JEE Main 2024 Paper 1 (B.E./B.Tech) Jan 30 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 1 (Jan-Feb), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 30 January 2024, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \frac{x}{(1+x^4)^{1/4}}$, and $g(x) = f(f(f(f(x))))$. Then, $18\int_0^{\sqrt{2\sqrt{5}}} x^2 g(x)dx$ is equal to [4 marks]

(A) 36

(B) 33

(C) 39

(D) 42

Correct Answer: (C) 39

Solution:

Concept:

- For $h_n = f \circ \cdots \circ f$, repeated application adds one copy of x^4 to the fourth-power denominator. The real fourth root is positive, so the sign is carried by the numerator.
- A substitution $u = 1 + 4x^4$ gives $du = 16x^3\,dx$. It converts the required integral to a power integral with directly computable endpoints.

Step 1: Find the fourfold composition First compute the fourth power of the inner function:

$$f(x)^4 = \frac{x^4}{1+x^4}.$$

Consequently,

$$f(f(x)) = \frac{x}{(1+2x^4)^{1/4}}.$$

More generally, if $h_n(x) = x/(1+nx^4)^{1/4}$, substitution into f gives $h_{n+1}(x) = x/(1+(n+1)x^4)^{1/4}$. Thus

$$g(x) = \frac{x}{(1+4x^4)^{1/4}}.$$

This calculation uses the composition four times, rather than multiplying the original function by four.

Step 2: Transform and evaluate the integral Put $a = \sqrt{2\sqrt{5}}$; then $a^4 = 20$. The integrand $x^2 g(x)$ is $x^3(1+4x^4)^{-1/4}$, so

$$\begin{aligned} 18\int_0^a x^2 g(x)\,dx &= \frac{18}{16}\int_1^{81} u^{-1/4}\,du \\ &= \frac{3}{2}\left(81^{3/4} - 1\right) \\ &= \frac{3}{2}(27-1) = 39. \end{aligned}$$

The value 81 is the transformed upper endpoint, not the value of the integral. The other proposed values 36, 33 and 42 do not equal this exact endpoint evaluation.

Step 3: Conclusion The required integral is 39. Hence, the correct option is (C).

Quick Tip to Solve: For this composition, track the fourth power: the coefficient of x^4 in the denominator counts the number of applications.

2. Let a and b be be two distinct positive real numbers. Let 11^{th} term of a GP, whose first term is a and third term is b, is equal to p^{th} term of another GP, whose first term is a and fifth term is b. Then p is equal to [4 marks]

(A) 20

(B) 24

(C) 21

(D) 25

Correct Answer: (C) 21

Solution:

Concept:

- The nth term of a GP with first term a and ratio r is ar^{n-1}. Positions in the sequence therefore correspond to exponents one less than the term number.
- Use the common endpoint ratio $b/a > 0$. Since $a \neq b$, this ratio is not 1, so equal powers with real exponents have equal exponents.

Step 1: Express the first target term Let the first GP have ratio r. Its third term gives $ar^2 = b$, hence $r^2 = b/a$. The eleventh term is

$$ar^{10} = a(r^2)^5 = a\left(\frac{b}{a}\right)^5.$$

This uses only an even power of r, so a possible negative real ratio produces the same eleventh term. There is no need to extract a square root and choose a sign.

Step 2: Match the second progression Let the second GP have ratio s. Its fifth term gives $s^4 = b/a$. For the positive ratio, its pth term equals

$$a\left(\frac{b}{a}\right)^{(p-1)/4}.$$

Equating the terms and using $b/a \neq 1$ gives

$$\frac{p-1}{4} = 5, \qquad p = 21.$$

The result also holds if the real ratio s is negative: $p - 1 = 20$ is even and $s^{20} = (s^4)^5$. More directly, the magnitude equation forces $p - 1 = 20$; the sign then agrees. The other positions 20, 24 and 25 have exponents 19, 23 and 24, whose magnitudes cannot give the fifth power of b/a.

Step 3: Conclusion The required term number is $p = 21$. Hence, the correct option is (C).

Quick Tip to Solve: Write every given term as a power of b/a; remember that the exponent of the ratio in the pth term is $p - 1$.

3. Let $y = f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $y = f(x)$ at $(1, f(1))$ and $(3, f(3))$ make angles $\pi/6$ and $\pi/4$, respectively with positive x-axis. If $27\int_1^3 \left((f'(t))^2 + 1\right) f''(t)dt = \alpha + \beta\sqrt{3}$ where α, β are integers, then the value of $\alpha + \beta$ equals [4 marks]

(A) 26

(B) -16

(C) 36

(D) -14

Correct Answer: (A) 26

Solution:

Concept:

- The derivative at a point is the slope of its tangent: $f'(x) = \tan\theta$, where θ is its inclination to the positive x-axis.
- The chain rule gives an exact antiderivative: $\frac{d}{dt}\left((f'(t))^3/3 + f'(t)\right) = ((f'(t))^2 + 1)f''(t)$.

Step 1: Convert the tangent data to endpoints The two given inclinations determine the derivative values, although the values of $f(1)$ and $f(3)$ themselves are not needed:

$$f'(1) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}, \qquad f'(3) = \tan\frac{\pi}{4} = 1.$$

The variable in the integral is t. Its inner function is $f'(t)$, and the accompanying factor $f''(t)\,dt$ is its differential. No explicit formula for f has to be reconstructed.

Step 2: Evaluate the exact endpoint difference Apply the antiderivative directly, which does not require f' to be monotone:

$$\begin{aligned} I &= 27\left[\frac{(f'(t))^3}{3} + f'(t)\right]_1^3 \\ &= 27\left[\frac{4}{3} - \left(\frac{1}{9\sqrt{3}} + \frac{1}{\sqrt{3}}\right)\right] \\ &= 36 - \frac{30}{\sqrt{3}} = 36 - 10\sqrt{3}. \end{aligned}$$

Therefore $\alpha = 36$ and $\beta = -10$. The requested quantity is their sum, not α alone or the value of the integral. Thus 36, -16 and -14 are ruled out by the coefficient comparison.

Step 3: Conclusion $\alpha + \beta = 36 - 10 = 26$. Hence, the correct option is (A).

Quick Tip to Solve: When an integral contains a polynomial in f' multiplied by f'', integrate with respect to f' and use the tangent slopes.

4. For $\alpha, \beta \in (0, \pi/2)$, let $3\sin(\alpha+\beta) = 2\sin(\alpha-\beta)$ and a real number k be such that $\tan\alpha = k\tan\beta$. Then, the value of k is equal to [4 marks]

(A) 5

(B) $-2/3$

(C) -5

(D) 2/3

Correct Answer: (C) -5

Solution:

Concept:

- The addition identities are $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$. Expanding both sides exposes the ratio of tangents.
- On $(0, \pi/2)$, both sine and cosine are positive. Division by $\cos\alpha\cos\beta$ is allowed, but positivity must also be checked after algebraic simplification.

Step 1: Expand and collect the two products Substitute the addition identities into the given relation:

$$3(\sin\alpha\cos\beta + \cos\alpha\sin\beta) = 2(\sin\alpha\cos\beta - \cos\alpha\sin\beta).$$

Move the right-hand side to the left and combine like products:

$$\sin\alpha\cos\beta + 5\cos\alpha\sin\beta = 0.$$

Dividing by the nonzero product of cosines gives

$$\tan\alpha + 5\tan\beta = 0.$$

Comparison with the requested relation formally gives $k = -5$. The coefficient is negative because the sine-difference term has a minus sign before its second product.

Step 2: Check the stated angle domain The printed interval makes both tangents strictly positive. Their sum $\tan\alpha + 5\tan\beta$ therefore cannot be zero. Accordingly no pair of angles satisfies every printed premise; the question contains a domain inconsistency. The formal coefficient obtained from its equation is nevertheless uniquely -5, the value selected by the official key. The positive choices 5 and $2/3$ do not follow from the expanded equation, while $-2/3$ has the wrong coefficient ratio. The interval conflict must not be concealed by claiming that actual acute angles realize the equation.

Step 3: Conclusion The formal algebraic value is $k = -5$, corresponding to option (C), as in the official key. However, no angles in the stated interval satisfy the premises; the printed question is inconsistent.

Quick Tip to Solve: After expanding sine sums and differences, check signs against the angle domain before interpreting the formal tangent ratio.

5. If z is a complex number, then the number of common roots of the equations $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$, is equal to [4 marks]

(A) 0

(B) 2

(C) 1

(D) 3

Correct Answer: (B) 2

Solution:

Concept:

- Factor the lower-degree polynomial first: every common root must be among its roots. A cubic can supply at most three distinct candidates.
- For a non-real cube root of unity ω, one has $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$. Powers can be reduced modulo 3.

Step 1: List all roots of the cubic Group its terms to factor completely:

$$z^3 + 2z^2 + 2z + 1 = (z + 1)(z^2 + z + 1).$$

Its roots are $-1, \omega, \omega^2$, where the last two are the distinct non-real cube roots of unity. These three candidates are all distinct, so counting which of them also annihilate the first polynomial gives the number of common roots. The real number 1 is not a root of the quadratic factor.

Step 2: Test the candidates in the high-degree equation At $z = -1$, the first polynomial has value

$$(-1)^{1985} + (-1)^{100} + 1 = -1 + 1 + 1 = 1,$$

so this candidate is excluded. For either non-real cube root z, reduce the powers using

$$1985 \equiv 2 \pmod 3, \qquad 100 \equiv 1 \pmod 3.$$

It follows that

$$z^{1985} + z^{100} + 1 = z^2 + z + 1 = 0.$$

Both non-real candidates therefore work. There can be no further common roots because all roots of the cubic have already been checked. Zero or one would omit a valid cube root, whereas three would incorrectly include -1.

Step 3: Conclusion There are exactly 2 common roots. Hence, the correct option is (B).

Quick Tip to Solve: For a common-root question with very large exponents, factor the smaller polynomial and reduce powers using the identities its roots satisfy.

6. Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be a function satisfying $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all $x, y, f(y) \neq 0$. If $f'(1) = 2024$, then [4 marks]

(A) $xf'(x) + 2024f(x) = 0$

(B) $xf'(x) - 2023f(x) = 0$

(C) $xf'(x) - 2024f(x) = 0$

(D) $xf'(x) + f(x) = 2024$

Correct Answer: (C) $xf'(x) - 2024f(x) = 0$

Solution:

Concept:

- The quotient functional equation gives $f(1) = 1$ by taking equal arguments at which f is nonzero. The nonzero derivative at 1 excludes the identically zero function.
- Fix a nonzero x and express nearby values as $f(x + h) = f(x)f(1 + h/x)$. A difference quotient then reduces differentiation at x to differentiation at 1.

Step 1: Normalize the function and obtain a local product rule There is a point y with $f(y) \neq 0$, since otherwise $f'(1)$ would be zero. Taking equal arguments in the given equation yields $f(1) = 1$. Differentiability at 1 implies continuity there, so f remains nonzero in a sufficiently small neighbourhood of 1. For a fixed $x \neq 0$ and sufficiently small h, use the denominator argument $1 + h/x$. Since $(x + h)/(1 + h/x) = x$, the functional equation gives

$$f(x + h) = f(x)f(1 + h/x).$$

This form is valid even before any differentiability at the chosen x has been established.

Step 2: Take the difference quotient Subtract $f(x)$, use $f(1) = 1$, and divide by h:

$$\frac{f(x + h) - f(x)}{h} = \frac{f(x)}{x}\frac{f(1 + h/x) - f(1)}{h/x}.$$

As $h \to 0$, the final quotient tends to $f'(1) = 2024$. Consequently

$$f'(x) = \frac{2024f(x)}{x}, \qquad xf'(x) - 2024f(x) = 0.$$

For a quick rejection of the other identities, substitute $x = 1$: options (A), (B) and (D) would respectively require $4048 = 0$, $1 = 0$, and $2025 = 2024$. Each contradicts the given derivative and $f(1) = 1$.

Step 3: Conclusion For every $x \neq 0$, $xf'(x) - 2024f(x) = 0$. Hence, the correct option is (C).

Quick Tip to Solve: To differentiate a quotient functional equation safely, write values near an arbitrary point in terms of values near 1 and take a difference quotient.

7. Let $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ **be a non-zero** 3×3 **matrix, where** $x \sin\theta = y \sin\left(\theta + \frac{2\pi}{3}\right) = z \sin\left(\theta + \frac{4\pi}{3}\right) \neq 0, \theta \in (0, 2\pi)$**. For a square matrix** M**, let trace** (M) **denote the sum of all the diagonal entries of** M**. Then, among the statements:**

(I) Trace $(R) = 0$

(II) If trace $(\text{adj}(\text{adj}(R)) = 0$**, then** R **has exactly one non-zero entry.** [4 marks]

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

Correct Answer: (D) Neither (I) nor (II) is true

Solution:

Concept:

- Write the common nonzero product as c. Then $x = c/\sin\theta$, with analogous expressions for y, z; in particular all three diagonal entries are nonzero.
- For three angles separated by $2\pi/3$, their sines sum to zero and the sum of their pairwise products is $-3/4$. For a 3×3 matrix, $\text{adj}(\text{adj}\, R) = (\det R)R$.

Step 1: Test the trace statement Put $u = \sin\theta$, $v = \sin(\theta + 2\pi/3)$, and $w = \sin(\theta + 4\pi/3)$. Addition identities give $u + v + w = 0$ and $u^2 + v^2 + w^2 = 3/2$. Hence

$$uv + vw + wu = -\frac{3}{4}.$$

All three sines and c are nonzero, so

$$\text{tr}\, R = c\left(\frac{1}{u} + \frac{1}{v} + \frac{1}{w}\right) = -\frac{3c}{4uvw} \neq 0.$$

Statement (I) is therefore false. Replacing the sum of reciprocal sines by the sum of the sines would incorrectly give zero.

Step 2: Examine the second condition carefully Since $xyz \neq 0$, the double-adjugate identity gives

$$\text{tr}(\text{adj}(\text{adj}\, R)) = xyz(x + y + z) \neq 0.$$

Thus no matrix satisfying the question's nonzero-product constraint can meet the antecedent in (II), and every admissible matrix has three nonzero entries, never exactly one. The official key takes (II) as an assertion of an attainable rank-one conclusion and marks it false, selecting (D). There is a logical qualification: read strictly as a material implication, (II) has an impossible antecedent and is vacuously true, which would instead make (B) defensible. A valid solution must preserve this distinction rather than invent a rank-one matrix satisfying the given constraint.

Step 3: Conclusion The official keyed answer is (D), with (I) false and the asserted rank-one situation in (II) impossible. Under strict material-implication logic, (II) is vacuously true, so (B) is also

defensible; the wording is logically ambiguous.

Quick Tip to Solve: Before inferring rank from a trace condition, check whether that trace can vanish under the original constraints; zero trace alone never forces rank one.

8. If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals

[4 marks]

(A) 8

(B) 14

(C) 29

(D) 6

Correct Answer: (B) 14

Solution:

Concept:

- The distance of (x, y) from $Ax + By + C = 0$ is $|Ax + By + C|/\sqrt{A^2 + B^2}$. Equating two such distances gives the pair of angle bisectors.
- After squaring the distance equation, multiply the entire equation by a common nonzero constant so that its x^2 coefficient agrees with the given normalized locus.

Step 1: Equate the perpendicular distances Both lines have normal-vector length $\sqrt{5}$. Thus the condition is

$$\frac{|x + 2y + 7|}{\sqrt{5}} = \frac{|2x - y + 8|}{\sqrt{5}}.$$

Squaring loses no points because both distances are nonnegative. Consequently,

$$(x + 2y + 7)^2 - (2x - y + 8)^2 = 0.$$

This is a pair of straight lines, although the given expression is written as a quadratic locus.

Step 2: Expand and normalize the locus Expansion gives

$$-3x^2 + 3y^2 + 8xy - 18x + 44y - 15 = 0.$$

Divide every term by -3, rather than comparing unnormalized coefficients:

$$x^2 - y^2 - \frac{8}{3}xy + 6x - \frac{44}{3}y + 5 = 0.$$

Comparing this with $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$, we get

$$h = -\frac{4}{3}, \qquad g = 3, \qquad f = -\frac{22}{3}, \qquad c = 5.$$

Step 3: Evaluate the required combination Substitute the coefficients with their signs:

$$g + c + h - f = 3 + 5 - \frac{4}{3} + \frac{22}{3} = 14.$$

In particular the subtraction of the negative f contributes positively. The other offered values $8, 29, 6$ do not equal this coefficient combination.

Step 4: Conclusion The required value is $\boxed{14}$. Hence, the correct option is (B).

Quick Tip to Solve: Normalize the coefficient of x^2 before comparing quadratic loci; also remember that the linear coefficients are $2g$ and $2f$.

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9. Let a and b be real constants such that the function f defined by $f(x) = \begin{cases} x^2 + 3x + a & , x \leq 1 \\ bx + 2 & , x > 1 \end{cases}$ be differentiable on $\mathbb{R}$. Then, the value of $\int_{-2}^{2} f(x)dx$ equals [4 marks]

(A) 21

(B) 19/6

(C) 17

(D) 15/6

Correct Answer: (C) 17

Solution:

Concept:

- A function differentiable at a joining point must first be continuous there, and its left and right derivatives must coincide.
- A definite integral of a piecewise function must be split at every point where the defining expression changes. A single point does not affect the integral value.

Step 1: Determine the constants at the joining point The only possible difficulty is at $x = 1$; away from it both branches are polynomials. Continuity requires

$$1 + 3 + a = b + 2, \qquad a = b - 2.$$

The left derivative is $2x + 3$, while the right derivative is b. Their equality at 1 yields

$$5 = b, \qquad a = 3.$$

These values satisfy both continuity and derivative matching, so they indeed make the function differentiable throughout the real line.

Step 2: Integrate on the correct intervals Use $x^2 + 3x + 3$ up to 1 and $5x + 2$ after 1:

$$I = \int_{-2}^{1} (x^2 + 3x + 3)\, dx + \int_{1}^{2} (5x + 2)\, dx.$$

The first contribution is

$$\left[\frac{x^3}{3}+\frac{3x^2}{2}+3x\right]_{-2}^{1}=\frac{29}{6}+\frac{8}{3}=\frac{15}{2}.$$

The second is

$$\left[\frac{5x^2}{2}+2x\right]_{1}^{2}=14-\frac{9}{2}=\frac{19}{2}.$$

Step 3: Add the two contributions Therefore $I = 15/2 + 19/2 = 17$. The value $19/6$ is not the second interval's contribution, and $15/6$ is not the first; neither fractional option includes the correctly evaluated antiderivatives. The value 21 likewise fails the direct integration check.

Step 4: Conclusion The integral equals $\boxed{17}$. Hence, the correct option is (C).

Quick Tip to Solve: At a piecewise joining point use derivative matching to find the slope, continuity to find the offset, and only then split and integrate.

10. Let $f:\mathbb{R}\to\mathbb{R}$ **be defined as** $f(x)=ae^{2x}+be^{x}+cx$**. If** $f(0)=-1, f'(\log_e 2)=21$ **and** $\int_0^{\log_e 4}(f(x)-cx)dx=\frac{39}{2}$**, then the value of** $|a+b+c|$ **equals** [4 marks]

(A) 16

(B) 12

(C) 8

(D) 10

Correct Answer: (C) 8

Solution:

Concept:

- The conditions on f, its derivative and its integral give three linear equations in the unknown constants a, b, c.
- Use $e^{\log_e k}=k$ and $\int e^{kx}\,dx=e^{kx}/k$. Subtracting cx in the integrand removes the polynomial term exactly.

Step 1: Use the function value and integral At zero, the linear term vanishes and both exponentials equal one, so

$$a+b=-1.$$

The integrand is $ae^{2x}+be^{x}$, giving

$$\left[\frac{a}{2}e^{2x}+be^{x}\right]_{0}^{\log_e 4}=\frac{15a}{2}+3b=\frac{39}{2}.$$

Multiplying by two gives $15a+6b=39$. Subtract six times $a+b=-1$ to obtain $9a=45$, hence

$$a=5,\qquad b=-6.$$

Step 2: Apply the derivative condition Differentiate before substituting the logarithm:

$$f'(x) = 2ae^{2x} + be^{x} + c.$$

At $x = \log_e 2$, the two exponential factors are 4 and 2, respectively. Therefore

$$8a + 2b + c = 21.$$

Using $a = 5, b = -6$, we have $40 - 12 + c = 21$, so $c = -7$. As a check, the integral condition becomes $75/2 - 18 = 39/2$, exactly as required.

Step 3: Take the absolute value at the end The sum itself is negative:

$$a + b + c = 5 - 6 - 7 = -8, \qquad |a + b + c| = 8.$$

Thus $16, 12, 10$ cannot be selected: each differs from the sum determined simultaneously by all three independent conditions. Taking absolute values of individual constants would not compute the expression asked.

Step 4: Conclusion We obtain $\boxed{|a + b + c| = 8}$. Hence, the correct option is (C).

Quick Tip to Solve: When an integral contains $f(x) - cx$, eliminate the linear term first; solve for the exponential coefficients before using the derivative condition.

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11. Let $L_1 : \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k}), \lambda \in \mathbb{R}$,

$$L_2 : \vec{r} = (\hat{j} - \hat{k}) + \mu(3\hat{i} + \hat{j} + p\hat{k}), \mu \in \mathbb{R}, \text{ and } L_3 : \vec{r} = \delta(\ell\hat{i} + m\hat{j} + n\hat{k}), \delta \in \mathbb{R}$$

be three lines such that L_1 **is perpendicular to** L_2 **and** L_3 **is perpendicular to both** L_1 **and** L_2**. Then, the point which lies on** L_3 **is** [4 marks]

(A) $(1, 7, -4)$

(B) $(1, -7, 4)$

(C) $(-1, 7, 4)$

(D) $(-, 1 - 7, 4)$

Correct Answer: (C) $(-1, 7, 4)$

Solution:

Concept:

- For lines written in vector parametric form, the coefficient of the parameter is the direction vector; the constant vector specifies a point and is not needed for direction perpendicularity.
- Two nonzero direction vectors are perpendicular when their dot product is zero. Their cross product supplies a direction perpendicular to both.

Step 1: Find the missing direction parameter The first two line directions are

$$\mathbf{u} = (1, -1, 2), \qquad \mathbf{v} = (3, 1, p).$$

Perpendicularity requires

$$\mathbf{u} \cdot \mathbf{v} = 3 - 1 + 2p = 0.$$

Thus $p = -1$ and $\mathbf{v} = (3, 1, -1)$. Both vectors are nonzero, and they are not parallel, so their common perpendicular direction is unique up to a nonzero scalar.

Step 2: Construct the direction of the third line Compute the cross product component by component:

$$\mathbf{u} \times \mathbf{v} = ((-1)(-1) - 2,\ 2 \cdot 3 - 1(-1),\ 1 - (-1)3) = (-1, 7, 4).$$

Since L_3 passes through the origin, its points are all scalar multiples of this vector. Taking the scalar equal to one gives the offered point $(-1, 7, 4)$. Its dot products with the first two directions are $-1 - 7 + 8 = 0$ and $-3 + 7 - 4 = 0$.

Step 3: Reject the other point choices For $(1, 7, -4)$, the dot product with $\mathbf{u}$ is $1 - 7 - 8 = -14$, so it fails. For $(1, -7, 4)$, that dot product is $1 + 7 + 8 = 16$, so it also fails. The last printed choice is malformed as a coordinate triple; it does not supply the verified common perpendicular point. Option (C) satisfies both equations directly, with no interpretation of that malformed distractor required.

Step 4: Conclusion The point on L_3 is $\boxed{(-1, 7, 4)}$. Hence, the correct option is (C).

Quick Tip to Solve: For a line through the origin perpendicular to two given directions, cross the directions and test the offered points by dot products.

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12. Let $\vec{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k}, \alpha, \beta \in \mathbb{R}$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{b}|^2 = 6$. If $\vec{a} \cdot \vec{b} = 3\sqrt{2}$, then the value of $\left(\alpha^2 + \beta^2\right)|\vec{a} \times \vec{b}|^2$ is equal to [4 marks]

(A) 85

(B) 90

(C) 75

(D) 95

Correct Answer: (B) 90

Solution:

Concept:

- The scalar product is $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$. Squaring it avoids carrying square roots through the norm calculation.
- The vector-product identity is $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$. Also $|\vec{a}|^2 = 1 + \alpha^2 + \beta^2$.

Step 1: Find the magnitude of the first vector The angle is $\pi/4$, so $\cos^2(\pi/4) = 1/2$. Square the

dot-product relation using the given numbers:

$$(3\sqrt{2})^2 = |\vec{a}|^2 \cdot 6 \cdot \frac{1}{2}.$$

Thus $18 = 3|\vec{a}|^2$, hence $|\vec{a}|^2 = 6$. Since its first Cartesian component is one,

$$\alpha^2 + \beta^2 = 6 - 1 = 5.$$

The individual values of α and β are unnecessary; only their squared sum is determined and requested.

Step 2: Evaluate the cross product magnitude Use the identity relating dot products and cross products:

$$|\vec{a} \times \vec{b}|^2 = 6 \cdot 6 - (3\sqrt{2})^2 = 36 - 18 = 18.$$

Equivalently, $\sin^2(\pi/4) = 1/2$ gives $6 \cdot 6/2 = 18$, independently confirming the same value. Both methods use the squared magnitude, not the magnitude itself, which is $3\sqrt{2}$.

Step 3: Form the requested product Multiplying the two quantities now gives

$$(\alpha^2 + \beta^2)|\vec{a} \times \vec{b}|^2 = 5 \cdot 18 = 90.$$

The other values $85, 75, 95$ are incompatible with the exact norm and cross-product identities. In particular replacing the squared cross-product magnitude by its square root would evaluate a different expression.

Step 4: Conclusion The required value is $\boxed{90}$. Hence, the correct option is (B).

Quick Tip to Solve: When only $\alpha^2 + \beta^2$ is needed, find the vector norm first instead of trying to determine the two components separately.

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13. Let P be a point on the hyperbola $H : \frac{x^2}{9} - \frac{y^2}{4} = 1$, in the first quadrant such that the area of triangle formed by P and the two foci of H is $2\sqrt{13}$. Then, the square of the distance of P from the origin is [4 marks]

(A) 26

(B) 22

(C) 20

(D) 18

Correct Answer: (B) 22

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, the foci are $(\pm c, 0)$, where $c^2 = a^2 + b^2$. Their separation is $2c$.
- Taking the focal segment as a triangle base, its altitude from (x, y) is $|y|$. The square of distance from the origin is $x^2 + y^2$, not the distance to either focus.

Step 1: Use the area to find the ordinate Here $a^2 = 9$ and $b^2 = 4$, so $c = \sqrt{13}$. Hence the distance between the two foci is $2\sqrt{13}$. Write $P = (x, y)$. Because the point is in the first quadrant, its height above the focal axis is $y > 0$. The triangle area is therefore

$$\frac{1}{2}(2\sqrt{13})y = \sqrt{13}\,y.$$

Equating it to $2\sqrt{13}$ gives $y = 2$. This uses the actual perpendicular height, regardless of where the foot of the perpendicular falls relative to the focal segment.

Step 2: Recover the abscissa from the hyperbola Substitute $y^2 = 4$ in the curve equation:

$$\frac{x^2}{9} - \frac{4}{4} = 1, \qquad \frac{x^2}{9} = 2.$$

Thus $x^2 = 18$, and the first-quadrant point has $x = 3\sqrt{2}$. Both its coordinate signs are consistent with the stated quadrant.

Step 3: Compute the squared distance The requested square of the distance is

$$OP^2 = x^2 + y^2 = 18 + 4 = 22.$$

The option 18 includes only the squared abscissa and omits the ordinate contribution. The alternatives 20 and 26 likewise do not equal the sum of the two squared coordinates fixed by the area and curve conditions.

Step 4: Conclusion The square of the distance from the origin is $\boxed{22}$. Hence, the correct option is (B).

Quick Tip to Solve: For a triangle whose base joins the foci of a standard horizontal hyperbola, the area immediately determines the ordinate through area $= c|y|$.

14. Let $f(x) = (x+3)^2(x-2)^3, x \in [-4, 4]$. If M and m are the maximum and minimum values of f, respectively in $[-4, 4]$, then the value of $M - m$ is [4 marks]

(A) 108

(B) 392

(C) 608

(D) 600

Correct Answer: (C) 608

Solution:

Concept:

- A polynomial is continuous on a closed bounded interval, so it attains its absolute maximum and minimum there. Compare endpoint values and every interior stationary value.
- The product rule and factorization give all zeros of the derivative, including repeated roots. A stationary point need not be an absolute extremum.

Step 1: Find all stationary candidates Differentiate $f(x) = (x+3)^2(x-2)^3$:

$$\begin{aligned} f'(x) &= 2(x+3)(x-2)^3 + 3(x+3)^2(x-2)^2 \\ &= (x+3)(x-2)^2[2(x-2) + 3(x+3)] \\ &= 5(x+3)(x-2)^2(x+1). \end{aligned}$$

The stationary candidates in $[-4, 4]$ are $-3, -1, 2$. Add both endpoints -4 and 4; omitting them would miss the global extremes in this problem.

Step 2: Evaluate the finite list of candidates Direct substitution gives

x	-4	-3	-1	2	4
$f(x)$	-216	0	-108	0	392

For instance, $f(-4) = (-1)^2(-6)^3 = -216$, while $f(4) = 7^2 \cdot 2^3 = 392$. The stationary value at -1 is $2^2(-3)^3 = -108$. Comparing the entire list proves $M = 392$ and $m = -216$.

Step 3: Subtract the minimum with its sign The range width is

$$M - m = 392 - (-216) = 608.$$

Option 392 is only the maximum, not the difference. The value 108 is the magnitude of an interior stationary value and does not describe the full interval. The value 600 fails the exact endpoint subtraction. A repeated derivative root at 2 adds a candidate but cannot replace endpoint checks.

Step 4: Conclusion The difference of the maximum and minimum is $\boxed{608}$. Hence, the correct option is (C).

Quick Tip to Solve: On a closed interval, make one candidate table containing derivative zeros and both endpoints, then subtract the smallest value from the largest.

15. Suppose $2-p, p, 2-\alpha, \alpha$ are the coefficients of four consecutive terms in the expansion of $(1+x)^n$. Then the value of $p^2 - \alpha^2 + 6\alpha + 2p$ equals [4 marks]

(A) 8

(B) 4

(C) 6

(D) 10

Correct Answer: No admissible finite binomial expansion has the stated four coefficients; the official key dropped this question.

Solution:

Concept:

- For a finite binomial expansion, the coefficients are positive integers: the coefficient of x^r is $\binom{n}{r}$, where n is a nonnegative integer and $0 \le r \le n$.
- Four consecutive terms require $n \ge 3$. Check that the given coefficient data are possible before

using their ratios.

Step 1: Test the first pair of coefficients The first two specified coefficients have sum $(2-p)+p=2$. Since each is a positive integer, both would have to be 1, giving $p=1$. Similarly, the last two coefficients sum to 2, forcing $\alpha=1$. Thus the premise would require four consecutive binomial coefficients all equal to 1.

Step 2: Compare with a finite binomial row For $n \geq 3$, only the two endpoint coefficients are 1; every interior coefficient is at least $n \geq 3$. Four consecutive terms cannot therefore all have coefficient 1. In fact any adjacent pair has sum at least $n+1 \geq 4$, whereas the stated pair has sum 2. The coefficient conditions have no admissible finite binomial expansion.

Step 3: Distinguish formal substitution from an admissible answer Substituting the forced but impossible values $p=\alpha=1$ into the requested expression produces $1-1+6+2=8$. This explains the tempting option (A), but numerical substitution cannot repair the contradictory coefficient premise. Options (B), (C) and (D) do not resolve that contradiction either.

Step 4: Conclusion The stated coefficient conditions are inconsistent, so no option is justified by an actual finite binomial expansion. The official answer key dropped this question and awarded bonus marks to all; 8 is only the formal value from the impossible coefficient data.

Quick Tip to Solve: For a binomial-coefficient question, first check positivity and integrality. A pair summing to 2 must be two endpoint-sized coefficients, which cannot be adjacent when four terms exist.

16. Consider the system of linear equations $x+y+z=5, x+2y+\lambda^2 z=9, x+3y+\lambda z=\mu$, where $\lambda, \mu \in \mathbb{R}$. Then, which of the following statement is NOT correct? [4 marks]

(A) System is consistent if $\lambda \neq 1$ and $\mu = 13$

(B) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$

(C) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

(D) System has infinite number of solutions if $\lambda = 1$ and $\mu = 13$

Correct Answer: (C) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

Solution:

Concept:

- Elementary row operations preserve the solution set of a linear system. A zero left side with a nonzero right side makes the system inconsistent.
- A nonzero coefficient determinant gives a unique solution; all parameter values that make it zero must be checked.

Step 1: Eliminate the first variable Subtract the first equation from the second, then eliminate x and y from the third using $R_3 - 2R_2 + R_1$. This gives

$$y+(\lambda^2-1)z=4, \qquad (1+\lambda-2\lambda^2)z=\mu-13.$$

The coefficient in the last equation factors as $-(2\lambda+1)(\lambda-1)$. Thus there are two singular values,

$\lambda = 1$ and $\lambda = -\frac{1}{2}$, not just one.

Step 2: Check each proposed statement If $\mu = 13$, the choice $(x, y, z) = (1, 4, 0)$ satisfies every equation for any λ, so (A) is true. At $\lambda = 1$, the last reduced equation is $0 = \mu - 13$; hence (B) is true when $\mu \neq 13$. If $\lambda = 1, \mu = 13$, only $y = 4$ and $x + z = 1$ remain, giving infinitely many solutions, so (D) is also true.

Step 3: Give a counterexample to the remaining claim Take $\lambda = -\frac{1}{2}$ and $\mu = 0$. These satisfy the conditions in (C), but the reduced equation becomes $0 = -13$, so there is no solution. A unique solution actually requires $\lambda \notin \{1, -\frac{1}{2}\}$, independently of the value of μ.

Step 4: Conclusion Statement (C) is NOT correct because it omits the singular value $\lambda = -\frac{1}{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Factor the parameter-dependent determinant completely before classifying consistency; one overlooked root can invalidate a uniqueness claim.

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17. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$. Then $|(\vec{b} \times \vec{a}) - \vec{b}|^2$ is equal to [4 marks]

(A) 1

(B) 3

(C) 5

(D) 4

Correct Answer: (C) 5

Solution:

Concept:

- The cross product $\vec{b} \times \vec{a}$ is perpendicular to $\vec{b}$, so their scalar product is zero.
- For any vectors $\vec{u}, \vec{v}$, $|\vec{u} - \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2\vec{u} \cdot \vec{v}$.

Step 1: Identify the perpendicular vectors Put $\vec{u} = \vec{b} \times \vec{a}$. The given magnitude is $|\vec{u}| = 2$, while $|\vec{b}| = 1$. By the defining direction of a cross product, $\vec{u} \cdot \vec{b} = 0$. No information about the component of $\vec{a}$ parallel to $\vec{b}$ is needed; that component contributes nothing to the cross product.

Step 2: Expand the squared magnitude Apply the scalar-product identity rather than subtracting the two magnitudes:

$$\begin{aligned} |(\vec{b} \times \vec{a}) - \vec{b}|^2 &= |\vec{u} - \vec{b}|^2 \\ &= |\vec{u}|^2 + |\vec{b}|^2 - 2\vec{u} \cdot \vec{b} \\ &= 2^2 + 1^2 - 0 = 5. \end{aligned}$$

The sign before $\vec{b}$ makes no numerical difference here because the mixed scalar product vanishes.

Step 3: Reject magnitude-only shortcuts Option (A), 1, could arise from squaring $2-1$, but magnitudes cannot be subtracted unless the vectors point along the same line in the appropriate direction. Here they are perpendicular. Option (B), 3, is their sum of magnitudes rather than the required squared norm, while (D), 4, ignores the unit-vector contribution. Only (C) includes both perpendicular squared lengths.

Step 4: Conclusion The required squared magnitude is 5. Hence, the correct option is (C).

Quick Tip to Solve: Whenever one vector is a cross product involving the other, check orthogonality first; the squared norm then reduces to a sum of squares.

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18. If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$, then the value of $5\beta - 4\alpha$ is equal to [4 marks]

(A) 9

(B) 12

(C) 11

(D) 10

Correct Answer: (B) 12

Solution:

Concept:

- A real logarithm requires a strictly positive argument, while a real inverse cosine requires its argument to lie in $[-1, 1]$.
- The domain of a sum is the intersection of the individual domains, including all denominator exclusions.

Step 1: Find the logarithm domain Factor $4x^2 + x - 3 = (4x - 3)(x + 1)$. The critical values are $-\frac{3}{2}, -1, \frac{3}{4}$. A sign chart gives

$$\frac{2x+3}{(4x-3)(x+1)} > 0 \quad \Longleftrightarrow \quad x \in (-\tfrac{3}{2}, -1) \cup (\tfrac{3}{4}, \infty).$$

All three critical values are excluded: the numerator zero gives logarithm of zero, and either denominator zero is undefined.

Step 2: Find the inverse-cosine domain For $x \neq -2$, impose $|(2x-1)/(x+2)| \leq 1$. Squaring and moving terms gives

$$(2x-1)^2 \leq (x+2)^2 \quad \Longleftrightarrow \quad (3x+1)(x-3) \leq 0.$$

Thus $x \in [-\frac{1}{3}, 3]$; the excluded denominator value -2 lies outside this interval anyway. Squaring is legitimate here because both squared sides are nonnegative.

Step 3: Intersect and evaluate The negative logarithm interval has no overlap with $[-\frac{1}{3}, 3]$. The common domain is therefore $(\frac{3}{4}, 3]$. Its left endpoint is open because the logarithm denominator vanishes there, while $x = 3$ is allowed and gives inverse-cosine argument 1. Hence $\alpha = \frac{3}{4}$, $\beta = 3$, and

$$5\beta - 4\alpha = 15 - 3 = 12.$$

Options 9, 11 and 10 do not follow from these endpoints.

Step 4: Conclusion The domain is $(3/4, 3]$ and the requested value is 12. Hence, the correct option is (B).

Quick Tip to Solve: For a logarithm plus an inverse trigonometric function, solve each domain separately and intersect only at the end; keep strict and non-strict endpoints distinct.

19. Bag A contains 3 white, 7 red balls and Bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn is white, is [4 marks]

(A) 1/4

(B) 1/3

(C) 3/10

(D) 1/9

Correct Answer: (B) 1/3

Solution:

Concept:

- Bayes' theorem reverses a conditional probability: $P(A \mid W) = P(A)P(W \mid A)/P(W)$.
- The total probability of a white ball sums the mutually exclusive routes through bags A and B, weighted by their selection probabilities.

Step 1: Write the two routes to a white ball Let A and B denote the selected bags and W the event of a white ball. Random selection gives $P(A) = P(B) = 1/2$. Bag A has 10 balls, of which 3 are white, while bag B has 5 balls, of which 3 are white. Therefore

$$P(W \mid A) = \frac{3}{10}, \qquad P(W \mid B) = \frac{3}{5}.$$

The two bags are equally likely initially, but white is more likely from B.

Step 2: Find the total evidence probability The selection events partition the sample space, so

$$P(W) = \frac{1}{2}\frac{3}{10} + \frac{1}{2}\frac{3}{5} = \frac{3}{20} + \frac{6}{20} = \frac{9}{20}.$$

The joint event of selecting A and then drawing white has probability $P(A \cap W) = 3/20$. This is not yet the desired probability because the observation restricts us to outcomes with a white ball.

Step 3: Condition on the observed white ball Normalize the A-and-white route by all white outcomes:

$$P(A \mid W) = \frac{3/20}{9/20} = \frac{1}{3}.$$

Option (C), $3/10$, reverses the conditioning and is merely $P(W \mid A)$. Options (A), $1/4$, and (D), $1/9$, do not equal the normalized route weight. A useful check is that B is twice as likely as A after white is observed, giving posterior odds $1 : 2$.

Step 4: Conclusion The probability that the white ball came from bag A is $\frac{1}{3}$. Hence, the correct option is (B).

Quick Tip to Solve: With equal prior bag probabilities, compare their white fractions directly: posterior weights are proportional to $3/10$ and $3/5$, hence $1 : 2$.

20. Let $A(\alpha, 0)$ and $B(0, \beta)$ be the points on the line $5x + 7y = 50$. Let the point P divide the line segment AB internally in the ratio $7 : 3$. Let $3x - 25 = 0$ be a directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S. If from S, the perpendicular on the x-axis passes through P, then the length of the latus rectum of E is equal to, [4 marks]

(A) $\frac{25}{3}$

(B) $\frac{25}{9}$

(C) $\frac{32}{5}$

(D) $\frac{32}{9}$

Correct Answer: (C) $\frac{32}{5}$

Solution:

Concept:

- For a horizontal ellipse, the focus is $(c, 0)$, the corresponding directrix is $x = a^2/c$, and $b^2 = a^2 - c^2$.
- The latus rectum has length $2b^2/a$. An internal division $AP : PB = m : n$ gives $P = (nA + mB)/(m + n)$.

Step 1: Find the intercepts and division point Setting $y = 0$ and $x = 0$ in the given line yields $A = (10, 0)$ and $B = (0, 50/7)$. Since $AP : PB = 7 : 3$,

$$P = \left(\frac{3(10) + 7(0)}{10}, \frac{3(0) + 7(50/7)}{10}\right) = (3, 5).$$

The weights are opposite the endpoints; using the weight 7 on A would reverse the specified ratio.

Step 2: Recover the ellipse parameters A perpendicular from the focus to the x-axis is a vertical line. Since it passes through P, the focus has x-coordinate 3, giving $S = (3, 0)$ and $c = 3$. The given vertical directrix corresponds to a horizontal major axis. Its equation $x = 25/3$ gives

$$\frac{a^2}{c} = \frac{25}{3} \quad \Longrightarrow \quad a^2 = 25, \quad a = 5.$$

Thus $b^2 = 25 - 9 = 16$, which is positive and smaller than a^2, as required.

Step 3: Calculate the full latus rectum Substitute the semiaxis values into the full-length formula:

$$L = \frac{2b^2}{a} = \frac{2(16)}{5} = \frac{32}{5}.$$

The number $25/3$ in (A) is the directrix coordinate, not a chord length. Option (B) confuses that coordinate with its ratio to the focal distance, and (D) uses 9 instead of the semimajor axis 5 in the latus-rectum calculation.

Step 4: Conclusion The length of the latus rectum is $\frac{32}{5}$. Hence, the correct option is (C).

Quick Tip to Solve: The focus and its matching directrix give $a^2 = c \times$ (directrix coordinate); then obtain b^2 by subtracting c^2.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections : A, B and C. A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is _____.

[4 marks]

Solution:

Concept:

- Selecting questions is an unordered choice, counted by binomial coefficients rather than permutations.
- Fixing how many questions are selected from each section creates disjoint cases; multiply choices within a case and add across cases.

Step 1: List the admissible section counts Let a, b, c be the numbers selected from sections A, B, C. They satisfy $a + b + c = 15$, $4 \le a \le 8$, and $4 \le b, c \le 6$. Since $b + c \ge 8$, one must have $a \le 7$, so $a = 8$ contributes no case. For each valid triple the number of selections is

$$\binom{8}{a}\binom{6}{b}\binom{6}{c}.$$

A selected set determines its own triple uniquely, so this counting does not duplicate selections.

Step 2: Evaluate the cases by the number from A Using the symmetry between B and C, the grouped contributions are

$$\begin{aligned}
a = 4: &\quad \binom{8}{4}\left[2\binom{6}{5}\binom{6}{6}\right] = 840,\\
a = 5: &\quad \binom{8}{5}\left[2\binom{6}{4}\binom{6}{6} + \binom{6}{5}^2\right] = 3696,\\
a = 6: &\quad \binom{8}{6}\left[2\binom{6}{4}\binom{6}{5}\right] = 5040,\\
a = 7: &\quad \binom{8}{7}\binom{6}{4}^2 = 1800.
\end{aligned}$$

The doubled terms correspond to interchanging unequal counts from B and C; equal counts are included once.

Step 3: Add the mutually exclusive totals The required count is

$$840 + 3696 + 5040 + 1800 = 11376.$$

For example, when $a = 4$, the remaining 11 questions must be divided as $(b, c) = (5, 6)$ or $(6, 5)$; when $a = 7$, only $(4, 4)$ is possible. These endpoint cases confirm that the lower bound of four in every section has been enforced rather than merely choosing any 15 of the 20 questions.

Step 4: Conclusion The total number of allowed selections is 11376 ways.

Quick Tip to Solve: Group the sum by the section with a different size. Here A has eight questions, while the two six-question sections provide an interchange symmetry that halves the case listing.

22. Let $Y = Y(X)$ be a curve lying in the first quadrant such that the area enclosed by the line $Y - y = Y'(x)(X - x)$ and the co-ordinate axes, where (x, y) is any point on the curve, is always $\frac{-y^2}{2Y'(x)} + 1, Y'(x) \neq 0$. If $Y(1) = 1$, then $12Y(2)$ equals _____. [4 marks]

Solution:

Concept:

- For slope $p = y\prime \neq 0$, the tangent has intercepts $a = x - y/p$ and $b = y - xp$. The geometric triangular area is $|ab|/2 = (y - xp)^2/(2|p|)$.
- A signed intercept product cannot automatically be used as geometric area. The sign of the slope and the printed condition $p \neq 0$ must also hold at the initial point.

Step 1: Recover the formal calculation behind the key Using the negative-slope intercept-area formula formally gives

$$-\frac{(y - xp)^2}{2p} = -\frac{y^2}{2p} + 1.$$

After cancellation and division by $p \neq 0$, this becomes

$$p - \frac{2y}{x} = -\frac{2}{x^2}.$$

The integrating factor is x^{-2}, so

$$\frac{d}{dx}\left(\frac{y}{x^2}\right) = -\frac{2}{x^4}, \qquad y = Cx^2 + \frac{2}{3x}.$$

The initial value gives $C = 1/3$, and this formal expression yields $y(2) = 5/3$, hence $12y(2) = 20$. However, its derivative is zero at $x = 1$, contrary to the printed condition. At $x = 2$, its slope is $7/6$ and the printed area expression is $-4/21$, which is negative.

Step 2: Check whether positive slope resolves the issue At $(1, 1)$, a negative slope would force $p = 0$, so any admissible curve must start with positive slope. The actual area formula for $p > 0$ instead gives

$$x^2p^2 - 2(xy + 1)p + 2y^2 = 0.$$

Put $z = xy$. For a positive first-quadrant solution continued from $x = 1$, the nonzero derivative cannot change sign; thus $y \geq 1$, $z \geq 1$, and

$$xp = \frac{z + 1 \pm \sqrt{1 + 2z - z^2}}{x}.$$

Reality requires $z \leq 1 + \sqrt{2}$. Since $\sqrt{1 + 2z - z^2} \leq \sqrt{2}$, either branch satisfies

$$z\prime \geq \frac{2z + 1 - \sqrt{2}}{x}.$$

Integrating this inequality with $z(1) = 1$ yields

$$z(x) \geq \frac{3-\sqrt{2}}{2}x^2 + \frac{\sqrt{2}-1}{2}.$$

At $x = 2$, this lower bound is $(11 - 3\sqrt{2})/2 > 1 + \sqrt{2}$, contradicting the required reality condition. Therefore the geometric condition cannot define such a curve all the way to $x = 2$.

Step 3: Conclusion The official numerical key is $\boxed{20}$, obtained by the formal signed-area calculation. Literally, the geometric-area and nonzero-slope conditions do not admit the required curve through to $x = 2$, so $Y(2)$ is not well-defined under all printed conditions.

Quick Tip to Solve: For tangent-area problems, take the absolute value of the intercept product and check that the resulting curve obeys the stated slope condition.

23. The number of real solutions of the equation $x\left(x^2 + 3|x| + 5|x-1| + 6|x-2|\right) = 0$ is _____.

[4 marks]

Solution:

Concept:

- For real x, every squared term and every absolute-value term is nonnegative. A sum of nonnegative quantities vanishes only when each term vanishes.
- The zero-product principle states that $AB = 0$ implies $A = 0$ or $B = 0$. Apply it before splitting the real line into intervals.

Step 1: Separate the two factors Write the equation as $xF(x) = 0$, where

$$F(x) = x^2 + 3|x| + 5|x-1| + 6|x-2|.$$

The first factor gives $x = 0$, and this really is a root of the original equation. To find whether any further roots exist, we must decide whether $F(x)$ can be zero; there is no need to expand the absolute values piecewise.

Step 2: Exclude a zero of the second factor All four summands of $F(x)$ are nonnegative. If their sum were zero, then $x^2 = 0$ would force $x = 0$. However,

$$F(0) = 0 + 0 + 5 + 12 = 17 > 0.$$

This contradiction proves that $F(x) > 0$ for every real x. Equivalently, the absolute-value terms centred at 1 and 2 cannot vanish together. Thus the second factor contributes no roots.

Step 3: Conclusion The only real solution is $x = 0$. Therefore the required number of real solutions is $\boxed{1}$, a dimensionless count.

Quick Tip to Solve: Before dividing by x, retain x=0; then use nonnegativity to rule out the entire bracket at once.

24. Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x - \alpha)^2 + y^2 = 16$, where $\alpha \in (5, 9)$. Let the angle between the two radii (one to each circle) drawn from one of the intersection points of C_1 and C_2 be $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$. If the length of common chord of C_1 and C_2 is β, then the value of $(\alpha\beta)^2$ equals _____.

[4 marks]

Solution:

Concept:

- The centres and an intersection point form a triangle with side lengths equal to the two radii and the distance between the centres.
- The common chord is perpendicular to the line of centres. If its length is β, its half-length is the altitude of the centres-intersection triangle.

Step 1: Determine the distance between the centres Let $O_1 = (0, 0)$, $O_2 = (\alpha, 0)$, and let P be an intersection point. Then $O_1P = 5$, $O_2P = 4$ and $O_1O_2 = \alpha$. The given angle θ at P is the principal inverse sine, so it is acute. Hence

$$\sin\theta = \frac{\sqrt{63}}{8}, \qquad \cos\theta = \frac{1}{8}.$$

The cosine rule gives

$$\alpha^2 = 25 + 16 - 2(5)(4)\frac{1}{8} = 36.$$

Since $\alpha \in (5, 9)$, this means $\alpha = 6$, consistent with the stated range.

Step 2: Relate the triangle area to the common chord Both intersection points are symmetric about the line of centres. Thus the perpendicular distance of P from O_1O_2 is $\beta/2$. Compute the same triangle area using its included angle and using this altitude:

$$\frac{1}{2}(5)(4)\sin\theta = \frac{1}{2}\alpha\frac{\beta}{2}.$$

Consequently,

$$\alpha\beta = 40\sin\theta = 5\sqrt{63}.$$

This avoids computing the chord midpoint separately. Squaring yields

$$(\alpha\beta)^2 = 25 \cdot 63 = 1575.$$

Step 3: Conclusion The required value is $\boxed{1575}$ (in the corresponding fourth power of the coordinate-length unit).

Quick Tip to Solve: Equate the two triangle-area formulae to find the product of centre distance and chord length directly.

25. Let a line passing through the point $(-1, 2, 3)$ intersect the lines $L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2}$ at $M(\alpha, \beta, \gamma)$ and $L_2 : \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4}$ at $N(a, b, c)$. Then, the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals _____. [4 marks]

Solution:

Concept:

- A point on a symmetric-form line is found by setting all three ratios equal to one parameter.
- For three collinear points P, M, N, their displacement vectors satisfy $\overrightarrow{PM} = k\overrightarrow{PN}$ for one scalar k.

Step 1: Parameterise the two intersection points Let the common ratio on L_1 be t, and that on L_2 be s. Then

$$M = (1 + 3t,\ 2 + 2t,\ -1 - 2t),$$

$$N = (-2 - 3s,\ 2 - 2s,\ 1 + 4s).$$

Relative to $P = (-1, 2, 3)$, the displacement vectors are

$$\overrightarrow{PM} = (2 + 3t,\ 2t,\ -4 - 2t),$$

$$\overrightarrow{PN} = (-1 - 3s,\ -2s,\ -2 + 4s).$$

The point P lies on neither given line, so these vectors must be nonzero and parallel.

Step 2: Impose collinearity and solve Set $\overrightarrow{PM} = k\overrightarrow{PN}$. The second component gives $t = -ks$. Substitute this into the first component:

$$2 - 3ks = -k - 3ks \quad \Rightarrow \quad k = -2.$$

Therefore $t = 2s$. The third component now gives

$$-4 - 4s = 4 - 8s \quad \Rightarrow \quad s = 2, \quad t = 4.$$

Thus

$$M = (13, 10, -9), \qquad N = (-8, -2, 9).$$

As a check, $\overrightarrow{PM} = (14, 8, -12) = -2(-7, -4, 6) = -2\overrightarrow{PN}$.

Step 3: Evaluate the ratio of coordinate sums The sums are

$$\alpha + \beta + \gamma = 13 + 10 - 9 = 14,$$

$$a + b + c = -8 - 2 + 9 = -1.$$

The denominator is nonzero. Therefore

$$\frac{(\alpha + \beta + \gamma)^2}{(a + b + c)^2} = \frac{14^2}{(-1)^2} = 196.$$

Step 4: Conclusion The required dimensionless ratio is $\boxed{196}$.

Quick Tip to Solve: Use one proportionality constant for the displacement vectors; the shared y-coordinate makes the algebra especially short.

26. The variance σ^2 of the data

x_i	0	1	5	6	10	12	17
f_i	3	2	3	2	6	3	3

is _____. [4 marks]

Solution:

Concept:

- For frequencies f_i, let $N = \sum f_i$. The population mean is $\bar{x} = (\sum f_i x_i)/N$.
- The variance is $\sigma^2 = (\sum f_i x_i^2)/N - \bar{x}^2$. Frequencies must weight both moments; this is not an unweighted seven-value data set.

Step 1: Calculate the total frequency and first moment The frequencies count repeated observations. Their total is

$$N = 3 + 2 + 3 + 2 + 6 + 3 + 3 = 22.$$

Weight each data value by its frequency:

$$\sum f_i x_i = 0 + 2 + 15 + 12 + 60 + 36 + 51 = 176.$$

Therefore

$$\bar{x} = \frac{176}{22} = 8.$$

The zero value contributes nothing to the first moment, but its frequency still contributes to N; omitting it would change the mean.

Step 2: Compute the second moment and variance Square each value before multiplying by its frequency:

$$\begin{aligned}\sum f_i x_i^2 &= 3(0)^2 + 2(1)^2 + 3(5)^2 + 2(6)^2 \\ &\quad + 6(10)^2 + 3(12)^2 + 3(17)^2 \\ &= 0 + 2 + 75 + 72 + 600 + 432 + 867 = 2048.\end{aligned}$$

Thus

$$\sigma^2 = \frac{2048}{22} - 64 = \frac{320}{11} \approx 29.0909.$$

The numerical-entry value, to the nearest integer, is 29. Retaining the exact fraction until the last step prevents premature rounding of the mean-square term.

Step 3: Conclusion The exact variance is $320/11$ square units, approximately 29.0909. The required integer numerical entry is $\boxed{29}$.

Quick Tip to Solve: For frequency tables, compute N, the weighted sum and the weighted square sum first; round only the final variance.

27. The area of the region enclosed by the parabola $(y-2)^2 = x-1$, the line $x-2y+4=0$ and the positive coordinate axes is _____. [4 marks]

Solution:

Concept:

- For horizontal strips, write the boundaries as x in terms of y; area is $\int (x_{\text{right}} - x_{\text{left}})\, dy$.
- The positive axes contribute boundary segments in the first quadrant. Split the integral when the left boundary changes from the y-axis to the line.

Step 1: Locate the boundary points and choose strips The parabola is $x = (y-2)^2 + 1 = y^2 - 4y + 5$, and the line is $x = 2y - 4$. The parabola meets the positive x-axis at $(5, 0)$, while the line meets the positive y-axis at $(0, 2)$. Their intersections satisfy

$$y^2 - 4y + 5 = 2y - 4 \quad \Rightarrow \quad (y-3)^2 = 0.$$

They touch at $(2, 3)$. The enclosed boundary runs from the origin along the x-axis to $(5, 0)$, then along the parabola to $(2, 3)$, then along the line to $(0, 2)$, and down the y-axis to the origin.

Step 2: Integrate the widths in two intervals For $0 \le y \le 2$, the left boundary is $x = 0$, since the line lies to the left of the y-axis. For $2 \le y \le 3$, it is $x = 2y - 4$. The parabola is the right boundary throughout. Therefore

$$A = \int_0^2 (y^2 - 4y + 5)\, dy + \int_2^3 (y-3)^2\, dy.$$

The two contributions are

$$\left[\frac{y^3}{3} - 2y^2 + 5y\right]_0^2 = \frac{14}{3},$$

$$\left[\frac{(y-3)^3}{3}\right]_2^3 = \frac{1}{3}.$$

Adding gives $A = 14/3 + 1/3 = 5$. Both integrands are nonnegative on their respective intervals, confirming the boundary order.

Step 3: Conclusion The enclosed area is $\boxed{5}$ square units.

Quick Tip to Solve: Use horizontal strips here: the parabola is a single function x(y), and only the left boundary changes at y=2.

28. The number of symmetric relations defined on the set $\{1, 2, 3, 4\}$ which are not reflexive is _____. [4 marks]

Solution:

Concept:

- A relation on a four-element set is a subset of its sixteen ordered pairs. Symmetry forces an off-diagonal pair and its reverse to be selected together.
- Reflexivity requires all four diagonal pairs. To count symmetric relations that fail reflexivity, subtract

the reflexive symmetric relations from all symmetric relations.

Step 1: Count all symmetric relations There are four diagonal pairs $(1,1),(2,2),(3,3),(4,4)$. Each may independently be included or omitted without violating symmetry, giving 2^4 choices. The twelve off-diagonal ordered pairs form

$$\binom{4}{2} = 6$$

unordered pairs of distinct elements. For each pair $\{i,j\}$, either include both (i,j) and (j,i), or include neither. This gives 2^6 independent off-diagonal choices. The total number of symmetric relations is therefore

$$2^4 \cdot 2^6 = 2^{10} = 1024.$$

Step 2: Remove the reflexive symmetric relations For a reflexive relation all four diagonal pairs must be present, leaving just one diagonal choice. The six off-diagonal decisions remain independent, so the number that are both symmetric and reflexive is

$$1 \cdot 2^6 = 64.$$

Subtracting these from all symmetric relations gives

$$1024 - 64 = 960.$$

Equivalently, choose any diagonal subset except the full set, in $2^4 - 1 = 15$ ways, and combine it with the 64 off-diagonal choices. This gives $15 \cdot 64 = 960$ again. Non-reflexive requires at least one missing diagonal pair, not that every diagonal pair be absent.

Step 3: Conclusion The required number of symmetric relations that are not reflexive is $\boxed{960}$, a dimensionless count.

Quick Tip to Solve: Not reflexive means at least one missing diagonal pair; count 15 diagonal patterns rather than only the empty pattern.

---

29. Let $\alpha = \sum_{k=0}^{n} \left(\frac{({}^nC_k)^2}{k+1} \right)$ **and** $\beta = \sum_{k=0}^{n-1} \left(\frac{{}^nC_k \, {}^nC_{k+1}}{k+2} \right)$ **If** $5\alpha = 6\beta$**, then** n **equals _____.** [4 marks]

Solution:

Concept:

- Absorb the denominator using $\binom{n}{k}/(k+1) = \binom{n+1}{k+1}/(n+1)$. This converts the weighted sum into a convolution of binomial coefficients.
- Vandermonde's identity is $\sum_k \binom{a}{k}\binom{b}{r-k} = \binom{a+b}{r}$; use symmetry $\binom{b}{r} = \binom{b}{b-r}$ to put each sum in this form.

Step 1: Evaluate the first sum For every term, replace one of the two identical binomial factors

together with its denominator. Then use symmetry in the second factor:

$$\alpha = \frac{1}{n+1}\sum_{k=0}^{n}\binom{n}{k}\binom{n+1}{k+1} = \frac{1}{n+1}\sum_{k=0}^{n}\binom{n}{k}\binom{n+1}{n-k} = \frac{1}{n+1}\binom{2n+1}{n}.$$

The range includes every nonzero term of this convolution.

Step 2: Evaluate the second sum Here the denominator belongs naturally with the factor whose lower index is $k+1$. Thus

$$\beta = \frac{1}{n+1}\sum_{k=0}^{n-1}\binom{n}{k}\binom{n+1}{k+2} = \frac{1}{n+1}\binom{2n+1}{n-1}.$$

In the last step the complementary lower index is $n-1-k$, so the convolution selects index $n-1$.

Step 3: Use the given relation The common positive factor cancels. The ratio of adjacent binomial coefficients gives

$$\frac{\beta}{\alpha} = \frac{\binom{2n+1}{n-1}}{\binom{2n+1}{n}} = \frac{n}{n+2}.$$

Therefore $5\alpha = 6\beta$ implies $5(n+2) = 6n$, and hence $n = 10$. This positive integer lies in the domain of both finite sums.

Step 4: Conclusion The required value is $\boxed{n = 10}$.

Quick Tip to Solve: For a denominator one larger than a binomial lower index, increase both the upper and lower indices before applying Vandermonde.

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30. Let S_n be the sum to n-terms of an arithmetic progression $3, 7, 11$, If $40 < \left(\frac{6}{n(n+1)}\sum_{k=1}^{n} S_k\right) < 42$, then n equals _____. [4 marks]

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, the first k terms sum to $S_k = k[2a + (k-1)d]/2$.
- The identities $\sum_{k=1}^{n} k = n(n+1)/2$ and $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$ reduce the sum of these partial sums to a polynomial.

Step 1: Find each partial sum The progression $3, 7, 11, \ldots$ has $a = 3$ and $d = 4$. Substitute these values before summing over the partial sums:

$$S_k = \frac{k}{2}[6 + 4(k-1)] = k(2k+1) = 2k^2 + k.$$

Here S_k is itself a sum of the first k progression terms; it is not just the k-th term.

Step 2: Sum the partial sums Apply the standard sums separately to the square and linear terms:

$$\sum_{k=1}^{n} S_k = 2\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$
$$= \frac{n(n+1)(4n+5)}{6}.$$

Because n counts terms, it is a positive integer, and cancelling the factor $n(n+1)$ is valid.

Step 3: Solve the strict inequality The middle expression in the question consequently equals $4n+5$. Hence

$$40 < 4n+5 < 42 \quad \Longrightarrow \quad 35 < 4n < 37 \quad \Longrightarrow \quad 8.75 < n < 9.25.$$

The only integer in this open interval is 9. Direct substitution gives $4(9)+5 = 41$, which lies strictly between 40 and 42. Neighbouring integers 8 and 10 give 37 and 45 respectively, so neither is possible.

Step 4: Conclusion The number of terms is $\boxed{n = 9}$.

Quick Tip to Solve: First express the partial sum as a polynomial in its own index; only then sum the partial sums and apply the integer restriction.

Physics – Section A: Multiple Choice Questions (80 marks)

31. If 50 Vernier divisions are equal to 49 main scale divisions of a traveling microscope and one smallest reading of main scale is 0.5 mm, the Vernier constant of traveling microscope is [4 marks]

(A) 0.01 mm

(B) 0.01 cm

(C) 0.1 mm

(D) 0.1 cm

Correct Answer: (A) 0.01 mm

Solution:

Concept:

- The Vernier constant, or least count, of a direct Vernier is the difference between one main-scale division (MSD) and one Vernier-scale division (VSD). Both lengths must use the same unit.
- When N Vernier divisions coincide with $N-1$ main-scale divisions, $1\,\text{VSD} = (N-1)\text{MSD}/N$, so the least count is MSD/N.

Step 1: Calculate one Vernier division The stated coincidence involves 50 Vernier divisions and 49 main-scale divisions. One main-scale division is $0.5\,\text{mm}$, so the common length is $49 \times 0.5 = 24.5\,\text{mm}$. Therefore

$$1\,\text{VSD} = \frac{24.5}{50}\,\text{mm} = 0.49\,\text{mm}.$$

The Vernier division is slightly smaller than a main-scale division, as expected for this direct Vernier arrangement.

Step 2: Take the difference and compare units The smallest measurable difference is

$$\text{VC} = 1\,\text{MSD} - 1\,\text{VSD} = 0.50 - 0.49 = 0.01\,\text{mm}.$$

Equivalently, $0.5/50 = 0.01\,\text{mm}$. This difference, rather than the full length of a Vernier division, is the instrument resolution. Option (B), $0.01\,\text{cm}$, equals $0.1\,\text{mm}$, and so is ten times too large, just like option (C). Option (D), $0.1\,\text{cm}$, is $1\,\text{mm}$, a hundred times too large. Keeping the supplied main-scale unit throughout avoids these conversion errors.

Step 3: Conclusion The Vernier constant is $\boxed{0.01\,\text{mm}}$. Hence, the correct option is (A).

Quick Tip to Solve: For N VSD equal to $N-1$ MSD, divide one MSD by N; retain its original unit until the final comparison.

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32. An electron revolving in n^{th} Bohr orbit has magnetic moment μ_n. If $\mu_n \propto n^x$, the value of x is [4 marks]

(A) 2

(B) 0

(C) 3

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- An electron in a circular orbit produces a current of magnitude $I = e/\tau = ev/(2\pi r)$, where e is the magnitude of its charge and τ is the orbital period.
- The magnetic moment magnitude is current times loop area, $\mu = I\pi r^2 = evr/2$. Bohr quantisation fixes angular momentum as $m_e vr = n\hbar$.

Step 1: Relate magnetic moment to angular momentum During each revolution the same charge magnitude e passes any fixed point on the orbit. Its equivalent conventional current has magnitude $ev/(2\pi r)$; multiplying by the circular area gives

$$\mu_n = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2} = \frac{e}{2m_e}L_n.$$

For an electron the magnetic moment vector is opposite to its orbital angular momentum, but that direction does not change the dependence of its magnitude on n.

Step 2: Insert the Bohr angular momentum Use $L_n = m_e vr = n\hbar$, giving

$$\mu_n = \frac{e\hbar}{2m_e}n = \mu_B n,$$

where μ_B is the Bohr magneton, independent of the orbit number. Thus the exponent in $\mu_n \propto n^x$ is one. As an independent scaling check, the Bohr radius varies as $r_n \propto n^2$ while speed varies as

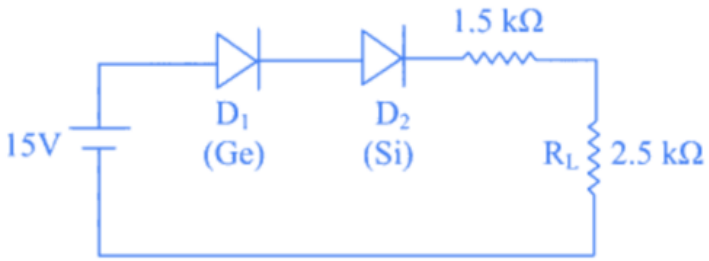
$v_n \propto n^{-1}$, so their product varies as n. Option (A) incorrectly retains the radius scaling alone. Option (C) ignores the inverse period dependence of current, and option (B) would require the orbital angular momentum to remain constant across all orbits, contrary to quantisation.

Step 3: Conclusion $\boxed{x = 1}$. Hence, the correct option is (D).

Quick Tip to Solve: Use $\mu = eL/(2m_e)$ directly: orbital magnetic moment has the same principal-quantum-number dependence as Bohr angular momentum.

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33. In the given circuit, the voltage across load resistance (R_L) is : [4 marks]



(A) 8.75 V

(B) 9.00 V

(C) 8.50 V

(D) 14.00 V

Correct Answer: (A) 8.75 V

Solution:

Concept:

- A forward-biased germanium diode has the usual threshold drop of approximately $0.3\,\text{V}$, while a silicon diode has approximately $0.7\,\text{V}$. The drops must be subtracted from the battery voltage.
- The two resistors are in series, so they carry the same current. The load voltage follows from Ohm's law, $V_L = IR_L$, after calculating the loop current.

Step 1: Find the voltage available to the resistors The positive terminal of the $15\,\text{V}$ source feeds the anodes in the direction allowed by both diode symbols, so the Ge and Si diodes are forward biased. Their total drop is

$$V_D = 0.3 + 0.7 = 1.0\,\text{V}.$$

Kirchhoff's loop law then leaves $15 - 1 = 14\,\text{V}$ across the series combination of $1.5\,\text{k}\Omega$ and $R_L = 2.5\,\text{k}\Omega$. This remaining voltage is not all across the load.

Step 2: Calculate the current and load voltage The total series resistance is $4.0\,\text{k}\Omega$, giving

$$I = \frac{14}{4.0 \times 10^3}\,\text{A} = 3.5\,\text{mA}.$$

Therefore

$$V_L = (3.5\,\text{mA})(2.5\,\text{k}\Omega) = 8.75\,\text{V}.$$

The other resistor drops $3.5 \times 1.5 = 5.25\,\text{V}$, and $8.75 + 5.25 + 1 = 15\,\text{V}$, checking the loop. Option (D) is the voltage across both resistors together. Options (B) and (C) do not satisfy the voltage divider with the stated forward drops and component values. Treating both diodes as ideal would also give a different load voltage and is inconsistent with identifying their materials.

Step 3: Conclusion The voltage across the load is $\boxed{8.75\,\text{V}}$. Hence, the correct option is (A).

Quick Tip to Solve: Subtract all forward diode drops first, then apply the resistor divider to the remaining supply voltage.

34. An alternating voltage $V(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50Ω. The time taken for the current to rise from half of the peak value to the peak value is: [4 marks]

(A) 7.2 ms

(B) 3.3 ms

(C) 5 ms

(D) 2.2 ms

Correct Answer: (B) 3.3 ms

Solution:

Concept:

- For a pure resistor, current is in phase with applied voltage: $i(t) = V(t)/R = I_0 \sin \omega t$, where $I_0 = V_0/R$.
- On the rising positive quarter-cycle, half the peak corresponds to phase $\pi/6$, while the positive peak corresponds to $\pi/2$. The time difference is phase difference divided by angular frequency.

Step 1: Write the current and identify its phases Using the supplied resistance,

$$i(t) = \frac{220}{50} \sin(100\pi t) = 4.4 \sin(100\pi t)\,\text{A}.$$

Thus $I_0 = 4.4\,\text{A}$ and $\omega = 100\pi\,\text{rad}\,\text{s}^{-1}$. At half the peak on the rising portion, $\sin\phi_1 = 1/2$, so $\phi_1 = \pi/6$. At the peak $\phi_2 = \pi/2$. The other half-current phase, $5\pi/6$, is on the falling portion and cannot be used for the requested rise.

Step 2: Convert the phase interval to time The relevant phase difference is $\pi/2 - \pi/6 = \pi/3$. Hence

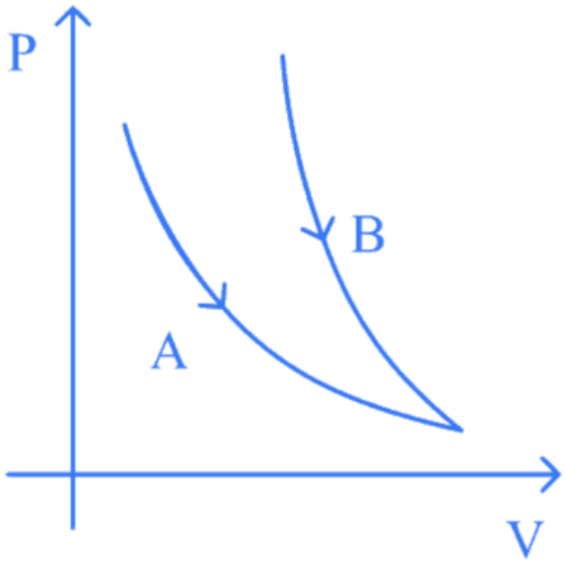
$$\Delta t = \frac{\Delta\phi}{\omega} = \frac{\pi/3}{100\pi}\,\text{s} = \frac{1}{300}\,\text{s} \simeq 3.3\,\text{ms}.$$

The period is $T = 2\pi/(100\pi) = 20\,\text{ms}$, so the interval is $T/6$. Option (C), $5\,\text{ms}$, is the full zero-to-peak quarter-period, rather than the half-peak-to-peak interval. Options (A) and (D) correspond to neither this phase change nor the given angular frequency. The amplitude cancels because the starting level is specified as a fraction of the peak.

Step 3: Conclusion The required time is $\boxed{3.3\,\text{ms}}$ approximately. Hence, the correct option is (B).

Quick Tip to Solve: A half-amplitude sine wave is at 30° on its rise; reaching 90° therefore takes one-sixth of a period.

35. Choose the correct statement for processes A & B shown in figure. [4 marks]



(A) $PV = k$ for process B and A.

(B) $\frac{P^{\gamma-1}}{T^{\gamma}} = k$ for process B and $T = k$ for process A.

(C) $\frac{T^{\gamma}}{P^{\gamma-1}} = k$ for process A and $PV = k$ for process B.

(D) $PV' = k$ for process B and $PV = k$ for process A.

Correct Answer: (B) $\frac{P^{\gamma-1}}{T^{\gamma}} = k$ for process B and $T = k$ for process A.

Solution:

Concept:

- For a fixed quantity of ideal gas undergoing an isothermal process, temperature is constant and $PV =$ constant. For a reversible adiabatic process, $PV^{\gamma} =$ constant, with $\gamma = C_P/C_V > 1$.
- At a common point in a pressure-volume plot, the adiabatic slope is $-\gamma P/V$, steeper in magnitude than the isothermal slope $-P/V$. This identifies which curve represents which process.

Step 1: Identify the two processes The two curves meet at the lower-right point of the plot. Curve B approaches this point with a steeper tangent than curve A. Accordingly B is the adiabatic process and A is the isothermal process. The arrows show expansion along both curves, but the distinction comes from their different pressure-volume dependence. Curve A obeys $T =$ constant and $PV =$ constant; curve B does not keep temperature constant.

Step 2: Convert the adiabatic relation to pressure and temperature Use the ideal-gas equation $V = \nu RT/P$ in $PV^{\gamma} = C$:

$$P\left(\frac{\nu RT}{P}\right)^{\gamma} = C \quad \Longrightarrow \quad \frac{T^{\gamma}}{P^{\gamma-1}} = C_1.$$

Its reciprocal is also constant, so $P^{\gamma-1}/T^{\gamma} = C_2$ for B. This, together with constant temperature for A, is exactly option (B). Option (A) wrongly treats both curves as isotherms. Option (C) reverses the assignments. The customary volume form for B is $PV^{\gamma} =$ constant, which is the adiabatic statement associated with the additionally accepted option (D).

Step 3: Conclusion Process A is isothermal and process B is adiabatic; option (B) gives the correct

pressure-temperature relation. Hence, the correct option is (B). The official final answer key accepts both B and D.

Quick Tip to Solve: At the same pressure-volume point, the steeper ideal-gas curve is adiabatic; its slope magnitude exceeds the isothermal slope by γ.

36. A block of mass m is placed on a surface having vertical crossection given by $y = x^2/4$. If coefficient of friction is 0.5, the maximum height above the ground at which block can be placed without slipping is: [4 marks]

(A) 1/2 m

(B) 1/3 m

(C) 1/6 m

(D) 1/4 m

Correct Answer: (D) 1/4 m

Solution:

Concept:

- The block tends to slide along the local tangent to the surface. Static equilibrium requires the tangential weight component to be no greater than the limiting friction: $mg\sin\theta \le \mu_s N$.
- The normal reaction is $N = mg\cos\theta$, so the condition is $|\tan\theta| \le \mu_s$. For a curve $y(x)$, its tangent slope is $\tan\theta = dy/dx$.

Step 1: Find the tangent inclination At any point of the given surface, the tangent behaves like a small inclined plane. Resolve the weight perpendicular and parallel to that tangent. There is no normal acceleration for a block at rest, so $N = mg\cos\theta$. The required friction is $mg\sin\theta$, directed up the slope. For $y = x^2/4$, differentiation gives

$$\frac{dy}{dx} = \frac{x}{2}.$$

On either side of the vertex, the magnitude of the slope is $|x|/2$.

Step 2: Apply the limiting-friction condition At the greatest possible height, static friction is at its maximum value. Thus

$$mg|\sin\theta| = \mu_s mg\cos\theta, \qquad \frac{|x|}{2} = 0.5.$$

Consequently $|x| = 1\,\text{m}$. Substitution in the surface equation gives

$$y_{\max} = \frac{1^2}{4} = \frac{1}{4}\,\text{m}.$$

Mass cancels because both the downslope weight and limiting friction scale with m.

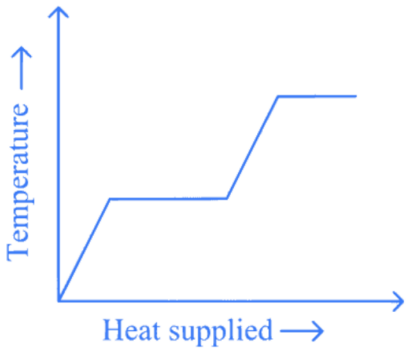
Step 3: Check the other proposed heights The surface gives $|\tan\theta| = \sqrt{y}$ when the stated coordinates are in metres. A height $1/2\,\text{m}$ or $1/3\,\text{m}$ needs a friction coefficient exceeding 0.5. The height $1/6\,\text{m}$ is stable but is below the limiting height, so it is not the maximum.

Step 4: Conclusion The maximum height is $\boxed{1/4\,\mathrm{m}}$. Hence, the correct option is (D).

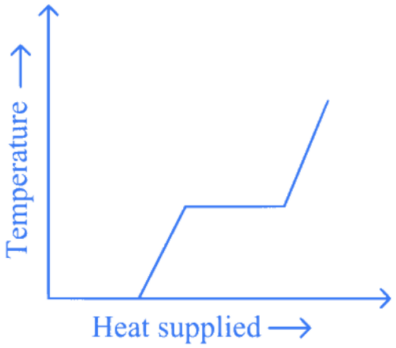
Quick Tip to Solve: For equilibrium on a curved surface, use the slope of its tangent at the block, then impose $|dy/dx| \leq \mu_s$.

37. A block of ice at $-10^{\circ}\mathrm{C}$ is slowly heated and converted to steam at $100^{\circ}\mathrm{C}$. Which of the following curves represent the phenomenon qualitatively: [4 marks]

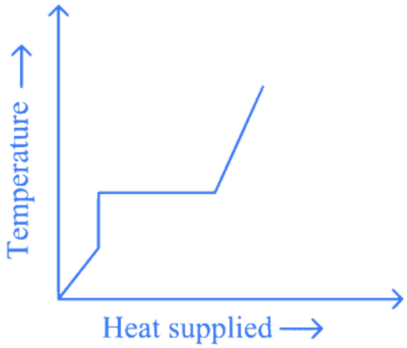
(A)



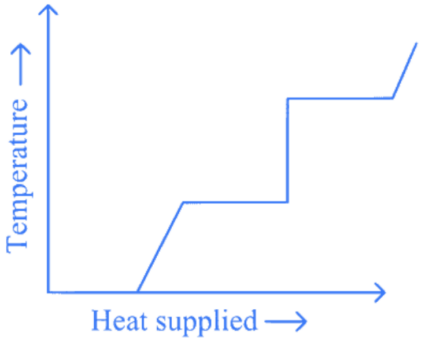
(B)



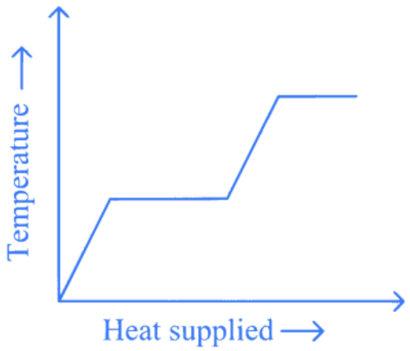
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- Within a single phase, supplied heat raises temperature according to $Q = mc\Delta T$; therefore temperature increases continuously with heat input.
- During melting or boiling at constant pressure, heat supplies latent energy, $Q = mL$, while temperature remains constant. Each phase transition produces a horizontal segment.

Step 1: Follow the ice up to melting The initial ice is below its melting point, at $-10^{\circ}\mathrm{C}$. Its first intake of heat raises its temperature to $0^{\circ}\mathrm{C}$. This is a rising segment, not a horizontal segment. At the melting point, additional energy breaks the solid structure and converts ice into water without increasing temperature. Thus the rising segment must be followed by a plateau. The plateau extends until all the ice has melted.

Step 2: Follow the water up to steam After melting, the water warms from $0^{\circ}\mathrm{C}$ to $100^{\circ}\mathrm{C}$, giving a second rising segment. At the boiling point, further heat provides the latent heat of vaporisation and the temperature again stays fixed while water becomes steam. The required sequence is therefore rising, horizontal, rising, horizontal. The process ends at steam at $100^{\circ}\mathrm{C}$; a final rising segment for superheated steam is not required.

Step 3: Compare all four curves Curve (A) has exactly the two rises separated and followed by the phase-change plateaus. Curve (B) starts with a plateau and then rises after its later plateau, so it omits the initial warming and final boiling behaviour required here. Curves (C) and (D) contain vertical temperature jumps at fixed heat input, which cannot describe slow heating with finite heat capacities. Their segment sequences also fail to show both phase changes correctly.

Step 4: Conclusion The qualitative heating curve is the one in option (A), with plateaus for melting and boiling. Hence, the correct option is (A).

Quick Tip to Solve: Count the phase changes first: ice to water and water to steam require two constant-temperature plateaus, with a rising segment before each plateau.

38. When a potential difference V is applied across a wire of resistance R, it dissipates energy at a rate W. If the wire is cut into two halves and these halves are connected mutually parallel across the same supply, the energy dissipation rate will become: [4 marks]

(A) 1/2W

(B) 4W

(C) 1/4W

(D) 2W

Correct Answer: (B) 4W

Solution:

Concept:

- For a resistor maintained at a fixed potential difference, its power is $P = V^2/R$. The original wire therefore dissipates $W = V^2/R$.
- A uniform wire has resistance $R = \rho L/A$. Cutting it into equal lengths leaves material and cross-sectional area unchanged, so each half has resistance $R/2$.

Step 1: Calculate the resistance of each piece The original resistance is proportional to the wire length. Each cut piece has half the original length, with the same resistivity and cross-sectional area. Therefore

$$R_1 = R_2 = \frac{R}{2}.$$

The cut does not halve the area. It is also essential to keep the supply voltage fixed, as stated: a constant-current source would lead to a different comparison.

Step 2: Combine the halves in parallel Both halves are connected directly across the same supply, so each sees the full voltage V. Their equivalent resistance satisfies

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R/2} + \frac{1}{R/2} = \frac{4}{R}, \qquad R_{\text{eq}} = \frac{R}{4}.$$

The new total dissipation is thus

$$P_{\text{new}} = \frac{V^2}{R/4} = 4\frac{V^2}{R} = 4W.$$

Step 3: Check by adding the branch powers Each branch individually dissipates $V^2/(R/2) = 2W$. With two identical branches, the combined power is $2W + 2W = 4W$, agreeing with the equivalent-resistance calculation. Option (D), $2W$, counts only one piece. Options (A) and (C) predict reduced power even though the equivalent resistance has decreased at fixed voltage, so both have the wrong trend.

Step 4: Conclusion The energy dissipation rate becomes $\boxed{4W}$. Hence, the correct option is (B).

Quick Tip to Solve: When a wire is cut and reconnected, calculate each piece resistance first, then the network resistance, before applying V^2/R.

39. Match List I with List II

	List I		List II
(A)	Gauss's law of magnetostatics	(I)	$\oint \vec{E} \cdot \vec{da} = \frac{1}{\varepsilon_0} \int \rho dV$
(B)	Faraday's law of electro magnetic induction	(II)	$\oint \vec{B} \cdot \vec{da} = 0$
(C)	Ampere's law	(III)	$\int \vec{E} \cdot \vec{dl} = \frac{-d}{dt} \int \vec{B} \cdot \vec{da}$
(D)	Gauss's law of electrostatics	(IV)	$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-IV, C-I, D-II

(B) A-IV, B-II, C-III, D-I

(C) A-II, B-III, C-IV, D-I

(D) A-I, B-III, C-IV, D-II

Correct Answer: (C) A-II, B-III, C-IV, D-I

Solution:

Concept:

- Gauss's laws describe flux through closed surfaces. Electric flux depends on enclosed charge, while net magnetic flux through a closed surface is zero.
- Faraday's law relates electric circulation to changing magnetic flux. Ampere's law relates magnetic circulation to the enclosed steady current.

Step 1: Identify the two flux laws Gauss's law of magnetostatics states

$$\oint \vec{B} \cdot d\vec{a} = 0.$$

Magnetic field lines form closed loops, so the net flux is zero; hence A matches II. Gauss's law of electrostatics gives

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enc}}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho \, dV.$$

Here ρ is charge density. Therefore D matches I.

Step 2: Identify the induction and circulation laws Faraday's induction law has a time derivative of magnetic flux and a minus sign expressing Lenz's law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}.$$

Thus B matches III. Ampere's law for steady current is

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I.$$

It follows that C matches IV.

Step 3: Select the complete matching The full correspondence is A-II, B-III, C-IV, D-I, which is precisely option (C). Option (A) assigns magnetic Gauss law to induction and fails the zero-flux

condition. Option (B) confuses the magnetic surface-flux law with Faraday's law and magnetic circulation. Option (D) swaps the two Gauss laws, giving electric charge as the source of net magnetic flux. Neither swap is consistent with the physical meanings.

Step 4: Conclusion The matching is A-II, B-III, C-IV, D-I. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish surface flux $d\vec{a}$ from loop circulation $d\vec{l}$; the changing-flux time derivative uniquely identifies Faraday's law.

40. In a nuclear fission reaction of an isotope of mass M, three similar daughter nuclei of same mass are formed. The speed of a daughter nuclei in terms of mass defect ΔM will be : [4 marks]

(A) $c\sqrt{\frac{3\Delta M}{M}}$

(B) $\frac{\Delta Mc^2}{3}$

(C) $c\sqrt{\frac{2\Delta M}{M}}$

(D) $\sqrt{\frac{2c\Delta M}{M}}$

Correct Answer: (C) $c\sqrt{\frac{2\Delta M}{M}}$

Solution:

Concept:

- A mass defect ΔM releases energy $Q = \Delta Mc^2$. In the stated fission model this becomes kinetic energy of the daughter nuclei.
- The standard nonrelativistic fission approximation takes each of the three equal daughter masses as $M/3$, since the mass defect is small compared with M. For equal speeds their total kinetic energy is $3 \times \frac{1}{2}(M/3)v^2$.

Step 1: Share the released energy The three similar daughters have the same mass and, in the symmetric breakup envisaged by the question, the same speed. Each receives one third of the released kinetic energy:

$$K_{\text{daughter}} = \frac{\Delta Mc^2}{3}.$$

This expression is an energy, not a speed, so it already rules out option (B). The energy is available because the products have a lower total rest mass than the original isotope.

Step 2: Relate the energy to the daughter speed Use $m_d \simeq M/3$ for one daughter in the usual small-mass-defect approximation. Equating its kinetic energy to the energy share gives

$$\frac{1}{2}\left(\frac{M}{3}\right)v^2 = \frac{\Delta Mc^2}{3}.$$

The factor of three cancels, leaving

$$v^2 = \frac{2\Delta Mc^2}{M}, \qquad v = c\sqrt{\frac{2\Delta M}{M}}.$$

Equivalently, summing the kinetic energies first gives $\frac{1}{2}Mv^2 = \Delta Mc^2$.

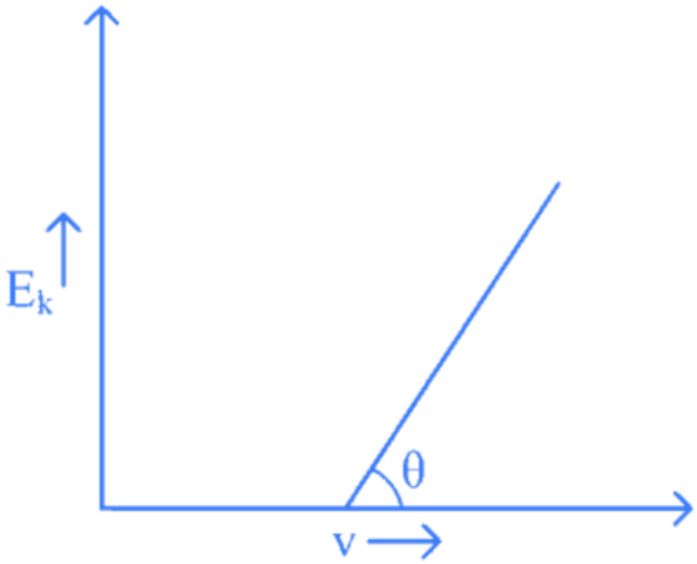
Step 3: Check the approximation and alternatives The speed tends to zero as the mass defect tends to zero, as it should. Option (A) contains an incorrect factor 3 inside the radical. Option (D) contains only one power of c inside the square root and is dimensionally not a speed. Retaining the tiny mass change in the daughter inertial masses would replace M by $M - \Delta M$ in this nonrelativistic model; the supplied options use the standard leading-order result.

Step 4: Conclusion In the standard small-mass-defect approximation, $\boxed{v = c\sqrt{2\Delta M/M}}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal fragments, the factors counting fragments cancel between the mass per fragment and the kinetic energy per fragment.

41. For the photoelectric effect, the maximum kinetic energy (E_k) of the photoelectrons is plotted against the frequency (v) of the incident photons as shown in figure. The slope of the graph gives

[4 marks]



(A) Planck's constant

(B) Work function of the metal

(C) Charge of electron

(D) Ratio of Planck's constant to electric charge

Correct Answer: (A) Planck's constant

Solution:

Concept:

- Einstein's photoelectric equation is $E_{k,\max} = h\nu - \phi$, where h is Planck's constant, ν is incident frequency and ϕ is the work function.
- A graph of maximum kinetic energy against frequency has the straight-line form $y = mx + b$. Its

gradient is $dE_{k,\max}/d\nu = h$.

Step 1: Identify the quantities on the axes The vertical axis in the figure is the maximum kinetic energy E_k, while the horizontal axis is photon frequency ν. They are not stopping potential and frequency. Photon energy is $h\nu$; the fixed energy ϕ is spent liberating an electron, and the remainder is its maximum kinetic energy. Therefore

$$E_k = h\nu - \phi.$$

The threshold frequency is the horizontal intercept $\nu_0 = \phi/h$, where the maximum kinetic energy is zero.

Step 2: Read the coefficient of frequency For any two points on the straight line, subtract their photoelectric equations:

$$E_{k,2} - E_{k,1} = h(\nu_2 - \nu_1).$$

Hence the physical slope is

$$\frac{\Delta E_k}{\Delta \nu} = h.$$

Its units are joule per hertz, or $\mathrm{J\,s}$, consistent with Planck's constant. The work function affects the intercept, not the gradient.

Step 3: Eliminate the other choices Option (B) is the magnitude of the extrapolated vertical intercept rather than the slope. The electron charge in option (C) does not appear in an energy-versus-frequency slope. The ratio h/e in option (D) would be the slope if the ordinate were stopping potential, because then $eV_s = h\nu - \phi$. The figure explicitly uses kinetic energy, so that conversion is not made.

Step 4: Conclusion The slope gives $\boxed{h}$, Planck's constant. Hence, the correct option is (A).

Quick Tip to Solve: Always read the ordinate: an energy-frequency graph has slope h, whereas a stopping-potential-frequency graph has slope h/e.

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42. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of emergent light is: [4 marks]

(A) $I_0/2$

(B) $I_0/8$

(C) $I_0/4$

(D) I_0

Correct Answer: (C) $I_0/4$

Solution:

Concept:

- An ideal polaroid transmits half the intensity of unpolarised incident light: $I_A = I_0/2$. The transmitted beam is plane polarised along its transmission axis.
- Malus's law applies to the second polaroid: $I_B = I_A \cos^2\theta$, where θ is the angle between the two

transmission axes.

Step 1: Calculate the intensity after the first polaroid The original light is unpolarised, so its electric-field orientations are randomly distributed in the transverse plane. The average squared projection on any chosen transmission direction is $1/2$. Consequently, an ideal first polaroid transmits

$$I_A = \frac{I_0}{2}.$$

This first reduction is required even though no angle is specified for polaroid A: the incident beam does not have one fixed polarisation direction.

Step 2: Apply Malus law to the second polaroid The light arriving at B is now linearly polarised along A. Since B is at 45° to A, its transmitted field amplitude is multiplied by $\cos 45^\circ$. Intensity is proportional to the square of field amplitude, so

$$I_B = I_A \cos^2 45^\circ = \frac{I_0}{2}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{4}.$$

There are two successive factors of $1/2$, one from the unpolarised beam and the other from the angle.

Step 3: Check the competing options Option (A), $I_0/2$, would follow if the two polaroids were parallel, or if one incorrectly omitted one of the two reductions. Option (B), $I_0/8$, adds an extra factor not present in Malus's law at 45°. Option (D), I_0, ignores the loss at the first polaroid and is impossible for an ideal absorbing polariser acting on unpolarised light.

Step 4: Conclusion The emergent intensity is $\boxed{I_0/4}$. Hence, the correct option is (C).

Quick Tip to Solve: For unpolarised light through two ideal polaroids, start with $I_0/2$, then multiply by $\cos^2\theta$.

43. If three moles of monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with two moles of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of adiabatic exponent γ for the mixture is [4 marks]

(A) 1.35

(B) 1.52

(C) 1.40

(D) 1.75

Correct Answer: (B) 1.52

Solution:

Concept:

- The adiabatic exponent is $\gamma = C_P/C_V$. Heat capacities, rather than their ratios, add when ideal gases are mixed without reaction.
- For one mole of an ideal gas, $C_P - C_V = R$, so $C_V = R/(\gamma - 1)$. Each component contributes its mole number times its molar heat capacity.

Step 1: Find the heat capacities of each gas For the monoatomic gas,

$$C_{V1} = \frac{R}{5/3 - 1} = \frac{3R}{2}, \qquad C_{P1} = \frac{5R}{2}.$$

For the diatomic gas,

$$C_{V2} = \frac{R}{7/5 - 1} = \frac{5R}{2}, \qquad C_{P2} = \frac{7R}{2}.$$

These capacities use the degrees of freedom already encoded in the given exponents; no additional temperature assumption is required.

Step 2: Add the extensive heat capacities and divide The mixture has three moles of the first gas and two of the second. Therefore,

$$C_{V,\text{mix}} = 3\left(\frac{3R}{2}\right) + 2\left(\frac{5R}{2}\right) = \frac{19R}{2}.$$

Similarly,

$$C_{P,\text{mix}} = 3\left(\frac{5R}{2}\right) + 2\left(\frac{7R}{2}\right) = \frac{29R}{2}.$$

Their difference is $5R$, consistent with the total five moles. Thus,

$$\gamma_{\text{mix}} = \frac{29R/2}{19R/2} = \frac{29}{19} = 1.5263\ldots.$$

Among the printed values, 1.52 is the matching approximation. Values 1.35 and 1.40 are too low; 1.75 even exceeds both component exponents. A direct arithmetic average of the exponents would not correctly weight the heat capacities.

Step 3: Conclusion The mixture has $\gamma = 29/19 \approx 1.53$, represented by the printed value 1.52. Hence, the correct option is (B).

Quick Tip to Solve: For mixtures, add nC_P and nC_V separately; never average the values of γ directly.

44. Three blocks A, B and C are pulled on a horizontal smooth surface by a force of 80 N as shown in figure

The tensions T_1 and T_2 in the string are respectively : [4 marks]



(A) 40N, 64N

(B) 60N, 80N

(C) 80N, 100N

(D) 88N, 96N

Correct Answer: (A) 40N, 64N

Solution:

Concept:

- On a smooth horizontal surface, friction is absent. All three blocks connected by taut light strings have the same horizontal acceleration.
- Apply $F_{\text{net}} = ma$ first to the complete system to eliminate internal tensions, then to a suitable block or subsystem to find each tension.

Step 1: Find the common acceleration The masses in the diagram are $m_A = 5\,\text{kg}$, $m_B = 3\,\text{kg}$ and $m_C = 2\,\text{kg}$. The force acts on block C to the right. For the whole system, both string tensions occur as internal action-reaction pairs and cancel. Thus,

$$a = \frac{80}{5+3+2} = 8\,\text{m s}^{-2}.$$

Weight and normal reaction balance vertically, so neither enters this horizontal equation.

Step 2: Isolate the blocks to determine both tensions For block A alone, the only horizontal force is T_1, directed to the right:

$$T_1 = 5a = 5(8) = 40\,\text{N}.$$

For the combined subsystem A+B, the tension between A and B is internal. The external horizontal pull on this subsystem is T_2, giving

$$T_2 = (5+3)a = 8(8) = 64\,\text{N}.$$

As a check, block C obeys

$$80 - T_2 = 2a \quad \Rightarrow \quad 80 - 64 = 2(8) = 16\,\text{N}.$$

The tensions differ because the second string accelerates both A and B, whereas the first accelerates A only. All other pairs in the options violate at least one of these force equations; a tension larger than the applied force is especially inconsistent with the rightward acceleration of C.

Step 3: Conclusion The tensions are $T_1 = 40\,\text{N}$ and $T_2 = 64\,\text{N}$, respectively. Hence, the correct option

is (A).

Quick Tip to Solve: For a string in a chain of blocks, multiply the common acceleration by the total mass behind that string.

45. A particle of charge '$-q$' and mass 'm' moves in a circle of radius 'r' around an infinitely long line charge of linear charge density '$+\lambda$'. Then time period will be given as :
(Consider k as Coulomb's constant) [4 marks]

(A) $T^2 = \frac{4\pi^2 m}{2k\lambda q} r^3$

(B) $T = \frac{1}{2\pi r}\sqrt{\frac{m}{2k\lambda q}}$

(C) $T = \frac{1}{2\pi}\sqrt{\frac{2k\lambda q}{m}}$

(D) $T = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$

Correct Answer: (D) $T = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$

Solution:

Concept:

- An infinite straight line of charge produces a field $E = 2k\lambda/r$, directed radially outward when $\lambda > 0$.
- The force on the negative charge is inward, so it can supply the centripetal force mv^2/r. The orbital period is circumference divided by speed: $T = 2\pi r/v$.

Step 1: Find the orbital speed from radial force balance At radius r, the magnitude of the electric force on the particle is

$$F = qE = \frac{2k\lambda q}{r}.$$

Here q denotes the positive magnitude of the particle's negative charge. Its sign determines the inward direction, rather than a negative speed. Equating the force magnitude to the centripetal requirement gives

$$\frac{mv^2}{r} = \frac{2k\lambda q}{r}.$$

Cancelling the common radius yields

$$v^2 = \frac{2k\lambda q}{m}, \qquad v = \sqrt{\frac{2k\lambda q}{m}}.$$

The speed is independent of orbital radius because both the line-charge force and the centripetal expression vary inversely with radius.

Step 2: Convert the speed to the period and compare the options One revolution covers the distance $2\pi r$, so

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{m}{2k\lambda q}}.$$

Thus the period is proportional to r, not to $r^{3/2}$. Option A has the radius dependence appropriate to a different force law. Option B incorrectly makes the period inversely proportional to radius. Option C contains a speed factor instead of circumference divided by speed and has no radius factor. Only option D has the correct dependence on all four physical parameters and the correct time dimension.

Step 3: Conclusion The required period is $T = 2\pi r\sqrt{m/(2k\lambda q)}$. Hence, the correct option is (D).

Quick Tip to Solve: A line charge gives $F \propto 1/r$, so circular speed is constant and orbital period grows linearly with radius.

46. Escape velocity of a body from earth is 11.2 km/s. If the radius of a planet be onethird the radius of earth and mass be one-sixth that of earth, the escape velocity from the planet is : [4 marks]

(A) 7.9 km/s

(B) 8.4 km/s

(C) 4.2 km/s

(D) 11.2 km/s

Correct Answer: (A) 7.9 km/s

Solution:

Concept:

- Escape speed from the surface of a spherical body is $v_e = \sqrt{2GM/R}$, neglecting atmosphere and the rotation of the body.
- The speed depends on the mass-to-radius ratio of the attracting planet, not on the mass of the object escaping. Comparing two bodies cancels the common factor $2G$.

Step 1: Express the planet parameters relative to Earth Write the mass and radius of Earth as M_E and R_E. The given planet has

$$M_P = \frac{M_E}{6}, \qquad R_P = \frac{R_E}{3}.$$

Hence its mass-to-radius ratio is

$$\frac{M_P}{R_P} = \frac{M_E/6}{R_E/3} = \frac{1}{2}\frac{M_E}{R_E}.$$

Although the smaller radius increases the gravitational binding per unit planet mass, the reduction in mass is greater. The combined effect is a halving of the quantity M/R.

Step 2: Take the square root to find the new escape speed Taking the ratio of the escape-speed formulas gives

$$\frac{v_{eP}}{v_{eE}} = \sqrt{\frac{M_P/R_P}{M_E/R_E}} = \frac{1}{\sqrt{2}}.$$

Using the stated terrestrial value,

$$v_{eP} = \frac{11.2}{\sqrt{2}} = 7.9196\ldots\ \mathrm{km\,s^{-1}}.$$

To the precision of the choices this is $7.9\,\mathrm{km\,s^{-1}}$. A direct factor of one-half would apply to the square

of escape speed, not to the speed itself. The value 11.2 would require the same M/R as Earth, and neither 8.4 nor 4.2 follows from the given mass and radius ratios. The units remain kilometres per second because the calculation uses a dimensionless ratio.

Step 3: Conclusion The planet has escape speed $v_{eP} \approx 7.9\,\mathrm{km\,s^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: When comparing escape speeds, first form the ratio of M/R, then take its square root.

47. Projectiles A and B are thrown at angles of 45° and 60° with vertical respectively from top of a $400\ \mathrm{m}$ high tower. If their ranges and times of flight are same, the ratio of their speeds of projection $v_A : v_B$ is :

[Take $g = 10\ \mathrm{ms^{-2}}$] [4 marks]

(A) $1 : 2$

(B) $\sqrt{2} : 1$

(C) $1 : \sqrt{2}$

(D) $1 : \sqrt{3}$

Correct Answer: The equal-range and equal-flight-time conditions are incompatible with the two distinct launch angles.

Solution:

Concept:

- For a projectile launched at angle θ to the vertical, horizontal speed is $v\sin\theta$, while the initial upward vertical component is $v\cos\theta$.
- With upward positive, launch height h, and flight time t, the landing condition is $-h = u_y t - gt^2/2$. Equal flight times from equal heights force equal initial vertical components.

Step 1: Apply the equal-range and equal-time conditions Let the common positive flight time be t and the common horizontal range be R. Horizontal motion gives

$$R = v_A \sin 45^\circ\, t = v_B \sin 60^\circ\, t.$$

Consequently the range condition alone requires

$$\frac{v_A}{v_B} = \frac{\sin 60^\circ}{\sin 45^\circ} = \sqrt{\frac{3}{2}}.$$

This ratio is already absent from the four printed options. It uses the angles to the vertical exactly as stated, so replacing sine by cosine would change the printed problem.

Step 2: Check the vertical motion for the same flight time Both projectiles fall through $400\,\mathrm{m}$. Their separate vertical equations are

$$-400 = v_A \cos 45^\circ\, t - 5t^2,$$

$$-400 = v_B \cos 60^\circ\, t - 5t^2.$$

Subtracting them and using $t > 0$ gives

$$v_A \cos 45^\circ = v_B \cos 60^\circ \quad \Rightarrow \quad \frac{v_A}{v_B} = \frac{1}{\sqrt{2}}.$$

This contradicts the necessary ratio $\sqrt{3/2}$ from horizontal motion. More generally, equal displacement and equal flight time in the same gravitational field require the same two initial velocity components. They cannot correspond to the distinct specified launch directions. Taking the launch directions downward instead leaves the same incompatibility in the component magnitudes.

Step 3: Conclusion As printed, the two conditions are incompatible, so no speed ratio satisfies the question and no option is correct. Option (C), $1 : \sqrt{2}$, agrees only with equal flight time and ignores the equal-range requirement; although the supplied key selects it, it does not solve the literal question.

Quick Tip to Solve: When both flight time and range are specified as equal, check horizontal and vertical components independently before choosing a ratio.

48. If the total energy transferred to a surface in time t is 6.48×10^5 J, then the magnitude of the total momentum delivered to this surface for complete absorption will be: [4 marks]

(A) 2.16×10^{-3} kg m/s

(B) 2.46×10^{-3} kg m/s

(C) 1.58×10^{-3} kg m/s

(D) 4.32×10^{-3} kg m/s

Correct Answer: (A) 2.16×10^{-3} kg m/s

Solution:

Concept:

- Electromagnetic radiation of total energy E carries momentum of magnitude $p = E/c$, where c is the speed of light.
- On complete absorption, the incident momentum is transferred to the surface. For normal complete reflection the transfer would be twice as large; that factor does not apply to absorption.

Step 1: Choose the transfer relation for absorption The radiation transfers its full energy $E = 6.48 \times 10^5$ J to the surface. Since it is absorbed rather than reflected, its outgoing radiation momentum is zero. The momentum delivered is therefore the incident momentum,

$$\Delta p = \frac{E}{c}.$$

The stated time t would be needed to calculate average force, $F_{\text{avg}} = E/(ct)$, but it is unnecessary when the total energy and total momentum transfer are requested. No division by time belongs in the impulse.

Step 2: Substitute the energy and compare the choices Using $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$,

$$\Delta p = \frac{6.48 \times 10^5}{3.00 \times 10^8} = 2.16 \times 10^{-3}\,\mathrm{kg\,m\,s^{-1}}.$$

The exponent follows from $10^5/10^8 = 10^{-3}$. The unit check is

$$\frac{\mathrm{J}}{\mathrm{m\,s^{-1}}} = \frac{\mathrm{kg\,m^2\,s^{-2}}}{\mathrm{m\,s^{-1}}} = \mathrm{kg\,m\,s^{-1}}.$$

The value 4.32×10^{-3} in option D is the result for complete reflection and therefore doubles the required transfer incorrectly. Options B and C have coefficients inconsistent with the given energy divided by the speed of light; neither is a rounding of 2.16.

Step 3: Conclusion The magnitude of momentum delivered is $2.16 \times 10^{-3}\,\mathrm{kg\,m\,s^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use E/c for absorbed radiation and $2E/c$ for normally reflected radiation; divide by time only when finding force.

49. If mass is written as $m = kc^{\mathrm{P}}G^{-1/2}h^{1/2}$ then the value of P will be : (Constants have their usual meaning with ka dimensionless constant) [4 marks]

(A) 1/3

(B) $-1/3$

(C) 1/2

(D) 2

Correct Answer: (C) 1/2

Solution:

Concept:

- The dimensions of the constants are $[c] = LT^{-1}$, $[G] = M^{-1}L^3T^{-2}$, and $[h] = ML^2T^{-1}$. The multiplier k is dimensionless.
- An expression representing mass must have net dimensions $M^1L^0T^0$. Powers of length and time must both cancel independently.

Step 1: Combine the dimensions of the gravitational and quantum factors Start with the two fixed powers:

$$[G^{-1/2}] = M^{1/2}L^{-3/2}T,$$

$$[h^{1/2}] = M^{1/2}LT^{-1/2}.$$

Multiplication gives

$$[G^{-1/2}h^{1/2}] = ML^{-1/2}T^{1/2}.$$

Thus this product already has the required single power of mass, but it still contains unwanted length and time dimensions. A suitable power of the speed of light must remove both of them.

Step 2: Match the length and time exponents Including the unknown speed factor gives

$$[kc^P G^{-1/2} h^{1/2}] = ML^{P-1/2} T^{-P+1/2}.$$

For this to equal $[m] = M$, we need

$$P - \frac{1}{2} = 0, \qquad -P + \frac{1}{2} = 0.$$

Both equations consistently give $P = 1/2$. Substitution reduces the original expression to

$$m = k\sqrt{\frac{hc}{G}},$$

whose dimensions are exactly those of mass. Choosing $P = 1/3$, $-1/3$, or 2 leaves nonzero residual powers of length and time, so all three other options fail the same dimensional test. The use of h, rather than a different quantum constant, affects a numerical multiplier but not this exponent.

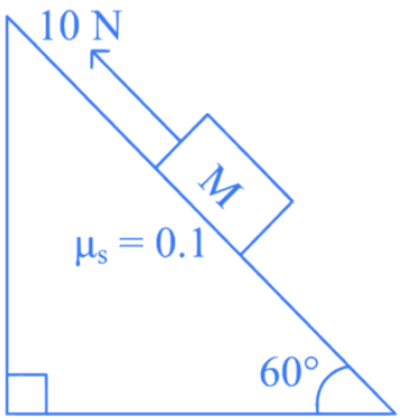
Step 3: Conclusion The required power is $P = 1/2$. Hence, the correct option is (C).

Quick Tip to Solve: In products of fundamental constants, collect the exponents of M, L, and T before solving for an unknown power.

50. A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of $60°$ by a force of 10 N parallel to the inclined surface as shown in figure. When the block is pushed up by 10 m along inclined surface, the work done against frictional force is :

$$[g = 10 \text{ m/s}^2]$$

[4 marks]



(A) $5\sqrt{3}$ J

(B) 5 J

(C) 5×10^3 J

(D) 10 J

Correct Answer: (B) 5 J

Solution:

Concept:

- The normal reaction on a block on an incline is $N = mg\cos\theta$ when the applied force is parallel to the plane; there is no perpendicular component of that force.
- The frictional resistance has magnitude $f = \mu N$. The positive work done against friction over distance s is $W_{\text{against}} = fs$, while friction itself does negative work.

Step 1: Find the normal reaction The figure gives the coefficient as 0.1, and the incline makes 60° with the horizontal. Gravity must be resolved perpendicular to the surface, rather than along it. Thus

$$N = 1 \times 10\cos 60^\circ = 5\,\text{N}.$$

The applied $10\,\text{N}$ force is parallel to the incline, so it does not increase or decrease this normal reaction.

Step 2: Calculate the work against friction Using the stated coefficient for the frictional resistance during the displacement,

$$f = 0.1 \times 5 = 0.5\,\text{N}.$$

The block moves $10\,\text{m}$ along the surface, so

$$W_{\text{against}} = 0.5 \times 10 = 5\,\text{J}.$$

The applied force does $100\,\text{J}$ of work, but that includes the gain in gravitational potential energy and any increase in kinetic energy; it is not all spent against friction. Option (A) introduces a spurious sine factor, option (C) is far too large, and option (D) would use twice the actual frictional resistance.

Step 3: Conclusion The work done against the frictional force is $\boxed{5\,\text{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a force parallel to an incline, first use $mg\cos\theta$ for the normal reaction, then multiply μN by distance along the incline.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A big drop is formed by coalescing 1000 small identical drops of water. If E_1 be the total surface energy of 1000 small drops of water and E_2 be the surface energy of single big drop of water, then $E_1 : E_2$ is $x : 1$ where $x =$ _____. [4 marks]

Solution:

Concept:

- The surface energy of a liquid drop is $E = TA$, where T is surface tension and A is its surface area. A water drop has one liquid-air interface.
- When identical incompressible drops coalesce without loss of water, total volume is conserved. For

spherical drops, volume varies as the cube of radius, whereas area varies as its square.

Step 1: Use volume conservation to find the big radius Let each small drop have radius r, and let the final drop have radius R. All 1000 small drops contribute their volume to the final sphere:

$$1000\frac{4\pi r^3}{3} = \frac{4\pi R^3}{3}.$$

Cancel the common factors to get $R^3 = 1000r^3$, giving $R = 10r$. The radius grows by a factor of ten, not by a factor of one thousand, because volume is proportional to radius cubed.

Step 2: Compare the total surface energies At the same surface tension, the initial and final surface energies are

$$E_1 = 1000T(4\pi r^2), \qquad E_2 = T(4\pi R^2).$$

Insert $R = 10r$ before taking the ratio:

$$\frac{E_1}{E_2} = \frac{1000r^2}{(10r)^2} = \frac{1000}{100} = 10.$$

The final area is smaller than the sum of the initial areas even though volume is unchanged. This decrease is the origin of the energy released on coalescence, and it confirms that the initial energy must exceed the final energy.

Step 3: Conclusion Thus $E_1 : E_2 = 10 : 1$, so the requested dimensionless value is $\boxed{x = 10}$.

Quick Tip to Solve: For n identical spherical drops merging into one, the initial-to-final surface-energy ratio is $n^{1/3} : 1$.

52. A power transmission line feeds input power at 2.3 kV to a step down transformer with its primary winding having 3000 turns. The output power is delivered at 230 V by the transformer. The current in the primary of the transformer is 5 A and its efficiency is 90%. The winding of transformer is made of copper. The output current of transformer is _____ A. [4 marks]

Solution:

Concept:

- Electrical power at a transformer terminal is $P = VI$, using the corresponding voltage and current on the same winding.
- The efficiency is $\eta = P_{\text{out}}/P_{\text{in}}$. For a nonideal transformer, output power is smaller than input power, so ideal power conservation cannot be used without the efficiency factor.

Step 1: Calculate input and useful output power The primary voltage is $2.3\,\text{kV} = 2300\,\text{V}$, and its current is $5\,\text{A}$. Therefore

$$P_{\text{in}} = V_p I_p = 2300 \times 5 = 11500\,\text{W}.$$

Convert the percentage efficiency into its fractional form, $\eta = 0.90$, and use it on the input power:

$$P_{\text{out}} = 0.90 \times 11500 = 10350\,\text{W}.$$

The remaining $1150\,\mathrm{W}$ is the power lost in the transformer, which is consistent with an efficiency below unity.

Step 2: Find the secondary current The load receives the output power at the given secondary voltage of $230\,\mathrm{V}$. Hence

$$I_s = \frac{P_{\text{out}}}{V_s} = \frac{10350}{230} = 45\,\mathrm{A}.$$

Equivalently, $I_s = \eta(V_p/V_s)I_p = 0.90 \times 10 \times 5$. The tenfold reduction in voltage allows a much larger secondary current, while the efficiency reduces it below the ideal value $50\,\mathrm{A}$. The number of primary turns and the copper winding material are not needed once both terminal voltages and the actual efficiency are specified.

Step 3: Conclusion The output current of the transformer is $\boxed{45\,\mathrm{A}}$; the numerical entry is 45.

Quick Tip to Solve: Combine efficiency and terminal power directly: $I_s = \eta V_p I_p / V_s$. Do not use the ideal current ratio when efficiency is given.

53. The current of $5\ \mathrm{A}$ flows in a square loop of sides $1\ \mathrm{m}$ is placed in air. The magnetic field at the centre of the loop is $X\sqrt{2} \times 10^{-7}T$. The value of X is _____. [4 marks]

Solution:

Concept:

- A finite straight current-carrying segment produces $B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$, where d is the perpendicular distance and the endpoint angles are measured from that perpendicular.
- At the centre of a square loop, the four sides give equal magnetic fields, all perpendicular to the loop in the same sense. Therefore their magnitudes add. In air, $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$.

Step 1: Calculate the field due to one side The side length is $a = 1\,\mathrm{m}$, so the centre is at perpendicular distance $d = a/2 = 0.5\,\mathrm{m}$ from every side. The line to either endpoint makes 45° with this perpendicular because both relevant distances are $a/2$. Hence

$$B_{\text{side}} = \frac{\mu_0 I}{4\pi(a/2)}(2\sin 45^\circ) = \frac{\mu_0 I\sqrt{2}}{2\pi a}.$$

Using the infinite-wire formula here would overestimate the contribution of a finite side.

Step 2: Add all four contributions and compare coefficients The right-hand rule shows that no side cancels another at the centre: all contributions have the same direction normal to the plane. Thus

$$B = 4B_{\text{side}} = \frac{2\sqrt{2}\mu_0 I}{\pi a}.$$

Substitute the current and side length:

$$B = \frac{2\sqrt{2}(4\pi \times 10^{-7})(5)}{\pi(1)} = 40\sqrt{2} \times 10^{-7}\,\mathrm{T}.$$

The question has already included $\sqrt{2} \times 10^{-7}\,\mathrm{T}$, so the remaining coefficient is the required number.

Step 3: Conclusion The magnetic field magnitude is $40\sqrt{2} \times 10^{-7}$ T, and hence $\boxed{X = 40}$ (dimensionless).

Quick Tip to Solve: At the centre of a square loop, use endpoint angles $45°, 45°$ for one side and multiply its field by four.

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54. Two discs of moment of inertia $I_1 = 4$ kg m^2 and $I_2 = 2$ kg m^2, about their central axes & normal to their planes, rotating with angular speeds 10 rad/s & 4 rad/s respectively are brought into contact face to face with their axes of rotation coincident. The loss in kinetic energy of the system in the process is _____ J. [4 marks]

Solution:

Concept:

- With no external torque about the common axis, the total angular momentum of the two discs is conserved during contact. Friction between their faces is an internal torque.
- Once slipping stops, both discs rotate with the same angular speed. Their kinetic energy need not be conserved: friction converts some of the initial rotational energy into heat.

Step 1: Find the common angular speed Take the initial rotations to be in the same sense, as used for the stated positive angular speeds. The total initial angular momentum is

$$L_i = I_1\omega_1 + I_2\omega_2 = 4(10) + 2(4) = 48\,\text{kg m}^2\,\text{s}^{-1}.$$

The combined moment of inertia is $I_1 + I_2 = 6\,\text{kg m}^2$. Conservation about the coincident axis gives

$$\omega_f = \frac{48}{6} = 8\,\text{rad s}^{-1}.$$

This value lies between the two initial angular speeds, as expected when the faster disc slows down and the slower one speeds up.

Step 2: Compare initial and final rotational energies Add the kinetic energies of the separate discs before contact:

$$K_i = \frac{1}{2}(4)(10)^2 + \frac{1}{2}(2)(4)^2 = 200 + 16 = 216\,\text{J}.$$

After both rotate together,

$$K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2 = \frac{1}{2}(6)(8)^2 = 192\,\text{J}.$$

Therefore the energy dissipated is $K_i - K_f = 216 - 192 = 24\,\text{J}$. As a check, the formula $\frac{1}{2}\frac{I_1 I_2}{I_1+I_2}(\omega_1 - \omega_2)^2$ also gives $\frac{1}{2}\frac{8}{6}(6)^2 = 24\,\text{J}$.

Step 3: Conclusion The loss in rotational kinetic energy is $\boxed{24\,\text{J}}$; the numerical entry is 24.

Quick Tip to Solve: For coaxial discs that lock together, conserve angular momentum first; calculate the kinetic-energy loss only after finding their common speed.

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55. A point source is emitting sound waves of intensity $16 \times 10^{-8}\ \mathrm{Wm}^{-2}$ at the origin. The difference in intensity (magnitude only) at two points located at a distances of $2m$ and $4m$ from the origin respectively will be _____ $\times 10^{-8}\ \mathrm{Wm}^{-2}$. [4 marks]

Solution:

Concept:

- For isotropic radiation from a point source, the same acoustic power crosses every spherical surface centred on the source. Its intensity decreases according to $I(r) = C/r^2$, with $C = P/(4\pi)$.
- For an ideal point source, intensity at the origin is undefined: the law $I = P/(4\pi r^2)$ is singular at $r = 0$. A finite intensity cannot literally be prescribed there. The numerical result below uses the conventional unit-distance interpretation of the stated intensity scale.

Step 1: Calculate the intensities at the two distances The wording does not specify a finite reference distance for the given intensity. Under the conventional interpretation that $16 \times 10^{-8}\,\mathrm{W\,m^{-2}}$ is the intensity at $1\,\mathrm{m}$, apply the inverse-square law at the two requested distances:

$$I_1 = \frac{16 \times 10^{-8}}{2^2} = 4 \times 10^{-8}\,\mathrm{W\,m^{-2}},$$

$$I_2 = \frac{16 \times 10^{-8}}{4^2} = 1 \times 10^{-8}\,\mathrm{W\,m^{-2}}.$$

Doubling the distance reduces intensity to one quarter, which checks the ratio. The normalization is an interpretation of the wording, rather than a finite intensity physically measurable at the source point.

Step 2: Take the magnitude of the difference The nearer point has the larger intensity, so subtract the farther-point value from it:

$$|I_1 - I_2| = (4 - 1) \times 10^{-8} = 3 \times 10^{-8}\,\mathrm{W\,m^{-2}}.$$

The intensity difference is positive because only its magnitude is requested. There is no interference calculation here: these are intensities at different positions due to a single point source. The factor $10^{-8}\,\mathrm{W\,m^{-2}}$ is already printed after the blank, so the number to enter is only its coefficient.

Step 3: Conclusion The conventional unit-distance interpretation gives $\boxed{3 \times 10^{-8}\,\mathrm{W\,m^{-2}}}$, so the numerical coefficient is $\boxed{3}$, matching the official key. Literally, a finite intensity at the origin of an ideal point source is undefined; a finite reference distance is needed to make the wording precise.

Quick Tip to Solve: For a point source, write the difference as $C(1/r_1^2 - 1/r_2^2)$, keeping the common intensity scale outside the subtraction.

56. Two resistance of 100Ω and 200Ω are connected in series with a battery of 4 V and negligible internal resistance. A voltmeter is used to measure voltage across 100Ω resistance, which gives reading as 1 V. The resistance of voltmeter must be _____ Ω. [4 marks]

Solution:

Concept:

- A voltmeter is connected in parallel with the resistance whose voltage is measured. Its finite resistance changes the equivalent resistance and therefore changes the circuit current.
- In a parallel combination, both branches have the same voltage. The current through the series resistor equals the sum of currents in the measured resistor and the voltmeter.

Step 1: Find the current in the series branch The $100\,\Omega$ resistor and the voltmeter together have the measured voltage $1\,\text{V}$. The remaining $200\,\Omega$ series resistor therefore has voltage

$$V_{200} = 4 - 1 = 3\,\text{V}.$$

The current through this resistor, which is the total current supplied to the parallel combination, is

$$I_{\text{total}} = \frac{3}{200} = 0.015\,\text{A}.$$

It would be incorrect to use $4/(100 + 200)$, because attaching the voltmeter has changed the effective resistance of the first branch.

Step 2: Find the voltmeter current and resistance The current through the $100\,\Omega$ resistor itself is

$$I_{100} = \frac{1}{100} = 0.010\,\text{A}.$$

By the junction rule, the voltmeter carries the difference:

$$I_V = I_{\text{total}} - I_{100} = 0.015 - 0.010 = 0.005\,\text{A}.$$

It also has $1\,\text{V}$ across it, so

$$R_V = \frac{1}{0.005} = 200\,\Omega.$$

For a consistency check, $100\,\Omega$ in parallel with $200\,\Omega$ is $200/3\,\Omega$; its division of the $4\,\text{V}$ supply with the series $200\,\Omega$ does indeed give $1\,\text{V}$.

Step 3: Conclusion The resistance of the voltmeter is $\boxed{200\,\Omega}$; the numerical entry is 200.

Quick Tip to Solve: When a voltmeter reading is lower than the unloaded divider voltage, include the meter as a parallel resistor before applying Ohm's law.

57. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 37° with each other. When suspended in a liquid of density $0.7\ \mathrm{g/cm^3}$, the angle remains same. If density of material of the sphere is $1.4\ \mathrm{g/cm^3}$, the dielectric constant of the liquid is _____ $\left(\tan 37^\circ = \frac{3}{4}\right)$ [4 marks]

Solution:

Concept:

- At equilibrium the horizontal electric repulsion is balanced by the horizontal component of string tension, while the vertical component balances the effective weight. Thus $\tan\theta = F_e/W_{\text{eff}}$, where θ is each string's angle to the vertical.
- In a dielectric of relative permittivity K, Coulomb repulsion at unchanged separation becomes F_e/K. Buoyancy reduces the sphere's weight from $\rho_s V g$ to $(\rho_s - \rho_l)Vg$.

Step 1: Compare the two equilibria The angle between the strings is unchanged, so each string has the same inclination in both situations. Equal string lengths therefore also give the same separation between the sphere centres. With the charges unchanged, only the dielectric factor changes the electrostatic force. In air, neglecting air buoyancy,

$$\tan\theta = \frac{F_e}{\rho_s V g}.$$

In the liquid, the upward buoyant force is $\rho_l V g$, so

$$\tan\theta = \frac{F_e/K}{(\rho_s - \rho_l)Vg}.$$

Step 2: Cancel the common quantities Equate the two expressions for the same inclination. The factors F_e, V and g cancel, giving

$$K(\rho_s - \rho_l) = \rho_s.$$

Hence

$$K = \frac{1.4}{1.4 - 0.7} = \frac{1.4}{0.7} = 2.$$

Both densities are in the same units, so their ratio is dimensionless. The supplied tangent is unnecessary because the angle cancels. The reduced weight is half its air value, requiring the electric force also to be halved.

Step 3: Conclusion The dielectric constant of the liquid is $\boxed{2}$ (dimensionless).

Quick Tip to Solve: When immersion leaves the angle unchanged, equate force-to-effective-weight ratios; the angle and charge cancel.

58. A simple pendulum is placed at a place where its distance from the earth's surface is equal to the radius of the earth. If the length of the string is $4m$, then the time period of small oscillations will be _____ s. [take $g = \pi^2 ms^{-2}$] [4 marks]

Solution:

Concept:

- Outside a spherical Earth, the gravitational acceleration is $g_r = GM/r^2$, where r is the distance from Earth's centre, not its surface.
- A simple pendulum undergoing small oscillations has period $T = 2\pi\sqrt{L/g_r}$. Use the local gravitational acceleration at the pendulum's location.

Step 1: Find the local acceleration The stated distance from the surface is an altitude $h = R$, where R is Earth's radius. The distance from the centre is therefore $r = R + h = 2R$. Since the surface acceleration is $g = GM/R^2$, the acceleration at the pendulum is

$$g_h = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{4} = \frac{\pi^2}{4}\,\mathrm{m\,s^{-2}}.$$

Using the surface acceleration directly would miss this reduction by a factor of four.

Step 2: Evaluate the period For the stated string length $L = 4\,\mathrm{m}$, insert the local acceleration into the small-angle pendulum formula:

$$T = 2\pi\sqrt{\frac{4\,\mathrm{m}}{(\pi^2/4)\,\mathrm{m\,s^{-2}}}} = 2\pi\sqrt{\frac{16}{\pi^2}\,\mathrm{s^2}} = 8\,\mathrm{s}.$$

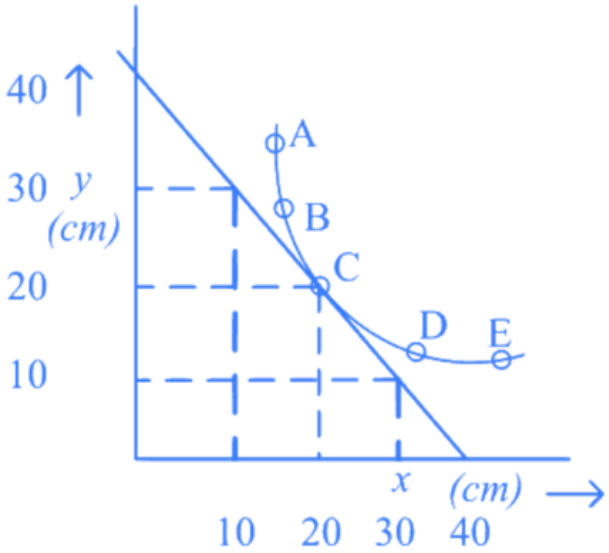
The square root has units of time. As a check, the same pendulum would have period $4\,\mathrm{s}$ at the surface with the supplied g. At altitude R, one-quarter the acceleration doubles the period, as required by $T \propto g_h^{-1/2}$.

Step 3: Conclusion The time period of the small oscillations is $\boxed{8\,\mathrm{s}}$.

Quick Tip to Solve: Convert altitude to distance from the centre first: at height R, gravity is g/4 and the pendulum period doubles.

59. In an experiment to measure the focal length (f) of a convex lens, the magnitude of object distance (x) and the image distance (y) are measured with reference to the focal point of the lens. The y-x plot is shown in figure.

The focal length of the lens is _____ cm. [4 marks]



Solution:

Concept:

- For a convex lens forming a real image, let the object and image distances from the lens be u_m and v_m, both positive magnitudes. Then $1/f = 1/u_m + 1/v_m$.
- When distances are instead measured from the corresponding focal points, $x = u_m - f$ and $y = v_m - f$. Newton's lens equation is therefore $xy = f^2$.

Step 1: Relate the plotted distances The plotted coordinates are not measured from the optical centre. Write $u_m = x + f$ and $v_m = y + f$ in the usual lens equation. Multiplication by the denominators gives

$$(x + f)(y + f) = f(x + y + 2f).$$

Expanding and cancelling the terms fx and fy yields

$$xy = f^2.$$

Thus the experimental curve is a rectangular hyperbola. A point on that curve determines the focal length from the square root of the product of its coordinates.

Step 2: Read the labelled point C From the dashed projections, point C on the experimental curve has $x = 20\,\text{cm}$ and $y = 20\,\text{cm}$. Hence

$$f^2 = (20)(20)\,\text{cm}^2 = 400\,\text{cm}^2, \qquad f = 20\,\text{cm}.$$

The positive root applies to the stated convex lens. The oblique straight line through C is tangent to the hyperbola; its intercepts of $40\,\text{cm}$ are not the focal length. Indeed the slope at C is $-f^2/x^2 = -1$,

consistent with that tangent.

Step 3: Conclusion The focal length of the convex lens is $\boxed{20\,\text{cm}}$.

Quick Tip to Solve: For distances measured from focal points use xy = f squared; read a point on the curve, not a tangent intercept.

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60. A vector has magnitude same as that of $\vec{A} = 3\hat{i} + 4\hat{j}$ and is parallel to $\vec{B} = 4\hat{i} + 3\hat{j}$. The x and y components of this vector in first quadrant are x and 3 respectively where $x =$ _____. [4 marks]

Solution:

Concept:

- The magnitude of a plane vector $a\hat{i} + b\hat{j}$ is $\sqrt{a^2 + b^2}$. Components along perpendicular axes must be combined quadratically.
- A vector parallel to a nonzero vector $\vec{B}$ is $\lambda\vec{B}$. Its scalar multiplier is positive when it points into the same quadrant as $\vec{B}$.

Step 1: Calculate the required magnitude The given vector has magnitude

$$|\vec{A}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5.$$

The direction vector also has magnitude

$$|\vec{B}| = \sqrt{4^2 + 3^2} = 5.$$

Write the desired vector as $\vec{C} = \lambda(4\hat{i} + 3\hat{j})$. Its magnitude must be 5, so $5|\lambda| = 5$, giving $|\lambda| = 1$. Magnitude alone would permit either direction along the same line.

Step 2: Use the quadrant and components The required vector lies in the first quadrant, where both components are positive. Therefore $\lambda = +1$, not -1, and

$$\vec{C} = 4\hat{i} + 3\hat{j}.$$

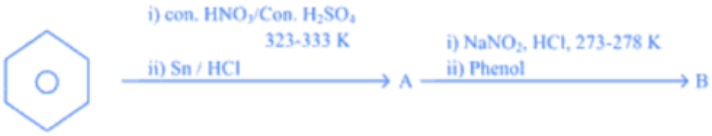
Its stated vertical component is indeed 3, and its horizontal component is $x = 4$. Independently, the prescribed components give $x^2 + 3^2 = 5^2$, hence $x^2 = 16$. The first-quadrant restriction chooses the positive root. The component ratio $x/3 = 4/3$ also confirms that $\vec{C}$ is parallel to $\vec{B}$, rather than merely having the correct magnitude.

Step 3: Conclusion The required horizontal component is $\boxed{x = 4}$, in the same component units as the given vectors.

Quick Tip to Solve: Write a parallel vector as a scalar multiple, then use the magnitude and quadrant to fix the scalar and its sign.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The products A and B formed in the following reaction scheme are respectively [4 marks]



(A) Aniline and 2-(phenyldiazenyl)phenol (ortho-hydroxyazobenzene).

(B) Aniline and 4-(phenyldiazenyl)phenol (para-hydroxyazobenzene).

(C) 1-Chloro-3-nitrobenzene and 3′-nitro[1,1′-biphenyl]-4-ol.

(D) Aniline and 2-amino[1,1′-biphenyl]-4′-ol.

Correct Answer: (B) Aniline and 4-(phenyldiazenyl)phenol (para-hydroxyazobenzene).

Solution:

Concept:

- Nitration of benzene with concentrated nitric and sulfuric acids gives nitrobenzene. Reduction of the nitro group by $\mathrm{Sn/HCl}$ gives aniline, $\mathrm{C_6H_5NH_2}$.
- Aniline undergoes diazotisation with $\mathrm{NaNO_2/HCl}$ at 273–278 K. The resulting benzenediazonium ion couples with phenol predominantly at the para position, giving an azo dye.

Step 1: Identify intermediate A The first reagent mixture introduces $\mathrm{NO_2}$ into the benzene ring by electrophilic aromatic substitution. The subsequent $\mathrm{Sn/HCl}$ step reduces that group to $\mathrm{NH_2}$. Thus the reaction sequence is

$$\mathrm{C_6H_6} \xrightarrow{\mathrm{HNO_3/H_2SO_4}} \mathrm{C_6H_5NO_2} \xrightarrow{\mathrm{Sn/HCl}} \mathrm{C_6H_5NH_2}.$$

Intermediate A is aniline. Chlorine from hydrochloric acid is not substituted onto the benzene ring in this reduction, so the chloronitro compound depicted in option C cannot be A.

Step 2: Identify product B and compare the options Cold nitrous acid generated from $\mathrm{NaNO_2/HCl}$ converts aniline into benzenediazonium chloride. Coupling with the activated phenol ring gives the para-hydroxy azo product,

$$\mathrm{C_6H_5{-}N{=}N{-}C_6H_4{-}OH},$$

with the hydroxy group para to the azo linkage. Option B shows aniline followed by this para-coupled product. Option A instead shows the ortho-coupled isomer, whereas the para position is free and favoured. Option C lacks the required diazonium-derived azo bridge and has the wrong first intermediate. Option D shows a direct carbon-carbon linkage with an amino group retained, rather than the characteristic $-\mathrm{N{=}N}-$ linkage.

Step 3: Conclusion A is aniline and B is 4-(phenyldiazenyl)phenol (para-hydroxyazobenzene). Hence, the correct option is (B).

Quick Tip to Solve: Track the functional group through nitration, reduction and diazotisation; a free para position in phenol favours para azo coupling.

62. The correct stability order of carbocations is [4 marks]

(A) $(CH_3)_3C^+ > CH_3CH_2^+ > (CH_3)_2CH^+ > CH_3^+$

(B) $(CH_3)_3C^+ > (CH_3)_2CH^+ > CH_3CH_2^+ > CH_3^+$

(C) $CH_3^+ > CH_3CH_2^+ > (CH_3)_2CH^+ > (CH_3)_3C^+$

(D) $CH_3^+ > (CH_3)_2CH^+ > CH_3CH_2^+ > (CH_3)_3C^+$

Correct Answer: (B) $(CH_3)_3C^+ > (CH_3)_2CH^+ > CH_3CH_2^+ > CH_3^+$

Solution:

Concept:

- A carbocation has an electron-deficient carbon with an empty p orbital. Alkyl groups stabilise that centre by the positive inductive effect and hyperconjugation.
- For these simple acyclic alkyl carbocations, with no competing resonance effects, stability decreases in the order tertiary, secondary, primary and methyl.

Step 1: Classify the four cations The cation $(CH_3)_3C^+$ has three methyl groups attached to the positively charged carbon and is tertiary. The cation $(CH_3)_2CH^+$ has two attached methyl groups and is secondary. Ethyl cation, $CH_3CH_2^+$, is primary. Methyl cation, CH_3^+, has no attached alkyl group. More attached alkyl groups donate electron density and disperse the positive charge, lowering the energy of the electron-deficient centre.

Step 2: Compare stabilisation and select the order The numbers of adjacent C–H bonds available for hyperconjugation in the tertiary, secondary and primary examples are nine, six and three respectively. Methyl cation has no adjacent carbon and therefore no such alkyl hyperconjugation. Together with the electron-releasing inductive effect, this gives

$$(CH_3)_3C^+ > (CH_3)_2CH^+ > CH_3CH_2^+ > CH_3^+.$$

Option B is the only order that follows this sequence throughout. Option A wrongly places primary ethyl cation above secondary isopropyl cation. Option C reverses the overall trend. Option D wrongly makes methyl the most stable and tertiary the least stable; both endpoints contradict alkyl stabilisation.

Step 3: Conclusion The stability order is tertiary $>$ secondary $>$ primary $>$ methyl. Hence, the correct option is (B).

Quick Tip to Solve: For simple alkyl carbocations without resonance, count alkyl groups attached to the charged carbon before comparing stability.

63. Which among the following purification methods is based on the principle of "Solubility" in two different solvents? [4 marks]

(A) Distillation

(B) Sublimation

(C) Column Chromatography

(D) Differential Extraction

Correct Answer: (D) Differential Extraction

Solution:

Concept:

- Differential extraction separates a solute according to its unequal solubilities in two immiscible solvents. The solute distributes itself between the two liquid phases.
- At a fixed temperature the distribution coefficient relates equilibrium concentrations in the two solvents. A solute preferentially enters the solvent in which it is more soluble.

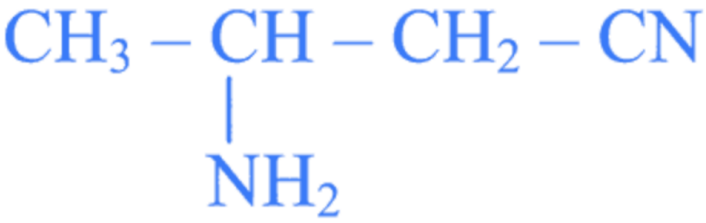
Step 1: Apply the two-solvent principle In differential extraction, a solution is shaken with a second solvent that is immiscible with the first. The dissolved compound moves into both layers but is enriched in the solvent where its solubility is greater. After the layers settle, a separating funnel allows them to be collected separately. Repeated extraction with suitable small portions of the extracting solvent improves recovery. The decisive feature in the question is therefore solubility in two different solvent phases, which directly describes differential extraction.

Step 2: Distinguish the other methods Distillation separates components through their different volatilities or boiling behaviour, involving vaporisation followed by condensation. Sublimation uses a solid's ability to pass directly into the vapour phase and then deposit as a solid. Column chromatography separates compounds through unequal interactions with a stationary phase and a mobile phase, commonly differential adsorption in the standard column method. It is not the liquid-liquid separation between two immiscible solvents described here. Accordingly, options A, B and C invoke different separation principles, while option D matches the stated one exactly.

Step 3: Conclusion The method based on unequal solubilities in two different solvents is differential extraction. Hence, the correct option is (D).

Quick Tip to Solve: Two immiscible liquid layers and preferential solubility point to extraction; boiling points point to distillation.

64. IUPAC name of following compound is : [4 marks]



(A) 2-Aminobutanenitrile

(B) 3-Aminopropanenitrile

(C) 2-Aminopentanenitrile

(D) 3-Aminobutanenitrile

Correct Answer: (D) 3-Aminobutanenitrile

Solution:

Concept:

- The nitrile group $-C\equiv N$ has priority over an amino group in suffix selection. Its carbon belongs to the parent chain and is assigned position 1.
- An amino group is named as the prefix amino when nitrile supplies the suffix. Count the longest chain containing the nitrile carbon, then locate the amino substituent.

Step 1: Identify the principal group and count the parent chain The displayed structure is $CH_3-CH(NH_2)-CH_2-CN$. The terminal nitrile is the principal characteristic group, so the parent name ends in nitrile. Starting at its carbon gives four consecutive carbon atoms: nitrile carbon, the adjacent methylene carbon, the amino-bearing methine carbon and the terminal methyl carbon. The saturated four-carbon parent is therefore butanenitrile; the nitrile nitrogen is not counted as a chain carbon.

Step 2: Assign the amino locant and check each name Numbering from the nitrile end puts CH_2 at C-2, $CH(NH_2)$ at C-3 and CH_3 at C-4. Thus the prefix is 3-amino. Option (A) incorrectly places amino at C-2. Option (B) omits the nitrile carbon from the four-carbon parent and gives a three-carbon chain. Option (C) gives a five-carbon parent and also the wrong amino locant. Only option (D) has both the correct parent and locant.

Step 3: Conclusion The IUPAC name is $\boxed{\text{3-aminobutanenitrile}}$. Hence, the correct option is (D).

Quick Tip to Solve: For an open-chain nitrile, write C-1 under the carbon of CN before numbering any substituent; this prevents the common one-carbon counting error.

65. The solution from the following with highest depression in freezing point/lowest freezing point is [4 marks]

(A) 180 g of acetic acid dissolved in benzene

(B) 180 g of acetic acid dissolved in water

(C) 180 g of benzoic acid dissolved in benzene

(D) 180 g of glucose dissolved in water

Correct Answer: (B) 180 g of acetic acid dissolved in water

Solution:

Concept:

- For a dilute solution, $\Delta T_f = iK_f m$, where i is the van't Hoff factor, K_f is the solvent cryoscopic constant and m is molality. The final freezing point is $T_f = T_f^\circ - \Delta T_f$.
- Carboxylic acids tend to associate into dimers in benzene, reducing particle number. Acetic acid partially ionizes in water, increasing particle number; glucose remains molecular.

Step 1: Compare the intended particle counts For the stated 180 g samples, acetic acid provides $180/60 = 3$ mol, benzoic acid provides $180/122 \simeq 1.48$ mol and glucose provides $180/180 = 1$ mol. Aqueous acetic acid undergoes $CH_3COOH \rightleftharpoons H^+ + CH_3COO^-$, so its effective particle amount is $3(1+\alpha)$, where α is its degree of ionization. In benzene, dimerization decreases the effective particle amount: completely dimerized acetic acid gives 1.5 mol of dimers and benzoic acid about 0.74 mol. Glucose in water gives 1 mol of solute particles.

Step 2: Select the intended answer and state the comparison limits The intended examination comparison selects aqueous acetic acid, option (B), because it combines the largest formula-unit amount with an increase in particle number rather than association. Options (A) and (C) involve association in benzene, and (D) has fewer particles without ionization. Strictly, however, the printed data do not give the solvent masses, so molalities cannot be calculated. Also, water and benzene have different K_f values and pure-solvent freezing points. Therefore largest depression and lowest absolute freezing point are not equivalent comparisons across these solvents; a rigorous universal ranking cannot follow from solute mass alone.

Step 3: Conclusion The official key selects 180 g of acetic acid dissolved in water: hence, the intended correct option is (B). The question as printed lacks the solvent quantities needed for an unconditional freezing-point ranking.

Quick Tip to Solve: Compare im only for the same solvent; when solvents differ, include both K_f and the pure-solvent freezing point before ranking the solutions.

66. Given below are two statements:

Statement - I: Since Fluorine is more electronegative than nitrogen, the net dipole moment of NF_3 is greater than NH_3.

Statement - II: In NH_3, the orbital dipole due to lone pair and the dipole moment of NH bonds are in opposite direction, but in NF_3 the orbital dipole due to lone pair and dipole moments of N-F bonds are in same direction.

In the light of the above statements, choose the most appropriate from the options given below:

[4 marks]

(A) Statement I is true but Statement II is false.

(B) Both Statement I and Statement II are true.

(C) Both Statement I and Statement II are false.

(D) Statement I is false but Statement II is true.

Correct Answer: (C) Both Statement I and Statement II are false.

Solution:

Concept:

- A molecular dipole moment is the vector sum of the bond dipoles and the electronic contribution associated with the lone pair; electronegativity alone does not determine its magnitude.
- Both NH_3 and NF_3 are trigonal pyramidal with one lone pair on nitrogen. Their different bond-dipole directions produce reinforcement in ammonia and cancellation in nitrogen trifluoride.

Step 1: Evaluate Statement I using the net dipole In ammonia, nitrogen is more electronegative than hydrogen, so the N-H bond dipoles point toward nitrogen. Their resultant reinforces the lone-pair contribution along the molecular symmetry axis. In nitrogen trifluoride, fluorine is more electronegative than nitrogen, so the N-F bond dipoles point toward fluorine; their resultant opposes the lone-pair contribution. Consequently $\mu(NH_3) \simeq 1.47\,\text{D}$ is appreciably larger than $\mu(NF_3) \simeq 0.24\,\text{D}$. Thus Statement I is false, even though its electronegativity premise is correct.

Step 2: Evaluate Statement II and eliminate the options Statement II reverses the actual reinforcement and opposition. The lone-pair and resultant bond contributions are in the same direction in NH_3, and in opposite directions in NF_3. Hence Statement II is also false. Option (A) wrongly calls Statement I true, option (B) wrongly calls both true, and option (D) wrongly calls Statement II true. The pair of false statements is represented only by option (C).

Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (C).

Quick Tip to Solve: For pyramidal molecules, first draw the direction of the bond-dipole resultant along the symmetry axis; then decide whether it adds to or opposes the lone-pair contribution.

67. A and B formed in the following reactions are:

$$\mathrm{CrO_2Cl_2 + 4NaOH \rightarrow A + 2NaCl + 2H_2O},$$
$$\mathrm{A + 2HCl + 2H_2O_2 \rightarrow B + 3H_2O}$$

[4 marks]

(A) $\mathrm{A = Na_2Cr_2O_7,\ B = CrO_5}$
(B) $\mathrm{A = Na_2CrO_4,\ B = CrO_5}$
(C) $\mathrm{A = Na_2Cr_2O_4,\ B = CrO_4}$
(D) $\mathrm{A = Na_2Cr_2O_7,\ B = CrO_3}$

Correct Answer: (B) $\mathrm{A = Na_2CrO_4,\ B = CrO_5}$

Solution:

Concept:

- Chromyl chloride $\mathrm{CrO_2Cl_2}$ gives chromate in aqueous alkali. The chromium remains in oxidation state +6; hydroxide replaces the chloride-containing environment with the chromate oxoanion.
- Acidified chromate reacts with hydrogen peroxide to give blue chromium peroxide, $\mathrm{CrO_5}$. This species contains one oxo oxygen and two peroxide groups.

Step 1: Identify A from the alkaline reaction The first reaction produces the yellow chromate salt, not dichromate. Its balanced equation is

$$\mathrm{CrO_2Cl_2 + 4\,NaOH \longrightarrow Na_2CrO_4 + 2\,NaCl + 2\,H_2O}.$$

One chromium atom enters and one chromium atom leaves in $\mathrm{Na_2CrO_4}$. The sodium count is four on both sides: two in chromate and two in sodium chloride. Dichromate contains two chromium atoms and is favoured by acidification of chromate, whereas the first reaction is strongly alkaline.

Step 2: Identify B from the peroxide reaction In acid, chromate and hydrogen peroxide form blue $\mathrm{CrO_5}$. Including the spectator sodium and chloride ions gives the complete balanced equation

$$\mathrm{Na_2CrO_4 + 2\,HCl + 2\,H_2O_2 \longrightarrow CrO_5 + 2\,NaCl + 3\,H_2O}.$$

The displayed question suppresses the spectator salt in its second line, but its intended chromium product is unchanged. Option (A) has the wrong first salt, option (C) has incorrect chromium species and option (D) has neither the correct first salt nor the peroxide product. Option (B) supplies the correct pair.

Step 3: Conclusion $\boxed{\mathrm{A{=}Na_2CrO_4,\quad B{=}CrO_5}}$. Hence, the correct option is (B).

Quick Tip to Solve: Remember the sequence: chromyl chloride plus alkali gives yellow chromate; acidified chromate plus hydrogen peroxide gives blue chromium peroxide.

68. If a substance 'A' dissolves in solution of a mixture of 'B' and 'C' with their respective number of moles as $\mathrm{n_A}, \mathrm{n_B}$ and $\mathrm{n_{C_3}}$. Mole fraction of C in the solution is [4 marks]

(A) $\frac{n_C}{n_A \times n_B \times n_C}$
(B) $\frac{n_B}{n_A + n_B}$
(C) $\frac{n_C}{n_A + n_B + n_C}$
(D) $\frac{n_C}{n_A - n_B - n_C}$

Correct Answer: (C) $\frac{n_C}{n_A + n_B + n_C}$

Solution:

Concept:

- The mole fraction of a component is its amount in moles divided by the total amount in moles of all components in the solution.
- For a mixture of A, B and C with no chemical reaction specified, the total amount is $n_{\text{tot}} = n_A + n_B + n_C$. All mole fractions are dimensionless and sum to unity.

Step 1: Form the total number of moles The solution contains three components, so each contributes to the denominator. Dissolving A in a pre-existing B-C mixture does not remove either solvent component from the mole balance. With the usual notation n_C for the stated amount of C,

$$n_{\text{tot}} = n_A + n_B + n_C.$$

The distinction between solute and mixed solvent does not affect this definition; the whole solution supplies the total moles.

Step 2: Place the moles of C in the numerator and check the alternatives The fraction belonging to C is therefore

$$x_C = \frac{n_C}{n_{\text{tot}}} = \frac{n_C}{n_A + n_B + n_C}.$$

As a consistency check, the analogous fractions for A and B add to give $x_A + x_B + x_C = 1$. Option (A) multiplies mole amounts and has the wrong dimensions. Option (B) is a fraction involving B in a binary A-B mixture and omits C. Option (D) subtracts mole amounts rather than summing them and can even give a negative fraction for a physical mixture. Only option (C) uses the required total.

Step 3: Conclusion The mole fraction is $\boxed{x_C = \frac{n_C}{n_A + n_B + n_C}}$. Hence, the correct option is (C).

Quick Tip to Solve: For any mole-fraction question, circle the named component for the numerator and sum every component present for the denominator.

69. The molecule / ion with square pyramidal shape is [4 marks]

(A) PCl_5

(B) $[Ni(CN)_4]^{2-}$

(C) PF_5

(D) BrF_5

Correct Answer: (D) BrF_5

Solution:

Concept:

- In VSEPR notation, a square pyramidal species has five bonded atoms and one lone pair around the central atom: AX_5E. Six electron domains have an octahedral electron-pair arrangement.
- Molecular shape counts the positions of bonded atoms only. Removing one vertex occupied by a lone pair from an octahedron leaves a square base and one axial bond, forming a square pyramid.

Step 1: Count electron domains in bromine pentafluoride Bromine has seven valence electrons. In BrF_5, it forms five single bonds to fluorine atoms and retains two nonbonding electrons, giving one lone pair. Thus its steric number is $5 + 1 = 6$, its electron-domain arrangement is octahedral and its VSEPR formula is AX_5E. Four fluorine atoms occupy the square base and the fifth occupies the apex; the lone pair occupies the opposite octahedral position.

Step 2: Check the shapes of the other species Both PCl_5 and PF_5 have five bonding pairs and no central lone pair, giving AX_5 trigonal bipyramidal molecules rather than square pyramids. In $[Ni(CN)_4]^{2-}$, nickel is in the $+2$ oxidation state with a d^8 configuration; the strong-field cyanide ligands give the familiar square planar complex. A square planar species has four ligand positions and no apical fifth ligand. Thus options (A), (B) and (C) are excluded, leaving BrF_5.

Step 3: Conclusion The square pyramidal molecule is $\boxed{BrF_5}$. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish five bonds alone from five bonds plus a lone pair: AX_5 is trigonal bipyramidal, whereas AX_5E is square pyramidal.

70. Given below are two statements:

Statement - I: High concentration of strong nucleophilic reagent with secondary alkyl halides which do not have bulky substituents will follow S_N2 mechanism.

Statement - II: A secondary alkyl halide when treated with a large excess of ethanol follows S_N1 mechanism.

In the light of the above statements, choose the most appropriate from the options given below:

[4 marks]

(A) Both Statement I and Statement II are true.

(B) Statement I is true but Statement II is false.

(C) Both Statement I and Statement II are false.

(D) Statement I is false but Statement II is true.

Correct Answer: (A) Both Statement I and Statement II are true.

Solution:

Concept:

- For a bimolecular nucleophilic substitution, $\text{rate} = k[\text{R}{-}\text{X}][\text{Nu}^-]$. A strong nucleophile at high concentration promotes direct backside attack when steric hindrance is modest.
- For unimolecular substitution, the slow step is ionization of the alkyl halide. A polar protic solvent stabilizes the developing ions, and a weak neutral nucleophile can capture the carbocation after it forms.

Step 1: Evaluate the strong-nucleophile conditions in Statement I A secondary carbon is more hindered than a primary carbon, but it can still undergo S_N2 substitution if the surrounding groups are not bulky. Raising the concentration of a strong nucleophile increases the frequency of productive backside collisions and increases the bimolecular rate. Under the substitution conditions described in the statement, direct nucleophilic displacement is therefore favoured. Statement I is true. This does not claim that all secondary halides always react by one pathway; bulky strong bases or heat can instead enhance elimination.

Step 2: Evaluate ethanol solvolysis in Statement II Ethanol is a polar protic solvent and a much weaker neutral nucleophile than a typical anionic reagent. In a large excess it acts mainly as the solvolysis medium, stabilizing ionic intermediates. A secondary alkyl halide can ionize to a secondary carbocation, which ethanol then attacks, followed by deprotonation to form the substitution product. This is the standard S_N1 description for the stated comparison, so Statement II is true. Option (B) incorrectly rejects Statement II, option (C) rejects both statements and option (D) incorrectly rejects Statement I.

Step 3: Conclusion Both Statement I and Statement II are true under the substitution conditions described. Hence, the correct option is (A).

Quick Tip to Solve: For a secondary halide, compare the reagent first: concentrated strong nucleophile supports S_N2, whereas weak neutral nucleophile in a protic solvolysis medium supports S_N1.

71. Choose the correct statements about the hydrides of group 15 elements.

A. The stability of the hydrides decreases in the order $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$**.**

B. The reducing ability of the hydride increases in the order $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$**.**

C. Among the hydrides, NH_3 **is strong reducing agent while** BiH_3 **is mild reducing agent.**

D. The basicity of the hydrides increases in the order $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$**.**

Choose the most appropriate from the options given below : [4 marks]

(A) C and D only

(B) A and D only

(C) A and B only

(D) B and C only

Correct Answer: (C) A and B only

Solution:

Concept:

- Group 15 forms covalent hydrides EH_3. Increasing atomic size down the group lengthens and weakens the E–H bonds.
- Thermal stability and reducing power generally vary oppositely: a less stable hydride releases hydrogen more readily. Basicity depends on the availability of the central atom's lone pair.

Step 1: Test the stability statement The overlap of the central atom's valence orbital with the hydrogen 1s orbital becomes poorer as the central atom gets larger. The E–H bond therefore becomes weaker down the group. Consequently the thermal stability follows

$$NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3.$$

This is exactly the order in statement A, so A is true.

Step 2: Test reducing power and basicity Because the hydrides decompose more readily down the group, their reducing power increases from ammonia to bismuthine. Statement B is true. Statement C reverses the end members: ammonia is the mild reducing agent compared with the much stronger reducing hydrides lower in the group, so C is false. The basicity also decreases down the group, rather than increasing as D claims:

$$NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3.$$

The lone pair on nitrogen is more available for donation than the more diffuse lone pairs on the heavier atoms. Thus D is false. Options A and D contain C, and option B contains D; each therefore contains a false statement.

Step 3: Conclusion Only statements A and B are correct. Hence, the correct option is (C).

Quick Tip to Solve: For group 15 hydrides, remember stability and basicity decrease down the group, while reducing power increases.

72. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: H_2Te **is more acidic than** H_2 S.

Reason R: Bond dissociation enthalpy of H_2Te **is lower than** H_2 S.

In the light of the above statements, choose the most appropriate from the options given below:

[4 marks]

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is false but R is true.

(D) A is true but R is false.

Correct Answer: (A) Both A and R are true and R is the correct explanation of A.

Solution:

Concept:

- For hydrides within group 16, acid strength increases down the group because the E–H bond becomes longer and weaker.
- Acidity describes release of a proton in solution. Bond dissociation enthalpy is not itself the aqueous acid dissociation enthalpy, but the falling E–H bond strength explains the dominant trend in this series.

Step 1: Evaluate the assertion Tellurium lies below sulfur in group 16 and has a larger atomic radius. The hydrogen telluride molecule therefore releases a proton more readily than hydrogen sulfide. The usual group trend is

$$H_2O < H_2S < H_2Se < H_2Te \quad \text{in acid strength.}$$

In particular, H_2Te is more acidic than H_2S, so assertion A is true.

Step 2: Evaluate the reason The larger tellurium atom forms a longer Te–H bond with less effective orbital overlap than the S–H bond. Its bond dissociation enthalpy is consequently smaller. Thus the stated comparison of the bond dissociation enthalpies is true, and reason R is true.

Step 3: Connect the two statements The weaker Te–H bond makes removal of hydrogen as a proton easier in the group comparison. This is the physical basis of the increasing acidity on moving from sulfur to tellurium, so R explains A. Option B incorrectly denies this link; option C incorrectly makes the assertion false; and option D incorrectly makes the reason false. Electronegativity alone should not be used to reverse the observed down-group trend.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: Compare bond strength for acidity down a hydride group: larger central atoms usually give weaker H–E bonds and stronger acids.

73. Alkaline oxidative fusion of MnO_2 gives "A" which on electrolytic oxidation in alkaline solution produces B. A and B respectively are [4 marks]

(A) Mn_2O_3 and MnO_4^{2-}

(B) Mn_2O_7 and MnO_4^-

(C) MnO_4^{2-} and MnO_4^-

(D) MnO_4^{2-} and Mn_2O_7

Correct Answer: (C) MnO_4^{2-} and MnO_4^-

Solution:

Concept:

- Alkaline oxidative fusion converts manganese dioxide, containing Mn(IV), into green manganate(VI).
- Further anodic oxidation removes one electron from manganate(VI), producing purple permanganate(VII). Oxidation means an increase in oxidation number.

Step 1: Identify the fusion product A On fusing MnO_2 with an alkali such as potassium hydroxide in the presence of an oxidant, manganese is oxidised from +4 to +6. For oxygen as the oxidant, a balanced equation is

$$2\,MnO_2 + 4\,KOH + O_2 \longrightarrow 2\,K_2MnO_4 + 2\,H_2O.$$

The potassium salt contains the manganate ion, ${MnO_4}^{2-}$. Thus A is manganate, not manganese(III) oxide or manganese(VII) oxide.

Step 2: Identify the electrolytic oxidation product B At the anode, the manganate ion loses one electron:

$${MnO_4}^{2-} \longrightarrow {MnO_4}^- + e^-.$$

In ${MnO_4}^{2-}$, manganese has oxidation number +6; in ${MnO_4}^-$, it has oxidation number +7. This electron-loss step therefore gives permanganate as B. The alkaline solution supplies the medium for the electrolytic conversion; it does not turn the product into the covalent acidic oxide Mn_2O_7. Option A begins with the wrong oxidation-state product. Option B incorrectly proposes an oxide as A, and option D incorrectly proposes that oxide as B.

Step 3: Conclusion A is ${MnO_4}^{2-}$ and B is ${MnO_4}^-$. Hence, the correct option is (C).

Quick Tip to Solve: Track Mn oxidation states: alkaline fusion gives +6 manganate, and a further one-electron oxidation gives +7 permanganate.

74. Salicylaldehyde is synthesized from phenol, when reacted with [4 marks]

(A) $HCCl_3,\ NaOH$

(B) $CO_2,\ NaOH$

(C) $HCOCl,\ NaOH$

(D) $CCl_4,\ NaOH$

Correct Answer: (A) $HCCl_3,\ NaOH$

Solution:

Concept:

- The Reimer–Tiemann reaction introduces a formyl group into phenol using chloroform and aqueous alkali, followed by hydrolysis and acidification.
- Phenoxide activates the aromatic ring toward electrophilic substitution, especially at its ortho and para positions. The principal product after work-up is salicylaldehyde, or 2-hydroxybenzaldehyde.

Step 1: Select the formylating reagent In strong alkali, chloroform $CHCl_3$, also written $HCCl_3$, generates dichlorocarbene, $:CCl_2$. Phenol is converted into phenoxide under the same conditions. The reactive carbene provides the carbon atom that ultimately becomes the aldehyde group. Thus the reagent pair required by the question is chloroform and sodium hydroxide.

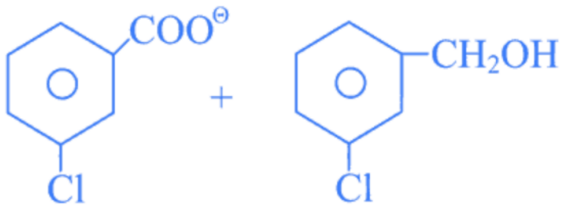
Step 2: Follow substitution and work-up The electron-rich phenoxide ring reacts mainly at an ortho position. Subsequent alkaline hydrolysis converts the introduced dichloromethyl group into a formyl group, and acid work-up regenerates the phenolic OH. The principal isolated compound is therefore $o\text{-}HOC_6H_4CHO$, salicylaldehyde. The para isomer can occur as a minor product, but does not change the reagent choice.

Step 3: Reject the remaining reagent pairs Carbon dioxide and alkali are associated with the Kolbe–Schmitt carboxylation, yielding a hydroxybenzoic acid after acidification, rather than the aldehyde. Formyl chloride is not the reagent used for the Reimer–Tiemann transformation. Carbon tetrachloride and alkali lead toward carboxylation products rather than the required formylation. Thus none of options B, C or D supplies the specified preparation.

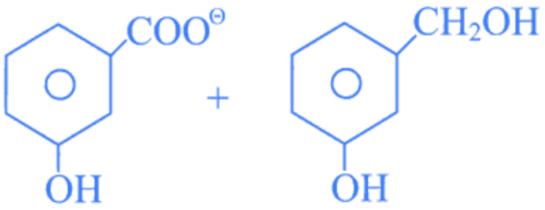
Step 4: Conclusion Phenol reacts with $HCCl_3$ and $NaOH$ in the Reimer–Tiemann reaction to give salicylaldehyde after work-up. Hence, the correct option is (A).

Quick Tip to Solve: Phenol plus chloroform/base gives an ortho aldehyde; phenoxide plus carbon dioxide gives an ortho carboxylic acid.

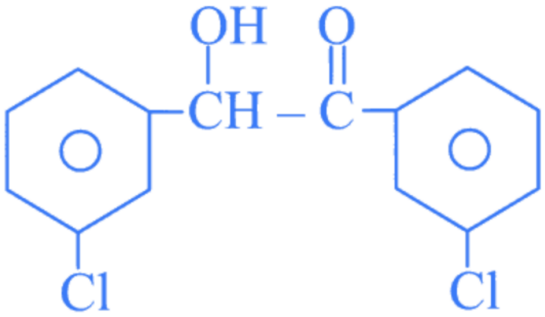
75. m-chlorobenzaldehyde on treatment with 50% KOH solution yields : [4 marks]

(A) 

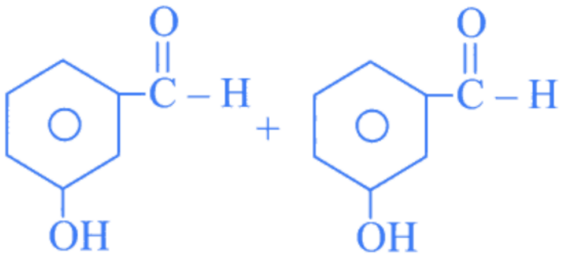
(B)



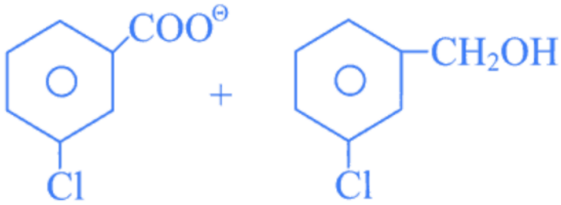
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- Aldehydes without an alpha hydrogen undergo the Cannizzaro reaction in concentrated aqueous alkali: two aldehyde molecules disproportionate into an alcohol and a carboxylate.
- In an aromatic aldehyde, the carbon directly adjacent to the carbonyl carbon is the aromatic ipso carbon. It has no hydrogen available for the usual aldol enolate formation.

Step 1: Choose the reaction pathway In m-chlorobenzaldehyde, the aldehyde group is attached directly to the benzene ring. There is no alpha hydrogen. With 50% KOH, the appropriate reaction is therefore Cannizzaro disproportionation rather than aldol condensation. The meta chlorine substituent remains on the aromatic ring; the reaction concerns the aldehyde groups.

Step 2: Identify the oxidation and reduction products One aldehyde molecule is oxidised to the carboxylate ion, while a second is reduced to the benzyl alcohol. Let $\mathrm{Ar} = m\text{-}\mathrm{ClC_6H_4}$. The overall

equation is

$$2\,\mathrm{ArCHO} + \mathrm{KOH} \longrightarrow \mathrm{ArCOOK} + \mathrm{ArCH_2OH}.$$

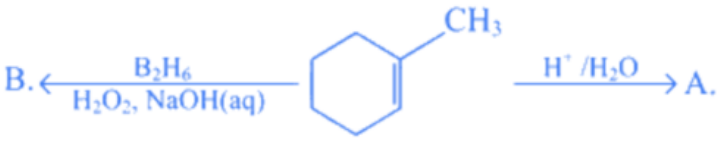
Hence the products are potassium 3-chlorobenzoate and 3-chlorobenzyl alcohol in equimolar amounts. In the strongly basic reaction mixture, the oxidised product is the carboxylate salt, not the free carboxylic acid.

Step 3: Compare the displayed alternatives Option A shows precisely the meta-chlorinated benzoate and benzyl alcohol. Option B replaces the chlorine substituents by hydroxyl groups and therefore depicts different compounds. Option C is a benzoin-type coupling product, which is not produced by concentrated potassium hydroxide under these conditions. Option D shows hydroxybenzaldehydes rather than the products of disproportionation. Neither substitution of aryl chlorine nor that coupling is the demanded transformation.

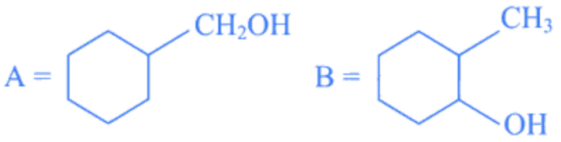
Step 4: Conclusion The products are m-chlorobenzoate and m-chlorobenzyl alcohol, with the potassium ion accompanying the carboxylate. Hence, the correct option is (A).

Quick Tip to Solve: Before choosing an aldol product, check for an alpha hydrogen: aromatic aldehydes without one give Cannizzaro products with concentrated alkali.

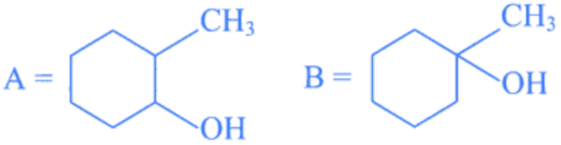
76. Products A and B formed in the following set of reactions are [4 marks]



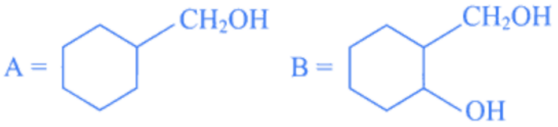
(A)



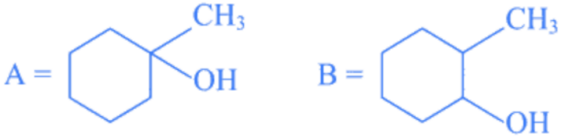
(B)



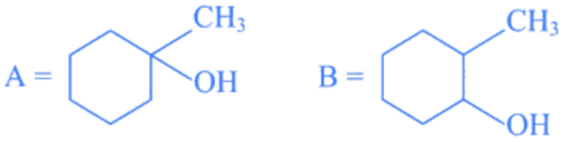
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- Acid-catalysed hydration follows Markovnikov orientation: water adds through the more stable carbocation, placing OH on the more substituted alkene carbon.
- Hydroboration–oxidation gives the anti-Markovnikov alcohol: boron initially bonds to the less substituted carbon, and oxidation replaces the C–B bond by C–OH without changing that carbon skeleton.

Step 1: Identify A from acid-catalysed hydration The starting alkene is 1-methylcyclohexene. Of its double-bond carbons, C1 carries the methyl group and is more substituted, while C2 is less substituted. Proton addition at C2 gives the more stable tertiary carbocation at C1. Water attacks that carbon, and deprotonation affords 1-methylcyclohexan-1-ol. Thus A has the methyl and hydroxyl groups attached to the same ring carbon.

Step 2: Identify B from hydroboration–oxidation In reaction with diborane, addition places boron at the less substituted double-bond carbon C2 and hydrogen at C1. Oxidation by H_2O_2 in aqueous $NaOH$ replaces boron by hydroxyl. The product connectivity is therefore 2-methylcyclohexan-1-ol: methyl and hydroxyl are on adjacent ring carbons. Hydroboration is syn addition; the options here distinguish connectivity and do not specify stereochemical wedges.

Step 3: Compare both products together Option D has A as the alcohol with OH and methyl on one carbon and B as the alcohol with those groups on neighbouring carbons. Option B swaps these hydration orientations. Options A and C introduce a hydroxymethyl substituent or otherwise change the carbon skeleton in a manner neither hydration pathway produces. Both reactions add the elements of water across the original double bond; neither moves the methyl carbon out of its original substituent position.

Step 4: Conclusion A is 1-methylcyclohexan-1-ol and B is 2-methylcyclohexan-1-ol. Hence, the correct option is (D).

Quick Tip to Solve: Label the more substituted alkene carbon first: acid hydration puts OH there, whereas hydroboration–oxidation puts OH on the other carbon.

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77. The orange colour of $K_2Cr_2O_7$ and purple colour of $KMnO_4$ is due to [4 marks]

(A) Charge transfer transition in both.

(B) $d \rightarrow d$ transitions in $KMnO_4$ and charge transfer transitions in $K_2Cr_2O_7$.

(C) $d \rightarrow d$ transitions in both

(D) $d \rightarrow d$ transitions in $K_2Cr_2O_7$ and charge transfer transitions in $KMnO_4$.

Correct Answer: (A) Charge transfer transition in both.

Solution:

Concept:

- For a transition-metal ion, determine its oxidation state and d-electron count before assigning a colour to a d–d transition.

- Charge-transfer absorption transfers electron density between ligand and metal orbitals. Oxoanions of high-oxidation-state metals can be intensely coloured through oxygen-to-metal charge transfer even when the metal is d-zero.

Step 1: Determine the metal electron counts In $K_2Cr_2O_7$, the dichromate ion is $Cr_2O_7^{2-}$. If chromium has oxidation number x,

$$2x + 7(-2) = -2 \quad \Rightarrow \quad x = +6.$$

Chromium(VI) has a $3d^0$ configuration. Similarly, in $KMnO_4$,

$$x + 4(-2) = -1 \quad \Rightarrow \quad x = +7,$$

so manganese(VII) also has a $3d^0$ configuration.

Step 2: Identify the allowed source of visible absorption A d–d transition requires a d electron that can be excited between split metal d orbitals. Neither Cr(VI) nor Mn(VII) has such an electron, so assigning the visible colours to d–d transitions is inappropriate. Instead, absorption involves charge transfer from filled oxygen ligand orbitals to suitable empty metal-centred orbitals. Such ligand-to-metal charge-transfer transitions account for the orange dichromate and purple permanganate colours.

Step 3: Eliminate the mixed assignments Option B wrongly assigns a d–d transition to permanganate. Option D wrongly assigns one to dichromate, and option C makes that wrong assignment for both. Option A alone recognises charge-transfer transitions in both oxoanions. The existence of a coloured compound therefore does not by itself establish that its metal must have a partly filled d subshell.

Step 4: Conclusion The orange colour of $K_2Cr_2O_7$ and the purple colour of $KMnO_4$ both arise from charge-transfer transitions. Hence, the correct option is (A).

Quick Tip to Solve: Calculate oxidation state before explaining colour: d-zero oxoanions such as permanganate and dichromate cannot show ordinary d–d transitions.

78. The coordination geometry around the manganese in decacarbonyldimanganese (0) is [4 marks]

(A) Trigonal bipyramidal

(B) Square pyramidal

(C) Square planar

(D) Octahedral

Correct Answer: (D) Octahedral

Solution:

Concept:

- Decacarbonyldimanganese(0), $Mn_2(CO)_{10}$, contains a direct metal–metal bond. Each manganese atom is attached to five terminal carbonyl ligands and to the other manganese atom.
- Coordination geometry describes the directions of the bonds around one metal centre. Counting only carbonyl ligands overlooks the sixth direction supplied by the metal–metal bond.

Step 1: Identify the bonds at one manganese centre The molecule consists of two $Mn(CO)_5$ units joined through an $Mn-Mn$ bond. For either manganese, four carbonyl groups occupy approximately an equatorial square and the fifth carbonyl occupies an axial position. The bond to the second manganese occupies the other axial position. Thus there are six approximately octahedrally directed bonds around each manganese, although only five of those bonds are to carbonyl ligands.

Step 2: Check the geometry and the alternatives The electron count supports this structure: manganese contributes seven valence electrons, five carbonyls donate ten electrons, and the metal–metal bond contributes one electron to each metal. The total is $7 + 10 + 1 = 18$ electrons per manganese. Trigonal bipyramidal and square pyramidal describe five-direction arrangements and omit the metal–metal bond in this question. Square planar has only four directions. Including all six bond directions gives an octahedral arrangement.

Step 3: Conclusion The geometry around each manganese atom, including the $Mn-Mn$ bond, is octahedral. Hence, the correct option is (D).

Quick Tip to Solve: In a metal carbonyl dimer, identify any metal–metal bond before deciding the geometry from the number of CO ligands.

79. Reduction potential of ions are given below:

ClO_4^-	IO_4^-	BrO_4^-
$E^\circ = 1.19$ V	$E^\circ = 1.65$ V	$E^\circ = 1.74$ V

The correct order of their oxidising power is : [4 marks]

(A) $IO_4^- > BrO_4^- > ClO_4^-$
(B) $BrO_4^- > ClO_4^- > IO_4^-$
(C) $ClO_4^- > IO_4^- > BrO_4^-$
(D) $BrO_4^- > IO_4^- > ClO_4^-$
Correct Answer: (D) $BrO_4^- > IO_4^- > ClO_4^-$

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. A larger standard reduction potential indicates a greater thermodynamic tendency for that reduction under standard conditions.
- Compare the given reduction potentials directly; the order of oxidising strength follows increasing E°, with the highest potential placed first.

Step 1: Arrange the supplied potentials The values supplied in the question are $E^\circ(ClO_4^-) = 1.19\,V$, $E^\circ(IO_4^-) = 1.65\,V$, and $E^\circ(BrO_4^-) = 1.74\,V$. In descending numerical order,

$$1.74\,V > 1.65\,V > 1.19\,V.$$

The bromine species therefore has the largest electron-accepting tendency among the three on the given data.

Step 2: Translate the potential order into the required order The oxidising agent with the more positive reduction potential is more readily reduced while oxidising another species. Consequently the strength decreases in the sequence

$$BrO_4^- > IO_4^- > ClO_4^-.$$

Option (A) incorrectly places the iodine species above the bromine species. Option (B) reverses the order of iodine and chlorine, while option (C) puts the lowest-potential species first. Option (D) alone follows all three stated values. This conclusion uses the supplied electrochemical data rather than a guessed periodic trend.

Step 3: Conclusion The correct oxidising-power order is $BrO_4^- > IO_4^- > ClO_4^-$. Hence, the correct option is (D).

Quick Tip to Solve: For oxidising strength, rank reduction potentials from most positive to least positive; do not reverse the list as for reducing agents.

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80. Given below are two statements:

Statement - I: Along the period, the chemical reactivity of the elements gradually increases from group 1 to group 18 .

Statement - II: The nature of oxides formed by group 1 elements is basic while that of group 17 elements is acidic.

In the light of the above statements, choose the most appropriate from the options given below:

[4 marks]

(A) Statement I is False but Statement II is true

(B) Both Statement I and Statement II are False

(C) Statement I is True But Statement II is False

(D) Both Statement I and Statement II are True

Correct Answer: (A) Statement I is False but Statement II is true

Solution:

Concept:

- Chemical reactivity across a period is not a uniformly increasing property. Metals react by losing electrons, whereas reactive nonmetals commonly gain or share electrons.
- Metallic oxides are generally basic and nonmetallic oxides generally acidic. The strongly electropositive group 1 metals and the oxide-forming halogens illustrate this contrast.

Step 1: Evaluate Statement I At the left of a period, alkali metals are highly reactive because they readily lose their single valence electron. As one moves across the metallic elements, electron loss becomes less favourable, so metallic reactivity generally decreases. Reactivity then rises among the nonmetals towards the halogens, but the noble gases in group 18 have filled valence shells and very low ordinary chemical reactivity. Thus the stated gradual increase all the way from group 1 to group 18 cannot hold. Statement I is false.

Step 2: Evaluate Statement II and select the pair Group 1 oxides are basic; for example, $Na_2O + H_2O \longrightarrow 2\,NaOH$. Halogen oxides characteristically act as acidic oxides, for example $Cl_2O_7 + H_2O \longrightarrow 2\,HClO_4$. This is the standard periodic comparison expressed in Statement II, which is true. The statement does not require every halogen to form an ordinary oxide; oxygen fluorides do not provide a contrary halogen oxide. Options (C) and (D) wrongly call Statement I true, and option (B) wrongly calls Statement II false. Only option (A) has the required truth values.

Step 3: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (A).

Quick Tip to Solve: Check the noble-gas end of a period before accepting any claim that chemical reactivity rises continuously across it.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The total number of correct statements, regarding the nucleic acids is _____.

A. RNA is regarded as the reserve of genetic information

B. DNA molecule self-duplicates during cell division

C. DNA synthesizes proteins in the cell

D. The message for the synthesis of particular proteins is present in DNA

E. Identical DNA strands are transferred to daughter cells. [4 marks]

Solution:

Concept:

- DNA stores hereditary information, replicates before cell division, and supplies the coded message that is expressed through RNA and protein synthesis.
- RNA participates in expression of this information. In the usual cellular context, DNA is the genetic-information reserve; proteins are assembled on ribosomes using the RNA machinery.

Step 1: Test the statements about information and synthesis Statement A is false in the cellular context of this question: DNA, rather than RNA, is regarded as the reserve of genetic information. Certain viruses have RNA genomes, but that does not make the given general statement correct. Statement C is also false as stated: DNA does not directly synthesize proteins. Its sequence is transcribed into RNA, and the message is translated at ribosomes, with transfer RNA supplying amino acids. Thus storage of the instructions must be distinguished from the machinery that assembles a protein.

Step 2: Test replication and count the correct statements Statement B is correct: DNA self-duplicates as part of the cell-division cycle, before the genetic material is partitioned. Statement D is correct because DNA carries the sequence information for synthesizing particular proteins. Statement E is correct in the ordinary description of faithful DNA replication and daughter-cell inheritance: equivalent genetic information is transmitted to the daughter cells. Replication is semiconservative, so each daughter DNA double helix contains one parental strand and one newly synthesized strand; this does not negate the transfer of identical sequence information. Therefore the correct statements

are B, D and E, giving

$$N = 1 + 1 + 1 = 3.$$

Step 3: Conclusion The correct statements are B, D and E. Their total number is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: Separate three roles: DNA stores the message, DNA replication copies it, and RNA plus ribosomes execute protein synthesis.

82. Number of metal ions characterized by flame test among the following is _____.

$$\mathrm{Sr}^{2+}, \mathrm{Ba}^{2+}, \mathrm{Ca}^{2+}, \mathrm{Cu}^{2+}, \mathrm{Zn}^{2+}, \mathrm{Co}^{2+}, \mathrm{Fe}^{2+}$$

[4 marks]

Solution:

Concept:

- A flame test identifies certain metal ions through characteristic emission colours. Heating excites electrons, and emission as they return to lower energy states produces the observed colour.
- The question asks about the ordinary qualitative flame test, not every possible atomic-emission measurement or a borax-bead test. A coloured ion or solution need not give a diagnostic flame colour.

Step 1: Identify the ions with characteristic flame colours Among the listed ions, Sr^{2+} gives a crimson-red flame, Ba^{2+} gives an apple-green flame, and Ca^{2+} gives a brick-red flame. Copper compounds, represented here by Cu^{2+}, impart a characteristic bluish-green colour. These are the four standard positive identifications in this list. The observed emission involves excited atomic or molecular species produced in the flame; it is not simply the visible colour of the original aqueous ion.

Step 2: Exclude other tests and count the qualifying ions Zn^{2+}, Co^{2+} and Fe^{2+} are not characterized by a distinctive ordinary flame colour in this qualitative-analysis list. Cobalt can give a blue borax bead, but that is a different test and must not be included. Likewise, the colour of an iron solution or sparks from burning iron do not constitute the characteristic ionic flame test being asked. The qualifying set is therefore

$$\{\mathrm{Sr}^{2+}, \mathrm{Ba}^{2+}, \mathrm{Ca}^{2+}, \mathrm{Cu}^{2+}\},$$

which contains four members.

Step 3: Conclusion The number of metal ions characterized by the flame test is $\boxed{4}$, a dimensionless count.

Quick Tip to Solve: Do not count cobalt merely because its borax bead is blue: distinguish flame colour, solution colour and bead-test colour.

83. Number of spectral lines obtained in He^+ spectra, when an electron makes transition from fifth excited state to first excited state will be [4 marks]

Solution:

Concept:

- He^+ is a one-electron ion. Every distinct downward transition between two allowed principal energy levels gives a spectral line with an energy fixed by those two levels.
- The ground state has $n = 1$, so the first excited state is $n = 2$ and the fifth excited state is $n = 6$. For all possible cascades among m levels, the number of distinct level pairs is $m(m-1)/2$.

Step 1: Identify the levels included in the cascade The electron starts in the fifth excited state, so its initial principal quantum number is six, not five. It ultimately reaches the first excited state, whose principal quantum number is two. Across the possible de-excitation paths, the relevant levels are

$$n = 6, 5, 4, 3, 2.$$

There are five levels in this interval. A direct transition produces one line for one path, but the question asks the number of lines obtainable in the spectrum from all possible transitions during the cascade.

Step 2: Count all downward transitions in this interval Transitions starting at $n = 6$ can end at four lower levels in the interval; those from $n = 5$ at three, from $n = 4$ at two, and from $n = 3$ at one. Thus

$$N = 4 + 3 + 2 + 1 = \binom{5}{2} = 10.$$

Levels below $n = 2$ are excluded because the specified final state is the first excited state. The nuclear charge of helium changes the line frequencies through the hydrogen-like energy formula, but does not change this count of transitions. Counting six levels down to the ground state would incorrectly give fifteen lines.

Step 3: Conclusion The number of obtainable spectral lines is $\boxed{10}$, a dimensionless count.

Quick Tip to Solve: Convert excited-state names to principal quantum numbers first, then count only the levels between the stated initial and final states.

84. Total number of species from the following which can undergo disproportionation reaction is _____.

$$H_2O_2, ClO_3^-, P_4, Cl_2, Ag, Cu^{+1}, F_2, NO_2, K^+$$

[4 marks]

Solution:

Concept:

- In disproportionation, the same element in one starting oxidation state is simultaneously oxidised and reduced. The reactant must have access to both a higher and a lower oxidation state.

- An intermediate oxidation state is a useful screening test, but an actual disproportionation pathway confirms the selection. Species already at a relevant limiting state do not qualify here.

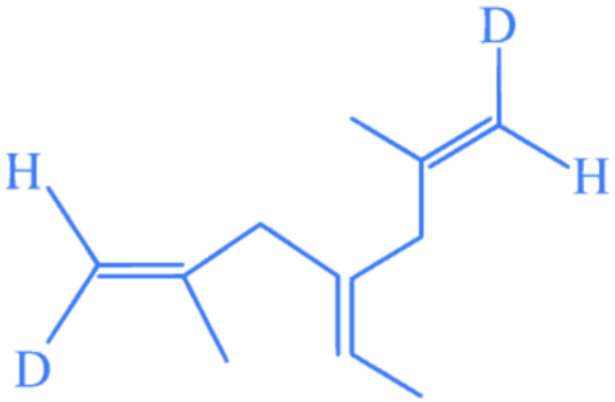
Step 1: Identify the species that undergo both changes H_2O_2 disproportionates into water and oxygen: oxygen changes from -1 to -2 and 0. In ClO_3^-, chlorine is $+5$, and chlorate can form perchlorate and chloride, with chlorine at $+7$ and -1. White phosphorus P_4 reacts in alkali to form phosphine and hypophosphite, changing phosphorus from 0 to -3 and $+1$. Chlorine similarly gives chloride and hypochlorite in cold dilute alkali. These four therefore qualify.

Step 2: Complete the list and exclude limiting states Cu^+ disproportionates by $2\,Cu^+ \longrightarrow Cu^{2+} + Cu$, with copper changing from $+1$ to $+2$ and 0. NO_2 reacts with water to give nitrous and nitric acids, so nitrogen changes from $+4$ to $+3$ and $+5$. These supply two more species. Metallic silver Ag does not undergo the required simultaneous oxidation and reduction under the usual chemistry considered here. F_2 can only be reduced to fluoride because fluorine has no positive oxidation states in its compounds. K^+ is already at potassium's stable highest oxidation state and cannot be oxidised further. Hence the qualifying species are H_2O_2, ClO_3^-, P_4, Cl_2, Cu^+ and NO_2.

Step 3: Conclusion Six listed species undergo disproportionation, so the required dimensionless count is $\boxed{6}$.

Quick Tip to Solve: For each candidate, write one product with a higher oxidation state and one with a lower state; count the reactant only when both are possible.

85. Number of geometrical isomers possible for the given structure is/are _____. [4 marks]



Solution:

Concept:

- An alkene has geometrical isomers only if each double-bonded carbon has two distinguishable substituents. Hydrogen and deuterium are distinguishable isotopes, so the terminal double bonds can each have two configurations.
- Equivalent ends must not be counted twice. When the two side chains on the central double-bonded carbon are different, that central double bond can also have two configurations.

Step 1: Identify the three double bonds The left and right terminal double bonds each have H and D

on one carbon, and a methyl group and the remaining chain on the other carbon. Thus each terminal bond admits E and Z configurations. The central double bond has H and a methyl group on its lower carbon. Its upper carbon is attached to two constitutionally identical side chains, whose configurations must be compared before counting the central bond.

Step 2: Group equivalent terminal configurations Interchanging the two equivalent side chains does not produce a new geometrical isomer. The terminal combinations therefore fall into three classes: EE, ZZ, and mixed EZ. The ordered descriptions EZ and ZE identify the same pair of side chains; counting them as separate arrangements would duplicate structures.

Step 3: Count the central configurations For EE, the central carbon carries two identical E-configured side chains, so the central double bond has no independent geometrical choice: one isomer results. The same argument gives one isomer for ZZ. For the mixed pair, the E-configured and Z-configured side chains are distinguishable, so their positions relative to the lower methyl group give two central geometrical arrangements. Therefore

$$N = 1 + 1 + 2 = 4.$$

Step 4: Conclusion The number of geometrical isomers is $\boxed{4}$ (a dimensionless count).

Quick Tip to Solve: For a symmetric polyene, first group equivalent terminal configurations; only then test whether the central double bond has distinguishable substituents.

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86. The pH of an aqueous solution containing $1\mathrm{M}$ benzoic acid $(\mathrm{pK_a} = 4.20)$ and $1\mathrm{M}$ sodium benzoate is 4.5. The volume of benzoic acid solution in $300\ \mathrm{mL}$ of this buffer solution is _____ mL. (given : $\log 2 = 0.3$) [4 marks]

Solution:

Concept:

- A buffer containing a weak acid HA and its conjugate base obeys $\mathrm{pH} = \mathrm{p}K_a + \log([\mathrm{A}^-]/[\mathrm{HA}])$. Benzoic acid and sodium benzoate supply this conjugate pair.
- Both stock solutions have the same molarity. After mixing, their concentration ratio is their mole ratio, which equals the ratio of the stock-solution volumes.

Step 1: Find the conjugate-base to acid ratio Substitute the specified buffer pH and acid constant in the Henderson–Hasselbalch equation:

$$4.50 = 4.20 + \log\left(\frac{[\mathrm{C_6H_5COO^-}]}{[\mathrm{C_6H_5COOH}]}\right).$$

Consequently the logarithm is 0.30. Using the given $\log 2 = 0.3$, the required concentration ratio is 2, with benzoate in excess because the pH exceeds the acid pKa.

Step 2: Convert the ratio to stock-solution volumes Let the volume of 1 M benzoic acid be $V\ \mathrm{mL}$. The volume of 1 M sodium benzoate is then $(300 - V)\ \mathrm{mL}$. The common stock molarity and common final

dilution volume cancel when the two final concentrations are divided. Thus

$$\frac{n_{\text{salt}}}{n_{\text{acid}}} = \frac{1(300 - V)}{1V} = 2.$$

This equation uses the amounts mixed; interpreting both quoted 1 M values as final concentrations would instead give equal amounts and the wrong pH.

Step 3: Solve and check the volumes From $300 - V = 2V$, obtain $3V = 300$ and $V = 100\,\text{mL}$. The complementary salt volume is $200\,\text{mL}$. Their ratio is 2, so substitution gives $4.20 + \log 2 = 4.50$, confirming both the required pH and the stated total volume.

Step 4: Conclusion The volume of benzoic acid solution required is $\boxed{100\,\text{mL}}$.

Quick Tip to Solve: With equal-molarity acid and salt stock solutions, use the volume ratio directly in the buffer equation; the larger volume belongs to the conjugate base when pH exceeds pKa.

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87. NO_2 **required for a reaction is produced by decomposition of** N_2O_5 **in** CCl_4 **as by equation**

$$2\,N_2O_{5(\,g)} \rightarrow 4NO_{2(\,g)} + O_{2(\,g)}$$

The initial concentration of N_2O_5 **is** $3\ \text{mol L}^{-1}$ **and it is** $2.75\ \text{mol L}^{-1}$ **after 30 minutes.**
The rate of formation of NO_2 **is** $x \times 10^{-3}\ \text{mol L}^{-1}\ \text{min}^{-1}$**, value of** x **is _____. (nearest integer)**

[4 marks]

Solution:

Concept:

- Over a finite time interval, the average disappearance rate of a reactant is its decrease in concentration divided by the elapsed time.
- For $2\,N_2O_5 \longrightarrow 4\,NO_2 + O_2$, two moles of NO_2 form per mole of N_2O_5 consumed. Species formation rates therefore differ from the stoichiometrically normalized reaction rate.

Step 1: Calculate the amount of reactant consumed The concentration decreases from 3.00 to $2.75\,\text{mol L}^{-1}$ in 30 minutes. Hence

$$-\Delta[N_2O_5] = 3.00 - 2.75 = 0.25\,\text{mol L}^{-1}.$$

Use the positive decrease when reporting a disappearance rate. The data specify a finite interval, so the calculation gives its average rate and requires no assumed rate law or rate constant.

Step 2: Apply the stoichiometric ratio to nitrogen dioxide The balanced equation gives $\Delta[NO_2] = \frac{4}{2}(-\Delta[N_2O_5])$. Thus the nitrogen dioxide concentration formed is

$$\Delta[NO_2] = 2(0.25) = 0.50\,\text{mol L}^{-1}.$$

Equivalently, the normalized rate satisfies $-\frac{1}{2}\Delta[N_2O_5]/\Delta t = \frac{1}{4}\Delta[NO_2]/\Delta t$. The requested quantity is the product formation rate itself, not this normalized rate.

Step 3: Express the rate in the requested form Dividing the product increase by the elapsed time,

$$\bar{r}_{NO_2} = \frac{0.50}{30} = \frac{1}{60}\,\text{mol}\,\text{L}^{-1}\,\text{min}^{-1} = 16.666\ldots \times 10^{-3}\,\text{mol}\,\text{L}^{-1}\,\text{min}^{-1}.$$

Therefore $x = 16.666\ldots$, whose nearest integer is 17. Keeping minutes throughout avoids an unnecessary factor of 60.

Step 4: Conclusion The average formation rate is $1.6667 \times 10^{-2}\,\text{mol}\,\text{L}^{-1}\,\text{min}^{-1}$; the requested nearest integer is $\boxed{x = 17}$.

Quick Tip to Solve: First multiply the reactant concentration loss by the product-to-reactant coefficient ratio, then divide by time; round only the final requested coefficient.

88. Number of complexes which show optical isomerism among the following is _____.

cis-$[Cr(ox)_2Cl_2]^{3-}$, $[Co(en)_3]^{3+}$,
cis-$[Pt(en)_2Cl_2]^{2+}$, **cis-**$[Co(en)_2Cl_2]^{+}$,
trans-$[Pt(en)_2Cl_2]^{2+}$, **trans-**$[Cr(ox)_2Cl_2]^{3-}$

[4 marks]

Solution:

Concept:

- An optically active coordination isomer is chiral: its mirror image cannot be superimposed on it. Bidentate ligands such as en (ethane-1,2-diamine) and ox (oxalato) form chelate rings that can give a handed octahedral arrangement.
- The tris-chelate form $[M(AA)_3]$ and the cis bis-chelate form $[M(AA)_2B_2]$ have optical isomers. The corresponding trans bis-chelate form with these symmetric bidentate ligands is achiral.

Step 1: Classify the first two complexes The cis dichloridobis(oxalato)chromate(III) complex, cis-$[Cr(ox)_2Cl_2]^{3-}$, has two adjacent chlorido ligands and two chelate rings arranged in a handed way. It therefore has a pair of optical isomers. The tris(ethane-1,2-diamine)cobalt(III) ion, $[Co(en)_3]^{3+}$, also has the familiar right-handed and left-handed chelate arrangements, denoted Δ and Λ. Both entries qualify.

Step 2: Classify the remaining cis and trans forms The third and fourth entries, cis-$[Pt(en)_2Cl_2]^{2+}$ and cis-$[Co(en)_2Cl_2]^{+}$, are also cis octahedral bis-chelate arrangements and are optically active. In the two trans entries, the chlorido ligands lie opposite each other; the symmetric chelate arrangement has an internal symmetry operation relating the two halves and is superimposable on its mirror image. Neither trans complex qualifies. The platinum complexes here are six-coordinate Pt(IV), so treating them as square-planar Pt(II) would misclassify them.

Step 3: Count complexes rather than individual enantiomers The first four listed complexes show optical isomerism and the last two do not. Thus

$$N_{\text{complexes}} = 1 + 1 + 1 + 1 + 0 + 0 = 4.$$

Each qualifying entry can have two enantiomers, but the question asks how many listed complexes show the phenomenon, not how many enantiomeric structures they collectively produce.

Step 4: Conclusion Exactly $\boxed{4}$ of the listed complexes show optical isomerism (a dimensionless count).

Quick Tip to Solve: For symmetric bidentate ligands in octahedral complexes, distinguish cis bis-chelates from trans bis-chelates, and count qualifying entries rather than doubling for their enantiomers.

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89. 2-chlorobutane $+Cl_2 \rightarrow C_4H_8Cl_2$ (isomers)

Total number of optically active isomers shown by $C_4H_8Cl_2$, obtained in the above reaction is _____.

[4 marks]

Solution:

Concept:

- Further substitution of 2-chlorobutane replaces a hydrogen by chlorine while retaining the four-carbon chain. Distinct substitution sites give distinct constitutional dichlorobutanes.
- A tetrahedral carbon is stereogenic when it has four different substituents. Enantiomeric forms count separately, but a meso structure is optically inactive and must be excluded.

Step 1: List the distinct dichlorinated products Write 2-chlorobutane as $CH_3{-}CHCl{-}CH_2{-}CH_3$. Replacing a hydrogen at each of its four carbon positions gives 1,2-dichlorobutane, 2,2-dichlorobutane, 2,3-dichlorobutane, and 1,3-dichlorobutane (the last name follows renumbering from the nearer end). This exhausts the carbon sites without introducing rearranged skeletons or products that require moving the original chlorine.

Step 2: Count the products with one stereogenic centre Consider 1,2-dichlorobutane:

$$CH_2Cl{-}CHCl{-}CH_2{-}CH_3.$$

Here carbon 2 is attached to H, Cl, chloromethyl and ethyl groups. These are four different substituents, giving two enantiomers. For 1,3-dichlorobutane,

$$CH_2Cl{-}CH_2{-}CHCl{-}CH_3,$$

the chlorinated methine likewise has four different substituents and gives two enantiomers. The geminal product 2,2-dichlorobutane has two identical chlorine substituents on the same carbon and no stereogenic centre, so contributes none.

Step 3: Handle the symmetric two-centre product For 2,3-dichlorobutane, the two stereogenic centres give the $(2R, 3R)$ and $(2S, 3S)$ enantiomeric pair and the meso $(2R, 3S)$ form. The meso form has internal symmetry and is optically inactive, so only two of these three stereoisomers enter the requested count. Hence

$$N_{\text{optically active}} = 2 + 0 + 2 + 2 = 6.$$

A racemic sample may have zero net rotation, but that does not make its individual enantiomeric structures achiral.

Step 4: Conclusion The total number of optically active isomers is $\boxed{6}$ (a dimensionless count).

Quick Tip to Solve: Enumerate substitution positions before counting stereoisomers, then remove meso forms; distinguish an achiral molecule from a racemic mixture of chiral molecules.

90. Two reactions are given below:

$$2Fe_{(s)} + \frac{3}{2}O_{2(\ g)} \rightarrow Fe_2O_{3(\ s)}, \Delta H^\circ = -822\ kJ/mol$$
$$C_{(s)} + \frac{1}{2}O_{2(\ g)} \rightarrow CO_{(g)}, \Delta H^\circ = -110\ kJ/mol$$

Then enthalpy change for following reaction $3C_{(s)} + Fe_2O_{3(\ s)} \rightarrow 2Fe_{(s)} + 3CO_{(g)}$ **is _____** kJ/mol**.**

[4 marks]

Solution:

Concept:

- Enthalpy is a state function, so the enthalpy of a target reaction is the sum of the enthalpies of reactions that add algebraically to it.
- Reversing a reaction changes the sign of its enthalpy. Multiplying a chemical equation by a coefficient multiplies its enthalpy by the same coefficient.

Step 1: Reverse the iron oxide formation reaction The target consumes one mole of iron(III) oxide and produces two moles of iron. Reverse the first supplied equation:

$$Fe_2O_3(s) \longrightarrow 2\,Fe\,(s) + \frac{3}{2}\,O_2(g).$$

Its enthalpy changes from -822 to $+822\,kJ\,mol^{-1}$, because decomposing the oxide reverses the direction of energy transfer for its formation. No further scaling is required for this equation.

Step 2: Scale the carbon monoxide formation reaction The target consumes three moles of carbon and produces three moles of carbon monoxide. Multiply the second supplied equation by three:

$$3\,C\,(s) + \frac{3}{2}\,O_2(g) \longrightarrow 3\,CO\,(g).$$

The associated enthalpy is

$$3(-110) = -330\,kJ\,mol^{-1}.$$

This uses carbon monoxide, not carbon dioxide; replacing CO by another oxide would correspond to a different reaction and energy change.

Step 3: Add the equations and their enthalpies The $\frac{3}{2}$ moles of oxygen produced in oxide decomposition cancel exactly with the oxygen consumed in carbon monoxide formation. The net equation is therefore

$$3\,C\,(s) + Fe_2O_3(s) \longrightarrow 2\,Fe\,(s) + 3\,CO\,(g),$$

which matches the target in every species and coefficient. Add the signed enthalpies:

$$\Delta H^\circ = +822 - 330 = +492\,\text{kJ mol}^{-1}.$$

The positive sign means the target reaction is endothermic as written, even though both supplied formation equations individually have negative enthalpies.

Step 4: Conclusion The enthalpy change for the reaction as written is $\boxed{+492\,\text{kJ mol}^{-1}}$.

Quick Tip to Solve: Write the scaled and reversed equations first, cancel every intermediate species, and only then add enthalpies with their new signs.