JEE Main 2024 Paper 1 (B.E./B.Tech) Jan 31 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 1 (Jan-Feb), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 31 January 2024, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is [4 marks]

(A) 130

(B) 136

(C) 142

(D) 406

Correct Answer: (B) 136

Solution:

Concept:

- The apples are identical, but the three children are distinct recipients. A distribution is therefore an ordered triple of integer allocations, rather than a choice of labelled apples.
- The number of nonnegative integer solutions of $u + v + w = N$ is $\binom{N+2}{2}$, obtained by placing two separators among N identical objects.

Step 1: Reserve the compulsory apples Let the children receive x, y, z apples. The restrictions are $x + y + z = 21$ and $x, y, z \geq 2$. First give two apples to each child. This uses six apples and leaves fifteen apples to distribute freely. Define $u = x - 2$, $v = y - 2$, and $w = z - 2$. Then

$$u + v + w = 15, \qquad u, v, w \geq 0.$$

Every allowed original distribution corresponds to exactly one such triple, so this change of variables neither omits nor repeats any possibility.

Step 2: Count the unrestricted allocations Represent the remaining fifteen apples by fifteen stars and separate the shares of the three children using two bars. Empty shares of these remaining apples are allowed, because each child already has the compulsory two. There are seventeen positions occupied by fifteen stars and two bars; choosing the two bar positions gives

$$\binom{17}{2} = \frac{17 \cdot 16}{2} = 136.$$

We do not divide by $3!$, since changing which child receives a given share generally changes the distribution.

Step 3: Check the listed alternatives The residual total is fifteen, not twenty-one, so applying stars and bars directly to all the apples would ignore the lower bounds. The count 136 agrees exactly with option (B); 130, 142, and 406 do not equal the binomial coefficient dictated by the one-to-one allocation model.

Step 4: Conclusion There are $\boxed{136}$ possible distributions. Hence, the correct option is (B).

Quick Tip to Solve: For identical objects with lower bounds, subtract the compulsory share of every recipient before applying stars and bars.

2. A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is [4 marks]

(A) $\frac{1}{9}$
(B) $\frac{2}{9}$
(C) $\frac{1}{27}$
(D) $\frac{2}{27}$

Correct Answer: (B) $\frac{2}{9}$

Solution:

Concept:

- Independent tosses multiply probabilities within a specified sequence. Mutually exclusive sequences leading to the same requested outcome have their probabilities added.
- If head has probability p and tail has probability q, then $p+q=1$. Exactly one head in three tosses has probability $\binom{3}{1}pq^2$.

Step 1: Determine the biased probabilities The condition that a head is twice as likely as a tail means $p=2q$, not $p=2$. Combining this relation with total probability gives

$$2q+q=1 \quad \Longrightarrow \quad q=\frac{1}{3}, \qquad p=\frac{2}{3}.$$

Both probabilities are positive and sum to one, as required. Using one-half for either outcome would incorrectly treat this biased coin as fair.

Step 2: Count and evaluate the favourable sequences Two tails and one head can occur as HTT, THT, or TTH. These are three distinct, mutually exclusive sequences. By independence, each has probability

$$pq^2=\frac{2}{3}\left(\frac{1}{3}\right)^2=\frac{2}{27}.$$

Thus the total probability is

$$P(\text{two tails and one head})=3\cdot\frac{2}{27}=\frac{2}{9}.$$

Equivalently, the factor three is $\binom{3}{1}$, the number of choices for the position of the single head.

Step 3: Distinguish the competing options Option (D), $2/27$, counts only one prescribed ordering instead of all three orderings. Option (C), $1/27$, is the probability of three tails and has the wrong event. Option (A), $1/9$, is the probability of tails on two prescribed tosses without imposing the required head on the remaining toss. Only option (B) includes both the bias and all allowed head positions.

Step 4: Conclusion The required probability is $\boxed{\frac{2}{9}}$. Hence, the correct option is (B).

Quick Tip to Solve: For an exact number of heads, first find the probability of one ordering and then multiply by the number of possible head positions.

3. Let A be a 3×3 real matrix such that

$$A\begin{pmatrix}1\\0\\1\end{pmatrix} = 2\begin{pmatrix}1\\0\\1\end{pmatrix}, A\begin{pmatrix}-1\\0\\1\end{pmatrix} = 4\begin{pmatrix}-1\\0\\1\end{pmatrix}, A\begin{pmatrix}0\\1\\0\end{pmatrix} = 2\begin{pmatrix}0\\1\\0\end{pmatrix}.$$

Then, the system $(A-3I)\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ **has :** [4 marks]

(A) exactly two solutions

(B) infinitely many solutions

(C) unique solution

(D) no solution

Correct Answer: (C) unique solution

Solution:

Concept:

- The stated action of a matrix on a basis determines the matrix uniquely. The vectors $(1,0,1)^T$, $(-1,0,1)^T$, and $(0,1,0)^T$ are linearly independent.
- A square system $B\mathbf{x} = \mathbf{c}$ has a unique solution for every right-hand side if $\det B \neq 0$. Here I is the identity matrix.

Step 1: Recover the columns of the matrix Write $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ for the standard unit vectors. The first two given relations imply

$$A\mathbf{e}_1 + A\mathbf{e}_3 = 2\mathbf{e}_1 + 2\mathbf{e}_3,$$

$$-A\mathbf{e}_1 + A\mathbf{e}_3 = -4\mathbf{e}_1 + 4\mathbf{e}_3.$$

Adding and subtracting these equations gives $A\mathbf{e}_3 = -\mathbf{e}_1 + 3\mathbf{e}_3$ and $A\mathbf{e}_1 = 3\mathbf{e}_1 - \mathbf{e}_3$. The third given relation gives $A\mathbf{e}_2 = 2\mathbf{e}_2$.

Step 2: Solve the resulting system These three column vectors give

$$A = \begin{pmatrix}3 & 0 & -1\\0 & 2 & 0\\-1 & 0 & 3\end{pmatrix}, \qquad A - 3I = \begin{pmatrix}0 & 0 & -1\\0 & -1 & 0\\-1 & 0 & 0\end{pmatrix}.$$

Therefore the printed system reduces to

$$-z = 1, \qquad -y = 2, \qquad -x = 3.$$

It fixes every unknown: $(x, y, z) = (-3, -2, -1)$. Direct substitution yields the right-hand side $(1,2,3)^T$.

Step 3: Establish uniqueness and eliminate alternatives The determinant of $A - 3I$ is 1, so this coefficient matrix is invertible. Consequently no free parameter remains, excluding infinitely many solutions, and the exhibited solution excludes no solution. A linear system cannot have exactly two

isolated solutions: two distinct solutions would generate infinitely many by affine combinations. Here there is precisely the one solution already computed.

Step 4: Conclusion The system has a unique solution, namely $(x, y, z) = (-3, -2, -1)$. Hence, the correct option is (C).

Quick Tip to Solve: When a matrix is specified on simple vector combinations, add and subtract those relations to obtain its columns before solving the system.

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4. Let (α, β, γ) be the mirror image of the point $(2, 3, 5)$ in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Then, $2\alpha + 3\beta + 4\gamma$ is equal to [4 marks]

(A) 32

(B) 31

(C) 33

(D) 34

Correct Answer: (C) 33

Solution:

Concept:

- For reflection of a point P in a line, the perpendicular foot H is the midpoint of P and its image P': $P' = 2H - P$.
- The displacement from a point to its reflected image is perpendicular to the reflecting line. Its scalar product with a direction vector of that line is therefore zero.

Step 1: Identify the scalar product that is preserved The line has a point $A = (1, 2, 3)$ and direction vector $\mathbf{d} = (2, 3, 4)$. If $P = (2, 3, 5)$ and $P' = (\alpha, \beta, \gamma)$, the midpoint construction gives $P' - P = 2(H - P)$. Since PH is perpendicular to the line,

$$\mathbf{d} \cdot (P' - P) = 0.$$

Thus reflection preserves the component parallel to $\mathbf{d}$, even though it reverses the perpendicular component.

Step 2: Evaluate the required expression Taking scalar products immediately gives

$$2\alpha + 3\beta + 4\gamma = \mathbf{d} \cdot P' = \mathbf{d} \cdot P.$$

Substitute the original point:

$$\mathbf{d} \cdot P = 2(2) + 3(3) + 4(5) = 4 + 9 + 20 = 33.$$

The coefficients in the requested expression are exactly the direction ratios of the reflecting line, which is why the full image coordinates are unnecessary.

Step 3: Verify through the perpendicular foot For a coordinate check put $H = A + t\mathbf{d}$. Orthogonality

gives

$$t = \frac{(P - A) \cdot \mathbf{d}}{\mathbf{d} \cdot \mathbf{d}} = \frac{2 + 3 + 8}{4 + 9 + 16} = \frac{13}{29}.$$

Hence $H = (55, 97, 139)/29$, and $P' = 2H - P = (52, 107, 133)/29$. Its required scalar product is $(104 + 321 + 532)/29 = 33$, confirming the invariant calculation. The other proposed values, 32, 31, and 34, violate the preserved parallel component.

Step 4: Conclusion The value of $2\alpha + 3\beta + 4\gamma$ is $\boxed{33}$. Hence, the correct option is (C).

Quick Tip to Solve: When the requested linear combination uses the line direction ratios, use preservation of the parallel projection instead of calculating the whole reflected point.

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5. If $a = \sin^{-1}(\sin(5))$ and $b = \cos^{-1}(\cos(5))$, then $a^2 + b^2$ is equal to [4 marks]

(A) 25

(B) $4\pi^2 + 25$

(C) $8\pi^2 - 40\pi + 50$

(D) $4\pi^2 - 20\pi + 50$

Correct Answer: (C) $8\pi^2 - 40\pi + 50$

Solution:

Concept:

- The principal range of $\sin^{-1}$ is $[-\pi/2, \pi/2]$, whereas the principal range of $\cos^{-1}$ is $[0, \pi]$. The input angle is in radians.
- Periodic identities preserve the sine or cosine value, but an inverse function returns the representative in its prescribed principal range.

Step 1: Reduce the sine argument to its principal range The angle 5 lies between $3\pi/2$ and 2π, so its sine is negative. Subtracting one full revolution gives

$$5 - 2\pi \in (-\pi/2, 0), \qquad \sin(5 - 2\pi) = \sin 5.$$

This reduced angle already lies in the principal range of inverse sine. Therefore

$$a = \sin^{-1}(\sin 5) = 5 - 2\pi.$$

Cancelling inverse sine and sine to obtain $a = 5$ would be invalid because 5 is outside that principal range.

Step 2: Reduce the cosine argument to its principal range Cosine is even and periodic. The angle $2\pi - 5$ lies in $(0, \pi/2)$, and

$$\cos(2\pi - 5) = \cos 5.$$

It is therefore the principal inverse-cosine value:

$$b = \cos^{-1}(\cos 5) = 2\pi - 5.$$

Notice that $b = -a$; the two principal angles have equal magnitude and opposite signs.

Step 3: Square and compare the options Using the opposite values,

$$a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2 = 2(5 - 2\pi)^2.$$

Expanding gives

$$a^2 + b^2 = 2(25 - 20\pi + 4\pi^2) = 8\pi^2 - 40\pi + 50.$$

Numerically this is about 3.29, consistent with principal angles of magnitude about 1.283. Option (A) wrongly retains the original angle; (B) and (D) have coefficients inconsistent with the expansion of both squares. Only (C) is the exact result.

Step 4: Conclusion We obtain $\boxed{a^2 + b^2 = 8\pi^2 - 40\pi + 50}$. Hence, the correct option is (C).

Quick Tip to Solve: Locate the original angle first, then choose the sine and cosine representatives in their different principal ranges before doing algebra.

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6. Let P be a parabola with vertex $(2, 3)$ and directrix $2x + y = 6$. Let an ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$, of eccentricity $\frac{1}{\sqrt{2}}$ pass through the focus of the parabola P. Then, the square of the length of the latus rectum of E, is [4 marks]

(A) $\frac{512}{25}$
(B) $\frac{656}{25}$
(C) $\frac{385}{8}$
(D) $\frac{347}{8}$

Correct Answer: (B) $\frac{656}{25}$

Solution:

Concept:

- The vertex of a parabola is halfway between its focus and the perpendicular foot on its directrix. Reflecting that foot in the vertex therefore gives the focus.
- For $x^2/a^2 + y^2/b^2 = 1$ with $a > b$, $e^2 = 1 - b^2/a^2$ and the full latus-rectum length is $L = 2b^2/a$.

Step 1: Find the parabola focus The vertex is $V = (2, 3)$, and the directrix is $2x + y - 6 = 0$. Its normal vector is $(2, 1)$. The perpendicular foot from V to the directrix is

$$D = (2, 3) - \frac{2(2) + 3 - 6}{2^2 + 1^2}(2, 1) = \left(\frac{8}{5}, \frac{14}{5}\right).$$

Since the vertex is the midpoint between this foot and the focus,

$$F = 2V - D = \left(\frac{12}{5}, \frac{16}{5}\right).$$

The displacement from the vertex toward the focus points away from the directrix, as required.

Step 2: Use the eccentricity and the point on the ellipse The given eccentricity satisfies $e^2 = 1/2$, hence

$$b^2 = a^2(1 - e^2) = \frac{a^2}{2}.$$

The ellipse passes through F, so substitution in its equation gives

$$\frac{144}{25a^2} + \frac{256}{25b^2} = 1.$$

Replacing b^2 by $a^2/2$ yields

$$\frac{144 + 512}{25a^2} = 1 \quad \Longrightarrow \quad a^2 = \frac{656}{25}, \qquad b^2 = \frac{328}{25}.$$

These values satisfy $a > b > 0$, consistent with the specified major axis.

Step 3: Square the full latus-rectum length The requested quantity is the square of the full latus rectum, rather than the semi-latus rectum. Therefore

$$L^2 = \left(\frac{2b^2}{a}\right)^2 = \frac{4b^4}{a^2} = \frac{4(a^2/2)^2}{a^2} = a^2 = \frac{656}{25}.$$

Thus option (B) agrees with both geometric conditions. Options (A), (C), and (D) do not equal the value forced by the focus and eccentricity; using the directrix foot in place of the focus or using half the latus rectum would produce the wrong ellipse or length.

Step 4: Conclusion The square of the length of the latus rectum is $\boxed{\frac{656}{25}}$. Hence, the correct option is (B).

Quick Tip to Solve: Obtain the focus by reflecting the directrix foot in the vertex, and distinguish the full latus rectum $2b^2/a$ from its half-length.

7. The number of solutions, of the equation $e^{\sin x} - 2e^{-\sin x} = 2$, is : [4 marks]

(A) 0

(B) 1

(C) 2

(D) more than 2

Correct Answer: (A) 0

Solution:

Concept:

- For real x, $-1 \le \sin x \le 1$. Consequently $t = e^{\sin x}$ is positive and belongs to $[e^{-1}, e]$.
- An algebraic root obtained after a substitution must lie in the range of the substituted expression. Positivity alone is insufficient when the exponent is bounded.

Step 1: Convert the equation into a quadratic Put $t = e^{\sin x}$. Then $e^{-\sin x} = 1/t$, and $t > 0$, so

multiplication by t is legitimate and reversible. The equation becomes

$$t - \frac{2}{t} = 2 \quad \Longleftrightarrow \quad t^2 - 2t - 2 = 0.$$

Applying the quadratic formula gives

$$t = \frac{2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3}.$$

The root $1 - \sqrt{3}$ is negative and cannot be an exponential. Thus any real solution would have to satisfy $e^{\sin x} = 1 + \sqrt{3}$.

Step 2: Check the remaining root against the exponential range The largest possible value of $e^{\sin x}$ is e, attained when $\sin x = 1$. However,

$$1 + \sqrt{3} \approx 2.73205 > e \approx 2.71828.$$

Hence the remaining positive root is also outside the allowed range. Equivalently, taking logarithms would demand $\sin x = \ln(1 + \sqrt{3}) > 1$, which is impossible for real x. The discrepancy is small, so checking the bound is essential.

Step 3: Confirm why no periodic families arise As an alternative check, $f(s) = e^s - 2e^{-s}$ has derivative $f'(s) = e^s + 2e^{-s} > 0$. On $[-1, 1]$ its maximum is therefore $f(1) = e - 2/e \approx 1.98252 < 2$. The original left side never reaches the required right side. Thus neither a single root nor two roots nor infinitely repeated trigonometric roots are possible; all the positive-count alternatives fail the same range test.

Step 4: Conclusion The equation has $\boxed{0}$ real solutions. Hence, the correct option is (A).

Quick Tip to Solve: After substituting $t = e^{\sin x}$, retain the upper bound $t \leq e$; accepting every positive quadratic root can create false trigonometric solutions.

8. The shortest distance, between lines L_1 and L_2, where $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line, passing through the points $\mathrm{A}(-4, 4, 3), \mathrm{B}(-1, 6, 3)$ and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$, is

[4 marks]

(A) $\frac{141}{\sqrt{221}}$
(B) $\frac{24}{\sqrt{117}}$
(C) $\frac{42}{\sqrt{117}}$
(D) $\frac{121}{\sqrt{221}}$

Correct Answer: (A) $\frac{141}{\sqrt{221}}$

Solution:

Concept:

- A line is specified by one point on it and a direction vector. The direction of the line through two points is obtained by subtracting their position vectors.

- For points P_1, P_2 on two nonparallel lines with directions $\mathbf{u}, \mathbf{v}$, their shortest distance is $d = |\overrightarrow{P_1P_2} \cdot (\mathbf{u} \times \mathbf{v})|/|\mathbf{u} \times \mathbf{v}|$.

Step 1: Identify the points and directions From the symmetric equations of L_1, take $P_1 = (1, -1, -4)$ and $\mathbf{u} = (2, -3, 2)$. For L_2, use $P_2 = A = (-4, 4, 3)$ and $\mathbf{v} = \overrightarrow{AB} = (3, 2, 0)$. The additional perpendicularity condition is consistent, since $(3, 2, 0) \cdot (-2, 3, 1) = -6 + 6 = 0$. Thus it does not require changing the direction of the line through A and B.

Step 2: Evaluate the scalar triple product The normal common to the two directions and the connecting vector are

$$\mathbf{u} \times \mathbf{v} = (-4, 6, 13), \qquad \overrightarrow{P_1P_2} = (-5, 5, 7).$$

Therefore

$$|\mathbf{u} \times \mathbf{v}| = \sqrt{16 + 36 + 169} = \sqrt{221},$$
$$|\overrightarrow{P_1P_2} \cdot (\mathbf{u} \times \mathbf{v})| = |20 + 30 + 91| = 141.$$

The cross product is nonzero, so the skew-line formula applies. The nonzero triple product also confirms the lines do not intersect.

Step 3: Compare with the listed distances Dividing gives $d = 141/\sqrt{221}$. In particular, the denominator is determined by the cross product, so the two options containing $\sqrt{117}$ cannot be correct. Option (D) has the correct denominator but an incorrect scalar triple product in its numerator.

Step 4: Conclusion The shortest distance is $\boxed{141/\sqrt{221}}$ units. Hence, the correct option is (A).

Quick Tip to Solve: Use AB as the second direction first; check the stated perpendicularity by a dot product before applying the skew-line formula.

9. The area of the region enclosed by the parabolas $y = 4x - x^2$ and $3y = (x-4)^2$ is equal to : [4 marks]

(A) $\frac{32}{9}$

(B) $\frac{14}{3}$

(C) 4

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- The area between two curves is the integral of the upper ordinate minus the lower ordinate between their intersection abscissae.
- Intersection points must be found before integrating; integrating only one curve would include area outside the enclosed region.

Step 1: Find the common abscissae Put the two values of y equal:

$$4x - x^2 = \frac{(x-4)^2}{3}.$$

Multiplication by 3 and collection of terms gives

$$4x^2 - 20x + 16 = 0, \qquad 4(x-1)(x-4) = 0.$$

Thus the required interval is $[1, 4]$. The corresponding ordinates are 3 and 0, respectively.

Step 2: Determine the upper curve The vertical separation is

$$\begin{aligned} h(x) &= 4x - x^2 - \frac{x^2 - 8x + 16}{3} \\ &= -\frac{4}{3}(x-1)(x-4) = \frac{4}{3}(x-1)(4-x). \end{aligned}$$

It is positive for $1 < x < 4$, so $y = 4x - x^2$ is the upper parabola throughout the enclosed region. No subdivision or absolute-value sign change is required inside the interval.

Step 3: Integrate the separation Let $u = x - 1$; the limits become 0 and 3. Then

$$\begin{aligned} A &= \frac{4}{3}\int_0^3 u(3-u)\,du \\ &= \frac{4}{3}\left[\frac{3u^2}{2} - \frac{u^3}{3}\right]_0^3 \\ &= \frac{4}{3}\left(\frac{27}{2} - 9\right) = 6. \end{aligned}$$

The integral is positive as an area must be. The values $32/9$, $14/3$, and 4 do not equal this integral and therefore exclude options (A), (B), and (C).

Step 4: Conclusion The enclosed area is $\boxed{6}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Factor the ordinate difference before integration; it reveals both the limits and which parabola lies above the other.

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10. Let $f, g : (0, \infty) \to \mathbb{R}$ be two functions defined by $f(x) = \int_{-x}^{x} \left(|t| - t^2\right) e^{-t^2} dt$ and $g(x) = \int_0^{x^2} t^{1/2} e^{-t} dt$. Then, the value of $9\left(f\left(\sqrt{\log_e 9}\right) + g\left(\sqrt{\log_e 9}\right)\right)$ is equal to : [4 marks]

(A) 10

(B) 9

(C) 8

(D) 6

Correct Answer: (C) 8

Solution:

Concept:

- An even integrand satisfies $\int_{-x}^{x} F(t)\,dt = 2\int_0^x F(t)\,dt$. Here both $|t|$ and t^2 are even.
- A substitution in one integral can put two integrals over identical limits, allowing cancellation

before either is evaluated separately.

Step 1: Use the symmetry in f For $x > 0$, the integrand defining f is even. On the positive half-interval $|t| = t$, hence

$$f(x) = 2\int_0^x (t - t^2)e^{-t^2}\,dt.$$

In particular, the quadratic contribution has a minus sign. Keeping this sign is essential because it will cancel the transformed integral defining g.

Step 2: Transform g to the same variable In $g(x)$, write $t = u^2$. Since $u \geq 0$, we have $t^{1/2} = u$ and $dt = 2u\,du$; the upper limit becomes $u = x$, not x^2. Therefore

$$g(x) = 2\int_0^x u^2 e^{-u^2}\,du.$$

Adding it to the expression for $f(x)$ cancels exactly the two quadratic terms.

Step 3: Evaluate the surviving integral The sum reduces to

$$\begin{aligned} f(x) + g(x) &= 2\int_0^x te^{-t^2}\,dt \\ &= \left[-e^{-t^2}\right]_0^x = 1 - e^{-x^2}. \end{aligned}$$

At $x = \sqrt{\log_e 9}$, $e^{-x^2} = 1/9$. Consequently the demanded quantity is $9(1 - 1/9) = 8$. Options 10 and 9 ignore the negative endpoint contribution, while 6 also fails the exact evaluation.

Step 4: Conclusion The required value is $\boxed{8}$. Hence, the correct option is (C).

Quick Tip to Solve: When one upper limit is the square of the other, try t=u squared before attempting separate antiderivatives.

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11. Let $f :\to \mathbb{R} \to (0, \infty)$ be strictly increasing function such that $\lim_{x\to\infty} \frac{f(7x)}{f(x)} = 1$. Then, the value of $\lim_{x\to\infty} \left[\frac{f(5x)}{f(x)} - 1\right]$ is equal to [4 marks]

(A) 0

(B) 4

(C) 1

(D) 7/5

Correct Answer: (A) 0

Solution:

Concept:

- A strictly increasing function preserves strict inequalities between its arguments. Its positive range permits division by $f(x)$ without reversing an inequality.
- The squeeze theorem determines a limit when a quantity lies between two expressions approaching the same value. No formula for the function is needed.

Step 1: Order the three arguments For all $x > 0$, the arguments satisfy $x < 5x < 7x$. Since f is strictly increasing,

$$f(x) < f(5x) < f(7x).$$

The limit concerns $x \to \infty$, so restricting to positive x loses no relevant values. This comparison uses only the given monotonicity; it does not presume that f is linear, differentiable, or a power function.

Step 2: Divide by the positive denominator The codomain is $(0, \infty)$, so $f(x) > 0$. Dividing and subtracting 1 gives

$$0 < \frac{f(5x)}{f(x)} - 1 < \frac{f(7x)}{f(x)} - 1.$$

The lower bound is exactly zero, and the given limit says the right-hand bound tends to $1 - 1 = 0$. Strict inequalities at finite arguments are fully compatible with equality of their limiting values.

Step 3: Apply the squeeze theorem The expression is trapped between zero and a positive quantity tending to zero. Hence

$$\lim_{x \to \infty} \left[\frac{f(5x)}{f(x)} - 1 \right] = 0.$$

Any positive proposed value, including 4, 1, or $7/5$, would contradict the upper bound for sufficiently large x. In particular, the constants 5 and 7 are arguments of the function and cannot be treated as multiplicative factors outside it.

Step 4: Conclusion The required limit is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: For a positive increasing function, trap an intermediate ratio between 1 and the given outer ratio; then subtract 1.

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12. The temperature $T(t)$ of a body at time $t = 0$ is $160°\text{F}$ and it decreases continuously as per the differential equation $\frac{dT}{dt} = -K(T - 80)$, where K is a positive constant. If $T(15) = 120°\text{F}$, then $T(45)$ is equal to [4 marks]

(A) 90° F

(B) 85° F

(C) 80° F

(D) 95° F

Correct Answer: (A) 90° F

Solution:

Concept:

- The differential equation $dT/dt = -K(T - 80)$ makes the excess temperature above $80°\text{F}$ decay exponentially.
- Separation of variables gives $T(t) - 80 = Ce^{-Kt}$. The constant C follows from the initial excess, while a later reading determines the decay factor.

Step 1: Solve for the excess temperature As the temperature is initially above 80, separate and

integrate:

$$\frac{dT}{T-80} = -K\,dt, \qquad \log(T-80) = -Kt + C_1.$$

Thus $T(t) - 80 = Ce^{-Kt}$. Substitution of $T(0) = 160$ gives $C = 80$, so

$$T(t) = 80 + 80e^{-Kt}.$$

The equilibrium value 80 must be added back after calculating the exponential decay.

Step 2: Use the reading after fifteen time units At $t = 15$, the temperature is 120, and hence

$$120 - 80 = 80e^{-15K}, \qquad e^{-15K} = \frac{1}{2}.$$

Thus the excess temperature halves in each interval of length 15. Equivalently, $K = (\log 2)/15 > 0$, in agreement with the required positive constant.

Step 3: Evaluate the temperature at t=45 The elapsed time 45 is three such intervals, giving

$$\begin{aligned} T(45) - 80 &= 80(e^{-15K})^3 \\ &= 80\left(\frac{1}{2}\right)^3 = 10. \end{aligned}$$

Therefore $T(45) = 90°\text{F}$. The option 85 would correspond to four halving intervals, while 80 is reached only as a limiting equilibrium. The option 95 does not satisfy the measured halving factor, so none of these alternatives fits the differential equation and readings.

Step 4: Conclusion The temperature is $\boxed{90°\text{F}}$. Hence, the correct option is (A).

Quick Tip to Solve: Halve the excess above ambient temperature, not the temperature itself; 45 corresponds to three intervals of length 15.

13. If the function $f : (-\infty, -1] \to (a, b]$ defined by $f(x) = e^{x^3-3x+1}$ is one - one and onto, then the distance of the point $P(2b + 4, a + 2)$ from the line $x + e^{-3}y = 4$ is : [4 marks]

(A) $2\sqrt{1+e^6}$

(B) $\sqrt{1+e^6}$

(C) $3\sqrt{1+e^6}$

(D) $4\sqrt{1+e^6}$

Correct Answer: (A) $2\sqrt{1+e^6}$

Solution:

Concept:

- An onto map has codomain equal to its actual range. Determine that range from the monotonicity and endpoint behaviour of the defining function.
- The distance from (x_0, y_0) to $Ax + By + C = 0$ is $|Ax_0 + By_0 + C|/\sqrt{A^2 + B^2}$.

Step 1: Find the exact range Write $h(x) = x^3 - 3x + 1$. Its derivative is $h\prime(x) = 3(x^2 - 1) > 0$ for $x < -1$. Hence h, and therefore $e^{h(x)}$, increases strictly on the given domain. Moreover $h(x) \to -\infty$ as $x \to -\infty$, whereas $h(-1) = -1 + 3 + 1 = 3$. The range of the exponential is consequently $(0, e^3]$, with the lower endpoint excluded and the upper endpoint attained.

Step 2: Identify the point P Because the function is onto $(a, b]$, the actual range forces $a = 0$ and $b = e^3$. Therefore

$$P = (2e^3 + 4, 2).$$

These endpoints also agree with the one-one requirement: the strict increase ensures no two domain values give the same output. Neither the exponent range nor the endpoint e^{-3} is the codomain upper endpoint.

Step 3: Apply and simplify the distance formula The line is $x + e^{-3}y - 4 = 0$. Thus

$$\begin{aligned} d &= \frac{|2e^3 + 4 + 2e^{-3} - 4|}{\sqrt{1 + e^{-6}}} \\ &= \frac{2(e^3 + e^{-3})}{\sqrt{1 + e^{-6}}} \\ &= 2\sqrt{1 + e^6}. \end{aligned}$$

For the last step multiply numerator and denominator by $e^3 > 0$. The resulting coefficient is 2; coefficients 1, 3, and 4 in the other options do not equal this distance.

Step 4: Conclusion The distance is $\boxed{2\sqrt{1 + e^6}}$ units. Hence, the correct option is (A).

Quick Tip to Solve: Find the range of the exponent first, exponentiate its endpoints, and only then substitute a and b into the point coordinates.

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14. Let 2^{nd}, 8^{th} and 44^{th} terms of a non-constant A. P. be respectively the 1^{st}, 2^{nd} and 3^{rd} terms of a G. P. If the first term of the A. P. is 1, then the sum of its first 20 terms is equal to - [4 marks]

(A) 990

(B) 980

(C) 960

(D) 970

Correct Answer: (D) 970

Solution:

Concept:

- For an arithmetic progression with first term 1 and common difference d, its nth term is $1+(n-1)d$. Non-constant means $d \neq 0$.
- Three consecutive geometric-progression terms u,v,w satisfy $v^2 = uw$. The sum of the first n arithmetic-progression terms is $S_n = n[2a + (n-1)d]/2$.

Step 1: Express the selected terms using d The second, eighth and forty-fourth AP terms are

respectively

$$1+d, \qquad 1+7d, \qquad 1+43d.$$

Since these are the first three terms of a GP in the stated order, the square of the middle one equals the product of the outside two:

$$(1+7d)^2 = (1+d)(1+43d).$$

The offsets are 1, 7 and 43 because the AP formula uses $n-1$, not n.

Step 2: Solve for the nonzero common difference Expansion gives

$$\begin{aligned} 1+14d+49d^2 &= 1+44d+43d^2, \\ 6d^2-30d &= 0, \\ 6d(d-5) &= 0. \end{aligned}$$

The root $d=0$ is inadmissible because the AP is non-constant. Thus $d=5$. As a check, the chosen AP terms become 6, 36 and 216, which indeed form a GP of common ratio 6.

Step 3: Calculate the required sum Using $a=1$, $n=20$, and $d=5$,

$$\begin{aligned} S_{20} &= \frac{20}{2}\{2(1)+19(5)\} \\ &= 10(2+95) = 970. \end{aligned}$$

Alternatively, the twentieth term is $1+19(5)=96$; twenty times the average of the first and last terms is $20(1+96)/2 = 970$. Both checks exclude the offered values 990, 980 and 960.

Step 4: Conclusion The sum of the first twenty AP terms is $\boxed{970}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert the GP condition to a square-equals-product equation, and explicitly discard zero common difference when the AP is non-constant.

15. Let z_1 and z_2 be two complex numbers such that $z_1+z_2=5$ and $z_1^3+z_2^3=20+15i$ Then, $|z_1^4+z_2^4|$ equals - [4 marks]

(A) $15\sqrt{15}$

(B) $30\sqrt{3}$

(C) $25\sqrt{3}$

(D) 75

Correct Answer: (D) 75

Solution:

Concept:

- The identities for symmetric powers express the required sum using $s=z_1+z_2$ and $p=z_1z_2$, without finding the individual complex numbers.
- Use $z_1^3+z_2^3=s^3-3ps$, $z_1^2+z_2^2=s^2-2p$, and $|u+iv|=\sqrt{u^2+v^2}$.

Step 1: Find the product of the two numbers Substitute the given sum and cubic-power sum into the cubic identity. As the sum is nonzero, division by it is valid:

$$20 + 15i = 125 - 15p, \qquad p = \frac{105 - 15i}{15} = 7 - i.$$

This determines the product uniquely even though the two individual numbers can be interchanged.

Step 2: Evaluate the fourth-power sum First compute the square-power sum:

$$z_1^2 + z_2^2 = 25 - 2(7 - i) = 11 + 2i.$$

Squaring this sum introduces twice the product of the squares, which must be subtracted:

$$\begin{aligned} z_1^4 + z_2^4 &= (11 + 2i)^2 - 2(7 - i)^2 \\ &= (117 + 44i) - 2(48 - 14i) \\ &= 21 + 72i. \end{aligned}$$

Here $i^2 = -1$; keeping this sign is essential in both squared expressions.

Step 3: Take the modulus and compare the options The modulus is a nonnegative real number, obtained from the real and imaginary components:

$$|21 + 72i| = \sqrt{21^2 + 72^2} = \sqrt{5625} = 75.$$

The squared values of the other three options are respectively 3375, 2700, and 1875, so none equals the required squared modulus 5625.

Step 4: Conclusion The required modulus is 75. Hence, the correct option is (D).

Quick Tip to Solve: Find the product from the cubic sum first; then use the square-power sum to reach the fourth powers without solving a quadratic for the individual complex numbers.

16. If for some m, n; ${}^6C_m + 2\left({}^6C_{m+1}\right) + {}^6C_{m+2} > {}^8C_3$ and ${}^{n-1}P_3 : {}^nP_4 = 1 : 8$, then ${}^nP_{m+1} + {}^{n+1}C_m$ is equal to [4 marks]

(A) 380

(B) 376

(C) 372

(D) 384

Correct Answer: (C) 372

Solution:

Concept:

- Pascal's identity is $\binom{r}{k} + \binom{r}{k+1} = \binom{r+1}{k+1}$. Apply it twice to the three-term expression.
- For nonnegative admissible integers, ${}^nP_r = n!/(n-r)!$. The given permutation ratio simplifies before numerical evaluation.

Step 1: Combine the three binomial coefficients Group the middle term into two equal terms and apply Pascal's identity to each neighbouring pair:

$$\binom{6}{m} + 2\binom{6}{m+1} + \binom{6}{m+2} = \binom{7}{m+1} + \binom{7}{m+2} = \binom{8}{m+2}.$$

Thus the strict inequality becomes $\binom{8}{m+2} > \binom{8}{3} = 56$.

Step 2: Determine the two integer parameters Because the original binomial coefficients must exist, $0 \le m \le 4$. For $m+2 = 2, 3, 4, 5, 6$, the corresponding eighth-row coefficients are

$$28, \quad 56, \quad 70, \quad 56, \quad 28.$$

Only the central value is strictly greater than 56, giving $m = 2$. Also,

$$\frac{{}^{n-1}P_3}{{}^{n}P_4} = \frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{n} = \frac{1}{8}.$$

The admissibility condition $n \ge 4$ allows cancellation, so $n = 8$.

Step 3: Evaluate the requested sum Now substitute both values, remembering that the first term is a permutation, whereas the second is a combination:

$${}^{8}P_3 + \binom{9}{2} = 8 \times 7 \times 6 + \frac{9 \times 8}{2} = 336 + 36 = 372.$$

Options (A), (B), and (D) give 380, 376, and 384, respectively; they do not equal this sum.

Step 4: Conclusion The value is 372. Hence, the correct option is (C).

Quick Tip to Solve: Combine the binomial terms before testing integer values; the strict inequality selects the unique central coefficient rather than either neighbouring equal coefficient.

17. Consider the function $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) = e^{-|\log_e x|}$. If m and n be respectively the number of points at which f is not continuous and f is not differentiable, then $m + n$ is [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- An absolute value is removed by considering the sign of its argument. Here $\log_e x$ changes sign only at $x = 1$ within the domain $(0, \infty)$.
- Continuity requires the two one-sided limits to equal the function value. Differentiability additionally requires equal one-sided derivatives.

Step 1: Write the function without the absolute value For $0 < x < 1$, the logarithm is negative, whereas for $x \geq 1$ it is nonnegative. Therefore,

$$f(x) = \begin{cases} e^{\log_e x} = x, & 0 < x < 1, \\ e^{-\log_e x} = 1/x, & x \geq 1. \end{cases}$$

Both formulas are smooth on the interiors of their respective intervals. The only possible exceptional point in the domain is $x = 1$; the endpoint 0 is excluded and is not counted.

Step 2: Test continuity at the joining point From the first branch the left-hand limit at 1 is 1, and from the second branch the right-hand limit is also 1. The definition gives $f(1) = e^0 = 1$. Hence,

$$\lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x) = 1.$$

There are no discontinuities anywhere in the stated domain, so $m = 0$. A change in the formula alone does not imply a discontinuity.

Step 3: Compare the one-sided derivatives The derivative on the left branch is 1, while on the right branch it is $-1/x^2$. At the join,

$$f'_-(1) = 1, \qquad f'_+(1) = -1.$$

These slopes differ, so the continuous graph has a corner at 1. Consequently $n = 1$ and $m + n = 0 + 1 = 1$. Option (A) misses the corner, while (C) and (D) overcount exceptional points.

Step 4: Conclusion The required sum is $m + n = 1$. Hence, the correct option is (B).

Quick Tip to Solve: For an absolute value involving a logarithm, split at the zero of the logarithm and test both continuity and slopes; do not count a domain endpoint that is excluded.

18. Let a variable line passing through the centre of the circle $x^2 + y^2 - 16x - 4y = 0$, meet the positive co-ordinate axes at the points A and B. Then the minimum value of $OA + OB$, where O is the origin, is equal to [4 marks]

(A) 12

(B) 20

(C) 24

(D) 18

Correct Answer: (D) 18

Solution:

Concept:

- Completing squares gives the centre of a circle. A line with positive intercepts a, b has equation $x/a + y/b = 1$.
- The Cauchy–Schwarz inequality gives $(a+b)(u^2/a + v^2/b) \geq (u+v)^2$ for positive a, b; its equality condition also checks attainability.

Step 1: Locate the centre and impose the line condition Complete squares in the circle equation:

$$(x-8)^2 + (y-2)^2 = 68.$$

Thus its centre is $(8, 2)$. Put $A = (a, 0)$ and $B = (0, b)$, with $a, b > 0$. Since the line through the two intercepts passes through the centre,

$$\frac{8}{a} + \frac{2}{b} = 1.$$

The requested quantity is the sum of the positive intercept lengths, $OA + OB = a + b$, rather than the segment length AB.

Step 2: Find a lower bound for the sum Apply Cauchy–Schwarz to the pairs $(\sqrt{a}, \sqrt{b})$ and $(\sqrt{8/a}, \sqrt{2/b})$:

$$(a+b)\left(\frac{8}{a} + \frac{2}{b}\right) \geq (\sqrt{8} + \sqrt{2})^2 = 18.$$

Using the line condition, this gives $a + b \geq 18$. The lower bound already excludes option (A), whose value 12 is too small.

Step 3: Check that equality can actually occur Equality requires $a/b = \sqrt{8}/\sqrt{2} = 2$. Substituting $a = 2b$ into the constraint gives

$$\frac{8}{2b} + \frac{2}{b} = 1 \quad \Longrightarrow \quad b = 6, \quad a = 12.$$

Both intercepts are positive, and $8/12 + 2/6 = 1$, so this line is admissible and attains the bound. Values 20 and 24 in (B) and (C) are larger than the attained minimum.

Step 4: Conclusion The minimum of $OA + OB$ is 18 coordinate units. Hence, the correct option is (D).

Quick Tip to Solve: For a line through a fixed first-quadrant point, use its intercept equation and Cauchy–Schwarz; always exhibit the positive intercepts that attain the lower bound.

19. Let $A(a,b), B(3,4)$ and $C(-6,-8)$ respectively denote the centroid, circumcentre and orthocentre of a triangle. Then, the distance of the point $P(2a+3, 7b+5)$ from the line $2x+3y-4=0$ measured parallel to the line $x-2y-1=0$ is [4 marks]

(A) $\frac{17\sqrt{5}}{6}$

(B) $\frac{15\sqrt{5}}{7}$

(C) $\frac{17\sqrt{5}}{7}$

(D) $\frac{\sqrt{5}}{17}$

Correct Answer: (C) $\frac{17\sqrt{5}}{7}$

Solution:

Concept:

- The centroid G, circumcentre O_c and orthocentre H satisfy $3\vec{G} = 2\vec{O}_c + \vec{H}$, including the equilateral case.
- A distance measured parallel to a line is found by intersecting a line in that direction through the given point with the target line. It is generally different from perpendicular distance.

Step 1: Determine the centroid and the point P Use the Euler-line relation with the stated circumcentre and orthocentre:

$$3(a,b) = 2(3,4) + (-6,-8) = (0,0).$$

Hence $a = b = 0$, and the specified point becomes

$$P = (2a+3, 7b+5) = (3,5).$$

The cancellation follows from the full vector relation; interchanging the roles of the centroid and circumcentre would produce the wrong point.

Step 2: Intersect the required parallel line with the target A direction vector for $x-2y-1=0$ is $(2,1)$, since its dot product with the normal $(1,-2)$ vanishes. Therefore a point on the line through P parallel to it is

$$Q = (3+2t, 5+t).$$

Impose the target-line equation:

$$2(3+2t) + 3(5+t) - 4 = 0 \quad \Longrightarrow \quad 17 + 7t = 0,$$

so $t = -17/7$. The negative sign selects the direction of travel and will disappear when taking a length.

Step 3: Compute the length and reject the other values The displacement from P to Q is $t(2,1)$, so

$$PQ = |t|\sqrt{2^2+1^2} = \frac{17\sqrt{5}}{7}.$$

Option (A) uses the wrong directional denominator 6; option (B) has the wrong residual 15 instead of 17; option (D) incorrectly puts the residual in the denominator. None gives the length of this displacement.

Step 4: Conclusion The distance in the prescribed direction is $17\sqrt{5}/7$ coordinate units. Hence, the correct option is (C).

Quick Tip to Solve: Use a direction vector of the parallel line and parametrize from the point; the absolute parameter times the vector length gives the requested oblique distance.

20. Let the mean and the variance of 6 observations $a, b, 68, 44, 48, 60$ be 55 and 194, respectively. If $a > b$, then $a + 3b$ is [4 marks]

(A) 180

(B) 210

(C) 190

(D) 200

Correct Answer: (A) 180

Solution:

Concept:

- For N observations, $\bar{x} = \sum x_i/N$ and $\sigma^2 = \sum x_i^2/N - \bar{x}^2$. The given variance uses all six observations.
- Once the sum and sum of squares of two unknowns are known, their product follows from $(a+b)^2 = a^2 + b^2 + 2ab$. The inequality $a > b$ fixes their order.

Step 1: Use the mean to find the sum of the unknowns The total of the six observations is $6 \times 55 = 330$. The four known observations sum to $68 + 44 + 48 + 60 = 220$. Hence,

$$a + b = 330 - 220 = 110.$$

This determines their sum but not their individual values, so the variance supplies the second independent relation.

Step 2: Convert the variance into a sum of squares Rearrange the population-variance formula before substituting:

$$\sum x_i^2 = 6(194 + 55^2) = 6(3219) = 19314.$$

The known squared values total

$$68^2 + 44^2 + 48^2 + 60^2 = 4624 + 1936 + 2304 + 3600 = 12464.$$

Therefore $a^2 + b^2 = 19314 - 12464 = 6850$, and

$$2ab = 110^2 - 6850 = 5250, \qquad ab = 2625.$$

Using $N - 1$ in this problem would give an incorrect second relation.

Step 3: Recover the observations in their stated order The two values are the roots of

$$t^2 - 110t + 2625 = (t - 75)(t - 35) = 0.$$

Since $a > b$, take $a = 75$, $b = 35$. These give squared deviations $400, 400, 169, 121, 49, 25$, whose sum 1164 divided by 6 is indeed 194. Thus,

$$a + 3b = 75 + 3(35) = 180.$$

Options (B), (C), and (D) fail this evaluated expression; merely knowing $a + b$ would not justify any of them.

Step 4: Conclusion The requested value is 180. Hence, the correct option is (A).

Quick Tip to Solve: Convert mean and variance into the sum and sum of squares first, then solve the quadratic; apply the given ordering only after finding both possible values.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$ and $(\vec{a} - \vec{b} + \hat{i}) \cdot \vec{c} = -3$. Then $|\vec{c}|^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For vectors (u_1, u_2, u_3) and (v_1, v_2, v_3), their cross product has components $(u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$.
- The cross-product equation gives linear component conditions on $\vec{c}$. A dot-product condition supplies the remaining information, after which $|\vec{c}|^2$ is the sum of its squared components.

Step 1: Evaluate the known vectors and the right-hand side Write $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$. From the given components,

$$\vec{a} + \vec{b} = (5, 1, 4), \qquad \vec{a} \times \vec{b} = (7, -7, -7).$$

Consequently the right-hand side of the cross-product equation is

$$2(7, -7, -7) + (0, 24, -6) = (14, 10, -20).$$

The signs in the second and third components follow from the cyclic order of the cross product.

Step 2: Solve the component equations using the dot product The cross-product equation now reads

$$(z - 4y,\ 4x - 5z,\ 5y - x) = (14, 10, -20).$$

Its first and third equations yield $z = 14 + 4y$ and $x = 20 + 5y$; these also satisfy the second equation. The cross product alone leaves a free parameter. For the other condition,

$$\vec{a} - \vec{b} + \hat{i} = (2, 3, -2),$$

so

$$2x + 3y - 2z = -3,$$
$$2(20 + 5y) + 3y - 2(14 + 4y) = -3,$$
$$12 + 5y = -3.$$

Thus $y = -3$, $x = 5$, and $z = 2$.

Step 3: Check the vector and compute its squared magnitude For $(x, y, z) = (5, -3, 2)$, the three cross-product components are $(14, 10, -20)$, and the dot product is $10 - 9 - 4 = -3$, so both constraints hold. Finally,

$$|\vec{c}|^2 = 5^2 + (-3)^2 + 2^2 = 25 + 9 + 4 = 38.$$

The question asks for the squared magnitude, so taking a square root would answer a different quantity.

Step 4: Conclusion The required numerical value is $\boxed{38}$.

Quick Tip to Solve: Use two independent cross-product components to express two unknowns in terms of the third, then use the dot product; check the unused component to catch sign errors.

22. Let $y = y(x)$ be the solution of the differential equation
$\sec^2 x dx + \left(e^{2y} \tan^2 x + \tan x\right) dy = 0, 0 < x < \frac{\pi}{2}, y(\pi/4) = 0$.
If $y(\pi/6) = \alpha$, then $e^{8\alpha}$ is equal to _____. [4 marks]

Solution:

Concept:

- Divide by $\tan^2 x$, which is nonzero on $0 < x < \pi/2$, to expose the derivative of $\cot x$.
- A linear differential equation $du/dy - u = e^{2y}$ has integrating factor e^{-y}. The given initial value determines its integration constant.

Step 1: Transform the equation On the stated interval, division by $\tan^2 x$ is valid and gives

$$\csc^2 x \, dx + (e^{2y} + \cot x) \, dy = 0.$$

Put $u = \cot x$, so that $du = -\csc^2 x \, dx$. Thus the differential equation becomes

$$-du + (e^{2y} + u) \, dy = 0, \qquad \frac{du}{dy} - u = e^{2y}.$$

This substitution accounts for both the differential term and the remaining trigonometric term.

Step 2: Integrate and impose the initial condition Multiply by the integrating factor e^{-y}. The left side is now a product derivative:

$$\frac{d}{dy}(ue^{-y}) = e^{y}.$$

Integration gives $ue^{-y} = e^{y} + C$, or

$$\cot x = e^{2y} + Ce^{y}.$$

At $x = \pi/4$, the given value is $y = 0$. Hence $1 = 1 + C$, so $C = 0$ and $\cot x = e^{2y}$. Equivalently, $y(x) = \frac{1}{2}\log(\cot x)$, which is real throughout the stated interval.

Step 3: Evaluate the requested exponential At $x = \pi/6$, $y = \alpha$, so

$$e^{2\alpha} = \cot\frac{\pi}{6} = \sqrt{3}.$$

Raise this equation to the fourth power:

$$e^{8\alpha} = (e^{2\alpha})^4 = (\sqrt{3})^4 = 9.$$

The positive exponential is consistent with $\cot x > 0$ on the prescribed interval.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{9}$.

Quick Tip to Solve: When a differential equation contains $\sec^2 x$ together with $\tan^2 x$, divide by $\tan^2 x$ and try $u = \cot x$.

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23. $\left|\frac{120}{\pi^3}\int_0^{\pi}\frac{x^2\sin x\cos x}{\sin^4 x+\cos^4 x}dx\right|$ **is equal to _____.** [4 marks]

Solution:

Concept:

- Use reflection $x \mapsto \pi - x$ to reduce a quadratic weight when the remaining factor changes sign.
- For $f(x) = \sin x\cos x/(\sin^4 x + \cos^4 x)$, the identities $f(\pi - x) = -f(x)$ and $f(x + \pi/2) = -f(x)$ control the integral.

Step 1: Reduce the quadratic weight by reflection Write

$$I = \int_0^{\pi} x^2 f(x)\,dx, \qquad J = \int_0^{\pi} x f(x)\,dx.$$

The denominator is always positive, so these integrals are proper. Reflection about $\pi/2$ gives

$$I = -\int_0^{\pi} (\pi - x)^2 f(x)\,dx.$$

Also $\int_0^{\pi} f(x)\,dx = 0$ by the same antisymmetry. Adding the two expressions for I therefore yields

$$2I = 2\pi J - \pi^2\int_0^{\pi} f(x)\,dx = 2\pi J, \qquad I = \pi J.$$

Step 2: Reduce the linear moment to an elementary integral Split J at $\pi/2$ and use $x = t + \pi/2$ in the second half. Since $f(t + \pi/2) = -f(t)$,

$$J = \int_0^{\pi/2} t f(t)\,dt - \int_0^{\pi/2} (t + \pi/2) f(t)\,dt = -\frac{\pi}{2}K,$$

where $K = \int_0^{\pi/2} f(t)\,dt$. Put $u = \sin^2 t$, with $du = 2\sin t\cos t\,dt$:

$$K = \frac{1}{2}\int_0^1 \frac{du}{u^2 + (1-u)^2}$$
$$= \int_0^1 \frac{du}{(2u-1)^2 + 1} = \frac{1}{2}[\arctan(2u-1)]_0^1 = \frac{\pi}{4}.$$

Step 3: Apply the absolute value at the end The preceding relations give

$$J = -\frac{\pi^2}{8}, \qquad I = -\frac{\pi^3}{8}.$$

Consequently,

$$\left|\frac{120}{\pi^3} I\right| = \left|-\frac{120}{8}\right| = 15.$$

The sign of I is negative because the negative half of f is weighted by larger values of x^2; the absolute value removes that sign only at the final stage.

Step 4: Conclusion The requested dimensionless value is $\boxed{15}$.

Quick Tip to Solve: Check both reflection and half-period shifts before attempting integration by parts on a polynomial times a trigonometric ratio.

24. Let $A(-2,-1), B(1,0), C(\alpha,\beta)$ and $D(\gamma,\delta)$ be the vertices of a parallelogram $ABCD$. If the point C lies on $2x - y = 5$ and the point D lies on $3x - 2y = 6$, then the value of $|\alpha + \beta + \gamma + \delta|$ is equal to _____.

[4 marks]

Solution:

Concept:

- The diagonals of a parallelogram bisect each other, so opposite vertex vectors satisfy $A+C = B+D$.
- Combine that vector identity with the two line equations to determine all four unknown coordinates without introducing a slope.

Step 1: Use the opposite-vertex identity For consecutive vertices A, B, C, D, the midpoint of AC equals the midpoint of BD. Thus

$$(-2,-1) + (\alpha,\beta) = (1,0) + (\gamma,\delta).$$

Equating coordinates gives

$$\gamma = \alpha - 3, \qquad \delta = \beta - 1.$$

These equations also say that the side vector DC is $(3,1)$, the same as AB, as required for a parallelogram. Using adjacent vertices in the diagonal identity would give an incorrect pair of signs.

Step 2: Enforce both line conditions Since C lies on $2x - y = 5$,

$$2\alpha - \beta = 5, \qquad \beta = 2\alpha - 5.$$

Since D lies on $3x - 2y = 6$, substitute its coordinates from the first step:

$$3(\alpha - 3) - 2(\beta - 1) = 6, \qquad 3\alpha - 2\beta = 13.$$

Now eliminate β:

$$3\alpha - 2(2\alpha - 5) = 13, \qquad -\alpha = 3.$$

Hence $\alpha = -3$ and $\beta = -11$. It follows that $\gamma = -6$ and $\delta = -12$.

Step 3: Check the coordinates and take the modulus The two required line checks are

$$2(-3) - (-11) = 5, \qquad 3(-6) - 2(-12) = 6.$$

Moreover, $A + C = (-5, -12) = B + D$, confirming the parallelogram condition. Therefore

$$\alpha + \beta + \gamma + \delta = -3 - 11 - 6 - 12 = -32.$$

The question asks for its absolute value, so the final numerical entry is positive.

Step 4: Conclusion $|\alpha + \beta + \gamma + \delta| = \boxed{32}$.

Quick Tip to Solve: For four consecutive parallelogram vertices, write $A + C = B + D$ first; it gives two linear coordinate equations immediately.

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25. Let a, b, c be the lengths of three sides of a triangle satistying the condition $\left(a^2 + b^2\right) x^2 - 2b(a + c)x + \left(b^2 + c^2\right) = 0$. If the set of all possible values of x is the interval (α, β), then $12\left(\alpha^2 + \beta^2\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- A sum of squares of real quantities is zero only when each square is zero.
- Three positive numbers are sides of a nondegenerate triangle precisely when each is less than the sum of the other two; the inequalities are strict.

Step 1: Factor the condition into a sum of squares The given quadratic can be regrouped exactly as

$$(ax - b)^2 + (bx - c)^2 = 0.$$

For real x, both squares are nonnegative. Therefore

$$ax = b, \qquad bx = c.$$

As a, b, c are positive side lengths, $x = b/a > 0$. The side lengths consequently have the geometric-progression form

$$a, \quad ax, \quad ax^2.$$

Conversely, these relations make both squares zero, so any positive x producing a valid triangle really

is attainable.

Step 2: Apply all three triangle inequalities Divide the triangle inequalities by $a > 0$. They become

$$1 + x > x^2, \qquad x + x^2 > 1, \qquad 1 + x^2 > x.$$

The last inequality holds for every real x, because

$$x^2 - x + 1 = (x - 1/2)^2 + 3/4 > 0.$$

For positive x, the first two inequalities respectively give

$$x < \frac{1 + \sqrt{5}}{2}, \qquad x > \frac{\sqrt{5} - 1}{2}.$$

The endpoints correspond to equality in a triangle inequality and hence to a degenerate triangle; they must be excluded.

Step 3: Compute the endpoint expression Thus

$$\alpha = \frac{\sqrt{5} - 1}{2}, \qquad \beta = \frac{\sqrt{5} + 1}{2}.$$

Every value strictly between them gives three positive lengths satisfying all triangle inequalities, so no additional restriction is required. Squaring and adding gives

$$\alpha^2 + \beta^2 = \frac{(\sqrt{5} - 1)^2 + (\sqrt{5} + 1)^2}{4} = \frac{12}{4} = 3.$$

Multiplying this result by the factor specified in the question completes the calculation.

Step 4: Conclusion The required dimensionless value is $12(\alpha^2 + \beta^2) = 12 \cdot 3 = \boxed{36}$.

Quick Tip to Solve: The cross terms $-2abx - 2bcx$ suggest completing two squares; then use the strict triangle inequalities to find the open interval.

---

26. If $\lim_{x \to 0} \frac{ax^2 e^x - b \log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$, **then** $16\left(a^2 + b^2 + c^2\right)$ **is equal to _____.** [4 marks]

Solution:

Concept:

- Because $x^2 \sin x = x^3 + O(x^5)$, a finite nonzero limit requires the numerator coefficients of x and x^2 to vanish.
- Use the expansions of e^x, e^{-x} and $\log(1 + x)$ through the terms contributing to x^3, then match the leading coefficients.

Step 1: Expand the numerator to cubic order For x near zero, all three functions have their usual real

expansions (in particular, $1 + x > 0$):

$$ax^2 e^x = ax^2 + ax^3 + O(x^4),$$
$$-b\log(1+x) = -bx + \frac{b}{2}x^2 - \frac{b}{3}x^3 + O(x^4),$$
$$cxe^{-x} = cx - cx^2 + \frac{c}{2}x^3 + O(x^4).$$

Adding like powers gives the numerator as

$$(c-b)x + (a + b/2 - c)x^2 + (a - b/3 + c/2)x^3 + O(x^4).$$

Step 2: Match the coefficients required by the limit The denominator is

$$x^2 \sin x = x^3 + O(x^5).$$

A nonzero coefficient of x or x^2 in the numerator would make the quotient divergent near zero. Therefore

$$c - b = 0, \qquad a + b/2 - c = 0.$$

These imply $c = b$ and $a = b/2$. For the limit to be exactly 1, the cubic coefficient must then equal the denominator's cubic coefficient:

$$a - b/3 + c/2 = 1.$$

Substitution yields $b/2 - b/3 + b/2 = 1$, or $2b/3 = 1$.

Step 3: Find the constants and verify the value We obtain

$$b = \frac{3}{2}, \qquad c = \frac{3}{2}, \qquad a = \frac{3}{4}.$$

With these values the lower-order terms cancel and the numerator is $x^3 + O(x^4)$, so division by $x^3 + O(x^5)$ indeed tends to 1. Finally,

$$16(a^2 + b^2 + c^2) = 16\left(\frac{9}{16} + \frac{9}{4} + \frac{9}{4}\right)$$
$$= 9 + 36 + 36 = 81.$$

The constants and the requested sum are dimensionless.

Step 4: Conclusion The requested numerical value is $\boxed{81}$.

Quick Tip to Solve: First identify the denominator's order of vanishing; cancel all lower powers in the numerator before matching the leading coefficient.

27. Let $A = \{1, 2, 3, \ldots\ldots\ldots\ldots, 100\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $2x = 3y$. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n. Then, the minimum value of n is _____. [4 marks]

Solution:

Concept:

- A relation is symmetric when every pair (x, y) is accompanied by (y, x). The smallest symmetric relation containing R is $R \cup R^{-1}$.
- For the integer equation $2x = 3y$, coprimality of 2 and 3 forces $x = 3k$ and $y = 2k$ with an integer k.

Step 1: Count all ordered pairs in the given relation If $2x = 3y$ with x, y positive integers, then 3 divides x and 2 divides y. Write

$$x = 3k, \qquad y = 2k.$$

The conditions $1 \le x, y \le 100$ require $k \ge 1$, $3k \le 100$ and $2k \le 100$. The tighter bound is $k \le 33$. Hence

$$R = \{(3k, 2k) : k = 1, 2, \ldots, 33\}, \qquad |R| = 33.$$

No other pairs satisfy the defining equation, and different values of k give distinct pairs.

Step 2: Add exactly the reverse pairs symmetry forces Every symmetric relation containing R must also contain

$$R^{-1} = \{(2k, 3k) : k = 1, 2, \ldots, 33\}.$$

There is no overlap between R and R^{-1}. Indeed, an overlapping pair would obey both $2x = 3y$ and $2y = 3x$. Substitution gives $4x = 9x$, so $x = 0$, which is excluded from A. Thus all 33 reverse pairs are new. In particular, no pair on the diagonal (x, x) occurs in R.

Step 3: Prove that the lower bound is attainable It follows that every admissible R_1 has at least

$$|R \cup R^{-1}| = 33 + 33 = 66$$

elements. The union itself is symmetric: reversing a pair in R places it in R^{-1}, and reversing a pair in R^{-1} places it in R. Therefore choosing $R_1 = R \cup R^{-1}$ attains this lower bound. It is a proper enlargement of R, so it also meets the printed containment condition. Reflexivity and transitivity were not requested, so no extra diagonal pairs or chains need be added.

Step 4: Conclusion The minimum possible number of elements is $n = \boxed{66}$.

Quick Tip to Solve: To build the smallest symmetric extension, add only reverse pairs and subtract any overlap; do not add diagonal pairs unless reflexivity is asked.

28. Let the coefficient of x^r in the expansion of $(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+\ldots\ldots\ldots+(x+2)^{n-1}$ be α_r. If $\sum_{r=0}^{n}\alpha_r=\beta^n-\gamma^n, \beta,\gamma\in\mathbb{N}$, then the value of $\beta^2+\gamma^2$ equals _____.

[4 marks]

Solution:

Concept:

- The sum of all coefficients of a polynomial $P(x)$ is $P(1)$. A missing highest-degree coefficient is simply zero.
- The finite identity $A^{n-1}+A^{n-2}B+\cdots+B^{n-1}=(A^n-B^n)/(A-B)$ follows by multiplying the sum by $A-B$.

Step 1: Collapse the finite polynomial sum Let $P(x)$ denote the expansion in the question, and put

$$A=x+3, \qquad B=x+2.$$

Then $A-B=1$. Multiplication of the sum by $A-B$ cancels its intermediate terms:

$$(A-B)P(x)=A^n-B^n.$$

Consequently the polynomial has the exact simpler form

$$P(x)=(x+3)^n-(x+2)^n.$$

The leading x^n terms cancel, consistently with the original sum having degree $n-1$.

Step 2: Evaluate the sum of coefficients Write $P(x)=\sum_{r=0}^{n}\alpha_r x^r$, with $\alpha_n=0$ because of the leading-term cancellation. Substituting $x=1$ makes every power equal to 1, so

$$\sum_{r=0}^{n}\alpha_r=P(1).$$

Using the collapsed form gives

$$P(1)=(1+3)^n-(1+2)^n=4^n-3^n.$$

The same result follows by setting $x=1$ directly in the original geometric sum $4^{n-1}+4^{n-2}3+\cdots+3^{n-1}$, whose ratio is $3/4$.

Step 3: Identify the bases in the stated identity Comparing this coefficient-sum identity with the question's form

$$\sum_{r=0}^{n}\alpha_r=\beta^n-\gamma^n$$

identifies the natural-number bases as $\beta=4$ and $\gamma=3$. These are the bases of the expression obtained for general n, so no separate expansion or choice of a particular degree is needed. The requested expression is therefore

$$\beta^2+\gamma^2=4^2+3^2=16+9=25.$$

Keeping the finite identity avoids unnecessarily calculating each individual α_r.

Step 4: Conclusion The requested dimensionless value is $\boxed{25}$.

Quick Tip to Solve: For a coefficient sum, set $x = 1$ immediately; a chain of mixed powers often collapses by the difference-of-powers identity.

29. Let A be a 3×3 matrix and $\det(A) = 2$. If $n = \det(\underbrace{\text{adj}(\text{adj}(\ldots..(\text{adj}\, A)))}_{2024-\text{ times}})$, then the remainder when n is divided by 9 is equal to _____. [4 marks]

Solution:

Concept:

- For a square matrix of order m, $\det(\text{adj}\, M) = (\det M)^{m-1}$. Here the order remains 3 after every adjugate, so taking an adjugate squares the determinant.
- The powers of 2 modulo 9 repeat with period 6, since $2^6 = 64 \equiv 1 \pmod 9$. Reduce the exponent modulo 6 before reducing the final power modulo 9.

Step 1: Set up the determinant recurrence Let $M_0 = A$ and $M_k = \text{adj}\, M_{k-1}$. Write $d_k = \det M_k$. The matrices are all of order 3, hence

$$d_0 = 2, \qquad d_k = d_{k-1}^2.$$

Thus the first few determinants are $2, 2^2, 2^4, 2^8$. Induction gives

$$d_k = 2^{2^k}.$$

There are exactly 2024 adjugates, so the number in the question is $n = 2^{2^{2024}}$. It is the exponent that doubles at each step; the determinant does not merely become 2^{2024}.

Step 2: Reduce the exponent and then the power To use the period 6 modulo 9, determine $2^{2024} \pmod 6$. Positive powers of 2 alternate as $2, 4, 2, 4, \ldots$ modulo 6. Since 2024 is even,

$$2^{2024} \equiv 4 \pmod 6.$$

Therefore

$$n = 2^{2^{2024}} \equiv 2^4 = 16 \equiv 7 \pmod 9.$$

Both reductions are needed: the first handles the huge exponent, while the second gives the remainder of the determinant itself. No explicit matrix entries or expansion of the adjugates is necessary.

Step 3: Conclusion The remainder when n is divided by 9 is $\boxed{7}$.

Quick Tip to Solve: For repeated adjugates, track the determinant recursively first; then reduce the outer power using its modular period, rather than expanding any matrix.

30. A line passes through $A(4,-6,-2)$ and $B(16,-2,4)$. The point $P(a,b,c)$, where a,b,c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the points $P(a,b,c)$ and $Q(4,-12,3)$ is equal to _____. [4 marks]

Solution:

Concept:

- A direction vector of the line through two points is their difference. A point at distance d from A on that line is $A \pm d\,\widehat{\boldsymbol{AB}}$, where the hat denotes a unit vector.
- The distance between points (x_1,y_1,z_1) and (x_2,y_2,z_2) is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$. The non-negative-coordinate condition selects one of the two line points.

Step 1: Find both possible points at the specified distance The displacement from A to B is

$$\boldsymbol{AB} = (16-4,-2+6,4+2) = (12,4,6) = 2(6,2,3).$$

The vector $(6,2,3)$ has length $\sqrt{36+4+9} = 7$. Write a general line point as

$$P = (4,-6,-2) + t(6,2,3).$$

Its distance from A is $7|t|$. The requirement $AP = 21$ therefore gives

$$7|t| = 21, \qquad t = 3 \text{ or } -3.$$

Retaining both signs initially is essential because a distance does not fix a direction along a line.

Step 2: Apply the coordinate restriction and calculate the distance For $t = 3$, the point is

$$P = (4+18,-6+6,-2+9) = (22,0,7).$$

For $t = -3$, the point would be $(-14,-12,-11)$, which violates the non-negative condition. Thus only $(22,0,7)$ is admissible; zero is allowed because the coordinates need only be non-negative. Using $Q = (4,-12,3)$,

$$\begin{aligned} PQ &= \sqrt{(22-4)^2+(0+12)^2+(7-3)^2} \\ &= \sqrt{324+144+16} = \sqrt{484} = 22. \end{aligned}$$

The selected point also has integer coordinates and $AP = \sqrt{18^2+6^2+9^2} = 21$, checking both original conditions.

Step 3: Conclusion The required distance is $\boxed{22 \text{ units}}$.

Quick Tip to Solve: Use a simplified integer direction vector and its length; keep both signs of the line parameter until the coordinate restrictions choose the admissible point.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Given below are two statements:

Statement I: Electromagnetic waves carry energy as they travel through space and this energy is equally shared by the electric and magnetic fields.

Statement II: When electromagnetic waves strike a surface, a pressure is exerted on the surface.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Statement I is incorrect but Statement II is correct.

(B) Both Statement I and Statement II are correct.

(C) Statement I is correct but Statement II is incorrect.

(D) Both Statement I and Statement II are incorrect.

Correct Answer: (B) Both Statement I and Statement II are correct.

Solution:

Concept:

- The electric and magnetic energy densities in vacuum are $u_E = \frac{1}{2}\varepsilon_0 E^2$ and $u_B = B^2/(2\mu_0)$. For a travelling electromagnetic wave, $E = cB$ and $c^2 = 1/(\mu_0\varepsilon_0)$.
- Electromagnetic radiation carries momentum as well as energy. Transfer of momentum to a surface gives a force; force per unit area is radiation pressure.

Step 1: Check the sharing of wave energy Using the wave relation in the electric energy density gives

$$u_E = \frac{1}{2}\varepsilon_0 c^2 B^2 = \frac{B^2}{2\mu_0} = u_B.$$

Therefore the electric field and magnetic field contribute equal amounts to the total energy density of the travelling wave. This equality holds instantaneously for the plane wave and also for the time-averaged densities. The energy travels with the wave through space; it does not belong only to one field. Statement I is consequently correct.

Step 2: Check the pressure and compare all options When incident radiation is absorbed or reflected, its momentum changes and the surface receives the opposite momentum. The rate of momentum transfer produces a force, so there is a pressure on the illuminated surface. For normal incidence, the pressure is

$$p = \frac{I}{c} \quad \text{for complete absorption,} \qquad p = \frac{2I}{c} \quad \text{for complete reflection,}$$

where I is intensity. These formulae demonstrate Statement II. Both statements are true, which selects (B). Option (A) wrongly rejects equal field-energy sharing; (C) wrongly rejects radiation pressure; and (D) rejects both valid physical properties. The question asks for the truth of the statements, rather than an assertion-reason explanatory relation.

Step 3: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (B).

Quick Tip to Solve: Check wave energy with $E = cB$, and check pressure by momentum transfer; these are two independent properties of electromagnetic radiation.

32. Consider two physical quantities A and B related to each other as $E = \frac{B-x^2}{At}$ where E, x and t have dimensions of energy, length and time respectively. The dimension of AB is [4 marks]

(A) $L^{-2}M^{1}T^{0}$

(B) $L^{2}M^{-1}T^{1}$

(C) $L^{0}M^{-1}T^{1}$

(D) $L^{-2}M^{-1}T^{1}$

Correct Answer: (B) $L^{2}M^{-1}T^{1}$

Solution:

Concept:

- Dimensional homogeneity requires terms that are added or subtracted to have the same dimensions. Since $B - x^2$ appears in the numerator and x is a length, $[B] = L^2$.
- Energy has dimensions $[E] = ML^2T^{-2}$, and time has dimensions $[t] = T$. Dimensions multiply and divide like algebraic powers, even when the actual numerical values are unknown.

Step 1: Determine the dimensions of each unknown The subtraction in the numerator immediately yields

$$[B] = [x^2] = L^2.$$

Now use the given equation $E = (B - x^2)/(At)$. Its dimensional form is

$$[E] = \frac{L^2}{[A]T}.$$

Rearranging,

$$[A] = \frac{L^2}{[E]T}$$
$$= \frac{L^2}{(ML^2T^{-2})T} = M^{-1}T.$$

The two length powers cancel in $[A]$. The negative mass power is unavoidable because energy is in the denominator after rearrangement.

Step 2: Multiply the dimensions and eliminate the alternatives The requested quantity is the product AB, not the quotient A/B. Hence

$$[AB] = [A][B] = (M^{-1}T)(L^2) = L^2M^{-1}T^1.$$

This agrees with option (B). Option (A) has a positive mass power and lacks the required time power. Option (C) would correspond to $[A]$ alone, omitting the L^2 contribution from B. Option (D) puts L^2 in the denominator, as though dividing by B. A quick consistency check is $[A][E][t] = (M^{-1}T)(ML^2T^{-2})T = L^2$, exactly the dimension of the numerator. Thus no information about numerical values of A or B is required.

Step 3: Conclusion The dimension of AB is $\boxed{L^2M^{-1}T^1}$. Hence, the correct option is (B).

Quick Tip to Solve: In a dimension question with a subtraction, identify the dimensions of the two subtracted terms first, then rearrange for the remaining unknown.

33. In a photoelectric effect experiment a light of frequency 1.5 times the threshold frequency is made to fall on the surface of photosensitive material. Now if the frequency is halved and intensity is doubled, the number of photo electrons emitted will be: [4 marks]

(A) doubled

(B) halved

(C) Zero

(D) quadrupled

Correct Answer: (C) Zero

Solution:

Concept:

- The threshold frequency ν_0 satisfies the work-function relation $\phi = h\nu_0$. A photon ejects an electron only when its energy $h\nu$ is at least the work function.
- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$. Increasing intensity increases the arrival rate of photons, but does not increase the energy of an individual photon when frequency is fixed.

Step 1: Compare the new frequency with the threshold The original frequency is $\nu = 1.5\nu_0$, so initially the photons have sufficient energy to eject electrons. After halving this frequency,

$$\nu' = \frac{1.5\nu_0}{2} = 0.75\nu_0.$$

Thus $\nu' < \nu_0$. Equivalently, each new photon has energy

$$h\nu' = 0.75h\nu_0 = 0.75\phi < \phi.$$

It cannot supply the minimum energy needed for a photoelectron to escape. Direct substitution into the kinetic-energy equation gives -0.25ϕ; this negative result signals that emission is impossible, rather than an emitted electron with negative kinetic energy.

Step 2: Assess the doubled intensity and the offered answers Doubling the intensity supplies more incident radiation energy per unit area per unit time, but every individual photon is still below threshold. In the ordinary photoelectric effect described by the question, the electron absorbs a single photon, so a larger photon arrival rate cannot compensate for insufficient photon energy. Consequently there are no emitted photoelectrons. Options (A), (B) and (D) all predict a non-zero count and overlook the threshold-frequency condition. Their possible scaling arguments would apply only after ensuring that the incident photons can eject electrons. Here that preliminary test already rules emission out.

Step 3: Conclusion The number of emitted photoelectrons is $\boxed{0}$. Hence, the correct option is (C).

Quick Tip to Solve: Always test the changed frequency against the threshold before using any relation between photoelectric current and intensity.

34. The mass number of nucleus having radius equal to half of the radius of nucleus with mass number 192 is : [4 marks]

(A) 32

(B) 24

(C) 20

(D) 40

Correct Answer: (B) 24

Solution:

Concept:

- The empirical nuclear-radius formula is $R = R_0 A^{1/3}$, where A is the mass number and R_0 is approximately the same for nuclei. Nuclear volume is therefore proportional to mass number.
- When comparing two nuclei, take the ratio of radii so that R_0 cancels. A change in radius must be cubed to obtain the corresponding change in mass number.

Step 1: Convert the radius relation to a mass-number ratio Let the unknown nucleus have mass number A_1 and radius R_1. The reference nucleus has $A_2 = 192$, radius R_2, and the condition is $R_1 = R_2/2$. Applying the radius law to both nuclei,

$$\frac{R_1}{R_2} = \frac{R_0 A_1^{1/3}}{R_0 (192)^{1/3}} = \left(\frac{A_1}{192}\right)^{1/3} = \frac{1}{2}.$$

The same R_0 cancels, so no numerical nuclear length scale needs to be inserted. The cube root, rather than a linear radius dependence, is the decisive feature of the law.

Step 2: Cube the ratio and select the matching mass number Cubing both sides gives

$$\frac{A_1}{192} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Hence

$$A_1 = \frac{192}{8} = 24.$$

The result is dimensionless, as a mass number counts nucleons. As a check, $(24/192)^{1/3} = (1/8)^{1/3} = 1/2$, recovering the prescribed radius ratio. Option (B) is therefore correct. The other offered values $32, 20, 40$ would give mass-number ratios $1/6, 5/48, 5/24$, none equal to $1/8$. A factor of two in radius corresponds to a factor of eight in volume and in mass number, not a factor of two in the nucleon count.

Step 3: Conclusion The required mass number is $\boxed{24}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $A_1/A_2 = (R_1/R_2)^3$: halving a nuclear radius reduces the mass number by a factor of eight.

35. A uniform magnetic field of 2×10^{-3} T acts along positive Y-direction. A rectangular loop of sides 20 cm and 10 cm with current of 5 A is in $Y - Z$ plane. The current is in anticlockwise sense with reference to negative X axis. Magnitude and direction of the torque is: [4 marks]

(A) 2×10^{-4} N- m along negative Z-direction

(B) 2×10^{-4} N - m along positive X-direction

(C) 2×10^{-4} N - m along positive Y-direction

(D) 2×10^{-4} N - m along positive Z-direction

Correct Answer: (A) 2×10^{-4} N- m along negative Z-direction

Solution:

Concept:

- The magnetic dipole moment of a single current loop is $\boldsymbol{\mu} = IA\,\hat{\mathbf{n}}$, with the normal fixed by the right-hand rule. Torque is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$.
- For a loop in the Y-Z plane, its normal lies on the X axis. An anticlockwise current as viewed from the negative X side has magnetic moment directed towards that observer, along $-\hat{\mathbf{x}}$.

Step 1: Calculate the loop moment in SI units Convert both side lengths to metres before multiplying them:

$$A = (20 \times 10^{-2})(10 \times 10^{-2}) = 0.020\,\text{m}^2.$$

With current $I = 5\,\text{A}$, the magnitude of the moment is

$$\mu = IA = 5(0.020) = 0.10\,\text{A m}^2.$$

The right-hand rule applied from the specified negative-X viewing direction gives

$$\boldsymbol{\mu} = -0.10\,\hat{\mathbf{x}}\,\text{A m}^2.$$

The field is $\mathbf{B} = 2 \times 10^{-3}\hat{\mathbf{y}}\,\text{T}$. These vectors are perpendicular because the field lies in the loop plane.

Step 2: Take the cross product and compare the directions The magnitude is

$$\tau = \mu B \sin 90^\circ = (0.10)(2 \times 10^{-3}) = 2 \times 10^{-4}\,\text{N m}.$$

For its direction, use $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$:

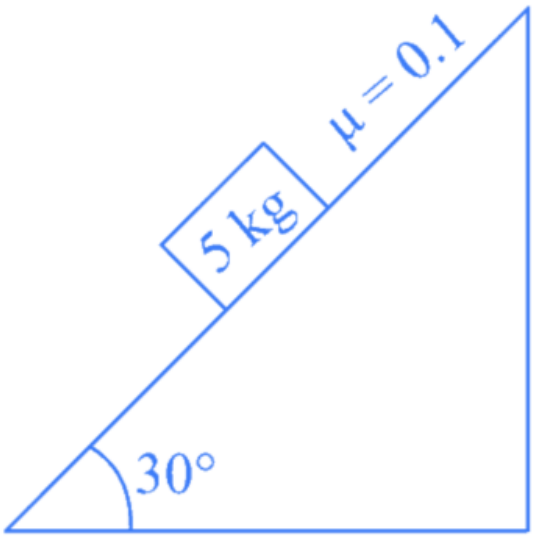
$$(-\hat{\mathbf{x}}) \times \hat{\mathbf{y}} = -\hat{\mathbf{z}}.$$

The torque therefore points along negative Z, matching (A). Options (B) and (C) point along the moment or the field, respectively, whereas a non-zero cross product is perpendicular to both. Option (D) has the opposite sign and would result from reversing the current or the viewing direction. The stated sense of current must therefore be used along with the loop plane; the plane alone does not determine the torque sign.

Step 3: Conclusion The torque is $\boxed{2 \times 10^{-4}\,\text{N m}}$ along negative Z-direction. Hence, the correct option is (A).

Quick Tip to Solve: Fix the normal using the stated viewing side before taking $\boldsymbol{\mu} \times \mathbf{B}$; changing the viewing side reverses the apparent current sense.

36. A block of mass $5\ \text{kg}$ is placed on a rough inclined surface as shown in the figure. If $\overrightarrow{F_1}$ is the force required to just move the block up the inclined plane and $\overrightarrow{F_2}$ is the force required to just prevent the block from sliding down, then the value of $\left|\overrightarrow{F_1}\right| - \left|\overrightarrow{F_2}\right|$ is : [Use $g = 10\ \text{m/s}^2$] [4 marks]



(A) $5\sqrt{3}N$

(B) $\frac{5\sqrt{3}}{2}N$

(C) $10\ \text{N}$

(D) $25\sqrt{3}N$

Correct Answer: (A) $5\sqrt{3}N$

Solution:

Concept:

- The component of weight along an incline of angle θ is $mg\sin\theta$; the normal reaction is $N = mg\cos\theta$ when the applied force is parallel to the plane.
- Limiting friction has magnitude $f = \mu N$. It opposes the impending motion, so its direction reverses between impending upward and downward motion.

Step 1: Resolve the weight and find limiting friction The figure gives $m = 5\,\text{kg}$, $\theta = 30^\circ$ and $\mu = 0.1$. Thus the downslope component and the normal reaction are

$$mg\sin 30^\circ = 25\,\text{N}, \qquad N = 50\cos 30^\circ = 25\sqrt{3}\,\text{N}.$$

Hence

$$f = 0.1(25\sqrt{3}) = \frac{5\sqrt{3}}{2}\,\text{N}.$$

The normal component is balanced, and only the forces parallel to the incline enter the sliding condition.

Step 2: Write both limiting balances and subtract To just move upward, the applied force must

balance both the downslope weight component and downslope friction:

$$|\vec{F}_1| = 25 + \frac{5\sqrt{3}}{2}\,\text{N}.$$

To just prevent downward sliding, friction acts upward, helping the applied force:

$$|\vec{F}_2| = 25 - \frac{5\sqrt{3}}{2}\,\text{N}.$$

The two applied-force magnitudes are approximately $29.33\,\text{N}$ and $20.67\,\text{N}$, respectively. This value is positive because the downslope weight exceeds the available friction. Subtracting cancels gravity and gives

$$|\vec{F}_1| - |\vec{F}_2| = 2f = 5\sqrt{3}\,\text{N}.$$

Option (B) is only one limiting-friction force, while (D) is the normal reaction; neither is the requested difference. The value in (C) also fails this balance.

Step 3: Conclusion The required difference is $\boxed{5\sqrt{3}\,\text{N}}$. Hence, the correct option is (A).

Quick Tip to Solve: For two opposite impending motions on the same incline, subtract the force balances: gravity cancels and the difference is twice the limiting friction.

37. If two vectors $\vec{A}$ and $\vec{B}$ having equal magnitude R are inclined at angle θ, then [4 marks]

(A) $|\vec{A}+\vec{B}| = 2R\cos\left(\frac{\theta}{2}\right)$
(B) $|\vec{A}-\vec{B}| = 2R\cos\left(\frac{\theta}{2}\right)$
(C) $|\vec{A}-\vec{B}| = \sqrt{2}R\sin\left(\frac{\theta}{2}\right)$
(D) $|\vec{A}+\vec{B}| = 2R\sin\left(\frac{\theta}{2}\right)$

Correct Answer: (A) $|\vec{A}+\vec{B}| = 2R\cos\left(\frac{\theta}{2}\right)$

Solution:

Concept:

- The scalar product gives $|\vec{A}+\vec{B}|^2 = A^2 + B^2 + 2AB\cos\theta$, where θ is the angle between the vectors.
- The half-angle identities $1+\cos\theta = 2\cos^2(\theta/2)$ and $1-\cos\theta = 2\sin^2(\theta/2)$ simplify the sum and difference when both magnitudes equal R.

Step 1: Calculate the magnitude of the sum Insert $A = B = R$ into the scalar-product relation:

$$|\vec{A}+\vec{B}|^2 = R^2 + R^2 + 2R^2\cos\theta = 2R^2(1+\cos\theta).$$

Use the half-angle identity to obtain

$$|\vec{A}+\vec{B}|^2 = 4R^2\cos^2\left(\frac{\theta}{2}\right).$$

The angle between two vectors lies between 0 and π, so $\cos(\theta/2)$ is nonnegative. Taking the positive

square root therefore gives

$$|\vec{A}+\vec{B}| = 2R\cos\left(\frac{\theta}{2}\right).$$

Step 2: Check the difference and reject the other forms Similarly,

$$|\vec{A}-\vec{B}|^2 = 2R^2(1-\cos\theta) = 4R^2\sin^2\left(\frac{\theta}{2}\right),$$

$$|\vec{A}-\vec{B}| = 2R\sin\left(\frac{\theta}{2}\right).$$

Thus option (B) uses cosine where the difference requires sine, and option (C) has an incorrect coefficient $\sqrt{2}$ instead of 2. Option (D) assigns the difference formula to the sum. As a quick limiting check, parallel equal vectors have a sum of magnitude $2R$ and zero difference, exactly as these derived expressions predict.

Step 3: Conclusion For equal vectors, $\boxed{|\vec{A}+\vec{B}| = 2R\cos(\theta/2)}$. Hence, the correct option is (A).

Quick Tip to Solve: Equal-vector sums use the cosine of the half-angle; differences use the sine. Check parallel vectors to avoid interchanging the two formulae.

38. By what percentage will the illumination of the lamp decrease if the current drops by 20%?

[4 marks]

(A) 26%

(B) 36%

(C) 46%

(D) 56%

Correct Answer: (B) 36%

Solution:

Concept:

- For a lamp treated as a fixed resistance R, the electrical power dissipated is $P = I^2R$. Its illumination is taken as proportional to this power.
- When the current changes, compare the ratio of final to initial powers before converting the fractional loss to a percentage.

Step 1: Express the reduced current as a fraction Let the initial current be I_0 and the initial illumination be L_0. A decrease of 20% means that 80% of the current remains:

$$I_1 = I_0 - 0.20I_0 = 0.80I_0.$$

This factor describes the new current, not the new power. Since electrical heating depends on the square of current, directly setting the illumination loss equal to 20% would omit the essential quadratic dependence. The resistance cancels in a comparison of the same lamp.

Step 2: Find the illumination ratio and the percentage loss Using $L \propto P = I^2R$,

$$\frac{L_1}{L_0} = \frac{I_1^2 R}{I_0^2 R} = \left(\frac{I_1}{I_0}\right)^2 = (0.80)^2 = 0.64.$$

The lamp therefore retains 64% of its original illumination. The decrease is

$$\frac{L_0 - L_1}{L_0} \times 100\% = (1 - 0.64) \times 100\% = 36\%.$$

Equivalently, a fractional current fall x gives a fractional power fall $2x - x^2$; at $x = 0.20$, this is $0.40 - 0.04 = 0.36$. The alternatives 26%, 46% and 56% do not give the required remaining-power fraction 0.64, so all three are excluded.

Step 3: Conclusion The illumination decreases by $\boxed{36\%}$. Hence, the correct option is (B).

Quick Tip to Solve: After a current decrease of $p\%$, square the remaining fraction $1 - p/100$; subtract that square from one to find the fractional power loss.

39. The speed of sound in oxygen at S.T.P. will be approximately: (given, $R = 8.3\ \text{JK}^{-1}, \gamma = 1.4$)

[4 marks]

(A) 341 m/s

(B) 333 m/s

(C) 325 m/s

(D) 315 m/s

Correct Answer: (D) 315 m/s

Solution:

Concept:

- The adiabatic speed of sound in an ideal gas is $v = \sqrt{\gamma RT/M}$, where M is its molar mass in kg mol^{-1}.
- At S.T.P., the absolute temperature is $273\,\text{K}$. Oxygen is diatomic O_2, with molar mass $32\,\text{g mol}^{-1}$.

Step 1: Convert the molar mass to consistent SI units The universal gas constant must be used with the molar mass, rather than the mass of one molecule. For oxygen,

$$M = 32\,\text{g mol}^{-1} = 0.032\,\text{kg mol}^{-1}.$$

Use the given $\gamma = 1.4$, $R = 8.3\,\text{J mol}^{-1}\,\text{K}^{-1}$ and $T = 273\,\text{K}$. Sound propagation involves rapid compressions and rarefactions, which are approximately adiabatic; this is why the factor γ belongs in the sound-speed expression.

Step 2: Substitute and select the closest speed The square of the sound speed is

$$v^2 = \frac{(1.4)(8.3)(273)}{0.032} = 9.9133125 \times 10^4\,\text{m}^2\,\text{s}^{-2}.$$

Therefore,

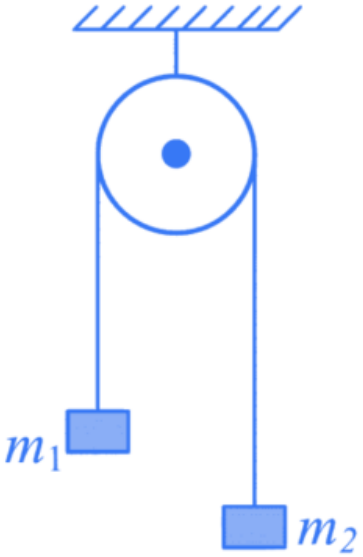
$$v = \sqrt{9.9133125 \times 10^4}\,\mathrm{m\,s^{-1}} \simeq 314.85\,\mathrm{m\,s^{-1}}.$$

Rounding to the nearest listed whole-number value gives $315\,\mathrm{m\,s^{-1}}$. The alternatives 325, 333 and $341\,\mathrm{m\,s^{-1}}$ all exceed the value calculated for oxygen with the supplied data. In particular, using the molar mass of air instead of oxygen would produce a larger speed and would answer a different question.

Step 3: Conclusion The speed is approximately $\boxed{315\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: In $v = \sqrt{\gamma RT/M}$, always convert molar mass from grams to kilograms and use absolute temperature; otherwise the numerical speed is wrong.

40. A light string passing over a smooth light fixed pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is $g/8$, then the ratio of masses is: [4 marks]



(A) $\frac{8}{1}$
(B) $\frac{9}{7}$
(C) $\frac{5}{3}$
(D) $\frac{4}{3}$

Correct Answer: (B) $\frac{9}{7}$

Solution:

Concept:

- A massless string over a smooth, massless pulley has the same tension T on both sides. The two connected masses have equal acceleration magnitudes.
- Adding Newton's second-law equations removes the internal tension, giving $a = g(m_h - m_l)/(m_h + m_l)$, where m_h and m_l are the heavier and lighter masses.

Step 1: Apply Newton's law to both blocks Let $m_h > m_l$, so the heavier block accelerates downward while the lighter block accelerates upward. Taking each block's direction of motion as positive gives

$$m_h g - T = m_h a, \qquad T - m_l g = m_l a.$$

Adding the equations cancels tension:

$$(m_h - m_l)g = (m_h + m_l)a.$$

Thus the acceleration depends on the difference of the masses divided by their sum, rather than on either mass alone. The ideal string and pulley assumptions allow the common tension used here.

Step 2: Solve the mass ratio and check the choices With the specified $a = g/8$,

$$\frac{m_h - m_l}{m_h + m_l} = \frac{1}{8}.$$

Cross-multiplying gives

$$8m_h - 8m_l = m_h + m_l, \qquad 7m_h = 9m_l,$$
$$\frac{m_h}{m_l} = \frac{9}{7}.$$

The ratio is written heavier to lighter, as in the listed choices. For comparison, the proposed ratios $8/1$, $5/3$ and $4/3$ give accelerations $7g/9$, $g/4$ and $g/7$, respectively. None equals $g/8$. The reciprocal ratio would describe the same two masses with their order reversed, but it is not among the choices.

Step 3: Conclusion The ratio of the heavier mass to the lighter mass is $\boxed{9:7}$. Hence, the correct option is (B).

Quick Tip to Solve: For an ideal Atwood machine, if $a/g = k$, the heavier-to-lighter mass ratio is $(1+k)/(1-k)$; here it is $(9/8)/(7/8) = 9/7$.

41. A gas mixture consists of 8 moles of argon and 6 moles of oxygen at temperature T. Neglecting all vibrational modes, the total internal energy of the system is: [4 marks]

(A) 29 RT

(B) 27 RT

(C) 20 RT

(D) 21 RT

Correct Answer: (B) 27 RT

Solution:

Concept:

- For n moles of an ideal gas with f active degrees of freedom, equipartition gives $U = (f/2)nRT$. Internal energies of the components of a mixture add.
- Monatomic argon has three translational degrees of freedom. Diatomic oxygen has three transla-

tional and two rotational degrees of freedom when vibrations are neglected.

Step 1: Calculate the argon contribution Argon atoms are monatomic, so only the three independent translational kinetic-energy terms contribute. Each quadratic degree of freedom contributes $RT/2$ per mole. For the eight moles of argon,

$$U_{\mathrm{Ar}} = \frac{3}{2}(8)RT = 12RT.$$

There is no rotational contribution for a monatomic ideal-gas particle in this treatment. The temperature T is common to both gases, but their degrees of freedom are not the same, so treating all fourteen moles as a single monatomic gas would be incorrect.

Step 2: Calculate the oxygen contribution and add With vibrational modes explicitly neglected, an oxygen molecule has $f = 5$. Thus for six moles,

$$U_{\mathrm{O_2}} = \frac{5}{2}(6)RT = 15RT.$$

Adding the two independent contributions gives

$$U_{\mathrm{total}} = U_{\mathrm{Ar}} + U_{\mathrm{O_2}} = 12RT + 15RT = 27RT.$$

The coefficient 21 would result from assigning three degrees of freedom to all fourteen moles and omitting oxygen's rotational energy. The coefficients 29 and 20 also fail the separate degree-of-freedom count. The result has energy units because R is in $\mathrm{J\,mol^{-1}\,K^{-1}}$, the mole amounts are included, and T is in kelvin.

Step 3: Conclusion The total internal energy is $\boxed{27RT}$, in joules with SI units. Hence, the correct option is (B).

Quick Tip to Solve: For a gas mixture, calculate nf separately for each species and add before multiplying by $RT/2$; do not average the degrees of freedom without mole weighting.

42. A small spherical ball of radius r, falling through a viscous medium of negligible density has terminal velocity 'v'. Another ball of the same mass but of radius $2r$, falling through the same viscous medium will have terminal velocity: [4 marks]

(A) 4v

(B) 2 V

(C) $\frac{v}{4}$

(D) $\frac{v}{2}$

Correct Answer: (D) $\frac{v}{2}$

Solution:

Concept:

- For a small sphere moving slowly through a viscous medium, Stokes' drag is $F_d = 6\pi\eta r v$, where η is the dynamic viscosity and r is the sphere radius.
- At terminal speed the net force is zero. Since the medium has negligible density, buoyancy is negligible and drag balances the sphere's weight mg.

Step 1: Write the terminal balance for the first ball For the first sphere, the downward force is its weight and the upward resistive force is Stokes' drag. Therefore,

$$mg = 6\pi\eta r v, \qquad v = \frac{mg}{6\pi\eta r}.$$

This expression is useful here because the two balls are stated to have the same mass. Their densities therefore need not be equal: increasing radius while keeping mass fixed makes the second sphere less dense. The common same-density formula $v \propto r^2$ cannot be used under this different constraint.

Step 2: Compare the second ball using the unchanged mass The second sphere has radius $2r$, mass m, and the same viscosity η. At its terminal speed v_2,

$$mg = 6\pi\eta(2r)v_2.$$

Divide this by the first balance, or substitute for mg:

$$v_2 = \frac{mg}{12\pi\eta r} = \frac{1}{2}\frac{mg}{6\pi\eta r} = \frac{v}{2}.$$

The doubled radius doubles the drag coefficient, so half the speed supplies the same force. Option (A), $4v$, would correspond to keeping the sphere density fixed rather than its mass. Neither $2v$ nor $v/4$ satisfies the terminal force balance for the stated equal masses; those would respectively quadruple or halve the required drag force.

Step 3: Conclusion The second ball's terminal speed is $\boxed{v/2}$. Hence, the correct option is (D).

Quick Tip to Solve: Before using the familiar $v_t \propto r^2$, check what stays fixed. With equal masses and negligible buoyancy, Stokes' law instead gives $v_t \propto 1/r$.

43. The measured value of the length of a simple pendulum is $20\ \mathrm{cm}$ with $2\ \mathrm{mm}$ accuracy. The time for 50 oscillations was measured to be 40 seconds with 1 second resolution. From these measurements, the accuracy in the measurement of acceleration due to gravity is $\mathrm{N}\%$. The value of N is: [4 marks]

(A) 6

(B) 5

(C) 4

(D) 8

Correct Answer: (A) 6

Solution:

Concept:

- For a simple pendulum undergoing small oscillations, $T = 2\pi\sqrt{L/g}$, so $g = 4\pi^2 L/T^2$. Here L is its length and T is the period.
- For a product or quotient involving powers, maximum fractional measurement errors add with the absolute powers: $\Delta g/g = \Delta L/L + 2\Delta T/T$.

Step 1: Find the fractional error in length The length is $L = 20\,\mathrm{cm}$, and the stated accuracy is $\Delta L = 2\,\mathrm{mm} = 0.2\,\mathrm{cm}$. Therefore

$$\frac{\Delta L}{L} = \frac{0.2}{20} = 0.01 = 1\%.$$

The error and measured length must be expressed in the same unit before taking their ratio.

Step 2: Find the fractional error in the period The measured time for 50 oscillations is $t_{50} = 40\,\mathrm{s}$, with uncertainty $\Delta t_{50} = 1\,\mathrm{s}$. The count 50 is exact, so $T = t_{50}/50 = 0.8\,\mathrm{s}$, and $\Delta T = 1/50\,\mathrm{s}$. Hence

$$\frac{\Delta T}{T} = \frac{\Delta t_{50}}{t_{50}} = \frac{1}{40} = 0.025 = 2.5\%.$$

Dividing the measured time by 50 divides its absolute uncertainty by 50 as well.

Step 3: Combine the uncertainties Since the period occurs squared in the denominator of g, its percentage uncertainty contributes twice:

$$\frac{\Delta g}{g} \times 100 = 1 + 2(2.5) = 6\%.$$

Thus the options 5, 4 and 8 do not include both the length contribution and the correctly doubled time contribution. The required accuracy is expressed as a percentage error, rather than as the numerical value of g.

Step 4: Conclusion The percentage uncertainty is 6%, so $N = 6$. Hence, the correct option is (A).

Quick Tip to Solve: Use the fractional uncertainty of the total time directly; dividing by an exact oscillation count does not change the percentage uncertainty.

44. When unpolarized light is incident at an angle of 60° on a transparent medium from air, the reflected ray is completely polarized. The angle of refraction in the medium is: [4 marks]

(A) 60°

(B) 90°

(C) 30°

(D) 45°

Correct Answer: (C) 30°

Solution:

Concept:

- When the reflected ray is completely plane polarized, the incidence angle equals Brewster's angle i_B. For incidence from air, $\tan i_B = \mu$, where μ is the refractive index of the medium.
- At Brewster incidence, the reflected and refracted rays are perpendicular. Consequently $i_B + r = 90^\circ$, where r is measured from the normal inside the transparent medium.

Step 1: Identify the incidence condition The incident light is unpolarized, but the reflected light is completely polarized. This is the characteristic Brewster condition. The given incidence angle is therefore $i_B = 60^\circ$. The angle of reflection is also 60°, by the law of reflection; it is not the requested angle of refraction.

Step 2: Use perpendicular reflected and refracted rays The angle between the reflected and transmitted directions is 90°. In terms of their angles with the normal, the Brewster relation gives

$$i_B + r = 90^\circ.$$

Substituting the incident angle,

$$r = 90^\circ - 60^\circ = 30^\circ.$$

The refracted ray bends towards the normal as light travels from air into this optically denser medium.

Step 3: Check using the refractive index Brewster's law also gives $\mu = \tan 60^\circ = \sqrt{3}$. Snell's law then gives

$$\sin r = \frac{\sin 60^\circ}{\sqrt{3}} = \frac{1}{2},$$

which again yields $r = 30^\circ$. An angle of 60° confuses refraction with reflection, 90° would describe a grazing refracted ray, and 45° would not satisfy either Snell's law or the perpendicular-ray condition for this incident angle.

Step 4: Conclusion The angle of refraction is 30°. Hence, the correct option is (C).

Quick Tip to Solve: Complete polarization on reflection signals Brewster incidence: immediately add the incidence and refraction angles to get 90°.

45. The resistance per centimeter of a meter bridge wire is r, with $X\Omega$ resistance in left gap. Balancing length from left end is at 40 cm with 25Ω resistance in right gap. Now the wire is replaced by another wire of $2r$ resistance per centimeter. The new balancing length for same settings will be at [4 marks]

(A) 10 cm

(B) 80 cm

(C) 40 cm

(D) 20 cm

Correct Answer: (C) 40 cm

Solution:

Concept:

- A balanced metre bridge is a Wheatstone bridge. With gap resistances X and R, its balance condition is $X/R = R_{\text{left}}/R_{\text{right}}$. No current passes through the galvanometer at balance.
- For a uniform bridge wire of length $100\,\text{cm}$ and resistance per centimetre r, the two wire resistances are $r\ell$ and $r(100-\ell)$. The common resistance per unit length cancels in their ratio.

Step 1: Apply the original balance condition The left gap contains $X\,\Omega$, the right gap contains $25\,\Omega$, and balance occurs $40\,\text{cm}$ from the left end. Thus the left and right wire lengths are 40 and $60\,\text{cm}$, respectively:

$$\frac{X}{25} = \frac{40r}{60r} = \frac{2}{3}.$$

If required, this gives $X = 50/3\,\Omega$. The ratio, rather than this absolute value, determines the balance point.

Step 2: Apply balance with the replacement wire Let the new balancing length from the left end be ℓ centimetres. The new uniform wire has resistance per centimetre $2r$, so

$$\frac{X}{25} = \frac{2r\ell}{2r(100-\ell)} = \frac{\ell}{100-\ell}.$$

Both wire arms have had their resistance multiplied by the same factor. Since the gap resistances are unchanged, their required ratio remains $2/3$.

Step 3: Solve and reject changes in length Equating the ratios gives

$$\frac{\ell}{100-\ell} = \frac{2}{3} \quad \Rightarrow \quad 3\ell = 200 - 2\ell \quad \Rightarrow \quad \ell = 40\,\text{cm}.$$

The proposed lengths 10, 80 and $20\,\text{cm}$ give wire ratios $1/9$, 4 and $1/4$, respectively; none equals $2/3$. Increasing the total resistance of the wire therefore does not shift this ideal balance point.

Step 4: Conclusion The new balancing length remains $40\,\text{cm}$ from the left end. Hence, the correct option is (C).

Quick Tip to Solve: In a uniform metre-bridge wire, a common change in resistance per unit length cancels from the balance ratio.

46. Force between two point charges q_1 and q_2 placed in vacuum at 'r' cm apart is F. Force between them when placed in a medium having dielectric constant $K = 5$ at '$r/5$' cm apart will be: [4 marks]

(A) $5F$

(B) $25F$

(C) $F/5$

(D) $F/25$

Correct Answer: (A) $5F$

Solution:

Concept:

- The magnitude of the electrostatic force between fixed point charges in vacuum is $F = k|q_1q_2|/d^2$, where $k = 1/(4\pi\varepsilon_0)$ and d is their separation.
- In a medium with relative permittivity (dielectric constant) K, the force at separation d becomes $k|q_1q_2|/(Kd^2)$. A change of separation must also be included through the inverse-square law.

Step 1: Write the reference and changed forces Let the original separation be $d = r\,\text{cm}$. The reference vacuum force is

$$F = \frac{k|q_1q_2|}{d^2}.$$

In the medium, $K = 5$, while the new separation is $d_{\text{new}} = d/5$. The charges themselves do not change, so the changed force is

$$F_{\text{new}} = \frac{k|q_1q_2|}{5(d/5)^2}.$$

The same distance unit can be retained in the ratio because its conversion factor cancels.

Step 2: Take the ratio Dividing the changed expression by the original expression removes the common charge and Coulomb-constant factors:

$$\frac{F_{\text{new}}}{F} = \frac{1}{5}\frac{d^2}{(d/5)^2} = \frac{25}{5} = 5.$$

Reducing the separation by five increases the force by 25, whereas inserting the dielectric reduces it by five. The combined effect is therefore a fivefold increase, with the attractive or repulsive character unchanged.

Step 3: Check the alternative factors The factor $25F$ would follow from the distance change alone and ignores the dielectric. The factor $F/5$ includes only the medium and ignores the reduction of separation. The factor $F/25$ incorrectly treats the closer separation as reducing the force. Applying both changes simultaneously gives $F_{\text{new}} = 5F$, which is consistent with the inverse-square dependence.

Step 4: Conclusion The force in the medium at separation $r/5$ is $5F$. Hence, the correct option is (A).

Quick Tip to Solve: For unchanged charges use $F_2/F_1 = (K_1/K_2)(d_1/d_2)^2$; account for the dielectric and distance in separate factors.

47. A body of mass $2\ \text{kg}$ begins to move under the action of a time dependent force given by $\vec{F} = \left(6t\hat{i} + 6t^2\hat{j}\right) N$. The power developed by the force at the time t is given by: [4 marks]

(A) $\left(3t^3 + 6t^5\right) W$

(B) $\left(9t^5 + 6t^3\right) W$

(C) $\left(6t^4 + 9t^5\right) W$

(D) $\left(9t^3 + 6t^5\right) W$

Correct Answer: (D) $\left(9t^3 + 6t^5\right) W$

Solution:

Concept:

- Newton's second law gives $\vec{a} = \vec{F}/m$. For a force depending on time, integrate each acceleration component to obtain the corresponding velocity component.
- The instantaneous power delivered by a force is the scalar product $P = \vec{F} \cdot \vec{v} = F_x v_x + F_y v_y$. Perpendicular component cross terms do not contribute.

Step 1: Find the acceleration components The mass is $2\,\text{kg}$, and the applied force is $\vec{F} = 6t\hat{i} + 6t^2\hat{j}$ in newtons. Hence

$$\vec{a} = \frac{\vec{F}}{2} = 3t\hat{i} + 3t^2\hat{j}.$$

The body begins to move under this force, so its initial velocity at $t = 0$ is zero in both directions.

Step 2: Integrate to obtain the velocity Integrate from zero to the specified time, using a dummy time variable τ:

$$v_x(t) = \int_0^t 3\tau\, d\tau = \frac{3}{2}t^2, \qquad v_y(t) = \int_0^t 3\tau^2\, d\tau = t^3.$$

Thus $\vec{v} = (3t^2/2)\hat{i} + t^3\hat{j}$ in SI units. The velocity is not simply the acceleration multiplied by time, because the acceleration is not constant.

Step 3: Take the scalar product and compare Using the matching Cartesian components,

$$\begin{aligned} P &= (6t)\left(\frac{3}{2}t^2\right) + (6t^2)(t^3) \\ &= 9t^3 + 6t^5. \end{aligned}$$

The coefficient of t^3 must be 9 from the horizontal contribution, and the coefficient of t^5 must be 6 from the vertical contribution. Option (A) has an incorrect horizontal coefficient, (B) exchanges the coefficients and powers, and (C) introduces the wrong power t^4. As a check, differentiating $K = \frac{1}{2}m(v_x^2 + v_y^2) = 9t^4/4 + t^6$ gives exactly the same power.

Step 4: Conclusion The instantaneous power is $P = (9t^3 + 6t^5)\,\text{W}$, with t in seconds. Hence, the correct option is (D).

Quick Tip to Solve: Integrate a time-dependent force to find velocity before using $P = \vec{F} \cdot \vec{v}$; multiplying force by acceleration does not give power.

48. An AC voltage $V = 20\sin 200\pi t$ is applied to a series LCR circuit which drives a current $I = 10\sin\left(200\pi t + \frac{\pi}{3}\right)$. The average power dissipated is: [4 marks]

(A) 21.6 W

(B) 200 W

(C) 173.2 W

(D) 50 W

Correct Answer: (D) 50 W

Solution:

Concept:

- For sinusoidal voltage and current with phase difference ϕ, the average power is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$. The factor $\cos\phi$ is the power factor.
- If the peak values are V_0 and I_0, then $V_{\text{rms}} = V_0/\sqrt{2}$ and $I_{\text{rms}} = I_0/\sqrt{2}$, giving $P_{\text{av}} = V_0 I_0 \cos\phi/2$.

Step 1: Identify amplitudes and phase The applied voltage is $V = 20\sin(200\pi t)$ and the current is $I = 10\sin(200\pi t + \pi/3)$. Thus the peak values are $V_0 = 20\,\text{V}$, $I_0 = 10\,\text{A}$, and the phase difference has magnitude $\phi = \pi/3$. The current leads the voltage, but the cosine is the same for positive and negative phase differences.

Step 2: Compute the average real power Convert the amplitudes to rms values and include the power factor:

$$P_{\text{av}} = \frac{20}{\sqrt{2}}\frac{10}{\sqrt{2}}\cos\left(\frac{\pi}{3}\right)$$
$$= \frac{200}{2}\frac{1}{2} = 50\,\text{W}.$$

Ideal inductors and capacitors alternately store and return energy; only the resistive contribution gives a nonzero average dissipation.

Step 3: Verify by averaging the instantaneous product Writing $\theta = 200\pi t$, the instantaneous power is $p = 200\sin\theta\sin(\theta + \pi/3)$. The product-to-sum identity gives

$$p = 100\left[\cos(\pi/3) - \cos(2\theta + \pi/3)\right].$$

The oscillating cosine averages to zero over a complete cycle, leaving $100/2 = 50\,\text{W}$. Option (B), $200\,\text{W}$, is the peak-voltage times peak-current product, and does not perform the necessary averaging. Options (A) and (C) likewise do not equal the rms product multiplied by $\cos 60^\circ$.

Step 4: Conclusion The average power dissipated is $50\,\text{W}$. Hence, the correct option is (D).

Quick Tip to Solve: When voltage and current are written as sine functions, their coefficients are peak values: include both the factor $1/2$ and the phase factor $\cos\phi$.

49. The mass of the moon is $\frac{1}{144}$ times the mass of a planet and its diameter is $\frac{1}{16}$ times the diameter of a planet. If the escape velocity on the planet is v, the escape velocity on the moon will be : [4 marks]

(A) $\frac{v}{4}$

(B) $\frac{v}{6}$

(C) $\frac{V}{12}$

(D) $\frac{v}{3}$

Correct Answer: (D) $\frac{v}{3}$

Solution:

Concept:

- The minimum escape speed from the surface of a spherical body is $v_e = \sqrt{2GM/R}$, neglecting atmospheric resistance and rotation. It follows by equating the launch kinetic energy to the magnitude of surface gravitational potential energy.
- For two bodies, escape-speed ratios depend on the square root of their mass-to-radius ratios. A diameter ratio is also the radius ratio because each radius is half the corresponding diameter.

Step 1: Express the mass and radius ratios Let the planet have mass M, radius R, and escape speed v. The moon has mass $M_m = M/144$. Its diameter is one sixteenth of the planet's diameter, so its radius is also $R_m = R/16$, not $R/32$. The given escape speed therefore satisfies

$$v = \sqrt{\frac{2GM}{R}}.$$

Step 2: Form the escape-speed ratio Write the escape formula for the moon and divide by the planet's expression:

$$\frac{v_m}{v} = \sqrt{\frac{M_m}{M}\frac{R}{R_m}} = \sqrt{\frac{1}{144}\,16}.$$

Both the universal gravitational constant and the factor 2 cancel. The smaller radius partly offsets the effect of the much smaller lunar mass.

Step 3: Simplify and compare the alternatives The squared-speed ratio is $16/144 = 1/9$, so the speed ratio itself is

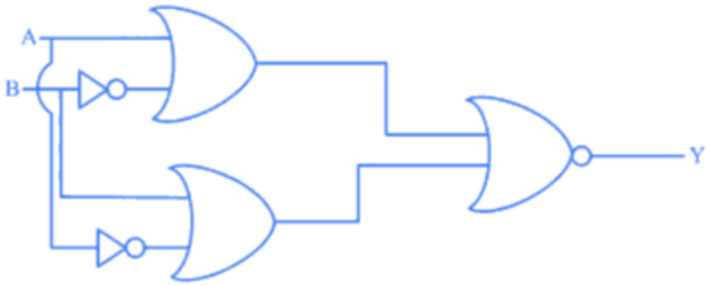
$$\frac{v_m}{v} = \sqrt{\frac{1}{9}} = \frac{1}{3}.$$

Consequently $v_m = v/3$. The value $v/12$ would follow from considering the mass ratio alone and forgetting the radius. The other proposed values, $v/4$ and $v/6$, would give squared-speed ratios $1/16$ and $1/36$, respectively, neither of which matches the required $1/9$. The escaping test object's own mass cancels from the energy equation and does not enter this comparison.

Step 4: Conclusion The escape velocity on the moon is $v/3$. Hence, the correct option is (D).

Quick Tip to Solve: For escape speed, first form $(v_1/v_2)^2 = (M_1/M_2)(R_2/R_1)$, then take the square root only at the end.

50. **The output of the given circuit diagram is -** [4 marks]



(A)

A	B	Y
0	0	0
1	0	0
0	1	1
1	1	0

(B)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

(C)

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

(D)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

Correct Answer: (D)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

Solution:

Concept:

- An OR gate produces the Boolean sum of its inputs. A NOT gate complements its input, so the upper intermediate output is $U = A + \overline{B}$.
- The lower OR output is $V = B + \overline{A}$. The final gate has an output bubble and is therefore NOR: $Y = \overline{U + V}$.

Step 1: Trace the two branches The upper OR receives the direct signal A and the inverted signal $\overline{B}$; the lower OR receives B and $\overline{A}$. The crossing with a bridge is not an electrical junction. Thus each input appears once directly and once complemented when the two intermediate outputs reach the final gate. Neither intermediate gate is AND: its curved input side identifies it as OR.

Step 2: Simplify the final NOR expression Combining the branches gives

$$Y = \overline{(A + \overline{B}) + (B + \overline{A})} = \overline{(A + \overline{A}) + (B + \overline{B})} = \overline{1} = 0.$$

The identities used are $A + \overline{A} = 1$ and $1 + X = 1$. For $A = B = 0$, both inverted inputs are high; for mixed inputs, one direct input and its counterpart ensure a high NOR input; for $A = B = 1$, both direct inputs are high. In every case at least one NOR input is 1.

Step 3: Compare all four tables In the printed row order $(0, 0), (1, 0), (0, 1), (1, 1)$, the output column is $0, 0, 0, 0$. Option (A) incorrectly gives a high output for $(0, 1)$, option (B) does so for $(1, 1)$, and option (C) does so for both mixed inputs. Only option (D) has zeros in every row.

Step 4: Conclusion The output is identically $Y = 0$. Hence, the correct option is (D).

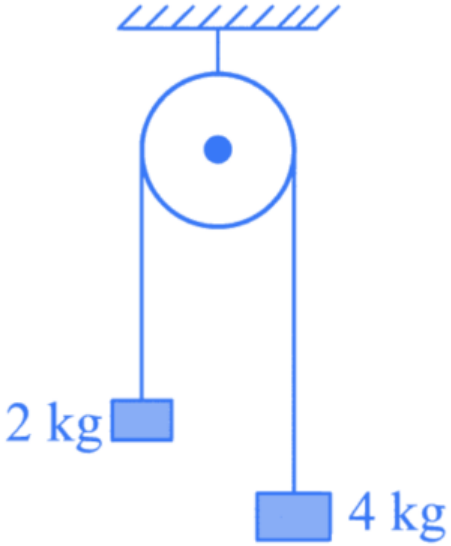
Quick Tip to Solve: Trace bubbles before simplifying: the last bubble complements the sum of both OR outputs, and a signal OR its complement is always 1. This circuit contains both A and its complement among the final NOR inputs.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Two blocks of mass $2\ \text{kg}$ and $4\ \text{kg}$ are connected by a metal wire going over a smooth pulley as shown in figure. The radius of wire is $4.0\times10^{-5}\ \text{m}$ and Young's modulus of the metal is $2.0\times10^{11}\ \text{N/m}^2$. The longitudinal strain developed in the wire is $\frac{1}{\alpha\pi}$. The value of α is _____. [Use $g = 10\ \text{m/s}^2$]

[4 marks]



Solution:

Concept:

- For a light wire over a smooth pulley, the tension T is the same on both sides. Apply Newton's second law separately to the two moving blocks.
- Young's modulus is stress divided by longitudinal strain: $Y = (T/A)/\epsilon$, where $A = \pi r^2$ and $\epsilon = \Delta L/L$.

Step 1: Find the common acceleration and tension The $4\,\text{kg}$ block moves downward and the $2\,\text{kg}$ block upward. Taking these directions as positive for the respective blocks,

$$40 - T = 4a, \qquad T - 20 = 2a.$$

Adding eliminates the internal tension and gives $20 = 6a$, so $a = 10/3\,\text{m s}^{-2}$. Consequently,

$$T = 20 + 2(10/3) = 80/3\,\text{N}.$$

Using either block's equation gives the same tension; it is neither block's weight because the system accelerates.

Step 2: Convert the radius into area and find the strain The supplied radius is already in metres. Therefore,

$$A = \pi(4.0\times10^{-5})^2 = 16\pi\times10^{-10}\,\text{m}^2, \qquad AY = 320\pi\,\text{N}.$$

Since strain is stress divided by Young's modulus,

$$\epsilon = \frac{T}{AY} = \frac{80/3}{320\pi} = \frac{1}{12\pi}.$$

The units cancel between tension and AY, as required for a strain. Both sides of the uniform wire experience this same strain because their tension and cross-sectional area agree.

Step 3: Identify the requested coefficient The question writes the strain as $1/(\alpha\pi)$. Comparing its denominator with $1/(12\pi)$ gives $\alpha = 12$; the coefficient is dimensionless.

Step 4: Conclusion The longitudinal strain is $1/(12\pi)$, and the required dimensionless value is $\boxed{\alpha = 12}$.

Quick Tip to Solve: Find the moving-system tension first; using either hanging weight directly would give the wrong stress and hence the wrong strain. Here the accelerating-system tension is 80/3 N.

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52. A body of mass 'm' is projected with a speed 'u' making an angle of 45° with the ground. The angular momentum of the body about the point of projection, at the highest point is expressed as $\frac{\sqrt{2}mu^3}{Xg}$. The value of 'X' is _____. [4 marks]

Solution:

Concept:

- Angular momentum about the launch point is $\boldsymbol{L} = \boldsymbol{r} \times m\boldsymbol{v}$. Its magnitude is momentum multiplied by the perpendicular distance from the point to the velocity line.
- For projectile motion without air resistance, $v_x = u\cos\theta$, and the maximum height is $H = u^2\sin^2\theta/(2g)$.

Step 1: Determine the velocity and height at the top The launch angle is 45°, so the initial components are both $u/\sqrt{2}$. The vertical component falls to zero at the highest point, while the horizontal component remains constant:

$$v_x = \frac{u}{\sqrt{2}}, \qquad v_y = 0, \qquad H = \frac{(u/\sqrt{2})^2}{2g} = \frac{u^2}{4g}.$$

The body is not at rest at this point: its remaining horizontal momentum is essential to the angular momentum.

Step 2: Take the cross product about the launch point Choose the horizontal launch direction as $+x$ and upward as $+y$. At the top the position is $\boldsymbol{r} = x\hat{\boldsymbol{i}} + H\hat{\boldsymbol{j}}$, whereas the momentum is $m(u/\sqrt{2})\hat{\boldsymbol{i}}$. Hence

$$\boldsymbol{L} = -\frac{muH}{\sqrt{2}}\hat{\boldsymbol{k}}, \qquad |L| = \frac{mu}{\sqrt{2}}\frac{u^2}{4g} = \frac{\sqrt{2}\,mu^3}{8g}.$$

The horizontal displacement does not contribute because it is parallel to the momentum. Equivalently, the perpendicular lever arm is simply the maximum height.

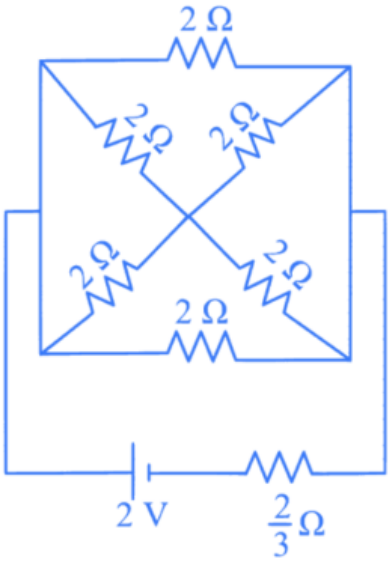
Step 3: Compare the denominator The stated form is $\sqrt{2}\,mu^3/(Xg)$. Matching it to the magnitude just obtained gives $X = 8$. The angular momentum points into the plane for motion toward the right;

the question uses its positive magnitude. The expression has dimensions $\mathrm{kg\,m^2\,s^{-1}}$, consistent with angular momentum.

Step 4: Conclusion The magnitude at the highest point is $\sqrt{2}\,mu^3/(8g)$. Thus $\boxed{X = 8}$, a dimensionless number.

Quick Tip to Solve: At the top of a projectile path, use the height as the lever arm for the horizontal momentum; do not multiply by the full displacement.

53. In the following circuit, the battery has an emf of 2 V and an internal resistance of $\frac{2}{3}\Omega$. The power consumption in the entire circuit is _____ W. [4 marks]



Solution:

Concept:

- Resistors connected between the same pair of nodes are in parallel. Ideal conducting vertical sides make each side of the square one equipotential node.
- The entire circuit includes the internal resistance r. Its total power consumption is $P = I^2(R_{\text{ext}} + r) = \mathcal{E}I$, where $\mathcal{E}$ is the emf.

Step 1: Reduce the external network Each horizontal edge is a $2\,\Omega$ branch between the left and right nodes, so their parallel combination is $1\,\Omega$. Each diagonal contains two $2\,\Omega$ resistors in series, hence each diagonal branch has resistance $4\,\Omega$. The two diagonals in parallel give $2\,\Omega$. Thus

$$\frac{1}{R_{\text{ext}}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = \frac{3}{2}, \qquad R_{\text{ext}} = \frac{2}{3}\,\Omega.$$

If the crossing is treated as a joined node, the two left $2\,\Omega$ resistors are parallel and so are the two right ones; this again gives a $2\,\Omega$ middle branch and the same external resistance.

Step 2: Include the battery resistance and calculate current The internal resistance is in series with

the entire external combination. It must not be placed in parallel with one of its branches. Therefore,

$$R_{\text{total}} = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}\,\Omega, \qquad I = \frac{\mathcal{E}}{R_{\text{total}}} = \frac{2}{4/3} = \frac{3}{2}\,\text{A}.$$

The terminal voltage is $IR_{\text{ext}} = 1\,\text{V}$; the other volt is the drop inside the battery.

Step 3: Calculate power in the entire circuit The total rate of dissipation includes both the outside network and the internal resistor:

$$P = I^2 R_{\text{total}} = \left(\frac{3}{2}\right)^2 \frac{4}{3} = 3\,\text{W}.$$

As an independent check, $\mathcal{E}I = 2(3/2) = 3\,\text{W}$. Each of the equal external and internal equivalent resistances dissipates $1.5\,\text{W}$, giving the same total.

Step 4: Conclusion The power consumption in the entire circuit is $\boxed{3\,\text{W}}$.

Quick Tip to Solve: For total circuit power use emf times current; terminal voltage times current counts only the external load and excludes internal heating. Here internal and external resistances are equal, so they dissipate equal powers.

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54. Two circular coils P and Q of 100 turns each have same radius of π cm. The currents in P and R are $1A$ and $2A$ respectively. P and Q are placed with their planes mutually perpendicular with their centers coincide. The resultant magnetic field induction at the center of the coils is $\sqrt{x}\ mT$, where $x =$ _____.

[Use $\mu_0 = 4\pi \times 10^{-7}\ \text{TmA}^{-1}$] [4 marks]

Solution:

Concept:

- A circular coil of N turns and radius R, carrying current I, produces the central field $B = \mu_0 NI/(2R)$, directed perpendicular to its plane.
- Fields add as vectors. Mutually perpendicular coil planes give mutually perpendicular central field vectors, so their resultant magnitude is $\sqrt{B_P^2 + B_Q^2}$.

Step 1: Convert the radius and evaluate the first field Both coils have $N = 100$ turns and radius $R = \pi\,\text{cm} = \pi \times 10^{-2}\,\text{m}$. For the first coil, the current is $1\,\text{A}$. Using the specified permeability,

$$B_P = \frac{(4\pi \times 10^{-7})(100)(1)}{2\pi \times 10^{-2}} = 2 \times 10^{-3}\,\text{T} = 2\,\text{mT}.$$

The factor π cancels; converting centimetres to metres before substitution prevents a factor-of-100 error.

Step 2: Find the second field and add vectorially The second of the two stated coils carries $2\,\text{A}$. With the same radius and turn count, its field is twice the first:

$$B_Q = 4\,\text{mT}.$$

The field vectors follow the coil axes. Since the planes are perpendicular and their centres coincide, these axes are perpendicular at the common centre. Therefore,

$$B_{\text{resultant}} = \sqrt{(2)^2 + (4)^2}\,\text{mT} = \sqrt{20}\,\text{mT}.$$

Adding the magnitudes to obtain $6\,\text{mT}$ would incorrectly treat the fields as parallel. Their directions along their respective axes do not change the resultant magnitude when the vectors are perpendicular.

Step 3: Read the requested number The paper expresses the resultant as $\sqrt{x}\,\text{mT}$. Comparison gives $x = 20$. Here x is dimensionless because the millitesla unit is written outside the radical.

Step 4: Conclusion The resultant magnetic field is $\sqrt{20}\,\text{mT}$, so the required dimensionless value is $\boxed{x = 20}$.

Quick Tip to Solve: First calculate each coil field in the same units, then use Pythagoras because the coil axes, and hence their central fields, are perpendicular.

55. Two identical spheres each of mass $2\ \text{kg}$ and radius $50\ \text{cm}$ are fixed at the ends of a light rod so that the separation between the centers is $150\ \text{cm}$. Then, moment of inertia of the system about an axis perpendicular to the rod and passing through its middle point is $\frac{x}{20}\ \text{kgm}^2$, where the value of x is _____.

[4 marks]

Solution:

Concept:

- A uniform solid sphere of mass M and radius R has moment of inertia $I_{\text{cm}} = 2MR^2/5$ about any diameter.
- The parallel-axis theorem gives $I = I_{\text{cm}} + Md^2$ about an axis parallel to a diameter and distance d from the sphere centre. Moments of inertia of separate bodies add.

Step 1: Locate the common rotation axis The axis is perpendicular to the rod through its midpoint. The centres are separated by $150\,\text{cm} = 1.50\,\text{m}$, so each centre is $d = 0.75\,\text{m}$ from this axis. The radius of each sphere is $R = 50\,\text{cm} = 0.50\,\text{m}$ and its mass is $M = 2\,\text{kg}$. The light rod contributes negligible inertia, but the spheres cannot be treated simply as point masses because their radii are finite.

Step 2: Apply the parallel-axis theorem to one sphere The diameter axis through each sphere centre, parallel to the required axis, has inertia

$$I_{\text{cm}} = \frac{2}{5}(2)(0.50)^2 = 0.20\,\text{kg}\,\text{m}^2.$$

The displacement correction is

$$Md^2 = 2(0.75)^2 = 1.125\,\text{kg}\,\text{m}^2.$$

Consequently one sphere contributes $0.20 + 1.125 = 1.325\,\text{kg}\,\text{m}^2$. The correction is positive because shifting an axis away from the centre increases the mean squared distance of the mass from it.

Step 3: Add both contributions and identify x The two spheres are identical and equally far from the

midpoint axis. Hence

$$I_{\text{system}} = 2(1.325) = 2.65\,\text{kg m}^2 = \frac{53}{20}\,\text{kg m}^2.$$

Comparing with the given form $x/20\,\text{kg m}^2$ gives $x = 53$. Using $1.50\,\text{m}$ as the distance for each sphere would incorrectly use the full separation instead of half of it.

Step 4: Conclusion The moment of inertia is $53/20\,\text{kg m}^2$, and the required dimensionless number is $\boxed{x = 53}$.

Quick Tip to Solve: For two identical extended bodies, add each central moment and each parallel-axis term; use half the centre separation for a midpoint axis.

---

56. Light from a point source in air falls on a convex curved surface of radius $20\ \text{cm}$ and refractive index 1.5. If the source is located at $100\ \text{cm}$ from the convex surface, the image will be formed at _____ cm from the object. [4 marks]

Solution:

Concept:

- For refraction at a spherical interface, $n_2/v - n_1/u = (n_2 - n_1)/R$. Distances are measured from the pole using the Cartesian sign convention.
- Take the incident light direction as positive. The real source in air has $u < 0$; for this convex refracting surface the centre of curvature lies in the second medium, giving $R > 0$.

Step 1: Assign the signed quantities The incident medium is air, so $n_1 = 1$, and the refracting medium has $n_2 = 1.5$. The source is $100\,\text{cm}$ in front of the surface, giving $u = -100\,\text{cm}$. The convex surface has $R = +20\,\text{cm}$. All lengths may remain in centimetres because the formula uses ratios with consistent length units. The requested distance is from the object, so it will not in general equal v, which is measured from the pole.

Step 2: Solve the spherical refraction equation Substitute the signed data:

$$\frac{1.5}{v} - \frac{1}{-100} = \frac{1.5 - 1}{20}.$$

Therefore,

$$\frac{1.5}{v} = \frac{0.5}{20} - \frac{1}{100} = 0.025 - 0.010 = 0.015\,\text{cm}^{-1}, \qquad v = \frac{1.5}{0.015} = 100\,\text{cm}.$$

The positive result places a real image $100\,\text{cm}$ inside the refracting medium on the opposite side of the pole from the object. A thin-lens formula would be inappropriate because only one curved interface is involved.

Step 3: Calculate the object-image separation The object has coordinate $-100\,\text{cm}$ and the image has coordinate $+100\,\text{cm}$, both measured from the same surface pole. Their separation is thus

$$|v - u| = |100 - (-100)| = 200\,\text{cm}.$$

The two positive magnitudes add because the object and image lie on opposite sides of the refracting surface.

Step 4: Conclusion The image is formed $\boxed{200\,\text{cm}}$ from the object.

Quick Tip to Solve: Distinguish image distance from object-image separation: solve for v from the surface first, then subtract the signed object coordinate.

---

57. A nucleus has mass number A_1 and volume V_1. Another nucleus has mass number A_2 and Volume V_2. If relation between mass number is $A_2 = 4A_1$, then $\frac{V_2}{V_1} =$ _____. [4 marks]

Solution:

Concept:

- The empirical nuclear radius is $R = R_0 A^{1/3}$, where A counts the nucleons and R_0 is approximately the same constant for ordinary nuclei.
- A nucleus is modelled as a sphere of volume $V = 4\pi R^3/3$. Consequently nuclear volume is proportional to the mass number, rather than to its cube root.

Step 1: Write the volume in terms of mass number Insert the nuclear radius law into the spherical volume formula. This gives the same multiplicative constant for both nuclei:

$$V = \frac{4\pi}{3}\left(R_0 A^{1/3}\right)^3 = \frac{4\pi R_0^3}{3} A.$$

The cube on the radius removes the cube-root dependence on mass number. Physically this expresses the approximately constant density of nuclear matter: adding more nucleons increases the occupied volume in the same proportion.

Step 2: Form the ratio and use the stated relation For the two nuclei, divide the second expression by the first so that both the geometric factor and R_0 cancel:

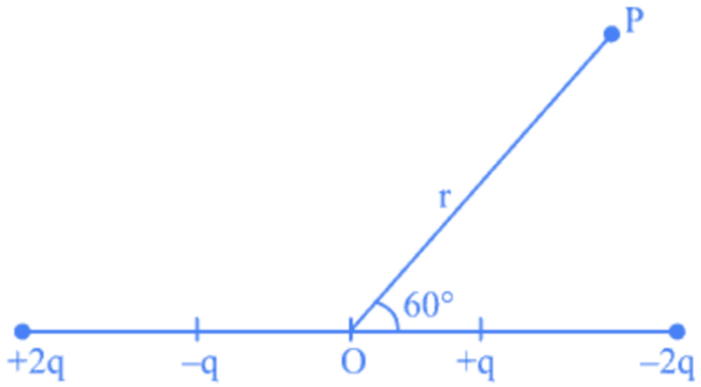
$$\frac{V_2}{V_1} = \frac{(4\pi R_0^3/3)A_2}{(4\pi R_0^3/3)A_1} = \frac{A_2}{A_1}.$$

The question states $A_2 = 4A_1$, hence the ratio is $4A_1/A_1 = 4$. A radius ratio of $4^{1/3}$ would also be consistent with these volumes; it must not be mistaken for the volume ratio itself. No conversion between length units is required because a ratio of like volumes is dimensionless.

Step 3: Conclusion The required volume ratio is $\boxed{V_2/V_1 = 4}$. The numerical entry is $\boxed{4}$, with no unit.

Quick Tip to Solve: Cube the nuclear radius law before comparing volumes: $R \propto A^{1/3}$ but $V \propto A$.

58. The distance between charges $+q$ and $-q$ is $2l$ and between $+2q$ and $-2q$ is $4l$. The electrostatic potential at point P at a distance r from center O is $-\alpha\left[\frac{ql}{r^2}\right]\times 10^9$ V, where the value of α is _____. (Use $\frac{1}{4\pi\varepsilon_0}=9\times 10^9\ \mathrm{Nm^2C^{-2}}$) [4 marks]



Solution:

Concept:

- The electric dipole moment points from negative charge to positive charge and has magnitude equal to charge times their separation.
- The dipole potential is $V = k\,\boldsymbol{p}\cdot\hat{\boldsymbol{r}}/r^2$, where $k = 1/(4\pi\varepsilon_0)$. Potentials from different dipoles add algebraically.

Step 1: Find the signed resultant dipole moment Take the horizontal direction from O towards $+q$ as positive. The inner pair consists of $-q$ on the left and $+q$ on the right, separated by $2l$. It therefore contributes $p_1 = +2ql$. The outer pair has $+2q$ on the left and $-2q$ on the right, separated by $4l$, so its moment points left and is $p_2 = -8ql$. Thus

$$p_x = 2ql - 8ql = -6ql.$$

Both pairs are centred at O; adding their moments before calculating the potential keeps the relative directions explicit.

Step 2: Project the moment towards the observation point The line OP makes 60° with the positive horizontal axis. Using the dipole potential represented by the stated r^{-2} form,

$$V = \frac{kp_x\cos 60^\circ}{r^2} = \frac{(9\times 10^9)(-6ql)(1/2)}{r^2} = -27\frac{ql}{r^2}\times 10^9\ \mathrm{V}.$$

This is the usual dipole, or far-field, expression; an exact finite-distance four-charge potential would contain separate charge-to-point distances rather than solely r^{-2}. The negative sign is essential: the larger outer dipole points opposite to the positive horizontal direction and dominates the smaller inner dipole. Compare the positive numerical coefficient of the minus sign with the expression given in the question.

Step 3: Conclusion Comparing with $-\alpha[ql/r^2]\times 10^9$ V gives $\boxed{\alpha = 27}$. The coefficient is dimensionless.

Quick Tip to Solve: Add dipole moments as vectors first. The outer pair has four times the inner moment and points the opposite way.

59. The magnetic flux ϕ (in weber) linked with a closed circuit of resistance 8Ω varies with time (in seconds) as $\phi = 5t^2 - 36t + 1$. The induced current in the circuit at $t = 2$ s is _____ A. [4 marks]

Solution:

Concept:

- Faraday's law gives the induced emf as $\mathcal{E} = -d\phi/dt$, when ϕ is the magnetic flux linked with the complete circuit.
- Ohm's law gives $I = \mathcal{E}/R$. A time derivative of flux in webers per second has the unit volt, so dividing it by resistance in ohms gives amperes.

Step 1: Differentiate the magnetic flux The flux is a function of time, so use its instantaneous derivative at the specified instant rather than dividing the flux itself by time. With $\phi = 5t^2 - 36t + 1$,

$$\frac{d\phi}{dt} = 10t - 36.$$

Therefore

$$\mathcal{E} = -(10t - 36) = 36 - 10t.$$

The constant term in the flux differentiates to zero: a constant flux contributes no induced emf. The minus sign is Lenz's law and says that the induced response opposes the change in linked flux.

Step 2: Evaluate the emf and divide by the resistance At the instant $t = 2\,\text{s}$,

$$\left.\frac{d\phi}{dt}\right|_2 = 20 - 36 = -16\,\text{Wb}\,\text{s}^{-1}, \qquad \mathcal{E} = 16\,\text{V}.$$

Use the full circuit resistance $R = 8\,\Omega$:

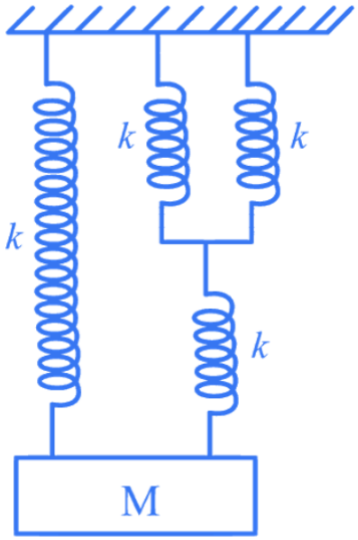
$$I = \frac{16}{8} = 2\,\text{A}.$$

The flux is decreasing in the reference sense at this instant, so the signed emf is positive in the corresponding circuit orientation. Since the question does not specify a drawn current orientation, the requested numerical current is its magnitude. There is no extra turn multiplier: the given quantity is already the flux linked with the closed circuit.

Step 3: Conclusion The induced current at $t = 2\,\text{s}$ is $\boxed{2\,\text{A}}$. The numerical entry is $\boxed{2}$.

Quick Tip to Solve: For a polynomial flux, differentiate first and then substitute time; ϕ/t is not the instantaneous induced emf.

60. The time period of simple harmonic motion of mass M in the given figure is $\pi\sqrt{\frac{\alpha M}{5k}}$, where the value of α is _____. [4 marks]



Solution:

Concept:

- Springs in parallel undergo the same extension and their spring constants add; springs in series carry the same force and their reciprocal spring constants add.
- For small vertical oscillations about equilibrium, $T = 2\pi\sqrt{M/k_{\text{eff}}}$. Gravity shifts the equilibrium position but does not change this period for ideal linear springs.

Step 1: Reduce the springs on the right The two upper right springs, each of constant k, connect the ceiling to the same junction. They are in parallel, so their combined constant is $k_{\text{upper}} = k + k = 2k$. That combination is in series with the lower right spring of constant k. For a force F, the extensions add as $F/(2k) + F/k$. Hence

$$\frac{1}{k_{\text{right}}} = \frac{1}{2k} + \frac{1}{k} = \frac{3}{2k}, \qquad k_{\text{right}} = \frac{2k}{3}.$$

It is the connectivity of the junctions, rather than the number of drawn coils, that determines which springs are in series.

Step 2: Add the left branch and compare the period The single left spring of constant k connects the same ceiling and mass as the complete right branch. They therefore act in parallel:

$$k_{\text{eff}} = k + \frac{2k}{3} = \frac{5k}{3}.$$

For a displacement x from static equilibrium the total restoring force is $-(5k/3)x$, so

$$T = 2\pi\sqrt{\frac{M}{5k/3}} = 2\pi\sqrt{\frac{3M}{5k}} = \pi\sqrt{\frac{12M}{5k}}.$$

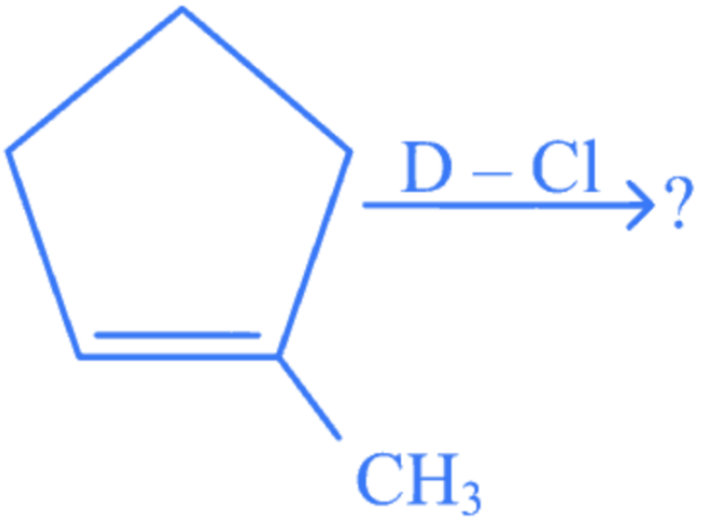
Moving the outside factor 2 inside the square root multiplies its argument by 4. Comparing this expression with the specified $\pi\sqrt{\alpha M/(5k)}$ determines the requested coefficient.

Step 3: Conclusion The time period is $\pi\sqrt{12M/(5k)}$, so $\boxed{\alpha = 12}$. This coefficient has no unit.

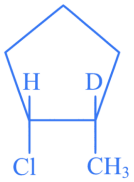
Quick Tip to Solve: Reduce the right-hand network first: $k \parallel k = 2k$, then series with k, and finally parallel with the left spring.

Chemistry – Section A: Multiple Choice Questions (80 marks)

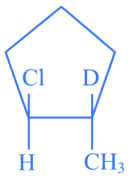
61. Major product of the following reaction is - [4 marks]



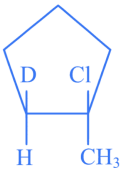
(A) Cl on the carbon without the methyl substituent; D on the methyl-bearing carbon (first drawing).



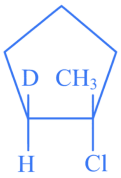
(B) Cl on the carbon without the methyl substituent; D on the methyl-bearing carbon (second drawing).



(C) D on the carbon without the methyl substituent; Cl on the methyl-bearing carbon (third drawing).



(D) D on the carbon without the methyl substituent; Cl on the methyl-bearing carbon (fourth drawing).



Correct Answer: Both C and D have the Markovnikov product connectivity; no wedge/dash stereochemistry distinguishes them.

Solution:

Concept:

- Deuterium chloride adds electrophilically across an alkene. Deuterium is the hydrogen isotope ^{2}H, so D^{+} plays the electrophilic role and chloride acts as the nucleophile.
- Markovnikov orientation is selected by the stability of the intermediate carbocation: deuterium attaches to the less substituted alkene carbon, leaving the more substituted carbon positively charged.

Step 1: Locate the more stable carbocation In 1-methylcyclopentene, one carbon of the double bond bears a methyl group as well as its two ring neighbours. Addition of D^{+} to the other alkene

carbon leaves a tertiary carbocation at this methyl-bearing carbon. The reverse addition would leave a secondary carbocation, so it is disfavoured. The carbon skeleton does not need a rearrangement because the directly formed carbocation is already tertiary. Chloride then bonds to the positively charged methyl-bearing carbon.

Step 2: Compare the structures without inventing stereochemical bonds The major constitutional product is therefore 1-chloro-2-deuterio-1-methylcyclopentane: D is attached to the formerly less substituted alkene carbon and Cl to the carbon bearing methyl. Options (A) and (B) instead place Cl on the less substituted carbon and D on the methyl-bearing carbon; these have the opposite regioselectivity. Both (C) and (D), as drawn, have the required Markovnikov connectivity. Their substituent lines are ordinary bonds, with no solid wedges or dashed bonds defining a stereochemical distinction. Merely exchanging the up/down positions of ordinary bond strokes in this schematic does not establish a unique stereoisomer. Moreover, capture of a planar carbocation is not constrained to a single stereospecific face.

Step 3: Conclusion The major product is 1-chloro-2-deuterio-1-methylcyclopentane. Both options (C) and (D) depict its connectivity without explicit stereochemical bonds; thus the printed drawings do not support a unique choice between them.

Quick Tip to Solve: Treat D like H for Markovnikov orientation. Check actual wedge/dash bonds before interpreting a planar drawing as a stereochemical restriction.

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62. $\mathrm{A}_{(g)} \rightleftharpoons \mathrm{B}_{(g)} + \frac{\mathrm{C}}{2}(g)$ **The correct relationship between** $\mathrm{K_P}, \alpha$ **and equilibrium pressure** P **is** [4 marks]

(A) $K_P = \frac{\alpha^{1/2}P^{3/2}}{(2+\alpha)^{3/2}}$

(B) $K_P = \frac{\alpha^{3/2}P^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$

(C) $K_P = \frac{\alpha^{1/2}P^{1/2}}{(2+\alpha)^{3/2}}$

(D) $K_P = \frac{\alpha^{1/2}P^{1/2}}{(2+\alpha)^{1/2}}$

Correct Answer: (B) $K_P = \frac{\alpha^{3/2}P^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$

Solution:

Concept:

- For $\mathrm{A}\,(g) \rightleftharpoons \mathrm{B}\,(g) + \frac{1}{2}\,\mathrm{C}\,(g)$, the pressure equilibrium constant is $K_P = p_B p_C^{1/2}/p_A$. Each partial pressure equals mole fraction times total equilibrium pressure.
- Let α be the fraction of initially pure A dissociated. Starting from one mole, the equilibrium amounts are $1-\alpha$, α and $\alpha/2$ for A, B and C respectively.

Step 1: Express the equilibrium partial pressures The total number of moles at equilibrium is

$$n_{\text{total}} = 1 - \alpha + \alpha + \frac{\alpha}{2} = 1 + \frac{\alpha}{2} = \frac{2+\alpha}{2}.$$

Dividing each amount by this total and multiplying by the equilibrium pressure P gives

$$p_A = \frac{2(1-\alpha)P}{2+\alpha}, \quad p_B = \frac{2\alpha P}{2+\alpha}, \quad p_C = \frac{\alpha P}{2+\alpha}.$$

The factor one-half in the reaction is a stoichiometric coefficient; it means C is formed in half the amount of B. The change in gas moles is $1/2$, so the final pressure power must be $P^{1/2}$.

Step 2: Substitute and simplify the equilibrium quotient Using these partial pressures,

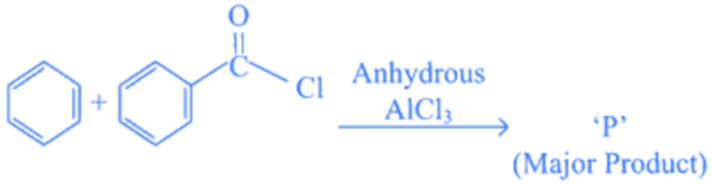
$$\begin{aligned} K_P &= \frac{p_B}{p_A}\sqrt{p_C} \\ &= \frac{\alpha}{1-\alpha}\sqrt{\frac{\alpha P}{2+\alpha}} \\ &= \frac{\alpha^{3/2}P^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}. \end{aligned}$$

Option (A) has the wrong pressure power. Options (C) and (D) omit the factor $\alpha/(1-\alpha)$ from the B-to-A partial-pressure ratio and thus cannot represent the quotient. In particular, depletion of A must leave a denominator $1-\alpha$; dropping it would incorrectly keep the quotient finite as dissociation approaches completion.

Step 3: Conclusion The relation is $\boxed{K_P = \alpha^{3/2}P^{1/2}/[(2+\alpha)^{1/2}(1-\alpha)]}$. Hence, the correct option is (B).

Quick Tip to Solve: Calculate p_B/p_A first so the total-mole factors cancel, then multiply by $\sqrt{p_C}$.

63. Identify major product 'P' formed in the following reaction. [4 marks]



(A) 3-Phenoxybenzoyl chloride, $C_6H_5OC_6H_4COCl$ (meta substitution).

(B) Benzaldehyde, C_6H_5CHO.

(C) Benzophenone, $C_6H_5COC_6H_5$.

(D) Acetophenone, $C_6H_5COCH_3$.

Correct Answer: (C) Benzophenone, $C_6H_5COC_6H_5$.

Solution:

Concept:

- Benzene reacts with an acyl chloride in the presence of anhydrous $AlCl_3$ by Friedel–Crafts acylation. The electrophile is an acylium ion, and aromatic hydrogen is replaced by an acyl group.
- The printed acyl chloride is benzoyl chloride, C_6H_5COCl. Its acyl fragment is $C_6H_5CO^-$, so bonding that fragment to benzene gives an aromatic ketone with two phenyl groups.

Step 1: Generate the benzoyl electrophile Anhydrous $AlCl_3$ is a Lewis acid and accepts an electron pair from benzoyl chloride, facilitating cleavage of the carbon–chlorine bond. The resulting electrophile is the resonance-stabilised benzoyl acylium ion $C_6H_5CO^+$, accompanied by $AlCl_4{}^-$. The carbonyl

carbon is the electrophilic centre. The phenyl group already attached to that carbon remains attached throughout acylation; it is not replaced by hydrogen or a methyl group.

Step 2: Carry out aromatic substitution and identify the ketone The benzene ring attacks the acylium carbon to form a sigma complex. Loss of a proton restores aromaticity, and work-up releases the ketone from its aluminium chloride complex. The overall transformation is

$$C_6H_6 + C_6H_5COCl \xrightarrow{AlCl_3} C_6H_5COC_6H_5 + HCl.$$

The product is benzophenone, also called diphenylmethanone, shown in (C). Option (A) introduces an ether linkage absent from this mechanism and retains an acyl chloride. Option (B), benzaldehyde, would require a formyl group rather than benzoyl. Option (D), acetophenone, would result from an acetylating reagent such as acetyl chloride; the supplied reagent contains a phenyl group instead of methyl.

Step 3: Conclusion The major product P is benzophenone, $\boxed{C_6H_5COC_6H_5}$. Hence, the correct option is (C).

Quick Tip to Solve: In Friedel–Crafts acylation retain the entire RCO^- group of $RCOCl$: benzoyl chloride transfers benzoyl, not acetyl.

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64. Given below are two statements:

Statement I : Group 13 trivalent halides get easily hydrolyzed by water due to their covalent nature.

Statement II : $AlCl_3$ **upon hydrolysis in acidified aqueous solution forms octahedral** $[Al(H_2O)_6]^{3+}$ **ion.**

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is false but statement II is true

(B) Both statement I and statement II are true

(C) Both statement I and statement II are false

(D) Statement I is true but statement II is false

Correct Answer: (B) Both statement I and statement II are true

Solution:

Concept:

- Group 13 trihalides have appreciable covalent character and behave as Lewis acids. Water donates an oxygen lone pair to the electron-deficient central atom, initiating hydrolysis.
- In acidified water, aluminium(III) is represented by the hexaaqua ion, $[Al(H_2O)_6]^{3+}$. Six oxygen donors give coordination number six and an octahedral arrangement.

Step 1: Evaluate Statement I The small, highly charged central atom polarises the halide ions, giving the trihalides covalent character. These compounds interact readily with water instead of remaining unchanged as molecular halides. For example, boron trichloride gives boric acid and hydrogen chloride:

$$BCl_3 + 3\,H_2O \longrightarrow H_3BO_3 + 3\,HCl.$$

This illustrates the hydrolysis characteristic described in the statement. Thus Statement I is true.

Step 2: Evaluate Statement II On adding aluminium chloride to acidified aqueous solution, water molecules coordinate through oxygen to the aluminium(III) centre. The hydrated cation is $[Al(H_2O)_6]^{3+}$, together with chloride counterions. Its six ligands occupy the vertices of an octahedron. The acidic medium suppresses further deprotonation of coordinated water and precipitation as aluminium hydroxide. Therefore the stated hexaaqua species and its geometry are correct.

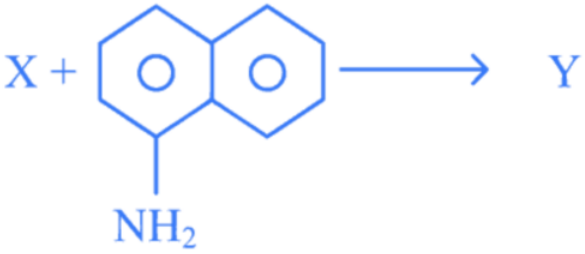
Step 3: Match the two truth values Both claims are true. Option (A) incorrectly rejects the hydrolysis statement; option (C) rejects both correct claims, and option (D) incorrectly rejects the hexaaqua ion. The statement pair is not asking whether one statement explains the other, so no additional assertion-reason condition is required.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

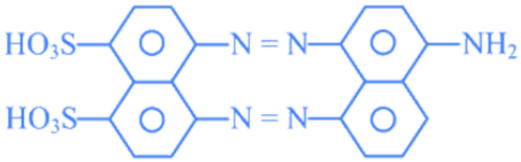
Quick Tip to Solve: Distinguish the covalent anhydrous halide from its hydrated cation: six coordinated water molecules around aluminium imply an octahedral ion.

65. The azo-dye (Y) formed in the following reactions is

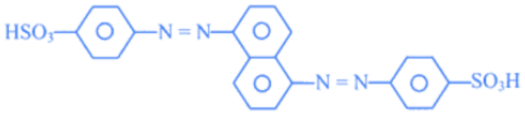
Sulphanilic acid $+NaNO_2 + CH_3COOH \longrightarrow X$**.** [4 marks]



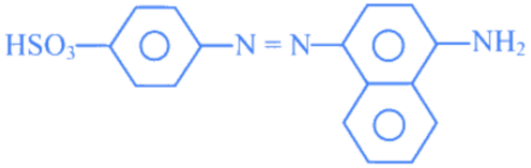
(A) Bis-azo product with two naphthalene systems, as shown.



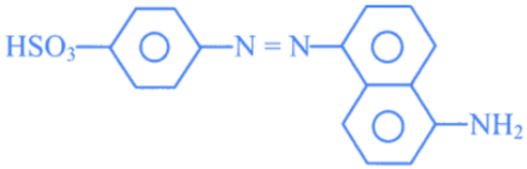
(B) Bis-azo product with two sulphophenyl groups on a naphthalene system, as shown.



(C) Mono-azo product with amino and azo groups on the same constituent ring of naphthalene, as shown.

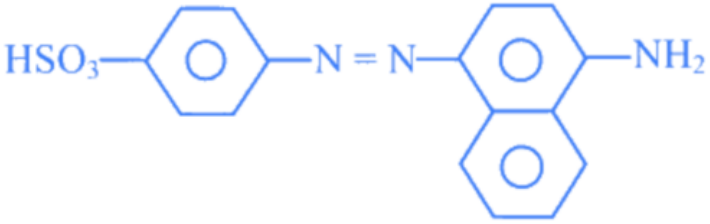


(D) Mono-azo product with amino and azo groups on different constituent rings of naphthalene, as shown.



Correct Answer: (C)

Mono-azo product with amino and azo groups on the same constituent ring of naphthalene, as shown.



Solution:

Concept:

- Diazotisation converts a primary aromatic amino group into a diazonium group. Sulphanilic acid is para-aminobenzenesulphonic acid, so its diazonium product retains the para sulphonic acid substituent.
- Azo coupling is electrophilic aromatic substitution of an activated aromatic ring by an aryl diazonium ion. The new linkage is $-N=N-$, and the amino group on the coupling component remains attached to that component.

Step 1: Identify the diazonium component Sulphanilic acid has the arrangement $H_2N-C_6H_4-SO_3H$, with the amino and sulphonic acid groups para to one another. Nitrous acid generated from sodium nitrite in acidic medium diazotises its amino group. The resulting electrophile is para-sulphobenzenediazonium, represented as $HO_3S-C_6H_4-N_2^+$. Its benzene ring is not converted into a fused bicyclic ring during this reaction.

Step 2: Locate coupling on the naphthylamine ring The second reactant in the scheme is 1-naphthylamine. Its amino group donates electron density by resonance and activates the same ring for substitution. The favourable para position, position 4 relative to the amino-bearing position 1, couples to the diazonium electrophile. Loss of the ring proton restores aromaticity and produces an azo linkage between the sulphophenyl ring and position 4 of the naphthalene system.

Step 3: Compare the four molecular structures The required product has one sulphophenyl group, one azo bridge and one naphthalene system with its amino group retained in the para position on the same constituent ring as the azo substituent. This is the structure in (C). Option (A) introduces an extra fused-ring unit and extra azo linkage; (B) has two azo linkages and does not retain the required amino substituent. In (D), the amino group lies on the other constituent ring of naphthalene, giving the wrong coupling connectivity.

Step 4: Conclusion The product is 4-[(4-sulphophenyl)azo]naphthalen-1-amine, represented by structure (C). Hence, the correct option is (C).

Quick Tip to Solve: Track both original rings: the diazonium benzene keeps its sulphonic acid group, while 1-naphthylamine keeps its amino group and couples at position 4.

66. Given below are two statements:

Statement I : S_8 **solid undergoes disproportionation reaction under alkaline conditions to form** S^{2-} **and** ${S_2O_3}^{2-}$**.**

Statement II : ClO_4^- **can undergo disproportionation reaction under acidic condition.**

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is correct but statement II is incorrect

(B) Both statement I and statement II are incorrect

(C) Statement I is incorrect but statement II is correct

(D) Both statement I and statement II are correct

Correct Answer: (A) Statement I is correct but statement II is incorrect

Solution:

Concept:

- Disproportionation requires one element in an intermediate oxidation state to undergo simultaneous oxidation and reduction. Oxidation numbers must be followed on that same element.
- Elemental sulphur has oxidation number zero. In sulphide it is -2, whereas in thiosulphate its average oxidation number is $+2$. Chlorine in perchlorate already has its maximum common oxidation number, $+7$.

Step 1: Test the sulphur reaction Hot alkali reacts with elemental sulphur to form sulphide and thiosulphate. A balanced equation using sulphur atoms is

$$4\,S + 6\,OH^- \longrightarrow 2\,S^{2-} + {S_2O_3}^{2-} + 3\,H_2O.$$

Equivalently, the equation can be doubled and written with S_8 on the left. Some sulphur is reduced from zero to -2, while some is oxidised into thiosulphate. The thiosulphate sulphur atoms are inequivalent, but their average oxidation number is positive, so the overall oxidation-reduction conclusion is unchanged. Statement I is correct.

Step 2: Test the perchlorate claim For perchlorate, let the chlorine oxidation number be x. Then

$$x + 4(-2) = -1 \quad \Rightarrow \quad x = +7.$$

A disproportionation would require some of this chlorine to move above $+7$ while another portion is reduced. There is no higher usual chlorine oxidation state available. Acidifying the medium does not create such an oxidation state, so perchlorate cannot undergo the stated disproportionation. Statement II is incorrect.

Step 3: Select the truth-value combination The required combination is true for the first statement and false for the second. Options (B) and (C) incorrectly deny the sulphur reaction, while option (D) incorrectly accepts disproportionation of chlorine at its maximum oxidation state.

Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: Before balancing a proposed disproportionation, check the starting oxidation state: an element already at its maximum cannot be oxidised further in that process.

67. Identify structure of 2,3-dibromo-1-phenylpentane. [4 marks]

(A) $C_6H_5-CH(Br)-CH_2-CH(Br)-CH_2-CH_3$

(B) $C_6H_5-CH(Br)-CH(Br)-CH_2-CH_2-CH_3$

(C) $C_6H_5-CH_2-CH(Br)-CH(Br)-CH_2-CH_3$

(D) $C_6H_5-CH_2-CH_2-CH(Br)-CH(Br)-CH_3$

Correct Answer: (C) $C_6H_5-CH_2-CH(Br)-CH(Br)-CH_2-CH_3$

Solution:

Concept:

- The parent name pentane specifies a continuous chain of five carbon atoms. The prefix phenyl denotes $C_6H_5{}^-$, attached through one ring carbon to the parent chain.
- The locants in 2,3-dibromo-1-phenylpentane place the phenyl group at carbon 1 and the two bromine atoms at carbons 2 and 3. Each carbon must retain total valency four.

Step 1: Construct the named carbon chain Number the pentane backbone from the phenyl end as C1 through C5. The phenyl substituent replaces one hydrogen at C1, giving $C_6H_5-CH_2{}^-$. Add bromine at each of C2 and C3; those two carbons are therefore $CH(Br)$ units. The remaining end of the chain consists of C4 as CH_2 and C5 as CH_3.

Step 2: Write and check the connectivity The complete condensed structure is

$$C_6H_5-CH_2-CH(Br)-CH(Br)-CH_2-CH_3.$$

Counting only the non-ring parent chain gives exactly five carbons. Starting at the end next to phenyl gives the substituent locants $1, 2, 3$; reversing the chain would give a higher locant sequence. The phenyl-bearing carbon is a methylene group, not a brominated methine carbon.

Step 3: Reject the displaced bromine patterns In (A), bromine is attached at C1 and C3, so it is not the 2,3-dibromo compound. In (B), bromine is at C1 and C2. In (D), numbering from the phenyl end places bromine at C3 and C4. Only (C) leaves C1 unbrominated and places bromine on both adjacent internal carbons C2 and C3. No stereochemical specification is given, so wedges or absolute configurations are not required to identify the connectivity.

Step 4: Conclusion The required structure is $C_6H_5-CH_2-CH(Br)-CH(Br)-CH_2-CH_3$, shown in option (C). Hence, the correct option is (C).

Quick Tip to Solve: Write C1–C5 beside the zigzag before checking bromine positions; the benzene ring is a phenyl substituent and is not part of the pentane count.

68. Choose the correct statements from the following

A. All group 16 elements form oxides of general formula EO_2 and EO_3, where $E = S, Se, Te$ and Po. Both the types of oxides are acidic in nature.

B. TeO_2 is an oxidising agent while SO_2 is reducing in nature.

C. The reducing property decreases from H_2S to H_2Te down the group.

D. The ozone molecule contains five lone pairs of electrons.

Choose the correct answer from the options given below: [4 marks]

(A) A and B only

(B) C and D only

(C) A and D only

(D) B and C only

Correct Answer: (A) A and B only

Solution:

Concept:

- The standard group-16 oxide trend treats the dioxides and trioxides as acidic oxides, with acidic character decreasing down the group as metallic character increases.
- Reducing ability of group-16 hydrides increases down the group because the hydrogen-element bond becomes weaker. Ozone has 18 valence electrons, which can be counted using either resonance structure.

Step 1: Evaluate the oxide statements A and B Statement A gives the standard group-16 generalisation for the oxides EO_2 and EO_3: they show acidic character, strongest for sulphur and decreasing down the group. Statement B is also correct. Sulphur dioxide is readily oxidised, for example to sulphate, so it acts as a reducing agent. Tellurium dioxide instead shows oxidising behaviour: reduction of tellurium(IV) becomes more favourable down the group. Thus the two dioxide examples have the contrasting redox behaviour stated.

Step 2: Check the hydride trend in C Thermal stability decreases from H_2S towards H_2Te. The larger central atoms form weaker bonds to hydrogen, making oxidation of the hydride easier. Consequently reducing power increases, rather than decreases, down this series. Statement C reverses the trend and is false.

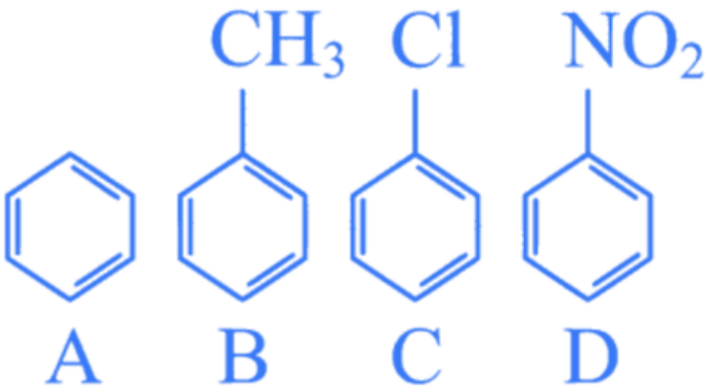
Step 3: Count the lone pairs for D An ozone resonance structure contains one single O–O bond and one double O=O bond. These bonds use six of the 18 valence electrons, leaving 12 nonbonding electrons, or six lone pairs. More explicitly, the singly bonded terminal oxygen has three lone pairs, the doubly bonded terminal oxygen has two, and the central oxygen has one. The total is $3+2+1=6$, not five. Statement D is false.

Step 4: Match the surviving statements The accepted statement combination is A and B only. Options (B), (C) and (D) each include at least one of the false claims C or D, so they cannot represent the required set.

Step 5: Conclusion A and B are the correct statements in the stated group-trend description. Hence, the correct option is (A).

Quick Tip to Solve: For ozone count nonbonding electrons after drawing one resonance form; for hydride reducing power, follow weakening of the E–H bond down group 16.

69. The correct order of reactivity in electrophilic substitution reaction of the following compounds is : [4 marks]



(A) B > C > A > D

(B) B > A > C > D

(C) D > C > B > A

(D) A > B > C > D

Correct Answer: (B) B > A > C > D

Solution:

Concept:

- Electrophilic aromatic substitution is faster when a substituent raises the electron density of the aromatic ring and stabilises the positively charged sigma complex.
- A methyl group activates benzene by electron donation and hyperconjugation. Chlorine is overall deactivating because its inductive withdrawal dominates its resonance donation; a nitro group strongly withdraws by both induction and resonance.

Step 1: Identify the labelled compounds The structures are A: benzene, B: toluene, C: chlorobenzene and D: nitrobenzene. Benzene provides the reference reactivity. The comparison concerns the overall rate of attack by an electrophile, rather than merely the ortho/para or meta position preferred within each ring.

Step 2: Compare methyl and chloro substitution The methyl group in toluene donates electron density through its positive inductive effect and hyperconjugation. It stabilises the sigma complex and makes B more reactive than A. Chlorine has lone pairs that donate by resonance, explaining its ortho/para direction, but its strong negative inductive effect decreases the overall ring reactivity. Therefore chlorobenzene C reacts more slowly than benzene A. Combining these facts gives $B > A > C$.

Step 3: Place nitrobenzene and eliminate alternatives The nitro group removes ring electron density by both its negative inductive and negative resonance effects. It strongly destabilises electrophilic attack and makes nitrobenzene the least reactive member. The full order is

$$B > A > C > D.$$

Option (A) wrongly puts chlorobenzene above benzene; option (C) places the strongly deactivated nitrobenzene first; and option (D) places benzene above activated toluene. Only option (B) respects

all three substituent effects.

Step 4: Conclusion The decreasing reactivity order is toluene $>$ benzene $>$ chlorobenzene $>$ nitrobenzene, that is $B > A > C > D$. Hence, the correct option is (B).

Quick Tip to Solve: Do not confuse direction with activation: chlorine directs to ortho/para positions but still makes its ring less reactive than benzene.

70. Choose the correct statements from the following

A. Mn_2O_7 **is an oil at room temperature**

B. V_2O_4 **reacts with acid to give** VO_2^{2+}

C. CrO **is a basic oxide**

D. V_2O_5 **does not react with acid**

Choose the correct answer from the options given below : [4 marks]

(A) A, B and D only

(B) A, B and C only

(C) A and C only

(D) B and C only

Correct Answer: (C) A and C only

Solution:

Concept:

- Transition-metal oxides in low oxidation states tend to be basic, whereas high oxidation states favour acidic or amphoteric behaviour. The actual aqueous product must also have the correct metal oxidation state.
- For oxide formulas and oxocations, assign oxygen oxidation number -2 and equate the sum to the total charge. Distinguish vanadyl VO^{2+} from dioxovanadium VO_2^{+} and the printed VO_2^{2+}.

Step 1: Evaluate A and C Mn_2O_7, manganese(VII) oxide, is a dark oily liquid at room temperature, so A is correct. Chromium in CrO has oxidation number $+2$, a low oxidation state with predominantly basic oxide behaviour. For example,

$$CrO + 2\,H^+ \longrightarrow Cr^{2+} + H_2O.$$

Thus C is also correct.

Step 2: Check the exact oxocation in B Vanadium in V_2O_4 is $+4$. Dissolution in acid gives the vanadyl ion VO^{2+}:

$$V_2O_4 + 4\,H^+ \longrightarrow 2\,VO^{2+} + 2\,H_2O.$$

The printed product is instead VO_2^{2+}, with two oxygen atoms and charge $+2$. It would require vanadium to be $+6$, since $x - 4 = +2$. This is not the vanadium(IV) oxocation formed by the acid reaction. B is therefore incorrect as printed.

Step 3: Test the nonreaction claim in D Vanadium(V) oxide is amphoteric and can react with acid,

forming the dioxovanadium(V) ion VO_2^+. Its acid dissolution may be represented by

$$V_2O_5 + 2\,H^+ \longrightarrow 2\,VO_2^+ + H_2O.$$

Hence the categorical claim that it does not react with acid is false. The surviving statements are A and C. Options (A), (B) and (D) each include the incorrect statement B, and (A) also includes D.

Step 4: Conclusion Only A and C are correct. Hence, the correct option is (C).

Quick Tip to Solve: Count oxygen atoms before assigning an oxocation: VO^{2+} contains V(IV), while VO_2^+ contains V(V). A subscript changes the chemistry.

71. Select the option with correct property - [4 marks]

(A) $[Ni(CO)_4]$ and $[NiCl_4]^{2-}$ both Paramagnetic

(B) $[Ni(CO)_4]$ and $[NiCl_4]^{2-}$ both Diamagnetic

(C) $[NiCl_4]^{2-}$ Diamagnetic, $[Ni(CO)_4]$ Paramagnetic

(D) $[Ni(CO)_4]$ Diamagnetic, $[NiCl_4]^{2-}$ Paramagnetic

Correct Answer: (D) $[Ni(CO)_4]$ Diamagnetic, $[NiCl_4]^{2-}$ Paramagnetic

Solution:

Concept:

- A species with unpaired electrons is paramagnetic; a species in which all electrons are paired is diamagnetic. Count the metal d electrons after determining its oxidation state.
- Carbon monoxide is a neutral strong-field ligand, while chloride is a weak-field ligand. Four-coordinate nickel complexes can therefore have different electronic configurations and magnetic behaviour.

Step 1: Count the electrons in nickel carbonyl In $Ni(CO)_4$, the four neutral carbonyl ligands and the neutral complex give nickel oxidation state zero. Free nickel has configuration $[Ar]3d^8 4s^2$. In forming this complex its valence electrons pair to give a filled $3d^{10}$ set. The four ligands form a tetrahedral complex using the available outer orbitals. There are no unpaired electrons, so nickel tetracarbonyl is diamagnetic.

Step 2: Count the unpaired electrons in the chloride complex For $[NiCl_4]^{2-}$, let the oxidation state of nickel be x. Then $x - 4 = -2$, so $x = +2$, giving a $3d^8$ ion. Chloride does not cause the pairing needed for a square-planar arrangement; the complex is tetrahedral. Its tetrahedral configuration is $e^4 t_2^4$, with two unpaired electrons. It is therefore paramagnetic. Options (A) and (B) wrongly give both complexes the same behaviour, while (C) reverses the two properties.

Step 3: Conclusion $Ni(CO)_4$ is diamagnetic and $[NiCl_4]^{2-}$ is paramagnetic. Hence, the correct option is (D).

Quick Tip to Solve: Find the oxidation state before counting d electrons: neutral CO gives Ni(0), whereas four chloride ligands give Ni(II) here.

72. Which of the following is least ionic? [4 marks]

(A) $CoCl_2$

(B) KCl

(C) $BaCl_2$

(D) AgCl

Correct Answer: (D) AgCl

Solution:

Concept:

- Ionic and covalent bonding are limiting descriptions. Greater distortion of an anion electron cloud by a cation increases covalent character and decreases ionic character.
- Fajans' rules compare cation size, charge and electronic configuration for a common anion. A cation with an 18-electron outer configuration generally polarises an anion more than a comparable noble-gas configuration does.

Step 1: Compare the common-anion salts All four compounds contain chloride, so the anion being polarised is the same. K^+ is a singly charged, relatively large ion with a noble-gas configuration, and KCl is strongly ionic. Ba^{2+} has a high charge but is large; $BaCl_2$ also has predominantly ionic bonding. $CoCl_2$ has some covalent character because the smaller transition-metal cation polarises chloride. The decision must nevertheless include electronic configuration, rather than cation charge alone.

Step 2: Identify the strongly polarising silver ion Silver(I) has the configuration $Ag^+ : [Kr]4d^{10}$, with an 18-electron outer-shell arrangement. The filled d electrons shield less effectively than the closed noble-gas arrangement in an alkali-metal cation, enhancing its polarising effect on chloride. As a result, AgCl has appreciable covalent character and is the least ionic among the listed salts. Thus neither the alkali-metal chloride, the large alkaline-earth chloride nor cobalt(II) chloride gives the required minimum ionic character.

Step 3: Conclusion Among $CoCl_2$, KCl, $BaCl_2$ and AgCl, silver chloride is the least ionic. Hence, the correct option is (D).

Quick Tip to Solve: For salts with the same anion, check cation polarisation and the filled d-shell effect; do not rank ionic character by charge alone.

73. The four quantum numbers for the electron in the outer most orbital of potassium (atomic no. 19) are [4 marks]

(A) $\mathrm{n}=3, l=0,\ \mathrm{m}=1,\ \mathrm{s}=+\frac{1}{2}$

(B) $\mathrm{n}=4, l=0,\ \mathrm{m}=0, s=+\frac{1}{2}$

(C) $\mathrm{n}=2, l=0,\ \mathrm{m}=0, s=+\frac{1}{2}$

(D) $\mathrm{n}=4, l=2,\ \mathrm{m}=-1, s=+\frac{1}{2}$

Correct Answer: (B) $\mathrm{n}=4, l=0,\ \mathrm{m}=0, s=+\frac{1}{2}$

Solution:

Concept:

- The principal quantum number n identifies the shell. The azimuthal quantum number l identifies the subshell: $l=0$ for s, 1 for p and 2 for d.
- For a given l, the magnetic quantum number runs from $-l$ to $+l$, while the spin projection is $m_s=\pm\frac{1}{2}$. The spin entry called s in the options denotes this projection.

Step 1: Write the ground-state configuration Potassium has atomic number 19, so a neutral atom contains 19 electrons. Filling the orbitals in ground-state order gives

$$\mathrm{K}: 1s^2 2s^2 2p^6 3s^2 3p^6 4s^1 = [\mathrm{Ar}]4s^1.$$

The electron in the outermost orbital is therefore the single $4s$ electron, not an electron in the filled second or third shell. The 4s orbital is occupied before the 3d orbitals in this ground-state atom.

Step 2: Assign the four allowed quantum numbers For the $4s$ electron, $n=4$ and $l=0$. Since m_l must lie between $-l$ and $+l$, only $m_l=0$ is possible. The listed choice with $m_s=+\frac{1}{2}$ is allowed. Option (A) has the wrong shell and also violates $m_l=0$ for an s orbital. Option (C) describes a 2s electron, which is not outermost in potassium. Option (D) gives $l=2$, a d subshell rather than the occupied 4s subshell. Thus only the second set describes the required electron.

Step 3: Conclusion The required set is $n=4,\ l=0,\ m_l=0,\ m_s=+\frac{1}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the last occupied subshell first: its leading number gives n, and the letter s immediately fixes both l and the magnetic quantum number to zero.

74. Identify the name reaction. [4 marks]



(A) Stephen Reaction

(B) Etard Reaction

(C) Gatterman - Koch Reaction

(D) Rosenmund Reduction

Correct Answer: (C) Gatterman - Koch Reaction

Solution:

Concept:

- Gattermann-Koch formylation introduces a formyl group, $-\mathrm{CHO}$, directly into an aromatic ring using CO and HCl with anhydrous $\mathrm{AlCl_3}$ and CuCl.
- To identify a named reaction, compare the starting functional group, reagent combination and product; several unrelated reactions can all produce an aldehyde.

Step 1: Recognise the transformation in the scheme The starting aromatic compound is benzene and the product is benzaldehyde. Thus a hydrogen on the benzene ring has been replaced by a formyl group. The arrow shows carbon monoxide and hydrogen chloride, together with anhydrous aluminium chloride and cuprous chloride. This combination generates the effective formylating electrophile for an electrophilic aromatic substitution. The net transformation can be represented as

$$\mathrm{C_6H_6 + CO \xrightarrow{HCl,\ AlCl_3/CuCl} C_6H_5CHO.}$$

Step 2: Distinguish the other aldehyde preparations This direct aromatic formylation is the Gattermann-Koch reaction. Stephen reduction instead converts a nitrile to an iminium intermediate with $\mathrm{SnCl_2/HCl}$, followed by hydrolysis to the aldehyde. Etard oxidation converts a methyl group on an aromatic ring into an aldehyde using chromyl chloride, $\mathrm{CrO_2Cl_2}$; benzene has no such methyl group here. Rosenmund reduction hydrogenates an acyl chloride over poisoned palladium, usually $\mathrm{Pd/BaSO_4}$. The scheme contains neither a nitrile, a benzylic methyl group nor an acyl chloride, so those three names do not fit.

Step 3: Conclusion The conversion of benzene into benzaldehyde with $\mathrm{CO/HCl}$ and anhydrous $\mathrm{AlCl_3/CuCl}$ is the Gattermann-Koch reaction. Hence, the correct option is (C).

Quick Tip to Solve: The reagent pair CO and HCl with aluminium chloride/cuprous chloride is the diagnostic clue for direct aromatic formylation.

75. Consider the following elements.

Which of the following is/are true about A', B', C' and D' ?

A. Order of atomic radii: $B' < A' < D' < C'$

B. Order of metallic character: $B' < A' < D' < C'$

C. Size of the element: $D' < C' < B' < A'$

D. Order of ionic radii: $B'^{+} < A^{1+} < D'^{+} < C^{+}$

Choose the correct answer from the options given below : [4 marks]



(A) A only

(B) B, C and D only

(C) A and B only

(D) A, B and D only

Correct Answer: (D) A, B and D only

Solution:

Concept:

- Across a period from left to right, increasing effective nuclear attraction generally decreases atomic radius and metallic character. Down a group, additional occupied shells increase size and metallic character.
- For the corresponding singly charged positive ions, compare like charges: radii generally decrease across a period and increase down a group. The arrangement in the figure determines which comparisons are relevant.

Step 1: Locate the four positions The figure puts A' and B' next to each other in the upper period, with A' to the left. C' is below A', and D' is below B'. Going right contracts the atom, while going down increases its size. The order represented by the question is therefore

$$B' < A' < D' < C'.$$

Statement A gives this atomic-radius order. Metallic character also decreases to the right and increases downwards, giving the same order, so statement B is true.

Step 2: Check the remaining statements and select the combination Statement C puts C' below B' in size and A' above all the others. That contradicts the down-group increase from A' to C'; it is false. Statement D compares the corresponding monovalent cations. Applying the same directional trends to like-charged ions gives $B'^{+} < A'^{+} < D'^{+} < C'^{+}$, so D is true. The correct set is A, B and D. Option (A) omits two true statements, (B) includes the false C, and (C) omits D.

Step 3: Conclusion Statements A, B and D are true, while statement C is false. Hence, the correct option is (D).

Quick Tip to Solve: Mark the left-right and top-bottom relations before testing the statements, and compare ionic sizes only after checking that the ionic charges match.

76. The fragrance of flowers is due to the presence of some steam volatile organic compounds called essential oils. These are generally insoluble in water at room temperature but are miscible with water vapour in vapour phase. A suitable method for the extraction of these oils from the flowers is - [4 marks]

(A) distillation

(B) steam distillation

(C) distillation under reduced pressure

(D) crystallisation

Correct Answer: (B) steam distillation

Solution:

Concept:

- Steam distillation separates a steam-volatile organic substance that is immiscible or only sparingly miscible with liquid water. Each immiscible liquid contributes its own vapour pressure.
- The mixture boils when the sum of the vapour pressures reaches atmospheric pressure: $p_{\text{water}} + p_{\text{oil}} = p_{\text{atm}}$. This can occur below the boiling point of the pure oil.

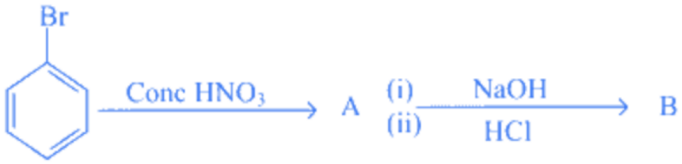
Step 1: Match the physical properties to the method The question states that the essential oils are steam volatile and generally insoluble in liquid water at room temperature. Passing steam through the flowers carries the oil into the vapour stream. Although the liquids form separate phases, both water vapour and oil vapour are present above them, so the oil can be distilled at a lower temperature than its normal boiling point. This is useful for fragrant organic compounds that can be damaged by prolonged strong heating.

Step 2: Condense the vapours and distinguish the alternatives The outgoing vapours are cooled in a condenser, giving water and essential oil. On cooling, the low liquid-phase miscibility allows the oil to separate and be collected, commonly with a separating funnel. Ordinary distillation relies on heating the liquid mixture and is not the specific method indicated by the stated steam volatility. Reduced-pressure distillation lowers the pressure and suits high-boiling substances, but the problem specifically gives the steam-distillation conditions. Crystallisation separates a dissolved solid and does not extract these volatile liquid oils from flowers.

Step 3: Conclusion The suitable extraction method is steam distillation. Hence, the correct option is (B).

Quick Tip to Solve: When an organic oil is water-insoluble yet steam volatile, think of co-distillation with water followed by separation of the condensed liquid layers.

77. Identify A and B in the following reaction sequence. [4 marks]



(A) A: nitrobenzene; B: 3-nitrophenol.

(B) A: 1-bromo-2,4-dinitrobenzene; B: 4-bromobenzene-1,3-diol.

(C) A: 1-bromo-2,4,6-trinitrobenzene; B: 2,4,6-trinitrophenol.

(D) A: 1-bromo-4-nitrobenzene; B: 2-bromo-5-nitrophenol.

Correct Answer: (C) A: 1-bromo-2,4,6-trinitrobenzene; B: 2,4,6-trinitrophenol.

Solution:

Concept:

- Halogens deactivate an aromatic ring but direct electrophilic substitution to ortho and para positions. Extensive nitration can place nitro groups at the two ortho positions and the para position relative to bromine.
- Electron-withdrawing nitro groups ortho or para to an aryl halide activate nucleophilic aromatic substitution by stabilising the anionic intermediate. Hydroxide replaces the halogen; acidification gives the phenol.

Step 1: Identify the extensively nitrated intermediate The starting compound is bromobenzene and the first reagent is concentrated nitric acid. The option showing exhaustive nitration has bromine at carbon 1 and nitro groups at carbons 2, 4 and 6: A is 1-bromo-2,4,6-trinitrobenzene. These positions are consistent with the ortho/para directing effect of bromine. Nitration replaces ring hydrogens by nitro groups; it does not remove bromine or directly replace nitro groups by hydroxyl groups.

Step 2: Replace bromine and acidify The three strongly electron-withdrawing nitro groups activate the carbon bonded to bromine towards attack by OH^-. Treatment with $NaOH$ replaces bromine by oxygen, forming the corresponding phenoxide under basic conditions. Subsequent HCl treatment protonates it to 2,4,6-trinitrophenol, also called picric acid. Option (C) alone shows this pair. Option (A) loses bromine during nitration and gives the wrong nitrophenol. Option (B) retains bromine while replacing nitro groups with hydroxyl groups, which is not the indicated substitution. Option (D) also retains bromine in the final product and does not show the activated trinitro pair.

Step 3: Conclusion A is 1-bromo-2,4,6-trinitrobenzene and B is 2,4,6-trinitrophenol (picric acid). Hence, the correct option is (C).

Quick Tip to Solve: In activated aryl-halide hydrolysis, follow the carbon bearing the halogen: OH replaces Br, and the ortho/para nitro groups remain on the ring.

78. Match List I with List II.

	List I (Complex ion)		List II (Configuration)
(A)	$[Cr(H_2O)_6]^{3+}$	**(I)**	$t_{2g}^2 e_g^0$
(B)	$[Fe(H_2O)_6]^{3+}$	**(II)**	$t_{2g}^3 e_g^0$
(C)	$[Ni(H_2O)_6]^{2+}$	**(III)**	$t_{2g}^3 e_g^2$
(D)	$[V(H_2O)_6]^{3+}$	**(IV)**	$t_{2g}^6 e_g^2$

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-I, C-II, D-III

(B) A-III, B-II, C-IV, D-I

(C) A-II, B-III, C-IV, D-I

(D) A-IV, B-III, C-I, D-II

Correct Answer: (C) A-II, B-III, C-IV, D-I

Solution:

Concept:

- In an octahedral complex the five metal d orbitals split into lower-energy t_{2g} and higher-energy e_g sets, containing three and two orbitals respectively.
- Water is neutral, so the oxidation state of each metal equals the charge of its complex ion. Remove the outer 4s electrons before counting the remaining 3d electrons. Water gives a high-spin iron(III) complex.

Step 1: Count the d electrons of each metal ion The neutral-atom configurations give $Cr^{3+} : 3d^3$, $Fe^{3+} : 3d^5$, $Ni^{2+} : 3d^8$, and $V^{3+} : 3d^2$. For chromium, remove its one 4s electron and then two 3d electrons. For vanadium remove its two 4s electrons and one 3d electron. These counts are the totals to distribute between the split d levels, not coordination numbers.

Step 2: Fill the octahedral levels and match the entries Chromium(III), with three electrons, singly occupies all three lower orbitals: $t_{2g}^3 e_g^0$, giving A–II. High-spin iron(III) places five electrons singly in the five d orbitals before pairing: $t_{2g}^3 e_g^2$, giving B–III. Nickel(II), with eight electrons, has $t_{2g}^6 e_g^2$, giving C–IV. Vanadium(III) has two lower-level electrons: $t_{2g}^2 e_g^0$, giving D–I. Option B wrongly gives chromium five d electrons; A and D give chromium eight. Thus each competing option already fails the chromium assignment, independently of its other mismatches.

Step 3: Conclusion The matching is A–II, B–III, C–IV, D–I. Hence, the correct option is (C).

Quick Tip to Solve: First count d electrons and match the unambiguous d2, d3 and d8 ions; only d5 iron then requires a high-spin versus low-spin decision.

79. Given below are two statements :

Statement I : Aniline reacts with con. H_2SO_4, followed by heating at $453-473$ K gives p-aminobenzene sulphonic acid, which gives blood red colour in the 'Lassaigne's test'.

Statement II : In Friedel - Craft's alkylation and acylation reactions, aniline forms salt with the $AlCl_3$ catalyst. Due to this, nitrogen of aniline aquires a positive charge and acts as deactivating group.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both statement I and statement II are true

(B) Statement I is true but statement II is false

(C) Statement I is false but statement II is true

(D) Both statement I and statement II are false

Correct Answer: (A) Both statement I and statement II are true

Solution:

Concept:

- Aniline undergoes sulphonation with concentrated H_2SO_4 on heating to form sulphanilic acid, p-aminobenzenesulphonic acid.
- In sodium-fusion analysis, a compound containing both nitrogen and sulphur can form NaSCN; ferric ions produce the characteristic blood-red thiocyanate complex. An amine lone pair also binds a Lewis acid such as $AlCl_3$.

Step 1: Check the formation and test of sulphanilic acid Aniline first forms anilinium hydrogensulphate with concentrated sulphuric acid. Heating at the stated $453-473\,K$ gives the para sulphonic acid. The product contains nitrogen in its amino group and sulphur in its sulphonic-acid group. During Lassaigne sodium fusion these heteroatoms can be converted to thiocyanate; the subsequent ferric-ion test gives a blood-red colour. This is the joint nitrogen-and-sulphur test, rather than the Prussian-blue test used for nitrogen alone. Thus both clauses of Statement I are true.

Step 2: Check the effect of the Friedel–Crafts catalyst The nitrogen lone pair of aniline coordinates to electron-deficient $AlCl_3$. In the resulting salt-like Lewis adduct, nitrogen acquires positive formal character and its lone pair is no longer freely available for resonance donation into the aromatic ring. The resulting group deactivates the ring toward electrophilic substitution. Consequently aniline does not undergo the usual Friedel–Crafts alkylation or acylation under these conditions, and Statement II is true. Options B and C each deny one true statement, while D denies both, so none of those alternatives is consistent with the chemistry.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: For a Lassaigne question, check whether nitrogen and sulphur occur together; their joint thiocyanate test gives blood red instead of the nitrogen-only blue test.

80. A sample of $CaCO_3$ and $MgCO_3$ weighed 2.21 g is ignited to constant weight of 1.152 g. The composition of mixture is :

(Given molar mass in g mol^{-1} $CaCO_3 : 100, MgCO_3 : 84$) [4 marks]

(A) 1.187 g $CaCO_3 + 1.187$ g $MgCO_3$

(B) 1.187 g $CaCO_3 + 1.023$ g $MgCO_3$

(C) 1.023 g $CaCO_3 + 1.187$ g $MgCO_3$

(D) 1.023 g $CaCO_3 + 1.023$ g $MgCO_3$

Correct Answer: (B) 1.187 g $CaCO_3 + 1.023$ g $MgCO_3$

Solution:

Concept:

- On strong heating, $CaCO_3 \longrightarrow CaO + CO_2$ and $MgCO_3 \longrightarrow MgO + CO_2$. The measured residue is the sum of the oxide masses; the mass lost is carbon dioxide.
- Using the supplied carbonate molar masses, the oxide molar masses are 56 and $40\,\text{g}\,\text{mol}^{-1}$, because each carbonate loses $44\,\text{g}\,\text{mol}^{-1}$ of CO_2.

Step 1: Express both oxide contributions using one unknown Let the initial mass of calcium carbonate be x g; then magnesium carbonate weighs $(2.21-x)$ g. Complete ignition converts them to $(56/100)x$ g of calcium oxide and $(40/84)(2.21-x)$ g of magnesium oxide. Therefore conservation of the residue mass gives

$$0.56x + \frac{40}{84}(2.21 - x) = 1.152.$$

It would be incorrect to equate the residue to either carbonate alone or to use the same oxide fraction for both carbonates.

Step 2: Solve the equation and check the choices Multiplying by 2100 clears the denominators:

$$1176x + 1000(2.21 - x) = 2419.2.$$

Thus $176x = 209.2$, so $x = 1.188636\ldots$ g and the other component is $1.021364\ldots$ g. The given masses have rounded experimental precision, and the matching listed composition is $1.187\,\text{g}$ and $1.023\,\text{g}$. Its calculated residue is $0.56(1.187) + (40/84)(1.023) = 1.15186\,\text{g}$, which rounds to $1.152\,\text{g}$. Options A and D do not even add to 2.21 g; C has the correct initial total but gives a residue of about $1.1381\,\text{g}$, inconsistent with the measurement.

Step 3: Conclusion The matching composition is $1.187\,\text{g}$ of $CaCO_3$ and $1.023\,\text{g}$ of $MgCO_3$, within the rounding of the measurements. Hence, the correct option is (B).

Quick Tip to Solve: Before solving a mixture equation, reject compositions whose masses do not sum to the original sample; then check the remaining choices using oxide mass fractions.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. From the vitamins $\mathrm{A}, \mathrm{B}_1, \mathrm{B}_6, \mathrm{B}_{12}, \mathrm{C}, \mathrm{D}, \mathrm{E}$ and K, the number of vitamins that can be stored in our body is _____. [4 marks]

Solution:

Concept:

- The fat-soluble vitamins A, D, E and K can be retained in fatty tissue and the liver; their storage is unlike the usual rapid excretion of excess water-soluble vitamins.
- Vitamin B_{12} is the important water-soluble exception: substantial liver stores can supply the body for a long period. Classifying a vitamin as water-soluble does not by itself prove that it cannot be stored.

Step 1: Separate the four fat-soluble vitamins From the supplied list A, B_1, B_6, B_{12}, C, D, E and K, select A, D, E and K. All four are fat-soluble and can be stored in the body. They contribute four to the count. The question asks for the number that can be stored, rather than simply the number that dissolve in fat, so the remaining water-soluble candidates still need examination.

Step 2: Include the storage exception among the B vitamins Vitamins B_1, B_6 and C are water-soluble vitamins for which a regular dietary supply is needed and are not included among the substantially stored vitamins in this classification. Vitamin B_{12}, however, is retained in the liver and provides the relevant exception to the general water-solubility rule. Add it once to the four fat-soluble members. The five selected entries are therefore A, B_{12}, D, E and K, giving $4 + 1 = 5$. Counting only A, D, E and K would answer a different question about fat solubility and miss the specified storage exception; counting every B vitamin would also be incorrect.

Step 3: Conclusion The number of listed vitamins that can be stored in the body is $\boxed{5}$ (a dimensionless count).

Quick Tip to Solve: For storage questions remember A, D, E, K plus the liver-storage exception B12; do not automatically exclude every water-soluble vitamin.

82. A diatomic molecule has a dipole moment of $1.2\ \mathrm{D}$. If the bond distance is $1\ \mathrm{A}^\circ$, then fractional charge on each atom is _____ $\times 10^{-1}$ esu.
(Given $1\ \mathrm{D} = 10^{-18}$ esucm) [4 marks]

Solution:

Concept:

- For equal and opposite charges separated by a distance r, the dipole-moment magnitude is $\mu = qr$. In electrostatic cgs units, μ is in esu cm, q in esu and r in cm.
- Use the stated conversion $1\,\mathrm{D} = 10^{-18}\,\mathrm{esu\,cm}$, together with $1\,\text{Å} = 10^{-8}\,\mathrm{cm}$. The coefficient requested in the printed power-of-ten form must be distinguished from the actual charge.

Step 1: Convert the given dipole moment and bond length The molecular dipole moment is

$$\mu = 1.2 \times 10^{-18}\,\text{esu cm}, \qquad r = 10^{-8}\,\text{cm}.$$

Both quantities are now in consistent cgs units. Using a bond distance of 1 without its centimetre conversion would give a charge smaller by eight powers of ten. No electronic-charge constant is needed to find a charge expressed directly in esu.

Step 2: Calculate the charge and express the printed coefficient Dividing the moment by the separation gives

$$q = \frac{1.2 \times 10^{-18}}{10^{-8}} = 1.2 \times 10^{-10}\,\text{esu}.$$

The two atoms carry opposite signs with this same magnitude. If the blank is denoted by x, its printed form is $q = x \times 10^{-1}\,\text{esu}$, so

$$x = \frac{1.2 \times 10^{-10}}{10^{-1}} = 1.2 \times 10^{-9}.$$

Thus the literal coefficient is extremely small and rounds to zero as an integer numerical entry. The phrase fractional charge here is accompanied by esu, so the requested quantity is a charge magnitude in those units; a dimensionless fraction of the electronic charge would instead require dividing by e and would answer a different demand. Preserve the printed exponent rather than silently changing it to a more convenient one.

Step 3: Conclusion The charge magnitude on each atom is $1.2 \times 10^{-10}\,\text{esu}$. The literal coefficient of $10^{-1}\,\text{esu}$ is 1.2×10^{-9}, whose nearest-integer numerical entry is $\boxed{0}$.

Quick Tip to Solve: Compute the physical charge first, then divide by the exact power of ten printed after the blank; the coefficient and charge need not have the same numerical value.

83. Number of isomeric products formed by monochlorination of 2-methylbutane in presence of sunlight is _____. [4 marks]

Solution:

Concept:

- Monochlorination replaces one hydrogen by chlorine without changing the carbon skeleton. Chemically equivalent hydrogens give the same constitutional product.
- Count stereoisomers as separate isomeric products when the question does not restrict the count to structural isomers. A product with one stereogenic carbon exists as an R/S pair.

Step 1: Identify the four distinct substitution sites Write 2-methylbutane as $CH_3{-}CH(CH_3){-}CH_2{-}CH_3$. Its two methyl groups bonded directly to the central CH are equivalent; substitution on either gives the same constitution. The terminal methyl at the other end is a second site, the methylene is a third, and the central tertiary CH is the fourth. Thus there are four constitutional monochloro derivatives, not five obtained by incorrectly distinguishing the two equivalent methyl groups.

Step 2: Check each product for chirality Substitution on a methyl group adjacent to the central carbon gives $ClCH_2{-}CH(CH_3){-}CH_2{-}CH_3$, namely 1-chloro-2-methylbutane. Its central carbon now

has four different substituents, so it gives two enantiomers. Substitution on the remote terminal methyl gives $CH_3-CH(CH_3)-CH_2-CH_2Cl$; the central carbon still bears two identical methyl groups, so this is one achiral product. Substitution on the methylene gives $CH_3-CH(CH_3)-CH(Cl)-CH_3$, with a chiral chlorine-bearing carbon, giving two enantiomers. Substitution on the tertiary CH gives $CH_3-C(Cl)(CH_3)-CH_2-CH_3$, which has two identical methyl groups and is achiral. Each chiral constitution has just one stereocentre, so no further diastereomers or meso forms occur. The total is $2+1+2+1=6$.

Step 3: Conclusion The total number of isomeric monochlorination products, including enantiomers, is $\boxed{6}$ (a dimensionless count).

Quick Tip to Solve: After grouping equivalent hydrogens, inspect every product again: substitution on an apparently ordinary methyl group can create a stereocentre on its neighbouring carbon.

84. In the reaction of potassium dichromate, potassium chloride and sulfuric acid (conc.), the oxidation state of the chromium in the product is $(+)$ _____. [4 marks]

Solution:

Concept:

- Potassium dichromate and a chloride salt with concentrated sulphuric acid produce chromyl chloride, CrO_2Cl_2, in the chromyl-chloride test.
- The oxidation numbers in a neutral molecule sum to zero. Oxygen is -2 and each chlorine is -1 in chromyl chloride; apply these values to determine chromium, without assuming every dichromate reaction reduces it.

Step 1: Identify the chromium-containing product The chloride salt is KCl, and concentrated sulphuric acid provides the strongly acidic dehydrating conditions for formation of red vapours of chromyl chloride. A balanced representation is

$$K_2Cr_2O_7 + 4\,KCl + 6\,H_2SO_4 \longrightarrow 2\,CrO_2Cl_2 + 6\,KHSO_4 + 3\,H_2O.$$

There are two chromium atoms on each side, and each product molecule contains one chromium, two oxygen and two chlorine atoms. The product sought is therefore not a chromium(III) salt from a different dichromate redox reaction.

Step 2: Use charge balance in chromyl chloride Let the chromium oxidation number in CrO_2Cl_2 be x. The molecule is neutral, hence

$$x + 2(-2) + 2(-1) = 0.$$

Combining the oxygen contribution -4 and chlorine contribution -2 gives $x-6=0$, or $x=+6$. The starting dichromate also contains chromium(VI), which is consistent with this reaction being a formation of a volatile oxychloride rather than a reduction of chromium. Concentrated sulphuric acid in the specified salt test is decisive: merely remembering that dichromate often forms green chromium(III) would select the wrong product. The printed positive sign is already supplied before the blank, so only the magnitude belongs in the numerical entry.

Step 3: Conclusion The oxidation state of chromium in the product CrO_2Cl_2 is $+6$; the required dimensionless entry is $\boxed{6}$.

Quick Tip to Solve: When dichromate, chloride salt and concentrated sulphuric acid occur together, identify chromyl chloride first; then sum the oxygen and chlorine oxidation numbers.

85. The values of conductivity of some materials at $298.15\ K^{-1}$ in Sm^{-1} are 2.1×10^3, $1.0 \times 10^{-16}, 1.2 \times 10, 3.91, 1.5 \times 10^{-2}, 1 \times 10^{-7}, 1.0 \times 10^3$.

The number of conductors among the materials is _____. [4 marks]

Solution:

Concept:

- Electrical conductivity σ measures the ability of a material to carry current. In SI units it is expressed in $S\,m^{-1}$, the reciprocal of resistivity in $\Omega\,m$.
- The intended numerical grouping treats the entries with conductivity of order unity or larger as conductors. This is an approximate classification for this list, not a universal physical boundary: conductivity ranges overlap, and material identity and charge carriers would be needed for a more detailed classification.

Step 1: Identify the conducting entries For the requested count, select the listed materials whose conductivity is at least of order $1\,S\,m^{-1}$. The qualifying values are

$$2.1 \times 10^3, \quad 1.2 \times 10 = 12, \quad 3.91, \quad 1.0 \times 10^3 \quad (S\,m^{-1}).$$

All four meet this intended numerical grouping. In particular, 1.2×10 means twelve, with an implicit exponent of one; it is not a small conductivity.

Step 2: Exclude the low-conductivity entries and count The remaining values are

$$1.0 \times 10^{-16}, \quad 1.5 \times 10^{-2}, \quad 1 \times 10^{-7} \quad (S\,m^{-1}).$$

Each is below the order-unity cutoff of this numerical grouping. The value 10^{-16} characterises an insulator; the middle value lies among the much lower conductivities which this grouping excludes. We do not count these three entries as conductors. Thus four of the seven given materials are counted.

Step 3: Conclusion With the intended approximate grouping, the number of conductors is $\boxed{4}$ (a dimensionless count). Conductivity alone does not define a universal sharp boundary between conductors and semiconductors.

Quick Tip to Solve: Convert every scientific-notation entry to its order of magnitude first; the implicit exponent in 1.2×10 is one, so that entry is counted.

86. A compound (x) with molar mass $108\ \text{g mol}^{-1}$ undergoes acetylation to give product with molar mass $192\ \text{g mol}^{-1}$. The number of amino groups in the compound (x) is _____. [4 marks]

Solution:

Concept:

- Acetylation of a primary amino group replaces one hydrogen on nitrogen by an acetyl group CH_3CO attached through its carbonyl carbon, converting $-NH_2$ into $-NHCOCH_3$.
- The increase in molar mass per acetylated amino group is the mass of C_2H_3O minus that of the displaced hydrogen, namely $43 - 1 = 42\,\text{g mol}^{-1}$.

Step 1: Calculate the molar-mass increase The same molecular framework is present before and after acetylation; only the acetyl substituents have been introduced at the amino groups. Therefore the total increase per mole of compound is

$$\Delta M = M_{\text{product}} - M_x = 192 - 108 = 84\,\text{g mol}^{-1}.$$

The mass change must be found by subtraction, rather than by dividing the product molar mass by the starting molar mass.

Step 2: Relate that increase to the number of amino groups Let n be the number of amino groups in one molecule of x. For each amino group, the change is

$$\text{R}-\text{NH}_2 \longrightarrow \text{R}-\text{NHCOCH}_3.$$

The added fragment is C_2H_3O, of mass $2(12) + 3(1) + 16 = 43$, while one hydrogen of mass 1 is removed. Hence

$$42n = 84, \qquad n = \frac{84}{42} = 2.$$

As a check, two acetylations increase the molar mass by $2 \times 42 = 84\,\text{g mol}^{-1}$, and $108 + 84 = 192$, exactly the stated product mass. The count concerns amino groups, not the total number of N–H bonds present initially.

Step 3: Conclusion The compound x contains $\boxed{2}$ amino groups per molecule.

Quick Tip to Solve: For ordinary amino-group acetylation, use a net molar-mass rise of 42 per group: one N–H hydrogen is replaced, so the rise is not 43.

87. Number of moles of H^+ ions required by 1 mole of MnO_4^- to oxidise oxalate ion to CO_2 is _____. [4 marks]

Solution:

Concept:

- In acidic solution, permanganate is reduced from manganese in oxidation state $+7$ to Mn^{2+}, gaining five electrons per permanganate ion.
- Oxalate is oxidised to carbon dioxide. Oxygen is balanced using water, hydrogen using H^+, and charge using electrons in the half-reaction method.

Step 1: Balance the reduction and oxidation half-reactions The reduction half-reaction is

$$\mathrm{MnO_4^-} + 8\,\mathrm{H^+} + 5\,\mathrm{e^-} \longrightarrow \mathrm{Mn^{2+}} + 4\,\mathrm{H_2O}.$$

Four oxygen atoms require four water molecules; their eight hydrogens require eight protons on the left. The left charge is $-1 + 8 - 5 = +2$, agreeing with the right side. The oxalate half-reaction is

$$\mathrm{C_2O_4^{2-}} \longrightarrow 2\,\mathrm{CO_2} + 2\,\mathrm{e^-}.$$

It requires no additional protons, because its carbon and oxygen atoms already balance.

Step 2: Cancel electrons and read the proton coefficient Multiply the reduction half-reaction by two and the oxidation half-reaction by five, so that ten electrons cancel. The resulting ionic equation is

$$2\,\mathrm{MnO_4^-} + 5\,\mathrm{C_2O_4^{2-}} + 16\,\mathrm{H^+} \longrightarrow 2\,\mathrm{Mn^{2+}} + 10\,\mathrm{CO_2} + 8\,\mathrm{H_2O}.$$

Thus two moles of permanganate consume sixteen moles of protons. For the one mole specified,

$$n(\mathrm{H^+}) = \frac{16}{2} \times 1 = 8\,\mathrm{mol}.$$

The total proton requirement comes from the balanced ionic equation, not from the five-electron change alone. Both atoms and total charge are conserved.

Step 3: Conclusion One mole of $\mathrm{MnO_4^-}$ requires $\boxed{8\,\mathrm{mol}}$ of $\mathrm{H^+}$; the requested numerical value is $\boxed{8}$.

Quick Tip to Solve: In acidic permanganate reductions to $\mathrm{Mn^{2+}}$, first write the four water molecules; they immediately fix the coefficient of $\mathrm{H^+}$ as eight.

88. If 5 moles of an ideal gas expands from 10 L to a volume of 100 L at 300 K under isothermal and reversible condition then work, w, is $-x$ J. The value of x is _____.
(Given R = 8.314 J $\mathrm{K^{-1}}$ $\mathrm{mol^{-1}}$) [4 marks]

Solution:

Concept:

- For reversible isothermal expansion of n moles of an ideal gas, the chemistry sign convention gives $w = -nRT\ln(V_2/V_1)$. Expansion work is negative because the gas does work on its surroundings.
- The volume ratio is dimensionless, so both volumes may be kept in litres. To match the usual numerical evaluation, write $\ln z \simeq 2.303\log_{10} z$.

Step 1: Substitute the isothermal data The mole number and temperature remain fixed during this process: $n = 5\,\mathrm{mol}$ and $T = 300\,\mathrm{K}$. The volume ratio is

$$\frac{V_2}{V_1} = \frac{100}{10} = 10.$$

Using the supplied gas constant,

$$w = -5 \times 8.314 \times 300 \ln 10 \, \text{J}.$$

No pressure value is needed because reversible pressure varies continuously as nRT/V. Using a constant-pressure work expression here would therefore give the wrong result.

Step 2: Evaluate the logarithm and identify the requested magnitude With the standard rounded factor 2.303 and $\log_{10} 10 = 1$,

$$w \simeq -2.303 \times 5 \times 8.314 \times 300 = -28720.713 \, \text{J}.$$

Since the question writes $w = -x$ J, x is the positive magnitude. Rounding this standard common-logarithm evaluation to the nearest integer gives $x = 28721$. If the unrounded natural logarithm $\ln 10 = 2.302585\ldots$ is used instead, the magnitude is 28715.54 J; the small difference comes solely from rounding the logarithm conversion factor. The quoted numerical entry uses the conventional 2.303 evaluation.

Step 3: Conclusion The expansion work is approximately -2.8721×10^4 J using 2.303; the requested numerical entry is $\boxed{x = 28721}$.

Quick Tip to Solve: Keep the minus sign on the work, but enter its positive magnitude for x. State the logarithm precision when a large numerical result depends on the rounded factor 2.303.

89. The molarity of 1 L orthophosphoric acid (H_3PO_4) having 70% purity by weight (specific gravity 1.54 g cm^{-3}) is _____ M.

(Molar mass of $H_3PO_4 = 98$ g mol^{-1}) [4 marks]

Solution:

Concept:

- Molarity is moles of solute per litre of solution: $M = n_{\text{solute}}/V_{\text{solution}}$. A mass percentage of 70 means 70 g of solute in every 100 g of solution.
- The supplied value $1.54 \, \text{g cm}^{-3}$ is used as the solution density. Since $1 \, \text{L} = 1000 \, \text{cm}^3$, it converts the given solution volume into mass.

Step 1: Find the mass of pure orthophosphoric acid The total mass of the one litre of solution is

$$m_{\text{solution}} = \rho V = 1.54 \times 1000 = 1540 \, \text{g}.$$

Only 70 percent of this mass is orthophosphoric acid; the remaining mass does not contribute to the number of moles of H_3PO_4. Hence

$$m(H_3PO_4) = \frac{70}{100} \times 1540 = 1078 \, \text{g}.$$

The density is applied to the whole solution before taking the purity fraction. Treating one litre as one kilogram would ignore the given density.

Step 2: Convert solute mass into molarity Use the stated molar mass $98\,\mathrm{g\,mol^{-1}}$:

$$n(\mathrm{H_3PO_4}) = \frac{1078}{98} = 11\,\mathrm{mol}.$$

The volume in the molarity denominator is the original solution volume of one litre, not the volume of water or of pure acid. Therefore

$$M = \frac{11\,\mathrm{mol}}{1\,\mathrm{L}} = 11\,\mathrm{mol\,L^{-1}}.$$

Orthophosphoric acid has three ionisable hydrogens, but this is a molarity question, so there is no factor of three; that factor could enter an acid-base equivalent-concentration calculation.

Step 3: Conclusion The molarity of the orthophosphoric acid solution is $\boxed{11\,\mathrm{M}}$.

Quick Tip to Solve: For mass percentage p, density d in $\mathrm{g\,mL^{-1}}$, and molar mass M_r, use molarity $= 10dp/M_r$; do not multiply by the acid basicity.

90. $\mathrm{r} = \mathrm{k[A]}$ **for a reaction,** 50% **of** A **is decomposed in 120 minutes. The time taken for** 90% **decomposition of** A **is _____ minutes.** [4 marks]

Solution:

Concept:

- The rate law $r = k[A]$ identifies a first-order reaction. Its integrated form is $kt = \ln([A]_0/[A]_t)$, where $[A]_t$ is the amount remaining at time t.
- For a first-order process, $t_{1/2} = \ln 2/k$ is independent of the initial concentration. A 50 percent decomposition means one half-life has passed.

Step 1: Determine the rate constant from the half-life The statement that half of A decomposes in 120 minutes gives

$$t_{1/2} = 120\,\mathrm{min}, \qquad k = \frac{\ln 2}{120}\,\mathrm{min^{-1}}.$$

Numerically this is approximately $5.776 \times 10^{-3}\,\mathrm{min^{-1}}$. The unit is inverse minutes because the time data are given in minutes. We retain the logarithmic form to avoid rounding the intermediate rate constant.

Step 2: Use the fraction remaining after 90 percent decomposition When 90 percent has decomposed, only 10 percent remains, so $[A]_t = 0.10[A]_0$. Substitution in the integrated first-order equation gives

$$t_{90} = \frac{\ln 10}{k} = 120\frac{\ln 10}{\ln 2} = 398.631\ldots\,\mathrm{min}.$$

The initial concentration cancels, so its numerical value is not needed. Rounding the required time to the nearest integer gives 399 minutes. The answer exceeds three half-lives (360 minutes), because after three half-lives 12.5 percent remains; reaching 10 percent requires a little more time. A linear scaling $120 \times 90/50$ would be incorrect because the reaction slows as A is consumed.

Step 3: Conclusion The time for 90 percent decomposition is approximately $398.63\,\mathrm{min}$, so the

requested integer is $\boxed{399 \text{ minutes}}$.

Quick Tip to Solve: Use the fraction remaining, not the fraction decomposed, in the logarithm: 90 percent decomposition gives $\ln(1/0.10) = \ln 10$.