JEE Main 2024 Paper 1 (B.E./B.Tech) Feb 1 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 1 (Jan-Feb), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 1 February 2024, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the domain of the function

$f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}\left(x^2+2x-15\right)$ **is** $(-\infty, \alpha) \cup [\beta, \infty)$**, then** $\alpha^2 + \beta^3$ **is equal to :** [4 marks]

(A) 140

(B) 175

(C) 125

(D) 150

Correct Answer: (D) 150

Solution:

Concept:

- A real square root requires its radicand to be nonnegative, while a real logarithm requires its argument to be strictly positive.
- The domain of a sum is the intersection of the domains of its terms. A denominator must also be nonzero; endpoints must be checked in every term.

Step 1: Apply the square-root and denominator conditions For the first term to be real, we require

$$x^2 - 25 \geq 0 \quad \Longrightarrow \quad x \leq -5 \text{ or } x \geq 5.$$

The denominator vanishes only at $x = \pm 2$. These two values already lie outside the intervals just obtained, so they cause no further exclusions. A zero numerator is allowed provided the other conditions hold.

Step 2: Impose positivity of the logarithm Factor its argument before making a sign chart:

$$x^2 + 2x - 15 = (x+5)(x-3) > 0.$$

Both factors have the same sign when $x < -5$ or $x > 3$. The roots themselves are excluded because the logarithm of zero is undefined. Intersecting with the square-root condition gives

$$D_f = (-\infty, -5) \cup [5, \infty).$$

In particular, -5 is excluded by the logarithm, whereas 5 satisfies the logarithmic inequality and is included.

Step 3: Identify the endpoints and evaluate Comparing with the given domain, $\alpha = -5$ and $\beta = 5$. Thus

$$\alpha^2 + \beta^3 = (-5)^2 + 5^3 = 25 + 125 = 150.$$

The other choices 140, 175 and 125 do not equal this endpoint expression; 125 alone would omit the square of the left endpoint.

Step 4: Conclusion The required value is $\boxed{150}$. Hence, the correct option is (D).

Quick Tip to Solve: Intersect all domain conditions before deciding whether endpoints are open or closed; the logarithm excludes -5 even though the square root permits it.

2. If z is a complex number such that $|z| \leqslant 1$, then the minimum value of $\left|z + \frac{1}{2}(3+4i)\right|$ is : [4 marks]

(A) 2

(B) $\frac{5}{2}$

(C) $\frac{3}{2}$

(D) 3

Correct Answer: (C) $\frac{3}{2}$

Solution:

Concept:

- The modulus is distance in the complex plane. The constraint $|z| \leq 1$ describes the closed unit disk, so boundary points are permitted.
- The reverse triangle inequality gives $|z+w| \geq \big||w|-|z|\big|$. A sharp lower bound must be accompanied by an admissible equality case.

Step 1: Obtain a lower bound Put $w = (3+4i)/2$. Its modulus is

$$|w| = \frac{\sqrt{3^2+4^2}}{2} = \frac{5}{2}.$$

Since $|z| \leq 1 < |w|$, the reverse triangle inequality yields

$$|z+w| \geq |w| - |z| \geq \frac{5}{2} - 1 = \frac{3}{2}.$$

Notice that the unit disk allows us to move towards the point $-w$ by at most one unit. The requested quantity is a distance, so the lower bound is positive rather than negative.

Step 2: Show that the bound is attained Choose the unit complex number in the direction opposite to w:

$$z = -\frac{w}{|w|} = -\frac{3+4i}{5}.$$

Indeed, $|z| = \sqrt{9+16}/5 = 1$, so the choice is allowed. Direct substitution now gives

$$z + w = (3+4i)\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{3}{10}(3+4i),$$

whose modulus is $3 \cdot 5/10 = 3/2$. Thus this is an actual minimum, not merely an unattained infimum. The value $5/2$ occurs at $z = 0$ but is not minimal; 2 and 3 are both larger than the attained lower bound.

Step 3: Conclusion The minimum is $\boxed{3/2}$. Hence, the correct option is (C).

Quick Tip to Solve: For a fixed point outside a closed disk, subtract the radius from its distance to the centre, then exhibit the boundary point directed towards it.

3. Consider a $\triangle ABC$ where $A(1,3,2), B(-2,8,0)$ and $C(3,6,7)$. If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\overrightarrow{AD}$ on the vector $\overrightarrow{AC}$ is : [4 marks]

(A) $\frac{37}{2\sqrt{38}}$

(B) $\sqrt{19}$

(C) $\frac{39}{2\sqrt{38}}$

(D) $\frac{\sqrt{38}}{2}$

Correct Answer: (A) $\frac{37}{2\sqrt{38}}$

Solution:

Concept:

- The internal angle-bisector theorem gives $BD/DC = AB/AC$. Equal adjacent sides make the angle bisector meet the opposite side at its midpoint.
- The length of the projection of a vector $\mathbf{u}$ on a nonzero vector $\mathbf{v}$ is $|\mathbf{u}\cdot\mathbf{v}|/|\mathbf{v}|$; it is a scalar, not the projected vector.

Step 1: Find the adjacent side lengths Subtract the coordinates of A from those of the other vertices:

$$\overrightarrow{AB} = (-3,5,-2), \qquad \overrightarrow{AC} = (2,3,5).$$

Consequently,

$$AB^2 = 9+25+4 = 38, \qquad AC^2 = 4+9+25 = 38.$$

Both lengths are $\sqrt{38}$. Therefore the internal angle bisector divides BC in the ratio $1:1$. This uses the given angle at A, rather than an angle at another vertex.

Step 2: Locate the midpoint and its displacement The midpoint of $B(-2,8,0)$ and $C(3,6,7)$ is

$$D = \left(\frac{1}{2}, 7, \frac{7}{2}\right).$$

Hence

$$\overrightarrow{AD} = D - A = \left(-\frac{1}{2}, 4, \frac{3}{2}\right).$$

Using D itself as a displacement would be incorrect because the initial point is A, not the origin.

Step 3: Calculate the scalar projection Taking the dot product with the direction specified in the question gives

$$\overrightarrow{AD}\cdot\overrightarrow{AC} = -1+12+\frac{15}{2} = \frac{37}{2}.$$

Divide its absolute value by $|\overrightarrow{AC}| = \sqrt{38}$:

$$\text{projection length} = \frac{37}{2\sqrt{38}}.$$

The numerator is 37, ruling out 39, while the other two proposed lengths do not give this dot product when multiplied by $\sqrt{38}$.

Step 4: Conclusion The projection length is $\boxed{37/(2\sqrt{38})}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the two sides at the bisected angle first: equal lengths turn the section-formula calculation into a midpoint calculation.

---

4. Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all $a, b \in \mathbf{R}$ and $(a, b)R_2(c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in \mathbf{N} \times \mathbf{N}$. Then : [4 marks]

(A) R_1 and R_2 both are equivalence relations

(B) Only R_1 is an equivalence relation

(C) Only R_2 is an equivalence relation

(D) Neither R_1 nor R_2 is an equivalence relation

Correct Answer: (C) Only R_2 is an equivalence relation

Solution:

Concept:

- An equivalence relation must be reflexive, symmetric and transitive on its entire underlying set. One failed property is enough to disqualify it.
- A relation given by equality of a quantity attached to each element is naturally tested by rewriting it in that form.

Step 1: Test the relation on the real numbers For R_1 to be reflexive, every real a would have to satisfy $a^2 + a^2 = 1$. This fails already for $a = 0$, because $0^2 + 0^2 = 0 \neq 1$. Although the equation is symmetric in a, b, symmetry alone is insufficient. It is not transitive either: $0R_11$ and $1R_10$, but 0 is not related to itself. Thus R_1 cannot be an equivalence relation on $\mathbf{R}$.

Step 2: Rewrite the relation on ordered pairs The condition for R_2 can be rearranged without changing it:

$$(a, b)R_2(c, d) \quad \Longleftrightarrow \quad a - b = c - d.$$

Reflexivity holds since each pair has the same difference as itself. Symmetry holds since equality of two differences can be reversed. For transitivity, suppose $(a, b)R_2(c, d)$ and $(c, d)R_2(e, f)$. Then

$$a - b = c - d = e - f,$$

so $(a, b)R_2(e, f)$. All three properties hold for every pair of natural-number coordinates; the differences themselves need not be natural numbers.

Step 3: Compare the statements in the options Only R_2 passes all three tests. The first two options incorrectly call R_1 an equivalence relation, despite its failure of reflexivity. The last option incorrectly rejects R_2, whose equivalence classes are precisely the pairs with a fixed coordinate difference.

Step 4: Conclusion Only R_2 is an equivalence relation. Hence, the correct option is (C).

Quick Tip to Solve: Rearrange $a + d = b + c$ as equality of the invariant $a - b$; reflexivity, symmetry and transitivity then follow directly from equality.

---

5. Let the system of equations $x + 2y + 3z = 5, 2x + 3y + z = 9, 4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to : [4 marks]

(A) 22

(B) 17

(C) 15

(D) 28

Correct Answer: (B) 17

Solution:

Concept:

- Infinitely many solutions require equal coefficient and augmented ranks, both less than the number of unknowns.
- If two equation rows are independent, the third row must be their linear combination, including the right-hand-side constant, for infinitely many solutions.

Step 1: Check independence and determine the combination The first two coefficient rows are $(1, 2, 3)$ and $(2, 3, 1)$. They are independent: the minor formed by their first two columns is $3 - 4 = -1 \neq 0$. Thus the coefficient rank is at least two. Write the third row as u times the first plus v times the second. Matching the first two coefficients gives

$$u + 2v = 4, \qquad 2u + 3v = 3.$$

Twice the first equation minus the second yields $v = 5$, and then $u = 4 - 10 = -6$.

Step 2: Match the remaining coefficient and constant The same combination must reproduce both the coefficient of z and the constant term:

$$\lambda = 3u + v = 3(-6) + 5 = -13,$$

$$\mu = 5u + 9v = 5(-6) + 9(5) = 15.$$

These values make the entire third equation exactly -6 times the first equation plus 5 times the second. Hence both ranks are two, with three unknowns; one free parameter remains. Matching only the coefficients without checking μ could instead produce an inconsistent system.

Step 3: Evaluate the requested combination Using the values forced by dependence and consistency,

$$\lambda + 2\mu = -13 + 2(15) = 17.$$

The coefficient λ is negative, even though the requested combination is positive. The remaining options 22, 15 and 28 do not equal this combination; 15 is only the value of μ.

Step 4: Conclusion We obtain $\boxed{17}$. Hence, the correct option is (B).

Quick Tip to Solve: When two coefficient columns already give a nonzero minor, express the third row using those columns and apply the same combination to every remaining entry.

6. If $\int_0^{\frac{\pi}{3}} \cos^4 x \, dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then $9a + 8b$ is equal to : [4 marks]

(A) 2

(B) 1

(C) 3

(D) $\frac{3}{2}$

Correct Answer: (A) 2

Solution:

Concept:

- The power-reduction identity $\cos^2 x = (1 + \cos 2x)/2$ converts even powers into a sum of cosines with elementary antiderivatives.
- In a definite integral, evaluate each antiderivative at both limits before collecting the rational coefficient of π and that of $\sqrt{3}$.

Step 1: Reduce the fourth power Squaring the double-angle expression and reducing $\cos^2 2x$ gives

$$\cos^4 x = \frac{1 + 2\cos 2x + \cos^2 2x}{4} = \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x.$$

The constant term is $3/8$, not $1/4$, because $\cos^2 2x$ contributes an additional constant. Integrating the three terms separately is now straightforward.

Step 2: Evaluate the integral at its limits An antiderivative is

$$\frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32}.$$

It is zero at the lower limit. At $x = \pi/3$, we have $\sin(2\pi/3) = \sqrt{3}/2$ and $\sin(4\pi/3) = -\sqrt{3}/2$. Therefore

$$I = \frac{\pi}{8} + \frac{\sqrt{3}}{8} - \frac{\sqrt{3}}{64} = \frac{\pi}{8} + \frac{7\sqrt{3}}{64}.$$

The negative sign in the last contribution is essential: its angle lies in the third quadrant.

Step 3: Read the coefficients and combine them Comparison with $a\pi + b\sqrt{3}$ gives $a = 1/8$ and $b = 7/64$. Thus

$$9a + 8b = \frac{9}{8} + \frac{56}{64} = \frac{9}{8} + \frac{7}{8} = 2.$$

The other choices 1, 3 and $3/2$ are inconsistent with these coefficients. In particular, dropping the $\sin 4x$ term changes b and cannot give the correct integral.

Step 4: Conclusion The required value is $\boxed{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Reduce $\cos^4 x$ completely before integrating, and check the quadrant of $4x$ at the upper limit to preserve the negative sine contribution.

7. Let α and β be the roots of the equation $px^2+qx-r=0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non constant G.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$, then the value of $(\alpha-\beta)^2$ is : [4 marks]

(A) 8

(B) 9

(C) $\frac{20}{3}$

(D) $\frac{80}{9}$

Correct Answer: (D) $\frac{80}{9}$

Solution:

Concept:

- For $px^2+qx-r=0$, Vieta's relations give $\alpha+\beta=-q/p$ and $\alpha\beta=-r/p$. The negative constant term must be retained.
- Consecutive terms of a geometric progression satisfy $q^2=pr$. The identity $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ avoids solving for individual roots.

Step 1: Use the reciprocal-root condition The stated reciprocals require $\alpha\beta \neq 0$, hence $r \neq 0$. By Vieta's relations,

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-q/p}{-r/p}=\frac{q}{r}=\frac{3}{4}.$$

Thus $q \neq 0$ as well. If t is the common ratio of the G.P., then $q=pt$, $r=pt^2$, and $q/r=1/t$. Consequently $t=4/3$, which is indeed nonconstant.

Step 2: Determine the sum and product of the roots The common ratio gives

$$\frac{q}{p}=\frac{4}{3}, \qquad \frac{r}{p}=\frac{16}{9}.$$

It follows that

$$\alpha+\beta=-\frac{4}{3}, \qquad \alpha\beta=-\frac{16}{9}.$$

The product is negative, not positive, because the original quadratic contains $-r$. This also shows that the real roots have opposite signs; the discriminant is positive.

Step 3: Compute the squared difference Using only the sum and product,

$$(\alpha-\beta)^2=\left(-\frac{4}{3}\right)^2-4\left(-\frac{16}{9}\right)=\frac{16}{9}+\frac{64}{9}=\frac{80}{9}.$$

Neither 8 nor 9 equals this exact fraction, and $20/3=60/9$ is different as well. Thus no extraction of individual square roots is necessary to distinguish the choices.

Step 4: Conclusion The value is $\boxed{80/9}$. Hence, the correct option is (D).

Quick Tip to Solve: Use reciprocal-root sums to obtain q/r, then the G.P. condition to obtain q/p; keep the minus sign in $\alpha\beta=-r/p$.

8. Let Ajay will not appear in JEE exam with probability $\mathrm{p} = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $\mathrm{q} = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is : [4 marks]

(A) $\frac{9}{35}$
(B) $\frac{3}{35}$
(C) $\frac{24}{35}$
(D) $\frac{18}{35}$

Correct Answer: (D) $\frac{18}{35}$

Solution:

Concept:

- For an event A, the probability of its complement is $P(A^c) = 1 - P(A)$. The events that Ajay appears and that he does not appear exhaust all possibilities.
- The event A is the disjoint union of $A \cap V$ and $A \cap V^c$, where V means Vijay appears. Thus $P(A \cap V^c) = P(A) - P(A \cap V)$; independence is not needed.

Step 1: Name the events and find the marginal probability Let A denote Ajay appearing and V denote Vijay appearing. The stated probability $2/7$ concerns Ajay not appearing, so it is $P(A^c)$, rather than $P(A)$. Taking its complement gives

$$P(A) = 1 - \frac{2}{7} = \frac{5}{7}.$$

The given joint probability is $P(A \cap V) = 1/5$. We do not need to determine Vijay's marginal probability.

Step 2: Separate the two possibilities for Vijay Whenever Ajay appears, either Vijay also appears or Vijay does not appear. These alternatives cannot occur together, so their probabilities add to $P(A)$:

$$P(A \cap V) + P(A \cap V^c) = P(A).$$

Consequently,

$$P(A \cap V^c) = \frac{5}{7} - \frac{1}{5} = \frac{25 - 7}{35} = \frac{18}{35}.$$

This is nonnegative and less than $P(A)$, as it must be. Options (A), (B) and (C) do not satisfy this partition identity with the given joint probability; multiplying the two given numbers would incorrectly assume independence and would also use the wrong marginal event.

Step 3: Conclusion The probability that Ajay appears while Vijay does not is $\boxed{18/35}$. Hence, the correct option is (D).

Quick Tip to Solve: First complement the probability of Ajay being absent, then subtract the probability that both appear. No independence assumption is required.

9. Let P be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let the line passing through P and parallel to y-axis meet the circle $x^2+y^2=9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that $PR:RQ=4:3$ as P moves on the ellipse, is : [4 marks]

(A) $\frac{13}{21}$

(B) $\frac{\sqrt{139}}{23}$

(C) $\frac{\sqrt{13}}{7}$

(D) $\frac{11}{19}$

Correct Answer: (C) $\frac{\sqrt{13}}{7}$

Solution:

Concept:

- A point on $x^2/9+y^2/4=1$ can be written as $P=(3\cos t, 2\sin t)$. Matching the horizontal coordinate on the circle gives $Q=(3\cos t, 3\sin t)$ on the same side of the horizontal axis.
- If $PR:RQ=m:n$, internal division gives $R=(nP+mQ)/(m+n)$. An ellipse with semiaxes $a>b$ has eccentricity $e=\sqrt{1-b^2/a^2}$.

Step 1: Parametrize the endpoints of the segment Choose $P=(3\cos t, 2\sin t)$. A vertical line through P has $x=3\cos t$; substitution in $x^2+y^2=9$ gives $y^2=9\sin^2 t$. The condition that P and Q lie on the same side of the horizontal axis selects $Q=(3\cos t, 3\sin t)$. Thus their vertical coordinates have the same sign, rather than opposite signs. The degenerate endpoint cases are included by continuity and do not affect the locus eccentricity.

Step 2: Apply the division ratio and identify the ellipse For the ratio $PR:RQ=4:3$, the coordinate of P has weight 3 and that of Q has weight 4. Hence

$$R=\frac{3P+4Q}{7}=\left(3\cos t, \frac{18}{7}\sin t\right).$$

Writing $R=(X,Y)$ and eliminating t,

$$\frac{X^2}{9}+\frac{Y^2}{324/49}=1.$$

The larger semiaxis is 3, since $18/7<3$. Thus

$$e^2=1-\frac{(18/7)^2}{3^2}=1-\frac{36}{49}=\frac{13}{49}.$$

Using the original ellipse semiaxis 2, or reversing the section weights, produces a different locus and cannot justify any of the other proposed eccentricities.

Step 3: Conclusion The required eccentricity is $\boxed{\sqrt{13}/7}$. Hence, the correct option is (C).

Quick Tip to Solve: For a vertical interpolation between an ellipse and its auxiliary circle, retain the same cosine coordinate and take the weighted average only of the sine coefficients.

10. Consider 10 observations $x_1, x_2, \ldots, x_{10}$ such that $\sum_{i=1}^{10}(x_i - \alpha) = 2$ and $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. Then $\frac{\beta}{\alpha}$ is equal to : [4 marks]

(A) 2

(B) 1

(C) $\frac{5}{2}$

(D) $\frac{3}{2}$

Correct Answer: (A) 2

Solution:

Concept:

- The arithmetic mean is $\bar{x} = (\sum_i x_i)/n$, so $\sum_i (x_i - a) = n(\bar{x} - a)$ for a constant a.
- The mean squared deviation from any number b satisfies $\frac{1}{n}\sum_i (x_i - b)^2 = \sigma^2 + (\bar{x} - b)^2$, where σ^2 is the variance about the mean.

Step 1: Use the sum of deviations to find alpha There are 10 observations and their mean is $6/5$, so their sum is $10(6/5) = 12$. The first condition therefore becomes

$$\sum_{i=1}^{10}(x_i - \alpha) = 12 - 10\alpha = 2.$$

Solving gives $\alpha = 1$, which satisfies the requirement that it be a positive integer. Notice that a sum of deviations is zero only when its reference number equals the mean; here that sum is 2, so α is not the mean.

Step 2: Shift the variance to the reference beta Expand $x_i - \beta = (x_i - \bar{x}) + (\bar{x} - \beta)$. On summing the squares, the cross term vanishes because $\sum_i (x_i - \bar{x}) = 0$. Dividing the stated squared-deviation sum by 10 gives

$$4 = \frac{84}{25} + \left(\frac{6}{5} - \beta\right)^2.$$

Therefore

$$\left(\frac{6}{5} - \beta\right)^2 = \frac{16}{25}, \qquad \beta = \frac{6}{5} \pm \frac{4}{5}.$$

The candidates are 2 and $2/5$, of which only 2 is a positive integer. Finally $\beta/\alpha = 2/1 = 2$. Substitution checks the squared-deviation average: $84/25 + 16/25 = 4$. The remaining options fail either this identity or the integer constraint.

Step 3: Conclusion We obtain $\alpha = 1$, $\beta = 2$, and $\boxed{\beta/\alpha = 2}$. Hence, the correct option is (A).

Quick Tip to Solve: To use a sum of squares about a shifted origin, add the squared distance from that origin to the mean to the variance; do not confuse the two sums.

11. Let $f(x) = \left|2x^2 + 5|x| - 3\right|$, $x \in \mathbf{R}$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then $m + n$ is equal to : [4 marks]

(A) 5

(B) 3

(C) 2

(D) 0

Correct Answer: (B) 3

Solution:

Concept:

- Polynomials and the absolute-value function are continuous, and sums and compositions of continuous functions remain continuous. A corner can nevertheless prevent differentiability.
- For $g(x) = 2x^2 + 5|x| - 3$, possible corners of $f(x) = |g(x)|$ occur at $x = 0$ from the inner modulus and at real zeros of g from the outer modulus. Each candidate must be checked.

Step 1: Establish continuity and locate the candidates Both x^2 and $|x|$ are continuous for every real number. Thus $g(x) = 2x^2 + 5|x| - 3$, and then $f(x) = |g(x)|$, are continuous everywhere. Hence $m = 0$. To find zeros of g, put $u = |x| \geq 0$. Then

$$2u^2 + 5u - 3 = (2u - 1)(u + 3) = 0.$$

Only $u = 1/2$ is admissible, giving $x = \pm 1/2$. Away from these two points and zero, each modulus has a fixed sign and the function is polynomial locally.

Step 2: Check the derivatives at all three candidates Near zero, $g(x) < 0$, so $f(x) = 3 - 2x^2 - 5|x|$. Its left derivative at zero is 5 and its right derivative is -5, so zero is a corner. At $x = 1/2$, g has derivative $4x + 5 = 7$, and it crosses zero; consequently $|g|$ has one-sided derivatives -7 and 7. At $x = -1/2$, g has derivative $4x - 5 = -7$ and again crosses zero, producing unequal one-sided derivatives for $|g|$. Therefore all three candidates are nondifferentiable and there are no others: $n = 3$. The answer 2 misses the inner-modulus corner, while 0 ignores the distinction between continuity and differentiability; 5 has no additional candidate points to support it.

Step 3: Conclusion Here $m = 0$, $n = 3$, so $\boxed{m + n = 3}$. Hence, the correct option is (B).

Quick Tip to Solve: For nested moduli, check both the zeros of the outer argument and the corner points introduced by any inner modulus; continuity is preserved throughout.

12. The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0; x \in [-2\pi, 2\pi]$ is : [4 marks]

(A) 0

(B) 3

(C) 1

(D) 2

Correct Answer: (A) 0

Solution:

Concept:

- For real x, $\sin^2 x \geq 0$ and $-1 \leq \cos x \leq 1$. In particular, both $\cos x$ and $\cos^3 x$ are at most 1.
- If a real expression is bounded below by a strictly positive constant throughout an interval, it cannot be zero anywhere in that interval. Endpoint inclusion cannot create a root when the bound holds globally.

Step 1: Bound each term with the correct inequality direction Let $F(x) = 4\sin^2 x - 4\cos^3 x + 9 - 4\cos x$. Since $\sin^2 x \geq 0$, its contribution is nonnegative. Since $\cos^3 x \leq 1$, multiplication by -4 reverses the inequality and gives $-4\cos^3 x \geq -4$. Similarly $-4\cos x \geq -4$. Adding these valid bounds yields

$$F(x) \geq 0 - 4 + 9 - 4 = 1.$$

All three lower bounds can occur simultaneously when $\cos x = 1$, showing that this bound is sharp rather than merely sufficient.

Step 2: Exclude roots on the entire stated interval The equation asks for $F(x) = 0$, whereas the estimate shows $F(x) \geq 1 > 0$ for every real number. Therefore there are no solutions in $[-2\pi, 2\pi]$, or indeed in any real interval. As a check, at $x = -2\pi, 0, 2\pi$ the value is exactly 1, so even those points where the negative cosine terms are largest in magnitude are not roots. At other values the same strict positivity argument still applies. Thus counts 1, 2 and 3 cannot arise from endpoint counting or periodicity: those procedures matter only after genuine zeros have been found. There is no need to solve a cubic in $\cos x$, which would obscure the immediate positivity condition.

Step 3: Conclusion The number of real solutions in the given interval is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: Before solving a trigonometric polynomial algebraically, compare the positive constant with the largest possible magnitude of its negative terms.

13. Let the locus of the midpoints of the chords of the circle $x^2 + (y-1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at P and Q. Then, the length of PQ is : [4 marks]

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{1}{\sqrt{2}}$

(D) $\sqrt{2}$

Correct Answer: (C) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- The origin lies on the circle $x^2 + (y-1)^2 = 1$. A chord drawn from the origin therefore has endpoints $O = (0,0)$ and another point T on this circle.
- If $M = (h,k)$ is the midpoint of OT, then $T = (2h, 2k)$. Substitution in the original circle gives the midpoint locus, whose intersection with the given line determines the required segment.

Step 1: Derive the midpoint locus from the endpoint relation The original circle has centre $(0,1)$ and radius 1, and the origin satisfies its equation. Let the other endpoint of a chord through the origin be $T = (a,b)$. Its midpoint is $(a/2, b/2)$. Writing this midpoint as (h,k), substitute $a = 2h, b = 2k$ into the circle:

$$(2h)^2 + (2k-1)^2 = 1.$$

On expansion and division by 4 this is

$$h^2 + k^2 - k = 0, \qquad h^2 + (k - \tfrac{1}{2})^2 = \tfrac{1}{4}.$$

Thus the midpoint locus is a circle of radius $1/2$, not the original radius-1 circle.

Step 2: Intersect the locus with the line and find the distance On $h + k = 1$, put $k = 1 - h$. Then the locus condition becomes

$$h^2 + (1-h)^2 - (1-h) = 0 \quad \Longrightarrow \quad 2h^2 - h = 0.$$

The roots are $h = 0$ and $h = 1/2$, yielding $P = (0,1)$ and $Q = (1/2, 1/2)$, in either order. Their Euclidean separation is

$$PQ = \sqrt{(\tfrac{1}{2} - 0)^2 + (\tfrac{1}{2} - 1)^2} = \sqrt{\tfrac{1}{2}} = \frac{1}{\sqrt{2}}.$$

The coordinate difference $1/2$ alone is not the length because both coordinates change. Values 1 and $\sqrt{2}$ likewise disagree with these actual intersection coordinates.

Step 3: Conclusion The length is $\boxed{PQ = 1/\sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: For chords with one endpoint fixed at the origin, obtain the midpoint locus by replacing each original endpoint coordinate by twice the midpoint coordinate.

14. Let α be a non-zero real number. Suppose $f : \mathbf{R} \to \mathbf{R}$ is a differentiable function such that $f(0) = 2$ and $\lim_{x \to -\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in \mathbf{R}$, then $f(-\log_e 2)$ is equal to : [4 marks]

(A) 7

(B) 9

(C) 3

(D) 5

Correct Answer: As printed, the hypotheses are inconsistent, so $f(-\ln 2)$ is not defined by any admissible function and none of the options is valid. The keyed value 9 would apply with a limit at positive infinity, but not with the stated negative-infinity limit.

Solution:

Concept:

- For a nonzero constant α, the general solution of $f\prime(x) = \alpha f(x) + 3$ is $f(x) = Ce^{\alpha x} - 3/\alpha$, obtained by shifting f by $3/\alpha$.
- A finite limit at negative infinity restricts the sign of the exponential rate, unless the exponential coefficient vanishes. The initial condition and the limiting value must both hold for the same function.

Step 1: Solve the differential equation with its initial condition Set $g(x) = f(x) + 3/\alpha$. Then $g\prime = \alpha g$, so

$$f(x) = Ce^{\alpha x} - \frac{3}{\alpha}.$$

Since $f(0) = 2$, we have $C = 2 + 3/\alpha$. Thus every function satisfying the equation and initial value has the form

$$f(x) = \left(2 + \frac{3}{\alpha}\right) e^{\alpha x} - \frac{3}{\alpha}.$$

It remains essential to test the stated limit $x \to -\infty$; it cannot simply be replaced by the limit in the opposite direction.

Step 2: Test every possible sign of alpha against the printed limit If $\alpha > 0$, the exponential tends to zero as $x \to -\infty$, so the limit is $-3/\alpha < 0$, which cannot be 1. If $\alpha < 0$ and $C \neq 0$, the exponential grows without bound as $x \to -\infty$, so there is no finite limit. The only remaining possibility is $C = 0$, giving $\alpha = -3/2$ and the constant function $f(x) = 2$, whose limit is 2 rather than 1. Therefore no nonzero real α and no differentiable function satisfy all the printed hypotheses. Formally setting $-3/\alpha = 1$ gives $\alpha = -3$ and $f(x) = 1 + e^{-3x}$, whose value at $-\ln 2$ is 9, but this function diverges at negative infinity. Thus the keyed value 9 cannot be justified by the limit as printed; the other options are unsupported as well.

Step 3: Conclusion As printed, the hypotheses are inconsistent, so $f(-\ln 2)$ is not defined by any admissible function and none of the options is valid. The keyed value 9 would apply with a limit at positive infinity, but not with the stated negative-infinity limit.

Quick Tip to Solve: After solving a linear differential equation, check the exponential at the specified end of the real line before substituting a limiting value.

15. Let P and Q be the points on the line $\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$ which are at a distance of 6 units from the point $\mathrm{R}(1,2,3)$. If the centroid of the triangle PQR is (α,β,γ), then $\alpha^2+\beta^2+\gamma^2$ is : [4 marks]

(A) 18

(B) 24

(C) 26

(D) 36

Correct Answer: (A) 18

Solution:

Concept:

- A point on a line in symmetric form is obtained by setting every ratio equal to one parameter. Here write $X(t)=(-3+8t,4+2t,-1+2t)$.
- The distance formula gives $|X(t)-R|^2=36$. The centroid of three vertices is their coordinatewise average, so both admissible points must be included.

Step 1: Find the two points on the line The coordinate differences from $R(1,2,3)$ are $8t-4, 2t+2, 2t-4$. Therefore

$$\begin{aligned}(8t-4)^2+(2t+2)^2+(2t-4)^2&=36,\\ 72t^2-72t+36&=36,\\ 72t(t-1)&=0.\end{aligned}$$

Thus the two parameter values are 0 and 1, giving $P=(-3,4,-1)$ and $Q=(5,6,1)$. The order in which we name the two points does not affect the centroid. As a direct check, their respective difference vectors from R are $(-4,2,-4)$ and $(4,4,-2)$; each has squared length 36.

Step 2: Average the vertices and evaluate the expression Include R as the third vertex rather than taking only the midpoint of PQ. The centroid is

$$\begin{aligned}(\alpha,\beta,\gamma)&=\left(\frac{-3+5+1}{3},\frac{4+6+2}{3},\frac{-1+1+3}{3}\right)\\ &=(1,4,1).\end{aligned}$$

Hence $\alpha^2+\beta^2+\gamma^2=1+16+1=18$. The other choices 24, 26 and 36 do not equal the squared length of this centroid; 36 is the given squared distance from R to either endpoint, a different quantity.

Step 3: Conclusion The required value is $\boxed{18}$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the line parameter until the distance equation is solved; then average P, Q and R, not just the two points on the line.

16. The value of $\int_0^1 \left(2x^3 - 3x^2 - x + 1\right)^{\frac{1}{3}}\ \mathrm{d}x$ is equal to : [4 marks]

(A) -1

(B) 2

(C) 0

(D) 1

Correct Answer: (C) 0

Solution:

Concept:

- For an integrable real function, reflection of the interval gives $\int_0^1 g(x)\,dx = \int_0^1 g(1-x)\,dx$.
- The real cube root is an odd function: $\sqrt[3]{-u} = -\sqrt[3]{u}$ for every real u. This remains true when the polynomial changes sign inside the interval.

Step 1: Check the polynomial under reflection Let $p(x) = 2x^3 - 3x^2 - x + 1$. Expanding after replacing x by $1 - x$ gives

$$\begin{aligned} p(1-x) &= 2(1 - 3x + 3x^2 - x^3) \\ &\quad - 3(1 - 2x + x^2) - (1 - x) + 1 \\ &= -2x^3 + 3x^2 + x - 1 \\ &= -p(x). \end{aligned}$$

Thus the integrand $g(x) = \sqrt[3]{p(x)}$ satisfies $g(1-x) = -g(x)$. We use the real cube root throughout; no positivity assumption about the expression inside the cube root is needed. The integrand is continuous on the closed interval, including its zero, so the definite integral exists.

Step 2: Pair equal and opposite contributions If the required integral is I, substitute $u = 1 - x$, reversing the endpoints and absorbing the minus sign in the differential. This yields

$$I = \int_0^1 g(1-u)\,du = -\int_0^1 g(u)\,du = -I.$$

Equivalently, adding the two representations gives $2I = \int_0^1 [g(x) + g(1-x)]\,dx = 0$. Hence $I = 0$. Each point to the left of $1/2$ is paired with a point to its right whose integrand has the same magnitude and opposite sign. The options -1, 2 and 1 are nonzero and cannot satisfy the identity $I = -I$.

Step 3: Conclusion The integral equals $\boxed{0}$. Hence, the correct option is (C).

Quick Tip to Solve: Before attempting an antiderivative on [0,1], test x replaced by 1-x; an odd outer function preserves a sign reversal of the inner expression.

17. If the mirror image of the point $P(3,4,9)$ in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α,β,γ), then 14 $(\alpha+\beta+\gamma)$ is : [4 marks]

(A) 102

(B) 138

(C) 132

(D) 108

Correct Answer: (D) 108

Solution:

Concept:

- The perpendicular foot H from P to a line has $(P-H)\cdot\mathbf{d}=0$, where $\mathbf{d}$ is a direction vector of the line.
- Reflection in the line places H at the midpoint of P and its image P', so $P'=2H-P$.

Step 1: Project P onto the line A fixed point on the line is $A=(1,-1,2)$, and its direction vector is $\mathbf{d}=(3,2,1)$. Set

$$H=(1+3t,-1+2t,2+t).$$

Since $P-A=(2,5,7)$, perpendicularity gives

$$\begin{aligned} t&=\frac{(P-A)\cdot\mathbf{d}}{\mathbf{d}\cdot\mathbf{d}}\\ &=\frac{2(3)+5(2)+7(1)}{3^2+2^2+1^2}=\frac{23}{14}. \end{aligned}$$

Consequently, $H=(83/14,16/7,51/14)$. The denominator 14 is the squared length of the direction vector; using its length instead would not give the correct projection parameter.

Step 2: Reflect across the perpendicular foot The required reflected point is

$$\begin{aligned} (\alpha,\beta,\gamma)&=2H-(3,4,9)\\ &=\left(\frac{62}{7},\frac{4}{7},-\frac{12}{7}\right). \end{aligned}$$

Its midpoint with P is H, and the segment from P to H is perpendicular to the line by construction, checking both requirements of the reflection. Add the coordinates only after the subtraction:

$$14(\alpha+\beta+\gamma)=14\left(\frac{62+4-12}{7}\right)=14\cdot\frac{54}{7}=108.$$

Neither 102, 138 nor 132 agrees with this projection and midpoint calculation; in particular, the negative z-coordinate must be retained in the sum.

Step 3: Conclusion The requested value is $\boxed{108}$. Hence, the correct option is (D).

Quick Tip to Solve: For reflection in a line, first find the perpendicular foot and use image = twice the foot minus the original point; keep signed coordinates until the final sum.

18. Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is $15 : 7$, then $S_{15} - S_5$ is equal to : [4 marks]

(A) 800

(B) 890

(C) 790

(D) 690

Correct Answer: (C) 790

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, $T_n = a + (n-1)d$ and $S_n = \frac{n}{2}[2a + (n-1)d]$.
- The difference $S_{15} - S_5$ is the sum of terms 6 through 15, which contains ten terms. Its first and last terms determine the sum.

Step 1: Use the ratio and the known sum The given ratio is a ratio of terms, not of sums. Therefore

$$\frac{a + 9d}{a + 4d} = \frac{15}{7}.$$

Cross-multiplication gives $7a + 63d = 15a + 60d$, or $3d = 8a$. Write $a = 3k$ and $d = 8k$. Substituting into the other condition gives

$$\begin{aligned} S_{10} &= 5(2a + 9d) \\ &= 5(6k + 72k) = 390k = 390. \end{aligned}$$

Hence $k = 1$, so $a = 3$ and $d = 8$. This also checks the ratio directly: $T_{10} = 75$, $T_5 = 35$, and $75 : 35 = 15 : 7$.

Step 2: Sum the correct block of ten terms Removing S_5 from S_{15} leaves $T_6 + T_7 + \cdots + T_{15}$, not just the first ten terms. The endpoint terms are

$$T_6 = 3 + 5(8) = 43, \qquad T_{15} = 3 + 14(8) = 115.$$

Thus

$$S_{15} - S_5 = \frac{10}{2}(43 + 115) = 5(158) = 790.$$

As an independent check with the full-sum formula, $S_{15} = \frac{15}{2}(6 + 112) = 885$, while $S_5 = \frac{5}{2}(6 + 32) = 95$; their difference is again 790. The choices 800, 890 and 690 do not match either calculation and cannot arise from the stated progression.

Step 3: Conclusion The required sum difference is $\boxed{790}$. Hence, the correct option is (C).

Quick Tip to Solve: Interpret a difference of partial sums as a consecutive block; count its terms and use the average of its first and last terms.

19. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is : [4 marks]

(A) $\frac{1}{9}$
(B) $\frac{1}{4}$
(C) $\frac{4}{9}$
(D) $\frac{9}{4}$

Correct Answer: (D) $\frac{9}{4}$

Solution:

Concept:

- In $(u+v)^{18}$, the $(r+1)$th term is $\binom{18}{r}u^{18-r}v^r$. Thus the seventh and thirteenth terms correspond to r = 6 and r = 12.
- The numerical coefficient excludes the power of x. Symmetry of binomial coefficients gives $\binom{18}{6} = \binom{18}{12}$, allowing cancellation before any large integer is calculated.

Step 1: Identify the two numerical coefficients Here $u = x^{1/3}/3$ and $v = 1/(2x^{2/3})$, so the general term is

$$T_{r+1} = \binom{18}{r}\frac{1}{3^{18-r}2^r}x^{(18-r)/3-2r/3} = \binom{18}{r}\frac{x^{6-r}}{3^{18-r}2^r}.$$

Consequently, the coefficient of the seventh term is $m = \binom{18}{6}/(3^{12}2^6)$, and the coefficient of the thirteenth term is $n = \binom{18}{12}/(3^6 2^{12})$. Although their x-powers are respectively 0 and -6, those powers are not part of the numerical coefficients requested in the question.

Step 2: Cancel and take the cube root Using the equality of the binomial coefficients,

$$\frac{n}{m} = \frac{3^{12}2^6}{3^6 2^{12}} = \left(\frac{3}{2}\right)^6,$$
$$\left(\frac{n}{m}\right)^{1/3} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}.$$

All coefficients here are positive, so the positive real cube root is unambiguous. The option $4/9$ is the reciprocal and would result from reversing n and m. The options $1/9$ and $1/4$ incorrectly retain only one of the two numerical factors. Keeping the powers of 3 and 2 separately until cancellation avoids both errors.

Step 3: Conclusion The value is $\boxed{\frac{9}{4}}$. Hence, the correct option is (D).

Quick Tip to Solve: Term number k means r = k-1 in the binomial formula; check for symmetric binomial coefficients before expanding their factorials.

20. Let $f(x) = \begin{cases} x-1, & x \text{ is even,} \\ 2x, & x \text{ is odd,} \end{cases} \quad x \in \mathbf{N}.$

If for some $\mathrm{a} \in \mathbf{N}, f(f(f(\mathrm{a}))) = 21$**, then** $\lim\limits_{x \to \mathrm{a}^-} \left\{ \frac{|x|^3}{\mathrm{a}} - \left[\frac{x}{\mathrm{a}}\right] \right\}$**, where** $[t]$ **denotes the greatest integer less than or equal to** t**, is equal to :** [4 marks]

(A) 169

(B) 121

(C) 225

(D) 144

Correct Answer: (D) 144

Solution:

Concept:

- The parity rule determines possible preimages: an odd output can only come from the even-input branch $f(n) = n - 1$, while an even output can only come from the odd-input branch $f(n) = 2n$.
- For a positive natural number a, approaching a from below makes x/a slightly less than 1. Hence $\lfloor x/a \rfloor = 0$ sufficiently close to a on that side.

Step 1: Work backwards through the three applications Since $f(f(f(a))) = 21$ is odd, its input must be even and one larger than 21. Thus $f(f(a)) = 22$. To obtain the even number 22, the input must be odd and doubled, giving $f(a) = 11$. Finally, the odd output 11 requires the even input $a = 12$. This produces the chain

$$12 \longmapsto 11 \longmapsto 22 \longmapsto 21.$$

Each step respects the required parity. There is no second preimage at any stage: subtracting 1 from an even input always gives an odd output, whereas doubling an odd input always gives an even output. Therefore the backward reconstruction uniquely determines the natural number a.

Step 2: Evaluate the left-hand limit Now take $x \to 12^-$. In a small left neighborhood, x is positive and $0 < x/12 < 1$. It follows that $|x|^3 = x^3$ and $\lfloor x/12 \rfloor = 0$ throughout that neighborhood. Hence

$$\lim_{x \to 12^-} \left\{ \frac{|x|^3}{12} - \left\lfloor \frac{x}{12} \right\rfloor \right\} = \frac{12^3}{12} - 0 = 144.$$

The floor value at exactly x = 12 is 1, but a left-hand limit uses nearby inputs below 12 and does not substitute that endpoint floor value. The alternatives 169, 121 and 225 do not match the unique a or its left-hand limit.

Step 3: Conclusion The limit is $\boxed{144}$. Hence, the correct option is (D).

Quick Tip to Solve: Invert a parity-defined function from the final output backwards, and determine the floor on the specified side before taking the limit.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Three points $\mathrm{O}(0,0), \mathrm{P}\left(\mathrm{a}, \mathrm{a}^2\right), \mathrm{Q}\left(-\mathrm{b}, \mathrm{b}^2\right), \mathrm{a}>0,\ \mathrm{b}>0$, are on the parabola $y=x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of $\frac{\mathrm{S}_1}{\mathrm{S}_2}$ is $\frac{\mathrm{m}}{\mathrm{n}}, \gcd(\mathrm{m}, \mathrm{n})=1$, then $\mathrm{m}+\mathrm{n}$ is equal to _____. [4 marks]

Solution:

Concept:

- The area between an upper curve and a lower curve is their difference integrated across the intersection interval. Here the chord PQ lies above $y=x^2$ between x = -b and x = a.
- The area of triangle OPQ is half the absolute determinant of the coordinate vectors OP and OQ. For positive a and b, $(a-b)^2 \geq 0$ gives $(a+b)^2 \geq 4ab$.

Step 1: Find the chord and the parabolic-segment area The chord slope is $(a^2-b^2)/(a+b)=a-b$, so its equation is $y=(a-b)x+ab$. Its vertical separation from the parabola factors as

$$(a-b)x+ab-x^2=(a-x)(x+b).$$

This is nonnegative on the complete interval $[-b,a]$. Set $u=x+b$ and $L=a+b$; then

$$\begin{aligned} S_1 &= \int_{-b}^{a}(a-x)(x+b)\,dx \\ &= \int_{0}^{L}(L-u)u\,du \\ &= \frac{L^3}{2}-\frac{L^3}{3}=\frac{(a+b)^3}{6}. \end{aligned}$$

This calculation includes the entire region between the two intersection points, so neither side of the y-axis is omitted.

Step 2: Compare with the triangle and minimize the ratio Using O as the origin, the triangle area is

$$S_2=\frac{1}{2}\left|ab^2-(-b)a^2\right|=\frac{ab(a+b)}{2}.$$

Because a and b are positive, this denominator is nonzero and

$$\frac{S_1}{S_2}=\frac{(a+b)^2}{3ab}=\frac{4}{3}+\frac{(a-b)^2}{3ab}\geq\frac{4}{3}.$$

Equality is attainable when $a=b>0$, so this bound is a minimum rather than merely an unattained infimum. The fraction $4/3$ is already in lowest terms because 4 and 3 have no common divisor other than 1. Thus the required relatively prime integers are $m=4$ and $n=3$.

Step 3: Conclusion The minimum ratio is $4/3$, so $\boxed{m+n=7}$.

Quick Tip to Solve: For a parabola chord, factor chord minus parabola using the two endpoints; shift the interval to [0,a+b] before integrating.

22. The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to : [4 marks]

Solution:

Concept:

- For curves written as $x = x(y)$, the enclosed area is the integral of their horizontal separation with respect to y. The separation must be nonnegative on the interval of integration.
- The area depends on k through k^2. Optimising in the nonnegative variable $u = k^2$ therefore finds both possible signs of k.

Step 1: Find the intersections First suppose $k \neq 0$, so that the first parabola can be written as $x = 2y^2/k$. The second is $x = y - ky^2/2$. Define $a = (k^2 + 4)/(2k)$. Their horizontal difference is

$$y - \frac{ky^2}{2} - \frac{2y^2}{k} = y - ay^2.$$

Thus their intersections have $y = 0$ and $y = 1/a$. For positive k, both a and the second endpoint are positive; the displayed difference is then the right curve minus the left curve.

Step 2: Integrate the bounded area For $k > 0$, integrate from the lower to the upper intersection:

$$A = \int_0^{1/a} (y - ay^2)\, dy = \left[\frac{y^2}{2} - \frac{ay^3}{3}\right]_0^{1/a} = \frac{1}{6a^2} = \frac{2k^2}{3(k^2 + 4)^2}.$$

When $k < 0$, the interval runs from $1/a$ to zero and the left and right curves interchange. Taking the positive horizontal separation gives exactly the same area formula. At $k = 0$, the first curve degenerates to $y = 0$; this cannot yield a larger bounded area.

Step 3: Maximise the expression Put $u = k^2 > 0$. Since $(u + 4)^2 - 16u = (u - 4)^2 \geq 0$,

$$A = \frac{2u}{3(u + 4)^2} \leq \frac{2u}{48u} = \frac{1}{24}.$$

Equality holds precisely when $u = 4$. Hence both $k = 2$ and $k = -2$ maximise the area; neither sign may be discarded.

Step 4: Conclusion The required sum is $2^2 + (-2)^2 = \boxed{8}$.

Quick Tip to Solve: After integrating a parameter-dependent area, look for dependence on the square of the parameter before differentiating; this preserves both signs.

---

23. If $y = \frac{(\sqrt{x}+1)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}\left(3\cos^2 x - 5\right)\cos^3 x$, then $96y'\left(\frac{\pi}{6}\right)$ is equal to : [4 marks]

Solution:

Concept:

- Factor the algebraic fraction before differentiating. With $t = \sqrt{x}$, the identity $t^3 - 1 = (t-1)(t^2+t+1)$ cancels its common factor.

- The chain rule differentiates powers of $\cos x$: $\frac{d}{dx}\cos^n x = -n\cos^{n-1} x \sin x$. The evaluation point is positive, so the square-root substitution is valid nearby.

Step 1: Simplify the first term Set $t = \sqrt{x} > 0$. Then the numerator contains $x^2 - \sqrt{x} = t^4 - t = t(t^3 - 1)$, while the denominator is $t^3 + t^2 + t = t(t^2 + t + 1)$. Therefore

$$\frac{(t+1)(t^4 - t)}{t^3 + t^2 + t} = \frac{(t+1)t(t-1)(t^2 + t + 1)}{t(t^2 + t + 1)} = t^2 - 1 = x - 1.$$

No denominator vanishes for positive t. This cancellation avoids an unnecessary quotient-rule calculation and is an identity in a neighbourhood of the point required.

Step 2: Differentiate the trigonometric term Write the whole function in the simpler form

$$y = x - 1 + \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x.$$

Differentiating each term gives

$$\begin{aligned} y'(x) &= 1 - \cos^4 x \sin x + \cos^2 x \sin x \\ &= 1 + \sin x \cos^2 x(1 - \cos^2 x) \\ &= 1 + \sin^3 x \cos^2 x. \end{aligned}$$

The minus sign from differentiating cosine is essential in both power terms.

Step 3: Evaluate the requested multiple At $x = \pi/6$, use $\sin(\pi/6) = 1/2$ and $\cos^2(\pi/6) = 3/4$. Thus

$$y'(\pi/6) = 1 + \left(\frac{1}{2}\right)^3 \frac{3}{4} = 1 + \frac{3}{32} = \frac{35}{32}.$$

The question asks for this derivative multiplied by 96, rather than for the derivative alone.

Step 4: Conclusion Consequently, $96y'(\pi/6) = 96(35/32) = \boxed{105}$.

Quick Tip to Solve: In expressions involving several powers of a square root, substitute its symbol temporarily and factor before applying any differentiation rule.

24. If $\frac{dx}{dy} = \frac{1+x-y^2}{y}, x(1) = 1$, then $5x(2)$ is equal to _____. [4 marks]

Solution:

Concept:

- Treat x as the dependent variable and y as the independent variable. The equation is linear in x, with the standard form $dx/dy + P(y)x = Q(y)$.
- An integrating factor is $e^{\int P(y)\,dy}$. The initial condition is at $y = 1$, so the relevant solution interval is $y > 0$, where the coefficient $1/y$ is defined.

Step 1: Put the equation in linear form Rearrange the printed equation without interchanging the

roles of the variables:

$$\frac{dx}{dy} - \frac{x}{y} = \frac{1}{y} - y.$$

Here $P(y) = -1/y$. On the positive interval containing both the initial point and $y = 2$, the integrating factor is

$$\text{IF} = e^{-\ln y} = \frac{1}{y}.$$

The given notation $x(1) = 1$ means that the value of x is 1 when the independent variable y equals 1.

Step 2: Integrate the exact derivative Multiply every term by $1/y$. The product rule then identifies the left side:

$$\frac{1}{y}\frac{dx}{dy} - \frac{x}{y^2} = \frac{1}{y^2} - 1, \qquad \frac{d}{dy}\left(\frac{x}{y}\right) = \frac{1}{y^2} - 1.$$

Integrating with respect to y gives

$$\frac{x}{y} = -\frac{1}{y} - y + C, \qquad x = -1 - y^2 + Cy.$$

The arbitrary constant multiplies y after clearing the denominator; it is not an additive constant in the final expression for x.

Step 3: Apply the initial condition and check At $y = 1$, the expression gives $1 = -1 - 1 + C$, hence $C = 3$. Therefore $x(y) = -1 - y^2 + 3y$. Its derivative is $-2y + 3$, while substitution into the original right side gives

$$\frac{1 + (-1 - y^2 + 3y) - y^2}{y} = 3 - 2y,$$

confirming the solution. Finally $x(2) = -1 - 4 + 6 = 1$.

Step 4: Conclusion The required value is $5x(2) = 5 \cdot 1 = \boxed{5}$.

Quick Tip to Solve: For an equation written as dx/dy, build the integrating factor in y and interpret x(1) as an initial value at y=1.

25. Let $f : (0, \infty) \to \mathbf{R}$ and $\mathrm{F}(x) = \int_0^x \mathrm{t} f(\mathrm{t})\mathrm{dt}$. If $\mathrm{F}\left(x^2\right) = x^4 + x^5$, then $\sum_{\mathrm{r}=1}^{12} f\left(\mathrm{r}^2\right)$ is equal to _____.

[4 marks]

Solution:

Concept:

- For $F(x) = \int_0^x tf(t)\,dt$, the fundamental theorem gives $F'(x) = xf(x)$, and the chain rule applies to $F(x^2)$.
- All arguments used here are positive. In particular, $\sqrt{r^2} = r$ for the integers $r = 1, \ldots, 12$, allowing the resulting finite sum to be evaluated directly.

Step 1: Differentiate the given identity Differentiate $F(x^2) = x^4 + x^5$ with respect to positive x. The argument x^2 contributes the chain-rule factor $2x$. Also, the integrand defining F shows that

$F'(x^2) = x^2 f(x^2)$. Consequently,

$$2x \cdot x^2 f(x^2) = 4x^3 + 5x^4.$$

This step uses both the fundamental theorem and the chain rule; omitting either the factor x^2 or $2x$ would change the answer.

Step 2: Find the values being summed Since $x > 0$, divide through by $2x^3$, obtaining

$$f(x^2) = 2 + \frac{5}{2}x.$$

Equivalently, if $u = x^2 > 0$, then $f(u) = 2 + (5/2)\sqrt{u}$. Substituting $x = r$ gives the exact values $f(r^2) = 2 + (5/2)r$ for every required positive integer r. As a check, integrating $t[2 + (5/2)\sqrt{t}]$ from zero to u returns $u^2 + u^{5/2}$, which at $u = x^2$ is the printed expression.

Step 3: Evaluate the arithmetic sum Separate the constant and linear terms:

$$\begin{aligned}\sum_{r=1}^{12} f(r^2) &= \sum_{r=1}^{12} \left(2 + \frac{5}{2}r\right) \\ &= 24 + \frac{5}{2}\frac{12 \cdot 13}{2} \\ &= 24 + 195 = 219.\end{aligned}$$

The 12 constant contributions are included, and the standard arithmetic-series sum is $1 + \cdots + 12 = 78$.

Step 4: Conclusion The requested sum is $\boxed{219}$.

Quick Tip to Solve: When an integral function is given at x squared, differentiate first and retain both the chain-rule factor and the original integrand factor.

---

26. Let ABC be an isosceles triangle in which A is at $(-1, 0), \angle A = \frac{2\pi}{3}, AB = AC$ and B is on the positve x-axis. If $\mathrm{BC} = 4\sqrt{3}$ and the line BC intersects the line $y = x + 3$ at (α, β), then $\frac{\beta^4}{\alpha^2}$ is _____.

[4 marks]

Solution:

Concept:

- The cosine rule relates the equal sides to the base opposite the included angle. Here $BC^2 = AB^2 + AC^2 - 2AB\,AC\cos(2\pi/3)$.
- Coordinates of a point a distance s from A, at angle ϕ to the positive horizontal direction, are $A + (s\cos\phi, s\sin\phi)$. Reflection across the x-axis must also be allowed.

Step 1: Locate the vertices Write $AB = AC = s > 0$. Since $\cos(2\pi/3) = -1/2$, the cosine rule gives $BC^2 = 3s^2$. With $BC = 4\sqrt{3}$, this yields $s = 4$. The vertex B lies on the positive x-axis, so $B = (3, 0)$, because its distance from $A = (-1, 0)$ is 4. The other vertex has coordinates

$$C = (-1 + 4\cos(2\pi/3),\ \pm 4\sin(2\pi/3)) = (-3,\ \pm 2\sqrt{3}).$$

The two signs describe reflected triangles and both satisfy the stated data.

Step 2: Find the intersection using the slope Let m denote the slope of BC. From these coordinates $m = \mp 1/\sqrt{3}$, so $m^2 = 1/3$. The line through B is $y = m(x-3)$. At its intersection with $y = x+3$, write the coordinates as (α, β). Solving the pair gives

$$\alpha = -\frac{3(1+m)}{1-m}, \qquad \beta = -\frac{6m}{1-m}.$$

Neither denominator vanishes for the two allowed slopes.

Step 3: Evaluate a reflection-independent ratio Instead of selecting a reflected triangle, simplify the required even-powered expression for either slope:

$$\begin{aligned}\frac{\beta^4}{\alpha^2} &= \frac{1296m^4}{(1-m)^4}\frac{(1-m)^2}{9(1+m)^2} \\ &= \frac{144m^4}{(1-m^2)^2} \\ &= \frac{144/9}{(1-1/3)^2} = 36.\end{aligned}$$

Only m^2 remains, proving that the upper and lower choices of C produce the same answer.

Step 4: Conclusion Therefore $\beta^4/\alpha^2 = \boxed{36}$.

Quick Tip to Solve: When a triangle has two reflected coordinate positions, keep the slope symbolic and use its square if the requested expression contains even powers.

---

27. Let $A = I_2 - 2MM^T$, where M is a real matrix of order 2×1 such that the relation $M^TM = I_1$ holds. If λ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix X of order 2×1, then the sum of squares of all possible values of λ is equal to _____. [4 marks]

Solution:

Concept:

- Let $P = MM^T$. Since $M^TM = I_1$, the column M is a unit vector and $P^2 = P$; thus P is the projection onto its one-dimensional span.
- For a nonzero vector X satisfying $AX = \lambda X$, a polynomial identity in A gives the same identity in λ. Both candidate values must be shown to occur.

Step 1: Square the matrix The given dimensions make M^TM a scalar 1 and MM^T a real matrix of order 2×2. Associativity gives

$$P^2 = M(M^TM)M^T = MM^T = P.$$

Consequently,

$$A^2 = (I_2 - 2P)^2 = I_2 - 4P + 4P^2 = I_2.$$

This identity follows directly from the unit-length condition and does not require the entries of M to

be specified.

Step 2: Restrict the possible eigenvalues If $AX = \lambda X$, applying A once more yields $A^2X = \lambda^2 X$. Since $A^2 = I_2$, we have $(\lambda^2 - 1)X = 0$. The hypothesis that $X \neq 0$ forces $\lambda^2 = 1$. Thus the only possible real values are 1 and -1; this proves necessity but still requires an existence check.

Step 3: Exhibit both eigendirections The nonzero column M itself satisfies

$$AM = M - 2M(M^T M) = -M,$$

so -1 occurs. To see that 1 occurs, write $M = (a, b)^T$, where $a^2 + b^2 = 1$, and take $N = (-b, a)^T$. This column is nonzero and $M^T N = 0$. Therefore $AN = N - 2M(M^T N) = N$. The two values are consequently both attainable for every admissible M, rather than merely being formal roots.

Step 4: Conclusion The sum of the squares of all possible values is $1^2 + (-1)^2 = \boxed{2}$.

Quick Tip to Solve: For a matrix built from a unit column and its transpose, test the column itself and a perpendicular column to identify both eigendirections.

---

28. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = -\hat{i}-8\hat{j}+2\hat{k}$ and $\vec{c} = 4\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is θ, then the greatest integer less than or equal to $\tan^2\theta$ is _____. [4 marks]

Solution:

Concept:

- The equality $\mathbf{b} \times \mathbf{a} = \mathbf{c} \times \mathbf{a}$ implies $(\mathbf{c} - \mathbf{b}) \times \mathbf{a} = 0$. Since $\mathbf{a} \neq 0$, the difference is parallel to $\mathbf{a}$.
- For nonzero vectors $\mathbf{u}, \mathbf{v}$, $\cos^2\theta = (\mathbf{u} \cdot \mathbf{v})^2/(|\mathbf{u}|^2|\mathbf{v}|^2)$, and $\tan^2\theta = 1/\cos^2\theta - 1$ when their dot product is nonzero.

Step 1: Determine the unknown components The vanishing cross product gives a scalar t with $\mathbf{c} - \mathbf{b} = t\mathbf{a}$. Comparing the first components gives $4 - (-1) = t$, hence $t = 5$. The remaining components then satisfy

$$c_2 - (-8) = 5, \qquad c_3 - 2 = 5,$$

so $c_2 = -3$, $c_3 = 7$, and $\mathbf{c} = (4, -3, 7)$. As a check, $\mathbf{c} - \mathbf{b} = (5, 5, 5) = 5\mathbf{a}$, which indeed has zero cross product with $\mathbf{a}$.

Step 2: Calculate the angle invariant Write the other vector as $\mathbf{d} = (3, 4, 1)$. Calculate the dot product and squared lengths explicitly:

$$\begin{aligned} \mathbf{c} \cdot \mathbf{d} &= 4 \cdot 3 + (-3) \cdot 4 + 7 \cdot 1 = 7, \\ |\mathbf{c}|^2 &= 16 + 9 + 49 = 74, \\ |\mathbf{d}|^2 &= 9 + 16 + 1 = 26. \end{aligned}$$

Thus $\cos^2\theta = 49/(74 \cdot 26) = 49/1924$. The nonzero dot product ensures that the tangent is defined; squaring avoids unnecessary square roots.

Step 3: Find the greatest integer Use the identity relating tangent and cosine:

$$\tan^2\theta = \frac{1924}{49} - 1 = \frac{1875}{49}.$$

To take the floor exactly, observe that $49 \cdot 38 = 1862$ and $49 \cdot 39 = 1911$. Hence

$$38 < \frac{1875}{49} < 39.$$

There is no need to approximate the angle numerically or round the tangent; the requested operation is the greatest integer, not nearest-integer rounding.

Step 4: Conclusion The greatest integer less than or equal to $\tan^2\theta$ is $\boxed{38}$.

Quick Tip to Solve: Turn an equality of cross products into a parallel-vector relation first, then obtain tangent squared from dot products and squared norms.

29. If three successive terms of a G.P. with common ratio $r(r > 1)$ are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r, then $3[r] + [-r]$ is equal to _____. [4 marks]

Solution:

Concept:

- Three successive positive terms of a geometric progression can be written as a, ar, ar^2, where $a > 0$. Since $r > 1$, their increasing order is exactly this order.
- A non-degenerate triangle requires the sum of the two shorter sides to exceed the longest side. The floor $[x]$ is the greatest integer not exceeding x; for negative nonintegers it rounds down, away from zero.

Step 1: Apply the restrictive triangle inequality The longest side is ar^2, so the necessary inequality is

$$a + ar > ar^2.$$

Dividing by the positive number a does not reverse the inequality. Thus

$$r^2 - r - 1 < 0.$$

The other two triangle inequalities follow automatically: adding a positive side to the largest side exceeds either smaller side. Therefore this single inequality contains the required restriction.

Step 2: Find the interval for the common ratio The roots of the quadratic are $(1-\sqrt{5})/2$ and $(1+\sqrt{5})/2$. A quadratic with positive leading coefficient is negative strictly between its roots. Combining this fact with the given restriction gives

$$1 < r < \frac{1+\sqrt{5}}{2} < 2.$$

Neither endpoint $r = 1$ nor a degenerate triangle is permitted. In particular, there is no integer in the allowed interval.

Step 3: Evaluate both greatest integers For every permitted value, $1 < r < 2$, hence $[r] = 1$. Negating reverses the inequalities: $-2 < -r < -1$, hence $[-r] = -2$, not -1. Substitute these fixed integer values:

$$3[r] + [-r] = 3(1) + (-2) = 1.$$

The answer is independent of the scale factor a and of the particular permitted ratio.

Step 4: Conclusion The required numerical value is $\boxed{1}$.

Quick Tip to Solve: First bound the ratio using the longest side. For a negative noninteger, the greatest integer is the next integer to its left on the number line.

30. The lines $L_1, L_2, \ldots, L_{20}$ are distinct. For $n = 1, 2, 3, \ldots, 10$ all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \ldots, L_{20}\}$ is equal to _____. [4 marks]

Solution:

Concept:

- The ten odd-indexed lines form one parallel family. Distinct parallel lines never meet, so this family contributes no mutual intersections.
- The ten even-indexed lines are distinct and concurrent at P. Counting intersection points means counting each location once, regardless of how many line pairs meet there.

Step 1: Count intersections between the two families Each of the ten odd-indexed lines can meet each of the ten even-indexed lines at most once. Thus there are at most

$$10 \times 10 = 100$$

cross-family intersection points. To attain this bound, place P away from all parallel lines and choose the ten concurrent lines so that none is parallel to the first family. Each concurrent line then crosses every parallel line.

Step 2: Check that these points can all be distinct Two distinct concurrent lines meet only at P. If they crossed one of the parallel lines at the same point, that common point would have to be P, contrary to the choice that P lies off the parallel family. Intersections lying on different parallel lines are also distinct, since parallel lines have no common point. Therefore all one hundred cross-family intersections are simultaneously achievable.

Step 3: Include the concurrence point once All pairs within the even-indexed family meet at the single point P, which has not been included in the cross-family count. Although there are $\binom{10}{2} = 45$ pairs in that family, they produce just one distinct location. There are no points from pairs of odd-indexed lines. Hence the maximum total is

$$100 + 1 = 101.$$

Choosing P on a parallel line, or choosing a concurrent line parallel to that family, would reduce the number and cannot improve the maximum.

Step 4: Conclusion The maximum number of distinct intersection points is $\boxed{101}$.

Quick Tip to Solve: Separate parallel-family pairs, concurrent-family pairs and cross-family pairs; count distinct locations rather than the number of pairs.

Physics – Section A: Multiple Choice Questions (80 marks)

31. From the statements given below :

(A) The angular momentum of an electron in n^{th} orbit is an integral multiple of $\hbar$.

(B) Nuclear forces do not obey inverse square law.

(C) Nuclear forces are spin dependent.

(D) Nuclear forces are central and charge independent.

(E) Stability of nucleus is inversely proportional to the value of packing fraction.

Choose the correct answer from the options given below : [4 marks]

(A) (B), (C), (D), (E) only

(B) (A), (C), (D), (E) only

(C) (A), (B), (C), (E) only

(D) (A), (B), (C), (D) only

Correct Answer: (C) (A), (B), (C), (E) only

Solution:

Concept:

- Bohr quantisation gives the orbital angular momentum $L = n\hbar$ in the nth allowed orbit, with $n = 1, 2, 3, \ldots$. This is a statement about the Bohr model.
- The nuclear force is short ranged, charge independent to a good approximation, and spin dependent. Its full interaction includes a noncentral component; charge independence does not imply that the force is purely central.

Step 1: Judge the first three statements Statement (A) is correct because $L = nh/(2\pi) = n\hbar$, an integral multiple of $\hbar$. Statement (B) is correct: the nuclear interaction does not follow the long-range inverse-square law of electrostatic or gravitational forces. It is strong over nuclear dimensions and falls rapidly beyond that range. Statement (C) is also correct: the interaction depends on the relative spins of the nucleons, as illustrated by the different behaviour of their spin configurations.

Step 2: Examine centrality and packing fraction Statement (D) joins two claims. Although the nuclear interaction is approximately charge independent, it is not purely central: a noncentral, spin-dependent tensor component is present. Therefore the combined statement (D) is false. In the standard packing-fraction comparison used here, a smaller packing fraction corresponds to greater binding and greater stability, so statement (E) is accepted. This is the usual qualitative stability criterion; it should not be treated as an exact reciprocal formula over signed packing-fraction values.

Step 3: Match the full set to an option The accepted set is therefore (A), (B), (C) and (E). The option containing this set excludes (D), as required. Every other offered set contains (D), and thus includes

the false claim that nuclear forces are purely central. Rejecting that claim is decisive even before comparing the remaining entries.

Step 4: Conclusion The correct statements are A, B, C and E only. Hence, the correct option is (C).

Quick Tip to Solve: Test every clause in a compound statement: charge independence can be true while the claim of a purely central nuclear force is false.

32. A body of mass $4\ \text{kg}$ experiences two forces $\vec{F}_1 = 5\hat{i}+8\hat{j}+7\hat{k}$ and $\vec{F}_2 = 3\hat{i}-4\hat{j}-3\hat{k}$. The acceleration acting on the body is : [4 marks]

(A) $2\hat{i}+\hat{j}+\hat{k}$

(B) $4\hat{i}+2\hat{j}+2\hat{k}$

(C) $-2\hat{i}-\hat{j}-\hat{k}$

(D) $2\hat{i}+3\hat{j}+3\hat{k}$

Correct Answer: (A) $2\hat{i}+\hat{j}+\hat{k}$

Solution:

Concept:

- Newton's second law applies to the resultant external force: $\vec{F}_{\text{net}} = m\vec{a}$. For constant mass, divide the net force vector by the scalar mass.
- Cartesian unit vectors $\hat{i}, \hat{j}, \hat{k}$ are independent directions. Add force components along the same direction, keeping the signs; do not add the magnitudes of nonparallel vectors.

Step 1: Add the two applied force vectors The component sums in the stated axes are

$$F_x = 5 + 3 = 8\ \text{N},$$
$$F_y = 8 - 4 = 4\ \text{N},$$
$$F_z = 7 - 3 = 4\ \text{N}.$$

Consequently the resultant is

$$\vec{F}_{\text{net}} = 8\hat{i} + 4\hat{j} + 4\hat{k}\ \text{N}.$$

The negative components of the second force partially cancel the positive components of the first force in the y and z directions. They do not reverse either resultant component.

Step 2: Divide each component by the mass The same mass $m = 4\ \text{kg}$ applies to all three components. Therefore

$$\vec{a} = \frac{8\hat{i} + 4\hat{j} + 4\hat{k}}{4} = 2\hat{i} + \hat{j} + \hat{k}\ \text{m s}^{-2}.$$

As a direct consistency check, multiplying this acceleration by $4\ \text{kg}$ recovers $8\hat{i} + 4\hat{j} + 4\hat{k}\ \text{N}$, the vector sum above. No component of acceleration is negative.

Step 3: Compare the alternatives Option (A) agrees with all three acceleration components. Option (B) is twice the required vector and would correspond to using half the given mass. Option (C) points exactly opposite to the net force, violating Newton's second law for a positive mass. Option (D) has

incorrect y and z components; those components must be one, not three, after adding the signed forces and dividing by four.

Step 4: Conclusion The acceleration is $\boxed{\vec{a} = (2\hat{i} + \hat{j} + \hat{k})\ \mathrm{m\,s^{-2}}}$. Hence, the correct option is (A).

Quick Tip to Solve: Combine signed force components first, then divide every component by the same mass. Check the result by multiplying back by the mass.

---

33. Monochromatic light of frequency 6×10^{14} Hz is produced by a laser. The power emitted is 2×10^{-3} W.

How many photons per second on an average, are emitted by the source ?

(Given $\mathrm{h} = 6.63 \times 10^{-34}\mathrm{Js}$ **)** [4 marks]

(A) 5×10^{15}

(B) 7×10^{16}

(C) 6×10^{15}

(D) 9×10^{18}

Correct Answer: (A) 5×10^{15}

Solution:

Concept:

- A photon of monochromatic light of frequency ν has energy $E_\gamma = h\nu$, where h is Planck's constant. Equal frequency means equal energy per photon.
- Power is energy emitted per unit time. If $\dot{N}$ photons are emitted per second, $P = \dot{N}h\nu$; hence $\dot{N} = P/(h\nu)$, with unit $\mathrm{s^{-1}}$.

Step 1: Calculate the energy of one photon Use the given frequency and Planck constant directly:

$$\begin{aligned} E_\gamma &= (6.63 \times 10^{-34})(6 \times 10^{14})\ \mathrm{J} \\ &= 39.78 \times 10^{-20}\ \mathrm{J} \\ &= 3.978 \times 10^{-19}\ \mathrm{J}. \end{aligned}$$

The frequency is already in hertz and requires no conversion. This is the energy of one photon; it is much smaller than the total energy emitted in a second.

Step 2: Divide the energy per second by the photon energy The laser emits 2×10^{-3} J each second. The average photon emission rate is therefore

$$\begin{aligned} \dot{N} &= \frac{2 \times 10^{-3}}{3.978 \times 10^{-19}}\ \mathrm{s^{-1}} \\ &= \frac{2}{3.978} \times 10^{16}\ \mathrm{s^{-1}} \\ &\approx 5.03 \times 10^{15}\ \mathrm{s^{-1}}. \end{aligned}$$

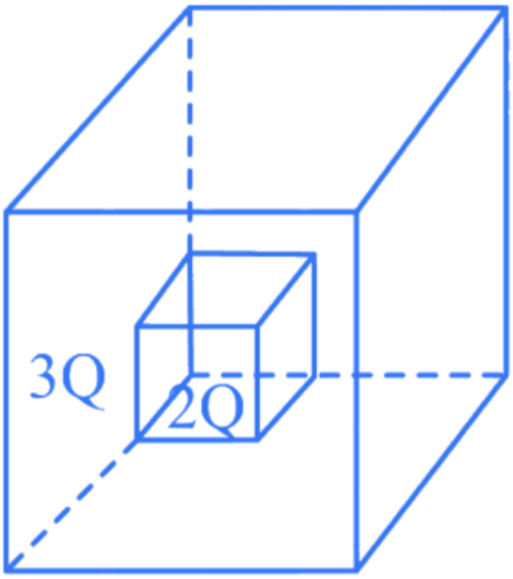
A dimensional check gives $(\mathrm{J\,s^{-1}})/\mathrm{J} = \mathrm{s^{-1}}$, as needed for the number emitted per second.

Step 3: Select the closest listed emission rate The offered value 5×10^{15} is the rounded result. The other listed values are not consistent with the calculated quotient: 7×10^{16} and 9×10^{18} have much larger orders of magnitude, while 6×10^{15} is not the nearest value. Multiplying 5×10^{15} by the single-photon energy gives approximately 1.99×10^{-3} W, consistent with the given power.

Step 4: Conclusion The source emits approximately $\boxed{5 \times 10^{15}}$ photons per second. Hence, the correct option is (A).

Quick Tip to Solve: Convert power into photons per second by dividing by the energy of one photon, not by the frequency alone; keep the exponents separate.

34. C_1 **and** C_2 **are two hollow concentric cubes enclosing charges** $2Q$ **and** $3Q$ **respectively as shown in figure. The ratio of electric flux passing through** C_1 **and** C_2 **is :** [4 marks]



(A) 3 : 2

(B) 5 : 2

(C) 2 : 5

(D) 2 : 3

Correct Answer: (C) 2 : 5

Solution:

Concept:

- Gauss's law states that the net electric flux through a closed surface is $\Phi = Q_{\text{enclosed}}/\varepsilon_0$. The surface need not be spherical or an equipotential.
- A larger closed surface that surrounds a smaller charged region encloses those charges as well as charges between the two surfaces. External charges can affect the field locally but contribute zero net flux through a closed surface.

Step 1: Identify the charge enclosed by each cube The figure places $2Q$ inside the inner cube C_1. The

charge $3Q$ is in the region outside C_1 but inside the larger cube C_2. Therefore

$$Q_{\text{enc},1} = 2Q, \qquad Q_{\text{enc},2} = 2Q + 3Q = 5Q.$$

The outer cube must include the inner charge; treating its enclosed charge as only $3Q$ would omit a charge that is visibly inside it.

Step 2: Apply Gauss's law independently to the cubes The net fluxes through the complete closed cubes are

$$\Phi_1 = \frac{2Q}{\varepsilon_0}, \qquad \Phi_2 = \frac{5Q}{\varepsilon_0}.$$

No cube side length or distribution of flux over individual faces is required. The charge outside the smaller cube may produce inward flux through some portions and outward flux through others, but its net contribution to that closed cube is zero. This is why the first expression contains only $2Q$.

Step 3: Form the ratio in the requested order The question asks for flux through C_1 compared with flux through C_2. Thus

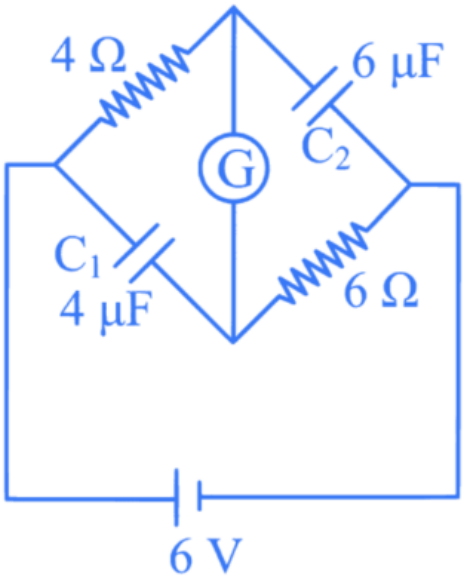
$$\frac{\Phi_1}{\Phi_2} = \frac{2Q/\varepsilon_0}{5Q/\varepsilon_0} = \frac{2}{5}.$$

Option (C) gives $2 : 5$. Option (B) reverses the requested order. Option (D) compares the two separately labelled charges without adding the inner charge for the larger surface, and option (A) similarly fails to use total enclosed charge.

Step 4: Conclusion The required ratio is $\boxed{\Phi_1 : \Phi_2 = 2 : 5}$. Hence, the correct option is (C).

Quick Tip to Solve: For nested Gaussian surfaces, add every charge inside the larger surface before taking a flux ratio; its shape and size do not enter.

35. A galvanometer (G) of 2Ω resistance is connected in the given circuit. The ratio of charge stored in C_1 and C_2 is : [4 marks]



(A) 1

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Concept:

- After a DC circuit reaches steady state, an ideal capacitor carries no conduction current. Its stored charge is nevertheless $Q = CV$, using the potential difference across its own terminals.
- The galvanometer is not an ideal wire: its stated $2\ \Omega$ resistance belongs in the conducting resistor path. Series resistors carry the same steady current, with voltage drops given by $V = IR$.

Step 1: Find the steady current through the resistors At steady state, both capacitor branches are open to conduction. The remaining path from the left battery terminal to the right terminal passes through the $4\ \Omega$ resistor, the $2\ \Omega$ galvanometer and the $6\ \Omega$ resistor in series. Consequently

$$R_{\text{total}} = 4 + 2 + 6 = 12\ \Omega, \qquad I = \frac{6}{12} = 0.5\ \text{A}.$$

The voltage drops across these three elements are respectively $2\ \text{V}$, $1\ \text{V}$ and $3\ \text{V}$.

Step 2: Locate the capacitor terminal voltages Capacitor $C_1 = 4\ \mu\text{F}$ lies between the left junction and the bottom junction. Its potential difference equals the combined drops across the upper-left resistor and galvanometer:

$$V_1 = I(4 + 2) = 3\ \text{V}.$$

Capacitor $C_2 = 6\ \mu\text{F}$ lies between the top junction and the right junction. Its potential difference

equals the combined drops across the galvanometer and lower-right resistor:

$$V_2 = I(2+6) = 4\ \text{V}.$$

The capacitors are neither a simple parallel pair nor a simple series pair.

Step 3: Calculate and compare the stored charges Use each capacitance with its corresponding terminal voltage:

$$Q_1 = 4 \times 3 = 12\ \mu\text{C}, \qquad Q_2 = 6 \times 4 = 24\ \mu\text{C}.$$

The required ratio is

$$\frac{Q_1}{Q_2} = \frac{12}{24} = \frac{1}{2}.$$

Option (D) agrees. The value $2/3$ would use only the capacitance ratio and incorrectly assume equal voltages; $3/2$ reverses that ratio, while unity incorrectly imposes equal charges.

Step 4: Conclusion The charge ratio is $\boxed{Q_1/Q_2 = 1/2}$. Hence, the correct option is (D).

Quick Tip to Solve: Open the capacitor branches for steady DC, solve the resistor path including the galvanometer resistance, then evaluate each capacitor's own terminal voltage.

---

36. Match List - I with List - II.

List I (Number)	List II (Significant figure)
(A) 1001	(I) 3
(B) 010.1	(II) 4
(C) 100.100	(III) 5
(D) 0.0010010	(IV) 6

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(B) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(C) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer: (A) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Solution:

Concept:

- Nonzero digits are significant, and zeros enclosed between nonzero digits also count. Leading zeros merely locate the decimal point and do not count.
- Trailing zeros in a number with an explicit decimal point indicate measured precision and are significant. Apply these rules to each number independently.

Step 1: Count digits in the first two numbers In 1001, both zeros lie between nonzero digits. Thus all four digits count, so (A) matches (II). In 010.1, the first zero is only a leading zero. The remaining digits are $1, 0, 1$, and the middle zero is enclosed between nonzero digits. Hence there are three

significant figures and (B) matches (I). A zero is not discarded just because its own numerical value is zero; its position determines whether it records precision.

Step 2: Count digits after locating the first nonzero digit For 100.100, the two zeros before the decimal are within the significant digit sequence, and the two terminal zeros after the decimal explicitly indicate precision. There are six significant figures, so (C) matches (IV). For 0.0010010, ignore the zeros preceding the first 1. The significant sequence is 10010: two enclosed zeros and the final decimal zero count. It contains five significant figures, so (D) matches (III).

Step 3: Match the complete code The counts, in the order (A), (B), (C), (D), are $4, 3, 6, 5$. Only the first answer choice has all four matches. The other codes assign an incorrect count already to 1001 and cannot represent the full list.

Step 4: Conclusion The significant-figure counts in the listed order are $4, 3, 6, 5$, giving the first matching code. Hence, the correct option is (A).

Quick Tip to Solve: Cross out leading zeros first; then count from the first nonzero digit through any explicitly written trailing decimal zeros.

37. A big drop is formed by coalescing 1000 small droplets of water. The surface energy will become :

[4 marks]

(A) $\frac{1}{100}$ th
(B) $\frac{1}{10}$ th
(C) 100 times
(D) 10 times

Correct Answer: (B) $\frac{1}{10}$ th

Solution:

Concept:

- For a liquid surface of area A, the surface energy is $U = SA$, where S is surface tension. For the same liquid under unchanged conditions, energy ratios equal area ratios.
- Coalescence conserves liquid volume, not surface area. A spherical drop of radius r has volume $4\pi r^3/3$ and surface area $4\pi r^2$.

Step 1: Use conservation of volume Let each of the 1000 identical droplets have radius r, and let the final large drop have radius R. No liquid is lost, so

$$\frac{4\pi R^3}{3} = 1000\frac{4\pi r^3}{3}.$$

Therefore $R^3 = 1000r^3$, giving $R = 10r$. The radius grows by the cube root of the number of droplets, rather than by that number itself.

Step 2: Compare final and initial total surface energies The initial energy includes the surfaces of all 1000 droplets:

$$U_i = 1000S(4\pi r^2).$$

The final drop has only one outer liquid surface, giving

$$U_f = S(4\pi R^2) = 100S(4\pi r^2).$$

Consequently,

$$\frac{U_f}{U_i} = \frac{100}{1000} = \frac{1}{10}.$$

This is a reduction in total surface energy. The factor 100 compares the final area with one initial droplet, not with all 1000 droplets together. The factor 1/100 therefore uses the wrong area scaling, while factors 100 and 10 wrongly predict an increase in total energy.

Step 3: Conclusion The final surface energy is $U_f = U_i/10$, or one tenth of its initial total value. Hence, the correct option is (B).

Quick Tip to Solve: For coalescence of N identical spherical drops, volume conservation gives an energy ratio of $N^{-1/3}$; compare with all initial drops.

38. A cricket player catches a ball of mass $120\ \mathrm{g}$ moving with $25\ \mathrm{m/s}$ speed. If the catching process is completed in $0.1\ \mathrm{s}$ then the magnitude of force exerted by the ball on the hand of player will be (in SI unit) : [4 marks]

(A) 30

(B) 24

(C) 12

(D) 25

Correct Answer: (A) 30

Solution:

Concept:

- The average force on a body equals its change in momentum divided by the interaction time: $\vec{F}_{\text{avg}} = m(\vec{v} - \vec{u})/\Delta t$.
- Newton's third law makes the force exerted by the ball on the hand equal in magnitude and opposite in direction to that exerted by the hand on the ball. Mass must be expressed in kilograms for SI force.

Step 1: Find the momentum change of the ball Convert the mass before substituting:

$$m = 120\,\mathrm{g} = 0.120\,\mathrm{kg}.$$

Take the initial motion of the ball as positive. Its initial speed is $u = 25\,\mathrm{m\,s^{-1}}$. Completing the catch brings the ball to rest, so $v = 0$. Its momentum change is

$$\Delta p = 0.120(0 - 25) = -3.0\,\mathrm{kg\,m\,s^{-1}}.$$

The negative sign shows that the stopping force on the ball opposes its original motion; it does not make the requested magnitude negative.

Step 2: Apply the impulse relation and the third law The stopping interval is $0.1\,\text{s}$. The average force on the ball is

$$F_{\text{avg}} = \frac{-3.0}{0.1} = -30\,\text{N}.$$

Thus its magnitude is $30\,\text{N}$. The ball exerts a force of the same magnitude on the player's hand in the opposite direction. The given time determines an average interaction force; an instantaneous peak force would require details of how the force varies during the catch. Options 24, 12 and 25 do not equal the required momentum change per unit time and therefore fail the impulse check.

Step 3: Conclusion The magnitude of the average force exerted by the ball on the hand is $30\,\text{N}$. Hence, the correct option is (A).

Quick Tip to Solve: Compute the ball's lost momentum in SI units, divide by the stopping time, and use Newton's third law for the force on the hand.

---

39. In a metre-bridge when a resistance in the left gap is 2Ω and unknown resistance in the right gap, the balance length is found to be 40 cm. On shunting the unknown resistance with 2Ω, the balance length changes by : [4 marks]

(A) 62.5

(B) 22.5 cm

(C) 20 cm

(D) 65 cm

Correct Answer: (B) 22.5 cm

Solution:

Concept:

- At balance in a uniform metre bridge, the gap-resistance ratio equals the corresponding wire-length ratio: $R_L/R_R = l/(100-l)$, with lengths in centimetres.
- A shunt is connected in parallel. Two resistances R and S have equivalent resistance $RS/(R+S)$. The asked change in balance length is the new length minus the original one.

Step 1: Determine the original unknown resistance Let the unknown right-gap resistance be X. Since the balance length is measured from the left end, the two wire lengths are 40 cm and 60 cm. Hence

$$\frac{2}{X} = \frac{40}{60}.$$

Cross-multiplication gives $X = 3\,\Omega$. This value is larger than the left resistance, consistent with the left balance length being less than half the wire.

Step 2: Replace the shunted right resistance A $2\,\Omega$ resistor in parallel with $3\,\Omega$ gives

$$X_{\text{eq}} = \frac{3\times 2}{3+2} = \frac{6}{5}\,\Omega.$$

Write the new bridge balance as

$$\frac{2}{6/5} = \frac{l\prime}{100 - l\prime} = \frac{5}{3}.$$

Thus $3l\prime = 5(100 - l\prime)$, so $8l\prime = 500$ and $l\prime = 62.5\,\text{cm}$.

Step 3: Calculate the shift rather than the new reading The change is

$$\Delta l = 62.5 - 40 = 22.5\,\text{cm}.$$

The balance point shifts towards the right because the right-gap resistance decreases while the left-gap resistance stays fixed. The number 62.5 is the final balance length, not its change. Neither 20 cm nor 65 cm follows from the parallel-resistance and bridge equations.

Step 4: Conclusion The balance length increases by $22.5\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: First find the unknown gap resistance, then apply the parallel formula for its shunt; subtract the original bridge reading only at the end.

40. A diatomic gas $(\gamma = 1.4)$ does 200 J of work when it is expanded isobarically. The heat given to the gas in the process is : [4 marks]

(A) 800 J

(B) 600 J

(C) 700 J

(D) 850 J

Correct Answer: (C) 700 J

Solution:

Concept:

- For an ideal gas in an isobaric process, $W = P\Delta V = nR\Delta T$, while the supplied heat is $Q = nC_P\Delta T$. Work done by the expanding gas is positive.
- Mayer's relation is $C_P - C_V = R$, and $\gamma = C_P/C_V$. Consequently, $C_P/R = \gamma/(\gamma - 1)$.

Step 1: Relate heat input to the work done The gas expands at constant pressure, so its rise in temperature produces both increased internal energy and mechanical work. Divide the expressions for heat and work:

$$\frac{Q}{W} = \frac{nC_P\Delta T}{nR\Delta T} = \frac{C_P}{R}.$$

The unknown gas quantity and temperature change cancel. Using $C_P = \gamma C_V$ in $C_P - C_V = R$ gives $(\gamma - 1)C_V = R$, and hence $C_P = \gamma R/(\gamma - 1)$.

Step 2: Substitute the heat-capacity ratio With $\gamma = 1.4$ and $W = 200\,\text{J}$,

$$Q = \frac{1.4}{1.4 - 1}(200) = \frac{1.4}{0.4}(200) = 700\,\text{J}.$$

As a first-law check, $\Delta U = Q - W = 500\,\text{J}$, and independently $\Delta U = W/(\gamma - 1) = 200/0.4 = 500\,\text{J}$.

Both heat input and internal-energy change are positive, as expected for isobaric heating accompanied by expansion. Values 800 J, 600 J and 850 J would give a heat-to-work ratio different from the required $C_P/R = 3.5$, so none satisfies the specified gas properties.

Step 3: Conclusion The heat supplied to the gas is $700\,\mathrm{J}$. Hence, the correct option is (C).

Quick Tip to Solve: For ideal-gas isobaric expansion, use $Q/W = \gamma/(\gamma - 1)$; the work alone is not the heat supplied.

41. Train A is moving along two parallel rail tracks towards north with speed $72\ \mathrm{km/h}$ and train B is moving towards south with speed $108\ \mathrm{km/h}$. Velocity of train B with respect to A and velocity of ground with respect to B are (in ms^{-1}): [4 marks]

(A) -50 and -30

(B) -50 and 30

(C) -30 and 50

(D) 50 and -30

Correct Answer: (B) -50 and 30

Solution:

Concept:

- In one-dimensional relative motion, the velocity of B relative to A is $v_{B/A} = v_B - v_A$. Specify a positive direction before inserting signed velocities.
- To convert kilometres per hour to metres per second, multiply by $5/18$. The stationary ground has zero velocity in the ground reference frame.

Step 1: Convert the two speeds and assign directions Choose north as positive. Train A moves north, so

$$v_A = 72\frac{5}{18} = +20\,\mathrm{m\,s^{-1}}.$$

Train B moves south, so

$$v_B = -108\frac{5}{18} = -30\,\mathrm{m\,s^{-1}}.$$

The minus sign on B is essential: the question gives speeds, which are magnitudes, but relative velocity must be calculated from signed velocities. The ground has $v_G = 0$.

Step 2: Find each requested relative velocity The velocity of B as observed from A is

$$v_{B/A} = v_B - v_A = -30 - 20 = -50\,\mathrm{m\,s^{-1}}.$$

Thus B approaches and passes A towards the south at relative speed 50 metres per second. For the ground as observed from B, subtract B's velocity from that of the ground:

$$v_{G/B} = v_G - v_B = 0 - (-30) = +30\,\mathrm{m\,s^{-1}}.$$

The ground therefore appears to move north from B. The pair $(-50, -30)$ has the second direction

reversed; $(-30, 50)$ mixes the train's ground speed with relative speed; and $(50, -30)$ reverses both directions under the chosen north-positive convention.

Step 3: Conclusion With north positive, the velocities are $-50\,\mathrm{m\,s^{-1}}$ and $+30\,\mathrm{m\,s^{-1}}$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: For every relative velocity, subtract the observer's signed velocity from the object's signed velocity; keep one direction convention throughout.

---

42. A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T. If the force of attraction between planet and star is proportional to $\mathrm{R}^{-3/2}$ then choose the correct option : [4 marks]

(A) $\mathrm{T}^2 \propto \mathrm{R}^{7/2}$

(B) $\mathrm{T}^2 \propto \mathrm{R}^3$

(C) $\mathrm{T}^2 \propto \mathrm{R}^{5/2}$

(D) $\mathrm{T}^2 \propto \mathrm{R}^{3/2}$

Correct Answer: (C) $\mathrm{T}^2 \propto \mathrm{R}^{5/2}$

Solution:

Concept:

- For circular motion of radius R and period T, the speed is $v = 2\pi R/T$, so the required centripetal force is $F = mv^2/R = 4\pi^2 mR/T^2$.
- The stated attractive force is proportional to $R^{-3/2}$. Its exponent must be used directly; the usual inverse-square gravitational law is not the force law specified here.

Step 1: Equate the attraction to the centripetal force Write the magnitude of the given force as $F = kR^{-3/2}$, where k is independent of orbital radius for this fixed star and planet. In a circular orbit this force supplies the radial acceleration:

$$\frac{4\pi^2 mR}{T^2} = kR^{-3/2}.$$

The planet is light compared with the star, so treating the star as the centre of the planet's orbit is consistent with the stated model.

Step 2: Solve for the radius dependence of the period Rearrange without changing the exponent:

$$T^2 = \frac{4\pi^2 m}{k}\frac{R}{R^{-3/2}} = \frac{4\pi^2 m}{k}R^{5/2}.$$

The prefactor is constant, so $T^2 \propto R^{5/2}$. More generally, a central force proportional to R^{-p} yields $T^2 \propto R^{p+1}$. Here $p = 3/2$, and the added power of R comes from the centripetal-force expression. Option B is the familiar result for an inverse-square force, so applying it would ignore the altered law in the question. The powers 7/2 and 3/2 similarly do not satisfy the circular-orbit equation when substituted.

Step 3: Conclusion The period obeys $T^2 \propto R^{5/2}$. Hence, the correct option is (C).

Quick Tip to Solve: Derive the orbital scaling from $F = 4\pi^2 mR/T^2$; for $F \propto R^{-p}$, simply add one to p to obtain the exponent of R in T^2.

43. A microwave of wavelength 2.0 cm falls normally on a slit of width 4.0 cm. The angular spread of the central maxima of the diffraction pattern obtained on a screen 1.5 m away from the slit, will be : [4 marks]

(A) 60°

(B) 45°

(C) 15°

(D) 30°

Correct Answer: (A) 60°

Solution:

Concept:

- For a slit of width a, the first diffraction minima satisfy $a \sin\theta = \lambda$. Here θ is the angle from the central axis to either first minimum, not the full central spread.
- The central maximum extends symmetrically from $-\theta$ to $+\theta$, so its angular spread is 2θ. The exact sine relation must be used when λ/a is not small.

Step 1: Locate the first minima The wavelength and slit width are in the same units, so their ratio can be evaluated directly. For the first minimum on either side,

$$\sin\theta = \frac{\lambda}{a} = \frac{2.0}{4.0} = \frac{1}{2}.$$

Thus $\theta = 30^\circ$. The negative-angle minimum is at -30°. A small-angle replacement of the sine by the angle is inappropriate for this fairly wide diffraction pattern.

Step 2: Find the full angular spread The required spread is the separation between these two minima, rather than the angle on one side:

$$\Delta\theta = 30^\circ - (-30^\circ) = 60^\circ.$$

The screen distance of 1.5 m would be relevant to a linear width on the screen, but it does not change these diffraction angles. Option (D) is only the half-angle; the angles 45° and 15° do not satisfy the first-minimum condition for the supplied ratio.

Step 3: Conclusion The angular spread of the central maximum is 60°. Hence, the correct option is (A).

Quick Tip to Solve: Compute the first-minimum half-angle from $\sin\theta = \lambda/a$, then double it; do not confuse a half-angle with the full central width.

44. If the root mean square velocity of hydrogen molecule at a given temperature and pressure is $2\ \mathrm{km/s}$, the root mean square velocity of oxygen at the same condition in $\mathrm{km/s}$ is : [4 marks]

(A) 1.0

(B) 1.5

(C) 2.0

(D) 0.5

Correct Answer: (D) 0.5

Solution:

Concept:

- The root mean square speed is $v_{\text{rms}} = \sqrt{3RT/M}$, where T is absolute temperature, R the gas constant and M the molar mass.
- At equal temperature, molecular rms speeds vary inversely as the square root of the molar mass. Molecular hydrogen is $\mathrm{H_2}$ and molecular oxygen is $\mathrm{O_2}$, with molar masses 2 and 32 in $\mathrm{g\,mol^{-1}}$.

Step 1: Compare the molecular masses For the two gases at the same temperature, dividing the speed formulas cancels $3RT$:

$$\frac{v_{\mathrm{O_2}}}{v_{\mathrm{H_2}}} = \sqrt{\frac{M_{\mathrm{H_2}}}{M_{\mathrm{O_2}}}} = \sqrt{\frac{2}{32}} = \frac{1}{4}.$$

Equal pressure is compatible with the stated conditions but is not an extra factor in this speed ratio. Oxygen molecules are sixteen times as massive, so their rms speed is four times smaller, not sixteen times smaller.

Step 2: Calculate the oxygen rms speed Use the supplied hydrogen speed without changing its units:

$$v_{\mathrm{O_2}} = \frac{1}{4} \times 2\,\mathrm{km\,s^{-1}} = 0.5\,\mathrm{km\,s^{-1}}.$$

This also equals $500\,\mathrm{m\,s^{-1}}$. Option (A) would require a speed ratio of one half, option (B) three quarters, and option (C) equal speeds. None matches the square-root mass relation at equal temperature.

Step 3: Conclusion The rms speed of oxygen is $0.5\,\mathrm{km\,s^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: At one temperature, use $v_1/v_2 = \sqrt{M_2/M_1}$ and use molecular masses, not atomic masses.

45. If frequency of electromagnetic wave is $60\ \mathrm{MHz}$ and it travels in air along z direction then the corresponding electric and magnetic field vectors will be mutually perpendicular to each other and the wavelength of the wave (in m) is : [4 marks]

(A) 2.5

(B) 5

(C) 10

(D) 2

Correct Answer: (B) 5

Solution:

Concept:

- For an electromagnetic wave in air, its speed is approximately $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$, and speed, frequency and wavelength obey $c = f\lambda$.
- An electromagnetic wave is transverse: $\mathbf{E}$, $\mathbf{B}$ and the propagation direction are mutually perpendicular, with $\mathbf{E} \times \mathbf{B}$ pointing along propagation.

Step 1: Express the frequency in SI units The prefix mega represents 10^6, so the stated frequency becomes

$$f = 60\,\mathrm{MHz} = 60 \times 10^6\,\mathrm{s^{-1}}.$$

The direction of propagation fixes the orientation of the transverse fields; it does not alter the scalar speed-frequency-wavelength relation. For propagation along the positive z-axis, one possible orientation is $\mathbf{E}$ along x and $\mathbf{B}$ along y, so their cross product is along z.

Step 2: Calculate the wavelength Rearrange the wave relation and substitute:

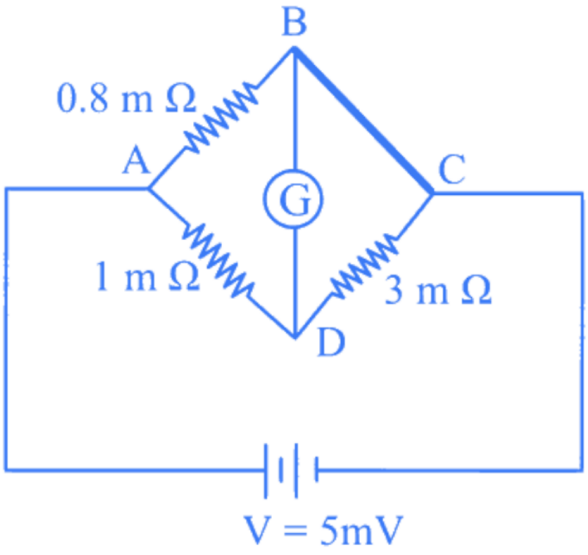
$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{60 \times 10^6}\,\mathrm{m} = 5\,\mathrm{m}.$$

As a direct check, $60 \times 10^6 \times 5 = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$. The lengths 2.5, 10 and $2\,\mathrm{m}$ would give different wave speeds at the stated frequency and therefore cannot be the wavelength in air.

Step 3: Conclusion The wavelength is $5\,\mathrm{m}$, with both fields transverse and mutually perpendicular. Hence, the correct option is (B).

Quick Tip to Solve: Convert MHz to 10^6 Hz first, then use $\lambda = c/f$; field directions do not enter this scalar calculation.

46. To measure the temperature coefficient of resistivity α of a semiconductor, an electrical arrangement shown in the figure is prepared. The arm BC is made up of the semiconductor. The experiment is being conducted at $25^{\circ}\mathrm{C}$ and resistance of the semiconductor arm is $3\ \mathrm{m}\Omega$. Arm BC is cooled at a constant rate of $2^{\circ}\mathrm{C/s}$. If the galvanometer G shows no deflection after $10\ \mathrm{s}$, then α is : [4 marks]



(A) $-1 \times 10^{-2}{}^{\circ}\mathrm{C}^{-1}$

(B) $-2 \times 10^{-2}{}^{\circ}\mathrm{C}^{-1}$

(C) $-2.5 \times 10^{-2}{}^{\circ}\mathrm{C}^{-1}$

(D) $-1.5 \times 10^{-2}{}^{\circ}\mathrm{C}^{-1}$

Correct Answer: No listed option: the printed bridge and cooling give a positive coefficient, $+1.0 \times 10^{-2}\,{}^{\circ}\mathrm{C}^{-1}$.

Solution:

Concept:

- At balance, a Wheatstone bridge with supply across A and C and galvanometer across B and D obeys $R_{AB}/R_{BC} = R_{AD}/R_{DC}$. No deflection means B and D are at the same potential.
- For a fixed semiconductor arm geometry, fractional changes in resistance and resistivity coincide. With the initial temperature as reference, $R_f = R_i[1 + \alpha(T_f - T_i)]$. A negative semiconductor coefficient would make its resistance increase on cooling.

Step 1: Use the printed bridge values The figure gives $R_{AB} = 0.8\,\mathrm{m}\Omega$, $R_{AD} = 1\,\mathrm{m}\Omega$ and $R_{DC} = 3\,\mathrm{m}\Omega$. Therefore at the stated no-deflection time,

$$R_f = R_{BC} = \frac{R_{AB}R_{DC}}{R_{AD}} = \frac{0.8 \times 3}{1} = 2.4\,\mathrm{m}\Omega.$$

The $5\,\mathrm{mV}$ battery does not affect this balance ratio. The initial value of the variable BC arm is independently supplied in the question as $R_i = 3\,\mathrm{m}\Omega$.

Step 2: Apply the stated cooling Cooling at $2^\circ\mathrm{C\,s^{-1}}$ for $10\,\mathrm{s}$ gives

$$\Delta T = -20^\circ\mathrm{C}, \qquad T_f = 5^\circ\mathrm{C}.$$

Consequently,

$$\alpha = \frac{R_f - R_i}{R_i \Delta T} = \frac{2.4 - 3}{3(-20)} = +1.0 \times 10^{-2}\,^\circ\mathrm{C}^{-1}.$$

The positive sign follows from a decrease in resistance during cooling. All four listed answers are negative, and each would instead predict an increase in resistance from its initial value. Thus none satisfies both the printed bridge and the stated temperature change; selecting a negative sign merely because the arm is called a semiconductor would contradict the numerical data.

Step 3: Conclusion The printed data give $\alpha = +1.0 \times 10^{-2}\,^\circ\mathrm{C}^{-1}$; none of options (A)–(D) is correct. The negative option (A) has the matching magnitude but the wrong sign.

Quick Tip to Solve: Keep the sign of ΔT: cooling means a negative temperature change. Determine the balanced resistance before assigning the sign of the coefficient.

---

47. Conductivity of a photodiode starts changing only if the wavelength of incident light is less than $660\ \mathrm{nm}$. The band gap of photodiode is found to be $\left(\frac{\mathrm{X}}{8}\right)\mathrm{eV}$. The value of X is :
(Given, $\mathrm{h} = 6.6 \times 10^{-34}\mathrm{Js}, \mathrm{e} = 1.6 \times 10^{-19}\mathrm{C}$) [4 marks]

(A) 11

(B) 13

(C) 15

(D) 21

Correct Answer: (C) 15

Solution:

Concept:

- A photon can generate electron-hole pairs across the semiconductor band gap when its energy reaches E_g. The longest effective wavelength is therefore the threshold wavelength λ_0, with $E_g = hc/\lambda_0$.
- Photon energy decreases as wavelength increases. An energy in joules is converted to electronvolts by dividing by $e = 1.6 \times 10^{-19}\ \mathrm{J/eV}$.

Step 1: Find the threshold photon energy The quoted wavelength is the threshold, so take $\lambda_0 = 660 \times 10^{-9}\ \mathrm{m}$, and use $c = 3 \times 10^8\ \mathrm{m\,s^{-1}}$:

$$E_g = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{660 \times 10^{-9}} = 3.0 \times 10^{-19}\ \mathrm{J}.$$

Shorter wavelengths supply higher-energy photons, consistent with the stated onset of photoconductivity. Using a longer wavelength would give photons unable to bridge this energy gap.

Step 2: Convert to electronvolts and identify X Divide the energy in joules by the energy of one

electronvolt:

$$E_g = \frac{3.0 \times 10^{-19}}{1.6 \times 10^{-19}}\,\text{eV} = 1.875\,\text{eV} = \frac{15}{8}\,\text{eV}.$$

Comparison with the specified form $E_g = (X/8)\,\text{eV}$ gives $X = 15$. For $X = 11, 13, 21$, the gaps would be $1.375, 1.625, 2.625\,\text{eV}$, respectively, and would not reproduce the supplied $660\,\text{nm}$ threshold.

Step 3: Conclusion The band gap is $1.875\,\text{eV}$, so $X = 15$. Hence, the correct option is (C).

Quick Tip to Solve: At a photodiode threshold use $E_g = hc/\lambda_0$, convert to eV, and only then compare with the requested fractional form.

48. A transformer has an efficiency of 80% and works at 10 V and 4 kW. If the secondary voltage is 240 V, then the current in the secondary coil is : [4 marks]

(A) 1.33 A

(B) 13.33 A

(C) 1.59 A

(D) 15.1 A

Correct Answer: (B) 13.33 A

Solution:

Concept:

- Transformer efficiency is the ratio of useful secondary output power to primary input power: $\eta = P_s/P_p$. An efficiency of 80% is the fraction 0.80.
- For the secondary coil in the stated transformer power calculation, $P_s = V_s I_s$. Losses reduce the available output power, so the input power cannot be used directly as the secondary power.

Step 1: Calculate the useful secondary power The transformer is supplied with $P_p = 4\,\text{kW} = 4000\,\text{W}$. Its secondary output is

$$P_s = \eta P_p = 0.80 \times 4000 = 3200\,\text{W}.$$

The remaining $800\,\text{W}$ represents the difference between input and useful output. The primary voltage of $10\,\text{V}$ could be used to calculate the primary current, but that current is not the quantity requested. The secondary voltage is a separate given value.

Step 2: Divide by the secondary voltage With $V_s = 240\,\text{V}$,

$$I_s = \frac{P_s}{V_s} = \frac{3200}{240}\,\text{A} = \frac{40}{3}\,\text{A} \simeq 13.33\,\text{A}.$$

A check gives $240 \times 13.33 \simeq 3200\,\text{W}$, allowing for rounding. The alternatives 1.33, 1.59 and $15.1\,\text{A}$ give secondary powers inconsistent with 80% of the supplied $4\,\text{kW}$. Ignoring efficiency would instead give $16.67\,\text{A}$, which is also too large.

Step 3: Conclusion The current in the secondary coil is approximately $13.33\,\text{A}$. Hence, the correct option is (B).

Quick Tip to Solve: Apply efficiency to input power before dividing by the secondary voltage; a voltage ratio alone cannot account for power loss.

49. In an ammeter, 5% of the main current passes through the galvanometer. If resistance of the galvanometer is G, the resistance of ammeter will be : [4 marks]

(A) $199\ \mathrm{G}$

(B) $200\ \mathrm{G}$

(C) $\frac{G}{20}$

(D) $\frac{\mathrm{G}}{199}$

Correct Answer: (C) $\frac{G}{20}$

Solution:

Concept:

- An ammeter contains a galvanometer of resistance G in parallel with a low-resistance shunt. The two branches have the same potential difference, while the total current is the sum of their currents.
- The resistance of the complete ammeter is the terminal voltage divided by the main current, $R_A = V/I$. It is different from either the galvanometer resistance or the shunt resistance separately.

Step 1: Express the common terminal voltage Let I be the main current. The question states that 5% passes through the galvanometer, hence

$$I_g = 0.05I = \frac{I}{20}.$$

Ohm's law in that branch gives the voltage across the entire parallel combination:

$$V = I_g G = \frac{IG}{20}.$$

The other 95% of the current passes through the shunt, but it does not change this common branch voltage. This use of the total current is essential when calculating the effective meter resistance.

Step 2: Calculate the effective ammeter resistance The complete instrument therefore has resistance

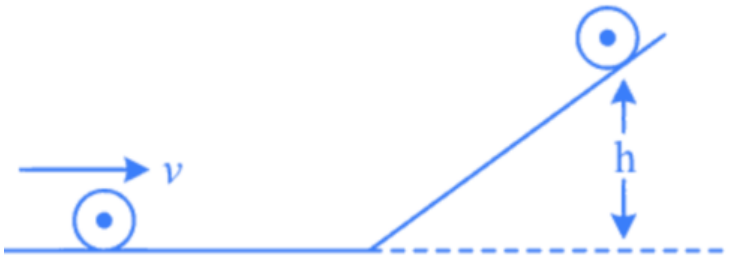
$$R_A = \frac{V}{I} = \frac{IG/20}{I} = \frac{G}{20}.$$

For a consistency check, the shunt alone would have $S = I_g G/(I - I_g) = G/19$, and $GS/(G + S) = G/20$. Thus the shunt and effective ammeter resistances must not be confused. Options $199G$ and $200G$ are larger than G, impossible for a parallel combination; $G/199$ also does not match the stated five-percent division.

Step 3: Conclusion The resistance of the ammeter is $G/20$. Hence, the correct option is (C).

Quick Tip to Solve: If a fraction f of the main current passes through a galvanometer of resistance G, the effective ammeter resistance is simply fG.

50. A disc of radius R and mass M is rolling horizontally without slipping with speed v. It then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is : [4 marks]



(A) $\frac{3}{4}\frac{v^2}{g}$

(B) $\frac{v^2}{g}$

(C) $\frac{2}{3}\frac{v^2}{g}$

(D) $\frac{1}{2}\frac{v^2}{g}$

Correct Answer: (D) $\frac{1}{2}\frac{v^2}{g}$

Solution:

Concept:

- The mechanical energy of the disc is conserved. Translational kinetic energy is $Mv^2/2$, rotational kinetic energy is $I\omega^2/2$, and gravitational potential energy gained by its centre is Mgh.
- On the smooth incline there is no friction. Both gravity and the normal reaction exert zero torque about the centre, so the angular speed remains constant even while the translational speed decreases.

Step 1: Identify the conserved rotational motion Before reaching the incline the disc rolls without slipping, so $\omega_0 = v/R$. Its moment of inertia is $I = MR^2/2$. On the smooth incline the normal reaction passes through the centre, as does the weight. Therefore neither force can change its angular velocity. The relation between translation and rotation on the initial rough horizontal surface must not be imposed throughout the subsequent smooth portion.

Step 2: Compare the initial and highest-point energies At the highest position the centre of mass is instantaneously at rest along the incline, but the disc continues spinning at ω_0. Conservation of energy gives

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega_0^2 = Mgh + \frac{1}{2}I\omega_0^2.$$

The identical rotational terms cancel, leaving

$$Mgh = \frac{1}{2}Mv^2, \qquad h = \frac{v^2}{2g}.$$

Thus only the initial translational kinetic energy is available for the increase in height. Option (A) would incorrectly convert the entire initial rolling energy into potential energy; options (B) and (C) likewise exceed the available translational energy.

Step 3: Conclusion The maximum vertical rise is $\boxed{h = v^2/(2g)}$. Hence, the correct option is (D). The question-sheet key selects the first option, but that value would require the rotational kinetic energy to vanish; the explicitly smooth incline provides no torque to stop the spin.

Quick Tip to Solve: For a rolling body entering a smooth incline, keep its spin energy unchanged; use only its translational kinetic energy to find the rise.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A mass m is suspended from a spring of negligible mass and the system oscillates with a frequency f_1. The frequency of oscillations if a mass 9 m is suspended from the same spring is f_2. The value of $\frac{f_1}{f_2}$i _____. [4 marks]

Solution:

Concept:

- For an ideal spring of force constant k carrying an oscillating mass m, the angular frequency is $\omega = \sqrt{k/m}$, and the frequency is $f = \omega/(2\pi)$.
- Gravity shifts the equilibrium extension of a vertical spring but does not change the frequency about that equilibrium. The same spring means the same force constant in both experiments.

Step 1: Write the equations about each equilibrium For a mass m, the static extension y_0 satisfies $ky_0 = mg$. If its displacement from equilibrium is z, gravity and the equilibrium spring force cancel, leaving $m\ddot{z} = -kz$. Consequently

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

For the larger suspended mass the static extension is greater, but displacement measured from its new equilibrium again obeys the same restoring-force law with the new inertial mass.

Step 2: Take the ratio with the mass increased ninefold Replacing the oscillating mass by $9m$ gives

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{9m}} = \frac{1}{3}\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{f_1}{3}.$$

Therefore

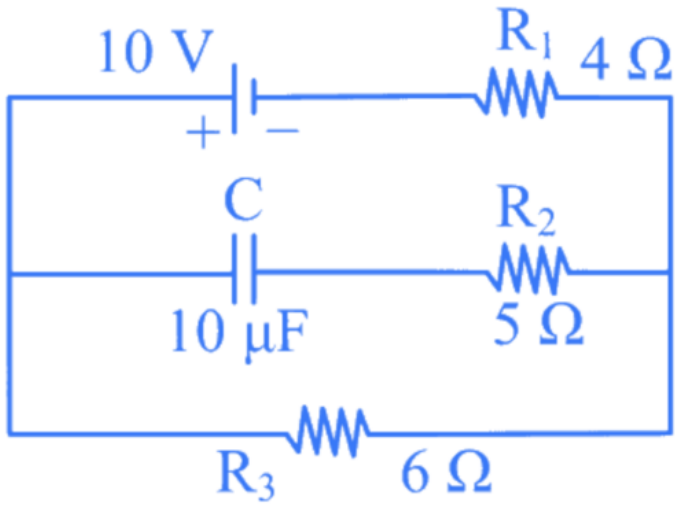
$$\frac{f_1}{f_2} = \sqrt{\frac{9m}{m}} = \sqrt{9} = 3.$$

The numerical factor is three, rather than nine, because frequency is inversely proportional to the square root of mass. The spring is stated to have negligible mass, so no contribution from the spring itself needs to be added to either suspended mass.

Step 3: Conclusion The required frequency ratio is $\boxed{f_1/f_2 = 3}$, a dimensionless quantity.

Quick Tip to Solve: For the same spring, compare masses inside a square root: increasing the mass by a factor of nine divides the frequency by three.

52. In an electrical circuit drawn below the amount of charge stored in the capacitor is _____ μ C. [4 marks]



Solution:

Concept:

- Once a capacitor has reached the steady state in a DC circuit, no current flows through its branch. Its final charge is $Q = CV_C$, where V_C is the potential difference across its plates.
- A resistor has potential drop IR. In particular, a resistor in the capacitor branch has zero final drop if the branch current is zero; the capacitor then takes the full voltage between the branch end nodes.

Step 1: Find the current in the conducting loop Let the left and right vertical wires in the figure be nodes L and R. At steady state the middle branch, containing the $10\,\mu$F capacitor and the $5\,\Omega$ resistor, carries no current. The remaining closed loop contains the $10\,$V battery, the top $4\,\Omega$ resistor and the bottom $6\,\Omega$ resistor. These two resistors are in series in this loop, so

$$I = \frac{10}{4+6} = 1\,\text{A}.$$

The ideal battery is the sole source of the current; the capacitor branch is not included as a conducting parallel resistance.

Step 2: Use the node voltage to calculate the charge The voltage between L and R is the voltage across the bottom resistor:

$$V_L - V_R = I(6\,\Omega) = 6\,\text{V}.$$

No current passes through the middle $5\,\Omega$ resistor, so its voltage drop is zero. Thus the right capacitor plate has the same potential as node R, and the left plate is at node L. Therefore

$$Q = CV_C = (10\,\mu\text{F})(6\,\text{V}) = 60\,\mu\text{C}.$$

The top resistor accounts for the other $4\,$V of the battery emf; assigning the entire battery voltage to the capacitor would overlook this drop.

Step 3: Conclusion The magnitude of the charge stored is $\boxed{60\,\mu\text{C}}$, so the required numerical entry is $\boxed{60}$.

Quick Tip to Solve: At steady state open the capacitor branch first, find the voltage between its end nodes, and then restore it using $Q = CV$.

---

53. A particle initially at rest starts moving from reference point $x = 0$ along x-axis, with velocity v that varies as $v = 4\sqrt{x}$ m/s. The acceleration of the particle is _____ ms^{-2}. [4 marks]

Solution:

Concept:

- When velocity is given as a function of position, acceleration follows from the chain rule: $a = dv/dt = (dv/dx)(dx/dt) = v\,dv/dx$.
- An equivalent useful form is $a = \frac{1}{2}\,d(v^2)/dx$. Squaring the velocity before differentiating avoids the square-root derivative at the initial endpoint.

Step 1: Differentiate the square of the stated velocity The position-dependent speed is $v = 4\sqrt{x}$ in SI units. Hence

$$v^2 = 16x.$$

Differentiating this relation along the motion, for a point with positive velocity, gives

$$2v\frac{dv}{dt} = 16\frac{dx}{dt} = 16v.$$

Cancellation of the common nonzero velocity yields $2a = 16$. This is a time acceleration obtained using the relation between position and velocity; differentiating with respect to position alone would have the wrong physical meaning.

Step 2: Check the acceleration and the starting condition Using the alternate chain-rule calculation for $x > 0$,

$$a = v\frac{dv}{dx} = 4\sqrt{x}\left(\frac{2}{\sqrt{x}}\right) = 8\,\text{m}\,\text{s}^{-2}.$$

This value is constant throughout the stated motion. At the initial point it is more convenient to use the squared-velocity relation, which is nonsingular. In fact constant acceleration $8\,\text{m}\,\text{s}^{-2}$ from rest gives $v = 8t$ and $x = 4t^2$, and substituting $t \geq 0$ into $4\sqrt{x}$ returns $8t$. Thus the result is consistent both with the initial rest condition and with the specified positive direction of motion.

Step 3: Conclusion The particle has acceleration $\boxed{8\,\text{m}\,\text{s}^{-2}}$ in the positive x-direction. The numerical entry is $\boxed{8}$.

Quick Tip to Solve: If velocity contains a square root of position, differentiate v^2 and use $a = \frac{1}{2}d(v^2)/dx$.

54. Suppose a uniformly charged wall provides a uniform electric field of 2×10^4 N/C normally. A charged particle of mass 2 g being suspended through a silk thread of length 20 cm and remain stayed at a distance of 10 cm from the wall.

Then the charge on the particle will be $\frac{1}{\sqrt{x}}\mu$C where $x =$ _____ . [use g $= 10$ m/s^2] [4 marks]

Solution:

Concept:

- A charged particle in the uniform electric field experiences a horizontal force of magnitude $|q|E$, normal to the wall. Its weight mg acts vertically downward.
- If the thread makes an angle θ with the vertical, resolving its tension gives $T\sin\theta = |q|E$ and $T\cos\theta = mg$. Therefore $\tan\theta = |q|E/(mg)$.

Step 1: Find the angle from the thread geometry The thread has length $L = 20\,\text{cm}$, and the particle is at a perpendicular distance $d = 10\,\text{cm}$ from the wall. Its horizontal displacement is therefore $L\sin\theta = d$, where θ is measured from the vertical wall. Hence

$$\sin\theta = \frac{10}{20} = \frac{1}{2}, \qquad \theta = 30^\circ, \qquad \tan\theta = \frac{1}{\sqrt{3}}.$$

The specified distance gives the sine of this angle, not its tangent; using the distance ratio directly as a force ratio would be incorrect.

Step 2: Balance the forces and identify the requested parameter Convert the mass before using the equilibrium equations: $m = 2 \times 10^{-3}\,\text{kg}$, so $mg = 2 \times 10^{-2}\,\text{N}$. Dividing the horizontal and vertical balance equations eliminates the unknown tension:

$$|q| = \frac{mg\tan\theta}{E} = \frac{(2\times10^{-3})(10)}{(2\times10^4)\sqrt{3}}\,\text{C} = \frac{10^{-6}}{\sqrt{3}}\,\text{C}.$$

Since $10^{-6}\,\text{C} = 1\,\mu\text{C}$, the charge magnitude is $1/\sqrt{3}\,\mu$C. Comparison with the stated form $1/\sqrt{x}\,\mu$C determines the dimensionless parameter.

Step 3: Conclusion The charge magnitude is $\boxed{|q| = 1/\sqrt{3}\,\mu\text{C}}$, giving $\boxed{x = 3}$.

Quick Tip to Solve: First obtain the angle using the horizontal displacement divided by thread length; then use its tangent in the force ratio.

55. A moving coil galvanometer has 100 turns and each turn has an area of $2.0\ \text{cm}^2$. The magnetic field produced by the magnet is $0.01\ \text{T}$ and the deflection in the coil is 0.05 radian when a current of $10\ \text{mA}$ is passed through it. The torsional constant of the suspension wire is $x \times 10^{-5}\ \text{N} - \text{m/rad}$. The value of x is _____ . [4 marks]

Solution:

Concept:

- In a moving-coil galvanometer with a radial magnetic field, the magnetic torque is $\tau_m = NIAB$, where N is the number of turns, A is the area per turn, and I is the coil current.
- The suspension wire produces restoring torque $\tau_r = \kappa\theta$. At the steady deflection these torques balance, so the torsional constant is $\kappa = NIAB/\theta$.

Step 1: Convert the area and current to SI units The area is given for each turn, so it must be used once in $NIAB$, with the separate factor $N = 100$. Squaring the centimetre conversion gives

$$A = 2.0\,\text{cm}^2 = 2.0 \times 10^{-4}\,\text{m}^2.$$

The coil current and magnetic field are

$$I = 10\,\text{mA} = 10^{-2}\,\text{A}, \qquad B = 0.01\,\text{T} = 10^{-2}\,\text{T}.$$

The observed angular displacement is already in radians: $\theta = 0.05\,\text{rad}$. No conversion to degrees is needed, since the torsional constant is specified per radian.

Step 2: Calculate the balanced torque and the torsional constant The magnetic torque acting on the coil at the observed current is

$$\tau_m = 100(10^{-2})(2.0 \times 10^{-4})(10^{-2}) = 2.0 \times 10^{-6}\,\text{N m}.$$

At equilibrium this equals the restoring torque. Therefore

$$\kappa = \frac{2.0 \times 10^{-6}}{0.05} = 4.0 \times 10^{-5}\,\text{N m rad}^{-1}.$$

The product $NIAB$ is a torque, whereas dividing by the measured angle gives torque per unit angular displacement. Comparing this result with $x \times 10^{-5}\,\text{N m rad}^{-1}$ gives the requested coefficient directly.

Step 3: Conclusion The torsional constant is $\boxed{4.0 \times 10^{-5}\,\text{N m rad}^{-1}}$, so $\boxed{x = 4}$.

Quick Tip to Solve: In $\kappa = NIAB/\theta$, convert square centimetres with a factor of 10^{-4} and milliamperes with 10^{-3} before multiplying.

56. One end of a metal wire is fixed to a ceiling and a load of $2\ \text{kg}$ hangs from the other end. A similar wire is attached to the bottom of the load and another load of $1\ \text{kg}$ hangs from this lower wire. Then the ratio of longitudinal strain of upper wire to that of the lower wire will be _____.

[Area of cross section of wire $= 0.005\ \text{cm}^2, \text{Y} = 2 \times 10^{11}\text{Nm}^{-2}$ and $\text{g} = 10\ \text{ms}^{-2}$] [4 marks]

Solution:

Concept:

- Young's modulus relates longitudinal stress and strain by $Y = (T/A)/\epsilon$. Thus the strain is $\epsilon = T/(AY)$, where T is the tensile force in the wire.
- Each wire supports every load below it. For similar wires the cross-sectional area and Young's modulus are the same, so their strain ratio equals their tension ratio.

Step 1: Find the tension carried by each wire The lower wire supports only the bottom $1\,\text{kg}$ load. Its tension is therefore

$$T_{\text{lower}} = (1)(10) = 10\,\text{N}.$$

The upper wire supports the $2\,\text{kg}$ load directly attached to it as well as the downward pull of the lower wire. Balance of the forces on the upper load gives

$$T_{\text{upper}} = 2g + T_{\text{lower}} = 20 + 10 = 30\,\text{N}.$$

It would be incorrect to assign the upper wire a tension of only $2g$, since this neglects the bottom load transmitted through the lower wire.

Step 2: Calculate and compare the two strains For the similar wires, $A = 0.005\,\text{cm}^2 = 5 \times 10^{-7}\,\text{m}^2$ and $Y = 2 \times 10^{11}\,\text{N}\,\text{m}^{-2}$. Hence $AY = 10^5\,\text{N}$, and

$$\epsilon_{\text{upper}} = \frac{30}{10^5} = 3 \times 10^{-4}, \qquad \epsilon_{\text{lower}} = \frac{10}{10^5} = 10^{-4}.$$

Both quantities are dimensionless fractional extensions. Their ratio is

$$\frac{\epsilon_{\text{upper}}}{\epsilon_{\text{lower}}} = \frac{30/(AY)}{10/(AY)} = 3.$$

The common material and area cancel, so the ratio could also be obtained without calculating the two individual strains. Wire length would be needed to find extension, but not strain from stress.

Step 3: Conclusion The ratio of upper-wire strain to lower-wire strain is $\boxed{3:1}$. The numerical entry is $\boxed{3}$.

Quick Tip to Solve: For vertically stacked loads, add all loads below a wire to find its tension; similar wires then have strains in the same ratio as tensions.

57. In Young's double slit experiment, monochromatic light of wavelength 5000 Å is used. The slits are 1.0 mm apart and screen is placed at 1.0 m away from slits. The distance from the centre of the screen where intensity becomes half of the maximum intensity for the first time is _____ $\times 10^{-6}$ m. [4 marks]

Solution:

Concept:

- For two identical coherent slits, the resultant intensity at phase difference ϕ is $I = I_{\max}\cos^2(\phi/2)$. The maximum occurs at zero path difference at the screen centre.
- At a small transverse displacement y, the path difference is $\delta = dy/D$ and the phase difference is $\phi = 2\pi\delta/\lambda$, where d is slit separation and D is the screen distance.

Step 1: Convert the given lengths The wavelength must be expressed in the same length unit as the other distances. Since one angstrom is $10^{-10}\,\mathrm{m}$, the data become

$$\lambda = 5000 \times 10^{-10} = 5.0 \times 10^{-7}\,\mathrm{m}, \quad d = 10^{-3}\,\mathrm{m}, \quad D = 1.0\,\mathrm{m}.$$

The screen distance is much larger than the slit separation, so the usual small-angle interference relation applies.

Step 2: Locate the first half-intensity point Setting the intensity to half the central maximum gives

$$\cos^2(\phi/2) = \frac{1}{2}.$$

As we move outwards from the centre, the first solution is $\phi/2 = \pi/4$, hence $\phi = \pi/2$. Therefore the required path difference is $\delta = \lambda/4$. This is a quarter of a fringe width from the centre, not a half-fringe displacement, which would give the first dark fringe.

Step 3: Calculate the transverse distance Use $dy/D = \lambda/4$ to find

$$y = \frac{\lambda D}{4d} = \frac{(5.0 \times 10^{-7})(1.0)}{4 \times 10^{-3}} = 1.25 \times 10^{-4}\,\mathrm{m}.$$

Writing this in the requested scale gives $y = 125 \times 10^{-6}\,\mathrm{m}$. The two symmetric points have the same distance from the central maximum.

Step 4: Conclusion The first half-maximum intensity occurs at a distance $125 \times 10^{-6}\,\mathrm{m}$. The required numerical value is $\boxed{125}$.

Quick Tip to Solve: For equal slit intensities, the first half-maximum is at one quarter of the fringe width: use $y = \lambda D/(4d)$.

58. A coil of 200 turns and area $0.20\ \mathrm{m}^2$ is rotated at half a revolution per second and is placed in uniform magnetic field of $0.01\ \mathrm{T}$ perpendicular to axis of rotation of the coil. The maximum voltage generated in the coil is $\frac{2\pi}{\beta}$ volt. The value of β is _____. [4 marks]

Solution:

Concept:

- The magnetic flux linkage of a rotating coil is $N\Phi = NBA\cos\theta$, where N is the number of turns, B is magnetic field magnitude and A is the area of each turn.
- Faraday's law gives $\mathcal{E} = -d(N\Phi)/dt$. With uniform angular speed ω, the sinusoidal emf has peak magnitude $\mathcal{E}_0 = NBA\omega$.

Step 1: Convert frequency to angular speed The coil rotates through half a revolution each second, so $f = 0.50\,\mathrm{Hz}$. One complete revolution corresponds to 2π radians. Therefore

$$\omega = 2\pi f = 2\pi(0.50) = \pi\,\mathrm{rad\,s^{-1}}.$$

The magnetic field is perpendicular to the rotation axis. Consequently the coil normal sweeps through directions parallel and perpendicular to the field, giving the full flux variation required for the usual peak-emf formula.

Step 2: Find the maximum induced voltage The instantaneous emf varies with the angle of the coil normal as $NBA\omega\sin\theta$. Its largest magnitude is obtained when $|\sin\theta| = 1$. Substituting the data gives

$$\mathcal{E}_0 = 200 \times 0.01 \times 0.20 \times \pi = 0.40\pi\,\mathrm{V}.$$

The factor of 200 counts flux linkage through all turns; the area supplied is already in square metres and needs no further conversion.

Step 3: Compare with the specified expression The question expresses the same maximum voltage as $2\pi/\beta$ volts. Thus

$$\frac{2\pi}{\beta} = 0.40\pi \quad\Longrightarrow\quad \beta = \frac{2}{0.40} = 5.$$

This is a comparison of peak voltages. Introducing the factor $1/\sqrt{2}$, which relates rms and peak values, would be inappropriate because the question specifically asks about the maximum voltage.

Step 4: Conclusion The peak voltage is $0.40\pi\,\mathrm{V} = 2\pi/5\,\mathrm{V}$, so $\boxed{\beta = 5}$.

Quick Tip to Solve: Convert revolutions per second to radians per second first, then use the peak formula $NBA\omega$; do not insert an rms factor.

59. A uniform rod AB of mass $2\ \text{kg}$ and length $30\ \text{cm}$ at rest on a smooth horizontal surface. An impulse of force $0.2\ \text{Ns}$ is applied to end B. The time taken by the rod to turn through at right angles will be $\frac{\pi}{x}$ s, where $x =$ _____ . [4 marks]

Solution:

Concept:

- An impulse at the end of a free rod produces both translation of its centre of mass and rotation about that centre. Angular impulse about the centre equals the change in spin angular momentum.
- For a uniform thin rod of mass M and length L, the moment of inertia about a perpendicular axis through its centre is $I_{\text{cm}} = ML^2/12$. After the impulse, the smooth surface supplies no torque about the vertical spin axis.

Step 1: Calculate the moment of inertia Take $M = 2\,\text{kg}$, $L = 30\,\text{cm} = 0.30\,\text{m}$ and impulse magnitude $J = 0.20\,\text{N s}$. The relevant spin axis passes through the rod's centre and is perpendicular to the horizontal surface. Hence

$$I_{\text{cm}} = \frac{ML^2}{12} = \frac{2(0.30)^2}{12} = 0.015\,\text{kg m}^2.$$

The end is a distance $L/2 = 0.15\,\text{m}$ from this axis.

Step 2: Apply angular impulse about the centre For the transverse impulse at the end, the angular impulse is $J(L/2)$. The rod was initially at rest, so

$$I_{\text{cm}}\omega = J\frac{L}{2} = (0.20)(0.15) = 0.030\,\text{kg m}^2\,\text{s}^{-1}.$$

Therefore $\omega = 0.030/0.015 = 2\,\text{rad s}^{-1}$. The impulse also gives the centre a speed $J/M = 0.10\,\text{m s}^{-1}$, but this translation does not affect the change in the rod's orientation.

Step 3: Find the time for a right-angle turn After the short impulse has ended, the angular velocity remains constant. A right angle corresponds to $\pi/2$ radians, so

$$t = \frac{\pi/2}{\omega} = \frac{\pi/2}{2} = \frac{\pi}{4}\,\text{s}.$$

Comparing this with the requested form $t = \pi/x\,\text{s}$ gives $x = 4$. Using the centre-of-mass moment of inertia avoids incorrectly treating the struck end as a fixed pivot.

Step 4: Conclusion The rod turns through a right angle in $\pi/4\,\text{s}$; hence $\boxed{x = 4}$.

Quick Tip to Solve: For a free rod use angular impulse about its centre, $JL/2 = I_{\text{cm}}\omega$, and then divide the required angle by ω.

60. A particular hydrogen-like ion emits the radiation of frequency 3×10^{15} Hz when it makes transition from $n=2$ to $n=1$. The frequency of radiation emitted in transition from $n=3$ to $n=1$ is $\frac{x}{9}\times10^{15}$ Hz, when $x=$ _____ . [4 marks]

Solution:

Concept:

- The bound-state energy of a hydrogen-like ion is $E_n=-CZ^2/n^2$, with C a positive constant and Z the nuclear charge. A transition emits a photon whose energy is the initial energy minus the final energy.
- From $hu=E_i-E_f$, the frequency for a transition to $n=1$ is proportional to $Z^2(1-1/n_i^2)$. For the same ion, both the nuclear-charge factor and the common constants cancel in a frequency ratio.

Step 1: Express the known transition frequency Write the common frequency constant as $K=CZ^2/h$. The transition from the second orbit to the first orbit then gives

$$u_{21}=K\left(1-\frac{1}{4}\right)=\frac{3}{4}K=3\times10^{15}\,\text{Hz}.$$

This identifies the scale of the energy levels without requiring a separate calculation of the ion's atomic number. The photon frequency is positive because the electron loses energy on moving to the lower orbit.

Step 2: Form the ratio for the other transition For a transition from the third orbit to the first orbit,

$$u_{31}=K\left(1-\frac{1}{9}\right)=\frac{8}{9}K.$$

Dividing by the known frequency eliminates K:

$$\frac{u_{31}}{u_{21}}=\frac{8/9}{3/4}=\frac{32}{27}.$$

This ratio is greater than one, as expected: the drop from the third orbit to the ground state releases more energy than the drop from the second orbit.

Step 3: Evaluate and match the requested coefficient Using the supplied u_{21},

$$u_{31}=\frac{32}{27}\,(3\times10^{15})=\frac{32}{9}\times10^{15}\,\text{Hz}.$$

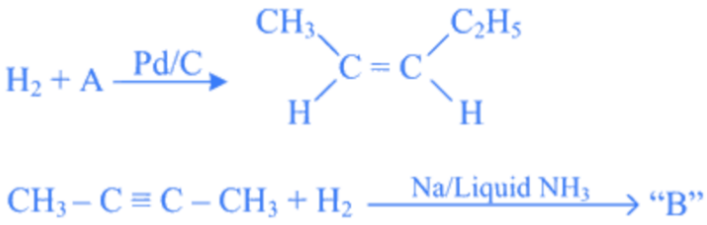
The question writes this frequency as $(x/9)\times10^{15}$ Hz. The denominators and powers of ten already match, so the numerator is directly $x=32$. There is no need to approximate the frequency to an integer.

Step 4: Conclusion The emitted frequency is $\frac{32}{9}\times10^{15}$ Hz, giving $\boxed{x=32}$.

Quick Tip to Solve: For two transitions of the same hydrogen-like ion, take the ratio of the $1/n_f^2-1/n_i^2$ factors so that the unknown nuclear charge cancels.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. In the given reactions identify A and B [4 marks]



(A) A : n-Pentane B : Cis - 2 - butene

(B) A : 2-Pentyne B : Cis-2-butene

(C) A : n-Pentane B : trans-2-butene

(D) A : 2-Pentyne B : trans-2-butene

Correct Answer: (D) A : 2-Pentyne B : trans-2-butene

Solution:

Concept:

- Adding one equivalent of hydrogen across a carbon-carbon triple bond converts an alkyne into an alkene without changing its carbon skeleton. The position of the remaining double bond identifies the position of the original triple bond.
- Dissolving-metal reduction with sodium in liquid ammonia gives anti addition overall and converts an internal alkyne to a trans alkene. This stereochemical outcome differs from syn catalytic addition.

Step 1: Identify the carbon skeleton of A The first displayed product has one double-bond carbon bonded to a methyl group and the other to an ethyl group. It therefore contains five carbon atoms and is pent-2-ene. In the depicted addition, the corresponding unsaturated precursor is

$$CH_3-C\equiv C-CH_2-CH_3,$$

namely pent-2-yne (2-pentyne). Its triple bond lies between the same two carbon atoms as the product's double bond. The methyl and ethyl groups are drawn on the same side in the product, indicating its cis configuration.

Step 2: Determine B using the reducing system The second starting compound is $CH_3-C\equiv C-CH_3$, but-2-yne. Sodium in liquid ammonia is the diagnostic dissolving-metal reduction system. Successive electron and proton additions give the anti-addition product, so the two methyl groups end up on opposite sides of the new double bond. Therefore B is trans-but-2-ene, not cis-but-2-ene. The four-carbon chain and the position of the unsaturation are retained.

Step 3: Compare the two identities with every option Options A and C require A to be n-pentane. A saturated alkane cannot give the displayed alkene merely by addition of hydrogen, so those identifications are excluded. Option B correctly names the five-carbon alkyne but gives the cis product for the sodium/liquid-ammonia reduction, which has the wrong stereochemistry. Only option D

combines A as 2-pentyne with B as trans-2-butene. The product skeleton and the second reagent system together decide the pair.

Step 4: Conclusion A is 2-pentyne and B is trans-2-butene. Hence, the correct option is (D).

Quick Tip to Solve: First preserve the carbon skeleton to identify the alkyne; then use sodium in liquid ammonia as the immediate signal for the trans alkene.

62. Solubility of calcium phosphate (molecular mass, M) in water is $\mathrm{W_g}$ per $100\ \mathrm{mL}$ at $25^\circ\mathrm{C}$. Its solubility product at $25^\circ\mathrm{C}$ will be approximately. [4 marks]

(A) $10^7\left(\frac{W}{M}\right)^3$

(B) $10^3\left(\frac{\mathrm{W}}{\mathrm{M}}\right)^5$

(C) $10^7\left(\frac{W}{M}\right)^5$

(D) $10^5\left(\frac{\mathrm{W}}{\mathrm{M}}\right)^5$

Correct Answer: (C) $10^7\left(\frac{W}{M}\right)^5$

Solution:

Concept:

- Calcium phosphate is $\mathrm{Ca_3(PO_4)_2}$. Its dissolution equilibrium is $\mathrm{Ca_3(PO_4)_2(s) \rightleftharpoons 3\,Ca^{2+}(aq) + 2\,PO_4{}^{3-}(aq)}$. The pure solid does not appear in the solubility-product expression.
- If the molar solubility is $s\ \mathrm{mol\,L^{-1}}$, the ion concentrations in pure water are $[\mathrm{Ca^{2+}}] = 3s$ and $[\mathrm{PO_4{}^{3-}}] = 2s$. The stoichiometric coefficients become exponents in K_{sp}.

Step 1: Convert the mass solubility to molar solubility The given solubility is W grams in $100\,\mathrm{mL} = 0.100\,\mathrm{L}$. Thus one litre contains $10W$ grams of dissolved salt. Dividing by its molecular mass M, interpreted as molar mass in grams per mole, gives

$$s = \frac{W/M}{0.100} = \frac{10W}{M}\ \mathrm{mol\,L^{-1}}.$$

The factor of ten is essential because the solubility is not quoted per litre.

Step 2: Build the solubility-product expression At the stated temperature, the concentration approximation gives

$$K_{sp} = [\mathrm{Ca^{2+}}]^3[\mathrm{PO_4{}^{3-}}]^2 = (3s)^3(2s)^2 = 108s^5.$$

There are five dissolved ions per formula unit in this stoichiometric expression, so the fifth power of s is expected. It would be incorrect to use a third power merely because three calcium ions are formed; both ion concentrations must be included.

Step 3: Substitute and compare the coefficients Substituting $s = 10W/M$,

$$K_{sp} = 108\left(\frac{10W}{M}\right)^5 = 1.08 \times 10^7\left(\frac{W}{M}\right)^5.$$

The question asks for an approximate expression, so 1.08×10^7 is represented by 10^7. Option A has

the wrong exponent. Options B and D have the fifth power but miss the coefficient generated by converting the volume and including both ionic stoichiometric factors.

Step 4: Conclusion Approximately, $K_{sp} = 10^7 \left(\frac{W}{M}\right)^5$. Hence, the correct option is (C).

Quick Tip to Solve: Convert grams per 100 mL into moles per litre before substituting; for $Ca_3(PO_4)_2$, remember $K_{sp} = 108s^5$.

---

63. Given below are two statements :

Statement (I) : SiO_2 and GeO_2 are acidic while SnO and PbO are amphoteric in nature.

Statement (II) : Allotropic forms of carbon are due to property of catenation and $p\pi - d\pi$ bond formation.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- The acidic character of group 14 oxides generally decreases down the group as metallic character increases. The lighter-element dioxides are acidic, while oxides of the heavier elements can behave amphoterically.
- Carbon exhibits strong catenation: carbon atoms form stable covalent bonds with other carbon atoms. Its allotropes differ in bonding arrangement and structure, involving carbon's valence $2s$ and $2p$ orbitals.

Step 1: Check the oxide classification in Statement I SiO_2 and GeO_2 are classified as acidic oxides in the group 14 trend. They react with suitable bases to form silicates and germanates. The monoxides SnO and PbO are amphoteric: they can react with acids as well as strong bases. This is consistent with the increasing metallic character of tin and lead compared with silicon and germanium. Thus the oxide classifications stated in Statement I are correct, and Statement I is true.

Step 2: Check the proposed bonding explanation in Statement II Carbon allotropy is indeed related to its ability to form carbon-carbon bonds and to adopt different networks. Diamond has a three-dimensional tetrahedral covalent structure, whereas graphite has sheets of bonded carbon atoms with delocalised electrons. However, carbon does not use valence d orbitals to form the claimed $p\pi - d\pi$ bonds. Its multiple bonds are described using $p\pi - p\pi$ overlap of $2p$ orbitals. Therefore the conjunction in Statement II includes a false bonding assertion, making the statement false overall.

Step 3: Select the matching truth-value combination The required combination is Statement I true

and Statement II false. Option A reverses these truth values, option B incorrectly rejects Statement I, and option C incorrectly accepts the d-orbital explanation. The fact that catenation is a valid property does not make all of Statement II correct: both parts of its proposed explanation must hold. Only option D states the correct combination.

Step 4: Conclusion Statement I is true, but Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: When a statement joins two explanations with "and", check both: carbon has catenation, but its ordinary pi bonds use p orbitals rather than d orbitals.

64. The set of meta directing functional groups from the following sets is : [4 marks]

(A) $-CN, -NH_2, -NHR, -OCH_3$

(B) $-CN, -CHO, -NHCOCH_3, -COOR$

(C) $-NO_2, -NH_2, -COOH, -COOR$

(D) $-NO_2, -CHO, -SO_3H, -COR$

Correct Answer: (D) $-NO_2, -CHO, -SO_3H, -COR$

Solution:

Concept:

- Electron-withdrawing groups that withdraw by resonance generally deactivate a benzene ring and direct electrophilic substitution predominantly to the meta position.
- An ortho/para attack on a ring bearing a resonance-withdrawing group gives a particularly unfavourable resonance contributor with positive charge at the carbon attached to that group. Meta attack avoids this contributor.

Step 1: Identify the entirely withdrawing set The groups $-NO_2$, $-CHO$, $-SO_3H$ and $-COR$ are all electron withdrawing and meta directing. In the carbonyl groups the ring is bonded to the carbonyl carbon, which accepts electron density from the aromatic ring by resonance. Nitro and sulphonic acid groups likewise strongly withdraw electron density. Therefore every member of the fourth set has the required directing effect.

Step 2: Eliminate the mixed sets In option (A), amino, alkylamino and methoxy groups donate a lone pair to the ring and are ortho/para directing; cyano alone is meta directing. In option (B), the acetamido group $-NHCOCH_3$ is attached through nitrogen and remains ortho/para directing, although its donation is weaker than that of an amino group. Option (C) contains $-NH_2$, another ortho/para director. Thus none of these three sets consists exclusively of meta directors. The point of attachment matters: $-NHCOCH_3$ and $-COR$ have different directing behaviour.

Step 3: Conclusion The required set is $-NO_2, -CHO, -SO_3H, -COR$. Hence, the correct option is (D).

Quick Tip to Solve: For an amide substituent, first identify whether the ring is attached to nitrogen or to the carbonyl carbon; this decides its resonance donation or withdrawal.

65. $[Co\,(NH_3)_6]^{3+}$ **and** $[CoF_6]^{3-}$ **are respectively known as :** [4 marks]

(A) Inner orbital Complex, Spin paired Complex

(B) Spin paired Complex, Spin free Complex

(C) Spin free Complex, Spin paired Complex

(D) Outer orbital Complex, Inner orbital Complex

Correct Answer: (B) Spin paired Complex, Spin free Complex

Solution:

Concept:

- For an octahedral complex, the five d orbitals split into lower-energy t_{2g} and higher-energy e_g sets. Their occupation depends on the splitting energy relative to the electron-pairing energy.
- Both complexes contain cobalt in oxidation state $+3$, so the metal-ion configuration is $3d^6$. Stronger-field ligands favour pairing; weak-field ligands favour the maximum number of unpaired electrons.

Step 1: Determine the state of the ammine complex Ammonia is a stronger-field ligand than fluoride. For $[Co(NH_3)_6]^{3+}$, the octahedral splitting for cobalt(III) is sufficient to pair the six d electrons in the lower set:

$$t_{2g}^6 e_g^0.$$

All electrons are paired, so this is a low-spin or spin-paired complex. In valence-bond terminology it is an inner-orbital d^2sp^3 complex and is diamagnetic.

Step 2: Determine the fluoride state and compare the options Fluoride produces a weak ligand field. In $[CoF_6]^{3-}$, the six d electrons occupy the orbitals as

$$t_{2g}^4 e_g^2,$$

leaving four electrons unpaired. It is a high-spin, or spin-free, outer-orbital complex, described by sp^3d^2 hybridisation in valence-bond theory. Option (A) wrongly calls the fluoride complex spin paired. Option (C) reverses both spin descriptions, and option (D) reverses the inner/outer orbital assignments. The ordered pair must therefore be spin paired followed by spin free.

Step 3: Conclusion $[Co(NH_3)_6]^{3+}$ is spin paired and $[CoF_6]^{3-}$ is spin free. Hence, the correct option is (B).

Quick Tip to Solve: Find the metal oxidation state before counting d electrons; then compare ligand-field strength with pairing energy for the same d-electron count.

66. The transition metal having highest 3^{rd} ionisation enthalpy is : [4 marks]

(A) Mn

(B) Fe

(C) Cr

(D) V

Correct Answer: (A) Mn

Solution:

Concept:

- The third ionisation enthalpy is the energy required for $M^{2+}(g) \longrightarrow M^{3+}(g) + e^-$. Thus it is the electronic configuration of the divalent ion that must be compared.
- A half-filled $3d^5$ subshell has extra exchange stability. Removing an electron from this stable configuration requires unusually high energy.

Step 1: Write the configurations before the third removal For transition-metal cations, the $4s$ electrons are removed before $3d$ electrons. The divalent ions in the options are therefore

$$Mn^{2+} : [Ar]\, 3d^5, \quad Fe^{2+} : [Ar]\, 3d^6,$$
$$Cr^{2+} : [Ar]\, 3d^4, \quad V^{2+} : [Ar]\, 3d^3.$$

The stability relevant to the question is the stability of each of these starting ions, rather than the familiar neutral-atom configuration alone.

Step 2: Compare the loss of the next electron The third electron removed from manganese changes $3d^5$ to $3d^4$, destroying the particularly stable half-filled arrangement. This gives manganese a large third ionisation enthalpy. In contrast, removal from Fe^{2+} gives Fe^{3+} with the stable $3d^5$ configuration, so this removal is comparatively favoured. Chromium and vanadium start from $3d^4$ and $3d^3$ divalent configurations respectively; neither loses the special half-filled stability at the third ionisation step. Consequently iron, chromium and vanadium do not exceed manganese for the stated comparison. The neutral chromium half-filled configuration is not the one undergoing this ionisation.

Step 3: Conclusion Manganese has the highest third ionisation enthalpy among the given metals. Hence, the correct option is (A).

Quick Tip to Solve: For the nth ionisation enthalpy, write the configuration of the ion with charge $(n-1)+$; do not compare neutral configurations directly.

67. Match List - I with List - II.

List I (Compound)	List II (Use)
(A) Carbon tetrachloride	(I) Paint remover
(B) Methylene chloride	(II) Refrigerators and air conditioners
(C) DDT	(III) Fire extinguisher
(D) Freons	(IV) Non Biodegradable insecticide

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(C) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(D) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Correct Answer: (B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- The characteristic uses of common halogen compounds follow their solvent properties, volatility, nonflammability or biological persistence. Matching requires identifying each compound independently.
- Carbon tetrachloride and methylene chloride are different chlorinated solvents; Freons are chlorofluorocarbon refrigerants, whereas DDT is a persistent organochlorine insecticide.

Step 1: Match the chlorinated solvents Carbon tetrachloride, CCl_4, was historically used in fire extinguishers because it is nonflammable and its vapour suppresses combustion. Thus (A) matches (III). This is the textbook historical use, rather than a recommendation for present-day handling. Methylene chloride, CH_2Cl_2, is a solvent used in paint removal, so (B) matches (I). These two assignments distinguish the fire-extinguishing use from the paint-removing solvent use.

Step 2: Match the insecticide and refrigerants DDT is a non-biodegradable insecticide. Its resistance to degradation allows persistence in the environment and accumulation along food chains, so (C) matches (IV). Freons were widely used as refrigerants in refrigerators and air conditioners, giving (D) matches (II). These are their characteristic historical applications; the environmental problems of DDT and chlorofluorocarbons do not interchange their identities or uses. Combining the four independent matches gives

$$\text{(A)–(III), (B)–(I), (C)–(IV), (D)–(II).}$$

Only the second option contains all four assignments. The other options wrongly allocate the refrigerant, solvent or insecticide applications.

Step 3: Conclusion All four compound-to-use assignments agree with the second matching set. Hence, the correct option is (B).

Quick Tip to Solve: Anchor this matching with Freons as refrigerants and DDT as the persistent insecticide, then distinguish the two chlorinated solvents.

68. Match List - I with List - II.

List I (Reactants)	List II (Product)
(A) Phenol, Zn/Δ	(I) Salicylaldehyde
(B) Phenol, $CHCl_3$, NaOH, HCl	(II) Salicylic acid
(C) Phenol, CO_2, NaOH, HCl	(III) Benzene
(D) Phenol, Conc. HNO_3	(IV) Picric acid

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(B) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(C) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(D) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Correct Answer: (B) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

Solution:

Concept:

- Phenol undergoes reduction with zinc dust, ortho formylation in the Reimer–Tiemann reaction, ortho carboxylation in the Kolbe–Schmitt reaction, and extensive nitration with concentrated nitric acid.
- The reagents distinguish the products: chloroform introduces a formyl group, carbon dioxide introduces a carboxyl group, and concentrated nitric acid introduces nitro groups.

Step 1: Identify the zinc and chloroform reactions Heating phenol with zinc dust removes the oxygen-containing group and gives benzene:

$$C_6H_5OH + Zn \xrightarrow{\Delta} C_6H_6 + ZnO.$$

Thus (A) matches (III). Treating phenol with chloroform and sodium hydroxide, followed by acidification, is the Reimer–Tiemann reaction. Its major ortho product is salicylaldehyde, *o*-hydroxybenzaldehyde, so (B) matches (I). The aldehyde group introduced in this reaction must not be confused with a carboxylic acid group.

Step 2: Identify carboxylation and nitration Phenol first forms sodium phenoxide in alkali. Reaction with carbon dioxide under the Kolbe–Schmitt conditions, followed by acidification, gives salicylic acid, *o*-hydroxybenzoic acid. Therefore (C) matches (II). Concentrated nitric acid nitrates the activated phenol ring at both ortho positions and the para position to give 2,4,6-trinitrophenol, known as picric acid; hence (D) matches (IV). The complete matching is

$$\text{(A)–(III), (B)–(I), (C)–(II), (D)–(IV).}$$

The second option alone has this sequence. Options (A) and (C) wrongly assign picric acid to zinc treatment, while option (D) interchanges the products of the other reactions.

Step 3: Conclusion All four reagent-to-product assignments agree with the second matching set. Hence, the correct option is (B).

Quick Tip to Solve: Remember the carbon source: chloroform gives phenol an aldehyde group, whereas carbon dioxide gives it a carboxyl group after acidification.

69. Given below are two statements :

Statement (I) : Dimethyl glyoxime forms a six-membered covalent chelate when treated with $NiCl_2$ solution in presence of NH_4OH.

Statement (II) : Prussian blue precipitate contains iron both in $(+2)$ and $(+3)$ oxidation states.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false

Correct Answer: (A) Statement I is false but Statement II is true

Solution:

Concept:

- A chelate ring includes the central metal and the atoms along the ligand chain connecting its two donor atoms. Ring size must be counted along this closed path.
- Prussian blue is a mixed-valence iron compound, conventionally represented as $Fe_4[Fe(CN)_6]_3$, containing ferric ions and ferrocyanide units.

Step 1: Test the dimethylglyoxime statement Dimethylglyoxime reacts with nickel(II) in ammoniacal medium to form the characteristic red nickel dimethylglyoximate precipitate. It binds through the two nitrogen donors. The metal-containing ring follows the path Ni–N–C–C–N back to Ni. Counting the nickel atom, two nitrogen atoms and two carbon atoms gives a five-membered chelate ring, not a six-membered one. Hydrogen bonding between oxime groups contributes to the stability of the complex but does not make the stated metal chelate ring six-membered. Therefore Statement (I) is false.

Step 2: Test the oxidation states in Prussian blue In the ferrocyanide ion $[Fe(CN)_6]^{4-}$, let the iron oxidation state be x. Since each cyanide ligand has charge -1,

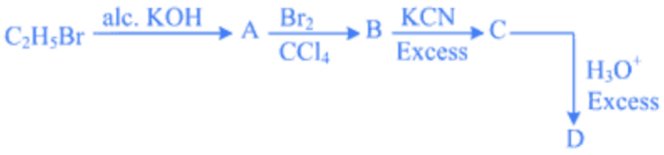
$$x + 6(-1) = -4 \quad \Rightarrow \quad x = +2.$$

In $Fe_4[Fe(CN)_6]_3$, three ferrocyanide units have total charge -12, balanced by four iron ions outside the brackets, each with charge $+3$. The solid therefore contains both iron(II) and iron(III), making Statement (II) true. Options claiming both statements true or both false are excluded, and the option reversing these truth values is also excluded.

Step 3: Conclusion Statement (I) is false, whereas Statement (II) is true. Hence, the correct option is (A).

Quick Tip to Solve: Count chelate-ring atoms explicitly along metal–donor–ligand–donor–metal; for mixed-valence salts, calculate the iron inside and outside brackets separately.

70. Acid D formed in above reaction is : [4 marks]



(A) Malonic acid

(B) Oxalic acid

(C) Succinic acid

(D) Gluconic acid

Correct Answer: (C) Succinic acid

Solution:

Concept:

- Alcoholic potassium hydroxide favours beta elimination from bromoethane, producing ethene. Bromine in carbon tetrachloride adds across its carbon–carbon double bond.
- Cyanide replaces alkyl bromide groups through its carbon atom, adding one carbon per substituted bromide. Acidic hydrolysis converts each nitrile group into a carboxylic acid group without removing that carbon.

Step 1: Obtain the vicinal dibromide Bromoethane loses hydrogen bromide on treatment with alcoholic potassium hydroxide:

$$CH_3CH_2Br + KOH \xrightarrow{\text{alcohol}} CH_2{=}CH_2 + KBr + H_2O.$$

Thus A is ethene. Addition of bromine across its double bond gives B, 1,2-dibromoethane:

$$CH_2{=}CH_2 + Br_2 \xrightarrow{CCl_4} BrCH_2CH_2Br.$$

Both bromine atoms are now on adjacent saturated carbon atoms and can undergo nucleophilic substitution.

Step 2: Introduce two nitrile carbons and hydrolyse Excess potassium cyanide replaces both bromide groups to form C, succinonitrile:

$$BrCH_2CH_2Br + 2\,KCN \longrightarrow NCCH_2CH_2CN + 2\,KBr.$$

Each cyanide contributes an extra carbon, so C contains four carbon atoms. Excess acid and water hydrolyse both terminal nitriles:

$$NCCH_2CH_2CN + 4\,H_2O + 2\,H^+ \longrightarrow HOOCCH_2CH_2COOH + 2\,NH_4{}^+.$$

Product D is therefore butanedioic acid, commonly called succinic acid. Malonic acid has only three carbons, oxalic acid has two, and gluconic acid has six carbons and several hydroxyl groups; none matches this sequence or the four-carbon dicarboxylic acid obtained here.

Step 3: Conclusion Acid D is succinic acid, $HOOCCH_2CH_2COOH$. Hence, the correct option is (C).

Quick Tip to Solve: Count carbons when using KCN: each replaced halogen adds a nitrile carbon, which remains as the carboxyl carbon after hydrolysis.

71. Lassaigne's test is used for detection of : [4 marks]

(A) Phosphorous and halogens only

(B) Nitrogen, Sulphur and Phosphorous only

(C) Nitrogen, Sulphur, phosphorous and halogens

(D) Nitrogen and Sulphur only

Correct Answer: (C) Nitrogen, Sulphur, phosphorous and halogens

Solution:

Concept:

- Lassaigne's sodium-fusion method converts covalently bonded heteroatoms in an organic compound into water-soluble ionic salts that can be tested in the aqueous extract.
- Nitrogen, sulphur, halogens and phosphorus can be detected after suitable conversion; carbon and hydrogen are not identified by these characteristic extract tests.

Step 1: Identify the purpose of sodium fusion Direct aqueous tests often fail for an organic compound because its heteroatoms are covalently attached to carbon. Fusion with sodium destroys that organic framework and transfers the heteroatoms into ionic compounds. Thus nitrogen forms sodium cyanide, sulphur forms sodium sulphide, and a halogen forms the corresponding sodium halide. These products dissolve in water to give the sodium-fusion extract.

Step 2: Include the phosphorus test Phosphorus is converted into sodium phosphate during the fusion/oxidation procedure. The extract is treated with nitric acid and ammonium molybdate: formation of a yellow ammonium phosphomolybdate precipitate confirms phosphorus. Nitrogen can be confirmed through Prussian-blue formation, sulphur through its sulphide tests, and halogens through silver-halide precipitation after appropriate acid treatment.

Step 3: Compare all the listed sets The complete set is nitrogen, sulphur, phosphorus and halogens. Option A omits nitrogen and sulphur. Option B omits halogens, while option D omits both phosphorus and halogens. Their use of 'only' therefore makes each set incomplete. The spelling 'phosphorous' in the options refers here to the element phosphorus, not to a separate detectable species.

Step 4: Conclusion Lassaigne's test is used for detection of nitrogen, sulphur, phosphorus and halogens. Hence, the correct option is (C).

Quick Tip to Solve: Remember the fusion products: cyanide for N, sulphide for S, halide for X and phosphate for P; the complete list follows immediately.

72. The strongest reducing agent among the following is : [4 marks]

(A) SbH_3

(B) NH_3

(C) BiH_3

(D) PH_3

Correct Answer: (C) BiH_3

Solution:

Concept:

- For group 15 hydrides, reducing character generally increases down the group as the central-atom–hydrogen bond becomes weaker.
- Thermal stability decreases down the series: the larger central atoms overlap less effectively with hydrogen, so their hydrides are more readily oxidised.

Step 1: Compare the central atoms The compounds are hydrides of group 15 elements. Nitrogen lies above phosphorus, which lies above antimony and bismuth. The relevant downward sequence among the choices is therefore ammonia, phosphine, stibine and bismuthine. As atomic size increases, overlap with the small hydrogen orbital becomes poorer, weakening the element–hydrogen bond.

Step 2: Relate bond weakness to reduction A reducing agent supplies electrons and is itself oxidised. The heavier group 15 hydrides are less stable and undergo oxidation more readily than the lighter members. Their reducing power therefore increases in the same downward order in which their thermal stability decreases. This is a hydride-stability trend, so it should not be confused with the trend in basic strength of ammonia and its heavier analogues.

Step 3: Rank the given choices The reducing-power order for the listed hydrides is

$$NH_3 < PH_3 < SbH_3 < BiH_3.$$

Consequently, stibine is stronger than phosphine and ammonia but is still weaker than bismuthine. Ammonia has the strongest central-atom–hydrogen bonds of this set and cannot be the strongest reducing agent. The last member in the ordered list, bismuthine, gives the requested maximum.

Step 4: Conclusion BiH_3 is the strongest reducing agent among the given hydrides. Hence, the correct option is (C).

Quick Tip to Solve: For group 15 hydrides, move down the group for stronger reducing power and weaker thermal stability; do not use the basicity trend.

73. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In aqueous solutions Cr^{2+} is reducing while Mn^{3+} is oxidising in nature.

Reason (R) : Extra stability to half filled electronic configuration is observed than incompletely filled electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) (A) is true but (R) is false

(B) Both (A) and (R) are true and (R) is the correct explanation of (A)

(C) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(D) (A) is false but (R) is true

Correct Answer: (B) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- A reducing ion loses an electron and is oxidised; an oxidising ion gains an electron and is reduced. Stable product configurations help explain the direction of these changes.
- A half-filled arrangement has extra stability. For aqueous chromium(III), the three electrons occupy the three lower-energy octahedral t_{2g} orbitals singly; manganese(II) has a half-filled d^5 subshell.

Step 1: Evaluate the assertion Chromium(II) is $3d^4$ and readily loses an electron to form chromium(III), $3d^3$. In the octahedral aqueous complex the product has the stable t_{2g}^3 arrangement, with one electron in each of the three lower-energy d orbitals. Thus Cr^{2+} acts as a reducing agent. Manganese(III) is $3d^4$ and readily gains an electron to form manganese(II), $3d^5$, with a half-filled d subshell. Hence Mn^{3+} acts as an oxidising agent. Both parts of assertion A are true.

Step 2: Evaluate the reason The reason states that half-filled electronic configurations possess extra stability compared with incompletely filled configurations. This is the relevant electronic-stability principle: the half-filled arrangement permits favourable exchange stabilisation. It applies to the half-filled lower-energy t_{2g} set in chromium(III), as well as to the complete half-filled d subshell in high-spin manganese(II). Reason R is therefore true.

Step 3: Connect the reason with both changes For chromium, the change $d^4 \rightarrow d^3$ gives t_{2g}^3: the three orbitals in this set are half-filled. For manganese, the change $d^4 \rightarrow d^5$ gives a half-filled d subshell. It would be too restrictive to interpret the reason as referring only to an entire d subshell and ignore the stable half-filled t_{2g} level. With the relevant octahedral orbital splitting included, R explains why chromium(II) is oxidised and manganese(III) is reduced. Options declaring either statement false, or denying the explanation, are therefore excluded.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: For chromium(III), check the split orbital set: d^3 means half-filled t_{2g}, even though the entire d subshell is not half-filled.

74. The functional group that shows negative resonance effect is : [4 marks]

(A) $-OH$

(B) $-OR$

(C) $-COOH$

(D) $-NH_2$

Correct Answer: (C) $-COOH$

Solution:

Concept:

- A negative resonance effect ($-R$ or $-M$) means withdrawal of electron density from a conjugated system through resonance.
- A lone pair on the atom directly attached to a conjugated system commonly gives a positive resonance effect, while an attached carbonyl carbon can accept electron density.

Step 1: Identify the electron-withdrawing group In a carboxyl group, $-COOH$, the atom directly attached to the conjugated system is the carbonyl carbon. The carbonyl unit is polarised towards oxygen and can accept electron density from an adjacent conjugated bond system. Resonance delocalisation therefore withdraws electron density from that system towards the carbonyl oxygen. The carboxyl group consequently shows a negative resonance effect.

Step 2: Check the oxygen-donor alternatives For $-OH$ and $-OR$, oxygen itself is directly attached to the conjugated system. A lone pair on that oxygen can overlap with adjacent p orbitals and be donated into the conjugated system. These groups accordingly display a positive resonance effect. Although electronegative oxygen also exerts a negative inductive effect, induction and resonance are different mechanisms and must not be interchanged.

Step 3: Check the nitrogen-donor alternative In $-NH_2$, the attached nitrogen has an available lone pair that can delocalise into a neighbouring conjugated system. The amino group therefore has a positive resonance effect as well. It is not an electron-withdrawing resonance group in its neutral form as listed. Among all four choices, only the carboxyl group has the accepting carbonyl arrangement required for the negative resonance effect.

Step 4: Conclusion The $-COOH$ group shows a negative resonance effect. Hence, the correct option is (C).

Quick Tip to Solve: Check the atom attached to the conjugated system: an attached lone-pair donor gives $+R$, while an attached carbonyl carbon gives $-R$.

75. The number of radial node/s for $3p$ orbital is : [4 marks]

(A) 3

(B) 2

(C) 1

(D) 4

Correct Answer: (C) 1

Solution:

Concept:

- For an atomic orbital with principal quantum number n and azimuthal quantum number l, the number of radial nodes is $n - l - 1$.
- The number of angular nodes is l, and the total number of nodes is $n - 1$. A p orbital always has $l = 1$.

Step 1: Read the quantum numbers The orbital is $3p$. Its leading number gives the principal quantum number, $n = 3$, and its letter p gives the azimuthal quantum number, $l = 1$. The letter is essential: the same principal shell contains orbitals with different values of l and therefore different divisions between radial and angular nodes.

Step 2: Calculate the radial node count Substitute these quantum numbers into the radial-node formula:

$$N_{\text{radial}} = n - l - 1 = 3 - 1 - 1 = 1.$$

A radial node is a spherical nodal surface at a finite radial distance where the radial wavefunction vanishes. It should not be confused with the nodal plane associated with the angular part of a p orbital. The probability density is zero on either kind of nodal surface, but the question specifically asks for radial nodes.

Step 3: Cross-check against the total number For this orbital, the angular-node count is $l = 1$, so radial plus angular nodes gives $1 + 1 = 2$, agreeing with $n - 1 = 2$. Thus the option 2 is the total-node count rather than the radial-node count. The value 3 simply repeats the principal quantum number and does not apply the formula, while 4 exceeds even the total number of nodes for this orbital. Only the value 1 satisfies both checks.

Step 4: Conclusion The $3p$ orbital has one radial node. Hence, the correct option is (C).

Quick Tip to Solve: Write both counts before choosing: radial $= n - l - 1$, angular $= l$. Their sum must be $n - 1$.

76. Which among the followng has highest boiling point? [4 marks]

(A) $CH_3CH_2CH_2CH_2 - OH$

(B) $CH_3CH_2CH_2CH_3$

(C) $CH_3CH_2CH_2CHO$

(D) $H_5C_2 - O - C_2H_5$

Correct Answer: (A) $CH_3CH_2CH_2CH_2 - OH$

Solution:

Concept:

- A liquid boils when its vapour pressure reaches the external pressure. Stronger intermolecular attractions generally require a higher temperature to separate molecules into the vapour.
- Among molecules of comparable size, extensive intermolecular hydrogen bonding can raise the boiling point above values produced by dipole interactions and dispersion forces alone.

Step 1: Identify the four compounds The options are butan-1-ol, butane, butanal and diethyl ether, respectively. They all contain four carbon atoms, so this comparison is especially suited to examining the functional group and the intermolecular forces it permits. The alcohol, aldehyde and ether have similar molecular masses, while butane is nonpolar and somewhat lighter.

Step 2: Identify the strongest association Butan-1-ol contains an O–H bond and lone pairs on oxygen. One molecule can donate a hydrogen bond through its hydroxyl hydrogen, while another accepts it through oxygen. Repeated intermolecular hydrogen bonding associates the molecules in the liquid and substantially increases the energy required for vaporisation. The carbon chain also supplies dispersion attractions, so these are present in addition to the hydrogen-bonding contribution.

Step 3: Eliminate the other choices Butane has only dispersion interactions and no permanent molecular dipole or hydrogen-bonding group. Butanal has a polar carbonyl group and therefore dipole–dipole attractions, but it has no O–H hydrogen and cannot form a self-associated hydrogen-bond network in the pure liquid. Diethyl ether likewise has an oxygen acceptor but no O–H donor, so ether molecules do not hydrogen-bond to one another. Neither can match the alcohol's strong self-association in this comparable-sized set.

Step 4: Conclusion Butan-1-ol, $CH_3CH_2CH_2CH_2OH$, has the highest boiling point. Hence, the correct option is (A).

Quick Tip to Solve: For similar carbon counts, first look for an O–H or N–H donor: an oxygen atom alone does not guarantee hydrogen bonding in the pure liquid.

77. Given below are two statements :

Statement (I) : A π bonding MO has lower electron density above and below the inter-nuclear axis.

Statement (II) : The π^* antibonding MO has a node between the nuclei.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Sidewise constructive overlap of parallel p orbitals forms a π bonding molecular orbital, with electron density in lobes above and below the internuclear axis.
- Destructive overlap forms a π^* antibonding molecular orbital, which has an additional node between the two nuclei.

Step 1: Evaluate statement I In a π bonding MO, constructive sidewise overlap concentrates electron density in the regions above and below the line joining the nuclei. This density between the atoms stabilises the bond. The internuclear axis itself lies in an angular nodal plane, so it is incorrect to describe the bonding orbital as having lower electron density above and below that axis. Those are precisely the regions where its bonding density occurs. Statement I is false.

Step 2: Evaluate statement II For the π^* antibonding combination, the interacting p-orbital lobes on adjacent atoms have opposite phases. Their destructive overlap produces a nodal surface between the nuclei, perpendicular to the internuclear axis. Electron density vanishes there, reducing density in the internuclear region and giving antibonding character. This additional node is distinct from the nodal plane containing the internuclear axis that is common to pi-type orbitals. Statement II is true.

Step 3: Match the truth values to the options The required combination is false for statement I and true for statement II. Option A would incorrectly accept the first statement; option B would incorrectly reject the second. Option C reverses the two truth values and also fails. The final option alone correctly identifies the density enhancement of a pi bonding MO and the extra internuclear node of a pi-star antibonding MO.

Step 4: Conclusion Statement I is false but statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish the two nodal planes: every pi orbital has one containing the bond axis; pi-star has an additional node separating the nuclei.

78. Given below are two statements :

Statement (I) : Both metals and non-metals exist in p and d-block elements.

Statement (II) : Non-metals have higher ionisation enthalpy and higher electronegativity than the metals.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are false

(B) Both Statement I and Statement II are true

(C) Statement I is false but Statement II is true

(D) Statement I is true but Statement II is false

Correct Answer: (C) Statement I is false but Statement II is true

Solution:

Concept:

- An element belongs to a block according to the subshell receiving the differentiating electron. The p-block includes metals, non-metals and metalloids; the d-block consists of metallic elements.
- Ionisation enthalpy measures the energy required to remove an electron from an isolated gaseous atom. Electronegativity expresses an atom's tendency to attract shared electrons in a chemical bond.

Step 1: Evaluate Statement I The p-block contains metals such as aluminium and tin and non-metals such as carbon, nitrogen and chlorine. However, d-block elements such as iron, copper and zinc are metals. Merely finding both categories in the p-block does not establish their presence in the d-block. Since the statement explicitly includes both blocks, its claim about the d-block makes Statement I false.

Step 2: Evaluate Statement II Non-metals generally lie towards the upper right of the periodic table. Their smaller atomic sizes and stronger effective nuclear attraction make electron removal more difficult than for metals, which readily lose electrons. Non-metals also tend to attract the shared electron pair more strongly. Thus the statement describes the standard general trends in ionisation enthalpy and electronegativity, and Statement II is true.

Step 3: Select the matching combination Option (A) incorrectly rejects Statement II. Option (B) incorrectly accepts Statement I, and option (D) reverses both truth values. The required combination is therefore false for Statement I and true for Statement II.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (C).

Quick Tip to Solve: When a statement mentions two blocks together, test each block separately; one counterexample to its claimed classification is enough to reject it.

79. Which of the following compounds show colour due to d-d transition? [4 marks]

(A) $K_2Cr_2O_7$

(B) $CuSO_4 \cdot 5H_2O$

(C) $KMnO_4$

(D) K_2CrO_4

Correct Answer: (B) $CuSO_4 \cdot 5H_2O$

Solution:

Concept:

- A d–d transition promotes an electron between ligand-field-split d orbitals. It requires a partially occupied d subshell, so neither a d^0 nor a d^{10} ion supports this mechanism.
- Oxidation state must be determined before counting d electrons. A strongly coloured compound need not owe its colour to a d–d transition; charge-transfer transitions can also absorb visible light.

Step 1: Count d electrons in the copper salt In $CuSO_4 \cdot 5\,H_2O$, sulphate is $SO_4{}^{2-}$ and water is neutral, so copper is in oxidation state $+2$. Neutral copper has configuration $[Ar]3d^{10}4s^1$; removal of two electrons gives Cu^{2+} the configuration $[Ar]3d^9$. This incomplete d subshell permits electronic excitation between levels split by the surrounding ligands, giving the hydrated salt its blue colour.

Step 2: Check chromium and manganese compounds For dichromate, $2x + 7(-2) = -2$ gives $x = +6$; chromate likewise has chromium in oxidation state $+6$. Chromium(VI) has a d^0 configuration. In permanganate, $x + 4(-2) = -1$ gives manganese(VII), also d^0. These ions have no d electron to promote in a d–d transition.

Step 3: Distinguish the other colours The orange dichromate, purple permanganate and yellow chromate colours arise chiefly from ligand-to-metal charge transfer, rather than transitions of an electron already in a d orbital. Their visible colours therefore do not make options (A), (C) or (D) suitable answers.

Step 4: Conclusion $CuSO_4 \cdot 5\,H_2O$ shows colour due to a d–d transition of its Cu^{2+} ion. Hence, the correct option is (B).

Quick Tip to Solve: Calculate the metal oxidation state and d-electron count before using colour as evidence for a d–d transition.

80. Select the compound from the following that will show intramolecular hydrogen bonding.

[4 marks]

(A) o-Nitrophenol (2-nitrophenol)

(B) H_2O

(C) C_2H_5OH

(D) NH_3

Correct Answer: (A) o-Nitrophenol (2-nitrophenol)

Solution:

Concept:

- Hydrogen bonding requires a hydrogen atom attached to an electronegative atom and a suitable lone-pair acceptor. Intramolecular hydrogen bonding occurs when donor and acceptor belong to the same molecule.
- An ortho arrangement on a benzene ring can bring a phenolic $O-H$ donor close to an oxygen acceptor in a neighbouring substituent, producing a favourable ring-like hydrogen-bonded arrangement.

Step 1: Identify the donor and acceptor in option A The compound in option (A) is o-nitrophenol, with OH and NO_2 on adjacent carbon atoms of the benzene ring. The phenolic hydrogen is the donor, and an oxygen of the nitro group provides a lone pair as acceptor. The ortho positions place these groups sufficiently close for a hydrogen bond within one molecule.

Step 2: Describe the intramolecular interaction The interaction can be represented in words as phenolic $O-H$ directed towards nitro oxygen, with $O-H\cdots O$ indicating the hydrogen bond. The covalent framework and this interaction form a favourable six-membered ring-like arrangement. No second molecule is needed, which is the defining feature asked in the question.

Step 3: Rule out the other compounds Water, ethanol and ammonia can all form hydrogen bonds between different molecules. However, none of these small molecules has the appropriately separated donor and acceptor groups needed to close a favourable intramolecular hydrogen-bonded ring. In water and ammonia, bonding to the same atom that carries the hydrogen is not a separate intramolecular hydrogen bond; ethanol similarly lacks a second suitable functional group.

Step 4: Conclusion o-Nitrophenol in option (A) exhibits intramolecular hydrogen bonding. Hence, the correct option is (A).

Quick Tip to Solve: Look for a donor and acceptor in the same molecule and check their geometry; adjacent nitro and hydroxyl groups on benzene are a standard example.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The number of tripeptides formed by three different amino acids using each amino acid once is _____.

[4 marks]

Solution:

Concept:

- A tripeptide contains three amino-acid residues joined by two peptide bonds. The sequence is read from its free amino terminus to its free carboxyl terminus.
- When three different amino acids must each occur exactly once, sequence counting is a permutation problem without repetition. Reversing the order generally gives a different peptide.

Step 1: Label the three different amino acids Write the amino acids as A, B and C. The condition that each is used once means that AAA, AAB and every other sequence with a repeated residue is excluded. Each allowed peptide therefore corresponds to one arrangement of the three distinct labels along the peptide chain.

Step 2: Count the choices at successive positions The N-terminal position can be occupied by any of the three amino acids. After this choice, two unused amino acids remain for the middle position. The final, C-terminal position is then fixed by the one amino acid that has not yet been used. By the multiplication rule, the number of distinct sequences is

$$N = 3 \times 2 \times 1 = 3! = 6.$$

Step 3: List the sequences and check their distinction The six arrangements are ABC, ACB, BAC, BCA, CAB and CBA. For example, ABC and CBA are different because the amino and carboxyl termini are chemically different; the chain cannot be treated as an undirected arrangement. No cyclic peptide or additional side-chain stereoisomerism is being introduced: the question counts sequences formed from the specified three amino acids.

Step 4: Conclusion The number of distinct tripeptides is $\boxed{6}$.

Quick Tip to Solve: For different residues used once each, count ordered peptide sequences with a factorial; do not divide by two for reversed chains.

82. Mass of ethylene glycol (antifreeze) to be added to 18.6 kg of water to protect the freezing point at -24°C is _____ kg (Molar mass in g mol^{-1} for ethylene glycol 62, K_f of water $= 1.86$ K kg mol^{-1})

[4 marks]

Solution:

Concept:

- For a non-electrolyte dissolved in water, freezing-point depression is $\Delta T_f = K_f m$, where m is the molality in moles of solute per kilogram of solvent.

- Ethylene glycol does not dissociate into ions in this calculation, so its van't Hoff factor is $i = 1$. Use the given solvent mass, not the final mass of solution, in calculating molality.

Step 1: Calculate the required freezing-point depression Pure water freezes at 0°C. To lower its freezing point to -24°C, the required depression is positive:

$$\Delta T_f = 0 - (-24) = 24 \text{ K}.$$

A temperature difference has the same numerical value in kelvin and degrees Celsius. The negative final freezing temperature must not be substituted as a negative depression.

Step 2: Find the moles of ethylene glycol Using the given cryoscopic constant,

$$m = \frac{\Delta T_f}{K_f} = \frac{24}{1.86} \text{ mol kg}^{-1}.$$

The solvent is 18.6 kg of water, so

$$n = m \times 18.6 = \frac{24 \times 18.6}{1.86} = 240 \text{ mol}.$$

The factor $18.6/1.86 = 10$ makes the arithmetic especially simple.

Step 3: Convert the solute amount into mass The molar mass of ethylene glycol is 62 g mol^{-1}. Therefore

$$w = 240 \times 62 = 14880 \text{ g} = 14.88 \text{ kg}.$$

This is the mass of glycol to add, not the total mass after mixing. Reporting the numerical entry to the nearest integer gives 15.

Step 4: Conclusion The calculated mass is 14.88 kg; the nearest-integer numerical answer is $\boxed{15}$ kg.

Quick Tip to Solve: Combine the relations as mass of solute $= (\Delta T_f/K_f) \times$ kilograms of solvent $\times$ molar mass, then convert grams to kilograms.

83. Total number of isomeric compounds (including stereoisomers) formed by monochlorination of 2-methylbutane is _____ . [4 marks]

Solution:

Concept:

- Monochlorination replaces one hydrogen by chlorine. Equivalent hydrogen positions give the same constitutional product, while inequivalent positions give distinct products.
- After identifying constitutional products, check whether the replacement creates a stereogenic carbon. A product with one stereogenic centre contributes two enantiomers when stereoisomers are included.

Step 1: Identify the distinct substitution positions The condensed structure is $(CH_3)_2CHCH_2CH_3$. The two methyl groups attached directly to the central methine carbon are equivalent before substitution. The remaining terminal methyl group, the methylene group and the central methine hydrogen are

all different environments. Thus there are four constitutional monochloro products, rather than five independent carbon positions.

Step 2: Count products from the two primary environments Replacement on either equivalent methyl group attached to the methine carbon gives 1-chloro-2-methylbutane. Its carbon 2 now bears H, methyl, ethyl and chloromethyl groups, so it is stereogenic and gives two enantiomers. Replacement on the other terminal methyl gives 1-chloro-3-methylbutane; its methine carbon still carries two identical methyl groups, so this product is achiral and contributes one.

Step 3: Count secondary and tertiary substitution products Replacing a methylene hydrogen gives 2-chloro-3-methylbutane. The chlorine-bearing carbon has four different groups and gives two enantiomers. Replacing the central methine hydrogen gives 2-chloro-2-methylbutane, whose chlorine-bearing carbon retains two identical methyl groups and is achiral. It contributes one product. The total is

$$N = 2 + 1 + 2 + 1 = 6.$$

Step 4: Conclusion The total number of monochlorination products, including stereoisomers, is $\boxed{6}$.

Quick Tip to Solve: First count four distinct hydrogen environments, then add one extra product for each constitutional product that forms an enantiomeric pair.

84. The following data were obtained during the first order thermal decomposition of a gas A at constant volume :

$$\mathrm{A(g)} \rightarrow 2\ \mathrm{B(\ g)} + \mathrm{C(g)}$$

S.No.	Time /s	Total pressure /(atm)
1.	0	0.1
2.	115	0.28

The rate constant of the reaction is _____ $\times 10^{-2}\ \mathrm{s}^{-1}$ (nearest integer) [4 marks]

Solution:

Concept:

- At constant temperature and volume, partial pressure is proportional to the number of moles of a gas. For $\mathrm{A(g)} \longrightarrow 2\,\mathrm{B(g)} + \mathrm{C(g)}$, decomposition of one mole of A increases the total amount of gas by two moles.
- The first-order integrated law can therefore use the reactant partial pressure: $k = t^{-1} \ln(P_{A,0}/P_{A,t})$. Total pressure must first be converted into the remaining partial pressure of A.

Step 1: Find the pressure relation Initially, A alone has pressure $P_0 = 0.10$ atm. Let x be the pressure decrease due to consumption of A. The partial pressures are $P_A = P_0 - x$, $P_B = 2x$ and $P_C = x$. Consequently,

$$P_t = (P_0 - x) + 2x + x = P_0 + 2x.$$

The total-pressure increase is not itself the pressure of remaining reactant.

Step 2: Calculate the remaining reactant pressure At $t = 115$ s, the total pressure is 0.28 atm, giving

$$x = \frac{0.28 - 0.10}{2} = 0.09 \text{ atm}, \qquad P_A = 0.10 - 0.09 = 0.01 \text{ atm}.$$

This also checks the physical limit: complete decomposition would produce total pressure $3P_0 = 0.30$ atm, so 0.28 atm represents nearly complete decomposition.

Step 3: Apply the integrated first-order law The ratio of initial to remaining reactant pressure is ten. Hence

$$k = \frac{1}{115} \ln\left(\frac{0.10}{0.01}\right) = \frac{\ln 10}{115} = 0.0200225 \text{ s}^{-1}.$$

Writing this as 2.00225×10^{-2} s^{-1}, the coefficient to the nearest integer is 2.

Step 4: Conclusion $k \approx 2.00 \times 10^{-2}$ s^{-1}, so the required numerical entry is $\boxed{2}$.

Quick Tip to Solve: For this 1-to-3 gaseous decomposition, use $P_A = (3P_0 - P_t)/2$ before applying the first-order logarithmic law.

85. For a certain reaction at 300 K, K $= 10$, then ΔG° for the same reaction is - _____ $\times 10^{-1}$ kJ mol^{-1}.

(Given R $= 8.314$JK^{-1} mol^{-1} **)** [4 marks]

Solution:

Concept:

- The standard Gibbs energy change and the dimensionless equilibrium constant are related by $\Delta G^\circ = -RT \ln K$. Here R is the gas constant and T is the absolute temperature.
- When R is in J K^{-1} mol^{-1}, the energy calculated is in J mol^{-1}. Convert to kilojoules before comparing with the requested coefficient.

Step 1: Insert the temperature and equilibrium constant The supplied values are $T = 300$ K, $K = 10$ and $R = 8.314$ J K^{-1} mol^{-1}. Use the natural logarithm, so $\ln 10 = 2.302585$; using $\log_{10} 10 = 1$ without the factor 2.303 would give the wrong magnitude. Therefore,

$$\Delta G^\circ = -(8.314)(300)(2.302585) = -5743.11 \text{ J mol}^{-1}.$$

The negative sign is consistent with $K > 1$: the forward reaction is favoured when reactants and products are in their standard states.

Step 2: Convert the energy into the form in the question Divide the value in joules by 1000 to obtain

$$\Delta G^\circ = -5.74311 \text{ kJ mol}^{-1} = -57.4311 \times 10^{-1} \text{ kJ mol}^{-1}.$$

The blank asks for the positive coefficient after the already printed minus sign, rather than the negative energy itself. The numerical-entry value is the nearest integer to 57.4311, namely 57. The factor 10^{-1} changes the coefficient by ten and must not be overlooked.

Step 3: Conclusion $\Delta G^\circ \approx -57.43 \times 10^{-1}$ kJ mol^{-1}. Hence the required integer entry is $\boxed{57}$.

Quick Tip to Solve: Use $-RT \ln K$ first, then convert units and match the printed power of ten; the minus sign is already outside this blank.

86. The amount of electricity in Coulomb required for the oxidation of $1\ \text{mol}$ of H_2O to O_2 is _____ $\times 10^5 C$. [4 marks]

Solution:

Concept:

- Faraday's law relates charge to electron transfer: $Q = n_e F$, where n_e is the amount of electrons in moles and $F \approx 96485\,\text{C mol}^{-1}$.
- Oxygen has oxidation number -2 in water and zero in oxygen gas. Thus each oxygen atom derived from water loses two electrons during oxidation.

Step 1: Balance the oxidation half-reaction In acidic notation, water is oxidised according to

$$2\,H_2O \longrightarrow O_2 + 4\,H^+ + 4\,e^-.$$

Both sides contain two oxygen atoms and four hydrogen atoms, and the total charge is zero on each side. The coefficient of electrons refers to two moles of water, not one. Dividing the stoichiometric amounts by two shows that oxidation of $1\,\text{mol}$ of water releases $2\,\text{mol}$ of electrons. The same electron count follows directly from changing the oxidation number of its one oxygen atom from -2 to zero.

Step 2: Calculate the charge and requested coefficient For complete oxidation of the stated amount of water, the charge magnitude is

$$Q = 2F = 2(96485) = 192970\,\text{C}.$$

This can be written as

$$Q = 1.92970 \times 10^5\,\text{C}.$$

The question requires the numerical coefficient of $10^5\,\text{C}$. Rounding 1.92970 to the nearest integer gives 2. Using four Faradays here would correspond to oxidation of two moles of water, or formation of one mole of oxygen gas; neither is the stated amount. Charge required is reported as a positive magnitude even though electrons are products of the oxidation half-reaction.

Step 3: Conclusion The required charge is $1.93 \times 10^5\,\text{C}$, so the required integer entry is $\boxed{2}$.

Quick Tip to Solve: Count electrons per mole of the species named in the question: one mole of water requires two Faradays, while one mole of oxygen corresponds to four.

87. Following Kjeldahl's method, $1\ \text{g}$ of organic compound released ammonia, that neutralised $10\ \text{mL}$ of $2\text{M}\ H_2SO_4$. The percentage of nitrogen in the compound is _____ %. [4 marks]

Solution:

Concept:

- In Kjeldahl's method, the nitrogen converted to ammonia is measured by the amount of acid neutralised. Each mole of NH_3 contains one mole of nitrogen atoms.
- Sulfuric acid supplies two acid equivalents per mole: $2\,NH_3 + H_2SO_4 \longrightarrow (NH_4)_2SO_4$. Conse-

quently one mole of this acid neutralises two moles of ammonia.

Step 1: Find the amount of ammonia from the acid Convert the acid volume to litres before using the molarity:

$$V = 10\,\text{mL} = 0.010\,\text{L}.$$

The amount of sulfuric acid neutralised is

$$n(\text{H}_2\text{SO}_4) = MV = (2)(0.010) = 0.020\,\text{mol}.$$

The balanced neutralisation equation gives a $2:1$ ammonia-to-acid ratio. Hence

$$n(\text{NH}_3) = 2(0.020) = 0.040\,\text{mol}.$$

Treating $2\,\text{M}$ sulfuric acid as if it were $2\,\text{N}$ would miss its second acidic proton and halve the result.

Step 2: Convert ammonia into nitrogen percentage Each ammonia molecule carries exactly one nitrogen atom, so the amount of nitrogen in the $1\,\text{g}$ compound sample is also $0.040\,\text{mol}$. With the molar mass of nitrogen atoms $14\,\text{g mol}^{-1}$,

$$m(\text{N}) = (0.040)(14) = 0.560\,\text{g}.$$

The required percentage is the nitrogen mass divided by the original compound mass, multiplied by one hundred:

$$\%\text{N} = \frac{0.560}{1.00} \times 100 = 56.0\%.$$

Use the mass of nitrogen, not the mass of ammonia, because hydrogen in the collected ammonia is not included in the requested elemental percentage.

Step 3: Conclusion The nitrogen content is $\boxed{56\%}$. Therefore the required numerical entry is $\boxed{56}$.

Quick Tip to Solve: For Kjeldahl calculations, convert acid moles to equivalents first; sulfuric acid neutralises twice its molar amount of ammonia.

88. Consider the following redox reaction :

$$MnO_4^- + H^+ + H_2C_2O_4 \rightleftharpoons Mn^{2+} + H_2O + CO_2$$

The standard reduction potentials are given as below (E^0_{red}) :

$$E^\circ_{MnO_4^-/Mn^{2+}} = +1.51 \text{ V}$$

$$E^\circ_{CO_2/H_2C_2O_4} = -0.49 \text{ V}$$

If the equilibrium constant of the above reaction is given as $K_{eq} = 10^x$, then the value of $x =$ _____ (nearest integer) [4 marks]

Solution:

Concept:

- For a balanced cell reaction, $\Delta G^\circ = -nFE^\circ_{cell} = -RT\ln K$, where n is the electron-transfer number of the whole reaction.
- At the standard electrochemical temperature 298 K, $\log_{10} K = nE^\circ_{cell}/0.0591$. Reduction potentials are intensive quantities and are not multiplied by stoichiometric coefficients.

Step 1: Balance the electron exchange Permanganate is reduced in acid and oxalic acid is oxidised:

$$MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O,$$

$$H_2C_2O_4 \longrightarrow 2\,CO_2 + 2\,H^+ + 2\,e^-.$$

The least common electron count is ten. Multiply the first half-reaction by two and the second by five, then add and cancel common species to obtain

$$2\,MnO_4^- + 6\,H^+ + 5\,H_2C_2O_4 \longrightarrow 2\,Mn^{2+} + 8\,H_2O + 10\,CO_2.$$

Thus the integer-balanced redox reaction has $n = 10$, not five or two. This count sets the equilibrium constant for the balanced reaction.

Step 2: Find the cell potential and logarithm of the constant Both given values are reduction potentials. Permanganate is the cathode couple; the carbon dioxide/oxalic acid couple runs in reverse at the anode. Therefore

$$E^\circ_{cell} = 1.51 - (-0.49) = 2.00\,V.$$

At the standard temperature,

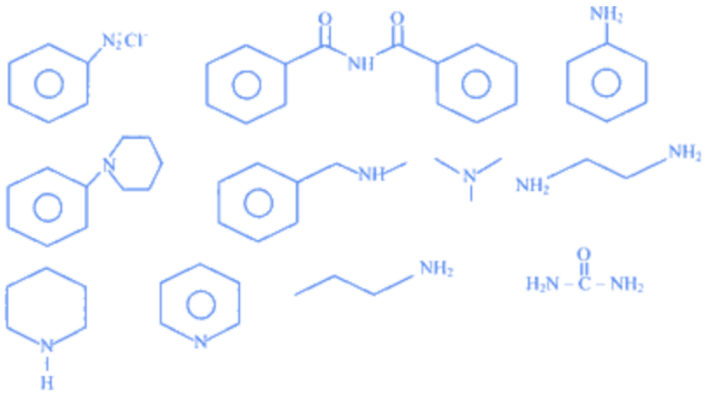
$$x = \log_{10} K = \frac{10(2.00)}{0.0591} \approx 338.41.$$

The large positive cell potential produces a very large equilibrium constant, consistent with a strongly favourable forward redox reaction. The nearest integer is 338. Multiplying either supplied potential by its balancing coefficient would be incorrect because potential is energy per unit charge, not total energy.

Step 3: Conclusion The equilibrium constant is approximately $10^{338.4}$; to the nearest integer, $\boxed{x = 338}$.

Quick Tip to Solve: Balance the half-reactions to get the common electron count, but subtract the original reduction potentials without scaling them.

89. Number of compounds which give reaction with Hinsberg's reagent is _____. [4 marks]



Solution:

Concept:

- Hinsberg's reagent is benzenesulfonyl chloride, $C_6H_5SO_2Cl$. Primary and secondary amines with an available nitrogen lone pair and an $N-H$ bond form sulfonamide derivatives.
- Tertiary amines have no $N-H$ bond and do not form the sulfonamide in this test. Amides and imides are not classified as ordinary primary or secondary amines because their lone pairs are delocalised into carbonyl groups.

Step 1: Identify the reactive amines in the figure Count compounds rather than individual nitrogen atoms. The reactive compounds are aniline (top right), the secondary benzyl methylamine (middle row), the diamine bearing two terminal NH_2 groups (middle right), piperidine (bottom left), and the straight-chain primary alkylamine (bottom middle). Aniline has a primary amino group; benzyl methylamine and piperidine are secondary amines with one $N-H$ bond each. The diamine contains two reactive amino groups but represents only one compound in the requested count. The terminal alkylamine has a free primary amino group and also reacts.

Step 2: Exclude the other nitrogen compounds The aryl diazonium salt is not an amine. The compound containing $-CO-NH-CO-$ is an imide with its lone pair delocalised over carbonyl groups, so it is excluded from the ordinary Hinsberg amine test. The phenyl-substituted cyclic amine in the middle left and trimethylamine are tertiary amines without an $N-H$ bond. Pyridine is a ring nitrogen base without a nitrogen-hydrogen bond and does not form the required sulfonamide. Urea is a carbonyl amide rather than a free primary amine and is excluded from this diagnostic reaction. Therefore exactly five distinct compounds in the displayed collection satisfy the test conditions.

Step 3: Conclusion The number of compounds that react in the Hinsberg test is $\boxed{5}$.

Quick Tip to Solve: Look for a free primary or secondary amino group, then count each molecule once; do not count amide nitrogens or multiply the count for a diamine.

90. $10\ \text{mL}$ **of gaseous hydrocarbon on combustion gives** $40\ \text{mL}$ **of** $CO_2(\text{g})$ **and** $50\ \text{mL}$ **of water vapour. Total number of carbon and hydrogen atoms in the hydrocarbon is _____ .** [4 marks]

Solution:

Concept:

- At the same temperature and pressure, gas volumes are proportional to amounts in moles. Therefore the measured volume ratios can be used directly as reaction coefficient ratios.
- Complete combustion of a hydrocarbon C_xH_y gives x moles of carbon dioxide and $y/2$ moles of water per mole of hydrocarbon: carbon and hydrogen atoms are conserved independently.

Step 1: Use carbon dioxide to find the carbon count Write the general combustion equation as

$$C_xH_y + (x^+y/_4)O_2 \longrightarrow xCO_2 + (y/_2)H_2O.$$

The volume of carbon dioxide produced from $10\,\text{mL}$ of hydrocarbon is $40\,\text{mL}$, so the product-to-reactant volume ratio is

$$x = \frac{V(CO_2)}{V(\text{hydrocarbon})} = \frac{40}{10} = 4.$$

Each molecule of hydrocarbon therefore contains four carbon atoms. The ratio uses hydrocarbon volume in the denominator; the oxygen volume is not needed to determine either atom count.

Step 2: Use water vapour to find the hydrogen count The water is measured as vapour, so the same volume-to-mole proportionality applies. The water coefficient per hydrocarbon molecule is

$$\frac{y}{2} = \frac{50}{10} = 5, \qquad y = 10.$$

Thus the molecular formula inferred from the combustion products is C_4H_{10}. The value five is the number of water molecules formed per hydrocarbon molecule, whereas each water molecule contains two hydrogen atoms. As a balance check,

$$2\,C_4H_{10} + 13\,O_2 \longrightarrow 8\,CO_2 + 10\,H_2O.$$

This produces the observed hydrocarbon:carbon-dioxide:water-vapour volume ratio $1 : 4 : 5$. The total number of carbon and hydrogen atoms in one hydrocarbon molecule is consequently $4 + 10 = 14$.

Step 3: Conclusion The hydrocarbon is C_4H_{10}, containing $\boxed{14}$ carbon and hydrogen atoms in total per molecule.

Quick Tip to Solve: Divide both product gas volumes by the hydrocarbon volume; the carbon dioxide ratio gives carbon atoms, while twice the water-vapour ratio gives hydrogen atoms.