

JEE Main 2024 Paper 1 (B.E./B.Tech) Apr 6 Shift 1

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 6 April 2024, Shift 1. The total duration is 3 hours.
- (ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n , respectively, then $m + n^2$ is equal to [4 marks]

(A) 408

(B) 414

(C) 312

(D) 396

Correct Answer: (A) 408

Solution:

Concept:

- For an equilateral triangle of side a , its altitude is $a\sqrt{3}/2$, and its inradius is one third of this altitude: $r = a\sqrt{3}/6$.
- A square inscribed in a circle has its diagonal equal to the diameter. If its side is s , then $s\sqrt{2} = 2r$, its area is s^2 , and its perimeter is $4s$.

Step 1: Find the radius of the inscribed circle The three angle bisectors of an equilateral triangle coincide with its medians. The common centre divides an altitude in the ratio 2 : 1; its shorter distance to a side is the inradius. With the given side $a = 12$,

$$h = \frac{\sqrt{3}}{2}(12) = 6\sqrt{3}, \quad r = \frac{h}{3} = 2\sqrt{3}.$$

Thus the circle has diameter $4\sqrt{3}$. Using the circumradius of the triangle instead would give twice the correct radius, so the distinction between the two circles matters.

Step 2: Calculate the square area and squared perimeter The square vertices are on the circle, so its diagonal passes through the centre and equals the diameter. Consequently,

$$s = \frac{4\sqrt{3}}{\sqrt{2}} = 2\sqrt{6}, \quad m = s^2 = 24.$$

The perimeter itself contains a radical, but the question asks for its square:

$$n = 4s = 8\sqrt{6}, \quad n^2 = 64 \times 6 = 384.$$

Therefore $m + n^2 = 24 + 384 = 408$. The other choices 414, 312 and 396 do not equal this sum; in particular one must square the whole perimeter rather than merely multiply the square area by four.

Step 3: Conclusion The required value is 408. Hence, the correct option is (A).

Quick Tip to Solve: For any square, perimeter squared is sixteen times its area. Here the required combination is therefore $16m$, once the inradius has given $m = 24$.

2. Let a variable line of slope $m > 0$ passing through the point $(4, -9)$ intersect the coordinate axes at the points A and B . The minimum value of the sum of the distances of A and B from the origin is

[4 marks]

- (A) 30
- (B) 15
- (C) 10
- (D) 25

Correct Answer: (D) 25

Solution:

Concept:

- A line through (x_0, y_0) with slope m has equation $y - y_0 = m(x - x_0)$. Set one coordinate to zero to find each intercept.
- Distance from the origin is nonnegative. For $m > 0$, AM–GM gives $4m + 9/m \geq 2\sqrt{36} = 12$, with equality when $4m = 9/m$.

Step 1: Express both distances in terms of the slope The equation of the variable line is

$$y + 9 = m(x - 4), \quad \text{or} \quad y = mx - 4m - 9.$$

Putting $y = 0$ gives its horizontal intercept $x = 4 + 9/m$, which is positive because $m > 0$. Putting $x = 0$ gives its vertical intercept $y = -4m - 9$, which is negative. Thus the distances, irrespective of the names assigned to the two intercepts, are

$$OA = 4 + \frac{9}{m}, \quad OB = 4m + 9.$$

Taking the absolute value of the negative vertical intercept is essential: a signed coordinate is not a distance.

Step 2: Minimise and check that equality is attainable Adding the positive distances yields

$$S = 13 + 4m + \frac{9}{m}.$$

Since both variable terms are positive, the arithmetic–geometric mean inequality gives

$$S \geq 13 + 2\sqrt{4m \cdot \frac{9}{m}} = 13 + 12 = 25.$$

Equality requires $4m = 9/m$, or $m^2 = 9/4$. The admissible solution is $m = 3/2$. At this slope the intercept distances are 10 and 15, so their sum is indeed 25. This verifies an attained minimum rather than only a lower bound. The numbers 10 and 15 are the individual distances at the minimiser, while 30 is larger than the achieved sum.

Step 3: Conclusion The minimum sum of distances is $\boxed{25}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the signs of the intercepts before adding distances. The resulting positive pair $4m$ and $9/m$ makes AM–GM immediate.

3. Let $A = \{n \in [100, 700] \cap \mathbb{N} : n \text{ is neither a multiple of 3 nor a multiple of 4}\}$. Then the number of elements in A is [4 marks]

- (A) 300
(B) 310
(C) 290
(D) 280

Correct Answer: (A) 300

Solution:

Concept:

- For integers in an inclusive interval $[L, U]$, the number of multiples of d is $\lfloor U/d \rfloor - \lfloor (L-1)/d \rfloor$.
- The condition neither a multiple of 3 nor a multiple of 4 means the complement of their union. Inclusion–exclusion counts the common multiples, which are multiples of 12, only once.

Step 1: Count the integers and the excluded multiples The full interval includes both endpoints, so it contains

$$700 - 100 + 1 = 601$$

integers. Let B contain the multiples of 3 in this interval and C the multiples of 4. Their counts are

$$|B| = \left\lfloor \frac{700}{3} \right\rfloor - \left\lfloor \frac{99}{3} \right\rfloor = 233 - 33 = 200,$$

$$|C| = \left\lfloor \frac{700}{4} \right\rfloor - \left\lfloor \frac{99}{4} \right\rfloor = 175 - 24 = 151.$$

The second count includes both 100 and 700 because both are divisible by 4.

Step 2: Restore the common multiples once and take the complement An integer divisible by both 3 and 4 is divisible by $\text{lcm}(3, 4) = 12$. There are

$$|B \cap C| = \left\lfloor \frac{700}{12} \right\rfloor - \left\lfloor \frac{99}{12} \right\rfloor = 58 - 8 = 50.$$

Therefore the number excluded is $200 + 151 - 50 = 301$, giving

$$|A| = 601 - 301 = 300.$$

Subtracting 200 and 151 without restoring 50 would exclude common multiples twice. The alternatives 310, 290 and 280 do not satisfy the complete inclusion–exclusion count with these inclusive endpoints.

Step 3: Conclusion The number of elements in the set is $\boxed{300}$. Hence, the correct option is (A).

Quick Tip to Solve: Use floor counts with the lower endpoint reduced by one, then subtract the union of the forbidden divisibility classes.

4. Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y = \sqrt{3}|x|$. Then, which one of the following points lies on the circle C ? [4 marks]

- (A) (1, 2)
- (B) (2, 2)
- (C) (1, 1)
- (D) (2, 4)

Correct Answer: For the equations as printed, the minimum circle has centre $(0, 3\sqrt{3} - 2)$ and radius $(3\sqrt{3} - 2)/2$. None of (A), (B), (C) or (D) lies on it, so the printed MCQ has no correct listed option.

Solution:

Concept:

- A circle tangent to both arms of $y = \sqrt{3}|x|$ has centre on the positive vertical angle bisector. Write its centre as $(0, h)$, with $h > 0$; the perpendicular distance to either supporting line gives $r = h/2$.
- For a circle to touch a parabola without cutting it, its radius is the minimum distance from its centre to the parabola. Minimise the squared distance before imposing tangency.

Step 1: Determine the least tangent circle from the literal equations The perpendicular feet must lie on the two upper rays, so the other angle bisectors do not supply admissible centres. At a parabola point $(x, 6 - x^2)$, put $t = x^2 \geq 0$. Its squared distance to $(0, h)$ is

$$D^2 = t + (6 - h - t)^2.$$

For $h \leq 11/2$, this quadratic is minimised at $t = 11/2 - h$, with minimum $23/4 - h$. Tangency therefore requires

$$\frac{h^2}{4} = \frac{23}{4} - h, \quad h^2 + 4h - 23 = 0.$$

The positive root is $h = 3\sqrt{3} - 2$, giving $r = (3\sqrt{3} - 2)/2$. It is in the required range and gives two symmetric points of contact. For $h > 11/2$, the minimum is at $t = 0$; the admissible tangency equation $(6 - h)^2 = h^2/4$ gives $h = 12$, a larger circle. Thus the smaller circle obtained above is the required minimum.

Step 2: Test the printed choices against the resulting circle The required circle is

$$x^2 + (y - h)^2 = \frac{h^2}{4}, \quad h = 3\sqrt{3} - 2.$$

Using $h^2 = 23 - 4h$, the residual of a candidate point is

$$F(x, y) = x^2 + y^2 + \frac{69}{4} - (2y + 3)h.$$

For the four choices, respectively, it is

$$\begin{aligned} F(1, 2) &= \frac{89}{4} - 7h, & F(2, 2) &= \frac{101}{4} - 7h, \\ F(1, 1) &= \frac{77}{4} - 5h, & F(2, 4) &= \frac{149}{4} - 11h. \end{aligned}$$

Every residual is nonzero because h is irrational and each displayed coefficient of h is a nonzero integer. Hence none of the four printed points lies on the minimum circle. In particular, using the vertex-tangent circle with centre $(0, 4)$ and radius 2 would overlook smaller off-axis tangencies and would not solve the stated minimum-area problem.

Step 3: Conclusion For the equations as printed, the minimum circle has centre $(0, 3\sqrt{3} - 2)$ and radius $(3\sqrt{3} - 2)/2$. None of (A), (B), (C) or (D) lies on it, so the printed MCQ has no correct listed option.

Quick Tip to Solve: For minimum-circle tangency to a parabola, minimise the distance to the whole parabola; assuming the point of contact is its vertex can miss a smaller tangent circle.

5. If $A(3, 1, -1)$, $B(\frac{5}{3}, \frac{7}{3}, \frac{1}{3})$, $C(2, 2, 1)$ and $D(\frac{10}{3}, \frac{2}{3}, \frac{-1}{3})$ are the vertices of a quadrilateral $ABCD$, then its area is [4 marks]

- (A) $\frac{4\sqrt{2}}{3}$
 (B) $\frac{2\sqrt{2}}{3}$
 (C) $\frac{5\sqrt{2}}{3}$
 (D) $2\sqrt{2}$

Correct Answer: (A) $\frac{4\sqrt{2}}{3}$

Solution:

Concept:

- The magnitude of the cross product of adjacent vectors is the area of the parallelogram they span: $S = |\mathbf{u} \times \mathbf{v}|$.
- Before using the parallelogram area formula for a quadrilateral, check that its opposite side vectors are equal. A triangle would require the extra factor $1/2$.

Step 1: Identify the adjacent vectors and verify the shape Using the vertices in their given order,

$$\overrightarrow{AB} = B - A = \frac{1}{3}(-4, 4, 4), \quad \overrightarrow{AD} = D - A = \frac{1}{3}(1, -1, 2).$$

Also,

$$\overrightarrow{BC} = C - B = \frac{1}{3}(1, -1, 2) = \overrightarrow{AD},$$

$$\overrightarrow{DC} = C - D = \frac{1}{3}(-4, 4, 4) = \overrightarrow{AB}.$$

The opposite sides are equal as vectors, so $ABCD$ is a parallelogram. This check justifies using one cross product directly for its whole area, rather than assuming every spatial quadrilateral has that formula.

Step 2: Evaluate the cross product and its magnitude Taking the product of the adjacent side vectors

gives

$$\vec{AB} \times \vec{AD} = \frac{1}{9} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 4 & 4 \\ 1 & -1 & 2 \end{vmatrix} = \frac{1}{9}(12, 12, 0).$$

Hence its area is

$$S = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{4}{3}\right)^2} = \frac{4\sqrt{2}}{3}.$$

The value $2\sqrt{2}/3$ would be the area of only one of the two triangles made by a diagonal. The values $5\sqrt{2}/3$ and $2\sqrt{2}$ do not equal the magnitude of the cross product. The side vectors are not parallel, as the product is nonzero.

Step 3: Conclusion The area is $\boxed{\frac{4\sqrt{2}}{3}}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: For four spatial vertices, first test $A + C = B + D$. When it holds, the quadrilateral is a parallelogram and one adjacent-side cross product gives its area.

6. Let the relations R_1 and R_2 on the set $X = \{1, 2, 3, \dots, 20\}$ be given by $R_1 = \{(x, y) : 2x - 3y = 2\}$ and $R_2 = \{(x, y) : -5x + 4y = 0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2 , respectively, in order to make the relations symmetric, then $M + N$ equals [4 marks]

- (A) 16
- (B) 12
- (C) 8
- (D) 10

Correct Answer: (D) 10

Solution:

Concept:

- A relation is symmetric when every ordered pair (x, y) is accompanied by (y, x) . Its minimum symmetric enlargement is $R \cup R^{-1}$.
- The number of pairs to add is $|R^{-1} \setminus R|$. Count the existing pairs within the stated finite set, then check whether any reversed pair is already present.

Step 1: List the first relation and its missing reverses For R_1 , the condition $2x - 3y = 2$ gives $x = (3y + 2)/2$. Thus y must be even. The bounds $1 \leq x, y \leq 20$ allow $y = 2, 4, 6, 8, 10, 12$, producing

$$R_1 = \{(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)\}.$$

For a reversed pair to be present as well we would need both $2x - 3y = 2$ and $2y - 3x = 2$. Solving gives $x = y = -2$, outside the set. Consequently none of the six required reversed pairs is already present, and $M = 6$.

Step 2: Count the second relation and combine the additions For R_2 , we have $4y = 5x$. Since 4 and

5 are coprime, write $x = 4k$, $y = 5k$. The bounds give $k = 1, 2, 3, 4$, so

$$R_2 = \{(4, 5), (8, 10), (12, 15), (16, 20)\}.$$

If the reverse of one pair also belonged to R_2 , both $4y = 5x$ and $4x = 5y$ would hold, forcing $x = y = 0$, which is not allowed. Hence all four reverse pairs must be added, giving $N = 4$. Each added pair then has its reverse in the original relation, so no further additions are needed. Thus $M + N = 6 + 4 = 10$, ruling out 16, 12 and 8.

Step 3: Conclusion The minimum combined number of additions is $\boxed{10}$. Hence, the correct option is (D).

Quick Tip to Solve: Count reverse pairs that are missing; symmetry does not require adding every diagonal pair. Diagonal pairs are required for reflexivity, a different property.

7. Let the area of the region enclosed by the curves $y = 3x$, $2y = 27 - 3x$ and $y = 3x - x\sqrt{x}$ be A . Then $10A$ is equal to [4 marks]

- (A) 172
- (B) 154
- (C) 162
- (D) 184

Correct Answer: (C) 162

Solution:

Concept:

- The area between two graphs on an interval is the integral of upper ordinate minus lower ordinate. The upper graph can change where two bounding lines intersect.
- For $x \geq 0$, the nonlinear boundary is $y = 3x - x^{3/2}$, and $\int x^{3/2} dx = 2x^{5/2}/5$.

Step 1: Find the three boundary intersections The line $y = 3x$ meets the nonlinear curve when $x^{3/2} = 0$, at $(0, 0)$. The two straight lines meet when

$$3x = \frac{27 - 3x}{2}, \quad \text{so } (x, y) = (3, 9).$$

For the other line and the nonlinear curve, put $t = \sqrt{x} \geq 0$. Their intersection equation becomes

$$2t^3 - 9t^2 + 27 = (t - 3)^2(2t + 3) = 0.$$

Its only admissible root is $t = 3$, hence $x = 9$ and $y = 0$. The repeated root expresses tangency but still gives the endpoint of the enclosed region.

Step 2: Integrate with the appropriate upper boundary From 0 to 3 the upper graph is $3x$, while from

3 to 9 it is $(27 - 3x)/2$. The curve is the lower boundary throughout. Therefore

$$A = \int_0^3 x^{3/2} dx + \int_3^9 \left(\frac{27}{2} - \frac{9x}{2} + x^{3/2} \right) dx.$$

Combine the power terms before evaluating:

$$A = \frac{2}{5}(9^{5/2}) + \left[\frac{27x}{2} - \frac{9x^2}{4} \right]_3^9 = \frac{486}{5} - 81 = \frac{81}{5}.$$

Thus $10A = 162$. Using a single straight line as the upper boundary over the whole interval would count a different region. The alternatives 172, 154 and 184 differ from the exact integral.

Step 3: Conclusion The area is $81/5$ square units and $10A = 162$. Hence, the correct option is (C).

Quick Tip to Solve: Split the integral where the two upper boundary lines cross; combine the identical $x^{3/2}$ terms over the full interval to shorten the calculation.

8.

$$\text{If } f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}, \text{ then}$$

[4 marks]

(A) $f''(0) = 0$

(B) $f''(0) = 1$

(C) $f''\left(\frac{2}{\pi}\right) = \frac{24-\pi^2}{2\pi}$

(D) $f''\left(\frac{2}{\pi}\right) = \frac{12-\pi^2}{2\pi}$

Correct Answer: (C) $f''\left(\frac{2}{\pi}\right) = \frac{24-\pi^2}{2\pi}$

Solution:

Concept:

- At a separately defined point, compute derivatives by their limits; a formula derived only for $x \neq 0$ cannot simply be substituted at zero.
- For $x \neq 0$, use the product and chain rules, including $\frac{d}{dx}(1/x) = -1/x^2$.

Step 1: Test the derivative at zero The derivative at zero is

$$f'(0) = \lim_{h \rightarrow 0} \frac{h^3 \sin(1/h)}{h} = \lim_{h \rightarrow 0} h^2 \sin(1/h) = 0.$$

For nonzero h , differentiation gives $f'(h) = 3h^2 \sin(1/h) - h \cos(1/h)$. Hence

$$\frac{f'(h) - f'(0)}{h} = 3h \sin(1/h) - \cos(1/h).$$

The first term tends to zero but the cosine oscillates. For example, along $h = 1/(2k\pi)$ and $h = 1/((2k+1)\pi)$ the limits are -1 and 1 , respectively. Thus $f''(0)$ does not exist; neither (A) nor (B) is

true.

Step 2: Differentiate away from zero and evaluate Differentiate the nonzero-point formula term by term:

$$f''(x) = 6x \sin(1/x) - 3 \cos(1/x) - \cos(1/x) - \frac{\sin(1/x)}{x}.$$

At $x = 2/\pi$, the sine is 1 and the cosine is 0, so

$$f''(2/\pi) = \frac{12}{\pi} - \frac{\pi}{2} = \frac{24 - \pi^2}{2\pi}.$$

Option (D) has 12 rather than 24 in the numerator and is therefore not this value.

Step 3: Conclusion The second derivative at zero does not exist, while $f''(2/\pi) = (24 - \pi^2)/(2\pi)$. Hence, the correct option is (C).

Quick Tip to Solve: For oscillating piecewise functions, apply the derivative definition at the joining point before using an ordinary derivative formula elsewhere.

9. The shortest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ is [4 marks]

- (A) $8\sqrt{3}$
- (B) $6\sqrt{3}$
- (C) $5\sqrt{3}$
- (D) $4\sqrt{3}$

Correct Answer: (D) $4\sqrt{3}$

Solution:

Concept:

- For lines through $\mathbf{a}_1, \mathbf{a}_2$ with nonparallel directions $\mathbf{b}_1, \mathbf{b}_2$, the shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)| / |\mathbf{b}_1 \times \mathbf{b}_2|$.
- The cross product supplies a common perpendicular; projecting the joining vector onto its unit direction removes components along either line.

Step 1: Read the points and directions The symmetric equations give

$$\begin{aligned}\mathbf{a}_1 &= (3, -15, 9), & \mathbf{b}_1 &= (2, -7, 5), \\ \mathbf{a}_2 &= (-1, 1, 9), & \mathbf{b}_2 &= (2, 1, -3).\end{aligned}$$

In particular, $y + 15$ corresponds to a point with $y = -15$, while $x + 1$ corresponds to $x = -1$. Thus the joining vector is $\mathbf{a}_2 - \mathbf{a}_1 = (-4, 16, 0)$. Its length itself is not the shortest distance: the joining segment need not be perpendicular to the lines.

Step 2: Project along the common perpendicular The cross product is

$$\mathbf{b}_1 \times \mathbf{b}_2 = (21 - 5, 10 + 6, 2 + 14) = (16, 16, 16).$$

Its magnitude is $16\sqrt{3}$, so the lines are nonparallel. The scalar triple product is

$$(-4, 16, 0) \cdot (16, 16, 16) = -64 + 256 = 192.$$

Therefore

$$d = \frac{192}{16\sqrt{3}} = \frac{12}{\sqrt{3}} = 4\sqrt{3}.$$

The absolute value guarantees a nonnegative distance independent of which joining vector is chosen. Options (A), (B) and (C) give larger multiples of $\sqrt{3}$ and disagree with this projection.

Step 3: Conclusion The shortest distance is $4\sqrt{3}$ units. Hence, the correct option is (D).

Quick Tip to Solve: When a cross product has a common factor, cancel it from the triple-product numerator and its magnitude before simplifying the distance.

10. Let $y = y(x)$ be the solution of the differential equation $(2x \log_e x) \frac{dy}{dx} + 2y = \frac{3}{x} \log_e x, x > 0$ and $y(e^{-1}) = 0$. Then, $y(e)$ is equal to [4 marks]

- (A) $-\frac{3}{e}$
- (B) $-\frac{3}{2e}$
- (C) $-\frac{2}{3e}$
- (D) $-\frac{2}{e}$

Correct Answer: The initial condition does not determine the value across the singular point $x = 1$; no listed option is forced.

Solution:

Concept:

- On an interval with $x > 0$ and $x \neq 1$, the linear equation $y' + Py = Q$ uses the integrating factor $\exp(\int P dx)$.
- Here a signed integrating factor $\ln x$ works on either interval, since its derivative is $1/x$; integration by parts evaluates $\int (\ln x)/x^2 dx$.

Step 1: Find the integrating factor and integrate Dividing by $2x \ln x$ gives

$$y' + \frac{y}{x \ln x} = \frac{3}{2x^2}.$$

Multiplication by $\ln x$ makes the left side a product derivative:

$$\frac{d}{dx}(y \ln x) = \frac{3 \ln x}{2x^2}.$$

Using integration by parts with $u = \ln x$, $dv = x^{-2}dx$, obtain

$$\int \frac{\ln x}{x^2} dx = -\frac{\ln x + 1}{x}.$$

Thus $y \ln x = -3(\ln x + 1)/(2x) + C$. This identity can also be differentiated directly to check the

signs and the factor $3/2$.

Step 2: Check the initial condition and the singular point At $x = e^{-1}$, $\ln x = -1$ and $y = 0$, so $C = 0$ on the interval $(0, 1)$. Hence

$$y = -\frac{3(\ln x + 1)}{2x \ln x} \quad (0 < x < 1).$$

This tends to $+\infty$ as $x \rightarrow 1^-$. There can therefore be no differentiable solution on the whole domain $x > 0$ satisfying the initial condition. If solutions are instead allowed separately on the two sides of 1, the constant C_+ on $(1, \infty)$ is independent of the initial condition. In that interval,

$$y(e) = -\frac{3}{e} + C_+,$$

so any real value is possible. Mechanically reusing $C = 0$ on the other interval produces $-3/e$, the value in (A), but the initial condition does not justify that reuse. Options (B), (C) and (D) could also be obtained by suitable independent constants; none is uniquely forced.

Step 3: Conclusion The initial condition cannot determine the requested value across the singular point $x = 1$. A global differentiable solution does not exist, and separate interval solutions leave the requested value arbitrary. No unique option is justified.

Quick Tip to Solve: Before transporting an initial condition to another point, check that both points lie in one interval where the normalized linear equation has continuous coefficients.

11. Let α, β be the distinct roots of the equation $x^2 - (t^2 - 5t + 6)x + 1 = 0, t \in \mathbb{R}$ and $a_n = \alpha^n + \beta^n$. Then the minimum value of $\frac{a_{2023} + a_{2025}}{a_{2024}}$ is [4 marks]

- (A) $-1/2$
- (B) $-1/4$
- (C) $1/4$
- (D) $1/2$

Correct Answer: (B) $-1/4$

Solution:

Concept:

- If α, β solve $z^2 - sz + 1 = 0$, then $\alpha + \beta = s$, $\alpha\beta = 1$, and each root satisfies $z^{n+1} + z^{n-1} = sz^n$.
- The required quotient therefore simplifies by a recurrence, with no need to compute enormous powers. Minimize the remaining quadratic by completing its square.

Step 1: Reduce the power-sum quotient Put $s = t^2 - 5t + 6$. Multiplying the two root equations by α^{2023} and β^{2023} , respectively, and adding gives

$$a_{2025} - sa_{2024} + a_{2023} = 0.$$

For values where the printed quotient is defined,

$$\frac{a_{2023} + a_{2025}}{a_{2024}} = s = t^2 - 5t + 6.$$

The distinct-root hypothesis excludes a repeated quadratic root; it does not require the roots to be real. Restricting α, β to real numbers would impose an extra condition absent from the question.

Step 2: Minimize and verify attainment Complete the square:

$$t^2 - 5t + 6 = \left(t - \frac{5}{2}\right)^2 - \frac{1}{4} \geq -\frac{1}{4}.$$

At $t = 5/2$, the discriminant is $1/16 - 4 = -63/16 \neq 0$, so the roots are distinct. The denominator is nonzero here. To check this exactly, put $b_n = 4^n a_n$. The recurrence becomes $b_n = -b_{n-1} - 16b_{n-2}$, starting with $b_0 = 2, b_1 = -1$. Every b_n for $n \geq 1$ is an odd integer by induction, so $b_{2024} \neq 0$. Hence the lower bound is attained. Option (A) is below this bound; (C) and (D) are positive and cannot be the minimum.

Step 3: Conclusion The minimum value is $-1/4$, attained at $t = 5/2$. Hence, the correct option is (B).

Quick Tip to Solve: For reciprocal quadratic roots, look for a recurrence linking adjacent power sums before trying to evaluate any large exponent.

12. For $\alpha, \beta \in \mathbb{R}$ and a natural number n , let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then $2A_{10} - A_8$ is [4 marks]

(A) $4\alpha + 2\beta$

(B) 0

(C) $2n$

(D) $2\alpha + 4\beta$

Correct Answer: (A) $4\alpha + 2\beta$

Solution:

Concept:

- Adding a multiple of one row to another leaves a determinant unchanged. Such operations can create zeros even when entries contain parameters.
- Expansion along a row with two zeros uses only one minor, whose cofactor sign is $(-1)^{i+j}$.

Step 1: Eliminate the repeated first two columns Write $c = n^2/2 + \alpha$ and $d = n(3n - 1)/2$. Apply $R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - 3R_1$. The determinant becomes

$$A_r = \begin{vmatrix} r & 1 & c \\ 0 & 0 & -\beta - 2\alpha \\ -2 & 0 & d - 3c \end{vmatrix}.$$

Indeed, the first two entries of the second row vanish because they were exactly twice those of the first row. In the third row, $(3r - 2) - 3r = -2$ and $3 - 3 = 0$. The third-column expressions need not be expanded further, which avoids unnecessary algebra involving n .

Step 2: Expand and combine the requested terms Expand along the second row. Its only nonzero entry is in column 3, with cofactor sign $(-1)^5 = -1$. Hence

$$A_r = (-\beta - 2\alpha)(-1) \begin{vmatrix} r & 1 \\ -2 & 0 \end{vmatrix} = 2\beta + 4\alpha.$$

This is independent of both r and n . Therefore

$$2A_{10} - A_8 = 2(4\alpha + 2\beta) - (4\alpha + 2\beta) = 4\alpha + 2\beta.$$

Option (B) is not identically zero; (C) wrongly leaves a dependence on n ; (D) interchanges the coefficients of α and β . These special expressions can coincide for individual parameter choices, but only (A) is the general identity.

Step 3: Conclusion For arbitrary allowed parameters, $2A_{10} - A_8 = 4\alpha + 2\beta$. Hence, the correct option is (A).

Quick Tip to Solve: When two columns are nearly proportional, subtract row multiples first; the remaining small difference often determines the entire determinant.

13. Let $f : (-\infty, \infty) - \{0\} \rightarrow \mathbb{R}$ be a differentiable function such that $f'(1) = \lim_{a \rightarrow \infty} a^2 f\left(\frac{1}{a}\right)$. Then $\lim_{a \rightarrow \infty} \frac{a(a+1)}{2} \tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2 \log_e a$ is equal to [4 marks]

- (A) $\frac{5}{2} + \frac{\pi}{8}$
- (B) $\frac{3}{8} + \frac{\pi}{4}$
- (C) $\frac{3}{4} + \frac{\pi}{8}$
- (D) $\frac{3}{2} + \frac{\pi}{4}$

Correct Answer: The limit is $+\infty$; all four printed options are finite.

Solution:

Concept:

- The limit of a sum with a positive quadratic term is controlled by that term when the remaining negative term grows only logarithmically.
- For $a \rightarrow +\infty$, $\arctan(1/a) = 1/a - 1/(3a^3) + O(a^{-5})$. This expansion follows from the Taylor series of inverse tangent at zero.

Step 1: Read the displayed expression literally The expression to be evaluated contains no f :

$$L = \lim_{a \rightarrow \infty} \left[\frac{a(a+1)}{2} \arctan(1/a) + a^2 - 2 \ln a \right].$$

The differentiability condition on f cannot remove any of these displayed terms. For positive a ,

the inverse tangent and its multiplier are both positive. Also $\ln a/a^2 \rightarrow 0$, for example by applying l'Hopital's rule to that quotient. Thus $a^2 - 2 \ln a \rightarrow +\infty$. Even discarding the positive inverse-tangent term already proves that the entire displayed expression diverges to $+\infty$.

Step 2: Check the growth and compare all options More explicitly, multiply the inverse-tangent expansion by $(a^2 + a)/2$:

$$\frac{a(a+1)}{2} \arctan(1/a) = \frac{a}{2} + \frac{1}{2} + O(1/a).$$

The full expression is therefore

$$a^2 + \frac{a}{2} - 2 \ln a + \frac{1}{2} + O(1/a),$$

which tends to positive infinity. Each listed answer, $5/2 + \pi/8$, $3/8 + \pi/4$, $3/4 + \pi/8$, and $3/2 + \pi/4$, is finite; consequently none equals the printed limit. This conclusion follows from the literal expression and does not require guessing a replacement involving the unused function.

Step 3: Conclusion The printed limit diverges to $+\infty$, so none of (A), (B), (C) or (D) is defensible as the answer.

Quick Tip to Solve: Before using an unrelated function condition, check whether the requested expression actually contains that function and compare the largest growth terms.

14.

$$\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{(\cos^3 x + \sin^3 x)^2} dx \text{ is equal to}$$

[4 marks]

- (A) 1/9
- (B) 1/6
- (C) 1/3
- (D) 1/12

Correct Answer: (B) 1/6

Solution:

Concept:

- When sine and cosine occur in matching homogeneous powers, $t = \tan x$ often converts a trigonometric integral into a rational integral.
- Under $t = \tan x$, $\sin x = t/\sqrt{1+t^2}$, $\cos x = 1/\sqrt{1+t^2}$, and $dx = dt/(1+t^2)$. Transform the endpoints along with the integrand.

Step 1: Convert to a rational integral Let the given integral be I . The interval $[0, \pi/4]$ lies where cosine is positive, so these square-root expressions have the stated signs. The numerator and denominator become

$$\cos^2 x \sin^2 x = \frac{t^2}{(1+t^2)^2}, \quad (\cos^3 x + \sin^3 x)^2 = \frac{(1+t^3)^2}{(1+t^2)^3}.$$

After also multiplying by $dx = dt/(1+t^2)$, all powers of $1+t^2$ cancel. The bounds are $t = 0$ and $t = 1$, yielding

$$I = \int_0^1 \frac{t^2}{(1+t^3)^2} dt.$$

Keeping the differential factor is essential: otherwise an extra $1+t^2$ would remain.

Step 2: Use the derivative of the denominator Now set $u = 1+t^3$, so $du = 3t^2 dt$. As t runs from 0 to 1, u runs from 1 to 2. Therefore

$$I = \frac{1}{3} \int_1^2 u^{-2} du = \frac{1}{3} \left[-\frac{1}{u} \right]_1^2 = \frac{1}{3} \left(1 - \frac{1}{2} \right) = \frac{1}{6}.$$

An equivalent check is the antiderivative $-1/[3(1+t^3)]$; differentiating it recovers $t^2/(1+t^3)^2$. The other values $1/9$, $1/3$, and $1/12$ disagree with the evaluated endpoint difference, so (A), (C) and (D) are rejected.

Step 3: Conclusion The definite integral equals $1/6$. Hence, the correct option is (B).

Quick Tip to Solve: For a squared sum of equal trigonometric powers, try tangent substitution and simplify the differential before attempting any expansion.

15. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is [4 marks]

- (A) 1.94
- (B) $\sqrt{3.96}$
- (C) $\sqrt{3.86}$
- (D) 1.8

Correct Answer: (B) $\sqrt{3.96}$

Solution:

Concept:

- The population variance of n observations is $\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$, where $\bar{x} = \frac{\sum x_i}{n}$. Both sums must be corrected when one observation changes.
- Replacing the wrongly recorded value 8 by 12 increases the sum by $12 - 8$, but increases the sum of squares by $12^2 - 8^2$. The number of observations remains 20.

Step 1: Recover the two recorded sums The given mean and standard deviation describe the erroneous data. Hence their sum is $20(10) = 200$. Since the given standard deviation is 2, the recorded variance is 4. Therefore

$$\frac{\sum x_i^2}{20} - 10^2 = 4, \quad \sum x_i^2 = 20(104) = 2080.$$

The standard deviation is not itself a sum, so it cannot be corrected by merely adding the difference between the two observations.

Step 2: Correct the sums and recompute the variance Remove the erroneous entry and insert the correct one in each sum:

$$\sum x_i = 200 - 8 + 12 = 204, \quad \bar{x} = \frac{204}{20} = 10.2.$$

$$\sum x_i^2 = 2080 - 64 + 144 = 2160.$$

Thus

$$\sigma^2 = \frac{2160}{20} - (10.2)^2 = 108 - 104.04 = 3.96.$$

The square root must be taken only after the corrected variance has been calculated. Values 1.94 and 1.8 do not square to 3.96, while the radicand 3.86 is also incorrect.

Step 3: Conclusion The correct standard deviation is $\sqrt{3.96}$, approximately 1.99, in the same units as the observations. Hence, the correct option is (B).

Quick Tip to Solve: For a replaced observation, update both $\sum x_i$ and $\sum x_i^2$; keeping the old mean in the variance formula gives an incorrect correction.

16. Let $y = y(x)$ be the solution of the differential equation $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$, $y(1) = 0$. Then $y(0)$ is [4 marks]

- (A) $\frac{1}{4} (e^{\pi/2} - 1)$
- (B) $\frac{1}{2} (1 - e^{\pi/2})$
- (C) $\frac{1}{4} (1 - e^{\pi/2})$
- (D) $\frac{1}{2} (e^{\pi/2} - 1)$

Correct Answer: (B) $\frac{1}{2} (1 - e^{\pi/2})$

Solution:

Concept:

- A first-order linear equation $y' + P(x)y = Q(x)$ has integrating factor $e^{\int P(x) dx}$. Multiplying by this factor turns the left-hand side into a product derivative.
- Here the substitution $t = \tan^{-1} x$ gives $dt = dx/(1+x^2)$, converting the equation into $dy/dt + y = e^t$. The condition $x = 1$ corresponds to $t = \pi/4$.

Step 1: Transform and integrate the linear equation Because $dt/dx = 1/(1+x^2)$, the chain rule gives $(1+x^2)dy/dx = dy/dt$. Thus

$$\frac{dy}{dt} + y = e^t.$$

Its integrating factor is e^t . Multiplying throughout gives

$$\frac{d}{dt}(ye^t) = e^{2t}.$$

Integrating, with a single arbitrary constant C ,

$$ye^t = \frac{1}{2}e^{2t} + C, \quad y = \frac{1}{2}e^t + Ce^{-t}.$$

Step 2: Apply the initial condition and evaluate at zero At $x = 1$, $t = \pi/4$ and $y = 0$. Substitution into the integrated equation gives

$$0 = \frac{1}{2}e^{\pi/2} + C, \quad C = -\frac{1}{2}e^{\pi/2}.$$

At $x = 0$, $t = 0$, so

$$y(0) = \frac{1}{2} - \frac{1}{2}e^{\pi/2}.$$

The value is negative since $e^{\pi/2} > 1$, ruling out the two positive options (A) and (D). Option (C) has the right sign but an erroneous coefficient $1/4$; integration of e^{2t} supplies $1/2$, not $1/4$.

Step 3: Conclusion

$$y(0) = \frac{1}{2} \left(1 - e^{\pi/2} \right).$$

Hence, the correct option is (B).

Quick Tip to Solve: When both $1 + x^2$ and $\tan^{-1} x$ occur, try $t = \tan^{-1} x$; transform the initial condition at the same time.

17. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is [4 marks]

- (A) 56
- (B) 16
- (C) 24
- (D) 48

Correct Answer: (B) 16

Solution:

Concept:

- Any three distinct vertices of a regular octagon form a triangle, because no three of its vertices are collinear. Therefore the unrestricted count is $\binom{8}{3}$.
- A forbidden triangle contains at least one adjacent pair of octagon vertices. Inclusion-exclusion corrects double counting when a triangle contains two octagon sides.

Step 1: Count all triangles and the side incidences The number of unrestricted triangles is

$$\binom{8}{3} = \frac{8 \cdot 7 \cdot 6}{6} = 56.$$

For each of the 8 octagon sides, a triangle containing that side is obtained by choosing its third vertex

among the remaining 6. Thus there are $8 \cdot 6 = 48$ side-triangle incidences. This is not yet the number of forbidden triangles, because triangles formed by three consecutive vertices are counted twice.

Step 2: Remove the double counting and take the complement A triangle with two octagon sides must consist of three consecutive vertices. There are exactly 8 such triples, one for each possible middle vertex. Each was counted twice in 48, so subtract it once. A triangle cannot contain three octagon sides, since an octagon has no three-edge cycle. Hence the number of forbidden triangles is

$$48 - 8 = 40.$$

The required number, with no octagon side, is

$$56 - 40 = 16.$$

Option (A) counts every triangle, while (D) counts side incidences before correcting repetitions. Option (C) does not equal the complement after the eight repeated triples have been restored.

Step 3: Conclusion There are 16 triangles satisfying the condition. Hence, the correct option is (B).

Quick Tip to Solve: Count forbidden adjacent pairs, but restore each triangle on three consecutive polygon vertices once: it was subtracted twice.

18. The function $f(x) = \frac{x^2+2x-15}{x^2-4x+9}$, $x \in \mathbb{R}$ **is**

[4 marks]

- (A) both one-one and onto.
- (B) onto but not one-one.
- (C) neither one-one nor onto.
- (D) one-one but not onto.

Correct Answer: (C) neither one-one nor onto.

Solution:

Concept:

- A real function is one-one only if distinct inputs always have distinct outputs. A single pair of distinct inputs giving the same value disproves injectivity.
- For a real-valued function on $\mathbb{R}$, being onto $\mathbb{R}$ means that every real number occurs as an output. A strict bound on all outputs disproves surjectivity.

Step 1: Check the domain and find a repeated output The denominator satisfies

$$x^2 - 4x + 9 = (x - 2)^2 + 5 > 0$$

for every real x , so the stated domain really is all of $\mathbb{R}$. Factor the numerator:

$$x^2 + 2x - 15 = (x + 5)(x - 3).$$

Consequently $f(-5) = f(3) = 0$. The two inputs are distinct and both denominators are nonzero.

Thus f is not one-one. This rules out options (A) and (D), without requiring a derivative calculation.

Step 2: Exhibit a real number outside the range Compare the output with 2, using the denominator already shown positive:

$$2 - f(x) = \frac{2(x^2 - 4x + 9) - (x^2 + 2x - 15)}{x^2 - 4x + 9}.$$

$$2 - f(x) = \frac{x^2 - 10x + 33}{x^2 - 4x + 9} = \frac{(x - 5)^2 + 8}{(x - 2)^2 + 5} > 0.$$

Therefore $f(x) < 2$ for every real input, so 2 is never attained. The function is not onto $\mathbb{R}$, ruling out option (B). Finding its entire range is unnecessary: one missing real output is sufficient.

Step 3: Conclusion The function is neither one-one nor onto. Hence, the correct option is (C).

Quick Tip to Solve: Use distinct zeros to disprove one-one behavior, then compare with a convenient constant to find an output the rational function can never attain.

19. The interval in which the function $f(x) = x^x, x > 0$, is strictly increasing is

[4 marks]

(A) $(0, \infty)$

(B) $(0, \frac{1}{e}]$

(C) $[\frac{1}{e^2}, 1)$

(D) $[\frac{1}{e}, \infty)$

Correct Answer: (D) $[\frac{1}{e}, \infty)$

Solution:

Concept:

- For $x > 0$, logarithmic differentiation handles $f(x) = x^x$: $\ln f(x) = x \ln x$. The derivative of $x \ln x$ is $1 + \ln x$.
- A positive derivative throughout the interior of an interval establishes strict increase there. A derivative equal to zero at a single endpoint does not prevent strict increase on the interval including that endpoint.

Step 1: Differentiate and determine the derivative sign Since x^x is positive for every allowed input, logarithmic differentiation is valid:

$$\frac{f'(x)}{f(x)} = \ln x + 1, \quad f'(x) = x^x (\ln x + 1).$$

The factor x^x is always positive, so the sign depends only on $\ln x + 1$. Solving the sign conditions gives

$$f'(x) < 0 \quad (0 < x < e^{-1}), \quad f'(x) > 0 \quad (x > e^{-1}).$$

At $x = e^{-1}$, the derivative is zero.

Step 2: Include the minimum endpoint and compare the intervals The function decreases up to $1/e$,

then increases after it; hence $1/e$ is its minimum point. On $[1/e, \infty)$, take any $a < b$. The mean value theorem supplies a point $c \in (a, b)$, so $c > 1/e$ and

$$f(b) - f(a) = f'(c)(b - a) > 0.$$

Thus strict increase holds even with the minimum endpoint included. Option (A) includes the decreasing part, (B) is the decreasing interval, and (C) crosses the minimum because $1/e^2 < 1/e < 1$. Only (D) lies entirely on the increasing side with its endpoint.

Step 3: Conclusion The function is strictly increasing on $[1/e, \infty)$. Hence, the correct option is (D).

Quick Tip to Solve: For x^x , differentiate $x \ln x$ and use its sign; a stationary minimum may be included as the left endpoint of a strictly increasing interval.

20. A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B . 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B , then $126p$ is [4 marks]

- (A) 54
- (B) 66
- (C) 56
- (D) 64

Correct Answer: (A) 54

Solution:

Concept:

- Let S denote standard quality. The production shares are prior probabilities: $P(A) = 0.6$, $P(B) = 0.4$. The quality rates are conditional probabilities $P(S | A) = 0.8$, $P(S | B) = 0.9$.
- The total probability is $P(S) = P(A)P(S | A) + P(B)P(S | B)$. Bayes theorem then gives $P(B | S) = P(B)P(S | B)/P(S)$.

Step 1: Find the standard-quality share from each plant The two plants exhaust all production and their contributions are disjoint. Therefore

$$P(A \cap S) = 0.6(0.8) = 0.48, \quad P(B \cap S) = 0.4(0.9) = 0.36.$$

Adding these contributions gives

$$P(S) = 0.48 + 0.36 = 0.84.$$

For example, in a proportional sample of 100 motorcycles, 48 are standard-quality motorcycles from plant A and 36 are standard-quality motorcycles from plant B . The conditioned sample therefore

contains 84 standard-quality motorcycles.

Step 2: Condition on the observed quality and multiply The selected motorcycle is already known to be standard quality, so the denominator must be the standard-quality total, not all production. Hence

$$p = P(B | S) = \frac{0.36}{0.84} = \frac{36}{84} = \frac{3}{7}.$$

The expression requested is consequently

$$126p = 126 \left(\frac{3}{7} \right) = 18 \cdot 3 = 54.$$

The other proposed values 66, 56 and 64 would correspond to posterior probabilities different from $3/7$, so none follows from the given production and quality rates.

Step 3: Conclusion The probability is $p = 3/7$, so the requested dimensionless value is $126p = 54$. Hence, the correct option is (A).

Quick Tip to Solve: In Bayes questions, multiply the production share by the quality rate first, then normalize the desired contribution by the total accepted production.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2) = \frac{125}{16}m$. Then the value of m is _____. [4 marks]

Solution:

Concept:

- For a polynomial $P(x) = 4 \prod_{j=1}^4 (x - x_j)$, evaluating at a suitable complex conjugate pair can recover a product involving $x_j^2 + 4$ without finding any root individually.
- The identity $(2i - x_j)(-2i - x_j) = x_j^2 + 4$ holds algebraically. Because P has real coefficients, $P(-2i) = \overline{P(2i)}$, so their product is a squared modulus.

Step 1: Express the desired product using polynomial values Set

$$P(x) = 4x^4 + 8x^3 - 17x^2 - 12x + 9.$$

Its leading coefficient is 4, so

$$P(2i)P(-2i) = 16 \prod_{j=1}^4 (2i - x_j)(-2i - x_j).$$

Each paired factor equals $x_j^2 + 4$. Therefore

$$\prod_{j=1}^4 (4 + x_j^2) = \frac{P(2i)P(-2i)}{16}.$$

The factor 16 is essential: both polynomial evaluations carry the leading coefficient 4. This argument also applies if roots repeat.

Step 2: Evaluate the conjugate pair and compare coefficients Using $(2i)^2 = -4$, $(2i)^3 = -8i$ and $(2i)^4 = 16$,

$$P(2i) = 64 - 64i + 68 - 24i + 9 = 141 - 88i.$$

Thus $P(-2i) = 141 + 88i$, and

$$P(2i)P(-2i) = 141^2 + 88^2 = 19881 + 7744 = 27625.$$

The stated product is $(125/16)m$, so comparison gives

$$\frac{27625}{16} = \frac{125}{16}m, \quad m = \frac{27625}{125} = 221.$$

No numerical approximation to the four roots has entered the calculation, and the equality is exact.

Step 3: Conclusion The required dimensionless integer is $m = 221$.

Quick Tip to Solve: For products $\prod (a^2 + x_j^2)$, evaluate the polynomial at ia and $-ia$, multiply, and divide by the square of the leading coefficient.

22. Let a conic C pass through the point $(4, -2)$ and $P(x, y)$, $x \geq 3$, be any point on C . Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and $(3, -5)$. If the focal distance of the point $(7, 1)$ on C is d , then $12d$ equals _____. [4 marks]

Solution:

Concept:

- The tangent slope is dy/dx , whereas the slope from $(3, -5)$ to (x, y) is $(y + 5)/(x - 3)$. Separation of variables reconstructs the conic.
- A parabola in the form $(y - k)^2 = 4a(x - h)$ has vertex (h, k) , focus $(h + a, k)$, and focal distance equal to the ordinary distance from the point to its focus.

Step 1: Form and integrate the slope equation The prescribed factor of one-half gives, for $x > 3$,

$$\frac{dy}{dx} = \frac{y + 5}{2(x - 3)}, \quad \frac{2 dy}{y + 5} = \frac{dx}{x - 3}.$$

Integrating yields $2 \log |y + 5| = \log(x - 3) + C$. Thus $(y + 5)^2 = K(x - 3)$. The given point fixes the constant: $9 = K(4 - 3)$, so $K = 9$. The resulting conic includes its vertex by continuity and is the right-opening parabola $(y + 5)^2 = 9(x - 3)$.

Step 2: Locate the focus and calculate the distance Comparison with the standard form gives $h = 3$, $k = -5$, and $a = 9/4$. Its focus is therefore $F = (21/4, -5)$. First check that the requested point belongs to the curve: at $(7, 1)$, both sides of the equation equal 36. Its focal distance is

$$d = \sqrt{\left(7 - \frac{21}{4}\right)^2 + (1 + 5)^2} = \sqrt{\frac{49}{16} + 36} = \frac{25}{4}.$$

This is positive, as a distance must be; the same result follows from the directrix $x = 3 - 9/4 = 3/4$.

Step 3: Conclusion The focal distance is $d = 25/4$ coordinate units, and the required value is $12d = \boxed{75}$.

Quick Tip to Solve: After integrating a tangent-slope condition, substitute the known point before locating the focus; the coefficient of the square is $4a$, not a .

23. If the second, third and fourth terms in the expansion of $(x + y)^n$ are 135, 30 and $\frac{10}{3}$, respectively, then $6(n^3 + x^2 + y)$ is equal to _____. [4 marks]

Solution:

Concept:

- The general term of $(x + y)^n$ is $T_{r+1} = \binom{n}{r} x^{n-r} y^r$. Ratios of consecutive nonzero terms eliminate the common high powers.
- For real x, y , the second, third and fourth terms determine n and then the ratio y/x . Use an original term to recover the actual scale.

Step 1: Use two consecutive term ratios The second, third and fourth terms are $nx^{n-1}y$, $n(n-1)x^{n-2}y^2/2$, and $n(n-1)(n-2)x^{n-3}y^3/6$. Their stated nonzero values show that division is valid. Put $t = y/x$. Then

$$\frac{n-1}{2}t = \frac{30}{135} = \frac{2}{9}, \quad \frac{n-2}{3}t = \frac{10/3}{30} = \frac{1}{9}.$$

Dividing these equations eliminates t : $3(n-1)/[2(n-2)] = 2$. Hence $3n-3 = 4n-8$, giving $n = 5$.

Step 2: Determine the variables and evaluate the expression The first ratio now gives $2t = 2/9$, so $y = x/9$. Substituting into the original second term, rather than only using ratios, gives

$$5x^4 \frac{x}{9} = 135, \quad x^5 = 243, \quad x = 3, \quad y = \frac{1}{3}.$$

The real fifth root is unique. As a check, the third term is $10 \cdot 27/9 = 30$, and the fourth is $10 \cdot 9/27 = 10/3$, precisely the supplied values. Finally,

$$6(n^3 + x^2 + y) = 6\left(125 + 9 + \frac{1}{3}\right) = 750 + 54 + 2.$$

Step 3: Conclusion The required numerical value is $\boxed{806}$; the corresponding parameters are $n = 5$, $x = 3$, $y = 1/3$.

Quick Tip to Solve: Use adjacent-term ratios to find the exponent first; ratios alone determine y/x , so retain one original term to find x and y .

24. Let $\alpha\beta\gamma = 45$; $\alpha, \beta, \gamma \in \mathbb{R}$. If $x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0)$ for some $x, y, z \in \mathbb{R}, xyz \neq 0$, then $6\alpha + 4\beta + \gamma$ is equal to _____. [4 marks]

Solution:

Concept:

- A homogeneous linear system with a nonzero solution has a singular coefficient matrix: its determinant must vanish. Here $xyz \neq 0$ certainly ensures a nonzero solution.
- The given vectors multiply the unknown scalars as columns of the coefficient matrix. Expand its determinant carefully, and only then use $\alpha\beta\gamma = 45$.

Step 1: Translate the vector identity into a singular system Equating the three coordinates of the vector identity gives

$$\begin{aligned}\alpha x + y + 2z &= 0, \\ x + \beta y + 3z &= 0, \\ 2x + 2y + \gamma z &= 0.\end{aligned}$$

Thus the matrix multiplying $(x, y, z)^T$ is

$$M = \begin{pmatrix} \alpha & 1 & 2 \\ 1 & \beta & 3 \\ 2 & 2 & \gamma \end{pmatrix}.$$

Because the supplied solution has all three components nonzero, it is not the zero vector. If this matrix were invertible, multiplication by its inverse would force all three components to vanish, a contradiction. Therefore $\det M = 0$.

Step 2: Expand and isolate the requested combination Expanding along the first row gives

$$\begin{aligned}0 &= \alpha(\beta\gamma - 6) - (\gamma - 6) + 2(2 - 2\beta) \\ &= \alpha\beta\gamma - 6\alpha - \gamma + 6 + 4 - 4\beta.\end{aligned}$$

The minus sign attached to the middle cofactor is essential. Now insert the known product $\alpha\beta\gamma = 45$:

$$0 = 45 + 10 - (6\alpha + 4\beta + \gamma).$$

This equation determines the requested linear combination directly; finding the three real parameters individually is unnecessary. The constant 10 consists of the two positive cross-product contributions 6 and 4.

Step 3: Conclusion The singularity condition gives $6\alpha + 4\beta + \gamma = \boxed{55}$.

Quick Tip to Solve: When a vector relation has a nontrivial solution, set the coefficient determinant to zero and look for the requested combination inside its expansion.

25. Let P be the point $(10, -2, -1)$ and Q be the foot of the perpendicular drawn from the point $R(1, 7, 6)$ on the line passing through the points $(2, -5, 11)$ and $(-6, 7, -5)$. Then the length of the line segment PQ is equal to _____.

[4 marks]

Solution:

Concept:

- A point on the line through A with direction vector $\mathbf{v}$ has position $A + t\mathbf{v}$. The perpendicular foot Q satisfies $(R - Q) \cdot \mathbf{v} = 0$.
- After locating the foot on the infinite line, use the Euclidean distance formula to obtain PQ . The foot need not lie between the two defining points.

Step 1: Choose a simple direction and find the foot Let $A = (2, -5, 11)$ and $B = (-6, 7, -5)$. Their difference is $(-8, 12, -16)$, so use the simpler parallel direction $\mathbf{v} = (-2, 3, -4)$. Write

$$Q = A + t\mathbf{v} = (2 - 2t, -5 + 3t, 11 - 4t).$$

Since $R - A = (-1, 12, -5)$, the perpendicularity equation becomes

$$0 = (R - A - t\mathbf{v}) \cdot \mathbf{v} = 58 - 29t.$$

Here $(R - A) \cdot \mathbf{v} = 2 + 36 + 20 = 58$ and $\mathbf{v} \cdot \mathbf{v} = 4 + 9 + 16 = 29$. Hence $t = 2$ and $Q = (-2, 1, 3)$.

Step 2: Check the projection and compute the required length The vector from the foot to R is $R - Q = (3, 6, 3)$. Its dot product with the line direction is $-6 + 18 - 12 = 0$, confirming that the constructed point is indeed the perpendicular foot. For $P = (10, -2, -1)$,

$$P - Q = (12, -3, -4), \quad PQ = \sqrt{12^2 + (-3)^2 + (-4)^2}.$$

Therefore $PQ = \sqrt{144 + 9 + 16} = \sqrt{169} = 13$. Notice that the required length is from P to the foot, rather than the perpendicular distance from R to the line; these are different segments.

Step 3: Conclusion The foot is $Q = (-2, 1, 3)$, so $PQ = \boxed{13}$ coordinate units.

Quick Tip to Solve: Reduce the line direction vector before taking dot products, and verify the final foot with a zero dot product before computing the requested distance.

26. Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3T_{r-1} + 6^r, r = 2, 3, \dots, n$. If the sum of the first n terms of this series is $\frac{1}{5}(n^2 - 12n + 39)(4 \cdot 6^n - 5 \cdot 3^n + 1)$, then n is equal to _____.

[4 marks]

Solution:

Concept:

- The recurrence $T_r = 3T_{r-1} + 6^r$ becomes a first-difference relation after division by 3^r . This removes the homogeneous multiplication factor.
- For a geometric progression, $\sum_{r=1}^n a^r = a(a^n - 1)/(a - 1)$. Sum the explicit formula for T_r , then compare it with the given total.

Step 1: Solve the recurrence by normalisation Define $u_r = T_r/3^r$. The initial value is $u_1 = 6/3 = 2$, and the recurrence becomes

$$u_r = u_{r-1} + 2^r.$$

Summing the successive differences for $r = 2, \dots, n$,

$$u_r = 2 + \sum_{j=2}^r 2^j = 2 + (2^{r+1} - 4) = 2^{r+1} - 2.$$

This expression also holds for $r = 1$. Consequently

$$T_r = 3^r(2^{r+1} - 2) = 2 \cdot 6^r - 2 \cdot 3^r.$$

For example, it gives $T_2 = 72 - 18 = 54$, consistent with $3T_1 + 6^2 = 18 + 36$.

Step 2: Sum and compare the common positive factor The total is therefore

$$\begin{aligned} S_n &= 2 \frac{6(6^n - 1)}{5} - 2 \frac{3(3^n - 1)}{2} \\ &= \frac{3}{5} (4 \cdot 6^n - 5 \cdot 3^n + 1). \end{aligned}$$

For every positive integer n , $6^n \geq 2 \cdot 3^n$; hence the bracket is at least $3 \cdot 3^n + 1 > 0$, and cancellation is legitimate. Comparing this formula with the stated sum gives $n^2 - 12n + 39 = 3$. Thus

$$n^2 - 12n + 36 = 0, \quad (n - 6)^2 = 0.$$

The unique solution is a positive integer, as required for a number of terms.

Step 3: Conclusion The number of terms is $n = \boxed{6}$.

Quick Tip to Solve: Divide a recurrence by the power of its multiplier to turn it into a telescoping difference, then sum the resulting geometric terms.

27. For $n \in \mathbb{N}$, if $\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{-1} n = \frac{\pi}{4}$, then n is equal to _____. [4 marks]

Solution:

Concept:

- For a positive number a , the principal angle $\cot^{-1} a$ equals $\tan^{-1}(1/a)$ and lies in $(0, \pi/2)$.
- The tangent addition formula is $\tan(A+B) = (\tan A + \tan B)/(1 - \tan A \tan B)$. Branch information must accompany this formula to identify the actual angle.

Step 1: Combine the first three positive angles Put $A = \cot^{-1} 3$, $B = \cot^{-1} 4$, and $C = \cot^{-1} 5$. Since $\tan A \tan B = 1/12 < 1$, their sum $A + B$ remains in the first quadrant. Thus

$$\tan(A + B) = \frac{1/3 + 1/4}{1 - 1/12} = \frac{7}{11}.$$

The next addition also has a positive denominator:

$$\tan(A + B + C) = \frac{7/11 + 1/5}{1 - 7/55} = \frac{46}{48} = \frac{23}{24}.$$

Because $A + B < \pi/2$ and its tangent times $\tan C$ is less than one, the three-angle sum is also in the first quadrant. Its tangent is below one, so it is below $\pi/4$.

Step 2: Find the residual angle and determine the integer Let $D = \cot^{-1} n$. The given equation requires $D = \pi/4 - (A + B + C)$, which is strictly between zero and $\pi/4$. Its tangent is

$$\tan D = \frac{1 - 23/24}{1 + 23/24} = \frac{1/24}{47/24} = \frac{1}{47}.$$

On the other hand, n is a positive integer, so $\tan(\cot^{-1} n) = 1/n$. Equating these values gives $1/n = 1/47$. The first-quadrant checks ensure that this tangent calculation satisfies the original angle equation, with no additional multiple of π .

Step 3: Conclusion The required natural number is $n = \boxed{47}$.

Quick Tip to Solve: Convert positive inverse cotangents to reciprocal tangents, combine them successively, and check the quadrant before equating the residual angle.

28. Let $r_k = \frac{\int_0^1 (1-x^7)^k dx}{\int_0^1 (1-x^7)^{k+1} dx}$, $k \in \mathbb{N}$. Then the value of $\sum_{k=1}^{10} \frac{1}{7(r_k-1)}$ is equal to _____. [4 marks]

Solution:

Concept:

- Write $I_k = \int_0^1 (1-x^7)^k dx$, so that $r_k = I_k/I_{k+1}$. Each integral is positive, since its integrand is positive throughout $(0, 1)$.
- The derivative of $x(1-x^7)^{k+1}$ relates adjacent integrals. Its boundary values vanish at both endpoints, giving a recurrence without evaluating the integrals individually.

Step 1: Obtain a relation between adjacent integrals For $k \geq 1$, differentiate the product:

$$\frac{d}{dx} [x(1-x^7)^{k+1}] = (1-x^7)^{k+1} - 7(k+1)x^7(1-x^7)^k.$$

Its integral from zero to one is zero: at zero the factor x vanishes, and at one the power of $1-x^7$ vanishes. Also,

$$x^7(1-x^7)^k = (1-x^7)^k - (1-x^7)^{k+1}.$$

Integrating this identity in the product-derivative equation yields

$$0 = I_{k+1} - 7(k+1)(I_k - I_{k+1}).$$

This is the needed relation, and involves only the two integrals already present in the question.

Step 2: Simplify the ratio and sum the terms Since $I_{k+1} > 0$, divide the recurrence by it:

$$1 = 7(k+1)(r_k - 1), \quad \frac{1}{7(r_k - 1)} = k + 1.$$

In particular, $r_k > 1$, which agrees with the fact that raising a number in $(0, 1)$ to a larger power reduces it. The requested sum is therefore

$$\sum_{k=1}^{10} (k+1) = \sum_{k=1}^{10} k + 10 = \frac{10 \cdot 11}{2} + 10 = 65.$$

Every denominator is nonzero by the same positive recurrence; no special functions or numerical quadrature are needed.

Step 3: Conclusion The value of the stated sum is $\boxed{65}$.

Quick Tip to Solve: For ratios of integrals with successive powers, integrate the derivative of x times the higher power; the vanishing boundary terms often give the ratio directly.

29. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and a vector $\vec{c}$ be such that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 13\hat{k}$. If $\vec{a} \cdot \vec{c} = 13$, then $(24 - \vec{b} \cdot \vec{c})$ is equal to _____. [4 marks]

Solution:

Concept:

- Distributivity of the cross product gives $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times \vec{c} = \vec{a} \times \vec{b} + (\vec{a} + \vec{b}) \times \vec{c}$.
- Write $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$. Equality of vectors gives three scalar component equations; the given dot product supplies the remaining constraint.

Step 1: Combine the cross products For the given vectors, direct evaluation of the determinant gives

$$\vec{a} \times \vec{b} = -\hat{i} + 22\hat{j} + 17\hat{k}, \quad \vec{a} + \vec{b} = 5\hat{i} + \hat{j} - \hat{k}.$$

Subtract the first cross product from the specified right-hand side. Therefore

$$(5\hat{i} + \hat{j} - \hat{k}) \times \vec{c} = 2\hat{i} - 14\hat{j} - 4\hat{k}.$$

The cross product with (x, y, z) has components $(y + z, -x - 5z, 5y - x)$. Equating components gives

$$y + z = 2, \quad x + 5z = 14, \quad 5y - x = -4.$$

Step 2: Apply the dot-product condition The first two equations give $y = 2 - z$ and $x = 14 - 5z$. Substitution into the third gives an identity, as expected because a cross product cannot determine the component parallel to its first vector. The additional condition fixes it:

$$2x - 3y + 4z = 13.$$

$$2(14 - 5z) - 3(2 - z) + 4z = 13 \Rightarrow 22 - 3z = 13.$$

Hence $z = 3$, $y = -1$, $x = -1$, and $\vec{c} = -\hat{i} - \hat{j} + 3\hat{k}$. Check that $\vec{a} \cdot \vec{c} = -2 + 3 + 12 = 13$.

Step 3: Evaluate the requested expression Using the components just found,

$$\vec{b} \cdot \vec{c} = 3(-1) + 4(-1) - 5(3) = -22.$$

The question asks for a shifted dot product, not for this dot product alone:

$$24 - \vec{b} \cdot \vec{c} = 24 - (-22) = 46.$$

Step 4: Conclusion The required numerical value is $\boxed{46}$ (dimensionless).

Quick Tip to Solve: A cross-product equation leaves freedom along its first vector. Use the independent dot-product condition before evaluating the final expression.

30. Let L_1, L_2 be the lines passing through the point $P(0, 1)$ and touching the parabola $9x^2 + 12x + 18y - 14 = 0$. Let Q and R be the points on the lines L_1 and L_2 such that the $\triangle PQR$ is an isosceles triangle with base QR . If the slopes of the lines QR are m_1 and m_2 , then $16(m_1^2 + m_2^2)$ is equal to _____.

[4 marks]

Solution:

Concept:

- A line $y = mx + 1$ through $P(0, 1)$ touches the parabola when substitution yields a quadratic with zero discriminant.
- In an isosceles triangle, the perpendicular from the apex to the base bisects the apex angle. Thus a possible base is perpendicular to an internal or external angle bisector of the two given lines.

Step 1: Find the tangent slopes Insert $y = mx + 1$ into the given parabola:

$$9x^2 + (12 + 18m)x + 4 = 0.$$

For tangency the two intersection roots coincide, so

$$(12 + 18m)^2 - 4(9)(4) = 0.$$

Thus $12 + 18m = \pm 12$, giving $m = 0$ or $m = -4/3$. The two tangents are therefore

$$y - 1 = 0, \quad 4x + 3y - 3 = 0.$$

A vertical line through P would be $x=0$, which meets this parabola transversely and is not a tangent. The two roots account for both tangent lines.

Step 2: Find both angle bisectors Points on an angle bisector have equal perpendicular distances

from the two lines. Their normal lengths are 1 and 5, respectively. Hence

$$|y - 1| = \frac{|4x + 3y - 3|}{5}.$$

Taking the two signed possibilities gives

$$5(y - 1) = 4x + 3y - 3 \Rightarrow y - 1 = 2x,$$

$$5(y - 1) = -(4x + 3y - 3) \Rightarrow y - 1 = -\frac{x}{2}.$$

The bisector slopes are 2 and $-1/2$. Allowing points on the full tangent lines gives both apex-angle choices, so both bisectors must be considered.

Step 3: Use perpendicularity to the base Since $PQ = PR$, the base QR is perpendicular to the appropriate apex-angle bisector. Negative reciprocal slopes give the same unordered pair:

$$\{m_1, m_2\} = \left\{-\frac{1}{2}, 2\right\}.$$

The base intercept can vary with the equal side length, but these two directions do not. Consequently,

$$16(m_1^2 + m_2^2) = 16\left(\frac{1}{4} + 4\right) = 68.$$

Step 4: Conclusion The required value is 68 (dimensionless).

Quick Tip to Solve: Find the two tangent equations first, normalize their distances, and use both angle bisectors; the possible base directions are perpendicular to those bisectors.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A bullet of mass 50 g is fired with a speed 100 m/s on a plywood and emerges with 40 m/s. The percentage loss of kinetic energy is : [4 marks]

- (A) 44%
- (B) 16%
- (C) 84%
- (D) 32%

Correct Answer: (C) 84%

Solution:

Concept:

- Translational kinetic energy is $K = \frac{1}{2}mv^2$, where m is the bullet mass and v its speed.
- Percentage loss is $100(K_i - K_f)/K_i$. The same bullet emerges, so the common mass cancels from the energy ratio.

Step 1: Calculate the initial and final energies Convert the bullet mass to SI units: $m = 50 \text{ g} = 0.050 \text{ kg}$. The incident and emergent speeds are $u = 100 \text{ m s}^{-1}$ and $v = 40 \text{ m s}^{-1}$. Therefore,

$$K_i = \frac{1}{2}(0.050)(100)^2 = 250 \text{ J},$$

$$K_f = \frac{1}{2}(0.050)(40)^2 = 40 \text{ J}.$$

The resisting force of the plywood does negative work on the bullet, decreasing its kinetic energy. Energy transferred to deformation, heating and sound is not part of the bullet's final kinetic energy.

Step 2: Form the percentage loss The loss is $250 - 40 = 210 \text{ J}$, measured relative to the initial energy:

$$\frac{K_i - K_f}{K_i} \times 100 = \frac{210}{250} \times 100 = 84\%.$$

Equivalently, the retained fraction is $(v/u)^2 = (40/100)^2 = 0.16$, so the lost fraction is $1 - 0.16 = 0.84$. This ratio method confirms the result without needing the mass.

Step 3: Check the alternatives Option (B), 16%, is the percentage retained, not lost. The speed decreases by 60%, but kinetic energy is quadratic in speed, so a speed percentage cannot be used directly. Options (A), 44%, and (D), 32%, do not follow from the squared-speed ratio and fail the explicit energy calculation.

Step 4: Conclusion The bullet loses 84% of its initial kinetic energy. Hence, the correct option is (C).

Quick Tip to Solve: For an unchanged mass, percentage kinetic-energy loss is $100[1 - (v/u)^2]$; square the speed ratio before subtracting from one.

32. The specific heat at constant pressure of a real gas obeying $PV^2 = RT$ equation is:

[4 marks]

- (A) R
- (B) $C_V + R$
- (C) $C_V + \frac{R}{2V}$
- (D) $\frac{R}{3} + C_V$

Correct Answer: (C) $C_V + \frac{R}{2V}$

Solution:

Concept:

- For a thermodynamically consistent simple compressible gas, $C_P - C_V = T(\partial P/\partial T)_V(\partial V/\partial T)_P$. Here V is the molar volume in the stated molar equation of state.
- The general relation follows from $dS = (C_V/T)dT + (\partial P/\partial T)_V dV$ and $C_P = T(\partial S/\partial T)_P$; the ideal-gas Mayer relation cannot simply be imposed on a different equation of state.

Step 1: Differentiate the equation of state Write the supplied relation as

$$P = \frac{RT}{V^2}.$$

At fixed volume, the temperature derivative of pressure is

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V^2}.$$

Next differentiate $PV^2 = RT$ at fixed pressure:

$$2PV \left(\frac{\partial V}{\partial T}\right)_P = R.$$

Thus $(\partial V/\partial T)_P = R/(2PV)$. Using the equation of state again, $R/(PV) = V/T$, so this derivative is also $V/(2T)$. The fixed-variable labels matter: these derivatives describe different heating constraints.

Step 2: Evaluate the heat-capacity difference Substitute both derivatives into the general identity:

$$C_P - C_V = T \left(\frac{R}{V^2}\right) \left(\frac{V}{2T}\right) = \frac{R}{2V}.$$

Hence

$$C_P = C_V + \frac{R}{2V}.$$

This uses the volume dependence of the actual equation given. It does not assume that internal energy depends only on temperature, an extra assumption that is not generally true for a real gas.

Step 3: Compare with the options Option (C) contains precisely the derived correction. Option (B) uses $C_P - C_V = R$, which applies to the ideal equation $PV = RT$, not the printed $PV^2 = RT$. Option (D) has neither the required coefficient nor volume dependence. Option (A) omits the independent contribution C_V and therefore is not the constant-pressure heat capacity.

Step 4: Conclusion $C_P = C_V + R/(2V)$. Hence, the correct option is (C).

Quick Tip to Solve: For an unfamiliar gas equation, differentiate it under the two fixed-variable conditions and use the general heat-capacity identity instead of assuming Mayer's ideal-gas result.

33. The correct truth table for the following logic circuit is :

[4 marks]



(A)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(B)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

(C)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(D)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

Correct Answer: (B)

Solution:

Concept:

- The first gate is AND, giving AB . The separate upper branch inverts input A , giving $\bar{A}$. The final OR gate combines these two outputs.
- Therefore $Y = \bar{A} + AB = \bar{A} + B$, where multiplication denotes AND and addition denotes OR. The absorption identity follows from $X + YZ = (X + Y)(X + Z)$.

Step 1: Read the two circuit branches Follow the wires before assigning the output. Both A and B enter the lower AND gate, so its output is AB . A wire also branches from A before this gate and goes through the NOT gate. Its output is $\bar{A}$, not the complement of the AND output. The two branch

outputs enter the final OR gate, giving

$$Y = \overline{A} + AB.$$

Simplifying with the distributive identity,

$$\overline{A} + AB = (\overline{A} + A)(\overline{A} + B) = \overline{A} + B.$$

The circuit output is high whenever $A = 0$, and when $A = 1$ it equals B .

Step 2: Evaluate every input pair For the row order $(A, B) = (0, 0), (0, 1), (1, 0), (1, 1)$, calculate both branch outputs and then their OR:

A	B	$\overline{A}$	AB	Y
0	0	1	0	1
0	1	1	0	1
1	0	0	0	0
1	1	0	1	1

Thus the required output column is 1, 1, 0, 1. This tests all possible binary inputs, so matching this column is sufficient.

Step 3: Identify the matching printed table Option (B) has exactly the output sequence 1, 1, 0, 1. Option (A) is merely the AND table and ignores the inverted upper branch. Option (C) fails at (1, 1), where the AND input to the final gate is high. Option (D) fails at (0, 0), where the NOT input to the final gate is high. Both of those failing rows can be checked directly on the circuit.

Step 4: Conclusion The correct output column is 1, 1, 0, 1, for input rows 00, 01, 10, 11. Hence, the correct option is (B).

Quick Tip to Solve: When a wire branches before a gate, follow the branch separately. A single row with $A = 0$ immediately rules out any table that ignores the NOT branch.

34. Electromagnetic waves travel in a medium with speed of $1.5 \times 10^8 \text{ m s}^{-1}$. The relative permeability of the medium is 2.0. The relative permittivity will be: [4 marks]

- (A) 4
- (B) 1
- (C) 2
- (D) 5

Correct Answer: (C) 2

Solution:

Concept:

- For a homogeneous medium, the electromagnetic-wave speed is $v = 1/\sqrt{\mu\varepsilon} = c/\sqrt{\mu_r\varepsilon_r}$.
- The relative permeability and permittivity are dimensionless: $\mu_r = \mu/\mu_0$, $\varepsilon_r = \varepsilon/\varepsilon_0$, with $c = 3.0 \times 10^8 \text{ m s}^{-1}$ in vacuum.

Step 1: Find the refractive-index factor The given speed is $v = 1.5 \times 10^8 \text{ m s}^{-1}$. Therefore the ratio of

vacuum speed to medium speed is

$$\frac{c}{v} = \frac{3.0 \times 10^8}{1.5 \times 10^8} = 2.$$

From the wave-speed relation, this ratio is $\sqrt{\mu_r \epsilon_r}$. Squaring both sides is essential because permeability and permittivity enter under a square root:

$$\mu_r \epsilon_r = \left(\frac{c}{v}\right)^2 = 4.$$

The common factor 10^8 cancels; no residual speed unit remains in this ratio.

Step 2: Solve for the relative permittivity Use the stated relative permeability $\mu_r = 2.0$:

$$\epsilon_r = \frac{4}{2.0} = 2.$$

As a substitution check, $c/\sqrt{2 \times 2} = c/2 = 1.5 \times 10^8 \text{ m s}^{-1}$, which exactly reproduces the supplied speed. The result is a relative material property and carries no physical unit.

Step 3: Reject the other choices Option (A), 4, is the product $\mu_r \epsilon_r$, not ϵ_r alone; it would give an incorrectly smaller speed if used as the permittivity. Option (B), 1, misses the square in the speed relation and gives $c/\sqrt{2}$. Option (D), 5, gives $c/\sqrt{10}$. Only option (C) reproduces the specified speed together with the specified permeability.

Step 4: Conclusion The relative permittivity is $\boxed{\epsilon_r = 2}$. Hence, the correct option is (C).

Quick Tip to Solve: First square the speed ratio to obtain $\mu_r \epsilon_r$; then divide by the given relative permeability.

35. To find the spring constant (k) of a spring experimentally, a student commits 2% positive error in the measurement of time and 1% negative error in measurement of mass. The percentage error in determining value of k is : [4 marks]

- (A) 5%
- (B) 3%
- (C) 1%
- (D) 4%

Correct Answer: (A) 5%

Solution:

Concept:

- For a mass on a spring, $T = 2\pi\sqrt{m/k}$, so $k = 4\pi^2 m/T^2$. The period is proportional to a measured time for any fixed number of oscillations.
- For small signed errors, logarithmic differentiation gives $\Delta k/k \simeq \Delta m/m - 2\Delta T/T$. A positive time error and a negative mass error both decrease the inferred spring constant.

Step 1: Express the measured quantity in terms of the readings Rearrange the oscillation-period

formula rather than treating time and mass symmetrically:

$$k = \frac{4\pi^2 m}{T^2}.$$

If the measured interval covers N oscillations, $T = t/N$; an exact count N does not change the fractional error in time. Thus the stated positive 2% time error is a positive 2% period error. The mass reading is 1% too small, so its signed relative error is -0.01 .

Step 2: Propagate the signed first-order errors Take the logarithmic differential:

$$\frac{\Delta k}{k} \simeq \frac{\Delta m}{m} - 2\frac{\Delta T}{T}.$$

Substitute the two signed percentages:

$$\frac{\Delta k}{k} \times 100 \simeq -1 - 2(2) = -5\%.$$

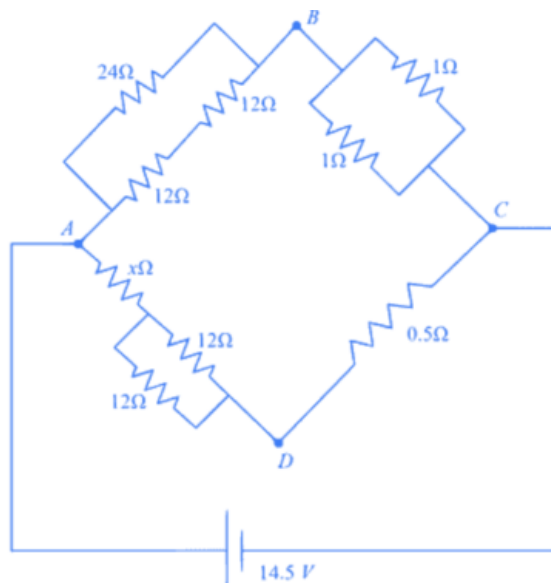
The inferred value is therefore too low by approximately 5%. Both effects reinforce each other: underestimating the numerator reduces k , while overestimating the squared denominator also reduces it. The requested percentage-error magnitude is 5%.

Step 3: Check the approximation and the alternatives Keeping finite factors would give $k_{\text{meas}}/k_{\text{true}} = 0.99/(1.02)^2$, or a decrease of about 4.84%, consistent with the standard first-order 5% answer among the choices. Option (B), 3%, incorrectly combines unsigned contributions by subtraction. Option (C), 1%, ignores the period error. Option (D), 4%, includes only the squared-time contribution and omits the mass error.

Step 4: Conclusion The calculated spring constant has an approximately 5% negative error; its percentage-error magnitude is 5%. Hence, the correct option is (A).

Quick Tip to Solve: Keep error signs until the end: a smaller mass and a larger period both lower m/T^2 , so their effects add in magnitude.

36. The value of unknown resistance (x) for which the potential difference between B and D will be zero in the arrangement shown, is : [4 marks]



- (A) $3\ \Omega$
- (B) $42\ \Omega$
- (C) $6\ \Omega$
- (D) $9\ \Omega$

Correct Answer: (C) $6\ \Omega$

Solution:

Concept:

- Resistors in series add, while parallel resistors obey $R^{-1} = R_1^{-1} + R_2^{-1}$. Reduce each bridge arm before applying balance.
- For a supply across A, C , equality of the junction potentials $V_B = V_D$ requires $R_{AB}/R_{BC} = R_{AD}/R_{DC}$.

Step 1: Reduce the upper arms Between A and B , the two inner $12\ \Omega$ resistors form a series branch of $24\ \Omega$. This is parallel to the outer $24\ \Omega$ resistor. Hence

$$R_{AB} = \frac{24 \times 24}{24 + 24} = 12\ \Omega.$$

Between B and C , the two $1\ \Omega$ resistors are parallel, so

$$R_{BC} = \frac{1 \times 1}{1 + 1} = 0.5\ \Omega.$$

Step 2: Reduce the lower arms and balance In the lower left arm, the two $12\ \Omega$ resistors are parallel and give $6\ \Omega$. They are in series with x , so $R_{AD} = x + 6$. The lower right arm is $R_{DC} = 0.5\ \Omega$. Therefore

$$\frac{12}{0.5} = \frac{x + 6}{0.5} \Rightarrow x = 6\ \Omega.$$

The equal right arms mean the left arms must also be equal. The supply voltage cancels from the balance equation; substituting $3, 42, 9 \Omega$ instead makes the lower left arm $9, 48, 15 \Omega$, respectively, none equal to 12Ω .

Step 3: Conclusion The unknown resistance is $x = 6 \Omega$. Hence, the correct option is (C).

Quick Tip to Solve: First simplify each arm separately. When the two right arms are equal, zero junction voltage requires equal left-arm resistances.

37. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (where $m_2 > m_1$). If the acceleration of the system is $\frac{g}{\sqrt{2}}$, then the ratio of the masses $\frac{m_1}{m_2}$ is: [4 marks]

- (A) $\frac{1+\sqrt{5}}{\sqrt{5}-1}$
 (B) $\frac{\sqrt{3}+1}{\sqrt{2}-1}$
 (C) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$
 (D) $\frac{1+\sqrt{5}}{\sqrt{2}-1}$

Correct Answer: (C) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

Solution:

Concept:

- A light inextensible string gives both blocks the same acceleration magnitude; a smooth light pulley gives equal tension T on both sides.
- Newton's second law for the two masses gives $a = g(m_2 - m_1)/(m_2 + m_1)$, with the heavier mass descending.

Step 1: Write and combine the force equations Take downward as positive for m_2 and upward as positive for m_1 . The equations along the string are

$$m_2 g - T = m_2 a, \quad T - m_1 g = m_1 a.$$

Adding eliminates the internal tension and gives

$$(m_2 - m_1)g = (m_2 + m_1)a.$$

The driving force is the difference of the weights, while the accelerated inertia is the sum of the masses.

Step 2: Solve for the mass ratio Let $r = m_1/m_2$. Dividing the combined equation by $m_2 g$, and using the stated acceleration,

$$\frac{1-r}{1+r} = \frac{1}{\sqrt{2}}.$$

Thus $\sqrt{2} - \sqrt{2}r = 1 + r$, giving

$$r = \frac{\sqrt{2}-1}{\sqrt{2}+1} = 3 - 2\sqrt{2} \approx 0.172.$$

This is positive and less than one, as required by $m_2 > m_1$. Substitution recovers $a/g = 1/\sqrt{2}$. Each of options (A), (B) and (D) is larger than one, contradicting the stated ordering of the masses; only option (C) satisfies both Newton's law and that ordering.

Step 3: Conclusion The dimensionless ratio is $m_1/m_2 = (\sqrt{2} - 1)/(\sqrt{2} + 1)$. Hence, the correct option is (C).

Quick Tip to Solve: For an Atwood machine write $r = (1 - a/g)/(1 + a/g)$; check that the lighter-to-heavier mass ratio is below one.

38. To project a body of mass m from earth's surface to infinity, the required kinetic energy is (assume, the radius of earth is R_E , g = acceleration due to gravity on the surface of earth): [4 marks]

- (A) $1/2mgR_E$
- (B) $4mgR_E$
- (C) mgR_E
- (D) $2mgR_E$

Correct Answer: (C) mgR_E

Solution:

Concept:

- Taking gravitational potential energy as zero at infinity, a mass m at a distance r from Earth's centre has $U = -GM_E m/r$.
- The minimum escape condition is arrival at infinity with zero speed; mechanical energy is conserved if resistance is neglected.

Step 1: Find the required increase in potential energy At the surface $r = R_E$, so $U_i = -GM_E m/R_E$. At infinity $U_f = 0$. The initial kinetic energy must supply the increase in potential energy:

$$K_i + U_i = K_f + U_f = 0, \quad K_i = \frac{GM_E m}{R_E}.$$

This is the threshold energy. A larger energy also permits escape but leaves nonzero kinetic energy at infinity and is not the required minimum.

Step 2: Express the result using surface gravity The given surface acceleration obeys $g = GM_E/R_E^2$. Consequently

$$K_i = \frac{m}{R_E}(gR_E^2) = mgR_E.$$

Its dimensions are $\text{kg m}^2 \text{s}^{-2}$, as required for energy. Equivalently $v_{\text{esc}}^2 = 2gR_E$, and multiplying by $m/2$ gives the same result. The expression $mgR_E/2$ is too small to remove the negative total energy; $2mgR_E$ and $4mgR_E$ exceed the threshold. Using constant g all the way to infinity would be invalid because gravity decreases with distance.

Step 3: Conclusion The minimum kinetic energy required is mgR_E , measured in joules. Hence, the correct option is (C).

Quick Tip to Solve: Use the potential-energy difference from R_E to infinity; insert $GM_E = gR_E^2$ only after writing the energy equation.

39. Match List I with List II

	LIST I		LIST II
A.	Torque	I.	$[M^1 L^1 T^{-2} A^{-2}]$
B.	Magnetic field	II.	$[L^2 A^1]$
C.	Magnetic moment	III.	$[M^1 T^{-2} A^{-1}]$
D.	Permeability of free space	IV.	$[M^1 L^2 T^{-2}]$

Choose the correct answer from the options given below:

[4 marks]

(A) A-I, B-III, C-II, D-IV

(B) A-IV, B-II, C-III, D-I

(C) A-III, B-I, C-II, D-IV

(D) A-IV, B-III, C-II, D-I

Correct Answer: (D) A-IV, B-III, C-II, D-I

Solution:

Concept:

- Build dimensions from defining physical equations. Here M, L, T, A denote mass, length, time and electric current.
- Torque is force times perpendicular distance; magnetic moment of a current loop is current times loop area.

Step 1: Match torque and magnetic field Force has dimensions $[MLT^{-2}]$, hence torque has

$$[\tau] = [F][r] = [ML^2T^{-2}],$$

which is entry IV. From the magnetic force on a perpendicular current element, $F = BIl$,

$$[B] = \frac{[F]}{[I][l]} = [MT^{-2}A^{-1}],$$

which is entry III. This already rules out options (A) and (C), which do not match torque correctly.

Step 2: Match moment and permeability For a current loop, magnetic moment is $\mu = IA$, where A is area. Thus

$$[\mu] = [AL^2],$$

giving entry II. For a long straight wire, $B = \mu_0 I / (2\pi r)$. Since numerical factors are dimensionless,

$$[\mu_0] = \frac{[B][r]}{[I]} = [MLT^{-2}A^{-2}],$$

giving entry I. Option (B) assigns the field the dimensions of moment and the moment those of field, so it is also excluded. The complete mapping is A-IV, B-III, C-II, D-I; each relation follows from a separate defining equation, rather than a similarity of symbols.

Step 3: Conclusion The matching is A-IV, B-III, C-II, D-I. Hence, the correct option is (D).

Quick Tip to Solve: Remember $B = F/(Il)$, magnetic moment $= I \times \text{area}$, and $\mu_0 = 2\pi r B / I$; these three equations reconstruct the magnetic dimensions.

40. Which of the following phenomena does not explain by wave nature of light.

- A. reflection
- B. diffraction
- C. photoelectric effect
- D. interference
- E. polarization

Choose the most appropriate answer from the options given below:

[4 marks]

- (A) C only
- (B) B, D only
- (C) A, C only
- (D) E only

Correct Answer: (A) C only

Solution:

Concept:

- Classical electromagnetic waves explain reflection, diffraction, interference and polarization through propagation, superposition and transverse oscillations.
- The photoelectric effect requires energy transfer in photons of energy $h\nu$, with $h\nu = \phi + K_{\max}$.

Step 1: Classify the wave phenomena Reflection (A) follows from wave propagation at a boundary and is consistent with the laws obtained from wavefronts. Diffraction (B) is wave spreading around edges or through apertures; interference (D) comes from coherent superposition. Polarization (E) restricts the direction of the electric field oscillation and is evidence that light is transverse. All four therefore have a wave explanation. In particular, reflection is not exclusively evidence for particles: a wave model explains it as well.

Step 2: Identify the exception and test the options For the photoelectric effect (C), electrons are emitted only above a threshold frequency $\nu_0 = \phi/h$, and their maximum kinetic energy follows $K_{\max} = h\nu - \phi$. Increasing intensity below threshold does not produce the emission predicted by a simple continuous wave-energy accumulation picture. The photon model instead supplies energy in individual quanta. Thus C is the sole phenomenon in the list that the classical wave nature does not explain. Option (B) selects two hallmark wave effects; option (C) incorrectly includes reflection, and option (D) incorrectly rejects polarization. None describes the required exception.

Step 3: Conclusion Only the photoelectric effect, listed as C, requires the photon explanation here. Hence, the correct option is (A).

Quick Tip to Solve: Interference and diffraction show superposition, polarization shows transverse waves, and the photoelectric threshold points to photons.

41. Four particles A, B, C, D of mass $\frac{m}{2}, m, 2m, 4m$, have same momentum, respectively. The particle with maximum kinetic energy is : [4 marks]

- (A) B
- (B) C
- (C) D
- (D) A

Correct Answer: (D) A

Solution:

Concept:

- For a nonrelativistic particle, momentum is $p = Mv$ and kinetic energy is $K = Mv^2/2$, where M is its mass.
- Eliminating speed gives $K = p^2/(2M)$. At fixed momentum, kinetic energy is inversely proportional to mass.

Step 1: Express all energies using the common momentum Let p be the common momentum magnitude. Substituting the four masses into $K = p^2/(2M)$ gives

$$K_A = \frac{p^2}{m}, \quad K_B = \frac{p^2}{2m},$$
$$K_C = \frac{p^2}{4m}, \quad K_D = \frac{p^2}{8m}.$$

The mass symbol in the denominator is the individual particle mass; for particle A that mass is $m/2$, so its factor of two cancels the two in the kinetic-energy formula.

Step 2: Compare and identify the printed option The ratios are

$$K_A : K_B : K_C : K_D = 8 : 4 : 2 : 1.$$

Thus A has the largest energy, B has half of A's energy, C one quarter and D one eighth. Physically, the lightest particle must move fastest to carry the same momentum, and the speed increase more than compensates for its lower mass. The printed choices name particles B, C, D, A in that order. Therefore the largest-energy particle A corresponds to option (D), not option (A). Options (A), (B) and (C) identify progressively heavier particles and hence smaller energies under the equal-momentum condition.

Step 3: Conclusion Particle A, of mass $m/2$, has maximum kinetic energy. Hence, the correct option is (D).

Quick Tip to Solve: Check what is held equal: at equal momentum use $K \propto 1/M$, whereas at equal speed use $K \propto M$.

42. In photoelectric experiment energy of 2.48 eV irradiates a photo sensitive material. The stopping potential was measured to be 0.5 V. Work function of the photo sensitive material is : [4 marks]

- (A) 1.98 eV
- (B) 1.68 eV
- (C) 2.48 eV
- (D) 0.5 eV

Correct Answer: (A) 1.98 eV

Solution:

Concept:

- Einstein's equation is $E_\gamma = \phi + K_{\max}$, where ϕ is the material work function and $K_{\max}$ is the largest emitted-electron kinetic energy.
- The stopping potential satisfies $K_{\max} = eV_s$. For an electron, a stopping potential of one volt corresponds to one electronvolt of kinetic energy.

Step 1: Convert stopping potential into kinetic energy The retarding field just stops the most energetic electrons when its electrical work equals their kinetic energy. With $V_s = 0.5$ V,

$$K_{\max} = e(0.5 \text{ V}) = 0.5 \text{ eV}.$$

The potential itself is not an energy; multiplying by the electron charge provides the energy. Expressing both photon energy and electron kinetic energy in electronvolts avoids an unnecessary conversion to joules.

Step 2: Subtract the emitted electron energy Of each incident photon's energy, the work function is spent freeing the electron and the remainder becomes kinetic energy. Therefore

$$\phi = E_\gamma - K_{\max} = 2.48 - 0.50 = 1.98 \text{ eV}.$$

The positive remainder is less than the incident photon energy, consistent with emission. Option (C) uses the entire photon energy and leaves no kinetic energy, contrary to the nonzero stopping voltage. Option (D) gives the kinetic energy rather than the work function. Option (B), 1.68 eV, would require 0.80 eV kinetic energy and a stopping potential of 0.80 V, which is not measured.

Step 3: Conclusion The work function is $\phi = 1.98$ eV. Hence, the correct option is (A).

Quick Tip to Solve: When photon energy is in eV, subtract the numerical stopping voltage in volts directly to obtain the work function in eV.

43. While measuring diameter of wire using screw gauge the following readings were noted. Main scale reading is 1 mm and circular scale reading is equal to 42 divisions. Pitch of screw gauge is 1 mm and it has 100 divisions on circular scale. The diameter of the wire is $\frac{x}{50}$ mm. The value of x is : [4 marks]

- (A) 42
- (B) 71
- (C) 21
- (D) 142

Correct Answer: (B) 71

Solution:

Concept:

- The least count of a screw gauge is the axial distance moved through one circular-scale division: $LC = \text{pitch}/N$, where N is the number of divisions.
- The measured diameter is the main-scale reading plus the circular-scale reading multiplied by the least count. No zero correction is specified, so no correction is applied.

Step 1: Calculate the least count The screw advances 1 mm in one complete rotation and the circular scale has 100 divisions. Therefore each division represents

$$LC = \frac{1}{100} = 0.01 \text{ mm.}$$

The number 42 is a count of divisions, not a diameter in millimetres; it must first be multiplied by this least count.

Step 2: Combine the scale readings The contribution from the circular scale is $42(0.01) = 0.42$ mm. Add it to the main-scale reading:

$$d = 1 + 0.42 = 1.42 \text{ mm.}$$

The main scale and circular scale measure complementary parts of the same reading, so neither should be omitted.

Step 3: Match the requested parameter The question writes the diameter as $x/50$ mm. Equating the two forms gives

$$\frac{x}{50} = 1.42 \Rightarrow x = 71.$$

Option A uses the circular division count directly; option C would follow from ignoring the main scale. Option D corresponds to a denominator of 100 rather than the printed denominator 50.

Step 4: Conclusion The diameter is 1.42 mm, giving $x = 71$. Hence, the correct option is (B).

Quick Tip to Solve: Find the least count before using the circular reading, then match the final diameter to the exact denominator in the question.

44. Given below are two statements :

Statement I : In an LCR series circuit, current is maximum at resonance.

Statement II : Current in a purely resistive circuit can never be less than that in a series LCR circuit when connected to same voltage source.

In the light of the above statements, choose the correct from the options given below : [4 marks]

- (A) Statement I is true but Statement II is false
- (B) Statement I is false but Statement II is true
- (C) Both Statement I and Statement II are true
- (D) Both Statement I and Statement II are false

Correct Answer: (C) Both Statement I and Statement II are true

Solution:

Concept:

- For a series LCR circuit driven by a sinusoidal voltage of rms value V , the impedance is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$, and the rms current is $I = V/Z$.
- At resonance the inductive and capacitive reactances cancel. A purely resistive circuit with the same resistance R has current V/R .

Step 1: Test Statement I using the impedance The squared reactance term is nonnegative, so $Z \geq R$. At resonance, $\omega L = 1/(\omega C)$, making that term zero. Thus

$$Z_{\text{res}} = R, \quad I_{\text{res}} = \frac{V}{R}.$$

This is the largest current for the given voltage and resistance. Statement I is true; resonance minimizes impedance rather than making it zero when resistance is present.

Step 2: Compare with the purely resistive circuit For the resistive circuit, using the same resistance as in the LCR comparison,

$$I_R = \frac{V}{R}, \quad I_{\text{LCR}} = \frac{V}{\sqrt{R^2 + (\omega L - 1/(\omega C))^2}} \leq I_R.$$

Equality holds at resonance; away from resonance the LCR current is smaller. Thus the resistive current can never be less in this standard comparison, and Statement II is true.

Step 3: Select the joint truth value Both statements follow from the same inequality $Z \geq R$. Options A and B wrongly classify one of the statements as false, while option D wrongly rejects both. The presence of two reactive elements does not allow the series current to exceed V/R , because their net reactance enters the impedance as a square.

Step 4: Conclusion Both Statement I and Statement II are true, with the resistance held the same in the circuit comparison. Hence, the correct option is (C).

Quick Tip to Solve: For series resonance, compare impedances first: $Z \geq R$ immediately gives the maximum current and the resistive-circuit comparison.

45. σ is the uniform surface charge density of a thin spherical shell of radius R . The electric field at any point on the surface of the spherical shell is :

[4 marks]

(A) $\sigma/(\epsilon_0 R)$

(B) σ/ϵ_0

(C) $\sigma/(2\epsilon_0)$

(D) $\sigma/(4\epsilon_0)$

Correct Answer: (B) σ/ϵ_0

Solution:

Concept:

- A uniformly charged spherical shell is spherically symmetric. Outside it, Gauss law gives $E(4\pi r^2) = Q/\epsilon_0$.
- The total charge is surface charge density times area: $Q = 4\pi R^2\sigma$. The conventional surface field of the shell is evaluated at $r = R^+$, immediately outside.

Step 1: Relate the total charge to the density The shell has radius R , so its area is $4\pi R^2$. Uniform surface density therefore gives

$$Q = \sigma 4\pi R^2.$$

A spherical Gaussian surface just outside the shell encloses this entire charge; no factor of one-half belongs in the enclosed charge.

Step 2: Evaluate the field at the surface Spherical symmetry makes the field radial and constant in magnitude on the Gaussian sphere. Hence

$$E(r) = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{\sigma R^2}{\epsilon_0 r^2}, \quad r > R.$$

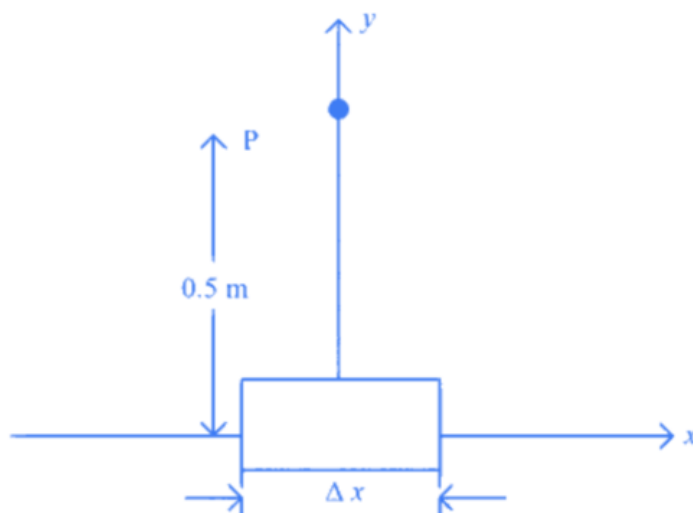
Taking the exterior limit $r \rightarrow R^+$ yields $E = \sigma/\epsilon_0$. The radius cancels because the total charge itself is proportional to R^2 .

Step 3: Distinguish the alternative expressions Option A leaves an incorrect factor $1/R$ and has the wrong dimensions for this surface field. The factor $1/2$ in option C belongs to a single infinite nonconducting plane sheet, not the exterior field of a spherical shell. Option D also introduces an unsupported factor. For an ideal zero-thickness charge sheet the interior and exterior limits differ; the shell formula here uses the exterior surface value.

Step 4: Conclusion The electric field immediately outside the spherical shell at its surface is σ/ϵ_0 , radially outward for positive charge. Hence, the correct option is (B).

Quick Tip to Solve: Write $Q = 4\pi R^2\sigma$ before substituting $r = R$ in the exterior field; this makes the radius cancellation explicit.

46. An element $\Delta l = \Delta x \hat{i}$ is placed at the origin and carries a large current $I = 10$ A. The magnetic field on the y -axis at a distance of 0.5 m from the elements Δx of 1 cm length is: [4 marks]



- (A) 10×10^{-8} T
- (B) 8×10^{-8} T
- (C) 4×10^{-8} T
- (D) 12×10^{-8} T

Correct Answer: (C) 4×10^{-8} T

Solution:

Concept:

- For a short current element, Biot-Savart law gives $\Delta B = (\mu_0/4\pi)I\Delta l \sin\theta/r^2$, where r is its distance from the observation point.
- The vector direction is $\Delta \mathbf{l} \times \hat{\mathbf{r}}$. Here the element is along $+x$ and the point is on $+y$, so the field is along $+z$.

Step 1: Identify the angle and convert the length The current element is $\Delta l = \Delta x \hat{i}$, while the observation point lies on the positive y -axis. Thus $\theta = 90^\circ$ and $\sin\theta = 1$. Convert the element length to SI units:

$$\Delta l = 1 \text{ cm} = 10^{-2} \text{ m}, \quad r = 0.5 \text{ m}, \quad I = 10 \text{ A}.$$

The element is much shorter than its distance from the point, consistent with treating it as a current element.

Step 2: Substitute in Biot-Savart law Using $\mu_0/(4\pi) = 10^{-7} \text{ T m/A}$,

$$\Delta B = 10^{-7} \frac{10(10^{-2})}{(0.5)^2} = 10^{-7} \frac{0.1}{0.25} = 4 \times 10^{-8} \text{ T}.$$

The inverse-square distance factor is essential; replacing it by an inverse-distance expression would describe a different geometry.

Step 3: Check the direction and the options The cross product $\hat{i} \times \hat{j} = \hat{k}$ places the field perpendicular

to the illustrated plane, outward from it. The requested options give magnitudes. Only option C equals the substituted value; A, B and D give larger values not produced by the stated current, element length and separation.

Step 4: Conclusion The magnetic field has magnitude $4 \times 10^{-8} \text{ T}$, directed along $+z$. Hence, the correct option is (C).

Quick Tip to Solve: For a short element use the r^{-2} Biot-Savart expression, and convert centimetres to metres before substituting.

47. A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ_0 . After sometime, the ball falls with constant velocity. The viscous force on the ball is : [4 marks]

- (A) $mg \left(1 - \frac{\rho_0}{\rho}\right)$
- (B) $mg \left(\frac{\rho_0}{\rho} - 1\right)$
- (C) $mg (1 - \rho\rho_0)$
- (D) $mg \left(1 + \frac{\rho}{\rho_0}\right)$

Correct Answer: (A) $mg \left(1 - \frac{\rho_0}{\rho}\right)$

Solution:

Concept:

- At terminal velocity the acceleration is zero, so the resultant force on the falling ball is zero.
- The weight mg acts downward. Buoyancy $\rho_0 V g$ and viscous drag act upward; the ball volume is $V = m/\rho$.

Step 1: Express the buoyant force in terms of the mass The density of the ball is its mass divided by its volume. Therefore

$$V = \frac{m}{\rho}, \quad F_b = \rho_0 V g = mg \frac{\rho_0}{\rho}.$$

Buoyancy depends on the density of the displaced liquid, not the density of the ball alone. Since the ball settles downward, its density exceeds that of the liquid in the usual terminal-fall situation.

Step 2: Balance forces at constant velocity Let F_v denote the magnitude of the upward viscous force. With downward taken positive, the force balance is

$$mg - F_b - F_v = 0.$$

Hence

$$F_v = mg - mg \frac{\rho_0}{\rho} = mg \left(1 - \frac{\rho_0}{\rho}\right).$$

There is no need to determine the terminal speed or the viscosity, because the force balance already fixes the required drag.

Step 3: Check the sign and reject the alternatives For $\rho > \rho_0$, the expression in option A is positive and smaller than the weight. Option B is its negative and therefore cannot be the upward drag

magnitude. Option C subtracts a product of densities from 1, which is dimensionally invalid. Option D makes the upward drag exceed the weight even though buoyancy also acts upward, contradicting equilibrium.

Step 4: Conclusion The viscous force acts upward and has magnitude $mg(1 - \rho_0/\rho)$. Hence, the correct option is (A).

Quick Tip to Solve: At terminal speed, drag equals effective weight: subtract buoyancy from weight before using any formula for viscous resistance.

48. A train starting from rest first accelerates uniformly up to a speed of 80 km/h for time t , then it moves with a constant speed for time $3t$. The average speed of the train for this duration of journey will be (in km/h) : [4 marks]

- (A) 70
- (B) 40
- (C) 30
- (D) 80

Correct Answer: (A) 70

Solution:

Concept:

- Average speed is total distance divided by total elapsed time, not the unweighted mean of the speeds in different stages.
- For uniform acceleration, the mean speed during a stage is $(u + v)/2$. For constant speed, distance equals speed multiplied by time.

Step 1: Find the distance during acceleration The train starts from rest and reaches 80 km h^{-1} uniformly in time t . Its mean speed in this stage is

$$\bar{v}_1 = \frac{0 + 80}{2} = 40 \text{ km h}^{-1}.$$

Writing t in hours for consistency, the distance covered is $s_1 = 40t \text{ km}$. A different consistent time unit gives the same final speed after conversion.

Step 2: Add the distance at constant speed The second stage lasts $3t$ at 80 km h^{-1} . Thus

$$s_2 = 80(3t) = 240t \text{ km}.$$

The total distance and total time are therefore

$$s = 40t + 240t = 280t \text{ km}, \quad T = t + 3t = 4t.$$

The constant-speed stage contributes most of the distance because it lasts three times as long as the first stage.

Step 3: Calculate the journey average Dividing the total distance by the total duration gives

$$\bar{v} = \frac{280t}{4t} = 70 \text{ km h}^{-1}.$$

Option B is only the first-stage mean, not the full-journey mean. Option D ignores the period below the final speed. Option C is even smaller than the first-stage mean although the later stage is faster, so it cannot be correct.

Step 4: Conclusion The average speed for the full duration is 70 km h^{-1} . Hence, the correct option is (A).

Quick Tip to Solve: Use the time-weighted average of the stage means: weights t and $3t$ give $(40 + 3 \times 80)/4$.

49. A sample contains mixture of helium and oxygen gas. The ratio of root mean square speed of helium and oxygen in the sample, is : [4 marks]

- (A) $\frac{1}{2\sqrt{2}}$
- (B) $\frac{1}{4}$
- (C) $\frac{2\sqrt{2}}{1}$
- (D) $\frac{1}{32}$

Correct Answer: (C) $\frac{2\sqrt{2}}{1}$

Solution:

Concept:

- The rms speed of an ideal gas is $v_{\text{rms}} = \sqrt{3RT/M}$, where T is the absolute temperature and M is the molar mass.
- Gas species in an equilibrium mixture have the same temperature. Their rms speeds are inversely proportional to the square roots of their molar masses.

Step 1: Identify the masses and common temperature Helium is monatomic with molar mass $M_{\text{He}} = 4 \text{ g mol}^{-1}$. Oxygen gas is diatomic, so $M_{\text{O}_2} = 32 \text{ g mol}^{-1}$, rather than the atomic value 16. Both species in the sample share the temperature T , so no temperature ratio remains when their speeds are divided.

Step 2: Form the rms-speed ratio Using the kinetic-theory expression for each gas,

$$\frac{v_{\text{He}}}{v_{\text{O}_2}} = \frac{\sqrt{3RT/M_{\text{He}}}}{\sqrt{3RT/M_{\text{O}_2}}} = \sqrt{\frac{M_{\text{O}_2}}{M_{\text{He}}}}.$$

The larger molar mass appears in the numerator after taking the ratio because speed varies inversely with its square root. Equal temperatures mean equal mean translational kinetic energies, not equal speeds.

Step 3: Evaluate and compare all options Substitute the molar masses in the same units:

$$\frac{v_{\text{He}}}{v_{\text{O}_2}} = \sqrt{\frac{32}{4}} = \sqrt{8} = 2\sqrt{2}.$$

Option A is the reciprocal ratio. Options B and D are also smaller than unity, whereas helium must have the larger rms speed because its particles are lighter. None of those options has the required square-root mass dependence.

Step 4: Conclusion The rms-speed ratio of helium to oxygen is $2\sqrt{2} : 1$, a dimensionless ratio. Hence, the correct option is (C).

Quick Tip to Solve: Use the molecular molar mass of oxygen, 32, and place the heavier gas mass on top when finding the lighter gas speed divided by the heavier gas speed.

50. The ratio of the shortest wavelength of Balmer series to the shortest wavelength of Lyman series for hydrogen atom is : [4 marks]

- (A) 1 : 2
- (B) 1 : 4
- (C) 2 : 1
- (D) 4 : 1

Correct Answer: (D) 4 : 1

Solution:

Concept:

- The Rydberg formula for a hydrogen transition from an upper level n_i to a lower level n_f is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where R_H is the Rydberg constant.
- The shortest wavelength of a series is its series limit: the photon has the greatest energy when $n_i \rightarrow \infty$. Balmer transitions end at $n_f = 2$; Lyman transitions end at $n_f = 1$.

Step 1: Take the Balmer series limit For a fixed lower level, increasing the upper principal quantum number increases the energy released. Therefore the limit, rather than the first line of the series, determines the shortest wavelength. Setting $n_f = 2$ and $1/n_i^2 \rightarrow 0$ gives

$$\frac{1}{\lambda_{B,\min}} = \frac{R_H}{4}, \quad \lambda_{B,\min} = \frac{4}{R_H}.$$

Step 2: Take the Lyman series limit For the Lyman series the electron ends in the ground state. Applying the same infinite-upper-level limit gives

$$\frac{1}{\lambda_{L,\min}} = R_H, \quad \lambda_{L,\min} = \frac{1}{R_H}.$$

The Lyman limit has a higher photon frequency and hence a smaller wavelength, consistent with its larger transition energy.

Step 3: Form the ratio in the requested order The question asks for Balmer divided by Lyman, so the common Rydberg constant cancels:

$$\lambda_{B,\min} : \lambda_{L,\min} = \frac{4}{R_H} : \frac{1}{R_H} = 4 : 1.$$

Options (A) and (B) reverse the physical order by making the Balmer wavelength shorter. Option (C) incorrectly uses the level numbers without their squares.

Step 4: Conclusion The ratio of the shortest Balmer wavelength to the shortest Lyman wavelength is $4 : 1$. Hence, the correct option is (D).

Quick Tip to Solve: At the series limit, $\lambda_{\min} = n_f^2 / R_H$. Compare the squares of the final levels, keeping the requested ratio order.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of 1000 droplets to that of energy of big drop is $\frac{10}{x}$. The value of x is _____. [4 marks]

Solution:

Concept:

- The surface energy of a liquid drop is $U = \gamma A$, where γ is surface tension and A is its surface area. A water drop has one liquid-air interface.
- Coalescence conserves liquid volume: if N equal spherical droplets of radius r form a drop of radius R , then $R^3 = Nr^3$.

Step 1: Conserve the volume of water Let the radius of each of the 1000 small droplets be r , and let the radius of the final spherical drop be R . No water is lost, so

$$1000 \left(\frac{4\pi r^3}{3} \right) = \frac{4\pi R^3}{3}.$$

Cancelling the common factor yields $R^3 = 1000r^3$, hence $R = 10r$. The radius increases by ten, rather than by a thousand, because volume depends on its cube.

Step 2: Calculate the two surface energies At the same temperature the surface tension is common to the droplets and the large drop. The total initial area and the final area are

$$A_i = 1000(4\pi r^2), \quad A_f = 4\pi(10r)^2 = 100(4\pi r^2).$$

Consequently,

$$\frac{U_i}{U_f} = \frac{\gamma A_i}{\gamma A_f} = \frac{1000}{100} = 10.$$

Surface energy decreases during coalescence because the same liquid volume is enclosed by a smaller total surface.

Step 3: Match the form supplied in the question The printed ratio is initial surface energy divided by final surface energy, written as $10/x$. Therefore

$$\frac{10}{x} = 10 \implies x = 1.$$

The reciprocal ratio would be $1/10$, but it is not the order requested. All surface-tension and radius factors cancel, so x has no unit.

Step 4: Conclusion $x = 1$ (dimensionless). The 1000 droplets initially have ten times the surface energy of the final drop.

Quick Tip to Solve: For N equal droplets coalescing, the initial-to-final surface-energy ratio is $N^{1/3}$; first conserve volume, then compare areas.

52. If the radius of earth is reduced to three-fourth of its present value without change in its mass then value of duration of the day of earth will be _____ hours 30 minutes. [4 marks]

Solution:

Concept:

- With no external torque, the rotational angular momentum of Earth remains constant: $I\omega = I'2\pi/T$, where T is its rotation period.
- For unchanged mass and the same mass-distribution shape, the moment of inertia scales as $I = kMR^2$. The common shape factor k cancels between initial and final states.

Step 1: Relate the moments of inertia Let the original radius be R , and let the contracted radius be $R' = 3R/4$. For the same mass and corresponding spherical mass distribution,

$$\frac{I'}{I} = \frac{kM(3R/4)^2}{kMR^2} = \frac{9}{16}.$$

For a uniform sphere $k = 2/5$; its explicit value is unnecessary because the two states have the same factor.

Step 2: Apply angular momentum conservation Contraction redistributes mass nearer the rotation axis and therefore increases the angular speed. Using angular momentum conservation gives

$$I\frac{2\pi}{T} = I'\frac{2\pi}{T'} \implies T' = \frac{I'}{I}T.$$

Thus the rotation period scales as the square of the radius, not as the radius itself. This also checks that the contracted Earth has a shorter day.

Step 3: Convert the new period into hours and minutes Using the original day length of 24 h,

$$T' = \frac{9}{16}(24 \text{ h}) = 13.5 \text{ h}.$$

The fractional hour is $0.5 \times 60 = 30$ minutes. Hence the new duration is 13 hours 30 minutes. Since

the question already prints the 30-minute part, the number entered in its blank is only the whole-hour component.

Step 4: Conclusion The new day lasts $\boxed{13 \text{ h } 30 \text{ min}}$; the required blank is $\boxed{13}$ hours.

Quick Tip to Solve: Without external torque, $T \propto I \propto R^2$. Square the radius ratio before multiplying the old day length, then convert the fractional hour.

53. A particle is doing simple harmonic motion of amplitude 0.06 m and time period 3.14 s. The maximum velocity of the particle is _____ cm/s. [4 marks]

Solution:

Concept:

- For simple harmonic motion, $x = A \sin(\omega t + \phi)$, where A is amplitude, ω is angular frequency and ϕ is phase.
- Differentiating displacement gives $v = A\omega \cos(\omega t + \phi)$, so the greatest speed is $v_{\max} = A\omega = 2\pi A/T$.

Step 1: Identify where the maximum speed occurs In SHM the particle moves fastest at the mean position, where its displacement is zero and all the oscillation energy is kinetic. Its speed becomes zero at either extreme displacement. Thus the amplitude 0.06 m is a distance scale; it is not itself a speed. The maximum value of the cosine factor in the velocity equation has magnitude one.

Step 2: Find the angular frequency and speed The stated time period is $T = 3.14 \text{ s}$. Taking $\pi \simeq 3.14$,

$$\omega = \frac{2\pi}{T} = \frac{2(3.14)}{3.14} = 2 \text{ rad s}^{-1}.$$

Then

$$v_{\max} = A\omega = (0.06 \text{ m})(2 \text{ s}^{-1}) = 0.12 \text{ m s}^{-1}.$$

Radians are dimensionless, so the product has the required dimensions of length per time.

Step 3: Express the speed in the requested unit The answer blank is followed by cm/s, so convert metres to centimetres:

$$0.12 \text{ m s}^{-1} = 0.12(100) \text{ cm s}^{-1} = 12 \text{ cm s}^{-1}.$$

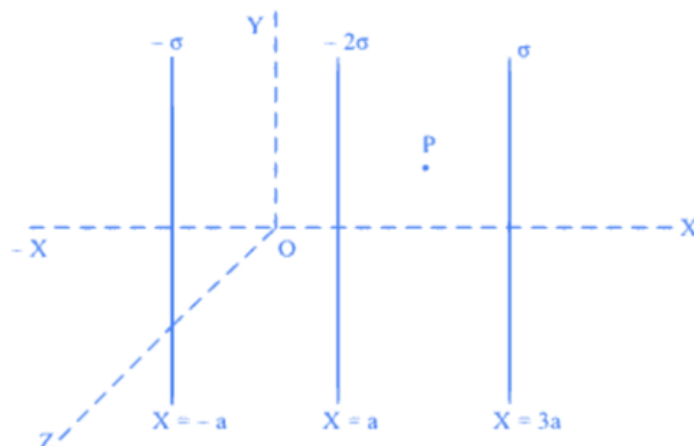
Equivalently, using the amplitude as 6 cm from the start gives $v_{\max} = 2 \times 6 = 12 \text{ cm/s}$. Do not use A/T : that omits the factor 2π .

Step 4: Conclusion The maximum speed is $\boxed{12 \text{ cm s}^{-1}}$, so the required numerical value is $\boxed{12}$.

Quick Tip to Solve: Use $v_{\max} = 2\pi A/T$, with amplitude converted to the final length unit before substitution to avoid a factor-of-100 error.

54. Three infinitely long charged thin sheets are placed as shown in figure. The magnitude of electric field at the point P is $\frac{x\sigma}{\epsilon_0}$. The value of x is _____ (all quantities are measured in SI units).

[4 marks]



Solution:

Concept:

- An infinite thin sheet with surface charge density σ_s produces a field of magnitude $|\sigma_s|/(2\epsilon_0)$ on either side, independent of distance.
- The field points away from a positive sheet and towards a negative sheet. The net field is the vector sum of the contributions of all sheets.

Step 1: Read the location of the point from the figure The sheets at $x = -a$, $x = a$, and $x = 3a$ have surface charge densities $-\sigma$, -2σ , and $+\sigma$, respectively. Point P lies between the sheets at a and $3a$. Take the positive horizontal direction as $+\hat{i}$. The vertical height of P does not affect an infinite-sheet field.

Step 2: Assign the field direction from each sheet Point P is to the right of the two negative sheets, so their fields point left, towards those sheets:

$$\vec{E}_1 = -\frac{\sigma}{2\epsilon_0}\hat{i}, \quad \vec{E}_2 = -\frac{2\sigma}{2\epsilon_0}\hat{i}.$$

It is to the left of the positive sheet, whose field points away from the sheet, also to the left:

$$\vec{E}_3 = -\frac{\sigma}{2\epsilon_0}\hat{i}.$$

Thus all three contributions reinforce one another; none is subtracted in the magnitude sum.

Step 3: Add the fields and identify the coefficient Adding the signed components gives

$$\vec{E}_P = -\frac{(1 + 2 + 1)\sigma}{2\epsilon_0}\hat{i} = -\frac{2\sigma}{\epsilon_0}\hat{i}.$$

The question asks for magnitude, so

$$|\vec{E}_P| = \frac{2\sigma}{\epsilon_0} = \frac{x\sigma}{\epsilon_0}.$$

The negative component specifies leftward direction, whereas the requested coefficient is positive.

Step 4: Conclusion The field magnitude at P is $2\sigma/\epsilon_0$; therefore $x = 2$ (dimensionless).

Quick Tip to Solve: For each sheet first mark whether its field at the observation point points left or right; only then add the signed $\sigma_s/(2\epsilon_0)$ contributions.

55. Radius of a certain orbit of hydrogen atom is $8.48 \text{ \AA}$. If energy of electron in this orbit is E/x , then $x = \underline{\hspace{2cm}}$ (Given $a_0 = 0.529 \text{ \AA}$, $E =$ energy of electron in ground state). [4 marks]

Solution:

Concept:

- For hydrogen in the Bohr model, the radius of the orbit with principal quantum number n is $r_n = a_0 n^2$, where a_0 is the ground-state radius.
- The electron energy in that orbit is $E_n = E_1/n^2$, with the ground-state energy $E_1 < 0$. The same factor n^2 increases radius and reduces the magnitude of binding energy.

Step 1: Find the principal quantum number from the radius The printed radius is $r_n = 8.48 \text{ \AA}$, and the given Bohr radius is $a_0 = 0.529 \text{ \AA}$. Their common length unit cancels, giving

$$n^2 = \frac{r_n}{a_0} = \frac{8.48}{0.529} \simeq 16.03.$$

The radii are quoted to limited precision, while the Bohr principal quantum number must be an integer. This ratio identifies $n = 4$, for which $n^2 = 16$.

Step 2: Use the corresponding energy level The symbol E in the question denotes the ground-state energy, not its magnitude. Therefore

$$E_n = \frac{E}{n^2} = \frac{E}{16}.$$

The excited-state energy remains negative, but its magnitude is smaller than that of the ground state. It is not $16E$, which would imply a more tightly bound state despite the larger orbit.

Step 3: Compare with the printed expression The requested energy has the form E/x . Comparing this expression with $E/16$ yields $x = 16$. One may also eliminate n directly: $E_n/E = a_0/r_n$. The small difference between 16.03 and 16 reflects the rounded radii supplied in the question; it does not represent a permissible nonintegral hydrogen energy level.

Step 4: Conclusion The electron occupies the fourth orbit and has energy $E/16$. Thus $x = 16$ (dimensionless).

Quick Tip to Solve: For hydrogen, $r_n/a_0 = n^2 = E_1/E_n$. Use the radius ratio to identify the integer level and keep the negative sign of both energies consistent.

56. For three vectors $\vec{A} = (-x\hat{i} - 6\hat{j} - 2\hat{k})$, $\vec{B} = (-\hat{i} + 4\hat{j} + 3\hat{k})$ **and** $\vec{C} = (-8\hat{i} - \hat{j} + 3\hat{k})$, **if** $\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$, **then value of** x **is** _____. [4 marks]

Solution:

Concept:

- The scalar triple product $\vec{A} \cdot (\vec{B} \times \vec{C})$ vanishes when the three vectors are coplanar. Algebraically it is a determinant of their components.
- Calculate the cross product first, respecting the negative sign in the middle component, and then take the dot product with $\vec{A}$.

Step 1: Compute the cross product of the known vectors The vectors are $\vec{B} = (-1, 4, 3)$ and $\vec{C} = (-8, -1, 3)$. Therefore

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}.$$

Its components are

$$(\vec{B} \times \vec{C})_x = 4(3) - 3(-1) = 15,$$

$$(\vec{B} \times \vec{C})_y = 3(-8) - (-1)(3) = -21,$$

$$(\vec{B} \times \vec{C})_z = (-1)(-1) - 4(-8) = 33.$$

Hence $\vec{B} \times \vec{C} = 15\hat{i} - 21\hat{j} + 33\hat{k}$.

Step 2: Take the dot product with the vector containing the unknown The components of $\vec{A}$ are $(-x, -6, -2)$. Multiplying corresponding components and adding gives

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (-x)(15) + (-6)(-21) + (-2)(33).$$

Using the stated zero value,

$$-15x + 126 - 66 = 0 \implies -15x + 60 = 0.$$

Both minus signs in the y -component product are essential; their product is positive.

Step 3: Solve and check the unknown The resulting linear equation gives

$$x = \frac{60}{15} = 4.$$

Substitution provides a direct check:

$$(-4)(15) + (-6)(-21) + (-2)(33) = -60 + 126 - 66 = 0.$$

Thus the calculated value satisfies the original scalar triple-product condition exactly, without introducing any additional geometric assumptions.

Step 4: Conclusion The required value is $x = 4$ (dimensionless).

Quick Tip to Solve: Write the cross-product components explicitly before taking the dot product. Checking the final value in the original triple product catches a middle-component sign error.

57. The refractive index of prism is $\mu = \sqrt{3}$ and the ratio of the angle of minimum deviation to the angle of prism is one. The value of angle of prism is _____°. [4 marks]

Solution:

Concept:

- At minimum deviation, the path through a prism is symmetric: the two internal refraction angles are equal, so each is $A/2$, where A is the prism angle.
- The minimum-deviation relation is $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$, with δ_m the minimum deviation and the prism surrounded by air.

Step 1: Use the stated ratio of angles The ratio of minimum deviation to prism angle is one, hence $\delta_m/A = 1$ and $\delta_m = A$. We substitute this equality into the prism relation, rather than using the small-angle approximation. Here the refractive index is large enough that an exact trigonometric calculation is necessary. Thus

$$\sqrt{3} = \frac{\sin[(A + A)/2]}{\sin(A/2)} = \frac{\sin A}{\sin(A/2)}.$$

Step 2: Simplify the prism relation Use the double-angle identity:

$$\sin A = 2 \sin(A/2) \cos(A/2).$$

A genuine prism has a nonzero angle, so $\sin(A/2) \neq 0$, and cancellation gives

$$\sqrt{3} = 2 \cos(A/2), \quad \cos(A/2) = \frac{\sqrt{3}}{2}.$$

$$A/2 = 30^\circ, \quad A = 60^\circ.$$

The physical range of the internal angle selects this acute value, rather than the additional periodic solutions of cosine. As a check, the incidence at minimum deviation is $(A + \delta_m)/2 = 60^\circ$, and Snell's law gives $\sin 60^\circ / \sin 30^\circ = \sqrt{3}$, as required.

Step 3: Conclusion The angle of the prism is 60° . The numerical entry is 60.

Quick Tip to Solve: When $\delta_m = A$, use the double-angle identity immediately: the prism formula reduces to $\mu = 2 \cos(A/2)$.

58. A wire of resistance R and radius r is stretched till its radius became $r/2$. If new resistance of the stretched wire is $x R$, then value of x is _____. [4 marks]

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/S$, where ρ is resistivity, L is length and $S = \pi r^2$ is cross-sectional area.
- For stretching without loss of material, the volume SL is conserved. The same material at

unchanged temperature has unchanged resistivity.

Step 1: Find the new area and length The final radius is $r/2$, so the new cross-sectional area is

$$S' = \pi(r/2)^2 = \frac{\pi r^2}{4} = \frac{S}{4}.$$

A reduction of radius by two therefore reduces area by four, not by two. Volume conservation then requires

$$S'L' = SL, \quad L' = \frac{S}{S'}L = 4L.$$

The wire becomes four times as long as well as four times smaller in cross-sectional area.

Step 2: Compare the final resistance with the original Insert both changes in the resistance formula. Since the stretching changes geometry but not the assumed resistivity,

$$R' = \rho \frac{L'}{S'} = \rho \frac{4L}{S/4} = 16\rho \frac{L}{S} = 16R.$$

Comparing this result with the stated form $R' = xR$ gives $x = 16$. The multiplier is dimensionless because it is a ratio of two resistances. Accounting only for the area change would incorrectly give a factor four; omitting the simultaneous increase in length is the missing contribution in that shortcut. The increase is physically sensible: a longer and thinner conductor offers more opposition to current.

Step 3: Conclusion The stretched wire has resistance $16R$, so $x = 16$ (dimensionless).

Quick Tip to Solve: For constant wire volume, $R \propto 1/S^2 \propto 1/r^4$. Halving the radius multiplies resistance by 2^4 .

59. When a dc voltage of 100 V is applied to an inductor, a dc current of 5 A flows through it. When an ac voltage of 200 V peak value is connected to inductor, its inductive reactance is found to be $20\sqrt{3}\Omega$. The power dissipated in the circuit is _____ W. [4 marks]

Solution:

Concept:

- A real inductor has winding resistance R as well as inductance. In steady direct current its inductive voltage is zero, so $R = V_{dc}/I_{dc}$.
- For sinusoidal alternating current, $Z = \sqrt{R^2 + X_L^2}$ and average power is $P = I_{rms}^2 R$. A peak voltage must first be converted using $V_{rms} = V_0/\sqrt{2}$.

Step 1: Find the winding resistance and impedance The direct-current measurement isolates the resistance of the winding:

$$R = \frac{100}{5} = 20 \Omega.$$

The inductor cannot be treated as ideal, since a finite steady current flows at a finite direct voltage.

With the given inductive reactance $X_L = 20\sqrt{3} \Omega$, its alternating-current impedance is

$$Z = \sqrt{20^2 + (20\sqrt{3})^2} = \sqrt{400 + 1200} = 40 \Omega.$$

Resistance and reactance are perpendicular components of impedance and must be added in quadrature, not directly.

Step 2: Use the rms current to calculate real power The alternating voltage has peak value 200 V, so

$$V_{\text{rms}} = \frac{200}{\sqrt{2}} = 100\sqrt{2} \text{ V}, \quad I_{\text{rms}} = \frac{100\sqrt{2}}{40} = \frac{5\sqrt{2}}{2} \text{ A}.$$

Only the resistance dissipates energy on average; the inductance stores and returns energy. Therefore

$$P = I_{\text{rms}}^2 R = \left(\frac{5\sqrt{2}}{2}\right)^2 (20) = \frac{25}{2}(20) = 250 \text{ W}.$$

Equivalently the power factor is $R/Z = 1/2$, giving the same result from $V_{\text{rms}} I_{\text{rms}} \cos \phi$.

Step 3: Conclusion The average power dissipated is $\boxed{250 \text{ W}}$. The required numerical entry is 250.

Quick Tip to Solve: Use the direct-current data to find winding resistance, then use rms quantities for alternating-current power; a peak voltage used as rms doubles the result.

60. A circular coil having 200 turns, $2.5 \times 10^{-4} \text{ m}^2$ area and carrying $100 \mu\text{A}$ current is placed in a uniform magnetic field of 1 T. Initially the magnetic dipole moment ($\vec{M}$) was directed along $\vec{B}$. Amount of work, required to rotate the coil through 90° from its initial orientation such that $\vec{M}$ becomes perpendicular to $\vec{B}$, is _____ μJ . [4 marks]

Solution:

Concept:

- The magnetic dipole moment of a coil is $M = NIA$, where N is its turn count, I its current and A the area of one turn.
- Its potential energy in a uniform magnetic field is $U = -MB \cos \theta$, where θ is the angle between the dipole moment and the field. Slow external rotation requires work equal to the increase in this potential energy.

Step 1: Calculate the magnetic moment in SI units The current is $100 \mu\text{A} = 100 \times 10^{-6} \text{ A} = 10^{-4} \text{ A}$. Using the area of each turn and multiplying by the number of turns,

$$M = 200(10^{-4})(2.5 \times 10^{-4}) = 5.0 \times 10^{-6} \text{ A m}^2.$$

The factor of 200 is essential: the moments of all turns add. No additional geometrical area calculation is required because the area is already supplied in the question.

Step 2: Calculate the change in potential energy Initially the moment points along the magnetic field, so $\theta_i = 0^\circ$. Finally it is perpendicular, giving $\theta_f = 90^\circ$. The work supplied against the magnetic torque

is

$$W = U_f - U_i = -MB \cos 90^\circ + MB \cos 0^\circ = MB.$$

With $B = 1 \text{ T}$,

$$W = (5.0 \times 10^{-6})(1) = 5.0 \times 10^{-6} \text{ J} = 5 \mu\text{J}.$$

The sign is positive because the aligned orientation is the lowest-energy position, and turning away from it raises the energy. Multiplying the maximum torque by $\pi/2$ would be incorrect since torque varies as $\sin \theta$ throughout the rotation.

Step 3: Conclusion The work required is $5 \mu\text{J}$. Thus the numerical entry in the requested unit is 5.

Quick Tip to Solve: For rotation from alignment to a perpendicular orientation, the work is exactly $MB = NIAB$; convert microamperes before multiplying.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The number of element from the following that do not belong to lanthanoids is Eu, Cm, Er, Tb, Yb and Lu [4 marks]

- (A) 3
- (B) 1
- (C) 4
- (D) 5

Correct Answer: (B) 1

Solution:

Concept:

- The lanthanoid series is the 4f series, conventionally including the elements from cerium through lutetium in the post-lanthanum sequence.
- Actinoids form the corresponding 5f series. Element symbols must be identified individually before counting which entries lie outside the requested series.

Step 1: Classify the elements in the list Eu is europium (atomic number 63), Er is erbium (68), Tb is terbium (65), Yb is ytterbium (70), and Lu is lutetium (71). All five belong to the lanthanoid series. Lutetium is included as the last member even though its ground-state configuration contains a filled 4f subshell; a filled subshell does not remove it from the series. By contrast, Cm is curium, atomic number 96, and belongs to the actinoid series in the seventh period.

Step 2: Count the exclusions and test the options There are six listed elements altogether. Five are lanthanoids and the remaining one, curium, is an actinoid. Consequently,

$$\text{Number not belonging to lanthanoids} = 6 - 5 = 1.$$

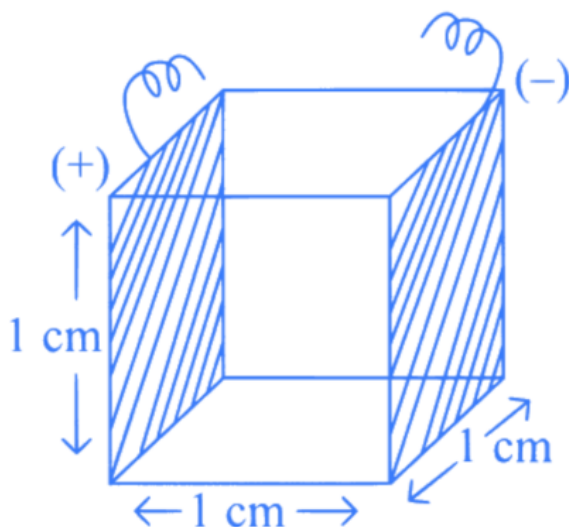
This is option (B). Option (A), three, would require incorrectly excluding two additional lanthanoids. Options (C) and (D), four and five, also contradict the individual classifications: Eu, Er, Tb, Yb and Lu

are all genuine members of the 4f series. The question asks for elements that do not belong, so the number five of members must not be mistaken for the required count of non-members.

Step 3: Conclusion Only Cm does not belong to the lanthanoids; the required number is 1. Hence, the correct option is (B).

Quick Tip to Solve: Locate curium in the 5f actinoid series first, and remember that lutetium still counts as a lanthanoid despite its filled 4f subshell.

62. A conductivity cell with two electrodes (dark side) are half filled with infinitely dilute aqueous solution of a weak electrolyte. If volume is doubled by adding more water at constant temperature, the molar conductivity of the cell will - [4 marks]



- (A) depend upon type of electrolyte
- (B) increase sharply
- (C) decrease sharply
- (D) remain same or can not be measured accurately

Correct Answer: (D) remain same or can not be measured accurately

Solution:

Concept:

- Molar conductivity is $\Lambda_m = \kappa/c$ in consistent units, where κ is conductivity and c is molar concentration.
- For a weak electrolyte, dilution increases ionisation until the limiting value Λ_m° is reached. At infinite dilution, the ions are effectively independent and further dilution does not produce another sharp increase.

Step 1: Apply the stated infinite-dilution limit The key qualification is that the solution is already infinitely dilute. In this limiting regime a weak electrolyte has reached its limiting molar conductivity

at the fixed temperature. Doubling the volume halves the analytical concentration, but it does not change the limiting ionic mobilities. In the ideal dilute limit,

$$c' = \frac{c}{2}, \quad \kappa' = \frac{\kappa}{2}, \quad \Lambda_m' = \frac{\kappa'}{c'} = \frac{\kappa/2}{c/2} = \Lambda_m^\circ.$$

The conductivity per unit volume decreases, whereas the conductivity per mole stays at its limiting value.

Step 2: Distinguish the cell geometry from the molar quantity The figure shows vertical electrodes in a half-filled cell. Adding water can increase the wetted electrode area, so the raw conductance depends on the active cell geometry. Molar conductivity is a property obtained after accounting for geometry and concentration; the area change does not itself create a sharp rise in that property. At extreme dilution the electrolyte contribution also becomes difficult to measure accurately relative to background conductivity. Thus option (D) describes the situation. Option (B) is the familiar trend for a weak electrolyte before the limiting regime, not after it; (C) confuses molar conductivity with conductivity. The stated limiting behaviour does not become an arbitrary choice between increases and decreases depending on electrolyte type, excluding (A).

Step 3: Conclusion At infinite dilution the molar conductivity remains at its limiting value, while accurate measurement can be difficult. Hence, the correct option is (D).

Quick Tip to Solve: Always distinguish κ from Λ_m , and check whether the question says merely dilute or already infinitely dilute before applying the weak-electrolyte trend.

63. Match List I with List II.

	List I (Compound)		List II (Uses)
A.	Iodoform	I.	Fire extinguisher
B.	Carbon tetrachloride	II.	Insecticide
C.	CFC	III.	Antiseptic
D.	DDT	IV.	Refrigerants

Choose the correct answer from the options given below:

[4 marks]

(A) A-II, B-IV, C-I, D-III

(B) A-III, B-I, C-IV, D-II

(C) A-III, B-II, C-IV, D-I

(D) A-I, B-II, C-III, D-IV

Correct Answer: (B) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Common halogen compounds have distinct textbook uses: iodoform as an antiseptic, carbon tetrachloride in historical fire extinguishers, chlorofluorocarbons as refrigerants, and DDT as an insecticide.

- A matching question is settled by assigning every compound to its specific use and then checking the complete four-part code, rather than accepting one correct pair.

Step 1: Match iodoform and carbon tetrachloride Iodoform, CHI_3 , has historically been used as an antiseptic, so A corresponds to III. Its antiseptic action is associated with the liberation of iodine; it is not the insecticide in this list. Carbon tetrachloride, CCl_4 , is nonflammable and was historically used in fire extinguishers, so B corresponds to I. This identifies the textbook association without suggesting that its old use overrides its well-known toxicity.

Step 2: Match the remaining compounds and eliminate incorrect codes Chlorofluorocarbons, abbreviated CFCs, were widely used as refrigerants because suitable members were volatile, stable working fluids; hence C corresponds to IV. DDT is an organochlorine insecticide, so D corresponds to II. The combined mapping is therefore



Option (B) alone contains all four pairs. Option (A) assigns iodoform to insecticide and CFCs to fire extinguishers, both wrong. Option (C) swaps the uses of carbon tetrachloride and DDT. Option (D) assigns all four compounds to the wrong roles. The familiar environmental effects of CFCs and DDT do not change the uses being matched here.

Step 3: Conclusion The matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (B).

Quick Tip to Solve: Start with the most distinctive pair, DDT-insecticide or CFC-refrigerant, then verify every remaining pair before selecting a matching code.

64. At -20°C and 1 atm pressure, a cylinder is filled with equal number of H_2 , I_2 and HI molecules for the reaction $\text{H}_2(\text{g}) + \text{I}_2(\text{g}) \rightleftharpoons 2\text{HI}(\text{g})$, the K_p for the process is $x \times 10^{-1}$.

$x = \underline{\hspace{2cm}}$.

[Given : $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$]

[4 marks]

- (A) 2
- (B) 1
- (C) 10
- (D) 0.01

Correct Answer: (C) 10

Solution:

Concept:

- For a gaseous equilibrium, K_p is the product of equilibrium partial pressures of products divided by that of reactants, each raised to its stoichiometric coefficient.
- In an ideal gas mixture, $p_i = y_i P$, where y_i is the mole fraction and P is the total pressure. Equal molecular populations mean equal mole fractions.

Step 1: Find the partial pressures The three gases have equal numbers of molecules, so their mole amounts are in the ratio 1 : 1 : 1. Each therefore contributes one third of the total pressure. Taking

the stated composition as the equilibrium composition for evaluating the requested constant,

$$p_{\text{H}_2} = p_{\text{I}_2} = p_{\text{HI}} = \frac{1}{3} \text{ atm.}$$

The temperature does not enter this mole-fraction calculation; it would matter if we needed to calculate pressure from an amount and a volume.

Step 2: Evaluate the equilibrium expression For $\text{H}_2(\text{g}) + \text{I}_2(\text{g}) \rightleftharpoons 2 \text{HI}(\text{g})$, the product pressure is squared, whereas each reactant pressure appears once:

$$K_p = \frac{(p_{\text{HI}})^2}{p_{\text{H}_2} p_{\text{I}_2}} = \frac{(1/3)^2}{(1/3)(1/3)} = 1.$$

Here $\Delta n_g = 2 - (1 + 1) = 0$, so the pressure factors cancel. There is no need to substitute the supplied gas constant or convert the temperature into kelvin.

Step 3: Extract the requested coefficient The question writes the constant as $x \times 10^{-1}$, so the value of the constant is not itself the requested coefficient:

$$x \times 10^{-1} = 1 \quad \implies \quad x = 10.$$

Option B gives K_p rather than x . Options A and D would give constants 0.2 and 0.001, respectively, which do not reproduce the pressure quotient.

Step 4: Conclusion $K_p = 1$ and the required coefficient is $x = 10$. Hence, the correct option is (C).

Quick Tip to Solve: Calculate the pressure quotient first, then compare it with the stated power-of-ten form; the coefficient need not equal the constant.

65. The electron affinity value are negative for

- A. $\text{Be} \rightarrow \text{Be}^-$
- B. $\text{N} \rightarrow \text{N}^-$
- C. $\text{O} \rightarrow \text{O}^{2-}$
- D. $\text{Na} \rightarrow \text{Na}^-$
- E. $\text{Al} \rightarrow \text{Al}^-$

Choose the most appropriate answer from the options given below :

[4 marks]

- (A) A, B, D and E only
- (B) D and E only
- (C) A and D only
- (D) A, B and C only

Correct Answer: (D) A, B and C only

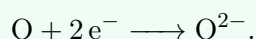
Solution:

Concept:

- Electron affinity, when defined as the energy released on electron attachment, has the opposite sign to electron gain enthalpy: $EA = -\Delta_{eg}H$.
- An energy-requiring electron addition has negative electron affinity under this convention. Filled and half-filled subshells resist electron addition, and adding an electron to an anion faces electrostatic repulsion.

Step 1: Fix the sign convention The question asks for negative electron affinity, rather than negative electron gain enthalpy. Using electron affinity as the energy released on attachment, an exothermic addition has positive affinity, while an endothermic addition has negative affinity. The enthalpy of attachment has the opposite sign. This distinction is essential: exchanging these two terms would reverse the selected set.

Step 2: Examine the three unfavourable processes Beryllium has the filled configuration $1s^2 2s^2$. An added electron must occupy the higher-energy $2p$ subshell, so attachment requires energy and A has negative affinity. Nitrogen has the stable half-filled configuration $1s^2 2s^2 2p^3$; adding an electron introduces pairing and disrupts that stability, so B likewise has negative affinity. For C, the final ion is O^{2-} . Although the first addition to neutral oxygen releases energy, forcing a second electron onto O^- requires more energy than the first addition releases:



The net attachment is endothermic, so its energy-release affinity is negative. Stabilization of oxide ions in a solid lattice is not part of this isolated-ion process.

Step 3: Exclude sodium and aluminium and select the list Sodium and aluminium accept an electron with release of energy. Therefore D and E have positive electron affinities under the energy-release convention and must be excluded. The required list is A, B and C only. Option A incorrectly includes D and E; option B instead selects the species with negative electron gain enthalpy; option C both includes D and omits the endothermic nitrogen and oxygen processes. The sign definition, together with the stable electronic configurations and the repulsion for second attachment, decides the result.

Step 4: Conclusion With electron affinity defined as energy released, Be, N and net formation of O^{2-} require energy and have negative affinity; the set is A, B and C only. Hence, the correct option is (D).

Quick Tip to Solve: Write $EA = -\Delta_{eg}H$ before choosing signs: negative electron affinity and negative electron gain enthalpy select opposite energy changes.

66. Consider the following complexes

(A) $[\text{CoCl}(\text{NH}_3)_5]^{2+}$, (B) $[\text{Co}(\text{CN})_6]^{3-}$, (C) $[\text{Co}(\text{NH}_3)_5(\text{H}_2\text{O})]^{3+}$, (D) $[\text{Cu}(\text{H}_2\text{O})_4]^{2+}$

The correct order of A, B, C and D in terms of wavenumber of light absorbed is :

[4 marks]

(A) $D < A < C < B$

(B) $B < C < A < D$

(C) $C < D < A < B$

(D) $A < C < B < D$

Correct Answer: (A) $D < A < C < B$

Solution:

Concept:

- The energy of absorbed light satisfies $\Delta E = hc\bar{\nu}$, where $\bar{\nu} = 1/\lambda$ is its wavenumber. Greater ligand-field splitting therefore requires a greater absorbed wavenumber.
- The relevant spectrochemical trend is $\text{Cl}^- < \text{H}_2\text{O} < \text{NH}_3 < \text{CN}^-$. Metal ion charge and identity also affect the splitting.

Step 1: Compare the three cobalt complexes Determine the oxidation states before comparing ligands. In A, one chloride contributes -1 , so cobalt is $+3$. In B, six cyanides contribute -6 , again making cobalt $+3$. In C all the ligands are neutral and the complex is $+3$, so cobalt is also $+3$. These three complexes contain cobalt(III), allowing the ligand strengths to decide their relative absorption energies.

Step 2: Rank the cobalt ligand fields A and C both contain five ammonia ligands. Replacing the chloride of A by the stronger-field water ligand in C increases the splitting, so $\Delta_A < \Delta_C$. B has six cyanide ligands, which are much stronger-field ligands than ammonia, water or chloride. Thus B has the greatest splitting among the cobalt complexes:

$$\Delta_A < \Delta_C < \Delta_B.$$

Since absorbed wavenumber is proportional to this transition energy, the same increasing order applies to their wavenumbers.

Step 3: Place the copper aqua complex and test the options D is a copper(II) aqua complex. Its lower-charge copper centre with water ligands gives a lower-energy absorption than the cobalt(III) ammine complexes listed. It therefore comes before A. Combining the comparisons gives

$$\bar{\nu}_D < \bar{\nu}_A < \bar{\nu}_C < \bar{\nu}_B.$$

Option B reverses the trend, option C incorrectly places C before the weaker-field A, and option D incorrectly makes D the highest-energy absorber. Only option A preserves all the comparisons.

Step 4: Conclusion The increasing absorbed-wavenumber order is $D < A < C < B$. Hence, the correct option is (A).

Quick Tip to Solve: Wavenumber increases with transition energy; wavelength goes in the opposite direction. Compare metal oxidation states before using ligand strength.

67. Functional group present in sulphonic acids is :

[4 marks]

- (A) $-\text{SO}_2$
- (B) $-\text{S}(=\text{O})-\text{OH}$
- (C) $-\text{SO}_3\text{H}$
- (D) $-\text{SO}_4\text{H}$

Correct Answer: (C) $-\text{SO}_3\text{H}$

Solution:

Concept:

- A functional group is the atom or group of atoms responsible for the characteristic chemistry of an organic compound. Sulphonic acids contain a sulphonic acid group attached to an organic residue.
- The general formula of a sulphonic acid is $\text{R}-\text{SO}_3\text{H}$, equivalently $\text{R}-\text{S}(=\text{O})_2-\text{OH}$, where R is an alkyl or aryl group.

Step 1: Count the oxygen atoms and identify the acidic group In the sulphonic acid group, sulphur is bonded to the organic residue, to two oxygen atoms through double bonds in the usual structural representation, and to a hydroxyl oxygen. Thus it contains three oxygen atoms in total, with the acidic hydrogen attached to oxygen. Removing the unspecified organic residue from $\text{R}-\text{SO}_3\text{H}$ leaves the functional group $-\text{SO}_3\text{H}$. The leading bond indicates its point of attachment to the rest of the molecule.

Step 2: Compare the first two options Option A, $-\text{SO}_2$, has only two oxygens and does not include the acidic hydroxyl hydrogen, so it is not the sulphonic acid group. Option B shows sulphur with one double-bonded oxygen and one hydroxyl group, $-\text{S}(=\text{O})-\text{OH}$. It therefore has only two oxygen atoms and corresponds to a sulphinic acid group rather than a sulphonic acid group. The two names differ by one oxygen atom, so the oxygen count is decisive.

Step 3: Choose the three-oxygen group Option C is $-\text{SO}_3\text{H}$, exactly the group required in $\text{R}-\text{SO}_3\text{H}$. For example, benzenesulphonic acid has the condensed formula $\text{C}_6\text{H}_5\text{SO}_3\text{H}$, which contains this group attached to the aromatic ring. Option D, $-\text{SO}_4\text{H}$, has an extra oxygen and is not the directly carbon-bonded sulphonic acid functional group. No additional substituent or ionization change is needed to select C.

Step 4: Conclusion The sulphonic acid functional group is $-\text{SO}_3\text{H}$. Hence, the correct option is (C).

Quick Tip to Solve: Remember sulphonic as three oxygens on sulphur: two oxo groups and one hydroxyl group. Sulphinic acid has only two oxygens.

68. Match List I with List II.

	LIST I (Precipitating reagent and conditions)		LIST II (Cation)
A.	$\text{NH}_4\text{Cl} + \text{NH}_4\text{OH}$	I.	Mn^{2+}
B.	$\text{NH}_4\text{OH} + \text{Na}_2\text{CO}_3$	II.	Pb^{2+}
C.	$\text{NH}_4\text{OH} + \text{NH}_4\text{Cl} + \text{H}_2\text{S}$ gas	III.	Al^{3+}
D.	dilute HCl	IV.	Sr^{2+}

Choose the correct answer from the options given below:

[4 marks]

(A) A-III, B-IV, C-I, D-II

(B) A-III, B-IV, C-II, D-I

(C) A-IV, B-III, C-II, D-I

(D) A-IV, B-III, C-I, D-II

Correct Answer: (A) A-III, B-IV, C-I, D-II

Solution:

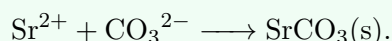
Concept:

- Selective precipitation separates cations using the solubility products of chlorides, hydroxides, sulphides and carbonates. The concentration of the precipitating anion is controlled by the solution conditions.
- An ammonium chloride–ammonium hydroxide mixture buffers hydroxide concentration; alkaline hydrogen sulphide supplies enough sulphide for cations whose sulphides do not precipitate in acidic medium.

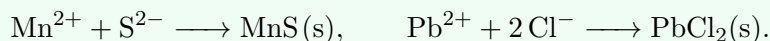
Step 1: Match hydroxide and carbonate precipitation For A, $\text{NH}_4\text{Cl} + \text{NH}_4\text{OH}$ gives the controlled hydroxide concentration used to precipitate group III hydroxides. Aluminium is precipitated as $\text{Al}(\text{OH})_3$, so A matches III:



For B, carbonate in ammoniacal medium precipitates the alkaline-earth cation strontium as SrCO_3 . Therefore B matches IV:



Step 2: Match sulphide and chloride precipitation For C, passing H_2S through the ammoniacal ammonium chloride mixture provides the alkaline sulphide conditions needed for manganese(II). It precipitates as MnS , giving C-I. For D, dilute hydrochloric acid supplies chloride ions; lead(II) forms sparingly soluble PbCl_2 , giving D-II:



The medium matters for manganese sulphide because acidic conditions suppress sulphide ion concentration.

Step 3: Assemble the full matching code The complete result is A-III, B-IV, C-I, D-II. Option B exchanges the last two cations and would assign manganese to dilute HCl. Options C and D incorrectly interchange aluminium and strontium in the first two pairs. The precipitates and the selective reagents establish every pair, so the matching does not rely merely on eliminating one unfamiliar ion.

Step 4: Conclusion A-III, B-IV, C-I, D-II. Hence, the correct option is (A).

Quick Tip to Solve: Match dilute HCl with lead chloride and ammoniacal carbonate with strontium carbonate first; then verify the hydroxide and sulphide pairs.

69. Given below are two statements :

Statement I : Gallium is used in the manufacturing of thermometers.

Statement II : A thermometer containing gallium is useful for measuring the freezing point (256 K) of brine solution.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Statement I is true but Statement II is false
- (C) Statement I is false but Statement II is true
- (D) Both Statement I and Statement II are false

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- A thermometric liquid must remain liquid throughout the temperature range being measured. A wide liquid range alone does not ensure suitability at every low temperature.
- Gallium melts near 303 K and boils at a much higher temperature. Its low melting point relative to most metals and very high boiling point motivate thermometric applications.

Step 1: Evaluate the use of gallium Statement I says that gallium is used in manufacturing thermometers. This is a recognized use associated with its unusually broad liquid-temperature range and high boiling point; gallium-containing thermometric materials can replace mercury in suitable instruments. Hence Statement I is true. This use does not mean that pure gallium stays liquid at any temperature we might want to measure.

Step 2: Compare the brine temperature with gallium melting The temperature in Statement II is 256 K. Compare it directly with the melting point of gallium:

$$256\text{ K} < 303\text{ K}.$$

The proposed measurement is therefore roughly 47 K below the melting point. Pure gallium would not be an equilibrium liquid there, so it cannot function as the ordinary expanding liquid in a thermometer measuring that freezing point. The high boiling point does not resolve a low-temperature freezing limitation.

Step 3: Select the truth-value combination Thus Statement I is true and Statement II is false, which is precisely option B. Option A incorrectly accepts the unsuitable low-temperature application. Option C reverses both truth values. Option D incorrectly rejects gallium's recognized use in thermometers. A gallium alloy can have a different melting point, but no particular low-melting alloy or altered

temperature range is specified in Statement II; the comparison here is the usual one for gallium itself.

Step 4: Conclusion Statement I is true, whereas Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: For liquid thermometers, check the melting point as well as the boiling point: the target temperature must lie inside the liquid range.

70. The density of ' x ' M solution (' x ' molar) of NaOH is 1.12 g mL^{-1} , while in molality, the concentration of the solution is 3 m (3 molal). Then x is

(Given : Molar mass of NaOH is 40 g/mol)

[4 marks]

- (A) 3.8
- (B) 3.0
- (C) 3.5
- (D) 2.8

Correct Answer: (B) 3.0

Solution:

Concept:

- Molality is moles of solute per kilogram of solvent; molarity is moles of solute per litre of solution. The denominators refer to different quantities.
- Density relates solution mass to its volume by $\rho = m_{\text{solution}}/V_{\text{solution}}$. Include the solute mass before using this relation.

Step 1: Choose a convenient solvent mass Take 1.000 kg of solvent. Since the solution is 3 mol kg^{-1} , the moles of sodium hydroxide in this sample are

$$n_{\text{NaOH}} = 3 \times 1.000 = 3.000 \text{ mol.}$$

Using the given molar mass, its mass is

$$m_{\text{NaOH}} = 3.000 \times 40 = 120 \text{ g.}$$

The chosen solvent basis fixes the solute amount without needing any initial assumption about volume.

Step 2: Find the mass and volume of solution The complete solution contains both solvent and solute, so

$$m_{\text{solution}} = 1000 + 120 = 1120 \text{ g.}$$

Using the density 1.12 g mL^{-1} , its volume is

$$V_{\text{solution}} = \frac{1120}{1.12} = 1000 \text{ mL} = 1.000 \text{ L.}$$

Using only the solvent mass in this division would confuse the solvent with the solution and give an incorrect molarity.

Step 3: Calculate and compare the molarity There are three moles of solute in one litre of solution:

$$x = \frac{n_{\text{NaOH}}}{V_{\text{solution}}(\text{L})} = \frac{3.000}{1.000} = 3.0.$$

Thus the molarity happens to have the same numerical value as the molality for these particular density and molar-mass values. This equality is not a general rule. The other choices, 3.8, 3.5 and 2.8, do not agree with the solution volume obtained from mass conservation and the supplied density.

Step 4: Conclusion The solution concentration is $x = 3.0 \text{ mol L}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Take one kilogram of solvent, add the solute mass, and divide the total mass by density before calculating molarity.

71. Match List I with List II.

	List I (Compound/Species)		List II (Shape/Geometry)
A.	SF ₄	I.	Tetrahedral
B.	BrF ₃	II.	Pyramidal
C.	BrO ₃ ⁻	III.	See saw
D.	NH ₄ ⁺	IV.	Bent T-Shape

Choose the correct answer from the options given below:

[4 marks]

(A) A-III, B-II, C-IV, D-I

(B) A-III, B-IV, C-II, D-I

(C) A-II, B-IV, C-III, D-I

(D) A-II, B-III, C-I, D-IV

Correct Answer: (B) A-III, B-IV, C-II, D-I

Solution:

Concept:

- VSEPR theory arranges electron domains to minimise repulsion. A multiple bond counts as one domain when predicting the molecular shape.
- The molecular shape describes the positions of bonded atoms; lone pairs influence that shape but are not included as visible vertices.

Step 1: Count the domains on sulfur and bromine In SF₄, sulfur has four bonding pairs and one lone pair, so its five domains have trigonal-bipyramidal electron geometry. The lone pair occupies an equatorial site, leaving a see-saw molecular shape: A-III. In BrF₃, bromine has three bonding pairs and two lone pairs. Both lone pairs occupy equatorial positions; the remaining fluorines form the bent T-shape listed as IV. Thus B-IV, not pyramidal.

Step 2: Assign the two remaining species In BrO₃⁻, bromine has three bonded oxygen atoms and one lone pair. The four domains have a tetrahedral arrangement, but removal of the lone-pair vertex gives a trigonal-pyramidal molecular shape. Thus C-II. In NH₄⁺, nitrogen forms four equivalent bonds and has no lone pair; its molecular geometry is tetrahedral, so D-I. The complete mapping is A-III, B-IV,

C–II, D–I. Option (A) incorrectly makes BrF_3 pyramidal and bromate T-shaped. Options (C) and (D) incorrectly make SF_4 pyramidal; (D) also assigns the wrong geometry to ammonium.

Step 3: Conclusion The matching is A–III, B–IV, C–II, D–I. Hence, the correct option is (B).

Quick Tip to Solve: For VSEPR matching, count central-atom lone pairs first: SF_4 has one, BrF_3 two, BrO_3^- one and NH_4^+ none.

72. Match List I with List II.

	List I (Molecule/Species)		List II (Property/Shape)
A.	SO_2Cl_2	I.	Paramagnetic
B.	NO	II.	Diamagnetic
C.	NO_2^-	III.	Tetrahedral
D.	I_3^-	IV.	Linear

Choose the correct answer from the options given below:

[4 marks]

(A) A–II, B–III, C–I, D–IV

(B) A–III, B–I, C–II, D–IV

(C) A–IV, B–I, C–III, D–II

(D) A–III, B–IV, C–II, D–I

Correct Answer: (B) A–III, B–I, C–II, D–IV

Solution:

Concept:

- A species is paramagnetic when it contains an unpaired electron; species with all electrons paired are diamagnetic.
- VSEPR treats each bonded atom as one electron domain. Three lone pairs in a trigonal-bipyramidal arrangement occupy the equatorial positions.

Step 1: Identify tetrahedral sulfuryl chloride and nitric oxide In SO_2Cl_2 , sulfur is attached to two oxygen and two chlorine atoms. Each double bond is one domain, giving four bonding domains with no lone pair on sulfur. The atom arrangement is tetrahedral, hence A–III. Nitric oxide, NO , has $5 + 6 = 11$ valence electrons. The odd electron count leaves an unpaired electron, making it paramagnetic: B–I. It cannot be assigned tetrahedral or linear as its distinctive match when the table explicitly provides its magnetic property.

Step 2: Determine nitrite magnetism and triiodide shape For NO_2^- , the total valence-electron count is $5 + 2(6) + 1 = 18$. Its ordinary closed-shell Lewis structures have paired electrons; it is diamagnetic, so C–II. The bent geometry of nitrite is not an offered shape in List II. In I_3^- , central iodine has two bonding domains and three lone pairs, an AX_2E_3 arrangement. The three lone pairs occupy equatorial sites and the two terminal iodine atoms occupy opposite axial positions, giving a linear species: D–IV. Option (A) wrongly calls NO tetrahedral; option (C) calls sulfuryl chloride linear and nitrite tetrahedral; option (D) wrongly calls NO linear and triiodide paramagnetic.

Step 3: Conclusion The correct correspondence is A–III, B–I, C–II, D–IV. Hence, the correct option is

(B).

Quick Tip to Solve: In mixed shape/property lists, first fix the unmistakable odd-electron species NO as paramagnetic, then use VSEPR to match the shapes.

73. Which of the following statements are correct?

- A. Glycerol is purified by vacuum distillation because it decomposes at its normal boiling point.
- B. Aniline can be purified by steam distillation as aniline is miscible in water.
- C. Ethanol can be separated from ethanol water mixture by azeotropic distillation because it forms azeotrope.
- D. An organic compound is pure, if mixed M.P. is remained same.

Choose the most appropriate answer from the options given below :

[4 marks]

- (A) A, C, D only
- (B) B, C, D only
- (C) A, B, D only
- (D) A, B, C only

Correct Answer: (A) A, C, D only

Solution:

Concept:

- Vacuum distillation lowers the boiling temperature by lowering external pressure; steam distillation is suited to steam-volatile compounds with very low water solubility.
- An unchanged sharp mixed melting point with an authentic reference supports purity and identity; ordinary impurities generally depress and broaden melting.

Step 1: Check vacuum and steam distillation statements Statement A is correct. Glycerol has a high normal boiling temperature and can decompose on strong heating. Reduced pressure lets it distil at a lower temperature, avoiding decomposition. Statement B gives the wrong reason: aniline can be steam-distilled because it is steam-volatile and only sparingly soluble in water, not because it is miscible with water. Water and the organic liquid contribute to the total vapour pressure, allowing distillation below the organic liquid's normal boiling temperature. Thus the statement as a whole is false.

Step 2: Check azeotropic distillation and the mixed melting point Statement C is correct. Ethanol and water form an azeotrope, so simple fractional distillation cannot produce completely water-free ethanol. Azeotropic distillation uses an appropriate additional component to overcome that limitation and separate the mixture. Statement D describes the standard mixed-melting-point test: mixing the sample with an authentic pure specimen of the same compound leaves the sharp melting point unchanged. It is therefore accepted as correct in this context. The true statements are A, C and D. Each of options (B), (C) and (D) includes the false statement B and omits one of the required true statements; only option (A) selects all three correctly.

Step 3: Conclusion Statements A, C and D are correct, whereas B is false because aniline is not

miscible with water. Hence, the correct option is (A).

Quick Tip to Solve: Check the reason after “because as carefully as the separation method: a valid method paired with a false reason makes the whole statement false.

74. Which of the following material is not a semiconductor.

[4 marks]

- (A) Copper oxide
- (B) Graphite
- (C) Silicon
- (D) Germanium

Correct Answer: (B) Graphite

Solution:

Concept:

- Semiconductors have a small energy gap between occupied and empty electronic bands, so thermal excitation can produce mobile charge carriers.
- Graphite conducts through delocalised electrons associated with its layered carbon network and is conventionally classified as a conductor (more precisely, a semimetal).

Step 1: Classify the three semiconductor choices Silicon and germanium are standard elemental semiconductors. Their small band gaps allow some electrons to be thermally excited into conducting states, leaving mobile holes in the valence band. Their conductivity can be increased substantially by controlled doping. Copper oxides are also semiconducting materials; copper vacancies and related defects commonly produce p-type behaviour. Thus copper oxide in option (A), silicon in option (C), and germanium in option (D) all belong to the semiconductor class intended in this question.

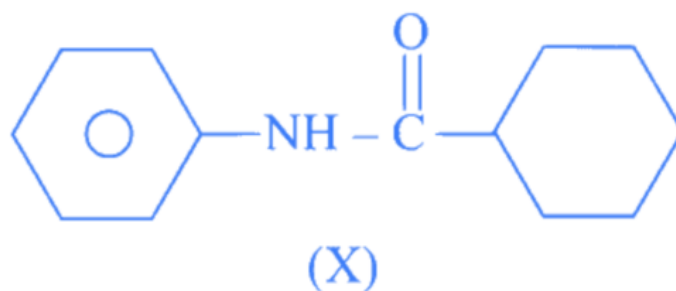
Step 2: Explain why graphite is the exception In graphite, each carbon atom is approximately sp^2 -hybridised and makes three sigma bonds within a hexagonal sheet. The remaining p orbital participates in a delocalised electronic system extending through the layer. These delocalised electrons can carry current along the sheets. The low-energy electronic structure is semimetallic rather than that of the ordinary gapped semiconductors silicon and germanium. Therefore graphite is the material that is not a semiconductor in this list. Its good conduction within sheets should not be confused with diamond, whose tetrahedral carbon network has a large gap and is insulating. Options (A), (C) and (D) cannot answer a question asking for the non-semiconductor because they are themselves semiconductor examples.

Step 3: Conclusion Graphite is the non-semiconductor among the listed materials. Hence, the correct option is (B).

Quick Tip to Solve: Remember the carbon allotropes separately: graphite has mobile delocalised electrons within layers, while diamond has a fully bonded insulating network.

75. Which of the following is metamer of the given compound (X) ?

[4 marks]



- (A) $p\text{-OHC}-\text{C}_6\text{H}_4-\text{NH}-\text{C}_6\text{H}_{11}$
(B) $\text{C}_6\text{H}_5-\text{NH}-\text{C}(=\text{O})-\text{C}_6\text{H}_5$
(C) $\text{C}_6\text{H}_{11}-\text{NH}-\text{C}(=\text{O})-\text{C}_6\text{H}_5$
(D) $\text{C}_6\text{H}_{11}-\text{NH}-\text{C}(=\text{O})-\text{C}_6\text{H}_{11}$

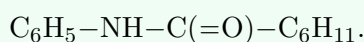
Correct Answer: (C) $\text{C}_6\text{H}_{11}-\text{NH}-\text{C}(=\text{O})-\text{C}_6\text{H}_5$

Solution:

Concept:

- Metamers have the same molecular formula and functional group but a different distribution of carbon groups on the two sides of a polyvalent functional linkage.
- In the printed structures, C_6H_5 denotes a phenyl group and C_6H_{11} a cyclohexyl group. An amide linkage has the connectivity $-\text{C}(=\text{O})-\text{NH}-$.

Step 1: Identify the amide The given structure is



The nitrogen bears phenyl and the carbonyl carbon bears cyclohexyl. It is an amide. Its total formula is obtained from phenyl C_6H_5 , cyclohexyl C_6H_{11} , one NH and one CO, giving $\text{C}_{13}\text{H}_{17}\text{NO}$. A valid metamer must preserve that formula as well as the amide linkage, rather than merely contain the same two rings somewhere in the molecule.

Step 2: Compare all four carbon distributions Option (C), $\text{C}_6\text{H}_{11}-\text{NH}-\text{C}(=\text{O})-\text{C}_6\text{H}_5$, interchanges the cyclohexyl and phenyl groups across the amide linkage. It retains $\text{C}_{13}\text{H}_{17}\text{NO}$ and the same amide functional group, so it is the required metamer. Option (A) is an aldehyde with an amino substituent on the benzene ring; it does not retain the amide functional group. Option (B) has phenyl groups on both sides and consequently has formula $\text{C}_{13}\text{H}_{11}\text{NO}$, not the formula of X. Option (D) has two cyclohexyl groups and has formula $\text{C}_{13}\text{H}_{23}\text{NO}$, again different from X. Therefore neither changing aromaticity nor relocating the oxygen into an aldehyde can produce the required metamer.

Step 3: Conclusion The metamer is $\text{C}_6\text{H}_{11}-\text{NH}-\text{C}(=\text{O})-\text{C}_6\text{H}_5$, with phenyl and cyclohexyl exchanged across the amide linkage. Hence, the correct option is (C).

Quick Tip to Solve: For metamerism, verify both the molecular formula and the functional linkage before comparing how the carbon groups are distributed.

76. In Reimer - Tiemann reaction, phenol is converted into salicylaldehyde through an intermediate. The structure of intermediate is _____.

[4 marks]

- (A) $o\text{-HO-C}_6\text{H}_4\text{-CHCl}_2$
(B) $o\text{-Na}^+ \text{ } ^-\text{O-C}_6\text{H}_4\text{-CHCl}_2$
(C) $o\text{-Na}^+ \text{ } ^-\text{O-C}_6\text{H}_4\text{-CH}_3$
(D) $o\text{-Na}^+ \text{ } ^-\text{O-C}_6\text{H}_4\text{-CHO}$

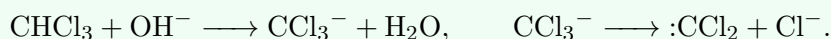
Correct Answer: (B) $o\text{-Na}^+ \text{ } ^-\text{O-C}_6\text{H}_4\text{-CHCl}_2$

Solution:

Concept:

- Phenol reacts with chloroform and aqueous alkali in the Reimer-Tiemann reaction; acidic work-up ultimately gives predominantly ortho-hydroxybenzaldehyde.
- Under the alkaline reaction conditions, phenol is present as phenoxide. Chloroform produces electrophilic dichlorocarbene, :CCl_2 .

Step 1: Generate the electrophile and activate the ring Hydroxide deprotonates phenol to phenoxide, which activates the ortho and para positions of the benzene ring. Base also removes the proton of chloroform, after which loss of chloride generates dichlorocarbene. The relevant sequence is



The electrophilic carbon of dichlorocarbene is attacked by the electron-rich ring, predominantly at the ortho position. Proton transfer and rearomatisation introduce a dichloromethyl group adjacent to the phenoxide oxygen.

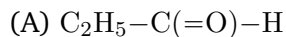
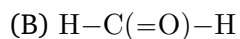
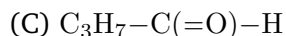
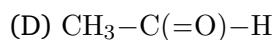
Step 2: Identify the intermediate before hydrolysis The resulting alkaline-stage intermediate is sodium ortho-dichloromethylphenoxide, $o\text{-Na}^+ \text{ } ^-\text{O-C}_6\text{H}_4\text{-CHCl}_2$, shown in option (B). Subsequent hydrolysis of the geminal dichloride gives the formyl group, and acidification restores phenolic OH, yielding salicylaldehyde. Option (A) shows the protonated phenol form rather than the sodium phenoxide present in the alkaline reaction mixture. Option (C) has a methyl group, which does not contain the two chlorine atoms needed for the hydrolysis route to an aldehyde. Option (D) already has the formyl group and corresponds to the aldehyde phenoxide after hydrolysis, rather than the dichloromethyl intermediate in the standard reaction sequence.

Step 3: Conclusion The intermediate is sodium ortho-dichloromethylphenoxide, bearing CHCl_2 next to $\text{O}^- \text{Na}^+$. Hence, the correct option is (B).

Quick Tip to Solve: Use the reaction medium to distinguish the close choices: before acidic work-up, the phenolic oxygen remains in its phenoxide salt form.

77. Which among the following aldehydes is most reactive towards nucleophilic addition reactions?

[4 marks]



Correct Answer: (B) $\text{H}-\text{C}(=\text{O})-\text{H}$

Solution:

Concept:

- Nucleophiles attack the electron-deficient carbon of the polar carbonyl group. Greater positive character at that carbon generally favours addition.
- Alkyl groups donate electron density by the positive inductive effect and also obstruct approach to the carbonyl carbon. Both effects reduce aldehyde reactivity relative to methanal.

Step 1: Identify the substituents on each carbonyl carbon Option (A) is propanal, $\text{C}_2\text{H}_5\text{CHO}$; option (B) is methanal, HCHO ; option (C) contains a propyl group, $\text{C}_3\text{H}_7\text{CHO}$; and option (D) is ethanal, CH_3CHO . Each is an aldehyde, but methanal has two hydrogen atoms on the carbonyl carbon and no alkyl substituent. The others each have one electron-donating alkyl group directly attached to that carbon. This difference affects the carbonyl electrophilicity as well as the space available for attack.

Step 2: Compare electronic and steric effects In methanal, there is no alkyl group to reduce the positive character of the carbonyl carbon through electron donation. Its two small hydrogen atoms also impose the least steric hindrance. A nucleophile can therefore approach and form the tetrahedral addition intermediate more readily than in any of the other listed aldehydes. The methyl group in (D), ethyl group in (A), and propyl group in (C) each introduce electron donation and greater crowding than hydrogen. None can be more reactive than methanal on these criteria. There is no need to establish a subtle order among the three alkyl aldehydes: both independent effects already select the unsubstituted carbonyl compound as the unique maximum.

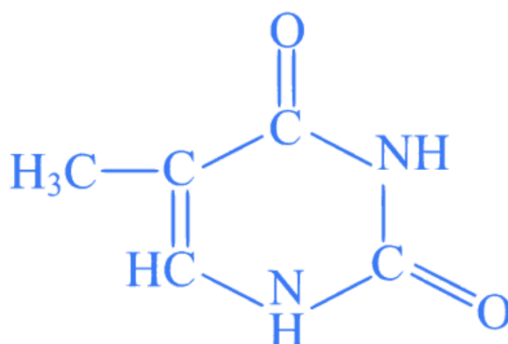
Step 3: Conclusion Methanal, HCHO , is most reactive towards nucleophilic addition because it has the least steric hindrance and no electron-donating alkyl group. Hence, the correct option is (B).

Quick Tip to Solve: When methanal appears among simple aldehydes, check its two hydrogen substituents: they maximise access to, and electrophilicity of, the carbonyl carbon.

78. DNA molecule contains 4 bases whose structure are shown below. One of the structures is not correct, identify the incorrect base structure.

[4 marks]

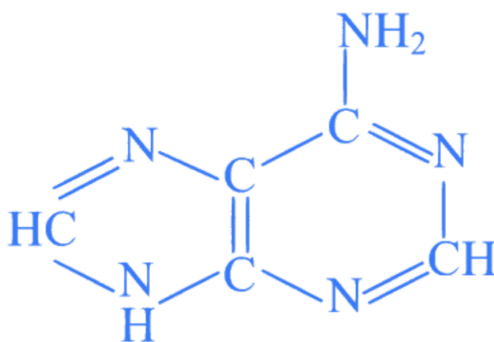
(A)



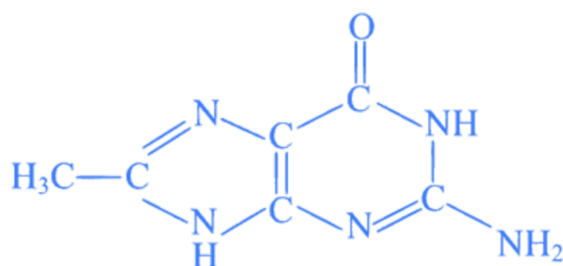
(B)



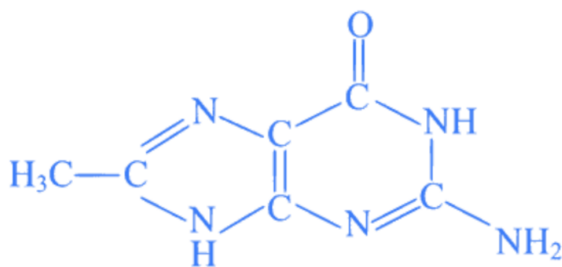
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- DNA contains adenine and guanine, which are purines, together with cytosine and thymine, which are pyrimidines. Identify both the ring skeleton and its substituents.
- Thymine is 5-methyluracil; cytosine has an amino group and one carbonyl group. Adenine is 6-aminopurine and guanine is 2-amino-6-oxopurine.

Step 1: Identify the two pyrimidines Option (A) has a six-membered ring with two ring nitrogens,

two carbonyl groups and a methyl group at carbon 5. These features identify thymine, a correct DNA base. Option (B) has the pyrimidine ring with an amino group at carbon 4 and a carbonyl group at carbon 2. It is cytosine and is also correct. The methyl group in thymine is a normal substituent and must not be rejected simply because another base lacks it.

Step 2: Inspect the fused purine structures Option (C) contains fused five- and six-membered nitrogen-containing rings with an amino group at carbon 6 and no carbonyl group: this is adenine. Option (D) has the amino and carbonyl substituents characteristic of guanine, but the carbon in its five-membered ring carries an extra CH_3 substituent. In ordinary guanine that position, carbon 8, bears hydrogen. The displayed methylated structure is therefore not the guanine base of DNA.

Step 3: Select the incorrect structure The criterion is agreement with the four standard DNA bases, rather than whether the drawn molecule could exist as some modified base. Options (A), (B) and (C) reproduce thymine, cytosine and adenine respectively. The additional methyl group in (D) prevents that structure from representing the required standard guanine.

Step 4: Conclusion The incorrect DNA-base structure is (D), which shows an extra methyl group on the purine ring. Hence, the correct option is (D).

Quick Tip to Solve: For DNA-base diagrams, first separate purines from pyrimidines, then check the amino, carbonyl and methyl substituents position by position.

79. Match List I with List II

	LIST I (Hybridization)		LIST II (Orientation in Shape)
A.	sp^3	I.	Trigonal bipyramidal
B.	dsp^2	II.	Octahedral
C.	sp^3d	III.	Tetrahedral
D.	sp^3d^2	IV.	Square planar

Choose the correct answer from the options given below:

[4 marks]

- (A) A-III, B-I, C-IV, D-II
 (B) A-III, B-IV, C-I, D-II
 (C) A-IV, B-III, C-I, D-II
 (D) A-II, B-I, C-IV, D-III

Correct Answer: (B) A-III, B-IV, C-I, D-II

Solution:

Concept:

- Hybrid orbitals point towards ligand positions. Their number and spatial orientation determine the ideal coordination geometry in the valence-bond description.
- Four hybrid orbitals may be tetrahedral or square planar; five point towards a trigonal bipyramid, while six point towards an octahedron.

Step 1: Match the four-orbital hybrids For A, one s orbital and three p orbitals combine to give four

equivalent sp^3 orbitals directed towards the vertices of a tetrahedron. Hence A matches III. For B, one d orbital, one s orbital and two p orbitals form four coplanar dsp^2 hybrids, directed along the corners of a square. Thus B matches IV. Equal numbers of hybrid orbitals alone do not distinguish these two arrangements; the specific orbitals and their orientation matter.

Step 2: Match the five- and six-orbital hybrids For C, sp^3d combines five orbitals and corresponds to trigonal bipyramidal orientation: three equatorial directions and two axial directions. Therefore C matches I. For D, sp^3d^2 combines six orbitals directed towards an octahedral set of ligand positions, so D matches II. These are the ideal orientations requested, without any separate lone-pair distortion specified in the question.

Step 3: Compare the complete matching with the options The complete sequence is A-III, B-IV, C-I, D-II. Only option (B) contains all four matches. Option (A) interchanges the square-planar and trigonal-bipyramidal assignments; option (C) assigns a square plane to sp^3 ; and option (D) assigns an octahedron to a four-orbital hybrid set. Each therefore fails at least one required pairing.

Step 4: Conclusion The matching is A-III, B-IV, C-I, D-II. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the two four-coordinate cases first: sp^3 is tetrahedral, whereas dsp^2 is square planar.

80. Given below are two statements :

Statement I : Picric acid is 2,4,6 - trinitrotoluene.

Statement II : Phenol - 2,4 - disulphonic acid is treated with Conc. HNO_3 to get picric acid.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

- (A) Statement I is incorrect but Statement II is correct
- (B) Both Statement I and Statement II are incorrect
- (C) Both Statement I and Statement II are correct
- (D) Statement I is correct but Statement II is incorrect

Correct Answer: (A) Statement I is incorrect but Statement II is correct

Solution:

Concept:

- Picric acid is 2,4,6-trinitrophenol: its benzene ring bears one hydroxyl group and three nitro groups. Trinitrotoluene instead has a methyl group.
- Phenol activates its ring towards electrophilic substitution. Sulphonation followed by nitration provides a route to picric acid.

Step 1: Evaluate Statement I from the functional group The defining parent compound for picric acid is phenol, C_6H_5OH , so its hydroxyl group remains attached to the aromatic ring. Its name is 2,4,6-trinitrophenol, with condensed formula $C_6H_2(NO_2)_3OH$. By contrast, 2,4,6-trinitrotoluene is derived from toluene and contains CH_3 in place of OH . These are different compounds despite having three nitro substituents at analogous positions. Statement I is therefore incorrect.

Step 2: Evaluate Statement II using the preparation route Phenol can first be converted into phenol-2,4-disulphonic acid by sulphonation. Treatment of this intermediate with concentrated nitric acid replaces the sulphonic-acid groups by nitro groups and introduces a further nitro group at the remaining activated position. The resulting product is 2,4,6-trinitrophenol, or picric acid. This is the stated preparation, so Statement II is correct. The sulphonation step is useful because direct vigorous nitration of phenol can also produce oxidation side products.

Step 3: Apply the truth pattern to every option The required pattern is I incorrect and II correct, exactly as in option (A). Option (B) wrongly rejects the valid nitration route; option (C) wrongly identifies picric acid as a toluene derivative; and option (D) reverses both truth values. The statements must be tested separately before selecting the combined answer.

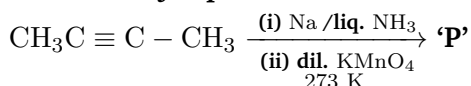
Step 4: Conclusion Statement I is incorrect but Statement II is correct. Hence, the correct option is (A).

Quick Tip to Solve: Keep the parent group in mind: picric acid retains phenolic OH, whereas TNT retains the methyl group of toluene.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The major product of the following reaction is P.



Number of oxygen atoms present in product 'P' is _____. (nearest integer)

[4 marks]

Solution:

Concept:

- Sodium in liquid ammonia reduces an alkyne to an alkene by dissolving-metal reduction. For an internal alkyne, the principal alkene has trans geometry.
- Cold dilute permanganate adds two hydroxyl groups across an alkene double bond, producing a vicinal diol rather than oxidative cleavage under these mild conditions.

Step 1: Reduce but-2-yne to the alkene The starting material is $\text{CH}_3-\text{C}\equiv\text{C}-\text{CH}_3$, but-2-yne. Sodium in liquid ammonia supplies electrons and protons in stages, reducing the triple bond to a double bond. The major intermediate is trans-but-2-ene. The carbon skeleton remains a chain of four carbon atoms; this step adds hydrogen but introduces no oxygen. It is important to stop at the alkene, because sodium/liquid-ammonia conditions are not the same as complete catalytic hydrogenation to an alkane.

Step 2: Apply the stated mild oxidation The second reagent is dilute KMnO_4 at 273 K. These cold, mild conditions convert the double bond into a vicinal diol. In condensed structural notation the product has the connectivity



The product is butane-2,3-diol. The stereochemical outcome of addition does not alter its number of

oxygen atoms. Hot, concentrated oxidant could cleave the carbon chain, but those conditions are not given here.

Step 3: Count oxygen atoms in one product molecule The product contains one hydroxyl group on carbon 2 and another on carbon 3. Each hydroxyl group contributes exactly one oxygen atom, so the total is $1 + 1 = 2$. Count the atoms in the organic product P, rather than oxygen atoms in the oxidizing reagent or inorganic by-products. The requested nearest-integer count is already exact.

Step 4: Conclusion Product P is butane-2,3-diol and contains $\boxed{2}$ oxygen atoms per molecule. The numerical entry is 2.

Quick Tip to Solve: Cold dilute permanganate converts an alkene into a diol; distinguish these conditions from hot oxidative cleavage before counting product atoms.

82. Time required for 99.9% completion of a first order reaction is _____ times the time required for completion of 90% reaction. (nearest integer) [4 marks]

Solution:

Concept:

- For a first-order reaction, $t = k^{-1} \ln([A]_0/[A]_t)$, where k is the rate constant and $[A]_t$ is the remaining reactant concentration.
- Convert percentage completion into the fraction remaining before applying the logarithm. Times for the same reaction have the same rate constant, which cancels in their ratio.

Step 1: Calculate the time for 90 percent completion At 90% completion, 10% of the initial reactant remains. Therefore $[A]_t = 0.10[A]_0$. Substituting in the integrated first-order law gives

$$t_{90} = \frac{1}{k} \ln\left(\frac{1}{0.10}\right) = \frac{\ln 10}{k}.$$

The expression uses the amount remaining, not the reacted amount $0.90[A]_0$. This distinction is necessary because the rate decreases as the reactant concentration falls.

Step 2: Calculate the time for 99.9 percent completion At 99.9% completion only 0.1%, or 0.001 of the initial amount, remains. Thus

$$t_{99.9} = \frac{1}{k} \ln\left(\frac{1}{0.001}\right) = \frac{\ln 1000}{k} = \frac{3 \ln 10}{k}.$$

There are three successive tenfold reductions in the amount remaining: from 1 to 0.1, then to 0.01, and finally to 0.001. Each tenfold reduction requires the same time for first-order kinetics.

Step 3: Take the ratio of the two times Both times refer to the same first-order reaction under the same conditions, so their common factor $1/k$ cancels:

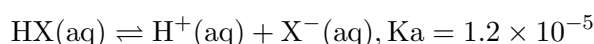
$$\frac{t_{99.9}}{t_{90}} = \frac{3 \ln 10/k}{\ln 10/k} = 3.$$

This is a dimensionless ratio of times. It is not the ratio of the completion percentages, since the first-order time depends logarithmically on the unreacted fraction. No approximation or numerical value of the rate constant is needed.

Step 4: Conclusion The required time is $\boxed{3}$ times the time for 90 percent completion. The nearest-integer numerical entry is 3 (dimensionless).

Quick Tip to Solve: For first-order reactions, 90, 99 and 99.9 percent completion take one, two and three times $\ln 10/k$, respectively.

83. Consider the dissociation of the weak acid HX as given below



[K_a : dissociation constant]

The osmotic pressure of 0.03M aqueous solution of HX at 300 K is _____ $\times 10^{-2}$ bar (nearest integer).

[Given : $R = 0.083 \text{ L bar mol}^{-1} \text{ K}^{-1}$]

[4 marks]

Solution:

Concept:

- For $\text{HX} \rightleftharpoons \text{H}^+ + \text{X}^-$, if α is the degree of dissociation at analytical concentration c , then $K_a = c\alpha^2/(1 - \alpha)$.
- The van't Hoff factor for a weak acid producing two ions is $i = 1 + \alpha$. Its osmotic pressure is $\pi = icRT$, including both undissociated molecules and ions.

Step 1: Determine the small degree of dissociation Here $c = 0.03 \text{ mol L}^{-1}$ and $K_a = 1.2 \times 10^{-5}$. The weak-dissociation estimate gives

$$\alpha \simeq \sqrt{\frac{K_a}{c}} = \sqrt{\frac{1.2 \times 10^{-5}}{0.03}} = 0.020.$$

Only about two percent dissociates, so replacing $1 - \alpha$ by 1 is justified for the requested integer precision. The exact quadratic, $0.03\alpha^2 + 1.2 \times 10^{-5}\alpha - 1.2 \times 10^{-5} = 0$, gives $\alpha \simeq 0.019801$, which will produce the same rounded answer.

Step 2: Include the increase in dissolved particles Of each initial mole of acid, $1 - \alpha$ mole remains as HX, while α mole each of H^+ and X^- is formed. The total is $1 + \alpha$ moles of solute particles per initial mole. Using $i \simeq 1.02$,

$$\pi = (1.02)(0.03)(0.083)(300) = 0.76194 \text{ bar}.$$

The supplied gas constant is in litre-bar units, consistent with the concentration in moles per litre and temperature in kelvin.

Step 3: Express the result in the requested scale The blank multiplies 10^{-2} bar, so write

$$0.76194 \text{ bar} = 76.194 \times 10^{-2} \text{ bar}.$$

The nearest integer is 76. Using the exact degree of dissociation gives approximately 0.76179 bar, again rounding to 76 in this scale. Treating the acid as fully dissociated would incorrectly use $i = 2$; ignoring dissociation entirely would omit the additional ions.

Step 4: Conclusion The osmotic pressure is approximately $0.762 \text{ bar} = 76.2 \times 10^{-2} \text{ bar}$. The required nearest-integer entry is 76.

Quick Tip to Solve: For a weak monoprotic acid, estimate $\alpha = \sqrt{K_a/c}$, use $i = 1 + \alpha$, then convert to the power of ten printed beside the blank.

84. An ideal gas, $\bar{C}_v = \frac{5}{2}R$, is expanded adiabatically against a constant pressure of 1 atm until it doubles in volume. If the initial temperature and pressure is 298 K and 5 atm, respectively then the final temperature is _____ K (nearest integer).

[$\bar{C}_v$ is the molar heat capacity at constant volume]

[4 marks]

Solution:

Concept:

- For an adiabatic process $q = 0$. With the chemistry sign convention, $\Delta U = q + w = w$, and expansion against constant external pressure gives $w = -P_{\text{ext}}(V_f - V_i)$.
- For an ideal gas with constant molar heat capacity, $\Delta U = n\bar{C}_v(T_f - T_i)$. The initial state obeys $P_i V_i = nRT_i$.

Step 1: Write the energy balance for the actual expansion The gas expands against a constant external pressure of 1 atm, so this is an irreversible expansion. The reversible adiabatic relation $TV^{\gamma-1} = \text{constant}$ does not apply. Since $V_f = 2V_i$, its volume change is $\Delta V = V_i$. Adiabaticity and the first law give

$$n \frac{5R}{2} (T_f - 298) = -P_{\text{ext}} V_i.$$

The negative right-hand side shows that the gas loses internal energy while doing work, so its temperature must decrease.

Step 2: Eliminate the unknown initial volume The initial ideal-gas equation gives

$$V_i = \frac{nR(298)}{P_i}, \quad P_i = 5 \text{ atm}.$$

Substitute this into the energy equation and cancel the common factor nR :

$$\frac{5}{2}(T_f - 298) = -\frac{P_{\text{ext}}}{P_i}(298) = -\frac{298}{5}.$$

The pressures appear only as a ratio, so both can be used in atmospheres consistently. There is no need to choose a numerical gas constant or a number of moles.

Step 3: Solve and round the final temperature Rearranging the preceding equation,

$$T_f - 298 = -\frac{2}{5} \frac{298}{5} = -23.84 \text{ K}, \quad T_f = 274.16 \text{ K}.$$

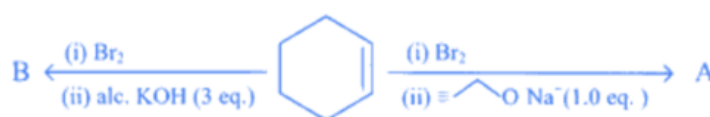
The nearest integer is 274. As a consistency check, the final gas pressure at the specified doubled volume is $P_f = P_i T_f / (2T_i) \simeq 2.30 \text{ atm}$. It need not equal the external pressure at this specified intermediate volume; setting it equal to 1 atm would impose an unstated final mechanical-equilibrium condition.

Step 4: Conclusion The final temperature at the specified doubled volume is 274.16 K, so the nearest-integer answer is 274 K.

Quick Tip to Solve: For expansion against a fixed external pressure, use $w = -P_{\text{ext}}\Delta V$; reserve the usual reversible adiabatic power laws for reversible paths.

85. The major products from the following reaction sequence are product A and product B.

The total sum of π electrons in product A and product B are _____ (nearest integer) [4 marks]



Solution:

Concept:

- Bromine adds across an alkene double bond to give a vicinal dibromide. The original alkene π bond is consumed during this addition.
- An alkoxide can replace a bromide through nucleophilic substitution, whereas alcoholic KOH promotes dehydrohalogenation. Each double bond contains two π electrons and each triple bond contains four.

Step 1: Identify the common bromination intermediate The ring in the reaction scheme is cyclohexene. Addition of Br_2 to its double bond produces 1,2-dibromocyclohexane. Thus both routes start from the same saturated vicinal dibromide; its carbon skeleton initially contains no π bond. The bromine atoms occupy adjacent ring carbons, which is important when deciding how the subsequent reagents act.

Step 2: Count the electrons retained in product A The reagent drawn on the right is sodium prop-2-yn-1-oxide, $\text{HC}\equiv\text{CCH}_2\text{O}^- \text{Na}^+$, used in one equivalent. The alkoxide oxygen replaces a bromide to form a propargyl ether, with the other bromide still on the ring. The ring remains saturated, but the introduced propargyl group retains its carbon-carbon triple bond. Consequently, A has two π bonds, hence $2 \times 2 = 4$ π electrons.

Step 3: Count the electrons in product B and add On the left, excess alcoholic KOH removes two molecules of HBr from the dibromide. Elimination toward the adjacent ring carbons gives the conjugated cyclohexa-1,3-diene; formation of a highly strained cyclohexyne is not the favoured outcome. B therefore contains two carbon-carbon double bonds and four π electrons. Only electrons in these π bonds are counted, not oxygen lone pairs or carbon-bromine σ bonds. Thus

$$N_{\pi}(A) + N_{\pi}(B) = 4 + 4 = 8.$$

Step 4: Conclusion The total number of π electrons in A and B is $\boxed{8}$. This is a dimensionless electron count, already an integer.

Quick Tip to Solve: Inspect the alkoxide carefully: its propargyl triple bond survives substitution and contributes four π electrons even though the cyclohexane ring is saturated.

86. The difference in the 'spin-only' magnetic moment values of KMnO_4 and the manganese product formed during titration of KMnO_4 against oxalic acid in acidic medium is _____ BM. (nearest integer) [4 marks]

Solution:

Concept:

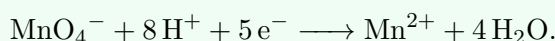
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}$ BM, where n is the number of unpaired electrons and BM denotes the Bohr magneton.
- In an acidic permanganate titration, manganese is reduced from oxidation state +7 in MnO_4^- to +2 in Mn^{2+} . Remove the $4s$ electrons before the $3d$ electrons when forming manganese ions.

Step 1: Find the magnetic moment of permanganate Potassium has oxidation state +1 and each oxygen has oxidation state -2. Charge balance in KMnO_4 gives

$$1 + x + 4(-2) = 0 \Rightarrow x = +7.$$

Neutral manganese has configuration $[\text{Ar}] 3d^5 4s^2$. Its +7 state is therefore $3d^0$, so no unpaired electron remains. Hence $n = 0$ and $\mu(\text{KMnO}_4) = 0$ BM.

Step 2: Identify the manganese product in acidic medium The relevant reduction half-reaction is



Oxalic acid supplies the reducing equivalents and is oxidised to carbon dioxide. The manganese product is consequently Mn^{2+} , rather than the MnO_2 obtained in neutral or weakly alkaline conditions. The $3d^5$ ion has five unpaired electrons under these conditions.

Step 3: Calculate and round the difference Substitute $n = 5$ into the spin-only expression:

$$\mu(\text{Mn}^{2+}) = \sqrt{5(5+2)} = \sqrt{35} \text{ BM} \approx 5.916 \text{ BM}.$$

The magnitude of the difference is $5.916 - 0 = 5.916$ BM. Rounding only this final quantity to the nearest integer gives 6 BM.

Step 4: Conclusion The required difference, rounded to the nearest integer, is $\boxed{6 \text{ BM}}$.

Quick Tip to Solve: Use the medium to identify the manganese product before counting unpaired electrons: acidic permanganate gives Mn^{2+} , a d^5 ion.

87. 9.3 g of pure aniline upon diazotisation followed by coupling with phenol gives an orange dye. The mass of orange dye produced (assume 100% yield/conversion) is _____ g. (nearest integer)

[4 marks]

Solution:

Concept:

- Diazotisation converts one mole of aniline into one mole of benzenediazonium salt; azo coupling with phenol then gives one mole of the orange azo dye.
- The principal coupling product is para-hydroxyazobenzene, $\text{C}_6\text{H}_5-\text{N}=\text{N}-\text{C}_6\text{H}_4-\text{OH}$, with molecular formula $\text{C}_{12}\text{H}_{10}\text{N}_2\text{O}$. Its mass follows from $m = nM$.

Step 1: Convert the aniline mass into moles Aniline has formula $\text{C}_6\text{H}_5\text{NH}_2$, or $\text{C}_6\text{H}_7\text{N}$. Using the usual atomic masses $\text{C} = 12$, $\text{H} = 1$, and $\text{N} = 14$, its molar mass is

$$M(\text{aniline}) = 6(12) + 7(1) + 14 = 93 \text{ g mol}^{-1}.$$

Therefore the amount initially present is

$$n(\text{aniline}) = \frac{9.3 \text{ g}}{93 \text{ g mol}^{-1}} = 0.100 \text{ mol}.$$

Step 2: Establish the dye stoichiometry Each aniline molecule supplies one phenyl group and one nitrogen atom to the diazonium species; nitrous acid supplies the second nitrogen atom. Coupling with one phenol molecule then creates the $-\text{N}=\text{N}-$ link between the two aromatic rings. The major product is para-hydroxyazobenzene. Thus the net mole ratio of aniline consumed to dye produced is 1 : 1, so the stated complete conversion produces 0.100 mol of dye. No yield correction is needed.

Step 3: Calculate the product mass The product contains two rings, two azo nitrogen atoms, and the phenolic oxygen. Its molar mass is

$$M(\text{C}_{12}\text{H}_{10}\text{N}_2\text{O}) = 12(12) + 10(1) + 2(14) + 16 = 198 \text{ g mol}^{-1}.$$

Consequently,

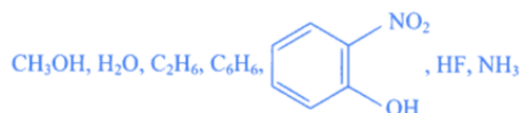
$$m = 0.100 \times 198 = 19.8 \text{ g}.$$

Rounding this calculated mass to the nearest integer, as requested, gives 20 g.

Step 4: Conclusion The orange dye produced has mass 19.8 g, so the required nearest-integer answer is 20 g.

Quick Tip to Solve: For azo coupling, count one dye molecule per aniline molecule. Determine the product formula before using its molar mass; the dye contains two nitrogen atoms.

88. Number of molecules from the following which can exhibit hydrogen bonding is _____.
(nearest integer) [4 marks]



Solution:

Concept:

- A conventional hydrogen bond requires a donor hydrogen directly bonded to a small, electronegative atom such as N, O, or F, and an acceptor lone pair.
- Hydrogen bonding may be intermolecular or intramolecular. A molecule with a suitable donor and acceptor can qualify even when its preferred hydrogen bond is within the same molecule.

Step 1: Check the oxygen-containing molecules Methanol, CH_3OH , has an O–H bond and oxygen lone pairs, so methanol molecules form intermolecular hydrogen bonds. Water, H_2O , also has O–H donors and lone-pair acceptors, and forms an extended hydrogen-bonded network. Both qualify. The aromatic structure shown is ortho-nitrophenol: its hydroxyl hydrogen can bond to an oxygen of the neighbouring nitro group, forming an intramolecular hydrogen bond. It therefore also qualifies; intramolecular bonding must not be excluded from the count.

Step 2: Check the remaining four molecules Hydrogen fluoride, HF , has a strongly polar H–F bond and lone pairs on fluorine, so it forms hydrogen bonds. Ammonia, NH_3 , has N–H bonds and a nitrogen lone pair, so it can also form intermolecular hydrogen bonds. In contrast, ethane, C_2H_6 , contains only carbon-carbon and carbon-hydrogen bonds. Benzene, C_6H_6 , likewise has no conventional N–H, O–H, or F–H donor. Neither hydrocarbon is included in this standard hydrogen-bonding count.

Step 3: Add the qualifying species The qualifying molecules are methanol, water, ortho-nitrophenol, hydrogen fluoride, and ammonia. There are five distinct entries, so

$$N = 1 + 1 + 1 + 1 + 1 = 5.$$

The requested quantity counts molecules capable of hydrogen bonding, rather than counting the individual hydrogen bonds each molecule can make.

Step 4: Conclusion The number of listed molecules that can exhibit hydrogen bonding is 5, a dimensionless count.

Quick Tip to Solve: Count ortho-nitrophenol: its intramolecular O–H $\cdots$ O interaction qualifies just as an intermolecular hydrogen bond does.

89. Among CrO , Cr_2O_3 and CrO_3 , the sum of spin-only magnetic moment values of basic and amphoteric oxides is _____ 10^{-2}BM (nearest integer).

(Given atomic number of Cr is 24)

[4 marks]

Solution:

Concept:

- Metal oxides become less basic and more acidic as the oxidation state of the metal increases. Thus CrO is basic, Cr_2O_3 is amphoteric, and CrO_3 is acidic.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)} \text{ BM}$, with n unpaired electrons. The question asks for the sum for the basic and amphoteric oxides only.

Step 1: Find the chromium oxidation states Taking oxygen as -2 , chromium is $+2$ in CrO , $+3$ in Cr_2O_3 , and $+6$ in CrO_3 . Only the first two compounds enter the required sum. Although Cr_2O_3 contains two chromium atoms per formula unit, the spin-only values in this question are the values associated with the respective chromium ions; they are not obtained by multiplying the ionic moment by the subscript two.

Step 2: Count the unpaired electrons Chromium has atomic number 24 and ground-state configuration $[\text{Ar}] 3d^5 4s^1$. Remove the $4s$ electron first. The Cr^{2+} ion is $3d^4$, with four unpaired electrons, whereas Cr^{3+} is $3d^3$, with three unpaired electrons. Thus

$$\mu(\text{Cr}^{2+}) = \sqrt{4(4+2)} = \sqrt{24} \text{ BM},$$

$$\mu(\text{Cr}^{3+}) = \sqrt{3(3+2)} = \sqrt{15} \text{ BM}.$$

Step 3: Add the moments and use the specified scale The sum is

$$\mu_{\text{sum}} = (\sqrt{24} + \sqrt{15}) \text{ BM} \approx 8.77196 \text{ BM}.$$

If the blank is x in units of 10^{-2} BM , then

$$x = \frac{8.77196}{10^{-2}} = 877.196.$$

Rounding this coefficient to the nearest integer gives 877. Rounding the moments separately before adding would unnecessarily reduce the accuracy.

Step 4: Conclusion The required sum is approximately $8.77196 \text{ BM} = 877.196 \times 10^{-2} \text{ BM}$; the blank is 877.

Quick Tip to Solve: Identify the oxide types first, then count the metal-ion d electrons. Convert BM into the printed 10^{-2} BM scale before rounding.

90. Frequency of the de-Broglie wave of electron in Bohr's first orbit of hydrogen atom is _____ $\times 10^{13}$ Hz (nearest integer).

[Given : R_H (Rydberg constant) = 2.18×10^{-18} J, h (Plank's constant) = 6.6×10^{-34} J.s.] [4 marks]

Solution:

Concept:

- For the orbiting electron, the de Broglie wavelength is $\lambda = h/(m_e v)$. The frequency associated with the electron traversing one such wavelength is $\nu = v/\lambda = m_e v^2/h$.
- In the first Bohr orbit of hydrogen, the electron kinetic energy is $K = R_H = 2.18 \times 10^{-18}$ J, while its total energy is $-R_H$. Therefore $m_e v^2 = 2R_H$.

Step 1: Relate the wavelength to the orbital speed Using the de Broglie relation,

$$\lambda = \frac{h}{m_e v}.$$

The orbital-wave frequency used here follows by dividing the electron speed by this wavelength:

$$\nu = \frac{v}{\lambda} = \frac{m_e v^2}{h} = \frac{2K}{h}.$$

This is also the orbital revolution frequency in the first Bohr orbit because its circumference contains exactly one de Broglie wavelength. It must not be confused with the frequency of a photon emitted during a transition between two energy levels.

Step 2: Use the first-orbit kinetic energy The Coulomb potential energy in a Bohr orbit is $U = -2K$, so the total energy is $E = K + U = -K$. Since the first-orbit energy is $-R_H$, its positive kinetic energy is R_H . Consequently,

$$\nu = \frac{2R_H}{h} = \frac{2(2.18 \times 10^{-18})}{6.6 \times 10^{-34}} \text{ s}^{-1}.$$

The factor of two comes from $m_e v^2 = 2K$; omitting it would halve the required frequency.

Step 3: Evaluate and round the coefficient Calculate with the constants supplied in the question:

$$\nu = \frac{4.36}{6.6} \times 10^{16} \text{ Hz} = 6.60606 \times 10^{15} \text{ Hz}.$$

In the printed scale this is

$$\nu = 660.606 \times 10^{13} \text{ Hz}.$$

The nearest integer coefficient is therefore 661. The value 658 follows approximately from more accurate standard constants, but it does not follow from the explicitly supplied $h = 6.6 \times 10^{-34}$ J.s.

Step 4: Conclusion Using the given constants, the frequency is 6.60606×10^{15} Hz, so the required coefficient is 661. The listed key value 658 differs because it is consistent with more precise constants rather than the rounded values printed here.

Quick Tip to Solve: Keep the given numerical constants throughout, and round only the coefficient of the requested power of ten at the end.