{K} : 39, \text{O} : 16, \text{Br} : 80, \text{I} : 127,\ \text{F} : 19, \text{Ca} : 40$)** [4 marks]

**Solution:**

**Concept:**

- Concentrated sulphuric acid can displace volatile acids from salts and can also act as an oxidising agent. Identify the vapour formed before using the supplied atomic masses.
- Bromine, iodine and nitrogen dioxide give coloured vapours. Hydrogen fluoride is colourless, so a fluoride salt can be distinguished from the other listed salts by this test.

**Step 1:** Compare the bromide, iodide and nitrate With $\text{NaBr}$, hydrogen bromide is first liberated and is then oxidised by concentrated $\text{H}_2\text{SO}_4$ to reddish-brown bromine vapour. Iodide is an even stronger reducing agent: KI leads to iodine, whose vapour is violet. For $\text{NaNO}_3$, the acid liberates nitric acid. On heating, nitric acid decomposes and produces brown nitrogen dioxide:

$$4\,\text{HNO}_3 \longrightarrow 4\,\text{NO}_2 + 2\,\text{H}_2\text{O} + \text{O}_2.$$

Thus these three choices can give coloured vapours under the specified heated conditions. Merely recognising an acid-displacement reaction is not enough; its subsequent heating or redox chemistry matters.

**Step 2:** Identify the salt giving colourless vapour Calcium fluoride reacts with concentrated sulphuric acid according to

$$\text{CaF}_2 + \text{H}_2\text{SO}_4 \longrightarrow \text{CaSO}_4 + 2\,\text{HF}.$$

The volatile product is hydrogen fluoride, a colourless gas. Fluoride is not oxidised to fluorine by concentrated sulphuric acid in this reaction. Consequently $\text{CaF}_2$ is the salt sought. Its behaviour

contrasts with bromide and iodide, whose hydrogen halides undergo oxidation and release coloured halogen vapours.

**Step 3:** Calculate its molar mass from the supplied values One formula unit of $CaF_2$ contains one calcium atom and two fluorine atoms. Using only the supplied values,

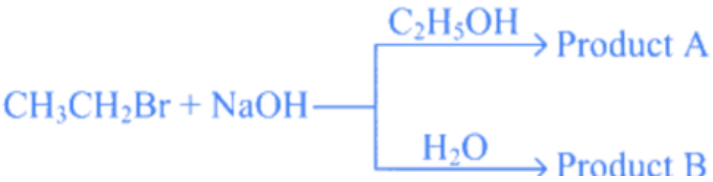
$$M(CaF_2) = M(Ca) + 2M(F) = 40 + 2(19) = 78\,g\,mol^{-1}.$$

The subscript two multiplies the fluorine contribution, not the calcium contribution. The question asks for the molar mass of the salt, so the mass of the evolved hydrogen fluoride is not the required result.

**Step 4:** Conclusion The salt is $CaF_2$, and its molar mass is $\boxed{78\,g\,mol^{-1}}$. The numerical entry is 78.

**Quick Tip to Solve:** Identify the gas before calculating a molar mass: bromide and iodide give coloured halogens, while fluoride gives colourless hydrogen fluoride.

**89. The total number of hydrogen atoms in product A and product B is _____.** [4 marks]




**Solution:**

**Concept:**

- The solvent changes the main reaction of an alkyl halide with alkali: alcoholic alkali favours elimination, while aqueous alkali favours nucleophilic substitution.
- Count hydrogen atoms from the molecular formula of each organic product. The question asks for the sum for products A and B, not for hydrogen in inorganic by-products or the solvent.

**Step 1:** Identify product A with alcoholic sodium hydroxide The starting haloalkane is bromoethane, $CH_3CH_2Br$. In the upper route the reagent is sodium hydroxide in ethanol. Alcoholic alkali removes a beta-hydrogen from the carbon adjacent to the carbon bearing bromine, while bromide leaves, so a carbon-carbon double bond forms. The elimination equation is

$$CH_3CH_2Br + NaOH \longrightarrow CH_2{=}CH_2 + NaBr + H_2O.$$

Thus product A is ethene, $C_2H_4$. Each of its two carbon atoms bears two hydrogen atoms, giving four hydrogen atoms per molecule. The ethanol written above the arrow is the solvent, not a fragment added to the organic product.

**Step 2:** Identify product B with aqueous sodium hydroxide In the lower route water is the solvent. Hydroxide acts as a nucleophile and replaces the bromine atom of the primary haloalkane:

$$CH_3CH_2Br + NaOH \longrightarrow CH_3CH_2OH + NaBr.$$

Product B is ethanol, $C_2H_6O$. The methyl group contributes three hydrogens, the methylene group contributes two, and the hydroxyl group contributes one. Hence there are $3 + 2 + 1 = 6$ hydrogen atoms in one molecule of B. Omitting the hydroxyl hydrogen would incorrectly give five.

**Step 3:** Add the hydrogen counts of the two products The required total refers to one molecule of each named organic product. Therefore

$$N_H(A) + N_H(B) = 4 + 6 = 10.$$

No extra hydrogen is counted from water or sodium hydroxide, because those are a by-product or reagent rather than the labelled products. The carbon skeleton remains two carbons in both routes, but elimination reduces the organic hydrogen count compared with substitution.

**Step 4:** Conclusion Product A is ethene with 4 hydrogen atoms and product B is ethanol with 6. Their total is $\boxed{10}$ hydrogen atoms.

**Quick Tip to Solve:** Check the solvent over each arrow first: alcoholic alkali gives an alkene, aqueous alkali gives an alcohol. Include the alcohol's O–H hydrogen in the count.

**90. The number of species from the following in which the central atom uses $\mathrm{sp}^3$ hybrid orbitals in its bonding is _____.**

$$NH_3, SO_2, SiO_2, BeCl_2, CO_2, H_2O, CH_4, BF_3$$

[4 marks]

**Solution:**

**Concept:**

- For a discrete molecular species, count electron domains around the central atom. Four domains correspond to $sp^3$, three to $sp^2$, and two to $sp$ hybridisation in the usual model.
- A multiple bond is one domain for determining hybridisation. Lone pairs also count as domains, even though they do not form bonds. The physical form of silica needs care because an extended network differs from a discrete molecule.

**Step 1:** Count the three tetrahedral electron-domain arrangements In $NH_3$, nitrogen has three sigma bonds and one lone pair: four domains, so it uses $sp^3$ orbitals. In $H_2O$, oxygen has two sigma bonds and two lone pairs, again giving four domains and $sp^3$ hybridisation. In $CH_4$, carbon has four sigma bonds and no lone pair, so its four bonding orbitals are $sp^3$. The molecular shapes differ (pyramidal, bent and tetrahedral), but their electron-domain counts are all four.

**Step 2:** Exclude the other molecular species For $SO_2$, sulfur has two bond directions and a lone pair in the usual molecular model, giving three domains and $sp^2$ hybridisation. Monomeric $BeCl_2$ and $CO_2$ are linear with two bond directions, so their central atoms use $sp$. Boron in $BF_3$ has three sigma bonds and no lone pair, giving trigonal planar $sp^2$ hybridisation. Treating the listed $SiO_2$ as the discrete molecular species O=Si=O likewise gives linear $sp$, so it is not counted under this molecular convention.

**Step 3:** Distinguish molecular silica from its network solid The formula $SiO_2$ also describes solid silica, in which each silicon is tetrahedrally bonded to four oxygen atoms in an extended network; that solid-state description is commonly associated with $sp^3$ silicon. It is different from the isolated molecular species used in the list's molecular hybridisation count. Under the discrete-species interpretation, the qualifying entries are exactly $NH_3$, $H_2O$ and $CH_4$, a total of three.

**Step 4:** Conclusion Using the molecular-species convention for the list, the $sp^3$ species are $NH_3$, $H_2O$ and $CH_4$. The requested count is $\boxed{3}$.

**Quick Tip to Solve:** Count sigma-bond directions plus lone pairs, treating a double bond as one domain. For silica, distinguish an isolated molecule from the tetrahedral network solid.