JEE Main 2024 Paper 1 (B.E./B.Tech) Feb 1 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 1 (Jan-Feb), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 1 February 2024, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is : [4 marks]

(A) $\frac{2}{5}$
(B) $\frac{2}{7}$
(C) $\frac{1}{7}$
(D) $\frac{1}{5}$

Correct Answer: (B) $\frac{2}{7}$

Solution:

Concept:

- Bayes' theorem updates the probability of a possible bag composition after observing the sample. Write H_k for the hypothesis that the bag contains k white balls and $8-k$ black balls.
- For sampling without replacement, the probability of two balls of each colour is hypergeometric: $P(E \mid H_k) = \binom{k}{2}\binom{8-k}{2}/\binom{8}{4}$.

Step 1: Identify the possible compositions The observation rules out fewer than two balls of either colour. Thus only $k = 2, 3, 4, 5, 6$ can produce the observed sample. In the usual equally likely composition model for this question, the original hypotheses $H_0, \ldots, H_8$ have equal prior probabilities. These equal factors cancel in Bayes' ratio. The surviving hypotheses are not equally likely after observing the sample; their likelihoods must be included.

Step 2: Calculate the likelihood weights Count the favourable four-ball subsets for each possible composition. The common denominator $\binom{8}{4} = 70$ cancels, leaving the weights

k	2	3	4	5	6
$\binom{k}{2}\binom{8-k}{2}$	15	30	36	30	15

The weights are symmetric on exchanging white and black, as they should be.

Step 3: Apply the conditional probability ratio Equal numbers of the two colours means H_4, not merely equal numbers in the sample. Therefore

$$P(H_4 \mid E) = \frac{36}{15+30+36+30+15} = \frac{36}{126} = \frac{2}{7}.$$

The value $1/5$ would incorrectly treat the five surviving compositions as equally likely after the observation. Neither $2/5$ nor $1/7$ equals the likelihood-normalised result.

Step 4: Conclusion The required probability, with the usual equally likely composition model, is $\boxed{2/7}$. Hence, the correct option is (B).

Quick Tip to Solve: For a hidden composition inferred from a sample, weight each possible composition by its sampling likelihood before normalising.

2. The value of the integral $\int_0^{\pi/4} \frac{x\ dx}{\sin^4(2x)+\cos^4(2x)}$ equals : [4 marks]

(A) $\frac{\sqrt{2}\pi^2}{8}$

(B) $\frac{\sqrt{2}\pi^2}{16}$

(C) $\frac{\sqrt{2}\pi^2}{32}$

(D) $\frac{\sqrt{2}\pi^2}{64}$

Correct Answer: (C) $\frac{\sqrt{2}\pi^2}{32}$

Solution:

Concept:

- For $f(a-x)=f(x)$, reflection of the interval gives $\int_0^a xf(x)\,dx = \frac{a}{2}\int_0^a f(x)\,dx$. This removes the numerator x without integration by parts.
- The identity $\sin^4 u + \cos^4 u = 1 - \frac{1}{2}\sin^2(2u)$ reduces the denominator, and $t = \tan v$ turns the remaining trigonometric integral into a rational one.

Step 1: Use reflection to remove the numerator Let the required integral be I, and set $a = \pi/4$. Replacing x by $a - x$ interchanges $\sin(2x)$ and $\cos(2x)$, leaving their fourth-power sum unchanged. Adding the original and reflected integrals yields

$$2I = \frac{\pi}{4}J, \qquad J = \int_0^{\pi/4} \frac{dx}{1-\frac{1}{2}\sin^2(4x)}.$$

Consequently $I = (\pi/8)J$.

Step 2: Evaluate the remaining symmetric integral Put $u = 4x$, so $dx = du/4$ and the limits become 0 and π. Reflection about $\pi/2$ gives

$$J = \frac{1}{2}\int_0^{\pi/2} \frac{du}{1-\frac{1}{2}\sin^2 u}.$$

With $t = \tan u$, we have $du = dt/(1+t^2)$, and therefore

$$J = \frac{1}{2}\int_0^{\infty} \frac{dt}{1+t^2/2} = \frac{\sqrt{2}}{2}\left[\tan^{-1}\frac{t}{\sqrt{2}}\right]_0^{\infty} = \frac{\sqrt{2}\pi}{4}.$$

The improper limit is finite because the rational integrand decays quadratically.

Step 3: Combine the factors and check the options Substituting this value into the reflected expression gives

$$I = \frac{\pi}{8}\cdot\frac{\sqrt{2}\pi}{4} = \frac{\sqrt{2}\pi^2}{32}.$$

Keeping both factors, $1/4$ from $u = 4x$ and the reflection factor, is essential. Options (A) and (B) are respectively four and two times this value, while option (D) is half of it.

Step 4: Conclusion The integral equals $\boxed{\sqrt{2}\pi^2/32}$. Hence, the correct option is (C).

Quick Tip to Solve: Before integrating an expression of the form x times a trigonometric function, test reflection about the midpoint of the interval.

3. If $\mathrm{A}=\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}, \mathrm{B}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}, \mathrm{C}=\mathrm{ABA}^{\mathrm{T}}$ **and** $\mathrm{X}=\mathrm{A}^{\mathrm{T}}\mathrm{C}^{2}\,\mathrm{A}$**, then** $\det \mathrm{X}$ **is equal to :** [4 marks]

(A) 243

(B) 729

(C) 27

(D) 891

Correct Answer: (B) 729

Solution:

Concept:

- Determinants satisfy $\det(PQ)=\det P\det Q$, $\det(A^{\mathrm{T}})=\det A$, and $\det(C^2)=(\det C)^2$. Thus matrix multiplication is unnecessary when only the final determinant is required.
- For a two-by-two matrix, $\det\begin{bmatrix}a & b\\ c & d\end{bmatrix}=ad-bc$. In particular the negative lower-left entry of A changes the sign of its contribution.

Step 1: Evaluate the determinants of the given matrices Using the printed entries gives

$$\det A=(\sqrt{2})(\sqrt{2})-(1)(-1)=2+1=3, \qquad \det B=(1)(1)-(0)(1)=1.$$

Do not replace $\det A$ by $2-1$; the off-diagonal product is negative. The transpose preserves the determinant, so $\det(A^{\mathrm{T}})=3$ as well.

Step 2: Find the determinant of the intermediate matrix The order in $C=ABA^{\mathrm{T}}$ is important for the matrix itself, but its determinant is the product of three scalar determinants:

$$\det C=(\det A)(\det B)(\det A^{\mathrm{T}})=3\cdot 1\cdot 3=9.$$

Squaring the matrix squares its determinant, since $C^2=CC$. Hence

$$\det(C^2)=9^2=81.$$

Step 3: Evaluate the determinant of X Apply the same product property to the two outer factors in $X=A^{\mathrm{T}}C^2A$:

$$\det X=3\cdot 81\cdot 3=729=3^6.$$

As a single check, the general expression is $\det X=(\det A)^6(\det B)^2$, which again gives 3^6. The value 27 misses determinant factors, 243 has only five factors of 3, and 891 is not the resulting product.

Step 4: Conclusion Thus $\boxed{\det X=729}$. Hence, the correct option is (B).

Quick Tip to Solve: For a nested matrix product whose determinant alone is asked, track determinant powers instead of expanding the matrices.

4. If $\tan A = \frac{1}{\sqrt{x(x^2+x+1)}}, \tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ **and** $\tan C = \left(x^{-3}+x^{-2}+x^{-1}\right)^{1/2}, 0 < A, B, C < \frac{\pi}{2}$**, then** $A + B$ **is equal to :** [4 marks]

(A) C

(B) $\pi - C$

(C) $2\pi - C$

(D) $\frac{\pi}{2} - C$

Correct Answer: (A) C

Solution:

Concept:

- The tangent addition identity is $\tan(A+B) = (\tan A + \tan B)/(1 - \tan A \tan B)$. The angle range must be checked after equating tangents.
- The square roots and the positive acute tangents imply $x > 0$. This allows $\sqrt{x^3} = x^{3/2}$ and ensures $x^2 + x > 0$.

Step 1: Simplify the sum and product of tangents Write $D = x^2 + x + 1$. The two given expressions become $\tan A = 1/\sqrt{xD}$ and $\tan B = x/\sqrt{xD}$. Therefore

$$\tan A + \tan B = \frac{x+1}{\sqrt{xD}}, \qquad \tan A \tan B = \frac{1}{D}.$$

The denominator of the addition formula is $1 - 1/D = (x^2 + x)/D$, which is positive for every permitted x.

Step 2: Compare the tangent of the sum with C Substitute and cancel the positive factor $x + 1$:

$$\tan(A+B) = \frac{x+1}{\sqrt{xD}} \frac{D}{x(x+1)} = \frac{\sqrt{D}}{x^{3/2}}.$$

Meanwhile the third printed condition gives

$$\tan C = \sqrt{x^{-3}+x^{-2}+x^{-1}} = \sqrt{\frac{1+x+x^2}{x^3}} = \frac{\sqrt{D}}{x^{3/2}}.$$

Thus $\tan(A+B) = \tan C$.

Step 3: Resolve the angle rather than just its tangent Both A and B are acute, so their sum lies in $(0, \pi)$. Also $\cos(A+B) = \cos A \cos B(1 - \tan A \tan B) > 0$; hence their sum is acute. Since C is acute too, equality of tangents gives $A + B = C$. The angles $\pi - C$ and $2\pi - C$ have negative tangent, while $\pi/2 - C$ generally has the reciprocal tangent.

Step 4: Conclusion The sum is $\boxed{A + B = C}$. Hence, the correct option is (A).

Quick Tip to Solve: After the tangent addition formula, use the sign of the cosine and the stated angle intervals to eliminate supplementary-angle errors.

5. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to : [4 marks]

(A) 47

(B) 53

(C) 51

(D) 43

Correct Answer: (C) 51

Solution:

Concept:

- Different employees placed in indistinguishable offices form a partition of five labelled people into at most four nonempty groups. Empty offices do not create additional arrangements.
- Count each possible group-size pattern separately. Divide by factorials for equal-sized groups, because exchanging two identical offices does not produce a new allocation.

Step 1: Count one or two occupied offices For one occupied office there is just one group containing everyone. For two occupied offices, the size patterns are $4+1$ and $3+2$. Choosing the singleton in the first case gives $\binom{5}{1}=5$; choosing the pair in the second gives $\binom{5}{2}=10$. The sizes distinguish the groups, so neither of these counts needs another division. The total so far is $1+5+10=16$.

Step 2: Count three occupied offices The patterns now are $3+1+1$ and $2+2+1$. In the first, choose the triple: $\binom{5}{3}=10$; the two remaining employees form their own singleton groups. In the second, choose the singleton and then divide the remaining four into two unlabelled pairs:

$$5\cdot\frac{\binom{4}{2}}{2}=15.$$

The division by 2 avoids counting the two pair-groups in both orders. Thus three occupied offices contribute $10+15=25$.

Step 3: Count four occupied offices and add With four occupied offices, the only pattern is $2+1+1+1$. Choosing the pair gives $\binom{5}{2}=10$. Five occupied offices are prohibited, even though there are five employees. Therefore

$$n=1+(5+10)+(10+15)+10=51.$$

This is also the sum of the Stirling numbers $S(5,1)+S(5,2)+S(5,3)+S(5,4)$. The alternatives 47, 53 and 43 do not match the exhaustive pattern counts; assigning labels to offices would overcount the arrangements.

Step 4: Conclusion There are $\boxed{51}$ allocations. Hence, the correct option is (C).

Quick Tip to Solve: For indistinguishable boxes and distinct objects, list group-size patterns and divide only when equal-sized groups are being interchanged.

6. Let $S = |z \in C : |z-1| = 1$ **and** $(\sqrt{2}-1)(z+\bar{z}) - i(z-\bar{z}) = 2\sqrt{2}\ |$**. Let** $z_1, z_2 \in S$ **be such that** $|z_1| = \max_{z \in s} |z|$ **and** $|z_2| = \min_{z \in S} |z|$**. Then** $\left|\sqrt{2}z_1 - z_2\right|^2$ **equals :** [4 marks]

(A) 1

(B) 4

(C) 3

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- Write $z = x + iy$, so $z + \bar{z} = 2x$ and $z - \bar{z} = 2iy$. The modulus condition $|z-1| = 1$ is the circle $(x-1)^2 + y^2 = 1$.
- The second condition becomes a straight line. Their intersection consists of the allowable complex numbers, and comparing $|z|^2 = x^2 + y^2$ identifies the maximum and minimum modulus.

Step 1: Convert the second condition to a line Let $r = \sqrt{2}$. Substitution into $(r-1)(z+\bar{z}) - i(z-\bar{z}) = 2r$ gives $2(r-1)x + 2y = 2r$, because $-i(2iy) = 2y$. Thus

$$y = r - (r-1)x.$$

The positive sign of the $2y$ term is essential; a sign error here would select a different line.

Step 2: Find both intersections with the circle Insert this expression for y into the circle equation and expand:

$$(4-2r)x^2 + (-6+2r)x + 2 = 0.$$

One root is $x = 1$, giving $y = 1$. The product of roots is $2/(4-2r) = 1 + r/2$, so the other point has $x = 1 + r/2$, $y = r/2$. Therefore the set contains exactly $1 + i$ and $1 + r/2 + i\,r/2$. Direct substitution confirms both satisfy the original two equations.

Step 3: Assign the extrema and evaluate the required modulus The squared moduli are 2 and $(1 + r/2)^2 + (r/2)^2 = 2 + r$. Thus

$$z_1 = 1 + \frac{r}{2} + i\frac{r}{2}, \qquad z_2 = 1 + i.$$

Since $r^2 = 2$, their required combination simplifies to

$$rz_1 - z_2 = (r + 1 + i) - (1 + i) = r.$$

Its squared modulus is $r^2 = 2$. The other values 1, 4 and 3 arise neither from this real difference nor from its modulus square; in particular, the modulus itself is $\sqrt{2}$, which must still be squared.

Step 4: Conclusion The required value is $\boxed{|\sqrt{2}z_1 - z_2|^2 = 2}$. Hence, the correct option is (D).

Quick Tip to Solve: For a complex-number set defined by a circle and line, find their intersections first and compare squared moduli to avoid unnecessary radicals.

7. Let the median and the mean deviation about the median of 7 observation $170, 125, 230, 190, 210$, a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is : [4 marks]

(A) 31

(B) 28

(C) 30

(D) 32

Correct Answer: (C) 30

Solution:

Concept:

- For seven observations the median is the fourth observation in increasing order. Mean deviation about a centre c is $\frac{1}{7}\sum|x_i - c|$.
- The given mean deviation about 170 specifies the total absolute deviation as 205. Use the median restriction to decide the signs of the unknown deviations before forming their sum.

Step 1: Use the fourth-position median restriction Three of the known observations, 190, 210 and 230, already exceed 170. If either a or b also exceeded 170, at least four observations would be larger than the stated median, which is impossible for seven values. Hence $a \leq 170$ and $b \leq 170$. The fixed observations contribute

$$|170 - 170| + |125 - 170| + |230 - 170| + |190 - 170| + |210 - 170| = 165.$$

Step 2: Find the sum of the unknown observations and the mean The two remaining deviations must total $205 - 165 = 40$. Their known signs give

$$(170 - a) + (170 - b) = 40, \qquad a + b = 300.$$

Individual values are not needed. The sum of the five printed numbers is 925, so the mean of all seven is

$$\bar{x} = \frac{925 + 300}{7} = 175.$$

Since both unknowns are at most 170, both also lie below this new centre, making their combined deviation from 175 easy to calculate.

Step 3: Recalculate the absolute deviations about 175 The five known observations contribute

$$5 + 50 + 55 + 15 + 35 = 160.$$

The two unknown observations contribute $(175 - a) + (175 - b) = 350 - 300 = 50$. Thus

$$\mathrm{MD}_{\bar{x}} = \frac{160 + 50}{7} = 30.$$

The total is independent of how the sum 300 is split between the two permissible unknown values. The other choices 31, 28 and 32 do not equal the recalculated total divided by seven; the old centre 170 must not be retained.

Step 4: Conclusion The mean deviation about the mean is $\boxed{30}$. Hence, the correct option is (C).

Quick Tip to Solve: When a median constrains unknown values to one side of it, remove the absolute values with those signs; their sum may determine the new deviation without finding each value.

8. Let $\overrightarrow{a} = -5\hat{i} + \hat{j} - 3\hat{k}$, $\overrightarrow{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\overrightarrow{c} = (((\overrightarrow{a} \times \overrightarrow{b}) \times \hat{i}) \times \hat{i}) \times \hat{i}$. Then $\vec{c} \cdot (-\hat{i} + \hat{j} + \hat{k})$ is equal to : [4 marks]

(A) -12

(B) -10

(C) -13

(D) -15

Correct Answer: (A) -12

Solution:

Concept:

- The cross product is evaluated from the inside out; changing the order of its factors changes its sign. It is not an associative operation.
- For a vector $\mathbf{v} = (v_1, v_2, v_3)$, right multiplication by $\hat{i}$ gives $\mathbf{v} \times \hat{i} = (0, v_3, -v_2)$. The scalar product is the sum of products of corresponding components.

Step 1: Find the first cross product Using the component formula, retain the sign of the middle component carefully:

$$\begin{aligned}\mathbf{a} \times \mathbf{b} &= (1(-4) - (-3)2,\ (-3)1 - (-5)(-4),\ (-5)2 - 1(1)) \\ &= (2, -23, -11).\end{aligned}$$

In particular, the second component is negative because both terms being subtracted have opposite signs before the subtraction is performed.

Step 2: Apply the three right cross products Apply the same map successively, following the parentheses in the question:

$$\begin{aligned}(2, -23, -11) \times \hat{i} &= (0, -11, 23), \\ (0, -11, 23) \times \hat{i} &= (0, 23, 11), \\ (0, 23, 11) \times \hat{i} &= (0, 11, -23).\end{aligned}$$

The first component vanishes after the first such operation. The remaining two components exchange positions, with a minus sign on the old second component each time.

Step 3: Take the requested scalar product Thus $\mathbf{c} = 11\hat{j} - 23\hat{k}$. Therefore

$$\mathbf{c} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 0(-1) + 11(1) - 23(1) = -12.$$

The alternatives $-10, -13, -15$ do not equal this component sum; no magnitude or absolute value is requested.

Step 4: Conclusion The required scalar product is $\boxed{-12}$. Hence, the correct option is (A).

Quick Tip to Solve: For repeated right cross products with $\hat{i}$, use $(u, v, w) \mapsto (0, w, -v)$ instead of expanding a new determinant each time.

9. Let $\mathbf{S} = \{x \in \mathbf{R} : (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$. Then the number of elements in S is : [4 marks]

(A) 4

(B) 0

(C) 2

(D) 1

Correct Answer: (C) 2

Solution:

Concept:

- The positive numbers $a = \sqrt{3}+\sqrt{2}$ and $b = \sqrt{3}-\sqrt{2}$ are reciprocals because $ab = 1$. Thus $b^x = a^{-x}$ for every real x.
- The map $x \mapsto a^x$ is strictly increasing and takes all positive real values when $a > 1$. Every positive root in the transformed variable therefore determines exactly one real value of x.

Step 1: Replace the reciprocal exponential Set $t = a^x > 0$. Since $a > 1$, this substitution neither identifies two distinct real values of x nor excludes any positive value of t. The equation becomes

$$t + \frac{1}{t} = 10.$$

Multiplication by t is legitimate because an exponential is never zero. It gives

$$t^2 - 10t + 1 = 0.$$

Step 2: Solve and test both roots By the quadratic formula,

$$t = \frac{10 \pm \sqrt{100-4}}{2} = 5 \pm 2\sqrt{6}.$$

Both roots are positive: $25 > 24$, so $5 - 2\sqrt{6} > 0$. They are also distinct. Moreover,

$$(\sqrt{3}+\sqrt{2})^2 = 5 + 2\sqrt{6}, \qquad (\sqrt{3}-\sqrt{2})^2 = 5 - 2\sqrt{6}.$$

The roots are therefore a^2 and a^{-2}, giving $x = 2$ and $x = -2$, respectively.

Step 3: Count the solutions without duplication The quadratic has no further roots, and the exponential substitution is one-to-one. Consequently $S = \{-2, 2\}$. Direct substitution also checks both values, as $(5 + 2\sqrt{6}) + (5 - 2\sqrt{6}) = 10$. Thus the counts 0 and 1 omit valid solutions, while 4 counts more values than the transformed quadratic can produce.

Step 4: Conclusion There are $\boxed{2}$ elements in S. Hence, the correct option is (C).

Quick Tip to Solve: When two positive bases multiply to 1, substitute one exponential by t and the other by $1/t$; check positivity before counting the roots.

10. The area enclosed by the curves $xy + 4y = 16$ and $x + y = 6$ is equal to : [4 marks]

(A) $28 - 30\log_e 2$

(B) $30 - 28\log_e 2$

(C) $30 - 32\log_e 2$

(D) $32 - 30\log_e 2$

Correct Answer: (C) $30 - 32\log_e 2$

Solution:

Concept:

- The area between two graphs on a common interval is the integral of upper ordinate minus lower ordinate. The intersection points determine the limits.
- Here the curves are $y = 16/(x + 4)$ and $y = 6 - x$. The hyperbola has a vertical asymptote at $x = -4$, outside the bounded region found below.

Step 1: Find the intersection limits At an intersection,

$$\frac{16}{x+4} = 6 - x \quad \Longrightarrow \quad 16 = (6 - x)(x + 4).$$

Expansion gives $x^2 - 2x - 8 = 0$, or $(x - 4)(x + 2) = 0$. Hence the intersection abscissae are -2 and 4. The corresponding ordinates are 8 and 2. The entire interval lies to the right of the vertical asymptote, so the integrand is continuous there.

Step 2: Identify the upper boundary On $[-2, 4]$,

$$(6 - x) - \frac{16}{x+4} = \frac{-(x-4)(x+2)}{x+4} \geq 0.$$

The denominator is positive, while $(x - 4)(x + 2) \leq 0$. Thus the line stays above the hyperbola throughout the region; the difference can be integrated without splitting the interval.

Step 3: Integrate the difference The area is

$$\begin{aligned} A &= \int_{-2}^{4} \left(6 - x - \frac{16}{x+4}\right) dx \\ &= \left[6x - \frac{x^2}{2} - 16\ln(x+4)\right]_{-2}^{4} \\ &= (16 - 16\ln 8) - (-14 - 16\ln 2) \\ &= 30 - 16\ln 4 = 30 - 32\ln 2. \end{aligned}$$

The logarithmic coefficient comes from the numerator 16 and the ratio of endpoint arguments $8/2 = 4$. The other three options have different constants or logarithmic coefficients and therefore do not match this evaluation.

Step 4: Conclusion The enclosed area is $\boxed{30 - 32\log_e 2}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Check which curve is above by factoring their difference; then use the ratio of the endpoint logarithm arguments to simplify the definite integral.

11. Let $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ be defined as

$f(x) = \begin{cases} \log_e x, & x > 0 \\ e^{-x}, & x \leq 0 \end{cases}$ **and**

$g(x) = \begin{cases} x, & x \geqslant 0 \\ e^{x}, & x < 0 \end{cases}$ **. Then, gof : $\mathbf{R} \to \mathbf{R}$ is :** [4 marks]

(A) one-one but not onto

(B) neither one-one nor onto

(C) onto but not one-one

(D) both one-one and onto

Correct Answer: (B) neither one-one nor onto

Solution:

Concept:

- To evaluate $(g \circ f)(x)$, first compute $f(x)$, then choose the branch of g according to the sign of that result. The sign of the original x alone does not choose the outer branch.
- A function is one-one if distinct inputs have distinct images. It is onto $\mathbf{R}$ only if every real number, including every negative number, occurs as an output.

Step 1: Write the composite on its actual intervals Let $h = g \circ f$. For $x \leq 0$, $f(x) = e^{-x} \geq 1$, and the nonnegative branch of g gives $h(x) = e^{-x}$. For $0 < x < 1$, $\ln x < 0$, so $h(x) = e^{\ln x} = x$. For $x \geq 1$, $\ln x \geq 0$, giving $h(x) = \ln x$. Thus

$$h(x) = \begin{cases} e^{-x}, & x \leq 0, \\ x, & 0 < x < 1, \\ \ln x, & x \geq 1. \end{cases}$$

At $x = 1$, the output is 0, as required by the nonnegative branch of g.

Step 2: Test whether distinct inputs can share an image The inputs 0 and e are distinct, but

$$h(0) = e^0 = 1, \qquad h(e) = \ln e = 1.$$

This explicit counterexample disproves injectivity. Each individual piece may be strictly monotone, but their images overlap, so piecewise monotonicity does not establish that the full function is one-one.

Step 3: Determine the range and compare the choices The first interval gives $[1, \infty)$, the middle interval gives $(0, 1)$, and the last gives $[0, \infty)$. Their union is $[0, \infty)$, which excludes every negative real number. Hence the composite is not onto the stated codomain $\mathbf{R}$. Options (A) and (D) wrongly claim injectivity; option (C) wrongly claims surjectivity.

Step 4: Conclusion The function $g \circ f : \mathbf{R} \to \mathbf{R}$ is neither one-one nor onto. Hence, the correct option is (B).

Quick Tip to Solve: In a piecewise composite, split the inner branch again wherever its output changes sign; then test injectivity with an explicit repeated image.

12. If the system of equations

$$2x + 3y - z = 5$$

$$x + \alpha y + 3z = -4$$

$$3x - y + \beta z = 7$$

has infinitely many solutions, then $13\alpha\beta$ is equal to : [4 marks]

(A) 1110

(B) 1120

(C) 1210

(D) 1220

Correct Answer: (B) 1120

Solution:

Concept:

- Infinitely many solutions require a consistent dependent equation: the third row of the augmented matrix must be a linear combination of the first two rows.
- The first two coefficient rows are independent for every α, since their x, z minor is $2 \cdot 3 - (-1) \cdot 1 = 7 \neq 0$. Therefore their rank is 2, and making the third augmented row dependent yields one free variable.

Step 1: Match the first coefficient and constant term Write equation 3 as λ times equation 1 plus μ times equation 2. The x-coefficient and the right-hand side give

$$2\lambda + \mu = 3, \qquad 5\lambda - 4\mu = 7.$$

Substitution of $\mu = 3 - 2\lambda$ gives $13\lambda = 19$. Consequently

$$\lambda = \frac{19}{13}, \qquad \mu = \frac{1}{13}.$$

Using the constant column here is essential: a zero coefficient determinant alone would not ensure consistency.

Step 2: Match the remaining coefficients For the y-coefficient, the printed third equation has coefficient -1. Hence

$$3\lambda + \alpha\mu = -1 \quad \Longrightarrow \quad \frac{57 + \alpha}{13} = -1,$$

so $\alpha = -70$. For the z-coefficient,

$$\beta = -\lambda + 3\mu = \frac{-19 + 3}{13} = -\frac{16}{13}.$$

These values make every entry of the third augmented row equal to the stated combination, not merely one or two entries.

Step 3: Check the rank condition and evaluate The first two rows stay independent by their nonzero x, z minor. The dependent third row therefore makes both the coefficient rank and augmented rank 2, below the number of unknowns 3. Thus there are indeed infinitely many solutions. Finally,

$$13\alpha\beta = 13(-70)\left(-\frac{16}{13}\right) = 1120.$$

Both negative signs cancel. The alternatives 1110, 1210 and 1220 differ from this exact product.

Step 4: Conclusion The required value is $\boxed{1120}$. Hence, the correct option is (B).

Quick Tip to Solve: For infinitely many solutions, match a linear combination of augmented rows; include the constant term to exclude inconsistent parallel equations.

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13. For $0 < \theta < \pi/2$, if the eccentricity of the hyperbola $x^2 - y^2\,\mathrm{cosec}^2\,\theta = 5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2\,\mathrm{cosec}^2\,\theta + y^2 = 5$, then the value of θ is : [4 marks]

(A) $\frac{\pi}{6}$
(B) $\frac{5\pi}{12}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Concept:

- For the hyperbola $x^2/a^2 - y^2/b^2 = 1$, $e^2 = 1 + b^2/a^2$. For an ellipse with major semiaxis a and minor semiaxis b, $e^2 = 1 - b^2/a^2$.
- The interval $0 < \theta < \pi/2$ gives $0 < \sin^2\theta < 1$. Thus the major axis of the ellipse here is the vertical axis, while the transverse axis of the hyperbola is horizontal.

Step 1: Put the hyperbola in standard form Since $\mathrm{cosec}^2\,\theta = 1/\sin^2\theta$, division by 5 gives

$$\frac{x^2}{5} - \frac{y^2}{5\sin^2\theta} = 1.$$

Thus $a_h^2 = 5$, $b_h^2 = 5\sin^2\theta$, and

$$e_h^2 = 1 + \sin^2\theta.$$

The plus sign is specific to a hyperbola; using the ellipse formula would reverse the required relationship.

Step 2: Identify the ellipse major semiaxis The ellipse becomes

$$\frac{x^2}{5\sin^2\theta} + \frac{y^2}{5} = 1.$$

Because $\sin^2\theta < 1$, the larger squared semiaxis is 5, under y^2. Therefore

$$e_e^2 = 1 - \frac{5\sin^2\theta}{5} = \cos^2\theta.$$

Using the denominator under x^2 as the major semiaxis would be incorrect for this interval.

Step 3: Use the stated ratio and the angle interval The condition $e_h = \sqrt{7}\, e_e$ may be squared because eccentricities are nonnegative. It yields

$$1 + \sin^2\theta = 7(1 - \sin^2\theta) \quad \Longrightarrow \quad \sin^2\theta = \frac{3}{4}.$$

In the first quadrant, $\sin\theta = \sqrt{3}/2$, so $\theta = \pi/3$ uniquely. The angles $\pi/6$ and $\pi/4$ give squared sines $1/4$ and $1/2$; $5\pi/12$ gives $(2+\sqrt{3})/4$. None equals $3/4$.

Step 4: Conclusion The angle is $\boxed{\theta = \pi/3}$. Hence, the correct option is (C).

Quick Tip to Solve: For an ellipse, identify the larger denominator before applying eccentricity; the major semiaxis need not lie along the horizontal axis.

14. Let $y = y(x)$ be the solution of the differential equation
$\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1, y(0) = 1$.
Then, $\left(\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals : [4 marks]

(A) $\frac{4}{4+\sqrt{e}}$
(B) $\frac{3}{3-\sqrt{e}}$
(C) $\frac{2}{1+\sqrt{e}}$
(D) $\frac{1}{2-\sqrt{e}}$

Correct Answer: (D) $\frac{1}{2-\sqrt{e}}$

Solution:

Concept:

- The repeated expression $x + y$ suggests substituting $z = x + y$, for which $z\prime = 1 + y\prime$. This removes the constant term in the given derivative.
- After this substitution, $z\prime + xz = 2xz^3$ is a Bernoulli equation. For its nonzero solution, $u = z^{-2}$ converts it to a first-order linear equation. The initial value $z(0) = 1$ selects this branch.

Step 1: Transform the differential equation Let $z = x + y$. Since $y\prime = 2xz^3 - xz - 1$, we have

$$z\prime = 2xz^3 - xz, \qquad z(0) = 1.$$

The required expression is exactly $z(1/\sqrt{2})^2$. Put $u = z^{-2}$. Differentiating with the chain rule gives

$$u\prime = -2z^{-3}z\prime = -4x + 2xz^{-2},$$

so $u\prime - 2xu = -4x$, with $u(0) = 1$.

Step 2: Solve the linear equation The integrating factor is

$$\exp\left(\int -2x\,dx\right) = e^{-x^2}.$$

Multiplication converts the left side to a product derivative:

$$\frac{d}{dx}(ue^{-x^2}) = -4xe^{-x^2}.$$

Since the derivative of e^{-x^2} is $-2xe^{-x^2}$, integration gives

$$ue^{-x^2} = 2e^{-x^2} + C, \qquad u = 2 + Ce^{x^2}.$$

The initial value gives $1 = 2 + C$, hence $C = -1$ and $z^2 = 1/(2 - e^{x^2})$.

Step 3: Evaluate on the initial solution branch On the interval from 0 to $1/\sqrt{2}$, the denominator stays positive because $e^{x^2} \le \sqrt{e} < 2$. Thus the positive branch through $z(0) = 1$ exists throughout this interval. Evaluation gives

$$\left(\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right)^2 = z\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2 - \sqrt{e}}.$$

The value is approximately 2.85. The expressions in (A), (B) and (C) are approximately 0.71, 2.22 and 0.76, respectively, so they do not match the value fixed by the initial condition.

Step 4: Conclusion The requested value is $\boxed{\frac{1}{2 - \sqrt{e}}}$. Hence, the correct option is (D).

Quick Tip to Solve: When a cubic in $x + y$ occurs with a constant -1 in $y\prime$, set $z = x + y$ first, then linearise using z^{-2}.

15. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as:

$$f(x) = \begin{cases} \dfrac{a - b\cos 2x}{x^2}, & x < 0, \\ x^2 + cx + 2, & 0 \le x \le 1, \\ 2x + 1, & x > 1. \end{cases}$$

If f is continuous everywhere in $\mathbf{R}$ and m is the number of points where f is NOT differential, then $m + a + b + c$ equals: [4 marks]

(A) 1

(B) 4

(C) 3

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- Continuity at a joining point requires the two one-sided limits to equal the assigned value. A finite limit of $(a - b\cos 2x)/x^2$ at zero first requires its constant numerator term to vanish.
- Differentiability must be tested separately after continuity: equal limiting values do not guarantee equal one-sided derivatives. Away from the joining points, each of the three displayed expressions is differentiable.

Step 1: Use continuity at zero At zero the middle piece gives $f(0) = 2$. Since $\cos 2x \to 1$, finiteness of the left limit requires $a = b$. Then the identity $1 - \cos 2x = 2\sin^2 x$ gives

$$\lim_{x \to 0^-} \frac{b(1 - \cos 2x)}{x^2} = 2b \lim_{x \to 0^-} \left(\frac{\sin x}{x}\right)^2 = 2b.$$

Thus $2b = 2$, so $a = b = 1$.

Step 2: Use continuity at one The middle piece takes the value $f(1) = 3 + c$, whereas the right limit is $2(1) + 1 = 3$. Equating these values gives $c = 0$. The continuous function therefore has middle piece $x^2 + 2$.

Step 3: Check both one-sided derivatives Near zero, the expansion $\sin x/x = 1 - x^2/6 + O(x^4)$ yields $f(x) = 2 - 2x^2/3 + O(x^4)$ on the left. Therefore $(f(x) - f(0))/x \to 0$. The right derivative is also $2(0) = 0$. At one, the middle derivative is $2(1) = 2$, equal to the right derivative of $2x + 1$. There are no other possible points of non-differentiability, so $m = 0$.

Step 4: Conclusion Thus $m + a + b + c = 0 + 1 + 1 + 0 = 2$. The values 1, 4 and 3 do not equal this sum. Hence, the correct option is (D).

Quick Tip to Solve: Find the continuity constants first, then check derivatives at both joins; counting a join automatically as a corner gives a wrong value of m.

16. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latusrectum is $\sqrt{14}$. Then the square of the eccentricity of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is : [4 marks]

(A) 3

(B) $\frac{7}{2}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

Correct Answer: (C) $\frac{3}{2}$

Solution:

Concept:

- For the ellipse $x^2/a^2+y^2/b^2=1$ with $a>b$, the eccentricity satisfies $e_E^2=1-b^2/a^2$, and its latus rectum has length $2b^2/a$.
- For the hyperbola $x^2/a^2-y^2/b^2=1$, the focal distance satisfies $c_H^2=a^2+b^2$; consequently $e_H^2=1+b^2/a^2$. The same letters describe the given positive semiaxes in both equations.

Step 1: Find the ratio of the squared semiaxes The ellipse has $e_E=1/\sqrt{2}$, hence

$$\frac{1}{2}=1-\frac{b^2}{a^2} \quad \Longrightarrow \quad \frac{b^2}{a^2}=\frac{1}{2}.$$

The eccentricity determines the shape ratio, which is the only information needed for the requested hyperbola eccentricity.

Step 2: Check the supplied size information The latus rectum condition is consistent with the ratio just found:

$$\frac{2b^2}{a}=\frac{2(a^2/2)}{a}=a=\sqrt{14}.$$

Thus $a^2=14$ and $b^2=7$. This confirms that the data give real positive semiaxes with $a>b$; the length condition sets their scale rather than changing their ratio.

Step 3: Apply the hyperbola formula Changing the plus sign between the squared coordinate terms to a minus sign changes the focal relation from a difference to a sum. For the hyperbola,

$$e_H^2=\frac{a^2+b^2}{a^2}=\frac{14+7}{14}=\frac{3}{2}.$$

The question asks for the square of the eccentricity, so no final square root is required. Options 3, $7/2$, and $5/2$ each contradict the ratio $b^2/a^2=1/2$.

Step 4: Conclusion The square of the hyperbola eccentricity is $\boxed{3/2}$. Hence, the correct option is (C).

Quick Tip to Solve: With identical denominators, the ellipse and hyperbola obey $e_E^2+e_H^2=2$; the latus rectum need not be evaluated to find the requested ratio.

17. Let $3, a, b, c$ be in A.P. and $3, a-1, b+1, c+9$ be in G.P. Then, the arithmetic mean of a, b and c is : [4 marks]

(A) -4

(B) -1

(C) 13

(D) 11

Correct Answer: (D) 11

Solution:

Concept:

- For three terms a, b, c in an arithmetic progression, the middle term is their arithmetic mean: $a + c = 2b$ and $(a + b + c)/3 = b$.
- A geometric progression with first term 3 and real common ratio r has successive terms $3, 3r, 3r^2, 3r^3$. Introduce this single ratio to impose the conditions on all four given GP terms together.

Step 1: Express the three unknowns using the GP ratio From the printed geometric progression $3, a-1, b+1, c+9$, write

$$a - 1 = 3r, \qquad b + 1 = 3r^2, \qquad c + 9 = 3r^3.$$

Therefore $a = 3r + 1$, $b = 3r^2 - 1$, and $c = 3r^3 - 9$. This parametrisation avoids selecting a ratio from only the first three terms and forgetting the fourth.

Step 2: Use the arithmetic-progression condition Substitute these expressions in $a + c = 2b$:

$$3r + 1 + 3r^3 - 9 = 2(3r^2 - 1).$$

Rearranging and factoring gives

$$3(r^3 - 2r^2 + r - 2) = 0, \qquad (r - 2)(r^2 + 1) = 0.$$

The terms are real, so $r^2 + 1$ cannot vanish. The only real possibility is $r = 2$.

Step 3: Compute and verify the arithmetic mean For $r = 2$, the values are $a = 7$, $b = 11$, and $c = 15$. The full sequence $3, 7, 11, 15$ has successive differences 4, verifying the entire printed AP. The corresponding GP is $3, 6, 12, 24$, whose ratio is consistently 2. Hence

$$\frac{a + b + c}{3} = \frac{7 + 11 + 15}{3} = 11.$$

Neither -4, -1, nor 13 equals this mean; checking the fourth GP term rules out spurious values.

Step 4: Conclusion The arithmetic mean of a, b, c is $\boxed{11}$. Hence, the correct option is (D).

Quick Tip to Solve: For three AP terms, the requested arithmetic mean is the middle term; parameterise the entire GP before using $a + c = 2b$.

18. Let $C : x^2 + y^2 = 4$ and $C' : x^2 + y^2 - 4\lambda x + 9 = 0$ be two circles. If the set of all values of λ so that the circles C and C′ intersect at two distinct points, is $\mathrm{R} - [\mathrm{a}, \mathrm{b}]$, then the point $(8\mathrm{a} + 12, 16\ \mathrm{b} - 20)$ lies on the curve : [4 marks]

(A) $x^2 + 2y^2 - 5x + 6y = 3$

(B) $5x^2 - y = -11$

(C) $x^2 - 4y^2 = 7$

(D) $6x^2 + y^2 = 42$

Correct Answer: (D) $6x^2 + y^2 = 42$

Solution:

Concept:

- The common points of two circles satisfy their radical-axis equation, obtained by subtracting the two circle equations. A line cuts a circle in two distinct points exactly when its distance from the centre is strictly less than the radius.
- For a vertical line $x = k$, the distance from the origin is $|k|$. The first circle has centre $(0, 0)$ and radius 2, so its secant condition is $|k| < 2$.

Step 1: Find the line through the intersection points At a point of C, we have $x^2 + y^2 = 4$. Substituting in C' gives

$$4 - 4\lambda x + 9 = 0 \quad \Longrightarrow \quad 4\lambda x = 13.$$

If $\lambda = 0$, there is no common point. For nonzero λ, the radical axis is the vertical line $x = 13/(4\lambda)$.

Step 2: Impose two distinct intersections The line must pass strictly inside the circle of radius 2:

$$\left|\frac{13}{4\lambda}\right| < 2 \quad \Longleftrightarrow \quad |\lambda| > \frac{13}{8}.$$

At equality the circles touch, so both endpoints are excluded from the admissible set. Thus that set is $\mathbf{R} - [-13/8, 13/8]$, and $a = -13/8$, $b = 13/8$. These values also give a genuine second circle, since $4\lambda^2 - 9 > 0$ throughout the admissible set.

Step 3: Evaluate the point and test the options The required point has coordinates

$$8a + 12 = -13 + 12 = -1, \qquad 16b - 20 = 26 - 20 = 6.$$

At $(-1, 6)$, the left sides of options (A), (B), and (C) are respectively 114, -1, and -143, which do not equal their stated right sides 3, -11, and 7. For option (D), $6x^2 + y^2 = 6(1) + 36 = 42$, exactly as required.

Step 4: Conclusion The point is $(-1, 6)$, and it lies on $\boxed{6x^2 + y^2 = 42}$. Hence, the correct option is (D).

Quick Tip to Solve: Subtract the circle equations first: the resulting radical axis turns a two-circle intersection condition into a simple line-circle distance inequality.

19. **If $5f(x)+4f\left(\frac{1}{x}\right)=x^2-2, \forall x \neq 0$ and $y=9x^2f(x)$, then y is strictly increasing in :** [4 marks]

(A) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(C) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

(D) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

Correct Answer: (B) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

Solution:

Concept:

- Replace x by $1/x$ in the functional equation to obtain a second linear equation in $f(x)$ and $f(1/x)$. The substitution is valid because the domain excludes zero.
- A differentiable function increases on intervals where its derivative is positive. A sign chart of the factored derivative finds the intervals, while the original domain restriction must still be retained.

Step 1: Eliminate the reciprocal argument The given equation and its reciprocal version are

$$5f(x)+4f(1/x)=x^2-2,$$
$$4f(x)+5f(1/x)=x^{-2}-2.$$

Multiply the first by 5 and the second by 4, then subtract. This yields

$$9f(x)=5x^2-4x^{-2}-2.$$

The coefficient determinant is $25-16=9$, so the elimination gives a unique value of $f(x)$ for every nonzero x.

Step 2: Form the function and differentiate Multiplying by x^2, as in $y=9x^2f(x)$, removes the reciprocal term:

$$y=5x^4-2x^2-4, \qquad y'=20x^3-4x=4x(5x^2-1).$$

Although this polynomial expression extends to zero, the function specified by the question still has $x \neq 0$. The derivative changes sign at $\pm 1/\sqrt{5}$.

Step 3: Read the derivative sign chart For $x<-1/\sqrt{5}$, the factors have signs negative and positive, so the derivative is negative. On $(-1/\sqrt{5}, 0)$, both factors are negative, so it is positive. On $(0, 1/\sqrt{5})$, the derivative is negative, and on $(1/\sqrt{5}, \infty)$, it is positive. Thus (A) and (C) include a decreasing positive interval, while (D) includes negative points below $-1/\sqrt{5}$ where the function decreases.

Step 4: Conclusion The strictly increasing intervals are $\boxed{(-1/\sqrt{5}, 0) \cup (1/\sqrt{5}, \infty)}$. Hence, the correct option is (B).

Quick Tip to Solve: Eliminate $f(1/x)$ before differentiating, and keep the excluded point $x=0$ even when multiplication produces a polynomial expression.

20. If the shortest distance between the lines

$\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ **and** $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ **is 1 , then the sum of all possible values of** λ **is :** [4 marks]

(A) 0

(B) $2\sqrt{3}$

(C) $3\sqrt{3}$

(D) $-2\sqrt{3}$

Correct Answer: (B) $2\sqrt{3}$

Solution:

Concept:

- For two nonparallel lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$ with directions $\mathbf{d}_1, \mathbf{d}_2$, their shortest distance is $D = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)|/|\mathbf{d}_1 \times \mathbf{d}_2|$.
- The cross product is perpendicular to both lines. Projecting the vector joining any one point on each line onto this normal removes the components parallel to the lines. The absolute value accounts for either orientation of that normal.

Step 1: Identify the points and direction vectors From the symmetric equations, take

$$\mathbf{a}_1 = (\lambda, 2, 1), \quad \mathbf{d}_1 = (-2, 1, 1),$$
$$\mathbf{a}_2 = (\sqrt{3}, 1, 2), \quad \mathbf{d}_2 = (1, -2, 1).$$

Thus $\mathbf{a}_2 - \mathbf{a}_1 = (\sqrt{3} - \lambda, -1, 1)$. The direction vectors are not proportional, so the standard nonparallel-line distance formula applies.

Step 2: Evaluate the scalar triple product Computing the cross product gives

$$\mathbf{d}_1 \times \mathbf{d}_2 = (3, 3, 3), \qquad |\mathbf{d}_1 \times \mathbf{d}_2| = 3\sqrt{3}.$$

The numerator is

$$|3(\sqrt{3} - \lambda) + 3(-1) + 3(1)| = 3|\sqrt{3} - \lambda|.$$

The last two terms cancel; keeping their signs correct is essential. Therefore $D = |\sqrt{3} - \lambda|/\sqrt{3}$.

Step 3: Solve both branches of the distance condition Setting $D = 1$ gives $|\sqrt{3} - \lambda| = \sqrt{3}$. Therefore $\sqrt{3} - \lambda = \sqrt{3}$ or $\sqrt{3} - \lambda = -\sqrt{3}$, giving $\lambda = 0$ or $\lambda = 2\sqrt{3}$. Both yield distance 1 on substitution. Their sum is $2\sqrt{3}$; 0 is only one individual value, and neither $3\sqrt{3}$ nor $-2\sqrt{3}$ is the sum.

Step 4: Conclusion The sum of all possible values of λ is $\boxed{2\sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the absolute value until the final equation, then solve both signs; the distance is unchanged by reversing the common normal.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If $x = x(t)$ is the solution of the differential equation $(t+1)\mathrm{d}x = \left(2x + (t+1)^4\right)\mathrm{dt}, x(0) = 2$, then, $x(1)$ equals _____. [4 marks]

Solution:

Concept:

- A first-order linear differential equation $x' + P(t)x = Q(t)$ is solved by multiplying by the integrating factor $\mu(t) = e^{\int P(t)\,dt}$, so that $(\mu x)' = \mu Q$.
- The initial condition selects the integration constant on an interval containing the initial time. Here the equation is singular at $t = -1$, so the relevant interval includes $t = 0$ and $t = 1$ and stays to the right of -1.

Step 1: Write the equation in linear form Divide the differential equation by $(t+1)\,dt$ for $t \neq -1$:

$$\frac{dx}{dt} - \frac{2}{t+1}x = (t+1)^3.$$

Thus $P(t) = -2/(t+1)$. On $t > -1$, an integrating factor is

$$\mu(t) = e^{-2\log(t+1)} = (t+1)^{-2}.$$

Step 2: Multiply by the integrating factor and integrate The product rule transforms the equation to

$$\frac{d}{dt}\left(\frac{x}{(t+1)^2}\right) = t+1.$$

Integrating gives

$$\frac{x}{(t+1)^2} = \frac{(t+1)^2}{2} + C, \qquad x(t) = \frac{(t+1)^4}{2} + C(t+1)^2.$$

Using an arbitrary constant here accounts for all solutions on the chosen interval; it must be fixed before evaluating the requested time.

Step 3: Apply the initial condition and evaluate At $t = 0$, the given condition becomes $2 = 1/2 + C$, hence $C = 3/2$. Therefore

$$x(1) = \frac{2^4}{2} + \frac{3}{2}(2^2) = 8 + 6 = 14.$$

As a check, differentiating $x = (t+1)^4/2 + 3(t+1)^2/2$ gives $x' = 2(t+1)^3 + 3(t+1)$; multiplying this by $t+1$ equals $2x + (t+1)^4$. The initial condition is also satisfied. No physical unit is assigned to x in the question.

Step 4: Conclusion The required numerical value is $\boxed{x(1) = 14}$.

Quick Tip to Solve: After applying the integrating factor, verify that the new right side is $t+1$, not $(t+1)^3$; the factor changes both sides of the equation.

22. The number of elements in the set $S = \{(x, y, z) : x, y, z \in \mathbf{Z}, x + 2y + 3z = 42, x, y, z \geqslant 0\}$ equals _____.

[4 marks]

Solution:

Concept:

- For each fixed nonnegative integer z, the equation determines x uniquely once y is chosen. The conditions on x therefore give the allowable range of y.
- The greatest-integer expression $\lfloor (42-3z)/2 \rfloor$ is easiest to sum after separating even and odd values of z.

Step 1: Fix the third coordinate Since all three variables are nonnegative, $3z \leq 42$, so $z = 0, 1, \ldots, 14$. For any one such value,

$$x + 2y = 42 - 3z.$$

The allowed integers are $y = 0, 1, \ldots, \lfloor (42 - 3z)/2 \rfloor$. Each gives exactly one nonnegative integer $x = 42 - 3z - 2y$. Thus there are

$$\left\lfloor \frac{42 - 3z}{2} \right\rfloor + 1$$

solutions for that value of z. The extra one includes $y = 0$; omitting it would lose a solution for every z.

Step 2: Sum separately over even and odd values For even $z = 2k$, we have $k = 0, \ldots, 7$, and the count is $22 - 3k$. Hence

$$N_{\text{even}} = \sum_{k=0}^{7} (22 - 3k) = 8(22) - 3\frac{7 \cdot 8}{2} = 92.$$

For odd $z = 2k + 1$, we have $k = 0, \ldots, 6$. Since the right side is odd, its half has fractional part one-half, giving the count $20 - 3k$. Therefore

$$N_{\text{odd}} = \sum_{k=0}^{6} (20 - 3k) = 7(20) - 3\frac{6 \cdot 7}{2} = 77.$$

These two cases are disjoint and cover every possible z, including the endpoint $z = 14$, which contributes just $(0, 0, 14)$.

Step 3: Conclusion The required number of ordered triples is $92 + 77 = \boxed{169}$. This is a dimensionless count.

Quick Tip to Solve: Fix the variable with the largest coefficient, then split the remaining floor sum by parity; remember to count the zero endpoint.

23. If the Coefficient of x^{30} in the expansion of $\left(1+\frac{1}{x}\right)^6\left(1+x^2\right)^7\left(1-x^3\right)^8; x \neq 0$ is α, then $|\alpha|$ equals _____. [4 marks]

Solution:

Concept:

- The binomial theorem gives $(1+x^{-1})^6 = \sum_{r=0}^{6} \binom{6}{r} x^{-r}$; negative powers must be included in the exponent equation.
- A product contributes to the coefficient of x^{30} precisely when its three exponents add to 30. Contributions with odd powers chosen from $(1-x^3)^8$ carry a minus sign.

Step 1: Set up the exponent condition Choose the terms indexed by r, s, t from the three factors. Their product is

$$\binom{6}{r}\binom{7}{s}\binom{8}{t}(-1)^t x^{-r+2s+3t},$$

where $0 \le r \le 6$, $0 \le s \le 7$, and $0 \le t \le 8$. We therefore require

$$-r + 2s + 3t = 30.$$

Since $2s \le 14$ and $r \ge 0$, this forces $t \ge 6$. Only $t = 6, 7, 8$ need to be considered. For each such value, solve $r = 2s + 3t - 30$ while retaining both index bounds.

Step 2: List and add all contributions The valid choices, grouped by t, are

t	(r, s)	$(-1)^t\binom{6}{r}\binom{7}{s}\binom{8}{t}$
6	$(0,6), (2,7)$	$196 + 420$
7	$(1,5), (3,6), (5,7)$	$-1008 - 1120 - 48$
8	$(0,3), (2,4), (4,5), (6,6)$	$35 + 525 + 315 + 7$

For example, $(r, s, t) = (1, 5, 7)$ contributes $-\binom{6}{1}\binom{7}{5}\binom{8}{7} = -1008$. The signs in the middle row are negative because $t = 7$ is odd. No other index triples obey the exponent condition and the bounds. Thus

$$\begin{aligned}\alpha &= (196 + 420) - (1008 + 1120 + 48) \\ &\quad + (35 + 525 + 315 + 7) \\ &= 616 - 2176 + 882 = -678.\end{aligned}$$

The question asks for the absolute value, rather than the signed coefficient.

Step 3: Conclusion Therefore $|\alpha| = \boxed{678}$, a dimensionless numerical value.

Quick Tip to Solve: First write the exponent equation with the negative exponent intact. Bound the final index before enumerating coefficient contributions.

24. Let $3, 7, 11, 15, \ldots, 403$ and $2, 5, 8, 11, \ldots, 404$ be two arithmetic progressions. Then the sum, of the common terms in them, is equal to _____. [4 marks]

Solution:

Concept:

- A number common to two arithmetic progressions must satisfy both residue conditions and lie inside both finite ranges.
- Once a common term is found, successive common terms differ by the least common multiple of the two common differences. Their sum is $n(a + l)/2$.

Step 1: Find the common progression The first progression has common difference 4, and every term is congruent to 3 modulo 4. The second has common difference 3, and every term is congruent to 2 modulo 3. A common term u must satisfy

$$u \equiv 3 \pmod 4, \qquad u \equiv 2 \pmod 3.$$

Testing the first few terms of the first progression, 3 and 7 do not belong to the second, while 11 does. Since $\text{lcm}(4, 3) = 12$, every subsequent common term is obtained by adding 12. Conversely adding 12 preserves both congruences, so no common terms are omitted. The common sequence is therefore $11, 23, 35, \ldots$.

Step 2: Enforce the endpoints and sum The common terms cannot exceed 403, the smaller of the two final terms. Write their general term as $11 + 12k$, with $k \geq 0$. Then

$$11 + 12k \leq 403 \quad \Longrightarrow \quad k \leq \frac{392}{12} = 32\frac{2}{3}.$$

Thus $k = 0, \ldots, 32$, giving 33 common terms. The last is

$$l = 11 + 12(32) = 395.$$

Indeed, 395 appears in both progressions, whereas the next candidate 407 exceeds both ranges. Applying the arithmetic-series formula gives

$$S = \frac{33}{2}(11 + 395) = 33 \cdot 203 = 6699.$$

The count uses both endpoints: the indices 0 through 32 represent 33 terms, not 32 terms.

Step 3: Conclusion The sum of all common terms is $\boxed{6699}$. It is a dimensionless numerical sum.

Quick Tip to Solve: Use the least common multiple for the common difference, then check the last term against the smaller endpoint before applying the sum formula.

25. Let $\{x\}$ denote the fractional part of x and $f(x) = \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, x \neq 0$. If L and R respectively denotes the left hand limit and the right hand limit of $f(x)$ at $x = 0$, then $\frac{32}{\pi^2}\left(\mathrm{L}^2 + \mathrm{R}^2\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- Near zero, $\{x\} = x$ for $x > 0$, while $\{x\} = 1 + x$ for $-1 < x < 0$. Thus the two one-sided limits correspond to different endpoints of the fractional part.
- Use $\sin^{-1} u \sim u$ as $u \to 0$ and $\cos^{-1}(1 - u^2) \sim \sqrt{2}\,u$ as $u \to 0^+$. The latter follows from $1 - \cos v \sim v^2/2$.

Step 1: Evaluate the right-hand limit Put $t = \{x\}$. As $x \to 0^+$, we have $t \to 0^+$, and

$$f(x) = \frac{\cos^{-1}(1 - t^2)}{t}\,\frac{\sin^{-1}(1 - t)}{1 - t^2}.$$

The first factor tends to $\sqrt{2}$. To see its sign and value, set $v = \cos^{-1}(1 - t^2)$; then $v \geq 0$, $v \to 0$, and $t^2 = 1 - \cos v \sim v^2/2$. The second factor tends to $\pi/2$, because the principal inverse sine is continuous at 1. Hence

$$R = \sqrt{2}\frac{\pi}{2} = \frac{\pi}{\sqrt{2}}.$$

Step 2: Evaluate the left-hand limit and combine As $x \to 0^-$, the fractional part is $1 + x$, so $t \to 1^-$. Set $u = 1 - t \to 0^+$. Since $t - t^3 = t(1 - t)(1 + t)$, we obtain

$$f(x) = \cos^{-1}(1 - t^2)\,\frac{\sin^{-1} u}{u}\,\frac{1}{t(1 + t)}.$$

These factors tend respectively to $\pi/2$, 1, and $1/2$. Thus

$$L = \frac{\pi}{4}.$$

Both limits are finite even though the defining denominator vanishes at the limiting endpoints. They differ because the fractional-part function jumps at zero. Substituting the two exact values,

$$\frac{32}{\pi^2}(L^2 + R^2) = \frac{32}{\pi^2}\left(\frac{\pi^2}{16} + \frac{\pi^2}{2}\right) = 18.$$

Step 3: Conclusion The requested dimensionless numerical value is $\boxed{18}$.

Quick Tip to Solve: For a fractional-part limit at an integer, replace the fractional part separately on each side before simplifying the inverse-trigonometric factors.

26. Let the line $\mathrm{L}: \sqrt{2}x + y = \alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle $\mathrm{PQ_1Q_2}$ is equal to _____. [4 marks]

Solution:

Concept:

- The intersection point is found by substituting the parabola equation into the circle equation and retaining the first-quadrant root.
- A circle is tangent to a line precisely when the perpendicular distance from its centre to the line equals its radius. A vertical side of a triangle allows its altitude to be read as a horizontal distance.

Step 1: Locate the intersection and the tangent line On the parabola, $x^2 = 2y$. Substitution into the circle gives

$$2y + y^2 = 3 \quad \Longrightarrow \quad (y-1)(y+3) = 0.$$

The first-quadrant condition selects $y = 1$ and $x = \sqrt{2}$, so

$$P = (\sqrt{2}, 1).$$

Because P lies on $\sqrt{2}x + y = \alpha$, it follows that $\alpha = 2 + 1 = 3$. Thus the common tangent line has equation

$$\sqrt{2}x + y - 3 = 0.$$

The other algebraic root $y = -3$ cannot give a first-quadrant point; the positive root for x is also essential.

Step 2: Find the two centres and the triangle area A centre on the y-axis is $(0, c)$. Its perpendicular distance to the line must equal the given radius $2\sqrt{3}$, giving

$$\frac{|c-3|}{\sqrt{(\sqrt{2})^2 + 1^2}} = 2\sqrt{3}.$$

Therefore $|c - 3| = 6$, and the two centres are $(0, 9)$ and $(0, -3)$, in either order. Their separation is 12. Taking Q_1Q_2 as the vertical base, the perpendicular altitude from P is its distance from the y-axis, namely $\sqrt{2}$. Hence

$$[PQ_1Q_2] = \frac{1}{2}(12)(\sqrt{2}) = 6\sqrt{2}.$$

The two centres must be taken from opposite distance signs: retaining only $c - 3 = 6$ would miss one of the tangent circles.

Step 3: Conclusion The square of the triangle area is $(6\sqrt{2})^2 = \boxed{72}$, in coordinate units to the fourth power.

Quick Tip to Solve: For centres constrained to an axis, use the point-to-line distance equation with both signs, then choose the centre-joining segment as the triangle base.

27. Let $P = \{z \in \mathbb{C} : |z+2-3i| \le 1\}$ and $Q = \{z \in \mathbb{C} : z(1+i)+\bar{z}(1-i) \le -8\}$. Let in $P \cap Q$, $|z-3+2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals _____. [4 marks]

Solution:

Concept:

- Write $z = x + iy$. The modulus inequality describes the closed unit disk with centre $C = (-2, 3)$, while the second condition is a real linear inequality.
- The quantity $|z - 3 + 2i|$ is distance from $A = (3, -2)$. Resolve a point relative to the disk centre along the direction toward A and a perpendicular direction to optimize that distance.

Step 1: Express the feasible region in orthogonal coordinates A direct multiplication gives

$$z(1 + i) + \bar{z}(1 - i) = 2(x - y).$$

Thus Q is $x - y \le -4$. Let $\mathbf{u} = (1, -1)/\sqrt{2}$, the unit vector from C toward A, and let $\mathbf{v} = (1, 1)/\sqrt{2}$. Write a feasible point as

$$(x, y) = C + s\mathbf{u} + t\mathbf{v}.$$

The disk condition is $s^2 + t^2 \le 1$. Also $x - y = -5 + \sqrt{2}s$, so the half-plane is $s \le 1/\sqrt{2}$. Since $A = C + 5\sqrt{2}\mathbf{u}$, the squared distance being optimized is

$$D^2 = (5\sqrt{2} - s)^2 + t^2.$$

Step 2: Determine the maximizing and minimizing points The largest distance in the full disk is $5\sqrt{2} + 1$, attained at $s = -1, t = 0$. This point satisfies the half-plane condition, so it remains the maximum in the intersection. It gives

$$z_1 = \left(-2 - \frac{1}{\sqrt{2}}\right) + i\left(3 + \frac{1}{\sqrt{2}}\right), \qquad |z_1|^2 = 14 + 5\sqrt{2}.$$

For the minimum, $s \le 1/\sqrt{2}$ implies

$$D^2 \ge (5\sqrt{2} - 1/\sqrt{2})^2.$$

Equality occurs at $s = 1/\sqrt{2}, t = 0$, which lies inside the disk and on the boundary of the half-plane. Therefore

$$z_2 = -\frac{3}{2} + \frac{5}{2}i, \qquad |z_2|^2 = \frac{9}{4} + \frac{25}{4} = \frac{17}{2}.$$

In particular the nearest point is the midpoint of the cut chord, rather than one of its endpoints. Finally,

$$|z_1|^2 + 2|z_2|^2 = 14 + 5\sqrt{2} + 17 = 31 + 5\sqrt{2}.$$

Step 3: Conclusion We obtain $\alpha = 31$ and $\beta = 5$; hence $\alpha + \beta = \boxed{36}$, a dimensionless value.

Quick Tip to Solve: Convert the complex inequality to a half-plane first. Resolve along the line from the disk centre to the fixed point to avoid incorrectly choosing chord endpoints.

28. If $\int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x\ dx}{(1+e^{\sin x})(1+\sin^4 x)} = \alpha\pi + \beta\log_e(3+2\sqrt{2})$**, where** α, β **are integers, then** $\alpha^2+\beta^2$ **equals :**

[4 marks]

Solution:

Concept:

- With $t = \sin x$, the factor $\cos x\,dx$ becomes dt and the integration limits become -1 and 1.
- The identity $1/(1+e^t) + 1/(1+e^{-t}) = 1$ removes the exponential on symmetric limits. The remaining denominator factors as $(t^2+\sqrt{2}t+1)(t^2-\sqrt{2}t+1)$.

Step 1: Use symmetry to eliminate the exponential factor Denote the given integral by I. The substitution $t = \sin x$ gives

$$I = 8\sqrt{2}\int_{-1}^{1} \frac{dt}{(1+e^t)(1+t^4)}.$$

Replace t by $-t$ in an identical copy of this integral and add the two expressions. The even factor $1+t^4$ is unchanged, and the two logistic factors add to 1. Therefore

$$2I = 8\sqrt{2}\int_{-1}^{1} \frac{dt}{1+t^4}, \qquad I = 8\sqrt{2}\int_{0}^{1} \frac{dt}{1+t^4}.$$

The second equality also uses the evenness of the remaining integrand.

Step 2: Evaluate the rational integral exactly Partial fractions give

$$\frac{1}{1+t^4} = \frac{1}{2}\left(\frac{1+t/\sqrt{2}}{t^2+\sqrt{2}t+1} + \frac{1-t/\sqrt{2}}{t^2-\sqrt{2}t+1}\right).$$

Integrating these two completed-square denominators yields the antiderivative

$$F(t) = \frac{1}{4\sqrt{2}}\log\frac{t^2+\sqrt{2}t+1}{t^2-\sqrt{2}t+1} + \frac{1}{2\sqrt{2}}\left[\tan^{-1}(\sqrt{2}t+1) + \tan^{-1}(\sqrt{2}t-1)\right].$$

Here $F(0) = 0$. At $t = 1$, the logarithm's argument is $3+2\sqrt{2}$, and the two principal arctangents are $3\pi/8$ and $\pi/8$. Thus

$$\int_0^1 \frac{dt}{1+t^4} = \frac{\pi + \log(3+2\sqrt{2})}{4\sqrt{2}}.$$

Multiplication by $8\sqrt{2}$ gives $I = 2\pi + 2\log(3+2\sqrt{2})$, identifying both required integer coefficients.

Step 3: Conclusion Thus $\alpha = 2$, $\beta = 2$, and $\alpha^2+\beta^2 = \boxed{8}$. The result is dimensionless.

Quick Tip to Solve: On symmetric limits, pair the integrand at opposite arguments before attempting integration; the exponential factors here collapse to one.

29. Let the line of the shortest distance between the lines

$$L_1 : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and}$$

$$L_2 : \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$$

intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the mid point of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to _____. [4 marks]

Solution:

Concept:

- A common perpendicular meets each of two skew lines at right angles. Thus its displacement vector has zero dot product with both line direction vectors.
- For endpoints P and Q, their midpoint is $(P + Q)/2$. Therefore twice the sum of midpoint coordinates equals the sum of all coordinates of the two endpoints.

Step 1: Parameterise the two endpoints Put $P = (1 + \lambda, 2 - \lambda, 3 + \lambda)$ and $Q = (4 + \mu, 5 + \mu, 6 - \mu)$. These parameterisations guarantee that each endpoint belongs to its required line. The vector from the first endpoint to the second is

$$\overrightarrow{PQ} = (3 + \mu - \lambda, 3 + \mu + \lambda, 3 - \mu - \lambda).$$

The direction vectors are $u = (1, -1, 1)$ and $v = (1, 1, -1)$; they are not parallel, so the common perpendicular endpoints are uniquely determined.

Step 2: Impose both perpendicularity conditions The shortest joining segment must be perpendicular to both lines, not merely to one. Taking the dot product with u gives

$$3 - \mu - 3\lambda = 0.$$

Taking the dot product with v gives

$$3 + 3\mu + \lambda = 0.$$

Hence $3\lambda + \mu = 3$ and $\lambda + 3\mu = -3$. Subtracting three times the second equation from the first gives $-8\mu = 12$, so $\mu = -3/2$, followed by $\lambda = 3/2$.

Step 3: Find the midpoint and required sum Substitution gives

$$P = (5/2, 1/2, 9/2), \qquad Q = (5/2, 7/2, 15/2).$$

Their joining vector is $(0, 3, 3)$, whose dot products with both u and v vanish, independently confirming the endpoint calculation. The midpoint is $(\alpha, \beta, \gamma) = (5/2, 2, 6)$. Therefore

$$2(\alpha + \beta + \gamma) = 2(5/2 + 2 + 6) = 21.$$

Step 4: Conclusion The required numerical value is $\boxed{21}$. It is a dimensionless coordinate expression.

Quick Tip to Solve: For a midpoint coordinate sum, add the endpoint coordinate sums directly; the factor of two cancels the midpoint denominator.

30. Let $A = \{1, 2, 3, \ldots, 20\}$. Let R_1 and R_2 two relation on A such that

$R_1 = \{(a, b) : b$ **is divisible by** $a\}$

$R_2 = \{(a, b) : a$ **is an integral multiple of** $b\}$.

Then, number of elements in $R_1 - R_2$ is equal to _____. [4 marks]

Solution:

Concept:

- The difference $R_1 - R_2$ contains ordered pairs belonging to R_1 but not to R_2; it is not the numerical difference of the two relation cardinalities.
- For a fixed positive integer $a \leq 20$, the multiples of a in A number $\lfloor 20/a \rfloor$. Two positive integers divide each other only when they are equal.

Step 1: Identify the pairs to remove In R_1, the condition is $a \mid b$. In R_2, it is $b \mid a$. If a pair belongs to both, write $b = ka$ and $a = lb$ with positive integers k, l. Then $kl = 1$, so $k = l = 1$ and $a = b$. Conversely every diagonal pair (a, a) belongs to both relations. Thus $R_1 \cap R_2$ has exactly 20 elements, and the required pairs are precisely the proper-multiple pairs with $b > a$.

Step 2: Count the relation by its first coordinate For $a = 1, 2, \ldots, 20$, list how many multiples lie within the set:

$$\lfloor 20/a \rfloor = 20, 10, 6, 5, 4, 3, 2, 2, 2, 2,$$
$$1, 1, 1, 1, 1, 1, 1, 1, 1, 1.$$

Each pair is counted exactly once because its first coordinate is fixed in one row. Consequently

$$|R_1| = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2 + 2 + 10 = 66.$$

In particular the ten final ones count the single self-multiple available when $a > 10$.

Step 3: Subtract only the common diagonal The set-difference formula now gives

$$|R_1 - R_2| = |R_1| - |R_1 \cap R_2| = 66 - 20 = 46.$$

A direct cross-check is $\sum_{a=1}^{20}(\lfloor 20/a \rfloor - 1)$, which removes one diagonal pair for each first coordinate and gives the same result. Although R_1 and R_2 have equal sizes, their set difference is not empty: for example $(1, 2)$ is in R_1 only.

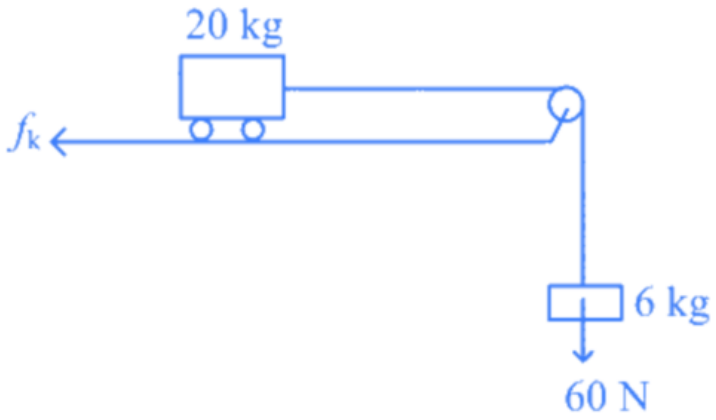
Step 4: Conclusion The number of ordered pairs in $R_1 - R_2$ is $\boxed{46}$, a dimensionless count.

Quick Tip to Solve: When positive integers divide each other, they must be equal. For this relation, remove one diagonal pair for each possible first coordinate.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Consider a block and trolley system as shown in figure. If the coefficient of kinetic friction between the trolley and the surface is 0.04 , the acceleration of the system in ms^{-2} is :
(Consider that the string is massless and unstretchable and the pulley is also massless and frictionless) :

[4 marks]



(A) 1.2

(B) 4

(C) 3

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- A massless unstretchable string over a frictionless massless pulley gives equal tension on both sides and equal acceleration magnitudes for the connected bodies.
- Kinetic friction is $f_k = \mu_k N$. For the trolley the vertical forces balance, giving $N = Mg$. Adding the two equations of motion eliminates the internal tension.

Step 1: Read the masses and calculate friction The figure gives trolley mass $M = 20\,\mathrm{kg}$, hanging mass $m = 6\,\mathrm{kg}$, and downward hanging weight $60\,\mathrm{N}$. Thus the indicated gravitational acceleration is $g = 60/6 = 10\,\mathrm{m\,s^{-2}}$. The trolley has no vertical acceleration, and the string force on it is horizontal. Hence

$$N = 20(10) = 200\,\mathrm{N}, \qquad f_k = 0.04(200) = 8\,\mathrm{N}.$$

Friction opposes the trolley motion toward the pulley.

Step 2: Write and add the two force equations Take the trolley acceleration to the right and the hanging acceleration downward as positive. If the common tension is T, Newton's second law gives

$$T - 8 = 20a, \qquad 60 - T = 6a.$$

Both bodies accelerate; treating the hanging body as if it were static would incorrectly set the tension equal to its weight. Adding the equations eliminates T:

$$60 - 8 = (20 + 6)a.$$

The external driving force is therefore $52\,\mathrm{N}$ acting on total inertial mass $26\,\mathrm{kg}$.

Step 3: Evaluate and check the options The acceleration is

$$a = \frac{52}{26} = 2\,\mathrm{m\,s^{-2}}.$$

As a consistency check, $T = 20(2) + 8 = 48\,\mathrm{N}$, and the hanging equation gives $60 - 48 = 6(2)$. Both equations are satisfied. Options (A), (B), and (C), giving 1.2, 4, and 3, respectively, fail the combined-force equation; the listed value 2 is option (D).

Step 4: Conclusion The acceleration is $\boxed{2\,\mathrm{m\,s^{-2}}}$, with the trolley moving toward the pulley and the hanging mass downward. Hence, the correct option is (D).

Quick Tip to Solve: Eliminate string tension by treating both bodies as one system: subtract trolley friction from the hanging weight and divide by the combined mass.

32. If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2\ \mathrm{m/s^2}$, then the length of the second's pendulum at a height $\mathrm{h} = 2R$ from the surface of earth will be, : [4 marks]

(A) $\frac{1}{9}\ \mathrm{m}$

(B) $\frac{8}{9}\ \mathrm{m}$

(C) $\frac{2}{9}\ \mathrm{m}$

(D) $\frac{4}{9}\ \mathrm{m}$

Correct Answer: (A) $\frac{1}{9}\ \mathrm{m}$

Solution:

Concept:

- At radial distance r from Earth's centre, gravitational acceleration is GM/r^2, where M is Earth's mass. Height above the surface must first be added to the radius.
- For small oscillations of a simple pendulum, $T = 2\pi\sqrt{\ell/g_h}$. A seconds pendulum has period $T = 2\,\mathrm{s}$, so each half-oscillation takes one second.

Step 1: Find gravity at the stated height The height is measured from the surface, so $h = 2R$ corresponds to radial distance $r = R + h = 3R$, not $2R$. Relating the local gravitational acceleration to the surface value gives

$$g_h = g\left(\frac{R}{R+h}\right)^2 = \pi^2\left(\frac{R}{3R}\right)^2 = \frac{\pi^2}{9}\,\mathrm{m\,s^{-2}}.$$

The mass of the pendulum bob does not enter this relation or its oscillation period.

Step 2: Solve the period formula for length Squaring the period equation and rearranging gives

$$T^2 = \frac{4\pi^2 \ell}{g_h}, \qquad \ell = \frac{g_h T^2}{4\pi^2}.$$

Use the local value of gravity because the pendulum operates at the specified height. A seconds pendulum has a complete period of two seconds; using one second here would reduce the computed length by a factor of four. Substituting the stated period gives

$$\ell = \frac{(\pi^2/9)(2)^2}{4\pi^2} = \frac{1}{9}\,\mathrm{m}.$$

Step 3: Check the length and eliminate alternatives At the surface, the same period with $g = \pi^2\,\mathrm{m\,s^{-2}}$ would require length $1\,\mathrm{m}$. Since gravity at the new location is one ninth of its surface value, length must also be one ninth to keep ℓ/g, and hence the period, unchanged. This independently confirms $1/9\,\mathrm{m}$. The values $8/9$, $2/9$, and $4/9\,\mathrm{m}$ do not maintain the required ratio and therefore rule out options (B), (C), and (D).

Step 4: Conclusion The seconds pendulum must have length $\boxed{\frac{1}{9}\,\mathrm{m}}$. Hence, the correct option is (A).

Quick Tip to Solve: For an unchanged pendulum period, length is proportional to local gravity. Always convert surface height into distance from Earth's centre first.

33. 10 divisions on the main scale of a Vernier calliper coincide with 11 divisions on the Vernier scale. If each division on the main scale is of 5 units, the least count of the instrument is : [4 marks]

(A) $\frac{5}{11}$
(B) $\frac{10}{11}$
(C) $\frac{50}{11}$
(D) $\frac{1}{2}$

Correct Answer: (A) $\frac{5}{11}$

Solution:

Concept:

- The least count of a direct vernier is the magnitude of the difference between one main-scale division and one vernier-scale division, expressed in the same units.
- A scale-coincidence statement compares lengths: if 10 main-scale divisions equal 11 vernier-scale divisions, divide their common length by 11 to obtain one vernier division.

Step 1: Convert the coincidence into a scale length Let one main-scale division have length s and one vernier-scale division have length v, both expressed in the given measurement units. The problem states

$$10s = 11v.$$

Since $s = 5$ units, the common span of the stated divisions is $10(5) = 50$ units. Therefore

$$v = \frac{50}{11} \text{ units.}$$

One vernier division is slightly shorter than one main-scale division, as expected because more vernier divisions cover the same length.

Step 2: Calculate the smallest resolvable difference As successive vernier marks are compared with main-scale marks, the mismatch changes by the difference between the division sizes. Hence

$$\text{LC} = s - v = 5 - \frac{50}{11} = \frac{55 - 50}{11} = \frac{5}{11} \text{ units.}$$

Equivalently, $v = (10/11)s$, so $\text{LC} = s/11$. The denominator is the number of vernier divisions in the stated coincidence, not the number of main-scale divisions.

Step 3: Distinguish the alternative quantities Option (C), $50/11$, is one vernier-scale division, which was only an intermediate quantity. Option (D), $1/2$, would arise from dividing the main-scale division by ten instead of eleven and does not follow the printed coincidence. Option (B), $10/11$, is twice the calculated least count and likewise fails the scale-difference equation. Since the instrument resolves the difference of division lengths, only option (A) has the required value.

Step 4: Conclusion The least count is $\boxed{\frac{5}{11} \text{ units}}$. Hence, the correct option is (A).

Quick Tip to Solve: From n main divisions equal to $n + 1$ vernier divisions, immediately use least count $= 1$ main division divided by $n + 1$.

34. Two moles a monoatomic gas is mixed with six moles of a diatomic gas. The molar specific heat of the mixture at constant volume is : [4 marks]

(A) $\frac{3}{2}$R

(B) $\frac{7}{4}$R

(C) $\frac{5}{2}$R

(D) $\frac{9}{4}$R

Correct Answer: (D) $\frac{9}{4}$R

Solution:

Concept:

- For ideal gases at a common temperature, internal energies and constant-volume heat capacities add. The molar heat capacity of a mixture is the total heat capacity divided by the total mole number.
- Equipartition gives $C_V = 3R/2$ for a monoatomic gas and $C_V = 5R/2$ for a diatomic gas with active translation and rotation and negligible vibration.

Step 1: Write the heat absorbed by both gases Let the mixture undergo a small common temperature increase ΔT at constant volume. The monoatomic component contains $n_1 = 2$ moles, and the

diatomic component contains $n_2 = 6$ moles. Their heat inputs are respectively

$$Q_1 = 2\left(\frac{3}{2}R\right)\Delta T = 3R\Delta T,$$

$$Q_2 = 6\left(\frac{5}{2}R\right)\Delta T = 15R\Delta T.$$

Translation and rotation contribute; vibration is neglected.

Step 2: Divide the total capacity by total moles The total heat input is $Q_1 + Q_2 = 18R\Delta T$. There are $2 + 6 = 8$ moles in the mixture. By the definition of its molar constant-volume specific heat,

$$Q_1 + Q_2 = (n_1 + n_2)C_{V,\text{mix}}\Delta T.$$

Consequently

$$C_{V,\text{mix}} = \frac{2(3R/2) + 6(5R/2)}{8} = \frac{18R}{8} = \frac{9}{4}R.$$

Use mole weights because the requested capacity is per mole.

Step 3: Check the weighting and compare the options The mixture value must lie between $3R/2$ and $5R/2$, and must lie closer to $5R/2$ because three quarters of the moles are diatomic. The result $9R/4 = 2.25R$ satisfies both checks. Options (A) and (C) are the pure-component values and ignore one constituent. Option (B), $7R/4 = 1.75R$, weights the monoatomic component too strongly and contradicts the six-to-two mole ratio. Only option (D) gives the mole-weighted result.

Step 4: Conclusion The molar specific heat at constant volume is $\boxed{\frac{9}{4}R}$, in $\text{J mol}^{-1}\,\text{K}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: For an ideal-gas mixture, average molar heat capacities using mole fractions; never add the component molar values without dividing by total moles.

35. The dimensional formula of angular impulse is : [4 marks]

(A) $\left[\text{ML}^2\ \text{T}^{-2}\right]$
(B) $\left[\text{ML}^{-2}\ \text{T}^{-1}\right]$
(C) $\left[\text{ML}^2\ \text{T}^{-1}\right]$
(D) $\left[\text{MLT}^{-1}\right]$

Correct Answer: (C) $\left[\text{ML}^2\ \text{T}^{-1}\right]$

Solution:

Concept:

- Angular impulse is the time integral of torque and equals the change in angular momentum: $J_{\text{ang}} = \int \tau\, dt = \Delta L$.
- Torque is the cross product of position and force. Its dimension is lever-arm length multiplied by force dimension; dimensions do not depend on the direction or angle of the vectors.

Step 1: Determine the dimensions of torque Force follows from Newton's second law, $F = ma$, so

$$[F] = [\mathrm{MLT}^{-2}].$$

Torque about a chosen origin is $\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$, where $\boldsymbol{r}$ is the position from that origin. The sine of the angle between the vectors is dimensionless. Therefore

$$[\tau] = [r][F] = [\mathrm{L}][\mathrm{MLT}^{-2}] = [\mathrm{ML}^2\mathrm{T}^{-2}].$$

This is a torque dimension, not yet an angular-impulse dimension.

Step 2: Include the duration of the torque Integrating torque over time adds one power of time to its dimension. Thus

$$[J_{\mathrm{ang}}] = [\tau][t] = [\mathrm{ML}^2\mathrm{T}^{-2}][\mathrm{T}] = [\mathrm{ML}^2\mathrm{T}^{-1}].$$

In SI units angular impulse can be written $\mathrm{N\,m\,s}$, equivalently $\mathrm{kg\,m^2\,s^{-1}}$. This result also applies to a time-varying torque, since integration over time carries the same dimensional factor as multiplication by a time interval.

Step 3: Verify through angular momentum and exclude distractors Independently, angular momentum is $\boldsymbol{L} = \boldsymbol{r} \times m\boldsymbol{v}$. Its dimensions are

$$[L] = [\mathrm{L}][\mathrm{M}][\mathrm{LT}^{-1}] = [\mathrm{ML}^2\mathrm{T}^{-1}],$$

agreeing with the impulse calculation. Option (A) omits the time integration and represents torque. Option (B) has an inverse-square length factor, inconsistent with a lever arm times linear momentum. Option (D) represents linear impulse, which lacks the additional lever-arm length. Hence only option (C) is consistent.

Step 4: Conclusion The dimensional formula is $\boxed{[\mathrm{ML}^2\mathrm{T}^{-1}]}$. Hence, the correct option is (C).

Quick Tip to Solve: Angular impulse changes angular momentum; obtain its dimensions by multiplying linear-momentum dimensions by one lever-arm length.

36. A monochromatic light of wavelength $6000\ \mathring{A}$ is incident on the single slit of width 0.01 mm. If the diffraction pattern is formed at the focus of the convex lens of focal length 20 cm, the linear width of the central maximum is : [4 marks]

(A) 12 mm

(B) 24 mm

(C) 60 mm

(D) 120 mm

Correct Answer: (B) 24 mm

Solution:

Concept:

- For a slit of width a, the first diffraction minima obey $a\sin\theta = \lambda$, where λ is the wavelength.
- The central maximum extends between the first minima on both sides. In the focal plane of a lens its paraxial linear width is $W = 2f\lambda/a$, where f is the focal length.

Step 1: Convert every length to SI units The wavelength, slit width and focal length must use the same length unit before substitution. Using $1\,\mathring{A} = 10^{-10}\,\text{m}$ gives

$$\lambda = 6000 \times 10^{-10} = 6.0 \times 10^{-7}\,\text{m}, \qquad a = 0.01 \times 10^{-3} = 1.0 \times 10^{-5}\,\text{m}.$$

The lens focal length is $f = 20\,\text{cm} = 0.20\,\text{m}$. The lens forms the angular diffraction pattern at its focus, so its focal length takes the place of the screen distance.

Step 2: Find the separation of the first minima Each first minimum is a distance $y_1 = f\lambda/a$ from the central axis. Thus

$$y_1 = \frac{0.20 \times 6.0 \times 10^{-7}}{1.0 \times 10^{-5}} = 0.012\,\text{m} = 12\,\text{mm}.$$

The required central maximum contains both halves, not just the distance from the centre to one minimum:

$$W = 2y_1 = 24\,\text{mm}.$$

Option (A) is only the half-width. Options (C) and (D) are respectively 2.5 and 5 times the calculated full width, so neither follows from the first-minimum condition.

Step 3: Conclusion The linear width of the central maximum is $\boxed{24\,\text{mm}}$. Hence, the correct option is (B).

Quick Tip to Solve: First calculate the centre-to-first-minimum distance, then double it. In a lens focal plane use f, and convert angstroms and millimetres before dividing.

37. A ball of mass $0.5\ \text{kg}$ is attached to a string of length $50\ \text{cm}$. The ball is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is $400\ \text{N}$. The maximum possible value of angular velocity of the ball in rad/s is, : [4 marks]

(A) 1600

(B) 20

(C) 40

(D) 1000

Correct Answer: (C) 40

Solution:

Concept:

- A mass moving in a horizontal circle requires inward force $m\omega^2 r$, where m is its mass, r its circular-path radius and ω its angular velocity.
- For a string of length l making angle θ with the vertical, $r = l\sin\theta$. Its horizontal tension component is $T\sin\theta$, giving $T = m\omega^2 l$ for a nonzero circle.

Step 1: Obtain the tension and angular-speed relation The relevant geometric length is the string length $l = 50\,\text{cm} = 0.50\,\text{m}$. For a conical rotation about the vertical axis, equate the inward component of tension to the centripetal force:

$$T\sin\theta = m\omega^2(l\sin\theta).$$

Cancelling $\sin\theta$ yields $T = m\omega^2 l$. Thus the result does not require assuming that the inclined string has the same length as the horizontal radius. The vertical component balances weight and fixes the angle, while the horizontal equation fixes the speed-tension relation.

Step 2: Apply the breaking-tension limit Because tension grows as ω^2, the largest allowed angular speed occurs at $T = T_{\text{max}} = 400\,\text{N}$. Therefore

$$\omega_{\text{max}}^2 = \frac{T_{\text{max}}}{ml} = \frac{400}{0.5 \times 0.50} = 1600\,\text{s}^{-2},$$

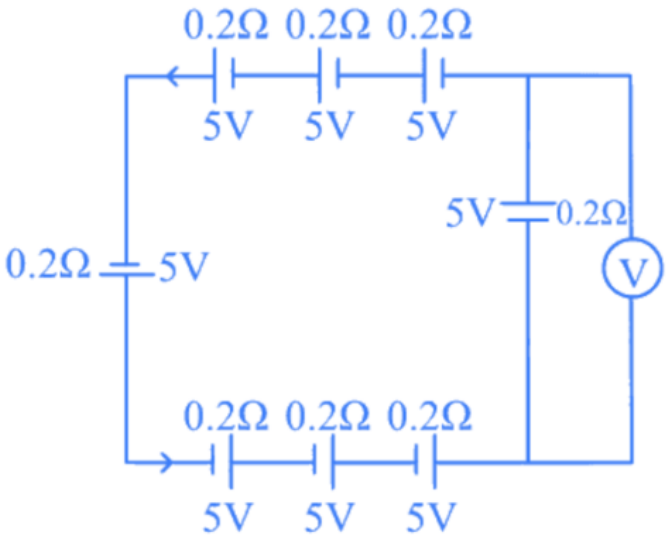
$$\omega_{\text{max}} = \sqrt{1600} = 40\,\text{rad}\,\text{s}^{-1}.$$

The value 1600 in option (A) is the squared angular speed, not the angular speed. At $20\,\text{rad}\,\text{s}^{-1}$, the tension would be only $100\,\text{N}$, so option (B) is possible but not the maximum. At $1000\,\text{rad}\,\text{s}^{-1}$, the required tension far exceeds the stated limit.

Step 3: Conclusion The maximum possible angular velocity is $\boxed{40\,\text{rad}\,\text{s}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a conical pendulum, combine $r = l\sin\theta$ with the horizontal tension equation: the angle cancels. Remember to take the square root of $T/(ml)$.

38. The reading in the ideal voltmeter (V) shown in the given circuit diagram is : [4 marks]



(A) 3 V

(B) 10 V

(C) 5 V

(D) 0 V

Correct Answer: (D) 0 V

Solution:

Concept:

- An ideal voltmeter has infinite resistance, so it measures a potential difference without drawing current. The current in the cell loop is determined by the cells and their internal resistances.
- For a discharging cell of emf E, internal resistance r and current I, its terminal potential difference is $V = E - Ir$. Kirchhoff's loop law gives $I = E_{\text{net}}/r_{\text{total}}$.

Step 1: Identify the active loop and cell polarities The voltmeter is connected between the top-right and bottom-right nodes, in parallel with the right-hand cell. Its own branch is open to current, but the eight-cell outer loop is closed. Reading the long plate as the positive terminal, traverse the loop in the indicated direction: leftwards along the top, down the left side, rightwards along the bottom, and upwards through the right cell. Every cell is crossed from its negative to its positive terminal. Thus all eight emfs aid one another; the right-hand cell is delivering current, rather than being charged.

Step 2: Calculate the loop current Each of the eight cells has emf $5\,\text{V}$ and internal resistance $0.2\,\Omega$. Their total emf and resistance are therefore

$$E_{\text{net}} = 8 \times 5 = 40\,\text{V}, \qquad r_{\text{total}} = 8 \times 0.2 = 1.6\,\Omega.$$

Consequently, $I = 40/1.6 = 25\,\text{A}$. No extra resistance is added by the ideal voltmeter because no current passes through it.

Step 3: Find the potential difference measured by the meter The terminal voltage of the right-hand

cell is

$$V = E - Ir = 5 - (25)(0.2) = 0\,\text{V}.$$

Its internal drop exactly equals its emf. A zero meter reading does not mean zero loop current. Options (A) and (B) do not satisfy this terminal-voltage equation. Option (C) would be the emf of an unloaded cell, whereas this cell carries the loop current.

Step 4: Conclusion The ideal voltmeter reads $\boxed{0\,\text{V}}$. Hence, the correct option is (D).

Quick Tip to Solve: Treat the ideal voltmeter branch as open, then calculate the current through the cell it spans. A loaded cell's terminal voltage is $E - Ir$, not automatically its emf.

39. A galvanometer has a resistance of $50\ \Omega$ and it allows maximum current of $5\ \text{mA}$. It can be converted into voltmeter to measure upto $100\ \text{V}$ by connecting in series a resistor of resistance :

[4 marks]

(A) 19500Ω

(B) 5975Ω

(C) 20050Ω

(D) 19950Ω

Correct Answer: (D) 19950Ω

Solution:

Concept:

- A galvanometer reaches full-scale deflection at its limiting current I_g. A voltmeter is formed by adding a large multiplier resistance R in series with its resistance G.
- At the desired full-scale voltage V, the series current must be I_g, so $V = I_g(R + G)$ and $R = V/I_g - G$.

Step 1: Calculate the total full-scale resistance The stated current is $I_g = 5\,\text{mA} = 5 \times 10^{-3}\,\text{A}$, not $5\,\text{A}$. The maximum voltage to be measured is $100\,\text{V}$. The total resistance required to restrict the current to the galvanometer limit is therefore

$$R_{\text{total}} = \frac{V}{I_g} = \frac{100}{5 \times 10^{-3}} = 20000\,\Omega.$$

This total includes both the galvanometer coil and the added resistor. A parallel shunt would be used to extend a current range; it would not perform the series voltage-range conversion required here.

Step 2: Subtract the resistance already present The galvanometer already contributes $G = 50\,\Omega$, so the additional series resistance is

$$R = R_{\text{total}} - G = 20000 - 50 = 19950\,\Omega.$$

As a check, at full scale the coil drop is $I_g G = 0.005 \times 50 = 0.25\,\text{V}$. The multiplier drops the remaining $99.75\,\text{V}$, giving $99.75/0.005 = 19950\,\Omega$. Option (C) adds the coil resistance instead of subtracting it. Options (A) and (B), when combined with the coil, give total resistances below $20000\,\Omega$, so they

would allow more than the limiting current at $100\,\mathrm{V}$.

Step 3: Conclusion A resistance of $\boxed{19950\,\Omega}$ must be connected in series. Hence, the correct option is (D).

Quick Tip to Solve: Compute the total resistance V/I_g first, then subtract the galvanometer resistance. Convert milliamperes to amperes before using Ohm's law.

40. The minimum energy required by a hydrogen atom in ground state to emit radiation in Balmer series is nearly : [4 marks]

(A) $13.6\mathrm{eV}$

(B) $1.5\mathrm{eV}$

(C) $12.1\mathrm{eV}$

(D) $1.9\mathrm{eV}$

Correct Answer: (C) $12.1\mathrm{eV}$

Solution:

Concept:

- The hydrogen energy levels are $E_n = -13.6/n^2\,\mathrm{eV}$, measured relative to the ionisation limit.
- Balmer photons arise from downward transitions ending at $n = 2$, with the initial level $n \geq 3$. An atom initially at $n = 1$ must first absorb enough energy to reach an appropriate emitting level.

Step 1: Choose the lowest level that can emit a Balmer photon The ground state is $n = 1$. Excitation merely to $n = 2$ does not allow Balmer emission because a downward transition from $n = 2$ ends at $n = 1$, in the Lyman series. The lowest excited state capable of a Balmer transition is $n = 3$, since the transition $3 \to 2$ belongs to the Balmer series. Therefore the minimum input is the excitation energy for $1 \to 3$, rather than the energy of the emitted Balmer photon itself.

Step 2: Calculate the minimum excitation energy The relevant levels are

$$E_1 = -13.6\,\mathrm{eV}, \qquad E_3 = -\frac{13.6}{9}\,\mathrm{eV}.$$

The energy that must be supplied is the positive difference

$$\Delta E = E_3 - E_1 = 13.6\left(1 - \frac{1}{9}\right) = 13.6 \times \frac{8}{9} = 12.088\ldots\,\mathrm{eV}.$$

Thus the nearest listed value is $12.1\,\mathrm{eV}$. The $13.6\,\mathrm{eV}$ in option (A) is the ground-state ionisation energy, which is larger than necessary. The $1.5\,\mathrm{eV}$ in option (B) is approximately the binding energy at $n = 3$. The $1.9\,\mathrm{eV}$ in option (D) is approximately the emitted $3 \to 2$ photon energy, not the energy needed to excite a ground-state atom.

Step 3: Conclusion The minimum energy required is nearly $\boxed{12.1\,\mathrm{eV}}$. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish excitation energy from emitted photon energy: first raise the ground-state atom to $n = 3$, then its $3 \to 2$ transition can produce a Balmer photon.

41. A parallel plate capacitor has a capacitance $C = 200\ \text{pF}$. It is connected to $230\ \text{V}$ ac supply with an angular frequency $300\ \text{rad/s}$. The rms value of conduction current in the circuit and displacement current in the capacitor respectively are : [4 marks]

(A) $14.3\ \mu\text{A}$ and $143\ \mu\text{A}$

(B) $13.8\ \mu\text{A}$ and $13.8\ \mu\text{A}$

(C) $13.8\ \mu\text{A}$ and $138\ \mu\text{A}$

(D) $1.38\ \mu\text{A}$ and $1.38\ \mu\text{A}$

Correct Answer: (B) $13.8\ \mu\text{A}$ and $13.8\ \mu\text{A}$

Solution:

Concept:

- For a capacitor on a sinusoidal supply, $i = C\,dv/dt$. Its rms current satisfies $I_{\text{rms}} = \omega C V_{\text{rms}}$, where ω is angular frequency.
- The displacement current in the gap is $I_d = dQ/dt$, equal to the conduction current in the leads at each instant. Their rms values are consequently equal.

Step 1: Convert the capacitance and identify the voltage convention The capacitance is

$$C = 200\,\text{pF} = 200 \times 10^{-12}\,\text{F} = 2.00 \times 10^{-10}\,\text{F}.$$

The quoted ac supply voltage $230\,\text{V}$ is its rms value, and the angular frequency is already $\omega = 300\,\text{rad}\,\text{s}^{-1}$. There is no need to multiply this frequency by another factor of 2π. Although differentiation introduces a factor of ω into the peak current, conversion of both peak quantities to rms leaves the same relation $I_{\text{rms}} = \omega C V_{\text{rms}}$.

Step 2: Calculate the conduction-current magnitude Substitute the given values:

$$I_{\text{rms}} = 300 \times (200 \times 10^{-12}) \times 230 = 1.38 \times 10^{-5}\,\text{A} = 13.8\,\mu\text{A}.$$

This is the conduction current carried by charges in the connecting wires. It changes the charge on the capacitor plates without requiring those charges to pass across the dielectric gap.

Step 3: Use current continuity across the capacitor The changing electric flux between the plates supplies an equal displacement current. Thus $I_{d,\text{rms}} = 13.8\,\mu\text{A}$ as well. Options (A) and (C) give unequal conduction and displacement currents, contrary to continuity for this capacitor. Option (D) gives equal currents but is too small by a factor of ten, as the direct substitution shows.

Step 4: Conclusion Both rms currents are $\boxed{13.8\,\mu\text{A}}$. Hence, the correct option is (B).

Quick Tip to Solve: When angular frequency is given, use $I_{\text{rms}} = \omega C V_{\text{rms}}$ directly. The displacement current between the plates equals the conduction current in the leads.

42. The de Broglie wavelengths of a proton and an α particle are λ and 2λ respectively. The ratio of the velocities of proton and α particle will be : [4 marks]

(A) $8 : 1$

(B) $1 : 2$

(C) $1 : 8$

(D) $4 : 1$

Correct Answer: (A) $8 : 1$

Solution:

Concept:

- The nonrelativistic de Broglie relation is $\lambda = h/p = h/(mv)$, where h is Planck's constant, m is particle mass and v is speed.
- An alpha particle has approximately four times the proton mass: $m_\alpha \simeq 4m_p$. At given wavelengths, velocity is inversely proportional to both mass and wavelength.

Step 1: Write the two momentum relations The proton wavelength is $\lambda_p = \lambda$, whereas the alpha-particle wavelength is $\lambda_\alpha = 2\lambda$. Therefore

$$m_p v_p = \frac{h}{\lambda}, \qquad m_\alpha v_\alpha = \frac{h}{2\lambda}.$$

The proton momentum is twice the alpha-particle momentum. This comparison alone is not the speed ratio, because the alpha particle is heavier. Equal or simply inverse wavelength ratios for speeds would be valid only for equal particle masses.

Step 2: Include the alpha-particle mass Dividing the momentum relations gives

$$\frac{m_p v_p}{m_\alpha v_\alpha} = 2.$$

Consequently,

$$\frac{v_p}{v_\alpha} = 2\frac{m_\alpha}{m_p} = 2 \times 4 = 8.$$

Equivalently, dividing $h/(m_p\lambda)$ by $h/(4m_p \cdot 2\lambda)$ gives the same factor of eight. Option (B) ignores the mass difference and reverses the relevant comparison. Option (C) reverses the requested proton-to-alpha speed ratio. Option (D) accounts for the approximate mass ratio but misses the additional factor of two due to the different wavelengths. Only option (A) satisfies both the de Broglie relation and the stated mass and wavelength ratios.

Step 3: Conclusion The velocity ratio is $\boxed{v_p : v_\alpha = 8 : 1}$. Hence, the correct option is (A).

Quick Tip to Solve: For two different particles, use $v_1/v_2 = m_2\lambda_2/(m_1\lambda_1)$. Check the particle masses before interpreting a wavelength ratio as a speed ratio.

43. The radius (r), length (l) and resistance (R) of a metal wire was measured in the laboratory as

$$\mathrm{r} = (0.35 \pm 0.05)\ \mathrm{cm}$$

$$\mathrm{R} = (100 \pm 10)\ \mathrm{ohm}$$

$$l = (15 \pm 0.2)\ \mathrm{cm}$$

The percentage error in resistivity of the material of the wire is : [4 marks]

(A) 37.3%

(B) 25.6%

(C) 35.6%

(D) 39.9%

Correct Answer: (D) 39.9%

Solution:

Concept:

- Resistance and resistivity are related by $R = \rho l/A$. For a circular wire, $A = \pi r^2$, so $\rho = R\pi r^2/l$.
- For maximum fractional uncertainty in a product or quotient, add the magnitudes of the fractional uncertainties, multiplied by their powers. The constant π contributes no measurement uncertainty.

Step 1: Express the required fractional error The radius occurs squared, so its fractional error must be counted twice. Although length appears in the denominator, its uncertainty adds positively when finding the maximum error:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} + 2\frac{\Delta r}{r} + \frac{\Delta l}{l}.$$

Here the symbols $\Delta R, \Delta r, \Delta l$ denote positive absolute uncertainties, not signed changes in the measurements.

Step 2: Substitute all three measured uncertainties The individual percentage contributions are

$$100\frac{\Delta R}{R} = 100\frac{10}{100} = 10\%,$$
$$200\frac{\Delta r}{r} = 200\frac{0.05}{0.35} = 28.5714\%,$$
$$100\frac{\Delta l}{l} = 100\frac{0.2}{15} = 1.3333\%.$$

Thus the required percentage error is $10 + 28.5714 + 1.3333 = 39.9047\%$.

Step 3: Compare with the printed choices To one decimal place this is 39.9%. Options (A), (B) and (C) all underestimate the sum. In particular, counting the radius uncertainty only once would miss its squared dependence in the area; subtracting the length uncertainty is also incorrect for a maximum-error calculation.

Step 4: Conclusion The percentage error in resistivity is 39.9%. Hence, the correct option is (D).

Quick Tip to Solve: Write resistivity in terms of the measured quantities before adding errors; a measured radius contributes twice because the area is proportional to r^2.

44. Two identical capacitors have same capacitance C. One of them is charged to the potential V and other to the potential 2 V. The negative ends of both are connected together. When the positive ends are also joined together, the decrease in energy of the combined system is : [4 marks]

(A) $\frac{1}{4}CV^2$

(B) $\frac{3}{4}CV^2$

(C) $\frac{1}{2}CV^2$

(D) $2CV^2$

Correct Answer: (A) $\frac{1}{4}CV^2$

Solution:

Concept:

- A capacitor of capacitance C at potential difference V has charge $Q = CV$ and energy $U = \frac{1}{2}CV^2$.
- Joining like plates conserves their total charge. The final capacitors are in parallel and therefore have the same potential difference; electric field energy need not be conserved during redistribution.

Step 1: Calculate the initial charge and energy The positive plates initially carry charges CV and $2CV$, since their voltages have the same polarity. Their combined positive charge is therefore $3CV$. The initial stored energy is

$$U_i = \frac{1}{2}CV^2 + \frac{1}{2}C(2V)^2 = \frac{5}{2}CV^2.$$

The second capacitor stores four times the first capacitor's energy, rather than twice its energy, because energy depends on the square of voltage.

Step 2: Find the common voltage after connection Once both pairs of like plates are joined, the equivalent capacitance across the common terminals is $2C$. Charge conservation gives

$$V_f = \frac{CV + 2CV}{C + C} = \frac{3}{2}V.$$

Consequently the final total energy stored in the two capacitors is

$$U_f = \frac{1}{2}(2C)\left(\frac{3}{2}V\right)^2 = \frac{9}{4}CV^2.$$

Step 3: Subtract the final energy from the initial energy The decrease is a positive quantity:

$$\Delta U = U_i - U_f = \left(\frac{10}{4} - \frac{9}{4}\right)CV^2 = \frac{1}{4}CV^2.$$

The missing electric field energy is dissipated during the transient current. Options (B), (C) and (D) do not equal this difference. Conserving energy to infer the common voltage would overlook this dissipation; charge conservation is the appropriate constraint.

Step 4: Conclusion The energy decreases by $\frac{1}{4}CV^2$. Hence, the correct option is (A).

Quick Tip to Solve: For two capacitors joined with like plates together, first find the common voltage from charge conservation; then subtract the two stored energies.

45. A simple pendulum of length $1\ \mathrm{m}$ has a wooden bob of mass $1\ \mathrm{kg}$. It is struck by a bullet of mass $10^{-2}\ \mathrm{kg}$ moving with a speed of $2 \times 10^2\ \mathrm{ms}^{-1}$. The bullet gets embedded into the bob. The height to which the bob rises before swinging back is. (use $\mathrm{g} = 10\ \mathrm{m/s}^2$) [4 marks]

(A) $0.20\ \mathrm{m}$

(B) $0.40\ \mathrm{m}$

(C) $0.30\ \mathrm{m}$

(D) $0.35\ \mathrm{m}$

Correct Answer: (A) $0.20\ \mathrm{m}$

Solution:

Concept:

- In a perfectly inelastic collision, the bullet and bob stick together. Horizontal linear momentum is conserved during the brief impact, whereas kinetic energy is not.
- After impact, the combined mass swings upward. Neglecting resistance, its kinetic energy immediately after impact converts into gravitational potential energy at the highest point.

Step 1: Find the common speed immediately after impact Let the bullet mass be $m = 0.01\,\mathrm{kg}$, the bob mass be $M = 1\,\mathrm{kg}$, and the incoming speed be $u = 200\,\mathrm{m\,s^{-1}}$. The bob is initially at rest. The short impact has negligible external horizontal impulse, so

$$mu = (M + m)v.$$

It follows that

$$v = \frac{0.01 \times 200}{1.01} = \frac{200}{101}\,\mathrm{m\,s^{-1}}.$$

Step 2: Apply energy conservation to the subsequent swing The bullet remains embedded during the ascent. At the turning point the speed is zero, so the loss of kinetic energy equals the gain in gravitational potential energy:

$$\frac{1}{2}(M + m)v^2 = (M + m)gh.$$

The common mass cancels, giving

$$h = \frac{v^2}{2g} = \frac{(200/101)^2}{20} = \frac{2000}{10201}\,\mathrm{m} \simeq 0.196\,\mathrm{m}.$$

Step 3: Choose the height at the precision of the options The height is approximately $0.20\,\mathrm{m}$, which is smaller than the pendulum length and is therefore physically possible. The other choices, 0.40, 0.30 and $0.35\,\mathrm{m}$, exceed the value allowed by the post-impact kinetic energy. Applying conservation of the bullet's initial kinetic energy through the collision would incorrectly count energy dissipated in embedding the bullet as energy available for the swing.

Step 4: Conclusion The bob rises through approximately $0.20\,\mathrm{m}$. Hence, the correct option is (A).

Quick Tip to Solve: Separate a ballistic-pendulum problem into two stages: conserve momentum during embedding, then conserve mechanical energy during the upward swing.

46. The pressure and volume of an ideal gas are related as $\mathrm{PV}^{\frac{3}{2}} = \mathrm{K}$ (Constant). The work done when the gas is taken from state $A\,(P_1, V_1, T_1)$ to state $B\,(P_2, V_2, T_2)$ is : [4 marks]

(A) $2\left(\mathrm{P_2}\sqrt{\mathrm{V_2}} - \mathrm{P_1}\sqrt{\mathrm{V_1}}\right)$

(B) $2\left(\sqrt{\mathrm{P_1}}\ \mathrm{V_1} - \sqrt{\mathrm{P_2}}\ \mathrm{V_2}\right)$

(C) $2\left(\mathrm{P_2}\ \mathrm{V_2} - \mathrm{P_1}\ \mathrm{V_1}\right)$

(D) $2\left(\mathrm{P_1}\ \mathrm{V_1} - \mathrm{P_2}\ \mathrm{V_2}\right)$

Correct Answer: (D) $2\left(\mathrm{P_1}\ \mathrm{V_1} - \mathrm{P_2}\ \mathrm{V_2}\right)$

Solution:

Concept:

- For a quasistatic change of volume, the work done by a gas is $W = \int_{V_1}^{V_2} P\,dV$. The pressure must be taken from the specified process equation.
- The given path satisfies $PV^{3/2} = K$, hence $P = KV^{-3/2}$. The constant K has the same value at both endpoints.

Step 1: Replace pressure using the path equation The pressure is not constant during this change, so $P\Delta V$ cannot be used with either endpoint pressure. Substitute the given volume dependence into the work integral:

$$W = K\int_{V_1}^{V_2} V^{-3/2}\,dV.$$

The antiderivative of $V^{-3/2}$ is $-2V^{-1/2}$. Its derivative is $V^{-3/2}$, which checks both the factor and the sign.

Step 2: Evaluate at the initial and final states Using the integration limits in their stated order gives

$$\begin{aligned} W &= \left[-2KV^{-1/2}\right]_{V_1}^{V_2} \\ &= 2K\left(\frac{1}{\sqrt{V_1}} - \frac{1}{\sqrt{V_2}}\right). \end{aligned}$$

At the endpoints, $K = P_1V_1^{3/2} = P_2V_2^{3/2}$. Therefore $K/\sqrt{V_1} = P_1V_1$ and $K/\sqrt{V_2} = P_2V_2$.

Step 3: Simplify and check the sign and dimensions Substitution yields

$$W = 2(P_1V_1 - P_2V_2).$$

For expansion, $V_2 > V_1$; the integral and this expression are both positive because $PV = K/\sqrt{V}$ decreases. Option (C) reverses the sign and represents the negative of the work done by the gas. Options (A) and (B) do not even have the dimensions of pressure times volume, so neither can be an energy.

Step 4: Conclusion The work done by the gas is $2(P_1V_1 - P_2V_2)$. Hence, the correct option is (D).

Quick Tip to Solve: For $PV^n = K$, integrate KV^{-n} and replace KV^{1-n} by PV; keep the initial and final limits in the correct order.

47. In series LCR circuit, the capacitance is changed from C to $4C$. To keep the resonance frequency unchanged, the new inductance should be: [4 marks]

(A) increased by 2 L

(B) reduced by $\frac{1}{4}$ L

(C) reduced by $\frac{3}{4}$ L

(D) increased to 4 L

Correct Answer: (C) reduced by $\frac{3}{4}$ L

Solution:

Concept:

- The resonance angular frequency of an ideal series LCR circuit is $\omega_0 = 1/\sqrt{LC}$, or $f_0 = 1/(2\pi\sqrt{LC})$.
- Resonance occurs when inductive and capacitive reactances cancel: $\omega_0 L = 1/(\omega_0 C)$. Keeping the resonance frequency unchanged therefore requires the product LC to remain constant.

Step 1: Relate the original and new circuit values Let the initial inductance be L, and write the new inductance as L'. The new capacitance is $C' = 4C$. Equality of the two resonance frequencies gives

$$\frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{L'(4C)}}.$$

Since all capacitances and inductances are positive, the products inside the square roots must be equal:

$$LC = 4L'C.$$

Step 2: Find both the final value and the reduction Cancel the common nonzero capacitance and solve:

$$L' = \frac{L}{4}.$$

The question describes the change in words, so distinguish this final value from the amount removed:

$$L - L' = L - \frac{L}{4} = \frac{3L}{4}.$$

Thus the inductance must be reduced by three quarters of its original value, leaving one quarter.

Step 3: Test the other proposed changes Increasing the inductance by $2L$ gives $L' = 3L$, so the product $L'C'$ becomes $12LC$. Reducing it by $L/4$ leaves $3L/4$, giving $3LC$. Increasing it to $4L$ gives $16LC$. None preserves LC. Only the reduction of $3L/4$ makes $L'C' = (L/4)(4C) = LC$, restoring the original frequency.

Step 4: Conclusion The new inductance is $L/4$, so it is reduced by $3L/4$. Hence, the correct option is (C).

Quick Tip to Solve: At fixed resonance frequency, inductance and capacitance vary inversely; distinguish a reduction by $3L/4$ from a final value of $L/4$.

48. A particle moving in a circle of radius R with uniform speed takes time T to complete one revolution.

If this particle is projected with the same speed at an angle θ to the horizontal, the maximum height attained by it is equal to $4R$. The angle of projection θ is then given by : [4 marks]

(A) $\sin^{-1}\left[\frac{2gT^2}{\pi^2R}\right]^{\frac{1}{2}}$

(B) $\sin^{-1}\left[\frac{\pi^2R}{2gT^2}\right]^{\frac{1}{2}}$

(C) $\cos^{-1}\left[\frac{\pi R}{2gT^2}\right]^{\frac{1}{2}}$

(D) $\cos^{-1}\left[\frac{2gT^2}{\pi^2R}\right]^{\frac{1}{2}}$

Correct Answer: (A) $\sin^{-1}\left[\frac{2gT^2}{\pi^2R}\right]^{\frac{1}{2}}$

Solution:

Concept:

- Uniform circular motion with radius R and period T has speed $u = 2\pi R/T$, since one complete revolution covers the circumference.
- For a projectile launched at angle θ to the horizontal, its initial vertical speed is $u\sin\theta$. Its maximum height above the launch point is $H = u^2\sin^2\theta/(2g)$, neglecting air resistance.

Step 1: Express the given speed in terms of the period The particle retains the magnitude of its circular-motion speed when it is projected:

$$u = \frac{2\pi R}{T}, \qquad u^2 = \frac{4\pi^2R^2}{T^2}.$$

Only its vertical component contributes to the rise against gravity. At the highest point that component becomes zero, while the horizontal component remains unchanged.

Step 2: Impose the required maximum height Use $H = 4R$ in the vertical-motion equation:

$$4R = \frac{4\pi^2R^2\sin^2\theta}{2gT^2}.$$

Cancel one positive factor R and rearrange:

$$\sin^2\theta = \frac{2gT^2}{\pi^2R}.$$

For an upward launch angle between 0 and $\pi/2$, the sine is nonnegative, so take the positive square root.

Step 3: Recover the angle and compare the alternatives The angle is

$$\theta = \sin^{-1}\left(\sqrt{\frac{2gT^2}{\pi^2R}}\right).$$

The assumed motion is possible only when the square-root argument is at most one. Option (B) uses

the reciprocal of the required quantity. Options (C) and (D) use an inverse cosine even though the height is controlled by the vertical, sine component; option (C) also has a different factor of π. These do not satisfy the derived height relation in general.

Step 4: Conclusion The projection angle is $\theta = \sin^{-1}\sqrt{2gT^2/(\pi^2 R)}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert the circular-motion period into speed first, then use only the vertical component in the maximum-height formula.

49. With rise in temperature, the Young's modulus of elasticity : [4 marks]

(A) changes erratically

(B) increases

(C) decreases

(D) remains unchanged

Correct Answer: (C) decreases

Solution:

Concept:

- Young's modulus is the ratio of longitudinal stress to longitudinal strain within the elastic limit: $Y = (F/A)/(\Delta l/l)$. It measures stiffness, rather than the amount of thermal expansion.
- The usual temperature dependence for a solid is a decrease in elastic stiffness as temperature rises. In the standard elastic model, stronger resistance of interatomic bonds to displacement gives a larger elastic modulus.

Step 1: Interpret the meaning of a larger modulus For a specimen under a given small longitudinal stress σ, the strain is

$$\epsilon = \frac{\sigma}{Y}.$$

A larger Y means less deformation under that stress and therefore a stiffer material. A smaller Y means the specimen is more easily deformed. This comparison concerns the elastic response about the equilibrium state at each temperature, not simply the change in its unstressed length.

Step 2: Apply the effect of heating on elastic stiffness With an increase in temperature, atoms vibrate more vigorously and their mean separation generally increases. For the usual interatomic potential, the effective restoring stiffness around the thermally shifted equilibrium decreases. Consequently a given applied stress produces a larger elastic strain, so the ratio of stress to strain, Young's modulus, decreases.

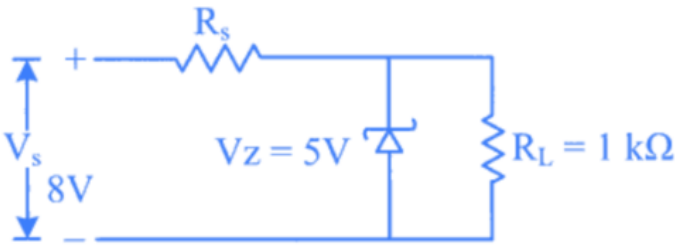
Step 3: Select the trend represented by the options The standard trend is a systematic softening of the solid with temperature, not an erratic change. Thus option (A) does not describe the expected behaviour. Option (B) would describe increased stiffness on heating and reverses the usual trend. Option (D) neglects the temperature dependence altogether. The decrease in option (C) is the qualitative result required; the question supplies no material-specific data for a numerical temperature coefficient.

Step 4: Conclusion With a rise in temperature, Young's modulus generally decreases. Hence, the

correct option is (C).

Quick Tip to Solve: Associate Young's modulus with stiffness: the usual thermal softening of a solid means a smaller modulus and more elastic strain for a fixed stress.

50. In the given circuit if the power rating of Zener diode is $10\ \mathrm{mW}$, the value of series resistance R_s to regulate the input unregulated supply is : [4 marks]



(A) $10\mathrm{k}\Omega$

(B) 10Ω

(C) $1\mathrm{k}\Omega$

(D) $\frac{3}{7}\mathrm{k}\Omega < \mathrm{R_s} < \frac{3}{5}\mathrm{k}\Omega$

Correct Answer: (D) $\frac{3}{7}\mathrm{k}\Omega < \mathrm{R_s} < \frac{3}{5}\mathrm{k}\Omega$

Solution:

Concept:

- A reverse-biased Zener diode in breakdown holds the load voltage at its Zener voltage V_Z. The series resistor carries both load and Zener currents: $I_s = I_L + I_Z$.
- The power limit sets $I_{Z,\max} = P_{Z,\max}/V_Z$. Regulation requires a positive Zener current, without exceeding this maximum.

Step 1: Find the load current and the allowable Zener current The circuit gives an input of $8\,\mathrm{V}$, a Zener voltage of $5\,\mathrm{V}$, and a load resistance of $1\,\mathrm{k}\Omega$. Hence the voltage across the series resistor is $8 - 5 = 3\,\mathrm{V}$. The current demanded by the load and the largest safe Zener current are

$$I_L = \frac{5}{1000} = 5\,\mathrm{mA}, \qquad I_{Z,\max} = \frac{10 \times 10^{-3}}{5} = 2\,\mathrm{mA}.$$

Thus the series current must lie between $5\,\mathrm{mA}$ and $7\,\mathrm{mA}$. At the lower limit the diode is just leaving breakdown; at the upper limit its power reaches the rating.

Step 2: Convert the current limits into resistance limits Using Ohm's law for the series resistor gives

$$I_Z = \frac{3}{R_s} - 5 \times 10^{-3}.$$

For regulation with a safety margin, impose $0 < I_Z < 2 \times 10^{-3}\,\text{A}$. Since all resistances are positive,

$$5 \times 10^{-3} < \frac{3}{R_s} < 7 \times 10^{-3} \quad \Longrightarrow \quad \frac{3}{7}\,\text{k}\Omega < R_s < \frac{3}{5}\,\text{k}\Omega.$$

A resistance of $10\,\text{k}\Omega$ or $1\,\text{k}\Omega$ cannot supply even the required load current at a regulated output. A resistance of $10\,\Omega$ supplies an excessive current and would exceed the Zener power limit.

Step 3: Conclusion The allowable series resistance is $\frac{3}{7}\,\text{k}\Omega < R_s < \frac{3}{5}\,\text{k}\Omega$. Hence, the correct option is (D).

Quick Tip to Solve: Calculate the load current first; the series current is the sum of load current and Zener current, not just the Zener current.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A rectangular loop of sides $12\ \text{cm}$ and $5\ \text{cm}$, with its sides parallel to the x-axis and y-axis respectively, moves with a velocity of $5\ \text{cm/s}$ in the positive x axis direction, in a space containing a variable magnetic field in the positive z direction. The field has a gradient of $10^{-3}\ \text{T/cm}$ along the negative x direction and it is decreasing with time at the rate of $10^{-3}\ \text{T/s}$. If the resistance of the loop is $6\ \text{m}\Omega$, the power dissipated by the loop as heat is _____ $\times 10^{-9}$ W. [4 marks]

Solution:

Concept:

- Faraday's law gives the induced emf magnitude as $|\mathcal{E}| = |d\Phi/dt|$. For a translating loop in a spatially varying field, the field experienced changes at the rate $\partial B/\partial t + v\,\partial B/\partial x$.
- The resistive power loss is $P = \mathcal{E}^2/R$. Spatial and temporal changes of flux must first be added with their signs, before squaring the emf.

Step 1: Combine the two rates of change of field The field points along positive z, but becomes weaker towards positive x, because the specified gradient points along negative x. Therefore $\partial B/\partial x = -10^{-3}\,\text{T cm}^{-1}$. The loop travels in positive x at $5\,\text{cm s}^{-1}$, giving

$$v\frac{\partial B}{\partial x} = -5 \times 10^{-3}\,\text{T s}^{-1}.$$

The explicit time variation is also negative, $\partial B/\partial t = -10^{-3}\,\text{T s}^{-1}$. Thus the total rate experienced by the loop is

$$\frac{dB}{dt} = -6 \times 10^{-3}\,\text{T s}^{-1}.$$

Both effects reduce the flux, so their magnitudes add rather than cancel.

Step 2: Find the induced emf and the heating power The loop area is $A = (0.12)(0.05) = 6 \times 10^{-3}\,\text{m}^2$. Its shape and orientation stay fixed, and the field gradient is uniform, so the flux change is the area

times the total field-change rate. Consequently,

$$|\mathcal{E}| = (6 \times 10^{-3})(6 \times 10^{-3}) = 36 \times 10^{-6}\,\text{V}.$$

Convert the loop resistance to $R = 6 \times 10^{-3}\,\Omega$. Then

$$P = \frac{(36 \times 10^{-6})^2}{6 \times 10^{-3}} = 216 \times 10^{-9}\,\text{W}.$$

The induced current opposes the decrease of flux, but its direction does not alter the positive Joule heating power.

Step 3: Conclusion The power dissipated is $216 \times 10^{-9}\,\text{W}$; the number to enter is $\boxed{216}$.

Quick Tip to Solve: For a moving loop in a changing field, use the total rate $\partial B/\partial t + v\partial B/\partial x$, retaining signs until the emf is found.

52. A plane is in level flight at constant speed and each of its two wings has an area of $40\ \text{m}^2$. If the speed of the air is $180\ \text{km/h}$ over the lower wing surface and $252\ \text{km/h}$ over the upper wing surface, the mass of the plane is _____ kg.

(Take air density to be $1\ \text{kg m}^{-3}$ and $g = 10\ \text{ms}^{-2}$) [4 marks]

Solution:

Concept:

- For steady flow at equal height, Bernoulli's equation gives $p + \frac{1}{2}\rho v^2 =$ constant. Faster air above a wing corresponds to lower pressure there.
- In level flight without vertical acceleration, upward lift balances weight: $(p_{\text{lower}} - p_{\text{upper}})A_{\text{total}} = mg$.

Step 1: Convert the speeds and calculate the pressure difference The speeds must be expressed in SI units because the density is in kg m^{-3}. Their values are

$$v_{\text{lower}} = 180\frac{5}{18} = 50\,\text{m s}^{-1}, \qquad v_{\text{upper}} = 252\frac{5}{18} = 70\,\text{m s}^{-1}.$$

At corresponding upper and lower surface points the height difference is neglected. Subtracting their Bernoulli equations therefore gives

$$\Delta p = p_{\text{lower}} - p_{\text{upper}} = \frac{1}{2}\rho(v_{\text{upper}}^2 - v_{\text{lower}}^2).$$

Substitute $\rho = 1\,\text{kg m}^{-3}$:

$$\Delta p = \frac{1}{2}(1)(70^2 - 50^2) = \frac{4900 - 2500}{2} = 1200\,\text{Pa}.$$

The result is positive, correctly predicting a net upward force.

Step 2: Use the combined area of both wings The area quoted is for each wing, and both wings

provide lift. The total effective area is therefore

$$A_{\text{total}} = 2 \times 40 = 80\,\text{m}^2.$$

The total upward force is $F_L = 1200 \times 80 = 96000\,\text{N}$. Since the plane is in level flight at constant speed, the net vertical force vanishes. Thus

$$mg = F_L, \qquad m = \frac{96000}{10} = 9600\,\text{kg}.$$

Using only one wing would halve the lift and give an incorrect mass; using a sum of the squared speeds would also misrepresent the pressure difference.

Step 3: Conclusion The mass of the plane is $\boxed{9600\,\text{kg}}$; enter 9600.

Quick Tip to Solve: Convert both airspeeds to metres per second, subtract their squares, and remember that the given wing area is for each of two wings.

53. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle θ with each other. When suspended in water the angle remains the same. If density of the material of the sphere is $1.5\ \text{g/cc}$, the dielectric constant of water will be _____.
(Take density of water $= 1\ \text{g/cc}$) [4 marks]

Solution:

Concept:

- For either sphere in equilibrium, the ratio of the horizontal electric force to its effective weight equals the tangent of the string's inclination to the vertical.
- Immersion reduces electrostatic repulsion by the dielectric constant K, while buoyancy reduces the weight to $(\rho_s - \rho_w)Vg$. The unchanged angle means the separation also stays unchanged.

Step 1: Write equilibrium in air and in water Let each sphere have volume V, density ρ_s, and charge q. By symmetry each string makes angle $\alpha = \theta/2$ with the vertical. Denote the electric repulsion in air by F. Taking the horizontal and vertical components of tension gives

$$\tan\alpha = \frac{F}{\rho_s V g}.$$

Air buoyancy is negligible. When the spheres are immersed, the same angle and string lengths imply the same separation. Their charges remain the same, so the electric force is F/K. The water exerts an upward buoyant force $\rho_w V g$, giving

$$\tan\alpha = \frac{F/K}{(\rho_s - \rho_w)Vg}.$$

Step 2: Equate the two force ratios and substitute densities The two tangents are equal because the

angle is unchanged. Cancelling F, V, and g from their equality gives

$$\frac{1}{\rho_s} = \frac{1}{K(\rho_s - \rho_w)}, \qquad K = \frac{\rho_s}{\rho_s - \rho_w}.$$

Both densities are given in the same units, so a common conversion factor would cancel in the ratio. Using $\rho_s = 1.5\,\mathrm{g\,cm^{-3}}$ and $\rho_w = 1\,\mathrm{g\,cm^{-3}}$,

$$K = \frac{1.5}{1.5 - 1} = \frac{1.5}{0.5} = 3.$$

Indeed, water reduces the apparent weight to one-third of its air value, so the repulsive force must also decrease to one-third to preserve the angle.

Step 3: Conclusion The required dielectric constant is $\boxed{3}$. It is dimensionless, so the numerical entry is 3.

Quick Tip to Solve: An unchanged suspension angle preserves the ratio of electric repulsion to apparent weight; include buoyancy as well as dielectric screening.

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54. A particle is moving in one dimension (along x axis) under the action of a variable force. It's initial position was $16\ \mathrm{m}$ right of origin. The variation of its position (x) with time (t) is given as $x = -3t^3 + 18t^2 + 16t$, where x is in m and t is in s.

The velocity of the particle when its acceleration becomes zero is _____ $\mathrm{m/s}$. [4 marks]

Solution:

Concept:

- The instantaneous velocity is the first derivative $v = dx/dt$, and acceleration is the second derivative $a = dv/dt$. The requested velocity must be evaluated at the time when acceleration vanishes.
- A constant offset in the position coordinate disappears on differentiation, so it cannot change either velocity or acceleration.

Step 1: Differentiate the position twice The stated position polynomial is

$$x(t) = -3t^3 + 18t^2 + 16t.$$

Apply the power rule to each term, keeping the numerical coefficients and powers of time distinct. The first derivative is

$$v(t) = \frac{dx}{dt} = -9t^2 + 36t + 16.$$

The second derivative is

$$a(t) = \frac{dv}{dt} = -18t + 36.$$

The introductory statement places the initial position $16\,\mathrm{m}$ to the right of the origin, whereas the displayed polynomial has zero constant term. An added initial-position constant would differentiate to zero; consequently that coordinate offset has no effect on the velocity requested here.

Step 2: Find the zero-acceleration instant and evaluate velocity Set the derived acceleration equal to

zero:

$$-18t + 36 = 0 \quad \Longrightarrow \quad t = 2\,\text{s}.$$

This is a physically admissible positive time after the motion starts. Substitute it into the velocity expression, not into the position polynomial:

$$v(2) = -9(2)^2 + 36(2) + 16 = -36 + 72 + 16 = 52\,\text{m s}^{-1}.$$

Zero acceleration means the slope of the velocity-time graph is zero at that instant; it does not mean that velocity itself is zero. Here the acceleration changes from positive to negative at $2\,\text{s}$, so this instant is the maximum of the quadratic velocity function.

Step 3: Conclusion The velocity when acceleration becomes zero is $\boxed{52\,\text{m s}^{-1}}$, directed along positive x. Enter 52.

Quick Tip to Solve: Differentiate position twice, solve $a(t) = 0$, then substitute that time into the first derivative; a position offset drops out.

55. The radius of a nucleus of mass number 64 is 4.8 fermi. Then the mass number of another nucleus having radius of 4 fermi is $\frac{1000}{x}$, where x is _____. [4 marks]

Solution:

Concept:

- The empirical nuclear radius relation is $R = R_0 A^{1/3}$, where A is the mass number and R_0 is approximately common to nuclei.
- For two nuclei this gives $A_2/A_1 = (R_2/R_1)^3$. Ratios of radii in the same unit can be used without converting fermi to metres.

Step 1: Relate the mass numbers through the radius ratio Let the first nucleus have mass number $A_1 = 64$ and radius $R_1 = 4.8\,\text{fm}$, and let the second have radius $R_2 = 4\,\text{fm}$. Dividing their radius laws cancels the common nuclear-radius constant:

$$\frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3}.$$

Cube both sides before substituting the radius ratio. This gives

$$A_2 = 64\left(\frac{4}{4.8}\right)^3 = 64\left(\frac{5}{6}\right)^3.$$

The cubic dependence reflects the nearly constant density of nuclear matter: nucleon number scales with nuclear volume rather than radius or surface area.

Step 2: Express the result in the form asked for Evaluate the ratio exactly instead of rounding the second mass number at an intermediate stage:

$$A_2 = 64\frac{125}{216} = \frac{8000}{216} = \frac{1000}{27}.$$

The question defines this mass number as $1000/x$. Comparing the two forms directly gives

$$\frac{1000}{x} = \frac{1000}{27} \quad \Longrightarrow \quad x = 27.$$

Equivalently, the first radius relation gives $R_0 = 4.8/\sqrt[3]{64} = 1.2\,\text{fm}$, and then $A_2 = (4/1.2)^3 = 1000/27$, confirming the same value. The requested blank is the denominator parameter x, rather than the value of A_2 itself.

Step 3: Conclusion The value of x is $\boxed{27}$. The parameter is dimensionless; enter 27.

Quick Tip to Solve: Cube the radius ratio, keep it as an exact fraction, and compare with the given $1000/x$ form before rounding anything.

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56. A regular polygon of 6 sides is formed by bending a wire of length 4π meter.
If an electric current of $4\pi\sqrt{3}$ A is flowing through the sides of the polygon, the magnetic field at the centre of the polygon would be $x \times 10^{-7}$ T.
The value of x is _____. [4 marks]

Solution:

Concept:

- The field at a point a perpendicular distance d from a finite straight wire is $B = \mu_0 I(\sin\alpha + \sin\beta)/(4\pi d)$, with the angles measured from the perpendicular to the wire.
- At the centre of a regular polygon, all side contributions have equal magnitude and the same direction normal to its plane; their magnitudes therefore add.

Step 1: Calculate the hexagon side and the centre-to-side distance The wire forms a regular six-sided polygon, so each side has length

$$s = \frac{4\pi}{6} = \frac{2\pi}{3}\,\text{m}.$$

In a regular hexagon, the perpendicular distance from the centre to any side is its apothem, $d = (\sqrt{3}/2)s$. Therefore

$$d = \frac{\sqrt{3}}{2}\frac{2\pi}{3} = \frac{\pi\sqrt{3}}{3}\,\text{m}.$$

The perpendicular bisects each side. Either endpoint subtends an angle 30° with this perpendicular, since

$$\tan\alpha = \frac{s/2}{d} = \frac{1}{\sqrt{3}}.$$

Thus both endpoint angles in the finite-wire formula are 30°.

Step 2: Add the fields of the six finite wire segments For a single side, $\sin 30^\circ + \sin 30^\circ = 1$, so

$$B_{\text{side}} = \frac{\mu_0 I}{4\pi d}.$$

The current circulates consistently around the polygon; the right-hand rule gives the same normal

direction for the field from every side at the centre. Hence

$$B = 6\frac{\mu_0 I}{4\pi d}.$$

Using $\mu_0/(4\pi) = 10^{-7}\,\mathrm{T\,m\,A^{-1}}$, $I = 4\pi\sqrt{3}\,\mathrm{A}$, and the distance found above,

$$B = 6 \times 10^{-7}\,\frac{4\pi\sqrt{3}}{\pi\sqrt{3}/3} = 72 \times 10^{-7}\,\mathrm{T}.$$

The circular-loop formula would give a different answer because the wire consists of straight sides. It is essential to use the apothem, not the centre-to-vertex distance, in the finite-wire formula.

Step 3: Conclusion The central magnetic field has magnitude $72 \times 10^{-7}\,\mathrm{T}$, so $\boxed{x = 72}$.

Quick Tip to Solve: For a regular hexagon use six identical finite-wire fields, each with endpoint angles 30° and perpendicular distance $\sqrt{3}s/2$.

57. The distance between object and its 3 times magnified virtual image as produced by a convex lens is $20\ \mathrm{cm}$. The focal length of the lens used is _____ cm. [4 marks]

Solution:

Concept:

- For a thin lens, the Cartesian sign convention gives $1/f = 1/v - 1/u$. Distances measured in the direction of incident light are positive.
- The transverse magnification is $m = v/u$. A virtual image of a real object formed by a convex lens is erect, so its magnification is positive.

Step 1: Assign signed object and image distances Place the lens at the origin and take incident light to travel from left to right. Let the object distance from the lens be $a > 0$, so $u = -a$. Since the image is virtual and three times the object size, it is on the object side of the lens and $m = +3$. Thus

$$v = mu = 3(-a) = -3a.$$

The two positions are both negative; their separation is the difference of their distances from the lens, rather than the sum. This observation prevents using the real-image geometry for this virtual-image problem.

Step 2: Use the separation to find both positions The given object-image separation is $20\,\mathrm{cm}$. Therefore

$$|v - u| = |-3a + a| = 2a = 20 \quad \Rightarrow \quad a = 10\,\mathrm{cm}.$$

Hence $u = -10\,\mathrm{cm}$ and $v = -30\,\mathrm{cm}$. The image is farther from the lens than the object, as required for an erect enlarged virtual image.

Step 3: Apply the lens equation Substitute the signed distances without changing the sign of the

virtual-image distance:

$$\frac{1}{f} = \frac{1}{-30} - \frac{1}{-10} = \frac{-1+3}{30} = \frac{1}{15}\,\text{cm}^{-1}.$$

Thus $f = +15\,\text{cm}$. The positive focal length confirms that the lens is converging; the object is within its focal length, consistent with the virtual enlarged image.

Step 4: Conclusion The focal length is $\boxed{15\,\text{cm}}$. The numerical entry is 15.

Quick Tip to Solve: For an erect virtual image, use positive magnification but negative image distance: here the separation is $(m-1)|u|$, not $(m+1)|u|$.

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58. Three identical spheres each of mass 2M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to $4\ \text{m}$ each. Taking point of intersection of these two sides as origin, the magnitude of position vector of the centre of mass of the system is $\frac{4\sqrt{2}}{x}$, where the value of x is _____ . [4 marks]

Solution:

Concept:

- The centre-of-mass position is $\vec{r}_{\text{cm}} = \sum_i m_i\vec{r}_i / \sum_i m_i$. Equal masses make it the arithmetic mean of the position vectors.
- For perpendicular Cartesian axes, the magnitude of a position vector is $|\vec{r}| = \sqrt{x^2+y^2}$; a common mass factor cancels from every coordinate.

Step 1: Choose coordinates along the perpendicular sides Take the vertex at the intersection of the two equal perpendicular sides as $O = (0,0)$. Choose the horizontal side as the x-axis and the vertical side as the y-axis. The other two sphere centres are then $P = (4,0)\,\text{m}$ and $Q = (0,4)\,\text{m}$. Each sphere has mass $2M$, so the total mass is $6M$. This coordinate choice uses the right angle explicitly and avoids introducing any trigonometric angle.

Step 2: Evaluate the mass-weighted coordinates The equal masses contribute equally, including the sphere at the origin whose coordinate contribution is zero:

$$x_{\text{cm}} = \frac{2M(0)+2M(4)+2M(0)}{6M} = \frac{4}{3}\,\text{m},$$

$$y_{\text{cm}} = \frac{2M(0)+2M(0)+2M(4)}{6M} = \frac{4}{3}\,\text{m}.$$

Thus $\vec{r}_{\text{cm}} = \frac{4}{3}\hat{i} + \frac{4}{3}\hat{j}$ metres. The equality of the two coordinates also follows from symmetry between the two perpendicular sides.

Step 3: Find the magnitude and identify the denominator Use the Pythagorean relation for the two perpendicular components:

$$|\vec{r}_{\text{cm}}| = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{4}{3}\right)^2} = \sqrt{\frac{32}{9}} = \frac{4\sqrt{2}}{3}\,\text{m}.$$

The question writes this same magnitude as $4\sqrt{2}/x$. Comparing denominators gives $x = 3$. The requested parameter is dimensionless; it is not either coordinate of the centre of mass.

Step 4: Conclusion The centre of mass is at distance $4\sqrt{2}/3\,\mathrm{m}$ from the origin, so $\boxed{x = 3}$.

Quick Tip to Solve: For three equal masses at triangle vertices, locate the centroid by averaging coordinates first; take the vector magnitude only after both coordinates are known.

59. A tuning fork resonates with a sonometer wire of length $1\ \mathrm{m}$ stretched with a tension of $6\ \mathrm{N}$. When the tension in the wire is changed to $54\ \mathrm{N}$, the same tuning fork produces 12 beats per second with it. The frequency of the tuning fork is _____ Hz. [4 marks]

Solution:

Concept:

- For a stretched sonometer wire of fixed vibrating length L and linear mass density μ, its fundamental frequency is $f = (2L)^{-1}\sqrt{T/\mu}$. Thus $f \propto \sqrt{T}$.
- The beat frequency between two nearby periodic sources is the absolute difference of their frequencies: $f_{\text{beat}} = |f_2 - f_1|$. At resonance the wire frequency equals the tuning-fork frequency.

Step 1: Express the initial resonance condition Let the tuning-fork frequency be f_0. With the wire of length $1\,\mathrm{m}$ under tension $T_1 = 6\,\mathrm{N}$, resonance means its frequency is $f_1 = f_0$. The wire itself is unchanged when its tension is altered, so its length and mass per unit length remain the same. Consequently those quantities cancel when we take a ratio; the value of μ is not needed.

Step 2: Calculate the new wire frequency The second tension is $T_2 = 54\,\mathrm{N}$. Therefore

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{54}{6}} = \sqrt{9} = 3.$$

Hence $f_2 = 3f_0$. Increasing the tension raises the wire frequency, while the tuning fork retains its own frequency f_0. The relevant frequency difference is therefore positive and equals $3f_0 - f_0$, rather than their sum.

Step 3: Use the observed beat frequency The stated 12 beats per second give

$$12 = |f_2 - f_0| = |3f_0 - f_0| = 2f_0.$$

Thus $f_0 = 6\,\mathrm{Hz}$. As a check, the wire frequencies before and after the tension change are $6\,\mathrm{Hz}$ and $18\,\mathrm{Hz}$, whose difference is $12\,\mathrm{Hz}$. Applying the tension ratio directly to frequency would incorrectly give a factor of nine instead of the required square-root factor of three.

Step 4: Conclusion The frequency of the tuning fork is $\boxed{6\,\mathrm{Hz}}$. The numerical entry is 6.

Quick Tip to Solve: First scale the wire frequency by $\sqrt{T_2/T_1}$; then subtract the unchanged tuning-fork frequency to form the beat frequency.

60. The current in a conductor is expressed as $I = 3t^2 + 4t^3$, where I is in Ampere and t is in second. The amount of electric charge that flows through a section of the conductor during $t = 1$ s to $t = 2$ s is _____ C. [4 marks]

Solution:

Concept:

- Current is the rate at which electric charge crosses a section: $I = dQ/dt$. The charge over a finite interval is therefore $\Delta Q = \int_{t_1}^{t_2} I(t)\,dt$.
- The area under a current-time curve has units $\mathrm{A\,s} = \mathrm{C}$. For a power of time, $\int t^n dt = t^{n+1}/(n+1)$.

Step 1: Set up the charge integral over the correct interval The current is $I(t) = 3t^2 + 4t^3$ amperes, with time measured in seconds. The required charge is only that passing between $1\,\mathrm{s}$ and $2\,\mathrm{s}$, so the lower limit must be 1 rather than 0:

$$\Delta Q = \int_1^2 (3t^2 + 4t^3)\,dt.$$

The current is positive throughout this interval, so this net charge is also the total amount of charge crossing the section.

Step 2: Integrate the two current contributions Apply the power rule separately to each term:

$$\int 3t^2 dt = t^3, \qquad \int 4t^3 dt = t^4.$$

The charge expression whose derivative reproduces the supplied current is consequently $t^3 + t^4$, apart from an additive constant. That constant cancels when evaluating a definite integral. Differentiating this expression gives $3t^2 + 4t^3$, providing a direct check on the integration.

Step 3: Evaluate both endpoints and subtract At the upper limit the expression is $2^3 + 2^4 = 8 + 16 = 24$, whereas at the lower limit it is $1^3 + 1^4 = 2$. Thus

$$\Delta Q = [t^3 + t^4]_1^2 = 24 - 2 = 22\,\mathrm{C}.$$

Equivalently the quadratic-current contribution is $8 - 1 = 7\,\mathrm{C}$, and the cubic-current contribution is $16 - 1 = 15\,\mathrm{C}$; their sum is $22\,\mathrm{C}$. Multiplying one endpoint current by the time interval is invalid because the current varies with time.

Step 4: Conclusion The charge flowing through the section is $\boxed{22\,\mathrm{C}}$. The numerical entry is 22.

Quick Tip to Solve: For a time-dependent current, integrate first and evaluate both time limits; the charge accumulated before the lower limit must be subtracted.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Which of the following reactions are disproportionation reactions?

(A) $Cu^{+} \rightarrow Cu^{2+} + Cu$

(B) $3MnO_4^{2-} + 4H^{+} \longrightarrow 2MnO_4^{-} + MnO_2 + 2H_2O$

(C) $2KMnO_4 \longrightarrow K_2MnO_4 + MnO_2 + O_2$

(D) $2MnO_4^{-} + 3Mn^{2+} + 2H_2O \longrightarrow 5MnO_2 + 4H^{+}$

Choose the correct answer from the options given below : [4 marks]

(A) (A), (B)

(B) (A), (D)

(C) (B), (C), (D)

(D) (A), (B), (C)

Correct Answer: (A) (A), (B)

Solution:

Concept:

- In disproportionation, atoms of the same element in one initial oxidation state undergo both oxidation and reduction, producing a higher and a lower oxidation state.
- Oxidation increases oxidation number and reduction decreases it. Oppositely, comproportionation combines two initial oxidation states of an element into a common intermediate state.

Step 1: Test copper and manganate reactions In reaction (A), copper begins as Cu^{+}, with oxidation number $+1$. It forms Cu^{2+}, where the number is $+2$, and elemental Cu, where it is zero. Copper is thus both oxidised and reduced: (A) is disproportionation. In reaction (B), manganese in ${MnO_4}^{2-}$ is $+6$. Its products contain Mn at $+7$ in ${MnO_4}^{-}$ and $+4$ in MnO_2. Again one initial state becomes both higher and lower states, so (B) also qualifies.

Step 2: Distinguish decomposition from disproportionation In reaction (C), manganese in $KMnO_4$ starts at $+7$. It changes to $+6$ in K_2MnO_4 and $+4$ in MnO_2. Both manganese changes are reductions, not one oxidation and one reduction. The oxygen that becomes O_2 is oxidised from -2 to zero. Thus this is a redox decomposition involving different elements; it is not disproportionation of manganese. The presence of two manganese-containing products alone does not establish disproportionation.

Step 3: Classify the remaining reaction and select the combination In reaction (D), Mn at $+7$ in ${MnO_4}^{-}$ and Mn at $+2$ in Mn^{2+} both end at $+4$ in MnO_2. Two different initial states converge, so this is comproportionation. Only (A) and (B) meet the definition. Option (B) includes the comproportionation reaction (D); option (C) includes both (C) and (D); option (D) includes the decomposition (C). Each therefore contains a reaction that fails the required test.

Step 4: Conclusion Only the copper(I) reaction and the manganate(VI) reaction are disproportionation reactions. Hence, the correct option is (A).

Quick Tip to Solve: Track one element: a single starting oxidation state must split into both a higher and a lower state. Two starting states converging means comproportionation.

62. Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following : [4 marks]

(A) $\mathrm{q} = 0, \Delta\mathrm{T} = 0, \mathrm{w} = 0$

(B) $\mathrm{q} = 0, \Delta\mathrm{T} \neq 0, \mathrm{w} = 0$

(C) $\mathrm{q} \neq 0, \Delta\mathrm{T} = 0, \mathrm{w} = 0$

(D) $\mathrm{q} = 0, \Delta\mathrm{T} < 0, \mathrm{w} \neq 0$

Correct Answer: (A) $\mathrm{q} = 0, \Delta\mathrm{T} = 0, \mathrm{w} = 0$

Solution:

Concept:

- An adiabatic process has no heat transfer, so $q = 0$. Free expansion is expansion against zero external pressure: $P_{\text{ext}} = 0$.
- With the chemistry sign convention, the first law is $\Delta U = q + w$, and pressure-volume work is $w = -\int P_{\text{ext}}\, dV$. The internal energy of an ideal gas depends only on temperature.

Step 1: Apply the adiabatic condition The boundary is thermally insulating under the given adiabatic condition. Therefore no energy enters or leaves the gas as heat:

$$q = 0.$$

This conclusion is about heat transfer, rather than temperature. An adiabatic expansion can have a temperature change when work is performed, so the temperature must be determined from the remaining information instead of being guessed from the word adiabatic.

Step 2: Evaluate the work in free expansion The gas expands into an evacuated region and encounters no external resisting pressure. Hence

$$w = -\int_{V_i}^{V_f} P_{\text{ext}}\, dV = -\int_{V_i}^{V_f} 0\, dV = 0.$$

The volume can increase even though no pressure-volume work is done. The gas pressure itself must not replace external pressure in this formula; free expansion is irreversible and is not a sequence of equilibrium states.

Step 3: Use the first law to determine temperature The first law now gives $\Delta U = q + w = 0$. For a fixed amount of ideal gas, $dU = nC_V dT$; its internal energy has no volume dependence. Thus equal initial and final internal energies require

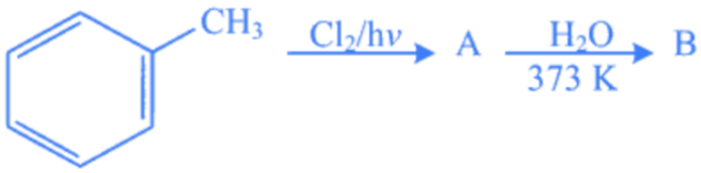
$$\Delta T = 0.$$

The complete set is $q = 0,\ \Delta T = 0,\ w = 0$. Option (B) contradicts the ideal-gas internal-energy condition. Option (C) contradicts the adiabatic condition. Option (D) incorrectly assigns nonzero work and cooling, as in a different expansion with external resistance.

Step 4: Conclusion For adiabatic free expansion of an ideal gas, $q = 0,\ \Delta T = 0,\ w = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Use two separate facts: adiabatic gives zero heat and vacuum gives zero work. The first law then fixes the ideal-gas temperature change.

63. Identify A and B in the following sequence of reaction [4 marks]



(A) $A = C_6H_5CHCl_2, \quad B = C_6H_5COOH$

(B) $A = C_6H_5CHCl_2, \quad B = C_6H_5CHO$

(C) $A = C_6H_5COCl, \quad B = C_6H_5CHO$

(D) $A = C_6H_5CH_2Cl, \quad B = C_6H_5CHO$

Correct Answer: (B) $A = C_6H_5CHCl_2, \quad B = C_6H_5CHO$

Solution:

Concept:

- Chlorination of toluene under light proceeds by a free-radical substitution at the benzylic side chain; it is distinct from ring chlorination with a Lewis-acid catalyst.
- A geminal dihalide has both halogens on one carbon. Hydrolysis replaces them with hydroxyl groups, giving an unstable geminal diol that loses water to form a carbonyl group.

Step 1: Identify the side-chain intermediate The starting compound in the reaction scheme is toluene, $C_6H_5CH_3$. Chlorine under $h\nu$ attacks the benzylic side chain. Successive substitution of two benzylic hydrogen atoms gives benzal chloride, $C_6H_5CHCl_2$. The pair of chlorines is on the same side-chain carbon, so the intermediate is a geminal dichloride:

$$C_6H_5CH_3 + 2\,Cl_2 \xrightarrow{h\nu} C_6H_5CHCl_2 + 2\,HCl.$$

The subsequent hydrolysis and the offered answer pairs identify this degree of side-chain chlorination.

Step 2: Hydrolyse the geminal dichloride With water at $373\,K$, the two carbon-chlorine bonds are hydrolysed to form the geminal diol $C_6H_5CH(OH)_2$. This unstable diol eliminates water to form benzaldehyde, retaining the carbon-hydrogen bond:

$$C_6H_5CHCl_2 + H_2O \longrightarrow C_6H_5CHO + 2\,HCl.$$

Thus A is benzal chloride and B is benzaldehyde. This hydrolysis creates the aldehyde group; it does not oxidise it further into a carboxylic acid.

Step 3: Compare all four proposed pairs Option (B) contains exactly $C_6H_5CHCl_2$ and C_6H_5CHO. Option (A) incorrectly makes benzoic acid from a geminal dichloride by simple hydrolysis. Option (C) starts with benzoyl chloride, whose hydrolysis instead forms benzoic acid. Option (D) starts with benzyl chloride, whose hydrolysis forms benzyl alcohol, $C_6H_5CH_2OH$, rather than benzaldehyde. The two chlorine substituents on the benzylic carbon are therefore the decisive structural feature.

Step 4: Conclusion $A = C_6H_5CHCl_2$ (benzal chloride) and $B = C_6H_5CHO$ (benzaldehyde). Hence, the correct option is (B).

Quick Tip to Solve: Count the benzylic chlorines: hydrolysis of $-CH_2Cl$ gives an alcohol, while hydrolysis of $-CHCl_2$ gives an aldehyde.

64. Which of the following complex is homoleptic? [4 marks]

(A) $[\mathrm{Ni}\,(\mathrm{NH_3})_2\,\mathrm{Cl_2}]$

(B) $[\mathrm{Co}\,(\mathrm{NH_3})_4\,\mathrm{Cl_2}]^+$

(C) $[\mathrm{Fe}\,(\mathrm{NH_3})_4\,\mathrm{Cl_2}]^+$

(D) $[\mathrm{Ni(CN)_4}]^{2-}$

Correct Answer: (D) $[\mathrm{Ni(CN)_4}]^{2-}$

Solution:

Concept:

- A homoleptic coordination entity contains only one kind of ligand attached to its central metal atom or ion. The number of ligands and the charge of the complex do not decide this classification.
- A heteroleptic coordination entity contains two or more chemically different ligand types. Both neutral molecules such as $\mathrm{NH_3}$ and anions such as $\mathrm{Cl^-}$ count as ligands when inside the coordination sphere.

Step 1: Identify the ligand types in each candidate In $[\mathrm{Ni(NH_3)_2Cl_2}]$, nickel is coordinated by ammonia and chloride, so there are two ligand types. The cobalt and iron complexes $[\mathrm{Co(NH_3)_4Cl_2}]^+$ and $[\mathrm{Fe(NH_3)_4Cl_2}]^+$ likewise each contain ammonia and chloride. Changing the metal from nickel to cobalt or iron does not remove the presence of these two different donor species. Thus options (A), (B) and (C) are heteroleptic.

Step 2: Check the cyanide complex In $[\mathrm{Ni(CN)_4}]^{2-}$, each of the four ligands is the same cyanide ion, $\mathrm{CN^-}$. All four donor groups therefore belong to one ligand type. Although the complex is charged and cyanide is an anionic ligand, neither fact prevents the coordination entity from being homoleptic. Do not confuse homoleptic with a neutral complex or with a particular coordination number: the decisive feature is the identity of its coordinated ligands.

Step 3: Conclusion Only $[\mathrm{Ni(CN)_4}]^{2-}$ has a single kind of ligand throughout its coordination sphere. Hence, the correct option is (D).

Quick Tip to Solve: List the distinct ligand formulae inside the square brackets; one distinct ligand type means homoleptic, even when several copies are present.

65. In Kjeldahl's method for estimation of nitrogen, $CuSO_4$ acts as : [4 marks]

(A) catalytic agent

(B) hydrolysis agent

(C) reducing agent

(D) oxidising agent

Correct Answer: (A) catalytic agent

Solution:

Concept:

- Kjeldahl analysis converts the nitrogen of a suitable organic compound into ammonium sulfate during digestion with concentrated sulfuric acid. Alkaline treatment then liberates ammonia for quantitative estimation.
- The digestion mixture contains reagents with different roles: concentrated H_2SO_4 carries out the digestion, K_2SO_4 raises the boiling point, and $CuSO_4$ accelerates the process catalytically.

Step 1: Locate copper sulfate in the digestion stage The organic material is heated with concentrated sulfuric acid. The carbonaceous material is digested and the nitrogen that can be estimated by this method is retained as ammonium ions in the acid medium. Copper sulfate is added to speed up this conversion, so the digestion reaches completion more efficiently. Its specified analytical role is therefore that of a catalyst; it is not the reagent used to release the ammonia in the subsequent stage.

Step 2: Distinguish the other proposed roles Copper sulfate is not added as a hydrolysis agent, so option (B) does not describe its function. Nor is it the reducing agent in this procedure, excluding (C). The oxidation and digestion of the organic matter occur in the concentrated sulfuric-acid medium; calling copper sulfate the oxidising reagent misses its catalytic role, so (D) is unsuitable. During the later alkaline step, ammonium ions give ammonia, for example $NH_4^+ + OH^- \longrightarrow NH_3 + H_2O$, which is collected and estimated.

Step 3: Conclusion In Kjeldahl digestion, $CuSO_4$ acts as a catalytic agent. Hence, the correct option is (A).

Quick Tip to Solve: Keep the digestion additives separate: copper sulfate catalyses the digestion, whereas potassium sulfate raises its boiling temperature.

66. Given below are two statements :

Statement (I) : Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution.

Statement (II) : In this titration phenolphthalein can be used as indicator.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct

(B) Statement I is incorrect but Statement II is correct

(C) Both Statement I and Statement II are incorrect

(D) Statement I is correct but Statement II is incorrect

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- A primary standard must have high purity, a stable composition and a sufficiently high molar mass so that an accurately weighed sample gives a reliable amount of substance.
- Potassium hydrogen phthalate, abbreviated KHP, is a stable monoprotic acid salt. Its neutralisation with sodium hydroxide has a one-to-one mole ratio, and the equivalence solution is alkaline.

Step 1: Assess potassium hydrogen phthalate as a standard KHP can be obtained pure and dried to constant composition. It is sufficiently stable for accurate weighing, and its relatively high molar mass reduces the relative weighing error. Sodium hydroxide, in contrast, absorbs moisture and carbon dioxide from air, so an accurately prepared concentration normally needs standardisation. KHP supplies a known number of acidic equivalents to determine that concentration. The neutralisation is

$$\mathrm{KHC_8H_4O_4 + NaOH \longrightarrow KNaC_8H_4O_4 + H_2O.}$$

Therefore Statement (I) is correct.

Step 2: Choose the indicator from the equivalence conditions The acidic hydrogen of KHP is neutralised by the strong base. At equivalence the resulting phthalate species hydrolyses to give a basic solution, so the pH is above seven. Phenolphthalein changes colour in the alkaline range, approximately pH 8.2 to 10, within the sharp endpoint region for this titration. Its first persistent faint pink colour is therefore a suitable endpoint. Statement (II) is also correct. Options (B), (C) and (D) each incorrectly reject at least one of these two true statements.

Step 3: Conclusion Both Statement (I) and Statement (II) are correct. Hence, the correct option is (A).

Quick Tip to Solve: For a weak-acid salt titrated with a strong base, check the alkaline equivalence region before choosing phenolphthalein as the endpoint indicator.

67. In case of isoelectronic species the size of F^-, Ne and Na^+ is affected by : [4 marks]

(A) Nuclear charge (z)

(B) None of the factors because their size is the same

(C) Electron-electron interaction in the outer orbitals

(D) Principal quantum number (n)

Correct Answer: (A) Nuclear charge (z)

Solution:

Concept:

- Isoelectronic species have the same number of electrons. When their electronic configurations are the same, increasing nuclear charge draws the electron cloud more strongly towards the nucleus.
- For F^-, Ne and Na^+, there are ten electrons with the configuration $1s^2 2s^2 2p^6$. Their nuclear charges are different even though their outer occupied shell is the same.

Step 1: Compare electron counts and nuclear charges Fluorine has atomic number 9 and its anion gains one electron, giving ten electrons. Neon has atomic number 10 and ten electrons as a neutral atom. Sodium has atomic number 11 and loses one electron to form its cation, again leaving ten electrons. The three nuclei therefore contain 9, 10 and 11 protons, respectively. Their identical electron counts do not imply identical electrostatic attraction, since the attracting nuclear charge increases across the sequence.

Step 2: Identify the factor causing the size change With the same shell occupancy, the stronger nuclear attraction contracts the cloud as Z increases. The usual isoelectronic-size trend is therefore

$$r(F^-) > r(Ne) > r(Na^+).$$

Option (B) is false because equal electron count does not force equal size. Electron-electron repulsion exists in all three, but their corresponding occupancies are the same and it is not the distinguishing factor requested in (C). The outer principal quantum number is $n = 2$ throughout, so (D) does not explain the variation either.

Step 3: Conclusion The changing nuclear charge Z is the decisive factor for the sizes of these isoelectronic species. Hence, the correct option is (A).

Quick Tip to Solve: In an isoelectronic series, first hold the electron count fixed; the ion or atom with more protons has the more contracted electron cloud.

68. Ionic reactions with organic compounds proceed through :

(A) homolytic bond cleavage

(B) heterolytic bond cleavage

(C) free radical formation

(D) primary free radical

(E) secondary free radical

Choose the correct answer from the options given below : [4 marks]

(A) (A) only

(B) (B) only

(C) (C) only

(D) (D) and (E) only

Correct Answer: (B) (B) only

Solution:

Concept:

- Heterolytic cleavage breaks a covalent bond so that both shared electrons go to one of the bonded atoms. It produces oppositely charged fragments and is characteristic of ionic reaction mechanisms.
- Homolytic cleavage divides the shared pair equally, leaving one electron on each fragment and producing free radicals. Radical mechanisms must be distinguished from ionic mechanisms.

Step 1: Identify the bond cleavage that produces ions Write a general polar bond as $\mathrm{R{-}X}$. On heterolytic cleavage, the more electron-attracting fragment can take the complete bonding electron pair:

$$\mathrm{R{-}X} \longrightarrow \mathrm{R^{+} + X^{-}}.$$

Depending on the bond and relative electron affinities, the opposite charge assignment is also possible; the essential point is that the electron pair is transferred together. The charged intermediates then react with nucleophiles or electrophiles. Thus statement (B), heterolytic bond cleavage, describes the requested ionic pathway.

Step 2: Exclude the radical pathways and select the combination In homolysis, the bonding pair is split rather than transferred as a pair:

$$\mathrm{R{-}X} \longrightarrow \mathrm{R\cdot + X\cdot}.$$

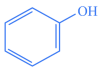
The species so produced have unpaired electrons and are radicals, not the ions that define an ionic mechanism. Hence statement (A) and the free-radical formation in (C) do not apply. Primary and secondary radicals in (D) and (E) are particular radical intermediates, so neither is the required ionic intermediate. Only statement (B) remains. In the answer list, the second option is the combination "(B) only".

Step 3: Conclusion Ionic reactions proceed through heterolytic bond cleavage, so the valid statement is (B) only. Hence, the correct option is (B).

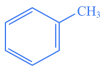
Quick Tip to Solve: Track the bonding electron pair: both electrons to one fragment gives ions; one electron to each fragment gives radicals.

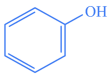
69. Which of the following compound will most easily be attacked by an electrophile? [4 marks]

(A) Chlorobenzene, C_6H_5Cl 

(B) Phenol, C_6H_5OH 

(C) Benzene, C_6H_6 

(D) Toluene, $C_6H_5CH_3$ 

Correct Answer: (B) Phenol, C_6H_5OH 

Solution:

Concept:

- An electrophile is electron deficient and attacks an aromatic ring more readily when substituents increase its electron density and stabilise the positively charged substitution intermediate.
- The hydroxyl group donates a lone pair into the aromatic ring by resonance. A methyl group activates more weakly by hyperconjugation and its inductive effect, while chlorine deactivates overall because its electron-withdrawing inductive effect dominates.

Step 1: Identify the four substituted rings The structures are chlorobenzene (A), phenol (B), benzene (C) and toluene (D). In phenol, an oxygen lone pair overlaps with the aromatic π system and donates electron density by resonance. This especially enhances electron density at the ortho and para positions and stabilises the arenium-ion intermediate formed during electrophilic substitution. Although oxygen also has a withdrawing inductive effect, its resonance donation is the controlling activating effect on the aromatic ring.

Step 2: Compare the alternative rings with phenol Toluene is more reactive than benzene because methyl donates electron density through hyperconjugation and its positive inductive effect, but it is a weaker activator than hydroxyl. Benzene has no activating substituent, so it is less readily attacked than phenol or toluene. In chlorobenzene, chlorine can donate by resonance and directs substitution ortho/para, yet its strong negative inductive effect deactivates the ring overall. Direction of substitution and rate of substitution are different properties. Thus (A), (C) and (D) are all less susceptible to electrophilic attack than (B).

Step 3: Conclusion Phenol is most readily attacked by an electrophile because the hydroxyl group strongly activates the ring by resonance donation. Hence, the correct option is (B).

Quick Tip to Solve: Compare net activation rather than orientation: hydroxyl strongly activates, methyl weakly activates, and halogens direct ortho/para while deactivating.

70. Given below are two statements :

Statement (I) : A solution of $[\mathrm{Ni}\,(\mathrm{H_2O})_6]^{2+}$ is green in colour.

Statement (II) : A solution of $[\mathrm{Ni(CN)_4}]^{2-}$ is colourless.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is incorrect but Statement II is correct

(B) Both Statement I and Statement II are correct

(C) Both Statement I and Statement II are incorrect

(D) Statement I is correct but Statement II is incorrect

Correct Answer: (D) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- Nickel is in oxidation state $+2$ in both complexes, so the central ion has the $3d^8$ configuration. The colour depends on the ligand field and the electronic transition energies, not on oxidation state alone.
- Water is a weaker-field ligand than cyanide. Changing the ligand and coordination geometry changes the splitting of the metal d orbitals and hence the wavelengths absorbed by the complex.

Step 1: Assess the aqueous hexaaquanickel ion The six water ligands give $[\mathrm{Ni(H_2O)_6}]^{2+}$ an octahedral ligand environment. For the d^8 ion its octahedral distribution is $t_{2g}^6 e_g^2$. Electronic transitions between ligand-field levels absorb parts of visible light, and the transmitted light gives the familiar green colour of the aqueous hexaaquanickel(II) ion. Thus Statement (I) is correct. The presence of water ligands is significant: the colour should not be inferred merely from the metal name.

Step 2: Assess the strong-field cyanide complex Cyanide gives the square-planar, diamagnetic d^8 complex $[\mathrm{Ni(CN)_4}]^{2-}$, but paired electrons do not make a complex necessarily colourless. Electrons can still be promoted from filled orbitals to empty levels. Tetracyanonickelate(II) has weak visible absorption and can give a pale yellow solution. For example, its aqueous absorption band near $22500\,\mathrm{cm^{-1}}$ corresponds to

$$\lambda = \frac{10^7}{22500}\,\mathrm{nm} \simeq 444\,\mathrm{nm},$$

which lies in the visible violet-blue region. Absorption there is consistent with a yellow transmitted colour. Thus Statement (II), which calls the solution colourless, is not strictly correct. Very weak absorption can make a dilute solution look nearly colourless, but that does not establish the unqualified statement. The visible absorption of the aqueous ion is documented by Oppenheim and co-workers, *Inorganic Chemistry* (2019), DOI: 10.1021/acs.inorgchem.9b02135.

Step 3: Conclusion Statement (I) is correct, whereas Statement (II) is incorrect for the literal chemical claim: $[\mathrm{Ni(CN)_4}]^{2-}$ has weak visible absorption and is pale coloured, despite being diamagnetic. Hence, the correct option is (D). This differs from the supplied final key, which accepts both statements as correct; that keyed choice uses the simplified colourless description and overlooks the weak visible absorption.

Quick Tip to Solve: Do not infer colourlessness from diamagnetism: even a fully paired ground state may have electronic excitations in the visible region.

71. We have three aqueous solutions of NaCl labelled as ' A ', ' B ' and ' C ' with concentration $0.1\mathrm{M}$, $0.01\mathrm{M}$ and $0.001\mathrm{M}$, respectively. The value of van 't Hoff factor(i) for these solutions will be in the order : [4 marks]

(A) $\mathrm{i_A} < \mathrm{i_C} < \mathrm{i_B}$

(B) $\mathrm{i_A} < \mathrm{i_B} < \mathrm{i_C}$

(C) $\mathrm{i_A} > \mathrm{i_B} > \mathrm{i_C}$

(D) $\mathrm{i_A} = \mathrm{i_B} = \mathrm{i_C}$

Correct Answer: (B) $\mathrm{i_A} < \mathrm{i_B} < \mathrm{i_C}$

Solution:

Concept:

- The van't Hoff factor i is the ratio of the observed colligative effect to the value calculated for an undissociated solute. It measures the effective number of solute particles.
- For dissociation of one formula unit into two ions, the ideal relation is $i = 1 + \alpha$, where α denotes the degree of dissociation. Ionic interactions reduce the effective particle count at appreciable concentrations.

Step 1: Identify the particles produced Sodium chloride is a strong electrolyte and produces two kinds of ions in water:

$$\mathrm{NaCl} \longrightarrow \mathrm{Na^+ + Cl^-}.$$

If all ions behaved independently, one mole of salt would give two moles of particles and the limiting factor would be $i = 2$. This limit is approached at infinite dilution; it need not be the observed value at each finite concentration.

Step 2: Compare the effect of dilution The concentrations satisfy $c_A > c_B > c_C$. At the larger concentration, electrostatic attractions and ion association make the ions behave less independently. Dilution weakens these interionic effects, so the effective van't Hoff factor increases towards its limiting value. Consequently the most dilute solution C has the greatest factor and the most concentrated solution A has the smallest:

$$i_A < i_B < i_C.$$

The order refers to the factors, not to the absolute osmotic pressures of the three solutions.

Step 3: Eliminate the alternatives Option (A) incorrectly places B above the more dilute C. Option (C) reverses the dilution trend. Option (D) would treat every solution as an ideal, completely independent-ion solution and ignores the concentration dependence tested here. Only option (B) has the increasing order with dilution.

Step 4: Conclusion The required order is $\boxed{i_A < i_B < i_C}$. Hence, the correct option is (B).

Quick Tip to Solve: For the same electrolyte at different concentrations, dilution makes the measured van't Hoff factor approach the number of ions per formula unit; for sodium chloride that limit is 2.

72. Given below are two statements :

Statement (I) : The NH_2 group in Aniline is ortho and para directing and a powerful activating group.

Statement (II) : Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation).

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- Aniline contains an amino group, $-NH_2$, whose nitrogen lone pair can conjugate with the aromatic ring. Its resonance donation increases electron density at the ortho and para positions.
- Friedel–Crafts alkylation and acylation normally require a Lewis acid such as $AlCl_3$. A basic amino group interacts strongly with this catalyst, preventing the usual ring substitution.

Step 1: Check Statement I In free aniline the lone pair on nitrogen overlaps with the benzene pi system. Resonance donation stabilises the intermediates for electrophilic attack at the ortho and para positions more effectively than for meta attack. The positive resonance effect outweighs the amino group's inductive withdrawal in this context. Thus the amino group is both strongly activating and ortho/para directing, so Statement I is correct.

Step 2: Check Statement II under Friedel–Crafts conditions The nitrogen atom is a Lewis base and donates its lone pair to the Lewis-acid catalyst, $AlCl_3$. Formation of the resulting salt-like Lewis acid–base complex ties up the lone pair and strongly deactivates the ring under the reaction conditions. Consequently aniline does not undergo the usual Friedel–Crafts alkylation or acylation. Statement II is also correct. This behaviour concerns the catalyst conditions; it does not contradict activation by the free amino group.

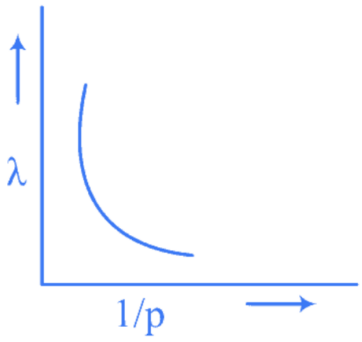
Step 3: Choose the combined truth value Both statements have been established independently as correct. Option (B) rejects both and is inconsistent with resonance donation and catalyst complexation. Option (C) incorrectly accepts Friedel–Crafts reaction of aniline, whereas option (D) incorrectly denies activation by the free amino group. The combination in option (A) is the only appropriate one.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

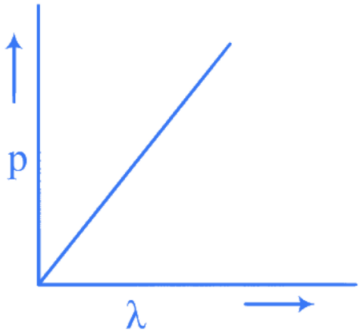
Quick Tip to Solve: For aniline distinguish the free amino group, which activates the ring, from its interaction with a Lewis-acid catalyst, which prevents ordinary Friedel–Crafts substitution.

73. According to the wave-particle duality of matter by de-Broglie, which of the following graph plot presents most appropriate relationship between wavelength of electron (λ) and momentum of electron (p)? [4 marks]

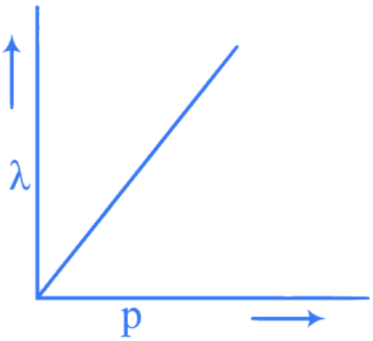
(A)



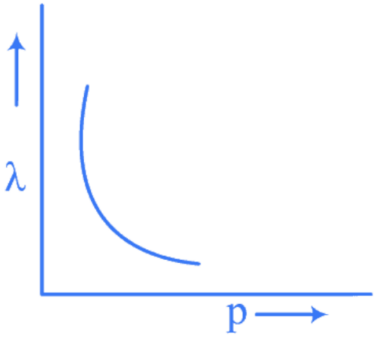
(B)



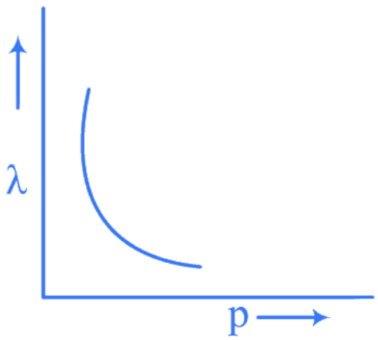
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- De Broglie's relation for a matter wave is $\lambda = h/p$, where λ is wavelength, p is the magnitude of momentum and h is Planck's constant.
- For positive momentum, wavelength is positive and inversely proportional to momentum. A plot of one variable against the reciprocal of the other must instead be a straight line through the origin.

Step 1: Determine the wavelength–momentum curve Rearrange the de Broglie relation as

$$\lambda p = h.$$

The product remains constant, so doubling the momentum halves the wavelength. In a graph of λ on the vertical axis against p on the horizontal axis, the curve decreases in the first quadrant and has the shape of a rectangular hyperbola. The curve approaches the axes without crossing them.

Step 2: Check the axes of every option Option (A) uses $1/p$ on the horizontal axis, but draws a decreasing curve. For those axes the equation is $\lambda = h(1/p)$, which requires a rising straight line, so (A) is wrong. Option (B) plots p against λ as a rising straight line, whereas $p = h/\lambda$ requires an inverse curve. Option (C) plots λ against p as a rising straight line, again contradicting inverse proportionality. Option (D) has the correct axes and the required decreasing hyperbola.

Step 3: Confirm the limiting behaviour As p becomes large, h/p becomes small and the graph tends towards the horizontal axis. As positive p becomes very small, the wavelength increases and the curve rises steeply. Both limiting behaviours agree with option (D). The distinction from option (A) depends on reading its reciprocal-momentum axis, rather than selecting a graph solely by its shape.

Step 4: Conclusion The decreasing rectangular hyperbola of λ against p represents $\lambda = h/p$. Hence, the correct option is (D).

Quick Tip to Solve: Read the horizontal-axis label before identifying a graph: λ versus p is a hyperbola, but λ versus $1/p$ is a straight line.

74. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): PH_3 **has lower boiling point than** NH_3**.**

Reason (R) : In liquid state NH_3 **molecules are associated through vander Waal's forces, but** PH_3 **molecules are associated through hydrogen bonding.**

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(B) (A) is not correct but (R) is correct

(C) (A) is correct but (R) is not correct

(D) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Correct Answer: (C) (A) is correct but (R) is not correct

Solution:

Concept:

- Boiling requires overcoming intermolecular attractions. Stronger attractions generally raise the boiling point when comparing related molecular substances.
- Conventional hydrogen bonding requires hydrogen bonded to a small, strongly electronegative atom such as nitrogen, oxygen or fluorine. Ammonia satisfies this condition; phosphine does not show comparable hydrogen bonding.

Step 1: Evaluate the assertion Ammonia contains polar $N-H$ bonds and a nitrogen lone pair, so neighbouring molecules associate through intermolecular hydrogen bonds. In phosphine, the $P-H$ bonds do not provide this strong association. Although phosphine is heavier and has dispersion attractions, ammonia's hydrogen bonding raises its boiling point above that of phosphine. Thus the assertion that PH_3 has a lower boiling point than NH_3 is correct.

Step 2: Evaluate the reason The reason assigns the important intermolecular forces the wrong way round. Liquid ammonia does have ordinary dispersion forces, as all molecules do, but its characteristic additional association is hydrogen bonding. Phosphine does not form the hydrogen-bonded network claimed in the reason. Therefore the stated reason is false; it cannot explain the true boiling-point comparison.

Step 3: Match the truth values to the options The result is A true and R false. Options (A) and (D) both require the reason to be correct and must be rejected. Option (B) reverses the two truth values, claiming a false assertion and a true reason. Option (C) states the required combination. A true assertion does not make a reason true merely because the reason mentions relevant intermolecular forces.

Step 4: Conclusion The assertion is correct, but the reason is not correct. Hence, the correct option is (C).

Quick Tip to Solve: In the group-15 hydrides, identify the exceptional hydrogen bonding of ammonia before applying a simple molar-mass trend to boiling points.

75. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Haloalkanes react with KCN to form alkyl cyanides as a main product while with $AgCN$ form isocyanide as the main product.

Reason (R) : KCN and $AgCN$ both are highly ionic compounds.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) (A) is correct but (R) is not correct

(B) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(C) (A) is not correct but (R) is correct

(D) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Correct Answer: (A) (A) is correct but (R) is not correct

Solution:

Concept:

- The cyanide ion is an ambident nucleophile: bond formation can occur through carbon or nitrogen. Carbon attack gives an alkyl cyanide, while nitrogen attack gives an isocyanide.
- The bonding in the cyanide reagent controls which end attacks. Potassium cyanide is predominantly ionic; silver cyanide is predominantly covalent, so the two reagents are not equivalent.

Step 1: Check the potassium cyanide reaction In solution, ionic KCN supplies cyanide ions. Attack through the carbon end gives the principal product $R-CN$, an alkyl cyanide or nitrile:

$$R-X + KCN \longrightarrow R-CN + KX.$$

The new bond links the alkyl carbon to the carbon of cyanide. Thus the first part of the assertion correctly describes the usual product of haloalkanes with potassium cyanide.

Step 2: Check the silver cyanide reaction Silver cyanide is predominantly covalent. Its carbon end is involved in bonding to silver, so nitrogen is the accessible end for attack on the haloalkane. This gives an isocyanide as the principal product:

$$R-X + AgCN \longrightarrow R-NC + AgX.$$

Here the alkyl group is bonded to nitrogen. Hence the second part is also correct and the assertion as a whole is true.

Step 3: Judge the reason and reject the other choices The reason says that both cyanides are highly ionic. This is incorrect because $AgCN$ is predominantly covalent. Indeed, the contrast in their bonding helps account for their different major products. Thus the reason is false. Options (B) and (D) incorrectly regard it as true, while option (C) also incorrectly rejects the assertion. The combination in option (A), assertion true and reason false, is the required one.

Step 4: Conclusion Assertion (A) is correct, but Reason (R) is not correct. Hence, the correct option is (A).

Quick Tip to Solve: Follow the connecting atom: $R-CN$ is a cyanide from ionic potassium cyanide, whereas $R-NC$ is an isocyanide from predominantly covalent silver cyanide.

76. If one strand of a DNA has the sequence ATGCTTCA, sequence of the bases in complementary strand is : [4 marks]

(A) CATTAGCT

(B) TACGAAGT

(C) ATGCGACT

(D) GTACTTAC

Correct Answer: (B) TACGAAGT

Solution:

Concept:

- The two DNA strands are held together by specific complementary base pairing: adenine pairs with thymine, and guanine pairs with cytosine.
- The strands run antiparallel. When the bases of one strand are written in a given left-to-right order, its aligned complementary bases are obtained position by position; reversing the order is a separate operation.

Step 1: Apply the base-pairing rule at each position The given sequence contains eight bases: A, T, G, C, T, T, C, A. Replace each A by T, each T by A, each G by C and each C by G without changing the order of positions. The first four bases ATGC therefore pair with TACG. The last four bases TTCA pair with AAGT. Combining these two blocks gives TACGAAGT.

Step 2: Check the complete alignment The complementary pairing can be displayed as

Given	A	T	G	C	T	T	C	A
Complement	T	A	C	G	A	A	G	T

Every column contains either an A–T pair or a G–C pair. If the upper strand is read from $5'$ to $3'$, the lower aligned strand is read from $3'$ to $5'$. The question asks for the complementary sequence in this aligned sense, as shown among the choices, rather than an explicitly requested reverse complement.

Step 3: Compare all four choices Option (B), TACGAAGT, agrees at all eight positions. Option (A) begins with C opposite the initial A, which violates A–T pairing. Option (C) begins with A opposite A and is also impossible. Option (D) begins with G opposite A and fails the same first-position test. Thus the first base already rejects the three distractors, and the full alignment verifies the selected sequence.

Step 4: Conclusion The complementary sequence is $\boxed{\text{TACGAAGT}}$. Hence, the correct option is (B).

Quick Tip to Solve: Write a partner directly under each DNA base before considering strand direction. Reverse the result only when both sequences are explicitly requested in the same $5'$-to-$3'$ direction.

77. In acidic medium, $K_2Cr_2O_7$ shows oxidising action as represented in the half reaction:

$$Cr_2O_7^{2-} + XH^+ + Ye^{\ominus} \rightarrow 2\ A + ZH_2O$$

X, Y, Z and A are respectively are : [4 marks]

(A) $14, 7, 6$ and Cr^{3+}

(B) $14, 6, 7$ and Cr^{3+}

(C) $8, 4, 6$ and Cr_2O_3

(D) $8, 6, 4$ and Cr_2O_3

Correct Answer: (B) $14, 6, 7$ and Cr^{3+}

Solution:

Concept:

- In acidic solution, dichromate is reduced from chromium(VI) to chromium(III). Each chromium atom gains three electrons, and one dichromate ion contains two chromium atoms.
- Balance a half-reaction by conserving each element first: add water for oxygen, hydrogen ions for hydrogen, and electrons last to balance the total electric charge.

Step 1: Identify the chromium product In $Cr_2O_7^{2-}$, the seven oxygen atoms contribute a total oxidation number of -14. Since the ion has charge -2, the two chromium atoms together contribute $+12$, or $+6$ each. The acidic reduction product is Cr^{3+}, so $A = Cr^{3+}$. Two product chromium ions conserve the two chromium atoms.

Step 2: Balance oxygen and hydrogen The reactant contains seven oxygen atoms. Add seven water molecules on the product side so that all oxygen atoms are conserved. These waters contain fourteen hydrogen atoms, requiring fourteen hydrogen ions on the reactant side:

$$Cr_2O_7^{2-} + 14\,H^+ \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

Comparing coefficients with the question gives $X = 14$ and $Z = 7$.

Step 3: Balance charge and test the alternatives Before electrons are included, the left-hand charge is $-2 + 14 = +12$, while the right-hand charge is $2(3) = +6$. Add six electrons to the left to lower its charge to $+6$:

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

Hence $Y = 6$, consistent with two chromium atoms each gaining three electrons. Option (A) interchanges the electron and water coefficients and fails charge and oxygen balance. Options (C) and (D) use the wrong chromium product and coefficients for this acidic half-reaction. Only (B) conserves all atoms and charge.

Step 4: Conclusion $\boxed{X = 14,\ Y = 6,\ Z = 7,\ A = Cr^{3+}}$. Hence, the correct option is (B).

Quick Tip to Solve: For one acidic dichromate reduction, remember the checks rather than only the numbers: seven oxygen atoms require seven waters, fourteen hydrogens require fourteen protons, and two Cr(VI) atoms require six electrons.

78. Given below are two statements :

Statement (I) : Aminobenzene and aniline are same organic compounds.

Statement (II) : Aminobenzene and aniline are different organic compounds.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct

(B) Statement I is incorrect but Statement II is correct

(C) Statement I is correct but Statement II is incorrect

(D) Both Statement I and Statement II are incorrect

Correct Answer: (C) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- An aromatic primary amine contains an amino group, $-NH_2$, directly attached to an aromatic ring. Its identity is determined by its connectivity, not by which valid name is used.
- Aniline is the retained common name for $C_6H_5NH_2$. Aminobenzene is a substitutive name describing a benzene ring bearing one amino group.

Step 1: Translate each name into a structure Begin with benzene, C_6H_6. Replacing one ring hydrogen by $-NH_2$ produces $C_6H_5NH_2$, called aminobenzene. The name aniline refers to exactly this same molecule: nitrogen is bonded directly to a ring carbon and carries two hydrogens. No different position or additional substituent is specified by either name.

Step 2: Judge both statements separately Statement I says the two names denote the same organic compound. This is true because both describe the same atomic connections and the same functional group. Statement II claims that they are different compounds. This is false: a common name and a systematic descriptive name do not create structural isomers. In particular, aminobenzene must not be confused with benzylamine, $C_6H_5CH_2NH_2$, which has an intervening methylene group and therefore is a different compound.

Step 3: Eliminate the other truth combinations Option A incorrectly treats Statement II as true. Option B reverses the two truth values. Option D incorrectly rejects Statement I. Only the combination "Statement I correct; Statement II incorrect" agrees with the structural identification above.

Step 4: Conclusion Statement I is correct and Statement II is incorrect. Both names denote $C_6H_5NH_2$. Hence, the correct option is (C).

Quick Tip to Solve: When comparing two organic names, first write their connectivity; aniline and aminobenzene both have nitrogen directly bonded to the ring.

79. Match List I with List II.

List I (Reactions)	List II (Reagents)
(A) $CH_3(CH_2)_5COOC_2H_5 \longrightarrow CH_3(CH_2)_5CHO$	**(I)** CH_3MgBr, H_2O
(B) $C_6H_5COC_6H_5 \longrightarrow C_6H_5CH_2C_6H_5$	**(II)** $Zn(Hg)$ **and conc.** HCl
(C) $C_6H_5CHO \longrightarrow C_6H_5CH(OH)CH_3$	**(III)** $NaBH_4$, H^+
(D) $CH_3COCH_2COOC_2H_5 \longrightarrow CH_3CH(OH)CH_2COOC_2H_5$	**(IV) DIBAL-H,** H_2O

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(B) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(C) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(D) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Correct Answer: (C) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Solution:

Concept:

- Identify the functional-group change before selecting a reagent: ester to aldehyde, carbonyl to methylene, carbon-carbon bond formation, and selective ketone reduction are different operations.
- DIBAL-H partially reduces an ester to an aldehyde under controlled conditions. Clemmensen reduction uses $Zn(Hg)$ with concentrated HCl.

Step 1: Match the ester and deoxygenation reactions In A the ester $CH_3(CH_2)_5COOC_2H_5$ becomes the aldehyde $CH_3(CH_2)_5CHO$, without changing the alkyl skeleton. DIBAL-H followed by hydrolysis effects this controlled partial reduction, so A matches IV. In B the carbonyl group of benzophenone becomes $-CH_2-$, giving diphenylmethane. That is Clemmensen reduction, so B matches II. A reagent that stops at an alcohol would not achieve B.

Step 2: Match carbon addition and selective reduction In C benzaldehyde becomes $C_6H_5CH(OH)CH_3$. A new methyl group has been added to the carbonyl carbon. CH_3MgBr supplies that carbon nucleophile; water then protonates the alkoxide, so C matches I. In D the ketone group of ethyl acetoacetate is converted into a secondary alcohol while its ester group remains intact. Mild $NaBH_4$, followed by acidic work-up, gives this selectivity, so D matches III.

Step 3: Compare the complete matching patterns The mapping is A-IV, B-II, C-I, D-III. Option A wrongly assigns borohydride to the ester-to-aldehyde transformation and DIBAL-H to B. Option B wrongly assigns borohydride to C, even though a new carbon-carbon bond must be made. Option D wrongly assigns Clemmensen conditions to D, which would not give the stated hydroxy ester. Only option C preserves all four transformations.

Step 4: Conclusion The matches are A-IV, B-II, C-I, and D-III. Hence, the correct option is (C).

Quick Tip to Solve: First check whether a new carbon-carbon bond is formed: that immediately identifies the Grignard step C-I; then separate full from partial reduction.

80. Arrange the bonds in order of increasing ionic character in the molecules. LiF, K_2O, N_2, SO_2 **and** ClF_3 **:** [4 marks]

(A) $N_2 < SO_2 < ClF_3 < K_2O < LiF$

(B) $ClF_3 < N_2 < SO_2 < K_2O < LiF$

(C) $LiF < K_2O < ClF_3 < SO_2 < N_2$

(D) $N_2 < ClF_3 < SO_2 < K_2O < LiF$

Correct Answer: The supplied key accepts A; modern Pauling electronegativities give D, so the close S-O versus Cl-F comparison is scale-dependent unless a table is specified.

Solution:

Concept:

- For the electronegativity criterion used to compare bond polarity, increasing $\Delta\chi = |\chi_A - \chi_B|$ gives increasing ionic character. Identical atoms have $\Delta\chi = 0$.
- Using modern Pauling values, $\chi(\mathrm{N}) = 3.04$, $\chi(\mathrm{Cl}) = 3.16$, $\chi(\mathrm{F}) = 3.98$, $\chi(\mathrm{S}) = 2.58$, $\chi(\mathrm{O}) = 3.44$, $\chi(\mathrm{K}) = 0.82$, and $\chi(\mathrm{Li}) = 0.98$.

Step 1: Compare the covalent bonds quantitatively The nitrogen-nitrogen bond is nonpolar because both atoms have the same electronegativity. The two close comparisons are

$$\Delta\chi(\mathrm{Cl} - \mathrm{F}) = 3.98 - 3.16 = 0.82,$$

$$\Delta\chi(\mathrm{S} - \mathrm{O}) = 3.44 - 2.58 = 0.86.$$

Thus modern Pauling values place the chlorine-fluorine bond slightly below the sulphur-oxygen bond in ionic character. The molecular geometry does not change this comparison of individual bond polarities; a zero molecular dipole and a nonpolar individual bond are different ideas.

Step 2: Compare the metal-nonmetal bonds For potassium oxide the electronegativity difference is $3.44 - 0.82 = 2.62$; for lithium fluoride it is $3.98 - 0.98 = 3.00$. Both exceed the nonmetal-nonmetal differences, and LiF is the more ionic of these two in this criterion. Combining all five gives

$$N_2 < ClF_3 < SO_2 < K_2O < LiF.$$

Option B puts a heteronuclear bond below nonpolar nitrogen; option C reverses the strongly ionic end; option A interchanges the two closely spaced nonmetal bonds.

Step 3: Distinguish the numerical result from the supplied key The supplied answer key selects A, which orders sulphur-oxygen below chlorine-fluorine. The question does not specify an electronegativity table, and these two differences are very close. Coarse tabulations can obscure this distinction. With the explicit modern Pauling values above, however, $0.82 < 0.86$, so the calculated order is D rather than A. The numerical comparison is stated openly instead of attributing the key's order to values that do not support it.

Step 4: Conclusion The official accepted response is (A). With modern Pauling electronegativities, however, the calculated order is $N_2 < ClF_3 < SO_2 < K_2O < LiF$, corresponding to (D), since $\Delta\chi(\mathrm{Cl} - \mathrm{F}) = 0.82 < 0.86 = \Delta\chi(\mathrm{S} - \mathrm{O})$. The unspecified electronegativity convention makes this close comparison ambiguous; the key and the explicitly calculated ordering are distinguished here.

Quick Tip to Solve: When two bond polarities are almost equal, write the electronegativity values and their differences explicitly; avoid deciding from the elements alone.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The number of white coloured salts, among the following is

(a) $SrSO_4$

(b) $Mg(NH_4)PO_4$

(c) $BaCrO_4$

(d) $Mn(OH)_2$

(e) $PbSO_4$

(f) $PbCrO_4$

(g) $AgBr$

(h) PbI_2

(i) CaC_2O_4

(j) $[Fe(OH)_2(CH_3COO)]$ [4 marks]

Solution:

Concept:

- The observed colour belongs to the complete precipitate or salt, not merely to whether its metal is a transition element. Qualitative analysis uses characteristic precipitate colours.
- Freshly precipitated $Mn(OH)_2$ is white or very pale and later darkens in air through oxidation; the initial precipitate is the one classified here.

Step 1: Identify the white precipitates Strontium sulphate, $SrSO_4$, is white. Magnesium ammonium phosphate, $MgNH_4PO_4$, is white and is the familiar confirmatory precipitate for magnesium. Fresh $Mn(OH)_2$ is classified as white; its later brown oxidation products should not be substituted for the compound asked about. Lead sulphate, $PbSO_4$, is white. Calcium oxalate, CaC_2O_4, is also white and is used in identifying calcium ions. Thus entries (a), (b), (d), (e), and (i) qualify.

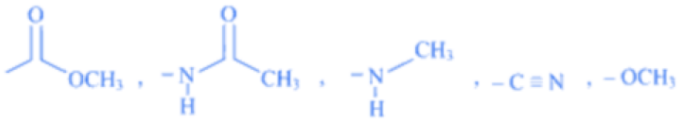
Step 2: Exclude the coloured compounds Barium chromate, $BaCrO_4$, and lead chromate, $PbCrO_4$, are yellow because of the chromate species. Silver bromide, $AgBr$, gives a pale-yellow or cream precipitate, rather than a white one; white silver chloride is a different salt. Lead iodide, PbI_2, is yellow. The basic ferric acetate species $[Fe(OH)_2(CH_3COO)]$ is reddish-brown, not white. These are entries (c), (f), (g), (h), and (j), so none adds to the required count.

Step 3: Count the selected entries There are five white compounds in the selected list. The count concerns the ten named substances, so a compound is counted once even when several of its constituent ions are colourless. No inference from solubility alone is needed: insoluble precipitates can be white or strongly coloured.

Step 4: Conclusion The white compounds are (a), (b), (d), (e), and (i). Their number is $\boxed{5}$ (a dimensionless count).

Quick Tip to Solve: Distinguish white AgCl from cream AgBr, and classify fresh manganese(II) hydroxide before its air oxidation darkens the precipitate.

82. Total number of deactivating groups in aromatic electrophilic substitution reaction among the following is _____ . [4 marks]



Solution:

Concept:

- An activating group donates electron density to the aromatic ring and accelerates electrophilic substitution relative to benzene. A deactivating group withdraws electron density and slows the reaction.
- The atom directly attached to the ring is crucial: $-\mathrm{COOCH_3}$ attaches through a carbonyl carbon, whereas $-\mathrm{NHCOCH_3}$ attaches through nitrogen and can donate a lone pair.

Step 1: Read the five substituents from the structures The groups shown, in order, are $-\mathrm{COOCH_3}$, $-\mathrm{NHCOCH_3}$, $-\mathrm{NHCH_3}$, $-\mathrm{C{\equiv}N}$, and $-\mathrm{OCH_3}$. In the first group, the ring is bonded to the carbonyl carbon; it is not bonded through oxygen. This distinction separates an electron-withdrawing ester substituent from an electron-donating alkoxy substituent.

Step 2: Classify withdrawal and donation The ester group $-\mathrm{COOCH_3}$ withdraws through its carbonyl system by resonance as well as induction, so it deactivates the ring. The cyano group $-\mathrm{C{\equiv}N}$ also withdraws by resonance and induction and is deactivating. The nitrogen lone pair of $-\mathrm{NHCOCH_3}$ can still donate into the ring, although delocalisation toward the carbonyl makes this donation weaker than in an amino group; it remains activating. The groups $-\mathrm{NHCH_3}$ and $-\mathrm{OCH_3}$ donate lone-pair density into the aromatic system and are activating under the usual electrophilic-substitution classification.

Step 3: Count only the deactivating entries Exactly the first and fourth substituents meet the requirement. Do not count every group containing oxygen or nitrogen as deactivating: resonance donation depends on which atom attaches to the ring and whether an available lone pair is conjugated with it.

Step 4: Conclusion The deactivating groups are $-\mathrm{COOCH_3}$ and $-\mathrm{C{\equiv}N}$, giving $\boxed{2}$ groups (a dimensionless count).

Quick Tip to Solve: Inspect the atom bonded to the aromatic ring: a carbonyl carbon withdraws, while directly attached N or O with an available lone pair generally donates.

83. The potential for the given half cell at $298\ \mathrm{K}$ is (-) _____ $\times 10^{-2}\ \mathrm{V}$

$$2\mathrm{H}^{+}_{(\mathrm{aq})} + 2\mathrm{e}^{-} \longrightarrow \mathrm{H}_2(\mathrm{g})$$

$$\left[\mathrm{H}^{+}\right] = 1\mathrm{M}, \mathrm{P}_{\mathrm{H}_2} = 2\ \mathrm{atm}$$

(Given : $2.303\mathrm{RT/F} = 0.06\ \mathrm{V}, \log 2 = 0.3$ **)** [4 marks]

Solution:

Concept:

- For a reduction half-cell at $298\,\mathrm{K}$, $E = E^\circ - (0.06/n)\log Q$ using the logarithmic factor specified in the question. Here $n = 2$.
- The standard hydrogen-electrode potential is $E^\circ = 0\,\mathrm{V}$. For $2\,\mathrm{H}^+ + 2\,\mathrm{e}^- \longrightarrow \mathrm{H}_2$, the reaction quotient is $Q = P_{\mathrm{H}_2}/[\mathrm{H}^+]^2$, with pressure and activity referred to their standard values.

Step 1: Form the reaction quotient The product is hydrogen gas and the reactant is the hydrogen ion. Electrons are omitted from the reaction quotient. At the stated acid concentration and hydrogen pressure,

$$Q = \frac{2}{1^2} = 2.$$

The larger-than-standard product pressure makes reduction less favourable, so the electrode potential should be negative. This sign check helps avoid accidentally inverting the quotient.

Step 2: Substitute into the Nernst equation Two electrons occur in the balanced reduction, so divide the given logarithmic factor by two:

$$E = 0 - \frac{0.06}{2}\log 2 = -0.03(0.3) = -0.009\,\mathrm{V}.$$

Thus the potential is $-9 \times 10^{-3}\,\mathrm{V}$. Keeping the factor of two is essential; using $n = 1$ would double the magnitude without changing the sign.

Step 3: Convert to the requested numerical coefficient The paper gives the form $(-)x \times 10^{-2}\,\mathrm{V}$. Therefore $x = 0.009/0.01 = 0.9$. For the numerical-value entry rounded to the nearest integer, this becomes 1. The negative sign and the power of ten are already supplied outside the blank, so the entered number is positive; it is not -1.

Step 4: Conclusion The calculated half-cell potential is $E = -0.009\,\mathrm{V} = -0.9 \times 10^{-2}\,\mathrm{V}$. The requested coefficient rounds to $\boxed{1}$, with the voltage unit already supplied in the question.

Quick Tip to Solve: For the hydrogen reduction electrode, put hydrogen pressure in the numerator and remember that the balanced half-reaction transfers two electrons.

84. Among the following oxides of p-block elements, number of oxides having amphoteric nature is _____.

$$Cl_2O_7, CO, PbO_2,\ N_2O, NO, Al_2O_3, SiO_2,\ N_2O_5, SnO_2$$

[4 marks]

Solution:

Concept:

- An amphoteric oxide reacts with both acids and bases to form salts. Acidic oxides react mainly with bases, while neutral oxides do not show the ordinary acid-base oxide behaviour.
- The familiar amphoteric oxides in this list are those of aluminium, tin(IV), and lead(IV). Their metal centres can participate in both salt formation with acids and complex anion formation in alkali.

Step 1: Identify the amphoteric members The three amphoteric compounds are PbO_2, Al_2O_3, and SnO_2. Aluminium oxide provides a standard illustration:

$$Al_2O_3 + 6\,HCl \longrightarrow 2\,AlCl_3 + 3\,H_2O,$$

$$Al_2O_3 + 2\,NaOH + 3\,H_2O \longrightarrow 2\,Na[Al(OH)_4].$$

These reactions show that the same oxide responds to an acid and a strong base. Tin and lead oxides similarly show amphoteric behaviour in the usual qualitative classification; the oxidation state alone does not make every oxide acidic.

Step 2: Classify the six remaining oxides Cl_2O_7 is acidic, corresponding to perchloric acid. N_2O_5 is acidic, corresponding to nitric acid. SiO_2 is an acidic oxide and reacts with strong alkali to form silicates; it is not classified as amphoteric merely because it has a network structure. The oxides CO, N_2O, and NO are neutral under the standard acid-base classification. Neutral is a separate category and must not be interpreted as amphoteric.

Step 3: Count the selected oxides From the nine entries, only lead dioxide, aluminium oxide, and tin dioxide satisfy the definition. Each formula is counted once. The six other entries are fully accounted for as three acidic and three neutral oxides, providing a check that no listed compound was omitted.

Step 4: Conclusion The amphoteric oxides are PbO_2, Al_2O_3, and SnO_2. Their number is $\boxed{3}$ (a dimensionless count).

Quick Tip to Solve: Keep neutral and amphoteric oxides separate: CO, NO, and nitrous oxide are neutral, whereas the listed Al, Sn, and Pb oxides react with both acids and bases.

85. Consider the following reaction :

$$3PbCl_2 + 2\,(NH_4)_3\,PO_4 \rightarrow Pb_3\,(PO_4)_2 + 6NH_4Cl$$

If 72 mmol **of** $PbCl_2$ **is mixed with** 50 mmol **of** $(NH_4)_3\,PO_4$**, then the amount of** $Pb_3\,(PO_4)_2$ **formed is _____ mmol (nearest integer).** [4 marks]

Solution:

Concept:

- A balanced equation gives the mole ratio of the reacting substances. The coefficients apply equally to moles and millimoles, provided the same amount unit is used throughout.
- The limiting reagent is identified by the smaller value of amount divided by stoichiometric coefficient. It fixes the maximum product amount, even when some of the other reactant remains.

Step 1: Write the required mole ratio The reaction is

$$3\,PbCl_2 + 2\,(NH_4)_3PO_4 \longrightarrow Pb_3(PO_4)_2 + 6\,NH_4Cl.$$

Thus three millimoles of lead(II) chloride react with two millimoles of ammonium phosphate to give one millimole of lead(II) phosphate. We must compare both reactants rather than simply choose the smaller given amount.

Step 2: Identify the limiting reagent The number of stoichiometric reaction units available from each reactant is

$$\frac{72}{3} = 24, \qquad \frac{50}{2} = 25.$$

The smaller value is 24, so $PbCl_2$ is limiting. Equivalently, consuming all 72 mmol of it requires

$$72\left(\frac{2}{3}\right) = 48 \text{ mmol}$$

of ammonium phosphate, less than the 50 mmol supplied. The latter is therefore in excess by 2 mmol.

Step 3: Calculate the product amount The coefficient of $Pb_3(PO_4)_2$ is one. Using the limiting reactant gives

$$n\big(Pb_3(PO_4)_2\big) = 72\left(\frac{1}{3}\right) = 24 \text{ mmol}.$$

Using the excess reactant alone would give 25 mmol and overestimate the yield, because there is insufficient lead chloride for that amount. The calculated amount is already an integer.

Step 4: Conclusion The amount of $Pb_3(PO_4)_2$ formed is $\boxed{24 \text{ mmol}}$. The required numerical entry is $\boxed{24}$.

Quick Tip to Solve: Divide each reactant amount by its balanced coefficient first; the smaller quotient, multiplied by the product coefficient, gives the product amount.

86. The number of molecules/ion/s having trigonal bipyramidal shape is _____.

$$PF_5, BrF_5, PCl_5, [PtCl_4]^{2-}, BF_3, Fe(CO)_5$$

[4 marks]

Solution:

Concept:

- Molecular shape describes the positions of atoms around the central atom; it is distinct from electron-pair geometry when lone pairs are present.
- A trigonal bipyramid has five bonded positions: three equatorial and two axial. Five-coordinate species must be distinguished from square-pyramidal species and from four- or three-coordinate species.

Step 1: Classify the phosphorus halides In PF_5, phosphorus forms five P–F bonds and has no lone pair on the central atom in the usual molecular description. Its five bonding domains adopt a trigonal bipyramidal arrangement. Molecular PCl_5 has the same five-coordinate arrangement and also belongs in the count. These contribute two species.

Step 2: Exclude the nonmatching shapes In BrF_5, bromine has five bonding pairs and one lone pair. Six electron domains have an octahedral electron-pair arrangement, but the molecular shape after omitting the lone-pair position is square pyramidal, not trigonal bipyramidal. The complex $[PtCl_4]^{2-}$ is square planar: Pt(II) is a four-coordinate d^8 metal centre. Finally, BF_3 has three B–F bonds and no central lone pair, so it is trigonal planar. None of these three species is counted.

Step 3: Include the metal carbonyl and total $Fe(CO)_5$ has five carbonyl ligands around iron and adopts trigonal bipyramidal geometry, with two axial and three equatorial ligands. It therefore contributes one further species. The complete matching set is

$$PF_5, \quad PCl_5, \quad Fe(CO)_5.$$

The total is $2 + 1 = 3$; having five ligands alone would not have justified including BrF_5.

Step 4: Conclusion There are $\boxed{3}$ trigonal bipyramidal molecules among the listed species. This count is dimensionless.

Quick Tip to Solve: Count central lone pairs before naming a shape: five bonds plus one lone pair gives square pyramidal, whereas five bonds without a lone pair gives trigonal bipyramidal.

87. The lowest oxidation number of an atom in a compound A_2B is -2 . The number of electrons in its valence shell is _____. [4 marks]

Solution:

Concept:

- For a main-group nonmetal with v valence electrons, completing an octet requires $8 - v$ additional electrons.

- Its lowest oxidation state is correspondingly $-(8-v) = v-8$. Oxidation number is formal electron bookkeeping; it is not the total number of electrons in the atom.

Step 1: Relate the negative oxidation state to electron gain The given lowest oxidation number is -2. A negative oxidation state corresponds to formal gain of electrons relative to the neutral atom. Here the magnitude 2 means that two additional electrons complete the stable valence octet. The two electrons gained must be distinguished from the electrons already present in the neutral atom's outer shell.

Step 2: Solve for the original valence-shell population Let the required number of valence electrons be v. The octet relation gives

$$v + 2 = 8,$$

or, using the lowest oxidation-state formula,

$$v - 8 = -2.$$

Both expressions give

$$v = 6.$$

This is the usual group-16 valence configuration ns^2np^4, containing two electrons in the outer s subshell and four in the outer p subshell.

Step 3: Check the result against the compound formula A group-16 atom can attain the formal oxidation state -2 by completing its ns^2np^6 octet. In the formula A_2B, assigning -2 to B requires the two A atoms to contribute a total of $+2$, or $+1$ each, for an electrically neutral compound. Thus the formula is consistent with the stated negative oxidation state. It does not mean that the atom has two valence electrons; the subscript counts atoms, while the requested quantity counts electrons in the neutral atom's valence shell.

Step 4: Conclusion The atom has $\boxed{6}$ electrons in its valence shell. The numerical answer is dimensionless.

Quick Tip to Solve: For the lowest negative oxidation state, subtract its magnitude from eight: an oxidation state of -2 corresponds to six original valence electrons.

88. K_a **for** CH_3COOH **is** 1.8×10^{-5} **and** K_b **for** NH_4OH **is** 1.8×10^{-5}**. The** pH **of ammonium acetate solution will be _____.** [4 marks]

Solution:

Concept:

- Ammonium acetate, CH_3COONH_4, is a salt of the weak acid acetic acid and the weak base ammonium hydroxide. Both of its ions undergo hydrolysis in water.
- For such a salt, $\text{pH} = \frac{1}{2}(pK_w + pK_a - pK_b)$, where $pK_a = -\log K_a$, $pK_b = -\log K_b$, and $pK_w = 14$ at the usual room-temperature reference.

Step 1: Identify the competing hydrolysis processes The ammonium ion acts as an acid, while the

acetate ion acts as a base:

$$NH_4^+ + H_2O \rightleftharpoons NH_3 + H_3O^+,$$

$$CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + OH^-.$$

The first process tends to lower the pH and the second tends to raise it. We must compare their strengths rather than classify this salt using only one of its ions.

Step 2: Compare the acid and base constants The given values are identical:

$$K_a(CH_3COOH) = K_b(NH_4OH) = 1.8 \times 10^{-5}.$$

Therefore $pK_a = pK_b$, and their difference cancels exactly. Equivalently, the acidic strength of ammonium is K_w/K_b, while the basic strength of acetate is K_w/K_a. Since the denominators are equal, the two hydrolysis tendencies balance.

Step 3: Calculate the pH Substitution in the weak-acid weak-base salt expression gives

$$\text{pH} = \frac{1}{2}(14 + pK_a - pK_b) = \frac{1}{2}(14) = 7.$$

The solution is neutral at this reference temperature, rather than acidic merely because it contains ammonium or alkaline merely because it contains acetate. No concentration is needed here because the equality of the two constants removes the imbalance between the conjugate ions.

Step 4: Conclusion The ammonium acetate solution has $\boxed{\text{pH} = 7}$. The required numerical entry is $\boxed{7}$; pH is dimensionless.

Quick Tip to Solve: For a salt of a weak acid and a weak base, compare the parent constants first: equal K_a and K_b give neutral pH at the reference temperature.

89. Number of optical isomers possible for 2-chlorobutane _____. [4 marks]

Solution:

Concept:

- A tetrahedral carbon attached to four different substituents is a stereogenic centre. Such a centre can give a pair of nonsuperimposable mirror-image configurations.
- The maximum number of configurations for n independent stereogenic centres is 2^n. Any reduction due to internal symmetry or a meso form must be checked before counting optical isomers.

Step 1: Locate the potentially stereogenic carbon The condensed structural formula of 2-chlorobutane is

$$CH_3{-}CH(Cl){-}CH_2{-}CH_3.$$

Carbon 2 is bonded to chlorine, hydrogen, a methyl group and an ethyl group. These are four distinct substituents: in particular, CH_3 and CH_2CH_3 are different even though both begin with carbon. Thus carbon 2 is a stereogenic centre.

Step 2: Check the remaining carbon atoms The terminal methyl carbons each have three identical

hydrogen substituents, so neither is stereogenic. Carbon 3 is a methylene carbon with two hydrogens and likewise is not stereogenic. There is consequently just one stereogenic centre in the molecule. Rotation about a single bond changes a conformation, but does not create an additional configurational optical isomer.

Step 3: Count the mirror-image configurations With $n = 1$, the configuration count is

$$2^n = 2^1 = 2.$$

They are the (R) and (S) configurations at carbon 2, which form an enantiomeric pair. A one-centre molecule of this kind has no meso form to subtract: its four different substituents preclude the internal symmetry needed for achirality. A racemic mixture contains both enantiomers and has no net rotation, but it is not a third optical isomer.

Step 4: Conclusion The number of optical isomers of 2-chlorobutane is $\boxed{2}$. This is a dimensionless count.

Quick Tip to Solve: At the carbon bearing chlorine, list all four groups explicitly. One carbon with four different groups gives two enantiomers; a racemic mixture does not add another isomer.

90. The ratio of $\frac{^{14}C}{^{12}C}$ in a piece of wood is $\frac{1}{8}$ part that of atmosphere. If half life of ^{14}C is 5730 years, the age of wood sample is _____ years. [4 marks]

Solution:

Concept:

- Radioactive carbon-14 decays by first-order kinetics: $N = N_0 e^{-\lambda t}$, with $\lambda = \ln 2 / t_{1/2}$. Carbon-12 is stable.
- For radiocarbon dating, the carbon-14 to carbon-12 ratio in living material matches the atmospheric reference. After the wood stops exchanging carbon, the relative decrease in that ratio measures the surviving fraction of carbon-14.

Step 1: Convert the isotope ratio into a surviving fraction Let R_0 be the atmospheric $^{14}C/^{12}C$ ratio and R the present ratio in the wood. The question states

$$\frac{R}{R_0} = \frac{1}{8}.$$

Since the carbon-12 reference is stable, this equals the surviving fraction N/N_0 of carbon-14. We use the ratio relative to its initial value, not the absolute abundance of either isotope.

Step 2: Determine the number of half-lives After n half-lives, the surviving fraction is $(1/2)^n$. Therefore

$$\left(\frac{1}{2}\right)^n = \frac{1}{8} = \left(\frac{1}{2}\right)^3,$$

giving $n = 3$. As a direct check, the successive fractions are $1/2$, $1/4$ and $1/8$ after one, two and three half-lives, respectively.

Step 3: Calculate the age in years Each half-life lasts 5730 years, so the elapsed time is

$$t = n\,t_{1/2} = 3(5730) = 17190 \text{ years}.$$

The integrated first-order formula gives the same result:

$$t = \frac{t_{1/2}}{\ln 2} \ln\left(\frac{N_0}{N}\right) = \frac{5730}{\ln 2} \ln 8 = 3(5730).$$

The ratio is dimensionless and the half-life supplies the unit of years; no conversion to seconds is needed.

Step 4: Conclusion The wood sample is $\boxed{17190 \text{ years}}$ old. The required numerical entry is $\boxed{17190}$.

Quick Tip to Solve: When the surviving fraction is a simple power of one-half, count half-lives directly: $1/8 = (1/2)^3$, so multiply the half-life by three.