

JEE Main 2024 Paper 1 (B.E./B.Tech) Apr 5 Shift 1

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 5 April 2024, Shift 1. The total duration is 3 hours.
- (ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let a rectangle ABCD of sides 2 and 4 be inscribed in another rectangle PQRS such that the vertices of the rectangle ABCD lie on the sides of the rectangle PQRS. Let a and b be the sides of the rectangle PQRS when its area is maximum. Then $(a+b)^2$ is equal to : [4 marks]

(A) 64

(B) 80

(C) 60

(D) 72

Correct Answer: (D) 72

Solution:

Concept:

- The side lengths of the outer rectangle are the horizontal and vertical projections of the inscribed rectangle. Let its side of length 4 make an angle θ with one outer side, where $0 \leq \theta \leq \pi/2$.
- The area is maximised using $\sin \theta \cos \theta \leq 1/2$, with equality at $\theta = \pi/4$. The requested expression must be evaluated at this area maximum.

Step 1: Express the outer dimensions The four vertices touch the four bounding sides, so the horizontal span and vertical span of the inner rectangle equal the dimensions of the outer one. The two perpendicular sides contribute their respective projections:

$$a = 4 \cos \theta + 2 \sin \theta, \quad b = 4 \sin \theta + 2 \cos \theta.$$

Swapping the roles of the outer sides simply interchanges these expressions and has no effect on the area or the required sum.

Step 2: Maximise the area Multiply the spans and use $\sin^2 \theta + \cos^2 \theta = 1$:

$$ab = 8 + 20 \sin \theta \cos \theta = 8 + 10 \sin 2\theta \leq 18.$$

Equality holds when $\sin 2\theta = 1$, hence at $\theta = \pi/4$. This is a global maximum over the permitted orientations; the endpoint orientations give the smaller area 8.

Step 3: Evaluate the requested sum At the maximum-area orientation,

$$a = b = \frac{4 + 2}{\sqrt{2}} = 3\sqrt{2}.$$

Therefore

$$(a + b)^2 = (6\sqrt{2})^2 = 72.$$

The other numerical choices do not result from the dimensions at the area maximum; in particular the maximum area 18 itself is not the expression asked for.

Step 4: Conclusion The sides at maximum area are both $3\sqrt{2}$, giving $(a + b)^2 = 72$. Hence, the correct option is (D).

Quick Tip to Solve: For a rotated rectangle, first write its two bounding spans as projections; maximise their product before evaluating a different expression in the spans.

2. Let $A = \{1, 3, 7, 9, 11\}$ and $B = \{2, 4, 5, 7, 8, 10, 12\}$. Then the total number of one-one maps $f : A \rightarrow B$, such that $f(1) + f(3) = 14$, is : [4 marks]

- (A) 120
- (B) 180
- (C) 240
- (D) 480

Correct Answer: (C) 240

Solution:

Concept:

- A one-one map assigns distinct images to distinct domain elements. The two special elements 1 and 3 have labelled, ordered images.
- Once their distinct images are fixed, the remaining three domain elements must be assigned distinct images from the five unused elements of B , in ${}^5P_3$ ways.

Step 1: List the admissible image pairs Write $u = f(1)$ and $v = f(3)$. We need $u, v \in B$, $u + v = 14$, and $u \neq v$. Checking the elements of B gives the unordered pairs $\{2, 12\}$ and $\{4, 10\}$. The candidate $\{7, 7\}$ is inadmissible because it repeats an image. Values 5 and 8 have complements 9 and 6, neither of which belongs to B .

Step 2: Assign the two labelled elements Each permitted pair has two orders, because the domain elements 1 and 3 are different. Thus the ordered choices are

$$(2, 12), (12, 2), (4, 10), (10, 4).$$

There are exactly four possibilities for $(f(1), f(3))$. Counting unordered pairs alone would lose the distinction between these different functions.

Step 3: Complete the injective map For each fixed ordered pair, five codomain elements remain. Assign images to 7, 9, 11 successively: there are 5, 4, and 3 choices. Hence

$$N = 4 \times 5 \times 4 \times 3 = 240.$$

The count 120 omits the two orders of the special pair; 180 differs from the required product 4×60 ; 480 counts each valid map twice.

Step 4: Conclusion The required number of one-one maps is 240. Hence, the correct option is (C).

Quick Tip to Solve: In function-counting problems, treat images of named inputs as ordered; discard equal-image pairs before filling the remaining inputs with a permutation.

3. Let two straight lines drawn from the origin O intersect the line $3x + 4y = 12$ at the points P and Q such that $\triangle OPQ$ is an isosceles triangle and $\angle POQ = 90^\circ$. If $l = OP^2 + PQ^2 + QO^2$, then the greatest integer less than or equal to l is : [4 marks]

- (A) 42
(B) 46
(C) 48
(D) 44

Correct Answer: (B) 46

Solution:

Concept:

- An isosceles triangle whose angle at O is 90° has equal perpendicular legs $OP = OQ = r$ and hypotenuse $PQ = \sqrt{2}r$.
- The perpendicular from the right-angle vertex to the hypotenuse bisects it. Its length is $r/\sqrt{2}$, and it also equals the distance of the origin from the line $3x + 4y = 12$.

Step 1: Find the altitude from the origin The side PQ lies on the given line. The perpendicular distance from $O = (0, 0)$ to this line is

$$h = \frac{|3(0) + 4(0) - 12|}{\sqrt{3^2 + 4^2}} = \frac{12}{5}.$$

This distance is fixed by the line; it is not the distance to either intercept. The equal-leg condition determines the endpoints P, Q symmetrically about the foot of this perpendicular.

Step 2: Relate the legs to the altitude Let M be the midpoint of PQ . In a right isosceles triangle, OM is both an altitude and a median. Equivalently, comparing the two area formulas yields

$$\frac{1}{2}r^2 = \frac{1}{2}(\sqrt{2}r)h, \quad r = \sqrt{2}h.$$

Thus

$$r^2 = 2 \left(\frac{12}{5} \right)^2 = \frac{288}{25}.$$

The hypotenuse therefore has square $2r^2 = 576/25$.

Step 3: Compute the sum and take the floor Add the three squared side lengths as requested:

$$l = r^2 + 2r^2 + r^2 = 4r^2 = \frac{1152}{25} = 46.08.$$

The greatest integer not exceeding this value is 46, since $46 \leq 46.08 < 47$. Neither rounding to a convenient side length nor replacing the floor by another nearby choice gives the required value.

Step 4: Conclusion We obtain $l = 46.08$, and hence $\lfloor l \rfloor = 46$. Hence, the correct option is (B).

Quick Tip to Solve: For a right isosceles triangle with hypotenuse on a known line, its altitude from the opposite vertex immediately determines every squared side length.

4. If the line $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4-z$ makes a right angle with the line $\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$, then $4\lambda + 9\mu$ is equal to : [4 marks]

- (A) 4
(B) 13
(C) 5
(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- The direction ratios of a line are the coefficients of its parameter in x, y, z , after each coordinate is isolated. Numerators such as $3y - 2$ must be divided by their coordinate coefficient.
- Two lines have perpendicular directions when the scalar product of their direction vectors is zero. Multiplying either entire direction vector by a nonzero constant does not affect this condition.

Step 1: Parametrise the first line Set the common ratio in the first line equal to t . Then

$$x = 2 - 3t, \quad y = \frac{2}{3} + \frac{4\lambda + 1}{3}t, \quad z = 4 - t.$$

A direction vector is therefore

$$\mathbf{d}_1 = \left(-3, \frac{4\lambda + 1}{3}, -1 \right).$$

The constant terms locate a point on the line, but they do not enter the angle calculation.

Step 2: Parametrise the second line Set its common ratio equal to s . Solving the three equations gives

$$x = -3 + 3\mu s, \quad y = \frac{1}{2} - 3s, \quad z = 5 - 7s.$$

Thus a direction vector is

$$\mathbf{d}_2 = (3\mu, -3, -7).$$

In particular, the direction ratio for y is -3 , not 6 , because the numerator is $1 - 2y$.

Step 3: Use the right-angle condition Take the scalar product and set it to zero:

$$(-3)(3\mu) + \frac{4\lambda + 1}{3}(-3) + (-1)(-7) = 0.$$

On simplifying,

$$-9\mu - (4\lambda + 1) + 7 = 0, \quad 4\lambda + 9\mu = 6.$$

This fixes the requested combination without needing individual values of λ and μ . The other three choices cannot satisfy this dot-product equation.

Step 4: Conclusion The perpendicularity condition gives $4\lambda + 9\mu = 6$. Hence, the correct option is (D).

Quick Tip to Solve: Always isolate each coordinate before reading direction ratios from a nonstandard symmetric equation; coordinate coefficients and minus signs both matter.

5. Consider the following two statements :

Statement I: For any two non-zero complex numbers z_1, z_2 , $(|z_1| + |z_2|) \left| \frac{z_1}{|z_1|} + \frac{z_2}{|z_2|} \right| \leq 2(|z_1| + |z_2|)$, and

Statement II : If x, y, z are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$, then $\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = 1$.

Between the above two statements,

[4 marks]

- (A) both Statement I and Statement II are incorrect.
(B) Statement I is correct but Statement II is incorrect.
(C) Statement I is incorrect but Statement II is correct.
(D) both Statement I and Statement II are correct.

Correct Answer: (B) Statement I is correct but Statement II is incorrect.

Solution:

Concept:

- For complex numbers u, v , the triangle inequality gives $|u + v| \leq |u| + |v|$. The normalised numbers $z_1/|z_1|$ and $z_2/|z_2|$ each have modulus one.
- For nonzero complex w , $|w|^2/w = \bar{w}$. This identity turns a ratio involving squared moduli into a conjugate and makes cyclic differences telescope.

Step 1: Test Statement I Because both complex numbers are nonzero, their normalisations are defined. Apply the triangle inequality:

$$\left| \frac{z_1}{|z_1|} + \frac{z_2}{|z_2|} \right| \leq 1 + 1 = 2.$$

Multiplying by the positive real number $|z_1| + |z_2|$ proves precisely the printed inequality. Hence Statement I is correct; equality is possible when the two complex numbers have the same argument.

Step 2: Evaluate the expression in Statement II Let the common positive ratio be k . Then

$$a = k|y - z|, \quad b = k|z - x|, \quad c = k|x - y|.$$

Since x, y, z are distinct, every denominator is nonzero. Therefore the stated sum equals

$$k^2 \left(\frac{|y - z|^2}{y - z} + \frac{|z - x|^2}{z - x} + \frac{|x - y|^2}{x - y} \right).$$

Using $|w|^2 = w\bar{w}$, this becomes

$$k^2 (\overline{y - z} + \overline{z - x} + \overline{x - y}) = 0.$$

Step 3: Compare the two truth values Statement II claims that the last expression is 1, whereas the assumptions force it to be 0. Thus Statement II is incorrect. The alternatives asserting that Statement I is false fail by the triangle inequality, and the alternative asserting both are true fails by the telescoping calculation.

Step 4: Conclusion Statement I is correct and Statement II is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: When squared moduli are divided by the same complex differences, use $|w|^2/w = \bar{w}$ before expanding coordinates.

6. Let A and B be two square matrices of order 3 such that $|A| = 3$ and $|B| = 2$. Then

$$\left| A^T A (\text{adj}(2A))^{-1} (\text{adj}(4B)) (\text{adj}(AB))^{-1} A A^T \right|$$

is equal to :

[4 marks]

- (A) 32
- (B) 81
- (C) 64
- (D) 108

Correct Answer: (C) 64

Solution:

Concept:

- For order-three matrices, $\det(kM) = k^3 \det M$ and $\det(\text{adj } M) = (\det M)^2$. These powers depend on the matrix order.
- The determinant of a product is the product of determinants; $\det(M^T) = \det M$ and $\det(M^{-1}) = 1/\det M$. The given nonzero determinants ensure the inverses here exist.

Step 1: Evaluate the adjugate determinants First account for scalar multiplication before taking the adjugate:

$$\det(2A) = 2^3 \cdot 3 = 24, \quad \det(4B) = 4^3 \cdot 2 = 128.$$

Also $\det(AB) = \det A \det B = 6$. Hence

$$\det(\text{adj}(2A)) = 24^2, \quad \det(\text{adj}(4B)) = 128^2, \quad \det(\text{adj}(AB)) = 6^2.$$

All three values are nonzero, as required for the two inverse factors.

Step 2: Take the determinant of the full product Keep the matrix factors in their printed expression and take determinants one by one. The factors $A^T A$ and $A A^T$ each contribute $3^2 = 9$. The inverse adjugates contribute reciprocals. Thus the requested determinant is

$$D = 9 \cdot \frac{1}{24^2} \cdot 128^2 \cdot \frac{1}{6^2} \cdot 9.$$

There is no need to commute matrices or calculate their individual entries.

Step 3: Simplify the numerical expression Combine the squared quantities to reduce arithmetic:

$$D = \frac{81 \cdot 128^2}{24^2 \cdot 36} = \left(\frac{9 \cdot 128}{24 \cdot 6} \right)^2 = 8^2 = 64.$$

The choices 32, 81, and 108 do not include all scalar powers and inverse determinants correctly. In particular, scalar multiplication inside an order-three determinant contributes a cube, not a single scalar.

Step 4: Conclusion The determinant of the displayed product is 64. Hence, the correct option is (C).

Quick Tip to Solve: For a determinant of a long matrix product, replace each factor by its determinant first; handle scalar and adjugate powers separately before cancelling.

7. The value of $\int_{-\pi}^{\pi} \frac{2y(1 + \sin y)}{1 + \cos^2 y} dy$ is :

[4 marks]

- (A) $\frac{\pi}{2}$
- (B) π^2
- (C) $\frac{\pi^2}{2}$
- (D) $2\pi^2$

Correct Answer: (B) π^2

Solution:

Concept:

- An odd function integrates to zero over $[-a, a]$. Here $2y/(1 + \cos^2 y)$ is odd, while $y \sin y/(1 + \cos^2 y)$ is even.
- If $g(\pi - y) = g(y)$, then $\int_0^{\pi} yg(y) dy = (\pi/2) \int_0^{\pi} g(y) dy$, obtained by adding the original and reflected integrals.

Step 1: Separate the odd and even contributions Write the integral as the sum of the terms containing 1 and $\sin y$. The first vanishes by oddness, and the second is even. Consequently

$$I = 4 \int_0^{\pi} \frac{y \sin y}{1 + \cos^2 y} dy.$$

The denominator is always positive, so the integral is well defined throughout the interval. Notice that $y \sin y$ is even; discarding it as an odd term would be incorrect.

Step 2: Apply reflection symmetry Let $g(y) = \sin y/(1 + \cos^2 y)$ and $J = \int_0^{\pi} yg(y) dy$. Since $\sin(\pi - y) = \sin y$ and $\cos^2(\pi - y) = \cos^2 y$, substitution $y \mapsto \pi - y$ yields

$$J = \int_0^{\pi} (\pi - y)g(y) dy.$$

Adding gives

$$2J = \pi \int_0^{\pi} g(y) dy, \quad I = 2\pi \int_0^{\pi} \frac{\sin y}{1 + \cos^2 y} dy.$$

Step 3: Evaluate the remaining integral Put $u = \cos y$, so $du = -\sin y dy$. The endpoints become 1 and -1 , respectively. Therefore

$$\int_0^{\pi} \frac{\sin y}{1 + \cos^2 y} dy = \int_{-1}^1 \frac{du}{1 + u^2} = [\tan^{-1} u]_{-1}^1 = \frac{\pi}{2}.$$

It follows that $I = 2\pi \cdot \pi/2 = \pi^2$. The other choices miss either the even-interval factor or the reflection factor, or leave only one power of π .

Step 4: Conclusion The value of the integral is π^2 . Hence, the correct option is (B).

Quick Tip to Solve: For a factor y multiplied by a kernel symmetric about $\pi/2$, reflect y to $\pi - y$ and average before integrating.

8. Let $f(x) = x^5 + 2x^3 + 3x + 1, x \in \mathbf{R}$, and $g(x)$ be a function such that $g(f(x)) = x$ for all $x \in \mathbf{R}$. Then $\frac{g(7)}{g'(7)}$ is equal to :

[4 marks]

- (A) 42
- (B) 7
- (C) 1
- (D) 14

Correct Answer: (D) 14

Solution:

Concept:

- The identity $g(f(x)) = x$ means that g is the inverse of f on its range. Differentiating the identity gives $g'(f(x))f'(x) = 1$.
- To evaluate an inverse and its derivative at 7, first find the input a for which $f(a) = 7$; then $g(7) = a$ and $g'(7) = 1/f'(a)$.

Step 1: Find the preimage of 7 At $x = 1$, the polynomial gives $f(1) = 1 + 2 + 3 + 1 = 7$. Also,

$$f'(x) = 5x^4 + 6x^2 + 3 > 0$$

for every real x , so f is strictly increasing and this preimage is unique. Thus $g(7) = 1$.

Step 2: Differentiate and evaluate the ratio Apply the chain rule at $x = 1$:

$$g'(7)(5 + 6 + 3) = 1.$$

Hence $g'(7) = 1/14$, not 14. The required ratio is therefore

$$\frac{g(7)}{g'(7)} = \frac{1}{1/14} = 14.$$

The other values 42, 7 and 1 do not equal this ratio; in particular, 1 is only the inverse value, not the requested quotient.

Step 3: Conclusion The required value is 14. Hence, the correct option is (D).

Quick Tip to Solve: For $g = f^{-1}$, evaluate $g(y)/g'(y)$ as $af'(a)$, where $f(a) = y$.

9. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are chosen from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. The probability of this equation having repeated roots is : [4 marks]

- (A) $\frac{1}{128}$
(B) $\frac{1}{64}$
(C) $\frac{3}{256}$
(D) $\frac{3}{128}$

Correct Answer: (B) $\frac{1}{64}$

Solution:

Concept:

- A real quadratic $ax^2 + bx + c = 0$ has repeated roots exactly when its discriminant vanishes: $b^2 - 4ac = 0$. Here every allowed a is nonzero.
- The coefficients are chosen independently from eight values, so the equally likely sample space consists of $8^3 = 512$ ordered triples. Interchanging a and c usually gives a different triple.

Step 1: List the possible middle coefficients The condition is $ac = b^2/4$. Since ac is an integer, b must be even. For $b = 2, 4, 6, 8$, the products ac are respectively 1, 4, 9, 16. Odd values of b cannot contribute because their squares are not divisible by 4.

Step 2: Count ordered factor pairs Within the allowed set, the pairs are

$$\begin{aligned}b = 2 &: (1, 1), \\b = 4 &: (1, 4), (2, 2), (4, 1), \\b = 6 &: (3, 3), \\b = 8 &: (2, 8), (4, 4), (8, 2).\end{aligned}$$

The count is $1 + 3 + 1 + 3 = 8$, giving

$$P = \frac{8}{512} = \frac{1}{64}.$$

The alternatives correspond to 4, 6 or 12 favourable triples out of 512, whereas the explicit enumeration gives eight.

Step 3: Conclusion The probability is $\boxed{1/64}$. Hence, the correct option is (B).

Quick Tip to Solve: Count ordered factor pairs of $b^2/4$; do not merge (a, c) with (c, a) when they differ.

10. If the system of equations

$$11x + y + \lambda z = -5$$

$$2x + 3y + 5z = 3$$

$$8x - 19y - 39z = \mu$$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to :

[4 marks]

- (A) 51
- (B) 45
- (C) 47
- (D) 49

Correct Answer: (C) 47

Solution:

Concept:

- Infinitely many solutions require the coefficient matrix and augmented matrix to have the same rank, smaller than the number of unknowns. A dependent third equation must also have the matching right-hand side.
- The first two equations are independent since their x, y coefficient determinant is $11 \cdot 3 - 2 \cdot 1 = 31 \neq 0$. Thus it suffices to express the third equation as a linear combination of these two.

Step 1: Match the first two coefficients Write the third row as u times the first plus v times the second. Matching x and y gives

$$11u + 2v = 8, \quad u + 3v = -19.$$

From $u = -19 - 3v$, the first equation gives $-209 - 31v = 8$, so $v = -7$ and $u = 2$.

Step 2: Enforce the same dependence in the remaining entries The z coefficient and right-hand side must obey

$$2\lambda - 7(5) = -39, \quad \mu = 2(-5) - 7(3).$$

Therefore $\lambda = -2$ and $\mu = -31$. The third equation is then exactly twice the first minus seven times the second, leaving one free parameter. Finally,

$$\lambda^4 - \mu = (-2)^4 - (-31) = 16 + 31 = 47.$$

The other numerical choices fail this forced pair of parameter values.

Step 3: Conclusion The required value is 47. Hence, the correct option is (C).

Quick Tip to Solve: When two equations are independent, match the third row using two coefficients first, then impose that same combination on its constant.

11. The integral $\int_0^{\pi/4} \frac{136 \sin x}{3 \sin x + 5 \cos x} dx$ is equal to :

[4 marks]

- (A) $3\pi - 50 \log_e 2 + 20 \log_e 5$
- (B) $3\pi - 25 \log_e 2 + 10 \log_e 5$
- (C) $3\pi - 10 \log_e(2\sqrt{2}) + 10 \log_e 5$
- (D) $3\pi - 30 \log_e 2 + 20 \log_e 5$

Correct Answer: (A) $3\pi - 50 \log_e 2 + 20 \log_e 5$

Solution:

Concept:

- For a denominator $D(x)$, writing the numerator as $AD(x) + BD'(x)$ separates the integrand into a constant and a logarithmic derivative.
- Here $D(x) = 3 \sin x + 5 \cos x$ and $D'(x) = 3 \cos x - 5 \sin x$. The denominator is positive throughout $[0, \pi/4]$, so the logarithm has no singularity on this interval.

Step 1: Decompose the numerator Choose constants A, B so that

$$136 \sin x = A(3 \sin x + 5 \cos x) + B(3 \cos x - 5 \sin x).$$

Matching sine and cosine coefficients gives $3A - 5B = 136$ and $5A + 3B = 0$. Solving yields $A = 12$, $B = -20$; direct substitution gives $36 + 100 = 136$ and $60 - 60 = 0$.

Step 2: Integrate and evaluate the bounds Thus

$$I = [12x - 20 \log_e(3 \sin x + 5 \cos x)]_0^{\pi/4}.$$

The denominator at the upper bound is $8/\sqrt{2} = 4\sqrt{2}$, and at the lower bound it is 5. Therefore

$$\begin{aligned} I &= 3\pi - 20 \log_e(4\sqrt{2}) + 20 \log_e 5 \\ &= 3\pi - 50 \log_e 2 + 20 \log_e 5. \end{aligned}$$

In particular, $\log_e(4\sqrt{2}) = \frac{5}{2} \log_e 2$; missing this factor gives incorrect logarithmic coefficients in the other alternatives.

Step 3: Conclusion The integral equals $3\pi - 50 \log_e 2 + 20 \log_e 5$. Hence, the correct option is (A).

Quick Tip to Solve: For a quotient of linear combinations of sine and cosine, express the numerator using the denominator and its derivative.

12. If $\frac{1}{\sqrt{1+\sqrt{2}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + \dots + \frac{1}{\sqrt{99+\sqrt{100}}} = m$ and $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{99 \cdot 100} = n$, then the point (m, n) lies on the line [4 marks]

- (A) $11(x - 1) - 100y = 0$
 (B) $11x - 100y = 0$
 (C) $11(x - 1) - 100(y - 2) = 0$
 (D) $11(x - 2) - 100(y - 1) = 0$

Correct Answer: (B) $11x - 100y = 0$

Solution:

Concept:

- Rationalising consecutive square roots gives $1/(\sqrt{k} + \sqrt{k+1}) = \sqrt{k+1} - \sqrt{k}$. Summing these differences leaves only endpoint terms.
- The identity $1/[k(k+1)] = 1/k - 1/(k+1)$ telescopes similarly. A point lies on a line if substituting its coordinates makes the line equation zero.

Step 1: Evaluate both sums For the first sum, rationalisation cancels the denominator because $(k+1) - k = 1$. Consequently,

$$m = \sum_{k=1}^{99} (\sqrt{k+1} - \sqrt{k}) = \sqrt{100} - \sqrt{1} = 9.$$

The intermediate square roots cancel pairwise. Likewise,

$$n = \sum_{k=1}^{99} \left(\frac{1}{k} - \frac{1}{k+1} \right) = 1 - \frac{1}{100} = \frac{99}{100}.$$

Step 2: Substitute the point in the proposed lines For $(x, y) = (9, 99/100)$,

$$11x - 100y = 99 - 99 = 0.$$

Hence the second line contains the point. The left sides of the first, third and fourth proposed equations evaluate respectively to -11 , 189 and 78 , all nonzero. This also distinguishes the shifted lines from the unshifted relation rather than relying only on their similar coefficients.

Step 3: Conclusion The point lies on $11x - 100y = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Before adding many reciprocal terms, look for a conjugate or a partial-fraction difference that leaves only the first and last terms.

13. Let d be the distance of the point of intersection of the lines $\frac{x+6}{3} = \frac{y}{2} = \frac{z+1}{1}$ and $\frac{x-7}{4} = \frac{y-9}{3} = \frac{z-4}{2}$ from the point $(7, 8, 9)$. Then $d^2 + 6$ is equal to : [4 marks]

- (A) 75
- (B) 78
- (C) 72
- (D) 69

Correct Answer: (A) 75

Solution:

Concept:

- Each symmetric line equation can be parametrised by setting its common ratio equal to a separate parameter. An intersection must satisfy all three coordinate equations simultaneously.
- The squared distance between (x_1, y_1, z_1) and (x_2, y_2, z_2) is $(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$. Computing the square directly avoids an unnecessary square root.

Step 1: Find and verify the intersection Use parameters t, s for the first and second lines:

$$P = (-6 + 3t, 2t, -1 + t), \quad P = (7 + 4s, 9 + 3s, 4 + 2s).$$

Equating the z coordinates gives $t = 5 + 2s$. Equating the y coordinates then gives $10 + 4s = 9 + 3s$, so $s = -1$ and $t = 3$. Both x formulas give 3, verifying an actual intersection $P = (3, 6, 2)$.

Step 2: Compute the requested expression From P to $Q = (7, 8, 9)$, the coordinate differences are 4, 2, 7. Thus

$$d^2 = 4^2 + 2^2 + 7^2 = 16 + 4 + 49 = 69.$$

The question asks for $d^2 + 6$, so its value is $69 + 6 = 75$. The choice 69 is only the squared distance, while 72 and 78 do not follow from these verified coordinates.

Step 3: Conclusion The required value is 75. Hence, the correct option is (A).

Quick Tip to Solve: After solving two coordinate equations for line parameters, check the third; skew lines can satisfy two coordinates without intersecting.

14. If the function $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$, $x \in \mathbf{R}$, is continuous at $x = 0$, then $f(0)$ is equal to :

[4 marks]

- (A) 4
- (B) -2
- (C) -4
- (D) 2

Correct Answer: (C) -4

Solution:

Concept:

- Continuity at zero requires a finite two-sided limit of the expression for nonzero x , equal to the assigned value $f(0)$. Lower-order nonzero numerator terms would make division by x^3 divergent.
- The exact triple-angle identity $\sin 3x = 3 \sin x - 4 \sin^3 x$, together with $\lim_{x \rightarrow 0} \sin x/x = 1$, evaluates the remaining cubic term without a power-series calculation.

Step 1: Determine the parameter values required for a finite limit The numerator tends to $-\beta$ as $x \rightarrow 0$. Therefore a finite limit first requires $\beta = 0$. With this value,

$$\sin 3x + \alpha \sin x = (3 + \alpha) \sin x - 4 \sin^3 x.$$

The first term divided by x^3 behaves as $(3 + \alpha)/x^2$, so finiteness also requires $\alpha = -3$.

Step 2: Evaluate the continuous extension With $\beta = 0$ and $\alpha = -3$, the formula for $x \neq 0$ becomes

$$f(x) = -4 \left(\frac{\sin x}{x} \right)^3.$$

Hence

$$f(0) = \lim_{x \rightarrow 0} f(x) = -4(1)^3 = -4.$$

The positive answer 4 has the wrong sign from the triple-angle identity, while the magnitudes 2 do not equal the cubic coefficient. The expression at zero is understood as the continuous extension required in the question.

Step 3: Conclusion The continuous value is $f(0) = -4$. Hence, the correct option is (C).

Quick Tip to Solve: For $\sin 3x - 3 \sin x$, use the exact identity $-4 \sin^3 x$ before applying any limit.

15. Let the line $2x + 3y - k = 0$, $k > 0$, intersect the x -axis and y -axis at the points A and B, respectively. If the equation of the circle having the line segment AB as a diameter is $x^2 + y^2 - 3x - 2y = 0$ and the length of the latus rectum of the ellipse $x^2 + 9y^2 = k^2$ is $\frac{m}{n}$, where m and n are coprime, then $2m + n$ is equal to [4 marks]

- (A) 12
(B) 13
(C) 11
(D) 10

Correct Answer: (C) 11

Solution:

Concept:

- The circle with endpoints $(u, 0)$ and $(0, v)$ of a diameter has equation $x(x - u) + y(y - v) = 0$. Its linear coefficients therefore identify the two intercepts.
- For an ellipse $x^2/a^2 + y^2/b^2 = 1$, with $a > b > 0$, the full length of a latus rectum is $2b^2/a$. The semi-major axis, rather than its square, occurs in the denominator.

Step 1: Determine the positive line constant The intercepts of $2x + 3y - k = 0$ are $A = (k/2, 0)$ and $B = (0, k/3)$. Hence its diameter circle is

$$x^2 + y^2 - \frac{k}{2}x - \frac{k}{3}y = 0.$$

Comparison with the printed equation gives $k/2 = 3$ and $k/3 = 2$. Both conditions consistently give $k = 6$, which also satisfies the stated positivity condition.

Step 2: Write the ellipse in standard form Substituting $k = 6$, the ellipse becomes

$$x^2 + 9y^2 = 36 \implies \frac{x^2}{36} + \frac{y^2}{4} = 1.$$

Thus $a = 6$ and $b = 2$, and its major axis lies along the x -axis. These are semi-axis lengths; using 36 or 4 as lengths would give an incorrect result.

Step 3: Reduce the latus-rectum length The full length is

$$\frac{2b^2}{a} = \frac{2(4)}{6} = \frac{4}{3}.$$

Since 4 and 3 are coprime, $m = 4$ and $n = 3$. Therefore $2m + n = 2(4) + 3 = 11$. The alternatives 12, 13 and 10 do not equal this uniquely determined integer; the reduced fraction fixes both integers without any further choice.

Step 4: Conclusion The required value is $\boxed{11}$. Hence, the correct option is (C).

Quick Tip to Solve: For a diameter joining the axes, read its intercepts directly from the circle's linear coefficients before computing the ellipse parameters.

16. Let a circle C of radius 1 and closer to the origin be such that the lines passing through the point $(3, 2)$ and parallel to the coordinate axes touch it. Then the shortest distance of the circle C from the point $(5, 5)$ is :

[4 marks]

- (A) $4\sqrt{2}$
- (B) 4
- (C) 5
- (D) $2\sqrt{2}$

Correct Answer: (B) 4

Solution:

Concept:

- A circle tangent to a line has its centre at perpendicular distance equal to the radius from that line. Here the two tangent lines are $x = 3$ and $y = 2$.
- The minimum distance from an external point to a circle equals its distance from the centre minus the radius. Select the circle nearest the origin before applying this formula.

Step 1: Find all possible centres Write the centre as (h, k) . Since the radius is 1, tangency to $x = 3$ requires $|h - 3| = 1$, while tangency to $y = 2$ requires $|k - 2| = 1$. Therefore $h = 2$ or 4 and $k = 1$ or 3 . The four possible centres are $(2, 1), (2, 3), (4, 1), (4, 3)$; no other centre satisfies both distance conditions.

Step 2: Choose the circle closer to the origin The squared distances of these centres from the origin are respectively

$$5, \quad 13, \quad 17, \quad 25.$$

Every circle has the same unit radius and the origin lies outside each one. Their nearest distances from the origin are consequently $\sqrt{5} - 1, \sqrt{13} - 1, \sqrt{17} - 1, 4$. The unique nearest circle has centre $O = (2, 1)$. Comparing centre distances is legitimate precisely because their radii are equal.

Step 3: Calculate the shortest distance from the given point For $P = (5, 5)$,

$$OP = \sqrt{(5 - 2)^2 + (5 - 1)^2} = \sqrt{9 + 16} = 5.$$

The closest point of the circle lies on the ray from O toward P , one unit from O . Thus the required distance is $5 - 1 = 4$. Option (C) is the centre distance rather than the distance to the circumference, and the two multiples of $\sqrt{2}$ do not follow from the unequal coordinate differences 3 and 4.

Step 4: Conclusion The shortest distance is 4 units. Hence, the correct option is (B).

Quick Tip to Solve: First use the two tangent lines to list the four centres; then select the nearest circle and subtract its radius from the point-to-centre distance.

17. Suppose $\theta \in [0, \frac{\pi}{4}]$ is a solution of $4 \cos \theta - 3 \sin \theta = 1$. Then $\cos \theta$ is equal to :

[4 marks]

(A) $\frac{6-\sqrt{6}}{(3\sqrt{6}-2)}$

(B) $\frac{4}{(3\sqrt{6}+2)}$

(C) $\frac{6+\sqrt{6}}{(3\sqrt{6}+2)}$

(D) $\frac{4}{(3\sqrt{6}-2)}$

Correct Answer: (D) $\frac{4}{(3\sqrt{6}-2)}$

Solution:

Concept:

- The identity $\sin^2 \theta + \cos^2 \theta = 1$ turns a linear combination of sine and cosine into a quadratic after one trigonometric function is isolated.
- On $[0, \pi/4]$, both sine and cosine are nonnegative, and $1/\sqrt{2} \leq \cos \theta \leq 1$. Any root introduced by squaring must satisfy the original equation and this interval.

Step 1: Form a quadratic for the cosine Put $c = \cos \theta$. The given relation gives $3 \sin \theta = 4c - 1$. Squaring and using $\sin^2 \theta = 1 - c^2$,

$$9(1 - c^2) = (4c - 1)^2,$$

$$25c^2 - 8c - 8 = 0.$$

Consequently

$$c = \frac{8 \pm \sqrt{864}}{50} = \frac{4 \pm 6\sqrt{6}}{25}.$$

This step produces two algebraic candidates, so the sign cannot be chosen arbitrarily.

Step 2: Select and check the admissible root The root with the minus sign is negative and cannot be a cosine in the stated interval. The positive root is approximately 0.748 and lies between $1/\sqrt{2}$ and 1. It also gives $\sin \theta = (4c - 1)/3 > 0$. Together with the quadratic identity this recovers the original relation, so it is not an extraneous solution caused by squaring.

Step 3: Match the exact printed option Rationalise option (D):

$$\frac{4}{3\sqrt{6}-2} = \frac{4(3\sqrt{6}+2)}{54-4} = \frac{6\sqrt{6}+4}{25}.$$

This is exactly the admissible root. For comparison, options (A), (B) and (C) have approximate values 0.664, 0.428 and 0.904, respectively. The first two violate the allowed cosine range, while the third does not equal the root of the derived quadratic.

Step 4: Conclusion $\cos \theta = \frac{4}{3\sqrt{6}-2}$. Hence, the correct option is (D).

Quick Tip to Solve: After squaring a trigonometric equation, use the given angle interval to discard impossible roots before rationalising the option expressions.

18. For the function

$$f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x), \text{ where } x \in \left[0, \frac{\pi}{2}\right],$$

consider the following two statements :

(I) f is increasing in $(0, \frac{\pi}{2})$.

(II) f' is decreasing in $(0, \frac{\pi}{2})$.

Between the above two statements,

[4 marks]

(A) only (I) is true.

(B) both (I) and (II) are true.

(C) only (II) is true.

(D) neither (I) nor (II) is true.

Correct Answer: (B) both (I) and (II) are true.

Solution:

Concept:

- A differentiable function is strictly increasing throughout an interval if its derivative is positive there. To decide whether the derivative itself decreases, inspect the second derivative.
- On $(0, \pi/2)$, $\sin x > 0$, so a negative constant added to $-\sin x$ remains negative. Endpoint values can bound a continuous monotone derivative.

Step 1: Differentiate the function twice From the printed expression,

$$f'(x) = \cos x + 3 - \frac{2}{\pi}(2x + 1),$$

$$f''(x) = -\sin x - \frac{4}{\pi}.$$

Since $\sin x > 0$ on the open interval and $4/\pi > 0$, we have $f''(x) < 0$ everywhere there. Thus f' is strictly decreasing, proving statement (II).

Step 2: Find the smallest limiting derivative A decreasing derivative takes its lowest limiting value at the right endpoint. The function is smooth on the closed interval, and

$$f'(\pi/2) = 0 + 3 - \frac{2}{\pi}(\pi + 1) = 1 - \frac{2}{\pi} > 0,$$

because $\pi > 2$. Therefore every interior value satisfies $f'(x) > 1 - 2/\pi > 0$. It is essential to check this sign: a decreasing derivative need not itself be negative.

Step 3: Decide both statements The positivity of f' proves that f is strictly increasing on $(0, \pi/2)$, so statement (I) is true as well. Equivalently, for any two distinct points in the interval, the mean value theorem gives a positive change in f from left to right. Options (A) and (C) each discard one established property, and option (D) contradicts both derivative tests.

Step 4: Conclusion Both statements (I) and (II) are true. Hence, the correct option is (B).

Quick Tip to Solve: For two monotonicity claims about a function and its derivative, use the first and second derivatives separately; a decreasing derivative may stay positive.

19. If $A(1, -1, 2)$, $B(5, 7, -6)$, $C(3, 4, -10)$ and $D(-1, -4, -2)$ are the vertices of a quadrilateral ABCD, then its area is :

[4 marks]

- (A) $24\sqrt{7}$
- (B) $48\sqrt{7}$
- (C) $24\sqrt{29}$
- (D) $12\sqrt{29}$

Correct Answer: (D) $12\sqrt{29}$

Solution:

Concept:

- Two adjacent side vectors determine a parallelogram. Its area is the magnitude of their cross product, whereas a triangle with those sides has half that area.
- For vertices given in order, verify that opposite sides are parallel and equal before treating the quadrilateral as a parallelogram. Cross products also establish the common plane.

Step 1: Identify the adjacent side vectors Take the two sides starting at A:

$$\overrightarrow{AB} = (4, 8, -8), \quad \overrightarrow{AD} = (-2, -3, -4).$$

Also $\overrightarrow{BC} = C - B = (-2, -3, -4) = \overrightarrow{AD}$ and $\overrightarrow{DC} = C - D = (4, 8, -8) = \overrightarrow{AB}$. Thus the vertices do form a parallelogram in the stated order, so its entire area is one cross-product magnitude.

Step 2: Evaluate the cross product Using the determinant expansion,

$$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 8 & -8 \\ -2 & -3 & -4 \end{vmatrix}.$$

Its components are

$$8(-4) - (-8)(-3) = -56,$$

$$(-8)(-2) - 4(-4) = 32,$$

$$4(-3) - 8(-2) = 4.$$

Therefore the area vector is $-56\mathbf{i} + 32\mathbf{j} + 4\mathbf{k}$. The middle component includes the alternating determinant sign.

Step 3: Take the vector magnitude The required scalar area is

$$\sqrt{(-56)^2 + 32^2 + 4^2} = \sqrt{3136 + 1024 + 16} = \sqrt{4176} = 12\sqrt{29}.$$

A factor of one half would give a triangle rather than the whole quadrilateral. Option (C) doubles this area, and options (A) and (B) have the wrong radical. Reversing the order of the two vectors would change the sign of the area vector but leave its magnitude unchanged.

Step 4: Conclusion The area is $12\sqrt{29}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Check opposite-side vectors first; once a quadrilateral is a parallelogram, take the full cross-product magnitude without a triangle factor of one half.

20. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + 2y = \sin(2x)$, $y(0) = \frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$ is equal to : [4 marks]

- (A) $e^{-\pi/8}$
- (B) $e^{\pi/4}$
- (C) $e^{-\pi/4}$
- (D) $e^{\pi/8}$

Correct Answer: (C) $e^{-\pi/4}$

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, the integrating factor is $\exp(\int P(x) dx)$, turning the left side into a product derivative.
- The initial condition determines the integration constant. Keep the complementary term Ce^{-2x} when a particular solution is used; the forcing term alone does not determine the solution.

Step 1: Multiply by the integrating factor Here $P(x) = 2$, so the integrating factor is e^{2x} . Multiplication gives

$$\frac{d}{dx}(e^{2x}y) = e^{2x} \sin(2x).$$

Integration yields

$$e^{2x}y = \frac{e^{2x}}{4}(\sin 2x - \cos 2x) + C,$$

and hence $y = (\sin 2x - \cos 2x)/4 + Ce^{-2x}$. The exponential-trigonometric integral can be checked by differentiating its right-hand side.

Step 2: Apply the prescribed initial value At $x = 0$, the sine term is zero, the cosine term is one, and the exponential is one. Therefore

$$\frac{3}{4} = -\frac{1}{4} + C \implies C = 1.$$

Thus

$$y(x) = \frac{\sin 2x - \cos 2x}{4} + e^{-2x}.$$

As a direct check, the derivative of the trigonometric term plus twice that term is $\sin 2x$, while e^{-2x} contributes zero to $y' + 2y$.

Step 3: Substitute the requested argument At $x = \pi/8$, we have $2x = \pi/4$, where sine and cosine are equal. The trigonometric contribution consequently vanishes, leaving

$$y(\pi/8) = e^{-2(\pi/8)} = e^{-\pi/4}.$$

The positive exponents in (B) and (D) conflict with the decay factor of the complementary solution. Option (A) omits the coefficient 2 multiplying x in that exponential.

Step 4: Conclusion $y(\pi/8) = e^{-\pi/4}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep the particular and complementary solutions separate; at the requested angle the sine and cosine terms cancel, so only the initial-condition term remains.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The area of the region enclosed by the parabolas $y = x^2 - 5x$ and $y = 7x - x^2$ is _____. [4 marks]

Solution:

Concept:

- The area between two curves is the integral of upper ordinate minus lower ordinate between their intersection abscissae. The integrand must be nonnegative over that interval.
- Equating the two quadratic expressions gives the limits. A numerical answer should follow the printed equations exactly; the width and height of the enclosed lens provide an independent scale check.

Step 1: Find the intersection limits Equate the printed parabolas:

$$x^2 - 5x = 7x - x^2 \implies 2x^2 - 12x = 0.$$

Thus $2x(x - 6) = 0$, giving intersections at $x = 0$ and $x = 6$. Their corresponding ordinates are 0 and 6. These are the only roots, so there is a single bounded region between the curves.

Step 2: Determine the upper boundary On $0 < x < 6$,

$$(7x - x^2) - (x^2 - 5x) = 12x - 2x^2 = 2x(6 - x) > 0.$$

Hence $y = 7x - x^2$ lies above $y = x^2 - 5x$ throughout the region. It is the vertical separation, rather than either individual ordinate or its distance from an axis, that must be integrated. No splitting at the roots of the separate parabolas is needed.

Step 3: Integrate the positive separation The enclosed area is

$$A = \int_0^6 (12x - 2x^2) dx = \left[6x^2 - \frac{2}{3}x^3 \right]_0^6.$$

Substitution gives

$$A = 216 - 144 = 72.$$

For a scale check, the separation is $18 - 2(x - 3)^2$, with width 6 and maximum height 18. The area 72 is smaller than the enclosing rectangle area $6 \times 18 = 108$, as required. A value 198 would even exceed that bounding rectangle and cannot represent this printed region.

Step 4: Conclusion The area enclosed by the printed parabolas is 72 square units. The supplied answer key lists 198, but that value disagrees with the printed equations; direct integration and the bounding-rectangle check both give 72.

Quick Tip to Solve: Always subtract the lower curve from the upper curve between their intersections; compare the result with width times maximum separation to catch an impossible area.

22. From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the variance of X is σ^2 , then $96\sigma^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Sampling is without replacement. Hence the number of defective items follows a hypergeometric distribution with population size $N = 10$, defective count $K = 3$, and sample size $n = 5$.
- For this distribution, $\text{Var}(X) = np(1-p)(N-n)/(N-1)$, where $p = K/N$. The final factor accounts for dependence between draws.

Step 1: Identify the parameters The probability that any specified draw is defective is $p = 3/10$, while the probability of a non-defective item is $7/10$. The population contains only ten items and none is replaced. Consequently the independent-trial binomial variance cannot be used directly. Here the finite-population correction is $(10-5)/(10-1) = 5/9$.

Step 2: Compute and scale the variance Substituting the three counts gives

$$\sigma^2 = 5 \left(\frac{3}{10} \right) \left(\frac{7}{10} \right) \frac{5}{9} = \frac{525}{900} = \frac{7}{12}.$$

As a check on the correction, if the whole population were sampled, its factor would become zero and the defective count would be fixed. For the present sample the mean is $np = 3/2$, and the variance is positive, as expected. Therefore

$$96\sigma^2 = 96 \cdot \frac{7}{12} = 56.$$

Step 3: Conclusion The required numerical value is 56.

Quick Tip to Solve: For a sample drawn without replacement, multiply the binomial variance by $(N-n)/(N-1)$; omitting this correction overestimates the variance.

23. Suppose AB is a focal chord of the parabola $y^2 = 12x$ of length l and slope $m < \sqrt{3}$. If the distance of the chord AB from the origin is d , then $l d^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The standard parabola $y^2 = 4ax$ has focus $(a, 0)$; here $a = 3$, so every focal chord lies on a line through $(3, 0)$.
- For endpoints on a line of slope m , the chord length is the difference in their x -coordinates multiplied by $\sqrt{1+m^2}$. Use the point-to-line formula for its distance from the origin.

Step 1: Find the chord length from the intersection equation The chord line is $y = m(x-3)$. A finite nondegenerate focal chord has $m \neq 0$, since the axis meets this parabola at only one finite point. Substitution gives

$$m^2 x^2 - (6m^2 + 12)x + 9m^2 = 0.$$

Its roots are the endpoint abscissae. The discriminant is

$$(6m^2 + 12)^2 - 36m^4 = 144(1 + m^2).$$

Thus $|x_B - x_A| = 12\sqrt{1 + m^2}/m^2$, and

$$l = \frac{12(1 + m^2)}{m^2}.$$

Step 2: Calculate the perpendicular distance and cancel the slope Writing the line as $mx - y - 3m = 0$, its perpendicular distance from $(0, 0)$ is

$$d = \frac{3|m|}{\sqrt{1 + m^2}}, \quad d^2 = \frac{9m^2}{1 + m^2}.$$

Therefore

$$ld^2 = \frac{12(1 + m^2)}{m^2} \cdot \frac{9m^2}{1 + m^2} = 108.$$

The cancellation shows that the value is independent of the admissible slope, so the stated bound on m does not alter this product.

Step 3: Conclusion The required value of ld^2 is $\boxed{108}$.

Quick Tip to Solve: For $y^2 = 4ax$, a focal chord satisfies $ld^2 = 4a^3$; derive it from the focus-line equation when the slope is given.

24. The number of ways of getting a sum 16 on throwing a dice four times is _____. [4 marks]

Solution:

Concept:

- Four successive throws are distinguishable: an outcome is an ordered quadruple (x_1, x_2, x_3, x_4) , where $1 \leq x_i \leq 6$. We must count those with total sixteen.
- Stars and bars counts nonnegative integer solutions. Inclusion-exclusion removes solutions that exceed the upper bound imposed by the six faces.

Step 1: Shift the face values to nonnegative integers Set $y_i = x_i - 1$. Then each y_i is an integer from zero through five, and

$$y_1 + y_2 + y_3 + y_4 = 16 - 4 = 12.$$

Without the upper bounds, stars and bars gives

$$\binom{12 + 4 - 1}{4 - 1} = \binom{15}{3} = 455.$$

These are ordered assignments, matching the order of the four throws; there is no division by a factorial.

Step 2: Exclude values above five For a particular variable with $y_i \geq 6$, subtract six from it. The

remaining sum is six, so there are $\binom{9}{3} = 84$ assignments. There are four choices of this variable. If two specified variables are at least six, subtracting six from each leaves sum zero, giving exactly one assignment. There are $\binom{4}{2} = 6$ such pairs. Three variables cannot each be at least six because their sum would exceed twelve. Hence the exact count is

$$455 - 4(84) + 6(1) = 455 - 336 + 6 = 125.$$

Pair intersections are added back because each was removed twice in the previous subtraction.

Step 3: Conclusion The number of ordered outcomes is $\boxed{125}$.

Quick Tip to Solve: After shifting die values to $0, \dots, 5$, a violation starts at six, not five. Use that threshold in inclusion-exclusion.

25. Let f be a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \rightarrow x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $2f(2) + 3f(3)$ is equal to _____. [4 marks]

Solution:

Concept:

- For fixed x , rewrite the numerator using differences in t . Differentiability gives $\lim_{t \rightarrow x} (f(t) - f(x))/(t - x) = f'(x)$.
- The resulting first-order linear differential equation becomes a direct derivative after division by x^4 : $(f(x)/x^2)' = f'(x)/x^2 - 2f(x)/x^3$. The domain $x > 0$ permits these divisions.

Step 1: Convert the limit to a differential equation Split the numerator as

$$t^2 f(x) - x^2 f(t) = (t^2 - x^2)f(x) - x^2(f(t) - f(x)).$$

Divide by $t - x$ and take the limit. Since $(t^2 - x^2)/(t - x) = t + x$, the stated condition becomes

$$2xf(x) - x^2 f'(x) = 1.$$

Rearranging and dividing by x^4 therefore gives

$$\left(\frac{f(x)}{x^2}\right)' = -\frac{1}{x^4}.$$

Step 2: Integrate and apply the initial value Integration on $(0, \infty)$ yields

$$\frac{f(x)}{x^2} = \frac{1}{3x^3} + C, \quad f(x) = \frac{1}{3x} + Cx^2.$$

The condition $f(1) = 1$ gives $C = 2/3$. This expression also satisfies $2xf - x^2 f' = 1$ upon substitution. Now

$$f(2) = \frac{1}{6} + \frac{8}{3} = \frac{17}{6}, \quad f(3) = \frac{1}{9} + 6 = \frac{55}{9}.$$

Consequently

$$2f(2) + 3f(3) = \frac{17}{3} + \frac{55}{3} = 24.$$

Step 3: Conclusion The required numerical value is $\boxed{24}$.

Quick Tip to Solve: Before applying a limit rule, split the numerator into a polynomial difference and a function difference. The signs then determine the correct differential equation.

26. If $S = \{a \in \mathbf{R} : |2a - 1| = 3[a] + 2\{a\}\}$, where $[t]$ denotes the greatest integer less than or equal to t and $\{t\}$ represents the fractional part of t , then $72 \sum_{a \in S} a$ is equal to _____. [4 marks]

Solution:

Concept:

- Write $n = [a]$ and $r = \{a\}$, so that $a = n + r$, $n \in \mathbb{Z}$, and $0 \leq r < 1$. Thus $3[a] + 2\{a\} = 2a + n$.
- The absolute value changes form at $a = 1/2$. Solve both cases and retain only candidates satisfying both the case condition and $n \leq a < n + 1$.

Step 1: Rule out the upper branch If $a \geq 1/2$, the defining equation becomes

$$2a - 1 = 2a + n, \quad n = -1.$$

But $[a] = -1$ requires $-1 \leq a < 0$, contradicting $a \geq 1/2$. Therefore no element of S occurs in this branch, including the boundary $a = 1/2$. The floor constraint is essential; solving the absolute-value equation alone is insufficient.

Step 2: Solve the lower branch with the floor constraint For $a < 1/2$, the equation is

$$1 - 2a = 2a + n, \quad a = \frac{1 - n}{4}.$$

The requirement that n is exactly the floor of a gives

$$n \leq \frac{1 - n}{4} < n + 1.$$

The left inequality implies $n \leq 1/5$, and the right implies $n > -3/5$. The only integer in this interval is $n = 0$, yielding $a = 1/4$. This value satisfies $a < 1/2$; directly, $|2a - 1| = 1/2 = 3(0) + 2(1/4)$. Hence $S = \{1/4\}$, and

$$72 \sum_{a \in S} a = 72 \cdot \frac{1}{4} = 18.$$

Step 3: Conclusion The requested numerical value is $\boxed{18}$.

Quick Tip to Solve: Always impose $n \leq a < n + 1$ after writing $n = [a]$. This excludes algebraic candidates that have the wrong integer part.

27. Let $a_1, a_2, a_3, \dots$ be in an arithmetic progression of positive terms.

Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \dots + a_{2k-1}^2 - a_{2k}^2$.

If $A_3 = -153$, $A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to _____.

[4 marks]

Solution:

Concept:

- Write $a_j = a + (j - 1)d$. Pair consecutive squared terms in A_k , using $u^2 - v^2 = (u - v)(u + v)$.
- The statement gives an arithmetic progression of positive terms throughout. A negative common difference would eventually give negative terms, so the admissible common difference is nonnegative.

Step 1: Obtain a formula for the alternating sum The j -th pair is

$$a_{2j-1}^2 - a_{2j}^2 = -d(2a + (4j - 3)d).$$

Summing for $j = 1, \dots, k$, and using $\sum_{j=1}^k (4j - 3) = k(2k - 1)$, gives

$$A_k = -kd(2a + (2k - 1)d).$$

Thus $A_3 = -153$ and $A_5 = -435$ imply

$$d(2a + 5d) = 51, \quad d(2a + 9d) = 87.$$

Step 2: Find the progression and evaluate the requested difference Subtracting the two equations gives $4d^2 = 36$, so $d^2 = 9$. Positivity of the progression selects $d = 3$. Then $3(2a + 15) = 51$, yielding $a = 1$. As an independent check, the first three terms are 1, 4, 7, whose squared sum is $1 + 16 + 49 = 66$, exactly the third condition. Now

$$a_{17} = 1 + 16(3) = 49,$$

$$A_7 = -7(3)(2 + 13(3)) = -21(41) = -861.$$

Finally, subtract the negative alternating sum:

$$a_{17} - A_7 = 49 - (-861) = 910.$$

Step 3: Conclusion The required value is 910.

Quick Tip to Solve: Pair adjacent squared terms before summing. Each pair has a common factor $-d$, turning the alternating-square sum into an ordinary arithmetic sum.

28. Let $\vec{a} = \hat{i} - 3\hat{j} + 7\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + 2\vec{b}) \times \vec{c} = 3(\vec{c} \times \vec{a})$. If $\vec{a} \cdot \vec{c} = 130$, then $\vec{b} \cdot \vec{c}$ is equal to _____.

[4 marks]

Solution:

Concept:

- The cross product is anti-commutative: $\vec{c} \times \vec{a} = -\vec{a} \times \vec{c}$. Hence all terms in the given relation can be collected with $\vec{c}$ as the second factor.
- If a nonzero vector has zero cross product with another vector, the two are parallel. Dot products then determine the proportionality constant.

Step 1: Identify the direction of the unknown vector Using anti-commutativity and distributivity,

$$(\vec{a} + 2\vec{b}) \times \vec{c} = -3\vec{a} \times \vec{c} \implies (4\vec{a} + 2\vec{b}) \times \vec{c} = \vec{0}.$$

Therefore $(2\vec{a} + \vec{b}) \times \vec{c} = \vec{0}$. The vector $2\vec{a} + \vec{b} = 4\hat{i} - 7\hat{j} + 15\hat{k}$ is nonzero, so write

$$\vec{c} = \lambda(2\vec{a} + \vec{b}).$$

The given nonzero dot product also rules out $\vec{c} = \vec{0}$.

Step 2: Use scalar products to determine the required value The needed products are

$$\vec{a} \cdot \vec{a} = 1 + 9 + 49 = 59, \quad \vec{a} \cdot \vec{b} = 2 + 3 + 7 = 12.$$

Thus

$$130 = \vec{a} \cdot \vec{c} = \lambda(2 \cdot 59 + 12) = 130\lambda,$$

so $\lambda = 1$. Also $\vec{b} \cdot \vec{b} = 4 + 1 + 1 = 6$. It follows that

$$\vec{b} \cdot \vec{c} = 2(\vec{b} \cdot \vec{a}) + \vec{b} \cdot \vec{b} = 2(12) + 6 = 30.$$

One may check directly with $\vec{c} = (4, -7, 15)$: its dot products with $\vec{a}$ and $\vec{b}$ are respectively 130 and 30, confirming both the constraint and the result.

Step 3: Conclusion The required scalar product is 30.

Quick Tip to Solve: Move cross products to a common side before expanding components. Zero cross product usually exposes the direction of the unknown vector immediately.

29. If the constant term in the expansion of $(1 + 2x - 3x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is p, then 108p is equal to _____.

[4 marks]

Solution:

Concept:

- The general term in a binomial expansion is obtained by choosing the second term a fixed number of times. We must collect the coefficient of x^0 only.

- Multiplication by 1, $2x$ and $-3x^3$ shifts powers by 0, 1 and 3, respectively. Thus the required binomial coefficients are those of x^0, x^{-1}, x^{-3} .

Step 1: Write the general term If the second term is chosen k times, where $0 \leq k \leq 9$, the corresponding term is

$$T_{k+1} = \binom{9}{k} \left(\frac{3}{2}\right)^{9-k} \left(-\frac{1}{3}\right)^k x^{18-3k}.$$

All exponents in this expansion are multiples of three. Consequently there is no x^{-1} term, and the factor $2x$ contributes nothing to the constant term.

Step 2: Find the two contributing coefficients For the constant coefficient take $18 - 3k = 0$, giving $k = 6$. For the coefficient of x^{-3} , take $18 - 3k = -3$, giving $k = 7$. Their values are

$$\begin{aligned} [x^0] &= \binom{9}{6} \left(\frac{3}{2}\right)^3 \left(\frac{1}{3}\right)^6 = \frac{7}{18}, \\ [x^{-3}] &= \binom{9}{7} \left(\frac{3}{2}\right)^2 \left(-\frac{1}{3}\right)^7 = -\frac{1}{27}. \end{aligned}$$

The negative sign of the second coefficient comes from the odd power seven; it must be retained when multiplying by the prefactor.

Step 3: Combine the contributions The constant term of the whole product is therefore

$$p = \frac{7}{18} + 2(0) - 3\left(-\frac{1}{27}\right) = \frac{7}{18} + \frac{1}{9} = \frac{1}{2}.$$

Only these terms can produce exponent zero, since the prefactor has no powers other than zero, one and three.

Step 4: Conclusion Thus $108p = 108 \times \frac{1}{2} = \boxed{54}$. The required numerical value is 54.

Quick Tip to Solve: Before calculating binomial coefficients, check the possible exponents: here every binomial power is a multiple of three, so the $2x$ term cannot contribute.

30. The number of distinct real roots of the equation $|x||x+2| - 5|x+1| - 1 = 0$ is _____. [4 marks]

Solution:

Concept:

- The identity $|a||b| = |ab|$ allows the product of two absolute values to be combined. Centering at $x = -1$ reveals the symmetry of this equation.
- With $t = |x+1| \geq 0$, we have $x(x+2) = (x+1)^2 - 1 = t^2 - 1$. Each positive value of t gives two real values of x , whereas $t = 0$ gives only one.

Step 1: Reduce the equation to one nonnegative variable The original equation becomes

$$|t^2 - 1| - 5t - 1 = 0, \quad t = |x+1| \geq 0.$$

The sign inside the remaining absolute value changes at $t = 1$, so solve separately on $[0, 1]$ and $[1, \infty)$. No sign information about x is lost, since both original absolute values were combined exactly.

Step 2: Solve on the inner interval For $0 \leq t \leq 1$, replace $|t^2 - 1|$ by $1 - t^2$. This gives

$$1 - t^2 - 5t - 1 = 0 \implies t(t + 5) = 0.$$

The value $t = -5$ is inadmissible; $t = 0$ is allowed and gives $x = -1$. In particular, this root is counted only once, even though the equation is symmetric about -1 .

Step 3: Solve on the outer interval and count For $t \geq 1$, the equation is

$$t^2 - 5t - 2 = 0, \quad t = \frac{5 \pm \sqrt{33}}{2}.$$

The minus choice is negative, whereas $t_+ = (5 + \sqrt{33})/2 > 1$ is valid. It gives the two different roots $x = -1 + t_+$ and $x = -1 - t_+$. Neither equals the inner-interval root. The boundary $t = 1$ does not solve the equation, so it adds no root.

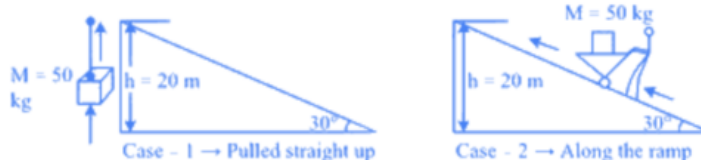
Step 4: Conclusion The equation has one central root and two symmetric outer roots. Hence the number of distinct real roots is $\boxed{3}$.

Quick Tip to Solve: For absolute values involving x , $x + 1$ and $x + 2$, shift the origin to $x = -1$. Count the value $|x + 1| = 0$ once and every admissible positive value twice.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A body of mass 50 kg is lifted to a height of 20 m from the ground in the two different ways as shown in the figures. The ratio of work done against the gravity in both the respective cases, will be :

[4 marks]



- (A) 2 : 1
- (B) $\sqrt{3} : 2$
- (C) 1 : 1
- (D) 1 : 2

Correct Answer: (C) 1 : 1

Solution:

Concept:

- Gravity is conservative, so the work done against it depends only on the change in height, not on the path taken between the endpoints.

- Near the surface of Earth, the gravitational potential-energy increase is $\Delta U = mgh$, with h the vertical rise. A ramp reduces the required force component but increases the distance in the same proportion.

Step 1: Evaluate the vertical-lift work In the first case, the 50 kg body rises vertically through 20 m. The work against gravity is

$$W_1 = mgh = 50g \times 20 = 1000g \text{ J.}$$

This result is the increase in gravitational potential energy. The question asks specifically for work against gravity, so any possible additional work associated with friction is outside this comparison.

Step 2: Evaluate the ramp work The second path is inclined at 30° . Its length for the same vertical rise is

$$s = \frac{h}{\sin 30^\circ} = 40 \text{ m.}$$

The gravitational component opposing motion along the ramp is $mg \sin 30^\circ$. Hence

$$W_2 = (mg \sin 30^\circ)s = mg \sin 30^\circ \frac{h}{\sin 30^\circ} = mgh = 1000g \text{ J.}$$

The smaller force along the ramp is exactly compensated by the greater path length.

Step 3: Compare the two works The initial and final heights are equal in the two cases, so

$$\frac{W_1}{W_2} = \frac{mgh}{mgh} = 1.$$

The ratios 2 : 1 and 1 : 2 incorrectly compare path lengths or force components alone. The factor $\sqrt{3}$ is associated with a horizontal component and cannot change the potential-energy increase.

Step 4: Conclusion The required ratio is $W_1 : W_2 = 1 : 1$. Hence, the correct option is (C).

Quick Tip to Solve: When only work against gravity is asked, compare vertical heights first. Equal masses raised through equal heights require equal gravitational work along any paths.

32. If G be the gravitational constant and u be the energy density then which of the following quantity have the dimensions as that of the $\sqrt{uG}$: [4 marks]

- (A) Gravitational potential
- (B) pressure gradient per unit mass
- (C) Energy per unit mass
- (D) Force per unit mass

Correct Answer: (D) Force per unit mass

Solution:

Concept:

- The gravitational constant has dimensions $[G] = M^{-1}L^3T^{-2}$, obtained from $F = Gm_1m_2/r^2$.
- Energy density is energy per unit volume: $[u] = (ML^2T^{-2})/L^3 = ML^{-1}T^{-2}$. Multiply the

dimensions first, then take their square root.

Step 1: Find the dimensions of the product Using the mass, length and time dimensions explicitly,

$$\begin{aligned}[uG] &= (ML^{-1}T^{-2})(M^{-1}L^3T^{-2}) \\ &= L^2T^{-4}.\end{aligned}$$

The mass dimensions cancel, while the length exponents add to two and the time exponents add to minus four. Thus the result is independent of the choice of a mass unit.

Step 2: Take the square root Taking the square root halves every exponent:

$$[\sqrt{uG}] = LT^{-2}.$$

This is the dimension of acceleration. Newton's second law gives the same dimensions for force divided by mass:

$$\left[\frac{F}{m}\right] = \frac{MLT^{-2}}{M} = LT^{-2}.$$

Step 3: Check the remaining alternatives Gravitational potential is potential energy per unit mass, with dimensions L^2T^{-2} , as is the energy-per-unit-mass option. Both contain one extra power of length compared with the required quantity. A pressure gradient has dimensions $ML^{-2}T^{-2}$; dividing it by mass gives $L^{-2}T^{-2}$, which also differs. Therefore only force per unit mass matches the required dimensions. Dimensional agreement identifies the requested option without requiring any numerical constants.

Step 4: Conclusion $\sqrt{uG}$ has the dimensions of force per unit mass, namely LT^{-2} . Hence, the correct option is (D).

Quick Tip to Solve: Write energy density as energy divided by volume before multiplying by G . The mass cancels, leaving an acceleration after the square root.

33. An alternating voltage of amplitude 40 V and frequency 4 kHz is applied directly across the capacitor of $12\mu\text{F}$. The maximum displacement current between the plates of the capacitor is nearly :

[4 marks]

- (A) 10 A
- (B) 8 A
- (C) 13 A
- (D) 12 A

Correct Answer: (D) 12 A

Solution:

Concept:

- For a capacitor of capacitance C , the plate charge is $q = CV$. A changing voltage therefore produces current $i = C dV/dt$.

- The displacement current between the plates equals the conduction current in the connecting wires. For sinusoidal voltage of peak amplitude V_0 , its peak value is $I_0 = \omega CV_0 = 2\pi f CV_0$.

Step 1: Differentiate the applied voltage Write the voltage as $V(t) = V_0 \sin \omega t$, choosing the time origin conveniently. Then

$$i_d = C \frac{dV}{dt} = \omega CV_0 \cos \omega t.$$

Since the largest magnitude of the cosine is one, the maximum displacement current is $I_0 = \omega CV_0$. The given voltage is already an amplitude, so it does not need an rms-to-peak conversion.

Step 2: Convert the data and substitute The supplied values in SI units are

$$f = 4 \times 10^3 \text{ Hz}, \quad C = 12 \times 10^{-6} \text{ F}, \quad V_0 = 40 \text{ V}.$$

Thus

$$\begin{aligned} I_0 &= 2\pi(4 \times 10^3)(12 \times 10^{-6})(40) \\ &= 3.84\pi \text{ A} \approx 12.06 \text{ A}. \end{aligned}$$

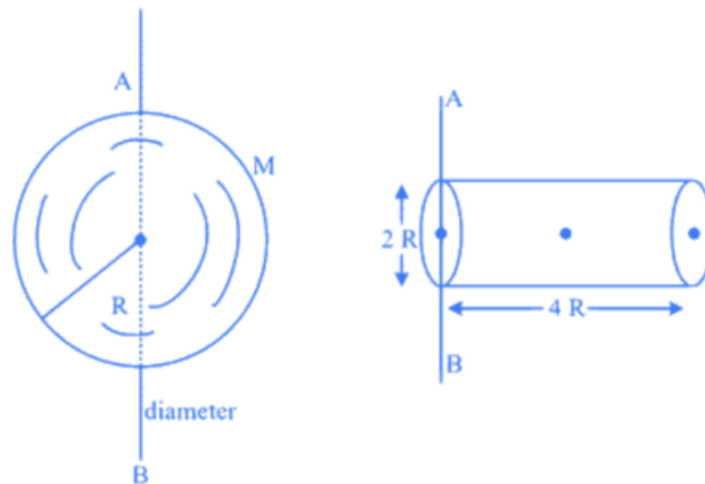
The powers 10^3 and 10^{-6} combine to 10^{-3} , which is essential for the correct scale.

Step 3: Select the nearest stated value The nearest value to 12.06 A is 12 A. The alternatives 8 A, 10 A and 13 A are farther from the calculated peak. An rms current would be smaller by $\sqrt{2}$, but the question asks for the maximum current, so that conversion would answer a different quantity.

Step 4: Conclusion The maximum displacement current is approximately 12 A. Hence, the correct option is (D).

Quick Tip to Solve: For a sinusoidally driven capacitor, use $I_0 = 2\pi f CV_0$. Keep peak quantities together; do not divide by $\sqrt{2}$ when the question asks for maximum current.

34. Ratio of radius of gyration of a hollow sphere to that of a solid cylinder of equal mass, for moment of Inertia about their diameter axis AB as shown in figure is $\sqrt{8/x}$. The value of x is : [4 marks]



- (A) 34
(B) 51
(C) 67
(D) 17

Correct Answer: (C) 67

Solution:

Concept:

- The radius of gyration is defined by $I = Mk^2$. Consequently, for equal masses, the ratio of radii of gyration is the square root of the ratio of moments of inertia.
- A thin hollow sphere about a diameter has $I_s = \frac{2}{3}MR^2$. For the cylinder, the indicated axis is a diameter through one end face, so the parallel-axis theorem must be applied to the central transverse axis.

Step 1: Read the cylinder geometry and axis The figure gives the solid cylinder diameter as $2R$, so its radius is R , and its length is $L = 4R$. About a transverse diameter through the cylinder center,

$$I_{\text{cm}} = \frac{M}{12}(3R^2 + L^2) = \frac{19}{12}MR^2.$$

The displayed axis AB passes through the center of the end face. It is parallel to this central transverse axis and lies a distance $L/2 = 2R$ from it.

Step 2: Shift to the end diameter By the parallel-axis theorem,

$$\begin{aligned} I_c &= I_{\text{cm}} + M(2R)^2 \\ &= \frac{19}{12}MR^2 + 4MR^2 \\ &= \frac{67}{12}MR^2. \end{aligned}$$

Using only the central-diameter formula would omit the displacement of the specified axis. Using the cylinder's symmetry-axis formula would also be incorrect because AB is perpendicular to its length.

Step 3: Compare radii of gyration The hollow sphere has $k_s^2 = 2R^2/3$, and the cylinder has $k_c^2 = 67R^2/12$. Therefore

$$\frac{k_s}{k_c} = \sqrt{\frac{(2/3)R^2}{(67/12)R^2}} = \sqrt{\frac{8}{67}}.$$

Comparing with the given form $\sqrt{8/x}$ fixes the denominator uniquely. The choices 34, 51 and 17 do not reproduce the end-axis inertia derived from the shown dimensions.

Step 4: Conclusion The comparison gives $x = 67$. Hence, the correct option is (C).

Quick Tip to Solve: Check where the cylinder axis actually lies. A diameter through an end face needs a parallel-axis shift of $M(L/2)^2$ from the central transverse diameter.

35. In hydrogen like system the ratio of coulombian force and gravitational force between an electron and a proton is in the order of : [4 marks]

- (A) 10^{19}
- (B) 10^{39}
- (C) 10^{29}
- (D) 10^{36}

Correct Answer: (B) 10^{39}

Solution:

Concept:

- The electron and proton attract electrically with magnitude $F_e = ke^2/r^2$, where $k = 1/(4\pi\epsilon_0)$. Their gravitational attraction is $F_g = Gm_em_p/r^2$.
- Both forces obey an inverse-square law. Their ratio is therefore independent of separation: $F_e/F_g = ke^2/(Gm_em_p)$.

Step 1: Form a distance-independent ratio For the same electron-proton pair at separation r ,

$$\frac{F_e}{F_g} = \frac{ke^2/r^2}{Gm_em_p/r^2} = \frac{ke^2}{Gm_em_p}.$$

The factor r^{-2} cancels exactly. Neither the orbital radius nor an assumed Bohr orbit is needed. Both forces are attractive here, but the question compares their magnitudes.

Step 2: Estimate the numerator and denominator Using standard SI values, $k \approx 9.0 \times 10^9$, $e \approx 1.60 \times 10^{-19}$ C, $G \approx 6.67 \times 10^{-11}$, $m_e \approx 9.11 \times 10^{-31}$ kg and $m_p \approx 1.67 \times 10^{-27}$ kg,

$$ke^2 \approx 2.30 \times 10^{-28},$$

$$Gm_em_p \approx 1.01 \times 10^{-67}.$$

Use the proton mass in the gravitational denominator; replacing it by a second electron mass describes

a different pair.

Step 3: Divide and identify the order The ratio is dimensionless and equals approximately

$$\frac{F_e}{F_g} \approx \frac{2.30 \times 10^{-28}}{1.01 \times 10^{-67}} \approx 2.3 \times 10^{39}.$$

Thus its power-of-ten scale is 10^{39} . The alternatives 10^{19} , 10^{29} and 10^{36} are respectively about twenty, ten and three powers of ten below this scale, so none represents the calculated ratio.

Step 4: Conclusion The Coulomb force is of order 10^{39} times the gravitational force. Hence, the correct option is (B).

Quick Tip to Solve: Cancel the common r^{-2} factor before substituting constants. Keep the electron and proton masses distinct when estimating the force ratio.

36. A simple pendulum doing small oscillations at a place R height above earth surface has time period of $T_1 = 4$ s. T_2 would be it's time period if it is brought to a point which is at a height $2R$ from earth surface. Choose the correct relation [R = radius of earth] : [4 marks]

(A) $3 T_1 = 2 T_2$

(B) $T_1 = T_2$

(C) $2 T_1 = 3 T_2$

(D) $2 T_1 = T_2$

Correct Answer: (A) $3 T_1 = 2 T_2$

Solution:

Concept:

- The time period of a simple pendulum of fixed length ℓ , for small oscillations, is $T = 2\pi\sqrt{\ell/g}$. Thus only the local gravitational acceleration changes when its location changes.
- At height h above a spherical Earth of radius R , $g(h) = GM/(R+h)^2$. The distance in this inverse-square law is measured from the centre of Earth, not from its surface.

Step 1: Convert heights to distances from the centre The first location is at height R , so its distance from Earth's centre is $r_1 = R + R = 2R$. The second is at height $2R$, giving $r_2 = R + 2R = 3R$. Therefore

$$g_1 = \frac{GM}{4R^2}, \quad g_2 = \frac{GM}{9R^2}.$$

Moving the pendulum farther away reduces gravity; the oscillations must therefore become slower, so we already expect $T_2 > T_1$.

Step 2: Compare the periods and test the options The pendulum length is unchanged, so the common factor $2\pi\sqrt{\ell}$ cancels:

$$\frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \sqrt{\frac{9}{4}} = \frac{3}{2}.$$

Using $T_1 = 4$ s, this also gives $T_2 = 6$ s. Hence $3T_1 = 12$ s $= 2T_2$. Equal periods would require equal

gravity; option (C) incorrectly predicts a shorter second period, while option (D) uses the ratio of heights instead of the ratio of distances from the centre.

Step 3: Conclusion The correct relation is $3T_1 = 2T_2$, with $T_2 = 6$ s. Hence, the correct option is (A).

Quick Tip to Solve: For a fixed-length pendulum outside Earth, substitute the inverse-square gravity law first: its period is directly proportional to $R + h$, not to height h .

37. Match List I with List II:

	LIST I		LIST II
A.	Kinetic energy of planet	I.	$-GMm/a$
B.	Gravitation Potential energy of sun-planet system	II.	$GMm/2a$
C.	Total mechanical energy of planet	III.	Gm/r
D.	Escape energy at the surface of planet for unit mass object	IV.	$-GMm/2a$

(Where a = radius of planet orbit, r = radius of planet, M = mass of Sun, m = mass of planet.)
Choose the correct answer from the options given below: [4 marks]

(A) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(B) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(C) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(D) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

Correct Answer: (A) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Solution:

Concept:

- For a circular orbit of radius a , gravity supplies centripetal force: $GMm/a^2 = mv^2/a$. This fixes the kinetic energy of the orbiting planet.
- With potential energy zero at infinity, the gravitational potential energy is $U = -GMm/a$, and the total mechanical energy is $E = K + U$. Escape energy is the positive energy needed to reach infinity with zero final speed.

Step 1: Match the three orbital energies From the circular-orbit force balance,

$$v^2 = \frac{GM}{a}, \quad K = \frac{1}{2}mv^2 = \frac{GMm}{2a}.$$

Thus entry A matches II. The potential energy is

$$U = -\frac{GMm}{a},$$

so B matches I. Adding these two energies gives

$$E = \frac{GMm}{2a} - \frac{GMm}{a} = -\frac{GMm}{2a}.$$

Therefore C matches IV. The negative total energy is the signature of a gravitationally bound orbit; it

must not be confused with the positive kinetic energy.

Step 2: Match escape energy and select the complete code For a unit-mass object initially at rest on the planet's surface, the initial gravitational potential energy is $-Gm/r$, where m is the planet's mass. The minimum final energy at infinity is zero. The energy that must be supplied is therefore

$$\Delta E = 0 - \left(-\frac{Gm}{r}\right) = \frac{Gm}{r}.$$

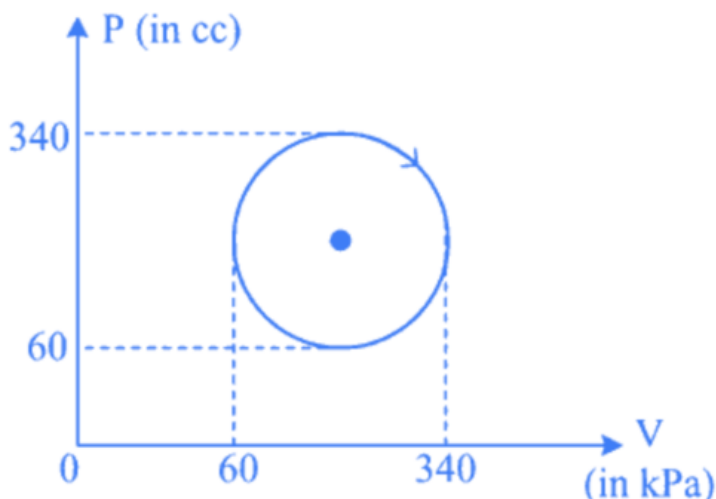
This matches D with III. Only the first code combines all four matches. The other codes fail already at A: kinetic energy cannot be negative and it depends on the Sun-planet orbital separation a , not on the planet's surface radius r .

Step 3: Conclusion The matches are A-II, B-I, C-IV and D-III. Hence, the correct option is (A).

Quick Tip to Solve: For a circular gravitational orbit, remember $K : U : E = 1 : -2 : -1$; then treat surface escape energy separately using the planet as the attracting mass.

38. The heat absorbed by a system in going through the given cyclic process is :

[4 marks]



- (A) 61.6 J
- (B) 431.2 J
- (C) 19.6 J
- (D) 616 J

Correct Answer: (A) 61.6 J

Solution:

Concept:

- Internal energy is a state function. Over a complete cycle the system returns to its initial state, so $\Delta U = 0$ and the first law gives net heat absorbed $Q_{\text{net}} = W_{\text{net}}$.
- The magnitude of cyclic work is the enclosed area on the pressure-volume diagram. A clockwise

cycle has positive work by the gas and therefore positive net heat absorbed.

Step 1: Read the semi-axes and cycle direction The boundary extends from 60 to 340 along both plotted coordinates. Hence the centre is at 200 and each numerical semi-axis is

$$\frac{340 - 60}{2} = 140.$$

The arrow on the upper-right portion points clockwise. The expansion branch is therefore at higher pressure than the compression branch, giving positive net work. The two coordinate units shown are cc and kPa; their product, irrespective of the interchanged unit labels beside the axes, is the pressure-volume product needed for energy.

Step 2: Evaluate the enclosed area in joules Although the curve is drawn as a circle, its area must carry the units of the two physical axes. Thus

$$W_{\text{net}} = \pi(140 \text{ cc})(140 \text{ kPa}).$$

Using $1 \text{ cc} = 10^{-6} \text{ m}^3$ and $1 \text{ kPa} = 10^3 \text{ Pa}$,

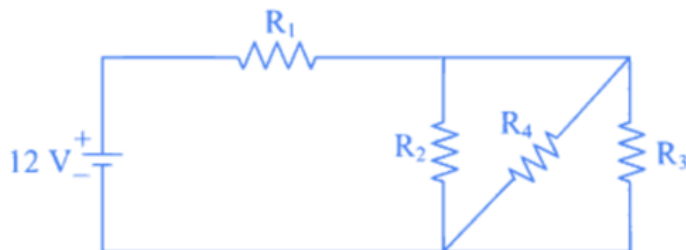
$$W_{\text{net}} = \pi \times 19600 \times 10^{-3} \text{ J} = 19.6\pi \text{ J} \simeq 61.6 \text{ J}.$$

Since $\Delta U = 0$, this is also the net heat absorbed. The value 19.6 J omits the factor π ; 616 J uses an incorrect power of ten, and 431.2 J does not equal the enclosed pressure-volume area.

Step 3: Conclusion The net heat absorbed over the complete cycle is $Q_{\text{net}} = 61.6 \text{ J}$. Hence, the correct option is (A).

Quick Tip to Solve: For a complete thermodynamic cycle, calculate the enclosed area, attach one pressure unit and one volume unit, and convert their product to joules before choosing an option.

39. In the given figure $R_1 = 10\Omega$, $R_2 = 8\Omega$, $R_3 = 4\Omega$ and $R_4 = 8\Omega$. Battery is ideal with emf 12 V. Equivalent resistant of the circuit and current supplied by battery are respectively : [4 marks]



- (A) 12Ω and 11.4 A
 (B) 10.5Ω and 1.14 A
 (C) 10.5Ω and 1 A
 (D) 12Ω and 1 A

Correct Answer: (D) 12Ω and 1 A

Solution:

Concept:

- Resistors are in parallel when both their terminals connect to the same pair of nodes. Their geometric placement in a drawing does not determine the connection type.
- For a parallel group, $1/R_p = \sum_i 1/R_i$. A series resistor adds directly to the group resistance, and the current supplied by an ideal battery is $I = \mathcal{E}/R_{eq}$.

Step 1: Identify the common nodes and reduce the parallel group After R_1 , the top wire connects the upper ends of R_2 , R_3 and the diagonal resistor R_4 . Their lower ends all join the bottom return wire. Consequently these three resistors are in parallel, even though R_4 is drawn diagonally:

$$\frac{1}{R_p} = \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \frac{1+2+1}{8} = \frac{1}{2} \Omega^{-1}.$$

Thus $R_p = 2\Omega$, which is smaller than each of the three individual parallel resistances, as required.

Step 2: Add the series resistance and calculate battery current All current supplied by the battery passes through $R_1 = 10\Omega$ before dividing among the parallel paths. Therefore

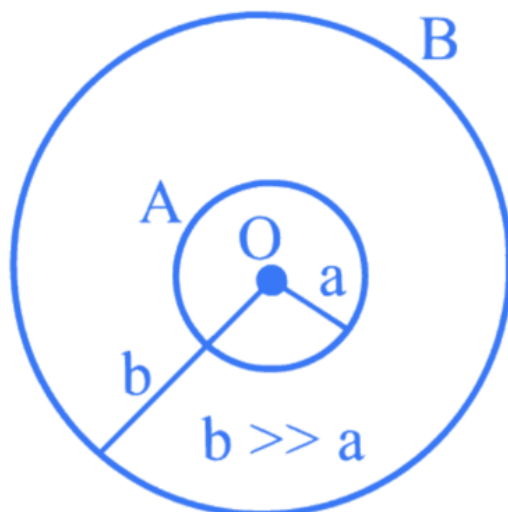
$$R_{eq} = R_1 + R_p = 10 + 2 = 12\Omega, \quad I = \frac{12\text{ V}}{12\Omega} = 1\text{ A}.$$

As a check, the voltage across the parallel group is $IR_p = 2\text{ V}$. Its branch currents are $2/8 = 0.25\text{ A}$, $2/4 = 0.50\text{ A}$ and $2/8 = 0.25\text{ A}$, which sum to the battery current. Options involving 10.5Ω use an incorrect reduction, while 11.4 A contradicts Ohm's law for a 12 V source and 12 ohm load.

Step 3: Conclusion The equivalent resistance and battery current are 12Ω and 1 A, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Mark equipotential wire nodes before reducing a circuit: a diagonal resistor is parallel to a vertical one whenever their endpoints share the same two nodes.

40. Two conducting circular loops A and B are placed in the same plane with their centres coinciding as shown in figure. The mutual inductance between them is: [4 marks]



- (A) $\frac{\mu_0}{2\pi} \cdot \frac{b^2}{a}$
 (B) $\frac{\mu_0 \pi a^2}{2b}$
 (C) $\frac{\mu_0 \pi b^2}{2a}$
 (D) $\frac{\mu_0}{2\pi} \cdot \frac{a^2}{b}$

Correct Answer: (B) $\frac{\mu_0 \pi a^2}{2b}$

Solution:

Concept:

- Mutual inductance is the magnetic flux through one loop per unit current in the other: $M = \Phi/I$. For two stationary loops in the same medium, the two mutual inductances are equal.
- The magnetic field at the centre of a circular loop of radius b carrying current I is $B = \mu_0 I / (2b)$. If the second loop has radius $a \ll b$, this field is approximately uniform over its area.

Step 1: Choose the large loop as the current source The figure gives the inner radius as a , the outer radius as b , and states $b \gg a$. Let current I flow in the outer loop B. The field at its common centre O is

$$B_O = \frac{\mu_0 I}{2b}.$$

Because the inner loop occupies only a small neighbourhood of O, the variation of the outer loop's field across it is negligible in the stated large-radius-ratio limit. The field is perpendicular to the common plane, so it is normal to the area of the inner loop and no cosine factor reduces the flux.

Step 2: Calculate the flux through the smaller loop The inner loop A has area πa^2 . Its magnetic flux due to the outer-loop current is therefore

$$\Phi_A \simeq B_O \pi a^2 = \frac{\mu_0 I \pi a^2}{2b}.$$

Dividing by the source current gives

$$M = \frac{\Phi_A}{I} \simeq \frac{\mu_0 \pi a^2}{2b}.$$

This has the expected scaling: reducing the small loop area reduces the linked flux, while increasing the outer radius weakens its central field. Options proportional to b^2/a reverse these roles. The option with $a^2/(2\pi b)$ misses the factor π in the small-loop area.

Step 3: Conclusion For $b \gg a$, the mutual inductance is $M \simeq \mu_0 \pi a^2/(2b)$. Hence, the correct option is (B).

Quick Tip to Solve: Use reciprocity to choose the easier source loop: the large loop gives an almost uniform field over the small loop, so multiply its central field by the smaller area.

41. If the collision frequency of hydrogen molecules in a closed chamber at 27°C is Z , then the collision frequency of the same system at 127°C is : [4 marks]

- (A) $\frac{\sqrt{3}}{2}Z$
- (B) $\frac{2}{\sqrt{3}}Z$
- (C) $\frac{3}{4}Z$
- (D) $\frac{4}{3}Z$

Correct Answer: (B) $\frac{2}{\sqrt{3}}Z$

Solution:

Concept:

- For a gas with molecular diameter d and number density n , the mean free path is $\lambda = 1/(\sqrt{2}\pi d^2 n)$. At fixed chamber volume and fixed number of molecules, n is unchanged.
- The collision frequency per molecule is $z = \bar{v}/\lambda$, where the mean molecular speed $\bar{v}$ is proportional to $\sqrt{T}$ for a fixed molecular mass. Absolute temperature must be used.

Step 1: Identify what remains fixed in the chamber The same hydrogen sample is heated in the closed chamber. Its molecular mass and collision diameter do not change, and the fixed chamber volume keeps its number density unchanged. Thus the mean free path remains constant in the ideal-gas kinetic description. The rise in collision rate comes from the higher speed of the molecules, not from a change in the number of available molecules per unit volume. The relevant temperatures in kelvin are

$$T_1 = 27 + 273 = 300 \text{ K}, \quad T_2 = 127 + 273 = 400 \text{ K}.$$

Step 2: Apply the square-root temperature dependence Since λ is unchanged, the ratio of collision frequencies equals the ratio of mean speeds:

$$\frac{z_2}{z_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{400}{300}} = \frac{2}{\sqrt{3}}.$$

With $z_1 = Z$, the new frequency is $z_2 = 2Z/\sqrt{3}$. Heating must increase the rate, which immediately excludes the two options below Z . The factor $4/3$ is the ratio of absolute temperatures; it describes quantities proportional to T , whereas molecular speed and collision frequency here scale as $\sqrt{T}$. Using the Celsius values directly would give a physically incorrect ratio.

Step 3: Conclusion The collision frequency at 127°C is $2Z/\sqrt{3}$. Hence, the correct option is (B).

Quick Tip to Solve: At fixed number density, the mean free path is constant: collision frequency follows molecular speed, so take the square root of the ratio of kelvin temperatures.

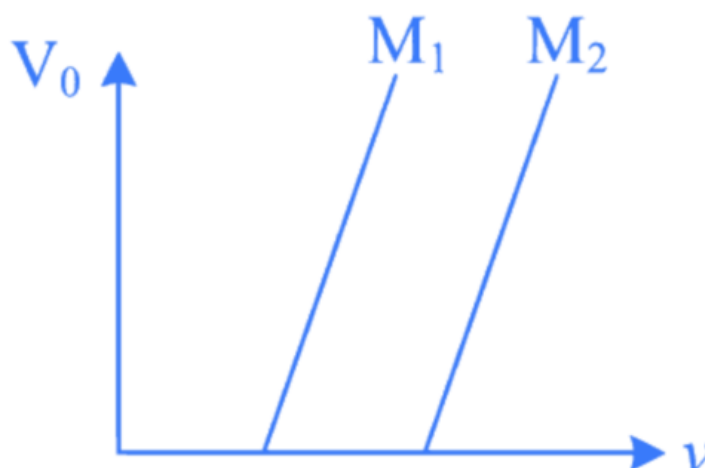
42. Given below are two statements :

Statement I : Figure shows the variation of stopping potential with frequency (ν) for the two photosensitive materials M_1 and M_2 . The slope gives value of $\frac{h}{e}$, where h is Planck's constant, e is the charge of electron.

Statement II : M_2 will emit photoelectrons of greater kinetic energy for the incident radiation having same frequency.

In the light of the above statements, choose the most appropriate answer from the options given below.

[4 marks]



- (A) Statement I is correct and Statement II is incorrect
- (B) Statement I is incorrect but Statement II is correct
- (C) Both Statement I and Statement II are correct
- (D) Both Statement I and Statement II are incorrect

Correct Answer: (A) Statement I is correct and Statement II is incorrect

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = eV_0$, where ϕ is the material's work function and V_0 is its stopping potential.
- A plot of stopping potential against frequency obeys $V_0 = (h/e)\nu - \phi/e$. Its slope is universal, while

its frequency-axis intercept $\nu_0 = \phi/h$ depends on the material.

Step 1: Check Statement I using the slope For either photosensitive material, differentiation of the straight-line equation gives

$$\frac{dV_0}{d\nu} = \frac{h}{e}.$$

Thus the slope of the stopping-potential versus frequency graph gives h/e , exactly as Statement I claims. Different work functions shift the straight lines horizontally without changing their slope. The two parallel rising lines in the figure are therefore consistent with this result, so Statement I is correct.

Step 2: Compare the kinetic energies at the same frequency The line for M_2 meets the frequency axis farther to the right than the line for M_1 . Consequently

$$\nu_{02} > \nu_{01}, \quad \phi_2 = h\nu_{02} > h\nu_{01} = \phi_1.$$

For a common incident frequency high enough to produce emission from both,

$$K_{\max,2} = h\nu - \phi_2 < h\nu - \phi_1 = K_{\max,1}.$$

The second material therefore emits electrons with smaller, not greater, maximum kinetic energy. If the frequency lies between the two thresholds, M_2 does not emit at all. Statement II is incorrect. Hence options that reject Statement I or accept Statement II cannot match the graph.

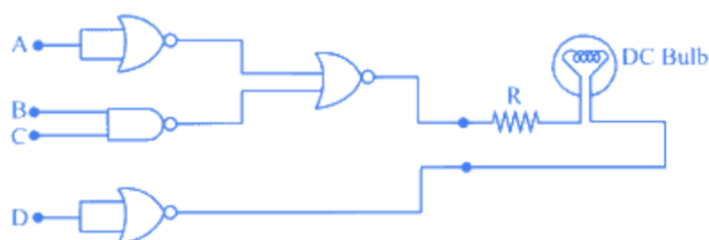
Step 3: Conclusion Statement I is correct and Statement II is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: For parallel photoelectric lines, the line with the larger threshold frequency has the larger work function and the smaller stopping potential at any common frequency.

43. Following gates section is connected in a complete suitable circuit.

For which of the following combination, bulb will glow (ON) :

[4 marks]



(A) $A = 1, B = 1, C = 1, D = 0$

(B) $A = 1, B = 0, C = 0, D = 0$

(C) $A = 0, B = 0, C = 0, D = 1$

(D) $A = 0, B = 1, C = 1, D = 1$

Correct Answer: (B) $A = 1, B = 0, C = 0, D = 0$

Solution:**Concept:**

- A NOR gate gives the complement of the OR of its inputs; a NAND gate gives the complement of their AND. Tying both inputs of a NOR gate together makes it a NOT gate.
- The bulb is connected between two logic outputs. It glows when their voltage levels differ, since a nonzero potential difference then drives current through it and the series resistor.

Step 1: Find the two terminal outputs For the top branch, the first NOR gate has both inputs connected to A, so its output is $\overline{A}$. The middle NAND output is $\overline{BC}$. Their outputs feed a NOR gate, giving the upper bulb-terminal level

$$X = \overline{\overline{A} + \overline{BC}} = ABC.$$

The lower NOR gate has both inputs connected to D, so the lower terminal level is $Y = \overline{D}$.

Step 2: Test the four input combinations In option (A), $ABC = 1$ and $\overline{D} = 1$, so the terminals have equal levels and the bulb is off. In option (B), $ABC = 0$ and $\overline{D} = 1$, so the bulb has a voltage across it and is on. Option (C) gives $X = 0, Y = 0$; option (D) also gives $X = 0, Y = 0$. Both leave the bulb off. The resistor limits the current; it does not alter which input combination supplies a potential difference.

Step 3: Conclusion The bulb glows for $A = 1, B = 0, C = 0, D = 0$, because its terminals are at opposite logic levels. Hence, the correct option is (B).

Quick Tip to Solve: For a bulb between two gate outputs, compare the two output levels; a high output alone does not guarantee that the bulb glows.

44. Given below are two statements :

Statement I : When a capillary tube is dipped into a liquid, the liquid neither rises nor falls in the capillary. The contact angle may be 0° .

Statement II : The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well.

In the light of the above statement, choose the correct answer from the options given below. [4 marks]

- (A) Statement I is true and Statement II is false
- (B) Statement I is false but Statement II is true
- (C) Both Statement I and Statement II are true
- (D) Both Statement I and Statement II are false

Correct Answer: (B) Statement I is false but Statement II is true

Solution:**Concept:**

- For a capillary of radius r , the equilibrium height is $h = 2T \cos \theta / (\rho g r)$, where T is surface tension and θ is the contact angle measured through the liquid.

- The contact angle depends on the interacting solid, liquid and surrounding medium. It expresses the balance of interfacial forces at the contact line.

Step 1: Check Statement I using capillary rise If the liquid neither rises nor falls, its capillary height relative to the outside surface is zero. With nonzero surface tension and finite tube radius,

$$h = 0 \implies \cos \theta = 0 \implies \theta = 90^\circ.$$

At $\theta = 0^\circ$, the cosine is one, so the upward component of surface tension is largest and the liquid rises. Thus a zero contact angle cannot describe the stated absence of rise or fall, and Statement I is false.

Step 2: Check Statement II and select the pair The material of the solid affects its adhesive interaction with the liquid, while the liquid determines its cohesive and interfacial properties. Consequently the contact angle is a property of the particular solid-liquid combination in the given surroundings. Statement II is therefore true. Option (A) reverses these truth values; option (C) wrongly accepts Statement I, and option (D) wrongly rejects Statement II.

Step 3: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (B).

Quick Tip to Solve: Use the sign of $\cos \theta$: acute angles cause rise, obtuse angles cause depression, and 90° gives neither.

45. Light emerges out of a convex lens when a source of light kept at its focus. The shape of wavefront of the light is : [4 marks]

- (A) cylindrical
- (B) spherical
- (C) plane
- (D) both spherical and cylindrical

Correct Answer: (C) plane

Solution:

Concept:

- A wavefront is a surface joining points at the same phase. In an isotropic medium, rays are perpendicular to their wavefronts.
- An ideal convex lens converts the diverging light from a point source at its principal focus into a parallel beam on the other side.

Step 1: Determine the emergent ray directions The source is at the principal focus of the convex lens. By the focal property of the lens, each paraxial ray emerging from it travels parallel to the principal axis. Equivalently, in the lens formula, placing the object at the focal distance makes the image distance infinite. Thus the emergent light neither converges to a finite point nor diverges from a finite point.

Step 2: Identify the surface of equal phase Since the emerging rays are parallel, their common normal surfaces are planes perpendicular to the beam. The lens changes the optical paths of the

incident spherical wave so that its emerging wavefront is plane. A spherical wavefront belongs to rays converging to or diverging from a finite point. A cylindrical wavefront is associated with a line-source geometry, which is absent here. Therefore neither the cylindrical choice nor the combined spherical-and-cylindrical choice describes this beam. This is the usual ideal, paraxial description; finite-aperture diffraction does not change the intended geometrical-optics answer.

Step 3: Conclusion The emergent wavefront is plane. Hence, the correct option is (C).

Quick Tip to Solve: A point source at a convex lens focus gives parallel rays; parallel rays always identify a plane wavefront in geometrical optics.

46. An electron rotates in a circle around a nucleus having positive charge Ze . Correct relation between total energy (E) of electron to its potential energy (U) is : [4 marks]

- (A) $2E = 3U$
- (B) $E = U$
- (C) $2E = U$
- (D) $E = 2U$

Correct Answer: (C) $2E = U$

Solution:

Concept:

- For an electron a distance r from a nucleus of charge $+Ze$, with zero potential energy at infinity, $U = -kZe^2/r$, where $k = 1/(4\pi\epsilon_0)$.
- In a circular orbit, the attractive Coulomb force supplies centripetal force. The total mechanical energy is $E = K + U$, with $K = mv^2/2$.

Step 1: Use the circular-orbit force balance Equating the Coulomb force magnitude to the required centripetal force gives

$$\frac{mv^2}{r} = \frac{kZe^2}{r^2}.$$

Multiplying by $r/2$, the kinetic energy becomes

$$K = \frac{1}{2}mv^2 = \frac{kZe^2}{2r} = -\frac{U}{2}.$$

The minus sign matters: the kinetic energy is positive whereas the electron-nucleus potential energy is negative for a bound orbit.

Step 2: Combine kinetic and potential energies Now add the two contributions without changing the potential-energy reference:

$$E = K + U = -\frac{U}{2} + U = \frac{U}{2}.$$

Hence $2E = U$. Both energies are negative and E is half of U , rather than twice its value. Option (A) would require a different kinetic-energy relation. Option (B) would make the kinetic energy zero, incompatible with a circular orbit. Option (D) would imply negative kinetic energy when $U < 0$, so it

also cannot hold.

Step 3: Conclusion The required relation is $2E = U$. Hence, the correct option is (C).

Quick Tip to Solve: For a bound circular Coulomb orbit, remember $K = -U/2$, then add $K + U$ to avoid a sign error in total energy.

47. Time periods of oscillation of the same simple pendulum measured using four different measuring clocks were recorded as 4.62 s, 4.632 s, 4.6 s and 4.64 s. The arithmetic mean of these readings in correct significant figure is : [4 marks]

- (A) 5 s
- (B) 4.6 s
- (C) 4.62 s
- (D) 4.623 s

Correct Answer: (B) 4.6 s

Solution:

Concept:

- The arithmetic mean of n measurements is their sum divided by n . Here the count of four readings is exact and does not introduce measurement uncertainty.
- The reported precision is limited by the least precise measurement. For addition, compare decimal places, keeping guard digits until the final rounding.

Step 1: Calculate the unrounded arithmetic mean Using all digits supplied in the four recorded readings, the sum is

$$4.62 + 4.632 + 4.6 + 4.64 = 18.492 \text{ s.}$$

Since exactly four clocks supplied readings,

$$\bar{T} = \frac{18.492}{4} = 4.623 \text{ s.}$$

This is the unrounded numerical mean; writing every digit as a final measured result would overstate the precision of the observations.

Step 2: Round to the precision supported by the readings The reading 4.6 s is only recorded to one decimal place, whereas the others have two or three decimal places. The mean should therefore be stated to the corresponding limited precision, giving 4.6 s. The guard digits determine the rounding but do not become additional justified digits. Option (D) is the raw mean with excessive precision, and option (C) still claims hundredths despite the coarser reading. Option (A) rounds unnecessarily to whole seconds and discards useful precision.

Step 3: Conclusion The arithmetic mean in the appropriate significant figures is 4.6 s. Hence, the correct option is (B).

Quick Tip to Solve: Keep extra digits while averaging, then round once using the coarsest recorded reading rather than the number of digits on the best clock.

48. The angle between vector $\vec{Q}$ and the resultant of $(2\vec{Q} + 2\vec{P})$ and $(2\vec{Q} - 2\vec{P})$ is :

[4 marks]

- (A) $\tan^{-1}(P/Q)$
- (B) 0°
- (C) $\tan^{-1} \frac{(2\vec{Q}-2\vec{P})}{2\vec{Q}+2\vec{P}}$
- (D) $\tan^{-1}(2Q/P)$

Correct Answer: (B) 0°

Solution:

Concept:

- The resultant of two vectors is their vector sum, obtained by adding corresponding components. Vector addition is commutative and associative.
- Multiplying a nonzero vector by a positive scalar changes its magnitude but preserves its direction. The angle between such parallel, codirectional vectors is zero.

Step 1: Add the two vectors before finding any angle Write the two given vectors as $\vec{V}_1 = 2\vec{Q} + 2\vec{P}$ and $\vec{V}_2 = 2\vec{Q} - 2\vec{P}$. Their resultant is

$$\vec{R} = \vec{V}_1 + \vec{V}_2 = (2\vec{Q} + 2\vec{P}) + (2\vec{Q} - 2\vec{P}) = 4\vec{Q}.$$

The two $\vec{P}$ contributions cancel exactly, irrespective of the relative orientation or magnitudes of $\vec{P}$ and $\vec{Q}$.

Step 2: Compare the resultant direction with the given vector Since the coefficient four is positive, $\vec{R}$ points in exactly the same direction as the nonzero vector $\vec{Q}$. For example, the dot-product relation gives $\cos \theta = (\vec{Q} \cdot 4\vec{Q}) / (|\vec{Q}| |4\vec{Q}|) = 1$, so $\theta = 0^\circ$. No ratio involving P/Q is needed. Options (A) and (D) generally predict an angle depending on P , even though that vector has vanished from the sum. Option (C) also does not represent the direction of the resultant and is not a scalar component ratio for this calculation.

Step 3: Conclusion The angle is 0° . Hence, the correct option is (B).

Quick Tip to Solve: Combine symmetric vector expressions first: the plus and minus terms cancel before any trigonometry is necessary.

49. In a co-axial straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero : [4 marks]

- (A) inside the outer conductor
- (B) outside the cable
- (C) in between the two conductors
- (D) inside the inner conductor

Correct Answer: (B) outside the cable

Solution:

Concept:

- For a long coaxial cable, cylindrical symmetry makes the magnetic field tangential to a concentric circular Amperian loop and constant in magnitude on that loop.
- Ampere's law gives $B(2\pi r) = \mu_0 I_{\text{enc}}$, where the enclosed current is an algebraic sum with opposite directions counted with opposite signs.

Step 1: Enclose both conductors with an outside loop Let the central conductor carry current $+I$ and the outer conductor carry $-I$. A circular loop outside the entire cable encloses both currents, so

$$I_{\text{enc}} = I - I = 0.$$

By cylindrical symmetry and Ampere's law,

$$B(2\pi r) = 0 \implies B = 0.$$

The cancellation depends on the equal magnitudes and opposite directions specified in the question.

Step 2: Check why the internal regions are different Between the two conductors, a loop encloses the full central current but none of the return current, giving $B = \mu_0 I / (2\pi r)$, which is nonzero. Within the outer conductor, a loop generally includes only part of the return current, so the enclosed current is not yet zero. Within the inner conductor, the enclosed fraction of the central current is generally nonzero as well; only the axis can have zero field by symmetry for a regular current distribution. Thus the field does not vanish throughout any of the internal regions named in the other options. The complete outside region is the one where both currents cancel.

Step 3: Conclusion The magnetic field is zero outside the cable. Hence, the correct option is (B).

Quick Tip to Solve: In a coaxial cable, count all enclosed current algebraically; cancellation is complete only after the loop encloses both full currents.

50. A wooden block of mass 5 kg rests on a soft horizontal floor. When an iron cylinder of mass 25 kg is placed on the top of the block, the floor yields and the block and the cylinder together go down with an acceleration of 0.1 ms^{-2} . The action force of the system on the floor is equal to : [4 marks]

- (A) 297 N
- (B) 291 N
- (C) 196 N
- (D) 294 N

Correct Answer: (B) 291 N

Solution:

Concept:

- Treat the block and cylinder together as one system of total mass $M = 5 + 25 = 30 \text{ kg}$; the contact forces between them are internal and cancel.
- Newton's second law gives the resultant external force $\sum F = Ma$. The floor's upward normal reaction is smaller than the weight when acceleration is downward.
- Newton's third law makes the downward action on the floor equal in magnitude to its upward reaction on the system.

Step 1: Identify the external forces The combined system has weight Mg downward and normal reaction N upward. Its downward acceleration is $a = 0.1 \text{ ms}^{-2}$. Choose downward as positive. The cylinder's force on the block and the block's force on the cylinder are not included separately, because they form an internal action-reaction pair. Thus the system equation is

$$Mg - N = Ma.$$

Step 2: Calculate the floor reaction Rearranging,

$$N = M(g - a).$$

Using the standard terrestrial value $g = 9.8 \text{ ms}^{-2}$,

$$N = 30(9.8 - 0.1) = 30(9.7) = 291 \text{ N}.$$

The floor is yielding, so the system is not in vertical equilibrium. Its weight is 294 N, but the contact force is smaller by $Ma = 3 \text{ N}$.

Step 3: Match the action force to the options The force exerted by the system on the floor is downward and has magnitude N , by Newton's third law. Option D, 294 N, would apply to zero acceleration. Option A, 297 N, follows from replacing g by 10 ms^{-2} , which is not the standard value used here. Option C does not satisfy the force balance for the combined 30 kg mass.

Step 4: Conclusion The system exerts a force of 291 N vertically downward on the floor. Hence, the correct option is (B).

Quick Tip to Solve: For a yielding support, first combine all supported masses, then use $N = M(g - a)$ for downward acceleration; do not equate the contact force to the weight.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The electric field between the two parallel plates of a capacitor of $1.5\mu\text{F}$ capacitance drops to one third of its initial value in $6.6\mu\text{s}$ when the plates are connected by a thin wire. The resistance of this wire is _____ Ω . (Given, $\log 3 = 1.1$) [4 marks]

Solution:

Concept:

- For a capacitor discharging through resistance R , the charge falls exponentially: $Q(t) = Q_0 e^{-t/(RC)}$, where RC is the time constant.
- For fixed parallel-plate geometry the electric field is proportional to the plate charge: $E = Q/(\epsilon_0 A)$. Therefore the field and charge have the same fractional decrease.

Step 1: Convert the field decrease to a discharge ratio The plates and their separation are unchanged during discharge, so a fall of the electric field to one third also means that the voltage and charge fall to one third. Hence

$$\frac{E(t)}{E_0} = \frac{Q(t)}{Q_0} = \frac{1}{3} = e^{-t/(RC)}.$$

Taking natural logarithms gives

$$\frac{t}{RC} = \ln 3.$$

The given value $\log 3 = 1.1$ is the natural-logarithm approximation needed in this exponential relation.

Step 2: Solve for the resistance in SI units Rearrange the decay equation before substituting:

$$R = \frac{t}{C \ln 3}.$$

Here $t = 6.6 \times 10^{-6} \text{ s}$ and $C = 1.5 \times 10^{-6} \text{ F}$, so

$$R = \frac{6.6 \times 10^{-6}}{(1.5 \times 10^{-6})(1.1)} = \frac{6.6}{1.65} = 4 \Omega.$$

The factors 10^{-6} cancel because both the time and capacitance carry the micro prefix. The units are $\text{s/F} = \Omega$.

Step 3: Check the exponential factor The corresponding time constant is

$$RC = 4(1.5 \times 10^{-6}) = 6.0 \times 10^{-6} \text{ s}.$$

At the stated time, $t/(RC) = 6.6/6.0 = 1.1$, so the fraction remaining is $e^{-1.1} \simeq 1/3$, exactly the prescribed decrease. A linear discharge law would not describe a resistor connected across a capacitor.

Step 4: Conclusion The resistance of the thin wire is $\boxed{4 \Omega}$. The numerical entry is 4.

Quick Tip to Solve: In a fixed capacitor, charge, potential difference and electric field decay with the same exponential; apply the given fractional fall directly to $e^{-t/(RC)}$.

52. If three helium nuclei combine to form a carbon nucleus then the energy released in this reaction is _____ $\times 10^{-2}$ MeV. (Given $1\text{u} = 931\text{ MeV}/c^2$, atomic mass of helium = 4.002603u) [4 marks]

Solution:

Concept:

- The triple-alpha fusion reaction is $3\text{}^4_2\text{He} \rightarrow \text{}^{12}_6\text{C} + \text{energy}$. The released energy is the decrease of rest mass multiplied by c^2 .
- Neutral atomic masses may be used because three helium atoms contain six electrons, the same number as one carbon atom; their electron rest masses cancel. The atomic mass of ^{12}C is exactly 12 u.

Step 1: Calculate the total initial and final masses The three identical helium nuclei form a carbon-12 nucleus while conserving nucleon number $3 \times 4 = 12$ and charge $3 \times 2 = 6$. Using the given helium atomic mass, the total initial atomic mass is

$$m_i = 3(4.002603) = 12.007809\text{ u.}$$

The final carbon-12 atomic mass is $m_f = 12\text{ u}$. Electron masses cancel between the two sides, so the atomic-mass difference gives the nuclear energy release to the accuracy required.

Step 2: Convert the mass defect into energy The mass lost in the reaction is

$$\Delta m = m_i - m_f = 12.007809 - 12 = 0.007809\text{ u.}$$

It is positive, so the products have lower rest mass and the reaction releases energy. The supplied conversion $1\text{ u} = 931\text{ MeV}/c^2$ gives

$$Q = \Delta mc^2 = 0.007809 \times 931 = 7.270179\text{ MeV.}$$

The factors of c^2 cancel; no conversion to joules is needed.

Step 3: Express the result in the required unit The blank is the coefficient of 10^{-2} MeV , rather than the energy directly in MeV. Therefore

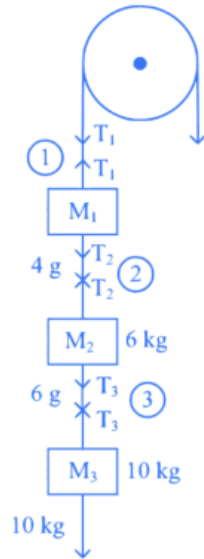
$$Q = 727.0179 \times 10^{-2}\text{ MeV.}$$

Rounding that coefficient to the nearest integer gives 727. This scaling is essential: entering 7 would represent 0.07 MeV, not the calculated fusion energy.

Step 4: Conclusion The energy released is approximately $727 \times 10^{-2}\text{ MeV}$. The required numerical entry is 727.

Quick Tip to Solve: For nuclear reactions, first check that electron counts cancel before using atomic masses; then convert the energy into the unit multiplying the answer blank.

53. Three blocks M_1, M_2, M_3 having masses 4 kg, 6 kg and 10 kg respectively are hanging from a smooth pulley using rope 1, 2 and 3 as shown in figure. The tension in the rope 1, T_1 when they are moving upward with acceleration of 2 ms^{-2} is _____ N (if $g = 10 \text{ m/s}^2$). [4 marks]



Solution:

Concept:

- For a collection of bodies moving with the same acceleration, Newton's second law can be applied to the whole collection, so the tensions between its members cancel.
- For upward acceleration a , the top supporting tension satisfies $T - Mg = Ma$, and hence $T = M(g + a)$. The top rope supports every mass below it.

Step 1: Choose the combined system below rope 1 The diagram shows rope 1 above M_1 , rope 2 between M_1 and M_2 , and rope 3 between M_2 and M_3 . Thus all three blocks are supported by rope 1. Their combined mass is

$$M = 4 + 6 + 10 = 20 \text{ kg.}$$

They move upward together with $a = 2 \text{ ms}^{-2}$. The pulley only changes the direction of the top rope; it does not reduce the load carried by this rope segment.

Step 2: Apply Newton's second law upward The external vertical forces on the three-block system are T_1 upward and its total weight Mg downward. The forces due to ropes 2 and 3 occur in internal opposite pairs and cancel in the combined equation. Therefore

$$T_1 - Mg = Ma.$$

With $g = 10 \text{ ms}^{-2}$,

$$T_1 = 20(10 + 2) = 240 \text{ N.}$$

The result exceeds the weight 200 N, as required to accelerate the blocks upward.

Step 3: Verify using the lower rope tensions For the lowest block alone,

$$T_3 = 10(10 + 2) = 120 \text{ N.}$$

For the lower two blocks together,

$$T_2 = (6 + 10)(10 + 2) = 192 \text{ N.}$$

For the top block, $T_1 - T_2 - 4g = 4a$, giving $T_1 = 192 + 40 + 8 = 240 \text{ N}$ again. This also confirms that one must not use just the mass of the top block in calculating T_1 .

Step 4: Conclusion The tension in rope 1 is $T_1 = 240 \text{ N}$. The numerical entry is 240.

Quick Tip to Solve: Cut the system mentally just below the rope whose tension is required: add every mass below that cut, then multiply by $g + a$ for upward acceleration.

54. Three capacitors of capacitances $25\mu\text{F}$, $30\mu\text{F}$ and $45\mu\text{F}$ are connected in parallel to a supply of 100 V . Energy stored in the above combination is E . When these capacitors are connected in series to the same supply, the stored energy is $\frac{9}{x}E$. The value of x is _____. [4 marks]

Solution:

Concept:

- Capacitances add in parallel: $C_p = C_1 + C_2 + C_3$. In series, the reciprocals add: $1/C_s = 1/C_1 + 1/C_2 + 1/C_3$.
- The energy stored by an equivalent capacitance connected to voltage V is $U = \frac{1}{2}CV^2$. At the same voltage, the energy ratio is therefore the capacitance ratio.

Step 1: Find the parallel equivalent capacitance All three capacitors have the same 100 V across them in the parallel combination. Hence

$$C_p = (25 + 30 + 45) \mu\text{F} = 100 \mu\text{F}.$$

The energy denoted by E in the question is

$$E = \frac{1}{2}C_p V^2 = \frac{1}{2}(100 \times 10^{-6})(100)^2 = 0.50 \text{ J.}$$

Here E means the total energy, not the electric field.

Step 2: Find the series equivalent capacitance For the series connection, the charge on each capacitor has equal magnitude and the voltage is shared between them. Using capacitances in microfarads throughout,

$$\frac{1}{C_s} = \frac{1}{25} + \frac{1}{30} + \frac{1}{45} = \frac{18 + 15 + 10}{450} = \frac{43}{450}.$$

Consequently

$$C_s = \frac{450}{43} \mu\text{F}.$$

This is smaller than each individual capacitance, as a series equivalent must be.

Step 3: Compare energies at the unchanged supply voltage The series combination is connected to the same 100 V supply, so the common factor $V^2/2$ cancels in the ratio:

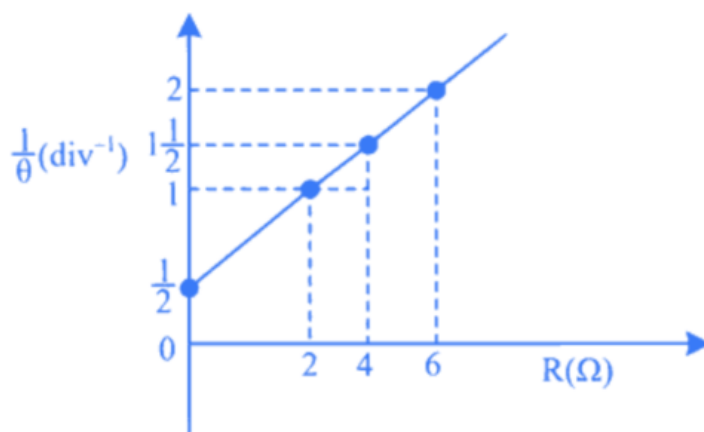
$$\frac{U_s}{E} = \frac{C_s}{C_p} = \frac{450/43}{100} = \frac{9}{86}.$$

The question writes this ratio as $9/x$, hence $9/x = 9/86$. The energy is reduced in series; using a fixed-charge energy relation would be inappropriate because the supply voltage, rather than the charge, is fixed.

Step 4: Conclusion The energy in series is $U_s = \frac{9}{86}E$, so the required value is $x = 86$.

Quick Tip to Solve: When both capacitor networks use the same voltage source, compare equivalent capacitances directly; the factor $V^2/2$ cancels from the energy ratio.

55. In the experiment to determine the galvanometer resistance by half-deflection method, the plot of $1/\theta$ vs the resistance (R) of the resistance box is shown in the figure. The figure of merit of the galvanometer is _____ $\times 10^{-1}$ A/ division. [The source has emf 2 V] [4 marks]



Solution:

Concept:

- The figure of merit k of a galvanometer is the current required for unit deflection: $I = k\theta$, where θ is measured in scale divisions.
- For a source of emf $\mathcal{E}$, an external series resistance R and galvanometer resistance G , the current is $I = \mathcal{E}/(R + G)$. Thus $1/\theta = (k/\mathcal{E})R + kG/\mathcal{E}$.

Step 1: Relate the graph slope to figure of merit Combining the deflection law $I = k\theta$ with the circuit current gives

$$k\theta = \frac{\mathcal{E}}{R + G}.$$

Taking the reciprocal of the deflection yields

$$\frac{1}{\theta} = \frac{k}{\mathcal{E}}R + \frac{kG}{\mathcal{E}}.$$

This is a straight line in the plotted variables $1/\theta$ and R . Its slope is $k/\mathcal{E}$; the nonzero intercept comes from the galvanometer's own resistance.

Step 2: Read two points and calculate the slope The graph passes through $(R, 1/\theta) = (2, 1)$ and $(6, 2)$, with R in ohms and $1/\theta$ in inverse divisions. Hence

$$s = \frac{2 - 1}{6 - 2} = \frac{1}{4} \Omega^{-1} \text{ division}^{-1}.$$

The intercept $1/2$ at $R = 0$ gives the same slope when combined with either point, providing a check that the scale has been read correctly.

Step 3: Use the source emf and scale the answer The source emf is $\mathcal{E} = 2 \text{ V}$, so

$$k = \mathcal{E}s = 2 \times \frac{1}{4} = \frac{1}{2} \text{ A/division}.$$

The units follow from $V/\Omega = \text{A}$. In the units requested by the blank,

$$0.5 \text{ A/division} = 5 \times 10^{-1} \text{ A/division}.$$

The slope, rather than the reciprocal slope or the intercept alone, determines the figure of merit when the emf is known.

Step 4: Conclusion The galvanometer's figure of merit is $5 \times 10^{-1} \text{ A/division}$, so the required numerical entry is 5.

Quick Tip to Solve: For a graph of reciprocal deflection against series resistance, multiply the slope by the source emf: $k = \mathcal{E} \Delta(1/\theta)/\Delta R$.

56. The density and breaking stress of a wire are $6 \times 10^4 \text{ kg/m}^3$ and $1.2 \times 10^8 \text{ N/m}^2$ respectively. The wire is suspended from a rigid support on a planet where acceleration due to gravity is $\frac{1}{3}$ rd of the value on the surface of earth. The maximum length of the wire with breaking is _____ m (take, $g = 10 \text{ m/s}^2$). [4 marks]

Solution:

Concept:

- The maximum tension in a uniform hanging wire is at its upper end, where the support carries the weight of the entire wire.
- For density ρ , length L , cross-sectional area A and local gravity g_p , the top stress is $\sigma = T/A = \rho L g_p$. The limiting length occurs when this equals the breaking stress.

Step 1: Find the gravitational acceleration on the planet The local gravitational acceleration is one

third of the given Earth value. Therefore

$$g_p = \frac{g}{3} = \frac{10}{3} \text{ m s}^{-2}.$$

For a uniform wire of area A and length L , its mass is $m = \rho AL$. The uppermost cross section carries its complete weight, so its tension is

$$T_{\text{top}} = mg_p = \rho ALg_p.$$

Sections farther down support less wire and consequently experience a smaller tensile stress.

Step 2: Set the top stress equal to the breaking stress Dividing the top tension by its cross-sectional area gives

$$\sigma_{\text{top}} = \frac{T_{\text{top}}}{A} = \rho Lg_p.$$

The area cancels because both the weight and the load-bearing area scale in the same proportion. At the limiting length, the uppermost section just reaches the given breaking stress $\sigma_b = 1.2 \times 10^8 \text{ N m}^{-2}$. Thus

$$L_{\text{max}} = \frac{\sigma_b}{\rho g_p}.$$

Step 3: Substitute the given material properties With $\rho = 6 \times 10^4 \text{ kg m}^{-3}$,

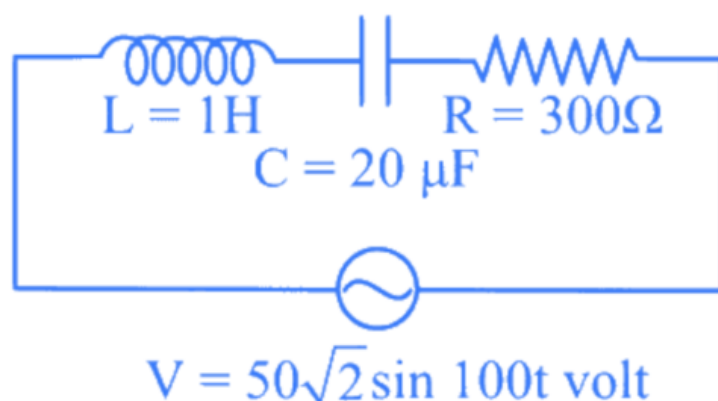
$$L_{\text{max}} = \frac{1.2 \times 10^8}{(6 \times 10^4)(10/3)} = \frac{1.2 \times 10^8}{2.0 \times 10^5} = 600 \text{ m}.$$

The denominator is a weight density, with units N m^{-3} , so the ratio is a length. A wire longer than this would have top stress above the breaking stress; at this length it is at the threshold.

Step 4: Conclusion The limiting length of the suspended wire is 600 m. The numerical entry is 600.

Quick Tip to Solve: For a uniform wire failing under its own weight, use $L_{\text{max}} = \sigma_b/(\rho g)$; its diameter cancels, and weaker gravity allows a greater limiting length.

57. An ac source is connected in given series LCR circuit. The rms potential difference across the capacitor of $20\mu\text{F}$ is _____ V. [4 marks]



Solution:

Concept:

- In a series LCR circuit the same rms current flows through all three components. The reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$.
- The impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, the rms current is $I_{\text{rms}} = V_{\text{rms}}/Z$, and the capacitor voltage is $V_{C,\text{rms}} = I_{\text{rms}}X_C$.

Step 1: Read the source and calculate the reactances The source is $v = 50\sqrt{2} \sin(100t)$ V, so its angular frequency is 100 rad s^{-1} , not 100 Hz. Its rms voltage is 50 V. With $L = 1 \text{ H}$ and $C = 20 \times 10^{-6} \text{ F}$,

$$X_L = 100(1) = 100 \Omega, \quad X_C = \frac{1}{100(20 \times 10^{-6})} = 500 \Omega.$$

Step 2: Find the current and capacitor voltage The inductive and capacitive voltage phasors oppose each other, so subtract their reactances before combining with the resistance:

$$Z = \sqrt{300^2 + (100 - 500)^2} = 500 \Omega.$$

Therefore,

$$I_{\text{rms}} = \frac{50}{500} = 0.10 \text{ A}, \quad V_{C,\text{rms}} = 0.10(500) = 50 \text{ V}.$$

The capacitor voltage is found from its own reactance; the resistance alone cannot be used to find the circuit current.

Step 3: Conclusion The rms potential difference across the capacitor is 50 V. The numerical entry is 50.

Quick Tip to Solve: First convert the sinusoidal source amplitude to rms; then multiply the common series rms current by the capacitor reactance.

58. A body moves on a frictionless plane starting from rest. If S_n is distance moved between $t = n - 1$ and $t = n$ and S_{n-1} is distance moved between $t = n - 2$ and $t = n - 1$, then the ratio $\frac{S_{n-1}}{S_n}$ is $(1 - \frac{2}{x})$ for $n = 10$. The value of x is _____. [4 marks]

Solution:

Concept:

- For motion from rest with uniform acceleration a , displacement after time t is $s(t) = \frac{1}{2}at^2$. Distances in consecutive one-second intervals are differences of these displacements.
- In the intended uniformly accelerated motion on the frictionless plane, the common acceleration cancels in the requested ratio; its numerical value is unnecessary.

Step 1: Express the two successive distances Taking time in seconds, the distance in the interval from $n - 1$ to n is

$$S_n = s(n) - s(n - 1) = \frac{a}{2}[n^2 - (n - 1)^2] = \frac{a}{2}(2n - 1).$$

Similarly, the preceding interval runs from $n - 2$ to $n - 1$, giving

$$S_{n-1} = \frac{a}{2}[(n - 1)^2 - (n - 2)^2] = \frac{a}{2}(2n - 3).$$

These are distances in individual seconds, rather than the total distances measured from the start.

Step 2: Form the ratio and compare with the given expression Cancel the common nonzero factor $a/2$:

$$\frac{S_{n-1}}{S_n} = \frac{2n - 3}{2n - 1} = 1 - \frac{2}{2n - 1}.$$

For $n = 10$, the numerator and denominator are 17 and 19. Comparing $17/19 = 1 - 2/19$ with $1 - 2/x$ gives $x = 19$. The ratio is less than one, as expected because the body covers more distance in each later second.

Step 3: Conclusion The required value is $x = 19$, a dimensionless numerical entry.

Quick Tip to Solve: Distances in successive seconds from rest follow the odd-number sequence; the ninth and tenth distances are proportional to 17 and 19.

59. In Young's double slit experiment, carried out with light of wavelength $5000 \text{ \AA}$, the distance between the slits is 0.3 mm and the screen is at 200 cm from the slits. The central maximum is at $x = 0 \text{ cm}$. The value of x for third maxima is _____ mm. [4 marks]

Solution:

Concept:

- A bright fringe occurs when the path difference between the two coherent waves is an integral multiple of the wavelength: $d \sin \theta = m\lambda$.
- For a distant screen and small angles, the displacement of the m th bright fringe from the central maximum is $x_m = m\lambda D/d$, where D is the slit-screen distance and d is slit separation.

Step 1: Convert all lengths to SI units The wavelength is 5000 angstrom , and one angstrom is 10^{-10} m .

Hence

$$\lambda = 5.0 \times 10^{-7} \text{ m}, \quad d = 0.3 \times 10^{-3} = 3.0 \times 10^{-4} \text{ m}, \quad D = 2.00 \text{ m}.$$

The central maximum is order zero. The third maximum away from the centre therefore corresponds to $m = 3$, not $m = 2$.

Step 2: Calculate the third bright-fringe position The separation of adjacent bright fringes is

$$\beta = \frac{\lambda D}{d} = \frac{(5.0 \times 10^{-7})(2.00)}{3.0 \times 10^{-4}} = \frac{10}{3} \text{ mm}.$$

The third maximum is three fringe widths from the central maximum:

$$|x_3| = 3\beta = 10 \text{ mm}.$$

A symmetric maximum also exists on the other side of the centre. The numerical blank asks for the positive displacement value; the two signed coordinates are $\pm 10 \text{ mm}$.

Step 3: Conclusion The third maximum is at a distance 10 mm from the central maximum. The required numerical entry is 10.

Quick Tip to Solve: Count the central bright fringe as order zero and convert angstrom to metres before evaluating $m\lambda D/d$.

60. A 2A current carrying straight metal wire of resistance 1Ω , resistivity $2 \times 10^{-6}\Omega\text{m}$, area of cross-section 10 mm^2 and mass 500 g is suspended horizontally in mid air by applying a uniform magnetic field $\vec{B}$. The magnitude of B is _____ $\times 10^{-1} \text{ T}$ (given, $g = 10 \text{ m/s}^2$). [4 marks]

Solution:

Concept:

- A uniform wire of length ℓ , resistivity ρ and cross-sectional area A has resistance $R = \rho\ell/A$.
- The magnetic force on a straight wire carrying current I perpendicular to a uniform field is $F = BIl$. For horizontal suspension the upward magnetic force balances the downward weight mg .

Step 1: Determine the wire length from its resistance Convert area and mass to SI units:

$$A = 10 \text{ mm}^2 = 10 \times 10^{-6} \text{ m}^2, \quad m = 0.500 \text{ kg}.$$

Rearranging the resistance formula gives

$$\ell = \frac{RA}{\rho} = \frac{1(10 \times 10^{-6})}{2 \times 10^{-6}} = 5 \text{ m}.$$

The millimetre-to-metre conversion is squared for an area; using 10^{-3} instead of 10^{-6} would give an incorrect length.

Step 2: Balance the magnetic force against weight Choose the field perpendicular to the horizontal

wire so that the magnetic force points upward. In equilibrium,

$$BI\ell = mg, \quad B = \frac{0.500(10)}{2(5)} = 0.50 \text{ T}.$$

The field must act in the direction that gives an upward force by the cross-product rule. A field parallel to the wire would exert no force and could not support its weight. The standard supporting configuration uses the perpendicular field above.

Step 3: Conclusion The required field is $0.50 \text{ T} = 5 \times 10^{-1} \text{ T}$. Hence the number to enter in the blank is 5.

Quick Tip to Solve: Find the wire length using $R = \rho\ell/A$ first, then balance $BI\ell$ with mg ; square the length conversion when converting area.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Cis form of alkene is found to be more polar than the trans form.

Reason (R) : Dipole moment of trans isomer of 2-butene is zero.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) is true but (R) is false

(B) is false but (R) is true

(C) Both (A) and (R) are true and (R) is the correct explanation of (A)

(D) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Correct Answer: (C) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- Molecular dipole moment is the vector sum of the individual bond moments. Equal opposing contributions cancel, whereas components directed towards the same side reinforce one another.
- In the usual comparison of corresponding cis and trans disubstituted alkenes, the cis arrangement has a nonzero resultant while the symmetric trans arrangement can have complete cancellation.

Step 1: Evaluate the assertion In the cis form of an alkene such as 2-butene, the corresponding substituent groups lie on the same side of the double bond. Their dipole contributions do not cancel completely. The resultant dipole moment is therefore greater than that of the corresponding symmetric trans form. Thus Assertion (A) is true in the stated cis-trans comparison.

Step 2: Evaluate the reason In trans-2-butene the two identical methyl groups are on opposite sides of the double bond. The molecule has a symmetric arrangement, so the equal dipole contributions cancel vectorially. Consequently its net dipole moment is zero, and Reason (R) is true.

Step 3: Connect the two statements and select the option The vanishing resultant in the trans form expresses the cancellation absent in the cis form. This accounts for the greater polarity of the cis isomer in this comparison, so the reason explains the assertion. Options (A) and (B) each incorrectly declare one true statement false. Option (D) acknowledges both truths but misses their connection through dipole cancellation.

Step 4: Conclusion The assertion and the reason are both true, and the reason correctly explains the assertion. Hence, the correct option is (C).

Quick Tip to Solve: Add bond moments as vectors: a symmetric trans arrangement cancels the contributions, while the corresponding cis arrangement retains a resultant.

62. Given below are two statement :

Statements I : Bromination of phenol in solvent with low polarity such as CHCl_3 or CS_2 requires Lewis acid catalyst.

Statements II : The Lewis acid catalyst polarises the bromine to generate Br^+ .

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Statement I is true but Statement II is false
- (B) Both Statement I and Statement II are true
- (C) Statement I is false but Statement II is true
- (D) Both Statement I and Statement II are false

Correct Answer: (C) Statement I is false but Statement II is true

Solution:

Concept:

- The hydroxyl group activates the benzene ring of phenol by resonance donation of an oxygen lone pair. Electron density increases particularly at the ortho and para positions.
- Lewis acids such as FeBr_3 can polarise Br_2 and generate an electrophilic brominating species in ordinary aromatic bromination. A sufficiently activated ring need not require this catalyst.

Step 1: Test Statement I using the activation of phenol Phenol reacts with bromine without a Lewis acid catalyst because its ring is already strongly activated by the hydroxyl group. In low-polarity solvents such as CHCl_3 or CS_2 , controlled bromination gives ortho- and para-bromophenol, with the para isomer usually predominant. The solvent changes the substitution conditions and product distribution; it does not create a requirement for a Lewis acid. Statement I is false.

Step 2: Test Statement II independently A Lewis acid accepts electron density from the bromine molecule, polarising the bromine-bromine bond. This creates an electrophilic brominating species, conventionally represented as Br^+ , that can attack an aromatic ring. This is the usual function of the catalyst in bromination of less activated aromatic substrates. Statement II is therefore true as a description of Lewis acid action, even though phenol does not need that catalyst.

Step 3: Eliminate the other combinations The first statement is false and the second is true. Thus

options (A) and (B) are excluded because they call Statement I true, while option (D) incorrectly calls Statement II false. A true general statement about catalyst action does not make the catalyst necessary for every aromatic substrate.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (C).

Quick Tip to Solve: Separate the catalyst's general role from whether it is needed: phenol's activated ring brominates without a Lewis acid.

63. The incorrect postulates of the Dalton's atomic theory are :

- (A) Atoms of different elements differ in mass.
- (B) Matter consists of divisible atoms.
- (C) Compounds are formed when atoms of different element combine in a fixed ratio.
- (D) All the atoms of given element have different properties including mass.
- (E) Chemical reactions involve reorganisation of atoms.

Choose the correct answer from the options given below :

[4 marks]

- (A) (B), (D) only
- (B) (A), (B), (D) only
- (C) (B), (D), (E) only
- (D) (C), (D), (E) only

Correct Answer: (A) (B), (D) only

Solution:

Concept:

- Dalton's theory describes matter as made of indivisible atoms. It assigns identical mass and properties to the atoms of one element and different masses to atoms of different elements.
- The question asks which statements are incorrect as postulates of Dalton's theory. Later discoveries of subatomic particles or isotopes must not be substituted for what Dalton originally proposed.

Step 1: Compare the statements about atoms with Dalton's postulates Statement (A) says atoms of different elements differ in mass. This is a postulate of Dalton's theory, so it is not selected. Statement (B) calls atoms divisible, directly contradicting his postulate of indivisible atoms. Statement (D) says atoms of a given element have different properties including mass; Dalton instead proposed that these atoms have identical mass and properties. Thus (B) and (D) are incorrect descriptions of his postulates.

Step 2: Check the statements about compounds and reactions Statement (C) says that atoms of different elements combine in a fixed ratio to form compounds. This agrees with Dalton's account of chemical combination. Statement (E) says chemical reactions involve reorganisation of atoms, also consistent with the theory: atoms are rearranged rather than created or destroyed in a chemical reaction. Neither statement belongs in the incorrect set.

Step 3: Match the selected set to the options Only the statement letters (B) and (D) have been selected. The second option wrongly includes statement (A); the third wrongly includes statement (E);

and the fourth includes the correct statements (C) and (E). The first option is the sole combination containing exactly the two contradictions.

Step 4: Conclusion The incorrect postulates are the statements labelled B and D only. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish an incorrect statement of Dalton's theory from a limitation discovered later; answer using his original postulates.

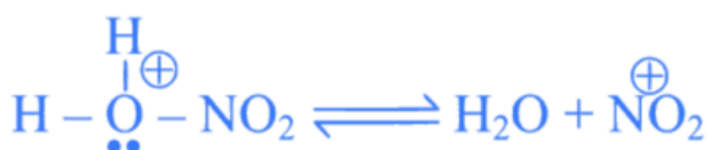
64. Given below are two statements :

Statement I : Nitration of benzene involves the following step -

Statement II : Use of Lewis base promotes the electrophilic substitution of benzene.

In the light of the above statements, choose the most appropriate answer from the options given below :

[4 marks]



- (A) Both Statement I and Statement II are incorrect
(B) Statement I is correct but Statement II is incorrect
(C) Statement I is incorrect but Statement II is correct
(D) Both Statement I and Statement II are correct

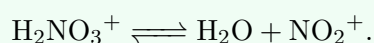
Correct Answer: (B) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- Nitration is an electrophilic aromatic substitution. The electrophile is the nitronium ion, NO_2^+ , which attacks the electron-rich benzene ring.
- Concentrated sulphuric acid protonates nitric acid. Loss of water from protonated nitric acid produces the electrophile; an electron-pair acceptor promotes electrophile formation.

Step 1: Check the nitronium-ion formation step The species drawn on the left is protonated nitric acid: the oxygen bonded to nitrogen also carries two hydrogen atoms and a positive charge. Breaking its nitrogen-oxygen bond with the electron pair going to oxygen releases neutral water and the nitronium ion. Thus the displayed step is chemically consistent with nitration:



The atoms and total positive charge are conserved. Statement I is therefore correct; it describes generation of the electrophile rather than direct substitution of a hydrogen on benzene.

Step 2: Evaluate the role of a Lewis base A Lewis base donates an electron pair, whereas a Lewis acid accepts one. Electrophilic substitution is promoted by reagents that generate or strengthen

an electrophile, such as Lewis acids in halogenation or Friedel–Crafts reactions. In the nitrating mixture, sulphuric acid acts as the proton donor toward nitric acid. A Lewis base would tend to bind an electrophile or consume the acid, and the claim that a Lewis base promotes this substitution is incorrect. Statement II confuses Lewis bases with Lewis acids. Consequently the choices making I false are rejected, and the choice making both statements true is also rejected.

Step 3: Conclusion Statement I is correct, but Statement II is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: For nitration, first identify NO_2^+ . Do not confuse an electron-pair-accepting Lewis acid catalyst with an electron-pair-donating Lewis base.

65. Which of the following gives a positive test with ninhydrin ?

[4 marks]

- (A) Polyvinyl chloride
- (B) Cellulose
- (C) Egg albumin
- (D) Starch

Correct Answer: (C) Egg albumin

Solution:

Concept:

- Ninhydrin is used to detect amino acids and proteins containing accessible free amino groups. A positive reaction typically develops a violet or blue-violet colour.
- Proteins are polymers of amino acids linked by peptide bonds. Carbohydrates and a polymer of vinyl chloride do not contain the amino-acid functionality responsible for this test.

Step 1: Identify the functional group being tested The ninhydrin test is a test for amino functionality, rather than a general test for every natural polymer. Free alpha-amino acids react with ninhydrin through oxidative deamination and related condensation steps to produce the characteristic coloured product. Proteins can also give this reaction because they possess available amino groups, including terminal amino groups and amino-containing side chains. Therefore a protein among the listed substances is the appropriate choice; the mere presence of a polymeric chain or hydroxyl groups is insufficient.

Step 2: Compare all four substances Egg albumin is a protein, so its amino-acid residues provide the amino functionality needed for a positive ninhydrin test. Polyvinyl chloride contains carbon, hydrogen and chlorine, with no amino groups. Cellulose is a polysaccharide built from beta-glucose units, and starch is a polysaccharide built from alpha-glucose units. Both carbohydrate polymers contain many hydroxyl groups but lack the required amino groups. Thus neither carbohydrate is selected, even though both are biological macromolecules. This functional-group comparison selects egg albumin without confusing the ninhydrin test with iodine testing for starch or other carbohydrate tests.

Step 3: Conclusion Egg albumin gives a positive ninhydrin test because it is a protein with accessible amino groups. Hence, the correct option is (C).

Quick Tip to Solve: Associate ninhydrin with amino groups, iodine with starch, and the biuret reaction with peptide bonds; identify the functionality before selecting a biomolecule.

66. Given below are two statements: One is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A) : Enthalpy of neutralisation of strong monobasic acid with strong monoacidic base is always -57 kJ mol^{-1}

Reason (R) : Enthalpy of neutralisation is the amount of heat liberated when one mole of H^+ ions furnished by acid combine with one mole of OH^- ions furnished by base to form one mole of water.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) (A) is true but (R) is false

(B) Both (A) and (R) are true and (R) is the correct explanation of (A)

(C) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(D) (A) is false but (R) is true

Correct Answer: (B) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- In dilute aqueous solution, a strong acid and a strong base are essentially completely ionised. Their neutralisation reduces to the same net ionic reaction, $\text{H}^+(\text{aq}) + \text{OH}^-(\text{aq}) \longrightarrow \text{H}_2\text{O}(\text{l})$.
- The molar enthalpy change for this reaction under the usual conditions is approximately -57 kJ mol^{-1} , per mole of water formed. The negative sign denotes release of heat to the surroundings.

Step 1: Evaluate the assertion A strong monobasic acid supplies one mole of hydrogen ions per mole of acid, and a strong monoacidic base supplies one mole of hydroxide ions per mole of base. For dilute solutions, no appreciable additional ionisation of a weak electrolyte is required during the reaction. The reacting ions are therefore the same for different strong acid–strong base pairs. For example, removing the spectator sodium and chloride ions from hydrochloric acid reacting with sodium hydroxide gives the net reaction stated in the concept block. In this standard textbook context, the assertion is true: the neutralisation enthalpy is approximately the quoted value.

Step 2: Evaluate the reason The reason correctly describes neutralisation on the scale of one mole of water: one mole of hydrogen ions from the acid combines with one mole of hydroxide ions from the base. This chemical process liberates heat, so the enthalpy change of the reacting system is negative. The definition is therefore true. It is not claiming that the spectator ions undergo a separate bond-forming reaction.

Step 3: Connect the common ionic reaction with the fixed enthalpy Because strong acids and strong bases furnish these ions completely, every such neutralisation in dilute aqueous solution represents the same ionic change. This is precisely why its molar enthalpy has the common approximate value rather than depending on the identities of the spectator ions. The reason thus explains the assertion. Options denying either statement, or denying the explanatory connection, do not describe the standard neutralisation process.

Step 4: Conclusion The assertion and the reason are true, and the reason correctly explains the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Cancel spectator ions first. A strong acid–strong base neutralisation gives the same ionic equation; weak-electrolyte ionisation introduces an additional enthalpy term.

67. Given below are two statements:

Statement I: In group 13, the stability of +1 oxidation state increases down the group.

Statement II : The atomic size of gallium is greater than that of aluminium.

In the light of the above statements, choose the most appropriate answer from the options given below :

[4 marks]

(A) Statement I is incorrect but Statement II is correct

(B) Statement I is correct but Statement II is incorrect

(C) Both Statement I and Statement II are correct

(D) Both Statement I and Statement II are incorrect

Correct Answer: (B) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- Group 13 elements have outer configuration ns^2np^1 . The lower +1 oxidation state retains the ns^2 pair, while the +3 state involves all three outer electrons.
- The inert-pair effect becomes stronger down the group. Filled inner d orbitals shield nuclear charge inefficiently, producing an important size irregularity between aluminium and gallium.

Step 1: Check the oxidation-state trend When the two outer s electrons remain less available for bonding, the oxidation state obtained by using only the p electron becomes more stable. This tendency increases toward the heavier elements of group 13. Hence the stability of the +1 state increases down the group, becoming especially prominent for thallium. Statement I is therefore correct. This is a relative stability trend; it does not assert that every lighter element must have +1 as its most stable or only oxidation state.

Step 2: Account for the size of gallium A simple increase in principal shell number would suggest that gallium should be larger than aluminium. However, gallium has a filled inner 3d subshell between the noble-gas core and its outer electrons. These d electrons shield the increased nuclear charge relatively poorly, so the outer electrons experience a greater effective nuclear attraction and are pulled inward. This d -block contraction makes gallium slightly smaller than aluminium in the standard atomic-radius comparison. The claim that its atomic size is greater is therefore incorrect.

Step 3: Match the truth values to the choices The statements concern two different effects: increasing retention of the s -electron pair, and incomplete shielding by filled d orbitals. Combining them gives I true and II false. The first and fourth options wrongly reject the inert-pair trend, while the third ignores the gallium–aluminium size anomaly. Only the second option has both truth values in the correct order.

Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: For group 13, remember two separate exceptions to simple trends: heavier elements favour +1, and gallium is slightly smaller than aluminium because of poor 3d shielding.

68. The metal that shows highest and maximum number of oxidation state is :

[4 marks]

- (A) Co
- (B) Mn
- (C) Fe
- (D) Ti

Correct Answer: (B) Mn

Solution:

Concept:

- Transition metals show variable oxidation states because their $(n-1)d$ and ns electrons have sufficiently similar energies to participate in bonding.
- Manganese has ground-state configuration $[\text{Ar}] 3d^5 4s^2$. Its seven electrons outside the argon core allow the highest familiar oxidation state +7 in the first transition series.

Step 1: Identify the available valence-electron count Manganese lies near the middle of the 3d series and has five d electrons and two s electrons. In compounds with strongly electronegative oxygen or fluorine, these electrons can participate to a large extent in bonding. In the permanganate ion, assigning oxygen its usual oxidation state of -2 gives

$$x + 4(-2) = -1 \Rightarrow x = +7.$$

This provides a concrete example of the high manganese oxidation state rather than treating the electron configuration alone as proof of every possible compound.

Step 2: Compare manganese with the other metals Manganese commonly exhibits a wide range including +2, +3, +4, +5, +6 and +7; examples include manganese(II) salts, manganese dioxide, manganate and permanganate. Titanium is mainly associated with +2, +3 and +4, iron predominantly with +2 and +3, and cobalt predominantly with +2 and +3 in the usual first-row comparison. These competitors neither reach the characteristic +7 state nor display the same broad familiar range as manganese. The question asks for the metal satisfying both a high oxidation state and a maximum variety, so a metal merely known for two variable states is insufficient. Manganese satisfies both requirements among the listed elements.

Step 3: Conclusion Manganese, Mn, shows the highest oxidation state and the widest familiar range among the listed metals. Hence, the correct option is (B).

Quick Tip to Solve: Use permanganate to recall Mn(+7): four oxygens contribute -8 and the ion has charge -1, so manganese must contribute +7.

69. Number of σ and π bonds present in ethylene molecule is respectively :

[4 marks]

- (A) 3 and 1
- (B) 5 and 1
- (C) 4 and 1
- (D) 5 and 2

Correct Answer: (B) 5 and 1

Solution:

Concept:

- Every covalent single bond consists of one sigma bond. A double bond consists of one sigma bond and one pi bond, not two pi bonds.
- Ethylene (ethene) has formula C_2H_4 and connectivity $H_2C=CH_2$. Each carbon is sp^2 -hybridised, leaving one unhybridised p orbital for sideways overlap.

Step 1: Count the carbon–hydrogen sigma bonds Each of the two carbon atoms is bonded to two hydrogen atoms. This gives four separate carbon–hydrogen single bonds in the molecule. Each is formed by head-on overlap of a carbon hybrid orbital with a hydrogen 1s orbital, so each contributes one sigma bond. Thus the carbon–hydrogen contribution alone is four sigma bonds. Hydrogen forms only one bond here, and there are no additional hydrogen–hydrogen bonds to include.

Step 2: Separate the components of the carbon double bond The carbon–carbon double bond contains one sigma bond formed by head-on overlap of carbon hybrid orbitals. Its second component is a pi bond formed by sideways overlap of the two unhybridised p orbitals. Therefore the totals are

$$N_{\sigma} = 4 + 1 = 5, \quad N_{\pi} = 1.$$

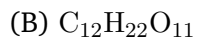
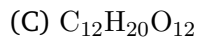
The option giving four sigma bonds omits the sigma component of the carbon–carbon bond. The choice with two pi bonds mistakes a double bond for two pi bonds; one sigma component is always present. The choice giving three sigma bonds undercounts the four carbon–hydrogen bonds even before the carbon–carbon bond is added. Only five sigma and one pi correctly counts all six bonding components, in the order requested.

Step 3: Conclusion Ethylene contains five σ bonds and one π bond, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Count one sigma bond for each connected atom pair, then add one pi bond for each double bond and two pi bonds for each triple bond.

70. An organic compound has 42.1% carbon, 6.4% hydrogen and remainder is oxygen. If its molecular weight is 342, then its molecular formula is :

[4 marks]



Correct Answer: (B) $C_{12}H_{22}O_{11}$

Solution:

Concept:

- An element's mass per mole of compound equals its mass fraction multiplied by the molar mass. Dividing by that element's atomic mass gives the number of its atoms in the molecular formula.
- Use atomic masses $C = 12$, $H = 1$ and $O = 16$. The percentages quoted to one decimal place should yield atom counts close to whole numbers.

Step 1: Find the oxygen percentage and atom counts Only carbon, hydrogen and oxygen are present, so the oxygen percentage is

$$100 - 42.1 - 6.4 = 51.5\%.$$

For one mole of molecules, the mass is 342 g. The carbon, hydrogen and oxygen atom counts are therefore

$$n_C = \frac{0.421(342)}{12} = 11.9985 \simeq 12,$$

$$n_H = \frac{0.064(342)}{1} = 21.888 \simeq 22,$$

$$n_O = \frac{0.515(342)}{16} = 11.0081 \simeq 11.$$

The small departures from integers result from the stated rounding of the percentages. They do not justify a fractional-atom molecular formula.

Step 2: Check the resulting formula against all given data The integer atom counts give $C_{12}H_{22}O_{11}$. Its formula mass is

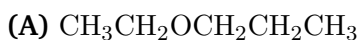
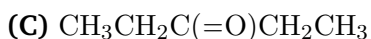
$$12(12) + 22(1) + 11(16) = 144 + 22 + 176 = 342.$$

The corresponding percentages are approximately 42.11% carbon and 6.43% hydrogen, agreeing with 42.1% and 6.4% after rounding. Its oxygen fraction is correspondingly about 51.46%. The formulas in options A, C and D have formula masses 348, 356 and 342 respectively; even though the fourth also has mass 342, it has only 11 carbons and 18 hydrogens, giving about 38.6% carbon and 5.3% hydrogen. Thus molecular mass alone cannot choose the answer: the composition check excludes the fourth as well as the first and third options.

Step 3: Conclusion The molecular formula is $C_{12}H_{22}O_{11}$, which satisfies both the composition and molecular mass. Hence, the correct option is (B).

Quick Tip to Solve: When molar mass is supplied, calculate atom counts directly from percentage times molar mass divided by atomic mass, then verify the candidate formula mass.

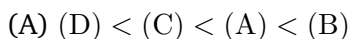
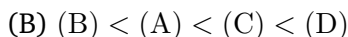
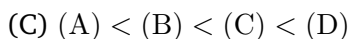
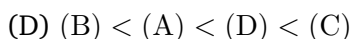
71. For the compounds:



The increasing order of boiling point is:

Choose the correct answer from the options given below:

[4 marks]



Correct Answer: (B) $(\text{B}) < (\text{A}) < (\text{C}) < (\text{D})$

Solution:

Concept:

- Boiling requires molecules to overcome intermolecular attractions. Stronger attractions usually give a higher boiling point when molecular sizes are comparable.
- These five-carbon compounds illustrate London dispersion forces, permanent dipole interactions and intermolecular hydrogen bonding. An alcohol can hydrogen-bond to itself; an ether or ketone cannot act as a hydrogen-bond donor.

Step 1: Identify the dominant attractions Compound B is pentane, a nonpolar hydrocarbon, so its molecules interact mainly through London forces. Compound A is ethyl propyl ether: its polar C–O bonds add dipole interactions, raising its boiling point above pentane. Compound C is pentan-3-one, whose strongly polar carbonyl group gives stronger dipole interactions than the ether.

Step 2: Compare the alcohol and eliminate alternatives Compound D is pentan-2-ol. Its O–H groups produce intermolecular hydrogen bonding, making its boiling point higher than that of the ketone. Thus B must come first and D last, eliminating the first and third sequences; the fourth incorrectly puts the alcohol below the ketone. The complete increasing sequence is



Step 3: Conclusion The sequence derived above matches the second listed sequence. Hence, the correct option is (B).

Quick Tip to Solve: For comparable five-carbon molecules, compare functional groups: alkane, ether, ketone, then alcohol in increasing boiling point.

72. The correct order of ligands arranged in increasing field strength.

[4 marks]

- (A) $F^- < Br^- < I^- < NH_3$
(B) $H_2O < ^-OH < CN^- < NH_3$
(C) $Br^- < F^- < H_2O < NH_3$
(D) $Cl^- < ^-OH < Br^- < CN^-$

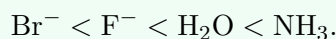
Correct Answer: (C) $Br^- < F^- < H_2O < NH_3$

Solution:

Concept:

- The spectrochemical series orders ligands by the crystal-field splitting they produce. Increasing field strength means increasing splitting energy Δ_o for a comparable octahedral complex.
- The relevant part is $I^- < Br^- < Cl^- < F^- < OH^- < H_2O < NH_3 < CN^-$. This is an empirical ligand order, not simply an order of charge or electronegativity.

Step 1: Select the increasing subsequence Among the four ligands in the third sequence, bromide is a weak-field halide, fluoride is stronger than bromide, water is stronger than fluoride, and ammonia is stronger than water. Removing the intervening ligands from the series gives



Every adjacent comparison therefore increases the field strength.

Step 2: Check the competing sequences The first sequence reverses the halide order: iodide is weaker than bromide and fluoride, not stronger. The second incorrectly places hydroxide above water and ammonia above cyanide. The fourth incorrectly puts bromide above hydroxide. These violations rule out the other three choices without requiring a numerical splitting energy.

Step 3: Conclusion The increasing field-strength order is $Br^- < F^- < H_2O < NH_3$. Hence, the correct option is (C).

Quick Tip to Solve: Memorise the halides as iodide < bromide < chloride < fluoride, and keep water below ammonia and cyanide.

73. The number of neutrons present in the more abundant isotope of boron is ' x '. Amorphous boron upon heating with air forms a product, in which the oxidation state of boron is ' y '. The value of $x + y$ is _____ . [4 marks]

(A) 9

(B) 6

(C) 4

(D) 3

Correct Answer: (A) 9

Solution:

Concept:

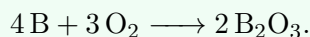
- For an isotope ${}^A_Z\text{B}$, the neutron number is $N = A - Z$. Boron has atomic number $Z = 5$; its more abundant naturally occurring isotope is boron-11.
- Amorphous boron burns on heating in air to form boron(III) oxide, B_2O_3 . In an ordinary oxide oxygen has oxidation state -2 , and the oxidation states sum to zero.

Step 1: Find the neutron number Use the mass number 11 of the more abundant isotope, rather than 10 of the less abundant isotope. Subtract the five protons:

$$x = N = 11 - 5 = 6.$$

Changing an isotope changes the neutron count; it does not change boron's atomic number or chemical identity.

Step 2: Determine the oxidation state and sum Oxidation in air is represented by



Let the oxidation state of each boron atom in the oxide be y . Electrical neutrality gives

$$2y + 3(-2) = 0, \quad y = +3.$$

Consequently $x + y = 6 + 3 = 9$. The value 6 counts only the neutrons, while 3 counts only the oxidation state; neither answers the requested sum. The remaining value 4 also fails this calculation.

Step 3: Conclusion Thus $x = 6$, $y = +3$, and $x + y = 9$. Hence, the correct option is (A).

Quick Tip to Solve: Use boron-11 for natural abundance, then apply charge neutrality to the oxide before adding the two numbers.

74. Which one of the following complexes will exhibit the least paramagnetic behaviour ? [Atomic number, Cr = 24, Mn = 25, Fe = 26, Co = 27] [4 marks]

- (A) $[\text{Fe}(\text{H}_2\text{O})_6]^{2+}$
(B) $[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$
(C) $[\text{Co}(\text{H}_2\text{O})_6]^{2+}$
(D) $[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$

Correct Answer: (C) $[\text{Co}(\text{H}_2\text{O})_6]^{2+}$

Solution:

Concept:

- For these first-row transition-metal aqua complexes, water is a weak-field ligand and the octahedral complexes are high spin. Neutral water leaves each metal in oxidation state +2.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)} \mu_B$, where n is the number of unpaired electrons. Fewer unpaired electrons means less paramagnetic behaviour in this comparison.

Step 1: Obtain the metal-ion configurations Remove the two outer $4s$ electrons to form the divalent ions. The resulting configurations are $\text{Fe}^{2+} : 3d^6$, $\text{Mn}^{2+} : 3d^5$, $\text{Co}^{2+} : 3d^7$, and $\text{Cr}^{2+} : 3d^4$. In a weak octahedral field, Hund's rule gives high-spin occupancy rather than forcing early pairing in the lower set.

Step 2: Count unpaired electrons The high-spin occupations and unpaired counts are

$$\begin{aligned}d^6 : t_{2g}^4 e_g^2, \quad n &= 4, \\d^5 : t_{2g}^3 e_g^2, \quad n &= 5, \\d^7 : t_{2g}^5 e_g^2, \quad n &= 3, \\d^4 : t_{2g}^3 e_g^1, \quad n &= 4.\end{aligned}$$

Cobalt(II) has the smallest count, three. Iron(II) and chromium(II) each have four, and manganese(II) has five; all three are more paramagnetic in the spin-count comparison.

Step 3: Conclusion $[\text{Co}(\text{H}_2\text{O})_6]^{2+}$ exhibits the least paramagnetic behaviour among the listed complexes. Hence, the correct option is (C).

Quick Tip to Solve: First determine oxidation state and d-electron count, then use high-spin filling for water before counting unpaired electrons.

75. Molar ionic conductivities of divalent cation and anion are $57 \text{ S cm}^2 \text{ mol}^{-1}$ and $73 \text{ S cm}^2 \text{ mol}^{-1}$ respectively. The molar conductivity of solution of an electrolyte with the above cation and anion will be:

[4 marks]

- (A) $187 \text{ S cm}^2 \text{ mol}^{-1}$
- (B) $260 \text{ S cm}^2 \text{ mol}^{-1}$
- (C) $65 \text{ S cm}^2 \text{ mol}^{-1}$
- (D) $130 \text{ S cm}^2 \text{ mol}^{-1}$

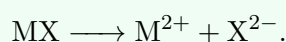
Correct Answer: (D) $130 \text{ S cm}^2 \text{ mol}^{-1}$

Solution:

Concept:

- The molar conductivity of an electrolyte is the sum of ionic contributions multiplied by the stoichiometric number of each ion per formula unit: $\Lambda_m = \nu_+ \lambda_+ + \nu_- \lambda_-$.
- The given quantities are molar ionic conductivities, not equivalent conductivities. Charge determines the neutral formula; it is not an additional multiplier for each given molar ionic contribution.

Step 1: Write the neutral electrolyte formula Let the cation be M^{2+} and the anion be X^{2-} . Equal and opposite charges require one of each, so the electrolyte is MX , dissociating as



One mole of electrolyte provides one mole of cations and one mole of anions. Hence both stoichiometric coefficients in the conductivity sum are 1.

Step 2: Add the two ionic contributions Substitute the supplied values directly:

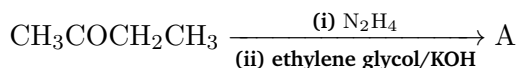
$$\Lambda_m = 1(57) + 1(73) = 130 \text{ S cm}^2 \text{ mol}^{-1}.$$

Multiplying this sum by two would double-count the ionic charges and give 260 incorrectly. Dividing it by two would convert the molar basis improperly, giving 65. The value 187 instead corresponds to the wrong ion ratio and is not compatible with a neutral divalent-divalent electrolyte.

Step 3: Conclusion The molar conductivity is $130 \text{ S cm}^2 \text{ mol}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: Balance the ionic charges to obtain the formula-unit ratio, then multiply each molar ionic conductivity only by that ratio.

76. Identify A in the following reaction:



[4 marks]

- (A) $\text{CH}_3\text{CH}(\text{OH})\text{CH}_2\text{CH}_3$
(B) $(\text{CH}_3)_2\text{C}=\text{NNH}_2$
(C) $\text{CH}_3\text{C}(=\text{NNH}_2)\text{CH}_2\text{CH}_3$
(D) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_3$

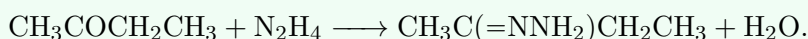
Correct Answer: (D) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_3$

Solution:

Concept:

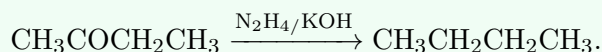
- Hydrazine followed by hot base in a high-boiling solvent is the Wolff-Kishner reduction. It converts a ketone carbonyl group, $\text{R}_2\text{C}=\text{O}$, into a methylene group, R_2CH_2 .
- The carbon skeleton is retained. Hydrazone formation is an intermediate stage; the final strongly basic treatment removes nitrogen and completes deoxygenation.

Step 1: Identify the ketone and its intermediate The starting compound is butan-2-one, $\text{CH}_3\text{COCH}_2\text{CH}_3$, containing four carbon atoms. Reaction with hydrazine initially forms its hydrazone:



This hydrazone corresponds to the third displayed structure, but it is not the product after both specified stages.

Step 2: Apply the hot-base reduction With potassium hydroxide in ethylene glycol, the hydrazone loses nitrogen and the carbonyl carbon becomes a methylene carbon. Thus the complete transformation is



The product is n-butane. The first structure is butan-2-ol, the product of carbonyl-to-alcohol reduction rather than deoxygenation. The second is an acetone hydrazone with the wrong carbon skeleton. The third is only the correct ketone's intermediate hydrazone, so none of these is the final product.

Step 3: Conclusion A is $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_3$, n-butane. Hence, the correct option is (D).

Quick Tip to Solve: Hydrazine plus hot KOH replaces the carbonyl oxygen by two hydrogens; count the carbons to avoid selecting a hydrazone intermediate.

77. The statement(s) that are correct about the species O^{2-} , F^{-} , Na^{+} and Mg^{2+} .

- (A) All are isoelectronic
- (B) All have the same nuclear charge
- (C) O^{2-} has the largest ionic radii
- (D) Mg^{2+} has the smallest ionic radii

Choose the most appropriate answer from the options given below :

[4 marks]

- (A) (C) and (D) only
- (B) (A), (C) and (D) only
- (C) (B), (C) and (D) only
- (D) (A), (B), (C) and (D)

Correct Answer: (B) (A), (C) and (D) only

Solution:

Concept:

- Isoelectronic species have the same electron count. For anions add the magnitude of the negative charge to atomic number; for cations subtract the positive charge.
- Within an isoelectronic series, increasing nuclear charge attracts the same electron cloud more strongly, so ionic radius decreases as atomic number increases.

Step 1: Count electrons and compare nuclear charges The electron counts are

$$\begin{aligned} O^{2-} : 8 + 2 &= 10, & F^{-} : 9 + 1 &= 10, \\ Na^{+} : 11 - 1 &= 10, & Mg^{2+} : 12 - 2 &= 10. \end{aligned}$$

All four are isoelectronic with neon, so statement (A) is true. Their nuclear charges are respectively $+8e$, $+9e$, $+11e$, $+12e$, which are unequal. Statement (B) therefore confuses electron count with proton count and is false.

Step 2: Order the radii and select the true statements With the same ten electrons, the weakest nuclear attraction occurs in oxide and the strongest in magnesium ion. The radius order is

$$O^{2-} > F^{-} > Na^{+} > Mg^{2+}.$$

Oxide consequently has the largest radius and magnesium ion the smallest, making (C) and (D) true. A combination containing (B) is incorrect, and the combination containing only (C) and (D) omits the true statement (A).

Step 3: Conclusion The true statements are isoelectronicity, the largest oxide radius and the smallest magnesium-ion radius. Hence, the correct option is (B).

Quick Tip to Solve: Equal electrons do not mean equal nuclear charge: list atomic numbers, then reverse that order to compare isoelectronic ionic radii.

78. The reaction at cathode in the cells commonly used in clocks involves.

[4 marks]

- (A) oxidation of Mn from +2 to +7
- (B) reduction of Mn from +4 to +3
- (C) oxidation of Mn from +3 to +4
- (D) reduction of Mn from +7 to +2

Correct Answer: (B) reduction of Mn from +4 to +3

Solution:

Concept:

- A primary dry cell has a zinc anode and a cathodic mixture containing manganese dioxide, MnO_2 , around a carbon current collector.
- Reduction is gain of electrons and always occurs at the cathode. The oxidation state of manganese decreases when the cell delivers current.

Step 1: Identify the electrode process The cells commonly used in clocks are dry cells. Zinc supplies electrons at the anode, while the manganese dioxide in the cathode mixture accepts electrons. The carbon rod primarily collects current; it is not the substance whose oxidation state changes in the stated alternatives. Therefore the required manganese process must be a reduction, eliminating both oxidation alternatives (A) and (C).

Step 2: Calculate the oxidation-state change In MnO_2 , oxygen contributes -4 , so manganese is $+4$. A simplified cathodic reduction can be represented as

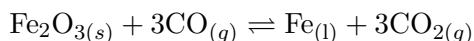


Manganese is $+3$ in Mn_2O_3 , since $2x - 6 = 0$. Thus each manganese atom accepts one electron, changing from $+4$ to $+3$. This is not permanganate reduction: no $+7$ manganese reactant is present, so the $+7$ to $+2$ change in (D) does not describe this dry-cell cathode.

Step 3: Conclusion The cathode reaction involves reduction of manganese from $+4$ to $+3$. Hence, the correct option is (B).

Quick Tip to Solve: First identify the cell chemistry; in a dry cell, follow the manganese in MnO_2 , rather than treating every manganese reduction as permanganate reduction.

79. The following reaction occurs in the Blast furnace where iron ore is reduced to iron metal



Using the Le-chatelier's principle, predict which one of the following will not disturb the equilibrium.

[4 marks]

- (A) Addition of CO_2
- (B) Removal of CO
- (C) Addition of Fe_2O_3
- (D) Removal of CO_2

Correct Answer: (C) Addition of Fe_2O_3

Solution:

Concept:

- In a heterogeneous equilibrium, the activities of pure solids and pure liquids are unity as long as those phases remain present.
- Le Chatelier's principle predicts a shift when a change alters the reaction quotient relative to its equilibrium value at the same temperature.

Step 1: Write the equilibrium expression For the printed reaction, iron(III) oxide is a pure solid and iron is a pure liquid. Their amounts do not occur as concentration factors in the equilibrium expression. For the gaseous components,

$$K_p = \frac{(p_{\text{CO}_2})^3}{(p_{\text{CO}})^3}.$$

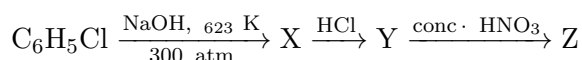
Consequently it is the carbon monoxide and carbon dioxide partial pressures, rather than the mass of a pure condensed phase, that determine the reaction quotient at fixed temperature.

Step 2: Test each proposed change Adding CO_2 raises the numerator of the quotient and shifts equilibrium to the left, consuming some added product. Removing CO lowers the denominator and also shifts the equilibrium to the left, regenerating reactant. Removing CO_2 lowers the numerator and shifts the reaction to the right. These eliminate (A), (B) and (D). Adding more pure Fe_2O_3 , while solid is already present, changes neither its unit activity nor the gas partial pressures. It may change the amount of solid available, but it does not change the equilibrium composition of the gases.

Step 3: Conclusion Addition of pure Fe_2O_3 does not disturb the equilibrium, provided that the solid phase is already present. Hence, the correct option is (C).

Quick Tip to Solve: Omit pure solids and pure liquids from the equilibrium quotient before deciding whether an added substance shifts a heterogeneous equilibrium.

80. Identify compound (Z) in the following reaction sequence.



[4 marks]

- (A) 4-Nitrophenol
(B) 2,4,6-Trinitrophenol
(C) 2-Nitrophenol
(D) 2,4-Dinitrophenol

Correct Answer: (B) 2,4,6-Trinitrophenol

Solution:

Concept:

- Chlorobenzene reacts with aqueous sodium hydroxide at high temperature and pressure to give sodium phenoxide, which yields phenol on acidification.
- The hydroxyl group activates the benzene ring and directs electrophilic substitution to its ortho and para positions. Concentrated nitric acid gives extensive nitration.

Step 1: Identify the intermediate compounds The severe conditions, 623 K and 300 atm, permit replacement of the aryl chlorine by the oxygen-containing group. In the alkaline medium the product is sodium phenoxide, not neutral phenol:

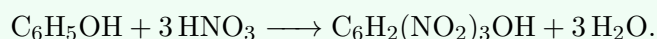


Thus $X = \text{C}_6\text{H}_5\text{ONa}$. Addition of hydrochloric acid protonates phenoxide:



Therefore Y is phenol.

Step 2: Apply the nitration conditions Phenol is strongly activated because the oxygen lone pair donates electron density into the ring. Its ortho and para positions, numbered 2, 4 and 6 relative to the hydroxyl carbon, undergo nitration with concentrated nitric acid:



The product is 2,4,6-trinitrophenol, also called picric acid. The para mononitro and ortho mononitro structures in (A) and (C) are associated with controlled dilute-acid nitration; they do not represent the stated concentrated-acid endpoint. The dinitro compound in (D) still lacks one of the three activated-position nitro groups.

Step 3: Conclusion Compound Z is 2,4,6-trinitrophenol (picric acid). Hence, the correct option is (B).

Quick Tip to Solve: In phenol nitration, read the acid concentration: dilute nitric acid gives ortho/para mononitro products, whereas concentrated nitric acid gives picric acid.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The spin-only magnetic moment value of the ion among Ti^{2+} , V^{2+} , Co^{3+} and Cr^{2+} , that acts as strong oxidising agent in aqueous solution is _____ BM (Near integer).

(Given atomic numbers : Ti : 22, V : 23, Cr : 24, Co : 27)

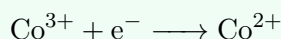
[4 marks]

Solution:

Concept:

- A strong oxidising agent accepts electrons readily. In aqueous solution Co^{3+} is readily reduced to the more stable Co^{2+} state.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}$ BM, where n is the number of unpaired electrons. Remove outer $4s$ electrons before $3d$ electrons when forming these ions.

Step 1: Choose the oxidising species Among the given ions, Ti^{2+} , V^{2+} and Cr^{2+} tend to lose electrons to form higher oxidation states and are reducing species. In particular Cr^{2+} is a familiar strong reducing agent because it forms stable Cr^{3+} . In contrast,



is favourable in aqueous solution. Hence the magnetic moment required is that of Co^{3+} , rather than the ion with the largest number of unpaired electrons in the entire list.

Step 2: Count the unpaired electrons and evaluate Cobalt has atomic number 27 and neutral configuration $[\text{Ar}]3d^74s^2$. Removing three electrons gives Co^{3+} the configuration $[\text{Ar}]3d^6$. For the free-ion spin-only calculation, Hund's rule places one electron in each of the five d orbitals before the sixth pairs, leaving four unpaired electrons. Therefore

$$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.90 \text{ BM.}$$

The question asks for the nearest integer, so round the magnetic moment only after evaluating the square root. A low-spin strong-field complex has not been specified in this question.

Step 3: Conclusion The oxidising ion is Co^{3+} , with spin-only moment $\sqrt{24} \approx 4.90$ BM. The required nearest-integer value is 5.

Quick Tip to Solve: Choose the ion from its redox behaviour first, then calculate its unpaired-electron count; the largest atomic number alone does not decide the answer.

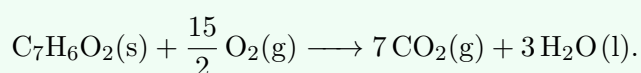
82. The heat of combustion of solid benzoic acid at constant volume is -321.30 kJ at 27°C . The heat of combustion at constant pressure is $(-321.30 - xR)$ kJ, the value of x is _____. [4 marks]

Solution:

Concept:

- At constant volume the heat of reaction equals ΔU ; at constant pressure it equals ΔH . For ideal gases, $\Delta H = \Delta U + \Delta n_g RT$.
- Only gaseous species contribute to Δn_g . At 27°C , take $T = 300$ K; combustion water is liquid under the standard combustion convention.

Step 1: Balance the combustion equation The molecular formula of benzoic acid is $\text{C}_7\text{H}_6\text{O}_2$. Complete combustion gives carbon dioxide and liquid water:



The oxygen balance is $2 + 15 = 17$ atoms on the left and $14 + 3 = 17$ on the right. The benzoic acid and water are condensed phases, so neither enters the gas-mole difference. For one mole of benzoic acid,

$$\Delta n_g = 7 - \frac{15}{2} = -\frac{1}{2}.$$

Step 2: Convert the constant-volume heat to constant-pressure heat Use the given constant-volume value as $\Delta U = -321.30$ kJ. With the gas constant expressed in units consistent with kilojoules, the correction is

$$\Delta H = -321.30 + \left(-\frac{1}{2}\right) R(300) = (-321.30 - 150R) \text{ kJ}.$$

Comparison with the expression $(-321.30 - xR)$ kJ gives $x = 150$. The sign is negative because gaseous reactant moles exceed gaseous product moles; the constant-pressure heat is therefore slightly more negative. If using $R = 8.314 \text{ J mol}^{-1}\text{K}^{-1}$ numerically, first convert the correction to kilojoules rather than mixing joules with the given heat.

Step 3: Conclusion The coefficient of R in the stated constant-pressure expression is $x = 150$.

Quick Tip to Solve: For a combustion $\Delta H - \Delta U$ correction, balance the equation and count only gas moles; liquid water must not be included in Δn_g .

83. In a borax bead test under hot condition, a metal salt (one from the given) is heated at point B of the flame, resulted in green colour salt bead. The spin-only magnetic moment value of the salt is _____ BM (Nearest integer) [Given atomic number of Cu = 29, Ni = 28, Mn = 25, Fe = 26] [4 marks]



Solution:

Concept:

- In the borax bead test, the reducing region of the flame converts an iron-containing bead to a green ferrous borate bead when hot.
- The spin-only moment is $\mu = \sqrt{n(n+2)}$ BM. For iron, Fe^{3+} is $3d^5$, whereas Fe^{2+} is $3d^6$; the initial salt oxidation state must be distinguished from that of the bead.

Step 1: Identify the iron salt from the bead test Point B is the inner reducing region of the flame shown. A hot green bead in that region identifies iron among the supplied metals. The green colour is associated with the reduced ferrous form in the bead. The test identifies the metal, but the oxidation state of an original iron salt need not be the oxidation state present after heating in the reducing flame. The accepted numerical answer uses a ferric starting salt. The wording does not explicitly supply that oxidation state, so this convention must be stated rather than inferred solely from the green bead.

Step 2: Calculate the ferric-ion spin-only moment Neutral iron has configuration $[\text{Ar}]3d^64s^2$. For the ferric salt intended by the accepted value, removal of three electrons gives Fe^{3+} with configuration $[\text{Ar}]3d^5$. Hund's rule gives five unpaired electrons, hence

$$\mu = \sqrt{5(5+2)} = \sqrt{35} = 5.92 \text{ BM,}$$

which rounds to 6. In contrast, a high-spin ferrous starting salt has $3d^6$, four unpaired electrons and $\mu = \sqrt{24} \approx 4.90 \text{ BM}$, rounding to 5. Both can yield the iron bead under reducing conditions. Thus the green bead alone cannot resolve the initial oxidation state; 6 is the accepted ferric-salt interpretation, not a unique deduction from an unspecified iron salt.

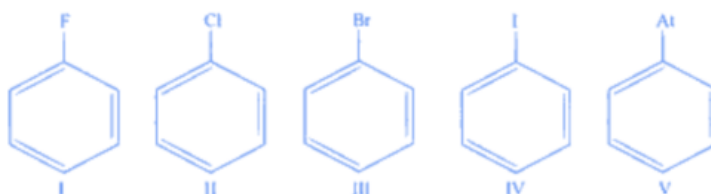
Step 3: Conclusion The accepted answer is 6, corresponding to a ferric starting salt with $\mu \approx 5.92 \text{ BM}$.

Strictly, the starting oxidation state is unstated: a ferrous salt would give a nearest-integer moment of 5, so the printed wording is ambiguous.

Quick Tip to Solve: Use the bead colour to identify the metal, then keep the starting salt's oxidation state separate from the oxidation state generated inside the flame.

84. The number of halobenzenes from the following that can be prepared by Sandmeyer's reaction is

[4 marks]



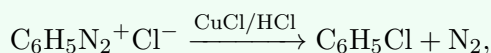
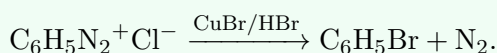
Solution:

Concept:

- Sandmeyer reactions replace an aromatic diazonium group by chlorine or bromine using the corresponding cuprous salt, with evolution of nitrogen.
- Not every replacement of a diazonium group by a halogen is called a Sandmeyer reaction: fluorination and iodination have distinct reagent requirements.

Step 1: Identify the compounds shown The five illustrated compounds are fluorobenzene (I), chlorobenzene (II), bromobenzene (III), iodobenzene (IV) and astatobenzene (V). The label At on the fifth compound is the symbol for astatine, not a functional-group abbreviation. For the usual Sandmeyer route, begin with benzenediazonium chloride, obtained by diazotising aniline at low temperature. The aromatic skeleton is retained while the diazonium group leaves as nitrogen gas.

Step 2: Apply the specific Sandmeyer reagents Cuprous chloride and cuprous bromide give the following conversions:



Therefore compounds II and III qualify. Fluorobenzene is normally prepared through the Balz-Schiemann reaction, using the diazonium tetrafluoroborate and heating; it is not included here. Iodobenzene can be obtained with iodide, but that direct iodide substitution is not the cuprous-halide Sandmeyer reaction. The standard reaction also does not include astatobenzene. Thus only the chlorine and bromine derivatives in this list are counted.

Step 3: Conclusion Chlorobenzene (II) and bromobenzene (III) can be prepared by Sandmeyer's reaction. The required number is 2.

Quick Tip to Solve: For the standard halobenzene Sandmeyer count, select Cl and Br; distinguish the fluorination and direct iodide routes by their named reagents.

85. An artificial cell is made by encapsulating 0.2 M glucose solution within a semipermeable membrane. The osmotic pressure developed when the artificial cell is placed within a 0.05 M solution of NaCl at 300 K is _____ $\times 10^{-1}$ bar. (nearest integer).

[Given : $R = 0.083 \text{ L bar mol}^{-1} \text{ K}^{-1}$]

Assume complete dissociation of NaCl

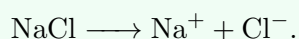
[4 marks]

Solution:

Concept:

- Osmotic pressure of a dilute solution is $\pi = iCRT$, where i is the van't Hoff factor and C is its molarity. Across a semipermeable membrane, the driving pressure is the difference between the two solution pressures.
- Glucose is a nonelectrolyte, so its particle concentration equals its molarity. Completely dissociated NaCl supplies two ions per formula unit, making its particle concentration twice its molarity.

Step 1: Find the effective concentrations The artificial cell contains 0.2 mol L^{-1} glucose, with $i = 1$. The surrounding solution contains 0.05 mol L^{-1} of the electrolyte. Its dissociation is



Thus the outside solution has effective particle concentration $2(0.05) = 0.10 \text{ mol L}^{-1}$, whereas the inside has 0.20 mol L^{-1} .

Step 2: Calculate the net osmotic pressure The common temperature and gas constant allow us to subtract particle concentrations directly. Using the supplied value of R ,

$$\begin{aligned}\Delta\pi &= (C_{\text{in}} - iC_{\text{out}})RT \\ &= (0.20 - 2 \times 0.05)(0.083)(300) \\ &= 2.49 \text{ bar}.\end{aligned}$$

The inside solution has the higher osmotic pressure, so water tends to enter the cell. Using only the glucose pressure would ignore the osmotic opposition from the outside solution.

Step 3: Convert to the requested scale The answer blank multiplies 10^{-1} bar, so write

$$2.49 \text{ bar} = 24.9 \times 10^{-1} \text{ bar}.$$

The required integer is the coefficient rounded to the nearest integer, rather than the pressure in bar rounded directly.

Step 4: Conclusion The developed osmotic pressure is 2.49 bar, or 24.9×10^{-1} bar. The numerical entry is therefore 25.

Quick Tip to Solve: For a cell surrounded by another solution, subtract the two values of iC before multiplying by RT ; then round the coefficient in the printed scale.

86. In the lewis dot structure for NO_2^- , total number of valence electrons around nitrogen is _____.

[4 marks]

Solution:

Concept:

- The Lewis structure counts all valence electrons of the ion, including the extra electron contributed by a negative charge. A bonding pair counts around each of the two atoms joined by that bond.
- For second-period nitrogen, the stable nitrite structure completes an octet: the central atom has one double bond, one single bond and one lone pair. Resonance exchanges the two oxygen positions.

Step 1: Count the electrons available to the ion Nitrogen contributes five valence electrons and each oxygen contributes six. The single negative charge adds one electron. Therefore the complete ion contains

$$5 + 2(6) + 1 = 18$$

valence electrons. This total belongs to the entire NO_2^- ion and is different from the number surrounding nitrogen alone.

Step 2: Construct the central-atom environment Place nitrogen between the two oxygen atoms. Starting with two single bonds and completing the oxygen octets uses all 18 electrons after one lone pair is placed on nitrogen. Nitrogen would then have only six surrounding electrons. Convert a lone pair on one oxygen into an additional bonding pair to form one $\text{N}=\text{O}$ double bond. The resulting contributors have the connectivity



Nitrogen retains its lone pair in both contributors.

Step 3: Count electrons around nitrogen The double bond supplies four shared electrons, the single bond supplies two, and the lone pair supplies two nonbonding electrons. Hence

$$N_{\text{around nitrogen}} = 4 + 2 + 2 = 8.$$

This is the octet count; it does not assign half of each bond pair to nitrogen, as a formal-charge calculation would. The formal charges are zero on nitrogen and the double-bonded oxygen, and minus one on the singly bonded oxygen, consistent with the total ionic charge.

Step 4: Conclusion The total number of valence electrons around nitrogen in the Lewis structure of NO_2^- is 8.

Quick Tip to Solve: Distinguish three counts: 18 electrons in the whole nitrite ion, 8 electrons around central nitrogen, and 5 electrons assigned to nitrogen when computing formal charge.

87. The value of Rydberg constant (R_H) is 2.18×10^{-18} J. The velocity of electron having mass 9.1×10^{-31} kg in Bohr's first orbit of hydrogen atom = _____ $\times 10^5$ ms^{-1} (nearest integer). [4 marks]

Solution:

Concept:

- In the Bohr model, $E_n = -R_H/n^2$ when the Rydberg quantity is supplied as an energy. Here $R_H = 2.18 \times 10^{-18}$ J, so the ground-state total energy is negative.
- For a circular orbit in the Coulomb field, potential energy is minus twice kinetic energy. Thus $E = -K$, and in the first hydrogen orbit $\frac{1}{2}m_e v^2 = R_H$.

Step 1: Relate kinetic and total energies The electrostatic attraction provides the centripetal force, giving

$$\frac{k_e e^2}{r^2} = \frac{m_e v^2}{r}.$$

Multiplying by r shows that the potential energy $-k_e e^2/r$ equals $-m_e v^2 = -2K$. Hence $E = K + U = -K$. For $n = 1$, $E_1 = -2.18 \times 10^{-18}$ J, so $K = 2.18 \times 10^{-18}$ J. The negative total energy must not be inserted as a negative kinetic energy.

Step 2: Substitute the given mass Using the supplied electron mass,

$$\begin{aligned} v &= \sqrt{\frac{2R_H}{m_e}} \\ &= \sqrt{\frac{2(2.18 \times 10^{-18})}{9.1 \times 10^{-31}}} \\ &= \sqrt{4.7912 \times 10^{12}} \\ &\simeq 2.189 \times 10^6 \text{ ms}^{-1}. \end{aligned}$$

The ratio of joules to kilograms has units of squared speed, so the square root gives the required velocity magnitude.

Step 3: Express the answer in the printed units The question asks for the coefficient of 10^5 ms^{-1} . Therefore

$$2.189 \times 10^6 \text{ ms}^{-1} = 21.89 \times 10^5 \text{ ms}^{-1}.$$

Rounding this coefficient to the nearest integer gives the numerical entry. No further factors of the principal quantum number occur because the orbit is the first orbit.

Step 4: Conclusion The electron speed is approximately $2.189 \times 10^6 \text{ ms}^{-1}$. The required nearest-integer coefficient is 22.

Quick Tip to Solve: When R_H is given in joules, use it directly as the magnitude of the first-orbit binding energy: $v = \sqrt{2R_H/m_e}$.

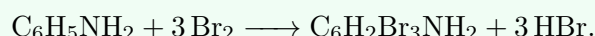
88. 9.3 g of pure aniline is treated with bromine water at room temperature to give a white precipitate of the product 'P'. The mass of product 'P' obtained is 26.4 g. The percentage yield is _____.
[4 marks]

Solution:

Concept:

- The strongly activating amino group makes aniline undergo bromination at both ortho positions and the para position in bromine water. The white precipitate is 2,4,6-tribromoaniline.
- Percentage yield is actual isolated mass divided by theoretical product mass, multiplied by 100. The theoretical mass follows from the balanced equation and the amount of pure aniline.

Step 1: Identify the product and its stoichiometry The amino group directs substitution to positions 2, 4 and 6. The aqueous bromine reaction is



Therefore one mole of aniline forms one mole of product P. The three bromine atoms replace three ring hydrogen atoms; they do not add to the unchanged aniline formula. The precipitate has molecular formula $\text{C}_6\text{H}_4\text{Br}_3\text{N}$.

Step 2: Calculate the theoretical precipitate mass Using the usual integer atomic masses $\text{C} = 12$, $\text{H} = 1$, $\text{N} = 14$, and $\text{Br} = 80$,

$$\begin{aligned}M_{\text{aniline}} &= 6(12) + 7(1) + 14 = 93 \text{ g mol}^{-1}, \\M_P &= 6(12) + 4(1) + 3(80) + 14 \\&= 330 \text{ g mol}^{-1}.\end{aligned}$$

The amount of aniline taken is $9.3/93 = 0.100 \text{ mol}$. For complete conversion it gives 0.100 mol P , corresponding to a theoretical mass of $0.100(330) = 33.0 \text{ g}$.

Step 3: Compare actual and theoretical masses The isolated product weighs 26.4 g, while 33.0 g could be obtained at complete conversion and recovery. Hence

$$\% \text{ yield} = \frac{26.4}{33.0} \times 100 = 80\%.$$

Dividing the product mass by the starting aniline mass would not calculate yield because bromination adds substantial bromine mass to the product.

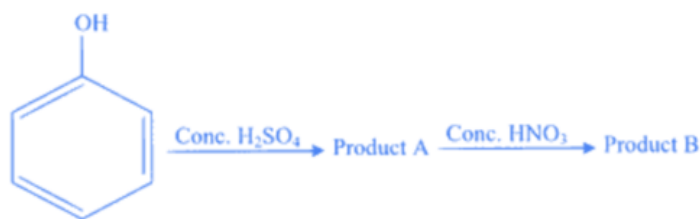
Step 4: Conclusion The white precipitate P is 2,4,6-tribromoaniline, and its percentage yield is 80%. The numerical entry is 80.

Quick Tip to Solve: For bromine water and aniline, use tribromination and a 1:1 aniline-to-product mole ratio; calculate theoretical product mass before forming the yield ratio.

89. Consider the given chemical reaction sequence :

Total sum of oxygen atoms in Product A and Product B are _____.

[4 marks]



Solution:

Concept:

- The conventional conversion of phenol to picric acid first treats phenol with concentrated sulphuric acid, forming phenol-2,4-disulphonic acid. Subsequent concentrated nitric acid produces 2,4,6-trinitrophenol.
- Count all oxygen atoms in each molecular product separately. A hydroxyl group contributes one oxygen, a sulphonic acid group $-\text{SO}_3\text{H}$ contributes three, and a nitro group $-\text{NO}_2$ contributes two.

Step 1: Identify product A The first reagent is concentrated H_2SO_4 . In this standard preparative sequence, phenol is converted to phenol-2,4-disulphonic acid, whose condensed composition may be written as $\text{C}_6\text{H}_3(\text{OH})(\text{SO}_3\text{H})_2$. Its ring carries the original hydroxyl group and two sulphonic acid groups. The number of oxygen atoms in A is therefore

$$N_O(A) = 1 + 2(3) = 7.$$

The hydroxyl oxygen remains present during the ring-substitution reactions and must be included in the count.

Step 2: Identify product B Treatment of the sulphonated phenol with concentrated HNO_3 gives picric acid, or 2,4,6-trinitrophenol. In this conversion the sulphonic acid groups are replaced by nitro groups, and the remaining activated position is also nitrated. The final product can be represented by $\text{C}_6\text{H}_2(\text{OH})(\text{NO}_2)_3$. This is the standard sequence used to avoid the poor yield associated with directly nitrating phenol under harsh conditions.

Step 3: Add the oxygen counts Product B has one hydroxyl oxygen plus six oxygens from its three nitro groups:

$$N_O(B) = 1 + 3(2) = 7.$$

The question asks for the sum across products A and B, rather than the number of atoms in only the final product or in the reagents. Thus

$$N_O(A) + N_O(B) = 7 + 7 = 14.$$

Neither water formed along the sequence nor excess acid is part of either named organic product.

Step 4: Conclusion Product A contains 7 oxygen atoms and product B contains 7 oxygen atoms. Their total is 14.

Quick Tip to Solve: In this picric-acid sequence, count $1 + 2 \times 3$ oxygens in phenol disulphonic acid and $1 + 3 \times 2$ in trinitrophenol; both products contain seven.

90. During Kinetic study of reaction $2A + B \rightarrow C + D$, the following results were obtained :

	A[M]	B[M]	initial rate of formation of D
I	0.1	0.1	6.0×10^{-3}
II	0.3	0.2	7.2×10^{-2}
III	0.3	0.4	2.88×10^{-1}
IV	0.4	0.1	2.40×10^{-2}

Based on above data, overall order of the reaction is _____.

[4 marks]

Solution:

Concept:

- For an experimentally determined rate law $r = k[A]^m[B]^n$, the overall order is $m+n$. Stoichiometric coefficients in the reaction equation do not by themselves give the exponents.
- Compare experiments in which one reactant concentration remains constant. The resulting rate ratio isolates the exponent of the other reactant, without requiring the value or units of the rate constant.

Step 1: Determine the exponent of B Experiments II and III have the same $[A] = 0.3 \text{ M}$. The concentration of B doubles from 0.2 to 0.4 M, while the initial rate increases by a factor of four:

$$\frac{r_{\text{III}}}{r_{\text{II}}} = \frac{2.88 \times 10^{-1}}{7.2 \times 10^{-2}} = 4 = \left(\frac{0.4}{0.2}\right)^n = 2^n.$$

Consequently the reaction is second order with respect to B, so $n = 2$.

Step 2: Determine the exponent of A Experiments I and IV maintain $[B] = 0.1 \text{ M}$. The concentration of A rises from 0.1 to 0.4 M, which is a fourfold change. The rate is also four times as large:

$$\frac{r_{\text{IV}}}{r_{\text{I}}} = \frac{2.40 \times 10^{-2}}{6.0 \times 10^{-3}} = 4 = \left(\frac{0.4}{0.1}\right)^m = 4^m.$$

Thus $m = 1$, and the rate law is $r = k[A][B]^2$.

Step 3: Check the law and calculate the overall order As a consistency check, going from experiment I to II triples A and doubles B. The predicted rate factor is $3 \times 2^2 = 12$, which agrees with

$$\frac{7.2 \times 10^{-2}}{6.0 \times 10^{-3}} = 12.$$

All four experimental rates therefore support the same law. Adding the experimentally found exponents gives

$$\text{overall order} = m + n = 1 + 2 = 3.$$

The coefficient 2 before A in the balanced equation is not the order in A.

Step 4: Conclusion The rate law is $r = k[A][B]^2$; hence the overall order of the reaction is 3.

Quick Tip to Solve: Use II and III to isolate B, then I and IV to isolate A. When B doubles and the rate quadruples, its order is two.