JEE Main 2024 Paper 1 (B.E./B.Tech) Apr 5 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 5 April 2024, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let the circle $C_1 : x^2 + y^2 - 2(x + y) + 1 = 0$ and C_2 be a circle having centre at $(-1, 0)$ and radius 2 . If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is: [4 marks]

(A) 4

(B) 6

(C) 2

(D) 1

Correct Answer: (C) 2

Solution:

Concept:

- The line of the common chord of two intersecting circles is their radical axis. Subtract their equations with equal coefficients of $x^2 + y^2$ to obtain this line.
- A circle $x^2 + y^2 - 2hx - 2ky + c = 0$ has centre (h, k). The squared distance from (h, k) to (u, v) is $(u - h)^2 + (v - k)^2$.

Step 1: Write both circle equations The first circle is already in general form. Completing squares gives $(x - 1)^2 + (y - 1)^2 = 1$, so its centre is $O_1 = (1, 1)$. The second circle has centre $(-1, 0)$ and radius 2, hence its equation is

$$(x + 1)^2 + y^2 = 4 \quad \Longrightarrow \quad x^2 + y^2 + 2x - 3 = 0.$$

Keep the radius squared as 4; using the radius itself here would produce the wrong chord.

Step 2: Find the common chord and its intercept Subtract the second equation from the first. The quadratic terms cancel because both equations have the same leading coefficients:

$$-4x - 2y + 4 = 0 \quad \Longrightarrow \quad 2x + y - 2 = 0.$$

Every common point satisfies this line, which is therefore the line of the common chord. On the y-axis, $x = 0$, giving $P = (0, 2)$.

Step 3: Calculate the requested squared distance Use the centre of C_1, not that of C_2:

$$O_1P^2 = (0 - 1)^2 + (2 - 1)^2 = 1 + 1 = 2.$$

The problem asks for the square, so no square root is needed. Options $4, 6, 1$ do not equal this coordinate-distance calculation.

Step 4: Conclusion The square of the distance is $\boxed{2}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a common-chord intercept, subtract the circle equations first; then impose the relevant axis coordinate before calculating distance.

2. Let $f, g : \mathbf{R} \to \mathbf{R}$ be defined as :

$$f(x) = |x - 1| \text{ and } g(x) = \begin{cases} e^x, & x \geq 0 \\ x + 1, & x \leq 0. \end{cases}$$

Then the function $f(g(x))$ is [4 marks]

(A) neither one-one nor onto.

(B) one-one but not onto.

(C) both one-one and onto.

(D) onto but not one-one.

Correct Answer: (A) neither one-one nor onto.

Solution:

Concept:

- Composition means substituting $g(x)$ into f: here $(f \circ g)(x) = |g(x) - 1|$. Evaluate the absolute value separately on each domain interval.
- A function is one-one if distinct inputs have distinct outputs. It is onto its stated codomain $\mathbf{R}$ only if every real number occurs as an output.

Step 1: Simplify the composite on both branches For $x \geq 0$, $g(x) = e^x \geq 1$, so the expression inside the absolute value is nonnegative. For $x \leq 0$, $g(x) = x + 1$, so the expression inside it is $x \leq 0$. Thus

$$(f \circ g)(x) = \begin{cases} e^x - 1, & x \geq 0, \\ -x, & x \leq 0. \end{cases}$$

At $x = 0$, both formulae give zero, so the overlapping endpoint definitions are consistent.

Step 2: Test whether it is one-one Choose one input on each side of zero. The distinct real numbers -1 and $\log 2$ give

$$(f \circ g)(-1) = 1, \qquad (f \circ g)(\log 2) = 2 - 1 = 1.$$

This explicit repeated output disproves injectivity. Monotonicity of each separate branch does not make the entire function one-one.

Step 3: Compare its range with its codomain Both branches are nonnegative and attain every value in $[0, \infty)$. For example, any $y \geq 0$ is obtained by choosing $x = -y$. Consequently the range is exactly $[0, \infty)$, which omits every negative real number in the codomain $\mathbf{R}$. The function is therefore not onto. Option B incorrectly claims injectivity, option C incorrectly claims both properties, and option D incorrectly claims surjectivity.

Step 4: Conclusion The composite is neither one-one nor onto $\mathbf{R}$. Hence, the correct option is (A).

Quick Tip to Solve: For an absolute-value composite with codomain all real numbers, check nonnegativity for onto and find equal outputs from its two branches for one-one.

3. Let ABCD and AEFG be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies : [4 marks]

(A) $r = 1$

(B) $2r^2 - 4r + 1 = 0$

(C) $2r^2 - 8r + 7 = 0$

(D) $r^2 - 8r + 8 = 0$

Correct Answer: (D) $r^2 - 8r + 8 = 0$

Solution:

Concept:

- A coordinate system may be chosen along the sides of a square without changing distances. For a circle tangent to two perpendicular sides, its centre is at distance r from each side.
- A point lies on a circle precisely when its squared distance from the centre equals r^2. This converts the tangency geometry into an algebraic equation.

Step 1: Place the squares in coordinates Take $A = (0,0), B = (4,0), C = (4,4), D = (0,4)$. Since $AE = 2$ and E lies on AB, $E = (2,0)$. The smaller square has its next vertex F on the diagonal AC, whose equation is $y = x$. Therefore $F = (2,2)$; the orientation below AB would not place F on that diagonal segment.

Step 2: Express the centre using tangency The sides BC and CD lie on $x = 4$ and $y = 4$, respectively. A circle touching both segments on the interior side has centre

$$O = (4 - r, 4 - r).$$

Indeed its perpendicular distances from both lines are r. A centre beyond either line would place one of the tangency feet beyond the corresponding segment, so it does not give the required configuration.

Step 3: Impose passage through the smaller square vertex Because the circle passes through $F = (2,2)$,

$$OF^2 = (4 - r - 2)^2 + (4 - r - 2)^2 = r^2.$$

Thus

$$2(2 - r)^2 = r^2 \quad \Longrightarrow \quad r^2 - 8r + 8 = 0.$$

The admissible root is $r = 4 - 2\sqrt{2}$, which lies between zero and four; the other root places the feet outside the side segments. Substitution shows $r = 1$ does not satisfy the distance condition. Options B and C give different polynomial relations, so neither expresses the required radius.

Step 4: Conclusion The radius satisfies $\boxed{r^2 - 8r + 8 = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: For a circle tangent to adjacent sides of a square, write its centre as the common corner minus r in each coordinate, then use the given point.

4. Let (α, β, γ) be the image of the point $(8, 5, 7)$ in the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}$. Then $\alpha + \beta + \gamma$ is equal to : [4 marks]

(A) 16

(B) 20

(C) 18

(D) 14

Correct Answer: (D) 14

Solution:

Concept:

- The reflected point Q has the perpendicular foot H as the midpoint of PQ. Hence its position vector is $Q = 2H - P$. The joining segment is perpendicular to the reflecting line.
- For a line $\mathbf{r} = \mathbf{a} + t\mathbf{d}$, the perpendicular foot satisfies $(P - \mathbf{a} - t\mathbf{d}) \cdot \mathbf{d} = 0$, giving $t = ((P - \mathbf{a}) \cdot \mathbf{d})/(\mathbf{d} \cdot \mathbf{d})$.

Step 1: Parameterise the reflecting line The given symmetric equations yield

$$H = (1 + 2t, -1 + 3t, 2 + 5t).$$

Thus a point on the line is $A = (1, -1, 2)$, its direction vector is $\mathbf{d} = (2, 3, 5)$, and the given point is $P = (8, 5, 7)$. The vector from A to P is $(7, 6, 5)$.

Step 2: Find the perpendicular foot Impose the perpendicularity condition using the dot product:

$$t = \frac{(7, 6, 5) \cdot (2, 3, 5)}{2^2 + 3^2 + 5^2} = \frac{14 + 18 + 25}{4 + 9 + 25} = \frac{57}{38} = \frac{3}{2}.$$

Therefore

$$H = \left(4, \frac{7}{2}, \frac{19}{2}\right).$$

As a check, $P - H = (4, 3/2, -5/2)$ has dot product $8 + 9/2 - 25/2 = 0$ with the line direction.

Step 3: Reflect through the foot and add the coordinates The perpendicular foot is the midpoint of the original point and its image, so

$$(\alpha, \beta, \gamma) = 2H - P = (8, 7, 19) - (8, 5, 7) = (0, 2, 12).$$

Consequently $\alpha + \beta + \gamma = 0 + 2 + 12 = 14$. Taking the coordinates of the foot itself would not give the image. The values $16, 20, 18$ in the other options differ from the sum of the reflected coordinates and are therefore excluded.

Step 4: Conclusion $\boxed{\alpha + \beta + \gamma = 14}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the perpendicular foot first, then double its position vector and subtract the original point to obtain reflection in a line.

5. Let $S_1 = \{z \in \mathbf{C} : |z| \le 5\}, S_2 = \left\{z \in \mathbf{C} : \text{Im}\left(\frac{z+1-\sqrt{3}i}{1-\sqrt{3}i}\right) \ge 0\right\}$ and $S_3 = \{z \in \mathbf{C} : \text{Re}(z) \ge 0\}$. Then the area of the region $S_1 \cap S_2 \cap S_3$ is : [4 marks]

(A) $\frac{125\pi}{24}$

(B) $\frac{125\pi}{6}$

(C) $\frac{125\pi}{12}$

(D) $\frac{125\pi}{4}$

Correct Answer: (C) $\frac{125\pi}{12}$

Solution:

Concept:

- Write $z = x + iy$. The modulus condition $|z| \le 5$ describes the closed disk $x^2 + y^2 \le 25$, and $\text{Re}\, z \ge 0$ restricts it to the right half-plane.
- To interpret an imaginary-part inequality, rationalise the complex denominator. A disk cut by rays through its centre has sector area $\frac{1}{2}R^2\theta$, with θ in radians.

Step 1: Convert the complex inequality into Cartesian form Use the identity

$$\frac{z+1-\sqrt{3}i}{1-\sqrt{3}i} = 1 + \frac{z}{1-\sqrt{3}i}.$$

The added real number 1 contributes no imaginary part. Multiplying by the conjugate gives

$$\frac{x+iy}{1-\sqrt{3}i} = \frac{(x+iy)(1+\sqrt{3}i)}{4} = \frac{x-\sqrt{3}y+i(\sqrt{3}x+y)}{4}.$$

Hence S_2 is the half-plane $y + \sqrt{3}x \ge 0$, with boundary passing through the origin.

Step 2: Find the angular width of the intersection In the right half-plane, the polar angle obeys $-\pi/2 \le \theta \le \pi/2$. The line $y = -\sqrt{3}x$ has its right-hand ray at angle $-\pi/3$. The inequality keeps points above this ray, so the intersection within the disk is the sector

$$-\frac{\pi}{3} \le \theta \le \frac{\pi}{2}.$$

Its angle is $\pi/2 + \pi/3 = 5\pi/6$. Boundary lines have zero area and do not affect this calculation.

Step 3: Evaluate the sector area and compare options With radius 5 and angular width $5\pi/6$,

$$\text{Area} = \frac{1}{2}(5)^2\frac{5\pi}{6} = \frac{125\pi}{12}.$$

Option A gives half this sector area. Option B is twice the result and exceeds the area of the right semicircle. Option D even exceeds the full disk area 25π, so neither can describe the required intersection.

Step 4: Conclusion The area is $\boxed{125\pi/12}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Before rationalising a complex fraction, separate any real constant; then translate the remaining imaginary-part inequality into a line through the origin.

6. If the constant term in the expansion of $\left(\frac{\sqrt[5]{3}}{x}+\frac{2x}{\sqrt[3]{5}}\right)^{12}, x \neq 0$, is $\alpha \times 2^8 \times \sqrt[5]{3}$, then 25α is equal to : [4 marks]

(A) 724

(B) 742

(C) 693

(D) 639

Correct Answer: (C) 693

Solution:

Concept:

- For $(a+b)^{12}$, the general term is $T_{k+1}=\binom{12}{k}a^{12-k}b^k$, where k runs from zero to twelve.
- A constant term is independent of x, so its total exponent of x must be zero. Fractional powers of numerical constants must be retained when matching its coefficient.

Step 1: Identify the term with zero power of the variable Here $a=3^{1/5}/x$ and $b=2x/5^{1/3}$. Therefore

$$T_{k+1}=\binom{12}{k}3^{(12-k)/5}2^k5^{-k/3}x^{-(12-k)+k}.$$

The exponent of x is $2k-12$. Setting it equal to zero gives $k=6$, so the constant term is the seventh term. No other value of k gives zero exponent.

Step 2: Simplify the constant term exactly Substitute $k=6$, using $\binom{12}{6}=924$:

$$T_7=924\,3^{6/5}\,2^6\,5^{-2}=\frac{924\cdot 3\cdot 2^6}{25}\,3^{1/5}.$$

The factor $3^{6/5}$ is $3\sqrt[5]{3}$, whereas the denominator $(\sqrt[3]{5})^6$ is 25. These simplifications keep the two different root indices distinct.

Step 3: Match the specified form and select the value The question writes this term as $\alpha\,2^8\sqrt[5]{3}$. Cancelling the common fifth root and dividing by 2^8,

$$\alpha=\frac{924\cdot 3}{25}\,2^{6-8}=\frac{924\cdot 3}{100}=\frac{693}{25}.$$

Thus $25\alpha=693$. This exact coefficient comparison rules out 724, 742, and 639; none equals $924\cdot 3/4$.

Step 4: Conclusion $\boxed{25\alpha=693}$. Hence, the correct option is (C).

Quick Tip to Solve: First set the exponent of x to zero; only afterwards simplify the radical factors and compare the numerical coefficient with the requested form.

7. Let $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}, \vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{c}$ be three vectors such that $(\vec{c} + \hat{i}) \times (\vec{a} + \vec{b} + \hat{i}) = \vec{a} \times (\vec{c} + \hat{i})$. If $\vec{a} \cdot \vec{c} = -29$, then $\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k})$ is equal to: [4 marks]

(A) 15

(B) 10

(C) 5

(D) 12

Correct Answer: (C) 5

Solution:

Concept:

- The cross product is distributive and anti-commutative: $\mathbf{a} \times \mathbf{u} = -\mathbf{u} \times \mathbf{a}$. A zero cross product with a nonzero vector implies parallel vectors.
- Once a vector is expressed as a scalar multiple of a known direction, a given dot product determines that scalar. The requested dot product can then be calculated component by component.

Step 1: Combine the cross products with consistent order Put $\mathbf{u} = \mathbf{c} + \hat{i}$. The given equation becomes

$$\mathbf{u} \times (\mathbf{a} + \mathbf{b} + \hat{i}) = \mathbf{a} \times \mathbf{u} = -\mathbf{u} \times \mathbf{a}.$$

Bringing the last term to the left and using distributivity gives

$$\mathbf{u} \times (2\mathbf{a} + \mathbf{b} + \hat{i}) = \mathbf{0}.$$

The sign change is essential: adding just $\mathbf{b}$ would lose one copy of $\mathbf{a}$.

Step 2: Express the unknown vector using parallelism Calculate the known vector:

$$2\mathbf{a} + \mathbf{b} + \hat{i} = (4, 10, -2) + (2, -2, 2) + (1, 0, 0) = (7, 8, 0).$$

It is nonzero, so $\mathbf{u} = \lambda(7, 8, 0)$, and therefore

$$\mathbf{c} = (7\lambda - 1, 8\lambda, 0).$$

The subtraction of $\hat{i}$ must be retained when applying the dot-product condition.

Step 3: Use the given dot product and evaluate the target Using $\mathbf{a} = (2, 5, -1)$,

$$\mathbf{a} \cdot \mathbf{c} = 2(7\lambda - 1) + 5(8\lambda) = 54\lambda - 2 = -29.$$

Hence $\lambda = -1/2$ and $\mathbf{c} = (-9/2, -4, 0)$. The required result is

$$\mathbf{c} \cdot (-2, 1, 1) = (-9/2)(-2) + (-4)(1) + 0(1) = 9 - 4 = 5.$$

The other proposed values $15, 10, 12$ fail this final component calculation; in particular the negative second component reduces the contribution 9 rather than increasing it.

Step 4: Conclusion The requested dot product is $\boxed{5}$. Hence, the correct option is (C).

Quick Tip to Solve: When cross products place the unknown on opposite sides, reverse one product using a minus sign before collecting the known vectors.

8. Let $\mathrm{A}(-1,1)$ and $\mathrm{B}(2,3)$ be two points and P be a variable point above the line AB such that the area of $\triangle\mathrm{PAB}$ is 10. If the locus of P is $\mathrm{a}x+\mathrm{by}=15$, then $5\mathrm{a}+2\,\mathrm{b}$ is : [4 marks]

(A) $-\frac{12}{5}$

(B) $-\frac{6}{5}$

(C) 6

(D) 4

Correct Answer: (A) $-\frac{12}{5}$

Solution:

Concept:

- The area of a triangle is half the product of its fixed base and perpendicular height. Consequently a fixed nonzero area gives two lines parallel to the base.
- For points $A(x_1,y_1), B(x_2,y_2), P(x,y)$, twice the signed area is $(x_2-x_1)(y-y_1)-(y_2-y_1)(x-x_1)$. Its sign identifies the side of the directed base.

Step 1: Write the fixed-area equation Here the displacement from A to B is $(3,2)$. Therefore the determinant for the triangle gives

$$2\,\mathrm{Area}(PAB)=|3(y-1)-2(x+1)|=|3y-2x-5|.$$

Since the area is 10, the absolute value must be 20. This gives the two possible parallel lines $3y-2x-5=20$ and $3y-2x-5=-20$.

Step 2: Choose the line above the base The base line is $y=(2x+5)/3$. Above it, $3y-2x-5>0$, so only the positive-area branch is permitted. Thus

$$3y-2x=25.$$

Multiplying this equation by $3/5$ puts its right-hand side into the stipulated value 15:

$$-\frac{6}{5}x+\frac{9}{5}y=15.$$

Hence $a=-6/5$ and $b=9/5$.

Step 3: Evaluate the requested combination Substitute these coefficients only after normalising the line equation:

$$5a+2b=5\left(-\frac{6}{5}\right)+2\left(\frac{9}{5}\right)=-6+\frac{18}{5}=-\frac{12}{5}.$$

The other three options do not equal this value. In particular, $-6/5$ is just the coefficient a, not the requested linear combination.

Step 4: Conclusion The required value is $\boxed{-12/5}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the side condition before normalising the line. The equation $ax+by=15$ fixes the scale of both coefficients.

9. The values of m, n, for which the system of equations

$$x + y + z = 4,$$
$$2x + 5y + 5z = 17,$$
$$x + 2y + \mathrm{m}z = \mathrm{n}$$

has infinitely many solutions, satisfy the equation : [4 marks]

(A) $\mathrm{m}^2 + \mathrm{n}^2 - \mathrm{m} - \mathrm{n} = 46$

(B) $\mathrm{m}^2 + \mathrm{n}^2 + \mathrm{mn} = 68$

(C) $\mathrm{m}^2 + \mathrm{n}^2 - \mathrm{mn} = 39$

(D) $\mathrm{m}^2 + \mathrm{n}^2 + \mathrm{m} + \mathrm{n} = 64$

Correct Answer: (C) $\mathrm{m}^2 + \mathrm{n}^2 - \mathrm{mn} = 39$

Solution:

Concept:

- A system has infinitely many solutions when its coefficient rank equals its augmented rank and both are smaller than the number of unknowns.
- Elimination is especially useful when two columns have identical coefficients in the first two equations: it exposes a free variable and the exact condition needed for the third equation.

Step 1: Eliminate x using the first two equations Subtract twice the first equation from the second:

$$(2x + 5y + 5z) - 2(x + y + z) = 17 - 8.$$

Thus $3y + 3z = 9$, so $y + z = 3$. Substituting this into the first equation gives $x = 1$. The first two equations are independent, but they leave one parameter free: we can write $z = t$, $y = 3 - t$, and $x = 1$.

Step 2: Impose dependence of the third equation Insert this family into $x + 2y + mz = n$:

$$1 + 2(3 - t) + mt = n, \qquad 7 + (m - 2)t = n.$$

For infinitely many values of t to work, its coefficient must vanish and the constants must agree. Consequently

$$m = 2, \qquad n = 7.$$

If $m \neq 2$, the third equation fixes one value of t, giving a unique solution. If $m = 2$ but $n \neq 7$, it gives a contradiction.

Step 3: Check the four proposed relations For $(m, n) = (2, 7)$, the left-hand sides in order are

$$m^2 + n^2 - m - n = 44,$$
$$m^2 + n^2 + mn = 67,$$
$$m^2 + n^2 - mn = 39,$$
$$m^2 + n^2 + m + n = 62.$$

The first, second and fourth do not equal their proposed right-hand sides 46, 68 and 64. The third is exactly 39, so it is the required relation.

Step 4: Conclusion The parameters are $m = 2, n = 7$, and they satisfy $\boxed{m^2 + n^2 - mn = 39}$. Hence, the correct option is (C).

Quick Tip to Solve: Parameterise the solutions of the first two equations; infinite consistency means the third equation must hold for every parameter value.

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10. If $y(\theta) = \frac{2\cos\theta + \cos 2\theta}{\cos 3\theta + 4\cos 2\theta + 5\cos\theta + 2}$, then at $\theta = \frac{\pi}{2}$, $y'' + y' + y$ is equal to : [4 marks]

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- Multiple-angle identities turn an expression involving $\cos\theta$, $\cos 2\theta$ and $\cos 3\theta$ into a rational function of $c = \cos\theta$.
- Cancel a common factor before differentiating, provided that factor is nonzero in a neighbourhood of the evaluation point. Then apply the chain rule to the simpler expression.

Step 1: Factor the denominator Use $\cos 2\theta = 2c^2 - 1$ and $\cos 3\theta = 4c^3 - 3c$. The numerator is $2c^2 + 2c - 1$, while the denominator becomes

$$4c^3 + 8c^2 + 2c - 2 = 2(c + 1)(2c^2 + 2c - 1).$$

At $\theta = \pi/2$, we have $c = 0$; the common factor is -1, not zero. It is therefore legitimate to cancel it locally, obtaining

$$y = \frac{1}{2(1 + \cos\theta)}.$$

Step 2: Differentiate the simplified expression Writing $s = \sin\theta$ and $c = \cos\theta$, the first derivative is

$$y' = \frac{s}{2(1 + c)^2}.$$

The product and chain rules then give

$$y'' = \frac{c}{2(1 + c)^2} + \frac{s^2}{(1 + c)^3}.$$

The second term is positive because differentiation of $(1 + c)^{-2}$ introduces the two negative signs from its exponent and from $c' = -s$.

Step 3: Evaluate all three terms At $\theta = \pi/2$, $s = 1$ and $c = 0$. Hence

$$y = \frac{1}{2}, \qquad y' = \frac{1}{2}, \qquad y'' = 1.$$

Adding gives $1 + 1/2 + 1/2 = 2$. The values $1/2$, 1 and $3/2$ omit at least one required contribution; none equals the full sum.

Step 4: Conclusion At the specified angle, $\boxed{y'' + y' + y = 2}$. Hence, the correct option is (D).

Quick Tip to Solve: Factor the trigonometric quotient in $\cos\theta$ first; the cancellation avoids differentiating a long quotient twice.

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11. Let $\beta(\mathrm{m},\mathrm{n}) = \int_0^1 x^{\mathrm{m}-1}(1-x)^{\mathrm{n}-1}\,\mathrm{d}x,\ \mathrm{m},\mathrm{n} > 0$**. If** $\int_0^1 \left(1-x^{10}\right)^{20}\,\mathrm{d}x = \mathrm{a} \times \beta(\mathrm{b},\mathrm{c})$**, then** $100(\mathrm{a}+\mathrm{b}+\mathrm{c})$ **equals _____.** [4 marks]

(A) 2012

(B) 1021

(C) 1120

(D) 2120

Correct Answer: (D) 2120

Solution:

Concept:

- The definition $\beta(m,n) = \int_0^1 t^{m-1}(1-t)^{n-1}\,dt$ identifies an integral by the powers of its two factors.
- When the integrand contains $1 - x^{10}$, set $t = x^{10}$. The differential contributes a power of t and a constant factor; both must be retained.

Step 1: Transform the variable and endpoints Let $t = x^{10}$, so that $x = t^{1/10}$. Differentiation gives

$$dx = \frac{1}{10}t^{1/10-1}\,dt = \frac{1}{10}t^{-9/10}\,dt.$$

The original endpoints $x = 0, 1$ become $t = 0, 1$, respectively. Since $x \mapsto x^{10}$ is increasing on this interval, the order of the limits is unchanged. Therefore

$$\int_0^1 (1-x^{10})^{20}\,dx = \frac{1}{10}\int_0^1 t^{-9/10}(1-t)^{20}\,dt.$$

Step 2: Match the exponents to the given definition Compare the transformed integral with the defining powers t^{b-1} and $(1-t)^{c-1}$. We obtain

$$b - 1 = -\frac{9}{10}, \qquad c - 1 = 20.$$

Thus $b = 1/10$, $c = 21$, and the factor outside the integral is $a = 1/10$. Both beta parameters are positive, so this use of the definition meets the condition in the question. No special-function identity or evaluation is needed.

Step 3: Compute the requested expression The required combination is

$$100(a+b+c) = 100\left(\frac{1}{10}+\frac{1}{10}+21\right) = 100\cdot\frac{106}{5} = 2120.$$

In particular the exponent 20 corresponds to the parameter 21, not 20. Losing this shift, or losing the differential factor $1/10$, gives a different value. Options 2012, 1021 and 1120 do not equal the correctly transformed expression.

Step 4: Conclusion The substitution gives $a = b = 1/10$ and $c = 21$, hence $\boxed{100(a+b+c) = 2120}$. Hence, the correct option is (D).

Quick Tip to Solve: In a beta integral, add one to each displayed exponent to find its parameter, and keep the substitution factor outside.

12. Let ,$f : [-1, 2] \to \mathbf{R}$ be given by $f(x) = 2x^2 + x + [x^2] - [x]$, where $[t]$ denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is : [4 marks]

(A) 5

(B) 6

(C) 4

(D) 3

Correct Answer: (C) 4

Solution:

Concept:

- The polynomial part $2x^2 + x$ is continuous everywhere. Discontinuities can therefore arise only from jumps in $\lfloor x^2 \rfloor - \lfloor x \rfloor$.
- At an interior point compare both one-sided limits with the function value. At an endpoint of the stated domain, continuity uses only the one-sided limit within that domain. Simultaneous jumps can cancel.

Step 1: List all possible jump points On $[-1, 2]$, $\lfloor x \rfloor$ can change at $-1, 0, 1, 2$. The function $\lfloor x^2 \rfloor$ can change when x^2 is a nonnegative integer. The distinct candidates in the domain are therefore

$$-1, \quad 0, \quad 1, \quad \sqrt{2}, \quad \sqrt{3}, \quad 2.$$

There are no other candidates because between consecutive such points both floor terms are constant.

Step 2: Check simultaneous and endpoint changes At $x = -1$, the value of the floor difference is $1 - (-1) = 2$, while its right-hand limit is $0 - (-1) = 1$; this endpoint is discontinuous. At $x = 0$, the left-hand floor difference is 1, whereas its value and right-hand limit are 0, so it is discontinuous. At $x = 1$, both floors jump up by 1, leaving their difference unchanged at 0. Thus f is continuous there.

Step 3: Count the remaining genuine jumps At $\sqrt{2}$ and $\sqrt{3}$, $\lfloor x^2 \rfloor$ increases by 1 while $\lfloor x \rfloor = 1$ remains unchanged. Both are discontinuities. At $x = 2$, the left-hand difference is $3 - 1 = 2$ and its value is

$4 - 2 = 2$, so the right endpoint is continuous in its domain. The discontinuity set is

$$\{-1, 0, \sqrt{2}, \sqrt{3}\}.$$

Counting all candidates would give 6, and ignoring endpoint -1 would give 3; neither is valid. The count is 4, also excluding 5.

Step 4: Conclusion There are exactly $\boxed{4}$ discontinuity points. Hence, the correct option is (C).

Quick Tip to Solve: For a difference of floor functions, test whether jumps cancel; also check continuity at both domain endpoints.

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13. Let the set $S = \{2, 4, 8, 16, \ldots, 512\}$ be partitioned into 3 sets A, B, C with equal number of elements such that $\mathrm{A} \cup \mathrm{B} \cup \mathrm{C} = \mathrm{S}$ and $\mathrm{A} \cap \mathrm{B} = \mathrm{B} \cap \mathrm{C} = \mathrm{A} \cap \mathrm{C} = \phi$. The maximum number of such possible partitions of S is equal to: [4 marks]

(A) 1640

(B) 1520

(C) 1710

(D) 1680

Correct Answer: (D) 1680

Solution:

Concept:

- The elements $2, 4, 8, \ldots, 512$ are the nine distinct powers $2^1, 2^2, \ldots, 2^9$. Equal sizes of three disjoint sets therefore mean three elements per set.
- The multinomial coefficient $9!/(3!3!3!)$ counts assignments of nine distinct elements to three specified sets, with three elements in each. Order inside an individual set does not matter.

Step 1: Identify the objects and group sizes Since $512 = 2^9$, the displayed set has 9 elements, not 8. The union condition puts every element into one of A, B, C, and the pairwise empty intersections prevent an element from occurring in two sets. Equal cardinalities then imply

$$|A| = |B| = |C| = \frac{9}{3} = 3.$$

The actual magnitudes of these powers play no further role; only their being distinct matters.

Step 2: Choose the contents of the named sets Choose any 3 of the 9 elements for A, then 3 of the remaining 6 for B. The last 3 necessarily form C. This gives

$$\binom{9}{3}\binom{6}{3}\binom{3}{3} = 84 \cdot 20 \cdot 1 = 1680.$$

There is no ordering of the selected elements inside a set, which is why each selection uses combinations rather than permutations.

Step 3: Distinguish named assignments from unlabelled groups The three sets are specified as

A, B, C, so exchanging the contents of A and B gives another assignment of those named sets. The requested maximum number is accordingly the labelled count. If the groups were unlabelled, one would divide by $3!$, obtaining 280, but that is a different convention and is not one of the choices. The three numbers 1640, 1520 and 1710 also differ from the exact product 1680.

Step 4: Conclusion The maximum number of assignments to A, B, C is $\boxed{1680}$. Hence, the correct option is (D).

Quick Tip to Solve: Choose the named groups successively. Divide by the number of group permutations only when the groups themselves are unlabelled.

14. The coefficients $\mathrm{a}, \mathrm{b}, \mathrm{c}$ in the quadratic equation $\mathrm{a}x^2 + \mathrm{b}x + \mathrm{c} = 0$ are from the set $\{1, 2, 3, 4, 5, 6\}$. If the probability of this equation having one real root bigger than the other is p, then 216p equals :

[4 marks]

(A) 38

(B) 76

(C) 57

(D) 19

Correct Answer: (A) 38

Solution:

Concept:

- A real quadratic has two unequal real roots exactly when its discriminant $D = b^2 - 4ac$ is strictly positive. Equality gives a repeated root and must be excluded.
- Each coefficient is chosen from six values. Thus there are $6^3 = 216$ equally likely ordered triples (a, b, c); the order of a and c matters in counting, even though their product is symmetric.

Step 1: Translate the root condition into a product bound The leading coefficient a is always nonzero, so every choice gives a quadratic. Having one real root bigger than the other means that the roots are real and distinct:

$$b^2 - 4ac > 0 \quad \Longleftrightarrow \quad ac < \frac{b^2}{4}.$$

We count favourable ordered pairs (a, c) separately for each possible b, keeping the inequality strict.

Step 2: Count pairs for each middle coefficient For $b = 1, 2$, the bound allows no pair because $ac \geq 1$. For $b = 3$, the allowed products are at most 2, giving counts 2 and 1 for $a = 1, 2$, hence 3 pairs. For $b = 4$, products must be below 4: the counts for $a = 1, 2, 3$ are 3,1,1, hence 5 pairs. For $b = 5$, products can be at most 6, giving

$$6 + 3 + 2 + 1 + 1 + 1 = 14.$$

For $b = 6$, products must be strictly below 9, so the counts for $a = 1, \ldots, 6$ are

$$6 + 4 + 2 + 2 + 1 + 1 = 16.$$

Step 3: Convert the favourable count into the requested value The total favourable count is

$$N = 0 + 0 + 3 + 5 + 14 + 16 = 38.$$

Hence $p = 38/216$, and multiplying by 216 recovers the count itself. No division by 2 is appropriate, since (a, c) and (c, a) are distinct coefficient choices. Thus 19 undercounts, 76 double-counts, and 57 is not the discriminant count.

Step 4: Conclusion The probability is $p = 38/216 = 19/108$, so $\boxed{216p = 38}$. Hence, the correct option is (A).

Quick Tip to Solve: For two distinct real roots use a strict discriminant inequality, and count ordered coefficient pairs for each fixed value of b.

15. Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of A, then $\det(AB)$ is equal to : [4 marks]

(A) 64

(B) 343

(C) 125

(D) 216

Correct Answer: (D) 216

Solution:

Concept:

- The cofactor at position (i, j) is $(-1)^{i+j}$ times the minor obtained by deleting row i and column j. Corresponding entries of the cofactor matrix therefore give equations in the unknown parameters.
- For a square matrix of order three, $\det(\text{adj } A) = (\det A)^2$. The cofactor matrix is the transpose of the adjoint, and transposition leaves its determinant unchanged.

Step 1: Use two convenient cofactors The first and third entries of the first row give particularly simple equations. As $\alpha \neq 0$, division by α is legitimate. Their signs are both positive.

$$C_{11} = 2\alpha^2 - \alpha\beta = 3\alpha \quad \Rightarrow \quad 2\alpha - \beta = 3.$$

$$C_{13} = \alpha^2 + \alpha\beta = 3\alpha \quad \Rightarrow \quad \alpha + \beta = 3.$$

Adding these equations yields $3\alpha = 6$, so $\alpha = 2$ and $\beta = 1$. The middle cofactor checks the signs: $C_{12} = -(2\alpha^2 + \beta^2) = -9$, as given.

Step 2: Expand the determinant along the first row Use the entries of A and their corresponding cofactors from B, rather than multiplying the two full matrices. This gives

$$\det A = \beta(3\alpha) + \alpha(-9) + 3(3\alpha).$$

$$\det A = 1(6) + 2(-9) + 3(6) = 6.$$

Although the adjoint is the transpose of the cofactor matrix, this distinction does not change the determinant required here.

Step 3: Apply the product determinant rule The determinant of a product is the product of the determinants. Hence

$$\det(AB) = \det A \det B = (\det A)^3 = 6^3 = 216.$$

The other choices, 64, 343, and 125, would require $\det A$ to be 4, 7, and 5, respectively; the direct expansion rules each of them out.

Step 4: Conclusion $\det(AB) = 216$. Hence, the correct option is (D).

Quick Tip to Solve: When a cofactor matrix is supplied, recover parameters from its simplest minors and expand the determinant using those cofactors; full matrix multiplication is unnecessary.

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16. For $x \geqslant 0$, the least value of K, for which $4^{1+x} + 4^{1-x}, \frac{\mathrm{K}}{2}, 16^x + 16^{-x}$ are three consecutive terms of an A.P., is equal to : [4 marks]

(A) 10

(B) 4

(C) 8

(D) 16

Correct Answer: (A) 10

Solution:

Concept:

- Three consecutive terms a, b, c of an arithmetic progression satisfy $2b = a + c$, even when their common difference is zero.
- For positive u, $u + u^{-1} \geq 2$, with equality exactly at $u = 1$. Also $u^2 + u^{-2} = (u + u^{-1})^2 - 2$.

Step 1: Translate the A.P. condition The middle term is $K/2$, so twice this term is K. Apply the A.P. identity before attempting to minimise:

$$K = 4^{1+x} + 4^{1-x} + 16^x + 16^{-x}.$$

The two terms with base 4 contribute $4(4^x + 4^{-x})$. The two terms with base 16 are squares of 4^x and 4^{-x}, respectively.

Step 2: Reduce to one bounded variable Put $t = 4^x + 4^{-x}$. Since 4^x is positive, the reciprocal-sum inequality gives $t \geq 2$. Then

$$16^x + 16^{-x} = t^2 - 2, \qquad K = t^2 + 4t - 2.$$

Compare this expression with its value at $t = 2$:

$$K - 10 = t^2 + 4t - 12 = (t - 2)(t + 6) \geq 0.$$

Both factors are nonnegative on the permitted domain, so this establishes a global lower bound without a calculus argument.

Step 3: Check attainment and the listed values Equality requires $t = 2$, which means $4^x = 1$, or $x = 0$. This value is allowed because $x \geq 0$. At $x = 0$, the actual three terms are

$$8, \quad K/2, \quad 2.$$

They form an A.P. for $K/2 = 5$, with common difference -3. Thus the bound 10 is attained. The values 4 and 8 lie below the bound, while 16 is larger than an attainable value and cannot be the least value.

Step 4: Conclusion The least value is $K = 10$, attained at $x = 0$. Hence, the correct option is (A).

Quick Tip to Solve: For expressions containing both $a^x + a^{-x}$ and $a^{2x} + a^{-2x}$, use the reciprocal sum as a single variable and remember to check equality.

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17. The area enclosed between the curves $y = x|x|$ and $y = x - |x|$ is : [4 marks]

(A) $\frac{8}{3}$

(B) $\frac{2}{3}$

(C) $\frac{4}{3}$

(D) 1

Correct Answer: (C) $\frac{4}{3}$

Solution:

Concept:

- Resolve each absolute value separately on $x \geq 0$ and $x < 0$; intersections and vertical ordering can change at zero.
- The bounded area between graphs is the integral of upper curve minus lower curve between consecutive intersection points. The result is nonnegative and is measured in square units.

Step 1: Find the intersections on each side of zero When $x \geq 0$, the two curves are $y = x^2$ and $y = 0$. They meet only at $x = 0$, and these branches do not enclose another bounded region. When $x < 0$, they become

$$y = -x^2, \qquad y = 2x.$$

Their intersections satisfy

$$-x^2 = 2x \quad \Rightarrow \quad x(x + 2) = 0.$$

The endpoints of the enclosed region are consequently $x = -2$ and $x = 0$, with corresponding points $(-2, -4)$ and $(0, 0)$.

Step 2: Determine the upper curve For $-2 < x < 0$, compute the difference rather than relying on a sketch:

$$(-x^2) - 2x = -x(x+2) > 0.$$

Here $-x > 0$ and $x + 2 > 0$, so the parabola $y = -x^2$ lies above the straight line $y = 2x$. The vertical strip therefore has height $-x^2 - 2x$. Using the positive-side formula x^2 over this interval would give the wrong region.

Step 3: Integrate the strip height The area is

$$\mathcal{A} = \int_{-2}^{0} (-x^2 - 2x)\,\mathrm{d}x.$$

$$\mathcal{A} = \left[-\frac{x^3}{3} - x^2\right]_{-2}^{0} = 0 - \left(\frac{8}{3} - 4\right) = \frac{4}{3}.$$

There is only one bounded lobe, so there is no extra symmetry factor. The choices $8/3$, $2/3$, and 1 do not equal the definite integral; in particular, doubling this value would incorrectly count a second lobe on the positive side.

Step 4: Conclusion The enclosed area is $\frac{4}{3}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Split a modulus-area question at zero first, and check whether each side actually contains a bounded region before applying any symmetry factor.

18. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50^{th} word is: [4 marks]

(A) OBBJH

(B) HBBJO

(C) OBBHJ

(D) JBBOH

Correct Answer: (A) OBBJH

Solution:

Concept:

- Dictionary order uses the alphabet order $B < H < J < O$. Count complete blocks with an earlier first letter before considering later positions.
- For arrangements of n letters with a letter repeated r times, the number of distinct words is $n!/r!$. The two copies of B are indistinguishable.

Step 1: Count the blocks before words beginning with O There are $5!/2! = 60$ distinct words in total, consistent with the question. A word beginning with B leaves the four distinct letters B, H, J, O, giving $4! = 24$ words. A word beginning with H leaves B, B, J, O, giving $4!/2! = 12$ words. The same count

applies to words beginning with J. Thus

$$24 + 12 + 12 = 48$$

words appear before the block beginning with O. The required 50th word is therefore the second word in the O block, not the second word of the entire dictionary list.

Step 2: Order the first words of the O block After fixing O, arrange the remaining letters B, B, H, J in increasing dictionary order. The smallest remaining letter is B, and the smallest possible next letter is the other B. Only H and J then remain. Their increasing and decreasing arrangements give

$$49\text{th}: \ \text{OBBHJ}, \qquad 50\text{th}: \ \text{OBBJH}.$$

There is no additional word between these two, since the two-letter suffix has exactly two permutations.

Step 3: Check the alternative words Option (C), OBBHJ, is the 49th word, immediately before the required word. Option (B), HBBJO, is in the H block and is in fact the first word there, at position 25. Option (D), JBBOH, is in the J block, which occupies positions 37 through 48, so it cannot have rank 50. These block checks independently distinguish the selected option from every distractor.

Step 4: Conclusion The 50th word is OBBJH. Hence, the correct option is (A).

Quick Tip to Solve: For a dictionary rank, subtract whole prefix blocks first, dividing by the factorial of each remaining repeated-letter count; only then list the last few suffixes.

19. Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} = \vec{b} \times \vec{c}$. If $\alpha \in \left[0, \frac{\pi}{3}\right]$ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then the minimum value of $27|\vec{c} - \vec{a}|^2$ is equal to: [4 marks]

(A) 124

(B) 110

(C) 121

(D) 105

Correct Answer: (A) 124

Solution:

Concept:

- A cross product is perpendicular to both of its factors. From $\vec{a} = \vec{b} \times \vec{c}$, it follows that $\vec{a} \cdot \vec{c} = 0$.
- The magnitude identity is $|\vec{b} \times \vec{c}| = |\vec{b}||\vec{c}| \sin \alpha$. On $[0, \pi/3]$, sine increases and has maximum $\sqrt{3}/2$.

Step 1: Express the magnitude of c using the angle Take magnitudes in the given cross-product equation. With $|\vec{a}| = 2$ and $|\vec{b}| = 3$, we obtain

$$2 = 3|\vec{c}| \sin \alpha, \qquad |\vec{c}| = \frac{2}{3 \sin \alpha}.$$

The endpoint $\alpha = 0$ cannot occur for any finite vector $\vec{c}$, since it would make the cross product zero.

Thus the admissible angles are effectively $0 < \alpha \leq \pi/3$.

Step 2: Use perpendicularity in the required expression Expand the square of the difference with the dot product. The cross term vanishes because $\vec{a}$ is perpendicular to $\vec{c}$:

$$|\vec{c} - \vec{a}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a} = |\vec{c}|^2 + 4.$$

Therefore

$$27|\vec{c} - \vec{a}|^2 = 108 + \frac{12}{\sin^2 \alpha}.$$

Using the angle between $\vec{b}$ and $\vec{c}$ as the angle between $\vec{a}$ and $\vec{c}$ would incorrectly retain a cross term.

Step 3: Minimise by maximising sine Since $\sin^2 \alpha \leq 3/4$, its reciprocal is at least $4/3$. Hence

$$108 + \frac{12}{\sin^2 \alpha} \geq 108 + 16 = 124.$$

Equality is attained when $\alpha = \pi/3$ and $|\vec{c}| = 4/(3\sqrt{3})$; these values satisfy the original magnitude relation. The other proposed values 110, 121 and 105 are all below this proven lower bound, so none is possible.

Step 4: Conclusion The minimum value is 124, attained at $\alpha = \pi/3$. Hence, the correct option is (A).

Quick Tip to Solve: Before expanding the square of a vector difference, look for a cross-product relation that makes its dot-product term zero.

20. The differential equation of the family of circles passing through the origin and having centre at the line $y = x$ is : [4 marks]

(A) $\left(x^2 - y^2 + 2xy\right) dx = \left(x^2 - y^2 + 2xy\right) dy$

(B) $\left(x^2 + y^2 - 2xy\right) dx = \left(x^2 + y^2 + 2xy\right) dy$

(C) $\left(x^2 + y^2 + 2xy\right) dx = \left(x^2 + y^2 - 2xy\right) dy$

(D) $\left(x^2 - y^2 + 2xy\right) dx = \left(x^2 - y^2 - 2xy\right) dy$

Correct Answer: (D) $\left(x^2 - y^2 + 2xy\right) dx = \left(x^2 - y^2 - 2xy\right) dy$

Solution:

Concept:

- A circle with centre (h, k) passing through the origin has radius squared $h^2 + k^2$. A centre on $y = x$ can be parameterised as (a, a).
- A one-parameter family produces a first-order differential equation by differentiation followed by elimination of the arbitrary parameter. Use total differentials to compare directly with the printed options.

Step 1: Write the one-parameter circle equation The centre is (a, a), and passing through $(0, 0)$ makes the squared radius $2a^2$. Thus

$$(x - a)^2 + (y - a)^2 = 2a^2.$$

Expanding and cancelling the constant terms gives

$$x^2 + y^2 = 2a(x + y).$$

Step 2: Differentiate and eliminate the parameter Treat a as constant while differentiating the family equation:

$$2x\,\mathrm{d}x + 2y\,\mathrm{d}y = 2a(\mathrm{d}x + \mathrm{d}y).$$

Eliminate $2a$ by cross multiplication with the original equation:

$$2(x + y)(x\,\mathrm{d}x + y\,\mathrm{d}y) = (x^2 + y^2)(\mathrm{d}x + \mathrm{d}y).$$

Step 3: Collect the differential coefficients The coefficient of $\mathrm{d}x$ on the left after subtraction is $2x(x+y) - (x^2+y^2) = x^2 - y^2 + 2xy$. The coefficient of $\mathrm{d}y$ is $2y(x+y) - (x^2+y^2) = -x^2 + y^2 + 2xy$. Moving the latter to the other side yields

$$(x^2 - y^2 + 2xy)\,\mathrm{d}x = (x^2 - y^2 - 2xy)\,\mathrm{d}y.$$

Option (A) has the wrong sign of the mixed term on the right, while (B) and (C) have $+y^2$ in both coefficients and fail this elimination.

Step 4: Conclusion The required equation is $(x^2 - y^2 + 2xy)\,\mathrm{d}x = (x^2 - y^2 - 2xy)\,\mathrm{d}y$. Hence, the correct option is (D).

Quick Tip to Solve: Parameterise a geometric constraint on the centre before differentiating; cross multiplication during elimination preserves a clean equation through special points.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $y = y(x)$ be the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2x}{(1+x^2)^2}y = xe^{\frac{1}{(1+x^2)}}; y(0) = 0.$$

Then the area enclosed by the curve $f(x) = y(x)e^{-\frac{1}{(1+x^2)}}$ and the line $y - x = 4$ is _____. [4 marks]

Solution:

Concept:

- For $y\prime + P(x)y = Q(x)$, the integrating factor is $\exp(\int P(x)\,\mathrm{d}x)$, and multiplication turns the left side into a product derivative.
- The area between a line and a curve is the integral of their positive vertical difference between intersection points. Solve the differential equation first to identify the actual curve.

Step 1: Find and apply the integrating factor Here $P(x) = 2x/(1+x^2)^2$. Since the derivative of

$(1+x^2)^{-1}$ is $-2x/(1+x^2)^2$, an integrating factor is

$$\mu(x) = \exp\left(-\frac{1}{1+x^2}\right).$$

Multiplication of the given equation by $\mu(x)$ cancels the exponential on the right and gives

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\mathrm{e}^{-1/(1+x^2)}\right) = x.$$

Thus the quantity called $f(x)$ in the question is exactly the product whose derivative is x.

Step 2: Use the initial condition and intersect the line Integrating yields $f(x) = x^2/2 + C$. The condition $y(0) = 0$ implies $f(0) = 0$, so $C = 0$. Consequently the required curve is

$$f(x) = \frac{x^2}{2}.$$

The line is $y = x + 4$. Equating their ordinates gives

$$\frac{x^2}{2} = x + 4 \quad \Rightarrow \quad (x-4)(x+2) = 0.$$

The intersection abscissae are -2 and 4.

Step 3: Integrate the positive vertical difference Between the two intersections, the line lies above the parabola: its ordinate exceeds the curve by $4 + x - x^2/2$, which is positive inside the interval. Therefore

$$\mathcal{A} = \int_{-2}^{4}\left(4 + x - \frac{x^2}{2}\right)\,\mathrm{d}x.$$

$$\mathcal{A} = \left[4x + \frac{x^2}{2} - \frac{x^3}{6}\right]_{-2}^{4} = \frac{40}{3} - \left(-\frac{14}{3}\right) = 18.$$

Step 4: Conclusion The enclosed area is 18 square units, so the required numerical value is 18.

Quick Tip to Solve: When the curve is defined as the unknown function times an exponential, compare that exponential with the integrating factor before finding the function separately.

22. Let the mean and the standard deviation of the probability distribution

X	α	**1**	**0**	**−3**
P(X)	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ, respectively. If $\sigma - \mu = 2$, then $\sigma + \mu$ is equal to _____. [4 marks]

Solution:

Concept:

- The probabilities of all distinct values of a discrete random variable sum to one. Its mean is $\mu = \sum xp(x)$.
- The variance is $\sigma^2 = E(X^2) - \mu^2$, where the standard deviation σ is nonnegative. The table lists distinct support values.

Step 1: Normalize the probabilities The unknown probability is found before calculating either moment. Thus

$$K = 1 - \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{4}\right) = \frac{1}{4}.$$

The total probability assigned to the two nonzero fixed entries is therefore one half.

Step 2: Calculate the first two moments Using the values in the table gives

$$\mu = \frac{\alpha}{3} + \frac{1}{4} - \frac{3}{4} = \frac{\alpha}{3} - \frac{1}{2}, \qquad E(X^2) = \frac{\alpha^2}{3} + \frac{1}{4} + \frac{9}{4} = \frac{\alpha^2}{3} + \frac{5}{2}.$$

Hence

$$\sigma^2 = \frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4}.$$

Step 3: Use the specified difference The equation $\sigma - \mu = 2$ gives $\sigma = \alpha/3 + 3/2$. Squaring and comparing with the variance yields

$$\frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4} = \frac{\alpha^2}{9} + \alpha + \frac{9}{4} \quad \Longrightarrow \quad \alpha(\alpha - 6) = 0.$$

The value $\alpha = 0$ would repeat the already listed support value zero with a different entry for its probability; it is excluded for the given probability-distribution table. Thus $\alpha = 6$.

Step 4: Evaluate the requested sum For this admissible value,

$$\mu = 2 - \frac{1}{2} = \frac{3}{2}, \qquad \sigma = \mu + 2 = \frac{7}{2}.$$

Both values satisfy the nonnegative-standard-deviation requirement, and their difference is exactly two.

Step 5: Conclusion The required dimensionless value is $\sigma + \mu = \frac{7}{2} + \frac{3}{2} = \boxed{5}$.

Quick Tip to Solve: Normalize the table first, and retain the requirement that its listed support values are distinct when solving for an unknown support point.

23. Let the maximum and minimum values of $\left(\sqrt{8x-x^2-12}-4\right)^2+(x-7)^2, x \in \mathbf{R}$ be M and m, respectively. Then $\mathrm{M}^2-\mathrm{m}^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The radical requires $8x-x^2-12 \geq 0$, or $2 \leq x \leq 6$. Put $u=x-4$ and $y=\sqrt{4-u^2}$, so $u^2+y^2=4$ and $y \geq 0$.
- Cauchy–Schwarz bounds $3u+4y$ by $\sqrt{3^2+4^2}\sqrt{u^2+y^2}=10$. The restriction to the upper semicircle must also be used for the other extreme.

Step 1: Rewrite the expression The expression is the squared distance from (x, y) to $(7, 4)$. On substituting $x=4+u$, it becomes

$$F=(y-4)^2+(u-3)^2=u^2+y^2-6u-8y+25=29-2(3u+4y).$$

This form reduces the problem to bounding one linear expression.

Step 2: Find the minimum value The largest possible value of $3u+4y$ is ten. Equality holds when (u, y) is proportional to $(3, 4)$, specifically $(u, y)=(6/5, 8/5)$. This point has $y>0$, so it lies on the required upper semicircle and is attainable. Therefore

$$m=29-2(10)=9.$$

Step 3: Find the maximum value Since $u \geq -2$ and $y \geq 0$, $3u+4y \geq -6$. Equality is possible at $(u, y)=(-2, 0)$, corresponding to $x=2$. Hence

$$M=29-2(-6)=41.$$

The lower-circle point opposite $(3, 4)$ is inadmissible because the radical cannot be negative.

Step 4: Calculate the difference of squares Using these actual extremes rather than unconstrained full-circle extremes gives

$$M^2-m^2=41^2-9^2=(41-9)(41+9)=32 \times 50=1600.$$

The endpoint $x=6$ gives seventeen and does not change either extreme.

Step 5: Conclusion The requested dimensionless value is $\boxed{1600}$.

Quick Tip to Solve: Interpret a square root of a circle equation as the upper semicircle; the full-circle farthest point may be outside the domain.

24. Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$. Then $(\alpha - \beta)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For skew lines with direction vectors $\mathbf{d}_1, \mathbf{d}_2$, their shortest joining segment is perpendicular to both direction vectors.
- Write a point on each line using separate parameters. Dot products of their joining vector with both directions determine the feet of the common perpendicular.

Step 1: Parameterize the two lines Take

$$A = (-2 - 3u,\ 2 + 4u,\ 5 + 2u), \qquad B = (-2 - v,\ -6 + 2v,\ 1).$$

The zero denominator in the second symmetric equation means that its z-coordinate is fixed at one; it does not prevent a parametric representation.

Step 2: Impose perpendicularity The joining vector is

$$B - A = (3u - v,\ -8 + 2v - 4u,\ -4 - 2u).$$

Its dot products with $(-3, 4, 2)$ and $(-1, 2, 0)$ must vanish. Simplification gives

$$29u - 11v = -40, \qquad 11u - 5v = -16.$$

Multiplying the first equation by five and the second by eleven, then subtracting, yields $24u = -24$. Thus $u = -1$ and $v = 1$.

Step 3: Write the common perpendicular The feet are therefore

$$A = (1, -2, 3), \qquad B = (-3, -4, 1).$$

Their joining direction is $(-4, -2, -2)$, which may be simplified to $(-2, -1, -1)$. It is perpendicular to both original lines, since the two dot products are $6 - 4 - 2 = 0$ and $2 - 2 = 0$, respectively.

Step 4: Locate the specified point A general point on the shortest-distance line is

$$(x, y, z) = (1, -2, 3) + s(-2, -1, -1).$$

Setting $x = -1$ gives $1 - 2s = -1$, so $s = 1$. Hence $\alpha = -3$ and $\beta = 2$. The desired difference is $\alpha - \beta = -5$.

Step 5: Conclusion The required dimensionless value is $(\alpha - \beta)^2 = (-5)^2 = \boxed{25}$.

Quick Tip to Solve: Find the feet by two perpendicularity equations, then use the given coordinate to locate the required point on the joining line.

25. If $1+\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}+\frac{5-2\sqrt{6}}{18}+\frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}}+\frac{49-20\sqrt{6}}{180}+\ldots$ **upto** $\infty=2+\left(\sqrt{\frac{b}{a}}+1\right)\log_e\left(\frac{a}{b}\right)$**, where a and b are integers with** $\gcd(a,b)=1$**, then** $11a+18b$ **is equal to _____.** [4 marks]

Solution:

Concept:

- Let $r=1-\sqrt{2/3}$. The successive nonconstant terms are $r^n/[n(n+1)]$ for $n=1,2,3,\ldots$; here $0<r<1$.
- For $|r|<1$, $-\log(1-r)=\sum_{n=1}^{\infty} r^n/n$. Also $1/[n(n+1)]=1/n-1/(n+1)$, which lets the sum be expressed using that logarithm.

Step 1: Identify the common power The first term after one is $r/2$. Direct squaring and cubing give

$$r^2=\frac{5-2\sqrt{6}}{3},\qquad r^3=\frac{9\sqrt{3}-11\sqrt{2}}{3\sqrt{3}},\qquad r^4=\frac{49-20\sqrt{6}}{9}.$$

Division by six, twelve and twenty reproduces the next three displayed terms. Thus

$$S=1+\sum_{n=1}^{\infty}\frac{r^n}{n(n+1)}.$$

Step 2: Sum by partial fractions Using the convergent logarithmic series,

$$\begin{aligned}\sum_{n=1}^{\infty}\frac{r^n}{n(n+1)}&=\sum_{n=1}^{\infty}\frac{r^n}{n}-\sum_{n=1}^{\infty}\frac{r^n}{n+1}\\&=-\log(1-r)-\frac{-\log(1-r)-r}{r}\\&=1+\frac{1-r}{r}\log(1-r).\end{aligned}$$

Consequently $S=2+[(1-r)/r]\log(1-r)$.

Step 3: Compare with the prescribed form Put $c=\sqrt{2/3}$, so $1-r=c$. Rationalizing the coefficient gives

$$\frac{c}{1-c}=2+\sqrt{6}.$$

Also $\log c=\frac{1}{2}\log(2/3)$, hence

$$S=2+\left(1+\sqrt{\frac{3}{2}}\right)\log\left(\frac{2}{3}\right).$$

This is exactly the form in the question, with the same logarithmic argument and square-root ratio.

Step 4: Use the coprime integers The positive coprime integers are $a=2$, $b=3$, since $a/b=2/3$ and $b/a=3/2$. Therefore

$$11a+18b=11(2)+18(3)=22+54=76.$$

The sign of the logarithm is negative, consistent with the original sum being slightly above one.

Step 5: Conclusion The requested integer is $\boxed{76}$.

Quick Tip to Solve: When denominators follow two, six, twelve, twenty, try $n(n+1)$; splitting its reciprocal converts a power series to logarithms.

26. Let $a > 0$ be a root of the equation $2x^2 + x - 2 = 0$. If $\lim_{x \to \frac{1}{a}} \frac{16\left(1-\cos\left(2+x-2x^2\right)\right)}{(1-ax)^2} = \alpha + \beta\sqrt{17}$, where $\alpha, \beta \in Z$, then $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- The positive root is $a = (\sqrt{17} - 1)/4$. Its reciprocal is $b = 1/a = (\sqrt{17} + 1)/4$, which satisfies $2b^2 - b - 2 = 0$.
- For $g \to 0$, $(1 - \cos g)/g^2 \to 1/2$. For two functions vanishing at the same point with nonzero denominator derivative, their first-order ratio is the ratio of their derivatives.

Step 1: Verify that the inner argument vanishes Write $g(x) = 2 + x - 2x^2$. The reciprocal-root equation gives

$$g(b) = 2 + b - 2b^2 = 0.$$

The denominator factor also vanishes at $x = b$, because $1 - ab = 0$. Thus the cosine expansion applies to the inner argument at precisely the required limiting point.

Step 2: Calculate the ratio of vanishing factors The derivative of g is $g'(x) = 1-4x$. At $b = (1+\sqrt{17})/4$, it is $-\sqrt{17}$. The derivative of $1 - ax$ is $-a$, which is nonzero. Hence

$$\lim_{x \to b} \frac{g(x)}{1 - ax} = \frac{-\sqrt{17}}{-a} = \frac{\sqrt{17}}{a}.$$

Step 3: Evaluate the original limit Factor the expression into the standard trigonometric limit and the squared ratio:

$$\begin{aligned} L &= 16 \lim_{x \to b} \frac{1 - \cos g(x)}{g(x)^2} \left(\frac{g(x)}{1 - ax}\right)^2 \\ &= 16 \cdot \frac{1}{2} \cdot \frac{17}{a^2} = 136b^2. \end{aligned}$$

All factors have finite limits, and the squared ratio removes the sign of the local linear factor.

Step 4: Extract the integer coefficients The square of the reciprocal root is

$$b^2 = \frac{(1 + \sqrt{17})^2}{16} = \frac{9 + \sqrt{17}}{8}.$$

Therefore

$$L = 17(9 + \sqrt{17}) = 153 + 17\sqrt{17}.$$

Comparing with $\alpha + \beta\sqrt{17}$ gives $\alpha = 153$, $\beta = 17$.

Step 5: Conclusion The required dimensionless integer is $\alpha + \beta = 153 + 17 = \boxed{170}$.

Quick Tip to Solve: Work with the reciprocal root directly; the derivative of the quadratic argument at that point simplifies to the discriminant square root.

27. If $f(t)=\int_0^{\pi}\frac{2x\,dx}{1-\cos^2 t\sin^2 x}, 0<t<\pi$, then the value of $\int_0^{\frac{\pi}{2}}\frac{\pi^2 dt}{f(t)}$ equals _____. [4 marks]

Solution:

Concept:

- If $h(\pi-x)=h(x)$, then $\int_0^{\pi} xh(x)\,dx=(\pi/2)\int_0^{\pi}h(x)\,dx$, obtained by adding the original integral and its reflected form.
- For $0<t<\pi$, $\sin t>0$. The substitution $u=\tan x$ on $[0,\pi/2)$ evaluates the remaining rational integral without a sign ambiguity.

Step 1: Apply reflection symmetry Let $h(x)=1/(1-\cos^2 t\sin^2 x)$, holding t fixed. Since $\sin(\pi-x)=\sin x$, the symmetry identity gives

$$f(t)=2\int_0^{\pi}xh(x)\,dx=\pi\int_0^{\pi}h(x)\,dx=2\pi\int_0^{\pi/2}\frac{dx}{1-\cos^2 t\sin^2 x}.$$

The last equality again uses symmetry about $\pi/2$.

Step 2: Evaluate the reduced integral On the half interval put $u=\tan x$, so $dx=du/(1+u^2)$ and $\sin^2 x=u^2/(1+u^2)$. Then

$$\begin{aligned}\int_0^{\pi/2}\frac{dx}{1-\cos^2 t\sin^2 x}&=\int_0^{\infty}\frac{du}{1+\sin^2 t\,u^2}\\&=\frac{1}{\sin t}\left[\arctan(u\sin t)\right]_0^{\infty}\\&=\frac{\pi}{2\sin t}.\end{aligned}$$

Positivity of $\sin t$ justifies the displayed factor rather than its absolute value.

Step 3: Substitute into the outer integral It follows that

$$f(t)=2\pi\frac{\pi}{2\sin t}=\frac{\pi^2}{\sin t}.$$

Thus for $0<t\le\pi/2$, $\pi^2/f(t)=\sin t$. At zero this reciprocal expression has the limiting value zero, so the integral with the lower endpoint zero is interpreted by its convergent limiting value.

Step 4: Integrate the remaining function The required quantity is therefore

$$\lim_{\varepsilon\downarrow 0}\int_{\varepsilon}^{\pi/2}\sin t\,dt=\lim_{\varepsilon\downarrow 0}\left[-\cos t\right]_{\varepsilon}^{\pi/2}=\lim_{\varepsilon\downarrow 0}\cos\varepsilon=1.$$

There is no endpoint divergence in the reciprocal integrand.

Step 5: Conclusion The required dimensionless value is $\boxed{1}$.

Quick Tip to Solve: Remove the factor of the integration variable by reflection symmetry before attempting a tangent substitution.

28. The number of real solutions of the equation $x|x+5|+2|x+7|-2=0$ is _____. [4 marks]

Solution:

Concept:

- The expressions inside the absolute values change sign at $x=-7$ and $x=-5$. Solve separately on the intervals determined by these two breakpoints.
- Every root of a resulting quadratic must lie in the interval whose sign choices produced that quadratic. Roots outside that interval are extraneous.

Step 1: Solve on the rightmost interval For $x \geq -5$, both $x+5$ and $x+7$ are nonnegative. The equation becomes

$$x(x+5)+2(x+7)-2=0 \quad \Longrightarrow \quad x^2+7x+12=0.$$

Factoring gives $(x+3)(x+4)=0$. Both roots -3 and -4 belong to this interval, so both are retained.

Step 2: Solve between the breakpoints For $-7 \leq x < -5$, the first absolute value is $-x-5$, while the second is $x+7$. Hence

$$-x^2-5x+2x+14-2=0 \quad \Longrightarrow \quad x^2+3x-12=0.$$

Its roots are

$$x=\frac{-3\pm\sqrt{57}}{2}.$$

Since $7<\sqrt{57}<8$, the negative root lies strictly between -5.5 and -5, within the required interval; the positive root lies outside it.

Step 3: Solve on the leftmost interval For $x<-7$, both interior expressions are negative. Thus

$$x(-x-5)+2(-x-7)-2=0 \quad \Longrightarrow \quad x^2+7x+16=0.$$

Its discriminant is $49-64=-15<0$, so this interval contributes no real root. The breakpoint -7 itself was included in the middle interval and is not a root.

Step 4: Count the distinct admissible values The point -5 was included in the rightmost interval; its substituted polynomial is $25-35+12=2$, so it is not a root either. The complete solution set is

$$\left\{-4,-3,\frac{-3-\sqrt{57}}{2}\right\}.$$

These three values are distinct and each satisfies the correct sign conditions.

Step 5: Conclusion The number of real solutions is $\boxed{3}$.

Quick Tip to Solve: Assign each breakpoint to exactly one interval, and filter every quadratic root through that interval before counting.

29. The number of solutions of $\sin^2 x + \left(2 + 2x - x^2\right)\sin x - 3(x-1)^2 = 0$, where $-\pi \le x \le \pi$, is _____. [4 marks]

Solution:

Concept:

- Factor the expression as a polynomial in $s = \sin x$, using $u = (x-1)^2$. Since $-1 \le \sin x \le 1$, any factor demanding a sine outside this range is impossible.
- A strictly concave differentiable function has at most two zeros on an interval when its derivative is strictly decreasing. Sign changes then establish that those two zeros actually exist.

Step 1: Factor the equation Write $2 + 2x - x^2 = 3 - (x-1)^2 = 3 - u$. The equation becomes

$$s^2 + (3-u)s - 3u = (s+3)(s-u) = 0.$$

The factor $s + 3$ cannot vanish for a real angle, because it would require $\sin x = -3$. Thus every admissible solution must satisfy $\sin x = (x-1)^2$. This factorisation is equivalent to the original equation and introduces no extra roots.

Step 2: Restrict the possible interval The right side is nonnegative, so sine must be nonnegative. On the prescribed interval $[-\pi, \pi]$, the open negative half has negative sine and supplies no solutions. Neither $-\pi$ nor 0 satisfies the equation. Also $(x-1)^2 \le 1$ requires $0 \le x \le 2$. Hence it is enough to count zeros of

$$f(x) = \sin x - (x-1)^2, \qquad 0 \le x \le 2.$$

Step 3: Count the zeros rigorously We have $f(0) = -1 < 0$, $f(1) = \sin 1 > 0$, and $f(2) = \sin 2 - 1 < 0$. Continuity gives a root in each of $(0, 1)$ and $(1, 2)$. On this interval,

$$f''(x) = -\sin x - 2 < 0.$$

Therefore f' is strictly decreasing. If three distinct zeros existed, Rolle's theorem would give two zeros of f', contradicting strict decrease. Thus the two roots already located are all the roots.

Step 4: Conclusion The number of solutions in $[-\pi, \pi]$ is $\boxed{2}$ (a dimensionless count).

Quick Tip to Solve: After factoring, use the sine range to discard the impossible factor; sign changes plus strict concavity count roots without approximating them.

30. Let a line perpendicular to the line $2x - y = 10$ touch the parabola $y^2 = 4(x-9)$ at the point P. The distance of the point P from the centre of the circle $x^2 + y^2 - 14x - 8y + 56 = 0$ is _____. [4 marks]

Solution:

Concept:

- For two nonvertical perpendicular lines, the product of slopes is -1. The tangent slope of $y^2 = 4(x-9)$ is found by implicit differentiation.
- The centre of $x^2 + y^2 + 2gx + 2fy + c = 0$ is $(-g, -f)$. The distance between two points is the

square root of the sum of squared coordinate differences.

Step 1: Find the required tangent slope The given line can be written as $y = 2x - 10$, so its slope is 2. A line perpendicular to it has slope $-1/2$. This is the slope of the tangent at P; the intercept of the given line does not affect perpendicularity. Differentiate the parabola with respect to x:

$$2y\frac{dy}{dx} = 4, \qquad \frac{dy}{dx} = \frac{2}{y}.$$

Step 2: Locate the point of contact At the point of contact the derivative must equal the tangent's slope. Hence

$$\frac{2}{y_P} = -\frac{1}{2} \quad \Longrightarrow \quad y_P = -4.$$

Substitute this ordinate into the parabola itself:

$$(-4)^2 = 4(x_P - 9), \qquad x_P = 13.$$

Thus $P = (13, -4)$. The negative ordinate is essential: using $y = 4$ would give a positive tangent slope and would not satisfy the perpendicularity requirement.

Step 3: Find the centre and distance Complete the squares in the circle equation:

$$(x - 7)^2 + (y - 4)^2 = 9.$$

Its centre is therefore $C = (7, 4)$. Its radius is not the quantity requested. The required distance is

$$CP = \sqrt{(13 - 7)^2 + (-4 - 4)^2} = \sqrt{36 + 64} = 10.$$

The displacement from the centre to the point has components 6 and -8, confirming the familiar 6-8-10 right triangle.

Step 4: Conclusion The distance of P from the centre of the circle is $\boxed{10}$ coordinate units.

Quick Tip to Solve: Use the tangent slope to get the ordinate first, then the parabola to get the abscissa; complete squares only to identify the circle centre.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A body is moving unidirectionally under the influence of a constant power source. Its displacement in time t is proportional to : [4 marks]

(A) $t^{2/3}$

(B) $t^{3/2}$

(C) t

(D) t^2

Correct Answer: (B) $t^{3/2}$

Solution:

Concept:

- Power is the rate of doing work: $P = dK/dt$, where $K = mv^2/2$ is the translational kinetic energy of a body of fixed mass m.
- For the displacement power law intended here, take the motion from rest. Integrating velocity, rather than acceleration, then gives the displacement dependence on time.

Step 1: Express speed using constant power Let the constant supplied power be $P_0 > 0$. The work-energy theorem gives

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = P_0.$$

For motion starting from rest at $t = 0$, integration yields

$$\frac{1}{2}mv^2 = P_0 t, \qquad v = \sqrt{\frac{2P_0}{m}}\, t^{1/2}.$$

The positive root is taken because the body moves in one direction. Constant power does not mean constant force: as speed increases, $F = P_0/v$ decreases.

Step 2: Integrate the speed Put $s(0) = 0$. Since $ds/dt = v$,

$$s(t) = \sqrt{\frac{2P_0}{m}} \int_0^t \tau^{1/2}\, d\tau = \frac{2}{3}\sqrt{\frac{2P_0}{m}}\, t^{3/2}.$$

Thus it is displacement, rather than speed, that contains the exponent $3/2$. The speed itself grows only as the square root of time.

Step 3: Compare the offered exponents Option (A) would give $v \propto t^{-1/3}$ and would not represent growth of kinetic energy at a constant positive rate. Option (C) gives constant velocity, hence constant kinetic energy and zero net power. Option (D) is the displacement dependence for uniform acceleration from rest, which requires power increasing with time. Only option (B) is consistent with $K \propto t$. For a nonzero initial speed, the general relation is $v^2 = v_0^2 + 2P_0 t/m$; the pure power law is the usual from-rest interpretation of this question.

Step 4: Conclusion Under the intended from-rest interpretation, $s \propto t^{3/2}$. Hence, the correct option is (B).

Quick Tip to Solve: Constant power makes kinetic energy linear in time: take one square root for speed and then integrate once for displacement.

32. What is the dimensional formula of ab^{-1} in the equation $\left(\mathrm{P}+\frac{\mathrm{a}}{\mathrm{V}^2}\right)(\mathrm{V}-\mathrm{b})=\mathrm{RT}$, where letters have their usual meaning. [4 marks]

(A) $[\mathrm{M}^6\ \mathrm{L}^7\ \mathrm{T}^4]$

(B) $[\mathrm{M}^{-1}\ \mathrm{L}^5\ \mathrm{T}^3]$

(C) $[\mathrm{M}^0\ \mathrm{L}^3\ \mathrm{T}^{-2}]$

(D) $[\mathrm{ML}^2\ \mathrm{T}^{-2}]$

Correct Answer: (D) $[\mathrm{ML}^2\ \mathrm{T}^{-2}]$

Solution:

Concept:

- Terms that are added or subtracted in a physical equation must have identical dimensions. Thus a/V^2 has pressure dimensions and b has volume dimensions.
- Pressure is force per unit area, so $[P] = [ML^{-1}T^{-2}]$, while ordinary volume has dimensions $[V] = [L^3]$.

Step 1: Determine the dimensions of a The first bracket contains $P+a/V^2$. Dimensional homogeneity requires

$$\left[\frac{a}{V^2}\right] = [P], \qquad [a] = [P][V]^2.$$

Using pressure and volume dimensions gives

$$[a] = [ML^{-1}T^{-2}][L^6] = [ML^5T^{-2}].$$

This step uses addition inside the bracket; equating a directly to pressure would ignore the factor V^{-2}.

Step 2: Determine b and form the ratio In the second bracket, $V-b$ is a subtraction, so $[b] = [V] = [L^3]$. Therefore

$$[ab^{-1}] = \frac{[a]}{[b]} = \frac{[ML^5T^{-2}]}{[L^3]} = [ML^2T^{-2}].$$

Equivalently, a/b has dimensions $[P][V]$, the same as pressure multiplied by volume, which is an energy. This provides an independent check of the dimensional result.

Step 3: Eliminate the other options Option (A) has mass raised to the sixth power and a positive time exponent, neither of which can result from pressure times volume. Option (B) incorrectly places mass in the denominator and also has a positive time exponent. Option (C) loses the mass dimension and retains an extra length power. Option (D) alone gives one mass power, two length powers and inverse time squared, exactly as obtained from both brackets. No numerical value of the gas constant or temperature is needed for this comparison.

Step 4: Conclusion The dimensional formula is $\boxed{[ML^2T^{-2}]}$. Hence, the correct option is (D).

Quick Tip to Solve: For a van der Waals equation, match dimensions inside each bracket first; the ratio a/b then immediately has dimensions of pressure times volume.

33. A particle moves in x-y plane under the influence of a force $\vec{F}$ such that its linear momentum is $\overrightarrow{\mathrm{p}}(\mathrm{t}) = \hat{i}\cos(\mathrm{kt}) - \hat{j}\sin(\mathrm{kt})$. If k is constant, the angle between $\overrightarrow{\mathrm{F}}$ and $\overrightarrow{\mathrm{p}}$ will be : [4 marks]

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

Correct Answer: (A) $\frac{\pi}{2}$

Solution:

Concept:

- Newton's second law in momentum form is $\vec{F} = d\vec{p}/dt$. Differentiate each Cartesian component, retaining the sign from the chain rule.
- The scalar product satisfies $\vec{F} \cdot \vec{p} = |\vec{F}||\vec{p}|\cos\theta$. A zero product for nonzero vectors means they are perpendicular.

Step 1: Differentiate the momentum vector The Cartesian unit vectors are constant. Differentiate the supplied momentum with respect to time:

$$\vec{p} = \hat{i}\cos(kt) - \hat{j}\sin(kt),$$

$$\vec{F} = -k\hat{i}\sin(kt) - k\hat{j}\cos(kt).$$

The minus sign in the second force component is retained because the derivative of $-\sin(kt)$ is $-k\cos(kt)$. Mixing up this sign would give a false time-dependent angle.

Step 2: Evaluate the scalar product Using $\hat{i} \cdot \hat{j} = 0$ and unit lengths for both basis vectors,

$$\begin{aligned}\vec{F} \cdot \vec{p} &= (-k\sin kt)(\cos kt) + (-k\cos kt)(-\sin kt) \\ &= -k\sin kt\cos kt + k\cos kt\sin kt = 0.\end{aligned}$$

Also $|\vec{p}| = \sqrt{\cos^2 kt + \sin^2 kt} = 1$, and $|\vec{F}| = |k|$. For the nonzero force implied by the question, the angle is therefore fixed at $\pi/2$.

Step 3: Check the interpretation and options The magnitude of momentum stays constant even though its direction rotates. Accordingly the force changes direction rather than speed, which is consistent with its being perpendicular to momentum. The alternatives $\pi/3$, $\pi/4$ and $\pi/6$ all have positive cosines, and would require a nonzero dot product; they contradict the exact cancellation above. If $k = 0$, force is the zero vector and its angle is undefined, so that degenerate case is outside the stated angle comparison.

Step 4: Conclusion The force is perpendicular to the momentum: $\theta = \boxed{\pi/2}$ radians. Hence, the correct option is (A).

Quick Tip to Solve: When momentum has sine and cosine components, differentiate it and use a dot product; constant momentum magnitude also signals a perpendicular force.

34. Match List I with List II :

	LIST I EM-Wave		LIST II Wavelength Range
A.	**Infra-red**	**I.**	**$< 10^{-3}$ nm**
B.	**Ultraviolet**	**II.**	**400 nm to 1 nm**
C.	**X-rays**	**III.**	**1 mm to 700 nm**
D.	**Gamma rays**	**IV.**	**1 nm to 10^{-3} nm**

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(B) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(C) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(D) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Correct Answer: (B) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Solution:

Concept:

- The electromagnetic spectrum in decreasing wavelength is radio, microwave, infrared, visible, ultraviolet, X-rays and gamma rays. Frequency increases as wavelength decreases because $c = \nu\lambda$.
- The question supplies approximate wavelength bands. Match each named radiation to one listed band rather than imposing a different convention for boundaries.

Step 1: Match the long wavelength bands Infrared lies just beyond the red end of visible light and extends approximately from $700\,\text{nm}$ to $1\,\text{mm}$. Thus entry A matches range III. Ultraviolet lies beyond the violet end at shorter wavelengths than visible light; the supplied band from $400\,\text{nm}$ to $1\,\text{nm}$ is its match. Therefore B matches II. Although the ranges are written in descending order in the table, the endpoints still describe the same wavelength interval.

Step 2: Match X-rays and gamma rays X-rays have shorter wavelengths than ultraviolet, and the table assigns the interval from $1\,\text{nm}$ down to $10^{-3}\,\text{nm}$. Hence C matches IV. Gamma rays are at the shortest-wavelength end of this list, below $10^{-3}\,\text{nm}$, so D matches I. The resulting sequence is

$$\text{A} \to \text{III}, \quad \text{B} \to \text{II}, \quad \text{C} \to \text{IV}, \quad \text{D} \to \text{I}.$$

Step 3: Compare the complete matching codes The second answer choice contains precisely this sequence. Choice (A) assigns infrared to the gamma-ray band, so it fails immediately. Choice (C) assigns infrared to the X-ray band and ultraviolet to the infrared band. Choice (D) assigns ultraviolet to the shortest wavelengths and gamma rays to the longest listed band, reversing their physical order. Checking all four assignments confirms that the second choice is the unique complete match; matching only X-rays would not distinguish all the alternatives.

Step 4: Conclusion The matching is A–III, B–II, C–IV, D–I. Hence, the correct option is (B).

Quick Tip to Solve: Order the four radiations by decreasing wavelength first, then align them with the supplied bands; infrared is longest and gamma rays shortest.

35. During an adiabatic process, if the pressure of a gas is found to be proportional to the cube of its absolute temperature, then the ratio of $\frac{C_P}{C_V}$ for the gas is : [4 marks]

(A) $\frac{5}{3}$
(B) $\frac{3}{2}$
(C) $\frac{7}{5}$
(D) $\frac{9}{7}$

Correct Answer: (B) $\frac{3}{2}$

Solution:

Concept:

- For a reversible adiabatic change of an ideal gas, $PV^\gamma = \text{constant}$, where $\gamma = C_P/C_V$. The ideal-gas law is $PV = nRT$ for a fixed quantity of gas.
- Eliminate the volume to relate pressure directly to temperature. If two nonconstant quantities follow power laws, equate their exponents only after deriving that relationship.

Step 1: Eliminate the volume From the ideal-gas law, $V = nRT/P$. Substitute this expression into the adiabatic relation:

$$P\left(\frac{nRT}{P}\right)^\gamma = \text{constant}.$$

Since n and R are fixed, their powers can be absorbed into the constant. Therefore

$$T^\gamma P^{1-\gamma} = \text{constant}, \qquad P^{\gamma-1} \propto T^\gamma.$$

This relation connects absolute temperature to pressure for the same adiabatic path.

Step 2: Use the given pressure law For an ideal gas with $\gamma > 1$, take the power $1/(\gamma - 1)$:

$$P \propto T^{\gamma/(\gamma-1)}.$$

The question states $P \propto T^3$, so

$$\frac{\gamma}{\gamma - 1} = 3 \quad \Longrightarrow \quad \gamma = 3\gamma - 3 \quad \Longrightarrow \quad \gamma = \frac{3}{2}.$$

The result is dimensionless because it is a ratio of two heat capacities measured in the same units.

Step 3: Check the choices by substitution For choice (A), $\gamma = 5/3$ would produce an exponent $(5/3)/(2/3) = 5/2$, not 3. Choice (C), $7/5$, gives $7/2$, and choice (D), $9/7$, gives $9/2$. For choice (B), $(3/2)/(1/2) = 3$, exactly the observed exponent. The dependence uses absolute temperature; replacing it with Celsius temperature would not preserve the power law.

Step 4: Conclusion The required ratio is $\boxed{C_P/C_V = 3/2}$. Hence, the correct option is (B).

Quick Tip to Solve: For an adiabatic ideal gas, remember the direct relation P proportional to T raised to gamma/(gamma minus 1); equate that exponent to the given one.

36. A heavy box of mass $50\ \mathrm{kg}$ is moving on a horizontal surface. If co-efficient of kinetic friction between the box and horizontal surface is 0.3 then force of kinetic friction is : [4 marks]

(A) 1470 N

(B) 147 N

(C) 1.47 N

(D) 14.7 N

Correct Answer: (B) 147 N

Solution:

Concept:

- Kinetic friction opposes relative sliding and has magnitude $f_k = \mu_k N$, where μ_k is the kinetic-friction coefficient and N is the normal reaction.
- On a horizontal surface with no additional vertical force, vertical acceleration is zero, so $N = mg$. Use $g = 9.8\,\mathrm{m\,s^{-2}}$ for the stated answer choices.

Step 1: Find the normal reaction The box is moving horizontally, but it does not accelerate vertically. Its weight acts downward and the supporting reaction acts upward. Consequently,

$$N - mg = 0, \qquad N = 50 \times 9.8 = 490\,\mathrm{N}.$$

The normal reaction is the weight here; the mass itself is not a force and cannot be substituted for the reaction without multiplying by gravitational acceleration.

Step 2: Calculate kinetic friction The coefficient is dimensionless, so multiplying it by the normal force gives a force in newtons:

$$f_k = \mu_k N = 0.3 \times 490 = 147\,\mathrm{N}.$$

This friction acts opposite to the sliding velocity. Its magnitude is determined by the contact and reaction, rather than by the box speed in the simple kinetic-friction model.

Step 3: Check the alternatives Option (A), $1470\,\mathrm{N}$, is ten times too large; options (C) and (D), $1.47\,\mathrm{N}$ and $14.7\,\mathrm{N}$, are respectively one hundred and ten times too small. All are excluded by direct substitution. The coefficient being below one also requires the friction to be smaller than the $490\,\mathrm{N}$ normal force.

Step 4: Conclusion The kinetic friction has magnitude $\boxed{147\,\mathrm{N}}$, opposite to the motion. Hence, the correct option is (B).

Quick Tip to Solve: For horizontal sliding without other vertical forces, first find mg, then multiply by μ_k; do not multiply the coefficient by mass alone.

37. A man carrying a monkey on his shoulder does cycling smoothly on a circular track of radius $9\ \mathrm{m}$ and completes 120 resolutions in 3 minutes. The magnitude of centripetal acceleration of monkey is (in $\mathrm{m/s^2}$) : [4 marks]

(A) $4\pi^2\ \mathrm{ms^{-2}}$

(B) $16\pi^2\ \mathrm{ms^{-2}}$

(C) $57600\pi^2\ \mathrm{ms^{-2}}$

(D) Zero

Correct Answer: (B) $16\pi^2\ \mathrm{ms^{-2}}$

Solution:

Concept:

- For uniform circular motion the angular speed is $\omega = 2\pi f$, where f is the number of revolutions per second.
- The centripetal acceleration has magnitude $a_c = \omega^2 r = 4\pi^2 f^2 r$. Constant speed does not imply zero acceleration because velocity changes direction.

Step 1: Convert the revolution rate The monkey travels with the cyclist, so it shares the circular motion around the track. Convert the observation interval into seconds before calculating frequency:

$$t = 3 \times 60 = 180\,\mathrm{s}, \qquad f = \frac{120}{180} = \frac{2}{3}\,\mathrm{s^{-1}}.$$

Thus each revolution takes $1/f = 1.5\,\mathrm{s}$. Using three as the time without converting minutes would incorrectly inflate the frequency by a factor of sixty.

Step 2: Calculate angular speed and acceleration The angle swept in one complete turn is 2π radians. Hence

$$\omega = 2\pi \times \frac{2}{3} = \frac{4\pi}{3}\,\mathrm{rad\,s^{-1}}.$$

Using the track radius $r = 9\,\mathrm{m}$,

$$a_c = \left(\frac{4\pi}{3}\right)^2 \times 9 = 16\pi^2\,\mathrm{m\,s^{-2}}.$$

This acceleration points toward the centre at every instant, although the question asks only for its magnitude.

Step 3: Eliminate the other values Option (A) is smaller by a factor of four, and cannot follow from the given revolution frequency. Option (C) results from leaving the interval in minutes when seconds are required, introducing a factor 60^2. Option (D) confuses uniform speed with uniform velocity: the changing direction makes centripetal acceleration nonzero.

Step 4: Conclusion The monkey has centripetal acceleration $\boxed{16\pi^2\,\mathrm{m\,s^{-2}}}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert revolutions per minute to revolutions per second before squaring the frequency in $a_c = 4\pi^2 f^2 r$.

38. A series LCR circuit is subjected to an ac signal of $200\ \text{V}, 50\ \text{Hz}$. If the voltage across the inductor $(\text{L} = 10\ \text{mH})$ is $31.4\ \text{V}$, then the current in this circuit is _____. [4 marks]

(A) 10 A

(B) 10 mA

(C) 68 A

(D) 63 A

Correct Answer: (A) 10 A

Solution:

Concept:

- The inductive reactance of an ideal inductor is $X_L = \omega L = 2\pi f L$, measured in ohms.
- The inductor voltage and circuit current satisfy $V_L = I X_L$ when both are rms values or both are amplitudes. Every element of a series circuit carries the same current.

Step 1: Convert the inductance and evaluate reactance The given inductance is in millihenries, so

$$L = 10\,\text{mH} = 10 \times 10^{-3}\,\text{H} = 0.010\,\text{H}.$$

At the applied frequency $f = 50\,\text{Hz}$,

$$X_L = 2\pi \times 50 \times 0.010 = \pi\,\Omega \simeq 3.14\,\Omega.$$

The factor 10^{-3} is essential: neglecting it would make the calculated reactance one thousand times too large.

Step 2: Use the inductor voltage The voltage across the inductor is already specified, so the unknown resistance and capacitance need not be found. The series current is

$$I = \frac{V_L}{X_L} = \frac{31.4}{3.14} = 10\,\text{A}.$$

The applied $200\,\text{V}$ is the phasor resultant of the element voltages, rather than the voltage across the inductor alone. Dividing that supply voltage by X_L would therefore be unjustified.

Step 3: Test the competing answers Option (B), $10\,\text{mA}$, would produce only $0.0314\,\text{V}$ across this inductor. Options (C) and (D), $68\,\text{A}$ and $63\,\text{A}$, would give roughly $214\,\text{V}$ and $198\,\text{V}$, not $31.4\,\text{V}$. Only $10\,\text{A}$ reproduces the specified inductor voltage.

Step 4: Conclusion The current in the series circuit is $\boxed{10\,\text{A}}$. Hence, the correct option is (A).

Quick Tip to Solve: When an individual inductor voltage is given, use $I = V_L/(2\pi f L)$; the source voltage is unnecessary for this calculation.

39. Which of the following statement is not true about stopping potential (V_0) ? [4 marks]

(A) It depends upon frequency of the incident light.

(B) It is $1/e$ times the maximum kinetic energy of electrons emitted.

(C) It increases with increase in intensity of the incident light.

(D) It depends on the nature of emitter material.

Correct Answer: (C) It increases with increase in intensity of the incident light.

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ν is incident frequency and ϕ is the emitter work function.
- The stopping-potential magnitude satisfies $eV_0 = K_{\max}$, with e the positive elementary charge. It measures maximum electron energy, not the number of emitted electrons.

Step 1: Identify the dependencies of stopping potential Combining the energy and stopping relations gives

$$V_0 = \frac{h\nu - \phi}{e}.$$

For photoemission above threshold, raising the frequency raises the available energy per photon and hence the maximum photoelectron energy. Thus statement (A) is true. Different emitter materials generally have different work functions, so statement (D) is also true for the same incident frequency.

Step 2: Check the energy statement and intensity statement The definition immediately yields

$$V_0 = \frac{K_{\max}}{e},$$

so statement (B) is true. At fixed frequency, increasing light intensity increases the number of incident photons per unit time and generally the photoelectric current. It does not increase the energy $h\nu$ of each photon. Therefore it cannot increase $K_{\max}$ or V_0 in the usual photoelectric experiment.

Step 3: Select the statement that is false The question asks for the statement that is not true, so the three genuine properties must be rejected as answer choices. Frequency dependence, the relation to maximum kinetic energy, and material dependence are all present in Einstein's equation. Intensity is absent from that equation, making the asserted increase of stopping potential with intensity the false statement.

Step 4: Conclusion Stopping potential does not increase with intensity when incident frequency and emitter material are fixed. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish photon energy from photon number: frequency controls stopping potential, whereas intensity mainly controls photoelectric current.

40. Given below are two statements :

Statement I : When the white light passed through a prism, the red light bends lesser than yellow and violet.

Statement II : The refractive indices are different for different wavelengths in dispersive medium. In the light of the above statements, chose the correct answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true but Statement II is false

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are false

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- In a dispersive medium the refractive index depends on wavelength: $n = n(\lambda)$. This wavelength dependence causes white light to separate into colours.
- For ordinary glass in the visible region, normal dispersion gives $n_{\text{violet}} > n_{\text{yellow}} > n_{\text{red}}$, so red is deviated least and violet most.

Step 1: Evaluate Statement I White light contains a range of visible wavelengths. In an ordinary prism, the red component has the longest wavelength of the three colours named, and experiences the smallest refractive index. Consequently it undergoes the smallest net deviation through the prism. Yellow is deviated more than red and less than violet. Thus the ordering asserted in Statement I is correct.

Step 2: Evaluate Statement II The defining optical feature of a dispersive medium is that its refractive index changes with wavelength. Since the speed within the material is $v = c/n$, different colours have different speeds there and refract by different amounts at an interface. Statement II correctly states this wavelength dependence; a wavelength-independent refractive index would give no separation of colours by this mechanism.

Step 3: Relate the statements to the answer choices For a thin prism the deviation can be written approximately as

$$\delta \simeq (n-1)A,$$

where A is its fixed prism angle. This makes the larger deviation associated with larger refractive index explicit. Both statements are true, so options (B) and (C), each declaring one statement false, are excluded. Option (D), declaring both false, is excluded as well. The question requires their truth values rather than a separate assertion-reason classification.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: For a usual glass prism, remember the order of net deviation: red least, violet most; connect it to wavelength-dependent refractive index.

41. Match List I with List II:

	LIST I		LIST II
A.	A force that restores an elastic body of unit area to its original state	I.	Bulk modulus
B.	Two equal and opposite forces parallel to opposite faces	II.	Young's modulus
C.	Forces perpendicular everywhere to the surface per unit area same everywhere	III.	Stress
D.	Two equal and opposite forces perpendicular to opposite faces Choose the correct answer from the options given below:	IV.	Shear modulus

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(C) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(D) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

Correct Answer: (B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Solution:

Concept:

- Stress is the restoring internal force per unit area, $\sigma = F/A$. Elastic moduli relate a particular type of stress to its corresponding strain.
- Tangential forces produce shear deformation; uniform normal pressure produces volume deformation; opposite longitudinal normal forces produce extension or compression.

Step 1: Match restoring force per unit area and tangential forces List I entry A describes the restoring force for unit area of an elastic body, which is stress. Therefore A matches III. Entry B describes equal and opposite forces parallel to opposite faces. These tangential forces change the shape by shearing, so the associated elastic coefficient is the shear modulus. Therefore B matches IV. The descriptions distinguish the general quantity stress from the particular deformation associated with shear.

Step 2: Match uniform pressure and longitudinal forces Entry C specifies equal normal force per unit area acting everywhere on the surface. This is uniform pressure, whose associated volume strain defines the bulk modulus:

$$K = -\frac{\Delta p}{\Delta V/V}.$$

Hence C matches I. Entry D specifies equal and opposite forces perpendicular to opposite faces, causing longitudinal extension or compression. The relevant modulus is Young's modulus,

$$Y = \frac{F/A}{\Delta l/l},$$

so D matches II.

Step 3: Compare the complete matching sequence The required sequence is

$$\mathrm{A} \to \mathrm{III}, \quad \mathrm{B} \to \mathrm{IV}, \quad \mathrm{C} \to \mathrm{I}, \quad \mathrm{D} \to \mathrm{II}.$$

Only option (B) gives all four pairings. Option (A) misidentifies stress and shear, option (C) interchanges the stress and Young's-modulus assignments, and option (D) mismatches shear, bulk and longitudinal responses.

Step 4: Conclusion The matching is A–III, B–IV, C–I, D–II. Hence, the correct option is (B).

Quick Tip to Solve: Classify the deformation first: tangential forces mean shear, uniform all-round pressure means bulk, and longitudinal stretching means Young's modulus.

42. A satellite revolving around a planet in stationary orbit has time period 6 hours. The mass of planet is one-fourth the mass of earth. The radius orbit of planet is :
(Given = Radius of geo-stationary orbit for earth is 4.2×10^4 km) [4 marks]

(A) 1.68×10^5 km

(B) 1.4×10^4 km

(C) 8.4×10^4 km

(D) 1.05×10^4 km

Correct Answer: (D) 1.05×10^4 km

Solution:

Concept:

- For a circular satellite orbit, gravity supplies centripetal force: $GMm/r^2 = mv^2/r$. Together with $v = 2\pi r/T$, this gives $r^3 = GMT^2/(4\pi^2)$.
- Comparing two central bodies eliminates G: $(r_p/r_E)^3 = (M_p/M_E)(T_p/T_E)^2$. The relevant radius is measured from the centre of the planet.

Step 1: Set up the comparison with Earth The Earth reference is its geostationary orbit, whose period is $T_E = 24\,\text{h}$ and radius is $r_E = 4.2 \times 10^4\,\text{km}$. For the other planet,

$$\frac{M_p}{M_E} = \frac{1}{4}, \qquad \frac{T_p}{T_E} = \frac{6}{24} = \frac{1}{4}.$$

Since only a ratio of periods is used, both periods may remain in hours; converting both to seconds would cancel identically.

Step 2: Calculate the cube of the radius ratio Using the circular-orbit relation,

$$\left(\frac{r_p}{r_E}\right)^3 = \frac{1}{4}\left(\frac{1}{4}\right)^2 = \frac{1}{64}.$$

Taking the positive cube root appropriate to a physical radius,

$$\frac{r_p}{r_E} = \frac{1}{4}.$$

Both the lower planetary mass and shorter orbital period reduce the required orbit radius, consistent with this result.

Step 3: Evaluate the radius and eliminate alternatives Therefore

$$r_p = \frac{4.2 \times 10^4}{4} = 1.05 \times 10^4 \text{ km}.$$

Option (A) is four times the Earth reference and option (C) is twice that reference, both contrary to the required smaller orbit. Option (B) corresponds to a radius ratio of one third, whose cube is $1/27$, rather than $1/64$. Only option (D) satisfies the period-mass relation.

Step 4: Conclusion The orbital radius is $\boxed{1.05 \times 10^4 \text{ km}}$. Hence, the correct option is (D).

Quick Tip to Solve: When comparing satellite orbits around different bodies, use $r \propto M^{1/3}T^{2/3}$, including both the mass ratio and period ratio.

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43. A vernier callipers has 20 divisions on the vernier scale, which coincides with 19^{th} division on the main scale. The least count of the instrument is 0.1 mm. One main scale division is equal to _____ mm. [4 marks]

(A) 5

(B) 2

(C) 1

(D) 0.5

Correct Answer: (B) 2

Solution:

Concept:

- A direct vernier measures fractions of a main-scale division by comparing almost equal division lengths. Its least count is the positive difference between one main-scale division and one vernier-scale division.
- If N vernier divisions equal $(N-1)$ main-scale divisions, then $\text{LC} = \text{MSD}/N$. Here MSD denotes the length of one main-scale division, not the total span of the vernier.

Step 1: Relate the two division lengths Let one main-scale division be s mm and one vernier-scale division be v mm. The stated coincidence refers to the total lengths of 20 vernier divisions and 19 main divisions, so

$$20v = 19s, \qquad v = \frac{19}{20}s.$$

Thus a vernier division is slightly shorter than a main division; the difference is what enables the instrument to resolve a smaller length.

Step 2: Use the given least count The least count is

$$s - v = s - \frac{19s}{20} = \frac{s}{20} = 0.1 \text{ mm}.$$

Multiplying by 20 gives

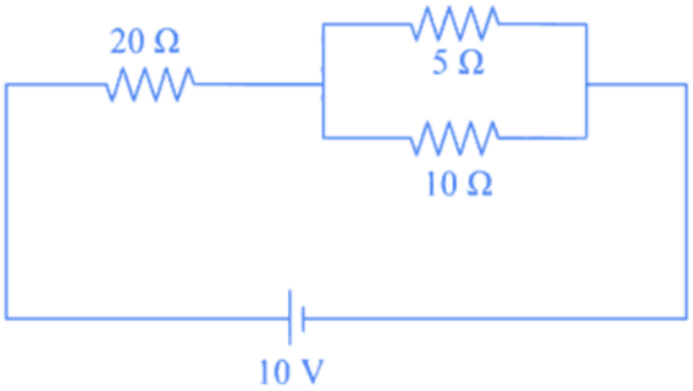
$$s = 20(0.1) = 2 \text{ mm}.$$

For a direct check, 20 vernier divisions then span 38 mm, giving one vernier division as 1.9 mm; the difference from 2 mm is exactly 0.1 mm. The proposed main divisions 5, 1 and 0.5 mm would instead give least counts 0.25, 0.05 and 0.025 mm, respectively, so none matches the data.

Step 3: Conclusion One main-scale division is $2\ \text{mm}$. Hence, the correct option is (B).

Quick Tip to Solve: For a direct vernier with 20 divisions matching 19 main divisions, multiply the least count by 20 to recover one main-scale division.

44. The ratio of heat dissipated per second through the resistance 5Ω and 10Ω in the circuit given below is: [4 marks]



(A) 4 : 1

(B) 2 : 1

(C) 1 : 2

(D) 1 : 1

Correct Answer: (B) 2 : 1

Solution:

Concept:

- The heat dissipated per second in a resistor is its electrical power: $P = VI = I^2R = V^2/R$. Choose the form that uses a quantity common to the resistors being compared.
- Resistors connected across the same two junctions are in parallel and have the same potential difference. A series resistor changes that common voltage, but not the ratio of the two branch powers.

Step 1: Identify the parallel branches The diagram connects the $5\ \Omega$ and $10\ \Omega$ resistors between the same two junctions. Denote their common voltage by V_p. The $20\ \Omega$ resistor is in series with this parallel combination, so it does not carry the same current as either individual branch. In particular, using I^2R with an identical branch current would be incorrect.

Step 2: Compare the powers at equal voltage Using $P = V_p^2/R$,

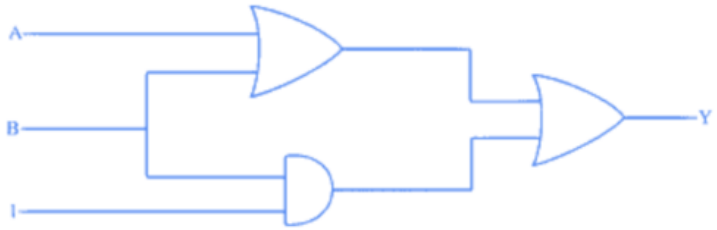
$$\frac{P_5}{P_{10}} = \frac{V_p^2/5}{V_p^2/10} = \frac{10}{5} = 2.$$

The ratio is therefore $2 : 1$. As a circuit check, the parallel equivalent is $10/3\ \Omega$, the total resistance is $70/3\ \Omega$, and the source current is $3/7$ A. The parallel voltage is then $10/7$ V, giving powers $20/49$ W and $10/49$ W. A ratio $4 : 1$ would incorrectly square the resistance ratio; $1 : 2$ reverses the equal-voltage relation, while $1 : 1$ ignores the unequal branch resistances.

Step 3: Conclusion The required heat-dissipation rate ratio is $P_5 : P_{10} = 2 : 1$. Hence, the correct option is (B).

Quick Tip to Solve: Before comparing resistor powers, identify the connection: equal voltage in parallel means power is inversely proportional to resistance.

45. The output (Y) of logic circuit given below is 0 only when : [4 marks]



(A) $\mathrm{A} = 0,\ \mathrm{B} = 0$

(B) $\mathrm{A} = 0,\ \mathrm{B} = 1$

(C) $\mathrm{A} = 1,\ \mathrm{B} = 0$

(D) $\mathrm{A} = 1, \mathrm{B} = 1$

Correct Answer: (A) $\mathrm{A} = 0,\ \mathrm{B} = 0$

Solution:

Concept:

- An OR gate produces 1 if at least one input is 1, whereas an AND gate produces 1 only if both inputs are 1. In Boolean algebra, $+$ denotes OR and a product denotes AND.
- The identities $B \cdot 1 = B$ and $B + B = B$ simplify repeated signals. Gate connections must be traced from the input wires before constructing the final truth table.

Step 1: Write the output of each first-stage gate The upper gate is an OR gate receiving inputs A and B, so its output is $A + B$. The lower gate is an AND gate receiving B and the fixed logical input 1, so its output is $B \cdot 1 = B$. The branching wire from B feeds both gates; the constant 1 is not an extra input to the upper OR gate.

Step 2: Simplify the final OR output The final gate combines those two outputs by OR:

$$Y = (A + B) + (B \cdot 1) = A + B + B = A + B.$$

Consequently, the circuit is equivalent, at its output, to a single OR gate. For input pairs $(A, B) = (0,0), (0,1), (1,0), (1,1)$, the resulting outputs are respectively $0, 1, 1, 1$. This explicitly tests every listed option: the second choice makes the B signal high, the third makes the A signal high, and the fourth makes both high. None of these three can give a zero final OR output.

Step 3: Conclusion The output is zero only for $A = 0$ and $B = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Reduce an AND input tied to logical 1 first: it simply passes the other input, often exposing a Boolean absorption or repetition identity.

46. The angular momentum of an electron in a hydrogen atom is proportional to : (Where r is the radius of orbit of electron) [4 marks]

(A) $\frac{1}{\mathrm{r}}$

(B) $\frac{1}{\sqrt{\mathrm{r}}}$

(C) $\sqrt{\mathrm{r}}$

(D) r

Correct Answer: (C) $\sqrt{\mathrm{r}}$

Solution:

Concept:

- For an electron in a circular hydrogen orbit, electrostatic attraction supplies the centripetal force: $mv^2/r = ke^2/r^2$, with $k = 1/(4\pi\varepsilon_0)$.
- The magnitude of orbital angular momentum is $L = mvr$. The electron mass and the electrostatic constants are fixed, so the radius dependence must include the change in orbital speed.

Step 1: Find how orbital speed depends on radius Equating the force magnitudes gives

$$\frac{mv^2}{r} = \frac{ke^2}{r^2}, \qquad v^2 = \frac{ke^2}{mr}.$$

Taking the positive speed,

$$v = \sqrt{\frac{ke^2}{mr}} \propto r^{-1/2}.$$

Thus a larger hydrogen orbit has a lower electron speed; treating the speed as a fixed constant would give the wrong dependence for angular momentum.

Step 2: Substitute the speed into angular momentum Using the magnitude of $m\vec{r} \times \vec{v}$, with velocity perpendicular to the orbital radius,

$$L = mrv = mr\sqrt{\frac{ke^2}{mr}} = \sqrt{mke^2 r}.$$

Hence $L^2 \propto r$, or $L \propto \sqrt{r}$. The same result follows from Bohr quantisation: $L_n = n\hbar$, while $r_n = a_0 n^2$, so eliminating n gives the square-root law. The inverse-radius and inverse-square-root options incorrectly predict decreasing angular momentum with increasing orbit size; the linear-radius

option would require constant speed and so contradicts the force equation.

Step 3: Conclusion The orbital angular momentum is proportional to $\sqrt{r}$. Hence, the correct option is (C).

Quick Tip to Solve: Combine $v \propto r^{-1/2}$ from the Coulomb force with $L = mvr$; the extra factor of radius changes the exponent to $+1/2$.

47. The vehicles carrying inflammable fluids usually have metallic chains touching the ground:

[4 marks]

(A) To protect tyres from catching dirt from ground

(B) It is a custom

(C) To alert other vehicles

(D) To conduct excess charge due to air friction to ground and prevent sparking

Correct Answer: (D) To conduct excess charge due to air friction to ground and prevent sparking

Solution:

Concept:

- A moving vehicle can accumulate static electric charge through friction with air and other contact processes. A charged conducting body develops a potential relative to Earth.
- Earthing provides a conducting path that allows excess charge to flow between the body and Earth, reducing the potential difference and the risk of a sudden electrical discharge.

Step 1: Identify the ignition hazard An inflammable liquid can produce vapour around a tanker. If the vehicle accumulates charge, its potential relative to nearby objects or the ground may become large enough for air to break down. A discharge across a small gap can then produce a spark, whose energy can ignite the vapour. Rubber tyres are poor conductors, so the tyres alone do not provide a reliable conducting path from the vehicle body to Earth.

Step 2: Explain the role of the metallic chain The chain is electrically connected to the vehicle and touches the ground, creating a conducting route for accumulated charge to leak to Earth rather than building up on the vehicle. In the simple capacitor description,

$$V = \frac{Q}{C}, \qquad U = \frac{Q^2}{2C},$$

so reducing the stored charge reduces both the vehicle potential and the energy available for a spark. The chain is therefore a protection against electrostatic discharge. It does not clean the tyres, and a customary practice or an audible warning does not explain why electrical contact with the ground is essential. These points rule out the first three options.

Step 3: Conclusion The chain conducts excess charge produced by friction to the ground and prevents sparking near inflammable vapour. Hence, the correct option is (D).

Quick Tip to Solve: When a metal chain or strip touches Earth on a fuel tanker, connect the observation to earthing: charge must be drained before a spark can ignite vapour.

48. A galvanometer of resistance 100Ω when connected in series with 400Ω measures a voltage of upto 10 V. The value of resistance required to convert the galvanometer into ammeter to read upto 10 A is $x \times 10^{-2}\Omega$. The value of x is : [4 marks]

(A) 2

(B) 20

(C) 800

(D) 200

Correct Answer: (B) 20

Solution:

Concept:

- A galvanometer reaches full-scale deflection at a fixed current I_g. Adding a series resistance makes a voltmeter, for which $V = I_g(R_g + R_s)$.
- To make an ammeter, connect a low shunt resistance S in parallel with the galvanometer. Equal parallel voltages give $I_g R_g = (I - I_g)S$, where I is the desired maximum total current.

Step 1: Determine the full-scale galvanometer current The galvanometer resistance is $R_g = 100\ \Omega$, and the series resistance used for the 10 V voltmeter is $400\ \Omega$. At full scale,

$$I_g = \frac{10}{100 + 400} = 0.020\ \text{A}.$$

The galvanometer itself therefore has a full-scale voltage of $I_g R_g = 2.0$ V; the remaining 8 V is across the series resistor.

Step 2: Calculate the parallel shunt For the 10 A ammeter, the galvanometer still carries only 0.020 A, while the shunt carries

$$I_s = 10 - 0.020 = 9.98\ \text{A}.$$

Thus

$$S = \frac{I_g R_g}{I_s} = \frac{2}{9.98} = 0.2004\ \Omega \simeq 20 \times 10^{-2}\ \Omega.$$

Therefore $x \simeq 20$ at the precision of the options. Choosing 2 would give a shunt ten times too small, while 200 and 800 give shunts much larger than needed and fail to bypass the required current. The shunt is parallel, not the 400-ohm series resistor used in the earlier voltmeter conversion.

Step 3: Conclusion The required shunt is approximately $0.20\ \Omega$, giving $x = 20$ to the stated option precision. Hence, the correct option is (B).

Quick Tip to Solve: First infer I_g from the voltmeter range; then keep that same current through the galvanometer and send the remaining ammeter current through a parallel shunt.

49. The electrostatic force $\left(\vec{F}_1\right)$ and magnetic force $\left(\vec{F}_2\right)$ acting on a charge q moving with velocity v can be written : [4 marks]

(A) $\vec{F}_1 = q\vec{B}, \vec{F}_2 = q(\vec{B} \times \vec{v})$

(B) $\vec{F}_1 = q\vec{V} \cdot \vec{E}, \vec{F}_2 = q(\vec{B} \cdot \vec{V})$

(C) $\vec{F}_1 = q\vec{E}, \vec{F}_2 = q(\vec{V} \times \vec{B})$

(D) $\vec{F}_1 = q\vec{E}, \vec{F}_2 = q(\vec{B} \times \vec{V})$

Correct Answer: (C) $\vec{F}_1 = q\vec{E}, \vec{F}_2 = q(\vec{V} \times \vec{B})$

Solution:

Concept:

- The electric part of the Lorentz force is $\vec{F}_E = q\vec{E}$. It acts on a charge even if that charge is at rest and has the direction of $\vec{E}$ for positive charge.
- The magnetic part is $\vec{F}_B = q(\vec{v} \times \vec{B})$. Its magnitude is $|q|vB\sin\theta$, and its direction depends on the ordered cross product as well as the sign of the charge.

Step 1: Identify the electric-force expression Electrostatic force depends on the charge and electric field, not on particle velocity or on magnetic field:

$$\vec{F}_1 = q\vec{E}.$$

This immediately rejects the first option, which uses $\vec{B}$ for electrostatic force. It also rejects the second option: a dot product of velocity and electric field is a scalar, so it cannot represent the vector electrostatic force shown on the left.

Step 2: Check the order of the magnetic cross product The complete Lorentz expression is

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}).$$

Therefore the magnetic contribution must be

$$\vec{F}_2 = q(\vec{v} \times \vec{B}).$$

For positive charge, the right-hand rule applied from velocity towards magnetic field gives its direction; a negative charge reverses that direction. Reversing the cross-product order also changes the sign, since $\vec{B} \times \vec{v} = -\vec{v} \times \vec{B}$. Thus the fourth option predicts the opposite magnetic force for a nonparallel velocity and field. A magnetic dot product, as in the second option, also fails because it is scalar and gives the wrong angular dependence.

Step 3: Conclusion The correct pair is $\vec{F}_1 = q\vec{E}$ and $\vec{F}_2 = q(\vec{v} \times \vec{B})$. Hence, the correct option is (C).

Quick Tip to Solve: Remember the magnetic force in the order charge, velocity, field: $q(\vec{v} \times \vec{B})$; reversing the last two reverses the force.

50. If n is the number density and d is the diameter of the molecule, then the average distance covered by a molecule between two successive collisions (i.e. mean free path) is represented by : [4 marks]

(A) $\frac{1}{\sqrt{2}\mathrm{n}\pi\mathrm{d}^2}$

(B) $\frac{1}{\sqrt{2}n^2\pi^2d^2}$

(C) $\frac{1}{\sqrt{2}n\pi d^2}$

(D) $\sqrt{2}\mathrm{n}\pi\mathrm{d}^2$

Correct Answer: (A) $\frac{1}{\sqrt{2}\mathrm{n}\pi\mathrm{d}^2}$

Solution:

Concept:

- The mean free path λ is the average distance a molecule travels between successive intermolecular collisions. It equals mean molecular speed divided by collision frequency.
- For identical hard-sphere molecules of diameter d, the collision cross-section is $\sigma = \pi d^2$. Number density n is the number of molecules per unit volume.

Step 1: Find the collision rate If other molecules were stationary, a molecule moving with speed $\bar{v}$ would sweep out a collision volume $\pi d^2 \bar{v}$ per unit time. Multiplying this by the number density gives the corresponding collision frequency. In a gas all molecules move; the mean relative speed introduces the factor $\sqrt{2}$. Thus, for identical molecules,

$$z = \sqrt{2}\, n\pi d^2 \bar{v}.$$

Step 2: Divide distance travelled by collisions The average time between collisions is $1/z$. Multiplying it by the molecular speed gives

$$\lambda = \frac{\bar{v}}{z} = \frac{1}{\sqrt{2}\, n\pi d^2}.$$

This predicts a shorter free path when the gas is denser or when its molecules have a larger collision diameter, as expected physically.

Step 3: Check the alternatives Since $[n] = L^{-3}$ and $[d] = L$, the product nd^2 has dimension L^{-1}, so its reciprocal has the required dimension of length. Option (B) instead has dimension L^4, option (C) has dimension $L^{1/2}$, and option (D) has dimension L^{-1}. None can describe a distance.

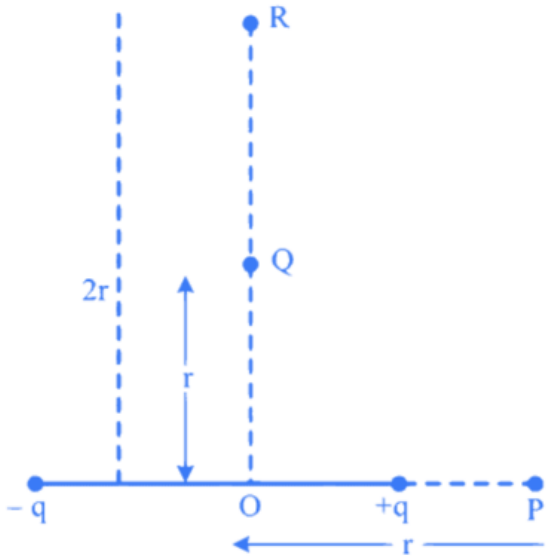
Step 4: Conclusion The mean free path is $\boxed{\lambda = 1/(\sqrt{2}\, n\pi d^2)}$. Hence, the correct option is (A).

Quick Tip to Solve: First check that a proposed free-path expression has units of length; then remember the relative-motion factor $\sqrt{2}$ in the collision rate.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The electric field at point p due to an electric dipole is E. The electric field at point R on equitorial line will be $\frac{\mathrm{E}}{x}$. The value of x : [4 marks]



Solution:

Concept:

- For a short electric dipole of moment magnitude p_d, the axial field magnitude at distance s from its centre is $E_{\text{ax}} = 2kp_d/s^3$, where $k = 1/(4\pi\varepsilon_0)$.
- The equatorial field magnitude at distance s is $E_{\text{eq}} = kp_d/s^3$; it points opposite to the dipole moment. Compare magnitudes when a question writes a scalar ratio.

Step 1: Read the distances from the figure The dipole centre is O. The axial point P is at $OP = r$, while the equatorial point R is at $OR = 2r$. The point Q, marked at distance r on the equatorial line, is not the requested point. Hence the required comparison changes both the direction of observation and the distance from the centre.

Step 2: Write both field magnitudes Using the standard short-dipole field expressions, the given axial field is

$$E = E_P = \frac{2kp_d}{r^3}.$$

At the equatorial point R, substitute its own distance $2r$:

$$E_R = \frac{kp_d}{(2r)^3} = \frac{kp_d}{8r^3}.$$

The dipole moment and electrostatic constant are identical in these two expressions and will cancel in their ratio.

Step 3: Identify the requested factor Dividing the equatorial field by the given axial field gives

$$\frac{E_R}{E} = \frac{kp_d/(8r^3)}{2kp_d/r^3} = \frac{1}{16}.$$

The factor of eight comes from doubling the distance in an inverse-cube field, while the additional factor of two comes from comparing equatorial and axial positions. Therefore $E_R = E/16 = E/x$.

Step 4: Conclusion The field magnitude at R is $E/16$, so $\boxed{x = 16}$ (dimensionless).

Quick Tip to Solve: For dipole comparisons, account for the axial/equatorial factor of two separately from the inverse-cube distance factor; here they multiply to 2×8.

---

52. A sonometer wire of resonating length 90 cm has a fundamental frequency of 400 Hz when kept under some tension. The resonating length of the wire with fundamental frequency of 600 Hz under same tension _____ cm. [4 marks]

Solution:

Concept:

- The fundamental frequency of a stretched wire fixed at both ends is $f = (1/2L)\sqrt{T/\mu}$, where L is its vibrating length, T its tension and μ its mass per unit length.
- For the same uniform wire at unchanged tension, wave speed $v = \sqrt{T/\mu}$ stays constant. The fundamental mode has wavelength $2L$, so $fL = v/2$ is constant.

Step 1: Identify what remains unchanged The two resonances refer to the same sonometer wire under the same tension. Adjusting its resonating length does not change the material, cross-section or mass per unit length. Consequently, the wave speed is the same in both cases. Both specified frequencies are fundamental frequencies, so the mode number is also unchanged; no harmonic multiplier is needed.

Step 2: Apply the inverse-length relation Write the fundamental-mode formula at the two lengths:

$$f_1 = \frac{v}{2L_1}, \qquad f_2 = \frac{v}{2L_2}.$$

Eliminating v gives

$$f_1 L_1 = f_2 L_2, \qquad L_2 = \frac{f_1 L_1}{f_2}.$$

The length units can remain centimetres because only the dimensionless frequency ratio multiplies the known length.

Step 3: Substitute and check the result With $f_1 = 400\,\text{Hz}$, $L_1 = 90\,\text{cm}$ and $f_2 = 600\,\text{Hz}$,

$$L_2 = \frac{400 \times 90}{600}\,\text{cm} = 60\,\text{cm}.$$

The frequency increases by a factor $3/2$, so the resonating length must decrease by the reciprocal factor $2/3$. The calculated value satisfies this check.

Step 4: Conclusion The required resonating length is $\boxed{60\,\text{cm}}$. The numerical entry is 60.

Quick Tip to Solve: For a sonometer at fixed tension, compare fL for the same harmonic; use inverse proportionality rather than direct proportionality.

53. In a single slit experiment, a parallel beam of green light of wavelength $550\ \text{nm}$ passes through a slit of width $0.20\ \text{mm}$. The transmitted light is collected on a screen $100\ \text{cm}$ away. The distance of first order minima from the central maximum will be $x \times 10^{-5}\ \text{m}$. The value of x is : [4 marks]

Solution:

Concept:

- Single-slit diffraction minima satisfy $a \sin\theta = m\lambda$, where a is slit width, λ wavelength and $m = 1, 2, \ldots$. The first minimum corresponds to $m = 1$.
- For a screen at distance D and small diffraction angles, $y = D\tan\theta \simeq D\sin\theta$. Thus the first minimum lies a distance $y_1 = D\lambda/a$ from the central maximum.

Step 1: Convert all distances to SI units The wavelength is $550\,\text{nm} = 550 \times 10^{-9}\,\text{m}$, the slit width is $0.20\,\text{mm} = 2.0 \times 10^{-4}\,\text{m}$, and the screen distance is $100\,\text{cm} = 1.00\,\text{m}$. Here $\lambda/a = 2.75 \times 10^{-3}$, so the angle is small and the usual screen-position approximation is justified.

Step 2: Locate the first dark fringe For the first-order minimum on either side of the centre, set $m = 1$. Then

$$y_1 = \frac{D\lambda}{a} = \frac{1.00 \times 550 \times 10^{-9}}{2.0 \times 10^{-4}}\,\text{m} = 2.75 \times 10^{-3}\,\text{m}.$$

This is the distance from the centre to one first minimum, not the full width of the central bright fringe. The latter would be twice this distance.

Step 3: Express the answer in the specified scale The question defines the distance as $x \times 10^{-5}\,\text{m}$. Rewriting the calculated result,

$$2.75 \times 10^{-3}\,\text{m} = 275 \times 10^{-5}\,\text{m}.$$

Hence the coefficient is 275. The two first minima are symmetrically placed at positive and negative y_1; the requested distance is their positive magnitude.

Step 4: Conclusion The first minimum is $2.75 \times 10^{-3}\,\text{m}$ from the centre. Therefore $\boxed{x = 275}$ (dimensionless).

Quick Tip to Solve: For a single slit, centre-to-first-minimum distance is $D\lambda/a$; double it only when the full central-maximum width is requested.

54. A hollow sphere is rolling on a plane surface about its axis of symmetry. The ratio of rotational kinetic energy to its total kinetic energy is $\frac{x}{5}$. The value of x is _____. [4 marks]

Solution:

Concept:

- For rolling without slipping, the speed of the centre of mass and angular speed obey $v = \omega R$. Total kinetic energy is the sum of translational and rotational contributions.
- A thin hollow sphere has moment of inertia $I = (2/3)MR^2$ about a diameter through its centre. Its rotational kinetic energy about this axis is $K_{\text{rot}} = I\omega^2/2$.

Step 1: Write the two energy contributions Let the hollow sphere have mass M, radius R and centre-of-mass speed v. Its translational energy is

$$K_{\text{trans}} = \frac{1}{2}Mv^2.$$

For pure rolling, use $\omega = v/R$ in the rotational energy:

$$K_{\text{rot}} = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\frac{v^2}{R^2} = \frac{1}{3}Mv^2.$$

The radius cancels because the angular speed is fixed by the rolling constraint.

Step 2: Form the total energy The sphere simultaneously translates and rotates. Therefore the total energy is

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \left(\frac{1}{2} + \frac{1}{3}\right)Mv^2 = \frac{5}{6}Mv^2.$$

The requested denominator is this total, rather than the translational energy alone. Using only the latter would give a different and inappropriate ratio.

Step 3: Compare with the given fraction The fraction of the total kinetic energy that is rotational is

$$\frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{(1/3)Mv^2}{(5/6)Mv^2} = \frac{2}{5}.$$

Both mass and speed cancel, so the result depends only on the shape through its moment of inertia. Equating $2/5$ to the given $x/5$ determines x directly.

Step 4: Conclusion The rotational energy is $2/5$ of the total energy, and $\boxed{x = 2}$ (dimensionless).

Quick Tip to Solve: For pure rolling with $I = kMR^2$, the rotational fraction of total energy is $k/(1+k)$; a hollow sphere has $k = 2/3$.

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55. A wire of resistance 20Ω is divided into 10 equal parts, resulting pairs. A combination of two parts are connected in parallel and so on. Now resulting pairs of parallel combination are connected in series. The equivalent resistance of final combination is _____ Ω. [4 marks]

Solution:

Concept:

- For a uniform wire, $R = \rho L/A$. Cutting it into equal lengths preserves resistivity ρ and area A, so each piece has resistance proportional to its length.
- Two equal resistances r in parallel have equivalent resistance $r/2$. Resistances connected in series add because each carries the same current.

Step 1: Find the resistance of each piece The original wire has resistance $R_0 = 20\,\Omega$ and is cut into ten equal parts. Each piece has one tenth of the original length, with the same cross-sectional area.

Hence

$$r = \frac{R_0}{10} = \frac{20}{10}\,\Omega = 2\,\Omega.$$

Cutting the wire changes the resistance of an individual piece; it does not give ten separate pieces each having $20\,\Omega$.

Step 2: Combine each pair in parallel The ten pieces make five disjoint pairs. For one pair,

$$\frac{1}{R_p} = \frac{1}{2\,\Omega} + \frac{1}{2\,\Omega} = \frac{1}{\Omega}, \qquad R_p = 1\,\Omega.$$

Each of the five pairs has this same resistance. The parallel pairing doubles the conducting paths across the terminals of that pair and therefore halves the resistance of one piece.

Step 3: Add the five pairs in series The resulting five parallel combinations are then joined in series, exactly in the order of operations stated in the question. Their equivalent resistance is

$$R_{\text{eq}} = 5R_p = 5 \times 1\,\Omega = 5\,\Omega.$$

This final connection is not a parallel connection of all ten pieces. As a check, each parallel pair is equivalent to a wire of length $L/10$ and area $2A$; five such sections in series have resistance $R_0/4$.

Step 4: Conclusion The final equivalent resistance is $\boxed{5\,\Omega}$. The numerical entry is 5.

Quick Tip to Solve: Count the final blocks: ten equal pieces give five parallel pairs; halve each piece resistance, then add the five resulting resistances.

56. The maximum height reached by a projectile is 64 m. If the initial velocity is halved, the new maximum height of the projectile is _____ m. [4 marks]

Solution:

Concept:

- For a projectile launched with speed u at angle θ, its initial vertical velocity is $u_y = u\sin\theta$. At the highest point the vertical velocity is zero.
- Under constant gravitational acceleration g, $H = u_y^2/(2g) = u^2\sin^2\theta/(2g)$. Thus maximum height is proportional to the square of launch speed when the direction remains fixed.

Step 1: Express the original maximum height Apply the vertical kinematic relation $v_y^2 = u_y^2 - 2gH$ at the highest point, where $v_y = 0$. This gives

$$H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = 64\,\text{m}.$$

The horizontal component of velocity does not enter the maximum-height relation because gravity changes only the vertical motion.

Step 2: Apply the change in launch velocity Halving the initial velocity leaves its direction unchanged

and halves its vertical component as well. Thus $u_y' = u_y/2$, while g is unchanged. The new height is

$$H' = \frac{(u_y/2)^2}{2g} = \frac{1}{4}\frac{u_y^2}{2g} = \frac{H}{4}.$$

No numerical value of g or launch angle is required because both cancel in the ratio.

Step 3: Calculate and check the new height Using the original height,

$$H' = \frac{64}{4}\,\mathrm{m} = 16\,\mathrm{m}.$$

The square dependence is essential: halving velocity reduces the associated vertical kinetic energy to one quarter, so the gravitational potential energy gained at the top, and therefore the rise in height, also become one quarter of their original values.

Step 4: Conclusion The new maximum height is $\boxed{16\,\mathrm{m}}$. The numerical entry is 16.

Quick Tip to Solve: With the launch direction unchanged, scale projectile height by the square of the speed factor: $u \to u/2$ implies $H \to H/4$.

---

57. The shortest wavelength of the spectral lines in the Lyman series of hydrogen spectrum is $915\,\overset{o}{A}$. The longest wavelength of spectral lines in the Balmer series will be _____ $\overset{o}{A}$. [4 marks]

Solution:

Concept:

- The Rydberg formula for hydrogen is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where the electron falls from n_i to n_f.
- The shortest wavelength in a series is its series limit, obtained as $n_i \to \infty$. The longest wavelength comes from the smallest allowed energy difference.

Step 1: Identify the Lyman limit The Lyman series ends at $n_f = 1$. At its shortest wavelength the initial level tends to infinity, so the second term in the Rydberg expression vanishes. Thus

$$\frac{1}{\lambda_{L,\min}} = R_H, \qquad \frac{1}{R_H} = 915\,\text{Å}.$$

Using the measured series limit directly avoids any need to substitute a separately rounded Rydberg constant.

Step 2: Choose the longest Balmer transition The Balmer series ends at $n_f = 2$, with initial levels $3, 4, 5, \ldots$. Its smallest frequency, and therefore its largest wavelength, belongs to the transition $3 \to 2$. A transition from a higher initial level releases more energy and gives a shorter wavelength. Hence

$$\frac{1}{\lambda_{B,\max}} = R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36}.$$

Step 3: Evaluate the wavelength Take the reciprocal and substitute the given Lyman limit:

$$\lambda_{B,\max} = \frac{36}{5R_H} = \frac{36}{5}(915\,\text{Å}) = 6588\,\text{Å}.$$

The factor $36/5$ is dimensionless, so the resulting wavelength remains in the requested angstrom unit.

Step 4: Conclusion The longest Balmer wavelength is $\boxed{6588\,\text{Å}}$; the required numerical entry is 6588.

Quick Tip to Solve: Use the Lyman limit as $1/R_H$, then multiply it by $36/5$ for the Balmer $3 \to 2$ line.

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58. The current in an inductor is given by $\mathrm{I} = (3\mathrm{t} + 8)$ where t is in second. The magnitude of induced emf produced in the inductor is $12\ \mathrm{mV}$. The self-inductance of the inductor _____ mH. [4 marks]

Solution:

Concept:

- An inductor develops an emf $\varepsilon = -L\,dI/dt$. Here L is its self-inductance; the minus sign expresses opposition to the change of current.
- For a constant inductance the magnitude relation is $|\varepsilon| = L|dI/dt|$. A henry is a volt-second per ampere.

Step 1: Differentiate the current The given current varies linearly with time:

$$I(t) = 3t + 8.$$

Because time is measured in seconds and current in amperes, the coefficient of time has units $\mathrm{A\,s^{-1}}$. Therefore

$$\frac{dI}{dt} = 3\,\mathrm{A\,s^{-1}}.$$

The constant term $8\,\mathrm{A}$ contributes no derivative: a steady current by itself does not generate a self-induced emf.

Step 2: Use the magnitude of the induced emf The question gives the positive magnitude of the emf rather than its polarity, so solve with magnitudes. Converting millivolts into volts gives

$$|\varepsilon| = 12 \times 10^{-3}\,\mathrm{V}.$$

Consequently

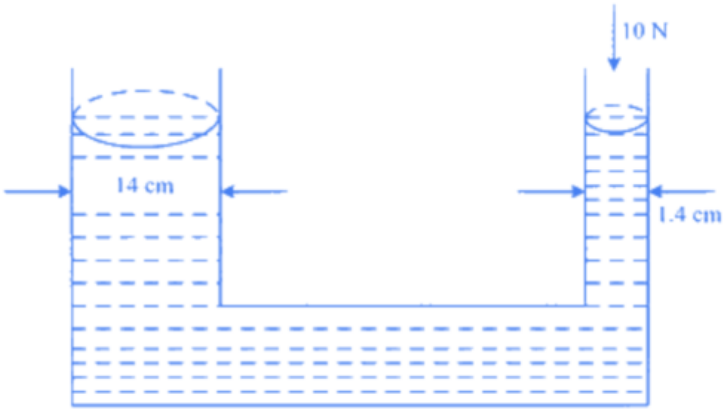
$$L = \frac{|\varepsilon|}{|dI/dt|} = \frac{12 \times 10^{-3}}{3}\,\mathrm{V\,s\,A^{-1}} = 4 \times 10^{-3}\,\mathrm{H}.$$

Step 3: Convert to the requested unit and check Since $1\,\mathrm{mH} = 10^{-3}\,\mathrm{H}$, the inductance equals $4\,\mathrm{mH}$. Substitution checks the result: $(4 \times 10^{-3}\,\mathrm{H})(3\,\mathrm{A\,s^{-1}}) = 12 \times 10^{-3}\,\mathrm{V}$, exactly the stated magnitude. No value of time is needed because the current has a constant slope.

Step 4: Conclusion The self-inductance is $\boxed{4\,\mathrm{mH}}$, so the numerical entry is 4.

Quick Tip to Solve: Differentiate the current first: its slope determines the emf, while a constant current offset has no effect.

59. A hydraulic press containing water has two arms with diameters as mentioned in the figure. A force of 10 N is applied on the surface of water in the thinner arm. The force required to be applied on the surface of water in the thicker arm to maintain equilibrium of water is _____ N. [4 marks]



Solution:

Concept:

- Pascal's law states that an applied pressure increment is transmitted throughout a confined fluid in equilibrium.
- For circular pistons, $A = \pi d^2/4$; equal transmitted pressures give $F_1/A_1 = F_2/A_2$. Thus the force ratio is the square of the diameter ratio.

Step 1: Read the two diameters The diagram labels the thin arm $d_1 = 1.4\,\text{cm}$ and the thick arm $d_2 = 14\,\text{cm}$. The applied force on the thin arm is $F_1 = 10\,\text{N}$. The two water surfaces are shown at the same level, so there is no additional hydrostatic head between them; atmospheric pressure is common to both surfaces and cancels from the balance.

Step 2: Equate the pressure increments For the water to remain at equilibrium, the downward pressure applied at one surface must be balanced by that at the other. Write

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}.$$

The cross sections are circular, hence

$$\frac{A_2}{A_1} = \frac{\pi d_2^2/4}{\pi d_1^2/4} = \left(\frac{14}{1.4}\right)^2 = 10^2 = 100.$$

Diameters can stay in centimetres because their units cancel in the ratio.

Step 3: Calculate and verify the balancing force The required force is therefore

$$F_2 = F_1 \frac{A_2}{A_1} = 10 \times 100 = 1000\,\text{N}.$$

This larger force acts downward on the thicker arm to balance the much smaller force on the thin arm. As a check, dividing $1000\,\text{N}$ by an area one hundred times larger produces the same pressure as $10\,\text{N}$ on the smaller area. Using a diameter ratio without squaring would confuse length with area.

Step 4: Conclusion The required force on the thicker arm is $\boxed{1000\,\mathrm{N}}$. The numerical entry is 1000.

Quick Tip to Solve: For a hydraulic press, square the diameter ratio before multiplying the applied force; pressure depends on area.

---

60. A solenoid of length 0.5 m has a radius of 1 cm and is made up of 'm' number of turns. It carries a current of 5 A. If the magnitude of the magnetic field inside the solenoid is 6.28×10^{-3} T then the value of m is _____. [4 marks]

Solution:

Concept:

- The magnetic field well inside an ideal long air-core solenoid is $B = \mu_0 nI = \mu_0 (m/\ell) I$. Here m is the total number of turns and n is turns per unit length.
- The permeability of free space is $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$. The radius does not enter the ideal internal-field formula.

Step 1: Identify the quantities in the field relation The solenoid has length $\ell = 0.5\,\mathrm{m}$, carries $I = 5\,\mathrm{A}$, and produces $B = 6.28 \times 10^{-3}\,\mathrm{T}$. Its radius is $0.01\,\mathrm{m}$, much smaller than its length, consistent with using the long-solenoid approximation away from the ends. The symbol m denotes a count of turns, not the metre unit.

Step 2: Solve for the total turns Rearrange the solenoid field equation before substituting:

$$B = \mu_0 \frac{m}{\ell} I \quad \Longrightarrow \quad m = \frac{B\ell}{\mu_0 I}.$$

With $\pi \simeq 3.14$,

$$m = \frac{(6.28 \times 10^{-3})(0.5)}{(4 \times 3.14 \times 10^{-7})(5)} = \frac{3.14 \times 10^{-3}}{6.28 \times 10^{-6}} = 500.$$

The units in numerator and denominator cancel, as required for a number of turns.

Step 3: Check the turn density The calculated turn density is $n = m/\ell = 500/0.5 = 1000\,\mathrm{m^{-1}}$. Substitution gives

$$B = (4\pi \times 10^{-7})(1000)(5) = 2\pi \times 10^{-3}\,\mathrm{T} \simeq 6.28 \times 10^{-3}\,\mathrm{T}.$$

This reproduces the supplied field; using the radius in place of length would describe a different magnetic geometry.

Step 4: Conclusion The value of m is $\boxed{500}$, i.e. the solenoid contains 500 turns.

Quick Tip to Solve: First distinguish total turns from turns per metre: multiply $B/(\mu_0 I)$ by the solenoid length.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The number of complexes from the following with no electrons in the t_2 orbital is _____.

$$TiCl_4, [MnO_4]^-, [FeO_4]^{2-}, [FeCl_4]^-, [CoCl_4]^{2-}$$

[4 marks]

(A) 4

(B) 2

(C) 3

(D) 1

Correct Answer: (C) 3

Solution:

Concept:

- In a tetrahedral field, the two e orbitals have lower energy than the three t_2 orbitals. The d electrons are assigned after finding the metal oxidation state.
- Tetrahedral complexes are normally high spin. An ion with d^0 or d^2 can have t_2^0; for d^2, the two electrons singly occupy the lower e orbitals.

Step 1: Determine the oxidation states and d counts Chloride is -1 and oxide is -2. Charge balance gives titanium $+4$ in $TiCl_4$, manganese $+7$ in MnO_4^-, iron $+6$ in FeO_4^{2-}, iron $+3$ in $FeCl_4^-$, and cobalt $+2$ in $CoCl_4^{2-}$. Removing the outer electrons gives the respective metal configurations d^0, d^0, d^2, d^5, d^7. In particular, iron(VI) has two d electrons, not six.

Step 2: Fill the tetrahedral levels The first two species have no d electrons and hence no t_2 occupancy. In ferrate(VI), the two electrons occupy the separate lower-energy e orbitals, giving $e^2t_2^0$. For the weak-field chloride complexes, the high-spin configurations are

$$FeCl_4^- : e^2t_2^3, \qquad CoCl_4^{2-} : e^4t_2^3.$$

Both therefore have electrons in the t_2 set and must be excluded.

Step 3: Count and compare the options The qualifying species are exactly $TiCl_4$, MnO_4^-, and FeO_4^{2-}, a total of three. A count of two overlooks the empty upper set of the d^2 ion; counts of one or four are also inconsistent with these individual occupancies. Thus only the option containing 3 matches the crystal-field analysis.

Step 4: Conclusion Three complexes have no electrons in the t_2 orbitals. Hence, the correct option is (C).

Quick Tip to Solve: Find oxidation state before d count, then remember tetrahedral splitting puts e below t_2, opposite to the octahedral ordering.

62. Match List I with List II.

	LIST I		LIST II
A.	ICl	**I.**	**T - shape**
B.	ICl_3	**II.**	**Square pyramidal**
C.	ClF_5	**III.**	**Pentagonal bipyramidal**
D.	IF_7	**IV.**	**Linear**

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

(B) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(C) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(D) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Correct Answer: (B) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Solution:

Concept:

- VSEPR theory assigns molecular shape from the number of bonding domains and lone pairs around the central atom. Multiple bonds, if present, count as one domain.
- Lone pairs occupy positions that minimize repulsions. Molecular shape describes the atom positions only, whereas electron-pair geometry also includes lone pairs.

Step 1: Assign the first two molecules ICl is a diatomic molecule, so its two nuclei necessarily form a straight line: A matches IV (linear). In ICl_3, iodine has seven valence electrons and forms three single bonds. Four electrons remain as two lone pairs, giving five electron domains, or AX_3E_2. The underlying trigonal-bipyramidal arrangement places both lone pairs equatorially and leaves a T-shaped molecule. Thus B matches I.

Step 2: Assign the higher-coordinate fluorides In ClF_5, chlorine forms five bonds and retains one lone pair. Six domains give an octahedral electron-pair arrangement; removing one vertex from the atomic shape gives a square pyramid, so C matches II. In IF_7, iodine forms seven bonds with no lone pairs on the central atom. Its seven-coordinate molecular geometry is pentagonal bipyramidal, so D matches III.

Step 3: Assemble the matching and reject the others The complete correspondence is

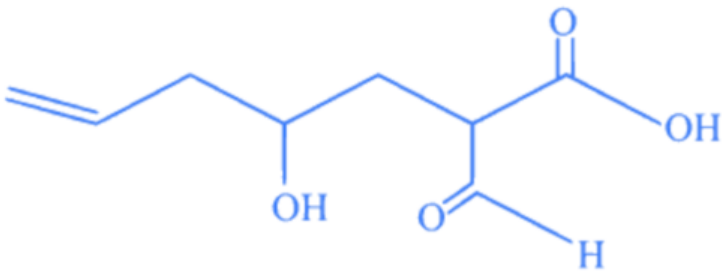
$$A: \text{IV}, \quad B: \text{I}, \quad C: \text{II}, \quad D: \text{III}.$$

This is option (B). Options (A) and (C) incorrectly make the diatomic ICl T-shaped; option (D) gives ICl_3 a seven-coordinate pentagonal-bipyramidal shape despite its having only three bonded atoms. These checks also distinguish molecular shape from electron geometry.

Step 4: Conclusion The matching is A-IV, B-I, C-II, D-III. Hence, the correct option is (B).

Quick Tip to Solve: Count central-atom lone pairs explicitly: AX_3E_2 is T-shaped, AX_5E is square pyramidal, and AX_7 is pentagonal bipyramidal.

63. The correct nomenclature for the following compound is : [4 marks]



(A) 2-carboxy-4-hydroxyhept-7-enal

(B) 2-formyl-4-hydroxyhept-7-enoic acid

(C) 2-carboxy-4-hydroxyhept-6-enal

(D) 2-formyl-4-hydroxyhept-6-enoic acid

Correct Answer: (D) 2-formyl-4-hydroxyhept-6-enoic acid

Solution:

Concept:

- For a molecule containing carboxylic acid, aldehyde and alcohol groups, the carboxylic acid has suffix priority. Its carbon must be included as carbon 1 of the parent chain.
- An aldehyde group outside the selected parent chain is named by the prefix formyl, while an alcohol group is represented by hydroxy when acid supplies the suffix.

Step 1: Select the principal group and parent chain The rightmost $-COOH$ group has higher nomenclature priority than the $-CHO$ and $-OH$ groups. Select the longest chain containing its carbon and the carbon-carbon double bond. Starting from the carboxyl carbon as C1 and tracing to the terminal alkene carbon gives seven carbons, so the parent is a heptenoic acid. The aldehyde carbon is a branch attached to C2 rather than part of this seven-carbon path.

Step 2: Locate the substituents and unsaturation Numbering from the acid end places the $-CHO$ substituent at C2, giving 2-formyl, and the $-OH$ substituent at C4, giving 4-hydroxy. The terminal double bond joins C6 and C7, so its locant is the lower number 6. The parent suffix is therefore hept-6-enoic acid; a double-bond locant of 7 would require a C8 atom and is impossible for this chain.

Step 3: Assemble the name and compare the alternatives List the prefixes in alphabetical order, formyl before hydroxy, and attach them to the unsaturated acid parent. This gives 2-formyl-4-hydroxyhept-6-enoic acid. Options (A) and (C) wrongly use the lower-priority aldehyde as the suffix and acid as carboxy. Option (B) retains the correct acid suffix but places the double bond at the impossible locant 7. Only option (D) respects both priority and numbering.

Step 4: Conclusion The correct name is 2-formyl-4-hydroxyhept-6-enoic acid. Hence, the correct option is (D).

Quick Tip to Solve: Choose the acid carbon as C1 before tracing the chain; for a terminal double bond in a seven-carbon chain, the highest possible locant is 6.

64. Given below are two statements :

Statement I : On passing $HCl_{(g)}$ through a saturated solution of $BaCl_2$, at room temperature white turbidity appears.

Statement II : When HCl gas is passed through a saturated solution of $NaCl$, sodium chloride is precipitated due to common ion effect.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct

(B) Statement I is incorrect but Statement II is correct

(C) Both Statement I and Statement II are incorrect

(D) Statement I is correct but Statement II is incorrect

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- A saturated salt solution is in dynamic equilibrium with its solid; adding a common ion decreases the amount that can remain dissolved.
- For $BaCl_2(s) \rightleftharpoons Ba^{2+}(aq) + 2\,Cl^-(aq)$, the ion product contains $[Cl^-]^2$; for sodium chloride it contains $[Cl^-]$.

Step 1: Evaluate Statement I Passing hydrogen chloride gas into water supplies additional chloride ions because HCl ionises essentially completely. In saturated barium chloride, these chloride ions are common to the dissolution equilibrium. The increased chloride concentration shifts that equilibrium towards undissolved $BaCl_2$. Fine crystals separate and scatter light, producing the white turbidity described. The turbidity is precipitation of the original salt, not formation of a new insoluble barium compound.

Step 2: Evaluate Statement II The same argument applies to $NaCl(s) \rightleftharpoons Na^+(aq) + Cl^-(aq)$. At a fixed temperature, adding chloride to a saturated solution makes the ion product exceed its equilibrium value. Sodium chloride therefore crystallises until saturation is re-established. Its high solubility does not prevent this common-ion precipitation from an already saturated solution.

Step 3: Compare the choices Both statements describe suppression of salt solubility by the chloride ion supplied by hydrogen chloride. Options (B) and (D) incorrectly reject one of these two applications, while option (C) rejects both. Neither observation requires a change in temperature or a reaction forming a different salt.

Step 4: Conclusion Both Statement I and Statement II are correct: HCl supplies the common chloride ion in both cases and causes precipitation from each saturated solution. The supplied key selects the statement-II-false choice, but that contradicts the printed common-ion equilibrium for sodium chloride. Hence, the correct option is (A).

Quick Tip to Solve: Identify the ion supplied by the added reagent first: chloride is common to both saturated chlorides, so precipitation is favoured in each case.

65. Match List I with List II.

	List I (Pair of compounds)		List II (Isomerism)
A.	n-Propanol and isopropanol	I.	Metamerism
B.	Methoxypropane and ethoxyethane	II.	Chain isomerism
C.	Propanone and propanal	III.	Position isomerism
D.	Neopentane and isopentane	IV.	Functional isomerism

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(B) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(D) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

Correct Answer: (C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- Structural isomers have the same molecular formula but differ in connectivity; the type is determined by what structural feature changes.
- Position isomerism changes a group position, metamerism changes the alkyl groups around a polyvalent functional atom, chain isomerism changes the carbon skeleton, and functional isomerism changes the functional group.

Step 1: Classify the two alcohols and ethers n-Propanol is propan-1-ol, $CH_3CH_2CH_2OH$, whereas isopropanol is propan-2-ol, $CH_3CH(OH)CH_3$. Their carbon skeleton and alcohol group agree; only the hydroxyl position changes. Therefore A matches III. Methoxypropane has a methyl and a propyl group on oxygen, whereas ethoxyethane has two ethyl groups. Both are ethers of formula $C_4H_{10}O$, but their alkyl distribution around oxygen differs. Thus B matches I, metamerism.

Step 2: Classify the carbonyl compounds and alkanes Propanone, CH_3COCH_3, is a ketone, while propanal, CH_3CH_2CHO, is an aldehyde. Their shared formula C_3H_6O but different functional groups give C–IV. Neopentane is 2,2-dimethylpropane and isopentane is 2-methylbutane. Both have formula C_5H_{12}, but their branching differs, so D–II is chain isomerism.

Step 3: Select the matching code The complete sequence is A–III, B–I, C–IV, D–II. Option (A) interchanges the alcohol and alkane classifications; option (B) misclassifies the aldehyde/ketone pair and the pentanes; option (D) interchanges the alcohol and ether classifications. Only option (C) preserves all four independently established matches.

Step 4: Conclusion The matching is A–III, B–I, C–IV and D–II. Hence, the correct option is (C).

Quick Tip to Solve: For each pair, compare the functional group first, then the skeleton, then the group position; ethers with different alkyl splits are metamers.

66. While preparing crystals of Mohr's salt, dil H_2SO_4 is added to a mixture of ferrous sulphate and ammonium sulphate, before dissolving this mixture in water, dil H_2SO_4 is added here to : [4 marks]

(A) prevent the hydrolysis of ferrous sulphate

(B) prevent the hydrolysis of ammonium sulphate

(C) increase the rate of formation of crystals

(D) make the medium strongly acidic

Correct Answer: (A) prevent the hydrolysis of ferrous sulphate

Solution:

Concept:

- Mohr's salt is the double salt $(NH_4)_2Fe(SO_4)_2 \cdot 6\,H_2O$, prepared by crystallising an equimolar mixture of ferrous sulphate and ammonium sulphate.
- Acid suppresses hydrolysis of hydrated metal ions by increasing the concentration of H^+, which shifts their hydrolysis equilibria backwards.

Step 1: Identify the hydrolysis to suppress Ferrous sulphate supplies hydrated Fe^{2+} ions. Their interaction with water can be represented schematically by

$$Fe^{2+} + H_2O \rightleftharpoons FeOH^+ + H^+.$$

Hydrolysis changes the composition of the ferrous solution and can lead to basic impurities. This is undesirable when a pure double salt is to be crystallised with a definite composition.

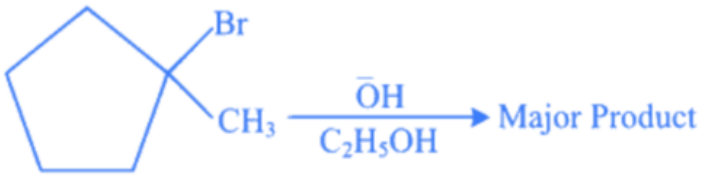
Step 2: Apply the effect of dilute sulphuric acid Adding dilute H_2SO_4 increases H^+ concentration and shifts the displayed equilibrium to the left by Le Chatelier's principle. Ferrous ions are thereby maintained in solution in the required form. Sulphuric acid is suitable because sulphate is already the anion in the salts; it does not introduce an unrelated counter-ion into the preparation.

Step 3: Reject the alternative purposes Ammonium sulphate is not the ferrous salt whose metal-ion hydrolysis is being prevented, so option (B) identifies the wrong component. Acid is not added primarily to accelerate crystal formation; crystallisation follows concentration and cooling, excluding (C). Although acid makes the solution acidic, making it strongly acidic is not the stated chemical purpose, so (D) is less appropriate than the precise hydrolysis explanation.

Step 4: Conclusion Dilute sulphuric acid is added to prevent the hydrolysis of ferrous sulphate. Hence, the correct option is (A).

Quick Tip to Solve: In preparing ferrous double salts, connect the addition of dilute acid with preservation of dissolved ferrous ions, rather than faster crystallisation.

67. Identify the major product in the following reaction. [4 marks]



(A) Methylcyclopentane

(B) 1-Methylcyclopentene

(C) Methylenecyclopentane

(D) 1-Bromocyclopentene

Correct Answer: (B) 1-Methylcyclopentene

Solution:

Concept:

- Alcoholic hydroxide favours dehydrohalogenation of an alkyl halide: a beta hydrogen and a halide leave to create a carbon–carbon double bond.
- With a small base, the more substituted alkene is generally the major product according to Saytzeff's rule; substitution and elimination must be distinguished using the reagent and substrate.

Step 1: Locate the reactive carbon and beta positions The substrate is 1-bromo-1-methylcyclopentane. The carbon carrying bromine is attached to two ring carbons and a methyl group, so it is a tertiary alkyl halide. Beta hydrogens are available on both adjacent ring carbons and on the methyl substituent. The two adjacent ring positions are equivalent by symmetry and lead to the same endocyclic alkene.

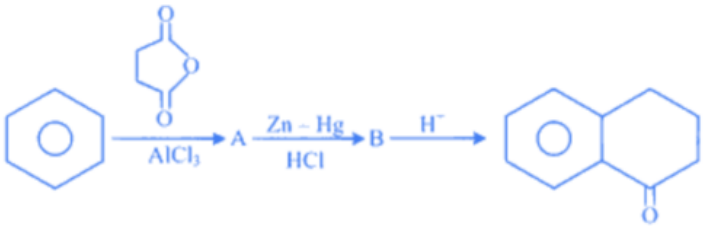
Step 2: Compare the possible elimination products Removal of a ring beta hydrogen gives 1-methylcyclopentene, with its double bond inside the ring. This alkene is trisubstituted: the two double-bond carbons collectively have three carbon substituents. Removal of a hydrogen from the methyl group instead gives methylenecyclopentane, an exocyclic, disubstituted alkene. Alcoholic hydroxide is not a bulky base, so the more substituted endocyclic alkene predominates.

Step 3: Match and exclude the remaining structures Option (B) shows 1-methylcyclopentene. Option (C) is the less substituted elimination product and is therefore not the major product. Option (A), methylcyclopentane, would require replacement of bromine by hydrogen rather than elimination and is not produced by this reagent. Option (D) retains bromine while losing a carbon substituent; it does not represent removal of hydrogen bromide from the given substrate.

Step 4: Conclusion The major product is 1-methylcyclopentene, the more substituted endocyclic alkene. Hence, the correct option is (B).

Quick Tip to Solve: For alcoholic hydroxide, mark every beta hydrogen and compare the substitution of the resulting double bonds before selecting the major alkene.

68. Identify A and B in the given chemical reaction sequence : [4 marks]



(A) A: 1,2,3,4-Tetrahydronaphthalene-1,4-dione; B: naphthalene

(B) A: $C_6H_5COCH_2CH_2COOH$; B: $C_6H_5CH_2CH_2CH_2COOH$

(C) A: 1,2,3,4-Tetrahydronaphthalene-1,4-dione; B: 1,4-dihydroxy-1,2,3,4-tetrahydronaphthalene

(D) A: $C_6H_5CH_2CH_2CH_2COOH$; B: $C_6H_5CH_2CH_2CH_2CH_2OH$

Correct Answer: (B) A: $C_6H_5COCH_2CH_2COOH$; B: $C_6H_5CH_2CH_2CH_2COOH$

Solution:

Concept:

- Succinic anhydride acylates benzene in the presence of anhydrous $AlCl_3$, opening the anhydride ring to give an aromatic keto acid.
- Clemmensen reduction, $Zn-Hg/HCl$, converts a ketone carbonyl group to methylene while the carboxylic acid group remains available for intramolecular acylation.

Step 1: Identify intermediate A Benzene attacks the acylating species generated from succinic anhydride and $AlCl_3$. Ring opening and work-up produce beta-benzoylpropionic acid, also called 4-oxo-4-phenylbutanoic acid:

$$A = C_6H_5COCH_2CH_2COOH.$$

A is therefore an open-chain keto acid attached to a benzene ring, not an already fused bicyclic diketone. The two methylene groups between the ketone and acid are retained from the succinic anhydride skeleton.

Step 2: Identify intermediate B Treatment of A with zinc amalgam and hydrochloric acid reduces its ketone group to CH_2. The carboxylic acid is not reduced by this reagent. Thus

$$C_6H_5COCH_2CH_2COOH \xrightarrow{Zn-Hg/HCl} C_6H_5CH_2CH_2CH_2COOH.$$

B is 4-phenylbutanoic acid. Its three methylene groups connect the aromatic ring to the carboxyl carbon.

Step 3: Check the final ring closure and options Acid-promoted intramolecular acylation of B at the ortho position closes a six-membered ring, giving the displayed tetralone. This final product independently confirms the chain length of B. Option (B) contains the correct keto acid followed by the correct reduced acid. Options (A) and (C) incorrectly begin with fused bicyclic compounds, and option (D) would require reduction of a carboxylic acid to an alcohol by Clemmensen reagent.

Step 4: Conclusion $A = C_6H_5COCH_2CH_2COOH$ and $B = C_6H_5CH_2CH_2CH_2COOH$. Hence, the correct option is (B).

Quick Tip to Solve: In a sequence containing Zn–Hg/HCl, replace the ketone C=O by methylene but keep COOH; then count the tether atoms to verify the final ring size.

69. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : NH_3 **and** NF_3 **molecule have pyramidal shape with a lone pair of electrons on nitrogen atom. The resultant dipole moment of** NH_3 **is greater than that of** NF_3**.**

Reason (R) : In NH_3**, the orbital dipole due to lone pair is in the same direction as the resultant dipole moment of the** $N - H$ **bonds.** F **is the most electronegative element.**

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) is false but (R) is true

(B) is true but (R) is false

(C) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(D) Both (A) and (R) are true and (R) is the correct explanation of (A)

Correct Answer: (D) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- Both NH_3 and NF_3 have three bond pairs and one lone pair around nitrogen; their molecular shape is trigonal pyramidal.
- The molecular dipole is a vector resultant. The direction of the bond contribution relative to the lone-pair contribution determines whether the two reinforce or oppose each other.

Step 1: Evaluate the assertion Four electron domains around nitrogen give a tetrahedral electron-pair arrangement. One domain is a lone pair, leaving a trigonal-pyramidal arrangement of the three attached atoms in both molecules. Ammonia has a dipole moment of about 1.47 D, whereas nitrogen trifluoride has a much smaller moment, about 0.24 D. Consequently both the shape description and the stated ordering in the assertion are correct.

Step 2: Evaluate the reason In ammonia, nitrogen is more electronegative than hydrogen, so the resultant of the three N–H bond contributions points towards the nitrogen/lone-pair side. It reinforces the orbital contribution associated with the lone pair. Fluorine is the most electronegative element, as the reason states; the N–F bond contributions therefore point towards fluorine, in the opposite sense to the lone-pair contribution. The two contributions largely cancel in nitrogen trifluoride.

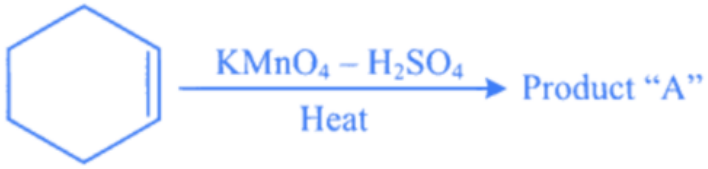
Step 3: Test the explanatory connection The reason supplies the directional comparison needed to explain the assertion: reinforcement in ammonia gives a larger net dipole than the partial cancellation in nitrogen trifluoride. Thus option (C), which denies the explanatory relation, is inappropriate even though it recognises that both statements are true. Options (A) and (B) wrongly classify one statement as false. A larger individual bond polarity alone does not guarantee a larger molecular dipole.

Step 4: Conclusion Both the assertion and the reason are true, and the reason correctly explains the assertion. Hence, the correct option is (D).

Quick Tip to Solve: Compare dipole vectors, not just electronegativity differences: lone-pair and bond contributions reinforce in ammonia but oppose in nitrogen trifluoride.

70. Consider the given chemical reaction :

Product "A" is : [4 marks]



(A) picric acid

(B) oxalic acid

(C) adipic acid

(D) acetic acid

Correct Answer: (C) adipic acid

Solution:

Concept:

- Hot acidified potassium permanganate cleaves an alkene double bond oxidatively; an alkene carbon bearing hydrogen becomes a carboxylic acid carbon.
- Cleavage of a double bond within a ring opens that ring. The remaining single-bond carbon chain is retained, so atom counting identifies the resulting dicarboxylic acid.

Step 1: Identify the double-bond carbon environments The starting compound is cyclohexene. Each of its two alkene carbons is bonded to one hydrogen and one other ring carbon. With hot $KMnO_4/H_2SO_4$, oxidation proceeds beyond a vicinal diol to cleavage of the carbon–carbon double bond. Since both alkene carbons bear hydrogen, each becomes a carboxyl group rather than a ketone carbonyl.

Step 2: Open the ring and count the chain Breaking only the original double bond leaves all six carbon atoms connected along the other side of the ring. The two former alkene carbons become the terminal carboxyl carbons and the other four remain methylene groups. Therefore the product is

$$HOOC-(CH_2)_4-COOH,$$

hexanedioic acid, commonly called adipic acid. The ring opens to one molecule containing two acid groups; it does not split into two independent three-carbon chains.

Step 3: Compare the named options Adipic acid is option (C). Picric acid is a trinitrophenol and would require an aromatic substrate and nitration chemistry, neither of which is present. Oxalic acid has only two carbon atoms, while acetic acid is a two-carbon monocarboxylic acid; neither preserves the six-carbon chain generated by cleavage of the single double bond in cyclohexene.

Step 4: Conclusion Product A is adipic acid, $HOOC(CH_2)_4COOH$. Hence, the correct option is (C).

Quick Tip to Solve: For oxidative cleavage of a cycloalkene, cut the double bond on paper and retain the other ring bonds; this immediately gives the open-chain carbon count.

71. The correct statements from the following are :

(A) The decreasing order of atomic radii of group 13 elements is $\mathrm{Tl} > \mathrm{In} > \mathrm{Ga} > \mathrm{Al} > \mathrm{B}$.

(B) Down the group 13 electronegativity decreases from top to bottom.

(C) Al **dissolves in dil.** HCl **and liberates** $\mathrm{H_2}$ **but conc.** $\mathrm{HNO_3}$ **renders** Al **passive by forming a protective oxide layer on the surface.**

(D) All elements of group 13 exhibits highly stable +1 oxidation state.

(E) Hybridisation of Al **in** $[\mathrm{Al(H_2O)_6}]^{3+}$ **ion is** $\mathrm{sp^3d^2}$.

Choose the correct answer from the options given below : [4 marks]

(A) (A), (B), (C) and (E) only

(B) (C) and (E) only

(C) (A), (C) and (E) only

(D) (A) and (C) only

Correct Answer: (B) (C) and (E) only

Solution:

Concept:

- Atomic size normally increases down a group, but poor shielding by the filled $3d$ shell contracts gallium relative to aluminium. Simple monotonic trends therefore require checking.
- The inert-pair effect makes the $+1$ state more stable toward the bottom of group 13. The coordination number and availability of outer orbitals determine the conventional hybridisation assignment.

Step 1: Test the size and electronegativity statements Statement (A) is false: gallium is slightly smaller than aluminium, so the printed $\mathrm{Ga} > \mathrm{Al}$ relation is incorrect. Statement (B) is also false. Electronegativity initially decreases from boron to aluminium, but then increases slightly for heavier members because of imperfect shielding. It does not decrease uniformly from top to bottom.

Step 2: Check the chemical behaviour and complex Statement (C) is true. Aluminium reacts with dilute hydrochloric acid:

$$2\,\mathrm{Al} + 6\,\mathrm{HCl} \longrightarrow 2\,\mathrm{AlCl_3} + 3\,\mathrm{H_2}.$$

Concentrated nitric acid passivates its surface by forming a protective oxide film. Statement (D) is false: the $+1$ state is not highly stable for every member; boron and aluminium favour $+3$, whereas the inert-pair effect strongly stabilises $+1$ in thallium.

Step 3: Assign the octahedral hybridisation The ion $[\mathrm{Al(H_2O)_6}]^{3+}$ has six coordinated water molecules and an octahedral arrangement. In the conventional valence-bond description used here its hybridisation is sp^3d^2, so statement (E) is true. Option A includes false statements (A) and (B), option C includes false (A), and option D includes false (A) while omitting true (E).

Step 4: Conclusion Only the aluminium-passivation and octahedral-hybridisation statements are correct. Hence, the correct option is (B).

Quick Tip to Solve: For group 13, check the Ga–Al size exception before accepting a radius order; do not extend the stability of Tl(I) to every member.

72. Consider the above reaction sequence and identify the major product P. [4 marks]

$$CH_3CH_2 - OH \xrightarrow[\text{(ii) } KMnO_4]{\text{(i) Jone's Reagent}} P$$
$$\text{(iii) } NaOH, CaO, \Delta$$

(A) Methanoic acid

(B) Methoxymethane

(C) Methane

(D) Methanal

Correct Answer: (C) Methane

Solution:

Concept:

- Jones reagent oxidises a primary alcohol to a carboxylic acid under aqueous acidic conditions. Ethanol therefore supplies the two-carbon ethanoic-acid framework.
- Heating a sodium carboxylate with soda lime removes its carboxyl carbon as carbonate: $RCOONa + NaOH \longrightarrow RH + Na_2CO_3$. The alkane has one fewer carbon atom.

Step 1: Follow the oxidation stage The starting material in the sequence is ethanol, CH_3CH_2OH. Oxidation changes the terminal alcohol group first toward an aldehyde and then to the acid. With Jones reagent the relevant product is ethanoic acid:

$$CH_3CH_2OH + 2\,[O] \longrightarrow CH_3COOH + H_2O.$$

The subsequent permanganate treatment does not turn this into a one-carbon ether or aldehyde; the carboxylic-acid stage is retained for the final soda-lime step.

Step 2: Apply soda-lime decarboxylation In the presence of sodium hydroxide the acid forms sodium ethanoate. Heating it with $NaOH$ and CaO gives

$$CH_3COONa + NaOH \xrightarrow{\Delta} CH_4 + Na_2CO_3.$$

Calcium oxide acts as the soda-lime support. The carbon attached to the carboxyl group becomes methane when the carboxyl carbon is removed.

Step 3: Eliminate the other products Methanoic acid and methanal retain an oxygen-containing functional group inconsistent with soda-lime decarboxylation to an alkane. Methoxymethane would require ether formation and preserves two carbons joined through oxygen; that is not what the listed reagents do. The one-carbon hydrocarbon methane is consequently the major final product.

Step 4: Conclusion The major product P is methane, CH_4. Hence, the correct option is (C).

Quick Tip to Solve: Count carbons immediately before the soda-lime step: an ethanoate salt loses its carboxyl carbon and gives methane.

73. Coagulation of egg, on heating is because of : [4 marks]

(A) Denaturation of protein occurs

(B) Biological property of protein remains unchanged

(C) Breaking of the peptide linkage in the primary structure of protein occurs

(D) The secondary structure of protein remains unchanged

Correct Answer: (A) Denaturation of protein occurs

Solution:

Concept:

- A protein has a primary amino-acid sequence and higher structural levels maintained by hydrogen bonding and other interactions. Its biological activity depends on its folded conformation.
- Heat can disrupt the interactions responsible for secondary and tertiary structures without hydrolysing the peptide bonds of the primary chain. This loss of native structure is denaturation.

Step 1: Identify what heating changes Egg white contains proteins such as albumin in their native folded forms. On heating, interactions stabilising those forms are disrupted. The polypeptide chains unfold and expose groups previously buried inside the protein. These chains then associate with one another, producing an insoluble network. This aggregation appears as the familiar coagulation or setting of the egg.

Step 2: Distinguish denaturation from hydrolysis The change is in the organisation of the chain, especially the secondary and tertiary structures. It is not the breaking of peptide linkages into individual amino acids. Ordinary cooking therefore illustrates denaturation followed by aggregation, rather than hydrolysis of the primary structure. Option (C) describes a different process and cannot explain the stated observation.

Step 3: Check the remaining alternatives Option (B) is incorrect because loss of the native conformation changes biological properties and usually destroys the original activity. Option (D) is incorrect because the secondary structure is among the structural levels disrupted by heating; it does not remain unchanged. Option (A) correctly names the structural event responsible for coagulation, and is the only defensible choice.

Step 4: Conclusion Coagulation of egg on heating is caused by denaturation of protein. Hence, the correct option is (A).

Quick Tip to Solve: In denaturation, remember: folding is disturbed, while the peptide-bond sequence normally remains intact. Peptide cleavage indicates hydrolysis instead.

74. Which one of the following reactions is NOT possible? [4 marks]

(A) Anisole $\xrightarrow{Cl_2/AlCl_3}$ p-chloroanisole

(B) Chlorobenzene $\xrightarrow[\text{high temperature, then } H^+]{NaOH}$ phenol

(C) Phenol $\xrightarrow{HCl}$ chlorobenzene

(D) Anisole $\xrightarrow{HBr}$ phenol

Correct Answer: (C) Phenol $\xrightarrow{HCl}$ chlorobenzene

Solution:

Concept:

- Phenol has a strong aryl carbon–oxygen bond with partial double-bond character because an oxygen lone pair conjugates with the aromatic ring. It does not undergo ordinary alcohol-like replacement of OH by chloride using HCl.
- Anisole activates its ring toward electrophilic substitution. Aryl alkyl ethers can also be cleaved by hydrogen halides at the alkyl–oxygen bond.

Step 1: Identify the impossible substitution Option (C) proposes conversion of phenol into chlorobenzene with hydrochloric acid. This is not possible under the stated conditions. The aromatic carbon is sp^2-hybridised, and the phenolic carbon–oxygen bond is strengthened by resonance. Ordinary S_N2 displacement at this carbon is unavailable; an aryl carbocation needed for a simple S_N1 route is also unfavourable.

Step 2: Verify the first two reactions In option (A), the methoxy group donates electron density to the ring and directs chlorination to the ortho and para positions. Formation of para-chloroanisole is therefore possible. In option (B), chlorobenzene reacts with sodium hydroxide under sufficiently harsh high-temperature conditions to form sodium phenoxide; subsequent acidification produces phenol. This is the industrial hydrolysis route and is a possible transformation.

Step 3: Check ether cleavage Option (D) is possible because protonation of the ether oxygen in anisole facilitates attack of bromide at its methyl group. The aryl–oxygen bond remains intact, giving phenol and methyl bromide:

$$C_6H_5OCH_3 + HBr \longrightarrow C_6H_5OH + CH_3Br.$$

Thus A, B and D can occur, while C treats phenol incorrectly as an ordinary aliphatic alcohol.

Step 4: Conclusion Phenol cannot be converted to chlorobenzene by the indicated HCl treatment. Hence, the correct option is (C).

Quick Tip to Solve: For anisole cleavage, break the methyl–oxygen bond; for phenol substitution, first recognise the resistant aryl carbon–oxygen bond.

75. The number of ions from the following that have the ability to liberate hydrogen from a dilute acid is _____.

$$\mathrm{Ti}^{2+}, \mathrm{Cr}^{2+} \textbf{ and } \mathrm{V}^{2+}$$

[4 marks]

(A) 1

(B) 2

(C) 3

(D) 0

Correct Answer: (C) 3

Solution:

Concept:

- An ion liberates hydrogen from a dilute non-oxidising acid if it can donate electrons to H^+. The relevant comparison is between its higher/lower oxidation-state reduction potential and $E^\circ(\mathrm{H}^+/\mathrm{H}_2) = 0\,\mathrm{V}$.
- For $\mathrm{M}^{3+} + \mathrm{e}^- \longrightarrow \mathrm{M}^{2+}$, a negative reduction potential means that the reverse oxidation of M^{2+}, coupled to proton reduction, is thermodynamically favourable under standard conditions.

Step 1: Assess each divalent ion The approximate standard reduction potentials for $\mathrm{Ti}^{3+}/\mathrm{Ti}^{2+}$, $\mathrm{Cr}^{3+}/\mathrm{Cr}^{2+}$ and $\mathrm{V}^{3+}/\mathrm{V}^{2+}$ are respectively -0.37, -0.41 and $-0.26\,\mathrm{V}$. Each is below the hydrogen-electrode value. Consequently each divalent ion acts as a reducing agent and can be oxidised to its trivalent state while protons are reduced.

Step 2: Write the common reaction For each of titanium, chromium and vanadium, the electron balance is

$$2\,\mathrm{M}^{2+} \longrightarrow 2\,\mathrm{M}^{3+} + 2\,\mathrm{e}^-, \qquad 2\,\mathrm{H}^+ + 2\,\mathrm{e}^- \longrightarrow \mathrm{H}_2.$$

Adding gives

$$2\,\mathrm{M}^{2+} + 2\,\mathrm{H}^+ \longrightarrow 2\,\mathrm{M}^{3+} + \mathrm{H}_2.$$

The cell potential is zero minus the negative metal-ion reduction potential, so it is positive in all three cases.

Step 3: Count the successful ions All three ions listed satisfy the required reducing-power criterion. The answer is therefore three, not one or two. Zero would require none of these couples to favour proton reduction, which contradicts their negative reduction potentials. This question concerns dissolved divalent ions undergoing oxidation, not neutral metal atoms dissolving directly.

Step 4: Conclusion All three ions can liberate hydrogen from dilute acid, so the number is 3. Hence, the correct option is (C).

Quick Tip to Solve: For a dissolved reducing ion, compare the higher/lower ion couple with zero volts; a negative reduction potential favours oxidation coupled to hydrogen evolution.

76. The number of moles of methane required to produce $11\ \text{g}\ CO_2(g)$ after complete combustion is : (Given molar mass of methane in g mol^{-1} : 16) [4 marks]

(A) 0.35

(B) 0.25

(C) 0.5

(D) 0.75

Correct Answer: (B) 0.25

Solution:

Concept:

- A balanced chemical equation gives mole ratios, not mass ratios. Complete combustion of methane produces one mole of carbon dioxide per mole of methane.
- The amount in moles is $n = m/M$, where m is mass in grams and M is molar mass in g mol^{-1}. Carbon dioxide has molar mass $12 + 2(16) = 44\,\text{g mol}^{-1}$.

Step 1: Balance the combustion equation Complete combustion means that the carbon in methane ends in carbon dioxide and the hydrogen ends in water. The balanced equation is

$$CH_4 + 2\,O_2 \longrightarrow CO_2 + 2\,H_2O.$$

Its coefficients show a $1 : 1$ ratio between methane consumed and carbon dioxide formed. The oxygen coefficient does not change that carbon-based mole relationship.

Step 2: Convert the product mass to moles The required mass of carbon dioxide is $11\,\text{g}$. Hence

$$n(CO_2) = \frac{11\,\text{g}}{44\,\text{g mol}^{-1}} = 0.25\,\text{mol}.$$

By the balanced equation,

$$n(CH_4) = n(CO_2) = 0.25\,\text{mol}.$$

As a check, this is $0.25 \times 16 = 4\,\text{g}$ of methane using the molar mass supplied.

Step 3: Check the alternative amounts An amount of $0.35\,\text{mol}$ methane would produce $15.4\,\text{g}$ carbon dioxide; $0.5\,\text{mol}$ would produce $22\,\text{g}$, and $0.75\,\text{mol}$ would produce $33\,\text{g}$. None matches $11\,\text{g}$. Using methane molar mass to divide the carbon-dioxide mass would mix two different substances and give a wrong result.

Step 4: Conclusion The required amount of methane is $0.25\,\text{mol}$. Hence, the correct option is (B).

Quick Tip to Solve: Balance first, convert the given substance to moles using its own molar mass, and only then use the coefficient ratio.

77. The quantity of silver deposited when one coulomb charge is passed through $AgNO_3$ solution :

[4 marks]

(A) 0.1 g atom of silver

(B) 1 chemical equivalent of silver

(C) 1 g of silver

(D) 1 electrochemical equivalent of silver

Correct Answer: (D) 1 electrochemical equivalent of silver

Solution:

Concept:

- Faraday's first law states that deposited mass is proportional to charge: $m = ZQ$, where Z is the electrochemical equivalent in $\mathrm{g\,C^{-1}}$.
- The electrochemical equivalent is the mass deposited by one coulomb. One chemical equivalent, in contrast, is deposited by one faraday, approximately $96485\,\mathrm{C}$.

Step 1: Apply the definition to silver At the cathode the silver-ion reduction is

$$\mathrm{Ag^+ + e^- \longrightarrow Ag.}$$

Thus one mole of electrons deposits one mole of silver. In terms of charge,

$$m = \frac{MQ}{F} = ZQ,$$

where M is the molar mass of silver and the electron number is one. With $Q = 1\,\mathrm{C}$, the deposited quantity is exactly one electrochemical equivalent of silver.

Step 2: Check the magnitude Using $M \approx 107.9\,\mathrm{g\,mol^{-1}}$ and $F \approx 96485\,\mathrm{C\,mol^{-1}}$,

$$Z = \frac{107.9}{96485} \approx 1.118 \times 10^{-3}\,\mathrm{g\,C^{-1}}.$$

Accordingly one coulomb deposits about $1.118\,\mathrm{mg}$. This numerical check makes clear that the answer is not one gram or a substantial fraction of a mole.

Step 3: Distinguish the other quantities Option (A), 0.1 gram-atom of silver, means $0.1\,\mathrm{mol}$, requiring about $9648.5\,\mathrm{C}$. Option (B), one chemical equivalent of monovalent silver, corresponds to about $107.9\,\mathrm{g}$ and requires one faraday. Option (C), one gram, also needs far more than one coulomb. Only option (D) uses the quantity defined specifically for one coulomb of charge.

Step 4: Conclusion The deposited mass is one electrochemical equivalent of silver, approximately $1.118\,\mathrm{mg}$. Hence, the correct option is (D).

Quick Tip to Solve: Associate one coulomb with electrochemical equivalent and one faraday with chemical equivalent; these differ by the Faraday constant.

78. The metal atom present in the complex MABXL (where A, B, X and L are unidentate ligands and M is metal) involves sp^3 hybridization. The number of geometrical isomers exhibited by the complex is : [4 marks]

(A) 0

(B) 2

(C) 3

(D) 4

Correct Answer: (A) 0

Solution:

Concept:

- Four equivalent sp^3 hybrid orbitals point towards the vertices of a tetrahedron. Thus a four-coordinate complex with this hybridization has tetrahedral geometry.
- Geometrical isomers require distinct relative ligand positions that cannot be interconverted by rotating the entire complex. Optical isomers are a separate category of stereoisomer.

Step 1: Identify the coordination geometry The ligands A, B, X and L are all unidentate, so each provides one donor atom and the coordination number of M is four. The stated sp^3 hybridization identifies a tetrahedral arrangement. A square-planar arrangement, for which distinct adjacent and opposite positions exist, would instead require the corresponding planar bonding description; it cannot be used for this question.

Step 2: Compare the possible ligand positions In a tetrahedron, every pair of vertices is adjacent and every ligand–metal–ligand angle has the same ideal value, approximately $109.5°$. There are no two opposite vertices defining a trans pair. Consequently, choosing any one ligand as a reference does not divide the other three sites into distinct cis and trans classes. Permuting four different ligands can produce mirror-image forms, but those are optical enantiomers rather than different geometrical arrangements.

Step 3: Separate geometrical and optical counts The tetrahedral complex MABXL can therefore be optically active because all four ligands differ. Its two mirror-image configurations must not be counted as two geometrical isomers. The value 2 in option (B) confuses these two types of isomerism; 3 and 4 in options (C) and (D) likewise do not describe distinct tetrahedral geometrical arrangements.

Step 4: Conclusion The complex exhibits no geometrical isomerism, so the requested number is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: For a four-coordinate complex, establish tetrahedral or square-planar geometry before counting isomers. Four different tetrahedral ligands allow optical, but not cis–trans, isomerism.

79. For the electro chemical cell

$$M\left|M^{2+}\right||X|X^{2-}$$

If $E^0_{(M^{2+}/M)} = 0.46$ V **and** $E^0_{(x/x^{2-})} = 0.34$ V**.**

Which of the following is correct? [4 marks]

(A) $E_{cell} = 0.80$ V

(B) $M + X \rightarrow M^{2+} + X^{2-}$ is a spontaneous reaction

(C) $E_{cell} = -0.80$ V

(D) $M^{2+} + X^{2-} \rightarrow M + X$ is a spontaneous reaction

Correct Answer: (D) $M^{2+} + X^{2-} \rightarrow M + X$ is a spontaneous reaction

Solution:

Concept:

- For a cell written with oxidation at the left and reduction at the right, $E^\circ_{cell} = E^\circ_{right,red} - E^\circ_{left,red}$. Electrode potentials are not added merely because two values are given.
- The relation $\Delta G^\circ = -nFE^\circ_{cell}$ connects the sign of the standard cell potential with the spontaneous direction under standard conditions.

Step 1: Write the two half-reactions The left-hand electrode in the written cell undergoes oxidation, $M \longrightarrow M^{2+} + 2e^-$. Its tabulated reduction potential for $M^{2+} + 2e^- \longrightarrow M$ is $+0.46$ V. At the right-hand electrode, the reduction is $X + 2e^- \longrightarrow X^{2-}$, with reduction potential $+0.34$ V. Both half-reactions involve two electrons, so no multiplication of an electrode potential is required.

Step 2: Calculate the potential in the written direction Adding the oxidation at M and reduction at X gives $M + X \longrightarrow M^{2+} + X^{2-}$. For this direction,

$$E^\circ_{cell} = 0.34 - 0.46 = -0.12\,\text{V}.$$

The negative value makes ΔG° positive. Therefore the forward reaction in option (B) is not spontaneous under standard conditions.

Step 3: Reverse the reaction and compare the options Reversing the reaction reverses the sign of the potential:

$$M^{2+} + X^{2-} \longrightarrow M + X, \qquad E^\circ = +0.12\,\text{V}.$$

This direction has negative standard Gibbs energy and is spontaneous. Neither $+0.80$ V in (A) nor -0.80 V in (C) follows from subtracting the given reduction potentials; both use an incorrect sum of their magnitudes.

Step 4: Conclusion The spontaneous standard reaction is $M^{2+} + X^{2-} \longrightarrow M + X$. Hence, the correct option is (D).

Quick Tip to Solve: Write both given values as reduction potentials, subtract left from right, and use the sign to determine whether to reverse the cell reaction.

80. Given below are two statements :

Statement I : The metallic radius of Na is $1.86\ \mathrm{A}^\circ$ and the ionic radius of Na^+ is lesser than $1.86\ \mathrm{A}^\circ$

Statement II : Ions are always smaller in size than the corresponding elements.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is incorrect but Statement II is true

(C) Both Statement I and Statement II are false

(D) Statement I is correct but Statement II is false

Correct Answer: (D) Statement I is correct but Statement II is false

Solution:

Concept:

- Forming a cation removes electrons while leaving nuclear charge unchanged. Loss of an outer occupied shell and reduced electron repulsion generally make a cation smaller than its neutral atom.
- Forming an anion adds electrons to the valence shell. Increased electron–electron repulsion makes an anion larger than the corresponding neutral atom.

Step 1: Evaluate the sodium statement A sodium atom has electronic configuration $\mathrm{Na} : 1s^2 2s^2 2p^6 3s^1$. Removal of the outer $3s$ electron produces $\mathrm{Na}^+ : 1s^2 2s^2 2p^6$. The highest occupied principal shell changes from $n = 3$ to $n = 2$, while the nucleus still contains eleven protons. Thus the ionic radius of Na^+ is smaller than the stated metallic radius $1.86\,\text{Å}$. Statement I is correct.

Step 2: Test the universal claim about ions Statement II applies the word always to both cations and anions. This is false: chlorine gains an electron to form chloride, $\mathrm{Cl} + \mathrm{e}^- \longrightarrow \mathrm{Cl}^-$. The extra electron enters the same outer shell and increases repulsion among electrons, whereas the nuclear charge remains unchanged. As a result, the chloride anion is larger than the neutral chlorine atom. One such counterexample is sufficient to disprove an assertion about all ions.

Step 3: Match the pair of truth values The correct combination is therefore Statement I true and Statement II false. Option (A) incorrectly accepts the universal second statement. Option (B) rejects the correct sodium comparison and accepts the false generalization. Option (C) also wrongly rejects Statement I. Only option (D) preserves the correct assessment of both statements.

Step 4: Conclusion Statement I is correct but Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: When a size statement says ions, check a cation and an anion separately. Cations contract, whereas anions expand relative to their own neutral atoms.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. In an atom, total number of electrons having quantum numbers $\mathrm{n} = 4, |\ \mathrm{m}_l| = 1$ and $\mathrm{m_s} = -\frac{1}{2}$ is _____. [4 marks]

Solution:

Concept:

- For a shell with principal quantum number n, the allowed orbital quantum numbers are $l = 0, 1, \ldots, n-1$, and each subshell has $m_l = -l, \ldots, +l$.
- An orbital is specified by (n, l, m_l). Pauli exclusion permits only one electron in an orbital with a specified spin projection m_s.

Step 1: List the allowed subshells The shell $n = 4$ contains $l = 0, 1, 2, 3$, namely $4s, 4p, 4d, 4f$. The condition $|m_l| = 1$ means that m_l can have either of the two values $+1$ and -1; it does not mean that only $+1$ is permitted. The $4s$ subshell has $l = 0$, so it offers only $m_l = 0$ and contributes no electron satisfying the question.

Step 2: Count the orbitals satisfying the magnetic condition Each of the subshells with $l \geq 1$ contains both magnetic states $m_l = -1$ and $m_l = +1$. Thus $4p$, $4d$ and $4f$ each supply exactly two eligible orbitals. Their additional states, such as $m_l = 0$ or $m_l = \pm 2$, do not meet the specified absolute value and are excluded. The eligible orbital count is

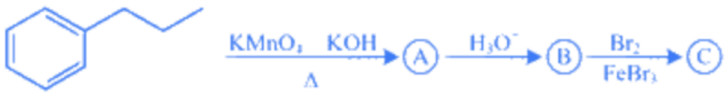
$$N_{\text{orbital}} = 2 + 2 + 2 = 6.$$

Step 3: Apply the specified spin restriction Although an orbital can accommodate two electrons with opposite spins, this question admits only $m_s = -\frac{1}{2}$. Hence each of the six selected orbitals contributes one permitted electron state, not two. Explicitly, these states have $l = 1, 2, 3$, each paired with $m_l = \pm 1$, and all have the one prescribed spin projection. Multiplying by a spin factor of two would incorrectly include $m_s = +\frac{1}{2}$.

Step 4: Conclusion The total allowed number is $3 \times 2 \times 1 = \boxed{6}$ electrons. The numerical entry is 6, a dimensionless count.

Quick Tip to Solve: Translate an absolute magnetic quantum number into both signs, then count only subshells with sufficiently large l. Apply a factor of one when spin is fixed.

82. The product C in the following sequence of reactions has _____ π bonds. [4 marks]



Solution:

Concept:

- Hot alkaline potassium permanganate oxidizes an alkylbenzene containing a benzylic hydrogen to the benzoate ion, regardless of the remaining alkyl-chain length.
- Acidification converts the carboxylate salt into a carboxylic acid. Bromination with $Br_2/FeBr_3$ is electrophilic aromatic substitution, which preserves the aromatic ring.

Step 1: Identify the oxidation product A The starting structure is an alkylbenzene with a benzylic CH_2 group and therefore contains the benzylic hydrogen needed for vigorous side-chain oxidation. On heating with $KMnO_4/KOH$, the side chain is oxidized to the carboxylate group. The carbon directly bonded to the aromatic ring becomes the carboxylate carbon. Thus A is potassium benzoate, C_6H_5COOK, rather than a product retaining the full original propyl chain.

Step 2: Follow acidification and ring bromination Treatment with H_3O^+ protonates A to give benzoic acid B, C_6H_5COOH. The $-COOH$ group withdraws electrons and directs electrophilic substitution predominantly to a meta position. Consequently $Br_2/FeBr_3$ converts B to m-bromobenzoic acid C. Bromine replaces an aromatic hydrogen; the aromatic system and carboxyl carbonyl remain intact.

Step 3: Count every pi bond in C The benzene ring contributes three π bonds, represented by its three delocalized double bonds in a Kekule structure. The carboxyl group contains one $C=O$ double bond and hence one additional π bond. The $C-Br$, $C-O$ and $O-H$ bonds are sigma bonds and contribute no additional pi bond. Therefore

$$N_\pi = 3 \text{ (aromatic)} + 1 \text{ (carbonyl)} = 4.$$

Step 4: Conclusion Product C is m-bromobenzoic acid and contains $\boxed{4}$ π bonds. The requested numerical entry is 4, a dimensionless count.

Quick Tip to Solve: After vigorous benzylic oxidation, replace the entire alkyl side chain by a carboxyl group. Count aromatic pi bonds and carbonyl pi bonds separately.

83. Using the given figure, the ratio of R_f values of sample A and sample C is $x \times 10^{-2}$. Value of x is _____.

[4 marks]

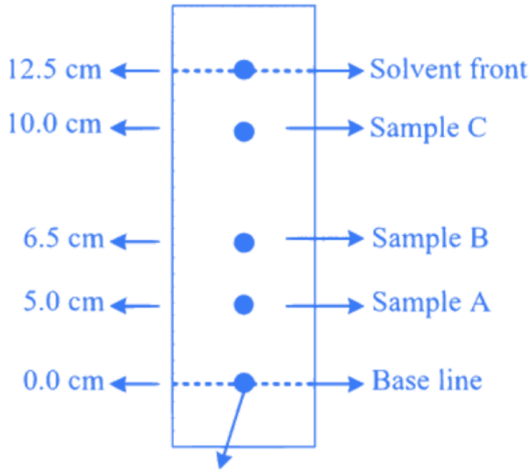


Fig : Paper chromatography of Samples

Solution:

Concept:

- The retention factor is $R_f = d_{\text{sample}}/d_{\text{solvent}}$, where both distances are measured from the same baseline along the chromatogram.
- For samples on the same chromatogram, the common solvent-front distance cancels in a ratio of retention factors. Each retention factor and their ratio are dimensionless.

Step 1: Read distances from the baseline The baseline in the figure is marked $0.0\,\text{cm}$. Sample A is at $5.0\,\text{cm}$, sample C is at $10.0\,\text{cm}$, and the solvent front is at $12.5\,\text{cm}$. These markings are distances measured from the baseline, so there is no need to subtract the A position from the C position. The intermediate sample B at $6.5\,\text{cm}$ does not enter the requested A-to-C ratio.

Step 2: Calculate the two retention factors Using the solvent front as the denominator gives

$$R_f(A) = \frac{5.0\,\text{cm}}{12.5\,\text{cm}} = 0.40, \qquad R_f(C) = \frac{10.0\,\text{cm}}{12.5\,\text{cm}} = 0.80.$$

Both values are below one, as expected because neither solute has travelled beyond the solvent front. The units cancel in each fraction, so no unit is attached to an individual retention factor.

Step 3: Express the ratio in the requested form The order in the question is A relative to C. Therefore

$$\frac{R_f(A)}{R_f(C)} = \frac{0.40}{0.80} = 0.50.$$

Equivalently the common solvent distance cancels directly, leaving $5.0/10.0$. Comparing this value with the specified expression gives

$$x \times 10^{-2} = 0.50, \qquad x = \frac{0.50}{0.01} = 50.$$

Reversing the sample order would give 2 and would answer a different ratio.

Step 4: Conclusion The required coefficient is $\boxed{x = 50}$. It is dimensionless because it is obtained from a ratio of retention factors.

Quick Tip to Solve: For two spots developed with the same solvent front, divide their distances from the baseline directly; then convert the decimal to the stated power of ten.

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84. Considering acetic acid dissociates in water, its dissociation constant is 6.25×10^{-5}. If 5 mL of acetic acid is dissolved in 1 litre water, the solution will freeze at $-x \times 10^{-2}{}^{\circ}\mathrm{C}$, provided pure water freezes at $0^{\circ}\mathrm{C}$.

$x =$ _____. (Nearest integer)

Given : $(\mathrm{K_f})_{\text{water}} = 1.86 \text{ K kg mol} - 1$

density of acetic acid is 1.2 g mol^{-1}.

molar mass of water $= 18 \text{ g mol}^{-1}$.

molar mass of acetic acid= 60 g mol^{-1}.

density of water $= 1 \text{ g cm}^{-3}$

Acetic acid dissociates as $\mathrm{CH_3COOH} \rightleftharpoons \mathrm{CH_3COO^{\ominus}} + \mathrm{H^{\oplus}}$ [4 marks]

Solution:

Concept:

- For a dilute solution, freezing-point depression is $\Delta T_f = iK_f m$, where m is solute molality and i counts the increase in particles due to dissociation.
- For the weak monoprotic acid $\mathrm{CH_3COOH} \rightleftharpoons \mathrm{CH_3COO^-} + \mathrm{H^+}$, a fraction α dissociated gives $i = 1 + \alpha$, with $K_a = c\alpha^2/(1-\alpha)$.

Step 1: Convert the acid volume to molality The stated density value 1.2 is used as a mass-per-volume density, $1.2\,\mathrm{g\,mL^{-1}}$; the printed $\mathrm{g\,mol^{-1}}$ unit is dimensionally a molar-mass unit rather than a density unit. Thus the acid mass and amount are

$$w = 5 \times 1.2 = 6\,\mathrm{g}, \qquad n = \frac{6}{60} = 0.100\,\mathrm{mol}.$$

One litre of water of density $1\,\mathrm{g\,cm^{-3}}$ has mass $1.000\,\mathrm{kg}$, giving $m = 0.100\,\mathrm{mol\,kg^{-1}}$. For this dilute solution take $c \simeq 0.100\,\mathrm{mol\,L^{-1}}$.

Step 2: Calculate the dissociation correction For a weak acid, the small-dissociation estimate gives

$$\alpha \simeq \sqrt{\frac{K_a}{c}} = \sqrt{\frac{6.25 \times 10^{-5}}{0.100}} = 0.025.$$

Thus $i = 1 + 0.025 = 1.025$. The exact quadratic $0.1\alpha^2 + K_a\alpha - K_a = 0$ gives approximately 0.02469, which changes the final temperature only slightly and does not change the required rounded integer.

Step 3: Obtain the freezing temperature and x Substitution gives

$$\Delta T_f = 1.025 \times 1.86 \times 0.100 = 0.19065\,\mathrm{K}.$$

A temperature depression in kelvin has the same numerical magnitude in Celsius degrees. Since pure water freezes at zero Celsius, the solution freezes at approximately $-0.19065\,^\circ\mathrm{C}$. Comparing with $-x \times 10^{-2}\,^\circ\mathrm{C}$,

$$x = 19.065 \simeq 19 \quad \text{(nearest integer)}.$$

Step 4: Conclusion The solution freezes at approximately $-0.191\,^\circ\mathrm{C}$, giving the requested dimensionless coefficient $\boxed{x = 19}$.

Quick Tip to Solve: Use kilograms of solvent for molality, and include the extra particles from weak-acid dissociation through i = 1 + alpha before converting to the requested temperature scale.

85. Consider the following single step reaction in gas phase at constant temperature.

$$2\,\mathrm{A_{(g)}} + \mathrm{B_{(g)}} \rightarrow \mathrm{C_{(g)}}$$

The initial rate of the reaction is recorded as $\mathrm{r_1}$ **when the reaction starts with** $1.5\ \mathrm{atm}$ **pressure of** A **and** $0.7\ \mathrm{atm}$ **pressure of B. After some time, the rate** r_2 **is recorded when the pressure of C becomes** $0.5\ \mathrm{atm}$**. The ratio** $\mathrm{r_1} : \mathrm{r_2}$ **is _____** $\times 10^{-1}$**. (Nearest integer)** [4 marks]

Solution:

Concept:

- For an elementary, single-step reaction, the powers in the rate law equal the molecular stoichiometric coefficients. Thus for $2\,\mathrm{A\,(g)} + \mathrm{B\,(g)} \longrightarrow \mathrm{C\,(g)}$, the rate is proportional to $p_A^2 p_B$ at constant temperature.
- At fixed temperature and volume, partial pressures change in the stoichiometric ratio. Formation of pressure x of C consumes $2x$ of A and x of B. The same rate constant applies at both observations.

Step 1: Write the initial rate The reaction is explicitly single step, so we use the elementary rate law rather than infer an arbitrary experimental order. With the pressure-based rate constant denoted by k_p,

$$r_1 = k_p(1.5)^2(0.7) = 1.575k_p.$$

All pressures are measured in the same unit, atm; the units contained in k_p will cancel in the ratio.

Step 2: Find the remaining reactant pressures Initially the reaction starts with A and B. When C reaches $0.5\,\mathrm{atm}$, the corresponding pressure decreases are $1.0\,\mathrm{atm}$ for A and $0.5\,\mathrm{atm}$ for B. Therefore,

$$p_A = 1.5 - 2(0.5) = 0.5\,\mathrm{atm}, \qquad p_B = 0.7 - 0.5 = 0.2\,\mathrm{atm}.$$

Both are positive, as required for this reaction extent.

Step 3: Calculate the ratio and its requested coefficient Substitute these remaining pressures in the

same rate law:

$$r_2 = k_p(0.5)^2(0.2) = 0.05k_p.$$

Consequently,

$$\frac{r_1}{r_2} = \frac{1.575}{0.05} = 31.5 = 315 \times 10^{-1}.$$

The requested blank multiplies 10^{-1}, so it is ten times the numerical ratio itself.

Step 4: Conclusion The ratio is $r_1 : r_2 = 31.5 : 1$. The requested dimensionless nearest-integer coefficient is $\boxed{315}$.

Quick Tip to Solve: Use the pressure of product formed as the reaction extent; subtract twice that value from A before squaring its pressure.

86. The fusion of chromite ore with sodium carbonate in the presence of air leads to the formation of products A and B along with the evolution of CO_2. The sum of spin-only magnetic moment values of A and B is _____ B.M. (Nearest integer)

[Given atomic number : $C : 6, Na : 11, O : 8, Fe : 26, Cr : 24$**]** [4 marks]

Solution:

Concept:

- Roasting or oxidative fusion of chromite, $FeCr_2O_4$, with sodium carbonate in air produces sodium chromate and iron(III) oxide. Oxidation states identify the metal-ion electron configurations.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$, where n is the number of unpaired electrons. The chromium(VI) ion is d^0, whereas high-spin iron(III) is d^5.

Step 1: Identify the two fusion products The balanced oxidative fusion reaction is

$$4\,FeCr_2O_4 + 8\,Na_2CO_3 + 7\,O_2 \longrightarrow 8\,Na_2CrO_4 + 2\,Fe_2O_3 + 8\,CO_2.$$

Hence A and B are sodium chromate and ferric oxide, in either naming order. Carbon dioxide is separately mentioned in the question and is not one of the two products whose magnetic moments are being added.

Step 2: Count the unpaired electrons In Na_2CrO_4, charge balance gives chromium oxidation state +6: $2(+1) + x + 4(-2) = 0$. Neutral chromium has configuration $[Ar]3d^5 4s^1$; loss of six electrons gives Cr^{6+} a $3d^0$ configuration. It has no unpaired electrons. In Fe_2O_3, iron is +3. Iron loses its two 4s electrons and one 3d electron, leaving $[Ar]3d^5$, with five unpaired electrons in the standard spin-only treatment.

Step 3: Add the spin-only moments and round Apply the formula to the metal ions associated with the two products:

$$\mu_A = \sqrt{0(0+2)} = 0\,\text{B.M.}, \qquad \mu_B = \sqrt{5(5+2)} = \sqrt{35}\,\text{B.M.}$$

$$\mu_A + \mu_B = \sqrt{35} = 5.916\ldots\ \text{B.M.}$$

The formula-unit coefficient two on iron in ferric oxide is not used to multiply the ionic spin-only moment in this question.

Step 4: Conclusion The sum of the spin-only magnetic moments rounds to $\boxed{6\,\text{B.M.}}$.

Quick Tip to Solve: First obtain Cr(VI) and Fe(III) from charge balance; their d counts, zero and five, immediately determine the spin-only result.

87. In the Claisen-Schmidt reaction to prepare 351 g of dibenzalacetone using 87 g of acetone, the amount of benzaldehyde required is _____ g. (Nearest integer) [4 marks]

Solution:

Concept:

- In the Claisen-Schmidt condensation, one mole of acetone condenses with two moles of benzaldehyde to produce one mole of dibenzalacetone and two moles of water.
- Convert each stated mass into moles using its molecular formula. Stoichiometric coefficients relate mole amounts, rather than directly relating masses.

Step 1: Write the balanced condensation Two alpha-hydrogen positions of acetone undergo condensation with two benzaldehyde molecules. The net reaction is

$$2\,C_6H_5CHO + CH_3COCH_3 \longrightarrow C_6H_5CH{=}CHCOCH{=}CHC_6H_5 + 2\,H_2O.$$

Dibenzalacetone has formula $C_{17}H_{14}O$. Each condensation loses one water molecule, so the two water molecules must be accounted for in the mass balance.

Step 2: Calculate the amount of dibenzalacetone Using the usual atomic masses C = 12, H = 1 and O = 16, its molar mass is

$$M(C_{17}H_{14}O) = 17(12) + 14(1) + 16 = 234\,\text{g mol}^{-1}.$$

Thus the required amount of product is

$$n(\text{dibenzalacetone}) = \frac{351}{234} = 1.5\,\text{mol}.$$

The given acetone mass independently gives $87/58 = 1.5\,\text{mol}$, matching the required one-to-one acetone-to-product ratio.

Step 3: Obtain the benzaldehyde mass The balanced reaction requires twice as many moles of benzaldehyde as of product:

$$n(C_6H_5CHO) = 2(1.5) = 3.0\,\text{mol}.$$

Benzaldehyde has formula C_7H_6O, and hence

$$M = 7(12) + 6 + 16 = 106\,\text{g mol}^{-1}, \qquad m = 3(106) = 318\,\text{g}.$$

As a mass check, $318 + 87 = 405\,\text{g}$, and the products contain 351 g of dibenzalacetone plus $3(18) = 54\,\text{g}$

of water.

Step 4: Conclusion The required benzaldehyde mass, already an integer, is $\boxed{318\,\text{g}}$.

Quick Tip to Solve: Remember the 2:1 benzaldehyde-to-acetone ratio; subtracting two water molecules gives the correct product molar mass, 234 g per mole.

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88. Combustion of 1 mole of benzene is expressed at

$$C_6H_6(l) + \frac{15}{2}O_2(\,g) \rightarrow 6CO_2(\,g) + 3H_2O(l).$$

The standard enthalpy of combustion of 2 mol **of benzene is** $-'x'$kJ. $x =$ _____.

Given :

1. standard Enthalpy of formation of 1 mol **of** $C_6H_6(l)$**, for the reaction** $6C$ **(graphite)** $+3H_2(g) \rightarrow C_6H_6(l)$ **is** 48.5 kJ mol^{-1}.

2. Standard Enthalpy of formation of 1 mol **of** $CO_2(g)$**, for the reaction** C **(graphite)** $+O_2(g) \rightarrow CO_2(g)$ **is** -393.5 kJ mol^{-1}.

3. Standard and Enthalpy of formation of 1 mol **of** $H_2O(l)$**, for the reaction** $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$ **is** -286 kJ mol^{-1}. [4 marks]

Solution:

Concept:

- Hess's law gives a reaction enthalpy as the stoichiometric sum of standard formation enthalpies of products minus that of reactants: $\Delta_r H^\circ = \sum \nu \Delta_f H^\circ(\text{products}) - \sum \nu \Delta_f H^\circ(\text{reactants})$.
- The standard formation enthalpy of elemental oxygen in its standard state is zero. Enthalpy is extensive, so doubling the amount of benzene doubles the heat change without changing its sign.

Step 1: Evaluate the product formation-enthalpy sum The stated equation burns one mole of liquid benzene to six moles of carbon dioxide and three moles of liquid water. Retain the liquid-water formation value supplied in the question, rather than using a gaseous-water value:

$$\sum \nu \Delta_f H^\circ(\text{products}) = 6(-393.5) + 3(-286).$$

The two contributions are $-2361\,\text{kJ}$ and $-858\,\text{kJ}$, respectively, so their sum is $-3219\,\text{kJ}$.

Step 2: Subtract the reactant formation-enthalpy sum The liquid benzene formation enthalpy is positive, $+48.5\,\text{kJ mol}^{-1}$, while the oxygen contribution is zero. For the equation as written,

$$\Delta_c H^\circ = -3219 - \left[48.5 + \frac{15}{2}(0)\right] = -3267.5\,\text{kJ mol}^{-1}.$$

The positive benzene formation enthalpy is subtracted; adding it would give the wrong combustion heat. The negative result is consistent with exothermic combustion.

Step 3: Scale to two moles and identify x The question asks for the heat change for $2\,\text{mol}$, not the

molar heat alone. Therefore,

$$\Delta H^\circ(2\,\text{mol}) = 2(-3267.5) = -6535\,\text{kJ}.$$

Comparing this with the given notation $-x\,\text{kJ}$, the numerical magnitude is $x = 6535$. The minus sign already appears outside the blank, so it must not be included again in x.

Step 4: Conclusion For two moles of benzene the standard combustion enthalpy is $-6535\,\text{kJ}$; hence $\boxed{x = 6535}$.

Quick Tip to Solve: Keep products minus reactants and multiply by two only after finding the molar combustion enthalpy; report x as a positive magnitude.

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89. X g **of ethanamine was subjected to reaction with** $NaNO_2/HCl$ **followed by hydrolysis to liberate** N_2 **and** HCl**. The** HCl **generated was completely neutralised by 0.2 moles of** NaOH.X **is _____ g.** [4 marks]

Solution:

Concept:

- A primary aliphatic amine reacts with nitrous acid to form an unstable aliphatic diazonium species, which hydrolyses to the alcohol with liberation of nitrogen.
- One mole of the diazonium chloride intermediate gives one mole of HCl on hydrolysis. Neutralisation follows $HCl + NaOH \longrightarrow NaCl + H_2O$, a one-to-one mole ratio.

Step 1: Relate the amine to the liberated acid Sodium nitrite and hydrochloric acid generate nitrous acid in situ. Ethanamine forms the unstable ethyldiazonium chloride intermediate. Its hydrolysis can be represented as

$$C_2H_5N_2Cl + H_2O \longrightarrow C_2H_5OH + N_2 + HCl.$$

Thus each mole of ethanamine entering this sequence corresponds to one mole of the HCl liberated in the hydrolysis step. The acid specified here is the generated acid, rather than an unspecified initial excess of the hydrochloric acid reagent.

Step 2: Use the neutralisation data The liberated HCl is completely neutralised by $0.2\,\text{mol}$ of sodium hydroxide. Since both are monofunctional in this acid-base reaction,

$$n(HCl) = n(NaOH) = 0.2\,\text{mol}.$$

The preceding one-to-one relation then gives

$$n(C_2H_5NH_2) = 0.2\,\text{mol}.$$

There is no factor of two from the two nitrogen atoms in the diazonium ion: one comes from the amine and the other from nitrous acid.

Step 3: Convert the amine amount into mass Ethanamine has molecular formula C_2H_7N. Taking C =

12, H = 1 and N = 14,

$$M = 2(12) + 7(1) + 14 = 45\,\text{g mol}^{-1}.$$

Consequently,

$$X = nM = 0.2(45) = 9\,\text{g}.$$

The units cancel as moles times grams per mole, leaving grams as requested.

Step 4: Conclusion The initial mass of ethanamine is $\boxed{X = 9\,\text{g}}$.

Quick Tip to Solve: Follow one mole of amine through diazonium hydrolysis to one mole of HCl, then use the one-to-one NaOH neutralisation ratio.

90. Number of compounds from the following with zero dipole moment is _____.

$$\text{HF}, \text{H}_2, \text{H}_2\text{ S}, \text{CO}_2, \text{NH}_3, \text{BF}_3, \text{CH}_4, \text{CHCl}_3, \text{SiF}_4, \text{H}_2\text{O}, \text{BeF}_2$$

[4 marks]

Solution:

Concept:

- A molecular dipole moment is the vector sum of all bond dipole moments. Polar bonds can therefore give a zero molecular dipole when the molecular geometry makes their vectors cancel.
- Homonuclear diatomic molecules have nonpolar bonds. Symmetric linear, trigonal-planar and tetrahedral molecules with identical terminal atoms also have zero resultant dipole.

Step 1: Identify the nonpolar and linear molecules In H_2, both bonded atoms have equal electronegativity, so the bond itself has zero dipole moment. In linear CO_2, the two equal C–O bond moments point in opposite directions and cancel. Linear BeF_2 similarly has two opposite, equal Be–F bond moments. These give three zero-dipole species. In contrast, heteronuclear HF has a single polar bond whose moment cannot be cancelled.

Step 2: Check the planar and tetrahedral symmetric species In trigonal-planar BF_3, the three equal B–F bond moments are separated by $120°$, so their vector sum vanishes. In CH_4, the four equivalent C–H bonds occupy the vertices of a regular tetrahedron and cancel. The four identical Si–F bonds in tetrahedral SiF_4 cancel in the same manner. These add three more zero-dipole species.

Step 3: Exclude the remaining unsymmetric molecules Both H_2O and H_2S are bent, so their two bond moments do not cancel. NH_3 is trigonal pyramidal and has a nonzero resultant moment along its symmetry axis. Although CHCl_3 is tetrahedral around carbon, its terminal atoms are not all identical: three C–Cl moments and one C–H moment do not sum to zero. Therefore none of these four belongs in the zero-dipole list. Count the selected species explicitly rather than equating the presence of polar bonds with molecular polarity.

Step 4: Conclusion The six zero-dipole species are H_2, CO_2, BF_3, CH_4, SiF_4 and BeF_2. The requested dimensionless count is $\boxed{6}$.

Quick Tip to Solve: Combine shape with identical terminal atoms: BF3, CO2 and SiF4 are nonpolar despite their polar bonds; CHCl3 fails the identical-terminal test.