

JEE Main 2024 Paper 1 (B.E./B.Tech) Apr 4 Shift 2

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 4 April 2024, Shift 2. The total duration is 3 hours.
- (ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Consider a hyperbola H having centre at the origin and foci on the x -axis. Let C_1 be the circle touching the hyperbola H and having the centre at the origin. Let C_2 be the circle touching the hyperbola H at its vertex and having the centre at one of its foci. If areas (in sq units) of C_1 and C_2 are 36π and 4π , respectively, then the length (in units) of latus rectum of H is [4 marks]

- (A) $\frac{28}{3}$
 (B) $\frac{11}{3}$
 (C) $\frac{14}{3}$
 (D) $\frac{10}{3}$

Correct Answer: (A) $\frac{28}{3}$

Solution:

Concept:

- For a horizontal hyperbola $x^2/a^2 - y^2/b^2 = 1$, the vertices are $(\pm a, 0)$ and the foci are $(\pm c, 0)$, where $c^2 = a^2 + b^2$.
- The circle centred at the origin that touches the hyperbola has radius a . A circle centred at a focus and touching the nearer vertex has radius $c - a$. The full latus rectum has length $2b^2/a$.

Step 1: Recover the transverse semiaxis The least distance of the hyperbola from its centre occurs at a vertex. Indeed, its equation gives $x^2 = a^2 + (a^2/b^2)y^2$, so $x^2 + y^2 \geq a^2$, with equality at $y = 0$. Therefore the first touching circle has radius a . Its given area yields

$$\pi a^2 = 36\pi \implies a = 6.$$

Step 2: Use the focus-centred circle The second circle has radius $\sqrt{4\pi/\pi} = 2$. The nearer vertex is a distance $c - a$ from the focus. The other vertex is $c + a > 6$ units away, so it cannot give the specified radius. Thus

$$c - a = 2, \quad c = 8.$$

The hyperbola identity now determines the other semiaxis:

$$b^2 = c^2 - a^2 = 64 - 36 = 28.$$

Step 3: Compute the complete focal chord The latus rectum is the chord through a focus perpendicular to the transverse axis. Substitution of $x = c$ gives its endpoints at $y = \pm b^2/a$, so its full length is

$$L = \frac{2b^2}{a} = \frac{2 \times 28}{6} = \frac{28}{3}.$$

Option (C) gives only the half-length b^2/a . Options (B) and (D) do not satisfy this focal-chord formula with the recovered semiaxes.

Step 4: Conclusion The length of the latus rectum is $\boxed{28/3}$ units. Hence, the correct option is (A).

Quick Tip to Solve: Convert both areas into radii first: they give a and $c - a$, after which $b^2 = c^2 - a^2$.

2. If the coefficients of x^4 , x^5 and x^6 in the expansion of $(1+x)^n$ are in the arithmetic progression, then the maximum value of n is: [4 marks]

- (A) 28
(B) 21
(C) 7
(D) 14

Correct Answer: (D) 14

Solution:

Concept:

- The coefficient of x^r in $(1+x)^n$ is $\binom{n}{r}$. Three consecutive coefficients form an arithmetic progression when twice the middle coefficient equals the sum of its neighbours.
- Adjacent binomial coefficients satisfy $\binom{n}{r+1}/\binom{n}{r} = (n-r)/(r+1)$. Here the required expansion has $n \geq 6$.

Step 1: Translate the progression condition The three specified coefficients are $\binom{n}{4}$, $\binom{n}{5}$, $\binom{n}{6}$, in that order. Their arithmetic-progression condition is

$$2\binom{n}{5} = \binom{n}{4} + \binom{n}{6}.$$

As $n \geq 6$, division by the positive quantity $\binom{n}{4}$ is legitimate and preserves the condition.

Step 2: Reduce the coefficient ratios Using factorial cancellation rather than evaluating large coefficients gives

$$\frac{\binom{n}{5}}{\binom{n}{4}} = \frac{n-4}{5}, \quad \frac{\binom{n}{6}}{\binom{n}{4}} = \frac{(n-4)(n-5)}{30}.$$

Consequently,

$$\frac{2(n-4)}{5} = 1 + \frac{(n-4)(n-5)}{30}.$$

Multiplying by 30 and collecting terms yields

$$12(n-4) = 30 + (n-4)(n-5),$$

$$n^2 - 21n + 98 = 0 = (n-7)(n-14).$$

Step 3: Select the larger admissible integer The two possible values are $n = 7$ and $n = 14$, both of which meet $n \geq 6$. At $n = 7$, the coefficients are 35, 21, 7, a decreasing arithmetic progression; increasing order was never required. The larger possible exponent is therefore 14. Options (A) and (B), 28 and 21, are not roots of the condition. Option (C), 7, is possible but is not the maximum.

Step 4: Conclusion The maximum value is $n = 14$. Hence, the correct option is (D).

Quick Tip to Solve: Divide an arithmetic-progression equation for binomial coefficients by the smallest-index coefficient; factorials then reduce to short ratios.

3. The value of $\frac{1 \times 2^2 + 2 \times 3^2 + \dots + 100 \times (101)^2}{1^2 \times 2 + 2^2 \times 3 + \dots + 100^2 \times 101}$ is

[4 marks]

- (A) $\frac{305}{301}$
- (B) $\frac{306}{305}$
- (C) $\frac{32}{31}$
- (D) $\frac{31}{30}$

Correct Answer: (A) $\frac{305}{301}$

Solution:

Concept:

- Write the numerator and denominator with the common index $k = 1, \dots, 100$: expand $k(k+1)^2$ and $k^2(k+1)$ before summing.
- The standard identities are $\sum k = n(n+1)/2$, $\sum k^2 = n(n+1)(2n+1)/6$, and $\sum k^3 = [n(n+1)/2]^2$.

Step 1: Expand both summands Let $S_j = \sum_{k=1}^{100} k^j$. The numerator is

$$N = \sum_{k=1}^{100} (k^3 + 2k^2 + k) = S_3 + 2S_2 + S_1,$$

whereas the denominator is

$$D = \sum_{k=1}^{100} (k^3 + k^2) = S_3 + S_2.$$

The squares occur on different factors in the two sums; interchanging them would invert the intended comparison.

Step 2: Evaluate the three standard sums With $n = 100$, the power-sum formulae give

$$S_1 = \frac{100 \times 101}{2} = 5050,$$

$$S_2 = \frac{100 \times 101 \times 201}{6} = 338350,$$

$$S_3 = 5050^2 = 25502500.$$

It follows that

$$N = 25502500 + 676700 + 5050 = 26184250,$$

$$D = 25502500 + 338350 = 25840850.$$

Step 3: Simplify and check the options Both totals have the factor 85850:

$$\frac{N}{D} = \frac{85850 \times 305}{85850 \times 301} = \frac{305}{301}.$$

As a useful check, $N - D = S_2 + S_1 = 343400 > 0$, so the ratio exceeds one, and its excess is exactly $4/301$. The other proposed ratios exceed one by $1/305$, $1/31$, and $1/30$, respectively; none has the required excess. Thus only option (A) agrees with the computed sums. A direct check is $301N = 305D$, confirming the reduced ratio exactly.

Step 4: Conclusion The required value is $\boxed{305/301}$. Hence, the correct option is (A).

Quick Tip to Solve: For products of neighbouring integers, expand into powers first; the same three power sums often evaluate both numerator and denominator.

4. If the function

$$f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}}, & x \neq 0 \\ a \log_e 2 \log_e 3, & x = 0 \end{cases}$$

is continuous at $x = 0$, then the value of a^2 is equal to

[4 marks]

- (A) 968
(B) 1250
(C) 1152
(D) 746

Correct Answer: (C) 1152

Solution:

Concept:

- Continuity at zero requires the two-sided limit of the nonzero branch to equal the assigned value $a \ln 2 \ln 3$.
- Use $(b^x - 1)/x \rightarrow \ln b$, $(1 - \cos x)/x^2 \rightarrow 1/2$, and rationalise a difference of square roots before taking its limit.

Step 1: Factor the exponential numerator Since $72 = 8 \times 9$, the numerator factors exactly as

$$72^x - 9^x - 8^x + 1 = (8^x - 1)(9^x - 1).$$

Both factors vanish at zero, so their product has a quadratic leading term. In particular,

$$\lim_{x \rightarrow 0} \frac{(8^x - 1)(9^x - 1)}{x^2} = \ln 8 \ln 9 = 6 \ln 2 \ln 3.$$

Step 2: Rationalise the denominator The square-root difference can be written without cancellation:

$$\sqrt{2} - \sqrt{1 + \cos x} = \frac{1 - \cos x}{\sqrt{2} + \sqrt{1 + \cos x}}.$$

Thus its ratio to x^2 tends to

$$\frac{1/2}{2\sqrt{2}} = \frac{1}{4\sqrt{2}}.$$

The denominator is positive for sufficiently small nonzero x , and this quadratic limit is the same from either side.

Step 3: Impose continuity and square the parameter Combining the two limits gives

$$\lim_{x \rightarrow 0} f(x) = 24\sqrt{2} \ln 2 \ln 3.$$

Continuity therefore implies

$$a \ln 2 \ln 3 = 24\sqrt{2} \ln 2 \ln 3.$$

Both logarithms are nonzero, so $a = 24\sqrt{2}$, and

$$a^2 = 24^2 \times 2 = 1152.$$

Options (A), (B), and (D) differ from this exact value. In particular, losing the factor from rationalisation would give an incorrect parameter.

Step 4: Conclusion The required value is $a^2 = 1152$. Hence, the correct option is (C).

Quick Tip to Solve: Factor $(ab)^x - a^x - b^x + 1$ as $(a^x - 1)(b^x - 1)$, and rationalise the radical difference before comparing orders.

5. Let P be the point of intersection of the lines $\frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1}$ and $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}$. Then, the shortest distance of P from the line $4x = 2y = z$ is [4 marks]

- (A) $\frac{3\sqrt{14}}{7}$
- (B) $\frac{5\sqrt{14}}{7}$
- (C) $\frac{\sqrt{14}}{7}$
- (D) $\frac{6\sqrt{14}}{7}$

Correct Answer: (A) $\frac{3\sqrt{14}}{7}$

Solution:

Concept:

- Introduce one parameter for each symmetric line and equate the three coordinates to locate their intersection.
- For a line through the origin with direction vector $\mathbf{d}$, the perpendicular distance of a point with position vector $\mathbf{p}$ is $|\mathbf{p} \times \mathbf{d}|/|\mathbf{d}|$.

Step 1: Find the intersection point Write the first line as

$$(x, y, z) = (2 + t, 4 + 5t, 2 + t),$$

and the second as

$$(x, y, z) = (3 + 2s, 2 + 3s, 3 + 2s).$$

Matching the first and second coordinates gives

$$t - 2s = 1, \quad 5t - 3s = -2.$$

Substitute $t = 1 + 2s$: $5 + 7s = -2$, so $s = -1$ and $t = -1$. Therefore $P = (1, -1, 1)$. The third coordinates also agree, confirming the intersection.

Step 2: Read the direction ratios correctly The line $4x = 2y = z$ passes through the origin. Put its common value equal to λ , giving

$$(x, y, z) = (\lambda/4, \lambda/2, \lambda).$$

Hence a direction vector is $\mathbf{d} = (1, 2, 4)$, rather than $(4, 2, 1)$. Use $\mathbf{p} = (1, -1, 1)$ for the vector from the origin to P .

Step 3: Calculate the perpendicular distance The cross product and its magnitude are

$$\mathbf{p} \times \mathbf{d} = (-6, -3, 3), \quad |\mathbf{p} \times \mathbf{d}| = \sqrt{54}.$$

Also $|\mathbf{d}| = \sqrt{21}$. Thus

$$d = \frac{\sqrt{54}}{\sqrt{21}} = \frac{3\sqrt{14}}{7}.$$

Its square is $18/7$; options (B), (C), and (D) instead have squares $50/7$, $2/7$, and $72/7$. None equals the perpendicular-distance square obtained from these vectors.

Step 4: Conclusion The shortest distance is $\boxed{3\sqrt{14}/7}$ units. Hence, the correct option is (A).

Quick Tip to Solve: For equations such as $4x = 2y = z$, set the common value to a parameter; the direction ratios are reciprocals of the coefficients.

6. Let $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of f , then $\alpha^2 + 2\beta^2$ is equal to [4 marks]

- (A) 42
- (B) 38
- (C) 24
- (D) 44

Correct Answer: (A) 42

Solution:

Concept:

- The radicands impose the closed domain $2 \leq x \leq 4$. A continuous function on this interval attains both a minimum and a maximum.
- For nonnegative u, v , Cauchy-Schwarz gives $(3u + v)^2 \leq (3^2 + 1^2)(u^2 + v^2)$, with equality when $u : v = 3 : 1$.

Step 1: Transform the radical constraint Let $u = \sqrt{x-2}$ and $v = \sqrt{4-x}$. Then $u, v \geq 0$,

$$u^2 + v^2 = (x-2) + (4-x) = 2, \quad f(x) = 3u + v.$$

This expresses the problem on a quarter-circle of fixed radius, so the coefficient vector $(3, 1)$ controls

the maximum.

Step 2: Establish the maximum and its attainment By Cauchy–Schwarz,

$$(3u + v)^2 \leq (9 + 1)(u^2 + v^2) = 20.$$

Equality requires $u = 3v$. The constraint then gives $v^2 = 1/5$, $u^2 = 9/5$, corresponding to $x = 19/5$, inside the allowed interval. Therefore the maximum is attained and $\beta^2 = 20$.

Step 3: Establish the minimum and combine the results Because $u, v \geq 0$,

$$3u + v \geq u + v \geq \sqrt{u^2 + v^2} = \sqrt{2}.$$

Equality in both inequalities occurs at $u = 0$, namely $x = 2$; hence $\alpha = \sqrt{2}$. At the other endpoint, $x = 4$, the value is $3\sqrt{2}$, so that endpoint is not the minimum. The requested combination is

$$\alpha^2 + 2\beta^2 = 2 + 2(20) = 42.$$

The values 38, 24, and 44 fail this combination of the attained extrema, eliminating (B), (C), and (D).

Step 4: Conclusion The required value is $\boxed{42}$. Hence, the correct option is (A).

Quick Tip to Solve: When two squared radicals add to a constant, use Cauchy–Schwarz for the maximum and check equality; obtain the minimum with a separate lower bound.

7. Let $f(x) = \int_0^x (t + \sin(1 - e^t)) dt$, $x \in \mathbb{R}$. Then, $\lim_{x \rightarrow 0} \frac{f(x)}{x^3}$ is equal to

[4 marks]

- (A) $\frac{1}{6}$
- (B) $-\frac{1}{6}$
- (C) $\frac{2}{3}$
- (D) $-\frac{2}{3}$

Correct Answer: (B) $-\frac{1}{6}$

Solution:

Concept:

- The fundamental theorem gives $f'(x) = x + \sin(1 - e^x)$. Near zero the linear terms cancel, so the leading integrand term is quadratic.
- The standard expansions $e^t = 1 + t + t^2/2 + O(t^3)$ and $\sin u = u + O(u^3)$ determine the first nonzero term needed for a cubic denominator.

Step 1: Find the first nonzero integrand term Put $u = 1 - e^t$. As $t \rightarrow 0$,

$$u = -t - \frac{t^2}{2} + O(t^3).$$

Since $u = O(t)$, its cube is $O(t^3)$, and the sine expansion gives

$$\sin(1 - e^t) = -t - \frac{t^2}{2} + O(t^3).$$

Adding the explicit t cancels the linear term exactly:

$$t + \sin(1 - e^t) = -\frac{t^2}{2} + O(t^3).$$

Step 2: Integrate the local expression Integrating from zero to x , with the orientation retained also when $x < 0$, yields

$$f(x) = \int_0^x \left(-\frac{t^2}{2} + O(t^3) \right) dt = -\frac{x^3}{6} + O(x^4).$$

The remainder bound is valid on both sides of zero, so division by x^3 gives

$$\frac{f(x)}{x^3} = -\frac{1}{6} + O(x).$$

Consequently the required two-sided limit exists.

Step 3: Check the sign and distinguish the choices The leading term of the integrand is negative and proportional to t^2 . For positive small x , its integral is therefore negative, ruling out the positive options (A) and (C). The coefficient comes from integrating $-t^2/2$, which divides it by three: it is $-1/6$, not $-2/3$ as in (D). This confirms both the magnitude and sign without mistaking the integrand limit for the integral limit.

Step 4: Conclusion The limit is $\boxed{-1/6}$. Hence, the correct option is (B).

Quick Tip to Solve: Expand the integrand only through its first surviving term; a quadratic term integrates to a cubic term, exactly matching this denominator.

8. The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x, \text{ and } y \geq 4x - 1\}$ is [4 marks]

- (A) $\frac{9}{32}$
- (B) $\frac{11}{12}$
- (C) $\frac{8}{9}$
- (D) $\frac{11}{32}$

Correct Answer: (A) $\frac{9}{32}$

Solution:

Concept:

- For horizontal strips, the area is $\int (x_{\text{right}} - x_{\text{left}}) dy$; the inequalities determine which boundary is on each side.
- The parabola $y^2 = 2x$ opens to the right. The condition $y \geq 4x - 1$ is equivalent to $x \leq (y + 1)/4$.

Step 1: Write the horizontal bounds The two inequalities give

$$\frac{y^2}{2} \leq x \leq \frac{y+1}{4}.$$

A horizontal strip exists only where the right endpoint is at least the left endpoint. Thus we must first determine the intersections, rather than integrate with respect to x across two different parabola branches.

Step 2: Find the intersection levels At an intersection,

$$\frac{y^2}{2} = \frac{y+1}{4} \implies 2y^2 - y - 1 = (2y+1)(y-1) = 0.$$

Therefore the lower and upper levels are $-1/2$ and 1 . Between these roots the strip width is nonnegative; outside them there are no points satisfying both inequalities.

Step 3: Integrate the strip width The required area is

$$A = \int_{-1/2}^1 \left(\frac{y+1}{4} - \frac{y^2}{2} \right) dy = \left[\frac{y^2}{8} + \frac{y}{4} - \frac{y^3}{6} \right]_{-1/2}^1.$$

The upper endpoint contributes $5/24$, while the lower contributes $-7/96$. Hence

$$A = \frac{20}{96} + \frac{7}{96} = \frac{9}{32}.$$

The other listed fractions do not equal this difference; in particular one must subtract the negative lower-endpoint contribution.

Step 4: Conclusion The bounded region has area $\boxed{9/32}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: For a sideways parabola bounded by a sloping line, horizontal strips often avoid splitting the region into separate integrals.

9. Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in $[-1, 1]$ such that $\cos^{-1} x - \sin^{-1} y = \alpha$, $\frac{-\pi}{2} \leq \alpha \leq \pi$. Then, the minimum value of $x^2 + y^2 + 2xy \sin \alpha$ is [4 marks]

- (A) 0
(B) -1
(C) $\frac{1}{2}$
(D) $\frac{-1}{2}$

Correct Answer: (A) 0

Solution:

Concept:

- The sine of every real angle lies in $[-1, 1]$, so $2xy \sin \alpha \geq -2|xy|$, regardless of the sign of xy .
- A minimum requires both a universal lower bound and a feasible pair attaining it; the principal-value condition must hold at that pair.

Step 1: Bound the mixed term Let $F = x^2 + y^2 + 2xy \sin \alpha$. Since $|\sin \alpha| \leq 1$,

$$F \geq x^2 + y^2 - 2|xy| = (|x| - |y|)^2 \geq 0.$$

This estimate applies to every allowed pair and hence to the smaller set satisfying the inverse-trigonometric relation. No negative value is possible, even if the mixed term itself is negative.

Step 2: Exhibit an admissible equality case Choose $x = 0$ and $y = 0$, both of which belong to $[-1, 1]$. Their principal inverse values give

$$\alpha = \cos^{-1} 0 - \sin^{-1} 0 = \frac{\pi}{2} - 0 = \frac{\pi}{2}.$$

This angle lies in the stated interval $[-\pi/2, \pi]$. Substitution in the target expression gives $F = 0$, so the lower bound is actually achieved and is not merely an infimum.

Step 3: Compare the proposed minima The proposed values -1 and $-1/2$ contradict the universal nonnegative bound. The positive value $1/2$ cannot be the minimum because the feasible origin gives the smaller value zero. Thus the choice is fixed without needing to eliminate the inverse trigonometric functions algebraically.

Step 4: Conclusion The minimum value is $\boxed{0}$, attained at $x = y = 0$ with $\alpha = \pi/2$. Hence, the correct option is (A).

Quick Tip to Solve: Before expanding inverse-trigonometric identities, bound the mixed term by its absolute value and test whether the origin gives equality.

10. For $\lambda > 0$, let θ be the angle between the vectors $\vec{a} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. If the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are mutually perpendicular, then the value of $(14 \cos \theta)^2$ is equal to [4 marks]

- (A) 25
(B) 50
(C) 20
(D) 40

Correct Answer: (A) 25

Solution:

Concept:

- Perpendicular vectors have zero scalar product. Expanding the product of a sum and a difference gives the difference of squared vector lengths.
- The angle satisfies $\cos \theta = (\vec{a} \cdot \vec{b}) / (|\vec{a}||\vec{b}|)$; retain its sign until the required squaring.

Step 1: Use perpendicularity to determine lambda The mixed products cancel because $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$. Therefore

$$0 = (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2.$$

From the given components,

$$|\vec{a}|^2 = 1 + \lambda^2 + 9 = 10 + \lambda^2, \quad |\vec{b}|^2 = 9 + 1 + 4 = 14.$$

Thus $\lambda^2 = 4$. The condition $\lambda > 0$ selects $\lambda = 2$, excluding the negative root.

Step 2: Calculate the angle cosine For this value of the parameter,

$$\vec{a} \cdot \vec{b} = (1)(3) + (2)(-1) + (-3)(2) = 3 - 2 - 6 = -5.$$

Both vector magnitudes equal $\sqrt{14}$. Consequently

$$\cos \theta = \frac{-5}{\sqrt{14}\sqrt{14}} = -\frac{5}{14}.$$

The negative sign is consistent with an obtuse angle and should not be discarded during the dot-product calculation.

Step 3: Evaluate exactly the requested expression The quantity asked for is the square of the whole product, not fourteen times the square of the cosine. Thus

$$(14 \cos \theta)^2 = \left(14 \cdot \frac{-5}{14}\right)^2 = (-5)^2 = 25.$$

The remaining options 50, 20 and 40 cannot result from this scalar-product identity and the positive permitted parameter.

Step 4: Conclusion $(14 \cos \theta)^2 = 25$. Hence, the correct option is (A).

Quick Tip to Solve: If a vector sum is perpendicular to its difference, immediately equate the squared lengths of the original vectors.

11. Let $y = y(x)$ be the solution of the differential equation $(x^2 + 4)^2 dy + (2x^3 y + 8xy - 2)dx = 0$. If $y(0) = 0$, then $y(2)$ is equal to [4 marks]

- (A) 2π
(B) $\frac{\pi}{8}$
(C) $\frac{\pi}{16}$
(D) $\frac{\pi}{32}$

Correct Answer: (D) $\frac{\pi}{32}$

Solution:

Concept:

- A first-order linear equation $dy/dx + P(x)y = Q(x)$ becomes a product derivative after multiplication by $I = e^{\int P(x)dx}$.
- The standard integral $\int 2/(x^2 + 4) dx = \tan^{-1}(x/2) + C$ supplies the remaining antiderivative.

Step 1: Put the equation in linear form Since $x^2 + 4 > 0$ for every real x , division introduces no forbidden points. Dividing the printed equation by $(x^2 + 4)^2 dx$ gives

$$\frac{dy}{dx} + \frac{2x}{x^2 + 4}y = \frac{2}{(x^2 + 4)^2}.$$

Here the terms $2x^3 y + 8xy$ combine as $2x(x^2 + 4)y$; the constant term moves to the right with positive sign.

Step 2: Multiply by the integrating factor The integrating factor is

$$I = \exp\left(\int \frac{2x}{x^2 + 4} dx\right) = \exp(\log(x^2 + 4)) = x^2 + 4.$$

Thus

$$\frac{d}{dx}((x^2 + 4)y) = \frac{2}{x^2 + 4}.$$

Integrating produces

$$(x^2 + 4)y = \tan^{-1}(x/2) + C.$$

Differentiating the left side confirms both the coefficient of y and the coefficient of its derivative.

Step 3: Impose the initial condition and evaluate At $x = 0$, the condition $y(0) = 0$ makes the left side zero, while $\tan^{-1} 0 = 0$, so $C = 0$. At $x = 2$,

$$8y(2) = \tan^{-1} 1 = \frac{\pi}{4}, \quad y(2) = \frac{\pi}{32}.$$

The factors 8 and 4 must both be retained; dropping either yields one of the larger distractors.

Step 4: Conclusion The initial-value solution gives $y(2) = \pi/32$. Hence, the correct option is (D).

Quick Tip to Solve: Factor the coefficient of y before finding the integrating factor; here its factor x squared plus four cancels one denominator immediately.

12. If the mean of the following probability distribution of a random variable X :

X	0	2	4	6	8
$P(X)$	a	$2a$	$a + b$	$2b$	$3b$

is $\frac{46}{9}$, then the variance of the distribution is

[4 marks]

- (A) $\frac{581}{81}$
(B) $\frac{566}{81}$
(C) $\frac{151}{27}$
(D) $\frac{173}{27}$

Correct Answer: (B) $\frac{566}{81}$

Solution:

Concept:

- The probabilities must sum to one, and $E(X) = \sum xp(x)$ equals the specified mean. These give two equations for a and b .
- Variance is $\text{Var}(X) = E(X^2) - [E(X)]^2$, with $E(X^2) = \sum x^2p(x)$; square the values of X , not their probabilities.

Step 1: Find the two parameter equations Adding the five probabilities gives

$$a + 2a + (a + b) + 2b + 3b = 4a + 6b = 1.$$

The mean condition yields

$$0a + 2(2a) + 4(a + b) + 6(2b) + 8(3b) = 8a + 40b = \frac{46}{9}.$$

These equations use each table entry once, including the probability $a+b$ attached to the value 4.

Step 2: Solve for a and b Twice the normalization equation is $8a + 12b = 2$. Subtracting it from the mean equation gives

$$28b = \frac{46}{9} - 2 = \frac{28}{9}, \quad b = \frac{1}{9}.$$

Then

$$4a = 1 - \frac{6}{9} = \frac{1}{3}, \quad a = \frac{1}{12}.$$

Both values are nonnegative, as required for the displayed probabilities.

Step 3: Compute the second moment and subtract the square of the mean Weighting the squared values gives

$$E(X^2) = 4(2a) + 16(a + b) + 36(2b) + 64(3b) = 24a + 280b.$$

Hence

$$E(X^2) = 2 + \frac{280}{9} = \frac{298}{9}.$$

Therefore

$$\text{Var}(X) = \frac{298}{9} - \left(\frac{46}{9}\right)^2 = \frac{2682 - 2116}{81} = \frac{566}{81}.$$

None of the other listed fractions equals this exact difference. In particular the mean must be squared before subtraction.

Step 4: Conclusion The variance is $\boxed{566/81}$. Hence, the correct option is (B).

Quick Tip to Solve: Build normalization, first moment and second moment in separate lines; this prevents confusing the mean with the mean square.

13. Let PQ be a chord of the parabola $y^2 = 12x$ and the midpoint of PQ be at $(4, 1)$. Then, which of the following point lies on the line passing through the points P and Q ? [4 marks]

- (A) $(3, -3)$
- (B) $(\frac{1}{2}, -20)$
- (C) $(2, -9)$
- (D) $(\frac{3}{2}, -16)$

Correct Answer: (B) $(\frac{1}{2}, -20)$

Solution:

Concept:

- For a parabola $y^2 = 4ax$, subtracting its endpoint equations relates a chord slope to the sum of the endpoint ordinates.
- The midpoint gives the average of both coordinates. Once the slope is known, the chord line follows from point-slope form.

Step 1: Express the endpoints using their midpoint Let the distinct chord endpoints be $P = (x_1, y_1)$ and $Q = (x_2, y_2)$. Since their midpoint is $(4, 1)$,

$$x_1 + x_2 = 8, \quad y_1 + y_2 = 2.$$

As each endpoint lies on $y^2 = 12x$, subtraction gives

$$(y_2 - y_1)(y_2 + y_1) = 12(x_2 - x_1).$$

A vertical chord would have endpoint ordinates opposite in sign, hence midpoint ordinate zero; that is impossible here.

Step 2: Find the slope and line equation Thus we can divide by $x_2 - x_1$. Using the sum of ordinates,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12}{y_2 + y_1} = 6.$$

The chord passes through its midpoint, so

$$y - 1 = 6(x - 4), \quad y = 6x - 23.$$

This equation is for the chord line, not the tangent at the midpoint, which is not on the parabola.

Step 3: Test the listed coordinates For option (B), $6(1/2) - 23 = 3 - 23 = -20$, exactly its listed ordinate. At the abscissae in options (A), (C) and (D), the line instead gives respectively -5 , -11 and -14 ; these disagree with their proposed ordinates -3 , -9 and -16 . The point need only lie on the infinite line through P and Q, so it is not required to lie on the parabola itself.

Step 4: Conclusion The point on the chord line is $(1/2, -20)$. Hence, the correct option is (B).

Quick Tip to Solve: For a parabola chord, subtract the endpoint equations; the known midpoint ordinate then determines the chord slope without solving for either endpoint.

14. Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line $x + y = 2$ intersects the circle C at the points P and Q. Let MN be a chord of C of length 2 unit and slope -1 . Then, a distance (in units) between the chord PQ and the chord MN is [4 marks]

- (A) $3 - \sqrt{2}$
- (B) $2 - \sqrt{3}$
- (C) $\sqrt{2} - 1$
- (D) $\sqrt{2} + 1$

Correct Answer: (A) $3 - \sqrt{2}$

Solution:

Concept:

- The perpendicular from a circle centre bisects a chord. If R is the radius and L the chord length, its distance d from the centre obeys $d^2 + (L/2)^2 = R^2$.
- A line of slope -1 has equation $x + y = c$; the distance from the origin is $|c|/\sqrt{2}$, and parallel-line distances use the difference of their constants.

Step 1: Determine the distance of MN from the centre The circle has radius $R = \sqrt{10}$, and chord MN has length 2. Each half-chord therefore has length 1. The right triangle formed by the radius, half-chord and centre perpendicular gives

$$d_{MN} = \sqrt{R^2 - 1^2} = \sqrt{10 - 1} = 3.$$

As MN has slope -1, it is parallel to PQ and can be written $x + y = c$. Thus $|c|/\sqrt{2} = 3$, so $c = \pm 3\sqrt{2}$.

Step 2: Find the possible parallel separations The given chord PQ lies on $x + y = 2$. The distance between this line and MN is

$$D = \frac{|c - 2|}{\sqrt{2}}.$$

For $c = 3\sqrt{2}$, the separation is

$$D = \frac{3\sqrt{2} - 2}{\sqrt{2}} = 3 - \sqrt{2}.$$

For the other possible chord, $c = -3\sqrt{2}$, it is $3 + \sqrt{2}$. The question asks for a distance, so it suffices to identify the listed attainable separation.

Step 3: Identify the listed attainable value Only $3 - \sqrt{2}$ appears among the four choices. The values $2 - \sqrt{3}$, $\sqrt{2} - 1$ and $\sqrt{2} + 1$ equal neither of the two separations derived from the radius and chord length. Both MN locations are geometrically possible, but their two different distances do not make any additional listed option correct.

Step 4: Conclusion An attainable distance between the chords is $3 - \sqrt{2}$ units. Hence, the correct option is (A).

Quick Tip to Solve: Use the half-chord in the radius triangle, then measure the gap perpendicular to the parallel chords rather than along an axis.

15. The area (in sq. units) of the region $S = \{z \in \mathbb{C} : |z - 1| \leq 2; (z + \bar{z}) + i(z - \bar{z}) \leq 2, \text{Im}(z) \geq 0\}$ is [4 marks]

- (A) $\frac{7\pi}{4}$
- (B) $\frac{3\pi}{2}$
- (C) $\frac{7\pi}{3}$
- (D) $\frac{17\pi}{8}$

Correct Answer: (B) $\frac{3\pi}{2}$

Solution:

Concept:

- Write $z = x + iy$, where x, y are real. Then $|z - 1| \leq 2$ describes the closed disc $(x - 1)^2 + y^2 \leq 4$, centred at $(1, 0)$ with radius 2.
- The area of a circular sector with radius r and angle ϕ in radians is $\frac{1}{2}r^2\phi$. Boundary curves have zero area, so closed inequalities do not change the area calculation.

Step 1: Translate the linear inequality Using $z + \bar{z} = 2x$ and $z - \bar{z} = 2iy$, the second restriction becomes

$$2x + i(2iy) \leq 2 \iff x - y \leq 1.$$

Thus the required portion of the disc lies above the line $y = x - 1$, and the final restriction also puts it above the real axis, $y \geq 0$. The two bounding lines pass through the centre of the circle.

Step 2: Measure the retained angle about the centre Use coordinates relative to the centre: $u = x - 1 = r \cos \phi$, $y = r \sin \phi$, with $0 \leq r \leq 2$. The upper-half-plane condition first restricts the angle to $0 \leq \phi \leq \pi$. The other condition is $y \geq u$, or $\sin \phi \geq \cos \phi$. Within that upper semicircle this means

$$\frac{\pi}{4} \leq \phi \leq \pi.$$

Consequently the retained angle is $\pi - \pi/4 = 3\pi/4$, rather than the angle of the entire upper semicircle.

Step 3: Calculate and compare the areas The region is therefore exactly a sector, since both straight

boundaries pass through the centre. Its area is

$$\frac{1}{2}(2)^2 \left(\frac{3\pi}{4} \right) = \frac{3\pi}{2}.$$

Equivalently, remove a sector of angle $\pi/4$ and area $\pi/2$ from the upper semicircle of area 2π . The values $7\pi/4$, $7\pi/3$, and $17\pi/8$ do not equal this sector area and so the other listed options fail.

Step 4: Conclusion The required area is $\boxed{\frac{3\pi}{2}}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Translate the origin to the centre of the disc first; half-plane boundaries through the centre then reduce the area problem to measuring an angle.

16. If the value of the integral $\int_{-1}^1 \frac{\cos \alpha x}{1+3^x} dx$ is $\frac{2}{\pi}$. Then, a value of α is

[4 marks]

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{3}$
- (D) $\frac{\pi}{6}$

Correct Answer: (A) $\frac{\pi}{2}$

Solution:

Concept:

- On a symmetric interval, substituting $x \mapsto -x$ preserves the integral. An even numerator such as $\cos(\alpha x)$ is unchanged by this substitution.
- The complementary weights satisfy $\frac{1}{1+3^x} + \frac{1}{1+3^{-x}} = 1$. Adding the two representations therefore removes the exponential denominator.

Step 1: Reflect the integral Let

$$I = \int_{-1}^1 \frac{\cos(\alpha x)}{1+3^x} dx.$$

Under $x = -t$, the limits reverse and the minus sign in $dx = -dt$ reverses them back. Since cosine is even, this gives

$$I = \int_{-1}^1 \frac{\cos(\alpha x)}{1+3^{-x}} dx.$$

The denominator is strictly positive everywhere, so all these integrals are well defined.

Step 2: Add the complementary expressions Adding the original and reflected forms and using the weight identity yields

$$2I = \int_{-1}^1 \cos(\alpha x) dx.$$

For the listed nonzero values of α , integration gives

$$2I = \left[\frac{\sin(\alpha x)}{\alpha} \right]_{-1}^1 = \frac{2 \sin \alpha}{\alpha}, \quad I = \frac{\sin \alpha}{\alpha}.$$

Thus the condition to check is $\sin \alpha / \alpha = 2/\pi$. This determines a suitable listed value without requiring a general solution of the transcendental equation.

Step 3: Test all four listed values For $\alpha = \pi/2$, the ratio is $1/(\pi/2) = 2/\pi$, exactly the prescribed value. For the other options the ratios are, respectively,

$$\frac{2\sqrt{2}}{\pi}, \quad \frac{3\sqrt{3}}{2\pi}, \quad \frac{3}{\pi}.$$

None equals $2/\pi$: their numerators after multiplication by π are $2\sqrt{2}$, $3\sqrt{3}/2$, and 3, each different from 2. The question asks for a value, so proving uniqueness over all real numbers is unnecessary.

Step 4: Conclusion A suitable value is $\alpha = \pi/2$. Hence, the correct option is (A).

Quick Tip to Solve: For an even numerator over $1 + a^x$ on symmetric limits, add the reflected integral to cancel the exponential weights.

17. Let three real numbers a, b, c be in arithmetic progression and $a + 1, b, c + 3$ be in geometric progression. If $a > 10$ and the arithmetic mean of a, b and c is 8, then the cube of the geometric mean of a, b and c is [4 marks]

- (A) 120
- (B) 316
- (C) 312
- (D) 128

Correct Answer: (A) 120

Solution:

Concept:

- Three numbers a, b, c in arithmetic progression satisfy $a + c = 2b$, so their arithmetic mean is the middle term b .
- For three numbers in geometric progression, the square of the middle term equals the product of the outer terms. The cube of the geometric mean of three positive numbers is their product.

Step 1: Use the arithmetic mean The given mean is 8, so $a + b + c = 24$. Because the numbers are in arithmetic progression, $a + c = 2b$, hence $3b = 24$. It follows that

$$b = 8, \quad c = 16 - a.$$

This uses both arithmetic conditions and leaves just one unknown. The condition $a > 10$ must be retained when the quadratic equation produces two possibilities.

Step 2: Apply the geometric progression condition The three numbers $a + 1, b, c + 3$ are in geometric progression. Consequently

$$b^2 = (a + 1)(c + 3).$$

Substituting $b = 8$ and $c = 16 - a$ gives

$$64 = (a + 1)(19 - a) = -a^2 + 18a + 19.$$

Rearrange and factor:

$$a^2 - 18a + 45 = 0, \quad (a - 15)(a - 3) = 0.$$

Only $a = 15$ satisfies $a > 10$; $a = 3$ is excluded by the explicitly given inequality, even though it satisfies the other equations.

Step 3: Compute the requested cube With $a = 15$, the original arithmetic progression is 15, 8, 1. Its successive differences are both -7 . The shifted triple is 16, 8, 4, whose successive ratios are both $1/2$, so the selected root satisfies the geometric condition as well. All original numbers are positive, and their geometric mean is

$$G = (15 \cdot 8 \cdot 1)^{1/3}, \quad G^3 = 120.$$

The question asks for G^3 , not for G itself or for the product of the shifted numbers. The alternatives 316, 312, and 128 are not the product abc .

Step 4: Conclusion The cube of the geometric mean is 120. Hence, the correct option is (A).

Quick Tip to Solve: For a three-term arithmetic progression, replace the given arithmetic mean directly by the middle term before imposing the geometric condition.

18. Let a relation R on $\mathbb{N} \times \mathbb{N}$ be defined as: $(x_1, y_1) R (x_2, y_2)$ if and only if $x_1 \leq x_2$ or $y_1 \leq y_2$. Consider the two statements:

(I) R is reflexive but not symmetric.

(II) R is transitive

Then which one of the following is true?

[4 marks]

(A) Only (II) is correct.

(B) Both (I) and (II) are correct.

(C) Neither (I) nor (II) is correct.

(D) Only (I) is correct.

Correct Answer: (D) Only (I) is correct.

Solution:

Concept:

- A relation is reflexive when every element is related to itself; symmetric when reversing any related pair preserves the relation; and transitive when two successive related pairs imply the direct pair.
- The word or means that either coordinate inequality is enough. The first relation in a chain and the second may hold because of different coordinates, which can obstruct transitivity.

Step 1: Verify reflexivity Take any $P = (x, y) \in \mathbb{N} \times \mathbb{N}$. In comparing P with itself, $x \leq x$ and $y \leq y$ are both true. In particular at least one is true, so $P R P$. Since this argument holds for every ordered pair, R is reflexive. This establishes the first part of statement (I) without restricting the chosen natural numbers.

Step 2: Disprove symmetry with one pair Take $P = (1, 1)$ and $Q = (2, 2)$. We have $P R Q$ because $1 \leq 2$. But $Q R P$ would require $2 \leq 1$ in at least one coordinate, and both comparisons are false. Thus Q is not related to P , so symmetry fails. The relation is therefore reflexive but not symmetric, making statement (I) true.

Step 3: Test transitivity independently Choose

$$P = (2, 2), \quad Q = (3, 1), \quad T = (1, 1).$$

Then $P R Q$ holds because the first coordinates satisfy $2 \leq 3$. Also $Q R T$ holds because the second coordinates satisfy $1 \leq 1$. However $P R T$ is false: it requires $2 \leq 1$ in the first or second coordinate, and neither holds. This is a complete counterexample to transitivity, so statement (II) is false. Options (A) and (B) wrongly accept (II), while (C) wrongly rejects the proved statement (I).

Step 4: Conclusion Only statement (I) is correct. Hence, the correct option is (D).

Quick Tip to Solve: For relations defined using or, try a transitivity counterexample where the first link uses one coordinate and the next link uses the other.

19. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = I + \text{adj}(A) + (\text{adj } A)^2 + \dots + (\text{adj } A)^{10}$. Then, the sum of all the elements of the matrix B is: [4 marks]

- (A) -110
- (B) 22
- (C) -124
- (D) -88

Correct Answer: (D) -88

Solution:

Concept:

- For a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, its adjoint is $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. Here this gives $\text{adj } A = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.
- If $N^2 = 0$, the binomial expansion truncates to $(I + N)^k = I + kN$ for every nonnegative integer k . This avoids multiplying ten matrices separately.

Step 1: Find the general matrix power Write

$$\text{adj } A = I + N, \quad N = \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}.$$

Direct multiplication gives $N^2 = 0$. All binomial terms containing two or more factors of N therefore vanish, so

$$(\text{adj } A)^k = \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix} \quad (k \geq 0).$$

The case $k = 0$ is the identity term already present in the definition of B .

Step 2: Sum all eleven terms The expression for B includes the powers from 0 through 10, inclusive. There are eleven diagonal contributions of 1 in each diagonal position, and the upper-right entries form an arithmetic sum. Thus

$$B = \sum_{k=0}^{10} \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 11 & -2 \sum_{k=0}^{10} k \\ 0 & 11 \end{pmatrix}.$$

Using $\sum_{k=0}^{10} k = 10 \cdot 11/2 = 55$, this becomes

$$B = \begin{pmatrix} 11 & -110 \\ 0 & 11 \end{pmatrix}.$$

Step 3: Add the four entries and reject distractors The required sum includes all four entries, not just the upper-right entry or the diagonal trace:

$$11 - 110 + 0 + 11 = -88.$$

Thus -110 counts only the upper-right entry and omits both diagonals. The value 22 counts only the diagonal entries and misses the negative contribution. The remaining alternative -124 also disagrees with the explicit matrix sum. Counting the identity as one of the eleven terms is essential to obtaining the correct diagonal entries.

Step 4: Conclusion The sum of all elements of B is $\boxed{-88}$. Hence, the correct option is (D).

Quick Tip to Solve: For an upper triangular matrix with equal diagonal entries, separate out the identity and check whether the remaining matrix squares to zero.

20. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k}$, $x \in \mathbb{R}$. If $\vec{d}$ is the unit vector in the direction of $\vec{b} + \vec{c}$ such that $\vec{a} \cdot \vec{d} = 1$, then $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to [4 marks]

- (A) 3
(B) 9
(C) 11
(D) 6

Correct Answer: (C) 11

Solution:

Concept:

- The unit vector in the direction of a nonzero vector $\mathbf{v}$ is $\mathbf{v}/|\mathbf{v}|$. The positive magnitude in the denominator must be retained when using a dot-product constraint.
- The scalar triple product $(\vec{a} \times \vec{b}) \cdot \vec{c}$ can be evaluated by finding the cross product and then taking its dot product with $\vec{c}$.

Step 1: Express the given unit vector Adding the two specified vectors gives

$$\vec{b} + \vec{c} = (x + 2)\hat{i} + 6\hat{j} - 2\hat{k}.$$

Its magnitude is $\sqrt{(x + 2)^2 + 40}$, which is positive for every real x . Consequently

$$\vec{d} = \frac{(x + 2)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(x + 2)^2 + 40}}.$$

Taking its dot product with $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ yields the exact condition

$$\frac{x + 6}{\sqrt{(x + 2)^2 + 40}} = 1.$$

Step 2: Solve and check the parameter The last equation requires $x + 6 > 0$. Squaring and expanding gives

$$\begin{aligned}(x + 6)^2 &= (x + 2)^2 + 40, \\ x^2 + 12x + 36 &= x^2 + 4x + 44, \quad 8x = 8, \quad x = 1.\end{aligned}$$

For this value, $\vec{b} + \vec{c} = (3, 6, -2)$ has magnitude 7, while the sum of its components is also 7. Therefore the original unsquared dot-product equation is satisfied, so the selected parameter is valid.

Step 3: Evaluate the signed triple product Using the component formula for the cross product,

$$\vec{a} \times \vec{b} = (1(-5) - 1(4), 1(2) - 1(-5), 1(4) - 1(2)) = (-9, 7, 2).$$

With $x = 1$, we have $\vec{c} = (1, 2, 3)$, hence

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = -9 + 14 + 6 = 11.$$

The order of the cross product matters; reversing it would change the sign. The other listed values 3,

9, and 6 do not equal the full dot product and are therefore excluded.

Step 4: Conclusion The scalar triple product is $\boxed{11}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the unit-vector constraint to determine the unknown component first, and check the unsquared equation before computing the triple product.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $S = \{\sin^2 2\theta : (\sin^4 \theta + \cos^4 \theta)x^2 + (\sin 2\theta)x + (\sin^6 \theta + \cos^6 \theta) = 0 \text{ has real roots}\}$. If α and β be the smallest and largest elements of the set S , respectively, then $3((\alpha - 2)^2 + (\beta - 1)^2)$ equals _____. [4 marks]

Solution:

Concept:

- Put $t = \sin^2 2\theta$. Since θ is real, $0 \leq t \leq 1$, and $\sin^2 \theta \cos^2 \theta = t/4$.
- The identities $\sin^4 \theta + \cos^4 \theta = 1 - t/2$ and $\sin^6 \theta + \cos^6 \theta = 1 - 3t/4$ express both even-power sums in one variable. A real quadratic has real roots exactly when its discriminant is nonnegative.

Step 1: Rewrite the discriminant using one variable The coefficient of x^2 becomes $A = 1 - t/2$, which is at least $1/2$ for $0 \leq t \leq 1$. Thus the equation is genuinely quadratic throughout the relevant range. Its linear coefficient is $\sin 2\theta$, whose square is t , and its constant is $C = 1 - 3t/4$. The discriminant is therefore

$$\Delta = t - 4 \left(1 - \frac{t}{2}\right) \left(1 - \frac{3t}{4}\right) = 6t - \frac{3}{2}t^2 - 4.$$

Step 2: Find the full admissible interval The condition $\Delta \geq 0$ is equivalent to

$$3t^2 - 12t + 8 \leq 0.$$

Its two boundary roots are

$$t = \frac{12 \pm \sqrt{144 - 96}}{6} = 2 \pm \frac{2\sqrt{3}}{3}.$$

Because this quadratic opens upward, the inequality holds between the roots. Intersecting that interval with $[0, 1]$, the range attainable by a squared sine, gives

$$S = \left[2 - \frac{2\sqrt{3}}{3}, 1\right].$$

Every value in this interval is attained by $\sin^2 2\theta$, and the nonnegative discriminant then guarantees real roots, including a repeated root at the lower endpoint.

Step 3: Insert the smallest and largest values It follows that

$$\alpha = 2 - \frac{2\sqrt{3}}{3}, \quad \beta = 1.$$

The larger algebraic boundary $2 + 2\sqrt{3}/3$ cannot be β , since it exceeds the maximum possible value 1 of a squared sine. Substitution now gives

$$3((\alpha - 2)^2 + (\beta - 1)^2) = 3 \left[\left(-\frac{2\sqrt{3}}{3} \right)^2 + 0^2 \right] = 3 \cdot \frac{4}{3} = 4.$$

Step 4: Conclusion The requested dimensionless numerical value is $\boxed{4}$.

Quick Tip to Solve: After solving a discriminant inequality in $\sin^2 2\theta$, intersect its solution with $[0, 1]$ before selecting endpoints.

22. If $\int \operatorname{cosec}^5 x dx = \alpha \cot x \operatorname{cosec} x (\operatorname{cosec}^2 x + \frac{3}{2}) + \beta \log_x |\tan \frac{x}{2}| + C$ where $\alpha, \beta \in \mathbb{R}$ and C is the constant of integration, then the value of $8(\alpha + \beta)$ equals _____. [4 marks]

Solution:

Concept:

- The reduction formula for an odd power of cosecant is obtained by integration by parts. In particular, $I_5 = -\frac{1}{4} \cot x \csc^3 x + \frac{3}{4} I_3$, where $I_n = \int \csc^n x dx$.
- The base of a logarithm matters: $\log_x u = \ln u / \ln x$, so a variable base introduces an additional derivative term. Work on $0 < x < 1$, where the printed logarithm is defined.

Step 1: Compute the actual antiderivative Applying the reduction formula once more gives $I_3 = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln |\tan(x/2)|$. Hence, with $H(x) = \cot x \csc x (\csc^2 x + 3/2)$, we obtain

$$I_5 = -\frac{1}{4} H(x) + \frac{3}{8} \ln |\tan(x/2)| + C.$$

Direct differentiation gives $H' = -4 \csc^5 x + \frac{3}{2} \csc x$, which verifies the coefficients without relying on a memorised sign.

Step 2: Test the logarithm actually printed The question prints $\log_x$, rather than a natural logarithm. Put $L(x) = \ln(\tan(x/2)) / \ln x$ on $0 < x < 1$. As $x \rightarrow 0^+$, we have $\ln(\tan(x/2)) = \ln x - \ln 2 + O(x^2)$, so $xL'(x) \rightarrow 0$. Differentiating the proposed identity and comparing its leading x^{-5} term forces $\alpha = -1/4$. The remaining equality would then be

$$\beta L'(x) = \frac{3}{8} \csc x.$$

Multiplication by x and passage to the limit gives $0 = 3/8$, a contradiction for every constant β .

Step 3: Conclusion For the printed variable-base logarithm, no real constants α, β satisfy the identity, so the requested value is undefined. The key value 1 results only if the logarithm is natural: then $\alpha = -1/4$, $\beta = 3/8$, and $8(\alpha + \beta) = 1$.

Quick Tip to Solve: Before comparing antiderivative coefficients, check the logarithm base; a variable base cannot be absorbed into a constant coefficient.

23. Let A be a 2×2 symmetric matrix such that $A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ and the determinant of A be 1. If $A^{-1} = \alpha A + \beta I$, where I is an identity matrix of order 2×2 , then $\alpha + \beta$ equals _____. [4 marks]

Solution:

Concept:

- A symmetric matrix of order two has the form $A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$. Multiplication by $(1, 1)^T$ gives its row sums.
- Cayley-Hamilton gives $A^2 - (\text{tr } A)A + (\det A)I = 0$. Since $\det A = 1$, multiplication by A^{-1} expresses the inverse as a linear combination of A and I .

Step 1: Use the row sums and determinant The given vector equation yields $a + b = 3$ and $b + d = 7$. Thus $a = 3 - b$, $d = 7 - b$, and the determinant condition becomes

$$(3 - b)(7 - b) - b^2 = 1.$$

Expanding gives $21 - 10b = 1$, hence $b = 2$, $a = 1$ and $d = 5$. Therefore

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}, \quad \text{tr } A = 6.$$

Step 2: Express the inverse and compare coefficients Substitution in Cayley-Hamilton gives $A^2 - 6A + I = 0$. The determinant is nonzero, so multiplication by A^{-1} is valid and gives

$$A - 6I + A^{-1} = 0, \quad A^{-1} = -A + 6I.$$

The matrices A and I are linearly independent because A has nonzero off-diagonal entries. Consequently coefficient comparison is unique: $\alpha = -1$, $\beta = 6$. As a direct check, the inverse is $\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$, which equals $-A + 6I$.

Step 3: Conclusion Thus $\alpha + \beta = -1 + 6 = \boxed{5}$, a dimensionless value.

Quick Tip to Solve: For a nonsingular two-by-two matrix with determinant one, use $A^{-1} = (\text{tr } A)I - A$ after finding the trace.

24. Consider a triangle ABC having the vertices $A(1, 2)$, $B(\alpha, \beta)$ and $C(\gamma, \delta)$ and angles $\angle ABC = \frac{\pi}{6}$ and $\angle BAC = \frac{2\pi}{3}$. If the points B and C lie on the line $y = x + 4$, then $\alpha^2 + \gamma^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The angles of a triangle sum to π . Equal angles have equal opposite sides, and the altitude from the vertex of an isosceles triangle bisects both its base and vertex angle.
- The foot of the perpendicular from A to BC is the midpoint of BC . A displacement along $y = x + 4$

has equal horizontal and vertical components.

Step 1: Find the foot of the altitude The third angle is $\angle ACB = \pi - 2\pi/3 - \pi/6 = \pi/6$. Thus $AB = AC$, and the perpendicular from $A(1, 2)$ to BC passes through its midpoint M . The base line has slope 1, so the perpendicular through A has equation $y - 2 = -(x - 1)$, or $y = -x + 3$. Solving simultaneously with $y = x + 4$ gives

$$M = (-1/2, 7/2), \quad AM = \frac{3}{\sqrt{2}}.$$

Step 2: Recover the two base coordinates The altitude bisects the vertex angle, so $\angle BAM = \pi/3$. In the right triangle AMB ,

$$MB = AM \tan(\pi/3) = 3\sqrt{3/2}.$$

Write the two endpoints as $B = M + (t, t)$, $C = M - (t, t)$, with their names interchangeable. Since $MB = \sqrt{2}|t|$, we find $t^2 = 27/4$. Their abscissae are therefore $-1/2 + t$ and $-1/2 - t$; adding their squares cancels the mixed terms:

$$\alpha^2 + \gamma^2 = 2(1/4 + t^2) = \frac{1}{2} + \frac{27}{2} = 14.$$

Step 3: Conclusion The required sum of squared abscissae is $\boxed{14}$.

Quick Tip to Solve: Use equal base angles to find the midpoint first; then add the squared endpoint coordinates so that the unknown displacement sign cancels.

25. In a tournament, a team plays 10 matches with probabilities of winning and losing each match as $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Let x be the number of matches that the team wins, and y be the number of matches that team loses. If the probability $P(|x - y| \leq 2)$ is p , then $3^9 p$ equals _____. [4 marks]

Solution:

Concept:

- With independent matches and fixed win probability $1/3$, the number of wins X has the binomial distribution $\text{Bin}(10, 1/3)$. Thus $P(X = k) = \binom{10}{k} (1/3)^k (2/3)^{10-k}$.
- Every match is either a win or a loss, so $Y = 10 - X$. Convert the restriction on their difference into an integer interval for X before summing probabilities.

Step 1: Identify all admissible numbers of wins Using $Y = 10 - X$, the event becomes

$$|X - Y| \leq 2 \iff |2X - 10| \leq 2 \iff 4 \leq X \leq 6.$$

Since a count takes integer values, the allowed cases are exactly $X = 4, 5, 6$. These cases are mutually exclusive, so their probabilities add. Counts outside this interval give a win-loss difference of at least four and do not belong to the required event.

Step 2: Evaluate the binomial sum For each admissible k , write the probability with the common

denominator 3^{10} :

$$p = \frac{\binom{10}{4}2^6 + \binom{10}{5}2^5 + \binom{10}{6}2^4}{3^{10}}.$$

The numerator terms are $210 \cdot 64 = 13440$, $252 \cdot 32 = 8064$, and $210 \cdot 16 = 3360$. Hence

$$p = \frac{24864}{3^{10}}, \quad 3^9 p = \frac{24864}{3} = 8288.$$

The different powers of two must be retained because wins and losses do not have equal probabilities.

Step 3: Conclusion The requested dimensionless value is $\boxed{8288}$.

Quick Tip to Solve: Eliminate losses using $Y = 10 - X$; then sum only the three binomial terms for four, five and six wins.

26. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{2x}{\sqrt{1+9x^2}}$. If the composition of f , $\underbrace{(f \circ f \circ f \circ \dots \circ f)}_{10 \text{ times}}(x) = \frac{2^{10}x}{\sqrt{1+9\alpha x^2}}$, then the value of $\sqrt{3\alpha+1}$ is equal to _____. [4 marks]

Solution:

Concept:

- To iterate $f(x) = 2x/\sqrt{1+9x^2}$, track the numerator multiplier and the coefficient of x^2 in the denominator separately.
- All square roots are positive for real x , so combining the nested denominators introduces no sign ambiguity. The resulting coefficient follows a geometric sum.

Step 1: Derive the recurrence for the denominator Let $f^{\circ n}(x) = 2^n x / \sqrt{1+9a_n x^2}$. For one application, $a_1 = 1$. Applying f to this expression gives

$$f^{\circ(n+1)}(x) = \frac{2^{n+1}x}{\sqrt{1+9a_n x^2 + 9 \cdot 4^n x^2}}.$$

Therefore $a_{n+1} = a_n + 4^n$. The coefficient is not multiplied by four alone: each new composition contributes the square of the existing numerator multiplier.

Step 2: Sum the recurrence through ten applications Beginning with $a_1 = 1$, the recurrence yields

$$a_n = 1 + 4 + 4^2 + \dots + 4^{n-1} = \frac{4^n - 1}{3}.$$

This also proves the claimed form by induction, since adding 4^n produces $(4^{n+1} - 1)/3$. For the tenfold composition, $\alpha = a_{10} = (4^{10} - 1)/3$. Consequently

$$\sqrt{3\alpha+1} = \sqrt{4^{10}} = 2^{10} = 1024.$$

The positive square root is selected, independently of the sign of the input x , because the requested expression contains only the positive constant $3\alpha + 1$.

Step 3: Conclusion The required dimensionless number is $\boxed{1024}$.

Quick Tip to Solve: For repeated maps of the form $ax/\sqrt{1+bx^2}$, preserve the same form and derive a geometric recurrence for the denominator coefficient.

27. Let $y = y(x)$ be the solution of the differential equation $(x + y + 2)^2 dx = dy, y(0) = -2$. Let the maximum and minimum values of the function $y = y(x)$ in $[0, \frac{\pi}{3}]$ be α and β , respectively. If $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}, \gamma, \delta \in \mathbb{Z}$, then $\gamma + \delta$ equals _____. [4 marks]

Solution:

Concept:

- The substitution $u = x + y + 2$ changes $y' = (x + y + 2)^2$ into the separable equation $u' = 1 + u^2$.
- After applying the initial condition, evaluate extrema on the closed interval by examining the derivative and endpoints. Here y' is a square and is nonnegative.

Step 1: Solve the initial-value problem Put $u = x + y + 2$, so $du/dx = 1 + dy/dx = 1 + u^2$. Separating and integrating gives

$$\arctan u = x + C.$$

At $x = 0$, the condition $y(0) = -2$ gives $u(0) = 0$, hence $C = 0$. On $[0, \pi/3]$, the tangent is finite and this branch gives $u = \tan x$. Therefore

$$y(x) = \tan x - x - 2.$$

Step 2: Find the extreme values and simplify the expression Differentiation gives $y' = \sec^2 x - 1 = \tan^2 x \geq 0$. The derivative is positive for $0 < x \leq \pi/3$, so the minimum is at zero and the maximum at $\pi/3$:

$$\beta = -2, \quad \alpha = \sqrt{3} - \frac{\pi}{3} - 2.$$

Thus $3\alpha + \pi = 3\sqrt{3} - 6$, and

$$(3\alpha + \pi)^2 + \beta^2 = (3\sqrt{3} - 6)^2 + 4 = 67 - 36\sqrt{3}.$$

Since the rational and irrational parts are unique, $\gamma = 67$ and $\delta = -36$. Their sum retains the negative sign of the irrational coefficient.

Step 3: Conclusion It follows that $\gamma + \delta = 67 - 36 = \boxed{31}$.

Quick Tip to Solve: Use $u = x + y + 2$ to separate the equation, then use the nonnegative derivative to locate both extrema at the interval endpoints.

28. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a thrice differentiable function such that $f(0) = 0, f(1) = 1, f(2) = -1, f(3) = 2$ and $f(4) = -2$. Then, the minimum number of zeros of $(3f'f'' + ff''')(x)$ is _____. [4 marks]

Solution:

Concept:

- Rolle's theorem guarantees a derivative zero between distinct zeros of a continuous function differentiable inside the interval. Repeated applications count guaranteed zeros.
- For $g = f^2$, differentiation gives $g' = 2ff'$ and $g''' = 2(3f'f'' + ff''')$. Thus the requested expression is $g'''/2$.

Step 1: Find seven distinct zeros of the first derivative The sign changes $f(1) = 1, f(2) = -1, f(3) = 2, f(4) = -2$ give roots $a \in (1, 2), b \in (2, 3), c \in (3, 4)$. Together with $f(0) = 0$, there are four ordered zeros $0 < a < b < c$. Rolle's theorem applied to f gives one zero of f' in each of $(0, a), (a, b), (b, c)$. These three points are distinct from the four endpoints. At all seven points, $g' = 2ff' = 0$.

Step 2: Differentiate the zero count twice Applying Rolle's theorem between the seven ordered zeros of g' yields at least six distinct zeros of g'' . Applying it again yields at least five distinct zeros of g''' . Consequently the required expression has at least five zeros. To check that this lower bound is attainable, take the polynomial of degree at most four interpolating the five specified values. Its fourth finite difference is $-20 \neq 0$, so it has degree four. For that polynomial, $3f'f'' + ff'''$ has degree five with nonzero leading coefficient. It has at least five distinct real roots by the argument just given and at most five by its degree. Thus exactly five is possible.

Step 3: Conclusion The minimum number of zeros is therefore 5.

Quick Tip to Solve: When $3f'f'' + ff'''$ appears, recognise half the third derivative of f^2 , then count zeros of $2ff'$ first.

29. Consider a line L passing through the points $P(1, 2, 1)$ and $Q(2, 1, -1)$. If the mirror image of the point $A(2, 2, 2)$ in the line L is (α, β, γ) , then $\alpha + \beta + 6\gamma$ is equal to _____. [4 marks]

Solution:

Concept:

- The mirror image of a point in a line is obtained by reflecting it about its perpendicular foot on that line. Thus, if H is the foot from A and B is its image, H is the midpoint of AB .
- A direction vector of the line through P and Q is $\mathbf{d} = \overrightarrow{PQ}$. Writing $H = P + t\mathbf{d}$, perpendicularity gives $(H - A) \cdot \mathbf{d} = 0$.

Step 1: Parametrise the line The given points are $P = (1, 2, 1)$ and $Q = (2, 1, -1)$, so $\mathbf{d} = (1, -1, -2)$, whose squared length is $1 + 1 + 4 = 6$. Hence a general point on the line is

$$H = (1 + t, 2 - t, 1 - 2t).$$

We must determine the value of t that makes AH perpendicular to the line; merely reflecting coordi-

nates individually would not preserve this condition.

Step 2: Locate the perpendicular foot With $A = (2, 2, 2)$, impose the zero dot product:

$$\begin{aligned}(H - A) \cdot \mathbf{d} &= (t - 1) + t - 2(-1 - 2t) \\ &= 6t + 1 = 0.\end{aligned}$$

Thus $t = -1/6$, and the foot is

$$H = \left(\frac{5}{6}, \frac{13}{6}, \frac{4}{3}\right).$$

The negative parameter simply locates the foot on the extension opposite the direction from P to Q.

Step 3: Reflect through the foot and evaluate The image B satisfies $B = 2H - A$. Therefore

$$(\alpha, \beta, \gamma) = \left(-\frac{1}{3}, \frac{7}{3}, \frac{2}{3}\right).$$

Its midpoint with A is exactly H, and the connecting direction is perpendicular to $\mathbf{d}$, verifying the reflection. Now

$$\alpha + \beta + 6\gamma = -\frac{1}{3} + \frac{7}{3} + 6\left(\frac{2}{3}\right) = 2 + 4 = 6.$$

Step 4: Conclusion The requested numerical value is $\boxed{6}$. This is a dimensionless combination of the given coordinates.

Quick Tip to Solve: Find the perpendicular foot first and use image = twice the foot minus the original point; check the midpoint and perpendicularity.

30. There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is _____.

[4 marks]

Solution:

Concept:

- The people in each group are distinct, so an unordered selection is counted using combinations: $\binom{n}{r}$ counts r people chosen from n .
- Exactly four people must come from each group as well as four men and four women overall. Split the selection according to the number k of men taken from Group A, then add disjoint cases.

Step 1: Express each case using k If Group A contributes k men, it contributes $4 - k$ women. Group B must then contribute $4 - k$ men and k women. The number for this case is

$$N_k = \binom{4}{k} \binom{5}{4-k} \binom{5}{4-k} \binom{4}{k}.$$

All integers $k = 0, 1, 2, 3, 4$ are possible with the stated group sizes. The factors refer to four independent selections from disjoint sets, so they multiply.

Step 2: Evaluate all five cases Use the small combination values to obtain

$$N_0 = 1 \times 5 \times 5 \times 1 = 25,$$

$$N_1 = 4 \times 10 \times 10 \times 4 = 1600,$$

$$N_2 = 6 \times 10 \times 10 \times 6 = 3600,$$

$$N_3 = 4 \times 5 \times 5 \times 4 = 400,$$

$$N_4 = 1 \times 1 \times 1 \times 1 = 1.$$

For instance, the middle case chooses two men and two women from each group. The extreme cases are different in size because the groups have opposite gender counts.

Step 3: Add the disjoint possibilities Every valid selection has exactly one value of k , so none is counted twice. Conversely, each case has four selections from each group and the required total gender split. Therefore

$$N = 25 + 1600 + 3600 + 400 + 1 = 5626.$$

Choosing four men from all nine men and four women from all nine women alone would also count selections with the wrong number of people from a particular group, so that shortcut does not enforce the question.

Step 4: Conclusion The number of selections is 5626 ways (a dimensionless count).

Quick Tip to Solve: Track the number of men in just one group; the other three category counts then follow automatically from the totals.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Applying the principle of homogeneity of dimensions, determine which one is correct, where T is time period, G is gravitational constant, M is mass, r is radius of orbit. [4 marks]

(A) $T^2 = \frac{4\pi^2 r}{GM^2}$

(B) $T^2 = 4\pi^2 r^3$

(C) $T^2 = \frac{4\pi^2 r^3}{GM}$

(D) $T^2 = \frac{4\pi^2 r^2}{GM}$

Correct Answer: (C) $T^2 = \frac{4\pi^2 r^3}{GM}$

Solution:

Concept:

- An equation between physical quantities must have the same dimensions on both sides. Pure numbers such as $4\pi^2$ are dimensionless and cannot repair a dimensional mismatch.
- Newton's gravitational law gives $[G] = \text{M}^{-1}\text{L}^3\text{T}^{-2}$. Here $[M] = \text{M}$, $[r] = \text{L}$, and the square of a time period has dimensions T^2 .

Step 1: Form the dimensions of GM Multiply the dimensions of the gravitational constant by those of

mass:

$$[GM] = (M^{-1}L^3T^{-2})M = L^3T^{-2}.$$

Consequently, division of a length cubed by GM leaves time squared. This observation fixes the radius exponent in the dimensionally permitted combination without using a numerical value of any quantity.

Step 2: Check the candidate that has radius cubed For option (C), the right-hand side has dimensions

$$\left[\frac{4\pi^2 r^3}{GM} \right] = \frac{L^3}{L^3T^{-2}} = T^2,$$

exactly matching $[T^2]$. Dimensional analysis tests the form of this expression; it does not by itself derive the dimensionless coefficient $4\pi^2$.

Step 3: Eliminate the other expressions Option (A) contains an extra factor of mass in its denominator: $[r/(GM^2)] = M^{-1}L^{-2}T^2$, so it fails. Option (B) has dimensions L^3 , with no time dimension at all. Option (D) gives $[r^2/(GM)] = L^{-1}T^2$, retaining an unwanted inverse length. None equals time squared, regardless of the common numerical factor.

Step 4: Conclusion The dimensionally correct relation is $T^2 = 4\pi^2 r^3 / (GM)$. Hence, the correct option is (C).

Quick Tip to Solve: First combine G and M: GM has dimensions of length cubed divided by time squared, making the needed power of r immediate.

32. A 2 kg brick begins to slide over a surface which is inclined at an angle of 45° with respect to horizontal axis. The co-efficient of static friction between their surfaces is: [4 marks]

- (A) 1
- (B) 1.7
- (C) 0.5
- (D) $\frac{1}{\sqrt{3}}$

Correct Answer: (A) 1

Solution:

Concept:

- A body just beginning to slide down a rough incline is at limiting equilibrium. The downhill component of weight equals the maximum static friction $f_{s,\max} = \mu_s N$.
- On an incline at angle θ , the components of weight are $mg \sin \theta$ parallel to the plane and $mg \cos \theta$ perpendicular to it. With no other perpendicular force, $N = mg \cos \theta$.

Step 1: Resolve the forces at impending motion The brick tends to move down the surface, so static friction acts up the surface. At the instant it begins sliding, friction has reached its limiting value. The balance along the incline is therefore

$$mg \sin \theta = \mu_s N.$$

This threshold uses the coefficient of static friction. A kinetic-friction formula after the brick is already

moving would not determine the stated threshold.

Step 2: Eliminate the normal reaction Insert $N = mg \cos \theta$ into the limiting equation:

$$\mu_s = \frac{mg \sin \theta}{mg \cos \theta} = \tan \theta.$$

The common factor mg cancels. Thus the given 2 kg mass does not affect the coefficient inferred from this angle; neither a value of g nor a separate numerical calculation of the reaction is needed.

Step 3: Substitute and compare the options The angle of the plane to the horizontal is 45° , so

$$\mu_s = \tan 45^\circ = 1.$$

This coefficient is a dimensionless ratio of two forces. The alternatives 1.7, 0.5 and $1/\sqrt{3}$ would correspond to different angles of repose. In particular, $1/\sqrt{3} = \tan 30^\circ$, whereas the angle here is explicitly 45° . A coefficient need not be less than one; unity is physically allowed.

Step 4: Conclusion The coefficient of static friction is $\boxed{\mu_s = 1}$. Hence, the correct option is (A).

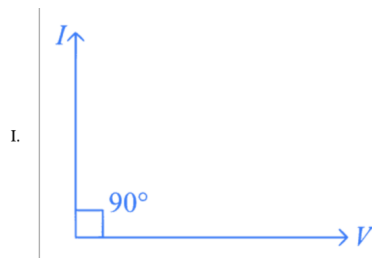
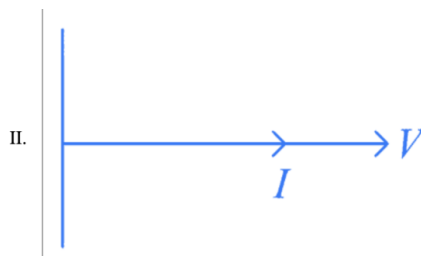
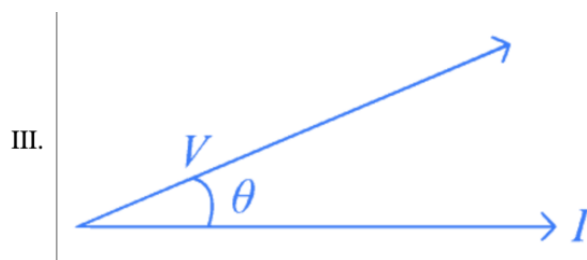
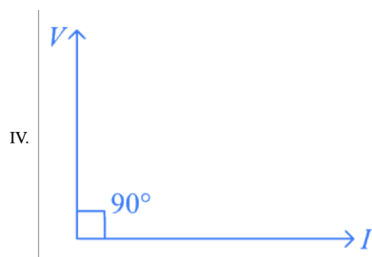
Quick Tip to Solve: For a block just starting to slip under gravity alone, use the angle of repose directly: the coefficient is the tangent of that angle.

33. Match List I with List II.

	LIST I
A.	Purely capacitive circuit
B.	Purely inductive circuit
C.	LCR series at resonance
D.	LCR series circuit

LIST II (I–IV, shown in order below). Choose the correct answer from the options given below:

[4 marks]



(A) A-IV, B-I, C-III, D-II

(B) A-I, B-IV, C-II, D-III

(C) A-IV, B-I, C-II, D-III

(D) A-I, B-IV, C-III, D-II

Correct Answer: (B) A-I, B-IV, C-II, D-III

Solution:

Concept:

- A phasor represents a sinusoidal voltage or current by a rotating vector. With the usual anticlockwise convention, the phasor farther ahead in angle leads the other; equal directions mean equal phases.
- For a pure capacitor, current leads voltage by $\pi/2$. For a pure inductor, current lags voltage by $\pi/2$. These relations follow from $i = C dv/dt$ and $v = L di/dt$, respectively.
- In a series LCR circuit, $\tan \phi = (X_L - X_C)/R$, where $X_L = \omega L$, $X_C = 1/(\omega C)$, and ϕ is the phase of voltage relative to current. At resonance the reactances cancel and $\phi = 0$.

Step 1: Identify the capacitive and inductive graphs In graph I, voltage lies along the positive horizontal axis and current lies vertically upward. Thus current is ahead of voltage by 90° , which is the purely capacitive relation. Therefore A matches I. In graph IV, current is horizontal and voltage is vertical. Voltage leads current by 90° , so current lags voltage by that angle. This is the purely inductive relation, giving B matches IV. The right-angle diagrams must therefore be distinguished by the labels on the arrows, rather than by their identical shapes.

Step 2: Match resonance and the general LCR circuit At resonance, $X_L = X_C$, so the impedance of the series circuit is purely resistive: $Z = R$. Current and applied voltage are in phase. Graph II shows both arrows in the same direction, and hence C matches II. Graph III shows a finite phase difference θ between voltage and current, with voltage leading. It represents a series LCR circuit with net inductive reactance, $X_L > X_C$. Consequently D matches III; it is not the resonant case because its phase angle is nonzero.

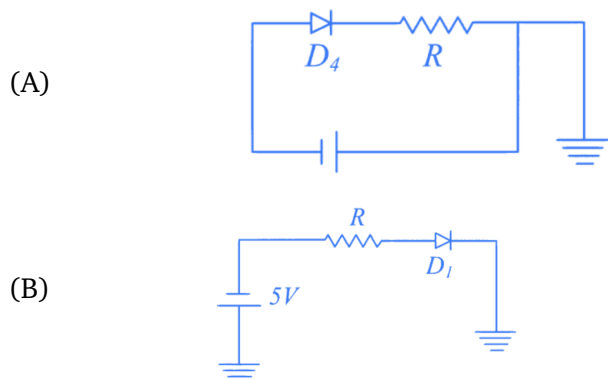
Step 3: Check the offered combinations The complete assignment is A-I, B-IV, C-II, D-III. Option A interchanges the pure-capacitor and pure-inductor relations and also misidentifies resonance. Option C retains the resonance match but reverses the two pure-reactance matches. Option D correctly pairs the pure circuits, but assigns the nonzero-phase graph to resonance. Only option B satisfies all four phase relations simultaneously.

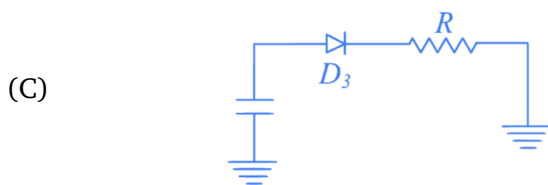
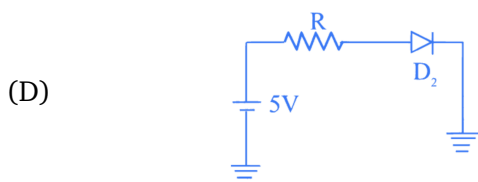
Step 4: Conclusion The matching is A-I, B-IV, C-II, D-III. Hence, the correct option is (B).

Quick Tip to Solve: Read the I and V labels before comparing right-angle phasors: current leads for a capacitor and voltage leads for an inductor. Parallel phasors immediately identify resonance.

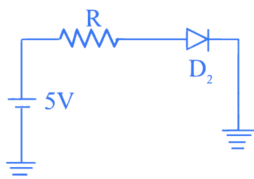
34. Which of the diode circuit shows correct biasing used for the measurement of dynamic resistance of p-n junction diode :

[4 marks]





Correct Answer: (D)



Solution:

Concept:

- To measure the usual forward dynamic resistance of a p-n junction diode, operate it in forward bias. The p-side (anode) must be at higher potential than the n-side (cathode).
- The bar in a diode symbol marks its cathode. The longer line of a cell is its positive terminal; all ground symbols denote a common reference potential. Dynamic resistance is the local slope $r_d = \Delta V_D / \Delta I_D$ of the diode characteristic.

Step 1: Identify the forward-bias connection In each option the diode points from left to right, so its anode is on the left and its cathode, identified by the vertical bar, is on the right. A positive left-side supply feeding the anode through R with the cathode grounded gives forward bias. The series resistor limits current while the supply setting is varied to obtain nearby voltage-current readings.

Step 2: Inspect the polarity in option D In option (D), the longer plate of the 5 V cell is at the top. Its lower, negative terminal is grounded, so the top supply node is positive relative to ground. That node connects through R to the anode of D_2 , while the cathode is grounded. The anode therefore lies at a positive potential relative to the cathode, producing the required forward-biased operating point.

Step 3: Rule out the reverse connections In option (A), the cell places the diode cathode side at the positive terminal and its anode side at the negative terminal, so D_4 is reverse biased. In (B), the longer cell plate is the lower, grounded one; hence the upper node feeding the anode of D_1 is negative. Option (C) similarly has the lower cell plate positive and grounded, leaving the anode of D_3 negative. Thus these three circuits have the opposite bias. The position of R before or after the diode is not the deciding feature; the polarity across the junction is.

Step 4: Conclusion Circuit D_2 , shown in option (D), has the correct forward bias. Hence, the correct option is (D).

Quick Tip to Solve: Mark the battery long plate positive and the diode bar cathode before tracing any circuit; resistor placement does not reverse the bias.

35. A sample of gas at temperature T is adiabatically expanded to double its volume. Adiabatic constant for the gas is $\gamma = 3/2$. The work done by the gas in the process is:

$$(\mu = 1 \text{ mole})$$

[4 marks]

(A) $RT[2\sqrt{2} - 1]$

(B) $RT[2 - \sqrt{2}]$

(C) $RT[1 - 2\sqrt{2}]$

(D) $RT[\sqrt{2} - 2]$

Correct Answer: (B) $RT[2 - \sqrt{2}]$

Solution:

Concept:

- For a reversible adiabatic ideal-gas process, $TV^{\gamma-1}$ is constant, where $\gamma = C_P/C_V$. During expansion the gas cools because heat exchange is zero.
- With work positive when done by the gas, the first law gives $W = nC_V(T_i - T_f)$. For an ideal gas $C_V = R/(\gamma - 1)$; R is the molar gas constant.

Step 1: Find the final temperature The initial temperature is T and the volume doubles from V_i to $V_f = 2V_i$. Since $\gamma = 3/2$, we have $\gamma - 1 = 1/2$. Apply the adiabatic relation:

$$T_f = T \left(\frac{V_i}{V_f} \right)^{\gamma-1} = T \left(\frac{1}{2} \right)^{1/2} = \frac{T}{\sqrt{2}}.$$

This is below T , as required for cooling during an expansion with no heat input.

Step 2: Use the decrease of internal energy There is one mole of gas, and $C_V = R/(1/2) = 2R$. Therefore

$$\begin{aligned} W &= 1 \times 2R \left(T - \frac{T}{\sqrt{2}} \right) \\ &= 2RT \left(1 - \frac{1}{\sqrt{2}} \right) \\ &= RT(2 - \sqrt{2}). \end{aligned}$$

Because RT has units of energy for one mole, the expression has the correct work units. The positive sign agrees with the gas doing work on its surroundings.

Step 3: Check the alternatives and the sign Options (C) and (D) are negative at positive absolute temperature, so they cannot describe work done by this expanding gas. Option (A) is positive but gives a factor $2\sqrt{2} - 1$, which is inconsistent with the computed final temperature. Only (B) has the factor $2 - \sqrt{2}$. Treating the process as isothermal would incorrectly keep T constant; the stated adiabatic relation must be used.

Step 4: Conclusion The work done by the gas is $W = RT(2 - \sqrt{2})$, in joules when R is in SI units. Hence, the correct option is (B).

Quick Tip to Solve: Use the volume ratio to find the final temperature first, then take the decrease in internal energy; expansion work must be positive.

36. The magnetic moment of a bar magnet is 0.5 Am^2 . It is suspended in a uniform magnetic field of $8 \times 10^{-2} \text{ T}$. The work done in rotating it from its most stable to most unstable position is: [4 marks]

- (A) $4 \times 10^{-2} \text{ J}$
- (B) $16 \times 10^{-2} \text{ J}$
- (C) $8 \times 10^{-2} \text{ J}$
- (D) Zero

Correct Answer: (C) $8 \times 10^{-2} \text{ J}$

Solution:

Concept:

- The potential energy of a magnetic dipole in a uniform field is $U = -\mu B \cos \theta$, where μ is its magnetic moment and θ is the angle between the moment and the field.
- Stable equilibrium occurs at minimum potential energy, with the dipole parallel to the field. Unstable equilibrium occurs at maximum potential energy, with the dipole antiparallel to the field.

Step 1: Identify the initial and final orientations In its most stable position the bar magnet has $\theta_i = 0$. In its most unstable position it has $\theta_f = \pi$. These are opposite orientations, so the rotation takes the magnet from the bottom to the top of its magnetic potential-energy range. Hence,

$$U_i = -\mu B, \quad U_f = +\mu B.$$

Step 2: Calculate the work against the magnetic torque The external work required for a slow rotation equals the increase in potential energy. Using $\mu = 0.5 \text{ A m}^2$ and $B = 8 \times 10^{-2} \text{ T}$,

$$W = U_f - U_i = 2\mu B = 2(0.5)(8 \times 10^{-2}) = 8 \times 10^{-2} \text{ J}.$$

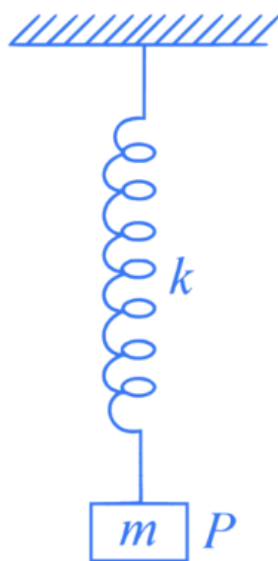
The field itself does negative work during this rotation; the positive value is the work required to turn the magnet against its restoring torque.

Step 3: Compare the options Option (A) equals μB , which is only half the required energy change. Option (B) is twice the required result. Zero work in (D) would require no change in potential energy, whereas stable and unstable equilibrium have different energies.

Step 4: Conclusion The work required is $8 \times 10^{-2} \text{ J}$. Hence, the correct option is (C).

Quick Tip to Solve: For a dipole turned from parallel to antiparallel, use the full energy difference $+\mu B - (-\mu B) = 2\mu B$.

37. In simple harmonic motion, the total mechanical energy of given system is E . If mass of oscillating particle P is doubled then the new energy of the system for same amplitude is: [4 marks]



- (A) $E/\sqrt{2}$
- (B) $2E$
- (C) $E\sqrt{2}$
- (D) E

Correct Answer: (D) E

Solution:

Concept:

- For a mass attached to a spring of stiffness k , the angular frequency satisfies $\omega^2 = k/m$. The amplitude is measured from the equilibrium position.
- The total energy of the oscillation is $E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$, where A is the amplitude. It depends on stiffness and amplitude, rather than on mass separately.

Step 1: Write the initial oscillation energy The diagram shows a mass suspended from the same spring, whose spring constant is k . At a turning point the speed is zero and all the oscillatory energy is stored as potential energy relative to equilibrium:

$$E = \frac{1}{2}kA^2.$$

For a vertical spring gravity changes the equilibrium extension, but its linear contribution is absorbed when displacement is measured from that equilibrium.

Step 2: Account for the change of angular frequency When the mass is doubled to $m' = 2m$, the spring stiffness does not change. Thus

$$\omega' = \sqrt{\frac{k}{2m}} = \frac{\omega}{\sqrt{2}}.$$

At the same amplitude,

$$E' = \frac{1}{2}(2m) \left(\frac{\omega^2}{2} \right) A^2 = \frac{1}{2}m\omega^2 A^2 = E.$$

The increase in mass is exactly cancelled by the decrease in the square of angular frequency.

Step 3: Eliminate the changing-energy options Option (B) incorrectly doubles the energy while keeping frequency fixed. Options (A) and (C) confuse the square-root change of frequency with a change of energy. Since k and A remain fixed, none of these factors appears in the final energy.

Step 4: Conclusion The new total oscillation energy is E , unchanged. Hence, the correct option is (D).

Quick Tip to Solve: For the same spring and amplitude, substitute $m\omega^2 = k$ before comparing energies; this removes the misleading mass dependence.

38. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Number of photons increases with increase in frequency of light.

Reason R: Maximum kinetic energy of emitted electrons increases with the frequency of incident radiation.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

- (A) A is not correct but R is correct.
- (B) A is correct but R is not correct.
- (C) Both A and R are correct and R is the correct explanation of A.
- (D) Both A and R are correct and R is NOT the correct explanation of A.

Correct Answer: (A) A is not correct but R is correct.

Solution:

Concept:

- Each photon of frequency ν has energy $h\nu$. For a fixed total radiant energy E_{rad} , the number of photons is $N = E_{\text{rad}}/(h\nu)$.
- Einstein's photoelectric equation is $K_{\text{max}} = h\nu - \phi$, where ϕ is the work function of the emitting material. Emission requires frequency at least equal to the threshold frequency.

Step 1: Evaluate the assertion about photon number Frequency specifies the energy of an individual photon, not how many photons a beam contains. If the same total light energy is compared,

$$N = \frac{E_{\text{rad}}}{h\nu} \propto \frac{1}{\nu}.$$

Therefore a larger frequency gives fewer photons for that fixed energy. Equivalently, at fixed beam power the photon arrival rate is $P/(h\nu)$, which decreases as frequency increases. Without specifying beam energy or power, frequency alone cannot determine photon number. The stated increase is therefore not correct.

Step 2: Evaluate the reason about electron kinetic energy For a fixed photoemissive material, ϕ is constant. Above threshold, increasing the incident frequency by $\Delta\nu$ increases the maximum electron kinetic energy by

$$\Delta K_{\max} = h\Delta\nu > 0.$$

Thus the reason correctly describes the dependence of maximum photoelectron energy on incident frequency.

Step 3: Select the logical combination The assertion is false and the reason is true, so (A) is the required combination. Option (B) reverses these truth values. Options (C) and (D) both require the assertion to be true; they are ruled out before asking whether the reason explains it.

Step 4: Conclusion Assertion A is not correct, whereas Reason R is correct. Hence, the correct option is (A).

Quick Tip to Solve: Keep photon energy, photon count and electron kinetic energy distinct: frequency fixes the first and affects the third, but count also requires beam energy or power.

39. A 90 kg body placed at $2R$ distance from surface of earth experiences gravitational pull of :

(R = Radius of earth, $g = 10 \text{ m s}^{-2}$)

[4 marks]

- (A) 300 N
- (B) 225 N
- (C) 100 N
- (D) 120 N

Correct Answer: (C) 100 N

Solution:

Concept:

- Outside a spherical Earth, gravitational attraction is $F = GMm/r^2$, where r is measured from the centre of the Earth.
- The surface acceleration is $g = GM/R^2$. At altitude h , it becomes $g_h = g[R/(R+h)]^2$; an altitude must first be added to the Earth's radius.

Step 1: Convert the stated height to a central distance The body is at a distance $2R$ from the surface, so its altitude is $h = 2R$. Its distance from the centre is therefore

$$r = R + h = R + 2R = 3R.$$

The inverse-square law uses this central distance. Substituting $2R$ directly for r would describe a different position, only one Earth radius above the surface.

Step 2: Find the local acceleration and force Use the given surface value $g = 10 \text{ m s}^{-2}$:

$$g_h = g \frac{R^2}{(3R)^2} = \frac{10}{9} \text{ m s}^{-2}.$$

For the body of mass $m = 90 \text{ kg}$, the magnitude of the pull is

$$F = mg_h = 90 \left(\frac{10}{9} \right) = 100 \text{ N}.$$

This force acts radially towards the centre of the Earth. The mass is unchanged by moving the body; only the gravitational field strength changes.

Step 3: Check the options against the distance dependence Option (B), 225 N, would follow from the incorrect central distance $2R$. Option (A), 300 N, corresponds to dividing the surface weight by three rather than nine. Option (D) does not satisfy the inverse-square reduction. The surface weight is 900 N, and its ninth part is 100 N.

Step 4: Conclusion The gravitational pull is 100 N, directed towards the Earth's centre. Hence, the correct option is (C).

Quick Tip to Solve: Underline whether a distance is from the surface or the centre. For height $2R$, the inverse-square denominator contains $(3R)^2$.

40. According to Bohr's theory, the moment of momentum of an electron revolving in 4th orbit of hydrogen atom is: [4 marks]

- (A) $2\frac{h}{\pi}$
- (B) $\frac{h}{2\pi}$
- (C) $\frac{h}{\pi}$
- (D) $8\frac{h}{\pi}$

Correct Answer: (A) $2\frac{h}{\pi}$

Solution:

Concept:

- The moment of momentum is the orbital angular momentum $L = m_e v r$, with SI unit $\text{kg m}^2 \text{s}^{-1}$, or equivalently J s.
- Bohr's quantisation condition is $L = n\hbar = nh/(2\pi)$, where n is the positive-integer orbit number and h is Planck's constant.

Step 1: Identify the quantum number in the question The electron is in the fourth orbit of the hydrogen atom, so the principal quantum number in Bohr's model is $n = 4$. The condition concerns angular momentum itself, rather than orbital energy or orbit radius. It is linear in n , even though Bohr radii and the magnitudes of the inverse binding energies have a dependence on n^2 .

Step 2: Substitute into the quantisation condition Apply the allowed angular momentum for this orbit:

$$L = \frac{nh}{2\pi} = \frac{4h}{2\pi} = \frac{2h}{\pi}.$$

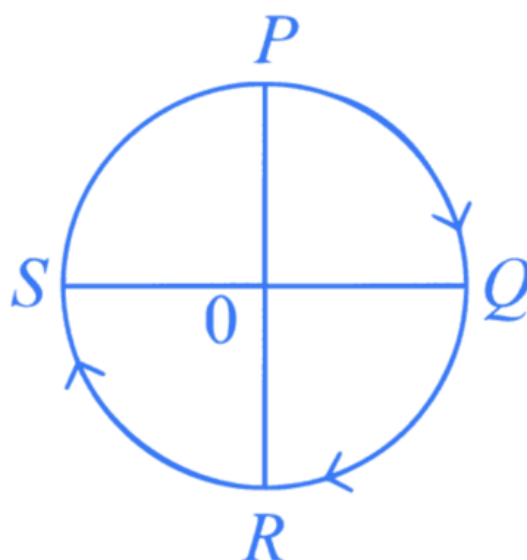
No electron speed or radius is required separately because their product with the electron mass is already specified by the quantisation rule. In terms of the reduced Planck constant the same result is $L = 4\hbar$, which confirms the factor of four relative to the first orbit.

Step 3: Check the other possible values Option (B), $h/(2\pi)$, is the first-orbit angular momentum. Option (C), $h/\pi = 2\hbar$, belongs to the second orbit. Option (D), $8h/\pi = 16\hbar$, would correspond to $n = 16$, not to the fourth orbit. Only (A) contains the correct factor $4/(2\pi)$.

Step 4: Conclusion The moment of momentum in Bohr's fourth orbit is $2h/\pi$, with units of J s. Hence, the correct option is (A).

Quick Tip to Solve: Convert each option to a multiple of $\hbar = h/(2\pi)$; the coefficient must equal the orbit number in Bohr's model.

41. A cyclist starts from the point P of a circular ground of radius 2 km and travels along its circumference to the point S . The displacement of a cyclist is: [4 marks]



- (A) $\sqrt{8}$ km
- (B) 4 km
- (C) 6 km
- (D) 8 km

Correct Answer: (A) $\sqrt{8}$ km

Solution:

Concept:

- Displacement is the vector joining the initial position directly to the final position. Its magnitude is the straight-line separation, independent of the path length travelled.
- The chord joining points whose radii enclose angle θ has length $d = 2R \sin(\theta/2)$. Perpendicular radii also allow the chord to be found using Pythagoras' theorem.

Step 1: Identify the endpoint geometry The figure places P at the top of the circle and S at its leftmost point. Thus the radii OP and OS are perpendicular. Each has length 2 km. Although the arrows show the cyclist moving clockwise around the circumference from P through Q and R to S ,

the displacement depends only on P and S .

Step 2: Calculate the chord joining the endpoints In right triangle POS , the displacement magnitude is the hypotenuse PS :

$$PS = \sqrt{OP^2 + OS^2} = \sqrt{2^2 + 2^2} = \sqrt{8} \text{ km.}$$

Equivalently, using a central angle of 90° ,

$$PS = 2(2) \sin 45^\circ = 2\sqrt{2} \text{ km.}$$

The displacement vector points from P towards S , diagonally left and down in the figure.

Step 3: Distinguish displacement from distance travelled The displacement cannot exceed the diameter 4 km, so (C) and (D) are immediately excluded. Option (B) would require diametrically opposite endpoints, whereas P and S are adjacent quadrantal points. The longer circular path does not increase their straight-line separation; hence (A) is the only correct magnitude.

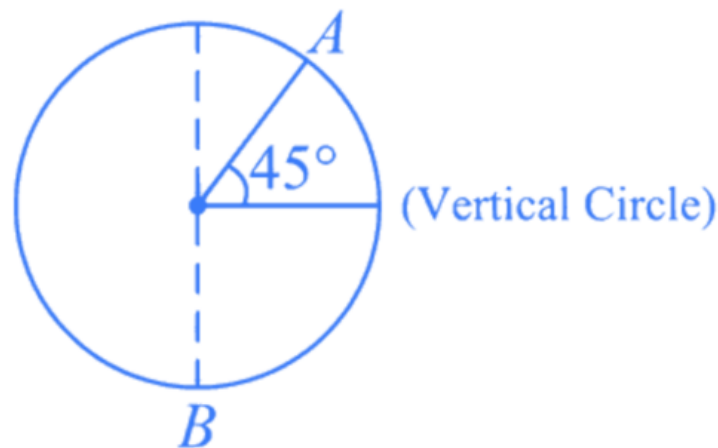
Step 4: Conclusion The displacement has magnitude $\sqrt{8} \text{ km} = 2\sqrt{2} \text{ km}$, directed from P to S . Hence, the correct option is (A).

Quick Tip to Solve: For circular motion, draw the straight chord mentally between the start and finish before calculating. The arrows determine the travelled arc, not the displacement magnitude.

42. A body of m kg slides from rest along the curve of vertical circle from point A to B in friction less path. The velocity of the body at B is:

(given, $R = 14 \text{ m}$, $g = 10 \text{ m/s}^2$ and $\sqrt{2} = 1.4$)

[4 marks]



(A) 10.6 m/s

(B) 19.8 m/s

(C) 16.7 m/s

(D) 21.9 m/s

Correct Answer: (D) 21.9 m/s

Solution:

Concept:

- On a frictionless stationary track, gravity converts the loss of potential energy into kinetic energy. The normal reaction is perpendicular to motion and does no work.
- For release from rest through a vertical drop Δh , energy conservation gives $mg\Delta h = \frac{1}{2}mv^2$, hence $v = \sqrt{2g\Delta h}$. The vertical drop is determined by the geometry, not by the arc length.

Step 1: Find the vertical drop from the diagram The radius towards A makes 45° above the horizontal through the centre. Therefore A is $R \sin 45^\circ = R/\sqrt{2}$ above the centre. The lowest point B is R below the centre. Thus

$$\Delta h = R + \frac{R}{\sqrt{2}}.$$

The two contributions add because the body passes from above the centre to the bottom of the circle.

Step 2: Apply energy conservation before substituting Initially the body is at rest, so

$$\frac{1}{2}mv_B^2 = mgR \left(1 + \frac{1}{\sqrt{2}}\right).$$

Cancel m and rationalise the geometric factor:

$$v_B^2 = 2gR \left(1 + \frac{1}{\sqrt{2}}\right) = gR(2 + \sqrt{2}).$$

Using the supplied approximation $\sqrt{2} = 1.4$ consistently in this simplified expression,

$$v_B^2 = 10(14)(2 + 1.4) = 476, \quad v_B = \sqrt{476} \simeq 21.8 \text{ m s}^{-1}.$$

Step 3: Choose the matching listed speed The value using the exact square root is about 21.86 m s^{-1} , so the listed answer is 21.9 m s^{-1} . Options (A), (B) and (C) all give too little kinetic energy for the full vertical drop $R + R/\sqrt{2}$; in particular 16.7 m s^{-1} corresponds approximately to a drop of only R .

Step 4: Conclusion The speed at B is approximately 21.9 m s^{-1} . Hence, the correct option is (D).

Quick Tip to Solve: Find the heights relative to the circle's centre and subtract them. Simplify expressions containing $\sqrt{2}$ before using a coarse supplied approximation.

43. The width of one of the two slits in a Young's double slit experiment is 4 times that of the other slit. The ratio of the maximum of the minimum intensity in the interference pattern is: [4 marks]

- (A) 1 : 1
- (B) 4 : 1
- (C) 16 : 1
- (D) 9 : 1

Correct Answer: (D) 9 : 1

Solution:

Concept:

- For equally illuminated slits, the transmitted intensity is proportional to slit width. Thus a width ratio of 4 gives individual intensities in the ratio $I_1 : I_2 = 4 : 1$.
- Interference adds amplitudes, while intensity is proportional to the square of amplitude: $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$.

Step 1: Convert intensities into amplitudes Write the intensity from the narrower slit as I . The wider slit then supplies intensity $4I$. The corresponding amplitudes are proportional to $\sqrt{I}$ and $2\sqrt{I}$, respectively. It is essential to take this square root before combining the waves: the intensity ratio is not itself the amplitude ratio.

Step 2: Calculate both interference extremes At a bright fringe the two disturbances are in phase, so their amplitudes add. At a dark fringe they have opposite phases, so their amplitudes subtract. Consequently,

$$I_{\max} = (2\sqrt{I} + \sqrt{I})^2 = 9I, \quad I_{\min} = (2\sqrt{I} - \sqrt{I})^2 = I.$$

The minimum is nonzero because the amplitudes are unequal; complete destructive cancellation requires equal amplitudes.

Step 3: Compare the choices The quotient is $I_{\max}/I_{\min} = 9$. The ratio 1 : 1 would imply no variation in intensity. The value 4 : 1 is the ratio of the individual slit intensities, while 16 : 1 incorrectly squares that ratio rather than combining the amplitudes at the two extremes.

Step 4: Conclusion The maximum-to-minimum intensity ratio is 9 : 1. Hence, the correct option is (D).

Quick Tip to Solve: Take the square root of the slit-intensity ratio first, then square the sum and difference of those amplitude factors.

44. The translational degrees of freedom (f_t) and rotational degrees of freedom (f_r) of CH_4 molecule are: [4 marks]

(A) $f_t = 2$ and $f_r = 2$

(B) $f_t = 3$ and $f_r = 3$

(C) $f_t = 3$ and $4f_r = 2$

(D) $f_t = 2$ and $f_r = 3$

Correct Answer: (B) $f_t = 3$ and $f_r = 3$

Solution:

Concept:

- Translational degrees of freedom count the independent coordinates of the centre of mass. A freely moving molecule in three-dimensional space has $f_t = 3$.
- A nonlinear rigid molecule has three independent rotational degrees of freedom, one for rotation about each principal axis. A linear molecule has only two appreciable rotational degrees of freedom.

Step 1: Identify the molecular geometry Methane, CH_4 , is tetrahedral: its four hydrogen atoms surround the carbon atom in three dimensions. The atoms do not lie along a single straight molecular axis. Therefore methane must be treated as a nonlinear polyatomic molecule, rather than as a linear diatomic molecule.

Step 2: Count the independent motions The centre of mass can translate independently along the x , y and z directions, giving $f_t = 3$. In addition, the tetrahedral arrangement has nonzero moments of inertia about all three principal axes, so rotation about each axis stores rotational kinetic energy. This gives $f_r = 3$. Vibrations are separate internal motions and are not included in either of the quantities requested.

Step 3: Check the option pairs The required pair is therefore $(f_t, f_r) = (3, 3)$. Options (A) and (D) incorrectly restrict free translation to two dimensions. Option (C) does not give the three rotational modes of a nonlinear molecule; using the two-mode rotational count would be appropriate to a linear molecule, not tetrahedral methane. Only option (B) contains both correct counts.

Step 4: Conclusion For methane, $f_t = 3$ and $f_r = 3$. Hence, the correct option is (B).

Quick Tip to Solve: First decide whether a molecule is linear: both linear and nonlinear molecules translate in three directions, but their rotational counts differ.

45. A charge q is placed at the center of one of the surface of a cube. The flux linked with the cube is:

[4 marks]

- (A) $\frac{q}{2\epsilon_0}$
(B) Zero
(C) $\frac{q}{4\epsilon_0}$
(D) $\frac{q}{8\epsilon_0}$

Correct Answer: (A) $\frac{q}{2\epsilon_0}$

Solution:

Concept:

- Gauss's law states that the net outward electric flux through a closed surface enclosing charge q is $\Phi = q/\epsilon_0$.
- A charge at the centre of a cube's face can be made an interior charge by reflecting the cube across that face. The two identical halves receive equal flux by reflection symmetry.

Step 1: Construct a symmetric closed surface Attach an identical cube on the other side of the face whose centre holds the charge. The shared face becomes an internal plane of the combined rectangular box, and the charge lies at the centre of that box. The charge is now strictly inside a closed Gaussian surface, so Gauss's law can be applied without assigning a boundary charge arbitrarily.

Step 2: Divide the total flux by symmetry The total flux through the outer surface of the double cube is q/ϵ_0 . Reflection in the shared face exchanges the two cubes while leaving the central charge unchanged. Hence the outer faces belonging to each cube receive exactly half this flux:

$$\Phi_{\text{cube}} = \frac{1}{2} \frac{q}{\epsilon_0} = \frac{q}{2\epsilon_0}.$$

On the shared plane the electric field is tangential away from the charge, so that face contributes no additional normal flux.

Step 3: Distinguish other boundary positions A face-centre charge is shared between two reflected volumes, not four or eight. The fractions $q/(4\epsilon_0)$ and $q/(8\epsilon_0)$ apply to corresponding edge and vertex constructions. Zero is also incorrect: field lines do leave the outer faces even though the charge is located on the original cube's boundary.

Step 4: Conclusion The flux linked with the cube is $q/(2\epsilon_0)$. Hence, the correct option is (A).

Quick Tip to Solve: For a charge on a symmetry boundary, reflect the volume until the charge is interior and divide the total flux among the identical pieces.

46. Correct formula for height of a satellite from earth's surface is :

[4 marks]

- (A) $\left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3} - R$
(B) $\left(\frac{T^2 R^2 g}{4\pi}\right)^{1/2} - R$
(C) $\left(\frac{T^2 R^2 g}{4\pi^2}\right)^{-1/3} + R$
(D) $\left(\frac{T^2 R^2}{4\pi^2 g}\right)^{1/3} - R$

Correct Answer: (A) $\left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3} - R$

Solution:

Concept:

- For a satellite in a circular orbit of radius r , gravity supplies the centripetal force: $GMm/r^2 = m\omega^2 r$, with $\omega = 2\pi/T$.
- Surface gravitational acceleration satisfies $g = GM/R^2$. The altitude above Earth's surface is $h = r - R$, whereas r is measured from Earth's centre.

Step 1: Relate the orbital radius to the period Let M be Earth's mass and m the satellite's mass. Substituting $\omega = 2\pi/T$ in the force balance and cancelling m gives

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}, \quad r^3 = \frac{GMT^2}{4\pi^2}.$$

The cube appears because the inverse-square gravitational field is combined with the radius in the centripetal acceleration.

Step 2: Replace the gravitational parameter and subtract the radius From the surface value of gravity, $GM = gR^2$. Substitution therefore gives

$$r = \left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3}, \quad h = \left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3} - R.$$

Subtracting R , rather than adding it, converts the distance from the centre into height above the surface.

Step 3: Check dimensions and the remaining options The product $T^2 R^2 g$ has dimensions L^3 , so its cube root is a length. Option (B) uses a square root and the wrong power of π . Option (C) gives an inverse length inside the addition and also adds R . Option (D) divides by g instead of multiplying by it, so its dimensional dependence is incorrect.

Step 4: Conclusion The satellite height is $h = (T^2 R^2 g / (4\pi^2))^{1/3} - R$. Hence, the correct option is (A).

Quick Tip to Solve: Kepler's relation gives distance from Earth's centre; subtract Earth's radius only after taking the cube root.

47. Given below are two statements :

Statement I : The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well.

Statement II : The rise of a liquid in a capillary tube does not depend on the inner radius of the tube.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are false.

(B) Both Statement I and Statement II are true.

(C) Statement I is false but Statement II is true.

(D) Statement I is true but Statement II is false.

Correct Answer: (D) Statement I is true but Statement II is false.

Solution:

Concept:

- The equilibrium contact angle θ is set by the balance of interfacial tensions for the solid, liquid and surrounding medium. It depends on the nature of the solid-liquid pair.
- For a capillary of inner radius r , liquid density ρ and surface tension S , the equilibrium rise is $h = 2S \cos \theta / (\rho g r)$.

Step 1: Evaluate the material-dependence statement At the contact line, the competing solid-air, solid-liquid and liquid-air interfacial energies determine the equilibrium angle. Their values change with the materials forming the interfaces. Thus the angle for water on clean glass need not equal that for another liquid on the same glass, or for water on a different solid. Statement I correctly identifies this dependence and is true.

Step 2: Test the radius-dependence statement The upward vertical surface-tension force is $2\pi r S \cos \theta$, and it balances the weight $\pi r^2 h \rho g$ of the liquid column. Equating them gives

$$2\pi r S \cos \theta = \pi r^2 h \rho g \quad \Rightarrow \quad h = \frac{2S \cos \theta}{\rho g r}.$$

For a fixed liquid and surface, h is inversely proportional to the inner radius. A narrower wetting capillary therefore produces a larger rise. Statement II denies this dependence and is false.

Step 3: Select the matching truth pair The combination is Statement I true and Statement II false. Option (A) wrongly rejects the first statement, option (B) wrongly accepts the second, and option (C) reverses both truth values. Option (D) alone agrees with the interfacial-energy argument and the explicit capillary-rise equation.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: Write the capillary-rise formula before judging dependence: the radius remains in the denominator after the force balance.

48. An electric bulb rated 50 W – 200 V is connected across a 100 V supply. The power dissipation of the bulb is:

[4 marks]

- (A) 100 W
- (B) 50 W
- (C) 12.5 W
- (D) 25 W

Correct Answer: (C) 12.5 W

Solution:

Concept:

- For a resistive bulb in the usual constant-resistance model, electrical power is $P = V^2/R$, where V is the applied potential difference and R is the filament resistance.
- A rating of 50 W at 200 V specifies the power at the rated voltage; it is not a fixed power at every supply voltage.

Step 1: Find the resistance from the rating Use the rated quantities together, since they describe one operating condition:

$$R = \frac{V_{\text{rated}}^2}{P_{\text{rated}}} = \frac{(200)^2}{50} = 800 \, \Omega.$$

This calculation avoids treating the rated current as constant when the voltage changes. The question uses the standard resistance-based model for the bulb.

Step 2: Calculate the power at the new supply voltage With 100 V across the same resistance, the current is $I = 100/800 = 0.125$ A. The resulting power is

$$P = \frac{100^2}{800} = 12.5 \, \text{W}.$$

Equivalently, the applied voltage is half its rated value, so the power is multiplied by $(1/2)^2 = 1/4$. Thus 50 W becomes 12.5 W.

Step 3: Reject the alternative values The value 50 W would require the rated voltage, so option (B) cannot apply. Option (D), 25 W, assumes that power falls linearly with voltage and overlooks the simultaneous fall in current. Option (A), 100 W, predicts an increase when the voltage has been reduced and contradicts $P \propto V^2$. Only option (C) satisfies both Ohm's law and the power equation.

Step 4: Conclusion The bulb dissipates 12.5 W on the 100 V supply. Hence, the correct option is (C).

Quick Tip to Solve: For the same resistance, multiply rated power by the square of the ratio of actual voltage to rated voltage.

49. Arrange the following in the ascending order of wavelength:

- A. Gamma rays (λ_1)
- B. x - rays (λ_2)
- C. Infrared waves (λ_3)
- D. Microwaves (λ_4)

Choose the most appropriate answer from the options given below

[4 marks]

- (A) $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$
- (B) $\lambda_2 < \lambda_1 < \lambda_4 < \lambda_3$
- (C) $\lambda_4 < \lambda_3 < \lambda_2 < \lambda_1$
- (D) $\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2$

Correct Answer: (A) $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$

Solution:

Concept:

- Electromagnetic waves in vacuum obey $c = \nu\lambda$. Frequency ν and wavelength λ are inversely related because all the waves travel at the same speed c .
- In order of increasing wavelength, the electromagnetic spectrum runs through gamma rays, X-rays, ultraviolet, visible light, infrared, microwaves and radio waves.

Step 1: Locate the four named regions Gamma rays occupy the shortest-wavelength, highest-frequency end of the conventional spectrum. X-rays follow gamma rays as wavelength increases. Infrared radiation lies beyond the red end of visible light, so it has a much longer wavelength than X-rays. Microwaves lie beyond infrared towards the long-wavelength radio region.

Step 2: Translate the spectrum order into the given symbols The question assigns λ_1 to gamma rays, λ_2 to X-rays, λ_3 to infrared radiation and λ_4 to microwaves. Reading from short to long wavelength gives

$$\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4.$$

The corresponding frequencies would occur in the opposite order, because $\nu = c/\lambda$; that reversal must not be applied to the wavelengths themselves.

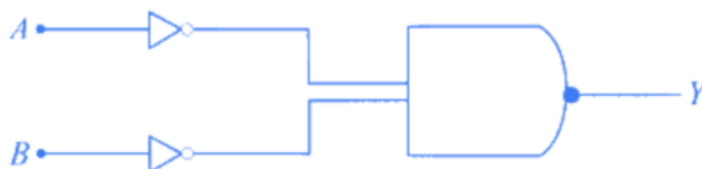
Step 3: Compare all listed sequences Option (A) gives exactly this ordering. Option (B) puts X-rays below gamma rays and microwaves below infrared in wavelength, reversing both pairs. Option (C) gives the descending wavelength sequence. Option (D) also starts with microwaves as the shortest and incorrectly places gamma rays after infrared. Those three alternatives conflict with the standard positions of the named spectral regions.

Step 4: Conclusion The ascending wavelength order is $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$. Hence, the correct option is (A).

Quick Tip to Solve: Recall the spectrum from gamma rays towards radio waves for increasing wavelength; reverse it only when increasing frequency is asked.

50. Identify the logic gate given in the circuit :

[4 marks]



- (A) OR gate
- (B) NAND gate
- (C) NOR gate
- (D) AND gate

Correct Answer: (A) OR gate

Solution:

Concept:

- A NOT gate complements its input: the two input signals reaching the final gate are $\bar{A}$ and $\bar{B}$. The output bubble on the final AND-shaped gate represents inversion, so that gate is NAND.
- De Morgan's theorem converts a complemented product into a sum of complements: $\overline{PQ} = \bar{P} + \bar{Q}$. In Boolean algebra, $+$ denotes OR and juxtaposition denotes AND.

Step 1: Write the Boolean expression Trace each signal through its own inverter before considering the final gate. The upper inverter changes A to $\bar{A}$; the lower changes B to $\bar{B}$. Their NAND output is therefore

$$Y = \overline{\bar{A}\bar{B}}.$$

The output inversion must be included; omitting the small output bubble would instead describe a different circuit.

Step 2: Apply De Morgan's theorem and test the outputs Complementing the product gives

$$Y = \overline{\bar{A}\bar{B}} = A + B.$$

For input pairs $(A, B) = (0, 0), (0, 1), (1, 0), (1, 1)$, the outputs are respectively 0, 1, 1, 1, which is the OR truth table. A NAND gate would give 1 for input 00 and 0 for input 11. A NOR gate would give 1 only for input 00. An AND gate would give 0 for input 01 or 10. Each of these alternatives fails the calculated truth table.

Step 3: Conclusion The circuit has output $Y = A + B$, so it acts as an OR gate. Hence, the correct option is (A).

Quick Tip to Solve: Follow every inversion bubble before simplifying. A NAND gate with both inputs inverted becomes OR by De Morgan's theorem.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. In a system two particles of masses $m_1 = 3$ kg and $m_2 = 2$ kg are placed at certain distance from each other. The particle of mass m_1 is moved towards the center of mass of the system through a distance 2 cm. In order to keep the center of mass of the system at the original position, the particle of mass m_2 should move towards the center of mass by the distance _____ cm. [4 marks]

Solution:

Concept:

- For two particles on a line, $x_{\text{CM}} = (m_1x_1 + m_2x_2)/(m_1 + m_2)$. With fixed masses, an unchanged centre of mass requires $m_1\Delta x_1 + m_2\Delta x_2 = 0$.
- The centre of mass lies between the two positive masses. Consequently their motions towards it have opposite signs along any one chosen coordinate axis. The weighted displacements, rather than the distances alone, must balance.

Step 1: Choose a direction and impose the fixed-centre condition Take the direction from the initial position of the 3 kg particle towards the 2 kg particle as positive. The given displacement of the first particle is $\Delta x_1 = +2$ cm. Let the second particle move a distance d towards the original centre of mass; its displacement is $\Delta x_2 = -d$. Subtract the initial centre-of-mass equation from the final one:

$$\Delta x_{\text{CM}} = \frac{3(2) + 2(-d)}{3 + 2} = 0.$$

It is consistent to use centimetres for both displacements because they occur in the same linear equation.

Step 2: Find and check the second displacement The numerator must vanish, so

$$6 - 2d = 0, \quad d = 3 \text{ cm}.$$

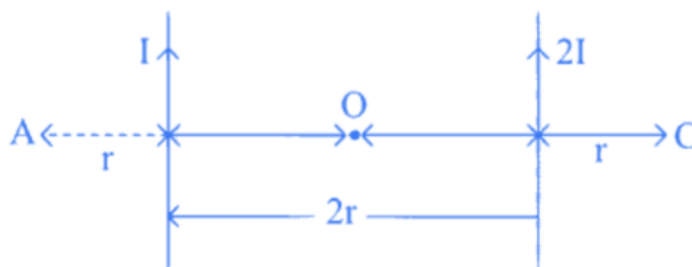
The first particle changes the mass-weighted position sum by +6 kg cm, and the second changes it by –6 kg cm. Their sum is zero, hence division by the total mass gives no shift of the centre of mass. The lighter particle must move farther, which agrees with the inverse relation between displacement magnitude and mass.

Step 3: Conclusion The 2 kg particle must move 3 cm towards the centre of mass. The numerical entry is 3.

Quick Tip to Solve: For a fixed centre of mass, set the two mass-times-displacement magnitudes equal and give the displacements opposite directions.

52. Two parallel long current carrying wire separated by a distance $2r$ are shown in the figure. The ratio of magnetic field at A to the magnetic field produced at C is $\frac{x}{7}$. The value of x is _____.

[4 marks]



Solution:

Concept:

- The magnetic field magnitude at a perpendicular distance d from a long straight wire is $B = \mu_0 I / (2\pi d)$. Fields from different wires add vectorially, with directions given by the right-hand thumb rule.
- Both currents in the figure point upwards, with magnitudes I and $2I$. At an exterior point on either side of the pair, both wire fields point in the same perpendicular direction and their magnitudes add.

Step 1: Find the magnetic field at A Point A is a distance r to the left of the wire carrying I . Since the wire separation is $2r$, its distance from the wire carrying $2I$ is $3r$. Both fields at A point out of the page. Thus

$$B_A = \frac{\mu_0}{2\pi} \left(\frac{I}{r} + \frac{2I}{3r} \right) = \frac{\mu_0 I}{2\pi r} \frac{5}{3}.$$

The two contributions must not be subtracted: A lies outside the wires, rather than between them.

Step 2: Find the field at C and form the ratio Point C is a distance r to the right of the $2I$ wire and $3r$ from the I wire. Both fields there point into the page, giving

$$B_C = \frac{\mu_0}{2\pi} \left(\frac{I}{3r} + \frac{2I}{r} \right) = \frac{\mu_0 I}{2\pi r} \frac{7}{3}.$$

The question compares field magnitudes. The common factor cancels:

$$\frac{B_A}{B_C} = \frac{5/3}{7/3} = \frac{5}{7} = \frac{x}{7}.$$

The field is larger at C because the larger current is the nearer wire there, providing a useful check on the ratio being below unity.

Step 3: Conclusion The ratio of the magnetic field magnitudes is $5/7$, hence $x = 5$ (dimensionless).

Quick Tip to Solve: For exterior points, add the two wire fields. Write both distances first: the farther wire is at $3r$, not $2r$.

53. A bus moving along a straight highway with speed of 72 km/h is brought to halt within 4 s after applying the brakes. The distance travelled by the bus during this time (Assume the retardation is uniform) is _____ m. [4 marks]

Solution:

Concept:

- For uniform acceleration, velocity changes linearly with time, so the average velocity is $(u + v)/2$, where u and v are the initial and final velocities. Displacement is this average velocity multiplied by the elapsed time.
- The equations $v = u + at$ and $s = ut + \frac{1}{2}at^2$ provide an independent check. Convert kilometres per hour to metres per second before substituting a time in seconds.

Step 1: Convert the speed and identify the final velocity The initial speed in SI units is

$$u = 72 \times \frac{1000}{3600} = 72 \times \frac{5}{18} = 20 \text{ m s}^{-1}.$$

The bus comes to rest, so $v = 0$, and the braking time is $t = 4$ s. Take the initial direction of motion as positive. The acceleration is negative, but velocity does not reverse during the stated interval; therefore distance travelled equals the positive displacement.

Step 2: Use the average velocity and verify the result The average velocity during the uniform braking is

$$v_{\text{av}} = \frac{20 + 0}{2} = 10 \text{ m s}^{-1}.$$

Hence the distance is

$$s = v_{\text{av}}t = 10 \times 4 = 40 \text{ m}.$$

For a check, the acceleration is $a = (0 - 20)/4 = -5 \text{ m s}^{-2}$. Substitution in the displacement equation gives

$$s = 20(4) + \frac{1}{2}(-5)(4)^2 = 80 - 40 = 40 \text{ m}.$$

Equivalently, the triangular area under the velocity-time graph is half the initial speed times the stopping time. Using the initial speed throughout would incorrectly ignore the braking.

Step 3: Conclusion The bus travels 40 m before stopping. The numerical entry is 40.

Quick Tip to Solve: For uniform braking to rest, the average speed is half the initial speed; convert to SI units before multiplying by the stopping time.

54. A rod of length 60 cm rotates with a uniform angular velocity 20 rads^{-1} about its perpendicular bisector, in a uniform magnetic field $0.5T$. The direction of magnetic field is parallel to the axis of rotation. The potential difference between the two ends of the rod is _____ V. [4 marks]

Solution:

Concept:

- In a conductor rotating perpendicular to a uniform magnetic field, the local magnetic force is $qv \times B$. For a point a signed distance x from the rotation axis, the speed magnitude is $\omega|x|$.
- The potential difference between the centre and an end at radius a has magnitude $B\omega a^2/2$. A rod rotating about its midpoint has two equal radial halves, whose ends acquire the same potential relative to the midpoint.

Step 1: Identify the equal radial halves The perpendicular bisector passes through the centre of the 60 cm rod. Thus each end is at radius

$$a = \frac{0.60}{2} = 0.30 \text{ m.}$$

For one sense of rotation and field orientation, $v \times B$ drives positive charges radially outwards on both halves. Reversing either the rotation or the magnetic field reverses both potential shifts together. It does not make one end high and the other low relative to the centre.

Step 2: Compare the two end potentials For either half, integrate the motional contribution from the centre to the end:

$$|V_{\text{end}} - V_{\text{centre}}| = \int_0^a B\omega x dx = \frac{1}{2}B\omega a^2.$$

With the supplied values this is

$$\frac{1}{2}(0.5)(20)(0.30)^2 = 0.45 \text{ V.}$$

Both ends have this same potential shift, with the same sign. Consequently

$$V_{\text{right}} - V_{\text{left}} = 0.45 - 0.45 = 0 \text{ V.}$$

The nonzero centre-to-end value is not the requested end-to-end potential difference. Applying the end-pivoted-rod formula to the full length would use the wrong rotation axis.

Step 3: Conclusion The two ends are equipotential, so their potential difference is 0 V. The numerical entry is 0.

Quick Tip to Solve: Locate the rotation axis before applying the rotating-rod formula: equal radii on opposite sides of a central pivot give equal end potentials.

55. The disintegration energy Q for the nuclear fission of $^{235}\text{U} \rightarrow ^{140}\text{Ce} + ^{94}\text{Zr} + n$ is _____ MeV. Given atomic masses of $^{235}\text{U} : 235.0439\text{u}$, $^{140}\text{Ce} : 139.9054\text{u}$, $^{94}\text{Zr} : 93.9063\text{u}$; $n : 1.0086\text{u}$, Value of $c^2 = 931 \text{ MeV/u}$. [4 marks]

Solution:

Concept:

- The energy released by a nuclear reaction is $Q = (m_{\text{initial}} - m_{\text{final}})c^2$. A positive mass decrease corresponds to a positive energy release, using the combined rest masses of all products.
- The supplied conversion is $1 \text{ u } c^2 = 931 \text{ MeV}$. For the requested numerical calculation, use the stated atomic masses and conversion factor directly. The calculation compares the listed initial and final mass sums.

Step 1: Sum the stated product masses Use exactly the listed reaction and include the free neutron along with both fission fragments. The total initial mass is 235.0439 u , whereas the stated final mass is

$$m_f = 139.9054 + 93.9063 + 1.0086 = 234.8203 \text{ u}.$$

The mass decrease for the stated mass data is therefore

$$\Delta m = 235.0439 - 234.8203 = 0.2236 \text{ u}.$$

Keep all four decimal places at this stage; rounding the individual masses before subtraction would produce a large fractional error in the small mass difference.

Step 2: Convert the mass decrease into released energy Multiply the mass decrease by the conversion factor supplied in the question:

$$Q = 0.2236 \times 931 = 208.1716 \text{ MeV}.$$

This is positive, so the products have lower total rest energy and the difference is released. To the nearest integer, the required numerical value is 208. The neutron mass cannot be omitted; doing so would change the mass difference by roughly one atomic mass unit. The calculation is a mass-energy evaluation using the stated reaction data.

Step 3: Conclusion The disintegration energy from the supplied masses is $Q = 208.1716 \text{ MeV}$, or 208 MeV to the nearest integer. The numerical entry is 208.

Quick Tip to Solve: Sum every product mass, including emitted neutrons, before subtracting from the initial mass; round the energy only after applying 931 MeV/u .

56. Mercury is filled in a tube of radius 2 cm up to a height of 30 cm. The force exerted by mercury on the bottom of the tube is _____ N.

(Given, atmospheric pressure = 10^5 Nm^{-2} , density of mercury = $1.36 \times 10^4 \text{ kg m}^{-3}$, $g = 10 \text{ m s}^{-2}$, $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- At depth h beneath a liquid surface exposed to the atmosphere, the absolute pressure is $P = P_{\text{atm}} + \rho gh$. The force exerted by the liquid on a horizontal base of area A is $F = PA$.
- The circular base has area $A = \pi r^2$. Use SI values for the radius and liquid height so that pressure is in pascals and the resulting force is in newtons. Atmospheric pressure is included in the pressure transmitted through the liquid.

Step 1: Calculate the absolute pressure at the bottom Convert the given dimensions:

$$r = 2 \text{ cm} = 0.02 \text{ m}, \quad h = 30 \text{ cm} = 0.30 \text{ m}.$$

The hydrostatic addition to atmospheric pressure is

$$\rho gh = (1.36 \times 10^4)(10)(0.30) = 40800 \text{ Pa}.$$

Therefore the mercury pressure at the bottom is

$$P = 100000 + 40800 = 140800 \text{ Pa}.$$

The quantity ρgh alone is the gauge pressure; it would represent the extra pressure above the atmosphere, rather than the total pressure exerted by the mercury.

Step 2: Multiply the pressure by the base area The base area, using the given value of π , is

$$A = \frac{22}{7}(0.02)^2 = \frac{22}{7} \times 0.0004 \text{ m}^2.$$

Thus

$$F = 140800 \times \frac{22}{7} \times 0.0004 = 177.0057 \dots \text{ N}.$$

The unit check is $\text{N m}^{-2} \times \text{m}^2 = \text{N}$. This is the downward force mercury exerts on the bottom, not the net force after subtracting atmospheric force on the opposite face of the base.

Step 3: Conclusion The force exerted by mercury on the bottom is 177 N to the nearest integer. The numerical entry is 177.

Quick Tip to Solve: If atmospheric pressure is supplied, distinguish total liquid pressure from gauge pressure before using $F = \pi r^2 P$.

57. A light ray is incident on a glass slab of thickness $4\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence is equal to the critical angle for the glass slab with air. The lateral displacement of ray after passing through glass slab is _____ cm.

(Given $\sin 15^\circ = 0.25$)

[4 marks]

Solution:

Concept:

- The critical angle C for glass-to-air propagation satisfies $\sin C = 1/\mu$, where μ is the refractive index of the glass relative to air.
- For incidence from air, Snell's law is $\sin i = \mu \sin r$. For a parallel slab of normal thickness t , the lateral displacement is $d = t \sin(i - r) / \cos r$.

Step 1: Find the angle of incidence The statement sets the incident angle equal to the glass-air critical angle. With $\mu = \sqrt{2}$,

$$\sin C = \frac{1}{\sqrt{2}} \Rightarrow C = 45^\circ.$$

Therefore $i = 45^\circ$. The ray is incident from air: equality to the critical-angle value does not mean total internal reflection takes place at this first surface.

Step 2: Calculate the refracted angle Apply Snell's law at the air-glass interface:

$$\sin r = \frac{\sin 45^\circ}{\sqrt{2}} = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}.$$

Hence $r = 30^\circ$. At the second parallel surface, the ray emerges into air at 45° ; its final direction is parallel to the incident ray, with a sideways shift.

Step 3: Evaluate the lateral displacement The angle difference inside the displacement formula is $i - r = 15^\circ$. Use the supplied value $\sin 15^\circ = 0.25$, rather than a more precise trigonometric value:

$$d = 4\sqrt{3} \frac{0.25}{\cos 30^\circ} = 4\sqrt{3} \frac{1/4}{\sqrt{3}/2} = 2 \text{ cm}.$$

The thickness is already in centimetres, so the displacement has the requested unit directly.

Step 4: Conclusion The lateral displacement is $\boxed{2 \text{ cm}}$, so the numerical entry is $\boxed{2}$.

Quick Tip to Solve: Find the internal angle with Snell's law before using the slab-shift formula; the stated critical angle gives the incident angle, not the refracted angle.

58. Two wires A and B are made up of the same material and have the same mass. Wire A has radius of 2.0 mm and wire B has radius of 4.0 mm. The resistance of wire B is 2Ω . The resistance of wire A is _____ Ω . [4 marks]

Solution:

Concept:

- A uniform wire has resistance $R = \rho L/A$, with resistivity ρ , length L , and cross-sectional area $A = \pi r^2$.
- Equal masses of the same material mean equal volumes: $A_A L_A = A_B L_B$. Therefore, at fixed volume and resistivity, $R \propto A^{-2} \propto r^{-4}$.

Step 1: Use the equal-mass condition Both wires have the same material density. Since mass equals density times volume, their equal masses give equal volumes. Thus

$$\pi r_A^2 L_A = \pi r_B^2 L_B.$$

It follows that

$$\frac{L_A}{L_B} = \frac{r_B^2}{r_A^2} = \left(\frac{4.0}{2.0}\right)^2 = 4.$$

The thinner wire is four times as long; its length cannot be treated as equal to that of the thicker wire.

Step 2: Form the resistance ratio The resistivity is identical because the material is the same, so it cancels in the ratio:

$$\frac{R_A}{R_B} = \frac{L_A}{L_B} \frac{A_B}{A_A} = 4 \left(\frac{4.0}{2.0}\right)^2 = 16.$$

One factor of four comes from the greater length of A ; another comes from its smaller area. Equivalently, substituting $L = V/A$ in the resistance formula gives $R = \rho V/A^2$.

Step 3: Substitute the known resistance Using $R_B = 2\Omega$,

$$R_A = 16R_B = 16 \times 2 = 32\Omega.$$

Radius units cancel in the ratio, so there is no need to convert millimetres to metres. The result is physically consistent: a longer, thinner wire has a substantially larger resistance.

Step 4: Conclusion The resistance of wire A is 32Ω . The numerical entry is 32 .

Quick Tip to Solve: For equal masses of one material, resistance varies as the inverse fourth power of radius; doubling radius reduces resistance by sixteen.

59. The displacement of a particle executing SHM is given by $x = 10 \sin\left(\omega t + \frac{\pi}{3}\right)$ m. The time period of motion is 3.14 s. The velocity of the particle at $t = 0$ is _____ m/s. [4 marks]

Solution:

Concept:

- For $x = A \sin(\omega t + \phi)$, differentiation gives the signed velocity $v = A\omega \cos(\omega t + \phi)$.

- The angular frequency and time period obey $\omega = 2\pi/T$. The phase must be evaluated at the stated time; $A\omega$ alone is only the maximum speed.

Step 1: Identify the amplitude and initial phase Compare the displacement with the standard sinusoidal form. The amplitude is $A = 10$ m, the initial phase is $\phi = \pi/3$, and the period is $T = 3.14$ s. At $t = 0$, the argument of the sine is $\pi/3$, not zero. The particle therefore starts away from equilibrium and its speed need not equal its maximum speed.

Step 2: Differentiate the displacement The time derivative introduces the angular frequency through the chain rule:

$$v(t) = \frac{dx}{dt} = 10\omega \cos\left(\omega t + \frac{\pi}{3}\right) \text{ m s}^{-1}.$$

With the stated period,

$$\omega = \frac{2\pi}{3.14} \simeq 2 \text{ rad s}^{-1}.$$

This is angular frequency, not the ordinary frequency $1/T$.

Step 3: Evaluate the initial velocity At $t = 0$, $\cos(\pi/3) = 1/2$. Therefore

$$v(0) = 10 \times 2 \times \frac{1}{2} = 10 \text{ m s}^{-1}.$$

Using a more precise value of π gives approximately 10.005 m s^{-1} , which has the same integer entry. The sign is positive because the cosine of the initial phase is positive, so the particle initially moves in the direction of increasing displacement.

Step 4: Conclusion The initial velocity is $\boxed{+10 \text{ m s}^{-1}}$, giving numerical entry $\boxed{10}$.

Quick Tip to Solve: Differentiate first and insert the initial phase second: displacement uses sine, while velocity uses cosine and includes a factor of angular frequency.

60. A parallel plate capacitor of capacitance 12.5 pF is charged by a battery connected between its plates to potential difference of 12.0 V . The battery is now disconnected and a dielectric slab ($\epsilon_r = 6$) is inserted between the plates. The change in its potential energy after inserting the dielectric slab is $\underline{\hspace{2cm}} \times 10^{-12} \text{ J}$. [4 marks]

Solution:

Concept:

- For a capacitor initially at voltage V , $U_i = \frac{1}{2}CV^2$ and its charge is $Q = CV$.
- After the battery is disconnected, charge stays fixed. Fully inserting a dielectric of relative permittivity K changes the capacitance to KC , so $U_f = Q^2/(2KC) = U_i/K$.

Step 1: Calculate the initial energy The initial capacitance and voltage are $C = 12.5 \times 10^{-12} \text{ F}$ and $V = 12.0 \text{ V}$. Thus

$$U_i = \frac{1}{2}(12.5 \times 10^{-12})(12.0)^2 = 900 \times 10^{-12} \text{ J}.$$

The battery establishes the initial charge before being disconnected; its voltage is not maintained during dielectric insertion.

Step 2: Keep the isolated charge fixed Here $K = \epsilon_r = 6$, so the final capacitance is $C_f = 6C$. Since there is no conducting path to or from the isolated plates, $Q_f = Q_i$. Consequently

$$U_f = \frac{U_i}{6} = 150 \times 10^{-12} \text{ J.}$$

The final voltage also falls to $V_f = V/6 = 2.0 \text{ V}$. Keeping 12 V in the final energy formula would incorrectly describe a capacitor still connected to the battery.

Step 3: Find the decrease and its sign The final-minus-initial energy change is

$$\Delta U = U_f - U_i = (150 - 900) \times 10^{-12} = -750 \times 10^{-12} \text{ J.}$$

Thus the stored electrical energy decreases by $750 \times 10^{-12} \text{ J}$. The positive numerical entry in the key represents the amount of the decrease; the signed change is negative. This is consistent with the dielectric being attracted between the charged plates.

Step 4: Conclusion The energy decreases by $750 \times 10^{-12} \text{ J}$, so the requested decrease has entry 750 . With the convention $\Delta U = U_f - U_i$, the signed change is $-750 \times 10^{-12} \text{ J}$.

Quick Tip to Solve: After disconnecting the battery, conserve charge. A dielectric then divides the stored energy by its relative permittivity, rather than multiplying it.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Fuel cell, using hydrogen and oxygen as fuels,

- A. has been used in spaceship
- B. has as efficiency of 40% to produce electricity
- C. uses aluminum as catalysts
- D. is eco-friendly
- E. is actually a type of Galvanic cell only

Choose the correct answer from the options given below:

[4 marks]

- (A) A, D, E only
- (B) A, B, C only
- (C) A, B, D only
- (D) A, B, D, E only

Correct Answer: (A) A, D, E only

Solution:

Concept:

- A fuel cell is a galvanic cell that converts the chemical energy of continuously supplied reactants directly into electrical energy.
- The hydrogen-oxygen fuel cell has overall reaction $2\text{H}_2 + \text{O}_2 \longrightarrow 2\text{H}_2\text{O}$. Its water product and relatively high conversion efficiency are key advantages.

Step 1: Check the application and efficiency statements Statement A is correct: hydrogen-oxygen fuel cells have been used in spacecraft, where the water formed is also useful. Statement B is incorrect in the textbook comparison: the hydrogen-oxygen fuel cell has an efficiency of about 70 percent, whereas roughly 40 percent is the comparison figure for conventional thermal power generation. The two efficiency figures must not be interchanged.

Step 2: Check the catalyst and environmental statements Statement C is incorrect. The electrodes use suitable finely divided catalysts such as platinum or palladium; aluminium is not the catalyst specified for this fuel cell. Statement D is correct: the principal reaction product is water, so the cell avoids the polluting combustion products associated with fossil-fuel power generation. The eco-friendly description concerns the operation of the hydrogen-oxygen cell.

Step 3: Classify the cell and select the combination Statement E is correct because the spontaneous redox reaction produces electricity, which is the defining operation of a galvanic cell. Continuous fuel supply distinguishes its practical operation without changing that classification. Therefore the correct statements are A, D and E. Options B, C and D all include the incorrect statement B; option B additionally includes the incorrect catalyst statement C. Only option A lists precisely the three true statements.

Step 4: Conclusion The correct statements are A, D and E only. Hence, the correct option is (A).

Quick Tip to Solve: Link a hydrogen-oxygen fuel cell with spacecraft, water and galvanic action; do not transfer the lower thermal-generation efficiency to the fuel cell.

62. Match List I with List II

	LIST I		LIST II
A.	α - Glucose and α - Galactose	I.	Functional isomers
B.	α - Glucose and β - Glucose	II.	Homologous
C.	α - Glucose and α - Fructose	III.	Anomers
D.	α - Glucose and α - Ribose	IV.	Epimers

Choose the correct answer from the options given below:

[4 marks]

- (A) A-III, B-IV, C-II, D-I
 (B) A-IV, B-III, C-I, D-II
 (C) A-III, B-IV, C-I, D-II
 (D) A-IV, B-III, C-II, D-I

Correct Answer: (B) A-IV, B-III, C-I, D-II

Solution:

Concept:

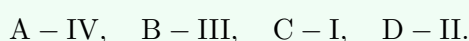
- Epimers differ in configuration at one stereogenic carbon, while anomers differ specifically at the anomeric carbon generated on cyclisation.
- Functional isomers have the same molecular formula but different functional groups. Members of a homologous sequence have the same functional type and differ by a characteristic composition

increment.

Step 1: Match the two stereochemical pairs Glucose and galactose differ in configuration at carbon 4 while retaining the same configurations at the other relevant stereocentres. Thus the pair of alpha forms in A is an epimer pair, giving A-IV. Alpha-glucose and beta-glucose differ at their anomeric carbon, carbon 1, so B is the anomer pair and gives B-III. The distinction is specific: anomerism concerns the newly formed hemiacetal centre.

Step 2: Match the functional and chain-length pairs Glucose and fructose both have molecular formula $C_6H_{12}O_6$, but their open-chain forms contain aldehyde and ketone functions respectively. Hence C matches functional isomers, C-I. Glucose and ribose are the aldohexose and aldopentose pair; their compositions differ by CH_2O . In the carbohydrate classification used in the listed alternatives, this is the homologous pair, so D-II.

Step 3: Compare the complete mapping with the options Together these identifications give



This is option B. Option A swaps the epimer and anomer assignments and misidentifies the last two pairs. Option C still swaps A and B. Option D gets the stereochemical pairs right but incorrectly interchanges functional isomerism and the homologous relationship for C and D. Matching all four rows prevents a partial match from misleading the selection.

Step 4: Conclusion The matching is A-IV, B-III, C-I and D-II. Hence, the correct option is (B).

Quick Tip to Solve: First identify the alpha/beta pair as anomers, then use the aldose/ketose pair for functional isomerism; these two anchors sharply constrain the matching.

63. If an iron (III) complex with the formula $[Fe(NH_3)_x(CN)_y]^-$ has no electron in its e_g orbital, then the value of $x + y$ is [4 marks]

- (A) 6
- (B) 4
- (C) 3
- (D) 5

Correct Answer: (A) 6

Solution:

Concept:

- Iron has atomic number 26. Removing two 4s electrons and one 3d electron gives the $3d^5$ configuration of Fe^{3+} .
- In an octahedral ligand field, the three t_{2g} orbitals lie below the two e_g orbitals. Strong-field ligands can produce a low-spin configuration by pairing electrons in t_{2g} .

Step 1: Determine the number of d electrons The metal is explicitly iron(III), so its oxidation state is +3 and its configuration is d^5 . For an octahedral high-spin d^5 ion, electrons occupy $t_{2g}^3 e_g^2$. That

arrangement would place two electrons in e_g , contrary to the condition in the question. The stated absence of e_g electrons instead selects the low-spin arrangement.

Step 2: Use the empty upper orbitals to find coordination In the low-spin octahedral case, the five d electrons occupy the lower set as $t_{2g}^5 e_g^0$. Cyanide is a strong-field ligand and supports this pairing. The notation t_{2g}, e_g and the stated empty upper set are consistent with this octahedral splitting. An octahedral complex has six ligand donor positions. Since both ammonia and cyanide are monodentate here, the total number of ligands is $x + y = 6$.

Step 3: Check the ligand numbers using charge balance Ammonia is neutral and each cyanide has charge -1. Charge balance gives

$$3 + x(0) - y = -1 \Rightarrow y = 4.$$

Therefore $x = 6 - 4 = 2$, consistent with a six-coordinate complex containing four strong-field cyanide ligands. Options 4, 3 and 5 do not give the required octahedral coordination number for the stated e_g^0 , low-spin d^5 description.

Step 4: Conclusion The coordination number is $x + y = \boxed{6}$, with $x = 2$ and $y = 4$. Hence, the correct option is (A).

Quick Tip to Solve: Use oxidation state to count d electrons, then use the stated orbital occupancy to identify the ligand-field geometry before balancing ligand charges.

64. The number of unpaired d-electrons in $[\text{Co}(\text{H}_2\text{O})_6]^{3+}$ is _____.

[4 marks]

- (A) 0
- (B) 2
- (C) 1
- (D) 4

Correct Answer: (A) 0

Solution:

Concept:

- Crystal-field splitting separates the five d orbitals in an octahedral complex into three lower t_{2g} and two higher e_g orbitals.
- For a low-spin complex, the octahedral splitting energy Δ_o exceeds the pairing energy, so electrons pair in the lower orbitals before entering the higher set.

Step 1: Find the oxidation state and d count Water is a neutral ligand. Therefore cobalt has oxidation state +3 in $[\text{Co}(\text{H}_2\text{O})_6]^{3+}$. Neutral cobalt has configuration $[\text{Ar}]3d^7 4s^2$. Removing two 4s electrons and one 3d electron gives Co^{3+} the configuration $[\text{Ar}]3d^6$. Thus six d electrons must be distributed among the octahedrally split orbitals.

Step 2: Choose the spin arrangement The high positive charge on cobalt gives a sufficiently large octahedral splitting in this cobalt(III) aqua complex. Consequently the complex is low spin: the six

electrons occupy the three lower orbitals as three pairs.



Each lower orbital contains two electrons with opposite spins; neither higher orbital contains an electron.

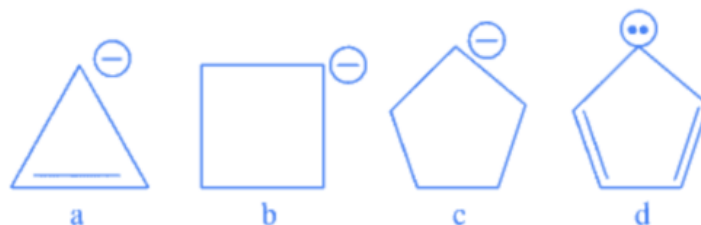
Step 3: Count the unpaired electrons All six d electrons are paired, so the count is zero and the complex is diamagnetic. The alternative count of four would correspond to the high-spin $t_{2g}^4 e_g^2$ arrangement, which is not the arrangement here. Counts of one or two also do not describe the fully paired low-spin d-six configuration.

Step 4: Conclusion The number of unpaired d electrons is 0. Hence, the correct option is (A).

Quick Tip to Solve: Find the metal oxidation state before the d count; the stronger splitting associated with Co^{3+} is essential when deciding the spin state.

65. Correct order of stability of carbanion is :

[4 marks]



(A) $d > c > b > a$

(B) $c > b > d > a$

(C) $d > a > c > b$

(D) $a > b > c > d$

Correct Answer: (A) $d > c > b > a$

Solution:

Concept:

- A carbanion is stabilized when its negative charge can be delocalized without producing an antiaromatic cyclic electron system.
- A planar, cyclic, fully conjugated system is aromatic with $4n + 2$ pi electrons and antiaromatic with $4n$ pi electrons.

Step 1: Identify the conjugated rings Structure d is the cyclopentadienyl anion. Its two double bonds contribute four pi electrons, and the negatively charged carbon contributes a lone pair to the cyclic conjugated system. The total is six pi electrons. Structure a is the cyclopropenyl anion: its double bond and lone pair give a four-pi-electron cyclic system.

Step 2: Compare aromatic and antiaromatic stabilization The six-electron system in d satisfies $4n + 2 = 6$ with $n = 1$, giving aromatic stabilization and making d the most stable. The four-electron

system in a has the antiaromatic electron count and is highly destabilized compared with the saturated-ring carbanions. Distortion can reduce antiaromaticity but does not supply the aromatic stabilization present in d.

Step 3: Place the saturated-ring carbanions Structures b and c are cyclobutyl and cyclopentyl carbanions, respectively. They have no continuous pi system, so neither is aromatic. The five-membered ring in c accommodates the carbanionic carbon with less small-ring strain than the four-membered ring in b. Thus c is more stable than b. Combining this with the conjugation comparison gives

$$d > c > b > a.$$

Options B and D fail by not putting aromatic d first, while C incorrectly puts destabilized a above the saturated-ring anions.

Step 4: Conclusion The stability order is $d > c > b > a$. Hence, the correct option is (A).

Quick Tip to Solve: Count the lone pair at the negatively charged ring carbon as two pi electrons before applying the aromaticity test.

66. Product P is

[4 marks]



- (A) $\text{C}_6\text{H}_5\text{CH}_2\text{CH}=\text{C}(\text{CH}_3)_2$
- (B) $\text{C}_6\text{H}_5\text{C}(=\text{CH}_2)\text{CH}(\text{CH}_3)_2$
- (C) $\text{C}_6\text{H}_5\text{CH}_2\text{CH}_2\text{C}(\text{CH}_3)=\text{CH}_2$
- (D) $\text{C}_6\text{H}_5\text{CH}=\text{CHCH}(\text{CH}_3)_2$

Correct Answer: (D) $\text{C}_6\text{H}_5\text{CH}=\text{CHCH}(\text{CH}_3)_2$

Solution:

Concept:

- Alcoholic KOH and heat favour beta-elimination of HBr from an alkyl bromide, producing an alkene.
- Conjugation of an alkene with an aromatic ring stabilizes the product; conjugation must be checked along with alkene substitution.

Step 1: Locate the beta hydrogens The substrate is $\text{C}_6\text{H}_5\text{CH}_2\text{CH}(\text{Br})\text{CH}(\text{CH}_3)_2$. The carbon bearing bromine is the alpha carbon. Its two adjacent carbons are beta carbons, and both have hydrogens available for elimination. Removing a beta hydrogen and bromide forms a double bond between the alpha carbon and the chosen beta carbon.

Step 2: Compare the two direct elimination products Removing a hydrogen from the benzylic CH_2 group gives $\text{C}_6\text{H}_5\text{CH}=\text{CHCH}(\text{CH}_3)_2$. This double bond is directly attached to the benzene ring and is

conjugated with it. Removing the hydrogen on the other side instead gives $\text{C}_6\text{H}_5\text{CH}_2\text{CH}=\text{C}(\text{CH}_3)_2$, whose double bond is separated from the benzene ring by a saturated carbon and is not conjugated.

Step 3: Choose the major product and reject the alternatives The conjugated alkene is favoured here because its pi system is delocalized with the aromatic ring. It is the structure shown in option D. Option A is the competing nonconjugated alkene. Options B and C place the new double bond at positions that are not obtained by direct beta-elimination from the given carbon-bromine bond; no carbon-skeleton rearrangement is required or supported under these basic elimination conditions.

Step 4: Conclusion The major product is $\text{C}_6\text{H}_5\text{CH}=\text{CHCH}(\text{CH}_3)_2$, the conjugated alkene. Hence, the correct option is (D).

Quick Tip to Solve: For elimination beside a benzene ring, check whether one beta hydrogen produces a double bond directly conjugated with the ring.

67. The correct order of the first ionization enthalpy is

[4 marks]

- (A) $\text{Ga} > \text{Al} > \text{B}$
- (B) $\text{Tl} > \text{Ga} > \text{Al}$
- (C) $\text{Al} > \text{Ga} > \text{Tl}$
- (D) $\text{B} > \text{Al} > \text{Ga}$

Correct Answer: (B) $\text{Tl} > \text{Ga} > \text{Al}$

Solution:

Concept:

- The first ionization enthalpy is the energy required to remove one electron from an isolated gaseous atom.
- Atomic size generally lowers ionization enthalpy down a group, but poor shielding by inner d and f electrons can increase the effective nuclear attraction.

Step 1: Compare aluminium and gallium Aluminium and gallium belong to group 13 and have an outer ns^2np^1 configuration. A simple size argument would predict a fall from aluminium to gallium. However, gallium contains a filled 3d subshell whose electrons shield the nuclear charge poorly. The resulting increased effective nuclear attraction makes gallium first ionization enthalpy slightly greater than that of aluminium.

Step 2: Place thallium relative to gallium Thallium has intervening d and f electrons, whose poor shielding enhances attraction for the outer electrons. Together with the contraction effects in the heavy atom, this gives thallium a larger first ionization enthalpy than gallium. Approximate values, useful as a consistency check, are 578, 579, and 589 kJ mol^{-1} for aluminium, gallium, and thallium, respectively.

Step 3: Test the printed orders Therefore the relevant order is

$$\text{Tl} > \text{Ga} > \text{Al}.$$

Option A fails because boron has a much higher ionization enthalpy than aluminium and gallium.

Option C reverses the order just established. Option D has aluminium above gallium, overlooking the small increase caused by poor d-electron shielding. Only option B agrees with all three elements in its comparison.

Step 4: Conclusion The correct order is $\boxed{\text{Tl} > \text{Ga} > \text{Al}}$. Hence, the correct option is (B).

Quick Tip to Solve: Do not apply a smooth downward trend to group 13: poor d- and f-electron shielding produces the important Ga/Al and Tl exceptions.

68. Given below are two statements :

Statement I : The correct order of first ionization enthalpy values of Li, Na, F and Cl is $\text{Na} < \text{Li} < \text{Cl} < \text{F}$.

Statement II : The correct order of negative electron gain enthalpy values of Li, Na, F and Cl is $\text{Na} < \text{Li} < \text{F} < \text{Cl}$

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Statement I is true but Statement II is false
- (B) Both Statement I and Statement II are false
- (C) Statement I is false but Statement II is true
- (D) Both Statement I and Statement II are true

Correct Answer: (D) Both Statement I and Statement II are true

Solution:

Concept:

- First ionization enthalpy generally increases across a period and decreases down a group because of changes in effective nuclear charge and atomic size.
- For exothermic electron gain, a larger magnitude of negative electron gain enthalpy means more energy is released; compact fluorine is an important exception to a simple size trend.

Step 1: Evaluate Statement I Sodium loses its outer 3s electron more easily than lithium loses its 2s electron, so sodium has the lower first ionization enthalpy. The halogens have much higher first ionization enthalpies than these alkali metals. Fluorine is smaller than chlorine and binds its outer electron more strongly. Thus

$$\text{Na} < \text{Li} < \text{Cl} < \text{F}$$

is the correct ionization-enthalpy order, making Statement I true.

Step 2: Evaluate Statement II Here the stated order of negative electron gain enthalpy compares the magnitude of energy released on accepting an electron. The alkali metals release much less energy than the halogens, and lithium releases slightly more than sodium. Chlorine releases more energy than fluorine because an added electron in fluorine enters a very compact 2p subshell and experiences stronger electron-electron repulsion. Chlorine has the less crowded 3p subshell. The magnitudes therefore increase as

$$\text{Na} < \text{Li} < \text{F} < \text{Cl}.$$

Statement II is also true in this conventional comparison of negative electron gain enthalpies.

Step 3: Select the combination Both statements give the correct respective trends. Options A and C each reject one correct statement, and option B rejects both, so none is appropriate. When writing signed thermodynamic values instead of their negative magnitudes, remember that the more exothermic value is numerically more negative.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (D).

Quick Tip to Solve: Keep ionization enthalpy and electron gain enthalpy separate: fluorine has higher ionization enthalpy, but chlorine has the more exothermic electron gain.

69. For a strong electrolyte, a plot of molar conductivity against (concentration)^{1/2} is a straight line, with a negative slope, the correct unit for the slope is [4 marks]

- (A) $\text{S cm}^2 \text{ mol}^{-3/2} \text{ L}$
- (B) $\text{S cm}^2 \text{ mol}^{-3/2} \text{ L}^{-1/2}$
- (C) $\text{S cm}^2 \text{ mol}^{-1} \text{ L}^{1/2}$
- (D) $\text{S cm}^2 \text{ mol}^{-3/2} \text{ L}^{1/2}$

Correct Answer: (D) $\text{S cm}^2 \text{ mol}^{-3/2} \text{ L}^{1/2}$

Solution:

Concept:

- For a strong electrolyte at low concentration, Kohlrausch's relation is $\Lambda_m = \Lambda_m^\circ - K\sqrt{c}$, where c is concentration.
- The unit of the slope of a graph is the vertical-axis unit divided by the horizontal-axis unit; the negative sign does not change the unit.

Step 1: Write the units on the two axes Molar conductivity Λ_m is given in $\text{S cm}^2 \text{ mol}^{-1}$. The concentration used here is in mol L^{-1} , so the horizontal coordinate has unit

$$[\sqrt{c}] = \text{mol}^{1/2} \text{ L}^{-1/2}.$$

The square root must act on both the amount-of-substance factor and the volume factor.

Step 2: Divide the ordinate unit by the abscissa unit For two points on the straight line, slope is $\Delta\Lambda_m/\Delta\sqrt{c}$. Hence

$$\begin{aligned} [\text{slope}] &= \frac{\text{S cm}^2 \text{ mol}^{-1}}{\text{mol}^{1/2} \text{ L}^{-1/2}} \\ &= \text{S cm}^2 \text{ mol}^{-3/2} \text{ L}^{1/2}. \end{aligned}$$

The molar exponent becomes $-1 - 1/2 = -3/2$, while dividing by $\text{L}^{-1/2}$ changes it to $\text{L}^{1/2}$.

Step 3: Compare the choices Option D has both exponents exactly as obtained. Option A has the first power of litre instead of its square root. Option B retains a negative half-power of litre rather than reversing it on division. Option C leaves the molar exponent at minus one and therefore neglects the molar factor in the horizontal-axis unit. A negative numerical slope simply expresses decreasing

conductivity with increasing square root of concentration; its dimensions remain those calculated.

Step 4: Conclusion The slope unit is $\boxed{\text{S cm}^2 \text{ mol}^{-3/2} \text{ L}^{1/2}}$. Hence, the correct option is (D).

Quick Tip to Solve: Write slope units as y-unit divided by x-unit before simplifying powers; taking a square root affects every concentration unit factor.

70. When MnO_2 and H_2SO_4 is added to a salt (A), the greenish yellow gas liberated as salt (A) is : [4 marks]

- (A) NH_4Cl
- (B) CaI_2
- (C) KNO_3
- (D) NaBr

Correct Answer: (A) NH_4Cl

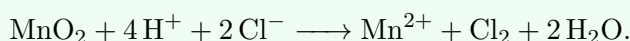
Solution:

Concept:

- Chlorine is a greenish-yellow gas. Oxidation of chloride ions by MnO_2 in strongly acidic conditions liberates chlorine.
- In the reaction, chlorine changes oxidation state from -1 to 0, while manganese is reduced from +4 to +2.

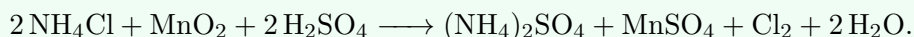
Step 1: Identify the liberated gas The greenish-yellow colour identifies Cl_2 . Therefore the original salt must supply chloride ions. Of the four salts listed, ammonium chloride contains chloride; calcium iodide contains iodide, potassium nitrate contains nitrate, and sodium bromide contains bromide. This immediately singles out NH_4Cl as the candidate.

Step 2: Explain the action of the reagents Sulfuric acid provides acidic conditions and converts the chloride salt into a source of hydrogen chloride. Manganese dioxide oxidizes chloride to molecular chlorine. The net ionic redox equation is



Two chloride ions together lose two electrons, and manganese gains those two electrons, so the atoms and charge are balanced.

Step 3: Confirm the salt and exclude the others An overall molecular representation is



Calcium iodide would lead to iodine rather than chlorine, and sodium bromide would lead to bromine; neither produces the characteristic greenish-yellow chlorine gas. Potassium nitrate contains no halide ion from which chlorine could be generated. Thus the identity of both the anion and the gas agrees only for ammonium chloride.

Step 4: Conclusion Salt (A) is $\boxed{\text{NH}_4\text{Cl}}$, which gives greenish-yellow Cl_2 . Hence, the correct option is

(A).

Quick Tip to Solve: Use the gas colour first: chlorine is greenish-yellow, bromine is reddish-brown, and iodine vapour is violet.

71. The correct statement/s about Hydrogen bonding is/are

- A. Hydrogen bonding exists when H is covalently bonded to the highly electro negative atom.
- B. Intermolecular H bonding is present in *o*-nitro phenol
- C. Intramolecular H bonding is present in HF.
- D. The magnitude of H bonding depends on the physical state of the compound.
- E. H-bonding has powerful effect on the structure and properties of compounds

Choose the correct answer from the options given below:

[4 marks]

(A) A, B, C only

(B) A only

(C) A, D, E only

(D) A, B, D only

Correct Answer: (C) A, D, E only

Solution:

Concept:

- Hydrogen bonding is attraction between a hydrogen attached to a strongly electronegative atom, usually N, O or F, and an electron-rich electronegative atom.
- Intermolecular hydrogen bonds connect different molecules; intramolecular hydrogen bonds occur between suitable groups within one molecule. Their extent depends on the molecular arrangement and physical state.

Step 1: Evaluate statements A and B Statement A gives the usual condition for the hydrogen-bond donor and is correct. The covalent bond is strongly polar, leaving hydrogen partially positive. In *o*-nitrophenol, the adjacent nitro oxygen accepts a hydrogen bond from the phenolic OH within the same molecule. Thus its characteristic hydrogen bond is intramolecular, so statement B is false.

Step 2: Evaluate statements C, D and E HF forms hydrogen-bonded associations between separate HF molecules; one HF molecule cannot make an internal hydrogen bond. Thus C is false. Molecular packing and separation change between gas, liquid and solid, affecting the extent and effective magnitude of hydrogen bonding, so D is correct. Hydrogen bonding changes boiling point, solubility, association and structures such as ice; E is also correct.

Step 3: Match the complete set The true statements are A, D and E. Options (A) and (D) include the false statement B, and option (A) also includes C. Option (B) omits the true statements D and E, so it is incomplete.

Step 4: Conclusion Only statements A, D and E are correct. Hence, the correct option is (C).

Quick Tip to Solve: First locate the donor H and acceptor atom: adjacent groups may bond within one molecule, while HF bonds between molecules.

72. Choose the Incorrect Statement about Dalton's Atomic Theory

[4 marks]

- (A) Compounds are formed when atoms of different elements combine in any ratio.
- (B) Matter consists of indivisible atoms.
- (C) chemical reactions involve reorganization of atoms
- (D) All the atoms of a given element have identical properties including identical mass.

Correct Answer: (A) Compounds are formed when atoms of different elements combine in any ratio.

Solution:

Concept:

- Dalton proposed that matter consists of atoms, and that chemical reactions rearrange atoms rather than create or destroy them.
- In this historical theory, atoms of each element are identical and atoms of different elements form compounds in fixed simple whole-number ratios.

Step 1: Test the composition statement Option (A) claims that atoms of different elements can combine in any ratio to form compounds. Dalton instead required simple whole-number ratios for atoms in a particular compound. For example, water contains hydrogen and oxygen atoms in the fixed ratio 2 : 1; an arbitrary mixing ratio describes a mixture and does not define the composition of water. Thus (A) contradicts a central postulate.

Step 2: Check the other postulates in their historical context Option (B), indivisibility of atoms, is a postulate of Dalton's model. Option (C), reorganization of atoms in chemical reactions, also agrees with that model. Option (D), identical properties and mass for all atoms of an element, is another original postulate. Later discoveries of subatomic particles and isotopes limit the modern validity of (B) and (D), but the question asks what Dalton's theory states.

Step 3: Select the requested incorrect statement The incorrect statement about the theory is therefore the unrestricted phrase "any ratio" in (A). Rejecting (B) or (D) on the basis of modern discoveries would answer a different question; those discoveries do not change which assumptions Dalton originally made.

Step 4: Conclusion Atoms form a given compound in a fixed simple whole-number ratio, not any ratio. Hence, the correct option is (A).

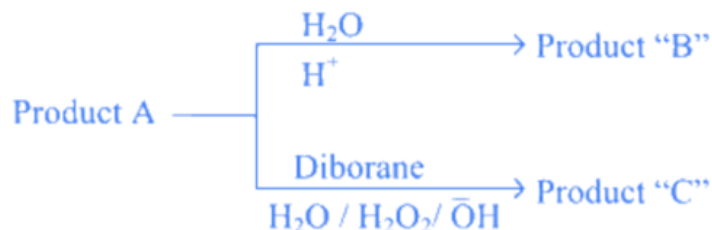
Quick Tip to Solve: For historical-theory questions, compare each choice with the original postulates; distinguish them from later experimental corrections.

73.



Consider the above reactions, identify product B and product C.

[4 marks]



- (A) B = C = 1-Propanol
 (B) B = 1-Propanol C = 2-Propanol
 (C) B = C = 2-Propanol
 (D) B = 2-Propanol C = 1-Propanol

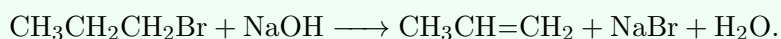
Correct Answer: (D) B = 2-Propanol C = 1-Propanol

Solution:

Concept:

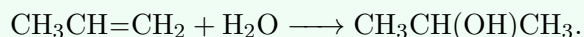
- Alcoholic alkali favours beta-elimination of hydrogen halide from a haloalkane, producing an alkene. Aqueous alkali instead commonly favours substitution.
- Acid-catalysed hydration of an unsymmetrical alkene gives the Markovnikov alcohol. Hydroboration followed by oxidation gives the anti-Markovnikov alcohol.

Step 1: Identify alkene A In the first reaction, ethanolic NaOH removes H from the beta carbon and Br from the terminal carbon of 1-bromopropane. There is only one possible alkene carbon skeleton:



Thus product A is propene. The alcoholic medium is essential: replacing it mentally by aqueous NaOH would give a different pathway.

Step 2: Find the acid-hydration product B For hydration with water and acid, protonation of propene favours the secondary carbocation rather than a primary one. Water then attacks the middle carbon, and deprotonation gives



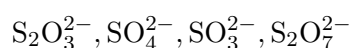
Therefore B is propan-2-ol, with OH attached to the more substituted carbon.

Step 3: Find the hydroboration product C and reject alternatives Diborane adds so that boron occupies the less substituted terminal carbon. Oxidation with alkaline hydrogen peroxide replaces the C-B bond by C-OH, giving $\text{CH}_3\text{CH}_2\text{CH}_2\text{OH}$, propan-1-ol. Consequently B and C are different alcohols, ruling out (A) and (C). Option (B) reverses their identities and is also incorrect.

Step 4: Conclusion B = propan-2-ol and C = propan-1-ol. Hence, the correct option is (D).

Quick Tip to Solve: After making the alkene, read the hydration reagent: acid puts OH on the more substituted carbon; hydroboration-oxidation puts it on the less substituted carbon.

74. The number of species from the following that have pyramidal geometry around the central atom is _____



[4 marks]

(A) 4

(B) 3

(C) 2

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- VSEPR predicts molecular geometry from the number of bonding directions and lone pairs around the central atom. A multiple bond counts as one bonding direction.
- Four bonding directions without a lone pair give tetrahedral geometry; three bonding directions with one lone pair give trigonal pyramidal molecular geometry.

Step 1: Classify sulfate and thiosulfate In sulfate, SO_4^{2-} , sulfur is bonded to four oxygen atoms with no lone pair on the central sulfur in its usual Lewis description. It is tetrahedral. Thiosulfate, $\text{S}_2\text{O}_3^{2-}$, has a central sulfur bonded to three oxygen atoms and one terminal sulfur atom. Replacing one peripheral oxygen of sulfate by sulfur preserves four bonding directions, so this central sulfur is tetrahedral as well.

Step 2: Classify sulfite and pyrosulfate In sulfite, SO_3^{2-} , sulfur has three sulfur–oxygen bonding directions and one lone pair. Its electron-pair arrangement is tetrahedral but its molecular shape is trigonal pyramidal. In pyrosulfate, $\text{S}_2\text{O}_7^{2-}$, two sulfur-centred tetrahedra share a bridging oxygen; each sulfur is bonded to four oxygens and is locally tetrahedral.

Step 3: Count only pyramidal species Only sulfite has the required lone-pair-induced pyramidal geometry. Do not count tetrahedral electron-pair geometry as pyramidal molecular geometry, or count the bridging oxygen as the central sulfur. The numbers 4, 3 and 2 would each require at least one tetrahedral sulfur species to be misclassified.

Step 4: Conclusion The number of species with pyramidal geometry is 1, namely SO_3^{2-} . Hence, the correct option is (D).

Quick Tip to Solve: Count bonded atoms and lone pairs around sulfur, treating every double bond as one direction; three bonds plus a lone pair identifies the pyramid.

75. A first row transition metal in its +2 oxidation state has a spin-only magnetic moment value of 3.86 BM. The atomic number of the metal is [4 marks]

- (A) 22
- (B) 23
- (C) 26
- (D) 25

Correct Answer: (B) 23

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)} \text{ BM}$, where n is the number of unpaired electrons.
- For first-row transition-metal cations, remove the outer 4s electrons before the 3d electrons, then distribute d electrons according to Hund's rule.

Step 1: Determine the unpaired-electron count The given moment is close to the spin-only value for three unpaired electrons:

$$\sqrt{3(3+2)} = \sqrt{15} = 3.873.$$

The quoted 3.86 BM identifies this nearest spin-only value among the listed ions. For comparison, two unpaired electrons give $\sqrt{8} = 2.83 \text{ BM}$, while four give $\sqrt{24} = 4.90 \text{ BM}$.

Step 2: Test the four atomic numbers At atomic number 22, titanium forms Ti^{2+} with $3d^2$, giving two unpaired electrons. At 23, vanadium forms V^{2+} with $3d^3$, giving three. At 26, iron forms the free-ion $3d^6$ configuration with four unpaired electrons, and at 25 manganese forms $3d^5$ with five unpaired electrons.

Step 3: Identify the matching metal Only the vanadium(II) ion among these choices has three unpaired electrons and hence the required moment. Titanium, iron and manganese predict approximately 2.83, 4.90 and 5.92 BM, respectively, so options (A), (C) and (D) do not match. The requested atomic number belongs to the neutral element, even though its ion supplies the measured magnetic behaviour.

Step 4: Conclusion The metal is vanadium, with atomic number 23. Hence, the correct option is (B).

Quick Tip to Solve: Memorise the spin-only sequence for one to five unpaired electrons: about 1.73, 2.83, 3.87, 4.90 and 5.92 BM, then check the ion configuration.

76. Common name of Benzene - 1,2 - diol is -

[4 marks]

- (A) catechol
- (B) quinol
- (C) o-cresol
- (D) resorcinol

Correct Answer: (A) catechol

Solution:

Concept:

- Benzene-1,2-diol is a dihydroxybenzene: two OH groups are attached to adjacent carbon atoms of the same benzene ring.
- The three positional dihydroxybenzene isomers have distinct common names: 1,2 is catechol, 1,3 is resorcinol, and 1,4 is hydroquinone or quinol.

Step 1: Translate the systematic name into positions The suffix “diol” indicates two hydroxyl groups. The locants 1 and 2 place them next to each other on the ring, so this is the ortho isomer of dihydroxybenzene. The benzene ring itself remains the parent framework; no methyl substituent is specified. Its condensed composition may be written $C_6H_4(OH)_2$, with the two OH groups adjacent.

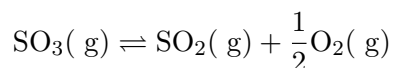
Step 2: Match the established common name The common name for this adjacent, ortho arrangement is catechol. Thus option (A) matches both the number of hydroxyl groups and their positions. Common names do not follow the same numbering form as systematic names, so identifying the positional pattern is more reliable than guessing from the sound of the name.

Step 3: Eliminate the remaining names Quinol refers to benzene-1,4-diol, the para isomer, so (B) has the wrong locants. Ortho-cresol is 2-methylphenol: it has one OH and a methyl group instead of two OH groups, excluding (C). Resorcinol is benzene-1,3-diol, the meta isomer, excluding (D). All three distractors are aromatic compounds, but none has the specified adjacent pair of hydroxyl groups.

Step 4: Conclusion The common name of benzene-1,2-diol is catechol. Hence, the correct option is (A).

Quick Tip to Solve: Use the ordered association 1,2-catechol; 1,3-resorcinol; 1,4-quinol, and remember that cresol contains a methyl group.

77. The equilibrium constant for the reaction



is $K_c = 4.9 \times 10^{-2}$. The value of K_c for the reaction given below is $2\text{SO}_2(\text{g}) + \text{O}_2(\text{g}) \rightleftharpoons 2\text{SO}_3(\text{g})$ is : [4 marks]

- (A) 49
- (B) 416
- (C) 41.6
- (D) 4.9

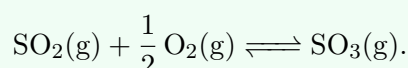
Correct Answer: (B) 416

Solution:

Concept:

- If a chemical reaction is reversed, its equilibrium constant becomes the reciprocal. If every stoichiometric coefficient is multiplied by m , its constant is raised to the power m .
- Apply these transformations to the balanced equation exactly as written; an equilibrium constant depends on the chosen stoichiometric form of the reaction.

Step 1: Relate the target equation to the given equation The given dissociation is $\text{SO}_3(\text{g}) \rightleftharpoons \text{SO}_2(\text{g}) + \frac{1}{2}\text{O}_2(\text{g})$, with $K_c = 4.9 \times 10^{-2}$. Reverse it to put sulfur trioxide on the product side:



The reversed constant is $1/K_c$. The target reaction has twice these coefficients, so its constant is K_c^{-2} .

Step 2: Evaluate the transformed constant First square the numerical value including its power of ten:

$$(4.9 \times 10^{-2})^2 = 24.01 \times 10^{-4} = 0.002401.$$

Then invert:

$$K_c (\text{target}) = \frac{1}{0.002401} = 416.49 \dots \simeq 416.$$

This is the value represented by the listed choice.

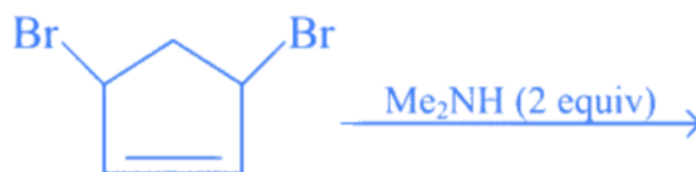
Step 3: Check the transformation and reject distractors The same result follows directly from the concentration expressions: the target ratio $[\text{SO}_3]^2/([\text{SO}_2]^2[\text{O}_2])$ is the reciprocal square of the original ratio. Merely changing the power of ten can produce 41.6, while copying 4.9 or choosing 49 ignores the reversal and doubling. Thus options (A), (C) and (D) fail the exact expression.

Step 4: Conclusion The required equilibrium constant is approximately 416. Hence, the correct option is (B).

Quick Tip to Solve: Write the reaction operation first: reverse means reciprocal, and doubling coefficients means square; here both give K^{-2} .

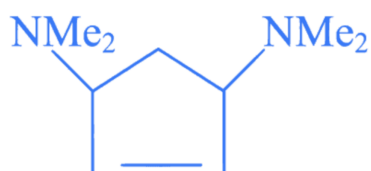
78. Find out the major product formed from the following reaction. [Me : $-\text{CH}_3$]

[4 marks]



- (A)
- (B)
- (C)
- (D)

Correct Answer: (D)



Solution:

Concept:

- Dimethylamine, $(\text{CH}_3)_2\text{NH}$, is a nucleophile because its nitrogen has an available lone pair. An allylic carbon bearing bromine can undergo nucleophilic substitution.
- In substitution, the carbon skeleton and the existing alkene are retained while a carbon–bromine bond is replaced by a carbon–nitrogen bond. Two equivalents permit replacement at both brominated positions.

Step 1: **Locate the two leaving groups** The starting compound is a cyclopentene with bromine on each of the two saturated carbons adjacent to the double bond. Number the double-bond carbons

1 and 2; the bromines are then at positions 3 and 5. Both carbon–bromine bonds are allylic. The saturated carbon between those two substituted positions is unchanged. This identifies the two places where dimethylamino groups must appear in the direct substitution product.

Step 2: Replace each bromine with dimethylamino Nitrogen attacks an allylic carbon and bromide leaves. Loss of the nitrogen-bound proton gives a tertiary amino substituent, $-\text{N}(\text{CH}_3)_2$. Repeating this at the other brominated carbon produces the 3,5-bis(dimethylamino)cyclopentene framework. The two methyl groups already attached to nitrogen are retained; the reagent does not introduce an extra carbon into the ring. No reagent in the sequence requires reduction or migration of the alkene.

Step 3: Compare the four drawings Option (D) places both NMe_2 groups exactly where the two bromines were, with the original double bond still between the two lower ring carbons. Option (A) puts the substituents on adjacent ring carbons, which changes their connectivity. Options (B) and (C) attach one amino group to a double-bond carbon, rather than replacing both allylic bromines in place. Thus those drawings do not represent the direct double substitution shown.

Step 4: Conclusion The major product has both allylic bromines replaced by NMe_2 , with the ring double bond retained. Hence, the correct option is (D).

Quick Tip to Solve: Number the original alkene first, then mark the leaving-group carbons. Replace Br at those marked carbons before comparing rotated product drawings.

79. The adsorbent used in adsorption chromatography is/are -

- A. silica gel
- B. alumina
- C. quick lime
- D. magnesia

Choose the most appropriate answer from the options given below :

[4 marks]

- (A) C and D only
- (B) A only
- (C) B only
- (D) A and B only

Correct Answer: (D) A and B only

Solution:

Concept:

- Adsorption chromatography separates components because they adsorb to different extents on a solid stationary phase. The mobile solvent carries the components past the adsorbent surface.
- Silica gel and alumina are the standard adsorbents used in the elementary adsorption-chromatography methods considered here. Their large surface areas provide many sites for intermolecular interactions.

Step 1: Identify the standard stationary phases Silica gel is a finely divided, porous material with a high surface area. Its surface can interact with solute molecules, so different solutes are retained

by different amounts. Alumina, Al_2O_3 , is another common solid stationary phase for adsorption chromatography. Therefore statement A, silica gel, and statement B, alumina, belong in the required combination.

Step 2: Distinguish the other materials Quick lime is CaO , commonly encountered as a drying or basic reagent. Magnesia is MgO . They are not the pair of standard adsorbents intended in this NCERT classification. The question asks for the most appropriate combination from its choices, rather than for every solid that could show any surface adsorption under a specialised experimental condition. The expected identification is consequently A and B.

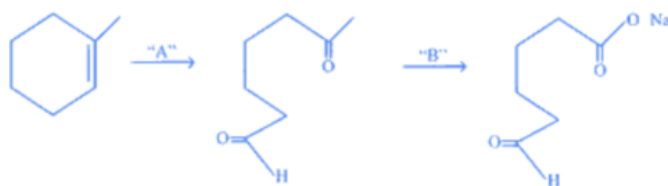
Step 3: Eliminate the alternative combinations Option (A), C and D only, omits both standard adsorbents. Option (B), A only, recognises silica gel but incorrectly excludes alumina. Option (C), B only, recognises alumina but incorrectly excludes silica gel. Only option (D), A and B only, includes both listed standard chromatographic adsorbents without substituting the two distractors. Differential adsorption on either standard stationary phase is what makes separation possible.

Step 4: Conclusion The standard adsorbents are silica gel and alumina, corresponding to statements A and B only. Hence, the correct option is (D).

Quick Tip to Solve: For the standard adsorption-chromatography question, recall the pair silica gel and alumina; identify both before selecting the combined statement option.

80. In the above chemical reaction sequence "A" and "B" respectively are

[4 marks]



- (A) $\text{H}_2\text{O}, \text{H}^+$ and $\text{NaOH}_{(\text{alc})}/\text{I}_2$
- (B) $\text{O}_3, \text{Zn}/\text{H}_2\text{O}$ and KMnO_4
- (C) $\text{H}_2\text{O}, \text{H}^+$ and KMnO_4
- (D) $\text{O}_3, \text{Zn}/\text{H}_2\text{O}$ and $\text{NaOH}_{(\text{alc})}/\text{I}_2$

Correct Answer: (D) $\text{O}_3, \text{Zn}/\text{H}_2\text{O}$ and $\text{NaOH}_{(\text{alc})}/\text{I}_2$

Solution:

Concept:

- Reductive ozonolysis, O_3 followed by $\text{Zn}/\text{H}_2\text{O}$, cleaves an alkene double bond into carbonyl groups without oxidising the aldehyde further.
- A methyl ketone, RCOCH_3 , gives the haloform reaction with iodine and alkali: $\text{RCOCH}_3 + 3\text{I}_2 + 4\text{NaOH} \longrightarrow \text{RCOONa} + \text{CHI}_3 + 3\text{NaI} + 3\text{H}_2\text{O}$.

Step 1: Recognise the first transformation The first structure is methyl-substituted cyclohexene. In the next structure the ring has opened and its two original double-bond carbons have become

carbonyl carbons. The alkene carbon carrying the methyl group becomes a ketone, whereas the alkene carbon carrying hydrogen becomes an aldehyde. These changes are diagnostic of reductive ozonolysis. Hydration by water and acid would add across the double bond to form an alcohol; it would not open the ring into the displayed ketoaldehyde.

Step 2: Identify the methyl-ketone reaction The intermediate contains a $-\text{COCH}_3$ group. In the final displayed product that group has become $-\text{COO}^- \text{Na}^+$, with loss of its methyl carbon through the haloform reaction. Iodine and sodium hydroxide perform this conversion, producing iodoform as the other organic product. The question focuses on this methyl-ketone transformation in the displayed sequence. Strong permanganate oxidation does not give this selective haloform conversion and can also oxidise the aldehyde.

Step 3: Match both reagents to one option Reagent A must therefore be $\text{O}_3, \text{Zn}/\text{H}_2\text{O}$, and reagent B must be I_2/NaOH , as written in the option. Option (A) has the correct type of second reagent but the wrong first reagent. Option (B) begins correctly but substitutes permanganate for the haloform reagent. Option (C) gets neither required transformation right. Option (D) is the only choice that fits the two diagnostic structural changes in order.

Step 4: Conclusion A is reductive ozonolysis, $\text{O}_3, \text{Zn}/\text{H}_2\text{O}$, and B is NaOH/I_2 . Hence, the correct option is (D).

Quick Tip to Solve: A ring opened into an aldehyde and a ketone signals reductive ozonolysis; a methyl ketone changing to a carboxylate signals the haloform reaction.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The maximum number of orbitals which can be identified with $n = 4$ and $m_l = 0$ is _____.

[4 marks]

Solution:

Concept:

- For a specified principal quantum number n , the azimuthal quantum number can take the integer values $l = 0, 1, \dots, n - 1$. Each value of l identifies one subshell.
- Within a subshell, $m_l = -l, -l + 1, \dots, 0, \dots, l$. One orbital is specified by the three quantum numbers (n, l, m_l) ; spin distinguishes electrons, not orbitals.

Step 1: List the allowed subshells Here the principal quantum number is fixed at $n = 4$. Hence the allowed values of the azimuthal quantum number are

$$l = 0, 1, 2, 3.$$

These correspond respectively to the $4s$, $4p$, $4d$ and $4f$ subshells. No higher value such as $l = 4$ is permitted because l must be smaller than n .

Step 2: Select the orbital with zero magnetic quantum number For $l = 0$, the only magnetic quantum

number is zero, so the $4s$ subshell contributes one orbital. For $l = 1$, the values are $-1, 0, 1$, and exactly one of them is zero. The same reasoning applies for $l = 2$ and $l = 3$: each of their allowed ranges contains zero exactly once. The required orbital specifications are thus

$$(4, 0, 0), \quad (4, 1, 0), \quad (4, 2, 0), \quad (4, 3, 0).$$

Step 3: Count orbitals rather than electrons There are four distinct triples in that list, giving four orbitals. Do not multiply the answer by two: two opposite spin values would count possible electrons occupying these orbitals. Also do not use $n^2 = 16$, which counts every orbital in the fourth shell irrespective of the imposed magnetic-quantum-number restriction. Fixing $m_l = 0$ leaves only one orbital from each of the four allowed subshells.

Step 4: Conclusion The maximum number of orbitals with $n = 4$ and $m_l = 0$ is $\boxed{4}$. This is a dimensionless count of orbitals.

Quick Tip to Solve: When n is fixed and the magnetic quantum number is zero, each allowed l contributes exactly one orbital; the answer is therefore n .

82. Number of compounds / species from the following with non-zero dipole moment is _____.



[4 marks]

Solution:

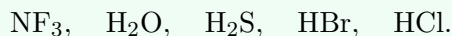
Concept:

- The molecular dipole moment is the vector sum of individual bond dipoles. Polar bonds can cancel in a symmetric molecular geometry, so bond polarity alone is insufficient.
- Bent and trigonal-pyramidal molecules generally have non-cancelling bond dipoles. A heteronuclear diatomic molecule has a non-zero bond dipole, whereas a homonuclear diatomic molecule does not.

Step 1: Remove the symmetric non-polar molecules BeCl_2 is linear with equal and opposite bond dipoles, while BCl_3 is trigonal planar with three equivalent dipoles cancelling. XeF_4 is square planar and its opposite bonds cancel. CCl_4 is a regular tetrahedral molecule with a zero vector sum. CO_2 is linear and symmetric. Finally, H_2 is homonuclear and has no permanent bond dipole in the first place. These six species have zero net dipole moment.

Step 2: Identify the polar molecules NF_3 is trigonal pyramidal. Although the bond dipoles oppose the contribution associated with its lone-pair arrangement, the cancellation is not exact, so its dipole moment is small but non-zero. H_2O and H_2S are bent molecules, so their two bond dipoles do not cancel. Both HBr and HCl are heteronuclear diatomic molecules, so each has a non-zero permanent dipole.

Step 3: Count the selected species The complete polar list is



It contains five species. Together with the six non-polar species identified above, it accounts for all eleven entries in the question. In particular, excluding NF_3 merely because its dipole is small would undercount the answer; the criterion is non-zero, not large.

Step 4: Conclusion There are 5 species with a non-zero dipole moment. This is a dimensionless count.

Quick Tip to Solve: First eliminate symmetric molecules by vector cancellation, then count bent, pyramidal and heteronuclear diatomic species. A small dipole still counts as non-zero.

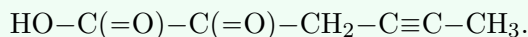
83. The total number of 'sigma' and 'pi' bonds in 2-oxohex-4-ynoic acid is _____. [4 marks]

Solution:

Concept:

- Every single bond is one sigma bond. A double bond contains one sigma and one pi bond, while a triple bond contains one sigma and two pi bonds.
- In 2-oxohex-4-ynoic acid the carboxyl carbon is carbon 1. The oxo group is a ketone at carbon 2 and the triple bond lies between carbons 4 and 5.

Step 1: Write the complete connectivity The condensed structure consistent with all parts of the name is



The acid carbon starts the six-carbon chain. Carbon 2 has an extra oxygen as a carbonyl group, carbon 3 is methylene, carbons 4 and 5 form the alkyne, and carbon 6 is methyl. This structure contains five carbon-bound hydrogens and the acidic hydroxyl hydrogen.

Step 2: Count the sigma framework The six-carbon chain has five carbon-carbon connections, each contributing one sigma bond, including the connection within the triple bond. There are three carbon-oxygen connections: two carbonyl sigma bonds and the single carbon-hydroxyl-oxygen bond. There are five carbon-hydrogen sigma bonds and one oxygen-hydrogen sigma bond. Therefore

$$N_\sigma = 5 + 3 + 5 + 1 = 14.$$

The two additional bonds of the triple bond are not added to this sigma count.

Step 3: Count pi bonds and add The two carbonyl groups contribute one pi bond each, and the carbon-carbon triple bond contributes two pi bonds. Thus

$$N_\pi = 1 + 1 + 2 = 4.$$

The question asks for the total number of sigma and pi bonds, so the required sum is

$$N_{\sigma} + N_{\pi} = 14 + 4 = 18.$$

Counting only the heavy-atom skeleton would miss the hydrogen bonds and give too small a result; counting the triple bond as three sigma bonds would also be incorrect.

Step 4: Conclusion The molecule contains 14 sigma bonds and 4 pi bonds, giving a total of 18 bonds. The requested numerical entry is 18.

Quick Tip to Solve: Draw the condensed structure first. Count one sigma per connected atom pair, include all hydrogen bonds, then add one pi per double bond and two per triple bond.

84. From 6.55 g of aniline, the maximum amount of acetanilide that can be prepared will be _____
 $\times 10^{-1}$ g. [4 marks]

Solution:

Concept:

- Acetylation converts aniline, $\text{C}_6\text{H}_5\text{NH}_2$, into acetanilide, $\text{C}_6\text{H}_5\text{NHCOCH}_3$. One mole of aniline produces one mole of acetanilide when the acetylating reagent is sufficient.
- The theoretical maximum yield follows the stoichiometric mole ratio, with no loss of material. Use molar masses $M(\text{aniline}) = 93 \text{ g mol}^{-1}$ and $M(\text{acetanilide}) = 135 \text{ g mol}^{-1}$.

Step 1: Establish the mass-to-mole conversion Aniline has molecular formula $\text{C}_6\text{H}_7\text{N}$, so its molar mass is

$$6(12) + 7(1) + 14 = 93 \text{ g mol}^{-1}.$$

Acetanilide has molecular formula $\text{C}_8\text{H}_9\text{NO}$, so its molar mass is

$$8(12) + 9(1) + 14 + 16 = 135 \text{ g mol}^{-1}.$$

The acetyl group adds mass; therefore the product mass need not be smaller than the starting mass of aniline.

Step 2: Apply the one-to-one reaction ratio For the given mass of aniline, the available amount is

$$n = \frac{6.55}{93} \text{ mol}.$$

Each molecule of aniline gives one molecule of acetanilide, so the maximum product mass is

$$m = \frac{6.55}{93} \times 135 = 9.5080645 \dots \text{ g}.$$

No percentage-yield factor is needed because the question asks for the maximum amount obtainable, with the other reactant available as required.

Step 3: Convert to the requested numerical coefficient Let the requested coefficient be x , so that

$m = x \times 10^{-1}$ g. Dividing by 0.1 g gives

$$x = \frac{9.5080645 \dots}{0.1} = 95.080645 \dots$$

The numerical entry to the nearest integer is consequently 95. The factor 10^{-1} belongs to the mass unit in the question, so entering 9.51 would give an answer ten times too small in that format.

Step 4: Conclusion The theoretical mass is approximately $9.51 \text{ g} = 95.08 \times 10^{-1} \text{ g}$. The required integer coefficient is 95.

Quick Tip to Solve: Use the 1:1 mole ratio, not a 1:1 mass ratio. Convert the calculated product mass into the question's power-of-ten format only at the end.

85. 2.7 kg of each of water and acetic acid are mixed. The freezing point of the solution will be $-x^\circ\text{C}$. Consider the acetic acid does not dimerise in water, nor dissociates in water. $x = \underline{\hspace{2cm}}$ (nearest integer)

[Given: Molar mass of water = 18 g mol^{-1} , acetic acid = 60 g mol^{-1}

$$K_f \text{H}_2\text{O} : 1.86 \text{ K kg mol}^{-1}$$

K_f acetic acid: $3.90 \text{ K kg mol}^{-1}$

freezing point: $\text{H}_2\text{O} = 273 \text{ K}$, acetic acid = 290 K]

[4 marks]

Solution:

Concept:

- For a non-electrolyte which neither associates nor dissociates, the van't Hoff factor is $i = 1$, so $\Delta T_f = K_f m$. Here m is moles of solute per kilogram of solvent.
- The question specifies acetic acid in water. Use water as the solvent and its cryoscopic constant; equal masses do not mean equal mole numbers.

Step 1: Find the amount of acetic acid The solute is acetic acid, whose molar mass is 60 g mol^{-1} . Convert its mass to grams before calculating its amount:

$$n = \frac{2700}{60} = 45 \text{ mol.}$$

The mass of water is 2.7 kg. Its molar mass is not needed for molality because the denominator is the solvent mass, not its number of moles.

Step 2: Calculate the molality and depression Use the kilograms of water directly in the denominator:

$$m = \frac{45}{2.7} = \frac{50}{3} \text{ mol kg}^{-1}.$$

There is no dissociation or dimerisation correction, as explicitly stated. Hence

$$\Delta T_f = 1 \times 1.86 \times \frac{50}{3} = 31.0 \text{ K}.$$

A temperature interval of one kelvin equals an interval of one degree Celsius.

Step 3: Find the final freezing point Water freezes at 273 K, taken as 0°C in the supplied data. Its freezing point decreases by 31 K:

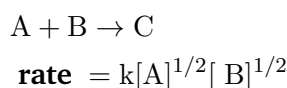
$$T_f = 273 - 31 = 242 \text{ K} = -31^\circ\text{C}.$$

The data for pure acetic acid would apply if it were designated the solvent. They are not combined with the water constant or averaged.

Step 4: Conclusion The solution freezes at -31°C . Therefore the requested nearest-integer value is $x = 31$ (dimensionless coefficient of the temperature in degrees Celsius).

Quick Tip to Solve: Identify the solvent before choosing K_f . Equal masses of the two liquids do not justify averaging their freezing points or cryoscopic constants.

86. Consider the following reaction, the rate expression of which is given below



The reaction is initiated by taking 1 M concentration of A and B each. If the rate constant (k) is $4.6 \times 10^{-2} \text{ s}^{-1}$, then the time taken for A to become 0.1 M is _____ sec. (nearest integer) [4 marks]

Solution:

Concept:

- For $\text{A} + \text{B} \rightarrow \text{C}$, equal initial concentrations and equal stoichiometric consumption imply $[\text{A}] = [\text{B}]$ throughout the reaction.
- The given law then becomes $-d[\text{A}]/dt = k[\text{A}]$. Integrate this first-order form to obtain $t = k^{-1} \ln([\text{A}]_0/[\text{A}]_t)$.

Step 1: Use the stoichiometric relation Let $z \text{ mol L}^{-1}$ of each reactant be consumed. Since both start at 1 M,

$$[\text{A}] = 1 - z, \quad [\text{B}] = 1 - z.$$

Substitution into the rate law gives

$$r = k(1 - z)^{1/2}(1 - z)^{1/2} = k(1 - z) = k[\text{A}].$$

Thus the square-root powers combine; treating either reactant concentration as constant would be inappropriate.

Step 2: Integrate the differential equation Separate variables and integrate from the initial concentra-

tion to its value at time t :

$$\int_1^{0.1} \frac{d[A]}{[A]} = -k \int_0^t dt.$$

Therefore

$$kt = \ln \frac{1}{0.1} = \ln 10 = 2.302585.$$

Although the individual partial orders are one-half, the equal-concentration trajectory obeys this first-order integral exactly.

Step 3: Substitute the rate constant and round The rate constant is $4.6 \times 10^{-2} \text{ s}^{-1} = 0.046 \text{ s}^{-1}$. Hence

$$t = \frac{2.302585}{0.046} = 50.0562 \text{ s}.$$

The logarithm contains a ratio of concentrations and is dimensionless, so dividing by s^{-1} correctly gives seconds. Round only this final value to the nearest integer.

Step 4: Conclusion The required reaction time is 50 s, so the numerical entry is 50.

Quick Tip to Solve: When reactants start equally and react in a 1:1 ratio, set their concentrations equal before integrating the rate law; the two half powers then add to one.

87. Three moles of an ideal gas are compressed isothermally from 60 L to 20 L using constant pressure of 5 atm. Heat exchange Q for the compression is - _____ Lit. atm. [4 marks]

Solution:

Concept:

- For pressure-volume work at constant external pressure, the chemistry sign convention is $w = -P_{\text{ext}}(V_f - V_i)$, with work on the gas positive.
- The internal energy of an ideal gas depends only on temperature. An isothermal process has $\Delta U = 0$, so the first law $\Delta U = Q + w$ gives $Q = -w$.

Step 1: Calculate the volume change The gas is compressed from 60 L to 20 L. Thus

$$\Delta V = V_f - V_i = 20 - 60 = -40 \text{ L}.$$

The negative sign describes a decrease in volume. The imposed external pressure is constant at 5 atm, so the work is obtained from the rectangular pressure-volume area, not from a reversible logarithmic expression.

Step 2: Find the work done on the gas Using the external pressure in the work equation,

$$w = -P_{\text{ext}}\Delta V = -(5)(-40) = +200 \text{ L atm}.$$

The positive sign is physically consistent with the surroundings compressing the system. The number of moles is not needed for this work calculation because both the pressure and volume change are already specified.

Step 3: Apply the first law The temperature remains constant and the gas is ideal. Therefore its internal energy does not change even though work is done on it:

$$0 = Q + w.$$

It follows that

$$Q = -200 \text{ L atm.}$$

Heat must leave the gas to prevent the compression work from increasing its temperature. No conversion to joules is required because the question requests litre-atmospheres. The minus sign is already printed before the answer blank.

Step 4: Conclusion The heat exchange is $Q = -200 \text{ L atm.}$ Enter 200 in the blank following the printed minus sign.

Quick Tip to Solve: Use $-P_{\text{ext}}\Delta V$ for constant external pressure. The reversible isothermal formula involving $\ln(V_f/V_i)$ does not describe this specified compression.

88. Vanillin compound obtained from vanilla beans, has total sum of oxygen atoms and π electrons is

[4 marks]

Solution:

Concept:

- Vanillin is 4-hydroxy-3-methoxybenzaldehyde. It contains a benzene ring bearing one aldehyde, one phenolic hydroxy group and one methoxy group.
- Count oxygen atoms from the functional groups, and count two π electrons for every formal double bond. Single bonds contain only σ bonding electrons.

Step 1: Identify the three oxygen-containing groups Write the substituents of vanillin as $-\text{CHO}$, $-\text{OH}$ and $-\text{OCH}_3$, attached to the same benzene ring. The aldehyde contributes one oxygen atom, the phenolic group contributes one, and the ether linkage in the methoxy group contributes one:

$$N_{\text{O}} = 1 + 1 + 1 = 3.$$

The methyl group in the methoxy substituent contains no additional oxygen.

Step 2: Count the pi electrons The benzene ring has an aromatic sextet, represented by three carbon-carbon double bonds in a Kekule structure. It therefore contributes six π electrons. The aldehyde carbonyl has one $\text{C}=\text{O}$ double bond and contributes two more:

$$N_{\pi \text{ electrons}} = 6 + 2 = 8.$$

The carbon-oxygen single bonds in the hydroxy and methoxy groups are not additional formal π bonds. Do not add their oxygen lone pairs to the requested double-bond electron count.

Step 3: Add the two requested counts The question asks for oxygen atoms plus π electrons, rather than oxygen atoms plus the number of π bonds. Thus the four formal π bonds correspond to eight

electrons, and

$$N_{\text{O}} + N_{\pi \text{ electrons}} = 3 + 8 = 11.$$

The result is a pure count and has no physical unit.

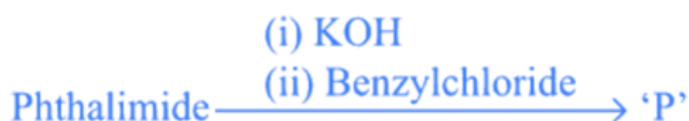
Step 4: Conclusion Vanillin contains 3 oxygen atoms and 8 π electrons. Their sum is 11 (dimensionless).

Quick Tip to Solve: For vanillin, separate the benzene sextet from the aldehyde carbonyl. Count electrons as twice the number of formal π bonds, not as the number of double bonds itself.

89. Phthalimide is made to undergo following sequence of reactions.

Total number of π bonds present in product 'P' is/are _____.

[4 marks]



Solution:

Concept:

- Potassium hydroxide removes the acidic N-H proton of phthalimide, forming potassium phthalimide. Its nitrogen anion undergoes nucleophilic substitution with a benzyl halide.
- A benzene ring has three formal π bonds and a carbonyl group has one. Benzylation replaces the nitrogen hydrogen without destroying the imide carbonyl groups.

Step 1: Form the nitrogen nucleophile Phthalimide contains an imide nitrogen bonded to hydrogen and to two carbonyl carbons. The two adjacent carbonyl groups make the N-H proton acidic. Reaction with KOH gives the potassium salt:



This acid-base step creates the nitrogen nucleophile; it does not reduce either carbonyl or alter the fused benzene ring.

Step 2: Identify product P after benzylation The nitrogen anion attacks the benzylic carbon of benzyl chloride, $\text{C}_6\text{H}_5\text{CH}_2\text{Cl}$, displacing chloride. Product P is N-benzylphthalimide, with an $\text{N-CH}_2\text{-C}_6\text{H}_5$ substituent. Only base treatment and benzylation are shown. No subsequent imide hydrolysis is specified, so P retains the entire phthalimide skeleton rather than becoming free benzylamine.

Step 3: Count all pi bonds in P The benzene ring originally present in phthalimide contributes three π bonds. Its two imide carbonyl groups contribute two more. The newly attached benzyl group has a second benzene ring, contributing another three:

$$N_{\pi \text{ bonds}} = 3 + 2 + 3 = 8.$$

The benzylic CH_2 and the new nitrogen-carbon bond are saturated single bonds and add no π bond.

Here the question asks for bonds, so do not double the count to obtain electrons.

Step 4: Conclusion Product P is N-benzylphthalimide and contains 8 π bonds (a dimensionless count).

Quick Tip to Solve: Stop the reaction sequence at the reagents actually shown: benzylation retains both imide carbonyls. Count the two benzene rings and the two carbonyl π bonds separately.

90. A first row transition metal with highest enthalpy of atomisation, upon reaction with oxygen at high temperature forms oxides of formula M_2O_n (where $n = 3, 4, 5$). The 'spin-only' magnetic moment value of the amphoteric oxide from the above oxides is _____ BM (near integer)

(Given atomic number: Sc : 21, Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26, Co : 27, Ni : 28, Cu : 29, Zn : 30)

[4 marks]

Solution:

Concept:

- Vanadium has the highest enthalpy of atomisation among the first-row transition metals. The listed oxides are V_2O_3 , V_2O_4 and V_2O_5 .
- The spin-only magnetic moment is $\mu = \sqrt{u(u+2)}$ BM, where u is the number of unpaired d electrons of the metal ion.

Step 1: Identify the metal and its oxidation states The first-row metal selected by the atomisation criterion is vanadium, $Z = 23$, whose neutral configuration is $[Ar]3d^34s^2$. In an oxide M_2O_n , oxygen has oxidation state -2 . Charge neutrality gives

$$2z - 2n = 0 \Rightarrow z = n.$$

Thus the three oxides correspond to vanadium in oxidation states $+3$, $+4$ and $+5$, respectively.

Step 2: Select the amphoteric oxide The relevant amphoteric highest oxide is V_2O_5 . It can react with acids as well as bases, forming vanadium(V) oxo species or vanadates. Its weak basic behaviour must not be confused with the strongly basic nature of a low-oxidation-state oxide. The oxidation state of each vanadium atom in this oxide is $+5$, irrespective of the two metal atoms per formula unit.

Step 3: Count unpaired electrons and calculate the moment Formation of V^{5+} removes the two $4s$ electrons and all three $3d$ electrons:



There are consequently no unpaired d electrons, so $u = 0$. Substitute this electron count into the spin-only expression:

$$\mu = \sqrt{0(0+2)} = 0 \text{ BM}.$$

The index $n = 5$ in the oxide formula is an oxygen subscript, not the number of unpaired electrons. The near-integer instruction leaves this zero unchanged.

Step 4: Conclusion The amphoteric oxide is V_2O_5 , with vanadium(V) in a d^0 configuration. The requested spin-only magnetic moment is 0 BM; enter 0.

Quick Tip to Solve: Determine the oxidation state before counting d electrons. In V_2O_5 , vanadium loses all five valence electrons and is d^0 , so its spin-only moment vanishes.